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ELEMENTS OF
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TRIGONOMETRY

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ELEMENTS OF PLANE AND SPHERICAL TRIGONOMETRY

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PREFACE

In this work the author has endeavored to prepare a text which would serve as a basis for a fifty- or sixty-hour course in Plane and Spherical Trigonometry as ordinarily presented in advanced secondary and elementary college courses.

Emphasis is placed upon drill work in the trigonometric identities, upon the applications of trigonometry to practical problems, and upon approximate calculations by means of natural functions. The more accurate results obtained by logarithmic calculations are emphasized in the solutions of oblique triangles; a uniform style of tabulating logarithmic calculations is suggested.

For the benefit of those who may wish to pursue advanced courses in mathematics, a brief discussion of analytic trigonometry is presented in Chapter IX. In Part II the elements of spherical trigonometry are developed in so far as to include the ordinary formulæ necessary in the solution of right and oblique spherical triangles.

DAVID A. ROTHROCK.

Bloomington, Indiana,
December, 1909.

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PART I

PLANE TRIGONOMETRY

CHAPTER I

TRIGONOMETRIC FUNCTIONS OF ACUTE ANGLES

1. Trigonometry. The word *Trigonometry* is derived from two Greek words, *triangle* (τρίγωνον) and *measurement* (μέτρον), which would suggest that the subject has to do with the measurement of the triangle. At the present time the subject of trigonometry, though treating of the measurement of the triangle, has a much wider scope, and includes all manner of investigations depending upon certain functions of angles called **Trigonometric Functions**. These functions are called *sine*, *cosine*, *tangent*, *cotangent*, *secant*, and *cosecant*. For any angle θ * these words are abbreviated into $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, $\csc\theta$, respectively. In the next section definitions are given for the trigonometric functions of an acute angle, but it should be borne in mind that similar definitions are applicable for any angle whatever, and later will be extended to angles varying from 0° to 360°.

* In trigonometric notation, angles are frequently denoted by Greek letters. The Greek alphabet is here inserted.

Letters	Names	Letters	Names	Letters	Names
Α α	Alpha	Ι ι	Iota	Ρ ρ	Rho
Β β	Beta	Κ κ	Kappa	Σ σ ς	Sigma
Γ γ	Gamma	Λ λ	Lambda	Τ τ	Tau
Δ δ	Delta	Μ μ	Mu	Υ υ	Upsilon
Ε ε	Epsilon	Ν ν	Nu	Φ φ	Phi
Ζ ζ	Zeta	Ξ ξ	Xi	Χ χ	Chi
Η η	Eta	Ο ο	Omicron	Ψ ψ	Psi
Θ θ	Theta	Π π ϖ	Pi	Ω ω	Omega

2. The Trigonometric Functions of an Acute Angle.

(1) *Definitions.* For an acute angle θ constructed in a right triangle, Fig. 1, with x, y, r, as the base, altitude, and hypotenuse, respectively, we define

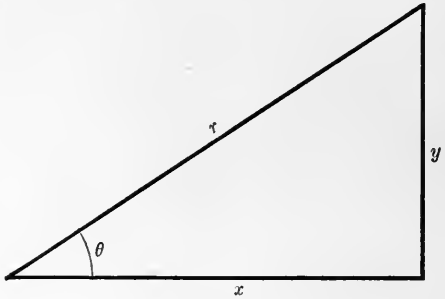


Fig. 1.

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{y}{r}, \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{1}{\sin\theta} = \frac{r}{y},$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{x}{r}, \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{1}{\cos\theta} = \frac{r}{x},$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{y}{x}, \qquad \cot\theta = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{1}{\tan\theta} = \frac{x}{y}.$$

To these six functions are sometimes added:

$$\text{versed sine } \theta = 1 - \cos\theta, \text{ written vers } \theta,$$

$$\text{coversed sine } \theta = 1 - \sin\theta, \text{ written covers } \theta.$$

These six functions of the angle θ are called trigonometric functions (trigonometric ratios) of that angle. The symbols $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, $\csc\theta$, are usually read *sine* θ, *cosine* θ, *tangent* θ, *cotangent* θ, *secant* θ, *cosecant* θ, *respectively.*

(2) *Elementary relations among the functions.* The six functions $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, $\csc\theta$, are not independent. By comparing the definitions we find:

$$(1)\quad \tan\theta = \frac{1}{\cot\theta}, \text{ or } \tan\theta\cdot\cot\theta = 1,$$

$$(2)\quad \cos\theta = \frac{1}{\sec\theta}, \text{ or } \cos\theta\cdot\sec\theta = 1,$$

$$(3)\quad \sin\theta = \frac{1}{\csc\theta}, \text{ or } \sin\theta\cdot\csc\theta = 1.$$

(3) *Exponents.* When the trigonometric functions are to be affected by exponents, the following notation is usually employed: (1) if the exponent be positive, the index is placed thus, $\sin^2\theta$, $\tan^3\theta$, $\sec^n\theta$, and read *sine square* θ, etc.; (2) if the exponent be negative, the index is usually attached to a bracket as in bracketed expressions in algebra, thus $(\sin\theta)^{-1} = \frac{1}{\sin\theta}$, $(\cot\theta)^{-2} = \frac{1}{\cot^2\theta}$, $(\cos\theta)^{-n} = \frac{1}{\cos^n\theta}$. These are read *sin* θ *exponent minus one, cot* θ *exponent minus two, cos* θ *exponent minus n.*

(4) *Functions are constant for any given angle.* The trigonometric functions of an acute angle are ratios of lines in a right triangle. They are constant for any fixed angle, and do not change value for different lengths of the sides of the triangle. Thus, Fig. 2,

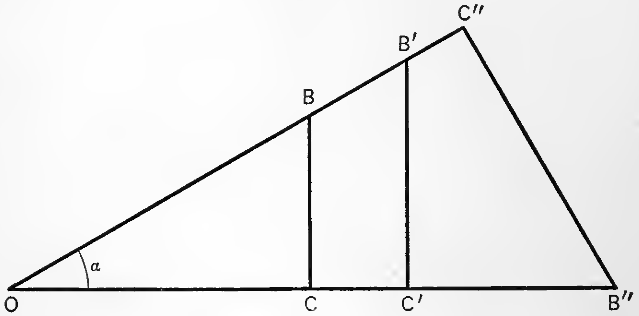


Fig. 2.

$$\sin\alpha = \frac{BC}{OB} = \frac{B'C'}{OB'} = \frac{B''C''}{OB''},$$

$$\cos\alpha = \frac{OC}{OB} = \frac{OC'}{OB'} = \frac{OC''}{OB''},$$

$$\tan\alpha = \frac{BC}{OC} = \frac{B'C'}{OC'} = \frac{B''C''}{OC''}.$$

Tables of these ratios have been constructed, showing their numerical values for all angles from 0° to 90° (see Table p. 15).

3. Functions of Complementary Angles. In Fig. 3, θ and ϕ are complementary angles: $\theta + \phi = 90°$.

$$\sin\theta = \frac{y}{r} = \cos\phi, \qquad \csc\theta = \frac{r}{y} = \sec\phi,$$

$$\cos\theta = \frac{x}{r} = \sin\phi, \qquad \sec\theta = \frac{r}{x} = \csc\phi,$$

$$\tan\theta = \frac{y}{x} = \cot\phi, \qquad \cot\theta = \frac{x}{y} = \tan\phi.$$

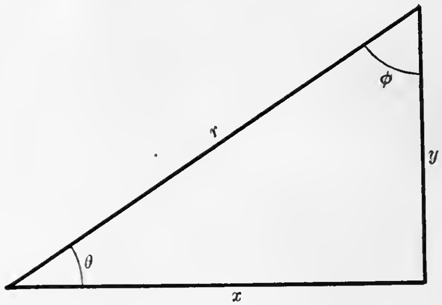


Fig. 3.

The cosine, cotangent, and cosecant of an angle are co-functions of the sine, tangent, and secant, respectively.

From the definitions we see that *any function of an angle equals the* **co-function** *of the complement of that angle.*

For example, $\sin 30° = \cos 60°$, $\tan 35° = \cot 55°$, $\csc 32° = \sec 58°$,

$$\sin(90° - A) = \cos A, \qquad \csc(90° - A) = \sec A,$$

$$\cos(90° - A) = \sin A, \qquad \sec(90° - A) = \csc A,$$

$$\tan(90° - A) = \cot A, \qquad \cot(90° - A) = \tan A.$$

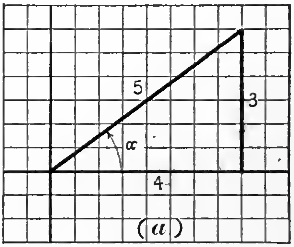
EXAMPLES. Fill the blanks in the following with the proper co-function:

1. $\sin 48° =$
2. $\sin 84° =$
3. $\tan 25° 48' =$
4. $\cot 87° 50' =$
5. $\cos 41° 50' =$
6. $\sec 10° 15' =$
7. $\cos(99° - x) =$
8. $\tan(90° - 10°) =$
9. $\sin(90° - 100°) =$
10. $\cot(80° - x) =$
11. $\csc 67° 10' =$
12. $\tan(90° - x + y) =$

EXERCISES

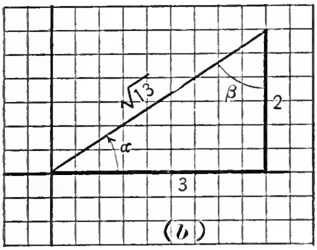
1. Construct an acute angle α such that $\sin\alpha = \frac{3}{5}$, and write the remaining trigonometric functions.

Solution. $\sin\alpha = \frac{3}{5}$ tells us that the side of the triangle opposite α is 3, and the hypotenuse is 5; hence, the construction is as shown. Writing the functions of α from the drawing, we have



$$\sin\alpha = \tfrac{3}{5},\qquad \cos\alpha = \tfrac{4}{5},$$
$$\tan\alpha = \tfrac{3}{4},\qquad \cot\alpha = \tfrac{4}{3},$$
$$\sec\alpha = \tfrac{5}{4},\qquad \csc\alpha = \tfrac{5}{3}.$$

2. Construct α when $\tan\alpha = \frac{2}{3}$, and write the other functions of α. Write the functions of $90° - \alpha$.



Solution. α is constructed as shown in figure.

$$\sin\alpha = \frac{2}{\sqrt{13}},\qquad \cos\alpha = \frac{3}{\sqrt{13}},$$
$$\tan\alpha = \frac{2}{3},\qquad \cot\alpha = \frac{3}{2},$$
$$\sec\alpha = \frac{\sqrt{13}}{3},\qquad \csc\alpha = \frac{\sqrt{13}}{2},$$
$$90° - \alpha = \beta;\ \sin\beta = \cos\alpha = \frac{3}{\sqrt{13}}, \text{ etc.}$$

Construct the angle α in the following cases and write the remaining functions:

3. $\sin\alpha = \frac{5}{13}$.

4. $\cos\alpha = \frac{3}{5}$.

5. $\cot\alpha = \frac{5}{3}$.

6. $\sec\alpha = \frac{7}{5}$.

7. $\tan\alpha = \frac{4}{5}$.

8. $\cot\alpha = \frac{8}{15}$.

9. $\sin\alpha = \frac{1}{2}$.

10. $\tan\alpha = \frac{3}{4}$.

11. $\sin\alpha = \frac{8}{17}$.

12. $\tan\alpha = 2$.

13. $\cot\alpha = 3$.

14. $\sin\alpha = \frac{1}{2}\sqrt{3}$.

15. $\tan\alpha = \frac{1}{3}\sqrt{3}$.

16. $\sec\alpha = 4$.

17. $\sin\alpha = 0.6$.

18. $\cos\alpha = \dfrac{2m}{1+m^2}$.

19. $\tan\alpha = \dfrac{\sqrt{1-m^2}}{m}$.

20. $\cot\alpha = \dfrac{3x}{4}$.

In the right triangle ABC, Fig. 4, if

21. $\sin A = \frac{2}{3}$, $c = 24$, find a.

22. $\sin A = \frac{3}{5}$, $c = 24$, find a.

23. $\tan A = \frac{3}{4}$, $b = 16$, find a.

24. $\tan A = \frac{4}{3}$, $c = 15$, find b.

25. $\tan A = 0.6$, $a = 10$, find b.

26. $\sin A = \frac{5}{8}$, $a = 15$, find c.

27. $\cos A = \frac{2}{3}$, $c = 21$, find b and a.

28. $\sec A = 5$, $b = 50$, find c and a.

29. $\tan A = 0.25$, $b = 40$, find the area.

30. $\csc A = 2.5$, $a = 100$, find c and the area.

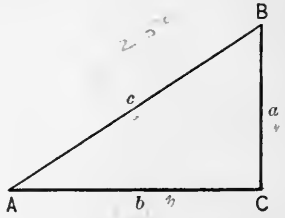


Fig. 4.

4. Fundamental Relations among the Trigonometric Functions. Let θ denote any acute angle, Fig. 5. From plane geometry,

$$x^2 + y^2 = r^2. \tag{1}$$

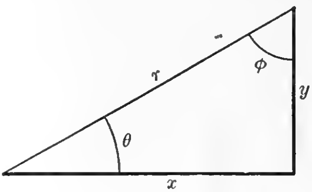


Fig. 5.

Defining $\sin\theta$ and $\cos\theta$,

$$\sin\theta = \frac{y}{r}, \quad \cos\theta = \frac{x}{r},$$

we have, on squaring and adding,

$$\sin^2\theta + \cos^2\theta = \frac{y^2 + x^2}{r^2} = 1. \tag{A}$$

From the same figure, $\sec\theta = \frac{r}{x}$, $\tan\theta = \frac{y}{x}$.

Squaring and subtracting,

$$\sec^2\theta - \tan^2\theta = \frac{r^2 - y^2}{x^2} = 1, \text{ from (1)}. \tag{B}$$

Combining $\csc\theta$ and $\cot\theta$ in a similar manner,

$$\csc^2\theta - \cot^2\theta = \frac{r^2 - x^2}{y^2} = 1. \tag{C}$$

Also,

$$\tan\theta = \frac{y}{x} = \frac{y \div r}{x \div r} = \frac{\sin\theta}{\cos\theta}, \tag{D}$$

$$\cot\theta = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}, \tag{E}$$

$$\sec\theta = \frac{r}{x} = \frac{1}{\cos\theta}, \quad (F) \qquad \csc\theta = \frac{r}{y} = \frac{1}{\sin\theta}. \quad (G)$$

As fundamental relations among the six functions, we may enumerate formulas *A*, *B*, *C*, above, and the identities coming directly from the definitions in § 2.

5. Fundamental Identities. Collecting important results, we have, for any angle α:

(A) $\sin^2 \alpha + \cos^2 \alpha = 1,$

(B) $\sec^2 \alpha - \tan^2 \alpha = 1,$

(C) $\csc^2 \alpha - \cot^2 \alpha = 1,$

(D) $\tan \alpha = \dfrac{\sin \alpha}{\cos \alpha},$

(E) $\cot \alpha = \dfrac{1}{\tan \alpha} = \dfrac{\cos \alpha}{\sin \alpha},$

(F) $\sec \alpha = \dfrac{1}{\cos \alpha},$

(G) $\csc \alpha = \dfrac{1}{\sin \alpha}.$

These identities should be memorized.

6. Variation of the Trigonometric Functions. It should be noticed that as the angle increases from 0° to 90°, the functions vary as follows:

(1) *sine increases from* 0 *to* 1,

(2) *cosine decreases from* 1 *to* 0,

(3) *tangent increases from* 0 *to* ∞,

(4) *cotangent decreases from* ∞ *to* 0,

(5) *secant increases from* 1 *to* ∞,

(6) *cosecant decreases from* ∞ *to* 1.

These facts may be observed from the above identities, as well as from the definitions of the functions in § 2.

7. Transformation of Identities. By means of the fundamental identities, § 5, any trigonometric function may be changed into various forms.

For example,

$$\sin \phi = \sqrt{1 - \cos^2 \phi} = \sqrt{1 - \frac{1}{\sec^2 \phi}} = \frac{\sqrt{\sec^2 \phi - 1}}{\sec \phi} = \frac{\tan \phi}{\sec \phi}, \text{ etc.}$$

These forms for the value of the sine of an angle are obtained by using identities (*A*), (*F*), (*B*), and algebraic manipulation.

Again,

$$\sin \alpha \cdot \sec \alpha \cdot \cot \alpha = \frac{\sin \alpha}{\cos \alpha} \cdot \cot \alpha = \tan \alpha \cdot \cot \alpha = 1.$$

In verifying an identity, we may proceed, (*a*) *by transforming the left member of the equation into the right,* (*b*) *by transforming the right into the left, or* (*c*) *by reducing each side to the same form.*

For example, show $\cos^4 x - \sin^4 x = 2\cos^2 x - 1$.

(*a*) Transform the left member into the right.

$$\begin{aligned} \cos^4 x - \sin^4 x &= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) \\ &= \cos^2 x - \sin^2 x & \text{by } (A) \\ &= \cos^2 x - (1 - \cos^2 x) & \text{by } (A) \\ &= 2\cos^2 x - 1. \end{aligned}$$

(*b*) Transform the right member to the form of the left.

$$\begin{aligned} 2\cos^2 x - 1 &= \cos^2 x - (1 - \cos^2 x) \\ &= \cos^2 x - \sin^2 x & \text{by } (A) \\ &= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x), \text{ multiplying by 1,} \\ &= \cos^4 x - \sin^4 x. \end{aligned}$$

(*c*) Transform each side to a common form.

$$\begin{aligned} \cos^4 x - \sin^4 x &= 2\cos^2 x - 1 \\ &= \cos^2 x - \sin^2 x. \end{aligned}$$

Remove the factor $\cos^2 x + \sin^2 x = 1$ from the left member, and we have

$$\cos^2 x - \sin^2 x = \cos^2 x - \sin^2 x.$$

The student should attain a good degree of skill in manipulating trigonometric functions; to this end is now inserted a list of the more common forms of identities involving a single angle.

EXERCISES

Verify the following identities:

1. $\cos^2 A\,(1 + \tan^2 A) = 1.$
2. $(\sec^2 A - 1)(\csc^2 A - 1) = 1.$
3. $\tan A + \cot A = \sec A \times \csc A.$
4. $\sin^2 A\,(\csc^2 A - 1) = \cos^2 A.$
5. $\cot^2 A - \cos^2 A = \cos^2 A \cot^2 A.$
6. $\dfrac{\cos A}{\sin A \cot^2 A} = \tan A.$
7. $\sin A \cos A\,(\tan A + \cot A) = 1.$

8. $(\tan A + \cot A)^2 = \sec^2 A + \csc^2 A.$

9. $\cos A \csc A \tan A = 1.$

10. $\cos A (\cot A + \tan A) = \csc A.$

11. $\cot A + \dfrac{\sin A}{1 + \cos A} = \csc A.$

12. $\dfrac{\sec A}{\cos A} - \dfrac{\tan A}{\cot A} = 1.$

13. $\sec^2 A - \sec^2 A \sin^2 A = 1.$

14. $(\csc A - \cot A)^2 = \dfrac{1 - \cos A}{1 + \cos A}.$

15. $\sec A - \cos A = \sin A \tan A.$

16. $\dfrac{\sin A}{\csc A} + \dfrac{\cos A}{\sec A} = 1.$

17. $\dfrac{\tan A - 1}{\tan A + 1} = \dfrac{1 - \cot A}{1 + \cot A}.$

18. $\dfrac{\sec A - \csc A}{\sec A + \csc A} = \dfrac{\tan A - 1}{\tan A + 1}.$

19. $\cos^4 A - \sin^4 A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1.$

20. $\dfrac{1 + \cot^2 A}{1 + \tan^2 A} = \cot^2 A.$

21. $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2.$

22. $\sec A + \tan A = (\sec A - \tan A)^{-1}.$

23. $\sin^4 A + \cos^4 A = 1 - 2\sin^2 A \cos^2 A.$

24. $\sin^3 A + \cos^3 A = (\sin A + \cos A)(1 - \sin A \cos A).$

25. $\sec^4 A - \tan^4 A = \sec^2 A + \tan^2 A = 1 + 2\tan^2 A.$

26.* Show $x^2 + y^2 = r^2$, when $x = r \cdot \cos \phi$, $y = r \cdot \sin \phi$.

27. Show $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, when $x = a \cdot \cos \phi$, $y = b \cdot \sin \phi$.

28. Show $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, when $x = a \cdot \sec \phi$, $y = b \cdot \tan \phi$.

29. Show $x^2 + y^2 + z^2 = r^2$, when $x = r \cdot \sin \theta \cdot \cos \phi$,
$y = r \cdot \sin \theta \cdot \sin \phi$, $z = r \cdot \cos \theta$.

* The algebraic equations in Exercises 26–29 are, respectively, the equations of a circle, ellipse, hyperbola, and sphere. The equations giving the values of x, y, z are the parametric equations of the same curves and sphere, respectively.

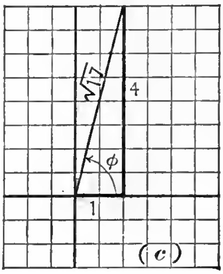
30. Show $x^2+y^2+z^2=r^2$,

$$\begin{aligned} \text{when } x &= r(\cos\theta\cdot\cos\phi+\sin\theta\cdot\sin\phi\cdot\cos\psi),\\ y &= r(\cos\theta\cdot\sin\phi\cdot\cos\psi-\sin\theta\cdot\cos\phi),\\ z &= r\cdot\sin\phi\cdot\sin\psi. \end{aligned}$$

31. Show $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$, when $x=a\cdot\cos^3\phi$, $y=a\cdot\sin^3\phi$.

EXERCISES

Simplify the following expressions, construct the angle ϕ in each case, and read the values of the remaining functions.



NOTE. When square roots are to be taken, use only the + sign.

1. $\sin\phi\cdot\sec\phi+1=5$.

In this exercise, we have

$$\sin\phi\cdot\sec\phi=4,$$

or,

$$\frac{\sin\phi}{\cos\phi}=\tan\phi=4.$$

Hence,

$$\sin\phi=\frac{4}{\sqrt{17}},\ \cos\phi=\frac{1}{\sqrt{17}},$$

$$\tan\phi=4,\ \cot\phi=\frac{1}{4},\ \sec\phi=\sqrt{17},\ \csc\phi=\frac{\sqrt{17}}{4}.$$

In the following the values of a single function should be found, then the angle may be constructed.

2. $\tan\phi=2-\sin\phi\cdot\sec\phi$. *Ans.* $\tan\phi=1$.

3. $\tan\phi\cdot\cos\phi=\frac{2}{3}$. *Ans.* $\sin\phi=\frac{2}{3}$.

4. $\sec\phi\cdot\cot\phi=\frac{4}{3}$. *Ans.* $\sin\phi=\frac{3}{4}$

5. $\sec\phi=4\cos\phi$. *Ans.* $\sec\phi=2$.

6. $\cot\phi=3\tan\phi$. *Ans.* $\tan\phi=\frac{\sqrt{3}}{3}$.

7. $\tan\phi=4\cot\phi$. *Ans.* $\tan\phi=2$.

8. $\cot\phi=\frac{4}{5}\csc\phi$. *Ans.* $\cos\phi=\frac{4}{5}$.

9. $\cos\phi\div\sin\phi=\frac{4}{3}$. *Ans.* $\cot\phi=\frac{4}{3}$.

10. $\cot\phi\cdot\cos\phi\cdot\tan\phi=\frac{a}{b}$. *Ans.* $\cos\phi=\frac{a}{b}$.

11. $\sin^2\phi\cdot\sec\phi\cdot\cos\phi\cdot\csc\phi=\frac{5}{13}$. *Ans.* $\sin\phi=\frac{5}{13}$.

12. $1-\cos^2\phi=\frac{4}{9}$. *Ans.* $\sin\phi=\frac{2}{3}$.

13. $\sec^2\phi - 1 = \frac{25}{144}$. *Ans.* $\tan\phi = \frac{5}{12}$.

14. $2 - 3\sin\phi = \frac{1}{2}$. *Ans.* $\sin\phi = \frac{1}{2}$.

15. $(1 + \sin\phi)(1 - \sin\phi) = \frac{9}{16}$. *Ans.* $\cos\phi = \frac{3}{4}$.

16. $\tan^2\phi + 1 = 2\sec\phi$. *Ans.* $\sec\phi = 2$.

17. $\tan\phi = 2 - 2\sec\phi \div \csc\phi$. *Ans.* $\tan\phi = \frac{2}{3}$.

18. $\tan\phi - \cos^2\phi = \sin^2\phi + 5$. *Ans.* $\tan\phi = 6$.

19. $\sin\phi = \frac{4}{25}\cos\phi \cdot \sec\phi \cdot \csc\phi$. *Ans.* $\sin\phi = \frac{2}{5}$.

20. $\dfrac{5\tan\phi \times \cos\phi \times \csc\phi}{4\cot\phi \times \sec\phi} = 1$. *Ans.* $\sin\phi = \dfrac{4}{5}$.

21. When $\tan\alpha = 4$, find value of $\dfrac{\sin\alpha \cdot \sec\alpha}{\cos^2\alpha}$.

22. When $\sin\alpha = \dfrac{5}{6}$, find value of $\dfrac{\cos^2\alpha + \sin^2\alpha}{\tan\alpha}$.

23. When $\cos\alpha = \dfrac{1}{2}$, find value of $\dfrac{\sin\alpha \cdot \tan\alpha}{\sec^2\alpha + 1}$.

24. When $\sec\alpha = 5$, find value of $\dfrac{\tan^2\alpha}{\cos\alpha \cdot \csc\alpha}$.

25. When $\cot\alpha = \dfrac{5}{8}$, find value of $\dfrac{\tan\alpha}{\csc^2\alpha - 1}$.

26. If $\sin\alpha = \dfrac{5}{13}$, what value has $\dfrac{\sin^2\alpha + \cos^2\alpha}{1 + \tan^2\alpha}$?

27. If $\tan\alpha = \frac{1}{3}\sqrt{3}$, what value has $3\sin\alpha - 4\sin^3\alpha$?

28. $\sin 30° = \frac{1}{2}$, find the value of $4\cos^3 30° - 3\cos 30°$.

29. $\sec 45° = \sqrt{2}$, find the value of $\dfrac{2\tan 45°}{1 - \tan^2 45°}$.

30. $\sin 30° = \frac{1}{2}$, $\cos 45° = \frac{1}{2}\sqrt{2}$, find the value of

$$\tan 45° \times \cot 30° \div (\sec 30° \times \sin 45°).$$

Change the following functions so that only $\sin\alpha$ appears:

31. $\sin\alpha\cos^2\alpha - \dfrac{\cot\alpha}{\cos\alpha\sin\alpha} + \dfrac{\tan\alpha}{\cos\alpha}$.

32. $\csc\alpha\sin\alpha + \cos\alpha\tan\alpha - \dfrac{\sin\alpha}{\cos^2\alpha}$.

33. $\tan^3\alpha + 1 - \csc^2\alpha + 2\sin\alpha\cos\alpha$.

34. $\sec^4\alpha - \tan^2\alpha + \cos^2\alpha - \cot^2\alpha$.

Change the following so that only $\tan\alpha$ appears:

35. $\cot\alpha + \sec\alpha - \csc\alpha$.

36. $(1+\cos\alpha)(1-\cos\alpha)+\tan\alpha(\cot^2\alpha - 1)$.

37. $\sin\alpha + \cos\alpha + \cos\alpha\tan\alpha + \cot\alpha\sin\alpha$.

38. $(1-\sin\alpha\cos\alpha)\div(1+\sin\alpha\cos\alpha) - \cot\alpha\sin\alpha$.

Change the following so that only $\cos\alpha$ appears:

39. $\dfrac{\sin^2\alpha}{\cos\alpha}+\dfrac{\tan\alpha}{\cot\alpha}$.

40. $\sec\alpha - \dfrac{1}{\cos\alpha}+\dfrac{\tan\alpha}{\sin\alpha}$.

41. $\cot^2\alpha + \tan^2\alpha - \sin^2\alpha - \cos^2\alpha$.

42. $\sin\alpha\cos\alpha\tan\alpha\cot\alpha$.

43. Express $\sin x$ in terms of each of the other functions.

Thus, (1) $\sin x = \sqrt{1-\cos^2 x}$, from identity (**A**), § 5.

(2) $\sin x = \tan x \div \sqrt{1+\tan^2 x}$, from (**D**), (**F**) and (**B**).

(3) $\sin x = \dfrac{1}{\sqrt{1+\cot^2 x}}$, from (**G**) and (**C**).

(4) $\sin x = \sqrt{1-\dfrac{1}{\sec^2 x}} = \sqrt{\dfrac{\sec^2 x - 1}{\sec x}}$, from (**A**) and (**F**).

(5) $\sin x = \dfrac{1}{\csc x}$, from (**G**).

44. Express $\cos x$ in terms of the other functions.

45. Express $\tan x$ in terms of the other functions.

46. Express each function in terms of $\sin x$.

[NOTE. Construct the angle x with hypotenuse unity, the opposite side $\sin x$, and the base $\sqrt{1-\sin^2 x}$.]

47. Express each function in terms of (1) $\cos x$, (2) $\tan x$, (3) $\cot x$.

48. Prove that $\sec A(\sin A - \cos A) + \csc A(\sin A + \cos A) = \sec A \csc A$.

49. Show $\sin\alpha + \cos\alpha \geqq 1$, $\quad 0 \leqq \alpha \leqq 90°$.

50. Show $\tan\alpha + \cot\alpha \geqq 2$.

51. Show $\sin\alpha + \cos\alpha \leqq \sqrt{2}$.

52. Show $\sin\alpha\cos\alpha \leqq \frac{1}{2}$.

8. Functions of Particular Angles. The trigonometric functions of certain angles may be found from geometrical drawings.

(1) *Functions of* 45°.

From the drawing, Fig. 6, we have

$$\sin 45° = \tfrac{1}{2}\sqrt{2}, \quad \csc 45° = \sqrt{2},$$

$$\cos 45° = \tfrac{1}{2}\sqrt{2}, \quad \sec 45° = \sqrt{2},$$

$$\tan 45° = 1, \qquad \cot 45° = 1.$$

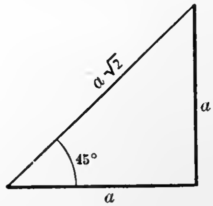


Fig. 6.

(2) *Functions of* 30° *and* 60°. From Fig. 7,

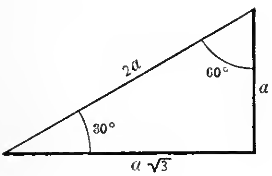


Fig. 7.

$$\sin 30° = \tfrac{1}{2} = \cos 60°,$$

$$\cos 30° = \frac{\sqrt{3}}{2} = \sin 60°,$$

$$\tan 30° = \frac{\sqrt{3}}{3} = \cot 60°,$$

$$\cot 30° = \sqrt{3} = \tan 60°,$$

$$\sec 30° = \tfrac{2}{3}\sqrt{3} = \csc 60°,$$

$$\csc 30° = 2 = \sec 60°.$$

(3) *Functions of* 0° *and* 90°. The trigonometric functions of 0° and 90° are limiting values which may be seen from the drawing, Fig. 8.

As the side y approaches 0, the opposite angle approaches 0 and the base x approaches r.

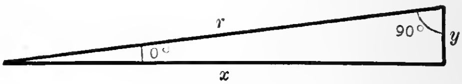


Fig. 8.

$$\sin 0° = \lim_{y=0} \frac{y}{r} = 0 = \cos 90°, \qquad \cot 0° = \lim_{y=0} \frac{x}{y} = \infty = \tan 90°,$$

$$\cos 0° = \lim_{x=r} \frac{x}{r} = 1 = \sin 90°, \qquad \sec 0° = \lim_{x=r} \frac{r}{x} = 1 = \csc 90°,$$

$$\tan 0° = \lim_{y=0} \frac{y}{x} = 0 = \cot 90°, \qquad \csc 0° = \lim_{y=0} \frac{r}{y} = \infty = \sec 90°.$$

The angles 0°, 30°, 45°, 60°, 90°, occur so frequently that it will be found convenient to keep in mind the numerical values of the trigonometric functions of each.

Tabulate the values of these functions as shown in the following table:

$\theta =$	0°	30°	45°	60°	90°
$\sin\theta$	0	.5	.707	.866	1
$\cos\theta$	1	.866	.707	.5	0
$\tan\theta$	0	.577	1	1.732	∞
$\cot\theta$	∞	1.732	1	.577	0
$\sec\theta$	1	1.155	1.414	2	∞
$\csc\theta$	∞	2	1.414	1.155	1

EXERCISES

Find the values of the following by inserting numerical values of the trigonometric functions and reducing to simplest form.

1. $5\cos 60° - 3\sin 30° + \tan 45°$. *Ans.* 2.
2. $5\cos 30° + 4\cos 60° - 5\sin 60°$. *Ans.* 2.
3. $8\tan 30° - 4\cot 45° + \cos 90° - 8\cot 60°$. *Ans.* -4.
4. $10\cos 30° + 16\cos 60° - 5\sqrt{3}$. *Ans.* 8.
5. $(4\tan 45° - 11\tan 60°)(4\cot 45° + 11\cot 30°)$. *Ans.* -347.
6. $(16\tan 0° + 10\cot 90° + 6\sin 30°)\cos 90°$. *Ans.* 0.
7. $\sin 30° \times \cos 0° \times \tan 45° \times \sec 60°$. *Ans.* 1.
8. $(x+y)\cos 0° - (x-y)\tan 45° - 2y\sin 30°$. *Ans.* y.
9. $(x+y)^2\sin 60° - (x-y)^2\cos 30° - 2xy\tan 60°$. *Ans.* 0.
10. $(a+b)\sec 60° + (a-b)\cos 90° + a\tan 90°$. *Ans.* ∞.
11. $a\sin 30° - (a-b)\tan 45° - b\sin 30°$. *Ans.* $\frac{1}{2}(b-a)$.
12. $(a\sin 60° - b\cos 30°) \times \csc 60°$. *Ans.* $a-b$.

9. Tables. For convenience in the numerical calculations which follow on p. 26 a condensed table of trigonometric functions, true to three decimals, will now be inserted. By use of this table approximate results may be obtained for numerical problems.

NATURAL TRIGONOMETRIC FUNCTIONS

Angle	Sin	Csc	Tan	Cot	Sec	Cos	
0°	0.000	∞	0.000	∞	1.000	1.000	90°
1	0.017	57.30	0.017	57.29	1.000	1.000	89
2	0.035	28.65	0.035	28.64	1.001	0.999	88
3	0.052	19.11	0.052	19.08	1.001	0.999	87
4	0.070	14.34	0.070	14.30	1.002	0.998	86
5°	0.087	11.47	0.087	11.43	1.004	0.996	85°
6	0.105	9.567	0.105	9.514	1.006	0.995	84
7	0.122	8.206	0.123	8.144	1.008	0.993	83
8	0.139	7.185	0.141	7.115	1.010	0.990	82
9	0.156	6.392	0.158	6.314	1.012	0.988	81
10°	0.174	5.759	0.176	5.671	1.015	0.985	80°
11	0.191	5.241	0.194	5.145	1.019	0.982	79
12	0.208	4.810	0.213	4.705	1.022	0.978	78
13	0.225	4.445	0.231	4.331	1.026	0.974	77
14	0.242	4.134	0.249	4.011	1.031	0.970	76
15°	0.259	3.864	0.268	3.732	1.035	0.966	75°
16	0.276	3.628	0.287	3.487	1.040	0.961	74
17	0.292	3.420	0.306	3.271	1.046	0.956	73
18	0.309	3.236	0.325	3.078	1.051	0.951	72
19	0.326	3.072	0.344	2.904	1.058	0.946	71
20°	0.342	2.924	0.364	2.747	1.064	0.940	70°
21	0.358	2.790	0.384	2.605	1.071	0.934	69
22	0.375	2.669	0.404	2.475	1.079	0.927	68
23	0.391	2.559	0.424	2.356	1.086	0.921	67
24	0.407	2.459	0.445	2.246	1.095	0.914	66
25°	0.423	2.366	0.466	2.145	1.103	0.906	65°
26	0.438	2.281	0.488	2.050	1.113	0.899	64
27	0.454	2.203	0.510	1.963	1.122	0.891	63
28	0.469	2.130	0.532	1.881	1.133	0.883	62
29	0.485	2.063	0.554	1.804	1.143	0.875	61
30°	0.500	2.000	0.577	1.732	1.155	0.866	60°
31	0.515	1.942	0.601	1.664	1.167	0.857	59
32	0.530	1.887	0.625	1.600	1.179	0.848	58
33	0.545	1.836	0.649	1.540	1.192	0.839	57
34	0.559	1.788	0.675	1.483	1.206	0.829	56
35°	0.574	1.743	0.700	1.428	1.221	0.819	55°
36	0.588	1.701	0.727	1.376	1.236	0.809	54
37	0.602	1.662	0.754	1.327	1.252	0.799	53
38	0.616	1.624	0.781	1.280	1.269	0.788	52
39	0.629	1.589	0.810	1.235	1.287	0.777	51
40°	0.643	1.556	0.839	1.192	1.305	0.766	50°
41	0.656	1.524	0.869	1.150	1.325	0.755	49
42	0.669	1.494	0.900	1.111	1.346	0.743	48
43	0.682	1.466	0.933	1.072	1.367	0.731	47
44	0.695	1.440	0.966	1.036	1.390	0.719	46
45°	0.707	1.414	1.000	1.000	1.414	0.707	45°
	Cos	Sec	Cot	Tan	Csc	Sin	Angle

CHAPTER II

SOLUTION OF RIGHT TRIANGLES

A right triangle is known when a side and any other part are known. The use of trigonometric functions enables us to compute the unknown parts.

10. Fundamental Formulas. We have, Fig. 9, the following relations:

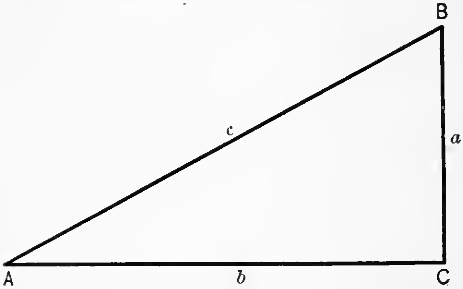


Fig. 9.

(1) $a^2 + b^2 = c^2$,

(2) $\sin A = \frac{a}{c} = \cos B$,

(3) $\cos A = \frac{b}{c} = \sin B$,

(4) $\tan A = \frac{a}{b} = \cot B$,

(5) $A + B = 90°$.

The first of these formulas is a statement of the Pythagorean Theorem; (2), (3), (4) are results from the definition of sine, cosine, tangent.

With any two dimensions given (not A and B) the other three dimensions may be found.

EXAMPLES. 1. Given $A = 37°$, $c = 12$ in.; find a and b.

Solution. From formula (2),

$$a = c \sin A = 12 \times \sin 37°.$$

From the approximate table, p. 15,

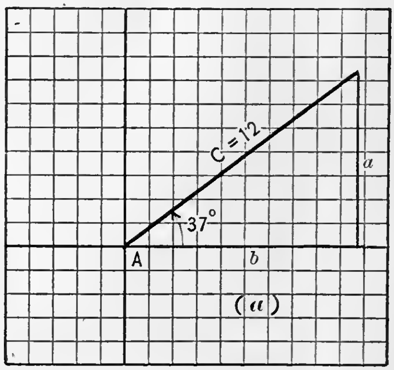
$$\sin 37° = .602.$$

Therefore,

$$a = 12 \times .602 = 7.2 \text{ in.}$$

From (3),

$$b = c \cos 37° = 12 \times .799 = 9.59 \text{ in.}$$



(a)

2. Given $a = 20$ ft., $c = 35$ ft., find angle A.

Solution. $\sin A = \frac{a}{c} = \frac{20}{35} = .571.$

From table, p. 15, the angle whose sine is .571 is 34° 50′ (approximately). Hence $A = 34° 50'$.

11. Projections. Let perpendiculars fall from two points, A and B (Fig. 10) upon any line L, intercepting PQ. PQ is called the projection of the line AB upon L. This manner of projection is called *orthogonal.*

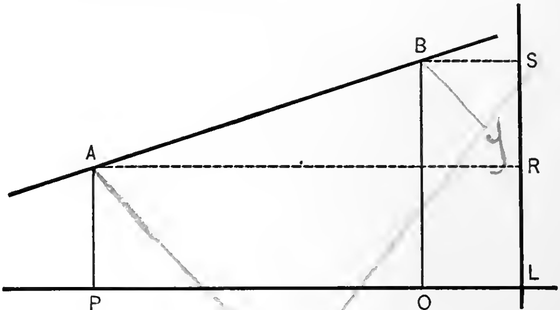


Fig. 10.

(1) *Horizontal and vertical projections.* Any line AB may be projected orthogonally in any direction; the usual projections, however, are upon the horizontal PQ, or upon the vertical RS.

(2) *Laws of orthogonal projection.* The orthogonal projection of any line upon another line involves the base and altitude of a right triangle, with the hypotenuse given. Hence,

(*a*) The horizontal projection,

$$x = r \cdot \cos \theta,$$

(*b*) The vertical projection,

$$y = r \cdot \sin \theta.$$

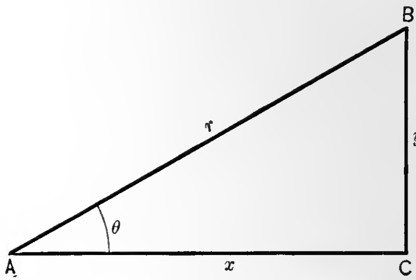


Fig. 11.

Theorem. *The horizontal projection of any line segment equals the length of the segment multiplied by the cosine of the* *angle of inclination; the vertical projection equals the length of the segment multiplied by the sine of the angle of inclination.*

12. Components. *Forces, velocities*, and *accelerations* may be represented by *directed lines.* A line of given length and fixed direction is sometimes called a *vector.* Thus, in Fig. 12, V is a vector, also V_x, V_y.

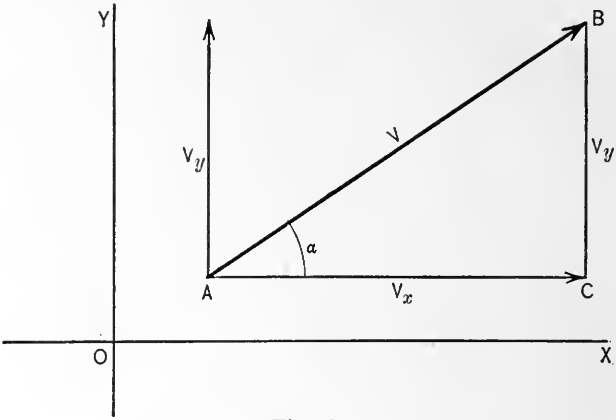


Fig. 12.

Suppose a body is moving with a velocity V in the path AB, Fig. 12, making an angle α with the horizontal OX; we may require the velocity in the horizontal AC (the horizontal component of the velocity, V_x), also the vertical velocity CB (the vertical component of the velocity, V_y). These components are projections of AB upon the horizontal and vertical, respectively.

$$AC = AB \cos \alpha, \text{ or } V_x = V \cos \alpha;$$

$$CB = AB \sin \alpha, \text{ or } V_y = V \sin \alpha.$$

Squaring and adding,

$$V_x^2 + V_y^2 = V^2(\cos^2 \alpha + \sin^2 \alpha) = V^2.$$

THEOREM. *The sum of the squares of the horizontal and vertical components of a velocity equals the square of the velocity.*

13. Resultant. If two forces act upon a particle at A, the one, AC, acting horizontally, the other, AD, acting vertically, the particle at A will be moved along the diagonal of the parallelogram to B, Fig. 13. The line AB is called the *re-*

sultant of the forces represented by the two lines AC and AD. The forces AC and AD need not act at right angles, but whatever their angle, the resultant will be the diagonal of the parallelogram constructed upon AC and AD.

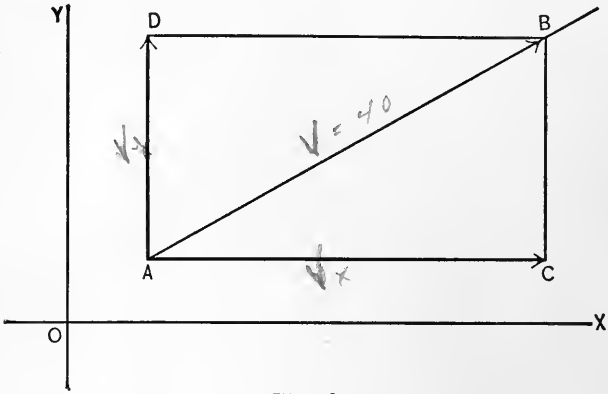


Fig. 13.

The relations between resultant and components which act at right angles to each other involve the simple trigonometric properties of right triangles.

EXAMPLES. **1.** A train of cars is moving northeast with a velocity V of 40 mi. per hour. Find its rates of travel east and north.

Solution. Let the vector V represent the given velocity. Then V_x, V_y are the velocities east and north, respectively. See Fig. 12.

$$V_x = V \cos 45^\circ = 40 \times \tfrac{1}{2}\sqrt{2} = 20\sqrt{2},$$

$$V_y = V \sin 45^\circ = 40 \times \tfrac{1}{2}\sqrt{2} = 20\sqrt{2}.$$

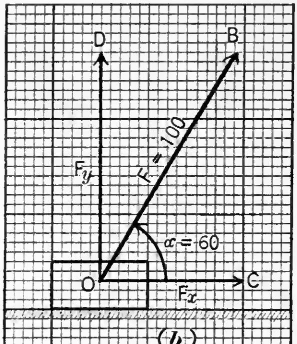
Hence, the velocities east and north are equal, $20\sqrt{2}$ mi. per hour.

2. A point is moving with a velocity of 25 ft. per second along a line making an angle of 38° with the horizontal. Find the horizontal component. *Ans.* 19.7 ft.

3. Find the velocity of a point moving at an angle of 64° with the horizontal, if the vertical component of the velocity be 100 ft. per second. *Ans.* 111.3 ft.

4. A point describes a circle of radius 20 inches at a uniform rate of two revolutions per minute. Find the distance from the centre of the circle to the projection of the point upon a horizontal diameter 5 seconds after passing the extremity of that diameter. *Ans.* 10 in.

5. A force F of 100 lb. is applied to a block at the point O, see Fig. (*b*). If the force makes an angle of 60° with the horizontal, what force tends to draw the block horizontally (OC)? What force tends to lift the block upwards (OD)?



(*b*)

Solution. Here the solutions called for are the horizontal component F_x and the vertical component F_y of the force F.

$$F_x = F \times \cos \alpha = 100 \cos 60° = 50 \text{ lb.},$$

$$F_y = F \times \sin \alpha = 100 \sin 60° = 86.6 \text{ lb.}$$

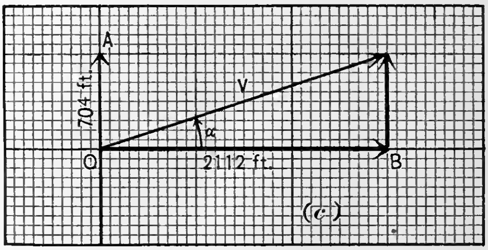
6. A balloon rising vertically at a uniform velocity of 704 ft. per minute encounters a wind blowing horizontally at the rate of 24 mi. per hour. Find the angle at which the balloon will rise and its velocity after meeting the wind.

Solution. The angle required is represented by α in Fig. *c*, the velocity of wind and vertical velocity of the balloon by OB and OA, respectively.

$$\tan \alpha = \frac{704}{2112} = \frac{1}{3},$$

$$\alpha = 18° \, 26'.$$

$$V = 2112 \times \sec \alpha$$
$$= 2112 \times 1.054 = 2226 \text{ ft.}$$



(*c*)

7. A force of 400 lb. acting in a direction inclined 40° from the horizontal is applied to a heavy block. Find the force which tends to move the block (1) horizontally, (2) the force which tends to lift the block vertically. *Ans.* (1) 306.4 lb., (2) 257.2 lb.

8. The horizontal and vertical components of a force acting upon a heavy block are 160 lb. and 120 lb. respectively. Find the force and its direction of action.

Ans. 200 lb., 36° 52′.

14. Projected Areas. Any plane area $ABCD$ inclined to a

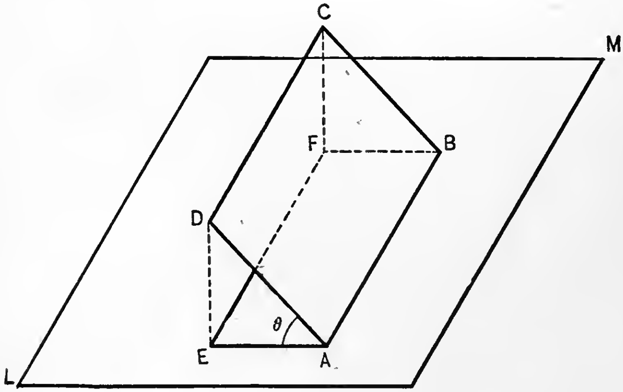


Fig. 14.

plane LM at an angle θ may be projected orthogonally upon LM into the area $ABFE$.

$$ABFE = ABCD \times \cos\theta$$

Law of projection. The projection of a given plane area upon a plane equals the given area multiplied by the cosine of the angle of inclination of the two planes.

EXAMPLES. 1. How much horizontal area will be covered by 60 sq. yd. of roofing, the roofing making an angle of 45° with the horizontal? *Ans.* $60 \cos 45° = 42.42$ sq. yd.

2. Find the roofing required to cover a horizontal space 15 ft. by 24 ft., the roofing rising at an angle of 40°.

Ans. $15 \times 24 \times \sec 40° = 469.8$ sq. ft.

15. Area of the Right Triangle. The area of a right triangle may be expressed in various ways. Let K = area; then we have,

(1) $K = \frac{1}{2} a \times b = \frac{1}{2} ac \cos A = \frac{1}{2} bc \sin A.$

(2) $K = \frac{1}{2} b^2 \tan A = \frac{1}{2} a^2 \tan B.$

(3) $K = \frac{1}{2} c^2 \sin A \cos A = \frac{1}{2} c^2 \sin B \cos B.$

In (1) $a \cos A = b \sin A = p$; and in (2) $b \tan A = a$, $a \tan B = b$.

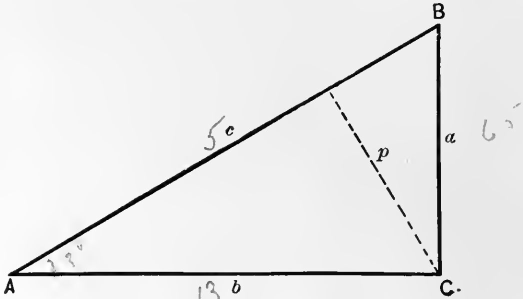


Fig. 15.

EXAMPLES. Find the area of each of the following right triangles, the lettering being shown in Fig. 15:

1. $a = 20$, $A = 35°$.
2. $c = 65$, $B = 28°$.
3. $a = 100$, $A = 30°$.
4. $b = 42$, $B = 67°$.
5. $c = 35$, $A = 3B$.
6. $c = 75$, $a = 2b$.
7. $p = 27$, $A = 48°$.
8. $p = 45$, $B = 75°$.
9. $b = 48$, $a = 52$.
10. $a = 65$, $5c = 13b$.

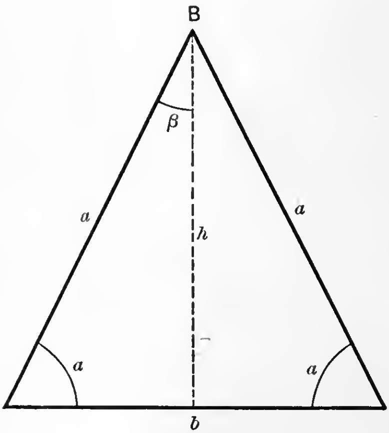


Fig. 16.

16. The Isosceles Triangle. Since the isosceles triangle may be divided into two equal right triangles by a perpendicular from the vertical angle B (see Fig. 16) upon the base, the solution depends only upon the solution of a right triangle.

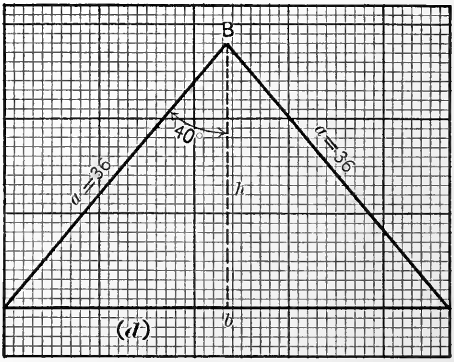
EXAMPLES. 1. Find the base b and area of an isosceles triangle when the vertical angle B is 80°, and each of the equal sides is 36 ft.

Solution. Construct an approximate triangle, Fig. *d*.

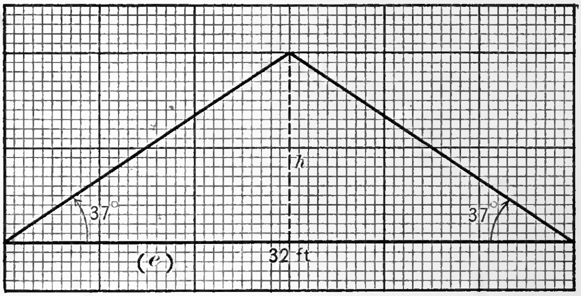
(1) b is the projection of the equal sides on the horizontal. Hence,

$b = 2\,a \times \sin\frac{B}{2} = 2 \times 36 \times \sin 40° = 2 \times 36 \times .643 = 46.3$ ft.

(2) Area $= \frac{1}{2}\,b \cdot h = \frac{1}{2}\,ab \times \cos 40° = 638.2$ sq. ft.



2. Find the length of rafters, height of ridge-pole, and area of gable from the data shown in the drawing.



(1) Length of rafter $= 16 \times \sec 37° = 16 \times 1.252 = 20.03$ ft.

(2) Height $h = 16 \times \tan 37° = 16 \times .754 = 12$ ft.

(3) Area of gable $= 16 \times h = 192$ sq. ft.

17. Notations of Direction. If OH, Fig 17, be a horizontal, the angle $POH = \phi$ is the *angle of elevation* of P; the angle $QOH = \psi$ is called the *angle of depression* of Q. A depression angle could also be considered as a negative elevation angle.

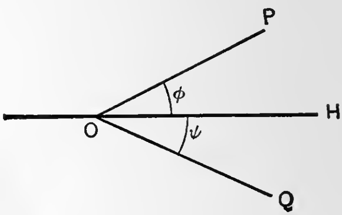


Fig. 17.

The direction of a line OP with respect to a north and south line NS, Fig. 18, is read North α degrees East, N. $\alpha°$ E.; the same direction with regard to the east and west line EW could be read East β degrees North, E. $\beta°$ N.

Similar readings for directions may be applied to south and

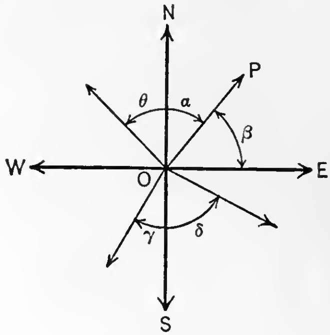
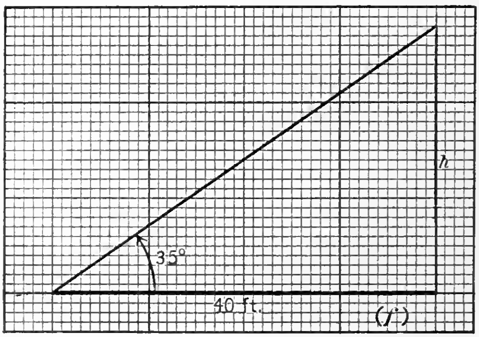


Fig. 18.

west lines. A direction of a line given as above is called the *bearing of the line when read from N. or S.*

EXAMPLES. 1. If the angle of elevation of the sun be 35°, how high is a pole whose shadow upon the ground is 40 ft.?



Solution. (1) Construct drawing showing data.

(2) Select formula (4), p. 16,

$$\tan 35^\circ = \frac{h}{40},$$

or
$$\begin{aligned} h &= 40 \times \tan 35^\circ \\ &= 40 \times 0.7 \\ &= 28 \text{ ft.} \end{aligned}$$

2. Find the radius of a circle in which a chord 30 in. long subtends a 20° arc. Also find the area of the triangle formed by the chord and the radii to its extremities.

Solution. (1) Construct figure; draw radius perpendicular to middle of chord. Let B be the middle point of the chord.

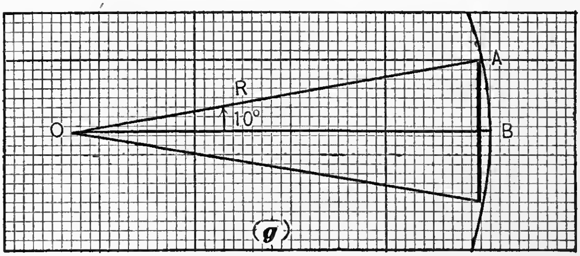
$$\text{Angle } AOB = 10^\circ, \quad AB = 15 \text{ in.}$$

(2)
$$\sin AOB = \frac{AB}{R}, \quad R = AB \div \sin AOB,$$

or,
$$R = AB \times \csc AOB.$$

(3) Substitute the values $AB = 15$, $AOB = 10°$, and we have

$$R = 15 \times \csc 10° = 15 \times 5.759 = 86.38 \text{ in.}$$



(g)

(4) The area is given by

$$K = \tfrac{1}{2} \text{ base} \times \text{altitude.} \quad OB = 15 \times \cot 10° = 85.06.$$

Then $$K = 15 \times OB = 1275.9 \text{ sq. in.}$$

18. To solve Right Triangles. In the solution of right triangles the beginner should provide himself with (1) a graduated ruler, (2) squared paper, and (3) a protractor graduated to degrees. With this equipment a triangle approximating very closely to any given data may be constructed. An approximate construction will enable the student to detect any considerable error in calculation.

In solving a right triangle the following directions are suggested: (1) *Draw, on squared paper, a figure as accurately as possible from the given data, and estimate approximate values for the unknown parts.* (2) *Select formulas,* § 10, *each of which contains one unknown.* (3) *Substitute the given data in the proper formula, using approximate values of the trigonometric functions,* p. 15, *and solve for the unknowns.* (4) *Check results by using some formula not employed in the calculation.*

In many ordinary measurements approximate results only are desired. The following list of exercises involving the right triangle is inserted to give practice in approximating results. The trigonometric functions found on p. 15 give approximations to three decimals, and when employed in solution will give results true to one or two decimals. When greater accuracy is desired, the more complete tables of the natural functions should be used. For still more accurate results, the logarithmic tables, explained in Chapter VI, should be employed.

EXERCISES

Solve the following right triangles, giving results to nearest tenth and nearest minute. Use Approximate Tables of trigonometric functions, p. 15, for solutions of Ex. 1 to 20, approximating angles to minutes. For areas employ formulas of § 15.

No.	Data		Answers			Area = K
1	$a = 13$	$b = 40$	$A = 18°$	$B = 72°$	$c = 42$	260
2	$a = 20$	$b = 8$	$A = 68° 12'$	$B = 21° 48'$	$c = 21.5$	80
3	$a = 80$	$b = 30$	$A = 69° 27'$	$B = 20° 33'$	$c = 85.4$	1200
4	$a = 16.16$	$b = 25.28$	$A = 32° 35'$	$B = 57° 25'$	$c = 30$	204.3
5	$a = 10$	$c = 50$	$A = 11° 32'$	$B = 78° 28'$	$b = 49$	245
6	$a = 71$	$c = 78$	$A = 65° 32'$	$B = 24° 28'$	$b = 32.3$	1146.6
7	$a = 84.9$	$c = 93.5$	$A = 65° 14'$	$B = 24° 46'$	$b = 39.17$	1662.8
8	$c = 42$	$A = 81° 30'$	$B = 8° 30'$	$a = 41.5$	$b = 6.2$	128.9
9	$c = 100$	$A = 36°$	$B = 54°$	$a = 58.8$	$b = 80.9$	2378.5
10	$c = 67.7$	$A = 23° 30'$	$B = 66° 30'$	$a = 27$	$b = 62.08$	838.1
11	$c = 250$	$B = 47°$	$A = 43°$	$a = 170.5$	$b = 182.8$	15583.7
12	$c = 400$	$b = 240$	$A = 53° 8'$	$B = 36° 52'$	$a = 320$	38400
13	$K = 55.42$	$A = 30°$	$a = 8$	$b = 8\sqrt{3}$	$c = 16$	
14	$K = 28.93$	$A = 26° 45'$	$a = 5.4$	$b = 10.7$	$c = 12$	
15	$K = 145.8$	$b = 18$	$A = 42°$	$c = 24.2$	$a = 16.2$	

16. A monument 283 ft. high casts a shadow 100 ft. long upon the ground. Find the angle of elevation of the sun at that instant. *Ans.* 70° 32′.

17. A ladder 30 ft. long resting upon the ground reaches a point 24 ft. high upon a vertical wall. Find the angle of elevation of the ladder. *Ans.* 53° 6′.

18. Gable rafters 20 ft. long project 2 ft. beyond the walls of a house and are set with a pitch (angle of elevation) of 40°. Find the height h of the ridge-pole and the width of the house. *Ans.* $h = 11.57$ ft., $w = 27.57$ ft.

19. Find the bearing of a road which leads to a point 5 mi. east and 8 mi. north. *Ans.* N. 32° E.

20. A rectangle is 98 by 148. Find the angle made by a diagonal with the longer side. *Ans.* 33° 30′.

21. The sides of an isosceles triangle are 30, 45, 45, respectively. Find the angles. [Note. Use four-place tables.] *Ans.* 70° 32′, 38° 56′.

22. Find the radius of a circle in which a 100 ft. chord subtends an angle of 18° at the centre. *Ans.* 319.69 ft.

23. A chord 50 in. long subtends an angle of 36° at the centre. Find the radius R of the circle and the area A of the inscribed square. *Ans.* $R = 80.9$ in., $A = 13{,}089.6$ sq. in.

24. Find the height of a tree which casts a horizontal shadow 60 ft. long when the sun's elevation is 65°. *Ans.* 128.67 ft.

25. Find the angle of inclination of the faces of a wedge whose base is 3 in. and whose face is 14 in. long. *Ans.* 12° 18′.

26. The exterior angle between two 500 ft. tangents is 72°. Find the radius of the circle. *Ans.* 688.2 ft.

27. The length of a kite-string is 200 m. Find the height of the kite when its angle of elevation is 34°. *Ans.* 111.84 m.

28. The radius of a circle is 5 cm. and the length of a chord is 4 cm. Find the angle subtended by the chord at the centre. *Ans.* 47° 9′ 20″.

29. Find the radius of a circle inscribed in an equilateral triangle whose perimeter is 42 cm. *Ans.* 4.04 cm.

30. What is the height of a tower if a 16 ft. flagpole upon the top of the tower subtends an angle of 4° at a point on the ground and the angle of elevation of the bottom of the pole is 40°? *Ans.* 106 ft.

31. At a certain point the angle of elevation of a mountain peak is 27°; at a distance of 2 mi. farther away in the same direction its elevation angle is 25°. Find the horizontal distance from the first point of observation to the peak. *Ans.* 21.58 mi.

32. Two consecutive milestones on a level road in the same vertical plane as a tower have depression angles of 42°, 3°, respectively, from the top of the tower. Find the height of the top of the tower and its horizontal distance from the nearer milestone.

Ans. $h = 293.8$ ft. or 261.4 ft., $d = 326.3$ ft. or 290.3 ft.

33. A tower stands upon the same plane as a house whose height is 60 ft. The elevation and depression of the top and bottom of the tower from the top of the house are 41° and 35°, respectively. Find the height of the tower. *Ans.* 134.49 ft.

34. From a point 10 ft. above the water, the angle of elevation of the top of a tree standing at the edge of the water is 46°, while the depression angle of its image in the water is 50°. Find the height of the tree, and its horizontal distance from the point of observation. *Ans.* $h = 142.5$ ft., $d = 128$ ft.

35. In measuring the width of a river a line AB is measured 240 ft. along one bank. A perpendicular to AB at A is erected which locates a point C upon the opposite bank, and the angle ABC is found to be 65°. Find the width AC of the stream.

Ans. 514.68 ft.

36. Two towers upon the same horizontal plane are of such heights that their elevation angles from a point midway between them are 40°, 60°, respectively. Find the ratio of their heights. *Ans.* $8391 \div 17{,}321$.

37. From each of two stations a mile apart upon a north and south road, the angle of elevation of a balloon is observed to be 30°, and its bearings are, respectively, N.E. and S.E. Find the height of the balloon. *Ans.* 2155.5 ft.

38. A balloon is exactly over the middle point between two cities. The balloon is a mile high and the distance between the cities subtends an angle of 136° at the balloon. Find the distance between the cities and the distance of the balloon from either of them. *Ans.* 4.95 mi., 2.67 mi.

39. Find the height of a tree if the angle of elevation of its top changed from 36° to 42° on walking toward it 75 ft. in a horizontal line through its base. *Ans.* 282.16 ft.

40. A hill rises uniformly 4 ft. in a horizontal distance of 85 ft. What is the difference in elevation of two points 3500 ft. apart, the distance being measured along the ground?

Ans. 164.1 ft.

41. What is the angle of incline of a railroad track if it rises 30 ft. in a horizontal distance of a mile? *Ans.* 19′ 32″.

42. What is the bearing of a road which leads to a point 14 mi. east and 8 mi. north? *Ans.* N. 60° 15′ 20″ E.

43. If the radius of the earth (3956 mi.) subtends 57′ at the moon, what is the moon's distance from the earth?

Ans. 238,300 mi.

44. Find the radius of one's horizon if he be located 1320 ft. above the earth. How large when located 2 mi. above the earth? *Ans.* 44.48 mi., 125.8 mi.

45. How high above the earth must one be in order to see a point located on the surface 50 mi. away? *Ans.* 1668.2 ft.

46. The radius of a circle is 240 ft. Find the perimeter of a regular inscribed pentagon. *Ans.* 1410.72 ft.

47. The radius of a circle is 85 ft. What is the area of the regular inscribed decagon? *Ans.* 21,234.27 sq. ft.

48. Find the perimeter of a regular dodecagon inscribed in a circle whose radius is 25 in. *Ans.* 155.25 in.

49. What is the radius of a circle inscribed in an equilateral triangle whose perimeter is 99 ft.? *Ans.* 9.52 ft.

50. The area of a regular pentagon inscribed in a circle is 475.53 sq. cm. Find the area of a regular decagon inscribed in the same circle. *Ans.* 587.8 sq. cm.

51. The area of a regular pentagon is 441 sq. ft. Find the apothem; also find the radius of the circumscribed circle.

Ans. 11 ft.; 13.6 ft.

52. What is the length of a diagonal joining the first and fourth vertices of a regular polygon of 12 sides inscribed in a circle whose radius is 30 ft.? *Ans.* 42.426 ft.

53. If R = radius of a circle, show that the area of a regular inscribed polygon of n sides is $A = nR^2 \sin \frac{180°}{n} \cos \frac{180°}{n}$.

54. From the result in Ex. 53 construct a table showing, in terms of R, areas of the regular inscribed polygons of 3, 4, 5, 9, 10, 12, sides.

55. Show that the area of a regular circumscribed polygon of n sides is given by $A = nR^2 \tan \frac{180°}{n}$.

56. A train moving at a uniform speed of 40 mi. per hour on a straight track passes a station A at noon; at 2:15 o'clock P.M. it has arrived at a station B, 56 mi. farther north. How far east of A is B, and what is the bearing of the road?

Ans. 70.46 mi.; N. 51° 31′ 30″ E.

57. A boat running at a rate of 10 mi. per hour starts to steam directly across a river one mile in width. If the water be flowing uniformly at a rate of 4 mi. per hour, find the point at which the boat will land, and its velocity in the water.

Ans. 0.4 mi. downstream, 10.77 mi. velocity.

58. Find the projection of the altitude of an equilateral triangle upon a side. Let $a =$ side. *Ans.* $\frac{3}{4}a$.

59. A line extends N. 20° E. 125 rd. from a point A. Find its projection upon a line extending N. 60° E. from A.

Ans. 95.75 rd.

60. Find the projection of a line 450 ft. long running N. 47° E. upon a line running E. 15° S. *Ans.* 238.45 ft.

61. Two forces of 160, 120 lb. act upon a heavy body, the first at an angle of 30° with the horizontal, the second at an angle of 75°. Find the total forces which tend to move the body (1) horizontally, (2) vertically.

Ans. (1) 169.6 lb., (2) 195.9 lb.

62. Find the number of square yards in a conical tent with circular base, the vertical angle being 60°, and the centre pole 12 ft. high. [Take $\pi = \frac{22}{7}$.] *Ans.* 33.5 sq. yd.

63. Find the area in acres of the following tract of land: starting from a point A, the boundary line runs N. 24° E. 80 rd. to B, thence N. 65° E. 120 rd. to C, thence S. 180 rd. to D, thence back to A. Find also the length and bearing of DA.

Ans. 99.157 acres; $DA = 152.07$ rd.; N. 68° 18′ 23″ W.

64. Find the area of the following described tract of land: starting from a point A, the boundary line runs N. 10° E. 100 rd., thence N. 47° E. 150 rd., thence E. 40 rd., thence S. 10° W. 100 rd., thence W. 40 rd., thence to A the place of beginning.

Ans. 81.04 acres.

CHAPTER III

TRIGONOMETRIC FUNCTIONS OF ANY ANGLE. GRAPHS

In § 2 the trigonometric functions have been defined for positive acute angles only. We shall now extend these definitions to include angles of any size whatever.

19. Axes. Quadrants. To locate an angle which may be either acute or obtuse it is convenient to employ *Coördinate Axes* as shown in Fig. 19. The horizontal line is called the X-axis, the vertical is called the Y-axis. These axes divide the plane into four *quadrants* marked I, II, III, IV.

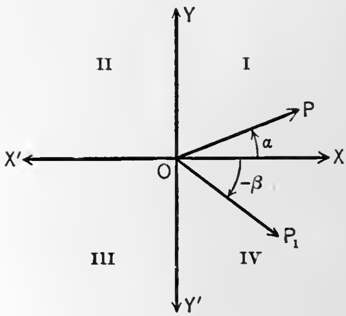


Fig. 19.

A *positive trigonometric angle* is described when a vector OP is rotated about O *counter-clockwise* from the *initial position* OX into a *terminal position* OP; $XOP = \alpha$. If the rotation be *clockwise* about O, the angle described is *negative*; $XOP_1 = -\beta$. See Fig. 19.

20. Coördinates. Abscissa, Ordinate. The position of a *terminal line* OP (Fig. 20 *a*, *b*, *c*, *d*) is determined by two measurements x, y, called *coördinates of the point P.* The *x-measurement* is called the *abscissa of the point P*, and is the *projection of OP on the X-axis.* If the projection of OP falls to the right of O (Fig. 20 *a*, *d*), the abscissa is *positive*, if to the left (Fig. 20 *b*, *c*), the abscissa is negative.

The *y-measurement* is called the *ordinate of the point P*, and is the *projection of OP upon the Y-axis.* The ordinate is positive (Fig. 20 *a*, *b*) when the projection of *OP* falls above the horizontal, and is *negative* (Fig. 20 *c*, *d*) when it falls below the horizontal.

In locating a point in the respective quadrants, the coördinates of any point are usually written in brackets. The symbol (a, b) means the abscissa $x = a$, the ordinate $y = b$.

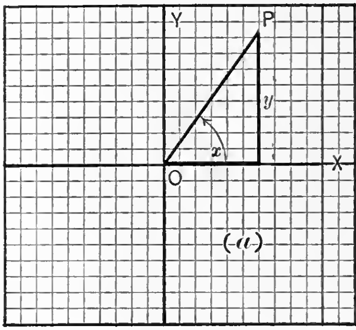


Fig. 20 *a*.

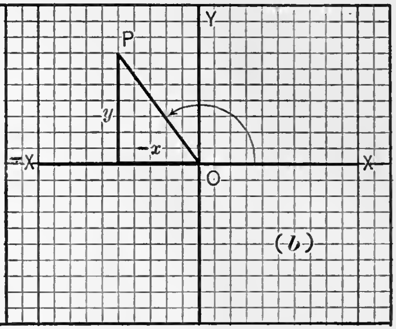


Fig. 20 *b*.

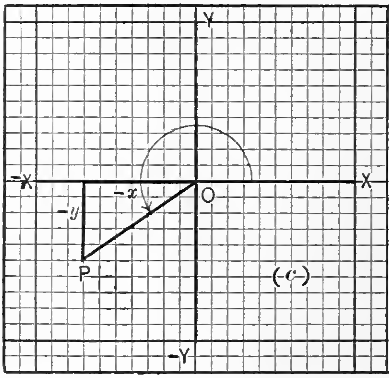


Fig. 20 *c*.

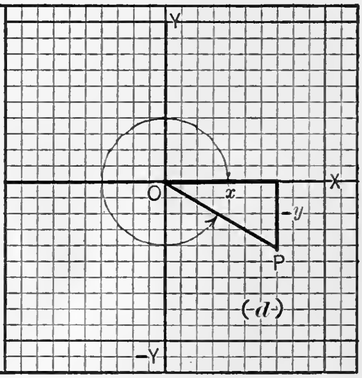


Fig. 20 *d*.

21. Definitions of the Functions. From the above figures, (*a*), (*b*), (*c*), (*d*), with $\angle XOP = \alpha$ and $OP = r$, we define in each case:

$$\sin \alpha = \frac{y}{r} = \frac{\text{ordinate}}{\text{distance}}, \qquad \csc \alpha = \frac{r}{y} = \frac{\text{distance}}{\text{ordinate}},$$

$$\cos \alpha = \frac{x}{r} = \frac{\text{abscissa}}{\text{distance}}, \qquad \sec \alpha = \frac{r}{x} = \frac{\text{distance}}{\text{abscissa}},$$

$$\tan \alpha = \frac{y}{x} = \frac{\text{ordinate}}{\text{abscissa}}, \qquad \cot \alpha = \frac{x}{y} = \frac{\text{abscissa}}{\text{ordinate}}.$$

In this notation the signs of the abscissa and ordinate determine the algebraic signs of the trigonometric functions for angles terminating in the respective quadrants.

22. Laws of Signs. (1) *For all angles terminating in the first quadrant, the functions are* ***positive.***

(2) *For angles terminating in the second quadrant, the six functions are* ***negative except sine and cosecant.***

(3) *For angles terminating in the third quadrant, the* ***tangent and cotangent are positive,*** *all others are* ***negative.***

(4) *For angles terminating in the fourth quadrant, the* ***cosine and secant are positive,*** *all others are* ***negative.***

These laws are shown in the following diagram:

	Sin	Cos	Tan	Cot	Sec	Csc
I	+	+	+	+	+	+
II	+	−	−	−	−	+
III	−	−	+	+	−	−
IV	−	+	−	−	+	−

It should be noticed that these general definitions apply to angles larger than 360°. For example, sin 400° is the sine of an angle terminating 40° above the initial line; hence $\sin 400° = \sin 40°$. $\tan 500° = \tan(360° + 140°)$ is the tangent of an angle terminating in the second quadrant. $\tan 500° = \tan 140° = -\tan 40°$. $\sin(n\,360° + \alpha) = \sin \alpha$.

The fundamental identities, § 5, *hold for any angle.*

$$\sin^2\alpha + \cos^2\alpha = 1, \qquad \sin\alpha\csc\alpha = 1,$$
$$\sec^2\alpha - \tan^2\alpha = 1, \qquad \cos\alpha\sec\alpha = 1,$$
$$\csc^2\alpha - \cot^2\alpha = 1, \qquad \tan\alpha\cot\alpha = 1.$$

Examples. 1. What values have the functions when $\sin\phi = \frac{3}{5}$?

Solution. The angle ϕ may be constructed in either of two positions, AOP_1 in the first quadrant, or the supplement AOP_2 in the second quadrant. Let $AOP_1 = \phi_1$, $AOP_2 = \phi_2$. In the first case, $\sin\phi_1 = \frac{3}{5}$, $\cos\phi_1 = \frac{4}{5}$, $\tan\phi_1 = \frac{3}{4}$, etc. In the second case, $\sin\phi_2 = \frac{3}{5}$, $\cos\phi_2 = -\frac{4}{5}$, $\tan\phi_2 = -\frac{3}{4}$, etc.

2. Find all the trigonometric functions when $\cos\phi = \frac{1}{2}$.

Solution. Locate angle ϕ as a positive angle in the first quadrant, and as an equal negative angle in the fourth quadrant. The angle whose cosine is $\frac{1}{2}$ is either ϕ or $-\phi$. Then, $\sin\phi = \pm\frac{\sqrt{3}}{2}$, $\cos\phi = \frac{1}{2}$, $\tan\phi = \pm\sqrt{3}$, $\cot\phi = \pm\frac{1}{\sqrt{3}}$, etc.

3. Locate the positive angle ϕ, when P has the following coördinates: (*a*) (4, 3); (*b*) (−4, 5); (*c*) (5, −3); (*d*) (8, −1); (*e*) (−5, −6); (*f*) (3, −4).

4. Give the values of each trigonometric function for each angle determined in Ex. 3.

EXERCISES

Name the quadrant (or quadrants) in which ϕ terminates when:

1. $\sin\phi = \frac{1}{2}$.

2. $\tan\phi = 4$.

3. $\cos\phi = \frac{3}{5}$.

4. $\sin\phi = -\frac{4}{5}$.

5. $\cot\phi = 0$.

6. $\tan\phi = -5$.

7. $\csc\phi = -2$.

8. $\sin\phi = \frac{2}{3}$, $\tan\phi < 0$.

9. $\cos\phi = \frac{1}{3}$, $\cot\phi < 0$.

10. $\tan\phi = 3$, $\sin\phi < 0$.

Express the following as functions of acute angles:

11. $\sin(440^\circ) = \sin(360^\circ + 80^\circ) = \sin 80^\circ$.

12. $\sin 370^\circ$.

13. $\tan 430^\circ$.

14. $\cos(2\pi + 20^\circ)$.

15. $\tan\left(\pi + \frac{\pi}{3}\right)$.

16. $\cot\left(n\pi + \frac{\pi}{6}\right)$.

17. $\sin\left(2n\pi + \frac{\pi}{4}\right)$.

18. $\sec 300^\circ$.

19. $\tan 700^\circ$.

20. $\sin 500^\circ$.

23. Functions of Negative Angles. To express the trigonometric functions of a negative angle in terms of an equal positive angle construct the negative angle and an equal positive angle, Figs. 21 and 22. In either drawing (θ acute, or θ obtuse) the triangles AOP and AOP_1 are equal,

$$x_1 = x, \quad y_1 = -y, \quad r_1 = r.$$

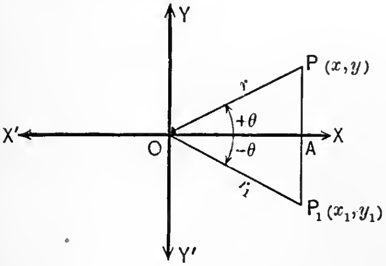


Fig. 21.

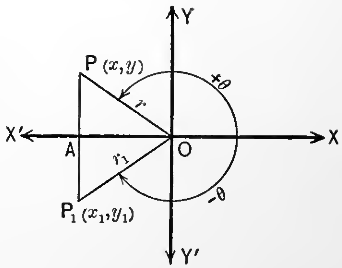


Fig. 22.

$$\sin(-\theta) = \frac{y_1}{r_1} = \frac{-y}{r} = -\sin\theta, \qquad \csc(-\theta) = -\csc\theta,$$

$$\cos(-\theta) = \frac{x_1}{r_1} = \frac{x}{r} = \cos\theta, \qquad \sec(-\theta) = \sec\theta,$$

$$\tan(-\theta) = \frac{y_1}{x_1} = \frac{-y}{x} = -\tan\theta, \qquad \cot(-\theta) = -\cot\theta.$$

THE LAWS GOVERNING FUNCTIONS OF NEGATIVE ANGLES ARE: (1) *Any trigonometric function of a negative angle equals the same function of an equal positive angle,* (2) *the sign being changed in all cases except for the cosine and secant.*

EXERCISES

1. Write equivalents of the following functions with the signs of the angles changed:

(1) $\cos(-48°)$; (4) $\cot(-87°)$; (7) $\sin 47°$;
(2) $\tan(-65°)$; (5) $\sec(-75°)$; (8) $\tan(\alpha - \beta)$;
(3) $\sin(-50°)$; (6) $\cot(-100°)$; (9) $\sin(\theta - \phi)$.

2. Write numerical values of the following:

(1) $\sin(-30°)$; (3) $\cot(-60°)$; (5) $\cos(-60°)$;
(2) $\tan(-45°)$; (4) $\sec(-45°)$; (6) $\tan(-90°)$.

3. Reduce to numerical values:

(1) $\sin 90° \times \sin(-90°) \div \tan(-45°)$.
(2) $\tan(-60°) \times \sin(-30°) \times \csc 60°$.
(3) $\sin^2(-45°) \times \cos(-60°) \times \csc(-45°)$.
(4) $\sin^2(-45°) \div \cos^2(-45°) + \tan(-45°)$.
(5) $\sec^2(120°) - \tan^2(120°) + \sin(-90°)$.
(6) $\cos(-80°) \times \sin(-20°) \times \sec 80° \times \csc(-20°)$.

24. Functions of 90° − θ and 90° + θ.

(1) *Complemental angles.* The functions of $90° - \theta$ with θ acute, Fig. 23 *a*, or θ obtuse, Fig 23 *b*, obey the same laws as have been developed in § 3.

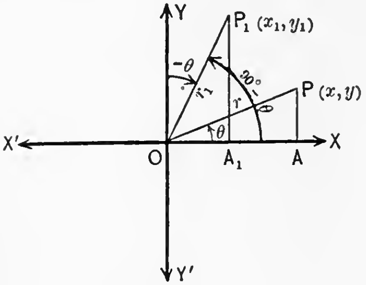


Fig. 23 *a*.

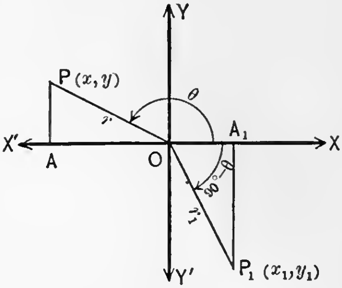


Fig. 23 *b*.

In the drawings, Figs. 23 *a*, *b*, the triangles OPA and OP_1A_1 are equal, and

$$x_1 = y, \quad y_1 = x, \quad r_1 = r.$$

Defining the functions, we have :

$$\sin(90° - \theta) = \frac{y_1}{x_1} = \frac{x}{r} = \cos\theta, \qquad \csc(90° - \theta) = \sec\theta,$$

$$\cos(90° - \theta) = \frac{x_1}{r_1} = \frac{y}{r} = \sin\theta, \qquad \sec(90° - \theta) = \csc\theta,$$

$$\tan(90° - \theta) = \frac{y_1}{x_1} = \frac{x}{y} = \cot\theta, \qquad \cot(90° - \theta) = \tan\theta.$$

LAW FOR COMPLEMENTAL ANGLES. *The functions of the complement of any angle equal the corresponding co-functions of the angle.*

(2) *Functions of* $90° + \theta$. The functions of $90° + \theta$ may be derived from the above functions of $90° - \theta$ by changing θ into $-\theta$ and noting the results of § 23.

$$\sin(90° + \theta) = \cos(-\theta) = \cos\theta, \qquad \csc(90° + \theta) = \sec\theta,$$

$$\cos(90° + \theta) = \sin(-\theta) = -\sin\theta, \qquad \sec(90° + \theta) = -\csc\theta,$$

$$\tan(90° + \theta) = \cot(-\theta) = -\cot\theta, \qquad \cot(90° + \theta) = -\tan\theta.$$

These results show that the same law holds for functions of $90° + \theta$ as for $90° - \theta$, except the algebraic sign is changed in all cases except that of the *sine and cosecant*.

25. Functions of $180^\circ - \theta$ and $180^\circ + \theta$. Supplemental Angles. The functions of the supplement of an angle θ may be expressed in terms of the functions of θ. Construct θ acute, Fig. 24 *a*; also construct θ obtuse, Fig. 24 *b*.

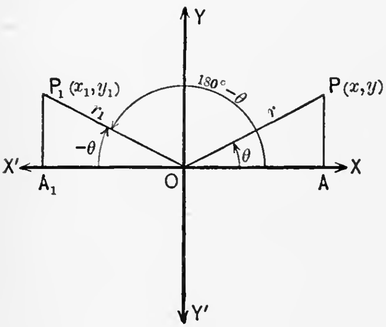


Fig. 24 *a*.

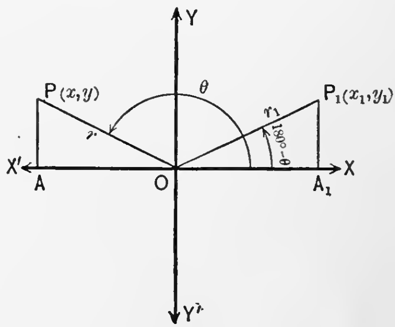


Fig. 24 *b*.

In either figure the triangle OAP equals the triangle OA_1P_1, and

$$x_1 = -x, \quad y_1 = y, \quad r_1 = r.$$

Hence,

$$\sin(180^\circ - \theta) = \frac{y_1}{r_1} = \frac{y}{r} = \sin\theta, \qquad \csc(180^\circ - \theta) = \csc\theta,$$

$$\cos(180^\circ - \theta) = \frac{x_1}{r_1} = \frac{-x}{r} = -\cos\theta, \qquad \sec(180^\circ - \theta) = -\sec\theta,$$

$$\tan(180^\circ - \theta) = \frac{y_1}{x_1} = \frac{y}{-x} = -\tan\theta, \qquad \tan(180^\circ - \theta) = -\tan\theta.$$

LAWS FOR SUPPLEMENTAL ANGLES. *Any trigonometric function of an angle equals the same function of its supplement, the algebraic sign being changed in all cases except the sine and cosecant.*

The functions of $180^\circ + \theta$ may be obtained from those of $180^\circ - \theta$ by changing θ to $-\theta$ in the above formulas. We find:

$$\sin(180^\circ + \theta) = \sin(-\theta) = -\sin\theta, \qquad \csc(180^\circ + \theta) = -\csc\theta,$$

$$\cos(180^\circ + \theta) = -\cos(-\theta) = -\cos\theta, \qquad \sec(180^\circ + \theta) = -\sec\theta,$$

$$\tan(180^\circ + \theta) = -\tan(-\theta) = \tan\theta, \qquad \cot(180^\circ + \theta) = \cot\theta.$$

LAW. *Any trigonometric function of* $180^\circ \pm \theta$ *equals the same function of the angle* θ, *regard being had for the algebraic sign.*

EXERCISES

1. Fill the blanks with the proper function of the supplement of each angle:

(1) $\sin 150° = \sin 30°$
(2) $\tan 97° 20' =$
(3) $\cos 160° 40' =$
(4) $\cot 175° 10' =$
(5) $\sec 120° 10' =$
(6) $\csc 100° 20' =$
(7) $\tan (90° + \phi) =$
(8) $\sin (90° - \phi) =$
(9) $\cos (45° - \phi) =$
(10) $\cot (60° + \phi) =$

2. By taking supplements of 0°, 30°, 45°, and 60° find the trigonometric functions of 180°, 150°, 135°, and 120°. Fill the blanks in the following table:

$\phi =$	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin \phi$									
$\cos \phi$									
$\tan \phi$									
$\cot \phi$									
$\sec \phi$									
$\csc \phi$									

3. Find numerical values of the following:

(1) $\sin 120° \times \sin 60°$.
(2) $\tan 45° \times \cot 12° \times \cos 90°$.
(3) $5 \times \cos 135° \times \sin 90° \times \cos 180°$.
(4) $\tan 135° \times \cot 130° \div \sin 60°$.
(5) $\tan 150° \times \cos 150° \div \sin 30°$.
(6) $2 \sin 120° + \cot 150°$.
(7) $\tan 135° + \cot 45° - \cos 180°$.
(8) $\cos 30° + \cos 150° + \tan 60° + \tan 120°$.
(9) $(\tan 120° - \tan 135°) \times (\tan 120° + \tan 135°)$.
(10) $\sec (180° - \theta) \times \cos \theta \times \tan (180° - a) \times \cot a$.

4. From proper drawings obtain values of the functions of $180° + \theta$ in terms of the functions of θ.

5. Verify $\sin 210° = -\frac{1}{2}$; $\cos 225° = -\frac{1}{2}\sqrt{2}$; $\tan 225° = 1$; $\tan 240° = \sqrt{3}$. [Note. Construct drawings.]

6. By adding 180° to the angle in the functions of $90° - \theta$ derive the functions of $270° - \theta$. Verify the results by constructing proper drawings.

7. Find the value of $\dfrac{\cos(90° + \alpha)}{\sin(-\alpha)} + \dfrac{\tan(-\alpha)}{\tan(180° + \alpha)}$.

8. Find the value of $\dfrac{\sin(180° - \phi)}{\cos(90° + \phi)} \times \dfrac{\tan(180° + \theta)}{\cot(90° + \theta)}$.

9. What sign has $\sin x + \cos x$ for the following values of x:
(1) $x = 90°$; (2) $x = 120°$; (3) $x = 135°$;
(4) $x = 210°$; (5) $x = 300°$?

10. Find all angles less than 360° which satisfy:
(1) $\tan\theta = -1$; (2) $\sin\theta = \frac{1}{2}\sqrt{3}$; (3) $\cos\theta = -\frac{1}{2}$.

26. Line-values of the Functions.

(1) *Acute angles.* Let the angle θ be constructed at the centre of a circle whose radius is unity, Fig. 25. Then arc $AB = \theta$.

$CB = \sin\theta$, $OT' = \csc\theta$,
$OC = \cos\theta$, $OT = \sec\theta$,
$AT = \tan\theta$, $DT' = \cot\theta$,
$CA = \text{vers}\,\theta$, $ED = \text{covrs}\,\theta$.

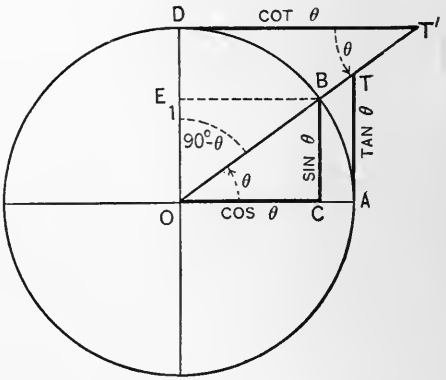


Fig. 25.

These lines represent graphically the values of the trigonometric functions when the radius of the circle is taken as unity.

It should be noticed that trigonometric functions considered as *line-values* may be described as follows: (1) *The sine of an angle is the length of the perpendicular let fall from the terminal end of the arc (B) upon the diameter through the initial end (A).* (2) *The cosine of an angle is the* *part of the radius (OC) cut off by the foot of the perpendicular (BC).* (3) *The tangent of an angle is the geometric tangent erected at the initial extremity (A) of the arc and terminated by the diameter produced through the terminal extremity (B) of the arc.* (4) *The secant is the line from O to the extremity of the tangent (OT).* (5) *The cotangent is the geometric tangent erected at the* 90° *point (D) of the circle, and terminated by the terminal radius produced to T′, (DT′).* (6) *The cosecant is the length of the line from the centre of the circle to the extremity of the cotangent (OT′).*

Convention of signs. In the above definitions the sines and tangents are verticals, and are taken *positive when drawn upward, negative when drawn downward.* The cosines and cotangents are horizontals, and are *positive when drawn to the right, negative when drawn to the left.* The secants and cosecants are *positive when drawn from O along the terminal boundary OB, negative when drawn from O backward*, as *OT* in Fig. 26 (*a*).

The older text-books defined the trigonometric functions from this line-value standpoint, but at the present time the definitions are usually given, as in § 2 and § 21, from the ratio standpoint. It is often convenient to use the line-values.

(2) *Angles larger than* 90°. Drawings are here inserted to show the line-values of the functions for angles larger than 90°, Figs. 26 (*a*), (*b*), (*c*). In each case $OT = \sec\theta$, $OT' = \csc\theta$.

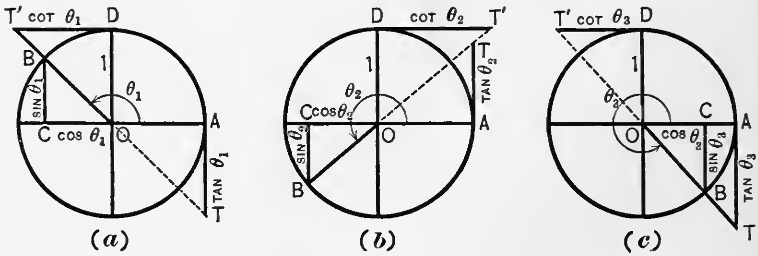


Fig. 26 (*a*), (*b*), (*c*).

These drawings show clearly that the fundamental relations, § 5, hold whatever the angle may be.

The following table shows the signs of the functions of θ, θ_1, θ_2. θ_3, shown in Figs. 25, 26 (*a*), (*b*), (*c*), respectively.

	Sin	Cos	Tan	Cot	Sec	Csc
θ	+	+	+	+	+	+
θ_1	+	−	−	−	−	+
θ_2	−	−	+	+	−	−
θ_3	−	+	−	−	+	−

It should be remarked that the functions of an angle between 270° and 360° are the same as those of a *negative acute angle*, and that the functions of an angle between 180° and 270° are the same as those of a *negative obtuse angle.*

EXERCISES

Draw figures and show:

1. $\sin 70° = \cos 20° = \sin 110°$.
2. $\sin 120° = \cos 30° = -\sin(-60°)$.
3. $\sin 117° = \cos(-27°) = \cos 27°$.
4. $\cos 300° = \cos(-60°) = \sin 30°$.
5. $\sin 220° = \sin(-40°) = -\sin 40°$.
6. $\cos 195° = -\cos(-15°) = -\cos 15°$.
7. $\sin(-10°) = \sin 190° = -\sin 10°$.
8. $\sin 60° = -\cos 150° = \cos 30°$.
9. $\tan 120° = -\tan 60° = \tan(-60°)$.
10. $\cot 165° = -\cot 15° = -\tan 75°$.

If $A + B + C = 180°$, show:

11. $\cos\frac{A}{2} = \sin\frac{B+C}{2}$.
12. $\sin\frac{B}{2} = \cos\frac{A+C}{2}$.
13. $\sin A = \sin(B + C)$.
14. $\cos B = -\cos(A + C)$.
15. $\tan A = -\tan(B + C)$.
16. $\sin A = -\sin(2A + B + C)$.
17. $\cos A = -\cos(2A + B + C)$.
18. $\cos\left(\frac{B-C}{2}\right) = \sin\left(\frac{A+2B}{2}\right) = \sin\left(\frac{A+2C}{2}\right)$.

19. $\sin\left(\frac{A-B}{2}\right) = -\cos\left(\frac{2A+C}{2}\right) = \cos\left(\frac{C+2B}{2}\right)$.

20. $\cos B = \sin\left(\frac{A+3B+C}{2}\right)$.

Change each of the following to functions of positive acute angles:

21. $\sin 340°$. **24.** $\cot(-160°)$. **27.** $\sec(-200°)$.

22. $\tan 540°$. **25.** $\sin 260°$. **28.** $\tan(-280°)$.

23. $\cos 450°$. **26.** $\cot 300°$. **29.** $\sin(-500°)$.

30. $\cos(-490°)$. **31.** $\sin(-40°) \times \tan 187°$.

27. Graphs of the Trigonometric Functions. The *line-values* of the functions furnish a means for constructing graphs which show to the eye the numerical values of $\sin x$, $\cos x$, $\tan x$ for any angle.

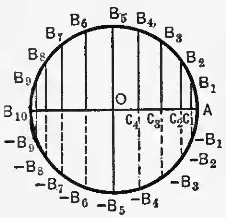


Fig. 27.

(1) *Graph of* $\sin x$. Let us take a unit circle, Fig. 27, and, beginning at A, locate points B_1, B_2, B_3, ⋯ at equal intervals, say 18°, along the circumference. Then draw perpendiculars B_1C_1, B_2C_2, ⋯. These lines are equal to the sines of the respective angles subtended by arcs AB_1, AB_2, ⋯. Now, place the point A at O, Fig. 28 (a), and straighten the circumference along the line OX, the points B_1, B_2, B_3, ⋯ falling at equal intervals along OX as indicated. Erect perpendiculars (+ or −) equal to B_1C_1, B_2C_2, B_3C_3, ⋯, respectively. These lines are the sines of the angles whose arc measures are AB_1, AB_2, AB_3, ⋯. In Figs. 28 (a) and (b) the scale is $\frac{1}{2}$ of that used in Fig. 27.

If a smooth curve be drawn through C_1, C_2, C_3, ⋯, we have a locus known as the *sine curve*, or the *graph of* $\sin x$.

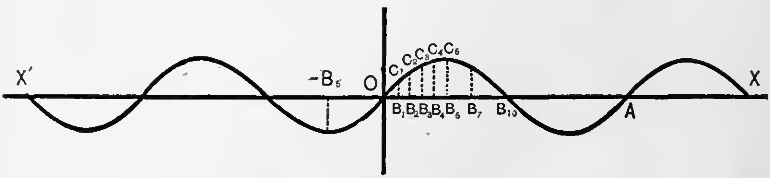


Fig. 28 (a). SINE CURVE.

If the circumference be laid off to the left of O, we have the part of the graph shown on OX'.

(2) *Graph of* $\cos x$. Taking the unit circle as before, Fig. 27, dividing its arc into equal intervals, laying the circumference upon OX, and erecting perpendiculars at O, B_1, B_2, B_3, $\cdots$ equal in length to OA, OC_1, OC_2, $\cdots$, we have lines which equal the cosines of the angles subtended by arcs AB_1, AB_2, $\cdots$, respectively. Now, connect the points A, C_1, C_2, $\cdots$ by a smooth curve, and we have the *graph of* $\cos x$, or the *cosine curve*, Fig. 28 (b).

