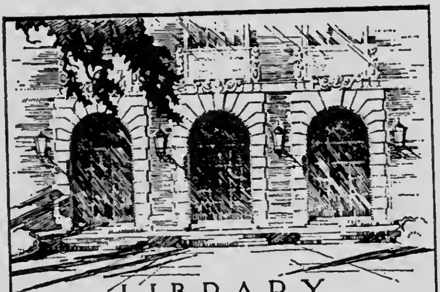
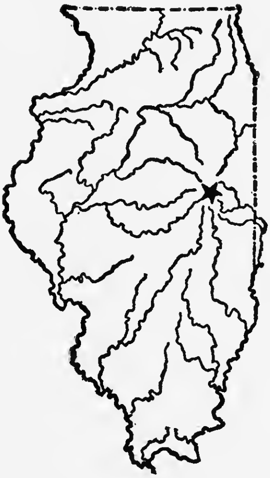


UNIVERSITY OF ILLINOIS
Agricultural Experiment Station

BULLETIN No. 308

THE ENERGY BASIS OF MEASURING MILK YIELD IN DAIRY COWS

By W. L. Gaines



URBANA, ILLINOIS, MAY, 1928

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THE ENERGY BASIS OF MEASURING MILK YIELD IN DAIRY COWS[a]

W. L. GAINES, Chief in Milk Production

INTRODUCTION

The dairy cow is maintained primarily as a milk animal, as a source of human food, and as such her worth depends upon her milk production. The most obvious measure of the milk production of the cow is the yield of milk itself, in terms of weight or volume.

It has come to be a somewhat prevalent practice to determine and record, by some systematic method, the weight of milk produced by individual cows in dairy herds. Also, since the advent of the Babcock test, it is usual to determine the percentage fat content of the milk by some system of sample taking and testing. There have developed, then, two common measures of the performance of the cow at the pail: (1) the milk yield in pounds over some definite period of time; (2) the butterfat yield in pounds over the same period of time. The average fat percentage is readily derived from the total milk and fat yields and is likewise commonly reported.

Milk is highly variable in composition, particularly as between different cows and breeds. It always contains a large proportion of water. When we measure production of the cow on the basis of milk alone, we place the water of the milk on a par with the solids. The water of milk has no more food value than water from any other source, and what is more pertinent, it seems that the production of the water fraction of the milk requires no particular expenditure of energy on the part of the cow. It is clear, therefore, that milk yield alone is not an entirely satisfactory measure of production.

When we measure production on the basis of fat alone, we ignore the other solids of the milk. These other solids have food value, and require the expenditure of energy on the part of the cow in their production. It is not quite proper to ignore them.

Another measure of production that has been used to a very limited extent is based on the total solids of the milk. The solids of milk consist mainly of lactose, fat, protein, and ash. Measuring yield on the basis of total solids attaches equal importance to these several constituents according to their amount.

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[a]Submitted for publication December 29, 1927.

In Bulletin 245[19*] of this Station it was proposed that the gross energy value of the milk solids be used as a measure of the yield of dairy cows, the energy value to be estimated in terms of 4-percent milk from the milk and fat yields. Additional evidence on the subject has accumulated in the meantime, which seems to support the equity of the energy measure. Inasmuch as the idea seems to be of some general import to those concerned with the milk yields of dairy cows, it is purposed in this paper to submit the evidence as it appears at the present time.

ESTIMATION OF ENERGY VALUE

From the Fat and Solids-Not-Fat.—Stocking and Brew[37*] have presented indirect evidence concerning the energy value of milk, based on 4,220 calories[a] per pound of fat and 1,860 calories per pound of solids-not-fat. Their figures lead to the equation

$$E = 49.64M\,(2.66 + f) \qquad (1)$$

where E is energy value in calories, M is the weight of milk in pounds, and f is the percentage fat content of the milk.

Equation (1) is readily transformed to

$$E' = .4M + 15F \qquad (2)$$

where E' is energy value in terms of pounds of average milk of 4-percent fat content, M is milk in pounds, and F is fat in pounds.[b]

In equation (2) E' may be designated "4-percent milk," or "fat-corrected milk," or "F.C.M.," and we may write

$$\text{F.C.M.} = .4M + 15F \qquad (3)$$

with the limitation only that the same unit of weight be used for each of the three terms.

Equation (3) is in convenient form for computation from the production record as it is usually kept to show the yield of milk and fat by weight. Where the average fat percentage is reported it may be convenient to employ the equivalent equation

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[a] Thruout this paper calorie refers to the large calorie, and 1,000 calories = 1 therm.

[b] Utilizing the mathematical relation, $Mf/100 = F$, or $Mf = 100F$, we have

$$E' = \frac{\text{total energy value of the entire quantity of milk}}{\text{energy value of 1 pound of 4-percent milk}}$$

$$= \frac{49.64M\,(2.66 + f)}{49.64\,(2.66 + 4)} = \frac{2.66M + Mf}{6.66} = \frac{2.66M + 100F}{6.66}$$

$$= .3994M + 15.015F$$

or, in round numbers, as in equation (2), and this gives 1 pound of 4-percent milk = 49.64 (2.66 + 4) = 330.6 calories. It will be observed that the coefficient (49.64) of M in equation (1) cancels out in the transformation to the form of equation (2). That is to say, equation (2) is independent of the absolute value of this coefficient.

$$\text{F.C.M.} = M\,(.4 + .15f) \qquad (4)$$

notation as before, using the same unit of weight for F.C.M. and M. Equation (4) is in useful form particularly for slide-rule computation, where one scale of the slide rule carries graduations of $(.4 + .15f)$ for various f values thru the range required (cf. Fig. 1. of Gaines[15*] and Fig. 3 of Gaines[16*]).

It will be clear that equations (1), (2), (3), and (4) are merely different forms to express the same thing. The question arises, with what degree of accuracy may we estimate the energy value of the milk of various individual cows from the weight of milk and its fat percentage by the use of these equations?

Five sets of data are available which show the percentages of fat and solids-not-fat for a considerable number of cows over part or all of a lactation period. The Minnesota Station[24*] has published the analyses of 543 samples of milk, each representing the milk yield for one week of 46 different cows in the Station herd. Various breeds and stages of lactation are represented. The solids were determined gravimetrically, the fat in part gravimetrically and in part by the Babcock method. The Connecticut Station[39*] has published 127 analyses, each representing a complete lactation period, for 50 cows of various dairy breeds in the Station herd. The Wisconsin Station[41*] has published analyses of the milk of 398 cows of various dairy breeds in the Wisconsin Cow Competition of 1909-1911. For the most part the period covered was 365 days within the same lactation. The Holstein-Friesian Association[26*] has published analyses of the milk of 458 registered Holstein cows in their yearly advanced-registry work. The American Jersey Cattle Club[1*] has published analyses of the milk of 70 registered Jersey cows for 120 days at the flush of lactation in the contest at the 1904 St. Louis Exposition. Analyses in all cases, except the Minnesota data, were by the Babcock and lactometer method.

By the use of Stocking's values it is possible to calculate the calories per pound of milk from the above analyses and then to determine the correlation between the fat percentage and energy value. The correlation surfaces and coefficients, together with the observed regressions and that of equation (1), give an index of the accuracy of the estimate by the equation. The correlation surfaces are given in Table 1 and the coefficients in Table 2. The mean energy values derived from Table 1 are given in Table 3, and are shown graphically in Fig. 1, together with the curve of equation (1).

It is clear from Fig. 1 that the equation conforms quite closely to the observations. The equation is being tested here against data not identical with those from which it was derived. The good agreement

TABLE 1.—CORRELATION BETWEEN FAT PERCENTAGE AND CALORIES PER POUND OF MILK

(Energy value estimated on the basis of 4,220 calories per pound of fat and 1,860 calories per pound of solids-not-fat)

The figures of the top row indicate the mid-values of the fat-percentage classes; the figures of the left-hand column indicate the mid-values of the caloric classes.

Data from Minnesota

	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0	7.2	Totsl
255	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2
265	7	4	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	11
275	2	16	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	18
285	..	5	10	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	17
295	..	..	4	15	8	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	27
305	..	..	2	2	12	8	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	25
315	..	..	..	1	6	5	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	14
325	..	..	..	..	1	4	9	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	15
335	..	..	..	..	..	2	10	9	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	22
345	..	..	..	..	..	..	1	12	18	2	..	..	..	..	..	..	..	..	..	..	..	..	..	33
355	..	..	..	..	..	..	1	4	26	11	..	..	..	..	..	..	..	..	..	..	..	..	..	42
365	..	..	..	..	..	..	..	..	7	27	17	..	..	..	..	..	..	..	..	..	..	..	..	51
375	..	..	..	..	..	..	..	..	..	1	24	18	..	..	..	..	..	..	..	..	..	..	..	43
385	..	..	..	..	..	..	..	..	..	..	1	19	26	1	..	..	..	..	..	..	..	..	..	47
395	..	..	..	..	..	..	..	..	..	..	..	6	16	14	5	..	..	..	..	..	..	..	..	41
405	..	..	..	..	..	..	..	..	..	..	..	..	2	6	19	1	..	..	..	..	..	..	..	28
415	..	..	..	..	..	..	..	..	..	..	..	..	1	3	18	9	4	..	..	..	..	..	..	35
425	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3	6	7	2	1	..	..	..	..	19
435	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	7	5	..	..	..	..	..	13
445	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3	3	1	..	..	..	7
455	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2	5	6	..	..	..	13
465	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2	2	3	..	1	8
475	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1	4	..	6
485	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1	3	5
495	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	1
Total	11	25	16	20	27	19	24	26	52	41	42	43	45	24	45	17	18	12	11	10	5	6	4	543

TABLE 1.—*Continued*

Data from Connecticut

	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6	6.8	7.0	7.2	Total
265	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
275	1	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2
285	..	..	3	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3
295	..	..	5	4	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	9
305	..	..	1	2	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	4
315	..	..	..	..	4	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	6
325	..	..	..	..	..	4	3	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	7
335	..	..	..	..	..	..	4	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	6
345	..	..	..	..	..	..	..	10	8	..	..	..	..	..	..	..	..	..	..	..	..	..	..	18
355	..	..	..	..	..	..	..	1	6	8	2	..	..	..	..	..	..	..	..	..	..	..	..	17
365	..	..	..	..	..	..	..	..	1	8	1	..	..	..	..	..	..	..	..	..	..	..	..	10
375	..	..	..	..	..	..	..	..	..	1	2	5	3	..	..	..	..	..	..	..	..	..	..	11
385	..	..	..	..	..	..	..	..	..	..	..	4	3	1	..	..	..	..	..	..	..	..	..	8
395	..	..	..	..	..	..	..	..	..	..	..	..	2	9	..	..	..	..	..	..	..	..	..	11
405	..	..	..	..	..	..	..	..	..	..	..	..	..	1	3	..	..	..	..	..	..	..	..	4
415	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	2	..	..	..	..	..	..	..	3
425	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1	..	..	..	..	..	..	2
435	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	1
445	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	2	..	..	..	..	3
455	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	1
Total	2	1	9	6	5	6	7	13	15	17	5	9	8	11	4	3	2	1	3	..	..	..	..	127

