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Estimating Replacement Cost of Fixed Assets

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Abstract

Various authors have argued for the use of the replacement cost valuation basis in accounting reports with such success that official bodies have either recommended or required that replacement cost figures be reported. This success is primarily due to the effectiveness of the conceptual arguments advanced for the use of replacement cost. Unfortunately the methods advocated for estimating the replacement cost of fixed assets are not as well developed as the conceptual arguments. First, we will review these methods in light of the authors' stated or implied desire to measure replacement cost as either the current cost of the asset in its current condition, or the availability of services equivalent to those currently contained in the asset. Then we will propose a method which will allow estimation of this replacement cost even in those situations where there is either no used asset market or the used asset market which exists is not sufficiently organized to allow ready estimation of used asset costs. Finally, we will then compare our method for estimating replacement cost to those previously proposed. We hope to point up that there are two salient advantages to our approach: it subsumes the work of previous proposals, and it permits greater generality.

Various authors have argued for the use of the replacement cost valuation basis in accounting reports¹ with such success that official bodies have either recommended or required that replacement cost figures be reported.² This success is primarily due to the effectiveness of the conceptual arguments advanced for the use of replacement cost. Unfortunately the methods advocated for estimating the replacement cost of fixed assets are not as well developed as the conceptual arguments. First, we will review these methods in light of the authors' stated or implied desire to measure replacement cost as either the current cost of the asset in its current condition, or the availability of services equivalent to those currently contained in the asset. Then, we will propose a method which will allow estimation of this replacement cost even in those situations where there is either no used asset market or the used asset market which exists is not sufficiently organized to allow ready estimation of used asset costs. Finally, we will then compare our method for estimating replacement cost to those previously proposed. We hope to point up that there are two salient advantages to our approach: it subsumes the work of previous proposals, and it permits greater generality.

In their first detailed discussion of their choice of replacement cost, Edwards and Bell (1972) conclude that "current cost"--cost currently of

¹Primary examples are: Bedford (1965), Edwards and Bell (1961), and Revsine (1973).

²Two examples are: American Institute of Certified Public Accountants, Study Group on the Objectives of Financial Statements, (1973), pp. 41-43, and Securities and Exchange Commission, (1976).

acquiring the inputs which the firm uses to produce the asset being valued-- is the appropriate replacement cost concept (pp. 91-92). However their discussion at this point concentrates on valuation of inventory. In their subsequent discussion of the valuation of fixed assets (which are not usually "produced" by the firm), they make it clear that they believe the RC should be based on the current cost of acquiring the existing asset in its current condition (p. 175, p. 186n). However they also suggest that this valuation may be impractical since they recommend that current cost of fixed assets be estimated at replacement cost new less accumulated depreciation (p. 186). (Nonetheless, this recommendation is qualified since they base it on the assumption that an accurate depreciation method is being used.)

In his original discussion of replacement cost, Revsine (1973) states: "Replacement cost balance sheet values represent the amount that a firm would have to pay, as of the balance sheet date, in order to replace the assets shown in the statement or to satisfy reported liabilities" (p. 69). Although this is fairly general and he does discuss the problem of choosing a depreciation method, his later statements and examples imply that he regards the acquisition cost new less depreciation as a surrogate for the current acquisition cost of the asset in current condition. (E.g., p. 100, "Realizable cost savings are equal to the change in the market price of assets held during the period." This will only be true if the "market price" is the acquisition cost of fixed assets in current condition).

Although Bedford (1965) does not specifically deal with the distinction between replacement cost new less depreciation and replacement cost of services

equivalent to those contained in the asset in its current condition, he co-authored a later statement implying that his concept of replacement cost would be "cash or cash equivalent that would have to be paid now to acquire resources capable of providing services equivalent to those currently expected to be extracted from the asset." (Bedford and McKeown, 1972).

From this examination it seems that the consensus of the literature is that the objective of an accounting system based upon replacement cost would best be met by relating those fixed assets currently held as directly as possible to the market. This would mean that those methods which estimate replacement cost as the cost new less depreciation should be viewed as surrogates for the former measure. This consensus view is supported by the definition of replacement cost given by the Objectives Study Group of the AICPA: "A valuation basis quantifying assets (and usually liabilities) in terms of present prices for items equivalent in capacity and services." (AICPA, 1973, p. 41).

A major advantage of the approach to measurement of replacement cost of fixed assets to be proposed in this paper is that it does not require selecting a depreciation method. Only in this way can the replacement cost system avoid arbitrary allocations. Any attempt to measure replacement cost of fixed assets as the replacement cost new less depreciation would be an arbitrary allocation and therefore subject to the same criticism that is applied to historical cost systems (Thomas, 1969, pp. 89, 91).

Previous Approaches

The approaches which have been proposed for estimation of replacement costs of fixed assets are of two basic types:

1. Estimation of replacement cost new then depreciating to a book value. The replacement cost new can be estimated by direct reference to the new asset market [Revsine (1973), p. 77, Edwards and Bell (1961), pp. 185-7], price indexing [Revsine (1973), p. 77, Edwards and Bell (1961), pp. 185-7, Brinkman (1977), p. 46-4]³ or expert appraisal (including engineering estimate) of current cost new [Revsine (1973), p. 77, Edwards and Bell (1961), pp. 185-7, Brinkman (1977), p. 46-4].

Under these suggested methods, the depreciation is usually computed from this estimate using a conventional accounting depreciation method. Since most conventional accounting depreciation methods are straight-line or accelerated and most theoretical calculations are accelerating (higher depreciation in later years), use of a conventional depreciation method applied to an estimate of replacement cost new seems unlikely to yield a good estimate of the market value (known or unknown) of the used asset. Therefore we must reject this method using conventional accounting depreciation methods.

Edwards and Bell (1961, pp. 175-176) suggest a depreciation method based on study of the patterns of decline of second-hand asset market values. This method should yield quite accurate results if specific second-hand asset value are available. However in a "thin" (relatively inactive) or non-existent used asset market,

³Brinkman especially notes the necessity of assuming the adjustment of index used to allow for technological change. [Brinkman, pp. 46-28 through 46-34].

the accountant would be unable to get good estimates of the pattern for specific assets. So although this method will probably perform well in the presence of a well organized used asset market, it provides little, if any, help in the absence of that market.

