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Estimation Biases in Discounted Cash Flow Analyses of Equity Capital Cost: A Pedagogical Note

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Capital Cost: A Pedagogical Note

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Abstract

The annual DCF models typically encountered in financial texts, rate hearings, and empirical financial research do not consider the time value of dividends. This note illustrates the bias arising from the conventional DCF analysis and presents a simple adjustment to the DCF model which eliminates the time value of dividend problem.

Estimation Biases in Discounted Cash Flow Analyses of Equity Capital Cost: A Pedagogical Note

The discounted cash flow (DCF) valuation model commonly found in finance textbooks, professional journals, public utility rate regulation testimony, and used by institutional investors underestimates a firm's cost of equity capital.[1] This underestimation of investor's required return depends on how closely the timing and amount of dividends expected to be paid by the firm match the assumptions inherent in the model. The underestimation can be particularly large when quarterly cash flows or dividends are forced, unadjusted, into an annual DCF framework. This bias is not trivial. It is sufficiently large that the revenue requirements of a large utility may be understated by several millions of dollars.[2] Institutional investors stock rankings based upon DCF expected returns may be altered. Also, DCF estimates of equity capital cost may be a source of bias in empirical financial research.

The purpose of this note is to illustrate this overlooked problem in the implementation of a DCF analysis of required return and to propose a simple solution to eliminate the systematic underestimation of equity capital cost.

THE PROBLEM

The DCF model envisions the value of an asset as being determined by the cash flows expected from the asset, and investors' required return which is determined by the time value of money and the required risk premium. Thus, for common stock, the value or price today is the present value of all future dividends expected, including any liquidating dividend or sale price. That is,

$$P_0 = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \ldots + \frac{D_\infty}{(1+k)^\infty} = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t}, \qquad (1)$$

or

$$P_0 = \frac{D_1}{k-g} \qquad (2)$$

where D_t is the dividend paid at the end of period t, k is the required rate of return of investors or the cost of equity capital, P_0 is the current price of the stock, and g is a constant expected dividend growth rate. These equations describe a generalized DCF model that may be used to analyze any periodic (annual, quarterly, monthly, etc.) cash flow. Solution of equations (1) or (2) will result in the correct annual (quarterly) cost of equity capital for firms which pay dividends annually (quarterly).

Problems arise when using the annual version of the model unless recognition is given to the fact that the quarterly dividends have an opportunity cost. Most firms pay dividends quarterly, and the price of the stock reflects both the timing and amount of the dividends. The typical application of the annual DCF model ignores the time value of dividends. Quarterly versions of equations (1) and (2) resolve the time value of dividends problem, but create a new problem related to the size of the dividends.[3]

PROBLEMS WITH THE ANNUAL GROWTH MODEL

DCF analyses of stock values should give recognition to the fact that firms commonly pay dividends quarterly and that, barring financial crises, firms change the quarterly dividend rate only periodically--generally at the beginning of the fiscal year. It is shown below that failure to adjust the quarterly dividend for the time value of money will cause the DCF estimate of the cost of equity capital to be understated.

Consider, for example, a firm that paid a \$.9432 annual dividend (quarterly dividends of \$.2358 per share) during the fiscal year just ended. Dividends are expected to increase 6.0 percent per annum or to \$.25 per share each quarter in the next fiscal year. The share price is \$8.00 The time configuration of the expected dividends is presented in Table 1. The implied annual dividends associated with the equation (1) and (2) annual models are also shown. The typical cost of equity capital estimate using the annual mode of equations (1) or (2) is 18.5 percent,

$$\$8.00 = \frac{4(\$.25)}{(1+.185)} + \frac{4[(\$.25)(1+.06)]}{(1+.185)^2} + \ldots + \frac{4[(\$.25)(1+.06)^{\infty}]}{(1+.185)^{\infty}}$$