Data from Wisconsin

	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	Total
255	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
265	1	..	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3
275	..	1	2	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	4
285	..	1	9	8	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	18
295	..	..	1	16	23	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	40
305	..	..	..	1	25	14	..	..	..	..	..	..	..	..	..	..	..	..	..	..	40
315	..	..	..	..	2	22	7	..	..	..	..	..	..	..	..	..	..	..	..	..	31
325	..	..	..	..	..	4	8	5	..	..	..	..	..	..	..	..	..	..	..	..	17
335	..	..	..	..	..	..	..	9	4	..	..	..	..	..	..	..	..	..	..	..	13
345	..	..	..	..	..	..	..	..	5	8	1	..	..	..	..	..	..	..	..	..	14
355	..	..	..	..	..	..	..	..	..	7	6	1	..	..	..	..	..	..	..	..	14
365	..	..	..	..	..	..	..	..	..	..	20	14	2	..	..	..	..	..	..	..	36
375	..	..	..	..	..	..	..	..	..	..	1	19	18	..	..	..	..	..	..	..	38
385	..	..	..	..	..	..	..	..	..	..	..	..	18	18	..	..	..	..	..	..	36
395	..	..	..	..	..	..	..	..	..	..	..	..	1	13	14	1	..	..	..	..	29
405	..	..	..	..	..	..	..	..	..	..	..	..	..	..	7	13	..	..	..	..	20
415	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	9	14	..	..	..	23
425	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	4	8	..	..	12
435	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2	5	..	7
445	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1	2
Total	1	2	15	26	50	40	15	14	9	15	28	34	39	31	21	23	18	10	6	1	398

TABLE 1.—*Concluded*

Data from Holstein-Friesian Association

	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2	6.4	Total
245	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
255	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0
265	1	1	3	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	5
275	..	4	14	3	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	28
285	..	2	17	37	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	51
295	..	..	3	55	41	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	100
305	..	..	..	10	77	30	..	..	..	..	..	..	..	..	..	..	..	..	..	..	117
315	..	..	..	..	7	63	16	..	..	..	..	..	..	..	..	..	..	..	..	..	86
325	..	..	..	..	..	11	32	6	..	..	..	..	..	..	..	..	..	..	..	..	49
335	..	..	..	..	..	1	3	8	2	..	..	..	..	..	..	..	..	..	..	..	14
345	..	..	..	..	..	..	..	1	3	..	..	..	..	..	..	..	..	..	..	..	4
355	..	..	..	..	..	..	..	..	..	3	..	..	..	..	..	..	..	..	..	..	3
Total	2	7	37	105	127	106	51	15	5	3	..	..	..	..	..	..	..	..	..	..	458

Data from American Jersey Cattle Club

275	..	..	..	3	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3
285	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
295	..	..	..	3	4	3	1	..	..	..	..	..	..	..	..	..	..	..	..	..	11
305	..	..	..	..	3	4	1	..	..	..	..	..	..	..	..	..	..	..	..	..	8
315	..	..	..	..	..	4	6	1	..	..	..	..	..	..	..	..	..	..	..	..	11
325	..	..	..	..	..	..	4	4	..	..	..	..	..	..	..	..	..	..	..	..	8
335	..	..	..	..	..	..	1	2	2	..	..	..	..	..	..	..	..	..	..	..	5
345	..	..	..	..	..	..	..	..	2	3	..	..	..	..	..	..	..	..	..	..	5
355	..	..	..	..	..	..	..	..	..	..	2	..	..	..	..	..	..	..	..	..	2
365	..	..	..	..	..	..	..	..	..	..	..	7	2	..	..	..	..	..	..	..	9
375	..	..	..	..	..	..	..	..	..	..	..	2	2	2	..	..	..	..	..	..	6
385	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	1
Total	..	..	..	6	8	11	13	7	4	3	2	9	4	3	..	..	..	..	..	..	70

Table 2.—Coefficients of Correlation Between Fat Percentage and Energy Value per Pound of Milk and Between Fat Percentage and Solids-Not-Fat Percentage

(Energy value estimated on the basis of 4,220 calories per pound of fat and 1,860 calories per pound of solids-not-fat)

Source of data	Fat percentage and energy value	Fat percentage and solids-not-fat percentage
Minnesota Station	.9882 ± .0007	.801 ± .007
Connecticut Station	.9878 ± .0015	.684 ± .032
Wisconsin Station	.9915 ± .0006	.794 ± .013
Holstein-Friesian Association	.9152 ± .0051	.533 ± .023
American Jersey Cattle Club	.9668 ± .0053	.517 ± .059

shown in Fig. 1 and the high coefficients of correlation of Table 2 may be taken to mean that the energy value of milk may be estimated with considerable accuracy from the fat percentage and weight of the milk.

Table 3.—Mean Energy Values per Pound of Milk at Various Fat Percentages

(Computed on the basis of 4,220 calories per pound of fat and 1,860 calories per pound of solids-not-fat)

Fat percentage	Computed by equation (1)[a]	Source of data				
		Minnesota	Connecticut	Wisconsin	Holstein	Jersey
	cals.	*cals.*	*cals.*	*cals.*	*cals.*	*cals.*
2.6	261.1	.....	.....	265.0	255.0	.....
2.8	271.0	265.0	270.0	280.0	276.4	.....
3.0	281.0	275.4	275.0	279.7	280.4	.....
3.2	290.9	290.0	292.8	291.5	291.9	285.0
3.4	300.8	296.0	298.3	300.8	302.0	297.5
3.6	310.7	305.0	313.0	312.5	313.2	305.9
3.8	320.7	315.0	321.7	320.3	322.5	317.3
4.0	330.6	329.6	330.7	331.4	331.7	326.4
4.2	340.5	342.3	344.2	340.6	341.0	340.0
4.4	350.5	352.5	350.3	349.7	355.0	345.0
4.6	360.4	361.6	360.9	362.5	.....	355.0
4.8	370.3	371.2	365.0	370.3	.....	367.2
5.0	380.2	382.2	379.4	379.6	.....	370.0
5.2	390.2	390.1	383.8	389.2	.....	378.3
5.4	400.1	399.6	395.0	398.3	.....	.....
5.6	410.0	409.2	407.5	408.5	.....	.....
5.8	420.0	419.1	418.3	417.2	.....	.....
6.0	429.9	426.7	430.0	427.0	.....	.....
6.2	439.8	439.2	445.0	436.7	.....	.....
6.4	449.7	451.4	448.3	445.0	.....	.....
6.6	459.7	458.0	.....	.....	.....	.....
6.8	469.6	471.0	.....	.....	.....	.....
7.0	479.5	480.0	.....	.....	.....	.....
7.2	489.5	480.0	.....	.....	.....	.....

[a] The gross energy value of one pound of milk at the various fat percentages (as will appear later) actually runs about 3 percent greater than the values given in this column.

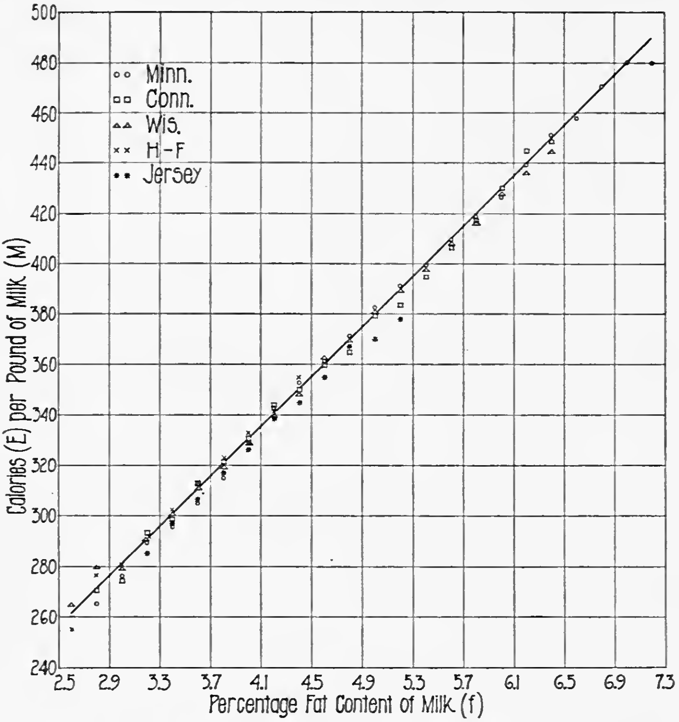

FIG. 1.—RELATION BETWEEN FAT PERCENTAGE AND ENERGY VALUE ACCORDING TO STOCKING'S METHOD OF ESTIMATION

The energy values were computed on the basis of 4,220 calories per pound of fat and 1,860 calories per pound of solids-not-fat. The smooth curve is that of equation (1), $E = 49.64M\ (2.66 + f)$, derived from Stocking's data. It will be observed that the equation conforms well to the observed energy values as estimated but, as will appear later, the estimates are about 3 percent lower than the actual energy values.

It is of course apparent that assigning a fixed energy value to a pound of butterfat necessitates a perfect correlation between fat percentage and energy value of the fat per pound of milk. We are testing consequently only the relation between fat percentage and the energy value of the solids-not-fat per pound of milk. The correlation between fat percentage and solids-not-fat percentage gives direct evidence of this relation, assuming, as we are, a fixed energy value per pound of solids-not-fat. The coefficients are given in Table 2, and they are all materially lower than the fat percentage and energy value coefficients.

From the Fat, Protein, and Lactose.—Andersen[3*] has approached the estimation of the energy value of milk in a more accurate manner than that of Stocking. Andersen has worked from the analyses of the milk of a large number of individual cows in Denmark, largely of the Red Danish and Jersey breeds. From his data he derived the relations: $p = 1.597 + .446f$ and $l = 5.23 - .1f$, where f, p and l are respectively percent of fat, protein, and lactose. In the next step he uses the values 4,132, 2,658, and 1,792 as representing the calories per pound of fat, protein, and lactose respectively. Using the form of equation (1), Andersen's results give $E = 51.48M\ (2.64 + f)$. Using the form of equation (3), we have F.C.M. = $.398M + 15.05F$, in which case 1 pound of F.C.M., or 4-percent milk, = 341.8 calories.

Hansson,[25*] working with cows in Swedish cow-testing associations, found that there is a close relation between fat percentage and energy value per unit of milk, the energy value being estimated in a manner similar to that outlined in the preceding paragraph. His published figures lead in the form of equation (1) to $E = 51.07M\ (2.72 + f)$, and in the form of equation (3) to F.C.M. = $.4048M + 14.88F$. In the latter case one pound of F.C.M., or 4-percent milk, = 343.2 calories.

