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ELEMENTS OF PHYSICS

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PREFACE

From an educational standpoint, sciences are taught in High Schools, and schools preparatory to Colleges, for two quite distinct reasons: to train the student in powers of observation and accurate description, and to cultivate habits of exact thought and statement.

Certain sciences can be learned directly in the laboratory, where the student himself performs the experiments, observes the phenomena, and draws his own conclusions. Such are Chemistry, Botany and Biology; and for these studies in their elementary stages a laboratory manual is the form of text-book which is most helpful. Other sciences must be studied in the open country where Nature herself is performing or has performed the experiments. Such are Geology and Physical Geography. But the phenomena of Physics are too complicated to be described by any student left unaided, or to be understood when demonstrated, unless he is guided by a suitable text-book containing the theory of the subject. In other words, Physics must first be taught in the class room, where the student may see demonstrated and explained those experiments on which the science is based. To teach Physics without lecture experiments is almost worse than useless. For a student beginning the subject, laboratory instruction in Physics is of secondary importance. It is extremely useful and a great aid to the student in understanding the subject; but he can receive the mental training and learn the fundamental facts and theories without himself performing the experiments. If possible, however, laboratory work should be required of all students; and the experiments should be in the main quantitative. Those performed on the lecture table need not be repeated by the student unless greater accuracy is desired.

All the principal facts and theories of Physics should be illustrated by lecture experiments; and attention should constantly be directed to the proper understanding of a "law of Nature" and its "verification." A law is a statement of our belief concerning certain phenomena; it is suggested by a number of observations and measurements, and is, in fact, a generalization of these. It is shown to be in accord with all observations, to within the range of error inherent in the experimental instruments used, but can never be perfectly *verified*. The experiments shown the student in the lecture room or performed by himself in the laboratory are to be considered as illustrations of the laws, not as attempts at verification.

In an elementary text-book designed for students unacquainted with Physics, the purpose should be to emphasize the most important laws of the various branches—Mechanics, Properties of Matter, Sound, etc. These laws should be illustrated in as many ways as possible; but the general principles should never be lost sight of. It seems a mistake to include in the body of the text—the portion which must be studied by the class—descriptions of all the demonstrations for the lecture room and of the experiments for the laboratory. The last should be the subject of a separate laboratory manual; but, if it is necessary to give these descriptions of class and laboratory experiments in the general text-book, they should certainly be placed in a section by themselves.

The arrangement of matter in the present book is as follows: Part I. Introduction, Mechanics, Properties of Matter, Sound, Heat, Magnetism, Electricity and Light; Part II. Suggestions to Teachers. In Part I. the subjects mentioned are treated in an elementary manner; and the amount given is no more than can easily be studied in the course of one school year. In Part II., descriptions are given of lecture demonstrations and laboratory experiments which are suited to illustrate the text of Part I.; and a few problems are added.

Our thanks are due to two of our former students, Dr. J. F. Merrill and Dr. C. W. Waidner, for their kindness in reading the manuscript and for many valuable suggestions.

J. S. Ames.
H. A. Rowland.

Johns Hopkins University, 1899.

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ELEMENTS OF PHYSICS

INTRODUCTION TO PHYSICS

1. Physics.—Each day we live, our attention is necessarily drawn to the fact that there is a certain regularity in events, that the same cause always produces the same effect. All facts of experience justify this belief, and the conduct of our lives is based upon it. We believe that if a stone is dropped from the hand, it will fall in a definite way and reach the earth in a definite time, depending upon the height from which it is dropped; we believe that day will succeed night, and night day, in a certain definite manner. This belief of ours is founded on our own experience and on that of past ages. It is equivalent to the statement that all the phenomena of nature take place according to fixed laws. To learn these laws and to express them accurately is the aim of NATURAL SCIENCE.

Physics is but a branch of Natural Science; and it is quite impossible to define its limits exactly. It may be said to include Astronomy; and in one direction it approaches Geology, in another Physical Geography, in still another Chemistry. In the main, such special subjects as Mechanics, Sound, Heat, Electricity, Magnetism and Light are included in Physics.

2. Matter and the Ether.—All things that appeal to our senses, in particular to our "muscle sense,"* are called "matter," e. g. a book, a stone, water, air, etc. As will be shown later, we can prove the existence of something which does not affect any of our senses directly, which is a medium filling all the space in the universe around us and permeating all matter, and which has certain properties in common with matter. It is called "the ether." Physics is primarily concerned with the properties of matter and the ether.

The difference between the purposes of Physics and Chemistry may be shown by an illustration. If two forms of matter, e. g. two kinds of sand, black and white, are mixed together, each keeping its individual properties unchanged, so that it is possible to separate them again, the change is called a "physical" one; whereas, if, as the result of bringing two things together, e. g. a piece of coal and the oxygen of the air, the properties of each are lost and an entirely new substance appears, it is called a "chemical" change. Physics is concerned mainly with physical changes. There are, however, many chemical phenomena which are of the greatest importance in the study of physics, such as solution and combustion.

3. Properties of Matter.—As stated above, the name "matter" is given to whatever our senses recognize. If we analyze the sensations by means of which we learn the properties of matter, we see that they are in most cases due to the "muscle sense." For instance, if a ball is stopped or thrown, if a chair is lifted, if a stick is bent or twisted, it is through our muscles that we receive the sensation. It should be noted that these three illustrations are fundamentally different, and that they therefore indicate three independent properties of matter.

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* By "muscle sense" is meant a definite sense to which sensations caused by the actions of the muscles are due.

4. Inertia.—The first illustration, that of stopping or throwing a ball, is one of a property called "inertia." If the motion of a piece of matter is changed in any way (in particular if it be set in motion), we always connect this change with our muscle sensations. Other illustrations are, setting a grindstone in motion, opening a door, holding one's hand under an open water tap, blowing against one's hand. The intensity of the sensation is known by experience to depend upon two things, the *quantity* of the substance and the *suddenness* of the change of motion. For instance, to tell which of two barrels or boxes is full and which empty, one has but to attempt to push one, and then the other; and a ball moving slowly is stopped with a sensation different in degree from that felt when a swift ball is caught.

5. Weight.—The second illustration given above, that of lifting a chair from the floor, has nothing in common with the one just discussed. It is not a question of changing the motion of the chair; for if the chair be held suspended by the arms, or be raised at a *uniform* rate, the sensation is the same. In this case the essential feature is the separation of the chair from the earth. This property of matter is called "weight;" and bodies are called "heavy" or "light" according to the intensity of the sensation felt when they are raised vertically away from the earth. The weight of a body is but a particular case of a much more general property of matter; viz. if our senses were more delicate, we should feel a similar sensation if we separated any two portions of matter, however small, e. g. two baseballs; and this more general property of the action of one body upon another has received the name "gravitation."

6. Size and Shape.—The third illustration, that of bending or twisting a stick, is one of a most numerous class. It includes such actions as stretching a rubber band or a spiral spring, plucking a stretched cord, squeezing out a lump of putty, and compressing a rubber bag full of water or inflated with air. It is seen that they all involve changes or properties of the *size* or the *shape* of the bodies concerned.

7. States of Matter.—Matter exists in many forms, which differ greatly in character. They may be divided conveniently into three groups: solids, liquids and gases, although it is sometimes difficult to tell in which group a body belongs. A *solid*, as ordinarily considered, has a definite volume and a definite shape; and in order to alter either volume or shape considerable effort is required. A *liquid*, if left to itself, forms spherical drops, but, if poured into a bottle or flask, takes its shape, keeping, however, its own volume. It requires great effort to change the volume of a liquid; but it will yield to the slightest attempt to change its shape. A *gas* distributes itself uniformly through the space offered by the vessel which contains it; and so it assumes the shape and volume of the vessel. A liquid, therefore, has a volume of its own, but can take any shape; while a gas has neither shape nor volume of its own. In general, a liquid only partially fills a bottle or goblet; there is a "free surface," separating the liquid from the air. Liquids and gases are both called "fluids," because they can be made to flow.

8. Mass and its Measurement.—The quantity of matter which an object has is called its "mass." To measure this, that is, to give a numerical value to it, several steps are necessary: (1) to choose some property of matter as a basis of comparison; (2) to define what is meant by saying "two objects have equal masses;" (3) to choose some standard of mass. Thus, it is possible to take as the basis of comparison the property of inertia, and to define two objects as having equal masses, if, when set in motion by the same cause, they have identical motions. For instance, let, in succession, the two objects be set in motion by means of the same compressed spring (like that of a toy musket) along a smooth, horizontal table, and if each receives the same velocity, that is, goes the same distance in the same time, their masses may be *defined* to be equal. Care would be necessary to guard against the influence of friction, resistance of air, etc.

Or, gravitation might be taken as the basis of comparison; and two bodies might be *defined* as being of equal mass if they had the same weight. Thus, if each body in turn is suspended by a rubber band (or a spring-balance), and if the elongation is the same in each case, they may be said to have equal masses.

There is no *a priori* reason why there should be any connection between these two definitions; but it is found by experiments (see Articles 30 and 31) that two bodies which have the same inertia also have the same weight. Consequently, it is immaterial which of *these two* properties is taken as the basis of comparison.

The next step is to choose some unit or standard. The scientific world and most of the civilized governments have agreed to accept as this standard a definite cylinder of platinum, known as the "Kilogramme des Archives," which is kept in Paris under the care of the French government. Copies of this have been made and are distributed over the world. Having accepted a standard, it is possible to make another body of almost equal mass, testing the equality by experiments as described above. (They can be made equal to within the degree of accuracy of the instruments used; or they may be slightly different, and the difference can be determined.) Thus having two bodies of equal mass, it is possible to make a third body whose mass shall be equal that of the two equal ones combined; its mass is said to be twice that of the standard. This process can evidently be continued, so as to construct masses three, four, five, etc., times that of the standard. Similarly, by repeated trials, two bodies of equal masses may be made, which when put together have the same mass as that of the standard; each of these two bodies is said to have a mass equal to one half that of the standard. This process of subdivision may be continued as far as desired. A body whose mass is one thousandth of that of the kilogram is said to have a mass of one "gram;" and other subdivisions have received suitable names. Thus, a "set of masses" may be constructed, extending from any great mass to any small one; and, if it is desired to know the mass of any body, e. g. a stone, it is possible by repeated trials to ascertain what combination of the members of the set of known masses has the same mass as that of the arbitrary body (the stone), to the desired degree of accuracy. In this way, the mass of any body may be measured. (In practice, masses are compared and measured by means of a balance, which is an instrument to measure *weight;* but, as said before, two objects which have the same weight also have the same inertia.)

9. **Conservation of Mass.**—By measurements of this kind, it may be proved that if two bodies are allowed to react on each other in any way, e. g. if salt is dissolved in water, the mass of the resulting substance equals the sum of the separate masses. This is known as the "Principle of the Conservation of Mass."

10. **Molecules and Atoms.**—All bodies are made up of smaller portions which have received various names according to their degree of smallness. If a piece of copper wire is cut in two, each piece is still like the original piece in all essential properties. We can imagine this process continued until a piece of copper is reached which is so small that, if in any way it is broken in two, its parts cease to have the properties of copper. This last piece of copper is called a

"molecule," and its fragments are called "atoms." Chemistry deals with methods of breaking down molecules into atoms and re-forming other molecules. Physics is concerned largely with groups of molecules, both those so small as to escape all microscopic investigation, and those so large as to be evident to our senses. (As will be shown later, these molecules and atoms are not at rest, even when the body is a solid, but are both vibrating very rapidly, and also moving about from point to point.)

11. Matter in Motion.—Since in no natural phenomenon is it possible to create or destroy matter, every change in nature must be one of position, for the minute, or for the large portions of matter. In a body falling to the ground, we have a large piece of matter moving; in heating bodies by means of a fire, we have the motion of extremely small portions of matter, as will be shown later. *Matter in motion* is then our fundamental conception of all those phenomena of Nature which involve only matter.

12. Motion.—Motion involves two ideas, a distance and an interval of time; we wish to know how far a portion of matter has moved in a definite time. To give a numerical value to motion, then, we must have methods of measuring distances and intervals of time. A distance has to our minds but one property; and two lengths are equal if they can be superimposed. Our standard of length is that of a platinum bar known as the "Mètre des Archives," which is kept in Paris, the length being measured when the bar is at the temperature of melting ice, i. e. 0° Centigrade. This length can be considered subdivided into smaller parts; and, in fact, one hundredth of a metre is called a "centimetre;" one thousandth, a "millimetre," etc.

Intervals of time have to our minds but one property, that of duration; and we have definite understanding as to what is meant by two "equal" intervals of time. The standard of time is the "mean solar second;" that is, the second of time which is referred to the average length of a solar day, the average being taken of all the days in one year. (Mean solar day = 24 mean solar hours = 1440 mean solar minutes = 86,400 mean solar seconds.

13. The C. G. S. System.—By means of these units, or by means of any other system, all the laws of motion of matter may receive mathematical expression. If the *centimetre*, the *gram* and the mean solar *second* form the system used, the quantities are said to be expressed in terms of the "C. G. S. System;" and this is used in all scientific writing and by most governments.

TABLE I

1 centimetre	=	0.3937 inch.
1 inch	=	2.540 centimetres.
1 gram	=	0.002205 pound.
1 pound	=	453.59 grams.

14. Density.—If the centimetre is the unit of length, the square centimetre is the standard of area, and the cubic centimetre that of volume. If a body is homogeneous, that is, if all its parts are exactly alike, its "density" is defined to be its mass expressed in grams divided by its volume stated in cubic centimetres, or the number of grams per cubic centimetre. (If it is not homogeneous, it is possible to find in the neighborhood of any point a small portion which is homogeneous; and the ratio of the mass of this portion to its volume is the "density at the point" considered.) The kilogram was so constructed that its mass is almost exactly that of 1000 cubic centimetres of pure water at the temperature 4° Centigrade. The difference is, in fact, so small that it can be observed only with the most delicate instruments (1000 cubic centimetres of water at 4° C. have a mass 999.998 grams). In all ordinary calculations, this difference may therefore be neglected. Consequently, the mass of 1 cubic centimetre of water at 4° C. is nearly 1 gram. The density of water, therefore, at 4° C., i. e. the mass of a definite quantity divided by its volume, is 1. (The density of mercury at 0° C. is found to be about 13.6, i. e. the mass of 1 cubic centimetre is 13.6 grams.) At higher temperatures the density becomes less, because the volume occupied by a given mass becomes greater.

TABLE II

DENSITIES

Solids

Brass . (about)	8.5	Lead	11.3
Copper	8.92	Platinum . . .	21.5
Ice at 0° C. . .	0.91	Silver	10.53
Iron, Cast . . .	7.4	Tin	7.29

Liquids

Alcohol at 20° C.	0.789	Sulphuric Acid .	1.85
Mercury . . .	13.6	Sea Water at 0°C.	1.03

Gases at 0° C. and 76 cm. of Mercury Pressure

Air, Dry . .	0.001293	Nitrogen . .	0.001257
Hydrogen .	0.0000895	Oxygen . .	0.001430

CHAPTER I

INTRODUCTION TO MECHANICS

THE science which is concerned with the laws of the motion of material bodies is called Mechanics. As an introduction to it, one must study the different kinds of motion which are possible.

15. Motion in General.—It should be recognized that when a motion of any kind is described, it is always done with reference to something which is considered as at rest. Thus, a body falling from the mast of a moving vessel is said to fall in a vertical straight line; but this is true only with reference to the vessel; the path of the body with reference to the earth is not vertical, but oblique. If, in Fig. 1, $A B$ is the position of the mast when the body starts to fall from A, and if $A' B'$ is its position when the body reaches the deck at B', the path of the body with reference to the vessel is vertical, down the mast; but with reference to the earth, it is the oblique line $A B'$.

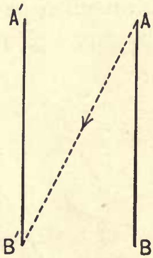

FIG. 1.

Motions are divided into two classes, which have received the names "translation" and "rotation."

Translation is motion such that all the points of the body have the same or equal parallel motions; therefore, all lines that can be imagined drawn in the body remain parallel to themselves during the motion. Motion of translation is that of a moving elevator or hoist, a body falling without twisting, a train moving along a straight track.

Rotation is motion such that all the points of the body are moving in circles which are in parallel planes, and whose centres all lie on the same straight line perpendicular to these parallel planes. This line is called the "axis" of rotation. Motion of rotation is that of a swinging door, a revolving fly wheel, a grindstone in motion.

Cases of pure translation or of pure rotation are uncommon. Almost all actual cases of motion are combinations of translation and rotation; for example, a stick thrown at random into the air, the wheel of a moving wagon, a baseball in motion, etc. If, however, the elementary principles of translation and rotation are understood, there is no difficulty in applying them to more complicated motions.

Translation

16. Displacement; Velocity; Acceleration.—Since in translation the motion of all points of the body is the same, we need consider the motion of one point only. If at any instant a point is at the position A, and later on is at B, the straight line AB is called the "displacement." It indicates by its direction and length the change in position of the body. If the actual motion of the body is such that the path traversed is the straight line $\overline{AB}$, and if the motion is at a uniform rate of speed, the line $\overline{AB}$ measured in centimetres, divided by the time taken for the motion, measured in seconds, is called the "linear velocity," i. e. if the motion is uniform, the

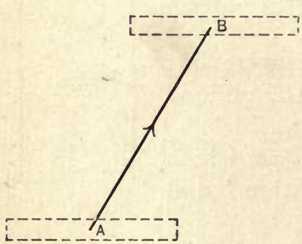

FIG. 2.

linear velocity is the distance traversed in one second in a particular direction. If the motion is not uniform, the "velocity at any instant" is the displacement the body *would* have in one second, *if* its motion were to remain during that time exactly as it is at the instant taken.

Thus, if the velocity of a railway train at any instant is said to be "40 miles an hour south," it is meant that the train is at that instant going in a southern direction at such a rate that, if this should not change, the train would in the next hour traverse forty miles. The linear velocity of a body can therefore be represented by a straight line of a definite length in a definite direction. Thus, in Fig. 3, the line $\overline{PQ}$ may indicate that at any instant a moving body has such a velocity that its motion is in the direction P to Q, and that its rate of motion is such that the length $\overline{PQ}$ equals, or is proportional to, the distance which is (or would be) traversed in the next second. The numerical value of the linear velocity, the element of *direction* being omitted, is called the "linear speed." Thus, velocity is speed-in-a-particular-direction.

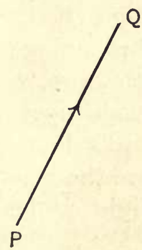

FIG. 3.

If the velocity is changing, either in direction or in speed, the motion is no longer uniform. If the *change*, however, is uniform, the amount of change in one second is called the "linear acceleration;" while if the change is not uniform, the "linear acceleration at any instant" is the change in the linear velocity which *would* take place in the next second, *if* the change were to be uniform during that time. (It should be noticed that "acceleration" does not imply an *increase* in speed necessarily, but any change in the velocity.) Since linear acceleration is a change in the velocity, there are two types: 1. When the direction is unchanged, but the speed altered, e. g. an elevator, a falling body. 2. When the speed is unchanged, but the direction altered, e. g. a stone whirled in a horizontal circle by means of a sling, a point on the rim of a fly wheel.

17. Composition and Resolution.—If a body is given in succession two displacements, $\overline{AB}$ and $\overline{BC}$ (Fig. 4), it is equivalent to the single displacement $\overline{AC}$. The line $\overline{AC}$ is called the "geometric sum" of the lines $\overline{AB}$ and $\overline{BC}$; or the line $\overline{AC}$ is said to be "resolved into" the lines $\overline{AB}$ and $\overline{BC}$. (It should be noted that it is immaterial which displacement comes first, $\overline{AB}$ or $\overline{BC}$. For, if $\overline{BC}$ comes first, the broken line giving the displacement is the opposite half of the parallelogram, viz. $\overline{AD}$ and $\overline{DC}$.) Similarly, any broken line, made up of two lines, starting from A and ending at C, is equivalent to $\overline{AC}$. Of these broken lines, however, those are the most important which are made up of two lines at right angles to each other, e. g. $\overline{AD}$ and $\overline{DC}$ in Fig. 4*a*, or

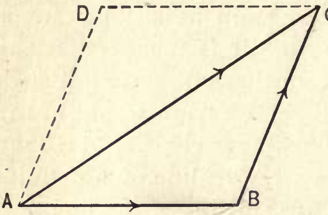

FIG. 4.

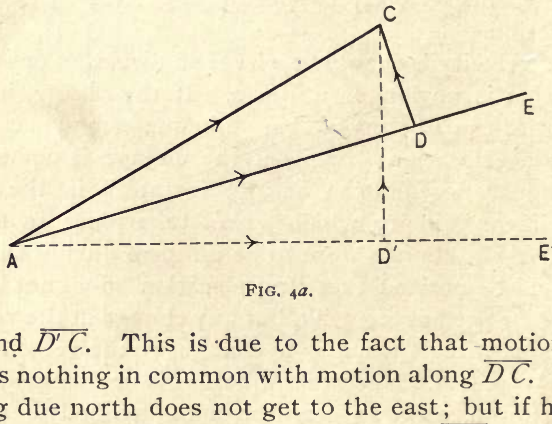

FIG. 4*a*.

$\overline{AD'}$ and $\overline{D'C}$. This is due to the fact that motion along $\overline{AD}$ has nothing in common with motion along $\overline{DC}$. (A man walking due north does not get to the east; but if he walks northeast, he goes both north and east.) $\overline{AD}$ is called the "component" of $\overline{AC}$ in the direction $\overline{AE}$; $\overline{AD'}$ is the component in the direction $\overline{AE'}$. It follows from geometry that

$$\overline{AC}^2 = \overline{AD}^2 + \overline{DC}^2 = \overline{AD'}^2 + \overline{D'C}^2$$

In a similar manner, velocities can be compounded. If in Fig. 4 $\overline{BC}$ represents the velocity of a railway carriage, and $\overline{AB}$ that of a man walking across it, $\overline{AC}$ is the velocity of the man with reference to the tracks; or, if $\overline{BC}$ is the velocity of a river current, and $\overline{AB}$ that which a boat would have if there were no current, $\overline{AC}$ will be the actual velocity of the boat. (Other illustrations are given by the path of a raindrop when it first strikes the window pane of a railway carriage in motion, by the direction in which the trail of smoke follows a steamer.) As before, $\overline{AC}$ is called the geometric sum of $\overline{AB}$ and $\overline{BC}$. Referring to Fig. 4*a*, if $\overline{AC}$ is a velocity, $\overline{AD}$ is called the component in the direction $\overline{AE}$.

18. Acceleration.—Linear acceleration is the rate of change of linear velocity; that is, it is the difference between two velocities, and can therefore be represented by a straight line. Thus, in the case of a falling body, let $\overline{AB}$, in Fig. 5, be the velocity at any instant, and let $\overline{CD}$ be the velocity "t" seconds later. The change in the velocity, $\overline{CD} - \overline{AB}$, is the line $\overline{BD}$; the acceleration has therefore the same direction as the two velocities, and has the numerical value $\dfrac{\overline{BD}}{t}$, if the change in velocity is uniform; otherwise the acceleration at the instant is the limiting value of this fraction as the interval of time t is made smaller and smaller.

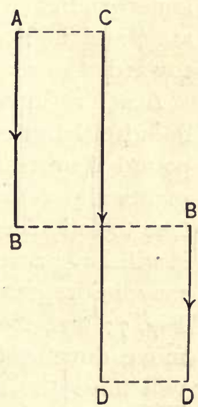

FIG. 5.

In the case of a point on the rim of a fly wheel in steady motion (see Fig. 6), the velocity at the instant when it is at P may be represented by the line $\overline{OA}$ which is parallel to the tangent at P; when t seconds later the point is at Q, its velocity will be $\overline{OB}$, a line which is parallel to the tangent at Q, and whose length equals that of $\overline{OA}$, since the speeds are the same. The change in the velocity, that is, the difference between the lines $\overline{OB}$ and $\overline{OA}$, is the line $\overline{AB}$, because $\overline{AB}$ added by geometry to $\overline{OA}$ gives $\overline{OB}$. The linear acceleration at P, therefore, is the limiting ratio of $\frac{\overline{AB}}{t}$ when t is made so small that Q is the point on the circle consecutive to P. In the limit, then, the direction of $\overline{AB}$ (and, consequently, that of the acceleration) is perpendicular to $\overline{OA}$, i. e. to the direction of the tangent at P, and hence is along the radius of the circle at P and towards the centre.

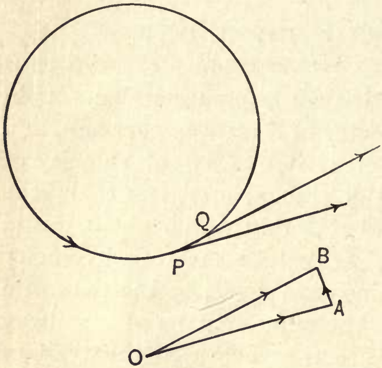

FIG. 6.

Accelerations being represented, as shown, by lines of definite lengths and in definite directions, can be compounded or resolved, just like displacements and velocities. Thus, if a body is free to fall vertically towards the earth, it will have a vertical acceleration which may be represented by the line $\overline{OP}$, Fig. 7; but, if the body is compelled to move down a smooth inclined plane, it will have the acceleration $\overline{OR}$, which is the component of $\overline{OP}$ in the direction down the plane, the angle (ORP) being

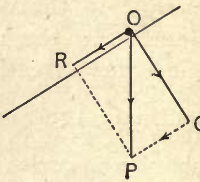

FIG. 7.

a right angle. The other component, $\overline{OQ}$, is neutralized by the plane down which the body falls.

SPECIAL CASES

19. Direction of Motion Unchanged, Acceleration Constant.—Let the numerical value of the acceleration be "a," and let the velocity of the body at any instant be "s_0"; then, since the acceleration is the change of velocity in one second, the velocity t seconds later will be $s_0 + at$. Call this s. Then,

$$s = s_0 + at. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

Since the acceleration is constant, the *average** velocity during these t seconds will be $\frac{s + s_0}{2}$; and this being the average distance traversed in one second, the distance traversed in the t seconds will be $t \times \frac{s + s_0}{2}$. Call this x. Then

$$x = t\,\frac{s + s_0}{2}.$$

or, substituting for s its value,

$$x = s_0 t + \tfrac{1}{2} a t^2 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

---

* To prove this statement, lay off a horizontal straight line $\overline{AB}$ of length t (t being a number); from A erect a perpendicular $\overline{AA'}$ of length s_0; and from B a perpendicular $\overline{BB'}$ of length s. Draw the straight line $\overline{A'B'}$ (Fig. 7*a*). Since the velocity changes at a *uniform* rate from s_0 to s during the t seconds, the velocity at any instant during these t seconds is given by the vertical distance between $\overline{AB}$ and $\overline{A'B'}$ at the proper point. The *average* velocity is thus the length of the line which is the average of all the lines drawn to $\overline{A'B'}$ perpendicular to $\overline{AB}$. If C is the middle point of $\overline{AB}$, the average velocity is $\overline{CC'}$, or $\frac{s_0 + s}{2}$.

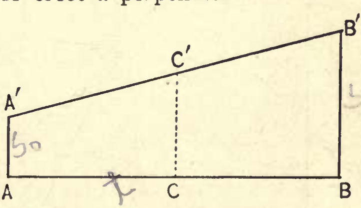

FIG. 7*a*.

$s_0 t$ is the distance the body would have gone if there had been no acceleration, i. e. if the velocity had remained constant; $\frac{1}{2} a t^2$ is the additional distance traversed, owing to the acceleration.

If the seconds are counted from the instant when the body starts from rest, i. e. if $s_0 = 0$,

$$\left.\begin{aligned} s &= a t \\ x &= \tfrac{1}{2} a t^2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If, therefore, it is found by experiment that the distance from rest which a body goes in t seconds varies as t^2, it follows that the acceleration is constant.

Illustrations of this motion are bodies falling freely, bodies falling down inclined planes, a train being brought to rest by means of friction (in which case "a" is negative), bodies thrown vertically upward, etc.

20. Projectiles.—If a body is projected from a height in a horizontal direction with a velocity v, it will in t seconds go in this direction a distance $t v$, if there is nothing to retard or hasten its horizontal motion; but, while it is traversing this distance, it is also falling a vertical distance $\frac{1}{2} a t^2$. Let the horizontal velocity be represented by $\overline{OA}$ in Fig. 8 and the vertical acceleration be "g," where $\frac{1}{2} g$ is represented by $\overline{Oa}$. Then at the end of the first second the body will be at P; at the end of the second at Q, where $\overline{OB}$ is twice $\overline{OA}$, and $\overline{BQ}$ is four times

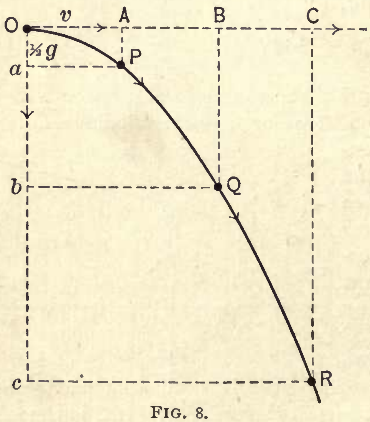

FIG. 8.

$\overline{Oa}$, etc. The actual path is seen to be a parabola. Similarly, a body thrown into the air from the earth will describe a parabola, e. g. path of jet of water, of a ball, of a bullet, etc. (This statement presupposes that the effect of the resistance of the air can be neglected. In all actual cases there is an effect due to the air, and the curve is not exactly a parabola.)

21. Uniform Motion in a Circle.—It has been shown (Article 18) that, when a point is moving in a circle with constant speed, the acceleration at any point is along the radius drawn to that point, and has for its numerical value the limiting value of $\frac{\overline{AB}}{t}$, where $\overline{OA}$ is the velocity at P, $\overline{OB}$ that at Q, and t is the time taken to traverse the path PQ. (See Figs. 6 and 9.) Let the radius $\overline{CP}$ be called r; the linear speed, s; the acceleration, a. Then $\overline{OA} = \overline{OB} = s$, because the numerical value of the velocity is the speed; and $a = \frac{\overline{AB}}{t}$ *in the limit*, i. e. when t is a very small fraction of a second. In the limit also, the arc PQ is a straight line, and, since the radii $\overline{CP}$ and $\overline{CQ}$ are perpendicular to the lines $\overline{OA}$ and $\overline{OB}$, which are parallel to the tangents at P and Q, the triangles CPQ and OAB are similar.

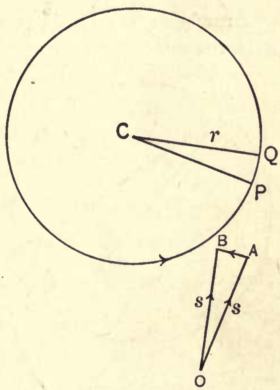

FIG. 9.

Hence $$\frac{\overline{AB}}{\overline{OA}} = \frac{\overline{PQ}}{\overline{CP}}.$$

But $\overline{OA} = s$; $\overline{CP} = r$; and $\overline{PQ}$, the path traversed in t seconds, equals st.

Therefore $$\frac{\overline{AB}}{s} = \frac{st}{r};$$

and so $$\overline{AB} = \frac{s^2 t}{r}.$$

The numerical value of the acceleration follows at once:

$$a = \frac{\overline{AB}}{t} = \frac{s^2}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It is seen to be independent of the position of the moving point. Its direction is, however, as shown above, towards the centre of the circle as the point moves.

Another method of proof is this: At P, in Fig. 10, the acceleration a is towards the centre (as shown above); therefore, in the t seconds taken to go to Q, the body would, if at rest in P, have fallen towards C a distance $\overline{PR}$, which equals $\frac{1}{2}at^2$. Since, however, the body has at P the speed s along the tangent, it will at the end of the t seconds be at Q if t is extremely small, where $\overline{PQ} = st$; and where $\overline{QR}$ is perpendicular to the radius $\overline{CP}$. (Compare the motion of a projectile.) If the radius $\overline{CP}$ is prolonged into a diameter $\overline{SP}$, the triangle SQP is a right-angled one; and by similar triangles

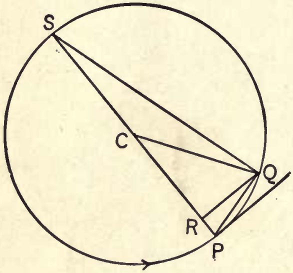

FIG. 10.

$$\overline{PQ}^2 = \overline{PR} \times \overline{SP}.$$

In the limit the straight line $\overline{PQ} = st$; further, $\overline{PR} = \frac{1}{2}at^2$, and $\overline{SP} = 2r$.

Hence $$s^2 t^2 = a t^2 r,$$

or $$a = \frac{s^2}{r}.$$

22. Angles.—The difference in direction between two straight lines which lie in one plane is called the "angle" between them. The numerical value to be given an angle is *defined* as follows: Let $\overline{PQ}$ and

$\overline{P' Q'}$ be the two straight lines; prolong them till they meet at O; with O as a centre and with any radius $\overline{O R}$ describe an *arc* of a *circle* intercepted by the two lines $\overline{O P Q}$ and $\overline{O P' Q'}$; then the ratio of the length of the arc $\overline{R R'}$ to the radius $\overline{O R}$ is the value of the angle (Fig. 11).

This ratio $\frac{R R'}{O R}$ is independent of the radius chosen, as is evident from geometry. Applying this to a point moving in a circle (Fig. 9), the angle

$$(P C Q) = \frac{P Q}{C P} = \frac{s t}{r}.$$

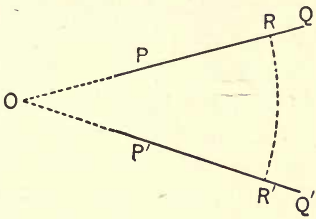

FIG. 11.

Hence

$$\frac{s}{r} = \text{angle traversed in one second.}$$

This is called the "angular speed," and may be written ω.

Then
$$\frac{s}{r} = \omega,$$

and
$$a = \frac{s^2}{r} = r \omega^2.$$

Since the circumference of a circle is $2 \pi r$, one quarter of a circumference is $\frac{\pi r}{2}$. Hence the numerical value of a right angle is $\frac{\pi}{2} r/r = \frac{\pi}{2}$; and, when a radius of a circle makes one complete revolution, it passes through an angle $\frac{2 \pi r}{r} = 2 \pi$. The angle $\frac{\pi}{2}$ is often called 90°; that of 2π, 360°; but these figures are perfectly arbitrary.

Rotation

23. Angular Velocity, etc.—The characteristic property of rotation is angular motion around a certain axis, e. g. a fly wheel in motion, a swinging door, a barrel rolling down an inclined plane, a top spinning, etc. These illustrations may be classified as follows:

1. Constant angular speed around a fixed axis. This is the case of a fly wheel in uniform motion.

2. Varying angular speed around a fixed axis. This is the case of a fly wheel when it is being set in motion or being stopped. (A barrel rolling down an inclined plane is

also an illustration, because its axis, although not fixed, is moving parallel to itself.)

3. Constant angular speed around an axis whose direction is varying. This is the case of the spinning top, if its axis of figure is not vertical. As every one knows who has played with tops, the axis under these conditions slowly changes its direction and describes a cone.

It should be noted that these three classes are perfectly analogous to the three classes of translation: 1. Constant linear velocity; 2. Varying speed, direction of motion unchanged, e. g. a falling body; 3. Constant speed, direction varying, e. g. a stone whirled in a horizontal circle by a sling. Thus there is complete analogy between "distances" and "direction of motion" in Translation, and "angles" and "direction of axis" in Rotation.

CHAPTER II

DYNAMICS

24. The fundamental properties of matter were discussed briefly in the Introduction, Articles 3–6; and methods were described for the measurement of quantity of matter or "mass." It remains, however, to express in mathematical language the properties of matter in motion, and to discuss the methods by which various motions may be produced. This branch of mechanics is called Dynamics.

TRANSLATION

25. Mutual Action of Two Bodies.—The motion of a single body is a special case of that of two bodies; and to have any change in the motion of a body, the presence of a second body is necessary. The commonest illustration of motion of matter in which only two bodies are concerned is a body falling towards the earth. Since the body falls towards the earth, the latter might be expected to move towards the body; and, therefore, to describe the phenomenon, it would be necessary to know the mass of the body and its velocity at any instant, and, in addition, the mass of the earth and its velocity. This shows that this case of motion is not by any means simple. A better illustration would be given by two balls rolling on a smooth table, colliding and rebounding; for here both masses and both velocities before and after impact may be measured. If the two balls are moving in the same straight line, and the faster overtakes the slower, it is known that the faster will go more slowly after impact, and the slower will go faster —one loses velocity, the other gains. Another simple illustration would be given by a man jumping horizontally off a chair which rests on rollers; the man would gain velocity in one direction; the chair, in the *opposite*. It is evident from experience that the heavier the chair, the less will be its velocity. Thus there is some connection between the masses and the velocities.

The general law which applies to two bodies may be stated as follows: If two bodies whose masses are m_1 and m_2 are moving in the same line with the linear velocities v_1 and v_2, and if they are *free from all external influences*, the sum $m_1 v_1 + m_2 v_2$ remains unchanged, no matter in what way one body influences the other. (It should be remembered that a velocity has the idea of *direction;* so that if two velocities are in opposite directions, one is +, the other is −.)

The following illustrations may be given:

1. Consider a rifle and its bullet. They are both at rest before the powder explodes, and therefore at that instant v_1 and v_2 are both zero. Consequently, the sum $m_1 v_1 + m_2 v_2 = 0$. After the explosion, the bullet whose mass may be called m_1 will have a velocity v_1; the rifle whose mass may be called m_2 will have a velocity v_2 in the *opposite* direction, if it is suspended by cords so as to be free to move; and it is known that the numerical value of v_2 is $\frac{m_1 v_1}{m_2}$.

Hence $$v_2 = -\frac{m_1 v_1}{m_2},$$

or $m_1 v_1 + m_2 v_2 = 0$; hence the sum is unchanged.

2. The common lawn sprinkler (a simple model of which is shown in Fig. 12), in which water issues horizontally in one direction and the wheel rotates in the opposite, is still

another illustration. This same apparatus may be used with air or any gas, as well as with liquids.

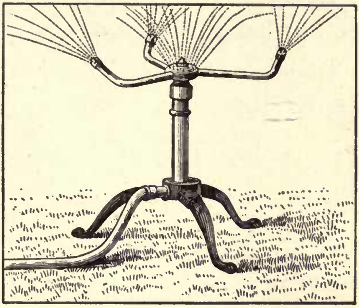

FIG. 12.

3. The law may be illustrated also by means of an "impact apparatus," a particular form of which is shown in Fig. 13. In this apparatus, bodies of different masses are put on the two swinging platforms, which are then drawn back and allowed to swing together and collide along a horizontal line. The velocities before and after impact may be measured; and it is found that the sum $m_1 v_1 + m_2 v_2$ remains unchanged.

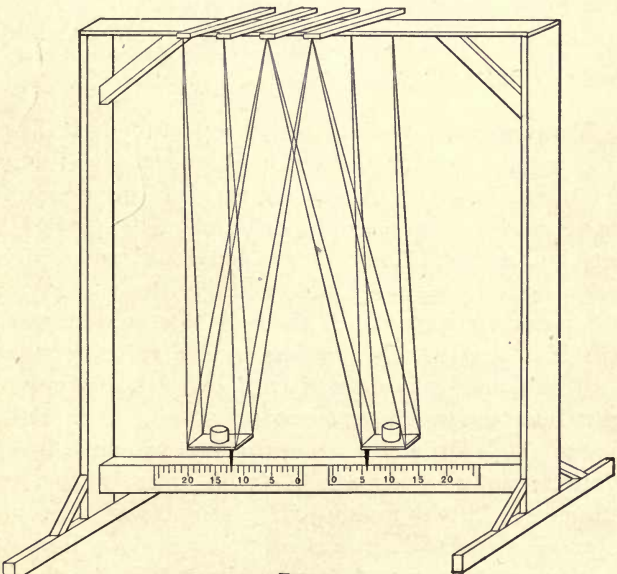

FIG. 13.

4. If a magnet and a piece of iron are floated on the surface of water, or are so suspended as to have freedom of motion, they will approach each other, their *speeds* at any instant being such that

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}.$$

Hence, since they are moving in *opposite* directions,

$$m_1 v_1 + m_2 v_2 \text{ is unchanged.}$$

5. Similarly, as a body falls towards the earth, the earth rises towards the body, but with a velocity so small that it is inappreciable.

It should be noted that in all these illustrations the velocities of both bodies are in the same straight line. If the velocities were in different directions, the law would be that the *geometric* sum of the products remains unchanged. Illustrations of this more general law are afforded by the impact of billiard balls.

26. Momentum.—Owing to the importance of this product $m v$, mass $\times$ linear velocity, it has been given a name, "linear momentum." A special case of the general law as stated above is when only one body is supposed to be present, i. e. $m = o$. Then $m v =$ constant; and, since the mass of a body is not subject to change, the velocity itself, v, must remain constant. This may be stated in words as follows: If a single body moving with a velocity v is free from all external influences, it will continue to move in a straight line and with a constant speed, i. e. with the velocity v. It is impossible to verify this principle by direct experiment; for no body can be freed entirely from external influences; but by making friction and resistance of the air as small as possible, it is seen that a body will maintain its velocity practically unchanged, e. g. a ball moving along a smooth horizontal table; a railway train, with steam off, moving on a level track.

27. **General Law of Momentum.**—A more general case than that stated in the above law is when more than two bodies are considered in the action. The principle in that case is as follows: If bodies whose masses are m_1, m_2, m_3, etc., and whose linear velocities at any instant are v_1, v_2, v_3, etc., form a system free from external influences, the geometrical sum $m_1 v_1 + m_2 v_2 + m_3 v_3 +$ etc., remains unchanged, regardless of what impacts, motions, explosions, etc., go on inside the system itself. Illustrations are afforded by billiard balls, by the planets forming the solar system, etc. This general principle may be shown by geometric considerations (see Ames's "Theory of Physics," p. 42) to be identical with the following: In any system of bodies free from external influences there is a point which moves in a straight line with a constant speed, no matter how the parts of the system move, impinge on each other, or affect each other. This point is known as the "centre of inertia" or the "centre of mass," and it coincides with the point commonly called the "centre of gravity." Thus, if a stick is sent whirling along a smooth table, there is one point which will describe a straight line on the table; this is the centre of inertia. In the case of any regular figure, the centre of inertia is the centre of figure, e. g. a sphere, cylinder, stick.

It should be noted that a single large body is a particular case of a system of small bodies; and, further, that the centre of inertia is not necessarily a point in the body itself, but is a point fixed with reference to it—thus the centre of inertia of a circular hoop is the centre of the circle.

28. **Kinetic Reaction.**—In the case of two bodies, the law under discussion states that the *algebraic* sum

$$m_1 v_1 + m_2 v_2 = \text{constant}$$

if the bodies are moving in the same straight line. Therefore, if v_1 changes in any way, v_2 must change so that $m_1 \times$ change in v_1 is equal and opposite to $m_2 \times$ change in v_2; otherwise the above sum would not remain unchanged. If the change in velocities in one second is considered, i. e. if the linear accelerations are concerned, this statement is, that

$$m_1 a_1 = - m_2 a_2,$$

if a_1 and a_2 are the accelerations of m_1 and m_2, respectively.

This product, mass $\times$ linear acceleration, $m a$, is called the "kinetic reaction" or the "inertia" of m with reference to the action of the other body. It in a way measures the opposition which a body of mass m offers to being given an acceleration a.

The above equation is sometimes expressed in words by saying that "action and reaction are equal and opposite;" for the reaction of m_1, as measured by $m_1 a_1$, is equal and opposite to that of m_2, as measured by $m_2 a_2$.

If a body whose mass is m is under the influence of several other bodies which separately would produce accelerations in m of values a_1, b_1, c_1, etc., the acceleration would be the geometric sum of a_1, b_1, c_1, etc. If this sum is a, the total kinetic reaction of m against all these actions is $m a$. This product measures what is called the "external force," implying that, owing to causes outside itself, the body whose mass is m is given an acceleration a, and that the proper measure of the effect of these causes is $m a$. Illustrations of external forces are given by magnets when near pieces of iron, electrically charged bodies, the action of weight, a man pulling or pushing, etc.

29. External Force.—The fact that $m a$, i. e. the product of the acceleration and the mass of the matter which is given the acceleration, is a proper measure of these external causes may be shown by various experiments. One simple form is as follows: Let a body whose mass is m_2 lie on a smooth* horizontal table and be joined by an inextensible cord running over a pulley to a body whose mass is m_1 and which hangs vertically. There will be motion, owing to the weight of m_1; the weight of m_2 has no influence on the motion, since it rests on the table. Call the acceleration a. The mass which has this acceleration is $m_1 + m_2$; hence the external force equals $(m_1 + m_2)\,a$. Now replace m_2 by a different mass m_3; there will be a different acceleration; call it b. The external force now equals $(m_1 + m_3)\,b$. But these two quantities should be the same, since the motion is due to the same external conditions, viz. the weight of m_1. And experiments show that they are equal. Therefore $m\,a$ is the right measure of external influences.

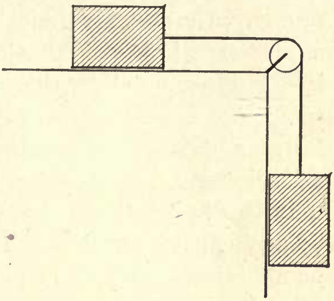

FIG. 14.

If the numerical value of $m\,a$ is f, there is said to be an external force of f "dynes;" that is, under the influence of 1 dyne a mass of 1 gram would have an acceleration of 1 cm. per second given to it in each second.

SPECIAL CASES

30. I. Falling Bodies.—If a body of any size, shape or mass is allowed to fall *freely* in a vertical direction towards the earth, it is found by experiment to have a constant acceleration of about 980 cm. per second in each second. (The motion must take place in a vacuum, so as to avoid the resistance of the air.) This acceleration is commonly called "g;" it is thus proved to be independent of the mass or material of the body, and to be constant at any

* By a "smooth" table is meant one whose surface offers no resistance to a body sliding over it, i. e. there is no friction.

one place on the earth's surface. Its value, though, varies with the latitude and with the height above sea level, and is affected by the neighborhood of mountains, etc. Consequently, the kinetic reaction of a body of mass m grams against the influence of the earth (or the "weight" of m) is mg dynes. Thus, the weight of 1 gram is 980 dynes; and so 1 dyne is nearly the same as the weight of 1 milligram. Since a dyne is so small, a larger unit is convenient; and a "kilodyne" or 1000 dynes may be used.

If the body whose mass is m is prevented from moving by any cause, e. g. by resting on a table or by being suspended by a cord, this cause, i. e. the table or the cord, must have an effect equal and opposite to the weight of the body. There is said to be a "pressure" on the table, or a "tension" in the cord. If the cord is hung from a peg or nail which is at rest, its action on the cord must be equal and opposite to that of the cord on it; that is, the peg experiences a downward pull mg dynes, and in turn produces an upward pull mg dynes. Thus, the cord simply transmits the action of the peg to the hanging body. This method is one of the simplest ways of producing a force.

31. Pendulums.—The fact that the acceleration "g" is the same for all kinds and quantities of matter is best shown by experiments on pendulums. A "simple pendulum" consists of a small, heavy bob suspended by a very fine cord in such a way that it can make vibrations in a vertical plane. The length of path traversed by the bob from the extreme position on one side to that on the other is called the "amplitude;" and the "period" of one complete vibration is the time required to swing from the extreme position on one side to the opposite extreme position, and back again to the original position. If the amplitude is small, e. g. less than one hundredth of the length of the supporting cord, the period is

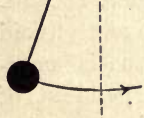

FIG. 15.

$$2\pi\sqrt{\frac{l}{g}},$$

where π is the number 3.1416, the ratio of the circumference of a circle to its diameter; l is the length of the pendulum; and g is the acceleration of a body falling freely. (See Ames's "Theory of Physics," page 59.)

It is seen that the period of a pendulum varies directly as the square root of its length, and so it may be altered as desired. Further, since both the period and the length can be measured, this gives a method for the determination of the acceleration g. By swinging pendulums of all kinds of matter and measuring their periods, it has been shown that g is a constant, as stated above.

Vibrations like those of a pendulum are called "harmonic." One of their properties is that the period does not depend upon the amplitude, provided it is small. Tuning-forks, vibrating spiral springs, watch-springs, etc., make harmonic vibrations. (See Article 60, Elastic Properties of Solids.)

