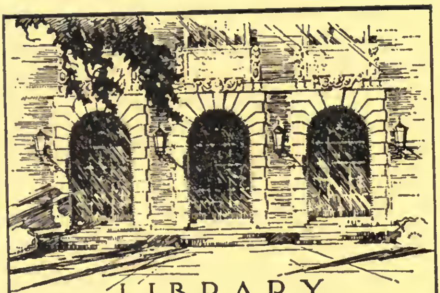

UNIVERSITY OF ILLINOIS

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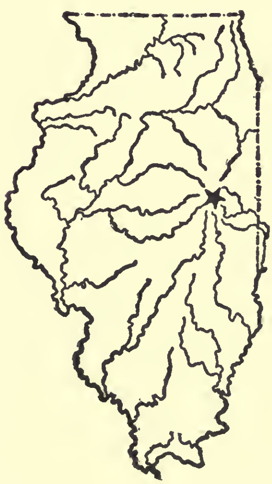
THE ENERGY VALUE OF MILK AS RELATED TO COMPOSITION

Formulas for the Computation of the Energy

By O. R. Overman and F. P. Sanmann



URBANA, ILLINOIS, DECEMBER, 1926

CONTENTS

	PAGES
EXPERIMENTAL PART	207-208
STATISTICAL PART	212-214
APPLICATION	215-218
CONCLUSIONS	218
LITERATURE CITED	218-219

THE ENERGY VALUE OF MILK AS RELATED TO COMPOSITION

Formulas for the Computation of the Energy

By O. R. Overman, Assistant Chief in Dairy Chemistry, and
F. P. Sanmann, Associate in Dairy Manufactures

A knowledge of the energy value[a] of milk or the number of calories of heat which can be produced by the complete combustion of a given quantity of milk is of interest and value both to students of human nutrition and to those of animal nutrition. Formulas have been developed for the computation of this value. Gaines and Davidson,[1] working from data compiled by Stocking and Brew,[2] have derived the formula $E = 49.64\ (2.66 + t)$, in which E is the energy of milk solids and t is the percentage of fat. This formula, which gives the energy value per pound of milk, is based on average analyses, and was used in estimating the energy yield of cows. Frederiksen[3] reports that Director A. C. Andersen gives the heat of combustion of milk to be equal to (percentage of fat $\times$ 113.5) + 290 calories per kilogram.

The present investigation was begun for the purpose of determining the energy values of milks of different composition, and of comparing such determined values with corresponding energy values as computed from the composition of the milk. As the work progressed, the possibility of applying statistical methods to these results, for the purpose of deriving formulas by which the energy value of milk might be computed from its composition, was recognized. This treatment of the data resulted in the derivation of formulas based, first, upon the complete analysis of the milk; second, on the fat, protein, and lactose content; and third, on the fat alone. The details of the statistical methods by which these formulas were derived, and a discussion of the application of the formulas, are presented in the following pages.

EXPERIMENTAL PART

The milks studied were obtained from the purebred and experimental herds of the Dairy Department of the University of Illinois.

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[a]Where the terms energy value or calorie are used it is to be understood that the large calorie is the unit of heat involved.

The samples represent milk from Ayrshire, Guernsey, Holstein, and Jersey cows, and from cows in the Guernsey-Holstein crossbred experimental herd. At the beginning of the work a few samples were taken from single milkings of individual cows. Later, each sample was a composite made from all the milk produced by one cow during three days. Samples were taken at all stages during the normal lactation periods of the cows.

The samples were preserved with formaldehyde, added in approximately the proportions recommended by Palmer,[4] and were kept in half-gallon glass fruit jars fitted with glass lids and rubber rings.

Determinations of percentages of fat, total protein, lactose, and total solids, and for specific gravity and energy value, were made for all samples studied.

The specific gravity was determined with a specific-gravity balance of the chainomatic type. The percentage of fat was determined by the Roese-Gottlieb method, using about 5 grams of the sample.

Total protein was determined by the Official Kjeldahl method. Lactose was determined by the Official Optical method, mercuric nitrate being used as the precipitant. A Schmidt-Haensch saccharimeter was used for all polarizations.

Total solids were determined by the Official method. A Parr adiabatic oxygen-bomb calorimeter was used for all determinations of energy value. The water equivalent of the calorimeter was carefully determined by several combustions of weighed quantities of pure benzoic acid.

About 10 grams of the milk sample were accurately weighed into an Illium combustion capsule and dried on a water bath. The dried samples were kept in a desiccator until the combustion could be made. Combustions were made in oxygen under a pressure of 25 to 30 atmospheres. Temperature changes were read to thousandths of a degree centigrade on a standardized Beckmann thermometer. The rise in temperature was corrected in every case for stem emergence and for the setting of the Beckmann thermometer. The total energy change in the combustion was then computed by multiplying the rise in temperature in degrees by the water equivalent of the calorimeter. This value was then corrected for the heat liberated by the combustion of the iron fuse wire and the heat liberated in the formation of nitric acid during the combustion. A total of 212 samples of milk were so studied.

All analytical results are tabulated in the first six columns of Table 1, arranged down the column in the order of increasing fat percentages. These results in each case are the means of closely corresponding duplicate determinations.