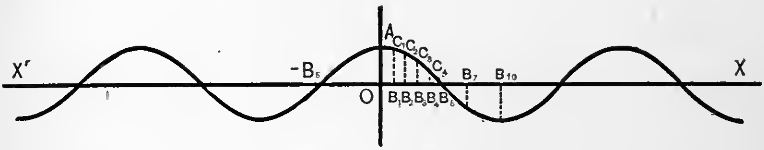


Fig. 28 (b). COSINE CURVE.

(3) *Graph of* $\tan x$ *and* $\cot x$. If at the points B_1, B_2, B_3, $\cdots$ on OX, Fig. 28 (c), we erect perpendiculars (+ or −) equal to the lengths of the tangents at A of the arcs AB_1, AB_2, AB_3, $\cdots$, the points T_1, T_2, $\cdots$ lie upon a curve known as the *tangent curve*, or the *graph of the* $\tan x$. This curve consists of a series of parallel graphs, each extending to infinity at 90°, 270°, 540°, $\cdots$.

The graph of $\cot x$ may be constructed in a similar manner. The graphs of $\tan x$ and $\cot x$ are shown in Fig. 28 (c).

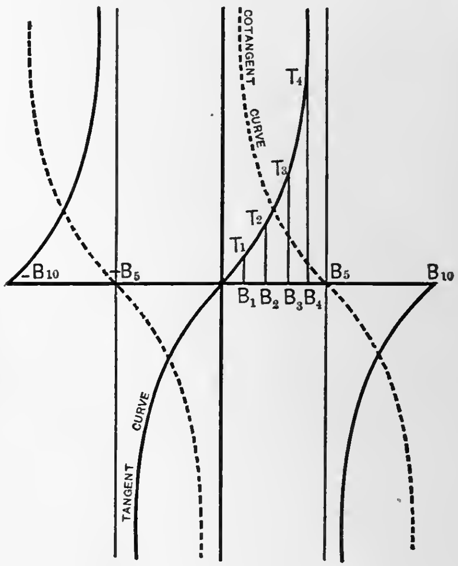


Fig. 28 (c).

CHAPTER IV

MEASUREMENT OF ANGLES

28. Units of Measure. In the measurement of angles, two units are in use, the *degree* and the *radian.*

(1) *Sexagesimal system.* The degree* (marked by °) is the $\frac{1}{90}$ of a right angle. It is the unit of *degree measure* or the *sexagesimal measure.* The degree is divided into 60 minutes (marked 60′), and the minute into 60 seconds (marked 60″).

$$48 \text{ degrees } 35 \text{ minutes } 24 \text{ seconds} = 48° \; 35' \; 24''.$$

Seconds are frequently written as decimals of a minute, and minutes and seconds may be written as decimals of a degree.

$$45° \; 21' \; 36'' = 45° \; 21.6' = 45.36°.$$

Degree measure is used in most of the practical calculations of astronomers, engineers, and surveyors.

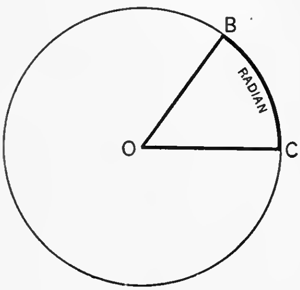


Fig. 29.

(2) *Radian measure* (*Circular measure*). In this system the unit of measure is the *radian, the angle subtended at the center of a circle by an arc equal to the radius.*

In Fig. 29, the arc $BC = OC$, hence the angle $COB = 1$ radian. Circular measure is used almost exclusively in the theoretical work of higher mathematics.

29. Relations between Degree and Radian Measure. The two systems of measurement may be compared. If C = circumference, R = radius, we have

$$C = 2\pi R, \; \pi = 3.14159265 \cdots.$$

* The degree could as well be defined as the $\frac{1}{60}$ of an angle of an equilateral triangle. In their efforts to determine direction, it is thought the ancient Babylonians used as a unit of measure the angle of an equilateral triangle; this angle was at first divided into 10 equal parts, and later into 60, which coincides with our sexagesimal system.

A circumference subtends 360° at the centre; if the radius be taken as the unit of measure (called a radian), we have 360° in angular measure equal to 2π radians in circular measure.

$$360° = 2\pi \text{ radians},$$

$$180° = \pi \text{ radians},$$

$$1° = \frac{\pi}{180} \text{ radian} = 0.0174532925199\cdots \text{ radian},$$

$$1' = 0.0002908882086\cdots \text{ radian},$$

$$1'' = 0.0000048481368\cdots \text{ radian}.$$

Conversely,

$$1 \text{ radian} = \frac{180°}{\pi} = 57.29577°\cdots\cdots = 57° \; 17' \; 44.8''.$$

CONVERSION RULES. (1) *To reduce degrees to radians multiply the number of degrees by* $\frac{\pi}{180} = .01745$, *and mark the result radians.* (2) *To reduce radians to degrees multiply the number of radians by* $\frac{180}{\pi} = 57.295$, *and mark the product degrees.*

For approximations take $\pi = \frac{22}{7}$.

EXAMPLES. **1.** Convert 63° 30′ to radians.

$$63.5 \times 0.01745 = 1.099 \text{ radians}.$$

2. Convert 148° 20′ 24″ to radians.

$$148° \; 20' \; 24'' = 148.34°.$$

$$148.34 \times 0.01745 = 2.588 \text{ radians}.$$

3. Change 2.5 radians to degrees.

$$2.5 \times 57.295 = 143.239 \text{ degrees}.$$

EXERCISES

1. Give degree measure of $\frac{\pi}{2}$; $\frac{\pi}{3}$; $\frac{\pi}{4}$; $\frac{\pi}{6}$; $\frac{2\pi}{3}$; $\frac{3\pi}{4}$.

2. Change from degrees to radians: 18°; 124°; 290°; 30°; 500° 45′.

3. Change from radian measure to degrees: 1.5; $\frac{3}{8}$; 1.65; 2.75; -0.4; $5\frac{1}{3}$.

4. Find the number of degrees in: (*a*) $1+\frac{\pi}{8}$; (*b*) $1-\frac{\pi}{6}$; (*c*) $140° - 1\frac{1}{2}$; (*d*) $30° + 2\pi$; (*e*) $40° - \frac{\pi}{10}$; (*f*) $3 - \frac{2}{3}\pi$.

5. What does each of the following symbols mean?

NOTE. *n* is any integer.

(*a*) $\pm\pi$; (*b*) $n\pi$; (*c*) $2n\pi$; (*d*) $n\frac{\pi}{2}$; (*e*) $(2n+1)\frac{\pi}{2}$; (*f*) $(2n-1)\frac{\pi}{2}$; (*g*) $n\pi \pm \frac{\pi}{4}$.

6. Name the quadrant in which each of the following angles terminates:

(*a*) $\pi - 1$; (*b*) $\pi + 48°$; (*c*) $240° + 2$; (*d*) $n\pi \pm \frac{\pi}{4}$, when $n = 0, 1, 2, 3, 4$.

7. Determine the smallest positive angle which has the same terminal boundary as:

(*a*) 440°; (*b*) 660°; (*c*) 378°; (*d*) $-200°$; (*e*) $-300°$; (*f*) $\pi - 40°$; (*g*) $2\pi + 120°$.

30. The Length of Any Arc. In geometry it is shown that in unequal circles arcs subtending equal angles are to each other as the corresponding radii. Thus, in the figure arc PQ : arc AB :: OP : OA. Hence, if OA be taken as a unit, the arc AB is the radian measure of the angle AOB; that is, $AB = \theta$; and $PQ = \theta \cdot OP = \theta \cdot R$.

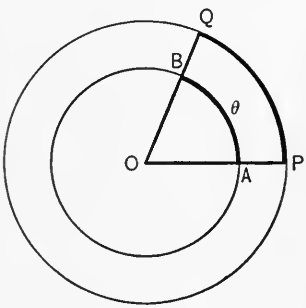


Fig. 30.

THEOREM. *The length of any arc equals the radius multiplied by the radian measure of the angle subtended by the arc.*

EXAMPLES. **1.** Find the length of a 20° arc upon a circle of radius 27 ft.

SOLUTION. $20° = 20 \times \frac{\pi}{180} = \frac{\pi}{9}$ radian.

$$\text{Arc of } 20° = \frac{\pi}{9} \times 27 \text{ ft.} = 3\pi \text{ ft.} = 9.42 \text{ ft.}$$

NOTE. Take $\pi = \frac{22}{7}$.

2. Find the length of an arc subtending 135° upon a circle whose radius is 16 in.

SOLUTION. $$135° = 135 \times \frac{\pi}{180} = \frac{3}{4}\pi \text{ radians.}$$

$$\text{Arc of } 135° = \tfrac{3}{4}\pi \times 16 = 12\pi \text{ in.} = \pi \text{ ft.}$$

3. Find the angle subtended at centre by a 6 ft. arc upon a circle whose radius is 15 ft.

SOLUTION. $$\text{Arc} = \theta \cdot R.$$

$$\theta = \frac{\text{arc}}{R} = \frac{6}{15} = \frac{2}{5} \text{ radian.}$$

Hence, θ expressed in degrees is

$$\tfrac{2}{5} \times 57.29° = 22.91° = 22° \, 54' \, 36''.$$

31. Segment and Sector Areas. The area of a sector of a circle, AOB, equals arc AB multiplied by one half of the radius, Fig. 31.

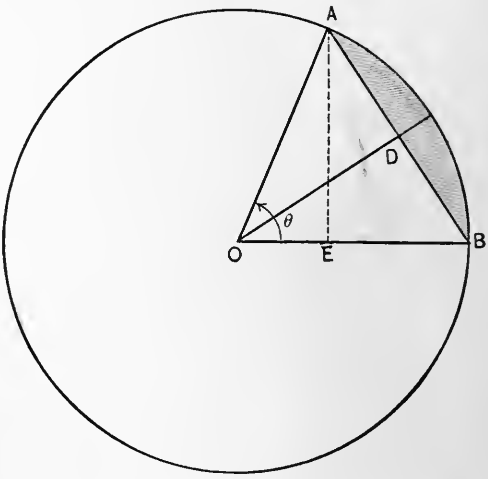


Fig. 31.

Sector $AOB = \frac{1}{2} \text{arc } AB \times OB = \frac{1}{2} R^2\theta.$

Area of triangle AOB

$$= \tfrac{1}{2} AB \times OD$$
$$= \tfrac{1}{2} AE \times OB$$
$$= \tfrac{1}{2} R^2 \sin\theta.$$

Area of shaded segment

$$ADB = \tfrac{1}{2}\theta R^2 - \tfrac{1}{2} R^2 \sin\theta$$
$$= \frac{R^2}{2}(\theta - \sin\theta).$$

NOTE. The angle θ is to be expressed in radian measure.

EXERCISES

1. Find the length of an arc of 63° on a circle of 20-in. radius. *Ans.* 22 in.

NOTE. In this list of exercises, take $\pi = \frac{22}{7}$, except otherwise indicated.

2. Through what distance does the extremity of a 7-in. minute hand of a clock move in 10 min.? *Ans.* $3\frac{2}{3}$ in.

3. Find the radius of a circle in which a 10-ft. arc subtends an angle of 2 radians at the centre. *Ans.* 5 ft.

4. Find the angle at the centre of a circle, radius 24 in., which is subtended by an 11-in. arc. *Ans.* 26° 15′.

5. Find the area of a 24° sector whose subtending arc is 8 ft. *Ans.* 76.36 sq. ft.

6. In a circle whose radius is 10 ft., a chord is 6 ft. from the centre. Find the area of the smaller segment cut off by the chord. *Ans.* 44.73 sq. ft.

NOTE. Take radian measure from Table IV in Ex. 6, 7, 8, 9, 10, 15, 16, 17.

7. A horizontal oil tank whose length is 30 ft. and radius 4 ft. is filled to a depth of 15 in. Find the number of gallons of oil in the tank. (231 cu. in. = 1 gal.) *Ans.* 1125.6 gal.

8. Find the length of a belt stretched around two pulleys whose radii are 5 ft. and $1\frac{1}{2}$ ft., respectively, the distance between the centres of the pulleys being 20 ft. Also find the length of the belt when crossed.

Ans. (*a*) 61.03 ft.; (*b*) 62.55 ft.

9. Find the distance at which a foot ruler must be held so that its length will subtend one degree at the eye.

Ans. 57.29 ft.

10. Find the length of a degree on a circle whose radius is 3956 mi. *Ans.* 69.045 mi.

11. What is the length of a degree of longitude on the 39th parallel of latitude, if one degree on the equator is 69.045 mi., the earth's radius being 3956 mi. *Ans.* 53.65 mi.

12. If m be the length of one degree upon the equator, show that the length of one degree upon any parallel of latitude is $m \times \cos \alpha$, when α is the latitude of the parallel.

13. In radian measure two angles of a triangle are $\frac{1}{3}$ and $\frac{2}{5}$. Find the third angle in degrees ($\pi = \frac{22}{7}$). *Ans.* 138°.

14. The end of a 30-in. pendulum swings through a 3-in. arc. Find the angle through which it swings. *Ans.* 5° 43.8′.

15. If the diameter of the moon be 2163 mi., find the moon's distance from the earth, assuming that its diameter subtends 31′ 7″ at the eye. See § 29. *Ans.* 238,900 mi.

16. If the sun's apparent diameter subtends an angle of 32′ 4″ at the eye, find its diameter, assuming the sun's distance from the earth to be 92,897,000 miles. *Ans.* 866,500 mi.

17. If the earth's equatorial radius (3963 mi.) subtends 8.8″ at the sun (the sun's parallax), find the distance of the sun from the earth. *Ans.* 92,890,000 mi.

CHAPTER V

FUNCTIONS OF TWO ANGLES. MULTIPLE ANGLES

We next develop the trigonometric functions of the sum and difference of two angles.

32. To develop sin $(\alpha+\beta)$ and cos $(\alpha+\beta)$. The angles α and β may be both acute and such that their sum is less than 90°, as in Fig. 32; or they may be such that their sum is greater than 90° and smaller than 180°, as shown in Fig. 33. In each construction, β is made to join α, the line PQ being erected perpendicular to OP and QN (QN'), drawn perpendicular to OM at N, or to OM produced negatively at N'.

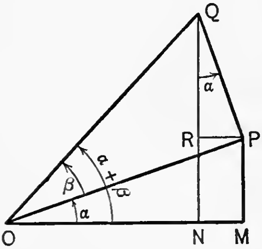


Fig. 32.

Then defining sin $(\alpha+\beta)$ from the figure, we have

$$\sin(\alpha+\beta)=\frac{NQ}{OQ}=\frac{MP+RQ}{OQ}$$

$$=\frac{MP}{OQ}\times\frac{OP}{OP}+\frac{QR}{OQ}\times\frac{QP}{QP}$$

$$=\sin\alpha\cos\beta+\cos\alpha\sin\beta.$$

Similarly,

$$\cos(\alpha+\beta)=\frac{ON}{OQ}=\frac{OM-RP}{OQ}$$

$$=\frac{OM}{OQ}\times\frac{OP}{OP}-\frac{RP}{OQ}\times\frac{QP}{QP}$$

$$=\cos\alpha\cos-\sin\alpha\sin\beta.$$

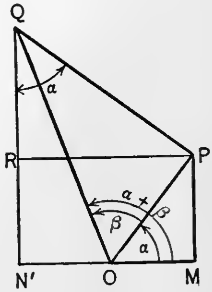


Fig. 33.

33. To develop tan $(\alpha+\beta)$. (1) We may derive $\tan(\alpha+\beta)$ from the drawing, thus:

$$\tan(\alpha+\beta)=\frac{QN}{ON}=\frac{PM+QR}{OM-PR}=\frac{\frac{PM}{OM}+\frac{QR}{OM}}{1-\frac{PR}{OM}}$$

$$=\frac{\tan\alpha+\frac{PQ}{OP}}{1-\frac{PM\cdot PQ}{OM\cdot OP}}=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}.$$

NOTE. The triangles OMP and QRP are similar.

(2) We may derive $\tan(\alpha+\beta)$ analytically by dividing $\sin(\alpha+\beta)$ by $\cos(\alpha+\beta)$.

$$\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}.$$

Dividing both numerator and denominator by $\cos\alpha\cos\beta$, we find

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}.$$

By taking the reciprocals of $\tan(\alpha+\beta)$, $\tan\alpha$, $\tan\beta$, we have

$$\cot(\alpha+\beta)=\frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}.$$

Since the $\sin(-\theta)=-\sin\theta$, $\cos(-\theta)=\cos\theta$, and $\tan(-\theta)=-\tan\theta$, we have, on changing β into $-\beta$ in each formula above:

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta,$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta,$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}.$$

NOTE. The above formulas have been derived from figures in which $\alpha+\beta$ is either acute, or lies between 90° and 180°. The same results are true for any values of α, β, but we shall not introduce any proof of this fact here.

34. Important Formulas. Collecting the results of the above demonstrations, we have the fundamental formulas:

(1) $$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta,$$

(2) $$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta,$$

(3) $$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta,$$

(4) $$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta,$$

(5) $$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta},$$

(6) $$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta},$$

(7) $$\cot(\alpha\pm\beta)=\frac{\pm\cot\alpha\cot\beta-1}{\cot\alpha\pm\cot\beta}.$$

learn

EXERCISES

1. Show $\sin 75^\circ=\sin(45^\circ+30^\circ)=\frac{1}{4}(\sqrt{6}+\sqrt{2})$.

SOLUTION. $\sin(45^\circ+30^\circ)=\sin 45^\circ\cos 30^\circ+\cos 45^\circ\sin 30^\circ$

$$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

Hence, $$\sin 75^\circ=\cos 15^\circ=\tfrac{1}{4}(\sqrt{6}+\sqrt{2}).$$

2. Show $\sin 15^\circ=\frac{1}{4}(\sqrt{6}-\sqrt{2})=\cos 75^\circ$.

3. Show $\tan 75^\circ=2+\sqrt{3}=\cot 15^\circ$.

4. Show $\tan 15^\circ=2-\sqrt{3}=\cot 75^\circ$.

5. Show $\sin(45^\circ+\alpha)=\dfrac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha)$.

NOTE. Apply formula (1).

6. Prove $\cos(30^\circ-\alpha)=\frac{1}{2}(\sqrt{3}\cos\alpha+\sin\alpha)$.

7. Prove $\tan(45^\circ+x)=\dfrac{1+\tan x}{1-\tan x}=\cot(45^\circ-x)$.

8. Prove $\dfrac{\sqrt{3}}{2}\cos x-\dfrac{1}{2}\sin x=\sin(60^\circ-x)$.

9. Prove $\dfrac{1}{2}\cos x+\dfrac{\sqrt{3}}{2}\sin x=\cos(60^\circ-x)$.

10. Prove $\tan(60^\circ-x)=\dfrac{\sqrt{3}-\tan x}{1+\sqrt{3}\tan x}$.

11. Derive the formula $\sin(\alpha+\beta+\gamma) = \sin\alpha\cos\beta\cos\gamma + \cos\alpha\sin\beta\cos\gamma + \cos\alpha\cos\beta\sin\gamma - \sin\alpha\sin\beta\sin\gamma$.

SUGGESTION. Write $\sin(\alpha+\beta+\gamma) = \sin(\alpha+[\beta+\gamma])$, and apply (1).

12. Derive $\cos(\alpha+\beta+\gamma) = \cos\alpha\cos\beta\cos\gamma - \cos\alpha\sin\beta\sin\gamma - \cos\beta\sin\alpha\sin\gamma - \cos\gamma\sin\alpha\sin\beta$.

13. Derive $\tan(x+y+z)$

$$= \frac{\tan x + \tan y + \tan z - \tan x\tan y\tan z}{1 - \tan x\tan y - \tan y\tan z - \tan z\tan x}.$$

14. If $\sin x = 0.5$ and $\cos y = 0.6$, find the value of $\sin(x+y)$, and $\cos(x+y)$.

SUGGESTION. Construct angles x, y, and expand $\sin(x+y)$, then substitute. Likewise with $\cos(x+y)$.

$$\sin(x+y) = \sin x\cos y + \cos x\sin y$$
$$= 0.5\times 0.6 + 0.5\sqrt{3}\times 0.8 = \frac{\sqrt{3}}{10}(\sqrt{3}+4).$$
$$\cos(x+y) = \cos x\cos y - \sin x\sin y.$$
$$= 0.5\sqrt{3}\times 0.6 - 0.5\times 0.8 = \frac{\sqrt{3}}{10}(\sqrt{3}-4).$$

15. If $\sin x = 0.8$, $\tan y = 1$, what value has $\sin(x-y)$? $\tan(x+y)$?

16. When $\tan x = \sqrt{3}$, $\cos y = 0.5$, find $\tan(x-y)$; also $\cos(x-y)$.

17. When $\cos x = 1$, $\tan y = 1$, find $\sin(x+y)$; also $\cot(x+y)$.

18. Show $\cos(x+y) = 0$, when $\tan x = 0.5$ and $\cot y = 0.5$.

19. Show $\sin(x+y) = 1$, when $\sin x = \cos y$.

20. Show $\tan(x+y) = \infty$, when $\cos x = \sin y$, or $\tan x = \cot y$.

FUNCTIONS OF MULTIPLE AND SUB-MULTIPLE ANGLES

The formulas enumerated in § 34 give rise to new and important results by making angle β equal to angle α.

35. Functions of 2 α. In the formulas,

(1) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$,

(3) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$,

(5) $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$,

let $\beta = \alpha$, and we derive:

(7) $$\sin 2\alpha = 2 \sin \alpha \cos \alpha,$$

(8) $$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2\cos^2 \alpha - 1 = 1 - 2\sin^2 \alpha,$$

(9) $$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}.$$

36. Functions of 3 α.

Let $\beta = 2\alpha$ in (1), (3), (5) above, and employ results of (7), (8), (9), and we obtain:

(10) $$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha,$$

(11) $$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha,$$

(12) $$\tan 3\alpha = \frac{3 \tan \alpha - \tan^3 \alpha}{1 - 3 \tan^2 \alpha}.$$

37. Half-angle Formulas. In (7), (8), (9) § 35, let $\alpha = \frac{x}{2}$, and we find:

(13) $$\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2},$$

(14) $$\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = 2\cos^2 \frac{x}{2} - 1 = 1 - 2\sin^2 \frac{x}{2},$$

(15) $$\tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}}.$$

Solving each of the latter two values of $\cos x$ for $\sin \frac{x}{2}$ and $\cos \frac{x}{2}$, we have the important formulas:

(16) $$\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}},$$

(17) $$\cos \frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}},$$

and dividing $\sin \frac{x}{2}$ by $\cos \frac{x}{2}$,

(18) $$\tan \frac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}.$$

(16), (17), (18) are known as the half-angle formulas.

EXERCISES

1. If $\sin x = \frac{3}{5}$, find $\sin 2x$; also $\tan 2x$.
2. If $\cos x = \frac{5}{13}$, find $\tan 2x$; also $\cos 2x$.
3. If $\csc x = 2$, find $\sin 2x$; also $\cos 2x$.
4. Find $\sin x$, if $\cos \frac{x}{2} = 0.3$.
5. Find $\tan x$, if $\sin \frac{x}{2} = \frac{5}{13}$.
6. Find $\sin x$, if $\cos 2x = \frac{\sqrt{3}}{2}$.
7. Find $\tan x$, if $\sin 2x = \frac{12}{13}$.
8. Find $\tan 60°$, knowing $\tan 30° = \frac{1}{3}\sqrt{3}$.
9. Find $\tan 53° 8'$, if $\tan 26° 34' = 0.5$.
10. Find $\cos 2x$, if $\sec x = \frac{3}{2}$.
11. If $\sin \alpha = 0.4$, find $\sin 3\alpha$.
12. When $\tan \alpha = 0.1$, find $\tan 3\alpha$.
13. Find $\sin 22\frac{1}{2}°$ and $\cos 22\frac{1}{2}°$ from the functions of 45°.
14. Find $\tan 15°$ from the functions of 30°.
15. Find the $\sin 67\frac{1}{2}°$ from the functions of $22\frac{1}{2}°$.

Verify the following identities:

16. $(\sin x \pm \cos x)^2 = 1 \pm \sin 2x$.
17. $\cos^4 x - \sin^4 x = \cos 2x$.
18. $\sin x = \dfrac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$.
19. $\cos x = \dfrac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$.
20. $2 \sin x + \sin 2x = \dfrac{2 \sin^3 x}{1 - \cos x}$.
21. $\tan x + \cot x = 2 \csc 2x$.
22. $\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) = \sec x + \tan x$.
23. $\cot x - \tan x = 2 \cot 2x$.

24. $\dfrac{\sin 2x}{\cot x} = 1 - \cos 2x.$ **26.** $\sin\dfrac{x}{2} = \sqrt{\dfrac{\text{vers } x}{2}}.$

25. $\dfrac{\sin 3x}{\sin x} = 1 + 2\cos 2x.$ **27.** $\tan\dfrac{x}{2} = \dfrac{1+\sin x - \cos x}{1 + \sin x + \cos x}.$

28. $\sin 4\theta = 4\sin\theta\cos\theta(\cos^2\theta - \sin^2\theta)$
$= 8\cos^3\theta\sin\theta - 4\cos\theta\sin\theta.$

29. $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1.$

30. When $\sin 3\theta - \sin 6\theta = 0$, show $\cos 3\theta = \frac{1}{2}$.

31. $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta.$

32. $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$

33. $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B).$

34. $\cos^2 A - \cos^2 B = \sin(A+B)\sin(B-A).$

35. $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B).$

36. $\sin\dfrac{\theta}{2} + \cos\dfrac{\theta}{2} = \pm\sqrt{1+\sin\theta}.$

37. $\sin\dfrac{\theta}{2} - \cos\dfrac{\theta}{2} = \pm\sqrt{1-\sin\theta}.$

(Explain use of double sign in 36, 37.)

SUM AND DIFFERENCE FORMULAS

38. Converting to Products. Taking the results (1), (2), (3), (4), § 34,

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta,$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta,$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta,$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta,$$

we have on adding and subtracting:

$$\left.\begin{aligned}
\sin(\alpha+\beta) + \sin(\alpha-\beta) &= 2\sin\alpha\cos\beta,\\
\sin(\alpha+\beta) - \sin(\alpha-\beta) &= 2\cos\alpha\sin\beta,\\
\cos(\alpha+\beta) + \cos(\alpha-\beta) &= 2\cos\alpha\cos\beta,\\
\cos(\alpha+\beta) - \cos(\alpha-\beta) &= -2\sin\alpha\sin\beta.
\end{aligned}\right\} \quad \text{(I)}$$

Let $\alpha+\beta = X$, $\alpha - \beta = Y$, then

$$\alpha = \frac{X+Y}{2}, \quad \beta = \frac{X-Y}{2}.$$

Making these substitutions, we have:

(19) $$\sin X + \sin Y = 2 \sin \frac{X+Y}{2} \cos \frac{X-Y}{2},$$

(20) $$\sin X - \sin Y = 2 \cos \frac{X+Y}{2} \sin \frac{X-Y}{2},$$

(21) $$\cos X + \cos Y = 2 \cos \frac{X+Y}{2} \cos \frac{X-Y}{2},$$

(22) $$\cos X - \cos Y = -2 \sin \frac{X+Y}{2} \sin \frac{X-Y}{2}.$$

These formulas should be remembered as *the sum of two sines equals twice the sine of the half sum, times the cosine of the half difference*, and so on for the remaining formulas. Formulas (19)–(22) are the so-called *Sum and Difference Formulas;* when read forward they convert a sum or difference of two sines or cosines into a product; when read backwards they convert a product of two sines or cosines into a sum or difference.

39. Converting to Sum or Difference. The formulas (19), (20), (21), (22) should be recognized when read conversely. Writing formulas (I), § 38, conversely, and replacing α by A, β by B, we have:

(19′) $$2 \sin A \cos B = \sin (A + B) + \sin (A - B),$$

(20′) $$2 \cos A \sin B = \sin (A + B) - \sin (A - B),$$

(21′) $$2 \cos A \cos B = \cos (A + B) + \cos (A - B),$$

(22′) $$2 \sin A \sin B = \cos (A - B) - \cos (A + B),$$

a set of important relations which should be recognized.

EXERCISES

Read the following exercises, applying the sum and difference formulas, reducing answers:

1. $\sin 10° + \sin 40° =$
2. $\sin 80° + \sin 30° =$
3. $\cos 60° + \cos 40° =$
4. $\sin 70° - \sin 40° =$
5. $\cos 28° + \cos 42° =$
6. $\cos 28° - \cos 42° =$
7. $\sin 35° + \cos 25° =$
8. $\sin (-10°) + \sin 40° =$
9. $\cos 80° - \cos (-20°) =$
10. $\cos \frac{\pi}{6} + \cos \frac{\pi}{3} =$
11. $\sin 4\theta + \sin 3\theta =$
12. $\sin 3\theta - \sin \theta =$

13. $\cos(n+1)\theta - \cos(n-1)\theta =$

14. $\sin(n+1)\frac{\pi}{4} - \sin(n-1)\frac{\pi}{4} =$

Express as a sum or difference:

15. $2\sin 40^\circ \cos 20^\circ =$

16. $2\cos 50^\circ \sin 40^\circ =$

17. $2\sin 20^\circ \sin 40^\circ =$

18. $2\cos 60^\circ \cos 10^\circ =$

19. $2\sin\frac{\pi}{4}\cos\frac{3\pi}{4} =$

20. $2\sin(n\theta)\cos(n-1)\theta =$

Prove the following identities:

21. $\dfrac{\sin 2x + \sin 2y}{\sin 2x - \sin 2y} = \dfrac{\tan(x+y)}{\tan(x-y)}.$

22. $\dfrac{\cos 3x - \cos 5x}{\sin 3x + \sin 5x} = \tan x.$

23. $\dfrac{\sin x + \sin y}{\cos x + \cos y} = \tan\dfrac{(x+y)}{2}.$

24. $\dfrac{\sin 9x + \sin x}{\cos 9x + \cos x} = \tan 5x.$

25. $\dfrac{\sin x + \sin y}{\sin x - \sin y} = \dfrac{\tan\frac{x+y}{2}}{\tan\frac{x-y}{2}}.$

26. $\dfrac{\cos x - \cos y}{\cos x + \cos y} = -\tan\dfrac{x+y}{2}\tan\dfrac{x-y}{2}.$

27. $\sin\left(x+\frac{\pi}{3}\right) + \sin\left(x-\frac{\pi}{3}\right) = \sin x.$

28. $\sin(30^\circ + A) + \sin(30^\circ - A) = \cos A.$

29. $\cos\left(\frac{\pi}{3}+A\right) + \cos\left(\frac{\pi}{3}-A\right) = \cos A.$

30. $\dfrac{\sin(n-2)x + \sin(nx)}{\cos(n-2)x - \cos(nx)} = \cot x.$

31. $2\sin\left(x+\frac{\pi}{4}\right)\sin\left(x-\frac{\pi}{4}\right) = \sin^2 x - \cos^2 x.$

32. $\cos x + \cos 3x + \cos 5x + \cos 7x = 4\cos x\cos 2x\cos 4x.$

33. $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x\cos 2x\sin 4x.$

34. $\sin\theta + \sin\phi + \sin(\theta+\phi) = 4\cos\frac{\theta}{2}\cos\frac{\phi}{2}\sin\frac{\theta+\phi}{2}$.

35. $\sin\theta + \sin\phi - \sin(\theta+\phi) = 4\sin\frac{\theta}{2}\sin\frac{\phi}{2}\sin\frac{\theta+\phi}{2}$.

36. $$\frac{\sin\theta + \sin\phi + \sin\psi - \sin(\theta+\phi+\psi)}{\cos\theta + \cos\phi + \cos\psi + \cos(\theta+\psi+\phi)}$$
$$= \tan\frac{\theta+\phi}{2}\tan\frac{\phi+\psi}{2}\tan\frac{\psi+\theta}{2}.$$

When $A + B + C = 180°$, prove:

37. $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$.

38. $\sin A + \sin B - \sin C = 4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$.

39. $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$.

40. $\cos A + \cos B - \cos C = -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$.

41. $\tan A + \tan B + \tan C = \tan A \tan B \tan C$.

42. Show $\frac{\sin(x+y)}{\cos(x-y)} = \frac{\cot x + \cot y}{1 + \cot x \cot y}$.

43. Show $\frac{\tan x + \tan y}{\tan x - \tan y} = \frac{\sin(x+y)}{\sin(x-y)}$.

44. Show $\frac{\tan 5x - \tan 3x}{1 + \tan 5x \tan 3x} = \frac{\tan 3x - \tan x}{1 + \tan 3x \tan x}$.

45. Prove that in a given circle the area of a regular inscribed pentagon is to the area of a regular inscribed decagon as cos 36° is to 1.

CHAPTER VI

LOGARITHMS

Calculations involving multiplication, division, raising to powers, and extracting roots may become quite laborious when performed in the ordinary way. To abbreviate the work of numerical calculation certain tables known as *Logarithmic Tables of Numbers* and of the *Trigonometric Functions* have been prepared. The use of such tables will now be explained.

40. The Index Laws. Let it be required to find an approximate value of the fraction

$$F = \frac{25 \times 78.6 \times \tan 65^\circ}{658.4 \times 4.75}.$$

Since $10^1 = 10$ and $10^2 = 100$, we may approximate a number between 1 and 2 which, when used as an exponent of 10, will produce 25. This exponent is approximately 1.39794. The number 10 may be affected by an exponent which will produce any one of the numbers in the fraction F.

We write the above fraction in exponential form thus:

$$F = \frac{25 \times 78.6 \times \tan 65^\circ}{658.4 \times 4.75} = \frac{10^{1.39794} \times 10^{1.89542} \times 10^{0.33133}}{10^{2.81849} \times 10^{0.67669}}$$

$$= \frac{10^{1.39794+1.89542+0.33133}}{10^{2.81849+0.67669}}$$

$$= \frac{10^{3.62469}}{10^{3.49518}} = 10^{3.62469-3.49518}$$

$$= 10^{0.12951}.$$

This final result will evidently be a number smaller than 10 since the exponent is less than unity; its approximate value is

$$F = 1.3474.$$

Thus, the operations of multiplication have been replaced by addition, and the operation of division has been performed by subtraction.

The exponents of 10 in this illustration obey the *index laws* of elementary algebra, viz. (1) the exponent of a product equals the *sum* of the exponents, (2) the exponent of a quotient equals the *difference* of the exponents (the exponent of the denominator subtracted from that of the numerator). To these two laws we attach a third, illustrated by

$$25^2 = (10^{1.39794})^2 = 10^{2.79588} = 625.$$

These three *index laws* are expressed by formula thus:

(1) $$a^x \times a^y = a^{x+y},$$

(2) $$a^x \div a^y = a^{x-y},$$

(3) $$(a^x)^m = a^{mx}.$$

41. Definition of Logarithms.*

(1) Common Logarithms. *The logarithm of a number N to the base* 10 *is the exponent by which* 10 *must be affected to produce the number N.* Logarithms constructed upon 10 as a base are called *Common logarithms* (Briggian logarithms).

As illustrations of logarithmic notation,

$$25 = 10^{1.39794}, \quad \log 25 = 1.39794,$$

$$78.6 = 10^{1.89542}, \quad \log 78.6 = 1.89542,$$

$$\tan 65° = 10^{0.33133}, \quad \log \tan 65° = 0.33133.$$

(2) Logarithms to Any Base. If we write

$$a^x = N,$$

we define $x =$ **the logarithm of** N **to base** a,

or more briefly $\log_a N = x$.

The logarithm of a number N to base a is the exponent by which a must be affected to produce N.

* Logarithm = ratio number. Logarithms were invented by John Napier (1550–1617), a native of Scotland. To his system of logarithms, published 1614, he applied the name *artificial numbers*. Napier's system of logarithms was not constructed upon 10 as a base. Henry Briggs (1556–1631), professor in Gresham College, London, first used 10 as a base (1617), and thus constructed a system adapted to the ordinary decimal notation. The tables first published by Briggs were constructed to 14 decimal places. Tables in ordinary use are constructed to *seven, six, five or even four decimals*. Greater degrees of accuracy will be obtained by using tables with greater numbers of decimals. The ordinary logarithms of the trigonometric functions were first constructed by Gunter, 1620.

EXAMPLES. **1.** If the base is 5, what is $\log_5 25$, $\log_5 125$, $\log_5(0.2)$, $\log_5(0.04)$? *Ans.* 2, 3, -1, -2.

2. With 8 as base, what is $\log_8 64$, $\log_8(\frac{1}{8})$, $\log_8 4$, $\log_8 16$, $\log_8 32$, $\log_8 128$? *Ans.* 2, -1, $\frac{2}{3}$, $\frac{4}{3}$, $\frac{5}{3}$, $\frac{7}{3}$.

3. With 10 as base, $\log 2 = 0.30103$, $\log 3 = 0.47712$, $\log 5 = 0.69897$. Find $\log 4$, $\log 6$, $\log 9$, $\log 12$, $\log 15$, $\log 18$, $\log 20$.

Ans. 0.60206, 0.77815, 0.95424, 1.07918, 1.17609, 1.25527, 1.30103.

4. From example 3 above find $\log(\frac{9}{4})$, $\log(\frac{15}{2})$, $\log 1.5$, $\log 1.2$. *Ans.* 0.35218, 0.87506, 0.17609, 0.07918.

5. With 16 as base, what numbers correspond to the following logarithms: 2, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{1}{4}$, $-\frac{1}{2}$, $-\frac{3}{4}$? *Ans.* 256, 4, 64, 2, $\frac{1}{4}$, $\frac{1}{8}$.

42. Systems of Logarithms. Any number (excluding 0 and 1) may be taken as a base for a logarithmic system. Two bases are in use, **10** and e, where e is defined by

$$e = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \cdots = 2.71828182\cdots.$$

Logarithms constructed with 10 as base are called *common logarithms* or *Briggian logarithms.* This system is adapted to all ordinary numerical calculations.

Logarithms constructed with e as base are called *natural logarithms*, or *Napierian logarithms*, or *hyperbolic logarithms.* This system is used in the theoretical work of higher mathematics. Tables of logarithms constructed to the base 10 will be found in the Tables compiled at the end of this book.

43. Laws Governing the Use of Logarithms. The interpretation of the Index Laws into logarithmic form furnishes the necessary rules for performing multiplication, division, raising to powers, and extracting roots by means of logarithms.

I. LAW OF PRODUCTS. *The logarithm of a product equals the sum of the logarithms of the factors.*

For, let $a^x = N, \quad a^y = M,$

then $x = \log_a N, \quad y = \log_a M.$

Also, $N \times M = a^x \times a^y = a^{x+y},$

$$\log_a(N \times M) = x + y = \log_a N + \log_a M,$$

which establishes the law.

EXAMPLE. $\log_{10}(67 \times 126) = \log_{10} 67 + \log_{10} 126$
$= 1.82607 + 2.10037 = 3.92644.$

II. LAW OF QUOTIENTS. *The logarithm of a quotient equals the logarithm of the numerator diminished by the logarithm of the denominator.*

For, let $a^x = N, \quad a^y = M,$

then $N \div M = a^x \div a^y = a^{x-y}.$

In logarithmic form

$$\log_a \left(\frac{N}{M}\right) = x - y = \log_a N - \log_a M.$$

EXAMPLE. $\log_{10} (184 \div 1.56) = \log_{10} 184 - \log_{10} 1.56$
$= 2.26482 - 0.19312 = 2.07170.$

III. LAW OF POWERS (ROOTS). *The logarithm of N^n equals n times the logarithm of N.* This law is true for any value of the exponent n, whether positive or negative, integer or fraction.

Let $a^x = N, \quad x = \log_a N;$

then, by the third index law,

$$(a^x)^n = a^{nx} = N^n.$$

In logarithmic notation,

$$\log_a (N^n) = nx = n \cdot \log_a N.$$

EXAMPLES.
$$\log 507^3 = 3 \log 507,$$
$$\log (148)^{\frac{1}{2}} = \tfrac{1}{2} \log 148,$$
$$\log (576)^{\frac{2}{3}} = \tfrac{2}{3} \log 576.$$

44. Characteristic and Mantissa. From the table

$$10^4 = 10000$$
$$10^3 = 1000$$
$$10^2 = 100$$
$$10^1 = 10$$
$$10^0 = 1$$
$$10^{-1} = .1$$
$$10^{-2} = .01$$

it is obvious that the logarithm of an integral power of 10 is an integer, either positive or negative. The logarithms of numbers between 1 and 10 are evidently between 0 and 1, logarithms of numbers between 10 and 100 are between 1 and 2, and so on. For example, $\log 8 = 0.90309$, $\log 80 = 1.90309$, $\log 800 = 2.90309$, $\log 8000 = 3.90309$.

The integral part of a logarithm is called the characteristic; the decimal part of the logarithm is called the mantissa.

I. LAW OF THE CHARACTERISTIC. From the above example, it is seen that log 8 has a characteristic 0, log 80 has 1, log 800 has 2, and so on. From this illustration, we see that *the characteristic of a logarithm of a whole number is one less than the number of digits in the number.*

Dividing a number by 10 reduces the characteristic by 1; dividing by 100 reduces its characteristic by 2, and so on. Hence, for a pure decimal, as .00567, *the characteristic is negative and equal to the number of places which the first significant figure occupies to the right of the decimal point.* Thus,

$$\log 0.00567 = -3 + .75358,$$

$$\log 0.0258 = -2 + .41162.$$

In such cases, the characteristic is negative, while the mantissa is positive. The usual notation is to place a negative sign (–) over the characteristic, leaving the form for the above illustrations thus:

$$\log 0.00567 = \bar{3}.75358,$$

$$\log 0.0258 = \bar{2}.41162.$$

II. LAW OF THE MANTISSA. *The mantissa is the same for any given sequence of digits, whatever may be the position of the decimal point.*

For example, $\log 4896 = 3.68984$, $\log 48.96 = 1.68984$, $\log 4.896 = 0.68984$, $\log 0.04896 = \bar{2}.68984$.

Logarithmic tables as usually constructed show only the mantissa. The characteristic is to be supplied according to Rule I.

45. Use of Tables. Tabulation of Logarithmic Work. (1) *To find the logarithm of a number.* Let us find the logarithm of a number of five figures.

Take for example log 58,769.

(*a*) The characteristic is 4.

(*b*) Look in column marked N, of the Tables, for the first three digits 587; follow the horizontal row opposite 587 to the right to column 6 at top, and we find the mantissa .76908.

(*c*) The correction for the last figure 9 is approximately .9 of the difference between the mantissa for 5876 and that for 5877, *i.e.* $.9 \times 8 = 7.2$. Hence, add 7 to the last figure of the mantissa already found.

$$\log 58769 = 4.76915.$$

(*d*) In the margin, the tabular differences between any two successive mantissas are indicated; on this page these differences are 7 and 8. The corrections for 1, 2, 3, ⋯, 9 are indicated in the columns under the 8 and 7, respectively.

(2) *To find the number corresponding to a logarithm.* A number whose logarithm is given is found by reversing the above process.

Required the number corresponding to the logarithm 2.57682.

$$\log N = 2.57682.$$

(*a*) In the logarithmic tables find the mantissa next smaller than .57682. This is .57680, leaving a difference 2.

(*b*) The digits in column N and at top of column in which .57680 is found are 3774.

(*c*) The difference between .57680 and the next tabular number is 12.

(*d*) In proportional parts column, under 12, the nearest number to proportional part 2 is 2.

(*e*) Hence, $N = 377.42$, the decimal point falling between 7 and 4, since the characteristic is 2.

(3) *To find the logarithm of trigonometric functions.*

(1) To illustrate, let us find the log sin 37° 48′ 15″.

(*a*) Sines and cosines are less than unity; hence, the characteristic in each case is negative. To avoid negative characteristics in the tables, 10 is added and subtracted.

(*b*) log sin 37° 48′ = 9.78739 − 10. Correction for 15″ = 4. Hence,

$$\log \sin 37° 48' 15'' = 9.78743 - 10.$$

(*c*) The tabular difference is found in column marked d, and proportional corrections for 6″, 7″, 8″, 9″, 10″, 20″, 30″, 40″, 50″ are indicated in the margin under proportional parts.

(2) Find log cos 49° 21′ 30″.

(*a*) The log cos 49° 21′ is found by looking at bottom of page 69, reading up the column marked log cos and taking the minutes, 21′, from the column to right of the page.

$$\log \cos 49° 21' = 9.81387 - 10.$$

(*b*) Since the cosine decreases as the angle increases, the correction for 30″ must be subtracted.

Correction 30″ = 7.5 (count as 8); see Prop. Pts. under 15.

(*c*) Hence, log cos 49° 21′ 30″ = 9.81379 − 10.

The use of the tables in finding logarithms of tangents and cotangents is similar to that explained for sines and cosines.

TABULATION OF WORK

In the work of computation by means of logarithms it is very important that the computor should economize in both time and labor. To accomplish this purpose the beginner should (1) make himself familiar with the mechanical construction of the logarithmic tables, especially the devices of marginal corrections, and (2) he should adopt and systematically carry out some convenient plan of tabulation of his work. A plan of tabulation is suggested in the following examples.

EXAMPLES. 1. Find by means of logarithms the value of $F = \frac{a \times b \times \tan \alpha}{c \times d}$, where $a = 25$, $b = 78.6$, $c = 658.4$, $d = 4.75$, and $\alpha = 65°$.

Let the numerator be called N, the denominator D. Then we may tabulate as follows:

$$F = \frac{a \times b \times \tan \alpha}{c \times d} = \frac{N}{D}.$$

Data and Results

a	25
b	78.6
α	65°
c	658.4
d	4.75
F	1.3474

Logarithms

log a	1.39794	add	
log b	1.89542	add	
log tan α	0.33133	add	
log N	3.62469		subtract
log c	2.81849	add	subtract
log d	0.67669	add	subtract
log D	3.49518		subtract
log F	0.12951		

2. Find $F=\dfrac{a\sin\alpha}{b\tan\beta}$, where $a=4694.5$, $b=37.6$, $\alpha=67°\,20'\,15''$ and $\beta=70°\,41'\,10''$.

Data and Results

a	4694.5
α	67° 20′ 15″
b	37.6
β	70° 41′ 10″
F	40.377

Logarithms

$\log a$	3.67154	
		5 correction
$\log\sin\alpha$	9.96509 – 1 0	
		1 correction
$\log a\sin\alpha$	13.63669 – 10	
$\log b$	1.57519	
$\log\tan\beta$	0.45529	
		6.7 correction
$\log b\tan\beta$	2.03055	
$\log F$	1.60614	

46. Conversion of Common to Napierian Logarithms. Logarithms of one system may easily be converted into logarithms of another system. Let us consider the systems to base 10 and e. Let

$$10^y=N,\ \text{then}\ y=\log_{10}N.$$

Take logarithm to base e of both members of $10^y=N$.

$$y\log_e 10=\log_e N,$$

or

$$\log_{10}N\cdot\log_e 10=\log_e N.$$

Now, $\log_e 10=2.302585$; hence,

(1) $$\log_e N=2.302585\times\log_{10}N,$$

(2) $$\log_{10}N=\frac{1}{2.302585}\times\log_e N=0.434294\times\log_e N.$$

EXERCISES IN USE OF LOGARITHMS

By means of logarithms find the value of each of the following:

1. $Q=\dfrac{78.54\times 9.6752}{8.269}$. *Ans.* 91.896.

2. $Q=\dfrac{(104.6)^{\frac{1}{3}}\times 0.2536}{(5.87)^{\frac{1}{2}}}$. *Ans.* 0.49319.

3. $Q = \frac{(1848)^{\frac{2}{3}} \times 10^{-2}}{0.7854}$. *Ans.* 1.9174.

4. $Q = \frac{1496 \times \tan 40° 17'}{\cos 10° 21'}$. *Ans.* 1288.9.

5. $A = 32.6 \times 5.87 \times \sin 56° 24' 15''$. *Ans.* 159.396.

6. $A = \pi r^2$, $\pi = 3.1416$, $r = 3.6$. *Ans.* 40.715.

7. The area of a triangle is given by $\Delta = \frac{1}{2} ab \sin C$, where a, b are sides and C is the included angle. Find Δ, when $a = 46.7$, $b = 6.91$, $C = 38° 24' 10''$. *Ans.* 100.23.

8. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$; find V when $\pi = 3.1416$, $r = 8.65$ cm. *Ans.* 2711.1 cu. cm.

9. Mass = volume × density. Find the radius of a sphere if its mass be 4.38×10^5 grams and its density 2.3. *Ans.* 35.69 cm.

10. The area of a segment of a circle is given by $S = \frac{1}{2} r^2 (\theta - \sin \theta)$, where the radius is r, and θ is the angle subtended by the segment at the centre of the circle. Find S, when $r = 4.6$ ft., $\theta = 126° 30'$. *Ans.* 14.85 sq. ft.

11. The volume of a right circular cone is given by $V = \frac{1}{3}\pi R^2 h$, where R is the radius of the base and h is the altitude. Find V, when $R = 1.5876$ m., and $h = 7.675$ m. *Ans.* 20.257 cu. m.

12. If the volume of a cone $V = 987.6$ cu. cm., and its height $h = 9.416$ cm., find the radius of the base. *Ans.* 10 cm.

13. The time of vibration of a pendulum is given by $t = \pi\sqrt{\frac{l}{g}}$, where l = length of the pendulum, g = the acceleration. Find the length l of a pendulum vibrating seconds if $g = 980.19$ cm. *Ans.* 99.314 cm.

14. What value must g have in order that a pendulum 1 m. long shall vibrate seconds? *Ans.* 986.96 cm.

15. Find the length of a pendulum l which makes 80 vibrations per minute; $g = 32.16$ ft. *Ans.* 1.8329 ft.

16. Find the time of vibration of a pendulum if $l = 8.04$ ft., $g = 32.16$ ft. *Ans.* 1.57 sec.

CHAPTER VII

SOLUTIONS OF TRIANGLES IN GENERAL

We shall now develop a number of theorems which are used in the solution of any plane triangle. The demonstrations of these theorems should be thoroughly mastered.

47. The Theorem of Sines.
(1) *First demonstration.*

Uniform lettering of a triangle as indicated in the drawings will be found convenient. Let the angles be indicated by capitals A, B, C, and let the sides opposite be called a, b, c, respectively. Draw a perpendicular p from the vertex upon the

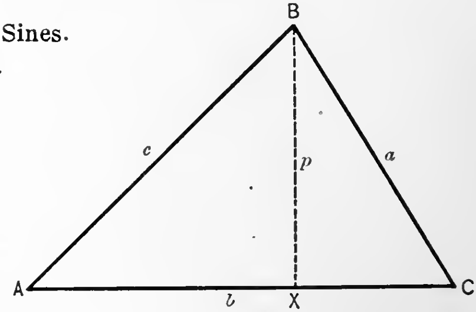


Fig. 34.

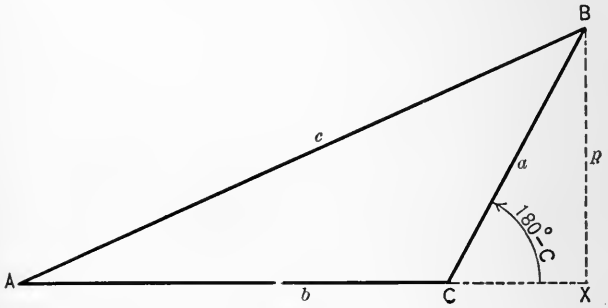


Fig. 35.

opposite side b, Fig. 34, or b produced, Fig. 35. Then, from the right triangles AXB and CXB, we have

$$\sin A = \frac{p}{c}, \quad \sin C = \frac{p}{a}.$$

Dividing $\sin A$ by $\sin C$,

$$\frac{\sin A}{\sin C} = \frac{a}{c}. \qquad \text{(I)}$$

In Fig. 35, we have

$$\sin(180° - C) = \sin C = \frac{p}{a}.$$

Drawing a perpendicular from angle A to side a, we would have, similarly,

$$\frac{\sin B}{\sin C} = \frac{b}{c}. \qquad \text{(II)}$$

Dividing (I) by (II),

$$\frac{\sin A}{\sin B} = \frac{a}{b}. \qquad \text{(III)}$$

These results may be written in the symmetrical form

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

THEOREM. *In any triangle the sines of the angles are proportional to the opposite sides.*

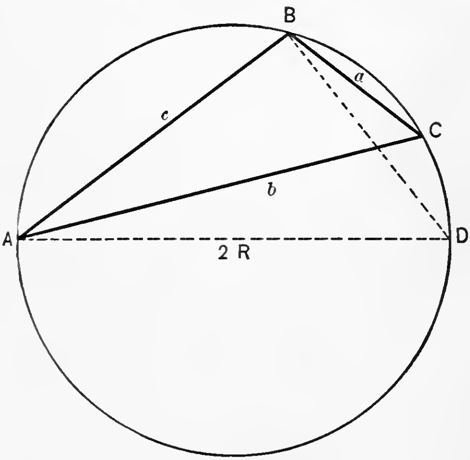


Fig. 36.

(2) *Second demonstration.* The theorem of sines may also be proved by circumscribing a circle about the given triangle, Fig. 36. Draw the diameter $AD = 2R$, and the line BD forming the right triangle ABD. Then, the angle ADB = angle C.

$$\sin ADB = \sin C = \frac{c}{2R}.$$

Similarly, $\quad \sin A = \frac{a}{2R}, \quad \sin B = \frac{b}{2R}.$

Hence, $\quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R},$

where R is the radius of the circumscribing circle.

It should be observed that the theorem of sines may be employed in the solution of a triangle when two angles and a side are given, or when two sides and an angle opposite one of them are given.

APPLICATIONS OF THEOREM OF SINES

CASE I. *Given a side and two angles.*

EXAMPLE. Given $a = 148.3$, $A = 37° 24'$, $C = 76° 48' 30''$, to find b and c.

Solution. (1) Since

$$A + B + C = 180°,$$
$$B = 180° - (A + C)$$
$$= 65° 47' 30''.$$

(2) Make an approximate construction of the triangle. (3) Select formulas, and tabulate the calculations.

$$b = \frac{a \times \sin B}{\sin A},$$

$$c = \frac{a \times \sin C}{\sin A}.$$

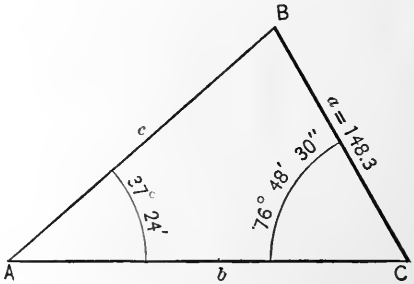


Fig. 37.

Data and Results

Given	a	148.3
	A	37° 24′
	C	76° 48′ 30″
	B	65° 47′ 30″
	b	222.695
	c	237.72

Logarithms

$\log a$	2.17114
$\log \sin A$	9.78346
$\log\left(\frac{a}{\sin A}\right)$	2.38768
$\log \sin B$	9.96003
$\log \sin C$	9.98839
$\log b$	2.34771
$\log c$	2.37607

CASE II. *Given two sides and an angle opposite one of the given sides.* This problem may have (1) one solution, (2) two

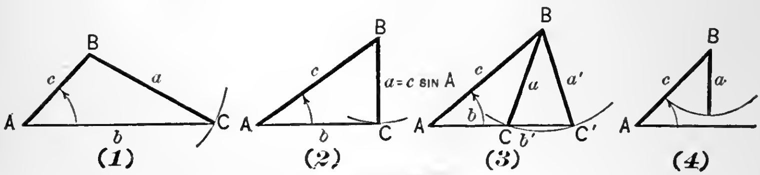


Fig. 38.

solutions, (3) or no solution. If the dimensions given be a, c, and A, where A is acute, *one solution* will result if $a \geq c$, or if $c \sin A = a$, Fig. (1), (2); *two solutions* (triangles ABC, ABC') will occur when $a < c$ and $a > c \sin A$, Fig. (3). If $a < c \sin A$, *no solution* will exist, Fig. (4). If A be obtuse, *one solution* will exist provided $a > c$, otherwise the construction is impossible.

EXAMPLE. Given $a = 556$, $b = 678.4$, $A = 31° 10' 30''$, to find B, C, c.

In this example the angle A is acute and the side $a < b$, hence

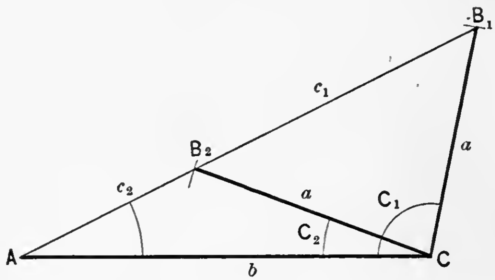


Fig. 39

two solutions will be found. In one solution the angle B_1 will be acute, in the other solution $B_2 = 180° - B_1$.

Solution. (1) Make an approximate construction of the data.

(2) Formulas and tabulation of the calculations.

$$\sin B = \frac{b \times \sin A}{a}, \quad C = 180° - (A + B), \quad c = \frac{a \times \sin C}{\sin A}.$$

Data and Results

Given	a	556
	b	678.4
	A	31° 10′ 30″
	B_1	39° 10′ 12″
	B_2	140° 49′ 48″
	C_1	109° 39′ 18″
	C_2	7° 59′ 42″
	c_1	1011.5
	c_2	149.39

Logarithms

log a	2.74507
log sin A	9.71404
log b	2.83149
log sin B	9.80046
log sin C_1	9.97393
log sin C_2	9.14329
log c_1	3.00496
log c_2	2.17432

EXERCISES

1. Given $a = 140.6$, $A = 48° 30' 10''$, $B = 76° 24'$; find b, c.
2. Given $b = 3875.4$, $A = 97° 24'$, $B = 40° 27' 15''$; find a, c.
3. Given $a = 148.6$, $b = 121.78$, $A = 69° 20' 10''$; find B, C, c.
4. Given $a = 2311$, $b = 1600.7$, $B = 34° 42' 29''$; find A, C, c.
5. Given $a = 1906$, $b = 224.8$, $A = 61° 24' 18''$; find B, C, c.
6. Given $b = 1009$, $c = 796.4$, $C = 85°$; find B, A, a.
7. Given $A = 67° 54'$, $B = 34° 52'$, $b = 4356.7$; find a, c.

48. The Theorem of Tangents. From the theorem of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R},$$

we have $a = 2R \sin A, \quad b = 2R \sin B.$

Adding and subtracting, and reducing by § 38,

(1) $a + b = 2R(\sin A + \sin B) = 4R \sin \frac{A+B}{2} \cos \frac{A-B}{2}$,

(2) $a - b = 2R(\sin A - \sin B) = 4R \cos \frac{A+B}{2} \sin \frac{A-B}{2}$.

Dividing (1) by (2),

$$\frac{a+b}{a-b} = \frac{\tan \frac{A+B}{2}}{\tan \frac{A-B}{2}}.$$

We may derive in a similar manner, or write by symmetry,

$$\frac{b+c}{b-c} = \frac{\tan \frac{B+C}{2}}{\tan \frac{B-C}{2}}, \qquad \frac{c+a}{c-a} = \frac{\tan \frac{C+A}{2}}{\tan \frac{C-A}{2}}.$$

THEOREM. *In any triangle the sum of two sides is to their difference as the tangent of the half sum of the opposite angles is to the tangent of their half difference.*

The theorem of tangents may be used in the solution of a triangle when two sides and their included angle are given. For example, let a, b, C, be given. Then we know

$$A + B + C = 180°.$$

$$\frac{A+B}{2} = \frac{180° - C}{2} = 90° - \frac{C}{2}.$$

Hence, $\tan\frac{A+B}{2}=\cot\frac{C}{2}$,

and the first formula above becomes

$$\frac{a+b}{a-b}=\frac{\cot\frac{C}{2}}{\tan\frac{A-B}{2}}.$$

This formula enables us to find the unknown $\tan\frac{A-B}{2}$, and then the angle $\frac{A-B}{2}$. The angle $\frac{A+B}{2}$ being known, we find at once the values of A and B:

$$A=\tfrac{1}{2}(A+B)+\tfrac{1}{2}(A-B),\quad B=\tfrac{1}{2}(A+B)-\tfrac{1}{2}(A-B).$$

The application of the theorem of sines now determines side c.

APPLICATIONS OF THEOREM OF TANGENTS

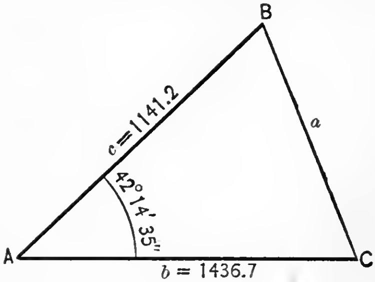


Fig. 40.

EXAMPLE.

Given $b=1436.7$,
$c=1141.2$,
$A=42°\ 14'\ 35''$;
find B, C, a.

Solution. (1) Make an approximate construction.

(2) Select formulas and tabulate the calculations.

$$\tan\frac{1}{2}(B-C)=\frac{b-c}{b+c}\tan\frac{1}{2}(B+C),\ B+C=180°-A,\ a=\frac{b\times\sin A}{\sin B}.$$

Data and Results

Given	b	1436.7
	c	1141.2
	A	42° 14′ 35″
	$b-c$	295.5
	$b+c$	2577.9
	$\frac{B+C}{2}$	68° 52′ 42″
	$\frac{B-C}{2}$	16° 31′ 40″
	B	85° 24′ 22″
	C	52° 21′ 2″
	a	969.0

Logarithms

$\log(b-c)$	2.47056
$\log\tan\frac{B+C}{2}$	0.41308
$\log(b+c)$	3.41126
$\log\tan\frac{B-C}{2}$	9.47238
$\log b$	3.15737
$\log\sin A$	9.82755
$\log\sin B$	9.99860
$\log a$	2.98632

EXERCISES

Solve for the unknown parts of the following triangles:

1. $a = 281$, $c = 153$, $B = 34° \ 42' \ 29''$.
2. $b = 296$, $c = 178$, $A = 78° \ 21' \ 40''$.
3. $b = 199.37$, $c = 642.75$, $A = 130° \ 9' \ 24''$.
4. $a = 101.47$, $b = 9936.7$, $C = 47° \ 48' \ 12''$.
5. $b = 1134.7$, $c = 2277.9$, $A = 19° \ 34' \ 24''$.
6. $a = 1434.2$, $b = 9767.2$, $C = 109° \ 19' \ 36''$.
7. $b = .538$, $c = 1.245$, $A = 62° \ 14' \ 40''$.
8. $b = 234.7$, $c = 185.4$, $A = 84° \ 36'$.
9. $a = 1896.9$, $b = 3463.7$, $C = 124° \ 10$.
10. $c = 9.876$, $a = 4.921$, $B = 76° \ 20.4'$.

49. The Theorem of Cosines. (1) *First derivation.*

Draw the perpendicular p from B to b, Fig 41. Then, from the right triangle BXC we have

$$a^2 = p^2 + \overline{CX}^2 = p^2 + (b - AX)^2 = p^2 + b^2 + \overline{AX}^2 - 2b \cdot AX.$$

But we have from the right triangle AXB

$$p^2 + \overline{AX}^2 = c^2,$$

and

$$AX = c \times \cos A.$$

Making these substitutions, we find

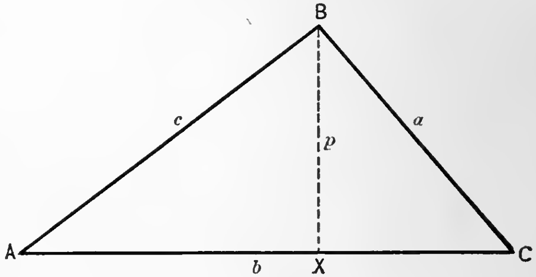


Fig. 41.

$$a^2 = b^2 + c^2 - 2\,bc \cos A,$$

and by symmetry,

$$b^2 = c^2 + a^2 - 2\,ca \cos B,$$

$$c^2 = a^2 + b^2 - 2\,ab \cos C.$$

If the point X should fall upon the base produced, it may readily be shown that the above results still hold.

THEOREM. *In any triangle the square of any side equals the sum of the squares of the other two sides diminished by twice their product into the cosine of their included angle.*

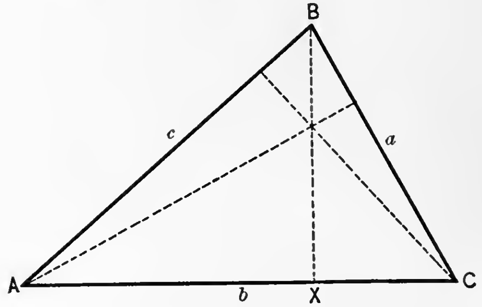


Fig. 42.

(2) *Second derivation.*

From Fig. 42, we have

$$AX = c \times \cos A, \quad CX = a \times \cos C,$$

and therefore,

$$\left.\begin{aligned} b &= c \times \cos A + a \times \cos C. \\ c &= a \times \cos B + b \times \cos A, \\ a &= b \times \cos C + c \times \cos B. \end{aligned}\right\} \quad \begin{aligned} &(-b) \\ &(-c) \\ &(a) \end{aligned}$$

Similarly,

Multiply these equations, as indicated, by $(-b)$, $(-c)$, (a), respectively, and add, giving

$$a^2 - b^2 - c^2 = -2\,bc \times \cos A,$$

or

$$\mathbf{a^2 = b^2 + c^2 - 2\,bc \times \cos A.}$$

By similar manipulation, the other formulas may be derived.

The theorem of cosines is not adapted to logarithmic calculation. When two sides and the included angle are given, this theorem gives the third side. When the given sides are simple numbers such that their squares may readily be known, the application of the cosine formula is to be advised. Otherwise, the theorem of tangents should be employed.

EXERCISES

Calculate from cosine theorem the following without using logarithms:

1. $b = 12$, $c = 15$, $A = 45° \, 34'$; find a.
2. $c = 10$, $a = 20$, $B = 37° \, 20'$; find b.
3. $a = 5$, $b = 7$, $c = 9$; find A.
4. $a = 25$, $b = 40$, $c = 60$; find C.
5. $A = 140°$, $b = 100$, $c = 300$; find a.
6. $a = 150$, $b = 180$, $C = 97°$; find c.
7. Find the perimeter of a triangle where $a = 200$ ft., $b = 300$ ft., $C = 37° \, 40'$.
8. Find the area of a square whose perimeter is the same as the triangle, where $a = 40$, $b = 60$, $C = 68° \, 10'$.

50. The Half-angle Theorems. From the formula

$$a^2 = b^2 + c^2 - 2\,bc \times \cos A \tag{I}$$

we may derive results suitable for logarithmic calculation of the angle A when the three sides, a, b, c, are given.

Adding, and subtracting $2\,bc$, we find

$$\begin{aligned} a^2 &= b^2 + 2\,bc + c^2 - 2\,bc - 2\,bc \times \cos A \\ &= (b + c)^2 - 2\,bc\,(1 + \cos A) \\ &= (b + c)^2 - 2\,bc\left(2\cos^2\frac{A}{2}\right). \qquad \S\ 37 \end{aligned}$$

Solving for $\cos^2\frac{A}{2}$, and factoring the right-hand member,

$$\cos^2\frac{A}{2} = \frac{(b + c + a)(b + c - a)}{4\,bc}.$$

Now, let $\quad a + b + c = 2\,s,$

then, $\quad b + c - a = 2\,s - 2\,a;$

substitute and extract the square root,

$$\cos\frac{A}{2} = \sqrt{\frac{s\,(s - a)}{bc}}.$$

Again, taking (I), subtracting and adding $2\,bc$,

$$a^2 = b^2 - 2\,bc + c^2 + 2\,bc - 2\,bc \times \cos A$$

$$= (b-c)^2 + 2\,bc(1-\cos A) = (b-c)^2 + 2\,bc\left(2\sin^2\frac{A}{2}\right), \qquad \S\ 37$$

and solving for $\sin^2\frac{A}{2}$,

$$\sin^2\frac{A}{2} = \frac{a^2-(b-c)^2}{4\,bc} = \frac{(a+b-c)(a-b+c)}{4\,bc}.$$

Now, take $\qquad a+b+c = 2\,s,$

then, $\qquad a-b+c = 2\,s - 2\,b,$

$$a+b-c = 2\,s - 2\,c;$$

substitute and extract the square root,

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}.$$

Dividing $\sin\frac{A}{2}$ by $\cos\frac{A}{2}$, we find

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

The form of this radical may be changed so as to make it symmetrical in a, b, c.

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{1}{s-a}\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

The radical here employed will be the same for $\tan\frac{B}{2}$, $\tan\frac{C}{2}$.

If we set $\qquad \sqrt{\frac{(s-a)(s-b)(s-c)}{s}} = r,$

$$\tan\frac{A}{2} = \frac{r}{s-a},\quad \tan\frac{B}{2} = \frac{r}{s-b},\quad \tan\frac{C}{2} = \frac{r}{s-c}.$$

Then, we have as results the following half-angle formulas.

(1) *The sines of the half-angles of a triangle.*

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}},$$

$$\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}},$$

$$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}.$$

(2) *The cosines of the half-angles of a triangle.*

$$\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}},$$

$$\cos\frac{B}{2}=\sqrt{\frac{s(s-b)}{ca}},$$

$$\cos\frac{C}{2}=\sqrt{\frac{s(s-c)}{ab}}.$$

(3) *The tangents of the half-angles of a triangle.*

$$\tan\frac{A}{2}=\frac{1}{s-a}\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}=\frac{r}{s-a},$$

$$\tan\frac{B}{2}=\frac{1}{s-b}\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}=\frac{r}{s-b},$$

$$\tan\frac{C}{2}=\frac{1}{s-c}\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}=\frac{r}{s-c}.$$

In calculating the angles of a triangle, the tangents of the half-angles should be used, as the complete calculation of A, B, C may be performed by taking only four logarithms from the tables, viz. $\log s$, $\log(s-a)$, $\log(s-b)$, $\log(s-c)$.

APPLICATION OF THE HALF-ANGLE THEOREMS

EXAMPLE. Given $a=65.43$, $b=58.26$, $c=49.35$; find A, B, C.

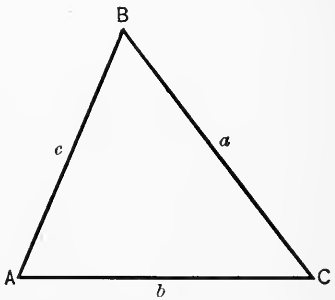


Fig. 43.

Solution. (1) An approximate construction with the given data will show angle $A>B>C$. (2) Select formulas and tabulate the calculations:

$$\tan\frac{A}{2}=\frac{r}{s-a},\quad \tan\frac{B}{2}=\frac{r}{s-b},\quad \tan\frac{C}{2}=\frac{r}{s-c},\quad r=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

Data and Results

Given	a	65.43
	b	58.26
	c	49.35
	$2s$	173.04
	s	86.52
	$s-a$	21.09
	$s-b$	28.26
	$s-c$	37.17
	$\frac{A}{2}$	37° 11′ 18″
	$\frac{B}{2}$	29° 31′ 10″
	$\frac{C}{2}$	23° 17′ 31″
	A	74° 22′ 36″
	B	59° 2′ 20″
	C	46° 35′ 2″

Logarithms

$\log(s-a)$	1.32408
$\log(s-b)$	1.45117
$\log(s-c)$	1.57019
$\log s$	1.93712
$\log r^2$	2.40832
$\log r$	1.20416
$\log\tan\frac{A}{2}$	9.88008
$\log\tan\frac{B}{2}$	9.75299
$\log\tan\frac{C}{2}$	9.63397

As a check,

$$A + B + C \stackrel{?}{=} 180°.$$

EXERCISES

Find the angles in each of the following triangles:

1. $a = 98.76,\quad b = 104.97,\quad c = 140.76.$
2. $a = 57{,}896,\quad b = 49{,}784,\quad c = 35{,}891.$
3. $a = 5.769,\quad b = 9.8764,\quad c = 11.675.$
4. $a = 0.0587,\quad b = 0.09765,\quad c = 0.1067.$
5. $a = 94.28,\quad b = 112.68,\quad c = 180.47.$

AREAS OF TRIANGLES

Many expressions may be obtained for the area of a triangle. Some of these will be enumerated.

51. Area in Terms of Sides and Angles. Let the area be denoted by K, and let the triangle be lettered as shown in the drawing. Then, from plane geometry,

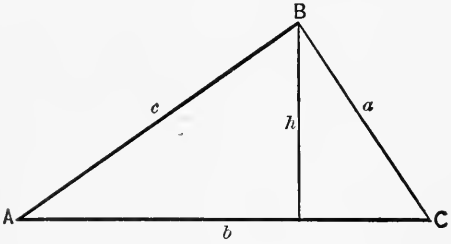


Fig. 44.

(1) $K = \frac{1}{2} b \times h.$

Making use of trigonometric relations

$$h = c \sin A,$$

(2) $K = \frac{1}{2} bc \sin A.$

THEOREM. *The area of any triangle equals one half the product of two sides by the sine of the included angle.*

(3) $$K = \frac{1}{2}\frac{c \times \sin B}{\sin C} \times (c \sin A) \qquad \text{§ 47}$$

$$= \frac{1}{2} \cdot \frac{c^2 \sin A \times \sin B}{\sin C}.$$

Since $$\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}, \qquad \text{§ 37}$$

and $$\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}, \quad \cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}, \qquad \text{§ 50}$$

we may express K directly in terms of the three sides.

(4) $$K = \frac{1}{2} bc \sin A = bc \sin \frac{A}{2} \cos \frac{A}{2}$$

$$= bc\sqrt{\frac{(s-b)(s-c)}{bc}} \times \sqrt{\frac{s(s-a)}{bc}}$$

$$= \sqrt{s(s-a)(s-b)(s-c)},$$

where $$2s = a + b + c.$$

52. Area in Terms of r. The area of a triangle may be

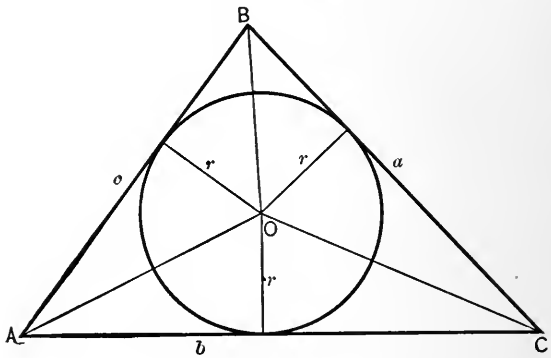


Fig. 45.

expressed in terms of the radius of the inscribed circle. Let a circle be inscribed in the triangle ABC. See Fig. 45.

Then, $$\triangle AOC = \tfrac{1}{2} br,$$
$$\triangle BOA = \tfrac{1}{2} cr,$$
$$\triangle COB = \tfrac{1}{2} ar.$$

Adding, we have $$\triangle ABC = K = \tfrac{1}{2}(a + b + c) r = sr,$$
$$s = \tfrac{1}{2}(a + b + c).$$

53. Expressions for the Area of a Triangle.

1. $K = \frac{1}{2} bc \times \sin A = \frac{1}{2} ca \times \sin B = \frac{1}{2} ab \times \sin C.$

2. $K = \dfrac{\frac{1}{2} c^2 \sin A \sin B}{\sin C}.$

3. $K = \sqrt{s(s-a)(s-b)(s-c)},\quad 2s = a + b + c.$

4. $K = s \times r,\quad r =$ radius of inscribed circle.

EXERCISES

Solve the following triangles for the unknown parts:

1. $a = 7950,\quad A = 79° 59',\quad B = 44° 41'.$
2. $a = 80.86,\quad A = 19° 29',\quad B = 33° 1'.$
3. $a = 62.65,\quad b = 89.81,\quad C = 55° 5'.$
4. $a = 2071,\quad b = 1887,\quad C = 55° 12' 3''.$
5. $a = 0.2034,\quad b = 0.1123,\quad C = 72° 15' 19''.$
6. $a = 48.5,\quad b = 84,\quad B = 21° 31'.$
7. $a = 838.56,\quad b = 841.53,\quad B = 68° 10' 24''.$
8. $a = 49,\quad b = 45,\quad B = 17° 41' 9''.$
9. $b = 117.4,\quad c = 726.3,\quad B = 80° 10'.$
10. $c = 1047.51,\quad a = 943.27,\quad A = 63° 17' 18''.$
11. $a = 5,\quad b = 7,\quad c = 8.$
12. $a = 341,\quad b = 260,\quad c = 158.$
13. $a = 40,\quad b = 50,\quad c = 60.$
14. $a = 0.654,\quad b = 1.5876,\quad c = 0.998.$
15. $a = 998.46,\quad b = 1004.5,\quad c = 1268.7.$

Find the area K in the following triangles:

16. $b = 96,\quad c = 108,\quad A = 65° 10'.$
17. $a = 480.6,\quad b = 396.4,\quad C = 110° 20'.$
18. $b = 494,\quad A = 114°,\quad C = 36°.$
19. $c = 493,\quad a = 540,\quad A = 76° 40' 10''.$
20. $a = 58,\quad b = 65,\quad c = 87.$

21. $a = 375.4$, $b = 460.24$, $c = 584.36$.

22. $a = 46.2$, $b = 65.8$, $s = 75$.

23. $b = 137.6$, $c = 184.5$, $A = 110° 24'$.

24. $a = 149.24$, $A = 65° 20'$, $B = 37° 28'$.

25. To find the distance from a point A to S, a base line $AB = 250$ ft., and the angles $SAB = 65° 10'$, $SBA = 48° 20'$ are measured. Find AS and the shortest distance h from S to the line AB. *Ans.* $AS = 203.65$ ft.; $h = 184.82$ ft.

26. A is a point 3 mi. due north of B; from A a point P bears N. 68° 40′ E., and from B it bears N. 35° 50′ E. Find AP, and the area of the triangle APB in square miles.
Ans. 3.239 mi.; 4.5268 sq. mi.

27. Find the sides of a parallelogram, when a diagonal 14.69 ft. long makes angles of 35° 48′, 64° 27′ with the sides.
Ans. 8.732 ft.; 13.468 ft.

28. A boat is steaming N.E. at a rate of 15 mi. per hour. At 10 o'clock a lighthouse bears N. 10° W.; at 12 o'clock it bears W. 31° S. Find the distance of the boat from the lighthouse at 10 o'clock. *Ans.* 7.774 mi.

29. The line $AB = 1460$ ft. upon shore subtends 64° 20′ at a lighthouse which is 985 ft from A. Find the distance of the lighthouse from B. *Ans.* 1585.7 ft.

30. A side and diagonal of a parallelogram are 690 ft. and 1248 ft. respectively. If the angle between the diagonals opposite the given side is 124° 10′, what is the length of the other diagonal? *Ans.* 214.635 ft.

31. To measure across a barrier from A to B a station C is taken, and the distances CA, CB, and the angle ACB are found to be 584.6 ft., 796.5 ft., and 87° 40′, respectively. Find the distance AB. *Ans.* 968.64 ft.

32. Find the length of a straight wall which subtends an angle of 110° 45′ at a point P, the distances of P being 487.5 ft. and 746.4 ft. from the ends of the wall. *Ans.* 1025.94 ft.

33. To measure an inaccessible distance XY a base line $AB = 550$ ft. is laid off and the angles $ABY = 75° 48'$, $ABX = 48° 25'$, $BAY = 58° 20'$, $BAX = 98° 24'$ are determined. Find the length XY. *Ans.* 512.05 ft.

34. If the sides of a triangle are to each other as 5 : 7 : 9, how do the angles compare?

35. The sides of a triangle are 15, 18, 21. Find the length of the perpendicular from the smallest angle upon the opposite side. *Ans.* 17.636.

36. Find the area of the circle circumscribing the triangle whose sides are 10, 20, 25. *Ans.* 544.

37. From the ridge of a mountain range the depression angles of the sides are 55° 40′, 68° 20′ respectively, and the corresponding distances from the ridge to the ends of a tunnel below are 3475 ft. and 2896 ft. Find the length of the tunnel through the mountain. *Ans.* 3034.4 ft.

38. Show that the median drawn to side a of the triangle whose sides are a, b, c is given by $m = \sqrt{\frac{1}{2}(b^2 + c^2) - \frac{1}{4}a^2}$.

39. A grass plot in the form of a triangle has its sides 48.5 ft., 65.4 ft., 84.2 ft., respectively. Find the radius and area of the largest circular bed that can be made in the plot. *Ans.* 15.969 ft.; 801.12 sq. ft.

40. The sides of a triangle are $AB = 24.5$ ft., $BC = 30$ ft., $CA = 36.5$ ft. If A, B, C be located upon level ground and vertical posts be erected to heights $AP_1 = 12$ ft., $BP_2 = 18$ ft., $CP_3 = 12$ ft., what is the area of the triangle $P_1P_2P_3$ formed by the tops of the posts? *Ans.* 381.15 sq. ft.

41. A triangle $b = 50$ ft., $A = 65° 24'$, $C = 76° 28'$, rests with its base b on a horizontal plane, the plane of the triangle being inclined to the horizontal at an angle 48° 49′. Find the orthogonal projection of the triangle upon the horizontal. *Ans.* 1178.3 sq. ft.

42. A parallelogram, whose sides are 48 ft., 27 ft., and included angle 58° 48′, rests with its longer side upon a horizontal plane and its own plane inclined to the horizontal at an angle 65° 50′. Find its area, and the area of its orthogonal projection upon the horizontal. *Ans.* 1108.5 sq. ft.; 453.8 sq. ft.

43. From the top of a lighthouse 160 ft. high, the depression angle of a ship at A is 15° 48′, one hour later its depression angle at a point B is 10° 25′, and the horizontal angle subtended at the lighthouse by AB is 100° 48′. Find the speed of the ship. *Ans.* 1123.2 ft. per hour.

44. To find the height h of a steeple above the plane through its base C, a line $AB = 237.5$ ft. is measured and the angles $CAB = 40° \ 10' \ 15''$, $CBA = 110° \ 20' \ 10''$ determined; the angle subtended by the steeple at B is 27° 18′. Find the height of the steeple. *Ans.* $h = 160.61$ ft.

45. In Ex. 44 let $AB = a$, $\angle\, CAB = \alpha$, $\angle\, CBA = \beta$, and the elevation angle of the steeple from B be γ. Show that

$$BC = \frac{a \sin \alpha}{\sin(\alpha + \beta)}, \text{ and } h = \frac{a \sin \alpha}{\sin(\alpha + \beta)} \times \tan \gamma.$$

46. At a horizontal distance a from a tower, the angle of elevation of the top of the tower is found to be α, the angle of depression of its base is found to be β. Show that the height of the tower is given by

$$h = a(\tan \alpha + \tan \beta) = a \frac{\sin (\alpha + \beta)}{\cos \alpha \cos \beta}.$$

47. At a certain point in a horizontal plane the angle of elevation of a peak is α, a feet farther away and in the same vertical plane the elevation angle is β. Show that the distance from the first point of observation to the foot of the perpendicular dropped from the peak to the plane is

$$d = \frac{a \tan \beta}{\tan \alpha - \tan \beta} = \frac{a \cos \alpha \sin \beta}{\sin(\alpha - \beta)},$$

and that the height of the peak is given by $h = d \times \tan \alpha$.

48. A tower 148 ft. high stands upon the top of a hill; from a point 1100 ft. down the hill the tower subtends an angle of 6° 48′ 12″. Find the angle of inclination of the hill. *Ans.* 21° 29′ 57″.

49. Two triangles are determined by $a = 120$, $b = 160$, $A = 35° \ 10'$. Find the difference of their areas without solving for the area of either of the given triangles.

Ans. 7083.

50. A ladder 40 ft. long is set with one end at a point 15 ft. from the base of a buttress, the other end reaches a point

35 ft. up its face. Find the angle of inclination of the face of the buttress from the vertical. *Ans.* 8° 12′ 44″.

51. Find the length of a window, if a pole resting upon the ground and making an angle of 64° 28′ with the horizontal just reaches the top of the window, and when its base is moved 20 ft. farther away, making an elevation angle of 39° 40′, its top reaches the window sill. *Ans.* 15.58 ft.

52. If the angle of elevation of a balloon from a point A due north is α, and at the same instant its angle of elevation from a point B due east of A is β, find the height h of the balloon if $AB = a$. *Ans.* $h = \dfrac{a \sin \alpha \sin \beta}{\sqrt{\sin(\alpha + \beta)\sin(\alpha - \beta)}}$.

CHAPTER VIII

INVERSE FUNCTIONS. TRIGONOMETRIC EQUATIONS

54. Inverse Notation. From Fig. 46 *a*, $\angle POX = \phi$,

$$\sin\phi = m, \quad \cos\phi = \sqrt{1-m^2}, \quad \tan\phi = \frac{m}{\sqrt{1-m^2}}.$$

These equations may be *expressed inversely*, *i.e. solved for* ϕ, thus:

$$\phi = \text{arc}\sin m = \text{arc}\cos\sqrt{1-m^2} = \text{arc}\tan\frac{m}{\sqrt{1-m^2}}$$

$$= \sin^{-1} m = \cos^{-1}\sqrt{1-m^2} = \tan^{-1}\frac{m}{\sqrt{1-m^2}}.$$

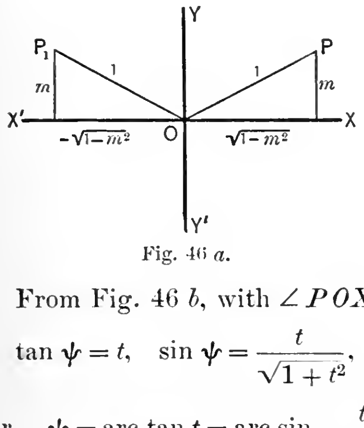


Fig. 46 *a*.

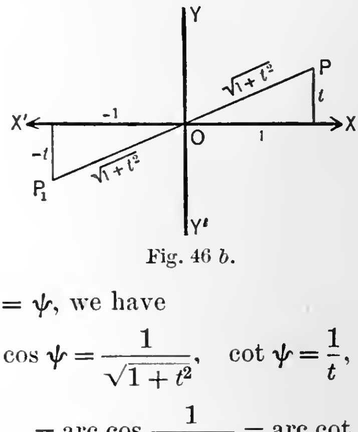


Fig. 46 *b*.

From Fig. 46 *b*, with $\angle POX = \psi$, we have

$$\tan\psi = t, \quad \sin\psi = \frac{t}{\sqrt{1+t^2}}, \quad \cos\psi = \frac{1}{\sqrt{1+t^2}}, \quad \cot\psi = \frac{1}{t},$$

or $$\psi = \text{arc}\tan t = \text{arc}\sin\frac{t}{\sqrt{1+t^2}} = \text{arc}\cos\frac{1}{\sqrt{1+t^2}} = \text{arc}\cot\frac{1}{t}$$

$$= \tan^{-1} t = \sin^{-1}\frac{t}{\sqrt{1+t^2}} = \cos^{-1}\frac{1}{\sqrt{1+t^2}} = \cot^{-1}\frac{1}{t}.$$

The symbols $\sin^{-1} m$, $\cos^{-1}\sqrt{1-m^2}$, etc., are sometimes called *anti-sine*, *anti-cosine*, etc., but a better reading is to call these *arc sin m*, *arc cosine* $\sqrt{1-m^2}$, etc.

From the above drawings it is to be noted that each inverse function has *two initial* or *primary values*. Thus

$$\sin\phi = m, \text{ or } \phi = \text{arc}\sin m$$

is satisfied by the angle ϕ or $\pi - \phi = \phi'$. The equation

$$\tan \psi = t, \text{ or } \psi = \text{arc} \tan t,$$

is satisfied by the angle ψ or $\pi + \psi = \psi'$.

In addition to the primary solutions, any number of solutions to a trigonometric equation may be obtained by adding any integral multiple of $2\pi = 360°$.

EXAMPLES. 1. $\sin x = \frac{1}{2}$.

Solution. $x = \text{arc} \sin \frac{1}{2} = 30°, 150°$, primary solutions.

$x = 2n\pi + \frac{\pi}{6}, \quad 2n\pi + \frac{5}{6}\pi$, multiple solutions,

where n = any integer.

2. $\cos \alpha = \frac{1}{2}\sqrt{2}$.

Solution. $\alpha = \cos^{-1} \frac{1}{2}\sqrt{2} = 45°, \; -45°$,

$\alpha = 2n\pi + \frac{\pi}{4}, \quad 2n\pi - \frac{\pi}{4}, \quad n$ = any integer.

3. $\tan \phi = \sqrt{3}$.

Solution. $\phi = \tan^{-1} \sqrt{3} = 60°, \; 240°$.

$\phi = n\pi + \frac{\pi}{3}, \quad n$ = any integer.

55. Inverse Identities. A number of important inverse identities will be introduced.

$$\text{I.} \quad \sin^{-1} x + \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}).$$

This identity may be established as follows: (1) construct an angle ϕ whose sine is x, also an angle ψ whose sine is y. (2) The left member of the identity is $\phi + \psi$. Take

$$\sin(\phi + \psi) = \sin \phi \cos \psi + \cos \phi \sin \psi$$
$$= x\sqrt{1-y^2} + y\sqrt{1-x^2}.$$

Hence,

$$\phi + \psi = \sin^{-1} x + \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}).$$

In a similar manner

$$\sin^{-1} x - \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2}).$$

$$\text{II.} \quad \cos^{-1} x \pm \cos^{-1} y = \cos^{-1}(xy \mp \sqrt{(1-x^2)(1-y^2)}).$$

This identity may be established by constructing two angles, $\phi = \cos^{-1} x$, $\psi = \cos^{-1} y$, taking the cosine of the sum and difference, substituting as above, and then passing to inverse notation.

Prove the following:

III. $\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 \mp xy}\right)$.

IV. $\sin^{-1} x = \frac{1}{2}\sin^{-1}(2x\sqrt{1-x^2}) = -\sin^{-1}(-x)$.

V. $\cos^{-1} x = \frac{1}{2}\cos^{-1}(2x^2-1)$.

EXERCISES

Construct the acute angles indicated in the inverse notation, and find values of the following:

1. $\cos(\sin^{-1}\frac{1}{2})$.

SUGGESTION. Construct $\phi = \sin^{-1}(\frac{1}{2})$, then $\cos\phi = \frac{1}{2}\sqrt{3}$.

2. $\tan(\sin^{-1}\frac{3}{5})$.
3. $\tan(\text{arc cot } \frac{17}{5})$.
4. $\cot\left(\text{arc sin } \frac{\sqrt{3}}{2}\right)$.
5. $\sin(\text{arc tan } 1)$.
6. $\cos(\text{arc cot } 0)$.
7. $\tan(\text{arc cot } 1)$.
8. $\sin(90° - \sin^{-1} 1)$.
9. $\cos(90° - \cos^{-1} a)$.
10. $\sin(\tan^{-1}\sqrt{3}) - \cos\frac{\pi}{6}$.
11. $\sin(2\cot^{-1}\sqrt{3})$.
12. $\cos(3\sin^{-1}\frac{1}{2})$.
13. $\tan(90° - \sec^{-1}\sqrt{2})$.
14. $\cos(90° - \sin^{-1}\frac{15}{17})$.

Verify the following:

15. $\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2} = 90°$.
16. $\text{arc tan } 1 + \text{arc cos } \frac{1}{\sqrt{2}} = 90°$.
17. $\text{arc sin } 1 - \text{arc tan } 1 = 45°$.
18. $\text{arc cos } 1 + \text{arc tan } \infty - \text{arc cot } 1 = 45°$.
19. $\text{arc vers } 1 - \text{arc sec } \sqrt{2} = 45°$.
20. $\sin(90° - \tan^{-1}\sqrt{3}) + \tan(90° - \sec^{-1}\sqrt{2}) = \frac{3}{2}$.
21. $\text{arc sin } \frac{3}{5} + \text{arc cos } \frac{3}{5} = \frac{\pi}{2}$.
22. $\text{arc sin } x + \text{arc cos } x = \frac{\pi}{2}$.
23. $\text{arc tan } \frac{1}{6} + \text{arc tan } \frac{5}{7} = \frac{\pi}{4}$.
24. $\sin\left(2 \text{ arc sin } \frac{1}{2}\right) = \frac{1}{2}\sqrt{3}$.

25. $\arctan\frac{1}{2}+\arctan\frac{1}{3}=\arctan 1=\frac{\pi}{4}.$

26. $\arctan\frac{2x}{1-x^2}=2\arctan x=\arcsin\frac{2x}{1+x^2}.$

27. $\sin^{-1}x=\frac{\pi}{2}-\cos^{-1}x=\frac{1}{2}\cos^{-1}(1-2x^2).$

28. $\cos^{-1}x=\frac{\pi}{2}-\sin^{-1}x=2\tan^{-1}\sqrt{\frac{1-x}{1+x}}.$

29. $\sin^{-1}(3x-4x^3)=3\sin^{-1}x.$

30. $\cos^{-1}(4x^3-3x)=3\cos^{-1}x.$

31. $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)=3\tan^{-1}x.$

32. $\tan^{-1}\left(\frac{x}{x-1}\right)-\tan^{-1}\frac{x+1}{x}=\tan^{-1}\left(\frac{1}{2x^2}\right).$

33. $2\sin^{-1}x=\sec^{-1}\frac{1}{1-2x^2}.$

34. $\tan^{-1}\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2=\frac{1}{2}\sin^{-1}x^2.$

TRIGONOMETRIC EQUATIONS

56. Definitions. An equation containing *one* or *more* trigonometric functions of an unknown angle is called a *trigonometric equation.* Thus,

(*a*) $\sin x=\frac{1}{2}$,

(*b*) $\tan x+\sin x=5$,

(*c*) $\sin x+\cos\frac{x}{2}=0$,

(*d*) $\begin{cases}\sin x+\cos y=\frac{1}{2},\\ 3\sin x-\cos y=1,\end{cases}$

are *trigonometric equations.* Equations (*d*) above are *simultaneous trigonometric equations.*

The operations of ordinary algebra — *clearing of fractions, transposing, multiplying by constants* — are applicable to trigonometric equations.

In addition to these operations, the transformations of trigonometric identities may also be brought into use. For example, the equation

$$\sin 2x=\cos x$$

may be changed trigonometrically into

$$2 \sin x \cos x = \cos x.$$

then transpose and factor,

$$\cos x (2 \sin x - 1) = 0.$$

Equating to zero each factor,

$$\begin{cases} \cos x = 0, & x = 90°, \text{ or } 270°. \\ 2 \sin x = 1, & x = 30°, \text{ or } 150°. \end{cases}$$

57. Solutions. Trigonometric equations differ from algebraic equations in one important particular, viz. they have a *multitude of solutions*, whereas algebraic equations have a *finite number of solutions*. As illustrations notice the following examples.

(1) $\sin x = \frac{1}{2}$ has $x = 30°$, $150°$, or $x = 2n\pi + \frac{\pi}{6}$, or

$$(2n+1)\pi - \frac{\pi}{6}, \quad (n = 0, \pm 1, \pm 2, \cdots).$$

(2) $\tan x = \sqrt{3}$ has for solutions $x = 60°$, $240°$, or $n\pi + \frac{\pi}{3}$,

when $n = \text{any integer}$.

We shall enumerate and illustrate some of the more important types of trigonometric equations.

58. Simple Equations. Under this heading may be included any equation which reduces readily to one of the forms:

(1) $\sin x = K$, (2) $\cos x = K$, where K is not greater numerically than 1.

(3) $\tan x = K$, $K = \text{any number}$.

EXAMPLES. **1.** Solve $4 \sin x = \csc x$ for the angle x.

Solution. Multiply by $\sin x$, and divide by 4, giving

$$\sin^2 x = \tfrac{1}{4},$$

or

$$\sin x = \pm \tfrac{1}{2}.$$

Hence,

$$x = \sin^{-1}(\pm \tfrac{1}{2}) = 30°, -30°;\ 150°, -150°;$$

or, in general notation,

$$x = n\pi \pm \frac{\pi}{6}, \quad n = 0, 1, 2, 3, \cdots.$$

2. Find x from $\tan^2 x + 4 = 2\sec^2 x$.

Solution. Replace $\sec^2 x$ by $1 + \tan^2 x$, transpose, collect, and change signs,

$$\tan^2 x = 2, \text{ or } \tan x = \pm\sqrt{2}.$$

Then,

$$x = \tan^{-1}(\pm\sqrt{2}) = \tan^{-}(\pm 1.4142) = 54° \, 44', \; -(54° \, 44'),$$

or

$$x = n\pi \pm (54° \, 44').$$

NOTE. Take the angle $\tan^{-1}(\sqrt{2})$ from the table of natural functions.

3. Find x, when $6\cot x + 5 = \tan x$.

Solution. Multiply by $\tan x$, transpose, and change signs,

$$\tan^2 x - 5\tan x - 6 = 0,$$

a quadratic equation with $\tan x$ as the variable, which factors into

$$(\tan x + 1)(\tan x - 6) = 0,$$

giving and

$$\begin{cases} \tan x = -1, \\ \tan x = 6. \end{cases}$$

Hence,

$$x_1 = 135°, \; -45°,$$

or

$$x_2 = \tan^{-1} 6 = 80° \, 32', \; 260° \, 32'.$$

The general values of x are

$$x_1 = n\pi - \frac{\pi}{4}, \quad x_2 = n\pi + 80° \, 32'.$$

EXERCISES

Solve the following trigonometric equations, giving only the solutions which are between $-180°$ and $+180°$, inclusive.

1. $3\sin x = 2$. *Ans.* $\sin^{-1}\frac{2}{3} = 41° \, 48' \, 35''$, or $138° \, 11' \, 25''$.

2. $\sin 2x - \cos x = 0$. [$\sin 2x = 2\sin x \cos x$.]

Ans. $\pm 90°$, $30°$, $150°$.

3. $\cos 2x + \sin x = 1$. *Ans.* $0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$.

4. $\cos 2x = \sin x$. *Ans.* $30°$, $-90°$, $150°$.

5. $\tan 2x + 2\sin x = 0$. *Ans.* 0, $\pm 60°$, $180°$.

6. $\cos 3x - \sin 2x = 0$.