32. 2. **An Elevator.**—Let the mass of the elevator be m, and let it be supported by a rope whose upper end may be supposed to be wound over a windlass. There are two external causes affecting the motion, the weight mg acting down and the cord whose tension may be called T, acting up. Let the acceleration upward be called a. Then the external force upward is $T-mg$, the kinetic reaction is ma. Hence

$$ma = T - mg,$$

or

$$T = mg + ma.$$

Therefore, if the elevator has acceleration upward, the tension in the cord is greater than the weight of the elevator; if there is an acceleration downward, i. e. if a is negative, the tension is less than the weight; if the elevator is moving with a constant velocity, either upward or downward; that is, if the acceleration is zero, the tension equals the weight.

It is evident, then, that if the elevator were to be jerked upward too suddenly, the required tension might be greater than the material of the supporting rope could stand, and it would break.

33. 3. Motion in a Circle.—If a body of mass m is made to move in a circle of radius r with a linear speed s, e. g. by means of a cord, as in a sling, or by being made to follow the inner side of a circular hoop, the acceleration is s^2/r and is always directed in towards the centre of the circle. (See Art. 21.) The kinetic reaction is therefore $m s^2/r$; and this must be the tension of the cord, or the pressure of the rim of the hoop, the direction being always towards the centre. If the cord is broken or the hoop removed, there will no longer be any acceleration, and the body will move with a constant speed s in the direction of its motion at the instant the restraint was removed; that is, along the tangent to the circle. Illustrations are afforded by a stone thrown from a sling; the sparks thrown off by a revolving emery wheel; the drying of clothes or of sugar by "centrifugals," which consist of cylinders revolving rapidly around their axes and having openings in their walls, thus allowing the water to escape when the speed is such that the cloth or the sugar no longer produces sufficient constraint to equal $m s^2/r$. When a bicycle rider passes rapidly around a curve, he leans in towards the centre of the curve so that his weight may exert the necessary force to keep moving in a circle. For a similar reason, the outer rail of the track of a railway is elevated at the curves, so as to make the train lean inward.

If the body is whirled by a cord whose other end is fastened to a peg at the centre, the external force on the moving body is $m s^2/r$ dynes, and this is towards the centre; similarly the peg experiences an equal and opposite pull outward. Thus the peg is pulled outward by the cord, but pulls the moving body inward, the action being transmitted by the cord.

34. Universal Gravitation.—Another illustration of this motion in a circle is the revolution of the moon around the earth, and, in fact, the revolution of any of the planets or their satellites. If m is the mass of the moon, s its linear speed, and r the radius of its path, there must be a force $m s^2/r$ acting towards the centre, i. e. the centre of the earth. Consequently, if the moon were stopped for an instant, it would begin to fall directly towards the earth with an acceleration s^2/r, s being the speed necessary to keep the moon moving in its orbit, in spite of the tendency to fall towards the earth. s is known; because, if T is the time of one complete revolution of the moon around the earth, $s = 2\pi r/T$; and r is known from astronomical calculations; consequently, s^2/r can be calculated. If this acceleration at the distance r from the centre of the earth is compared with g, the acceleration at the surface of the earth, i. e. at a distance from the centre of the earth of R, the radius of the earth, it is found that their ratio is equal to the inverse ratio of r^2 and R^2. That is, acceleration at r: acceleration at $R = \frac{1}{r^2} : \frac{1}{R^2}$.

Therefore, it may be *assumed*, as a general law applying to all bodies and to all distances, that the external force on a small body of mass m whose centre is at a distance r from the centre of a second small body of mass m', owing to the action of this body, varies directly as the product of the masses and inversely as the square of their distance apart, i. e. F varies as $\frac{m m'}{r^2}$,

or

$$F = c \frac{m m'}{r^2},$$

where c is a constant; it is called the "gravitation constant." (A spherical body, if homogeneous, acts as if all its mass were concentrated at its centre.) This law is known as "Newton's Law of Universal Gravitation," and accounts for all observed facts concerning the motions of the planets, satellites, double stars, etc.

35. 4. **System of Bodies.**—If a system of bodies, in particular if a single large body, is subject to external influences, it may be proved (see Ames's "Theory of Physics," p. 42) that the centre of inertia (see Article 27) will move with an acceleration exactly the same as that which a minute body would have whose mass was equal to that of the entire system and which was subjected to the same external force. Illustrations are afforded by the fact that when a beam falls from a building, its centre of inertia falls vertically with the acceleration g; when a man jumps over

a fence, his centre of inertia describes a parabola; when a bombshell explodes, the fragments move in such a way that their centre of inertia follows the same path which the shell would have followed if it had not exploded. If a stick lying on a smooth table is struck a blow perpendicular to its length, the centre of inertia will move in a straight line in the direction of the blow, entirely independently of the point where the blow was struck (naturally, the rotation is different for different points of striking, but the translation is not). Fig. 16 represents the motion of a stick thrown upward obliquely.

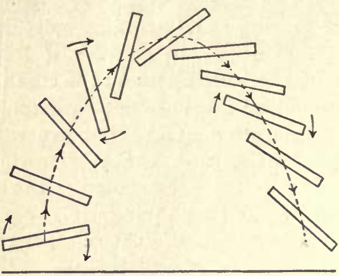

FIG. 16.

36. 5. Composition of Forces.—Forces can be added and resolved, since they are measured by accelerations. Let a body be so situated that there is a force F_1 in one direction and a force F_2 in a different one. (These forces may be produced by cords pulling on the body.) The numerical value of F_1 is ma_1, where a_1 is the acceleration which the body of mass m would have if the second force were absent. In particular, if F_1 is produced by the weight of M grams, its numerical value is Mg. Lay off a straight line $\overline{OA}$ in the direction of this acceleration, and of a length proporportional to F_1 (i. e. to ma_1). See Fig. 17. Similarly lay off a line $\overline{OB}$ proportional to F_2. Their geometric sum is the line $\overline{OC}$, the diagonal of the parallelogram $OACB$; and this represents the combined action of F_1 and F_2. It gives

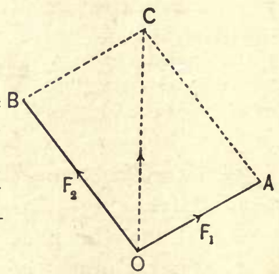

FIG. 17.

the actual acceleration of the body whose mass is m, and is called the "resultant."

This may be shown in a slightly different way (see Fig. 18). The force F_1, represented by $\overline{OA}$, may be resolved into two components (see Article 17), $\overline{OE}$ along the diagonal and $\overline{EA}$ at right angles to it. Similarly, the force F_2, represented by $\overline{OB}$, may be resolved into the two components $\overline{OD}$ along the diagonal and $\overline{DB}$ at right angles to it. But by principles of plane geometry, the lines $\overline{EA}$ and $\overline{DB}$ are equal and opposite, and therefore these two components neutralize each other; further, $\overline{OE}$ equals $\overline{DC}$; hence the two components $\overline{OE}$ and $\overline{OD}$ are equivalent to $\overline{OD}$ and $\overline{DC}$, that is, to $\overline{OC}$, the diagonal. In a similar manner three and more forces may be compounded.

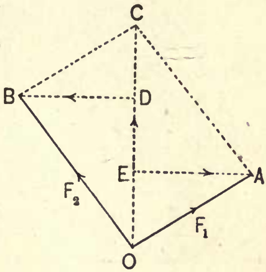

FIG. 18.

ROTATION

37. Moment of a Force.—It has been shown that in translation the proper measure of an external force is ma; but this says nothing about the rotation. The rotation depends evidently upon two things, the force and its line of application. Thus, a push near the hinges of an open door produces comparatively little effect; but, if delivered near the edge of the door, rotation is produced. The exact law may be found by considering a simple case: Let a board be pivoted by a peg at P, and let two forces, F_1 and F_2, be applied at points N_1 and N_2 in the plane of the board. Let the forces be so chosen that there is no rotation of the board. The action of the force F_1 (e. g. a string pulling on a nail fastened in the board) is exactly the same as if it were applied at any point in its "line of action," i. e. in the line $\overline{N_1 O}$; similarly, F_2 may be considered as applied at any point in its line of action $\overline{N_2 O}$. If these lines when prolonged meet at the point O, lay off from it $\overline{OA_1}$, and $\overline{OA_2}$ proportional to F_1 and F_2. Let $\overline{OB}$ be their geometric sum. Since there is no rotation of the board, this line must pass through P, the axis of rotation, (otherwise there would be rotation, as in the case of a push to a door). Draw perpendiculars from P to the lines $\overline{ON_1}$ and $\overline{ON_2}$. Call them $\overline{PQ_1}$ and $\overline{PQ_2}$. Then, by a well-known theorem of geometry,

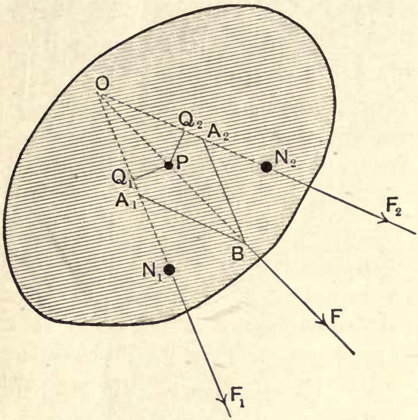

FIG. 19.

$$\overline{OA_1} \times \overline{PQ_1} = \overline{OA_2} \times \overline{PQ_2}.$$

That is, the condition that the rotating effect of F_1 should be neutralized by that of F_2 is that $F_1 \times \overline{PQ_1} = F_2 \times \overline{PQ_2}$. $\overline{PQ_1}$ and $\overline{PQ_2}$ are called the "lever arms" of the forces F_1 and F_2 around the axis through P. The product of a force by the perpendicular distance between its line of action and the direction of the axis is called the "moment of the force around the axis." Therefore the measure of the rotating power of an external force about any axis is the moment of the force around that axis.

The moment of a force around an axis is necessary in order to produce any change in the existing rotation; just as a force is necessary to produce a change in the existing translation. Therefore, if there is no moment, there is no change in the rotation; a "*twist*" of some kind is required to change either the angular speed or the direction of the axis of rotation. Thus, a quoit or a book, set spinning in its own plane when it is thrown, keeps its axis of rotation parallel to itself as it moves, the same side stays up; a projectile fired from a rifled-gun is set spinning and maintains the direction of its axis, because in neither of these cases does the action of the earth do anything but *pull* the body down; it does not produce rotation of any kind. (The action of the resistance of the air is neglected.) But, if a brake is applied to a fly wheel, or if the crank of the driving wheel of a locomotive is set in motion, there is a moment around the axis, and consequently a change in the rotation. Similarly, if a top were not spinning, it would turn over and fall on the ground; i. e. the action of gravity in this case produces a twist around an axis at right angles to the axis of figure of the top; so, when the top is set spinning, the direction of its axis is constantly changing. (See Article 23.)

The "curves" of a baseball are due to the fact that, as it spins through the air, there is a difference in the friction and pressure of the air on opposite sides; and so a "push" is given sidewise or up or down, according to the way in which the ball is thrown. There is also a "twist" which alters the angular speed and the direction of the axis of rotation.

38. Equilibrium.—Since in the case of the pivoted board there is no translation, it is evident that the resultant of F_1 and F_2, i. e. the force represented by $\overline{OB}$, must be equal and opposite to the action of the peg on the board. Consequently, the peg could be replaced by a force equal in amount to the geometric sum of F_1 and F_2, in a direction opposite to $\overline{OB}$, and applied at any point in the line $\overline{OB}$. Thus, if a body has neither translation nor rotation (that is, if it is at rest) under the action of three forces, F_1, F_2, F_3, the following conditions are satisfied:

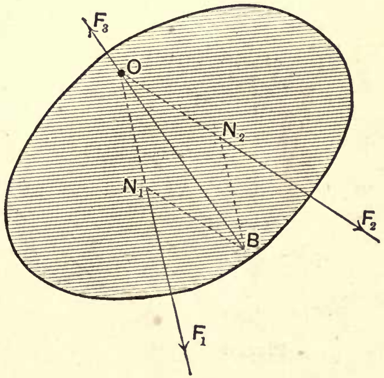

FIG. 20.

1. Their lines of action lie in the same plane;

2. These lines intersect in a common point;

3. One force is equal and opposite to the geometric sum of the other two, i. e. the geometric sum of all three is zero.

There is no difficulty in seeing that the effect of having the moment of one force balance that of another is not necessarily to stop all rotation, but is to leave any existing rotation unaltered; and, if one force neutralizes another, the result is not necessarily no translation, but no *change* in the existing translation. Hence the conditions just given are true of a state where there is no alteration either of translation or of rotation; this state is called "equilibrium;" rest is a particular case.

Special Cases of Equilibrium

39. 1. A Single Small Body.—If a point is in equilibrium under the influence of the three forces F_1, F_2, F_3; F_3 must be equal and opposite to the geometric sum of F_1 and F_2 (or, what is the same thing, F_1 must be equal and opposite to the geometric sum of F_2 and F_3, etc.).

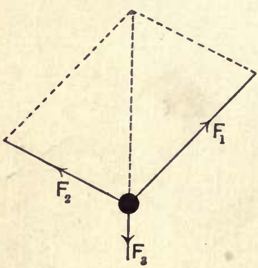

FIG. 21.

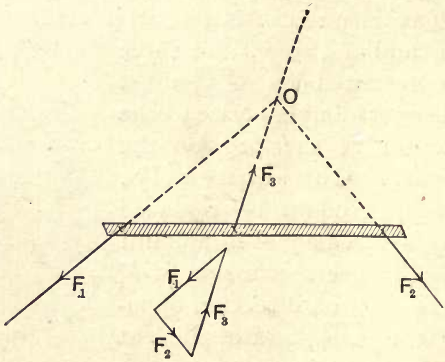

FIG. 22.

40. 2. Non-parallel Forces; Large Body.—This is the general case which has just been considered in Article 38. An illustration is offered by three men pulling at a stick. If the stick does not move, F_1, F_2, F_3 must all lie in a plane, must meet, if prolonged, in a point O, and must, if added geometrically, equal zero, i. e. they must form a closed triangle. All kinds of levers are illustrations of this. (See Article 48.)

41. 3. Parallel Forces.—Let three parallel forces, F_1, F_2, F_3, be applied at points A_1, A_2, A_3; and let them be in equilibrium. They must lie in a plane; they must meet in a point (at infinity); and their geometric sum must be zero; that is, since they are parallel, any one force, e. g. F_3, must be equal and opposite to the *algebraic* sum of the other two, $F_1 + F_2$, i. e. $F_1 + F_2 = - F_3$. But these conditions do not determine the relation between the points of application. These must be so chosen that the moments of any two forces around any axis must be equal and opposite to the moment of the third, because there is no rotation. Prolong the line of action of F_3 and draw a line, $\overline{B_1 B_2 B_3}$, perpendicular to the parallel lines. Imagine an axis drawn through the point A_3 perpendicular to the plane of the three forces. The moment of F_3 about this axis is zero; that of F_2 is $F_2 \times \overline{B_3 B_2}$; that of F_1 is $F_1 \times \overline{B_3 B_1}$, but in an opposite sense. Hence

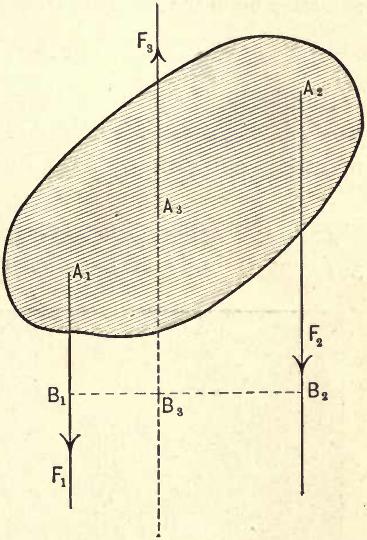

FIG. 23.

$$F_1 \times \overline{B_3 B_1} \text{ is equal numerically to } F_2 \times \overline{B_3 B_2},$$

i. e.
$$\frac{\overline{B_3 B_1}}{\overline{B_3 B_2}} = \frac{F_2}{F_1};$$

or, in words, F_3 must be so placed that B_3 divides the line $\overline{B_1 B_2}$ into two portions inversely proportional to F_1 and F_2.

As an illustration, let $F_1 = 20{,}000$ dynes, $F_2 = 10{,}000$ dynes, $\overline{B_1 B_2} = 30$ cm.; then F_3 must equal 30,000 dynes and must be in an opposite direction to F_1 and F_2, and B_3 must be such that $\overline{B_1 B_3} = 10$ cm. Or, let $F_1 = 40{,}000$ dynes, $F_3 = 60{,}000$ dynes and be opposite to F_1; and $\overline{B_1 B_3} = 20$ cm.; then F_2 must equal $40{,}000 - 60{,}000$, i. e. 20,000 dynes, and must be in the same direction as F_1; and B_2 must be such that

$$F_2 \times \overline{B_3 B_2} = F_1 \times \overline{B_1 B_3}, \text{ i. e. } \overline{B_3 B_2} = 40 \text{ cm.}$$

Similarly, if there are four or more parallel forces in equilibrium, their *algebraic* sum must equal zero; and their moments around any axis must balance each other.

42. 4. **Forces Due to Gravity.**—It has been noted (Article 30) that a body of mass m is subject, owing to its weight, to a force $m\,g$, vertically down; therefore any large body considered made up of smaller parts is under the action of a great number of parallel forces. Consequently, if such a body is suspended at rest by a cord or on a knife-edge, the supporting force must be vertical and must be so placed, i. e. its point of attachment must be such, that the moments of all the gravity forces around an axis through it shall balance each other. Imagine a line marked in the body so as to coincide with the line of action of the suspending force;

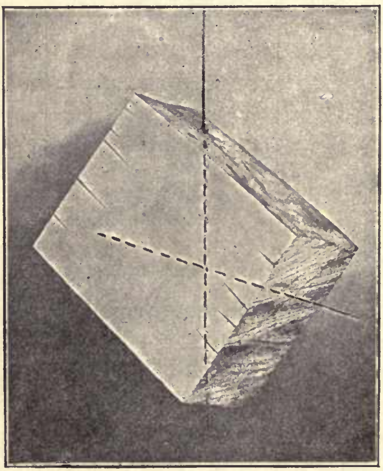

FIG. 24.

then attach the cord to another point of the body and suspend it, marking again a line in the body, which coincides with the new line of action of the suspending force. These two lines in the body will meet in a point, which is the same for all points of suspension, and which is called the "centre of gravity." The action of gravity is therefore just the same as if all the matter were concentrated at this point, because the entire action is neutralized by an upward force through it. This point is identical with the "centre of inertia."

This may be illustrated by the following experiment: By means of a cord suspend a rod (or a broom) in such a manner that there is equilibrium. The centre of gravity is in the rod at the point of support. Strike the rod an upward blow by means of a hammer or stick. The point of support will move *vertically* up, regardless of where the blow is struck. If the blow is struck at the point itself, there will be translation, no rotation. Therefore this point is the centre of inertia also. (See Articles 27 and 35.)

In certain simple cases, the position of the centre of gravity is known. It is the centre of figure of a sphere, a cylinder or a rod. If a rod of

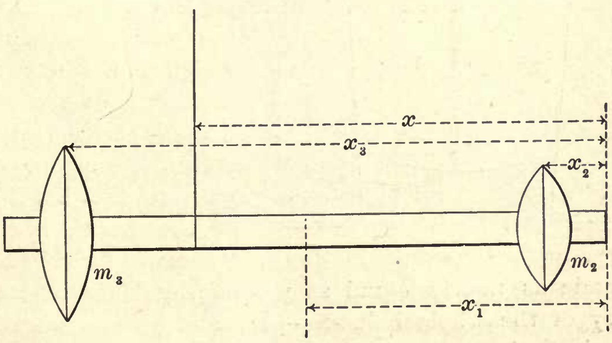

FIG. 25.

mass m_1 is loaded with heavy bobs of mass m_2 and m_3 (as in Fig. 25), the position of the centre of gravity of the system may be calculated; for, if the rod is suspended by a cord through its centre of gravity, the upward force must be $(m_1 + m_2 + m_3)\ g$, and its moment around any axis—e. g. one through the end of the rod—must exactly balance the sum of the moments of the separate weights around this same axis, as there is no rotation. If x_1 is the distance of the centre of figure of the rod from its end (that is, one half the length of the rod); x_2 the distance of the centre of the bob whose mass is m_2 from the end of the bar; x_3 the distance of the other bob from the same end; the moments of the weights are $m_1 g x_1 + m_2 g x_2 + m_3 g x_3$.

If x be the distance of the centre of gravity from this same end, the moment the upward force is $(m_1 + m_2 + m_3) g x$.

Hence $$(m_1 + m_2 + m_3) g x = m_1 g x_1 + m_2 g x_2 + m_3 g x_3,$$

and $$x = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3},$$

and may be calculated.

43. 5. Chemical Balance.—A chemical balance is an instrument designed to measure the mass of a body. It consists essentially of a light, rigid metal beam so supported on a knife-edge as to be free to turn around a horizontal axis perpendicular to its length, and of two pans of equal mass, which hang one from each end of the beam. The distances from the points of support of the pans to the knife-edge of the beam, i. e. the "arms" of the balance, are made as nearly equal as possible, and the centre of gravity of the whole balance—pans and beam—is made to lie vertically below the knife-edge when the beam is horizontal. The body of unknown mass m is placed on one pan, and bodies selected from a set of standard masses are placed on the other until the beam is again horizontal. Let the mass of the standards be m_1. There will now be three forces

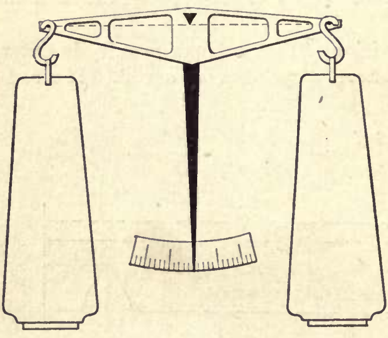

FIG. 26.

acting on the balance-beam. The force down on one pan is mg; the force down on the other pan is $m_1 g$; these are neutralized by the force from the knife-edge. (The weight of the beam and pans is of no effect, because the centre of gravity lies *vertically* below the knife-edge.) Since there is equilibrium, the moments, around the knife-edge, of the forces on the two pans must be equal; but the balance-arms are of equal length; hence the two forces must be equal.

Therefore $$mg = m_1 g, \quad \text{or} \quad m = m_1.$$

Consequently, owing to the fact that g is a constant at any one place for all kinds and amounts of matter, a chemical balance measures masses.

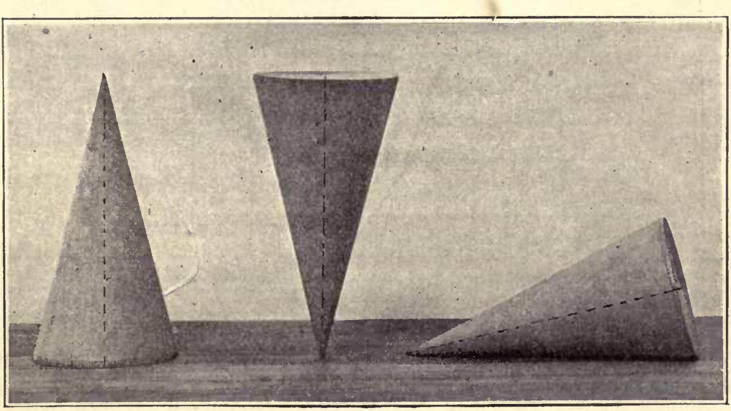

FIG. 27.

44. Stability of Equilibrium.—Systems of bodies which are in equilibrium behave differently when subjected to slight displacements. Thus, a cone resting on its base will, if slightly tipped, return to its previous position; but, if balanced on its point, it will, if touched, fall over on its side; and if, while resting on its side, it is tapped, it will roll with constant speed.

Equilibrium is called "stable" if the system is in such a condition that, after a slight velocity is given it by a sudden blow, the velocity decreases quickly, becomes zero, and is then reversed, the system continuing to make oscillations about its position of equilibrium until brought to rest by friction. Illustrations are afforded by all solid bodies pivoted around axes *above* their centres of gravity, or by those whose centres of gravity rise when the blow is struck, e. g. an ordinary pendulum, an elastic solid considered by itself, a spiral spring carrying a weight.

Equilibrium is called "unstable" if the system is in such a condition that, when a slight velocity is given it by a blow, the velocity increases. Illustrations are afforded by all solid bodies pivoted around axes *below* their centres of gravity, e. g. the cone balanced on its point.

Equilibrium is called "neutral" if the velocity given by a blow remains unchanged. It is illustrated by solid bodies pivoted around axes passing *through* their centres of gravity, and by solid bodies whose centres of gravity remain at the same horizontal level, e. g. a cone or a sphere rolling on a table.

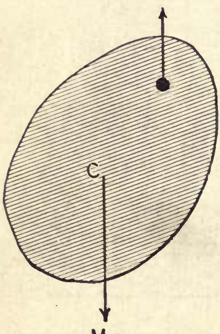

FIG. 28.

In connection with this, attention may be called to the fact that unless a vertical line through the centre of gravity of a body passes through the point of support, if it is suspended, or falls within the area of support, if it rests on the table or on the earth, the body will be under the action of a moment and will rotate.

If the body is pivoted around a horizontal axis, there will be a moment as shown in Fig. 28; and the body will rotate unless *C*, the centre of gravity, lies vertically above or below the pivot.

If the body rests on a table as shown in Fig. 29, and if the vertical line through *G*, its centre of gravity, falls outside the base, it will topple over; while if the line passes inside, it will be in stable equilibrium.

The degree of stability of a body is measured by the displacement which must be given it in order to make it pass from stable into unstable equilibrium.

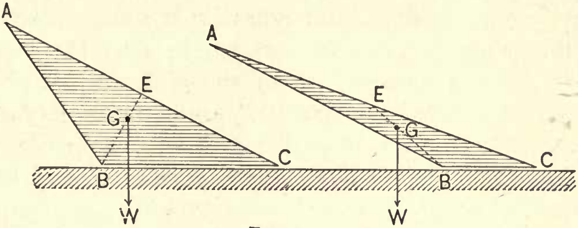

FIG. 29.

WORK AND ENERGY

45. Definition of Work and Energy; Conservation of Energy.—If a body of mass m is moving with a speed s, and is met by a resistance in its line of motion equivalent to a force F, the speed will decrease, and there will be a negative acceleration equal to F/m (for $F = m\,a$). Therefore, the body will continue to move for t seconds only, where

$$t = \frac{s}{a} = \frac{s}{\frac{F}{m}} = \frac{m\,s}{F},$$

because a is the decrease of s in one second.

And the distance it will go before it comes to rest is $x = \frac{1}{2}\,a\,t^2$. (See Article 19, equation 2.)

But $$a = F/m,\ t^2 = s^2/a^2$$

$$\therefore x = \tfrac{1}{2}\frac{s^2}{a} = \tfrac{1}{2}\frac{m\,s^2}{F}.$$

Thus, for example, the *distance* a trolley-car in motion goes, after brakes are applied, before stopping is $\frac{1}{2}\frac{m\,s^2}{F}$; the *time* taken to come to rest is $\frac{m\,s}{F}$, where F is the resistance due to the friction of the brakes.

The momentum, $m s$, therefore, is a measure of the time-resisting quality of a given motion; $\frac{1}{2} m s^2$ is a measure of the space-resisting quality. $\frac{1}{2} m s^2$ is called the "kinetic energy" of the body whose mass is m when its speed is s.

As just shown, $\frac{1}{2} m s^2 = F x$, where F is the resistance overcome for a distance x in the direction of F. This product $F x$ is called the "work;" and if a force of F dynes acts over a distance x centimetres, the work done is called $F x$ "ergs;" e.g. a force of 1 dyne acting over 1 cm. does the work of 1 erg. The equation may then be stated: owing to the work $F x$ expended in overcoming opposition to its motion, the body whose mass is m is brought to rest and loses kinetic energy $\frac{1}{2} m s^2$, the quantities $F x$ and $\frac{1}{2} m s^2$ being numerically equal.

Similarly, if a body whose mass is m is set in motion by a constant force F, at the end of t seconds the speed will be s, where

$$t = \frac{s}{a} = \frac{m s}{F},$$

and the distance it takes to secure this speed is x, where

$$x = \frac{1}{2} a t^2 = \frac{1}{2} \frac{m s^2}{F}.$$

Consequently, $F x = \frac{1}{2} m s^2$; and the equation may be stated: owing to the work $F x$ done on the body whose mass is m, it is given acceleration and gains kinetic energy $\frac{1}{2} m s^2$, where the quantities $F x$ and $\frac{1}{2} m s^2$ are equal.

It is seen, therefore, that work can be done in two ways: in overcoming a resistance due to the opposition of a force, and in producing acceleration, i. e. in overcoming the inertia of matter. As illustrations, consider a body thrown vertically upward from the earth by means of the release of a compressed spiral spring; or this same body falling from a height upon a spiral spring, thus compressing it and being finally brought to rest itself. In the first case, the spring does work upon the body in giving it kinetic energy; in the second case, the body does work upon the spring in compressing it. It is seen that the body in motion has the power of doing work; so has the compressed spring. The spring is said to have "potential energy." Thus, "energy" may be defined as the power of doing work; and, if the body owes its energy to its being in motion, its energy is *kinetic*, and its numerical value is $\frac{1}{2} m s^2$; while if it owes its energy to the fact that work has been done upon it in overcoming some opposing resistance, its energy is called *potential*, and its numerical value is Fx, the work done. Thus, in the first of the above cases the spring loses potential energy, and the body thrown upward gains kinetic energy. As it rises, it goes more and more slowly, and thus loses kinetic energy; but it gains potential energy, because the force of gravity which opposes its motion is being overcome; and, when it comes to rest at its highest point, its energy is entirely potential. As it falls, it loses potential energy and gains kinetic; and, when it strikes the compressed spring, it does work upon it, loses its energy, and the spring gains potential energy. Two things should be noted: (1) When work is done, one body loses energy and another gains energy; (2) the amount of energy lost by one equals that gained by the other, i. e. work is a transfer of energy. This is equivalent to stating that no energy is lost; and the statement is called the "Principle of the Conservation of Energy."

ILLUSTRATIONS.—A body whose mass is m is raised through a vertical height h from the floor to a table; it gains potential energy, mgh. A swinging pendulum loses kinetic energy as it swings out to the end of its path, but gains potential; then, as it swings back, it loses potential, but gains kinetic, etc. If a piece of iron is drawn away from a magnet, work is done; there is potential energy, which is changed into kinetic when the iron is allowed to move back towards the magnet. In some cases where work is done against a resisting force, there is *apparently* no gain of energy by the body which offers this resistance; for instance, when one board is pushed over another, work being done against "friction." In these cases of the *apparent* violation of the conservation of energy, the energy is gained, not by the body as a whole, but by its minute parts. This increase in energy of the smallest parts is rendered evident to our senses by some heat-effect, such as rise in temperature, melting, etc. Thus, if a bullet enters a target, it loses kinetic energy, and the molecules of the target gain energy, as is shown by the rise of temperature; the hammer of a pile-driver falling on a pile loses kinetic energy, work is done in overcoming the friction of the earth into which the pile is driven, energy is gained by the minute portions of the matter; if two pieces of ice are rubbed together, thus overcoming friction, the ice melts. (See Article 99.)

46. Potential Energy.—As was explained in Article 45, work is defined as the product of force and the distance through which it acts, both being measured in the same direction. Thus, if a body whose mass is m is raised through a vertical height h, the work done is, as shown above, $m\,g\,h$, because the distance h is in the direction of the force. If the body is raised along an oblique line, $\overline{AB}$ (see Fig. 30), through the same *vertical* height h, the work done is also $m\,g\,h$. This may be shown as follows: In moving along $\overline{AB}$, the force overcome is the *component* of $m\,g$ along $\overline{AB}$; therefore, if $\overline{PQ}$ represents mg, $\overline{PR}$ is this component; and the work done is $\overline{AB} \times \overline{PR}$;

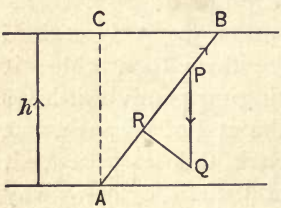

FIG. 30.

but $$\overline{AB} : \overline{AC} = \overline{PQ} : \overline{PR};$$

hence $$\overline{AB} \times \overline{PR} = \overline{AC} \times \overline{PQ} = m\,g\,h.$$

Similarly, if the body had been carried from A to C and then to B, no work would be done in going from C to B, because this motion is at right angles to the force of gravity; but in being raised from A to C, the work $m g h$ is done. Hence the amount of work required to raise the body from A to B is the same along both paths, and therefore the same along all paths.

Conversely, if the body falls from B vertically down a height h, or along $\overline{BA}$, or along *any* path through the *vertical* height h, it loses potential energy to the amount $m g h$ and gains an equal amount of kinetic energy $\frac{1}{2} m s^2$. Consequently, the *speed* which a body has after falling through a vertical distance h is $\sqrt{2 g h}$, and is independent of the path traversed.

It should be noticed that if a system has potential energy, it will tend to lose it, if left to itself; a body will, if free to move, fall towards the earth; a compressed spring will relax; a clock-spring if wound up will tend to unwind; a piece of iron separated from a magnet tends to approach it, etc. A system does not come into stable equilibrium until its potential energy is as small as it can possibly become; e. g. a system suspended near the earth is not in stable equilibrium unless its centre of gravity is as low as is possible, consistent with the restraints; thus, a pendulum comes to rest finally in its lowest position.

47. Machines.—Since work is product of force by distance moved in the direction of the force, it is evident that the same work may be done by different forces, if the distances moved are different; and various mechanisms known as "machines" have been devised to allow a small force to be transformed into a large one (or *vice versa*), and to change the direction in which motion is produced. Such machines are levers, pulleys, screws, wedges, windlasses, etc. Machines do not create energy. The work done by a machine can never be greater than that done on it; in fact, it is always less, for there is friction to overcome if there is motion.

48. Levers.—Consider a rigid beam in equilibrium under the action of three forces, F_1, F_2, F_3, applied at points A_1, A_2, A_3.

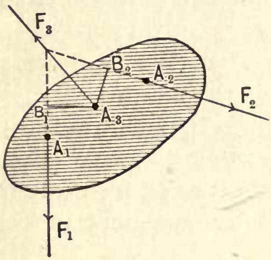

FIG. 31.

If the beam is pivoted around an axis at A_3, the moment of F_1 around this axis must be equal and opposite to that of F_2 around the same axis. Call the perpendicular distances from A_3 to the lines of action of F_1 and F_2, i. e. $\overline{A_3 B_1}$ and $\overline{A_3 B_2}$, l_1 and l_2; then

$$F_1 l_1 = F_2 l_2,$$

or

$$F_2 = F_1 l_1 / l_2.$$

Thus, if $\frac{l_1}{l_2}$ is large, F_2 is much greater than F_1. But, if F_1 is increased by an *extremely* small amount, equilibrium will be destroyed and motion will be produced in the direction of F_1 (provided there is no friction); and, after this motion is begun, it will continue, owing to inertia, even if the small increase in F_1 is removed. Work will thus be done by F_1 against F_2. Let the motion be considered for a very small interval of time; let x_1 be the distance which B_1 moves; and let x_2 be the distance that B_2 moves. Then, by geometry,

$$x_1 : x_2 = l_1 : l_2.$$

Hence

$$F_1 x_1 = F_2 x_2,$$

or the work done by F_1 equals that done against F_2.

Illustrations of this kind of lever, in which the pivot is between the points of application of the two forces, are furnished by a steelyard, a crowbar, a pump handle, etc.

Similarly, if the axis of rotation passes through A_1, the moment of F_3 around it must be equal and opposite to that of F_2, and if k_3 and k_2 are the perpendicular distances from A_1 upon the lines of action of F_3 and F_2,

$$F_3 k_3 = F_2 k_2.$$

Further, if there is motion, the work done by one force must equal that done against the other. Illustrations of this kind of lever, where the pivot is beyond the points of application of the two forces, are given by a lemon-squeezer, tongs, nut-crackers, a shovel, sheep-shears, etc.

A good illustration of levers is furnished by the muscles of the arm, as is shown in the drawing (Fig. 32), which is copied roughly from the treatise on "The Motion of Animals," by Borelli, 1685. The pull in the muscle F has a lever-arm $\overline{OI}$ around O; while the weight R has the lever-arm $\overline{KO}$ around O.

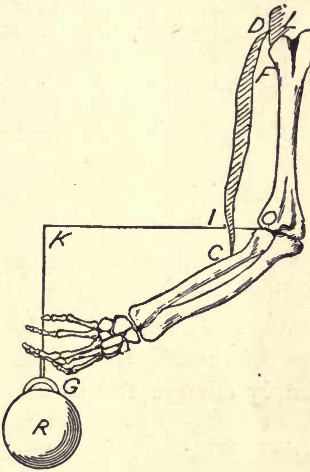

FIG. 32.

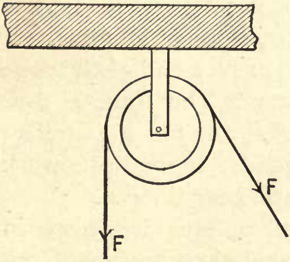

FIG. 33.

49. Pulleys.—A pulley consists of a circular wheel pivoted on an axle through its centre, and with a groove cut in its edge, so as to receive a cord. It is used to change the direction of the line of action of a force. Thus, if a cord is passed over a single pulley and a force F applied to one end, it will be balanced by a force of an equal amount applied at the other, no matter what the direction is, because the moments around the axle will be equal, as the wheel is circular. (See Fig. 33.)

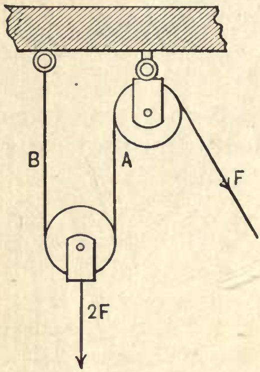

FIG. 34.

If two pulleys are arranged as in Fig. 34, the upper fixed and the lower movable in the bight of the cord which passes over them both, a force of 2 F applied in a vertical direction to the axle of the free pulley will balance a force F applied to the free end of the cord passing over the fixed pulley (neglecting the weight of the lower pulley and friction of the wheels). For, in the right-hand branch, A, there is a force F; also in the left-hand branch, B, since pulleys simply change the direction of a force; hence, if the two branches are parallel, a force 2 F acts upon the lower pulley; and to balance it a force 2 F must be applied downward. Or, if a slight motion is produced and the free end of the rope moves a distance x, the force F thus doing a work Fx, the movable pulley will rise a distance $\frac{1}{2}x$, and the work done against 2 F is again Fx.

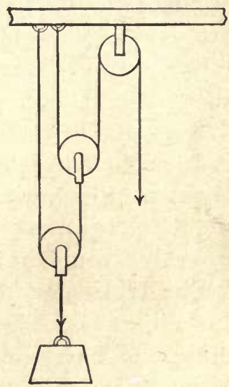
FIG. 35.

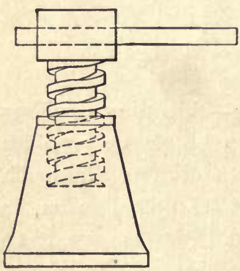
FIG. 36.

Another form of combination of pulleys is shown in Fig. 35. The principle may be extended to any combination of pulleys.

50. Screws.—Screws, as used in jack-screws (see Fig. 36), book-presses, elevators, vises, etc., are machines by which a small force moving a long distance in turning the screw makes it advance through its nut a small distance, and thus overcome a large force.

A screw is in reality a special case of an "inclined plane," the principle of which was discussed in Article 18. To raise a body along an inclined plane against the force of gravity—weight—the same work is required for a definite vertical height h, viz. $m\,g\,h$; but, if the plane is very inclined, the actual path is very long, and hence the *force* which is overcome is small.

51. Power.—The usefulness of a machine or "motive power" depends largely upon how *fast* it can do work; a child can do any amount of work, however large, if time enough is given for it; but to have a large amount of work done quickly a powerful engine is necessary. The rate at which work is done, i. e. the number of ergs done in one second, is called the "power." A power of 10000000, or 10^7, ergs per second is called a "watt;" thus, a "kilowatt" machine does 1000×10^7 or 10^{10} ergs per second. One "horse-power" is the power equivalent to raising 33,000 pounds 1 foot in a minute, and is therefore equal to 746 watts.

TABLE III

FORCE

Weight of 1 gram	= 980 dynes
" " 1 pound	= 44,518 dynes

WORK

1 Foot-pound	$= 1.383 \times 10^7$ ergs
	= 0.1383 kilogram-metres

POWER

1 Horse-power	= 746 watts
1 Watt	= 0.0013406 horse-power

CHAPTER III

PROPERTIES OF MATTER

SIZE AND SHAPE

General Properties of Size and Shape.—As stated in the Introduction, Article 7, names are given to certain forms of matter; viz. solids, liquids and gases.

52. Solids.—A solid body has a definite volume and shape, which can, however, be changed by suitable forces; thus, solids may be compressed, twisted, bent, squeezed, etc. Different solids vary greatly in the way they yield to these forces. Some solids, e. g. iron, copper, platinum, are "ductile," that is, can be drawn out into wires; some are "malleable," that is, can be beaten out into thin sheets, e. g. gold, silver, aluminium; some are "plastic," that is, yield easily to forces which change their shape, and do not spring back into their previous shape, e. g. putty, soft plaster, lead; some are easily compressible, e. g. rubber, cork; some are almost incompressible, e. g. glass, steel; some are rigid and do not suffer a change in shape, except under a great force, e. g. glass, steel, blocks of wood, etc. Most solids return to their original volume and shape when the applied forces are removed, unless these forces are too great; that is, for small deformations such solids are "elastic." A solid may thus have two kinds of elasticity, one corresponding to a change in volume, the other to a change in shape. Certain solids slowly diffuse into each other; in particular, such soft metals as gold and lead diffuse, so that if a piece of gold is placed under a piece of lead, traces of gold may soon be found throughout the lead.

53. Liquids.—A liquid, if left to itself, takes the shape of a spherical drop; but, when placed in a hollow vessel, it assumes the shape of the latter, keeping its own volume. Liquids differ greatly in their power of flowing; e. g. water, alcohol, etc., flow easily; but molasses, oils, etc., flow slowly and with considerable friction—they are called "viscous." Some liquids are more compressible than others; water suffers hardly any change in volume unless the compressing force is enormous. (To produce a change of one ten thousandth of its volume, a pressure of two atmospheres is required, i. e. a force of nearly 30 pounds to the square inch. See Article 79.) A liquid will yield to any force, no matter how small, which tends to change its shape, that is, to produce a "shear" or slipping sidewise of one layer over another; thus, pitch is a liquid, but a most viscous one, so is shoemaker's wax. Therefore, a liquid which is not viscous has no elasticity corresponding to a change in shape; but for a change in volume it is perfectly elastic.

If two liquids are put in contact, there is, in general, a gradual intermixture at the surface of contact, they are said to "diffuse;" but some liquids diffuse easily, e. g. water and alcohol, while others do so only slightly, e. g. water and kerosene oil, and still others practically not at all, e. g. mercury and water.

54. Gases.—A gas distributes itself uniformly through the vessel that contains it, and so has no shape or size of its own. If the volume of a gas is changed, it tends to return to its previous volume, and is perfectly elastic in this sense; thus, if a hollow rubber ball is compressed, the enclosed gas tends to oppose the compression and to restore the ball to its original volume.

All gases are more or less viscous, although less so than liquids. Two gases, if put together, diffuse rapidly through each other until the mixture is uniform throughout.

We picture to ourselves, therefore, the parts of a solid as held together more or less rigidly, so that the minute parts can oscillate, but cannot move about, in general, from one position of the solid to another. In a non-viscous liquid, we consider that these restraints which hold the parts fixed together do not exist, and that therefore the parts can move about with a moderate degree of freedom. In a gas we imagine the particles moving around rapidly, and being so far apart with reference to their own size that they are practically unaffected by each other except when they collide. If by suitable air-pumps the gas is so exhausted from a closed vessel (like a glass bulb) that the average distance apart of the minute portions of matter is one or two centimetres, the matter no longer has the properties of a gas; and this condition is sometimes called the "Fourth State of Matter."

In all forms of matter, the individual molecules and atoms are making rapid vibrations (many million a second), besides having these varying degrees of freedom of motion from one part to another of the body; that is, each molecule is, during its motion as a whole, vibrating, owing to the motion of its parts, just like a bell or tuning-fork. This statement will be discussed more fully in Article 103.

55. Elasticity.—All bodies are elastic in one sense or another, but to different degrees; and it has been shown that, if the deformation, whatever it is, is small, its amount is proportional to the force which produces it; i. e. "strain is proportional to stress," or "*Ut tensio sic vis*," as Robert Hooke expressed it in 1676. This is called "Hooke's law."

It requires work to produce these deformations, and therefore a body which is deformed (or strained) gains energy from the agency which has produced the strain; thus, a stretched wire, a bent rod, a twisted wire, or a compressed liquid has potential energy. (In the case of inelastic deformations, e. g. change of shape of putty, of lead, of water, of a gas as involved in flowing, the work done is spent in giving energy to the smallest parts of the bodies, i. e. in internal friction; and heat-effects such as rise in temperature are produced.)

56. Waves Due to Elasticity.—If one end of a rope or stretched cord is vibrated sidewise rapidly, a series of disturbances or "pulses" pass along the rope or cord; if the vibrations are regular and if the cord is long, this is a "train of waves." (These disturbances are reflected when they reach the further end of the cord, and thus interfere with the advancing waves if the cord is short.) Similarly, if one end of a long wire is twisted quickly, first in one direction, then in another, there will be waves of a torsional kind. If a series of balls be suspended by long cords

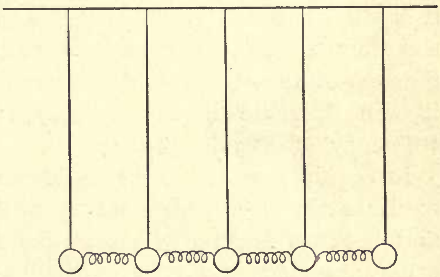

FIG. 37.

and be separated by spiral springs, as shown in Fig. 37, the apparatus can be made to carry waves of two different kinds. If the end ball is vibrated sidewise, there will be a train of waves like that in the rope; this is called a "transverse" train, because the vibrations of the balls are at right angles to the direction in which the train of waves is advancing. If the end ball is vibrated rapidly in the line joining the balls and springs, there will be a train of waves, owing to the alternate compression and extension of the springs; this is called a "longitudinal" train, because the vibrations of the balls are in the line of advance of the train of waves. It is evident that both kind of waves are due to the elastic nature of the springs or connections of the system; and, since a solid, e. g. a long rubber cord, has two kinds of elasticity, corresponding to change of shape and volume, it can carry the two kinds of trains of waves. Whereas a fluid, either a liquid or a gas, which has only one kind of elasticity, viz. that corresponding to change in volume, can carry only those waves which are due to compressions and extensions, viz. longitudinal trains. Thus, a tuning-fork vibrating under water or in air will produce compressional waves. (It is evident that, in order to produce waves in this way in water or air, the vibrations must be so rapid that the water or the air is actually *compressed*, and does not simply *flow around* the vibrating body, as it does when an ordinary pendulum vibrates.) Further, it is evident that the velocity of any train of waves, that is, the distance the disturbance advances in one second, must increase if the "stiffness" of the medium increases, while it will decrease if the density increases. (See Ames's "Theory of Physics," p. 153.)

It requires work to produce waves, both because the medium through which waves are advancing is in motion and because the parts are strained with reference to each other, twisted, bent, compressed, etc. As the waves advance, energy is carried on; and the body producing the waves must supply it.

57. Intensity.—If the source of the waves is a vibrating point, e. g. a tuning-fork vibrating in air, the waves spread out in all directions; if the medium is "isotropic," i. e. has the same properties in all directions, the "wave-front" is spherical. If two spheres are drawn around the vibrating point with radii r_1 and r_2, the energy going through each sphere is the same (unless there is absorption—see Article 103); but the amount per square centimetre of surface is different. The area of the spherical surface of radius r_1 is $4\pi r_1^2$; that of the surface of radius r_2 is $4\pi r_2^2$. Hence, if E is the total energy, the energy per sq. cm. at the sphere of radius r_1 is $\frac{E}{4\pi r_1^2}$; that at the sphere of radius r_2 is $\frac{E}{4\pi r_2^2}$. The amount of energy which passes through one square centimetre in one second is called the "intensity" of the waves. So, if I_1 is the intensity at r_1, and I_2 the intensity at r_2,

$$I : I_2 = \frac{1}{r_1^2} : \frac{1}{r_2^2};$$

or, the intensity of waves radiating from a point varies inversely as the square of the distance.

