BEBR

FACULTY WORKING
PAPER NO. 740

An Evaluation of the Distributional and Causal Relationships Between the Stock and Commodity Futures Market Indices

Cheng F. Lee, Raymond M. Leuthold, and Jean E. Cordier

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

FACULTY WORKING PAPER NO. 740

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

January 1981

An Evaluation of the Distributional and Causal Relationships Between the Stock and Commodity Futures Market Indices

Cheng F. Lee, Professor
Department of Finance

Raymond M. Leuthold, Professor
Department of Agricultural Economics

Jean E. Cordier, Graduate Student
Department of Agricultural Economics

Acknowledgment: The authors thank Richard W. McEnally for critical review and stimulating comments on an earlier draft.

The purpose of this paper is to estimate the distributional and causal relationships between the stock and commodity-futures market indices. Three major findings are: (1) the first three moments of the rates of return for both indices are generally not independent of the investment horizon, (2) empirical results from regression and parametric time-series technique have shown that virtually no relationship exists between the rates of return for the two indices, and (3) inclusion of commodity future contracts in an equity portfolio has a strong opportunity to reduce the risks and enhance the performance of the portfolio.

An Evaluation of the Distributional and Causal Relationships Between the Stock and Commodity Futures Market Indices

I. Introduction

Security analysts and portfolio managers have in the past devoted much time to studying the behavior of the stock market, and more recently they have become interested in the behavior of the commodity futures market. Generally, these studies utilize a market price index to indicate the overall behavior of a market. An examination of the relationships between the commodity futures market index and stock market index would be of interest to both security analysts and portfolio managers in determining the appropriate combination of funds to invest in each market.

The purpose of this paper is to evaluate the distributional and causal relationships between the stock and commodity futures market indices. In the second section the distributional characteristics of both the stock market index and futures market index are investigated and compared in terms of 22 different investment horizons. In the third section regression relationships in terms of the market model as developed by Sharpe (1963) are used to investigate the causal relationship between future market rates of return and stock market rates of return. Impacts of investment horizon on this kind of causal relationship analysis are also explored. A time-series analysis is performed in the fourth section to investigate the lead-lag relationship between the stock market index and futures market index in terms of the univariate residual cross-correlation technique. In the fifth section the implications of this study are explored and results of the paper are summarized.

II. Distributional Characteristics of the Stock Market Index and the Futures Market Index over 22 Horizons

The daily stock index used in this paper is the Standard and Poor Composite Index of 500 industrial common stocks. The daily commodity futures index is based on 27 commodities and is constructed by the Commodity Research Bureau, Inc. The sample period is January 1, 1972 through December 31, 1977.[1]

The distributional characteristics of an index can be described by the first four moments. Impacts of investment horizon on the moments of the distribution of the stock market rates of return have been investigated by Brenner (1974), Fogler and Radcliff (1974), and Lee (1976). Other empirical investigations of the impact of investment horizon on estimated expected rates of return for common stocks have been done by Cheng and Deets (1973), Levhari and Levy (1977), and Lee and Morimune (1978). Similar analyses have not been performed on commodity futures market data. Here, the first four moments in terms of daily data for the stock market index (S&P) and the commodity futures market index (CFI) are calculated and analyzed over 22 horizons. The rates of return are computed assuming one is on the "long" side of the market.

The relative skewness and kurtosis are defined as:

$$\text{skewness } (g_1) = \frac{\Sigma(X-\bar{X})^3/n}{[\Sigma(X-\bar{X})^2/n]^{3/2}} \tag{1}$$

$$\text{kurtosis } (g_2) = \frac{\Sigma(X-\bar{X})^4/n}{[\Sigma(X-\bar{X})^2/n]^2} - 3 \tag{2}$$

Following Snedecor and Cochran (1956), the standard errors used to test the significance of g_1 and g_2 are defined as:

$$Sg_1 = [6n(n-1)/(n-2)(n+1)(n+3)]^{1/2} \tag{3}$$

$$Sg_2 = [24n(n-1)^2/(n-3)(n-2)(n+3)(n+5)]^{1/2} \tag{4}$$

where:

Sg_1 = the standard error for g_1,

Sg_2 = the standard error for g_2, and

n = the sample size.

Equations (1-4) can be used to test the degree of symmetry and the degree of normality for a time series. The first four moments of percentage returns from the CFI and S&P index are listed in Tables 1 and 2. Each statistic is calculated for all horizons from 1 to 22 in order to investigate in detail the impact of horizon on the four measures. The 22-day horizon approximates one month in trading days and is selected as the limit.

