PHYSICAL MANIPULATION

PICKERING

ELEMENTS

OF

PHYSICAL MANIPULATION.

BY

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TO

Prof. Wm. B. Rogers,

THE FIRST TO PROPOSE A PHYSICAL LABORATORY,

THIS WORK IS MOST AFFECTIONATELY INSCRIBED

BY HIS

SINCERE FRIEND AND PUPIL,

THE AUTHOR.

PREFACE.

THE rapid spread of the Laboratory System of teaching Physics, both in this country and abroad, seems to render imperative the demand for a special text-book, to be used by the student. To meet this want the present work has been prepared, based on the experience gained in the Massachusetts Institute of Technology during the past four years. The preliminary chapter is devoted to general methods of investigation, and the more common applications of the mathematics to the discussion of results. The graphical method does not seem to have attracted the attention it deserves; it is accordingly compared here with the analytical method. Some new developments of it are moreover inserted. It is of fundamental importance that the student should clearly understand how to deal with his observations, and reduce them, and that he should be familiar with the various kinds of errors present in all physical experiments. A short description is also given of the various methods of measuring distances, time and weights, which, in fact, form the basis of all physical investigation. This chapter is intended as the ground-work of a short course of lectures, given to the students before they begin their work in the laboratory. It should be so far extended by the instructor, as to render them familiar with the general principles on which all physical instruments are constructed, thus greatly aiding them when they have occasion to devise apparatus for their own work.

The remainder of the volume is devoted to a series of experiments which it is intended that the student shall perform in the laboratory. Each experiment is divided into two parts; the first called *Apparatus*, giving a description of the instruments required, and designed to aid the instructor in preparing the laboratory for the class. The student should read this over, and with it the second part, entitled *Experiment*, which explains in detail what he is to do.

Perhaps the greatest advantage to be derived from a course of physical manipulation, is the means it affords of teaching a student to think for himself. This should be encouraged by allowing him to carry out any ideas that may occur to him, and so far as possible devise and construct, with his own hands, the apparatus needed. Many such investigations are suggested in connection with some of the experiments, for instance Nos. 13, 37, 48, 69, 77, 93 and others. To aid in this work, a room adjoining the laboratory should be fitted up with a lathe and tools for working in metals and wood, as most excellent results may sometimes be attained at very small expense, by apparatus thus constructed by students.

The method of conducting a Physical Laboratory, for which this book is especially designed, and which has been in daily use with entire success at the Institute, is as follows. Each experiment is assigned to a table, on which the necessary apparatus is kept, and where it is always used. A board called an indicator is hung on the wall of the room, and carries two sets of cards opposite each other, one bearing the names of the experiments, the other those of the students. When the class enters the laboratory, each member goes to the indicator, sees what experiment is assigned to him, then to the proper table where he finds the instruments required, and by the aid of the book performs the experiment.

Any additional directions needed are written on a card also placed on the table. As soon as the experiment is completed, he reports the results to the instructor, who furnishes him with a piece of paper divided into squares if a curve is to be constructed, or with a blank to be filled out, when single measurements only have been taken. In either case a blank form is supplied, as a copy. New work is then assigned to him by merely moving his card opposite any unoccupied experiment. By following this plan an instructor can readily superintend classes of about twenty at a time, and is free to pass continually from one to another, answering questions and seeing that no mistakes are made. He can also select such experiments as are suited to the requirements or ability of each student, the order in which they are performed being of little importance, as the class is supposed to have previously attained a moderate familiarity with the general principles of physics. Moreover, the apparatus never being moved, the danger of injury or breakage is thus greatly lessened and much time is saved. To avoid delay, the number of experiments ready at any time should be greater than that of the students, and the easier ones should be gradually replaced by those of greater difficulty.

Among these experiments several novelties, here published for the first time, have been introduced. For instance, the apparatus for ruling scales, p. 59, the photometers, pp. 132 and 134, and the polarimeter, p. 221. It is also believed that the directions for weighing, p. 47, and the adjustments for the optical circle, p. 142, if not new, at least present the subject in a more concise and practical form than that commonly given. In fact it has been the object throughout to give definite directions, so far as possible, as if addressing the student in person. English weights and measures are occasionally used as well as French to familiarize the student with both systems, as in many of the practical applications of physics the general prevalence of the foot and pound as units seems to render premature the exclusive introduction of the metric system. The second volume of this work, including Heat, Electricity, a list of books of reference, and other matters of general interest to the physicist, will be issued at as early a date as possible.

It is difficult to give credit for all the aid rendered in preparing this work, as the author has for years made it a practice to collect for it information from all available sources. He is much indebted to Mr. Alvan Clark for the method of testing telescope lenses, and to Prof. F. E. Stimpson for advice and aid on photometry and other matters. The course in photography is essentially that given by Mr. Whipple to the students at the Institute. His especial thanks are due to his friend Prof. Cross, whose careful examination of the proof sheets, and whose excellent judgment has been of great assistance. Finally, if this volume, notwithstanding its shortcomings, aids in any way those engaged in physical investigations, either the student in the laboratory or the amateur experimenter, the object of the author will have been accomplished.

E. C. P.

April 29th, 1873.

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GENERAL METHODS

OF

PHYSICAL INVESTIGATION.

THE object of all Physical Investigation is to determine the effects of certain natural forces, such as gravity, cohesion, heat, light and electricity. For this purpose we subject various bodies to the action of these forces, and note under what circumstances the desired effect is produced; this is called an experiment. Investigations may be of several kinds. First, we may simply wish to know whether a certain effect can be produced, and if so, what are the necessary conditions. To take a familiar example, we find that water when heated boils, and that this result is attained whether the heat is caused by burning coal, wood or gas, or by concentrating the sun's rays; also whether the water is contained in a vessel of metal or glass, and finally that the same effect may be produced with almost all other liquids. Such work is called Qualitative, since no measurements are needed, but only to determine the quality or kind of conditions necessary for its fulfilment. Secondly, we may wish to know the magnitude of the force required, or the temperature necessary to produce ebullition. This we should find to be about 100° C. or 212° F., but varying slightly with the nature of the vessel and the pressure of the air. Thirdly, we often find two quantities so related that any change in one produces a corresponding change in the other, and we may wish to find the law by which we can compute the second, having given any value of the first. Thus by changing the pressure to which the water is subjected, we may alter the temperature of boiling, and to determine the law by which these two quantities are connected, hundreds of experiments have been made by physicists in all parts of the world. The last two classes of experiments are called Quantitative, since accurate measurements must be made of the quantity or magnitude of the forces involved. Most of the following experiments are of this nature, since they require more skill in their performance, and we can test with more certainty how accurately they have been done. Having obtained a number of measurements, we next proceed to discuss them by the aid of the mathematical principles described below, and finally to draw our conclusions from them. It is by this method that the whole science of Physics has been built up step by step.

Errors. In comparing a number of measurements of the same quantity, we always find that they differ slightly from one another, however carefully they may be made, owing to the imperfection of all human instruments, and of our own senses. These deviations or errors must not be confounded with mistakes, or observations where a number is recorded incorrectly, or the experiment improperly performed; such results must be entirely rejected, and not taken into consideration in drawing our conclusions.

If we knew the true value, and subtracted it from each of our measurements, the differences would be the errors, and these may be divided into two kinds. We have first, constant errors, such as a wrong length of our scale, incorrect rate of our clock, or natural tendency of the observer to always estimate certain quantities too great, and others too small. When we change our variables these errors often alter also, but generally according to some definite law. When they alternately increase and diminish the result at regular intervals they are called periodic errors. If we know their magnitude they do no harm, since we can allow for them, and thus obtain a value as accurate as if they did not exist. The second class of errors are those which are due to looseness of the joints of our instruments, impossibility of reading very small distances by the eye, &c., which sometimes render the result too large, sometimes too small. They are called accidental errors, and are unavoidable; they must be carefully distinguished from the mistakes referred to above.

Analytical and Graphical Methods. There are two ways of discussing the results of our experiments mathematically. By the first, or Analytical Method, we represent each quantity by a letter, and then by means of algebraic methods and the calculus draw our conclusions. By the Graphical Method quantities are represented by lines or distances, and are then treated geometrically.

The former method is the most accurate, and would generally be the best, were it not for the accidental errors, and were all physical laws represented by simple equations. The Graphical Method has, however, the advantage of quickness, and of enabling us to see at a glance the accuracy of our results.

ANALYTICAL METHOD.

Mean. Suppose we have a number of observations, A_1, A_2, A_3, A_4, &c., differing from one another only by the accidental errors, and we wish to find what value A is most likely to be correct. If A was the true value, $A_1 - A$, $A_2 - A$, &c., would be the errors of each observation, and it is proved by the Theory of Probabilities that the most probable value of A is that which makes the sum of the squares of the errors a minimum. Also that this property is possessed by the arithmetical mean. Hence, when we have n such observations, we take $A = (A_1 + A_2 + A_3 + \&c.) \div n$, or divide their sum by n. Thus the mean of 32, 33, 31, 30, 34, is $160 \div 5 = 32$. It is often more convenient to subtract some even number from all the observations, and add it to the mean of the remainder; thus, to find the mean of 1582, 1581, 1583, 1581, 1582, subtract 1580 from each, and we have the remainders 2, 1, 3, 1, 2. Their mean is $9 \div 5 = 1.8$, which added to 1580 gives 1581.8. Where many numbers are to be added, Webb's Adder may be used with advantage.

Probable Error. Having by the method just given, found the most probable value of A, we next wish to know how much reliance we may place on it. If it is just an even chance that the true value is greater or less than A by E, then E is called its probable error. To find this quantity, subtract the mean from each of the observed values, and place $A_1 - A = e_1$, $A_2 - A = e_2$, &c. Now the theory of probabilities shows that $E = .67\sqrt{e_1^2 + e_2^2 + \&c.,} \div n$, from which we can compute E in any special case. As an example, suppose we have measured the height of the barometer twenty-five times, and find the mean 29.526 with a probable error of .001 inches. Then it is an even chance that the true reading is more than 29.525, and less than 29.527. Now let us suppose that some other day we make a single reading, and wish to know its probable error. The theory of probabilities shows that the accuracy is proportional to the square root of the number of observations, or that the mean of four, is only twice as accurate as a single reading, the mean of a hundred, ten times as accurate as one. Hence in our example we have $1 : \sqrt{25} = .001 : .005$, the probable error of a single reading. Substituting in the formula, we have the probable error of a single reading, $E' = E \times \sqrt{n} = .67\sqrt{e_1^2 + e_2^2 + \&c.} \div \sqrt{n}$. It is generally best to compute E' as well as E, and thus learn how much dependence can be placed on a single reading of our instrument.

Weights. We have assumed in the above paragraph that all our observations are subject to the same errors, and hence are equally reliable. Frequently various methods are used to obtain the same result, and some being more accurate than others are said to have greater *weight.* Again, if one was obtained as the mean of two, and the second of three similar observations, their weights would be proportional to these numbers, and the simplest way to allow for the weights of observations is to assume that each is duplicated a number of times proportional to its weight. From this statement it evidently follows that instead of the mean of a series of measurements, we should multiply each by its weight, and divide by the sum of the weights. Calling A_1, A_2, &c., the measurements, and w_1, w_2, &c., their weights, the best value to use will be $A = (A_1 w_1 + A_2 w_2 + \&c.) \div (w_1 + w_2 + \&c.)$. We may always compute the weight of a series of n observations, if we know the errors e_1, e_2, &c., using the formula $w = n \div 2(e_1^2 + e_2^2 + e_3^2 + \&c.)$. Substituting this value in the equation for probable error, we deduce $E = .477 \div \sqrt{nw}$ if all the observations have the same weight, or $E = .477 \div \sqrt{w_1 + w_2 + \&c.}$, if their weights are w_1, w_2, &c.

Probable Error of Two or More Variables. Suppose we have a number of observations of several quantities, x, y, z, and know that they are so connected that we shall always have $0 = 1 + ax + by + cz$. If the first term of the equation does not equal 1, we may make it so, by dividing each term by it. Call the various values x assumes x', x'', x''', those of y, y', y'', y''', and those of z, z', z'', z''', and so on for any other variables which may enter. If we have more observations than variables, it will not in general be possible to find any values of a, b and c which will satisfy them all, but we shall always find the left hand side of our equation instead of being zero will become some small quantity, e', e'', e''', so that we shall have:—

$$e' = 1 + ax' + by' + cz',$$
$$e'' = 1 + ax'' + by'' + cz'',$$
$$e''' = 1 + ax''' + by''' + cz''',$$

and so on, one equation corresponding to each observation. These are called *equations of condition.* Now we wish to know what are the most probable values of a, b and c, that is, those which will make the errors e', e'', e''', as small as possible. As before, we must have the sum of the squares of the errors a minimum. We therefore square each equation of condition, and take their sum; differentiate this with regard to a, b and c, successively, and place each differential coefficient equal to zero. These last are called *normal equations*, and correspond to each of the quantities a, b and c, respectively. The practical rule for obtaining the normal equations is as follows:—*Multiply each equation of condition by its value of x (or coefficient of a), take their sum and equate it to zero.* Thus $x'(1 + ax' + by' + cz') + x''(1 + ax'' + by'' + cz'') +$ &c. $= 0$, is the first normal equation. Do the same with regard to y, and each other variable in turn. We thus obtain as many equations as there are quantities a, b and c to be determined. Solving them with regard to these last quantities, and substituting in the original formula $0 = 1 + ax + by + cz$, we have the desired equation.

As an example, suppose we have the three points, Fig. 1, whose coördinates are $x' = 1$, $y' = 1$, $x'' = 2$, $y'' = 2$, $x''' = 3$, $y''' = 4$, and we wish to pass a straight line as nearly as possible through them all. We have for our equations of conditions: $0 = 1 + a + b$, $0 = 1 + 2a + 2b$, $0 = 1 + 3a + 4b$. Applying our rule, we multiply the first equation by 1, the second by 2, and the third by 3, the three values of x, and take their sum, which gives $1 + a + b + 2 + 4a + 4b + 3 + 9a + 12b = 6 + 14a + 17b = 0$. For our second normal equation we multiply by 1, 2 and 4,

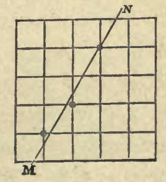


Fig. 1.

respectively, and obtain in the same way $7 + 17a + 21b = 0$. Solving, we find $a = -1.4$, $b = .8$, and substituting in our original equation $0 = 1 + ax + by$, we have $0 = 1 - 1.4x + .8y$, or $y = 1.75x - 1.25$. Constructing the line thus found, we obtain *MN*, Fig. 1, which will be seen to agree very well with our original conditions.

For a fuller description of the various applications of the Theory of Probabilities to the discussion of observations, the reader is referred to the following works. *Méthode des Moindres Carrées par Ch. Fr. Gauss, trad. par J. Bertrand, Paris*, 1855, *Watson's Astronomy*, 360, *Chauvenet's Astronomy*, II, 500, and *Todhunter's History of the Theory of Probabilities.* A good brief description is given in *Davies' and Peck's Math. Dict.*, 454, 536 and 590, also in *Mayer's Lecture Notes on Physics*, 29.

Peirce's Criterion. It has already been stated that all observations affected by errors not accidental, or mistakes, should be at once rejected. But it is generally difficult to detect them, and hence various *Criteria* have been suggested to enable us to decide whether to reject an observation which appears to differ considerably from the rest. One of the best known of these is Peirce's Criterion, which may be defined as follows: — The proposed observations should be rejected when the probability of the system of errors obtained by retaining them is less than that of the system of errors obtained by their rejection, multiplied by the probability of making so many and no more abnormal observations. Or, to put it in a simpler but less accurate form, reject any observations which increase the probable error, allowing for the chances of making so many and no more erroneous measurements. Without this last clause we might reject all but one, when the probable error by the formula would become zero. See *Gould's Astron. Journ.*, 1852, II, 161; IV, 81, 137, 145.

Another criterion has been proposed by Chauvenet, which, though less accurate than the above, is much more easily applied. It is fully described in *Watson's Astronomy*, 410.

Differences. To determine the law by which a change in any quantity *A* alters a second quantity *B*, we frequently measure *B* when *A* is allowed to alter continually by equal amounts. Thus in the example of the boiling of water, we measure the pressure corresponding to temperatures of 0°, 10°, 20°, 30°, &c. Writing these numbers in a table, by placing the various values of A in the first column, those of B in the second, we form a third column, in which each term is found by subtracting the value of B from that preceding it; the remainders are called the first differences D'. In the same way we obtain the second differences D'', by subtracting each first difference from that which follows it, and so on.

A.	*B.*	*D'.*	*D''.*	*D'''.*
0°	4.6			
		+ 4.6		
10°	9.2		+ 3.6	
		+ 8.2		+ 2.3
20°	17.4		+ 5.9	
		+ 14.1		+ 3.4
30°	31.5		+ 9.3	
		+ 23.4		
40°	54.9			

Interpolation. One of the most common applications of differences is to determine the value of B for any intermediate value of A. This is done by the formula,

$$B = B_m + n\,D_m' + \frac{n(n-1)}{1.\,2}D_m'' + \frac{n(n-1)(n-2)}{1.\,2.\,3}D_m''' + \&c,$$

in which B_m is the measurement next preceding B; D_m', D_m'', D_m''', the 1st, 2d, 3d, differences, and n a fraction equal to $(A - A_m) \div (A_{m+1} - A_m)$, in which A, A_m correspond to B, B_m, and A_{m+1} is the next term of the series to A_m. The use of this formula is best shown by an example. Suppose, from the accompanying table, we wish to find the value of B corresponding to $A = 12.5$. We have $B_m = 1728$, $D_m' = 469$, $D_m'' = 78$, $D_m''' = 6$, $A_m = 12$, $A_{m+1} = 13$, $A = 12.5$ and $n = (12.5 - 12) \div (13 - 12) = .5$. Hence,

A.	*B.*	*D'.*	*D''.*	*D'''.*	*D''''.*
10	1000				
		+ 331			
11	1331		+ 66		
		+ 397		+ 6	
12	1728		+ 72		0
		+ 469		+ 6	
13	2197		+ 78		0
		+ 547		+ 6	
14	2744		+ 84		0
		+ 631		+ 6	
15	3375		+ 90		
		+ 721			
16	4096				

$$B = 1728 + .5(469) + \frac{.5(-.5)}{1.\,2}(78) + \frac{.5(-.5)(-1.5)}{1.\,2.\,3}(6) + 0,$$

$$B = 1728 + 234.5 - 9.75 + .375 + 0 = 1953.125.$$

In this particular case B is always the cube of A, and it may be seen that our formula gives an exact result. The reason is that the 4th, and all following differences, equal zero.

Inverse Interpolation. Next suppose that in the above example we desired the value of A for some given value of B, as B'; that is, in the equation,

$$B' = B_m + nD_m' + \frac{n(n-1)}{1.\,2}D_m'' + \frac{n(n-1)(n-2)}{1.\,2.\,3}D_m''' + \&c.$$

we wish to find n. Evidently it is impossible to determine this exactly, but an approximate value may be found by the method of successive corrections. Neglect all terms after the third, and deduce n from the equation,

$$B' = B_m + nD_m' + \frac{n(n-1)}{1.\,2}D_m'',$$

which is a simple quadratic equation. Substitute this value of n in the terms we have neglected, and call the result N, then

$$B' = B_m + nD_m' + \frac{n(n-1)}{1.\,2}D_m'' + N,$$

from which again we may deduce a more accurate valne of n. This again gives a new value of N, and by continuing this process we finally deduce n with any required degree of accuracy.

It is sometimes more convenient to neglect the third term, and deduce n from the equation $B' = B_m + nD_m'$, which saves solving a quadratic equation, but requires more approximations. The values of $n(n-1) \div 1.\,2$, $n(n-1)(n-2) \div 1.\,2.\,3$, &c., may be more readily obtained from Interpolation tables than by computation. A good explanation of this subject is given in the *Assurance Magazine*, XI, 61, XI, 301, and XII, 136, by Woolhouse.

When the terms are not equidistant the method of interpolation by differences cannot be applied. In this case, if we wish to find values of B corresponding to known values of A, we assume the equation, $B = a + bA + cA^2 + dA^3 + \&c.$, and see what values of a, b, c, &c., will best satisfy these equations. If we have a great many corresponding values of A and B, the method of least squares should be applied. In general, however, it is much more convenient to solve this problem by the Graphical Method described below. See *Cauchy's Calculus*, I, 513, and an article in the *Connaissance des Temps*, for 1852, by *Villarceau*.

Numerical Computation. Where much arithmetical work is necessary to reduce a series of observations, a great saving of time is effected by making the computation in a systematic form. In general, measurements of the same quantity should be written in a column, one below the other, instead of on the same line, and plenty of room should always be allowed on the paper. When the same computations must be made for several values of one of the variables, instead of completing one before beginning the next, it is better to carry all on together, as in the following example. Suppose, as in the experiment of the Universal Joint, we wish to compute the values of b in the formula, $\tan b = \cos A \tan a$, in which $A = 45°$, and a in turn 5°, 10°, 15°, &c. Construct a table thus:—

a	5°	10°	15°	20°	25°	30°
log tan a	8.94195	9.24632	9.42805	9.56107	9.66867	9.76144
log cos A	9.84948	9.84948	9.84948	9.84948	9.84948	9.84948
log tan b	8.79143	9.09580	9.27753	9.41055	9.41815	9.61092
b	3° 32′	7° 6′	10° 44′	14° 26′	14° 41′	15′

In the first line write the various values of a, in the second the corresponding values of its log tan, and so on throughout the computation. An error is purposely committed in the above table to show how easily it may be detected. It will be noticed that the values of b increase pretty regularly, except that when $a = 25°$, and that this is but little greater than that corresponding to $a = 20°$. Following the column up we find that the same is the case for log tan b but not for log tan a, hence the error is between the two. In fact, in the addition of the logarithms we took 6 and 4 equal to 10, and omitted to carry the 1; log tan b then really equals 9.51815, and $b = 18° 15′$. If the error is not found at once this value of b should be recomputed. Besides these advantages, this method is much quicker and less laborious. When we have to multiply, or divide by, the same number A a great many times, it is often shorter to obtain at once $1A$, $2A$, $3A$, $4A$, &c., and use these numbers instead of making the multiplication each time. This is useful in reducing metres to inches, &c. There are many other arithmetical devices, but their consideration would lead us too far from our subject.

Significant Figures. One of the most common mistakes in reducing observations is to retain more decimal places than the experiment warrants. For instance, suppose we are measuring a distance with a scale of millimetres, and dividing them into tenths by the eye, we find it 32.7 mm. Now to reduce it to inches we have 1 metre = 39.37 in., hence 32.7 mm. = 1.287399. But it is absurd to retain the last three figures, since in our original measurement, as we only read to tenths of a millimetre, we are always liable to an error of one half this amount, or .002 of an inch. Then we merely know that our distance lies between 1.2894 and 1.2854 inches, showing that even the thousandths are doubtful. It is worse than useless to retain more figures, since they might mislead a reader by making him think greater accuracy of measurement had been attained.

If we are sure that our errors do not exceed one per cent. of the quantity measured, we say that we have two significant figures, if one tenth of a per cent. three, if one hundredth, four. Thus in the example given above, if we are sure the distance is nearer 32.7 than 32.8 or 32.6, we have three significant figures, and it would be the same if the number was 327,000, or .00327. In general, count the figures, after cutting off the zeros at either end, unless they are obtained by the measurement; thus 300,000 has three significant figures if we know that it is more correct than 301,000 or 299,000. In reducing results we should never retain but one more significant figure than has been obtained in the first measurement, and must remember that the last of these figures is sometimes liable to an error of several units.

Successive Approximations. This method is also known as that of trial and error. It consists in assuming an approximate value of the magnitude to be constructed, measuring the error, correcting by this amount as nearly as we can, measuring again, and so on, until the error is too small to do any harm. As an example, suppose we wish to cut a plate of brass so that its weight shall be precisely 100 grammes. We first cut a piece somewhat too large, weigh it and measure its area. If its thickness and density were perfectly uniform we could at once, by the rule of three, determine the exact amount to be cut off. As, however, it will not do to make it too light, we cut off a somewhat less quantity and weigh again; by a few repetitions of this process we may reduce the error to a very small amount.

This method is sometimes the only one available, but it should not be too generally used, as it encourages guessing at results, and tends to destroy habits of accuracy.

GRAPHICAL METHOD.

Suppose that we have any two quantities, x and y, so connected that a change in one alters the other. Then we may construct a curve, in which abscissas represent various values of x, and ordinates the corresponding values of y. Thus suppose we know that y is always equal to twice x. Take a piece of paper divided into squares by equidistant vertical and horizontal lines. Select one of each of these lines to start from. The vertical one is called the axis of Y, the other the axis of X, and their intersection, the origin. Make $x = 1$, y will equal 2, since it is double x; now construct a point distant 1 space from the origin horizontally, and 2 vertically. Make $x = 2$, $y = 4$, and we have a second point; $x = -1$, gives $y = -2$, &c., and $x = 0$, gives $y = 0$. Connecting these points we get a straight line passing through the origin, as is evident by analytical geometry from its equation, $y = 2x$. Again, let y always equal the square of x, and we have the corresponding values $x = 0, y = 0$; $x = 1, y = 1$; $x = -1, y = 1$; $x = 2$, $y = 4$; connecting all the points thus found we obtain a parabola with its apex at the origin, and tangent to the axis of X. As another example, suppose we have made a series of experiments on the volumes of a given amount of air corresponding to different pressures. Construct points making horizontal distances volumes, and vertical distances pressures. It will be found that a smooth curve drawn through these points approaches closely to an equilateral hyperbola with the two axes as asymptotes. Now this curve has the equation $xy = a$, or $y = a \div x$, that is, the volume is inversely proportional to the pressure, which is Mariotte's law. Owing to the accidental errors the points will not all lie on the curve, but some will be above it and others below, and this will be true however many points may be observed.

In general, then, after observing any two quantities, A and B, construct points such that their ordinates and abscissas shall be these quantities respectively. Draw a smooth curve as nearly as possible through them, and then see if it coincides with any common curve, or if its form can be defined in any simple way. To acquire practice in using the Graphical Method it is well to construct a number of curves representing familiar phenomena, as the variation in the U. S. debt during the late war, the strength of horses moving at different rates, and the alterations of the thermometer during the day or year. It is by no means necessary that the same scale should be used for vertical, as for horizontal distances, but this should depend on the size of paper, making the curve as large as possible. The greatest accuracy is attained when the latter is about equally inclined to both axes.

It is sometimes better when one of the variables is an angle to use polar coördinates. In this case paper must be used with a graduated circle printed on it. The points are constructed by drawing lines from the centre in the direction represented by one variable, and measuring off on them distances equal to the other. For ordinary purposes circles may easily be drawn, and divided with sufficient accuracy by hand. Laying off the radius on the circumference divides it to 60°; bisecting these spaces gives 30°, and a second bisection 15°. By trial these angles may be divided into three equal parts, which is generally small enough, as the observations are usually taken at intervals of 5°.

Interpolation. All kinds of interpolation are very readily performed by the Graphical Method. After constructing one curve to find the value of y, for any given value of x as x', we have only to draw a line parallel to the axis of Y, at a distance x', and note the ordinate of the point where it meets the curve. Inverse interpolation is performed in the same manner, and this method is equally applicable, whether the observations are at equal intervals or not. As by drawing a smooth curve the accidental errors are in a great measure corrected, this method of interpolation is often more accurate than that by differences.

Residual Curves. The principal objection to the Graphical Method, as ordinarily used, is its inaccuracy, as by it we can rarely obtain more than three significant figures, although Regnault, by using a large plate of copper and a dividing engine to construct his points, attained a higher degree of precision.

It will be found, however, that in many of the most carefully conducted researches the fourth figure is doubtful, as for example, in Regnault's measurements of the pressure of steam, and even in Angström's and Van der Willingen's determinations of wave-lengths.

By the following device the accuracy of the Graphical Method may be increased almost indefinitely. After constructing our points, assume some simple curve passing nearly through them. From its equation compute the value of y for each observed value of x, and construct points whose ordinates shall equal the difference between the point and curve on an enlarged scale, while the abscissas are unchanged. Thus let x', y' be the observed coördinates, and $y = f(x)$, the assumed curve. Construct a new point, whose coördinates are x' and $a\,[y' - f(x')]$, in which a equals 5, 10, or 100, according to the enlargement desired.

Do the same for all the other points, and a curve drawn through them is called a residual curve. In this way the accidental errors are greatly enlarged, and any peculiarities in the form of the curve rendered much more marked. If the points still fall pretty regularly, we may construct a second residual curve, and thus keep on until the accidental errors have attained such a size that they may be easily observed. To find the value of y corresponding to any given value of x, as x_n, we add $f(x_n)$ to the ordinate of the corresponding point of the residual curve, first reducing them to the same scale. Most of the *singular points* of a curve are very readily found by the aid of a residual curve. See an article by the author, *Journal of the Franklin Institute*, LXI, 272.

Maxima and Minima. To find the highest point of a curve, use, as an approximation, a straight line parallel to the axis of X, and nearly tangent to the curve. Construct a residual curve, which will show in a marked manner the position of the required point. The same plan is applicable to any other maximum or minimum.

Points of Inflexion. Draw a line approximately tangent to the curve at the required point. In the residual curve the change of curvature becomes very marked.

Asymptotes. Asymptotes present especial difficulties to the Graphical Method, as ordinarily used. Suppose our curve asymptotic to the axis of X; construct a new curve with ordinates unchanged, and abscissas the reciprocals of those previously used, that is equal to $1 \div x$. It will contain between 0 and 1 all the points in the original curve between 1 and ∞. It will always pass through the origin, and unless tangent to the axis of X at this point the area included between the curve and its asymptote will be infinite. When this space is finite, it may be measured by constructing another curve with abscissas as before equal to $1 \div x$, and ordinates equal to the area included between the curve and axis, as far as the point under consideration. Find where this curve meets the axis of Y, and its ordinate gives the required area. A problem in Diffraction is solved by this device in the *Journal of the Franklin Institute*, LIX, 264.

Curves of Error. This very fruitful application of the Graphical Method is best explained by an example. Suppose we wish to draw a tangent to the curve $B'A$, Fig. 2, at the point A. Describe a circle with A as a centre, through which pass a series of lines, as AB, AD, AE. Now construct C by laying off BC equal to AB', the part of the curve cut off by the line. We thus get a curve CD, called the *curve of error*, intersecting the circle at D, and the line AD is the required tangent. This is evident, since if we made our construction at this point we should have no distance intercepted, or the line AD touching, but not cutting, the curve. A similar method may be applied to a great variety of problems, such as drawing a tangent parallel to a given line, or through a point outside the curve.

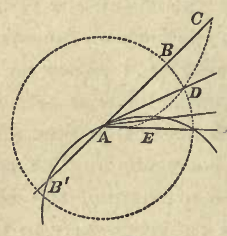


Fig. 2.

Three Variables. The Graphical Method may also be applied where we have three connected variables. If we construct points whose coördinates in space equal these three variables, a surface is generated whose properties show the laws by which they are connected. To represent this surface the device known as contour lines may be used, as in showing the irregularities of the ground in a map. First, generate a surface by constructing points in which ordinates and abscissas shall correspond to two of the variables, and mark near each in small letters the magnitude of the third variable, which represents its distance from the plane of the paper. If now we pass a series of equidistant planes parallel to the paper, their intersections with the surface will give the required contour lines. To find these intersections, connect each pair of adjacent points by a straight line, and mark on it its intersections with the intervening parallel planes. Thus if two adjacent points have elevations of 28 and 32, we may regard the point of the surface midway between them, as at the height 30, or as lying on the 30 contour line. Construct in this way a number of points at the same height, and draw a smooth curve approximately through them; do the same for other heights, and we thus obtain as many contours as we please.

They give an excellent idea of the general form of the surface, and by descriptive geometry it is easy to construct sections passing through the surface in any direction. An easy way to understand the contours on a map is to imagine the country flooded with water, when the contours will represent the shore lines when the water stands at different heights. This method is constantly used in Meteorology to show what points have equal temperature, pressure, magnetic variation, &c. Contour lines follow certain general laws which are best explained by regarding them as shore lines, as described above. Thus contour lines have no terminating points; they must either be ovals, or extend to infinity. Two contours never touch unless the surface becomes vertical, nor cross, unless it overhangs. A single contour line cannot lie between two others, both greater or both smaller, unless we have a ridge or gulley perfectly horizontal, and at precisely the height of the contour. In general, such lines should be drawn either as a series of long ovals, or as double throughout. There will be no angles in the contour lines unless there are sharp edges in the original surface. A contour line cannot cross itself, forming a loop, unless the highest point between two valleys, or the lowest point between two hills, is exactly at the height of the contour.

The value of contour lines in showing the relation between any three connected variables, is well illustrated in a paper by Prof. J.

Thomson, *Proc. of the Royal Society*, Nov., 1871, also in *Nature*, V, 106.

To acquire facility in using the Graphical Method, it is well to apply it to some numerical examples. Thus take the equation $y = ax^3 + bx^2 - cx + d$, assume certain values of a, b, c and d, and compute the value of y for various values of x. We thus get a curve with two maxima or minima, and a point of inflexion. Find their position first by residual curves, and then by the calculus, and see if they agree. In the same way the curve $yx^2 - 2ayx + a^2y = b$, has the axis of X for an asymptote. Assume, as before, positive values of a and b, and determine the area between the curve and asymptote, first by construction and then analytically.

PHYSICAL MEASUREMENTS.

The measurement of all physical constants may be divided into the determination of time, of weight and of distance, the apparatus used varying with the magnitude of the quantity to be measured and the degree of accuracy required.

Measurement of Time. A good clock with a second hand, and *beating seconds*, should be placed in the laboratory, where it can be used in all experiments in which the time is to be recorded. Watches with second-hands do not answer as well, as they generally give five ticks in two seconds, or some other ratio which renders a determination of the exact time difficult. The true time may be measured by a sextant or transit, as described in Experiment 16. This should be done, if possible, every clear day by different students, and a curve constructed, in which abscissas represent days, and ordinates errors of the clock, or its deviations from true time. Short intervals of time may be roughly measured by a pendulum, made by tying a stone to a string, or better, by a tape-measure drawn out to a fixed mark. We can thus measure such intervals as the time of flight of a rocket or bomb-shell, the distance of a cannon or of lightning, by the time required by sound to traverse the intervening space, or the velocity of waves, by the time they occupy in passing over a known distance. After the experiment we reduce the vibrations to seconds by swinging our pendulum, and counting the number of oscillations per minute.

By graduating the tape properly, we may readily construct a very serviceable metronome.

Where the greatest accuracy is required, as in astronomical observations, a chronograph is used. A cylinder covered with paper is made to revolve with perfect uniformity once in a minute. A pen passes against this, and receives a motion in the direction of the axis of the cylinder, of about a tenth of an inch a minute, causing it to draw a long helical line. An electro-magnet also acts on the pen, so that when the circuit is made and broken, the latter is drawn sideways, making a jog in the line. To use this apparatus a battery is connected with the electro-magnet, and the pendulum of the observatory clock included in the circuit, so that every second, or more commonly every alternate second, the circuit is made for an instant and then broken. Wires are carried to the observer, who may be in any part of the building, or even at a distance of many miles, and whenever he wishes to mark the time of any event, as the transit of a star, he has merely, by a finger key (such as is used in a telegraph office), to close the circuit, when it is instantly recorded on the cylinder. When the observations are completed the paper is unrolled from the cylinder, and is found to be traversed by a series of parallel straight lines, Fig. 3, one corresponding to each minute, with indentations corresponding to every two seconds. The time may be taken directly from it, the fractions of a second being measured by a graduated scale. One great difficulty in making this apparatus was to render the motion of the cylinder perfectly uniform, as if driven by clock-work it would go with a jerk each second. This is avoided by a device known as Bond's spring governor, in which a spring alternately retards and accelerates a revolving axle when it moves faster or slower than the desired rate. The seconds marks form a very delicate test for the regularity of this motion, since in consecutive minutes they should lie precisely in line, and the least variation is very marked in the finished sheet. It is a very simple matter by this apparatus to measure the difference in longitude of two points. It is merely necessary that an observer should be placed at each station,

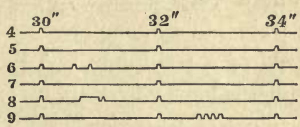


Fig. 3.

with a transit and finger key, a telegraph connecting them with the chronograph. They watch the same star as it approaches their meridian, and each taps on his finger key the instant it crosses the vertical line of his transit. Two marks are thus made on the chronograph, and the interval between them gives the difference in longitude. The advantage of this method of taking transits is not so much its accuracy, as the ease and rapidity with which it is used. Observers can work much longer with it without fatigue, and can use many more transit wires, thus greatly increasing the number of their observations. It is called the American or telegraphic method, in distinction from the old, or "eye and ear" method of observing transits, where the fractions of a second were estimated, as described in Experiment 15.

The chronograph is exceedingly convenient in all physical investigations where time is to be measured, and nothing but its expense prevents its more general application.

A simple means of measuring small intervals of time with accuracy, is to allow a fine stream of mercury to flow from a small orifice, and collect and weigh the amount passed during the time to be measured. Comparing this with the flow per minute we obtain the time. A less accurate, but much more convenient, liquid for this purpose is water, using, in fact, a kind of clepsydra. Where very minute intervals of time are to be measured they are commonly compared with the vibrations of a tuning-fork instead of a pendulum. A fine brass point is attached to the fork which is kept vibrating by an electro-magnet. If a plate of glass or piece of paper covered with lampblack, is drawn rapidly past the brass point, a sinuous line is drawn, the sinuosities denoting equal intervals of time, whose magnitude is readily determined when we know the pitch of the fork. A second brass point is placed by the side of the fork and depressed from the beginning to the end of the time to be measured. The length of the line thus drawn, compared with the sinuosities, gives the time with great accuracy. Recently a clock has been constructed, in which the pendulum is replaced by a reed vibrating one thousand times a second. The clock is started and stopped, so that it is going only during the time to be measured, and the hands record the number of vibrations made. The reed produces a musical note, and any irregularity is at once detected by a change in its pitch.

Measurement of Weight. This is done almost exclusively by the ordinary balance, whose principle is so fully explained in any good text-book of Physics that a detailed description is unnecessary here. We test the equality in length of its arms by double weighing, that is, placing any heavy body first in one pan and then in the other, and seeing if the same weights are required to counterpoise it in each case. The center of gravity should be very slightly below the knife-edges. If too low the sensibility is diminished, if too high the balance will overturn, and if coincident with them the beam, if inclined, will not return to a horizontal position The three knife-edges must be in line, otherwise the centre of gravity will vary with the weight in the scale pans, and of course the friction must be reduced to a minimum. A high degree of accuracy may be obtained with even an ordinary balance by first counterpoising the body to be weighed, then removing it and noting what weights are necessary to bring the beam again to a horizontal position. A spring balance is sometimes convenient for rough work, from the rapidity with which it can be used. It may be rendered quite accurate, though wanting in delicacy, by noting the weight required to bring its index to a certain point, first when the body to be weighed is on the scale pan, and then when it is removed.

Measurement of Length. Distances are most commonly measured by a scale of equal parts, that is, one with divisions at regular intervals, as millimetres, tenths of an inch, &c. This scale is then placed opposite the distance to be measured, and the reading taken directly. To obtain greater accuracy than within a single division, we may divide them into tenths by the eye, as in Experiment 1. The steel scales of Brown & Sharpe are good for common measurements, and may be obtained with either English or French graduation. Instead of dividing into tenths by the eye, a vernier is frequently used. Thus to read a millimetre scale to tenths, nine spaces are divided into ten equal parts, each of which will be a tenth of a millimetre less than the divisions of the scale, as in Experiment 2.

One of the best devices for measuring very minute quantities is the micrometer screw. A divided circle is attached to the head of a carefully made screw, so that a large motion of the former corresponds to a very minute motion of the latter. Thus if the pitch of the screw is one millimetre, and the circle is divided into one hundred parts, turning it completely around will move the screw but one millimetre, or turning it through one division only one hundredth of a millimetre. One of the best examples of this instrument is the dividing engine, which consists of a long and very perfect micrometer screw with a movable nut. See Experiment 21, also *Jamin's Physics*, I, 25. It is much used in engraving scales, but it has certain defects which are unavoidable, and have caused some of our best mechanicians to give it up. For example, it is impossible to make a screw perfectly accurate, and every joint, of which there are several, is a source of constantly varying error. For these reasons, and owing to its expense, the instrument described in Experiment 22 is for many purposes preferable. Two blocks of wood are drawn forward alternately step by step, through distances regulated by the play of a peg between a plate of brass and the end of a screw. As all joints are thus avoided, and the interval is determined by the direct contact of two pieces of metal, great accuracy is attainable by it.

Where several scales are to be made with the utmost accuracy, one should first be divided as correctly as possible, and its errors carefully studied by comparing the different parts with one another, or with a standard. It may then be copied by laying it on the same support with one of the other scales, and moving both so that one shall pass under a reading microscope, the other under a graver. We may thus copy any scale with great accuracy, but the process is very laborious. A good way to construct the first scale is by continual bisection with beam compasses, as is done in graduating circles. The finest scales are ruled with a diamond on glass. M. Nobert has succeeded in making them with divisions of less than a hundred thousandth of an inch. The intervals are so minute that until within a few years no microscope could separate the lines. The method of making them is kept a secret. Mr. Peters, by a combination of levers, has succeeded in reducing writings or drawings to less than one six thousandth their original size. He exhibited some writing done by this machine, which was so minute that the whole Bible might be written twenty-seven times in a square inch. Finally, it is claimed that Mr. Whitworth was able to detect differences of one millionth of an inch with a micrometer screw he has constructed.

To measure very minute distances a microscope is often used with a scale inserted in its eyepiece, which is used like a common rule. The absolute size of the divisions must be determined beforehand by measuring with it a standard millimetre, or hundredth of an inch. A more accurate method, however, is the spider-line micrometer, in which a fine thread is moved across the field of view by a micrometer screw, and small distances thus measured with the greatest precision. By using two of these instruments, which are then called reading microscopes, larger distances may be measured, or standards of length compared, as in Experiment 20.

Small distances are also sometimes measured by a lever, with one arm much longer than the other, so that a slight motion of the latter is shown on a greatly magnified scale. Instead of a long arm it is better to use a mirror, and view in it the image of a scale by a telescope. An exceedingly small deviation is thus readily perceptible, and this arrangement, sometimes known as Saxton's pyrometer, has been applied to a great variety of uses. Where we wish to bring the lever always into the same position a level may be substituted for the mirror, forming the instrument called the *contact level.* Small distances are also sometimes measured by a wedge with very slight taper, but this plan is objectionable on many accounts. In geodesy all the measurements are dependent on the accurate determination in the first place of a distance of five or ten miles, called a base line. Most of the above devices have been tried on such lines; thus the reading microscope was used by Colby in the Irish survey, the wedge in Hanover, and by Bessel in Prussia, the lever by Struvé in Russia, and the contact level is now in use on our Coast Survey. The principle in all is to use two long bars alternately, which are either brought in contact, or the distance between their ends measured each time they are laid down.

Many other physical constants are really determined by a measure of length. Thus temperatures are determined by a scale of equal parts in the thermometer, and here sufficient accuracy is obtained by reading with the unaided eye. Pressures of air and water are also measured by the height of a column of mercury or water. Where great accuracy is required, as in the barometer, a vernier is commonly used.

The instrument known as the cathetometer is so much used for measuring heights that it needs a notice here. It consists of a small telescope, capable of sliding up and down a vertical rod to which a scale is attached. The difference in height of any two objects is readily obtained by bringing the telescope first on a level with one, and then with the other, and taking the difference in the readings. A level should be attached to the telescope to keep it always horizontal, but the great objection to the instrument is that a very slight deviation in its position, which may be caused by focussing or turning it, is greatly magnified in a distant object. A good substitute for this instrument may be made by attaching a common telescope to a vertical brass tube, the scale being placed near the object to be measured instead of on the tube, as in Experiment 12.

Although the measurement of the following quantities is directly dependent on the above, yet their importance justifies a separate notice.

Measurement of Areas. It is difficult in general to determine an area with accuracy, especially where it forms the boundary of a curved surface. If plane, any of the methods of mensuration used in surveying may be adopted. Of these the best are division into triangles, Simpson's rule, and drawing the figure on rectangular paper and counting the number of enclosed squares, allowing for the fractions. Another method sometimes useful is to cut the figure out of sheet lead, tin foil, or even card board, and compare its weight with that of a square decimetre of the same material.

Measurement of Volumes. These are generally determined by the weight of an equal bulk of water or mercury, using the latter if the space is small. The interior capacity of a vessel is measured by weighing it first when empty, and then when filled with the liquid, as in Experiment 19. The difference in grammes gives the volume in cubic centimetres when water is used, but with mercury we must divide by 13.6, its specific gravity. In the same way we may determine the exterior volume of any body by immersing it and measuring its loss of weight, as when determining its specific gravity.

An easier, but less accurate, method is by a graduated vessel. These are made by adding equal weights or volumes of liquid, successively, and marking the height to which it rises after each addition. The volume of any space may then be found by filling it with water, emptying it into the graduated vessel and reading the scale attached to the side of the latter.

Measurement of Angles. Angles are measured by a circle divided into equal parts, the small divisions being determined by verniers or reading microscopes, as in measuring lengths. A great difficulty arises from the centre of the graduation not coinciding with that of the circle, and on this account it is best to have two or more at equal intervals around the circumference. By taking their mean we eliminate the eccentricity.

The precision of modern astronomy is almost entirely due to the methods of determining angles with accuracy. This is dependent on two things; first, a good graduated circle, and secondly, a means of pointing a telescope in a given direction, as towards a star, with great exactness. The latter is accomplished by placing cross-hairs at the common focus of the object glass and eye-piece, so that they may be distinctly seen in the centre of the field at the same time as the object. Most commonly two cross-hairs are used at right angles, one being horizontal, the other vertical. When, however, we are to bring them to coincide with a straight line, as in the spectroscope, or in a reading microscope, they are sometimes inclined at an angle of about 60°, that is, each making an angle of 30° with the line to be observed. The latter is then brought to the point of the V formed by their intersection. Still another method is to use two parallel lines very near together, the line to be observed being brought midway between them. The lines may be made of the thread of a spider, of filaments of silk, of platinum wire, or better for most purposes, by ruling fine lines on a plate of thin glass with a diamond, and inserting it at the focus.

There are two methods of graduating circles with accuracy. The first, which is used in Germany, consists in a direct comparison with an accurately divided circle, as when copying scales as described above. That is, both circles are mounted on the same axis, and the divisions of the first being successively brought under the cross-hairs of a microscope, the graver cuts lines on the second at precisely the same angular intervals. In the second method, which is much quicker but less accurate, the circle is laid on a toothed wheel which is turned through equal intervals by a tangent screw. Both methods are really only means of copying an *originally divided* circle, as it is called, and the construction of this with accuracy is a matter of extreme difficulty. It is dependent on the following principles. Any arc or distance may be accurately bisected by beam compasses; the chord of 60° equals the radius, and the angle 85° 20′, whose chord is 1.3554, by ten bisections is reduced to 5′. By constructing an accurate scale, laying off 1.3554 times the radius on the circumference, and repeatedly bisecting the arc, we finally divide the circle into 5′ divisions. Where great accuracy is not required we may divide circles approximately by hand, as described under the Graphical Method, or more accurately by a table of chords and a pair of beam compasses. When the divisions of the circle are very large we may subdivide them by a scale instead of a vernier. Thus if *AB*, Fig. 4, is part of a circle divided into degrees, we may attach a scale *CD*, divided to ten minutes, and subdivide these into single minutes by the eye. Thus in Fig. 4 the reading is 2° 35′. Much labor is thus saved where the circles have to be divided by hand.

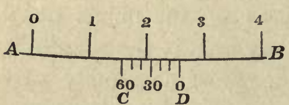


Fig. 4.

Saxton's pyrometer, described above, is of the utmost value in measuring small angular changes. As the reflected beam moves twice as fast as the mirror, the accuracy is doubled on this account. If the scale is flat, allowance must be made for the greater distance of its ends than the centre. To reduce the reading to degrees and minutes, the formula, $\tan 2a = s \div d$ is used, or $a = .5 \tan^{-1} s \div d$, in which a is the angle through which the mirror turns, s the reading, and d the distance of the scale taken in the same units. Instead of a telescope a light shining through a narrow slit is sometimes used, and an image projected on the scale by a lens, or the mirror itself may be made concave. This plan is adopted in the Thomson's Galvanometer, and other instruments for measuring the deviations of the magnetic needle.

Very small angles may also be measured by a spider line micrometer attached to the eye-piece of a telescope. This is used to determine the distance apart of the double stars, and other minute astronomical magnitudes. There are other methods, such as divided lenses, double image prisms, &c., but they will be considered in connection with the particular experiments which serve to illustrate them.

Measurement of Curvature. To measure the radius of a sphere, as the surface of a lens, an instrument called the spherometer is used. It consists of a micrometer screw at the centre of a tripod, whose three legs and central point are brought in contact with the surface. By noting the position of the screw, the radius is readily computed, as in Experiment 14.

When the surface is of glass, and the curvature very slight, a much more delicate method is as follows: Focus a telescope on a distant object, and then view the image reflected in the surface to be tested. If the latter is concave, it will render the ray less divergent, and hence the eye-piece will have to be pushed in. The opposite effect is produced by a convex mirror. The amount of change affords a rough measure of the curvature. This method is so delicate as to show a curvature whose radius is several miles.

GENERAL EXPERIMENTS.

1. Estimation of Tenths.

Apparatus. Two scales, *N* and *M*, are placed side by side, one being divided into millimetres, the other into tenths of an inch. Also a steel rule *A*, Fig. 5, divided into millimetres, and so arranged that it may be pushed past a fixed index *B*, by a micrometer screw, *C*. A spring, *D*, is used to bring it back, when the screw is turned the other way.

Experiment. Read the position of each tenth of an inch mark of scale *M*, in tenths of a millimetre, estimating the fractions by the eye. Thus if the interval is one half, call it .5, if a little less, .4, if not quite a third, .3, and so on for the other fractions. The .3 and .7 are the hardest to estimate correctly, as we are liable to imagine the former too great, the latter too small. They should always be compared with the fractions one and two thirds. Record your observations in five columns, placing in the first the readings of the scale *M*, in the second the corresponding readings of *N*, and in the third the first differences of *N*. Next, subtract the first from the last number in column two, and divide the difference by the number of spaces measured, that is, the number of readings minus one. This gives the average difference, and should be equal to each number of column three. Subtract it from these numbers, and place the results or errors, with proper signs, in column four. Next, compute the probable

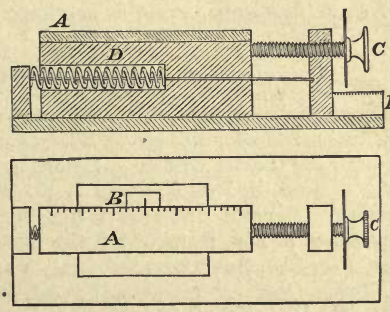


Fig. 5.

error (see page 3) of a single observation, using the fifth column for the squares of column four. In this way you can read any scale much more accurately than by its single divisions, and your computed probable error shows how closely you may rely on the result.