or

$$k = \frac{\$1.00}{\$8.00} + .06 = .185 = 18.5\%$$

This formulation is correct <u>only</u> if the entire annual dividend is paid at year end as shown in the second row of Table 1. But the present value of four quarterly dividends is greater than the present value of one year-end dividend. Indeed, the cost of equity capital is 19.375 percent when the timing and amount of dividends embodied in the market price of the stock are considered. That is, 19.375 percent is the iterative solution to

$$\$8.00 = \sum_{Q=1}^{4} \frac{\$.25}{(1+.19375)^{.25Q}} + \sum_{Q=1}^{4} \frac{\$.25(1.06)}{(1+.19375)^{1+.25Q}} + \ldots$$

$$= \sum_{t=0}^{\infty} \sum_{Q=1}^{4} \frac{\$.25(1+.06)^{t}}{(1+.19375)^{t+.25Q}}$$

The same equity cost estimate is obtained from the reduced form equation (2) DCF annual model if the D_1 measure is adusted for the time value of dividends. The D_1 value called for in the reduced form annual model is \$1.06998 [$\$1.06998 = \sum_{Q=1}^{4} \$.25(1 + .19375)^{1-.25Q}$] assuming a 19.375 percent opportunity cost to shareholders. The cost of equity after adjusting for the time value of dividends is

$$k = \frac{1.06998}{\$8.00} + .06 = .19375 \text{ or } 19.375\%.$$

Hence, the customary use of the annual DCF growth model understates the cost of equity capital for this firm by 88 basis points [19.375% - 18.50% = 0.875%] because the time value of money associated with the quarterly dividends and embodied in the market price of the stock is ignored.

PROBLEMS WITH THE QUARTERLY GROWTH MODEL

As indicated above, one method of considering the timing of the quarterly dividends is to use the equation (1) model in a quarterly mode. This formulation eliminates the time value of money problem associated with the unadjusted annual growth model. Unfortunately, common usage of a quarterly DCF model introduces a dividend bias since quarterly DCF models typically are formulated as;

$$P_0 = \sum_{Q=1}^{\infty} \frac{D_{Q-1}(1+g_Q)}{(1+k_q)^Q}, \tag{3}$$

or

$$P_0 = \frac{D_1}{k_Q-g_Q} = \frac{D_0(1+g_Q)}{k_Q-g_Q} \tag{4}$$

These formulations assume dividends are increased quarterly rather than periodically (annually). Thus, the quarterly dividend model correctly handles the time value of dividends but the quarterly dividend growth causes the cost of equity capital to be understated or overstated.

The data in Table 1 indicate clearly the reason for the bias in the quarterly model's equity cost estimates. The bottom rows of Table 1 present the implied quarterly dividends associated with a six percent annual dividend growth rate. The dividend stream denoted Q_1 assumes the analysis occurs at t=0 or fiscal year end; stream Q_2 assumes the analysis is made after the first quarterly dividend, etc. The top row of Table 1 shows the quarterly dividends actually expected. The discrepancy between the expected quarterly dividends (top row) and the dividends implied by the quarterly growth model (bottom rows) depends upon when the DCF analysis is made relative to the fiscal year dividend policy change. For example, if the analysis is made immediately following the fiscal year-end, t_0, the implied quarterly dividend is <u>less</u> than the actual dividend in three of four quarters. However, if the analysis is made at the end of the first quarter, the implied quarterly dividend will be <u>greater</u> than the expected dividend in three of the four quarters. Similar discrepancies occur if the analysis is performed at the end of Q_2 or Q_3.

The present value of the implied dividend stream will be greater or less than the present value of the expected dividend stream. As can be seen in Table 2, two of the quarterly equity cost estimates are above the correct 19.375 percent cost, and two are below. The largest bias using this quarterly model is approximately 32 basis points. This is substantially less than the 88 basis point bias of the unadjusted annual model because the quarter model correctly considers the time value of dividends. The bias remaining is caused by assuming that dividends increase quarterly rather than annually. A simple model is presented in the next section that properly considers both the timing and amount of dividends.