From the tables it is found that the standard deviation for the CFI rates of return are all similar to those for the S&P rates of return. However, the average rates of return for CFI are always higher than those for S&P. This means that the futures market has outperformed the stock market with higher returns at comparable levels of risks over the sample period analyzed.[2]

It is well known that both relative skewness and relative kurtosis are important statistics in the generating process for rates of return.[3] Utilizing Equations (1-4), standard t-tests can be used to determine if the third and fourth moments are significantly different from zero.

Table 1. Descriptive Rate of Return Statistics on the Commodity Futures Index

Arithmetic or Logarithmic	Horizon (Days)	Mean Return	Standard Deviation	Skewness	Kurtosis
Arithmetic	1	.00048	.00922	-.00312	.87387*
Logarithmic	1	.00044	.00921	-.04281	.86979*
Arithmetic	2	.00097	.01350	-.09602	.98391*
Logarithmic	2	.00088	.01350	-.15660	1.02293*
Arithmetic	3	.00145	.01626	-.07602	1.47243*
Logarithmic	3	.00132	.01625	-.16068	1.49834*
Arithmetic	4	.00191	.01831	.12001	.93593*
Logarithmic	4	.00174	.01827	.04019	.90076*
Arithmetic	5	.00242	.02189	.25195	.99445*
Logarithmic	5	.00218	.02179	.15702	.90092*
Arithmetic	6	.00291	.02428	.29518*	1.10883*
Logarithmic	6	.00261	.02415	.18564	1.04814*
Arithmetic	7	.00341	.02674	.35490*	1.34737*
Logarithmic	7	.00305	.02655	.22704	1.24212*
Arithmetic	8	.00383	.02725	.47544*	1.57408*
Logarithmic	8	.00346	.02701	.34061*	1.41756*
Arithmetic	9	.00431	.02973	.18233	1.93687*
Logarithmic	9	.00387	.02958	.00917	1.88856*
Arithmetic	10	.00498	.03366	.92030*	3.60634*
Logarithmic	10	.00442	.03307	.68999*	3.03296*
Arithmetic	11	.00534	.03269	.01848	.21136
Logarithmic	11	.00480	.03255	-.09060	.23557
Arithmetic	12	.00586	.03541	-.28138	.85024*
Logarithmic	12	.00522	.03546	-.43436*	1.12205*
Arithmetic	13	.00644	.03865	.39445	1.23765*
Logarithmic	13	.00570	.03821	.21946	1.06356*
Arithmetic	14	.00700	.04301	.60348*	2.37921*
Logarithmic	14	.00609	.04233	.34924	2.10891*
Arithmetic	15	.00754	.04465	.28244	2.23402*
Logarithmic	15	.00654	.04423	.00269	2.24823*
Arithmetic	16	.00779	.04075	.46124*	.03209
Logarithmic	16	.00696	.04013	.35022	-.04767
Arithmetic	17	.00859	.04771	.44039*	1.75309*
Logarithmic	17	.00746	.04701	.19748	1.37873*
Arithmetic	18	.00900	.04513	.36907	.46066
Logarithmic	18	.00798	.04448	.21548	.29821
Arithmetic	19	.00945	.04776	.79613*	.76064
Logarithmic	19	.00833	.04656	.65332*	.38624
Arithmetic	20	.01001	.04947	.34662	1.47065*
Logarithmic	20	.00879	.04878	.10800	1.17776*
Arithmetic	21	.01047	.05071	.16421	.48557
Logarithmic	21	.00918	.05016	-.01771	.34930
Arithmetic	22	.01114	.05227	.67443*	.45586
Logarithmic	22	.00979	.05096	.52164	.23219

*Significantly different from zero at the 95 percent level of confidence.

Table 2. Descriptive Rate of Return Statistics on the Standard and Poor Stock Index

