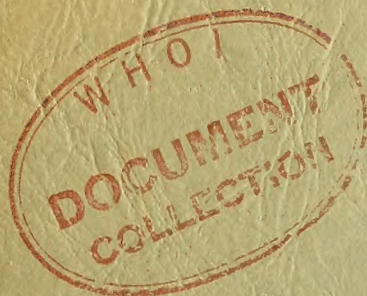# Columbia University
in the City of New York

HUDSON LABORATORIES
DOBBS FERRY, N.Y.

Technical Report

No. 67

An Estimate
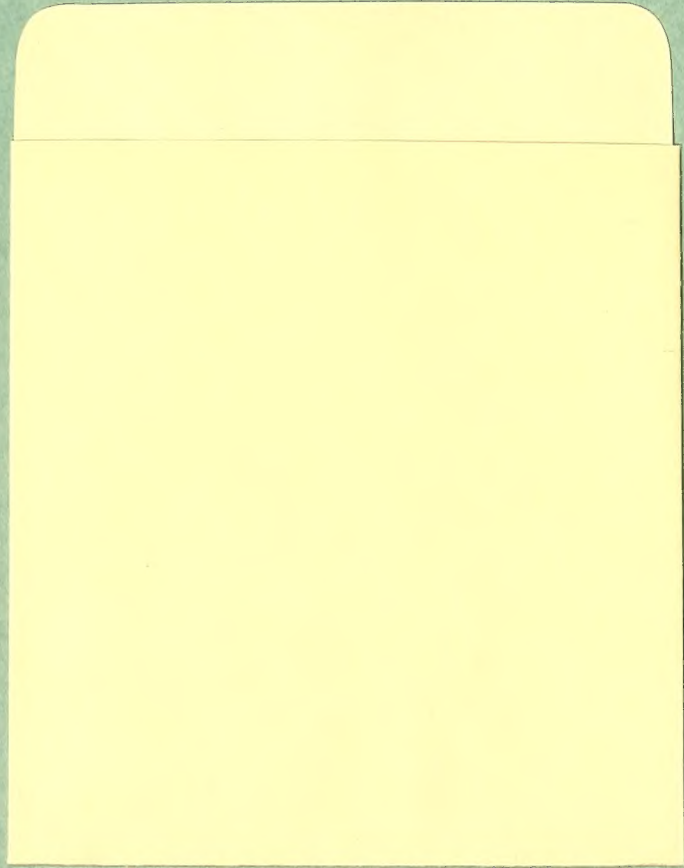## Of Turbulent Velocities
In the Ocean

by

T. E. Pochapsky

MARCH 1, 1959

Contract

N6-ONR-27135

Columbia University
Hudson Laboratories
Dobbs Ferry, N. Y.

R. A. Frosch
Director

Technical Report No. 67

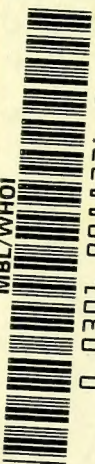AN ESTIMATE OF TURBULENT VELOCITIES IN THE OCEAN

by

T. E. Pochapsky

March 1, 1959

This report consists of   Copy No. 24
23 pages   of 75 copies

Abstract

Past eddy diffusion measurements have suggested that turbulent velocity fluctuations occur in the deep ocean. An attempt has been made to determine the magnitude and scale of the fluctuations from estimates of shear velocities in the stably stratified ocean. The rate at which the kinetic energy of these fluctuations is converted into heat by viscosity is compared with an estimate of the rate at which kinetic energy is supplied to the water by forces acting on the boundaries of the ocean.

The primary purpose of this report is to obtain an order of magnitude estimate of turbulent fluctuations which would be useful in designing experiments for turbulence measurements at sea. It is shown that vertical velocity fluctuations of less than 0.2 cm sec^{-1} can be expected in the deep ocean and that characteristic fluctuating vertical displacements would be about 50 cm . Corresponding magnitudes for the horizontal fluctuations are 2 cm sec^{-1} and 5 km .

Introduction

The general motion of ocean waters has features which are similar to those of atmospheric circulation. Details of this motion, however, are not known and predictions based on hypothetical models do not have the easy verification possible for a weather forecast.

The purpose of this report is to assemble a brief description of the mechanisms relating to oceanic turbulence so as to obtain a better picture of conditions to be dealt with experimentally. Ideally, one would wish to measure the random turbulent velocities as distinguished from steady or periodic motions and then determine the cause of the turbulence. The arguments used here suggest, however, that the turbulent motions in the ocean are so slow and weak that it may be impractical to obtain accurate direct velocity measurements. There also seems to be no suitable model of the ocean which can serve as a basis for deriving turbulent velocities from, say, mean flow or transport measurements. Nevertheless, turbulent fields can be specified in terms of velocity correlations (Batchelor, 1953), and measurements of this type are feasible. They could be done, for instance, by measuring the pair correlation separation between free neutrally-buoyant containers. It is hoped that the eddy lengths and time scales derived in this report on the basis of rather primitive arguments will be useful in the design of such experiments.