By Direct Calorimetry.—Overman and Sanmann[33,34*] have investigated the energy value of milk by direct and precise methods. They have reported the analyses of 212 samples of milk representing in part single milkings and in part 3-days' milk of individual purebred and crossbred cows at various stages of lactation in the University of Illinois dairy herds. The analyses included a gravimetric determination of the fat and a direct calorimetric determination of the energy value. The correlation between the fat percentage and the calories per unit weight of milk was found to be $r = .9814 \pm .0017$, Table 2.[34*] This direct evidence shows, therefore, very clearly that the energy value may be estimated from the weight of milk and its fat percentage with a high degree of accuracy. Overman and Sanmann's results in the form of equation (1) give, $E = 52.312M\ (2.5064 + f)$, and in the form of equation (3), F.C.M. = $.3852M + 15.37F$. In the latter case 1 pound of F.C.M., or 4-percent milk, = 340.3 calories.

In estimating the energy yield of cows we are often concerned with the entire lactation or a considerable portion of it, rather than with the short periods represented by single samples. Dr. Overman has generously allowed the writer to use his analytical data for application to the appropriate milk yields of the cows in order to secure a figure applying to a longer period of the lactation. Records were selected of all those cows having three or more 3-day composite samples. This selection provided 76 samples representing sections of three to six months of single lactations of 21 different cows. Some of the cows were purebred and some crossbred dairy stock.

When treated individually, the 76 samples show a correlation between fat percentage and energy value per unit of milk of $r = .9761$, with the regression equation $E = 51.38M\ (2.69 + f)$. When the samples are treated as composites so as to deal with the 21 cows as individuals, the correlation works out at $r = .9860$, with the regression equation $E = 51.34M\ (2.70 + f)$. The figures for the 21 cows as individuals give, in the form of equation (3), F.C.M. $= .403M + 14.925F$, and 1 pound of F.C.M., or 4-percent milk, = 344.0 calories.

The observed fat percentages and energy values for the 21 cows are shown graphically in Fig. 2. The smooth curve is that of equation (3) adjusted to give 344.0 calories at 4 percent fat. It has a slope of 51.60 as compared with the slope of 51.34 derived above from the coefficient of correlation and standard deviations, or least-squares fit. The difference in slope of the two curves is too small to be shown in the scale of Fig. 2. The correlation, $r = .9860$, and the graphic presentation of Fig. 2 show that, so far as these analyses indicate, the energy value of the milk of the cows concerned is very accurately determined in terms of 4-percent milk by equation (3). One or two cows show a deviation of about 3 percent from the formula, but most of them lie very close to it.

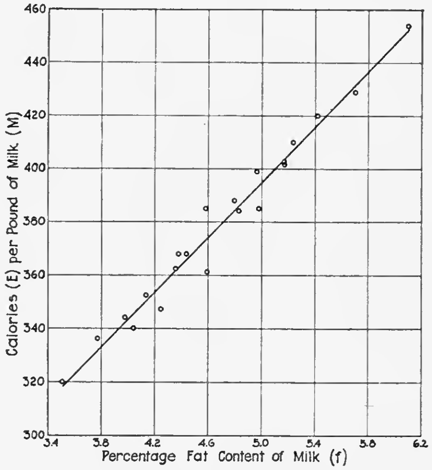

FIG. 2.—RELATION BETWEEN FAT PERCENTAGE AND ENERGY VALUE DETERMINED CALORIMETRICALLY

Each circle represents a section of 3 to 6 months of a single lactation of an individual cow. The smooth curve is that of equation (3) in which 1 pound F.C.M. = 344.0 calories, $E = 344\ (.4M + .15Mf) = 51.6M\ (2.66 + f)$. The regression equation derived from the coefficient of correlation and standard deviations is $E = 51.34M\ (2.70 + f)$. The curve of this equation is indistinguishable from the one given, in the scale of the figure.

Summary of Estimates.—Equation (3) was first proposed on the basis of Stocking's figures. These with the additional data mentioned above may be summarized:

Authority	*In form of equation (3)*	*Calories per pound F.C.M.*
Andersen	F.C.M. = $.3980M + 15.050F$	341.8
Hansson	F.C.M. = $.4048M + 14.880F$	343.1
Overman[a]	F.C.M. = $.3852M + 15.370F$	340.3
Overman[b]	F.C.M. = $.4030M + 14.925F$	344.0
Stocking	F.C.M. = $.3994M + 15.015F$	330.6

Discussion of Estimates.—Clearly, the formula $.4M + 15F$ originally derived from Stocking's figures is justified by the later evidence, altho the absolute value in calories is lower than that indicated by the other evidence. Stocking may have intended his figures to represent metabolizable energy. We may say that the gross energy value of 1 pound of F.C.M., or 4-percent milk is about 340 calories.

Overman and Sanmann[33, 34*] have shown that the energy value of milk may be estimated with greater accuracy if the percentages of protein and lactose are known in addition to the percentage of fat. They found for their 212 samples a multiple correlation, based on fat, protein, and lactose, of $R = .9917$; as compared with $r = .9814$, where only the fat percentage was used.[c] The protein and lactose as well as the fat were determined by accurate chemical methods.

It is a question whether an estimate of the solids-not-fat by the use of the lactometer would permit any closer estimate of energy value than may be made from the fat percentage alone. Overman *et al*[32*] have shown that even where the specific gravity of the milk is determined with greater precision than is possible by the ordinary application of the lactometer, the results in terms of solids-not-fat are apt to be wide of the facts as determined gravimetrically.

One may note also in Table 2 that the correlation between fat percentage and solids-not-fat percentage is much lower where the solids-

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[a] From 212 samples of the milk of purebred and crossbred dairy cows.

[b] From three-months to six-months sections of single lactations of 21 individual cows, purebred and crossbred dairy stock.

[c] The difference of .01 in the two coefficients, while small numerically, has a pronounced meaning in terms of the probable errors of the estimates by the regression equations. In theory a coefficient of $r_{xy} = .98$ means that the standard deviation of y at any fixed value of x is 20 percent of the standard deviation of y thru the whole range of x, $(\sqrt{1 - .98^2} = .20)$. Where $r_{xy} = .99$ the corresponding figure is 14 percent, $(\sqrt{1 - .99^2} = .14)$. The probable error of the y estimate when $r_{xy} = .98$ is therefore reduced by 30 percent for $r_{xy} = .99$, $[(.20 - .14)/.20 = .30]$.

not-fat were determined by lactometer than where determined gravimetrically. The lower coefficients are presumably a consequence of the inaccuracies of the lactometer method.

Everything considered, it appears that the energy yield of cows may be estimated with sufficient accuracy from the weight of milk and fat produced. If greater precision is desired, it is necessary to use direct calorimetric methods. The use of the lactometer as an aid in the accuracy of the energy estimate seems to be entirely unwarranted. Even accurate chemical determination of the protein and lactose contribute comparatively little over and above the accuracy attainable by the use of the fat determination alone. It will be understood, of course, that we are dealing with the unaltered normal milk of the cow.

If the energy yield is to be estimated in terms of 4-percent milk the $.4M + 15F$ formula answers very well. This basis of estimation is independent of the absolute energy value of the milk, and it was partly for this reason that it was first used, since, at the time, the absolute energy value did not appear to be any too well established. The straightforward scientific procedure would be to determine energy yield calorimetrically, and while the calorimetric determination is readily carried out in the chemical laboratory, it is not adapted to use in the ordinary and extensive keeping of production records of cows on the farm; hence the original procedure seems still to be justified. If it is desired to express the energy yield in terms of the customary unit of the calorie, we may say that 1 pound of 4-percent milk equals 340 calories, and, in accord with the $.4M + 15F$ formula, $E = 51M\ (2\frac{2}{3} + f)$, notation as before. The 4-percent milk formula has the advantage of ease of computation and of dealing with a familiar unit. The F.C.M. yield, however, is subject to confusion with the actual milk yield, which possibility of confusion is avoided by the calorie formula. There would be some justification in reserving the expression of yield in calories for such precise experimental work as actually determines the energy value calorimetrically, and using the "F.C.M.," or "4-percent milk," designation for the indirect estimate by the fat-percentage formula.

Interpretation of Formula.—The use of equation (3), F.C.M. $= .4M + 15F$, is *not* to be regarded as a process of assigning weight or importance to the milk and fat respectively. It is merely an algebraic device adapted to the computation of the energy yield, in terms of 4-percent milk, from the record of milk and fat yield as ordinarily reported.

The source of the energy value of milk of different fat percentages is shown diagrammatically in Fig. 3. This presents Andersen's[3*] scheme of estimation, namely: $E_F = 41.32f$, $E_P = 42.45 + 11.85f$ and $E_L =$

$93.72 - 1.79f$, where E_F is the calories in 1 pound of milk due to the fat; E_P, due to the protein; and E_L, due to the lactose. In a rough way one might say that $.4M$ gives a constant of 136 ($= .4 \times 340$) calories per pound of milk due to a nearly constant lactose together with a nearly constant portion of the protein; while $15F$ gives the variable calories per pound of milk due to the variable fat and the remaining, variable, portion of the protein associated with the variable fat.[a]

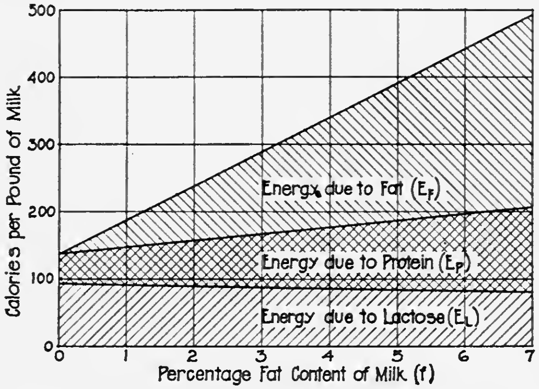

FIG. 3.—SOURCE OF THE ENERGY VALUE OF MILK ACCORDING TO ANDERSEN'S FORMULAS

The fraction of the total energy value of the milk represented by the fat, according to the formulas above, is $41.32f/(136.17 + 51.38f)$. The fractions at various fat percentages are:

f	2	3	4	4.36	5	6	7
Fraction	.346	.427	.484	.500	.526	.558	.583

That is, in 2-percent milk 34.6 percent of the energy value of the milk is in the fat; in 4.36-percent milk one-half of the energy value is in the fat; while in 7-percent milk 58.3 percent of the energy value is in the fat.

FAT PERCENTAGE AND FEED REQUIREMENTS

Data from Minnesota Station.—Milk production feeding standards, expressing the result of much experimental and practical observation, are adjusted to the weight of the cow for maintenance requirements and to the amount and fat percentage of the milk for lactation requirements. We may consider the system based on digestible nutrients as formulated by Haecker.[24*] From Haecker's published results it is possible to determine the correlation between the percentage fat content of the milk and the pounds of nutrients for lactation (maintenance re-

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[a] The secretion of fat and the secretion of protein are in some manner quite intimately related in the general physiology of milk secretion (cf. Gaines,[14*] Fig. 3).

TABLE 4.—CORRELATION BETWEEN FAT PERCENTAGE AND NUTRIENTS FOR LACTATION PER POUND OF MILK

The figures of the top row indicate the mid-values of the fat-percentage classes; the figures of the left-hand column indicate the mid-values of the classes in pounds of nutrients for lactation per pound of milk produced.