58. Wave-Length.—The "wave-length" of a train of waves is the distance from any point in the medium which is carrying the waves to the next point in the direction of the waves at which the condition of motion is identically

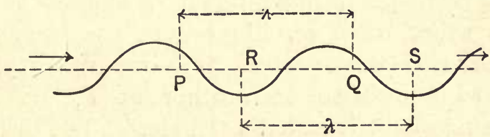

FIG. 38.

the same. Thus, if Fig. 38 represents the advance towards the right of a transverse wave in a cord, the distance from P to Q or from R to S is a wave-length (λ). The number of wave-lengths which pass a given point in one second is called the "wave-number" (n) or the "frequency." It is evidently equal to the number of the vibrations per second of the end of the rope, or, in general, of the "centre of disturbance." Further, if n wave-lengths pass in one second, and each has the length λ, the distance the disturbance goes in one second, i. e. the velocity of the waves, is n times λ; or $v = n\lambda$. If the medium carrying the waves is homogeneous, the velocity is the same for all trains of waves, of long or of short wave-lengths; but, if it is irregular, e. g. a cord having knots in it, the velocity is different for trains of waves of different wave-length.

59. Stationary Vibrations.—When waves reach the end of a medium or reach a new medium, *reflected* waves are produced in general. These, passing back through the medium, are superimposed upon the advancing waves, and thus alter the nature of the motion. Consider a rope fastened at one end to a wall, and carrying transverse waves, these will be reflected; and the motion of any point of the rope will be the *algebraic sum* of the motions which it would have due to the two waves separately. It will happen that, at some point,

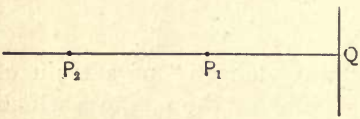

FIG. 39.

P_1 (Fig. 39), near the wall, the action of one wave would be to make the point move down at a particular instant, while the other wave would produce the opposite effect; and, as the two waves have the same velocity, they will continue to neutralize each other at P_1; it will be at rest. Such a point is called a "node." The distance from P_1 to the fixed end Q must be such that the wave goes from P_1 to Q and returns to P_1, reaching there just as a second wave comes up to P_1 from the other direction, i. e. $\overline{P_1 Q} + \overline{Q P_1}$ equals the wave-length. Hence

$$\overline{P_1 Q} = \lambda / 2.$$

Since P_1 is at rest, it may be regarded as held clamped, and so there will be another node at P_2 where $\overline{P_2 P_1} = \overline{P_1 Q} = \frac{\lambda}{2}$. Consequently, the whole rope, if it is of suitable length, will be divided into vibrating sections. It is then said to make "stationary vibrations." A point half way between two nodes is called a "loop."

If there are N vibrating sections in a distance L,

$$L = N\frac{\lambda}{2};$$

but $\lambda = v/n$, where v is the velocity of the component waves and n is the wave-number (or the frequency of the vibrating source). Hence

$$L = \frac{Nv}{2n} \text{ or } N = \frac{2nL}{v}.$$

If the rope is fastened at both ends, e. g. one end to a wall, the other in the hand of the person maintaining the waves, the length of the rope must be a whole number of half-wave-lengths. That is, if L is fixed, n must be so chosen as to make N a whole number; for v depends upon the properties of the rope, its ten sion, etc. Further, if v is constant, N varies directly as L; while, if n is constant, N varies inversely as v and directly as the distance L.

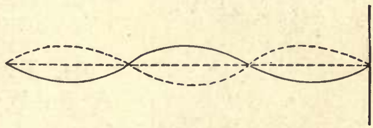

FIG. 40.

SOLIDS AND FLUIDS

60. Elastic Properties of Solids.—A solid, as distinct from a fluid, may be defined to be a body which requires a force exceeding a certain limit in order permanently to change its shape. Solids also, as explained before, have *elastic* properties corresponding to changes in both size and shape; they may be stretched, compressed, bent, twisted, etc.; and to all these strains the same law applies: the deformation is proportional to the force. Illustrations are given by stretching a steel wire, twisting a shaft, bending a beam, compressing a pillar or rod, stretching a spiral spring, as in a spring-balance (see Fig. 41), which therefore gives readings proportional to the weight hanging on the hook. If the force producing these deformations is suddenly removed, the body will proceed to make vibrations which are exactly analogous to those of a pendulum. (See Article 31.) Thus, there are vibrations of a piano or guitar cord, of a tuning-fork, of a spiral spring, of a flat coiled spring, like a watch-spring, etc. These all vibrate with a constant period. Any body which obeys Hooke's law (see Article 55) will make these harmonic, pendulum-like vibrations (Fig. 42.)

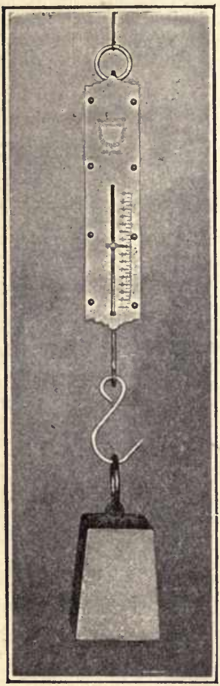

FIG. 41.

61. Fluids at Rest.—A fluid yields to any force, however small, which tends to make one portion slide over another, i. e. to "flow." If a fluid, either liquid or gas, is enclosed in a vessel of any kind, there is a force produced by it against the walls. Thus, let the fluid be in a cylinder (Fig. 43) closed by a piston which is pressed in with a force F until there is equilibrium, and let there be no flowing or currents. There is a force against the walls due to two things: to this force of the piston, F, and to the weight of the fluid.

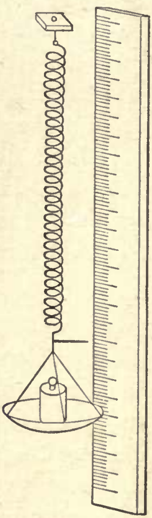

FIG. 42.

62. Fluid Pressure.—At any point on the wall of the cylinder the fluid tends to push the wall outward; and, if there is no flowing, the force must be perpendicular to the wall. If it were oblique, there would be a component along the wall, which would cause the fluid to flow. If a *small* area drawn around any point on the wall be considered, this perpendicular force, or "thrust," is uniform over it; and the value of the force divided by the area in square centimetres is called the "pressure" at that point of the wall; i. e. pressure is force per unit area, if the force is uniform over the area.

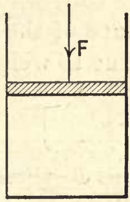

FIG. 43.

Again, if any small volume of the fluid be considered as distinct from the rest of the fluid, e. g. if a minute cube is imagined described around some point in the midst of the fluid, the action of the surrounding fluid is to press in this cubical volume on all sides, a tendency resisted by the matter inside the cube. The pressure at this point around which the cube is taken is the force on any side divided by the area of that side; and the pressure is the same in all directions, up, down, sidewise, etc.; for, if it were not, there would be a flowing of the fluid.

This pressure on the wall, and the pressure in the fluid also, is due, as said before, to two causes, the force of contraction of the containing walls (e. g. piston, as above; rubber bag, etc.) and the force of gravity.

Consider each separately. If one imagines the cylinder carried far away from the earth, so that there is no weight, there is still the pressure due to the contraction of the walls, i.e. to the force F in the case of the piston already described. This pressure must, moreover, be the same at all points throughout the fluid and on the walls; for, if there were a difference of pressure between two points, the fluid would flow from high to low pressure, since in a fluid there is nothing to prevent flowing, as there is in the case of a solid. Hence, if A is the area of the piston on which the force F is acting, there will be an equal force on an area A

everywhere through the fluid; in other words, there will be a pressure equal to F/A at every point of the fluid.

But, when the cylinder enclosing the fluid is on the surface of the earth, there is an additional pressure in the fluid due to weight. Consider two horizontal planes, each of area a sq. cm., one vertically above the other at a distance h. If ρ is the density of the fluid, the mass of the fluid included between these two areas is the product of ρ and the volume, i. e. $\rho a h$. Therefore the weight is g times this, i. e. $g \rho a h$; and this weight, in addition to the downward force on the upper area, must be borne by the upward force of the fluid against the lower plane. If the upward pressure required to balance this is p_2, the force is $p_2 a$; therefore, if the downward pressure on the upper area is p_1 (and hence the force $p_1 a$),

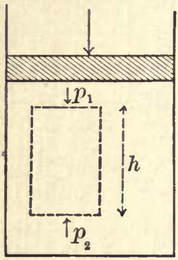

FIG. 44.

$$p_2 a = p_1 a + \rho g a h.$$

Therefore, $p_2 - p_1 = \rho g h$, or the difference in pressure due to a vertical height h is $\rho g h$.

Thus, in the bottle shown in Fig. 45 the pressure at a point h cm. below the piston is

$$p = \frac{F}{A} + \rho g h ;$$

and this is, therefore, the same at all points in a horizontal plane at this level. Similarly, in a vessel of *any* shape, the pressure is the same at all points in a horizonal plane; otherwise there would be flowing of the fluid. This applies equally to connecting tubes. The pressure is the same at all points of the plane at a depth h below the piston, viz.

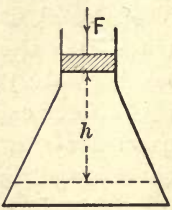

FIG. 45.

$$p = \frac{F}{A} + \rho g h;$$

and hence, at points in any other plane h' cm. above it (see Fig. 46), the pressures must be the same, viz. less by an amount $\rho g h'$ than at the depth h.

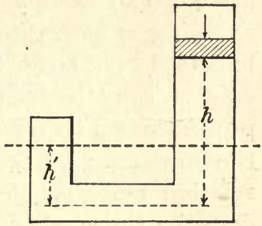

FIG. 46.

63. Hydraulic Press.—Again, as an illustration of the fact that the pressure due to the walls is the same throughout, let a fluid be enclosed in a vessel which is closed by two pistons, of areas A_1 and A_2. Let a force, F_1, be applied to the piston whose area is A_1; then, in order to keep the piston A_2 from being pressed out, a force F_2 must be applied, such that

$$F_2 = F_1 \frac{A_2}{A_1}.$$

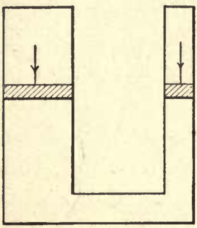

FIG. 47.

For, the pressure on the two pistons is the same (if the pistons are at the same level, so as to do away with considerations of pressures due to weight), and hence

$F_1 = p A_1$, $F_2 = p A_2$; and, therefore, $F_2/A_2 = F_1/A_1$.

A great pressure cannot be produced easily if the fluid is a gas; because it is compressed so readily, and the volume

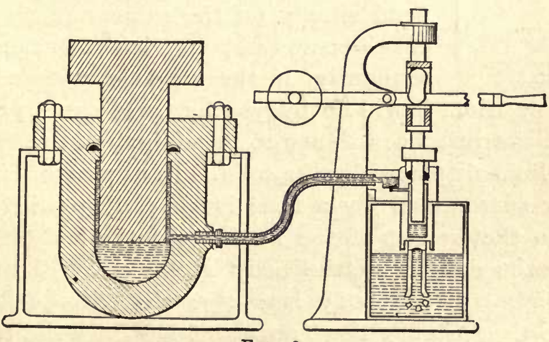

FIG. 48.

is diminished so much; but, if a liquid (e. g. water, which is nearly incompressible) is used, as great a pressure as is desired can readily be produced. This may be done by applying a moderate force over a small area, and may be balanced by a large force over a large area. Thus, in the case of the "hydraulic press" (Fig. 48), the small piston is pressed down by a small force, and there is produced over the large area of the large piston an enormous force, the relation being as given above, $F_1/A_1 = F_2/A_2$. The *work* done by the smaller piston is, however, equal to that done by the large one.

64. Principle of Archimedes.—An illustration of fluid pressure is afforded by the case of a solid *entirely* immersed in a fluid, e. g. a piece of iron suspended under water, or surrounded by air. Imagine it replaced by a portion of the fluid of exactly the same size and shape, which is separated from the rest of the fluid by a massless envelope.

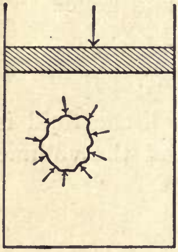

FIG. 49.

Since there is no flowing of the fluid, this enclosed portion must be buoyed up with a force equal to its weight; this buoyant force being due to minute pressures exerted by the surrounding fluid on the envelope. (See Fig. 49.) These minute pressures depend simply on the shape and size of the envelope and not on what is inside. Consequently, if the solid is restored to its former position, it will be buoyed by these same pressures, which have just been shown to be equivalent to the weight of the fluid displaced by the solid. In other words, a body entirely surrounded by a fluid is buoyed up with a force equal to the weight of the fluid which the body displaces. If ρ_1 is the density of the body and ρ_2 that of the fluid, the weight of the body is $\rho_1 v g$; that of the fluid displaced, $\rho_2 v g$; hence the "apparent loss in weight" of the body is $\rho_2 v g$, and the entire downward force on it is $(\rho_1 - \rho_2) v g$.

This is known as the "Principle of Archimedes," and is illustrated by the "loss in weight" of bodies immersed in liquids, by the floating of soap-bubbles and balloons in the air, etc. If $\rho_1 = \rho_2$, the downward force is zero; so that in this case the immersed body just floats.

This principle also leads to a method of measuring the densities of bodies. Neglecting the buoyancy of the air, let w_1 be the weight of a solid in the air, let w_2 be its "apparent" weight when suspended in water at 4° Centigrade; then, if ρ is the density of the solid, and v its volume, $w_1 = \rho v g$, $w_2 = \rho v g - v g$, because the density of the water is 1 (see Article 14); therefore, $\frac{w_1}{w_2} = \frac{\rho}{\rho - 1}$, and so ρ may be calculated. If the water used has a temperature different from 4° C., a slight correction must be applied in order to obtain accurate results. Similarly, in refined weighing, a correction is made for the buoyancy of the air when the solid is weighed in it.

65. Motion of a Solid in a Fluid.—If the solid is moving through the fluid, it meets with opposition, the nature of which depends upon its shape, etc. Thus, if a board is moving obliquely through a fluid, the direction of the flow of the fluid past it is such as to be equivalent to a force applied near the end which is in advance, and opposite in direction to that of the motion of the board. This force will therefore tend to make the board turn and place itself at right angles to the direction of motion. Thus, an oyster-shell falling through the water or a piece of paper falling through the air always tends to fall with its plane face horizontal; a boat set adrift on a lake will place itself at right angles to the wind which is driving it. Fig. 50 represents the flowing of a fluid past a board which is moving towards the left.

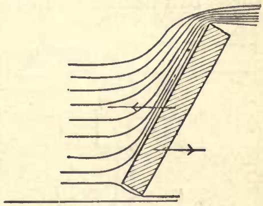

FIG. 50.

66. Fluids in Motion.—A difference of pressure is necessary to produce the flow of a fluid, and the direction of flow is from high to low pressure. If the flow continues for some time, it may become "steady;" that is, it no longer changes its character. If such a flow takes place through a tube of decreasing cross-section, there will be a uniform fall of pressure from one end to the other, the pressure being greatest at the end where the fluid enters. If the tube has a varying cross-section, the velocity will, in general terms, be greatest where the tube is narrowest and least where it is widest, if the flow is uniform; because the volume which goes past any section of area A in 1 sec. is $A\,v$, if v is the velocity at that area. The mass flowing by is therefore $A\,v\,\rho$; and, if this remains constant throughout the tube, v must vary inversely as A. This is therefore true for a liquid, and approximately so for a gas flowing slowly. But, if the velocity is greater at one point than another, there must be a force acting in the direction from the point of low velocity to that of high (because to produce an increase in velocity a force is necessary); therefore the *fall* of pressure must be in this same direction (because the force producing flow is always from high to low pressure); hence the direction of *fall* of pressure is that of *rise* of velocity. It follows that, if the velocity is varying, the pressure is greatest where the velocity is least, and *vice versa*.

Illustrations are afforded by the "atomizer" or "injector," and by the "ball-nozzle." In the former a blast of air or gas is blown across the opening of a tube which dips in a vessel of water or other liquid. Owing to the great velocity of the blast, the pressure is diminished over the mouth of the tube, and the liquid will rise up in the tube. In the "ball-nozzle" (Fig. 52) a blast of air or water is forced out into a cuplike opening containing a solid ball held loosely. After the motion becomes steady, the velocity of the stream is greater at the side where the fluid first meets the ball than

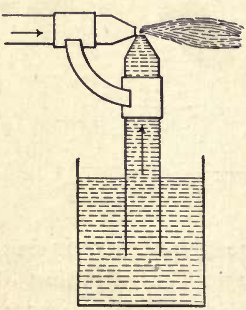

FIG. 51.

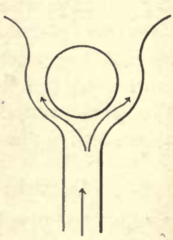

FIG. 52.

on the opposite side; hence the pressure is less there, and so the ball is kept pressed into the cup by the fluid flowing around it. Still another simple illustration is furnished when a piece of writing-paper about 3 cm. square is placed in the palm of one's hand, so as to cover the narrow opening between the first two fingers when they are closed, and an attempt is made, by blowing with one's mouth through the opening from the other side, to force the paper away. It will stick tight, because the pressure between it and the hand is so diminished by the current of air.

If a fluid issues from a vessel through a small opening in a thin wall, e. g. water running out from a small hole in the bottom of a tank, or gas escaping from an elastic rubber bag through a pin-hole, the velocity of "efflux" (v) is greatest for fluids which have the least density (ρ), and varies directly as the difference of pressure (p) producing the flow; the approximate relation being $v^2 = 2\,p/\rho$. (See Ames's "Theory of Physics," p. 121.)

The fact that the rate of efflux of lighter gases is greater than that of heavier ones can be shown by a simple lecture experiment, in which hydrogen, the lightest known gas, is made to flow through small openings faster than does air. A long glass tube is fitted, by means of a tight-fitting cork, into an unglazed porcelain jar (such as is used in making Daniell's cells); the tube is now placed vertical, with its open end dipping in a glass of water. The porcelain jar is, of course, full of air; but, if an inverted bell-jar filled with *hydrogen* is lowered quickly over the porcelain jar, the hydrogen will rush into the latter through the minute pores faster than the air inside can pass out. Therefore, the pressure of the gas inside is increased, and bubbles of air will be forced out through the lower end of the tube which is under the water. If the bell-jar is now removed, the hydrogen inside will pass out quickly, leaving a diminished pressure, and water will rise in the glass tube. (This experiment succeeds with gases, but not with liquids; for, when the bell-jar of hydrogen is lowered over the porous jar, the pressure of

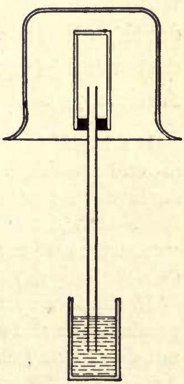

FIG. 53.

the hydrogen inside the latter is zero, and hence the hydrogen is *forced* in; similarly, the air inside is forced out, because there is no air outside in the bell-jar, and the two gases thus diffuse into each other; while, if liquids were inside and out, each would prevent the other from flowing out, but there might be a slow diffusion.)

Liquids

67. Free Surface of Liquids.—That property of a liquid which distinguishes it from a gas is the fact that it keeps a definite volume; if left to itself, it assumes the shape of a sphere, e. g. rain-drops; if contained in an open, hollow vessel, it has a free surface in contact with the air. If there is equilibrium, this free surface must be perpendicular to the forces acting on the liquid; for, if it were not, these forces would have components along the surface, and the liquid would flow in that direction. Thus, on the surface of the earth, all free surfaces of liquids are horizontal unless disturbed (except near solids dipping into them; see Article 73).

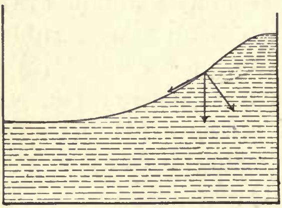

Fig. 54.

If a liquid is contained in a cylindrical vessel which is made to rotate around a vertical axle coinciding with the axis of the cylinder, the free surface will assume a parabolic form, so as to be at right angles to the force acting, which is the resultant of the force of gravity and the "centrifugal" forces owing to the motion of each portion in a circle. (See Article 33.)

If the free surface of a liquid is disturbed in any way, waves are produced on the surface and a short distance down. These waves are not due to any elastic force, as in the case of the waves discussed in Article 56. These are due to the fact that the force of gravity tends to bring the disturbed liquid back to its horizontal level; but, owing to its momentum, it goes further than this, and thus oscillates and spreads the disturbance to the neighboring portions of the surface. The velocity of the surface-waves varies as the square-root of the wave-length if the liquid is deep; therefore, long waves on the sea go faster than short ones. Very minute waves or "ripples" are not due to gravity, but to capillarity, and have properties different from those of the long "surface-waves."

68. Liquid Pressure.—The pressure in a liquid is, as shown in Article 62, due to two causes, the pressure of the walls of the containing vessel, and the weight of the liquid. The pressure due to the walls is felt everywhere through the liquid and is the same at all points. In particular, if a vessel containing a liquid is open to the atmosphere, there is a force pressing on the free surface due to the weight of

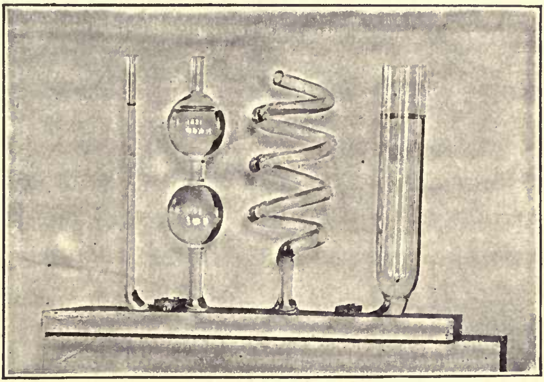

FIG. 55.

the atmosphere. If the *pressure* in the air above the surface is P, this same pressure is felt throughout the liquid.

In addition to this pressure, there is that due to the weight of the liquid, which amounts to $\rho g h$ for a vertical depth h. Thus, at a depth h in a liquid below the free surface, the pressure is $P + \rho g h$. If A is the area of the surface against which there is this pressure, the force is $(P + \rho g h) A$.

Since the pressure at all points of a liquid which are in the same horizontal plane is the same (see Article 62), the height above the earth to which a liquid will rise in wide, open, connecting tubes is the same; for, if any horizontal plane be taken through the tubes, the pressure at each point is as shown, $P + \rho g h$, where h is the vertical depth below the free surface; hence, since the pressure is the same at each point, h is the same for each. This fact is sometimes expressed, "water seeks its level," and is illustrated by springs, artesian wells, systems of water distribution in cities, etc.

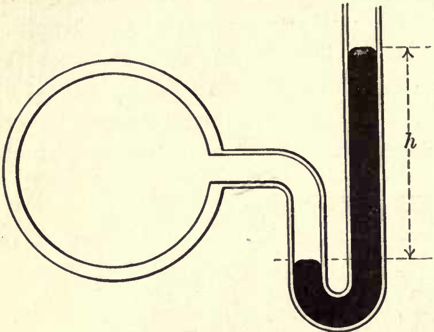

FIG. 56.

The principle is used, too, in the measurement of gaseous pressure. A U-shaped tube, containing some liquid, e. g. mercury, is joined, as shown in Fig. 56, to the vessel containing the gas whose pressure is desired. If h is the difference in height of the two surfaces of the liquid, and ρ its density, the pressure of the gas is $P + \rho g h$ (where P is the atmospheric pressure), if the liquid stands lower in the arm entering the vessel than in the open arm. For, in the liquid in the open arm at the level of the free surface in the other, the pressure is $P + \rho g h$. Similarly, if the surface in the open arm is the lower, the pressure in the gas is $P - \rho g h$. Such an apparatus for measuring pressure is called an "open manometer."

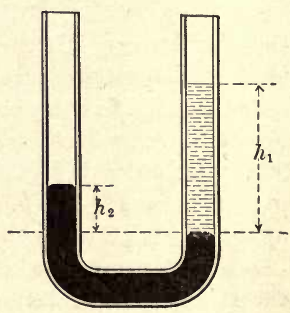

FIG. 57.

69. "Balancing Columns."—If two liquids which do not mix are poured into a U-tube, so that they stand as shown in Fig. 57, they have a free surface in common. Let a horizontal plane be drawn through this surface, and measure the vertical heights h_1 and h_2 of the other two free surfaces above this. The pressure at the common free surface due to the liquid above it is $P + \rho_1 g h_1$, if its density is ρ_1; this is balanced by a pressure $P + \rho_2 g h_2$ due to the second liquid, whose density is ρ_2.

Hence $$P + \rho_1 g h_1 = P + \rho_2 g h_2,$$

or $$\rho_1 h_1 = \rho_2 h_2.$$

Therefore, the heights at which the liquids stand vary inversely as their densities. This gives a method of comparing the densities of two liquids which do not mix, e. g. mercury and water. (For other methods see Ames's "Theory of Physics," p. 116.)

70. Thrust.—This pressure of liquids is exerted against the surfaces of the solids which contain them, and is illustrated by the thrust against dams, tanks, etc. The pressure at the free surface is P, the atmospheric pressure; that at the bottom is $P + \rho g h$, if h is the depth. The *average* pressure from top to bottom depends upon the shape of the wall which receives the thrust; if it is rectangular (or cylindrical), the average pressure is $P + \frac{\rho g h}{2}$, and so the total thrust sidewise is $(P + \frac{\rho g h}{2}) A$, if A is the area of the wall. Similarly, the force over the bottom of the tank is $(P + \rho g h) A$ if its area is A.

71. Floating Bodies.—If a body floats on the surface of a liquid, e. g. a block of wood on water, it displaces a certain volume of the liquid; and there is, therefore, a buoyant force (see Article 64) equal to the weight of the liquid displaced. The force *down* is the weight of the floating body; hence, since the body is in equilibrium, these two forces must be equal; that is, a floating body displaces its own weight of liquid.

The equilibrium of a floating body is not necessarily stable. A long stick floating on its side is stable; but, when set floating in an upright position, it is unstable. A boat with a heavy ballast is stable, etc.

72. Properties of a Liquid Surface.—As has been noted several times, a portion of a liquid if left to itself assumes a spherical shape, as is shown by rain-drops, lead shot, etc. The surface of a sphere has the least area for any geometric figure of an equal volume; so this proves that the surface of a liquid tends to become as small as it can. There is thus a contracting force in a liquid surface, which is in this respect analogous to the properties of a stretched rubber sheet. This is shown also by the following illustrations: If a soap-bubble is blown on the end of a glass tube (see Fig. 58), work is required to overcome the contracting tendency; and, if the bubble be given the chance, it will contract, expelling the air through the tube. Many animals, if light enough, can move over the surface of water, being held up by the surface-"skin."

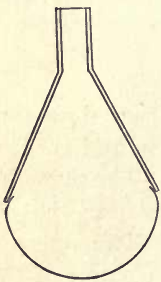

FIG. 58.

If a soap-film is blown on any wire frame, it will assume the smallest area possible; if such a film is placed between a frame and a thread in its plane, as shown in Fig. 59, it will require force to keep the film from contracting. This contracting force is due to the fact that the molecules of liquid in or near the free surface are not acted upon by "forces of cohesion" on all sides—as are the molecules in the interior of the liquid—

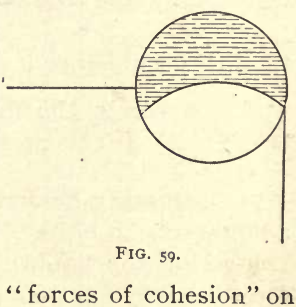

FIG. 59.

but only on the lower side. There is thus a tension in the surface. If the surface is made larger, e. g. by pouring the liquid into a wider vessel, the surface is not stretched—as in the case of a rubber sheet which is made larger—there is simply more surface, some liquid comes up from the interior to the surface.

73. Surface-Tension.—This "surface-tension," as it is called, is illustrated, too, by the rise of liquid in tubes of small bore—so-called "capillary" tubes—if the liquid wets the solid, e. g. water and clean glass; and by the sinking of the liquid, if it does not wet the solid, e. g. mercury and glass. Thus, if a glass tube whose inner surface is moistened with water is lowered into a vessel of water, the shape of the surface of the water inside the tube is like the inside of the finger of a glove, having its end in the water of the vessel and its side in the film adhering to the walls of the tube. (See Fig. 60.) This surface contracts, pulling the liquid in the vessel up the tube.

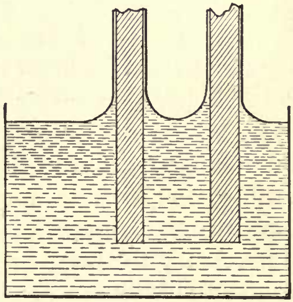

FIG. 60.

The smaller the bore, the further will the liquid rise. For a similar reason, water rises up along the surface of a plate of glass which dips in it. Expressed in terms of pressure, this fact may be stated: There is a pressure in a liquid if its surface is curved, due to the tendency of this surface to contract; if the surface is convex, as in a drop or bubble, the pressure is balanced by the fluid or gas inside; if it is concave, as in water rising in a glass tube, it makes the pressure at the surface less than it would be naturally, and less than at the same level elsewhere in the liquid. (It may be proved that this pressure due to the surface forces varies inversely as the radius of the curved surface. (See Ames's "Theory of Physics," p. 125.) This diminution of pressure due to a concave surface is shown in the familiar experiment of pressing together two wet plates of glass and then trying to separate them; at the edges of the plates the liquid surface is concave, and therefore the pressure between the plates is less than in the atmosphere outside; and so the two plates are held together, owing to the atmospheric pressure. The rise of liquids in capillary tubes is shown by the action of blotting-paper, lumps of sugar, thread, etc.

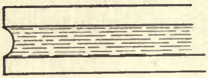

FIG. 61.

The sinking of mercury in glass tubes is explained in a perfectly similar way. The mercury does not wet the glass; therefore its free surface inside the tube is like the outside of the finger of a glove; and by its contraction it draws the surface down. (See Fig. 62.)

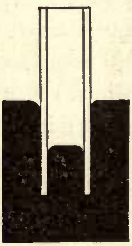

FIG. 62.

74. Effect of Points and Nuclei.—Drops and bubbles require nuclei for their formation, because, to produce a surface of such great curvature, i. e. of such a small radius, as would be necessary to start a drop or bubble, would require an infinite force; but layers of liquid can be deposited on solid nuclei, and thus form drops. Bubbles are generally started around fine points; or, in the case of bubbles of steam formed in boiling, they start from nuclei of minute bubbles of dissolved gases. Illustrations are given by the following facts: Dew forms on points of grass, etc., more readily than on plain surfaces; rain-drops practically always have particles of dust inside them; the process of boiling is assisted by the presence of fine points, such as broken glass, and by the introduction of a lump of sugar, which always contains a great deal of air; bubbles are formed in effervescent liquids at those places in the bottles or glasses where the surface is roughened.

75. Effect of Impurities, etc.—This tendency of the surface of a liquid to contract is different for different liquids, as is shown by several simple experiments. If a drop of alcohol is placed on a glass plate which has been previously wet with water, the edge of contact of the alcohol and water will be rapidly dragged away from the drop, thus spreading the drop over the surface (or perhaps leaving the plate dry). This shows that the surface-tension of pure water is greater than that of a mixture of alcohol and water. In a similar way, a drop of oil is spread over the surface of water. (The quieting action of oil upon waves comes from the fact that the film of oil destroys the "ripples," which are due to surface-tension; and there are thus no little ridges for the wind to catch and blow up into spray or magnify into large waves.) If a piece of camphor is put on a *clean* surface of water, it will make a number of erratic to-and-fro motions over the surface, because it dissolves irregularly, and thus weakens the surface-tension of the water more at one side than at another; it is therefore pulled towards the side of greater tension.

Increase in temperature diminishes the surface-tension. Thus, if a grease-spot is to be removed from a piece of cloth, a hot iron should be applied on the *under* side, and a piece of absorbent paper on the grease-spot side; the heat makes the tension so small on the under side that the grease gathers in a drop on the upper side, being pulled there by the surface forces.

Gases

76. Gaseous Pressure.—A gas is distinguished from a liquid in that it has no volume or shape of its own, but assumes those of the containing vessel. The pressure in a gas is due (see Article 62) to the pressure of the walls and to weight; but since the density of all gases is so very small, the main pressure in a gas enclosed in any receiver is that corresponding to the walls. The gas presses against the walls; and when the two pressures are equal, that of the gas pressing out and that of the wall pressing in, there is equilibrium.

77. **Mixture of Gases**.—If more than one gas is placed in a receiver, each acts as if the others were absent; and so the total pressure is the sum of the pressures which each by itself would exert. This is known as Dalton's Law.

78. **Boyle's Law.**—If the gas is compressed, and its volume thus diminished, it is found that the pressure increases. The experiment was first tried in the following way: A glass tube, closed at one end, was bent into the form of a J and placed vertical; mercury was poured into the open end, so as to trap some air in the shorter closed branch. (See Fig. 63.) The pressure of the air was $P + \rho g h$, where P was the atmospheric pressure, h the difference in level of the two free surfaces of the mercury, and ρ the density of the mercury; and the volume of the air could be measured easily. More mercury was poured in slowly, the new pressure and corresponding volume were measured; and it was seen that, *if the temperature* of the air in the tube was *kept constant*, the connection between pressure (p) and volume (v) could be expressed by saying that the product *pv remains constant*. This is known as Boyle's Law, and was first stated by Robert Boyle in 1662. It is equivalent to saying that the pressure of air varies directly as its density, if the temperature is kept constant; for the density is mass divided by volume. That is, if ρ is the density of air,

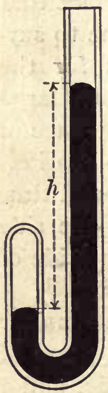

FIG. 63.

$$p = k\rho, \text{ temperature constant.}$$

This law holds true for air and other gases to a considerable degree of accuracy for great ranges of pressure.

79. Atmospheric Pressure; Barometer.—In order to have any appreciable pressure due to weight ($p = \rho g h$), it is necessary to have great heights of a gas, because the density is so small. The best illustration of such a condition is given by the pressure due to the atmosphere itself. The fact that there is such a pressure is shown by the following experiment: If a long (over 80 cm.) glass tube, closed at one end, is *filled* with mercury, and then carefully inverted, allowing no mercury to escape; and, if the open end is placed beneath the surface of mercury in a wide open vessel, and then left to itself, the mercury in the tube will not run out, but will stand at a certain height in the tube. There is no gas in the space above the mercury column; so, if the free surface in the tube is h cm. above the free surface in the basin, and if ρ is the density of the mercury, the pressure required to balance the mercury column, i. e. the atmospheric pressure, is $\rho g h$. (It is evident that, since fluid pressure depends only upon the

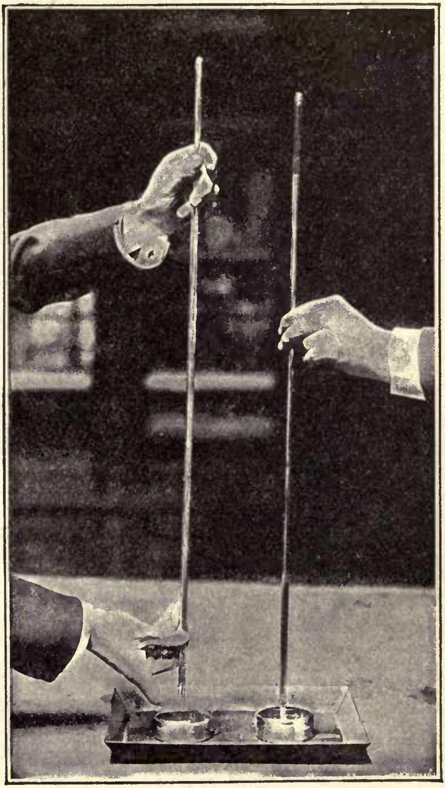

FIG. 64.

vertical height of the fluid, not upon the cross-section of the tube, this height h would be the same for wide tubes of any cross-section, or any shape.) Such an instrument is called a "barometer," and measures any fluctuations of the atmospheric pressure. (If a barometer is carried up one or more flights of stairs of a building, it will indicate differences in pressure, owing to the varying depths of air. If ρ' is the average density of the air, h' the vertical height through which the barometer is carried, the change in pressure is $\rho' g h'$, which would be shown by a change in the barometer.)

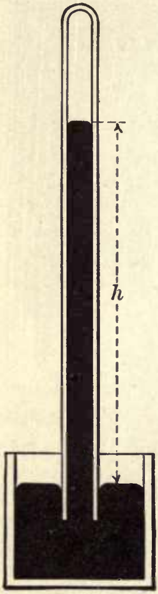

FIG. 65.

The pressure of the atmosphere at sea-level is about that of 76 cm. of mercury; that is, $P\ (= \rho g h) = 13.6 \times 980 \times 76 = 1{,}013{,}000$ dynes per square centimetre,

$$444{,}518 \text{ dynes} = \text{weight of 1 pound},$$

$$1 \text{ sq. cm.} = 0.1550 \text{ sq. inches}.$$

Hence $\qquad P = 14.7$ pounds per sq. in.

Similarly, if any other liquid than mercury was used, its height (h_1) in a barometer would be given by the same formula $P = \rho_1 g h_1$, where ρ_1 is its density. Hence, if ρ and h are the density and barometric height of mercury,

$$P = \rho g h = \rho_1 g h_1 \therefore h_1 = \frac{\rho h}{\rho_1}.$$

If air is exhausted from a hollow metal sphere which is divided into two hemispheres, it requires a great force to pull them apart; if air is exhausted from a glass bulb, it may break, etc.

80. Siphon.—If a glass tube, open at both ends, is bent into the form of a J, is filled with water, and then inverted, so that its shorter arm dips into a basin of water, no water being allowed to escape in the process, the apparatus is called a "siphon." The end of the longer arm must have been held closed by a stopper of some kind, e. g. the finger. Remove this, and the water from the vessel will continue to flow out until its level is below that of the end of the tube. Before the stopper is removed from the opening A, the pressure of the water there is equal to that at B plus $\rho g h_1$, where h_1 is the height of the longer arm; but that at B equals that at C, and is therefore equal to that at D minus $\rho g h_2$, where h_2 is the length of the shorter arm. The pressure at D is the atmospheric pressure (P), and therefore the pressure at A is

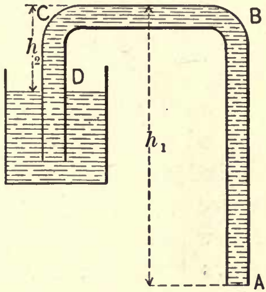

FIG. 66.

$$P - \rho g h_2 + \rho g h_1,$$

or

$$P + \rho g (h_1 - h_2),$$

which is greater than P. Consequently, when the stopper is removed, the atmospheric pressure is not sufficient to keep the water in the tube; and so it runs out; and the velocity of outflow will vary directly as $(h_1 - h_2)$.

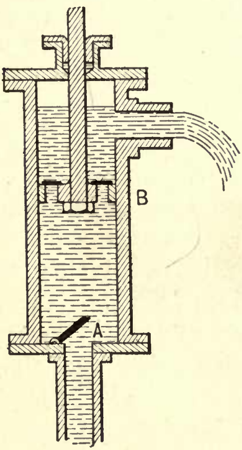

FIG. 67.

81. Pumps.—The action of the ordinary lift-pump depends upon the pressure of the atmosphere. A cylinder is connected at one end, A, with the well or cistern by means of a pipe which must not be over a certain length (viz. that height to which the liquid would stand in a barometer in which it was used in place of mercury). At the end where the pipe enters the cylinder there is a valve opening upward.

In the cylinder there works a piston, *B*, driven by the pump handle; the piston has openings through it containing valves, also opening upward. The process is as follows: Let the piston be at the bottom of its path; as it is raised, the pressure below it is diminished; the water in the well or cistern, being under atmospheric pressure, is forced up through the pipe, raising the valve at *A*, into the cylinder; when the piston reaches its highest point, the valve at *A* drops and closes; then, as the piston is lowered, the water in the cylinder raises the valves in the piston and flows through; so, when the piston gets to the bottom of the cylinder and begins to rise again, the valves in the piston close, that at *A* opens, water enters again through *A*, but the water on top the piston is lifted and may be made to flow out a spout. (The atmospheric pressure at sea-level is equivalent to a height 76 cm. of mercury whose density is 13.6 times that of water; hence, it is equivalent to a height of 76×13.6, i.e. 1034 cm. of water, or about 34 feet; consequently, the length of the pipe connecting a cistern of water to the cylinder is limited by this consideration. There is always also a certain amount of leakage of air.)

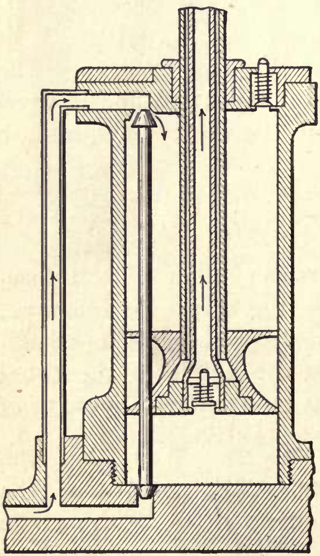

FIG. 68.

A pump of this same nature can be used to exhaust a gas out of a closed vessel. The only alteration is made necessary by the fact that the pressure of the gas soon becomes too small to lift the valves; and so automatic devices must be used to open and close them. These are apparent from the details of Fig. 68.

A cut is given of a so-called force-pump, consisting, as shown, of a piston working in a cylinder, out of which go two pipes, one to the cistern, the other to a bell-shaped receiver, into which enters from the top a pipe long enough to nearly reach the bottom. At the upper end of the cistern pipe, and at the entrance of the side pipe into the receiver, there are valves opening upward. The action is as follows: As the piston rises from the bottom of the cylinder, the cistern-valve opens and water enters; when the piston descends, the cistern-valve closes, the water is forced into the receiver; as the process continues, enough water enters the receiver to reach the open end of the pipe which comes in through the top; and, as more and more water is forced in, the air in the receiver is compressed and forces water up the tube to practically any height. The chief advantage of the air-receiver is to render the output of water continuous instead of intermittent, because the compressed air acts like a kind of spring or cushion.

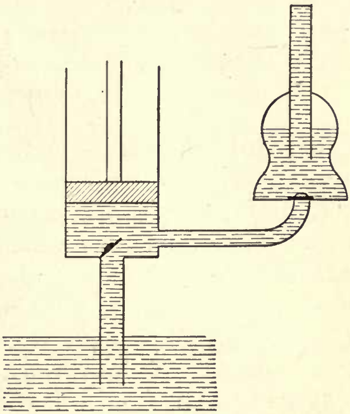

FIG. 69.

CHAPTER IV

NATURE OF SOUND

82. Introductory.—A "sound" is the name given to a particular sensation which is familiar to every one who is not deaf. Its physical cause can be traced in every case to some elastic substance which is making vibrations. Thus, the blow of a hammer, the rolling of a cart over cobble-stones, the blowing of a horn, the plucking of a guitar-string, etc., all produce vibrations, and our ears, in general, hear sounds. If the vibrating body is in a vacuum, e. g. in a large bell-jar from which the air has been exhausted, no sound is heard. Consequently, the presence of matter between the vibrating body and the ear is necessary as a medium of transfer. The loudness of the sound depends largely upon the medium connecting the ear and the vibrating body. (This is known to any one who has held his head under water and listened to the sound produced by knocking two stones together below the surface.) If the vibrations of the elastic body exceed in number a certain limit (see Art. 56), compressional waves, i. e. longitudinal ones, are produced in the surrounding material medium, if this is a fluid; these then spread out, and the wave-fronts advance with a velocity depending upon the elasticity and density of the medium, but not upon the wave-length. (See Art. 56.) If the vibrating body makes a series of regular vibrations at the rate of N per second, i. e. if its "frequency" is N or its period $\frac{1}{N}$, and if the velocity of the waves in the medium is v and λ is the wave-length of this particular train of waves, $v = N\lambda$. When the waves reach the ear, a sound is heard as a result of the vibrations of the ear-drum, provided that N, the frequency of these vibrations, lies between certain limits which vary with different people. 20 and 40,000 vibrations per second are about the extreme limits; but in music, frequencies from 40 to 4000 only are used in general.

These compressional waves have other properties, naturally, than that of causing certain sensations in the human ear. They affect the senses of most animals throughout various ranges of frequencies, which are probably different from the ranges audible to man. They produce mechanical motions if they fall on a thin disc, such as a telephone diaphragm. They affect a so-called "sensitive" flame, which is simply a flame of gas issuing from a circular opening under high pressure. The flame is thus a narrow, long one; but, if waves of great frequency, i. e. of short wave-length, strike it, it collapses into a broad, short one.

83. Simple and Complex Vibrations.—One simple method of studying the nature of the vibrations of an elas-

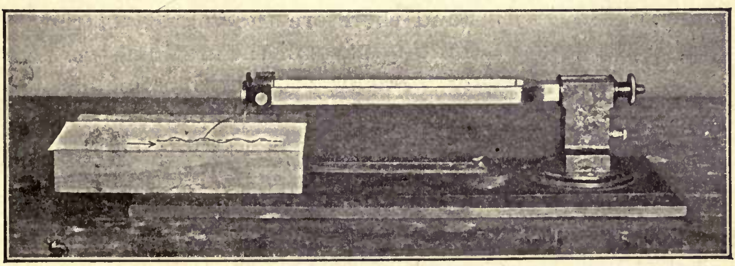

FIG. 70.

tic solid is to fasten a stiff light pointer to it; and, as the solid vibrates, to draw under the pointer a piece of smoked glass, so that the pointer leaves a trace on the glass. (See Figs. 70 and 71.) By this means it is found that, when the ear perceives a "musical" note, the vibrations are continuous and regular; but, when a "noise" is heard, the vibrations are discontinuous and abrupt. The simplest kind of vibration is that called "harmonic" or pendulumlike. (See

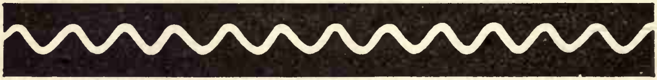

FIG. 71.

Article 31.) Tuning-forks and most musical instruments can be made to give harmonic vibrations. Such a vibration has a definite amplitude and a definite frequency, which may be studied by the smoked-glass methods. (See Fig. 71.) Different vibrations may have different amplitudes and frequencies, and may pass through their positions of equilibrium at different instants, i. e. may differ in "phase." It is evident that the energy of the vibration must increase with the amplitude, for, with a large amplitude, the strain is greater (and therefore the velocity of the vibrating parts of the medium—*not that of the waves*) than with a small amplitude. If a vibrating system is constructed, as shown in Fig. 72, of three pendulums suspended in series, it is called a "complex" pendulum; and the vibration is complex. Each pendulum of itself, if its point of support were fixed, would make harmonic vibrations; but, as it is, the motion is more complicated. The motion of all musical instruments is of this nature, unless special precautions are taken.

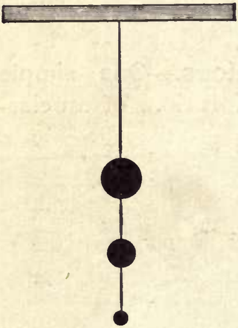

FIG. 72.

To analyze a complex vibration, therefore, it is necessary to learn what harmonic vibrations compose it, and then to determine the amplitude, frequency and phase of each of these component vibrations.

84. Resonance.—To learn what harmonic vibrations are present in any complex vibration, the simplest method is to make use of the principle of "resonance." If one wishes to set in vibration a boy sitting in a swing, he has but to give the swing a series of pushes which are so timed as to be at intervals equal to the natural period of vibration of the swing; that is, the swing is set in vibration if the force applied has the same period as it itself has. In general, if a small periodic force is applied to an elastic body whose period of vibration is the same as that of the force, the body will be set in intense vibrations; while, if the two periods differ, even slightly, this will not occur—there will be forced vibrations, but they will be comparatively feeble. If a man sings a note near a piano, he will set in vibration that particular string of the instrument which has the same frequency as the note. If a tuning-fork is held over the mouth of a suitable bottle, and, if water is poured in slowly, it will be noticed that for a given depth of water the sound of the fork will be greatly increased. This is because the air in the bottle can now vibrate with the same frequency as that of the fork; consequently, it is set in vibration, and the sound is due to both the vibrating bodies. The air in a bottle like this vibrates in a harmonic manner with a definite

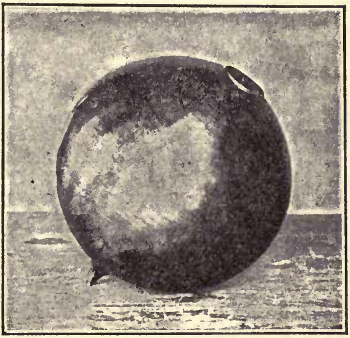

FIG. 73.

frequency. It forms what is called a "resonator." (In Fig. 73 is shown a common form of resonator.) If a series of such resonators, of different frequencies, is made, and, if the body which is making complex vibrations is brought near the resonators, each one that has a frequency equal to one of those of the components of the complex vibration will respond. In this manner, the vibration may be analyzed. (The sound heard on putting a sea-shell or bottle to the ear is due to the strengthening of some sound in the room which has the same frequency as that of the air in the shell or bottle.) It is thus found that in many cases, e. g. with stretched cords, organ-pipes, etc., the complex vibration is made up of harmonic vibrations which have frequencies n, $2n$, $3n$, $4n$, etc. The vibration of frequency n is called the "fundamental;" the others, the "partials." This is not true in general; for, with most instruments the frequencies of the partials do not bear any simple numerical relation with the fundamental.