The procedure adopted to estimate the turbulent structure is first to determine in a general way the mean motion field associated with the ocean. This is done so that the extent of turbulence-producing shear flows can be appreciated and so that an upper limit can be set on the velocity fluctuations.

The mean motion would be damped out unless it were maintained by external forces. Because an equilibrium, of sorts exists, the dissipation rate associated with the motion of the water must equal the rate at which energy is supplied to the water in <u>mechanical</u> form from the exterior so as to cause it to move. Although this energy eventually appears as heat, it first contributes to the kinetic energy of large scale motions and is transferred to a viscous loss mechanism by means of intermediate turbulent motions. Consequently this dissipation rate is an important index of the intensity of the turbulence and calculations are made to establish an upper limit for this quantity.

Finally, expected turbulent velocities and dissipation rates are incorporated into models which consider shear flow, stability criteria, etc., so as to try to determine eddy sizes and time scales which are, hopefully, of an order of magnitude of those that actually occur in the ocean.

For simplicity, all data will refer to the North Atlantic between 10°N and 60°N.

Energy of the Ocean

If the velocity distribution in the mean flow throughout the ocean were known, the problem of estimating the degree of turbulence would be

considerably simplified. Sufficient data of this type does not exist. Consequently, isolated measurements and somewhat more extensive measurements across interesting areas, such as the Gulf Stream, must serve as guides to guesses for the kinetic energy in any desired region.

The energy that must be supplied continuously from outside the ocean to maintain the existing general circulation is ultimately of lunar or solar origin. Much of it, however, enters indirectly through the atmosphere. In the calculations below, only an upper limit is established for this rate of doing mechanical work on the ocean which in turn is equated to the rate at which the general motion produces viscously generated heat.

It will be necessary to consider the effect of the inhomogeneous properties of the ocean on the flow primarily because work must be done to overcome adverse density gradients. Fortunately, oceanographers have always attached importance to a knowledge of the inhomogeneous structure and adequate data is available for present purposes. This subject will be considered in the subsequent section dealing with the stability of the ocean.

Total Energy

There is a large, almost steady, reservoir of kinetic energy in ocean movements which range from the high velocities in the Gulf Stream to the almost stationary deep ocean. An analysis was made of the impressive Gulf Stream flow to determine the extent of the movement of water in that flow relative to surrounding waters. Flow profiles obtained by Wust (S, 674)[*] were used for the Florida Straits and data obtained by

---

[*] Available information on the ocean can often be found in The Oceans by Sverdrup, Johnson, and Fleming (1942). References to that book in this report will be abbreviated to read "(S, page number)".

-3-

Worthington (1953) for flow off Montauk Point. The results of these calculations showed

	Stream Area	Average Velocity	Flow of Kinetic Energy	Volume Flow
	cm^2	$cm\ sec^{-1}$	$ergs\ sec^{-1}$	$meter^3 sec^{-1} \times 10^{-6}$
Florida Strait	360×10^9	66	12×10^{16}	24
Montauk Point	870×10^9	69	38×10^{16}	60

Flow maps suggested that the Gulf Stream circuit is about 15,000 km long, so that roughly 13×10^{20} cc of water is involved in the flow.

The area of water of the North Atlantic from the equator to 60°N, including the Mediterranean, is about $4 \times 10^7 km^2$, and the total volume is about $1.6 \times 10^{23}\ cm^3$. Consequently, the Gulf Stream circuit involves about 1 per cent of the North Atlantic water and has an average kinetic energy of approximately 6.3×10^3 ergs, as deduced from the Montauk data. The velocity associated with this kinetic energy is $110\ cm\ sec^{-1}$. Peak velocities in the swiftest part of the current reach about $250\ cm\ sec^{-1}$ (Worthington, 1953).

Turbulent velocities are random and have statistical features which are relatively stationary in time. The large streaming motions can be considered to be a shear flow which generates turbulence. Meanderings of this flow, however, may be difficult to separate from turbulent fluctuations. If the results for usual jet flows are an indication, (Townsend 1956) the fluctuating turbulent velocities within the stream should not exceed 5 per cent of the stream velocity and so are probably below $10cm\ sec^{-1}$.