Both Edwards and Bell (1961, pp. 176-177) and Weil (1977, pp. 46-35 to 46-43) discuss methods which are quite similar to the annuity or sinking fund depreciation methods. These are based on use of the internal rate of return and assume equal return from the asset (or its replacement) during each year of its life.⁴ These depreciation methods are probably the best of those proposed and if used consistently would yield better approximations to the replacement cost of the asset in the used asset market (or to the theoretical estimates of what that value would be if there were a market) than use of conventional depreciation methods. In fact the distinction

⁴Weil calls this method functional pricing. He does not apply it consistently to all five cases in his example. The reader may note that Weil could have applied the same method used in cases IV-V to Cases I and II yielding replacement cost of functional capacity in current condition of \$12,339 and \$7,386 respectively.

The solutions to Cases I and II which would be consistent with Weil's solutions to the other cases would be:

	I	II
Cost new	\$20,000	\$20,000
÷ present value of annuity 10 periods at 10%	÷ <u>6.14457</u> \$ 3,254.91	<u>6.14457</u> \$ 3,254.91
+ operating cost of new asset	+ <u>1,100</u>	<u>1,100</u>
x ratio of capacities	x <u>$\frac{700}{700}$</u>	x <u>$\frac{700}{1000}$</u>
	\$ 4,354.91	\$ 3,048.43
- operating cost of existing asset	- <u>1,100</u> \$ 3,254.91	- <u>1,100</u> \$ 1,948.43
x present value of annuity 5 periods at 10%	x <u>3.79079</u>	x <u>3.79079</u>
replacement cost of existing capacity	<u>\$12,338.67</u>	<u>\$ 7,386.11</u>

between this method and the one that will be proposed in this paper is that the former fails to consider the additional flexibility inherent in owning (or keeping) an asset with fewer remaining years of life.

2. Appraisal of the existing asset to estimate directly the current cost of replacing the asset in its existing condition. Depending on the accuracy of the appraisal, this approach could yield very good estimates of the replacement cost of the asset in its current condition. However, since the appraisals are likely to be most accurate in those cases where the used asset market is active and organized, the pattern method of depreciation suggested by Edwards and Bell should also work well and with less expense. On the other hand, in cases where the used asset market is not well-organized, the appraisals would probably tend to be less accurate and more costly. In general this approach would appear to be practical only in the case of very large assets if it is practical at all.

Since the approaches above either do not approximate the current cost of the asset in its current condition, work only in the presence of a well-organized used asset market, or ignore the increased flexibility of owning assets with shorter remaining lives, it seems appropriate to propose an approach which does not suffer from these deficiencies.

Definition of Replacement Cost

In order to provide a rigorous definition for the analysis and development of estimation methods, consider the relationship between the purchase price of new and used assets when an orderly used asset market

exists. We will regard the purchase price of an asset as including the full cost necessary to put the asset into service. In an orderly used asset market we would expect the purchase price of a used asset to "perfectly adjust" relative to that of a new asset such that the expected cost associated with purchasing the used asset is identical to the expected cost associated with purchasing a new asset. For example, assume that the purchase price of a new asset is \$10,000, the purchase price of a used asset is X, and a firm wishes to acquire services that can be performed by either asset. If X were too high (low), the firm would determine that the expected cost associated with paying \$10,000 for the new asset was lower (higher) than the expected cost associated with paying X for the used asset. Thus the market would induce X to decrease (increase) until the expected costs were identical. This perfect adjustment of the purchase price of a used asset relative to that of a new asset will be the basis of our definition. Specifically, we will define the replacement cost of a used asset to be that price at which the expected costs associated with "purchasing" the used asset which the firm currently holds, or purchasing a new asset (at a known price), are identical.

Proposed Approach

In practice, an orderly used asset market which perfectly adjusts prices may not exist. Nonetheless, we will use the definition of replacement cost suggested above to derive these prices. Two different situations will be considered: (1) one in which some sort of used asset market exists in that used assets can be purchased or sold, but the prices at which these transactions can be arranged are not easily observable; and (2) one in which no used asset market exists at all--used assets cannot be purchased or sold.

The need to distinguish between these two market situations is based upon two factors. First and most obvious, if a used asset market does exist, the asset can be sold at some future date if a decision is made to discontinue its use. This means that the "cost" of abandoning an asset with a relatively large proportion of its life remaining is lower than if there were no used asset market. Thus the added flexibility associated with holding used assets is reduced. (The amount of this difference is related to the amount which can be recovered from sale of a discarded asset.)

The second distinction between the two types of market situations is that if there is no used asset market, the firm cannot buy a used asset. Thus at the end of the life of an old asset, the firm can only choose between either abandoning the use of this type of asset or buying a new asset. This will be explored in more detail when the no used asset market situation is discussed below.

The Used Asset Market Model

Assume the firm holds a used asset with k remaining years of life. This asset could be replaced by a new asset with N years of life at cost P_N ($k < N$). Each asset performs the same level of services at the same cost during each year of life and each is worthless at the end of its life. Either asset can be sold at any time for a fraction, B , of its replacement cost at that time. The firm assesses the probability that it will abandon the use of this asset's services at the end of any year (given the use of the asset during the year) as θ . Thus we have:

P_N = the purchase price of a new asset with N years of life remaining.
(P_N is assumed known.)

P_k = the purchase price of a used asset with k years of life remaining,
 $1 \leq k < N$. (P_k is assumed unknown.)

- B = the fraction of the replacement cost for which a used asset may be sold, $0 \leq B \leq 1$.
- BP_k = the price for which a used asset with k remaining years of life could be sold.
- i = the appropriate discount rate for the firm
- θ = the probability that the firm will abandon use of this asset's services at the end of any year given that the services were used during that year, $0 \leq \theta < 1$.

On the basis of our definition, the replacement cost of the asset with k years of life remaining can be derived (see Appendix) to be:

$$P_k = \sum_{\ell=1}^k \left\{ \left[\frac{BP_{k-\ell}}{1+i} \right] \theta + P_1 \right\} \frac{1}{(1+i)^{\ell-1}} (1-\theta)^{\ell-1} \quad (1)$$

This equation can be explained easily. The term in brackets is the cost avoided at the beginning of year ℓ by having an asset with k years of life remaining on hand. That is, if the company uses this asset it will avoid paying the price of an asset with one year of life remaining, P_1 . In addition it will realize the exit value of the asset, $BP_{k-\ell}$, if use of the asset is terminated at the end of year ℓ because this is what the asset can be sold for. Assuming that use of the asset is not terminated before year ℓ , the probability the company will terminate it at the end of year ℓ is θ , and the factor to discount to the beginning of year ℓ is $\frac{1}{1+i}$. The remaining factors compute the probability the use of the asset is not terminated before year ℓ and discount costs incurred at the beginning of year ℓ back to the balance sheet date.

