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# **Faculty Working Papers**

An Empirical Investigation into a Contingency Rule for Selecting Firms for Which Statistical Models Will Provide More Accurate Earnings Forecasts Than Those Generated by Financial Analysts

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**#507** 

College of Commerce and Business Administration University of Illinois at Urbana-Champaign



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(Not for quotation)

## Summary:

The present study dealt with the hypothesis that, for some firms, a statistical model will generate forecasts of EPS which are more accurate than those of financial analysts. Guided by an explicit set of assumptions, a rule was developed for predicting which firms should have statistical forecasts that are more accurate than financial analysts forecasts. The rule was applied to a sample of firms and it was found to be highly applicable to one quarter ahead forecasts. The rule, however, was not found to be applicable for two quarter ahead forecasts.

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In recent years there has been an increased emphasis on the forecasting of earnings. This increased emphasis is largely a result of
a widespread recognition that future earnings is an important factor in
investor decision making and research. This is evidenced by the fact
that the Financial Accounting Standards Board has made the importance of
future earnings a primary consideration in the theoretical framework
underlying their recent proposed objectives of financial reporting and
elements of financial statements of business enterprises [8]. In addition
the SEC has recently been considering requiring earnings forecasts in
external reports [11].

The increased emphasis on future earnings has led to an increased emphasis on methods of predicting future earnings. In particular considerable attention has been given to statistical methods with respect to predicting future earnings (see [1, 4, 6, 7, 9, 10, 12]). The reason for this is that the accuracy of forecasts is largely dependent on the forecast method employed; and in particular, if a statistical method leads to misspecified or suboptimal forecasts then dysfunctional or suboptimal decision making can result from the use of such forecasts.

Recent research has implied that financial analysts make quarterly earnings forecasts which are superior to those of certain statistical models. In particular, Brown and Rozeff [3], using quarterly comparisons found financial analyst forecasts to be more accurate than those of Box-Jenkins [2], Martingale and Submartingale models. A basic reason for expecting such results is that the analyst can utilize a broad data base in projecting earnings whereas the statistical models often utilize data relating only to past earnings.

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Notable, however, is that the Brown and Rozeff study demonstrated the superiority of the analysts on the average for a given sample of firms. It did not demonstrate that for all firms the analysts are superior. Conclusion for such research must therefore be limited to the average for a given sample of firms. Of course given two forecast methods, an individual (ignoring cost considerations) in general would prefer the method that has the best average performance unless he has a rule for predicting exactly which firms are best forecasted by the other method. In this case he would use one method for some firms and another method for other firms. The purpose of the present study is to develop such a rule: one for predicting under what circumstances a statistical model will outperform a financial analyst in the forecasting of quarterly earnings per share. In addition, this rule is subjected to an empirical test.

The paper is divided into three sections. Section one discusses:

(1) reasons why, for a given firm, a statistical model might be superior,
and (2) the conditions under which this superiority would be expected to
exist. Section two develops a rule for selecting those firms for which
the statistical models would be expected to be superior (henceforth called
FSR for firm selection rule). The rule is then tested on a sample of
firms. Section three presents a summary followed by conclusions, limitations and suggestions for future research.

## 1.0 Analysts vs. Statistical Models

1.1 Reasons Why a Statistical Model Might Outperform a Financial Analyst in the Forecasting of Earnings

To gain an explicit understanding of when a given statistical model would outperform an analyst, it is first necessary to know the exact



modeling processes employed by both the analyst and statistical procedure. In addition, it is also necessary to know the underlying statistical process for which forecasts are required. Unfortunately little is empirically known about the financial analyst's modeling process and the time series process can only be statistically described. Notwithstanding this limitation, however, it is possible to specify an infinite number of cases where a statistical model would outperform an analyst.