SUGGESTION. Change $\cos 3x$ to $4\cos^3 x - 3\cos x$, $\sin 2x = 2\sin x \cos x$, giving $4\cos^3 x - 3\cos x = 2\sin x \cos x$; factor and solve. Or, $\cos 3x = \sin(3x + 90°)$; then

$$\cos 3x - \sin 2x = \sin(3x + 90°) - \sin 2x$$

$$= 2\cos\left(\frac{5}{2}x + 45°\right) \cdot \sin\left(\frac{x}{2} + 45°\right) = 0.$$

Hence, $\cos\left(\frac{5}{2}x + 45°\right) = 0$, and $\sin\left(\frac{x}{2} + 45°\right) = 0$,

giving $\left.\begin{array}{l}\frac{5}{2}x + 45° = 90°, 270°, 450°,\\ \\ \frac{x}{2} + 45° = 0°, 180°.\end{array}\right\}$

and

Then, $x = 18°, 90°, 162°,$

and $x = -90°.$

7. $\cos 3x + \sin 2x - \cos x = 0.$ *Ans.* $0°, 30°, 90°, 150°, 180°.$

8. $\cos(x + 60°) - \sin(x + 30°) = \frac{1}{2}\sqrt{3}.$ *Ans.* $-30°, -150°.$

9. $\sin(x + 60°) - \sin(x - 60°) = -\frac{1}{2}\sqrt{3}.$ *Ans.* $\pm 120°.$

10. $\sin 4x = 2 \sin 2x.$ *Ans.* $0°, 90°, 180°.$

59. Equations of the Form

$$\begin{cases} r\cos\phi = a, \\ r\sin\phi = b. \end{cases}$$

Here is a set of two simultaneous equations in two unknowns, r, ϕ.

(1) To find ϕ, divide the second equation by the first,

$$\tan\phi = \frac{b}{a}, \quad \phi = \tan^{-1}\left(\frac{b}{a}\right).$$

Or, $$\phi = n\pi + \tan^{-1}\left(\frac{b}{a}\right).$$

(2) To find r, square and add, recalling the identity $\sin^2\phi + \cos^2\phi = 1$,

$$r^2 = a^2 + b^2,$$

$$r = \pm\sqrt{a^2 + b^2}.$$

The proper sign of r must be chosen so that $r\cos\phi = a$, $r\sin\phi = b$.

Examples. **1.** Find r, ϕ from

$$\begin{cases} r\cos\phi = 3, \\ r\sin\phi = 4. \end{cases}$$

We have $\tan\phi = \frac{4}{3}, \quad \phi = \tan^{-1}(\frac{4}{3}) = 53° 8'.$

Squaring and adding, $r^2 = 4^2 + 3^2 = 25,$

$$r = \pm 5.$$

Find r, ϕ in the following:

2. $\begin{cases} r\cos\phi = 12, \\ r\sin\phi = 5. \end{cases}$ **4.** $\begin{cases} r\sin\phi = 5, \\ r\cos\phi = 5\sqrt{3}. \end{cases}$

3. $\begin{cases} r\cos\phi = 6, \\ r\sin\phi = 12. \end{cases}$ **5.** $\begin{cases} r\sin\phi = 4.876, \\ r\cos\phi = 2.396. \end{cases}$

60. Equations in the Form

$$\begin{cases} r\sin\theta\cos\phi = a, \\ r\sin\theta\sin\phi = b, \\ r\cos\theta = c, \end{cases}$$

where r, θ, ϕ are variables.

Divide the second equation by the first,

$$\tan\phi = \frac{b}{a}, \quad \phi = \tan^{-1}\frac{b}{a}.$$

Squaring all three equations and adding, we have

$$r^2(\sin^2\theta\,[\cos^2\phi + \sin^2\phi] + \cos^2\theta) = a^2 + b^2 + c^2,$$

or

$$r^2 = a^2 + b^2 + c^2,$$

$$r = \pm\sqrt{a^2 + b^2 + c^2}.$$

From the third equation,

$$\cos\theta = \frac{c}{r} = \frac{c}{\pm\sqrt{a^2 + b^2 + c^2}},$$

$$\theta = \cos^{-1}\left(\frac{\pm c}{\sqrt{a^2 + b^2 + c^2}}\right).$$

EXERCISES

Find r, θ, ϕ in the following:

1. $\begin{cases} r\sin\theta\cos\phi = 2, \\ r\sin\theta\sin\phi = 2, \\ r\cos\theta = 1. \end{cases}$ **4.** $\begin{cases} r\sin\theta\cos\phi = 12, \\ r\sin\theta\sin\phi = 12, \\ r\cos\theta = 1. \end{cases}$

2. $\begin{cases} r\sin\theta\cos\phi = 6, \\ r\sin\theta\sin\phi = 3, \\ r\cos\theta = 2. \end{cases}$ **5.** $\begin{cases} r\sin\theta\cos\phi = 1, \\ r\sin\theta\sin\phi = 4, \\ r\cos\theta = 8. \end{cases}$

3. $\begin{cases} r\sin\theta\cos\phi = 2, \\ r\sin\theta\sin\phi = 6, \\ r\cos\theta = 9. \end{cases}$ **6.** $\begin{cases} r\sin\theta\cos\phi = 5, \\ r\sin\theta\sin\phi = 2, \\ r\cos\theta = 0. \end{cases}$

61. To solve $a \sin x + b \cos x = c$. Divide this equation by $\sqrt{a^2+b^2}$.

(1) $$\frac{a \quad \sin x}{\sqrt{a^2+b^2}} + \frac{b \cdot \cos x}{\sqrt{a^2+b^2}} = \frac{c}{\sqrt{a^2+b^2}}.$$

Now let $$\frac{a}{\sqrt{a^2+b^2}} = \sin \phi, \qquad \frac{b}{\sqrt{a^2+b^2}} = \cos \phi,$$

and hence, $$\frac{a}{b} = \tan \phi, \qquad \phi = \tan^{-1}\frac{a}{b}.$$

Then equation (1) becomes

(2) $$\sin \phi \sin x + \cos \phi \cos x = \frac{c}{\sqrt{a^2+b^2}},$$

or $$\cos (x - \phi) = \frac{c}{\sqrt{a^2+b^2}}, \qquad x - \phi = \pm \cos^{-1}\left(\frac{c}{\sqrt{a^2+b^2}}\right),$$

$$x = \phi \pm \cos^{-1}\left(\frac{c}{\sqrt{a^2+b^2}}\right).$$

EXAMPLES. 1. Solve $5 \sin x + 12 \cos x = 6.5$.

Divide both members by 13,

(1) $$\tfrac{5}{13} \sin x + \tfrac{12}{13} \cos x = 0.5.$$

Write $$\tfrac{5}{13} = \sin \phi, \quad \tfrac{12}{13} = \cos \phi, \quad \tan \phi = \tfrac{5}{12}.$$

Hence, $$\phi = \tan^{-1}(\tfrac{5}{12}) = 22° \, 37'.$$

Now equation (1) becomes

(2) $$\sin \phi \sin x + \cos \phi \cos x = 0.5,$$

or $$\cos (x - \phi) = 0.5,$$

$$x = \phi \pm \cos^{-1}(0.5) = 22° \, 37' \pm 60° = 82° \, 37', \quad -37° \, 23'.$$

2. Solve $3 \cos \; x + 5 \sin x = 4$.

3. Solve $12 \sin x + 5 \cos x = 3.9$.

4. Solve $8 \cos x + 15 \sin x = 5.1$.

5. Solve $3 \cos x - 2 \sin x = \frac{3}{5}\sqrt{13}$.

6. Solve $5 \sin x - 6 \cos x = \frac{3}{8}\sqrt{61}$.

62. To solve $\sin (x + \phi) = a \sin x$.

We have $$\frac{\sin (x + \phi)}{\sin x} = \frac{a}{1}.$$

Take by composition and division,

$$\frac{\sin(x+\phi)+\sin x}{\sin(x+\phi)-\sin x}=\frac{a+1}{a-1},$$

$$\frac{2\sin\left(x+\frac{\phi}{2}\right)\cos\frac{\phi}{2}}{2\cos\left(x+\frac{\phi}{2}\right)\sin\frac{\phi}{2}}=\frac{a+1}{a-1},$$

or
$$\tan\left(x+\frac{\phi}{2}\right)=\frac{a+1}{a-1}\tan\frac{\phi}{2}.$$

The right member is now known, and the solution may be written out.

EXAMPLES. 1. Solve $\sin(x+37°\,14')=2\sin x$.

Substituting in the above result, and retaining the smallest angle,

$$\tan\left(x+\frac{37°\,14'}{2}\right)=3\tan\frac{37°\,14'}{2}=3(0.3369),$$

$$x=\tan^{-1}(1.0107)-\frac{37°\,14'}{2}=45°\,18'\,20''-(18°\,37')=26°\,41'\,20''.$$

2. Solve $\sin(x+65°\,21')=3\sin x$.

3. Solve $\sin(x-28°\,40')=\frac{2}{3}\sin x$.

4. Solve $\sin(x+56°\,24')=5\sin(x-10°\,20')$.

5. Solve $\sin(x+94°\,10')=4\sin x$.

6. Solve $\sin(x-124°)=\frac{3}{4}\sin x$.

63. To solve $\tan(x+\phi)=a\tan x$.

Divide by $\tan x$, and take by composition and division,

$$\frac{\tan(x+\phi)}{\tan x}=a,$$

$$\frac{\tan(x+\phi)+\tan x}{\tan(x+\phi)-\tan x}=\frac{a+1}{a-1}.$$

Simplifying,
$$\frac{\sin(2x+\phi)}{\sin\phi}=\frac{a+1}{a-1},$$

$$\sin(2x+\phi)=\frac{a+1}{a-1}\sin\phi.$$

EXAMPLES. 1. Solve $\tan(x + 20°) = 5 \tan x$.

Comparing with the above result,

$$\begin{aligned} \phi &= 20°,\ a = 5, \\ \sin(2x + \phi) &= \tfrac{3}{2} \sin 20° \\ &= \tfrac{3}{2}(0.342) = 0.513, \\ 2x + \phi &= \sin^{-1}(0.513) = 30°\,52', \\ 2x &= 30°\,52' - 20° = 10°\,52', \\ x &= 5°\,26'. \end{aligned}$$

2. Solve $\tan(x + 30°) = 6 \tan x$.

3. Solve $\tan(x + 47°\,20') = 7 \tan x$.

4. Solve $\tan(x + 25°\,10') = 10 \tan x$.

5. Solve $\tan(x + 60°) = 13 \tan x$.

64. To solve $x = \alpha + \beta \sin x$.

In this equation α, β are usually given as angles (degrees or radians); β expressed in radian measure is smaller than unity. Two plans of solution will be sketched.

(1) *Trial solution.* Let α, β be expressed in degrees. Then an upper limit to x will be shown by $\alpha + \beta$, since the multiplier β is smaller than unity. Take a trial solution, substitute in the equation, note the error, make another approximation, and so continue until the required degree of accuracy is attained.

(2) *Graphical solution.* Let α, β be expressed in radian measure: $\alpha° = a$ radians, $\beta° = b$ radians. Then we have to determine x so that

$$x - a = b \sin x.$$

Let $$y_1 = x - a, \quad y_2 = b \sin x.$$

Construct upon rectangular axes a curve representing each of these equations. The first is a *straight line* through the point a on the *X-axis*; the second is a *modified sine curve*. The x of the point of section of these two graphs is the required solution.

As an illustration of the graphical solution let us solve

$$x = 57°\,17'\,44'' + 45° \times \sin x.$$

Here, $57°\,17'\,44'' = 1$ radian, $45° = 0.785$ radians.

Then we are to solve $x = 1 + 0.785 \times \sin x$.

Let $$y_1 = x - 1, \quad y_2 = 0.785 \sin x.$$

Now construct the *straight line* $y_1 = x - 1$, and the modified sine curve $y_2 = 0.785 \sin x$. The abscissa of the point of intersection of the straight line and curve is approximately $x = 1.769$ radian $= 101° 27'$.

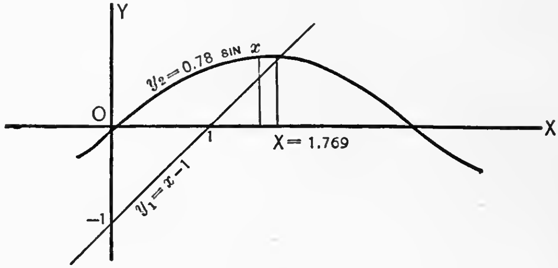


Fig. 47.

EXAMPLES. **1.** Solve $x = 24° + 30° \times \sin x$.

2. Solve $x = 64° + 28° \times \sin x$.

3. Solve $x = 30° + 50° \times \sin x$.

4. Solve $x = 10° 20' + 40° \times \sin x$.

5. Solve $x = 45° + (37° 30') \times \cos x$.

6. Show $\sin x \geqq \frac{2}{\pi} x, \quad 0 \leqq x \leqq \frac{\pi}{2}$.

CHAPTER IX

COMPLEX NUMBERS. DE MOIVRE'S THEOREM. TRIGONOMETRIC SERIES. EXPONENTIAL AND HYPERBOLIC FUNCTIONS

65. Roots of Quadratic Equations. In ordinary algebra we have such equations as

$$x^2 - 6x + 25 = 0,$$

whose roots,

$$z_1 = 3 + 4\sqrt{-1},\ z_2 = 3 - 4\sqrt{-1},$$

are called *complex numbers*. These numbers contain a *real unit*, 1, and a so-called *imaginary unit*, $\sqrt{-1}$.

(1) *Properties of* $\sqrt{-1}$. The imaginary unit is usually represented by i. We may easily show that when $\sqrt{-1} = i$,

$$i^2 = -1,\ i^3 = -i,\ i^4 = 1,\ i^5 = i \cdots,$$

and generally,

$$i^{4k} = 1,\ i^{4k+1} = i,\ i^{4k+2} = -1,\ i^{4k+3} = -i.$$

(2) *Graphical representation of* $x + yi$. To any complex number as $3 + 4i$, $3 - 4i$, $x + yi$ corresponds a *point in a plane.* If we multiply a real number a by i, and this product again by i, the result is $-a$. Thus, multiplying twice by i changes a number to its negative. *Multiplying a number by i may be interpreted as turning its direction through* 90°. To locate $3 + 4i$ upon a plane, lay off 3 units, OQ, along the real axis (horizontal in Fig. 48), then at Q erect a perpendicular 4 units long; the point P_1 represents the complex number $3 + 4i$. Lay off -4 perpendicular to OQ at Q, and we locate $3 - 4i$, at P_2.

Any complex number $x + yi$ may be represented upon a plane as shown in Fig. 49. The point P may be anywhere in the plane.

(3) *Modulus, arc* $(x + yi)$. The line $OP = r$ is a vector equal in length to the *modulus* of $x + yi$, or the *absolute value* of

$x + yi$. The angle $XOP = \phi$ is called the *arc* of $x + yi$, or *amplitude*, or *argument* of $x + yi$. As abbreviations

$$r = \text{mod}\ (x + yi),\ \phi = \text{amp}\ (x + yi).$$

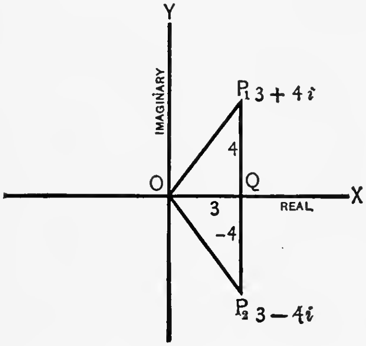


Fig. 48.

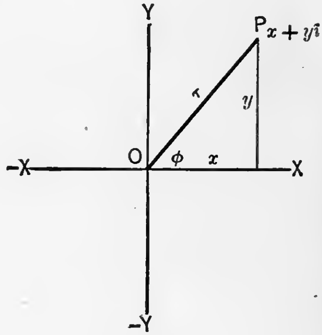


Fig. 49.

The following notation should be recognized:

$$\textbf{Modulus of } x + yi = r = \sqrt{x^2 + y^2} = |x + yi|.$$

$$\textbf{Arc of } x + yi = \phi = \tan^{-1} \frac{y}{x} = \textbf{amplitude } (x + yi).$$

EXERCISES

1. Locate the following complex numbers:

(1) $3 + 2i$; (2) $2 + 6i$; (3) $-3 + 3i$; (4) $-4 + i$;

(5) $-3 - 4i$; (6) $-6i$; (7) $4 - 5i$; (8) $\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) \times 5$.

2. Find the modulus and arc of each of the numbers in Ex. 1.

3. Solve the following equations and locate the roots as complex numbers:

(1) $z^2 - 4z + 13 = 0$; (2) $z^2 + 6z + 13 = 0$; (3) $z^2 + z + 1 = 0$; (4) $z^3 - 1 = 0$; (5) $z^3 + 1 = 0$.

4. Locate the following products: (1) $i \times (2 + 4i)$; (2) $i \times (-3 + 2i)$; (3) $i \times (5 - 3i)$; (4) $i \times i \times i(2 - i)$.

66. Complex Numbers expressed Trigonometrically. From Fig. 49, we have

$$x = r \cos \phi,\ y = r \sin \phi,\ r = \sqrt{x^2 + y^2},\ \phi = \tan^{-1} \frac{y}{x}.$$

Hence,

$$x + yi = r(\cos\phi + i\sin\phi) = \sqrt{x^2 + y^2}(\cos\phi + i\sin\phi).$$

THEOREM. *A complex number equals its modulus multiplied by the expression* $\cos\phi + i\sin\phi$ *where* ϕ *is its amplitude.*

EXAMPLES. **1.** $z^2 - z + 1 = 0$ has roots,

$$z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i, \quad z_2 = \frac{1}{2} - \frac{\sqrt{3}}{2}i.$$

The modulus of $z_1 = 1$, mod $z_2 = 1$; the amplitude of

$$z_1 = \tan^{-1}\left(\frac{\sqrt{3}}{2} \div \frac{1}{2}\right) = \tan^{-1}\sqrt{3} = 60°,$$

$$\text{am}\, z_2 = \tan^{-1}(-\sqrt{3}) = -60°.$$

Hence,

$$z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i = \cos 60° + i\sin 60°,$$

$$z_2 = \frac{1}{2} - \frac{\sqrt{3}}{2}i = \cos 60° - i\sin 60°.$$

2. Express the roots of the following equations in trigonometric form:

(1) $z^2 + 1 = 0$; (2) $z^2 - 2z + 2 = 0$; (3) $z^2 - \sqrt{3}z + 1 = 0$;
(4) $z^2 + 2z - 8 = 0$; (5) $z^2 + \sqrt{2}z + 1 = 0$; (6) $z^2 + z + 1 = 0$.

67. DeMoivre's Theorem. Let us take a complex number in trigonometric form,

$$z = x + yi = r(\cos\phi + i\sin\phi).$$

Squaring z, and recalling that $i^2 = -1$,

$$z^2 = r^2(\cos\phi + i\sin\phi)^2 = r^2(\cos^2\phi - \sin^2\phi + 2i\sin\phi\cos\phi),$$

(1) $z^2 = r^2(\cos 2\phi + i\sin 2\phi)$, § 35, (7), (8).

Now, multiply (1) by $z = r(\cos\phi + i\sin\phi)$.

$$z^3 = r^3[\cos 2\phi\cos\phi - \sin 2\phi\sin\phi + i(\sin 2\phi\cos\phi + \cos 2\phi\sin\phi)].$$

(2) $z^3 = r^3(\cos 3\phi + i\sin 3\phi)$, § 34.

The law of exponents shown in (1), (2) would indicate that for $n =$ any positive integer,

(3) $z^n = r^n(\cos n\phi + i \sin n\phi)$.

Assuming law (3) to hold for any integral n, let us see if it holds when n is replaced by $n + 1$.

Multiply (3) by

$$z = r(\cos \phi + i \sin \phi),$$

$$z^{n+1} = r^{n+1}[\cos n\phi \cos \phi - \sin n\phi \sin \phi + i(\sin n\phi \cos \phi + \cos n\phi \sin \phi)].$$

(4) $z^{n+1} = r^{n+1}[\cos (n+1)\phi + i \sin (n+1)\phi]$.

Hence, the law assumed in (3) for the integer n holds for $n + 1$. We see this law holds for $n = 2$ and $n = 3$, hence it holds for $n = 4, 5, \cdots, n =$ any positive integer. Hence, we have

DeMoivre's Theorem: $(\cos \phi + i \sin \phi)^n = \cos n\phi + i \sin n\phi$.

DeMoivre's Theorem holds when n is an integer, a fraction, or a negative number.

$$(\cos \phi + i \sin \phi)^{\frac{1}{n}} = \cos \frac{\phi}{n} + i \sin \frac{\phi}{n},$$

$$(\cos \phi + i \sin \phi)^{\frac{p}{q}} = \cos \frac{p}{q}\phi + i \sin \frac{p}{q}\phi,$$

$$(\cos \phi + i \sin \phi)^{-m} = \cos(-m\phi) + i \sin(-m\phi)$$

$$= \cos m\phi - i \sin m\phi.$$

68. Raising to Powers and Extracting Roots. DeMoivre's Theorem enables us to raise

$$z = x + iy = r(\cos \phi + i \sin \phi)$$

to any power, or to extract any root of z.

Thus, $$z^2 = r^2(\cos 2\phi + i \sin 2\phi).$$

Hence, to square a complex number, square its modulus and double its amplitude. To cube a complex number, cube its modulus and multiply its amplitude by three.

EXAMPLES. **1.** Raise $z = 3 + 4i$ to the 2d power; to the 3d power.

$$z = 3 + 4i = 5(\cos\phi + i\sin\phi),\ \phi = \tan^{-1}\tfrac{4}{3} = 53^\circ\ 8', \text{ nearly.}$$
$$z^2 = (3 + 4i)^2 = 25(\cos 2\phi + i\sin 2\phi).$$
$$z^3 = (3 + 4i)^3 = 125(\cos 3\phi + i\sin 3\phi).$$

2. Raise $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ to 2d, 3d, rth powers, and locate these respective numbers on a diagram. See Fig. 49.

3. Find z^2, z^4, z^6, when $z = \frac{-\sqrt{3}}{2} + \frac{1}{2}i$.

4. Find z^5, z^{10}, when $z = -1 - i$.

The extraction of roots may be performed by use of DeMoivre's Theorem:

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{\phi}{n} + i\sin\frac{\phi}{n}\right).$$

This formula seems to give but one of the n nth roots of z, but we may obtain n different roots by writing

$$z = r[\cos(\phi + 2k\pi) + i\sin(\phi + 2k\pi)];$$

then will

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\frac{\phi + 2k\pi}{n} + i\frac{\phi + 2k\pi}{n}\right],\ k = 1, 2, 3, \cdots, n - 1.$$

EXAMPLES **1.** Extract the square root of i.

Here, $$z = i = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$
$$= \cos\left(\frac{\pi}{2} + 2k\pi\right) + i\sin\left(\frac{\pi}{2} + 2k\pi\right).$$

Then, $$z^{\frac{1}{2}} = i^{\frac{1}{2}} = \cos\left(\frac{\frac{\pi}{2} + 2k\pi}{2}\right) + i\sin\left(\frac{\frac{\pi}{2} + 2k\pi}{2}\right).$$

Let z_1, z_2 be the two roots, then

$$z_1 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, \qquad k = 0,$$

$$z_2 = \cos\left(\frac{\pi}{4} + \pi\right) + i\sin\left(\frac{\pi}{4} + \pi\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i,\ k = 1.$$

2. Extract the cube root of -8, and locate the roots on a diagram.

In this case $z = -8 = 8(\cos\pi + i\sin\pi)$

$$= 8[\cos(\pi + 2k\pi) + i\sin(\pi + 2k\pi)].$$

Extract the cube root,

$$z^{\frac{1}{3}} = (-8)^{\frac{1}{3}} = 8^{\frac{1}{3}}\left[\cos\frac{\pi + 2k\pi}{3} + i\sin\frac{\pi + 2k\pi}{3}\right].$$

Giving k values 0, 1, 2, we find the three roots, z_1, z_2, z_3,

$$z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 1 + \sqrt{3}\,i, \qquad k = 0,$$

$$z_2 = 2(\cos\pi + i\sin\pi) = -2, \qquad k = 1,$$

$$z_3 = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) = 1 - \sqrt{3}\,i, \quad k = 2.$$

3. Find the five 5th roots of 32; of i.

4. Find the six 6th roots of -1; the cube roots of $\dfrac{-1+\sqrt{3}\,i}{2}$.

69. Value of sin *x*, cos *x* in Terms of *x*.

(1) *Value of* $\dfrac{\sin\phi}{\phi}$ *when* $\phi \doteq 0$.

When an angle ϕ *is small,* $\sin\phi$ *approaches arc* ϕ.

From the tables of natural functions and radian measure we have:

$\sin 0° = 0.00000$	$0° = 0.00000$ radian
$\sin 10' = 0.00291$	$10' = 0.00291$ radian
$\sin 40' = 0.01164$	$40' = 0.01164$ radian
$\sin 1° = 0.01745$	$1° = 0.01745$ radian
$\sin 2° = 0.03490$	$2° = 0.03491$ radian

which show that $\sin\phi = \phi$ for values of ϕ from 0° to near 2°, true to five decimals.

To show this property generally, we have from geometry

$$BC < BA < DA, \ \phi < 90°,$$

or, if $OA = 1$, Fig. 50,

$$\sin\phi < \phi < \tan\phi.$$

Divide this inequality by $\sin\phi$,

$$1 < \frac{\phi}{\sin\phi} < \frac{1}{\cos\phi}.$$

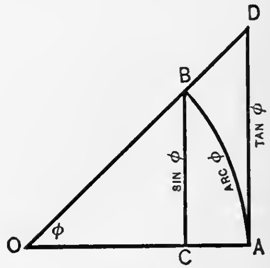


Fig. 50.

Now let ϕ approach zero, $\phi \doteq 0$, and $\cos\phi \doteq 1$; hence,

$$1 < \left[\frac{\phi}{\sin\phi}\right]_{\phi \doteq 0} < 1,$$

or,

$$\underset{\phi=0}{\text{limit}}\left[\frac{\phi}{\sin\phi}\right] = 1.$$

$$\mathbf{\underset{\phi=0}{limit}\left[\frac{\sin\phi}{\phi}\right] = 1,\ \sin\phi \doteq \phi \text{ as } \phi \doteq 0.}$$

(2) *Value of sin nφ, cos nφ in terms of sin φ, cos φ.*
In algebra it is shown that if

$$x + yi = a + bi,$$

then $$x = a, \quad y = b.$$

THEOREM. *If two complex numbers are equal, the real parts are equal and the imaginary parts are equal.*

By DeMoivre's Theorem,

$$(A) \qquad (\cos\phi + i\sin\phi)^n = \cos n\phi + i\sin n\phi.$$

But by the Binomial Theorem,

$$(B) \quad (\cos\phi + i\sin\phi)^n = \cos^n\phi + n\cos^{n-1}\phi\,(i\sin\phi)$$
$$+ \frac{n(n-1)}{1\cdot 2}\cos^{n-2}\phi\,(i\sin\phi)^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}\cos^{n-3}\phi\,(i\sin\phi)^3$$
$$+ \frac{n(n-1)(n-2)(n-3)}{1\cdot 2\cdot 3\cdot 4}\cos^{n-4}\phi\,(i\sin\phi)^4 + \cdots,$$

the series terminating if n is a positive integer, and becoming infinite if n be a fraction or negative. Now $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, $\cdots$; hence, the first, third, fifth, $\cdots$, terms of the right member of (B) are free of i, and the even terms contain i. The right members of (A) and (B) are equal. Equating the real and imaginary parts, respectively, we have

$$(C) \quad \cos n\phi = \cos^n\phi - \frac{n(n-1)}{1\cdot 2}\cos^{n-2}\phi\sin^2\phi$$
$$+ \frac{n(n-1)(n-2)(n-3)}{1\cdot 2\cdot 3\cdot 4}\cos^{n-4}\phi\sin^4\phi - \cdots,$$

$$(D)\quad \sin n\phi = n\sin^{n-1}\phi\cos\phi - \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}\cos^{n-3}\phi\sin^3\phi$$

$$+\frac{n(n-1)(n-2)(n-3)(n-4)}{1\cdot 2\cdot 3\cdot 4\cdot 5}\cos^{n-5}\phi\sin^5\phi - \cdots.$$

If in (C), (D) we give n the values 2, 3, 4, ⋯, we may obtain the ordinary expressions for $\sin 2\phi$, $\cos 2\phi$; $\sin 3\phi$, $\cos 3\phi$; etc., in terms of $\sin\phi$, $\cos\phi$. See §§ 35, 36.

EXAMPLES. **1.** Show from (C) $\cos 2\phi = \cos^2\phi - \sin^2\phi$, $n = 2$.

2. Show from (D) $\sin 2\phi = 2\sin\phi\cos\phi$.

3. Show $\cos 3\phi = 4\cos^3\phi - 3\cos\phi$.

4. Show $\sin 3\phi = 3\sin\phi - 4\sin^3\phi$.

(3) *Trigonometric series.* In formulas (C), (D) above, let us substitute $n\phi = x$, or $n = \frac{x}{\phi}$,
and we have

$$(C')\quad \cos x = \cos^n\phi - \frac{\frac{x}{\phi}\left(\frac{x}{\phi}-1\right)}{1\cdot 2}\cos^{n-2}\phi\sin^2\phi + \cdots$$

$$= \cos^n\phi - \frac{x(x-\phi)}{1\cdot 2}\cos^{n-2}\phi\left(\frac{\sin\phi}{\phi}\right)^2 + \cdots,$$

$$(D')\quad \sin x = \frac{x}{\phi}\cos^{n-1}\phi\sin\phi - \frac{\frac{x}{\phi}\left(\frac{x}{\phi}-1\right)\left(\frac{x}{\phi}-2\right)}{1\cdot 2\cdot 3}\cos^{n-3}\phi\sin^3\phi + \cdots$$

$$= x\cos^{n-1}\phi\left(\frac{\sin\phi}{\phi}\right) - \frac{x(x-\phi)(x-2\phi)}{1\cdot 2\cdot 3}\cos^{n-3}\phi\left(\frac{\sin\phi}{\phi}\right)^3 + \cdots.$$

Now let $n \doteq \infty$ in such manner that $n\phi \doteq x$, then $\frac{\sin\phi}{\phi} \doteq 1$, see (1) above, and (C'), (D') become the *infinite trigonometric series*

$$\textbf{I.}\quad \cos x = 1 - \frac{x^2}{\underline{|2}} + \frac{x^4}{\underline{|4}} - \frac{x^6}{\underline{|6}} + \cdots,$$

$$\textbf{II.}\quad \sin x = x - \frac{x^3}{\underline{|3}} + \frac{x^5}{\underline{|5}} - \frac{x^7}{\underline{|7}} + \cdots,$$

where $\underline{|2} = 1 \cdot 2$, $\underline{|3} = 1 \cdot 2 \cdot 3$, $\underline{|4} = 1 \cdot 2 \cdot 3 \cdot 4$, etc., these symbols being read *factorial two, factorial three, factorial four*, etc.

Dividing $\sin x$ by $\cos x$ and $\cos x$ by $\sin x$, we find

$$\text{III.}\quad \tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots,$$

$$\text{IV.}\quad \cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \cdots.$$

The series for $\cos x$ and $\sin x$ are convergent for any value of x. In the expansion for any trigonometric function the *radian measure of x* must be used in the series.

Tables of the numerical values of the trigonometric functions for any set of angles may be computed by means of the series I, II, III.

EXAMPLES. **1.** Compute $\sin 10°$ correct to four decimals. $\left(x = \frac{\pi}{18} = 0.17453\right)$.

2. Compute $\sin 12°$. $\left(x = \frac{\pi}{15} = 0.20944\right)$.

3. Compute $\cos 10°$, $\tan 10°$.

4. Compute $\sin 20°$ from Ex. 1 and 3.

5. Find $\sin 80°$, $\cos 80°$, $\cos 78°$, $\cos 70°$.

(4) *Trigonometric products.* $\sin x = 0$ when

$$x = 0,\ x = \pm\pi,\ x = \pm 2\pi,\ x = \pm 3\pi,\ \cdots.$$

This fact suggests that $\sin x$ can be expressed as a product of factors, and indeed an infinite number of factors. Likewise, $\cos x = 0$, when

$$x = \pm\frac{\pi}{2},\ x = \pm\frac{3\pi}{2},\ x = \pm\frac{5\pi}{2},\ \cdots.$$

Arranging the factors properly,

$$\sin x = x\left\{1 - \left(\frac{x}{\pi}\right)^2\right\}\left\{1 - \left(\frac{x}{2\pi}\right)^2\right\}\left\{1 - \left(\frac{x}{3\pi}\right)^2\right\}\cdots,$$

$$\cos x = \left\{1 - \left(\frac{2x}{\pi}\right)^2\right\}\left\{1 - \left(\frac{2x}{3\pi}\right)^2\right\}\left\{1 - \left(\frac{2x}{5\pi}\right)^2\right\}\cdots.$$

70. Summation of Series. In analysis it sometimes becomes necessary to sum the following series:

$$(1)\quad S_1 = \sin\theta + \sin(\theta+\alpha) + \sin(\theta + 2\alpha) + \cdots + \sin[\theta + (n-1)\alpha],$$

$$(2)\quad S_2 = \cos\theta + \cos(\theta+\alpha) + \cos(\theta + 2\alpha) + \cdots + \cos[\theta + (n-1)\alpha].$$

S_1 is a series of n sines in which the angles are in arithmetical progression, the common difference being α. S_2 is a similar series of cosines.

To find the sum S_1, multiply both members by $2\sin\frac{\alpha}{2}$.

$$2\sin\frac{\alpha}{2}S_1 = 2\sin\frac{\alpha}{2}\sin\theta + 2\sin\frac{\alpha}{2}\sin(\theta+\alpha) + \cdots + 2\sin\frac{\alpha}{2}\sin[\theta+(n-1)\alpha]$$

$$= \left\{\cos\left(\theta - \frac{\alpha}{2}\right) - \cos\left(\theta + \frac{\alpha}{2}\right)\right\} + \left\{\cos\left(\theta + \frac{\alpha}{2}\right) - \cos\left(\theta + \frac{3}{2}\alpha\right)\right\}$$

$$+ \left\{\cos\left(\theta + \frac{3}{2}\alpha\right) - \cos\left(\theta + \frac{5}{2}\alpha\right)\right\} + \cdots\ \cdots\ \cdots\ \cdots$$

$$+ \left\{\cos\left(\theta + \frac{2n-3}{2}\alpha\right) - \cos\left(\theta + \frac{2n-1}{2}\alpha\right)\right\}, \qquad \S\ 39\ (22')$$

$$= \cos\left(\theta - \frac{\alpha}{2}\right) - \cos\left(\theta + \frac{2n-1}{2}\alpha\right)$$

$$= 2\sin\left[\theta + (n-1)\frac{\alpha}{2}\right]\sin\frac{n\alpha}{2}.$$

Dividing by $2\sin\frac{\alpha}{2}$, we have

$$S_1 = \frac{\sin\left[\theta + (n-1)\frac{\alpha}{2}\right]\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}}.$$

In a similar manner, by multiplying S_2 by $2\sin\frac{\alpha}{2}$ and separating the double products as in S_1, we may reduce the value of S_2 to

$$S_2 = \cos\theta + \cos(\theta+\alpha) + \cos(\theta+2\alpha) + \cdots + \cos[\theta+(n-1)\alpha]$$

$$= \frac{\cos\left[\theta+(n-1)\frac{\alpha}{2}\right]\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}}.$$

EXAMPLES

Verify the following:

1. $\sin x + \sin\left(x+\frac{\pi}{3}\right) + \sin\left(x+\frac{2\pi}{3}\right) + \cdots + \sin\left[x+(n-1)\frac{\pi}{3}\right]$

$$= 2\sin\left[x+(n-1)\frac{\pi}{6}\right]\sin\frac{n\pi}{6}.$$

[SUGGESTION. Common difference $=\frac{\pi}{3}$, compare S_1.]

2. $\cos x + \cos\left(x-\frac{\pi}{3}\right) + \cos\left(x-\frac{2\pi}{3}\right) + \cdots + \cos\left[x-(n-1)\frac{\pi}{3}\right]$

$$= -2\cos\left[x-(n-1)\frac{\pi}{6}\right]\sin\left(-\frac{n\pi}{6}\right).$$

[SUGGESTION. The common difference is $-\frac{\pi}{3}$; apply S_2 with $\alpha = -\frac{\pi}{3}$.]

3. $\sin x + \sin 2x + \sin 3x + \cdots + \sin nx = \dfrac{\sin(n+1)\frac{x}{2}\sin\frac{nx}{2}}{\sin\frac{x}{2}}.$

4. $\cos x + \cos 2x + \cos 3x + \cdots + \cos nx = \dfrac{\cos(n+1)\frac{x}{2}\sin\frac{nx}{2}}{\sin\frac{x}{2}}.$

5. $\sin x + \sin 3x + \sin 5x + \cdots + \sin(2n-1)x = \dfrac{\sin^2 nx}{\sin x}.$

6. $\cos x + \cos 3x + \cos 5x + \cdots + \cos(2n-1)x = \dfrac{\cos nx \sin nx}{\sin x}$

$$= \frac{\sin 2nx}{2 \sin x}.$$

7. $\sin 2x + \sin 4x + \sin 6x + \cdots + \sin 2nx = \dfrac{\sin(n+1)x \sin nx}{\sin x}.$

8. $\cos 2x + \cos 4x + \cos 6x + \cdots + \cos 2nx$

$$= \frac{\cos(n+1)x \sin nx}{\sin x}.$$

9. $\sin^2 x + \sin^2(x+\alpha) + \sin^2(x+2\alpha) + \cdots + \sin^2[x+(n-1)\alpha]$

$$= \frac{n}{2} - \frac{\cos[2x+(n-1)\alpha]\sin n\alpha}{2 \sin \alpha}.$$

[SUGGESTION. Multiply this series by 2 and separate each term thus:

$$2\sin^2 x = 1 - \cos 2x,\quad 2\sin^2(x+a) = 1 - \cos(2x+2\alpha),\ \cdots$$
$$2\sin^2[x+(n-1)\alpha] = 1 - \cos[2x+(n-1)2\alpha];$$

add and sum the cosines as in Ex. 8.]

10. $\cos^2 x + \cos^2(x+a) + \cos^2(x+2\alpha) + \cdots$

$$+ \cos^2[x+(n-1)\alpha] = \frac{n}{2} + \frac{\cos[2x+(n-1)\alpha]\sin n\alpha}{2\sin\alpha}.$$

71. The Exponential Series. If we take the Binomial Series,

$$(1+z)^n = 1 + nz + \frac{n(n-1)}{\underline{|2}} z^2 + \frac{n(n-1)(n-2)}{\underline{|3}} z^3 + \cdots,$$

and substitute

$$z = \frac{1}{m},\quad n = mx,$$

we find

$$\left(1+\frac{1}{m}\right)^{mx} = \left\{\left(1+\frac{1}{m}\right)^m\right\}^x = 1 + mx\left(\frac{1}{m}\right) + \frac{mx(mx-1)}{\underline{|2}}\left(\frac{1}{m}\right)^2 + \cdots.$$

Now, divide the numerator and denominator of the respective fractions by m, m^2, m^3, $\cdots$, and finally let $m = \infty$; then we have

$$\underset{m=\infty}{\text{limit}}\left(1+\frac{1}{m}\right)^{mx} = 1 + x + \frac{x^2}{\underline{|2}} + \frac{x^3}{\underline{|3}} + \frac{x^4}{\underline{|4}} + \cdots.$$

The numerical value of this series, when $x = 1$, is denoted by e.

$$(A)\quad e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \quad = 2.7182818 \cdots$$

$$(B)\quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = (2.7182818 \cdots)^x$$

Series (B) is called the *exponential series;* e is the base of the *Napierian Logarithmic system.*

72. Euler's Formulas. Series (B) readily identifies the exponential functions with the trigonometric functions. In (B) substitute

$$x = i\theta,\text{ where } i = \sqrt{-1},\ i^2 = -1, \cdots, \qquad \S\ 65$$

$$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{i\theta^5}{5!} - \cdots$$

$$= 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right)$$

$$= \cos\theta + i\sin\theta, \qquad \S\ 69,\ \text{I, II.}$$

Changing i to $-i$,

$$e^{-i\theta} = \cos\theta - i\sin\theta.$$

Subtracting and dividing by $2i$, and adding and dividing by 2, respectively, we have Euler's Formulas for $\sin\theta$ and $\cos\theta$. $\tan\theta = \sin\theta \div \cos\theta$.

$$\text{V.}\quad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}.$$

$$\text{VI.}\quad \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}.$$

$$\text{VII.}\quad \tan\theta = \frac{e^{i\theta} - e^{-i\theta}}{i(e^{i\theta} + e^{-i\theta})}.$$

The reciprocals of these fractions define the $\csc\theta$, $\sec\theta$, $\cot\theta$, respectively.

These analytic definitions of $\sin\theta$, $\cos\theta$, $\tan\theta$, may be employed instead of the ratio definitions given in § 2.

EXERCISES

Prove the following identities by use of Euler's definitions, V, VI, VII.

1. $\sin 2\theta = 2\sin\theta\cos\theta$.

2. $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

3. $\sin(x+y) = \sin x\cos y + \cos x\sin y$.

4. $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$.

5. $\sin^2 x + \cos^2 x = 1$.

6. $\sec^2 x - \tan^2 x = 1$.

7. Write trigonometric values for each of the following:

(1) $e^{i\frac{\pi}{2}}$; (2) $e^{i\pi}$; (3) $e^{i\frac{2}{3}\pi}$; (4) $e^{i\frac{\pi}{4}}$; (5) $e^{-i\pi}$; (6) $e^{-i\frac{\pi}{3}}$;

(7) $e^{-i\frac{\pi}{2}}$; (8) $e^{i(\pi+\alpha)}$.

8. Express in exponential notation: (1) $\cos 30° + i\sin 30°$;

(2) $\frac{1+i}{\sqrt{2}}$; (3) $\frac{-1+\sqrt{3}\,i}{2}$; (4) $\frac{\sqrt{3}+i}{2}$; (5) $\frac{-1+i}{\sqrt{2}}$;

(6) $e^2\left(\frac{1-\sqrt{3}\,i}{2}\right)$.

9. Prove the following: (1) $e^{i(\alpha+2\pi)} = e^{i\alpha}$; (2) $e^{2+i\pi} = -e^2$;

(3) $e^{2+i\frac{\pi}{2}} = ie^2$; (4) $e^{2+2\pi i} = e^2$.

10. Find approximately the value of $\sqrt{e}$ by substituting $x = \frac{1}{2}$ in (*B*), § 71.

73. The Hyperbolic Functions. In V, VI, VII, § 72, let $\theta = ix$,

$$\sin ix = \frac{e^{-x} - e^{x}}{2i} = i\frac{e^{x} - e^{-x}}{2}, \quad \cos ix = \frac{e^{-x} + e^{x}}{2},$$

$$\tan ix = \frac{e^{-x} - e^{x}}{i(e^{-x} + e^{x})} = i\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}.$$

The fractions $\frac{e^x - e^{-x}}{2}$, $\frac{e^x + e^{-x}}{2}$, $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ are taken as definitions of the *hyperbolic sine of x*, *hyperbolic cosine of x*, and *hyperbolic tangent of x*, respectively. These functions are written:

VIII. $$\sinh x = \frac{e^x - e^{-x}}{2}.$$

IX. $$\cosh x = \frac{e^x + e^{-x}}{2}.$$

X. $$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

(1) *Relations between trigonometric and hyperbolic functions.* From the above definitions it is seen that

$\sin ix = i \sinh x$, $\qquad \cot ix = -i \coth x$,

$\cos ix = \cosh x$, $\qquad \sec ix = \operatorname{sech} x$,

$\tan ix = i \tanh x$, $\qquad \csc ix = -i \operatorname{csch} x$.

This table of relations enables us to determine the identities existing among the hyperbolic functions, corresponding to the trigonometric identities.

(2) *Identities.* Let the student verify the following identities among the hyperbolic functions:

(1) $\cosh^2 x - \sinh^2 x = 1$.

(2) $\operatorname{sech}^2 x + \tanh^2 x = 1$.

(3) $\coth^2 x - \operatorname{csch}^2 x = 1$.

(4) $\sinh 2x = 2 \sinh x \cosh x$.

(5) $\cosh 2x = \cosh^2 x + \sinh^2 x$.

(6) $\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$.

(7) $\sinh(-x) = -\sinh x$.

(8) $\cosh(-x) = \cosh x$.

(9) $\sinh(x + y) = \sinh x \cosh y \pm \cosh x \sinh y$.

(10) $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$.

(11) $\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$.

(12) $e^x = \cosh x + \sinh x$.

(13) $\sinh x + \sinh y = 2 \sinh \frac{x+y}{2} \cosh \frac{x-y}{2}$.

(14) $\cosh x - \cosh y = 2 \sinh \frac{x+y}{2} \sinh \frac{x-y}{2}$.

(3) *Infinite series forms for sinh x, cosh x.* Take the series for $\sin x$, $\cos x$, $\tan x$, II, I, III, § 69,

$$\sin x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \cdots,$$

$$\cos x = 1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \cdots,$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots,$$

and replace x by ix, giving

$$\sinh x = x + \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} + \cdots,$$

$$\cosh x = 1 + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} + \cdots,$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \cdots.$$

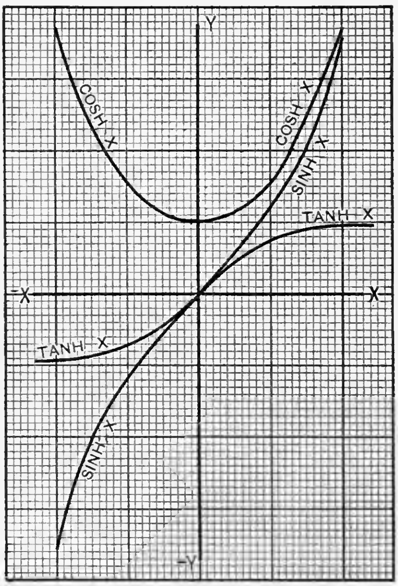


Fig. 51.

(4) *Graphs of the hyperbolic functions.* The numerical values of the hyperbolic functions for various values of the variable x are collected in Tables of Hyperbolic Functions. The approximate values of $\sinh x$, $\cosh x$, $\tanh x$ are shown upon the following graphs, Fig. 51, where the ordinates parallel to the Y-axis are the values of these functions for the corresponding values of x. The graphs of the $\operatorname{csch} x$, $\operatorname{sech} x$, and $\coth x$ are not shown, but these may be easily constructed upon the same diagram by taking reciprocals of the ordinates of the $\sinh x$, $\cosh x$, and $\tanh x$, respectively.

(5) *The inverse hyperbolic functions.* If we write

$$y = \sinh x = \frac{e^x - e^{-x}}{2},$$

we may express x in terms of y by an inverse notation similar to that employed in inverse trigonometric notation.

$$y = \sinh x, \quad x = \sinh^{-1} y.$$

By means of the exponential value of $\sinh x$, we may obtain another expression for x in terms of y. Thus,

$$\frac{e^x - e^{-x}}{2} = y, \quad e^{2x} - 1 = 2\,ye^x,$$

or

$$e^{2x} - 2\,ye^x - 1 = 0, \text{ a quadratic in } e^x.$$

Solving,

$$e^x = y \pm \sqrt{y^2 + 1},$$

the negative sign being excluded since e^x is positive. Take $\log_e$ of each member,

$$x = \log_e(y + \sqrt{y^2 + 1}) = \sinh^{-1} y.$$

EXERCISES

Prove the following:

1. $\cosh^{-1} x = \log_e(x + \sqrt{x^2 - 1})$.

2. $\tanh^{-1} x = \frac{1}{2} \log_e\left(\frac{1 + x}{1 - x}\right)$.

3. $\operatorname{sech}^{-1} x = \cosh^{-1}\frac{1}{x} = \log_e \frac{1 + \sqrt{1 - x^2}}{x}$.

4. $\sinh^{-1} x = \cosh^{-1}\sqrt{1 + x^2} = \tanh^{-1}\frac{x}{\sqrt{1 + x^2}}$.

5. $\tanh^{-1} x + \tanh^{-1} y = \tanh^{-1} \frac{x + y}{1 + xy}$.

6. $\cosh 0 = 1, \ \cosh \frac{\pi i}{2} = 0, \ \sinh \pi i = 0$.

7. $\sinh \frac{\pi i}{2} = i, \ \cosh \pi i = -1, \ \tanh 0 = 0$.

8. $\sinh 2\,n\pi i = 0, \ \cosh 2\,n\pi i = 1, \ \tanh n\pi i = 0$.

74. The Gudermannian. If an angle θ is related to a number x, so that

$$\sec\theta = \cosh x,$$

then θ is defined by

$$\theta = \text{gudermannian of } x,$$

or briefly,

$$\theta = \text{gd}\, x = \sec^{-1}(\cosh x).$$

Inversely,

$$x = \text{inverse gudermannian of } \theta,$$

or

$$x = \text{gd}^{-1}\,\theta = \cosh^{-1}(\sec\theta).$$

When

$$\sec\theta = \cosh x,$$

the following relations are true:

$$\cos\theta = \text{sech}\, x, \qquad \sin\theta = \tanh x,$$

$$\tan\theta = \sinh x, \qquad \csc\theta = \coth x,$$

$$\cot\theta = \text{csch}\, x, \qquad \tan\frac{\theta}{2} = \tanh\frac{x}{2}.$$

Any one of these relations defines θ as the gudermannian of x.

PART II

SPHERICAL TRIGONOMETRY

CHAPTER X

GENERAL DEFINITIONS. THE RIGHT TRIANGLE

75. Definitions and Geometric Properties. From spherical geometry we recall the following facts:

(1) The intersection of the surface of a sphere by a plane through its centre is a great circle.

(2) A great circle divides the surface of a sphere into two equal parts called hemispheres.

(3) Two great circles upon the same sphere divide its surface into four parts each of which is called a **Lune**. In Fig. 52, $BCB'AB$ is a lune, bounded by the semi-circumferences BCB' and BAB'.

(4) The angles of a lune are equal, and are measured by the diedral angle between the planes of the great circles forming the sides of the lune. In the drawing, angle B is measured by the diedral angle $CBB'A$. Since tangents to arcs BC, BA at B are perpendicular to the edge of the diedral angle, the spherical angle B is measured by the plane angle between the tangents Bt, Bt' to the arcs BC and BA at B.

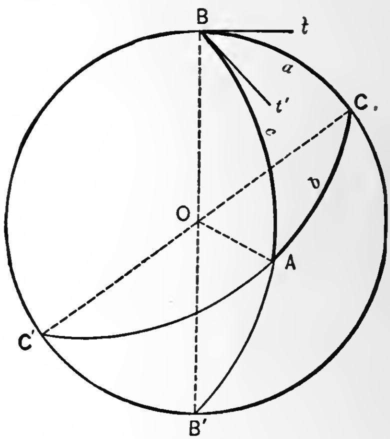


Fig. 52.

(5) Any third great circle will divide a lune into two parts called **spherical triangles**. Thus, arc AC, Fig. 52, divides the lune into the spherical triangles ABC and $AB'C$.

(6) O is the apex of a **spherical pyramid** whose base is the spherical triangle ABC. The face angles of this pyramid are equal to the corresponding sides (measured in degrees) of the spherical triangle; in Fig. 52, $a = \angle BOC$, $b = \angle COA$, $c = \angle AOB$. The diedral angles of the pyramid are equal to the corresponding angles A, B, C, of the spherical triangle.

(7) The length of a side a of a spherical triangle expressed in linear units is given by arc a = radius × radian measure of a. The length of the radius of the sphere is usually not considered in the trigonometric discussion of a spherical triangle.

(8) A side of a spherical triangle lies between 0° and 180°; likewise an angle lies between 0° and 180°. The following limitations should be recognized (see notation, Fig. 52):

(1) $$0° < a + b + c < 360°,$$

(2) $$180° < A + B + C < 540°.$$

(9) The angles and sides of a spherical triangle are in the same order of magnitude; if $a > b > c$, then will $A > B > C$.

76. The Polar Triangle. To any spherical triangle ABC there corresponds another called its polar which may be constructed as follows: take each vertex A, B, C, as a pole and describe arcs (90° away) $B'C'$, $C'A'$, $A'B'$, forming the spherical triangle $A'B'C'$, Fig. 53. The triangle $A'B'C'$ is the polar of the triangle ABC.

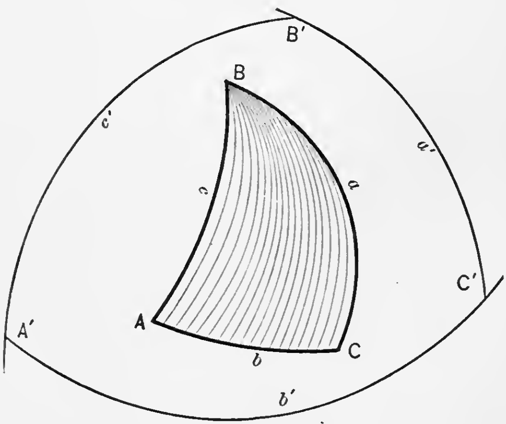


Fig. 53.

In geometry it is shown that the sides and angles of a spherical triangle ABC and the angles and sides of its polar $A'B'C'$ are related by the following equations:

(1) $$A = 180° - a',\ B = 180° - b',\ C = 180° - c',$$

(2) $$a = 180° - A',\ b = 180° - B',\ c = 180° - C'.$$

THEOREM. *The angles of a spherical triangle are the supplements of the corresponding sides of its polar; the sides of the given triangle are the supplements of the corresponding angles of the polar triangle.*

Polar triangles are so related that the vertices A, B, C, of one are poles of the corresponding sides a', b', c' of the other, and the vertices A', B', C' are poles of a, b, c.

THE RIGHT SPHERICAL TRIANGLE

77. Definitions. If one angle C, Fig. 54, be 90°, the triangle ABC is called a *right spherical triangle.* The side c opposite the right angle is called the *hypotenuse.*

(1) The angles A, B, may both be acute, both obtuse, or one acute and the other obtuse.

(2) The sides a, b, lie in the same quadrant as the opposite angles. If $A > 90°$, $a > 90°$; if $B > 90°$, $b > 90°$; if $A < 90°$, $a < 90$; if $B < 90°$, $b < 90°$.

(3) The hypotenuse c is smaller than 90° if A and B are both smaller or both larger than 90°; c is larger than 90° if A is smaller than 90° and B larger than 90°, or inversely. See § 81.

78. Trigonometric Relations. In the spherical triangle ABC, Fig. 54, let $C = 90°$, and let A, B, be acute. To measure the angle A draw at any point A' in the edge of the diedral angle $CAOB$, a plane perpendicular to OA; the traces of this plane upon planes OAC and OAB are the lines $A'C'$, $A'B'$, and the angle $B'A'C'$ equals the angle A. Draw $B'C'$. The following are plane right-angled triangles: $A'C'B'$, $OA'C'$, $OA'B'$, $OC'B'$.

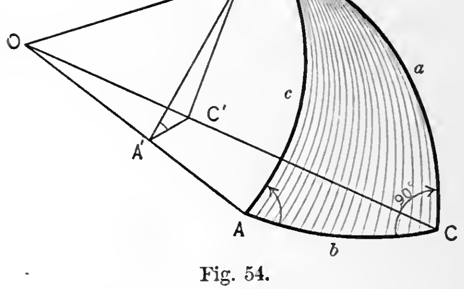


Fig. 54.

Defining sin A, cos A, and tan A from the drawing, we have

$$\sin A = \frac{B'C'}{A'B'} = \frac{B'C'}{A'B'} \times \frac{OB'}{OB'} = \frac{\sin a}{\sin c},$$

$$\cos A = \frac{A'C'}{A'B'} = \frac{A'C'}{A'B'} \times \frac{OA'}{OA'} = \frac{\tan b}{\tan c},$$

$$\tan A = \frac{B'C'}{A'C'} = \frac{B'C'}{A'C'} \times \frac{OC'}{OC'} = \frac{\tan a}{\sin b}.$$

Also, from triangle $B'OA'$,

$$\cos B'OA' = \cos c = \frac{OA'}{OB'} = \frac{OA'}{OB'} \times \frac{OC'}{OC'} = \cos a \cos b.$$

By drawing perpendiculars to the edge OB and constructing a measure of angle B, we could derive similar results for the functions of B, namely :

$$\sin B = \frac{\sin b}{\sin c}, \ \cos B = \frac{\tan a}{\tan c}, \ \tan B = \frac{\tan b}{\sin a}.$$

Taking the reciprocals of tan A and tan B, and multiplying their values together,

$$\cot A \cot B = \frac{\sin b}{\tan a} \times \frac{\sin a}{\tan b} = \cos a \cos b.$$

Hence, $\cos c = \cos a \cos b = \cot A \cot B.$

Again,

$$\cos A = \frac{\tan b}{\tan c} = \frac{\sin b}{\cos b} \times \frac{\cos c}{\sin c} = \frac{\sin b}{\cos b} \times \frac{\cos a \cos b}{\sin c} = \frac{\sin b}{\sin c} \times \cos a$$
$$= \sin B \cos a.$$

Similarly, $\cos B = \sin A \cos b.$

79. Important Formulas. Collecting the results of § 78, we have the following formulas relating to the right spherical triangle :

(1) $\sin A = \dfrac{\sin a}{\sin c}$, (5) $\sin B = \dfrac{\sin b}{\sin c}$,

(2) $\cos A = \dfrac{\tan b}{\tan c}$, (6) $\cos B = \dfrac{\tan a}{\tan c}$,

(3) $\tan A = \dfrac{\tan a}{\sin b}$, (7) $\tan B = \dfrac{\tan b}{\sin a}$,

(4) $\cos A = \sin B \cos a$, (8) $\cos B = \sin A \cos b$,

(9) $\cos c = \cos a \cos b = \cot A \cot B.$

These formulas are true for any right spherical triangle, whether the angles and sides be acute or obtuse.

It should be noticed that the values of sine, cosine, and tangent, formulas (1), (2), (3), (5), (6), (7), are very similar to the definitions of those functions in a plane triangle. This similarity enables one to remember the trigonometric relations of the angles and sides of the right spherical triangle.

A right spherical triangle may be completely solved for all its parts when any two parts, other than the right angle, are known.

80. Napier's Rules of Circular Parts. Another method of remembering the formulas (1) to (9) of § 79 is embraced in what are called *Napier's Rules of Circular Parts.*

Let a right spherical triangle be given with the usual notation, Fig. 54, the right angle being C. Then write a, b, *complement of* A, *complement of* c, *complement of* B, Fig. 55. The notation shown on the drawing consists of five parts, known as *Napier's Circular Parts.*

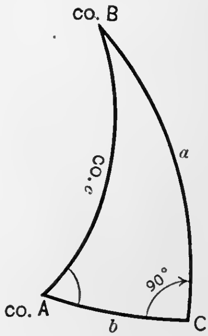


Fig. 55.

If any part as *co. c* be taken as a *middle part*, the next two parts to the right and left, *co. A*, *co. B* are called the *adjacent parts*, and the remaining two parts, a, b, are called the *opposite parts.*

NAPIER'S RULES. (1) *The sine of the middle part is equal to the product of the tangents of the adjacent parts.*

$$\sin(\text{mid. pt.}) = \tan(\text{adj. pt.}) \times \tan(\text{adj. pt.}).$$

(2) *The sine of the middle part is equal to the product of the cosines of the opposite parts.*

$$\sin(\text{mid. pt.}) = \cos(\text{op. pt.}) \times \cos(\text{op. pt.}).$$

As illustration of these rules, identify each of the following with some one of the formulas (1) to (9), § 79:

1. $\sin(co.\ c) = \cos c =$
2. $\sin(a) =$
3. $\sin(co.\ B) = \cos B =$
4. $\sin(b) =$
5. $\sin(co.\ A) = \cos A =$

81. Relative Dimensions of Sides and Angles. (1) Given $a < 90°$, $b < 90°$. In this case,

$$\overset{(+)}{\cos c} = \overset{+}{\cos a}\ \overset{+}{\cos b}$$

shows $\cos c$ *positive;* hence, $c < 90°$.

Also,
$$\overset{(+)}{\cos A} = \frac{\overset{+}{\tan b}}{\overset{+}{\tan c}}, \quad \overset{(+)}{\cos B} = \frac{\overset{+}{\tan c}}{\overset{+}{\tan c}}$$

show $\cos A$ and $\cos B$ *positive;* hence $A < 90°$, $B < 90°$.

(2) Given $a > 90°$, $b < 90°$. Formula (9), § 79,

$$\overset{(-)}{\cos c} = \overset{-}{\cos a}\ \overset{+}{\cos b},$$

shows $\cos c$ *negative;* hence, $c > 90°$, the supplement of the tabular angle determined by formula (9).

Formulas (2), (6), § 79,

$$\overset{(-)}{\cos A} = \frac{\overset{+}{\tan b}}{\overset{-}{\tan c}}, \quad \overset{+}{\cos B} = \frac{\overset{-}{\tan a}}{\overset{-}{\tan c}},$$

show $A > 90°$, $B < 90°$.

(3) Given $a > 90°$, $b > 90°$. Formula (9),

$$\overset{(+)}{\cos c} = \overset{-}{\cos a}\ \overset{-}{\cos b},$$

determines $\cos c$ *positive;* hence $c < 90°$.

Also,
$$\overset{(-)}{\cos A} = \frac{\overset{-}{\tan a}}{\overset{+}{\tan c}}, \quad \overset{(-)}{\cos B} = \frac{\overset{-}{\tan b}}{\overset{+}{\tan c}}$$

show that $\cos A$, $\cos B$ are negative; hence, $A > 90°$, $B > 90°$.

THEOREM. *In any right spherical triangle the hypotenuse c is acute if a and b lie in the same quadrant; c is obtuse if a and b lie in opposite quadrants.*

82. The Isosceles and Quadrantal Triangles. If two sides of a spherical triangle be equal, the opposite angles are equal, and the triangle is *isosceles.* An isosceles spherical triangle may be divided into two symmetrical right triangles, *ADB*, *CDB*, by drawing the arc of a great circle from *B*, Fig. 56, perpendicular upon side *b*. This perpendicular *p* bisects the angle *B* and the opposite side *b*. The solution of an isosceles triangle *ABC* depends upon the solution of the two right triangles *ADB*, *CDB*.

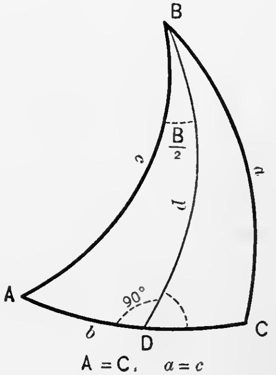


Fig. 56.

If a side be 90°, the triangle is called a *quadrantal spherical triangle.* The solution of a quadrantal triangle may be made to depend upon the solution of a right spherical triangle by taking the polar triangle, one of whose angles will be 90°, the supplement of the given side. See § 76.

To solve a quadrantal triangle, solve its polar and take the supplement of each part of the polar.

83. Solution of Right Spherical Triangles. Six cases arise in the solution of right spherical triangles.

Case I. Given two sides, *a*, *b*.

Case II. Given one side and the hypotenuse, *a*, *c*.

Case III. Given two angles, *A*, *B*.

Case IV. Given one angle and the adjacent side, *A*, *b*.

Case V. Given one angle and the opposite side, *A*, *a*. Double solution.

Case VI. Given one angle and the hypotenuse, *A*, *c*.

That Case V has a double solution may be seen from the drawing, Fig. 57.

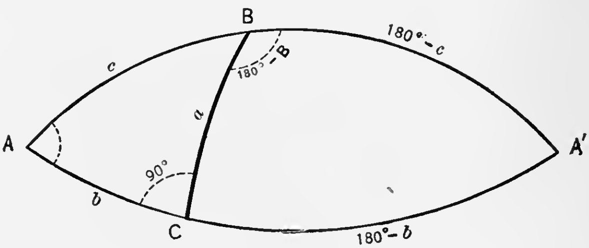


Fig. 57.

In calculating the unknown parts of a triangle falling under any one of the above Cases, (1) either select a proper set of formulas, § 79, or employ Napier's Rules to obtain the required trigonometric relations; (2) use logarithms in performing the operations of multiplication and division.

EXERCISES

Solve the following right spherical triangles for each unknown part:

1. $a = 64° 20'$, $b = 70° 24'$.
2. $a = 49° 28'$, $c = 65° 25'$.
3. $b = 100° 10'$, $A = 38° 47'$.
4. $a = 120° 30' 10''$, $B = 98° 27' 10''$.
5. $c = 110° 30' 10''$, $A = 46° 21' 30''$.
6. $A = 41° 48'$, $B = 80° 30'$.
7. $b = 140° 28'$, $c = 110° 29'$.
8. $b = 72° 20' 10''$, $B = 80° 10' 20''$.
9. $a = 130° 20'$, $b = 105° 48'$.
10. $A = 98° 25'$, $B = 104° 10'$.

11. Find the perimeter of the right triangle in which $a = 48°$, $b = 60°$, the radius of the sphere being 15 in.

12. Find the area of the right spherical triangle determined by $a = 36°$, $A = 36°$, the radius of the sphere being 30 in.

NOTE. The surface of a sphere = $4\pi R^2$.

13. If a spherical triangle have two right angles, show that the opposite sides are quadrants, and explain a method for finding its area when the radius is known.

14. Solve the isosceles triangles:

(1) $A = B = 54° 28'$, $C = 68°$.
(2) $A = 75° 34'$, $a = 38° 28'$, $b = c$.
(3) $B = 98° 24'$, $a = c = 124°$.

15. Solve the quadrantal triangles :

(1) $a = 90°$, $B = 48°$, $C = 38°$.
(2) $A = 100°$, $c = 98°$, $b = 90°$.

16. A vessel sails directly east from Sandy Hook (latitude, 40° 28′ N.; longitude, 74° 1′ W.) and continues upon the arc of a great circle. Find the latitude at which it crosses the meridian of 65° W. What will be the course of the vessel when crossing that meridian?

NOTE. See § 91 for definitions.

17. If a vessel sails directly west along an arc of a great circle from San Francisco (latitude, 37° 48′ N.; longitude, 122° 28′ W.), what distance will it have sailed and what direction will be its course on arriving at the 180th meridian?

NOTE. Take the radius of the earth as 3956 mi.

18. A ship sails due east from Boston (latitude, 42° 21′ N.; longitude, 71° 4′ W.) upon the arc of a great circle at the rate of 16 knots per hour. Find the latitude and longitude of the ship after 48 hours' sailing.

NOTE. 1 knot = 1 nautical mile = 1 geographical mile = 1′ arc of a great circle upon the earth.

19. A triangular pyramid $O\text{-}ABC$ has the diedral angle along the edge $OC = 90°$, and the face angles $AOC = 48°$, and $BOC = 69°$. Find the face angle BOA, and the diedral angles along the edges OA and OB.

20. If the three edges of a cube through a point O are OA, OB, and OC, find the diedral angle made by the plane ABC with either face of the cube.

CHAPTER XI

THE OBLIQUE SPHERICAL TRIANGLE

84. The Theorem of Sines. Let a spherical triangle be represented by ABC, Fig. 58. If no angle be a right angle, the triangle is called *oblique*.

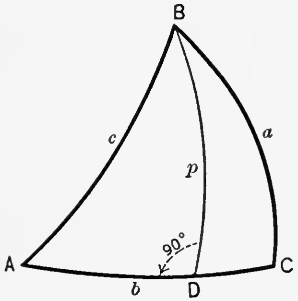


Fig. 58.

Draw through angle B the arc of a great circle to meet the side b at right angles in D. Then ADB, CDB are right spherical triangles. From § 79,

$$\sin A = \frac{\sin p}{\sin c}, \quad \sin C = \frac{\sin p}{\sin a}.$$

Dividing, we have,

$$\frac{\sin A}{\sin C} = \frac{\sin a}{\sin c}.$$

By drawing an arc through A perpendicular to a, we may derive similarly,

$$\frac{\sin C}{\sin B} = \frac{\sin c}{\sin b}.$$

Rearranging these two formulas,

$$(1) \qquad \frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$$

THEOREM OF SINES. *In any spherical triangle the sines of the angles are proportional to the sines of the opposite sides.*

Observe the similarity to the corresponding theorem in plane trigonometry.

The theorem of sines enables us to compute a fourth part, when two angles and a side opposite one of the angles are known, or when two sides and an angle opposite one of the given sides are known.

85. The Theorem of Cosines. Sides and One Angle. The angle B in the triangle ABC, Fig. 59, is measured by the plane angle between the tangents to AB and CB at B. Let these tangents intersect the radii OA and OC produced in L and M.

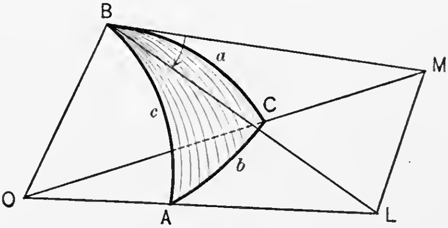


Fig. 59.

Then from the plane triangles LOM and LBM, we have

$$\overline{LM}^2 = \overline{LO}^2 + \overline{MO}^2 - 2\,LO \times MO \times \cos b,$$
$$\overline{LM}^2 = \overline{LB}^2 + \overline{MB}^2 - 2\,LB \times MB \times \cos B$$

Subtracting,

$$0 = \overbrace{\overline{LO}^2 - \overline{LB}^2} + \overbrace{\overline{MO}^2 - \overline{MB}^2} - 2\,LO \times MO \times \cos b + 2\,LB \times MB \times \cos B,$$

or,

$$0 = \overline{OB}^2 + \overline{OB}^2 - 2\,LO \times MO \times \cos b + 2\,LB \times MB \times \cos B.$$

Transposing the negative term and dividing by $2\,LO \times MO$, we find,

$$\cos b = \frac{\overline{OB}^2}{LO \times MO} + \frac{LB}{LO} \times \frac{MB}{MO} \times \cos B,$$

and substituting the ratios from the right triangles LBO, MBO,

(2) $$\mathbf{\cos b = \cos c \cos a + \sin c \sin a \cos B.}$$

Making symmetrical interchanges of the letters,

(3) $$\mathbf{\cos c = \cos a \cos b + \sin a \sin b \cos C.}$$

(4) $$\mathbf{\cos a = \cos b \cos c + \sin b \sin c \cos A.}$$

THEOREM OF COSINES. *In any spherical triangle, the cosine of any side equals the product of the cosines of the other sides increased by the product of their sines multiplied by the cosine of the included angle.*

Formulas (2), (3), or (4) will determine a side when the other two sides and their included angle are given; or they will determine the angles when the three sides are given. These formulas are not adapted to logarithmic calculation.

MODIFIED FORMULA. If one side only be required, with two sides and the included angle given, it is sometimes convenient to modify formulas (2), (3), (4) so as to permit logarithmic calculation.

For example, let b, c, A be given, then a is determined by (4),

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

Let

$$\tan \theta = \frac{\cos b}{\sin b \cos A} = \frac{\cot b}{\cos A}; \tag{A}$$

then,

$$\sin \theta = \frac{\cos b}{(\cos^2 b + \sin^2 b \cos^2 A)^{\frac{1}{2}}}.$$

With this substitution, $\cos a$ may be written

$$\cos a = \frac{\cos b}{\sin \theta} \sin(\theta + c). \tag{B}$$

To find side a, first find θ by equation (A), then substitute θ in (B) and determine a. Use logarithms.

86. The Theorem of Cosines. Angles and One Side. By substituting in

$$\cos b = \cos c \cos a + \sin c \sin a \cos B, \qquad \text{§ 85 (2)}$$

the values

$$b = 180° - B', \ c = 180° - C', \ a = 180° - A', \ B = 180° - b',$$

where B', C', A', b' belong to the polar triangle, we find

$$\cos(180° - B') = \cos(180° - C')\cos(180° - A') + \sin(180° - C')\sin(180° - A')\cos(180° - b');$$

reducing, and changing signs,

$$\cos B' = -\cos C' \cos A' + \sin C' \sin A' \cos b'.$$

Omitting accents, we have

(5) $$\mathbf{\cos B = -\cos C \cos A + \sin C \sin A \cos b.}$$

Formulas (3) and (4), § 85, by similar substitutions become

(6) $$\mathbf{\cos C = -\cos A \cos B + \sin A \sin B \cos c.}$$

(7) $$\mathbf{\cos A = -\cos B \cos C + \sin B \sin C \cos a,}$$

87. The Half-angle Formulas. By transformations similar to those employed in § 50, formulas (2), (3), (4), § 85, and (5), (6), (7), § 86, may be changed so that logarithmic computation may be used to determine the angles of a spherical triangle in terms of the sides, or the sides in terms of the angles.

(1) *To derive tan $\frac{1}{2}B$, tan $\frac{1}{2}C$, tan $\frac{1}{2}A$.*

Taking (2), § 85,

$$\cos b = \cos c \cos a + \sin c \sin a \cos B,$$

and solving for cos B, we find

$$(A) \qquad \cos B = \frac{\cos b - \cos c \cos a}{\sin c \sin a}.$$

Subtracting both members of equation (A) from unity, and recalling that

$$1 - \cos B = 2 \sin^2 \tfrac{1}{2} B, \qquad \text{§ 37}$$

we have

$$2 \sin^2 \tfrac{1}{2} B = \frac{\cos c \cos a + \sin c \sin a - \cos b}{\sin c \sin a}$$

$$= \frac{\cos (c - a) - \cos b}{\sin c \sin a}, \qquad \text{§ 34}$$

$$= \frac{2 \sin \frac{1}{2}(b + c - a) \sin \frac{1}{2}(a + b - c)}{\sin c \sin a}. \qquad \text{§ 38 (22)}$$

Dividing by 2, substituting

$$a + b + c = 2s, \quad b + c - a = 2(s - a), \quad a + b - c = 2(s - c),$$

and extracting the square root,

$$(B) \qquad \sin \tfrac{1}{2} B = \sqrt{\frac{\sin (s - a) \sin (s - c)}{\sin c \sin a}}.$$

Adding unity to both members of equation (A), we have

$$1 + \cos B = 2 \cos^2 \tfrac{1}{2} B = \frac{\cos b - (\cos c \cos a - \sin c \sin a)}{\sin c \sin a}$$

$$= \frac{\cos b - \cos (c + a)}{\sin c \sin a}$$

$$= \frac{2 \sin \frac{1}{2}(a + b + c) \sin \frac{1}{2}(c + a - b)}{\sin c \sin a}.$$

Hence,

$$(C) \qquad \cos \tfrac{1}{2} B = \sqrt{\frac{\sin s \sin (s - b)}{\sin c \sin a}}.$$

Dividing (B) by (C), and writing symmetric results for the angles C and A, we find:

$$(8)\qquad \tan\tfrac{1}{2}B = \sqrt{\frac{\sin(s-a)\sin(s-c)}{\sin s\sin(s-b)}} = \frac{k}{\sin(s-b)},$$

$$(9)\qquad \tan\tfrac{1}{2}C = \sqrt{\frac{\sin(s-b)\sin(s-a)}{\sin s\sin(s-c)}} = \frac{k}{\sin(s-c)},$$

$$(10)\qquad \tan\tfrac{1}{2}A = \sqrt{\frac{\sin(s-c)\sin(s-b)}{\sin s\sin(s-a)}} = \frac{k}{\sin(s-a)},$$

where k is defined by

$$k = \sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}.$$

Formulas (8), (9), (10) are well adapted to logarithmic calculations of the angles when the sides are given.

(2) *To derive* $\tan\frac{1}{2}b$, $\tan\frac{1}{2}c$, $\tan\frac{1}{2}a$. Values for $\tan\frac{1}{2}b$, $\tan\frac{1}{2}c$, $\tan\frac{1}{2}a$, may be derived directly from (5), (6), (7), § 86, by manipulation similar to that employed for $\tan\frac{1}{2}B$, $\tan\frac{1}{2}C$, $\tan\frac{1}{2}A$.

Another method consists in transforming (8), (9), and (10) directly into the required values of $\tan\frac{1}{2}b$, $\tan\frac{1}{2}c$, $\tan\frac{1}{2}a$ by means of the polar triangle. In (8) let us substitute

$$B = 180° - b',\; a = 180° - A',\; b = 180° - B',\; c = 180° - C',$$

$$\tfrac{1}{2}B = 90° - \tfrac{1}{2}b',\; s = \tfrac{1}{2}(a+b+c) = 270° - S,$$

$$s - b = 90° - (S - B'),\; s - c = 90° - (S - C'),$$

$$s - a = 90° - (S - A'),$$

where

$$S = \tfrac{1}{2}(A' + B' + C') > 90.$$

Then we have

$$\tan(90° - \tfrac{1}{2}b') = \sqrt{\frac{\sin[(90° - (S - A')]\sin[90° - (S - C')]}{\sin(270° - S)\sin[90° - (S - B')]}}$$

or,

$$\cot\tfrac{1}{2}b' = \sqrt{\frac{\cos(S - A')\cos(S - C')}{-\cos S\cos(S - B')}},$$

with similar expressions for $\cot\frac{1}{2}c'$, and $\cot\frac{1}{2}a'$.

Taking the reciprocal, and omitting accents, we have

$$(11)\quad \tan\tfrac{1}{2}b = \sqrt{\frac{-\cos S\cos(S-B)}{\cos(S-A)\cos(S-C)}} = K\cos(S-B),$$

(12) $$\tan\frac{1}{2}c = \sqrt{\frac{-\cos S\cos(S-C)}{\cos(S-B)\cos(S-A)}} = K\cos(S-C),$$

(13) $$\tan\frac{1}{2}a = \sqrt{\frac{-\cos S\cos(S-A)}{\cos(S-C)\cos(S-B)}} = K\cos(S-A),$$

where

$$K = \sqrt{\frac{-\cos S}{\cos(S-A)\cos(S-B)\cos(S-C)}}.$$

NOTE. It may be shown that k, used in (8), (9), (10), is equal to the tangent of the radius of the *small* circle inscribed in the spherical triangle ABC, and that K, used in (11), (12), (13), is equal to the tangent of the radius of the *small* circle circumscribing the triangle ABC.

88. Napier's Analogies. Dividing $\tan\frac{1}{2}B$ by $\tan\frac{1}{2}C$, formulas (8), (9), § 87, we have

$$\frac{\tan\frac{1}{2}B}{\tan\frac{1}{2}C} = \frac{\sin(s-c)}{\sin(s-b)}.$$

Taking this equality by composition and division,

$$\frac{\tan\frac{1}{2}B+\tan\frac{1}{2}C}{\tan\frac{1}{2}B-\tan\frac{1}{2}C} = \frac{\sin(s-c)+\sin(s-b)}{\sin(s-c)-\sin(s-b)},$$

which reduces to

$$\frac{\sin\frac{1}{2}(B+C)}{\sin\frac{1}{2}(B-C)} = \frac{\sin\frac{1}{2}(2s-b-c)\cos\frac{1}{2}(b-c)}{\cos\frac{1}{2}(2s-b-c)\sin\frac{1}{2}(b-c)},$$

and finally,

(14) $$\frac{\sin\frac{1}{2}(B+C)}{\sin\frac{1}{2}(B-C)} = \frac{\tan\frac{1}{2}a}{\tan\frac{1}{2}(b-c)}.$$

Similarly,

(15) $$\frac{\sin\frac{1}{2}(C+A)}{\sin\frac{1}{2}(C-A)} = \frac{\tan\frac{1}{2}b}{\tan\frac{1}{2}(c-a)},$$

(16) $$\frac{\sin\frac{1}{2}(A+B)}{\sin\frac{1}{2}(A-B)} = \frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a-b)}.$$

Another formula may be obtained by multiplying $\tan\frac{1}{2}B$ by $\tan\frac{1}{2}C$.

$$\tan\frac{1}{2}B\tan\frac{1}{2}C = \frac{\sin(s-a)}{\sin s}.$$

Write $\tan \frac{1}{2} B$ and $\tan \frac{1}{2} C$ in terms of sines and cosines, and take the equation by division and composition,

$$\frac{\cos \frac{1}{2} B \cos \frac{1}{2} C - \sin \frac{1}{2} B \sin \frac{1}{2} C}{\cos \frac{1}{2} B \cos \frac{1}{2} C + \sin \frac{1}{2} B \sin \frac{1}{2} C} = \frac{\sin s - \sin (s-a)}{\sin s + \sin (s-a)},$$

and reduce, giving

(17) $$\frac{\cos \frac{1}{2}(B+C)}{\cos \frac{1}{2}(B-C)} = \frac{\tan \frac{1}{2} a}{\tan \frac{1}{2}(b+c)}.$$

Similarly,

(18) $$\frac{\cos \frac{1}{2}(C+A)}{\cos \frac{1}{2}(C-A)} = \frac{\tan \frac{1}{2} b}{\tan \frac{1}{2}(c+a)},$$

(19) $$\frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2} c}{\tan \frac{1}{2}(a+b)}.$$

The formulas (14) to (19) are known as *Napier's Analogies.* Numbers (14) and (17) enable us to compute the sides b, c, when the opposite angle B, C, and the included side a are known. The Theorem of Sines, § 84, then determines the remaining angle A.

Napier's Analogies appear also in another form which may be derived by means of the polar triangle.

By substituting $a = 180° - A'$, $A = 180° - a'$, and so on, let the student deduce from (14) to (19) the following:

(20) $$\frac{\sin \frac{1}{2}(b+c)}{\sin \frac{1}{2}(b-c)} = \frac{\cot \frac{1}{2} A}{\tan \frac{1}{2}(B-C)},$$

(21) $$\frac{\sin \frac{1}{2}(c+a)}{\sin \frac{1}{2}(c-a)} = \frac{\cot \frac{1}{2} B}{\tan \frac{1}{2}(C-A)},$$

(22) $$\frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A-B)},$$

and

(23) $$\frac{\cos \frac{1}{2}(b+c)}{\cos \frac{1}{2}(b-c)} = \frac{\cot \frac{1}{2} A}{\tan \frac{1}{2}(B+C)},$$

(24) $$\frac{\cos \frac{1}{2}(c+a)}{\cos \frac{1}{2}(c-a)} = \frac{\cot \frac{1}{2} B}{\tan \frac{1}{2}(C+A)},$$

(25) $$\frac{\cos\frac{1}{2}(a+b)}{\cos\frac{1}{2}(a-b)}=\frac{\cot\frac{1}{2}C}{\tan\frac{1}{2}(A+B)}.$$

89. The Area of a Spherical Triangle. The sum of the three angles of a spherical triangle is greater than 180°. If A, B, C be the angles,

$$A+B+C-180^\circ=E$$

determines the *spherical excess* of the triangle.

In spherical geometry it is shown that the areas of spherical triangles are to each other as their spherical excesses. A tri-rectangular triangle has an area $\frac{1}{2}\pi R^2$ and its excess is 90°. Hence, if $\triangle$ denote the area of any spherical triangle whose spherical excess is E°, and R be the radius of the sphere, we have

$$\triangle : \tfrac{1}{2}\pi R^2 :: E : 90,$$

or

$$\triangle=\frac{\pi R^2}{180}\times E.$$

The spherical excess E is readily found when the angles of a spherical triangle are known.

The value of E may be determined directly when the three sides are known. The resulting formula is known as *L'Huilier's Theorem*, and is given by

$$\tan\frac{E}{4}=\sqrt{\tan\tfrac{1}{2}s\tan\tfrac{1}{2}(s-a)\tan\tfrac{1}{2}(s-b)\tan\tfrac{1}{2}(s-c)}.$$

The derivation of this formula will not be given here.

90. Solution of Oblique Spherical Triangles. When any three parts of a spherical triangle are given, the remaining parts may be found by use of the Theorem of Sines, the Half-angle Formulas, or by Napier's Analogies. Six cases occur.

Case I. *Given the three sides, a, b, c.* To determine the angles A, B, C, we use the half-angle formulas

$$\tan\tfrac{1}{2}A=\frac{k}{\sin(s-a)},\quad \tan\tfrac{1}{2}B=\frac{k}{\sin(s-b)},\quad \tan\tfrac{1}{2}C=\frac{k}{\sin(s-c)},$$

where $$k=\sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}.$$

Employ logarithms in the calculation, tabulating work after the form suggested in the solution of plane triangles.

Case II. *Given the three angles, A, B, C.* Determine the sides a, b, c by use of

$$\tan \tfrac{1}{2} a = K \cos (S - A),$$
$$\tan \tfrac{1}{2} b = K \cos (S - B),$$
$$\tan \tfrac{1}{2} c = K \cos (S - C),$$

where
$$K = \sqrt{\frac{-\cos S}{\cos (S - A) \cos (S - B) \cos (S - C)}}.$$

In this case $\cos S$ is always negative.

Case III. *Given two sides and the included angle, c, a, B.* To find C and A use Napier's Analogies:

$$\tan \tfrac{1}{2} (C + A) = \frac{\cos \frac{1}{2} (c - a)}{\cos \frac{1}{2} (c + a)} \cot \tfrac{1}{2} B,$$

$$\tan \tfrac{1}{2} (C - A) = \frac{\sin \frac{1}{2} (c - a)}{\sin \frac{1}{2} (c + a)} \cot \tfrac{1}{2} B.$$

In the first of these formulas $\cot \frac{1}{2} B$ is positive, $\cos \frac{1}{2} (c - a)$ is positive; if $\cos \frac{1}{2} (c + a)$ be positive, the $\tan \frac{1}{2} (C + A)$ is positive, and $\frac{1}{2} (C + A) < 90°$. If $\cos \frac{1}{2} (c + a)$ be negative, the $\tan \frac{1}{2} (C + A)$ is negative, and $\frac{1}{2} (C + A) > 90°$.

After the angles C and A have been found, the side b may be determined by means of the Theorem of Sines,

$$\sin b = \frac{\sin a \sin B}{\sin A}.$$

In using the Theorem of Sines care must be exercised in determining whether b lies in the first or second quadrant.

Case IV. *Given two angles and the included side, B, C, a.* Formulas (17) and (14) of Napier's Analogies determine b and c.

$$\tan \tfrac{1}{2} (b + c) = \frac{\cos \frac{1}{2} (B - C)}{\cos \frac{1}{2} (B + C)} \tan \tfrac{1}{2} a,$$

$$\tan \tfrac{1}{2} (b - c) = \frac{\sin \frac{1}{2} (B - C)}{\sin \frac{1}{2} (B + C)} \tan \tfrac{1}{2} a.$$

The first of these formulas shows that if $\frac{1}{2}(B + C) > 90°$, then will $\frac{1}{2} (b + c) > 90°$, and inversely. The angle A may be determined by the Theorem of Sines.

CASE V. *Given two sides and an angle opposite one of them,* b, c, B. Here, the angle C is given by

$$\sin C = \frac{\sin c \sin B}{\sin b}.$$

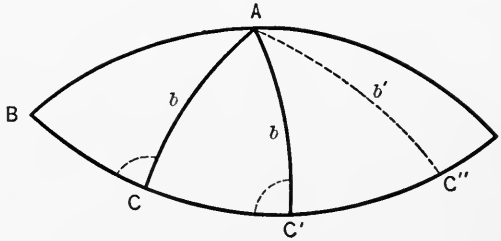


Fig. 60.

(1) If $\sin c \sin B > \sin b$, no solution exists.

(2) If $\sin c \sin B = \sin b$, the angle $C = 90°$.

(3) If $\sin c \sin B < \sin b$, either one or two solutions will occur; see Fig. 60.

After C has been found, angle A and side a may be obtained from

$$\cot \tfrac{1}{2} A = \frac{\cos \frac{1}{2}(b+c)}{\cos \frac{1}{2}(b-c)} \tan \tfrac{1}{2}(B+C),$$

$$\tan \tfrac{1}{2} a = \frac{\cos \frac{1}{2}(B+C)}{\cos \frac{1}{2}(B-C)} \tan \tfrac{1}{2}(b+c).$$

CASE VI. *Given two angles and a side opposite one of them,* B, C, b. The side c is given by

$$\sin c = \frac{\sin b \sin C}{\sin B}.$$

This case may also have no solution, one solution, or two solutions. After c has been determined the remaining parts, a, A, may be derived from Napier's Analogies, as in Case V.

EXERCISES

Solve the following spherical triangles for the unknown parts:

1. $A = 100°$, $B = 75°$, $C = 65°$.
2. $a = 27° 40'$, $b = 48°$, $c = 50° 40'$.
3. $a = 65° 48'$, $b = 120° 21'$, $c = 84° 21'$.
4. $A = 96° 50'$, $B = 75° 10'$, $C = 96° 50'$.
5. $B = 72° 30'$, $A = 41° 27'$, $c = 49° 17'$.
6. $b = 118° 48'$, $c = 71° 24'$, $A = 51° 24'$.
7. $a = 72° 48' 12''$, $b = 41° 38' 10''$, $C = 33° 24' 10''$.

8. $B = 129° 24' 10''$, $C = 31° 24' 30''$, $a = 42° 50' 40''$.

9. $a = 112° 40' 15''$, $b = 121° 10' 10''$, $C = 128° 44' 15''$.

10. $A = 100° 14' 20''$, $B = 140° 40' 40''$, $c = 39° 28' 15''$.

11. $b = 54° 21'$, $c = 31° 48'$, $B = 100° 10'$.

12. $A = 101° 40'$, $c = 62° 21'$, $a = 104° 24'$.

13. $b = 40° 16' 10''$, $c = 46° 48' 40''$, $B = 56° 21' 50''$.

14. $a = 37° 10' 20''$, $b = 112° 48'$, $A = 34° 28' 15''$.

15. $C = 123° 38'$, $b = 150° 40'$, $c = 10° 21' 30''$.

16. $A = 140° 29'$, $C = 24° 40'$, $a = 135° 40'$.

17. $B = 128° 40'$, $C = 141° 25'$, $c = 125° 47'$.

18. $A = 48° 37'$, $C = 34° 48'$, $c = 41° 10'$.

19. $B = 99° 27'$, $C = 89° 21'$, $a = 45° 37'$.

20. $a = 71° 47'$, $b = 145° 47'$, $C = 130° 50'$.

21. When $b = 37° 28'$, $c = 65° 21'$, and $A = 87° 40'$, find side a directly by *Modified Formula* (B), § 85.

22. Find c directly, (B) § 85, when $a = 101° 48'$, $b = 100°$, $C = 68° 41'$.

23. Find in inches the perimeter of a spherical triangle with sides 68°, 96°, 120°, on a sphere whose radius is 15 in.

24. Find the perimeter of a spherical triangle with angles 69°, 84°, 100°, upon a sphere whose radius is 10 in.

25. Find the area of the spherical triangle with angles 120°, 84°, 72°, upon a sphere whose radius is 20 in.

Find the areas of the following spherical triangles:

26. $A = 24° 30'$, $B = 140°$, $C = 80°$, radius = 12 in.

27. $a = 65°$, $b = 47°$, $c = 60°$, $r = 15$ in.

28. $A = 100°$, $B = 80°$, $c = 60°$, $r = 12$ in.

29. $b = 37°$, $c = 84°$, $A = 65°$, $r = 3956$ mi.

30. $B = 104°$, $C = 128°$, $b = 138°$, $r = 3956$ mi.

APPLICATIONS OF SPHERICAL TRIGONOMETRY

91. The Earth as a Sphere. Definitions and Notation. In what follows the earth is assumed to be a sphere with a radius of 3956 mi. The *shortest distance* between two points upon the earth is the length of the shorter arc of a great circle drawn through the two points.

(1) *The Geographical Mile and Statute Mile.* The *geographical mile* (called *nautical mile*, also *knot*) is a unit of distance along the arc of a great circle upon the earth; it is the length of an arc of one minute of a great circle, hence sixty geographical miles equal one degree. The *statute mile* is our ordinary unit of measurement equivalent to 5280 ft. The length of one degree of the arc of a great circle of the earth is given by

$$1° = \frac{\pi \times 3956}{180} \text{ mi.}, \ \pi = 3.14159.$$

(2) *Meridian, Longitude, Latitude.* A great circle drawn through the poles N, S, and through any point B, Fig. 61, is called the *Meridian* of the point B.

Meridians are numbered to the east and west with reference to some *zero meridian.* The meridian passing through Greenwich (near London), England, is usually taken as the zero meridian, $NGPS$, Fig. 61. The meridian of Greenwich intersects the earth's *equator* $ELMW$ in P.

The *Longitude* of any point B upon the earth is the number of degrees between the *zero meridian* and the *meridian through* B. This angle is measured by the arc of the equator PEM, or the spherical angle GNB. The longitude of F is the arc PEL, or the angle GNF.

The meridians are numbered by degrees, minutes, and seconds from 0° to 180° *east* (marked E.) and *west* (marked W.) of the *Greenwich* or *zero meridian.* Thus the point B is in W. longitude, also the point F.

The *Latitude* of a point B is the arc of the meridian of the point intercepted by the equator and the point. The *latitude* of B is the arc MB. Latitude is counted *positive* or *negative* according as the point is north or south of the equator; the direction is usually indicated by attaching N. or S. to the number of degrees in the latitude.

As samples of the notation of latitude and longitude we may write:

(1)	New York,	lat. 40° 43′ N., long. 74° W.
(2)	Boston,	lat. 42° 21′ N., long. 71° 4′ W.
(3)	Greenwich,	lat. 51° 29′ N., long. 0°.
(4)	Liverpool,	lat. 53° 24′ N., long. 3° 4′ W.
(5)	San Francisco,	lat. 37° 48′ N., long. 122° 28′ W.
(6)	Valparaiso,	lat. 33° 2′ S., long. 71° 41′ W.
(7)	Calcutta,	lat. 22° 33′ N., long. 88° 19′ E.
(8)	Sandy Hook,	lat. 40° 28′ N., long. 74° 1′ W.

92. The Terrestrial Triangle. When the *latitude* and *longitude* of any two points upon the earth are known, the distance

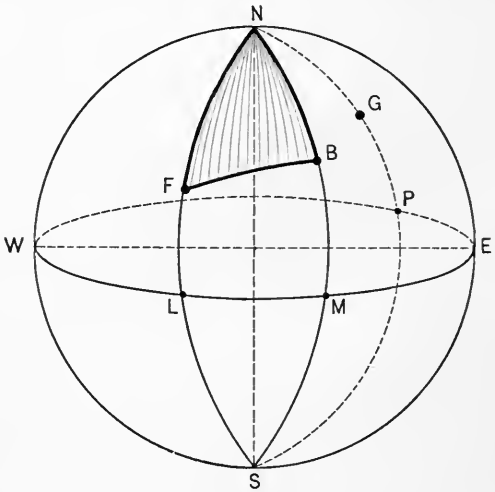


Fig. 61.

between the points may be found. Thus, in Fig. 61, if the point B be in lat. $\alpha°$ N., long. $\beta°$ W., and F be in lat. $\alpha_1°$ N., long. $\beta_1°$ W., the spherical triangle NFB is completely determined as follows:

arc $NB = 90^\circ - \alpha^\circ$ **= complement of the latitude of** B**,**

arc $NF = 90^\circ - \alpha_1{}^\circ$ **= complement of the latitude of** F**,**

angle $BNF = \beta_1{}^\circ - \beta^\circ$ **= difference in longitude of** B **and** F**.**

This data gives *two sides and the included angle of the triangle NFB.*

(1) *To find the distance between two points whose latitude and longitude are given.* If the distance BF, Fig. 61, alone be required, we have to solve the spherical triangle BNF for BF. This may be done by the use of (B), § 85, in which logarithmic calculation may be employed.

$$\cos BF = \frac{\cos NF}{\sin \theta} \times \sin(\theta + NB) = \frac{\sin \alpha_1}{\sin \theta} \times \cos(\alpha - \theta),$$

$$\tan \theta = \frac{\cot NF}{\cos BNF} = \frac{\tan \alpha_1}{\cos(\beta_1 - \beta)}.$$

The arc BF may also be obtained by use of the Cosine Theorem (2), § 85,

$$\cos BF = \cos NB \cos NF + \sin NB \sin NF \cos BNF,$$
$$= \sin \alpha \sin \alpha_1 + \cos \alpha \cos \alpha_1 \cos(\beta_1 - \beta),$$

but the use of this formula is not to be recommended except perhaps when the angles are such that the products $\sin \alpha \sin \alpha_1$, $\cos \alpha \cos \alpha_1 \cos(\beta_1 - \beta)$ reduce readily by ordinary multiplication.

(2) *To find the bearing and distance of two points whose latitude and longitude are known.* The angle NBF, Fig. 61, is the *bearing* of F from B; the angle NFB is the *bearing* of B from F. The bearing of a course at a given period is usually considered as the *smaller* angle which that course makes with the meridian through the point. Thus in the figure, the bearing of F from B is N. γ° W., if $\gamma = \angle NBF < 90^\circ$; the bearing of F from B is S. $\gamma_1{}^\circ$ W., if $\gamma > 90^\circ$ and $\gamma_1 = \angle SBF = 180^\circ - \gamma$.

When the bearing, or bearing and distance, of two points whose latitude and longitude are given, is required, Napier's Analogies, (20), (23), § 88, may be used to find the angles NBF and NFB. The side BF may then be found by use of the Theorem of Sines, (1), § 84.

EXERCISES

1. Find the distance in knots between Boston, lat. 42° 21′ N., long. 71° 4′ W., and Liverpool, lat. 53° 24′ N., long. 3° 4′ W. Find also the bearing of each port from the other.

2. Find the distance in statute miles along the arc of a great circle from New York, lat. 40° 43′ N., long. 74° W., to San Francisco, lat. 37° 48′ N., long. 122° 28′ W.

3. Find the distance in nautical miles from New York, lat. 40° 43′ N., long. 74° W., to Greenwich, lat. 51° 29′ N., and the bearing of each point from the other.

4. If a vessel sails directly east from Sandy Hook, lat. 40° 28′ N., long. 74° 1′ W., along the arc of a great circle at the uniform rate of 16 knots per hour, find its latitude and longitude (1) at the end of 48 hours' sailing, (2) at the end of five days' sailing.

5. Find the distance in statute miles from San Francisco, lat. 37° 48′ N., long. 122° 28′ W., to Calcutta, lat. 22° 33′ N., long. 88° 19′ E., and the bearing of each point from the other.

93. The Celestial Sphere. Astronomical problems furnish many applications of spherical trigonometry. One class of these problems will be noticed here.

The daily rotation of the earth upon its axis from *west* to *east* causes the stars to *seem* to rotate from *east* to *west* upon the surface of an immense sphere named the *celestial sphere.* To a person located at any point upon the earth one-half of this sphere is visible. The celestial sphere is represented in Fig. 62, the earth being a mere point at the centre.

(1) *The Horizon* of any point upon the earth is the intersection of the horizontal plane through the point with the celestial sphere. In Fig. 62, the horizon is the great circle HLH'.

(2) *The Zenith* of any point is the intersection of the perpendicular erected to the plane of the horizon at the point. The point on the celestial sphere diametrically opposite the zenith is the *Nadir.* In the figure Z is the zenith, Z' is the nadir.