On the west side of the Atlantic, this high level of motion would take place in a strip about 50 miles wide and 800 meters deep. The velocity gradients in this strip would generally not exceed 0.0002 sec^{-1} in a horizontal direction and 0.005 sec^{-1} vertically.

Additional energy concentrations are found in "eddies" outside the general course of the Gulf Stream as well as in the counter current flowing beneath it. In both cases, measurements by Swallow and Worthington (1957) showed that mean velocities of about 10 cm sec^{-1} were not the exception. These eddies are not considered to be included in the random background turbulence. On the other hand, Swallow (1957) has obtained deep current velocities of the order of 1 cm sec^{-1} near Salvage Island.

Outside the relatively small volume of strong streaming motion, the highest velocities are probably tidally induced. These velocities, of the order of 10 cm sec^{-1} (Bowden 1954), have the additional feature of being periodic.

The above considerations do not give a very firm basis for estimating the turbulent velocity fluctuations in deep ocean. For want of a better guess, it will be assumed that they do not exceed about 1 cm sec^{-1}.

Energy Dissipation

An estimate of the maximum rate at which mechanical energy is supplied to the ocean by the moon can be made on the basis of the secular acceleration of the moon. Jeffries (1952) estimates that lunar forces do work on the earth at the rate of 1.4×10^{19} ergs sec^{-1}. If the assumption is made that this energy is delivered uniformly over the

oceans alone, 3.9 ergs sec^{-1} would be supplied to each square centimeter of ocean surface. The total area of ocean involved is 361×10^6 km^2, (S, 15). When discussing Taylor's method of computing tidal dissipation, Jeffries shows that practically all of this energy is lost in the form of heat in shallow seas. Thus, very little of it could be converted into turbulent motion in deep ocean.

The sun produces motion in the sea by a variety of processes. Direct heating of the water produces mechanical effects as does heating of the atmosphere with its concomitant winds.

The energy supplied by the winds was obtained from the product of the stress τ exerted by wind on the surface of water and the associated water velocity. Stresses were calculated from the equation

$$\tau = 3.2 \times 10^{-6} W^2 \, ,$$

where W is the wind velocity, and W and τ are in c.g.s. units. A marine atlas (U.S. Navy 1955) was used to estimate average winds for each month in the North Atlantic. An average wind stress of 1.1 dynes cm^{-2} was obtained. The surface velocity imparted to the water was then assumed to be 3 per cent of the wind velocity (Hughes, 1957) and the shear stress-velocity products for each month were obtained. The average rate of energy input to the water during a year was found to be 32 ergs cm^{-2} sec^{-1}. Knauss (1956) used a value of 15 ergs cm^{-2} sec^{-1} which he obtained from averaged surface currents. Most of this energy is probably converted to motion in the layer of water above the thermocline.

An approximate value for the kinetic energy obtained from the sun can be obtained by not questioning details of the process. Although the yearly average incidence of radiation on the North Atlantic is approximately .004 cal sec^{-1}cm^{-2} (S, 103), only a small fraction of that energy can be

transformed to a motional form other than heat. In fact, motion is produced only as a consequence of heating or cooling which supplies mechanical energy by 1) evaporating water over large areas of the ocean so that a return flow from other areas is induced, 2) increasing the salinity or density of surface waters which then tend to sink, and 3) heating surface waters so that they expand above the equilibrium water level and gain potential energy.

The importance of a given process depends on the season (S, 122). At 47°N, 12°W, for instance, most evaporation takes place from September to November and the latent heat is supplied more from the thermal energy stored during the summer months than from solar heating at the time.

To estimate the magnitude of the mechanical energy that could be obtained as a result of evaporation, assume that steady evaporation has lowered the depth of the ocean over a large area by the amount h and raised the level by precipitation in another area by a similar amount. Water flowing from the high areas to the low would then lose potential or gain kinetic energy at the rate

$$2 \; h \; p \; g \; \frac{dh}{dt} = 4.8 \times 10^{-3} \; h \; \text{ergs} \; \text{cm}^{-2} \; \text{sec}^{-1} \; .$$

The quantities p and g are the density of sea water and the acceleration of gravity respectively. Substitution of these values and the value for the evaporation rate $\frac{dh}{dt}$, of 2.9×10^{-6} cm sec^{-1} per unit area of ocean into the above expression yields the quantity on the right of the equation. The evaporation rate was determined from the annual rate of evaporation over all oceans, $334 \times 10^{3} \text{km}^{3} \; \text{yr}^{-1}$(S, 120). The kinetic energy gained in

this process would be comparable to the maximum supplied by the moon only if h exceeded, say, 10^3 cm, but the time required to evaporate this quantity of water without its being replaced is much too long to be realistic.