85. Harmonic and Complex Waves.—The waves produced in the surrounding fluid medium by the vibrating body are longitudinal ones, consisting of compressions and rarefactions. An attempt is made in Fig. 74 to represent the advance of such a train of waves towards the right. Each individual particle makes harmonic vibration, and such a train of waves is called a harmonic train. The wave-length (λ) and wave-number (n) are connected by the relation $v = n\lambda$, where v is the velocity of the train. The amplitude of the waves is that of each individual particle, and may vary with different waves; but the greater the amplitude, so much the more is the energy which is being carried by the waves. The velocity of the waves is, however, independent of the wave-length or amplitude. The characteristics of a train of harmonic waves are therefore amplitude and wave-number (or wave-length, since $\lambda = v/n$). A harmonic vibration of the vibrating body will send out a harmonic train of waves; while a complex vibration will emit a complex train, corresponding to the complexity of the vibration. A complex train of waves, therefore, may be regarded as the sum of several harmonic trains. In Fig. 75 are shown the results of adding several harmonic transverse trains, e. g. in a stretched string. The two components are in dotted lines, their sum in a full line. *A* is the sum of two harmonic waves, whose wave-lengths are in the ratio of 1 : 1, and which are in the same phase, i. e. both begin at the same time. *B* and *C* refer to the same waves, but in different phases. *D* and *E* are the sums of two waves whose wave-lengths are in the ratio 1 : 2. It will be seen that the "form" of the resultant wave depends not alone upon the relative wave-lengths and amplitudes of the components, but also upon their phase with relation

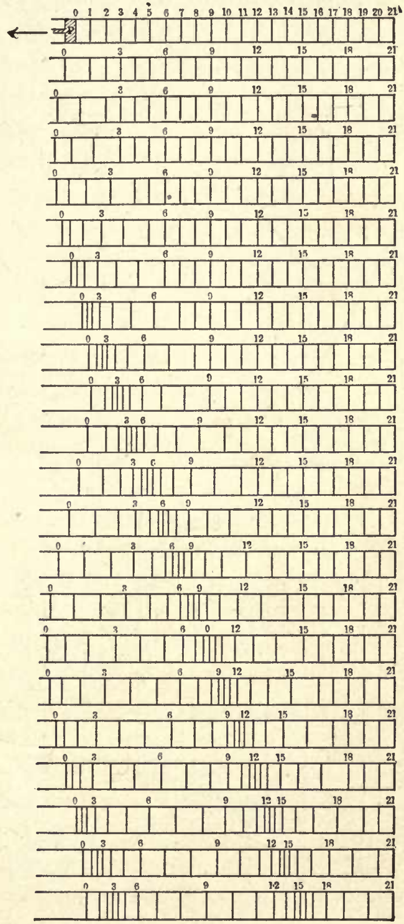

FIG. 74.

A compressional train of waves advancing towards the right. The left-hand particle —marked *o*—can be considered as kept in vibration by a piston moving to and fro.

to each other. As these component waves travel with the same velocity—if the medium is homogeneous—the complex wave maintains the same "form" as it advances.

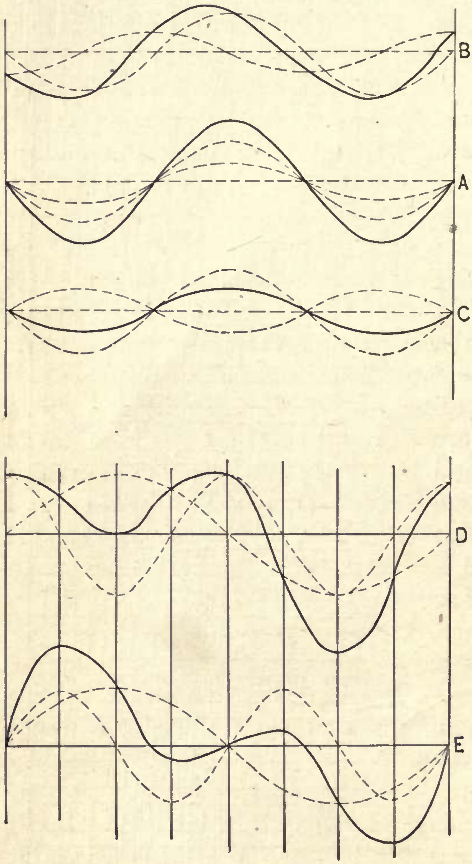

FIG. 75.

These compressional waves which produce sound when they reach the ear are generally in the air; and they can be reflected like water-waves and other kinds of waves. This is shown by echoes which are caused by the reflection of the waves from some large object, such as the side of a building or a rocky ledge.

The velocity of these waves in air is 332.5 metres per second at 0° C. At higher temperatures, it is greater, increasing about 70 cm. for a degree centigrade. The frequency of "middle C" on a piano is 256 vibrations per second; therefore, the wave-length in this case is 130 cm.

86. Characteristics of a Sound.—When these compressional waves reach the ear, a sound is heard, if the frequency falls within certain limits, viz. 20 and 40,000 approximately. (The limits of audibility vary greatly with different people, entirely apart from deafness. Some people can hear the shrill sounds made by the wings of certain insects, while others cannot.) The ear recognizes certain sounds as noises, and others as musical, as noted in Article 83. It also recognizes certain musical tones as being simple, others as being complex. Thus, the sound due to a tuning-fork is simple; that due to a banjo-string very complex. A simple musical sensation has a certain loudness and a certain "pitch" or shrillness; and different simple sounds vary greatly, both in loudness and in pitch. By means of proper attention, a trained musician can detect in a complex note the presence of certain component simple tones, of different pitch and loudness. This complex character of most musical notes is said to be their "quality," and is different for different musical instruments. Thus, almost any one can tell if a sound is due to a horn, to an organ-pipe, to a violin, to a drum, etc. The characteristics of a complex musical tone are, therefore, its quality, and the pitch and loudness of the component simple tones. If the attempt is made to identify these characteristics of a musical note with those of the vibrations of the elastic body, the following facts are discovered:

1. A harmonic vibration causes a simple tone; a complex vibration, a complex note.

2. The quality of a note depends upon the components of the complex vibration; if there are many of these, the quality of the tone is harsh and "twangy." (The quality, however, does not vary with the *phase* of the component vibrations.)

3. The pitch of a simple tone varies directly as the frequency of the harmonic vibration.

If the vibrating body is approaching the ear, the pitch is raised, because more waves enter the ear in one second than would if the body were at rest; if the body is receding from the ear, the pitch is lowered. (This is illustrated by the increased shrillness of the whistle of an approaching locomotive, etc.)

4. The loudness of the note increases with the amplitude of the vibration.

87. Beats.—If two bodies whose frequencies differ only three or four a second are set in vibration, a pulsating sound is heard. The loudness rises and falls at regular intervals, thus producing so-called "beats;" the number of beats in a second being equal to the difference of the frequencies of the two vibrating bodies. For, this "beating" is due to the interference of the two trains of waves which are sent out by the two bodies; and, since they are of different wave-length, it will happen at regular distances that one wave neutralizes the other, while at points half way between, each reinforces the other. Consequently, in a distance equal to v, the velocity of the waves, one train of waves will have n_1 wave-lengths, the other will have n_2; and therefore in this distance there will be $n_1 - n_2$ points, where one wave will neutralize the other, if they have the same amplitude.

88. Harmony and Musical Scales.—If the number of beats in a second exceeds in number about twenty, the sensation becomes disagreeable to the ear, just as a twinkling light is unpleasant to the eye, or the tickling of a feather to the skin. If two bodies, then, make complex vibrations which are so chosen that there are no beats between any of the many component vibrations, the two tones should be pleasant to the ear or in harmony; and, conversely, if there are beats between any of the partial vibrations, there should be discord. Such is observed to be the case. Musical scales and compositions are based upon groupings of notes which are in harmony.

A body which makes, when vibrating at its lowest fundamental rate, n vibrations per second, will have in many cases, as stated above, partial vibrations of frequencies $2n$, $3n$, $4n$, etc. If two bodies have the fundamentals n and $2n$, they will have the partials $2n$, $3n$, etc., $4n$, $6n$, etc. There will be no beats, and the two complex nodes are in harmony. They form in this case what is called the "octave."

Similarly, two bodies whose fundamentals are in the ratios of 1 : 3; 2 : 3; 1 : 4; 3 : 4; give rise to two notes which are more or less in harmony. Thus, frequencies may be chosen which are suited for musical composition. One particular scale, called the "diatonic," consists of a series of frequencies, such that in the interval of an octave there are seven notes so chosen that they are given by the following ratios:

1st note,	C, 24 n	vibrations	per	second.		
2d	"	D, 27 n	"	"	"	
3d	"	E, 30 n	"	"	"	
4th	"	F, 32 n	"	"	"	
5th	"	G, 36 n	"	"	"	
6th	"	A, 40 n	"	"	"	
7th	"	B, 45 n	"	"	"	
8th	"	$2C$, 48 n	"	"	"	etc.

The value of n is arbitrary. At the present time, it is so chosen that A (40 n) equals 435; hence, $n = 10\frac{7}{8}$.

This "diatonic" scale is not, however, used extensively at the present time; a "tempered" scale has been adopted by most instrument-makers, in which in the interval of an octave twelve notes are introduced, at equal intervals apart, i. e. the ratio of one frequency to the next is the same throughout the scale. Thus, if A is the starting-point of both scales, the notes C on the two would have slightly different frequencies.

CHAPTER V

SOUNDING BODIES

89. Nodes and Loops; Frequency.—It has been shown in Article 59 that, if transverse waves are sent along a stretched string and reflected at a fixed end, there will be a "stationary vibration" of the cord, if its length bears a certain relation to the frequency; it will be divided into vibrating sections limited by "nodes" or points of no motion. The wave-length (λ) of the *waves* is equal to twice the distance (d) between two nodes; $\lambda = 2\,d$. If v is the velocity of the waves (transverse in this case), and n is their wave-number, $v = n\,\lambda$. Therefore, $v = 2\,n\,d$ or $n = v\,/\,2\,d$. The wave-number of the waves is the frequency of the vibration of each particle of the medium carrying the waves, and is, therefore, the frequency of the stationary vibration caused by the waves. The sections between the nodes vibrate transversely like a cord whose length is that of the section. Consequently, the string is divided by the nodes into vibrating sections of equal length; and half way between the nodes are the points of greatest motion, called "loops."

In a perfectly similar manner, stationary vibrations may be set up in an elastic wire or in a column of air by *compressional* or longitudinal waves. In this case, the nodes are the points where the motion is least, and the fluctuations of pressure the greatest; the loops are the points where the pressure remains constant. (See Fig. 76.) As before, $\lambda = 2\,d$, and $n = v/2\,d$, where v is the velocity of the *compressional* waves.

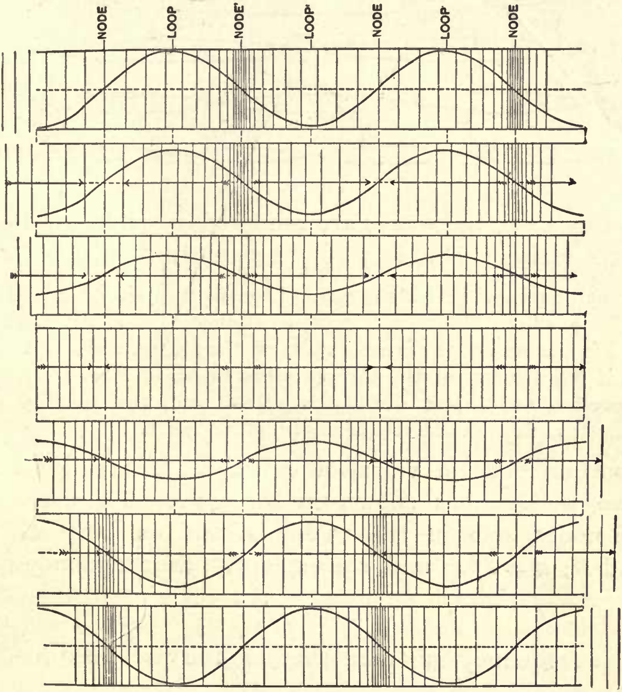

FIG. 76.

Stationary vibration in a column of gas. Vertical lines represent positions of layers of gas. *Curves* represent by their *vertical* displacements the *horizontal* displacements of the layers of gas from their positions of equilibrium. Arrows represent the directions of motion of the layers of gas.

90. Transverse Vibrations of a Stretched String.— One mode of vibration is when the string vibrates as a whole, as shown in Fig. 77 *a*. The only nodes are at the ends A and B; therefore, if l is the length of the string, $d = l$; and the frequency (n) is given by the formula

$$n = v/2\,l,$$

where v is the velocity of *transverse* waves in the string. This is the fundamental vibration.

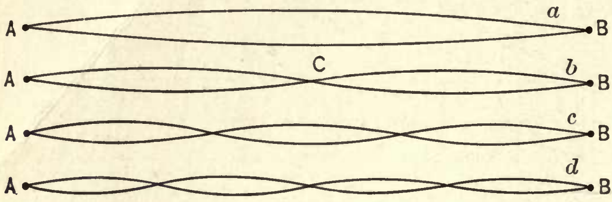

FIG. 77.

In this case, the waves are rendered possible by the fact that the string is held stretched by external tension.

It may be proved that if the string is perfectly uniform and flexible, $v^2 = T/a\rho$, where T is the tension in the string, i. e. the stretching force, a is the area of the cross-section of the string, and ρ its density. Thus, if the tension of the string is increased, or if one of smaller cross-section or of less density is substituted, the velocity (v) is increased; and hence the frequency (n) is increased.

Another mode of vibration is when the string vibrates in two sections, as shown in Fig. 77 *b*. There are thus three nodes, one in the middle C and one at each end, A and B; $d = l/2$; and, consequently, the frequency

$$n = v/l;$$

or the frequency is twice that of the previous mode of vibration.

Another mode is when the string vibrates in three sections (see Fig. 77 *c*); there are then four nodes; $d = l/3$; and, consequently,

$$n = 3v/2l;$$

or the frequency is three times that of the first mode of vibration.

Other simple methods are evidently possible, with frequencies, 4, 5, 6, etc., times that of the first node. If the string is set vibrating by a random blow, the vibration will not be one of these simple nodes, but a complex one, made up of two or more of them. In Fig. 78 the superposition of two modes, one and three, is shown.

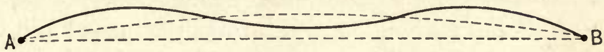

FIG. 78.

Pianos, violins, and all stringed instruments give illustrations of these vibrating cords.

91. Longitudinal Vibrations of Wires.—If a wire stretched between two fixed supports is rubbed lengthwise, it is set in vibration; and the individual portions of the wire move to and fro along the direction of the wire. There is a node at each end; $d = l$; and the frequency

$$n = v / 2l,$$

where v is the velocity of *compressional* waves in the wire, and has no connection with the velocity of *transverse* waves.

92. Vibrations of Rods.—If a solid rod is held clamped at its middle point and set in vibration, there is a node at that point and a loop at each end. Since the distance between a node and loop is half the distance between the two nodes, d in this case is the length of the rod (l). Therefore, the frequency

$$n = v / 2l,$$

where v is the velocity of transverse waves, if the vibrations are transverse, and that of compressional waves, if the vibrations are longitudinal. (These vibrations are due to the elastic properties of the solid itself; because it is not stretched.) Another illustration of transverse vibrations is given by a tuning-fork (Fig. 79), which is a rod bent into the form

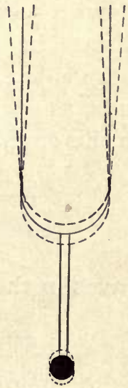

FIG. 79.

of a U, and having a heavy shank attached at its middle point. The vibrations are exactly analogous to those shown in Fig. 80, which represents the vibrations of a straight rod having two nodes. In the tuning-fork the two nodes are closer together, owing to the weight at the middle of the rod, which can be imagined bent into the shape of a U.

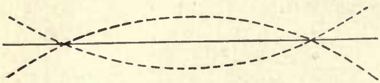

FIG. 80.

93. Vibrations of a Column of Air.—If a tuning-fork is held over a bottle or tube of suitable size (see Article 84), the column of air will be set in longitudinal vibrations; similarly, if one blows across the mouth of the bottle or tube, the air is set in vibration. This is illustrated by whistles, horns, flutes, organ-pipes, and all wind instruments. In the common organ-pipe, which is open at both ends, the air is forced across an opening by means of a bellows and sets the air in the pipe vibrating. (See Fig. 81, which shows a section of one.) The column of air may vibrate in many ways. The simplest is when there is but one node; for there are loops at the ends, as each is open to the air. (See Fig. 82 *a*.) In this case, $d = l$, and $n = v / 2\,l$, where v is the velocity of waves in the air in the pipe. The next simplest case is when there are two nodes. (See Fig. 82 *b*.) Then, $d = l / 2$, and

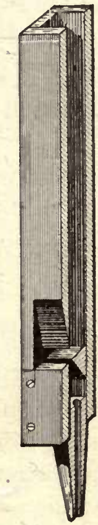

FIG. 81.

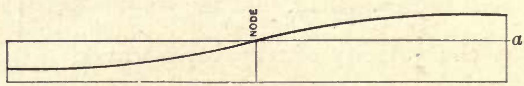

FIG. 82*a*.

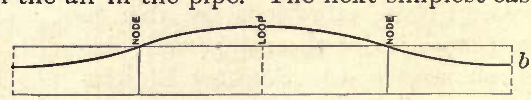

FIG. 82*b*.

$n = v/l$; that is, the frequency is twice that of the previous or fundamental vibration. If there are three nodes, $d = l/3$, and $n = 3v/2l$; etc. etc. If an opening is made in the side of the pipe, there must be a loop there; and so the frequency is changed, as in a flute or flageolet.

"Reed-pipes" have a stiff metal lip closing the opening between the bellows and the column of air; and, as the lip vibrates, it admits puffs of air which maintain the vibrations of the column of air in the pipe. In the playing of horns the human lips take the part of the metal lip.

94. **Vibrations of Plates, Bells, etc.**—The vibrations of metal plates, drum-heads, etc., may be studied by sprinkling light sand over the surfaces. It will gather into certain lines—called "nodal lines"—where there is no motion of the plate or membrane. The vibrations of a bell may be studied by suspending pith-balls in contact with different points; there will be found to be nodes and loops at regular intervals.

95. **Human Voice.**—The human voice is due to vibrations of various portions of the mouth and throat, and of the air in the cavity of the mouth. Vowel sounds, such as "a," "ah," "ee," etc., are due in the main to vibrations of the air in the cavity of the mouth, as is evident when these sounds are whispered; but, if spoken or sung, the sound is modified by the vibrations of the membranes of the larynx, which impress upon the sound a definite pitch. This pitch may be regulated at will, by making the membranes of the larynx more or less rigid by means of the muscles in the throat.

96. **Velocity of Compressional Waves.**—The velocity of the waves, which produce sound when they reach the ear, may be measured in many ways. If the waves are in air, there are two good methods. One is the direct one of hav-

ing the observers stationed a known distance apart, and then measuring the time required for the waves to pass from one to the other. For instance, one observer may fire a pistol at a noted time; the other may observe the time when he hears it. (Or the waves may be reflected by a wall, and the time of return of the echo noted.) The other method is to measure the frequency of the vibrations of air in an open organ-pipe. Then, since, as was shown, $n = v / 2l$ if the vibration is the fundamental, v, the velocity, can be deduced. A known frequency can be obtained by altering the length of the column of air until the pitch of the sound is the same as that of a standard tuning-fork whose frequency is known; for, if two sounds have the same pitch, the frequencies of the two vibrating bodies are the same. (This experiment must be slightly modified in practice, owing to the fact that the loop at the end of an open organ-pipe is not exactly at the end, but slightly outside.)

The velocity of compressional waves in other gases may be found by a modification of the second method just described; and their velocity in solids and liquids may also be easily determined. (See Ames's "Theory of Physics," p. 183.)

TABLE IV

VELOCITY OF SOUND

CM. PER SEC.

Air,	0° C.,	33,250	Brass,	350,000
Hydrogen,	0° C.,	128,600	Glass,	506,000
Water,	4° C.,	140,000	Iron,	509,000

CHAPTER VI

NATURE OF HEAT

97. Sources of Heat.—If one exposes his hand to the sun's rays, or puts it near a fire, a definite sensation is felt, known as the "sensation of heat," whereas, if the hand is put near a block of ice, there is a different sensation, known as the "sensation of cold." One knows from experience, too, that under conditions such that the hand would receive sensations of heat or cold, material bodies would experience certain effects—such as change in volume, melting, freezing, etc. These are called "heat-effects." Further, the name "source of heat" is given the object or operation which produces heat-effects: the sun, a fire, friction, sudden compression of a gas, etc., are "sources of heat."

98. Temperature.—The hot and cold sensations which our hands experience are due to what is called our "temperature" sense. If we put a hand in succession into two vessels of water, we can probably notice a difference of sensation; and we say that "one is warmer than the other," or that "the temperature of one is higher than that of the other." The heat-effect which is most easily noticeable to us depends upon our temperature sense; viz. if a material body is exposed to a "source of heat," it becomes "warmer," in the sense just explained, or, in other words, its "temperature becomes higher." In studying heat-effects, it is important for us to have some method of giving a numerical value to the temperature, so that we can express changes in temperature numerically, and compare these quantities with other changes, such as those of volume. The method of giving a number to temperature depends upon the following facts: If a glass bulb with a capillary stem is placed with its tube horizontal, and if a small drop of mercury is inserted in the tube, we have an instrument which responds immediately to any source of heat or cold, owing to the change in volume of the air in the bulb as indicated by the motion of the drop of mercury. (The pressure of the enclosed air equals that of the atmosphere outside, which is supposed to remain constant during the experiment; and the instrument is called a "constant-pressure air-thermometer.") The volume of the bulb and of each centimetre along the stem can be considered known from previous measurements. If this thermometer is placed in a mixture of ice and water, the air in the bulb will assume a definite volume (v_0), which is found, by repeated experiments with the same instrument, to be always the same. This is equivalent to saying that the temperature of a mixture of ice and water is always the same. If the thermometer is placed next in a bath of steam boiling off from water, the air in the bulb will assume a greater volume (v_1); and by repeated trials, it is found that this volume is always the same with the same instrument *if the atmospheric pressure is the same*, e. g. if it is equivalent to 76 cm. of mercury. Let the volume v_1 be measured, then, under this condition of pressure. Now place the thermometer in the space for whose temperature a number is desired, e. g. a vessel of water; measure the volume of the air in the bulb, and call it v. We can agree to choose numbers for temperatures, such that changes in temperatures are

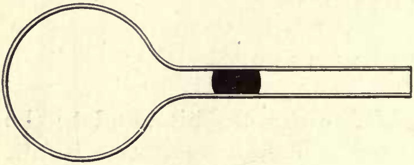

FIG. 83.

proportional to these changes in volume of the air-thermometer. Thus, call, for a moment,

t_0, the temperature of ice and water;
t_1, the temperature of steam at 76 cm. pressure;
t, the temperature of the vessel of water.

Then $$t_1 - t_0 : t - t_0 = v_1 - v_0 : v - v_0,$$

or $$t = t_0 + \frac{(v - v_0)(t_1 - t_0)}{v_1 - v_0}.$$

We can give any arbitrary numbers we wish to t_0 and t_1; and then we have a definite number for t corresponding to a definite volume (v) of the air in the bulb of the thermometer. On the Celsius or Centigrade scale, t_0 is put 0; and t_1, 100. Hence

$$t = 100 \frac{v - v_0}{v_1 - v_0},$$

and the temperature of the vessel of water is called t degrees Centigrade.

FIG. 84.

On the Fahrenheit scale t_0 is put 32; t_1, 212;

hence $$t = 32 + 180 \frac{v - v_0}{v_1 - v_0}.$$

On the Réaumur scale t_0 is 0; t_1 is 80;

hence $$t = 80 \frac{v - v_0}{v_1 - v_0}.$$

If some other fluid than air had been used in the thermometer, a different numerical value for t would, in general, have been obtained; and there is no fundamental reason why one fluid should be preferred to another. There is, however, one particular advantage in favor of a gas. (See Article 105, GASES.) Only, if one system based on the use of a definite substance is adopted, it must be used always in stating comparative results. Mercury-in-glass-thermometers (see Fig. 84) are used for ordinary measurements, because they have so many practical advantages. Their readings do not give, however, the true numerical values for the temperature, but only approximate ones, meaning by the "true" values those given by a constant pressure air-thermometer.

We are thus able, not to measure temperature, but to give it a numerical value.

99. Connection Between Heat-Effects and Energy.— If we consider various heat-effects and the methods by which they are produced, it is at once evident that in every case energy is given the minute portions, the molecules, of the body which undergoes the change. Thus, if two pieces of metal are rubbed together, or if a paddle is turned rapidly in water so as to make currents in the water and thus cause friction, work is done, and heat-effects, viz. rise of temperature, are produced. If two pieces of ice are rubbed together, work is done, and the ice melts. If a gas is suddenly compressed, work has to be done, and the temperature is raised. If a piece of lead is hammered, it is deformed permanently, work has been done, and the temperature is raised, etc., etc. In a flame or fire there are energy changes of the molecules of the burning gas; they "combine" with the oxygen of the air, and, as a result, the temperature is raised. These heat-effects are all due to the transfer of the energy to the minute portions of matter which make up the body showing the heat-effect. This is equivalent to saying that heat-effects are due to the minute portions of matter receiving energy; and we should expect, therefore, that the work done in producing the heat-effect was proportional to the quantity of matter which undergoes the change. All experiments are in accord with this idea. It is, therefore, important to know how much work is required to cause a definite amount of matter to experience a given heat-effect, e. g. melting from the solid to the liquid state. This experiment cannot be performed directly with any accuracy; but it can be indirectly. It is first determined that to make one gram of the substance melt requires an amount of heat-energy which if spent in raising the temperature of water from 10° to 11° C. would heat *m* grams. Then, one measures the amount of work, i. e. the number of ergs (see Article 45) which is required to turn a paddle in a cylinder containing a known mass of water until the temperature is raised a measured number of degrees. It is found that to raise the temperature of 1 gram of water from 10° to 11° C. requires the expenditure of 4.2×10^7 (i. e. 42,000,000) ergs. This number is called the "mechanical equivalent of heat;" and the amount of energy is called the "calorie." Thus, *m* calories are required to melt 1 gram of the solid.

CHAPTER VII

TRANSFER OF HEAT-ENERGY.

100. Introductory.—If two bodies of different temperatures are placed in contact or near each other, it will be noticed that in time they will be at the same temperature; the one at the higher temperature will have had its temperature fall, the other will have had its temperature rise. Since rise in temperature is due to the addition of energy (unless there are internal redistributions of energy), and fall in temperature to loss of energy, this shows that, if two bodies at different temperatures are left to themselves, the one at high temperature loses energy, the other gains it. (This is perfectly analogous to the fact that a fluid flows from high to low pressure.) There are several ways in which bodies gain or lose energy, as shown by heat-effects. These arc called Convection, Conduction, and Radiation.

101. Convection.—An illustration of convection is given when a pail of water is placed over a fire. The temperature of the water at the bottom of the pail is raised, the water expands, i. e. its density becomes less, and, consequently, it rises towards the top, owing to the action of gravity, which always makes the less dense liquid float above a denser liquid. As the hot water rises, it sets up currents, warms the upper portions of the liquid by contact, and also drives them down near the bottom, where they, in turn, are warmed. It will be seen that convection can take place only in a *fluid* which is warmed at the *bottom*, and that the direct cause of the process is gravity. Winds are largely due to convection currents in the atmosphere; the draught of chimneys and systems of hot-water heating depend upon convection.

102. Conduction.—An illustration of conduction is given when one end of an iron poker is put into a fire. The temperature of that end is raised rapidly; and, after some time, it may be noticed that the temperature of points some distance from the fire is rising. The rate at which this rise takes place measures the conducting power of the body. Silver is the best conductor, then come copper and aluminium; woods, woollen cloths, paper and glass are very poor conductors. The conduction in fluids (except liquid metals) is very small.

The process of conduction consists in the passing on of energy from the hot end of the body to the neighboring portions. One can picture the molecules at the hot end vibrating faster and with increased amplitude, and so causing the molecules next them to vibrate faster, etc.

If a piece of wire gauze is lowered over a flame, the temperature of the gas is so lowered by the conduction away of the energy that it falls below the temperature of combustion, and so there is no flame above the gauze, only cold gases rising. A piece of metal often appears colder to the touch than a piece of wood, even though they are at the same temperature, because the metal conducts away the heat from the hand or body so rapidly.

103. Radiation.—An illustration of radiation will be given if a mercury thermometer, sealed in a glass bulb, out of which all the air was exhausted, is brought near a block of ice. The temperature of the thermometer will be observed to fall; hence, energy has left it. But by what process has this taken place? There can be no question of convection or conduction, for all ordinary matter has been removed from the bulb surrounding the thermometer. There must, therefore, be some *medium* in the bulb which carries energy —this is called "the ether"—and the process by which the energy is carried through it is called radiation, and will be shown later (see Art. 135) to be wave-motion. In order to have waves, a vibrating centre is necessary. We can therefore picture the process as follows: The particles of matter making up the thermometer are making minute and very rapid vibrations; these cause waves in the ether, which pass across the space to the glass walls of the bulb; a certain proportion of these waves are reflected, some are transmitted, and the others are "absorbed" by the walls. By "absorption" is meant that the energy is taken away from the ether-waves, and is gained by the molecules of the body which absorbs the waves—the ether-waves will set in vibration those molecules of the body whose frequency of vibration is the same as that of the waves, on the general principle of resonance (see Art. 84).

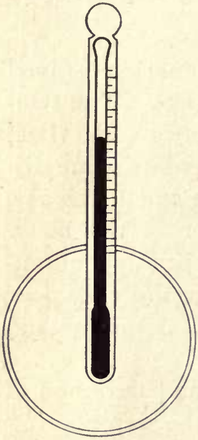

FIG. 85.

It is evident that the amount of energy radiated by a body—and all bodies do radiate energy—depends on the body itself, not on what becomes of the energy. Thus, if two bodies are put near each other, and radiation takes place, each will lose energy by emission, and each will gain energy by absorption; and the reason why the one at higher temperature has its temperature fall is because it loses more energy than it gains. There is thus seen to be some connection between radiation and temperature. The higher the temperature of a given body, so much the more intense is the radiation. Further, the waves emitted by a body are of different wave-numbers depending on the frequencies of the molecules of the body, which are vibrating; and it is found by experiment that, as the temperature is raised, there are waves of greater and greater wave-number emitted (i. e. of shorter and shorter wave-length). This is shown roughly by the fact that, if the temperature of a piece of iron is raised, it will finally appear colored to our eyes; and, as will be shown later (see Art. 136), the immediate cause of the sensation of color is the presence of ether-waves of immensely great wave-number.

To measure the energy carried by these ether-waves, some instrument like a thermometer must be used, which will absorb the energy of the waves. But, of course, heat-effects are not the only things which these ether-waves can cause. When they fall upon the human eye, they produce color-sensations, if their wave-number lies between certain limits. If they fall upon certain chemicals, changes are produced, e. g. photographic action. If the waves are very long, they produce electrical effects. In other words, "heat-waves," "light-waves," "chemical waves," "electrical waves," are all names for the same thing, viz. ether-waves, and are used to emphasize one feature of the various effects which are produced when the waves are absorbed. These waves—of all lengths—may be reflected, refracted, etc., as will be shown more fully in treating Light.

104. Absorption.—As shown above, absorption of the energy from ether-waves takes place when the vibrations of the matter have the same frequency as the waves. In other words, a body absorbs waves of the same frequency as those it would emit. (This energy which is thus absorbed is not all spent in increasing the vibrations of the molecules—as would be the case if it were simply a phenomenon of resonance; the energy is spread by conduction, etc., through the whole mass, and there are various internal changes. The energy in time becomes associated with the *parts* of the molecules, i. e. the "atoms.") Thus the word "transparent" has no definite meaning unless it is stated what the frequency of the waves is which pass through the body.

Ordinary glass is transparent to waves which produce sensations of color, but non-transparent to longer waves.

This explains the action of a "greenhouse" or "conservatory." The waves from the sun fall upon the glass roof, and the shorter waves —those which produce the sensation of light—are transmitted; these reach the ground and are absorbed. The temperature of the ground rises slightly; but the waves it emits are so long that they cannot pass through the glass; they are reflected and absorbed by the ground, etc. Thus, the energy which comes through the glass roof is "trapped" and cannot escape.

Similarly, on a cloudy night the energy stored up in the ground during the day is radiated from the surface, but is reflected by the clouds; or absorbed by them, in which case they emit radiation back towards the earth. Consequently, the temperature does not fall so rapidly as on a clear night.

The air absorbs some of the energy of the waves from the sun as they pass through it; but the rarer the air, the less the absorption. Thus, if a person is exposed to the sun's rays at a high altitude, the heat is intense; but, if he is in the shade, the cold is intense, because there are no large bodies near, which have absorbed the energy from the rays and are in a condition to emit radiation.

A body which is polished absorbs but very little radiation and reflects most of it; whereas, if it is blackened, e. g. with lampblack, it absorbs most of the radiation of short wave-lengths and reflects little. A blackened body must therefore emit a greater amount of radiation at a given temperature than a polished one. This fact is illustrated by the blackening of stoves, boilers, etc.

CHAPTER VIII

HEAT-EFFECTS

105. Change of Volume.—If, as the result of transferring energy to the molecules of a body, i. e. by warming it, its temperature rises, the volume in general will also change—it will increase, except in a very few cases.

If the volume is measured at 0° C., and again at t° C. (as determined on a constant-pressure air-thermometer), it is found (except in certain cases) that the change in volume is proportional to the change in temperature, to within a fair degree of exactness. Thus, if v_0 is the volume at 0° C.; v at t° C.; it is found that $v - v_0 = \beta v_0 t$, or $v = v_0 (1 + \beta t)$, where "β" is a constant for any one kind of matter (e. g. iron), and is called the "coefficient of cubical expansion;" or, it is the change in volume which one cubic centimetre of the substance would experience if its temperature were raised from 0° to 1° C. For solids β is small; for liquids it is larger; and for gases still larger.

If the body expanding is a cube, each of whose edges has the length l_0 at 0° C., and l at t°; $v_0 = l_0^3$, $v = l^3$;

hence $$l^3 - l_0^3 = \beta l_0^3 t,$$

or $$l^3 = l_0^3 (1 + \beta t).$$

Hence $$l = l_0 (1 + \beta t)^{\frac{1}{3}}$$

$$= l_0 \left(1 + \frac{\beta}{3} t\right),$$

if β is very small in comparison with 1, as it is in the case of solids. Hence the "coefficient of *linear* expansion" is one-third that of cubical expansion.

Different solids expand at different rates, i. e. β is different; and the fact may be made use of in constructing "compensating pendulums" (see Fig. 86), in which, as the pendulum lengthens with rise of temperature, there is an expansion produced upward of some bar fastened to the bottom of the pendulum, so that the centre of gravity remains unchanged. Illustrations of expansions of solids are given by the cracking of tumblers when dipped in hot water, the change in length of metal bridges and rails, by the fact that wagon-tires are always raised to a high temperature and then shrunk on the wooden parts, etc.

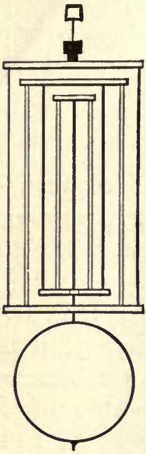

FIG. 86.

Liquids.—Different liquids expand differently also; and one liquid—water—behaves irregularly as the temperature is raised. Starting with a definite quantity of water at 0° C., its volume *decreases* as the temperature *rises*, until 4° C. is reached; after which the volume increases with rise in temperature. In other words, the density of water is greater at 4° C. than at any other temperature.

This fact is most important in the economy of Nature, because it is the direct cause of the formation of ice on the surface of a lake or pond instead of on the bottom. As the temperature of the water in the lake falls, the densest water goes to the bottom, the lightest to the top; hence water at 0° C. will float on that at 4° C.; and so, when the ice is formed, it is on the surface.

The fact that the volume of a liquid, and therefore its density, changes shows that a barometer (see Article 79) responds to changes of temperature as well as to those of pressure. Consequently, if an exact knowledge of the change in pressure is desired, it is necessary to make correction for the effect of change in temperature.

Gases.—In measuring the expansion of gases when the temperature is raised, it must be noticed that the change in volume depends also upon conditions of pressure. If the pressure is kept constant, it is found that the change in volume obeys a most simple law: if a certain quantity of gas has the volume v_0 at 0° C. (the temperature of a mixture of ice and water), and v at 100° C. (the temperature of steam coming off a surface of water at a pressure of 76 cm. of mercury), the ratio $\frac{v - v_0}{v_0}$ is *the same for all gases, and for all quantities of the gas.* (The apparatus by which this was proved is shown in Fig. 87.) Expressed in

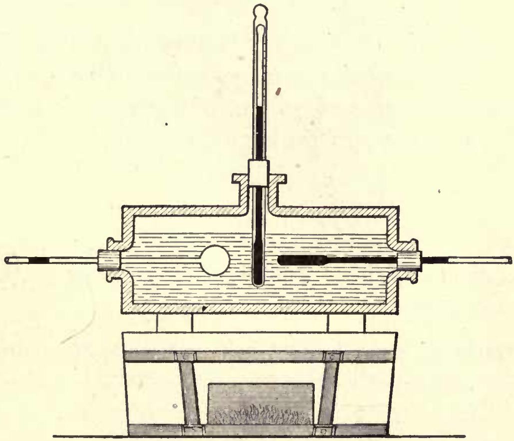

FIG. 87.

terms of the previous formula $v = v_0 (1 + \beta t)$, we have $v = v_0 (1 + 100 \beta)$; hence $\frac{v - v_0}{v_0} = 100 \beta$. Consequently, β is the same for all gases. (The formula $v = v_0 (1 + \beta t)$ is not *exactly* true; but the deviations from truth are very small for a gas.) The value of β for all gases is very nearly 0.003663, or $\frac{1}{273}$. (The gas must be perfectly dry.)

If the pressure and temperature both change, the resulting change in volume may be determined thus: Let the gas be at pressure p and temperature 0°, and let its volume be called v_0; keep the pressure constant, and change the temperature to t°, the resulting volume will be v, where $v = v_0 (1 + \beta t)$; now keep the temperature constant, and change the pressure from p to P, the volume will change from v to V, where $PV = pv$. (Boyle's law. See Article 78.) Hence

$$PV = pv_0 (1 + \beta t),$$

or

$$V = \frac{pv_0}{P} (1 + \beta t).$$

If the volume (V) of a gas is measured at t° C., and at a pressure of h cm. of mercury, the volume it would have at 0° and at a pressure of 76 cm. of mercury is seen on substitution in the above formula to be

$$v_0 = \frac{h}{76} \quad \frac{V}{1 + 0.003663\, t}.$$

This is known as the "corrected" volume of the gas, or its "volume under standard conditions."

If the volume of a gas is kept constant while the temperature is changed, the pressure will change. Thus, if in the above formula $V = v_0$,

$$P = p(1 + \beta t),$$

or

$$\beta t = \frac{P - p}{p};$$

i. e. the coefficient of increase of pressure, the volume being constant, is the same as that of expansion at constant pressure. An instrument, therefore, which measures the pressure of a gas at constant volume, as the temperature changes, can be used to determine the value of t, the temperature, if the pressure is known at 0° and 100° C. Such an instrument is called a "*constant-volume* air-thermometer," and one form is shown in Fig. 88.

If $t = -\frac{1}{\beta}$, i. e. if $t = -273°$ C., $P = 0$,

if the volume is constant; or $V = 0$, if the pressure is constant. Neither statement has any meaning; as, in fact, all gases would be liquefied at temperatures as low as $-273°$ C. But this number, 273, is sometimes called "absolute zero" on the Centigrade scale. Thus, 0° C. is called "273° absolute;" 100° C., 373° absolute, etc.

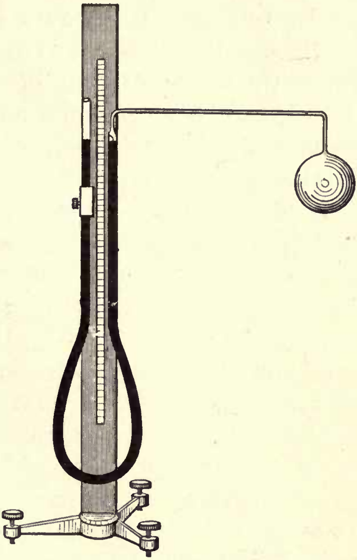

FIG. 88.

106. Change in State.—One class of heat-effects of most common occurrence is that of change from solid to the liquid state; liquid to the gaseous state, etc. Illustrations of this are melting, solidifying, boiling, condensation, etc.

Fusion.—"Fusion" is the process of passing from a solid into a liquid state. If a piece of lead is warmed continuously over a fire, its temperature will slowly rise until it begins to melt; but, as long as the melting goes on, the temperature remains unchanged; then, after it is all melted, the temperature will again rise. (Some bodies do not melt at a fixed temperature, but melt gradually, becoming soft, as the temperature rises, e. g. various waxes, plumber's solder.) This temperature at which a solid melts is, in general, the same as that at which the liquid would begin to solidify; it is, in fact, the temperature at which the solid and liquid can exist together in equilibrium, and is called the temperature of fusion, or the "melting-point."

It is often necessary to have a nucleus for the solid to form around when solidification is desired. This is specially true if the solid is a crystal; in which case the nucleus should be a crystal of the same shape as that of the ones to be formed. If pure water is cooled below 0° C., as it easily can be by means of a freezing mixture, it will freeze instantly if a small bit of ice is thrown in. (Similarly, "rock candy" is formed around strings which dip into a saturated solution of sugar in water.) Violent shaking or jarring often hastens solidification.

At fusion the volume of the substance changes. As ice melts, the volume decreases; similarly, "cast iron" contracts; but when lead, gold, silver, etc., melt, their volumes increase. Conversely, as water freezes it expands; but, as lead, gold or silver solidify, they contract. (For this reason gold and silver coins are not "cast," but are stamped.) Since ice contracts on melting, an increase in pressure would aid the process by making the volume less. If, therefore, the pressure on a piece of ice is made very great, it will not be necessary to have a temperature so high as 0° in order to make it melt. That is, the "melting-point" is lowered by the increase in pressure. Similarly, the melting-point of lead would be raised by an increase in pressure, because this would retard the action. (The change of the melting-point is, however, very small, e. g. in the case of ice, 0°.007 C. for an increased pressure of one atmosphere, i. e. 14.7 lb. to the sq. inch.) But for a fixed pressure there is a definite temperature of fusion.

This effect of change in melting-point is very easily shown in the case of ice. If two pieces of ice are pressed together (or a snowball squeezed tight), the *pressure* may be enormous, owing to the small surface of contact of the points which meet. The temperature of the ice and of the film of water over it is 0° under the atmospheric pressure; therefore, at the point where the pressure is so great, the ice, being at 0°, is higher than its fusion-point, which is possibly — 0.°01 C., and it will melt; the water thus produced flowing out around the point, at a temperature lower than 0°. Being relieved from the pressure, the water will now freeze again. This process is called "regelation;" the motion of glaciers depends largely upon it.

In order to make a solid melt, it must be warmed continuously; that is, energy must be added to it from the source of heat. The number of calories (see Art. 99) that must be added in order to melt one gram of a substance, the temperature remaining constant, is called the "latent heat of fusion" of that substance at that temperature. For instance, the latent heat of fusion of ice at 0° is 80; which means that the heat required to melt 1 gram of ice at 0° would raise the temperature of 80 grams of water from 10° to 11° C.

This quantity is determined by placing a known mass of ice in a known mass of water and noting the fall in temperature of the water; for, to make one gram of water fall in temperature one degree centigrade, one calorie must be taken away, the quantity of energy required to change the temperature of water one degree being nearly the same at all temperatures.

Conversely, if a liquid solidifies, energy is given out by conduction or radiation; and, if 1 gram solidifies, a number of calories equal to the latent heat of fusion is emitted.

An illustration of these facts is given by "freezing-mixtures," e. g. common salt and ice. A solution of salt in water can be made to solidify as a *homogeneous mixture* at a temperature which depends upon the nature of the salt; e. g. this temperature for common salt is —22° C. Therefore, if salt and ice are thoroughly mixed at 0°, the mixture is at a temperature higher than its melting-point; and so the ice will melt, and the salt will dissolve in the water. For each gram of ice melted, 80 calories of energy are taken from the surrounding bodies; and, for each gram of salt dissolved, a certain number of calories is absorbed also; therefore, the temperature of surrounding objects is lowered. In general, however, if a solution of a salt in water freezes, pure ice is formed, and the salt is left in the water which is unfrozen, or held in between portions of the ice. The freezing-point is not, however, that of pure water, but is always lower than 0° C.

Evaporation and Condensation.—Similarly, if a liquid is warmed continuously, its temperature will rise until it begins to "boil," that is, until bubbles of its vapor are formed in the liquid, rise to the surface, and break. As long as this process continues, the temperature remains constant, unless the pressure on the liquid changes. If the pressure increases, the temperature of boiling, i. e. the "boiling-point," rises; if it decreases, the boiling-point is lowered, as would be expected, because a greater force has to be overcome if the pressure increases, and *vice versa*. But for a definite pressure there is a definite boiling-point for any liquid.

At a pressure of 76 cm. of mercury the boiling-point of water is, by definition, called 100° C. If the pressure is lowered, as on a mountain-top, the boiling-point is lowered; so that many articles cannot be prepared for eating. But, if the pressure is greater than 76 cm. of mercury, the boiling-point is raised. The change in the boiling-point of water near 100° is 0°.1 for an increase in pressure of 2.68 millimetres of mercury. Boiling consists in the formation of bubbles filled with the vapor of the liquid. Nuclei of some kind are required (see Art. 74); otherwise, the liquid would never boil, but would explode finally as the temperature rose. (Boiling takes place with "bumping," if the supply of nuclei is nearly exhausted; it is a miniature explosion.) Further, the pressure of the vapor inside the bubble must be equal to (or slightly greater than) the pressure on the surface of the liquid. A thermometer immersed in the vapor rising from the liquid as it boils measures, then, the temperature of the vapor which is in equilibrium with its liquid at the pressure under which the boiling takes place. The experiments mentioned above show that for a definite pressure there is a definite temperature of equilibrium. If a solution of salt in water is made to boil at a definite pressure, practically pure water vapor is formed, but the temperature of boiling, as measured by a thermometer in the liquid, is higher than that for pure water.)

This fact as to the connection between pressure and temperature of equilibrium of a pure liquid may be shown also by a different experiment. If a few drops of the liquid are inserted under the open end of a mercury barometer tube dipping in a basin of mercury, they will rise up through the mercury and float on top (see Fig. 89). Some of the liquid will evaporate, and thus there will be a pressure downward on the mercury, which can be measured by the difference between the height of the mercury column in this tube and that of a barometer. ($p = \rho g h$.) The vapor which fills the space above the liquid is called "saturated" vapor and is in equilibrium with the liquid; if a portion of liquid evaporates, an equal mass of vapor condenses. Experiments show that the pressure of this vapor is constant, so long as the temperature is kept constant, entirely regardless of the mass of the liquid or the volume occupied by the vapor. If the tube is lowered in its tank so as to make the volume of the vapor less, some of it will condense and be liquefied; while, if the tube is raised so as to increase the volume of the vapor, some of the liquid will evaporate. (See Fig. 90.) If the temperature is changed, the pressure of equilibrium is altered. If the temperature is raised, some of the liquid evaporates, but, if it is lowered, some vapor condenses.