(3) *The Celestial Poles* are the intersections of the line of the earth's axis with the celestial sphere. The point N is the

north pole of the celestial sphere, S is its south pole. The celestial sphere rotates (apparently) about the axis NS once in 24 hours.

(4) *The Celestial Equator* is the intersection of the plane of the earth's equator with the celestial sphere. In the figure, EME' is the celestial equator, N and S are its poles.

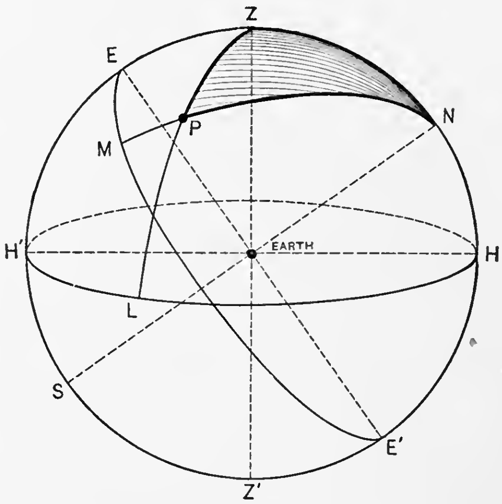


Fig. 62.

(5) *The Celestial Meridians* are the great circles through N and S. The celestial meridian through any star is called the *hour circle* of that star. Thus, NPM is the hour circle of P.

94. The Celestial Triangle. The position P of any star is known if we know its distance PL above the horizon, and its distance PM north or south of the celestial equator.

(1) *The Altitude* PL of a star at P is the arc of a zenith circle ZL intercepted between the point P and the horizon. See Fig. 62.

(2) *The Declination* of a star P is the arc PM of the celestial meridian NPM between the point and the celestial equator.

(3) *The Latitude of the Zenith* of any point on the earth is the arc EZ and is equal to the latitude of the observer.

(4) *The Hour Angle* of any star is the angle between the

meridian through the star and the zenith meridian. In the figure PNZ is the hour angle.

Since the celestial sphere apparently rotates through 360° in 24 hr., or 15° in one hour of time, we may express the hour angle in hours, minutes, and seconds, and thus determine the time required for a star to rotate into the meridian of the zenith.

The points P, N, Z determine the celestial triangle PNZ, sometimes called the astronomical triangle. This triangle is determined when the *latitude of the observer* EZ, *the altitude of the star* PL, and the *declination of the star* PM are known. We have:

$$NZ = 90° - \text{latitude of the observer},$$

$$ZP = 90° - \text{altitude of the star},$$

$$PN = 90° - \text{declination of the star}.$$

If an observation of the sun's altitude be made in the forenoon, let us say, and the declination of the sun upon that day of the year be known, then we may compute the hour angle provided the latitude of the observer be known. Having the hour angle, we may find the corresponding equivalent in time, and thus determine the hour of day at which the observation was made.

The sun's declination varies from near 23° 30′ south to near 23° 30′ north declination. At the vernal equinox and at the autumnal equinox the sun's declination is zero.

EXERCISES

1. At San Francisco, lat. 37° 48′ N., a forenoon observation shows the sun's altitude as 40° 21′. If the sun's declination be 10° 41′ N., what is the time of observation?

2. In latitude 40° 21′ N., the sun's altitude in the afternoon was found to be 35° 40′. What was the time of observation, if the sun's declination is 8° 4′ S.?

3. Find the time of sunrise at Boston, lat. 42° 21′ N., the sun's declination being 20° 31′ N.

4. Find the time of sunrise (approximately) at a point whose latitude is 39° 10′ N. twenty days after the vernal equinox.

CHAPTER XII

FORMULAS

PLANE TRIGONOMETRY

Fundamental Identities.

1. $\sin x \csc x = \cos x \sec x = \tan x \cot x = 1.$

2. $\sin^2 x + \cos^2 x = 1, \quad \sec^2 x - \tan^2 x = 1, \quad \csc^2 x - \cot^2 x = 1.$

3. $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{1}{\cot x} = \dfrac{\sec x}{\csc x} = \dfrac{\sqrt{1-\cos^2 x}}{\cos x}.$

4. $\sin x = \cos x \tan x = \dfrac{1}{\sqrt{1+\cot^2 x}} = \dfrac{\tan x}{\sqrt{1+\tan^2 x}}.$

5. $\cos x = \sin x \cot x = \dfrac{1}{\sqrt{1+\tan^2 x}} = \dfrac{\cot x}{\sqrt{1+\cot^2 x}}.$

Sum and Difference Formulas.

6. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y.$

7. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y.$

8. $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}.$

9. $\cot(x \pm y) = \dfrac{\cot x \cot y \mp 1}{\cot y \pm \cot x}.$

10. $\sin x + \sin y = 2 \sin \tfrac{1}{2}(x+y) \cos \tfrac{1}{2}(x-y).$

11. $\sin x - \sin y = 2 \cos \tfrac{1}{2}(x+y) \sin \tfrac{1}{2}(x-y).$

12. $\cos x + \cos y = 2 \cos \tfrac{1}{2}(x+y) \cos \tfrac{1}{2}(x-y).$

13. $\cos x - \cos y = -2 \sin \tfrac{1}{2}(x+y) \sin \tfrac{1}{2}(x-y).$

14. $\tan x \pm \tan y = \dfrac{\sin(x \pm y)}{\cos x \cos y}.$

15. $\cot x \pm \cot y = \pm \dfrac{\sin(x \pm y)}{\sin x \sin y}.$

16. $\dfrac{\sin x + \sin y}{\cos x + \cos y} = \tan \tfrac{1}{2}(x+y).$

17. $\dfrac{\sin x + \sin y}{\sin x - \sin y} = \dfrac{\tan \frac{1}{2}(x+y)}{\tan \frac{1}{2}(x-y)}.$

18. $\sin^2 x - \sin^2 y = \sin(x+y)\sin(x-y).$

19. $\cos^2 x - \cos^2 y = -\sin(x+y)\sin(x-y).$

20. $\cos^2 x - \sin^2 y = \cos(x+y)\cos(x-y).$

Half Angle and Multiple Angle Formulas.

21. $\sin x = 2\sin\frac{1}{2}x\cos\frac{1}{2}x = \dfrac{2\tan\frac{1}{2}x}{1+\tan^2\frac{x}{2}}.$

22. $\cos x = \cos^2\frac{1}{2}x - \sin^2\frac{1}{2}x = 2\cos^2\frac{1}{2}x - 1 = 1 - 2\sin^2\frac{1}{2}x$
$= \dfrac{1-\tan^2\frac{1}{2}x}{1+\tan^2\frac{1}{2}x}.$

23. $\tan x = \dfrac{2\tan\frac{1}{2}x}{1-\tan^2\frac{1}{2}x} = \dfrac{2\cot\frac{1}{2}x}{\cot^2\frac{1}{2}x - 1} = \dfrac{2}{\cot\frac{1}{2}x - \tan\frac{1}{2}x}.$

24. $\cot x = \dfrac{\cot^2\frac{1}{2}x - 1}{2\cot\frac{1}{2}x} = \frac{1}{2}(\cot\frac{1}{2}x - \tan\frac{1}{2}x).$

25. $\sin\frac{1}{2}x = \sqrt{\frac{1}{2}(1-\cos x)}.$

26. $\cos\frac{1}{2}x = \sqrt{\frac{1}{2}(1+\cos x)}.$

27. $\tan\frac{1}{2}x = \sqrt{\dfrac{1-\cos x}{1+\cos x}} = \dfrac{\sin x}{1+\cos x} = \dfrac{1-\cos x}{\sin x}.$

28. $\sin 2x = 2\sin x\cos x,\ \cos 2x = \cos^2 x - \sin^2 x.$

29. $\sin 3x = 3\sin x - 4\sin^3 x,\ \cos 3x = 4\cos^3 x - 3\cos x.$

30. $\sin 4x = \sin x(8\cos^3 x - 4\cos x),$
$\cos 4x = 8\cos^4 x - 8\cos^2 x + 1.$

31. $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x},\quad \tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}.$

Plane Triangles.

32. Theorems of sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = \dfrac{1}{2R}.$

33. Theorem of cosines: $a^2 = b^2 + c^2 - 2bc\cos A.$

34. Theorem of tangents:

$$\frac{a+b}{a-b} = \frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}.$$

35. The half angles:

$$\left.\begin{aligned}&(1)\ \sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}},\\&(2)\ \cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}},\\&(3)\ \tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.\end{aligned}\right\},\ \text{where } 2s=a+b+c.$$

36. Area: $K=\frac{1}{2}bc\sin A=\frac{1}{2}c^2\frac{\sin A\sin B}{\sin(A+B)}$

$$=\sqrt{s(s-a)(s-b)(s-c)}=sr.$$

Analytic Trigonometry.

37. Inverse identities:

(1) $\sin^{-1}x\pm\sin^{-1}y=\sin^{-1}(x\sqrt{1-y^2}\pm y\sqrt{1-x^2})$.

(2) $\cos^{-1}x\pm\cos^{-1}y=\cos^{-1}(xy\mp\sqrt{1-x^2}\sqrt{1-y^2})$.

(3) $\tan^{-1}x\pm\tan^{-1}y=\tan^{-1}\left(\frac{x\pm y}{1\mp xy}\right)$.

(4) $\sin^{-1}x=\frac{1}{2}\sin^{-1}(2x\sqrt{1-x^2})=\frac{1}{2}\pi-\cos^{-1}x$

$$=\csc^{-1}\frac{1}{x}=-\sin^{-1}(-x).$$

(5) $\cos^{-1}x=\frac{1}{2}\cos^{-1}(2x^2-1)=\frac{1}{2}\pi-\sin^{-1}x$

$$=2\tan^{-1}\sqrt{\frac{1-x}{1+x}}=\sec^{-1}\frac{1}{x}.$$

38. $\sin x=\frac{1}{2i}(e^{ix}-e^{-ix}),\ \cos x=\frac{1}{2}(e^{ix}+e^{-ix})$,

$$\tan x=\frac{1}{i}\left(\frac{e^{ix}-e^{-ix}}{e^{ix}+e^{-ix}}\right).$$

39. $\sinh x=\frac{1}{2}(e^x-e^{-x}),\ \cosh x=\frac{1}{2}(e^x+e^{-x})$,

$$\tanh x=\frac{e^x-e^{-x}}{e^x+e^{-x}}.$$

40. $e^{ix}=\cos x+i\sin x,\ e^{-ix}=\cos x-i\sin x$.

41. $e^x=\cosh x+\sinh x,\ e^{-ix}=\cosh x-\sinh x$.

42. $\sin(ix)=\frac{i}{2}(e^x-e^{-x})=i\sinh x$,

$$\cos(ix)=\frac{1}{2}(e^x+e^{-x})=\cosh x,\ \tan(ix)=i\frac{e^x-e^{-x}}{e^x+e^{-x}}=i\tanh x.$$

43. $(\cos x + i \sin x)^n = (e^{ix})^n = e^{nix} = \cos nx + i \sin nx.$

44. $\sin(x \pm iy) = \sin x \cos(iy) \pm \cos x \sin(iy)$
$= \sin x \cosh y \pm i \cos x \sinh y.$

45. $\cos(x \pm iy)x = \cos x \cosh y \mp i \sin x \sinh y.$

46. $\cosh^2 x - \sinh^2 x = \text{sech}^2 x + \tanh^2 x = \coth^2 x - \text{csch}^2 x = 1.$

47. Infinite series:

(1) $(1 + x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2} x^2 + \frac{n(n-1)(n-2)}{\lfloor 3} x^3 + \cdots.$

(2) $e^x = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} + \frac{x^5}{\lfloor 5} + \cdots.$

(3) $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots.$

(4) $\sin x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 7} + \cdots.$

(5) $\cos x = 1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \frac{x^6}{\lfloor 6} + \cdots.$

(6) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots.$

(7) $\sinh x = x + \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} + \frac{x^7}{\lfloor 7} + \cdots.$

(8) $\cosh x = 1 + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} + \frac{x^6}{\lfloor 6} + \cdots.$

(9) $\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \cdots.$

48. $\sin x = x \left\{1 - \left(\frac{x}{\pi}\right)^2\right\} \left\{1 - \left(\frac{x}{2\pi}\right)^2\right\} \left\{1 - \left(\frac{x}{3\pi}\right)^2\right\} \cdots.$

49. $\cos x = \left\{1 - \left(\frac{2x}{\pi}\right)^2\right\} \left\{1 - \left(\frac{2x}{3\pi}\right)^2\right\} \left\{1 - \left(\frac{2x}{5\pi}\right)^2\right\} \cdots.$

50. If $\sec \theta = \cosh x$, $\theta =$ gudermannian of $x = gdx$.

(1) $\theta = gdx = \sec^{-1}(\cosh x).$

(2) $x = gd^{-1}\theta = \cosh^{-1}(\sec \theta).$

SPHERICAL TRIGONOMETRY

The Right Spherical Triangle.

51. $\sin A = \frac{\sin a}{\sin c}$. **52.** $\cos A = \frac{\tan b}{\tan c}$. **53.** $\tan A = \frac{\tan a}{\sin b}$.

54. $\cos c = \cos a \cos b = \cot A \cot B$.

55. $\cos A = \sin B \cos a$.

56. Napier's Rules:

(1) sin (mid. pt.) = tan (adj. pt.) · tan (adj. pt.).

(2) sin (mid. pt.) = cos (op. pt.) · cos (op. pt.).

The Oblique Spherical Triangle.

57. $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$.

58. $\cos a = \cos b \cos c + \sin b \sin c \cos A$.

59. $\cos A = -\cos B \cos C + \sin B \sin C \cos a$.

60. $\tan \frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s \sin(s-a)}}$,

$\tan \frac{a}{2} = \sqrt{\frac{-\cos S \cos(S-A)}{\cos(S-B)\cos(S-C)}}$.

61. $\frac{\sin \frac{1}{2}(A+B)}{\sin \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2} c}{\tan \frac{1}{2}(a-b)}$,

$\frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2} c}{\tan \frac{1}{2}(a+b)}$.

62. $\frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A-B)}$,

$\frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A+B)}$.

63. Area, $\Delta = \frac{\pi R^2}{180} E$, where E is the spherical excess.

64. $\tan \frac{E}{4} = \sqrt{\tan\frac{1}{2}s \tan\frac{1}{2}(s-a) \tan\frac{1}{2}(s-b) \tan\frac{1}{2}(s-c)}$.

LOGARITHMIC, TRIGONOMETRIC, AND OTHER TABLES

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PREFACE

THE following Tables have been prepared to accompany the author's "Plane and Spherical Trigonometry." The beginner in trigonometric calculation should be taught the use of tables supplied with the most convenient means of interpolation. To secure this end, proportional parts have been inserted in the Tables of Logarithms of Numbers and Logarithms of the Trigonometric Functions. A proper use of these devices is especially recommended.

Numerical calculation is a laborious process at best. By the professional computer, as well as by the beginner in elementary trigonometric calculation, every facility for obtaining the required degree of accuracy with a minimum of time and labor should be sought. To this end it is recommended that (1) tables with a minimum of decimals consistent with accuracy be used, (2) formulas be arranged in a systematic manner, (3) data as well as logarithms be systematically tabulated, (4) a skilful use of proportional parts be acquired, (5) logarithmic calculations upon scraps of paper be at no time permitted.

CONTENTS

INTRODUCTION

Logarithms may be used to diminish the labor of numerical calculations involving *multiplication*, *division*, *involution*, and *evolution* by replacing these operations, respectively, by *addition*, *subtraction*, *multiplication* and *division.*

1. Definitions. (1) *The exponent by which a number* $\boldsymbol{a}$ *must be affected to produce a number* $\boldsymbol{N}$ *is called the logarithm of* $\boldsymbol{N}$ *to base* $\boldsymbol{a}$*.*

In symbols, if

$$a^x = N,$$
$$\log_a N = x.$$

(2) There are two systems of logarithms in use:

(*a*) The *Common System* (*Briggian System*) in which 10 is the base,

(*b*) The *Natural System* (*Napierian System*) in which $e = 2.71828\cdots$ is the base.

Examples.
$$10^2 = 100, \quad \log_{10} 100 = 2.$$
$$10^{2.23300} = 171, \quad \log_{10} 171 = 2.23300.$$
$$e^{3.40120} = 30, \quad \log_e 30 = 3.40120.$$

All ordinary calculations are performed by use of logarithms to the base 10. See Tables I, II.

Theoretical calculations in the higher mathematics sometimes require the use of *natural logarithms.* See Tables IV, 2, (*A*), (*B*).

(3) A logarithm of a number usually consists of an *integer* and a *decimal. The integral part of a logarithm is called the characteristic, the decimal part is called the mantissa.*

2. Laws for the Use of Logarithms.

(1) Law of Factors. *The logarithm of a product equals the sum of the logarithms of the factors.*

$$\log(A \times B \times C) = \log A + \log B + \log C.$$

(2) Law of Quotients. *The logarithm of a quotient equals the logarithm of the numerator diminished by that of the denominator.*

$$\log\left(\frac{A}{B}\right) = \log A - \log B.$$

(3) Law of Involution (or Evolution). *The logarithm of a number affected by an exponent equals the exponent multiplied by the logarithm of the number.*

$$\log (A)^n = n \times \log A,$$

$$\log (A)^{\frac{p}{q}} = \frac{p}{q} \times \log A.$$

The laws of *products, quotients,* and *involution* (or *evolution*) hold for logarithms to any base.

(4) Law of the Characteristic. *The common logarithm of a number has a characteristic one unit smaller than the number of digits to the left of the decimal point.*

Thus, $\log 476.8 = 2.67834, \quad \log 0.4768 = \bar{1}.67834,$
$\log 47.68 = 1.67834, \quad \log 0.04768 = \bar{2}.67834.$

In case of a pure decimal the same law of characteristic holds, the number of digits being counted as negative. A negative characteristic is usually indicated by a dash over the integer.

$$\log 0.04768 = \bar{2}.67834.$$

In calculations involving *negative characteristics*, 10 is usually added, making the characteristic positive.

Thus, $\log 0.917 = \bar{1}.96237 = 9.96237 - 10.$

In case of root extraction, it is convenient to add 10 multiplied by the index of the root to be extracted. Thus,

$$\log (0.0732)^{\frac{1}{3}} = \tfrac{1}{3} \times \log 0.0732 = \tfrac{1}{3}(\bar{2}.86451) = \tfrac{1}{3}(28.86451 - 30)$$
$$= 9.62150 - 10 = \bar{1}.62150.$$

In this example, 3 or any multiple of 3 could have been added and subtracted.

(5) Law of the Mantissa. *The mantissæ of the logarithms of all numbers composed of the same sequence of digits are the same.*

Thus, $\log 5297 = 3.72403, \quad \log 0.5297 = \bar{1}.72403,$
$\log 52.97 = 1.72403, \quad \log 0.05297 = \bar{2}.72403.$

3. The Co-Logarithm (Arithmetical Complement). The *co-logarithm* of a number is the logarithm of that number subtracted from unity or from 10. Thus,

$$\text{co-log } 824 = 10 - \log 824 = 10 - 2.91593 = 7.08407.$$

The co-logarithm of a number may be written down by beginning at the left-hand digit of its logarithm and subtracting each digit from 9 *except the last significant one, which must be taken from* 10.

In performing division by means of logarithms, the logarithm of the denominator must be taken from that of the numerator. Instead of subtracting the logarithm of the denominator, its co-logarithm may be added to the logarithm of the numerator.

EXAMPLE.
$$\begin{aligned} \log(564 \div 328) &= \log 564 - \log 328 \\ &= \log 564 = 2.75128 \\ + \text{co-log } 328 &= \underline{7.48413} \\ &= 0.23541 \end{aligned}$$

The common logarithms shown in Tables I and II are approximations to five decimal places of mantissæ. Table III shows the *natural values* of the trigonometric functions to four decimal places. Table IV contains a miscellaneous collection of short tables each of which is indicated by its title.

In any one of these Tables a dash underneath a terminal 5, thus $\underline{5}$, indicates that the true value is smaller than 5.

EXAMPLE. $\log 7292 = 3.8628\underline{5}$ to five decimals, but from six-place tables, $\log 7292 = 3.862847$.

TABLE I

This Table, pages 1–19, shows the *common* or *Briggian* logarithms of numbers from 1 to 10,009. On page 1 the characteristics and mantissæ of the logarithms of numbers from 1 to 100 are shown; pages 2–19 show the mantissæ only, the characteristics being supplied by *Law of the Characteristic* explained above.

Pages 2–19 show the first three digits of the number, whose logarithm may be required, in column marked **N**; the next digit is in a column with the proper heading, **0**, **1**, **2**, · · ·, **9**, shown in the horizontal row at top and bottom of the page. If a number consists of more than *four* digits its logarithm may be approximated by using the column marked *proportional parts* (**Prop. Pts.**). The first two digits of the mantissæ are shown in column marked **0**; an asterisk * in this Table indicates that the first two digits of the mantissæ must be taken from the following horizontal row.

EXAMPLES. **1.** Find log 4876.

(1) Find first three figures 487 in column **N**, p. 9.
(2) Follow along the horizontal row of 487 to column 6.
(3) We now have the mantissa, 68806.
(4) Determine the characteristic, 3.

$$\therefore \log 4876 = 3.68806.$$

2. Find log 54973.

(1) Find the first four figures, p. 10, as above.
(2) Determine the characteristic, 4.

$$\log 54970 = 4.74013.$$

The first two figures of the mantissa come from line below.
(3) On the same page, log 54980 = 4.74020,
showing an increase of 7 in the mantissa for 10 in the number.
(4) An increase of 3 in the number $= \frac{3}{10} \times 7 = 2.1$ in the mantissa.

$$\therefore \log 54973 = 4.74013 + 2 = 4.74015.$$

The difference in successive mantissæ on page 10 will be found to be 7, 8, or 9; and the corrections for a fifth figure 1, 2, 3, 4, 5, 6, 7, 8, 9 are shown in the marginal column marked **Prop. Pts.**

3. Find log 34.4764.

We find the first four digits on p. 6,

$$\begin{aligned} \log 34.4700 &= 1.53744, \text{ tabular difference } 13, \\ \text{correction } 6 &= \quad 7.8, \text{ from Prop. Pts. column}, \\ \text{correction } 0.4 &= \underline{\quad\quad .5,} \\ \therefore \log 34.4764 &= 1.53752 \end{aligned}$$

4. Find N, when $\log N = 3.57824$.

(1) The characteristic 3 shows that N has four digits to the left of the decimal point.
(2) Find the mantissa nearest to 57824, p. 7. This is 57818, which is 6 smaller than the given mantissa.
(3) From p. 7,

$$N = 3786 + \text{corrections for 6 in a tabular difference of 12.}$$
$$\therefore N = 3786.5.$$

The Prop. Pts. column shows this correction at once. A little practice will enable the computor to make the corrections mentally, merely writing down the corrected result.

Table II

Table II, pages 22–73, contains five-place logarithms of the trigonometric sine, cosine, tangent, and cotangent for angles 0° to 90°. In this Table 10 has been added throughout to each characteristic.

Page 22 shows the common logarithm of the sine and tangent for each second from 0″ to 3′, and the logarithms of cosine and cotangent for every second from 89° 57′ to 90°.

Pages 23–28 show the logarithmic sine, tangent, and cosine for every 10″ for angles 0° to 2°, also the logarithmic cosine, cotangent, and sine for every 10″ for angles from 88° to 90°. Corrections for intermediate seconds may be easily interpolated.

Thus,
log sin 1° 6′ 36″ = log sin 1° 6′ 30″ + correction for 6″. See p. 26.

$$\log\sin 1^\circ\ 6'\ 40'' = 8.28761$$
$$\log\sin 1^\circ\ 6'\ 30'' = 8.28652$$
$$\tfrac{6}{10} \text{ of difference} = \tfrac{6}{10} \times (109) = 65.4$$

Hence, $\log\sin 1^\circ\ 6'\ 36'' = 8.28652 + 65 = 8.28717.$

Pages 29–73 contain the logarithms of the sine, tangent, cotangent and cosine for each minute of arc from 0° to 90°. The degrees are indicated at the top and bottom of each page, the minutes are in columns at the left and right of the page.

In columns marked **d.** are the tabular differences of the log sine and log cosine for each minute. In column marked **c. d.** are the tabular differences for the log tangent (log cotangent). These tabular differences show the correction for 60″; any smaller number of seconds, for example, 35″, would have $\frac{35}{60}$ of this tabular correction. In proportional parts (**Prop. Pts.**) column, these corrections are reduced to scale of 1″, pages 29–34. On pages 35–73 the tabular corrections are reduced so as to show the corrections for 6″, 7″, 8″, 9″, 10″, 20″, 30″, 40″, 50″. By pointing off the corrections for 10″, 20″, 30″, 40″, 50″, corresponding corrections for 1″, 2″, 3″, 4″, 5″ may be obtained.

The corrections for seconds should be made *mentally* by aid of the proportional parts column, and the result of the corrected logarithm may be written at once.

Examples. **1.** Find log sin 28° 37′ 21″.

From p. 57, log sin 28° 37′ = 9.68029, tabular difference 23.
Prop. Pts. for 20″ = 7.7
Prop. Pts. for 1″ = .38
∴ log sin 28° 37′ 21″ = 9.68037

2. Find log cos 16° 48′ 24″.

On p. 45, log cos 16° 48′ = 9.98106, tabular difference 4.
Prop. Pts. for 20″ = 1.3 } to be subtracted.
Prop. Pts. for 4″ = .27 }
9.98104

Correction for log cosines and log cotangents are to be subtracted.

3. Find the angle x, when $\log\tan x = 9.65948$.

(1) Find in log tan column a log as near the given one as possible, p. 53.

(2) log tan 24° 32′ = 9.65937,

with a difference of 11 to a tabular difference of 34.

(3) In Prop. Pts. column 11 to a scale of 34 shows a 20″ correction.

$$\therefore x = 24^\circ\ 32'\ 20''.$$

4. Find the angle x, when $\log \cot x = 9.48726$.

(1) On p. 46, in log cot column,

$$\log \cot 72° \; 56' = 9.48714,$$

with a difference of 12 to a tabular difference of 45.

(2) In Prop. Pts. column 45,

$$\begin{aligned} \text{Correction for } 10'' &= 7.5 \\ \text{Correction for } 6'' &= \underline{4.5} \\ \text{Hence, correction for } 16'' &= 12.0 \end{aligned}$$

(3) Since cotangents increase with decrease of angle, subtract the 16″ correction, giving $x = 72° \; 55' \; 44''$.

In addition to the acute angle set at the top and bottom of each page in the Table, pages 29–73, 90° + the acute angle, 180° + the angle, and 270° + the angle are printed in smaller type. Since the trigonometric functions of $90° + x$ and $270° + x$ change to co-functions of x, and the functions of $180° + x$ retain their names, we may find the logarithms of the trigonometric functions of any one of these larger angles. In each case the sign of the function must be noted as shown in the following table:

x	$\sin x$	$\cos x$	$\tan x$	$\cot x$	$\sec x$	$\csc x$
$90° + x$	$+ \cos x$	$- \sin x$	$- \cot x$	$- \tan x$	$- \csc x$	$+ \sec x$
$180° + x$	$- \sin x$	$- \cos x$	$+ \tan x$	$+ \cot x$	$- \sec x$	$- \csc x$
$270° + x$	$- \cos x$	$+ \sin x$	$- \cot x$	$- \tan x$	$+ \csc x$	$- \sec x$

In the Table the angles marked by an asterisk (*) are made up of $90° + x$, or $270° + x$; this mark indicates that the co-function of x is to be taken. Otherwise the direct function of x is to be taken, proper regard being had for the algebraic sign in each case.

Table III

Pages 75–93 contain four-place tables of the natural trigonometric functions for each minute. The sines and cosines appear upon the left-hand pages, the tangents and cotangents upon the right.

Table IV

Table IV, pages 95–99, contains (1) a table of radian measure 0° to 180° for each degree; also for 1′ to 60′ and for 1″ to 60″; (2) two tables of natural logarithms of numbers from 1 to 200 and 1 to 9.9 by tenths; (3) a table of hyperbolic functions, $\sinh x$, $\cosh x$, x varying from 0.01 to 1 by hundredths.

LOGARITHMIC, TRIGONOMETRIC, AND OTHER TABLES

log of a number to a new
base equals log to old base times
log of old base to new base

TABLE I

THE

COMMON OR BRIGGIAN LOGARITHMS

OF

NUMBERS

From 1 to 10009

NOTE. A * in Table I indicates that the first two digits of the mantissa are to be taken from the following line.
A dash underneath a terminal 5, thus 5̲, indicates that the true value is less than 5.

1–100

N	log	N	log	N	log	N	log	N	log
1	0. 00 000	**21**	1. 32 222	**41**	1. 61 278	**61**	1. 78 533	**81**	1. 90 849
2	0. 30 103	22	1. 34 242	42	1. 62 325̲	62	1. 79 239	82	1. 91 381
3	0. 47 712	23	1. 36 173	43	1. 63 347	63	1. 79 934	83	1. 91 908
4	0. 60 206	24	1. 38 021	44	1. 64 345	64	1. 80 618	84	1. 92 428
5	0. 69 897	25	1. 39 794	45	1. 65 321	65	1. 81 291	85	1. 92 942
6	0. 77 815	**26**	1. 41 497	**46**	1. 66 276	**66**	1. 81 954	**86**	1. 93 450
7	0. 84 510	27	1. 43 136	47	1. 67 210	67	1. 82 607	87	1. 93 952
8	0. 90 309	28	1. 44 716	48	1. 68 124	68	1. 83 251	88	1. 94 448
9	0. 95 424	29	1. 46 240	49	1. 69 020	69	1. 83 885̲	89	1. 94 939
10	1. 00 000	30	1. 47 712	50	1. 69 897	70	1. 84 510	90	1. 95 424
11	1. 04 139	**31**	1. 49 136	**51**	1. 70 757	**71**	1. 85 126	**91**	1. 95 904
12	1. 07 918	32	1. 50 515̲	52	1. 71 600	72	1. 85 733	92	1. 96 379
13	1. 11 394	33	1. 51 851	53	1. 72 428	73	1. 86 332	93	1. 96 848
14	1. 14 613	34	1. 53 148	54	1. 73 239	74	1. 86 923	94	1. 97 313
15	1. 17 609	35	1. 54 407	55	1. 74 036	75	1. 87 506	95	1. 97 772
16	1. 20 412	**36**	1. 55 630	**56**	1. 74 819	**76**	1. 88 081	**96**	1. 98 227
17	1. 23 045̲	37	1. 56 820	57	1. 75 587	77	1. 88 649	97	1. 98 677
18	1. 25 527	38	1. 57 978	58	1. 76 343	78	1. 89 209	98	1. 99 123
19	1. 27 875	39	1. 59 106	59	1. 77 085	79	1. 89 763	99	1. 99 564
20	1. 30 103	40	1. 60 206	60	1. 77 815	80	1. 90 309	100	2. 00 000
N	log	N	log	N	log	N	log	N	log

100–150

N.	L. 0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
01	432	475	518	561	604	647	689	732	775	817
02	860	903	945	988	*030	*072	*115	*157	*199	*242
03	01 284	326	368	410	452	494	536	578	620	662
04	703	745	787	828	870	912	953	995	*036	*078
05	02 119	160	202	243	284	325	366	407	449	490
06	531	572	612	653	694	735	776	816	857	898
07	938	979	*019	*060	*100	*141	*181	*222	*262	*302
08	03 342	383	423	463	503	543	583	623	663	703
09	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
11	532	571	610	650	689	727	766	805	844	883
12	922	961	999	*038	*077	*115	*154	*192	*231	*269
13	05 308	346	385	423	461	500	538	576	614	652
14	690	729	767	805	843	881	918	956	994	*032
15	06 070	108	145	183	221	258	296	333	371	408
16	446	483	521	558	595	633	670	707	744	781
17	819	856	893	930	967	*004	*041	*078	*115	*151
18	07 188	225	262	298	335	372	408	445	482	518
19	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
21	08 279	314	350	386	422	458	493	529	565	600
22	636	672	707	743	778	814	849	884	920	955
23	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
24	09 342	377	412	447	482	517	552	587	621	656
25	691	726	760	795	830	864	899	934	968	*003
26	10 037	072	106	140	175	209	243	278	312	346
27	380	415	449	483	517	551	585	619	653	687
28	721	755	789	823	857	890	924	958	992	*025
29	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
31	727	760	793	826	860	893	926	959	992	*024
32	12 057	090	123	156	189	222	254	287	320	352
33	385	418	450	483	516	548	581	613	646	678
34	710	743	775	808	840	872	905	937	969	*001
35	13 033	066	098	130	162	194	226	258	290	322
36	354	386	418	450	481	513	545	577	609	640
37	672	704	735	767	799	830	862	893	925	956
38	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
39	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
41	922	953	983	*014	*045	*076	*106	*137	*168	*198
42	15 229	259	290	320	351	381	412	442	473	503
43	534	564	594	625	655	685	715	746	776	806
44	836	866	897	927	957	987	*017	*047	*077	*107
45	16 137	167	197	227	256	286	316	346	376	406
46	435	465	495	524	554	584	613	643	673	702
47	732	761	791	820	850	879	909	938	967	997
48	17 026	056	085	114	143	173	202	231	260	289
49	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869

N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

Prop. Pts.

	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

Prop. Pts.

150–200

N.	L. 0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
51	898	926	955	984	*013	*041	*070	*099	*127	*156
52	18 184	213	241	270	298	327	355	384	412	441
53	469	498	526	554	583	611	639	667	696	724
54	752	780	808	837	865	893	921	949	977	*005
55	19 033	061	089	117	145	173	201	229	257	285
56	312	340	368	396	424	451	479	507	535	562
57	590	618	645	673	700	728	756	783	811	838
58	866	893	921	948	976	*003	*030	*058	*085	*112
59	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
61	683	710	737	763	790	817	844	871	898	925
62	952	978	*005	*032	*059	*085	*112	*139	*165	*192
63	21 219	245	272	299	325	352	378	405	431	458
64	484	511	537	564	590	617	643	669	696	722
65	748	775	801	827	854	880	906	932	958	985
66	22 011	037	063	089	115	141	167	194	220	246
67	272	298	324	350	376	401	427	453	479	505
68	531	557	583	608	634	660	686	712	737	763
69	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
71	300	325	350	376	401	426	452	477	502	528
72	553	578	603	629	654	679	704	729	754	779
73	805	830	855	880	905	930	955	980	*005	*030
74	24 055	080	105	130	155	180	204	229	254	279
75	304	329	353	378	403	428	452	477	502	527
76	551	576	601	625	650	674	699	724	748	773
77	797	822	846	871	895	920	944	969	993	*018
78	25 042	066	091	115	139	164	188	212	237	261
79	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
81	768	792	816	840	864	888	912	935	959	983
82	26 007	031	055	079	102	126	150	174	198	221
83	245	269	293	316	340	364	387	411	435	458
84	482	505	529	553	576	600	623	647	670	694
85	717	741	764	788	811	834	858	881	905	928
86	951	975	998	*021	*045	*068	*091	*114	*138	*161
87	27 184	207	231	254	277	300	323	346	370	393
88	416	439	462	485	508	531	554	577	600	623
89	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
91	28 103	126	149	171	194	217	240	262	285	307
92	330	353	375	398	421	443	466	488	511	533
93	556	578	601	623	646	668	691	713	735	758
94	780	803	825	847	870	892	914	937	959	981
95	29 003	026	048	070	092	115	137	159	181	203
96	226	248	270	292	314	336	358	380	403	425
97	447	469	491	513	535	557	579	601	623	645
98	667	688	710	732	754	776	798	820	842	863
99	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	L. 0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
1	2.5
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	23
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

Prop. Pts.

200–250

N.	L. 0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
01	320	341	363	384	406	428	449	471	492	514
02	535	557	578	600	621	643	664	685	707	728
03	750	771	792	814	835	856	878	899	920	942
04	963	984	*006	*027	*048	*069	*091	*112	*133	*154
05	31 175	197	218	239	260	281	302	323	345	366
06	387	408	429	450	471	492	513	534	555	576
07	597	618	639	660	681	702	723	744	765	785
08	806	827	848	869	890	911	931	952	973	994
09	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
11	428	449	469	490	510	531	552	572	593	613
12	634	654	675	695	715	736	756	777	797	818
13	838	858	879	899	919	940	960	980	*001	*021
14	33 041	062	082	102	122	143	163	183	203	224
15	244	264	284	304	325	345	365	385	405	425
16	445	465	486	506	526	546	566	586	606	626
17	646	666	686	706	726	746	766	786	806	826
18	846	866	885	905	925	945	965	985	*005	*025
19	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
21	439	459	479	498	518	537	557	577	596	616
22	635	655	674	694	713	733	753	772	792	811
23	830	850	869	889	908	928	947	967	986	*005
24	35 025	044	064	083	102	122	141	160	180	199
25	218	238	257	276	295	315	334	353	372	392
26	411	430	449	468	488	507	526	545	564	583
27	603	622	641	660	679	698	717	736	755	774
28	793	813	832	851	870	889	908	927	946	965
29	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
31	361	380	399	418	436	455	474	493	511	530
32	549	568	586	605	624	642	661	680	698	717
33	736	754	773	791	810	829	847	866	884	903
34	922	940	959	977	996	*014	*033	*051	*070	*088
35	37 107	125	144	162	181	199	218	236	254	273
36	291	310	328	346	365	383	401	420	438	457
37	475	493	511	530	548	566	585	603	621	639
38	658	676	694	712	731	749	767	785	803	822
39	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
41	202	220	238	256	274	292	310	328	346	364
42	382	399	417	435	453	471	489	507	525	543
43	561	578	596	614	632	650	668	686	703	721
44	739	757	775	792	810	828	846	863	881	899
45	917	934	952	970	987	*005	*023	*041	*058	*076
46	39 094	111	129	146	164	182	199	217	235	252
47	270	287	305	322	340	358	375	393	410	428
48	445	463	480	498	515	533	550	568	585	602
49	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	L. 0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	22	21	20	19	18	17
1	2.2	2.1	2.0	1.9	1.8	1.7
2	4.4	4.2	4.0	3.8	3.6	3.4
3	6.6	6.3	6.0	5.7	5.4	5.1
4	8.8	8.4	8.0	7.6	7.2	6.8
5	11.0	10.5	10.0	9.5	9.0	8.5
6	13.2	12.6	12.0	11.4	10.8	10.2
7	15.4	14.7	14.0	13.3	12.6	11.9
8	17.6	16.8	16.0	15.2	14.4	13.6
9	19.8	18.9	18.0	17.1	16.2	15.3

250–300

N.	L. 0	1	2	3	4	5	6	7	8	9
250	39 794	811	829	846	863	881	898	915	933	950
51	967	985	*002	*019	*037	*054	*071	*088	*106	*123
52	40 140	157	175	192	209	226	243	261	278	295
53	312	329	346	364	381	398	415	432	449	466
54	483	500	518	535	552	569	586	603	620	637
55	654	671	688	705	722	739	756	773	790	807
56	824	841	858	875	892	909	926	943	960	976
57	993	*010	*027	*044	*061	*078	*095	*111	*128	*145
58	41 162	179	196	212	229	246	263	280	296	313
59	330	347	363	380	397	414	430	447	464	481
260	497	514	531	547	564	581	597	614	631	647
61	664	681	697	714	731	747	764	780	797	814
62	830	847	863	880	896	913	929	946	963	979
63	996	*012	*029	*045	*062	*078	*095	*111	*127	*144
64	42 160	177	193	210	226	243	259	275	292	308
65	325	341	357	374	390	406	423	439	455	472
66	488	504	521	537	553	570	586	602	619	635
67	651	667	684	700	716	732	749	765	781	797
68	813	830	846	862	878	894	911	927	943	959
69	975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	185	201	217	233	249	265	281
71	297	313	329	345	361	377	393	409	425	441
72	457	473	489	505	521	537	553	569	584	600
73	616	632	648	664	680	696	712	727	743	759
74	775	791	807	823	838	854	870	886	902	917
75	933	949	965	981	996	*012	*028	*044	*059	*075
76	44 091	107	122	138	154	170	185	201	217	232
77	248	264	279	295	311	326	342	358	373	389
78	404	420	436	451	467	483	498	514	529	545
79	560	576	592	607	623	638	654	669	685	700
280	716	731	747	762	778	793	809	824	840	855
81	871	886	902	917	932	948	963	979	994	*010
82	45 025	040	056	071	086	102	117	133	148	163
83	179	194	209	225	240	255	271	286	301	317
84	332	347	362	378	393	408	423	439	454	469
85	484	500	515	530	545	561	576	591	606	621
86	637	652	667	682	697	712	728	743	758	773
87	788	803	818	834	849	864	879	894	909	924
88	939	954	969	984	*000	*015	*030	*045	*060	*075
89	46 090	105	120	135	150	165	180	195	210	225
290	240	255	270	285	300	315	330	345	359	374
91	389	404	419	434	449	464	479	494	509	523
92	538	553	568	583	598	613	627	642	657	672
93	687	702	716	731	746	761	776	790	805	820
94	835	850	864	879	894	909	923	938	953	967
95	982	997	*012	*026	*041	*056	*070	*085	*100	*114
96	47 129	144	159	173	188	202	217	232	246	261
97	276	290	305	319	334	349	363	378	392	407
98	422	436	451	465	480	494	509	524	538	553
99	567	582	596	611	625	640	654	669	683	698
300	712	727	741	756	770	784	799	813	828	842
N.	L. 0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	18	17	16	15	14
1	1.8	1.7	1.6	1.5	1.4
2	3.6	3.4	3.2	3.0	2.8
3	5.4	5.1	4.8	4.5	4.2
4	7.2	6.8	6.4	6.0	5.6
5	9.0	8.5	8.0	7.5	7.0
6	10.8	10.2	9.6	9.0	8.4
7	12.6	11.9	11.2	10.5	9.8
8	14.4	13.6	12.8	12.0	11.2
9	16.2	15.3	14.4	13.5	12.6

300–350

N.	L. 0	1	2	3	4	5	6	7	8	9
300	47 712	727	741	756	770	784	799	813	828	842
01	857	871	885	900	914	929	943	958	972	986
02	48 001	015	029	044	058	073	087	101	116	130
03	144	159	173	187	202	216	230	244	259	273
04	287	302	316	330	344	359	373	387	401	416
05	430	444	458	473	487	501	515	530	544	558
06	572	586	601	615	629	643	657	671	686	700
07	714	728	742	756	770	785	799	813	827	841
08	855	869	883	897	911	926	940	954	968	982
09	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
11	276	290	304	318	332	346	360	374	388	402
12	415	429	443	457	471	485	499	513	527	541
13	554	568	582	596	610	624	638	651	665	679
14	693	707	721	734	748	762	776	790	803	817
15	831	845	859	872	886	900	914	927	941	955
16	969	982	996	*010	*024	*037	*051	*065	*079	*092
17	50 106	120	133	147	161	174	188	202	215	229
18	243	256	270	284	297	311	325	338	352	365
19	379	393	406	420	433	447	461	474	488	501
320	515	529	542	556	569	583	596	610	623	637
21	651	664	678	691	705	718	732	745	759	772
22	786	799	813	826	840	853	866	880	893	907
23	920	934	947	961	974	987	*001	*014	*028	*041
24	51 055	068	081	095	108	121	135	148	162	175
25	188	202	215	228	242	255	268	282	295	308
26	322	335	348	362	375	388	402	415	428	441
27	455	468	481	495	508	521	534	548	561	574
28	587	601	614	627	640	654	667	680	693	706
29	720	733	746	759	772	786	799	812	825	838
330	851	865	878	891	904	917	930	943	957	970
31	983	996	*009	*022	*035	*048	*061	*075	*088	*101
32	52 114	127	140	153	166	179	192	205	218	231
33	244	257	270	284	297	310	323	336	349	362
34	375	388	401	414	427	440	453	466	479	492
35	504	517	530	543	556	569	582	595	608	621
36	634	647	660	673	686	699	711	724	737	750
37	763	776	789	802	815	827	840	853	866	879
38	892	905	917	930	943	956	969	982	994	*007
39	53 020	033	046	058	071	084	097	110	122	135
340	148	161	173	186	199	212	224	237	250	263
41	275	288	301	314	326	339	352	364	377	390
42	403	415	428	441	453	466	479	491	504	517
43	529	542	555	567	580	593	605	618	631	643
44	656	668	681	694	706	719	732	744	757	769
45	782	794	807	820	832	845	857	870	882	895
46	908	920	933	945	958	970	983	995	*008	*020
47	54 033	045	058	070	083	095	108	120	133	145
48	158	170	183	195	208	220	233	245	258	270
49	283	295	307	320	332	345	357	370	382	394
350	407	419	432	444	456	469	481	494	506	518
N.	L. 0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	15	14	13	12
1	1.5	1.4	1.3	1.2
2	3.0	2.8	2.6	2.4
3	4.5	4.2	3.9	3.6
4	6.0	5.6	5.2	4.8
5	7.5	7.0	6.5	6.0
6	9.0	8.4	7.8	7.2
7	10.5	9.8	9.1	8.4
8	12.0	11.2	10.4	9.6
9	13.5	12.6	11.7	10.8

350–400

N.	L. 0	1	2	3	4	5	6	7	8	9
350	54 407	419	432	444	456	469	481	494	506	518
51	531	543	555	568	580	593	605	617	630	642
52	654	667	679	691	704	716	728	741	753	765
53	777	790	802	814	827	839	851	864	876	888
54	900	913	925	937	949	962	974	986	998	*011
55	55 023	035	047	060	072	084	096	108	121	133
56	145	157	169	182	194	206	218	230	242	255
57	267	279	291	303	315	328	340	352	364	376
58	388	400	413	425	437	449	461	473	485	497
59	509	522	534	546	558	570	582	594	606	618
360	630	642	654	666	678	691	703	715	727	739
61	751	763	775	787	799	811	823	835	847	859
62	871	883	895	907	919	931	943	955	967	979
63	991	*003	*015	*027	*038	*050	*062	*074	*086	*098
64	56 110	122	134	146	158	170	182	194	205	217
65	229	241	253	265	277	289	301	312	324	336
66	348	360	372	384	396	407	419	431	443	455
67	467	478	490	502	514	526	538	549	561	573
68	585	597	608	620	632	644	656	667	679	691
69	703	714	726	738	750	761	773	785	797	808
370	820	832	844	855	867	879	891	902	914	926
71	937	949	961	972	984	996	*008	*019	*031	*043
72	57 054	066	078	089	101	113	124	136	148	159
73	171	183	194	206	217	229	241	252	264	276
74	287	299	310	322	334	345	357	368	380	392
75	403	415	426	438	449	461	473	484	496	507
76	519	530	542	553	565	576	588	600	611	623
77	634	646	657	669	680	692	703	715	726	738
78	749	761	772	784	795	807	818	830	841	852
79	864	875	887	898	910	921	933	944	955	967
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081
81	58 092	104	115	127	138	149	161	172	184	195
82	206	218	229	240	252	263	274	286	297	309
83	320	331	343	354	365	377	388	399	410	422
84	433	444	456	467	478	490	501	512	524	535
85	546	557	569	580	591	602	614	625	636	647
86	659	670	681	692	704	715	726	737	749	760
87	771	782	794	805	816	827	838	850	861	872
88	883	894	906	917	928	939	950	961	973	984
89	995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59 106	118	129	140	151	162	173	184	195	207
91	218	229	240	251	262	273	284	295	306	318
92	329	340	351	362	373	384	395	406	417	428
93	439	450	461	472	483	494	506	517	528	539
94	550	561	572	583	594	605	616	627	638	649
95	660	671	682	693	704	715	726	737	748	759
96	770	780	791	802	813	824	835	846	857	868
97	879	890	901	912	923	934	945	956	966	977
98	988	999	*010	*021	*032	*043	*054	*065	*076	*086
99	60 097	108	119	130	141	152	163	173	184	195
400	206	217	228	239	249	260	271	282	293	304
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	13	12	11	10
1	1.3	1.2	1.1	1.0
2	2.6	2.4	2.2	2.0
3	3.9	3.6	3.3	3.0
4	5.2	4.8	4.4	4.0
5	6.5	6.0	5.5	5.0
6	7.8	7.2	6.6	6.0
7	9.1	8.4	7.7	7.0
8	10.4	9.6	8.8	8.0
9	11.7	10.8	9.9	9.0

400-450

N.	L. 0	1	2	3	4	5	6	7	8	9
400	60 206	217	228	239	249	260	271	282	293	304
01	314	325	336	347	358	369	379	390	401	412
02	423	433	444	455	466	477	487	498	509	520
03	531	541	552	563	574	584	595	606	617	627
04	638	649	660	670	681	692	703	713	724	735
05	746	756	767	778	788	799	810	821	831	842
06	853	863	874	885	895	906	917	927	938	949
07	959	970	981	991	*002	*013	*023	*034	*045	*055
08	61 066	077	087	098	109	119	130	140	151	162
09	172	183	194	204	215	225	236	247	257	268
410	278	289	300	310	321	331	342	352	363	374
11	384	395	405	416	426	437	448	458	469	479
12	490	500	511	521	532	542	553	563	574	584
13	595	606	616	627	637	648	658	669	679	690
14	700	711	721	731	742	752	763	773	784	794
15	805	815	826	836	847	857	868	878	888	899
16	909	920	930	941	951	962	972	982	993	*003
17	62 014	024	034	045	055	066	076	086	097	107
18	118	128	138	149	159	170	180	190	201	211
19	221	232	242	252	263	273	284	294	304	315
420	325	335	346	356	366	377	387	397	408	418
21	428	439	449	459	469	480	490	500	511	521
22	531	542	552	562	572	583	593	603	613	624
23	634	644	655	665	675	685	696	706	716	726
24	737	747	757	767	778	788	798	808	818	829
25	839	849	859	870	880	890	900	910	921	931
26	941	951	961	972	982	992	*002	*012	*022	*033
27	63 043	053	063	073	083	094	104	114	124	134
28	144	155	165	175	185	195	205	215	225	236
29	246	256	266	276	286	296	306	317	327	337
430	347	357	367	377	387	397	407	417	428	438
31	448	458	468	478	488	498	508	518	528	538
32	548	558	568	579	589	599	609	619	629	639
33	649	659	669	679	689	699	709	719	729	739
34	749	759	769	779	789	799	809	819	829	839
35	849	859	869	879	889	899	909	919	929	939
36	949	959	969	979	988	998	*008	*018	*028	*038
37	64 048	058	068	078	088	098	108	118	128	137
38	147	157	167	177	187	197	207	217	227	237
39	246	256	266	276	286	296	306	316	326	335
440	345	355	365	375	385	395	404	414	424	434
41	444	454	464	473	483	493	503	513	523	532
42	542	552	562	572	582	591	601	611	621	631
43	640	650	660	670	680	689	699	709	719	729
44	738	748	758	768	777	787	797	807	816	826
45	836	846	856	865	875	885	895	904	914	924
46	933	943	953	963	972	982	992	*002	*011	*021
47	65 031	040	050	060	070	079	089	099	108	118
48	128	137	147	157	167	176	186	196	205	215
49	225	234	244	254	263	273	283	292	302	312
450	321	331	341	350	360	369	379	389	398	408
N.	L. 0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	11
1	1.1
2	2.2
3	3.3
4	4.4
5	5.5
6	6.6
7	7.7
8	8.8
9	9.9

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

Prop. Pts.

450–500

N.	L. 0	1	2	3	4	5	6	7	8	9	Prop. Pts.
450	65 321	331	341	350	360	369	379	389	398	408	
51	418	427	437	447	456	466	475	485	495	504	
52	514	523	533	543	552	562	571	581	591	600	
53	610	619	629	639	648	658	667	677	686	696	
54	706	715	725	734	744	753	763	772	782	792	
55	801	811	820	830	839	849	858	868	877	887	
56	896	906	916	925	935	944	954	963	973	982	
57	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	
58	66 087	096	106	115	124	134	143	153	162	172	
59	181	191	200	210	219	229	238	247	257	266	
460	276	285	295	304	314	323	332	342	351	361	
61	370	380	389	398	408	417	427	436	445	455	
62	464	474	483	492	502	511	521	530	539	549	
63	558	567	577	586	596	605	614	624	633	642	
64	652	661	671	680	689	699	708	717	727	736	
65	745	755	764	773	783	792	801	811	820	829	
66	839	848	857	867	876	885	894	904	913	922	
67	932	941	950	960	969	978	987	997	*006	*015	
68	67 025	034	043	052	062	071	080	089	099	108	
69	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
71	302	311	321	330	339	348	357	367	376	385	
72	394	403	413	422	431	440	449	459	468	477	
73	486	495	504	514	523	532	541	550	560	569	
74	578	587	596	605	614	624	633	642	651	660	
75	669	679	688	697	706	715	724	733	742	752	
76	761	770	779	788	797	806	815	825	834	843	
77	852	861	870	879	888	897	906	916	925	934	
78	943	952	961	970	979	988	997	*006	*015	*024	
79	68 034	043	052	061	070	079	088	097	106	115	
480	124	133	142	151	160	169	178	187	196	205	
81	215	224	233	242	251	260	269	278	287	296	
82	305	314	323	332	341	350	359	368	377	386	
83	395	404	413	422	431	440	449	458	467	476	
84	485	494	502	511	520	529	538	547	556	565	
85	574	583	592	601	610	619	628	637	646	655	
86	664	673	681	690	699	708	717	726	735	744	
87	753	762	771	780	789	797	806	815	824	833	
88	842	851	860	869	878	886	895	904	913	922	
89	931	940	949	958	966	975	984	993	*002	*011	
490	69 020	028	037	046	055	064	073	082	090	099	
91	108	117	126	135	144	152	161	170	179	188	
92	197	205	214	223	232	241	249	258	267	276	
93	285	294	302	311	320	329	338	346	355	364	
94	373	381	390	399	408	417	425	434	443	452	
95	461	469	478	487	496	504	513	522	531	539	
96	548	557	566	574	583	592	601	609	618	627	
97	636	644	653	662	671	679	688	697	705	714	
98	723	732	740	749	758	767	775	784	793	801	
99	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
N.	L. 0	1	2	3	4	5	6	7	8	9	Prop. Pts.

Prop. Pts.

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

500–550

N.	L. 0	1	2	3	4	5	6	7	8	9
500	69 897	906	914	923	932	940	949	958	966	975
01	984	992	*001	*010	*018	*027	*036	*044	*053	*062
02	70 070	079	088	096	105	114	122	131	140	148
03	157	165	174	183	191	200	209	217	226	234
04	243	252	260	269	278	286	295	303	312	321
05	329	338	346	355	364	372	381	389	398	406
06	415	424	432	441	449	458	467	475	484	492
07	501	509	518	526	535	544	552	561	569	578
08	586	595	603	612	621	629	638	646	655	663
09	672	680	689	697	706	714	723	731	740	749
510	757	766	774	783	791	800	808	817	825	834
11	842	851	859	868	876	885	893	902	910	919
12	927	935	944	952	961	969	978	986	995	*003
13	71 012	020	029	037	046	054	063	071	079	088
14	096	105	113	122	130	139	147	155	164	172
15	181	189	198	206	214	223	231	240	248	257
16	265	273	282	290	299	307	315	324	332	341
17	349	357	366	374	383	391	399	408	416	425
18	433	441	450	458	466	475	483	492	500	508
19	517	525	533	542	550	559	567	575	584	592
520	600	609	617	625	634	642	650	659	667	675
21	684	692	700	709	717	725	734	742	750	759
22	767	775	784	792	800	809	817	825	834	842
23	850	858	867	875	883	892	900	908	917	925
24	933	941	950	958	966	975	983	991	999	*008
25	72 016	024	032	041	049	057	066	074	082	090
26	099	107	115	123	132	140	148	156	165	173
27	181	189	198	206	214	222	230	239	247	255
28	263	272	280	288	296	304	313	321	329	337
29	346	354	362	370	378	387	395	403	411	419
530	428	436	444	452	460	469	477	485	493	501
31	509	518	526	534	542	550	558	567	575	583
32	591	599	607	616	624	632	640	648	656	665
33	673	681	689	697	705	713	722	730	738	746
34	754	762	770	779	787	795	803	811	819	827
35	835	843	852	860	868	876	884	892	900	908
36	916	925	933	941	949	957	965	973	981	989
37	997	*006	*014	*022	*030	*038	*046	*054	*062	*070
38	73 078	086	094	102	111	119	127	135	143	151
39	159	167	175	183	191	199	207	215	223	231
540	239	247	255	263	272	280	288	296	304	312
41	320	328	336	344	352	360	368	376	384	392
42	400	408	416	424	432	440	448	456	464	472
43	480	488	496	504	512	520	528	536	544	552
44	560	568	576	584	592	600	608	616	624	632
45	640	648	656	664	672	679	687	695	703	711
46	719	727	735	743	751	759	767	775	783	791
47	799	807	815	823	830	838	846	854	862	870
48	878	886	894	902	910	918	926	933	941	949
49	957	965	973	981	989	997	*005	*013	*020	*028
550	74 036	044	052	060	068	076	084	092	099	107

| N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Prop. Pts. |

Prop. Pts.

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

550-600

N.	L. 0	1	2	3	4	5	6	7	8	9
550	74 036	044	052	060	068	076	084	092	099	107
51	115	123	131	139	147	155	162	170	178	186
52	194	202	210	218	225	233	241	249	257	265
53	273	280	288	296	304	312	320	327	335	343
54	351	359	367	374	382	390	398	406	414	421
55	429	437	445	453	461	468	476	484	492	500
56	507	515	523	531	539	547	554	562	570	578
57	586	593	601	609	617	624	632	640	648	656
58	663	671	679	687	695	702	710	718	726	733
59	741	749	757	764	772	780	788	796	803	811
560	819	827	834	842	850	858	865	873	881	889
61	896	904	912	920	927	935	943	950	958	966
62	974	981	989	997	*005	*012	*020	*028	*035	*043
63	75 051	059	066	074	082	089	097	105	113	120
64	128	136	143	151	159	166	174	182	189	197
65	205	213	220	228	236	243	251	259	266	274
66	282	289	297	305	312	320	328	335	343	351
67	358	366	374	381	389	397	404	412	420	427
68	435	442	450	458	465	473	481	488	496	504
69	511	519	526	534	542	549	557	565	572	580
570	587	595	603	610	618	626	633	641	648	656
71	664	671	679	686	694	702	709	717	724	732
72	740	747	755	762	770	778	785	793	800	808
73	815	823	831	838	846	853	861	868	876	884
74	891	899	906	914	921	929	937	944	952	959
75	967	974	982	989	997	*005	*012	*020	*027	*035
76	76 042	050	057	065	072	080	087	095	103	110
77	118	125	133	140	148	155	163	170	178	185
78	193	200	208	215	223	230	238	245	253	260
79	268	275	283	290	298	305	313	320	328	335
580	343	350	358	365	373	380	388	395	403	410
81	418	425	433	440	448	455	462	470	477	485
82	492	500	507	515	522	530	537	545	552	559
83	567	574	582	589	597	604	612	619	626	634
84	641	649	656	664	671	678	686	693	701	708
85	716	723	730	738	745	753	760	768	775	782
86	790	797	805	812	819	827	834	842	849	856
87	864	871	879	886	893	901	908	916	923	930
88	938	945	953	960	967	975	982	989	997	*004
89	77 012	019	026	034	041	048	056	063	070	078
590	085	093	100	107	115	122	129	137	144	151
91	159	166	173	181	188	195	203	210	217	225
92	232	240	247	254	262	269	276	283	291	298
93	305	313	320	327	335	342	349	357	364	371
94	379	386	393	401	408	415	422	430	437	444
95	452	459	466	474	481	488	495	503	510	517
96	525	532	539	546	554	561	568	576	583	590
97	597	605	612	619	627	634	641	648	656	663
98	670	677	685	692	699	706	714	721	728	735
99	743	750	757	764	772	779	786	793	801	808
600	815	822	830	837	844	851	859	866	873	880
N.	L. 0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

Prop. Pts.

600–650

N.	L. 0	1	2	3	4	5	6	7	8	9
600	77 815	822	830	837	844	851	859	866	873	880
01	887	895	902	909	916	924	931	938	945	952
02	960	967	974	981	988	996	*003	*010	*017	*025
03	78 032	039	046	053	061	068	075	082	089	097
04	104	111	118	125	132	140	147	154	161	168
05	176	183	190	197	204	211	219	226	233	240
06	247	254	262	269	276	283	290	297	305	312
07	319	326	333	340	347	355	362	369	376	383
08	390	398	405	412	419	426	433	440	447	455
09	462	469	476	483	490	497	504	512	519	526
610	533	540	547	554	561	569	576	583	590	597
11	604	611	618	625	633	640	647	654	661	668
12	675	682	689	696	704	711	718	725	732	739
13	746	753	760	767	774	781	789	796	803	810
14	817	824	831	838	845	852	859	866	873	880
15	888	895	902	909	916	923	930	937	944	951
16	958	965	972	979	986	993	*000	*007	*014	*021
17	79 029	036	043	050	057	064	071	078	085	092
18	099	106	113	120	127	134	141	148	155	162
19	169	176	183	190	197	204	211	218	225	232
620	239	246	253	260	267	274	281	288	295	302
21	309	316	323	330	337	344	351	358	365	372
22	379	386	393	400	407	414	421	428	435	442
23	449	456	463	470	477	484	491	498	505	511
24	518	525	532	539	546	553	560	567	574	581
25	588	595	602	609	616	623	630	637	644	650
26	657	664	671	678	685	692	699	706	713	720
27	727	734	741	748	754	761	768	775	782	789
28	796	803	810	817	824	831	837	844	851	858
29	865	872	879	886	893	900	906	913	920	927
630	934	941	948	955	962	969	975	982	989	996
31	80 003	010	017	024	030	037	044	051	058	065
32	072	079	085	092	099	106	113	120	127	134
33	140	147	154	161	168	175	182	188	195	202
34	209	216	223	229	236	243	250	257	264	271
35	277	284	291	298	305	312	318	325	332	339
36	346	353	359	366	373	380	387	393	400	407
37	414	421	428	434	441	448	455	462	468	475
38	482	489	496	502	509	516	523	530	536	543
39	550	557	564	570	577	584	591	598	604	611
640	618	625	632	638	645	652	659	665	672	679
41	686	693	699	706	713	720	726	733	740	747
42	754	760	767	774	781	787	794	801	808	814
43	821	828	835	841	848	855	862	868	875	882
44	889	895	902	909	916	922	929	936	943	949
45	956	963	969	976	983	990	996	*003	*010	*017
46	81 023	030	037	043	050	057	064	070	077	084
47	090	097	104	111	117	124	131	137	144	151
48	158	164	171	178	184	191	198	204	211	218
49	224	231	238	245	251	258	265	271	278	285
650	291	298	305	311	318	325	331	338	345	351
N.	L. 0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

650–700

N.	L. 0	1	2	3	4	5	6	7	8	9	Prop. Pts.
650	81 291	298	305	311	318	325	331	338	345	351	
51	358	365	371	378	385	391	398	405	411	418	
52	425	431	438	445	451	458	465	471	478	485	
53	491	498	505	511	518	525	531	538	544	551	
54	558	564	571	578	584	591	598	604	611	617	
55	624	631	637	644	651	657	664	671	677	684	
56	690	697	704	710	717	723	730	737	743	750	
57	757	763	770	776	783	790	796	803	809	816	
58	823	829	836	842	849	856	862	869	875	882	
59	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	
61	82 020	027	033	040	046	053	060	066	073	079	
62	086	092	099	105	112	119	125	132	138	145	7
63	151	158	164	171	178	184	191	197	204	210	1 0.7
64	217	223	230	236	243	249	256	263	269	276	2 1.4
65	282	289	295	302	308	315	321	328	334	341	3 2.1
66	347	354	360	367	373	380	387	393	400	406	4 2.8
67	413	419	426	432	439	445	452	458	465	471	5 3.5
68	478	484	491	497	504	510	517	523	530	536	6 4.2
69	543	549	556	562	569	575	582	588	595	601	7 4.9
670	607	614	620	627	633	640	646	653	659	666	8 5.6
71	672	679	685	692	698	705	711	718	724	730	9 6.3
72	737	743	750	756	763	769	776	782	789	795	
73	802	808	814	821	827	834	840	847	853	860	
74	866	872	879	885	892	898	905	911	918	924	
75	930	937	943	950	956	963	969	975	982	988	
76	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
77	83 059	065	072	078	085	091	097	104	110	117	
78	123	129	136	142	149	155	161	168	174	181	
79	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	
81	315	321	327	334	340	347	353	359	366	372	
82	378	385	391	398	404	410	417	423	429	436	6
83	442	448	455	461	467	474	480	487	493	499	1 0.6
84	506	512	518	525	531	537	544	550	556	563	2 1.2
85	569	575	582	588	594	601	607	613	620	626	3 1.8
86	632	639	645	651	658	664	670	677	683	689	4 2.4
87	696	702	708	715	721	727	734	740	746	753	5 3.0
88	759	765	771	778	784	790	797	803	809	816	6 3.6
89	822	828	835	841	847	853	860	866	872	879	7 4.2
690	885	891	897	904	910	916	923	929	935	942	8 4.8
91	948	954	960	967	973	979	985	992	998	*004	9 5.4
92	84 011	017	023	029	036	042	048	055	061	067	
93	073	080	086	092	098	105	111	117	123	130	
94	136	142	148	155	161	167	173	180	186	192	
95	198	205	211	217	223	230	236	242	248	255	
96	261	267	273	280	286	292	298	305	311	317	
97	323	330	336	342	348	354	361	367	373	379	
98	386	392	398	404	410	417	423	429	435	442	
99	448	454	460	466	473	479	485	491	497	504	
700	510	516	522	528	535	541	547	553	559	566	
N.	L. 0	1	2	3	4	5	6	7	8	9	Prop. Pts.

700–750

N.	L. 0	1	2	3	4	5	6	7	8	9
700	84 510	516	522	528	535	541	547	553	559	566
01	572	578	584	590	597	603	609	615	621	628
02	634	640	646	652	658	665	671	677	683	689
03	696	702	708	714	720	726	733	739	745	751
04	757	763	770	776	782	788	794	800	807	813
05	819	825	831	837	844	850	856	862	868	874
06	880	887	893	899	905	911	917	924	930	936
07	942	948	954	960	967	973	979	985	991	997
08	85 003	009	016	022	028	034	040	046	052	058
09	065	071	077	083	089	095	101	107	114	120
710	126	132	138	144	150	156	163	169	175	181
11	187	193	199	205	211	217	224	230	236	242
12	248	254	260	266	272	278	285	291	297	303
13	309	315	321	327	333	339	345	352	358	364
14	370	376	382	388	394	400	406	412	418	425
15	431	437	443	449	455	461	467	473	479	485
16	491	497	503	509	516	522	528	534	540	546
17	552	558	564	570	576	582	588	594	600	606
18	612	618	625	631	637	643	649	655	661	667
19	673	679	685	691	697	703	709	715	721	727
720	733	739	745	751	757	763	769	775	781	788
21	794	800	806	812	818	824	830	836	842	848
22	854	860	866	872	878	884	890	896	902	908
23	914	920	926	932	938	944	950	956	962	968
24	974	980	986	992	998	*004	*010	*016	*022	*028
25	86 034	040	046	052	058	064	070	076	082	088
26	094	100	106	112	118	124	130	136	141	147
27	153	159	165	171	177	183	189	195	201	207
28	213	219	225	231	237	243	249	255	261	267
29	273	279	285	291	297	303	308	314	320	326
730	332	338	344	350	356	362	368	374	380	386
31	392	398	404	410	415	421	427	433	439	445
32	451	457	463	469	475	481	487	493	499	504
33	510	516	522	528	534	540	546	552	558	564
34	570	576	581	587	593	599	605	611	617	623
35	629	635	641	646	652	658	664	670	676	682
36	688	694	700	705	711	717	723	729	735	741
37	747	753	759	764	770	776	782	788	794	800
38	806	812	817	823	829	835	841	847	853	859
39	864	870	876	882	888	894	900	906	911	917
740	923	929	935	941	947	953	958	964	970	976
41	982	988	994	999	*005	*011	*017	*023	*029	*035
42	87 040	046	052	058	064	070	075	081	087	093
43	099	105	111	116	122	128	134	140	146	151
44	157	163	169	175	181	186	192	198	204	210
45	216	221	227	233	239	245	251	256	262	268
46	274	280	286	291	297	303	309	315	320	326
47	332	338	344	349	355	361	367	373	379	384
48	390	396	402	408	413	419	425	431	437	442
49	448	454	460	466	471	477	483	489	495	500
750	506	512	518	523	529	535	541	547	552	558

N. | L. 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

Prop. Pts.

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Prop. Pts.

750–800

N.	L. 0	1	2	3	4	5	6	7	8	9
750	87 506	512	518	523	529	535	541	547	552	558
51	564	570	576	581	587	593	599	604	610	616
52	622	628	633	639	645	651	656	662	668	674
53	679	685	691	697	703	708	714	720	726	731
54	737	743	749	754	760	766	772	777	783	789
55	795	800	806	812	818	823	829	835	841	846
56	852	858	864	869	875	881	887	892	898	904
57	910	915	921	927	933	938	944	950	955	961
58	967	973	978	984	990	996	*001	*007	*013	*018
59	88 024	030	036	041	047	053	058	064	070	076
760	081	087	093	098	104	110	116	121	127	133
61	138	144	150	156	161	167	173	178	184	190
62	195	201	207	213	218	224	230	235	241	247
63	252	258	264	270	275	281	287	292	298	304
64	309	315	321	326	332	338	343	349	355	360
65	366	372	377	383	389	395	400	406	412	417
66	423	429	434	440	446	451	457	463	468	474
67	480	485	491	497	502	508	513	519	525	530
68	536	542	547	553	559	564	570	576	581	587
69	593	598	604	610	615	621	627	632	638	643
770	649	655	660	666	672	677	683	689	694	700
71	705	711	717	722	728	734	739	745	750	756
72	762	767	773	779	784	790	795	801	807	812
73	818	824	829	835	840	846	852	857	863	868
74	874	880	885	891	897	902	908	913	919	925
75	930	936	941	947	953	958	964	969	975	981
76	986	992	997	*003	*009	*014	*020	*025	*031	*037
77	89 042	048	053	059	064	070	076	081	087	092
78	098	104	109	115	120	126	131	137	143	148
79	154	159	165	170	176	182	187	193	198	204
780	209	215	221	226	232	237	243	248	254	260
81	265	271	276	282	287	293	298	304	310	315
82	321	326	332	337	343	348	354	360	365	371
83	376	382	387	393	398	404	409	415	421	426
84	432	437	443	448	454	459	465	470	476	481
85	487	492	498	504	509	515	520	526	531	537
86	542	548	553	559	564	570	575	581	586	592
87	597	603	609	614	620	625	631	636	642	647
88	653	658	664	669	675	680	686	691	697	702
89	708	713	719	724	730	735	741	746	752	757
790	763	768	774	779	785	790	796	801	807	812
91	818	823	829	834	840	845	851	856	862	867
92	873	878	883	889	894	900	905	911	916	922
93	927	933	938	944	949	955	960	966	971	977
94	982	988	993	998	*004	*009	*015	*020	*026	*031
95	90 037	042	048	053	059	064	069	075	080	086
96	091	097	102	108	113	119	124	129	135	140
97	146	151	157	162	168	173	179	184	189	195
98	200	206	211	217	222	227	233	238	244	249
99	255	260	266	271	276	282	287	293	298	304
800	309	314	320	325	331	336	342	347	352	358
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Prop. Pts.

800–850

N.	L. 0	1	2	3	4	5	6	7	8	9	Prop. Pts.
800	90 309	314	320	325	331	336	342	347	352	358	
01	363	369	374	380	385	390	396	401	407	412	
02	417	423	428	434	439	445	450	455	461	466	
03	472	477	482	488	493	499	504	509	515	520	
04	526	531	536	542	547	553	558	563	569	574	
05	580	585	590	596	601	607	612	617	623	628	
06	634	639	644	650	655	660	666	671	677	682	
07	687	693	698	703	709	714	720	725	730	736	
08	741	747	752	757	763	768	773	779	784	789	
09	795	800	806	811	816	822	827	832	838	843	
810	849	854	859	865	870	875	881	886	891	897	
11	902	907	913	918	924	929	934	940	945	950	
12	956	961	966	972	977	982	988	993	998	*004	
13	91 009	014	020	025	030	036	041	046	052	057	
14	062	068	073	078	084	089	094	100	105	110	
15	116	121	126	132	137	142	148	153	158	164	
16	169	174	180	185	190	196	201	206	212	217	
17	222	228	233	238	243	249	254	259	265	270	
18	275	281	286	291	297	302	307	312	318	323	
19	328	334	339	344	350	355	360	365	371	376	
820	381	387	392	397	403	408	413	418	424	429	
21	434	440	445	450	455	461	466	471	477	482	
22	487	492	498	503	508	514	519	524	529	535	
23	540	545	551	556	561	566	572	577	582	587	
24	593	598	603	609	614	619	624	630	635	640	
25	645	651	656	661	666	672	677	682	687	693	
26	698	703	709	714	719	724	730	735	740	745	
27	751	756	761	766	772	777	782	787	793	798	
28	803	808	814	819	824	829	834	840	845	850	
29	855	861	866	871	876	882	887	892	897	903	
830	908	913	918	924	929	934	939	944	950	955	
31	960	965	971	976	981	986	991	997	*002	*007	
32	92 012	018	023	028	033	038	044	049	054	059	
33	065	070	075	080	085	091	096	101	106	111	
34	117	122	127	132	137	143	148	153	158	163	
35	169	174	179	184	189	195	200	205	210	215	
36	221	226	231	236	241	247	252	257	262	267	
37	273	278	283	288	293	298	304	309	314	319	
38	324	330	335	340	345	350	355	361	366	371	
39	376	381	387	392	397	402	407	412	418	423	
840	428	433	438	443	449	454	459	464	469	474	
41	480	485	490	495	500	505	511	516	521	526	
42	531	536	542	547	552	557	562	567	572	578	
43	583	588	593	598	603	609	614	619	624	629	
44	634	639	645	650	655	660	665	670	675	681	
45	686	691	696	701	706	711	716	722	727	732	
46	737	742	747	752	758	763	768	773	778	783	
47	788	793	799	804	809	814	819	824	829	834	
48	840	845	850	855	860	865	870	875	881	886	
49	891	896	901	906	911	916	921	927	932	937	
850	942	947	952	957	962	967	973	978	983	988	
N.	L. 0	1	2	3	4	5	6	7	8	9	Prop. Pts.

Prop. Pts.

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

850–900

N.	L. 0	1	2	3	4	5	6	7	8	9
850	92 942	947	952	957	962	967	973	978	983	988
51	993	998	*003	*008	*013	*018	*024	*029	*034	*039
52	93 044	049	054	059	064	069	075	080	085	090
53	095	100	105	110	115	120	125	131	136	141
54	146	151	156	161	166	171	176	181	186	192
55	197	202	207	212	217	222	227	232	237	242
56	247	252	258	263	268	273	278	283	288	293
57	298	303	308	313	318	323	328	334	339	344
58	349	354	359	364	369	374	379	384	389	394
59	399	404	409	414	420	425	430	435	440	445
860	450	455	460	465	470	475	480	485	490	495
61	500	505	510	515	520	526	531	536	541	546
62	551	556	561	566	571	576	581	586	591	596
63	601	606	611	616	621	626	631	636	641	646
64	651	656	661	666	671	676	682	687	692	697
65	702	707	712	717	722	727	732	737	742	747
66	752	757	762	767	772	777	782	787	792	797
67	802	807	812	817	822	827	832	837	842	847
68	852	857	862	867	872	877	882	887	892	897
69	902	907	912	917	922	927	932	937	942	947
870	952	957	962	967	972	977	982	987	992	997
71	94 002	007	012	017	022	027	032	037	042	047
72	052	057	062	067	072	077	082	086	091	096
73	101	106	111	116	121	126	131	136	141	146
74	151	156	161	166	171	176	181	186	191	196
75	201	206	211	216	221	226	231	236	240	245
76	250	255	260	265	270	275	280	285	290	295
77	300	305	310	315	320	325	330	335	340	345
78	349	354	359	364	369	374	379	384	389	394
79	399	404	409	414	419	424	429	433	438	443
880	448	453	458	463	468	473	478	483	488	493
81	498	503	507	512	517	522	527	532	537	542
82	547	552	557	562	567	571	576	581	586	591
83	596	601	606	611	616	621	626	630	635	640
84	645	650	655	660	665	670	675	680	685	689
85	694	699	704	709	714	719	724	729	734	738
86	743	748	753	758	763	768	773	778	783	787
87	792	797	802	807	812	817	822	827	832	836
88	841	846	851	856	861	866	871	876	880	885
89	890	895	900	905	910	915	919	924	929	934
890	939	944	949	954	959	963	968	973	978	983
91	988	993	998	*002	*007	*012	*017	*022	*027	*032
92	95 036	041	046	051	056	061	066	071	075	080
93	085	090	095	100	105	109	114	119	124	129
94	134	139	143	148	153	158	163	168	173	177
95	182	187	192	197	202	207	211	216	221	226
96	231	236	240	245	250	255	260	265	270	274
97	279	284	289	294	299	303	308	313	318	323
98	328	332	337	342	347	352	357	361	366	371
99	376	381	386	390	395	400	405	410	415	419
900	424	429	434	439	444	448	453	458	463	468
N.	L. 0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

900–950

N.	L. 0	1	2	3	4	5	6	7	8	9	Prop. Pts.
900	95 424	429	434	439	444	448	453	458	463	468	
01	472	477	482	487	492	497	501	506	511	516	
02	521	525	530	535	540	545	550	554	559	564	
03	569	574	578	583	588	593	598	602	607	612	
04	617	622	626	631	636	641	646	650	655	660	
05	665	670	674	679	684	689	694	698	703	708	
06	713	718	722	727	732	737	742	746	751	756	
07	761	766	770	775	780	785	789	794	799	804	
08	809	813	818	823	828	832	837	842	847	852	
09	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	
11	952	957	961	966	971	976	980	985	990	995	
12	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	
13	96 047	052	057	061	066	071	076	080	085	090	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
14	095	099	104	109	114	118	123	128	133	137	
15	142	147	152	156	161	166	171	175	180	185	
16	190	194	199	204	209	213	218	223	227	232	
17	237	242	246	251	256	261	265	270	275	280	
18	284	289	294	298	303	308	313	317	322	327	
19	332	336	341	346	350	355	360	365	369	374	
920	379	384	388	393	398	402	407	412	417	421	
21	426	431	435	440	445	450	454	459	464	468	
22	473	478	483	487	492	497	501	506	511	515	
23	520	525	530	534	539	544	548	553	558	562	
24	567	572	577	581	586	591	595	600	605	609	
25	614	619	624	628	633	638	642	647	652	656	
26	661	666	670	675	680	685	689	694	699	703	
27	708	713	717	722	727	731	736	741	745	750	
28	755	759	764	769	774	778	783	788	792	797	
29	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	
31	895	900	904	909	914	918	923	928	932	937	
32	942	946	951	956	960	965	970	974	979	984	4 1 0.4 2 0.8 3 1.2 4 1.6 5 2.0 6 2.4 7 2.8 8 3.2 9 3.6
33	988	993	997	*002	*007	*011	*016	*021	*025	*030	
34	97 035	039	044	049	053	058	063	067	072	077	
35	081	086	090	095	100	104	109	114	118	123	
36	128	132	137	142	146	151	155	160	165	169	
37	174	179	183	188	192	197	202	206	211	216	
38	220	225	230	234	239	243	248	253	257	262	
39	267	271	276	280	285	290	294	299	304	308	
940	313	317	322	327	331	336	340	345	350	354	
41	359	364	368	373	377	382	387	391	396	400	
42	405	410	414	419	424	428	433	437	442	447	
43	451	456	460	465	470	474	479	483	488	493	
44	497	502	506	511	516	520	525	529	534	539	
45	543	548	552	557	562	566	571	575	580	585	
46	589	594	598	603	607	612	617	621	626	630	
47	635	640	644	649	653	658	663	667	672	676	
48	681	685	690	695	699	704	708	713	717	722	
49	727	731	736	740	745	749	754	759	763	768	
950	772	777	782	786	791	795	800	804	809	813	
N.	L. 0	1	2	3	4	5	6	7	8	9	Prop. Pts.

950–1000

N.	L. 0	1	2	3	4	5	6	7	8	9	Prop. Pts.
950	97 772	777	782	786	791	795	800	804	809	813	
51	818	823	827	832	836	841	845	850	855	859	
52	864	868	873	877	882	886	891	896	900	905	
53	909	914	918	923	928	932	937	941	946	950	
54	955	959	964	968	973	978	982	987	991	996	
55	98 000	005	009	014	019	023	028	032	037	041	
56	046	050	055	059	064	068	073	078	082	087	
57	091	096	100	105	109	114	118	123	127	132	
58	137	141	146	150	155	159	164	168	173	177	
59	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	
61	272	277	281	286	290	295	299	304	308	313	
62	318	322	327	331	336	340	345	349	354	358	
63	363	367	372	376	381	385	390	394	399	403	
64	408	412	417	421	426	430	435	439	444	448	
65	453	457	462	466	471	475	480	484	489	493	
66	498	502	507	511	516	520	525	529	534	538	
67	543	547	552	556	561	565	570	574	579	583	
68	588	592	597	601	605	610	614	619	623	628	
69	632	637	641	646	650	655	659	664	668	673	
970	677	682	686	691	695	700	704	709	713	717	
71	722	726	731	735	740	744	749	753	758	762	
72	767	771	776	780	784	789	793	798	802	807	
73	811	816	820	825	829	834	838	843	847	851	
74	856	860	865	869	874	878	883	887	892	896	
75	900	905	909	914	918	923	927	932	936	941	
76	945	949	954	958	963	967	972	976	981	985	
77	989	994	998	*003	*007	*012	*016	*021	*025	*029	
78	99 034	038	043	047	052	056	061	065	069	074	
79	078	083	087	092	096	100	105	109	114	118	
980	123	127	131	136	140	145	149	154	158	162	
81	167	171	176	180	185	189	193	198	202	207	
82	211	216	220	224	229	233	238	242	247	251	
83	255	260	264	269	273	277	282	286	291	295	
84	300	304	308	313	317	322	326	330	335	339	
85	344	348	352	357	361	366	370	374	379	383	
86	388	392	396	401	405	410	414	419	423	427	
87	432	436	441	445	449	454	458	463	467	471	
88	476	480	484	489	493	498	502	506	511	515	
89	520	524	528	533	537	542	546	550	555	559	
990	564	568	572	577	581	585	590	594	599	603	
91	607	612	616	621	625	629	634	638	642	647	
92	651	656	660	664	669	673	677	682	686	691	
93	695	699	704	708	712	717	721	726	730	734	
94	739	743	747	752	756	760	765	769	774	778	
95	782	787	791	795	800	804	808	813	817	822	
96	826	830	835	839	843	848	852	856	861	865	
97	870	874	878	883	887	891	896	900	904	909	
98	913	917	922	926	930	935	939	944	948	952	
99	957	961	965	970	974	978	983	987	991	996	
1000	00 000	004	009	013	017	022	026	030	035	039	
N.	**L. 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

Prop. Pts.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

TABLE II

FIVE-PLACE LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS

Table II contains the common logarithms of the trigonometric functions for angles from 0° to 90° as follows:

(1) log sin α = log tan α, for every second 0° to 3′, and log cos β = log cot β, for every second, 89° 57′ to 90°, page 22.

(2) logarithms of sine, tangent, and cosine for every 10″ from 0° to 2°; also the logarithms of sine, cosine, and cotangent for every 10″ from 88° to 90°, pages 23–28.