For example, assume that the value of a quarterly time-series variable  $\mathbf{x}_t$  depends on  $\mathbf{x}_{t-1}$ ,  $\mathbf{x}_{t-4}$ ,  $\mathbf{x}_{t-8}$  and an industry variable  $\mathbf{I}(t)$ . Further assume that the analyst has a modeling process that utilizes only the value of  $\mathbf{x}_{t-1}$ ,  $\mathbf{x}_{t-4}$  and  $\mathbf{I}(t)$ , while the statistical model utilizes only  $\mathbf{x}_{t-1}$ ,  $\mathbf{x}_{t-4}$  and  $\mathbf{x}_{t-8}$ . Under these assumptions, the statistical model would be expected to outperform the analyst for firms which have  $\mathbf{x}_t$  that depend heavily on  $\mathbf{x}_{t-8}$  and little on  $\mathbf{I}(t)$ ; the opposite would be true with the dependence reversed.

1.2 Conditions Under Which a Statistical Model Might Outperform a Financial Analyst in the Forecasting of Quarterly EPS.

As previously stated, there are many possible conditions which a statistical model could be expected to outperform a financial analyst. One special case of general interest can be derived from three assumptions:

- (1) The time series variable is a stationary Gaussian process.
- (2) The statistical modeling process is fixed and depends on the sample joint density function of  $D = f(x_1, x_2, ..., x_n)$ .
- (3) The analysts modeling process is fixed and depends on the sample joint density function  $\hat{D}_{\bullet}$



Assumption one can be mathematically stated by requiring that the joint probability distribution associated with m observations  $z_{t1}$ ,  $z_{t2}$ ,..., $z_{tm}$  made at any set of times t1,t2,...,tm is the same as that associated with m observations  $z_{t1+k}$ , $z_{t2+k}$ ,..., $z_{tm+k}$ , made at times t1+k,t2+k,...,tm+k.

Assumptions two and three can be symbolically represented by a composite mapping  $P: \hat{D} \to M \to F$  where P is a time invariant modeling process which maps the sample joint density function  $\hat{D}$  into a model M and set of forecasts F.

In order to derive implications, notationally let U represent the utility function of the user of the earnings forecasts,  $P_1 = F_1(M_1(\hat{D}))$  represent the statistical modeling process and  $P_2 = F_2(M_2(\hat{D}))$  represent the financial analyst's modeling process. Furthermore assume that  $EU(P_1,\hat{D}) > EU(P_2,\hat{D})$  for some sample interval containing periods i to j. This implies that one would expect  $EU(P_1,D)) > EU(P_2,D)$  for any sample interval containing periods i+k to j+L where k and L are arbitrary. This follows immediately since expected values of the arguments of M are time invariant by assumption.

The main implication for purposes of specifying conditions when, for a given firm, a statistical analyst would outperform an analyst is:

If, for a given firm, a statistical model outperforms a financial analyst in periods t,t+l,...,t+n, then it is expected to do the same in periods t+R,t+R+l,t+R+2,...,t+R+s.



## 2.0 Empirical Analysis

# 2.1 A Firm Selection Rule (FSR)

The result of the previous section is to suggest the following FSR for determining when a statistical model might outperform a financial analyst in the forecasting of EPS.

For a given firm, observe the performance given statistical model over a set of periods t,t+1,...,t+R. If it outperforms the analyst over these periods, use it to forecast periods t+R+1,t+R+2,...,t+R+s.