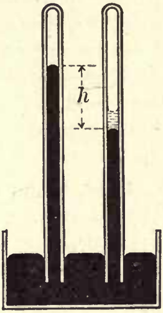

FIG. 89.

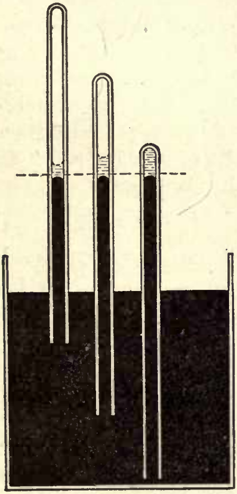

FIG. 90.

Liquefaction.—There are thus two methods of making a vapor condense into a liquid: one by lowering the temperature, the other by decreasing the volume. (There is no essential difference between what are ordinarily called gases and vapors. A "saturated vapor" is matter in the gaseous state which is in equilibrium with its liquid, as just explained; but a vapor is simply another name for a gas when it is in such a condition that it can be liquefied easily.) If the temperature of any gas is lowered sufficiently and if its volume is then made smaller, it may be liquefied. It is found, however, that there is a certain "critical" temperature for each gas, below which it must be before diminution of volume will produce liquefaction. (The critical temperature of water is 365° C.; of oxygen, − 119° C.; of nitrogen, − 146° C.; of carbonic acid gas, 31° C.; etc.)

There are three ways of lowering the temperature of a gas: one is to surround it by a freezing mixture; another is to surround it by some very cold liquid; and another is to compress it and then allow it to expand. In the last case, the gas which causes the expansion does work in expelling the expanded portions, producing violent currents; it therefore loses energy and suffers a fall in temperature. If the gas is under a high pressure before it is allowed to expand, the fall in temperature may be sufficient to liquefy the gas which remains behind. In fact, hydrogen, oxygen, air, and all gases may be liquefied in this manner.

Dew, Fog, etc.—One of the most common illustrations of condensation of vapor is dew. It is the formation of water from the moisture present in the air in its gaseous condition. The air contains, generally, the water-vapor as a gas, not as saturated vapor. But as the temperature of the ground is lowered owing to radiation, that of the air falls also, the amount of the water-vapor remaining constant; and a temperature is obtained finally at which the vapor is saturated, i. e. the temperature of saturation for the existing pressure of the water-vapor has been reached. This is called the "dew-point." If the temperature is lowered still more, some of the water-vapor will condense; and dew is formed. Water-vapor is a transparent gas; but if chilled it condenses into drops and forms a cloud, mist, fog, etc., thus becoming visible. If a woollen blanket is suspended in a damp place, moisture will deposit on it more quickly than elsewhere, because at a given temperature vapor will condense in capillary spaces at a less pressure than on a plane surface, if the liquid wets the walls of the body. The importance of nuclei for the drops to form on has been referred to in Article 74.

In order to make a liquid pass into the form of vapor, heat is required; as is shown by the fact that a liquid can be made to boil by being warmed continuously over a fire, by the cold felt when there is evaporation of liquids from the skin, by the fact that by rapid evaporation a liquid becomes frozen, the energy required for the evaporation being taken from the liquid left behind, etc. The number of calories required to make 1 gram of a substance pass from the liquid to the gaseous state at a definite temperature is called the "latent heat of evaporation" of that substance at that temperature. For water at 100° C. it is 536; i. e. the energy required to make 1 gram of water boil at the temperature of 100° C. would be sufficient to raise the temperature of 536 grams of water through one degree centigrade.

If a small amount of water is dropped lightly upon a metal surface which is at a temperature much higher than 100°, the drop of water will send out vapor so rapidly that there will be a cushion of it between the drops and the metal surface. The drop will therefore be suspended, and will evaporate without being in contact with the plate. This is called the "spheroidal state," and is illustrated by drops of water running over a hot stove.

107. Change in Temperature; "Specific Heat."— Substances vary in the amount of heat they require in order to experience a rise of temperature, and, therefore, in the amount they give off when their temperature is lowered. The number of calories required to raise the temperature of one gram of a substance from $t°$ to $(t+1)°$ is called the "specific heat" of that substance at the temperature $t°$. This quantity is different for different temperatures, but the variations are slight, and may be neglected in ordinary use. Thus, if the temperature of a body whose mass is m and whose specific heat is c is raised through t degrees centigrade, a number of calories equal to mct must have been added; similarly, if it falls t degrees, mct calories must be taken away.

This suggests at once a method of determining the specific heat of a solid or a liquid. Let its mass be m, and let it be at the temperature $t_0°$; then put it into a vessel of water, the mass of the water being M, and its temperature $T_0°$; the two bodies will finally come to the same temperature; call it $T°$. Then, if no energy has been received from surrounding bodies, and if none has escaped, the heat-energy lost by the substance inserted in the water, i. e. $mc(t_0 - T)$ calories, must equal that gained by the water, i. e. $M(T - T_0)$ calories, if it is assumed that the amount of heat required to raise the temperature of water one degree, anywhere between 0° and 100°, is one calorie—which is nearly true. That is,

$$mc(t_0 - T) = M(T - T_0).$$

In practice, allowance must be made for the heat received or lost by the vessel which contains the water; but this can be done easily. A cut is given of a simple form of "calorimeter," as such a vessel is called. (Fig. 91.)

Another method to determine the specific heat of a metal is to raise it to a temperature $t°$, then place it in a vessel surrounded by ice, so as to melt some of it. If M grams of ice are melted at 0°, 80 M calories are required; and as the metal has had its temperature lowered to 0° from $t°$, it has lost mct calories. Hence $80 M = mct$.

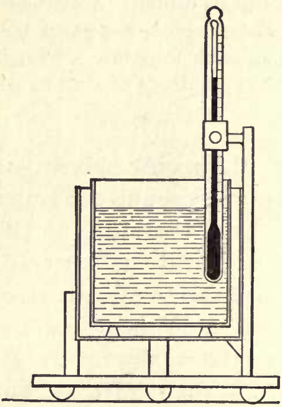

FIG. 91.

The specific heat of a substance represents not only the energy required to produce those *internal* changes which accompany changes of temperature, but also that required for *external* effects, because, in general, the bodies expand, and so force back whatever presses against them (e. g. the weight of the atmosphere). The energy is spent in general, then, in three ways: *external* work, internal work in producing *kinetic* energy, internal work in producing *potential* energy.

If other units than the C. G. S. system and the Centigrade scale are used (e. g. pounds and Fahrenheit scale), the definitions of specific heat and latent heat would have to be changed. A "thermal unit" being defined as the "heat required to raise the temperature of a unit mass of water one degree," the specific heat is the number of "thermal units" required to raise a unit mass of the substance one degree, etc. It is evident that the number for the specific heat is the same on all systems.

TABLE V

HEAT CONSTANTS

Coefficients of Linear Expansion

Brass . . .	0.000018	Platinum . .	0.000009
Glass . . .	0.000009	Steel	0.000011
Iron, cast . .	0.000011		

Coefficients of Cubical Expansion

Alcohol . . .	0.00105	Mercury . .	0.0001818

Fusion Constants

	Fusion Point	Latent Heat
Copper	1,050° C.	
Ice	0°	80.
Iron	1,400°–1,600°	
Lead	325°	5.86
Mercury	– 39°	2.82
Sulphur	115°	9.37

Vaporization Constants

	Boiling-point at pressure of 76 cm. of Mercury	Latent Heat
Alcohol	78.4 C.	209.
Chloroform . . .	61.2	58.5
Ether	34.9	90.
Hydrogen	– 243.	
Mercury	357.	62.
Oxygen	– 184.	
Water	100.	536.

Specific Heats

Brass	0.09	Iron	0.1130
Copper	0.0933	Lead	0.0315
Glass	0.20	Mercury . . .	0.0333
Ice	0.504	Silver	0.0568

CHAPTER IX

MAGNETISM

108. Magnets.—Certain minerals occurring in nature have the power of strongly attracting pieces of iron even when there are no electrical charges. (See Article 116.) It is possible, too, to make certain solid bodies have this power. All such bodies are called "magnets." Some iron ores are magnets, but they are not convenient for experiment; and so magnets are generally made artificially. Given one magnet, it is possible to make any number of others. The simplest method is to take a steel needle or bar and to draw the magnet slowly along it; the needle or bar then becomes a magnet. The following facts may now be observed.

109. Attraction and Repulsion.—A magnet attracts iron strongly, also ordinary steel; it attracts nickel and cobalt feebly; it repels bismuth and many substances *very* feebly.

110. North and South Poles.—If the magnetized needle or bar is pivoted so as to be free to turn around a vertical axis (or if it floated on a cork), it will place itself in a direction nearly north and south. The end of the needle or bar towards the north is called the "north pole;" the other, the "south pole." Every magnetic needle has thus a north and a south pole; and, if a second magnetic needle is brought near the pivoted magnet, it will be seen that "like poles repel each other," "unlike poles attract each other." This shows, too, that the earth itself has somewhat the properties of a powerful bar-magnet placed at the centre of the earth, nearly along the axis of the earth and with its magnetic south pole turned towards the geographical north pole of the earth.

111. Equality of Poles.—If a magnetic needle is placed on a cork which is floating on a tank of water, it will, as said above, turn and place itself nearly north and south; and it may be observed, further, that it will not move towards the north or south. This proves that the force pulling the north pole towards the north is *exactly* equal and opposite to that pulling the south pole towards the south. This is sometimes expressed by saying that the "strengths of the two poles of a magnetic needle are equal in amount."

112. Molecular Nature of Magnetism.—If a magnetic needle is broken in two, each of the halves will be found to be a magnet with a north and a south pole which are also of equal strength. This process may be continued indefinitely, and shows that it is impossible to have a north pole without an equal south pole attached. Therefore magnetism is undoubtedly a property of the molecules of a body.

This idea is confirmed by the fact that anything which affects the molecular structure of a magnetic needle affects its magnetism. Jarring, hammering, twisting, etc., all weaken the magnetism; so does rise in temperature.

113. Magnetic Induction.—If an iron bar is brought near a bar-magnet, it will be seen that the iron bar acquires magnetic properties. Thus, if in Fig. 92, *A* is the magnet with its north pole on the left, and its south pole on the right; and if *B* is the iron bar, it will be found that the end towards the magnet is a north pole, and the other a south

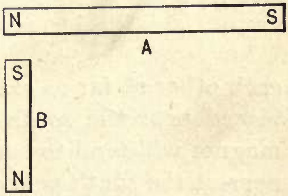

FIG. 92.

pole. This is called "magnetic induction," and explains why a magnet attracts a piece of iron: the pole of the magnet nearest the iron induces in it a pole opposite to itself, and hence attracts it. If the magnet is taken away, the iron bar loses almost all its magnetic properties. If a steel bar were used in place of the iron one, it would be magnetized also, and more of its magnetism would persist after the magnet was removed.

This phenomenon of magnetic induction in iron and steel (and nickel and cobalt also) may be accounted for if it is assumed that each molecule of iron (or nickel or cobalt) is itself a magnet. Before the iron bar is brought near the magnet, the molecular magnets may be considered as distributed at random so that north and south poles neutralize

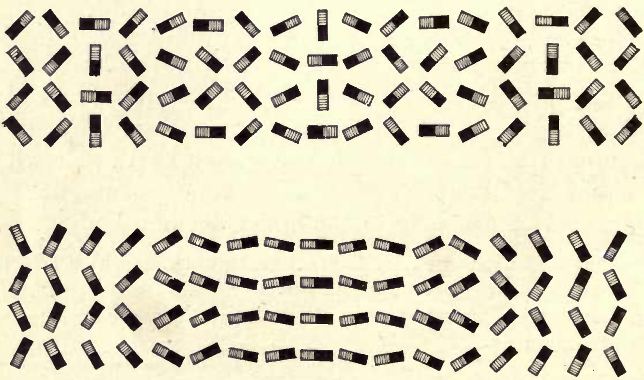

FIG. 93.

each other so far as external action is concerned; but, when placed near the south pole of the magnet, each molecular magnet will tend to turn and place itself with its north pole nearest the south pole of the bar-magnet. In other words, the molecular magnets will try to place themselves all in the same direction, so that in the body of the iron bar the poles neutralize each other's action, while at the face near the magnet only north poles appear, and at the opposite face there are only south poles. (In the figures, Fig. 93, the molecular magnets are drawn as if they were tiny bar-magnets; there is no justification, of course, for this; but the figures illustrate the idea.)

If a bar-magnet be slowly drawn along a steel bar so that the south pole touches the bar as shown, the bar will become a magnet with its north pole at the end at which the south pole ends its path, i. e. at the right end in the figure. The steel bar may be hammered with advantage as the magnet is being moved, so as to help the molecules turn and arrange themselves. The explanation is, obviously, that it is a case of induction.

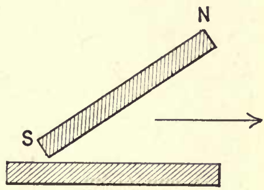

FIG. 94.

114. Lines of Magnetic Force.— Magnetic actions may be described also in terms of "lines of magnetic force" (which are analogous to the lines of force in the case of charged bodies, see Article 123). A line of magnetic force is

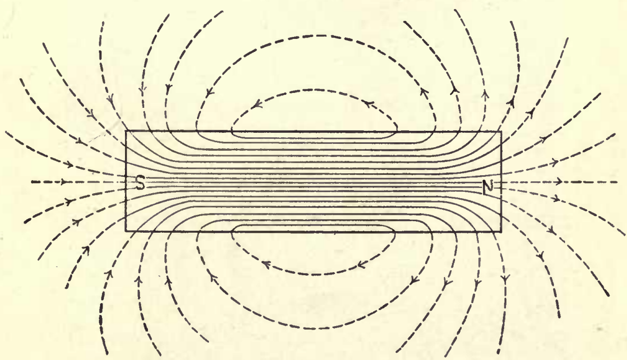

FIG. 95.

defined to be such a line that at any point its direction is that which a small magnetic needle would take if pivoted there perfectly free to turn, the north pole of the magnet indicating the direction of the line. If it were possible to get a north pole by itself, free from its south pole, it would move along a line of magnetic force. These lines may be determined by placing a small magnetic needle at different

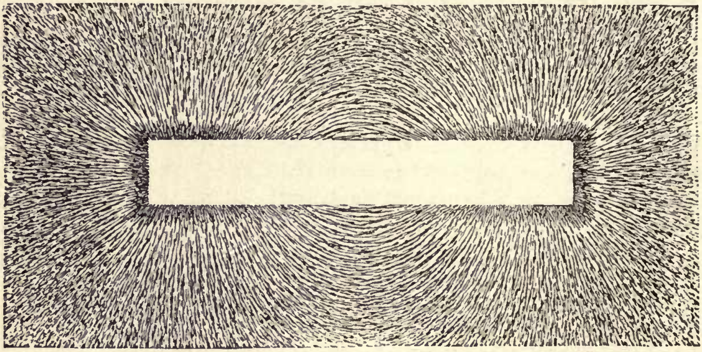

FIG. 96. BAR-MAGNET.

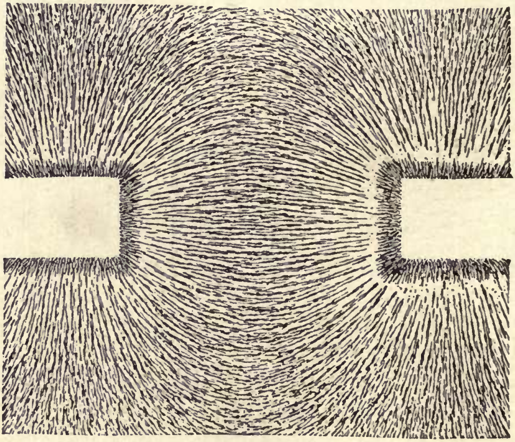

FIG. 97. TWO UNLIKE POLES.

points near a magnet and thus obtaining the direction of the lines ; or, by placing the magnet under a piece of paper or glass and sprinkling iron filings loosely over the paper or glass, in which case each minute bit of iron becomes a mag-

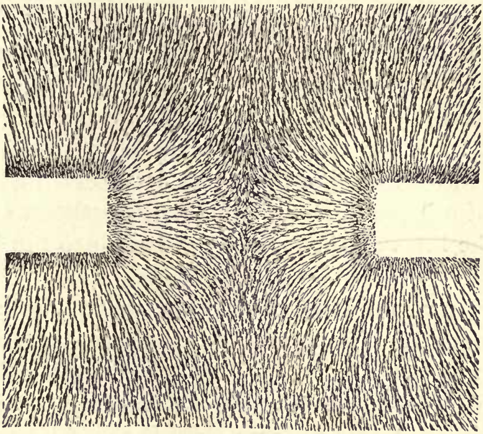

FIG. 98. TWO SIMILAR POLES.

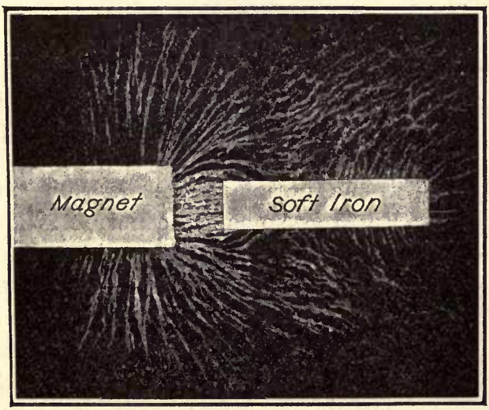

FIG. 99.

net by induction, and so will take the direction of the line of force. It is necessary to jar the paper or glass slightly so as to give the filings a chance to turn. Several illustrations are given. It will be noticed that lines of magnetic force pass in the air from the north poles to south poles; and that if the magnets were allowed to move freely, the lines would get shorter; or the action may be described by saying that they tend to contract. The fact that, when motion is not permitted, the lines of force do not actually thus shorten and come together may be described by saying that they "repel each" other sidewise. As is shown in Fig. 99, the presence of a piece of iron in the neighborhood of a magnet disturbs the lines of force; they crowd into the iron out of the air.

The region around magnets, or a space where there are lines of magnetic force, is called a "magnetic field," or a "field of magnetic force." The field is said to "increase" if the number of lines of force in a given area increase; thus, if a magnet is brought near a point, the field increases there; or, if two magnets take the place of one, the field is increased everywhere.

115. Magnetic Action of the Earth.—As shown above, the earth has the properties of a magnet; it is therefore possible to draw the lines of magnetic force due to the earth. If a magnetic needle is supported as in Fig. 100, so as to be perfectly free to turn in any way, it will point along the line of force. At any point on the earth's surface such a needle will take a definite position in a vertical plane and inclined to the horizontal. The vertical plane which includes the needle is called the "magnetic meridian;" and the angle which this plane makes with the geographical meridian is called the "declina-

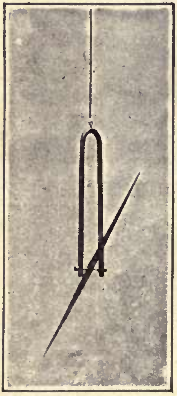

FIG. 100.

tion" or "compass variation." The angle the needle makes with a horizontal plane is called the "inclination" or "dip." The strength of the magnetic force is called its "intensity;" and the intensity, inclination and declination at any one place are called the "magnetic elements" at that place. These are all varying slightly. There are daily and hourly fluctuations; there is also a slow change of the declination and inclination which is apparently due to a gradual shifting of the magnetic axis of the earth, i. e. the direction of the hypothetical bar-magnet at the centre of the earth.

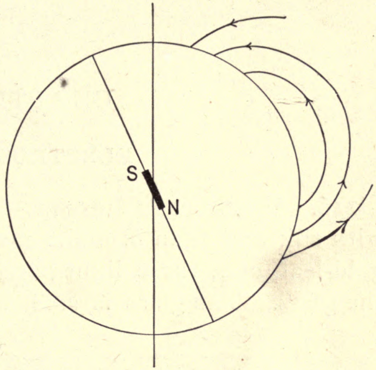

FIG. 101.

A drawing is given of a few of the lines of force which would be due to a bar-magnet at the centre of a sphere.

CHAPTER X

ELECTROSTATICS

116. Electrical Charges.—If a piece of glass is rubbed with silk and then brought near small bits of paper, dust, gold-leaf, etc., these light portions of matter move towards the glass. The glass is said to be "charged" or "electrified;" and it is said to "attract" the bits of paper, etc., using the word "attract" simply to *describe* the motion. The silk, itself, after having been rubbed on the glass, has this same power of attraction. Similarly, if any two different kinds of matter are brought closely in contact and then separated, each is charged, e. g. glass and silk, glass and cat-skin, ebonite and cat-skin, brass and silk, etc. If melted chocolate is poured into a glass tumbler, and then poured out, the glass will be found to be charged. When a stick of wood is whittled by a knife, the shavings are charged. If a glass rod which has been rubbed with silk is suspended by a

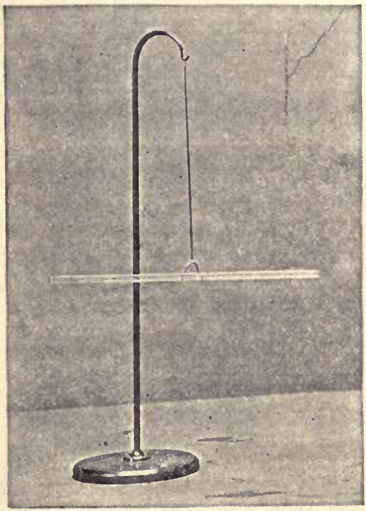

FIG. 102.

stirrup of paper as in Fig. 102, it will turn so as to move away from a piece of glass similarly charged, i. e. there is "repulsion;" but it will move towards a piece of vulcanized rubber which has been charged by cat-skin, i. e. there is "attraction." Similarly, other charged bodies may be tried; some will repel the glass, the others will attract it. The former are called "positively" charged, the latter, "negatively," to call attention to the fact that there is the same difference between their action on a charged body as between a positive and negative force. Experiments show that two "like" charges, either two positive or two negative, repel each other, while two "unlike" charges attract each other.

117. Conductors.—It may be shown that all bodies may be divided into two classes: those which manifest the forces of attraction and repulsion only near the portions of their surfaces which were originally charged by rubbing with the silk, cat-skin, etc.; and those on which the effect can be felt at all points of the body, even though but one portion was rubbed. That is, in one case the "charge" remains localized; in the other it spreads over the whole body. Bodies of the first kind are called "non-conductors" or "insulators," and glass, silk, wool, dry wood, ebonite are illustrations; bodies of the second kind are called "conductors," and all metals, water with acids or salts in solution, damp thread are illustrations. If a brass rod held in the hand is rubbed with silk, it will manifest no charge, because the charge produced has spread over the rod, the hand and body, and the whole earth; but, if one end of the rod is held in a silk cloth, and the other end rubbed with silk, it will be noticed that the rod is charged.

If an insulated charged conductor, e. g. a sphere on a glass stand, is made to touch an uncharged conductor also insulated, the latter becomes charged, as may be shown by bringing near it bits of paper, or a suspended pith-ball.

118. Gold-Leaf Electroscope.—An instrument based upon these facts is the "gold-leaf electroscope," which consists of a metal rod carrying a metal plate at one end and two light gold leaves at the other. These leaves are surrounded by a glass cylinder, so as to protect them from air-currents, e. g. by a lamp-chimney. If the plate is charged, the leaves will become charged also, since the rod, plate and leaves are all conductors; and therefore, being charged alike, the two leaves will repel each other and stand separated. If a positively charged body is brought near the plate, the leaves will either diverge more or else collapse; while, if a negatively charged body is brought near, the opposite effect is produced.

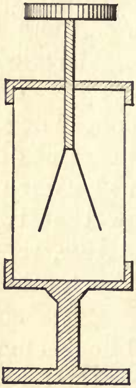

FIG. 103.

119. Positive and Negative Charges.—A "positive charge" is the name given to the condition of glass after it has been rubbed with silk; and "negative" to that of ebonite after being rubbed with cat-skin. It must not be thought, however, that a piece of glass is charged positively under all conditions; because if glass is rubbed against cat-skin it comes away negatively charged. In every case, however, when two pieces of matter are brought close together and then separated, they are found to be charged oppositely; and, if brought near an electroscope, one will produce an effect equal but opposite to that produced by the other. The charges are then *defined* as being equal and opposite, meaning simply that they produce equal and opposite effects. If they are brought near the electroscope together, without being separated, there is no effect. Therefore, whenever a charge of any kind is produced, an *equal* but *opposite* one appears at the same time. In particular, if two conductors have equal and opposite charges and are made to touch, the charges disappear; one neutralizes the other.

120. Charged Conductors.—If a hollow closed conductor is charged, e. g. a hollow sphere, or a wire cage made of fine gauze, it may be observed that no forces can be felt in the interior, i. e. "there is no charge inside a closed conductor." This can be shown by placing a gold-leaf electroscope inside a metal cage and joining it to the latter by a wire. When the cage is charged, there is no effect on the electroscope. The charge is entirely on the outer surface. This may be regarded as due to the fact that similar charges get as far apart as possible; and so a charge put on the surface will, as it were, "expand" as much as it can.

121. Electrostatic Induction.—If a small charged body, e. g. a piece of glass or brass, is suspended by a non-conducting thread (such as silk), and lowered into the hollow conductor, as shown in Fig. 104, so as not to touch it, the hollow conductor becomes charged, as may be shown by having it joined to a gold-leaf electroscope by a wire, or by bringing it near the electroscope. The conductor is charged both inside and out; on the inside the charge is opposite to the kind on the small body which has been lowered into the sphere; while on the outside it is of the same kind, as may be shown by a pith-ball carrying a charge of a definite kind and suspended by a thread. The *amounts* of the charge on the outside and inside are equal; for, if the small charged body is removed, the conductor shows no effect. Further, these charges on the two sides of the surrounding conductor may be shown to be equal in

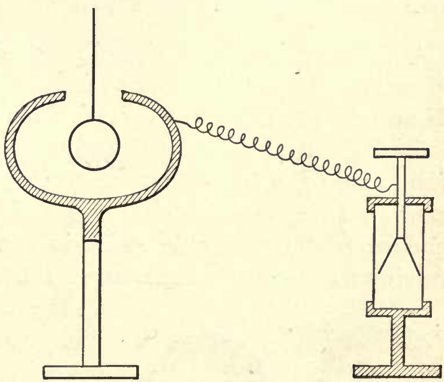

FIG. 104.

amount to that on the body which was inserted. For, if this last is a conductor and is allowed to touch the inner surface of the hollow conductor, the charge on the outside is unaffected, showing that the charge on the inner surface is equal and opposite to that on the inserted body, because the two have neutralized each other. It should be observed, too, that if, before the inserted body touches the inner surface, it is moved around at random inside the hollow conductor, the charge on the outer surface remains entirely unaffected, as indicated on the gold-leaf electroscope.

Thus, however the charged body inside is moved around, no change is noticeable outside; and, as was seen in the previous article, if there is no charged body inside the closed conductor, there are no forces felt inside owing to any charges outside. Thus a closed conductor serves as a screen separating the space inside from any action outside, and *vice versa*. This is one of the purposes of a system of lightning-rods enclosing a house.

This experiment of the charged body lowered inside a closed conductor is but one of a class of similar phenomena. If an uncharged conductor ($B\ C$) insulated from the earth is brought near a charged body A (see Fig. 105), it will be found that the conductor which was originally uncharged is now charged, positively on one side (C) and negatively on the other (B), that side which is nearest the charged body having a charge of an opposite kind. The amounts of charge on the originally uncharged conductor are, of course, equal and opposite, but they are not equal to that of the charged body. This phenomenon is called "electrostatic induction." The explanation is that the unlike charges tend to come close together, while like ones tend to get far apart (see Article 116). If the conductor which has the opposite charges at its two ends is touched at *any* point

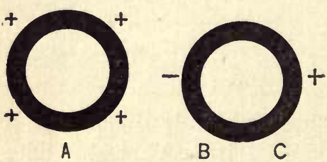

FIG. 105.

with a conductor leading to the earth, the charge which was on the farther side (C) can now escape still farther, and does so, spreading over the earth, while the other charge remains on the conductor. If the conductor leading to the earth is now removed, a charge will be left on the originally uncharged conductor, of a kind opposite to that on the charged body.

There are thus three ways of charging a conductor: one, by rubbing with silk, cotton, etc.; another by bringing it in contact with another charged conductor, in which case the charge will spread over the two (see Article 117); and last, by induction, by bringing it *near* a charged body, joining it to the earth and breaking the connection.

Electrophorus.—A simple form of electric machine consists in a conducting plate, joined to the earth, on which is fastened a non-conducting plate, e. g. a sheet of ebonite or resin, and a second metal plate or cover with an insulating handle, which can be placed on top of the non-conducting plate and then removed. The action is as follows: The non-conducting plate is charged by rubbing with silk or cat-skin, and the movable metallic cover is placed on it; the charge on the non-conducting plate induces charges on the cover; the latter is touched by the hand, and thus joined to the earth; there is thus left only one kind of charge on the cover, which may then be lifted by the insulating handle and can be carried away so as to give up its charge to any suitable body. This process may be repeated indefinitely. The charge does not pass off from the non-conductor to the cover except in very small amounts, since the two surfaces touch at only a few points; further, the charges induced on the lower conducting base tend to prevent any escape of the charge on the non-conducting plate. Such an instrument is called an electrophorus.

There are several types of machines which perform these steps automatically, and accumulate the charge on large conductors.

122. Electrostatic Strain; Sparks.—When two bodies, e. g. glass and silk, are rubbed together and then separated, they are oppositely charged; and therefore, since there is "attraction" between them, work must be done in order to separate them. What becomes of the energy? Neither of the bodies gains kinetic energy, nor is it deformed or strained; therefore the energy must be in the medium between and around the charged bodies. The fact that this medium is strained is shown by the passing of "sparks" through it, whenever the charges become too great. The charges disappear; just as, when a wire is twisted too much, it may break, and the strain in the wire become nil. (A spark may be said to act like a perfect conductor; and when it passes the electrical energy is converted into energy of radiation and heat.) If the spark is in air, the molecules of oxygen and nitrogen gas are torn apart and rendered luminous, the temperature being always raised; if the spark passes in glass or in paper, there is a hole made in it, the matter is torn away. This shows that the non-conducting matter, e. g. air, glass, paper, which is between the two charged bodies must be strained greatly. The energy is where the strain is, therefore in the medium between and around the charged bodies. Consequently, the main features of electrification are to be found not on the bodies which are charged, but in the medium around them. For this reason this non-conducting medium has been given a special name, the "dielectric."

Charges may exist in a vacuum, where there is no ordinary matter to be strained. Thus the energy is in the "ether" in this case (see Article 103); and even if matter is introduced around the charged bodies a certain amount of strain remains in the ether. The phenomena of radiation and absorption (see Article 104) show that motion of ether influences matter; and so the strain in the air, glass or paper may be regarded as due to the strain in the ether.

The amount of strain, and therefore of energy, is different for different media—the charges being kept the same—as is shown by the fact that the force of attraction or repulsion between two charged bodies depends upon the dielectric; for, if the force is less, less work is required to separate the charges. In air the force is greater than if turpentine, oil, etc., were substituted. The strain is greatest, naturally, near the charged body, because the force is greatest there.

Points.—It may easily be observed that the strain in a medium is greatest near any projecting points that the charged conductor may have. This is proved by the fact that sparks pass so easily off points; and there is always a slow leakage off them. This action consists in minute sparks passing from the points to particles of dust, etc., in the air near the conductor; they, being charged in the same manner as the conductor, are repelled and new particles come to take their place.

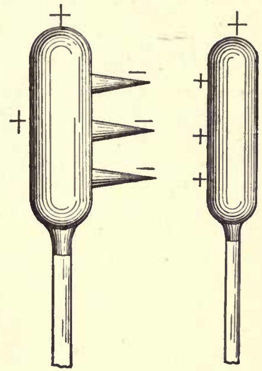

FIG. 106.

(This may be expressed by saying that the force, that is, the charge, is greatest on points.) For this same reason a conductor with points will, if placed near a charged body, discharge it and become charged itself. The charged body induces charges in the pointed conductor. (See Fig. 106.) The charges on the pointed side leak off and are drawn across to the originally charged body and discharge it, thus leaving the pointed conductor charged.

123. Lines of Force.—All these phenomena of charged bodies may be described in a different way. Let a line of "electrostatic force" be defined as such a line that at every point its direction is that in which a positively charged body would move if placed there. (These are therefore perfectly analogous to lines of magnetic force around magnets.) Thus, around a spherical conductor positively charged the lines of force are radial, beginning at the surface of the sphere.

(See Fig. 107.) Lines of force can never pass through a conductor, for there is no force inside; they always start from a positive charge and end on a negative one. In drawing them, it is customary to place them at such distances apart that by their closeness together at any point they indicate the intensity of the force at that point. A few cases are given in the accompanying figures. It will be seen that, if the charged bodies were free to move, the lines of force would all get shorter; in other words, "lines of force tend to contract." Since, when the charged bodies are held apart, the lines of force do not all place themselves along the

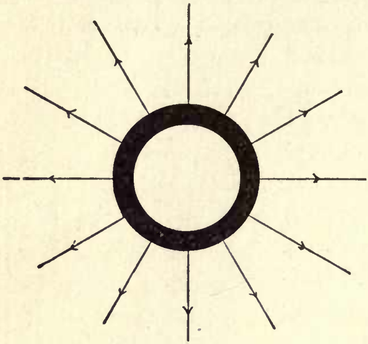

FIG. 107.

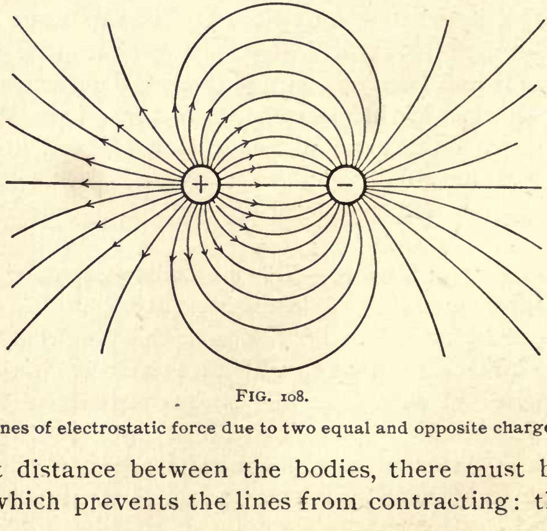

FIG. 108.

Lines of electrostatic force due to two equal and opposite charges.

shortest distance between the bodies, there must be some action which prevents the lines from contracting: this may be described as a "repulsion" sidewise between the lines. It is evident from the various figures. These expressions are to be regarded as simply *descriptions* of what is seen in the drawings. The region around charged bodies is often called the "field of force."

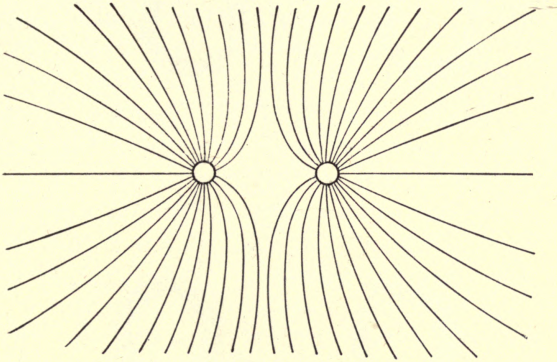

FIG. 109.

Lines of electrostatic force due to two equal charges of the same kind, either two positive or two negative charges.

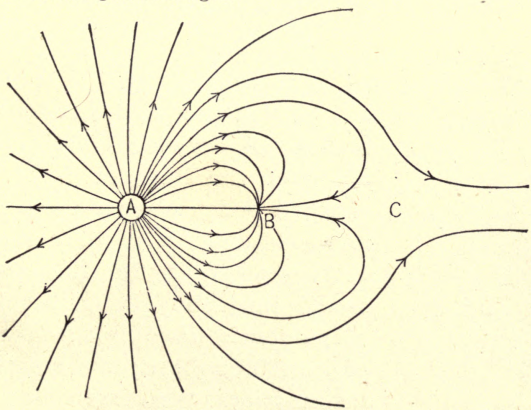

FIG. 110.

Lines of electrostatic force when *A* is charged with a quantity of positive electricity four times as great as the negative charge at *B*. At *C* the force is zero, as the effects due to *A* and *B* neutralize one another.

Drawings (Fig. 111) are given of the lines of force in the case of induction before and after the second conductor is

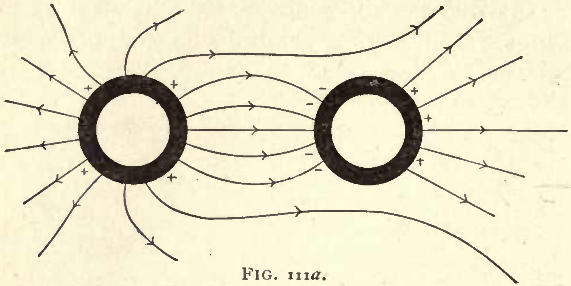

FIG. 111*a*.

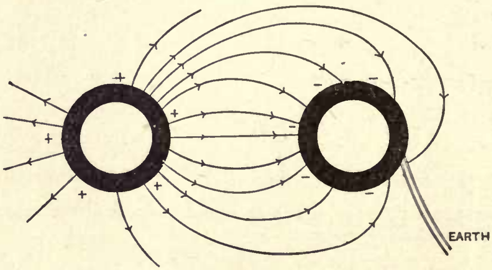

FIG. 111*b*.

joined to the earth; and it is evident that if the lines of force contract, there will be "attraction" between the two bodies. A drawing (Fig. 112) is also given of the lines of force from a charged body when a piece of some non-conductor, e. g. glass, is near it. It is seen that the lines of force crowd into the glass, just as lines of magnetic force crowd into a piece of iron brought near a magnet; and "attraction" therefore follows.

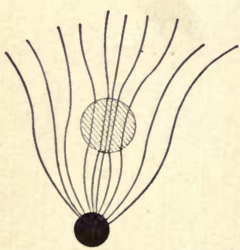

FIG. 112.

124. Condensers.—A special case of induction is given by a so-called "condenser," which consists of two conductors of similar shape placed close together but separated by a non-conductor; e. g. two metal plates separated by a piece of glass or a sheet of paper. Another case would be two metal cylinders separated by a cylinder of glass, as in the "Leyden jar," Fig. 113, which consists of a glass bottle with a sheet of tin-foil around the outside, and one also around the inside—this inner coating is connected by a conductor with a knob, which projects through the mouth of the bottle. The action is as follows: One conductor is charged, e. g. positively (see Fig. 114); it induces charges on the opposite conductor which is joined to the earth, thus producing on it a charge opposite to that on the first conductor (see Article 121). These opposite charges come as close together as possible by collecting on the faces nearest each other; therefore nearly all the strain is in between the two conductors. If, now, a piece of glass is inserted, *keeping the charges the same*, the strain is made less than when air was between.

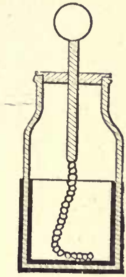

FIG. 113.

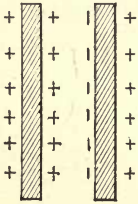

FIG. 114.

A spark will not pass between charged bodies unless the strain between them is great; and so, in general, whenever there is a great charge, there is danger of a spark. But in the apparatus just described the strain is so lowered by the second conductor and the glass plate that more charge may be added to the first conductor without liability of a spark passing; and for this reason the apparatus is called a "condenser."

When a condenser is charged there are nearly equal amounts of opposite charges on the two plates (or conducting surfaces), and the lines of electrostatic force are shown in Fig. 115. (Methods for the measurement of electrical quantities, such as "amount of charge," force, etc., are described in higher text-books.)

Unit Jar.—One convenient form of apparatus for giving a condenser a definite charge is known as the "unit jar." It is shown in Fig. 116, and its action is as follows: *A* and *B* are the inner and outer

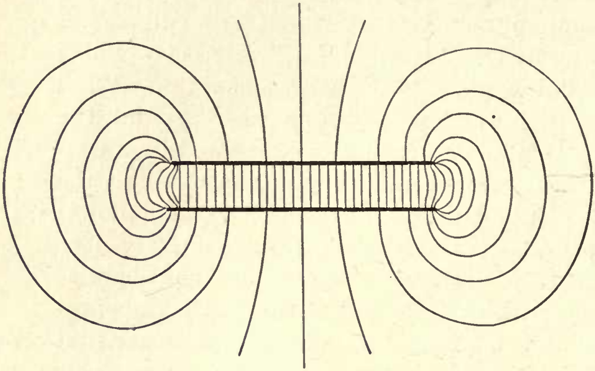

FIG. 115.

Lines of electrostatic force in the case of a plate condenser, the dielectric being air.

coatings of a Leyden jar, which is placed on an insulating stand *C*; *D* and *E* are the inner and outer coatings of a second smaller Leyden jar. *F* is a metal ball, carried on a metal rod *G*, which connects *F* with the

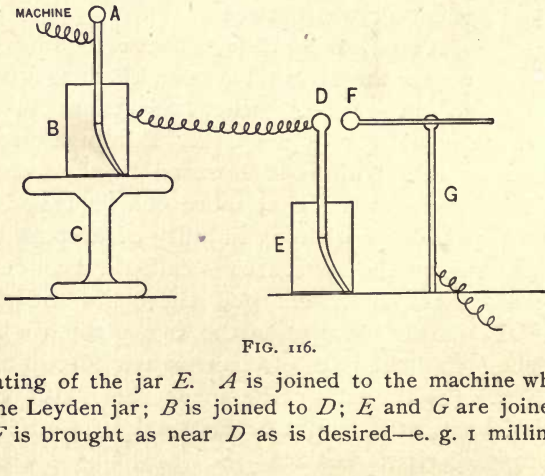

FIG. 116.

outer coating of the jar *E*. *A* is joined to the machine which is to charge the Leyden jar; *B* is joined to *D*; *E* and *G* are joined to the earth; *F* is brought as near *D* as is desired—e. g. 1 millimetre; 2; etc.

When the inner coating *A* is charged by means of the machine, *B* and *D* become charged by induction; and consequently if the charge on *A* is increased sufficiently, a spark will pass between *D* and the earth, through the knob *F*. This will discharge the small jar; and in order to make a second spark pass, more charge must be added to the large jar from the machine until it has twice as much as it had when the first spark passed, etc. Thus the charge given the large Leyden jar by the machine varies directly as the number of sparks passed at the small "unit jar."

CHAPTER XI

ELECTRIC CURRENTS

125. An Electric Current.—Around a charged condenser made up of two metal plates (see Article 124), the lines of electric force are as shown in Fig. 115; most of the strain is between the plates. If, now, the two plates are joined by a conductor, e. g. a wire, it will be found that the strain disappears—the plates are no longer charged; and, further, it may be shown that the temperature of the wire is raised. This entire phenomenon of discharge is called an "electric current;" there is said to be a "current through the wire," from the plate positively charged to the other. The whole process may be made continuous if the charges are replaced on the plates as fast as they disappear. To do this work is necessary, and therefore it requires energy to maintain an electric current.

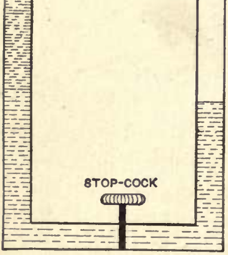

FIG. 117.

The reason why the phenomenon is called an electric "current" is from analogy with the flow of a liquid. If a vertical *U* tube is fitted with a stop-cock in the branch tube at the bottom (see Fig. 117) and is filled to different heights in the two arms with a liquid, there will be a greater pressure on that side of the stop-cock on which there is the greater height of the liquid. Consequently, if the cock is opened, the liquid will flow through to the other side, and everything finally will come to equilibrium, and the levels on the two sides will be the same. But if, as fast as the liquid disappears from one side, more is added from above, and, as fast as it comes through on the other side, an equal amount is removed, there will be a steady current. The one condition for such a current is constant difference of pressure.

One simple method of maintaining a steady electric current in a wire is to connect it between the two poles of an electric machine, which can be made to produce positive and negative charges at its two poles. The necessary energy for the current comes in this case from the power which works the machine.

As the current continues, the wire joining the two poles becomes gradually warmer; but this effect is not noticeable unless the current is large in proportion to the size of the conductor (see Article 129). Another property of the current, however, is known, which lends itself more easily to observation. If the wire is wound on a short spool or bobbin; and, if a small magnetic needle is pivoted inside the bobbin so as to be free to turn around an axis which is a diameter of the bobbin (see Fig. 118), it will be deflected when there is an electric current through the wire. If the direction of the current is reversed, so is that of the deflection of the magnetic needle. Since the "direction" of the current in a wire is by *definition* that from the plate or pole positively charged to the one negatively charged, the connection between direction of current and direction of deflection of the magnet may be found, once for all, by discharging a condenser through the wire around the magnet, for one knows which plate of the condenser is negative and which positive from the method of charging it. It is found that, if the current around the bobbin appears to an observer to be in the direction of the motion of the hands of a clock, the north

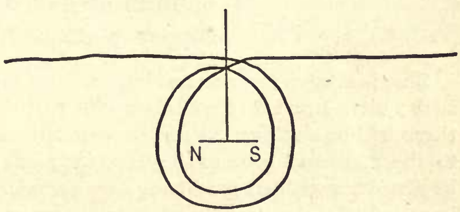

FIG. 118.

pole of the magnet is forced away from the observer, the south pole being drawn out towards him. A simpler rule is given in Article 128.

126. Voltaic Cell.—Another method for the production of a current is to dip a copper plate and a zinc plate into a vessel containing a dilute solution of sulphuric acid in water and to join these plates outside by means of a wire—this forms a "Voltaic cell" (see Fig. 119). There will be a steady current in the wire (as indicated by the deflection of the magnet in the apparatus described above), in the direction from copper to zinc; and in the liquid, from zinc to copper. As the process continues, the zinc dissolves away, that is the mass of the plate becomes less and less; and bubbles of hydrogen gas collect on the copper plate and rise to the surface.

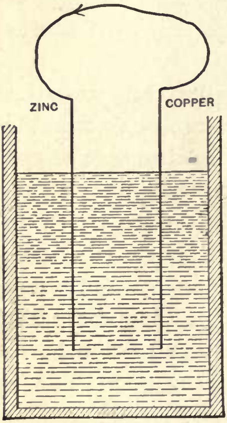

FIG. 119.

Zinc is always more or less impure, and contains particles of iron. If such a bit of iron is in the surface which is in contact with the dilute acid, there will be a minute electric current from the iron to the zinc, then to the acid, and back to the iron, etc.; and thus some of the zinc will be dissolved without producing any current in the wire outside. This is called "local action," and may be prevented largely by "amalgamating" the surface of the zinc with mercury. This is done by rubbing mercury over the surface, thus making the whole plate practically uniform.

The presence of the bubbles of hydrogen at the copper will be explained later (see Article 130); but their immediate action is to decrease the current; the cell is said to be "polarized." There are many ways of preventing it. The best is to have the copper plate dip into a porous jar which contains copper sulphate in solution in water, instead of having the copper dip directly into the dilute acid. As the current now passes, the zinc dissolves as before, but copper is deposited out of the copper sulphate solution upon the copper plate. This arrangement is called a "Daniell's cell."

A hydraulic analogy may make the explanation of the Voltaic cell somewhat simpler. Let a *U* tube be placed vertical, and let a water-wheel or pump be placed in the horizontal branch so as to force water through from one side to the other—the motive power may be a steam-engine turning the wheel (see Fig. 120). If the water is poured into the two arms, and the pump set going, water will be pumped through from one arm to the other; and, if there is a tube connecting the two arms, there will be a continuous flow of water as long as work is applied to the wheel. In the case of the Voltaic cell, energy is furnished for the driving of the current by the dissolving of the zinc in the dilute acid; for, if acid is put in a tumbler and pieces of zinc added, the temperature rises, showing that the process of solution of zinc in acid furnishes energy. As long as the zinc continues to dissolve, the current lasts; and the intensity of the current is proportional to the rate at which the zinc dissolves.

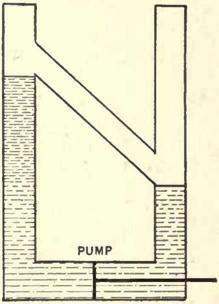

FIG. 120.

If *pure* copper and *pure* zinc are dipped in dilute acid, there is chemical action for a very short time only; but, if the two metals are joined by a wire, as in the Voltaic cell, a current begins and the zinc dissolves.