TABLE 1.—ANALYSES OF 212 MILK SAMPLES WITH COMPUTED AND DETERMINED ENERGY VALUES

Fat	Protein	Lactose	Total solids	Specific gravity	Calories per quart						
					Determined	Computed					
						Total	Metabolizable	Formula 1	Formula 2	Formula 3	Formula 4
A	B	C	D	E	F						
perct.	perct.	perct.	perct.			cals.	cals.				
2.68	2.93	4.72	10.96	1.0298	580.4	585.8	533.3	582.2	582.4	536.8	583.6
2.72	3.27	3.92	10.58	1.0291	565.8	577.1	518.5	577.7	586.9	541.0	584.3
2.76	2.86	4.89	11.13	1.0310	589.2	596.3	544.8	591.3	591.5	545.2	591.6
2.82	2.72	5.05	11.07	1.0312	585.3	600.3	541.2	589.5	598.3	551.5	593.7
2.85	2.91	4.81	11.04	1.0317	591.1	604.6	551.9	593.7	601.7	554.7	600.3
2.86	2.66	4.78	11.18	1.0301	592.0	589.5	541.0	595.5	602.8	555.8	586.9
2.94	3.10	4.60	11.48	1.0319	618.9	615.3	559.2	618.8	611.9	564.2	613.2
2.94	3.18	4.82	11.55	1.0329	622.6	628.8	571.4	622.3	611.9	564.2	623.4
2.94	2.92	4.46	11.10	1.0310	607.3	599.4	546.2	601.8	611.9	564.2	599.7
2.94	3.03	4.80	11.20	1.0306	602.9	618.4	563.5	606.5	611.9	564.2	614.8
2.96	3.06	4.70	11.15	1.0316	604.8	618.6	563.1	606.5	614.2	566.3	615.5
2.98	2.88	4.48	11.09	1.0316	603.0	601.9	549.3	602.9	616.5	568.4	601.7
2.98	3.02	4.84	11.37	1.0321	606.6	623.8	569.1	614.8	616.5	568.4	618.9
2.99	2.82	4.71	11.08	1.0310	595.0	607.9	556.4	601.4	617.6	569.4	605.5
2.99	2.94	4.62	11.25	1.0313	606.2	611.3	557.8	610.1	617.6	569.4	609.6
3.01	2.97	5.16	11.71	1.0318	625.2	635.9	582.1	627.6	619.9	571.6	627.5
3.02	2.97	4.75	11.48	1.0319	623.8	621.1	567.0	620.5	621.0	572.6	617.4
3.04	3.13	4.84	11.56	1.0317	614.0	635.1	578.4	627.0	623.3	574.7	630.3
3.04	3.16	5.01	11.90	1.0325	638.1	644.0	586.7	639.9	623.3	574.7	636.5
3.04	3.18	4.92	11.76	1.0326	639.0	641.5	585.7	635.2	623.3	574.7	635.2
3.06	3.15	5.11	12.23	1.0353	659.7	650.7	593.5	653.2	625.6	576.8	640.4
3.08	2.87	5.21	11.68	1.0318	624.4	638.6	586.3	628.4	627.9	578.9	629.7
3.09	2.91	4.96	11.66	1.0324	630.9	632.4	579.3	629.5	629.0	580.0	626.1
3.10	2.92	5.03	11.50	1.0308	621.4	635.5	582.4	623.9	630.1	581.0	629.4
3.10	3.00	4.96	11.88	1.0325	643.4	638.4	583.7	639.8	630.1	581.0	631.9
3.12	2.88	5.10	11.79	1.0321	653.7	638.9	586.0	635.1	632.4	583.1	631.0
3.12	3.12	4.93	11.70	1.0333	634.4	646.3	589.5	636.2	632.4	583.1	639.4
3.14	2.90	4.75	11.49	1.0316	633.0	627.7	574.6	626.1	634.7	585.2	624.5
3.14	2.91	4.64	11.39	1.0314	631.1	624.0	570.6	622.7	634.7	585.2	622.1
3.16	2.98	5.08	11.56	1.0312	623.8	646.5	592.2	630.2	637.0	587.3	639.5
3.17	3.11	4.48	11.72	1.0313	635.8	631.6	574.8	640.5	638.1	588.4	631.4
3.18	2.89	4.85	11.55	1.0298	628.6	633.6	580.6	629.9	639.2	589.4	630.3
3.18	3.03	5.02	11.82	1.0312	634.2	648.8	593.5	642.1	639.2	589.4	642.4
3.18	3.11	4.88	11.74	1.0329	637.4	649.0	592.2	640.8	639.2	589.4	643.0
3.19	3.05	4.74	11.63	1.0302	632.8	639.4	583.7	636.4	640.4	590.5	636.9
3.20	2.88	4.84	11.42	1.0294	625.1	634.2	581.4	625.9	641.5	591.6	631.3
3.20	3.26	4.78	11.80	1.0327	644.7	655.1	595.8	646.9	641.5	591.6	650.3
3.22	3.00	4.74	11.77	1.0313	638.7	641.9	587.0	642.4	643.8	593.7	636.9
3.22	3.18	4.48	11.70	1.0315	651.3	639.4	582.0	643.6	643.8	593.7	639.7
3.25	2.83	5.18	11.96	1.0317	646.3	650.5	598.4	647.4	647.2	596.8	642.2
3.29	3.00	4.94	11.94	1.0310	647.4	653.8	598.8	652.1	651.7	601.0	648.6
3.32	3.09	5.17	12.30	1.0323	667.8	671.2	614.7	668.2	655.1	604.2	662.4
3.35	2.88	5.26	12.01	1.0319	652.1	665.4	612.4	655.1	658.6	607.3	656.1
3.37	3.20	5.03	12.14	1.0317	671.3	676.1	617.6	667.0	660.8	609.5	669.1
3.39	3.07	4.69	11.68	1.0315	652.8	657.4	600.8	649.4	663.1	611.6	654.8
3.41	2.90	5.24	12.17	1.0329	669.6	671.8	618.3	664.8	665.4	613.7	662.1
3.42	3.11	4.94	11.90	1.0322	670.0	672.5	615.2	659.3	666.5	614.7	666.4
3.42	3.38	4.83	12.34	1.0316	682.7	682.8	621.1	680.7	666.5	614.7	678.1
3.43	3.13	5.27	12.64	1.0327	693.1	687.5	630.1	687.3	667.7	615.8	677.2
3.45	3.06	4.58	11.66	1.0314	653.3	658.0	601.4	652.0	670.0	617.9	656.7
3.46	3.14	5.15	12.24	1.0318	679.9	685.6	627.9	674.2	671.1	618.9	677.2
3.47	3.48	4.77	12.37	1.0322	684.1	691.0	627.4	686.3	672.2	620.0	686.4
3.49	3.02	4.98	12.32	1.0322	678.1	675.3	619.4	677.3	674.5	622.1	668.9
3.50	3.10	4.87	12.04	1.0310	671.7	675.6	618.4	668.8	675.6	623.1	671.2
3.50	3.64	5.26	12.87	1.0351	704.0	723.6	657.3	708.2	675.6	623.1	710.9
3.51	2.98	5.14	12.26	1.0319	676.3	680.8	635.4	675.0	676.8	624.2	672.8
3.52	2.90	5.26	12.22	1.0320	670.1	682.0	628.2	672.4	677.9	625 2	672.6
3.52	3.01	4.86	12.10	1.0333	676.1	673.5	617.6	670.8	677.9	625.2	667.9
3.52	3.12	4.90	12.10	1.0322	673.0	680.5	622.8	672.5	677.9	625.2	674.9
3.56	2.94	5.21	12.20	1.0312	676.3	685.4	630.8	674.5	682.4	629.5	677.0
3.56	3.55	5.16	12.97	1.0338	703.4	719.2	654.3	713.9	682.4	629.5	708.8
3.58	3.22	5.14	12.58	1.0325	694.4	700.9	641.6	694.8	684.7	631.6	692.2
3.59	3.10	5.39	12.72	1.0326	703.4	704.9	647.6	698.0	685.9	632.6	693.3
3.60	3.05	4.82	12.24	1.0308	677.8	679.7	623.2	681 0	687.0	633.7	676.2
3.60	3.08	5.05	12.16	1.0310	678.4	690.4	633.4	677.8	687.0	633.7	684.0
3.60	3.52	4.73	12.57	1.0331	698.3	704.0	639.4	701.5	687.0	633.7	699.3
3.61	3.12	5.31	12.66	1.0322	697.2	704.4	646.8	697.3	688.1	634.7	694.0