The equation (1) has several intuitively appealing properties:

1. As the ratio of exit value to replacement cost, B, decreases, the replacement cost, P_k , increases relative to the price of the new asset, P_N . That is, the significance of the added

flexibility which results from buying the used asset (with fewer years of life remaining) increases as the proportion of replacement cost realized from the sale of an unneeded asset decreases. An alternative way to view this is that the penalty for having to dispose of an asset before it is fully utilized becomes larger as B becomes smaller.

2. As the probability, θ , of discontinuing use of the asset in any year decreases, the replacement cost of a used asset, P_k , decreases relative to the cost of a new asset, P_N . This is consistent because the flexibility of holding the used asset becomes less important as the probability of discontinuing use of the asset decreases. In fact, when $\theta = 0$, the ratio $\frac{P_k}{P_N}$ is equal to the ratio of the present value of the annuities due (for the given discount rate) for k and N years respectively. That is:

$$\frac{P_k}{P_N} = \frac{A(k,i)}{A(N,i)}$$

where: $A(m,i)$ = present value of annuity due for m periods
discounted at the rate of i per period.

This is the result Edwards and Bell (1961, pp. 176-7) get and Weil (1977) should get for his Case I. Thus if the value of flexibility is 0 (i.e., the firm will never discontinue use of the asset), the adjustment for flexibility is 0 and the result is identical to those suggested approaches which ignored flexibility.

3. If the probability, θ , of discontinuing use of the asset is 0 and the discount rate, i, also 0, the ratio of prices is:

$$\frac{P_k}{P_N} = \frac{k}{N}$$

This is also the result which would be obtained if straight line depreciation were used. (Accelerated depreciation would have an even lower P_k/P_N ratio.) While a probability of termination, θ , of 0 might be reasonable in some cases, it is unlikely that a discount rate of 0 is appropriate for any case. Thus, we must conclude that use of straight line (or accelerated) depreciation applied to the replacement cost new, P_N , will understate the replacement cost of the used asset.

To illustrate the application of this approach, consider the following situation (Case I from Weil (1977), p. 46-30):

Cost new = \$20,000 = P_N

Life new = 10 years = N

Remaining life = 5 years = k

Discount rate = .10 = i

Probability of termination in any year⁵ = .10 = θ

Exit value/replacement cost⁶ = .75 = B

⁵Weil did not have this parameter specified. His solution is equivalent to assigning value of 0 to θ . The sensitivity of the results of this value of θ will be examined later.

⁶Weil did not need this parameter. Examination of equation (1) will show that if $\theta = 0$, the value of B is irrelevant. A value of B of .75 is true of some of the better organized markets, but typical values of B would probably be considerably lower---particularly since we are assuming here that we are not dealing with a well-organized used asset market. The effect of the value of B will also be examined later (even to the extent of considering the case where no used asset market exists).

The replacement cost of this asset is \$12,927.⁷ That is, if a market existed in which this used asset could be obtained, the management of this firm would be indifferent between paying \$20,000 for the new asset and paying \$12,927 for the used asset. Weil's solution (with $\theta = 0$) should have been \$12,339.

Having dealt with the case where the replacement asset is the same as the used asset (no technological change), the obvious question is: What happens if there has been technological improvement and a new asset is available in improved form? The answer is derived by considering the forms which the improvement could take. The primary possibilities appear to be: longer operating life, increased capacity, or lower operating costs.

1. Longer operating life does not require any change in the previously stated approach. The previous derivation did not assume the original life of the new asset was equal to the life of the new asset. The replacement cost of an asset is not affected by the number of years of previous use, only by the number of years of remaining use.⁸ Therefore, if the new asset has a longer useful life than the original life of the used asset, the life of the new asset is simply N and the remaining life of the used asset is k .

⁷No claim is made that the determination of P_k is a simple 30 second computation with a hand calculator. The details of solution are not shown here simply to avoid boring the reader. The contention is made, however, that the solution is straightforward and can be (and was) determined by a simple computer program--or even one of the more powerful hand calculators.

⁸Of course for a given asset, the longer the past use, the shorter the remaining use. The point is that the longer past use affects the replacement cost only if it is tied to remaining use.

2. Increased capacity of the new asset is considered in some detail by Weil (1977, pp. 46-36 to 46-37). His discussion there applies here as well. There are two subcases: the indivisible case where the firm can not make use of the increased capacity, and the divisible case where the firm can make use of the increased capacity (either through use of fewer machines, rental of service to external entities, increasing the operating life, etc.). We feel strongly that the divisible case should be assumed. (It seems likely that a company which held the asset with larger capacities would receive benefit from the increased capacity. Furthermore, the indivisible case would require reporting on the balance sheet a replacement cost representing a larger capacity than that currently available to the company. This seems inappropriate.) Under the divisible case the simplest way to adjust Equation (1) for the difference in capacity is to simply multiply the price of the new asset by the ratio of the used asset capacity to new capacity before entering the replacement cost new into the solution:

$$P_N^* = P_N \cdot \frac{V_u}{V_N}$$

where P_N^* = adjusted replacement cost new (to be used in solution)
 P_N = full replacement cost of new asset with larger capacity
 V_u = capacity of used asset
 V_N = capacity of new asset

Under the divisible case, the capacities of assets whose remaining lives are between those of the used and new assets

do not affect the replacement cost of the used asset. The only information needed is the capacity of the currently held asset and the capacity of the new asset.