	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0	Total
.21	..	1	..	..	..	1	1	..	..	..	..	..	..	..	..	..	..	..	3
.23	1	..	2	1	..	..	..	..	1	..	..	..	..	..	..	..	..	..	5
.25	..	..	..	..	..	2	..	..	..	..	..	..	..	..	..	..	..	..	2
.27	1	2	2	..	..	2	..	..	..	..	3	2	..	..	..	1	..	..	13
.29	..	..	1	1	3	1	1	1	1	3	..	1	..	..	..	..	..	..	13
.31	..	..	1	3	2	..	1	2	..	2	2	2	..	1	..	..	..	..	16
.33	..	..	1	..	2	1	..	..	..	3	4	..	1	..	..	..	..	..	12
.35	..	..	1	..	..	..	3	1	2	3	1	2	1	..	1	1	..	..	16
.37	..	..	..	..	..	1	3	..	..	..	2	4	2	1	..	2	..	..	15
.39	..	..	..	..	..	..	..	1	..	..	1	3	1	4	2	2	..	..	14
.41	2[a]	..	..	..	1	..	..	..	2	1	..	2	1	..	2	4	1	..	14
.43	..	..	..	..	1	..	..	..	..	..	..	2	1	..	2	..	..	1	7
.45	..	..	..	..	..	..	..	..	..	..	1	1	..	..	2	1	..	1	6
.47	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2	..	..	..	2
.49	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	1
.51	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	1
Total	2	3	8	5	9	8	9	5	6	12	14	19	8	6	11	11	2	2	140

[a] These two cases are for the same cow, Lou II, stated to have been out of condition at the time of the experiment. Because of this statement and the wide divergence of the results, the values are excluded in the present computations.

quirements excluded) per pound of milk yielded. The correlation surface is given in Table 4, and leads to the following constants:

	Fat percentage	*Nutrients for lactation per pound of milk*
Mean	4.399	.3463 pounds
Standard deviation	.852	.0636 pounds
Coefficient of correlation[a]	.648 ± .033	

The mean observed values of nutrients for lactation per pound of milk at the various fat-percentage classes are given in Fig. 4. The regression equation, $y = .1334 + .0484f$, derived from the above constants, shows that the production of 1 pound of 4-percent milk requires

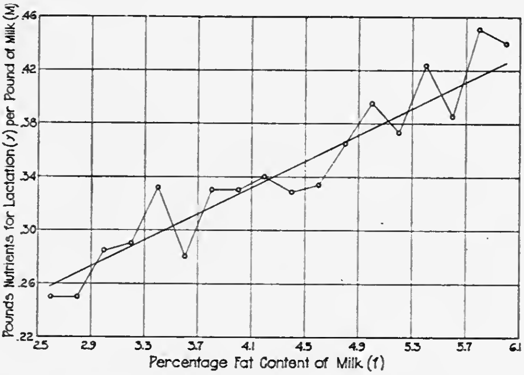

FIG. 4.—RELATION BETWEEN FAT PERCENTAGE AND NUTRIENTS FOR LACTATION PER POUND OF MILK DERIVED FROM HAECKER'S DATA

The equation of the smooth curve is that of equation (3), in which 1 pound F.C.M. requires .327 pounds of nutrients for lactation, $y = .327(.4M + .15Mf) = .049(2.66 + f)$. The regression equation derived from the coefficient of correlation and standard deviations is $y = .0484(2.76 + f)$ and gives a curve of slightly less slope than that of equation (3), but the difference is too small to be shown in the scale of the figure.

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[a] This coefficient has been computed by the use of four different groupings. As a matter of interest in statistical method the results are given below, in which f represents fat percentage and y represents pounds of nutrients for lactation per pound of milk:

Class interval, f	.01	.1	.2	.5
Class interval, y	.001	.01	.02	.03
Number of classes, f	349	36	18	8
Number of classes, y	317	34	16	11
Standard deviation, f	.8571	.8596	.8520	.8806
Standard deviation, y	.06380	.06404	.06363	.06479
$SD_f \times SD_y$	.05469	.05505	.05422	.05705
Coefficient of correlation	.6386	.6411	.6479	.6596

It works out in this particular case that the coarser the grouping, the higher the coefficient of correlation. In the finest grouping the number of classes actually represented does not, of course, exceed the total number of observations, 140.

.327 pounds of digestible nutrients for lactation (maintenance excluded). If we adjust equation (3) to this value, we have the smooth curve of Fig. 4. It is clear at once that, while the observed values of Fig. 4 are somewhat irregular, their trend is in the direction of the energy curve. The energy curve crosses the least-squares curve at $f = 4$, but its slope, .0491, is so nearly the same as that of the least-squares curve, .0484, as to make the two practically coincident in the scale of Fig. 4.

The feed energy required for lactation is directly proportional to the energy value of the milk solids, a relationship which should be known as HAECKER'S LAW. Therefore, when we measure production in terms of F.C.M. by equation (3), we measure it also in terms of the nutrients required for lactation. By indirect, but altogether straightforward, methods it has been shown[13*] that, within the same breed, and so far as affected by the percentage fat content of the milk, the maintenance requirements per pound of milk are also proportional to the energy value of the milk solids per pound of milk.

Data from Copenhagen Station.—Frederiksen[11*] has presented data which indicate that the total feed consumption of dairy cows is proportional to the yield of F.C.M., or 4-percent milk. His figures are of so much interest that they are given in Table 5, adapted to the present purpose. This table gives a summary of ten years' (1909-1919) results of an extensive breeding experiment conducted by the Counts Ahlefeldt-Laurvig in cooperation with the Copenhagen Experiment Station. Two pure breeds, the Red Danish and the Jersey, were used at the start in this experiment. These two breeds were intermated, creating a third

TABLE 5.—PERCENTAGE FAT CONTENT OF MILK AND FEED CONSUMPTION PER POUND OF MILK

Summary of ten years' (1909-1919) results of the Danish crossbreeding experiment, adapted from Frederiksen. The pertinent point of interest is the feed consumption per pound of F.C.M. This is remarkably constant, as measured in feed units. The Danish feed unit is 1 kilogram (2.2 pounds) of barley or its equivalent.

Breed of cows	Red Danish	Crossbred	Jersey
Number of cows	368	350	353
Age of cows, in years	5.6	5.8	5.7
Live weight per cow, in pounds	1021	913	796
Percentage fat content of milk	3.60	4.28	5.34
Milk per cow per year, in pounds	7934	6389	5018
Fat per cow per year, in pounds	286	273	268
F.C.M. per cow per year, in pounds	7458	6657	6027
F.C.M. per pound live weight, in pounds	7.30	7.29	7.57
Feed units per cow per year	3079	2748	2484
Feed units per pound milk	.388	.430	.495
Feed units per pound F.C.M.	**.413**	**.413**	**.412**

breed class designated as crossbreds. The primary purpose of the investigation was to determine the amount and economy of production of the three breed classes.

Table 5 shows that the three breed classes differ markedly in weight, in milk yield, in the percentage of fat in the milk, in amount of feed consumed, and in amount of feed per unit of milk. The last line of the table shows, however, that the feed consumption per pound of F.C.M. is the same for each of the three breed classes. Therefore, when we measure production in terms of F.C.M., we also measure it in terms of total feed consumption, so far as the average results of these three groups of cows indicate.[a]

FAT PERCENTAGE AND YIELD OF MILK

Correlation Between Fat Percentage and Milk Yield.—Dairy literature contains some confusion of thought relative to the relation between the richness of the milk and the amount of milk yielded by individual cows. In general it is recognized that these two variables show a small negative correlation. Some investigators have contended, however, that in certain breeds the correlation is zero. There are many factors which have a very great effect on milk yield, and this makes it difficult to determine precisely the relation between fat percentage and milk yield, independently of all other variables.

Gaines and Davidson,[19*] studying the regression of milk yield on fat percentage as shown by a large number of yearly and 7-day records, reached the conclusion that milk yield is affected by the fat percentage (composition) of the milk, and that, "*so far as affected by fat percentage, the milk yield is inversely proportional to the energy value of the milk solids per unit of milk.*" That is, the energy yield is not affected by the fat percentage of the milk. Fig. 5 presents the method of attack and the results for one set of Holstein data. Concordant results were obtained also for the Ayrshire, Brown Swiss, Guernsey, and Jersey breeds. To the evidence from the records of these breeds may now be added similar evidence from the Milking Shorthorn and Red Danish breeds.

Shorthorn Advanced Registry.—The correlation surface for fat percentage and milk yield for the Milking Shorthorns[2*] is given in Table 6. The coefficient of correlation works out at $r = -.227 \pm .020$. Fig. 6 shows graphically the mean milk yields at the various fat percentage classes, and the constant energy curve. The energy curve conforms

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[a] It is of interest to note that the energy yield of the three groups of cows is nearly proportional to their weight. Is this a general rule? (cf. footnote on page 597 of Gaines[13*]).

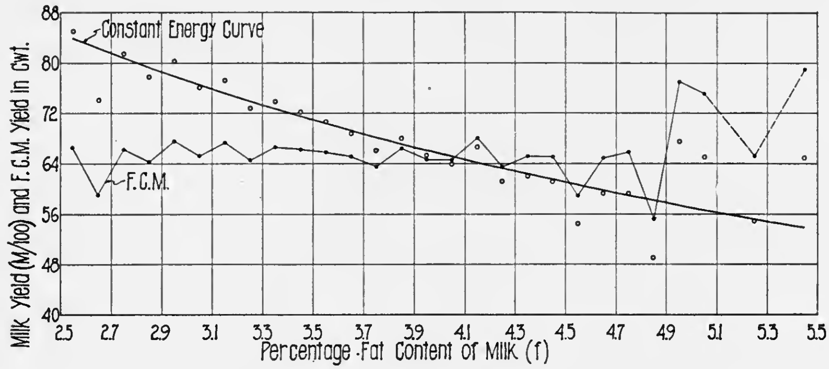

FIG. 5.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: HOLSTEIN RECORDS

This figure is based on 2,773 yearly records of milk yield and fat percentage of grade and purebred Holstein cows in Illinois cow-testing associations. The method of studying the records was to correlate the milk yield and fat percentage values, which gives, $r = -.229 \pm .012$. The correlation ratio for milk yield and fat percentage is, $\eta = .242 \pm .012$. The difference between the two is, $\eta^2 - r^2 = .0061 \pm .0020$, statistical evidence that the regression of milk yield on fat percentage deviates significantly from a straight line.

The mean milk yields, indicated by the open circles, have been derived from the correlation table. Each circle represents the mean milk yield of a group of cows, each cow of the group lying within .05 of the fat percentage indicated by the base line scale. It appears that fat percentage is not correlated with any of the other important factors affecting milk yield (condition of the cow at calving is in certain cases a disturbing exception to this statement). Hence we may assume that all factors other than fat percentage which affect milk yield are equal or counterbalanced in each fat-percentage group, and that the differences in milk yield between groups are due to differences in fat percentage. That is, inherent lactation capacity, size, age, feed supply, days in milk, etc., are assumed to average the same in each group, or advantages in some particulars are counterbalanced by disadvantages in the other particulars. The ideal of this assumption will be realized only if the groups are large, and practically we may expect considerable irregularity in the observed yields, which in fact occurs. We are warranted in taking a smooth curve which represents the trend of the observed milk yields to represent the true relation between fat percentage (composition of the milk as measured by fat percentage) and milk yield.