Next bring one of the millimetre marks of *A*, Fig. 5, opposite the index *B*. Read its position, as described on page 20. The scale *E* gives units, or number of revolutions, and the divided circle hundredths. Move the screw, set again, and repeat several times. Take the mean and compute the probable error of a single observation. Do the same with the next millimetre mark. Now move the scale until the reading shall be in turn .1, .2, .3, &c., of a millimetre, taking care to move the screw after each, so that you will not be biassed by your previous reading. Next compute what should be the true readings in these various positions. Thus let m' be the mean for the first millimetre, m'' for the second; the reading for one tenth would be $m' + (m'' - m') \div 10$, for two tenths $m' + 2(m'' - m') \div 10$, and so on. See how these readings agree with those previously found. If any differ by a considerable amount repeat them until you can estimate any fraction with accuracy. This work must be carefully distinguished from guessing, since there should be no element of chance in it, but an accurate division of the spaces by the eye. By practice one can read these fractions almost as accurately as by a vernier.

2. Verniers.

Apparatus. A number of verniers and scales along which they slide are made of large size. The best material is metal or wood, although cardboard will do. By making them on a large scale, as a foot or more in length, there is no trouble in attaining sufficient accuracy. Several different forms are given in *Gillespie's Land Surveying*, p. 228, from which the following may be selected.

1st, Fig. 225, Scale divided to .1, Vernier reads to .01; 2d, Fig. 227, Same Vernier retrograde; 3d, Fig. 228, Scale .05; Vernier .002; 4th, Fig. 229, Scale 1°, Vernier 5′; 5th, Fig. 230, Scale 30′, Vernier 1′; 6th, Fig. 233, scale 20′, Vernier 30″; 7th, Fig. 239, Scale 30′, Vernier 1′; Double Compass Vernier.

Experiment. A vernier may be regarded as a simple enlargement of one division of the scale. Thus if the scale is divided into tenths of an inch, and the vernier into ten parts, it will read to hundredths of an inch. Always read approximately by the zero of the vernier, taking the division of the scale next below it. The fraction to be added is found by seeing what line of the vernier coincides most nearly with some line of the scale. Thus in the first example, we obtain inches and tenths by seeing what division of the scale falls next below the zero of the vernier. If this is 8.6, and the division marked 7 of the vernier coincides with a line of the scale, the true reading is $8.6 + .07 = 8.67$. To prove this, set the zero of the vernier at 8.6 exactly. Nine divisions of the scale equal ten of the vernier. Hence each division of the latter equals .09, or is shorter by .01 than one division of the scale. Accordingly the line marked 1 of the vernier falls short by .01 of the scale-division, the 2 line .02, and so on. If we move the vernier forward by these amounts these lines will coincide in turn. Hence when the 7 line coincides, as in the above example, it denotes that the vernier has been pushed forward .07 beyond the 8.6 mark. This method may be applied to reading any vernier. To find the magnitude of the divisions of the latter, divide one division of the scale by the number of parts contained in the vernier.

Read and record the verniers as now set. Then set them as follows: 1st, 8.03; 2d, 29.9; 3d, 30.866; 4th, $4° 10'$; 5th, $0° 17'$; 6th, $2° 58' 30''$; 7th, $2° 51'$.

The last vernier is a double one, reading either way, the left hand upper figures being the continuation of those on the lower right hand. This is best understood by moving it $5'$ at a time and noting what lines coincide.

After each exercise the instructor should set all the verniers, and compare the record of the student with his own.

3. Insertion of Cross-Hairs.

Apparatus. Some common sewing silk, card-board and mucilage, also a pair of dividers, ruler and triangle.

Experiment. A great portion of the accuracy attained in modern astronomical work is dependent on the exactness with which we can point a telescope, or other similar instrument, in a given direction. This is accomplished by inserting two filaments of silk or spider's web at right angles to each other, at the point within the telescope where the image of the object is formed. In the astronomical telescope, where a positive eye-piece is used, this point lies just beyond the eye-piece, that is between it and the object-glass. A ring is placed at this point on which the lines are stretched. In telescopes rendering objects upright, as in most surveyor's transits, the lines are commonly placed between the object-glass and erecting lenses, and close to the latter. In the microscope, and other instruments where a negative eye-piece only is used, the lines have to be placed on the diaphragm between the field- and eye-lenses. This plan is objectionable, since the lines should be very accurately focussed, which can then only be done by screwing the eye-lens in or out. In the other cases the whole eye-piece may be slid in or out until the lines are perfectly distinct, and do not appear to move over the object when the eye is moved from side to side.

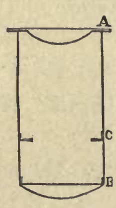


Fig. 6.

It is comparatively easy to insert the lines on their ring, where a positive eye-piece is used. The following experiment therefore includes the others. Take a negative eye-piece, Fig. 6, from a microscope or telescope, and unscrew the eye-lens *A*. *C* is the diaphragm which limits the field of view, and on which the lines should be placed. Cut from the cardboard a ring, Fig. 7, whose inner diameter is a little greater than the opening of the diaphragm, and the outer diameter such that it will easily rest on *C*. Mark on it two lines at right angles to each other passing through its centre. Unravel a short piece of the silk thread until you have separated a single filament. This is best done by holding the thread with the forceps over a sheet of white paper. We now wish to stretch two of these filaments over the lines marked on the cardboard circle. Put a little mucilage on the latter, dip one end of the silk into it, and press it down with one of the radial strips of paper shown in Fig. 7. When this is nearly dry fasten the other end in the same way, taking care to stretch it so that it shall be straight, or the twist in the thread will give it a sinuous form. Attach the

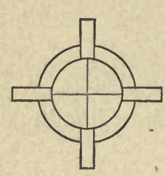


Fig. 7.

other thread in the same way, and bending the four strips of paper down lay the cardboard on the diaphragm. To hold it in place cut a strip of cardboard or brass, and bending it into a circle push it into the tube. By its elasticity it will hold the paper strips firmly against the sides of the tube. If the experiment has been well performed, on replacing the eye-lens we see two straight lines at right angles, dividing the field of view into four equal parts. The cardboard should not project beyond the diaphragm, or it will give a rough edge to the field of view, and we must be careful that no mucilage adheres to the visible portions of the threads.

4. Suspension by Silk Fibres.

Apparatus. The best method of suspending a light object so that it shall move very freely is by a single filament of silk. The only apparatus needed is a stand seven or eight inches high, some unspun silk (common silk thread will do, but is not so good) and some fine copper wire. We also need two pairs of forceps, such as come with cheap microscopes, some bees-wax and a sheet of white paper.

Experiment. Lay the silk on the paper and pick out a single fibre a little over six inches long. Bend pieces of the wire into the shapes *A* and *B*, Fig. 8. Pass one end of the filament through the ring of *B*, and fasten it with a little wax, twisting or tying it to prevent slipping. Fasten the other end to *A* in the same way, making the distance from *A* to *B* just six inches. Hook *A* into the stand, and lay the object to be suspended, as a needle on *B*.

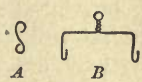


Fig 8.

5. Temperature Curve.

Apparatus. A beaker, stand and burner, by which water can be heated, a Centigrade thermometer, and a clock or watch giving seconds.

Experiment. Place the thermometer in the water and record the temperature, dividing the degrees to tenths, as described in Experiment 1. Place the burner under the beaker at the beginning of a minute, and at the end record the temperature; repeat at the end of each minute, as the water is warmed, until the thermometer stands at 95°; at the end of the next minute remove the burner and the temperature will at first continue to rise, and will then fall rapidly. Record the time (in minutes and seconds) of attaining 95°, 90°, 85°, &c., taking shorter intervals as the temperature becomes lower, and the cooling less rapid. Record your results in two columns, one giving times, the second temperatures. Finally construct a curve in which abscissas represent times, and ordinates temperatures, making in the former case, one space equal one minute, in the latter, one degree.

When two students, *A* and *B*, are engaged in this experiment, the following system should be used. *A* observes the watch and records, while *B* attends to the thermometer. Five seconds before the minute begins *A* says, *Ready!* and at the exact beginning, *Now!* *B* then gives the reading which *A* records. This plan saves much trouble, and greatly increases the accuracy of any observations which must be made at regular intervals of time.

6. Testing Thermometers.

Apparatus. An accurate Centigrade thermometer is hung upon a stand, and close to it a Fahrenheit thermometer, which is to be tested, their bulbs being at the same height, and close together. A telescope with which they can be read more accurately is placed on a stand at a short distance, and their temperature may be altered at will by immersing their bulbs in a beaker of water, which may be either cooled by ice, or heated by a Bunsen burner. Some arrangement is desirable for stirring the water to keep it at a uniform temperature. One way is to use a circular disk of tin with holes cut in it, which may be raised or lowered in the beaker by a cord passing over a pulley, so that the observer, while looking through the telescope, can stir the water by alternately tightening and loosening the cord. A simple glass stirring rod may be used instead, if preferred.

Experiment. The problem is to determine the error of the Fahrenheit thermometer at different temperatures, by comparing it with the Centigrade thermometer, which is regarded as a standard. By means of the telescope read them as they hang in the air, estimating the fractions of a degree in tenths. Do the same when their bulbs are immersed in water, then cool them with ice and read again. This observation is important, as it shows the absolute error of each instrument. Next heat the water a few degrees with the burner, and then remove the latter. The temperature will still rise for a short time, then become stationary and fall. Read each thermometer at its highest point, stirring the water meanwhile. Repeat at intervals of about 10° until the water boils, and finally immerse again in the ice water, and see if the reading is the same as before.

We have now two columns of figures, the first giving the temperature of the Centigrade, the second that of the Fahrenheit, thermometer. Reduce the first to the second, recollecting that 0° C. = 32° F., and 100° C. = 212° F.; hence $F. = \frac{9}{5} C. + 32°$, calling C and F the corresponding temperatures on the Centigrade and Fahrenheit scales respectively. Write the numbers thus found in a third column, and the errors will equal the differences between them and the readings given in column two. If the Centigrade thermometer does not stand at zero when immersed in ice water, all its readings should be corrected by the amount of the deviation, taking care to retain the proper sign. Now construct a curve whose ordinates shall represent the errors on an enlarged scale, and abscissas the temperatures.

7. Eccentricity of Graduated Circles.

Apparatus. A circle divided into degrees carries a pointer with an index at each end, which turns eccentrically, that is, the centres of the pointer and circle do not coincide. It may be made in a variety of ways. One of the simplest is to place a pivot on one side of the centre of the circle, and on it a rod with a needle projecting from each end. Another way is to let the circle turn and cover it with a plate of glass, on which are marked two fine lines, with a diamond or India ink. The indices may also be made of fine wire, or horsehair. Lines of considerable length must be used, since the edge of the circle advances and recedes as it is turned. If greater accuracy is desired the plan shown in Fig. 9 may be adopted. The two indices (which may have verniers) are connected with the centre by the arms AC and CB. The circle turns around the pin D, and a rod passing through the guides EF, keeps the verniers in the proper position. Another good instrument for this experiment is the form of compass described under Magnetism in the latter part of the present work.

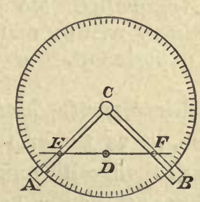


Fig. 9.

Experiment. Set the index A at 0° by turning the circle, and read B. Repeat moving A 10° at a time, until a complete revolution has been made. We have now two columns, giving the corresponding readings of A and B. Subtract 180° from the latter, and $\frac{1}{2}(A + B - 180°)$, or $\frac{1}{2}(A + B) - 90°$ will be the true reading; write this in column three; in the same way the error of each index is $\frac{1}{2}(A - B) - 90°$, which should be written in the fourth column. Construct a curve with abscissas equal to the numbers in column three, and ordinates equal to those in column four, enlarged. At the highest and lowest parts of the curve the indices differ most from their true position, or the absolute error, if we read one only, is here greatest. Find these points by Curves of Error, p. 14. On the other hand, where the curve cuts the axis the two indices are opposite each other, and the abscissa gives the azimuth of the line CD. As the ordinates alter most rapidly at these points, the error, when reading a small angle by one index, is here a maximum. Draw tangents, as before, by Curves of Error, and from their direction we can compute the amount of variation. It is a very good exercise to deduce by trigonometry the theoretical curve, and constructing it on the same sheet of paper to compare the results with those obtained by your measurement.

We have heretofore supposed that the line connecting the indices passed through the axis around which they turned, or that D lies on EF. If, as often happens in practice, this is not the case, a second correction is necessary.

8. Contour Lines.

Apparatus. No apparatus is needed for this experiment, except ordinary writing materials. It is, in fact, an exercise rather than an experiment.

Experiment. Mark in your note book nine rows of six points each, so as to form forty squares of about one inch on a side. Mark them with numbers taken from the adjoining table A. Now suppose these numbers represent the heights of the points to which they are attached, and we wish to draw contour lines to show the form of the surface passing through them. As the points are pretty near together we may assume that a line connecting any two that are adjacent will lie nearly in the surface. Now regard your drawing as a map, as on p. 15, and suppose the ground

		A						*B*						*C*			
83	79	73	79	79	74	46	56	67	84	86	84	65	76	68	57	40	28
82	78	70	81	84	76	29	52	73	94	86	73	73	72	50	29	28	52
78	76	66	83	88	73	39	31	65	82	70	56	66	48	31	11	27	41
74	73	58	78	82	63	60	62	68	72	57	49	59	29	29	42	38	29
70	61	50	73	82	74	69	73	81	81	65	48	38	46	38	72	61	39
71	58	61	82	96	75	80	94	80	81	73	50	27	35	70	99	70	28
70	59	70	83	84	72	80	58	58	65	70	49	21	46	87	96	60	29
67	65	72	79	73	69	67	58	58	67	62	46	33	63	95	81	49	31
66	67	72	76	69	75	74	68	72	80	49	37	44	71	86	64	47	27

flooded with water to a height of 80. Evidently all the points in the upper line will be submerged except that on the left, and the shore line will come between 79 and 83, about a fourth way from the former. Also midway between 82 and 78 in the second line, two fifths of the way from 78 to 83, and a third way from 79 to 82. Several points are thus obtained in each square through which the contour line passes. After obtaining as many as possible, draw a smooth curve nearly coinciding with them all, paying special attention to the rules given under the Graphical Method. Construct in the same way other contours at intervals of ten units. Do the same with the numbers in table *B* or *C*.

This work is very well supplemented by procuring from the U. S. Signal Bureau at Washington, some of their blank maps (issued at $2.75 per 100), and filling them out from the weather reports for the day, according to their published directions. These maps may also be used for drawing isothermals, isogonals, &c., if a list is prepared in the first place of the temperature, magnetic variation, &c., of a large number of stations in the United States. The method adopted for drawing these lines is essentially the same as that given above, only the points are irregularly spaced.

9. Cleaning Mercury.

Apparatus. But little apparatus is needed for this experiment, except such as is found in every chemical laboratory. Some bottles, funnels, &c., should be placed on the table, and the student should try as many of the following methods of purification as he can, and record in his note-book his opinion of their comparative value.

Experiment. Mercury is so much used in physical experiments that every student should know how to clean it. The impurities may be divided into three classes: first, mixture with metals, especially lead, zinc and tin; secondly, common dust and dirt; and thirdly, water or other liquids.

Redistillation is almost the only way to remove the metals, and even this is not perfectly effectual, especially in the case of zinc. Moreover, by long boiling a small amount of oxide is formed, which is dissolved by the metal. The mercury used for amalgamating battery plates should therefore be kept separate from the rest and used for this purpose only. If but little of the metal is present it may be removed by agitating with dilute nitric acid. The best way to do this is to fill a long vertical tube with the acid and allow the mercury to flow into it from a funnel, in which is a paper filter with a fine hole in the bottom. The mercury falls through the long column of liquid in minute globules, and is thus readily and thoroughly cleaned. It may be drawn out below by a glass stopcock, or by a bent tube in which a short column of mercury shall balance a long column of acid. As the mercury collects it flows out of the end of the tube into a vessel placed to receive it. Instead of nitric acid a solution of nitrate of mercury may be used, if preferred. Another method is to fill a bottle about a quarter full of mercury, add a quantity of finely powdered loaf sugar, and shake violently. The metallic impurities are oxidized at the expense of the air, which must be renewed by a pair of bellows.

A great variety of devices are used to remove the mechanical impurities of mercury. For example, pouring it into a bag of chamois leather and squeezing the latter until the mercury comes through in fine globules. Or, making a needle hole in the point of a paper filter, placing it in a funnel and letting the mercury run through. The mercury may be washed directly with water, by shaking them together in a bottle, or better, filling a jar half full of mercury and letting the water from the hydrant bubble up through it. This is an excellent way to remove most liquids.

Next, to remove the water, pour the mixture into a small bottle, when the mercury will settle to the bottom, and the water overflow from the top. When the mercury fills the bottle transfer it to another vessel and repeat. If there is only mercury enough to half fill the bottle the second time, pour back some of the mercury already dried to displace the remaining water. Another way is to close the end of a funnel with the finger and pour in the mixture, drawing off the mercury below and leaving the water above. Care must be taken that the mercury does not spurt out on one side and escape. An inverted bottle, or better, a vessel with a tube and stopcock below, is more convenient for this purpose.

When only a few drops of water are present they may be removed by blotting paper, or a camel's hair brush. Also by applying heat; but in this case a stain will be left when the water evaporates, unless it has been previously distilled.

To see if the mercury is pure pour it into a porcelain evaporating dish. If lead is present it will tarnish the sides. A thin film will also, after a short time, form on its surface, due to oxidation; zinc and tin produce a similar effect. The surface when at rest should be very bright and almost invisible, and small globules, if detached, should be perfectly spherical, and not adhere to the glass but roll over it when the surface is inclined.

10. Calibration by Mercury.

Apparatus. The best way to perform this experiment is that given by Bunsen in his *Gasometry*, p. 27. This method is substantially as follows: Select a glass tube, about 2 cm. in diameter, and 40 cm. long, closed at one end. Fasten to it a paper millimetre scale. This is placed upright in a stand, at a short distance from a small telescope, by which the scale may be read with accuracy. On another stand is placed a vessel containing about two kilogrammes of pure mercury, covered with a layer of concentrated sulphuric acid, with a stopcock below, by which it may be drawn off. A small glass tube, also closed at one end, is used to receive it, which should contain, when filled, about 10 $cm.^3$ Its open end is ground flat, and it may be closed with a plate of ground glass, which is fastened to the thumb by a piece of rubber.

Experiment. Both mercury and tube should be perfectly clean, but if not, a few drops of water may be placed in the longer tube, provided great accuracy is not required. Fill the small tube with mercury, holding it with the fingers of the left hand, and remove the surplus by pressing the glass plate, which should be attached to the left thumb, down on to it. Take care that no air bubbles are imprisoned. Empty the mercury into the large tube, and read its height on the scale by the telescope, measuring from the top of the curved surface of the liquid. A clean wooden rod may be used to remove any bubbles of air or globules of mercury which adhere to the sides of the tube. Repeat this operation until the large tube is full of mercury. We now wish to know the volume of the small tube, as this is the unit in terms of which the larger one has been calibrated. The most accurate way to do this is to weigh the whole amount of mercury transferred, and divide by the number of times the smaller tube has been filled. But as it is generally difficult to weigh so heavy a body accurately, the contents of the smaller tube had better be weighed alone, repeating two or three times to see how much the quantity used will vary in consecutive fillings. The volume is then obtained by dividing the weight by 13.6, the specific gravity of mercury. Multiplying the quotient by 1, 2, 3, 4, &c., we obtain the volumes corresponding to our observed readings of the mercury column in the long tube.

Represent the results by a residual curve, as follows: Let s be the scale reading when the small tube has been emptied once into the long tube, and s' when the latter is full, or has received n times this volume of mercury, which we will call v. Then $(n - 1)v$ of mercury will fill the space $s' - s$, and the average volume per unit of length will equal $(n - 1)v \div (s' - s) = a$. If the tube was perfectly cylindrical we could find the volume V for any scale reading S by the formula, $V = a(S - s) + v$. In reality the tube is probably a little larger in some places than in others, it is therefore better to retain only two significant figures in a, and then compute by the formula the volumes corresponding to the various scale readings that have been observed. Subtract each of these from the corresponding volumes 1, 2, 3, &c., times v, and construct a residual curve in which ordinates equal these differences on an enlarged scale, and abscissas the scale readings. We can now obtain the volume with the greatest accuracy for any scale reading by adding to the value of V given by the formula, the ordinate of the corresponding point of the curve. A table may thus be constructed, giving the volume corresponding to each millimetre mark of the scale. But it is generally sufficiently accurate to make a simple interpolation from the original measurements, using only the first differences, as when employing logarithmic tables.

11. Calibration by Water.

Apparatus. A Mohr's burette *B*, Fig. 10, on a stand, and the vessel to be graduated *A*, which should be about six inches high, and an inch and a half in diameter. A paper scale divided into tenths of an inch should be attached to *A* with gum tragacanth, although shellac, or even mucilage, answers tolerably. A long string wound spirally around the vessel will keep the scale in place until the gum is dry.

Experiment. Fill the burette *B* to the zero mark. This is done by adding a little too much water, and drawing it off by the stopcock *C* into another vessel, until it stands at precisely the right level. Next, let the water flow into *A* until it reaches the one tenth of an inch mark, and read *B*. Do the same for each tenth of an inch, until the one inch mark is reached, and then for every half inch to the top. Do not let the water level in *B* fall below the 100 cm.^{3} mark, but when it reaches this point refill as before, and add 100 to the volume measured. Care should be taken not to get too much water into *A*; should this happen, a little may be drawn out with a pipette and replaced in *B*, but a slight error is thus introduced.

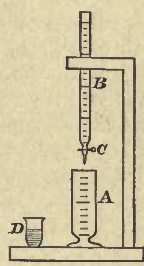


Fig. 10.

We have now a series of volumes corresponding to various scale readings. Construct a curve with these two quantities as co-ordinates. Find the point of the curve for which the volume is in turn 10, 20, 30, &c., cm.^{3}, and record the corresponding scale-reading. If the vessel is to be used for the measurement of volumes cover it with wax and draw horizontal lines on the latter, having the scale readings just found. Subject it to the fumes of fluorhydric acid, formed by mixing powdered fluor spar and concentrated sulphuric acid. The lines will thus be permanently etched on the glass.

12. Cathetometer.

Apparatus. A Cathetometer may be made by using as a base the tripod of a music stand or photographer's head-rest, and screwing into it a tube or solid rod of brass. To this is attached a small telescope with a clamp and set screw, and some form of *slow motion.* The latter may be obtained by placing the telescope on a hinge and raising and lowering one end by a screw. The slight deviation from a horizontal position will not affect the results, as the instrument is here used.

At a distance of five or ten feet is placed a U tube, open at both ends, with one arm about ten inches long, the other forty. The bend in the tube is filled with mercury, and water is poured into the long arm. We then have a long column of water sustaining a short column of mercury, the heights being inversely as the densities. By the side of this tube is a barometer, made by closing a common glass tube at one end, filling with mercury, and inverting over a cistern containing the same liquid. The precautions and details will be found under Experiment No. 55. By the side of this tube is placed a rod about ten inches long, sharply pointed at both ends, and capable of moving up and down so as to touch the surface of the mercury in the barometer cistern. A steel scale divided into millimetres is adjacent to both tubes, so that it can be read at the same time as the mercury columns.

Experiment. Focus the telescope so that both scale and mercury are distinctly visible. Then raise it until it is nearly on a level with A, the top of the column of water, and bring its horizontal cross-hair exactly to coincide by the slow motion. Read the scale, dividing the millimetres into tenths by the eye. Do the same at B and C; then the difference in height of A and C, divided by that of C and B, will equal the specific gravity of the mercury, which should be compared with its true value. As the surface of mercury is curved upwards, that of water downwards, the cross-hairs should be brought to the top of the former, and to the bottom of the latter. If great accuracy is required in this experiment, allow for the meniscus, or curved portion at the top of the

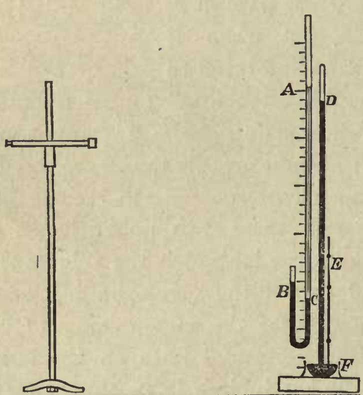


Fig. 11.

water, by adding one half its thickness to the height of the water column.

Next raise the rod *EF*, and read the height, first of the top and then of the bottom. The difference will be its length. It is safer to test the result by moving it and repeating. Then bring the rod so that it shall just touch the surface of the mercury, that is, so that the point and its reflection shall coincide, and read the height of *D*, and of the top of the rod. Their difference added to the length of the rod gives the height of the column. Read the height of the standard barometer placed among the meteorological instruments. Reduce this to millimetres, and subtract from it the other measurement. The difference will be the depression caused by air and the other errors in the barometer *D*.

13. Hook Gauge.

Apparatus. A stand, Fig. 12, on which may be placed a vessel of water *A*, and a micrometer screw *B*, by which we can raise or lower a rod carrying two points, one turned upwards, the other downwards.

Experiment. Fill up the vessel until the water just covers the point of the hook. Then turn the screw so that upon looking at the reflection on the surface of some object as a window sash, a slight distortion is produced by the elevation of the water above the hook. Make ten measurements, moving the screw after each, take their mean and compute the probable error of a single observation. When the point is raised it draws the liquid with it. Screw it down until it touches the liquid, and read the micrometer, then raise it until the liquid separates, and take ten readings in each position. Compute, as before, the probable error, and reduce to fractions of a millimetre, which is easily done if the pitch of the screw is known. This gives a measure of the comparative accuracy of the hook and simple point. Both are used for determining the exact height of any liquid surface, the hook being employed most frequently in this country, the point abroad. When the surface of a liquid is

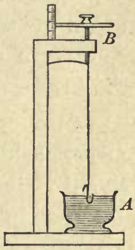


Fig. 12.

rising or falling, and we wish to know the exact time when it reaches a given level, we should use the hook when it descends, otherwise the point; because the former should always be brought up to the surface, the latter down to it.

This instrument is so extremely delicate that it will show the lowering of a surface of water in a few minutes by evaporation. A variety of interesting researches may be conducted with it, by the different students of a class. Thus its comparative accuracy with water, mercury and other liquids, may be measured, their rate of evaporation, and the effect of impurities, such as a drop of oil. The height to which a liquid may be raised by the point, is also a test of its viscosity.

14. Spherometer.

Apparatus. Two lenses, one convex, the other concave, a piece of thick plate glass and a spherometer. The latter consists of a tripod, with a micrometer screw in the centre, whose point may be moved to any desired distance above or below the plane of the three legs on which it rests. The most important qualities are lightness and stiffness, and on this account a very cheap, and quite efficient spherometer may be made with the nut and tripod of wood, using for legs, pieces of knitting needles.

Experiment. Stand the spherometer on the sheet of plate glass and turn the screw until its point is in contact with it. There are three ways of determining the exact position of contact. The first method is dependent on the fact that if the point of the screw is too low the spherometer will stand unsteadily, like a table with one leg too short. The screw is therefore depressed until the instrument rattles, when its top is moved gently from side to side. An exceedingly small motion of this kind is perceptible to the hand. The screw is then turned up and down until the exact point of contact is found. The second, and probably the best method, is to turn the screw slowly, taking care that no greater pressure is exerted on one leg than on the other; as soon as the point touches the glass the pressure is removed from the legs, and the friction of the nut at once makes the whole instrument revolve. Care must be taken not to press on the top of the screw, or the tripod will be bent, and an incorrect reading obtained. The third method of determining contact depends on the sound produced when the instrument slides over the glass, which changes when the screw touches the surface. It should be moved but a short distance and without pressure, for fear of scratching the glass.

Having determined this point with accuracy, read the position of the screw, taking the number of revolutions from the index on one side, and the fraction from the divided circle.

Place the spherometer on each face of the two lenses and measure the position of the point of contact as before. Of course the screw must be raised when the surface is convex, and depressed when it is concave. Subtract each of these readings from that taken on the plate glass, and the difference gives the height of a segment of the sphere to be measured, whose base is a circle passing through the three feet of the spherometer. Call this height h the radius of the circle r, and the radius of the sphere R; then we have, Fig. 13, $AB = h$, $BD = r$, and $AC = R$. But by similar triangles $AB : DB = DB : BE$, or $h : r = r : 2R - h$, or $R = \frac{r^2}{2h} + \frac{h}{2}$. Compute in this way the radius of each surface of the lenses, remembering that a negative radius denotes a concave surface. To determine r, measure the distance of each leg of the spherometer from the axis of the screw, and take their mean. Measure also the distances of the three legs from each other and take their mean. They form the three sides of an equilateral triangle; compute by geometry the radius of the circumscribed circle, and see if this value of r agrees with that previously found. Both r and h must be taken in the same unit, as millimetres or inches, and great care should be taken to make no mistake in the position of the decimal point. The reduction of h is effected by multiplying it by the pitch of the screw.

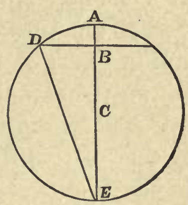


Fig. 13.

Finally, compute the principal focal distance, F, by the formula $\frac{1}{F} = (n - 1)\left[\frac{1}{R} + \frac{1}{R'}\right]$, in which R and R' are the radii of the two surfaces, as computed above, and n the index of refraction of the glass. The latter varies in different specimens, but in common lenses is about 1.53.

15. Estimation of Tenths of a Second.

Apparatus. A heavy body carrying a small vertical mirror is suspended by a wire, so that it will swing by torsion, about once in half a minute. A small telescope with cross hairs in its eye-piece, is pointed towards the mirror, and a plate with a pin hole in it is placed in such a position that when the mirror swings, the image of the hole will pass slowly across the field of view of the telescope, like a star. It may be made bright by placing a mirror behind it and reflecting the light of the window. The whole apparatus should be enclosed so as to cut off stray light. A good clock beating seconds is also needed.

Experiment. Twist the mirror slightly, so that it shall turn slowly. On looking through the telescope a point of light or star will be seen to cross the field of view, at equal intervals of about half a minute. Note the hour and minute, and as the star approaches the vertical line take the seconds from the clock and count the ticks of the pendulum. Fix the eye on the star and note its position the second before, and that after, it passes the wire. Subdividing the interval by the eye we may estimate the true time of transit within a tenth of a second. Take twenty or thirty such observations and write them in a column, and in a second column give their first differences. Take their mean and compute the probable error. It will show how accurately you can estimate these fractions of seconds.

This is called the eye and ear method of taking transits, which form the basis of our knowledge of almost all the motions of the heavenly bodies. It is still much used abroad, although in this country superseded in a great measure by the electric chronograph described on p. 16.

16. Rating Chronometers.

Apparatus. Two timekeepers giving seconds, one, which may be the laboratory clock, to be taken as a standard, and a second to be compared with it. For the latter a cheap watch may be kept expressly for the purpose, or the student may use his own. If the true time is also to be obtained, a transit or sextant is needed in addition.

Experiment. First, to obtain the true time. As this problem belongs to astronomy rather than physics, a brief description only will be given. It may be done in two ways, with a transit or a sextant; the former being used in astronomical observations, the latter at sea. A transit is a telescope, mounted so that it will move only in the meridian. With it note by the clock the minute and second when the eastern and western edges of the sun cross its vertical wire, and take their mean. Correct this by the amount that the sun is slow or fast, as given in the Nautical Almanac, and we have the instant of true noon. The interval between this and twelve, as given by the clock, is the error of the latter.

The sextant may be used at any time when the sun is not too near either the meridian or the horizon. A vessel containing mercury is used, called an artificial horizon, and the distance between the sun and its image in this is measured. Since the surface of the mercury is perfectly horizontal, this distance evidently equals exactly twice the sun's altitude. If the observation is made in the morning, when the sun is ascending, the sextant is set at somewhat too great an angle, if after noon at too small an angle, and the precise instant when the two images touch is noted by the clock. The sun's altitude, after allowing for its diameter, is thus obtained. We then have a spherical triangle, formed by the zenith Z, the pole P, and the sun S. In this, PZ is given, being the complement of the latitude; PS, the sun's north polar distance, is obtained from the Nautical Almanac, and ZS is the complement of the altitude just measured. From these data we can compute the angle ZPS, which corrected as before and reduced to hours, minutes and seconds, gives the time before or after noon. The practical directions for doing this will be found given in full in *Bowditch's Navigator.*

By these methods we obtain the mean solar time, which is that used in every day life. For astronomical purposes sidereal time, or that given by the apparent motion of the stars, is preferable. It is found by similar methods, using a star instead of the sun.

In an astronomical observatory it is found best not to attempt to make the clock keep perfect time, but only to make sure that its rate, or the amount it gains or loses per day, shall be as nearly as possible constant. We can then compute the error E at any given time very easily by the formula $E = E' + tr$, in which E' was the error t days ago, and r the rate. By transposing we may also obtain r, when we know the errors E and E', at two times separated by an interval t. Take the last two observations of the clock-error, which should be recorded in a book kept for the purpose, and compute the error at the time of your observation, and see how it agrees with your measurement.

If the day is cloudy, or no instruments are provided for determining the true time, the experiment may be performed as follows. Compute, as above, the rate and error of the clock. Next take the difference in minutes and seconds between the clock and the watch to be compared. To obtain the exact interval, a few seconds before the beginning of the minute by the watch, note the time given by the clock, and begin counting seconds by the ticks of the pendulum. Then fixing your eyes on the watch, mark the number counted when the seconds' hand is at zero. Repeat two or three times, until you get the interval within a single second. Now correcting this by the error of the clock, taking care to give the proper signs, we get the error of the watch. The next thing is to set the watch so that it shall be correct within a second. For this purpose it must be stopped, by opening it and touching the rim of the balance wheel very carefully with a piece of paper, or other similar object. Set the minute hand a few minutes ahead to allow for the following computation. Subtract the clock-error from the time now given by the watch. It will give the time by the clock, at which if the watch is started it will be exactly right. A few seconds before this time hold the watch horizontally, with the fingers around the rim, and at the precise second turn to the right and then back. The impulse starts the balance-wheel, and the watch will now go, differing from the clock by an amount just equal to the error of the latter.

17. Making Weights.

Apparatus. A very delicate balance and set of weights, some sheet metal, a pair of scissors, a millimetre scale, and a small piece of brass, A, weighing about 18.4 grammes. The weights are best made of platinum and aluminium foil; but where expense is a consideration, sheet brass may be used for the heavier, and tin foil for the lighter weights. To improve the appearance of the brass and prevent its rusting it may be tinned, or dipped in a silvering solution, or perhaps better still, coated with nickel. Some steel punches for marking the numbers 0, 1, 2 and 5, a mallet and sheet of lead should also be provided.

PROPER METHOD OF WEIGHING.

A good balance is so delicate an instrument that the utmost care is needed in using it. The student should thoroughly understand its principle, and know how to test both its accuracy and delicacy. See Measurement of Weights, p. 19. The beam should never be left resting on its knife-edges, or they will become dulled. It is therefore commonly made so that it may be lifted off of them by turning a milled head in front of the balance. A second milled head is also added to raise supports under each scale-pan. To weigh any object the following plan must be pursued. To see if the balance is in good order, lower the supports under the scale-pans, then those under the beam, by turning the two milled heads. The long pointer attached to the beam should now swing very slowly from side to side, and finally come to rest at the zero. Replace the supports, and open the glass case which protects the balance from currents of air. The object to be weighed, if metallic and perfectly dry, may be placed directly on the scale-pan, otherwise it should be weighed in a watch-glass whose weight is afterwards determined separately. Now place one of the weights in the opposite scale-pan, and remove the supports first from the pans and then from the beam. This must be done very slowly and carefully. Students are liable to let the beam fall with a jerk on the knife-edges, by which the latter are soon dulled and ruined. An accurate weighing is necessarily a slow process and should never be attempted when one is in a hurry. Moreover, by removing the supports quickly the scale-pans are set swinging, and the beam itself vibrating through a large arc, so that it will not come to rest for a long time. It is better while using the larger weights to lower the supports a very small amount only, and notice which way the index moves. As it is below the beam it always moves towards the lighter side. The smaller weights must be touched only with a pair of forceps, as the moisture of the fingers would soon rust them. Those over 100 grms. may be taken up in the hand by the knob, but no other part of them should be touched. Weights should never be laid down except on the scale-pans, or in their places in the box. Now try weighing the piece of brass *A*. Lay it on one scale-pan, and a 10 gr. weight on the opposite side. The index moves towards the latter when the supports are removed, as described above. Replace the 10 grs. by 20 grs. This is too heavy, and the index moves the other way. Try the 10 grs. and 5 grs. – too light; add 2 grs. – still too light; another 2 grs. – too heavy; replace the latter by 1 gr. – too light. The weight evidently lies between 18 and 19 grammes. Add the .5 gr., or 500 mgr. – too heavy; substitute the 200 mgr. – too light, and so go on, always following the rule of taking the weights in the order of their sizes, and never adding small weights by guess, or much time will be lost. Having determined the weight within .01 gr., the milligrammes are most easily found by a rider. This consists of a small wire whose weight is just 10 mgr. It is placed on different parts of the beam, which is divided like a steel-yard into ten equal parts, which represent milligrammes. Thus if the rider is placed at the point marked 6, or at a distance of .6 the length of one arm of the balance, it produces the same effect as if 6 mgrs. were placed in the scale-pan. It is generally arranged so that it can be moved along the beam without opening the glass case, which protects the latter from dust and currents of air. By taking care to lower the supports of the beam slowly, as recommended above, the swing of the index is made very small; it is sufficient to see if it moves an equal distance on each side of the zero, instead of waiting for it to come absolutely to rest. To make sure that no errror is made in counting the weights, their sum should be taken as they lie in the scale-pan, and also from their vacant places in the box.

Decimal weights are made in the ratio of 1, 2 and 5, and their multiples by 10, and its powers. To obtain the 4 and the 9 it is necessary to duplicate either the 1 or the 2. The English adopt the former method, the French the latter. Comparing the two mathematically, we find that using the weights 5, 2, 2, 1, we shall, on an average in ten weighings, remove a weight from box to scale-pan 34 times, of which it will be put back 17 times during the weighing, and the remaining 17 times after the weighing is completed. In the English method, with the weights 5, 2, 1, 1, under the same circumstances the weights are again used 34 times, replaced 15 times during the weighing, and 19 after it. There is therefore no difference in rapidity of one plan over the other. The French system has, however, the great advantage that we may at any time test our weights against one another, since $1 + 2 + 2$ should equal the 5 weight, and sometimes in weighing, if a mistake is suspected a test may be applied by using the additional weight instead of putting back all the small weights, and adding a larger one, as is necessary in the English system. To meet this difficulty a third 1 gramme weight is sometimes added by English makers.

Experiment. To make a set of weights for weighing fractions of a gramme. Four are needed of platinum or brass weighing 500, 200, 200 and 100 mgrs., and four of aluminum, or thick tin foil, weighing 50, 20, 20 and 10 mgrs. The latter should be made first, since being the lightest they are the easiest to adjust. Cut a rectangle of the foil about 3 or 4 centimetres on a side, and weigh it within a milligramme. Now determine its area by measuring its four sides and taking the product of its length by its breadth. If the opposite sides are not equal, take their mean. Let A equal the area, and W the weight of the foil. Evidently $W \div A$ will equal w, the weight per square millimetre, and 50, 20 and 10 divided by w will give the areas of the required weights. Cut pieces somewhat too large and reduce them to the proper size by the Method of Successive Corrections, p. 10. This is accomplished by weighing each and dividing its excess by w. The quotient shows how much should be cut off. As they cannot easily be enlarged if made too small, and the thickness of the foil may not be the same throughout, pieces should be cut off smaller than the computed excess. Small amounts may be taken from the corners, and when completed one of the latter should be turned up to make it easier to pick them up with the forceps. Finally, lay them on the plate of lead, and stamp their weight in milligrammes on them, with the steel punches and mallet. Do the same with the heavier foil, thus making the 500, 200, 200 and 100 mgrs. weight. More care is needed with them, and the last part of the reduction should be effected with a file. Unless great care is taken, two or three will be spoiled by making them too light, before one of the right weight is obtained.

18. Decanting Gases.

Apparatus. A pneumatic trough, which is best made of wood lined with lead, and painted over with paraffine varnish. A graduated glass tube *D*, Fig. 14, closed at one end, and holding about 100 cm.^{3}, a tubulated bell-glass *B* containing about a litre, with stop-cock *C* attached, and two or three dry Florence flasks. The mouths of the latter should be ground, so that they may be closed by a plate of ground glass; to remove the moisture they should be heated in a large sand bath, or over steam pipes. A thermometer is also needed.

Experiment. Measure the temperature of the air in the flask *A* by the thermometer, also its moisture, or rather its dew-point. The latter may be assumed to be the same as that of the room, and obtained from the student, using the meteorological instruments. Now close the flask with the plate of glass, and immerse it neck downwards in the pneumatic trough. It may be kept in this position for any length of time, as the water prevents the air from escaping. Next fill the large graduated vessel *B* with water, by opening its stop-cock *C* and immersing, then close *C* and raise it. Now decant the gas into it by pouring, just as you would pour water, only that it ascends instead of falling. When all has been transferred read very carefully the volume, as given by the graduation, also the approximate height of the water inside above that outside the jar. Dividing this difference by 13.6 the specific gravity of mercury, and subtracting the quotient from the height of the barometer, gives the pressure to which the enclosed air is subjected. Its temperature may be assumed equal to that of the water, and it may be regarded as saturated with moisture. Next, to transfer it into the graduated tube *D*, attach a rubber tube to *C*, and after fill-

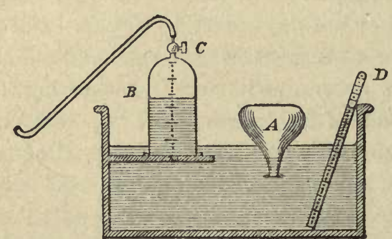


Fig. 14.

ing D with water and inverting it in the trough, let the air bubble into it from the tube by opening C and lowering B. When D is nearly full, close C so as to prevent the escape of the air, and read and record the volume as given by the graduation on D. Now decant the air from D into the flask A. Great care is necessary in this operation to prevent spilling, and it is best to practise a few times beforehand, until it can be transferred without allowing a single bubble to escape. Continue to empty B until all the air has been passed into A. The latter will then be nearly full of air, unless some has been lost. In the latter case do not give up the experiment, but keep on, retaining as much air as possible. Now holding the neck of the flask in the hand press the ground glass against it with the thumb, so as to retain what water is still in it, and taking it out of the trough stand it on the table right side up. Wipe the outside dry, and weigh it in its present condition; also when full of water, and when empty. Call the three weights m, n and o, respectively. The volume of air in $cm.^3$, at the beginning of the experiment, will equal $n - o$ in grammes; that at the end $n - m$. There are now four volumes of air to be compared. First the volume at the beginning of the experiment, when the air was moist and the dew-point was given; secondly, when transferred to B; thirdly, that found by adding the readings of D; and fourthly, that at the close of the experiment. Reduce all of these to the standard pressure and temperature by the method given below, when they should be equal if no air has escaped, otherwise the difference shows the amount of the loss. Great accuracy must not be expected, owing to the absorption of the air by the water, and for various other reasons.

REDUCTION OF GASES TO STANDARD TEMPERATURE AND PRESSURE.

I. Dry Gas. Given a volume V_{tP} of dry gas at temperature t, and barometric pressure P, to find what would be its volume V_{0H} if cooled to 0° C, and the pressure altered to the standard $H =$ 760 m.m. Suppose that it is first cooled to 0°, without changing the pressure, and call its new volume V_{0P}. We have by Gay Lussac's law for the expansion of gases, $V_{tP} = V_{0P}\ (1 + at)$, in which $a = \frac{1}{273}$, the coefficient of expansion of gas. Again, by

Mariotte's law we have, $V_{0P} : V_{0H} = H : P$. Hence $V_{0P} = V_{0H} \cdot \frac{H}{P}$, or substituting, $V_{tP} = V_{0H} \cdot (1 + at) \cdot \frac{H}{P}$, or,

$$V_{0H} = V_{tP} \frac{P}{(1+at)H} = V_{tP} \frac{273\, P}{(273+t)\, 760} \dots (1).$$

For any other temperature t', and pressure P', we have,

$$V_{0H} = V_{t'P'} \frac{273\, P'}{(273+t')\, 760},\ \text{hence}\ V_{t'P'} = V_{tP} \frac{P}{P'} \cdot \frac{273+t'}{273+t} \dots (2)$$

The first formula is used to determine the true quantity of gas present, that is, the volume at the standard temperature and pressure. The second, to compute the new volume when we alter both temperature and pressure.

II. Gas saturated with Moisture. Call p the pressure of aqueous vapor at the temperature t. Then of the total pressure P we have p due to the vapor, and $P - p$ to the gas; substitute, therefore, $P - p$ for P in equation (1), and we have,

$$V_{0H} = V_{tP} \frac{273\,(P-p)}{(273+t)760} \dots (3).$$

III. Gas moist, but not saturated. Let the gas be gradually cooled, until the temperature becomes so low that the moisture can no longer be retained as vapor, but begins to condense on the walls of the vessel. This temperature T is called the dew-point; let p' be the corresponding pressure of the vapor. Then $p : p' = 1 + at : 1 + aT$, or $p = p'(1+at) \div (1 + aT)$, and substituting this value in equation (3), we have

$$V_{0H} = V_{tP} \cdot \frac{273}{(273+t)\, 760} \cdot \left[P - p' \frac{(1+at)}{(1+aT)}\right] \dots (4).$$

19. Standards of Volume.

Apparatus. A balance *AB*, Fig. 15, capable of sustaining 5 kgrs. on each side, and turning with a tenth of a gramme under this load. Remarkably good results may be obtained with common balances, such as are used for commercial purposes, by attaching a long index to the beam, as in the figure. Several pounds of distilled water should be provided, a thermometer, a set of weights, and a rubber tube and funnel. Instead of a scale-pan, a counterpoise *C* is attached to one arm of the balance as a method of double weighing is to be used. The standard to be graduated, which we will suppose to be a tenth of a cubic foot, consists of a glass vessel *D*, whose capacity somewhat exceeds this amount. A

$\frac{1}{4}''$ steam valve is screwed into the cap closing the lower end which also carries a sharp brass point to form the lower limit of the volume. A ring is attached to the cap closing the upper end of the vessel, by which the whole is supported. A brass hook with the point turned upwards passes through this cap, in which a hole has been drilled to allow the air to pass in or out. The hook may be raised or lowered, and clamped at any height by a conical nut surrounding it, or by a set screw. Finally a millimetre scale should be attached to the upper end of *D*.

Experiment. Note the height of the barometer, the temperature of the room, also that of the distilled water. Fill *D*, by attaching the rubber tube, as in the figure, opening *E* and pouring in the water. When the vessel is full, close *E* and remove the rubber tube. Take care that no air bubbles adhere to the side of the glass. Open *E* and draw off the water until it stands just on a level with the top of the scale attached to the glass. Counterpoise by adding weights to the scale-pan *F*, until the index stands at zero, first reading the directions for weighing, given on page 47. Draw off enough water to lower its level just one centimetre, counterpoise again, and repeat until the surface reaches the bottom of the scale. If too much water is removed at any time refill the vessel above the mark, and draw off the water again. Now bring the water level just above the point of the hook, and close *E*, so that the flow shall take place drop by drop. Use the hook as in Experiment 13, and as soon as the point becomes visible close *E*. Read the level of the water and counterpoise as before. Repeat two or three times, adding a little water after each measurement. Now open *E*, and let the water run out until the lower point just touches the surface. Measure the temperature of the water as it escapes. To counterpoise the beam nearly three

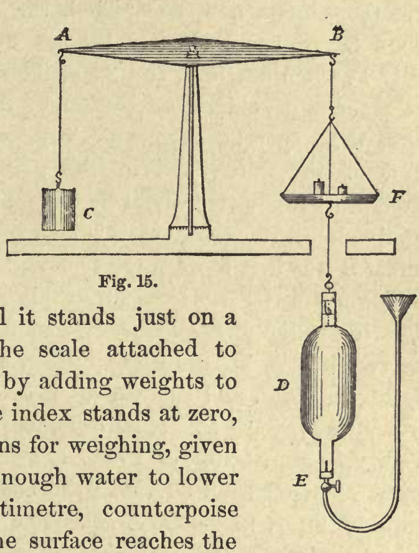


Fig. 15.

kilogrammes additional must be added to F. Make this weighing with care, and repeat two or three times, as when observing the upper point. Subtract each of the weights when the vessel was full, from the mean of those last taken, and the difference gives the weight of the water contained between the lower point and each of the other observed levels.

Now to determine the volume, we have given by Kater, the weight of 1 cubic inch of distilled water at 62° F., and 30 inches pressure, equals 252.456 grains, and 1 gramme equals 15.432 grains. From this compute the weight of one tenth of a cubic foot. Two corrections must now be applied, the first for temperature, the second for pressure. Water has an expansion of about .00009 per 1° F. when near 62°, and glass .000008 linear, or three times this amount of cubical expansion at the same temperature; of course the apparent change of volume is the difference of increase of the water, and of the glass. Evidently at a high temperature less water would be required, hence this correction is negative if the temperature is above 62°. Practically in making standards it is best to keep the temperature exactly at 62°, adding ice or warm water if necessary, as this correction is a little doubtful, owing to the unequal expansion of different specimens of glass. The vessel D is buoyed up by the air, by an amount equal to the weight displaced, and this weight is evidently proportional to the barometric pressure H. Now 100 cubic inches of air at 30 inches weigh 2.1 grms., hence at 1 inch it would be $\frac{2.1}{30}$, and if the pressure is changed from 30 to H, the change in weight would evidently be $2.1 \times (30 - H) \div 30$. The weights, however, are also buoyed up in the same way, but as the specific gravity of brass is about 8, the effect is only one-eighth as great. The true correction is then seven-eighths of this amount. The higher the barometer the greater the buoyancy, and the lighter the water will appear, or this correction will be negative for pressures above 30 inches. Both the corrections will be small, and in most cases can be neglected; but it is well to make them, in order to be sure to understand the principle. Having thus computed how much the tenth of a cubic foot ought to weigh, see if the distance between the points is correct, and if not, determine by interpolation where the water level should be in order to render the capacity exact.

20. Reading Microscopes.

Apparatus. Three cheap French microscopes mounted on moveable stands, as in *AB*, Fig. 16. Two should have cross-hairs in their eye-pieces, while the third should contain a thin plate of glass with a very fine scale ruled on it. An accurate scale divided into millimetres is required as a standard of comparison, and since the division marks of those in common use are too broad for exact measurements, it is better to have one made to order, with very fine lines cut on the centre of one face instead of on the edge. The best material is glass, but copper or steel will do, especially if coated with nickel or silver. Several objects to be measured should be selected, as a rod pointed at each end, the two needle points of a beam-compass, and a scale divided into tenths of an inch, whose correctness is to be tested. Under the microscopes is placed a board *D*, on which the object to be measured *C*, is laid, and which may be raised or lowered gradually by screws, or folding wedges. Another method of supporting the microscopes, superior in some respects, will be found described under the Experiment of Dilatation of Solids by Heat.