3. Operating cost decreases require a more complex adjustment than the two preceding types of technological improvements. (It should be mentioned that it is unlikely that a capacity change would be made without a change in operating cost.) The complicating factor is that the operating cost saving is only effective in those years in which the asset's services will be used by the firm. Since we are assuming that there is some probability the firm may discontinue use of the asset, there is similarly some probability that the cost savings of some future years will not be realized. Thus, equation (1) must be modified to:⁹

$$P_k = \sum_{\ell=1}^k \left[\left(\frac{B\theta P_{k-\ell}}{1+i} + P_1 + c_1^1 - c_{k+1-\ell}^k \right) \left(\frac{1-\theta}{1+i} \right)^{\ell-1} \right] \quad (2)$$

⁹Equation (2) requires the assumption that the total cost of acquiring and operating an asset with one year of life remaining is constant over the life of the new asset. That is as the operating cost decreases, the cost of acquiring the asset increases. This assumption should be valid so long as additional unexpected technological change does not occur. Equations (1) and (2) can be simplified for computational purposes (although some intuitive interpretability may be lost) to:

$$P_k = \sum_{j=1}^k \left\{ P_1 \left[\frac{1-\theta(1-B)}{1+i} \right]^{k-j} \right\} \quad (1a)$$

and:

$$P_k = \sum_{j=1}^k \left\{ (P_1 + c_1^1 - c_j^k) \left[\frac{1-\theta(1-B)}{1+i} \right]^{k-j} \right\} \quad (2a)$$

where c_j^i = operating cost of an asset which had i years of life remaining at the valuation date, but the cost of which is measured when the asset has j years of life remaining. This cost is assumed discounted to the beginning of the year.

All other variables are as defined for equation (1).

Please note that equation (2) handles not only the case where operating cost for the new asset is different from (presumably lower than) the operating cost of the used asset, but it also allows for situations where the operating cost for either asset is different for different years of that asset's life. Therefore through use of equation (2) we are able to drop the assumption--made for equation (1)--that the services of the assets are provided at the same cost for each year of their lives. The solution is general as far as pattern of operating cost is concerned.

Armed with these adjustments, it is now possible to compare the results obtained under the approach proposed here with the approach proposed by Weil. Table 1 presents the calculated results for the independent cases considered by Weil (1977, p. 46-36) first under Weil's method (assuming probability of discontinuing use of asset is 0), then under a variety of combinations of θ (probability of discontinuing use of the asset in any year) and B (ratio of exit value to replacement cost). The values used for θ are .10, .03, and .02 which correspond to expected number of years of use of the asset of 10, 33 1/3, and 50 years respectively. (Expected number of years of use is $\frac{1}{\theta}$.) A value of .10 is probably fairly high for a stable industry, but might be appropriate for an industry where product lives and processes change rapidly. Values of .75, .40, and .00 for B cover

the range of reasonable values. The value of .75 is probably too high since the markets we are considering are not well-organized. The middle value (.40) is also somewhat high for a poorly organized market. Alternatively, the value .00 represents the situation in which a firm would realize nothing on the sale of a used asset. This might occur for one of two reasons. Either there is no market for used assets or the expense of selling a used asset is likely to be greater than the amount that will be recovered in a sale. An illustration of the latter case would be when firms find it very expensive to use a broker to find likely purchasers. This situation would probably occur when a whole industry was changing product lines or processes since the number of prospective sellers of used assets would be far greater than the number of prospective purchasers.

Examination of Table 1 discloses the relationships mentioned above: replacement cost of the used asset increases with increasing θ and decreasing E . Please remember that the cases represent independent situations where different replacement assets are available. The various columns are presented so that the reader may see the effect of the methods as applied to situations where the replacement asset differs from the existing asset in different ways (operating cost, capacity, years of life). Consideration of the different columns demonstrates that there is a difference between an assumption of $\theta = .00$ and a θ even as small as .02 (50 expected years of use). This provides strong support for the use of the proposed method rather than one which requires the assumption that $\theta = .00$.

No Used Asset Market

Situations where no used asset market exists can be handled by using equation (1) or (2) above with E set equal to .00. However it is possible

Table 1

Results of Weil's Cases⁺

Existing asset: 5 years of life remaining, capacity: 700 units,
operating cost: \$1,100

Replacement Assets

	I	II	III	IV	V
Cost New	\$20,000	\$20,000	\$20,000	\$20,000	\$20,000
Life when new = N	10 years	10 years	10 years	12 years	12 years
Capacity	700 units	1,000 units	700 units	700 units	1,000 units
Operating cost	\$ 1,100	\$ 1,100	\$ 1,000	\$ 1,100	\$ 1,000
Cost new of existing capacity = P _N [©]	\$20,000	\$14,000	\$20,000	\$20,000	\$14,000
Operating cost for existing capacity [©]	\$ 1,100	\$ 770	\$ 1,000	\$ 1,000	\$ 700
Weil's method*	\$12,339	\$ 7,386	\$11,960	\$11,127	\$ 6,273
P ₅ (θ = .1, B = .75)	\$12,927	\$ 7,854	\$12,565	\$11,843	\$ 6,481
P ₅ (θ = .1, B = .40)	13,739	8,495	13,399	12,832	7,622
P ₅ (θ = .1, B = .00)	14,634	9,199	14,318	13,920	8,477
P ₅ (θ = .03, B = .75)	12,516	7,527	12,142	11,342	6,444
P ₅ (θ = .03, B = .40)	12,763	7,723	13,396	11,643	6,682
P ₅ (θ = .03, B = .00)	13,045	7,946	12,685	11,986	6,954
P ₅ (θ = .02, B = .75)	12,457	7,480	12,081	11,270	6,387
P ₅ (θ = .02, B = .40)	12,622	7,611	12,251	11,471	6,546
P ₅ (θ = .02, B = .00)	12,810	7,761	12,445	11,700	6,728

⁺This table is adopted from Weil (1977), p. 46-36.

*These are the results from Weil's method reported in his paper as modified in footnote 4. This is also P₅(θ = .00).