Arithmetic or Logarithmic	Horizon (Days)	Mean Return	Standard Deviation	Skewness	Kurtosis
Arithmetic	1	-.00000	.00908	.24941*	1.80911*
Logarithmic	1	-.00004	.00907	.19885	1.73499*
Arithmetic	2	.00001	.01434	.07198	1.16126*
Logarithmic	2	-.00009	.01434	.00394	1.16638*
Arithmetic	3	.00002	.01779	.17259	1.84945*
Logarithmic	3	-.00014	.01778	.07372	1.63310*
Arithmetic	4	.00001	.02106	.07639	1.52164*
Logarithmic	4	-.00022	.02106	-.03229	1.38393*
Arithmetic	5	.00005	.02536	.09514	1.59151*
Logarithmic	5	-.00027	.02536	-.03629	1.36398*
Arithmetic	6	-.00000	.02547	.47584*	3.52461*
Logarithmic	6	-.00032	.02536	.28676*	2.82738*
Arithmetic	7	-.00007	.02723	.11676	1.19606*
Logarithmic	7	-.00044	.02723	-.01281	1.15429*
Arithmetic	8	-.00002	.02953	.20256	1.35553*
Logarithmic	8	-.00045	.02949	.05881	1.19908*
Arithmetic	9	-.00004	.02965	.43946*	1.92588*
Logarithmic	9	-.00047	.02951	.28078	1.55201*
Arithmetic	10	.00001	.03501	-.05783	2.26143*
Logarithmic	10	-.00060	.03514	-.27886	2.21771*
Arithmetic	11	-.00004	.03135	.09828	.46065
Logarithmic	11	-.00053	.03135	-.01735	.44276
Arithmetic	12	-.00017	.03346	-.02920	-.29563
Logarithmic	12	-.00073	.03368	-.37448	-.24236
Arithmetic	13	.00023	.03849	.25672	2.74160*
Logarithmic	13	-.00050	.03843	-.00651	2.47752*
Arithmetic	14	.00003	.03713	-.05241	.16293
Logarithmic	14	-.00066	.03723	-.17401	.19058
Arithmetic	15	.00013	.04136	-.17014	.95456*
Logarithmic	15	-.00072	.04162	-.34911	.92057*
Arithmetic	16	-.00014	.04098	-.17174	.66303
Logarithmic	16	-.00098	.04125	-.33664	.77235
Arithmetic	17	.00034	.04462	-.11579	.67893
Logarithmic	17	-.00065	.04487	-.29527	.72361
Arithmetic	18	.00012	.04292	-.34513	.48800
Logarithmic	18	-.00081	.04337	-.51532*	.79409
Arithmetic	19	.00027	.04481	-.09677	.38256
Logarithmic	19	-.00073	.04503	-.26265	.54651
Arithmetic	20	.00023	.04333	-.12169	1.19901*
Logarithmic	20	-.00071	.04360	-.34019	1.53172*
Arithmetic	21	.00014	.05329	-.21290	1.22500*
Logarithmic	21	-.00128	.05388	-.46878	1.33961*
Arithmetic	22	.00016	.04599	.51843*	1.17372*
Logarithmic	22	-.00087	.04559	.31812	1.01042*

*Significantly different from zero at the 95 percent level of confidence.

First, the results show that the logarithmic transformation generally reduces positive skewness and increases negative skewness. The logarithmic transformation generally does not affect kurtosis. Secondly, based on discrete rates of return, the CFI has significant positive skewness for 6, 7, 8, 10, 14, 16, 17, 19 and 22-day horizons. S&P has significant positive skewness for 1, 6, 9, and 22-day horizons. Based on continuous rates of return, significant negative skewness exists for CFI at the 12-day horizon and for S&P at the 18-day horizon. These results demonstrate that the rates of return for CFI have more positive skewness than S&P rates of return, and this positive skewness occurs beyond the 5-day horizon. Finance theory suggests that investors prefer return and positive skewness and dislike risk and negative skewness. Again, this provides some evidence that futures performed better than stocks over the time period analyzed.

Finally, the tables show that relative kurtosis for both indexes is mostly significant, especially for horizons of 10 days or less. The implications of relative kurtosis in determining the performance of investments are still not clear. In the data analyzed here, the rates of return for the two indexes are generally not normally distributed.

III. Relationship Between the Stock Market Index and the Futures Market Index

As a further investigation of the relationship between the two indexes, the CFI is regressed on the S&P index to test for the existence of systematic risk in the CFI. The equation is:

$$R_{ct} = \alpha_j + \beta_j R_{mt} + \varepsilon_{jt} \tag{5}$$

where:

R_{ct} = rates of return for CFI,

R_{mt} = rates of return for S&P,

j = 1, ..., 22.

This model relates the percentage return of the CFI to the percentage return on S&P. The larger the β, the greater the systematic (nondiversifiable) risk. Systematic risk is the portion of total risk which hinders rather than helps diversification, meaning investors would require more return to induce them to include commodity futures in a portfolio (if β is large) since futures would not eliminate risks through diversification. A small β indicates primarily unsystematic (diversifiable) risk, or risks caused by factors peculiar to that particular investment.

The regression results for each of 22 horizons are shown in Table 3. The β coefficient is significantly different from zero only for the 12-day horizon, where the coefficient is negative. That is, there is little to no relationship, or systematic risk, between the two indexes.[4] These results imply that commodities in the CFI can be included in an equity portfolio to reduce risk and improve performance of the portfolio, regardless of horizon. Futures contracts as a whole have no systematic risk relative to stocks, and would serve to provide diversification within a portfolio composed of stocks.