The constant energy curve, $M = A/(2.66 + f)$, has been adjusted to the mean milk yields, thus: $A = \Sigma n\, M_0(2.66 + f)/\Sigma n$, where M_0 is the observed milk yield and n is the frequency at each fat percentage (f) class. This gives $A = 43{,}668$, which is simply the average energy yield shown by the 2,773 records, in units of 51 calories. The average energy yield is therefore 2,227 therms, or 6,550 pounds F.C.M. The constant energy curve, $M = 43{,}668/(2.66 + f)$, shows the milk yield required to give this average energy value at the various fat percentages. It describes the trend of the observed milk yields about as closely as could be expected of any simple smooth curve. We generalize, then, by saying that the milk yield changes with the fat percentage in such a manner that the energy yield remains constant, that is, the milk yield is inversely proportional to the calories per pound of milk.

Perhaps the matter may be presented more clearly by considering the energy yields directly, instead of the milk yields. The energy yields are represented in the figure in terms of F.C.M. by the solid circles. They show, of course, the same sort of irregularity as the milk yields, but unlike the milk yields they show no consistent variation with the fat percentage. The correlation between the F.C.M. and fat percentage values is $r = -.010 \pm .013$. That is to say, the F.C.M. yields fluctuate independently of the fat percentage.

TABLE 6.—CORRELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: MILKING SHORTHORN RECORDS

The figures of the top row indicate the lower values of the fat-percentage classes; the figures of the left-hand column indicate the mid-values in hundredweights of the milk-yield classes.

	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	Total
45	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2	..	..	..	2
55	..	..	1	1	2	7	5	13	17	11	16	10	7	12	5	3	1	..	..	..	111
65	..	..	1	1	3	16	13	27	20	20	17	10	14	4	3	3	4	..	..	1	157
75	..	1	..	1	4	8	11	19	27	21	15	8	6	12	8	2	1	..	..	1	145
85	4	2	4	12	11	17	24	32	44	35	17	24	13	6	6	..	1	2	..	..	254
95	2	1	2	5	9	17	15	19	25	13	11	10	3	3	3	..	..	1	1	..	140
105	2	1	..	8	9	9	11	11	10	11	11	6	4	5	1	..	..	1	..	1	101
115	1	1	4	1	2	5	6	3	3	8	7	4	1	..	2	1	..	..	..	..	49
125	..	..	..	1	3	4	1	6	4	3	3	2	1	..	..	..	..	..	1	..	29
135	..	..	1	3	1	2	2	1	..	2	3	..	..	..	..	..	..	..	..	..	15
145	..	..	..	1	1	1	1	2	1	1	..	1	..	1	..	..	..	..	..	..	10
155	..	..	..	..	..	1	..	..	2	..	1	1	..	1	..	..	..	..	..	..	6
165	..	..	..	..	1	1	2	..	..	..	..	..	..	..	..	..	..	..	..	..	4
175	..	1	..	1	1	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	4
185	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
Total	9	7	13	36	47	88	92	133	153	125	101	76	49	44	28	9	9	4	2	3	1028

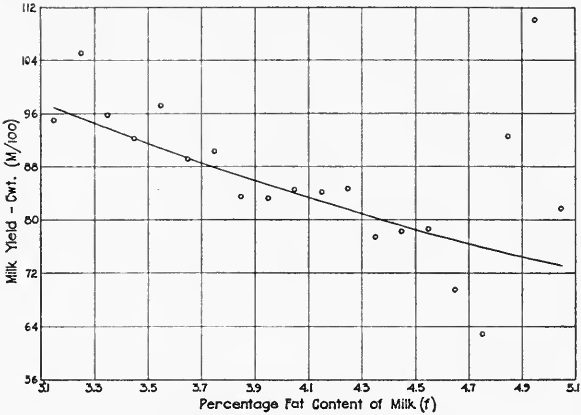

FIG. 6.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: SHORTHORN RECORDS

This figure is based on 1,028 advanced-registry yearly records of Milking Shorthorn cows. The plan is similar to that of Fig. 5. Equation of the curve is $M = 56{,}293/(2.66 + f)$. The average energy yield is 2,871 therms or 8,444 pounds F.C.M.

fairly well to the trend of the observations. The relation between fat percentage and milk yield in these Shorthorn advanced-registry records closely resembles that found previously[19*] for the Guernsey and Jersey advanced-registry records.

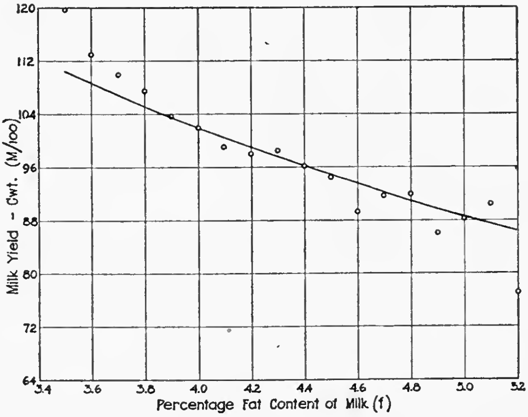

FIG. 7.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: RED DANISH ADVANCED-REGISTRY RECORDS

This figure is based on the average yearly records of 1,140 cows in the Red Danish advanced registry. The plan is similar to that of Fig. 5. Equation of the curve is $M = 67{,}926/(2.66 + f)$. The average energy yield is 3,464 therms or 10,189 pounds F.C.M.

Red Danish Advanced Registry.—Similar material for the Red Danish breed taken from the herd books[36*] (Vols. 1-4, 1921-1924, Cows Nos. 1000 to 2139) is presented in Table 7 and Fig. 7. The records of Table 7 represent the average yearly performance of the cow over a period of 3 to 14 years, excluding records disturbed by abortion or records otherwise misrepresentative. As a rule, calving occurs regularly every year. The requirement for admission to the herd book is a minimum average yield for three years, or more, of 160 kilograms of ''butter'' (about 315 pounds of fat) and a fat test of not less than 3.6 percent; or, 175 kilograms of ''butter'' (about 345 pounds of fat) and a fat test of not less than 3.45 percent. The records resemble our advanced-registry records in that a certain minimum production is required for admission to the herd book. But as above noted, the cows reproduce regularly every year, and in this respect the records resemble our cow-testing association records, being quite different from the usual advanced-registry record.

The following constants are derived from Table 7:

	Fat percentage	*Milk yield*
Mean	4.095	10,068 pounds
Standard deviation	.272	1,171 pounds
Coefficient of variability	6.64	11.63
Coefficient of correlation	−.438 ± .016	

It will be noted that the standard deviation or variability for both variables is very low, a result presumably of the entrance restrictions and the use of average records for the individual cows. The correlation between fat percentage and milk yield, $r = -.438$, is closer than any found heretofore in dealing with yearly records. This high correlation is probably a result of dealing here with three-year-or-more averages.

The mean milk yields are shown graphically in Fig. 7, together with the constant energy curve. The latter is based on the average energy yield of the 1,140 records, 3,464 therms or 10,189 pounds F.C.M. This curve plainly describes the general trend of the observed milk yields quite well except at the lower fat percentages. It has been shown elsewhere (page 587 of Gaines and Davidson[19*]) that the fixed fat-production requirement of the advanced registry is the more severe the lower the fat percentage, and hence tends to raise the mean milk yields at the lower fat percentages. In the present case we have also a marked increase in the fat-yield entrance requirement of the two classes at $f = 3.5$ and 3.6, which of course makes these two observations unduly high.

TABLE 7.—CORRELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: RED DANISH RECORDS FROM HERD BOOKS

The figures of the top row indicate the mid-values of the fat-percentage classes; the figures of the left-hand column indicate the mid-values in hundred kilograms of the milk-yield classes.

	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	Total
31	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	1
33	..	..	..	..	..	..	..	..	1	1	..	1	..	..	1	..	..	..	4
35	..	..	..	..	..	1	1	1	1	..	2	2	3	..	..	..	..	1	12
37	..	..	..	..	1	1	3	6	3	5	3	5	4	1	..	1	..	..	33
39	..	..	2	2	9	6	18	18	9	13	10	6	3	1	2	..	..	..	99
41	..	2	5	6	12	12	22	27	16	15	9	5	1	2	..	..	1	..	135
43	..	3	3	9	19	29	35	18	17	19	12	4	3	1	2	1	..	..	175
45	..	2	12	20	32	26	41	18	18	13	4	1	2	..	1	..	..	..	190
47	2	2	8	15	21	25	24	26	12	10	6	3	3	..	..	..	..	..	157
49	..	2	7	15	21	19	24	8	10	7	7	1	1	1	..	..	..	..	123
51	..	6	9	13	19	13	10	6	5	2	1	..	..	..	..	..	..	..	84
53	..	4	6	11	11	6	4	3	5	3	..	..	1	..	..	..	..	..	54
55	..	..	5	2	7	4	2	2	..	..	..	..	..	..	..	..	..	..	22
57	..	4	6	7	3	2	1	1	1	1	..	..	..	..	..	..	..	..	26
59	..	2	2	3	..	1	..	..	..	..	..	..	..	..	..	..	..	..	8
61	..	..	1	1	2	..	1	1	1	..	..	..	..	..	..	..	..	..	7
63	..	1	1	2	..	..	..	2	..	..	..	..	..	..	..	..	..	..	6
65	..	1	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2
67	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
69	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
Total	3	29	69	106	157	145	186	137	99	89	54	28	21	6	7	2	1	1	1140

Red Danish Herd Records.—Records of the Red Danish breed which correspond closely to our cow-testing association records are given in Table 8 and graphically in Fig. 8. The data of Table 8 are derived from the published[6*] records of the Count Ahlefeldt herd referred to above in connection with the relation between fat percentage and feed consumption per pound of milk. Yearly records of less than 250 days of lactation have been excluded in the present computations. Otherwise, all records were used of the Red Danish breed that appear in the first to fourteenth annual reports (1905-1919), except the third report (1907-1908), which latter was not available.

The constants derived from Table 8 are as follows:

	Fat percentage	*Milk yield*
Mean	3.544	7,527 pounds
Standard deviation	.295	1,679 pounds
Coefficient of variability	8.31	22.31
Coefficient of correlation	$-.185 \pm .029$	

A comparison of Tables 7 and 8 and of the derived constants gives some indication of the effect of the selection practiced in the advanced-registry records. The advanced-registry data show a higher milk yield and fat percentage and lower variability. A large number of the lower-fat-percentage cows are cut out in the advanced-registry records.