127. Thermo-electric Currents.—If wires of two different metals, e. g. iron and copper, are joined in series so as to form a closed circuit (see Fig. 121), there will be no current so long as the temperature is the same throughout; but, if one "junction," i. e. place of contact of the two metals, has a temperature higher than that of the other, there will be a current. The reason for this depends upon the fact that, when two bodies of any kind are

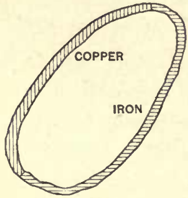

FIG. 121.

brought in contact, there are electric forces between their surfaces. This contact force varies with the temperature; and so, when there is a difference of temperature at the junctions, the electric force at one junction may be greater than that at the other; and thus there will be a current. The energy to maintain this current comes from the source of heat which raises the temperature of the warmer junction. This combination of two metals forms a "thermo-couple." Several "couples" of two metals may be arranged in series as shown in Fig. 122; and, if one face of the junctions is at a higher temperature than the other face, there will be a current, which may be made large by increasing the number of couples. Such an apparatus is called a "thermopile," and is very sensitive to slight differences of temperature.

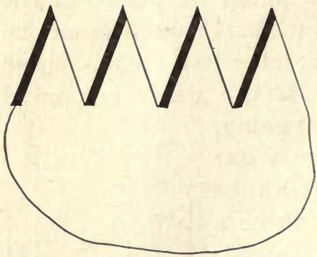
FIG. 122.

128. Magnetic Effect of a Current.— The property of an electric current which is observed most easily is the "magnetic effect" mentioned in Article 125. If a wire carrying a steady current is held either above or below a magnetic needle which is pivoted free to turn around a vertical axis, the needle will turn and tend to place itself east and west. If the electric current is flowing from south to north above the magnet, the north pole of the magnet moves towards the west (see Fig. 123); but, if the current is in the opposite direction, the north pole of the magnet moves towards the east. Similarly, if the wire is placed *below* the magnet, and if the current is flowing from south to north in the wire, the north pole of the magnet will move towards the east. There-

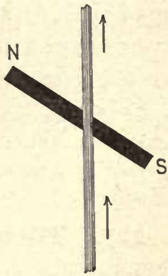

FIG. 123.

fore, if the wire is bent into a rectangle, with one long side above the needle and the other below, the upper and lower wires help each other, no matter which direction the current has in the circuit; and, if several rectangular loops are joined in series, each helps the other, and the apparatus is very sensitive.

The instrument may be made more sensitive by the use of an "astatic" combination (see Fig. 124), which consists of two magnetic needles placed parallel, but with their poles in opposite directions, and joined rigidly together by some solid, such as a glass rod. The wire which is to carry the rod is wound around the lower needle only, as

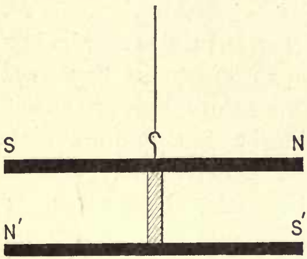

FIG. 124.

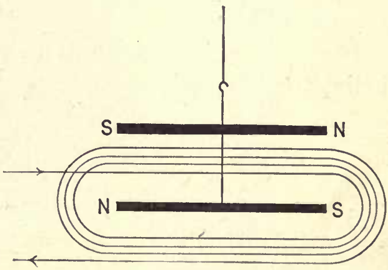

FIG. 125.

shown in Fig. 125. It will be seen that the action of the current on the upper needle helps that on the lower. Further, the direct action of the earth on the two magnets is almost neutralized, because the opposite poles come so close together. Such an instrument as this is called a "galvanometer."

In one form of instrument the wire is wound around in a flat circular coil of many turns, which is placed with its faces in the magnetic meridian (see Fig. 126). If a very small magnetic needle is suspended at the centre of the coil, it will, as seen before in Article 125, be deflected by a current.

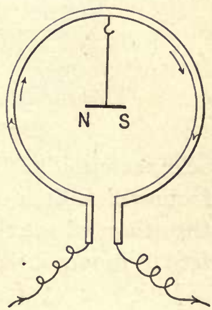

FIG. 126.

If in Fig. 126 the current is in the direction of the arrows, i. e. in the direction of the motion of the hands of a clock, the north pole of the magnet will move in a direction perpendicular to the plane of the coil and, as represented here, "down" into the page, while the south pole will move perpendicularly "up." This is equivalent to saying that an electric current in a closed circuit produces a magnetic field such that the lines of magnetic force *inside the circuit* pass perpendicular to the plane of the coil and in the direction in which a right-hand screw would enter a board if it were turned in the direction indicated by the current around the coil. *Outside* the circuit, the lines of magnetic force are in the opposite direction; in fact, they form closed loops around the conductor carrying the current.

If the current is through a conductor wound in a spiral or helix, the lines of magnetic force are as shown in Fig. 127.

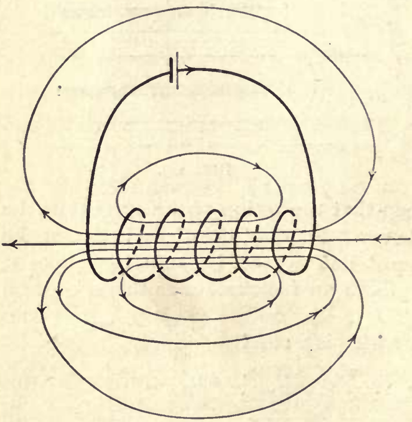

FIG. 127.

Therefore, it ought to act like a bar-magnet with its north pole towards the left, because lines of magnetic force come out from a north pole and enter a south pole. Such an apparatus is called a "solenoid." If suspended free to turn, it will point north and south in the magnetic meridian; and it has actions of attractions and repulsions on magnets or other solenoids. Similarly, that face of the flat coil in Fig. 126 from which lines of force *come out* is called its "north face;" the other, where the lines go in, the "south face." If the coil is free to move, it may be repelled or attracted by a bar-magnet.

If a bar of iron is placed inside the solenoid it becomes magnetized, because each molecular magnet (see Article 112) tends to place itself along the lines of force (this is, in fact, the easiest method of magnetizing a rod of iron or steel). Therefore the magnetic action of a solenoid is greatly increased by winding the wire around a bar of iron, but insulated from it. Such an arrangement is called an "electro-magnet."

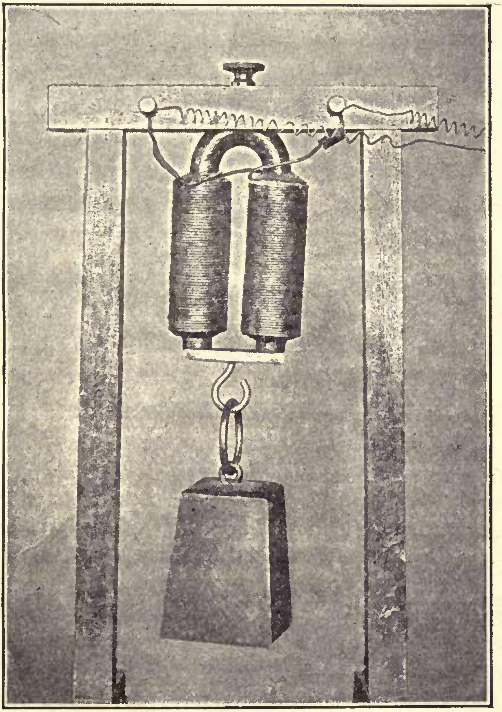

FIG. 128.

Ordinarily, the iron rod is bent into the form of a U or a "horseshoe," as is shown in Fig. 128. In this form they are used extensively in practical applications, e. g. electric call-bells, telegraphic instruments, etc.

The common electric motor, as used on electric elevators, electric cars, etc., depends also upon this magnetic action of a current: the revolving part of the motor, the "armature" so called, consists of coils of wire which are joined to the "commutator bars" on the axle in such a way that, as the armature turns, the electric current passes first

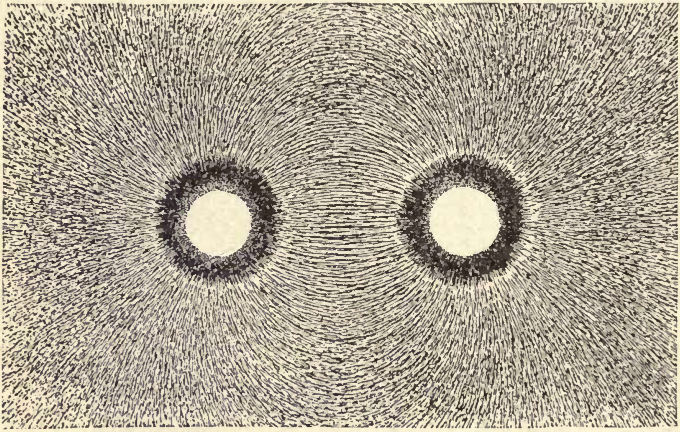

FIG. 128*a*.

Lines of magnetic force around the ends of an electro-magnet made in the form of a "horseshoe." Either pole may be called "north."

through one set of coils, then another, etc. The armature is placed between the poles of an electro-magnet; and the coils which are carrying the current at any one instant are always those on which the magnets exert the greatest pull or rotating effect.

129. Heating Effect of a Current.—Whenever a conductor carries a current, its temperature is raised, because the work done in maintaining the current is spent in overcoming forces which are concerned with the molecules of the conductor. The greater the opposition to the passage of the current at any one point of a circuit, so much the greater is the heat-effect there. Thus, wherever there is a bad joint, a poor connection, or a poor conductor, the temperature is raised more than elsewhere.

The ordinary "glow-lamp" and the electric arc-lamp are illustrations of this heating effect of currents. The glow-lamp, Fig. 129, consists of an exhausted glass bulb into which enter two platinum wires connected inside the bulb by a fine carbon filament. As the current passes, the temperature of the filament is raised to incandescence, because it is such a poor conductor. (It does not burn up, because there is no oxygen with which it can combine.)

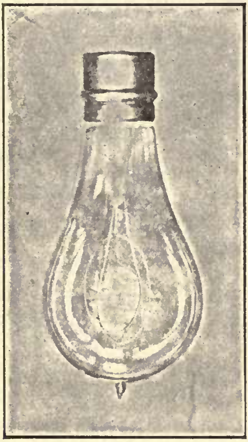

FIG. 129.

The arc-light, Fig. 130, consists of two carbon rods in the open air, which are kept a short distance apart. When the current is first applied, the two rods are made to touch; but they are separated immediately afterward. This is done because the air is not a conductor until its temperature is very high (or unless there are sparks). The passage between the carbon poles when they are in contact offers such resistance to the current that the temperature is greatly raised, and the air becomes a conductor; then the poles can be drawn apart. The passage now from the carbon pole to the air and from the air to the other pole offers a great resistance; the temperature of the carbon poles is raised, and they become "white hot."

FIG. 130.

130. Electrolysis.—In order to have a current pass through a vessel containing a liquid, two wires connected with the two poles of the electric machine or other source of currents are dipped into the liquid, one at each end of the vessel; then, if the liquid is a conductor and if the power of the machine is great enough, there will be a current. When a current is passed through a liquid conductor (other than a metal in the liquid state, such as mercury, molten iron, etc.),

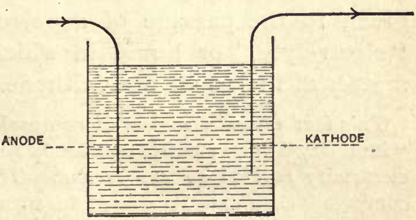

FIG. 131.

certain chemical changes are seen to occur at the places where the current enters and where it leaves. As a simple case, let the liquid conductor be a dilute solution of sulphuric acid in water, and let the current be led in and out by means of platinum wires, Fig. 131. The wire which leads the current in is called the "anode;" that which leads it out, the "kathode." In this case, as the current continues, hydrogen gas will form on the kathode and bubble off at the surface of the liquid; and oxygen gas will form on the anode and bubble off there. If the liquid conductor is a solution of copper sulphate in water, and if the anode and kathode are copper plates, copper will deposit on the kathode out of the solution, and an equal amount of copper will dissolve off the anode.

This is obviously the principle of "copper-plating" and similar processes. If the object to be copper-plated is made the kathode, copper will dissolve off the anode and will be deposited on the object forming the kathode, if it is suitably prepared—otherwise the copper might not adhere to it.

Careful experiments have shown that the quantity of matter (i. e. the mass) deposited or set free at both kathode and anode varies directly as the quantity of electricity carried through the liquid owing to the current. This is known as Faraday's first law of "electrolysis," a name given to the passage of an electric current through an "electrolyte," or liquid in which there are liberations of matter at the anode and kathode.

This fact offers a means of comparing two electric currents. The "intensity" of an electric current is the name given the quantity of electricity carried *in one second.* Thus, if two currents are to be compared, each may be passed in turn through the copper sulphate solution mentioned above, and the quantities of copper deposited in certain definite times may be measured. If the mass m_1 grams is deposited in t_1 seconds by the first current whose intensity may be called i_1; and the mass m_2 grams by the second current of intensity i_2 in t_2 seconds, the *quantity* of electricity in the first case is $i_1 t_1$; in the second, $i_2 t_2$. Therefore, in accordance with Faraday's Law, $m_1 : m_2 = i_1 t_1 : i_2 t_2$, and so i_1 and i_2 may be compared. Such an instrument for comparing currents is called a "voltameter." Another form is that shown in Fig. 132, in which dilute sulphuric acid in water is the electrolyte, and the gases formed, hydrogen and oxygen, are collected in suitable tubes and measured. If the volumes of the gases are known, and the temperature and pressure observed, the masses may be calculated from a knowledge of the densities as given in tables.

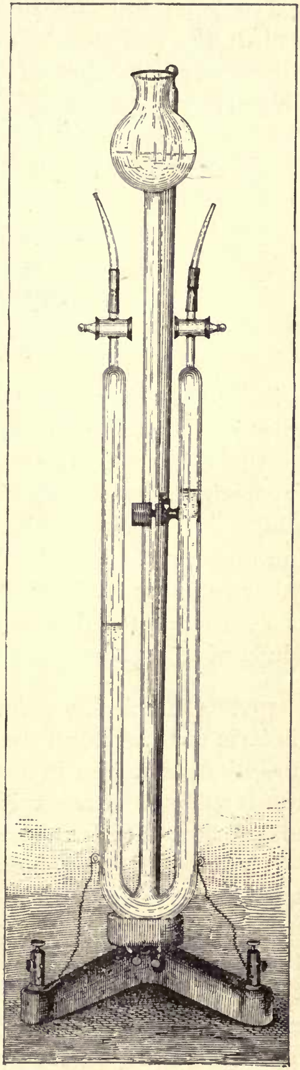

FIG. 132.

Further, if the same current is passed through several electrolytes arranged in series, e. g. first into the dilute sulphuric acid, then out and into the copper sulphate solution, out and back to the source of the current, the quantities of matter liberated at the various places bear simple relations to each other. (See Ames's "Theory of Physics," p. 316.)

131. Discharge Through Gases.—If a glass bulb into which two wires are sealed is sufficiently exhausted by means of an air-pump, a current may be passed through the tube from wire to wire by means of an electric machine or an induction coil (see Article 134). Many interesting phenomena occur. The gas is luminous in places, the bright spaces being in "striations" across the tube near the anode; and, if the exhaustion is carried far enough, one can detect the apparent passage of *something* in straight lines perpendicularly away from the surface of the kathode, whatever its shape is. These are the "kathode rays." They are really minute portions of matter, negatively charged, travelling with a velocity of about 25,000 miles per second. Where these charged particles strike any solid, tremendous disturbances

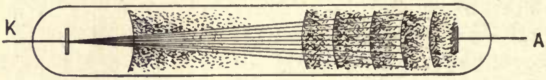

FIG. 133.

arise in the ether, and the solid matter has its temperature raised greatly and is otherwise affected. The disturbances in the ether which are thus caused give rise to Röntgen rays, which have such remarkable properties. The solid which is struck by the kathode rays will often become luminous, not from rise in temperature necessarily, but for other reasons; and this "fluorescent" color, as it is called, is characteristic of the material of the solid. For most kinds of soft glass it is a greenish yellow.

132. Ohm's Law for Steady Currents.—If a steady electric current flows through any number of conductors of any kind, arranged in series, it may be proved that the intensity of the current is uniform throughout; that is, the magnetic action is the same at all points of the circuit.

The intensity of the current in a given conductor of definite length, size and material depends upon a property of the cell which causes the current. Exactly as in the case of the flow of a liquid (Article 126), the current was proportional to the difference of pressure, so in a Voltaic cell we may say that the current is due to a "difference of pressure" at the zinc and copper poles, using the words in an electrical sense. This difference is called the "electro-motive force" because, without such a difference, there is no current. Call its numerical value E; it is evident that we may have great or small values of E by joining together various numbers of cells. If i is the intensity of the current through a definite conductor joining the two poles of the cells, experiments show that, if the current is steady, i is proportional to E. Or, expressed mathematically,

$$i = E/R,$$

where R is a constant for one definite conductor, and is called its "resistance." It depends upon its material, length, cross-section and temperature. This is known as "Ohm's Law." Further, if any two points A and B in the conductor are chosen, the same law applies to this portion of the conductor; E being the "difference of pressure" between A and B, and R being the resistance of the conductor between the same points, $i = E/R$. R varies directly as the length of the conductor and inversely as the cross-section, because,

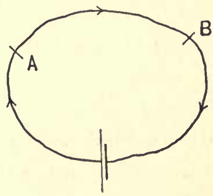

FIG. 134.

for a given difference of pressure, the resistance to the flow is twice as great in a conductor twice as long as another conductor of the same size and material; while, if the cross-section is doubled, the resistance offered is only half as great.

Thus, if several conductors of resistances R_1, R_2, R_3, etc. are joined "in series," and the current of intensity i forced through

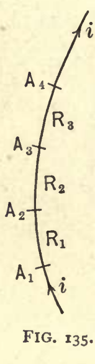

FIG. 135.

$$i = \frac{E_1}{R_1} = \frac{E_2}{R_2} = \frac{E_3}{R_3} = \text{etc.};$$

if E_1, E_2, E_3 are the "differences in pressure" at the terminals of the different sections of the conductor, $A_1 A_2$, $A_2 A_3$, etc. (see Fig. 135);

therefore $$i = \frac{E_1 + E_2 + \ldots .}{R_1 + R_2 + \ldots .} = \frac{E}{R},$$

where E is the total "difference in pressure" and R the total resistance.

If these same conductors are joined "in parallel" between two points A and B (see Fig. 136), and if a current of intensity i is forced through, there will be the same difference

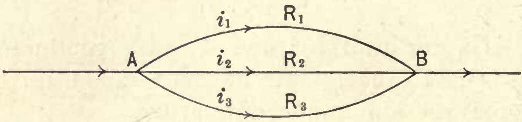

FIG. 136.

of pressure at the terminals of all the conductors; and consequently the currents in the different conductors will have the intensities i_1, i_2, i_3, etc., where

$$i_1 = \frac{E}{R_1},\ i_2 = \frac{E}{R_2},\ i_3 = \frac{E}{R_3},\ \text{etc.}$$

Hence $$i_1 : i_2 = \frac{1}{R_1} : \frac{1}{R_2},$$

or the currents in any two branches vary inversely as the resistances of those branches. (If R_1 is ten times R_2, i_2 is ten times i_1.

Further, the total current

$$i = i_1 + i_2 + i_3 = E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots .\right).$$

Thus, if R is the total resistance,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots .$$

133. Wheatstone's Bridge.—Another illustration of Ohm's Law is furnished by "Wheatstone's bridge," so-called. It consists of a network of wires, as shown in Fig. 137: two points, A and D, are connected by three conductors, $A\,B\,D$, $A\,C\,D$, $A\,E\,D$; two points of $A\,B\,D$ and $A\,C\,D$, viz. B and C, are joined by a conductor. In the branch $A\,E\,D$ a cell or some source of an electric current is introduced, and in the branch $B\,C$ is inserted a galvanometer. If the current flows from A across the two branches $A\,B\,D$ and $A\,C\,D$ to D, there will be a steady fall of "pressure" down each conductor; and therefore it will be possible by moving the point C to find some point which will have the same pressure as B, and consequently there will then be no current across and no deflection of the galvanometer. If this condition is secured, there is the same pressure between A and B as between A and C; call it E_1; also the same between B and D as between C and D; call it E_2. Further, the same current flows from A to B as does from B to D, because there is no branching of the current across from B to C; call the intensity of this current i_1. For a similar reason, there is the same current from A to C as from C to D; call its intensity i_2. Then, if R_1, R_2, R_3 and R_4 are the resistances in the four "arms" of the bridge, $A\,B$, $B\,D$, $A\,C$, $C\,D$,

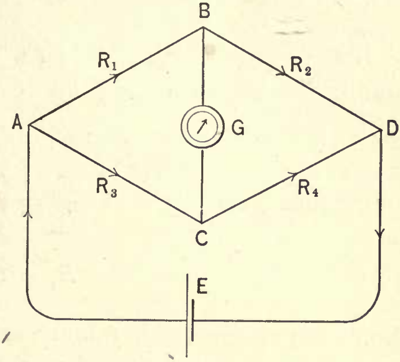

FIG. 137.

$$i_1 = \frac{E_1}{R_1} = \frac{E_2}{R_2},$$

$$i_2 = \frac{E_1}{R_3} = \frac{E_2}{R_4}.$$

Hence
$$\frac{R_1}{R_3} = \frac{R_2}{R_4},$$

or
$$R_1 = R_2 \frac{R_3}{R_4}.$$

So, if three resistances are known, a fourth may be measured.

If the conductor *A C D* is a wire of uniform cross-section and if the lengths of *A C* and *C D* are l_1 and l_2 respectively,

$$R_3 : R_4 = l_1 : l_2;$$

and hence
$$\frac{R_1}{R_2} = \frac{l_1}{l_2},$$

or
$$R_1 = R_2 \frac{l_1}{l_2}.$$

So, if any conductor is taken as a standard of resistance, the value of the resistance of any other conductor may be determined. (An actual form of the bridge as used in laboratories is given in Fig. 138.)

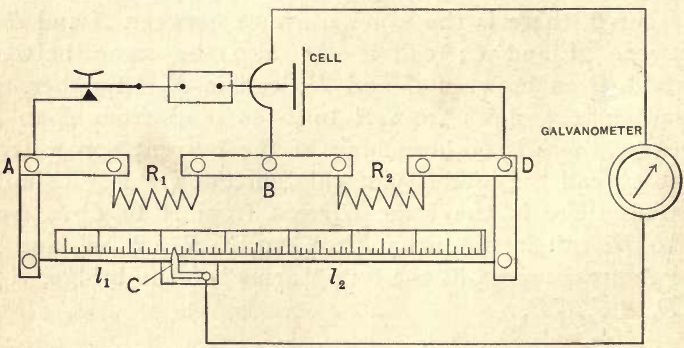

FIG. 138.

134. Induced Currents.—It has been shown in Articles 113 and 121 that a piece of iron becomes magnetized by induction if it is brought near a magnet, and that a conductor becomes charged by induction if it is brought near a charged body. Currents may also be induced, but not in a similar manner.

It has been shown by experiment that, if the number of lines of magnetic force which pass through a closed conducting circuit are changed in any way, i. e. if the magnetic field is varied, an electric current will be produced in this circuit. This current will be transient, unless the "field" continues *to be changed.* An illustration will help to make the matter clearer. If a wire is bent into a circle, and a bar-magnet is brought near it with its axis perpendicular to the plane of the circle (see Fig. 139), the field of magnetic force enclosed by the wire is altered, and an electric current is produced *so long as the magnet is approaching.* When it stops, the current ceases. If the magnet is drawn away, thus again *changing* the field of magnetic force, there is a current, but in the opposite direction to that of the former one. These currents are called "induced currents;" and the entire phenomenon is called "electro-magnetic induction." The direction of the induced current is such as by its own magnetic action (see Article 128) to oppose the motion which is causing the current (otherwise "perpetual motion" would be possible). Thus, if the magnet is approaching, north pole foremost, the field of magnetic force which passes through inside the wire circle from the right side to the left (the direction referring to Fig. 139) is increasing; the induced current will be in such a direction as to oppose this, i. e. as to tend to make the number of lines less; it will therefore send out lines of its own from left to right, and its direction is given by the right-hand screw law (see Article 128), viz. to a person looking at the wire circle from the magnet the current will be opposite to

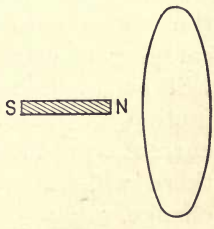

FIG. 139.

the direction of the hands of a watch. (Expressed differently, as the north pole of the magnet approaches the circuit, the induced current must be in such a direction as to oppose the motion; that is, it must be in such a direction as to have its "north" side—that from which the lines of magnetic force come out—facing the magnet. This will therefore tend to repel the approaching magnet.) The energy required to produce the current comes from the work done in bringing up the magnet against the opposing force; therefore, as soon as the motion ceases, the supply of energy ceases, and the current dies down, its energy being dissipated in heat-effects in the wire.

If there is a steady current in the wire before the magnet is brought up, the induced current will be added to or subtracted from this, according as the two currents are in the the same or in opposite directions. Similarly, if a coil carrying a current is brought near a closed circuit, there will be an induced current in the latter so long as the motion continues. And if, when one coil is near another, a current is produced in one, or if an existing current is varied, there will be an induced current in the other during the change.

The intensity of the induced current depends upon two things, the *amount* of the change in the magnetic field of force, and the *rate* at which this change takes place; the quicker the change, so much the more intense is the current.

One simple experiment which illustrates these facts is as follows: Place a bar of soft iron inside a bobbin of wire, like a solenoid, only wound in several layers, whose ends are joined to a galvanometer; bring a bar-magnet up close to the iron bar inside the bobbin, and notice the deflection of the galvanometer. The action is much more intense than when there is no iron bar inside the bobbin; for the iron becomes magnetized by induction from the magnet, thus making a great change in the magnetic field inside the bobbin. If the magnet is brought up quickly, the induced currents will be very intense. The effect of having many turns and layers of wire around the bobbin is to increase the inductive action; for there will be the same action in each turn, and so these will all be superimposed.

Another illustration of induced currents is when a wire in the form of a helix, having its ends joined to a Voltaic or other cell, is broken at any point. Minute sparks are observed at the point where the connection is broken. This is due to the fact that, as long as the current from the cell is flowing, there is a magnetic field inside the helix; and so, when the connection is broken, this field immediately decreases, thus inducing a current in the surrounding helix. It is this *induced* current which causes the spark. The inductive action is very great, and is used in many pieces of apparatus.

Transformer.—There are numerous commercial applications of this principle of induced currents. One of the simplest is the so-called "transformer" (see Fig. 140). It consists of an iron ring, or a ring made up of iron wires,

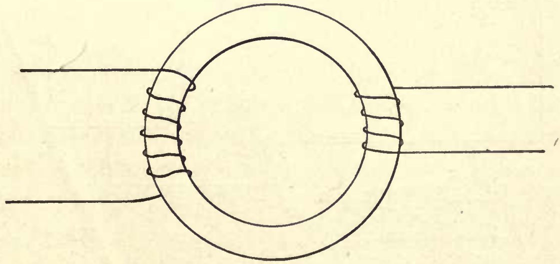

FIG. 140.

around two portions of which are wound two insulated conducting circuits (or helices) as shown. If a current is passed through one of these, it produces a magnetic field inside the iron ring, and thus through the second circuit;

consequently, if the current in the first conductor (the "primary" coil) is varied, there will be an induced current in the other (the "secondary" coil). In particular, if the primary current is first driven one way and then the other, that is, if it is an "alternating" current, there will be an alternating current induced in the second conductor. The intensity of the induced current and its "pressure" will depend upon the intensity and rate of alternation of the primary current, the number of turns the wires make around the iron ring, and the size of the ring.

Induction Coil.—Another instrument of the same construction as the transformer is an "induction coil," which is represented in Fig. 141. The current in the primary coil is made intermittent by means of a vibrator which alternately opens and closes the circuit.

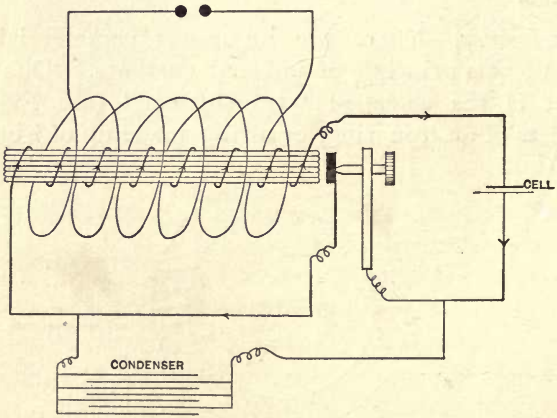

FIG. 141.

Currents are induced, therefore, in the secondary circuit, which is made in the form of a helix wound around the primary helix. By means of this instrument, sparks of great length may be obtained between the open ends of the secondary circuit.

Dynamo.—A dynamo is another illustration of induced currents. A simple type is shown in Fig. 142. The revolving part, or "armature," is a ring of iron wires around which are wound coils of insulated wire joined in series. It is

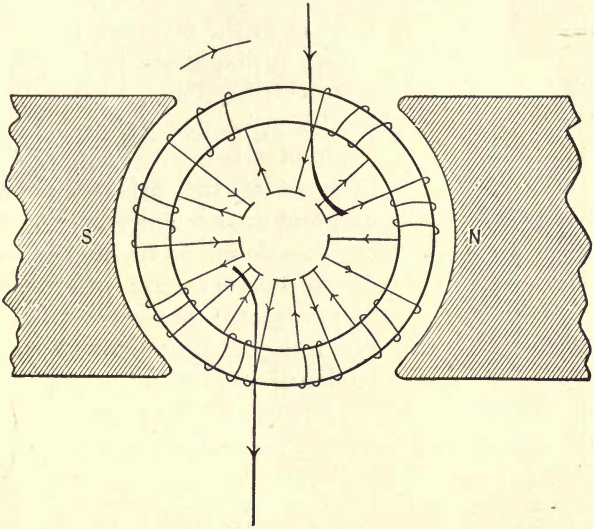

FIG. 142.

placed between the poles of an electro-magnet; and as the armature is made to revolve by a belt or gearing, the magnetic field inside the coils of wire varies, and currents are induced in them as shown.

Telephone.—The action of the "Bell telephone" depends also upon induced currents. It consists of a steel magnet, one end of which is surrounded by a bobbin of wire, and a thin iron diaphragm which is placed close to the end of the magnet carrying the bobbin (see Fig. 143). This iron plate is magnetized, therefore, by induction; and, if it approaches or recedes from the steel magnet, the lines of force inside the bobbin will be altered; and corresponding currents will be induced in the wire of the bobbin. If, therefore, the wire around the bobbin is joined in series with the bobbin of a second telephone, these fluctuating induced currents will alter the magnetic force of the magnet in the second telephone, and so its iron diaphragm will be attracted or repelled. Thus, when a "sounding body," e. g. a violin or the vocal cords of a person speaking, vibrates in front of the first telephone, its diaphragm is set in vibration by the compressional waves in the air; induced currents are produced; the diaphragm of the second telephone vibrates accordingly, and sounds will be heard by a listening ear.

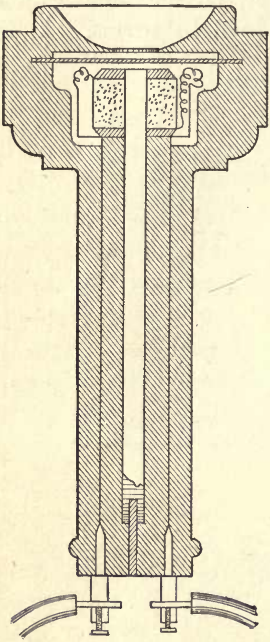

FIG. 143.

Microphone. — The "microphone" or "Blake transmitter," which is used with the telephone in practice, consists of a carbon button pressing lightly against a metal diaphragm. The button is joined to a wire running to one terminal of a telephone, and the metal diaphragm is joined to a cell, the other pole of which is joined to the second terminal of the telephone. The contact between the button and the diaphragm is poor; and its resistance varies with the pressure of the button against the diaphragm. Consequently, as the diaphragm vibrates, the resistance has corresponding fluctuations, and the intensity of the current from the cell will vary, and thus produce vibrations of the telephone diaphragm. Therefore, as the diaphragm of the transmitter vibrates, there are corresponding vibrations of the telephone diaphragm; and so "speech" may be transmitted.

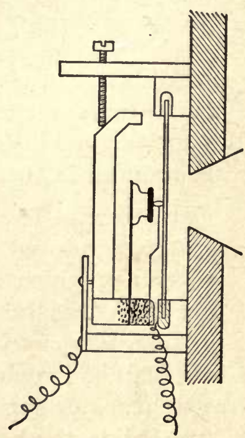

FIG. 144.

CHAPTER XII

NATURE OF LIGHT

135. Ether-Waves.—To anyone who was not born blind the word "light" conveys a definite meaning, applied to the sensation perceived by means of the eye. The sensation when traced to its source is found to be due to what is called a "source of light," such as a candle, a gas-flame, an electric light, the sun, etc. Such a source may be seen through a vacuum; hence the presence of ordinary matter is not essential for the propagation of the effect. In other words, we are dealing with the ether, and with "radiation" (see Article 103). As a rule, the temperature of a source of light is very high; but there are cases where this is not so. In every case, though, there is evidence that minute portions of matter (the portions of molecules) are vibrating very rapidly; and they thus produce waves in the ether. The fact that radiation is a wave-process may be proved by many experiments. One simple form is due to Thomas Young, and is called "Young's interference experiment."

A screen opaque to light has cut in it two narrow parallel slits, extremely close together; a second screen has also a narrow slit, which is placed parallel to the other two and equally distant from each (see Fig. 145); a source of light of one color, e. g. a flame colored with salt, is placed beyond the second screen and next the slit, so that the two slits in the first screen are equally illuminated by the bright slit in the second one. These slits are all extremely narrow, so that the effect to one looking at the two slits from the side removed from the source of light is that of two bright lines. These can be seen not only directly in front of the screen, but also when looking obliquely. The two slits act like two sources of light placed side by side. A distant screen placed so as to be illuminated by these two sources receives at each of its points, therefore, light from both. It might be expected that the illumination would be uniform over this third screen, but it is seen on examination by a microscope that this is not so. The line on the screen (marked by *O* in the diagram) which is equally distant from the two slits *A* and *B* is bright; but on either side of it there is darkness, then comes another bright line, etc. In other words, the screen is covered by a series of dark and bright lines parallel to the two slits. (In the actual experiment, it is best to introduce a lens between the first and third screen, so as to focus the light on the screen; or, better still, focus the light directly in the eye.) This can be accounted for easily if light is a wave-process, otherwise not. In the figure, let *P* be a point on the screen near *O*; it receives illumination from *A* and *B*; if the time taken for the ether-waves to pass from *A* to *P* is greater than that required to pass from *B* to *P* by half a period of the wave, or by three halves, or by five halves, the two trains of waves from *A* and *B* will reach *P* exactly half a wave-length apart, so that one train of waves will neutralize the effect of the other (see Articles 59 and 87). Therefore, in this case *P* will be

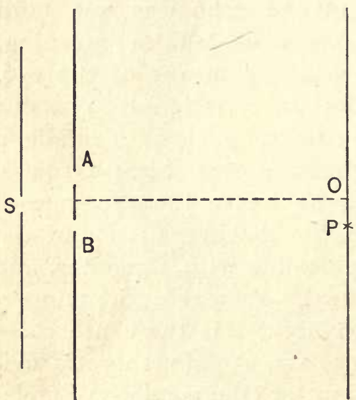

FIG. 145.

a point of darkness. If, however, *P* is such a point that the time taken from *A* to *P* differs from that taken from *B* to *P* by a whole period, or any whole number of periods, the two trains of waves will reach *P* in the same phase, and so they will help each other. *P* is, therefore, in this case a bright point.

The distance along the screen between two bright lines depends evidently upon the length of the waves; for in this distance the difference in the paths from *A* and *B* to the screen has changed by a wave-length. The greater the distance apart of the bands on the screen, the greater, therefore, is the wave-length.

This same experiment can be performed using a non-luminous source instead of the light; but some thermometer must be used to detect the presence of the "bright" and "dark" bands, which in this case would be simply hot and cold bands.

136. Colors.—If blue light is used as the source, and then red, it will be seen that the red bands are farther apart than the blue ones. This shows that those ether-waves which produce in the eye the sensation "red" have a longer wave-length than those which produce the sensation "blue." The common colors may be arranged in the order of increasing wave-length as follows: violet, blue, green, yellow, red.

(The wave-length of "red" light is about 0.000066 cm.; that of "green" about 0.000052 cm.; that of "blue" about 0.000042 cm. The average value is about 0.00005 cm., or $\frac{1}{50000}$th of an inch.)

If "white" light, e. g. sun-light, is used as the source, there are no *dark* bands at all on the screen, only colored ones. The effect is exactly as if the colored bands of all colors had been superimposed, each color having its own set of bands. This proves that in "white light" waves of all wave-lengths are present.

When white light is used, the central line on the screen (that through *O* parallel to the slits) is white, because all the colors have bright bands there. If a thin piece of glass is introduced, however, between one of the slits, e. g. *A*, and the screen, the central white band on the screen is moved sidewise along the screen towards the side facing *A*. This proves that ether-waves travel more slowly through the glass than through the air. In other words, waves travel more slowly when the ether is loaded with glass than when it is loaded with air.

There are several other, more familiar, experiments which prove that radiation is a wave-process. If a thin film of oil is spread over water, or if a soap-bubble is blown thin, it is seen to have beautiful colors when viewed in white light. These are due to the varying thicknesses of the film, and to the fact that, when the light falls upon the film, some is reflected directly from the first surface, while the rest enters the film, is partially reflected at the second surface, and comes out at the first surface, thus combining with the light reflected from that surface. These two trains of waves which are thus superimposed differ in phase, one is retarded behind the other; and, if this retardation amounts to an odd number of half wave-lengths of any definite train, the two will interfere and neutralize each other; thus the color which corresponds to this vanished train of waves will be absent from white light, and the reflected light will be colored. Different thicknesses will evidently cut out different colors. (These waves which are absent from the reflected light are *transmitted*, if there is no absorption, and may be observed on the other side of the film.) The colors of thin air films between pieces of glass, of thin glass films, and of opals depend upon this principle.

137. Velocity.—The velocity of ether-waves may be determined by astronomical observations, or by direct experiment. The satellites of a planet, e. g. Jupiter, make revolutions around it at regular intervals; and so, if the eclipses of a satellite behind the planet's disc were observed from a point in space at a fixed distance from the planet, they would occur at equal intervals of time. Similarly, if the observations were made from the earth at any one point of its orbit the eclipses would occur regularly. It is noticed, however, that, when an observation of an eclipse is made at that point of the earth's orbit which is nearest the planet and calculations are then made as to when an eclipse will be noted six months later, i. e. when the earth is farthest away from the planet, there is a discrepancy between the predicted and observed time of eclipse, unless allowance is made for the time taken for the ether-waves to cross the earth's orbit. The allowance necessary demands a velocity of 30,000,000,000 or 3×10^{10} cm. per sec. This is the same value practically as that found by direct observation on the surface of the earth.

138. Intensity.—A source of light is to be considered, then, as a source radiating energy in the form of ether-waves, of various wave-lengths, spreading out in the region around the source. These waves all travel with the same velocity—regardless of wave-length—in ether unloaded with matter; but the presence of matter not only slows up all the waves, but affects the shortest waves most (see Article 146). The amount of energy radiated by a point-source in one second is called its intensity, and the intensities of two sources of the same color may be compared in many ways. Let two blocks of paraffin of the same thickness be placed side by side, and then introduced between the two sources S_1 and S_2 (see Fig. 146). By moving these, distances x_1 and x_2 may be found, for which the two halves of the paraffin block appear equally illuminated. This means that the illumination at a distance x_1 from S_1 is the same as that at a distance x_2 from S_2. But illumination varies inversely as the square of the distance (see Article 57); therefore the

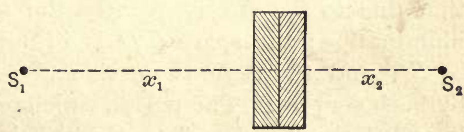

FIG. 146.

intensity of S_1 (I_1) must be such that at x, $\frac{I_1}{x_1^2}$ is equal to $\frac{I_2}{x_2^2}$ where I_2 is the intensity of S_2.

That is, $$\frac{I_1}{x_1^2} = \frac{I_2}{x_2^2}, \quad \text{or} \quad \frac{I_1}{I_2} = \frac{x_1^2}{x_2^2}.$$

139. Shadows.—These ether-waves spread out from a source in such a way as to cast sharp shadows; or, as sometimes expressed, "light travels in straight lines." If S_1 and

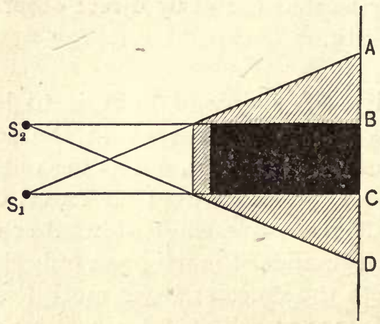

FIG. 147.

S_2 are two sources (or points of a single large source), each will cast on a screen a sharp shadow of an opaque object. Thus, in the figure, AC is the shadow due to S_1 and BD that due to S_2. Consequently the space BC receives no illumination; the space CD is illuminated by S_1, and AB by S_2; and the regions outside A and D are illuminated by both sources. The region which receives no illumination is called the "umbra;" that which is partially illuminated the "penumbra."

A "point-source," such as S_1 or S_2, sends out waves in an isotropic medium with a spherical wave-front, and so disturbances advance in all directions along the radii of the spherical surface. Such a line of disturbance is called a "ray." Thus the line $S_2 B$ is a ray from S_2. The experimental fact that light casts sharp shadows proves that light advances along these rays. There is no difficulty in showing that this is a necessary consequence of the fact that light is due to waves whose wave-length is extremely small. (See Ames's "Theory of Physics," p. 401.)

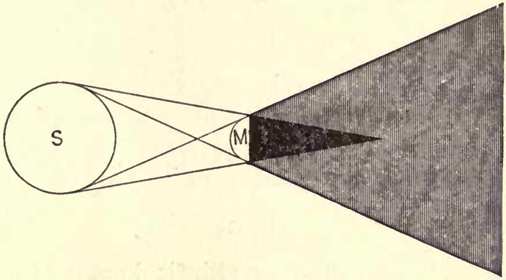

FIG. 148.

If in Fig. 148 S represents the sun and M the moon, there will be a total eclipse on the earth if it moves through the "umbra," and a partial eclipse if it moves through the "penumbra."

140. Pin-Hole Images.—The formation of images in "pin-hole photography" is due to the sharpness of shadows from a point-source of light. If a small hole is made in an opaque screen, any luminous object—e. g. a building in sunshine—situated on one side of it will produce on a screen on the other side an inverted image of itself, which is comparatively sharply defined. Thus, if there is a small opening at O in a screen (see Fig. 149), and A is a point of an illuminated figure, there will be a cone of light from A passing through the opening. If this meets a screen, it will make at point B a bright spot, which will have the shape of the opening. If the opening is small and the two screens are close together, the spot at B will be practically a point of light; and hence, as each point of the illuminated figure produces a point of light on the screen, there will be formed a well-defined image. As the images of the various points of the illuminated object overlap, the general appearance of the image is almost independent of the shape of the opening,

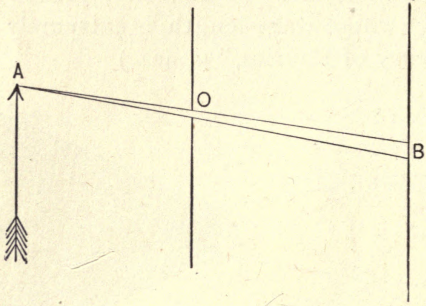

FIG. 149.

if it is small. (The round or elliptical spots of light which are seen on the floors of rooms near curtained windows or under trees are images of the sun formed by minute openings in the curtains or leaves.)

CHAPTER XIII

REFLECTION OF LIGHT

141. Introduction.—When waves of any kind meet the boundary separating the medium in which they are travelling from one in which they have a different velocity, they are always in part reflected and in part transmitted by the second medium. Since the velocity of ether-waves depends upon the matter which is embedded in the ether, reflection will take place when radiation in air falls upon glass, water, metal surfaces, etc. If the surface is irregular, the waves are scattered in all directions, e. g. from an ordinary sheet of paper, from the walls and floor of a room, from a cloud of dust, etc. This is called irregular reflection.

The blue color of the sky depends upon this kind of reflection. If an object, e. g. a particle of dust, is very small, it will reflect only very short waves, the others will pass around it. Thus, if there is a cloud of fine particles in the upper atmosphere, they will reflect the short ether-waves which produce the sensation blue in our eyes, but will allow the other waves to pass on. (Perhaps even a cloud of particles is not necessary; the portions of air may be sufficient.) Other illustrations are the blue color of distant mountains, the blue color of the ocean—at least partially, the blue color of fine smoke when seen by reflected light.

142. Spherical Waves; Plane Mirror.—If, however, the surface is smooth in the sense of having no inequalities comparable with the lengths of the ether-waves, the reflection will be regular and will obey certain laws. Such a surface is called a mirror.

The simplest case, perhaps, is that of waves from a point-source falling upon a plane surface. A point-source emits waves which advance in the form of ever-expanding spherical surfaces—they are therefore said to have a spherical "wave-front," or are called "spherical waves" (see Article 56). Let such a spherical wave-front proceed out from the point-source O (Fig. 150); it will meet the surface at A, where $\overline{OA}$ is perpendicular to the plane surface. The waves will then be reflected. If there had been no reflecting surface, the wave-front *would* have reached the position

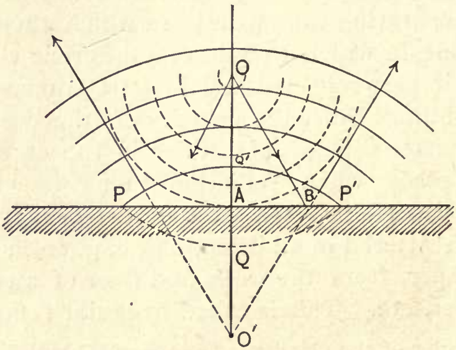

FIG. 150.

PQP' after a definite time; but owing to reflection, the disturbance has the wave-front $PQ'P'$, where this circular arc is simply the arc PQP' inverted. This new wave-front will therefore proceed back from the surface exactly as if it came from a point O' below the surface where $\overline{OO'}$ is a line perpendicular to the surface and bisected by it. O' is called the "image" of O. It is said, further, to be a "virtual" image, because the waves diverging from O do not converge towards O', but diverge from it after reflection.

This statement of the laws of reflection must not be considered a proof or an explanation. It is simply a *description* of what takes place. A proof could be given showing that this law is a necessary consequence of wave-motion; but it is too complicated for discussion here.

It should be noticed that by reflection the path of a ray $\overline{OB}$ is changed into $\overline{BC}$, which is a prolongation of $\overline{O'B}$. And since $\overline{OA} = \overline{AO'}$, it follows from geometry that the angles OBA and CBP' are equal.

Thus, if there is a luminous object AB near a plane mirror (see Fig. 151), each of its points will have virtual images below the surface, forming an image $A'B'$ of the object, which will be of the same size as the object, and symmetrically placed with reference to the surface. An observer looking at the mirror, therefore, will see the reflected image, on the opposite side apparently.

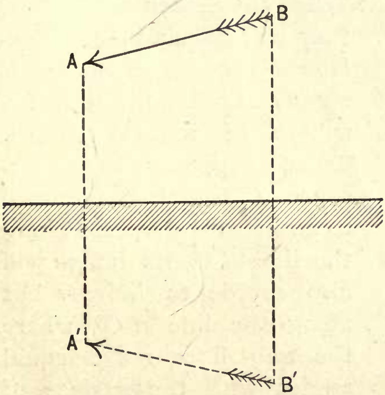

FIG. 151.

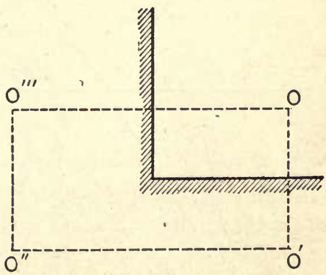

FIG. 152.