TABLE 1.—ANALYSES OF 212 MILK SAMPLES WITH COMPUTED AND DETERMINED ENERGY VALUES—(*Continued*)

Fat	Protein	Lactose	Total solids	Specific gravity	Calories per quart						
					Determined	Computed					
						Total	Metabolizable	Formula 1	Formula 2	Formula 3	Formula 4
A	*B*	*C*	*D*	*E*	*F*						
perct.	*perct.*	*perct.*	*perct.*			*cals.*	*cals.*				
3.62	2.93	5.10	12.18	1.0324	692.8	686.7	632.1	677.1	689.3	635.8	679.0
3.62	3.25	5.24	12.51	1.0325	696.3	710.1	650.2	694.5	689.3	635.8	700.1
3.64	3.06	4.50	12.06	1.0308	675.6	671.5	614.8	677.3	691.5	637.9	671.8
3.64	3.14	4.83	12.10	1.0333	674.4	690.5	632.1	679.4	691.5	637.9	685.0
3.64	3.62	5.16	12.84	1.0348	708.6	731.0	664.7	714.4	691.5	637.9	719.9
3.68	3.08	4.56	11.85	1.0313	649.1	679.0	621.5	671.7	696.1	642.1	678.1
3.70	3.00	4.99	12.16	1.0329	684.1	694.0	637.9	682.0	698.4	644.2	687.1
3.70	3.50	4.81	12.77	1.0322	711.9	714.5	650.0	713.8	698.4	644.2	709.4
3.71	2.84	5.34	12.33	1.0328	689.6	699.3	646.2	685.4	699.5	645.3	688.7
3.71	3.23	4.79	12.48	1.0328	699.8	699.9	639.9	699.0	699.5	645.3	695.2
3.72	3.08	4.85	12.29	1.0317	681.9	694.6	636.6	689.7	700.6	646.3	689.5
3.72	3.11	5.02	12.36	1.0314	693.6	702.0	644.2	692.9	700.6	646.3	695.7
3.75	3.17	4.99	12.38	1.0308	695.5	706.5	647.6	695.7	704.1	649.5	700.9
3.77	2.99	4.95	12.40	1.0324	700.8	697.8	641.8	694.8	706.3	651.6	691.8
3.78	3.10	4.50	12.12	1.0310	683.1	686.6	628.5	687.7	707.5	652.6	686.7
3.80	3.15	4.76	12.32	1.0329	693.9	702.4	643.6	696.5	709.7	654.7	698.2
3.82	2.79	5.02	12.20	1.0331	681.8	694.4	641.6	686.4	712.0	656.8	687.4
3.86	3.02	4.50	12.28	1.0311	691.8	689.4	632.5	696.6	716.6	661.0	689.6
3.86	3.36	5.41	13.21	1.0338	731.5	745.3	683.1	735.0	716.6	661.0	732.4
3.89	3.42	4.74	12.70	1.0314	718.2	723.9	660.3	720.0	720.0	664.2	720.4
3.89	3.64	5.07	13.06	1.0336	738.9	750.4	683.2	736.5	720.0	664.2	741.2
3.91	3.43	5.24	13.23	1.0332	739.5	746.7	683.2	740.0	722.3	666.3	736.2
3.92	2.99	4.42	12.24	1.0308	694.6	689.8	633.3	698.0	723.4	667.4	691.3
3.93	3.14	5.16	12.96	1.0342	724.5	730.0	671.1	726.4	724.5	668.4	720.1
3.94	3.37	5.02	12.78	1.0329	718.2	737.4	674.7	724.2	725.7	669.5	729.8
3.96	3.47	4.01	12.24	1.0314	698.4	704.7	639.9	709.1	727.9	671.6	710.0
3.97	3.71	5.15	13.36	1.0351	754.8	765.7	697.2	753.1	729.1	672.6	754.4
3.98	3.00	4.88	12.36	1.0313	706.4	713.8	657.2	704.6	730.0	673.7	709.6
3.98	3.16	4.88	12.49	1.0331	708.1	724.1	664.6	712.3	730.0	673.7	718.2
3.99	3.57	4.58	12.83	1.0314	727.4	735.0	668.7	733.1	731.4	674.7	733.4
4.02	3.29	5.22	13.26	1.0339	745.5	748.6	687.1	744.8	734.8	677.9	738.0
4.02	3.66	4.50	12.84	1.0318	723.7	740.0	672.0	736.8	734.8	677.9	738.8
4.04	3.62	5.44	13.54	1.0355	760.1	778.6	711.5	761.3	737.0	680.0	763.6
4.05	3.41	5.12	13.26	1.0328	751.4	753.4	689.8	748.5	738.2	681.0	744.6
4.07	4.32	4.98	13.97	1.0365	790.5	803.4	724.2	792.0	740.4	683.2	792.0
4.08	3.34	5.17	13.17	1.0319	743.9	753.4	691.0	745.4	741.6	684.2	744.8
4.09	3.05	5.17	12.84	1.0306	725.0	737.2	679.7	728.6	742.7	685.3	730.0
4.10	3.74	4.71	13.06	1.0308	749.4	758.9	689.7	750.1	743.9	686.3	756.1
4.12	3.41	3.96	12.14	1.0306	711.0	713.3	649.2	712.9	746.1	688.4	719.9
4.13	3.16	5.18	12.95	1.0330	738.6	749.0	689.5	736.8	747.3	689.5	739.9
4.16	3.06	5.01	12.84	1.0312	726.7	738.4	680.4	732.9	750.7	692.6	732.6
4.16	3.34	4.84	12.99	1.0317	748.6	747.8	685.0	743.7	750.7	692.6	743.3
4.16	3.36	5.36	13.42	1.0336	757.4	770.3	707.4	759.0	750.7	692.6	758.2
4.17	3.67	5.07	13.36	1.0335	765.9	777.3	709.0	763.1	751.8	693.7	768.2
4.18	3.70	4.76	13.47	1.0322	773.0	766.9	699.0	769.1	753.0	694.7	762.5
4.20	3.21	5.07	13.30	1.0332	753.4	754.1	693.4	754.8	755.2	696.8	746.0
4.25	3.62	5.46	13.80	1.0341	792.1	797.2	729.8	782.1	760.9	702.1	783.2
4.28	3.71	4.99	13.66	1.0341	782.0	786.8	717.5	781.2	764.3	705.3	778.2
4.32	3.39	4.68	13.19	1.0328	743.7	759.6	695.5	761.0	768.9	709.5	756.2
4.32	4.08	4.81	13.96	1.0338	799.9	803.9	728.3	801.2	768.9	709.5	797.1
4.38	3.14	4.94	12.92	1.0328	748.4	761.1	701.2	749.1	775.7	715.8	755.0
4.38	3.76	4.91	13.70	1.0327	792.6	794.5	724.2	788.9	775.7	715.8	787.9
4.39	3.71	5.41	14.06	1.0350	803.4	813.7	743.1	801.0	776.8	716.8	799.4
4.41	3.22	5.18	13.35	1.0325	762.8	777.3	716.1	767.7	779.1	719.0	768.5
4.41	3.68	5.10	13.91	1.0334	797.0	800.6	731.6	796.7	779.1	719.0	791.3
4.42	3.24	5.29	13.55	1.0324	779.7	783.5	722.0	775.8	780.3	720.0	773.4
4.42	3.29	5.01	13.31	1.0311	768.7	774.5	712.1	768.2	780.3	720.0	768.7
4.45	3.72	4.78	13.74	1.0326	794.8	793.5	723.6	793.8	783.7	723.2	788.6
4.46	3.36	4.86	13.32	1.0312	760.8	776.3	712.6	772.3	784.8	724.2	772.1
4.46	3.50	5.06	13.78	1.0338	792.7	793.8	727.7	791.6	784.8	724.2	785.0
4.47	3.94	4.77	13.90	1.0328	807.3	807.3	733.7	804.6	785.9	725.3	802.1
4.48	3.48	5.32	13.70	1.0336	791.0	804.4	738.7	788.5	787.1	726.3	792.7
4.50	3.32	5.26	13.50	1.0337	775.2	795.0	731.9	779.6	789.4	728.4	784.2
4.52	3.22	4.41	12.94	1.0311	756.8	756.5	694.8	760.0	791.6	730.5	757.9
4.52	3.33	4.82	13.38	1.0315	773.8	778.7	715.3	777.3	791.6	730.5	774.8
4.52	3.40	5.00	13.35	1.0313	767.4	789.4	725.0	776.9	791.6	730.5	783.4
4.53	3.56	5.24	13.84	1.0334	798.1	810.1	743.0	798.0	792.8	731.6	799.4