Experiment. If a measurement within a tenth of a millimetre is sufficiently exact, use the two microscopes with cross-hairs. Place them at such a distance apart that each shall be over the end of the object to be measured, which should be laid on *D*. They should be raised or lowered until in focus, and then set so that their cross-hairs shall exactly coincide with the two given points. Remove the object very carefully, so as not to disturb their position, and replace it by the standard scale, bringing the zero to coincide with one of the cross-hairs. Now looking through the other microscope read the position of its cross-hairs on the scale, estimating the fractions of a millimetre in tenths. If the image of the scale is not distinct it may be focussed by slowly raising or lowering the board on which it is placed, taking great care not to disturb the microscopes. To get the whole number of millimetres, a needle may be laid down on the scale, and the right division distinguished by its point.

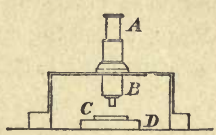


Fig. 16.

If greater accuracy is desired, use the third microscope, finding the magnitude of the divisions of its scale in the following manner; focus it on the steel scale, placing it so that two divisions of the latter shall be in the field at the same time. Read each of them by the scale in the eye-piece, and take the difference; the reciprocal is the magnitude of one division in millimetres. Repeat a number of times and take the mean. To make any measurement, place this microscope with one of the others over the points to be determined, and take the reading with its scale, estimating tenths of a division; then substitute the steel scale as before, and read the millimetre mark preceding, also that following. By a simple interpolation the distance is obtained from these three readings with great accuracy.

Try both these methods with the objects to be measured, and then test the scale of tenths of an inch by measuring the distance of each inch mark from the zero, and reducing the millimetres to inches. Measure also in the same way the ten divisions of one of the inches.

One of the best ways to measure off a large distance, as ten or twenty metres, with accuracy, is by means of a couple of reading microscopes. A steel rule is used, the ends being marked by the microscopes, as they are in rough measurements, by the finger. In all cases where the graduation extends to the end of the rule it is better to use the mark next to it, both as being more accurate, and as affording a better object to focus on.

21. Dividing Engine.

Apparatus. This instrument rests on a substantial stand *ABED*, Fig. 17, like the bed-plate of a lathe. A carefully constructed micrometer screw moves in this, and pushes a nut *C* from end to end. The screw should have a pitch of about a millimetre, or a twentieth of an inch, if English measures are preferred. The head of the screw is divided into one hundred parts, and turns past an index which is again divided into ten parts, as in Fig. 4, p. 24. The screw may be turned by a milled head or a crank. The nut must have a bearing of considerable length, a decimetre is scarcely too much, as any irregularities are thus compensated. It should be split so that it may be tightened by screws, or better, by a spring, and slides along two guides, *AB* formed like an inverted V, and *DE*, which is flat. A scale is cut on the latter to give the whole number of revolutions of the screw. The nut should move with perfect smoothness from end to end, but not too freely. A certain amount of *back-lash* is unavoidable (that is, the screw may always be turned a short distance backwards or forwards without moving the nut), but this does no harm, as when in use it should always be moved in the same direction. A second screw similar to the other, but smaller, and at right angles to it, is attached to *C*, so that its nut may be moved backwards or forwards about one decimetre. It carries a reading microscope *R*, made of a piece of light brass tubing, by inserting an eye-piece above, and screwing a microscope objective into the lower end. It may be focussed by sliding the tube up and down by a rack and pinion. Cross-hairs should be placed in the eye-piece, but in some cases a fine scale, or eye-piece micrometer, is preferable.

To use this instrument as a dividing engine, the microscope must be made movable, so that it can be replaced by a graver for metals, or a pen for paper. The micrometer head *F* has ten equidistant holes cut in it, in which steel pins can be inserted. These strike against a stop which they cannot pass unless it is pushed down by the finger. A sheet of thick plate glass *DSTE* serves as a stand on which to lay objects, and under it is a large mirror to illuminate them, but it may be removed when desired.

Experiment. This instrument may be applied to a great variety of purposes. Several experiments with it will therefore be described.

1st. *To test the screw.* Lay a glass plate divided into tenths of a millimetre on *DSTE*, and bring the microscope over it. Use a moderately high power, as a ¼″ objective, and focus on the scale; the want of a fine adjustment may be partly remedied by varying the distance of the eye-piece from the objective. Bring the first division of the scale to coincide with the cross-hairs of the microscope by turning the micrometer-head *F*. Read the whole number of turns from the scale on *DE*, and the fraction from *F*. Move it one or two turns to the right, and set again; repeat several times, and compute the probable error

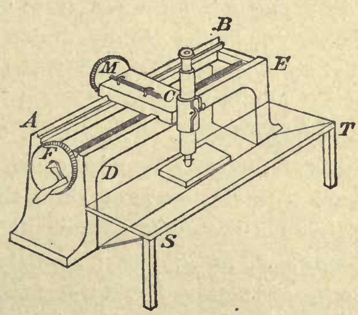


Fig. 17.

of one observation. It equals the error of setting. Turn the screw the other way, and bring it back to the line. The difference between this reading or the mean of ten such readings, and that previously obtained, gives the back-lash. Set in turn on several successive points of the scale. The first differences should be equal. Mark two crosses on a plate of glass with a diamond, three or four centimetres apart. Measure the interval between them with different portions of the screw, and see if they agree. If not, the defect in the screw must be carefully examined, and corrections computed. The screw M should be similarly tested.

2d. *Determination of the pitch of the screw.* Procure a standard decimetre (or other measure of length) and measure the distance between its ends. The temperature should be nearly that taken as a standard, or if great accuracy is required, allowance made for the difference of expansion of the screw and decimetre. From this measurement, which should be repeated several times, compute the true pitch of the screw, and the correction which must be applied when distances are measured with it.

3d. *To measure any distance.* Lay the object on the glass plate and bring the cross-hairs of the microscope to coincide first with one end of it, and then with the other. The difference in the readings is the length. Apply to it the correction previously determined.

4th. *To determine the form of any curved line.* For example, use one of the curves drawn by a tuning fork, in the Experiment on Acoustic Curves. Bring the cross-hairs to coincide with several points in turn of one of the sinuosities, and read both micrometer heads. These give two coördinates, from which the points of the curve may be constructed on a large scale, and compared with the curve of sines, the form given by theory. The relative positions of a number of detached points may also be thus determined, as in the photographs of the Pleiades and other groups of stars by Mr. Rutherford.

5th. *Graduation.* For a first attempt, make a scale on paper with a pencil or pen. Replace the microscope by a hard pencil with a flat, but very sharp point. It must be arranged so that it can be moved backwards or forwards a limited distance, but not sideways. Every fifth line should be longer than the rest, which should be exactly equal to each other in length. Fasten the paper securely on the glass plate so that it shall not slip. Suppose now lines are to be drawn at intervals of half a millimetre. Insert a pin in one of the holes in *F*, and turn the latter to the stop. Draw a line with the pencil for the beginning of the scale, depress the stop to let the pin pass, give *F* one turn, bring the pin again to the stop and draw a second line, and so on. If the lines are to be a millimetre apart, draw one line for every two turns. In the same way, by inserting more pins a finer graduation may be obtained. Instead of using the pins a table may be computed beforehand, giving the reading of the screw for each line to be drawn, allowing for the errors of the screw, if great accuracy is required. The scale is then ruled by bringing the nut successively into the various positions marked in the table, and drawing a line after each.

A most important application of this instrument is to the measurement of photographs of the sun taken during eclipses. The position of the moon at any instant is thus obtained, with a degree of precision otherwise unattainable. In this, and other cases where angles must also be measured, the plate of glass *ES* should be removed, and the object laid on a rotary stand, with a graduation showing the angle through which it is turned.

22. Ruling Scales.

Apparatus. In Fig. 18, two strips of wood *A* and *B*, rest on a smooth board, and are held in place by the weights *C* and *D*. The ends of a string are attached to them, which is stretched by means of a weight *F*, so that if *C* and *D* are raised *A* and *B* will slide. A peg is inserted in *B*, which moves between two steel plates fastened to *A*, one being fixed, the other movable by means of a screw *G*. If, then, either weight is raised, the strip of wood on which it rests will be drawn forward by *F*, but will be free to move through a space equal to the difference of the diameter of the peg and the interval between the two steel plates. If desired, *G* may be a micrometer screw, by which this interval may always be accurately determined. It may be fastened in any position by a clamp or set screw. A steel rod *H* is used to draw the division lines. It is fixed at one end, and carries at the other a pencil, pen, graver or diamond, according as the lines are to be drawn on paper, metal or glass. By this arrangement there is little or no lateral motion of the graver, but unfortunately it draws a curved line. To remedy this defect, the rod may be replaced by a stretched wire, to the centre of which the graver is attached, or the latter may slide past a guide against which it is pressed by a spring.

Experiment. For many purposes in using a scale, it makes but little difference what the divisions are, provided that they are all equal, and this is especially the case in all accurate measurements, since as a correction must always be made for temperature, we can readily at the same time correct for the size of the divisions. The instrument here described will probably give divisions more nearly equal than those obtained by a micrometer screw, but it is more difficult to make them of any exact magnitude, since any deviation is multiplied by the number of divisions.

To draw a scale, lay a piece of paper on *B* and fasten it with tacks or clips. To secure uniformity in the length of the long and short division marks, rule three parallel lines as limits, attach a sharp flat-pointed pencil to *H*, and slide *A* and *B* until the beginning of the scale is under *H*. Draw a line with the latter, and make one stroke with the machine. This is done by raising *C*, when *F* will draw *A* forward a distance equal to the interval between the two plates near *G*, *minus* the thickness of the peg. Lay *C* down and raise *D*. *A* will now remain at rest, but *B* will move through the same distance. Draw a second line with the pencil, and repeat, making every fifth line about twice as long as the others. They will be found spaced at distances which may be regulated by the screw *G*. Try making short scales in the same way, with large and small divisions. It is always safer to keep the hand on one weight while the other is lifted. The magnitude of *F* should be such that the strain on the cord will be greater than the friction of repose when the weights are up, but less than the friction of motion when they are down. If *F* is too light, when *C* is raised *A* will not start; if too heavy, it will strike so hard that it will move *B*. To test the accuracy of the machine draw a single line, take a hundred

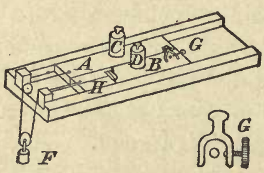


Fig. 18.

strokes and draw another. Then without moving *G* push the slides back and draw a third line close to the first; take a hundred strokes and draw a fourth line near the second. Measure the interval between the first and third, and the second and fourth. They should be equal, but if not, the difference divided by an hundred gives the average difference in length of a stroke the second time, compared with the first.

Instead of a pencil, a pen may be used to draw the lines, or a graver, if a metallic scale is desired. The finest scales are ruled on glass by a diamond. Instead of using the natural edge of the gem, as when cutting glass, an engraver's diamond should be employed, which is ground with a conical point; the direction in which it should be held, and the proper pressure, being obtained by trial. Scales may also be etched by covering the surface with a thin coating of wax or varnish, and the lines marked with a graver. If metallic, it is then subjected to the action of nitric acid; if of glass, to the fumes of fluorhydric acid. It is possible that the new method of cutting glass by a sand-blast may prove applicable to this purpose with a great saving of time and trouble.

MECHANICS OF SOLIDS.

23. Composition of Forces.

Apparatus. Two pulleys A and B, Fig. 19, are attached to a board which is hung vertically against a wall. Two threads pass over them, and a third C, is fastened to their ends at D. Three forces may now be applied by attaching weights to the ends of the cords. The weights of an Atwood's machine are of a convenient form, but links of a chain, picture hooks, cents, or any objects of nearly equal weight may be used. Small beads are attached to the three threads at distances of just a decimetre from D.

Experiment. Attach weights 2, 3 and 4 to the three cords, and let D assume its position of equilibrium. Owing to friction it will remain at rest in various neighboring positions, their centre being the true one. Now measure the distance of each bead from the other two with a millimetre scale, and obtain the angle directly from a table of chords. If these are not at hand, dividing the distance by two, gives the natural sine of one half the required angle. By the law of the parallelogram of forces, the latter are proportional to the sides of a triangle having the directions of the forces. But these sides are proportional to the sines of the opposite angles, hence the sines of the angles included between the threads should be proportional to the forces or weights applied. Divide the two larger forces by the smaller, and do the same with the sines of the angles, and see if the ratios are the same. The angles themselves should first be tested by taking their sum, which should equal 360°. If either angle is nearly 180°, it cannot be accurately measured in this way, but must be found by subtracting the sum of the other two from 360°, or measuring one side from

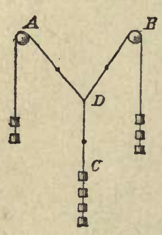


Fig. 19.

the prolongation of the other. It is well to draw the forces from the measurement, and see if a geometrical construction gives the same result as that obtained by calculation. Repeat with forces in several other ratios, as 3, 4, 5; 2, 2, 3; 3, 5, 7; taking care in all cases to include in the weights the supports on which they rest.

24. Moments.

Apparatus. A board *AB*, Fig. 20, is supported at its centre of gravity on the pin *C.* It should revolve freely, and come to rest in all positions equally. Two forces may be applied to it by the weights *D* and *E*, attached by threads to the pins *F* and *G.* Their magnitudes may be varied from 1 to 10 by different weights, and their points of application by using different pins, as *H*, *I* and *J.* To measure their perpendicular distances from the pin *C*, a wooden right-angled triangle or square is provided, one edge of which is divided into millimetres, or tenths of an inch.

Experiment. Various laws of forces may be proved with this apparatus. 1st. When a single force acts on a body *AB* fixed at one point, as *C*, there will be equilibrium only when it passes through this point. Remove *FD* and attach a weight *E* to *G.* It will be found that the body will remain at rest only when the point *G* is in line with *M* and *C.* 2d. A force produces the same effect if applied at any point along the line in which it tends to move the body. Apply the two weights *D* and *E*, which tend to turn the board in opposite directions. Make their ratio such that *MG* shall be in line with *G*, *H*, *J.* Now transfer the end of the thread from *G* to *H*, *I* and *J* in turn, when it will be found that the position of the board will be unchanged. It should be noticed, however, that in the last case the board is in unstable equilibrium, since *FJ* falls beyond the point of support *C.* 3d. The moment of a force, or its tendency to make a body revolve, is proportional to the product of its magnitude by its perpendicular distance from the point of support. Make *D* equal 2, and attach it to *K*, so that the thread rests over the edge of the board, which is the arc of a circle with centre at *C*, and radius .6. Its tendency to make the board revolve is therefore the same, what-

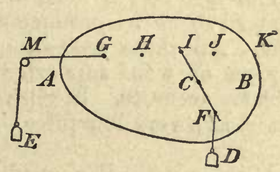


Fig. 20.

ever the position of the latter. Make E successively 1, 2, 3, 4, 5, 6, and measure the perpendicular distance of the thread to which it is attached in each case from C. This distance is measured by resting the triangle against the thread and measuring the distance of C by its graduated edge. In each case the moment of E will be found to be the same, and equal to 2×6, the moment of D. 4th. When two forces hold the body in equilibrium their resultant must pass through the fixed point. Make D equal 2, and attach it to F, and E equal 3, applied at G. Lay a sheet of paper on the right hand portion of AB, making holes for F, C and J to pass. Draw on it with a ruler the direction of the two threads prolonged, and then removing it, construct their resultant geometrically by means of the parallelogram of forces. It will be found to pass through C. Repeat two or three times with different weights and points of application.

25. Parallel Forces.

Apparatus. The apparatus used is shown in Fig. 21. AB is a straight rod about two feet long, with a paper scale divided into tenths of an inch attached to it. It is supported by a scale-beam CD with a counterpoise, so that it is freely balanced, and remains horizontal. Weights formed like those of a platform scale may be attached to it at any point, by riders, as at E, F and G. Taking each rider as unity, four sets of weights are required of magnitudes 10, 5, 2, 2, 1, .5, .2, .2, .1. Two other beams, like CD, should also be provided, to which these weights may be attached, as at E, so as to produce an upward force of any desired magnitude. All these scale-beams may be very roughly made, even a piece of wood supported at the centre by a cord, being sufficiently accurate. English beams of iron may, however, be obtained at a very low price.

Experiment. The resultant of any system of parallel forces lying in one plane may be found by this apparatus. Thus suppose we have a force of 15.7 acting upwards, and two of 8.3 and 1.4 acting downwards, and distant from the first 6.4 and 8.7 inches respectively. Produce the upward force by adding the weights 14.7 to E, and the

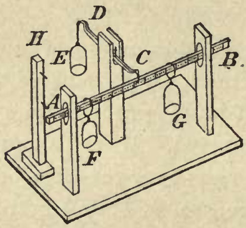


Fig. 21.

two downward forces by weights 7.3 and .4 (allowing 1 for each of the scale-pans) at F and G, setting them at the points of the beam marked 3.6 and 18.7. They are then at the proper distance from C, which is at 10 inches from the end. We now find that A goes down and B up; by placing the finger on the beam we see that it can be balanced only by applying a downward force to the right of C. Now place a rider in this position, and move it backwards and forwards, varying the weight on it until the beam is exactly balanced. The magnitude of this weight will be found to be 6, and its position 16.8, or 6.8 inches from C. The resultant of the three forces will be just equal and opposite to this. Had the force required to balance them acted upwards, we should have used one of the auxiliary scale-beams. To test the correctness of this result we compute the resultant thus: $R = 15.7 - (8.3 + 1.4) = 6$, and taking moments around C we have $8.3 \times 6.4 - 1.4 \times 8.7 = 6 \times x$, or $x = 6.8$ the observed distances.

Determine the position and magnitude of the resultant in several similar cases, as for example the following, in which U means an upward, and D an downward force, and each is followed first by its magnitude, and then by the point on the bar at which it is to be applied.

1. D, 5.0, 4.3; U, 10.0, 10.0.
2. D, 2.6, 3.2; U, 7.8, 10.0.
3. D, 7.4, 3.7; U, 17.1, 10.0.
4. D, 11.1, 2.1; D, 6.5, 5.6; U, 2.3, 18.4.
5. D, 5.2, 1.9; U, 15.2, 10.0; D, 8.4, 12.6; U, 3.0, 18.1.

Two equal parallel forces acting in opposite directions and not in the same line, form what is called a couple, and have no single resultant. Thus apply the two forces D, 12.0, 5.0, and U, 12.0, 10.0. No single force will balance the beam. Equilibrium is obtained only by a second couple having the same moment, and turning in the opposite direction; thus the moment being $12.0 \times 5.0 = 60.0$, the beam may be balanced by two forces of 10.0, each distant 6 inches from one another, placing the upward force to the *left.* Find in the same way some equivalent to D, 4.3, 7.6, and U, 4.3, 10.0, and notice that it makes no difference to what part of the beam the two forces are applied, provided their distance apart remains unchanged.

This same apparatus may be applied to illustrate the case of a body with one point fixed, acted on by parallel forces, as, for example, the lever, by using a stand *H* with two pins, between which the beam may turn. This stand is also useful in finding the point of application of the resultant in the above cases.

26. Centre of Gravity.

Apparatus. Several four-sided pieces of cardboard (not rectangles) and a plumb line, made by suspending a small leaden weight by a thread, from a needle with sealing wax head.

Experiment. Make four holes in the cardboard, two *AB*, Fig. 22, close to two adjacent corners, the others in any other part not too near the centre. Pass the needle through *A* and support the cardboard by it; the thread will hang vertically downwards, and the centre of gravity must lie somewhere in this line, or it would not be in equilibrium. Mark a point on this line as low down as possible, and connect it with the pin hole. Do the same with *B*; the intersection of the two is the centre of gravity. Turn the cardboard over and repeat with the other holes. This gives two determinations of the centre of gravity. To see if the two points are opposite one another, prick through one and see if the hole coincides with the other. By suspending at any other points, the same result should be obtained. Be careful that the holes are large enough to enable the card to swing freely.

Next, lay the card down on your note book and mark the four points *A*, *B*, *C*, *D*. Connecting them with lines gives a duplicate of the cardboard. On this construct the centre of gravity geometrically. Divide into two triangles by connecting *AC*. Bisect *AD* in *E*, and *CD* in *F*. The centre of gravity of *ACD* must lie in *AF*, also in *CE*, hence at *G*. Obtain *G′* by a similar construction with *ABC*. The centre of gravity of the whole figure must lie in *GG′*. Make a second construction by connecting *BD*, making the triangles *ABD* and *BCD*; the intersection of *GG′* and its corresponding line gives the centre of gravity. Lay the piece of cardboard on the figure and prick

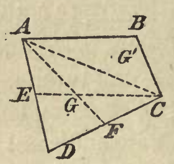


Fig. 22.

through the two centres of gravity previously found. They should agree closely with that found geometrically.

27. Catenary.

Apparatus. A chain three or four yards long, each link of which is a sphere, known in the trade as a ball link chain. Every tenth link should be painted black, and the fiftieths red. A horizontal scale ABC, Fig. 23, attached to the wall, also a number of pins to which the chain may be fastened by short wire hooks, and its length altered at will. A graduated rod BD is used to measure the vertical height of any point of the chain.

Experiment. First, to determine the average length of the links. Let the chain hang vertically from A, measure the length of each hundred links, and take their mean. A simple proportion gives the number of links to which AC is equal. Suspend the chain at A and C, making the flexure at the centre about half a foot. Measure it exactly, and increase the original length 10 links at a time to 100. Increase it also by 17 links, by 63 and by 48, and measure as before. Write the results in a column and take the first, second and third differences of the first measurements. Now obtain by interpolation the three values for 17, 63 and 48 links, and compare with their measured values.

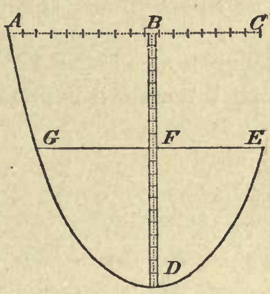


Fig. 23.

Next suspend the chain as in ADE, and measure the deflection at intervals of five inches horizontally. This is best done by passing a pin through the graduated rod at the zero point, letting it hang vertically, then measuring by it. Taking differences as before, those of the first order will be at first negative, then increase until they become positive. Where the first difference is zero, is evidently the lowest point of the curve. By the method of inverse interpolation find this point, treating the first differences as if they were the original variable, and recollecting that each difference belongs approximately to the point midway between the two terms from which it was obtained. Thus the difference obtained from the 5 and 10 inches corresponds to $7\frac{1}{2}$. Obtain this point also by measurement, by laying off BF equal to CE, prolonging EF to G and measuring GF. AC minus one-half GE will equal the required distance. Repeat with several points below E, and compare with the computed position of the lowest point.

28. Crank Motion.

Apparatus. A steel scale AB, Fig. 24, divided into millimetres, slides in a groove so that its position may be read by an index E. It is connected by the rod AD to the arm of the protractor, whose centre is C. On turning CD, which carries a vernier F, AB moves backwards and forwards. Several holes are cut in AD so that its length may be altered at will.

Experiment. Make AD as long as possible. Measure CD by turning it until D is in line with C and A, and read E; then turn it 180°, and read again. One-half the difference of these readings equals CD. Next, to find the reading of the vernier when CD and DA are in line. Make ACD about 90° and read E and F. Turn CD until the reading of E is again the same and read F. The mean of these two readings gives the required point. Repeat two or three times, and take the mean.

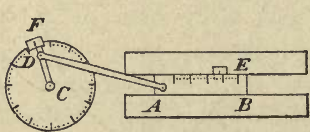


Fig. 24.

Let AB represent the piston rod of an engine, and CD the crank attached to the fly-wheel. The problem is to determine the relative positions of these two, during one revolution. Bring D in line with CA, and move it 10° at a time through one revolution, reading E in each case. Do the same, using a shorter connecting rod, so that AD shall be about two or three times CD. To compare these results with theory, first suppose the rod CD infinitely long. The distance of AB from the mean position will then always equal $CD \times \cos ACD$. This is readily computed from the accompanying table of natural cosines. If, as is most convenient, CD is made just equal to 1 decimetre, the distances are given directly in the second column of the table by moving the decimal point two places to the right. Compare these results with your observations. Construct a curve in which abscissas represent the computed positions of AB, and ordinates the difference between the observed and computed results, enlarging the scale ten times. If a smooth curve is thus obtained it is probably due to the short length of AD. The correction due to this is readily proved to be $AD - \sqrt{AD^2 - CD^2 \sin^2 ACD}$, or calling the ratio $AD \div CD = n$, it is $AD\,(n - \sqrt{n^2 - \sin^2 ACD})$. Compute this correction for every 30°, knowing that $\sin^2 30° = .25$, $\sin^2 60° = .75$. The points thus obtained should lie on the residual curve found above. Do the same with the shorter arm AD.

Angle.	*Cosine.*
0°	1.000
10°	.985
20°	.937
30°	.866
40°	.766
50°	.643
60°	.500
70°	.342
80°	.174
90°	.000

29. Hook's Universal Joint.

Apparatus. A model of this joint with graduated circles attached to its axles. The latter should be so connected that they may be set at any angle.

Experiment. Set the axes at an angle of 45°, and bringing one index to 0°, the reading of the other will be the same. Now move the first 5° at a time to 180°, and read the other in each position. Record the results in columns, giving in the first the reading of one index, in the second that of the other, and in the third their difference, which will be sometimes positive, and sometimes negative. Construct a curve with abscissas taken from the first column, and ordinates from the third, enlarging the latter ten times. It shows how much one wheel gets behind, or in advance of, the other. To compare this result with theory, let Fig 25 represent a plan of the joint, AC and CB being the two axes. Describe a sphere with their intersection C as a centre. The great circle CD is the path described by the ends of one hook, CE that described by the other. D and E must, by the construction of the apparatus, always be 90° apart. Then in the spherical triangle CDE we have given $DE = 90°$, $ECD = 45°$, the angle between the axes, and one side as CD, and we wish to compute

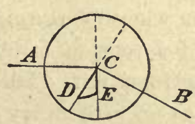


Fig. 25.

CE. But by spherical trigonometry, tang *CE* = tang *CD* cos *ECD*. Substituting in turn *CD* = 5°, 10°, 15°, 20°, &c., we compute the corresponding angle through which the second wheel has been turned. Construct a second curve on the same sheet as the other, using the same scale. Their agreement proves the correctness of both.

Experiments like Nos. 28 and 29 may be multiplied almost indefinitely. Thus various forms of parallel motion, the conversion of rotary into rectilinear motion by cams, link motion, gearing, and, in fact, almost all mechanical devices for altering the path of a moving body may be tested and compared with theory.

30. Coefficient of Friction.

Apparatus. *AB*, Fig. 26, is a board along which a block *C* is drawn by a cord passing over a pulley *D*, and stretched by weights placed in the scale pan *E*. The friction is produced between the surfaces of *C* and *AB*, which should be made so that they may be covered with thin layers of various substances as different kinds of wood, iron, brass, glass, leather, &c. *C* is made of such a shape that by turning it over the area of the surface in contact may be altered. The pressure on *C* and the tension of the cord may also be varied at will, by weights.

Experiment. Weights are added to *E* in regular order, as when weighing, and the tension in each case compared with the friction of *C*. Friction may be of two kinds; first, that required to start a body at rest, called the friction of repose, and secondly, the friction of motion, or that produced when the bodies are moving. To measure the friction of repose, see if the weight is capable of starting the body when at rest, if so, stop it and repeat, varying the weight until a tension is obtained sufficient sometimes to start it and sometimes not. This friction is very irregular, varying with different parts of every surface, and with the time during which the two substances have been in contact. It is but little used practically, since the least jar converts it into the friction of motion. The latter is much less than the friction of repose, and more uni-

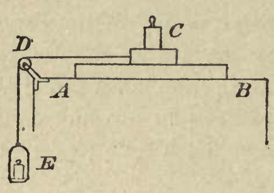


Fig. 26.

form. It is found by tapping the body so that it will move, and seeing if the velocity increases or diminishes. In the first case the weight in E is too large, in the second too small.

The first law of friction is that the friction is proportional to the pressure. The ratio of these two quantities is called the coefficient of friction. Make the load on C, including its own weight, equal to 1, 2, 3, 4, 5 kgs. in turn, and measure the friction. The latter equals the weight of E plus the load added to it minus the friction of the pulley. If great accuracy is required, a table should be prepared, giving the magnitude of the latter for different loads. Compute the coefficient of friction from the observations, and if the law is correct they should all give the same result. Measure, in each case, the friction of repose and of motion, and notice that the latter is always much the smaller.

Secondly, the friction is independent of the extent of the surfaces in contact. This law follows from the preceding, but it is well to prove it independently by turning C on its different sides, so as to vary the areas in contact. The friction will be found to be the same in each case. Finally, measure the coefficients in a number of cases, and compare the results with those given in the tables of friction.

31. Angle of Friction.

Apparatus. In Fig. 27, AB is a stand with an upright BC. AD is a board hinged at A, which may be set at any angle by a cord passing over the pulley C. The hinge is best made of soft leather held by a strip of brass, and its distance from the upright should be just one metre. A scale of millimetres is attached to the upright, and a wire parallel to AD serves as an index. Evidently the reading of the scale gives the natural tangent of the angle of inclination DAB. A cord attached to D passes over the pulley C, around the wheel F, and is stretched by the counterpoise G. F may be clamped in any position, or turned by a crank attached to it. AD may thus be set at any angle, and its position is to be determined when so inclined that any given body, as E, is just on the point of sliding. E may be made exactly like the sliding mass in Experiment 30, but to measure its friction of motion a fine wire should be attached to it and wound around the axle of F. When the crank is turned raising AD, the body E is thus drawn slowly down the inclined plane. In order that it may not move too rapidly this portion of the axle should be much smaller than that around which the cord CF is wound. EF should be a wire, as if a thread is used it will stretch, giving E an irregular motion, alternately starting and stopping.

Experiment. To measure the coefficient of friction of repose, turn the crank until the body begins to slide; the reading of the scale gives the tangent of the angle of inclination. But decomposing the weight of E into two parts, parallel and perpendicular to the plane, their ratio will equal the coefficient of friction, and also the tangent of the inclination. Hence the coefficient of friction is given directly by the scale. Measure again the coefficients found in Experiment 30, and see if the results agree with those then obtained.

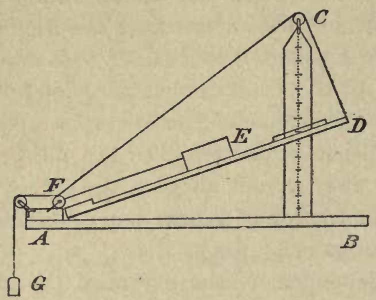


Fig. 27.

32. Breaking Weight.

Apparatus. In Fig. 28, B is a thumb screw, by which a spring balance A may be drawn back so as to exert a strain on the wire CD. Near C is placed a spring buffer so that when the wire breaks, the jar may be diminished. D may be a simple peg to which the wire is attached, or a spool with a clamp by which it is held at any point. A convenient method of connection is to attach one end of the wire to a chain, either link of which may be passed over the peg D, according to the length employed. To test the accuracy of the balance a cord may be substituted for CD, passed over a pulley E, and stretched by weights. Some cord and fine copper or iron wire of various sizes should be furnished, also a gauge to measure its diameter.

Experiment. Fasten a cord to C, and pulling it over E, record the reading of A, when by attaching weights, the strain is made in turn 0, 5, 10, 15, 20, &c., pounds. To eliminate the friction of the pulley, turn B first in one direction and then in the other, and take the mean of the readings of A in each case. Now construct a residual curve, in which abscissas represent the reading of A, and ordinates the difference between this reading and the weight applied. From this curve we can readily determine the true strain, knowing the reading of A, however inaccurate the latter may be.

To measure the breaking weight of any body, attach one end of it to C and the other to one link of the chain. Pass the latter over the peg D, so that C shall be a short distance from its spring buffer. Turn B and watch the index of A, until the cord breaks. A small block of wood may be placed in front of the index to show the greatest tension attained, but care must be taken that it is not disturbed by the recoil. Repeat several times with other portions of the same cord and take the mean of the observed maximum tensions. Do the same with some specimens of wire, and compute their tenacity T, or strength per square inch. For this purpose measure their diameter d with accuracy, by the gauge, and calling W the breaking weight, we have,

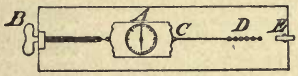


Fig. 28.

$$\frac{\pi d^2}{4} : 1 = W : T, \text{ or } T = \frac{4W}{\pi d^2}.$$

33. Laws of Tension.

Apparatus. In Fig. 29, A is a cast-iron bracket firmly fastened to the wall. A hook is attached at B, and from it the scale pan E is hung by the wire or rod to be tested. Weights may then be applied so as to give any desired tension to the latter. A brass rod BD hangs by the side of BC, being fastened to it by a small clamp at the top. Fine lines are drawn on both rods, and their relative change in position measures the elongation of BC. G is a reading microscope made of a brass tube about 6″ long, with the Microscopical Society's screw cut in one end, so that any microscope objective may be used with it. A positive eyepiece with a scale at its focus is slipped into the other end. This microscope is mounted like a cathetometer, by fastening it to a vertical brass tube screwed into the base of a music stand. It may be raised or lowered, and held at any point by a set screw. The following additional apparatus is also needed. Several wires or rods of various materials, as wood, copper, brass, iron, lead, and some of the same material but different diameters. A millimetre scale to measure their lengths, and a Brown & Sharpe's sheet metal gauge to give their diameters. Also a set of large weights to vary the tension. To prevent too sudden a jar on the wire, a board should be placed under *E* to support it when a weight is added, then lowering it by means of a screw.

Experiment. To measure the extension in any case, attach the wire to be tried to the hooks at *B* and *C*, and clamp *BD* to its upper end. Draw a line on *BC* opposite one of those on *BD*, and focus the microscope *G* on them. Read their relative position by means of the scale in *G*, then apply the weight and read again. The length of *BC* is thus increased, while that of *BD* is unaltered; hence the change in their relative distances equals the extension. To reduce this to millimetres, focus the microscope on a standard millimetre, and thus measure the scale in *G* directly. The distance from *F* to the clamp is measured by the millimetre scale, and the diameter of *BC* by the gauge.

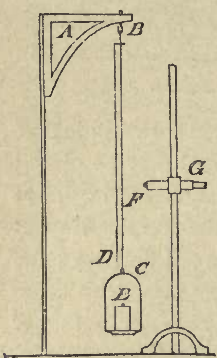


Fig. 29.

The laws of tension may now be determined.

1st. The extension is proportional to the length. Use a copper wire about a millimetre in diameter, and mark on it a number of lines at different heights. Measure the extension for each with a load of 20 kgs. It will be found proportional to the distances from the clamp. 2d. The extension is proportional to the weight applied. Measure the deflection for the lower lines, increasing the weight 2 kg. at a time, from 0 to 20 kilogrammes. Removing the latter weight, see if the wire has returned to its original length; any increase is called the permanent set. See if the results agree with the law. 3d. The elongations are inversely proportional to the cross-section. Try the series of wires of the same material, measuring the diameter of each with the gauge, and using the same weight for all. The product of the square of the diameters by the elongation should be constant. The modulus of elasticity is the force which would be required to double the length of a body of cross-section unity, supposing this could be done without breaking it, or changing the law which holds for small weights. To compute it, suppose d the diameter, l the length, and e the elongation of a wire under a tension T. If the cross-section was unity, to produce the same elongation we should increase the force in the same ratio as the two sections, or make it $\frac{4T}{\pi d^2}$; hence we have $e : l = \frac{4T}{\pi d^2} : M$, or $M = \frac{4lT}{\pi e d^2}$. Measure this modulus for the various substances provided, and compare the results with those given in the books. Finally, with an undue load the wire will take a permanent set, which increases if the wire is stretched for a considerable time. Study the laws regulating this property in the case of lead, in which the set is very marked.

34. Change of Volume by Tension.

Apparatus. A rubber tube AB, Fig. 30, about a metre long, and two or three centimetres in diameter, is closed above and below by plugs. The upper one is perforated, and carries a glass tube with graduated scale attached. A scale is also placed by the side of the rubber tube, and a number of points are marked on the latter. A cord and friction pin C (like that of a violin) is fastened to the lower plug, by which the tension of the tube may be varied. On the other side of the scale is a square rod DE of elastic rubber, about the size of the tube, and similarly marked. A pair of outside calipers capable of measuring objects as large as the rod to within a tenth of a millimetre, is also needed.

Experiment. The tube should be calibrated by weighing it when empty, and when filled with water to the zero, or beginning of the glass tube, also when filled to some division n, near its top. Call these three weights w', w'', w'''. Then $w'' - w' =$ the weight of a cylinder of water just filling the tube, or $\pi r^2 l$, in which l is the known length, and r the radius. From the equation $w'' - w' = \pi r^2 l$, we obtain $r = \sqrt{\frac{w'' - w'}{\pi l}}$. The volume per unit of length is $\frac{w'' - w'}{l}$, and in the same way for the glass tube it is $\frac{w''' - w''}{n}$. Call the ratio of these two, or $\frac{w''' - w''}{w'' - w'} \cdot \frac{l}{n} = b$. So much of the work may be done once for all. Any change in volume of the interior of the tube can be accurately measured by noting the change of level in the glass tube.

Fill the tube with water to the point marked n, and read the position of the marks. Stretch it by turning the pin below so as to lengthen the tube. Read each mark in its new position together with the water level, and so proceed, taking a number of readings under different tensions.

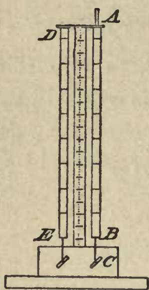


Fig. 30.

Construct a curve in which abscissas represent readings of the water level, and ordinates the changes which take place in the length of the rubber, draw also other curves, in which abscissas represent the water level, and ordinates the change of lengths of each section of the tube. To do this it is most convenient to make a table giving the readings of each point, a second giving the difference of reading of each two consecutive marks, and a third giving the increase of length they undergo when the cord is stretched. The first of the curves will be nearly a straight line, and the tangent of the angle it makes with the horizontal line, or the ratio of its vertical to its horizontal progression multiplied by b, gives the ratio of the increase of volume compared with the increase of length. In the same way by the other curves, determine the change in length of each section of the tube compared with the whole change.

Next measure the height of each marked point of the rubber rod, also its diameter at these points. Stretch it and measure again, and take four series of observations in this way. Now construct curves, in which abscissas represent scale readings, and ordinates alterations in thickness as given by the gauge. The scale for the latter must be greatly enlarged. Measure the area enclosed by this space, and reduce it to square millimetres, allowing for the change of scale. Multiplying this area by four times the thickness gives approximately the diminution in volume due to the contraction in the centre. If the rod is much altered in form, the change in cross-section may be obtained more accurately by taking the difference of the squares of the thickness before and after extension. Using them as ordinates of the curve the volume is given by the enclosed area. Construct such a curve for each extended position of the rod, and compare the decrease of volume thus found with the increase due to the change of length, or the product of the cross-section by the change of reading of the lower index mark.

35. Deflection of Beams. I.

Apparatus. In Fig. 31, *AB* is a rectangular bar of steel resting on two knife-edges, with a load applied to its centre by a weight placed in the scale-pan *D*. To measure the flexure, a micrometer screw *C* is placed over the bar, and turned until its point touches the latter. *E* is a galvanic battery having one pole connected with the bar, the other with *C* through an electro-magnet *F*. When the screw touches the bar the circuit is completed, and the magnet draws down its armature with a click. This gives a very accurate test of the exact position of the screw when contact takes place. The length of the beam may be altered by changing the position of *A* and *B*. *C* and *D* are also movable, and a set of weights is provided to vary the deflection. To ensure contact the wire should be soldered in position, and the point of *C* tipped with a piece of sheet platinum. A convenient size of bar for laboratory purposes is about half a metre long, a centimetre wide, and half a centimetre thick, using weights from 100 to 2000 grammes. Instead of the electro-magnet *F*, a galvanometer may be used if preferred.

Experiment. Set *A* and *B* 50 cm. apart, and *C* and *D* midway between them. Turn *C* until it touches the bar, when instantly a current will pass from the battery *E* through the magnet *F*, making a click, or if a galvanometer is used, swinging the needle to the right or left. Read the position of the micrometer screw, taking the whole number of turns from the index on one side, and the fraction from the graduated circle. Add 1000 grammes to *D*, and bring the screw again in contact. The difference in the readings gives the deflection with great accuracy. The general formula for the deflection a of a beam of length l, breadth b, and depth d, under a load W, is $a = \frac{Wl^3}{Dbd^3}$, in which D is the modulus of transverse elasticity, or the weight required to make a equal one, when b, d and l, all equal unity. From this formula the following laws may be deduced. 1st. The deflection, when small, is proportional to the weight applied. Measure the deflec-

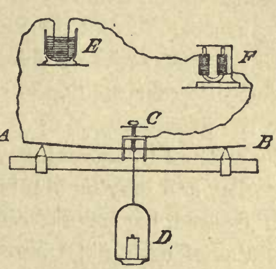


Fig. 31.

tion for every hundred grammes, from zero to two kilogrammes, and see if the beam returns to its original position when the load is removed. If not, the change is the permanent set. Construct a curve with abscissas proportional to the load, and ordinates to the corresponding deflection. Evidently, according to the law, this should be a straight line, and the near agreement proves conclusively its correctness. 2d. The deflection is proportional to the cube of the length. Measure the deflection with a load of 2 kgs., changing the length of beam 5 cm. at a time, from 50 cm. to 0, keeping C and D always at the middle of the beam. Care must be taken in each case to first measure the micrometer-reading when no load is applied, as this point will vary, owing to irregularities in the bar or stand. To compare the results with theory, construct a curve in which abscissas represent lengths of the beam, and ordinates deflections. According to the law this should be a cubic parabola, having the equation $y = ax^3$. To find the value of a, suppose one of the earlier readings gave a deflection of 5.8 mm., for a length of 40 cm.; then $5.8 = a\,40^3$, or $a = .00009$. Substituting this value in the equation, construct the curve $y = .00009\,x^3$ on the same sheet with the experimental curve, and see if they agree. To find the value of D, draw a line nearly coincident with the first series of observations. Deduce from it the increase of deflection for each added kilogramme. Substitute this value for a in the formula, making $W = 1$, and giving l, b and d, their proper values, found by measuring the bar. D is now the only unknown quantity, and may be obtained by solving the equation.

If desired, bars of different materials may be provided, and the modulus of each determined by measuring the deflection with a given load, and substituting these values in the formula. The law that the deflection is inversely proportional to the breadth, may be proved with bars alike except in breadth, and the law of the thickness in a similar manner. The form of the beam when bent is found by shifting the micrometer C and measuring the deflection at various points between A and B, and the effect of a change in position of the load by moving D. The same apparatus may be applied to the case of a beam supported at more than two points, or to beams built in at one or both ends. This experiment may also be almost indefinitely extended by using circular and triangular bars, hollow or solid, also those of a T or I shape, or, in fact, girders of any form.

36. Deflection of Beams. II.

Apparatus. *AB*, Fig. 32, is the bar to be tested, which may be one of the square rods described in the next experiment. It is clamped at one end by placing it between two similar rods, one of which is nailed to the wall, and the other pressed down upon it by two or three clamps, like those of a quilting frame, as at *C*. A small mirror may be placed upon it at any point, and the deflection measured by reading in it the reflection of a scale *F* by the telescope *E*. The beam is bent by weights placed in the scale-pan *D*. By attaching a finely divided scale to *A*, the absolute deflection may be read by the telescope *E*, using it like a cathetometer.

Experiment. Clamp *AB* so that its length shall be 50 cm., and place the mirror at its end. Place the telescope *E* opposite *A* and focus it, so that on looking through it the image of the scale *F* shall be distinctly seen reflected in the mirror. Read the position of the cross-hair in the telescope to tenths of a division of the scale. Now place a kilogramme in *D*. The beam is at once bent, and although the mirror moves but little, the scale-reading is greatly altered. The work described under the last experiment may be repeated with this apparatus. Or to vary it, let the following measurements be made. Place 2 kgs. in *D*, and take the scale-readings when placed at distances of 5, 10, 15, 20, &c., centimetres from *B*. Again, take the scale-reading very carefully when no load is applied. Now place in *D* as great a weight as the beam will safely bear, and take readings every half minute for five or ten minutes. Then remove the load and see if the beam returns to its original position. The permanent set of the beam may be studied in this manner.

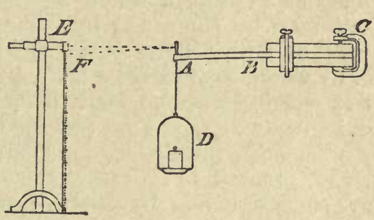


Fig. 32.

To compare the results with theory the scale-readings must be reduced to angular deviations by the formula given on page 24. For small deflections, however, it is sufficient to divide the change in scale-reading by twice the distance EA, to obtain the tangent of the angle through which the mirror moves. In deducing the form of an elastic beam by Analytical Mechanics, we have $EI\,\frac{d^2y}{dx^2} = P(l - x)$, the moment of the deflecting force P. In this E is the modulus of elasticity, I the moment of inertia of the cross-section, y the deflection of any point at a distance x from the end, and l the length of the beam. Integrating, we obtain $\frac{dy}{dx} = \frac{P}{EI}(lx - \frac{x^2}{2})$, in which $\frac{dy}{dx}$ may be compared directly with the measurement.

37. Trusses.

Apparatus. A number of deal rods, as nearly alike as possible, half an inch square and five or six feet long. These are to form the beams or units of which all the trusses are to be composed. They may be connected by clamps like those used for quilting frames, by boring holes in them and fastening by wire, or better still by using small carriage-bolts, $\frac{1}{8}''$ in diameter. A scale-pan and set of weights serve to apply a strain not exceeding one or two hundred pounds to any part of the truss to be tested. By attaching a fine scale the deflection of any point may be read by a telescope mounted like a cathetometer. The strain on any portion may be determined by inserting a small spring balance, as will be described below.

Experiment. This apparatus may be procured at very small expense, while with it almost all the laws of elasticity may be proved, and the strength of a great variety of trusses for bridges and roofs tested. Although the following work resembles that of Experiment 35, yet its importance, and the different method of measurement employed, justifies its repetition.

The flexure of a beam is proportional to the load. Set two knife-edges 40 inches apart, cut a rod a little longer than this, and lay it on them. Attach a fine scale to its centre point, focus the telescope on it and record the reading. Add weights, a pound at a time, until the beam breaks. The increase of reading in each case over that given at first, is proportional to the load. It must be noticed, however, that when the beam is much bent a new law holds. Repeat these measurements with rods 30, 20, and 10 inches long. The flexure is proportional to the cube, the breaking weight inversely as the square of the length.

In the same way the effect of applying a load at different points, or the deflection at different points with a given load, may be measured. A long beam may also be supported at several points, and the effect of a moving load noted. To measure the strength of a beam built in at one or both ends, clamp it at those points between two similar beams, one above, the other below, and fasten them by bolts or clamps to a fixed upright. Having thus fully determined the strength of the single beams, they should be examined when combined. Join two beams together so as to form a T, and measure the strain necessary to pull the vertical one off. Different kinds of joints may thus be compared. Some form of truss should now be built, and its strength tested. Let the king-post, Fig. 33, be the form selected, and make the span *AB* 40 inches. Cut two rods a little longer than this, and bore the holes *A*, *B* and *D*. Cut two more rods *CD* with holes distant 10 inches, and attach them to the others by bolts at *D*. Add *AC* and *CB*, and connect the two trusses thus formed by cross pieces at *A*, *B*, *C* and *D*, so that they shall be ten inches apart. A small bridge is thus made whose stiffness is remarkably great. Such a structure should bear a weight of fifty, or even a hundred pounds, with little flexure. Measure the deflection at different points under varying loads, trying the effect of applying the latter at the centre, on one side, or distributing it uniformly.

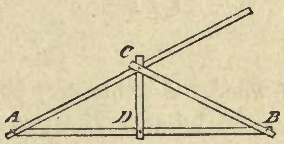


Fig. 33.

The force acting on any beam is readily determined by replacing it by a spring balance, with a screw and nut which may be made to take up the whole strain, without distorting the structure. If the force is one producing compression, it is best to elongate the beams, as at *CE*, and insert the balance between *C* and *E*. Compare the various results with those obtained by computation. It must be remembered that this method of connecting the beams of a truss by bolts is not employed in actual practice, but it is very convenient, and sufficiently strong for a model. If preferred, proper tools may be supplied, and the student may frame his truss as in a real bridge or roof. Further, any beam, as *CD*, subjected only to tension, may be replaced by a wire.

The rods will also be found useful for a great variety of purposes. Thus one of the easiest ways to make large screens is to fasten four of them together at the ends, and attach thick paper to them by double-pointed tacks. Again, a convenient way to make a large rectangular box to cut off light, or to protect an instrument from dust, is to connect twelve of these rods at the ends by slipping them into corner pieces of tin, Fig. 34, and covering them with black paper. In the same way all the principles of framing may be taught, and quite complicated structures built. It is well to have some of the latter loaded until they break, to determine the weakest points. They are then easily repaired by inserting new pieces of wood. Another very good object to construct and test is a suspension bridge. Use two stout wires or chains for the suspension cables, and build the roadway of the rods, hanging it from the chains by wires with screw threads cut on their ends, so that their lengths may be adjusted by nuts. Test the strain on the chains by inserting a spring balance like that known as the German ice-balance, and measure the deflections of the different parts under varying loads.

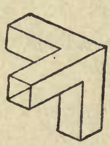


Fig. 34.

38. Laws of Torsion.

Apparatus. Let *AB*, Fig. 35, represent the bar whose torsion is to be measured. The further end *B* is firmly fastened to a piece of wood *C*, which can turn around the axis of the beam, but may be clamped in any position to the semicircle immediately behind it. *A* in the same way is attached to *D*, which carries two brass rods, acting like the dog of a lathe. *EF* is a long rod mounted on an axle, which may be turned by placing weights in the scale-pan *G*. The latter is supported by a cord passing over a curved block at the end of *E*, so that the moment of the weight, or its tendency to twist the bar is unchanged, whatever the position of *EF*. To measure the angle of torsion, two mirrors are attached to *AB*, and the scales *H*, *H′* reflected in them are viewed by telescopes *I*, *I′*. By making *H*, *H′* arcs of circles, with centres at *A* and *B*, the angle of torsion may be obtained directly.