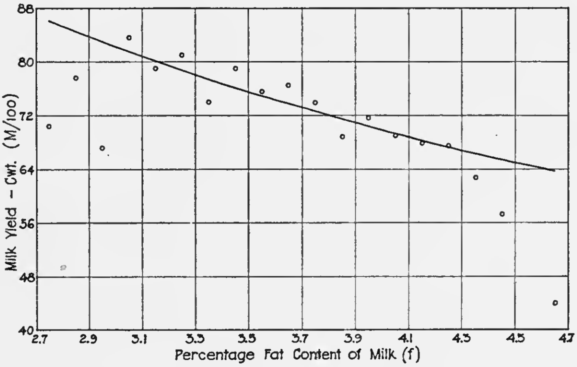

FIG. 8.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: RED DANISH HERD RECORDS

This figure is based on 511 yearly records of Red Danish cows in the herd of Count Ahlefeldt. The plan is similar to that of Fig. 5. Equation of the curve is $M = 46{,}568/(2.66 + f)$. The average energy yield is 2,375 therms, or 6,985 pounds F.C.M.

The correlation between fat percentage and milk yield in these herd records, $r = -.185$, is of about the same order of magnitude as usually found for yearly data of these variables in the lower testing breeds. Of

TABLE 8.—CORRELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: RED DANISH RECORDS FROM HERD OF COUNT AHLEFELDT

The figures of the top row indicate the lower values of the fat-percentage classes; the figures of the left-hand column indicate the mid-values in hundred kilograms of the milk-yield classes.

	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	Total
14	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	1
16	..	..	..	..	..	..	..	1	..	1	..	1	1	..	..	..	..	..	..	..	4
18	..	..	..	..	..	1	3	..	2	1	1	2	1	..	1	..	1	..	..	..	13
20	..	..	..	..	..	..	1	1	2	1	2	..	1	1	..	..	..	..	..	1	10
22	..	..	1	..	..	..	..	2	1	3	1	4	2	..	..	..	..	..	..	..	14
24	..	..	1	..	..	1	5	4	1	4	2	1	1	1	..	1	..	..	..	..	22
26	..	..	1	..	3	2	2	2	6	2	3	6	..	2	..	..	1	1	..	..	31
28	..	1	2	..	1	2	2	2	4	5	5	3	1	..	..	..	..	..	..	..	28
30	..	..	1	1	4	7	..	3	2	3	3	3	3	2	2	..	..	..	..	..	34
32	1	1	..	2	2	3	7	8	6	9	7	7	6	3	..	1	..	..	..	..	63
34	..	1	..	1	7	3	8	6	3	8	4	8	1	2	1	..	1	..	..	..	54
36	..	..	..	..	6	2	8	6	8	3	6	3	1	1	..	1	1	..	..	..	46
38	..	..	1	2	5	4	5	9	11	2	3	..	3	1	..	..	..	..	..	..	46
40	..	1	..	..	3	6	5	10	8	7	6	5	4	..	..	..	..	..	..	..	55
42	..	1	..	4	5	1	3	5	3	4	5	3	2	..	1	..	..	..	..	..	37
44	..	..	..	..	2	3	2	6	2	4	2	1	..	1	..	..	..	..	..	..	23
46	..	..	..	1	1	2	..	4	2	5	1	..	..	..	..	..	..	..	..	..	16
48	..	..	1	..	..	3	..	..	..	..	..	..	..	..	..	..	..	..	..	..	4
50	..	..	..	..	..	..	..	..	1	1	..	..	1	..	..	..	..	..	..	..	3
52	..	..	..	..	..	..	..	..	..	1	1	..	..	..	..	..	..	..	..	..	2
54	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
56	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	1
58	..	..	..	..	..	1	..	1	..	1	..	..	..	..	..	..	..	..	..	..	3
Total	1	5	8	11	39	42	52	70	62	65	53	47	28	14	5	3	4	1	0	1	511

principal interest is the regression of milk yield on fat percentage as shown in Fig. 8. The average energy yield of the 511 records is 2,375 therms, or 6,985 pounds, F.C.M. The constant energy curve of Fig. 8 represents this value thruout, and, except for some deviations at either fat percentage extreme, it conforms reasonably well with the observed milk yields.

Nature of the Relationship.—Milk secretion, as it has become quantitatively developed in the dairy cow, requires a very great expenditure of energy. It seems reasonable to suppose that on the average a 3-percent cow should do as much work as, and no more than, a 4-percent cow, or a 5-percent cow, if all factors such as size and age are equalized. It seems reasonable further that the work performed by the cow in milk secretion should be proportional to the product of that work as measured in terms of calorific value.[a] Haecker's work and the results of the Copenhagen experiment confirm the latter proposition.

The nature of the relationship between fat percentage and milk yield seems to indicate that the energy required in milk secretion is a limiting factor in the amount of milk secreted, and the amount of energy devoted to milk secretion is independent of the particular *proportions* in which the several milk constituents are elaborated. On the basis of such a physiological interpretation we should expect to find no exceptions so far as any particular breed of cows is concerned.

In this connection the negative correlation between fat percentage and milk yield found above for the Red Danish breed is of special interest because of the fact that Ellinger[9*] has reported a correlation of $r = .055 \pm .044$ for this breed in the Count Ahlefeldt herd. He dealt, however, with only the first ten weeks of the lactation and it seems

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[a] In some cases glandular activity is partly manifest in a difference in the osmotic pressure of the secretion and the osmotic pressure of the blood. The urine, for example, may be of much higher osmotic pressure than the blood and in such case the kidney has expended energy and performed work in this particular (cf. Baylis,[5*] pages 339-343). A striking characteristic of the milk of the cow (and probably all mammals) is that its osmotic pressure is the same as that of the blood which nourishes the gland; and the osmotic pressure of the blood in health varies only within very narrow limits. In milk secretion there is no balance of osmotic energy with which to reckon, unless it requires energy on the part of the cell to maintain, for instance, a lower concentration of sodium chlorid in the milk than exists in the blood. If the quantity of water in the milk is determined by osmotic forces (as seems likely) without net expenditure of energy, then it seems clear enough that the water of the milk should be ignored in any quantitative measure of milk yield for biological study of dairy capacity. To do otherwise places undue emphasis on the lactose of the milk (cf.[12, 27, 28, 29, 38*]).

that his peculiar result is in some way connected with this early stage of lactation, since as shown above, the usual negative correlation obtains when dealing with the yearly records in the same breed and herd.[a]

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[a] Some other investigators have reported practically zero correlation, or even positive correlation, between fat percentage and milk yield, but so far as the writer has observed, there was always some plausible reason to believe that extraneous factors were entering in to disturb the true relation. For example, in certain Holstein records special feeding and management practices may render the records unsuitable to reveal the true relationship. This is especially the case in the later 7-day records made shortly after calving (cf. Fig. 4 of Gaines[18*]).

Langmack[30*] has published an extensive series of correlations between fat percentage and milk yield for the separate lactation periods of the same cow. If we could have the same cow produce alternately, for certain periods, milk of a prescribed fat percentage, with other conditions remaining constant, we would have a direct way of measuring the influence of fat percentage (composition of the milk) on the yield of milk. Unfortunately this is a condition impossible of experimental control, and hence we must depend on the indirect evidence of some such statistical device as that of Fig. 5. In Langmack's procedure there is some fluctuation in fat percentage from one lactation to another. But the age of the cow is certainly changing, and, for immature ages, her weight (size) also. Age and weight are both powerful factors affecting productive capacity, and hence it is not permissible to attribute changes in milk yield from lactation to lactation as due entirely, or even to any important extent, to differences in fat percentage. About two-thirds of Langmack's coefficients were negative and one-third positive. The negative correlation is in accord with the known pronounced tendency for milk yield to increase with age (in cows under 8 or 9 years of age, which constitute the great majority of the population), and the slight tendency for the fat percentage to decrease with age. The positive correlations indicate, however, that a considerable proportion of the individual cows have a slight tendency for the fat percentage to increase with age. It would not be proper to interpret Langmack's positive correlations as conflicting with the theory that, so far as affected by the fat percentage (composition) of the milk, the milk yield is inversely proportional to the energy value per pound of milk. Neither do his negative correlations lend any support to the theory.

Since this paper was prepared there has come to hand Missouri Research Bulletin 105, by Samuel Brody, entitled "Growth and Development with Special Reference to Domestic Animals: X, The Relation Between the Course of Growth and the Course of Senescence with Special Reference to Age Changes in Milk Secretion." On the relation between fat percentage and milk yield Brody uses the equation $M = Cf^{-k}$ in which M and f are in the present notation and C and k are constants. This is a very interesting form of expression and undoubtedly capable of describing the relation with accuracy. Just what biological meaning may be attached to the constants is not clear.

SOME ILLUSTRATIVE APPLICATIONS

Lactation Curves.—An illustration of the application of energy values to the lactation curves of farrow Guernsey cows has been presented heretofore (Fig.1[20*]). It was there shown that the energy lactation curve is more regular than the milk or fat lactation curves. Similar material is presented here in Fig. 9 taken from Ellinger's[10*] data on the Red Danish breed.[a] It is evident from the graph and from the numerical values given in the legend, that the energy lactation curve is much more regular than the milk or fat lactation curves.

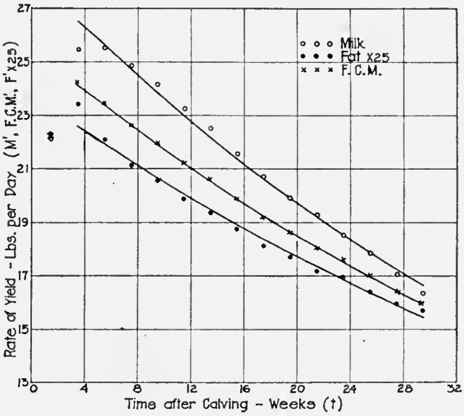

FIG. 9.—LACTATION CURVES OF RED DANISH COWS

This figure shows for the first lactation periods of Red Danish cows the rate of milk yield (M'), the rate of F.C.M. yield (F.C.M.'), and the rate of fat yield (F') in pounds per day with advance in lactation. The curves are of the exponential type, *rate of yield* $= Ae^{-kt}$. They have been fitted by the method of least squares, excluding the first observation. (For details of method of fitting cf. Gaines[15, 20*]). The equations are: $M' = 28.25e^{-.01799t}$; $\text{F.C.M.}' = 25.55e^{-.01468t}$; and $F' \times 25 = 23.78e^{-.01468t}$. The relative root-mean square errors weighted by $1/A$ are: F.C.M., 100; F, 487, and M, 476. The F.C.M. observations thus agree much more closely with their smooth curve than do the fat or milk observations with theirs.

The constant of proportionality, k, shows the rate of decrease per week in the rate of yield. The rate of decrease per month would be $4.345k$. The rate of decrease in energy yield is therefore 6.38 percent per month. This figure is for young cows; for older, higher-yielding cows the rate of decrease would undoubtedly be considerably greater (cf. Fig. 28 of Gaines[16*]).

If we choose to regard the cow as a machine, the energy lactation curve may be translated directly as representing the horsepower delivered by the machine. This point of view may be justified by the fact, as above pointed out, that the feed energy required for lactation is proportional to the milk energy. On the basis that 1 pound of

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[a] The writer is indebted to Dr. Ellinger for the numerical data, which were presented only graphically in the reference given.

F.C.M. = 340 calories, and on the basis of the mechanical equivalent of heat that 1 calorie = 3,084 foot-pounds, we have 1 pound F.C.M. = 1,048,560 foot-pounds and 1 pound F.C.M. per day = .022 horsepower. Accordingly, in Fig. 9 we would have a maximum power output of .533 (= 24.23 × .022) horsepower, which gradually declines with time, finally reaching zero with the cessation of lactation.