TABLE 1.—ANALYSES OF 212 MILK SAMPLES WITH COMPUTED AND DETERMINED ENERGY VALUES—(*Continued*)

Fat	Pro- tein	Lactose	Total solids	Specific gravity	Calories per quart						
					Deter- mined	Computed					
						Total	Meta- boliz- able	Form- ula 1	Form- ula 2	Form- ula 3	Form- ula 4
A	*B*	*C*	*D*	*E*	*F*						
perct.	*perct.*	*perct.*	*perct.*			*cals.*	*cals.*				
4.55	3.85	4.92	13.87	1.0340	803.6	816.2	744.0	805.9	795.0	733.7	808.4
4.55	3.97	5.38	14.69	1.0355	845.7	842.0	767.8	837.8	795.0	733.7	827.2
4.58	3.80	4.96	13.82	1.0325	807.3	816.5	745.2	804.5	798.5	736.9	809.5
4.60	3.41	3.91	12.71	1.0307	747.7	754.7	689.4	760.0	800.7	739.0	762.1
4.62	2.96	4.60	13.03	1.0322	769.1	759.1	701.6	763.9	803.0	741.1	757.9
4.63	3.54	5.26	14.06	1.0337	809.8	819.1	752.0	811.2	804.1	742.1	808.0
4.64	3.36	5.41	14.18	1.0331	812.8	815.2	751.3	812.9	805.3	743.2	803.1
4.65	3.80	4.62	13.73	1.0313	807.8	808.8	737.2	805.7	806.4	744.2	806.8
4.67	3.68	3.64	12.83	1.0297	773.0	764.9	694.9	773.3	807.7	746.3	775.9
4.68	3.80	4.69	14.09	1.0304	827.3	813.5	741.9	820.7	809.8	747.4	811.4
4.74	4.38	4.72	14.74	1.0344	858.3	855.6	773.9	858.1	816.7	753.7	849.1
4.74	4.04	5.02	14.47	1.0351	845.8	848.8	772.9	841.5	816.7	753.7	838.7
4.75	3.46	5.42	14.14	1.0338	821.7	831.7	765.8	818.9	817.8	754.7	818.8
4.76	4.08	5.02	14.41	1.0330	834.3	851.1	774.6	840.8	818.9	755.8	842.6
4.82	3.36	5.54	14.42	1.0320	842.3	835.6	771.4	831.1	825.7	762.1	822.9
4.87	3.91	4.59	14.19	1.0326	843.7	834.6	760.6	836.7	831.4	767.4	831.9
4.88	3.58	5.24	14.06	1.0332	822.5	842.6	774.4	825.1	832.6	768.4	832.3
4.88	3.70	5.24	14.20	1.0344	826.9	850.4	780.0	832.5	832.6	768.4	838.8
4.88	4.10	5.06	14.66	1.0346	856.8	864.9	788.8	856.9	832.6	768.4	855.7
4.94	3.54	5.59	14.58	1.0348	842.5	860.7	793.0	846.4	839.4	774.8	844.9
4.94	4.12	4.75	14.33	1.0340	842.6	859.9	782.2	848.8	839.4	774 8	853.9
4.96	3.23	5.08	13.78	1.0319	808.7	823.1	760.5	813.4	841.7	776.9	816.2
4.96	3.93	4.88	14.58	1.0338	860.4	856.1	781.5	855.8	841.7	776.9	848.9
4.97	3.48	5.37	14.30	1.0329	836.7	850.0	783.3	837.0	842.8	777.9	838.5
4.98	4.26	5.08	14.83	1.0358	874.7	885.7	805.6	871.3	843.9	779.0	874.0
5.05	4.09	4.76	14.51	1.0331	856.4	867.9	790.5	860.8	851.9	785.3	862.5
5.07	3.84	4.79	14.38	1.0330	853.9	856.8	783.5	852.8	854.2	788.4	851.6
5.07	4.04	5.09	14.60	1.0334	868.9	880.7	804.2	863.6	854.2	788.4	870.4
5.10	4.01	4.89	14.76	1.0337	872.9	873.5	797.3	871.3	857.6	791.6	866.2
5.10	4.02	4.86	14.61	1.0328	871.6	872.0	795.8	865.8	857.6	791.6	865.9
5.11	4.02	4.90	14.67	1.0320	870.0	873.8	797.6	868.5	858.7	792.7	867.9