## 3.0 An Empirical Test

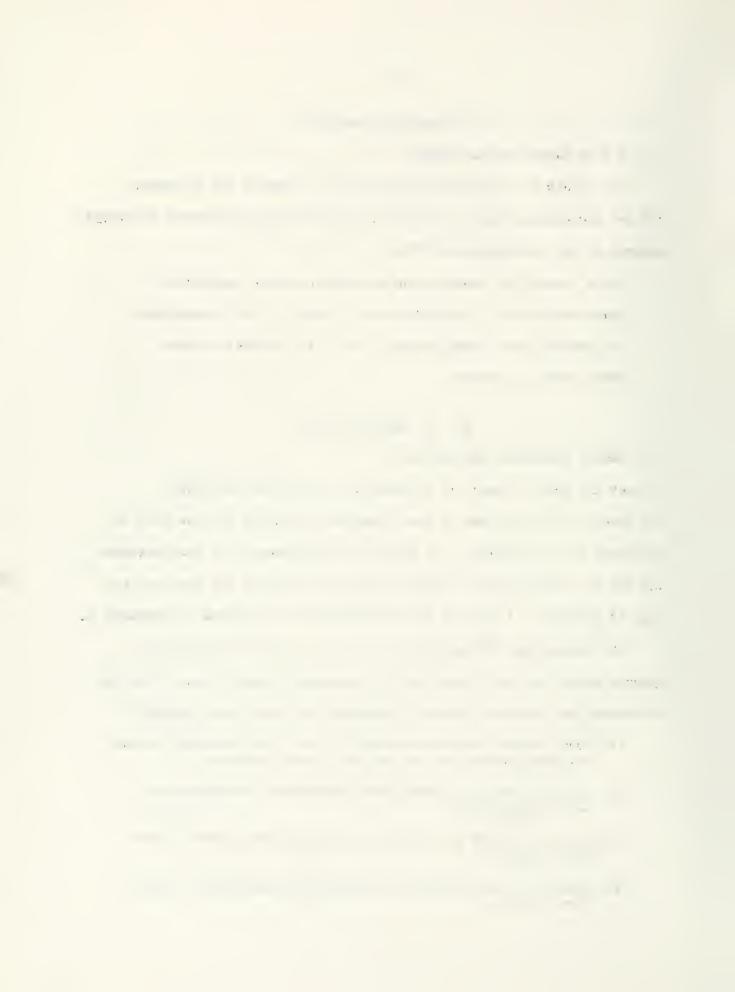
## 3.1 Sample Selection and Notation

To test the FSR, a sample of 50 firms was taken from the NYSE.

The sample was non-random in that firms were required to have been in existence for 96 quarters. An additional requirement was the existence one and two quarter ahead financial analyst forecasts for the most recent 19 quarters. A list of the sample firms is presented in Appendix 1.

The historical EPS data were then used to generate one and two quarter ahead forecasts made from 19 consecutive time origins using the following four forecast methods (henceforth referred to by number).<sup>2</sup>

- (1) Firm specific Box-Jenkins models using both reidentification and reestimation at each of the 19 time origins.
- (2)  $(1,0,0) \times (0,1,1)_3$  models with parameters reestimated at each time origin.
- (3) (1,0,0) (0,1,0) models with parameters reestimated at each time origin.
- (4) (0,1,1) (0,1,1) models with parameters reestimated at each time origin.



The next step was to partition the forecasts corresponding to the 19 time origins into two groups with the first containing forecasts corresponding to origins 1,2,...,R and the second containing forecasts corresponding to R+1,R+2,...,R+s.

Let each forecast be represented by F<sub>i,j,k,L</sub> where the subscripts are defined as:

- (1) Subscript i(i=1,2,3,4) refers to the forecast method used.
- (2) Subscript j(j=1,2,...,19) refers to the time origin that the forecast is made from.
- (3) Subscript k(k=1,2,...,50) refers to the number of the particular firm that the forecast is associated with.
- (4) Subscript L(L=1,2) refers to the number of quarters into the future that a forecast is made relative to its origin point.

Similarly, let the actual EPS values be represented as  $A_{j,k,L}$  and the analysts forecasts as  $N_{j,k,L}$  where the letter subscripts have the same meaning as above.

Furthermore define a zero-one variable  $D_{i,j,k,L}$  to be one for a given i,j,k,L when  $|F_{i,j,k,L} - A_{j,k,L}| < |N_{j,k,L} - A_{j,k,L}|$ , and zero otherwise. When  $D_{i,j,k,L}$  equals one we shall say that  $F_{i,j,k,L}$  dominates the corresponding financial analyst's forecast. Stated differently, a particular statistical model forecast dominates its corresponding financial analyst forecast when it is closer in absolute value to the actual EPS than the financial analyst forecast.