In a similar way it can be shown that the motion associated with the density changes produced by heating or cooling are very small.

Finally, it is necessary to consider the potential energy of the 8×10^{-8} grams of salt left behind at the surface each second by the evaporating water. If this salt fell 4000 meters to the bottom of the ocean without mixing and transformed all its potential into kinetic energy, it would release 28 ergs $sec^{-1} cm^{-2}$. Obviously, such complete penetration cannot occur, but this mechanism could induce some small stirring of deep waters.

Mention was not made of the effect of returning rain waters and a multitude of other possible processes. In any event, it would be hard to imagine that more energy could be obtained than in a process in which the sun served as a means of pumping distilled water from the surface of the ocean and returning it to the bottom of the ocean from which it eventually returns to the surface. Then, the same result would be obtained as was calculated for the falling salt precipitate. Thus the maximum kinetic energy realizable from solar heating of the water is below 56 ergs $sec^{-1} cm^{-2}$ of ocean surface, as obtained by adding the hypothetical salt and fresh water contributions.

It should be noted that the maximum value of 56 ergs $sec^{-1} cm^{-2}$ for the kinetic energy imparted to the water by solar heat is very small

compared to the incident radiation of approximately 10^5 ergs $sec^{-1} cm^{-2}$. Conversion is thus very inefficient and there remains the possibility that some subtle and unrecognized mechanism supplies more motion to the ocean than calculated above. Lacking this evidence of another process, the present value for the rate of supply of motional energy can certainly be considered an overestimate, probably by at least one order of magnitude.

Heat flows from the ocean floor at the rate of approximately 10^{-6} cal cm^{-2} sec^{-1} , (Bullard, 1954). Mechanical effects associated with this flow rate can be ignored except possibly in parts of the deep ocean which may be practically unaffected by motion imparted by the other sources mentioned.

Summarizing, the maximum average kinetic energy possible produced below a unit area of ocean surface by the various sources is:

Tide	3.9 ergs sec^{-1} cm^{-2}	
Wind	32	"
Direct Sun	56	"

Most of this energy might only be associated with water in the layer approximately 100 meters deep over the thermocline. In that event, steady turbulence could not dissipate energy faster than about 100 ergs sec^{-1} per unit area of ocean surface of 10^{-2} ergs sec^{-1} per unit volume of water above the thermocline. Some of the energy must appear in deep ocean, but it is hard to believe that the fraction could exceed 1/10 . In that event, 2×10^{-5} ergs cm^{-3} sec^{-1} would be the maximum supplied to maintain turbulence in a 4000-meter ocean, (i.e., $10^2 \times 10^{-1} \times (4\times10^5)^{-1}$.

Homogeneous Isotropic Turbulence

For turbulent motion to be homogeneous in a medium, it is neces-
sary that the statistical properties of the velocity fluctuations be
identical at all points of the medium. Because the ocean has boundaries,
the turbulence could not be homogeneous unless most of the eddies had a
size that is small compared to, say, the depth of the ocean. If, in ad-
dition, the ocean were a homogeneous fluid, with no temperature or density
gradients, it is possible that these smaller eddies would produce velocity
fluctuations which are similar in all directions; i.e., the turbulence
would be isotropic as well as homogeneous.

The ocean is not a homogeneous liquid. Nevertheless, if the turbu-
lent eddies are small enough, the effect of the density gradients in turn
might prove to be so small that the existing theory of homogeneous isotropic
turbulence can be applied to obtain a useful first approximation of the
turbulent motion.

Instead of trying to determine whether homogeneous isotropic turbu-
lence takes place in the ocean, or not, it would be more convenient to
assume that it does and then determine whether the associated size of the
eddies is consistent with known conditions in the ocean. This could be
done if the total energy, E , of the turbulent motion were known together
with the rate, ε , at which heat is produced as a result of the viscous
motion. Upper limits for these two quantities were established in the
preceding section.

The theory of homogeneous turbulence (Batchelor, 1953) shows the
energy density, $E(k,t)$, associated with particular eddy sizes at high eddy