5.19	4.18	4.82	14.83	1.0338	881.8	888.4	809.2	881.7	867.8	801.1	881.7
5.21	3.56	4.96	14.19	1.0321	847.1	859.6	790.9	847.9	870.1	803.2	853.6
5.21	3.59	5.17	15.04	1.0345	883.6	871.4	802.1	880.1	870.1	803.2	860.9
5.21	4.22	4.88	14.89	1.0346	881.6	895.5	815.5	885.5	870.1	803.2	887.3
5.22	3.70	5.22	14.65	1.0340	867.0	880.0	808.9	867.5	871.2	804.2	869.1
5.22	4.28	4.96	15.10	1.0349	899.1	903.1	822.1	894.8	871.2	804.2	893.6
5.23	3.82	5.16	14.64	1.0340	870.0	885.3	812.1	869.8	872.4	805.3	874.9
5.23	4.45	4.78	15.16	1.0353	897.9	906.9	822.9	900.9	872.4	805.3	898.9
5.25	3.68	5.25	14.72	1.0338	869.6	882.6	811.7	871.3	874.7	807.4	871.5
5.27	3.91	5.34	14.90	1.0345	881.9	901.3	826.6	882.7	876.9	809.5	888.2
5.28	3.66	4.97	14.64	1.0329	880.2	872.5	802.0	870.2	878.1	810.6	865.7
5.28	4.07	5.15	15.24	1.0352	901.0	904.4	826.8	899.3	878.1	810.6	892.7
5.34	3.45	5.59	14.75	1.0346	879.7	891.7	824.6	872.5	884.9	816.9	876.3
5.34	4.13	4.90	15.04	1.0350	905.2	903.4	824.5	896.6	884.9	816.9	894.7
5.36	4.24	4.81	15.05	1.0337	898.9	906.6	826.0	900.0	887.2	819.0	900.1
5.42	3.95	5.14	15.08	1.0360	902.2	910.6	834.7	898.7	894.0	825.3	898.6
5.44	4.06	5.14	15.37	1.0358	921.5	918.5	840.6	912.6	896.3	827.4	906.4
5.49	4.78	4.45	15.68	1.0356	950.7	936.3	846.1	940.6	902.0	832.7	931.6
5.50	3.71	5.04	14.70	1.0321	887.3	897.2	825.3	884.7	903.1	833.7	890.2
5.50	3.79	5.00	14.72	1.0328	886.2	900.7	827.5	886.9	903.1	833.7	893.5
5.50	3.90	5.08	15.06	1.0340	909.0	911.0	835.9	901.4	903.1	833.7	901.6
5.52	3.90	5.07	15.00	1.0349	898.7	913.3	838.0	900.3	905.4	835.8	903.1
5.62	3.87	4.95	14.98	1.0337	902.8	914.9	839.9	904.6	916.7	846.3	907.3
5.62	4.21	4.84	15.30	1.0359	913.8	931.6	850.7	922.7	916.7	846.3	922.9
5.66	3.62	5.25	15.02	1.0326	909.0	915.2	844.5	903.2	921.2	850 6	905.4
5.66	4.21	5.14	15.59	1.0355	951.9	946.4	865.7	935.0	921.2	850.6	934.5
5.72	3.83	5.20	15.28	1.0351	927.4	932.6	858.1	920.0	928.1	856.9	920.9
5.76	3.52	4.84	14.98	1.0336	916.8	903.6	834.2	906.5	932.7	861.1	898.1
5.76	4.08	5.33	15.57	1.0367	924.0	956.8	877.9	936.9	932.7	861.1	941.6
5.79	4.00	5.18	15.47	1.0342	928.5	946.8	869.4	933.7	936.1	864.2	936.0
5.86	3.73	4.96	15.04	1.0328	911.0	928.3	855.2	917.2	944.0	871.6	921.8
5.87	4.19	4.90	15.70	1.0339	951.9	953.6	872.7	950.5	945.2	872.7	946.0
5.88	3.84	4.97	15.61	1.0323	944.7	936.2	861.3	941.6	946.3	873.7	929.8
5.88	4.10	4.94	15.54	1.0357	940.8	952.7	873.1	943.4	946.3	873.7	943.1
5.90	3.98	5.34	15.86	1.0361	957.7	963.7	886.2	953.6	948.6	875.8	949.1
5.94	4.26	5.26	16.03	1.0355	963.6	979.3	897.0	966.9	953.1	880.0	965.8