[©]The cost new of existing capacity and operating cost are each multiplied by the following ratio: existing capacity/capacity of new asset. Also since the operating cost given by Weil was assumed to occur at the end of the period, the operating costs inputed into equation (2) were those shown discounted to the beginning of the period (divided by 1.10).

to make use of the absence of a used asset market to develop an approach which allows some generalization of the conditions regarding the probability of discontinuing use of the asset. Lack of a used asset market means that despite the firm's preferences it will both be unable to sell or buy a used asset. Thus the replacement cost of the existing asset may be computed by comparing the firm's only two alternatives: "buy" the existing asset or "buy" a new asset. Furthermore, when either asset expires, it may only be replaced by a new asset. To deal with this case, define Δ to be the probability that a new asset just purchased will not be replaced. This would occur because the firm's need for the asset's services has ended by the end of the new asset's life. That is, $1 - \Delta$ is the probability another new asset will be purchased when this one expires. Similarly, define Δ' to be the probability that the existing asset currently in use will not be replaced. Then $1 - \Delta'$ is the probability a new asset will be purchased when the one currently in use expires. In this case, the replacement cost of an asset with k years of life remaining is given by (proof is in Appendix):

$$P_k = \frac{\left[1 - \frac{(1 - \Delta')}{(1 + i)^k} \right]}{\left[1 - \frac{(1 - \Delta)}{(1 + i)^N} \right]} P_N \quad (3)$$

Equation (3) yields exactly the same result as Equation (1) with B set to .00 in those situations where equation (1) applies--that is where the probability of abandoning the service in a given year is constant over time (proof in Appendix). However Equation (3) can be used in many situations where the probability of abandonment in a year is not constant over time. All that is required is that Δ , the probability of abandoning

the asset's services within the lifetime of the (new) asset just purchased, remain constant. The distribution of probability between years is not constrained. In particular it may be that the probability of discontinuing use of an asset's services will depend on the age of the asset. As an asset grows older, the services it provides are more likely to be abandoned because the services have a higher probability of reaching obsolescence and because a smaller portion of the cost of the asset (with remaining life) would be lost. This latter point is particularly important since in the no used asset market situation the amount recovered from an abandoned asset is zero.

In a similar fashion to Equation (1), Equation (3) can handle technological improvements such as increased life and capacity. However, as stated, Equation (3) cannot handle operating costs which are not constant. It could be modified to handle different operating costs, but this is hardly seems worthwhile since it would be necessary to specify the year by year distribution of probabilities within Δ and Δ' .

Equivalent Services or Identical Asset

Two distinct concepts of the objective of replacement cost measurement of fixed assets have been proposed. Edwards and Bell (1962, p. 196n) clearly favor measurement of the cost of replacement of the identical asset. Bedford (1965) and Revsine (1973) favor measurement of the cost of acquisition of services equivalent to those contained in the existing asset. The approach proposed above can be applied to either concept.

If the replacement cost of the identical asset is desired, the new asset whose price, life and operating cost are identical to the existing asset should be used as the standard. This may be difficult if such an

asset is not available new, but this problem is to a certain extent inherent in the identical asset concept. (A suggestion for handling this problem will be made below.)

If the equivalent services concept is desired, the price, life and operating cost of various new assets which could provide services equivalent to those provided by the existing asset should be used as the standard. (There may, of course, be problems in the identification of those assets which provide equivalent services.) This concept of replacement cost would appear to require that the lowest of the replacement cost calculations (based on various new assets) be used as the measurement of replacement cost. This would be consistent with the assumption that if the firm were to replace the existing capacity with an asset capable of providing equivalent services, its management should select the mode of replacement which has the lowest cost.

An interesting feature of the proposed method of estimation is that the estimate of the replacement cost of the existing asset would be the same whether the identical asset or equivalent services concept is followed. This equality would occur because firms which are considering both alternatives (replacement with the same or an improved asset) would presumably buy whichever one is "underpriced". This action across the market would thus adjust the prices of the two (or more) new assets so that the expected costs associated with each are identical. If this is the case, the estimate of replacement cost will be the same when computed using any of the new assets which provide equivalent services as a standard. (In fact the estimation approach proposed could be used to compute perfectly adjusted prices for the various assets.)

Therefore the recommended procedure would be to use the parameters of the identical asset new (if it is available new) as a basis for the

computation. This recommendation is made not because the identical asset approach is favored on theoretical grounds, but simply because if two methods yield the same result, the easier one should be used, and the easier method here is obviously the one that avoids wherever possible the problem of identifying assets which provide equivalent services. If the identical asset is not available new, the parameters of an asset which can provide equivalent services should be used in the computation. Again the result should be the same as that which would be obtained if the parameters for an identical new asset were known and used. Thus the proponents of the identical asset concept can arrive at the estimate of the replacement cost of the identical asset when no used asset market exists and the identical asset is not available new.

Summary

An approach has been proposed which builds on the work of previous authors to develop a method of estimating the replacement cost of an asset in its current condition or the replacement cost of the services which can be provided by that asset even in the absence of readily available used asset market prices. The proposed approach allows adjustment for the probability of discontinuing use of the asset's services as well as technological changes such as increase in asset life, increase in asset capacity or decrease in operating costs. Although used asset market prices should be used if readily available, it is recommended that the proposed approach be used in other cases for estimation of replacement cost under either the identical asset or equivalent services concepts.

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1. The first part of the document discusses the importance of maintaining accurate records of all transactions.

2. It is essential to ensure that all data is entered correctly and consistently.

3. The following table provides a summary of the key findings from the study.

4. The results indicate that there is a significant correlation between the variables studied.

5. This suggests that the model used is a good fit for the data.

6. The data shows that the system is highly effective in handling the workload.

7. The analysis also shows that the system is able to handle a wide range of inputs.

8. This is a key feature of the system and is essential for its success.

9. The system is able to handle a large volume of data and is highly accurate.

10. The system is able to handle a wide range of inputs and is highly accurate.

11. The system is able to handle a large volume of data and is highly accurate.

12. The system is able to handle a wide range of inputs and is highly accurate.

13. The system is able to handle a large volume of data and is highly accurate.

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APPENDIX

1. Prices of Current Assets When a Used Asset Market Exists.

Let

- P_{ℓ}^{T+k} = The purchase price of a used asset which becomes available on the market T years from the balance sheet date, (at which time it had k years of potentially useful life, $1 \leq k \leq N$, $0 \leq T$) but which only has ℓ years of life remaining, $\ell \leq k$. (P_{ℓ}^{T+k} is, in general, assumed to be unknown).
- P_N^N = The purchase price of a new asset which is available at the balance sheet date and has N years of life remaining. (P_N^N is assumed known).
- B = The fraction of the replacement cost for which a used asset may be sold, $0 \leq B \leq 1$.
- Bp_{ℓ}^{T+k} = The price for which a used asset which became available T years from the balance sheet date, was used for $k-\ell$ years, and has ℓ years of life remaining can be sold.
- i = The appropriate discount rate for the firm.
- $Q = (1+i)^{-1}$
- θ = The probability that the firm will abandon use of this asset's services at the end of any year given that the services were used during the year, $0 \leq \theta < 1$.

c_l^{T+k} = The operating cost of an asset which became available T years from the balance sheet date and originally had k years of potentially useful life, but the cost of which is measured when the asset has l years of life remaining. This cost is assumed discounted to the beginning of the year.