Reynolds number is

$$E(k,t) = \alpha \, \varepsilon^{2/3} \, k^{-5/3}$$

where $E(k,t)$ and ε are on a unit weight basis, $\alpha \approx 0.3$, and $\lambda = \frac{2\pi}{k}$ is a measure of the eddy size. Because most of the energy is concentrated in the larger eddies, this expression can be integrated over all $k > k_{min}$ to give the total energy without particular concern for the small amount of energy attributed to those smaller eddies having a low Reynolds number. Then this may be written

$$E = 0.14(\varepsilon \, \lambda)^{2/3} \ .$$

In deep water, it was shown that $\varepsilon \leq 2\times10^{-5}$ ergs $gm^{-1}sec^{-1}$ and $E \leq \frac{1}{2}$ ergs gm^{-1} (for velocities below 1 cm/sec). Substitution shows a λ of 3.4×10^{5} cm to be representative of the size of the larger eddies if the equality signs hold for ε and E . The fact that λ turns out to be almost equal to the depth of the ocean is completely fortuitous, and these calculations do no more than indicate that a homogeneous ocean would be continuously well stirred. This conclusion would also hold for reasonable reductions in the values of ε and E used. Inasmuch as the ocean is not well stirred, the theory of homogeneous isotropic turbulence is generally inapplicable. If water above the thermocline is treated similarly, now, with $E \leq \frac{1}{2}$ and $\varepsilon = 10^{-2}$, λ becomes 675 cm. This layer is actually about 10^{4} cm deep and can be considered fairly well stirred by the wind so that the theory may have possible general application in this case.

Energy is lost from the larger eddies by transfer to smaller ones. Eventually eddies become small enough for velocity gradients to be large

enough to permit normal viscous loss and little further diminution in
size takes place. Dissipation by fluid viscosity occurs at lengths
characterized by (Batchelor, 1953):

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}} ,$$

where ν , the kinematic viscosity, is .015 (in c.g.s. units) for water.
For $\varepsilon = 2\times10^{-5}$ ergs, $\eta = 0.7$ cm. The velocities associated with these
eddies would be extremely small.

The Stably Stratified Ocean

In the previous section it was shown that if turbulent velocities
are approximately 1cm sec^{-1} in homogeneous water, eddy sizes would be
almost equal in magnitude to the depth of the ocean. However, if a unit
volume of water could be moved isothermally along a vertical eddy from
the cold bottom of the ocean, the density difference between it and the
surrounding water would necessitate doing work against gravity in the amount
of 70,000 ergs in order to reach the surface. Because the kinetic energy
at depth is only of the order of an erg, the sizes of the vertical eddies
must be several orders of magnitude less than the depth of the ocean.

The ability of turbulent stresses to move a fluid against gravi-
tational forces can be expressed quantitatively by Richardson's number
(Proudman 1953):

$$R_i = \frac{g\,\frac{\partial \rho}{\partial z}}{\rho\left(\frac{\partial U}{\partial z}\right)^2}$$

where the denominator represents work supplied by large scale motion
working against the Reynolds stresses of turbulent motion. In this equation,

g is the gravitational force; ρ , the fluid density; z , the height
of the layer considered; U , the velocity of mean, or non-turbulent,
flow. The implication of this equation is that a horizontal shearing
motion, $\frac{\partial U}{\partial z}$, generates turbulent energy at a rate which must be larger
than some critical rate at which energy is lost by mixing at the higher
gravitational potential near the top of an eddy. In order to have turbu-
lence, R_i must not exceed some critical value. Some type of horizontal
shearing motion is needed for continuation of the turbulent motion and
this may be the result of tidally induced boundary flow over the bottom
of the ocean, wind driven surface currents, or even gradients in the
large scale horizontal eddy motions.

Values of the stability, $\frac{1}{\rho}\frac{\partial \rho}{\partial z}$, were calculated for the North
Atlantic from data obtained on the Crawford cruise 16, 1 Oct. -11 Dec.,
1957 (Metcalf, 1957). The averages were

depth, meters	stability, $\frac{1}{\rho}\frac{\partial \rho}{\partial z}$, cm^{-1}
1000 - 2000	2.5×10^{-9}
2000 - 3000	4×10^{-10}
3000 - 4000	2×10^{-10}

Townsend (1958) showed that turbulence in the developed shear
flow of a stably stratified medium will collapse if R_i is more than
about 0.1. For a stability of 2×10^{-10} cm^{-1} , the gradient required

to maintain turbulence must be greater than $1.4 \times 10^{-3} \text{ sec}^{-1}$ Multi-

plying this by the depth of the ocean, 4×10^{5} cm, yields a value of

560 cm/sec for the change in velocity of the top relative to the bottom,

exclusive of the sharp drops occurring in the boundary layers. Tidal

currents are of the order of 10 cm/sec and wind induced velocities

rarely exceed 30 cm/sec. Consequently it is doubtful that wind or tide

could produce vertical turbulent fluctuations in the deep ocean, if Town-

send's criterion is applicable. ·It should be noted that the largest

gradients in the Gulf Stream are only about $5 \times 10^{-3} \text{ sec}^{-1}$.