TABLE 1.—ANALYSES OF 212 MILK SAMPLES WITH COMPUTED AND DETERMINED ENERGY VALUES—(Concluded)

Fat	Protein	Lactose	Total solids	Specific gravity	Calories per quart						
					Determined	Computed					
						Total	Metabolizable	Formula 1	Formula 2	Formula 3	Formula 4
A	*B*	*C*	*D*	*E*	*F*						
perct.	*perct.*	*perct.*	*perct.*			*cals.*	*cals.*				
5.94	4.32	5.00	15.97	1.0342	964.2	971.3	888.1	966.3	953.1	880.0	962.1
5.96	4.05	4.66	15.33	1.0336	950.2	944.3	865.5	939.5	955.4	882.1	940.2
5.98	3.88	4.90	15.43	1.0342	937.0	946.5	870.5	941.0	957.8	884.3	939.2
5.99	4.12	5.26	15.98	1.0353	962.6	975.7	895.8	965.3	958.8	885.3	962.7
6.00	3.69	5.18	15.33	1.0323	935.0	946.7	874.2	934.3	960.0	886.4	938.2
6.04	4.31	5.14	16.08	1.0349	981.0	985.9	902.6	975.2	964.5	890.6	974.4
6.05	4.83	4.14	15.87	1.0352	988.7	977.4	884.9	979.1	965.6	891.6	976.8
6.16	3.69	4.86	15.16	1.0326	939.7	949.1	876.0	937.2	978.2	903.2	944.1
6.42	3.95	5.24	16.27	1.0351	992.7	1004.2	926.1	996.3	1007.7	903.6	992.0
6.56	4.34	5.02	16.58	1.0338	1020.2	1028.8	943.9	1022.3	1023.6	945.3	1020.0
7.59	3.23	5.00	16.69	1.0314	1070.7	1056.6	988.1	1062.6	1140.8	1053.8	1052.6

STATISTICAL PART[a]

The total computed calories per quart for each of the 212 samples tabulated in Table 1, were obtained with a computing machine by completing the following computations: total calories per quart = $(A \times 9.23 + B \times 5.71 + C \times 3.95)\ (946.36 \times E) \div 100$. In this expression the values 9.23, 5.71, and 3.95[5] are large calories of heat evolved by the complete combustion of one gram of butterfat, one gram of protein, and one gram of lactose, respectively; 946.36 is the volume of one quart in milliliters.

The metabolizable calories per quart given in the eighth column in Table 1 were obtained by completing the indicated computation: $[A \times 9.00 + (B + C)\ 4.00]\ [946.36 \times E] \div 100$. In this expression the values[6] 9.00 and 4.00 are the metabolizable energy values, in large calories, of one gram of butterfat and one gram of protein and carbohydrate, respectively.

The means, standard deviations, coefficients of correlations, and partial regression coefficients of the analytical results in columns *A* to *F* inclusive are given in Table 2. The partial regression coefficients were computed by the method given by Wallace and Snedecor.[7]

From the results given in Table 2, regression equations were developed by substitution in the general regression equations. The re-

[a]In the statistical part, the following symbols are used:
A = percentage of fat in the sample.
B = percentage of protein in the sample.
C = percentage of sugar (lactose) in the sample.
D = percentage of total solids in the sample.
E = specific gravity of the sample at 20°C.
F = determined calories per quart.
F_M = computed calories per quart (metabolizable).
F_T = computed calories per quart (total).

gression equation of F, based upon the other variables, is $F = M_F + \beta_{FA} \cdot \frac{\sigma F}{\sigma A}(A - M_A) + \beta_{FB} \cdot \frac{\sigma F}{\sigma B}(B - M_B) + \beta_{FC} \cdot \frac{\sigma F}{\sigma C}(C - M_C) + \beta_{FD} \cdot \frac{\sigma F}{\sigma D}(D - M_D) + \beta_{FE} \cdot \frac{\sigma F}{\sigma E}(E - M_E)$. Substituting the values given in Table 2, and simplifying, this equation becomes,

$$F_1 = 52.78A + 16.41B + 37.87D + 46.91E - 2.75C - 57.70$$

calories per quart (Formula 1).

TABLE 2.—MEANS, STANDARD DEVIATIONS, COEFFICIENTS OF CORRELATIONS AND PARTIAL REGRESSION COEFFICIENTS DERIVED FROM ANALYSES OF 212 MILK SAMPLES SHOWN IN TABLE 1

Column	Means	Standard deviations	Coefficients of correlations
A	$4.2717 \pm .0452$	$.9761 \pm .0320$	$r_{AB} = .7923 \pm .0172$
B	$3.4620 \pm .0211$	$.4558 \pm .0149$	$r_{AC} = .1837 \pm .0448$
C	$4.9519 \pm .0140$	$.3021 \pm .0099$	$r_{AD} = .9640 \pm .0033$
D	$13.3000 \pm .0663$	$1.4316 \pm .0469$	$r_{AE} = .5657 \pm .0315$
E	$1.0328 \pm .00007$	$.0015 \pm .00005$	$r_{AF} = .9813 \pm .0017$
F	763.02 ± 5.17	111.681 ± 3.658	$r_{BC} = .0426 \pm .0462$
F_M	704.386 ± 4.839	104.463 ± 3.422	$r_{BD} = .8653 \pm .0116$
	(β for six variables)		$r_{BE} = .6842 \pm .0246$
	$\beta_{FA} = .46132$		$r_{BF} = .8525 \pm .0127$
	$\beta_{FB} = .06696$		$r_{CD} = .3025 \pm .0421$
	$\beta_{FC} = .00745$		$r_{CE} = .4687 \pm .0361$
	$\beta_{FD} = .48542$		$r_{CF} = .2274 \pm .0439$
	$\beta_{FE} = .00063$		$r_{DE} = .6965 \pm .0238$
	(β for variables A, B, C, and F		
	$\beta_{FA} = .7925$		$r_{DF} = .9862 \pm .0013$
	$\beta_{FB} = .2215$		$r_{EF} = .6419 \pm .0272$
	$\beta_{FC} = .0723$		$r_{AF_M} = .9838 \pm .0015$

The regression equation[a] of F on A is $F = M_F + r_{AF} \cdot \frac{\sigma F}{\sigma A}(A - M_A)$; which becomes,

$$F_2 = 113.7334\ (A + 2.4404) \text{ calories per quart (Formula 2).}$$

[a]As the regression equation of F on A is most likely to be of practical value, it was determined, without grouping, from the data given in columns A and F, Table 1, by the method set forth by Wallace and Snedecor.[7] The means, the standard deviations, and the coefficient of correlation are as follows: $M_A = 4.2686$, $M_F = 763.036$, $\sigma_A = .969632$, $\sigma_F = 111.7665$, $r_{AF} = .9867$.