For convenience, superscripts were suppressed in the earlier discussion.

However, by defining P_k , $1 \leq k \leq N$, as

$$P_k \equiv P_k^k,$$

the analysis in the body of the paper will be consistent with that in the appendix. Finally, it will be assumed that for all $T > 0$,

$$p_1^{T+k} + c_1^{T+k} = p_1^1 + c_1^1.$$

This assumption simply states that the purchase price plus operating cost of a used asset with one year of life remaining remains constant over time. This is not an unreasonable assumption concerning a short lived used asset, and will considerably facilitate the analysis.

Suppose that the firm assesses the probability that the asset's services will be terminated T years from the balance sheet date, $1 \leq T < \infty$, to be $\theta(1-\theta)^{T-1}$. Then the cumulative probability is

$$\text{Probability of termination in } T \text{ or fewer years} = \sum_{l=1}^T \theta(1-\theta)^{l-1}$$

Thus the probability of termination in T or fewer years is distributed as a geometric distribution with unknown parameter θ and moment generating function

$$M(t) = \frac{\theta e^t}{[1 - (1-\theta) e^t]}$$

A stream of purchases of new and used assets is any combination of purchases such that a firm can secure the service of one, and only one, of these assets in any given year. The price system will be derived by assuming that the expected cost associated with any conceivable stream of purchases over the life of the asset's service is equal to the expected cost associated with all other alternative streams. To derive these prices, consider two possible streams of purchases. Both streams are identical until T years from the balance sheet date, at which time a used asset with k years of useful life remaining is purchased in the first stream, while in the second an asset with one-year-of-life remaining is purchased annually from T years from the balance sheet date until T+k years. After the end of the year T+k, both streams continue to be identical. Since these two streams are identical except between years T and T+k, the expected cost associated with both will be identical if and only if the expected costs which result from the purchase decisions between years T and T+k are identical. For example, in the first stream the expected cost that results from the decision to purchase a used asset with k years of life remaining T years from the balance sheet date is

$$\begin{aligned} & \sum_{\ell=T+1}^{T+k} \left[P_k^{T+k} Q^T - BQ^\ell P_{T+k-\ell}^{T+k} + \sum_{j=T+1}^{\ell} c_{T+k+1-j}^{T+k} Q^{j-1} \right] \theta (1-\theta)^{\ell-1} \\ & + \left[P_k^{T+k} Q^T + \sum_{j=T+1}^{T+k} c_{T+k+1-j}^{T+k} Q^{j-1} \right] (1-\theta)^{T+k} \end{aligned} \quad (A1)$$

The first term in (A1) is the cost when the use of the asset's services is terminated at the end of year ℓ times the probability of this occurring, summed over all ℓ between $T+1$ and $T+k$. The second expression is the cost times the probability the use of the asset's services will not be terminated before the end of year $T+k$. Similarly, the expected cost associated with purchasing an asset with one-year-of-life remaining between years T and $T+k$ is

$$\begin{aligned} & \sum_{\ell=T+1}^{T+k} \left(\sum_{j=T+1}^{\ell} \{ p_1^j Q^{j-1} + c_1^j Q^{j-1} \} \right) \theta(1-\theta)^{\ell-1} \\ & + \left(\sum_{j=T+1}^{T+k} \{ p_1^j Q^{j-1} + c_1^j Q^{j-1} \} \right) \theta(1-\theta)^{T+k}, \end{aligned} \quad (A2)$$

with the same logic applying.

Equating these two expected costs yields

$$\begin{aligned} & \sum_{\ell=T+1}^{T+k} \left(p_k^{T+k} Q^T - BQ^\ell p_{T+k-\ell}^{T+k} + \sum_{j=T+1}^{\ell} c_{T+k+1-j}^{T+k} Q^{j-1} \right) \theta(1-\theta)^{\ell-1} \\ & + \left(p_k^{T+k} Q^T + \sum_{j=T+1}^{T+k} c_{T+k+1-j}^{T+k} Q^{j-1} \right) (1-\theta)^{T+k} \\ & = \sum_{\ell=T+1}^{T+k} \left(\sum_{j=T+1}^{\ell} \{ p_1^j Q^{j-1} + c_1^j Q^{j-1} \} \right) \theta(1-\theta)^{\ell-1} \\ & + \left(\sum_{j=T+1}^{T+k} \{ p_1^j Q^{j-1} + c_1^j Q^{j-1} \} \right) (1-\theta)^{T+k}. \end{aligned}$$

Eliminating a factor of $\{Q(1-\theta)\}^T$ from both sides and letting $m=l-T$ yields

$$\begin{aligned}
 p_k^{T+k} &= \sum_{m=1}^k \left(BQ^m p_{k-m}^{T+k} \right) \theta(1-\theta)^{m-1} \\
 &+ \sum_{m=1}^k \left\{ \sum_{j=1}^m \{p_1^{T+j} Q^{j-1} + c_1^{T+j} Q^{j-1} - c_{k+1-j}^{T+k} Q^{j-1}\} \right\} \theta(1-\theta)^{m-1} \\
 &+ \left(\sum_{j=1}^k \{p_1^{T+j} Q^{j-1} + c_1^{T+j} Q^{j-1} - c_{k+1-j}^{T+k} Q^{j-1}\} \right) (1-\theta)^k
 \end{aligned}$$

Reversing the summation signs permits

$$\begin{aligned}
 p_k^{T+k} &= \sum_{m=1}^k BQ^m p_{k-m}^{T+k} \theta(1-\theta)^{m-1} \\
 &+ \sum_{j=1}^k \left\{ \{p_1^{T+j} + c_1^{T+j} - c_{k+1-j}^{T+k}\} Q^{j-1} \right\} \left(\sum_{m=j}^k \theta(1-\theta)^{m-1} + (1-\theta)^k \right)
 \end{aligned}$$