This interpretation may serve to indicate the broad nature of the energy measure. It clearly puts the performance of the cow on a dynamical basis.

Nutrition Investigations.—Hansson[25*] says, "Die beste Grundlage zur Berechnung des Nahrungsbedarfs der Kühe bei der Produktion von Milch mit verschiedenem Fettgehalte ist der Kalorienwert der Milch je Kilogramm." (*The best basis of reckoning the food requirement of cows for the production of milk of different fat content is the calorific value of the milk per kilogram.*) The advantage of measuring milk yield on an energy basis for nutritional studies seems to be so plain as to require no extended discussion. For very refined investigations it would be desirable to have direct determinations of the energy value of the milk by the calorimeter. For the usual feeding trials equation (3), page 404 or the equivalent calorie formula, page 414 would seem to be amply accurate.[a]

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[a] It may be of interest to take any of the dairy feeding standards and compute the nutrients required at various fat percentages for one pound F.C.M. The results will be found substantially constant.

Haecker's data as above analyzed lead to the formula, Pounds of digestible nutrients for lactation = .327 F.C.M. His standard for maintenance is, Pounds digestible nutrients for maintenance per year = 2.893 W, where W is live weight of the cow in pounds. It is of interest to compare the observed feed consumption of Table 5 (1 feed unit = 1 kilogram of barley = 1.75 pounds of digestible nutrients) with the requirements computed by Haecker's formulas:

	Red Danish	*Crossbred*	*Jersey*
Pounds digestible nutrients consumed, observed	5,388	4,809	4,347
Pounds digestible nutrients required, computed	5,393	4,818	4,274

This is a rather remarkable agreement between theory and observation.

Since this paper was prepared there has come to hand Wisconsin Research Bulletin 79, by M. J. B. Ezekiel, P. E. McNall, and F. B. Morrison, entitled "Practices Responsible for Variations in Physical Requirements and Economic Costs of Milk Production on Wisconsin Dairy Farms." Fig. 6 of this Wisconsin bulletin presents three sets of data from farm records from the states of Wisconsin, Virginia, and Pennsylvania, showing the relation between fat percentage and the yield and feed cost of the milk. The three sets of data are not in the closest agreement among themselves, a result possibly of the difficulty of accurately evaluating all the factors involved, particularly pasture. The results from the Wisconsin farms agree fairly well with the proposition that feed cost in nutrients is proportional to the energy value of the milk.

Economic Interpretation.—Certain economic aspects of equation (3) have been presented in detail elsewhere[13, 17*] and it is sufficient here to say that the cost of producing milk, so far as affected by the fat percentage of the milk, is proportional to the energy value of the milk. There is a growing disposition on the part of whole-milk buyers to adjust the price of milk according to its fat test, and there seems to be a tendency for this adjustment to align more or less closely with the energy value of the milk (cf. Fig. 2 of Gaines[13*]).

Genetic Investigations.—A possible use of energy yield in genetical analysis may be illustrated by a hypothetical case. Suppose we have two pure races of cattle, one of which has a genetic capacity of an annual yield of 10,000 pounds of 3.5-percent milk, and the other 7,551 pounds of 5.5-percent milk. If these two races are hybridized, we might anticipate obtaining a certain proportion of F_2 segregates which had a capacity of 10,000 pounds of 5.5-percent milk. But if we take the energy yield as the measure of production, we find the two races have the same capacity, namely, 9,250 pounds F.C.M., and consequently we should expect all the F_2 generation to be of the 9,250-pound class, assuming that the capacity of 9,250 pounds F.C.M. of the two original pure races was determined by similar factors.

While we are speculating, let us consider a still wider divergence in the orginal stock. The reindeer produces milk containing 22 percent of fat.[4*] It is not extraordinary to conceive of a Holstein cow producing in 365 days 20,000 pounds of 3.3-percent milk. Many cows have exceeded that performance. Suppose such a race is crossed with the reindeer (assuming for the sake of the argument that such a cross would be fertile) should we obtain in the F_2 generation an occasional female segregate producing 20,000 pounds of 22-percent milk in 365 days under favorable environmental conditions? Considered from the energy viewpoint, we should expect nothing of the kind, for the reindeer's energy-yield capacity is a mere fraction of that of the Holstein, and we should be lucky, therefore, to recover in F_2 even the original capacity of the Holstein ancestor.[a]

Various breeding operations have been entered into at various times by various people with the idea of obtaining an improved dairy cow as a high-milk and high-fat-percentage segregate, on the basis that milk yield and fat percentage are not correlated. The above noted crossing of the Red Danish and Jersey breeds is an example, and much wider

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[a] The foregoing speculation assumes inheritance and segregation in accordance with Mendelian principles. The actual inheritance of milk yield and composition of the milk is a very complicated affair, necessitating, on the Mendelian interpretation, the assumption of multiple factors (cf. [7, 8, 12, 21, 22, 23, 35, 43*]).

crosses have been made, as mating a zebu male with Holstein females. The results of such crossings do not appear to promise the realization of increased capacity by such methods. To a certain extent, therefore, these hybridizing results may be regarded as experimental evidence that the energy yield is a more fundamental measure of performance than is the milk yield.

An experiment to demonstrate the possibility of improving the dairy production of the daughters of scrub cows by the use of purebred dairy bulls, has been carried out at the Iowa Experiment Station. In one case the average of three lactations of a certain scrub cow and six lactations of a certain daughter of the cow by a Holstein bull are given[31*] as:

	M	*F*	*f*	F.C.M.
Dam	3,874.6	192.62	4.97	4,439
Daughter	6,955.5	266.25	3.83	6,776
Daughter/dam	1.795	1.382	.771	1.526

The daughter's production shows thus an increase of 80 percent in milk and 38 percent in fat over and above that of the dam. The actual increase in work accomplished (F.C.M. yield) by the daughter is 53 percent. This may appeal to the reason as being a better expression of the improvement effected by the sire.[a]

The dairyman who is selling whole milk at a fixed price per hundredweight may argue that he is concerned only with the increase in milk yield. Likewise, the one who is selling cream may argue that he is concerned only with the increase in fat yield. But if the biologically important measure of activity of the mammary gland is the energy value of the milk solids, as seems to be sufficiently evident from the citations of the foregoing pages, then what the dairyman needs first of all is a high-energy-yielding (hard-working) cow.[b] If economic conditions are such that milk has money value only according to weight, then he will naturally want that high-energy-yielding cow which gives milk with a minimum energy value per pound, that is the cow with low fat percentage. If economic conditions are such that only the fat of the milk has money value, then he will want that high-energy-yielding cow which devotes the largest part of the energy of lactation to the production of fat. This is the cow with high fat percentage, as is clearly shown in Fig. 3.

SIGNIFICANCE OF FAT PERCENTAGE

The percent of fat in milk has been very extensively determined because of economic reasons, and has become a very familiar characteristic

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[a] It should be noted that this ratio method is not a good way of measuring the potential dairy capacity of the sire (cf. Yapp[42*]).

[b] High yield is essential to efficiency of production (cf. Fig. 29 of Gaines[16*]).

of breeds and individuals. Fat percentage is a universally used measure of the chemical quality of milk.

Many investigators seem to regard the fat yield of a cow as being due to the milk yield and fat percentage. Thus Winters[40*] cites the records of two purebred full sisters:

	M	F	f	F.C.M.
No. 1................	8,735.2	401.55	4.60	9,517
No. 2................	8,345.5	479.30	5.73	10,528

and says of them: "Sister number 1 has a greater production of milk, but number 2 has a greater production of fat, *due to the greater percent of fat in her milk.* This is a case of physiological variation where the *quantitative* variation favored one sister but the *qualitative* favored the other one." The quoted statement seems to involve the same sort of conception as is involved in the idea of securing a high-milk and high-fat-percentage cow by the crossbreeding methods mentioned in the previous section. The italics are the present writer's.

From a biological standpoint it is not proper to regard fat yield, at a given milk yield, as "due to" fat percentage. Fat yield is the direct result of the rate of fat secretion by the milk secreting cells and the time over which secretion continues. Likewise, milk yield is the result of the rate of milk secretion. In one case we are considering a particular part of the activity of the mammary gland, namely, its elaboration of milk fat; in the other case we are considering the entire mass of the secretory product. Obviously fat percentage is merely a mathematical expression of the ratio ($\times$ 100) of the average rate of fat secretion to the average rate of milk secretion. At a given rate of milk yield fat percentage is due to the rate of fat yield; certainly the rate of fat yield is not, in any biological sense, due to the fat percentage. The criticism is offered, not so much for the special case quoted, as for its bearing on the general point of view.

While the determination of fat percentage has been stimulated primarily by economic forces, it so happens, fortunately, that it is a very good biological measure of the properties of the entire and normal milk of the cow. There is a high correlation between fat percentage and both protein and water percentage,[14*] (r = .812 and .916, respectively) while, roughly, the percentage of lactose and ash may be regarded as constant. We have noted from Overman and Sanmann's[33, 34*] work that fat percentage is very highly correlated with energy value per unit of milk (r = .9814). From the milk yield and fat percentage we have warrant, therefore, to estimate energy yield.

The energy yield affords an inclusive, and fundamentally well-grounded measure of the *amount* of work performed by the cow in milk secretion. The fat percentage gives a reliable index of the *direction* in which the work is performed, that is, the relative extent to which it is directed toward the elaboration of fat, of protein, and of lactose, as shown in Fig. 3. Energy yield may be regarded as the primary variable in dairy production, expressing the quantity of production. Fat percentage may be regarded as a secondary variable, expressing the kind of production.

SUMMARY AND CONCLUSIONS

The performance of the dairy cow at the pail is commonly measured by one or both of two expressions—milk yield and fat yield. In Bulletin 245 of this Station (1923) it was suggested that the energy yield, that is, the gross energy value of the milk, is a better measure of yield than either the milk or the fat. This paper is intended to bring the evidence on the subject up to date. It is based largely on prior pertinent literature.

Energy yield may be estimated with a reasonable degree of accuracy from the milk yield and fat percentage. The correlation between fat percentage and energy value per unit of milk is of the order, $r = .98$ to $.99$. The formula used in Bulletin 245 was $\text{F.C.M.} = .4M + 15F$, where F.C.M. ("fat-corrected milk") is gross energy value in terms of normal average cows' milk of 4-percent fat content, M is actual milk and F is fat, all in the same unit of weight. Evidence since accumulated quite fully substantiates the accuracy of this formula, but indicates that the energy value of 1 pound of 4-percent milk is about 340 large calories (instead of 330.6, as previously given). The corresponding calorie formula becomes $E = 51M\ (2\ 2/3 + f)$, where E is energy in large calories, M is milk in pounds, and f is fat percentage. Use of the F.C.M. formula is continued because of facility of computation and with the idea that the expression of energy yield in calories might be reserved for refined experimental work, where the energy value is determined by direct calorimetry. The F.C.M. formula appears to be sufficiently accurate for ordinary work.