It is interesting to observe that the regression equation of F on A, based on the statistical values obtained by grouping the data into classes given in Table 2, is $F_2 = 112.276\ (A + 2.524)$. In this case the slope of the line constructed by plotting the estimated F's against their corresponding percentages of fat is less than the slope of the line obtained when the equation derived without grouping into classes is used. The other regression equations are derived from the statistical values obtained by grouping the data.

The correlation coefficient of A and F_M, the metabolizable energy value (Table 1, eighth column), was computed. The regression equation in this case becomes,

$$F_3 = 105.287\ (A + 2.4185) \text{ calories per quart (metabolizable)}$$
(Formula 3).

The regression equation for F, based on A, B, and C, computed from the data given in Table 2, becomes,

$$F_4 = 90.67A + 54.27B + 26.73C + 55.44 \text{ calories per quart}$$
(Formula 4).

These formulas may be used for estimating the energy value of a quart of milk. The values of the 212 samples used in this investigation, ranging in fat percentage from 2.68 to 7.59, so computed, are included in Table 1 in the columns headed by the respective formula numbers.

The multiple correlation coefficients may be computed by substituting the values from Table 2 in the equation,[7] $R^2 = \beta_{FA} \cdot r_{AF} + \beta_{FB} \cdot r_{BF} +$ (etc.). In the case of the six variables this becomes $R_1 = \sqrt{.98726} = .9936$; and for the four variables, A, B, C, and F, it becomes $R_2 = \sqrt{.9830} = .9915$.

The standard error of estimate in the energy values computed by the use of Formula 1 is $\sigma F \cdot ABCDE = \sigma F \sqrt{1 - R_1^2} = \sigma F \sqrt{1 - .98726} = .1128\ \sigma F$, or 11.28 percent of the standard deviation of F. In the case of Formula 4 the standard error of estimate is $\sigma F \cdot ABC = \sigma F \sqrt{1 - R_2^2} = \sigma F \sqrt{1 - .9830} = .1304$, or 13.04 percent of the standard deviation of F.

For Formulas 2[a] and 3 the standard errors of estimate are, respectively, $\sigma F \cdot A = \sigma F \sqrt{1 - r^2_{AF}} = \sigma F \sqrt{1 - (.9867)^2} = .1626\ \sigma F$, or 16.26 percent of the standard deviation of F, and $\sigma F_M \cdot A = \sigma F_M \sqrt{1 - r^2_{AF_M}} = \sigma F_M \sqrt{1 - (.9838)^2} = .1793\ \sigma F_M$, or 17.93 percent of the standard deviation of F_M.

The differences were found between the determined energy values (Column F, Table 1) and the total computed values given in the columns headed Total, Formula 1, Formula 2, and Formula 4, respectively; and also between the computed metabolizable energy and Formula 3. The means, standard deviations, and limits at odds of 30:1 and at odds of 100:1 for these five sets of differences are given in Table 3.

[a]Using for Formula 2 the statistical values given in the preceding footnote, page 213.

APPLICATION

The regression equations give a convenient method for use in computing the energy value of a quart of milk if the percentage composition of the milk is known. If A, B, C, D, and E are known, Formula 1 may be used. When A, B, and C are known, Formula 4 is to be used, and Formula 2 when A only is known. To compute the metabolizable energy value of a quart of milk when A only is known, Formula 3 is to be used.

Table 3, giving means, standard deviations, and limits within which the results computed by the various formulas will lie, gives a measure of the reliability of the computed values.

TABLE 3.—MEANS AND STANDARD DEVIATIONS OF THE DIFFERENCES AND LIMITS

Differences	Number of differences	Mean	Standard deviation	Limits at odds of 30:1	Limits at odds of 100:1
		cals.	cals.	cals.	cals.
$F_1 - F$	212	+.1416	5.7081 ± .1870	± 12.2153	± 14.7269
$F_2 - F$*	212	−.0071	18.1755 ± .5953	± 38.9056	± 46.8928
$F_{2_1} - F$**	212	−.4437	18.2448 ± .5976	± 39.0439	± 47.0267
$F_3 - F_M$	212	−.4436	14.9076 ± .4882	± 31.9023	± 38.4616
$F_4 - F$	212	−.3702	8.3356 ± .2730	± 17.8382	± 21.5058
$F_T - F$	212	+6.3584	8.5812 ± .2811	± 18.3638	± 22.1395

F = calories per quart, determined.
F_1 = calories per quart, computed by Formula 1.
F_2 = calories per quart, computed by Formula 2.
F_{2_1} = calories per quart, computed from regression equation obtained from grouped data.
F_3 = calories per quart, computed by Formula 3.
F_4 = calories per quart, computed by Formula 4.
F_M = calories per quart metabolizable.
F_T = calories per quart, computed from energy value of constituents.

*The mean and the standard deviation of the differences $F_2 - F$ were determined without grouping the data into classes; all other means and standard deviations in this table were determined from grouped data.

**The values F_{2_1} were obtained by the use of the regression equation given in the footnote on page 213. The standard deviations and limits for $F_2 - F$ and for $F_{2_1} - F$ show that the results obtained by grouping are only slightly different from those obtained without grouping.

Formula 1 gives results which at odds of 30:1[a] will lie within ± 12.2153 calories per quart of the determined value. These limits are within about ± 2 percent of the determined values for milk with low fat content, and within ± 1.25 percent for milk with high fat content.

[a]The mean of each group of differences ± 2.14 times the corresponding standard deviation gives limits such that the chances are 30:1 that any single difference determined in the same way will fall within them. For odds of 100:1 the standard deviation is to be multiplied by ± 2.58.

The constants ± 2.14 and ± 2.58 were determined from the equation given on page 28 of Karl Pearson's Tables for Statisticians and Biometricians.