A PROPOSED SOLUTION

Investors are fully aware of the quarterly payment schedule of dividends. Thus, the price, P_0, reflects the timing of the dividends as well as the amount of the dividends. If $(D_{t-1,Q1})$, $(D_{t-1,Q2})$, $(D_{t-1,Q3})$, and $(D_{t-1,Q4})$ represent the quarterly dividend payments at the end of the quarters in the year preceding the (t_0) date of analysis, and dividends are expected to grow at an annual rate g, then P_0 can be written as

$$P_0 = \frac{(D_{t-1,Q1})(1+g)}{(1+k)^{.25}} + \frac{(D_{t-1,Q2})(1+g)}{(1+k)^{.50}} + \frac{(D_{t-1,Q3})(1+g)}{(1+k)^{.75}} + \frac{(D_{t-1,Q4})(1+g)}{(1+k)} + \sum_{t=1}^{\infty}\sum_{Q=1}^{4} \frac{D_{t,Q}(1+g)}{(1+k)^{t+.25Q}} \quad (5)$$

or

$$P_0 = \frac{(D_{t-1,Q1})(1+g)}{(1+k)^{.25}} + \frac{(D_{t-1,Q2})(1+g)}{(1+k)^{.50}} + \frac{(D_{t-1,Q3})(1+g)}{(1+k)^{.75}} + \frac{(D_{t-1,Q4})(1+g)}{(1+k)} + \frac{P_0(1+g)}{(1+k)} \quad (5a)$$

The last term of equation (5) represents the present value of all the future dividends in years 2...∞, or the price of the stock at the end of year one (P_1) which also can be expressed as $P_1 = P_0(1+g)$ as shown in equation (5a). The equation (5) formulation considers correctly both the timing and the amount of dividends. The discounting to present value reflects the fact that dividends are paid quarterly. The dividend in year t, quarter Qx is estimated as $[D_{t,Qx} = (D_{t-1,Qx})(1+g)]$ which gives recognition to the fact that dividends follow a (fiscal year) step function process.

Because it is both customary and convenient to think in terms of annual and not quarterly returns and growth rates, the equation (5) model is reformulated below as a simple annual model which correctly considers the timing and amount of dividends. Multiplying both sides of equation (5a) by (1+k) yields

$$(1+k)P_0 = (D_{t-1,Q1})(1+g)(1+k)^{.75} + (D_{t-1,Q2})(1+g)(1+k)^{.50}$$
$$(D_{t-1,Q3})(1+g)(1+k)^{.25} + (D_{t-1,Q4})(1+g) + P_0(1+g). \qquad (6)$$

Letting $[D_{t,Q} = (D_{t-1,Q})(1+g)]$ and rearranging terms in equation (6) reveals

$$k = \frac{(D_{t,Q1})(1+k)^{.75} + (D_{t,Q2})(1+k)^{.50} + (D_{t,Q3})(1+k)^{.25} + (D_{t,Q4})}{P_0} + g. \qquad (7)$$

Equation (7) shows that the DCF model expressed in an annual mode must include a "time value of money adjustment to dividends" when applied to the real world where dividends are paid quarterly rather than once a year.[4] Applying the equation (7) annual model to the firm discussed earlier shows investors' required rate of return is correctly assessed as 19.375 percent,[5]

$$.19375 = \frac{\$.25(1+.19375)^{.75} + \$.25(1+.19375)^{.50} + \$.25(1+.19375)^{.25} + \$.25}{\$8.00} + .$$

or

$$.19375 = \frac{\$1.070}{\$8.00} + .06$$

when quarterly dividends are adjusted to reflect the time value of money. This adjustment raises the estimate of the example firm's cost of equity some 88 bases points or from 18.50% to 19.375 percent. Thus the "time value of money adjustment to dividends" is not trivial.