(3) logarithms of each function for every 1′ from 0° to 90°, pages 29–73.

log sin = log tan **0°**

″	0′	1′	2′	″	″	0′	1′	2′	″
0	– ∞	6.46 373	6.76 476	**60**	**30**	6.16 270	6.63 982	6.86 167	**30**
1	4.68 557	6.47 090	6.76 836	59	31	6.17 694	6.64 462	6.86 455	29
2	4.98 660	6.47 797	6.77 193	58	32	6.19 072	6.64 936	6.86 742	28
3	5.16 270	6.48 492	6.77 548	57	33	6.20 409	6.65 406	6.87 027	27
4	5.28 763	6.49 175	6.77 900	56	34	6.21 705	6.65 870	6.87 310	26
5	5.38 454	6.49 849	6.78 248	**55**	**35**	6.22 964	6.66 330	6.87 591	**25**
6	5.46 373	6.50 512	6.78 595	54	36	6.24 188	6.66 785	6.87 870	24
7	5.53 067	6.51 165	6.78 938	53	37	6.25 378	6.67 235	6.88 147	23
8	5.58 866	6.51 808	6.79 278	52	38	6.26 536	6.67 680	6.88 423	22
9	5.63 982	6.52 442	6.79 616	51	39	6.27 664	6.68 121	6.88 697	21
10	5.68 557	6.53 067	6.79 952	**50**	**40**	6.28 763	6.68 557	6.88 969	**20**
11	5.72 697	6.53 683	6.80 285	49	41	6.29 836	6.68 990	6.89 240	19
12	5.76 476	6.54 291	6.80 615	48	42	6.30 882	6.69 418	6.89 509	18
13	5.79 952	6.54 890	6.80 943	47	43	6.31 904	6.69 841	6.89 776	17
14	5.83 170	6.55 481	6.81 268	46	44	6.32 903	6.70 261	6.90 042	16
15	5.86 167	6.56 064	6.81 591	**45**	**45**	6.33 879	6.70 676	6.90 306	**15**
16	5.88 969	6.56 639	6.81 911	44	46	6.34 833	6.71 088	6.90 568	14
17	5.91 602	6.57 207	6.82 230	43	47	6.35 767	6.71 496	6.90 829	13
18	5.94 085	6.57 767	6.82 545	42	48	6.36 682	6.71 900	6.91 088	12
19	5.96 433	6.58 320	6.82 859	41	49	6.37 577	6.72 300	6.91 346	11
20	5.98 660	6.58 866	6.83 170	**40**	**50**	6.38 454	6.72 697	6.91 602	**10**
21	6.00 779	6.59 406	6.83 479	39	51	6.39 315	6.73 090	6.91 857	9
22	6.02 800	6.59 939	6.83 786	38	52	6.40 158	6.73 479	6.92 110	8
23	6.04 730	6.60 465	6.84 091	37	53	6.40 985	6.73 865	6.92 362	7
24	6.06 579	6.60 985	6.84 394	36	54	6.41 797	6.74 248	6.92 612	6
25	6.08 351	6.61 499	6.84 694	**35**	**55**	6.42 594	6.74 627	6.92 861	**5**
26	6.10 055	6.62 007	6.84 993	34	56	6.43 376	6.75 003	6.93 109	4
27	6.11 694	6.62 509	6.85 289	33	57	6.44 145	6.75 376	6.93 355	3
28	6.13 273	6.63 006	6.85 584	32	58	6.44 900	6.75 746	6.93 599	2
29	6.14 797	6.63 496	6.85 876	31	59	6.45 643	6.76 112	6.93 843	1
30	6.16 270	6.63 982	6.86 167	**30**	**60**	6.46 373	6.76 476	6.94 085	**0**
″	59′	58′	57′	″	″	59′	58′	57′	″

89° **log cos = log cot**

0°

′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
0 0	— ∞	— ∞	10.00000	0 **60**	**10** 0	7.46 373	7.46 373	10.00000	0 **50**
10	5.68 557	5.68 557	10.00000	50	10	7.47 090	7.47 091	10.00000	50
20	5.98 660	5.98 660	10.00000	40	20	7.47 797	7.47 797	10.00000	40
30	6.16 270	6.16 270	10.00000	30	30	7.48 491	7.48 492	10.00000	30
40	6.28 763	6.28 763	10.00000	20	40	7.49 175	7.49 176	10.00000	20
50	6.38 454	6 38 454	10.00000	10	50	7.49 849	7.49 849	10.00000	10
1 0	6.46 373	6.46 373	10.00000	0 **59**	**11** 0	7.50 512	7.50 512	10.00000	0 **49**
10	6.53 067	6.53 067	10.00000	50	10	7.51 165	7.51 165	10.00000	50
20	6.58 866	6.58 866	10.00000	40	20	7.51 808	7.51 809	10.00000	40
30	6.63 982	6.63 982	10.00000	30	30	7.52 442	7.52 443	10.00000	30
40	6.68 557	6.68 557	10.00000	20	40	7.53 067	7.53 067	10.00000	20
50	6.72 697	6.72 697	10.00000	10	50	7.53 683	7.53 683	10.00000	10
2 0	6.76 476	6.76 476	10.00000	0 **58**	**12** 0	7.54 291	7.54 291	10.00000	0 **48**
10	6.79 952	6.79 952	10.00000	50	10	7.54 890	7.54 890	10.00000	50
20	6.83 170	6.83 170	10.00000	40	20	7.55 481	7.55 481	10.00000	40
30	6.86 167	6.86 167	10.00000	30	30	7.56 064	7.56 064	10.00000	30
40	6.88 969	6.88 969	10.00000	20	40	7.56 639	7.56 639	10.00000	20
50	6.91 602	6.91 602	10.00000	10	50	7.57 206	7.57 207	10.00000	10
3 0	6.94 085	6.94 085	10.00000	0 **57**	**13** 0	7.57 767	7.57 767	10.00000	0 **47**
10	6.96 433	6.96 433	10.00000	50	10	7.58 320	7.58 320	10.00000	50
20	6.98 660	6.98 661	10.00000	40	20	7.58 866	7.58 867	10.00000	40
30	7.00 779	7.00 779	10.00000	30	30	7.59 406	7.59 406	10.00000	30
40	7.02 800	7.02 800	10.00000	20	40	7.59 939	7.59 939	10.00000	20
50	7.04 730	7.04 730	10.00000	10	50	7.60 465	7.60 466	10.00000	10
4 0	7.06 579	7.06 579	10.00000	0 **56**	**14** 0	7.60 985	7.60 986	10.00000	0 **46**
10	7.08 351	7.08 352	10.00000	50	10	7.61 499	7.61 500	10.00000	50
20	7.10 055	7.10 055	10.00000	40	20	7.62 007	7.62 008	10.00000	40
30	7.11 694	7.11 694	10.00000	30	30	7.62 509	7.62 510	10.00000	30
40	7.13 273	7.13 273	10.00000	20	40	7.63 006	7.63 006	10.00000	20
50	7.14 797	7.14 797	10.00000	10	50	7.63 496	7.63 497	10.00000	10
5 0	7.16 270	7.16 270	10.00000	0 **55**	**15** 0	7.63 982	7.63 982	10.00000	0 **45**
10	7.17 694	7.17 694	10.00000	50	10	7.64 461	7.64 462	10.00000	50
20	7.19 072	7.19 073	10.00000	40	20	7.64 936	7.64 937	10.00000	40
30	7.20 409	7.20 409	10.00000	30	30	7.65 406	7.65 406	10.00000	30
40	7.21 705	7.21 705	10.00000	20	40	7.65 870	7.65 871	10.00000	20
50	7.22 964	7.22 964	10.00000	10	50	7.66 330	7.66 330	10.00000	10
6 0	7.24 188	7.24 188	10.00000	0 **54**	**16** 0	7.66 784	7.66 785	10.00000	0 **44**
10	7.25 378	7.25 378	10.00000	50	10	7.67 235	7.67 235	10.00000	50
20	7.26 536	7.26 536	10.00000	40	20	7.67 680	7.67 680	10.00000	40
30	7.27 664	7.27 664	10.00000	30	30	7.68 121	7.68 121	10.00000	30
40	7.28 763	7.28 764	10.00000	20	40	7.68 557	7.68 558	9.99999	20
50	7.29 836	7.29 836	10.00000	10	50	7.68 989	7.68 990	9.99999	10
7 0	7.30 882	7.30 882	10.00000	0 **53**	**17** 0	7.69 417	7.69 418	9.99999	0 **43**
10	7.31 904	7.31 904	10.00000	50	10	7.69 841	7.69 842	9.99999	50
20	7.32 903	7.32 903	10.00000	40	20	7.70 261	7.70 261	9.99999	40
30	7.33 879	7.33 879	10.00000	30	30	7.70 676	7.70 677	9.99999	30
40	7.34 833	7.34 833	10.00000	20	40	7.71 088	7.71 088	9.99999	20
50	7.35 767	7.35 767	10.00000	10	50	7.71 496	7.71 496	9.99999	10
8 0	7.36 682	7.36 682	10.00000	0 **52**	**18** 0	7.71 900	7.71 900	9.99999	0 **42**
10	7.37 577	7.37 577	10.00000	50	10	7.72 300	7.72 301	9.99999	50
20	7.38 454	7.38 455	10.00000	40	20	7.72 697	7.72 697	9.99999	40
30	7.39 314	7.39 315	10.00000	30	30	7.73 090	7.73 090	9.99999	30
40	7.40 158	7.40 158	10.00000	20	40	7.73 479	7.73 480	9.99999	20
50	7.40 985	7.40 985	10.00000	10	50	7.73 865	7.73 866	9.99999	10
9 0	7.41 797	7.41 797	10.00000	0 **51**	**19** 0	7.74 248	7.74 248	9.99999	0 **41**
10	7.42 594	7.42 594	10.00000	50	10	7.74 627	7.74 628	9.99999	50
20	7.43 376	7.43 376	10.00000	40	20	7.75 003	7.75 004	9.99999	40
30	7.44 145	7.44 145	10.00000	30	30	7.75 376	7.75 377	9.99999	30
40	7.44 900	7.44 900	10.00000	20	40	7.75 745	7.75 746	9.99999	20
50	7.45 643	7.45 643	10.00000	10	50	7.76 112	7.76 113	9.99999	10
10 0	7.46 373	7.46 373	10.00000	0 **50**	**20** 0	7.76 475	7.76 476	9.99999	0 **40**
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	″ ′

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′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
20 0	7.76 475	7.76 476	9.99 999	0 **40**	**30** 0	7.94 084	7.94 086	9.99 998	0 **30**
10	7.76 836	7.76 837	9.99 999	50	10	7.94 325	7.94 326	9.99 998	50
20	7.77 193	7.77 194	9.99 999	40	20	7.94 564	7.94 566	9.99 998	40
30	7.77 548	7.77 549	9.99 999	30	30	7.94 802	7.94 804	9.99 998	30
40	7.77 899	7.77 900	9.99 999	20	40	7.95 039	7.95 040	9.99 998	20
50	7.78 248	7.78 249	9.99 999	10	50	7.95 274	7.95 276	9.99 998	10
21 0	7.78 594	7.78 595	9.99 999	0 **39**	**31** 0	7.95 508	7.95 510	9.99 998	0 **29**
10	7.78 938	7.78 938	9.99 999	50	10	7.95 741	7.95 743	9.99 998	50
20	7.79 278	7.79 279	9.99 999	40	20	7.95 973	7.95 974	9.99 998	40
30	7.79 616	7.79 617	9.99 999	30	30	7.96 203	7.96 205	9.99 998	30
40	7.79 952	7.79 952	9.99 999	20	40	7.96 432	7.96 434	9.99 998	20
50	7.80 284	7.80 285	9.99 999	10	50	7.96 660	7.96 662	9.99 998	10
22 0	7.80 615	7.80 615	9.99 999	0 **38**	**32** 0	7.96 887	7.96 889	9.99 998	0 **28**
10	7.80 942	7.80 943	9.99 999	50	10	7.97 113	7.97 114	9.99 998	50
20	7.81 268	7.81 269	9.99 999	40	20	7.97 337	7.97 339	9.99 998	40
30	7.81 591	7.81 591	9.99 999	30	30	7.97 560	7.97 562	9.99 998	30
40	7.81 911	7.81 912	9.99 999	20	40	7.97 782	7.97 784	9.99 998	20
50	7.82 229	7.82 230	9.99 999	10	50	7.98 003	7.98 005	9.99 998	10
23 0	7.82 545	7.82 546	9.99 999	0 **37**	**33** 0	7.98 223	7.98 225	9.99 998	0 **27**
10	7.82 859	7.82 860	9.99 999	50	10	7.98 442	7.98 444	9.99 998	50
20	7.83 170	7.83 171	9.99 999	40	20	7.98 660	7.98 662	9.99 998	40
30	7.83 479	7.83 480	9.99 999	30	30	7.98 876	7.98 878	9.99 998	30
40	7.83 786	7.83 787	9.99 999	20	40	7.99 092	7.99 094	9.99 998	20
50	7.84 091	7.84 092	9.99 999	10	50	7.99 306	7.99 308	9.99 998	10
24 0	7.84 393	7.84 394	9.99 999	0 **36**	**34** 0	7.99 520	7.99 522	9.99 998	0 **26**
10	7.84 694	7.84 695	9.99 999	50	10	7.99 732	7.99 734	9.99 998	50
20	7.84 992	7.84 993	9.99 999	40	20	7.99 943	7.99 946	9.99 998	40
30	7.85 289	7.85 290	9.99 999	30	30	8.00 154	8.00 156	9.99 998	30
40	7.85 583	7.85 584	9.99 999	20	40	8.00 363	8.00 365	9.99 998	20
50	7.85 876	7.85 877	9.99 999	10	50	8.00 571	8.00 574	9.99 998	10
25 0	7.86 166	7.86 167	9.99 999	0 **35**	**35** 0	8.00 779	8.00 781	9.99 998	0 **25**
10	7.86 455	7.86 456	9.99 999	50	10	8.00 985	8.00 987	9.99 998	50
20	7.86 741	7.86 743	9.99 999	40	20	8.01 190	8.01 193	9.99 998	40
30	7.87 026	7.87 027	9.99 999	30	30	8.01 395	8.01 397	9.99 998	30
40	7.87 309	7.87 310	9.99 999	20	40	8.01 598	8.01 600	9.99 998	20
50	7.87 590	7.87 591	9.99 999	10	50	8.01 801	8.01 803	9.99 998	10
26 0	7.87 870	7.87 871	9.99 999	0 **34**	**36** 0	8.02 002	8.02 004	9.99 998	0 **24**
10	7.88 147	7.88 148	9.99 999	50	10	8.02 203	8.02 205	9.99 998	50
20	7.88 423	7.88 424	9.99 999	40	20	8.02 402	8.02 405	9.99 998	40
30	7.88 697	7.88 698	9.99 999	30	30	8.02 601	8.02 604	9.99 998	30
40	7.88 969	7.88 970	9.99 999	20	40	8.02 799	8.02 801	9.99 998	20
50	7.89 240	7.89 241	9.99 999	10	50	8.02 996	8.02 998	9.99 998	10
27 0	7.89 509	7.89 510	9.99 999	0 **33**	**37** 0	8.03 192	8.03 194	9.99 997	0 **23**
10	7.89 776	7.89 777	9.99 999	50	10	8.03 387	8.03 390	9.99 997	50
20	7.90 041	7.90 043	9.99 999	40	20	8.03 581	8.03 584	9.99 997	40
30	7.90 305	7.90 307	9.99 999	30	30	8.03 775	8.03 777	9.99 997	30
40	7.90 568	7.90 569	9.99 999	20	40	8.03 967	8.03 970	9.99 997	20
50	7.90 829	7.90 830	9.99 999	10	50	8.04 159	8.04 162	9.99 997	10
28 0	7.91 088	7.91 089	9.99 999	0 **32**	**38** 0	8.04 350	8.04 353	9.99 997	0 **22**
10	7.91 346	7.91 347	9.99 999	50	10	8.04 540	8.04 543	9.99 997	50
20	7.91 602	7.91 603	9.99 999	40	20	8.04 729	8.04 732	9.99 997	40
30	7.91 857	7.91 858	9.99 999	30	30	8.04 918	8.04 921	9.99 997	30
40	7.92 110	7.92 111	9.99 998	20	40	8.05 105	8.05 108	9.99 997	20
50	7.92 362	7.92 363	9.99 998	10	50	8.05 292	8.05 295	9.99 997	10
29 0	7.92 612	7.92 613	9.99 998	0 **31**	**39** 0	8.05 478	8.05 481	9.99 997	0 **21**
10	7.92 861	7.92 862	9.99 998	50	10	8.05 663	8.05 666	9.99 997	50
20	7.93 108	7.93 110	9.99 998	40	20	8.05 848	8.05 851	9.99 997	40
30	7.93 354	7.93 356	9.99 998	30	30	8.06 031	8.06 034	9.99 997	30
40	7.93 599	7.93 601	9.99 998	20	40	8.06 214	8.06 217	9.99 997	20
50	7.93 842	7.93 844	9.99 998	10	50	8.06 396	8.06 399	9.99 997	10
30 0	7.94 084	7.94 086	9.99 998	0 **30**	**40** 0	8.06 578	8.06 581	9.99 997	0 **20**
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	″ ′

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′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
40 0	8.06 578	8.06 581	9.99 997	0 **20**	**50** 0	8.16 268	8.16 273	9.99 995	0 **10**
10	8.06 758	8.06 761	9.99 997	50	10	8.16 413	8.16 417	9.99 995	50
20	8.06 938	8.06 941	9.99 997	40	20	8.16 557	8.16 561	9.99 995	40
30	8.07 117	8.07 120	9.99 997	30	30	8.16 700	8.16 705	9.99 995	30
40	8.07 295	8.07 299	9.99 997	20	40	8.16 843	8.16 848	9.99 995	20
50	8.07 473	8.07 476	9.99 997	10	50	8.16 986	8.16 991	9.99 995	10
41 0	8.07 650	8.07 653	9.99 997	0 **19**	**51** 0	8.17 128	8.17 133	9.99 995	0 **9**
10	8.07 826	8.07 829	9.99 997	50	10	8.17 270	8.17 275	9.99 995	50
20	8.08 002	8.08 005	9.99 997	40	20	8.17 411	8.17 416	9.99 995	40
30	8.08 176	8.08 180	9.99 997	30	30	8.17 552	8.17 557	9.99 995	30
40	8.08 350	8.08 354	9.99 997	20	40	8.17 692	8.17 697	9.99 995	20
50	8.08 524	8.08 527	9.99 997	10	50	8.17 832	8.17 837	9.99 995	10
42 0	8.08 696	8.08 700	9.99 997	0 **18**	**52** 0	8.17 971	8.17 976	9.99 995	0 **8**
10	8.08 868	8.08 872	9.99 997	50	10	8.18 110	8.18 115	9.99 995	50
20	8.09 040	8.09 043	9.99 997	40	20	8.18 249	8.18 254	9.99 995	40
30	8.09 210	8.09 214	9.99 997	30	30	8.18 387	8.18 392	9.99 995	30
40	8.09 380	8.09 384	9.99 997	20	40	8.18 524	8.18 530	9.99 995	20
50	8.09 550	8.09 553	9·99 997	10	50	8.18 662	8.18 667	9.99 995	10
43 0	8.09 718	8.09 722	9.99 997	0 **17**	**53** 0	8.18 798	8.18 804	9.99 995	0 **7**
10	8.09 886	8.09 890	9.99 997	50	10	8.18 935	8.18 940	9.99 995	50
20	8.10 054	8.10 057	9.99 997	40	20	8.19 071	8.19 076	9.99 995	40
30	8.10 220	8.10 224	9.99 997	30	30	8.19 206	8.19 212	9.99 995	30
40	8 10 386	8.10 390	9.99 997	20	40	8.19 341	8.19 347	9.99 995	20
50	8.10 552	8.10 555	9.99 996	10	50	8.19 476	8.19 481	9.99 995	10
44 0	8.10 717	8.10 720	9.99 996	0 **16**	**54** 0	8.19 610	8.19 616	9.99 995	0 **6**
10	8.10 881	8.10 884	9.99 996	50	10	8.19 744	8.19 749	9.99 995	50
20	8.11 044	8.11 048	9.99 996	40	20	8.19 877	8.19 883	9.99 995	40
30	8.11 207	8.11 211	9.99 996	30	30	8.20 010	8.20 016	9.99 995	30
40	8.11 370	8.11 373	9.99 996	20	40	8.20 143	8.20 149	9.99 995	20
50	8.11 531	8.11 535	9.99 996	10	50	8.20 275	8.20 281	9.99 994	10
45 0	8.11 693	8.11 696	9.99 996	0 **15**	**55** 0	8.20 407	8.20 413	9.99 994	0 **5**
10	8.11 853	8.11 857	9.99 996	50	10	8.20 538	8.20 544	9.99 994	50
20	8.12 013	8.12 017	9.99 996	40	20	8.20 669	8.20 675	9.99 994	40
30	8.12 172	8.12 176	9.99 996	30	30	8.20 800	8.20 806	9.99 994	30
40	8.12 331	8.12 335	9.99 996	20	40	8.20 930	8.20 936	9.99 994	20
50	8.12 489	8.12 493	9.99 996	10	50	8.21 060	8.21 066	9.99 994	10
46 0	8.12 647	8.12 651	9.99 996	0 **14**	**56** 0	8.21 189	8.21 195	9.99 994	0 **4**
10	8.12 804	8.12 808	9.99 996	50	10	8.21 319	8.21 324	9.99 994	50
20	8.12 961	8.12 965	9.99 996	40	20	8.21 447	8.21 453	9.99 994	40
30	8.13 117	8.13 121	9.99 996	30	30	8.21 576	8.21 581	9.99 994	30
40	8.13 272	8.13 276	9.99 996	20	40	8.21 703	8.21 709	9.99 994	20
50	8.13 427	8.13 431	9.99 996	10	50	8.21 831	8.21 837	9.99 994	10
47 0	8.13 581	8.13 585	9.99 996	0 **13**	**57** 0	8.21 958	8.21 964	9.99 994	0 **3**
10	8.13 735	8.13 739	9.99 996	50	10	8.22 085	8.22 091	9.99 994	50
20	8.13 888	8.13 892	9.99 996	40	20	8.22 211	8.22 217	9.99 994	40
30	8.14 041	8.14 045	9.99 996	30	30	8.22 337	8.22 343	9.99 994	30
40	8.14 193	8.14 197	9.99 996	20	40	8.22 463	8.22 469	9.99 994	20
50	8.14 344	8.14 348	9.99 996	10	50	8.22 588	8.22 595	9.99 994	10
48 0	8.14 495	8.14 500	9.99 996	0 **12**	**58** 0	8.22 713	8.22 720	9.99 994	0 **2**
10	8.14 646	8.14 650	9.99 996	50	10	8.22 838	8.22 844	9.99 994	50
20	8.14 796	8.14 800	9.99 996	40	20	8.22 962	8.22 968	9.99 994	40
30	8.14 945	8.14 950	9.99 996	30	30	8.23 086	8.23 092	9.99 994	30
40	8.15 094	8.15 099	9.99 996	20	40	8.23 210	8.23 216	9.99 994	20
50	8.15 243	8.15 247	9.99 996	10	50	8.23 333	8.23 339	9.99 994	10
49 0	8.15 391	8.15 395	9.99 996	0 **11**	**59** 0	8.23 456	8.23 462	9.99 994	0 **1**
10	8.15 538	8.15 543	9.99 996	50	10	8.23 578	8.23 585	9.99 994	50
20	8.15 685	8.15 690	9.99 996	40	20	8.23 700	8.23 707	9.99 994	40
30	8.15 832	8.15 836	9.99 996	30	30	8.23 822	8.23 829	9.99 993	30
40	8.15 978	8.15 982	9.99 995	20	40	8.23 944	8.23 950	9.99 993	20
50	8.16 123	8.16 128	9.99 995	10	50	8.24 065	8.24 071	9.99 993	10
50 0	8.16 268	8.16 273	9.99 995	0 **10**	**60** 0	8.24 186	8.24 192	9.99 993	0 0
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	″ ′

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′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
0 0	8.24 186	8.24 192	9.99 993	0 **60**	**10** 0	8.30 879	8.30 888	9.99 991	0 **50**
10	8.24 306	8.24 313	9.99 993	50	10	8.30 983	8.30 992	9.99 991	50
20	8.24 426	8.24 433	9.99 993	40	20	8.31 086	8.31 095	9.99 991	40
30	8.24 546	8.24 553	9.99 993	30	30	8.31 188	8.31 198	9.99 991	30
40	8.24 665	8.24 672	9.99 993	20	40	8.31 291	8.31 300	9.99 991	20
50	8.24 785	8.24 791	9.99 993	10	50	8.31 393	8.31 403	9.99 991	10
1 0	8.24 903	8.24 910	9.99 993	0 **59**	**11** 0	8.31 495	8.31 505	9.99 991	0 **49**
10	8.25 022	8.25 029	9.99 993	50	10	8.31 597	8.31 606	9.99 991	50
20	8.25 140	8.25 147	9.99 993	40	20	8.31 699	8.31 708	9.99 991	40
30	8.25 258	8.25 265	9.99 993	30	30	8.31 800	8.31 809	9.99 991	30
40	8.25 375	8.25 382	9.99 993	20	40	8.31 901	8.31 911	9.99 991	20
50	8.25 493	8.25 500	9.99 993	10	50	8.32 002	8.32 012	9.99 991	10
2 0	8.25 609	8.25 616	9.99 993	0 **58**	**12** 0	8.32 103	8.32 112	9.99 990	0 **48**
10	8.25 726	8.25 733	9.99 993	50	10	8.32 203	8.32 213	9.99 990	50
20	8.25 842	8.25 849	9.99 993	40	20	8.32 303	8.32 313	9.99 990	40
30	8.25 958	8.25 965	9.99 993	30	30	8.32 403	8.32 413	9.99 990	30
40	8.26 074	8.26 081	9.99 993	20	40	8.32 503	8.32 513	9.99 990	20
50	8.26 189	8.26 196	9.99 993	10	50	8.32 602	8.32 612	9.99 990	10
3 0	8.26 304	8.26 312	9.99 993	0 **57**	**13** 0	8.32 702	8.32 711	9.99 990	0 **47**
10	8.26 419	8.26 426	9.99 993	50	10	8.32 801	8.32 811	9.99 990	50
20	8.26 533	8.26 541	9.99 993	40	20	8.32 899	8.32 909	9.99 990	40
30	8.26 648	8.26 655	9.99 993	30	30	8.32 998	8.33 008	9.99 990	30
40	8.26 761	8.26 769	9.99 993	20	40	8.33 096	8.33 106	9.99 990	20
50	8.26 875	8.26 882	9.99 993	10	50	8.33 195	8.33 205	9.99 990	10
4 0	8.26 988	8.26 996	9.99 992	0 **56**	**14** 0	8.33 292	8.33 302	9.99 990	0 **46**
10	8.27 101	8.27 109	9.99 992	50	10	8.33 390	8.33 400	9.99 990	50
20	8.27 214	8.27 221	9.99 992	40	20	8.33 488	8.33 498	9.99 990	40
30	8.27 326	8.27 334	9.99 992	30	30	8.33 585	8.33 595	9.99 990	30
40	8.27 438	8.27 446	9.99 992	20	40	8.33 682	8.33 692	9.99 990	20
50	8.27 550	8.27 558	9.99 992	10	50	8.33 779	8.33 789	9.99 990	10
5 0	8.27 661	8.27 669	9.99 992	0 **55**	**15** 0	8.33 875	8.33 886	9.99 990	0 **45**
10	8.27 773	8.27 780	9.99 992	50	10	8.33 972	8.33 982	9.99 990	50
20	8.27 883	8.27 891	9.99 992	40	20	8.34 068	8.34 078	9.99 990	40
30	8.27 994	8.28 002	9.99 992	30	30	8.34 164	8.34 174	9.99 990	30
40	8.28 104	8.28 112	9.99 992	20	40	8.34 260	8.34 270	9.99 989	20
50	8.28 215	8.28 223	9.99 992	10	50	8.34 355	8.34 366	9.99 989	10
6 0	8.28 324	8.28 332	9.99 992	0 **54**	**16** 0	8.34 450	8.34 461	9.99 989	0 **44**
10	8.28 434	8.28 442	9.99 992	50	10	8.34 546	8.34 556	9.99 989	50
20	8.28 543	8.28 551	9.99 992	40	20	8.34 640	8.34 651	9.99 989	40
30	8.28 652	8.28 660	9.99 992	30	30	8.34 735	8.34 746	9.99 989	30
40	8.28 761	8.28 769	9.99 992	20	40	8.34 830	8.34 840	9.99 989	20
50	8.28 869	8.28 877	9.99 992	10	50	8.34 924	8.34 935	9.99 989	10
7 0	8.28 977	8.28 986	9.99 992	0 **53**	**17** 0	8.35 018	8.35 029	9.99 989	0 **43**
10	8.29 085	8.29 094	9.99 992	50	10	8.35 112	8.35 123	9.99 989	50
20	8.29 193	8.29 201	9.99 992	40	20	8.35 206	8.35 217	9.99 989	40
30	8.29 300	8.29 309	9.99 992	30	30	8.35 299	8.35 310	9.99 989	30
40	8.29 407	8.29 416	9.99 992	20	40	8.35 392	8.35 403	9.99 989	20
50	8.29 514	8.29 523	9.99 992	10	50	8.35 485	8.35 497	9.99 989	10
8 0	8.29 621	8.29 629	9.99 992	0 **52**	**18** 0	8.35 578	8.35 590	9.99 989	0 **42**
10	8.29 727	8.29 736	9.99 991	50	10	8.35 671	8.35 682	9.99 989	50
20	8.29 833	8.29 842	9.99 991	40	20	8.35 764	8.35 775	9.99 989	40
30	8.29 939	8.29 947	9.99 991	30	30	8.35 856	8.35 867	9.99 989	30
40	8.30 044	8.30 053	9.99 991	20	40	8.35 948	8.35 959	9.99 989	20
50	8.30 150	8.30 158	9.99 991	10	50	8.36 040	8.36 051	9.99 989	10
9 0	8.30 255	8.30 263	9.99 991	0 **51**	**19** 0	8.36 131	8.36 143	9.99 989	0 **41**
10	8.30 359	8.30 368	9.99 991	50	10	8.36 223	8.36 235	9.99 988	50
20	8.30 464	8.30 473	9.99 991	40	20	8.36 314	8.36 326	9.99 988	40
30	8.30 568	8.30 577	9.99 991	30	30	8.36 405	8.36 417	9.99 988	30
40	8.30 672	8.30 681	9.99 991	20	40	8.36 496	8.36 508	9.99 988	20
50	8.30 776	8.30 785	9.99 991	10	50	8.36 587	8.36 599	9.99 988	10
10 0	8.30 879	8.30 888	9.99 991	0 **50**	**20** 0	8.36 678	8.36 689	9.99 988	0 **40**
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	″ ′

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′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
20 0	8.36 678	8.36 689	9.99 988	0 **40**	**30** 0	8.41 792	8.41 807	9.99 985	0 **30**
10	8.36 768	8.36 780	9.99 988	50	10	8.41 872	8.41 887	9.99 985	50
20	8.36 858	8.36 870	9.99 988	40	20	8.41 952	8.41 967	9.99 985	40
30	8.36 948	8.36 960	9.99 988	30	30	8.42 032	8.42 048	9.99 985	30
40	8.37 038	8.37 050	9.99 988	20	40	8.42 112	8.42 127	9.99 985	20
50	8.37 128	8.37 140	9.99 988	10	50	8.42 192	8.42 207	9.99 985	10
21 0	8.37 217	8.37 229	9.99 988	0 **39**	**31** 0	8.42 272	8.42 287	9.99 985	0 **29**
10	8.37 306	8.37 318	9.99 988	50	10	8.42 351	8.42 366	9.99 985	50
20	8.37 395	8.37 408	9.99 988	40	20	8.42 430	8.42 446	9.99 985	40
30	8.37 484	8.37 497	9.99 988	30	30	8.42 510	8.42 525	9.99 985	30
40	8.37 573	8.37 585	9.99 988	20	40	8.42 589	8.42 604	9.99 985	20
50	8.37 662	8.37 674	9.99 988	10	50	8.42 667	8.42 683	9.99 985	10
22 0	8.37 750	8.37 762	9.99 988	0 **38**	**32** 0	8.42 746	8.42 762	9.99 984	0 **28**
10	8.37 838	8.37 850	9.99 988	50	10	8.42 825	8.42 840	9.99 984	50
20	8.37 926	8.37 938	9.99 988	40	20	8.42 903	8.42 919	9.99 984	40
30	8.38 014	8.38 026	9.99 987	30	30	8.42 982	8.42 997	9.99 984	30
40	8.38 101	8.38 114	9.99 987	20	40	8.43 060	8.43 075	9.99 984	20
50	8.38 189	8.38 202	9.99 987	10	50	8.43 138	8.43 154	9.99 984	10
23 0	8.38 276	8.38 289	9.99 987	0 **37**	**33** 0	8.43 216	8.43 232	9.99 984	0 **27**
10	8.38 363	8.38 376	9.99 987	50	10	8.43 293	8.43 309	9.99 984	50
20	8.38 450	8.38 463	9.99 987	40	20	8.43 371	8.43 387	9.99 984	40
30	8.38 537	8.38 550	9.99 987	30	30	8.43 448	8.43 464	9.99 984	30
40	8.38 624	8.38 636	9.99 987	20	40	8.43 526	8.43 542	9.99 984	20
50	8.38 710	8.38 723	9.99 987	10	50	8.43 603	8.43 619	9.99 984	10
24 0	8.38 796	8.38 809	9.99 987	0 **36**	**34** 0	8.43 680	8.43 696	9.99 984	0 **26**
10	8.38 882	8.38 895	9.99 987	50	10	8.43 757	8.43 773	9.99 984	50
20	8.38 968	8.38 981	9.99 987	40	20	8.43 834	8.43 850	9.99 984	40
30	8.39 054	8.39 067	9.99 987	30	30	8.43 910	8.43 927	9.99 984	30
40	8.39 139	8.39 153	9.99 987	20	40	8.43 987	8.44 003	9.99 984	20
50	8.39 225	8.39 238	9.99 987	10	50	8.44 063	8.44 080	9.99 983	10
25 0	8.39 310	8.39 323	9.99 987	0 **35**	**35** 0	8.44 139	8.44 156	9.99 983	0 **25**
10	8.39 395	8.39 408	9.99 987	50	10	8.44 216	8.44 232	9.99 983	50
20	8.39 480	8.39 493	9.99 987	40	20	8.44 292	8.44 308	9.99 983	40
30	8.39 565	8.39 578	9.99 987	30	30	8.44 367	8.44 384	9.99 983	30
40	8.39 649	8.39 663	9.99 987	20	40	8.44 443	8.44 460	9.99 983	20
50	8.39 734	8.39 747	9.99 986	10	50	8.44 519	8.44 536	9.99 983	10
26 0	8.39 818	8.39 832	9.99 986	0 **34**	**36** 0	8.44 594	8.44 611	9.99 983	0 **24**
10	8.39 902	8.39 916	9.99 986	50	10	8.44 669	8.44 686	9.99 983	50
20	8.39 986	8.40 000	9.99 986	40	20	8.44 745	8.44 762	9.99 983	40
30	8.40 070	8.40 083	9.99 986	30	30	8.44 820	8.44 837	9.99 983	30
40	8.40 153	8.40 167	9.99 986	20	40	8.44 895	8.44 912	9.99 983	20
50	8.40 237	8.40 251	9.99 986	10	50	8.44 969	8.44 987	9.99 983	10
27 0	8.40 320	8.40 334	9.99 986	0 **33**	**37** 0	8.45 044	8.45 061	9.99 983	0 **23**
10	8.40 403	8.40 417	9.99 986	50	10	8.45 119	8.45 136	9.99 983	50
20	8.40 486	8.40 500	9.99 986	40	20	8.45 193	8.45 210	9.99 983	40
30	8.40 569	8.40 583	9.99 986	30	30	8.45 267	8.45 285	9.99 983	30
40	8.40 651	8.40 665	9.99 986	20	40	8.45 341	8.45 359	9.99 982	20
50	8.40 734	8.40 748	9.99 986	10	50	8.45 415	8.45 433	9.99 982	10
28 0	8.40 816	8.40 830	9.99 986	0 **32**	**38** 0	8.45 489	8.45 507	9.99 982	0 **22**
10	8.40 898	8.40 913	9.99 986	50	10	8.45 563	8.45 581	9.99 982	50
20	8.40 980	8.40 995	9.99 986	40	20	8.45 637	8.45 655	9.99 982	40
30	8.41 062	8.41 077	9.99 986	30	30	8.45 710	8.45 728	9.99 982	30
40	8.41 144	8.41 158	9.99 986	20	40	8.45 784	8.45 802	9.99 982	20
50	8.41 225	8.41 240	9.99 986	10	50	8.45 857	8.45 875	9.99 982	10
29 0	8.41 307	8.41 321	9.99 985	0 **31**	**39** 0	8.45 930	8.45 948	9.99 982	0 **21**
10	8.41 388	8.41 403	9.99 985	50	10	8.46 003	8.46 021	9.99 982	50
20	8.41 469	8.41 484	9.99 985	40	20	8.46 076	8.46 094	9.99 982	40
30	8.41 550	8.41 565	9.99 985	30	30	8.46 149	8.46 167	9.99 982	30
40	8.41 631	8.41 646	9.99 985	20	40	8.46 222	8.46 240	9.99 982	20
50	8.41 711	8.41 726	9.99 985	10	50	8.46 294	8.46 312	9.99 982	10
30 0	8.41 792	8.41 807	9.99 985	0 **30**	**40** 0	8.46 366	8.46 385	9.99 982	0 **20**
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	″ ′

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′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
40 0	8.46 366	8.46 385	9.99 982	0 **20**	**50** 0	8.50 504	8.50 527	9.99 978	0 **10**
10	8.46 439	8.46 457	9.99 982	50	10	8.50 570	8.50 593	9.99 978	50
20	8.46 511	8.46 529	9.99 982	40	20	8.50 636	8.50 658	9.99 978	40
30	8.46 583	8.46 602	9.99 981	30	30	8.50 701	8.50 724	9.99 978	30
40	8.46 655	8.46 674	9.99 981	20	40	8.50 767	8.50 789	9.99 977	20
50	8.46 727	8.46 745	9.99 981	10	50	8.50 832	8.50 855	9.99 977	10
41 0	8.46 799	8.46 817	9.99 981	0 **19**	**51** 0	8.50 897	8.50 920	9.99 977	0 **9**
10	8.46 870	8.46 889	9.99 981	50	10	8.50 963	8.50 985	9.99 977	50
20	8.46 942	8.46 960	9.99 981	40	20	8.51 028	8.51 050	9.99 977	40
30	8.47 013	8.47 032	9.99 981	30	30	8.51 092	8.51 115	9.99 977	30
40	8.47 084	8.47 103	9.99 981	20	40	8.51 157	8.51 180	9.99 977	20
50	8.47 155	8.47 174	9.99 981	10	50	8.51 222	8.51 245	9.99 977	10
42 0	8.47 226	8.47 245	9.99 981	0 **18**	**52** 0	8.51 287	8.51 310	9.99 977	0 **8**
10	8.47 297	8.47 316	9.99 981	50	10	8.51 351	8.51 374	9.99 977	50
20	8.47 368	8.47 387	9.99 981	40	20	8.51 416	8.51 439	9.99 977	40
30	8.47 439	8.47 458	9.99 981	30	30	8.51 480	8.51 503	9.99 977	30
40	8.47 509	8.47 528	9.99 981	20	40	8.51 544	8.51 568	9.99 977	20
50	8.47 580	8.47 599	9.99 981	10	50	8.51 609	8.51 632	9.99 977	10
43 0	8.47 650	8.47 669	9.99 981	0 **17**	**53** 0	8.51 673	8.51 696	9.99 977	0 **7**
10	8.47 720	8.47 740	9.99 980	50	10	8.51 737	8.51 760	9.99 976	50
20	8.47 790	8.47 810	9.99 980	40	20	8.51 801	8.51 824	9.99 976	40
30	8.47 860	8.47 880	9.99 980	30	30	8.51 864	8.51 888	9.99 976	30
40	8.47 930	8.47 950	9.99 980	20	40	8.51 928	8.51 952	9.99 976	20
50	8.48 000	8.48 020	9.99 980	10	50	8.51 992	8.52 015	9.99 976	10
44 0	8.48 069	8.48 090	9.99 980	0 **16**	**54** 0	8.52 055	8.52 079	9.99 976	0 **6**
10	8.48 139	8.48 159	9.99 980	50	10	8.52 119	8.52 143	9.99 976	50
20	8.48 208	8.48 228	9.99 980	40	20	8.52 182	8.52 206	9.99 976	40
30	8.48 278	8.48 298	9.99 980	30	30	8.52 245	8.52 269	9.99 976	30
40	8.48 347	8.48 367	9.99 980	20	40	8.52 308	8.52 332	9.99 976	20
50	8.48 416	8.48 436	9.99 980	10	50	8.52 371	8.52 396	9.99 976	10
45 0	8.48 485	8.48 505	9.99 980	0 **15**	**55** 0	8.52 434	8.52 459	9.99 976	0 **5**
10	8.48 554	8.48 574	9.99 980	50	10	8.52 497	8.52 522	9.99 976	50
20	8.48 622	8.48 643	9.99 980	40	20	8.52 560	8.52 584	9.99 976	40
30	8.48 691	8.48 711	9.99 980	30	30	8.52 623	8.52 647	9.99 975	30
40	8.48 760	8.48 780	9.99 979	20	40	8.52 685	8.52 710	9.99 975	20
50	8.48 828	8.48 849	9.99 979	10	50	8.52 748	8.52 772	9.99 975	10
46 0	8.48 896	8.48 917	9.99 979	0 **14**	**56** 0	8.52 810	8.52 835	9.99 975	0 **4**
10	8.48 965	8.48 985	9.99 979	50	10	8.52 872	8.52 897	9.99 975	50
20	8.49 033	8.49 053	9.99 979	40	20	8.52 935	8.52 960	9.99 975	40
30	8.49 101	8.49 121	9.99 979	30	30	8.52 997	8.53 022	9.99 975	30
40	8.49 169	8.49 189	9.99 979	20	40	8.53 059	8.53 084	9.99 975	20
50	8.49 236	8.49 257	9.99 979	10	50	8.53 121	8.53 146	9.99 975	10
47 0	8.49 304	8.49 325	9.99 979	0 **13**	**57** 0	8.53 183	8.53 208	9.99 975	0 **3**
10	8.49 372	8.49 393	9.99 979	50	10	8.53 245	8.53 270	9.99 975	50
20	8.49 439	8.49 460	9.99 979	40	20	8.53 306	8.53 332	9.99 975	40
30	8.49 506	8.49 528	9.99 979	30	30	8.53 368	8.53 393	9.99 975	30
40	8.49 574	8.49 595	9.99 979	20	40	8.53 429	8.53 455	9.99 975	20
50	8.49 641	8.49 662	9.99 979	10	50	8.53 491	8.53 516	9.99 974	10
48 0	8.49 708	8.49 729	9.99 979	0 **12**	**58** 0	8.53 552	8.53 578	9.99 974	0 **2**
10	8.49 775	8.49 796	9.99 979	50	10	8.53 614	8.53 639	9.99 974	50
20	8.49 842	8.49 863	9.99 978	40	20	8.53 675	8.53 700	9.99 974	40
30	8.49 908	8.49 930	9.99 978	30	30	8.53 736	8.53 762	9.99 974	30
40	8.49 975	8.49 997	9.99 978	20	40	8.53 797	8.53 823	9.99 974	20
50	8.50 042	8.50 063	9.99 978	10	50	8.53 858	8.53 884	9.99 974	10
49 0	8.50 108	8.50 130	9.99 978	0 **11**	**59** 0	8.53 919	8.53 945	9.99 974	0 **1**
10	8.50 174	8.50 196	9.99 978	50	10	8.53 979	8.54 005	9.99 974	50
20	8.50 241	8.50 263	9.99 978	40	20	8.54 040	8.54 066	9.99 974	40
30	8.50 307	8.50 329	9.99 978	30	30	8.54 101	8.54 127	9.99 974	30
40	8.50 373	8.50 395	9.99 978	20	40	8.54 161	8.54 187	9.99 974	20
50	8.50 439	8.50 461	9.99 978	10	50	8.54 222	8.54 248	9.99 974	10
50 0	8.50 504	8.50 527	9.99 978	0 **10**	**60** 0	8.54 282	8.54 308	9.99 974	0 **0**
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	″ ′

88°

0°   *90°   180°   *270°

′	log sin	d.	log tan	c. d.	log cot	log cos	′
0	—		—		—	0.00 000	60
1	6.46 373	30103	6.46 373	30103	3.53 627	0.00 000	59
2	6.76 476	17609	6.76 476	17609	3.23 524	0.00 000	58
3	6.94 085	12494	6.94 085	12494	3.05 915	0.00 000	57
4	7.06 579	9691	7.06 579	9691	2.93 421	0.00 000	56
5	7.16 270	7918	7.16 270	7918	2.83 730	0.00 000	55
6	7.24 188	6694	7.24 188	6694	2.75 812	0.00 000	54
7	7.30 882	5800	7.30 882	5800	2.69 118	0.00 000	53
8	7.36 682	5115	7.36 682	5115	2.63 318	0.00 000	52
9	7.41 797	4576	7.41 797	4576	2.58 203	0.00 000	51
10	7.46 373	4139	7.46 373	4139	2.53 627	0.00 000	50
11	7.50 512	3779	7.50 512	3779	2.49 488	0.00 000	49
12	7.54 291	3476	7.54 291	3476	2.45 709	0.00 000	48
13	7.57 767	3218	7.57 767	3219	2.42 233	0.00 000	47
14	7.60 985	2997	7.60 986	2996	2.39 014	0.00 000	46
15	7.63 982	2802	7.63 982	2803	2.36 018	0.00 000	45
16	7.66 784	2633	7.66 785	2633	2.33 215	0.00 000	44
17	7.69 417	2483	7.69 418	2482	2.30 582	9.99 999	43
18	7.71 900	2348	7.71 900	2348	2.28 100	9.99 999	42
19	7.74 248	2227	7.74 248	2228	2.25 752	9.99 999	41
20	7.76 475	2119	7.76 476	2119	2.23 524	9.99 999	40
21	7.78 594	2021	7.78 595	2020	2.21 405	9.99 999	39
22	7.80 615	1930	7.80 615	1931	2.19 385	9.99 999	38
23	7.82 545	1848	7.82 546	1848	2.17 454	9.99 999	37
24	7.84 393	1773	7.84 394	1773	2.15 606	9.99 999	36
25	7.86 166	1704	7.86 167	1704	2.13 833	9.99 999	35
26	7.87 870	1639	7.87 871	1639	2.12 129	9.99 999	34
27	7.89 509	1579	7.89 510	1579	2.10 490	9.99 999	33
28	7.91 088	1524	7.91 089	1524	2.08 911	9.99 999	32
29	7.92 612	1472	7.92 613	1473	2.07 387	9.99 998	31
30	7.94 084	1424	7.94 086	1424	2.05 914	9.99 998	30
31	7.95 508	1379	7.95 510	1379	2.04 490	9.99 998	29
32	7.96 887	1336	7.96 889	1336	2.03 111	9.99 998	28
33	7.98 223	1297	7.98 225	1297	2.01 775	9.99 998	27
34	7.99 520	1259	7.99 522	1259	2.00 478	9.99 998	26
35	8.00 779	1223	8.00 781	1223	1.99 219	9.99 998	25
36	8.02 002	1190	8.02 004	1190	1.97 996	9.99 998	24
37	8.03 192	1158	8.03 194	1159	1.96 806	9.99 997	23
38	8.04 350	1128	8.04 353	1128	1.95 647	9.99 997	22
39	8.05 478	1100	8.05 481	1100	1.94 519	9.99 997	21
40	8.06 578	1072	8.06 581	1072	1.93 419	9.99 997	20
41	8.07 650	1046	8.07 653	1047	1.92 347	9.99 997	19
42	8.08 696	1022	8.08 700	1022	1.91 300	9.99 997	18
43	8.09 718	999	8.09 722	998	1.90 278	9.99 997	17
44	8.10 717	976	8.10 720	976	1.89 280	9.99 996	16
45	8.11 693	954	8.11 696	955	1.88 304	9.99 996	15
46	8.12 647	934	8.12 651	934	1.87 349	9.99 996	14
47	8.13 581	914	8.13 585	915	1.86 415	9.99 996	13
48	8.14 495	896	8.14 500	895	1.85 500	9.99 996	12
49	8.15 391	877	8.15 395	878	1.84 605	9.99 996	11
50	8.16 268	860	8.16 273	860	1.83 727	9.99 995	10
51	8.17 128	843	8.17 133	843	1.82 867	9.99 995	9
52	8.17 971	827	8.17 976	828	1.82 024	9.99 995	8
53	8.18 798	812	8.18 804	812	1.81 196	9.99 995	7
54	8.19 610	797	8.19 616	797	1.80 384	9.99 995	6
55	8.20 407	782	8.20 413	782	1.79 587	9.99 994	5
56	8.21 189	769	8.21 195	769	1.78 805	9.99 994	4
57	8.21 958	755	8.21 964	756	1.78 036	9.99 994	3
58	8.22 713	743	8.22 720	742	1.77 280	9.99 994	2
59	8.23 456	730	8.23 462	730	1.76 538	9.99 994	1
60	8.24 186		8.24 192		1.75 808	9.99 993	0
	log cos	d.	log cot	c. d.	log tan	log sin	′

Prop. Pts.

d	pp 1″	d	pp 1″
30103	501.72	915	15.25
17609	293.48	914	15.23
12494	208.23	896	14.93
9691	161.52	895	14.92
7918	131.97	878	14.63
6694	111.57	877	14.62
5800	96.67	860	14.33
5115	85.25	843	14.05
4576	76.27	828	13.80
4139	68.98	827	13.78
3779	62.98	812	13.53
3476	57.93	797	13.28
3219	53.65	782	13.03
3218	53.63	769	12.82
2997	49.95	756	12.60
2996	49.93	755	12.58
2803	46.72	743	12.38
2802	46.70	742	12.37
2633	43.88	730	12.17
2483	41.38		
2482	41.37		
2348	39.13		
2228	37.13		
2227	37.12		
2119	35.32		
2021	33.68		
2020	33.67		
1931	32.18		
1930	32.17		
1848	30.80		
1773	29.55		
1704	28.40		
1639	27.32		
1579	26.32		
1524	25.40		
1473	24.55		
1472	24.53		
1424	23.73		
1379	22.98		
1336	22.27		
1297	21.62		
1259	20.98		
1223	20.38		
1190	19.83		
1159	19.32		
1158	19.30		
1128	18.80		
1100	18.33		
1072	17.87		
1047	17.45		
1046	17.43		
1022	17.03		
999	16.65		
998	16.63		
976	16.27		
955	15.92		
954	15.90		
934	15.57		

*179°   269°   *359°   **89°**

1° *91° 181° *271°

′	log sin	d.	log tan	c. d.	log cot	log cos	
0	8.24 186		8.24 192		1.75 808	9.99 993	60
1	8.24 903	717	8.24 910	718	1.75 090	9.99 993	59
2	8.25 609	706	8.25 616	706	1.74 384	9.99 993	58
3	8.26 304	695	8.26 312	696	1.73 688	9.99 993	57
4	8.26 988	684	8.26 996	684	1.73 004	9.99 992	56
5	8.27 661	673	8.27 669	673	1.72 331	9.99 992	55
6	8.28 324	663	8.28 332	663	1.71 668	9.99 992	54
7	8.28 977	653	8.28 986	654	1.71 014	9.99 992	53
8	8.29 621	644	8.29 629	643	1.70 371	9.99 992	52
9	8.30 255	634	8.30 263	634	1.69 737	9.99 991	51
10	8.30 879	624	8.30 888	625	1.69 112	9.99 991	50
11	8.31 495	616	8.31 505	617	1.68 495	9.99 991	49
12	8.32 103	608	8.32 112	607	1.67 888	9.99 990	48
13	8.32 702	599	8.32 711	599	1.67 289	9.99 990	47
14	8.33 292	590	8.33 302	591	1.66 698	9.99 990	46
15	8.33 875	583	8.33 886	584	1.66 114	9.99 990	45
16	8.34 450	575	8.34 461	575	1.65 539	9.99 989	44
17	8.35 018	568	8.35 029	568	1.64 971	9.99 989	43
18	8.35 578	560	8.35 590	561	1.64 410	9.99 989	42
19	8.36 131	553	8.36 143	553	1.63 857	9.99 989	41
20	8.36 678	547	8.36 689	546	1.63 311	9.99 988	40
21	8.37 217	539	8.37 229	540	1.62 771	9.99 988	39
22	8.37 750	533	8.37 762	533	1.62 238	9.99 988	38
23	8.38 276	526	8.38 289	527	1.61 711	9.99 987	37
24	8.38 796	520	8.38 809	520	1.61 191	9.99 987	36
25	8.39 310	514	8.39 323	514	1.60 677	9.99 987	35
26	8.39 818	508	8.39 832	509	1.60 168	9.99 986	34
27	8.40 320	502	8.40 334	502	1.59 666	9.99 986	33
28	8.40 816	496	8.40 830	496	1.59 170	9.99 986	32
29	8.41 307	491	8.41 321	491	1.58 679	9.99 985	31
30	8.41 792	485	8.41 807	486	1.58 193	9.99 985	30
31	8.42 272	480	8.42 287	480	1.57 713	9.99 985	29
32	8.42 746	474	8.42 762	475	1.57 238	9.99 984	28
33	8.43 216	470	8.43 232	470	1.56 768	9.99 984	27
34	8.43 680	464	8.43 696	464	1.56 304	9.99 984	26
35	8.44 139	459	8.44 156	460	1.55 844	9.99 983	25
36	8.44 594	455	8.44 611	455	1.55 389	9.99 983	24
37	8.45 044	450	8.45 061	450	1.54 939	9.99 983	23
38	8.45 489	445	8.45 507	446	1.54 493	9.99 982	22
39	8.45 930	441	8.45 948	441	1.54 052	9.99 982	21
40	8.46 366	436	8.46 385	437	1.53 615	9.99 982	20
41	8.46 799	433	8.46 817	432	1.53 183	9.99 981	19
42	8.47 226	427	8.47 245	428	1.52 755	9.99 981	18
43	8.47 650	424	8.47 669	424	1.52 331	9.99 981	17
44	8.48 069	419	8.48 089	420	1.51 911	9.99 980	16
45	8.48 485	416	8.48 505	416	1.51 495	9.99 980	15
46	8.48 896	411	8.48 917	412	1.51 083	9.99 979	14
47	8.49 304	408	8.49 325	408	1.50 675	9.99 979	13
48	8.49 708	404	8.49 729	404	1.50 271	9.99 979	12
49	8.50 108	400	8.50 130	401	1.49 870	9.99 978	11
50	8.50 504	396	8.50 527	397	1.49 473	9.99 978	10
51	8.50 897	393	8.50 920	393	1.49 080	9.99 977	9
52	8.51 287	390	8.51 310	390	1.48 690	9.99 977	8
53	8.51 673	386	8.51 696	386	1.48 304	9.99 977	7
54	8.52 055	382	8.52 079	383	1.47 921	9.99 976	6
55	8.52 434	379	8.52 459	380	1.47 541	9.99 976	5
56	8.52 810	376	8.52 835	376	1.47 165	9.99 975	4
57	8.53 183	373	8.53 208	373	1.46 792	9.99 975	3
58	8.53 552	369	8.53 578	370	1.46 422	9.99 974	2
59	8.53 919	367	8.53 945	367	1.46 055	9.99 974	1
60	8.54 282	363	8.54 308	363	1.45 692	9.99 974	0
	log cos	d.	log cot	c. d.	log tan	log sin	′

Prop. Pts.

d	pp 1″	d	pp 1″
718	11.97	485	8.08
717	11.95	480	8.00
706	11.77	475	7.92
696	11.60	474	7.90
695	11.58	470	7.83
684	11.40	464	7.73
673	11.22	460	7.67
663	11.05	459	7.65
654	10.90	455	7.58
653	10.88	450	7.50
644	10.73	446	7.43
643	10.72	445	7.42
634	10.57	441	7.35
625	10.42	437	7.28
624	10.40	436	7.27
617	10.28	433	7.22
616	10.27	432	7.20
608	10.13	428	7.13
607	10.12	427	7.12
599	9.98	424	7.07
591	9.85	420	7.00
590	9.83	419	6.98
584	9.73	416	6.93
583	9.72	412	6.87
575	9.58	411	6.85
568	9.47	408	6.80
561	9.35	404	6.73
560	9.33	401	6.68
553	9.22	400	6.67
547	9.12	397	6.62
546	9.10	396	6.60
540	9.00	393	6.55
539	8.98	390	6.50
533	8.88	386	6.43
527	8.78	383	6.38
526	8.77	382	6.37
520	8.67	380	6.33
514	8.57	379	6.32
509	8.48	376	6.27
508	8.47	373	6.22
502	8.37	370	6.17
496	8.27	369	6.15
491	8.18	367	6.12
486	8.10	363	6.05

*178° 268° *358° **88°**

2°   *92°   182°   *272°

′	log sin	d.	log tan	c. d.	log cot	log cos	
0	8.54 282		8.54 308		1.45 692	9.99 974	60
1	8.54 642	360	8.54 669	361	1.45 331	9.99 973	59
2	8.54 999	357	8.55 027	358	1.44 973	9.99 973	58
3	8.55 354	355	8.55 382	355	1.44 618	9.99 972	57
4	8.55 705	351	8.55 734	352	1.44 266	9.99 972	56
5	8.56 054	349	8.56 083	349	1.43 917	9.99 971	55
6	8.56 400	346	8.56 429	346	1.43 571	9.99 971	54
7	8.56 743	343	8.56 773	344	1.43 227	9.99 970	53
8	8.57 084	341	8.57 114	341	1.42 886	9.99 970	52
9	8.57 421	337	8.57 452	338	1.42 548	9.99 969	51
10	8.57 757	336	8.57 788	336	1.42 212	9.99 969	50
11	8.58 089	332	8.58 121	333	1.41 879	9.99 968	49
12	8.58 419	330	8.58 451	330	1.41 549	9.99 968	48
13	8.58 747	328	8.58 779	328	1.41 221	9.99 967	47
14	8.59 072	325	8.59 105	326	1.40 895	9.99 967	46
15	8.59 395	323	8.59 428	323	1.40 572	9.99 967	45
16	8.59 715	320	8.59 749	321	1.40 251	9.99 966	44
17	8.60 033	318	8.60 068	319	1.39 932	9.99 966	43
18	8.60 349	316	8.60 384	316	1.39 616	9.99 965	42
19	8.60 662	313	8.60 698	314	1.39 302	9.99 964	41
20	8.60 973	311	8.61 009	311	1.38 991	9.99 964	40
21	8.61 282	309	8.61 319	310	1.38 681	9.99 963	39
22	8.61 589	307	8.61 626	307	1.38 374	9.99 963	38
23	8.61 894	305	8.61 931	305	1.38 069	9.99 962	37
24	8.62 196	302	8.62 234	303	1.37 766	9.99 962	36
25	8.62 497	301	8.62 535	301	1.37 465	9.99 961	35
26	8.62 795	298	8.62 834	299	1.37 166	9.99 961	34
27	8.63 091	296	8.63 131	297	1.36 869	9.99 960	33
28	8.63 385	294	8.63 426	295	1.36 574	9.99 960	32
29	8.63 678	293	8.63 718	292	1.36 282	9.99 959	31
30	8.63 968	290	8.64 009	291	1.35 991	9.99 959	30
31	8.64 256	288	8.64 298	289	1.35 702	9.99 958	29
32	8.64 543	287	8.64 585	287	1.35 415	9.99 958	28
33	8.64 827	284	8.64 870	285	1.35 130	9.99 957	27
34	8.65 110	283	8.65 154	284	1.34 846	9.99 956	26
35	8.65 391	281	8.65 435	281	1.34 565	9.99 956	25
36	8.65 670	279	8.65 715	280	1.34 285	9.99 955	24
37	8.65 947	277	8.65 993	278	1.34 007	9.99 955	23
38	8.66 223	276	8.66 269	276	1.33 731	9.99 954	22
39	8.66 497	274	8.66 543	274	1.33 457	9.99 954	21
40	8.66 769	272	8.66 816	273	1.33 184	9.99 953	20
41	8.67 039	270	8.67 087	271	1.32 913	9.99 952	19
42	8.67 308	269	8.67 356	269	1.32 644	9.99 952	18
43	8.67 575	267	8.67 624	268	1.32 376	9.99 951	17
44	8.67 841	266	8.67 890	266	1.32 110	9.99 951	16
45	8.68 104	263	8.68 154	264	1.31 846	9.99 950	15
46	8.68 367	263	8.68 417	263	1.31 583	9.99 949	14
47	8.68 627	260	8.68 678	261	1.31 322	9.99 949	13
48	8.68 886	259	8.68 938	260	1.31 062	9.99 948	12
49	8.69 144	258	8.69 196	258	1.30 804	9.99 948	11
50	8.69 400	256	8.69 453	257	1.30 547	9.99 947	10
51	8.69 654	254	8.69 708	255	1.30 292	9.99 946	9
52	8.69 907	253	8.69 962	254	1.30 038	9.99 946	8
53	8.70 159	252	8.70 214	252	1.29 786	9.99 945	7
54	8.70 409	250	8.70 465	251	1.29 535	9.99 944	6
55	8.70 658	249	8.70 714	249	1.29 286	9.99 944	5
56	8.70 905	247	8.70 962	248	1.29 038	9.99 943	4
57	8.71 151	246	8.71 208	246	1.28 792	9.99 942	3
58	8.71 395	244	8.71 453	245	1.28 547	9.99 942	2
59	8.71 638	243	8.71 697	244	1.28 303	9.99 941	1
60	8.71 880	242	8.71 940	243	1.28 060	9.99 940	0
	log cos	d.	log cot	c. d.	log tan	log sin	′

Prop. Pts.

d	pp 1″	d	pp 1″
361	6.02	291	4.85
360	6.00	290	4.83
358	5.97	289	4.82
357	5.95	288	4.80
355	5.92	287	4.78
352	5.87	285	4.75
351	5.85	284	4.73
349	5.82	283	4.72
346	5.77	281	4.68
344	5.73	280	4.67
343	5.72	279	4.65
341	5.68	278	4.63
338	5.63	277	4.62
337	5.62	276	4.60
336	5.60	274	4.57
333	5.55	273	4.55
332	5.53	272	4.53
330	5.50	271	4.52
328	5.47	270	4.50
326	5.43	269	4.48
325	5.42	268	4.47
323	5.38	267	4.45
321	5.35	266	4.43
320	5.33	264	4.40
319	5.32	263	4.38
318	5.30	261	4.35
316	5.27	260	4.33
314	5.23	259	4.32
313	5.22	258	4.30
311	5.18	257	4.28
310	5.17	256	4.27
309	5.15	255	4.25
307	5.12	254	4.23
305	5.08	253	4.22
303	5.05	252	4.20
302	5.03	251	4.18
301	5.02	250	4.17
299	4.98	249	4.15
298	4.97	248	4.13
297	4.95	247	4.12
296	4.93	246	4.10
295	4.92	245	4.08
294	4.90	244	4.07
293	4.88	243	4.05
292	4.87	242	4.03

Prop. Pts.

*177°   267°   *357°   **87°**

3° *93° 183° *273°

′	log sin	d.	log tan	c. d.	log cot	log cos	
0	8.71 880		8.71 940		1.28 060	9.99 940	**60**
1	8.72 120	240	8.72 181	241	1.27 819	9.99 940	59
2	8.72 359	239	8.72 420	239	1.27 580	9.99 939	58
3	8.72 597	238	8.72 659	239	1.27 341	9.99 938	57
4	8.72 834	237	8.72 896	237	1.27 104	9.99 938	56
5	8.73 069	235	8.73 132	236	1.26 868	9.99 937	55
6	8.73 303	234	8.73 366	234	1.26 634	9.99 936	54
7	8.73 535	232	8.73 600	234	1.26 400	9.99 936	53
8	8.73 767	232	8.73 832	232	1.26 168	9.99 935	52
9	8.73 997	230	8.74 063	231	1.25 937	9.99 934	51
10	8.74 226	229	8.74 292	229	1.25 708	9.99 934	**50**
11	8.74 454	228	8.74 521	229	1.25 479	9.99 933	49
12	8.74 680	226	8.74 748	227	1.25 252	9.99 932	48
13	8.74 906	226	8.74 974	226	1.25 026	9.99 932	47
14	8.75 130	224	8.75 199	225	1.24 801	9.99 931	46
15	8.75 353	223	8.75 423	224	1.24 577	9.99 930	45
16	8.75 575	222	8.75 645	222	1.24 355	9.99 929	44
17	8.75 795	220	8.75 867	222	1.24 133	9.99 929	43
18	8.76 015	220	8.76 087	220	1.23 913	9.99 928	42
19	8.76 234	219	8.76 306	219	1.23 694	9.99 927	41
20	8.76 451	217	8.76 525	219	1.23 475	9.99 926	**40**
21	8.76 667	216	8.76 742	217	1.23 258	9.99 926	39
22	8.76 883	216	8.76 958	216	1.23 042	9.99 925	38
23	8.77 097	214	8.77 173	215	1.22 827	9.99 924	37
24	8.77 310	213	8.77 387	214	1.22 613	9.99 923	36
25	8.77 522	212	8.77 600	213	1.22 400	9.99 923	35
26	8.77 733	211	8.77 811	211	1.22 189	9.99 922	34
27	8.77 943	210	8.78 022	211	1.21 978	9.99 921	33
28	8.78 152	209	8.78 232	210	1.21 768	9.99 920	32
29	8.78 360	208	8.78 441	209	1.21 559	9.99 920	31
30	8.78 568	208	8.78 649	208	1.21 351	9.99 919	**30**
31	8.78 774	206	8.78 855	206	1.21 145	9.99 918	29
32	8.78 979	205	8.79 061	206	1.20 939	9.99 917	28
33	8.79 183	204	8.79 266	205	1.20 734	9.99 917	27
34	8.79 386	203	8.79 470	204	1.20 530	9.99 916	26
35	8.79 588	202	8.79 673	203	1.20 327	9.99 915	25
36	8.79 789	201	8.79 875	202	1.20 125	9.99 914	24
37	8.79 990	201	8.80 076	201	1.19 924	9.99 913	23
38	8.80 189	199	8.80 277	201	1.19 723	9.99 913	22
39	8.80 388	199	8.80 476	199	1.19 524	9.99 912	21
40	8.80 585	197	8.80 674	198	1.19 326	9.99 911	**20**
41	8.80 782	197	8.80 872	198	1.19 128	9.99 910	19
42	8.80 978	196	8.81 068	196	1.18 932	9.99 909	18
43	8.81 173	195	8.81 264	196	1.18 736	9.99 909	17
44	8.81 367	194	8.81 459	195	1.18 541	9.99 908	16
45	8.81 560	193	8.81 653	194	1.18 347	9.99 907	15
46	8.81 752	192	8.81 846	193	1.18 154	9.99 906	14
47	8.81 944	192	8.82 038	192	1.17 962	9.99 905	13
48	8.82 134	190	8.82 230	192	1.17 770	9.99 904	12
49	8.82 324	190	8.82 420	190	1.17 580	9.99 904	11
50	8.82 513	189	8.82 610	190	1.17 390	9.99 903	**10**
51	8.82 701	188	8.82 799	189	1.17 201	9.99 902	9
52	8.82 888	187	8.82 987	188	1.17 013	9.99 901	8
53	8.83 075	187	8.83 175	188	1.16 825	9.99 900	7
54	8.83 261	186	8.83 361	186	1.16 639	9.99 899	6
55	8.83 446	185	8.83 547	186	1.16 453	9.99 898	5
56	8.83 630	184	8.83 732	185	1.16 268	9.99 898	4
57	8.83 813	183	8.83 916	184	1.16 084	9.99 897	3
58	8.83 996	183	8.84 100	184	1.15 900	9.99 896	2
59	8.84 177	181	8.84 282	182	1.15 718	9.99 895	1
60	8.84 358	181	8.84 464	182	1.15 536	9.99 894	0
	log cos	d.	log cot	c. d.	log tan	log sin	′

Prop. Pts.

d	pp 1″	d	pp 1″
241	4.02	209	3.48
240	4.00	208	3.47
239	3.98	207	3.45
238	3.97	206	3.43
237	3.95	205	3.42
236	3.93	204	3.40
235	3.92	203	3.38
234	3.90	202	3.37
233	3.88	201	3.35
232	3.87	200	3.33
231	3.85	199	3.32
230	3.83	198	3.30
229	3.82	197	3.28
228	3.80	196	3.27
227	3.78	195	3.25
226	3.77	194	3.23
225	3.75	193	3.22
224	3.73	192	3.20
223	3.72	191	3.18
222	3.70	190	3.17
221	3.68	189	3.15
220	3.67	188	3.13
219	3.65	187	3.12
218	3.63	186	3.10
217	3.62	185	3.08
216	3.60	184	3.07
215	3.58	183	3.05
214	3.57	182	3.03
213	3.55	181	3.02
212	3.53		
211	3.52		
210	3.50		

*176° 266° *356° **86°**

4°	*94°	184°	*274°				
′	log sin	d.	log tan	c. d.	log cot	log cos	
0	8.84 358		8.84 464		1.15 536	9.99 894	60
1	8.84 539	181	8.84 646	182	1.15 354	9.99 893	59
2	8.84 718	179	8.84 826	180	1.15 174	9.99 892	58
3	8.84 897	179	8.85 006	180	1.14 994	9.99 891	57
4	8.85 075	178	8.85 185	179	1.14 815	9.99 891	56
5	8.85 252	177	8.85 363	178	1.14 637	9.99 890	55
6	8.85 429	177	8.85 540	177	1.14 460	9.99 889	54
7	8.85 605	176	8.85 717	177	1.14 283	9.99 888	53
8	8.85 780	175	8.85 893	176	1.14 107	9.99 887	52
9	8.85 955	175	8.86 069	176	1.13 931	9.99 886	51
10	8.86 128	173	8.86 243	174	1.13 757	9.99 885	50
11	8.86 301	173	8.86 417	174	1.13 583	9.99 884	49
12	8.86 474	173	8.86 591	174	1.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	1.13 237	9.99 882	47
14	8.86 816	171	8.86 935	172	1.13 065	9.99 881	46
15	8.86 987	171	8.87 106	171	1.12 894	9.99 880	45
16	8.87 156	169	8.87 277	171	1.12 723	9.99 879	44
17	8.87 325	169	8.87 447	170	1.12 553	9.99 879	43
18	8.87 494	169	8.87 616	169	1.12 384	9.99 878	42
19	8.87 661	167	8.87 785	169	1.12 215	9.99 877	41
20	8.87 829	168	8.87 953	168	1.12 047	9.99 876	40
21	8.87 995	166	8.88 120	167	1.11 880	9.99 875	39
22	8.88 161	166	8.88 287	167	1.11 713	9.99 874	38
23	8.88 326	165	8.88 453	166	1.11 547	9.99 873	37
24	8.88 490	164	8.88 618	165	1.11 382	9.99 872	36
25	8.88 654	164	8.88 783	165	1.11 217	9.99 871	35
26	8.88 817	163	8.88 948	165	1.11 052	9.99 870	34
27	8.88 980	163	8.89 111	163	1.10 889	9.99 869	33
28	8.89 142	162	8.89 274	163	1.10 726	9.99 868	32
29	8.89 304	162	8.89 437	163	1.10 563	9.99 867	31
30	8.89 464	160	8.89 598	161	1.10 402	9.99 866	30
31	8.89 625	161	8.89 760	162	1.10 240	9.99 865	29
32	8.89 784	159	8.89 920	160	1.10 080	9.99 864	28
33	8.89 943	159	8.90 080	160	1.09 920	9.99 863	27
34	8.90 102	159	8.90 240	160	1.09 760	9.99 862	26
35	8.90 260	158	8.90 399	159	1.09 601	9.99 861	25
36	8.90 417	157	8.90 557	158	1.09 443	9.99 860	24
37	8.90 574	157	8.90 715	158	1.09 285	9.99 859	23
38	8.90 730	156	8.90 872	157	1.09 128	9.99 858	22
39	8.90 885	155	8.91 029	157	1.08 971	9.99 857	21
40	8.91 040	155	8.91 185	156	1.08 815	9.99 856	20
41	8.91 195	155	8.91 340	155	1.08 660	9.99 855	19
42	8.91 349	154	8.91 495	155	1.08 505	9.99 854	18
43	8.91 502	153	8.91 650	155	1.08 350	9.99 853	17
44	8.91 655	153	8.91 803	153	1.08 197	9.99 852	16
45	8.91 807	152	8.91 957	154	1.08 043	9.99 851	15
46	8.91 959	152	8.92 110	153	1.07 890	9.99 850	14
47	8.92 110	151	8.92 262	152	1.07 738	9.99 848	13
48	8.92 261	151	8.92 414	152	1.07 586	9.99 847	12
49	8.92 411	150	8.92 565	151	1.07 435	9.99 846	11
50	8.92 561	150	8.92 716	151	1.07 284	9.99 845	10
51	8.92 710	149	8.92 866	150	1.07 134	9.99 844	9
52	8.92 859	149	8.93 016	150	1.06 984	9.99 843	8
53	8.93 007	148	8.93 165	149	1.06 835	9.99 842	7
54	8.93 154	147	8.93 313	148	1.06 687	9.99 841	6
55	8.93 301	147	8.93 462	149	1.06 538	9.99 840	5
56	8.93 448	147	8.93 609	147	1.06 391	9.99 839	4
57	8.93 594	146	8.93 756	147	1.06 244	9.99 838	3
58	8.93 740	146	8.93 903	147	1.06 097	9.99 837	2
59	8.93 885	145	8.94 049	146	1.05 951	9.99 836	1
60	8.94 030	145	8.94 195	146	1.05 805	9.99 834	0
	log cos	d.	log cot	c. d.	log tan	log sin	′
*175°	265°	*355°	85°				

Prop. Pts.

d	pp 1″
182	3.03
181	3.02
180	3.00
179	2.98
178	2.97
177	2.95
176	2.93
175	2.92
174	2.90
173	2.88
172	2.87
171	2.85
170	2.83
169	2.82
168	2.80
167	2.78
166	2.77
165	2.75
164	2.73
163	2.72
162	2.70
161	2.68
160	2.67
159	2.65
158	2.63
157	2.62
156	2.60
155	2.58
154	2.57
153	2.55
152	2.53
151	2.52
150	2.50
149	2.48
148	2.47
147	2.45
146	2.43
145	2.42

Prop. Pts.

5°								*95° 185° *275°
′	log sin	d.	log tan	c. d.	log cot	log cos		Prop. Pts.
0	8.94 030	144	8.94 195	145	1.05 805	9.99 834	60	
1	8.94 174	143	8.94 340	145	1.05 660	9.99 833	59	
2	8.94 317	144	8.94 485	145	1.05 515	9.99 832	58	
3	8.94 461	142	8.94 630	143	1.05 370	9.99 831	57	
4	8.94 603	143	8.94 773	144	1.05 227	9.99 830	56	
5	8.94 746	141	8.94 917	143	1.05 083	9.99 829	55	
6	8.94 887	142	8.95 060	142	1.04 940	9.99 828	54	
7	8.95 029	141	8.95 202	142	1.04 798	9.99 827	53	
8	8.95 170	140	8.95 344	142	1.04 656	9.99 825	52	
9	8.95 310	140	8.95 486	141	1.04 514	9.99 824	51	
10	8.95 450	139	8.95 627	140	1.04 373	9.99 823	50	
11	8.95 589	139	8.95 767	141	1.04 233	9.99 822	49	
12	8.95 728	139	8.95 908	139	1.04 092	9.99 821	48	
13	8.95 867	138	8.96 047	140	1.03 953	9.99 820	47	
14	8.96 005	138	8.96 187	138	1.03 813	9.99 819	46	
15	8.96 143	137	8.96 325	139	1.03 675	9.99 817	45	
16	8.96 280	137	8.96 464	138	1.03 536	9.99 816	44	
17	8.96 417	136	8.96 602	137	1.03 398	9.99 815	43	
18	8.96 553	136	8.96 739	138	1.03 261	9.99 814	42	
19	8.96 689	136	8.96 877	136	1.03 123	9.99 813	41	
20	8.96 825	135	8.97 013	137	1.02 987	9.99 812	40	
21	8.96 960	135	8.97 150	135	1.02 850	9.99 810	39	
22	8.97 095	134	8.97 285	136	1.02 715	9.99 809	38	
23	8.97 229	134	8.97 421	135	1.02 579	9.99 808	37	
24	8.97 363	133	8.97 556	135	1.02 444	9.99 807	36	
25	8.97 496	133	8.97 691	134	1.02 309	9.99 806	35	
26	8.97 629	133	8.97 825	134	1.02 175	9.99 804	34	
27	8.97 762	132	8.97 959	133	1.02 041	9.99 803	33	
28	8.97 894	132	8.98 092	133	1.01 908	9.99 802	32	
29	8.98 026	131	8.98 225	133	1.01 775	9.99 801	31	
30	8.98 157	131	8.98 358	132	1.01 642	9.99 800	30	
31	8.98 288	131	8.98 490	132	1.01 510	9.99 798	29	
32	8.98 419	130	8.98 622	131	1.01 378	9.99 797	28	
33	8.98 549	130	8.98 753	131	1.01 247	9.99 796	27	
34	8.98 679	129	8.98 884	131	1.01 116	9.99 795	26	
35	8.98 808	129	8.99 015	130	1.00 985	9.99 793	25	
36	8.98 937	129	8.99 145	130	1.00 855	9.99 792	24	
37	8.99 066	128	8.99 275	130	1.00 725	9.99 791	23	
38	8.99 194	128	8.99 405	129	1.00 595	9.99 790	22	
39	8.99 322	128	8.99 534	128	1.00 466	9.99 788	21	
40	8.99 450	127	8.99 662	129	1.00 338	9.99 787	20	
41	8.99 577	127	8.99 791	128	1.00 209	9.99 786	19	
42	8.99 704	126	8.99 919	127	1.00 081	9.99 785	18	
43	8.99 830	126	9.00 046	128	0.99 954	9.99 783	17	
44	8.99 956	126	9.00 174	127	0.99 826	9.99 782	16	
45	9.00 082	125	9.00 301	126	0.99 699	9.99 781	15	
46	9.00 207	125	9.00 427	126	0.99 573	9.99 780	14	
47	9.00 332	124	9.00 553	126	0.99 447	9.99 778	13	
48	9.00 456	125	9.00 679	126	0.99 321	9.99 777	12	
49	9.00 581	123	9.00 805	125	0.99 195	9.99 776	11	
50	9.00 704	124	9.00 930	125	0.99 070	9.99 775	10	
51	9.00 828	123	9.01 055	124	0.98 945	9.99 773	9	
52	9.00 951	123	9.01 179	124	0.98 821	9.99 772	8	
53	9.01 074	122	9.01 303	124	0.98 697	9.99 771	7	
54	9.01 196	122	9.01 427	123	0.98 573	9.99 769	6	
55	9.01 318	122	9.01 550	123	0.98 450	9.99 768	5	
56	9.01 440	121	9.01 673	123	0.98 327	9.99 767	4	
57	9.01 561	121	9.01 796	122	0.98 204	9.99 765	3	
58	9.01 682	121	9.01 918	122	0.98 082	9.99 764	2	
59	9.01 803	120	9.02 040	122	0.97 960	9.99 763	1	
60	9.01 923		9.02 162		0.97 838	9.99 761	0	
	log cos	d.	log cot	c. d.	log tan	log sin	′	Prop. Pts.
*174° 264° *354° 84°								

Prop. Pts.

d	pp 1″
145	2.42
144	2.40
143	2.38
142	2.37
141	2.35
140	2.33
139	2.32
138	2.30
137	2.28
136	2.27
135	2.25
134	2.23
133	2.22
132	2.20
131	2.18
130	2.17
129	2.15
128	2.13
127	2.12
126	2.10
125	2.08
124	2.07
123	2.05
122	2.03
121	2.02
120	2.00

6° *96° 186° *276°

′	log sin	d.	log tan	c. d.	log cot	log cos	
0	9.01 923	120	9.02 162	121	0.97 838	9.99 761	60
1	9.02 043	120	9.02 283	121	0.97 717	9.99 760	59
2	9.02 163	120	9.02 404	121	0.97 596	9.99 759	58
3	9.02 283	119	9.02 525	120	0.97 475	9.99 757	57
4	9.02 402	118	9.02 645	121	0.97 355	9.99 756	56
5	9.02 520	119	9.02 766	119	0.97 234	9.99 755	55
6	9.02 639	118	9.02 885	120	0.97 115	9.99 753	54
7	9.02 757	117	9.03 005	119	0.96 995	9.99 752	53
8	9.02 874	118	9.03 124	118	0.96 876	9.99 751	52
9	9.02 992	117	9.03 242	119	0.96 758	9.99 749	51
10	9.03 109	117	9.03 361	118	0.96 639	9.99 748	50
11	9.03 226	116	9.03 479	118	0.96 521	9.99 747	49
12	9.03 342	116	9.03 597	117	0.96 403	9.99 745	48
13	9.03 458	116	9.03 714	118	0.96 286	9.99 744	47
14	9.03 574	116	9.03 832	116	0.96 168	9.99 742	46
15	9.03 690	115	9.03 948	117	0.96 052	9.99 741	45
16	9.03 805	115	9.04 065	116	0.95 935	9.99 740	44
17	9.03 920	114	9.04 181	116	0.95 819	9.99 738	43
18	9.04 034	115	9.04 297	116	0.95 703	9.99 737	42
19	9.04 149	113	9.04 413	115	0.95 587	9.99 736	41
20	9.04 262	114	9.04 528	115	0.95 472	9.99 734	40
21	9.04 376	114	9.04 643	115	0.95 357	9.99 733	39
22	9.04 490	113	9.04 758	115	0.95 242	9.99 731	38
23	9.04 603	112	9.04 873	114	0.95 127	9.99 730	37
24	9.04 715	113	9.04 987	114	0.95 013	9.99 728	36
25	9.04 828	112	9.05 101	113	0.94 899	9.99 727	35
26	9.04 940	112	9.05 214	114	0.94 786	9.99 726	34
27	9.05 052	112	9.05 328	113	0.94 672	9.99 724	33
28	9.05 164	111	9.05 441	112	0.94 559	9.99 723	32
29	9.05 275	111	9.05 553	113	0.94 447	9.99 721	31
30	9.05 386	111	9.05 666	112	0.94 334	9.99 720	30
31	9.05 497	110	9.05 778	112	0.94 222	9.99 718	29
32	9.05 607	110	9.05 890	112	0.94 110	9.99 717	28
33	9.05 717	110	9.06 002	111	0.93 998	9.99 716	27
34	9.05 827	110	9.06 113	111	0.93 887	9.99 714	26
35	9.05 937	109	9.06 224	111	0.93 776	9.99 713	25
36	9.06 046	109	9.06 335	110	0.93 665	9.99 711	24
37	9.06 155	109	9.06 445	111	0.93 555	9.99 710	23
38	9.06 264	108	9.06 556	110	0.93 444	9.99 708	22
39	9.06 372	109	9.06 666	109	0.93 334	9.99 707	21
40	9.06 481	108	9.06 775	110	0.93 225	9.99 705	20
41	9.06 589	107	9.06 885	109	0.93 115	9.99 704	19
42	9.06 696	108	9.06 994	109	0.93 006	9.99 702	18
43	9.06 804	107	9.07 103	108	0.92 897	9.99 701	17
44	9.06 911	107	9.07 211	109	0.92 789	9.99 699	16
45	9.07 018	106	9.07 320	108	0.92 680	9.99 698	15
46	9.07 124	107	9.07 428	108	0.92 572	9.99 696	14
47	9.07 231	106	9.07 536	107	0.92 464	9.99 695	13
48	9.07 337	105	9.07 643	108	0.92 357	9.99 693	12
49	9.07 442	106	9.07 751	107	0.92 249	9.99 692	11
50	9.07 548	105	9.07 858	106	0.92 142	9.99 690	10
51	9.07 653	105	9.07 964	107	0.92 036	9.99 689	9
52	9.07 758	105	9.08 071	106	0.91 929	9.99 687	8
53	9.07 863	105	9.08 177	106	0.91 823	9.99 686	7
54	9.07 968	104	9.08 283	106	0.91 717	9.99 684	6
55	9.08 072	104	9.08 389	106	0.91 611	9.99 683	5
56	9.08 176	104	9.08 495	105	0.91 505	9.99 681	4
57	9.08 280	103	9.08 600	105	0.91 400	9.99 680	3
58	9.08 383	103	9.08 705	105	0.91 295	9.99 678	2
59	9.08 486	103	9.08 810	104	0.91 190	9.99 677	1
60	9.08 589		9.08 914		0.91 086	9.99 675	0
	log cos	d.	log cot	c. d.	log tan	log sin	′

Prop. Pts.

″	121	120	119
6	12.1	12.0	11.9
7	14.1	14.0	13.9
8	16.1	16.0	15.9
9	18.2	18.0	17.8
10	20.2	20.0	19.8
20	40.3	40.0	39.7
30	60.5	60.0	59.5
40	80.7	80.0	79.3
50	100.8	100.0	99.2

″	118	117	116
6	11.8	11.7	11.6
7	13.8	13.6	13.5
8	15.7	15.6	15.5
9	17.7	17.6	17.4
10	19.7	19.5	19.3
20	39.3	39.0	38.7
30	59.0	58.5	58.0
40	78.7	78.0	77.3
50	98.3	97.5	96.7

″	115	114	113
6	11.5	11.4	11.3
7	13.4	13.3	13.2
8	15.3	15.2	15.1
9	17.2	17.1	17.0
10	19.2	19.0	18.8
20	38.3	38.0	37.7
30	57.5	57.0	56.5
40	76.7	76.0	75.3
50	95.8	95.0	94.2

″	112	111	110
6	11.2	11.1	11.0
7	13.1	13.0	12.8
8	14.9	14.8	14.7
9	16.8	16.6	16.5
10	18.7	18.5	18.3
20	37.3	37.0	36.7
30	56.0	55.5	55.0
40	74.7	74.0	73.3
50	93.3	92.5	91.7

″	109	108	107
6	10.9	10.8	10.7
7	12.7	12.6	12.5
8	14.5	14.4	14.3
9	16.4	16.2	16.0
10	18.2	18.0	17.8
20	36.3	36.0	35.7
30	54.5	54.0	53.5
40	72.7	72.0	71.3
50	90.8	90.0	89.2

″	106	105	104
6	10.6	10.5	10.4
7	12.4	12.2	12.1
8	14.1	14.0	13.9
9	15.9	15.8	15.6
10	17.7	17.5	17.3
20	35.3	35.0	34.7
30	53.0	52.5	52.0
40	70.7	70.0	69.3
50	88.3	87.5	86.7

Prop. Pts.

*173° 263° *353° **83°**

7° *97° 187° *277°

′	log sin	d.	log tan	c. d.	log cot	log cos	
0	9.08 589	103	9.08 914	105	0.91 086	9.99 675	60
1	9.08 692	103	9.09 019	104	0.90 981	9.99 674	59
2	9.08 795	102	9.09 123	104	0.90 877	9.99 672	58
3	9.08 897	102	9.09 227	103	0.90 773	9.99 670	57
4	9.08 999	102	9.09 330	104	0.90 670	9.99 669	56
5	9.09 101	101	9.09 434	103	0.90 566	9.99 667	55
6	9.09 202	102	9.09 537	103	0.90 463	9.99 666	54
7	9.09 304	101	9.09 640	102	0.90 360	9.99 664	53
8	9.09 405	101	9.09 742	103	0.90 258	9.99 663	52
9	9.09 506	100	9.09 845	102	0.90 155	9.99 661	51
10	9.09 606	101	9.09 947	102	0.90 053	9.99 659	50
11	9.09 707	100	9.10 049	101	0.89 951	9.99 658	49
12	9.09 807	100	9.10 150	102	0.89 850	9.99 656	48
13	9.09 907	99	9.10 252	101	0.89 748	9.99 655	47
14	9.10 006	100	9.10 353	101	0.89 647	9.99 653	46
15	9.10 106	99	9.10 454	101	0.89 546	9.99 651	45
16	9.10 205	99	9.10 555	101	0.89 445	9.99 650	44
17	9.10 304	98	9.10 656	100	0.89 344	9.99 648	43
18	9.10 402	99	9.10 756	100	0.89 244	9.99 647	42
19	9.10 501	98	9.10 856	100	0.89 144	9.99 645	41
20	9.10 599	98	9.10 956	100	0.89 044	9.99 643	40
21	9.10 697	98	9.11 056	99	0.88 944	9.99 642	39
22	9.10 795	98	9.11 155	99	0.88 845	9.99 640	38
23	9.10 893	97	9.11 254	99	0.88 746	9.99 638	37
24	9.10 990	97	9.11 353	99	0.88 647	9.99 637	36
25	9.11 087	97	9.11 452	99	0.88 548	9.99 635	35
26	9.11 184	97	9.11 551	98	0.88 449	9.99 633	34
27	9.11 281	96	9.11 649	98	0.88 351	9.99 632	33
28	9.11 377	97	9.11 747	98	0.88 253	9.99 630	32
29	9.11 474	96	9.11 845	98	0.88 155	9.99 629	31
30	9.11 570	96	9.11 943	97	0.88 057	9.99 627	30
31	9.11 666	95	9.12 040	98	0.87 960	9.99 625	29
32	9.11 761	96	9.12 138	97	0.87 862	9.99 624	28
33	9.11 857	95	9.12 235	97	0.87 765	9.99 622	27
34	9.11 952	95	9.12 332	96	0 87 668	9.99 620	26
35	9.12 047	95	9.12 428	97	0.87 572	9 99 618	25
36	9.12 142	94	9.12 525	96	0.87 475	9.99 617	24
37	9.12 236	95	9.12 621	96	0.87 379	9.99 615	23
38	9.12 331	94	9.12 717	96	0.87 283	9.99 613	22
39	9.12 425	94	9.12 813	96	0.87 187	9.99 612	21
40	9.12 519	93	9.12 909	95	0.87 091	9.99 610	20
41	9.12 612	94	9.13 004	95	0.86 996	9.99 608	19
42	9.12 706	93	9.13 099	95	0.86 901	9.99 607	18
43	9.12 799	93	9.13 194	95	0.86 806	9.99 605	17
44	9.12 892	93	9.13 289	95	0.86 711	9.99 603	16
45	9.12 985	93	9.13 384	94	0.86 616	9.99 601	15
46	9.13 078	93	9.13 478	95	0.86 522	9.99 600	14
47	9.13 171	92	9.13 573	94	0.86 427	9.99 598	13
48	9.13 263	92	9.13 667	94	0.86 333	9.99 596	12
49	9.13 355	92	9.13 761	93	0.86 239	9.99 595	11
50	9.13 447	92	9.13 854	94	0.86 146	9.99 593	10
51	9.13 539	91	9.13 948	93	0.86 052	9.99 591	9
52	9.13 630	92	9.14 041	93	0.85 959	9.99 589	8
53	9.13 722	91	9.14 134	93	0.85 866	9.99 588	7
54	9.13 813	91	9.14 227	93	0.85 773	9.99 586	6
55	9.13 904	90	9.14 320	92	0.85 680	9.99 584	5
56	9.13 994	91	9.14 412	92	0.85 588	9.99 582	4
57	9.14 085	90	9.14 504	93	0.85 496	9.99 581	3
58	9.14 175	91	9.14 597	91	0.85 403	9.99 579	2
59	9.14 266	90	9.14 688	92	0.85 312	9.99 577	1
60	9.14 356		9.14 780		0.85 220	9.99 575	0
	log cos	d.	log cot	c. d.	log tan	log sin	′

Prop. Pts.

″	105	104	103
6	10.5	10.4	10.3
7	12.3	12.1	12.0
8	14.0	13.9	13.7
9	15.8	15.6	15.4
10	17.5	17.3	17.2
20	35.0	34.7	34.3
30	52.5	52.0	51.5
40	70.0	69.3	68.7
50	87.5	86.7	85.8

″	102	101	100
6	10.2	10.1	10.0
7	11.9	11.8	11.7
8	13.6	13.5	13.3
9	15.3	15.2	15.0
10	17.0	16.8	16.7
20	34.0	33.7	33.3
30	51.0	50.5	50.0
40	68.0	67.3	66.7
50	85.0	84.2	83.3

″	99	98	97
6	9.9	9.8	9.7
7	11.6	11.4	11.3
8	13.2	13.1	12.9
9	14.8	14.7	14.6
10	16.5	16.3	16.2
20	33.0	32.7	32.3
30	49.5	49.0	48.5
40	66.0	65.3	64.7
50	82.5	81.7	80.8

″	96	95	94
6	9.6	9.5	9.4
7	11.2	11.1	11.0
8	12.8	12.7	12.5
9	14.4	14.2	14.1
10	16.0	15.8	15.7
20	32.0	31.7	31.3
30	48.0	47.5	47.0
40	64.0	63.3	62.7
50	80.0	79.2	78.3

″	93	92	91
6	9.3	9.2	9.1
7	10.9	10.7	10.6
8	12.4	12.3	12.1
9	14.0	13.8	13.6
10	15.5	15.3	15.2
20	31.0	30.7	30.3
30	46.5	46.0	45.5
40	62.0	61.3	60.7
50	77.5	76.7	75.8

″	90	2	1
6	9.0	0.2	0.1
7	10.5	0.2	0.1
8	12.0	0.3	0.1
9	13.5	0.3	0.2
10	15.0	0.3	0.2
20	30.0	0.7	0.3
30	45.0	1.0	0.5
40	60.0	1.3	0.7
50	75.0	1.7	0.8

Prop. Pts.

*172° 262° *352° **82°**

8° *98° 188° *278°

′	log sin	d.	log tan	c. d.	log cot	log cos	
0	9.14 356	89	9.14 780	92	0.85 220	9.99 575	60
1	9.14 445	90	9.14 872	91	0.85 128	9.99 574	59
2	9.14 535	89	9.14 963	91	0.85 037	9.99 572	58
3	9.14 624	90	9.15 054	91	0.84 946	9.99 570	57
4	9.14 714	89	9.15 145	91	0.84 855	9.99 568	56
5	9.14 803	88	9.15 236	91	0.84 764	9.99 566	55
6	9.14 891	89	9.15 327	90	0.84 673	9.99 565	54
7	9.14 980	89	9.15 417	91	0.84 583	9.99 563	53
8	9.15 069	88	9.15 508	90	0.84 492	9.99 561	52
9	9.15 157	88	9.15 598	90	0.84 402	9.99 559	51
10	9.15 245	88	9.15 688	89	0.84 312	9.99 557	50
11	9.15 333	88	9.15 777	90	0.84 223	9.99 556	49
12	9.15 421	87	9.15 867	89	0.84 133	9.99 554	48
13	9.15 508	88	9.15 956	90	0.84 044	9.99 552	47
14	9.15 596	87	9.16 046	89	0.83 954	9.99 550	46
15	9.15 683	87	9.16 135	89	0.83 865	9.99 548	45
16	9.15 770	87	9.16 224	88	0.83 776	9.99 546	44
17	9.15 857	87	9.16 312	89	0.83 688	9.99 545	43
18	9.15 944	86	9.16 401	88	0.83 599	9.99 543	42
19	9.16 030	86	9.16 489	88	0.83 511	9.99 541	41
20	9.16 116	87	9.16 577	88	0.83 423	9.99 539	40
21	9.16 203	86	9.16 665	88	0.83 335	9.99 537	39
22	9.16 289	85	9.16 753	88	0.83 247	9.99 535	38
23	9.16 374	86	9.16 841	87	0.83 159	9.99 533	37
24	9.16 460	85	9.16 928	88	0.83 072	9.99 532	36
25	9.16 545	86	9.17 016	87	0.82 984	9.99 530	35
26	9.16 631	85	9.17 103	87	0.82 897	9.99 528	34
27	9.16 716	85	9.17 190	87	0.82 810	9.99 526	33
28	9.16 801	85	9.17 277	86	0.82 723	9.99 524	32
29	9.16 886	84	9.17 363	87	0.82 637	9.99 522	31
30	9.16 970	85	9.17 450	86	0.82 550	9.99 520	30
31	9.17 055	84	9.17 536	86	0.82 464	9.99 518	29
32	9.17 139	84	9.17 622	86	0.82 378	9.99 517	28
33	9.17 223	84	9.17 708	86	0.82 292	9.99 515	27
34	9.17 307	84	9.17 794	86	0.82 206	9.99 513	26
35	9.17 391	83	9.17 880	85	0.82 120	9.99 511	25
36	9.17 474	84	9.17 965	86	0.82 035	9.99 509	24
37	9.17 558	83	9.18 051	85	0 81 949	9.99 507	23
38	9.17 641	83	9.18 136	85	0.81 864	9.99 505	22
39	9.17 724	83	9.18 221	85	0.81 779	9.99 503	21
40	9.17 807	83	9.18 306	85	0.81 694	9.99 501	20
41	9.17 890	83	9.18 391	84	0.81 609	9.99 499	19
42	9.17 973	82	9.18 475	85	0.81 525	9.99 497	18
43	9.18 055	82	9.18 560	84	0.81 440	9.99 495	17
44	9.18 137	83	9.18 644	84	0.81 356	9.99 494	16
45	9.18 220	82	9.18 728	84	0.81 272	9.99 492	15
46	9.18 302	81	9.18 812	84	0.81 188	9.99 490	14
47	9.18 383	82	9.18 896	83	0.81 104	9.99 488	13
48	9.18 465	82	9.18 979	84	0.81 021	9.99 486	12
49	9.18 547	81	9.19 063	83	0.80 937	9.99 484	11
50	9.18 628	81	9.19 146	83	0.80 854	9.99 482	10
51	9.18 709	81	9.19 229	83	0.80 771	9.99 480	9
52	9.18 790	81	9.19 312	83	0.80 688	9.99 478	8
53	9.18 871	81	9.19 395	83	0.80 605	9.99 476	7
54	9.18 952	81	9.19 478	83	0.80 522	9.99 474	6
55	9.19 033	80	9.19 561	82	0.80 439	9.99 472	5
56	9.19 113	80	9.19 643	82	0.80 357	9.99 470	4
57	9.19 193	80	9.19 725	82	0.80 275	9.99 468	3
58	9.19 273	80	9.19 807	82	0.80 193	9.99 466	2
59	9.19 353	80	9.19 889	82	0.80 111	9.99 464	1
60	9.19 433		9.19 971		0.80 029	9.99 462	0
	log cos	d.	log cot	c. d.	log tan	log sin	′

Prop. Pts.

″	92	91	90
6	9.2	9.1	9.0
7	10.7	10.6	10.5
8	12.3	12.1	12.0
9	13.8	13.6	13.5
10	15.3	15.2	15.0
20	30.7	30.3	30.0
30	46.0	45.5	45.0
40	61.3	60.7	60.0
50	76.7	75.8	75.0

″	89	88	87
6	8.9	8.8	8.7
7	10.4	10.3	10.2
8	11.9	11.7	11.6
9	13.4	13.2	13.1
10	14.8	14.7	14.5
20	29.7	29.3	29.0
30	44.5	44.0	43.5
40	59.3	58.7	58.0
50	74.2	73.3	72.5

″	86	85	84
6	8.6	8.5	8.4
7	10.0	9.9	9.8
8	11.5	11.3	11.2
9	12.9	12.8	12.6
10	14.3	14.2	14.0
20	28.7	28.3	28.0
30	43.0	42.5	42.0
40	57.3	56.7	56.0
50	71.7	70.8	70.0

″	83	82	81
6	8.3	8.2	8.1
7	9.7	9.6	9.5
8	11.1	10.9	10.8
9	12.5	12.3	12.2
10	13.8	13.7	13.5
20	27.7	27.3	27.0
30	41.5	41.0	40.5
40	55.3	54.7	54.0
50	69.2	68.3	67.5

″	80	2	1
6	8.0	0.2	0.1
7	9.3	0.2	0.1
8	10.7	0.3	0.1
9	12.0	0.3	0.2
10	13.3	0.3	0.2
20	26.7	0.7	0.3
30	40.0	1.0	0.5
40	53.3	1.3	0.7
50	66.7	1.7	0.8

Prop. Pts.

*171° 261° *351° 81°

9° *99° 189° *279°

′	log sin	d.	log tan	c. d.	log cot	log cos	
0	9.19 433	80	9.19 971	82	0.80 029	9.99 462	60
1	9.19 513	79	9.20 053	81	0.79 947	9.99 460	59
2	9.19 592	80	9.20 134	82	0.79 866	9.99 458	58
3	9.19 672	79	9.20 216	81	0.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	0.79 622	9.99 452	55
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54
7	9.19 988	79	9.20 540	81	0.79 460	9.99 448	53
8	9.20 067	78	9.20 621	80	0.79 379	9.99 446	52
9	9.20 145	78	9.20 701	81	0.79 299	9.99 444	51
10	9.20 223	79	9.20 782	80	0.79 218	9.99 442	50
11	9.20 302	78	9.20 862	80	0.79 138	9.99 440	49
12	9.20 380	78	9.20 942	80	0.79 058	9.99 438	48
13	9.20 458	77	9.21 022	80	0.78 978	9.99 436	47
14	9.20 535	78	9.21 102	80	0.78 898	9.99 434	46
15	9.20 613	78	9.21 182	79	0.78 818	9.99 432	45
16	9.20 691	77	9.21 261	80	0.78 739	9.99 429	44
17	9.20 768	77	9.21 341	79	0.78 659	9.99 427	43
18	9.20 845	77	9.21 420	79	0.78 580	9.99 425	42
19	9.20 922	77	9.21 499	79	0.78 501	9.99 423	41
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	40
21	9.21 076	77	9.21 657	79	0.78 343	9.99 419	39
22	9.21 153	76	9.21 736	78	0.78 264	9.99 417	38
23	9.21 229	77	9.21 814	79	0.78 186	9.99 415	37
24	9.21 306	76	9.21 893	78	0.78 107	9.99 413	36
25	9.21 382	76	9.21 971	78	0.78 029	9.99 411	35
26	9.21 458	76	9.22 049	78	0.77 951	9.99 409	34
27	9.21 534	76	9.22 127	78	0.77 873	9.99 407	33
28	9.21 610	75	9.22 205	78	0.77 795	9.99 404	32
29	9.21 685	76	9.22 283	78	0.77 717	9.99 402	31
30	9.21 761	75	9.22 361	77	0.77 639	9.99 400	30
31	9.21 836	76	9.22 438	78	0.77 562	9.99 398	29
32	9.21 912	75	9.22 516	77	0.77 484	9.99 396	28
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27
34	9.22 062	75	9.22 670	77	0.77 330	9.99 392	26
35	9.22 137	74	9.22 747	77	0.77 253	9.99 390	25
36	9.22 211	75	9.22 824	77	0.77 176	9.99 388	24
37	9.22 286	75	9.22 901	76	0.77 099	9.99 385	23
38	9.22 361	74	9.22 977	77	0.77 023	9.99 383	22
39	9.22 435	74	9.23 054	76	0.76 946	9.99 381	21
40	9.22 509	74	9.23 130	76	0.76 870	9.99 379	20
41	9.22 583	74	9.23 206	77	0.76 794	9.99 377	19
42	9.22 657	74	9.23 283	76	0.76 717	9.99 375	18
43	9.22 731	74	9.23 359	76	0.76 641	9.99 372	17
44	9.22 805	73	9.23 435	75	0.76 565	9.99 370	16
45	9.22 878	74	9.23 510	76	0.76 490	9.99 368	15
46	9.22 952	73	9.23 586	75	0.76 414	9.99 366	14
47	9.23 025	73	9.23 661	76	0.76 339	9.99 364	13
48	9.23 098	73	9.23 737	75	0.76 263	9.99 362	12
49	9.23 171	73	9.23 812	75	0.76 188	9.99 359	11
50	9.23 244	73	9.23 887	75	0.76 113	9.99 357	10
51	9.23 317	73	9.23 962	75	0.76 038	9.99 355	9
52	9.23 390	72	9.24 037	75	0.75 963	9.99 353	8
53	9.23 462	73	9.24 112	74	0.75 888	9.99 351	7
54	9.23 535	72	9.24 186	75	0.75 814	9.99 348	6
55	9.23 607	72	9.24 261	74	0.75 739	9.99 346	5
56	9.23 679	73	9.24 335	75	0.75 665	9.99 344	4
57	9.23 752	71	9.24 410	74	0.75 590	9.99 342	3
58	9.23 823	72	9.24 484	74	0.75 516	9.99 340	2
59	9.23 895	72	9.24 558	74	0.75 442	9.99 337	1
60	9.23 967		9.24 632		0.75 368	9.99 335	0
	log cos	d.	log cot	c. d.	log tan	log sin	′

*170° 260° *350° **80°**

Prop. Pts.

″	80	79	78
6	8.0	7.9	7.8
7	9.3	9.2	9.1
8	10.7	10.5	10.4
9	12.0	11.9	11.7
10	13.3	13.2	13.0
20	26.7	26.3	26.0
30	40.0	39.5	39.0
40	53.3	52.7	52.0
50	66.7	65.8	65.0

″	77	76	75
6	7.7	7.6	7.5
7	9.0	8.9	8.8
8	10.3	10.1	10.0
9	11.6	11.4	11.3
10	12.8	12.7	12.5
20	25.7	25.3	25.0
30	38.5	38.0	37.5
40	51.3	50.7	50.0
50	64.2	63.3	62.5

″	74	73	72
6	7.4	7.3	7.2
7	8.6	8.5	8.4
8	9.9	9.7	9.6
9	11.1	11.0	10.8
10	12.3	12.2	12.0
20	24.7	24.3	24.0
30	37.0	36.5	36.0
40	49.3	48.7	48.0
50	61.7	60.8	60.0

″	71	3	2
6	7.1	0.3	0.2
7	8.3	0.4	0.2
8	9.5	0.4	0.3
9	10.7	0.5	0.3
10	11.8	0.5	0.3
20	23.7	1.0	0.7
30	35.5	1.5	1.0
40	47.3	2.0	1.3
50	59.2	2.5	1.7

10° *100° 190° *280°

′	log sin	d.	log tan	c. d.	log cot	log cos	
0	9.23 967	72	9.24 632	74	0.75 368	9.99 335	60
1	9.24 039	71	9.24 706	73	0.75 294	9.99 333	59
2	9.24 110	71	9.24 779	74	0.75 221	9.99 331	58
3	9.24 181	72	9.24 853	73	0.75 147	9.99 328	57
4	9.24 253	71	9.24 926	74	0.75 074	9.99 326	56
5	9.24 324	71	9.25 000	73	0.75 000	9.99 324	55
6	9.24 395	71	9.25 073	73	0.74 927	9.99 322	54
7	9.24 466	70	9.25 146	73	0.74 854	9.99 319	53
8	9.24 536	71	9.25 219	73	0.74 781	9.99 317	52
9	9.24 607	70	9.25 292	73	0.74 708	9.99 315	51
10	9.24 677	71	9.25 365	72	0.74 635	9.99 313	50
11	9.24 748	70	9.25 437	73	0.74 563	9.99 310	49
12	9.24 818	70	9.25 510	72	0.74 490	9.99 308	48
13	9.24 888	70	9.25 582	73	0.74 418	9.99 306	47
14	9.24 958	70	9.25 655	72	0.74 345	9.99 304	46
15	9.25 028	70	9.25 727	72	0.74 273	9.99 301	45
16	9.25 098	70	9.25 799	72	0.74 201	9.99 299	44
17	9.25 168	69	9.25 871	72	0.74 129	9.99 297	43
18	9.25 237	70	9.25 943	72	0.74 057	9.99 294	42
19	9.25 307	69	9.26 015	71	0.73 985	9.99 292	41
20	9.25 376	69	9.26 086	72	0.73 914	9.99 290	40
21	9.25 445	69	9.26 158	71	0.73 842	9.99 288	39
22	9.25 514	69	9.26 229	72	0.73 771	9.99 285	38
23	9.25 583	69	9.26 301	71	0.73 699	9.99 283	37
24	9.25 652	69	9.26 372	71	0.73 628	9.99 281	36
25	9.25 721	69	9.26 443	71	0.73 557	9.99 278	35
26	9.25 790	68	9.26 514	71	0.73 486	9.99 276	34
27	9.25 858	69	9.26 585	70	0.73 415	9.99 274	33
28	9.25 927	68	9.26 655	71	0.73 345	9.99 271	32
29	9.25 995	68	9.26 726	71	0.73 274	9.99 269	31
30	9.26 063	68	9.26 797	70	0.73 203	9.99 267	30
31	9.26 131	68	9.26 867	70	0.73 133	9.99 264	29
32	9.26 199	68	9.26 937	71	0.73 063	9.99 262	28
33	9.26 267	68	9.27 008	70	0.72 992	9.99 260	27
34	9.26 335	68	9.27 078	70	0.72 922	9.99 257	26
35	9.26 403	67	9.27 148	70	0.72 852	9.99 255	25
36	9.26 470	68	9.27 218	70	0.72 782	9.99 252	24
37	9.26 538	67	9.27 288	69	0.72 712	9.99 250	23
38	9.26 605	67	9.27 357	70	0.72 643	9.99 248	22
39	9.26 672	67	9.27 427	69	0.72 573	9.99 245	21
40	9.26 739	67	9.27 496	70	0.72 504	9.99 243	20
41	9.26 806	67	9.27 566	69	0.72 434	9.99 241	19
42	9.26 873	67	9.27 635	69	0.72 365	9.99 238	18
43	9.26 940	67	9.27 704	69	0.72 296	9.99 236	17
44	9.27 007	66	9.27 773	69	0.72 227	9.99 233	16
45	9.27 073	67	9.27 842	69	0.72 158	9.99 231	15
46	9.27 140	66	9.27 911	69	0.72 089	9.99 229	14
47	9.27 206	67	9.27 980	69	0.72 020	9.99 226	13
48	9.27 273	66	9.28 049	68	0.71 951	9.99 224	12
49	9.27 339	66	9.28 117	69	0.71 883	9.99 221	11
50	9.27 405	66	9.28 186	68	0.71 814	9.99 219	10
51	9.27 471	66	9.28 254	69	0.71 746	9.99 217	9
52	9.27 537	65	9.28 323	68	0.71 677	9.99 214	8
53	9.27 602	66	9.28 391	68	0.71 609	9.99 212	7
54	9.27 668	66	9.28 459	68	0.71 541	9.99 209	6
55	9.27 734	65	9.28 527	68	0.71 473	9.99 207	5
56	9.27 799	65	9.28 595	67	0.71 405	9.99 204	4
57	9.27 864	66	9.28 662	68	0.71 338	9.99 202	3
58	9.27 930	65	9.28 730	68	0.71 270	9.99 200	2
59	9.27 995	65	9.28 798	67	0.71 202	9.99 197	1
60	9.28 060		9.28 865		0.71 135	9.99 195	0
	log cos	d.	log cot	c. d.	log tan	log sin	′

Prop. Pts.

″	74	73	72
6	7.4	7.3	7.2
7	8.6	8.5	8.4
8	9.9	9.7	9.6
9	11.1	11.0	10.8
10	12.3	12.2	12.0
20	24.7	24.3	24.0
30	37.0	36.5	36.0
40	49.3	48.7	48.0
50	61.7	60.8	60.0

″	71	70	69
6	7.1	7.0	6.9
7	8.3	8.2	8.0
8	9.5	9.3	9.2
9	10.7	10.5	10.4
10	11.8	11.7	11.5
20	23.7	23.3	23.0
30	35.5	35.0	34.5
40	47.3	46.7	46.0
50	59.2	58.3	57.5

″	68	67	66
6	6.8	6.7	6.6
7	7.9	7.8	7.7
8	9.1	8.9	8.8
9	10.2	10.0	9.9
10	11.3	11.2	11.0
20	22.7	22.3	22.0
30	34.0	33.5	33.0
40	45.3	44.7	44.0
50	56.7	55.8	55.0

″	65	3	2
6	6.5	0.3	0.2
7	7.6	0.4	0.2
8	8.7	0.4	0.3
9	9.8	0.5	0.3
10	10.8	0.5	0.3
20	21.7	1.0	0.7
30	32.5	1.5	1.0
40	43.3	2.0	1.3
50	54.2	2.5	1.7

Prop. Pts.