Formula 2 gives results which at odds of 30:1 will lie within ± 38.8956 calories per quart of the determined value, or within about ± 6.5 percent for milk with low fat content, and ± 3.9 percent for milk with high fat content.

Formula 3 gives values which at odds of 30:1 will lie within ± 31.9023 calories per quart of the computed metabolizable energy value, or within about ± 6 percent of the computed value for milk with low fat content, and ± 3.4 percent for milk with high fat content.

Formula 4 gives values which at odds of 30:1 will lie within ± 17.8382 calories per quart of the determined value, or within about ± 3.4 percent for milk with low fat content, and ± 2 percent for milk with high fat content.

The mean and standard deviation of the differences between the total energy per quart as computed and as determined show that for the values for fat, for protein, and for lactose, as used in this work, the computed values are, on the average, 6.3584 calories per quart higher than the determined values. They also show that if 6.3584 calories are substracted from the computed values, the chances are 30:1 that such values will then lie within ± 18.3638 calories per quart of the determined values.[a]

The standard errors of estimate show that the standard deviations of computed values from determined values are, for Formula 1, 11.28 percent of the standard deviation of F; for Formulas 2 and 4, 16.26 percent and 13.04 percent, respectively, of the standard deviation of F; and for Formula 3 the standard deviation of values computed by the formula, from those obtained by the use of the metabolizable energy value of the constituents (F_M), is 17.93 percent of the standard deviation of F_M.

For the purpose of comparing results obtained by their use, with results obtained by using Formula 2 of this work, the formulas of Gaines and Davidson and of Andersen were changed to the basis of the average weight of a quart of milk (977.4 grams). These formulas thus become, $E = 106.9643\ (2.66 + t)$ calories per quart, and $E = 110.9349$ (percentage of fat $+ 2.555$) calories per quart, respectively.

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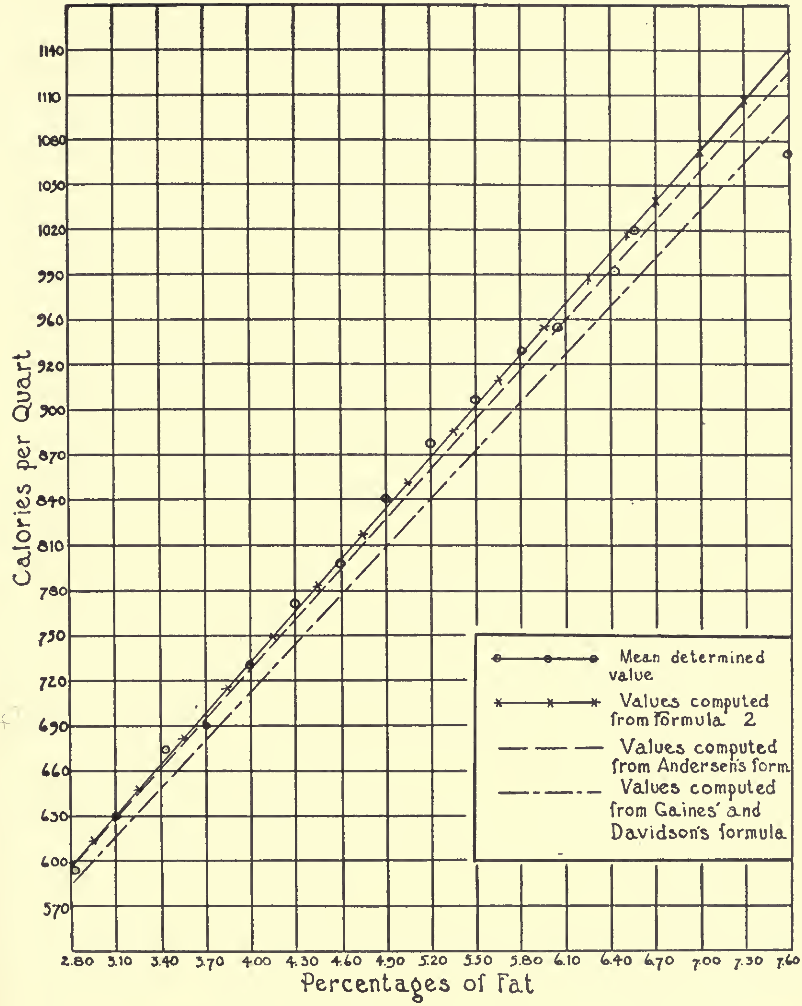
[a]If the values given by Frederiksen as found by Andersen[3] (butterfat, 9.11 calories per gram; milk protein, 5.86 calories; and lactose, 3.76 calories per gram) are used, the results are as follows: mean of the differences between the determined energy value per quart and the value computed by Andersen's constants, − 2.8302 calories; standard deviation of these differences, 8.5485 calories. That is, the energy of a quart of milk computed in this way is on the average 2.8302 calories less than the determined energy. At odds of 30:1, the limits within which computed values may be expected to fall are ± 18.2938; and at odds of 100:1, the limits are ± 22.0551. This work of Andersen's was not at hand when the statistical study shown in the tables was made.



DETERMINED AND COMPUTED ENERGY VALUES OF MILK

The energy value of milk of a given fat content, computed by formula, corresponds very closely to the value determined in the calorimeter.

The results as computed by each formula are compared graphically with each other and with the mean determined results in the accompanying figure. These graphs show that the energy values computed by the formulas correspond very closely with the determined values.

CONCLUSIONS

1. The heat of combustion of a quart of milk may be computed in one of the following ways:

a. If the percentages of fat, protein, lactose and total solids, and the specific gravity are known, the formula $F = 52.78A + 16.41B + 37.87D + 46.91E - 2.75C - 57.70$ should be used.

b. If the percentages of fat, protein, and lactose are known, the formula $F = 90.67A + 54.27B + 26.73C + 55.44$ should be used.

c. If only the percentage of fat is known, the formula $F = 113.7334 (A + 2.4404)$ should be used.

d. If only the percentage of fat is known and the metabolizable energy is wanted, the formula $F_M = 105.287 (A + 2.4185)$ should be used.

2. The true heats of combustion per gram of butterfat, milk protein, and lactose probably lie between the values given by Abderhalden[5] and by Hammarsten[5] and those given by Frederiksen as found by Andersen.[3]

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