The annual DCF models typically encountered in financial texts, rate hearings, and empirical financial research do not consider the time value of dividends. This note illustrates the bias arising from the conventional DCF analysis and presents a simple adjustment to the DCF model which eliminates the time value of dividend problem.

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TABLE 1

Expected Dividends Versus the Dividends Implied by the Annual and Quarterly Growth Models (annual growth rate = 6%; quarterly growth rate = 1.46738%)

	t_0 Fiscal Year End	Fiscal Year t=1 Dividend at End of...				Fiscal Year t=2 Dividend at End of ...			
		Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
Expected Quarterly Dividends	$.2358[a]	$.25	$.25	$.25	$.25	$.265	$.265	$.265	$.265
Implied Annual Dividends[b]	$.9432				$1.00				$1.06
Quarterly Model Implied Dividends (Q1)	$.2358[a]	$.239	$.243	$.246	$.250				
(Q2)		$.25[a]	$.254	$.257	$.261	$.265			
(Q3)			$.25[a]	$.254	$.257	$.261	$.265		
(Q4)				$.25[a]	$.254	$.257	$.261	$.265	

[a]Actual dividend in quarter preceding analysis.

[b]Total annual dividend (4 x Quarterly Dividend).

TABLE 2

Implied Dividends and Equity Capital Cost Estimates Based on Quarterly DCF Model

Time of Analysis	Stock Price At Time of Analysis[a] (P_0)	Dividends Over Next 4 Quarters Implied By Quarterly Dividend Growth Model[b]				Stock Price at End of 4th Quarter[a] (P_1)	Estimated k $P_0 = \sum_{Q=1}^{4} \frac{D_Q}{(1+k)^{.25Q}} + \frac{P_1}{(1+k)}$	Actual k (19.375%) Versus Estimated k
t_0	$8.0000	$.2393	$.2428	$.2464	$.2500	$8.4800	19.05%	-.325%
Q_1	8.1174	.2537	.2574	.2612	.2650	8.6044	19.67%	+.295%
Q_2	8.2365	.2537	.2574	.2612	.2650	8.7307	19.47%	+.095%
Q_3	8.3574	.2537	.2574	.2612	.2650	8.8588	19.24%	-.135%
Q_4	8.4800	.2537	.2574	.2612	.2650	8.9885	19.05%	-.325%

[a]The price of the stock increases at the six percent dividend growth rate or 1.4674 percent per quarter [$.014674 = (1+.06)^{.25}-1$].

[b]D_Q = Dividend in Quarter Preceding Analysis $\times (1+.06)^{.25Q}$

Footnotes

[1] See [1, Chapter 15; and 9, Chapter 8] for the standard textbook DCF treatment. Examples of empirical research using annual growth estimates and/or annual dividend values include [2, 3, 6].

[2] Most DCF analyses presented in rate regulatory hearings fail to recognize this bias. However, in recent years several academic rate of return witnesses have recognized this source of estimation bias in a DCF analysis. For example, see [4, 5, 7, 8].

[3] Some analysts (particularly intervenors in utility rate hearings) advocate the use of the continuous compounding DCF model [$k = (D_0/P_0) + g$] to estimate a firm's cost of equity capital. The continuous compounding model is not appropriate for estimating k since this model is derived under the assumption that the future stream of dividends is received continuously. Estimates of the cost of equity capital based upon the continuous compounding DCF model are biased downward.

[4] The mathematical complexity of estimating k via equation (7) can be reduced substantially by approximating the k in the numerator of the expression in brackets using $k = [4(D_{Q1,t})/P] + g$. This approximation technique causes k to be understated slightly. Additional iterations can determine the exact required return.

[5] Equation (7) is developed under the assumption that the analysis date occurs immediately after a dividend payment. Given quarterly dividend payments, the time periods for which time value of dividend adjustments are required are .75 year, .50 year, .25 year, and .00 year. A different set of time periods would be involved if the analysis occurred between dividend payment dates.

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