Eddy Diffusivity

A turbulent fluid can transport properties such as momentum or

temperature at a much higher rate than a non-turbulent liquid as a result

of the eddy motion. Measured values of the eddy viscosity for momentum

transport or eddy diffusion for the transport of impurities can be used

to get a measure of the associated turbulent velocities.

Eddy diffusion coefficients have been obtained at various locations

in the ocean (S, 481) by measurements of changes in temperature, salinity,

concentration of dissolved gases, etc; eddy viscosity measurements, which

represent momentum transport, are generally based on relations assumed to

exist in the velocity field being studied. In a horizontal direction

both types of coefficient are approximately equal,[*] but in the vertical

direction the viscosity coefficient is ten or more times larger than the

diffusion coefficient. A given coefficient changes in going from the

vertical to the horizontal direction by the staggering factor of about one

million.

---

[*] Both coefficients result from a mass transport and they can have equal c.g.s.
numerical values because water conveniently has unit specific heat (S92).

Recent determinations of deep ocean vertical eddy diffusion coefficients, A_v , were made by Koczy (1956) and were based on measurements of the concentration of radium. Values of about 10 gm cm^{-1}sec^{-1} obtained near the bottom fell to values below 1 gm cm^{-1}sec^{-1} at a height of 3 km. The viscosity of non-turbulent water is about .01 gm cm^{-1} sec^{-1} . Therefore, the presence of turbulent motion in deep ocean seems certain.

These measurements allow an estimate of the energy lost by the velocity gradient previously calculated to be necessary to maintain turbulence. This energy is of the order of (Proudman, 1953)

$$A_v \ (\frac{dU}{dz})^2 = 10 \times (1.4 \times 10^{-3})^2 \quad .$$

or 2×10^{-5} ergs cm^{-3} . Again, the agreement obtained with the same value established for the maximum rate of energy supplied from outside the ocean to maintain the turbulence is fortuitous. Nevertheless, the high gradient is not inconsistent with the maximum rate of energy dissipation even though it is inconsistent with the expected wind-induced or tidal gradients. If the turbulence is not induced by a mean horizontal shear flow, two other mechanisms come to mind: 1) large and small scale horizontal motions interact to produce vertical fluctuations, and 2) internal waves in the stratified medium interact with the horizontal eddies to produce an effective vertical turbulent component of motion (Townsend, 1958).

In shear flows, the eddy viscosity is equal to (Townsend, 1956)

$$A_v = \frac{\rho \ \overline{uw}}{|\frac{dU}{dz}|}$$

where $\bar{u}$ and $\bar{w}$ are the root-mean-square turbulent components in the horizontal and vertical directions. In such flows $\overline{uw} \approx 0.4 \, u^2$, (Townsend, 1956), so that the r.m.s. velocity in the minimum velocity gradient required to maintain turbulence is about 0.2 cm sec^{-1} for $A_v = \text{gm cm}^{-1} \text{sec}^{-1}$.

Further insight into the turbulent diffusion can be obtained by noting that particles moving in a one-dimensional random walk have a distribution after a sufficient time which is described by the diffusion equation. The diffusion coefficient is replaced by $\frac{1}{2}n\ell^2$, where n is the jump rate and ℓ is the constant jump distance. If ℓ is associated with a characteristic eddy size, $\frac{1}{2}n\ell$ has a value which approximates the velocity. For an A_v of $10 \text{ gm cm}^{-1} \text{ sec}^{-1}$ for a velocity of 0.2 cm sec^{-1}, ℓ must be 50 cm, the order of size of the eddies needed in the vertical direction. The time required for a particle to circulate around an eddy is then about 250 seconds.

An attempt was made to apply the random walk method to horizontal diffusion for the case where the horizontal motion consists primarily of a shear flow. Vertical fluctuations in position would move the diffusing particles in random steps through layers of differing horizontal velocities. Any particle leaving a reference plane as a result of vertical motion would be replaced by another particle having a horizontal velocity characteristic of a plane separated from the reference plane by a distance equal to the vertical eddy scale. Then many particles in the reference plane would have velocities differing from that of the mean velocity of the plane by $\pm \, \ell_v \frac{dU}{dz}$. As an approximation, it was assumed that all particles in this plane were engaged in a random walk

with a step length of $\ell_v \frac{dU}{dz} \tau$ where $\tau = \frac{1}{n}$ is the vertical time scale. The jump rate for conditions described in the previous paragraph, $.008$ sec^{-1}, then requires that a velocity gradient of 4 sec^{-1} be present to obtain diffusion coefficients of 10^6 gm cm^{-1} sec^{-1}. Such a gradient obviously cannot be present, and so diffusion probably takes place as a result of existing horizontal eddies having a large scale.