Experiment. To measure the torsion of the beam AB, attach it to the stand, as in the figure, placing the mirrors at a distance apart equal to the length to be examined. Focus the telescopes I, I' so that the scales H, H' shall be distinctly visible, and read the position of the cross-hairs. Place a weight in the scale-pan G which will twist both the mirrors A and B, deviating the former the most. See how much each scale-reading has changed; their difference measures the angular twist of AB. This may be reduced to degrees from the radius of curvature of the scale, and the magnitude of its divisions, recollecting that the motion of the mirror is only one-half that of the reflected ray. From the theory of torsion the following laws are readily deduced. The angle of torsion is proportional to the moment of the deflecting force. To prove this law, measure the torsion with several different weights in G, and see if the angles are proportional to the weight. The distance of the point of application of G from the axis may also be varied, and it will be found that the torsion is proportional to the product of this distance multiplied by G. The torsion is proportional to the length of the bar. Prove this by varying the distance between the mirrors, leaving the bar unchanged. By using a variety of bars it may be proved that in those having similar sections the torsion is proportional to the fourth power of the similar dimensions. In rectangular bars of breadth b, and depth d, it is proportional to $\frac{1}{12}bd\ (b^2 + d^2)$, and in tubes to $\frac{1}{2}\pi(r^4 - r'^4)$ calling r and r' the outer and inner radii. In practice, owing to the warping of the surfaces, these formulæ undergo slight modifications.

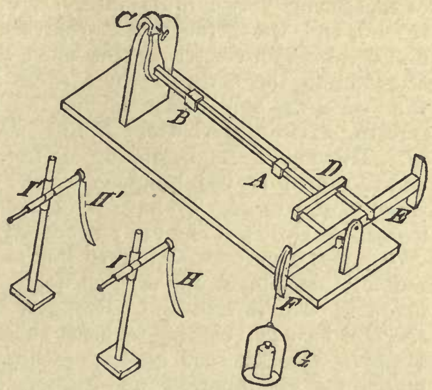


Fig. 35.

39. Falling Bodies.

Apparatus. This consists of two parts, a chronograph capable of measuring very minute intervals of time, as hundredths of a second, and the arrangement represented in Fig. 36, for making and breaking an electric circuit when the body falls. A ball *A* is attached to the spring *D* by a short thread and wire. Burning the thread the ball is released, and the spring rising allows the current to pass from the battery *B*, to *C*, *D*, *E*, *F*, and the chronograph *G*. This marks the beginning of the time to be recorded. Its end is shown by the breaking of the circuit, which occurs at *F* when *A* strikes *E*. To have the current broken during this time, instead of closed, it is merely necessary to place the points *E* and *F* below the springs. Another method of releasing the ball is to put an iron pin in its upper side, and support it by an electro-magnet. The instant the current is broken it will fall. One of the best forms of chronograph for this purpose is that devised by Hipp, in which a reed making a thousand vibrations a second replaces the pendulum of an ordinary clock. It therefore ticks a thousand times a second, and measures small intervals of time with great precision. Very good results may be obtained with a common marine clock, removing the hairspring and replacing it by an elastic bar of steel. An electro-magnet is placed close to the pallet, so that when the current passes the spring is bent. The clock, therefore, starts the instant the circuit is broken, and stops as soon as it is closed.

Experiment. The time of falling of any body through a height h, equals $\sqrt{\frac{2h}{g}}$, in which $g = 9.80$ metres. Measure the time of fall through various heights by altering the length of the wire *AD*, noting the height in each case with care, repeating several times and taking the mean. Compare this with the result given by theory. In vacuo the time would be independent of the material or magnitude of the ball. In air, however, this is not the case, owing to the resistance. The latter may therefore be determined by measuring the time of fall of bodies having the same form but different weights. This apparatus may also be applied to the measurement of the velocity of curve-motion and personal equation, or the coefficient of friction

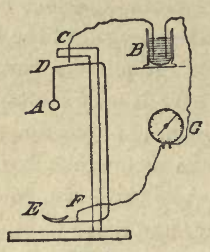


Fig. 36.

of a body may be found with great accuracy by measuring its time of descent down an inclined plane.

40. Metronome Pendulum.

Apparatus. A light deal rod is provided, to which two leaden weights may be attached at any point by set screws. A knife-edge passes through the centre of the rod, so that it may be swung like a pendulum. Either weight may also be attached to the knife-edge by a fine wire. A millimetre scale is used to measure the distance of the weights from the knife-edge, and a bracket fastened to the wall and carrying a steel plate serves as a support.

Experiments. First, to prove the law of the single pendulum. Attach the heavier weight to the knife-edge by the wire, using as great a length as possible. Measure the time in seconds of making 100 single, or 50 double vibrations, also the distance from the centre of the lead to the knife-edge. Repeat with several lengths of wire.

Now compare the observed time with that given by the formula $t = \pi\sqrt{\frac{l}{g}}$, in which $g = 9.80$ m., and l is the measured length. Repeat one of these measurements with the lighter weight, using the same value of l. It should give the same result as the other. Next place both weights on the deal rod at opposite ends. Measure, as before, their time of vibration, also their distances from the knife-edge. Compute the time of vibration as above, merely substituting for l the value $\frac{w'l'^2 + w''l''^2}{w'l' + w''l''}$, in which w', w'' are the weights, and l', l'' their distances from the knife-edge. If greater accuracy is required, a third term must be introduced for the weight of the rod. Repeat with various positions of the two weights, and compare the results with theory.

41. Borda's Pendulum.

Apparatus. In Fig. 37, CD is the pendulum, formed by attaching a ball of lead C, to a wire nearly four feet long, and supporting it on a knife-edge D. A sheet of platinum is fastened below the ball, so that when at rest it dips edgewise into a mercury cup, making electrical connection with the battery B. E is a clock whose pendulum F dips in a second mercury cup. When both pendulums are at rest the current passes from B through C, D, E

and F, to G, which is an electric bell arranged so as to strike whenever the circuit is closed.

Experiment. Connect the battery B with the wires attached to C and G. The bell will instantly strike. Start the pendulum DC. Whenever it passes through the mercury cup, that is, with every swing, the electric current passes through G and makes the bell strike. Stop CD and start the clock. The strokes now occur at intervals of exactly one second. Now set both pendulums going. The bell will strike only when both are vertical at the same instant. This will occur at regular intervals, equal to the time required by the longer pendulum to lose just one vibration. Record the minute and second of each stroke for ten or fifteen minutes consecutively. The first differences give the intervals, and from the mean of the latter the time of vibration may be computed with great accuracy. For example, if the interval is 47 seconds it denotes that in this time CD made one less, or 46 vibrations, hence the time of a single vibration would be $\frac{47}{46}$=1.0217 seconds. An error of one second in the mean of the interval would make the time $\frac{48}{47}$=1.0213 seconds, or alter the result less than a two-thousandth of a second. The method, therefore, is one of extreme precision. Sometimes, especially when the pendulum is swinging through a small arc, the bell will strike for several consecutive seconds, owing to the considerable interval of time during which contact is made at C, so that for several seconds the circuit is closed at F before it is broken at C. In this case the time of the first stroke should be recorded and their number; the true time being taken as the mean of the first and last. To make CD vibrate in one plane instead of describing an ellipse, attach a fine thread to the ball C; draw it to one side about ten inches; let it come to rest, and then burn the thread. Finally measure the length l of the pendulum, or the distance from the knife-edge to the centre of the

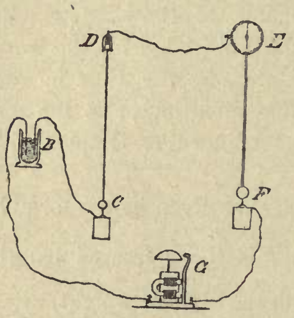


Fig. 87.

ball, and compute the force of gravity g from the formula; $t = \pi\sqrt{\frac{l}{g}}$, in which t equals the time of vibration, and $\pi = 3.1416$.

This experiment may be repeated with a different length of pendulum, or it may be varied so as to prove that the time increases with the amplitude. In the latter case the arc through which the ball swings should be as large as possible, and it should be measured as it progressively diminishes. To compute the theoretical time of swinging through any arc a, divide versin $\frac{1}{2}a$, or the vertical distance through which the ball moves, by its length, and call the quotient x. Then the time t' for any value of x may be found from the equation $t' = (1 + \frac{1}{8}x + \frac{9}{256}x^2 + \&c.)\,t$, in which t is the time when the arc is very small. When $a = 180°$, or the ball swings through a semicircle, $t' = 1.180\,t$, when $a = 30°$, $t' = 1.0063\,t$, when $a = 10°$, $t' = 1.00067\,t$, hence for small arcs the correction for this cause is very small. If great accuracy is required in this experiment the suspending wire should be very light, and with the knife-edge should vibrate in about one second when the ball is removed, or a correction may be applied for them as described in Experiment 40.

42. Torsion Pendulum.

Apparatus. AB, Fig. 38, is a vertical wire with an index C, which moves over a graduated circle. Weights of a cylindrical form, as D, may be attached below in such a manner that the wire cannot twist without turning them. To vary the length of the wire it is passed around several small brass tubes E, F, G, placed at different heights, so that it may be clamped at these points by inserting a pin G passing into a hole bored behind them. A scale and clock beating seconds are also needed for this experiment.

Experiment. 1st. The time is independent of the amplitude of the vibration. Use the whole length of the pendulum, and apply such a weight that the time of a single vibration shall be about one second. Twist the index through a small arc, and take the time of one hundred oscillations by noting the position of the index at the beginning of a minute, and the exact time, when after making one hundred single oscillations, it

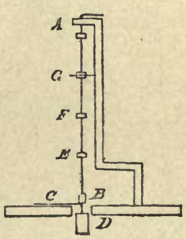


Fig. 38.

again reaches the same point. Dividing the interval by one hundred gives the time of a single oscillation. Repeat two or three times with arcs of different magnitudes, and compare the results. 2d. The time is proportionate to the length of the wire. Make the same experiment, first with the wire of its full length, then, passing the pin through the different tubes *E*, *F*, clamping it at these points. Measure their distances from *B*, and compare with the law. In the same way the relation of the time to the diameter of the weight, or to its length, may be tested and compared with theory.

MECHANICS OF LIQUIDS AND GASES.

43. Principle of Archimedes.

Apparatus. An inverted receiver *A*, Fig. 39, with a stopcock, or better, an $\frac{1}{8}''$ gas valve below. Near the top is placed a hook *C* with a sharp point, which is used to mark the level of the liquid. The whole may be hung from the scale-pan *D* of a large balance, *EF*, which has a counterpoise attached to the other end. *G* is a beaker to collect the water drawn off, and *H* a stand by which *A* may be supported if necessary. A set of weights is needed, also two bodies *M* and *N*, one heavier, the other lighter than water. They may be made of metal and wood, or, if preferred, of glass, and loaded so that one shall float, the other sink.

Experiment. 1st. A heavy body when immersed is buoyed up by a force equal to the weight of the displaced liquid. Place the receiver on the stand, fill it with water and draw out the latter until the point of the hook just touches the surface, observing the point of contact, as in Experiment 13. Place the beaker *G* on the scale-pan *D*, suspend *M* below it, and add weights to the other side so as to bring the beam into equilibrium. If now the receiver is brought up under *M* the water will rise, and the equilibrium will be destroyed. Open *B* and draw off the water into *G* until *M*, being immersed, the level is again exactly at *C*. Now replacing *G* on *D* it will be found that the equilibrium is restored. Hence the loss of weight of *M* equals the gain of *G*, or the weight of the displaced liquid, since the level is unchanged.

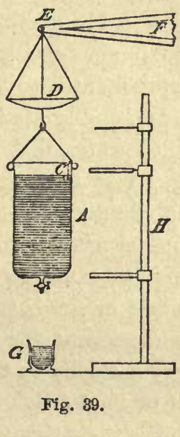


Fig. 39.

2d. Since action and reaction are equal, the vessel appears to gain in weight by an amount just equal to the loss of *M*. Suspend *A* from the scale-pan, and *M* from the stand. Bring the water-level to *C* and counterpoise as before. Immerse *M*, when the water will rise, and the weight apparently increase. Open *B* therefore, and draw out the water until the level is restored, when it will be found that the beam is again balanced, showing that it was necessary to draw out a volume of liquid equal to that of *M*.

3d. A floating body displaces a weight of liquid just equal to its own. Rest *A* on its stand and restore the water level to *C*. Place *G* and *N* on the scale-pan and counterpoise. Let *N* float in *A*, open *B* until the proper level is attained, collect the water in *G*, and replacing the latter on the scale it will be found that the equilibrium is restored. That is, the weight of the displaced water, or the increase of *G*, equals the weight of *N*.

44. Relation of Weights and Measures.

Apparatus. A delicate balance with a counterpoise on one side and scale-pan on the other, below which a small cube of brass is suspended by a very fine platinum wire. In addition, a beaker containing distilled water, a thermometer and a set of weights, must be provided.

Experiment. By definition a gramme is the weight in vacuo of a cubic centimetre of distilled water, at the temperature of maximum density, that is, about 4° C.; the object of the present experiment is to test this relation. Add weights to the scale-pan until equilibrium is established; then immerse the cube in the distilled water, first washing it with caustic potash to remove the air, then very thoroughly with common water to remove the potash, and finally with distilled water. The weight now required to counterpoise it will be greater than that previously taken, by an amount equal to the weight of the displaced water. Record the height of the barometer and the temperature of the water. Next, to determine the volume of the cube, measure the twelve edges very carefully to tenths of a millimetre, and take the mean of each set of four which are parallel. The product of these three means equals the volume. The dividing engine should be used to attain sufficient accuracy in this measurement. Add to this the volume of the wire found by multiplying its cross section by the length submerged. Correct the weight found above for the buoyancy of the air, and the volume for the dilatation of the water, as in Experiment 19. Only in this case the whole weight of the displaced air must be added, since by definition the weight must be taken in vacuo. The density D of the water at any ordinary temperature t, is given by the formula $D = 1 - .000006\,(t - 4)^2$, its density at 4° being unity. After applying these two corrections, see if the volume of the water in cm^3 equals its weight in grammes.

By using English weights and measures instead of French, the relation between the inch and pound may be established in a similar manner.

45. Hydrometers.

Apparatus. This consists of four tall jars, two containing water, the third some light liquid, as alcohol, and the fourth a saturated solution of salt, or other heavy liquid. A variety of hydrometers, some giving the specific gravity directly, others with the scales of Beaumé, Cartier and Beck, &c. In one of the jars of water, which should be larger than the other, is a Nicholson's hydrometer, Fig. 40, and on the table a box of weights, a small stone and a piece of hard wood. Near by should be a sink, with a large jar in it, through which water is continually flowing, to wash the hydrometers.

Experiment. Float each hydrometer in turn in the jar containing water, and record the reading of the point of the scale on its stem just at the surface. This point is determined most accurately by bringing the eye nearly on a level with the top of the water, but a little below it. All should give a specific gravity of very nearly unity, the difference being partly due to error in the instrument, and partly to expansion of the water by heat. Next immerse each in the alcohol, take the reading and wash by plunging it in the large vessel of water. Do the same with the solution of salt. If any hydrometer sinks lower than the top of its scale, the liquid is lighter than it can measure; if it floats too high the liquid is too heavy. Finally, reduce all the readings to specific gravities by the hydrometer tables. These instruments being of glass are easily broken, and must be handled with care.

Turning now to the Nicholson's hydrometer, place weights on the upper scale-pan A, until it sinks to the mark scratched on its stem. Record their sum, and replace them in their box, taking care (as must always be done with delicate weights) never to touch them with the fingers, but only with forceps. Moreover they must never be laid down on the table, and to prevent their falling into the water, the piece of metal *C* must be kept over the mouth of the jar. Place the stone, or other object whose specific gravity is to be measured, on *A*, and add weights, as before. Call their sum in the first case w, in the second w'. Raise the hydrometer out of the water (of course first replacing the weights in their box), and place the stone on the lower scale-pan *B*. Immerse it, taking care that there are no adhering air bubbles. If these cannot be detached with the finger, remove the stone and wash it first with caustic soda, and then with pure water. Call w'' the weight required to immerse the hydrometer when the stone is on *B*. Then $w'' - w'$ is the apparent diminution of weight of the stone when immersed, or the weight of an equal bulk of water. As $w - w'$ is the weight of the stone, its specific gravity is $\frac{w - w'}{w'' - w'}$. Perform the same experiment with the piece of wood, only placing it below *B* to keep it down, and noticing that w'' will be greater than w.

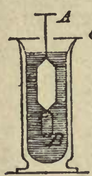


Fig. 40.

46. Specific Gravity Bottle.

Apparatus. A balance weighing up to 100 grms., and turning with two or three milligrammes, a set of weights and a specific gravity bottle, or as a substitute, two glass stoppered bottles, the neck of one being large, of the other, small. They should be carefully selected, with stoppers fitting smoothly, and a scratch should be made both on the neck and stopper, so that the latter may always be turned into the same position. As objects for determination of specific gravity any liquid may be used, as a solution of salt, and two or three solids, as stones, coins, gold ornaments, sand, &c.

Experiment. Weigh the empty bottle and stopper, and call their weight w_a. Fill the bottle with water, insert the stopper and wipe off the liquid which has overflowed, taking care that the exterior of both bottle and stopper are perfectly dry. Call this weight w_w.

Fill with the liquid to be tested in the same way, taking care that the stopper is inserted in the same position as before, and that no liquid adheres to the exterior. Let the weight be w_l, then $w_l - w_a$ and $w_w - w_a$ are the weights of equal bulks of the liquid and of water, and the specific gravity of the liquid is $= \dfrac{w_l - w_a}{w_w - w_a}$.

To find the specific gravity of a solid, use the bottle with the larger neck. Call w the weight of the solid, w_w the weight of the bottle filled with water, and w_s the weight when the solid is inserted, and the remaining space filled with liquid; then $w + w_w - w_s$ equals the weight of a volume of water equal to that of the solid, and the specific gravity $= \dfrac{w}{w + w_w - w_s}$. This method is applicable to solids heavier or lighter than water. The principal precaution is to take care that no bubbles adhere to the solid or sides of the bottle, and that the stopper is always pressed in by the same amount. Use the same devices for removing the air as with the Nicholson's hydrometer, Experiment 45. With metals these precautions are especially important, or large errors will be introduced. Another good method is to place the flask containing the solid and water under the receiver of an air-pump and exhaust two or three times. This method is not applicable to wood, as it removes the air from the cells, and increases the apparent specific gravity. The same effect is produced by long immersion, and finally when waterlogged, the specific gravity becomes greater than unity, and the wood sinks.

47. Hydrostatic Balance.

Apparatus. A complete apparatus for this purpose, known as Mohr's Balance, may be obtained, and the following description is especially applicable to it. A common balance may, however, be substituted, raising one scale-pan and attaching a hook below. Instead of riders it is then generally more convenient to use ordinary weights. Some solids and liquids are also needed as substances whose specific gravity is to be determined.

Experiment. Attach the small scale-pan to the left, and the glass counterpoise to the right end of the beam. The weighing is done by riders, of which there are three sizes, whose weights are in the ratio 10, 100 and 1000. The beam is divided into 10 equal parts, so that when balanced the weight may be read off directly to three places of decimals. Fill the small jar with water, and see what weight is necessary to immerse the counterpoise. It will be found to be very nearly 1000, and evidently equals the weight of the water displaced. Next, fill the jar with the liquid to be tested, and see what weights are now required. The ratio in the two cases is the specific gravity. The temperature should be recorded in each case by the thermometer contained in the counterpoise, and if great accuracy is required a correction applied for it, or better, the liquids may be cooled to the standard temperature.

To find the specific gravity of a solid, wind a piece of fine wire around it, and suspend from the left hand end of the beam. Counterpoise by adding lead, sand or paper to the scale-pan at the other end until a perfect balance is obtained. Immerse in a vessel of water, and balance by adding the riders; their weight equals that of an equal volume of water. Then remove the solid, and again bring the beam to a horizontal position by the riders; this gives the weight of the solid, which divided by the weight of the water displaced, gives the specific gravity. If more convenient, the weight of the body may be obtained directly by the riders without counterpoising it.

Next, find the specific gravity of a piece of wood, or other solid lighter than water. Attach a piece of lead, or other body heavy enough to sink it, and measure, as above, the following quantities. Weight of solid in air w_s, weight of lead in air w_l, weight of lead in water w_l', weight of solid and lead in water w_{ls}'. Then $w_l - w_l$ $=$ weight of a bulk of water equal to that of the lead. $w_l + w_s$ $- w_{ls}' =$ weight of a bulk of water equal to lead and solid. Hence their difference, or $w_l + w_s - w_{ls}' - w_l + w_l' = w_s + w_l' - w_{ls}' =$ weight of water equal in bulk to solid, and weight of solid divided by this, equals specific gravity, or $S.\ G. = \dfrac{w_s}{w_s + w_l' - w_{ls}'}$.

The same precautions are necessary, as with the gauge flask, regarding air bubbles, and the riders should never be touched with the fingers, but always with a small bent wire.

48. Efflux of Liquids.

Apparatus. In Fig. 41, *A* and *B* are two reservoirs of tin, or wooden boxes lined with lead, each containing two or three cubic feet. Water is admitted by a valve at *C*, and passes through a cylinder of perforated tin *D*, to break up the stream and prevent much motion of the water in *A*. An outlet is made at *E*, which may be closed by a stick of wood with a rubber flap on its end *K*, which is held in place by the pressure of the water. To keep the level constant, a funnel *F* is connected with the interior by a rubber tube, so that it may be raised or lowered, and serve as an overflow, or a simple straight tube may be used, passing through the bottom of *A* to the surface. The height of the water is read by a hook gauge *G* with an index attached, moving over a scale. A number of brass plates fitting into *E* are provided with orifices of various shapes and sizes, some circular, rectangular and triangular, and others furnished with projecting cylindrical or conical tubes.

The second reservoir *B* has also a hook gauge and scale *H* to show the amount of water in it, and an outlet *I* closed by a plug. To prevent motion of the surface of the water around *H*, a diaphragm is placed in the centre of the reservoir, on which the water impinges, a number of holes being bored in the lower portion to equalize the level on each side.

Experiment. When water flows through an aperture in a thin plate the amount per minute is much less than that given by theory, owing to the contraction of the liquid vein immediately after leaving the orifice. The ratio of the two is called the coefficient of efflux, and the whole science of hydraulics is based on this constant. To determine it, water is allowed to flow from *A* under a given head through an orifice *E*, and the quantity measured by the scale attached to *H*. Place one of the circular orifices in *E*, and measure its height by bringing the water just on a level with it, and using the hook gauge. This is done as is described in Experiment 13, by bringing the point of the hook just to the surface of the liquid, so as slightly to distort the image of outside objects, and reading the position by the scale. Close *E* with the rod *K* and open the valve *C*, first raising the funnel *F* nearly to the top

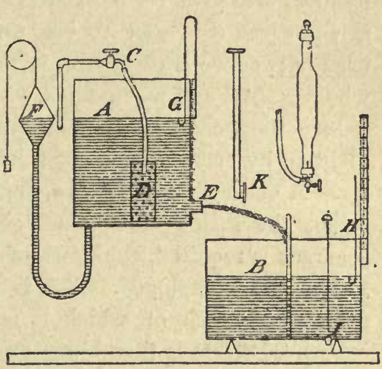


Fig. 41.

of the reservoir. When the water begins to escape over the edge of the funnel close the cock, and read very carefully the level by the gauge. Read also the height of the liquid in *B*, which should be nearly empty. At the beginning of a minute open *E* by removing the rod *K*, when the water will begin to flow into *B* in a clear transparent steam, marked, when the aperture is not circular, by alternate swellings and contractions. As the liquid will at once descend in *A*, the valve *C* should be opened at the same time, and adjusted so that the water shall slowly trickle over the edge of the funnel, or outlet tube, or the latter may be dispensed with, and the surface kept just at the point of the hook. When *B* is nearly full, which should take at least five minutes, close *E* and note the time. It is best to make this come at the end of a minute. Now read the height of the water in *B*, empty it, and repeat to see if the same results are obtained twice in succession. Make the experiment again with other pressures, also changing the orifices.

To reduce the scale-readings of *H* to cubic inches, the reservoir *B* must next be calibrated. If nearly rectangular, a direct measurement will give its horizontal cross-section, but if the sides are at all curved it is safer to use some other method. A plan much used in practice is to mount it on a platform scale and weigh it when empty, and when filled with water to various heights, and reduce the weight of the water in each case to cubic inches, by dividing by .03614, the weight in pounds of one cubic inch of water. A curve should then be constructed, in which ordinates represent the scale-readings and abscissas the volumes. If greater accuracy is required, the tenth of a cubic foot used in Experiment 19 should be employed. A T is placed between its valve and the glass, the branch of which is connected with the hydrant by a rubber tube. It is then hung over the reservoir *B*, as in the figure. To use it, admit water until it is filled to the top of the hook in its upper end. Shut off the water, and open the valve below. When the water level has reached the lower point, close the valve and read the gauge in *B*, thus taking a series of readings which will correspond to intervals of precisely one tenth of a cubic foot. In this case it is best to construct a residual curve to show more clearly the irregularities in form of the reservoir.

The area of the orifices must next be measured with a fine scale, reading to tenths of a division by the eye, or if greater accuracy is required, using the dividing engine.

Finally, to compute the theoretical flow, we have the following data. By the theorem of Torricelli the velocity $= \sqrt{2gh}$, in which $g = 32.2$ ft., or the acceleration of gravity, and h is the height of the liquid above the centre of pressure of the orifice. This equals the difference in the two readings of the hook gauge in A, before and after the experiment, correcting for the position of the centre of pressure, which will sensibly coincide with the centre of gravity of the orifice. Thus with a circular orifice one-half its diameter must be subtracted. A stream of water will then flow out having a volume equal to that of a prism with cross-section s equal to that of the orifice, and a length v for each second, or in t seconds, the observed time, the volume V should be $stv = st\sqrt{2gh}$. The observed volume is obtained directly from the calibration of B, of which either a curve or a table should be furnished. This quantity divided by V gives m, the coefficient of efflux.

49. Jets of Water.

Apparatus. A cylindrical brass tube is used as an orifice, and is mounted at a height of three or four feet from the floor, with a hinge and graduated circle, so that it can be set at any given angle. A deal rod divided into inches is attached to it to measure the range, and the whole is connected with the hydrant by a rubber tube and valve, so that water may flow through it at any required velocity. The water is collected as it escapes in a large vessel, which is weighed in a spring balance before and after the experiment, and thus the amount of water determined. A second scale of inches is also required to measure the vertical descent of the curve.

Experiment. Almost all the laws of projectiles may be proved by this apparatus. 1st. The form of the jet is a parabola. Set the tube horizontal, and allow the water to flow through it, with such a velocity that in moving three feet horizontally it will descend about the same distance. Take care that this velocity is unchanged during the experiment by noticing that the horizontal range remains the same. Now measure the vertical fall of the jet for every two inches on the horizontal scale, and construct a curve with these distances as coördinates. Next, to measure the velocity, allow the water to flow into the vessel for one minute, and weigh it. The weight in grammes equals the number of cubic centimetres, and this divided by the area of the orifice (found by measuring the diameter of the tube), gives the velocity of the water per minute. Divide this by 60, for the velocity per second, and construct the parabola given by theory, in which $x = vt$, and $y = \frac{gt^2}{2} = \frac{gx^2}{2v^2}$, and the acceleration of gravity $g = 386$ inches. Great care must be taken to reduce all these quantities to the same measure, as inches or metres, several different units being purposely employed in these measurements. Repeat the latter part of this experiment with three or four different velocities, and see if for a given value of y, x is proportional to v.

2d. The horizontal range for a velocity v, and angle of projection a, equals $\frac{v^2}{g} \sin 2a$. Prove this by measuring the range for every 5° from 0° to 90°. Evidently the maximum is when $x =$ 45°. In a similar manner we may prove that the maximum range on an inclined plane is attained when the direction of the jet bisects the angle between it and the vertical, and again, that the curve of safety or envelope to all the parabolas formed with a given velocity when the jet is turned in different directions, is a parabola, with the orifice for a focus.

50. Resistance of Pipes.

Apparatus. A $\frac{3}{8}''$ brass tube six feet in length has five holes drilled in it at intervals of exactly a foot, taking care that no burr or roughness remains on the inside. Short pieces of brass tubing are soldered on over them, and long glass tubes are attached by pieces of rubber hose. The whole is mounted on a stand, so that the brass pipe is horizontal, and the glass tubes vertical and a foot apart. Each tube is graduated, or has a paper scale attached, to show the height at which the water stands in it. Water may be passed through the brass pipe at different velocities by connecting it with the hydrant, and regulating the flow by the faucet. To keep the pressure regular, it is better to connect with a separate reservoir, and to measure the velocity, the water may be received in a large graduated vessel.

Experiment. When water flows through the brass pipe it will rise in the glass tubes owing to the friction, and the latter may be very accurately measured by the height of the liquid. On trying the experiment it will be noticed that the top of the liquid columns lie very nearly in a straight line, passing through the open end of the pipe, where of course the pressure is zero. The exact pressure should be measured by the attached scale, and observations of all of them taken for several different heights. A second series of experiments should also be made to determine the velocity corresponding to these heights. In this case the escaping liquid is received in the graduated vessel for a known time, or the time required to fill it is noted, and from this, knowing the volume and cross-section of the pipe, the velocity is readily determined. The results should be represented by curves, first making abscissas distances, and ordinates pressures, and secondly, using velocities as abscissas, and the heights of the liquid in the most distant tube for ordinates. From these curves the laws and coefficients of liquid friction are readily determined.

51. Flow of Liquids through Small Orifices.

Apparatus. A Mariotte's flask is placed about three feet above the table and a rubber tube is connected with its outlet. To this is fastened a brass tube with a perforated screw cap, so arranged that small circles of platinum foil may be inserted, with holes of various sizes. A vertical scale shows the height of the orifice, and a balance serves to measure the quantity of water received.

Experiment. Fill the Mariotte's flask with water. For this purpose it is often convenient to have a third tube, which is closed by a rubber cap, except when the flask is to be filled. It is then opened to allow the air to escape, and water is admitted by one of the other tubes. Raise the orifice so that water is just on the point of flowing out of it, and measure its height. Insert one of the platinum diaphragms and lower it, so that the water shall flow out drop by drop. Collect what escapes during a minute, and weigh it. Lower the orifice and repeat at intervals, until it is as low as possible. Measure also at the point where the drops begin to unite into a continuous stream. For all lower points measure the length of the stream, that is, the distance before it begins to divide into drops.

Compute the coefficient of efflux by means of the usual formula,

$V = mstv = mst\sqrt{2gh}$, hence $m = \frac{V}{st\sqrt{2gh}}$, in which s equals the cross section $= \frac{\pi d^2}{4}$, calling d the diameter of the orifice, t = the time of flow $= 60$, h the head, or the reading first taken minus that corresponding to the given observation, and V is obtained from the weight, remembering that 1 gramme of water $= 1\ cm^3 =$ .061 inches. Finally, construct a curve in which ordinates represent the coefficients m, and abscissas the heads h.

By this simple apparatus, interesting results could be obtained by measuring the flow of various liquids with different pressures and orifices. Their relative viscosity might thus be compared.

52. Capillarity.

Apparatus. In Fig. 42, A is a tall bell-glass set in a glass jar B containing water. C is a glass tube drawn out to a point and connected with A by a rubber tube; it is immersed in a test tube D, containing the liquid to be tried. A may be filled with air by blowing through the bent tube E. Paper scales divided into millimetres are attached to B and D to measure the pressure, and D is supported in such a way that it may be raised or lowered at will.

Experiment. Draw out a piece of glass tubing to a fine point, break off a small piece and grind the end flat so that the orifice shall be circular and smooth. Connect it as at C, by a rubber tube, with the bell-glass A, and fill the latter with air by blowing into E. Raise the test-tube D containing the liquid to be employed, so that the air escaping from C shall bubble up through it. Soon the pressure in A is so far diminished that it becomes insufficient to overcome the resistance opposed to it, the flow will then stop, and the top of the liquid in C will be found to be very much curved. Record the pressure of the air in A which equals the difference in level of the water within and without it. Call it h, and call h' the difference in level within and without C. Repeat this observation several times, either by blowing into E, or by lowering D so that the flow shall recommence. Next remove the tube from the liquid, break off the end, and stick it carefully into

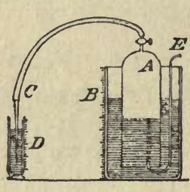


Fig. 42.

a cork. Grind down the end of C until it is again flat, and repeat until observations have been obtained with orifices of five or six different sizes. Now place the cork on the dividing engine, Experiment 21, and measure the diameter of each of the ground ends. This may be obtained with great accuracy by placing the axis of the tube vertical so as to look down through it.

Let s be the specific gravity of the liquid and x the height to which it would tend to rise in the tube, if the bore were the same throughout as at the end. The pressure due to this force will then be sx, and in the same way the pressure due to the column h' will be sh'. Both of these pressures will be in equilibrium with the force h of the water in B. In other words, $h = sx + sh'$, or $x = \frac{h - sh'}{s}$, from which x may be calculated in the various cases. If the liquid in D is water, $s = 1$, and $x = h - h'$. This method of studying capillarity was first proposed by M. Simon, who, however, found that his results did not agree with those obtained by direct measurement. It has, however, the great advantage that the diameter may be obtained with accuracy, even with very minute tubes, and the latter being heated to redness are rendered chemically clean.

53. Plateau's Experiment.

Apparatus. Some of Plateau's soap-bubble mixture, formed by mixing pure oleate of soda with 30 parts of water, and adding two thirds its bulk of glycerine. The oleate is made of olive oil and soda, which is then filtered. Common soap may however be used. Wires are bent into the following forms and soldered at the corners. A tetrahedron with a single wire as a handle, a cube, a circle, two triangles hinged along one side, and two squares, made in the same way, also a small vertical stand arranged so that two circles may be placed on it at any height. To measure the figures obtained, an upright C, Fig. 43, is attached to the table to support a sheet of paper and at a distance of about two feet is a second support, A, in which is a small hole to look through. A third stand, B, serves to hold the wire figures in any desired position.

Experiment. Dip the tetrahedron into the liquid, and on drawing it out, films will be found extending from each of the six edges, and meeting in the centre. This point is a fourth of the distance from each face to the opposite angle. Attach the tetrahedron to B, so that one face shall be nearly horizontal, and one edge perpendicular to the line through ABC. On looking through A it is projected as a triangle on C. Move B, if necessary, so that its lower face shall be projected as a straight line. Attach a piece of paper to C and mark on it the corners of the tetrahedron, also the intersection of the films. Measure on the paper the distance of this point from the top and bottom of the figure. Their ratio should be one to three. Turn the tetrahedron around, and repeat the measurement with one of the other sides. It should be the same for all.

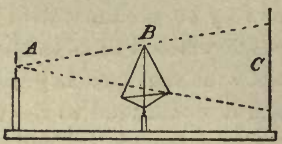


Fig. 43.

A general law of these films is that they are always subjected to tension and continually tend to contract, owing to the molecular attraction of the particles. This may be shown in various ways. Attach a loop of the finest silk thread to the circle of wire. Dip it in the liquid, and a film will be obtained in which the loop will float, irregular in shape and in any position. Break the film inside the loop, and instantly by the contraction of the film around it, it will be drawn out into a perfect circle, leaving of course a hole in the centre. Inclining the circle from side to side the loop moves freely over the film, presenting the curious appearance of a sheet of liquid containing a moveable hole.

Immerse the tetrahedron again in the liquid. The six films pulling equally in opposite directions, hold the centre point in equilibrium. Now break one of the films, and the remainder contracts, forming a curious curved surface drawn towards one side by a single plane film. On breaking this second film, the surfaces again contract and form the warped surface known as the hyperbolic paraboloid.

Immersing the cube in the same way twelve plane surfaces are obtained, meeting in a small square in the centre. This square may be parallel to either face, aud may be made to alter its position by gently blowing, so as in appearance to split it. See how many different figures can be obtained by breaking one or more films, and draw them in your note book. The whole number is twelve, not including a single plane attached to one face only. If the films attached to two opposite parallel sides are broken, a plane is obtained supported between two curved surfaces, the intersections being curved lines. Draw these lines by attaching the cube to *B* and see if they are hyperbolas. Another curious effect is obtained by blowing a small bubble and attaching it to the centre square, when it assumes a cubical form with curved sides; in the same way a four-sided bubble may be formed with the tetrahedron. Similar figures may be obtained with an octahedron, or other figures, but they are more complex.

On dipping the two triangles into the liquid a film forms over both, and on increasing the angle between them a single plane film is found attached to their common side, which is split as they separate. Breaking this film the curve springs back as before, forming a very beautiful hyperbolic paraboloid. This is probably the best way of producing this warped surface, and its properties are well shown by it. Varying the angle between the triangles, its form, or, more strictly, its parameter may be altered at will. Make the angle between the triangles about 30°, and draw the curve of intersection of the plane film with the other, also a section through the centre at right angles to it. Try and determine the form of the first of these curves, and see if it is a circle, parabola or hyperbola. Now break the film and draw the enveloping curves on the same sheet as before, to show how the films have contracted. Do the same with the jointed squares. Place the two circles on their stand near together, blow a bubble and lay it on them. Then draw them apart, and a hyperboloid of revolution of one nappe will be obtained.

54. Pneumatics.

Apparatus. The object of this experiment is to familiarize the student with the ordinary lecture-room apparatus in pneumatics, and is therefore chiefly of value to those who propose to adopt teaching as a profession. The apparatus needed will depend on the objects of each student, but may be made to include almost all the instruments used in a full course of lectures on this branch of physics. The following description, however, applies only to such experiments as could properly be introduced in any common school. The most important instrument is of course the air pump, which need not be of large size, or (for most of these experiments) capable of producing a very high degree of exhaustion. The other apparatus needed is best determined from the following list of experiments, which may be varied almost indefinitely.

Experiment. Place a receiver on the pump-plate, taking care that no dust or grit is retained under the edge, which should be freely supplied with sperm oil, or tallow, to ensure contact. Open communication between the pump and receiver, and close that leading to the outer air. Exhaust, by working the handle of the pump, and see if any leakage takes place around the bottom of the receiver, in which case air bubbles will be seen forcing their way through the oil. The greatest trouble in using the air-pump is to make this joint tight, especially if the plate or receiver is not ground perfectly true. When the exhaustion is nearly complete the pump handle will work freely, until the very end of the stroke, when a slight hissing will be heard, due to the expulsion of the remaining air. For this reason the piston must be moved until it strikes the end of the cylinder each time, and the strokes must be taken steadily, and not too fast. When the air is removed the exterior pressure becomes so great that it is impossible to move the receiver without breaking the glass. On opening communication with the outer air, the latter rushes in, and the receiver is easily removed. To determine the degree of exhaustion, a syphon vacuum gauge may be employed. This consists of a bent glass tube like a syphon barometer, with the closed end only about half a foot in length, and containing mercury, which of course rises to the top. Place it under a receiver and exhaust, when it will be found that as soon as the pressure inside is reduced to less than six inches the mercury begins to fall, until in a perfect vacuum it would stand at the same height in both branches of the tube. Read the difference in level, which in a common pump should not exceed two or three millimetres. If a barometer gauge, or long tube dipping in mercury, is attached to the pump, subtract its reading from that of the standard barometer, and the difference should equal that of the syphon gauge. Place a beaker of water on the pump-plate with a bolt head (or tube with a bulb blown at one end) in it, cover with a receiver and exhaust slowly. The air will now bubble up through the water, owing to its tendency to expand when the outer pressure is removed. If the pump is a very nice one, this experiment, and others requiring water, should be omitted, as the vapor may rust the interior of the pump. On readmitting the air the water will rush up into the bolt-head until but a small bubble of air remains. The ratio of the volume of this bubble to the whole interior of the bolt-head, shows the degree of exhaustion. When nearly all the pressure of the air is removed from the surface, the water bubbles make their appearance in it, due to the dissolved air. Carrying the exhaustion still farther, vapor begins to be formed so rapidly that the water enters into ebullition. This effect is more easily obtained if the water is somewhat warm. Select two tubes about three feet long, and closed at one end, fill one, *B*, with mercury (Experiment 58), the other, *A*, with air, and dip both into a small vessel containing mercury. Cover them with a tall receiver and exhaust. The mercury will descend in *B* until nearly on a level with that in the cistern, the air meanwhile escaping from *A* in bubbles. Readmit the air and the mercury will rise in both tubes, that in *A* being the lowest. Any leakage in the pump is well shown in these experiments, as it will cause the liquid to begin to rise slowly as soon as the pumping stops. To see if the leak is in the pump, or under the receiver, close the connection between them when leaks in the latter only will be perceptible. The great pressure of the air may be shown in various ways. Thus the palm-glass is a cylindrical vessel open at both ends, which is placed on the pump-plate and closed above by the hand; after exhaustion the latter is removed only with difficulty. Replacing the hand by a sheet of rubber, a single stroke of the pump will draw it strongly inwards, and in the same way a tightly stretched bladder may be made to burst with a loud report. In the upward pressure apparatus the air, being withdrawn above a piston, the latter, with a heavy weight attached, is raised by the pressure of the air below. The Magdeburg hemispheres consist of two brass hemispheres, accurately ground together, which require a great force to separate them when the air is withdrawn from the interior. Great care is needed in handling this apparatus as a slight blow will bend the brass sufficiently to cause leakage. Bursting squares are sealed rectangular vessels of glass, which explode when placed under an exhausted receiver. To prevent injury they should be covered with wire gauze, and the orifice leading to the pump, protected by a brass cap and valve. The porosity of wood may be shown by the mercury funnel, in which mercury is driven lengthwise through a piece of wood which passes through the top of a receiver. If a piece of wood held under water is covered with a receiver, and the air exhausted, torrents of bubbles imprisoned in its pores will pour from it. Now on admitting the air the water enters the wood, which becomes water-logged, and no longer floats. The revolving jet is a bent brass tube, like a Barker's Mill, which when placed under a receiver turns rapidly in one direction when the air is exhausted, and in the other when it is readmitted. The effect of the resistance of the air is shown by two fan wheels with vanes set flatwise and edgewise, respectively. If set in motion, the former stops first in air, but both revolve for nearly the same time in a vacuum. The same effect may be shown by a feather and guinea placed in a long glass tube from which the air is removed. They then fall with nearly equal velocity from end to end. An important experiment is the proof of the weight of the air. A glass sphere is weighed when full of air and when exhausted, and the difference gives approximately the required weight. The exact weight is obtained only by an accurate correction for temperature, pressure and moisture. The two following experiments, though properly belonging to other branches of physics, are inserted here for convenience. Both require a very high degree of exhaustion. If a bell is rung in a vacuum, no sound is heard. An electric bell is most convenient for this experiment. It should be carefully supported, so that the sound shall not be transmitted directly to the pump plate. For this purpose it is sometimes hung by threads; a rubber support is also recommended. The experiment is generally more successful, if after exhaustion hydrogen gas is admitted, and the exhaustion repeated. It is, however, almost impossible to destroy all sound. The latent heat of aqueous vapor is well shown by the experiment of freezing water in vacuo. A shallow pan of concentrated sulphuric acid [1] is placed on the pump plate, and on this a wire triangle which supports a flat metallic

[1] In all cases where sulphuric acid is used to absorb moisture in the presence of metallic surfaces, it should be freed from nitric fumes by boiling it for some time with sulphate of ammonia.

dish holding the water to be frozen. The whole is covered with a small receiver, and exhausted quickly. On removing the pressure from its surface the water is rapidly converted into vapor, which is absorbed by the sulphuric acid as fast as formed; the action therefore continues, the latent heat being obtained at the expense of the water, which accordingly cools until it is converted into ice. By substituting for water more volatile substances, as a mixture of solid carbonic acid and ether, and adding protoxide of nitrogen, the most intense cold yet observed is attained. In the best pumps water may be frozen by its own evaporation, without employing acid to absorb its vapor.

55. Mariotte's Law.

Apparatus. A modification of Regnault's apparatus may be made chiefly with steam fittings, as shown in Fig. 44. *C* is a tall mercury gauge formed of glass tubes, connected together by a steam pipe coupling with red sealing-wax. If very high, all the joints must be made like those of Regnault's gauge, but this is unnecessary for pressures below one hundred pounds. *A* and *B* are two similar tubes about three feet long, closed above by "pet-cocks," and attached below by "unions," so that they may be easily removed. *E* is a reservoir made of 3 inch pipe with caps, to hold the mercury, and with an $\frac{1}{8}''$ valve below, so that it may be emptied if necessary. It is filled by removing the plug in the T at *G*. *I* is a small force-pump such as is used in testing gauges, by which water may be drawn from the reservoir *K*, and forced into *E*. The water is allowed to flow back by opening the valve *H*. Remove the plug *G*, and pour into *E* enough mercury to fill *A*, *B* and *C*. Work the pump slowly until *E* is full of water. Then close *G* and expel any air that may remain by working *I* and opening *H* alternately, until no air bubbles rise up through the water in *K*. Scales are attached to *A*, *B* and *C*, and the first two should be carefully calibrated (Experiment 10). *B* may be permanently filled with dry carbonic acid, or other gas.

Experiment. *A* must first be filled with dry air. For this purpose connect it above with a U-tube containing chloride of lime, open the pet-cock and pump up the mercury nearly to the top, thus forcing out the air. Open *H* a very little, and let the mercury slowly descend. The air is thus drawn into *A*, first being dried by passing over the lime. Repeat several times to expel all the moisture that may remain, and finally, when full of dry air, close the pet-cock. Read very carefully the height of the mercury *A*, *B* and *C*, and record in three columns. Work the pump a few times, and take readings at intervals of about 10 inches, until the mercury has nearly reached the top of *AC*. Note the height of the barometer and the readings of *C* and *A*, when the latter is open to the atmosphere, also the height of the mercury in *A* when standing at the same level as in *C*. Write in the 4th and 5th columns the pressure in each case, found by adding to the height of the barometer the difference in level of the mercury columns. In the 6th and 7th columns give the volume of the gas in each case, deduced from the table of calibration of *A* and *B*. Next take the product of the pressure and volume, which would be constant if Mariotte's law were correct, or the volumes inversely as the pressures. Finally, construct curves for the two gases, making abscissas represent these products, and ordinates pressures. These results will be only approximate, owing to the change of temperature the gas undergoes when rarefied or condensed. To diminish this error an interval should be allowed for the gas to attain the temperature of the air of the room, or better, *A* and *B* should be surrounded with a water jacket, the temperature carefully noted, and a correction applied.

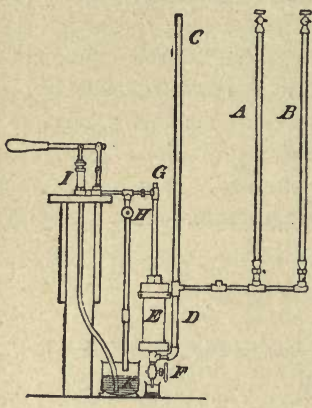


Fig. 44.

Much greater accuracy is attained by the following arrangement. A third tube is employed, in the upper part of which two platinum wires pointing upwards are inserted, the volume above them being determined very accurately by inverting, and weighing the mercury required to fill it. This portion of the tube is then enclosed in a larger one, through which water is kept circulating, and its temperature noted by a thermometer. Fill the tube in the usual way with dry gas, then condense it until the mercury is just on a level with the upper platinum point. The mercury in *C* should now stand near the top of the tube. Open

H, and let the pressure diminish until the mercury in the third tube is exactly on a level with the lower platinum point. Record the pressure in each case. To bring the mercury to the exact level, raise it by the pump and lower by opening *H* until the point is just perceptible by the slight distortion it produces in the image of objects reflected in the surface of the mercury. Let the pressure diminish to a few inches of mercury, let out a little gas and repeat. The law may be tested for pressures less than one atmosphere by merely drawing off the mercury in *C* until it stands below that in the other tube. The ratio of the volumes being constant in this experiment, the ratio of the pressures would be its reciprocal, if Mariotte's law were correct. The deviation may be shown by a curve in which abscissas represent the smaller pressure, and ordinates the ratio of the two pressures.

56. Gas-holder.

Apparatus. A good gas-holder containing three, or better, five cubic feet, with scale attached, the bell properly counterpoised, and most important of all, the friction reduced to a minimum. To calibrate it, the standard tenth of a cubic foot of Experiment 19 is employed, and to measure the pressure some very delicate form of gauge should be provided.

Experiment. The gas-holder consists of a large bell, *AB*, Fig. 45, suspended in a circular trough of water, and counterpoised by the weight *F* attached to a cord passing over the pulley *D*. A curved piece of metal *E*, called the cycloid, is attached to this pulley, and carries a second weight *G*, which acts at a longer and longer arm as the bell rises. It thus compensates for the diminution of weight of the bell when submerged, nd renders the pressure nearly the same whether the holder is full or empty. The proper form for *E* is the involute, a curve in which the perpendicular on the tangent is proportional to the angle described by the radius

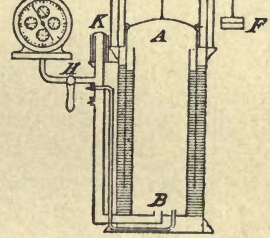


Fig. 45.

vector. The gas is drawn out by a large tube opening into the bottom of the holder at *B*, and covered at *K* by a cap with a water seal. A large tube with a stopcock *H*, also opens out of it, through which the gas may be drawn, or if preferred, through a small tube just below it. To fill the holder with air, remove *K* and press down on *F*, when the bell will rise to the top; it may be kept there by replacing *K*, and closing *H*. If gas is to be used, it must be filled through *H*, but this is a much slower process.

A variety of experiments may be performed with this apparatus. First, test the holder. Fill the bell nearly full of air, depress it a little by the hand, let it return, and record the reading of the scale. Then raise it, let it descend, and again read. Repeat several times, and the difference in the results shows the greatest error due to friction. Do the same with other parts of the scale. Next, calibrate the bell. The same method is employed as in Experiment 48, only the air is collected instead of the water. In this case, after emptying the holder, add one tenth of a cubic foot of air at a time, and read the scale after each addition. Repeat drawing out one tenth of a cubic foot from the holder into the glass standard, and see if the readings are the same as before. The great difficulty in this experiment is the change of volume of the air due to changes of temperature. As the bell rises from the water the adhering moisture evaporates, and sometimes lowers its temperature very rapidly. It is, however, customary to assume that the air is saturated with moisture, and at the same temperature as the water with which it is in contact.

Next, measure the pressure for different parts of the scale to see if the compensation is exact. The gauge is attached to a small tube just below *H*, with an independent outlet from the bell. To save time the pressure may be observed after adding each tenth of a foot in the last experiment. Various forms of gauges may be employed. The simplest is a large U-tube, with a scale attached to each branch. The pressure may thus be determined within a hundredth of an inch. For greater accuracy, a bell glass standing in water may be connected with the holder, and the difference in level of the water within and without it gives the pressure. By using two hook gauges for this purpose great accuracy may be attained. A method in common use is in principle similar to the wheel barometer. A small bell is connected with the interior of the holder, and its rise and fall is measured by a cord passing over a pulley which moves a pointer over a graduated circle. If the pressure increases as the holder rises, the weight *G* should be increased, and the contrary if it diminishes. The pressure to which the gas is subjected is varied by changing the weight *F*. Prove this, and determine the law. Do the same for different parts of the scale. To test the above work, fill the holder with air, and open *H* very slightly, or better, allow the air to escape through a minute aperture. The holder will now slowly descend, and by noting the time the index passes each tenth of a foot mark, a series of numbers is obtained whose first differences would be constant, if the apparatus was perfect. By varying the pressures, the orifices and the kind of gas in the holder, all the laws of the flow of gases may be verified.