If two plane mirrors are inclined to each other at an angle, e. g. if they form a right angle, and if a point-source, such as a candle, is placed in the angle between them at O, Fig. 152, there will be several reflections and several images. The spherical waves falling upon one surface are reflected and proceed back as if they came from O'''. These reflected waves fall upon the second mirror; and, since they apparently come from O''', they will be reflected from this second surface and proceed back as if they came from O'', where $\overline{O'''\,O''}$ is a line perpendicular to the second surface and bisected by its plane. Similarly, waves from O will fall upon the second surface, be reflected, and proceed as if they came from O'; then they will fall upon the first mirror and form an image at O''. (The two "second-images" coincide because the angle between the mirrors is an aliquot part of 360°.) Thus, if a person looks into the angle between the mirrors, the source O is seen and three images; that is, apparently, four sources. If the angle between the mirrors had been $\frac{360°}{n}$, where n is any whole number, there would have been seen n apparent sources, the source itself and $n-1$ images. This is the principle of the kaleidoscope, in which two plane mirrors are inclined to each other at an angle of 60°.

143. Plane Waves; Plane Mirror.—A particular case of spherical waves is that of waves proceeding from a point at an infinite distance away. When they reach the mirror, they will have a plane wave-front, because a plane is simply a small portion of a sphere of infinite radius. Thus, suppose the point-source O has been removed to an infinite distance along the line $\overline{AO}$, its image will also recede to infinity but along the line $\overline{AO'}$, where the two lines make equal angles with the surface of the mirror, because O' is always as far below the surface as O is above it. The plane waves from O, therefore, which fall upon the mirror will all be perpendicular to $\overline{OA}$ before reflection and to $\overline{O'AP}$ after reflection, where $\overline{AP}$ is the prolongation of $\overline{O'A}$. ($\overline{OA}$ is called a "wave-normal" of the incident waves, because it is perpendicular to the wave-fronts; and similarly $\overline{AP}$ is a wave-normal of the reflected waves.) The angle made between the normal to the incident waves and the normal to

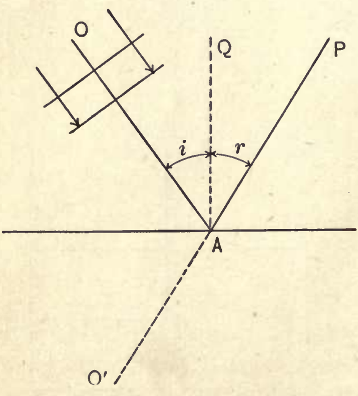

FIG. 153.

the surface, i. e. the angle OAQ, is called the "angle of incidence" (i); and that between the normal to the reflected waves and the normal to the surface, i. e. PAQ, is called the "angle of reflection" (r). Since the lines $\overline{OA}$ and $\overline{O'A}$ make equal angles with the plane surface, the angles i and r are equal. This is the same law as that noted in the preceding article for the reflection of a "ray." A simple illustration of this kind of reflection is that of sunlight falling directly on a plane mirror.

144. Spherical Mirror.—Another interesting case of reflection is that of spherical waves reflected from a spherical mirror. Let a point-source be at O, and let the spherical

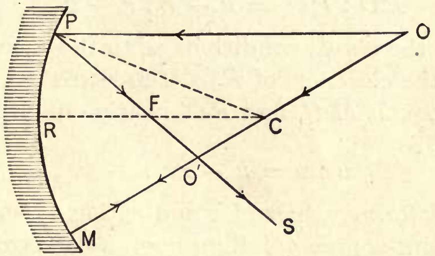

FIG. 154.

surface be a concave one, with its centre at C. Draw the straight line $\overline{OC}$, and let it meet the surface at M. From P, a point near M, draw the lines $\overline{PO}$, $\overline{PC}$, and $\overline{PS}$, making the angle SPC equal to the angle OPC. The point O', the intersection of $\overline{OM}$ and $\overline{PS}$, is the image of O; that is, it is the point to which the waves diverging from O converge after reflection at the surface. This may be proved as follows: The portion of the waves from O which meet the surface at P may be considered as a ray incident on a tiny plane surface; and therefore it will be reflected at an angle such that the angles of incidence and reflection are equal, viz. along $\overline{PS}$. For the radius $\overline{PC}$ is perpendicular to the tiny plane surface at P; and $\overline{PS}$ has been drawn so that the angles $O\,P\,C$ and $S\,P\,C$ are equal. The ray along $\overline{O\,M}$ will be reflected directly back, because the line is perpendicular to the mirror. Hence O', the intersection of $\overline{P\,S}$ and $\overline{O\,M}$, is the point to which the two rays come. Further, *all* rays from O, after reflection at other points of the surface, have directions which pass through O', provided the surface is only slightly curved and that only a small portion of the mirror around M is used. For, since the line $\overline{P\,C}$ bisects the angle $O'\,P\,O$,

$$\overline{PO} : \overline{PO'} = \overline{C\,O} : \overline{O'\,C};$$

or, putting $$\overline{M\,O} = u,\ \overline{M\,O'} = v,\ \overline{M\,C} = r,$$

$$\overline{P\,O} : \overline{P\,O'} = u - r : r - v.$$

If, however, the above conditions as to the curvature of the mirror and the closeness of P to M are satisfied, the distance $\overline{P\,O}$ nearly equals $\overline{M\,O}$, and $\overline{P\,O'}$ nearly equals $\overline{M\,O'}$. That is,

$$u : v = u - r : r - v.$$

Hence for definite values of u and r, that is for waves from a definite point-source O falling upon a concave mirror with the radius r, the value of v, that is, the position of the image O', is independent of P and may be calculated.

The equation for v may be put in the form

$$\frac{u - r}{u} = \frac{r - v}{v},$$

or $$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}.$$

A simple geometrical method for determining O' is as follows: Draw $\overline{O\,C\,M}$ through the centre of the mirror C; draw $\overline{O\,P}$ to any point P near M, and $\overline{C\,R}$ through C parallel to it; draw a line $\overline{PF}$ so as to bisect $\overline{C\,R}$ at F; where this line intersects the line $\overline{O\,M}$ is the image O'. For, as has just been shown, O' lies on the line $\overline{O\,M}$ and on $\overline{PS}$, where the angles SPC and OPC are equal; and the intersection of $\overline{PS}$ with $\overline{CR}$ may be proved to bisect the line $\overline{CR}$. The angles RFP and FPO are equal, and RFP equals the sum of FPC and FCP; hence FCP equals the difference between FPO and FPC, i. e. CPO. But CPO and FPC are equal (angles of incidence and reflection): therefore FCP equals FPC, the triangle (CFP) is isosceles, and the sides $\overline{FC}$ and $\overline{FP}$ are equal. P is supposed to be close to M, and therefore to R; and $\overline{FP}$ nearly equals $\overline{FR}$. Consequently $\overline{FC}$ equals $\overline{FR}$. Q. E. D.

A special case is when u is infinite; that is, when the incident waves are plane, with their wave-normal parallel to $\overline{OCM}$; hence $v = \frac{r}{2}$, or O' bisects the line $\overline{CM}$. This point is called the "principal focus." Conversely, if a point-source is at the middle point of $\overline{CM}$, i. e. if $u = \frac{r}{2}$, the reflected waves will be plane and will have their wave-normal along $\overline{CM}$.

Thus, if there is an illuminated object $\overline{ON}$, its image will be $\overline{O'N'}$, as shown in Fig. 155, where F bisects the line $\overline{CR}$. Each point of $\overline{ON}$ gives rise to an image at a point of $\overline{O'N'}$;

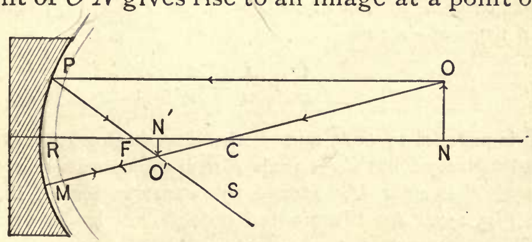

FIG. 155.

and if $\overline{ON}$ is perpendicular to the line $\overline{NCR}$, $\overline{O'N'}$ will be also, if $\overline{ON}$ is small. The image in this case is called a "real" one, because the waves converge towards it after reflection; and, if a screen is placed at $\overline{O'N'}$, a sharp image will be formed on it. The distances of object and image from the mirror satisfy the equation $\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$. If, then, $u < \frac{r}{2}$, v is negative, which means that O' is on the other side of the mirror; and the image is then virtual, for the waves from O diverge, after reflection, as if they came from O'.

The same proof may be carried through for a convex mirror. Let O be the source, C the centre of the spherical surface, P any point of the

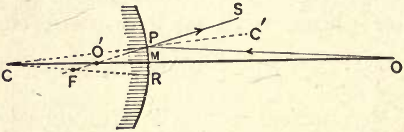

FIG. 156.

surface near M, the intersection of $\overline{OC}$ with the surface; draw $\overline{OP}$ and $\overline{PS}$ so as to make equal angles with the radius $\overline{CPC'}$. Prolong $\overline{SP}$ backward and let it meet $\overline{OC}$ in O'; then O' is the virtual image of O. Further, by geometry,

$$\overline{PO} : \overline{PO'} = \overline{CO} : \overline{O'C}.$$

Call, as before, $\overline{MC} = r$, $\overline{MO'} = v$, $\overline{MO} = u$; and the equation becomes $u : v = u - r : r - v$.

Hence, $$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}.$$

If $\overline{CR}$ is parallel to $\overline{OP}$, and F the intersection of $\overline{SO'}$ and $\overline{CR}$, it is proved with ease that $\overline{CF}$ equals $\overline{FR}$, if the curvature of the surface is slight and P is near M. Hence the construction of images is as shown in Fig. 157. An illuminated object $\overline{ON}$ has a virtual image $\overline{O'N'}$. The distances of the object and image from the mirror are given, as before, by the relation

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r},$$

or $$\frac{1}{v} = \frac{2}{r} - \frac{1}{u}.$$

Hence, so long as u is negative, i. e. so long as the waves diverge from a point on the *convex* side of the mirror, v is positive or the image lies on the same side as the centre of the mirror C. A special case is

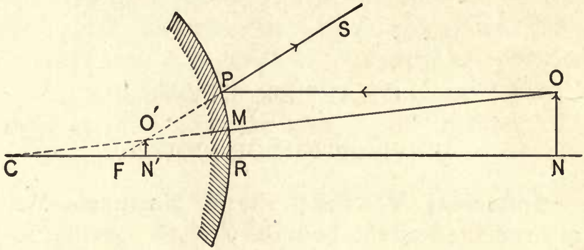

FIG. 157.

when u is infinite, that is, when plane waves fall upon the mirror with the wave-normal $\overline{OMC}$; $v = \frac{r}{2}$, and the image is therefore a point bisecting the line $\overline{CM}$. This is called the "principal focus."

If the curvature of the mirror is not small, the waves reflected from different portions of the surface do not all have the same image; and their different images lie on a

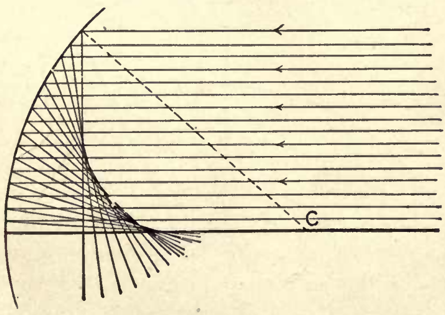

FIG. 158.

surface called a "caustic." This is noticeable in the reflections formed by the concave surfaces of cups and goblets, as seen when nearly full of milk or coffee. This phenomenon is called "spherical aberration." An illustration is given in Fig. 158 of plane waves falling upon a large concave mirror.

CHAPTER XIV

REFRACTION OF LIGHT

145. Spherical Waves; Plane Surface.—Whenever waves are reflected at the boundary of two media, in which therefore the waves have different velocities, a disturbance of some kind is always produced in the second medium. If the waves are absorbed in the second medium, the disturbance is confined to the surface; but, if the medium is transparent to the waves, they will proceed on and be transmitted. These entering waves are called "refracted" waves, for reasons to be explained later.

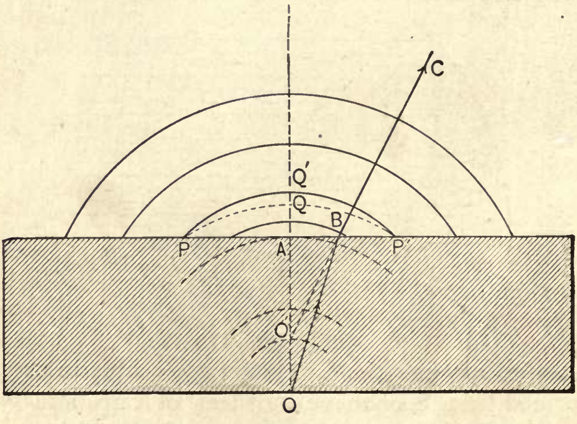

FIG. 159.

One simple case is that of spherical waves from a point-source falling upon a plane surface. Let the source O be in the medium in which the waves have the less velocity, which may therefore be called the "slow medium;" and let the waves meet the surface of the "faster medium" at A (see Fig. 159); $\overline{OA}$ is therefore perpendicular to the surface. If the velocity of the waves in the two media was the same, the wave-front would advance in a definite time to the position PQP'; but, since the new medium is "faster," the disturbance reaches Q' instead of Q, and the actual curve of the entering waves is $PQ'P'$, an arc of a circle whose centre O' is on the line $\overline{OA}$, but nearer the surface than O. O' is then the virtual image of O.

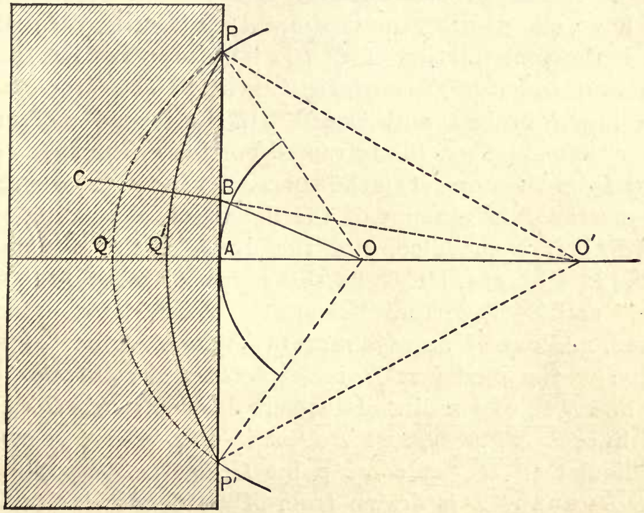

FIG. 160.

This, again, is not a proof or an explanation, but merely a description of what happens. If one looks into a glass of water from above at a small object lying on the bottom, it appears to be at a less distance below the surface than the actual depth of the water; its image is therefore virtual and nearer the surface. The exact law of refraction can be deduced from the theory of wave-motion, but it is too complicated for discussion here.

Consider also the case where the source O is in the "faster medium;" the spherical waves meet the surface at A, the foot of the perpendicular from O upon the plane surface (see Fig. 160). The waves entering the other medium would have the form PQP' if the velocity were unchanged; but, since the medium is "slower," the actual curve is $PQ'P'$, an arc of a circle whose centre is O', a point on $\overline{OA}$, but farther away from the surface than O. O' is, as before, the virtual image of O.

146. Index of Refraction.—A ray from O, e. g. $\overline{OB}$, will have, on entering the medium, the direction $\overline{BC}$, where $\overline{BC}$ is the prolongation of $\overline{O'B}$. The ray thus has its direction changed at the surface; it is said to be "refracted," meaning "broken and bent." If the second medium is faster than the first, the refracted portion of the ray is bent *away* from the normal to the surface at the point where the ray meets it, the amount of the deflection depending upon the difference in velocity of the waves of the two media. Thus, in Fig. 161, let O be the source of waves, the "object," and O' its virtual "image." O sends out spherical waves, and one of its rays meets the plane surface of separation of the media at B, is refracted and continues on in the line $\overline{BC}$, as a radius of a sphere drawn around O'. Draw the lines $\overline{CA'}$, $\overline{A'D}$, and $\overline{BD}$ as in Fig. 161; $\overline{CA'}$ is perpendicular to $\overline{BC}$ at some point C; $\overline{BD}$ is perpendicular to $\overline{OB}$; and $\overline{A'D}$ is drawn from A' perpendicular to $\overline{BD}$. $A'D$ is parallel to $\overline{OB}$, and is the distance the disturbance goes in the slower medium in the direction $\overline{OB}$, while the disturbance in the faster medium goes $\overline{BC}$.*

$$\text{Therefore} \quad \frac{\overline{BC}}{A'D} = \frac{\text{velocity in faster medium}}{\text{velocity in slower medium}}.$$

* For $\overline{BD}$ can be considered as the trace of a plane-wave incident on the surface in the direction $\overline{OB}$; after refraction it has the direction $\overline{O'B}$ and the wave-front $\overline{CA'}$. Therefore while the disturbance has gone from D to A', that at B has gone to C, as the velocities in the two media are different.

This fraction is called the "index of refraction," and may be measured by many methods. (See Ames's "Theory of Physics," p. 430.) It is ordinarily given the symbol μ.

That is, $$\mu = \frac{CB}{A'D}.$$

But the triangles BCA' and BAO' are similar, as are also the triangles $BA'D$ and ABO. Hence

$$\frac{BC}{BA'} = \frac{AB}{O'B} \text{ and } \frac{BA'}{A'D} = \frac{BO}{AB}.$$

Therefore $$\mu \left(= \frac{CB}{A'D}\right) = \frac{BO}{O'B}.$$

But if B is very near A, $O'B = O'A$ and $OB = OA$.

Hence $$\mu = \frac{AO}{AO'}.$$

Thus, if μ is known, and the position of the "source" O below the surface is given, the position of O' may be calculated. A ray OB and its prolongation may therefore be drawn. Further,

$$OO' = AO - AO'$$
$$= AO\left(1 - \frac{1}{\mu}\right).$$

This is the apparent elevation of an object owing to refraction. If μ is large, OO' is large; and *vice versa*.

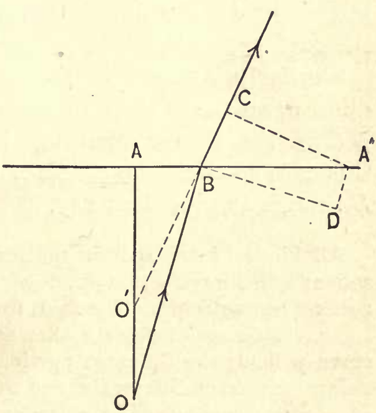

FIG. 161.

The angle which the incident "ray" $\overline{OB}$ makes with the normal to the surface at B—an angle equal to $A'BD$—is called the "angle of incidence;" the angle which the refracted ray $\overline{BC}$ makes with the normal to the surface at *B*—an angle equal to *A' B C*—is called the "angle of refraction."

If the angle of incidence is increased gradually, an angle will be reached for which the ray emerging into the faster medium will just graze the surface, i. e. the angle of refraction is 90°. If the incident ray is inclined still more to the surface there will be no *refracted* ray; the incident ray will be entirely *reflected* at the surface. The angle at which this "total reflection" begins to take place is called the "critical angle" for the two media. It is evidently a property of the index of refraction. This phenomenon is observed if one looks into the side of a tumbler of water, holding it so that the eye looks up towards the free surface of the water; it will be noted that nothing will be seen *through* the surface, and that it acts like a perfect mirror.

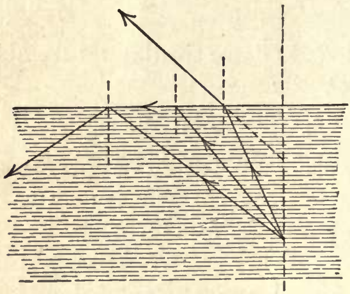

FIG. 162.

Similarly, a ray $\overline{OB}$ (Fig. 160) incident in the faster medium upon the surface of separation will have the direction $\overline{BC}$ in the slower medium, the refracted ray being bent in this case so as to come nearer the normal to the surface at *B* than is the incident ray.

All kinds of transparent matter refract light—gases, liquids and solids; but the refraction due to air and all gases is small, and may in general be neglected. The fact, though, that air refracts is shown by the unsteadiness of objects when seen through air rising from a hot stove or field; the different portions of the air being at different temperatures refract differently, and the effect is the same as if a pane of glass which is uneven and "streaky" is moved up and down in front of the eye. The refraction of air is important too in all astronomical observations.

The fact that the waves travel with different velocities in layers of air at different temperatures is shown by the fact that light is *reflected* by such layers. The explanation of mirage and similar phenomena depends upon this.

147. **Dispersion.**—If white light is used in any of these refraction experiments, it will be seen that the images and refracted waves appear colored. This is because waves of different periods (or wave-lengths)—which therefore produce color sensations in our eyes (as shown in Article 136)—have different velocities when the ether is loaded with matter. If the matter is glass or water, the longer waves (i. e. "red ones") have a greater velocity than the shorter ones (i. e. "blue ones"), as is shown by the fact that the red waves are bent less than the "blue" ones on passing from air into glass or water. (If two objects, one painted red, the other blue, are placed on the bottom of a pail of water, the blue one will appear to be the higher of the two—nearer the surface.) Consequently, the images in the case of spherical waves incident on a plane surface will be at different points for different colors; and, in terms of rays, the "indices of refraction" will be different. Further, each train of waves of definite wave-length will have its own critical angle; and, if white light is incident from the slower medium at an angle which is the average of the critical angles of all the waves which produce color sensation, some of the waves may emerge grazing along the surface, while the others will be totally reflected—so both beams will appear colored.

This dependence of the index of refraction upon the period (or wave-length) of the waves is best shown by a "prism," which is a portion of transparent matter bounded in part by two plane surfaces. The line in which these surfaces meet (produced if necessary) is called the "edge" of the prism, and a section at right angles to the edge is given in Fig. 163. If a ray is incident upon one of the plane faces of the prism, as shown, the refracted ray will be bent towards the normal to the surface (if the velocity is slower in the prism than in the air, as it is in glass prisms); and, where the ray emerges from the second face, it is bent away from the normal to that face. But the amount of the change of direction between the incident and the emerging rays, the "deviation" so called, the angle D in Fig. 163, is different for waves of different wave-length, being greater for short

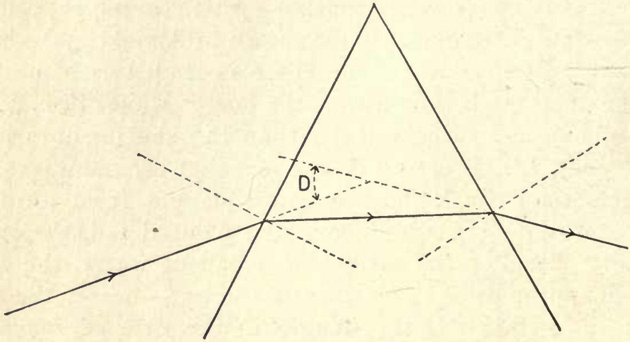

FIG. 163.

waves than for long ones, as is easily shown. Consequently, when white light is incident upon the prism, the emerging light is broken up into colors, the red being less deviated than the blue. This breaking up of white light into its components is called "dispersion;" and the "dispersion of two colors" is the difference between their deviations.

148. Lenses.—The most interesting case of spherical waves refracted at a spherical surface is that of the passage

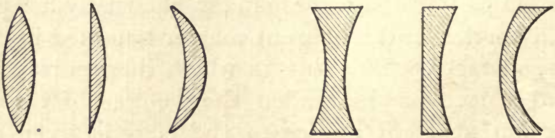

FIG. 164.

of these waves into and out of lenses. A spherical lens is a portion of transparent matter—generally glass—which is bounded by two spherical surfaces, the line joining the centres of the two spheres being an axis of symmetry for the lens. Several illustrations are given. (A plane is a sphere of an infinite radius.)

If there is a point-source of light on or near the axis of a lens, i. e. the line joining the centres of the two spherical

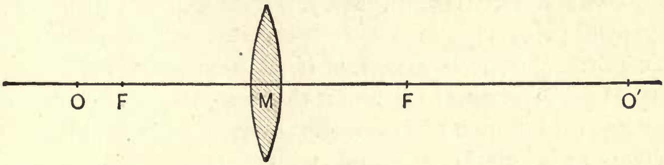

FIG. 165.

surfaces, the waves, after passing through the lens, will cause a point-image, also on or near the axis. (See Ames's "Theory of Physics," p. 440.) The proof of the formulæ for lenses is quite complicated, and there is no attempt made here to give it. One formula, however, holds for all cases, if a

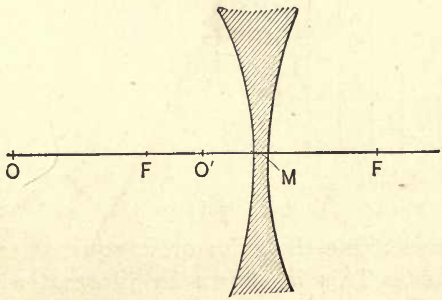

FIG. 166.

proper interpretation is given the quantities. The lens is supposed to be very thin; so it makes no difference from which side of the lens distances are measured. If O is the point-source at a distance u $(= \overline{OM})$ from the lens, the image O' will be at a distance v $(= \overline{O'M})$ where

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f},$$

f being a constant for any one lens and for any one definite wave-length. u is positive if O is on the opposite side of the lens from the centre of the surface which the waves meet first, i.e. if O is on the convex side of the surface; v is taken positive if O' is on the opposite side of the lens from the centre of the second surface; f is with ordinary lenses essentially a positive quantity. (See Ames's "Theory of Physics," p. 441.)

In particular, if $u = \infty$, i. e. if plane waves with the axis of the lens as normal fall upon the lens, $v = f$; v is therefore positive, and hence O' is on the convex side of the second surface at a distance equal to f. (Similarly, if $v = \infty$, $u = f$.) Hence, there are two points (F, F) on the axis at equal distances from a lens, such that they are the images

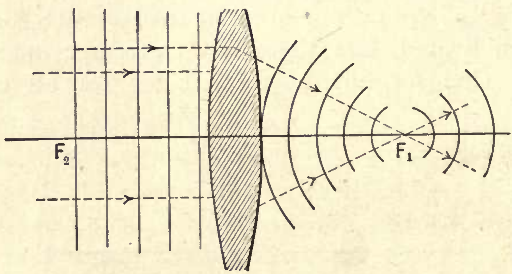

FIG. 167

for plane waves along the axis; or, as sources, they give rise to plane waves. This is shown by illustrations, Figs. 167, 168. In these figures the course of the wave-fronts before and after meeting the lens is shown; and lines at right angles to the wave-fronts show the paths of disturbance, or "rays."

1. Double convex lens, $u = \infty$; $v = f$, i. e. O' (or F_1) is on the convex side of the second surface. F_1 is a real image of the point-at-infinity; and if F_1 is a real source, it will produce plane waves along the axis. It is seen also that a "line

of disturbance," as indicated by the arrow-heads, will, if parallel to the axis of the lens, pass through the principal focus on the opposite side; and, conversely, a "ray" through the principal focus on one side of the lens becomes parallel to the axis on the opposite side.

2. Double concave lens, $u = \infty$; hence $v = f$, a positive quantity, and O' (or F_1) is on the convex side of the second surface (i.e. on the concave side of the first). F_1 is a *virtual* image of the point-at-infinity; and, conversely, if waves are made to converge from the right *apparently* towards F_1, they will be modified by the lens so as to emerge from the lens as

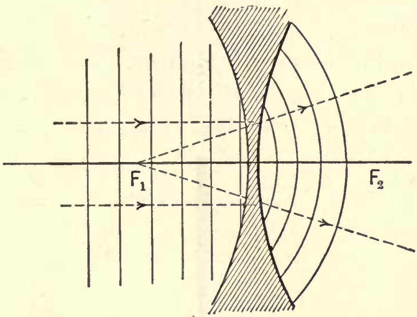

FIG. 168.

plane waves and with their wave-normal parallel to the axis —F_1 would be called a "virtual source" in this case. A line of disturbance parallel to the axis on one side is pointed on the other side *as if* it came from the principal focus on the first side; and, conversely, a ray pointed towards the focus on the opposite side of the lens will emerge on that side parallel to the axis.

Diagrams may be drawn in the same manner for parallel waves proceeding from the opposite directions; these would have images F_2 at the same distance from the lens as is F_1. These points are called "principal foci," and their distances from the lens are called the "focal length." A lens of the first kind is called "converging," and one of the second kind "diverging," for reasons apparent from the figures. Several illustrations are given (Figs. 169, 170, 171, 172, 173)

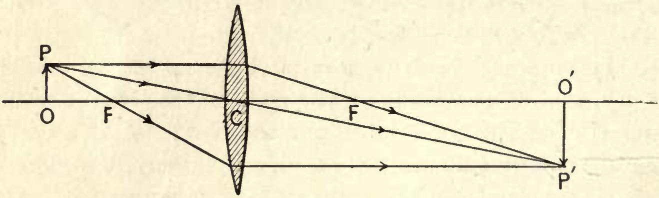

FIG. 169.

Converging lens.
Real object, $\overline{OP}$.
Real image, $\overline{O'P'}$.

of the formation of images by lenses. The general principle followed is to trace lines of disturbances parallel to the axis and pointed towards principal foci, for their paths are

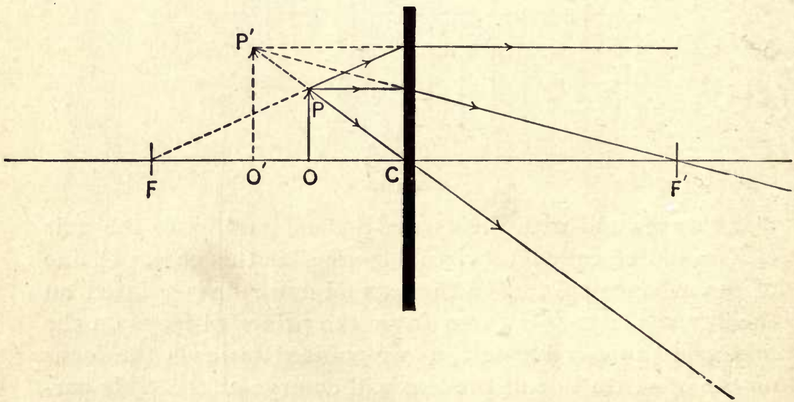

FIG. 170.

Converging lens, represented by a heavy line.
Real object, $\overline{OP}$.
Virtual image, $\overline{O'P'}$.

known from Figs. 166 and 167. Another line of disturbance is known also, viz. the one through the point where the lens is cut by the axis; for at that point the two surfaces of the lens are parallel and form a transparent plate with parallel faces very close together, if the lens is thin. Therefore a line of disturbance through this point does not change its direction. It suffers no refraction.

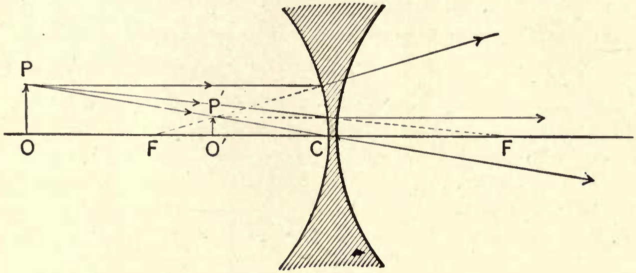

FIG. 171.

Diverging lens.
Real object, $\overline{OP}$.
Virtual image, $\overline{O'P'}$.

In this manner the image of a point P is determined; and corresponding to each point of the object $\overline{OP}$ there will be a point in the image. Thus, if $\overline{OP}$ is perpendicular to the

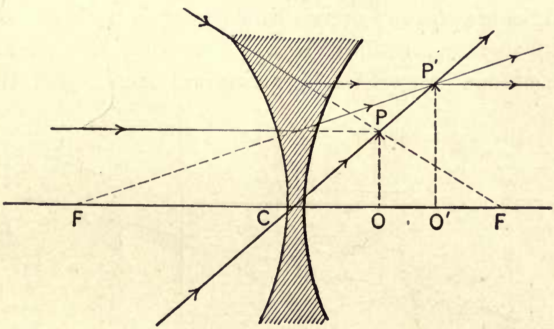

FIG. 172.

Diverging lens.
"Virtual object," $\overline{OP}$; that is. the incident waves would form this image if the lens were removed; but, owing to its presence, they form a real image, $\overline{O'P'}$.

axis of the lens, the image $\overline{O'P'}$ will also be perpendicular, if the object and image are both small.

Many applications are made of lenses, either singly or in combinations, e. g. microscopes, telescopes, photographic lenses, etc.

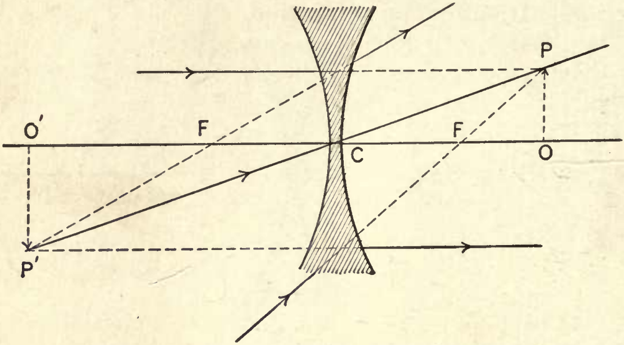

FIG. 173.

Diverging lens.
Virtual object, $\overline{OP}$.
Virtual image, $\overline{O'P'}$.

149. Microscope. — A microscope is an instrument designed to increase the apparent size of a small object. The simple magnifier, or reading-glass, is a single converging lens, used as in Fig. 170. The object to be viewed must be placed between the principal focus and the lens,

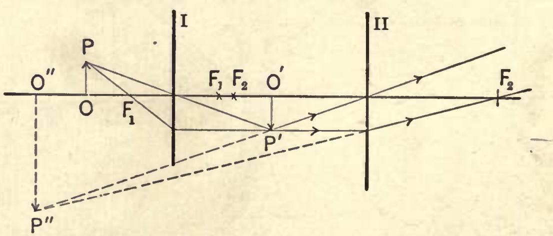

FIG. 174.

and then a virtual magnified object of the image will be seen; the image is not inverted.

The compound microscope is a combination of two converging lenses (or systems of lenses) arranged as shown in

Fig. 174. The object $\overline{OP}$ is placed just beyond the principal focus F_1 of lens I, which forms a real inverted image inside the focus of lens II; this therefore causes a virtual image to be seen. If the object $\overline{OP}$ is near F_1, the principal focus of the first lens, the first image $\overline{O'P'}$ will be magnified; and therefore the second image $\overline{O''P''}$ will be greatly magnified.

150. Telescope.—A telescope is an instrument designed to make distant objects appear nearer, that is, to increase

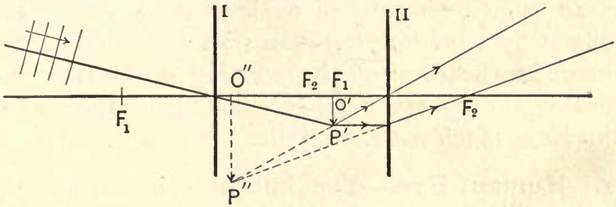

FIG. 175.

their apparent size. There are two forms of refracting telescopes, as shown in the two figures, 175, 176.

The first form consists of two double convex lenses. A real inverted image of the distant object is formed by the first lens

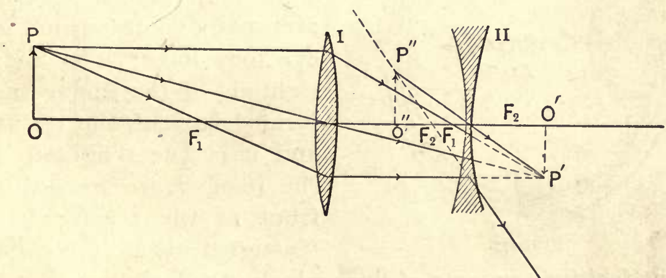

FIG. 176.

inside the principal focus F_2 of the second lens; this is then magnified into a virtual image $\overline{O''P''}$, which is still inverted.

The second form consists of a double convex lens, *I*, and a double concave one, *II*, as shown in Fig. 176. The first lens would form a real image at $\overline{O' P'}$, but the waves meet the diverging lens *II*, and the virtual image $\overline{O'' P''}$ is formed. (This is a combination of Figs. 169 and 173.) The final image $\overline{O'' P''}$ in this case is not inverted. This is the construction of the ordinary opera-glass.

Telescopes may also be formed by the combination of a concave mirror and a lens. One plan is to have the image due to the concave mirror reflected out one side of the telescope tube by means of a plane mirror, and then to magnify it by a magnifying-glass; another plan is to form the image at the edge of the opening of the tube, so that the head of the observer who is looking at the image will not interfere much with the light.

151. Human Eye.—The human eye consists fundamentally of a lens whose real focal length can be varied by muscles (see Fig. 177). An image of an object may thus be formed on the retina, and the sensation of sight is produced. There are many faults which an eye may have; it is "far-sighted" if the images are formed back of the retina; and it is "near-sighted" if the images are formed in front of the retina; it is "astigmatic" if it has different foci for two sets of lines which are at right angles to each other; then there are muscular faults, etc.

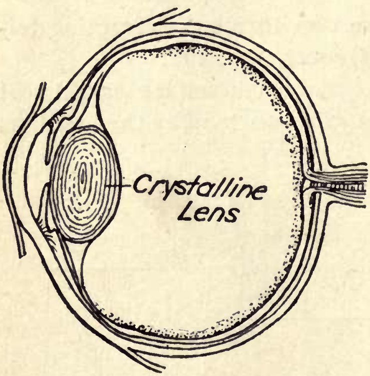

FIG. 177.

152. Chromatic Aberration.—As was stated in Article 148, the focal length of a lens, the quantity *f*, is a constant for a given lens and for a definite train of waves; but, if waves of different wave-length are used, there will be different values of f. In other words, different colors have different focal lengths, and therefore different foci. So, if white light is used, the image will be a series of colored points along the axis, the end nearest the lens being blue; that farthest away, red. This is known as "chromatic aberration." All lenses have "spherical aberration" also (see Article 144) if other portions of the lens are used than those close to the axis. (For this reason diaphragms are used in photographic lenses.)

Chromatic aberration may be partially corrected in a converging lens by combining it with a diverging lens of a different material whose dispersive action (see Article 147) for two particular colors is the same as that of the first lens. Thus the separation of these colors by the first lens will be neutralized by the action of the second; and the two corresponding trains of waves will be brought to one focus, while the other colors will be dispersed slightly. Such a lens—made up of the two—is called an "achromatic" one.

CHAPTER XV

DISPERSION

153. Pure Spectrum.—As explained in the last chapter, white light is broken up into component parts on passing through a prism; the different trains of waves are said to be dispersed. If these dispersed waves are received on a screen, the colored image is called a "spectrum." If light from any large object, such as a round opening in an illuminated screen,

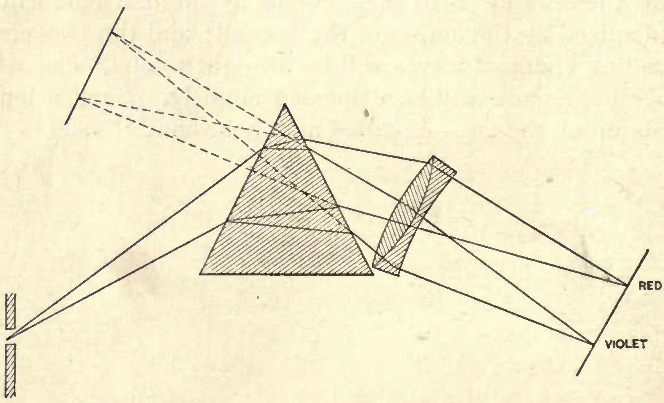

FIG. 178.

is allowed to fall on a prism, the spectrum is "impure;" that is, different colors will overlap in the spectrum, as each point of the luminous object produces a spectrum of its own. Three things are essential for the production of a *pure* spectrum, viz. a prism, a *small* opening, e. g. a narrow slit parallel to the edge of the prism, and a converging lens to focus on the screen the waves after they come through the prism. The arrangement is as shown in Fig. 178. Such an apparatus is called a spectroscope, and enables one to analyze various sources of light and discover their component trains of waves.

An improved form of spectroscope is shown in Fig. 179. The slit S is placed at the principal focus of an achromatic

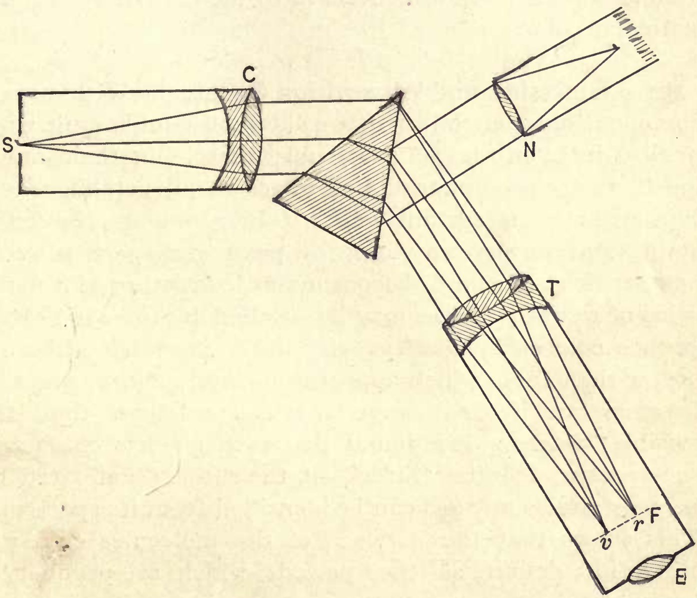

FIG. 179.

converging lens C, so that the waves falling on the prism have a *plane* wave-front; after passing through the prism and being dispersed by it, they meet an achromatic converging lens T which focuses the spectrum along a line F in the tube which holds T; this line F is just inside the principal focus of a magnifying eye-piece at E; and so the spectrum is greatly magnified. There is also a tube N which carries at one end a transparent scale—a line divided into equal parts and numbered—and at the other a converging lens at whose principal focus the scale is. When a lamp is placed close behind the transparent scale plane waves fall upon the face of the prism, are *reflected*, and brought to a focus in the same line *F*. Consequently, crossing the spectrum along that line there will be a series of parallel, equally spaced and numbered scale-lines, which serve to identify or locate any particular color.

154. Emission and Absorption Spectra.—With a spectroscope it may be shown that solids and liquids emit what is called a "continuous" spectrum; that is, in their radiation there are present trains of waves of all possible wave-lengths between certain limits. If gases are rendered luminescent in any way, e. g. by passing a spark through them, their spectra are discontinuous, consisting of isolated trains of waves. These may be studied by the eye if they produce colors; by photography if their wave-lengths are shorter than those which appeal to the eye; and by suitable thermometers if their wave-lengths are longer than the "visible" waves. It is found that each gas has characteristic spectra, definite "lines"—if the slit is illuminated by the gas; and so any gas can be identified from its spectrum. (This shows that the portions of the molecules of a gas vibrate in a definite series of periods, which are peculiar to that gas.)

If between the slit and a source of white light, e. g. a white hot carbon of an arc-lamp, there is interposed some liquid or gas, certain waves may be absorbed by the liquid or gas; and the waves which are absent in the resulting spectrum are called the "absorption spectrum" of that liquid or gas. As might be expected from the ideas of resonance (see Articles 84 and 104), the absorption spectrum of a gas is identical with its emission spectrum if this is due to rise of temperature. For instance, if sodium (or common salt) is put in a hot flame, it will vaporize and form sodium vapor. Its spectrum is mainly two brilliant lines in the yellow; if, however, white light from a source which is at a *higher temperature* than the vapor is passed through the vapor, its absorption spectrum is seen to be due to the absence of these same two lines in the yellow—their waves have been absorbed, while the other waves have been transmitted. The spectrum of sunlight is found to be an absorption spectrum, for nearly all the "dark lines"—the absent ones—agree exactly with the bright lines in the spectra of well-known vapors, e. g. iron, sodium, etc. This proves that the sun consists of a nucleus emitting a continuous spectrum and surrounded by an atmosphere of vapors of iron, sodium, etc., the temperature of the nucleus being higher than that of the atmosphere. Some stars have absorption spectra; others emission, i. e. bright line spectra. Some lines in the spectrum of sunlight are due to absorption by the water-vapor and oxygen in the atmosphere of the earth, through which the waves have passed.

CHAPTER XVI

OTHER PHENOMENA OF LIGHT

155. Color.—The color of an object may depend upon several things, meaning by the word "color" the sensation which an eye gets when looking at the object. The general illumination of the room affects colors; a cloth may appear red in white light, but in green light it will appear black. This is because the color of almost every body depends upon the fact that when the ether-waves fall upon it they enter it a slight distance (unless it is highly polished or metallic); absorption of certain waves takes place inside; and only the unabsorbed waves are reflected, return to the surface and emerge. Thus the color depends upon the nature of the light which falls upon the body and the nature of the absorption in the interior of the body. This is termed "volume absorption." The colors of carpets, flowers, woods, etc., are due to this absorption.

If white light passes through one kind of colored glass or colored liquid, only certain waves are transmitted; and, if they in turn fall upon a second and different piece of glass or layer of liquid, the emerging waves are those which are left over after the two absorptions. Thus the color obtained by mixing two colored paints depends upon the waves which are not absorbed when white light has been made to pass through the two in succession; a paint which appears yellow does not necessarily absorb all the other colors—faint traces of others may be present, or two or more others which of themselves would combine to form yellow may be in the mixture.

Objects whose color is due to volume absorption should appear the same if light is reflected from them or if light is transmitted through them; e. g. red glass, a green leaf, etc.

There are objects, however, which appear differently in transmitted and in reflected light; such are thin films of all metals and the aniline dyes used in inks and dye-stuffs. When "white light" falls upon a metallic surface, some of the waves are reflected at the *surface*, *without entering;* the others enter, and are transmitted if the film is thin. Thus the reflected light has one color and the transmitted light a different one. For instance, gold foil appears yellow in reflected light, and greenish blue in transmitted. These colors are said to be "complementary," meaning that they are due to waves which, if combined, would produce the sensation of white. Such absorption as this is called "surface absorption." (It may be that the transmitted color is due to "fluorescence." See below.)

In the case of volume-absorption, certain waves lose all their energy; it is given up to the minute portions of matter to which the absorption is due. This energy is generally spent in producing heat-effects, such as rise of temperature, etc. It may happen, however, that the energy lost by the absorbed waves is used in sending out other ether-waves, without rise of temperature; bodies in which this occurs are called "fluorescent." A solution of quinine sulphate is such a body, so are various uranium compounds. If this secondary radiation continues after the absorption ceases, the bodies are said to be "phosphorescent."

As was explained in Article 141, clouds of *fine* particles scatter the short "blue" waves, and thus transmit the others. White light, therefore, passing through such a cloud would have a reddish color, e. g. twilight colors of the western clouds.

Finally, the color of an object in any light may depend upon individual peculiarities of the eye itself; for certain eyes are "color-blind" to some colors, viz. red, green, blue.

156. Polarization.—There are certain phenomena of ether-waves, called "polarization" phenomena, which deserve mention, but which are too complicated for full discussion. (See Ames's "Theory of Physics," p. 487.) They prove that all ether-waves—visible and invisible—are due to *transverse* disturbances; the ether-waves themselves are therefore transverse, i. e. the vibrations of the portions of the ether which carry the waves are in planes at right angles to the direction in which the waves are advancing. If these vibrations are all in parallel straight lines, and, consequently, if through any one line of disturbance, or ray, a plane can be drawn which will include all the vibrations along that line (just like a rope vibrating transversely in one plane), the ether-waves are said to be "plane-polarized." If the vibrations of the individual portions of the ether are in circles, the waves are "circularly-polarized."

TABLE VI

INDICES OF REFRACTION

Air	1.0029
Alcohol	1.363
Glass, soft	1.52
——, hard	1.66
Water	1.34

PART II

SUGGESTIONS TO TEACHERS

INTRODUCTION

In Part II there will be found brief descriptions of demonstrations which are designed to illustrate, not prove, the questions considered in the text proper, together with suggestions as to laboratory experiments and problems.

Demonstrations should accompany every lecture, as far as possible, and should be arranged to instruct rather than to attract attention by any spectacular feature. As a rule, quantitative experiments on the lecture table are not satisfactory, for obvious reasons; but it is possible by means of exceedingly simple apparatus to illustrate nearly all the elementary laws and facts of Physics. This should be the purpose of the lecture.

Laboratory experiments, on the other hand, should be, almost without exception, quantitative; the student should be taught how to measure quantities and to form an idea of the value of his measurements. It is not advisable for him to repeat many of the experiments which he has seen performed in the lecture room, unless it is for the purpose of doing them with increased care and accuracy. There are several good laboratory manuals for elementary students; and the teacher may advise the class to use any book which best suits its needs or the equipment of the laboratory.