Some idea of the characteristics of the horizontal eddies can be obtained from the horizontal diffusion coefficient, A_H, and from the maximum rate of energy dissipation, 2×10^{-5} ergs gm^{-1} sec^{-1}. Thus,

$$\left\{ \begin{array}{l} \frac{1}{2} n\ell_H^{\,2} = u_H \, \ell_H \approx 10^6 = A_H \\[2em] A_H \left(\frac{dU}{dx}\right)^2 \approx A_H \left(\frac{u_H}{\ell_H}\right)^2 < 2 \times 10^{-5} \end{array} \right.$$

lead to

$$\left\{ \begin{array}{l} u_H < 2 \text{ cm sec}^{-1} \\[1.5em] \ell_H > 5 \times 10^5 \text{ cm} \end{array} \right.$$

and a time scale of the order of several days. The value of these results should be tempered by noting that 1) the expression for the energy dissipation applies to shear flows and may be appreciably in error when applied to the ocean, 2) the resulting length scale may be too large to have a meaningful random walk approximation, and 3) the horizontal diffusion coefficient used is based on experiments which were probably obtained over a smaller geographical area than required to obtain good statistical data for the large eddies expected.

Discussion

The preceding computations suggest that the vertical component of the background turbulent motion in deep ocean has a typical velocity less than 0.2 cm sec^{-1}, a displacement of more than 50 cm and a fluctuation time greater than 250 sec. Corresponding figures for the horizontal fluctuations are 2 cm sec^{-1}, 5km, and 2×10^5 seconds. These figures were based on the assumption that the turbulence is generated by comparatively steady shear flows in the ocean for which the Richardson's number does not exceed 0.1. They are consistent with observed diffusion rates and the maximum possible rate at which external forces at the boundaries of the ocean could supply energy to maintain the motion.

The requirement on the Richardson's number is a stringent one. It can be criticized as applying to a developed shear flow and not to uncertain oceanic conditions. In fact, other observers have reported numbers ranging from 10 to 100. Reference should be made to Townsend (1958) for a discussion of this point.

If the Richardson's number is assumed to be 10 instead of 0.1, the shear velocity gradients would be one-tenth as large and the vertical velocities would be reduced by a factor of about 3. Both the length and time scales would increase. The smaller Richardson's number was used in the preceding computations so as to obtain an upper limit for the velocity fluctuations.

It was necessary to attribute horizontal diffusion to a horizontal eddy structure. The origin of this motion remains unknown. It is possible

that alternating shear stresses, caused by wind or moon, in a Coriolis field may build up such a velocity field. On the other hand, it may somehow be induced by major currents such as the Gulf Stream. These possibilities, however, must remain speculative until more attention is given the problem.

The structure of the field of horizontal motion is important because it probably determines the vertical fluctuating velocities. So far, these vertical turbulent motions were postulated to explain vertical diffusion, and gradients in the horizontal motion were then postulated as necessary to maintain them. There remains the possibility that the relation between horizontal motion and vertical diffusion is not quite so indirect.

In effect, ideas derived from studies of turbulent motion in shear flow have led to a picture of an ocean in which the role of shear flow is uncertain. Despite this, it is probable that the approach used was simple enough for the limits established on the motion to be valid to within an order of magnitude.

Much continued experimental work on "ambient" oceanic turbulence is obviously needed, but, if the present results are any indication, this research will be difficult. Diffusion measurements on a large scale are particularly convenient because they can be made on material introduced naturally, or artificially well in advance of the gradient measurements. The outlook for making direct measurements of the turbulent velocities or associated pressure fluctuations is discouraging. Fluctuations in the horizontal velocity of the order of a centimeter per second over a day would be difficult to separate from periodic tidal

or other large-scale motions. Variations on the vertical motion are also small enough to pose severe measuring problems. Pressure fluctuations are almost numerically equal to the square of the velocity and so are of the order of 1 dyne cm^{-2} . These pressure changes would be difficult to detect at the expected frequencies of 0.004 sec^{-1} or lower.

In most cases the detection equipment could not be tied to a ship. Free floating devices, such as Swallow's neutrally buoyant cylinder, offer exceptional opportunities for quiet measurements and invaluable data could be collected if telemetering devices were developed to send up information directly from such cylinders.

A promising method of studying the turbulent motion is to follow the motion of two or more neutral cylinders at depth. Measurements of changes in their separation with time should give detailed information on the velocity structure of the water.

References

Batchelor, G. K.
    The Theory of Homogeneous Turbulence, Cambridge, University Press (1953).

Bowden, K. F.
    Deep-Sea Res. 2, 33-47 (1954).

Bullard, E. C.
    Deep-Sea Res. 1, 65-66 (1954).