57. Gas-meters.

Apparatus. Two gas-meters, one wet and the other dry, both graduated so as to read to thousandths of a foot. They are connected together so that the gas will pursue the following course. It leaves the pipe through a $\frac{1}{8}''$ valve, passes through a T to the wet meter, thence through a second T to the dry meter, and by a stopcock and third T to a fishtail burner. A short piece of pipe is screwed into the open end of each T, which may be closed by a cap, or connected with a gauge formed of a U-tube by a piece of rubber tubing.

Experiment. Gas-meters are of two kinds, wet and dry. The former consists of a cylindrical vessel half full of water, in which is placed a rotary drum with four compartments. As these are filled in turn, the drum revolves, and the amount of gas consumed is measured by the number of revolutions. The dry meter resembles in principle a blacksmith's bellows reversed in such a way that air being forced into the nozzle, the handle moves up and down. The number of strokes is then recorded by clockwork and dials. The wet meter was first used, but is now superseded in houses by the dry meter, owing to the error introduced by any increase or diminution in the amount of water present. The former is, however, still in vogue for experiments, as by it small amounts of gas may be measured with much greater accuracy. To determine the amount of gas which has passed through the meter, subtract the reading at the beginning from that at the end of the experiment, or if the rate of flow is required, take readings at intervals of one minute, as in Experiment 5. Usually in the best meters, one revolution of the large hand equals one tenth of a cubic foot, and the dial being divided into a hundred parts gives thousandths. In meters intended to be used in houses, one revolution of the hands of the three lower dials equals 100,000, 10,000 and 1000 cubic feet, respectively, and a fourth dial is placed above, whose hand makes one revolution for every five cubic feet, and which is used in testing the metre.

The common method of testing a meter is to bring the upper hand to the zero, connect it with a gas-holder, and force air or gas through it until the reading is the same as before. The change of reading of the holder should now be just five feet, and the difference is the error of the meter. This experiment should be repeated two or three times. If the meter reads to thousandths of a foot an additional test is needed to see if the divisions of the large dial correspond to equal quantities of gas. For this purpose, allow the gas to flow very slowly through both meters, turn it off and read them, dividing the thousandths into tenths by the eye. Allow a few thousandths to pass and read again, and so take a series of readings, until two complete revolutions have been made. Represent the results by a residual curve, in which abscissas represent the readings of the large hand of the wet meter, and ordinates the difference between the two enlarged, moving the origin down so as to bring the points on the paper. Two curves are thus obtained, one for each revolution, which should be coincident except for the accidental errors, and their deviation from a straight line shows the inequality of the thousandths, as given by the dry meter.

The next thing to be determined is the loss of pressure due to each meter. Evidently a certain amount of power is necessary to overcome the friction, and this power is obtained at the expense of the pressure of the gas, which therefore leaves the meter with less pressure than it enters it. To measure this, connect the first and second T with the two arms of the gauge, and allow the gas to pass through the meter. The difference in level of the two tubes shows the loss due to the meter. See if this varies with different pressures and with different positions of the revolving drum. By using the second and third T, the dry meter may be tested in the same way.

When a metre is placed between the outlet and the valve by which the gas is turned off, an error is introduced whenever the latter is opened or closed. This is due to the difference of pressure within and without the revolving drum, produced by gas flowing in or out without in some cases moving the hand. To show this, close both the valve and the stopcock near the third T. Open the valve, the gas will rush into both meters, moving the hands a small amount. Close the valve and open the stopcock, when the gas will rush out until the pressure within the meter equals that of the outer air. Take a number of readings, opening them thus alternately. Each meter is here affected by the error caused by the other, and by the intermediate pipe, to eliminate which the valve and stopcock should be placed close to the meter to be tested. The error may then be determined for different pressures and different positions of the hand. This error is not cumulative, and seldom exceeds one or two thousandths of a foot.

To measure the amount of gas consumed by any burner under different pressures, connect it with one of the meters, and attach the gauge to the third T. Turn on the gas and take a five minute observation; that is, take six consecutive readings of the meter at intervals of one minute, also the pressure as given by the gauge. Do the same with several other pressures, and see if the flow is proportional to the square root of the pressure, or if the curve formed by the readings of the gauge and volumes of gas burnt is a parabola. A similar experiment may be performed with an aperture in a plate of platinum, and the height of the flame measured corresponding to different pressures and rates of consumption. A regulator of some form, such as will be described under the photometer, should be introduced to prevent accidental variations in the pressure, if great accuracy is expected in the last experiment. The laws of the efflux of gases may then be tested, or the uniform division of the meter, by allowing the gas to escape very slowly, and seeing if the volume is proportional to the time.

58. Barometer.

Apparatus. Two barometer tubes, one already filled and placed in its cistern, some pure mercury, and a stand by which the height of the mercury column may be measured. This stand may be made in a variety of ways. Thus a half metre steel bar divided into millimetres is fastened to an upright, and a slider attached to it, so that it may be set at any desired height. This slider carries, first a steel point about 40 centimetres below, to determine the height of the mercury in the cistern; secondly, a vernier or a brass plate with a single line cut on it and resting against the steel scale, and finally two index plates of brass between which the tube is placed. The slider is raised or lowered until a thin line of light is just visible between the top of the mercury and the bottom of the index plates, and the reading then taken by the vernier. The tubes are held in position either by rings of brass, or by strips fastened by hinges. A steel rod three or four decimetres long and a tall jar of water, are also needed.

Experiment. First, to find the distance from the steel point to the index plates. This may be done by the cathetometer, or by the second steel rod. Place the jar of water close to the upright, and bring the points of both rod and slider just in contact with the surface of the liquid. Read the vernier, lower the slider until the index plates are just on a level with the top of the steel rod, and read again. The difference added to the length of the rod equals the distance from the index plates to the lower point. The length of the steel rod is found by bringing the index plate first to its upper and then to its lower end.

Now place the filled barometer in its proper position between the index plates, move the slider down until the point just touches its reflection in the mercury, and read the vernier. Raise the slider, until placing the eye on a level with the index plates the light is just cut off between them and the mercury. The difference between these readings is the height through which the slider has been raised, and this added to the distance from the plates to the point gives the height of the mercury column. Now measure the height of the standard barometer placed with the other meteorological instruments. Reduce it to millimetres (1 metre = 39.37 inches), and the difference of the two is the error of the barometer first measured. It is probably due to a little air in the top of the tube.

Now fill the empty tube in the following manner. If the mercury is not perfectly pure, it must be cleaned as described in Experiment 9. Hold the tube with the left hand in an inclined position, the closed end resting on the table. Pour in mercury slowly to within a few inches of the top. To prevent spilling, the stream should be guided by the forefinger and thumb of the left hand held at the opening of the tube. Next close this opening with the finger and raise the closed end so that the bubble of air shall move slowly along the tube. Make it pass from end to end, until all the small adhering air-bubbles are removed. Then fill it full of mercury, and closing the end again invert it, and immerse in the cistern, removing the finger under the surface of the mercury. The latter will now descend in the tube until its height is about 30 inches, leaving a vacuum at the top. Its pressure at the bottom of the tube is then just equal to that of the atmosphere, which by pressing on the outside mercury, supports the inner column. The vacuum at the top is known from its discoverer as the Torricellian vacuum, and is one of the most perfect that can be obtained artificially. To see if any air has entered, incline the tube, and notice if the mercury rises to the top, remaining at the same level throughout, and if when made to oscillate gently it strikes the top with a sharp click; if not, air has entered, and the experiment must be repeated. Next, put this tube in the place of that previously filled, measure its height and determine the error.

Allow a bubble of air to enter one of the barometer tubes (the one in which the error is the greatest), and notice that it increases in volume as it rises until it reaches the top, when it causes a considerable depression of the mercury column. Repeat, until this has fallen eight or ten inches. Then measure the height with care, and suppose it to be an observation taken at the top of a mountain. Compute the height on this supposition by the method given below. The temperature of the air and mercury at the upper station may be assumed equal to that of the thermometer outside the window, and at the lower station to those obtained by direct observation. This work may well be supplemented by a determination of the altitude of a real mountain. For this purpose the pressure of the air must be measured at the top and bottom, either by an aneroid, or more accurately by a mercurial mountain barometer. When going on such an expedition, it is well to take also a hypsometer, and other instruments, so as to determine the dew-point, solar radiation, temperature of the air, etc. These will be described in detail under Meteorological Instruments. On reaching the foot of the mountain, observations should be taken, and again on the return, and the mean of these compared with those taken at the top. Or better, one observer with a barometer is left below to take readings at regular intervals, as every quarter of an hour, during the whole time of the ascent. These are afterwards compared with those taken at the same time at the summit. Of course the lower barometer is compared carefully with the others at the beginning and end of the trip, and the errors corrected. If only one barometer is at hand, and time allows, a series of observations should be taken before and after the ascent, a curve constructed, and the intermediate readings obtained by interpolation. Accuracy is to be expected only from a long series of observations above and below, by which accidental errors are eliminated; any sudden change in the weather, as a thunder-storm, is especially liable to affect the result.

A small aneroid which may be easily carried in the pocket, is often very serviceable in preliminary surveys; by using it in connection with a pedometer, an approximate profile of the country may be constructed. In the same way the variations in the grade of a railway may be determined. The delicacy of these barometers is such that they will show the difference of the level of the different parts of a house, or even the rise and fall of a vessel at sea. For such observations the height is obtained with sufficient accuracy by allowing 87 feet for every tenth of an inch fall of the barometer.

MEASUREMENT OF HEIGHTS BY THE BAROMETER.

On ascending from the surface of the earth, the barometric pressure continually diminishes, and this is due to the fact that being caused by the weight of the superincumbent air, the greater the height the less the load to be borne. The law of diminution is easily deduced by the calculus; call p the pressure in inches at any height H. The decrease of pressure, or $-dp$, in any interval dH, is evidently due to a column of air of this height, whose weight is proportional to p. Hence $-dp = apd H$, $dH = \frac{-dp}{ap}$, or $H = \frac{1}{a} \log p + C$. The constant a equals the pressure due to a column of air of height unity and under pressure unity, or its reciprocal equals 60,300. The elevation E, or difference in height of two points H and H' is therefore $H - H' = 60{,}300\ (\log p' - \log p)$.

In order to obtain the true height by the formula, it is necessary to apply several corrections, of which the most important are the following.

I. *Capillarity.* The effect of this force is to depress the mercury column by an amount dependent on the diameter of the tube. A constant quantity should therefore be added to each reading. Unfortunately this result is modified by the adhesion of the liquid to the tube, which renders this correction uncertain; sometimes, therefore, the height of the meniscus or curved portion is allowed for, but the best way is to use a very large tube, when the effect of capillarity becomes inappreciable.

II. *Temperature of the Mercury.* The standard pressure assumes the temperature to be 0° C.; at higher temperatures the mercury would be lighter, and the pressure less. Let p be the observed height at temperature T, and P the true height with mercury at zero, then $p = P\ (1 + \alpha T)$, or $P = \frac{p}{1 + \alpha T}$, in which α equals the expansion of the mercury per degree. As the scale expands also, allowance must be made for this, which gives $\alpha = .00009$, when the scale is of brass. The temperature T is given by the thermometer attached to the instrument, and this correction should always be applied to a mercurial barometer, but not to an aneroid, when the height is wanted.

III. *Temperature of the Air.* In the above discussion the air also is supposed to be at zero. If warmer it will be lighter, and the elevation greater than that here assumed. Call t and t' the temperatures above and below, and their mean $t'' = \frac{1}{2}\ (t + t')$. The true elevation E' will then equal $E\ (1 + \alpha\ t'')$ in which $\alpha = \frac{1}{273}$ the coefficient of expansion of air. This is the most important correction of all, and should always be applied, or large errors will be introduced.

IV. *Latitude.* Still another correction may be applied when great accuracy is required, owing to the diminution of the force of gravity as we approach the equator. The computed elevation should be multiplied by $(1 + .0026 \cos 2l)$, in which l is the latitude, since the force of gravity varies according to this law.

Introducing these corrections into the formula and reducing, it may be written in the following form,

$$E = 120(\log p - \log p')\ (502 + t + t'),$$

which may be applied directly to observations taken with an aneroid. For a mercurial barometer, p and p', must be corrected, first for capillarity, and then divided by $(1 + .00009\ T)$ and $(1 + .00009\ T')$. The correction for latitude is always small, and becomes 0 at 45°.

59. Bunsen Pump.

Apparatus. Fig. 46 represents a Bunsen filter-pump, such as is used in chemical laboratories. A is a valve in the supply-pipe, by which the water is admitted to the bulb B. From this it withdraws a portion of the air, which passes down the pipe E with the water. The vessel to be exhausted C, is connected with B by the long pipe CB, through which the air is drawn. Above C is placed a U mercury-gauge, and below it a wide tube, designed to prevent the pressure from exceeding a certain amount. A fine hole is made near the bottom of this tube, and it dips into a mercury-cistern D. As the pressure diminishes, the mercury rises in D and falls in the outer vessel until below this hole, and the air rushes in and increases the pressure; by varying the height of the cistern any pressure may be maintained. This device, though excellent in theory, often gives trouble in practice from the jumping of the mercury, unless the tube is large and the hole small. Instead, therefore, two or more valves may be used, or the tube nearly closed, and thus the air admitted so slowly as to keep up the required pressure. If too much water is passed through B it sometimes overflows into C. An arm and stop should therefore be attached to A, so that it cannot be opened too far. The water escaping from E is received in a Florence flask, F, which is fitted with a second tube G, passing nearly to the bottom, while E opens near the top. To measure the amount of water expended, a balance and weights should be provided, or a large graduated vessel.

Experiment. Open A and the water will flow through B, and there encountering the air, will carry it in bubbles through E. If now C is closed, the air will gradually be carried out of B, producing a rarefaction, and the air-bubbles in E will be found to occupy less and less space compared with the water, until the limit of exhaustion is reached, and the tube carries off nothing but water. The diminution in pressure thus obtained should nearly equal that of a column of water of height BE, or if this is made 40 feet, nearly all the air should be withdrawn. The aqueous vapor, however, always remains, and for other reasons the exhaustion is never perfect; it nevertheless forms a very convenient method of producing a partial vacuum.

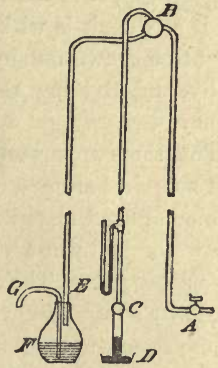


Fig. 46.

To test the working of the pump and its efficiency, the following experiment should be performed. Pour water through G until F is filled up to the end of the tube E. Empty it by blowing through E, collect the water escaping from G, and weigh it. The weight in grammes gives V, the volume in cubic centimetres of the portion of F included between the ends of E and G. Measure the temperature of the water and the height of the barometer. Fill F as before, take C out of the mercury, and open A slightly. A large amount of air and a small amount of water will now enter the flask. Water will flow from G until a volume of air equal to V has entered the flask, and air begins to bubble up through G. Collect the water that has escaped, and weigh it. Calling its volume V', the amount of water brought from B is evidently $V' - V$, while in the same time V centimetres of air have been brought down. Record also the time required to empty F. Repeat the experiment several times with a larger flow of water. Try also the effect of a flow under pressure by connecting the end of C in the mercury, which will then rise in it when the water is turned on, and may be kept at any desired height by raising or lowering D. As in these experiments it takes some time for the mercury to attain its normal level, it is well to connect a third tube with the flask, which may then be filled without disconnecting it from B. It will be seen that the best results are attained when the smallest amount of water is used, but as the exhaustion then takes place very slowly, it is often best to begin with a large flow, and diminish it as the air is withdrawn. The maximum amount of air that might be drawn out by the apparatus may be determined analytically, and dividing the observed amount by this, gives the efficiency.

This same apparatus may also be employed with advantage, to test aneroid barometers. *C* is attached to an air-tight chamber, formed of a tubulated receiver placed on an air-pump plate. When the water is turned on, the air is gradually withdrawn, and the barometer falls. The reading is compared with the true pressure found by subtracting the reading of the U gauge from the height of the standard barometer. Different results will be attained according as the barometer is placed vertically or horizontally, or if the friction is reduced by gently tapping on the instrument. To render the test more complete this experiment should be tried at different temperatures, which is best effected by a water jacket, which may be filled either with hot or cold water.

60. Air-Meter.

Apparatus. An organ bellows, such as is described in the next experiment, and an air-meter, of which a very convenient form is that manufactured by Casella. It consists of a very light fan-wheel, like a wind-mill, with a counter to record the number of revolutions. The vanes are set at such an angle that the divisions of the dial shall represent the number of feet traversed by the air.

Experiment. Work the bellows and allow the air to escape through an orifice, so as to produce a constant current of air. Measure its velocity at intervals of ten inches from the orifice, until it becomes imperceptible. Measure also the velocity on each side of the central line. A spring catch serves to throw the gearing in or out of connection with the fan-wheel. To make an observation, therefore, the meter should be placed in the current, and when it has attained a uniform velocity thrown in gear for exactly one minute. As the hands move only during this time, the difference of readings taken before and after, give the distance traversed, or dividing by 60, the velocity of the air per second.

A table of corrections accompanies the instrument, showing how much should be added or subtracted from the observed readings to get the true distance.

Interesting results may be attained with this instrument on the velocity of the wind, especially during gales, the air currents in buildings from registers, ventilators or doors slightly open. This forms one of the most efficient means of studying the ventilation of large halls and churches.

SOUND.

61. Sirene.

Apparatus. An organ bellows capable of giving a perfectly constant current of air under various pressures. One of the best forms is that made by Cavaillé Coll (sold by König) with regulator attached. If preferred, a large gas-regulator may be attached to any bellows. A set of organ-pipes well tuned, giving the notes of the scale from C_3 to C_4, two or three tuning forks, one giving the French normal pitch, etc. and a sirene. The latter need not be of large size, as good results may be obtained with a single moving disk with one circle of holes.

Experiment. Place the organ-pipe, C_3, in its hole on the bellows, and connect the sirene so that the air shall pass through it. Work the bellows, and the perforated disk will begin to revolve, at first slowly, giving a rustling or humming sound, and then faster, producing a note of low musical pitch. As the speed increases the pitch rises, until it is about that of the pipe. Sound the latter, and increase or diminish the pressure of the air, so that they shall be precisely together. A slight deviation produces beats, that is, an alternate increase and diminution in the intensity of the sound for every vibration gained or lost by the sirene. By a little practice these beats may be made to take place very slowly, or not at all. The wheelwork of the sirene may be thrown in or out of gear with the revolving shaft, so that the hands may or may not register the number of turns of the perforated disk. Throw it out of gear, and read the position of the hands. Bring the two sounds in unison, and keep them together for a minute, during which time the shaft is thrown in gear, and the hands are moving. The difference of the readings before and after, gives the number of turns, and this multiplied by the number of holes in the perforated disk, give the number of complete vibrations. Dividing by 60 gives the number per second. Repeat with the other pipes, and see if this ratio is that given by theory. Do the same with the tuning forks. This is more difficult as they cannot be sounded continuously. The best method of sounding a tuning fork is by means of a violin bow. The latter should be held near the end of the fork, nearly parallel to the two prongs, but touching only one, and drawn with considerable pressure, and not too rapidly To prevent slipping it should be well rubbed with resin.

62. Kundt's Experiment.

Apparatus. Several glass tubes two or three inches in diameter, and six feet long, one open at both ends, the others closed and filled with different gases, and also containing a little lycopodium powder. A number of rods of brass, steel, glass and wood, and a clamp by which they may be held at the centre. Three of the rods should be of the same material, but one of double diameter, the second half the length, of the third. Cloths which may be wet or covered with resin should be provided to set them in vibration, also some lycopodium powder.

Experiment. Place a little lycopodium powder in the open tube, hold it horizontally by the middle, and rub it lengthwise with a wet cloth. A clear musical note of high pitch is at once produced, and the powder arranges itself in about fifteen to twenty groups at regular intervals along the tube. The reason is, that the air in the interior of the tube vibrates with the same rapidity as the glass, but as the velocity of sound in it is much less, the wave-length is less in the same proportion. Hence dividing the length of the tube by the distance apart of the lycopodium groups gives the relative velocity of sound in glass and air, or multiplying this number by 332 gives the velocity of sound in glass in metres.

If the tube is filled with any other gas than air the interval will be proportional to the velocity. Thus knowing the velocity in glass, the velocity in the gas may be obtained. Make this measurement with the other tubes, and see if the law holds that the velocity is inversely proportional to the square root of the density.

This same method may be applied to the accurate determination of the velocity of sound in solids. One of the rods is clamped at the centre, and the end inserted in the open glass tube. The air in the latter is confined by a cork at one end, and a disk somewhat smaller than the tube is attached to the rod. The latter is now set in vibration by a cloth moistened with water, for glass, or covered with resin, for wood or metal. The vibrations of the rod are transmitted to the air, and the heaps of sand formed. In general these will not be clearly defined, because the whole length of the air space is not an exact multiple of half a wave-length. The rod should therefore be moved in or out until the heaps are distinctly marked. The velocity of sound in the rod is then obtained by the following calculation. If L is the length of rod, l the distance between the heaps of lycopodium, and $V = 333\,(1 + .0037\ t)^{\frac{1}{2}}$, the velocity of sound in air at any temperature t, then the velocity in the rod $= \frac{L\,V}{l}$. The temperature t may be taken as equal to that of the room, and measured with a Centigrade thermometer.

63. Melde's Experiment.

Apparatus. A tuning-fork projecting horizontally from a vertical wall, and tuned to give a low note, as C_1. Four weights in the ratio 1, $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, made of brass rods cut to these lengths respectively, some fine silk thread, and a millimetre scale. A piece of brass with a hole in it should be fastened to the end of one prong of the fork, and a fine wire hook attached to the silk to support the weights. A violin bow is also needed to excite the fork, or bette, ran electro-magnetic attachment, by which the vibrations may be maintained continuously.

Experiment. By this apparatus the various laws for the vibrations of cords may be proved. 1st. The time of vibration is proportional to the length. Place the fork so that its two prongs shall lie in the same vertical plane, and suspend the largest weight from it by the silk thread. Sound the fork, as described in Experiment 61, and vary the length of the thread until its time of vibration corresponds with that of the fork. When this is the case it will form a loop or spindle, fixed at the ends and swelling out at the centre through several inches. As this occurs only when the cord is very nearly the right length it may be tuned quite accurately by the eye alone. Make three or four observations in this way, measuring the length in each case with the millimetre scale. Next turn the fork 90°, so that the prongs shall lie in the same horizontal plane. The cord will now make as many vibrations as the fork, while in the former case it made but half as many. This is evident if the relative positions of the prong and cord are compared. When the prong is in its highest position the cord is straight or central. As the prong descends it moves to the right, and as it ascends again becomes central. At the next descent of the prong it moves to the left, becoming central a second time, when the prong has reached the top. It thus makes only one complete vibration, while the fork makes two. Accordingly when the fork is turned around 90°, the cord will be set vibrating exactly twice as fast as before. On trying the experiment it will be found that the new length of cord required will be just one half that in the first case. That is, a double length requires a double time.

2d. The time of vibration is inversely proportional to the square root of the tension. Applying the four weights in succession, the corresponding lengths are proportional to 1, 2, 3 and 4, or since for equal tensions the times are proportional to the lengths, the law is proved.

3d. The time is proportional to the diameter of the cord. A second cord of precisely twice the diameter of the first, may be made by twisting four strands of the former. It will be found that the length must be reduced one half to obtain the same effect as before.

All these laws may also be proved by preparing a string of such a length that it will vibrate as a whole, when the larger weight is applied, then attaching the other weights it divides into 2, 3 or 4 loops, separated by fixed points or nodes, and corresponding to the harmonics of the cord. A second string of double thickness serves to prove the 3d law. A simple proof of the first law may also be obtained by a second fork an octave higher than the other.

64. Acoustic Curves.

Apparatus. In Fig. 47, *A* is a large tuning-fork capable of giving out at least one harmonic besides its fundamental note, and carrying on the end of one of its prongs a piece of sheet brass cut to a point. *B* is a little carriage on which a piece of smoked glass may be laid and drawn under the brass point or style by a cord passing over the pulley *C*. Two weights, *D* and *E*, are attached below, the upper one, *D*, being just equal to the friction of the carriage. Some pieces of glass about three inches by four, are needed, and a gas-burner, by which they may be covered with lampblack. By using the size of glass employed in the lantern for projections, the curves may be thrown on the screen on a greatly enlarged scale.

Experiment. Cover one of the plates of glass with a layer of lampblack by holding it by one corner over the gas-flame, and moving it about so that the coating shall be uniform, and *very thin.* Instead of lampblack, collodion may be used, pouring it on in the usual way, as when taking a photograph. Care must be taken to select such collodion as will give an opaque and very tender film, when results of extreme beauty and delicacy will be obtained. Lay the glass down on the carriage, and raise it so that when passed under the style, the latter will just touch its surface. This may be accomplished by wedges or levelling screws under the glass. Draw *B* back a short distance beyond the style, and release it, when it will begin to move under the action of the two weights *D* and *E*. The length of the cord should be such that when the wagon reaches the style, *E* will touch the floor so that the carriage will move with a uniform motion by its inertia, the friction being just compensated by *D*. The style will accordingly draw a fine unbroken straight line over the glass. Now sound the fork by the violin bow (see Experiment 61), and again pass the carriage under, when the line, instead of being straight, will be marked by sinuosities, one corresponding to each vibration of the fork.

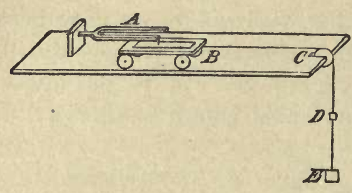


Fig. 47.

Next sound the harmonic, by drawing the bow somewhat more rapidly, and with less pressure than before, at a point about two-thirds of the distance from the end of the prong to the handle. The sound sometimes comes out more readily by lightly touching the intermediate one-third point or node with the finger. A high, clear note is thus produced, and on drawing the carriage back the same distance as before, and letting it again pass under, another curve is obtained, with indentations much nearer together, owing to the greater rapidity of the undulations. Of course the plate is moved sideways a short distance each time, to prevent the curves from overlapping. Produce the fundamental note, and while it is sounding draw the bow so as to give the harmonic, and immediately let go the carriage. A curve is thus obtained, resulting from these two systems of vibrations, and consisting of small sinuosities superimposed on larger ones. Determine their ratio by seeing how many of the former correspond to an exact number of the latter. Write on the lampblack your name and the date, and if all the curves are good, varnish the plates to render them permanent. For this purpose expose the blackened surface to the vapor of boiling alcohol to remove the grease, then holding it by one corner pour amber varnish over it precisely as when varnishing a photographic negative.

To compare the lines with theory, place the glass in a magic lantern, and project an image of it on the screen. If the sun is used as a source of light, it is scarcely necessary to darken the room. Place a sheet of paper so that three or four undulations of the curve of the fundamental note shall fall on it. Trace them carefully with a pencil and an enlarged reproduction of the original is obtained. Draw lines tangent to the waves above and below, and bisect the space between them by a line. It will intersect the curve in points at regular intervals b, any one of which may be taken as the origin of coördinates. If a is the height of the wave, or one half the distance between the two tangent lines, the theoretical equation will be $y = a \sin \frac{\pi x}{b}$. Construct points of this curve by dividing the space between two consecutive intersections of the curve into six equal parts, and lay off vertical distances equal to a multiplied by sin 15°, 30°, 45°, etc., to 180°. These sines have the following values: — sin 15° = .259, sin 30° = .500, sin 45° = .707, sin 60° = .866, sin 75° = .966. Draw a smooth curve through the points thus obtained, and compare it with that given by the forks. To test the combination of the two systems of vibrations is more difficult, but it may be done by taking their equations separately, $y' = a \sin \frac{\pi x}{b}$, and $y'' = a' \sin \frac{\pi x}{b'}$, and adding them so that $y = y' + y'' = a \sin \frac{\pi x}{b} + a' \sin \frac{\pi x'}{b}$.

An immense variety of curves may be obtained by mounting the plate on a second tuning-fork, which is also set vibrating. Different curves are thus obtained, according as the motion of the style is parallel or perpendicular to the vibrations of the plate, also with every change in the interval between the two forks. With this arrangement it is much better to maintain the vibrations of one or both forks continuously by electricity. Better effects are also obtained in this way in Melde's and Lissajous' experiments.

Instead of projecting the curve on the screen it may be measured by the Dividing Engine, Experiment 21, or enlarged by a microscope and drawn by a camera lucida. The length of the waves gives a very delicate test of the uniformity of the motion of the car, a difference of a ten thousandth of a second being easily perceived.

65. Lissajous' Experiment.

Apparatus. Mirrors are attached to the ends of the prongs of two tuning-forks, and the image of a spot of light reflected in them is viewed in a telescope. The planes of the tuning-forks must be perpendicular, that is, one must vibrate in a vertical, the other in a horizontal plane. It is best to have a series of forks with sliding weights, so that all the intervals in the octave may be obtained. A good spot of light is produced by a gas flame shining through a small aperture in a metallic plate, or a mirror may be used to reflect the light of the sky.

Experiment. On looking through the telescope a minute spot of light should be visible. When one of the tuning-forks is sounded the mirror is moved from side to side, carrying the image of the spot with it so rapidly as to make it appear like a horizontal line of light. In the same way the motion of the other fork produces a vertical line. When both sound, a curve is formed, which remains unchanged if the concord is exact, but continually alters if the forks are not a perfect tune. Bring the forks in unison by placing the weights on the corresponding points of each. They are best sounded by a bass-viol bow, drawing it slowly and with pressure over the end of one prong nearly parallel to, but not touching, the other. As the bows soon wear out by the horsehair giving way, a convenient and cheap substitute is made by covering a strip of wood of proper shape with leather, which when rubbed with resin, answers very well.

On sounding both forks, having brought them in unison as above, the point of light is in general converted into an ellipse which, as it is impossible to tune them exactly by the ear, gradually changes into a straight line, then into an ellipse, a circle, an ellipse turned the other way, a straight line and so on. Raise the pitch of one of the forks slightly, by moving the weight towards the handle, and if the changes take place more slowly the unison is more perfect. By trial, first moving the weights one way and then the other, they may be brought in tune with any desired degree of exactness, and far nearer than is possible by the ear alone, as the complete change of the curve from one line to the other denotes that one fork has advanced only a single vibration.

Next make one fork the octave of the other, and a curve is obtained, changing from the parabola to the lemnescata, or figure 8. A simple rule serves to determine the interval in all cases from the curve. Count the number of points where the latter touches the sides of the rectangle bounding it, also the number of points where it touches its top or bottom; the ratio of these two is the interval between the forks. When the curve terminates in either corner this point must be counted as one half on the horizontal, and half on the vertical, bounding line. Thus in Figure 48, both *A* and *B* correspond to the ratio of 2 : 3, or the interval of the fifth.

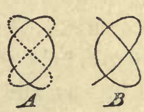


Fig. 48.

The more perfect the concord the more slowly will the curves alter their form, and the simpler the ratio of the number of vibrations the simpler the curve. When the forks are not quite in unison, beats will be heard, and the curve will then be seen to alter its form so as to keep time with them. Next try some other ratios, as $\frac{3}{4}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{1}{3}$, $\frac{1}{4}$; also some more complex curves, as $\frac{5}{8}$, $\frac{8}{9}$, and $\frac{7}{11}$.

66. Chladni's Experiment.

Apparatus. A number of brass plates attached to a stand, a violin bow and some sand. A good series of plates consists of three circles, whose diameters are as 2 : 2 : 1, and their thickness as 1 : 2 : 1. Also three square plates, similarly proportioned.

Experiment. The plates are sounded by touching them at certain points and drawing the bow across their edges, holding it nearly vertical, and moving it slowly and with considerable pressure. A sound is thus produced, and certain lines are formed on the plate called nodal lines, which remain at rest, the other parts vibrating. If sand is sprinkled uniformly over the plate, that on the nodal lines will remain there, the rest being thrown up and down, so that finally it will all collect on these lines. The higher the note the more complex the nodal lines, and the nearer they are together.

Taking first the largest and thinnest circular plate, touch it at any point of the circumference, and bow it at a point about 45° distant. The sand will collect on two lines at right angles. Next bow it at a point 90° distant, and it will divide into six parts. By touching the plate at two points distant 45° with the thumb and middle finger of the left hand, and bowing the point midway between them, a division into eight equal parts is obtained. In the same way 10, 12, or any even number of parts are formed, until the divisions become so small that they cannot be sounded.

Next try the first square plate. The lowest sound this will give is obtained by touching the centre of one side, and bowing the corner. The next note, a fifth above, is produced, when the corner is held and the centre bowed. By altering the position of the fingers and bow, a great variety of figures may be obtained, which may be still further extended by changing the points of support, or the form of the plate. Moreover, among plates of the same shape some seem to give out certain curves more easily than others, owing probably to peculiarities in their internal structure. A square plate generally gives readily, besides the curves described above, one formed of two diagonal lines, and four half ovals on its edges. The pitch is three octaves above that of the diagonal lines alone. Another curve of extreme beauty consists of a circle with eight radial lines, and the intermediate spaces marked by eight half ovals. It will be noticed that the points touched are necessarily at rest, and hence lie on the nodal lines.

By using the three plates of the same form, some of the laws of the vibrations of plates may be proved. 1st. The number of vibrations is proportional to the thickness. Form the same curve on the two plates, of which one is double the thickness of the other, and it will be noticed that the pitch is always an octave higher. 2d. In similar plates of equal thickness, the number of vibrations is inversely as the square of the homologous parts. Hence the small plate gives a note two octaves above the large one of the same thickness.

LIGHT.

67. Photometer for Absorption.

Apparatus. In Fig. 59, A is the source of light, which may be a candle in a spring candle-stick, or a small gas jet. B and C are two mirrors set at such an angle that they will form images of A, just 50 inches apart, and making ABC a right-angled triangle. D is a Bunsen photometer disk, made by placing a circular piece of thick paper in a lathe, and painting all but the centre with the best melted sperm-candle wax. It then possesses the property when placed between two lights, of changing its appearance according as one or the other is the brighter. It is mounted on a slide and carries an index which moves over a graduated scale. F is a screen so placed as to protect D and the eyes of the observer from the direct light of A, while it leaves the scale illuminated so that it can be easily read. A stand with a graduated circle is also provided, on which one or more plates of glass may be set and inclined at any angle to the ray of light AB. Some observers prefer a disk with only the central part covered with wax, and instead of a circular spot use some other form. The great difficulty in these cases is to distribute the wax uniformly, and prevent its accumulating at the edges. Still another method is to punch figures in a sheet of thick paper, and cover both sides of it with tissue paper, taking care that no wrinkles remain. The whole apparatus must be used in a darkened room.

Experiment. The disk D possesses the curious property of appearing bright in the centre when a strong light is in front of it, but dark when the brighter light is behind. When placed between two lights there is therefore a certain position where the spot will disppear, in which case it is so much nearer the fainter light that the illumination on its two sides are equal. Their relative brightness may then be readily

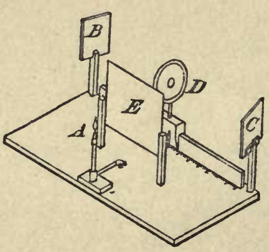


Fig 49.

determined from the law that the intensity is inversely as the square of the distance. Now the two images of A act like two precisely similar lights, any change in one affecting the other equally. By moving D the centre spot may be made either light or dark, and there will be a certain intermediate position in which it will disappear. The exact point of disappearance can be determined only by long practice, noticing that it varies with the position of the eye, and with the two sides of the disk. Find this point as nearly as possible, read the index, move D a short distance, set again and take the mean of several such observations. Compute the probable error in inches, and the result multiplied by 4 gives the error in percentage. Let x be the mean observed reading, or $AB + BD$. Then the distance of the other image of the light equals $DC + CA$, or $50 - x$. Calling B and C their intensities at a distance unity, their intensities at the distance of the disk will be $\frac{B}{x^2}$ and $\frac{C}{(50 - x)^2}$; or since these quantities are equal, their relative intensities $I = \frac{B}{C} = \left(\frac{x}{50 - x}\right)^2$. Next place a piece of plate glass carefully cleaned on the stand between A and B, and at right angles to the line connecting them. To make this adjustment remove D, and place the eye beyond the light. Then turn the stand until A, its reflection in the plate glass, and its reflection in B and C, all lie in the same straight line. Now set the disk as before, record the mean reading, and compute the relative intensities. Increase the number of plates one at a time, and compute the intensity in each case. This number divided by that when no plates were interposed, gives the percentage transmitted.

This same apparatus is well adapted to determine the amount of light transmitted at different angles of incidence, that cut off by ground glass, the effect of the snuff of a candle, or the overhanging portion of the wick, and the comparative brilliancy of the edge and side of a flat flame. In the last case it is only necessary to set the flame so that it shall shine edgewise, first into B and then into C, and compare the position of the disk in the two cases. It will thus be found that the prevalent impression that flame is perfectly transparent, is erroneous.

68. Daylight Photometer.

Apparatus. In Fig. 50, *AB* is a box about six feet long, a foot wide, and a foot and a half high. It may be made of a light wooden frame covered with black paper or cloth. A circular hole about four inches in diameter is cut in the end *B*, and covered with blue glazed paper with the white side out, and made into a Bunsen disk by a drop of melted candle-wax in the centre. A long wooden rod rests on the bottom of the box, and has a standard wax candle, *A*, in a spring candle-stick attached to one end. The distance of the candle from the disk may thus be varied at will, and measured by a scale attached to the rod. The box should be ventilated by suitable holes cut in it, or the air will become so impure that the candle will not burn properly.

Experiment. This instrument is intended to compare the amount of light in different portions of a room, or its brightness at different times. When the candle is placed at a distance from the photometer disk, the latter will appear dark in the centre, while by making *AB* very small, so that the strongest light shall be inside, the centre will be bright. The color of the candle flame being of a reddish tint compared with daylight, is first passed through the blue paper, which thus renders the colors more nearly alike. When the distance of the candle is such that the illumination is equal on both sides of the disk, the spot will nearly disappear, and unity divided by the square of this distance gives a measure of the comparative brightness under various circumstances.

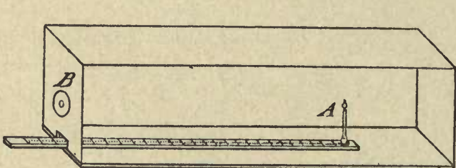


Fig. 50.

An excellent experiment with this instrument is to measure the fading of the light at twilight. Light the candle and place it at such a distance from the disk that the spot shall disappear, as in the last experiment. As the light diminishes, the distance *AB* must be increased. Take readings at intervals of one minute, and construct a curve with ordinates equal to one divided by the square of this distance, and abscissas equal to the time. The amount of light for different distances of the sun below the horizon may be obtained directly from this curve. In the same way the brightness of different parts of the laboratory may be measured, the effect of drawing the window curtains, and the comparative brightness of clear and cloudy days. This apparatus was used during the Total Eclipse of 1870, to measure the amount of light during totality, possessing the advantage that on returning, the precise degree of darkness could be reproduced artificially. Comparisons may also be made with moonlight, the light of the aurora or other similar sources of light.

69. Bunsen Photometer.

Apparatus. A photometer room forms a most valuable addition to a Physical Laboratory, both on account of the great variety of original investigations which may easily be conducted in it by students, and also owing to the practical value of the instrument, and the excellent training it affords in the use of various forms of gas apparatus. If a separate room cannot be obtained, a part of the laboratory may be partitioned off by paper or cloth screens blackened on the interior, so as to leave a space about twelve feet long by five wide and eight feet high, which should be nearly dark, and supplied with some means of ventilation. In this is a table ten feet long, a foot and a half wide, and three high, and over its centre, at a height of five feet from the floor, the photometer bar, *AB*, Fig. 51, is placed. The latter is 100 inches in length, and divided on one side into inches and tenths, and on the other into candle powers. To make this graduation, calling x the distance from one end of the bar in inches, and C the corresponding candle power, we have, as will be seen below, $x^2 : (100 - x)^2 = C : 1$, or $x = 100 \frac{\sqrt{C}}{1 + \sqrt{C}}$. By making $C = 1, 2, 3$, etc., the bar may be graduated as desired. At one end of the bar is placed a sperm candle, *A*, supported in a balance for determining its loss of weight as it burns. The best form is that invented by Prof. F. E. Stimpson, on the principle of the bent-lever balance, in which the motion of a long arm over a scale shows the number of grains consumed.

At the other end of the bar, gas is admitted, and its brightness when burned compared with that of the candle. The pipe supplying the gas passes through the meter *F*, which is of the form known as the wet meter, and indicates the volume to one thousandth of a foot. To read it, a separate burner *E* is provided, supplied with gas, which does not pass through the meter. Of course it is turned down when setting the disk. The gas passes from the meter to the regulator *G*, by which the pressure is rendered perfectly constant. This consists of a bell resting in water, like a gas-holder, with a long conical rod attached to its centre, which, when raised, cuts off the supply of gas. Whenever the pressure becomes too great, the bell rises and reduces the flow of gas, while too small a pressure makes the bell descend and admit more gas. Beyond the regulator a ┬ is inserted, and connected with a gauge *I*, which gives the pressure. In its simplest form this is a bent tube containing water, which should be of considerable size, if accuracy is required. Sometimes a floating bell is used, which rises and falls as the pressure varies, and moves a long index over a graduated scale. The gas next passes to the burner *B*, first traversing one or two stopcocks, *H*, to regulate the quantity consumed. A variety of burners should be procured, which may be used in turn and compared.

A slide *G* is placed on the photometer bar, carrying a Bunsen photometer disk (Experiment 67). When this is placed between two lights, if the brightest is in front, the small circle looks light on a dark ground, if the brightest is behind, it appears dark.

A clock *D*, marking seconds, is also needed in this room, the best form striking a bell at the beginning of each minute, also five seconds before it, and having what is called a centre seconds' hand. It is often convenient to have a separate gas-pipe, meter and burner at *A*, the candle-end of the photometer bar, and to have the latter arranged so that it can be swung horizontally to it. In fact, in almost every new research some change of arrangement will be found desirable, and the apparatus should therefore not be fixed, but arranged so that the connections can be easily altered.

Experiment. To measure the candle power of burning gas. The law in the State of Massachusetts requires that the gas fur-

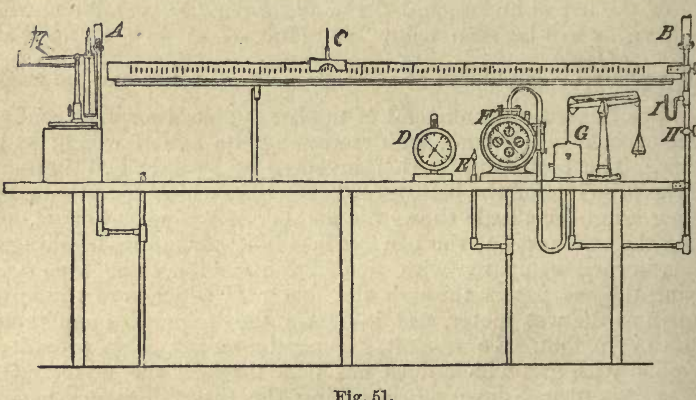


Fig. 51.

nished, when burnt at the rate of five feet per hour, under the most favorable circumstances, shall give a light at least equal to twelve sperm candles (6 to the pound), when consuming 120 grains per hour. To make the experiment the gas and candle are burnt at opposite ends of the photometer bar, their relative intensities compared, and the consumption of each measured. The amount of wax burnt is measured by the candle-balance *A*. This consists of a sort of steelyard, with a light weight or rider *K*, moving over its longer arm, which is divided so as to give grains. The centre of gravity of the beam is at such a distance from the point of suspension that the sensibility shall not be very great, and an index is attached, which moves over a scale, each of whose divisions corresponds to a change in weight of one grain.

Attach the candle to the shorter end of the beam and light it; set the rider at zero, and place weights in the scale-pan, until that end of the balance is somewhat the heaviest. Now as the candle burns it becomes lighter, and soon begins to rise, its diminution grain by grain being shown by the index moving over the scale. Light the gas also at the other end of the beam, and place the photometer-disk on the bar ready for use. Precisely at the beginning of a minute, as given by the clock, read the gas-meter, recording the feet and thousandths, also the position of the index of the candle-balance. The observations commonly extend over five minutes, and in this time 10 grains of wax should be consumed; set the rider therefore at 10, and if the candle is burning at the standard rate, the position of the index at the end of that time will be the same as at the beginning, if not, the difference shows the correction to be applied.

Next measure the intensity of the two lights by the photometer disk. As stated above, this possesses the property of appearing light on a dark background, or the contrary, according as the brightest light is in front of, or behind it. By moving it backwards and forwards therefore, a point will be found where the spot will disappear almost completely, owing to the equality of the two lights. Read its position by the graduation and record, then move it, and set again several times. At the end of the minute read the meter, and then take some more readings of the disk. Try also setting the disk so that the spot shall be first slightly brighter, and then equally darker than the adjacent paper. This is called taking limits, and the mean gives the true reading. Proceed in this way for five minutes, reading the meter at the end of each minute, and taking two or three intermediate settings of the disk. At the end of the time read the index of the candle-balance also.

From these data the candle power may be computed as follows. The consumption of the candle is obtained by subtracting the first reading of the index from the last, and adding the difference to 10. This gives the consumption in 5 minutes, and multiplying it by twelve gives C, the number of grains per hour. It is, however, safer to extend the reading of the candle-balance to a longer time, as fifteen or twenty minutes, to diminish the errors. The consumption of gas per minute is obtained by subtracting each reading of the meter from that which follows it, and multiplying by 60 gives G, the rate per hour. Call I the ratio of the two lights, as given by the mean of the readings of the scale attached to the bar, and apply the following corrections. First, for rate of candle, it is assumed that the light is proportional to the consumption. Hence the corrected candle power $L' : L = C : 120$, $L' = L\frac{C}{120}$. Again it is assumed that the light of the gas is proportional to its consumption, or to $\frac{G}{5}$, and dividing by this fraction gives what would be the candle power if just 5 feet were burned, or the true candle power $L'' = L'\frac{5}{G}$; hence $L'' = L \cdot \frac{C}{120} \cdot \frac{5}{G}$.

This example serves to show how the photometer is ordinarily used, but it may be applied to a great variety of investigations. For instance, different burners may be compared, or a single burner under varying consumption. The amount of light cut off by plain and ground glass at various angles may be measured, and the effect of changes in moisture, in temperature, or in barometric pressure studied.

70. Law of Reflection.

Apparatus. In Fig. 52 a circle divided into degrees is attached to a stand, and carries two arms with verniers, or simple pointers, C and D. The first is attached to a centre plate, which carries a vertical mirror placed at right angles to BC. This mirror is silvered on its front surface, or may be made of blackened glass, and a vertical line is ruled on it, which is brought to coincide exactly with the centre of the circle. A vertical rod or needle is attached to *D*, whose reflection in *B* is to be observed at different angles of incidence. *A* is a piece of brass with a small hole pierced in it to look through.

Experiment. Bring *C* in line with *B* and *A*, and turn it so that on looking through the latter the reflection of the hole may be brought to the centre of the mirror and bisected by the line marked on it. The reading of the index *C* gives the zero, or starting point. This observation should be repeated two or three times, dividing the degrees into tenths by the eye. Turn *C* a few degrees and bring *D* into such a position that the reflection of its needle shall coincide with the line on *C*. Now the difference in reading of *D* and *C* will equal the angle of incidence, and the difference between the reading of *C* and the zero equals the angle of reflection. By the law of reflection these two angles should be equal. Repeat this observation with different parts of the graduated circle, at intervals of about fifteen or twenty degrees. Small deviations from the law serve well to exemplify the different kinds of errors of observations. Thus if the needle is not exactly on the line connecting its index with *C*, a constant error will be introduced. If the mirror is not exactly over the centre of the circle, the difference will vary in different parts of the circle, causing a periodic error. If the differences between the angles of incidence and reflection are sometimes positive and sometimes negative, they are probably due to accidental errors, such as errors in graduation, in reading, unequal fitting of the parts, etc. Finally, if a single observation gives a large error, it is probably due to a mistake, or totally erroneous reading.

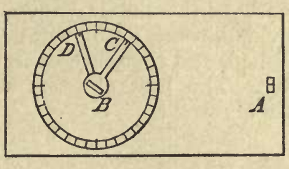


Fig. 52.

71. Angles of Crystals.

Apparatus. The instrument most commonly employed to measure the angles of crystals is Wollaston's reflecting goniometer, represented in Fig. 53. *A* is a vertical circle divided into degrees, and turned by a milled head *B* through any given angle, which is measured by a vernier *C*. A second milled head *D* is attached to a rod passing through the axis of this circle with friction. The crystal is fastened by wax to a small brass plate *E*, bent at right angles and resting in the split end of the pin *F*. By this it may be turned horizontally, and the joint *G* gives a vertical motion. The whole is mounted on a stand, which should be placed on a table opposite the window, and ten or twelve feet from it. A spring stop is attached to the stand, and a pin placed in the graduated circle, so that when the latter is turned forward it cannot pass the 0°, or 180° mark, but may be brought by the milled head exactly to this point. Several crystals to be measured should be provided, some, as quartz, galena, alum or salt, well formed and polished, and therefore easily measured, and others of greater difficulty. The best material with which to attach them to *E* is a little beeswax.

Experiment. The crystal must be fastened to the stand in such a way that the edge to be measured shall lie exactly in the axis of the instrument prolonged, and the main difficulty in the experiment is to make this adjustment with accuracy. Attach the crystal to the plate *E* by a little piece of wax, and adjust the edge as nearly as possible by the eye, turning it horizontally by the pin *F*, and vertically around the joint *G*. It is thus brought parallel to the axis, and may be made to coincide with it by sliding the plate in the pin *F*. Select now two parallel lines, one of which may be a bar of the window, and the other the further edge of the table, or a line ruled on paper, and set the axis of the instrument parallel to them. On bringing the eye near the crystal an image of the window will be seen reflected in one of its faces, and by turning either milled head the image of the bar may be brought to coincide with the second line. If they are not parallel it shows that the face is not parallel to the axis of the instrument, and the crystal must be moved. Do the same with the other face to be measured, and when both images are parallel to the line on the table, both faces, and consequently their intersection, are parallel to the axis. This adjustment is most readily made by placing one face as nearly as possible perpendicular to the pin *F*, when the image in this face may be rendered parallel by turning *G*, that in the other by turning *F*.

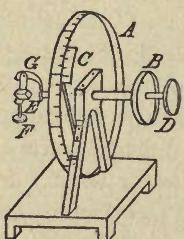


Fig. 53.

Now turn B until the circle stops at 180°, and turn D until the image in the further face of the crystal coincides exactly with the line on the table. Then turn B in the other direction until the second image coincides, when the reading of the vernier will give the correct angle. Evidently the two faces are in turn brought into exactly the same position, and the angle between them equals 180° minus the amount through which the circle has been turned. It is sometimes more accurate, though a little more troublesome, to turn the crystal into any position by D, and bring first one image to coincide, and then the other. 180° minus the difference in the readings of the vernier give the required angle. Try this with different parts of the circle. Remove the crystal, attach it a second time to E, and see if the same result is attained as before. Repeat until readings are obtained differing from each other but a few minutes. Also measure some of the crystals less highly polished. An excellent test of the work is to measure the angles completely around a crystal, and see if their sum equals $180°(n - 2)$, in which n is their number.