## 3.2 A Test of the FSR

Since the FSR rule defined above calls for the observation of model performance over a fixed number of time origins, we partition v tv =tv, in the first time of the second o

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 $\{D_{i,j,k,L}^{R}\}$  into the nonintersecting subsets  $\{D_{i,j,k,L}^{R}\}$  and  $\{D_{i,j,k,L}^{S}\}$  where the first subset contains  $D_{i,j,k,L}$  with  $j=1,2,\ldots,R$  and the second subset contains the remaining  $D_{i,j,k,L}$  (with  $j=R+1,R+2,\ldots,19$ ).

We employ the FSR by observing the performance of firms in time partition one (i.e., in  $\{D^R_{i,j,k,L}\}$ ) and selecting these firms as the most likely for high performance in time partition two (i.e., in  $\{D^S_{i,j,k,L}\}$ ). We operationalize this procedure by defining a firm k for step ahead L as a high performer in time partition one if

(1) 
$$\frac{\sum_{j=1}^{R} \sum_{i=1}^{4} \sum_{j=1}^{R} \sum_{i,j,k,L}}{4} \ge \frac{R}{2}$$

This definition requires that on the average the statistical models must dominate a number of times equal to or greater than one half of the R time origins in time partition one. We shall refer to firms that satisfy inequality (1) as average time partition one superior firms, and the corresponding  $\{D^R_{i,j,k,L}\}$  and  $\{D^S_{i,j,k,L}\}$  will be individually referred to as  $D^{R*}_{i,j,k,L}$  and  $D^{S*}_{i,j,k,L}$  firms respectively.

According to the FSR, D<sup>s\*</sup> firms should have models that outperform D<sup>s</sup> firms as a whole. This hypothesis can now be tested by making the following operational definitions:

(A) Time partition two unconditional superior (TPTUS) firms are those that satisfy:

(2) 
$$\sum_{j=R+1}^{19} D^{s}_{i,j,k,L} \ge \frac{19-R}{2}, (i, k \text{ and } L \text{ are fixed})$$

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(B) <u>Time partition two conditional superior</u> (TPTCS) firms are those that satisfy:

(3) 
$$\sum_{j=R+1}^{19} D_{i,j,k,L}^{s*} \ge \frac{19-R}{2}$$
 (i, k and L are fixed)

Both of these definitions are similar to that of average time partition one firms but differ in that they relate to time partition two and are not averaged across methods. For a firm to satisfy (2) it must have dominant forecasts for a number of origins equal to or greater than one half of the number of origins in time partition two. For a firm to satisfy (3) it must pass a much stricter test, namely it must satisfy (2) and be an average time partition one superior firm. The FSR implies that these TPTCS firms should have statistical models which outperform the analysts. (Note that these firms are the ones which are average time partition one superior and therefore the ones that should have statistical models which are superior to the analysts in time partition two.)

The formal hypothesis can now be stated using the following notation:

- (A) Let T equal 50 which is the total number of firms in the sample.
- (B) Let  $T_{R,L}$  represent the number of average time partition one firms for a given R and L as defined above.
- (C) Let T\*U
  i,R,L represent the number of TPTUS firms for a
  given i, R and L as defined above.
- (D) Let T\*C represent the number of TPTCS for a given i, R and L.

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The hypothesis can be stated as

(4) 
$$\frac{T_{1,R,L}^{*C}}{T_{R,L}} > \frac{T_{1,R,L}^{*U}}{T}$$

The hypothesis states if a restriction is made to a subsample of firms (namely those which are average time partition one superior) then the percentage of these firms which are time partition two superior is greater and the percentage of time partition two firms in the sample as a whole.

Table 1 presents the results of applying the FSR to the sample data for R=3,4,...,17, i=1,2,3,4 and L=1.

Each cell is composed of a 2x2 submatrix containing four ratios. Denoting these as  $R_{A,B}$  (where A and B represent row and column positions in the submatrix respectively)  $R_{1,1}$  and  $R_{1,2}$  represent  $\frac{T_{1,R,L}}{T}$  and  $\frac{T_{1,R,L}}{T_{R,L}}$  respectively. Also  $R_{2,1}$  and  $R_{2,2}$  represent the same ratios but with (2) and (3) restricted to equalities.