*169° 259° *349° **79°**

11° *101° 191° *281°

′	log sin	d.	log tan	c. d.	log cot	log cos	
0	9.28 060	65	9.28 865	68	0.71 135	9.99 195	60
1	9.28 125	65	9.28 933	67	0.71 067	9.99 192	59
2	9.28 190	64	9.29 000	67	0.71 000	9.99 190	58
3	9.28 254	65	9.29 067	67	0.70 933	9.99 187	57
4	9.28 319	65	9.29 134	67	0.70 866	9.99 185	56
5	9.28 384	64	9.29 201	67	0.70 799	9.99 182	55
6	9.28 448	64	9.29 268	67	0.70 732	9.99 180	54
7	9.28 512	65	9.29 335	67	0.70 665	9.99 177	53
8	9.28 577	64	9.29 402	66	0.70 598	9.99 175	52
9	9.28 641	64	9.29 468	67	0.70 532	9.99 172	51
10	9.28 705	64	9.29 535	66	0.70 465	9.99 170	50
11	9.28 769	64	9.29 601	67	0.70 399	9.99 167	49
12	9.28 833	63	9.29 668	66	0.70 332	9.99 165	48
13	9.28 896	64	9.29 734	66	0.70 266	9.99 162	47
14	9.28 960	64	9.29 800	66	0.70 200	9.99 160	46
15	9.29 024	63	9.29 865	66	0.70 134	9.99 157	45
16	9.29 087	63	9.29 932	66	0.70 068	9.99 155	44
17	9.29 150	64	9.29 998	66	0.70 002	9.99 152	43
18	9.29 214	63	9.30 064	66	0.69 936	9.99 150	42
19	9.29 277	63	9.30 130	65	0.69 870	9.99 147	41
20	9.29 340	63	9.30 195	66	0.69 805	9.99 145	40
21	9.29 403	63	9.30 261	65	0.69 739	9.99 142	39
22	9.29 466	63	9.30 326	65	0.69 674	9.99 140	38
23	9.29 529	62	9.30 391	66	0.69 609	9.99 137	37
24	9.29 591	63	9.30 457	65	0.69 543	9.99 135	36
25	9.29 654	62	9.30 522	65	0.69 478	9.99 132	35
26	9.29 716	63	9.30 587	65	0.69 413	9.99 130	34
27	9.29 779	62	9.30 652	65	0.69 348	9.99 127	33
28	9.29 841	62	9.30 717	65	0.69 283	9.99 124	32
29	9.29 903	63	9.30 782	64	0.69 218	9.99 122	31
30	9.29 966	62	9.30 846	65	0.69 154	9.99 119	30
31	9.30 028	62	9.30 911	64	0.69 089	9.99 117	29
32	9.30 090	61	9.30 975	65	0.69 025	9.99 114	28
33	9.30 151	62	9.31 040	64	0.68 960	9.99 112	27
34	9.30 213	62	9.31 104	64	0.68 896	9.99 109	26
35	9.30 275	61	9.31 168	65	0.68 832	9.99 106	25
36	9.30 336	62	9.31 233	64	0.68 767	9.99 104	24
37	9.30 398	61	9.31 297	64	0.68 703	9.99 101	23
38	9.30 459	62	9.31 361	64	0.68 639	9.99 099	22
39	9.30 521	61	9.31 425	64	0.68 575	9.99 096	21
40	9.30 582	61	9.31 489	63	0.68 511	9.99 093	20
41	9.30 643	61	9.31 552	64	0.68 448	9.99 091	19
42	9.30 704	61	9.31 616	63	0.68 384	9.99 088	18
43	9.30 765	61	9.31 679	64	0.68 321	9.99 086	17
44	9.30 826	61	9.31 743	63	0.68 257	9.99 083	16
45	9.30 887	60	9.31 806	64	0.68 194	9.99 080	15
46	9.30 947	61	9.31 870	63	0.68 130	9.99 078	14
47	9.31 008	60	9.31 933	63	0.68 067	9.99 075	13
48	9.31 068	61	9.31 996	63	0.68 004	9.99 072	12
49	9.31 129	60	9.32 059	63	0.67 941	9.99 070	11
50	9.31 189	61	9.32 122	63	0.67 878	9.99 067	10
51	9.31 250	60	9.32 185	63	0.67 815	9.99 064	9
52	9.31 310	60	9.32 248	63	0.67 752	9.99 062	8
53	9.31 370	60	9.32 311	62	0.67 689	9.99 059	7
54	9.31 430	60	9.32 373	63	0.67 627	9.99 056	6
55	9.31 490	59	9.32 436	62	0.67 564	9.99 054	5
56	9.31 549	60	9.32 498	63	0.67 502	9.99 051	4
57	9.31 609	60	9.32 561	62	0.67 439	9.99 048	3
58	9.31 669	59	9.32 623	62	0.67 377	9.99 046	2
59	9.31 728	60	9.32 685	62	0.67 315	9.99 043	1
60	9.31 788		9.32 747		0.67 253	9.99 040	0
	log cos	d.	log cot	c. d.	log tan	log sin	′

Prop. Pts.

″	68	67	66
6	6.8	6.7	6.6
7	7.9	7.8	7.7
8	9.1	8.9	8.8
9	10.2	10.0	9.9
10	11.3	11.2	11.0
20	22.7	22.3	22.0
30	34.0	33.5	33.0
40	45.3	44.7	44.0
50	56.7	55.8	55.0

″	65	64	63
6	6.5	6.4	6.3
7	7.6	7.5	7.4
8	8.7	8.5	8.4
9	9.8	9.6	9.4
10	10.8	10.7	10.5
20	21.7	21.3	21.0
30	32.5	32.0	31.5
40	43.3	42.7	42.0
50	54.2	53.3	52.5

″	62	61	60
6	6.2	6.1	6.0
7	7.2	7.1	7.0
8	8.3	8.1	8.0
9	9.3	9.2	9.0
10	10.3	10.2	10.0
20	20.7	20.3	20.0
30	31.0	30.5	30.0
40	41.3	40.7	40.0
50	51.7	50.8	50.0

″	59	3	2
6	5.9	0.3	0.2
7	6.9	0.4	0.2
8	7.9	0.4	0.3
9	8.9	0.5	0.3
10	9.8	0.5	0.3
20	19.7	1.0	0.7
30	29.5	1.5	1.0
40	39.3	2.0	1.3
50	49.2	2.5	1.7

Prop. Pts.

*168° 258° *348° **78°**

12° *102° 192° *282°

′	log sin	d.	log tan	c. d.	log cot	log cos	
0	9.31 788	59	9.32 747	63	0.67 253	9.99 040	60
1	9.31 847	60	9.32 810	62	0.67 190	9.99 038	59
2	9.31 907	59	9.32 872	61	0.67 128	9.99 035	58
3	9.31 966	59	9.32 933	62	0.67 067	9.99 032	57
4	9.32 025	59	9.32 995	62	0.67 005	9.99 030	56
5	9.32 084	59	9.33 057	62	0.66 943	9.99 027	55
6	9.32 143	59	9.33 119	61	0.66 881	9.99 024	54
7	9.32 202	59	9.33 180	62	0.66 820	9.99 022	53
8	9.32 261	58	9.33 242	61	0.66 758	9.99 019	52
9	9.32 319	59	9.33 303	62	0.66 697	9.99 016	51
10	9.32 378	59	9.33 365	61	0.66 635	9.99 013	50
11	9.32 437	58	9.33 426	61	0.66 574	9.99 011	49
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	48
13	9.32 553	59	9.33 548	61	0.66 452	9.99 005	47
14	9.32 612	58	9.33 609	61	0.66 391	9.99 002	46
15	9.32 670	58	9.33 670	61	0.66 330	9.99 000	45
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	44
17	9.32 786	58	9.33 792	61	0.66 208	9.98 994	43
18	9.32 844	58	9.33 853	60	0.66 147	9.98 991	42
19	9.32 902	58	9.33 913	61	0.66 087	9.98 989	41
20	9.32 960	58	9.33 974	60	0.66 026	9.98 986	40
21	9.33 018	57	9.34 034	61	0.65 966	9.98 983	39
22	9.33 075	58	9.34 095	60	0.65 905	9.98 980	38
23	9.33 133	57	9.34 155	60	0.65 845	9.98 978	37
24	9.33 190	58	9.34 215	61	0.65 785	9.98 975	36
25	9.33 248	57	9.34 276	60	0.65 724	9.98 972	35
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	34
27	9.33 362	58	9.34 396	60	0.65 604	9.98 967	33
28	9.33 420	57	9.34 456	60	0.65 544	9.98 964	32
29	9.33 477	57	9.34 516	60	0.65 484	9.98 961	31
30	9.33 534	57	9.34 576	59	0.65 424	9.98 958	30
31	9.33 591	56	9.34 635	60	0.65 365	9.98 955	29
32	9.33 647	57	9.34 695	60	0.65 305	9.98 953	28
33	9.33 704	57	9.34 755	59	0.65 245	9.98 950	27
34	9.33 761	57	9.34 814	60	0.65 186	9.98 947	26
35	9.33 818	56	9.34 874	59	0.65 126	9.98 944	25
36	9.33 874	57	9.34 933	59	0.65 067	9.98 941	24
37	9.33 931	56	9.34 992	59	0.65 008	9.98 938	23
38	9.33 987	56	9.35 051	60	0.64 949	9.98 936	22
39	9.34 043	57	9.35 111	59	0.64 889	9.98 933	21
40	9.34 100	56	9.35 170	59	0.64 830	9.98 930	20
41	9.34 156	56	9.35 229	59	0.64 771	9.98 927	19
42	9.34 212	56	9.35 288	59	0.64 712	9.98 924	18
43	9.34 268	56	9.35 347	58	0.64 653	9.98 921	17
44	9.34 324	56	9.35 405	59	0.64 595	9.98 919	16
45	9.34 380	56	9.35 464	59	0.64 536	9.98 916	15
46	9.34 436	55	9.35 523	58	0.64 477	9.98 913	14
47	9.34 491	56	9.35 581	59	0.64 419	9.98 910	13
48	9.34 547	55	9.35 640	58	0.64 360	9.98 907	12
49	9.34 602	56	9.35 698	59	0.64 302	9.98 904	11
50	9.34 658	55	9.35 757	58	0.64 243	9.98 901	10
51	9.34 713	56	9.35 815	58	0.64 185	9.98 898	9
52	9.34 769	55	9.35 873	58	0.64 127	9.98 896	8
53	9.34 824	55	9.35 931	58	0.64 069	9.98 893	7
54	9.34 879	55	9.35 989	58	0.64 011	9.98 890	6
55	9.34 934	55	9.36 047	58	0.63 953	9.98 887	5
56	9.34 989	55	9.36 105	58	0.63 895	9.98 884	4
57	9.35 044	55	9.36 163	58	0.63 837	9.98 881	3
58	9.35 099	55	9.36 221	58	0.63 779	9.98 878	2
59	9.35 154	55	9.36 279	57	0.63 721	9.98 875	1
60	9.35 209		9.36 336		0.63 664	9.98 872	0
	log cos	d.	log cot	c. d.	log tan	log sin	′

Prop. Pts.

″	63	62	61
6	6.3	6.2	6.1
7	7.4	7.2	7.1
8	8.4	8.3	8.1
9	9.4	9.3	9.2
10	10.5	10.3	10.2
20	21.0	20.7	20.3
30	31.5	31.0	30.5
40	42.0	41.3	40.7
50	52.5	51.7	50.8

″	60	59	58
6	6.0	5.9	5.8
7	7.0	6.9	6.8
8	8.0	7.9	7.7
9	9.0	8.8	8.7
10	10.0	9.8	9.7
20	20.0	19.7	19.3
30	30.0	29.5	29.0
40	40.0	39.3	38.7
50	50.0	49.2	48.3

″	57	56	55
6	5.7	5.6	5.5
7	6.6	6.5	6.4
8	7.6	7.5	7.3
9	8.6	8.4	8.3
10	9.5	9.3	9.2
20	19.0	18.7	18.3
30	28.5	28.0	27.5
40	38.0	37.3	36.7
50	47.5	46.7	45.8

″	3	2
6	0.3	0.2
7	0.4	0.2
8	0.4	0.3
9	0.5	0.3
10	0.5	0.3
20	1.0	0.7
30	1.5	1.0
40	2.0	1.3
50	2.5	1.7

Prop. Pts.

*167° 257° *347° **77°**

13° *103° 193° *283°

′	log sin	d.	log tan	c. d.	log cot	log cos	
0	9.35 209	54	9.36 336	58	0.63 664	9.98 872	60
1	9.35 263	55	9.36 394	58	0.63 606	9.98 869	59
2	9.35 318	55	9.36 452	57	0.63 548	9.98 867	58
3	9.35 373	54	9.36 509	57	0.63 491	9.98 864	57
4	9.35 427	54	9.36 566	58	0.63 434	9.98 861	56
5	9.35 481	55	9.36 624	57	0.63 376	9.98 858	55
6	9.35 536	54	9.36 681	57	0.63 319	9.98 855	54
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	53
8	9.35 644	54	9.36 795	57	0.63 205	9.98 849	52
9	9.35 698	54	9.36 852	57	0.63 148	9.98 846	51
10	9.35 752	54	9.36 909	57	0.63 091	9.98 843	50
11	9.35 806	54	9.36 966	57	0.63 034	9.98 840	49
12	9.35 860	54	9.37 023	57	0.62 977	9.98 837	48
13	9.35 914	54	9.37 080	57	0.62 920	9.98 834	47
14	9.35 968	54	9.37 137	56	0.62 863	9.98 831	46
15	9.36 022	53	9.37 193	57	0.62 807	9.98 828	45
16	9.36 075	54	9.37 250	56	0.62 750	9.98 825	44
17	9.36 129	53	9.37 306	57	0.62 694	9.98 822	43
18	9.36 182	54	9.37 363	56	0.62 637	9.98 819	42
19	9.36 236	53	9.37 419	57	0.62 581	9.98 816	41
20	9.36 289	53	9.37 476	56	0.62 524	9.98 813	40
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	39
22	9.36 395	54	9.37 588	56	0.62 412	9.98 807	38
23	9.36 449	53	9.37 644	56	0.62 356	9.98 804	37
24	9.36 502	53	9.37 700	56	0.62 300	9.98 801	36
25	9.36 555	53	9.37 756	56	0.62 244	9.98 798	35
26	9.36 608	52	9.37 812	56	0.62 188	9.98 795	34
27	9.36 660	53	9.37 868	56	0.62 132	9.98 792	33
28	9.36 713	53	9.37 924	56	0.62 076	9.98 789	32
29	9.36 766	53	9.37 980	55	0.62 020	9.98 786	31
30	9.36 819	52	9.38 035	56	0.61 965	9.98 783	30
31	9.36 871	53	9.38 091	56	0.61 909	9.98 780	29
32	9.36 924	52	9.38 147	55	0.61 853	9.98 777	28
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	27
34	9.37 028	53	9.38 257	56	0.61 743	9.98 771	26
35	9.37 081	52	9.38 313	55	0.61 687	9.98 768	25
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	24
37	9.37 185	52	9.38 423	56	0.61 577	9.98 762	23
38	9.37 237	52	9.38 479	55	0.61 521	9.98 759	22
39	9.37 289	52	9.38 534	55	0.61 466	9.98 756	21
40	9.37 341	52	9.38 589	55	0.61 411	9.98 753	20
41	9.37 393	52	9.38 644	55	0.61 356	9.98 750	19
42	9.37 445	52	9.38 699	55	0.61 301	9.98 746	18
43	9.37 497	52	9.38 754	54	0.61 246	9.98 743	17
44	9.37 549	51	9.38 808	55	0.61 192	9.98 740	16
45	9.37 600	52	9.38 863	55	0.61 137	9.98 737	15
46	9.37 652	51	9.38 918	54	0.61 082	9.98 734	14
47	9.37 703	52	9.38 972	55	0.61 028	9.98 731	13
48	9.37 755	51	9.39 027	55	0.60 973	9.98 728	12
49	9.37 806	52	9.39 082	54	0.60 918	9.98 725	11
50	9.37 858	51	9.39 136	54	0.60 864	9.98 722	10
51	9.37 909	51	9.39 190	55	0.60 810	9.98 719	9
52	9.37 960	51	9.39 245	54	0.60 755	9.98 715	8
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	7
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	6
55	9.38 113	51	9.39 407	54	0.60 593	9.98 706	5
56	9.38 164	51	9.39 461	54	0.60 539	9.98 703	4
57	9.38 215	51	9.39 515	54	0.60 485	9.98 700	3
58	9.38 266	51	9.39 569	54	0.60 431	9.98 697	2
59	9.38 317	51	9.39 623	54	0.60 377	9.98 694	1
60	9.38 368		9.39 677		0.60 323	9.98 690	0
	log cos	d.	log cot	c. d.	log tan	log sin	′

Prop. Pts.

″	58	57	56
6	5.8	5.7	5.6
7	6.8	6.6	6.5
8	7.7	7.6	7.5
9	8.7	8.6	8.4
10	9.7	9.5	9.3
20	19.3	19.0	18.7
30	29.0	28.5	28.0
40	38.7	38.0	37.3
50	48.3	47.5	46.7

″	55	54	53
6	5.5	5.4	5.3
7	6.4	6.3	6.2
8	7.3	7.2	7.1
9	8.3	8.1	8.0
10	9.2	9.0	8.8
20	18.3	18.0	17.7
30	27.5	27.0	26.5
40	36.7	36.0	35.3
50	45.8	45.0	44.2

″	52	51	4
6	5.2	5.1	0.4
7	6.1	6.0	0.5
8	6.9	6.8	0.5
9	7.8	7.7	0.6
10	8.7	8.5	0.7
20	17.3	17.0	1.3
30	26.0	25.5	2.0
40	34.7	34.0	2.7
50	43.3	42.5	3.3

″	3	2
6	0.3	0.2
7	0.4	0.2
8	0.4	0.3
9	0.5	0.3
10	0.5	0.3
20	1.0	0.7
30	1.5	1.0
40	2.0	1.3
50	2.5	1.7

*166° 256° *346° **76°**

14° *104° 194° *284°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	′
0	9.38 368		9.39 677		0.60 323	9.98 690		60
1	9.38 418	50	9.39 731	54	0.60 269	9.98 687	3	59
2	9.38 469	51	9.39 785	54	0.60 215	9.98 684	3	58
3	9.38 519	50	9.39 838	53	0.60 162	9.98 681	3	57
4	9.38 570	51	9.39 892	54	0.60 108	9.98 678	3	56
5	9.38 620	50	9.39 945	53	0.60 055	9.98 675	3	55
6	9.38 670	50	9.39 999	54	0.60 001	9.98 671	4	54
7	9.38 721	51	9.40 052	53	0.59 948	9.98 668	3	53
8	9.38 771	50	9.40 106	54	0.59 894	9.98 665	3	52
9	9.38 821	50	9.40 159	53	0.59 841	9.98 662	3	51
10	9.38 871	50	9.40 212	53	0.59 788	9.98 659	3	50
11	9.38 921	50	9.40 266	54	0.59 734	9.98 656	3	49
12	9.38 971	50	9.40 319	53	0.59 681	9.98 652	4	48
13	9.39 021	50	9.40 372	53	0.59 628	9.98 649	3	47
14	9.39 071	50	9.40 425	53	0.59 575	9.98 646	3	46
15	9.39 121	50	9.40 478	53	0.59 522	9.98 643	3	45
16	9.39 170	49	9.40 531	53	0.59 469	9.98 640	3	44
17	9.39 220	50	9.40 584	53	0.59 416	9.98 636	4	43
18	9.39 270	50	9.40 636	52	0.59 364	9.98 633	3	42
19	9.39 319	49	9.40 689	53	0.59 311	9.98 630	3	41
20	9.39 369	50	9.40 742	53	0.59 258	9.98 627	3	40
21	9.39 418	49	9.40 795	53	0.59 205	9.98 623	4	39
22	9.39 467	49	9.40 847	52	0.59 153	9.98 620	3	38
23	9.39 517	50	9.40 900	53	0.59 100	9.98 617	3	37
24	9.39 566	49	9.40 952	52	0.59 048	9.98 614	3	36
25	9.39 615	49	9.41 005	53	0.58 995	9.98 610	4	35
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34
27	9.39 713	49	9.41 109	52	0.58 891	9.98 604	3	33
28	9.39 762	49	9.41 161	52	0.58 839	9.98 601	3	32
29	9.39 811	49	9.41 214	53	0.58 786	9.98 597	4	31
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	30
31	9.39 909	49	9.41 318	52	0.58 682	9.98 591	3	29
32	9.39 958	49	9.41 370	52	0.58 630	9.98 588	3	28
33	9.40 006	48	9.41 422	52	0.58 578	9.98 584	4	27
34	9.40 055	49	9.41 474	52	0.58 526	9.98 581	3	26
35	9.40 103	48	9.41 526	52	0.58 474	9.98 578	3	25
36	9.40 152	49	9.41 578	52	0.58 422	9.98 574	4	24
37	9.40 200	48	9.41 629	51	0.58 371	9.98 571	3	23
38	9.40 249	49	9.41 681	52	0.58 319	9.98 568	3	22
39	9.40 297	48	9.41 733	52	0.58 267	9.98 565	3	21
40	9.40 346	49	9.41 784	51	0.58 216	9.98 561	4	20
41	9.40 394	48	9.41 836	52	0.58 164	9.98 558	3	19
42	9.40 442	48	9.41 887	51	0.58 113	9.98 555	3	18
43	9.40 490	48	9.41 939	52	0.58 061	9.98 551	4	17
44	9.40 538	48	9.41 990	51	0.58 010	9.98 548	3	16
45	9.40 586	48	9.42 041	51	0.57 959	9.98 545	3	15
46	9.40 634	48	9.42 093	52	0.57 907	9.98 541	4	14
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3	13
48	9.40 730	48	9.42 195	51	0.57 805	9.98 535	3	12
49	9.40 778	48	9.42 246	51	0.57 754	9.98 531	4	11
50	9.40 825	47	9.42 297	51	0.57 703	9.98 528	3	10
51	9.40 873	48	9.42 348	51	0.57 652	9.98 525	3	9
52	9.40 921	48	9.42 399	51	0.57 601	9.98 521	4	8
53	9.40 968	47	9.42 450	51	0.57 550	9.98 518	3	7
54	9.41 016	48	9.42 501	51	0.57 499	9.98 515	3	6
55	9.41 063	47	9.42 552	51	0.57 448	9.98 511	4	5
56	9.41 111	48	9.42 603	51	0.57 397	9.98 508	3	4
57	9.41 158	47	9.42 653	50	0.57 347	9.98 505	3	3
58	9.41 205	47	9.42 704	51	0.57 296	9.98 501	4	2
59	9.41 252	47	9.42 755	51	0.57 245	9.98 498	3	1
60	9.41 300	48	9.42 805	50	0.57 195	9.98 494	4	0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	54	53	52
6	5.4	5.3	5.2
7	6.3	6.2	6.1
8	7.2	7.1	6.9
9	8.1	8.0	7.8
10	9.0	8.8	8.7
20	18.0	17.7	17.3
30	27.0	26.5	26.0
40	36.0	35.3	34.7
50	45.0	44.2	43.3

″	51	50	49
6	5.1	5.0	4.9
7	6.0	5.8	5.7
8	6.8	6.7	6.5
9	7.7	7.5	7.4
10	8.5	8.3	8.2
20	17.0	16.7	16.3
30	25.5	25.0	24.5
40	34.0	33.3	32.7
50	42.5	41.7	40.8

″	48	47
6	4.8	4.7
7	5.6	5.5
8	6.4	6.3
9	7.2	7.0
10	8.0	7.8
20	16.0	15.7
30	24.0	23.5
40	32.0	31.3
50	40.0	39.2

″	4	3
6	0.4	0.3
7	0.5	0.4
8	0.5	0.4
9	0.6	0.5
10	0.7	0.5
20	1.3	1.0
30	2.0	1.5
40	2.7	2.0
50	3.3	2.5

Prop. Pts.

*165° 255° *345° **75°**

15° *105° 195° *285°

'	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.41 300	47	9.42 805	51	0.57 195	9.98 494	3	60
1	9.41 347	47	9.42 856	50	0.57 144	9.98 491	3	59
2	9.41 394	47	9.42 906	51	0.57 094	9.98 488	4	58
3	9.41 441	47	9.42 957	50	0.57 043	9.98 484	3	57
4	9.41 488	47	9.43 007	50	0.56 993	9.98 481	4	56
5	9.41 535	47	9.43 057	51	0.56 943	9.98 477	3	55
6	9.41 582	46	9.43 108	50	0.56 892	9.98 474	3	54
7	9.41 628	47	9.43 158	50	0.56 842	9.98 471	4	53
8	9.41 675	47	9.43 208	50	0.56 792	9.98 467	3	52
9	9.41 722	46	9.43 258	50	0.56 742	9.98 464	4	51
10	9.41 768	47	9.43 308	50	0.56 692	9.98 460	3	50
11	9.41 815	46	9.43 358	50	0.56 642	9.98 457	4	49
12	9.41 861	47	9.43 408	50	0.56 592	9.98 453	3	48
13	9.41 908	46	9.43 458	50	0.56 542	9.98 450	3	47
14	9.41 954	47	9.43 508	50	0.56 492	9.98 447	4	46
15	9.42 001	46	9 43 558	49	0.56 442	9.98 443	3	45
16	9.42 047	46	9.43 607	50	0.56 393	9.98 440	4	44
17	9.42 093	47	9.43 657	50	0.56 343	9.98 436	3	43
18	9.42 140	46	9.43 707	49	0.56 293	9.98 433	4	42
19	9.42 186	46	9.43 756	50	0.56 244	9.98 429	3	41
20	9.42 232	46	9.43 806	49	0.56 194	9.98 426	4	40
21	9.42 278	46	9.43 855	50	0.56 145	9.98 422	3	39
22	9.42 324	46	9.43 905	49	0.56 095	9.98 419	4	38
23	9.42 370	46	9.43 954	50	0.56 046	9.98 415	3	37
24	9.42 416	45	9.44 004	49	0.55 996	9.98 412	3	36
25	9.42 461	46	9.44 053	49	0.55 947	9.98 409	4	35
26	9.42 507	46	9.44 102	49	0.55 898	9.98 405	3	34
27	9.42 553	46	9.44 151	50	0.55 849	9.98 402	4	33
28	9.42 599	45	9.44 201	49	0.55 799	9.98 398	3	32
29	9.42 644	46	9.44 250	49	0.55 750	9.98 395	4	31
30	9.42 690	45	9.44 299	49	0.55 701	9.98 391	3	30
31	9.42 735	46	9.44 348	49	0.55 652	9.98 388	4	29
32	9.42 781	45	9.44 397	49	0.55 603	9.98 384	3	28
33	9.42 826	46	9.44 446	49	0.55 554	9.98 381	4	27
34	9.42 872	45	9.44 495	49	0.55 505	9.98 377	4	26
35	9.42 917	45	9.44 544	48	0.55 456	9.98 373	3	25
36	9.42 962	46	9.44 592	49	0.55 408	9.98 370	4	24
37	9.43 008	45	9.44 641	49	0.55 359	9.98 366	3	23
38	9.43 053	45	9.44 690	48	0.55 310	9.98 363	4	22
39	9.43 098	45	9.44 738	49	0.55 262	9.98 359	3	21
40	9.43 143	45	9.44 787	49	0.55 213	9.98 356	4	20
41	9.43 188	45	9.44 836	48	0.55 164	9.98 352	3	19
42	9.43 233	45	9.44 884	49	0.55 116	9.98 349	4	18
43	9.43 278	45	9.44 933	48	0·55 067	9.98 345	3	17
44	9.43 323	44	9.44 981	48	0.55 019	9.98 342	4	16
45	9.43 367	45	9.45 029	49	0.54 971	9.98 338	4	15
46	9.43 412	45	9.45 078	48	0.54 922	9.98 334	3	14
47	9.43 457	45	9.45 126	48	0.54 874	9.98 331	4	13
48	9.43 502	44	9.45 174	48	0.54 826	9.98 327	3	12
49	9.43 546	45	9.45 222	49	0.54 778	9.98 324	4	11
50	9.43 591	44	9.45 271	48	0.54 729	9.98 320	3	10
51	9.43 635	45	9.45 319	48	0.54 681	9.98 317	4	9
52	9.43 680	44	9.45 367	48	0.54 633	9.98 313	4	8
53	9.43 724	45	9.45 415	48	0.54 585	9.98 309	3	7
54	9.43 769	44	9.45 463	48	0.54 537	9.98 306	4	6
55	9.43 813	44	9.45 511	48	0.54 489	9.98 302	3	5
56	9.43 857	44	9.45 559	47	0.54 441	9.98 299	4	4
57	9.43 901	45	9.45 606	48	0.54 394	9.98 295	4	3
58	9.43 946	44	9.45 654	48	0.54 346	9.98 291	3	2
59	9.43 990	44	9.45 702	48	0.54 298	9.98 288	4	1
60	9.44 034		9.45 750		0.54 250	9.98 284		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	'

Prop. Pts.

"	51	50	49
6	5.1	5.0	4.9
7	6.0	5.8	5.7
8	6.8	6.7	6.5
9	7.7	7.5	7.4
10	8.5	8.3	8.2
20	17.0	16.7	16.3
30	25.5	25.0	24.5
40	34.0	33.3	32.7
50	42.5	41.7	40.8

"	48	47	46
6	4.8	4.7	4.6
7	5.6	5.5	5.4
8	6.4	6.3	6.1
9	7.2	7.0	6.9
10	8.0	7.8	7.7
20	16.0	15.7	15.3
30	24.0	23.5	23.0
40	32.0	31.3	30.7
50	40.0	39.2	38.3

"	45	44
6	4.5	4.4
7	5.3	5.1
8	6.0	5.9
9	6.8	6.6
10	7.5	7.3
20	15.0	14.7
30	22.5	22.0
40	30.0	29.3
50	37.5	36.7

"	4	3
6	0.4	0.3
7	0.5	0.4
8	0.5	0.4
9	0.6	0.5
10	0.7	0.5
20	1.3	1.0
30	2.0	1.5
40	2.7	2.0
50	3.3	2.5

Prop. Pts.

*164° 254° *344° **74°**

16° *106° 196° *286°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	′
0	9.44 034	44	9.45 750	47	0.54 250	9.98 284	3	60
1	9.44 078	44	9.45 797	48	0.54 203	9.98 281	4	59
2	9.44 122	44	9.45 845	47	0.54 155	9.98 277	4	58
3	9.44 166	44	9.45 892	48	0.54 108	9.98 273	3	57
4	9.44 210	43	9.45 940	47	0.54 060	9.98 270	4	56
5	9.44 253	44	9.45 987	48	0.54 013	9.98 266	4	55
6	9.44 297	44	9.46 035	47	0.53 965	9.98 262	3	54
7	9.44 341	44	9.46 082	48	0.53 918	9.98 259	4	53
8	9.44 385	43	9.46 130	47	0.53 870	9.98 255	4	52
9	9.44 428	44	9.46 177	47	0.53 823	9.98 251	3	51
10	9.44 472	44	9.46 224	47	0.53 776	9.98 248	4	50
11	9.44 516	43	9.46 271	48	0.53 729	9.98 244	4	49
12	9.44 559	43	9.46 319	47	0.53 681	9.98 240	3	48
13	9.44 602	44	9.46 366	47	0.53 634	9.98 237	4	47
14	9.44 646	43	9.46 413	47	0.53 587	9.98 233	4	46
15	9.44 689	44	9.46 460	47	0.53 540	9.98 229	3	45
16	9.44 733	43	9.46 507	47	0.53 493	9.98 226	4	44
17	9.44 776	43	9.46 554	47	0.53 446	9.98 222	4	43
18	9.44 819	43	9.46 601	47	0.53 399	9.98 218	3	42
19	9.44 862	43	9.46 648	46	0.53 352	9.98 215	4	41
20	9.44 905	43	9.46 694	47	0.53 306	9.98 211	4	40
21	9.44 948	44	9.46 741	47	0.53 259	9.98 207	3	39
22	9.44 992	43	9.46 788	47	0.53 212	9.98 204	4	38
23	9.45 035	42	9.46 835	46	0.53 165	9.98 200	4	37
24	9.45 077	43	9.46 881	47	0.53 119	9.98 196	4	36
25	9.45 120	43	9.46 928	47	0.53 072	9.98 192	3	35
26	9.45 163	43	9.46 975	46	0.53 025	9.98 189	4	34
27	9.45 206	43	9.47 021	47	0.52 979	9.98 185	4	33
28	9.45 249	43	9.47 068	46	0.52 932	9.98 181	4	32
29	9.45 292	42	9.47 114	46	0.52 886	9.98 177	3	31
30	9.45 334	43	9.47 160	47	0.52 840	9.98 174	4	30
31	9.45 377	42	9.47 207	46	0.52 793	9.98 170	4	29
32	9.45 419	43	9.47 253	46	0.52 747	9.98 166	4	28
33	9.45 462	42	9.47 299	47	0.52 701	9.98 162	3	27
34	9.45 504	43	9.47 346	46	0.52 654	9.98 159	4	26
35	9.45 547	42	9.47 392	46	0.52 608	9.98 155	4	25
36	9.45 589	43	9.47 438	46	0.52 562	9.98 151	4	24
37	9.45 632	42	9.47 484	46	0.52 516	9.98 147	3	23
38	9.45 674	42	9.47 530	46	0.52 470	9.98 144	4	22
39	9.45 716	42	9.47 576	46	0.52 424	9.98 140	4	21
40	9.45 758	43	9.47 622	46	0.52 378	9.98 136	4	20
41	9.45 801	42	9.47 668	46	0.52 332	9.98 132	3	19
42	9.45 843	42	9.47 714	46	0.52 286	9.98 129	4	18
43	9.45 885	42	9.47 760	46	0.52 240	9.98 125	4	17
44	9.45 927	42	9.47 806	46	0.52 194	9.98 121	4	16
45	9.45 969	42	9.47 852	45	0.52 148	9.98 117	4	15
46	9.46 011	42	9.47 897	46	0.52 103	9.98 113	3	14
47	9.46 053	42	9.47 943	46	0.52 057	9.98 110	4	13
48	9.46 095	41	9.47 989	46	0.52 011	9.98 106	4	12
49	9.46 136	42	9.48 035	45	0.51 965	9.98 102	4	11
50	9.46 178	42	9.48 080	46	0.51 920	9.98 098	4	10
51	9.46 220	42	9.48 126	45	0.51 874	9.98 094	4	9
52	9.46 262	41	9.48 171	46	0.51 829	9.98 090	3	8
53	9.46 303	42	9.48 217	45	0.51 783	9.98 087	4	7
54	9.46 345	41	9.48 262	45	0.51 738	9.98 083	4	6
55	9.46 386	42	9.48 307	46	0.51 693	9.98 079	4	5
56	9.46 428	41	9.48 353	45	0.51 647	9.98 075	4	4
57	9.46 469	42	9.48 398	45	0.51 602	9.98 071	4	3
58	9.46 511	41	9.48 443	46	0.51 557	9.98 067	4	2
59	9.46 552	42	9.48 489	45	0.51 511	9.98 063	3	1
60	9.46 594		9.48 534		0.51 466	9.98 060		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	48	47	46
6	4.8	4.7	4.6
7	5.6	5.5	5.4
8	6.4	6.3	6.1
9	7.2	7.0	6.9
10	8.0	7.8	7.7
20	16.0	15.7	15.3
30	24.0	23.5	23.0
40	32.0	31.3	30.7
50	40.0	39.2	38.3

″	45	44	43
6	4.5	4.4	4.3
7	5.3	5.1	5.0
8	6.0	5.9	5.7
9	6.8	6.6	6.4
10	7.5	7.3	7.2
20	15.0	14.7	14.3
30	22.5	22.0	21.5
40	30.0	29.3	28.7
50	37.5	36.7	35.8

″	42	41
6	4.2	4.1
7	4.9	4.8
8	5.6	5.5
9	6.3	6.2
10	7.0	6.8
20	14.0	13.7
30	21.0	20.5
40	28.0	27.3
50	35.0	34.2

″	4	3
6	0.4	0.3
7	0.5	0.4
8	0.5	0.4
9	0.6	0.5
10	0.7	0.5
20	1.3	1.0
30	2.0	1.5
40	2.7	2.0
50	3.3	2.5

Prop. Pts.

*163° 253° *343° **73°**

17° *107° 197° *287°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	′
0	9.46 594	41	9.48 534	45	0.51 466	9.98 060	4	60
1	9.46 635	41	9.48 579	45	0.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	0.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	0.51 331	9.98 048	4	57
4	9.46 758	42	9.48 714	45	0.51 286	9.98 044	4	56
5	9.46 800	41	9.48 759	45	0.51 241	9.98 040	4	55
6	9.46 841	41	9.48 804	45	0.51 196	9.98 036	4	54
7	9.46 882	41	9.48 849	45	0.51 151	9.98 032	3	53
8	9.46 923	41	9.48 894	45	0.51 106	9.98 029	4	52
9	9.46 964	41	9.48 939	45	0.51 061	9.98 025	4	51
10	9.47 005	40	9.48 984	45	0.51 016	9.98 021	4	50
11	9.47 045	41	9.49 029	44	0.50 971	9.98 017	4	49
12	9.47 086	41	9.49 073	45	0.50 927	9.98 013	4	48
13	9.47 127	41	9.49 118	45	0.50 882	9.98 009	4	47
14	9.47 168	41	9.49 163	44	0.50 837	9.98 005	4	46
15	9.47 209	40	9.49 207	45	0.50 793	9.98 001	4	45
16	9.47 249	41	9.49 252	44	0.50 748	9.97 997	4	44
17	9.47 290	40	9.49 296	45	0.50 704	9.97 993	4	43
18	9.47 330	41	9.49 341	44	0.50 659	9.97 989	3	42
19	9.47 371	40	9.49 385	45	0.50 615	9.97 986	4	41
20	9.47 411	41	9.49 430	44	0.50 570	9.97 982	4	40
21	9.47 452	40	9.49 474	45	0.50 526	9.97 978	4	39
22	9.47 492	41	9.49 519	44	0.50 481	9.97 974	4	38
23	9.47 533	40	9.49 563	44	0.50 437	9.97 970	4	37
24	9.47 573	40	9.49 607	45	0.50 393	9.97 966	4	36
25	9.47 613	41	9.49 652	44	0.50 348	9.97 962	4	35
26	9.47 654	40	9.49 696	44	0.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	0.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	0.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	0.50 128	9.97 942	4	30
31	9.47 854	40	9.49 916	44	0.50 084	9.97 938	4	29
32	9.47 894	40	9.49 960	44	0.50 040	9.97 934	4	28
33	9.47 934	40	9.50 004	44	0.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	25
36	9.48 054	40	9.50 136	44	0.49 864	9.97 918	4	24
37	9.48 094	39	9.50 180	43	0.49 820	9.97 914	4	23
38	9.48 133	40	9.50 223	44	0.49 777	9.97 910	4	22
39	9.48 173	40	9.50 267	44	0.49 733	9.97 906	4	21
40	9.48 213	39	9.50 311	44	0.49 689	9.97 902	4	20
41	9.48 252	40	9.50 355	43	0.49 645	9.97 898	4	19
42	9.48 292	40	9.50 398	44	0.49 602	9.97 894	4	18
43	9.48 332	39	9.50 442	43	0.49 558	9.97 890	4	17
44	9.48 371	40	9.50 485	44	0.49 515	9.97 886	4	16
45	9.48 411	39	9.50 529	43	0.49 471	9.97 882	4	15
46	9.48 450	40	9.50 572	44	0.49 428	9.97 878	4	14
47	9.48 490	39	9.50 616	43	0.49 384	9.97 874	4	13
48	9.48 529	39	9.50 659	44	0.49 341	9.97 870	4	12
49	9.48 568	39	9.50 703	43	0.49 297	9.97 866	5	11
50	9.48 607	40	9.50 746	43	0.49 254	9.97 861	4	10
51	9.48 647	39	9.50 789	44	0.49 211	9.97 857	4	9
52	9.48 686	39	9.50 833	43	0.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	0.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	0.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	0.49 038	9.97 841	4	5
56	9.48 842	39	9.51 005	43	0.48 995	9.97 837	4	4
57	9.48 881	39	9.51 048	44	0.48 952	9.97 833	4	3
58	9.48 920	39	9.51 092	43	0.48 908	9.97 829	4	2
59	9.48 959	39	9.51 135	43	0.48 865	9.97 825	4	1
60	9.48 998		9.51 178		0.48 822	9.97 821		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	45	44	43
6	4.5	4.4	4.3
7	5.3	5.1	5.0
8	6.0	5.9	5.7
9	6.8	6.6	6.4
10	7.5	7.3	7.2
20	15.0	14.7	14.3
30	22.5	22.0	21.5
40	30.0	29.3	28.7
50	37.5	36.7	35.8

″	42	41
6	4.2	4.1
7	4.9	4.8
8	5.6	5.5
9	6.3	6.2
10	7.0	6.8
20	14.0	13.7
30	21.0	20.5
40	28.0	27.3
50	35.0	34.2

″	40	39
6	4.0	3.9
7	4.7	4.6
8	5.3	5.2
9	6.0	5.9
10	6.7	6.5
20	13.3	13.0
30	20.0	19.5
40	26.7	26.0
50	33.3	32.5

″	5	4	3
6	0.5	0.4	0.3
7	0.6	0.5	0.4
8	0.7	0.5	0.4
9	0.8	0.6	0.5
10	0.8	0.7	0.5
20	1.7	1.3	1.0
30	2.5	2.0	1.5
40	3.3	2.7	2.0
50	4.2	3.3	2.5

Prop. Pts.

*162° 252° *342° **72°**

18° *108° 198° *288°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	′
0	9.48 998	39	9.51 178	43	0.48 822	9.97 821	4	60
1	9.49 037	39	9.51 221	43	0.48 779	9.97 817	5	59
2	9.49 076	39	9.51 264	42	0.48 736	9.97 812	4	58
3	9.49 115	38	9.51 306	43	0.48 694	9.97 808	4	57
4	9.49 153	39	9.51 349	43	0.48 651	9.97 804	4	56
5	9.49 192	39	9.51 392	43	0.48 608	9.97 800	4	55
6	9.49 231	38	9.51 435	43	0.48 565	9.97 796	4	54
7	9.49 269	39	9.51 478	42	0.48 522	9.97 792	4	53
8	9.49 308	39	9.51 520	43	0.48 480	9.97 788	4	52
9	9.49 347	38	9.51 563	43	0.48 437	9.97 784	5	51
10	9.49 385	39	9.51 606	42	0.48 394	9.97 779	4	50
11	9.49 424	38	9.51 648	43	0.48 352	9.97 775	4	49
12	9.49 462	38	9.51 691	43	0.48 309	9.97 771	4	48
13	9.49 500	39	9.51 734	42	0.48 266	9.97 767	4	47
14	9.49 539	38	9.51 776	43	0.48 224	9.97 763	4	46
15	9.49 577	38	9.51 819	42	0.48 181	9.97 759	5	45
16	9.49 615	39	9.51 861	42	0.48 139	9.97 754	4	44
17	9.49 654	38	9.51 903	43	0.48 097	9.97 750	4	43
18	9.49 692	38	9.51 946	42	0.48 054	9.97 746	4	42
19	9.49 730	38	9.51 988	43	0.48 012	9.97 742	4	41
20	9.49 768	38	9.52 031	42	0.47 969	9.97 738	4	40
21	9.49 806	28	9.52 073	42	0.47 927	9.97 734	5	39
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	4	38
23	9.49 882	38	9.52 157	43	0.47 843	9.97 725	4	37
24	9.49 920	38	9.52 200	42	0.47 800	9.97 721	4	36
25	9.49 958	38	9.52 242	42	0.47 758	9.97 717	4	35
26	9.49 996	38	9.52 284	42	0.47 716	9.97 713	5	34
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	4	33
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32
29	9.50 110	38	9.52 410	42	0.47 590	9.97 700	4	31
30	9.50 148	37	9.52 452	42	0.47 548	9.97 696	5	30
31	9.50 185	38	9.52 494	42	0.47 506	9.97 691	4	29
32	9.50 223	38	9.52 536	42	0.47 464	9.97 687	4	28
33	9.50 261	37	9.52 578	42	0.47 422	9.97 683	4	27
34	9.50 298	38	9.52 620	41	0.47 380	9.97 679	5	26
35	9.50 336	38	9.52 661	42	0.47 339	9.97 674	4	25
36	9.50 374	37	9.52 703	42	0.47 297	9.97 670	4	24
37	9.50 411	38	9.52 745	42	0 47 255	9.97 666	4	23
38	9.50 449	37	9.52 787	42	0.47 213	9.97 662	5	22
39	9.50 486	37	9.52 829	41	0.47 171	9.97 657	4	21
40	9.50 523	38	9.52 870	42	0.47 130	9.97 653	4	20
41	9.50 561	37	9.52 912	41	0.47 088	9.97 649	4	19
42	9.50 598	37	9.52 953	42	0.47 047	9.97 645	5	18
43	9.50 635	38	9.52 995	42	0.47 005	9.97 640	4	17
44	9.50 673	37	9.53 037	41	0.46 963	9.97 636	4	16
45	9.50 710	37	9.53 078	42	0.46 922	9.97 632	4	15
46	9.50 747	37	9.53 120	41	0.46 880	9.97 628	5	14
47	9.50 784	37	9 53 161	41	0.46 839	9.97 623	4	13
48	9.50 821	37	9.53 202	42	0.46 798	9.97 619	4	12
49	9.50 858	38	9.53 244	41	0.46 756	9.97 615	5	11
50	9.50 896	37	9.53 285	42	0.46 715	9.97 610	4	10
51	9.50 933	37	9.53 327	41	0.46 673	9.97 606	4	9
52	9.50 970	37	9.53 368	41	0.46 632	9.97 602	5	8
53	9.51 007	36	9.53 409	41	0.46 591	9.97 597	4	7
54	9.51 043	37	9.53 450	42	0.46 550	9.97 593	4	6
55	9.51 080	37	9.53 492	41	0.46 508	9.97 589	5	5
56	9.51 117	37	9.53 533	41	0.46 467	9.97 584	4	4
57	9.51 154	37	9.53 574	41	0.46 426	9.97 580	4	3
58	9.51 191	36	9.53 615	41	0.46 385	9.97 576	5	2
59	9.51 227	37	9.53 656	41	0.46 344	9.97 571	4	1
60	9.51 264		9.53 697		0.46 303	9.97 567		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	43	42	41
6	4.3	4.2	4.1
7	5.0	4.9	4.8
8	5.7	5.6	5.5
9	6.4	6.3	6.2
10	7.2	7.0	6.8
20	14.3	14.0	13.7
30	21.5	21.0	20.5
40	28.7	28.0	27.3
50	35.8	35.0	34.2

″	39	38	37
6	3.9	3.8	3.7
7	4.6	4.4	4.3
8	5.2	5.1	4.9
9	5.9	5.7	5.6
10	6.5	6.3	6.2
20	13.0	12.7	12.3
30	19.5	19.0	18.5
40	26.0	25.3	24.7
50	32.5	31.7	30.8

″	36	5	4
6	3.6	0.5	0.4
7	4.2	0.6	0.5
8	4.8	0.7	0.5
9	5.4	0.8	0.6
10	6.0	0.8	0.7
20	12.0	1.7	1.3
30	18.0	2.5	2.0
40	24.0	3.3	2.7
50	30.0	4.2	3.3

Prop. Pts.

*161° 251° *341° **71°**

19° *109° 199° *289°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4	60
1	9.51 301	37	9.53 738	41	0.46 262	9.97 563	5	59
2	9.51 338	36	9.53 779	41	0.46 221	9.97 558	4	58
3	9.51 374	37	9.53 820	41	0.46 180	9.97 554	4	57
4	9.51 411	36	9.53 861	41	0.46 139	9.97 550	5	56
5	9.51 447	37	9.53 902	41	0.46 098	9.97 545	4	55
6	9.51 484	36	9.53 943	41	0.46 057	9.97 541	5	54
7	9.51 520	37	9.53 984	41	0.46 016	9.97 536	4	53
8	9.51 557	36	9.54 025	40	0.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	41	0.45 935	9.97 528	5	51
10	9.51 629	37	9.54 106	41	0.45 894	9.97 523	4	50
11	9.51 666	36	9.54 147	40	0.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	41	0.45 813	9.97 515	5	48
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	4	47
14	9.51 774	37	9.54 269	40	0.45 731	9.97 506	5	46
15	9.51 811	36	9.54 309	41	0.45 691	9.97 501	4	45
16	9.51 847	36	9.54 350	40	0.45 650	9.97 497	5	44
17	9.51 883	36	9.54 390	41	0.45 610	9.97 492	4	43
18	9.51 919	36	9.54 431	40	0.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	41	0.45 529	9.97 484	5	41
20	9.51 991	36	9.54 512	40	0.45 488	9.97 479	4	40
21	9.52 027	36	9.54 552	41	0.45 448	9.97 475	5	39
22	9.52 063	36	9.54 593	40	0.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	5	37
24	9.52 135	36	9.54 673	41	0.45 327	9.97 461	4	36
25	9.52 171	36	9.54 714	40	0.45 286	9.97 457	4	35
26	9.52 207	35	9.54 754	40	0.45 246	9.97 453	5	34
27	9.52 242	36	9.54 794	41	0.45 206	9.97 448	4	33
28	9.52 278	36	9.54 835	40	0.45 165	9.97 444	5	32
29	9.52 314	36	9.54 875	40	0.45 125	9.97 439	4	31
30	9.52 350	35	9.54 915	40	0.45 085	9.97 435	5	30
31	9.52 385	36	9.54 955	40	0.45 045	9.97 430	4	29
32	9.52 421	35	9.54 995	40	0.45 005	9.97 426	5	28
33	9.52 456	36	9.55 035	40	0.44 965	9.97 421	4	27
34	9.52 492	35	9.55 075	40	0.44 925	9.97 417	5	26
35	9.52 527	36	9.55 115	40	0.44 885	9.97 412	4	25
36	9.52 563	35	9.55 155	40	0.44 845	9.97 408	5	24
37	9.52 598	36	9.55 195	40	0.44 805	9.97 403	4	23
38	9.52 634	35	9.55 235	40	0 44 765	9.97 399	5	22
39	9.52 669	36	9.55 275	40	0.44 725	9.97 394	4	21
40	9.52 705	35	9.55 315	40	0.44 685	9.97 390	5	20
41	9.52 740	35	9.55 355	40	0.44 645	9.97 385	4	19
42	9.52 775	36	9.55 395	39	0.44 605	9.97 381	5	18
43	9.52 811	35	9.55 434	40	0.44 566	9.97 376	4	17
44	9.52 846	35	9.55 474	40	0.44 526	9.97 372	5	16
45	9.52 881	35	9.55 514	40	0.44 486	9.97 367	4	15
46	9.52 916	35	9.55 554	39	0.44 446	9.97 363	5	14
47	9.52 951	35	9.55 593	40	0.44 407	9.97 358	5	13
48	9.52 986	35	9.55 633	40	0.44 367	9.97 353	4	12
49	9.53 021	35	9.55 673	39	0.44 327	9.97 349	5	11
50	9.53 056	36	9.55 712	40	0.44 288	9.97 344	4	10
51	9.53 092	34	9.55 752	39	0.44 248	9.97 340	5	9
52	9.53 126	35	9.55 791	40	0.44 209	9.97 335	4	8
53	9.53 161	35	9.55 831	39	0.44 169	9.97 331	5	7
54	9.53 196	35	9.55 870	40	0.44 130	9.97 326	4	6
55	9.53 231	35	9.55 910	39	0.44 090	9.97 322	5	5
56	9.53 266	35	9.55 949	40	0.44 051	9.97 317	5	4
57	9.53 301	35	9.55 989	39	0.44 011	9.97 312	4	3
58	9.53 336	34	9.56 028	39	0.43 972	9.97 308	5	2
59	9.53 370	35	9.56 067	40	0.43 933	9.97 303	4	1
60	9.53 405		9.56 107		0.43 893	9.97 299		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	41	40	39
6	4.1	4.0	3.9
7	4.8	4.7	4.6
8	5.5	5.3	5.2
9	6.2	6.0	5.9
10	6.8	6.7	6.5
20	13.7	13.3	13.0
30	20.5	20.0	19.5
40	27.3	26.7	26.0
50	34.2	33.3	32.5

″	37	36	35
6	3.7	3.6	3.5
7	4.3	4.2	4.1
8	4.9	4.8	4.7
9	5.6	5.4	5.3
10	6.2	6.0	5.8
20	12.3	12.0	11.7
30	18.5	18.0	17.5
40	24.7	24.0	23.3
50	30.8	30.0	29.2

″	34	5	4
6	3.4	0.5	0.4
7	4.0	0.6	0.5
8	4.5	0.7	0.5
9	5.1	0.8	0.6
10	5.7	0.8	0.7
20	11.3	1.7	1.3
30	17.0	2.5	2.0
40	22.7	3.3	2.7
50	28.3	4.2	3.3

Prop. Pts.

*160° 250° *340° **70°**

20° *110° 200° *290°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.53 405	35	9.56 107	39	0.43 893	9.97 299	5	60
1	9.53 440	35	9.56 146	39	0.43 854	9.97 294	5	59
2	9.53 475	34	9.56 185	39	0.43 815	9.97 289	4	58
3	9.53 509	35	9.56 224	40	0.43 776	9.97 285	5	57
4	9.53 544	34	9.56 264	39	0.43 736	9.97 280	4	56
5	9.53 578	35	9.56 303	39	0.43 697	9.97 276	5	55
6	9.53 613	34	9.56 342	39	0.43 658	9.97 271	5	54
7	9.53 647	35	9.56 381	39	0.43 619	9.97 266	4	53
8	9.53 682	34	9.56 420	39	0.43 580	9.97 262	5	52
9	9.53 716	35	9.56 459	39	0.43 541	9.97 257	5	51
10	9.53 751	34	9.56 498	39	0.43 502	9.97 252	4	50
11	9.53 785	34	9.56 537	39	0.43 463	9.97 248	5	49
12	9.53 819	35	9.56 576	39	0.43 424	9.97 243	5	48
13	9.53 854	34	9.56 615	39	0.43 385	9.97 238	4	47
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	5	46
15	9.53 922	35	9.56 693	39	0.43 307	9.97 229	5	45
16	9.53 957	34	9.56 732	39	0.43 268	9.97 224	4	44
17	9.53 991	34	9.56 771	39	0.43 229	9.97 220	5	43
18	9.54 025	34	9.56 810	39	0.43 190	9.97 215	5	42
19	9.54 059	34	9.56 849	38	0.43 151	9.97 210	4	41
20	9.54 093	34	9.56 887	39	0.43 113	9.97 206	5	40
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	5	39
22	9.54 161	34	9.56 965	39	0.43 035	9.97 196	4	38
23	9.54 195	34	9.57 004	38	0.42 996	9.97 192	5	37
24	9.54 229	34	9.57 042	39	0.42 958	9.97 187	5	36
25	9.54 263	34	9.57 081	39	0.42 919	9.97 182	4	35
26	9.54 297	34	9.57 120	38	0.42 880	9.97 178	5	34
27	9.54 331	34	9.57 158	39	0.42 842	9.97 173	5	33
28	9.54 365	34	9.57 197	38	0.42 803	9.97 168	5	32
29	9.54 399	34	9.57 235	39	0.42 765	9.97 163	4	31
30	9.54 433	33	9.57 274	38	0.42 726	9.97 159	5	30
31	9.54 466	34	9.57 312	39	0.42 688	9.97 154	5	29
32	9.54 500	34	9.57 351	38	0.42 649	9.97 149	4	28
33	9.54 534	33	9.57 389	39	0.42 611	9.97 145	5	27
34	9.54 567	34	9.57 428	38	0.42 572	9.97 140	5	26
35	9.54 601	34	9.57 466	38	0.42 534	9.97 135	5	25
36	9.54 635	33	9.57 504	39	0.42 496	9.97 130	4	24
37	9.54 668	34	9.57 543	38	0.42 457	9.97 126	5	23
38	9.54 702	33	9.57 581	38	0.42 419	9.97 121	5	22
39	9.54 735	34	9.57 619	39	0.42 381	9.97 116	5	21
40	9.54 769	33	9.57 658	38	0.42 342	9.97 111	4	20
41	9.54 802	34	9.57 696	38	0.42 304	9.97 107	5	19
42	9.54 836	33	9.57 734	38	0.42 266	9.97 102	5	18
43	9.54 869	34	9.57 772	38	0.42 228	9.97 097	5	17
44	9.54 903	33	9.57 810	39	0.42 190	9.97 092	5	16
45	9.54 936	33	9.57 849	38	0.42 151	9.97 087	4	15
46	9.54 969	34	9.57 887	38	0.42 113	9.97 083	5	14
47	9.55 003	33	9.57 925	38	0.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	5	12
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11
50	9.55 102	34	9.58 039	38	0.41 961	9.97 063	4	10
51	9.55 136	33	9.58 077	38	0.41 923	9.97 059	5	9
52	9.55 169	33	9.58 115	38	0.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	38	0.41 847	9.97 049	5	7
54	9.55 235	33	9.58 191	38	0.41 809	9.97 044	5	6
55	9.55 268	33	9.58 229	38	0.41 771	9.97 039	4	5
56	9.55 301	33	9.58 267	37	0.41 733	9.97 035	5	4
57	9.55 334	33	9.58 304	38	0.41 696	9.97 030	5	3
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	38	0.41 620	9.97 020	5	1
60	9.55 433		9.58 418		0.41 582	9.97 015		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	40	39
6	4.0	3.9
7	4.7	4.6
8	5.3	5.2
9	6.0	5.9
10	6.7	6.5
20	13.3	13.0
30	20.0	19.5
40	26.7	26.0
50	33.3	32.5

″	38	37
6	3.8	3.7
7	4.4	4.3
8	5.1	4.9
9	5.7	5.6
10	6.3	6.2
20	12.7	12.3
30	19.0	18.5
40	25.3	24.7
50	31.7	30.8

″	35	34
6	3.5	3.4
7	4.1	4.0
8	4.7	4.5
9	5.3	5.1
10	5.8	5.7
20	11.7	11.3
30	17.5	17.0
40	23.3	22.7
50	29.2	28.3

″	33	5	4
6	3.3	0.5	0.4
7	3.8	0.6	0.5
8	4.4	0.7	0.5
9	5.0	0.8	0.6
10	5.5	0.8	0.7
20	11.0	1.7	1.3
30	16.5	2.5	2.0
40	22.0	3.3	2.7
50	27.5	4.2	3.3

Prop. Pts.

*159° 249° *339° **69°**

21° *111° 201° *291°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	′
0	9.55 433		9.58 418		0.41 582	9.97 015		60
1	9.55 466	33	9.58 455	37	0.41 545	9.97 010	5	59
2	9.55 499	33	9.58 493	38	0.41 507	9.97 005	5	58
3	9.55 532	33	9.58 531	38	0.41 469	9.97 001	4	57
4	9.55 564	32	9.58 569	38	0.41 431	9.96 996	5	56
5	9.55 597	33	9.58 606	37	0.41 394	9.96 991	5	55
6	9.55 630	33	9.58 644	38	0.41 356	9.96 986	5	54
7	9.55 663	33	9.58 681	37	0.41 319	9.96 981	5	53
8	9.55 695	32	9.58 719	38	0.41 281	9.96 976	5	52
9	9.55 728	33	9.58 757	38	0.41 243	9.96 971	5	51
10	9.55 761	33	9.58 794	37	0.41 206	9.96 966	5	50
11	9.55 793	32	9.58 832	38	0.41 168	9.96 962	4	49
12	9.55 826	33	9.58 869	37	0.41 131	9.96 957	5	48
13	9.55 858	32	9.58 907	38	0.41 093	9.96 952	5	47
14	9.55 891	33	9.58 944	37	0.41 056	9.96 947	5	46
15	9.55 923	32	9.58 981	37	0.41 019	9.96 942	5	45
16	9.55 956	33	9.59 019	38	0.40 981	9.96 937	5	44
17	9.55 988	32	9.59 056	37	0.40 944	9.96 932	5	43
18	9.56 021	33	9.59 094	38	0.40 906	9.96 927	5	42
19	9.56 053	32	9.59 131	37	0.40 869	9.96 922	5	41
20	9.56 085	32	9.59 168	37	0.40 832	9.96 917	5	40
21	9.56 118	33	9.59 205	37	0.40 795	9.96 912	5	39
22	9.56 150	32	9.59 243	38	0.40 757	9.96 907	5	38
23	9.56 182	32	9.59 280	37	0.40 720	9.96 903	4	37
24	9.56 215	33	9.59 317	37	0.40 683	9.96 898	5	36
25	9.56 247	32	9.59 354	37	0.40 646	9.96 893	5	35
26	9.56 279	32	9.59 391	37	0.40 609	9.96 888	5	34
27	9.56 311	32	9.59 429	38	0.40 571	9.96 883	5	33
28	9.56 343	32	9.59 466	37	0.40 534	9.96 878	5	32
29	9.56 375	32	9.59 503	37	0.40 497	9.96 873	5	31
30	9.56 408	33	9.59 540	37	0.40 460	9.96 868	5	30
31	9.56 440	32	9.59 577	37	0.40 423	9.96 863	5	29
32	9.56 472	32	9.59 614	37	0.40 386	9.96 858	5	28
33	9.56 504	32	9.59 651	37	0.40 349	9.96 853	5	27
34	9.56 536	32	9.59 688	37	0.40 312	9.96 848	5	26
35	9.56 568	32	9.59 725	37	0.40 275	9.96 843	5	25
36	9.56 599	31	9.59 762	37	0.40 238	9.96 838	5	24
37	9.56 631	32	9.59 799	37	0.40 201	9.96 833	5	23
38	9.56 663	32	9.59 835	36	0.40 165	9.96 828	5	22
39	9.56 695	32	9.59 872	37	0.40 128	9.96 823	5	21
40	9.56 727	32	9.59 909	37	0.40 091	9.96 818	5	20
41	9.56 759	32	9.59 946	37	0.40 054	9.96 813	5	19
42	9.56 790	31	9.59 983	37	0.40 017	9.96 808	5	18
43	9.56 822	32	9.60 019	36	0.39 981	9.96 803	5	17
44	9.56 854	32	9.60 056	37	0.39 944	9.96 798	5	16
45	9.56 886	32	9.60 093	37	0.39 907	9.96 793	5	15
46	9.56 917	31	9.60 130	37	0.39 870	9.96 788	5	14
47	9.56 949	32	9.60 166	36	0.39 834	9.96 783	5	13
48	9.56 980	31	9.60 203	37	0.39 797	9.96 778	5	12
49	9.57 012	32	9.60 240	37	0.39 760	9.96 772	6	11
50	9.57 044	32	9.60 276	36	0.39 724	9.96 767	5	10
51	9.57 075	31	9.60 313	37	0.39 687	9.96 762	5	9
52	9.57 107	32	9.60 349	36	0.39 651	9.96 757	5	8
53	9.57 138	31	9.60 386	37	0.39 614	9.96 752	5	7
54	9.57 169	31	9.60 422	36	0.39 578	9.96 747	5	6
55	9.57 201	32	9.60 459	37	0.39 541	9.96 742	5	5
56	9.57 232	31	9.60 495	36	0.39 505	9.96 737	5	4
57	9.57 264	32	9.60 532	37	0.39 468	9.96 732	5	3
58	9.57 295	31	9.60 568	36	0.39 432	9.96 727	5	2
59	9.57 326	31	9.60 605	37	0.39 395	9.96 722	5	1
60	9.57 358	32	9.60 641	36	0.39 359	9.96 717	5	0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	38	37	36
6	3.8	3.7	3.6
7	4.4	4.3	4.2
8	5.1	4.9	4.8
9	5.7	5.6	5.4
10	6.3	6.2	6.0
20	12.7	12.3	12.0
30	19.0	18.5	18.0
40	25.3	24.7	24.0
50	31.7	30.8	30.0

″	33	32	31
6	3.3	3.2	3.1
7	3.9	3.7	3.6
8	4.4	4.3	4.1
9	5.0	4.8	4.6
10	5.5	5.3	5.2
20	11.0	10.7	10.3
30	16.5	16.0	15.5
40	22.0	21.3	20.7
50	27.5	26.7	25.8

″	6	5	4
6	0.6	0.5	0.4
7	0.7	0.6	0.5
8	0.8	0.7	0.5
9	0.9	0.8	0.6
10	1.0	0.8	0.7
20	2.0	1.7	1.3
30	3.0	2.5	2.0
40	4.0	3.3	2.7
50	5.0	4.2	3.3

*158° 248° *338° **68°**

22° *112° 202° *292°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.57 358	31	9.60 641	36	0.39 359	9.96 717	6	60
1	9.57 389	31	9.60 677	37	0.39 323	9.96 711	5	59
2	9.57 420	31	9.60 714	36	0.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57
4	9.57 482	32	9.60 786	37	0.39 214	9.96 696	5	56
5	9.57 514	31	9.60 823	36	0.39 177	9.96 691	5	55
6	9.57 545	31	9.60 859	36	0.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	0.39 069	9.96 676	6	52
9	9.57 638	31	9.60 967	37	0.39 033	9.96 670	5	51
10	9.57 669	31	9.61 004	36	0.38 996	9.96 665	5	50
11	9.57 700	31	9.61 040	36	0.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47
14	9.57 793	31	9.61 148	36	0.38 852	9.96 645	5	46
15	9.57 824	31	9.61 184	36	0.38 816	9.96 640	6	45
16	9.57 855	30	9.61 220	36	0.38 780	9.96 634	5	44
17	9.57 885	31	9.61 256	36	0.38 744	9.96 629	5	43
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42
19	9.57 947	31	9.61 328	36	0.38 672	9.96 619	5	41
20	9.57 978	30	9.61 364	36	0.38 636	9.96 614	6	40
21	9.58 008	31	9.61 400	36	0.38 600	9.96 608	5	39
22	9.58 039	31	9.61 436	36	0.38 564	9.96 603	5	38
23	9.58 070	31	9.61 472	36	0.38 528	9.96 598	5	37
24	9.58 101	30	9.61 508	36	0.38 492	9.96 593	5	36
25	9.58 131	31	9.61 544	35	0.38 456	9.96 588	6	35
26	9.58 162	30	9.61 579	36	0.38 421	9.96 582	5	34
27	9.58 192	31	9.61 615	36	0.38 385	9.96 577	5	33
28	9.58 223	30	9.61 651	36	0.38 349	9.96 572	5	32
29	9.58 253	31	9.61 687	35	0.38 313	9.96 567	5	31
30	9.58 284	30	9.61 722	36	0.38 278	9.96 562	6	30
31	9.58 314	31	9.61 758	36	0.38 242	9.96 556	5	29
32	9.58 345	30	9.61 794	36	0.38 206	9.96 551	5	28
33	9.58 375	31	9.61 830	35	0.38 170	9.96 546	5	27
34	9.58 406	30	9.61 865	36	0.38 135	9.96 541	6	26
35	9.58 436	31	9.61 901	35	0.38 099	9.96 535	5	25
36	9.58 467	30	9.61 936	36	0.38 064	9.96 530	5	24
37	9.58 497	30	9.61 972	36	0.38 028	9.96 525	5	23
38	9.58 527	30	9.62 008	35	0.37 992	9.96 520	6	22
39	9.58 557	31	9.62 043	36	0.37 957	9.96 514	5	21
40	9.58 588	30	9.62 079	35	0.37 921	9.96 509	5	20
41	9.58 618	30	9.62 114	36	0.37 886	9.96 504	6	19
42	9.58 648	30	9.62 150	35	0.37 850	9.96 498	5	18
43	9.58 678	31	9.62 185	36	0.37 815	9.96 493	5	17
44	9.58 709	30	9.62 221	35	0.37 779	9.96 488	5	16
45	9.58 739	30	9.62 256	36	0.37 744	9.96 483	6	15
46	9.58 769	30	9.62 292	35	0.37 708	9.96 477	5	14
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	36	0.37 638	9.96 467	6	12
49	9.58 859	30	9.62 398	35	0.37 602	9.96 461	5	11
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	10
51	9.58 919	30	9.62 468	36	0.37 532	9.96 451	6	9
52	9.58 949	30	9.62 504	35	0.37 496	9.96 445	5	8
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7
54	9.59 009	30	9.62 574	35	0.37 426	9.96 435	6	6
55	9.59 039	30	9.62 609	36	0.37 391	9.96 429	5	5
56	9.59 069	29	9.62 645	35	0.37 355	9.96 424	5	4
57	9.59 098	30	9.62 680	35	0.37 320	9.96 419	6	3
58	9.59 128	30	9.62 715	35	0.37 285	9.96 413	5	2
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1
60	9.59 188		9.62 785		0.37 215	9.96 403		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	37	36	35
6	3.7	3.6	3.5
7	4.3	4.2	4.1
8	4.9	4.8	4.7
9	5.6	5.4	5.3
10	6.2	6.0	5.8
20	12.3	12.0	11.7
30	18.5	18.0	17.5
40	24.7	24.0	23.3
50	30.8	30.0	29.2

″	32	31	30
6	3.2	3.1	3.0
7	3.7	3.6	3.5
8	4.3	4.1	4.0
9	4.8	4.6	4.5
10	5.3	5.2	5.0
20	10.7	10.3	10.0
30	16.0	15.5	15.0
40	21.3	20.7	20.0
50	26.7	25.8	25.0

″	29	6	5
6	2.9	0.6	0.5
7	3.4	0.7	0.6
8	3.9	0.8	0.7
9	4.4	0.9	0.8
10	4.8	1.0	0.8
20	9.7	2.0	1.7
30	14.5	3.0	2.5
40	19.3	4.0	3.3
50	24.2	5.0	4.2

Prop. Pts.

*157° 247° *337° **67°**

23° *113° 203° *293°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.59 188	30	9.62 785	35	0.37 215	9.96 403	6	60
1	9.59 218	29	9.62 820	35	0.37 180	9.96 397	5	59
2	9.59 247	30	9.62 855	35	0.37 145	9.96 392	5	58
3	9.59 277	30	9.62 890	36	0.37 110	9.96 387	6	57
4	9.59 307	29	9.62 926	35	0.37 074	9.96 381	5	56
5	9.59 336	30	9.62 961	35	0.37 039	9.96 376	6	55
6	9.59 366	30	9.62 996	35	0.37 004	9.96 370	5	54
7	9.59 396	29	9.63 031	35	0.36 969	9.96 365	5	53
8	9.59 425	30	9.63 066	35	0.36 934	9.96 360	6	52
9	9.59 455	29	9.63 101	34	0.36 899	9.96 354	5	51
10	9.59 484	30	9.63 135	35	0.36 865	9.96 349	6	50
11	9.59 514	29	9.63 170	35	0.36 830	9.96 343	5	49
12	9.59 543	30	9.63 205	35	0.36 795	9.96 338	5	48
13	9.59 573	29	9.63 240	35	0.36 760	9.96 333	6	47
14	9.59 602	30	9.63 275	35	0.36 725	9.96 327	5	46
15	9.59 632	29	9.63 310	35	0.36 690	9.96 322	6	45
16	9.59 661	29	9.63 345	34	0.36 655	9.96 316	5	44
17	9.59 690	30	9.63 379	35	0.36 621	9.96 311	6	43
18	9.59 720	29	9.63 414	35	0.36 586	9.96 305	5	42
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	6	41
20	9.59 778	30	9.63 484	35	0.36 516	9.96 294	5	40
21	9.59 808	29	9.63 519	34	0.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	35	0.36 447	9.96 284	6	38
23	9.59 866	29	9.63 588	35	0.36 412	9.96 278	5	37
24	9.59 895	29	9.63 623	34	0.36 377	9.96 273	6	36
25	9.59 924	30	9.63 657	35	0.36 343	9.96 267	5	35
26	9.59 954	29	9.63 692	34	0.36 308	9.96 262	6	34
27	9.59 983	29	9.63 726	35	0.36 274	9.96 256	5	33
28	9.60 012	29	9.63 761	35	0.36 239	9.96 251	6	32
29	9.60 041	29	9.63 796	34	0.36 204	9.96 245	5	31
30	9.60 070	29	9.63 830	35	0.36 170	9.96 240	6	30
31	9.60 099	29	9.63 865	34	0.36 135	9.96 234	5	29
32	9.60 128	29	9.63 899	35	0.36 101	9.96 229	6	28
33	9.60 157	29	9.63 934	34	0.36 066	9.96 223	5	27
34	9.60 186	29	9.63 968	35	0.36 032	9.96 218	6	26
35	9.60 215	29	9.64 003	34	0.35 997	9.96 212	5	25
36	9.60 244	29	9.64 037	35	0.35 963	9.96 207	6	24
37	9.60 273	29	9.64 072	34	0.35 928	9.96 201	5	23
38	9.60 302	29	9.64 106	34	0.35 894	9.96 196	6	22
39	9.60 331	28	9.64 140	35	0.35 860	9.96 190	5	21
40	9.60 359	29	9.64 175	34	0.35 825	9.96 185	6	20
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	5	19
42	9.60 417	29	9.64 243	35	0.35 757	9.96 174	6	18
43	9.60 446	28	9.64 278	34	0.35 722	9.96 168	6	17
44	9.60 474	29	9.64 312	34	0.35 688	9.96 162	5	16
45	9.60 503	29	9.64 346	35	0.35 654	9.96 157	6	15
46	9.60 532	29	9.64 381	34	0.35 619	9.96 151	5	14
47	9.60 561	28	9.64 415	34	0.35 585	9.96 146	6	13
48	9.60 589	29	9.64 449	34	0.35 551	9.96 140	5	12
49	9.60 618	28	9.64 483	34	0.35 517	9.96 135	6	11
50	9.60 646	29	9.64 517	35	0.35 483	9.96 129	6	10
51	9.60 675	29	9.64 552	34	0.35 448	9.96 123	5	9
52	9.60 704	28	9.64 586	34	0.35 414	9.96 118	6	8
53	9.60 732	29	9.64 620	34	0.35 380	9.96 112	5	7
54	9.60 761	28	9.64 654	34	0.35 346	9.96 107	6	6
55	9.60 789	29	9.64 688	34	0.35 312	9.96 101	6	5
56	9.60 818	28	9.64 722	34	0.35 278	9.96 095	5	4
57	9.60 846	29	9.64 756	34	0.35 244	9.96 090	6	3
58	9.60 875	28	9.64 790	34	0.35 210	9.96 084	5	2
59	9.60 903	28	9.64 824	34	0.35 176	9.96 079	6	1
60	9.60 931		9.64 858		0.35 142	9.96 073		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	36	35	34
6	3.6	3.5	3.4
7	4.2	4.1	4.0
8	4.8	4.7	4.5
9	5.4	5.2	5.1
10	6.0	5.8	5.7
20	12.0	11.7	11.3
30	18.0	17.5	17.0
40	24.0	23.3	22.7
50	30.0	29.2	28.3

″	30	29	28
6	3.0	2.9	2.8
7	3.5	3.4	3.3
8	4.0	3.9	3.7
9	4.5	4.4	4.2
10	5.0	4.8	4.7
20	10.0	9.7	9.3
30	15.0	14.5	14.0
40	20.0	19.3	18.7
50	25.0	24.2	23.3

″	6	5
6	0.6	0.5
7	0.7	0.6
8	0.8	0.7
9	0.9	0.8
10	1.0	0.8
20	2.0	1.7
30	3.0	2.5
40	4.0	3.3
50	5.0	4.2

Prop. Pts.

*156° 246° *336° **66°**

24° *114° 204° *294°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.60 931	29	9.64 858	34	0.35 142	9.96 073	6	60
1	9.60 960	28	9.64 892	34	0.35 108	9.96 067	5	59
2	9.60 988	28	9.64 926	34	0.35 074	9.96 062	6	58
3	9.61 016	29	9.64 960	34	0.35 040	9.96 056	6	57
4	9.61 045	28	9.64 994	34	0.35 006	9.96 050	5	56
5	9.61 073	28	9.65 028	34	0.34 972	9.96 045	6	55
6	9.61 101	28	9.65 062	34	0.34 938	9.96 039	5	54
7	9.61 129	29	9.65 096	34	0.34 904	9.96 034	6	53
8	9.61 158	28	9.65 130	34	0.34 870	9.96 028	6	52
9	9.61 186	28	9.65 164	33	0.34 836	9.96 022	5	51
10	9.61 214	28	9.65 197	34	0.34 803	9.96 017	6	50
11	9.61 242	28	9.65 231	34	0.34 769	9.96 011	6	49
12	9.61 270	28	9.65 265	34	0.34 735	9.96 005	5	48
13	9.61 298	28	9.65 299	34	0.34 701	9.96 000	6	47
14	9.61 326	28	9.65 333	33	0.34 667	9.95 994	6	46
15	9.61 354	28	9.65 366	34	0.34 634	9.95 988	6	45
16	9.61 382	29	9.65 400	34	0.34 600	9.95 982	5	44
17	9.61 411	27	9.65 434	33	0.34 566	9.95 977	6	43
18	9.61 438	28	9.65 467	34	0.34 533	9.95 971	6	42
19	9.61 466	28	9.65 501	34	0.34 499	9.95 965	5	41
20	9.61 494	28	9.65 535	33	0.34 465	9.95 960	6	40
21	9.61 522	28	9.65 568	34	0.34 432	9.95 954	6	39
22	9.61 550	28	9.65 602	34	0.34 398	9.95 948	6	38
23	9.61 578	28	9.65 636	33	0.34 364	9.95 942	5	37
24	9.61 606	28	9.65 669	34	0.34 331	9.95 937	6	36
25	9.61 634	28	9.65 703	33	0.34 297	9.95 931	6	35
26	9.61 662	27	9.65 736	34	0.34 264	9.95 925	5	34
27	9.61 689	28	9.65 770	33	0.34 230	9.95 920	6	33
28	9.61 717	28	9.65 803	34	0.34 197	9.95 914	6	32
29	9.61 745	28	9.65 837	33	0.34 163	9.95 908	6	31
30	9.61 773	27	9.65 870	34	0.34 130	9.95 902	5	30
31	9.61 800	28	9.65 904	33	0.34 096	9.95 897	6	29
32	9.61 828	28	9.65 937	34	0.34 063	9.95 891	6	28
33	9.61 856	27	9.65 971	33	0.34 029	9.95 885	6	27
34	9.61 883	28	9.66 004	34	0.33 996	9.95 879	6	26
35	9.61 911	28	9.66 038	33	0.33 962	9.95 873	5	25
36	9.61 939	27	9.66 071	33	0.33 929	9.95 868	6	24
37	9.61 966	28	9.66 104	34	0.33 896	9.95 862	6	23
38	9.61 994	27	9.66 138	33	0.33 862	9.95 856	6	22
39	9.62 021	28	9.66 171	33	0.33 829	9.95 850	6	21
40	9.62 049	27	9.66 204	34	0.33 796	9.95 844	5	20
41	9.62 076	28	9.66 238	33	0.33 762	9.95 839	6	19
42	9.62 104	27	9.66 271	33	0.33 729	9.95 833	6	18
43	9.62 131	28	9.66 304	33	0.33 696	9.95 827	6	17
44	9.62 159	27	9.66 337	34	0.33 663	9.95 821	6	16
45	9.62 186	28	9.66 371	33	0.33 629	9.95 815	5	15
46	9.62 214	27	9.66 404	33	0.33 596	9.95 810	6	14
47	9.62 241	27	9.66 437	33	0.33 563	9.95 804	6	13
48	9.62 268	28	9.66 470	33	0.33 530	9.95 798	6	12
49	9.62 296	27	9.66 503	34	0.33 497	9.95 792	6	11
50	9.62 323	27	9.66 537	33	0.33 463	9.95 786	6	10
51	9.62 350	27	9.66 570	33	0.33 430	9.95 780	5	9
52	9.62 377	28	9.66 603	33	0.33 397	9.95 775	6	8
53	9.62 405	27	9.66 636	33	0.33 364	9.95 769	6	7
54	9.62 432	27	9.66 669	33	0.33 331	9.95 763	6	6
55	9.62 459	27	9.66 702	33	0.33 298	9.95 757	6	5
56	9.62 486	27	9.66 735	33	0.33 265	9.95 751	6	4
57	9.62 513	28	9.66 768	33	0.33 232	9.95 745	6	3
58	9.62 541	27	9.66 801	33	0.33 199	9.95 739	6	2
59	9.62 568	27	9.66 834	33	0.33 166	9.95 733	5	1
60	9.62 595		9.66 867		0.33 133	9.95 728		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	34	33
6	3.4	3.3
7	4.0	3.9
8	4.5	4.4
9	5.1	5.0
10	5.7	5.5
20	11.3	11.0
30	17.0	16.5
40	22.7	22.0
50	28.3	27.5

″	29	28	27
6	2.9	2.8	2.7
7	3.4	3.3	3.2
8	3.9	3.7	3.6
9	4.4	4.2	4.1
10	4.8	4.7	4.5
20	9.7	9.3	9.0
30	14.5	14.0	13.5
40	19.3	18.7	18.0
50	24.2	23.3	22.5

″	6	5
6	0.6	0.5
7	0.7	0.6
8	0.8	0.7
9	0.9	0.8
10	1.0	0.8
20	2.0	1.7
30	3.0	2.5
40	4.0	3.3
50	5.0	4.2

Prop. Pts.

*155° 245° *335° **65°**

25° *115° 205° *295°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.62 595	27	9.66 867	33	0.33 133	9.95 728	6	60
1	9.62 622	27	9.66 900	33	0.33 100	9.95 722	6	59
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	0.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	55
6	9.62 757	27	9.67 065	33	0.32 935	9.95 692	6	54
7	9.62 784	27	9.67 098	33	0.32 902	9.95 686	6	53
8	9.62 811	27	9.67 131	32	0.32 869	9.95 680	6	52
9	9.62 838	27	9.67 163	33	0.32 837	9.95 674	6	51
10	9.62 865	27	9.67 196	33	0.32 804	9.95 668	5	50
11	9.62 892	26	9.67 229	33	0.32 771	9.95 663	6	49
12	9.62 918	27	9.67 262	33	0.32 738	9.95 657	6	48
13	9.62 945	27	9.67 295	32	0.32 705	9.95 651	6	47
14	9.62 972	27	9.67 327	33	0.32 673	9.95 645	6	46
15	9.62 999	27	9.67 360	33	0.32 640	9.95 639	6	45
16	9.63 026	26	9.67 393	33	0.32 607	9.95 633	6	44
17	9.63 052	27	9.67 426	32	0.32 574	9.95 627	6	43
18	9.63 079	27	9.67 458	33	0.32 542	9.95 621	6	42
19	9.63 106	27	9.67 491	33	0.32 509	9.95 615	6	41
20	9.63 133	26	9.67 524	32	0.32 476	9.95 609	6	40
21	9.63 159	27	9.67 556	33	0.32 444	9.95 603	6	39
22	9.63 186	27	9.67 589	33	0.32 411	9.95 597	6	38
23	9.63 213	26	9.67 622	32	0.32 378	9.95 591	6	37
24	9.63 239	27	9.67 654	33	0.32 346	9.95 585	6	36
25	9.63 266	26	9.67 687	32	0.32 313	9.95 579	6	35
26	9.63 292	27	9.67 719	33	0.32 281	9.95 573	6	34
27	9.63 319	26	9.67 752	33	0.32 248	9.95 567	6	33
28	9.63 345	27	9.67 785	32	0.32 215	9.95 561	6	32
29	9.63 372	26	9.67 817	33	0.32 183	9.95 555	6	31
30	9.63 398	27	9.67 850	32	0.32 150	9.95 549	6	30
31	9.63 425	26	9.67 882	33	0.32 118	9.95 543	6	29
32	9.63 451	27	9.67 915	32	0.32 085	9.95 537	6	28
33	9.63 478	26	9.67 947	33	0.32 053	9.95 531	6	27
34	9.63 504	27	9.67 980	32	0.32 020	9.95 525	6	26
35	9.63 531	26	9.68 012	32	0.31 988	9.95 519	6	25
36	9.63 557	26	9.68 044	33	0.31 956	9.95 513	6	24
37	9.63 583	27	9.68 077	32	0.31 923	9.95 507	7	23
38	9.63 610	26	9.68 109	33	0.31 891	9.95 500	6	22
39	9.63 636	26	9.68 142	32	0.31 858	9.95 494	6	21
40	9.63 662	27	9.68 174	32	0.31 826	9.95 488	6	20
41	9.63 689	26	9.68 206	33	0.31 794	9.95 482	6	19
42	9.63 715	26	9.68 239	32	0.31 761	9.95 476	6	18
43	9.63 741	26	9.68 271	32	0.31 729	9.95 470	6	17
44	9.63 767	27	9.68 303	33	0.31 697	9.95 464	6	16
45	9.63 794	26	9.68 336	32	0.31 664	9.95 458	6	15
46	9.63 820	26	9.68 368	32	0.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	0.31 600	9.95 446	6	13
48	9.63 872	26	9.68 432	33	0.31 568	9.95 440	6	12
49	9.63 898	26	9.68 465	32	0.31 535	9.95 434	7	11
50	9.63 924	26	9.68 497	32	0.31 503	9.95 427	6	10
51	9.63 950	26	9.68 529	32	0.31 471	9.95 421	6	9
52	9.63 976	26	9.68 561	32	0.31 439	9.95 415	6	8
53	9.64 002	26	9.68 593	33	0.31 407	9.95 409	6	7
54	9.64 028	26	9.68 626	32	0.31 374	9.95 403	6	6
55	9.64 054	26	9.68 658	32	0.31 342	9.95 397	6	5
56	9.64 080	26	9.68 690	32	0.31 310	9.95 391	7	4
57	9.64 106	26	9.68 722	32	0.31 278	9.95 384	6	3
58	9.64 132	26	9.68 754	32	0.31 246	9.95 378	6	2
59	9.64 158	26	9.68 786	32	0.31 214	9.95 372	6	1
60	9.64 184		9.68 818		0.31 182	9.95 366		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	33	32
6	3.3	3.2
7	3.9	3.7
8	4.4	4.3
9	5.0	4.8
10	5.5	5.3
20	11.0	10.7
30	16.5	16.0
40	22.0	21.3
50	27.5	26.7

″	27	26
6	2.7	2.6
7	3.2	3.0
8	3.6	3.5
9	4.1	3.9
10	4.5	4.3
20	9.0	8.7
30	13.5	13.0
40	18.0	17.3
50	22.5	21.7

″	7	6	5
6	0.7	0.6	0.5
7	0.8	0.7	0.6
8	0.9	0.8	0.7
9	1.1	0.9	0.8
10	1.2	1.0	0.8
20	2.3	2.0	1.7
30	3.5	3.0	2.5
40	4.7	4.0	3.3
50	5.8	5.0	4.2

Prop. Pts.

*154° 244° *334° **64°**

26° *116° 206° *296°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.64 184	26	9.68 818	32	0.31 182	9.95 366	6	60
1	9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	0.31 118	9.95 354	6	58
3	9.64 262	26	9.68 914	32	0.31 086	9.95 348	7	57
4	9.64 288	25	9.68 946	32	0.31 054	9.95 341	6	56
5	9.64 313	26	9.68 978	32	0.31 022	9.95 335	6	55
6	9.64 339	26	9.69 010	32	0.30 990	9.95 329	6	54
7	9.64 365	26	9.69 042	32	0.30 958	9.95 323	6	53
8	9.64 391	26	9.69 074	32	0.30 926	9.95 317	7	52
9	9.64 417	25	9.69 106	32	0.30 894	9.95 310	6	51
10	9.64 442	26	9.69 138	32	0.30 862	9.95 304	6	50
11	9.64 468	26	9.69 170	32	0.30 830	9.95 298	6	49
12	9.64 494	25	9.69 202	32	0.30 798	9.95 292	6	48
13	9.64 519	26	9.69 234	32	0.30 766	9.95 286	7	47
14	9.64 545	26	9.69 266	32	0.30 734	9.95 279	6	46
15	9.64 571	25	9.69 298	31	0.30 702	9.95 273	6	45
16	9.64 596	26	9.69 329	32	0.30 671	9.95 267	6	44
17	9.64 622	25	9.69 361	32	0.30 639	9.95 261	7	43
18	9.64 647	26	9.69 393	32	0.30 607	9.95 254	6	42
19	9.64 673	25	9.69 425	32	0.30 575	9.95 248	6	41
20	9.64 698	26	9.69 457	31	0.30 543	9.95 242	6	40
21	9.64 724	25	9.69 488	32	0.30 512	9.95 236	7	39
22	9.64 749	26	9.69 520	32	0.30 480	9.95 229	6	38
23	9.64 775	25	9.69 552	32	0.30 448	9.95 223	6	37
24	9.64 800	26	9.69 584	31	0.30 416	9.95 217	6	36
25	9.64 826	25	9.69 615	32	0.30 385	9.95 211	7	35
26	9.64 851	26	9.69 647	32	0.30 353	9.95 204	6	34
27	9.64 877	25	9.69 679	31	0.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	32	0.30 290	9.95 192	7	32
29	9.64 927	26	9.69 742	32	0.30 258	9.95 185	6	31
30	9.64 953	25	9.69 774	31	0.30 226	9.95 179	6	30
31	9.64 978	25	9.69 805	32	0.30 195	9.95 173	6	29
32	9.65 003	26	9.69 837	31	0.30 163	9.95 167	7	28
33	9.65 029	25	9.69 868	32	0.30 132	9.95 160	6	27
34	9.65 054	25	9.69 900	32	0.30 100	9.95 154	6	26
35	9.65 079	25	9.69 932	31	0.30 068	9.95 148	7	25
36	9.65 104	26	9.69 963	32	0.30 037	9.95 141	6	24
37	9.65 130	25	9.69 995	31	0.30 005	9.95 135	6	23
38	9.65 155	25	9.70 026	32	0.29 974	9.95 129	7	22
39	9.65 180	25	9.70 058	31	0.29 942	9.95 122	6	21
40	9.65 205	25	9.70 089	32	0.29 911	9.95 116	6	20
41	9.65 230	25	9.70 121	31	0.29 879	9.95 110	7	19
42	9.65 255	26	9.70 152	32	0.29 848	9.95 103	6	18
43	9.65 281	25	9.70 184	31	0.29 816	9.95 097	7	17
44	9.65 306	25	9.70 215	32	0.29 785	9.95 090	6	16
45	9.65 331	25	9.70 247	31	0.29 753	9.95 084	6	15
46	9.65 356	25	9.70 278	31	0.29 722	9.95 078	7	14
47	9.65 381	25	9.70 309	32	0.29 691	9.95 071	6	13
48	9.65 406	25	9.70 341	31	0.29 659	9.95 065	6	12
49	9.65 431	25	9.70 372	32	0.29 628	9.95 059	7	11
50	9.65 456	25	9.70 404	31	0.29 596	9.95 052	6	10
51	9.65 481	25	9.70 435	31	0.29 565	9.95 046	7	9
52	9.65 506	25	9.70 466	32	0.29 534	9.95 039	6	8
53	9.65 531	25	9.70 498	31	0.29 502	9.95 033	6	7
54	9.65 556	24	9.70 529	31	0.29 471	9.95 027	7	6
55	9.65 580	25	9.70 560	32	0.29 440	9.95 020	6	5
56	9.65 605	25	9.70 592	31	0.29 408	9.95 014	7	4
57	9.65 630	25	9.70 623	31	0.29 377	9.95 007	6	3
58	9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2
59	9.65 680	25	9.70 685	32	0.29 315	9.94 995	7	1
60	9.65 705		9.70 717		0.29 283	9.94 988		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

*153° 243° *333° **63°**

Prop. Pts.

″	32	31
6	3.2	3.1
7	3.7	3.6
8	4.3	4.1
9	4.8	4.6
10	5.3	5.2
20	10.7	10.3
30	16.0	15.5
40	21.3	20.7
50	26.7	25.8

″	26	25	24
6	2.6	2.5	2.4
7	3.0	2.9	2.8
8	3.5	3.3	3.2
9	3.9	3.8	3.6
10	4.3	4.2	4.0
20	8.7	8.3	8.0
30	13.0	12.5	12.0
40	17.3	16.7	16.0
50	21.7	20.8	20.0

″	7	6
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.0	0.9
10	1.2	1.0
20	2.3	2.0
30	3.5	3.0
40	4.7	4.0
50	5.8	5.0

27° *117° 207° *297°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.65 705	24	9.70 717	31	0.29 283	9.94 988	6	60
1	9.65 729	25	9.70 748	31	0.29 252	9.94 982	7	59
2	9.65 754	25	9.70 779	31	0.29 221	9.94 975	6	58
3	9.65 779	25	9.70 810	31	0.29 190	9.94 969	7	57
4	9.65 804	24	9.70 841	32	0.29 159	9.94 962	6	56
5	9.65 828	25	9.70 873	31	0.29 127	9.94 956	7	55
6	9.65 853	25	9.70 904	31	0.29 096	9.94 949	6	54
7	9.65 878	24	9.70 935	31	0.29 065	9.94 943	7	53
8	9.65 902	25	9.70 966	31	0.29 034	9.94 936	6	52
9	9.65 927	25	9.70 997	31	0.29 003	9.94 930	7	51
10	9.65 952	24	9.71 028	31	0.28 972	9.94 923	6	50
11	9.65 976	25	9.71 059	31	0.28 941	9.94 917	6	49
12	9.66 001	24	9.71 090	31	0.28 910	9.94 911	7	48
13	9.66 025	25	9.71 121	32	0.28 879	9.94 904	6	47
14	9.66 050	25	9.71 153	31	0.28 847	9.94 898	7	46
15	9.66 075	24	9.71 184	31	0.28 816	9.94 891	6	45
16	9.66 099	25	9.71 215	31	0.28 785	9.94 885	7	44
17	9.66 124	24	9.71 246	31	0.28 754	9.94 878	7	43
18	9.66 148	25	9.71 277	31	0.28 723	9.94 871	6	42
19	9.66 173	24	9.71 308	31	0.28 692	9.94 865	7	41
20	9.66 197	24	9.71 339	31	0.28 661	9.94 858	6	40
21	9.66 221	25	9.71 370	31	0.28 630	9.94 852	7	39
22	9.66 246	24	9.71 401	30	0.28 599	9.94 845	6	38
23	9.66 270	25	9.71 431	31	0.28 569	9.94 839	7	37
24	9.66 295	24	9.71 462	31	0.28 538	9.94 832	6	36
25	9.66 319	24	9.71 493	31	0.28 507	9.94 826	7	35
26	9.66 343	25	9.71 524	31	0.28 476	9.94 819	6	34
27	9.66 368	24	9.71 555	31	0.28 445	9.94 813	7	33
28	9.66 392	24	9.71 586	31	0.28 414	9.94 806	7	32
29	9.66 416	25	9.71 617	31	0.28 383	9.94 799	6	31
30	9.66 441	24	9.71 648	31	0.28 352	9.94 793	7	30
31	9.66 465	24	9.71 679	30	0.28 321	9.94 786	6	29
32	9.66 489	24	9.71 709	31	0.28 291	9.94 780	7	28
33	9.66 513	24	9.71 740	31	0.28 260	9.94 773	6	27
34	9.66 537	25	9.71 771	31	0.28 229	9.94 767	7	26
35	9.66 562	24	9.71 802	31	0.28 198	9.94 760	7	25
36	9.66 586	24	9.71 833	30	0.28 167	9.94 753	6	24
37	9.66 610	24	9.71 863	31	0.28 137	9.94 747	7	23
38	9.66 634	24	9.71 894	31	0.28 106	9.94 740	6	22
39	9.66 658	24	9.71 925	30	0.28 075	9.94 734	7	21
40	9.66 682	24	9.71 955	31	0.28 045	9.94 727	7	20
41	9.66 706	25	9.71 986	31	0.28 014	9.94 720	6	19
42	9.66 731	24	9.72 017	31	0.27 983	9.94 714	7	18
43	9.66 755	24	9.72 048	30	0.27 952	9.94 707	7	17
44	9.66 779	24	9.72 078	31	0.27 922	9.94 700	6	16
45	9.66 803	24	9.72 109	31	0.27 891	9.94 694	7	15
46	9.66 827	24	9.72 140	30	0.27 860	9.94 687	7	14
47	9.66 851	24	9.72 170	31	0.27 830	9.94 680	6	13
48	9.66 875	24	9.72 201	30	0.27 799	9.94 674	7	12
49	9.66 899	23	9.72 231	31	0.27 769	9.94 667	7	11
50	9.66 922	24	9.72 262	31	0.27 738	9.94 660	6	10
51	9.66 946	24	9.72 293	30	0.27 707	9.94 654	7	9
52	9.66 970	24	9.72 323	31	0.27 677	9.94 647	7	8
53	9.66 994	24	9.72 354	30	0.27 646	9.94 640	6	7
54	9.67 018	24	9.72 384	31	0.27 616	9.94 634	7	6
55	9.67 042	24	9.72 415	30	0.27 585	9.94 627	7	5
56	9.67 066	24	9.72 445	31	0.27 555	9.94 620	6	4
57	9.67 090	23	9.72 476	30	0.27 524	9.94 614	7	3
58	9.67 113	24	9.72 506	31	0.27 494	9.94 607	7	2
59	9.67 137	24	9.72 537	30	0.27 463	9.94 600	7	1
60	9.67 161		9.72 567		0.27 433	9.94 593		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	32	31	30
6	3.2	3.1	3.0
7	3.7	3.6	3.5
8	4.3	4.1	4.0
9	4.8	4.6	4.5
10	5.3	5.2	5.0
20	10.7	10.3	10.0
30	16.0	15.5	15.0
40	21.3	20.7	20.0
50	26.7	25.8	25.0

″	25	24	23
6	2.5	2.4	2.3
7	2.9	2.8	2.7
8	3.3	3.2	3.1
9	3.8	3.6	3.5
10	4.2	4.0	3.8
20	8.3	8.0	7.7
30	12.5	12.0	11.5
40	16.7	16.0	15.3
50	20.8	20.0	19.2

″	7	6
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.1	0.9
10	1.2	1.0
20	2.3	2.0
30	3.5	3.0
40	4.7	4.0
50	5.8	5.0

Prop. Pts.