But since

$$\begin{aligned}
 \sum_{m=j}^k \theta(1-\theta)^{m-1} + (1-\theta)^k &= \sum_{m=j}^k \theta(1-\theta)^{m-1} + \sum_{m=k+1}^{\infty} \theta(1-\theta)^{m-1} = \sum_{m=j}^{\infty} \theta(1-\theta)^{m-1} = \\
 &(1-\theta)^{j-1},
 \end{aligned}$$

$$p_k^{T+k} = \sum_{m=1}^k \left(\{BQ\theta\} p_{k-m}^{T+k} + p_1^{T+m} + c_1^{T+m} - c_{k+1-m}^{T+k} \right) \{Q(1-\theta)\}^{m-1}.$$

Finally the assumption that for all $T \geq 0$,

$$p_1^{T+1} + c_1^{T+1} = p_1^1 + c_1^1$$

implies

$$p_k^{T+k} = \sum_{m=1}^k \left(\{BQ\theta\} p_{k-m}^{T+k} + p_1^1 + c_1^1 - c_{k+1-m}^{T+k} \right) \{Q(1-\theta)\}^{m-1} \quad (A3)$$

As a means of simplifying (A3), let us propose that

$$p_k^{T+k} = \sum_{m=1}^k \{B\theta Q + Q(1-\theta)\}^{k-m} \left(p_1^1 + c_1^1 - c_m^{T+k} \right) \quad (A4)$$

We will demonstrate that (A3) implies (A4) using an inductive argument. When $T=0, k=1$, (A3) implies

$$p_1^1 = \left(p_1^1 + c_1^1 - c_1^1 \right) = p_1^1 .$$

This is consistent with (A4). When $T=1, k=1$, (A3) implies

$$p_1^2 = p_1^1 + c_1^1 - c_1^2$$

This is also consistent with (A4). When $T=0, k=2$, (A3) implies

$$p_2^2 = \{B\theta Q\} p_1^2 + \{p_1^1 + c_1^1 - c_2^2\} + \{p_1^1 + c_1^1 - c_1^2\} ,$$

but since $p_1^2 = p_1^1 + c_1^1 - c_1^2$,

$$p_2^2 = \{B\theta Q + Q(1-\theta)\} \left(p_1^1 + c_1^1 - c_2^2 \right) + p_1^1 + c_1^1 - c_2^2 .$$

$$= \sum_{m=1}^2 \{BQ\theta + Q(1-\theta)\}^{2-m} \left(p_1^1 + c_1^1 - c_m^2 \right) .$$

Therefore (A3) implies (A4) when $k=1, 2, T+k=1, 2$. Thus suppose that (A4) holds for P_{k-1}^ℓ :

$$P_{k-1}^\ell = \sum_{m=1}^{k-1} \{BQ\theta + Q(1-\theta)\}^{(k-1)-m} \left(p_1^1 + c_1^1 - c_m^\ell \right)$$

We will show that this along with (A3) implies

$$P_k^\ell = \sum_{m=1}^k \{BQ\theta + Q(1-\theta)\}^{k-m} \left(p_1^1 + c_1^1 - c_m^\ell \right)$$

From (A3)

$$\begin{aligned} P_k^\ell &= \sum_{m=1}^k \left[\{BQ\theta\} P_{k-m}^\ell + p_1^1 + c_1^1 - c_{k+1-m}^\ell \right] \{Q(1-\theta)\}^{m-1} \\ &= \sum_{m=2}^k \left[\{BQ\theta\} P_{k-m}^\ell + p_1^1 + c_1^1 - c_{k+1-m}^\ell \right] \{Q(1-\theta)\}^{m-1} \\ &\quad + \{BQ\theta\} P_{k-1}^\ell + p_1^1 + c_1^1 - c_k^\ell \\ &= \{Q(1-\theta)\} \sum_{m=2}^k \left[\{BQ\theta\} P_{k-m}^\ell + p_1^1 + c_1^1 - c_{k+1-m}^\ell \right] \{Q(1-\theta)\}^{m-2} \\ &\quad + \{BQ\theta\} P_{k-1}^\ell + p_1^1 + c_1^1 - c_k^\ell \end{aligned}$$

$$= \{Q(1-\theta)\} \sum_{j=1}^{k-1} \left(\{BQ\theta\} P_{(k-1)-j}^{\ell} + P_1^1 + c_1^1 - c_{(k-1)+1-j}^{\ell} \right) \{Q(1-\theta)\}^{j-1} \\ + \{BQ\theta\} P_{k-1}^{\ell} + P_1^1 + c_1^1 - c_k^{\ell}$$

But using (A3) again,

$$= \{Q(1-\theta)\} P_{k-1}^{\ell} + \{BQ\theta\} P_{k-1}^{\ell} + P_1^1 + c_1^1 - c_k^{\ell} \\ = \{Q(1-\theta) + BQ\theta\} P_{k-1}^{\ell} + P_1^1 + c_1^1 - c_k^{\ell} \\ = \{BQ\theta + Q(1-\theta)\} \sum_{m=1}^{k-1} \{BQ\theta + Q(1-\theta)\}^{(k-1)-m} \left(P_1^1 + c_1^1 - c_m^{\ell} \right) \\ + \{P_1^1 + c_1^1 - c_k^{\ell}\} \\ = \sum_{m=1}^{k-1} \{BQ\theta + Q(1-\theta)\}^{k-m} \left(P_1^1 + c_1^1 - c_m^{\ell} \right) + \{P_1^1 + c_1^1 - c_k^{\ell}\} \\ = \sum_{m=1}^k \{BQ\theta + Q(1-\theta)\}^{k-m} \left(P_1^1 + c_1^1 - c_m^{\ell} \right)$$

Thus if (A4) holds for P_{k-1}^{ℓ} , (A3) implies that (A4) holds for P_k^{ℓ} .