Inspection of Table 1 demonstrates that in virtually every case  $R_{1,1} < R_{2,2}$  as hypothesized. In fact, only five of the sixty cells are in the wrong direction (i=1 with R=14,15,15; and i=2 with R=14,15).

Of particular importance is the question as to whether or not an individual would prefer to use the statistical model or financial analyst forecasts for firms selected by the FSR. Note that a method to be preferred, in the present context,  $R_{1,B}$  must exceed  $\frac{R_{2,B}+1}{2}$  (proven in Appendix 2). In Table 1, those cases where a given statistical method is preferred are noted by an "\*". Note that for TPTCS firms the statistical models are preferred for 100% of all possible time partitions while the TPUCS firms have statistical models which are preferred for an average of 47%, 40%, 27%, 73%

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Table l
Application of FSR to Sample Data
for One Quarter Ahead Forecasts (L=1)

	Method								T <sub>R,1</sub>
R	1			2		3		4	
3	.500 .160	* .813 .313	.520 .140	* •750 •125	.480	* .688 .063	.580	* .813 .188	16
4	.420	* .643 .000	.420	* .714 .000	.400	* .714 .000	.440	* .714 .000	14
5	.500 .220	* .769 .385	.520 .220	* .692 .308	.480	* .615 .154	.560	* .692 .231	13
6	.440	* .643 .000	.480	* •714 •000	.340	* .643 .000	.480	* .643 .000	14
7	.560 .220	* .769 .308	.560 .180	* .692 .231	.520	* .692 .154	.620 .200	* .769 .231	13
8 .	.480	.600 .000	.480	.600 .000	.460	* .600 .000	.560 .000	* .667 .000	15
9	.560 .120	* .733 .200	.540 .140	.600 .133	.580	* .867 .267	.660 .200	* .733 .133	15
10	* .540 .000	.833 .000	.520 .000	* .750 .000	.420	* .667 .000	.600 .000	* .833 .000	12 .
11	.660 .180	.769 .000	.580 .180	.692 .077	.560 .200	.846 .308	.680	.846 .231	13
12	* .540 .000	* .769 .000	.500	.692 .000	.480	* .846 .000	.560 .000	* .769 .000	13
13	.680 .240	* .688 .188	.640 .220	.813 .313	.620 .180	* .750 .125	* .740 .280	* .750 .188	16
14	* .580 .000	* .533 .000	.520 .000	.533 .000	.500	* .667 .000	.620 .000	* .600 .000	15
15	* .700 .200	.611 .111	.680 .280	.722 .333	.740 .380	.722 .278	.760 .360	.722 .333	18
16	.600 .000	* • 556 • 000	.560 .000	.677 .000	* .540 .000	.667 .000	.600 .000	* .667 .000	18
17	* .800 .460	.895 .526	.660 .300	* .842 .368	.680 .300	.842 .263	.760 .320	.789 .211	19
rounded % cases where pre- ferred	47%	100%	40%	100%	27%	100%	73%	100%	T <sub>R,1</sub> =



respectively. Further note that in the right hand column  $T_{R,1}$  represents the number of firms selected by the FSR as candidates for statistical model superiority in time partition two.

In summary the results are strongly in favor of the validity of the FSR. This is true regardless of the statistical method (L=1,2,3,4) or time partition chosen.

Table 2 presents an identical analysis for forecasts made two quarters into the future (i.e., 1=2). Note that in this case the FSR does not bring about the same percentage improvements as it did in the case of the one quarter ahead forecast. In fact, the number of cases where a statistical method is preferred is actually decreased from 20 to 16 after applying the FSR.

As a secondary point of interest, it is notable that method four is preferred for the majority of the time partitions. This is true whether or not the FSR is employed. Also inspection of Table reveals that this holds for both one and two quarter ahead forecasts.