Hughes, P.
    Quart. J. R. Met. Soc. 82, 494-502 (1956).

Jeffries, H.
    The Earth, Cambridge, University Press (1952).

Knauss, J. A.
    J. Acoust. Soc. Am., 28, 443-446 (1956).

Koczy, F. F.
    Nature, Lond., 178, 585-586, (1956).

Metcalf, W. G.
    Woods Hole Ocean. Inst., Ref. 58-31 (1957).

Proudman, J.
    Dynamical Oceanography, John Wiley, New York (1953).

Sverdrup, H. U., Johnson, M. W., and Fleming, R. H.
    The Oceans, Prentice-Hall, Inc., Englewood Cliffs, N. J. (1942).

Swallow, J. C.
    Deep-Sea Res. 4, 93-104 (1957).

Swallow, J. C. and Worthington, L. V.
    Nature, Lond., 180, 1183-1184 (1957).

Townsend, A. A.
    The Structure of Turbulent Shear Flow, Cambridge, University Press (1956).

Townsend, A. A.
    J. Fluid Mech. 3, 361-372 (1958).

U. S. Navy
    Marine Climatic Atlas of the World, Vol. 1, North Atlantic Ocean,
    Pub. by Direction of the Chief of Naval Operations, Washington, D. C. (1955).

Worthington, L. V.
    Woods Hole Ocean. Inst., Ref. 53-85 (1953).

DISTRIBUTION

Office of Naval Research (Code 466)
Navy Department
Washington 25, D. C.
copies 1-2

Commanding Officer and Director
U. S. Navy Underwater Sound Laboratory
Fort Trumbull
New London, Connecticut
copies 3-4

Director
U. S. Naval Ordnance Laboratory
White Oak
Silver Springs 19, Maryland
copies 5-6

Commander
U. S. Naval Air Ordnance Test Station
China Lake, California
copy 7

Director, Marine Physical Laboratory
University of California
Scripps Institute of Oceanography
San Diego, California
copy 8

Director
Bell Telephone Laboratories
Whippany, New Jersey
copies 9-10

Committee on Undersea Warfare
Naval Research Council
National Academy of Sciences
2101 Constitution Avenue
Washington 25, D. C.
copies 11-12

Chief, Bureau of Ships (Code 688)
Navy Department
Washington 25, D. C.
copies 13-14

The Hydrographer
U. S. Navy Hydrographic Office
Washington 25, D. C.
copies 15-16

Director
Naval Research Laboratory
Washington 20, D. C.
Attn: Code 5500, Dr. H. L. Saxton
copies 17-18

Commanding Officer and Director
U. S. Naval Electronics Laboratory
Point Loma
San Diego 52, California
copies 19-20

Commander
U. S. Naval Air Development Center
Johnsville, Pennsylvania
copies 21-22

Dr. F. V. Hunt
Harvard University
Cambridge, Massachusetts
copy 23

Director
Woods Hole Oceanographic Institution
Woods Hole, Massachusetts
copy 24

Director
Lamont Geological Observatory
Torrey Cliff
Palisades, New York
copies 25-26

Chief, Bureau of Aeronautics (AV-43)
Navy Department
Washington 25, D. C.
copies 27-28

Chief, Bureau of Ordnance
Navy Department
Washington 25, D. C.
copies 29-30

Chief of Naval Operations
Navy Department
Washington 25, D. C.
    (Op-001)   copy 31
    (Op-91)   copies 32-33
    (Op-312) copies 34-35
    (Op-316) copies 36-37

Commanding Officer
ONR Branch Office
Navy No. 100
Fleet Post Office
New York, New York
copies 38-39

British Joint Services Mission
Main Navy Building, Room 4924
Washington 25, D. C.
via
Chief of Naval Operations (Op-316d)
Navy Department
Washington 25, D. C.
copies 40-42

Admiralty Research Laboratory
Teddington-Middlesex, England
via
Chief of Naval Operations (Op-316)
Navy Department
Washington 25, D. C.
copy 43

Commander
Armed Services Technical
    Information Agency
Arlington Hall Station
Arlington, Virginia
copies 44-53

Commander
Submarine Development Group II
U. S. Naval Submarine Base
Box 70
New London, Connecticut
copies 54-55

Weapons Systems Evaluation Group
Office of Sec'y of Defense
Room 2E1006- Pentagon Building
Washington 25, D. C.
copy 56

Commanding Officer
ONR Branch Office
346 Broadway
New York 13, New York
copy 57

Canadian Joint Staff
2450 Massachusetts Avenue, N.W.
Washington 25, D. C.
via
Chief of Naval Operations
(Op-316)
Navy Department
Washington 25, D. C.
copies 58-60