72. Angle of Prisms.

Apparatus. One of the most valuable instruments in a Physical Laboratory is the Optical Circle, or Babinet's goniometer. This instrument may be used as a goniometer for measuring the angles of crystals, to find the index of refraction of liquids or solids, to study dispersion, or, as a spectrometer, to measure wavelengths. It is therefore often desirable to duplicate it, or perhaps better, to procure one large and very accurate instrument, and others of smaller size for work requiring less precision.

This instrument, Fig. 54, consists of a graduated circle on a stand, with two telescopes, A and B, attached to it. A is the collimator, or a telescope in which the eye-piece is replaced by a fine slit, whose width may be varied by a screw resting against a spring, and whose distance from the object-glass may be altered by a sliding tube with a rack and pinion. This telescope is attached permanently to the stand, while B, which is a common telescope with cross hairs in its focus, is fastened to an arm revolving around the centre of the graduated circle. It may be held in any desired position by a clamp D, moved slowly by a tangent screw E, and the angle

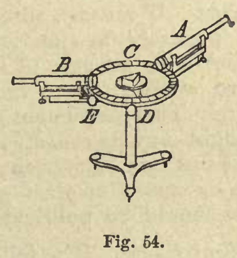


Fig. 54.

through which it has been turned, accurately measured by a vernier. To eliminate errors of eccentricity a second vernier is sometimes placed opposite the first, in which case the mean of their readings is always employed. For great accuracy a spider-line micrometer should be attached to B to measure small angles, as will be described more in detail in Experiment 77. It is often convenient to have both telescopes mounted on conical bearings so that they may be turned away from the centre of the circle when desired. They should also be supported in such a way that one end of each may be raised or lowered a little, so as to bring their axes perpendicular to that of the instrument. This is most readily accomplished by placing an adjusting screw under one of the Y's carrying them. C is a small circular stand on which prisms may be placed, and which may be turned around the centre of the circle and clamped in any position. Sometimes an arm and vernier is attached to measure its angular motion, but this is not absolutely necessary. Its principal use is to measure the angle of crystals, and by it the law of reflection may also be proved with great accuracy. The graduated circle is sometimes made to revolve, and the angle measured by one or more fixed verniers.

The whole is commonly mounted on a tripod with levelling screws, as shown in the figure. These are ornamental rather than useful, however, as in common experiments it makes no difference, except in appearance, if the circle is not properly levelled. In any case, except to raise or lower the instrument, only two screws are needed, and the third may be replaced by a fixed point. In this, as in all instruments mounted on three legs, the best form of support is that represented in Fig. 55. One leg rests in a conical hole A, a second in a wedge-shaped groove B, and the third on a plane surface C. A fixes the position of the tripod, which is prevented from turning by the groove B, while if the three legs change their relative positions, B can slide back and forth in its groove, and C move freely over the plane surface. If instead, three conical holes were used, and these were not precisely in the right position, or the distance of the legs varied with changes of temperature, the whole instrument might be so strained as to introduce serious errors in the graduated circle. This instrument should be placed near the window so that sunlight may be reflected through it by means of a mirror, or if preferred, the light from an Argand or Bunsen burner employed. One or more flint glass prisms are also needed, all three of whose faces should be polished and inclined at angles of 60°.

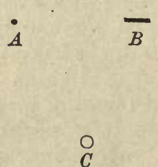


Fig. 55.

Experiment. The following adjustment must always be made when the optical circle is used. Draw out the eye-piece of B,

Fig. 54, until the cross-hairs are seen with perfect distinctness. Then turn the telescope towards some distant object and focus it, moving both eye-piece and cross-hairs. Now both the object and cross-hairs should be perfectly distinct, and not change their relative positions as the eye is moved from side to side so as to look through different portions of the eye-lens. Sometimes the objective alone moves, and sometimes the distance is permanently fixed, so that it is in adjustment for parallel rays. Now turn the two telescopes towards each other and illuminate the slit either by placing an Argand burner behind it, or reflecting the light of the sky through it by means of a mirror. On looking through the observing telescope an image of the slit will now be visible. Focus it, moving it towards or from its objective, when its distance will equal the principal focal distance of the collimator, and the beam of light between the two telescopes will be parallel, or as if coming from a slit placed at a very great distance. Bring the image of the slit to coincide exactly with the vertical cross-hairs in B by the tangent screw, first clamping the telescope. If it is not vertical the slit may be turned, and if it is too high or too low it should be brought to the centre of the field by raising or lowering one end of one of the telescopes, as described above. Having rendered the coincidence exact, read the vernier and repeat the setting two or three times, as it gives the zero from which most of the following measurements are made.

To measure the angle of a prism, stand it on the centre-plate with its edges vertical, and with the faces whose angle is to be determined about equally inclined to the axis of the collimator. To eliminate parallax in case the telescopes have not been accurately focussed for parallel rays, it is better to place the edge of the prism over the centre of the graduated circle. To prevent motion of the prism when B is turned, C should be clamped. Let ABC, Fig. 56, represent the prism whose angle A is to be measured, and DD' the axis of the collimator prolonged. Turn the observing telescope into the position AF, when an image of the slit will be seen on looking through. Bring it to coincide

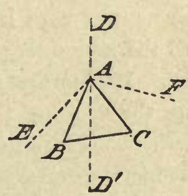


Fig. 56.

with the cross-hairs by the tangent-screw, first clamping the telescope, and read the vernier. Then turn the telescope into the position AE and set again. The difference in the readings divided by two, equals the angle of the prism. For $D'AC$ equals $90°$ — the angle of incidence, and FAC, $90°$ — the angle of reflection; hence they are equal. In the same way, $EAB = D'AB$, or $FAC + EAB = BAC$, and $FAE = 2BAC$. Move the prism a little, repeat the measurement, and see if the same result is obtained as before. Determine in the same way the three angles of the prism, and their sum should equal 180°. If either of the reflected images of the slit is too high or too low, the base of the prism is not perpendicular to the edges. In this case it must be adjusted by placing pieces of paper, or tinfoil, under one or two of its corners, until both images are in the centre. If either is out of focus when the telescopes have been adjusted for parallel rays the reflecting surface is curved instead of plane, while a distortion of the image shows that the surface is irregular. In either case, an accurate measurement is impossible, since the angle will vary for different parts of each face.

By the plan just described, the angle of a prism may be found if, as is often the case, the centre plate has no vernier attached to it. With such a vernier, however, the angle may be determined more readily, as follows. Set the telescopes nearly at right angles, and stand the prism on the centre-plate, as before, with its faces vertical, and the edge to be measured over the axis of the instrument. Turn the centre-plate until one of the faces is equally inclined to the axes of both telescopes, when the image of the slit reflected in this face will be seen in the field on looking through the observing telescope. Bring it to coincide with the cross-hairs by the clamp and tangent-screw, and read the vernier. Turn the centre-plate, taking great care not to disturb the position of the prism on it, until the image reflected in the other face coincides with the cross-hairs. 180° minus the difference in the readings of the vernier gives the angle of the prism. Repeat as before, and also measure the three angles and see if their sum equals 180°.

When a vernier is attached to the centre-plate this instrument serves to prove the law of reflection with great exactness. For this purpose it is only necessary to turn the centre-plate into various positions, bring the reflection of the slit to coincide with the cross-hairs of the observing telescope, and read the verniers attached to each; or in fact, to repeat Experiment 70, replacing the sight-hole by the slit and collimator, and the needle by the observing telescope.

73. Law of Refraction. I.

Apparatus. In Fig. 57, *DBCG* is a tank, like that of an aquarium, with the side *BD* of glass. Two horizontal scales are attached to *CG*, one over the other, and the tank filled so that one shall be seen above, the other below, the liquid. *A* is a plate of brass with a vertical slit in it, larger above, and tapering to a point. It is used as a sight, and is placed at a distance *AB* equal to *BC*. A small plumb-line may be hung in front of *DB* to serve as an index. A tank without glass sides may be employed instead, by regarding Fig. 57 as a vertical instead of a horizontal section, and placing one scale at the surface of the water, *BD*, the other at the bottom, *CG*. The divisions of the upper scale should then be one half those of the lower.

Experiment. On looking through *A* the lower scale will be seen through the water, the upper through the air only. The divisions of the former will therefore, by refraction, appear larger than those of the other, and from the amount of this increase the law of refraction may be deduced. Placing the plumb-line at *D*, and looking through *A*, it will be seen projected on the upper scale at *G*, but on the lower, owing to the bending of the ray at the surface *BD*, at *F*. If placed at *B*, however, the reading on both scales will be the same, since the incidence being normal there is no bending of the ray. To find this point, read both scales, and if the reading of the upper scale is the greatest, move *B* to the right, otherwise to the left, until both read alike. The object of the varying width of the slit is to read approximately through the upper part, and then lowering the eye to eliminate parallax, and read more exactly by the lower portion. Move the plumb-line a short distance, read both scales again, and thus take ten or fifteen readings between *B* and *D*.

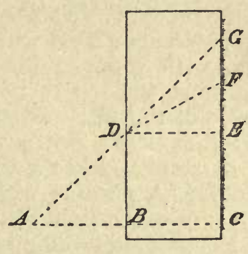


Fig. 57.

Now from these readings to prove that the ratio of the sines of the angles of incidence and refraction is always constant and equal to the index of refraction. In the figure, the angle of incidence equals 90° — ADB, and the angle of refraction EDF. To find these angles, subtract the reading of C from that of G, which gives GC, and dividing by two gives EG, or CE, since ABD equals DEG. In the same way, subtracting the reading of C from that of F gives CF, and subtracting CE, found above, gives FE. Dividing EG and EF by DE gives the tangents of the angles of incidence and reflection, from which these angles may be found. The ratio of their sines, or the difference of their logarithmic sines, should then be constant, and give the index of refraction. This in the case of water equals 1.33. If preferred, AB need not equal BC, but it is more convenient to have them both equal to some simple number, as 10 inches.

74. Law of Refraction. II.

Apparatus. The instrument represented in Fig. 52 may, by a slight change, be employed to prove the law of refraction. The graduated circle is mounted vertically, the needle D replaced by a narrow slit, the mirror B removed, and a test-tube attached to the index C. This test-tube is held by a strip of brass, whose top is just on a level with the centre of the circle. If, then, it is filled with water, so that the bottom of the meniscus is just above the brass, the top of the water will be just on a level with the centre of the circle, even if the tube is inclined. To mark the direction of the ray in the liquid, two diaphragms with slits in them are placed in the tube, one at the bottom, the other in the middle. A piece of white paper, or a mirror, should be placed below to reflect light up through the tube, and the whole should be mounted on levelling-screws, and placed in a good light opposite the window.

Experiment. By the following method, the law of refraction, that the ratio of the sines of the angles of incidence and refraction is a constant, may be proved more directly than in the last experiment. Set the index C at 90°, so that the test-tube shall be vertical, and move the other index carrying the slit, to 270°. The three slits will now be in the same vertical line, and on looking through the upper one, light will be seen through the other two. If not, they must be brought into this position by moving the test-tube. Fill the latter with water until light is just visible above the brass strip. If now the test-tube is inclined by moving its index, the other index must be moved by a larger amount to bring the three slits again apparently in line, owing to the refraction at the surface of the water. And the angles through which these indices have been moved will equal the angles of refraction and incidence, respectively. Before making the measurement, however, the line connecting the indices in their first position must be brought at right angles to the surface of the water. For this purpose turn the upper index 70° or 80°, or as far as readings can be conveniently taken, and turn the lower index until light passes through the three slits. Read its position and turn each index as much on the other side. If light is again visible through the slits, no further correction is necessary. If not, turn the levelling screws through one half the distance required to bring them apparently in line. By repeating this correction the adjustment may be made exact. Then take a number of readings of the two indices, moving the upper one a few degrees, and turning the lower one until light is visible through the slits. Subtracting from these readings 90° and 270°, gives the angles of incidence and refraction. The difference of the logarithm of their natural sines will equal the logarithm of the index of refraction.

75. Index of Refraction.

Apparatus. The instrument devised by Wollaston to measure the index of refraction is represented in Fig. 58. A cube or right-angled prism of glass A, rests on a plate of glass in which a slight depression has been ground. The system of jointed bars BC, CE and DF, is attached to this, so that when C is raised, F and B slide towards E. BC is exactly 10 inches long, and carries two sights through which A may be viewed. EC equals 10 inches multiplied by the index of refraction of the prism, and $DC = DE = DF$; hence E is always vertically under C, because if a circle is described with centre D and radius DC, CFE will be a right-angle, being inscribed in a semicircle. EF is divided into inches, but the graduation need extend only from about 12 to $15\frac{1}{2}$ inches, if the index of refraction is 1.55. Bottles containing several liquids to be measured are also needed, as water, alcohol, turpentine and various oils, and some solids with polished surfaces, as mica, quartz, and marble.

Experiment. Place a drop of water in the hollow under the prism and raise C. On looking through the sights the spot where the prism rests on the drop is at first bright, but after passing a certain position, becomes dark. The bounding line is marked by colors, and bringing it into the middle of the field of view and reading F should give 13.35, which divided by 10, or 1.335, is the index of refraction of water. The explanation is, that when C is low, total reflection takes place, and the spot appears bright; but when raised the light is mostly transmitted. The colored line appears at the angle of total reflection, in which case the sine of the angle of incidence equals the ratio of the indices of refraction of the two media. Call n and n' the indices of the glass and given liquid, i the angle of incidence of the ray through the sights upon the prism, r its angle of refraction, and $90° - r$ its angle of incidence on the reflecting surface of glass and liquid. Then being at the angle of total reflection, $\sin(90° - r) = \cos r = \frac{n'}{n}$, or $n' = n \cos r$, and the problem is to prove that this equals EF, calling CB, or 10 inches, equal to unity. Now $\sin CBE : \sin CEB = CE : CB = n : 1$. But $CBE = i$, hence $CEB = r$. Again, since F is vertically under C, $EF = CE \cos CEF = n \cos r$, or equals n'.

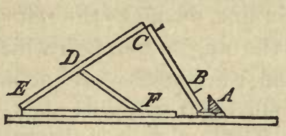


Fig. 58.

Wipe the drop of water carefully from under A, and replace it by other liquids in turn. Next measure the indices of one or more solids, by cementing them to the glass by a drop of some liquid of higher index, as balsam of tolu.

76. Chemical Spectroscope.

Apparatus. A common chemical spectroscope with one prism, and a photographed scale to measure the position of the lines. Two Bunsen burners, an Argand burner, some platinum wires sealed into the ends of glass tubes, and two stands to hold them a little lower than the slit of the spectroscope. A dozen small vials are set in a stand formed by boring holes in a block of wood, and filled with the substances to be tested. Part contain salts of sodium, lithium, strontium, calcium, barium, thallium, etc., and the remainder labelled A, B, C, etc., contain mixtures of these substances. In Fig. 59, the light enters the instrument through a slit in the end of the tube B. At the other end of this tube is a lens, whose focus equals the length of the tube, so that the rays emerging from it are parallel, that is, the same effect is produced as if the slit were placed at an infinite distance. The width of the slit is varied by means of a screw acting against a spring, so that more or less light may be admitted. The rays next encounter the prism *D*, by which they are refracted, the different colors being bent unequally, and then enter the observing telescope *A*. A series of images of the slit are thus produced, one for each color, forming a continuous band of colored light, red at one end, and violet at the other. To measure the position of the different parts of this spectrum, a third tube, *C*, is employed, which carries at its outer end a fine scale photographed on glass, and the rays from it are rendered parallel by a lens, as in the case of *B*. *C* is set at such an angle that the image of the scale reflected in the face of the prism is visible through the observing telescope *A*, at the same time as the spectrum. To exclude the stray light, *D* must either be enclosed in a box, or covered with a black cloth.

Experiment. Turn *B* towards the window, or better, reflect a ray of sunlight through it, and nearly close the slit. A brilliant band of color becomes visible through *A*, red and yellow at one end, and blue and violet at the other. Slide the eye-piece of *A* in or out, until the edges are sharply marked, when fine lines will be seen at right angles to its length. These must be focussed with care, noticing that with a wide slit they disappear entirely, while with a very narrow one they are obscured by other lines at right angles to them, due to irregularities in the slit, or dust on its edges. Light the Argand burner and place it near *C*, drawing the scale in or out, until its image is distinctly visible through *A*. This adjustment is aided by closing the slit, or covering it up. Both the lines and scale should now be distinctly visible, and the position of the former may be accurately determined by the latter. Record in this way the position of a number of the more prominent lines in the solar spectrum.

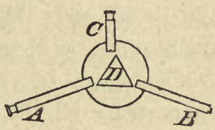


Fig. 59.

Turn the slit away from the window, so that the field of view shall be dark, and light one of the Bunsen burners, placing it opposite the slit, and three or four inches distant. Heat one of the platinum wires in it, until it ceases to color the flame. Then dip it in the vial containing soda, and place it on its stand in the flame. On looking through *A*, a brilliant yellow line is visible, which, with a more powerful instrument, is seen to be double, or to consist of two fine lines very near together. This line is very characteristic, and by it an almost infinitesimal amount of soda may be detected. In fact, it shows itself in all ordinary substances. Record the position of this line, burn the wire clean, and repeat with lithia and the other substances. Most of them give several lines, which sometimes become more visible after the wire has remained in the flame for some time. Thus strontia gives a blue line, a strongly marked orange line, and six red lines, all of whose positions should be accurately recorded. To save time, it is best to use two wires, observing with one, while the other is being cleaned by heating it with the second Bunsen burner. It may sometimes be cleaned more quickly by heating it to redness, and dipping it in cold water, or by crushing the bead formed with a pair of flat-nosed pliers; but care must then be taken not to break the wire. If the salt will not adhere to the wire, the latter may be moistened with distilled water, or a loop made in its end. Having measured, and become familiar with these spectra, try some of the contents of vial *A*. See first if its ingredients can be recognized from its spectrum by the eye, and then measure all the lines, and compare with the measurements previously taken. The substances present may be found by the law that the spectrum of a mixture of any bodies contains all the lines they give separately.

All measurements made with a spectroscope must be reduced to some common scale, in order that they may be of real value, as the scales attached to these instruments are quite arbitrary, no two being alike. To make such a reduction, the lines measured in the solar spectrum must first be identified. The table given in Experiment No. 77, on p. 152, may be employed for this purpose. If the light used is only that of the sky, instead of sunlight, the visible spectrum will only extend from near *B* to *G*. Having identified the intermediate lines, construct points with ordinates equal to their observed position, and abscissas equal to their wave-lengths. A curve is thus obtained, from which any readings of the spectroscope may be reduced to wave-lengths by inspection. Apply it to the lines of the metals measured above.

77. Solar Spectroscope.

Apparatus. The optical circle, a 60° prism of dense flint glass, and a mirror capable of turning horizontally or vertically, by which a ray of sunlight may be reflected in any desired direction. This is accomplished more perfectly by a heliostat, in which the apparent motion of the sun is corrected by moving the mirror by clockwork. The instrument should be placed near a window into which the sun is shining, or if the day is cloudy, the experiment may first be performed with a Bunsen burner and a wire on which is a little borax, and afterwards concluded by the aid of sunlight.

Experiment. Adjust the optical circle as described in Experiment 72, so that both telescopes shall be focussed for parallel rays, and when placed opposite each other the cross-hairs and slit shall be distinctly visible. Bring them to coincide, and read the vernier. Turn the observing telescope about 45°, and place the prism on the centre plate, so that its back shall be equally inclined to the axes of the telescope and collimator, as at *D*, Fig. 59. Place the mirror in the sunlight, and turn the collimator towards it, and distant only a few inches. Nearly close the slit, and reflect the light through it by turning the mirror. On moving the telescope to one side or the other, if necessary, a brilliant spectrum will be visible, any part of which may be brought to coincide with the cross-hairs, and its position determined by the vernier.

To obtain the best results, the position of the mirror must be accurately adjusted. This may be done in two ways. Most simply by opening the slit wide, when the position of the beam of sunlight may be seen in its passage through the object-glass of the collimator, forming a bright spot on it. The mirror should then be turned until this spot falls in the centre. Holding a sheet of thin paper against the object-glass renders the spot more visible. The slit must be nearly closed before looking through the telescope, or the eye may be injured by the intense light. A more accurate method of adjustment is to remove the eye-piece and look through the tube, when an image of objects reflected in the mirror will be faintly visible. Turn the mirror until the image of the sun falls in the centre of the object-glass and the light will then pass through the axes of both telescopes. At the same time the prism should be placed so that it shall cover as much of the object-glass as possible. If no light is seen, even if the slit is opened wide, probably the telescopes are not set at the right angle. The brilliancy of the image depends on the width of the slit, and when the latter is very narrow, the image of the sun will widen out by diffraction. Having set the mirror correctly, it will remain right only for a few minutes, owing to the apparent motion of the sun, and hence must be readjusted every little while. This may commonly be done with sufficient accuracy without removing the eye-piece. Much trouble may be saved by noting the point on the opposite wall where the reflected beam falls, and resetting the mirror by this. Or, a small mirror may be attached, and the direction of its reflected beam noticed. If the shadow of a window-sash falls on the mirror, move the latter across it, so that the further motion of the sun may separate them instead of again bringing them together.

Next bring the prism to the minimum of deviation, that is, so that its back shall be equally inclined to both telescopes. Turn the prism while looking through the telescope, and the spectrum will be seen to move a certain distance toward the red end, and then return. As a considerable motion of the prism corresponds to but a slight motion of spectrum, this point may be found with sufficient accuracy by the hand alone. Now focussing the telescope with care, the spectrum will be seen to be traversed by a multitude of fine vertical lines known as Fraunhofer's lines. Bring one of these to coincide with the cross-hairs after setting the prism at the minimum of deviation, and read the vernier. It will be found that the minimum for one line is not the minimum for another. Measure in the same way the position of several of the more prominent lines; which may then be identified by the accompanying table. The first column gives the names, the second their wave-lengths, and the third their position on the map of Kirchhoff, which is still much used as a standard. The line A is about the extreme limit of the red end of the spectrum, and can only be seen in strong sunlight.

Name.	*W.L.*	*K.*
A	7605	405
a	7185	500
B	6867	594
C	6562	694
a	6276	810
D	5892	1005
E	5269	1528
b_1	5183	1634
F	4861	2080
G	4307	2855
h	4101	3364
H_1	3968	3779

B is therefore often mistaken for it. C is a sharply marked, but fine line in the red, caused by hydrogen. a is due to aqueous vapor in the air, and is most conspicuous about sunset. It then bears a marked resemblance to B. D is a double line in the yellow, due to soda. The fine lines between its two components were often used as tests, until it was shown by Prof. Cooke that most of them were due to aqueous vapor. E is a close double line in the green in the midst of a group of double lines, some of them very close. b consists of four very strongly marked lines, three of them due to magnesium, of which the least refrangible is b_1. They contain several fine lines, which form good tests of the power of a spectroscope. F is a strong line in the blue, like C, due to hydrogen. G lies in the midst of a group, among a multitude of lines. h is fine and due to hydrogen, and H consists of two very broad lines, almost at the limit of the visible spectrum.

To determine the indices of refraction for these lines, subtract from the reading of the vernier in each case the reading when the telescopes were opposite, and the difference D gives the deviation. If i is the angle of incidence, and r the angle of refraction for the first surface, since the prism is at the minimum of deviation, the angle of incidence at the second surface will equal r, and the angle of refraction i, as both faces are equally inclined to the light. Again, it may be proved by Geometry, that calling A the angle of the prism, $A = 2r$. The ray is deviated at each surface $i - r$, hence the total deviation $D = 2(i - r) = 2i - A$, or $i = \frac{1}{2}(A + D)$. If the index of refraction equals n, $\sin i = n \sin r$ or $n = \dfrac{\sin \frac{1}{2}(A+D)}{\sin \frac{1}{2}A}$. Compute in this way the index of refraction n for each of the lines, and see if they satisfy the theoretical formula of Cauchy, $n = A + \dfrac{B}{\lambda^2} + \dfrac{C}{\lambda^4} = A + Bx + Cx^2$, calling x equal to the reciprocal of the square of the wave-length, and A, B and C, constants depending on the particular material of which the prism is composed. By the method of least squares, p. 4, the most probable values of A, B and C, may be found, and compared with observation by a residual curve. To insure accuracy, it is safer to remeasure the indices again, using the other side of the graduated circle and employing the mean. By using a hollow prism bounded by two plates of glass, the indices of liquids may be measured, and with a prism of quartz the relation of the ordinary and extraordinary indices to the wave-length, established.

To obtain really valuable results in this experiment, great care is necessary, and an instrument of the finest construction. The more powerful spectroscopes contain a number of prisms, thus giving a much longer spectrum. In some the light passes twice through each prism, the collimator being placed immediately over the observing telescope, or better, united with it. With such an instrument a vast number of lines may be seen and identified by comparison with the maps of Angström or Kirchhoff. To measure the exact place of those near together, it is better to determine accurately the position of two or three, measure the rest by a spider-line micrometer, and then reduce to wave-lengths by interpolation.

The distance between the two components of a double line may also be readily determined by the same instrument. It consists of an eye-piece, in which are two vertical spider lines, one fixed, the other movable by a micrometer screw the number of whose turns is commonly measured by notches in the upper part of the field of view, and the fraction of a turn by a circle divided into one hundred parts, attached to the screw-head. Both wires may be moved by a second screw, and illuminated by a light placed opposite a piece of glass inserted in one side. It is used when the field is dark, to render the lines visible. The distance between two lines may be measured as follows. Call the screw with divided head, *A*, and the other, *B*. Bring the two cross-hairs to coincide, and read the micrometer-screw, *A*, repeat several times, and take the mean. Then turn *B* until the fixed hair coincides with one line, and turn *B* until the movable hair coincides with the other. The reading of *A* minus that previously taken, gives their distance apart. After setting both hairs, their position is sometimes reversed, and the distance through which *A* has been turned equals double the distance between the lines. It is a good exercise to measure all the lines visible in a small portion of the spectrum, and then compare with one of the charts mentioned above.

If the sun is not shining, the Bunsen burner may be employed instead, using the platinum wire with a borax bead at its end. This will give a bright, double line, coinciding exactly with the dark line, D, in the solar spectrum. Its position, and the interval between its components, should be accurately determined. Spectra of great beauty may also be obtained with an induction coil by allowing the spark to pass between terminals of different metals placed in front of the slit. Still finer effects are obtained with the electric light.

78. Law of Lenses.

Apparatus. In Fig. 60, A is a fishtail burner, attached to the end of a bar eleven feet long, and divided into tenths of an inch. B is a lens of two feet focus, by which an image of A may be projected on the screen C. Both B and C are movable, and carry pointers to show their distance from A.

Experiment. Place C at the end of the bar, and B just 100 inches from A. An image of the flame will be formed on C, which is then moved backwards or forwards until the exact focus is found. When the screen is too near, it will be noticed that owing to chromatic aberration, the edges of the image are red, while if too distant they are blue. The intermediate position may thus be found with great accuracy. Read and record the distance AC, and repeat, making AB successively 95, 90, 85, etc., inches. C will approach A up to a certain point, until $AB = BC$ equals twice the focal length of the lens. Determine this point more exactly by taking a number of readings, moving B an inch at a time. Then continue to diminish AB two inches at a time, until the image falls off the bar.

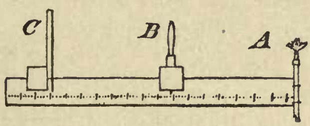


Fig. 60.

Write the results in a table in which the first column contains AB, the second AC, and the third their difference, or BC. Now compute the true value of BC in each case, and insert in the fourth column. Calling u and v the conjugate foci, or AB and

BC, and f the principal focus of the lens, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{1}{AB} + \frac{1}{BC}$. In this formula AB is successively made equal to 100, 95, etc., inches, and f equals one fourth the minimum value of AC. By a table of reciprocals the calculation is easily made by subtracting the reciprocal of AB from the reciprocal of f, and the reciprocal of the difference gives BC. Construct two curves on the same sheet of paper with the same abscissas AB, and ordinates equal to the observed and computed values of BC, respectively, and their agreement proves the correctness of the formula.

79. Microscope.

Apparatus. The importance of this instrument renders it desirable that each student should devote considerable time to its use. For this reason, in a large laboratory two or three microscopes should be procured, and it is well to have them from different makers, so that the student may be accustomed to all forms. For example, a "Student's Microscope," by Tolles or Zentmayer, to represent the American instrument, a binocular "Popular Microscope," by Beck, for the English, and a third instrument by Nachet or Hartnack, for the Continental form. The latter is very cheap and good, but not having the Microscropical Society's screw, common objectives cannot be used on it without an adapter. It is also well, if it can be afforded, to have one first-class microscope-stand for work of a higher nature. The usual appurtenances described below should be added, but need not be duplicated, also a number of objectives and objects.

The following description will serve for all the common forms of instrument. A brass tube or body is attached to a heavy stand, so that it can be set at an angle, or moved up or down. In its lower end the objectives are screwed, and the eye-pieces slide into the upper end. The objectives are made of three achromatic lenses, by which a short focus is attained, with great freedom from aberration. The eye-piece is of the form known as the negative eye-piece, and consists of two plano-convex lenses, with their plane surfaces turned upwards. Below is placed the stage, on which the object is laid and kept in place, either by a ledge, or by spring clips. In the larger stands the object may be moved by two racks and pinions in directions at right angles, or revolved by turning the stage. It is very desirable that this rotation should take place around the axis of the instrument, as is done in the English, but not in the American instrument mentioned above. Under the stage is the diaphragm, a plate of brass with a number of circular holes in it of different sizes, to admit light more or less obliquely. Below it is a mirror, plane on one side, and concave on the other, by which light may be reflected upon the object.

It is very important that the body of the instrument may be raised and lowered with precision. There are generally two adjustments to effect this, one the coarse adjustment to move it rapidly, which is commonly a rack and pinion, or a simple sliding motion effected by hand, and a fine adjustment which is used for getting the exact focus, and is made in a variety of ways. One of the best is by a movable nose-piece, or the lower end of the tube made free to slide, and acted on by a lever, which may be moved by a screw. In a second form, the screw acts directly on a part of the tube itself, and sometimes the stage is raised or lowered. If the tube is moved, it should be raised only by the screw, the lowering being effected by a spring, so as to prevent the objective from being pressed forcibly against the object.

To show the use of each instrument used in connection with the microscope, one or more objects should be selected suited to each, and numbered as in the following examples. They may then be distributed among the various microscopes, according to the means or requirements of each laboratory. In this way a student acquires a better knowledge of the apparatus, and of the proper objects to which each appliance is best suited, than he could attain in weeks of unsystematic work. It is also well to examine several of the objects described below, with various methods of illumination, to learn how much their appearance may be thus altered. When studying a new object it should always be illuminated in various ways, and viewed first with a low, and afterwards with a higher power. There is, however, no more common mistake than to suppose that objects will be seen better, the higher the power. On account of the difficulty with which they are used, the want of distinctness and of sufficient illumination, the highest magnifying powers must be reserved for special occasions, and the lower powers commonly employed, especially in the preliminary observation of common objects. To save the eyes, it is better, at first, not to use a microscope very long at a time, and for the same reason, they should both be kept open. If possible, sometimes one eye should be used, and sometimes the other.

The applications of the microscope have been so extensive that it is impossible in a short article like the present, to give more than a general description of the most important. The student who wishes to make a specialty of this instrument is therefore referred at once to some of the works devoted especially to this subject. For instance, the treatises of Carpenter, Hogg and Beale, particularly the work by the latter author, entitled "How to Work with the Microscope." The same remarks apply with even more force to Experiments 80 and 81.

Experiment. 1. *Ordinary Method of using the Microscope.* Set the microscope in an inclined position, at such an angle that it can be used with comfort. The tube carries an objective below, by means of which an enlarged image of the object is formed, and magnified a second time by the eye-piece. Slip into the upper end of the tube the lowest power eye-piece, that is, the longest. The objectives are contained in brass, cylindrical boxes, with screw covers. They must be handled with care, as they are very expensive; the higher powers consist of very minute lenses, and the glass surfaces must never be touched, lest they be tarnished or scratched; the lower surface, which is plane, is particularly exposed to injury. Remove the cover of an objective whose focus is one or two inches, and screw it into the tube. Now turn the mirror so that the light from the window shall be reflected along the axis of the instrument, and on looking in, a bright circle of light will be visible.

Place object No. 1, eye of a fly, on the stage, and raise or lower the tube until it is distinctly visible. The distance between the objective and object should be somewhat less than the focal length of the former. Notice that the eye is composed of a multitude of facets, like the meshes of a net, each one containing a separate lens. Sketch some of them in your note book. Try the other eye-piece, which will give a somewhat higher power. Then remove the objective, putting it back in its box, and replace it by one whose focal length is ¼ inch. The use of this is attended with somewhat greater difficulty. It must be brought very near the object, but not in contact, or it would very likely be scratched, or even broken. It is therefore safest to bring it as near as possible without touching, by the coarse adjustment, then looking through the instrument to withdraw it until the distance is about right, and finally focus exactly, by the fine adjustment. A great increase of magnifying power is thus attained; add to the description and drawing whatever additional is visible. Do the same with a second object, foot of the *Dytiscus*.

2. *Diaphragm.* Immediately under the stage is a brass plate pierced with a number of holes of different sizes. Its object is to vary the amount of light and the direction in which it comes. When a small aperture is used, all the light comes in nearly the

same direction, and thus renders the shadows of minute objects more distinct. The structure of delicate objects is thus sometimes brought out very beautifully, where a large aperture conceals everything. To show this, try object No. 2, proboscis of a horse-fly, and see how much more distinct the fine hairs at the end are, with the small aperture. Also the diatom *Isthmia nervosa*, in which the markings, although perfectly distinct with a small aperture, almost disappear when the diaphragm is turned so as to admit a large cone of light.

3. *Oblique Illumination.* Microscope objectives are made so that they will transmit rays of light not only along their axis, but also when falling obliquely on them, that is, they will receive a cone, the angle of whose vertex is called the angular aperture of the objective. For the higher powers this angle is sometimes very great, 170°, 175°, or even 177°. With them, instead of placing the mirror immediately underneath, it may be placed on one side, and the object illuminated obliquely. A better plan is by an Amici's prism which is placed below the object, and throws the rays obliquely like the mirror. The advantage in this case is like that of a diaphragm, only greater, shadows being strongly cast, and very delicate structure rendered visible. This effect is well shown with many diatoms, minute siliceous shells, on which are markings or very fine parallel lines, used frequently as tests. Try the quarter inch objective on specimens No. 3, *Pleurosigma formosum* and *Pleurosigma hippocampus.* First use direct light and then an oblique illumination, and see how much more distinctly the markings are visible in the second case.

4. *Opaque Objects.* Some objects, especially those of large size, cannot be rendered transparent, and sometimes the surface only of a body is to be examined. In this case remove the mirror from below its object, and place it above on a stand, turning it so that the light shall be thrown down upon the object. A second method is to use for the same purpose a large lens of short focus called a condenser. If the observer is facing the window, it is generally necessary in this case, to place the object nearly horizontal, in order to get light upon it. Try both these methods on objects No. 4, wing of a butterfly, and section of bone or tooth, viewing the latter also by transmitted light.

5. *Lieberkuhn.* Another method of illuminating opaque objects is by a parabolic mirror, with a hole in its centre, through which the objective is passed. This device, called a lieberkuhn, is used on small opaque objects, the light being thrown from the mirror below upon the lieberkuhn, and by it reflected upon the object. Try specimen, No. 4, wing of butterfly, thus illuminated, also some common objects, as a bit of paper, a steel scale, etc.

6. *Wenham's Parabolic Condenser.* This consists of a block of glass, plane below and parabolic above. It is placed, instead of the diaphragm, just below the object, which is at its focus, so that all light reflected upon it by the mirror below, will fall on the object illuminating it obliquely. The central rays are cut off by a circle of metal attached to the condenser. Objects are thus shown bright on a dark background, sometimes producing an excellent effect, though generally more beautiful than useful. See No. 6, *Arachnoidiscus Ehrenbergii.*

7. *Achromatic Condenser.* The mirror below the object is commonly plane on one side, and concave on the other, the former reflecting light on a given point from various directions, the latter concentrating that received from a single point. The second form is more commonly used, especially with artificial light, as any point may thus be selected as the source of illumination. The same effect is much better attained by placing below the object an objective similar to that above it, which allows only those rays parallel to its axis to pass through both. As it costs too much to duplicate all the objectives, each may be used as an achromatic condenser to that of next lowest power. This is a very favorite method of illumination, especially when using high powers on difficult objects. Try it on No. 7, fragment of hair and *Surirella gemma.*

8. *Polariscope.* One other method of illumination remains to be described; namely, that by polarized light. To use this to the greatest advantage, Experiment 88 should first be performed. The light is polarized by a Nicol's prism, placed under the object to be examined instead of the diaphragm, and a second prism or analyzer is placed above it, either slipping it over the eye-piece, or screwing it onto, and just above, the objective. On rotating either analyzer or polarizer, the field becomes dark when their planes are at right

angles, and brightest when they are parallel. Sometimes a plate of selenite is inserted just above the polarizer, when the field will assume a brilliant color, which may be changed into its complementary tint, by revolving one of the Nicol's prisms. To examine any object by polarized light, focus it as usual, and turn one of the prisms; the most marked effect is generally attained when they are at right angles, as with common objects, the field would be perfectly black, while any doubly refracting medium will appear bright in places, bringing out the structure with great beauty. Now insert the plate of selenite, and the uniform tint will, in many cases, be replaced by a gorgeous display of colors. Examine the following objects, No. 8, section of cuttle-fish bone, quill, sulphate of magnesia, nitre and salicine. In fact, any crystalline substances not in the monometric system, affect polarized light, and the same may be said of many organic structures, as starch, bone, hair and horn.

9. *Binocular Microscope.* To avoid the fatigue of using one eye only, and to obtain the stereoscopic effect due to two, this instrument is employed. A small prism is placed just above the objective, which divides the light into two portions, which pass along two tubes, one for each eye. Two eye-pieces are of course used, and on looking through them, the object is seen as in the stereoscope, standing out in its true form. The distance between the eye-pieces may be altered by lengthening or shortening their tubes by a pinion acting on two racks, and the microscope may be rendered monocular by merely pushing back the binocular prism. This instrument is best suited to opaque objects not requiring a high power. Apply it to objects No. 9, head of a bee, and an injected preparation of the upper portion of the lung of a frog, using the 1 or 2 inch objectives and the lieberkuhn.

30 51	31 51	32 51
30 52	31 52	32 52
30 52	31 52	32 52

Fig. 61.

10. *Maltwood's Finder.* Some objects are so minute that they are found only with difficulty, and it is also sometimes desirable to refer back to a certain point of an object and reëxamine it. A Maltwood's finder consists of a photograph on glass, one inch square, of a series of squares numbered as in Fig.

61, all the lower numbers in the same horizontal row being the same, also all the upper numbers in the same vertical column. In other words, the upper numbers give the abscissa, the lower the ordinate, of each square. This photograph is then mounted on a slip of glass like any other object. A pin or stop should be attached to the stage, so that both finder and object may always be placed in precisely the same position. Lay the object on the stage and bring the point to be recorded exactly in the centre of the field, using a moderately low power, if the object is not too minute. Now put the finder in the place of the object, taking care not to move the stage, and record the numbers of the square in the centre of the field. If at any time the same point of the object is again wanted, the finder is placed on the stage, and the latter moved until the square bearing these numbers is again in the centre of the field. Replacing it by the object, the desired point should be at once visible. Apply this method to objects No. 10. First, with the preparation of an entire insect, as a gnat, record the numbers corresponding to several prominent points, as the eye and the end of the proboscis. Then move the stage and see if they can be found again. Do the same with a slide containing a single minute object as No. 7, *Surirella gemma*. Now, placing on the stage an object containing a collection of diatoms, select a good specimen, sketch its position and that of the adjacent ones, and see if it can be found again from its numbers. The numbers corresponding to the marked points of some of these objects should be recorded on a card placed with the microscope, and these points then found by the student.

11. *Micrometers.* There are several forms of this instrument, which is used for measuring minute objects. First, the stage-micrometer, which consists of a plate of glass ruled with fine lines at equal intervals of $\frac{1}{1000}$ in. or $\frac{1}{100}$ mm. The lines are so delicate that it is often difficult to find them, as a slight difference of focus throws them out of sight. Their visibility may be increased by oblique illumination, or by using a small aperture in the diaphragm. The eye-piece micrometer consists of a coarser scale on glass, inserted at the focus of the eye-piece, which, in the negative form, lies between the lenses. An enlarged image of it is thus formed in the field, which is used like an ordinary scale to measure the dimensions

of objects. To reduce them to fractions of a millimetre, lay a stage micrometer on the stage and measure its divisions. For instance, if 7 hundredths of a millimetre equal 53.4 divisions of the micrometer, one division of the latter will equal .00131 mm. From this any measurement made with the micrometer may be reduced to millimetres. If the microscope has a draw-tube, this reduction may be simplified. In this case, the tube of the microscope is made double, so that its length may be altered, a scale showing the extent of the change. The power is increased by drawing out the tube, and the divisions of the stage-micrometer being enlarged, they may be made exactly equal to any desired number of divisions of the eye-piece micrometer. Thus, in the above example, make the 7 divisions equal to 56, instead of 53.4, when one will equal 8, or each division of the eye-piece micrometer equal one eight hundredth of a millimetre. In the same way make it one thousandth, and altering the objective, give it other values, recording in each case the reading of the draw-tube.

Now make a number of measurements of objects No. 11, as directed on a card, which should accompany the specimens, and reduce them to millimetres. Determine also the thickness of a hair, of a filament of silk, and the diameter of some minute holes made in a piece of tin-foil with a fine cambric needle. In the spider-line micrometer, the two hairs are brought to coincide with the ends of the distance to be measured, and the interval determined, as described in Experiment 77. The readings should be reduced to fractions of a millimetre, in the same way as the eye-piece micrometer. Repeat the above measurements, and see if the same results are obtained as before. The magnitude of the divisions of both the spider-line and eye-piece micrometers, depends only on the distance from the objective and its focus, and not at all on the eye-piece, unless a negative eye-piece is used, and the micrometer inserted between the field- and eye-lenses.

12. *Goniometers.* A spider-web, or filament of silk, is stretched in the eye-piece across the field of view, and turned so as to coincide first with one side, and then with the other, of the angle to be measured. The number of degrees through which it is moved gives the required angle. A graduated circle is therefore attached to the tube, and an index is fastened to the eye-piece. Sometimes

the object is moved instead, the graduated circle being attached to the stage. To test this instrument, form a triangle by selecting three points on any object, and suppose them connected by right lines. Measure each angle two or three times, displacing the object and eye-piece after each measurement; take the mean, and see if the sum of the three equals 180°. Now measure several angles of the crystals, No. 12, chlorate of potash, taking care that they lie flat, otherwise too small a value of the angle will be obtained.

13. *Camera Lucida.* It is often important, when studying minute objects by the aid of the microscope, to be able to draw them correctly. For this purpose, the enlarged image must be thrown on the paper in such a way that both may be distinctly seen at the same time. This is done most simply by keeping both eyes open, and directing one towards the paper the other through the microscope, when the image and paper, may be brought together so that the outlines of the former may be marked on the latter. A better method however, is by the camera lucida, which consists of a minute right-angled prism of glass, having its acute angles equal to 45°. Place the microscope horizontally, adjust the mirror so that the field shall be bright, and apply a low power to object No. 13, a flea. Attach the camera to the eye-piece so that on looking down into the mirror, a reflection of the object shall be thrown on a sheet of paper, placed immediately beneath, in which case one face of the prism will be horizontal and turned upwards, the other vertical and turned towards the microscope. The prism is of small size, so that it will cover only a part of the pupil of the eye, and bringing the latter over its edge, both paper and object may be seen simultaneously. With practice, the outline may be marked out very accurately, but at first it is difficult to see the pencil at the same time as the object. As the latter is often too bright, it is sometimes better to incline the mirror until the field is darker. Better results are also sometimes obtained if the eye is raised an inch or so above the prism.

As with this instrument, the brilliancy of the object generally renders it difficult to distinguish the pencil, the prism is sometimes replaced by a small piece of plate glass, which reflects the image in the same way as the back of the prism, while its transparency renders the pencil visible through it. The latter is in fact generally too bright, so that it is often better either to cast a shadow on the paper, or to make the glass of a neutral tint, to render it less transparent. Try making drawings with each of these instruments, first of some well marked object as the flea, and afterwards of some fainter object as the anchor-like spines of the *Synapta.* Generally only the outlines should be drawn with the camera, and the details filled in by the eye. After drawing the object, replace it by a stage-micrometer, and draw some of the divisions of the latter, which thus serve as a scale, by which the magnitude of different parts of the object may be determined. This also furnishes the best means of measuring the magnifying power, dividing the dimensions of the scale as drawn, by its real size. This will be correct, however, only when the distance from the camera to the paper is just 10 inches. In other cases it must be divided by the distance in inches, and multiplied by 10, to reduce it to the standard. Make this measurement with the 1 inch and the ¼ inch objectives, and two of the eye-pieces, also if there is a draw-tube, make the magnifying power some simple number, by varying the distance between the objective and eye-piece.

14. *Spectrum Microscope.* This consists of a spectroscope inserted in the eye-piece of a microscope in such a way, that the spectrum of very minute objects may be obtained. The form in most common use is that devised by Sorby. The slit replaces the diaphragm, and is partly covered by a right-angled prism, by which a second spectrum, from a light at one side of the eye-piece, may be compared with the other. The prisms are placed over the eye-lens, and are of the form known as direct vision, in which the deviation of two prisms of heavy flint glass is compensated by that of three crown-glass prisms, while the dispersion is only partly neutralized. Accordingly a spectrum of considerable length is obtained, while there is no deviation of the central portion. The width of the slit may be varied by a screw, and its length by a sliding stop. An ingenious scale is provided, formed of two Nicol's prisms, with a plate of quartz between them, and placed in the path of the rays reflected by the right-angled prism. They absorb from the visible spectrum twelve black bands, at regular intervals, and from their position, that of any line may be readily determined. To use this

instrument, slip it on the end of the microscope in the place of the eye-piece, and place object No. 14, human blood, on the stage with a one or two inch objective. The spectrum will now be seen to be traversed by two marked black lines in the red, which form an excellent test for the presence of blood. Their position may be measured with the scale, by attaching the latter to the side of the eye-piece, and adjusting the prism so that the spectrum for one half its breadth shall be traversed by strongly marked black bands. Other objects, such as nitrate of didymium, permanganate of potash and aniline violet, may be observed in a similar manner. Care should be taken to make all the light pass through the object, which is generally best accomplished by placing a cardboard diaphragm with a small hole in it, on the stage under the object. Liquids are placed in glass tubes or cells, which may be closed hermetically.

15. *Test-Objects.* The principal efforts of microscope makers are now directed towards the objectives, since it is by perfecting them that the greatest improvements are to be expected. The best method of judging of the excellence of an objective, or of comparing those of different makers, is by trying them on a number of objects called test-objects, some parts of which can be seen only with difficulty. To obtain the best results great skill is needed, especially in arranging the illumination, and it must not be forgotten that some objectives give the best results with one class of objects, others with another. For instance, some with a large angular aperture, give fine effects with objects requiring a very oblique illumination, but are not suited to those of considerable thickness, requiring great depth of focus.

When an objective is perfectly corrected for chromatic aberration, and a plate of thin glass is interposed between it and the object, a new correction for color becomes necessary, in amount depending on the thickness of the glass. This is commonly effected by varying the distance of the front lens from the other two, which is accomplished by turning a milled head near the end of the objective. A divided circle and index serve to mark the position, which will of course vary with each different object, according to the thickness of the covering glass. To make this correction, adjust the objective for an uncovered object, that is, set the index at zero and focus it on the object. Then turn the milled head until the dust on the upper surface of the covering glass is in focus, when the proper correction will have been applied. Focussing again on the object, the latter will be more sharply defined than before. The correction for covering glass, as it is called, must be applied to all objectives of higher power than ½ inch, to get the best effects, especially when they have a large angular aperture. Instead of moving the front lens, it is better to have it fixed, and to have the other two movable, as all danger of scratching or breaking the objective and object by bringing them in contact, is thus avoided.

Try some of the higher power objectives with the test-objects No. 15. One of the most common tests for defining power is the marking of the scales of the wood-flea (*Podura plumbea*), which are covered with delicate epithelial scales, like the tiles of a roof. Try also the hair of the Indian bat, and of the larva of the *Dermestes*. Some of the *Diatoms* described above, form excellent test-objects. The valves of the genus *Pleurosigma* are covered with fine markings, which form an excellent test for separating or penetrating power. For instance, the three species, *formosum*, *hippocampus* and *angulatum*, form a series of increasing difficulty, well adapted to test objectives of ordinary power. The marking of the first and third are apparently covered by three series of fine parallel lines, dividing the surface into hexagons, and of *hippocampus* by two series, forming squares, but in reality probably due to a multitude of very minute hemispheres with which the surface is covered. The same effect may be seen on an enlarged scale, in a common form of book-cover. Probably the best test of this kind is a plate of glass with very fine lines ruled on it. M. Nobert of Griefswold has made such plates with a series of bands formed of lines at various intervals up to a 112,000th of an inch.

80. Preparation of Objects.

Apparatus. A microscope with objectives and eye-pieces, several vials containing the substances to be examined, a number of glass slips three inches long and an inch wide, some of which have concave centres, that is, a concavity ground out on one side, and some circles of very thin glass.

Experiment. To examine a liquid under the microscope, dip a glass rod or tube into it, and place a small drop on one of the glass slides. Cover it with a circle of very thin glass, which will be held in place by capillarity, and wipe off the superfluous liquid carefully. A concavity is commonly ground in the centre of the slide to hold more liquid, and to keep the cover in place. Examine the following objects in this way, describe and sketch them, and compare their appearance with that given in the works on the Microscope, referred to in the last experiment. A drop of vinegar viewed with a low power, is seen to be full of eels in active motion. Milk contains multitudes of oil globules, which when united form butter, and organic matter whose appearance furnishes an excellent test of its purity. Blood is a curious object under the microscope. It is most readily obtained from the finger just below the nail. With a quarter-inch objective, it is seen to consist of a clear liquid or serum, in which a vast number of blood-corpuscles are floating. These are circular disks, thicker around the edge, and interspersed with larger white globules. In its natural state the blood is too thick to be conveniently observed, the corpuscles overlap, and soon begin to shrivel up, as the blood dries. If diluted with water osmotic action ensues; they swell up and sometimes burst. Salt water is therefore preferable, or better still, the serum or liquid portion which separates from the clot when blood coagulates.

Powders are sometimes viewed dry, but generally it is better to wet them, as they are thus rendered more transparent. Place a very minute quantity of starch on a slide, add a drop of water, and cover with a piece of thin glass. Viewed by polarized light, each grain is seen to contain a black cross, which changes to white on rotating the analyzer. This cross is characteristic of starch, and often serves to detect its presence. It is best seen in the larger grains, as those of potato starch, and assumes brilliant colors if a plate of selenite is interposed. The adulterations of coffee, cocoa, etc., are readily detected by examining them in powder under the microscope.