#### 4.0 Summary and Conclusions

The present study dealt with the hypothesis that, for some firms, a statistical model will generate forecasts of EPS which are more accurate than those of financial analysts. Guided by an explicit set of assumptions, a rule was developed for predicting which firms should have statistical forecasts that are more accurate than financial analysts forecasts. The rule was applied to a sample of firms and it was found to be highly applicable to one quarter ahead forecasts. The rule, however, was not found to be applicable for two quarter ahead forecasts.

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Table 2

Application of FSR to Sample Data for Two Quarter Ahead Forecasts (L=2)

	Method							T <sub>R</sub> , 2	
R	1		2		3		4		1,2
3	.400 .120	.320	.480	.440	.340	.360 .120	.640	* .600 .120	25
4	.320	.321	.320 .000	.250	.260	.286	.440	.429 .000	28
5	.420 .220	.400 .200	.420 .200	.360 .160	.360 .140	.320 .080	.600 .260	.560 .240	25
6	.320	.318	.320	.318	.300	.273 .000	.520 .000	.500	22
7	.520 .320	.476 .238	.380	.286	.360	.286 .048	.660 .300	.571	21
8	.400	.471 .000	.320	.294 .000	.320	.294 .000	.620 .000	* .588 .000	17
9	.600 .240	.467 .133	.540	.400 .200	.460	.400 .133	.700 .200	* .667 .267	15
10	.440 .000	.471 .000	.460	.353 .000	.400	.294 .000	.580 .000	* •529 •000	17
11	.580 .240	.500 .188	.560	.500 .215	.500	.438 .125	.700 .260	* .625 .188	16
12	.460 .000	* •533 •000	.440	.400 .000	.420	.400 .000	.600 .000	* .533 .000	15
13	* .660 .300	* .857 .429	.620 .280	.571 .214	* .660 .280	.643 .286	* .740 .220	* .714 .286	14
14	.500	.467 .000	.520 .000	.400	* • 540 • 000	.467 .000	.640 .000	* .600 .000	15
15	.680 .360	.765 .529	.680 .320	* .765 .353	.680 .320	* .706 .294	* .800 .340	* .765 .412	17
16	.480	.438	.500	* .563 .000	* .520 .000	* .563 .000	.660 .000	* .563 .000	16
17	.660 .420	.538 .462	.680 .420	.538 .385	.680 .420	.538 .308	* •840 •540	.692 .538	13
rounded % of cases where pre- ferred	7%	13%	20%	13%	27%	13%	80%	67%	T <sub>R,2</sub> = 18.4



The conclusion is the results indicate that, for some firms, financial analysts historically employed statistical forecast procedures which could be improved upon via certain statistical models. This conclusion, of course, is conditional on the error metric and sampling procedure used.



#### NOTES

We use the word "superior" in a general sense and recognize that the superiority of a given forecast method is necessarily conditioned upon a given error metric and utility function. The error metric measures error and the utility function determines when a method is preferred given a fixed error metric. In the present study we will operationally define both an error metric and a method of determining when a method is preferred.