*152° 242° *332° **62°**

28° *118° 208° *298°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	′
0	9.67 161	24	9.72 567	31	0.27 433	9.94 593	6	60
1	9.67 185	23	9.72 598	30	0.27 402	9.94 587	7	59
2	9.67 208	24	9.72 628	31	0.27 372	9.94 580	7	58
3	9.67 232	24	9.72 659	30	0.27 341	9.94 573	6	57
4	9.67 256	24	9.72 689	31	0.27 311	9.94 567	7	56
5	9.67 280	23	9.72 720	30	0.27 280	9.94 560	7	55
6	9.67 303	24	9.72 750	30	0.27 250	9.94 553	7	54
7	9.67 327	23	9.72 780	31	0.27 220	9.94 546	6	53
8	9.67 350	24	9.72 811	30	0.27 189	9.94 540	7	52
9	9.67 374	24	9.72 841	31	0.27 159	9.94 533	7	51
10	9.67 398	23	9.72 872	30	0.27 128	9.94 526	7	50
11	9.67 421	24	9.72 902	30	0.27 098	9.94 519	6	49
12	9.67 445	23	9.72 932	31	0.27 068	9.94 513	7	48
13	9.67 468	24	9.72 963	30	0.27 037	9.94 506	7	47
14	9.67 492	23	9.72 993	30	0.27 007	9.94 499	7	46
15	9.67 515	24	9.73 023	31	0.26 977	9.94 492	7	45
16	9.67 539	23	9.73 054	30	0.26 946	9.94 485	6	44
17	9.67 562	24	9.73 084	30	0.26 916	9.94 479	7	43
18	9.67 586	23	9.73 114	30	0.26 886	9.94 472	7	42
19	9.67 609	24	9.73 144	31	0.26 856	9.94 465	7	41
20	9.67 633	23	9.73 175	30	0.26 825	9.94 458	7	40
21	9.67 656	24	9.73 205	30	0.26 795	9.94 451	6	39
22	9.67 680	23	9.73 235	30	0.26 765	9.94 445	7	38
23	9.67 703	23	9.73 265	30	0.26 735	9.94 438	7	37
24	9.67 726	24	9.73 295	31	0.26 705	9.94 431	7	36
25	9.67 750	23	9.73 326	30	0.26 674	9.94 424	7	35
26	9.67 773	23	9.73 356	30	0.26 644	9.94 417	7	34
27	9.67 796	24	9.73 386	30	0.26 614	9.94 410	6	33
28	9.67 820	23	9.73 416	30	0.26 584	9.94 404	7	32
29	9.67 843	23	9.73 446	30	0.26 554	9.94 397	7	31
30	9.67 866	24	9.73 476	31	0.26 524	9.94 390	7	30
31	9.67 890	23	9.73 507	30	0.26 493	9.94 383	7	29
32	9.67 913	23	9.73 537	30	0.26 463	9.94 376	7	28
33	9.67 936	23	9.73 567	30	0.26 433	9.94 369	7	27
34	9.67 959	23	9.73 597	30	0.26 403	9.94 362	7	26
35	9.67 982	24	9.73 627	30	0.26 373	9.94 355	6	25
36	9.68 006	23	9.73 657	30	0.26 343	9.94 349	7	24
37	9.68 029	23	9.73 687	30	0.26 313	9.94 342	7	23
38	9.68 052	23	9.73 717	30	0.26 283	9.94 335	7	22
39	9.68 075	23	9.73 747	30	0.26 253	9.94 328	7	21
40	9.68 098	23	9.73 777	30	0.26 223	9.94 321	7	20
41	9.68 121	23	9.73 807	30	0.26 193	9.94 314	7	19
42	9.68 144	23	9.73 837	30	0.26 163	9.94 307	7	18
43	9.68 167	23	9.73 867	30	0.26 133	9.94 300	7	17
44	9.68 190	23	9.73 897	30	0.26 103	9.94 293	7	16
45	9.68 213	24	9.73 927	30	0.26 073	9.94 286	7	15
46	9.68 237	23	9.73 957	30	0.26 043	9.94 279	6	14
47	9.68 260	23	9.73 987	30	0.26 013	9.94 273	7	13
48	9.68 283	22	9.74 017	30	0.25 983	9.94 266	7	12
49	9.68 305	23	9.74 047	30	0.25 953	9.94 259	7	11
50	9.68 328	23	9.74 077	30	0.25 923	9.94 252	7	10
51	9.68 351	23	9.74 107	30	0.25 893	9.94 245	7	9
52	9.68 374	23	9.74 137	29	0.25 863	9.94 238	7	8
53	9.68 397	23	9.74 166	30	0.25 834	9.94 231	7	7
54	9.68 420	23	9.74 196	30	0.25 804	9.94 224	7	6
55	9.68 443	23	9.74 226	30	0.25 774	9.94 217	7	5
56	9.68 466	23	9.74 256	30	0.25 744	9.94 210	7	4
57	9.68 489	23	9.74 286	30	0.25 714	9.94 203	7	3
58	9.68 512	22	9.74 316	29	0.25 684	9.94 196	7	2
59	9.68 534	23	9.74 345	30	0.25 655	9.94 189	7	1
60	9.68 557		9.74 375		0.25 625	9.94 182		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

*151° 241° *331° **61°**

Prop. Pts.

″	31	30	29
6	3.1	3.0	2.9
7	3.6	3.5	3.4
8	4.1	4.0	3.9
9	4.6	4.5	4.4
10	5.2	5.0	4.8
20	10.3	10.0	9.7
30	15.5	15.0	14.5
40	20.7	20.0	19.3
50	25.8	25.0	24.2

″	24	23	22
6	2.4	2.3	2.2
7	2.8	2.7	2.6
8	3.2	3.1	2.9
9	3.6	3.5	3.3
10	4.0	3.8	3.7
20	8.0	7.7	7.3
30	12.0	11.5	11.0
40	16.0	15.3	14.7
50	20.0	19.2	18.3

″	7	6
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.0	0.9
10	1.2	1.0
20	2.3	2.0
30	3.5	3.0
40	4.7	4.0
50	5.8	5.0

29° *119° 209° *299°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.		Prop. Pts.
0	9.68 557	23	9.74 375	30	0.25 625	9.94 182	7	60	
1	9.68 580	23	9.74 405	30	0.25 595	9.94 175	7	59	
2	9.68 603	22	9.74 435	30	0.25 565	9.94 168	7	58	
3	9.68 625	23	9.74 465	29	0.25 535	9.94 161	7	57	
4	9.68 648	23	9.74 494	30	0.25 506	9.94 154	7	56	
5	9.68 671	23	9.74 524	30	0.25 476	9.94 147	7	55	
6	9.68 694	22	9.74 554	29	0.25 446	9.94 140	7	54	
7	9.68 716	23	9.74 583	30	0.25 417	9.94 133	7	53	
8	9.68 739	23	9.74 613	30	0.25 387	9.94 126	7	52	
9	9.68 762	22	9.74 643	30	0.25 357	9.94 119	7	51	
10	9.68 784	23	9.74 673	29	0.25 327	9.94 112	7	50	
11	9.68 807	22	9.74 702	30	0.25 298	9.94 105	7	49	
12	9.68 829	23	9.74 732	30	0.25 268	9.94 098	8	48	
13	9.68 852	23	9.74 762	29	0.25 238	9.94 090	7	47	
14	9.68 875	22	9.74 791	30	0.25 209	9.94 083	7	46	
15	9.68 897	23	9.74 821	30	0.25 179	9.94 076	7	45	
16	9.68 920	22	9.74 851	29	0.25 149	9.94 069	7	44	
17	9.68 942	23	9.74 880	30	0.25 120	9.94 062	7	43	
18	9.68 965	22	9.74 910	29	0.25 090	9.94 055	7	42	
19	9.68 987	23	9.74 939	30	0.25 061	9.94 048	7	41	
20	9.69 010	22	9.74 969	29	0.25 031	9.94 041	7	40	
21	9.69 032	23	9.74 998	30	0.25 002	9.94 034	7	39	
22	9.69 055	22	9.75 028	30	0.24 972	9.94 027	7	38	
23	9.69 077	23	9.75 058	29	0.24 942	9.94 020	8	37	
24	9.69 100	22	9.75 087	30	0.24 913	9.94 012	7	36	
25	9.69 122	22	9.75 117	29	0.24 883	9.94 005	7	35	
26	9.69 144	23	9.75 146	30	0.24 854	9.93 998	7	34	
27	9.69 167	22	9.75 176	29	0.24 824	9.93 991	7	33	
28	9.69 189	23	9.75 205	30	0.24 795	9.93 984	7	32	
29	9.69 212	22	9.75 235	29	0.24 765	9.93 977	7	31	
30	9.69 234	22	9.75 264	30	0.24 736	9.93 970	7	30	
31	9.69 256	23	9.75 294	29	0.24 706	9.93 963	8	29	
32	9.69 279	22	9.75 323	30	0.24 677	9.93 955	7	28	
33	9.69 301	22	9.75 353	29	0.24 647	9.93 948	7	27	
34	9.69 323	22	9.75 382	29	0.24 618	9.93 941	7	26	
35	9.69 345	23	9.75 411	30	0.24 589	9.93 934	7	25	
36	9.69 368	22	9.75 441	29	0.24 559	9.93 927	7	24	
37	9.69 390	22	9.75 470	30	0.24 530	9.93 920	8	23	
38	9.69 412	22	9.75 500	29	0.24 500	9.93 912	7	22	
39	9.69 434	22	9.75 529	29	0.24 471	9.93 905	7	21	
40	9.69 456	23	9.75 558	30	0.24 442	9.93 898	7	20	
41	9.69 479	22	9.75 588	29	0.24 412	9.93 891	7	19	
42	9.69 501	22	9.75 617	30	0.24 383	9.93 884	8	18	
43	9.69 523	22	9.75 647	29	0.24 353	9.93 876	7	17	
44	9.69 545	22	9.75 676	29	0.24 324	9.93 869	7	16	
45	9.69 567	22	9.75 705	30	0.24 295	9.93 862	7	15	
46	9.69 589	22	9.75 735	29	0.24 265	9.93 855	8	14	
47	9.69 611	22	9.75 764	29	0.24 236	9.93 847	7	13	
48	9.69 633	22	9.75 793	29	0.24 207	9.93 840	7	12	
49	9.69 655	22	9.75 822	30	0.24 178	9.93 833	7	11	
50	9.69 677	22	9.75 852	29	0.24 148	9.93 826	7	10	
51	9.69 699	22	9.75 881	29	0.24 119	9.93 819	8	9	
52	9.69 721	22	9.75 910	29	0.24 090	9.93 811	7	8	
53	9.69 743	22	9.75 939	30	0.24 061	9.93 804	7	7	
54	9.69 765	22	9.75 969	29	0.24 031	9.93 797	8	6	
55	9.69 787	22	9.75 998	29	0.24 002	9.93 789	7	5	
56	9.69 809	22	9.76 027	29	0.23 973	9.93 782	7	4	
57	9.69 831	22	9.76 056	30	0.23 944	9.93 775	7	3	
58	9.69 853	22	9.76 086	29	0.23 914	9.93 768	8	2	
59	9.69 875	22	9.76 115	29	0.23 885	9.93 760	7	1	
60	9.69 897		9.76 144		0.23 856	9.93 753		0	
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′	Prop. Pts.

Prop. Pts.

″	30	29
6	3.0	2.9
7	3.5	3.5
8	4.0	3.9
9	4.5	4.4
10	5.0	4.8
20	10.0	9.7
30	15.0	14.5
40	20.0	19.3
50	25.0	24.2

″	23	22
6	2.3	2.2
7	2.7	2.6
8	3.1	2.9
9	3.4	3.3
10	3.8	3.7
20	7.7	7.3
30	11.5	11.0
40	15.3	14.7
50	19.2	18.3

″	8	7
6	0.8	0.7
7	0.9	0.8
8	1.1	0.9
9	1.2	1.0
10	1.3	1.2
20	2.7	2.3
30	4.0	3.5
40	5.3	4.7
50	6.7	5.8

*150° 240° *330° **60°**

30° *120° 210° *300°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	60
1	9.69 919	22	9.76 173	29	0.23 827	9.93 746	8	59
2	9.69 941	22	9.76 202	29	0.23 798	9.93 738	7	58
3	9.69 963	21	9.76 231	30	0.23 769	9.93 731	7	57
4	9.69 984	22	9.76 261	29	0.23 739	9.93 724	7	56
5	9.70 006	22	9.76 290	29	0.23 710	9.93 717	8	55
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	7	54
7	9.70 050	22	9.76 348	29	0.23 652	9.93 702	7	53
8	9.70 072	21	9.76 377	29	0.23 623	9.93 695	8	52
9	9.70 093	22	9.76 406	29	0.23 594	9.93 687	7	51
10	9.70 115	22	9.76 435	29	0.23 565	9.93 680	7	50
11	9.70 137	22	9.76 464	29	0.23 536	9.93 673	8	49
12	9.70 159	21	9.76 493	29	0.23 507	9.93 665	7	48
13	9.70 180	22	9.76 522	29	0.23 478	9.93 658	8	47
14	9.70 202	22	9.76 551	29	0.23 449	9.93 650	7	46
15	9.70 224	21	9.76 580	29	0.23 420	9.93 643	7	45
16	9.70 245	22	9.76 609	30	0.23 391	9.93 636	8	44
17	9.70 267	21	9.76 639	29	0.23 361	9.93 628	7	43
18	9.70 288	22	9.76 668	29	0.23 332	9.93 621	7	42
19	9.70 310	22	9.76 697	28	0.23 303	9.93 614	8	41
20	9.70 332	21	9.76 725	29	0.23 275	9.93 606	7	40
21	9.70 353	22	9.76 754	29	0.23 246	9.93 599	8	39
22	9.70 375	21	9.76 783	29	0.23 217	9.93 591	7	38
23	9.70 396	22	9.76 812	29	0.23 188	9.93 584	7	37
24	9.70 418	21	9.76 841	29	0.23 159	9.93 577	8	36
25	9.70 439	22	9.76 870	29	0.23 130	9.93 569	7	35
26	9.70 461	21	9.76 899	29	0.23 101	9.93 562	8	34
27	9.70 482	22	9.76 928	29	0.23 072	9.93 554	7	33
28	9.70 504	21	9.76 957	29	0.23 043	9.93 547	8	32
29	9.70 525	22	9.76 986	29	0.23 014	9.93 539	7	31
30	9.70 547	21	9.77 015	29	0.22 985	9.93 532	7	30
31	9.70 568	22	9.77 044	29	0.22 956	9.93 525	8	29
32	9.70 590	21	9.77 073	28	0.22 927	9.93 517	7	28
33	9.70 611	22	9.77 101	29	0.22 899	9.93 510	8	27
34	9.70 633	21	9.77 130	29	0.22 870	9.93 502	7	26
35	9.70 654	21	9.77 159	29	0.22 841	9.93 495	8	25
36	9.70 675	22	9.77 188	29	0.22 812	9.93 487	7	24
37	9.70 697	21	9.77 217	29	0.22 783	9.93 480	8	23
38	9.70 718	21	9.77 246	28	0.22 754	9.93 472	7	22
39	9.70 739	22	9.77 274	29	0.22 726	9.93 465	8	21
40	9.70 761	21	9.77 303	29	0.22 697	9.93 457	7	20
41	9.70 782	21	9.77 332	29	0.22 668	9.93 450	8	19
42	9.70 803	21	9.77 361	29	0.22 639	9.93 442	7	18
43	9.70 824	22	9.77 390	28	0.22 610	9.93 435	8	17
44	9.70 846	21	9.77 418	29	0.22 582	9.93 427	7	16
45	9.70 867	21	9.77 447	29	0.22 553	9.93 420	8	15
46	9.70 888	21	9.77 476	29	0.22 524	9.93 412	7	14
47	9.70 909	22	9.77 505	28	0.22 495	9.93 405	8	13
48	9.70 931	21	9.77 533	29	0.22 467	9.93 397	7	12
49	9.70 952	21	9.77 562	29	0.22 438	9.93 390	8	11
50	9.70 973	21	9.77 591	28	0.22 409	9.93 382	7	10
51	9.70 994	21	9.77 619	29	0.22 381	9.93 375	8	9
52	9.71 015	21	9.77 648	29	0.22 352	9.93 367	7	8
53	9.71 036	22	9.77 677	29	0.22 323	9.93 360	8	7
54	9.71 058	21	9.77 706	28	0.22 294	9.93 352	8	6
55	9.71 079	21	9.77 734	29	0.22 266	9.93 344	7	5
56	9.71 100	21	9.77 763	28	0.22 237	9.93 337	8	4
57	9.71 121	21	9.77 791	29	0.22 209	9.93 329	7	3
58	9.71 142	21	9.77 820	29	0.22 180	9.93 322	8	2
59	9.71 163	21	9.77 849	28	0.22 151	9.93 314	7	1
60	9.71 184		9.77 877		0.22 123	9.93 307		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

59° *149° 239° *329°

Prop. Pts.

″	30	29	28
6	3.0	2.9	2.8
7	3.5	3.4	3.3
8	4.0	3.9	3.7
9	4.5	4.4	4.2
10	5.0	4.8	4.7
20	10.0	9.7	9.3
30	15.0	14.5	14.0
40	20.0	19.3	18.7
50	25.0	24.2	23.3

″	22	21
6	2.2	2.1
7	2.6	2.4
8	2.9	2.8
9	3.3	3.2
10	3.7	3.5
20	7.3	7.0
30	11.0	10.5
40	14.7	14.0
50	18.3	17.5

″	8	7
6	0.8	0.7
7	0.9	0.8
8	1.1	0.9
9	1.2	1.0
10	1.3	1.2
20	2.7	2.3
30	4.0	3.5
40	5.3	4.7
50	6.7	5.8

31° *121° 211° *301°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.71 184	21	9.77 877	29	0.22 123	9.93 307	8	60
1	9.71 205	21	9.77 906	29	0.22 094	9.93 299	8	59
2	9.71 226	21	9.77 935	28	0.22 065	9.93 291	7	58
3	9.71 247	21	9.77 963	29	0.22 037	9.93 284	8	57
4	9.71 268	21	9.77 992	28	0.22 008	9.93 276	7	56
5	9.71 289	21	9.78 020	29	0.21 980	9.93 269	8	55
6	9.71 310	21	9.78 049	28	0.21 951	9.93 261	8	54
7	9.71 331	21	9.78 077	29	0.21 923	9.93 253	7	53
8	9.71 352	21	9.78 106	29	0.21 894	9.93 246	8	52
9	9.71 373	20	9.78 135	28	0.21 865	9.93 238	8	51
10	9.71 393	21	9.78 163	29	0.21 837	9.93 230	7	50
11	9.71 414	21	9.78 192	28	0.21 808	9.93 223	8	49
12	9.71 435	21	9.78 220	29	0.21 780	9.93 215	8	48
13	9.71 456	21	9.78 249	28	0.21 751	9.93 207	7	47
14	9.71 477	21	9.78 277	29	0.21 723	9.93 200	8	46
15	9.71 498	21	9.78 306	28	0.21 694	9.93 192	8	45
16	9.71 519	20	9.78 334	29	0.21 666	9.93 184	7	44
17	9.71 539	21	9.78 363	28	0.21 637	9.93 177	8	43
18	9.71 560	21	9.78 391	28	0.21 609	9.93 169	8	42
19	9.71 581	21	9.78 419	29	0.21 581	9.93 161	7	41
20	9.71 602	20	9.78 448	28	0.21 552	9.93 154	8	40
21	9.71 622	21	9.78 476	29	0.21 524	9.93 146	8	39
22	9.71 643	21	9.78 505	28	0.21 495	9.93 138	7	38
23	9.71 664	21	9.78 533	29	0.21 467	9.93 131	8	37
24	9.71 685	20	9.78 562	28	0.21 438	9.93 123	8	36
25	9.71 705	21	9.78 590	28	0.21 410	9.93 115	7	35
26	9.71 726	21	9.78 618	29	0.21 382	9.93 108	8	34
27	9.71 747	20	9.78 647	28	0.21 353	9.93 100	8	33
28	9.71 767	21	9.78 675	29	0.21 325	9.93 092	8	32
29	9.71 788	21	9.78 704	28	0.21 296	9.93 084	7	31
30	9.71 809	20	9.78 732	28	0.21 268	9.93 077	8	30
31	9.71 829	21	9.78 760	29	0.21 240	9.93 069	8	29
32	9.71 850	20	9.78 789	28	0.21 211	9.93 061	8	28
33	9.71 870	21	9.78 817	28	0.21 183	9.93 053	7	27
34	9.71 891	20	9.78 845	29	0.21 155	9.93 046	8	26
35	9.71 911	21	9.78 874	28	0.21 126	9.93 038	8	25
36	9.71 932	20	9.78 902	28	0.21 098	9.93 030	8	24
37	9.71 952	21	9.78 930	29	0.21 070	9.93 022	8	23
38	9.71 973	21	9.78 959	28	0.21 041	9.93 014	7	22
39	9.71 994	20	9.78 987	28	0.21 013	9.93 007	8	21
40	9.72 014	20	9.79 015	28	0.20 985	9.92 999	8	20
41	9.72 034	21	9.79 043	29	0.20 957	9.92 991	8	19
42	9.72 055	20	9.79 072	28	0.20 928	9.92 983	7	18
43	9.72 075	21	9.79 100	28	0.20 900	9.92 976	8	17
44	9.72 096	20	9.79 128	28	0.20 872	9.92 968	8	16
45	9.72 116	21	9.79 156	29	0.20 844	9.92 960	8	15
46	9.72 137	20	9.79 185	28	0.20 815	9.92 952	8	14
47	9.72 157	20	9.79 213	28	0.20 787	9.92 944	8	13
48	9.72 177	21	9.79 241	28	0.20 759	9.92 936	7	12
49	9.72 198	20	9.79 269	28	0.20 731	9.92 929	8	11
50	9.72 218	20	9.79 297	29	0.20 703	9.92 921	8	10
51	9.72 238	21	9.79 326	28	0.20 674	9.92 913	8	9
52	9.72 259	20	9.79 354	28	0.20 646	9.92 905	8	8
53	9.72 279	20	9.79 382	28	0.20 618	9.92 897	8	7
54	9.72 299	21	9.79 410	28	0.20 590	9.92 889	8	6
55	9.72 320	20	9.79 438	28	0.20 562	9.92 881	7	5
56	9.72 340	20	9.79 466	29	0.20 534	9.92 874	8	4
57	9.72 360	21	9.79 495	28	0.20 505	9.92 866	8	3
58	9.72 381	20	9.79 523	28	0.20 477	9.92 858	8	2
59	9.72 401	20	9.79 551	28	0.20 449	9.92 850	8	1
60	9.72 421		9.79 579		0.20 421	9.92 842		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	29	28
6	2.9	2.8
7	3.5	3.3
8	3.9	3.7
9	4.4	4.2
10	4.8	4.7
20	9.7	9.3
30	14.5	14.0
40	19.3	18.7
50	24.2	23.3

″	21	20
6	2.1	2.0
7	2.4	2.3
8	2.8	2.7
9	3.2	3.0
10	3.5	3.3
20	7.0	6.7
30	10.5	10.0
40	14.0	13.3
50	17.5	16.7

″	8	7
6	0.8	0.7
7	0.9	0.8
8	1.1	0.9
9	1.2	1.0
10	1.3	1.2
20	2.7	2.3
30	4.0	3.5
40	5.3	4.7
50	6.7	5.8

Prop. Pts.

*148° 238° *328° **58°**

32° *122° 212° *302°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	′
0	9.72 421	20	9.79 579	28	0.20 421	9.92 842	8	60
1	9.72 441	20	9.79 607	28	0.20 393	9.92 834	8	59
2	9.72 461	21	9.79 635	28	0.20 365	9.92 826	8	58
3	9.72 482	20	9.79 663	28	0.20 337	9.92 818	8	57
4	9.72 502	20	9.79 691	28	0.20 309	9.92 810	7	56
5	9.72 522	20	9.79 719	28	0.20 281	9.92 803	8	55
6	9.72 542	20	9.79 747	29	0.20 253	9.92 795	8	54
7	9.72 562	20	9.79 776	28	0.20 224	9.92 787	8	53
8	9.72 582	20	9.79 804	28	0.20 196	9.92 779	8	52
9	9.72 602	20	9.79 832	28	0.20 168	9.92 771	8	51
10	9.72 622	21	9.79 860	28	0.20 140	9.92 763	8	50
11	9.72 643	20	9.79 888	28	0.20 112	9.92 755	8	49
12	9.72 663	20	9.79 916	28	0.20 084	9.92 747	8	48
13	9.72 683	20	9.79 944	28	0.20 056	9.92 739	8	47
14	9.72 703	20	9.79 972	28	0.20 028	9.92 731	8	46
15	9.72 723	20	9.80 000	28	0.20 000	9.92 723	8	45
16	9.72 743	20	9.80 028	28	0.19 972	9.92 715	8	44
17	9.72 763	20	9.80 056	28	0.19 944	9.92 707	8	43
18	9.72 783	20	9.80 084	28	0.19 916	9.92 699	8	42
19	9.72 803	20	9.80 112	28	0.19 888	9.92 691	8	41
20	9.72 823	20	9.80 140	28	0.19 860	9.92 683	8	40
21	9.72 843	20	9.80 168	27	0.19 832	9.92 675	8	39
22	9.72 863	20	9.80 195	28	0.19 805	9.92 667	8	38
23	9.72 883	19	9.80 223	28	0.19 777	9.92 659	8	37
24	9.72 902	20	9.80 251	28	0.19 749	9.92 651	8	36
25	9.72 922	20	9.80 279	28	0.19 721	9.92 643	8	35
26	9.72 942	20	9.80 307	28	0.19 693	9.92 635	8	34
27	9.72 962	20	9.80 335	28	0.19 665	9.92 627	8	33
28	9.72 982	20	9.80 363	28	0.19 637	9.92 619	8	32
29	9.73 002	20	9.80 391	28	0.19 609	9.92 611	8	31
30	9.73 022	19	9.80 419	28	0.19 581	9.92 603	8	30
31	9.73 041	20	9.80 447	27	0.19 553	9.92 595	8	29
32	9.73 061	20	9.80 474	28	0.19 526	9.92 587	8	28
33	9.73 081	20	9.80 502	28	0.19 498	9.92 579	8	27
34	9.73 101	20	9.80 530	28	0.19 470	9.92 571	8	26
35	9.73 121	19	9.80 558	28	0.19 442	9.92 563	8	25
36	9.73 140	20	9.80 586	28	0.19 414	9.92 555	9	24
37	9.73 160	20	9.80 614	28	0.19 386	9.92 546	8	23
38	9.73 180	20	9.80 642	27	0.19 358	9.92 538	8	22
39	9.73 200	19	9.80 669	28	0.19 331	9.92 530	8	21
40	9.73 219	20	9.80 697	28	0.19 303	9.92 522	8	20
41	9.73 239	20	9.80 725	28	0.19 275	9.92 514	8	19
42	9.73 259	19	9.80 753	28	0.19 247	9.92 506	8	18
43	9.73 278	20	9.80 781	27	0.19 219	9.92 498	8	17
44	9.73 298	20	9.80 808	28	0.19 192	9.92 490	8	16
45	9.73 318	19	9.80 836	28	0.19 164	9.92 482	9	15
46	9.73 337	20	9.80 864	28	0.19 136	9.92 473	8	14
47	9.73 357	20	9.80 892	27	0.19 108	9.92 465	8	13
48	9.73 377	19	9.80 919	28	0.19 081	9.92 457	8	12
49	9.73 396	20	9.80 947	28	0.19 053	9.92 449	8	11
50	9.73 416	19	9.80 975	28	0.19 025	9.92 441	8	10
51	9.73 435	20	9.81 003	27	0.18 997	9.92 433	8	9
52	9.73 455	19	9.81 030	28	0.18 970	9.92 425	9	8
53	9.73 474	20	9.81 058	28	0.18 942	9.92 416	8	7
54	9.73 494	19	9.81 086	27	0.18 914	9.92 408	8	6
55	9.73 513	20	9.81 113	28	0.18 887	9.92 400	8	5
56	9.73 533	19	9.81 141	28	0.18 859	9.92 392	8	4
57	9.73 552	20	9.81 169	27	0.18 831	9.92 384	8	3
58	9.73 572	19	9.81 196	28	0.18 804	9.92 376	9	2
59	9.73 591	20	9.81 224	28	0.18 776	9.92 367	8	1
60	9.73 611		9.81 252		0.18 748	9.92 359		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	29	28	27
6	2.9	2.8	2.7
7	3.5	3.3	3.2
8	3.9	3.7	3.6
9	4.4	4.2	4.1
10	4.8	4.7	4.5
20	9.7	9.3	9.0
30	14.5	14.0	13.5
40	19.3	18.7	18.0
50	24.2	23.3	22.5

″	21	20	19
6	2.1	2.0	1.9
7	2.4	2.3	2.2
8	2.8	2.7	2.5
9	3.2	3.0	2.9
10	3.5	3.3	3.2
20	7.0	6.7	6.3
30	10.5	10.0	9.5
40	14.0	13.3	12.7
50	17.5	16.7	15.8

″	9	8	7
6	0.9	0.8	0.7
7	1.1	0.9	0.8
8	1.2	1.1	0.9
9	1.4	1.2	1.1
10	1.5	1.3	1.2
20	3.0	2.7	2.3
30	4.5	4.0	3.5
40	6.0	5.3	4.7
50	7.5	6.7	5.8

Prop. Pts.

*147° 237° *327° **57°**

33° *123° 213° *303°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.73 611	19	9.81 252	27	0.18 748	9.92 359	8	60
1	9.73 630	20	9.81 279	28	0.18 721	9.92 351	8	59
2	9.73 650	19	9.81 307	28	0.18 693	9.92 343	8	58
3	9.73 669	20	9.81 335	27	0.18 665	9.92 335	9	57
4	9.73 689	19	9.81 362	28	0.18 638	9.92 326	8	56
5	9.73 708	19	9.81 390	28	0.18 610	9.92 318	8	55
6	9.73 727	20	9.81 418	27	0.18 582	9.92 310	8	54
7	9.73 747	19	9.81 445	28	0.18 555	9.92 302	9	53
8	9.73 766	19	9.81 473	27	0.18 527	9.92 293	8	52
9	9.73 785	20	9.81 500	28	0.18 500	9.92 285	8	51
10	9.73 805	19	9.81 528	28	0.18 472	9.92 277	8	50
11	9.73 824	19	9.81 556	27	0.18 444	9.92 269	9	49
12	9.73 843	20	9.81 583	28	0.18 417	9.92 260	8	48
13	9.73 863	19	9.81 611	27	0.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	28	0.18 362	9.92 244	9	46
15	9.73 901	20	9.81 666	27	0.18 334	9.92 235	8	45
16	9.73 921	19	9.81 693	28	0.18 307	9.92 227	8	44
17	9.73 940	19	9.81 721	27	0.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	28	0.18 252	9.92 211	9	42
19	9.73 978	19	9.81 776	27	0.18 224	9.92 202	8	41
20	9.73 997	20	9.81 803	28	0.18 197	9.92 194	8	40
21	9.74 017	19	9.81 831	27	0.18 169	9.92 186	9	39
22	9.74 036	19	9.81 858	28	0.18 142	9.92 177	8	38
23	9.74 055	19	9.81 886	27	0.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	28	0.18 087	9.92 161	9	36
25	9.74 093	20	9.81 941	27	0.18 059	9.92 152	8	35
26	9.74 113	19	9.81 968	28	0.18 032	9.92 144	8	34
27	9.74 132	19	9.81 996	27	0.18 004	9.92 136	9	33
28	9.74 151	19	9.82 023	28	0.17 977	9.92 127	8	32
29	9.74 170	19	9.82 051	27	0.17 949	9.92 119	8	31
30	9.74 189	19	9.82 078	28	0.17 922	9.92 111	9	30
31	9.74 208	19	9.82 106	27	0.17 894	9.92 102	8	29
32	9.74 227	19	9.82 133	28	0.17 867	9.92 094	8	28
33	9.74 246	19	9.82 161	27	0.17 839	9.92 086	9	27
34	9.74 265	19	9.82 188	27	0.17 812	9.92 077	8	26
35	9.74 284	19	9.82 215	28	0.17 785	9.92 069	9	25
36	9.74 303	19	9.82 243	27	0.17 757	9.92 060	8	24
37	9.74 322	19	9.82 270	28	0.17 730	9.92 052	8	23
38	9.74 341	19	9.82 298	27	0.17 702	9.92 044	9	22
39	9.74 360	19	9.82 325	27	0.17 675	9.92 035	8	21
40	9.74 379	19	9.82 352	28	0.17 648	9.92 027	9	20
41	9.74 398	19	9.82 380	27	0.17 620	9.92 018	8	19
42	9.74 417	19	9.82 407	28	0.17 593	9.92 010	8	18
43	9.74 436	19	9.82 435	27	0.17 565	9.92 002	9	17
44	9.74 455	19	9.82 462	27	0.17 538	9.91 993	8	16
45	9.74 474	19	9.82 489	28	0.17 511	9.91 985	9	15
46	9.74 493	19	9.82 517	27	0.17 483	9.91 976	8	14
47	9.74 512	19	9.82 544	27	0.17 456	9.91 968	9	13
48	9.74 531	18	9.82 571	28	0.17 429	9.91 959	8	12
49	9.74 549	19	9.82 599	27	0.17 401	9.91 951	9	11
50	9.74 568	19	9.82 626	27	0.17 374	9.91 942	8	10
51	9.74 587	19	9.82 653	28	0.17 347	9.91 934	9	9
52	9.74 606	19	9.82 681	27	0.17 319	9.91 925	8	8
53	9.74 625	19	9.82 708	27	0.17 292	9.91 917	9	7
54	9.74 644	18	9.82 735	27	0.17 265	9.91 908	8	6
55	9.74 662	19	9.82 762	28	0.17 238	9.91 900	9	5
56	9.74 681	19	9.82 790	27	0.17 210	9.91 891	8	4
57	9.74 700	19	9.82 817	27	0.17 183	9.91 883	9	3
58	9.74 719	18	9.82 844	27	0.17 156	9.91 874	8	2
59	9.74 737	19	9.82 871	28	0.17 129	9.91 866	9	1
60	9.74 756		9.82 899		0.17 101	9.91 857		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

56° *146° 236° *326°

Prop. Pts.

″	28	27	20
6	2.8	2.7	2.0
7	3.3	3.2	2.3
8	3.7	3.6	2.7
9	4.2	4.1	3.0
10	4.7	4.5	3.3
20	9.3	9.0	6.7
30	14.0	13.5	10.0
40	18.7	18.0	13.3
50	23.3	22.5	16.7

″	19	18
6	1.9	1.8
7	2.2	2.1
8	2.5	2.4
9	2.9	2.7
10	3.2	3.0
20	6.3	6.0
30	9.5	9.0
40	12.7	12.0
50	15.8	15.0

″	9	8
6	0.9	0.8
7	1.1	0.9
8	1.2	1.1
9	1.4	1.2
10	1.5	1.3
20	3.0	2.7
30	4.5	4.0
40	6.0	5.3
50	7.5	6.7

34°								*124° 214° *304°
′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.74 756	19	9.82 899	27	0.17 101	9.91 857	8	60
1	9.74 775	19	9.82 926	27	0.17 074	9.91 849	9	59
2	9.74 794	18	9.82 953	27	0.17 047	9.91 840	8	58
3	9.74 812	19	9.82 980	28	0.17 020	9.91 832	9	57
4	9.74 831	19	9.83 008	27	0.16 992	9.91 823	8	56
5	9.74 850	18	9.83 035	27	0.16 965	9.91 815	9	55
6	9.74 868	19	9.83 062	27	0.16 938	9.91 806	8	54
7	9.74 887	19	9.83 089	28	0.16 911	9.91 798	9	53
8	9.74 906	18	9.83 117	27	0.16 883	9.91 789	8	52
9	9.74 924	19	9.83 144	27	0.16 856	9.91 781	9	51
10	9.74 943	18	9.83 171	27	0.16 829	9.91 772	9	50
11	9.74 961	19	9.83 198	27	0.16 802	9.91 763	8	49
12	9.74 980	19	9.83 225	27	0.16 775	9.91 755	9	48
13	9.74 999	18	9.83 252	28	0.16 748	9.91 746	8	47
14	9.75 017	19	9.83 280	27	0.16 720	9.91 738	9	46
15	9.75 036	18	9.83 307	27	0.16 693	9.91 729	9	45
16	9.75 054	19	9.83 334	27	0.16 666	9.91 720	8	44
17	9.75 073	18	9.83 361	27	0.16 639	9.91 712	9	43
18	9.75 091	19	9.83 388	27	0.16 612	9.91 703	8	42
19	9.75 110	18	9.83 415	27	0.16 585	9.91 695	9	41
20	9.75 128	19	9.83 442	28	0.16 558	9.91 686	9	40
21	9.75 147	18	9.83 470	27	0.16 530	9.91 677	8	39
22	9.75 165	19	9.83 497	27	0.16 503	9.91 669	9	38
23	9.75 184	18	9.83 524	27	0.16 476	9.91 660	9	37
24	9.75 202	19	9.83 551	27	0.16 449	9.91 651	8	36
25	9.75 221	18	9.83 578	27	0.16 422	9.91 643	9	35
26	9.75 239	19	9.83 605	27	0.16 395	9.91 634	9	34
27	9.75 258	18	9.83 632	27	0.16 368	9.91 625	8	33
28	9.75 276	18	9.83 659	27	0.16 341	9.91 617	9	32
29	9.75 294	19	9.83 686	27	0.16 314	9.91 608	9	31
30	9.75 313	18	9.83 713	27	0.16 287	9.91 599	8	30
31	9.75 331	19	9.83 740	28	0.16 260	9.91 591	9	29
32	9.75 350	18	9.83 768	27	0.16 232	9.91 582	9	28
33	9.75 368	18	9.83 795	27	0.16 205	9.91 573	8	27
34	9.75 386	19	9.83 822	27	0.16 178	9.91 565	9	26
35	9.75 405	18	9.83 849	27	0.16 151	9.91 556	9	25
36	9.75 423	18	9.83 876	27	0.16 124	9.91 547	9	24
37	9.75 441	18	9.83 903	27	0.16 097	9.91 538	8	23
38	9.75 459	19	9.83 930	27	0.16 070	9.91 530	9	22
39	9.75 478	18	9.83 957	27	0.16 043	9.91 521	9	21
40	9.75 496	18	9.83 984	27	0.16 016	9.91 512	8	20
41	9.75 514	19	9.84 011	27	0.15 989	9.91 504	9	19
42	9.75 533	18	9.84 038	27	0.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	27	0.15 935	9.91 486	9	17
44	9.75 569	18	9.84 092	27	0.15 908	9.91 477	8	16
45	9.75 587	18	9.84 119	27	0.15 881	9.91 469	9	15
46	9.75 605	19	9.84 146	27	0.15 854	9.91 460	9	14
47	9.75 624	18	9.84 173	27	0.15 827	9.91 451	9	13
48	9.75 642	18	9.84 200	27	0.15 800	9.91 442	9	12
49	9.75 660	18	9.84 227	27	0.15 773	9.91 433	8	11
50	9.75 678	18	9.84 254	26	0.15 746	9.91 425	9	10
51	9.75 696	18	9.84 280	27	0.15 720	9.91 416	9	9
52	9.75 714	19	9.84 307	27	0.15 693	9.91 407	9	8
53	9.75 733	18	9.84 334	27	0.15 666	9.91 398	9	7
54	9.75 751	18	9.84 361	27	0.15 639	9.91 389	8	6
55	9.75 769	18	9.84 388	27	0.15 612	9.91 381	9	5
56	9.75 787	18	9.84 415	27	0.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	0.15 558	9.91 363	9	3
58	9.75 823	18	9.84 469	27	0.15 531	9.91 354	9	2
59	9.75 841	18	9.84 496	27	0.15 504	9.91 345	9	1
60	9.75 859		9.84 523		0.15 477	9.91 336		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′
*145° 235° *325°					55°			

Prop. Pts.

″	28	27	26
6	2.8	2.7	2.6
7	3.3	3.2	3.0
8	3.7	3.6	3.5
9	4.2	4.0	3.9
10	4.7	4.5	4.3
20	9.3	9.0	8.7
30	14.0	13.5	13.0
40	18.7	18.0	17.3
50	23.3	22.5	21.7

″	19	18
6	1.9	1.8
7	2.2	2.1
8	2.5	2.4
9	2.8	2.7
10	3.2	3.0
20	6.3	6.0
30	9.5	9.0
40	12.7	12.0
50	15.8	15.0

″	9	8
6	0.9	0.8
7	1.0	0.9
8	1.2	1.1
9	1.4	1.2
10	1.5	1.3
20	3.0	2.7
30	4.5	4.0
40	6.0	5.3
50	7.5	6.7

Prop. Pts.

35° *125° 215° *305°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.75 859	18	9.84 523	27	0.15 477	9.91 336	8	60
1	9.75 877	18	9.84 550	26	0.15 450	9.91 328	9	59
2	9.75 895	18	9.84 576	27	0.15 424	9.91 319	9	58
3	9.75 913	18	9.84 603	27	0.15 397	9.91 310	9	57
4	9.75 931	18	9.84 630	27	0.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	9	55
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	9	54
7	9.75 985	18	9.84 711	27	0.15 289	9.91 274	8	53
8	9.76 003	18	9.84 738	26	0.15 262	9.91 266	9	52
9	9.76 021	18	9.84 764	27	0.15 236	9.91 257	9	51
10	9.76 039	18	9.84 791	27	0.15 209	9.91 248	9	50
11	9.76 057	18	9.84 818	27	0.15 182	9.91 239	9	49
12	9.76 075	18	9.84 845	27	0.15 155	9.91 230	9	48
13	9.76 093	18	9.84 872	27	0.15 128	9.91 221	9	47
14	9.76 111	18	9.84 899	26	0.15 101	9.91 212	9	46
15	9.76 129	17	9.84 925	27	0.15 075	9.91 203	9	45
16	9.76 146	18	9.84 952	27	0.15 048	9.91 194	9	44
17	9.76 164	18	9.84 979	27	0.15 021	9.91 185	9	43
18	9.76 182	18	9.85 006	27	0.14 994	9.91 176	9	42
19	9.76 200	18	9.85 033	26	0.14 967	9.91 167	9	41
20	9.76 218	18	9.85 059	27	0.14 941	9.91 158	9	40
21	9.76 236	17	9.85 086	27	0.14 914	9.91 149	8	39
22	9.76 253	18	9.85 113	27	0.14 887	9.91 141	9	38
23	9.76 271	18	9.85 140	26	0.14 860	9.91 132	9	37
24	9.76 289	18	9.85 166	27	0.14 834	9.91 123	9	36
25	9.76 307	17	9.85 193	27	0.14 807	9.91 114	9	35
26	9.76 324	18	9.85 220	27	0.14 780	9.91 105	9	34
27	9.76 342	18	9.85 247	26	0.14 753	9.91 096	9	33
28	9.76 360	18	9.85 273	27	0.14 727	9.91 087	9	32
29	9.76 378	17	9.85 300	27	0.14 700	9.91 078	9	31
30	9.76 395	18	9.85 327	27	0.14 673	9.91 069	9	30
31	9.76 413	18	9.85 354	26	0.14 646	9.91 060	9	29
32	9.76 431	17	9.85 380	27	0.14 620	9.91 051	9	28
33	9.76 448	18	9.85 407	27	0.14 593	9.91 042	9	27
34	9.76 466	18	9.85 434	26	0.14 566	9.91 033	10	26
35	9.76 484	17	9.85 460	27	0.14 540	9.91 023	9	25
36	9.76 501	18	9.85 487	27	0.14 513	9.91 014	9	24
37	9.76 519	18	9.85 514	26	0.14 486	9.91 005	9	23
38	9.76 537	17	9.85 540	27	0.14 460	9.90 996	9	22
39	9.76 554	18	9.85 567	27	0.14 433	9.90 987	9	21
40	9.76 572	18	9.85 594	26	0.14 406	9.90 978	9	20
41	9.76 590	17	9.85 620	27	0.14 380	9.90 969	9	19
42	9.76 607	18	9.85 647	27	0.14 353	9.90 960	9	18
43	9.76 625	17	9.85 674	26	0.14 326	9.90 951	9	17
44	9.76 642	18	9.85 700	27	0.14 300	9.90 942	9	16
45	9.76 660	17	9.85 727	27	0.14 273	9.90 933	9	15
46	9.76 677	18	9.85 754	26	0.14 246	9.90 924	9	14
47	9.76 695	17	9.85 780	27	0.14 220	9.90 915	9	13
48	9.76 712	18	9.85 807	27	0.14 193	9.90 906	10	12
49	9.76 730	17	9.85 834	26	0.14 166	9.90 896	9	11
50	9.76 747	18	9.85 860	27	0.14 140	9.90 887	9	10
51	9.76 765	17	9.85 887	26	0.14 113	9.90 878	9	9
52	9.76 782	18	9.85 913	27	0.14 087	9.90 869	9	8
53	9.76 800	17	9.85 940	27	0.14 060	9.90 860	9	7
54	9.76 817	18	9.85 967	26	0.14 033	9.90 851	9	6
55	9.76 835	17	9.85 993	27	0.14 007	9.90 842	10	5
56	9.76 852	18	9.86 020	26	0.13 980	9.90 832	9	4
57	9.76 870	17	9.86 046	27	0.13 954	9.90 823	9	3
58	9.76 887	17	9.86 073	27	0.13 927	9.90 814	9	2
59	9.76 904	18	9.86 100	26	0.13 900	9.90 805	9	1
60	9.76 922		9.86 126		0.13 874	9.90 796		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	27	26	18
6	2.7	2.6	1.8
7	3.2	3.0	2.1
8	3.6	3.5	2.4
9	4.1	3.9	2.7
10	4.5	4.3	3.0
20	9.0	8.7	6.0
30	13.5	13.0	9.0
40	18.0	17.3	12.0
50	22.5	21.7	15.0

″	17	10
6	1.7	1.0
7	2.0	1.2
8	2.3	1.3
9	2.6	1.5
10	2.8	1.7
20	5.7	3.3
30	8.5	5.0
40	11.3	6.7
50	14.2	8.3

″	9	8
6	0.9	0.8
7	1.1	0.9
8	1.2	1.1
9	1.4	1.2
10	1.5	1.3
20	3.0	2.7
30	4.5	4.0
40	6.0	5.3
50	7.5	6.7

Prop. Pts.

*144° 234° *324° **54°**

36° *126° 216° *306°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.76 922	17	9.86 126	27	0.13 874	9.90 796	9	60
1	9.76 939	18	9.86 153	26	0.13 847	9.90 787	10	59
2	9.76 957	17	9.86 179	27	0.13 821	9.90 777	9	58
3	9.76 974	17	9.86 206	26	0.13 794	9.90 768	9	57
4	9.76 991	18	9.86 232	27	0.13 768	9.90 759	9	56
5	9.77 009	17	9.86 259	26	0.13 741	9.90 750	9	55
6	9.77 026	17	9.86 285	27	0.13 715	9.90 741	10	54
7	9.77 043	18	9.86 312	26	0.13 688	9.90 731	9	53
8	9.77 061	17	9.86 338	27	0.13 662	9.90 722	9	52
9	9.77 078	17	9.86 365	27	0.13 635	9.90 713	9	51
10	9.77 095	17	9.86 392	26	0.13 608	9.90 704	10	50
11	9.77 112	18	9.86 418	27	0.13 582	9.90 694	9	49
12	9.77 130	17	9.86 445	26	0.13 555	9.90 685	9	48
13	9.77 147	17	9.86 471	27	0.13 529	9.90 676	9	47
14	9.77 164	17	9.86 498	26	0.13 502	9.90 667	10	46
15	9.77 181	18	9.86 524	27	0.13 476	9.90 657	9	45
16	9.77 199	17	9.86 551	26	0.13 449	9.90 648	9	44
17	9.77 216	17	9.86 577	26	0.13 423	9.90 639	9	43
18	9.77 233	17	9.86 603	27	0.13 397	9.90 630	10	42
19	9.77 250	18	9.86 630	26	0.13 370	9.90 620	9	41
20	9.77 268	17	9.86 656	27	0.13 344	9.90 611	9	40
21	9.77 285	17	9.86 683	26	0.13 317	9.90 602	10	39
22	9.77 302	17	9.86 709	27	0.13 291	9.90 592	9	38
23	9.77 319	17	9.86 736	26	0.13 264	9.90 583	9	37
24	9.77 336	17	9.86 762	27	0.13 238	9.90 574	9	36
25	9.77 353	17	9.86 789	26	0.13 211	9.90 565	10	35
26	9.77 370	17	9.86 815	27	0.13 185	9.90 555	9	34
27	9.77 387	18	9.86 842	26	0.13 158	9.90 546	9	33
28	9.77 405	17	9.86 868	26	0.13 132	9.90 537	10	32
29	9.77 422	17	9.86 894	27	0.13 106	9.90 527	9	31
30	9.77 439	17	9.86 921	26	0.13 079	9.90 518	9	30
31	9.77 456	17	9.86 947	27	0.13 053	9.90 509	10	29
32	9.77 473	17	9.86 974	26	0.13 026	9.90 499	9	28
33	9.77 490	17	9.87 000	27	0.13 000	9.90 490	10	27
34	9.77 507	17	9.87 027	26	0.12 973	9.90 480	9	26
35	9.77 524	17	9.87 053	26	0.12 947	9.90 471	9	25
36	9.77 541	17	9.87 079	27	0.12 921	9.90 462	10	24
37	9.77 558	17	9.87 106	26	0.12 894	9.90 452	9	23
38	9.77 575	17	9.87 132	26	0.12 868	9.90 443	9	22
39	9.77 592	17	9.87 158	27	0.12 842	9.90 434	10	21
40	9.77 609	17	9.87 185	26	0.12 815	9.90 424	9	20
41	9.77 626	17	9.87 211	27	0.12 789	9.90 415	10	19
42	9.77 643	17	9.87 238	26	0.12 762	9.90 405	9	18
43	9.77 660	17	9.87 264	26	0.12 736	9.90 396	10	17
44	9.77 677	17	9.87 290	27	0.12 710	9.90 386	9	16
45	9.77 694	17	9.87 317	26	0.12 683	9.90 377	9	15
46	9.77 711	17	9.87 343	26	0.12 657	9.90 368	10	14
47	9.77 728	16	9.87 369	27	0.12 631	9.90 358	9	13
48	9.77 744	17	9.87 396	26	0.12 604	9.90 349	10	12
49	9.77 761	17	9.87 422	26	0.12 578	9.90 339	9	11
50	9.77 778	17	9.87 448	27	0.12 552	9.90 330	10	10
51	9.77 795	17	9.87 475	26	0.12 525	9.90 320	9	9
52	9.77 812	17	9.87 501	26	0.12 499	9.90 311	10	8
53	9.77 829	17	9.87 527	27	0.12 473	9.90 301	9	7
54	9.77 846	16	9.87 554	26	0.12 446	9.90 292	10	6
55	9.77 862	17	9.87 580	26	0.12 420	9.90 282	9	5
56	9.77 879	17	9.87 606	27	0.12 394	9.90 273	10	4
57	9.77 896	17	9.87 633	26	0.12 367	9.90 263	9	3
58	9.77 913	17	9.87 659	26	0.12 341	9.90 254	10	2
59	9.77 930	16	9.87 685	26	0.12 315	9.90 244	9	1
60	9.77 946		9.87 711		0.12 289	9.90 235		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	27	26	18
6	2.7	2.6	1.8
7	3.2	3.0	2.1
8	3.6	3.5	2.4
9	4.0	3.9	2.7
10	4.5	4.3	3.0
20	9.0	8.7	6.0
30	13.5	13.0	9.0
40	18.0	17.3	12.0
50	22.5	21.7	15.0

″	17	16
6	1.7	1.6
7	2.0	1.9
8	2.3	2.1
9	2.6	2.4
10	2.8	2.7
20	5.7	5.3
30	8.5	8.0
40	11.3	10.7
50	14.2	13.3

″	10	9
6	1.0	0.9
7	1.2	1.0
8	1.3	1.2
9	1.5	1.4
10	1.7	1.5
20	3.3	3.0
30	5.0	4.5
40	6.7	6.0
50	8.3	7.5

Prop. Pts.

*143° 233° *323° **53°**

37° *127° 217° *307°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.77 946	17	9.87 711	27	0.12 289	9.90 235	10	60
1	9.77 963	17	9.87 738	26	0.12 262	9.90 225	9	59
2	9.77 980	17	9.87 764	26	0.12 236	9.90 216	10	58
3	9.77 997	16	9.87 790	27	0.12 210	9.90 206	9	57
4	9.78 013	17	9.87 817	26	0.12 183	9.90 197	10	56
5	9.78 030	17	9.87 843	26	0.12 157	9.90 187	9	55
6	9.78 047	16	9.87 869	26	0.12 131	9.90 178	10	54
7	9.78 063	17	9.87 895	27	0.12 105	9.90 168	9	53
8	9.78 080	17	9.87 922	26	0.12 078	9.90 159	10	52
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	10	51
10	9.78 113	17	9.87 974	26	0.12 026	9.90 139	9	50
11	9.78 130	17	9.88 000	27	0.12 000	9.90 130	10	49
12	9.78 147	16	9.88 027	26	0.11 973	9.90 120	9	48
13	9.78 163	17	9.88 053	26	0.11 947	9.90 111	10	47
14	9.78 180	17	9.88 079	26	0.11 921	9.90 101	10	46
15	9.78 197	16	9.88 105	26	0.11 895	9.90 091	9	45
16	9.78 213	17	9.88 131	27	0.11 869	9.90 082	10	44
17	9.78 230	16	9.88 158	26	0.11 842	9.90 072	9	43
18	9.78 246	17	9.88 184	26	0.11 816	9.90 063	10	42
19	9.78 263	17	9.88 210	26	0.11 790	9.90 053	10	41
20	9.78 280	16	9.88 236	26	0.11 764	9.90 043	9	40
21	9.78 296	17	9.88 262	27	0.11 738	9.90 034	10	39
22	9.78 313	16	9.88 289	26	0.11 711	9.90 024	10	38
23	9.78 329	17	9.88 315	26	0.11 685	9.90 014	9	37
24	9.78 346	16	9.88 341	26	0.11 659	9.90 005	10	36
25	9.78 362	17	9.88 367	26	0.11 633	9.89 995	10	35
26	9.78 379	16	9.88 393	27	0.11 607	9.89 985	9	34
27	9.78 395	17	9.88 420	26	0.11 580	9.89 976	10	33
28	9.78 412	16	9.88 446	26	0.11 554	9.89 966	10	32
29	9.78 428	17	9.88 472	26	0.11 528	9.89 956	9	31
30	9.78 445	16	9.88 498	26	0.11 502	9.89 947	10	30
31	9.78 461	17	9.88 524	26	0.11 476	9.89 937	10	29
32	9.78 478	16	9.88 550	27	0.11 450	9.89 927	9	28
33	9.78 494	16	9.88 577	26	0.11 423	9.89 918	10	27
34	9.78 510	17	9.88 603	26	0.11 397	9.89 908	10	26
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	10	25
36	9.78 543	17	9.88 655	26	0.11 345	9.89 888	9	24
37	9.78 560	16	9.88 681	26	0.11 319	9.89 879	10	23
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22
39	9.78 592	17	9.88 733	26	0.11 267	9.89 859	10	21
40	9.78 609	16	9.88 759	27	0.11 241	9.89 849	9	20
41	9.78 625	17	9.88 786	26	0.11 214	9.89 840	10	19
42	9.78 642	16	9.88 812	26	0.11 188	9.89 830	10	18
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	10	17
44	9.78 674	17	9.88 864	26	0.11 136	9.89 810	9	16
45	9.78 691	16	9.88 890	26	0.11 110	9.89 801	10	15
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	10	14
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13
48	9.78 739	17	9.88 968	26	0.11 032	9.89 771	10	12
49	9.78 756	16	9.88 994	26	0.11 006	9.89 761	9	11
50	9.78 772	16	9.89 020	26	0.10 980	9.89 752	10	10
51	9.78 788	17	9.89 046	27	0.10 954	9.89 742	10	9
52	9.78 805	16	9.89 073	26	0.10 927	9.89 732	10	8
53	9.78 821	16	9.89 099	26	0.10 901	9.89 722	10	7
54	9.78 837	16	9.89 125	26	0.10 875	9.89 712	10	6
55	9.78 853	16	9.89 151	26	0.10 849	9.89 702	9	5
56	9.78 869	17	9.89 177	26	0.10 823	9.89 693	10	4
57	9.78 886	16	9.89 203	26	0.10 797	9.89 683	10	3
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1
60	9.78 934		9.89 281		0.10 719	9.89 653		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	27	26	17
6	2.7	2.6	1.7
7	3.2	3.0	2.0
8	3.6	3.5	2.3
9	4.0	3.9	2.6
10	4.5	4.3	2.8
20	9.0	8.7	5.7
30	13.5	13.0	8.5
40	18.0	17.3	11.3
50	22.5	21.7	14.2

″	16	10	9
6	1.6	1.0	0.9
7	1.9	1.2	1.0
8	2.1	1.3	1.2
9	2.4	1.5	1.4
10	2.7	1.7	1.5
20	5.3	3.3	3.0
30	8.0	5.0	4.5
40	10.7	6.7	6.0
50	13.3	8.3	7.5

*142° 232° *322° **52°**

38° *128° 218° *308°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.78 934	16	9.89 281	26	0.10 719	9.89 653	10	60
1	9.78 950	17	9.89 307	26	0.10 693	9.89 643	10	59
2	9.78 967	16	9.89 333	26	0.10 667	9.89 633	9	58
3	9.78 983	16	9.89 359	26	0.10 641	9.89 624	10	57
4	9.78 999	16	9.89 385	26	0.10 615	9.89 614	10	56
5	9.79 015	16	9.89 411	26	0.10 589	9.89 604	10	55
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53
8	9.79 063	16	9.89 489	26	0.10 511	9.89 574	10	52
9	9.79 079	16	9.89 515	26	0.10 485	9.89 564	10	51
10	9.79 095	16	9.89 541	26	0.10 459	9.89 554	10	50
11	9.79 111	17	9.89 567	26	0.10 433	9.89 544	10	49
12	9.79 128	16	9.89 593	26	0.10 407	9.89 534	10	48
13	9.79 144	16	9.89 619	26	0.10 381	9.89 524	10	47
14	9.79 160	16	9.89 645	26	0.10 355	9.89 514	10	46
15	9.79 176	16	9.89 671	26	0.10 329	9.89 504	9	45
16	9.79 192	16	9.89 697	26	0.10 303	9.89 495	10	44
17	9.79 208	16	9.89 723	26	0.10 277	9.89 485	10	43
18	9.79 224	16	9.89 749	26	0.10 251	9.89 475	10	42
19	9.79 240	16	9.89 775	26	0.10 225	9.89 465	10	41
20	9.79 256	16	9.89 801	26	0.10 199	9.89 455	10	40
21	9.79 272	16	9.89 827	26	0.10 173	9.89 445	10	39
22	9.79 288	16	9.89 853	26	0.10 147	9.89 435	10	38
23	9.79 304	15	9.89 879	26	0.10 121	9.89 425	10	37
24	9.79 319	16	9.89 905	26	0.10 095	9.89 415	10	36
25	9.79 335	16	9.89 931	26	0.10 069	9.89 405	10	35
26	9.79 351	16	9.89 957	26	0.10 043	9.89 395	10	34
27	9.79 367	16	9.89 983	26	0.10 017	9.89 385	10	33
28	9.79 383	16	9.90 009	26	0.09 991	9.89 375	11	32
29	9.79 399	16	9.90 035	26	0.09 965	9.89 364	10	31
30	9.79 415	16	9.90 061	25	0.09 939	9.89 354	10	30
31	9.79 431	16	9.90 086	26	0.09 914	9.89 344	10	29
32	9.79 447	16	9.90 112	26	0.09 888	9.89 334	10	28
33	9.79 463	15	9.90 138	26	0.09 862	9.89 324	10	27
34	9.79 478	16	9.90 164	26	0.09 836	9.89 314	10	26
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	25
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24
37	9.79 526	16	9.90 242	26	0.09 758	9.89 284	10	23
38	9.79 542	16	9.90 268	26	0.09 732	9.89 274	10	22
39	9.79 558	15	9.90 294	26	0.09 706	9.89 264	10	21
40	9.79 573	16	9.90 320	26	0.09 680	9.89 254	10	20
41	9.79 589	16	9.90 346	25	0.09 654	9.89 244	11	19
42	9.79 605	16	9.90 371	26	0.09 629	9.89 233	10	18
43	9.79 621	15	9.90 397	26	0.09 603	9.89 223	10	17
44	9.79 636	16	9.90 423	26	0.09 577	9.89 213	10	16
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	15
46	9.79 668	16	9.90 475	26	0.09 525	9.89 193	10	14
47	9.79 684	15	9.90 501	26	0.09 499	9.89 183	10	13
48	9.79 699	16	9.90 527	26	0.09 473	9.89 173	11	12
49	9.79 715	16	9.90 553	25	0.09 447	9.89 162	10	11
50	9.79 731	15	9.90 578	26	0.09 422	9.89 152	10	10
51	9.79 746	16	9.90 604	26	0.09 396	9.89 142	10	9
52	9.79 762	16	9.90 630	26	0.09 370	9.89 132	10	8
53	9.79 778	15	9.90 656	26	0.09 344	9.89 122	10	7
54	9.79 793	16	9.90 682	26	0.09 318	9.89 112	11	6
55	9.79 809	16	9.90 708	26	0.09 292	9.89 101	10	5
56	9.79 825	15	9.90 734	25	0.09 266	9.89 091	10	4
57	9.79 840	16	9.90 759	26	0.09 241	9.89 081	10	3
58	9.79 856	16	9.90 785	26	0.09 215	9.89 071	11	2
59	9.79 872	15	9.90 811	26	0.09 189	9.89 060	10	1
60	9.79 887		9.90 837		0.09 163	9.89 050		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	26	25
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

″	17	16
6	1.7	1.6
7	2.0	1.9
8	2.3	2.1
9	2.6	2.4
10	2.8	2.7
20	5.7	5.3
30	8.5	8.0
40	11.3	10.7
50	14.2	13.3

″	15	11
6	1.5	1.1
7	1.8	1.3
8	2.0	1.5
9	2.2	1.6
10	2.5	1.8
20	5.0	3.7
30	7.5	5.5
40	10.0	7.3
50	12.5	9.2

″	10	9
6	1.0	0.9
7	1.2	1.0
8	1.3	1.2
9	1.5	1.4
10	1.7	1.5
20	3.3	3.0
30	5.0	4.5
40	6.7	6.0
50	8.3	7.5

Prop. Pts.

*141° 231° *321° **51°**

39° *129° 219° *309°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.79 887		9.90 837		0.09 163	9.89 050		60
1	9.79 903	16	9.90 863	26	0.09 137	9.89 040	10	59
2	9.79 918	15	9.90 889	26	0.09 111	9.89 030	10	58
3	9.79 934	16	9.90 914	25	0.09 086	9.89 020	10	57
4	9.79 950	16	9.90 940	26	0.09 060	9.89 009	11	56
5	9.79 965	15	9.90 966	26	0.09 034	9.88 999	10	55
6	9.79 981	16	9.90 992	26	0.09 008	9.88 989	10	54
7	9.79 996	15	9.91 018	26	0.08 982	9.88 978	11	53
8	9.80 012	16	9.91 043	25	0.08 957	9.88 968	10	52
9	9.80 027	15	9.91 069	26	0.08 931	9.88 958	10	51
10	9.80 043	16	9.91 095	26	0.08 905	9.88 948	10	50
11	9.80 058	15	9.91 121	26	0.08 879	9.88 937	11	49
12	9.80 074	16	9.91 147	26	0.08 853	9.88 927	10	48
13	9.80 089	15	9.91 172	25	0.08 828	9.88 917	10	47
14	9.80 105	16	9.91 198	26	0.08 802	9.88 906	11	46
15	9.80 120	15	9.91 224	26	0.08 776	9.88 896	10	45
16	9.80 136	16	9.91 250	26	0.08 750	9.88 886	10	44
17	9.80 151	15	9.91 276	26	0.08 724	9.88 875	11	43
18	9.80 166	15	9.91 301	25	0.08 699	9.88 865	10	42
19	9.80 182	16	9.91 327	26	0.08 673	9.88 855	10	41
20	9.80 197	15	9.91 353	26	0.08 647	9.88 844	11	40
21	9.80 213	16	9.91 379	26	0.08 621	9.88 834	10	39
22	9.80 228	15	9.91 404	25	0.08 596	9.88 824	10	38
23	9.80 244	16	9.91 430	26	0.08 570	9.88 813	11	37
24	9.80 259	15	9.91 456	26	0.08 544	9.88 803	10	36
25	9.80 274	15	9.91 482	26	0.08 518	9.88 793	10	35
26	9.80 290	16	9.91 507	25	0.08 493	9.88 782	11	34
27	9.80 305	15	9.91 533	26	0.08 467	9.88 772	10	33
28	9.80 320	15	9.91 559	26	0.08 441	9.88 761	11	32
29	9.80 336	16	9.91 585	26	0.08 415	9.88 751	10	31
30	9.80 351	15	9.91 610	25	0.08 390	9.88 741	10	30
31	9.80 366	15	9.91 636	26	0.08 364	9.88 730	11	29
32	9.80 382	16	9.91 662	26	0.08 338	9.88 720	10	28
33	9.80 397	15	9.91 688	26	0.08 312	9.88 709	11	27
34	9.80 412	15	9.91 713	25	0.08 287	9.88 699	10	26
35	9.80 428	16	9.91 739	26	0.08 261	9.88 688	11	25
36	9.80 443	15	9.91 765	26	0.08 235	9.88 678	10	24
37	9.80 458	15	9.91 791	26	0.08 209	9.88 668	10	23
38	9.80 473	15	9.91 816	25	0.08 184	9.88 657	11	22
39	9.80 489	16	9.91 842	26	0.08 158	9.88 647	10	21
40	9.80 504	15	9.91 868	26	0.08 132	9.88 636	11	20
41	9.80 519	15	9.91 893	25	0.08 107	9.88 626	10	19
42	9.80 534	15	9.91 919	26	0.08 081	9.88 615	11	18
43	9.80 550	16	9.91 945	26	0.08 055	9.88 605	10	17
44	9.80 565	15	9.91 971	26	0.08 029	9.88 594	11	16
45	9.80 580	15	9.91 996	25	0.08 004	9.88 584	10	15
46	9.80 595	15	9.92 022	26	0.07 978	9.88 573	11	14
47	9.80 610	15	9.92 048	26	0.07 952	9.88 563	10	13
48	9.80 625	15	9.92 073	25	0.07 927	9.88 552	11	12
49	9.80 641	16	9.92 099	26	0.07 901	9.88 542	10	11
50	9.80 656	15	9.92 125	26	0.07 875	9.88 531	11	10
51	9.80 671	15	9.92 150	25	0.07 850	9.88 521	10	9
52	9.80 686	15	9.92 176	26	0.07 824	9.88 510	11	8
53	9.80 701	15	9.92 202	26	0.07 798	9.88 499	11	7
54	9.80 716	15	9.92 227	25	0.07 773	9.88 489	10	6
55	9.80 731	15	9.92 253	26	0.07 747	9.88 478	11	5
56	9.80 746	15	9.92 279	26	0.07 721	9.88 468	10	4
57	9.80 762	16	9.92 304	25	0.07 696	9.88 457	11	3
58	9.80 777	15	9.92 330	26	0.07 670	9.88 447	10	2
59	9.80 792	15	9.92 356	26	0.07 644	9.88 436	11	1
60	9.80 807	15	9.92 381	25	0.07 619	9.88 425	11	0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	26	25
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

″	16	15
6	1.6	1.5
7	1.9	1.8
8	2.1	2.0
9	2.4	2.2
10	2.7	2.5
20	5.3	5.0
30	8.0	7.5
40	10.7	10.0
50	13.3	12.5

″	11	10
6	1.1	1.0
7	1.3	1.2
8	1.5	1.3
9	1.6	1.5
10	1.8	1.7
20	3.7	3.3
30	5.5	5.0
40	7.3	6.7
50	9.2	8.3

Prop. Pts.

*140° 230° *320° **50°**

40° *130° 220° *310°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.80 807	15	9.92 381	26	0.07 619	9.88 425	10	60
1	9.80 822	15	9.92 407	26	0.07 593	9.88 415	11	59
2	9.80 837	15	9.92 433	25	0.07 567	9.88 404	10	58
3	9.80 852	15	9.92 458	26	0.07 542	9.88 394	11	57
4	9.80 867	15	9.92 484	26	0.07 516	9.88 383	11	56
5	9.80 882	15	9.92 510	25	0.07 490	9.88 372	10	55
6	9.80 897	15	9.92 535	26	0.07 465	9.88 362	11	54
7	9.80 912	15	9.92 561	26	0.07 439	9.88 351	11	53
8	9.80 927	15	9.92 587	25	0.07 413	9.88 340	10	52
9	9.80 942	15	9.92 612	26	0.07 388	9.88 330	11	51
10	9.80 957	15	9.92 638	25	0.07 362	9.88 319	11	50
11	9.80 972	15	9.92 663	26	0.07 337	9.88 308	10	49
12	9.80 987	15	9.92 689	26	0.07 311	9.88 298	11	48
13	9.81 002	15	9.92 715	25	0.07 285	9.88 287	11	47
14	9.81 017	15	9.92 740	26	0.07 260	9.88 276	10	46
15	9.81 032	15	9.92 766	26	0.07 234	9.88 266	11	45
16	9.81 047	14	9.92 792	25	0.07 208	9.88 255	11	44
17	9.81 061	15	9.92 817	26	0.07 183	9.88 244	10	43
18	9.81 076	15	9.92 843	25	0.07 157	9.88 234	11	42
19	9.81 091	15	9.92 868	26	0.07 132	9.88 223	11	41
20	9.81 106	15	9.92 894	26	0.07 106	9.88 212	11	40
21	9.81 121	15	9.92 920	25	0.07 080	9.88 201	10	39
22	9.81 136	15	9.92 945	26	0.07 055	9.88 191	11	38
23	9.81 151	15	9.92 971	25	0.07 029	9.88 180	11	37
24	9.81 166	14	9.92 996	26	0.07 004	9.88 169	11	36
25	9.81 180	15	9.93 022	26	0.06 978	9.88 158	10	35
26	9.81 195	15	9.93 048	25	0.06 952	9.88 148	11	34
27	9.81 210	15	9.93 073	26	0.06 927	9.88 137	11	33
28	9.81 225	15	9.93 099	25	0.06 901	9.88 126	11	32
29	9.81 240	14	9.93 124	26	0.06 876	9.88 115	10	31
30	9.81 254	15	9.93 150	25	0.06 850	9.88 105	11	30
31	9.81 269	15	9.93 175	26	0.06 825	9.88 094	11	29
32	9.81 284	15	9.93 201	26	0.06 799	9.88 083	11	28
33	9.81 299	15	9.93 227	25	0.06 773	9.88 072	11	27
34	9.81 314	14	9.93 252	26	0.06 748	9.88 061	10	26
35	9.81 328	15	9.93 278	25	0.06 722	9.88 051	11	25
36	9.81 343	15	9.93 303	26	0.06 697	9.88 040	11	24
37	9.81 358	14	9.93 329	25	0.06 671	9.88 029	11	23
38	9.81 372	15	9.93 354	26	0.06 646	9.88 018	11	22
39	9.81 387	15	9.93 380	26	0.06 620	9.88 007	11	21
40	9.81 402	15	9.93 406	25	0.06 594	9.87 996	11	20
41	9.81 417	14	9.93 431	26	0.06 569	9.87 985	10	19
42	9.81 431	15	9.93 457	25	0.06 543	9.87 975	11	18
43	9.81 446	15	9.93 482	26	0.06 518	9.87 964	11	17
44	9.81 461	14	9.93 508	25	0.06 492	9.87 953	11	16
45	9.81 475	15	9.93 533	26	0.06 467	9.87 942	11	15
46	9.81 490	15	9.93 559	25	0.06 441	9.87 931	11	14
47	9.81 505	14	9.93 584	26	0.06 416	9.87 920	11	13
48	9.81 519	15	9.93 610	26	0.06 390	9.87 909	11	12
49	9.81 534	15	9.93 636	25	0.06 364	9.87 898	11	11
50	9.81 549	14	9.93 661	26	0.06 339	9.87 887	10	10
51	9.81 563	15	9.93 687	25	0.06 313	9.87 877	11	9
52	9.81 578	14	9.93 712	26	0.06 288	9.87 866	11	8
53	9.81 592	15	9.93 738	25	0.06 262	9.87 855	11	7
54	9.81 607	15	9.93 763	26	0.06 237	9.87 844	11	6
55	9.81 622	14	9.93 789	25	0.06 211	9.87 833	11	5
56	9.81 636	15	9.93 814	26	0.06 186	9.87 822	11	4
57	9.81 651	14	9.93 840	25	0.06 160	9.87 811	11	3
58	9.81 665	15	9.93 865	26	0.06 135	9.87 800	11	2
59	9.81 680	14	9.93 891	25	0.06 109	9.87 789	11	1
60	9.81 694		9.93 916		0.06 084	9.87 778		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	26	25
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

″	15	14
6	1.5	1.4
7	1.8	1.6
8	2.0	1.9
9	2.3	2.1
10	2.5	2.3
20	5.0	4.7
30	7.5	7.0
40	10.0	9.3
50	12.5	11.7

″	11	10
6	1.1	1.0
7	1.3	1.2
8	1.5	1.3
9	1.7	1.5
10	1.8	1.7
20	3.7	3.3
30	5.5	5.0
40	7.3	6.7
50	9.2	8.3

Prop. Pts.

*139° 229° *319° **49°**

41° *131° 221° *311°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	′
0	9.81 694		9.93 916		0.06 084	9.87 778		60
1	9.81 709	15	9.93 942	26	0.06 058	9.87 767	11	59
2	9.81 723	14	9.93 967	25	0.06 033	9.87 756	11	58
3	9.81 738	15	9.93 993	26	0.06 007	9.87 745	11	57
4	9.81 752	14	9.94 018	25	0.05 982	9.87 734	11	56
5	9.81 767	15	9.94 044	26	0.05 956	9.87 723	11	55
6	9.81 781	14	9.94 069	25	0.05 931	9.87 712	11	54
7	9.81 796	15	9.94 095	26	0.05 905	9.87 701	11	53
8	9.81 810	14	9.94 120	25	0.05 880	9.87 690	11	52
9	9.81 825	15	9.94 146	26	0.05 854	9.87 679	11	51
10	9.81 839	14	9.94 171	25	0.05 829	9.87 668	11	50
11	9.81 854	15	9.94 197	26	0.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	25	0.05 778	9.87 646	11	48
13	9.81 882	14	9.94 248	26	0.05 752	9.87 635	11	47
14	9.81 897	15	9.94 273	25	0.05 727	9.87 624	11	46
15	9.81 911	14	9.94 299	26	0.05 701	9.87 613	11	45
16	9.81 926	15	9.94 324	25	0.05 676	9.87 601	12	44
17	9.81 940	14	9.94 350	26	0.05 650	9.87 590	11	43
18	9.81 955	15	9.94 375	25	0.05 625	9.87 579	11	42
19	9.81 969	14	9.94 401	26	0.05 599	9.87 568	11	41
20	9.81 983	14	9.94 426	25	0.05 574	9.87 557	11	40
21	9.81 998	15	9.94 452	26	0.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	25	0.05 523	9.87 535	11	38
23	9.82 026	14	9.94 503	26	0.05 497	9.87 524	11	37
24	9.82 041	15	9.94 528	25	0.05 472	9.87 513	11	36
25	9.82 055	14	9.94 554	26	0.05 446	9.87 501	12	35
26	9.82 069	14	9.94 579	25	0.05 421	9.87 490	11	34
27	9.82 084	15	9.94 604	25	0.05 396	9.87 479	11	33
28	9.82 098	14	9.94 630	26	0.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	25	0.05 345	9.87 457	11	31
30	9.82 126	14	9.94 681	26	0.05 319	9.87 446	11	30
31	9.82 141	15	9.94 706	25	0.05 294	9.87 434	12	29
32	9.82 155	14	9.94 732	26	0.05 268	9.87 423	11	28
33	9.82 169	14	9.94 757	25	0.05 243	9.87 412	11	27
34	9.82 184	15	9.94 783	26	0.05 217	9.87 401	11	26
35	9.82 198	14	9.94 808	25	0.05 192	9.87 390	11	25
36	9.82 212	14	9.94 834	26	0.05 166	9.87 378	12	24
37	9.82 226	14	9.94 859	25	0.05 141	9.87 367	11	23
38	9.82 240	14	9.94 884	25	0.05 116	9.87 356	11	22
39	9.82 255	15	9.94 910	26	0.05 090	9.87 345	11	21
40	9.82 269	14	9.94 935	25	0.05 065	9.87 334	11	20
41	9.82 283	14	9.94 961	26	0.05 039	9.87 322	12	19
42	9.82 297	14	9.94 986	25	0.05 014	9.87 311	11	18
43	9.82 311	14	9.95 012	26	0.04 988	9.87 300	11	17
44	9.82 326	15	9.95 037	25	0.04 963	9.87 288	12	16
45	9.82 340	14	9.95 062	25	0.04 938	9.87 277	11	15
46	9.82 354	14	9.95 088	26	0.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	25	0.04 887	9.87 255	11	13
48	9.82 382	14	9.95 139	26	0.04 861	9.87 243	12	12
49	9.82 396	14	9.95 164	25	0.04 836	9.87 232	11	11
50	9.82 410	14	9.95 190	26	0.04 810	9.87 221	11	10
51	9.82 424	14	9.95 215	25	0.04 785	9.87 209	12	9
52	9.82 439	15	9.95 240	25	0.04 760	9.87 198	11	8
53	9.82 453	14	9.95 266	26	0.04 734	9.87 187	11	7
54	9.82 467	14	9.95 291	25	0.04 709	9.87 175	12	6
55	9.82 481	14	9.95 317	26	0.04 683	9.87 164	11	5
56	9.82 495	14	9.95 342	25	0.04 658	9.87 153	11	4
57	9.82 509	14	9.95 368	26	0.04 632	9.87 141	12	3
58	9.82 523	14	9.95 393	25	0.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	25	0.04 582	9.87 119	11	1
60	9.82 551	14	9.95 444	26	0.04 556	9.87 107	12	0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	26	25
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

″	15	14
6	1.5	1.4
7	1.8	1.6
8	2.0	1.9
9	2.3	2.1
10	2.5	2.3
20	5.0	4.7
30	7.5	7.0
40	10.0	9.3
50	12.5	11.7

″	12	11
6	1.2	1.1
7	1.4	1.3
8	1.6	1.5
9	1.8	1.7
10	2.0	1.8
20	4.0	3.7
30	6.0	5.5
40	8.0	7.3
50	10.0	9.2

Prop. Pts.

*138° 228° *318° **48°**

42° *132° 222° *312°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	
0	9.82 551	14	9.95 444	25	0.04 556	9.87 107	11	60
1	9.82 565	14	9.95 469	26	0.04 531	9.87 096	11	59
2	9.82 579	14	9.95 495	25	0.04 505	9.87 085	12	58
3	9.82 593	14	9.95 520	25	0.04 480	9.87 073	11	57
4	9.82 607	14	9.95 545	26	0.04 455	9.87 062	12	56
5	9.82 621	14	9.95 571	25	0.04 429	9.87 050	11	55
6	9.82 635	14	9.95 596	26	0.04 404	9.87 039	11	54
7	9.82 649	14	9.95 622	25	0.04 378	9.87 028	12	53
8	9.82 663	14	9.95 647	25	0.04 353	9.87 016	11	52
9	9.82 677	14	9.95 672	26	0.04 328	9.87 005	12	51
10	9.82 691	14	9.95 698	25	0.04 302	9.86 993	11	50
11	9.82 705	14	9.95 723	25	0.04 277	9.86 982	12	49
12	9.82 719	14	9.95 748	26	0.04 252	9.86 970	11	48
13	9.82 733	14	9.95 774	25	0.04 226	9.86 959	12	47
14	9.82 747	14	9.95 799	26	0.04 201	9.86 947	11	46
15	9.82 761	14	9.95 825	25	0.04 175	9.86 936	12	45
16	9.82 775	13	9.95 850	25	0.04 150	9.86 924	11	44
17	9.82 788	14	9.95 875	26	0.04 125	9.86 913	11	43
18	9.82 802	14	9.95 901	25	0.04 099	9.86 902	12	42
19	9.82 816	14	9.95 926	26	0.04 074	9.86 890	11	41
20	9.82 830	14	9.95 952	25	0.04 048	9.86 879	12	40
21	9.82 844	14	9.95 977	25	0.04 023	9.86 867	12	39
22	9.82 858	14	9.96 002	26	0.03 998	9.86 855	11	38
23	9.82 872	13	9.96 028	25	0.03 972	9.86 844	12	37
24	9.82 885	14	9.96 053	25	0.03 947	9.86 832	11	36
25	9.82 899	14	9.96 078	26	0.03 922	9.86 821	12	35
26	9.82 913	14	9.96 104	25	0.03 896	9.86 809	11	34
27	9.82 927	14	9.96 129	26	0.03 871	9.86 798	12	33
28	9.82 941	14	9.96 155	25	0.03 845	9.86 786	11	32
29	9.82 955	13	9.96 180	25	0.03 820	9.86 775	12	31
30	9.82 968	14	9.96 205	26	0.03 795	9.86 763	11	30
31	9.82 982	14	9.96 231	25	0.03 769	9.86 752	12	29
32	9.82 996	14	9.96 256	25	0.03 744	9.86 740	12	28
33	9.83 010	13	9.96 281	26	0.03 719	9.86 728	11	27
34	9.83 023	14	9.96 307	25	0.03 693	9.86 717	12	26
35	9.83 037	14	9.96 332	25	0.03 668	9.86 705	11	25
36	9.83 051	14	9.96 357	26	0.03 643	9.86 694	12	24
37	9.83 065	13	9.96 383	25	0.03 617	9.86 682	12	23
38	9.83 078	14	9.96 408	25	0.03 592	9.86 670	11	22
39	9.83 092	14	9.96 433	26	0.03 567	9.86 659	12	21
40	9.83 106	14	9.96 459	25	0.03 541	9.86 647	12	20
41	9.83 120	13	9.96 484	26	0.03 516	9.86 635	11	19
42	9.83 133	14	9.96 510	25	0.03 490	9.86 624	12	18
43	9.83 147	14	9.96 535	25	0.03 465	9.86 612	12	17
44	9.83 161	13	9.96 560	26	0.03 440	9.86 600	11	16
45	9.83 174	14	9.96 586	25	0.03 414	9.86 589	12	15
46	9.83 188	14	9.96 611	25	0.03 389	9.86 577	12	14
47	9.83 202	13	9.96 636	26	0.03 364	9.86 565	11	13
48	9.83 215	14	9.96 662	25	0.03 338	9.86 554	12	12
49	9.83 229	13	9.96 687	25	0.03 313	9.86 542	12	11
50	9.83 242	14	9.96 712	26	0.03 288	9.86 530	12	10
51	9.83 256	14	9.96 738	25	0.03 262	9.86 518	11	9
52	9.83 270	13	9.96 763	25	0.03 237	9.86 507	12	8
53	9.83 283	14	9.96 788	26	0.03 212	9.86 495	12	7
54	9.83 297	13	9.96 814	25	0.03 186	9.86 483	11	6
55	9.83 310	14	9.96 839	25	0.03 161	9.86 472	12	5
56	9.83 324	14	9.96 864	26	0 03 136	9.86 460	12	4
57	9.83 338	13	9.96 890	25	0.03 110	9.86 448	12	3
58	9.83 351	14	9.96 915	25	0.03 085	9.86 436	11	2
59	9.83 365	13	9.96 940	26	0.03 060	9.86 425	12	1
60	9.83 378		9.96 966		0.03 034	9.86 413		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	26	25
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

″	14	13
6	1.4	1.3
7	1.6	1.5
8	1.9	1.7
9	2.1	2.0
10	2.3	2.2
20	4.7	4.3
30	7.0	6.5
40	9.3	8.7
50	11.7	10.8

″	12	11
6	1.2	1.1
7	1.4	1.3
8	1.6	1.5
9	1.8	1.6
10	2.0	1.8
20	4.0	3.7
30	6.0	5.5
40	8.0	7.3
50	10.0	9.2

Prop. Pts.

*137° 227° *317° **47°**

43° *133° 223° *313°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	′
0	9.83 378	14	9.96 966	25	0.03 034	9.86 413	12	60
1	9.83 392	13	9.96 991	25	0.03 009	9.86 401	12	59
2	9.83 405	14	9.97 016	26	0.02 984	9.86 389	12	58
3	9.83 419	13	9.97 042	25	0.02 958	9.86 377	11	57
4	9.83 432	14	9.97 067	25	0.02 933	9.86 366	12	56
5	9.83 446	13	9.97 092	26	0.02 908	9.86 354	12	55
6	9.83 459	14	9.97 118	25	0.02 882	9.86 342	12	54
7	9.83 473	13	9.97 143	25	0.02 857	9.86 330	12	53
8	9.83 486	14	9.97 168	25	0.02 832	9.86 318	12	52
9	9.83 500	13	9.97 193	26	0.02 807	9.86 306	11	51
10	9.83 513	14	9.97 219	25	0.02 781	9.86 295	12	50
11	9.83 527	13	9.97 244	25	0.02 756	9.86 283	12	49
12	9.83 540	14	9.97 269	26	0.02 731	9.86 271	12	48
13	9.83 554	13	9.97 295	25	0.02 705	9.86 259	12	47
14	9.83 567	14	9.97 320	25	0.02 680	9.86 247	12	46
15	9.83 581	13	9.97 345	26	0.02 655	9.86 235	12	45
16	9.83 594	14	9.97 371	25	0.02 629	9.86 223	12	44
17	9.83 608	13	9.97 396	25	0.02 604	9.86 211	11	43
18	9.83 621	13	9.97 421	26	0.02 579	9.86 200	12	42
19	9.83 634	14	9.97 447	25	0.02 553	9.86 188	12	41
20	9.83 648	13	9.97 472	25	0.02 528	9.86 176	12	40
21	9.83 661	13	9.97 497	26	0.02 503	9.86 164	12	39
22	9.83 674	14	9.97 523	25	0.02 477	9.86 152	12	38
23	9.83 688	13	9.97 548	25	0.02 452	9.86 140	12	37
24	9.83 701	14	9.97 573	25	0.02 427	9.86 128	12	36
25	9.83 715	13	9.97 598	26	0.02 402	9.86 116	12	35
26	9.83 728	13	9.97 624	25	0.02 376	9.86 104	12	34
27	9.83 741	14	9.97 649	25	0.02 351	9.86 092	12	33
28	9.83 755	13	9.97 674	26	0.02 326	9.86 080	12	32
29	9.83 768	13	9.97 700	25	0.02 300	9.86 068	12	31
30	9.83 781	14	9.97 725	25	0.02 275	9.86 056	12	30
31	9.83 795	13	9.97 750	26	0.02 250	9.86 044	12	29
32	9.83 808	13	9.97 776	25	0.02 224	9.86 032	12	28
33	9.83 821	13	9.97 801	25	0.02 199	9.86 020	12	27
34	9.83 834	14	9.97 826	25	0.02 174	9.86 008	12	26
35	9.83 848	13	9.97 851	26	0.02 149	9.85 996	12	25
36	9.83 861	13	9.97 877	25	0.02 123	9.85 984	12	24
37	9.83 874	13	9.97 902	25	0.02 098	9.85 972	12	23
38	9.83 887	14	9.97 927	26	0.02 073	9.85 960	12	22
39	9.83 901	13	9.97 953	25	0.02 047	9.85 948	12	21
40	9.83 914	13	9.97 978	25	0.02 022	9.85 936	12	20
41	9.83 927	13	9.98 003	26	0.01 997	9.85 924	12	19
42	9.83 940	14	9.98 029	25	0.01 971	9.85 912	12	18
43	9.83 954	13	9.98 054	25	0.01 946	9.85 900	12	17
44	9.83 967	13	9.98 079	25	0.01 921	9.85 888	12	16
45	9.83 980	13	9.98 104	26	0.01 896	9.85 876	12	15
46	9.83 993	13	9.98 130	25	0.01 870	9.85 864	13	14
47	9.84 006	14	9.98 155	25	0.01 845	9.85 851	12	13
48	9.84 020	13	9.98 180	26	0.01 820	9.85 839	12	12
49	9.84 033	13	9.98 206	25	0.01 794	9.85 827	12	11
50	9.84 046	13	9.98 231	25	0.01 769	9.85 815	12	10
51	9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9
52	9.84 072	13	9.98 281	26	0.01 719	9.85 791	12	8
53	9.84 085	13	9.98 307	25	0.01 693	9.85 779	13	7
54	9.84 098	14	9.98 332	25	0.01 668	9.85 766	12	6
55	9.84 112	13	9.98 357	26	0.01 643	9.85 754	12	5
56	9.84 125	13	9.98 383	25	0.01 617	9.85 742	12	4
57	9.84 138	13	9.98 408	25	0.01 592	9.85 730	12	3
58	9.84 151	13	9.98 433	25	0.01 567	9.85 718	12	2
59	9.84 164	13	9.98 458	26	0.01 542	9.85 706	13	1
60	9.84 177		9.98 484		0.01 516	9.85 693		0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	26	25
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

″	14	13
6	1.4	1.3
7	1.6	1.5
8	1.9	1.7
9	2.1	2.0
10	2.3	2.2
20	4.7	4.3
30	7.0	6.5
40	9.3	8.7
50	11.7	10.8

″	12	11
6	1.2	1.1
7	1.4	1.3
8	1.6	1.5
9	1.8	1.6
10	2.0	1.8
20	4.0	3.7
30	6.0	5.5
40	8.0	7.3
50	10.0	9.2

Prop. Pts.

*136° 226° *316° 46°

44° *134° 224° *314°

′	log sin	d.	log tan	c. d.	log cot	log cos	d.	′
0	9.84 177		9.98 484		0.01 516	9.85 693		60
1	9.84 190	13	9.98 509	25	0.01 491	9.85 681	12	59
2	9.84 203	13	9.98 534	25	0.01 466	9.85 669	12	58
3	9.84 216	13	9.98 560	26	0.01 440	9.85 657	12	57
4	9.84 229	13	9.98 585	25	0.01 415	9.85 645	12	56
5	9.84 242	13	9.98 610	25	0.01 390	9.85 632	13	55
6	9.84 255	13	9.98 635	25	0.01 365	9.85 620	12	54
7	9.84 269	14	9.98 661	26	0.01 339	9.85 608	12	53
8	9.84 282	13	9.98 686	25	0.01 314	9.85 596	12	52
9	9.84 295	13	9.98 711	25	0.01 289	9.85 583	13	51
10	9.84 308	13	9.98 737	26	0.01 263	9.85 571	12	50
11	9.84 321	13	9.98 762	25	0.01 238	9.85 559	12	49
12	9.84 334	13	9.98 787	25	0.01 213	9.85 547	12	48
13	9.84 347	13	9.98 812	25	0.01 188	9.85 534	13	47
14	9.84 360	13	9.98 838	26	0.01 162	9.85 522	12	46
15	9.84 373	13	9.98 863	25	0.01 137	9.85 510	12	45
16	9.84 385	12	9.98 888	25	0.01 112	9.85 497	13	44
17	9.84 398	13	9.98 913	25	0.01 087	9.85 485	12	43
18	9.84 411	13	9.98 939	26	0.01 061	9.85 473	12	42
19	9.84 424	13	9.98 964	25	0.01 036	9.85 460	13	41
20	9.84 437	13	9.98 989	25	0.01 011	9.85 448	12	40
21	9.84 450	13	9.99 015	26	0.00 985	9.85 436	12	39
22	9.84 463	13	9.99 040	25	0.00 960	9.85 423	13	38
23	9.84 476	13	9.99 065	25	0.00 935	9.85 411	12	37
24	9.84 489	13	9.99 090	25	0.00 910	9.85 399	12	36
25	9.84 502	13	9.99 116	26	0.00 884	9.85 386	13	35
26	9.84 515	13	9.99 141	25	0.00 859	9.85 374	12	34
27	9.84 528	13	9.99 166	25	0.00 834	9.85 361	13	33
28	9.84 540	12	9.99 191	25	0.00 809	9.85 349	12	32
29	9.84 553	13	9.99 217	26	0.00 783	9.85 337	12	31
30	9.84 566	13	9.99 242	25	0.00 758	9.85 324	13	30
31	9.84 579	13	9.99 267	25	0.00 733	9.85 312	12	29
32	9.84 592	13	9.99 293	26	0.00 707	9.85 299	13	28
33	9.84 605	13	9.99 318	25	0.00 682	9.85 287	12	27
34	9.84 618	13	9.99 343	25	0.00 657	9.85 274	13	26
35	9.84 630	12	9.99 368	25	0.00 632	9.85 262	12	25
36	9.84 643	13	9.99 394	26	0.00 606	9.25 250	12	24
37	9.84 656	13	9.99 419	25	0.00 581	9.85 237	13	23
38	9.84 669	13	9.99 444	25	0.00 556	9.85 225	12	22
39	9.84 682	13	9.99 469	25	0.00 531	9.85 212	13	21
40	9.84 694	12	9.99 495	26	0.00 505	9.85 200	12	20
41	9.84 707	13	9.99 520	25	0.00 480	9.85 187	13	19
42	9.84 720	13	9.99 545	25	0.00 455	9.85 175	12	18
43	9.84 733	13	9.99 570	25	0.00 430	9.85 162	13	17
44	9.84 745	12	9.99 596	26	0.00 404	9.85 150	12	16
45	9.84 758	13	9.99 621	25	0.00 379	9.85 137	13	15
46	9.84 771	13	9.99 646	25	0.00 354	9.85 125	12	14
47	9.84 784	13	9.99 672	26	0.00 328	9.85 112	13	13
48	9.84 796	12	9.99 697	25	0.00 303	9.85 100	12	12
49	9.84 809	13	9.99 722	25	0.00 278	9.85 087	13	11
50	9.84 822	13	9.99 747	25	0.00 253	9.85 074	13	10
51	9.84 835	13	9.99 773	26	0.00 227	9.85 062	12	9
52	9.84 847	12	9.99 798	25	0.00 202	9.85 049	13	8
53	9.84 860	13	9.99 823	25	0.00 177	9.85 037	12	7
54	9.84 873	13	9.99 848	25	0.00 152	9.85 024	13	6
55	9.84 885	12	9.99 874	26	0.00 126	9.85 012	12	5
56	9.84 898	13	9.99 899	25	0.00 101	9.84 999	13	4
57	9.84 911	13	9.99 924	25	0.00 076	9.84 986	13	3
58	9.84 923	12	9.99 949	25	0.00 051	9.84 974	12	2
59	9.84 936	13	9.99 975	26	0.00 025	9.84 961	13	1
60	9.84 949	13	0.00 000	25	0.00 000	9.84 949	12	0
	log cos	d.	log cot	c. d.	log tan	log sin	d.	′

Prop. Pts.