Under each of the following chapters there will be found lists of experiments which will be useful in general. The problems and questions which are added to the different sections are to be regarded as suggestions rather than as anything more definite. Teachers should from time to time alter the numbers used in the various problems, and should change their wording. Lists of problems are given in almost every text-book; and they can, in fact, be devised by the teacher as easily as they can be copied from a book. References are made frequently to Ames & Bliss's "Manual of Experiments in Physics," and many additional problems will be found in its articles.

The following books may be recommended to the teacher as suitable for study in preparation for lectures:

P. G. Tait, "Properties of Matter," The Macmillan Co.

T. Preston, "Theory of Heat," The Macmillan Co.

J. J. Thomson, "Elements of Electricity and Magnetism," University Press, Cambridge.

C. A. Perkins, "Outlines of Electricity and Magnetism," Henry Holt & Co.

T. Preston, "Theory of Light," The Macmillan Co.

Poynting and Thomson, "Sound," Griffiths & Co.

Great help may often be obtained in the preparation of lecture-experiments by referring to catalogues of various instrument-makers.

EQUIPMENT OF LECTURE ROOM

A lecture room should be well lighted by windows which can be covered when necessary by black curtains so as to darken the room. It should be fitted with a table, provided with gas and water pipes, with a tank, and with various electrical connections, in case the laboratory is equipped with dynamos or storage batteries.

There should be a sensitive lecture-galvanometer in a suitable place; this should be so arranged as to reflect from its mirror the image of a wire stretched across a flame produced by a lamp, such as an acetylene one, or of the filament of an incandescent electric light, and to focus it upon a translucent screen made of ground glass or waxed paper. (If the space between the screen and galvanometer is curtained, so that the back of the screen is dark, the room itself may be as light as desired. The focusing may be done by means of a lens or by using a concave mirror in the galvanometer.) The lecture apparatus should be kept in adjoining rooms and brought in when required.

A lantern and suitable screen should be procured if possible; otherwise a "porte-lumière" is necessary.

EQUIPMENT OF LABORATORY

The various pieces of apparatus needed for the different experiments should be kept in convenient cases, where the students may obtain them and to which they should be returned.

Many supplies and chemicals are necessary. (For details see Ames and Bliss's "Manual of Experiments in Physics," Appendix I.)

A machine-shop fitted with a lathe and other tools will be found most useful.

The following books will be found useful on the laboratory shelves:

Glazebrook and Shaw, "Practical Physics," Longmans & Co.

Stewart and Gee, "Experimental Physics," The Macmillan Co.

Barnes, C. L., "Practical Acoustics," The Macmillan Co.

Hastings and Beach, "General Physics," Ginn & Co.

Hopkins, "Experimental Science," Munn & Co.

Nichols, Smith and Turton, "Manual of Experimental Physics." Ginn & Co.

INTRODUCTION

LECTURE DEMONSTRATIONS

Article 2. An illustration of a chemical change is given by striking a match. Lifting or moving a book, or mixing iron filings and sand, is a physical change.

Articles 4 and 5. Inertia may be illustrated by two boxes of the same size, one empty, the other full of sand—more muscular effort is required to move one than the other; also by having a heavy body suspended by a cord and by having a horizontal thread attached to it—if the body be jerked sidewise too quickly, the thread will break; for inertia varies as the quantity of matter and as the *suddenness* of the change of motion.

A spring-balance indicates by its reading the muscular effort used to stretch its spring; it also indicates weights when it is used to lift or support heavy bodies, and the heavier the body the greater the extension of the spring; if this same balance is used to pull sidewise the suspended body mentioned in the preceding section it will be found that the more sudden the pull (i. e. the greater the inertia) the greater will be the extension of the spring. Hence, inertia depends upon both quantity of matter and suddenness of motion.

Article 7. A cork or a piece of glass illustrates a solid. Water poured from a glass into a bottle illustrates a liquid.

A colored gas can be formed in a glass jar by putting warm nitric acid in the jar and then adding a few pieces of copper; the oxides of nitrogen formed will spread uniformly through the jar.

Article 8. Suspend by means of long cords, side by side, two light balls or pails, separate them by a compressed spring, allow the spring to expand. If the bodies have the same mass they will acquire the same velocity. (See Ames and Bliss's "Manual of Experiments in Physics," Experiments 14 and 15.*)

Show a box of weights.

* Hereafter this book will be referred to as "Manual."

Article 12. Show a standard metre bar and a standard mass.

Article 14. Show two bodies of the same volume but different mass, e. g. a block of wood and a block of marble.

Laboratory Experiments

Measurement of length, mass and time,	"Manual,"	Experiment	1.
Determine the number of centimetres in one inch,	"	"	3.
Learn the method of using a vernier,	"	"	4.
Use of vernier caliper,	"	"	5.
Use of micrometer caliper,	"	"	6.

Problems

1. A sphere has a diameter 10 cm., and a mass 50 kilograms; what is its density?

2. If a rectangular block of iron has the dimensions $10 \times 20 \times 5$ cm., and if its mass is 7500 grams; what is its density?

3. Standard gold is an alloy—11 parts gold, 1 part copper. The density of gold is 19.3; that of copper, 8.92; calculate the density of the alloy.

CHAPTER I

MECHANICS

Introduction to Mechanics

Lecture Demonstrations

Article 15. To show that any description of motion is relative, hold a piece of chalk in the hand, move the hand sidewise rapidly, allow the chalk to escape; it will fall obliquely towards the earth, but would remain vertically below the hand if the latter continued its motion.

To show translation, lift or move a book or a stick.

To show rotation, open or close a door.

To show both translation and rotation, throw a stick into the air.

Article 16. See "Manual," Experiment 12.

Articles 18, 19, 21. As an illustration of linear acceleration such that there is no change in direction, show a body falling freely, one falling down an inclined plane, one thrown vertically in the air, and one thrown along a rough table or floor.

As an illustration of linear acceleration such that there is no change in speed, revolve a bullet in a horizontal circle by means of a string.

Article 20. To show this parabolic motion, attach a rubber pipe to a water-tap and let the water escape under pressure; the stream may be pointed sidewise or inclined upward.

Article 23. Various illustrations of rotation, including those in the text, should be shown. (See J. Perry, "Spinning Tops," Romance of Science Series.)

Laboratory Experiments

Determine linear velocity and acceleration. "Manual," Experiment 12.

Problems

1. A train goes 40 miles in 45 minutes; what is its average speed in the C. G. S. system?

2. If this train is running due east and a ball is thrown from it at right angles to the motion of the train with a speed 75 ft. per second; what is the velocity of the ball with reference to the earth?

3. The speed of a body is observed to decrease uniformly from 2000 cm. per second to 500 cm. per second in 10 seconds; what is the acceleration?

4. A body is thrown upward with a speed 1000 cm. per second; how high will it rise? What speed must it have to rise 2 seconds?

5. A stone is thrown horizontally from a cliff 300 ft. high, with a speed 100 ft. per second; how far will it go before striking the earth?

6. A stone is dropped over a cliff into water, the sound of the splash is heard 8 seconds later; what is the height of the cliff? (The velocity of sound is 33,300 cm. per second.)

CHAPTER II

DYNAMICS

Translation

Lecture Demonstrations

Article 25. Perform as many of the experiments described in the text as is possible, especially the impact ones. (See "Manual," Experiment 15.)

Article 27. To show the fundamental property of the centre of inertia, throw a hammer vertically up, and notice the point which seems to move in a straight line.

Show certain regular figures, e. g. a ball, a cylinder, a hoop, a stick, etc., and call attention to the position of the centres of inertia.

Article 29. The formula, force equals mass times acceleration, may be verified by Atwood's machine (see "Manual," Experiment 16), or by rolling a bicycle ball down a groove in an inclined plank.

Article 30. The fact that g is the same for all bodies may be shown by allowing a bullet and a piece of paper to fall in a vacuum (secured by exhausting a tube about 4 ft. long made for the purpose).

Suspend a heavy body by a spring-balance, and then place it on the pan of a platform-balance, illustrating "tension" and "pressure."

Article 31. Show different pendulums—different lengths, different amplitudes, different masses. A weight can also be suspended by a spiral spring, and this set in motion, to illustrate harmonic motion. (See "Manual," Experiment 18.)

Article 32. The change in the tension of the elevator rope on ascending and descending may be illustrated by suddenly lifting a body by a spring-balance and then quickly lowering it; there will be a marked difference in the readings on the balance.

Article 33. Centrifugal motion and its laws may be illustrated by the experiments with a "whirling table." (See "Manual," Experiment 17, also J. Perry, "Spinning Tops.")

Article 35. This property of the centre of inertia may be illustrated as described in the text, or by throwing a stick into the air, as shown in Fig. 16.

Article 36. Composition of forces may best be illustrated by balancing two by means of a third. The forces may be produced by spring-balances or by weights. (See "Manual," Experiment 20.)

Laboratory Experiments

Verify the principle of the conservation of momentum. ("Manual," Experiment 15.)

Verify the laws of harmonic motion. ("Manual," Experiment 18, Part 1.)

Study the composition of two forces. ("Manual,' Experiment 20.)

Problems

1. A body whose mass is 1000 grams is attached to a spring-balance and is raised vertically at such a rate that the reading on the balance is 1100 instead of 1000; what is the acceleration?

2. A body whose mass is 15 kilograms hangs by a cord which passes over a pulley and is fastened to a body whose mass is 10 kilograms and which rests on a smooth table, i.e. one which offers no friction; what is the acceleration?

3. What would be the value of g if the period of a simple pendulum 100 cm. long were 2 seconds?

4. What is the length of a simple pendulum which loses 2 seconds a day at a place where $g = 980$?

5. A body whose mass is 100 kilograms rests on an inclined plane, 3 metres long, one end of which is 50 cm. higher than the other; what force parallel to the plane will keep it from sliding down? what force parallel to the earth's surface would do the same?

6. A telegraph pole has three wires fastened to it, pulling in different directions; one pulls north with a force of 5 *kilograms' weight*, another pulls east with a force of 10 *kilograms' weight;* what must be the di-

rection and the amount of the third force which balances these two? (Express it in dynes.)

7. A cord is attached to two nails which are at the same level and 6 metres apart; a body whose mass is 50 kilograms is knotted to the middle of the cord, which hangs 3 metres below the level of the nails; what is the tension in each branch of the cord?

8. When a man sits in a hammock, what is the tension in each rope?

9. Show that the three forces 500, 600, 1200 dynes, cannot be in equilibrium.

10. A body is moving with a speed of 4 kilometres per hour; what force in dynes will bring it to rest in 5 seconds?

11. If a body whose mass is 500 grams, moving with a speed 1000 cm. per second, meets a body whose mass is 100 grams, moving in the opposite direction with the speed 500 cm. per second, what will be the speed after impact, if they stick together (i. e. are perfectly inelastic)?

12. If a man weighs 150 lbs. on the surface of the earth, what would he weigh at a distance from the earth equal to the radius of the earth?

13. The gravitation constant "c" is 0.000,000,066 or 6.6×10^{-8} on the C. G. S. system; what is the greatest force which two lead spheres, each of radius 1 metre, can exert on each other? Express the result in the weight of a certain number of grams.

CHAPTER II (*Continued*)

ROTATION AND EQUILIBRIUM

Lecture Demonstrations

Article 37. The law of moments can be verified by the method of Experiment 19, "Manual." (See Worthington, "Dynamics of Rotation.")

Articles 38–41. The illustrations mentioned in the text should be shown. For details, see "Manual," Experiments 20, 21 and 22.

Article 42. The facts in regard to the centre of gravity should be illustrated as described in the text; viz. A body should be supported by a cord from two different points in succession; one should be balanced on a knife-edge; the centre of gravity should be shown to coincide with the centre of inertia.

Article 43. A chemical balance should be shown and explained. (See "Manual," Experiment 26.)

Article 44. Stability of equilibrium should be illustrated by the methods described in the text; particular attention being given the question of the position of the centre of gravity.

Laboratory Experiments

Verify the law of moments. ("Manual," Experiment 19.)

Verify the laws of parallel forces. ("Manual," Experiment 21.)

Verify the laws of equilibrium of three forces. ("Manual," Experiment 22.)

To determine the centre of gravity. ("Manual," Experiment 23.)

Problems

1. A body whose mass is 100 kilograms is suspended from a pole resting on two supports at the same level; if the point of suspension is

1 metre from one support and 2 metres from the other, what is the force acting on each support?

2. A rod whose mass is 5 kilograms and whose length is 100 cm. is supported on a smooth peg at one end and by a vertical string 15 cm. from the other end. Calculate the pressure on the peg and the tension in the string.

3. A body whose mass is 100 grams is suspended from one end of a horizontal massless rod 100 cm. long; 160 grams is suspended from the other end, and 140 grams at a point 30 cm. from this end. Where is the centre of gravity? Where is it if the rod has a mass of 10 grams per centimetre of length?

CHAPTER II (*Continued*)

WORK AND ENERGY

Lecture Demonstrations

Article 45. Various methods of doing work should be shown, e. g. setting a pendulum in motion, opening a door, compressing a spring, bending a stick, raising a weight, driving a nail into a plank, pulling a body along a floor or table.

Article 46. A special apparatus can be purchased to show that the velocity of a falling body depends only upon the vertical height through which it falls.

Articles 47–50. Different machines should be shown, e. g. levers, pulleys, screws, inclined planes, etc.; and quantitative experiments may be performed, showing how allowance is made for friction. (" Manual," p. 142.)

Laboratory Experiments

Determine the mechanical advantage of a combination of pulleys. (" Manual," Experiment 24.)

Set up a train of cog-wheels or any machines, and let work be done on it by a falling body while it does work raising a body; if the system is in equilibrium, there will be no kinetic energy; and the work done by the falling body will equal that done on the rising body.

Show that if l is the *length* of an inclined plane and h its vertical height, the force required to raise a body of mass m along the plane is $F = \frac{m g h}{l}$; because the *work* must be $m g h$. Friction may be largely avoided by putting the body in a carriage with rollers; and in other ways the same precautions must be taken as in Experiment 24 of the " Manual."

Problems

1. The diameter of the piston of a steam-engine is 1 ft.; the pressure of the steam 100 lbs. per sq. inch; the length of stroke 2 ft. How much work is done each stroke? What is the power, if there are 100 strokes per minute?

2. If a car going at the rate of 400 metres per minute is stopped by the brakes in a distance of 60 metres, how far would it have gone if the speed had been 800 metres per minute? How long a time is taken to come to rest in each case?

3. If a body whose mass is 1000 grams, sliding down a rough inclined plane, acquires a speed of 300 cm. per sec. when it has fallen a vertical distance 100 cm., what is its kinetic energy, its loss of potential energy, and the work done against friction?

4. A bullet weighing 30 grams is shot horizontally into a block of wood suspended as a pendulum. If the block has a mass 3000 grams and rises a vertical height 30 cm., what is the speed of the bullet?

5. The hammer of a pile-driver weighs 500 kilograms. It falls 7 metres and drives in a pile a distance 40 centimetres. Calculate the average force of resistance.

6. In the case of a combination of two pulleys, one movable and one fixed, what would be the effect if the different branches of the string were not parallel? if the axle were not in the centre of the pulley?

7. If an inclined plane has the length 1 metre and the height 20 cm., what force parallel to the plane is required to raise a body of mass 1000 grams along the plane at a uniform rate? at an acceleration 100 cm. per sec. in each second? (Neglect friction.) If the force is applied parallel to the earth, what is its amount?

CHAPTER III

PROPERTIES OF MATTER

Solids, Liquids and Gases

Lecture Demonstrations

Article 52. The following solids should be shown: glass, iron, copper wire, gold-leaf, putty, lead, rubber, cork.

Their elastic properties should be demonstrated. (The porosity of wood may be shown by forcing mercury through a short piece along the grain.)

Article 53. The following liquids should be shown: water, kerosene, mercury, molasses. They should be poured from one vessel into another.

A stick of sealing-wax should be supported horizontally upon two corks placed at its ends, and then heated by a flame. It will bend under its own weight.

Diffusion of liquids may be shown as follows: Fill a beaker half full of a dilute solution of ammonia colored with litmus, and introduce beneath the ammonia by means of a pipette enough sulphuric acid also colored with litmus to fill the beaker. In a few hours the liquids will diffuse, as will be evident by the changes in color.

Article 54. Show a toy balloon, or a hollow rubber ball.

Show diffusion of gases by allowing some illuminating gas or strongly scented gas to escape in the room.

Article 56. One end of a long flexible rubber cord or a long spiral spring should be fastened to the wall high above the floor, and transverse waves produced in it by vibrations of the hand.

Wave-models, which can be bought, should also be shown and explained.

Article 59. Stationary vibrations should be made in the long cord by suitably timing the vibrations of the hand. (See " Manual," Experiment 38, Part I.)

Melde's form of the experiment should also be shown. This consists in using an electrically driven tuning-fork to set in vibration a long thread, one end of which is fastened to one prong of the fork, and the other is either held in the hand or passed over a pulley and kept stretched by a scale-pan carrying suitable weights. These weights should be small and may be varied, thus altering the tension in the cord, and therefore the velocity of the waves. This will change the number of nodes.

Laboratory Experiments

Study stationary vibrations. (" Manual," Experiment 38, Part I.)

CHAPTER III (*Continued*)

SOLIDS AND FLUIDS

Lecture Demonstrations

Article 60. Show that a spring-balance stretches in proportion to the weights applied. (See "Manual," p. 125.)

Suspend by one end a spiral spring (e. g. such as is used to close doors) and hang weights from the other; when the weight is suddenly diminished, the spring makes harmonic vibrations.

A lath may be laid flat across two supports near its ends, and bent by hanging weights from its middle point.

A wire may be stretched between two fixed points, as on a monochord or sonometer, and plucked one side.

A rod may be clamped vertically by a vise placed at one end, and the other end may be twisted by fastening a transverse stick to it and applying a moment.

All these furnish illustrations of Hooke's Law, and also of harmonic vibrations when the restraint is released.

Article 61. Show a toy balloon and a bottle *filled* with water and closed with a loose-fitting cork on which rests a weight.

Article 62. The upward pressure in a liquid may be shown by special apparatus which can be bought, or by the method of floating cans at different depths. (See "Manual," Experiment 29.)

"Pascal's vases" can be used for the purpose of showing that the pressure due to gravity varies with the vertical depth.

Article 63. A hydraulic press should be shown and used; or the "hydrostatic bellows" can be shown.

Article 64. The principle of Archimedes should be illustrated both for air and for water. A balloon may be shown; or the special apparatus in which a large hollow sphere is balanced against a solid cylinder and which may be placed under the bell-jar of an air-pump; when the air is exhausted the sphere is seen to be the heavier.

A piece of iron or brass may be suspended by a cord from a pan of a balance and weighted first in air, then in water. A special piece of lecture apparatus sometimes used consists of a hollow cylinder closed at one end and carrying from this end a solid cylinder of the same volume as the cavity in the former; if the hollow cylinder carrying the solid one is suspended from the pan of the balance and balanced by weights in the other pan, and if the solid cylinder is then immersed in water, the equilibrium will be disturbed, but may be restored by filling the hollow cylinder with water.

A "platform balance" may also be used; a beaker of water is placed on one pan and balanced by weights on the other, the solid is suspended by a cord from a fixed support and lowered into the water; the solid is buoyed up, and consequently, owing to the reaction, the beaker of water is forced down; the extra weight required to restore equilibrium is the buoyant force of the water.

Article 65. A piece of paper or sheet of tin should be allowed to fall through the air.

Article 66. An atomizer and a ball-nozzle should be shown as described in the text; the latter can be made to use with gas instead of water, the ball being made of pith.

Efflux of a liquid may be shown by making a small hole in the side of a bottle or metal can and pouring in water up to a suitable depth.

The "lawn-sprinkler" should be shown again, and any models of turbines, "hydraulic rams," etc., that are available.

Perform the experiment showing the efflux of hydrogen and air, as described in the text.

Laboratory Experiments

Verify Hooke's Law for a spiral spring. ("Manual," Experiment 18.)
Verify the laws of fluid pressure. ("Manual," Experiment 29.)
Study the principle of Archimedes. ("Manual," Experiments 31, 32, 33.)

Problems

1. A block of wood whose mass is 50 grams is immersed in water by the aid of a sinker whose "apparent weight" in water is 80 grams; if the "apparent weight" of the wood and sinker combined is 40 grams, what is the density of the wood?

2. A solid block $5 \times 4 \times 3$ cm. has the "apparent weight" 400 grams in water at 4° c.; what is its mass?

3. Let a vessel containing water be made of a tube 2 cm. in diameter entering a cylindrical jar from the top through a tight-fitting cork. If the jar has a diameter 10 cm. and a height 20 cm., and if the water rises in the tube to a height 30 cm. above the top of the jar, calculate the *force* over the bottom, sides and top of the jar, calling P the atmospheric pressure.

4. A sphere 1 metre in diameter is just immersed under water. What is the force upward? What is the force upward when the sphere is surrounded by atmospheric air at 0° C.?

CHAPTER III. (*Continued*)

LIQUIDS

Lecture Demonstrations

Article 67. Water standing in an open vessel should be shown; also the effect of revolution may be demonstrated if a whirling machine is available.

A water-wave model should be shown.

Article 68. A vessel as described in the text should be shown; also an open manometer as described.

Article 69 Balancing columns of mercury and water as described in the text should be shown.

Article 70. Pascal's vases may be shown again.

Article 71. Various bodies such as bottles and blocks of wood should be shown floating on water, and attention should be called to the fact that in certain positions the floating body is in stable equilibrium, in others in unstable. Float a piece of iron on a mercury surface.

Articles 72–75. Consult C. V. Boys, "Soap Bubbles," Romance of Science Series, for full directions as to experiments in capillarity.

Laboratory Experiments

Verify law of balancing columns. ("Manual," Experiment 30.)
Study floating bodies. ("Manual," Experiment 34.)
Repeat some of the capillary experiments described in the text.

Problems

1. Referring to Table II. for the densities of ice and salt water, what proportion of an iceberg floats below the surface of the sea?

2. A ship with its cargo has the mass 10,000 tons; how many cubic feet of fresh water, how many of salt water will it displace?

3. A cubical block of iron 30 cm. on an edge is lowered to the bottom of a tank of water 2 metres deep. If the barometer reading is 76 cm., calculate the force on all sides of the block. (See Article 79.)

4. A cubical box, 10 cm. on each edge, is filled half with water, half with mercury. Calculate the thrust on each side and the force on the bottom. Why is it unnecessary to take into account the pressure of the atmosphere?

CHAPTER III (*Continued*)

GASES

Lecture Demonstrations

Article 76. A toy balloon should be shown, or a soap-bubble filled with illuminating gas or with hydrogen.

Article 78. Boyle's law should be demonstrated as described in the text. (See "Manual," Experiment 37.)

Article 79. A barometer should be shown as described in the text.

"Bursting squares" may be broken under an exhausted bell jar.

A hollow sphere, divided into two hemispheres, may be exhausted, as described in the text.

Article 80. A siphon of glass should be used to draw water out of one beaker into another.

Article 81. Various pumps, both water and air, should be shown and explained.

Laboratory Experiments

Verify Boyle's Law. ("Manual," Experiment 37.)

Make and use a siphon as described in the text.

Problems

1. A glass tube 100 cm. long closed at one end is filled with air when the barometer reads 76 cm.; it is then forced—open end downward—into a tank of mercury. When the column of air is 40 cm. long, what is the position of the mercury inside the tube with reference to the free surface of the tank?

2. In a vessel whose volume is 1 cubic metre there are placed the following amounts of gas: (1) Hydrogen, which occupies 1 cubic metre at

atmospheric pressure, i. e. at 76 cm. of mercury; (2) nitrogen, which occupies 3 cubic metres at atmospheric pressure; (3) oxygen, which occupies 2 cubic metres at 3 atmospheric pressure. Calculate the pressure of the mixture.

3. A glass tube, 60 cm. long, closed at one end, is sunk, open end down, to the bottom of the ocean. When drawn up it is found that the water has penetrated to within 5 cm. of the top. Calculate the depth of the ocean, assuming that the density is constant.

CHAPTER IV

SOUND

Lecture Demonstrations

Article 82. Pluck a stretched cord and tear a sheet of paper, to show different kinds of vibrations and sounds.

Arrange an electric bell so as to be suspended in a vacuum under a large bell jar; have as little contact as possible between the bell and the pump or jar.

Show by means of steel rods or whistles the limits of audible vibrations.

Produce a sensitive flame, as described in the text, and shake a bunch of keys near it.

Demonstrate the vibrations of a tuning-fork by holding near it a suspended pith-ball.

Article 83. Show the experiment described in the text, of drawing a tuning-fork over a piece of smoked glass. (See " Manual," Experiment 12.)

Show several simple pendulums, a complex pendulum, a tuning-fork and a monochord.

Article 84. Illustrate resonance by tuning-forks and bottles into which water is poured until they respond.

If there are two tuning-forks of the same frequency, show that the vibrations of one will cause the other to vibrate if they are close together.

Illustrate the action of a sea-shell or bottle.

Article 85. Show a model of compressional waves.

Article 86. Show tuning-forks of different frequencies, also give them small, then great amplitudes, so as to demonstrate differences in pitch and loudness.

Pluck a stretched string, sound an organ pipe, thus showing a complex sound.

Article 87. Demonstrate beats by two organ pipes, or two tuning-forks whose frequencies are nearly the same. Or use two tuning-forks which have the same frequency, and load one by putting some wax on one of its prongs.

Article 88. Various harmonics or consonances may be demonstrated if there are suitable organ pipes or a piano.

Laboratory Experiments

A study should be made of resonance. (See "Manual," Experiments 40 and 43.)

CHAPTER V

SOUNDING BODIES

Lecture Demonstrations

Article 89. Repeat the experiments on stationary vibrations described under Article 59.

Article 90. Demonstrate transverse vibrations in a wire stretched on a monochord; show nodes and loops by means of little saddles of paper; show effect of touching the wire, when it is being bowed, at points ½, ⅓, ¼, etc., the distance between the fixed ends.

Show the effect of increasing the tension, and of changing the length of the wire.

(See "Manual," Experiment 39.)

Article 91. Show longitudinal vibrations in stretched wires by stroking them lengthwise by a resined cloth.

Show effect of change in length.

Article 92. Show vibrations of a rod: (1) by clamping it in a vise at one end and pulling the other end sidewise; (2) by clamping it in the middle and stroking it by a resined cloth.

Show the vibrations of a large tuning-fork.

Article 93. Sound various organ pipes, and by suitable blowing produce different sounds from the same pipe.

Article 95. Demonstrate the vibrations of plates and bells as described in the text.

Article 96. Measure the velocity of compressional waves in air and in brass by the methods described in the "Manual," Experiments 40 and 41.

Laboratory Experiments

Study the vibrations of a stretched wire and of a column of air. ("Manual," Experiments 39, Part I., and 40.)

Problems

1. What would be the length of an organ pipe, closed at one end, which would respond most loudly to a tuning-fork having a frequency of 320 vibrations per second: (1) when the pipe is filled with air? (2) when it is filled with hydrogen?

2. How will the frequency of a stretched string making 320 vibrations per second be changed if the tension is made twice as great?

What would happen if the string was held clamped at its middle point?

What other string of the same length and under the same tension would have the same frequency if its density were twice as great?

CHAPTERS VI AND VII

NATURE OF HEAT AND TRANSFER OF HEAT-ENERGY.

Lecture Demonstrations

Article 97. Show various methods of producing sensations of heat and cold, as described in the text, and show how the volume of air in an air thermometer is affected by them. Make an air thermometer by introducing a fine glass tube into a bottle or flask through a tight-fitting cork (see Fig. 180), and hold it inverted with its open end under the surface of a beaker of colored water. If the flask is warmed by the hand for a moment, some air will escape, and the water will rise in the tube.

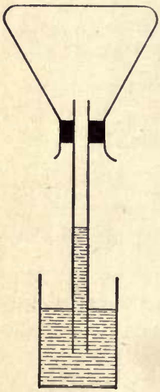

FIG. 180.

Now apply a flame to the thermometer, put ice on it, moisten it with water or alcohol, and blow over it so as to hasten evaporation, etc.

If the apparatus is at hand, boil water by means of friction.

Compress some gas suddenly and show rise of temperature, e. g. use a small bicycle pump to compress the gas in some tube.

Article 98. Show an air thermometer like the one described in the text, if one is at hand. Show a mercury thermometer, and demonstrate how it gives a constant reading if held in steam. (See "Manual," Experiment 44.)

Article 101. Convection currents can be shown by placing particles of coloring matter in a glass vessel full of water and applying a flame to one point of the bottom; also by holding some smoking substance over a flame or lamp—brown or filter paper soaked in a solution of potassium nitrate and dried gives a strong smoke when ignited.

A good lecture experiment is to place a candle in the middle of a tin dish, pour in ¼ inch of water, then light the candle and lower a tall "student-lamp" chimney over it. The flame will go out almost instantly, owing to the consumption of the oxygen in the air; but, if a piece of cardboard or tin is introduced into the top of the lamp-chimney, dividing it into two half cylinders, a draft will be established, down current on one side of the partition, up current on the other, and the flame will continue to burn, but will flicker, owing to the draft over it. These down and up currents may be demonstrated by smoke-paper.

Article 102. The difference in the conducting power of different substances, e. g. copper and iron, may be shown by arranging a rod or a wire each of the same cross-section, end to end, placing a Bunsen burner at their junction, and sticking tacks or nails along the rods by means of some kind of wax, e. g. "Universal." (See "Manual," p. 496.) (Or find at what point of each rod a match may be lighted by contact.)

The effect of wire-gauze on a flame, as described in the text, should be shown.

A cylinder may be made of a wooden rod and a brass or copper tube —the rod having the same diameter as the outer diameter of the tube, except for the short distance near its end where it is inserted in the tube; and if a piece of paper is wrapped closely around the cylinder and a flame applied to the paper at the junction of the wood and metal, the paper will char over the wood, but not over the metal.

The poor conductivity of water may be shown by applying a Bunsen burner to the upper portion of a test-tube filled with water, thus making the upper layers of water boil, while the lower portion will be still cool, as shown by a thermometer.

A flat-iron brought near a "fish-tail" gas flame chills it so that the flame becomes very smoky.

Article 103. If any vacuum-tubes are available, a thermometer held on one side of a tube is greatly affected by a flame on the other.

The reflection of the ether-waves may be shown by allowing the waves from a piece of steel heated white-hot to fall upon a plane mirror made of a plate of tin and be reflected to a thermopile; or by placing it in the focus of a concave mirror, and putting a match in the focus of a second concave mirror, so arranged as to stop the waves reflected from the first.

Absorption is shown by an experiment as follows: Fasten two strips of thin brass or iron to the ceiling or to a horizontal frame, at a distance of several feet apart (see Fig. 181); join their free ends by a cord; and from each strip suspend a simple pendulum *of equal lengths.* If one pendulum is set vibrating in the plane which includes the two, the second will soon be set in vibration; and as its amplitude increases, that of the first decreases, until finally it comes to rest. Then the process repeats itself. If, however, a sheet of paper of the proper size is fastened to the second pendulum, the vibration of the first will produce only feeble vibrations in it, and they will die down quickly.

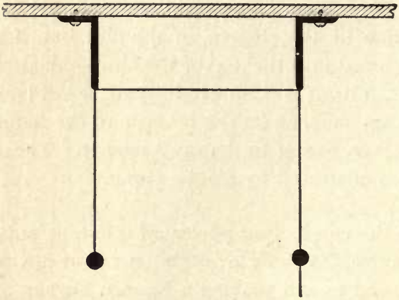

FIG. 181.

Article 104. The difference between the emission of polished and blackened bodies may be shown as described in the "Manual," Experiment 56.

The transparency of various substances may be shown, if a thermopile is available, by interposing pieces of glass, paper, etc., between it and a flame.

Laboratory Experiments

Test the fixed points of a mercury thermometer. ("Manual," Experiment 44.)

Study the radiation of black and polished bodies. ("Manual," Experiment 56.)

Problems

1. What are the temperatures 100° and 80° Fahrenheit on the Centigrade scale?

CHAPTER VIII

HEAT-EFFECTS

Lecture Demonstrations

Article 105. The expansion of a solid may be shown by clamping one end and letting the other press against a delicate lever. (See "Manual," Experiment 45.)

The expansion of a liquid is best shown by filling with the liquid a bulb which has a capillary stem or a bottle through whose cork a small tube enters; if the surface of the liquid stands half way up the tube at the temperature of the room, it will drop suddenly if the bulb or bottle is lowered into a basin of hot water, and will then rise rapidly. The first *apparent* contraction of the liquid is due to the expansion of the glass, which has its temperature raised before that of the liquid inside. (See "Manual," Experiments 46 and 47.)

If ice is left floating in a high glass of water, it will be observed that the temperature at the surface is lower than that at the bottom.

The change in volume of a gas accompanying changes in temperature can be shown by any air thermometer, e. g. the simple one described by Fig. 180. (See "Manual," Experiment 48.)

Article 106. *Fusion.*—The constancy of the fusion point of ice can be shown by means of an ordinary thermometer.

Change in volume on fusion may be illustrated by freezing the water in a closed glass bulb by means of a freezing mixture of ice and salt, and thus bursting the glass.

Regelation may be shown by supporting a block of ice on two trestles, and hanging over it a wire which carries 20 lbs. at each end. The wire will cut its way through, and the two pieces of ice thus made will be found frozen together.

An experiment referring to latent heat of fusion as described in the text should be performed. (See "Manual," Experiment 52.)

Evaporation.—The law of saturated vapor—pressure depends upon temperature alone—should be shown as described in the text. (See "Manual," Experiment 55.)

That heat-energy is required to produce evaporation may be shown by the "cryophorus;" by making water evaporate so fast under a bell jar which is exhausted by an air-pump that the water left behind freezes; or by moistening a thermometer bulb and then drying it by a draft of dry air.

The effect of dissolved air and of points upon boiling may be shown by warming water until it *almost* boils, and then dropping in a lump of sugar or some bits of broken glass.

The spheroidal state may be shown as described in the text.

Article 107. The method of measuring specific heat by the method of mixture may be shown. (See "Manual," Experiment 49.)

"Tyndall's experiment" is interesting; small spheres of different metals carried on a wire frame are all warmed to the same temperature in a bath of oil, e.g. 150° C., and are then taken out at the same time and placed on a thin cake of beeswax. Their temperature falls to the same final value; but different amounts of heat-energy are liberated, and so some balls may melt through while others do not.

Laboratory Experiments

Measure the expansion of a solid, liquid and gas, if possible. (See "Manual," Experiments 45, 46, 48.)

Measure the specific heat of brass. ("Manual," Experiment 49.)

Problems

1. A steel chain is 25 metres long at 0° C. What will be its length at 20° C?

2. A cast-iron sphere 5.005 cm. in diameter rests upon a copper ring 5 cm. in diameter, the lengths being measured at 0°. At what temperature will the sphere just pass through the ring?

3. If 50 grams of ice are mixed with 250 grams of water at 20° C., what will be the final temperature? What would be the final result if there had been only 150 grams of water?

4. Steam at 100° is passed into water which is originally at 15° C. until the temperature is 25° C. If the mass of water at the beginning is 100 grams, what is it at the end?

5. With what speed must a lead bullet strike a target in order to raise its temperature 50° C., if half the "heat generated" goes into the bullet?

6. A definite mass of air has a volume of 300 cubic cm. at 0° C. and under a pressure of 50 cm. of mercury; its temperature is raised, the pressure being constant, until the volume is 350 cc.; what is the final temperature? If the volume had been kept constant, and the temperature raised this amount, what would be the pressure?

7. Under what pressure would a litre (i. e. 1000 cc.) of hydrogen have a mass of 2 grams at the temperature 0° C.?

CHAPTER IX

MAGNETISM

Lecture Demonstrations

Articles 108–113. Repeat the experiments described in the text.

Article 114. Demonstrate lines of magnetic force as described in text. They may be shown a large number of people as follows: Soak the paper in paraffin before sprinkling on it the iron filings, and then, after the lines of force are formed, pass a Bunsen flame quickly over the paper. The paraffin will melt, and thus the iron filings will become fixed; and the paper may be passed around the room.

Filings made with a clean file from soft Norway iron are the best.

Article 115. Show the declination and dip of a magnetic needle. Also hold in one hand a rod of soft *unmagnetized* iron, incline it so as to have the direction of the dip, and hit one end a blow with a hammer; the iron becomes magnetized, owing to the action of the earth, as may be shown by bringing it near a pivoted magnet.

Laboratory Experiments

Map lines of magnetic force. ("Manual," Experiment 63.)
Measure the angle of dip. ("Manual," Experiment 64.)

Problems

1. Being given a pivoted magnetic needle and two bars of steel—one a magnet, the other not—how is it possible to tell which of the two is the magnet?

2. Being given simply the magnet and the bar of iron, how is it possible to tell which is the magnet, without suspending them?

CHAPTER X

ELECTROSTATICS

Lecture Demonstrations

Articles 116–119. Repeat the experiments described in the text. To prove that equal and opposite charges are produced, a convenient experiment is as follows: Fasten a small metal cup to the top of the gold-leaf electroscope; insert in it a roll of flannel or silk, leaving in the centre space to receive a glass or ebonite rod; rotate the rod against the friction of the flannel or silk, thus producing charges; there will, however, be no action of the gold leaves until the rod is removed.

Article 120. Surround a gold-leaf electroscope with a cylindrical wire cage, and place metal plates below and above the cage so as to close it. If the cage is charged to any degree, there will be no action on the gold leaves, even if the electroscope is joined by a wire to the cage.

Another method is to charge the hollow conductor and to lower into it an uncharged brass ball suspended by a silk thread. Let the ball touch the interior, and be then removed; it will be found to be uncharged.

Article 121. Repeat the experiments described in the text. The charges produced by induction may be demonstrated by touching the small brass ball suspended by a silk thread to different portions of the conductor and carrying it to an electroscope—care must be taken to discharge the brass ball before each contact with the conductor.

Article 122. Produce sparks by an electric machine, make them pass through pieces of paper.

Show the action of points by proving that on a pointed conductor the charge taken off by the small brass ball—as described above—is greater at a point than on a plane portion. Also, set up a wire "whirligig" like a lawn-sprinkler, and show that, if it is joined by a wire to an electric machine, it will rotate backward, i. e. in a direction opposite to that in which the ends of the wires point. This rotation is due to the fact that at the points of the wire sparks pass to particles in the air, thus charging them with the same kind of charge as is on the points. There is thus repulsion : a stream of charged particles flows away from the points, and the wire frame turns in the opposite direction. A candle-flame may easily be blown out by this wind from a pointed wire joined to a machine.

Article 123. A simple method of demonstrating lines of force is described in the "Manual," Experiment 57.

Article 124. The action of a condenser should be demonstrated as described in the text. Charge a conductor and join it by a wire to a gold-leaf electroscope ; bring up an uncharged insulated conductor ; join the latter to the earth by a wire ; insert between the two conductors a piece of glass *which has been discharged thoroughly* by repeatedly passing it through a Bunsen flame. Each of these three steps will cause the gold leaves to collapse further and further.

Charge and discharge Leyden jars, showing the nature of the sparks.

If there is a Leyden jar with movable inner and outer coatings, it may be charged from a machine, the outer coating being held in the hand, then placed on a table ; now remove the inner coating by means of a glass rod and discharge it ; remove the glass part by the hand and place on an insulating stand, e. g. a pane of glass ; discharge the outer coating by joining to the earth. Now replace the glass part in the outer coating, and the inner coating in its place, using a glass rod again. It may be shown that the jar is still charged, for a spark can be obtained. This proves that the essential feature of the charge is the strain in the glass. In fact, in this case of the movable coatings, the charges are drawn off the metal coatings across to the surface of the glass—the minute sparks which accompany this may be both seen and heard.

An excellent description of electrical machines is given in Perkins's "Electricity and Magnetism."

Laboratory Experiments

Study electrostatic phenomena. ("Manual," Experiments 58 and 59.)

Map lines of force. ("Manual," Experiment 57.)

Study an induction machine. ("Manual," Experiment 60.)

CHAPTER XI

ELECTRIC CURRENTS

Lecture Demonstrations

Article 125. By means of the lecture-room galvanometer show the current obtained when a condenser is discharged, also when the two poles of an electric machine are joined to it. Demonstrate which plate or which pole is positively charged and which negatively, and thus show the *direction* of the current in the wire, and the corresponding deflection of the galvanometer magnet.

Article 126. Show a voltaic cell, first before the cell is made up, then in use.

By means of the galvanometer show that it produces a current in the direction from copper to zinc. Show the effect of using first common zinc, and then amalgamated zinc; in the former case, bubbles of gas gather on the zinc, in the latter they do not. Construct a Daniell's cell.

(A lantern may be used with great advantage in these demonstrations.)

Article 127. Construct a simple thermo-couple by joining an iron wire and a brass wire to the galvanometer terminals and soldering the two free ends of the wires, thus having an iron-brass junction. If this is held in the fingers, a current should be indicated.

Show a thermopile if one is at hand.

Article 128. Repeat the experiments described in the text. Show an astatic needle, and a galvanometer.

Demonstate the lines of magnetic force near a current by suspending a *small* magnet, e. g. 1 cm. long, from a fibre and bringing near it the wire carrying the current. Show the action near a solenoid.

Magnetize a bar of iron by means of a solenoid, and show various types of electro-magnets. If a solenoid carrying a large current is freely suspended, the current entering and leaving through cups of mercury in which the two ends of the wire dip, it will take a north and south direction like a magnet; and attractions and repulsions with a magnet or with another solenoid may be shown.

A voltaic cell can be arranged so as to float in water. If its copper and zinc plates are joined outside the cell by a thin helix of rather heavy copper wire, it will be attracted or repelled if a bar-magnet is brought near it. Thus show which is the "north" and which the "south" face of the helix carrying the current.

Article 129. Show the two forms of electric lights. Pass a heavy current through a fine iron wire, and thus melt it; pass it also through a chain whose links are made of different metals and of wires of different sizes.

Article 130. Repeat the experiments described in the text. The difference between the gases given off in the water voltameter (Fig. 132) may be demonstrated by letting sufficient volumes gather and then allowing them to escape through the stop-cocks at the top; the hydrogen will burn with a bluish flame if a lighted match is applied (the yellow color which sometimes comes is due to the sodium in the glass); the oxygen will make burst into a flame a splinter of wood which has a spark at its end—light a splinter of pine, blow out the flame, and a spark will in general remain.

Article 131. If an induction coil and vacuum tubes are available, the effect of electrical discharges should be shown, as described in the text.

Article 132. If a steady current is sent through a number of conductors in series, the varying resistance of different sections may be shown as described in "Manual," Experiment 68.

Article 133. Wheatstone's bridge should be shown in various forms. ("Manual," Experiments 69, 70, 71.)

Article 134. Induced currents should be demonstrated by having a bobbin of wire joined to a galvanometer and making a magnet or a second coil carrying a current approach and then recede, first with one end foremost, then the other.

Various instruments should be shown and explained.

Laboratory Experiments

Study the field of force about a wire carrying a current. Set up a Daniell's cell.

Map a current sheet. (" Manual," Experiment 62.)

Measure a current by means of a water voltameter. (" Manual," Experiment 75.)

Measure a resistance. (" Manual," Experiment 69 or 70.)

Study the resistance of a uniform conductor. (" Manual," Experiment 67.)

Problems

1. If three wires, of resistances 20, 40, 60, are joined in parallel, what is the total resistance?

2. What " shunt " must be put in parallel with a coil of wire which has a resistance 1000, in order that only one-tenth of the current may flow through the coil?

CHAPTER XII

NATURE OF LIGHT

Lecture Demonstrations

Article 135. A lamp should be shown behind a vacuum-tube. Young's Interference Experiment should be performed; fine slits cut in the films on glass photographic plates which have been exposed to the light and not "developed" are satisfactory; the two parallel slits in the second should be as close together as possible; an ordinary magnifying-glass can be used to focus the light directly in the eye.

Article 136. Different colors may be obtained by using intense white light and pieces of colored glass.

Show the colors of soap-films and also Newton's rings.

Article 138. Show the action of a paraffin photometer (or a grease-spot one). (See "Manual," Experiment 79.)

Article 139. Show shadows cast by a small source of light, e. g. a small opening in a screen placed in front of a lamp; also those cast by a large opening.

Article 140. Repeat the experiment described in the text.

Laboratory Experiments

Repeat Young's Interference Experiment, and the one on pin-hole images.

Compare the illuminating powers of two lights. (See "Manual," Experiment 79.)

Problems

1. What is the height of a flag-staff which casts a shadow 200 ft. long, when a vertical rod 6 ft. long casts a shadow 8 ft. long?

CHAPTER XIII

REFLECTION OF LIGHT

Lecture Demonstrations

Article 141. Regular and irregular reflection may be shown by using a clean and a dusty mirror.

Article 142. The formation of images may be shown by any plane mirror. Show a kaleidoscope.

Article 143. The laws of plane reflection may be shown by the method given in the "Manual," Experiment 80.

Another method is to show the path of a beam of sunlight which enters a darkened room through a small opening, falls upon a plane mirror and is reflected; this may be done by scattering dust in the path, or by clapping together two blackboard erasers.

Article 144. Concave and convex mirrors should be shown and explained. (See "Manual," Experiment 81.)

Spherical aberration may be shown by nearly filling a glass with milk and holding it near a candle.

Laboratory Experiments

Study reflection from plane mirrors and from spherical ones. ("Manual," Experiments 80 and 81.)

Problems

1. If the sun is shining obliquely on a vertical plane mirror and an object is placed in front of the mirror, how many shadows will be cast on a screen?

2. If the radius of curvature of a concave mirror is 100 cm., and if an object is placed 100 cm. from the mirror, describe the image. Do this also in the case of the mirror being convex.

3. What is the smallest plane mirror in which a man may see his entire figure?

4. Draw images formed of an object by a concave and a convex mirror, when the object is at a distance from the mirror less than the focal distance.

CHAPTER XIV

REFRACTION OF LIGHT

Lecture Demonstrations

Article 145. Refraction may be shown as described in the text.

Article 146. The phenomena of plane refraction may be shown by causing a beam of sunlight, coming into a darkened room through a small opening, to fall upon a mirror and be reflected to the surface of a tank of water with glass sides. The path of the light outside may be shown by the dust in the air; and in the water it may be made evident by using water filled with sediment or by putting into the water a few grains of some fluorescent powder, e. g. "uranine."

Total reflection may be shown by placing a second mirror on the table beside the glass tank, and causing the light from the first mirror to be reflected from this second mirror and to enter the side of the tank below the free surface of the water, at such an angle as to be totally reflected.

Article 147. Dispersion should be shown by causing the beam of sunlight from the small opening (preferably a slit parallel to the edge of the prism) to pass through a prism. Show that a second prism identical with the first can be used to combine the colors again.

(Minimum deviation might be shown, and the difference in dispersion and deviation of different kinds of glass.)

Article 148. Two kinds of lenses, a converging and a diverging, should be used. Real images of illuminated objects may be thrown upon a screen.

The effect of the lenses upon beams of light can be demonstrated by making the air dusty.

Microscopes and telescopes should be shown taken apart, and should be explained.

Laboratory Experiments

Study refraction at a plane surface. ("Manual," Experiments 82 and 83.)

Study lenses. ("Manual," Experiment 84.)

Construct a telescope and a microscope. ("Manual," Experiments 85 and 86.)

Problems

1. What is the apparent depth of a lake 30 feet deep?

2. What is the focal length of a converging lens which forms an image at a distance 40 cm. in front of the lens when the object is 90 cm. behind it?

3. A converging lens at a distance of 5 cm. from an object forms an image on a screen; when it is moved nearer the screen through a distance 25 cm. it again forms an image on the screen. What is its focal length?

4. What advantage has a long-focus lens over one of short focus in the magnification of a very distant object?

CHAPTER XV

DISPERSION

Lecture Demonstrations

Article 153. A pure spectrum should be shown as described in the text.

Article 154. If a spectrometer is available, the spectra of various gases should be shown on the lecture table. Sparks should be passed by means of an induction coil through gases contained in vacuum-tubes or between two iron or copper wires.

Laboratory Experiments

The student may be required to map the spectrum of some gas.

CHAPTER XVI

OTHER PHENOMENA OF LIGHT

Lecture Demonstrations

Article 155. The colors of colored glass and colored liquids (e. g. litmus or potassium permanganate in water) should be shown by reflected and transmitted light; similarly a thin leaf of gold-foil held between two thin glass plates. Fluorescent and phosphorescent substances should be shown, if they are available (rub a match in a darkened room).

Throw some common salt on a pan of hot coals, or dissolve the salt in an alcohol lamp and light it, and notice the change in the color of objects.

Article 156. Some one experiment in polarization should be demonstrated, e. g. allow sunlight to pass from the small opening through two Nicols prisms in succession and show the alternate extinction and reappearance of the light on a screen as one Nicol is rotated around its axis.

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