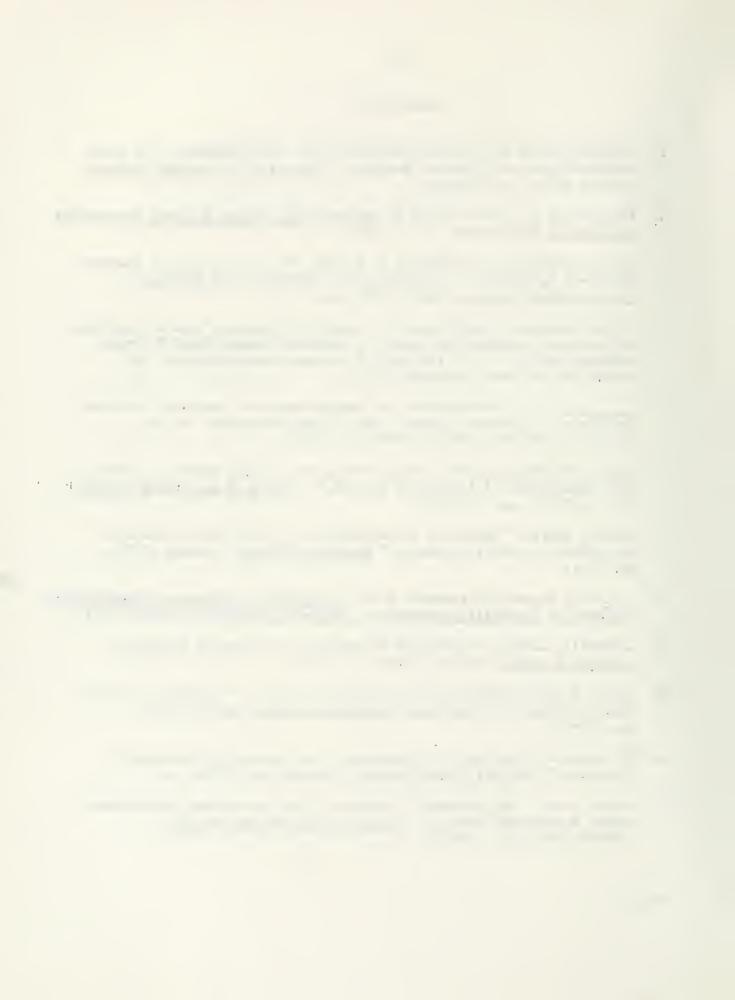
<sup>2</sup>Since there were 7,600 models (i.e., 19x2x50x4) it is not possible to present the models in an appendix. The models, however, will be furnished upon written request to the authors.

The structural models that we have considered for investigation are those which have been suggested in the literature (by several sources) as premier models. We refer the reader to Brown and Rozeff [5] for a bibliography and discussion of these models. In addition, the notation of Box and Jenkins [2] is used.



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## APPENDIX 1

## Listing of Sample Firms

Abbott Laboratories Allied Chemical American Cyanamid American Seating American Smelting Bethlehem Steel Borg-Warner Bucyrus-Erie Clark Equipment Consolidated Natural Gas Cooper Industries Cutler - Hammer Dr. Pepper Dupont Eastman Kodak Eaton Corporation Federal - Mogul Freeport Minerals Co. General Electric Gulf Oil Hercules, Inc. Hershey Foods Ingersol1 - Rand International Business Machines International Nickel Co. Lamsas City Southern Industries Lehigh - Portland Mead Corporation Merck and Company Mohasco Corp. Moore McCormack Nabisco, Inc. National Gypsum National Steel Northwest Arilines Peoples Drug Stores Pepsico, Inc. Rohm and Haas Safeway Stores Scott Paper Square D Stewart - Warner Texaco, Inc.



Trans World Airlines
Union Carbine
Union Oil (Cal.)
U.S. Tobacco
Westinghouse Electric
Weyerhaeuser, Inc.
Zenith Radio



## APPENDIX 2

Proof of Preference Theorem

Let: 
$$C = \frac{T_{i,R,L}^{*c}}{T_{R,L}}$$
 such that  $\sum_{j=R+1}^{19} D_{i,j,K,L}^{**} \ge \frac{19-R}{2}$ ,

$$A = \frac{T_{i,R,L}^{*c}}{T_{R,L}} \text{ such that } \sum_{j=R+1}^{19} D_{j,j,K,L}^{s*} < \frac{19-R}{2}$$

$$E = \frac{T_{i,R,L}^{*c}}{T_{R,L}} \text{ such that } \sum_{j=R+1}^{19} D_{i,j,K,L}^{**} = \frac{19-R}{2}$$

 $B = \frac{T_{i,R,L}^{*c}}{T_{R,I}} \text{ such that } \sum_{i=R+1}^{19} D_{i,j,K,L}^{s*} > \frac{19-R}{2}$ 

then the statistical model would be preferred if B > A. Substituting this gives

$$C - E > 1 - C$$

or 
$$2C > 1 + E$$

or 2C > 1 + E  
or C > 
$$\frac{1 + E}{2}$$

Q.E.D.

The proof for  $\frac{T_{i,R,L}^{*U}}{T}$  is identical.