The nutrients required for lactation (maintenance excluded) per pound of milk are directly proportional to the energy value of the milk as computed by the above formula. The correlation between fat percentage and nutrients for lactation per pound of milk is $r = .648$. It seems probable that the total feed consumption is also closely proportional to the energy value of the milk. This relation means practically, in terms of money, that the cost of milk production is proportional to the energy value of the milk. In nutritional work the energy value of the milk affords a comprehensive and well-grounded expression of yield.

It was previously concluded that in so far as milk yield is affected by the composition of the milk, the yield is inversely proportional to the energy value per unit of milk, that is, the energy yield is not affected by the fat percentage of the milk. This conclusion is here supported by additional evidence from the records of the Milking Shorthorn and Red Danish breeds. The correlation between fat percentage and milk yield is of the order, $r = -.2$ to $-.4$; but between fat percentage and energy yield, $r = 0$. As between different cows a certain amount of variability in milk yield is due to differences in the composition of the milk. This source of variability is eliminated when the yield is measured on the energy basis. Energy yields are directly comparable so far as fat percentage of the milk is concerned.

The lactation curve (rate of yield with time after calving) is more regular when expressed in terms of energy than in terms of milk or fat. Utilizing the mechanical equivalent of heat (1 calorie = 3084 foot-pounds) the energy lactation curve may be translated directly in terms of power: 1 pound F.C.M. per day = .022 horsepower. Measuring milk yield on an energy basis puts the performance of the cow on a dynamical basis.

Considering milk yield on an energy basis exposes the fallacy of attempting to breed increased dairy capacity by hybridizing a high-milk, low-fat-percentage race with a low-milk, high-fat-percentage race, in the expectation of obtaining a high-milk, high-fat-percentage, F_2 segregate. The results of several crossbreeding experiments do not promise improvement in energy yield capacity, and this may be taken as experimental evidence that energy yield is a more fundamental measure of performance than is milk yield.

The biological significance of fat percentage is as a measure of the relative rates of secretion of the fat as a part and of the milk as a whole. At a given milk yield the fat yield is not, in any biological sense, due to or caused by the fat percentage. Fat percentage is a good index of the composition of the milk and of its energy value.

Energy yield may be regarded as the primary measure of yield, showing the amount of work done in milk secretion. This work may be done in different directions, that is, to variable degrees in the elaboration of fat, protein and lactose. Fat percentage may be regarded as a secondary measure of yield, showing the direction in which the work is done.

From a biological point of view the essential measures of performance of the cow at the pail are the energy yield and fat percentage.

LITERATURE CITED

1. AMERICAN JERSEY CATTLE CLUB. The dairy cow demonstration at the Louisiana Purchase Exposition, St. Louis, Mo., U. S. A., 1904. 1905.
2. AMERICAN SHORTHORN BREEDERS' ASSOCIATION. Milking Shorthorn Year Book **5-7**. 1920-1922.
3. ANDERSEN, A. C. Undersøgelser over Komaelkens Sammensaetning. (Investigations on the composition of cow's milk.) Nord. Jordbrugsforsk. 4/7: 133-145, (Beret. Nord. Jordbrugsforsk. For. 3d. Kong. Oslo, Juni, 1926.) 1926.
4. BARTHEL, CHR., AND BERGMAN, A. M. Renntiermilch und Renntierkäse. (Reindeer milk and reindeer cheese.) Ztschr. Untersuch. Nahr. u. Genussmtl. **26**, 238-241. 1913.
5. BAYLIS, W. M. Principles of general physiology. Longmans, Green and Co. 1920.
6. BECK, N. Første-fjortende Meddelelse fra Forsøgslaboratoriet om Resultaterne fra sammenlignende Forsøg med forskellige Kvaegracer paa Tranekjaer, 1905-1906—1918-1919. (First-fourteenth communication from the experimental laboratories on the results of cooperative experiments with various breeds of cattle at Tranekjaer, 1905-1906—1918-1919.) Andelsbogtrykkeriet i Odense, 1906-1920.

(For Brody reference, see page 438.)

7. CASTLE, W. E. Inheritance of quantity and quality of milk production in dairy cattle. Nat. Acad. Sci. Proc. **5**, 428-434. 1919.
8. COLE, L. J. The Wisconsin experiment in cross-breeding cattle. Cattle breeding: Proceedings of the Scottish Cattle Breeding Conference. 1924. 315-327. Oliver and Boyd. 1925.
9. ELLINGER, TAGE. The variation and inheritance of milk characters. Nat. Acad. Sci. Proc. **9**, 111-116. 1923.
10. ————. Causes of variations in milk secretion and their bearing on practical breeding methods. Proc. of the World's Dairy Congress 1923, **2**, 1396-1401. 1924.

(For Ezekiel reference see page 438.)

11. FREDERICKSEN, LARS. Foreløbige Oplysninger om nogle Forsøg med Malkekøer i Vinteren 1923-1924. Plan for nogle Forsøg med Malkekøer i Vinteren 1924-1925. (Preliminary information on some experiments with milk cows in the winter of 1923-1924. Plan for some experiments with milk cows in the winter of 1924-1925.) Meddel. Forsøgslab. Husdyrbrug. K. Vet. og Landbohøjskole (Denmark), **2**. 1925.
12. GAINES, W. L. Inheritance of fat content of milk in dairy cattle. Amer. Soc. Anim. Prod. Proc. 1922, 29-32. 1923.
13. ————. Feed cost of milk production as affected by the percentage fat content of the milk. Jour. Agr. Res. **29**, 593-601. 1924.
14. ————. Relative rates of secretion of various milk constituents. Jour. Dairy Sci. **8**, 486-496. 1925.
15. ————. Measures of persistency of lactation. Jour. Agr. Res. **34**, 373-383. 1927.
16. ————. Persistency of lactation in dairy cows. Ill. Agr. Exp. Sta. Bul. 288. 1927.

17. ————. A price differential for whole milk. Ill. Agr. Exp. Sta. Circ. 318. 1927.

18. ————. The deferred short-time test as a measure of the performance of dairy cows. Jour. Agr. Res. **35**, 237-249. 1927.

19. ———— and DAVIDSON, F. A. Relation between percentage fat content and yield of milk. Ill. Agr. Exp. Sta. Bul. 245. 1923.

20. ———— ————. Rate of milk secretion as affected by advance in lactation and gestation. Ill. Agr. Exp. Sta. Bul. 272. 1926.

21. GOWEN, JOHN W. Studies in inheritance of certain characters of crosses between dairy and beef breeds of cattle. Jour. Agr. Res. **15**, 1-57. 1918.

22. ————. Inheritance in crosses of dairy and beef breeds of cattle. II—On the transmission of milk yield to the first generation. Jour. Heredity **11**. 300-316. 1920.

23. ————. Inheritance in crosses of dairy and beef breeds of cattle. III—Transmission of butterfat percentage to the first generation. Jour. Heredity **11**, 365-376. 1920.

24. HAECKER, T. L. Investigations in milk production. Minn. Agr. Exp. Sta. Bul. 140. 1914.

25. HANSSON, NILS. Fütterung der Haustiere. (Feeding domestic animals.) Verlag von Theodor Steinkopff, Dresden. 1926.

26. Holstein-Friesian Association of America. Holstein-Friesian Advanced Register Year Book **21-26**. 1910-1915.

27. JACKSON, L. C., and ROTHERA, A. C. H. Milk: its milk sugar, conductivity, and depression of freezing point. Biochem. Jour. **8**, 1-27. 1914.

28. VAN DER LAAN, H. F. Het osmotisch Evenwicht tusschen Bloed en Melk. (The osmotic equilibrium between blood and milk.) Chem. Weekbl. **12**, 522-541. 1915.

29. ————. Das osmotische Gleichgewicht zwischen Blut, Milch und Galle (The osmotic equilibrium between blood, milk and bile). Biochem. ztschr. **71**, 289-305. 1915.

30. LANGMACK, P. V. F. P. Anden Beretning om Undersøgelse af de enkelte Koers Maelk. (Second report on the investigation of the milk of individual cows.) Beret. Forsøgslab. K. Vet. og Landbohøjskole (Denmark) **107**. 1921.

31. MCCANDLISH, A. C., GILLETTE, L. S., and KILDEE, H. H. Influence of environment and breeding in increasing dairy production, II. Iowa Agr. Exp. Sta. Bul. 188. 1919.

32. OVERMAN, O. R., DAVIDSON, F. A., and SANMANN, F. P. Relation of solids in milk to fat and specific gravity of the milk. Ill. Agr. Exp. Sta. Bul. 263. 1925.

33. ———— and SANMANN, F. P. The energy value of milk as related to composition. Ill. Agr. Exp. Sta. Bul. 282. 1926.

34. ———— ————. Valeur du lait en calories. (Value of milk in calories.) Le Lait. **7**, 149-161. 1927.

35. VON PATOW, C. F. Studien über die Vererbung der Milchergiebigkeit an Hand von fünfzigjährigen Probemelkaufzeichnungen. (Studies on the inheritance of milk yield on the basis of 50-years' milk records.) Zeitschr. f. Tierzücht. u. Züchtungsbiol. **4**, 253-329. 1925.

36. SAMVIRKENDE DANSKE LANDBOFORENINGER. Stambog over Køer af Rød Dansk Malkerace. (Cooperating Danish Agricultural Societies. Herdbook of cows of the Red Danish dairy breed.) **1-4**. 1921-1924.
37. STOCKING, W. A., and BREW, J. D. Milk—the essential food. The Dairymen's League News. Jan. 10, 1920.
38. TAYLOR, WILLIAM, and HUSBAND, A. D. The effect on the percentage composition of the milk of (a) variations in the daily volume and (b) variations in the nature of the diet. Jour. Agr. Sci. **12**, 111-124. 1922.
39. WHITE, G. C., and JUDKINS, H. F. Variations in the fat, solids-not-fat, and total solids in cow's milk. Conn. (Storrs) Agr. Exp. Sta. Bul. 94. 1918.
40. WINTERS, L. M. Animal breeding. Wiley and Sons. 1925.
41. WOLL, F. W. Studies in dairy production. Wis. Agr. Exp. Sta. Res. Bul. 26 1912.
42. YAPP, W. W. Transmitting ability of dairy sires. Amer. Soc. Anim. Prod. Proc. 1924, 90-92. 1925.
43. ————. The inheritance of percent fat content and other constituents of milk in dairy cattle. Cattle breeding: Proceedings of the Scottish Cattle Breeding Conference. 1924, 328-334. Oliver and Boyd. 1925.
44. BRODY, SAMUEL. Growth and development with special reference to domestic animals. X. The relation between the course of growth and the course of senescence with special reference to age changes in milk secretion. Mo. Agr. Exp. Sta. Res. Bul. 105. 1927.[a]
45. EZEKIEL, M. J. B., MCNALL, P. E., and MORRISON, F. B. Practices responsible for variations in physical requirements and economic costs of milk production on Wisconsin dairy farms. Wis. Agr. Exp. Sta. Res. Bul. 79. 1927.[a]

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[a] The last two references were added to the list just before the manuscript was printed, and therefore are not in alphabetical order.