It is often necessary to pick up small objects under water, or to capture a minute animal without injuring it. A good example of this kind is the little *Cyclops*, often found in great numbers in common pond water, especially in the spring. Collecting the water in a white porcelain vessel, as a large evaporating dish, a close examination will often reveal dozens of them. Their number may also be increased by filtering a considerable quantity of the water through a cloth, which retains them, and from which they are easily washed into the dish. To place one on the slide, take a small glass tube about half a foot long, close one end by the finger, and immerse the other in the water. Bring it near the *Cyclops* and suddenly remove the finger, when the water will rush in, carrying the animal with it. Replacing the finger, the tube may be removed, the water allowed to escape a drop at a time, and the *Cyclops* finally deposited on the slide. Instead of a slide with concave centre, it is better for so large an object as this, to use an Animalcule-Cage. This consists of a small circle of glass, on which a drop of water containing the object is laid, and the cover pressed down upon it by means of a brass ring, so as to leave a space of any desired degree of thickness. Delicate objects are thus protected from injury by crushing. A wonderful variety of animalcule and of fungoid growths may be found in stagnant water, or sour flour-paste, in fact in almost any decaying animal or vegetable matter.

Minute air-bubbles are often found in various objects. To become familiar with their appearance, examine a drop of soap-suds, or gum-water containing them. They look like black, highly polished, metallic balls, with a broad, dark outline, and bright centre.

The formation of crystals is readily watched under the microscope by placing a drop of the hot saturated solution on a slide, and allowing it to cool. Try in this way sugar, phosphate of soda, and oxalate of ammonia, first using ordinary, and then polarized light.

A most instructive experiment is to watch the circulation of the blood in the foot of a frog. The animal is first rendered insensible by means of ether or chloroform, then put in a linen bag and well wet with water. Draw one of the hind legs out of the bag and tie it down upon the slide, supporting the frog on a piece of wood or frog-plate. Tie threads to three of the toes, so as to stretch the membranes between them, and on examining it with a half-inch objective the blood corpuscles will readily be seen passing from the arteries through the capillaries to the veins. By putting alcohol or salt on the foot, all the phenomena of inflammation and its cure may be observed. The black spots distributed over the membrane are due to the pigment. The circulation may also be observed in the tail of a stickle-back, or other small fish, or of a tadpole. The latter animal, when very small, forms a beautiful object with a low power and binocular microscope, as it is sufficiently transparent to render visible the action of the heart, and other internal organs. The effect is also improved by keeping the tadpole for some time previously without food.

Another interesting experiment is to watch the ciliary action, which in many of the lower animals takes the place of the circulation. Cilia consist of minute hairs, which vibrate rapidly back and forth, and thus establish currents in the liquid in contact with them. They may be seen by scraping a little of the mucus from the roof of the mouth of a frog, or better, from the gills of an oyster or mussel.

Most solid substances, like wood or bone, are best seen in thin sections, which are made as will be described in the next Experiment. Fine filaments, as silk, wool or hair, are viewed by transmitted light, and generally give better effects when wet with water or oil. Some solids, especially when highly colored, are best seen as opaque objects, with a condenser or lieberkuhn.

81. Mounting Objects.

Apparatus. Boxes may be obtained containing all the apparatus needed for mounting objects, such as glass slips, thin glass covers, Canada balsam, gold size, a stand on which slides may be heated, a whirling table for making cells, section-cutting apparatus, and other devices which will be described below.

Experiment. Objects are mounted in various ways, according to their size, and whether they are best seen dry, or immersed in some liquid. They are protected by a circular piece of glass, made very thin on account of the short working distance of high-power objectives. These circles are cut from a sheet of thin glass with an instrument like a very small beam-compass, the point which serves as a centre, being replaced by a flat disk, and the pencil, by a diamond. Only a faint scratch is needed, but some skill is required, or many of them will be broken.

Try each of the following methods of mounting objects, and if successful, cover the slides with paper and label them, giving also your name and the date. Unless the object is very thin, or if it is liable to be injured by pressure, it must be protected by a cell. This consists of a circular or square enclosure, on which the thin glass plate is laid, so as to leave a space between it and the slide. Cells may be made of various materials, as paper, cardboard, or tinfoil, and fastened to the glass by gum. These are very convenient for mounting objects dry, especially such as are not injured by the air. Mount in this way some crystals of bichromate of potash. Shallow cells may be made of Brunswick black, applying it with a brush. They are best made in a circular form by Shadbolt's apparatus, in which the slide is placed on a small turn-table, which is made to revolve rapidly by drawing the forefinger of the left hand over a milled head attached below, while the brush is held in the right hand. If the plate is warmed, the black will dry rapidly, and the thickness of the cell may be increased by applying successive layers. Make several such cells for some of the objects to be mounted in balsam, as described below. To preserve a liquid, or an object of considerable size, thick cells are employed, which may be procured ready-made of glass. They may be cemented to the slide by marine glue, warming them sufficiently to melt it, removing the superfluous glue by a sharp knife, and washing it clean with a solution of potash. Fill such a cell with some liquid, as vinegar, and fasten on the cover with marine glue. Take great care thatno air bubbles enter, and that the joints are perfectly tight.

The best method of mounting the parts of insects, sections of wood or bone, and in fact most substances, is in Canada balsam. The object, as the foot of a fly, must first be dried and freed from air-bubbles. For this purpose it should be heated nearly to the boiling point of water, or placed under a bell-glass containing concentrated sulphuric acid. To remove the air it should be soaked in turpentine and gently warmed; a much more effective method is to place the whole under the receiver of an air-pump and exhaust. Now lay the slip of glass on a little stand of brass, and heat it by means of a spirit-lamp, or Bunsen burner. Take up a little Canada balsam on the end of an iron wire, and lay it on the slide, when the heat will render it perfectly fluid. Pick up the object on the point of a needle, immerse it in the balsam, and then cover it with a piece of thin glass. Great care must be taken that both slide and covering-glass are perfectly clean, and that no dust gets into the balsam, as otherwise the object will be much disfigured when viewed under the microscope. The main difficulty is to prevent air-bubbles remaining on the slide. If present, they may be removed by a cold wire, or burst by touching them with a hot needle. The covering-glass must be lowered into place very slowly, or bubbles will adhere to its surface. The whole is then put away to harden under pressure, and the superfluous balsam afterwards removed by the aid of a little turpentine.

The structure of objects of large size is generally best seen by cutting thin sections of them, so that they may be rendered nearly transparent, and be viewed by transmitted light. Soft substances, as vegetable or animal tissues, may be cut with a sharp knife or scissors, or better, with a Valentin's knife, which has two parallel blades whose distance apart may be varied by a screw. They should be well wet with water or glycerine, or the section will adhere to them.

Harder substances, as wood or horn, are cut in thin sections by forcing them through a hole in a thick brass plate, cutting off the projecting portion, pushing it through a little farther, and cutting again. By means of a screw, sections of any desired thickness may thus be obtained. Cut longitudinal and transverse sections of a piece of pine wood, first soaking it in water, and mount them in Canada balsam. Cut also some thin transverse sections of hair by fastening a number of them together with gum so as to form a solid mass; cut a thin section, and then dissolve the gum in water.

To cut a thin section of still harder substances, as bone, quite a different method must be employed. A thin piece is first cut off with a fine saw, such as is used for cutting metals; it is then filed thinner, and finally ground down to the required thickness with water between two hones. On examining the section thus obtained, it will be found covered with scratches, which must be removed by grinding it on a dry hone, and afterwards polishing it on a sheet of plate glass. Prepare two such sections, soak one in turpentine and remove the air by means of a pump, and then mount both in Canada balsam. The difference in their appearance will be very marked, the one from which the air has not been removed appearing full of black spots or lacunæ, formerly called bone corpuscles. They are really cavities filled with air, which in the second specimen is replaced by the turpentine.

This experiment is well supplemented by performing some dissections of animal and vegetable substances, injecting tissues, and mounting thin sections of them.

82. Foci and Aperture of Objectives.

Apparatus. Two instruments are needed for this Experiment. First, a microscope with a positive eye-piece, a spider-line or eye-piece-micrometer, and a stage-micrometer, also several objectives to be measured. To measure the angular aperture, a graduated circle is employed with an arm and index, to which is attached a short brass tube, like the body of a microscope. The objective to be tested is screwed into one end of this tube, and a positive eye-piece slipped into the other. The tube is made so short that when the objective is directed towards a distant object, the image formed may be viewed by the eye-piece. To obtain a higher magnifying power, the eye-piece may be replaced by a compound microscope, like that used in Experiment No. 20. To obtain an accurate measurement when the object observed is not very distant, it is essential that the end of the objective should lie in the axis of the circle. This is most readily accomplished by means of a ledge, on which a vertical plate of glass may be placed with its front face over the axis of the circle. The objective is then brought up in contact with it, the tube clamped, and the glass removed.

Experiment. To measure the focal length of an objective it is assumed that two of the laws of simple lenses hold for a compound lens. First, that the sum of the reciprocals of the conjugate foci equals the reciprocal of the principal focus, and secondly, that the ratio of the magnitudes of the object and image equals the ratio of the conjugate foci. This assumption is not strictly correct, and valuable work might be done in determining the amount of the deviation. Screw the objective to be measured upon the microscope, and measure the divisions of the stage-micrometer, with the spider-line micrometer. Reduce to absolute measurements from the magnitude of the parts of the micrometers, or if these are not given, determine them from the Dividing Engine, Experiment No. 21. This reduction may be avoided by using two similar eye-piece micrometers, A and B. Measure several divisions of A with B, and call the mean of the readings m. Measure, in the same way, B with A, and call the mean reading m'. The true reading, n, will be the mean proportional of these two. Of course if the micrometers are precisely alike, m will equal m'. Now measure the distance between them, and call the distance D. Then if f equals the focal length of the objective, and f', f'' its conjugate foci, $f' + f'' = D$, $\frac{1}{f'} + \frac{1}{f''} = \frac{1}{f}$, and $\frac{f'}{f''} = n$. From which $f = D\frac{n}{(n+1)^2}$, and knowing D and n, f may be deduced. The number given by the maker is generally greater than the true focal length of the objective, and this experiment affords an excellent means of correcting it. To show the value of such measurements, and the accuracy attainable by them, see an article by Prof. Cross, Journ. Frank. Inst., Vol. LIX., p. 401. Useful work might also be done by varying D, and noting the effect on f, also by changing the correction for cover, or distance between the lenses.

To measure the angular aperture of an objective, screw it into the end of the tube attached to the graduated circle, set a plate of glass on the ledge, and bring the objective against it. The front surface of the lens will then be just over the axis of the circle. Now clamp the tube, remove the plate of glass, and slide the eye-piece or small compound microscope into place. Bringing it near the objective, an image of outside objects is seen, the whole in fact, forming a telescope with the objective for an object-glass. The field of view is seen to be bounded by a circle whose true angular diameter gives the aperture of the objective. Select some strongly marked vertical line, as the sash of the window, and notice that as the objective is turned from side to side, the image of this line moves also. Bring it to coincide first with one edge of the circle, and then with the other. The difference in the reading of the index in the two cases equals the angular aperture. Repeat this measurement with several other objects.

83. Testing Plane Surfaces.

Apparatus. A stand carrying two telescopes, which may be placed opposite each other, or set at right angles. The eye-piece of one, which acts as a collimator, is replaced by a plate of brass pierced with a very fine hole. This is placed exactly at the focus of the object-glass, and being illuminated by a lamp, forms a bright point of light or artificial star. The optical circle might be used for this experiment, but the graduated circle is not needed, and it is better to have telescopes of larger size. A millimetre scale is also wanted, a prism, a sextant-glass, a piece of plate or window glass, and a lens of very long focus.

Experiment. Make the same adjustment for parallel rays as is described in Experiment 72. That is, focus the observing telescope carefully on some distant object as a star, and turn it toward the collimator. An image of the hole or artificial star at the further end of the latter will now be visible, but it will generally be out of focus. Draw it towards, or from its object-glass until accurately focussed, when it should appear as a very minute circle of light, like a star. Measure with the millimetre scale the distance between two points, one on the eye-piece, the other on the end of the tube in which it slides. Throw the star out of focus by moving the eye-piece, and focus again; repeat ten times, and take the mean of the distances between the two points. Now set the telescopes at right angles, and place the surface to be tested at the intersection of their axes, equally inclined to each, and vertical. The image of the star reflected in the surface, will then fall in the centre of the field, and if the surface is perfectly plane will be as distinct as that previously obtained, although fainter. In general, however, it will be a little out of focus, due to the curvature of the surface. In this case move the eye-piece, focus ten times as before, and take the mean reading of the distance between the two marked points. Measure the focal length of the observing telescope, or the distance from its object-glass to the cross-hairs, also the angle between the axes of the collimator and observing telescope, unless this is fixed at 90°. Call F the focal length, d the change in position of the eye-piece, or difference of the means of the two sets of ten observations, v the angle between the axes, a the distance from the objective of the telescope to the plane surface, and R the required radius of curvature. Then $R = 2 \cdot \frac{F^2 + d(F - a)}{d \cos \frac{1}{2}v}$, or as d is generally very small compared with F, it is often sufficiently accurate, if $v = 90°$, to take $R = 2.83\frac{F^2}{d}$. If the surface is concave, the eye-piece has to be pushed in, if convex, out. Test in the same way the other plane surfaces, also the two sides of the lens. Any distortion of the image is due to irregularity of the surfaces, as is well shown by trying a piece of window glass.

The parallelism of two plane surfaces, like those of the sextant-glass, is well tested in the same way. If both surfaces are perfectly plane and parallel, only a single image is formed, otherwise there are two, one from each face. The angle A, between them, may be determined from the divergence of the images, by the formula $A = \frac{D \cos \frac{1}{2}v}{2n \cos r}$, in which n is the index of refraction, D the angle between the images, and r the angle of refraction of the light in the glass. The latter is known from the equation $\sin \frac{1}{2}v = n \sin r$, in which v is the angle between the telescopes. If $v = 90°$, $A = .267\,D$, and if $v = 0°$, $A = .33\,D$. If the surfaces are curved, it is also possible to determine the curvature of both, from the two images, but the problem is then much more complex.

Another method is to place the telescopes opposite each other, and cover half their object-glasses with the plate to be tested. If the two surfaces are plane and parallel, no effect will be produced. If they are inclined, they form a prism, and cause a second image. If D is the angular interval between the two images, and A the angle between the two faces, $A = \frac{D}{n - 1}$, or if $n = 1.5$, $A = 2D$. Comparing this formula with that given above, it is evident this method possesses only about one-seventh the delicacy of the other, since for a given value of A, the divergence of the two images is only a seventh part as great. The delicacy of the method by reflection may also be increased indefinitely by increasing v. If the surfaces are curved they act like a lens, and throw the image out of focus. The problem now becomes indeterminate, as there is only one equation to determine the curvature of both faces. The focus of the equivalent lens may, however, be found by measuring, as before, the change in position of the eye-piece, when the focus f will equal $\frac{F^2 + d(F - a)}{d}$. If d is very small, $f = \frac{F^2}{d}$, which is the best method of measuring the focus of a very flat lens. Thus, if $F = 24$ inches, and $d =$ one inch, f will equal nearly fifty feet. As in the case of reflection, any irregularity of the surfaces produces a distortion of the image. Test in this way the plates of glass, and the lens.

Still another method of testing the flatness of a glass plate is to form Newton's rings, using the monochromatic light of a soda flame. Very slight irregularities in the surface will then appear covered with yellow and black rings, like contour lines.

As it is often desirable to increase the reflecting power of a plane surface of glass when it is to be used as a mirror, the most common methods of silvering are here appended. A looking-glass is made by covering the back of the glass with an amalgam of mercury and tin, as follows. Lay a sheet of tinfoil the size of the glass to be silvered on a level surface, and pour some mercury upon it, making it spread over the whole surface with a hare's foot. Lay a sheet of paper on it, and the glass over all. Then draw the paper slowly out, when the mercury, as it is exposed, will unite with the glass, and the paper will remove any adhering dust. Special care is needed that the tin, mercury, paper and glass, should be perfectly clean, and that no bubbles remain under the glass. Sometimes the paper is dispensed with, and the glass slid on over the mercury, bringing it first in contact at one corner. It is then subjected to pressure, and set up on edge to drain. It is best to keep this mercury by itself, as if used for other purposes, it is difficult to remove the tin, which gives much trouble by adhering to any glass surfaces with which it is brought in contact. When a bright light is viewed in such a mirror, holding it very obliquely, a large number of images is seen. The first, reflected from the front surface is faint, the second from the mercury is strongly marked, and these are succeeded by many others, caused by successive reflections, and growing fainter and fainter until they finally become invisible.

To obtain a single image only, sometimes a plate of black glass is used, or the lower surface is covered with black paint, or better, since much light is lost in this way, the front surface is covered with a deposit of metallic silver. One method of doing this is by dissolving 10 grms. of pure nitrate of silver in 20 grms. of water, and adding 5 grms. of ammonia. Filter, add 35 grms. of alcohol of specific gravity .842, and 10 drops of oil of cassia. Cover the plate with this mixture to a depth of quarter or half an inch, and add 6 to 12 drops at a time of a mixture of 1 part of oil of cloves to 3 of alcohol, until the whole surface is covered with the precipitated silver. The plate is then dried, cleaned and polished. Various other receipts are recommended, some using starch, sugar, or tartaric acid, instead of oil of cloves to precipitate the silver. Probably much more depends on the practice and skill of the experimenter than on the details of the different formulas. Liebig employs a liquid formed by adding soda-ley of sp. gr. 1.035 to 45 $cm.^3$ of an ammoniacal solution of fused nitrate of silver, and dissolving the precipitate formed by adding ammonia until the volume equals 145 $cm.^3$. Add 5 $cm.^3$ of water, and shortly before using it, mix with one sixth to one eighth of its volume of a solution of sugar of milk, containing 1 part of sugar, to 10 of water. Flood the glass to a depth of half an inch, and it will soon become covered with a thick coating of silver.

Another method of making reflectors, is by platinizing glass, or covering it with a layer of metallic platinum. This is accomplished by covering the surface with chloride of platinum with a brush, reducing it to a metallic state by oil of lavender, and heating it in a muffle.

84. Testing Telescopes.

Apparatus. A long darkened chamber with a small aperture at the farther end, through which the light of the sky, or of a lamp, shines. A long empty water-pipe, or unused flue, answers very well for this purpose, but if this cannot be obtained, a large black screen with a small hole in it may be placed at the farther end of the room, and a short tube blackened on the interior, used to cut off the stray light. A double length may be obtained by placing a plane mirror at the farther end of the room, and the screen close to the observer. A telescope to be tested, which should have an object-glass at least three inches in diameter, is also needed. It is composed of two lenses, one concave, of flint, the other convex, of crown glass. The focus of the latter will be about three-fifths that of the two together. Suppose this is three feet, then the focal length of the crown glass will be about 22 inches. Procure two similar lenses of 20 and 24 inches focus, respectively. Combining the first with the flint gives an under-corrected, while the other gives an over-corrected lens.

Experiment. The principal defects to be sought for, are striæ or irregularity of the glass, spherical aberration or incorrect form, chromatic aberration or imperfect correction for color, imperfect annealing of the lenses, and wrong centering or want of coincidence of their axes with that of the telescope.

First, to test for striæ, direct the telescope towards the artificial star or minute point of light at the farther end of the room. Then remove the eye-piece, and placing the eye in the axis of the instrument a bright circle of light will be seen, which will cover the whole object-glass when the eye is exactly at the principal focus. If now the glass is free from veins, striæ, or other imperfections, this circle will appear perfectly uniform, otherwise the striæ are shown in a very marked manner. To determine whether they are caused by the crown or flint lens, remove the latter, and see if they still remain. Test in the same way the other two convex lenses, and sketch any striæ that may be present. Some cheap cosmorama lenses are made of common plate-glass, in which case they are often full of parallel striæ, running in the direction in which the glass was rolled.

To test for spherical aberration, place the eye a little beyond the focus, and pass a card through the latter. Since all the rays would intersect at the focus if there were no spherical aberration, the light would be instantly extinguished when the card covered this point. In practice, however, the bright circle of light assumes the appearance of a curiously shaped surface of revolution, from which the form of the lens is readily determined.

To test for chromatic aberration, examine the image of the artificial star with an eye-piece, precisely as when looking through the telescope at a real star. If the lens was perfectly achromatic, a very minute circle of light would be obtained, which would enlarge on pushing the eye-piece in or out, remaining all the time perfectly colorless. The change in size is due to the fact, that the rays of light form a cone of which the object-glass is the base, and the focus the apex. The field of view is really a section of this cone at right angles to its axis. If an uncorrected lens is used, since the violet rays are more bent than the red, they form a cone with vertex nearer the object-glass; accordingly, if the eye-piece is pushed in, the centre of the circle will be violet, and the exterior red. Owing to the unequal dispersion of different parts of the spectrum by the two glasses, it is impossible ever to obtain entire freedom from color, but the best correction is obtained when the eye-piece being pushed in, the circle has a bluish purple exterior, and when pulled out, a lemon green exterior. In the same way an under corrected lens should give inside the focus a pure purple, and outside a yellowish margin; an over-corrected lens will give a blue or violet color inside, and outside an orange margin. Use the three convex lenses in turn, and note the colors in each instance. Many other things may be learnt from the appearance of the artificial star. Thus if part of the object-glass is covered, the circle assumes the shape of the uncovered portion. Spherical aberration also shows itself by the formation of rings in the image of unequal brilliancy, and imperfect centering or obliquity of the lenses, by converting the circle into an ellipse, or throwing out a ray of light on one side. This effect is greatly increased by using the monochromatic light of a soda flame.

One other test remains to be applied, that for imperfect annealing of the glass. Lay a plate of glass horizontally in front of the window, so that the light reflected from it shall be polarized. Interpose the lens between it and the eye, and examine the transmitted light by a Nicol's prism, as will be described more in detail in Experiment 88. Any inequality in density of the glass will at once show itself by dark patches, which change their position as the prism is turned. Of course all these tests must be regarded as secondary to the real test of every large telescope by trying it on various celestial objects of known difficulty, and comparing the results with those obtained with other instruments of the same size.

Next measure the magnifying power of the different eye-pieces furnished with the telescope. The power of a small telescope or opera-glass is readily measured by looking simultaneously at the object with one eye, and at its image with the other, and comparing their relative magnitudes. The best object to be used is a large scale of inches, noting how many divisions as viewed by the eye, equal one or two as seen through the telescope. For high powers the best method is by the dynameter. Focus the telescope on a distant object, and turn it towards the sky, or other bright light. On holding a sheet of paper near the eye-piece, a bright circle of light is seen projected on it, which is really an image of the object-glass formed by the eye-piece. The diameter of the object-glass, divided by that of its image, equals the magnifying power. To measure accurately the diameter of the small circle, a spider line, or eye-piece micrometer, may be used, or a small reading microscope, whose objective is divided in two parts, which may be moved past each other a known amount by a micrometer-screw. Two images of the circle are thus formed, which may be rendered tangent to each other by turning the screw. The parts of the objective have then been moved a distance proportional to the diameter of the circle, which is thus measured with great precision. In cheap telescopes a diaphragm is sometimes inserted near the objective, thus reducing the available aperture, and increasing the sharpness of definition of a poor lens, though diminishing the amount of light without apparently reducing its size. This is detected by turning the eye-piece towards the light, and seeing if it is visible when looking through the very edge of the objective. If not, the diaphragm should be removed or an incorrect value of the magnifying power will be obtained. If focussed for near objects, the magnifying power is much increased; hence for purposes of comparison, objects at a great distance should always be selected when making this measurement.

85. Photography. I. Glass Negatives.

Apparatus. A small darkened chamber or closet is needed for this Experiment. In this, a sink is placed with an abundant supply of water, and over it a shelf for the bottles containing the various reagents described below. A glass or porcelain vessel, shaped somewhat like a card-case, is employed to hold the solution of nitrate of silver, and a flat dish for the hyposulphite of soda. A large number of plates of glass are needed on which the photographs are to be taken, and racks for holding them. In preparing the solutions, funnels, filter-paper, and other similar chemical apparatus are also required. The closet should be lighted by a gas-burner covered with a yellow shade, to cut off the actinic rays. The camera in which the photographs are taken consists of a blackened box with a convex lens in front, and closed behind by a plate of ground glass, or by the plate-holder which carries the prepared plate of glass.

Great care has been bestowed on various forms of lenses for cameras, and the best forms are somewhat expensive. Three kinds are commonly employed. First, portrait lenses which have a large aperture, and admitting much light work very quickly, but they only take in a cone with an angle of about 60°, and have not much depth of focus. That is, when focussed for a given distance, objects a little nearer or a little farther off will be indistinct. The second kind of lens is adapted for views; a small concave lens is inserted between two which are convex, thus giving a greater depth of focus, but not working so quickly as the preceding. The third class, as the globe and the Zentmayer lenses, take in a large cone of light, as 90°, but work very slowly, requiring one or two minutes even in strong sunlight. They have a great depth of focus, andmay be placed very near the object; but when this is a building, the perspective will be bad if the camera is brought too near. For the same reason, when taking a building, the glass plate should always be vertical, or a distortion will be produced. As no lens is perfectly achromatic, the focus of the actinic rays will not coincide exactly with the visual focus, being less for an under, and more for an over-corrected lens. The latter should always be avoided, and it is best to get one in which the two foci coincide as nearly as possible.

Experiment. Almost all photographic processes depend on first taking a glass negative, that is, a picture in which the bright portions shall be transparent, the dark parts, opaque. For this purpose the plate of glass is first prepared or rendered sensitive, then exposed in a camera obscura so that an image of the object shall fall on it. The plate is then developed, by which the image is rendered visible, and fixed or rendered permanent. These operations are easily performed when all the apparatus is ready, and the baths employed in good condition. The real difficulty in taking photographs, is in preparing the various solutions used, and in renewing them as they deteriorate; accordingly the following receipts are given, which the student should, if possible, try for himself.

The collodion used for coating the plates is made as follows. To 8 oz. of ether add 96 grains of gun-cotton, and then 8 oz. of alcohol. The strength of the latter must be 95 per cent., and it must contain no fusel oil, or the cotton will not be well dissolved. If made in the evening be very careful about lights, as the mixture may take fire from a lamp several feet distant. As the vapor is much heavier than air, there is more danger from lights on the floor than from those above. Dissolve 24 grains of bromide of potassium, in as little water as possible, add 64 grains of iodide of ammonium, and more water if necessary, and pour this into the collodion to iodize it. Pure iodide of ammonium will do, but it is better to have the iodine a little in excess, in which case, the salt will be dark colored instead of white. Too much iodide and too little bromide give a hard picture. In about two days the collodion will be ready for use, and it will keep in good condition about two weeks. Iodide of cadmium, which is commonly used in that which is sold, makes collodion keep better, but renders it less sensitive. If the weather is very hot, there is difficulty from the collodion drying too rapidly. In this case, less ether and more alcohol must be used, as the latter is much less volatile.

The silver-bath by which the plate is rendered sensitive, is made by adding 1 oz. of nitrate of silver to 12 oz. of water, and acidulating it with 30 or 40 drops of pure nitric acid. If used directly it would dissolve the iodides in the plate. Accordingly, a coated plate should be left standing in it over night. Filter, and in twenty-four hours it can be used. More depends on the condition of the silver-bath than on that of any other liquid employed. It should be kept nearly neutral, but always slightly acid. If alkaline the picture is fogged or blurred, and if too acid the action is much retarded. Organic matter is very injurious, and dust should therefore be carefully excluded by a cover. Whenever practicable, it should be exposed in a glass bottle to strong sunlight, which precipitates the organic matter in black flakes, and the latter removed by filtering. After using the bath for some time, it becomes covered with a scum, due to the alcohol and ether from the collodion plate, which is the first indication that the solution is becoming too weak. The iodide of silver is then liable to be precipitated on the plate, forming little spots in it like pinholes. One half its bulk of water should, in this case, be added, to precipitate the iodide, the solution filtered and boiled down to its original strength.

To develope the picture when taken from the camera, a solution of 1 oz. of proto-sulphate of iron, in about 20 oz. of water is used, and containing 1 or 2 oz. of pure acetic acid, No. 8. This liquid will keep only about three days. The object of the acetic acid is to retard the process, as otherwise the silver would blacken instantly.

The liquid used to fix the picture, is formed by dissolving 1 oz. of hyposulphite of soda in 5 oz. of water.

Be careful that the glass is not rusty or iridescent, as in that case the collodion is liable to cleave off. Double thick glass is preferable on account of its greater strength, unless the plates are small. There must be no "knobs" or glass dust on its surface, nor deep scratches, because these will appear in the positive if the printing is done by sunlight. Almost all plates are slightly curved, and it is essential that the hollow or concave side should be coated. The plate will then conform more nearly to the image in the camera, is easier to coat, is less liable to be scratched if laid down on its face, and is not so likely to be broken in printing.

To clean the glass, mix equal parts of alcohol and ammonia, and add enough rotton-stone to render it viscid. Pour a few drops on the glass and scour with wash leather, letting the plate rest on one corner. Clean also the edges carefully. Let it dry, and then rub off all the rotton-stone with clean flannel. Breathe on the plate, and if clean, the moisture will pass off rapidly and evenly from the surface. Thus cleaned, they will keep for two days. A better method is the following. Soak the plates over night in strong nitric acid, and wash thoroughly under a faucet. Mix up the white of an egg with an egg-beater, taking care that none of the yolk gets in, add 12 ounces of water and 3 or 4 drops of concentrated ammonia, which keeps the egg from souring, and neutralizes any acid that may remain on the plate. Filter through paper or cloth and keep the filtrate in a bottle. Pour it over the plate, and a uniform film is produced, which will last for six months if kept dry and free from dust. Another method is to soak the glass over night in a strong solution of caustic potash and then wash it, but this is liable to injure the silver bath. If any nitric acid gets in, it merely retards the action, but alkali will cause fogging, or the image will look smoky, as if under a veil.

To coat a plate properly with collodion requires considerable skill and practice, as it is very essential that the coating should be perfectly uniform. First, remove any films of collodion that may have dried on the mouth of the bottle, and take care never to disturb the sediment at the bottom. Regarding the plate as a map, it is held by the *S. W.* corner with the thumb and forefinger of the left hand, and the collodion poured on just *N. E.* of the centre. Enough is added to cover about half the plate which is then inclined, so that the liquid shall flow successively to the *N. E.*, the *N. W.*, the *S. W.* and the *S. E.* corners, and then tipped so that it will run off into the bottle. Allow it to drain for a few seconds and incline it gently from side to side, to prevent its forming streaks. The ether soon evaporates, and as soon as the film becomes sticky and consistent, the plate is immersed in the silver bath, by laying it on its holder, and lowering it into the liquid. This must be done slowly and steadily, or streaks will appear across the plate. If the plate is too large to be held by one corner while coating it, lay it on the palm of the hand, and interpose a sheet of card-board to prevent the warmth from drying the collodion too rapidly. Still larger plates must be placed on a board and rested on a point attached to the top of a tripod.

After remaining a few seconds in the bath, the plate should be raised gently out of it, when its surface will present a greasy appearance, due to the ether still remaining in the film; soon however, it will appear to be perfectly wet by the solution, and is then ready to be transferred to the camera. The plate-holder consists of a frame closed in front by a slide, and with a hinged back. The latter is opened, the plate put in with the coated side next the slide, and the latter then closed. A black cloth is thrown over it to cut off any stray light that may leak in through cracks in the holder, and it is then carried to the camera. The object to be taken having been placed in the proper position, and in a good light, the camera is turned towards it, when an image will be formed on the ground glass. This is best seen by standing behind the instrument, and cutting off the light by a black cloth thrown over the head. The size of the image may be varied by altering the distance of the camera, and the latter focussed by changing the distance between the ground glass and lens. For near objects the focus is greater and the size of the image larger than for those more distant. The image may be brought to the centre of the glass, by turning the camera or inclining it. When the image is satisfactory and carefully focussed, remove the ground glass, and replace it by the holder. Cover the lens with a cap or black cloth. Then draw the slide of the holder, and when all is ready, expose the plate to the light by removing the cap. The time of exposure varies with the light, the object, and the kind of lens, and must be learned by experience. When the time has expired, replace first the cap, and then the slide, and taking out the holder carry it into the dark room. Take care and never turn the plate-holder over after the plate is in it, as the silver collects along the lower edge and would, if inverted, flow over the glass, forming streaks.

On taking the plate out of the holder no trace of the picture is visible, but the film merely appears of a creamy white throughout. Pour on some of the developer so as to cover the plate at one flow (or streaks will be formed), holding the glass horizontal with the prepared side up, and in a few seconds the picture will appear, the portions acted on by light turning dark, the others remaining unchanged. The plate is then washed under the faucet to remove the developer. If the exposure has been too long, the picture will develop instantly, giving a dense blurred negative. With too short an exposure or too feeble a light, a faint transparent picture is obtained, which developes only very slowly. In this case, it may be improved by re-developing, or pouring on a mixture of the silver bath and developer as when developing. Pryrogallic acid is sometimes used instead of sulphate of iron.

The picture is then rendered permanent by immersing it in the solution of hyposulphite of soda, which dissolves the iodide of of silver, where unacted on by light, rendering these parts of the plate transparent.

It is still easily scratched, and should be varnished if it is to be handled much. For this purpose, it is warmed over a lamp until hot to the touch, and amber varnish poured on precisely as when coating a plate. If not warmed, the varnish gives a precipitate. Be careful that it does not catch fire, as it then dries in ridges. Warm again gently, to harden the varnish. The completed negatives are best kept side by side and vertical, in racks.

The most common difficulty in taking photographs is fogging, or the picture appearing misty or indistinct. In this case, test the silver bath, and if it is alkaline add a little nitric acid. If too much is added the process is retarded, and it must be nearly neutralized with ammonia. Great care must be taken that the plate is not exposed to stray light before or after placing it in the camera, and that the latter and the plate-holder have no cracks in them, through which the light may enter.

When taking views, the camera should never face the sun if it can possibly be avoided. If this is unavoidable, hold a hat or board so that its shadow shall fall on the camera, thus cutting off the direct rays of the sun from the lens. When light and dark objects, as the sky and deep shadows, are to be taken at the same time, hold a screen in front of the lens, to cut off the brighter objects until near the end of the exposure, and then remove it.

As objects for the stereoscope or lantern, glass positives are needed, that is, photographs on glass, in which the light portions shall be transparent, and the dark parts opaque. A negative is first taken and then photographed like any other object, reflecting the light of the sky through it by a sheet of paper, and cutting off all stray light in front. Porcelain photographs were formerly taken in this way, but both are now often printed like paper positives. Ambrotypes are negatives exposed for a short time, so that the dark portions are very transparent, and are rendered black by placing a sheet of tin covered with black varnish behind them.

86. Photography. II. Paper Positives.

Apparatus. Some albumenized paper, and the reagents needed to render it sensitive and to make the picture permanent. Also some negatives to be copied, and several printing-frames in which to expose the paper to the light. Strong sunlight is requisite for this Experiment, and it is much better to place the printing-frames outside the window, rather than let the sunlight first pass through the glass. Three flat porcelain dishes are needed to hold the liquids in which the paper is immersed, spring clothes-pins by which to hold it when drying, and an abundant supply of water in which to wash it.

Experiment. It is difficult to judge of the true appearance of an object from the glass negative, as the lights and shades are inverted, and, moreover, several copies of a photograph are often wanted for distribution. It is therefore customary to print positives on paper, by the process described below. A fine grained paper is employed, covered with a thin layer of albumen, which fills up the pores and gives a good surface. Excellent albumenized paper may now be procured ready-made, as it is manufactured on a large scale, for photographic purposes. When the picture is to be touched up with India ink, or colored, common paper is often preferred, as the paint will not adhere to the albumen. To render it sensitive, a solution of nitrate of silver is prepared, of a strength of about 40 grains to the ounce, but depending on the amount of chloride of sodium in the paper. It should also be somewhat stronger in summer than in winter. Render it alkaline by a few drops of ammonia, and pour it into a flat porcelain dish. If any scum appears on the surface, remove it by a piece of tissue paper.

Take a piece of the paper, somewhat smaller than the dish, by the two opposite edges, with the albumenized side down, and dip it into the dish so that the centre line shall touch the liquid. Lower the edges so that the paper shall float on the liquid, taking care that no air-bubbles are imprisoned under it, or they will form white circles on the picture. Breathe on the edges of the paper to prevent their curling up, by the warping due to the expansion of the lower side, and take great care that no silver gets on the upper side of the sheet. The time of immersion must be found by trial; it is generally one or two minutes. If too long, the silver works through the albumen into the paper, turning it yellow on exposure, while if too short, the salt does not decompose and fill up the surface with the silver, so that a mottled appearance is produced. To obtain the best results, various devices are employed, such as fuming the paper with ammonia, which gives a clearer picture. After taking the paper out of the bath it should be hung up by the corners by spring clothes-pins, in the dark. It will only keep a short time, in winter for a day or two, in summer scarcely over night. The prepared surface should never be touched by the fingers, or their mark will appear in the finished picture.

The paper must now be exposed to sunlight under the negative. The pressure frame in which this is effected is made somewhat like the frame of a picture, only it is much more substantial, and so arranged that the back can be removed; the latter, when in place, is pressed strongly against the glass in front, by springs. The back is lined with felt, and hinged, so that one half can be turned back, and part of the picture examined without disturbing the remainder. Having cut the paper of the right size, the pressure-frame is placed face downwards, and the back removed. A plate of glass is first inserted, like the glass of a picture, then the negative with the prepared side up, next the paper with the albumenized side down, and finally the back of the frame is restored to its place. Great care must be taken always to place the prepared sides of the glass and paper in contact, as otherwise only a blurred picture will be obtained.

Now turn the frame over, and place it in strong sunlight, and inclined at such an angle that the light shall fall on it nearly normally. It will soon be seen that the unprotected portions of the paper, that is, those under the transparent parts of the negative, are turning under the influence of sunlight from white to black. After a few minutes, take the frame out of the light, and opening half the back, bend the paper so as to examine it, when the picture will appear in its true aspect with shades dark, and the bright portions, light. If the picture is not dark enough, replace the back, and thus proceed, until the paper is somewhat darker than is desired for the finished photograph. The picture thus obtained is of a purplish color, and if exposed to light would turn completely dark. It must therefore first be toned, or its color altered, and then fixed, or rendered permanent.

The bath for toning is made of a solution of chloride of gold. Procure some pure gold from a gold beater, dissolve it in *aqua regia* and evaporate to dryness. Dissolve 5 grains of the yellow chloride of gold thus obtained, in 1 oz. of water, and neutralize it with carbonate of soda. This must then be mixed with about ten times its bulk of water, using a stronger solution in cold than in warm weather. Wash the picture thoroughly in running water for five or ten minutes, to remove all the free silver which would otherwise precipitate the gold and give a foggy picture, and then dip it face downwards into the toning solution. The effect of this will be to alter its color until a certain reddish blue tint is attained. The best effects are obtained with a lukewarm solution, of such a strength as to tone in about ten minutes. If not toned sufficiently the color is reddish, and if this process is carried too far it takes the life out of a picture, and gives it a chalky appearance. In the same way, too rapid a toning gives a mealy look to the photograph. As the paper withdraws the gold gradually from the solution, rendering it weaker, more should be added, first neutralizing it with soda. It is well to move the bath gently, swaying it from side to side. Be very careful that no hyposulphite of soda gets into the gold solution, as it would spoil it, forming a precipitate. As the paper is still sensitive to light, the toning should be carried on in a dark room, just light enough to distinguish clearly the color attained. To stop the toning, the photograph is next washed in cold water to remove the gold; it is then rendered permanent by immersion in a solution of hyposulphite of soda, 1 oz. of the salt to 8 oz. of water. As the picture is rendered lighter and redder by this process, allowance must be made, in the printing and toning. The photograph must now be washed very thoroughly in running water for an hour or two, or better, over night, to remove every trace of hyposulphite. Otherwise a black sulphide of silver is formed, which turns in time to red, and spoils the picture. To see if all the hyposulphite is removed, hold the paper up to the light, when it should appear clear; if mottled, the washing must be continued.

It now only remains to mount the photograph, that is, to paste it on to cardboard, to make it flat and stiff. Lay it face upwards on a plate of glass, and lay on it a second plate, of the size to which it is to be cut. It is thus easily centered, and cut to the right size by drawing a sharp knife along the edges of the glass, holding the blade always at the same angle of inclination. Attach it to the cardboard by paste made of the best wheat starch. The latter is first moistened with cold water, and boiling water then poured upon it; if it does not come thick enough it may be heated for a moment to the boiling point, but should not be boiled, otherwise it becomes watery and loses its adhesiveness. To prevent the cardboard from curling up, it is well to moisten it before attaching the photograph. The whole should then be passed through heavy rollers to give a good finish to the picture.

87. Testing the Eye.

Apparatus. A set of test-types, or letters of various sizes, should be placed at distances from the student proportional to their size. On the table are placed the optometers described below, a reading microscope on a stand, for the experiment of Cramer and Helmholtz, a gas flame, tests for astigmatism, and a set of concave, convex and cylindrical lenses of various curvatures, and prisms of various angles.

Experiment. The eye is formed like a camera obscura, in which the retina takes the place of the screen on which the image is received. In front of the lens is a delicate curtain, called the iris, which gives to the eye its color, and in this is a circular hole, the pupil. The iris is formed of fibres, some circular, others radial, the contraction of the first diminishing, of the second increasing the size of the pupil, and hence the amount of light admitted into the eye. These changes are readily seen by covering the eye with the hand, removing the latter, and looking in a mirror; the pupil will then be seen to contract slowly, having dilated in the dark. The pupil of the other eye will also contract a little, as they both commonly act together.

The image of objects at various distances, may be brought to a focus on the retina by varying the form of the lens, while in the camera it is effected by varying its position. By this change which is called accommodation, objects may be seen with perfect distinctness, with a normal or perfect eye, at any distance, from about 4 inches to infinity. Call P the nearer distance, and R the farther, for any eye, then $\frac{1}{A} = \frac{1}{P} - \frac{1}{R}$ is called the range of accommodation, and is much employed in studying defects of the eye. In the case of the normal eye, the range of accommodation evidently equals $\frac{1}{4}$. The most common defect to which the eye is subject, is that the ball is not spherical. If it is elongated, the retina is carried too far off, and objects must be brought nearer the eye, to render them distinctly visible. Such an eye is called myopic, or near-sighted. If the ball is flattened, near objects cannot be easily seen, and the eye is then hypermetropic, or far-sighted. This must not be confounded with the effect of age, which renders the lens harder and thus diminishes the range of accommodation, so that distant objects alone can be seen. The eye is then said to be presbyopic. The normal eye is called emmetropic.

To measure the far and near points optometers are used. One of the simplest of these consists of a board on which a straight line is ruled. At one end is a sheet of metal, with two fine slits very near together. The eye is placed close to the slits, so as to look through both, when it will be noticed, that the nearer end of the line appears double, since the images formed by the two slits cannot be brought together by the eye, on account of the short distance. Sometimes, also, the farther end will appear double, if the eye is myopic. The points, where the line divides, give the far and near limits of distinct vision. A better form of optometer, resembles the apparatus represented in Fig. 60, only it is much smaller. The lens, which should have a focus of 6 inches, is fixed at the end of the rod, in the place of the gas-burner, A, and some very fine print, or other minute object, is attached to the screen, C. Now measure the greatest and least distance at which the print can be read, when the eye is placed near the lens. Call these distances P' and R'. Then $\frac{1}{P} = \frac{1}{P'} - \frac{1}{6}$, and $\frac{1}{R} = \frac{1}{R'} - \frac{1}{6}$, which gives P and R, the far and near distances of accommodation. For the normal eye, as $P = 10$, $R = \infty$, P' should equal 2.67, and $R = 6$.

Another excellent form of optometer is very simply made of a sheet of cardboard or brass. This is pierced with three sets of holes, the first a single hole 1 mm. in diameter, the second several smaller holes near together, for instance three rows of three each at intervals of a millimetre, and thirdly two holes 3 mm. apart, one of which may be covered with a plate of red glass. View a small distant point of light, as a candle or gas flame, through them, and the appearance will vary according as the eye is normal, far or near sighted. Looking through the single hole and moving the card rapidly from side to side, the light will appear to remain stationary, if the eye is normal, otherwise it will appear to move as the rays pass through different portions of the pupil. In the same way the nine holes will give nine images, if the eye is not normal. If the two holes are used, two circles are seen which overlap as the card is brought near the eye. If the eye is not normal two images will be formed in the overlapping part, since the rays falling on different parts of the lens are not brought together to the same point on the retina. Now cover the right hand hole with the red glass, and if the eye is far-sighted the left hand one will be colored, if near-sighted the right one, since in the latter case the rays cross, coming to a focus before reaching the retina. From the distance between the images, the amount of the defect may be measured. Thus bring a second candle flame near the first, until two of the images overlap, forming three instead of four; the distance between the candles then equals the interval between the images, and from it the lens required to render the eye normal, may be determined. Let D be the interval between the images, d that between the two holes, and B the distance from the light to the eye. Then, $D : d = B : f$ in which f is the focal length of the lens required to produce distinct vision. By turning the card the two images will appear to revolve around each other, and if the eye is astigmatic their distance apart will vary. If the eye is normal all these effects may still be observed by putting on convex, concave, or cylindrical glasses. When observing one's own eye it is often more convenient to view the reflection of two lights near at hand in a distant mirror, so that their distance apart may be more easily varied.

The best test, however, for the eye, is to see if all the test-letters can be read easily. To understand how objects look to a near-sighted person, put on a pair of convex glasses, and repeat these observations with them. Do the same with concave glasses, which give the effect of hypermetropia. See also if the foci of the glasses can be determined correctly from these observations.

Another defect present in some eyes is astigmatism, or unequal focus for horizontal and vertical lines. For example, the eye may be normal for vertical, and near-sighted for horizontal lines. It is detected by looking at a test made of several series of strongly marked equidistant lines, running in various directions. This defect is corrected by using cylindrical lenses, or if, as often happens, the eye is myopic or hypermetropic at the same time, by means of lenses cylindrical on one side, and convex or concave on the other. Many persons who could never see well with common glasses, experience wonderful relief from such lenses. Sometimes the axes of the eyes are not quite parallel, a defect remedied by the aid of prisms, with very acute angles.

Many theories have been advanced to explain accommodation, some supposing that the retina was drawn back, others, that the lens moved, and others, that the ball of the eye changed its shape. The true explanation is deduced from the following experiment, devised and worked out quantitively, by Kramer and Helmholtz. Two persons are required; one, whose eye is to be examined, sits facing a candle, or gas-burner, while the other examines with the reading microscope the reflection of the light in his eye. Three images will be seen, as shown in Fig. 62, in which V is intended to represent the reflection of the candle flame.

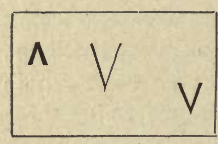


Fig. 62.

The eye being directed towards a distant object, the first image to the right is formed by reflection in the cornea, or front surface of the eye. It is bright and upright, as the surface is convex. The second is formed by the front surface of the lens. It is much fainter and larger, but also upright. The third being formed in the posterior and concave surface of the lens, is minute and inverted. Now let the eye be directed towards a near object. The first and third images will remain unchanged both in size and position, showing that the cornea and rear surface of the lens are not altered, either in position or curvature. But the second image, as shown in Fig.

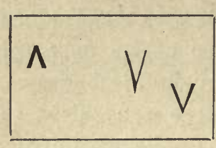


Fig. 63.

63, approaches the first, and diminishes in size, showing that the front surface of the lens is pushed forward, and becomes more curved. Measurements also show, that the amount of the change is just sufficient to account for the required difference in focus. This experiment is very conclusive, as each of the other hypotheses is disproved by it. If the cornea altered, the first image only should move. If the lens moved, the second and third images should approach the first without altering their size, and if the form of the ball altered, the relative position of all three should remain unchanged.

All parts of the retina are not equally sensitive; although the eye can perceive objects through an angle of about 150° horizontally, and 120° vertically, yet the portion where vision is most distinct is quite small, not more than 3° or 4° in diameter. This portion of the retina, which is called the macula lutea, is used almost exclusively whenever objects are carefully examined, and probably on this account, is not quite as sensitive to very faint objects as the adjacent parts. At any rate, it is very customary with astronomers, when trying to see very faint objects, to direct the eye a short distance from their supposed place, and try to catch sight of them, when not in the centre of the field of view. A short distance from the macula lutea, on the side towards the nose, is a small circle where the optic nerve enters. This space, although so near the most sensitive portion of the retina, is totally insensible to light. It is called the papilla, or sometimes the punctum cœcum, or blind spot. To observe it, mark two points on a sheet of paper, about 4 inches apart, and closing the left eye, direct the other to the left hand point, and then moving the paper to and fro, a certain distance will be found, at which the other point will completely disappear. By using two lights, this experiment may be rendered still more striking, as even a bright light may be made to completely disappear, although objects all around it are visible.

A great variety of experiments may be made, depending on the stereoscopic effects obtained with two eyes, or on the persistence of vision, using such instruments as the thaumatrope, chromatrope, and phenakistascope.

88. Opthalmoscope.

Apparatus. The instrument known as the "Opthalmoscopic Eye of Dr. Perrin," is admirably adapted as a substitute for a human eye, on which to use the opthalmoscope. It consists of a brass ball, on a stand, representing the globe of the eye, a series of cups, painted to represent different diseases of the retina, which may be inserted in its rear portion, and lenses, representing the cornea and lens, which may be screwed on in front. To these, diaphragms, representing the change in diameter of the pupil, or aperture in the iris, may be attached. An Argand burner is needed, and a Gräfe's opthalmoscope, also some plates of glass, and a small mirror, with the silvering removed from a circle a quarter of an inch in diameter. This Experiment should be performed in a darkened room, but instead, the light may be cut off by a large screen of black cloth.

Experiment. The opthalmoscope which is used in studying the interior of the eye, has caused a complete revolution in this branch of medical science. Its inventor, Helmholtz, reflected light into the eye, by a piece of plate glass, and then looking through it, found the interior sufficiently illuminated to be visible.

Take the model of the eye from its box and place it on its stand. Insert the retina marked 1, which represents the normal or healthy retina. Screw on in front the lens marked *E. M.*, and light the burner, placing it by the side of the model eye, and about a foot distant. By reflecting the light into the eye by a plate of glass, and looking through the latter, Helmholtz' experiment may be repeated, and a view of the interior obtained. This is more easily accomplished by using several plates, or better still, with the mirror from which the silvering has been partially removed.

The opthalmoscope consists of a circular, concave mirror, with the silvering removed from the centre. Just behind it is placed a fork, in which either of the five small lenses may be placed. They should be numbered on their edges from 1 to 5, the former being the most concave, the latter the most convex. The retina of the normal eye is placed at a distance from the lens, equal to its principal focus, hence its image is formed at an infinite distance, or the rays emerge parallel. It can therefore be viewed without any lens, using the mirror precisely as in the previous experiment. The image is better seen, if one of the lenses 1, 2, or 3 is inserted in the fork, as the distance of the image is then diminished, so that the rays diverge, instead of emerging parallel.