″	26	25	14
6	2.6	2.5	1.4
7	3.0	2.9	1.6
8	3.5	3.3	1.9
9	3.9	3.8	2.1
10	4.3	4.2	2.3
20	8.7	8.3	4.7
30	13.0	12.5	7.0
40	17.3	16.7	9.3
50	21.7	20.8	11.7

″	13	12
6	1.3	1.2
7	1.5	1.4
8	1.7	1.6
9	2.0	1.8
10	2.2	2.0
20	4.3	4.0
30	6.5	6.0
40	8.7	8.0
50	10.8	10.0

*135° 225° *315° 45°

TABLE III

NATURAL TRIGONOMETRIC FUNCTIONS

FOUR PLACES

′	0°		1°		2°		3°		4°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	.0000	1.000	.0175	.9998	.0349	.9994	.0523	.9986	.0698	.9976	**60**
1	03	.000	77	98	52	94	26	86	0700	75	59
2	06	.000	80	98	55	94	29	86	03	75	58
3	09	.000	83	98	58	94	32	86	06	75	57
4	12	.000	86	98	61	93	35	86	09	75	56
5	.0015	1.000	.0189	.9998	.0364	.9993	.0538	.9986	.0712	.9975	**55**
6	17	.000	92	98	66	93	41	85	15	74	54
7	20	.000	95	98	69	93	44	85	18	74	53
8	23	.000	98	98	72	93	47	85	21	74	52
9	26	.000	0201	98	75	93	50	85	24	74	51
10	.0029	1.000	.0204	.9998	.0378	.9993	.0552	.9985	.0727	.9974	**50**
11	32	.000	07	98	81	93	55	85	29	73	49
12	35	.000	09	98	84	93	58	84	32	73	48
13	38	.000	12	98	87	93	61	84	35	73	47
14	41	.000	15	98	90	92	64	84	38	73	46
15	.0044	1.000	.0218	.9998	.0393	.9992	.0567	.9984	.0741	.9973	**45**
16	47	.000	21	98	96	92	70	84	44	72	44
17	49	.000	24	97	98	92	73	84	47	72	43
18	52	.000	27	97	0401	92	76	83	50	72	42
19	55	.000	30	97	04	92	79	83	53	72	41
20	.0058	1.000	.0233	.9997	.0407	.9992	.0581	.9983	.0756	.9971	**40**
21	61	.000	36	97	10	92	84	83	58	71	39
22	64	.000	39	97	13	91	87	83	61	71	38
23	67	.000	41	97	16	91	90	83	64	71	37
24	70	.000	44	97	19	91	93	82	67	71	36
25	.0073	1.000	.0247	.9997	.0422	.9991	.0596	.9982	.0770	.9970	**35**
26	76	.000	50	97	25	91	99	82	73	70	34
27	79	.000	53	97	27	91	0602	82	76	70	33
28	81	.000	56	97	30	91	05	82	79	70	32
29	84	.000	59	97	33	91	08	82	82	69	31
30	.0087	1.000	.0262	.9997	.0436	.9990	.0610	.9981	.0785	.9969	**30**
31	90	.000	65	96	39	90	13	81	87	69	29
32	93	.000	68	96	42	90	16	81	90	69	28
33	96	.000	70	96	45	90	19	81	93	68	27
34	99	.000	73	96	48	90	22	81	96	68	26
35	.0102	.9999	.0276	.9996	.0451	.9990	.0625	.9980	.0799	.9968	**25**
36	05	99	79	96	54	90	28	80	0802	68	24
37	08	99	82	96	57	90	31	80	05	68	23
38	11	99	85	96	59	89	34	80	08	67	22
39	13	99	88	96	62	89	37	80	11	67	21
40	.0116	.9999	.0291	.9996	.0465	.9989	.0640	.9980	.0814	.9967	**20**
41	19	99	94	96	68	89	42	79	16	67	19
42	22	99	97	96	71	89	45	79	19	66	18
43	25	99	0300	96	74	89	48	79	22	66	17
44	28	99	02	95	77	89	51	79	25	66	16
45	.0131	.9999	.0305	.9995	.0480	.9988	.0654	.9979	.0828	.9966	**15**
46	34	99	08	95	83	88	57	78	31	65	14
47	37	99	11	95	86	88	60	78	34	65	13
48	40	99	14	95	88	88	63	78	37	65	12
49	43	99	17	95	91	88	66	78	40	65	11
50	.0145	.9999	.0320	.9995	.0494	.9988	.0669	.9978	.0843	.9964	**10**
51	48	99	23	95	97	88	71	77	45	64	9
52	51	99	26	95	0500	87	74	77	48	64	8
53	54	99	29	95	03	87	77	77	51	64	7
54	57	99	32	95	06	87	80	77	54	63	6
55	.0160	.9999	.0334	.9994	.0509	.9987	.0683	.9977	.0857	.9963	**5**
56	63	99	37	94	12	87	86	76	60	63	4
57	66	99	40	94	15	87	89	76	63	63	3
58	69	99	43	94	18	87	92	76	66	62	2
59	72	99	46	94	20	86	95	76	69	62	1
60	.0175	.9998	.0349	.9994	.0523	.9986	.0698	.9976	.0872	.9962	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	89°		88°		87°		86°		85°		′

′	0°		1°		2°		3°		4°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	.0000	Infinite	.0175	57.2900	.0349	28.6363	.0524	19.0811	.0699	14.3007	60
1	03	3437.75	77	56.3506	52	3994	27	18.9755	0702	2411	59
2	06	1718.87	80	55.4415	55	1664	30	8711	05	1821	58
3	09	1145.92	83	54.5613	58	27.9372	33	7678	08	1235	57
4	12	859.436	86	53.7086	61	7117	36	6656	11	0655	56
5	.0015	687.549	.0189	52.8821	.0364	27.4899	.0539	18.5645	.0714	14.0079	55
6	17	572.957	92	0807	67	2715	42	4645	17	13.9507	54
7	20	491.106	95	51.3032	70	0566	44	3655	20	8940	53
8	23	429.718	98	50.5485	73	26.8450	47	2677	23	8378	52
9	26	381.971	0201	49.8157	75	6367	50	1708	26	7821	51
10	.0029	343.774	.0204	49.1039	.0378	26.4316	.0553	18.0750	.0729	13.7267	50
11	32	312.521	07	48.4121	81	2296	56	17.9802	31	6719	49
12	35	286.478	09	47.7395	84	0307	59	8863	34	6174	48
13	38	264.441	12	0853	87	25.8348	62	7934	37	5634	47
14	41	245.552	15	46.4489	90	6418	65	7015	40	5098	46
15	.0044	229.182	.0218	45.8294	.0393	25.4517	.0568	17.6106	.0743	13.4566	45
16	47	214.858	21	2261	96	2644	71	5205	46	4039	44
17	49	202.219	24	44.6386	99	0798	74	4314	49	3515	43
18	52	190.984	27	0661	0402	24.8978	77	3432	52	2996	42
19	55	180.932	30	43.5081	05	7185	80	2558	55	2480	41
20	.0058	171.885	.0233	42.9641	.0407	24.5418	.0582	17.1693	.0758	13.1969	40
21	61	163.700	36	4335	10	3675	85	0837	61	1461	39
22	64	156.259	39	41.9158	13	1957	88	16.9990	64	0958	38
23	67	149.465	41	4106	16	0263	91	9150	67	0458	37
24	70	143.237	44	40.9174	19	23.8593	94	8319	69	12.9962	36
25	.0073	137.507	.0247	40.4358	.0422	23.6945	.0597	16.7496	.0772	12.9469	35
26	76	132.219	50	39.9655	25	5321	0600	6681	75	8981	34
27	79	127.321	53	5059	28	3718	03	5874	78	8496	33
28	81	122.774	56	0568	31	2137	06	5075	81	8014	32
29	84	118.540	59	38.6177	34	0577	09	4283	84	7536	31
30	.0087	114.589	.0262	38.1885	.0437	22.9038	.0612	16.3499	.0787	12.7062	30
31	90	110.892	65	37.7686	40	7519	15	2722	90	6591	29
32	93	107.426	68	3579	42	6020	17	1952	93	6124	28
33	96	104.171	71	36.9560	45	4541	20	1190	96	5660	27
34	99	101.107	74	5627	48	3081	23	0435	99	5199	26
35	.0102	98.2179	.0276	36.1776	.0451	22.1640	.0626	15.9687	.0802	12.4742	25
36	05	95.4895	79	35.8006	54	0217	29	8945	05	4288	24
37	08	92.9085	82	4313	57	21.8813	32	8211	08	3838	23
38	11	90.4633	85	0695	60	7426	35	7483	10	3390	22
39	13	88.1436	88	34.7151	63	6056	38	6762	13	2946	21
40	.0116	85.9398	.0291	34.3678	.0466	21.4704	.0641	15.6048	.0816	12.2505	20
41	19	83.8435	94	0273	69	3369	44	5340	19	2067	19
42	22	81.8470	97	33.6935	72	2049	47	4638	22	1632	18
43	25	79.9434	0300	3662	75	0747	50	3943	25	1201	17
44	28	78.1263	03	0452	77	20.9460	53	3254	28	0772	16
45	.0131	76.3900	.0306	32.7303	.0480	20.8188	.0655	15.2571	.0831	12.0346	15
46	34	74.7292	08	4213	83	6932	58	1893	34	11.9923	14
47	37	73.1390	11	1181	86	5691	61	1222	37	9504	13
48	40	71.6151	14	31.8205	89	4465	64	0557	40	9087	12
49	43	70.1533	17	5284	92	3253	67	14.9898	43	8673	11
50	.0145	68.7501	.0320	31.2416	.0495	20.2056	.0670	14.9244	.0846	11.8262	10
51	48	67.4019	23	30.9599	98	0872	73	8596	49	7853	9
52	51	66.1055	26	6833	0501	19.9702	76	7954	51	7448	8
53	54	64.8580	29	4116	04	8546	79	7317	54	7045	7
54	57	63.6567	32	1446	07	7403	82	6685	57	6645	6
55	.0160	62.4992	.0335	29.8823	.0509	19.6273	.0685	14.6059	.0860	11.6248	5
56	63	61.3829	38	6245	12	5156	88	5438	63	5853	4
57	66	60.3058	40	3711	15	4051	90	4823	66	5461	3
58	69	59.2659	43	1220	18	2959	93	4212	69	5072	2
59	72	58.2612	46	28.8771	21	1879	96	3607	72	4685	1
60	.0175	57.2900	.0349	28.6363	.0524	19.0811	.0699	14.3007	.0875	11.4301	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	89°		88°		87°		86°		85°		′

′	5°		6°		7°		8°		9°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	.0872	.9962	.1045	.9945	.1219	.9925	.1392	.9903	.1564	.9877	60
1	74	62	48	45	22	25	95	02	67	76	59
2	77	61	51	45	24	25	97	02	70	76	58
3	80	61	54	44	27	24	1400	01	73	76	57
4	83	61	57	44	30	24	03	01	76	75	56
5	.0886	.9961	.1060	.9944	.1233	.9924	.1406	.9901	.1579	.9875	55
6	89	60	63	43	36	23	09	00	82	74	54
7	92	60	66	43	39	23	12	00	84	74	53
8	95	60	68	43	42	23	15	9899	87	73	52
9	98	60	71	42	45	22	18	99	90	73	51
10	.0901	.9959	.1074	.9942	.1248	.9922	.1421	.9899	.1593	.9872	50
11	03	59	77	42	50	22	23	98	96	72	49
12	06	59	80	42	53	21	26	98	99	71	48
13	09	59	83	41	56	21	29	97	1602	71	47
14	12	58	86	41	59	20	32	97	05	70	46
15	.0915	.9958	.1089	.9941	.1262	.9920	.1435	.9897	.1607	.9870	45
16	18	58	92	40	65	20	38	96	10	69	44
17	21	58	94	40	68	19	41	96	13	69	43
18	24	57	97	40	71	19	44	95	16	69	42
19	27	57	1100	39	74	19	46	95	19	68	41
20	.0929	.9957	.1103	.9939	.1276	.9918	.1449	.9894	.1622	.9868	40
21	32	56	06	39	79	18	52	94	25	67	39
22	35	56	09	38	82	17	55	94	28	67	38
23	38	56	12	38	85	17	58	93	30	66	37
24	41	56	15	38	88	17	61	93	33	66	36
25	.0944	.9955	.1118	.9937	.1291	.9916	.1464	.9892	.1636	.9865	35
26	47	55	20	37	94	16	67	92	39	65	34
27	50	55	23	37	97	16	69	91	42	64	33
28	53	55	26	36	99	15	72	91	45	64	32
29	56	54	29	36	1302	15	75	91	48	63	31
30	.0958	.9954	.1132	.9936	.1305	.9914	.1478	.9890	.1650	.9863	30
31	61	54	35	35	08	14	81	90	53	62	29
32	64	53	38	35	11	14	84	89	56	62	28
33	67	53	41	35	14	13	87	89	59	61	27
34	70	53	44	34	17	13	90	88	62	61	26
35	.0973	.9953	.1146	.9934	.1320	.9913	.1492	.9888	.1665	.9860	25
36	76	52	49	34	23	12	95	88	68	60	24
37	79	52	52	33	25	12	98	87	71	59	23
38	82	52	55	33	28	11	1501	87	73	59	22
39	85	51	58	33	31	11	04	86	76	59	21
40	.0987	.9951	.1161	.9932	.1334	.9911	.1507	.9886	.1679	.9858	20
41	90	51	64	32	37	10	10	85	82	58	19
42	93	51	67	32	40	10	13	85	85	57	18
43	96	50	70	31	43	09	15	84	88	57	17
44	99	50	72	31	46	09	18	84	91	56	16
45	.1002	.9950	.1175	.9931	.1349	.9909	.1521	.9884	.1693	.9856	15
46	05	49	78	30	51	08	24	83	96	55	14
47	08	49	81	30	54	08	27	83	99	55	13
48	11	49	84	30	57	07	30	82	1702	54	12
49	13	49	87	29	60	07	33	82	05	54	11
50	.1016	.9948	.1190	.9929	.1363	.9907	.1536	.9881	.1708	.9853	10
51	19	48	93	29	66	06	38	81	11	53	9
52	22	48	96	28	69	06	41	80	14	52	8
53	25	47	98	28	72	05	44	80	16	52	7
54	28	47	1201	28	74	05	47	80	19	51	6
55	.1031	.9947	.1204	.9927	.1377	.9905	.1550	.9879	.1722	.9851	5
56	34	46	07	27	80	04	53	79	25	50	4
57	37	46	10	27	83	04	56	78	28	50	3
58	39	46	13	26	86	03	59	78	31	49	2
59	42	46	16	26	89	03	61	77	34	49	1
60	.1045	.9945	.1219	.9925	.1392	.9903	.1564	.9877	.1736	.9848	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	84°		83°		82°		81°		80°		′

′	5°		6°		7°		8°		9°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	.0875	11.4301	.1051	9.5144	.1228	8.1443	.1405	7.1154	.1584	6.3138	60
1	78	3919	54	4878	31	1248	08	1004	87	3019	59
2	81	3540	57	4614	34	1054	11	0855	90	2901	58
3	84	3163	60	4352	37	0860	14	0706	93	2783	57
4	87	2789	63	4090	40	0667	17	0558	96	2666	56
5	.0890	11.2417	.1066	9.3831	.1243	8.0476	.1420	7.0410	.1599	6.2549	55
6	92	2048	69	3572	46	0285	23	0264	1602	2432	54
7	95	1681	72	3315	49	0095	26	0117	05	2316	53
8	98	1316	75	3060	51	7.9906	29	6.9972	08	2200	52
9	0901	0954	78	2806	54	9718	32	9827	11	2085	51
10	.0904	11.0594	.1080	9.2553	.1257	7.9530	.1435	6.9682	.1614	6.1970	50
11	07	0237	83	2302	60	9344	38	9538	17	1856	49
12	10	10.9882	86	2052	63	9158	41	9395	20	1742	48
13	13	9529	89	1803	66	8973	44	9252	23	1628	47
14	16	9178	92	1555	69	8789	47	9110	26	1515	46
15	.0919	10.8829	.1095	9.1309	.1272	7.8606	.1450	6.8969	.1629	6.1402	45
16	22	8483	98	1065	75	8424	53	8828	32	1290	44
17	25	8139	1101	0821	78	8243	56	8687	35	1178	43
18	28	7797	04	0579	81	8062	59	8548	38	1066	42
19	31	7457	07	0338	84	7882	62	8408	41	0955	41
20	.0934	10.7119	.1110	9.0098	.1287	7.7704	.1465	6.8269	.1644	6.0844	40
21	36	6783	13	8.9860	90	7525	68	8131	47	0734	39
22	39	6450	16	9623	93	7348	71	7994	50	0624	38
23	42	6118	19	9387	96	7171	74	7856	53	0514	37
24	45	5789	22	9152	99	6996	77	7720	55	0405	36
25	.0948	10.5462	.1125	8.8919	.1302	7.6821	.1480	6.7584	.1658	6.0296	35
26	51	5136	28	8686	05	6647	83	7448	61	0188	34
27	54	4813	31	8455	08	6473	86	7313	64	0080	33
28	57	4491	33	8225	11	6301	89	7179	67	5.9972	32
29	60	4172	36	7996	14	6129	92	7045	70	9865	31
30	.0963	10.3854	.1139	8.7769	.1317	7.5958	.1495	6.6912	.1673	5.9758	30
31	66	3538	42	7542	19	5787	97	6779	76	9651	29
32	69	3224	45	7317	22	5618	1500	6646	79	9545	28
33	72	2913	48	7093	25	5449	03	6514	82	9439	27
34	75	2602	51	6870	28	5281	06	6383	85	9333	26
35	.0978	10.2294	.1154	8.6648	.1331	7.5113	.1509	6.6252	.1688	5.9228	25
36	81	1988	57	6427	34	4947	12	6122	91	9124	24
37	83	1683	60	6208	37	4781	15	5992	94	9019	23
38	86	1381	63	5989	40	4615	18	5863	97	8915	22
39	89	1080	66	5772	43	4451	21	5734	1700	8811	21
40	.0992	10.0780	.1169	8.5555	.1346	7.4287	.1524	6.5606	.1703	5.8708	20
41	95	0483	72	5340	49	4124	27	5478	06	8605	19
42	98	0187	75	5126	52	3962	30	5350	09	8502	18
43	1001	9.9893	78	4913	55	3800	33	5223	12	8400	17
44	04	9601	81	4701	58	3639	36	5097	15	8298	16
45	.1007	9.9310	.1184	8.4490	.1361	7.3479	.1539	6.4971	.1718	5.8197	15
46	10	9021	87	4280	64	3319	42	4846	21	8095	14
47	13	8734	89	4071	67	3160	45	4721	24	7994	13
48	16	8448	92	3863	70	3002	48	4596	27	7894	12
49	19	8164	95	3656	73	2844	51	4472	30	7794	11
50	.1022	9.7882	.1198	8.3450	.1376	7.2687	.1554	6.4348	.1733	5.7694	10
51	25	7601	1201	3245	79	2531	57	4225	36	7594	9
52	28	7322	04	3041	82	2375	60	4103	39	7495	8
53	30	7044	07	2838	85	2220	63	3980	42	7396	7
54	33	6768	10	2636	88	2066	66	3859	45	7297	6
55	.1036	9.6493	.1213	8.2434	.1391	7.1912	.1569	6.3737	.1748	5.7199	5
56	39	6220	16	2234	94	1759	72	3617	51	7101	4
57	42	5949	19	2035	97	1607	75	3496	54	7004	3
58	45	5679	22	1837	99	1455	78	3376	57	6906	2
59	48	5411	25	1640	1402	1304	81	3257	60	6809	1
60	.1051	9.5144	.1228	8.1443	.1405	7.1154	.1584	6.3138	.1763	5.6713	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	84°		83°		82°		81°		80°		′

′	10°		11°		12°		13°		14°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	.1736	.9848	.1908	.9816	.2079	.9781	.2250	.9744	.2419	.9703	**60**
1	39	48	11	16	82	81	52	43	22	02	59
2	42	47	14	15	85	80	55	42	25	02	58
3	45	47	17	15	88	80	58	42	28	01	57
4	48	46	20	14	90	79	61	41	31	00	56
5	.1751	.9846	.1922	.9813	.2093	.9778	.2264	.9740	.2433	.9699	**55**
6	54	45	25	13	96	78	67	40	36	99	54
7	57	45	28	12	99	77	69	39	39	98	53
8	59	44	31	12	2102	77	72	38	42	97	52
9	62	43	34	11	05	76	75	38	45	97	51
10	.1765	.9843	.1937	.9811	.2108	.9775	.2278	.9737	.2447	.9696	**50**
11	68	42	39	10	10	75	81	36	50	95	49
12	71	42	42	10	13	74	84	36	53	94	48
13	74	41	45	09	16	74	86	35	56	94	47
14	77	41	48	08	19	73	89	34	59	93	46
15	.1779	.9840	.1951	.9808	.2122	.9772	.2292	.9734	.2462	.9692	**45**
16	82	40	54	07	25	72	95	33	64	92	44
17	85	39	57	07	27	71	98	32	67	91	43
18	88	39	59	06	30	70	2300	32	70	90	42
19	91	38	62	06	33	70	03	31	73	89	41
20	.1794	.9838	.1965	.9805	.2136	.9769	.2306	.9730	.2476	.9689	**40**
21	97	37	68	04	39	69	09	30	78	88	39
22	99	37	71	04	42	68	12	29	81	87	38
23	1802	36	74	03	45	67	15	28	84	87	37
24	05	36	77	03	47	67	17	28	87	86	36
25	.1808	.9835	.1979	.9802	.2150	.9766	.2320	.9727	.2490	.9685	**35**
26	11	35	82	02	53	65	23	26	93	84	34
27	14	34	85	01	56	65	26	26	95	84	33
28	17	34	88	00	59	64	29	25	98	83	32
29	19	33	91	00	62	64	32	24	.2501	82	31
30	.1822	.9833	.1994	.9799	.2164	.9763	.2334	.9724	.2504	.9681	**30**
31	25	32	97	99	67	62	37	23	07	81	29
32	28	31	99	98	70	62	40	22	09	80	28
33	31	31	2002	98	73	61	43	22	12	79	27
34	34	30	05	97	76	60	46	21	15	79	26
35	.1837	.9830	.2008	.9796	.2179	.9760	.2349	.9720	.2518	.9678	**25**
36	40	29	11	96	81	59	51	20	21	77	24
37	42	29	14	95	84	59	54	19	24	76	23
38	45	28	16	95	87	58	57	18	26	76	22
39	48	28	19	94	90	57	60	18	29	75	21
40	.1851	.9827	.2022	.9793	.2193	.9757	.2363	.9717	.2532	.9674	**20**
41	54	27	25	93	96	56	66	16	35	73	19
42	57	26	28	92	98	55	68	15	38	73	18
43	60	26	31	92	2201	55	71	15	40	72	17
44	62	25	34	91	04	54	74	14	43	71	16
45	.1865	.9825	.2036	.9790	.2207	.9753	.2377	.9713	.2546	.9670	**15**
46	68	24	39	90	10	53	80	13	49	70	14
47	71	23	42	89	13	52	83	12	52	69	13
48	74	23	45	89	15	51	85	11	54	68	12
49	77	22	48	88	18	51	88	11	57	67	11
50	.1880	.9822	.2051	.9787	.2221	.9750	.2391	.9710	.2560	.9667	**10**
51	82	21	54	87	24	50	94	09	63	66	9
52	85	21	56	86	27	49	97	09	66	65	8
53	88	20	59	86	30	48	99	08	69	65	7
54	91	20	62	85	33	48	.2402	07	71	64	6
55	.1894	.9819	.2065	.9784	.2235	.9747	.2405	.9706	.2574	.9663	**5**
56	97	18	68	84	38	46	08	06	77	62	4
57	1900	18	71	83	41	46	11	05	80	62	3
58	02	17	73	83	44	45	14	04	83	61	2
59	05	17	76	82	47	44	16	04	85	60	1
60	.1908	.9816	.2079	.9781	.2250	.9744	.2419	.9703	.2588	.9659	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	79°		78°		77°		76°		75°		′

′	10°		11°		12°		13°		14°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	.1763	5.6713	.1944	5.1446	.2126	4.7046	.2309	4.3315	.2493	4.0108	**60**
1	66	6617	47	1366	29	6979	12	3257	96	0058	59
2	69	6521	50	1286	32	6912	15	3200	99	0009	58
3	72	6425	53	1207	35	6845	18	3143	2503	3.9959	57
4	75	6329	56	1128	38	6779	21	3086	06	9910	56
5	.1778	5.6234	.1959	5.1049	.2141	4.6712	.2324	4.3029	.2509	3.9861	**55**
6	81	6140	62	0970	44	6646	27	2972	12	9812	54
7	84	6045	65	0892	47	6580	30	2916	15	9763	53
8	87	5951	68	0814	50	6514	33	2859	18	9714	52
9	90	5857	71	0736	53	6448	36	2803	21	9665	51
10	.1793	5.5764	.1974	5.0658	.2156	4.6382	.2339	4.2747	.2524	3.9617	**50**
11	96	5671	77	0581	59	6317	42	2691	27	9568	49
12	99	5578	80	0504	62	6252	45	2635	30	9520	48
13	1802	5485	83	0427	65	6187	49	2580	33	9471	47
14	05	5393	86	0350	68	6122	52	2524	37	9423	46
15	.1808	5.5301	.1989	5.0273	.2171	4.6057	.2355	4.2468	.2540	3.9375	**45**
16	11	5209	92	0197	74	5993	58	2413	43	9327	44
17	14	5118	95	0121	77	5928	61	2358	46	9279	43
18	17	5026	98	0045	80	5864	64	2303	49	9232	42
19	20	4936	2001	4.9969	83	5800	67	2248	52	9184	41
20	.1823	5.4845	.2004	4.9894	.2186	4.5736	.2370	4.2193	.2555	3.9136	**40**
21	26	4755	07	9819	89	5673	73	2139	58	9089	39
22	29	4665	10	9744	93	5609	76	2084	61	9042	38
23	32	4575	13	9669	96	5546	79	2030	64	8995	37
24	35	4486	16	9594	99	5483	82	1976	68	8947	36
25	.1838	5.4397	.2019	4.9520	.2202	4.5420	.2385	4.1922	.2571	3.8900	**35**
26	41	4308	22	9446	05	5357	88	1868	74	8854	34
27	44	4219	25	9372	08	5294	92	1814	77	8807	33
28	47	4131	28	9298	11	5232	95	1760	80	8760	32
29	50	4043	31	9225	14	5169	98	1706	83	8714	31
30	.1853	5.3955	.2035	4.9152	.2217	4.5107	.2401	4.1653	.2586	3.8667	**30**
31	56	3868	38	9078	20	5045	04	1600	89	8621	29
32	59	3781	41	9006	23	4983	07	1547	92	8575	28
33	62	3694	44	8933	26	4922	10	1493	95	8528	27
34	65	3607	47	8860	29	4860	13	1441	99	8482	26
35	.1868	5.3521	.2050	4.8788	.2232	4.4799	.2416	4.1388	.2602	3.8436	**25**
36	71	3435	53	8716	35	4737	19	1335	05	8391	24
37	74	3349	56	8644	38	4676	22	1282	08	8345	23
38	77	3263	59	8573	41	4615	25	1230	11	8299	22
39	80	3178	62	8501	44	4555	28	1178	14	8254	21
40	.1883	5.3093	.2065	4.8430	.2247	4.4494	.2432	4.1126	.2617	3.8208	**20**
41	87	3008	68	8359	51	4434	35	1074	20	8163	19
42	90	2924	71	8288	54	4373	38	1022	23	8118	18
43	93	2839	74	8218	57	4313	41	0970	27	8073	17
44	96	2755	77	8147	60	4253	44	0918	30	8028	16
45	.1899	5.2672	.2080	4.8077	.2263	4.4194	.2447	4.0867	.2633	3.7983	**15**
46	1902	2588	83	8007	66	4134	50	0815	36	7938	14
47	05	2505	86	7937	69	4075	53	0764	39	7893	13
48	08	2422	89	7867	72	4015	56	0713	42	7848	12
49	11	2339	92	7798	75	3956	59	0662	45	7804	11
50	.1914	5.2257	.2095	4.7729	.2278	4.3897	.2462	4.0611	.2648	3.7760	**10**
51	17	2174	98	7659	81	3838	65	0560	51	7715	9
52	20	2092	2101	7591	84	3779	69	0509	55	7671	8
53	23	2011	04	7522	87	3721	72	0459	58	7627	7
54	26	1929	07	7453	90	3662	75	0408	61	7583	6
55	.1929	5.1848	.2110	4.7385	.2293	4.3604	.2478	4.0358	.2664	3.7539	**5**
56	32	1767	13	7317	96	3546	81	0308	67	7495	4
57	35	1686	16	7249	99	3488	84	0257	70	7451	3
58	38	1606	19	7181	2303	3430	87	0207	73	7408	2
59	41	1526	23	7114	06	3372	90	0158	76	7364	1
60	.1944	5.1446	.2126	4.7046	.2309	4.3315	.2493	4.0108	.2679	3.7321	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	79°		78°		77°		76°		75°		′

′	15°		16°		17°		18°		19°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	.2588	.9659	.2756	.9613	.2924	.9563	.3090	.9511	.3256	.9455	**60**
1	91	59	59	12	26	62	93	10	58	54	59
2	94	58	62	11	29	61	96	09	61	53	58
3	97	57	65	10	32	60	98	08	64	52	57
4	99	56	68	09	35	60	3101	07	67	51	56
5	.2602	.9655	.2770	.9609	.2938	.9559	.3104	.9506	.3269	.9450	**55**
6	05	55	73	08	40	58	07	05	72	49	54
7	08	54	76	07	43	57	10	04	75	49	53
8	11	53	79	06	46	56	12	03	78	48	52
9	13	52	82	05	49	55	15	02	80	47	51
10	.2616	.9652	.2784	.9605	.2952	.9555	.3118	.9502	.3283	.9446	**50**
11	19	51	87	04	54	54	21	01	86	45	49
12	22	50	90	03	57	53	23	9500	89	44	48
13	25	49	93	02	60	52	26	9499	91	43	47
14	28	49	95	01	63	51	29	98	94	42	46
15	.2630	.9648	.2798	.9600	.2965	.9550	.3132	.9497	.3297	.9441	**45**
16	33	47	2801	00	68	49	34	96	3300	40	44
17	36	46	04	9599	71	48	37	95	02	39	43
18	39	46	07	98	74	48	40	94	05	38	42
19	42	45	09	97	77	47	43	93	08	37	41
20	.2644	.9644	.2812	.9596	.2979	.9546	.3145	.9492	.3311	.9436	**40**
21	47	43	15	96	82	45	48	92	13	35	39
22	50	42	18	95	85	44	51	91	16	34	38
23	53	42	21	94	88	43	54	90	19	33	37
24	56	41	23	93	90	42	56	89	22	32	36
25	.2658	.9640	.2826	.9592	.2993	.9542	.3159	.9488	.3324	.9431	**35**
26	61	39	29	91	96	41	62	87	27	30	34
27	64	39	32	91	99	40	65	86	30	29	33
28	67	38	35	90	3002	39	68	85	33	28	32
29	70	37	37	89	04	38	70	84	35	27	31
30	.2672	.9636	.2840	.9588	.3007	.9537	.3173	.9483	.3338	.9426	**30**
31	75	36	43	87	10	36	76	82	41	25	29
32	78	35	46	87	13	35	79	81	44	24	28
33	81	34	49	86	15	35	81	80	46	23	27
34	84	33	51	85	18	34	84	80	49	23	26
35	.2686	.9632	.2854	.9584	.3021	.9533	.3187	.9479	.3352	.9422	**25**
36	89	32	57	83	24	32	90	78	55	21	24
37	92	31	60	82	26	31	92	77	57	20	23
38	95	30	62	82	29	30	95	76	60	19	22
39	98	29	65	81	32	29	98	75	63	18	21
40	.2700	.9628	.2868	.9580	.3035	.9528	.3201	.9474	.3365	.9417	**20**
41	03	28	71	79	38	27	03	73	68	16	19
42	06	27	74	78	40	27	06	72	71	15	18
43	09	26	76	77	43	26	09	71	74	14	17
44	12	25	79	77	46	25	12	70	76	13	16
45	.2714	.9625	.2882	.9576	.3049	.9524	.3214	.9469	.3379	.9412	**15**
46	17	24	85	75	51	23	17	68	82	11	14
47	20	23	88	74	54	22	20	67	85	10	13
48	23	22	90	73	57	21	23	66	87	09	12
49	26	21	93	72	60	20	25	66	90	08	11
50	2728	.9621	.2896	.9572	.3062	.9520	.3228	.9465	.3393	.9407	**10**
51	31	20	99	71	65	19	31	64	96	06	9
52	34	19	2901	70	68	18	34	63	98	05	8
53	37	18	04	69	71	17	36	62	3401	04	7
54	40	17	07	68	74	16	39	61	04	03	6
55	.2742	.9617	.2910	.9567	.3076	.9515	.3242	.9460	.3407	.9402	**5**
56	45	16	13	66	79	14	45	59	09	01	4
57	48	15	15	66	82	13	47	58	12	00	3
58	51	14	18	65	85	12	50	57	15	9399	2
59	54	13	21	64	87	11	53	56	17	98	1
60	.2756	.9613	.2924	.9563	.3090	.9511	.3256	.9455	.3420	.9397	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	74°		73°		72°		71°		70°		′

′	15°		16°		17°		18°		19°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	.2679	3.7321	.2867	3.4874	.3057	3.2709	.3249	3.0777	.3443	2.9042	60
1	83	7277	71	4836	60	2675	52	0746	47	9015	59
2	86	7234	74	4798	64	2641	56	0716	50	8987	58
3	89	7191	77	4760	67	2607	59	0686	53	8960	57
4	92	7148	80	4722	70	2573	62	0655	56	8933	56
5	.2695	3.7105	.2883	3.4684	.3073	3.2539	.3265	3.0625	.3460	2.8905	55
6	98	7062	86	4646	76	2506	69	0595	63	8878	54
7	2701	7019	90	4608	80	2472	72	0565	66	8851	53
8	04	6976	93	4570	83	2438	75	0535	69	8824	52
9	08	6933	96	4533	86	2405	78	0505	73	8797	51
10	.2711	3.6891	.2899	3.4495	.3089	3.2371	.3281	3.0475	.3476	2.8770	50
11	14	6848	2902	4458	92	2338	85	0445	79	8743	49
12	17	6806	05	4420	96	2305	88	0415	82	8716	48
13	20	6764	08	4383	99	2272	91	0385	86	8689	47
14	23	6722	12	4346	3102	2238	94	0356	89	8662	46
15	.2726	3.6680	.2915	3.4308	.3105	3.2205	.3298	3.0326	.3492	2.8636	45
16	29	6638	18	4271	08	2172	3301	0296	95	8609	44
17	33	6596	21	4234	11	2139	04	0267	99	8582	43
18	36	6554	24	4197	15	2106	07	0237	3502	8556	42
19	39	6512	27	4160	18	2073	10	0208	05	8529	41
20	.2742	3.6470	.2931	3.4124	.3121	3.2041	.3314	3.0178	.3508	2.8502	40
21	45	6429	34	4087	24	2008	17	0149	12	8476	39
22	48	6387	37	4050	27	1975	20	0120	15	8449	38
23	51	6346	40	4014	31	1943	23	0090	18	8423	37
24	54	6305	43	3977	34	1910	27	0061	22	8397	36
25	.2758	3.6264	.2946	3.3941	.3137	3.1878	.3330	3.0032	.3525	2.8370	35
26	61	6222	49	3904	40	1845	33	0003	28	8344	34
27	64	6181	53	3868	43	1813	36	2.9974	31	8318	33
28	67	6140	56	3832	47	1780	39	9945	35	8291	32
29	70	6100	59	3796	50	1748	43	9916	38	8265	31
30	.2773	3.6059	.2962	3.3759	.3153	3.1716	.3346	2.9887	.3541	2.8239	30
31	76	6018	65	3723	56	1684	49	9858	44	8213	29
32	80	5978	68	3687	59	1652	52	9829	48	8187	28
33	83	5937	72	3652	63	1620	56	9800	51	8161	27
34	86	5897	75	3616	66	1588	59	9772	54	8135	26
35	.2789	3.5856	.2978	3.3580	.3169	3.1556	.3362	2.9743	.3558	2.8109	25
36	92	5816	81	3544	72	1524	65	9714	61	8083	24
37	95	5776	84	3509	75	1492	69	9686	64	8057	23
38	98	5736	87	3473	79	1460	72	9657	67	8032	22
39	2801	5696	91	3438	82	1429	75	9629	71	8006	21
40	.2805	3.5656	.2994	3.3402	.3185	3.1397	.3378	2.9600	.3574	2.7980	20
41	08	5616	97	3367	88	1366	82	9572	77	7955	19
42	11	5576	3000	3332	91	1334	85	9544	81	7929	18
43	14	5536	03	3297	95	1303	88	9515	84	7903	17
44	17	5497	06	3261	98	1271	91	9487	87	7878	16
45	.2820	3.5457	.3010	3.3226	.3201	3.1240	.3395	2.9459	.3590	2.7852	15
46	23	5418	13	3191	04	1209	98	9431	94	7827	14
47	27	5379	16	3156	07	1178	3401	9403	97	7801	13
48	30	5339	19	3122	11	1146	04	9375	3600	7776	12
49	33	5300	22	3087	14	1115	08	9347	04	7751	11
50	.2836	3.5261	.3026	3.3052	.3217	3.1084	.3411	2.9319	.3607	2.7725	10
51	39	5222	29	3017	20	1053	14	9291	10	7700	9
52	42	5183	32	2983	23	1022	17	9263	13	7675	8
53	45	5144	35	2948	27	0991	21	9235	17	7650	7
54	49	5105	38	2914	30	0961	24	9208	20	7625	6
55	.2852	3.5067	.3041	3.2879	.3233	3.0930	.3427	2.9180	.3623	2.7600	5
56	55	5028	45	2845	36	0899	30	9152	27	7575	4
57	58	4989	48	2811	40	0868	34	9125	30	7550	3
58	61	4951	51	2777	43	0838	37	9097	33	7525	2
59	64	4912	54	2743	46	0807	40	9070	36	7500	1
60	.2867	3.4874	.3057	3.2709	.3249	3.0777	.3443	2.9042	.3640	2.7475	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	74°		73°		72°		71°		70°		′

′	20°		21°		22°		23°		24°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	.3420	.9397	.3584	.9336	.3746	.9272	.3907	.9205	.4067	.9135	60
1	23	96	86	35	49	71	10	04	70	34	59
2	26	95	89	34	51	70	13	03	73	33	58
3	28	94	92	33	54	69	15	02	75	32	57
4	31	93	95	32	57	67	18	00	78	31	56
5	.3434	.9392	.3597	.9331	.3760	.9266	.3921	.9199	.4081	.9130	55
6	37	91	3600	30	62	65	23	98	83	28	54
7	39	90	03	28	65	64	26	97	86	27	53
8	42	89	05	27	68	63	29	96	89	26	52
9	45	88	08	26	70	62	31	95	91	25	51
10	.3448	.9387	.3611	.9325	.3773	.9261	.3934	.9194	.4094	.9124	50
11	50	86	14	24	76	60	37	92	97	22	49
12	53	85	16	23	78	59	39	91	99	21	48
13	56	84	19	22	81	58	42	90	4102	20	47
14	58	83	22	21	84	57	45	89	05	19	46
15	.3461	.9382	.3624	.9320	.3786	.9255	.3947	.9188	.4107	.9118	45
16	64	81	27	19	89	54	50	87	10	16	44
17	67	80	30	18	92	53	53	86	12	15	43
18	69	79	33	17	95	52	55	84	15	14	42
19	72	78	35	16	97	51	58	83	18	13	41
20	.3475	.9377	.3638	.9315	.3800	.9250	.3961	.9182	.4120	.9112	40
21	78	76	41	14	03	49	63	81	23	10	39
22	80	75	43	13	05	48	66	80	26	09	38
23	83	74	46	12	08	47	69	79	28	08	37
24	86	73	49	11	11	45	71	78	31	07	36
25	.3488	.9372	.3651	.9309	.3813	.9244	.3974	.9176	.4134	.9106	35
26	91	71	54	08	16	43	77	75	36	04	34
27	94	70	57	07	19	42	79	74	39	03	33
28	97	69	60	06	21	41	82	73	42	02	32
29	99	68	62	05	24	40	85	72	44	01	31
30	.3502	.9367	.3665	.9304	.3827	.9239	.3987	.9171	.4147	.9100	30
31	05	66	68	03	30	38	90	69	50	9098	29
32	08	65	70	02	32	37	93	68	52	97	28
33	10	64	73	01	35	35	95	67	55	96	27
34	13	63	76	00	38	34	98	66	58	95	26
35	.3516	.9362	.3679	.9299	.3840	.9233	.4001	.9165	.4160	.9094	25
36	18	61	81	98	43	32	03	64	63	92	24
37	21	60	84	97	46	31	06	62	65	91	23
38	24	59	87	96	48	30	09	61	68	90	22
39	27	58	89	95	51	29	11	60	71	89	21
40	.3529	.9356	.3692	.9293	.3854	.9228	.4014	.9159	.4173	.9088	20
41	32	55	95	92	56	27	17	58	76	86	19
42	35	54	97	91	59	25	19	57	79	85	18
43	37	53	3700	90	62	24	22	55	81	84	17
44	40	52	03	89	64	23	25	54	84	83	16
45	.3543	.9351	.3706	.9288	.3867	.9222	.4027	.9153	.4187	.9081	15
46	46	50	08	87	70	21	30	52	89	80	14
47	48	49	11	86	72	20	33	51	92	79	13
48	51	48	14	85	75	19	35	50	95	78	12
49	54	47	16	84	78	18	38	48	97	77	11
50	.3557	.9346	.3719	.9283	.3881	.9216	.4041	.9147	.4200	.9075	10
51	59	45	22	82	83	15	43	46	02	74	9
52	62	44	24	81	86	14	46	45	05	73	8
53	65	43	27	79	89	13	49	44	08	72	7
54	67	42	30	78	91	12	51	43	10	70	6
55	.3570	.9341	.3733	.9277	.3894	.9211	.4054	.9141	.4213	.9069	5
56	73	40	35	76	97	10	57	40	16	68	4
57	76	39	38	75	99	08	59	39	18	67	3
58	78	38	41	74	3902	07	62	38	21	66	2
59	81	37	43	73	05	06	65	37	24	64	1
60	.3584	.9336	.3746	.9272	.3907	.9205	.4067	.9135	.4226	.9063	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	69°		68°		67°		66°		65°		′

′	20° tan	20° cot	21° tan	21° cot	22° tan	22° cot	23° tan	23° cot	24° tan	24° cot	′
0	.3640	2.7475	.3839	2.6051	.4040	2.4751	.4245	2.3559	.4452	2.2460	**60**
1	43	7450	42	6028	44	4730	48	3539	56	2443	59
2	46	7425	45	6006	47	4709	52	3520	59	2425	58
3	50	7400	49	5983	50	4689	55	3501	63	2408	57
4	53	7376	52	5961	54	4668	58	3483	66	2390	56
5	.3656	2.7351	.3855	2.5938	.4057	2.4648	.4262	2.3464	.4470	2.2373	**55**
6	59	7326	59	5916	61	4627	65	3445	73	2355	54
7	63	7302	62	5893	64	4606	69	3426	77	2338	53
8	66	7277	65	5871	67	4586	72	3407	80	2320	52
9	69	7253	69	5848	71	4566	76	3388	84	2303	51
10	.3673	2.7228	.3872	2.5826	.4074	2.4545	.4279	2.3369	.4487	2.2286	**50**
11	76	7204	75	5804	78	4525	83	3351	91	2268	49
12	79	7179	79	5782	81	4504	86	3332	94	2251	48
13	83	7155	82	5759	84	4484	89	3313	98	2234	47
14	86	7130	85	5737	88	4464	93	3294	4501	2216	46
15	.3689	2.7106	.3889	2.5715	.4091	2.4443	.4296	2.3276	.4505	2.2199	**45**
16	93	7082	92	5693	95	4423	4300	3257	08	2182	44
17	96	7058	95	5671	98	4403	03	3238	12	2165	43
18	99	7034	99	5649	4101	4383	07	3220	15	2148	42
19	3702	7009	3902	5627	05	4362	10	3201	19	2130	41
20	.3706	2.6985	.3906	2.5605	.4108	2.4342	.4314	2.3183	.4522	2.2113	**40**
21	09	6961	09	5583	11	4322	17	3164	26	2096	39
22	12	6937	12	5561	15	4302	20	3146	29	2079	38
23	16	6913	16	5539	18	4282	24	3127	33	2062	37
24	19	6889	19	5517	22	4262	27	3109	36	2045	36
25	.3722	2.6865	.3922	2.5495	.4125	2.4242	.4331	2.3090	.4540	2.2028	**35**
26	26	6841	26	5473	29	4222	34	3072	43	2011	34
27	29	6818	29	5452	32	4202	38	3053	47	1994	33
28	32	6794	32	5430	35	4182	41	3035	50	1977	32
29	36	6770	36	5408	39	4162	45	3017	54	1960	31
30	.3739	2.6746	.3939	2.5386	.4142	2.4142	.4348	2.2998	.4557	2.1943	**30**
31	42	6723	42	5365	46	4122	52	2980	61	1926	29
32	45	6699	46	5343	49	4102	55	2962	64	1909	28
33	49	6675	49	5322	52	4083	59	2944	68	1892	27
34	52	6652	53	5300	56	4063	62	2925	71	1876	26
35	.3755	2.6628	.3956	2.5279	.4159	2.4043	.4365	2.2907	.4575	2.1859	**25**
36	59	6605	59	5257	63	4023	69	2889	78	1842	24
37	62	6581	63	5236	66	4004	72	2871	82	1825	23
38	65	6558	66	5214	69	3984	76	2853	85	1808	22
39	69	6534	69	5193	73	3964	79	2835	89	1792	21
40	.3772	2.6511	.3973	2.5172	.4176	2.3945	.4383	2.2817	.4592	2.1775	**20**
41	75	6488	76	5150	80	3925	86	2799	96	1758	19
42	79	6464	79	5129	83	3906	90	2781	99	1742	18
43	82	6441	83	5108	87	3886	93	2763	4603	1725	17
44	85	6418	86	5086	90	3867	97	2745	07	1708	16
45	.3789	2.6395	.3990	2.5065	.4193	2.3847	.4400	2.2727	.4610	2.1692	**15**
46	92	6371	93	5044	97	3828	04	2709	14	1675	14
47	95	6348	96	5023	4200	3808	07	2691	17	1659	13
48	99	6325	4000	5002	04	3789	11	2673	21	1642	12
49	3802	6302	03	4981	07	3770	14	2655	24	1625	11
50	.3805	2.6279	.4006	2.4960	.4210	2.3750	.4417	2.2637	.4628	2.1609	**10**
51	09	6256	10	4939	14	3731	21	2620	31	1592	9
52	12	6233	13	4918	17	3712	24	2602	35	1576	8
53	15	6210	17	4897	21	3693	28	2584	38	1560	7
54	19	6187	20	4876	24	3673	31	2566	42	1543	6
55	.3822	2.6165	.4023	2.4855	.4228	2.3654	.4435	2.2549	.4645	2.1527	**5**
56	25	6142	27	4834	31	3635	38	2531	49	1510	4
57	29	6119	30	4813	34	3616	42	2513	52	1494	3
58	32	6096	33	4792	38	3597	45	2496	56	1478	2
59	35	6074	37	4772	41	3578	49	2478	60	1461	1
60	.3839	2.6051	.4040	2.4751	.4245	2.3559	.4452	2.2460	.4663	2.1445	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	69°		68°		67°		66°		65°		′

′	25°		26°		27°		28°		29°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	.4226	.9063	.4384	.8988	.4540	.8910	.4695	.8829	.4848	.8746	**60**
1	29	62	86	87	42	09	97	28	51	45	59
2	31	61	89	85	45	07	4700	27	53	43	58
3	34	59	92	84	48	06	02	25	56	42	57
4	37	58	94	83	50	05	05	24	58	41	56
5	.4239	.9057	.4397	.8982	.4553	.8903	.4708	.8823	.4861	.8739	**55**
6	42	56	99	80	55	02	10	21	63	38	54
7	45	54	4402	79	58	01	13	20	66	36	53
8	47	53	05	78	61	8899	15	19	68	35	52
9	50	52	07	76	63	98	18	17	71	33	51
10	.4253	.9051	.4410	.8975	.4566	.8897	.4720	.8816	.4874	.8732	**50**
11	55	50	12	74	68	95	23	14	76	31	49
12	58	48	15	73	71	94	26	13	79	29	48
13	60	47	18	71	74	93	28	12	81	28	47
14	63	46	20	70	76	92	31	10	84	26	46
15	.4266	.9045	.4423	.8969	.4579	.8890	.4733	.8809	.4886	.8725	**45**
16	68	43	25	67	81	89	36	08	89	24	44
17	71	42	28	66	84	88	38	06	91	22	43
18	74	41	31	65	86	86	41	05	94	21	42
19	76	40	33	64	89	85	43	03	96	19	41
20	.4279	.9038	.4436	.8962	.4592	.8884	.4746	.8802	.4899	.8718	**40**
21	81	37	39	61	94	82	49	01	4901	16	39
22	84	36	41	60	97	81	51	8799	04	15	38
23	87	35	44	58	99	79	54	98	07	14	37
24	89	33	46	57	4602	78	56	96	09	12	36
25	.4292	.9032	.4449	.8956	.4605	.8877	.4759	.8795	.4912	.8711	**35**
26	95	31	52	55	07	75	61	94	14	09	34
27	97	30	54	53	10	74	64	92	17	08	33
28	4300	28	57	52	12	73	66	91	19	06	32
29	02	27	59	51	15	71	69	90	22	05	31
30	.4305	.9026	.4462	.8949	.4617	.8870	.4772	.8788	.4924	.8704	**30**
31	08	25	65	48	20	69	74	87	27	02	29
32	10	23	67	47	23	67	77	85	29	01	28
33	13	22	70	45	25	66	79	84	32	8699	27
34	16	21	72	44	28	65	82	83	34	98	26
35	.4318	.9020	.4475	.8943	.4630	.8863	.4784	.8781	.4937	.8696	**25**
36	21	18	78	42	33	62	87	80	39	95	24
37	23	17	80	40	36	61	89	78	42	94	23
38	26	16	83	39	38	59	92	77	44	92	22
39	29	15	85	38	41	58	95	76	47	91	21
40	.4331	.9013	.4488	.8936	.4643	.8857	.4797	.8774	.4950	.8689	**20**
41	34	12	91	35	46	55	4800	73	52	88	19
42	37	11	93	34	48	54	02	71	55	86	18
43	39	10	96	32	51	53	05	70	57	85	17
44	42	08	98	31	54	51	07	69	60	83	16
45	.4344	.9007	.4501	.8930	.4656	.8850	.4810	.8767	.4962	.8682	**15**
46	47	06	04	28	59	49	12	66	65	81	14
47	50	04	06	27	61	47	15	64	67	79	13
48	52	03	09	26	64	46	18	63	70	78	12
49	55	02	11	25	66	44	20	62	72	76	11
50	.4358	.9001	.4514	.8923	.4669	.8843	.4823	.8760	.4975	.8675	**10**
51	60	8999	17	22	72	42	25	59	77	73	9
52	63	98	19	21	74	40	28	57	80	72	8
53	65	97	22	19	77	39	30	56	82	70	7
54	68	96	24	18	79	38	33	55	85	69	6
55	.4371	.8994	.4527	.8917	.4682	.8836	.4835	.8753	.4987	.8668	**5**
56	73	93	30	15	84	35	38	52	90	66	4
57	76	92	32	14	87	34	40	50	92	65	3
58	78	90	35	13	90	32	43	49	95	63	2
59	81	89	37	11	92	31	46	48	97	62	1
60	.4384	.8988	.4540	.8910	.4695	.8829	.4848	.8746	.5000	.8660	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	64°		63°		62°		61°		60°		′

′	25°		26°		27°		28°		29°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	.4663	2.1445	.4877	2.0503	.5095	1.9626	.5317	1.8807	.5543	1.8040	60
1	67	1429	81	0488	99	9612	21	8794	47	8028	59
2	70	1413	85	0473	5103	9598	25	8781	51	8016	58
3	74	1396	88	0458	06	9584	28	8768	55	8003	57
4	77	1380	92	0443	10	9570	32	8755	58	7991	56
5	.4681	2.1364	.4895	2.0428	.5114	1.9556	.5336	1.8741	.5562	1.7979	55
6	84	1348	99	0413	17	9542	40	8728	66	7966	54
7	88	1332	4903	0398	21	9528	43	8715	70	7954	53
8	91	1315	06	0383	25	9514	47	8702	74	7942	52
9	95	1299	10	0368	28	9500	51	8689	77	7930	51
10	.4699	2.1283	.4913	2.0353	.5132	1.9486	.5354	1.8676	.5581	1.7917	50
11	4702	1267	17	0338	36	9472	58	8663	85	7905	49
12	06	1251	21	0323	39	9458	62	8650	89	7893	48
13	09	1235	24	0308	43	9444	66	8637	93	7881	47
14	13	1219	28	0293	47	9430	69	8624	96	7868	46
15	.4716	2.1203	.4931	2.0278	.5150	1.9416	.5373	1.8611	.5600	1.7856	45
16	20	1187	35	0263	54	9402	77	8598	04	7844	44
17	23	1171	39	0248	58	9388	81	8585	08	7832	43
18	27	1155	42	0233	61	9375	84	8572	12	7820	42
19	31	1139	46	0219	65	9361	88	8559	16	7808	41
20	.4734	2.1123	.4950	2.0204	.5169	1.9347	.5392	1.8546	.5619	1.7796	40
21	38	1107	53	0189	72	9333	96	8533	23	7783	39
22	41	1092	57	0174	76	9319	99	8520	27	7771	38
23	45	1076	60	0160	80	9306	5403	8507	31	7759	37
24	48	1060	64	0145	84	9292	07	8495	35	7747	36
25	.4752	2.1044	.4968	2.0130	.5187	1.9278	.5411	1.8482	.5639	1.7735	35
26	55	1028	71	0115	91	9265	15	8469	42	7723	34
27	59	1013	75	0101	95	9251	18	8456	46	7711	33
28	63	0997	79	0086	98	9237	22	8443	50	7699	32
29	66	0981	82	0072	5202	9223	26	8430	54	7687	31
30	.4770	2.0965	.4986	2.0057	.5206	1.9210	.5430	1.8418	.5658	1.7675	30
31	73	0950	89	0042	09	9196	33	8405	62	7663	29
32	77	0934	93	0028	13	9183	37	8392	65	7651	28
33	80	0918	97	0013	17	9169	41	8379	69	7639	27
34	84	0903	5000	1.9999	20	9155	45	8367	73	7627	26
35	.4788	2.0887	.5004	1.9984	.5224	1.9142	.5448	1.8354	.5677	1.7615	25
36	91	0872	08	9970	28	9128	52	8341	81	7603	24
37	95	0856	11	9955	32	9115	56	8329	85	7591	23
38	98	0840	15	9941	35	9101	60	8316	88	7579	22
39	4802	0825	19	9926	39	9088	64	8303	92	7567	21
40	.4806	2.0809	.5022	1.9912	.5243	1.9074	.5467	1.8291	.5696	1.7556	20
41	09	0794	26	9897	46	9061	71	8278	5700	7544	19
42	13	0778	29	9883	50	9047	75	8265	04	7532	18
43	16	0763	33	9868	54	9034	79	8253	08	7520	17
44	20	0748	37	9854	58	9020	82	8240	12	7508	16
45	.4823	2.0732	.5040	1.9840	.5261	1.9007	.5486	1.8228	.5715	1.7496	15
46	27	0717	44	9825	65	8993	90	8215	19	7485	14
47	31	0701	48	9811	69	8980	94	8202	23	7473	13
48	34	0686	51	9797	72	8967	98	8190	27	7461	12
49	38	0671	55	9782	76	8953	5501	8177	31	7449	11
50	.4841	2.0655	.5059	1.9768	.5280	1.8940	.5505	1.8165	.5735	1.7437	10
51	45	0640	62	9754	84	8927	09	8152	39	7426	9
52	49	0625	66	9740	87	8913	13	8140	43	7414	8
53	52	0609	70	9725	91	8900	17	8127	46	7402	7
54	56	0594	73	9711	95	8887	20	8115	50	7391	6
55	.4859	2.0579	.5077	1.9697	.5298	1.8873	.5524	1.8103	.5754	1.7379	5
56	63	0564	81	9683	5302	8860	28	8090	58	7367	4
57	67	0549	84	9669	06	8847	32	8078	62	7355	3
58	70	0533	88	9654	10	8834	35	8065	66	7344	2
59	74	0518	92	9640	13	8820	39	8053	70	7332	1
60	.4877	2.0503	.5095	1.9626	.5317	1.8807	.5543	1.8040	.5774	1.7321	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	64°		63°		62°		61°		60°		′

′	30°		31°		32°		33°		34°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	.5000	.8660	.5150	.8572	.5299	.8480	.5446	.8387	.5592	.8290	60
1	03	59	53	70	5302	79	49	85	94	89	59
2	05	57	55	69	04	77	51	84	97	87	58
3	08	56	58	67	07	76	54	82	99	85	57
4	10	54	60	66	09	74	56	80	5602	84	56
5	.5013	.8653	.5163	.8564	.5312	.8473	.5459	.8379	.5604	.8282	55
6	15	52	65	63	14	71	61	77	06	81	54
7	18	50	68	61	16	70	63	76	09	79	53
8	20	49	70	60	19	68	66	74	11	77	52
9	23	47	73	58	21	67	68	72	14	76	51
10	.5025	.8646	.5175	.8557	.5324	.8465	.5471	.8371	.5616	.8274	50
11	28	44	78	55	26	63	73	69	18	72	49
12	30	43	80	54	29	62	76	68	21	71	48
13	33	41	83	52	31	60	78	66	23	69	47
14	35	40	85	51	34	59	80	64	26	68	46
15	.5038	.8638	.5188	.8549	.5336	.8457	.5483	.8363	.5628	.8266	45
16	40	37	90	48	39	56	85	61	30	64	44
17	43	35	93	46	41	54	88	60	33	63	43
18	45	34	95	45	44	53	90	58	35	61	42
19	48	32	98	43	46	51	93	56	38	59	41
20	.5050	.8631	.5200	.8542	.5348	.8450	.5495	.8355	.5640	.8258	40
21	53	30	03	40	51	48	98	53	42	56	39
22	55	28	05	39	53	46	5500	52	45	54	38
23	58	27	08	37	56	45	02	50	47	53	37
24	60	25	10	36	58	43	05	48	50	51	36
25	.5063	.8624	.5213	.8534	.5361	.8442	.5507	.8347	.5652	.8249	35
26	65	22	15	32	63	40	10	45	54	48	34
27	68	21	18	31	66	39	12	44	57	46	33
28	70	19	20	29	68	37	15	42	59	45	32
29	73	18	23	28	71	35	17	40	62	43	31
30	.5075	.8616	.5225	.8526	.5373	.8434	.5519	.8339	.5664	.8241	30
31	78	15	27	25	75	32	22	37	66	40	29
32	80	13	30	23	78	31	24	36	69	38	28
33	83	12	32	22	80	29	27	34	71	36	27
34	85	10	35	20	83	28	29	32	74	35	26
35	.5088	.8609	.5237	.8519	.5385	.8426	.5531	.8331	.5676	.8233	25
36	90	07	40	17	88	25	34	29	78	31	24
37	93	06	42	16	90	23	36	28	81	30	23
38	95	04	45	14	93	21	39	26	83	28	22
39	98	03	47	13	95	20	41	24	86	26	21
40	.5100	.8601	.5250	.8511	.5398	.8418	.5544	.8323	.5688	.8225	20
41	03	00	52	10	5400	17	46	21	90	23	19
42	05	8599	55	08	02	15	48	20	93	21	18
43	08	97	57	07	05	14	51	18	95	20	17
44	10	96	60	05	07	12	53	16	98	18	16
45	.5113	.8594	.5262	.8504	.5410	.8410	.5556	.8315	.5700	.8216	15
46	15	93	65	02	12	09	58	13	02	15	14
47	18	91	67	00	15	07	61	11	05	13	13
48	20	90	70	8499	17	06	63	10	07	11	12
49	23	88	72	97	20	04	65	08	10	10	11
50	.5125	.8587	.5275	.8496	.5422	.8403	.5568	.8307	.5712	.8208	10
51	28	85	77	94	24	01	70	05	14	07	9
52	30	84	79	93	27	8399	73	03	17	05	8
53	33	82	82	91	29	98	75	02	19	03	7
54	35	81	84	90	32	96	77	00	21	02	6
55	.5138	.8579	.5287	.8488	.5434	.8395	.5580	.8298	.5724	.8200	5
56	40	78	89	87	37	93	82	97	26	8198	4
57	43	76	92	85	39	91	85	95	29	97	3
58	45	75	94	84	42	90	87	94	31	95	2
59	48	73	97	82	44	88	90	92	33	93	1
60	.5150	.8572	.5299	.8480	.5446	.8387	.5592	.8290	.5736	.8192	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	59°		58°		57°		56°		55°		′

′	30° tan	30° cot	31° tan	31° cot	32° tan	32° cot	33° tan	33° cot	34° tan	34° cot	′
0	.5774	1.7321	.6009	1.6643	.6249	1.6003	.6494	1.5399	.6745	1.4826	60
1	77	7309	13	6632	53	5993	98	5389	49	4816	59
2	81	7297	17	6621	57	5983	6502	5379	54	4807	58
3	85	7286	20	6610	61	5972	06	5369	58	4798	57
4	89	7274	24	6599	65	5962	11	5359	62	4788	56
5	.5793	1.7262	.6028	1.6588	.6269	1.5952	.6515	1.5350	.6766	1.4779	55
6	97	7251	32	6577	73	5941	19	5340	71	4770	54
7	5801	7239	36	6566	77	5931	23	5330	75	4761	53
8	05	7228	40	6555	81	5921	27	5320	79	4751	52
9	08	7216	44	6545	85	5911	31	5311	83	4742	51
10	.5812	1.7205	.6048	1.6534	.6289	1.5900	.6536	1.5301	.6787	1.4733	50
11	16	7193	52	6523	93	5890	40	5291	92	4724	49
12	20	7182	56	6512	97	5880	44	5282	96	4715	48
13	24	7170	60	6501	6301	5869	48	5272	6800	4705	47
14	28	7159	64	6490	05	5859	52	5262	05	4696	46
15	.5832	1.7147	.6068	1.6479	.6310	1.5849	.6556	1.5253	.6809	1.4687	45
16	36	7136	72	6469	14	5839	60	5243	13	4678	44
17	40	7124	76	6458	18	5829	65	5233	17	4669	43
18	44	7113	80	6447	22	5818	69	5224	22	4659	42
19	47	7102	84	6436	26	5808	73	5214	26	4650	41
20	.5851	1.7090	.6088	1.6426	.6330	1.5798	.6577	1.5204	.6830	1.4641	40
21	55	7079	92	6415	34	5788	81	5195	34	4632	39
22	59	7067	96	6404	38	5778	85	5185	39	4623	38
23	63	7056	6100	6393	42	5768	90	5175	43	4614	37
24	67	7045	04	6383	46	5757	94	5166	47	4605	36
25	.5871	1.7033	.6108	1.6372	.6350	1.5747	.6598	1.5156	.6851	1.4596	35
26	75	7022	12	6361	54	5737	6602	5147	56	4586	34
27	79	7011	16	6351	58	5727	06	5137	60	4577	33
28	83	6999	20	6340	63	5717	10	5127	64	4568	32
29	87	6988	24	6329	67	5707	15	5118	69	4559	31
30	.5890	1.6977	.6128	1.6319	.6371	1.5697	.6619	1.5108	.6873	1.4550	30
31	94	6965	32	6308	75	5687	23	5099	77	4541	29
32	98	6954	36	6297	79	5677	27	5089	81	4532	28
33	5902	6943	40	6287	83	5667	31	5080	86	4523	27
34	06	6932	44	6276	87	5657	36	5070	90	4514	26
35	.5910	1.6920	.6148	1.6265	.6391	1.5647	.6640	1.5061	.6894	1.4505	25
36	14	6909	52	6255	95	5637	44	5051	99	4496	24
37	18	6898	56	6244	99	5627	48	5042	6903	4487	23
38	22	6887	60	6234	6403	5617	52	5032	07	4478	22
39	26	6875	64	6223	08	5607	57	5023	11	4469	21
40	.5930	1.6864	.6168	1.6212	.6412	1.5597	.6661	1.5013	.6916	1.4460	20
41	34	6853	72	6202	16	5587	65	5004	20	4451	19
42	38	6842	76	6191	20	5577	69	4994	24	4442	18
43	42	6831	80	6181	24	5567	73	4985	29	4433	17
44	45	6820	84	6170	28	5557	78	4975	33	4424	16
45	.5949	1.6808	.6188	1.6160	.6432	1.5547	.6682	1.4966	.6937	1.4415	15
46	53	6797	92	6149	36	5537	86	4957	42	4406	14
47	57	6786	96	6139	40	5527	90	4947	46	4397	13
48	61	6775	6200	6128	45	5517	94	4938	50	4388	12
49	65	6764	04	6118	49	5507	99	4928	54	4379	11
50	.5969	1.6753	.6208	1.6107	.6453	1.5497	.6703	1.4919	.6959	1.4370	10
51	73	6742	12	6097	57	5487	07	4910	63	4361	9
52	77	6731	16	6087	61	5477	11	4900	67	4352	8
53	81	6720	20	6076	65	5468	15	4891	72	4344	7
54	85	6709	24	6066	69	5458	20	4882	76	4335	6
55	.5989	1.6698	.6228	1.6055	.6473	1.5448	.6724	1.4872	.6980	1.4326	5
56	93	6687	33	6045	78	5438	28	4863	85	4317	4
57	97	6676	37	6034	82	5428	32	4854	89	4308	3
58	6001	6665	41	6024	86	5418	37	4844	93	4299	2
59	05	6654	45	6014	90	5408	41	4835	98	4290	1
60	.6009	1.6643	.6249	1.6003	.6494	1.5399	.6745	1.4826	.7002	1.4281	0
′	59° cot	59° tan	58° cot	58° tan	57° cot	57° tan	56° cot	56° tan	55° cot	55° tan	′

′	35°		36°		37°		38°		39°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	.5736	.8192	.5878	.8090	.6018	.7986	.6157	.7880	.6293	.7771	**60**
1	38	90	80	88	20	85	59	78	95	70	59
2	41	88	83	87	23	83	61	77	98	68	58
3	43	87	85	85	25	81	63	75	6300	66	57
4	45	85	87	83	27	79	66	73	02	64	56
5	.5748	.8183	.5890	.8082	.6030	.7978	.6168	.7871	.6305	.7762	**55**
6	50	81	92	80	32	76	70	69	07	60	54
7	52	80	94	78	34	74	73	68	09	59	53
8	55	78	97	76	37	72	75	66	11	57	52
9	57	76	99	75	39	71	77	64	14	55	51
10	.5760	.8175	.5901	.8073	.6041	.7969	.6180	.7862	.6316	.7753	**50**
11	62	73	04	71	44	67	82	60	18	51	49
12	64	71	06	70	46	65	84	59	20	49	48
13	67	70	08	68	48	64	86	57	23	48	47
14	69	68	11	66	51	62	89	55	25	46	46
15	.5771	.8166	.5913	.8064	.6053	.7960	.6191	.7853	.6327	.7744	**45**
16	74	65	15	63	55	58	93	51	29	42	44
17	76	63	18	61	58	56	96	50	32	40	43
18	79	61	20	59	60	55	98	48	34	38	42
19	81	60	22	58	62	53	6200	46	36	37	41
20	.5783	.8158	.5925	.8056	.6065	.7951	.6202	.7844	.6338	.7735	**40**
21	86	56	27	54	67	49	05	42	41	33	39
22	88	55	30	52	69	48	07	41	43	31	38
23	90	53	32	51	71	46	09	39	45	29	37
24	93	51	34	49	74	44	11	37	47	27	36
25	.5795	.8150	.5937	.8047	.6076	.7942	.6214	.7835	.6350	.7725	**35**
26	98	48	39	45	78	41	16	33	52	24	34
27	5800	46	41	44	81	39	18	32	54	22	33
28	02	45	44	42	83	37	21	30	56	20	32
29	05	43	46	40	85	35	23	28	59	18	31
30	.5807	.8141	.5948	.8039	.6088	.7934	.6225	.7826	.6361	.7716	**30**
31	09	39	51	37	90	32	27	24	63	14	29
32	12	38	53	35	92	30	30	22	65	13	28
33	14	36	55	33	95	28	32	21	68	11	27
34	16	34	58	32	97	26	34	19	70	09	26
35	.5819	.8133	.5960	.8030	.6099	.7925	.6237	.7817	.6372	.7707	**25**
36	21	31	62	28	6101	23	39	15	74	05	24
37	24	29	65	26	04	21	41	13	76	03	23
38	26	28	67	25	06	19	43	12	79	01	22
39	28	26	69	23	08	18	46	10	81	00	21
40	.5831	.8124	.5972	.8021	.6111	.7916	.6248	.7808	.6383	.7698	**20**
41	33	23	74	19	13	14	50	06	85	96	19
42	35	21	76	18	15	12	52	04	88	94	18
43	38	19	79	16	18	10	55	02	90	92	17
44	40	17	81	14	20	09	57	01	92	90	16
45	.5842	.8116	.5983	.8013	.6122	.7907	.6259	.7799	.6394	.7688	**15**
46	45	14	86	11	24	05	62	97	97	87	14
47	47	12	88	09	27	03	64	95	99	85	13
48	50	11	90	07	29	02	66	93	6401	83	12
49	52	09	93	06	31	00	68	92	03	81	11
50	.5854	.8107	.5995	.8004	.6134	.7898	.6271	.7790	.6406	.7679	**10**
51	57	06	97	02	36	96	73	88	08	77	9
52	59	04	6000	00	38	94	75	86	10	75	8
53	61	02	02	7999	41	93	77	84	12	74	7
54	64	00	04	97	43	91	80	82	14	72	6
55	.5866	.8099	.6007	.7995	.6145	.7889	.6282	.7781	.6417	.7670	**5**
56	68	97	09	93	47	87	84	79	19	68	4
57	71	95	11	92	50	85	86	77	21	66	3
58	73	94	14	90	52	84	89	75	23	64	2
59	75	92	16	88	54	82	91	73	26	62	1
60	.5878	.8090	.6018	.7986	.6157	.7880	.6293	.7771	.6428	.7660	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	54°		53°		52°		51°		50°		′

′	35°		36°		37°		38°		39°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	.7002	1.4281	.7265	1.3764	.7536	1.3270	.7813	1.2799	.8098	1.2349	60
1	06	4273	70	3755	40	3262	18	2792	8103	2342	59
2	11	4264	74	3747	45	3254	22	2784	07	2334	58
3	15	4255	79	3739	49	3246	27	2776	12	2327	57
4	19	4246	83	3730	54	3238	32	2769	17	2320	56
5	.7024	1.4237	.7288	1.3722	.7558	1.3230	.7836	1.2761	.8122	1.2312	55
6	28	4229	92	3713	63	3222	41	2753	27	2305	54
7	32	4220	97	3705	68	3214	46	2746	32	2298	53
8	37	4211	7301	3697	72	3206	50	2738	36	2290	52
9	41	4202	06	3688	77	3198	55	2731	41	2283	51
10	.7046	1.4193	.7310	1.3680	.7581	1.3190	.7860	1.2723	.8146	1.2276	50
11	50	4185	14	3672	86	3182	65	2715	51	2268	49
12	54	4176	19	3663	90	3175	69	2708	56	2261	48
13	59	4167	23	3655	95	3167	74	2700	61	2254	47
14	63	4158	28	3647	7600	3159	79	2693	65	2247	46
15	.7067	1.4150	.7332	1.3638	.7604	1.3151	.7883	1.2685	.8170	1.2239	45
16	72	4141	37	3630	09	3143	88	2677	75	2232	44
17	76	4132	41	3622	13	3135	93	2670	80	2225	43
18	80	4124	46	3613	18	3127	98	2662	85	2218	42
19	85	4115	50	3605	23	3119	7902	2655	90	2210	41
20	.7089	1.4106	.7355	1.3597	.7627	1.3111	.7907	1.2647	.8195	1.2203	40
21	94	4097	59	3588	32	3103	12	2640	99	2196	39
22	98	4089	64	3580	36	3095	16	2632	8204	2189	38
23	7102	4080	68	3572	41	3087	21	2624	09	2181	37
24	07	4071	73	3564	46	3079	26	2617	14	2174	36
25	.7111	1.4063	.7377	1.3555	.7650	1.3072	.7931	1.2609	.8219	1.2167	35
26	15	4054	82	3547	55	3064	35	2602	24	2160	34
27	20	4045	86	3539	59	3056	40	2594	29	2153	33
28	24	4037	91	3531	64	3048	45	2587	34	2145	32
29	29	4028	95	3522	69	3040	50	2579	38	2138	31
30	.7133	1.4019	.7400	1.3514	.7673	1.3032	.7954	1.2572	.8243	1.2131	30
31	37	4011	04	3506	78	3024	59	2564	48	2124	29
32	42	4002	09	3498	83	3017	64	2557	53	2117	28
33	46	3994	13	3490	87	3009	69	2549	58	2109	27
34	51	3985	18	3481	92	3001	73	2542	63	2102	26
35	.7155	1.3976	.7422	1.3473	.7696	1.2993	.7978	1.2534	.8268	1.2095	25
36	59	3968	27	3465	7701	2985	83	2527	73	2088	24
37	64	3959	31	3457	06	2977	88	2519	78	2081	23
38	68	3951	36	3449	10	2970	92	2512	83	2074	22
39	73	3942	40	3440	15	2962	97	2504	87	2066	21
40	.7177	1.3934	.7445	1.3432	.7720	1.2954	.8002	1.2497	.8292	1.2059	20
41	81	3925	49	3424	24	2946	07	2489	97	2052	19
42	86	3916	54	3416	29	2938	12	2482	8302	2045	18
43	90	3908	58	3408	34	2931	16	2475	07	2038	17
44	95	3899	63	3400	38	2923	21	2467	12	2031	16
45	.7199	1.3891	.7467	1.3392	.7743	1.2915	.8026	1.2460	.8317	1.2024	15
46	7203	3882	72	3384	47	2907	31	2452	22	2017	14
47	08	3874	76	3375	52	2900	35	2445	27	2009	13
48	12	3865	81	3367	57	2892	40	2437	32	2002	12
49	17	3857	85	3359	61	2884	45	2430	37	1995	11
50	.7221	1.3848	.7490	1.3351	.7766	1.2876	.8050	1.2423	.8342	1.1988	10
51	26	3840	95	3343	71	2869	55	2415	46	1981	9
52	30	3831	99	3335	75	2861	59	2408	51	1974	8
53	34	3823	7504	3327	80	2853	64	2401	56	1967	7
54	39	3814	08	3319	85	2846	69	2393	61	1960	6
55	.7243	1.3806	.7513	1.3311	.7789	1.2838	.8074	1.2386	.8366	1.1953	5
56	48	3798	17	3303	94	2830	79	2378	71	1946	4
57	52	3789	22	3295	99	2822	83	2371	76	1939	3
58	57	3781	26	3287	7803	2815	88	2364	81	1932	2
59	61	3772	31	3278	08	2807	93	2356	86	1925	1
60	.7265	1.3764	.7536	1.3270	.7813	1.2799	.8098	1.2349	.8391	1.1918	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	54°		53°		52°		51°		50°		′

′	40°		41°		42°		43°		44°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	.6428	.7660	.6561	.7547	.6691	.7431	.6820	.7314	.6947	.7193	60
1	30	59	63	45	93	30	22	12	49	91	59
2	32	57	65	43	96	28	24	10	51	89	58
3	35	55	67	41	98	26	26	08	53	87	57
4	37	53	69	39	6700	24	28	06	55	85	56
5	.6439	.7651	.6572	.7538	.6702	.7422	.6831	.7304	.6957	.7183	55
6	41	49	74	36	04	20	33	02	59	81	54
7	43	47	76	34	06	18	35	00	61	79	53
8	46	45	78	32	09	16	37	7298	63	77	52
9	48	44	80	30	11	14	39	96	65	75	51
10	.6450	.7642	.6583	.7528	.6713	.7412	.6841	.7294	.6967	.7173	50
11	52	40	85	26	15	10	43	92	70	71	49
12	55	38	87	24	17	08	45	90	72	69	48
13	57	36	89	22	19	06	48	88	74	67	47
14	59	34	91	20	22	04	50	86	76	65	46
15	.6461	.7632	.6593	.7518	.6724	.7402	.6852	.7284	.6978	.7163	45
16	63	30	96	16	26	00	54	82	80	61	44
17	66	29	98	15	28	7398	56	80	82	59	43
18	68	27	6600	13	30	96	58	78	84	57	42
19	70	25	02	11	32	94	60	76	86	55	41
20	.6472	.7623	.6604	.7509	.6734	.7392	.6862	.7274	.6988	.7153	40
21	75	21	07	07	37	90	65	72	90	51	39
22	77	19	09	05	39	88	67	70	92	49	38
23	79	17	11	03	41	87	69	68	95	47	37
24	81	15	13	01	43	85	71	66	97	45	36
25	.6483	.7613	.6615	.7499	.6745	.7383	.6873	.7264	.6999	.7143	35
26	86	12	17	97	47	81	75	62	7001	41	34
27	88	10	20	95	49	79	77	60	03	39	33
28	90	08	22	93	52	77	79	58	05	37	32
29	92	06	24	91	54	75	81	56	07	35	31
30	.6494	.7604	.6626	.7490	.6756	.7373	.6884	.7254	.7009	.7133	30
31	97	02	28	88	58	71	86	52	11	30	29
32	99	00	31	86	60	69	88	50	13	28	28
33	6501	7598	33	84	62	67	90	48	15	26	27
34	03	96	35	82	64	65	92	46	17	24	26
35	.6506	.7595	.6637	.7480	.6767	.7363	.6894	.7244	.7019	.7122	25
36	08	93	39	78	69	61	96	42	22	20	24
37	10	91	41	76	71	59	98	40	24	18	23
38	12	89	44	74	73	57	6900	38	26	16	22
39	14	87	46	72	75	55	03	36	28	14	21
40	.6517	.7585	.6648	.7470	.6777	.7353	.6905	.7234	.7030	.7112	20
41	19	83	50	68	79	51	07	32	32	10	19
42	21	81	52	66	82	49	09	30	34	08	18
43	23	79	54	64	84	47	11	28	36	06	17
44	25	78	57	63	86	45	13	26	38	04	16
45	.6528	.7576	.6659	.7461	.6788	.7343	.6915	.7224	.7040	.7102	15
46	30	74	61	59	90	41	17	22	42	00	14
47	32	72	63	57	92	39	19	20	44	7098	13
48	34	70	65	55	94	37	21	18	46	96	12
49	36	68	67	53	97	35	24	16	48	94	11
50	.6539	.7566	.6670	.7451	.6799	.7333	.6926	.7214	.7050	.7092	10
51	41	64	72	49	6801	31	28	12	53	90	9
52	43	62	74	47	03	29	30	10	55	88	8
53	45	60	76	45	05	27	32	08	57	85	7
54	47	59	78	43	07	25	34	06	59	83	6
55	.6550	.7557	.6680	.7441	.6809	.7323	.6936	.7203	.7061	.7081	5
56	52	55	83	39	11	21	38	01	63	79	4
57	54	53	85	37	14	19	40	7199	65	77	3
58	56	51	87	35	16	18	42	97	67	75	2
59	58	49	89	33	18	16	44	95	69	73	1
60	.6561	.7547	.6691	.7431	.6820	.7314	.6947	.7193	.7071	.7071	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	49°		48°		47°		46°		45°		′

′	40°		41°		42°		43°		44°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	.8391	1.1918	.8693	1.1504	.9004	1.1106	.9325	1.0724	.9657	1.0355	**60**
1	96	1910	98	1497	09	1100	31	0717	63	0349	59
2	8401	1903	8703	1490	15	1093	36	0711	68	0343	58
3	06	1896	08	1483	20	1087	41	0705	74	0337	57
4	11	1889	13	1477	25	1080	47	0699	79	0331	56
5	.8416	1.1882	.8718	1.1470	.9030	1.1074	.9352	1.0692	.9685	1.0325	**55**
6	21	1875	24	1463	36	1067	58	0686	91	0319	54
7	26	1868	29	1456	41	1061	63	0680	96	0313	53
8	31	1861	34	1450	46	1054	69	0674	9702	0307	52
9	36	1854	39	1443	52	1048	74	0668	08	0301	51
10	.8441	1.1847	.8744	1.1436	.9057	1.1041	.9380	1.0661	.9713	1.0295	**50**
11	46	1840	49	1430	62	1035	85	0655	19	0289	49
12	51	1833	54	1423	67	1028	91	0649	25	0283	48
13	56	1826	59	1416	73	1022	96	0643	30	0277	47
14	61	1819	65	1410	78	1016	9402	0637	36	0271	46
15	.8466	1.1812	.8770	1.1403	.9083	1.1009	.9407	1.0630	.9742	1.0265	**45**
16	71	1806	75	1396	89	1003	13	0624	47	0259	44
17	76	1799	80	1389	94	0996	18	0618	53	0253	43
18	81	1792	85	1383	99	0990	24	0612	59	0247	42
19	86	1785	90	1376	9105	0983	29	0606	64	0241	41
20	.8491	1.1778	.8796	1.1369	.9110	1.0977	.9435	1.0599	.9770	1.0235	**40**
21	96	1771	8801	1363	15	0971	40	0593	76	0230	39
22	8501	1764	06	1356	21	0964	46	0587	81	0224	38
23	06	1757	11	1349	26	0958	51	0581	87	0218	37
24	11	1750	16	1343	31	0951	57	0575	93	0212	36
25	.8516	1.1743	.8821	1.1336	.9137	1.0945	.9462	1.0569	.9798	1.0206	**35**
26	21	1736	27	1329	42	0939	68	0562	9804	0200	34
27	26	1729	32	1323	47	0932	73	0556	10	0194	33
28	31	1722	37	1316	53	0926	79	0550	16	0188	32
29	36	1715	42	1310	58	0919	84	0544	21	0182	31
30	.8541	1.1708	.8847	1.1303	.9163	1.0913	.9490	1.0538	.9827	1.0176	**30**
31	46	1702	52	1296	69	0907	95	0532	33	0170	29
32	51	1695	58	1290	74	0900	9501	0526	38	0164	28
33	56	1688	63	1283	79	0894	06	0519	44	0158	27
34	61	1681	68	1276	85	0888	12	0513	50	0152	26
35	.8566	1.1674	.8873	1.1270	.9190	1.0881	.9517	1.0507	.9856	1.0147	**25**
36	71	1667	78	1263	95	0875	23	0501	61	0141	24
37	76	1660	84	1257	9201	0869	28	0495	67	0135	23
38	81	1653	89	1250	06	0862	34	0489	73	0129	22
39	86	1647	94	1243	12	0856	40	0483	79	0123	21
40	.8591	1.1640	.8899	1.1237	.9217	1.0850	.9545	1.0477	.9884	1.0117	**20**
41	96	1633	8904	1230	22	0843	51	0470	90	0111	19
42	8601	1626	10	1224	28	0837	56	0464	96	0105	18
43	06	1619	15	1217	33	0831	62	0458	9902	0099	17
44	11	1612	20	1211	39	0824	67	0452	07	0094	16
45	.8617	1.1606	.8925	1.1204	.9244	1.0818	.9573	1.0446	.9913	1.0088	**15**
46	22	1599	31	1197	49	0812	78	0440	19	0082	14
47	27	1592	36	1191	55	0805	84	0434	25	0076	13
48	32	1585	41	1184	60	0799	90	0428	30	0070	12
49	37	1578	46	1178	66	0793	95	0422	36	0064	11
50	.8642	1.1571	.8952	1.1171	.9271	1.0786	.9601	1.0416	.9942	1.0058	**10**
51	47	1565	57	1165	76	0780	06	0410	48	0052	9
52	52	1558	62	1158	82	0774	12	0404	54	0047	8
53	57	1551	67	1152	87	0768	18	0398	59	0041	7
54	62	1544	72	1145	93	0761	23	0392	65	0035	6
55	.8667	1.1538	.8978	1.1139	.9298	1.0755	.9629	1.0385	.9971	1.0029	**5**
56	72	1531	83	1132	9303	0749	34	0379	77	0023	4
57	78	1524	88	1126	09	0742	40	0373	83	0017	3
58	83	1517	94	1119	14	0736	46	0367	88	0012	2
59	88	1510	99	1113	20	0730	51	0361	94	0006	1
60	.8693	1.1504	.9004	1.1106	.9325	1.0724	.9657	1.0355	1.0000	1.0000	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	49°		48°		47°		46°		45°		′

TABLE IV

MISCELLANEOUS TABLES

1. Radian Measure, 0° to 180°.
2. Natural Logarithms:
   - (*A*) Numbers from 1 to 200.
   - (*B*) Numbers from 1 to 9.9.
3. The Hyperbolic Functions, Sinh x, Cosh x.

TABLE IV. 1. RADIAN MEASURE, 0° TO 180°, RADIUS = 1

DEGREES						MINUTES		SECONDS	
0°	0.00000 00	**60°**	1.04719 76	**120°**	2.09439 51	**0′**	0.00000 00	**0″**	0.00000 00
1	0.01745 33	61	1.06465 08	121	2.11184 84	1	0.00029 09	1	0.00000 48
2	0.03490 66	62	1.08210 41	122	2.12930 17	2	0.00058 18	2	0.00000 97
3	0.05235 99	63	1.09955 74	123	2.14675 50	3	0.00087 27	3	0.00001 45
4	0.06981 32	64	1.11701 07	124	2.16420 83	4	0.00116 36	4	0.00001 94
5	0.08726 65	65	1.13446 40	125	2.18166 16	5	0.00145 44	5	0.00002 42
6	0.10471 98	66	1.15191 73	126	2.19911 49	6	0.00174 53	6	0.00002 91
7	0.12217 30	67	1.16937 06	127	2.21656 82	7	0.00203 62	7	0.00003 39
8	0.13962 63	68	1.18682 39	128	2.23402 14	8	0.00232 71	8	0.00003 88
9	0.15707 96	69	1.20427 72	129	2.25147 47	9	0.00261 80	9	0.00004 36
10	0.17453 29	**70**	1.22173 05	**130**	2.26892 80	**10**	0.00290 89	**10**	0.00004 85
11	0.19198 62	71	1.23918 38	131	2.28638 13	11	0.00319 98	11	0.00005 33
12	0.20943 95	72	1.25663 71	132	2.30383 46	12	0.00349 07	12	0.00005 82
13	0.22689 28	73	1.27409 04	133	2.32128 79	13	0.00378 15	13	0.00006 30
14	0.24434 61	74	1.29154 36	134	2.33874 12	14	0.00407 24	14	0.00006 79
15	0.26179 94	75	1.30899 69	135	2.35619 45	15	0.00436 33	15	0.00007 27
16	0.27925 27	76	1.32645 02	136	2.37364 78	16	0.00465 42	16	0.00007 76
17	0.29670 60	77	1.34390 35	137	2.39110 11	17	0.00494 51	17	0.00008 24
18	0.31415 93	78	1.36135 68	138	2.40855 44	18	0.00523 60	18	0.00008 73
19	0.33161 26	79	1.37881 01	139	2.42600 77	19	0.00552 69	19	0.00009 21
20	0.34906 59	**80**	1.39626 34	**140**	2.44346 10	**20**	0.00581 78	**20**	0.00009 70
21	0.36651 91	81	1.41371 67	141	2.46091 42	21	0.00610 87	21	0.00010 18
22	0.38397 24	82	1.43117 00	142	2.47836 75	22	0.00639 95	22	0.00010 67
23	0.40142 57	83	1.44862 33	143	2.49582 08	23	0.00669 04	23	0.00011 15
24	0.41887 90	84	1.46607 66	144	2.51327 41	24	0.00698 13	24	0.00011 64
25	0.43633 23	85	1.48352 99	145	2.53072 74	25	0.00727 22	25	0.00012 12
26	0.45378 56	86	1.50098 32	146	2.54818 07	26	0.00756 31	26	0.00012 61
27	0.47123 89	87	1.51843 64	147	2.56563 40	27	0.00785 40	27	0.00013 09
28	0.48869 22	88	1.53588 97	148	2.58308 73	28	0.00814 49	28	0.00013 57
29	0.50614 55	89	1.55334 30	149	2.60054 06	29	0.00843 58	29	0.00014 06
30	0.52359 88	**90**	1.57079 63	**150**	2.61799 39	**30**	0.00872 66	**30**	0.00014 54
31	0.54105 21	91	1.58824 96	151	2.63544 72	31	0.00901 75	31	0.00015 03
32	0.55850 54	92	1.60570 29	152	2.65290 05	32	0.00930 84	32	0.00015 51
33	0.57595 87	93	1.62315 62	153	2.67035 38	33	0.00959 93	33	0.00016 00
34	0.59341 19	94	1.64060 95	154	2.68780 70	34	0.00989 02	34	0.00016 48
35	0.61086 52	95	1.65806 28	155	2.70526 03	35	0.01018 11	35	0.00016 97
36	0.62831 85	96	1.67551 61	156	2.72271 36	36	0.01047 20	36	0.00017 45
37	0.64577 18	97	1.69296 94	157	2.74016 69	37	0.01076 29	37	0.00017 94
38	0.66322 51	98	1.71042 27	158	2.75762 02	38	0.01105 38	38	0.00018 42
39	0.68067 84	99	1.72787 60	159	2.77507 35	39	0.01134 46	39	0.00018 91
40	0.69813 17	**100**	1.74532 93	**160**	2.79252 68	**40**	0.01163 55	**40**	0.00019 39
41	0.71558 50	101	1.76278 25	161	2.80998 01	41	0.01192 64	41	0.00019 88
42	0.73303 83	102	1.78023 58	162	2.82743 34	42	0.01221 73	42	0.00020 36
43	0.75049 16	103	1.79768 91	163	2.84488 67	43	0.01250 82	43	0.00020 85
44	0.76794 49	104	1.81514 24	164	2.86234 00	44	0.01279 91	44	0.00021 33
45	0.78539 82	105	1.83259 57	165	2.87979 33	45	0.01309 00	45	0.00021 82
46	0.80285 15	106	1.85004 90	166	2.89724 66	46	0.01338 09	46	0.00022 30
47	0.82030 47	107	1.86750 23	167	2.91469 99	47	0.01367 17	47	0.00022 79
48	0.83775 80	108	1.88495 56	168	2.93215 31	48	0.01396 26	48	0.00023 27
49	0.85521 13	109	1.90240 89	169	2.94960 64	49	0.01425 35	49	0.00023 76
50	0.87266 46	**110**	1.91986 22	**170**	2.96705 97	**50**	0.01454 44	**50**	0.00024 24
51	0.89011 79	111	1.93731 55	171	2.98451 30	51	0.01483 53	51	0.00024 73
52	0.90757 12	112	1.95476 88	172	3.00196 63	52	0.01512 62	52	0.00025 21
53	0.92502 45	113	1.97222 21	173	3.01941 96	53	0.01541 71	53	0.00025 70
54	0.94247 78	114	1.98967 53	174	3.03687 29	54	0.01570 80	54	0.00026 18
55	0.95993 11	115	2.00712 86	175	3.05432 62	55	0.01599 89	55	0.00026 66
56	0.97738 44	116	2.02458 19	176	3.07177 95	56	0.01628 97	56	0.00027 15
57	0.99483 77	117	2.04203 52	177	3.08923 28	57	0.01658 06	57	0.00027 63
58	1.01229 10	118	2.05948 85	178	3.10668 61	58	0.01687 15	58	0.00028 12
59	1.02974 43	119	2.07694 18	179	3.12413 94	59	0.01716 24	59	0.00028 60
60	1.04719 76	**120**	2.09439 51	**180**	3.14159 27	**60**	0.01745 33	**60**	0.00029 09
DEGREES						MINUTES		SECONDS	

A. THE NATURAL LOGARITHMS

OF

INTEGERS FROM 1 TO 200

Base $e = 2.7182818284\cdots$
$\log_{10} e = 0.4342944819\cdots$
$\log_e 10 = 2.3025850929\cdots$
$\log_e \pi = 1.1447298858\cdots$

Conversion Laws:

$\log_e N = \log_e 10 \times \log_{10} N.$
$\log_{10} N = \log_{10} e \times \log_e N.$

N	log_e	N	log_e	N	log_e	N	log_e	N	log_e
0	− ∞	**40**	3.68 888	**80**	4.38 203	**120**	4.78 749	**160**	5.07 517
1	0.00 000	41	3.71 357	81	4.39 445	121	4.79 579	161	5.08 140
2	0.69 315	42	3.73 767	82	4.40 672	122	4.80 402	162	5.08 760
3	1.09 861	43	3.76 120	83	4.41 884	123	4.81 218	163	5.09 375
4	1.38 629	44	3.78 419	84	4.43 082	124	4.82 028	164	5.09 987
5	1.60 944	45	3.80 666	85	4.44 265	125	4.82 831	165	5.10 595
6	1.79 176	46	3.82 864	86	4.45 435	126	4.83 628	166	5.11 199
7	1.94 591	47	3.85 015	87	4.46 591	127	4.84 419	167	5.11 799
8	2.07 944	48	3.87 120	88	4.47 734	128	4.85 203	168	5.12 396
9	2.19 722	49	3.89 182	89	4.48 864	129	4.85 981	169	5.12 990
10	2.30 259	**50**	3.91 202	**90**	4.49 981	**130**	4.86 753	**170**	5.13 580
11	2.39 790	51	3.93 183	91	4.51 086	131	4.87 520	171	5.14 166
12	2.48 491	52	3.95 124	92	4.52 179	132	4.88 280	172	5.14 749
13	2.56 495	53	3.97 029	93	4.53 260	133	4.89 035	173	5.15 329
14	2.63 906	54	3.98 898	94	4.54 329	134	4.89 784	174	5.15 906
15	2.70 805	55	4.00 733	95	4.55 388	135	4.90 527	175	5.16 479
16	2.77 259	56	4.02 535	96	4.56 435	136	4.91 265	176	5.17 048
17	2.83 321	57	4.04 305	97	4.57 471	137	4.91 998	177	5.17 615
18	2.89 037	58	4.06 044	98	4.58 497	138	4.92 725	178	5.18 178
19	2.94 444	59	4.07 754	99	4.59 512	139	4.93 447	179	5.18 739
20	2.99 573	**60**	4.09 434	**100**	4.60 517	**140**	4.94 164	**180**	5.19 296
21	3.04 452	61	4.11 087	101	4.61 512	141	4.94 876	181	5.19 850
22	3.09 104	62	4.12 713	102	4.62 497	142	4.95 583	182	5.20 401
23	3.13 549	63	4.14 313	103	4.63 473	143	4.96 284	183	5.20 949
24	3.17 805	64	4.15 888	104	4.64 439	144	4.96 981	184	5.21 494
25	3.21 888	65	4.17 439	105	4.65 396	145	4.97 673	185	5.22 036
26	3.25 810	66	4.18 965	106	4.66 344	146	4.98 361	186	5.22 575
27	3.29 584	67	4.20 469	107	4.67 283	147	4.99 043	187	5.23 111
28	3.33 220	68	4.21 951	108	4.68 213	148	4.99 721	188	5.23 644
29	3.36 730	69	4.23 411	109	4.69 135	149	5.00 395	189	5.24 175
30	3.40 120	**70**	4.24 850	**110**	4.70 048	**150**	5.01 064	**190**	5.24 702
31	3.43 399	71	4.26 268	111	4.70 953	151	5.01 728	191	5.25 227
32	3.46 574	72	4.27 667	112	4.71 850	152	5.02 388	192	5.25 750
33	3.49 651	73	4.29 046	113	4.72 739	153	5.03 044	193	5.26 269
34	3.52 636	74	4.30 407	114	4.73 620	154	5.03 695	194	5.26 786
35	3.55 535	75	4.31 749	115	4.74 493	155	5.04 343	195	5.27 300
36	3.58 352	76	4.33 073	116	4.75 359	156	5.04 986	196	5.27 811
37	3.61 092	77	4.34 381	117	4.76 217	157	5.05 625	197	5.28 320
38	3.63 759	78	4.35 671	118	4.77 068	158	5.06 260	198	5.28 827
39	3.66 356	79	4.36 945	119	4.77 912	159	5.06 890	199	5.29 330
40	3.68 888	**80**	4.38 203	**120**	4.78 749	**160**	5.07 517	**200**	5.29 832

B. NATURAL LOGARITHMS 1 TO 9.9

The following table shows the Natural or Napierian logarithms, for each tenth, of numbers 1 to 9.9. Interpolation may be made for hundredths. The logarithms of numbers larger than 9.9 may be found as shown in the following illustration: *Let us find the* $\log_e 450$:

$$\log_e 450 = \log_e (4.5 \times 10^2) = \log_e 4.5 + 2 \log_e 10$$
$$= 1.5041 + 2\,(2.3026) = 6.1093.$$

No.	$\log_e$	No.	$\log_e$	No.	$\log_e$	No.	$\log_e$	No.	$\log_e$	No.	$\log_e$
1.0	0.0000	2.5	0.9163	4.0	1.3863	5.5	1.7048	7.0	1.9459	8.5	2.1401
1.1	.0953	2.6	.9555	4.1	.4110	5.6	.7228	7.1	.9601	8.6	.1518
1.2	.1823	2.7	.9933	4.2	.4351	5.7	.7405	7.2	.9741	8.7	.1633
1.3	.2624	2.8	1.0296	4.3	.4586	5.8	.7579	7.3	.9879	8.8	.1748
1.4	.3365	2.9	.0647	4.4	.4816	5.9	.7750	7.4	2.0015	8.9	.1861
1.5	0.4055	3.0	1.0986	4.5	1.5041	6.0	1.7917	7.5	2.0149	9.0	2.1972
1.6	.4700	3.1	.1314	4.6	.5261	6.1	.8083	7.6	.0282	9.1	.2083
1.7	.5306	3.2	.1632	4.7	.5476	6.2	.8246	7.7	.0412	9.2	.2192
1.8	.5878	3.3	.1939	4.8	.5686	6.3	.8406	7.8	.0541	9.3	.2300
1.9	.6419	3.4	.2238	4.9	.5892	6.4	.8563	7.9	.0669	9.4	.2407
2.0	0.6932	3.5	1.2528	5.0	1.6094	6.5	1.8718	8.0	2.0794	9.5	2.2513
2.1	.7419	3.6	.2809	5.1	.6292	6.6	.8871	8.1	.0919	9.6	.2618
2.2	.7885	3.7	.3083	5.2	.6487	6.7	.9021	8.2	.1041	9.7	.2721
2.3	.8329	3.8	.3350	5.3	.6677	6.8	.9169	8.3	.1163	9.8	.2824
2.4	.8755	3.9	.3610	5.4	.6864	6.9	.9315	8.4	.1282	9.9	.2925

TABLE IV

3. Hyperbolic Sinh $x = \frac{e^x - e^{-x}}{2}$, Cosh $x = \frac{e^x + e^{-x}}{2}$.

x	sinh x	cosh x	x	sinh x	cosh x
0.00	0.0000	1.0000	0.50	0.5211	1.1276
.01	.0100	1.0000	.51	.5324	1.1329
.02	.0200	1.0002	.52	.5438	1.1383
.03	.0300	1.0004	.53	.5552	1.1438
.04	.0400	1.0008	.54	.5666	1.1494
.05	.0500	1.0013	.55	.5782	1.1551
.06	.0600	1.0018	.56	.5897	1.1609
.07	.0701	1.0025	.57	.6014	1.1669
.08	.0801	1.0032	.58	.6131	1.1730
.09	.0901	1.0041	.59	.6248	1.1792
.10	.1002	1.0050	.60	.6367	1.1855
.11	.1102	1.0061	.61	.6485	1.1919
.12	.1203	1.0072	.62	.6605	1.1984
.13	.1304	1.0085	.63	.6725	1.2051
.14	.1405	1.0098	.64	.6846	1.2119
.15	.1506	1.0113	.65	.6967	1.2188
.16	.1607	1.0128	.66	.7090	1.2258
.17	.1708	1.0145	.67	.7213	1.2330
.18	.1810	1.0162	.68	.7336	1.2402
.19	.1911	1.0181	.69	.7461	1.2476
.20	.2013	1.0201	.70	.7586	1.2552
.21	.2115	1.0221	.71	.7712	1.2628
.22	.2218	1.0243	.72	.7838	1.2706
.23	.2320	1.0266	.73	.7966	1.2785
.24	.2423	1.0289	.74	.8094	1.2865
.25	.2526	1.0314	.75	.8223	1.2947
.26	.2629	1.0340	.76	.8353	1.3030
.27	.2733	1.0367	.77	.8484	1.3114
.28	.2837	1.0395	.78	.8615	1.3199
.29	.2941	1.0423	.79	.8748	1.3286
.30	.3045	1.0453	.80	.8881	1.3374
.31	.3150	1.0484	.81	.9015	1.3464
.32	.3255	1.0516	.82	.9150	1.3555
.33	.3360	1.0549	.83	.9286	1.3647
.34	.3466	1.0584	.84	.9423	1.3740
.35	.3572	1.0619	.85	.9561	1.3835
.36	.3678	1.0655	.86	.9700	1.3932
.37	.3785	1.0692	.87	.9840	1.4029
.38	.3892	1.0731	.88	.9981	1.4128
.39	.4000	1.0770	.89	1.0122	1.4229
.40	.4108	1.0811	.90	1.0265	1.4331
.41	.4216	1.0852	.91	1.0409	1.4434
.42	.4325	1.0895	.92	1.0554	1.4539
.43	.4434	1.0939	.93	1.0700	1.4645
.44	.4543	1.0984	.94	1.0847	1.4753
.45	.4653	1.1030	.95	1.0995	1.4862
.46	.4764	1.1077	.96	1.1144	1.4973
.47	.4875	1.1125	.97	1.1294	1.5085
.48	.4986	1.1174	.98	1.1446	1.5199
.49	.5098	1.1225	.99	1.1598	1.5314
0.50	0.5211	1.1276	1.00	1.1752	1.5431