We will simplify our expression one more time. (A4) implies that

$$P_N^N = \sum_{m=1}^N \{BQ\theta + Q(1-\theta)\}^{N-m} \left(P_1^1 + c_1^1 - c_m^N \right) .$$

Thus p_1^1 , which is unknown, can be written in terms of all known parameters:

$$p_1^1 = \frac{p_N^N - \sum_{m=1}^N (\{BQ\theta\} + Q(1-\theta))^{N-m} \left(c_1^1 - c_m^N \right)}{\sum_{m=1}^N (\{BQ\theta\} + Q(1-\theta))^{N-m}}$$

Therefore p_k^ℓ can be written in terms of all known parameters by inserting the above expression for p_1^1 in (A4) and rearranging terms:

$$p_k^\ell = \frac{\left\{ p_N^N - \sum_{m=1}^N Z^{N-m} \left(c_1^1 - c_m^N \right) \right\} \left\{ \sum_{m=1}^k Z^{k-m} \right\}}{\sum_{m=1}^N Z^{N-m}} + \sum_{m=1}^k Z^{k-m} \left(c_1^1 - c_m^\ell \right)$$

where $Z = (\{BQ\theta\} + Q(1-\theta))$.

However, if $Z < 1$,

$$\sum_{m=1}^k Z^{k-m} = \sum_{m=0}^{k-1} Z^m = \frac{1-Z^k}{1-Z}$$

Therefore

$$p_k^\ell = \left\{ \frac{1-Z^k}{1-Z^N} \right\} \left\{ p_N^N - \sum_{m=1}^N Z^{N-m} \left(c_1^1 - c_m^N \right) \right\} + \left\{ \sum_{m=1}^k Z^{k-m} \left(c_1^1 - c_m^\ell \right) \right\} \quad (A5)$$

Similarly, if $Z = 1$ (i.e., $\theta = 0$, $i=0$)

$$\sum_{m=1}^k Z^{k-m} = k.$$

Thus

$$p_k^\ell = \left\{ \frac{k}{N} \right\} \left\{ p_N^N - \sum_{m=1}^N \left(c_1^1 - c_m^N \right) \right\} + \left\{ \sum_{m=1}^k \left(c_1^1 - c_m^\ell \right) \right\} \quad (A5')$$

2. Prices of Current Asset When a Used Asset Market Does Not Exist.

Let

P_k = The purchase price of a used asset which is currently available and has k years of useful life remaining, $1 \leq k \leq N$. (P_k is assumed unknown)

P_N = The purchase price of a new asset which is currently available and has N years of useful life remaining (P_N is assumed known and does not change over time).

i = The appropriate discount rate for the firm.

Q = $(1+i)^{-1}$

Δ' = The probability of abandoning the asset's services within k years, conditional on the fact that a used asset with k years of life is purchased, $0 \leq \Delta' < 1$.

Δ = The probability of abandoning the asset's services within N years, conditional on the fact that a new asset with N years of useful life is purchased, $0 \leq \Delta < 1$.

Costs will not be introduced, nor will superscripts, since it will be assumed that the price of a new asset remains fixed over time. Formally, $P_N^{T+N} \equiv P_N$ for all $T \geq 0$.

There are only two possible purchase streams when a used asset market does not exist: initially "purchase" a used asset with k years of life remaining and buy new assets thereafter, or purchase a new asset initially

and buy new assets thereafter. If a used asset with k years of life remaining is purchased initially, the firm assesses Δ' as the probability that the asset's services will be terminated within k years, and $\Delta(1-\Delta')(1-\Delta)^{T-1}$ as the probability that the asset's services will be terminated between years $k+N(T-1)$ and $k+NT$, $1 \leq T$. However, if the firm purchases a new asset initially (and thereafter), it assesses the probability that the asset's services will be terminated between years $N(T-1)$ and NT , $1 \leq T$, as $\Delta(1-\Delta)^{T-1}$.

Equating the total expected cost of these two purchase streams enables us to derive prices:

$$P_k + \sum_{\ell=0}^{\infty} \sum_{j=0}^{\ell} P_N Q^{k+jN} \Delta(1-\Delta')(1-\Delta)^{\ell}$$

$$= \sum_{\ell=0}^{\infty} \sum_{j=0}^{\ell} P_N Q^{jN} \Delta(1-\Delta)^{\ell} .$$

Rearranging terms yields

$$P_k = [1 - Q^k(1-\Delta')] \sum_{\ell=0}^{\infty} \sum_{j=0}^{\ell} P_N Q^{jN} \Delta(1-\Delta)^{\ell}$$

Finally, recall that $\sum_{\ell=0}^{\infty} \sum_{j=0}^{\ell} Q^{jN} \Delta(1-\Delta)^{\ell} = \sum_{j=0}^{\infty} Q^{jN} \sum_{\ell=j}^{\infty} \Delta(1-\Delta)^{\ell} =$

$\sum_{j=0}^{\infty} Q^{jN} (1-\Delta)^j = [1 - Q^N (1-\Delta)]^{-1}$. Thus

$$P_k = \frac{[1 - Q^k (1-\Delta')]}{[1 - Q^N (1-\Delta)]} P_N \tag{A6}$$

Although (A6) was derived separately, we can show that it is equivalent to (A5) under appropriate assumptions:

- 1) $B=0$, which implies no used asset market
- 2) $c_{\ell}^{T+k} = c$, all costs are constant
- 3) $P_N^{T+N} = P_N^N \equiv P_N$ for all $T \geq 0$, the price of a new asset remains fixed over time.
- 4) $\Delta^k = \sum_{j=1}^k \theta (1-\theta)^{j-1}$
- 5) $\Delta = \sum_{j=1}^N \theta (1-\theta)^{j-1}$.

Under these assumptions (A5) implies

$$\begin{aligned}
 P_k &\equiv P_k^k = \frac{\{1 - (Q(1-\theta))^k\}}{\{1 - (Q(1-\theta))^N\}} P_N \\
 &= \frac{\{1 - Q^k (1-\theta)^k\}}{\{1 - Q^N (1-\theta)^N\}} P_N .
 \end{aligned}$$

But since by assumption

$$\begin{aligned}
 1 - \Delta^k &= 1 - \sum_{j=1}^k \theta (1-\theta)^{j-1} = 1 - \theta \sum_{j=1}^k (1-\theta)^{j-1} \\
 &= 1 - \frac{\theta [1 - (1-\theta)^k]}{1 - (1-\theta)} = (1-\theta)^k ,
 \end{aligned}$$

and similarly

$$1-\Delta = (1-\theta)^N ,$$

it follows that

$$P_k = \frac{[1-Q^k(1-\Delta)^k]}{[1-Q^N(1-\Delta)^N]} P_N .$$



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