This method has the objection of showing only a small portion of the retina at a time, and of bringing the observer too near the eye for convenience. Another method is therefore more commonly used, in which an aerial image is formed in front of the eye, by a convex lens, and this is viewed either directly with the eye, or with a second convex lens of long focus.

Hold the large lens, or objective, two or three inches in front of the model, with the left hand, steadying it with the little finger, which, in the case of the real eye, rests on the forehead of the patient. The mirror should be held a foot or more distant, and turned into such a position as to reflect the light of the gas flame into the model. After a few trials a very beautiful view of the retina will be obtained. The image will be inverted, and may be made as large as the objective by removing the latter to a distance equal to its focal length. To view the other portions of the retina, the model must be turned from side to side, or the patient requested to direct his eye towards various points in turn. The image is improved by placing lens 4 or 5 behind the mirror. In all these experiments, if near-sighted, use a lens with a number lower than that here recommended; thus, instead of 2, use 1, for 5, use 4, etc. The first of the above methods, that is, without the objective, is called the direct, the second the indirect method.

Now screw the larger diaphragm over the lens, and try once more to view the image; then replace it by the small diaphragm, with which it is about as difficult to observe the retina as with the eye in its normal condition. The larger diaphragm corresponds to the case where the pupil is expanded by belladonna. With the small diaphragm it is easier to look a little obliquely into the eye, thus avoiding the light reflected from the lens, which gives bright reflected images of the mirror.

When the eye is near-sighted, or myopic, the retina is beyond the principal focus of the lens. This effect is produced in the model by partly unscrewing the lens, taking care that it does not fall out. An image of the retina is thus formed in front of the lens, or the rays from it converge. Hence when employing the direct method, a concave lens, as 1 or 2, must be placed behind the mirror, to render the image visible. When the eye is far-sighted, or hypermetropic, and incapable of viewing near objects, the retina is nearer the lens than its focus. Hence the image is formed at a considerable distance behind the lens, and can readily be viewed by the direct method without any lens. This effect is imitated by using the lens marked *H*, which has a longer focus than the other. The difference is not perceptible by the indirect method, since it is neutralized by a slight motion of the mirror.

Sometimes the cornea has a different curvature in horizontal and vertical planes; it is then said to be astigmatic. The third lens marked *A*, shows this defect. With this, it is a little difficult to view the retina clearly, since the focus is different in different planes; it will be noticed, however, that the papilla assumes an elliptical, instead of a circular form.

Now replace the emmetropic lens, and insert the various diseased retinas in turn, using the small diaphragm, or, if necessary, the larger one. The retinas are numbered, and represent the following conditions of the eye: —

1. Normal retina.
2. Atrophy of the papilla and retina.
3. Atrophy of the choroid.
4. Staphyloma posterior, an old case; blood focus near the macula lutea.
5. Hemorrhage of the retina.
6. Alteration of the retina.
7. Staphyloma posterior. Separation of the retina.
8. Infiltration of the papilla with blood.
9. Exudation of serous fluid, between the choroid and retina.
10. Glaucoma, with the circle of atrophy of the choroid around the papilla.
11. Glaucoma and hemorrhage of the retina.
12. Atrophy of the papilla, and of the choroid around it.

The papilla is the point of entrance of the optic nerve. The macula lutea, the point of the retina most used.

Atrophy means the gradual wasting away, and absorption of any substance. Staphyloma, a thinning of the covering of the eyeball, especially around the optic nerve, allowing this portion of the ball to extend backwards. Glaucoma is an increase in quantity of the vitreous humor within the eye, causing a distention of the eyeball, accompanied with acute pain.

89. Interference of Light.

Apparatus. To observe the interference of light, a diffraction bank is employed, which consists of a long horizontal bar divided into millimetres, and carrying sliding uprights, to which the following instruments may be attached, and placed at any desired distance apart. A cylindrical lens to produce a bright line of light, and a brass plate with a slit in it of variable width, like that of a spectroscope. A biprism, or prism of glass with a very obtuse angle, by means of which two closely adjacent images of any object will be formed, and a double mirror designed for the same purpose, whose two halves are inclined at a very small angle, which may be varied by means of adjusting screws. To observe the various effects produced, one of the uprights carries a spider-line micrometer, or a simple eye-piece with cross hairs, which may be moved laterally, and its position determined by a millimetre scale and index. Or, this may be replaced by a small direct vision spectroscope, to analyze any portion of the light passing through the instrument. A screen of ground glass, or paper, may also be substituted for the eye-piece. Although some of the simpler phenomena are visible by ordinary light, yet to obtain the best results sunlight is indispensible. An arrangement is also desirable by which an intense monochromatic light may be obtained, which may be done roughly by interposing colored glasses, but much better by placing a prism in front of the slit, throwing a ray of sunlight through it, and projecting the spectrum thus obtained on the slit. When the day is cloudy, a soda or lithium flame may be employed. A cover should be placed over the whole to cut off the stray light, or a simple piece of black cloth may be employed for the same purpose.

Experiment. According to the Undulatory Theory all space is supposed to be filled with a very rare medium, called ether, whose, vibrations give rise to the phenomena of light. A luminous point throws out concentric spherical waves, whose diameters increase with very great velocity, and each of whose radii is called a ray of light. The direction of the vibrations is transverse, that is, perpendicular to the ray, as is the case of waves of water, and the terms crest and trough are here also used to denote the two opposite positions of any portion of the ether. The distance from one crest to the next determines the color of the ray, and is called the wave-length. In the same way, the intensity of the light depends the height of the wave, or distance traversed by each particle. A particle of ether can receive any number of systems of vibrations, whatever their wave-length, intensity, direction, or plane of vibration, and will transmit each precisely as if the others did not exist.

If a particle receives two rays of light, precisely similar in every respect, under the influence of both, its motion will be increased, and a more intense light produced. Now suppose one of the rays is retarded by half a wave-length ; its crests will coincide with, and neutralize the troughs of the other ray; accordingly the particle will not move at all, and the result will be darkness. The same effect will also be produced if one ray is retarded three, five, or in fact any odd number of half wave-lengths, while if the retardation is an even number of half wave-lengths, crest will fall upon crest, and the light will be increased. This neutralization of one ray by another, or light added to light, producing darkness, is called interference, and by means of it many most important laws have been established.

To produce interference, two precisely similar sources of light are needed, at a very short distance apart. For this purpose, place the cylindrical lens on a support at one end of the diffraction bank, and throw a beam of sunlight through it. A very narrow line of light is thus produced at its focus. Place the biprism on a support, at a short distance in front of it, and two images will be formed very near together, and precisely alike. If, now, a screen is placed near the other end of the bank, its centre will appear bright, since being equidistant from both images it will receive simultaneously the crests and troughs of them both. If, however, a point is taken on one side of the centre, it will be nearer one image than the other, and, accordingly, the crests and troughs will not arrive simultaneously. If, then, the difference of path is an odd number of half wave-lengths, darkness will be produced, while an even number will give brightness.

The consequence will be a series of vertical black bands, corresponding to 1, 3, 5, etc., half wave-lengths. As, however, the light is white, and is composed of rays of all colors, and various wave-lengths, the bright and dark spaces will be at different distances for each, consequently a series of colored bands will be produced, with a white centre. These bands are much more visible with the eye-piece, and their number is increased by employing monochromatic lights.

Their position affords a means of determining approximately, the length of a wave of light, as follows. Bring the cross-hairs to coincide successively, with each of the visible bright bands, when monochromatic light is used, and read the position of the eye-piece. Measure, also, the distance from the slit, or focus of the cylindrical lens to the cross-hair. If, now, the distance between the two images can be determined, a simple calculation will give the difference in their distances from the bright band, that is, one, two or three times the wave-length, according to the number of the band. The distance between the images may be found by the following device. Insert a lens between them and the eye-piece, having a focal length about one fourth their distance apart. There will be two positions, in which the images will be distinctly seen. Bring the cross hairs to coincide with their images, as formed with the lens, by moving it laterally, taking care to move the lens only, when focussing. In one case, the distance between the two luminous lines will be magnified, and in the other, diminished, in precisely the same ratio. Accordingly, the mean proportional of these two measurements will give the true distance with accuracy.

To calculate the wave-lengths from these measurements, let D be the distance from the slit to the cross hairs, d the distance apart of the two images, and b the distance of the first band from the centre, equals one half the distance between the images on opposite sides of the centre line. Then by similar triangles, since b and d are always very small, compared with D, it follows that $D : b = d : \lambda$ the required wave-length. In the same way the other bands give 2λ, 3λ, etc. Repeat this measurement, with other positions of the eye-piece, and rays of other colors, and notice that the more refrangible the ray, the shorter the wave-length.

As the position of the eye-piece is varied, any given band will evidently lie on a hyperbola with the two images as foci, since it is the locus of a point, whose distances from two others differs by a constant amount. To prove this, observe the position of one of the outer bands, varying D five centimetres at a time, and represent the results by a curve, in which D gives the abscissas, and an enlarged value of b, the ordinates. Construct also the theoretical curve on the same sheet.

Now replace the eye-piece by the spectroscope, and moving it laterally, the bright and dark bands, in turn, fall on the slit, and the colors of which they are composed may then be determined with precision. Thus starting with the central bright band, it will be found to give a continuous spectrum, traversed, of course, by the usual solar lines. In the first black band, all the colors disappear, then all the colors reappear, beginning with the violet, and as the slit is moved still further, a dark band will enter the violet end of the spectrum, and will traverse it, soon to be followed by a succession of others, whose distance apart becomes less and less, until finally, many are visible in the spectrum at the same time.

Similar effects may also be obtained with the double mirror, taking care that the two halves are slightly inclined, and that their edges meet exactly. This is accomplished by means of the adjusting screws. It will then form two closely adjacent images of any object reflected in it. Place the mirror in such a position, that it shall reflect two images of the slit (which is moved a short distance to one side) along the bank, when the bands are formed by interference, precisely as with the biprism. A diaphragm must be interposed, to cut off the direct rays of the slit from the eye-piece. As the interval between the two images diminishes, the bands become more spread out, as may be shown by diminishing the inclination of the two halves of the mirror, by means of the adjusting screws.

90. Diffraction.

Apparatus. Besides the diffraction bank employed in Experiment 89, a number of brass plates are needed, which may be inserted in the sliding uprights, and which are perforated with apertures of various shapes and sizes. Thus one will carry a slit of adjustable width, a second a large aperture half covered by a plate with a vertical edge, others with two closely adjacent slits, a vertical wire, circular holes of various sizes, and some with two holes a short distance apart. An immense variety of effects may be obtained by using apertures of different shapes, and sometimes a number of these are photographed on a plate of glass and brought successively into the axis of the instrument. A plate of glass on which a large number of equidistant fine lines are ruled is also needed, and pieces of wire gauze and lace with square and hexagonal meshes. The effect of these various apertures is best shown by placing them in front of a telescope directed toward an artificial star, as in Experiment 84, or the optical circle may be used, replacing the slit by a minute circular aperture.

Experiment. The phenomena of diffraction are best explained by a comparison with the similar effects produced by water. Suppose a long straight wave, like a breaker rolling in on a beach, encounters in its passage a rock, or the end of a breakwater. After passing, instead of leaving the water quite at rest behind the obstacle, the end of the wave will spread inwards, so that if the rock is small the two portions will meet after moving some distance, and no portion of the surface not close to the rock will remain perfectly level. When the undulatory theory was first proposed, it was claimed by its opponents that the waves of light would, in like manner, pass around an obstacle, so that shadows could not exist, and again that a beam of light could have no definite edges, since it would spread out on all sides like a wave of water after entering the narrow inlet to a bay. In point of fact, this spreading actually takes place, but as each wave is followed by millions of others similar to it, interference takes place so that but little remains, except in the direction of the beam. In fact, it is only by taking special precautions that the remaining rays can be detected, and the phenomena then observed are known by the name of diffraction.

Place the cylindrical lens, or the slit, at one end of the diffraction bank, and throw a beam of sunlight through it. Place a screen at the other end, and between them the plate with the aperture half closed by a piece of sheet brass having a vertical edge, or a slit with one side removed. The shadow will now be cast upon the latter, and examining its edge closely, it will be found that a small amount of light has been bent inwards or inflected. Outside of the edge, and parallel to it, a series of colored bands or diffraction fringes appear, due to the partial interference of these rays. They are seen on a much larger scale by means of the eye-piece, and their constitution is well shown by means of the direct-vision spectroscope, as is the case of the interference fringes.

Now replace the other side of the slit, and as it is gradually narrowed new fringes will appear in the shadow, which finally will quite obscure the others, leaving an appearance very like that produced by interference with a biprism. When the slit is moderately narrow, both system of fringes are visible, those in the interior being either bright or dark-centered, according to the distance of the screen. Next use a narrow screen, as a wire, instead of the slit, in the middle of the bank, when a series of fringes will be obtained, both inside and outside the shadow, and varying their distance apart with the diameter of the wire. By using two slits near together, fringes are also produced by interference as with the biprism. Strangely enough, these efforts are quite independent of the material of the slit, its thickness, or physical state. After verifying the above facts, measure the form of some of the fringes, as in the case of interference, and see if their relative distances from the centre agree with theory.

When rays from a real or artificial star pass through an aperture, those striking the edges are diffracted, so that they are thrown off obliquely in directions dependent on their wave-lengths and the form of the aperture. If, therefore, they are received in a telescope, the direct rays will form a bright spot, or image of the star, which will be surrounded with colored fringes or bands of very various forms.

To begin with the simplest case, suppose the aperture circular and of considerable size, as in a common telescope. The diffraction will be very slight, but quite perceptible, with a good instrument and a high power, although with a poor lens it is sometimes obscured by the aberration. The true angular diameter of the fixed stars is so exceedingly small that it would be quite impossible to observe their disks with any power yet employed. With a good telescope, however, small bright circles are seen, called spurious disks, which increase in diameter as the aperture is diminished, and which are surrounded by one or more colored rings, due also to diffraction. If, now, a triangular aperture is interposed in front of the objective, the star is seen to be surrounded with six diverging rays, corresponding to the angles and centres of the sides of the triangle, while a square aperture gives a star with four rays. Try, in the same way, the other apertures and the gauze, with which a vast variety of curious effects may be obtained. On interposing the series of equidistant lines, a number of colored bands are seen on each side of the star, with their violet ends towards the centre, and their direction perpendicular to the lines on the glass. This effect is most important, as it affords a means of determining the length of a ray of light with the utmost precision, as will be described in the next experiment.

In all cases the distances of the colored rays will depend on their wave-lengths, being greatest for the red and yellow, then for green, and least for the blue and violet. This may be shown by employing monochromatic light, which may be obtained by illuminating the slit or artificial star by different parts of the spectrum, formed by allowing a ray of sunlight to pass through a prism, or more simply by interposing colored glass, or employing a soda flame.

Many familiar phenomena are due to diffiraction; for example, the halo around a distant light seen through a window covered with moisture or frost, or the same effect produced on the sun or moon by fog. When the particles are all of nearly the same size, colors become visible, which may be distinguished from the ordinary large halos, both by their small size, and by noticing that in halos produced by diffraction the red is always outside, while in halos caused by refraction in minute crystals of ice, the red is always inside, since this color has a smaller index of refraction, but greater wave-length. This effect may be produced artificially by scattering on a plate of glass, lycopodium, or any fine powder whose particles are all of nearly the same size.

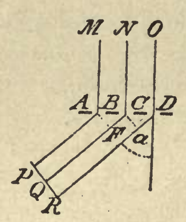
91. Wave Lengths.

Apparatus. The optical circle, and a glass plate, on which are ruled several thousand, very fine, equidistant lines, at intervals of about one hundredth of a millimetre. Sunlight is desirable for this Experiment, but if the day is cloudy, a soda and lithium flame may be employed instead.

Experiment. One of the most important applications of diffraction, is to the measurement of the lengths of waves of light of various colors. The fringes obtained with screens traversed by very fine lines, or gratings, as they are called, are employed for this purpose, and give very accurate results.

Set the two telescopes of the optical circle opposite each other, adjust them for parallel rays, and place the glass plate on the centre stand between them, and at right angles to their axes. Reflect a ray of sunlight through the slit of the collimator by means of the mirror, and on looking through the observing telescope, the following appearances will be visible as it is moved from side to side. In the centre will be a brilliant white image of the slit, and on each side spectra will be seen, with their violet ends turned towards it. The first will be bright and short, and each in turn fainter and longer, until finally they overlap, forming a continuous band of light. Focus with care, and nearly close the slit, when the solar lines will become visible in each spectrum, as in a spectroscope, except that in the present case the red end is much more extended, the violet more crowded together. Care must be taken that the lines are vertical, as the spectra being perpendicular to them will otherwise fall out of the field, above on one side, and below on the other. The plate should also be perpendicular to the axis of the collimator, or an error will be introduced in the angular distance of the spectra.

The formation of these spectra, is explained as follows. Let *M*, *N*, *O*, Fig. 64, represent the rays from the slit, after being rendered parallel by the lens of the collimator, so that they shall all be in the same phase, or state of vibration, when they strike the plate of glass. The lines on the latter, are shown on a greatly enlarged scale, at *A*, *B*, *C*, *D*, which represent a section at right angles to their length. Being opaque, or at least only translucent, they divide the surface of the glass, into a number of equidistant narrow apertures, through which the light passes. As was proved by Huyghens, each of these may be regarded as a new source of light, from



No. 64.

which the rays pass out in all directions, and in general, interfere and neutralize each other. There are, however, certain directions, as AP, in which the difference of path of AP and BQ, equals exactly one wave-length λ, and in this case they will unite, and light will be produced. The ray CR will add its effect, since it differs by exactly two wave-lengths, and in the same way, all the other rays from the other apertures unite and produce a bright light in the direction AP. The direction of PR is that of the front of the wave, or perpendicular to AP, and drawing AF parallel to it, the condition that light may be produced is evidently that in the right-angled triangle AFB, FB shall equal λ. Call d the distance between the lines, in fractions of a millimetre, and a the angle the ray AP makes with the axis of the collimator, equals FAB. Then $\lambda = d \sin a$, from which, knowing d and a, λ may be computed. To find a, bring the cross-hairs of the observing telescope successively to coincide with the image of the slit and the given ray in the first spectrum, and the difference in the readings of the vernier gives the required angle; or better, read the position of the corresponding rays in the spectra on the right and left of the central image, and divide the difference by two. Make a similar observation with three or four of the prominent lines. Suppose, now, that a is so much increased that $AF = 2\lambda$; evidently light is again produced, which gives rise to the second spectrum on each side. The third and fourth spectra are accounted for in the same way. Measure the position of the lines before observed in all of them, and compute λ from each, taking care to divide by two for the second, by three for the third, etc., to get the true wave-length. The mean of these observations should then agree closely with that given on page 152. If d is not given, it should be determined on the dividing engine, or with the microscope and spider-line micrometer.

If the lines on the glass plate are well ruled, very beautiful spectra may be obtained, in some cases almost equal to those formed by the best spectroscopes. Generally the spectra on one side are better than those on the other, probably owing to some want of symmetry in the two sides of the lines. Again, often one of the spectra will be fainter than the next beyond it, or even wanting altogether. It may be proved analytically that the mth spectrum will be wanting, when the ratio of the width of the lines to the spaces between them, is as $n : n'$, and $m = n + n'$. That is, if the dark spaces are half as broad as the bright, $n = 1$, $n' = 2$, $m = 3$, and the third spectrum will be wanting.

92. Polarized Light.

Apparatus. A rhomb of Iceland spar, and examples of the five methods of polarizing light, that is, by reflection, by refraction or by a bundle of plates set at an angle of 55°, and by the three methods of double refraction. These consist of a double-image prism, a Nicol's prism, and a tourmaline plate. Fig. 65 represents a form of polariscope which will be found both simple and effective. *B* is a plate of glass resting on a piece of black velvet, *A* a screen of ground glass, and *D* a Nicol's prism. This is so placed that the angle of incidence of the ray reflected from the centre of *B* shall be 55°, and consequently shall be totally polarized. *C* is a plate of glass on which the object to be examined is laid. Various objects should accompany this instrument, as a plate of selenite, some figures made of the same material, some pieces of unannealed glass, and two small screw-presses by which small squares or rods of glass may be subjected to longitudinal or transverse strain. Also some lenses, glass stoppers, and other articles imperfectly annealed, and some spectacle lenses of quartz and glass. Plates of the following series of crystals should be provided. 1. Iceland spar cut perpendicular to the axis; 2, quartz; 3, arragonite; 4, topaz; 5, borax; 6, nitre; 7, double plate of quartz giving hyperbolas; 8, Savart's bands. This list may be greatly extended, and it is well to add any novelties that can be found, with a written description appended. All these objects will appear to much greater advantage if the outside light is cut off, either by a black cloth, or by a cover fitting over the polariscope, and extending from *A* to *D* in the figure.

Experiment. According to the Undulatory Theory, light is produced by vibrations of the ether at right angles to the direction of the ray. If, then, the latter moves vertically, all the motions will be horizontal, and in common light, some north and south, others east and west, and others in various intermediate directions, that is, in all planes passing through the ray. If, now, the vibrations can in any way be confined to a single plane, the light is said to be polarized, and this plane is called the plane of vibration of the ray. A plane perpendicular to this and passing through the ray, is called the plane of polarization. It would be much better to have taken the latter plane as coincident with the former, but, unfortunately, the name was given before the direction of the vibrations was known.

Although it is impossible to detect by the eye alone whether light is polarized or not, yet many substances affect it differently, according to the direction its plane bears to some line in them, so that when it emerges from them, it no longer possesses the properties of plane polarized light. To examine the effect produced, the ray is first passed through the polarizer, as it is called, by which all its vibrations are brought into one plane, it is then allowed to pass through the substance under examination, and finally tested by the analyzer, which may be made precisely like the polarizer, and is used to detect any change effected in the ray.

There are five forms of polarizers in common use. First, by reflection. When a ray of polarized light impinges on a plane surface of a transparent medium, the amount reflected depends on the direction of the plane of polarization, and the angle of incidence. If the plane is perpendicular to the plane of incidence, and the angle is such that the reflected and refracted rays shall be perpendicular, all the light is transmitted. The angle of incidence is then called the angle of total polarization, and its value may be determined as follows. Let n be the index of refraction of the medium, i the angle of incidence, which equals the angle of reflection, and r the angle of refraction. Then, since the reflected and refracted rays are at right angles, $i + r = 90°$, but $\sin i = n \sin r = n \sin (90 - i) = n \cos r$, and $\tan i = n$. Common light being composed of rays polarized in every plane passing through the beam, may be regarded as composed of two equal rays, polarized at right angles, just as all forces acting on a point in a plane may be divided into two components at right angles to each other. Regarding, then, the light as composed of two beams, one A, polarized in the plane of incidence, and the other, B, polarized at right angles to it, evidently none of the latter will be reflected; hence the reflected ray will be entirely composed of light polarized in the plane of incidence, or will be totally polarized. The value of i is about 55° for glass, and 53° for water. The simplest form of polarizer is therefore a plate of glass, on which the light impinges at an angle of 55°. Commonly the lower surface is blackened, or black glass is employed, but there is no advantage in this, in fact it is better to use several plates of clear glass, to increase the light.

As a portion of A is turned back by the glass, evidently the refracted beam will be partially polarized, being composed of the whole of B, and part of A. By using a number of plates, each will reflect a portion of A, leaving B unaffected; the latter may thus be almost completely freed from A, or the light nearly perfectly polarized.

The other three polarizers depend on double refraction. If a ray of light is allowed to pass through any crystal not of the monometric system, it will be divided into two parts, one called the ordinary ray, which will follow the usual laws of refraction, and the other, the extraordinary ray, which will follow new laws. To show this, lay a crystal of Iceland spar on a piece of paper on which is marked a single dot. The latter will now appear double, and if the crystal is turned, one image, the extraordinary, will revolve around the other. These two rays are found to be polarized in planes at right angles to each other, but in the present case are not sufficiently separated to be conveniently employed. They, moreover, emerge parallel, that is, they are no more separated for distant objects than for near, since the plate being bounded by parallel faces, the second surface neutralizes the angular divergence produced by the first. To remedy this defect, prisms of glass and spar are cemented together in such a way that the refraction of one ray shall be compensated, while the other will pass out obliquely, giving two images separated by an angular amount of two or three degrees. This combination is called a double-image prism.

Another arrangement is the Nicol's prism, which consists of a rhomb of Iceland spar cut diagonally, and the two parts cemented together again with Canada balsam. This substance has an index of refraction greater than the extraordinary, but less than the ordinary ray in spar, consequently the former will pass through unchanged, while the latter being totally reflected will be thrown out on one side, and will be absorbed by the black paint covering the prism. The light passing through will therefore be polarized in a plane passing through the ray and the longer diagonal of the rhombus at the end of the prism. This is called the principal plane, or simply the plane, of the prism.

The fifth form of polarizer is a plate of tourmaline, cut parallel to the axis, which posseses the curious property of absorbing the ordinary ray, so that the emergent light is polarized in a plane parallel to its axis, or greater diameter.

Either of these instruments may be employed as a polarizer, but each has its special advantages and defects. The method of reflection is the simplest, and a beam of any size, perfectly polarized may be obtained by it, but there is much loss of light from the transmitted rays, and the change of direction is often an objection. The bundle of glass plates give a large beam, but the polarization is not very perfect unless a large number of plates is employed, and then the loss by absorption is considerable. By the other methods very large beams cannot be obtained. The double image prism gives excellent results when the presence of the second beam is not objectionable, or when, as sometimes happens, it can be thrown out to one side of the apparatus. The Nicol's prism is more employed than any other polarizer, but when of large size it is very expensive. A tourmaline plate is also good, but if very thin the ordinary ray is not wholly absorbed, and the polarization is not complete; while if thick, the ray is strongly colored. Colorless tourmalines exist, but unfortunately are not opaque to the ordinary ray, and hence do not polarize the transmitted light. Examples of these various polarizers will be found on the table.

When a ray of polarized light is viewed through a Nicol's prism, or other polarizer, the amount of light transmitted varies as the prism is turned. Thus allow a ray of light to pass through a Nicol's prism with its principal plane vertical, that is, so that the transmitted light shall be polarized in a vertical plane. If this beam is viewed with a second Nicol's prism, it will be found that as the latter is turned, the amount of light transmitted varies, being greatest when its plane is vertical, and nothing or all the light cut off, when the plane is horizontal. This evidently follows, since the prism then transmits only light polarized vertically. In intermediate positions, the amount of light is determined by decomposing the ray into two at right-angles, as in the case of forces, only it will be proportional not to the cosine, but to the square of the cosine of the angles. Try the other polarizer, in the same way, and it will be found in all cases, that when their planes are parallel, light is transmitted, but when turned at right-angles, or crossed, as it is called, the light is cut off. Accordingly, to test for polarized light, view the beam through a Nicol's prism or tourmaline, which is then called an analyzer. If there is no change of brightness of the transmitted ray, the light is unpolarized, while if in certain positions all the light is cut off, the polarization is complete. Next, turning the analyzer, find the position in which the field is darkest, when its plane will be perpendicular to the plane of polarization.

To apply this to some familiar objects, examine the light reflected from the top of a varnished table, and it will be found to be strongly polarized in a vertical plane. Moreover, when this light is cut off, the color of the wood and its grain is much better seen. It has been proposed to use Nicol's prisms in this way for viewing oil pantings, thus cutting off the troublesome reflection. Sometimes the light reflected from the front of glass cases renders it difficult to distinguish objects within them. A Nicol's prism is then often very serviceable. Again, it has been proposed to use a Nicol's prism to cut off the light reflected by water, to render rocks or other objects beneath its surface more visible. To show this effect, place a coin at the bottom of a vessel of water, or under several plates of glass, and allow a strong light to fall on it. It may then be easily seen when the polarized light is cut off, although otherwise quite invisible. As another example, view the two dots seen through a crystal of Iceland spar, and they will be found polarized in planes at right-angles. If the two images are connected by a line, it will lie in the plane of the ordinary image, or fixed dot, around which the other appears to revolve.

To view any body by polarized light, the instrument represented in Fig. 65, will be found both simple and effective. *B* is the polarizer, consisting of one or more plates of glass, and *D* a Nicol's prism, serving as an analyzer which may be turned by any desired amount, and which is set at such an angle that the light reflected from the centre of *B* shall be totally polarized; *C* is a plate of glass on which the object to be examined may be laid, and *A* a piece of ground glass, to cut off the reflection of outside objects, and to render the field of view bright and uniform. The light reflected from *B* is polarized vertically; accordingly, when *D* has its plane vertical, the field is bright, when horizontal, the field is dark.

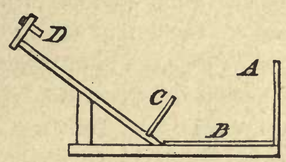


Fig. 65.

Suppose now, any doubly refracting medium is inserted between the analyzer and polarizer; for instance, a plate of selenite laid upon *C*. The ray on entering the selenite is divided into two, the ordinary and extraordinary, polarized at right-angles, the plane of the ordinary passing through the axis of the crystal. The relative intensities of the two will depend on the position of this axis with regard to the plane of polarization of the ray, and may always be obtained by decomposing the latter into two parts; one the ordinary ray, coinciding with the axis, the other at right angles to it. If the axis is perpendicular to the plane of polarization, evidently all the light will pass into the extraordinary ray, while if they coincide all becomes ordinary. The two intensities are equal when the angle between the axis and plane is 45°. Now the two rays travel through the crystal with unequal velocities, as is shown by their different indices of refraction. On emerging, one ray will be behind the other by an amount dependent on the thickness of the plate; for instance, one half wave-length of yellow light. The two rays, however, cannot interfere, since they are polarized, and are therefore vibrating, at right-angles. Therefore the crystal will still appear colorless to the eye. If, however, it is viewed with a Nicol's prism with its plane vertical, the two rays are again decomposed, the horizontal components cut off, and the vertical portions brought together so that they can interfere. If, then, white light is employed, which is composed of rays of all colors, the yellow portion will be stricken out, and the remainder will be of the complementary color, or purple. Now turn the analyzer 90°. The rays before cut off will now be transmitted, and *vice* *versa*, accordingly the color of the light will be yellow. As the analyzer is turned from these points the colors become fainter and fainter, until at the 45° points all the rays are equally affected, and the light becomes white.

In the same way, on turning the selenite plate, the two components are equal only when the axis is inclined at an angle of 45°, in which case the interference is complete. In other positions one component is larger than the other, the interference is only partial, and the colors are fainter, a part of the light being white. When the angle becomes 0°, or the axis lies in the plane of polarization, all of the light passes into the ordinary ray, none into the extraordinary, and consequently there is no interference, and the light is white. In the same way, when the axis is perpendicular to the plane of polarization, all the light enters the extraordinary ray, and the result is again white light. Accordingly as the selenite is turned, the color becomes fainter and fainter, and disappears at the 0° and 90° points, but on passing them does not assume its complementary tint.

By varying the thickness of the plate, the amount of retardation may be varied at will, and with it the wave-length of the ray stricken out, and the color produced. Figures are sometimes made of selenite to represent birds or flowers, each portions having such a thickness that when viewed by polarized light they will assume their proper colors. They are then mounted in Canada balsam between two plates of glass, and by ordinary light being transparent are almost invisible. Placing them on *C*, however, they appear in gorgeous colors, which disappear as the analyzer *D* is turned, and again reappear in complementary tints, as the rotation is continued. If, however, the selenite is turned, the colors fade, but reappear unchanged.

All transparent bodies will produce double refraction, and affect polarized light when subjected to unequal strains in different directions. In the cases mentioned above, this is effected by crystallization, but it may also be produced mechanically. To show this, place a square of glass in the small press, and lay it on *C*, turning *D* so as to cut off the light. No effect is now produced, but as soon as the glass is compressed by turning the screw, bright spaces appear at the points where the pressure is greatest, and which as the screw is turned, increase in size, and finally become colored. Care must be taken not to exert too great a pressure or the glass may be fractured. Apply in the same way a transverse strain to a rod of glass with the other press, and sketch the appearance. Still more care is needed in this case not to break the glass.

When glass is cooled suddenly the exterior contracts, and when the interior cools, the whole is subjected to great strain, rendering it very brittle. To remedy this, glass vessels intended for common use are annealed, that is, heated and allowed to cool very slowly. Place the pieces of unannealed glass on *C*, and very curious and beautiful markings will appear, which vary with the form of the specimen, and the position of the analyzer. Common glass objects, as stoppers to bottles, may be tested in the same way, to see if they have been properly annealed. The best practical application made of this principle, is to test lenses, as already mentioned in Experiment 84.

When a ray of light passes through a crystal, not of the monometric system, the effect produced varies with the direction. In the dimetric and hexagonal systems, when the ray passes through the crystal in the direction of its principal axis, it is not divided into two, or more properly, the two follow the same path but with different velocities. This direction is called the optic axis of the crystal, and such crystals are called uniaxial. Now place a rhomb of Iceland spar with its principal axis vertical, that is, so that the corner formed by the angles of 120° shall be uppermost, and the three adjacent faces equally inclined to the vertical. Now, as in the case of crystallographic axes, not only the line through the centre of the crystal, but any vertical line will be an optic axis. The principal section of any plane of this crystal is a vertical plane perpendicular to it. If the incident ray lies in a principal section, the extraordinary ray will lie in the plane of incidence, otherwise to one side of it. Crystals of the trimetric, monoclinic, and triclinic system, have two optic axes which may be inclined at any angle with each other. Such crystals are called biaxial.

When light slightly inclined to the optic axis passes through the crystal, interference takes place, producing brightness or darkness according to the amount of retardation, or angle of inclination. If rays are allowed to pass in all directions through the crystal, the optic axes will be seen to be surrounded with circles alternately bright and dark, and colored, owing to the unequal wave-lengths of the different rays. To observe them, it will not do to lay the crystal on *C*, as the rays would then all be nearly parallel, but it must be held close to *D*, and inclined from side to side. Object No. 1 is a crystal of Iceland spar, cut perpendicular to the axis, and gives readily the series of rings. A back cross is also formed with its centre in the axis which changes to white when the analyzer is turned 90°. In observing all these crystals, it will be noticed that the rings change only when they are inclined, and not when moved parallel to themselves, showing that the optic axis, as stated above, has no particular position, but is a certain direction. A common method of observing the rings is by tourmaline tongs, or two plates of tourmaline with the crystal placed between them, and the nearer one, or analyzer free to turn. Finer effects may be obtained by lenses forming a sort of microscope, but this arrangement is less simple than the above. Object No. 2 is a plate of quartz cut in the same manner. Quite a different effect is here produced, partly because the retardation is much less, unless the plate is very thick, and hence the rings much more widely spread, and partly because quartz produces what is called rotary polarization, that is, it twists the plane of polarization from its original position by an amount depending on the color, and proportional to the thickness. Accordingly the plate will appear colored, the tint varying as the analyzer is turned. Next try some biaxial crystals. No. 3 is specimen of arragonite, in which the two axes are visible, surrounded with colored rings, and with a double cross passing through them, which changes into a hyperbola as the crystal is turned. No. 4 and 5 are crystals of topaz and borax, in which the axes are so far separated that only one can be seen at a time. The other may sometimes be found by inclining the crystal. No. 6 is a crystal of nitre, in which the axes are only separated about 5°, and hence are both easily seen together. The separation of the rings depends on the thickness of the plate, and the difference of the ordinary and extraordinary indices of refraction, and is therefore quite independent of the axes. Some crystals give peculiar systems of rings which vary with each different specimen. Curious effects may be obtained by combining two or more plates in various ways. The most important are the following. No. 7, two plates of quartz cut parallel to the axis, turned at right angles, and then cemented together. A system of equilateral hyperbolas is thus obtained with a common centre. No. 8 is formed of two plates cut at an angle of 45° with the axis, and crossed in the same way. They give a series of rectilinear bands, forming in fact the ends of the hyperbolas of No. 7 with their asymptote, to which they are now parallel. This combination is known as Savart's plate, and is important, as forming one of the most delicate tests for polarized light. The centre band may be rendered either white or black by turning the analyzer 90°. They are most intense, when either parallel or perpendicular to the planes of both analyzer and polarizer.

93. Polariscope.

Apparatus. A stand is employed like a theodolite, or altitude and azimuth instrument, only the circles need be divided no finer than to degrees. The various forms of polariscopes and polarimeters described below, may be attached to this, so that they are free to turn around their axes, the angle of rotation being measured by a graduated circle and index. When the object to be examined is very minute a telescope is needed, with a positive eye-piece, in which is a Nicol's prism. In front of this, a slide is placed, by means of which a biquartz or Babinet's wedges may be interposed at the focus. The circle is attached to the eye-piece, and acts like an eye-piece goniometer (p. 163). For common objects, the telescope is replaced either by a Nicol's prism, in front of which a Savart's plate may be inserted, or by an Arago's polariscope.

Two forms of polarimeter are employed, the first or common form, proposed by Arago, consisting of a Savart's polariscope, in front of which one or more plates of glass may be inserted, and turned at any desired angle so that the light may be more or less strongly polarized by refraction. A graduated circle serves to determine the angle through which they are turned. The second form of polarimeter consists of an Arago's polariscope, in which the selenite plate is removed, and a Nicol's prism with a graduated circle placed in front of the double-image prism, so that it may be turned through any desired angle with regard to the latter. These may also be mounted on the stand like the polariscopes, so that they may be pointed in any desired direction.

Experiment. The simplest method of detecting the presence of polarized light is by a Nicol's prism, or other polarizer, as described in the last Experiment. Examine in this way the light reflected from the surface of the table, from a glass plate and other sources. Turn the Nicol until the least light is transmitted, and the direction of the shorter diagonal of the face of the prism will give the plane of polarization. This method, however, is not very sensitive, as the variation in intensity of the light is not perceptible, unless a considerable proportion is polarized. A more delicate instrument is the Arago polariscope. This consists of a tube with a square aperture at one end, and a double-image prism and plate of selenite at the other. The size of the aperture is such, that the two images of it shall be just in contact, but not overlapping. If, now, a ray of polarized light is viewed through the prism, the two images will in general assume complementary colors. When the line of separation of the two images is parallel or perpendicular to the plane of polarization, the colors are most strongly marked, and they disappear when the angle of inclination is 45°. To determine the plane of polarization of any ray, first direct the instrument towards the light reflected from a polished horizontal surface, turn the line of separation of the two images until it is vertical, or parallel to the plane of polarization, and note the color of the right hand image. Now direct the tube towards the source of light and turn it until this image has the same color as before. The plane of polarization is then parallel to the line of separation. It is well to make a notch in one side of the square, which will then appear in a different part of the two images and thus serve to distinguish them. The exact direction of the plane of polarization may be found by noting when the colors are most marked, or, more accurately, by bisecting the two positions where they disappear, each of which is 45° distant from it. The delicacy of this instrument is much greater than that of a simple Nicol's prism, as with it about three or four per cent. of polarized light can be detected.

Sometimes the Arago polariscope is used without the tube. For instance, in observing the polarization of the solar corona during a total eclipse, doubt was cast on the results by the strong polarization of the sky. To eliminate this, the tube was removed, in which case the two images of the sky overlapped, producing unpolarized light, while the images of the corona were separated so that they appeared on a white unpolarized back-ground.

A still more delicate form of polariscope is that proposed by Savart, which consists of a Nicol's prism with a double plate of quartz, giving bands as described in Experiment 92, specimen No. 8. The plate is attached to the Nicol so that the bands shall be perpendicular to its principal plane, in which case, when parallel to the plane of polarization they will be black-centred, and when perpendicular to it, white-centred. If the bands were parallel to the plane of the Nicol, this effect would be reversed. It is now very easy to determine the plane of polarization of a given ray. The instrument is turned until the bands are black-centred, when their direction marks that of the plane. The position of the latter is then found more precisely by bisecting the two points of disappearance of the bands. This instrument is more sensitive than either of the preceding, as by it one or two per cent. of polarized light can be detected. Try these different instruments on various sources of polarized light, and see if all give the same results for the direction of the plane of polarization. For instance, see if the light reflected by paper, wood or cloth is polarized in the plane of incidence, and if that transmitted obliquely through glass is polarized in a plane perpendicular to the plane of refraction.

Now direct the telescope towards some source of polarized light, and observe its plane with the simple Nicol's prism. Then push the slide so as to interpose the biquartz, which consists of two pieces of quartz joined together, one turning the ray to the right, the other to the left. The two halves will then assume complementary colors, unless the plane of polarization is parallel or perpendicular to their line of junction. In the first case, the color of both is a sort of pale violet, in the second, yellowish brown. Make a number of observations of the angle of the plane, and compute the probable error. Now try the effect of the other quartz plate. This is composed of two wedges of quartz cemented together, one turning the ray to the right, the other to the left. Accordingly a series of bands are produced, which disappear when parallel or perpendicular to the plane of polarization. Repeat the observations with this plate, and compare its probable error with that of the biquartz. It will be noticed that both these devices require a parallel beam, while the Savart's polariscope, which needs a converging beam, cannot be attached to a telescope in this way, but must be placed in front of the eye-piece. In this case it cuts down the field of view, and is therefore inconvenient to use.

To measure the proportion of polarized light in a given beam, the polarimeter is employed. This consists of a Savart, or other form of polariscope, in front of which some plates of glass are placed, free to turn, so that the transmitted light may pass through them at any desired angle. It will thus be polarized by refraction to a greater or less extent, depending on the number of plates, and the angle through which they are turned. To measure this, a graduated circle is attached, which may be divided either into degrees, or so as to give the percentage directly. The bands should be placed parallel to the axis around which the plates turn. To use this instrument, set the plates at 0° and direct it towards a source of unpolarized light. The field will now be perfectly uniform. Turn the plates, and the bands will appear faintly, dark-centred and increasing in strength with the angle. Turn the plates back to 0°, and direct the instrument towards the light to be examined, find its plane of polarization and bring the bands to a position at right angles to it, that is, so that they shall be most strongly light-centred. Now on turning the plates, they tend to neutralize the polarization, since they tend to polarize it in a plane passing through this axis, while it is already polarized in a plane perpendicular to this. As they are turned, the bands therefore become fainter and fainter, then disappear and reappear dark-centred, when the angle becomes too great. At the point of disappearance, the polarization produced by the plates is just equal to that already present in the beam, the transmitted light is therefore unpolarized, and gives no bands. Take a number of readings of the point of disappearance, first turning the plates to the right, and then to the left, and reduce to percentages by means of a table which should accompany the instrument.

The difficulty of computing this table with accuracy, greatly diminishes the value of this instrument. The theoretical formulas are quite complex, and of little use on account of the difficulty of allowing for the light absorbed by the glass. It must therefore be determined experimentally by observing with it a beam in which the percentage of polarized light may be regulated at will. This may be accomplished either by setting a plate of glass at an angle of 45°, and varying the relative intensities of the reflected and refracted beams, or by reuniting two beams of variable intensity by means of a double-image prism. Again, if the beam is strongly polarized it is impossible to make the bands disappear, unless a large number of plates are used, in which case the transmitted beam is very feeble.

These various difficulties are obviated by the other form of polarimeter. As in the Arago, two adjacent images of the square are formed, one polarized horizontally, the other vertically, which will have equal intensities if the light is unpolarized, but one of which will be in general, brighter than the other, when viewed by polarized light. If now the two images are seen through a Nicol's prism, their relative intensities will vary as it is turned, each disappearing when the plane of the Nicol is perpendicular to its own. Accordingly, certain positions can always be found, in which the two images will have precisely the same brightness, and the angle through which the Nicol has been turned, gives a measure of their true relative intensities, and hence the percentage of polarized light present. To make the reduction, call a the angle through which the Nicol has been turned, A the amount of light polarized in a vertical plane, and B that polarized horizontally. Thus if the plane is vertical, A is greater than B, and $A - B$ is the amount of free polarized light. $A + B$ being the total intensity of the light, their ratio gives the percentage of polarization. When the plane of the Nicol is vertical, A retains its full brilliancy, which, at any other angle is reduced in the ratio $\cos^2 a$. B is in like manner proportional to $\sin^2 a$. The percentage of polarized light n therefore equals $\frac{A - B}{A + B} = \frac{\cos^2 a - \sin^2 a}{\cos^2 a + \sin^2 a}$ $= \cos^2 a - \sin^2 a = \cos 2a$. The reduction may then be effected by a table of natural cosines, or by the accompanying table which gives the corresponding values of a and n.

a.	n.
0	100.0
5	98.5
10	94.0
15	86.6
20	76.6
25	64.3
30	50.0
35	34.2
40	17.4
45	0

To use this instrument direct it towards the source of light to be examined, and turn it so that the line separating the two squares shall be parallel to the plane of polarization. Then turn the Nicol until the two images are equally bright, when the angle will give, by means of the table, the percentage of polarized light present. It is best to take readings on each side of the 0° and employ the mean, thus eliminating any error in the 0° point. Otherwise, care must be taken that the circle is fixed in such a position that when the Nicol is turned so that one image shall completely disappear, the reading of the index shall be precisely 0° or 90°.

Now measure with the polarimeters the amount of polarized light contained in the rays whose plane of polarization was previously determined. Next throw a beam of sunlight upon a sheet of paper, and measure the percentage of polarization of the light thrown out in various directions. Observations of this kind are much needed for various substances at different angles of incidence and reflection. It will be found that it is extremely difficult to obtain a beam from a large surface entirely free from all traces of polarization, and hence much care is needed to obtain really accurate results.

When the sky is clear its light is found to be strongly polarized in planes passing through the sun, the effect being most marked, at a distance of 90° from that body. Beyond 90° the polarization again diminishes, and becomes zero at a point called the neutral point in the same vertical plane as the sun, but 150° distant, below this point the plane of polarization becomes horizontal. Two other neutral points exist, one 17° below the sun, the other 8½° above it, but both much more difficult to observe. Even a faint cloud alters these effects, and when the sky is entirely covered with clouds, no polarization is perceptible. Very valuable work might be done by measuring the plane and amount of polarization of the light in different parts of the sky.

94. SACCHARIMETER.

Apparatus. A Soleil saccharimeter, some pure sugar and some unrefined, or brown sugar. Also some chlorhydric acid and subacetate of lead, a flask containing just 100 cm.^{3}, a funnel, filter paper, a balance and weights.

Experiment. The most important practical application of polarized light is to saccharimetry, or the measurement of the strength of a solution of sugar. This depends on the property of such a solution of producing rotary polarization, or of turning the plane of a beam of polarized light by an amount proportional to the amount of sugar present. The saccharimeter is merely an instrument for measuring the angular change of the plane, or more strictly, the thickness of a plate of quartz, rotating it in the opposite direction, required to bring it back to its primitive position. The liquid is contained in a brass tube closed at each end with plates of glass, which are held in place by screw caps. The light first passes through a circular aperture, two polarized images of which are formed by a double-image prism, and one transmitted through the instrument, the other thrown off to one side. It next passes through a double plate of quartz formed of two semicircles, one of which turns the ray to the right, the other to the left. It next passes through the column of sugar by which both rays are turned, by a certain amount, to the right. It is brought back to its primitive position by a compensater, formed of a plate and two wedges of quartz, the latter being turned in opposite directions, and carrying racks which are acted on by a pinion so that they may be moved past each other by any desired amount. The thickness of the layer of quartz may thus be varied at will, and accurately determined by a scale attached to one wedge, and an index to the other. The light next passes through a Nicol's prism which serves as an analyzer, and then through a small Galilean telescope by which an enlarged image of the biquartz is formed. In front of the eye-piece of the telescope is an additional Nicol's prism and plate of quartz, the latter being free to turn. The object of this is to vary the tint of the two halves of the biquartz, so that the color to which the eye is most sensitive may be selected, and also to neutralize any color already present in the solution.

To use this instrument, weigh out 16.47 grammes of pure sugar and dissolve it in enough water to make the solution occupy 100 cm^3. Unscrew the cap from one of the tubes, fill it with water and slide on the glass plate, taking great care that no air-bubbles are imprisoned under it. Replace the cap and wipe the exterior dry. Fill a second tube in the same manner with the solution of sugar. Turn the stand towards the light, lay the tube containing water in place on it, and focus the telescope on the biquartz by drawing out the eye-piece until the line of separation is distinctly visible. The two semicircles will now, in general, appear of different colors which may be changed by moving the wedges by the milled head below. A certain position will be found, however, in which they are alike, and the reading of the scale should then be zero.

When the two halves appear of the same tint, turn the quartz plate in the eye-piece, by which their color is altered. There is a peculiar purplish brown color, different for different eyes, from which the two halves change more rapidly than from any other, when the wedges are moved. Consequently, when of this color, which is called the sensitive tint, they can be set more accurately than in any other case. To obtain this sensitive tint bring the two halves as nearly alike as possible, then turn the quartz and see if any difference is perceptible; if so, set again, until no difference can be detected in the two halves, however the the plate is turned. Take a number of observations, and reading the scale to tenths of a division, take the mean. If not zero, it may be brought to this position by means of a small screw, which moves the scale without affecting the wedges.

Now replace the tube containing water by that containing a solution of sugar, when it will be found that the semicircles have very different colors, and on making them alike, the reading of the scale becomes 100, if the sugar is perfectly pure. As before, take the mean of several readings, and turn the quartz each time to obtain the most sensitive tint.

Next make a solution of one half the strength by mixing some of the standard solution with exactly its own volume of water, and see if the reading is 50. Then dilute again one half, to get a solution of strength one fourth, and see if the reading is 25. If kept for some time, the solution will ferment, and the reading diminish, especially during warm weather, or if exposed to the air. In general, a solution of impure sugar is not transparent, and is often so opaque, that the semicircles cannot be observed through it. In this case it must be clarified by adding some sub-acetate of lead and then filtering. Animal charcoal was at one time used, but it is found that this absorbs some of the sugar with the impurities. In practice the problem generally is complicated by the fact, that the molasses and other impurities commonly found in sugar, also turn the plane of polarization to the right, and thus render the results uncertain. This effect must therefore be eliminated by adding to the solution one tenth of its bulk of pure chlorhydric acid, and heating to 68° C. The cane-sugar is thus converted into grape-sugar, which turns the plane of polarization by an equal amount to the left, while it does not effect the molasses and uncrystallizable sugar. After heating, the solution is poured into the larger tube, which has an aperture in one side to contain a thermometer. The length of the column in this case is one tenth greater than before, which just compensates for the dilution due to the addition of the chlorhydric acid. The reading should then be taken, and the temperature noted. As this reading gives the difference of the amount of crystallizable and uncrystallizable sugar, and the first reading gives their sum, the amount of crystallizable sugar may be obtained by taking half the sum of the two readings, and the amount of uncrystallizable, by taking half their difference. Thus if this first reading is 80 and the second 30, it denotes that there is 55 per cent. of crystallizable, and 25 per cent of syrup. A correction must be applied for temperature, which is best done by means of a table, which accompanies the instrument.

101.50	219.75	1.00
43.25	141.75	15.00
220.25	16.00	1.50
16.00	377.50	3.00
378	119.50	119.50
	258.00	

373.75