The coefficient of variation measures the magnitude of the risk relative to the average level of returns. In order to test whether the

Table 3. β Coefficient from Regressing Commodity Futures Index on Stock Index over 22 Horizons

Horizon (Days)	Arithmetic	Logrithmic
1	.02346 (.02618)[a]	.02398 (.02620)
2	.01729 (.03438)	.01722 (.03439)
3	.00680 (.04090)	.00597 (.04092)
4	-.01695 (.04502)	-.01751 (.04491)
5	-.01233 (.05000)	-.01555 (.04978)
6	-.01261 (.06052)	-.01385 (.06046)
7	.07265 (.06724)	.07144 (.06680)
8	-.03491 (.06780)	-.03968 (.06728)
9	.02278 (.07828)	.01853 (.07826)
10	-.08071 (.07902)	-.08287 (.07732)
11	-.10044 (.09000)	-.09666 (.08966)
12	-.18874* (.09427)	-.18932* (.09377)
13	-.00982 (.09487)	-.00877 (.09394)
14	.02091 (.11357)	.01368 (.11147)
15	-.06781 (.10940)	-.06855 (.10768)

Table 3. (continued)

Horizon (Days)	Arithmetic	Logrithmic
16	-.11442 (.10353)	-.10945 (.10135)
17	.03702 (.11589)	.03135 (.11359)
18	-.08141 (.11720)	-.08550 (.11426)
19	-.19662 (.12017)	-.19332 (.11652)
20	-.04084 (.13445)	-.03520 (.13179)
21	.01932 (.11536)	.02425 (.11285)
22	-.23524 (.13792)	-.23051 (.13567)

[a]The standard error is in parenthesis.

*Significantly different from zero at the 95 percent level of confidence.

coefficient of variation for each index is independent of the time horizon, the following model is examined:

$$CV_T = a + bT \tag{6}$$

where:

CV = coefficient of variation,

T = time, 1, ..., 22.

The results of Equation (6) in Table 4 show that the coefficient of variation is in general not independent of the investment horizon. That is, the longer the horizon, the more (less if negative sign) relative risk is assumed. Thus, the selection of horizon is important.

In similar tests, the mean rates of return and the standard deviation of returns for the two indices are also significantly related to horizon in both the arithmetric and logarithmic cases. The skewness of the rates of return for the S&P index when regressed against investment horizon is negative and significant only in the logarithmic case, and skewness of CFI rates of return is positive and significantly related to investment horizon in both arithmetic and logarithmic instances. The only kurtosis measure not independent of horizon is the one associated with arithmetic S&P rates of return where the relationship is negative. Beedles (1979) found there exists some skewness for stock market rates of return in both logarithmic and arithmetic cases, but he did not investigate the impact of horizon. Brenner (1974) used the stable distribution concept to investigate the impact of investment

Table 4. Slope Coefficients from Regressing the Coefficient of Variation Against Time ($CV_T = a + bT$)

	S&P Index	CFI
Arithmetic	-36.913 (40.370)[a]	-.431* (.070)
Logarithmic	4.722* (1.053)	-.496* (.087)

[a]The standard error is in parenthesis.

*The coefficient is significantly different from zero at the 95 percent level of confidence.

horizon on the distribution of stock market rates of return and found them not to be independent of each other. Those results are consistent with ours where both the S&P and CFI indexes are analyzed.

IV. The Lead-Lag Relationship Between the Stock Market Index and Commodity Futures Market Index

The general purpose of the univariate residual cross-correlation analysis is to determine how two time series are related to each other. It is a useful tool to determine links of causality between two series, by exhibiting a lead-lag relationship from one series X_t to another series Y_t.

We might expect that the link of causality between S&P and CFI, called X_t and Y_t, respectively, would be revealed through their sample cross-correlations:

$$r_{xy}(k) = \frac{\Sigma(X_{t-k}-\bar{X})(Y_t-\bar{Y})}{[\Sigma(X_t-\bar{X})^2\Sigma(Y_t-\bar{Y})^2]^{1/2}} . \tag{7}$$

Alternatively, we might consider regressing Y_t on past and present X_t, or vice versa, and performing an F test on the appropriate set of regression coefficients.

However, in practice, both of these procedures (correlation and regression) can be misleading if the autocorrelation in the series is not properly taken into account. Ignoring the autocorrelation results in overestimating the significance of the tests and asserting relationships that do not exist. Granger and Newbold (1974), in a discussion of spurious regressions, emphasize the adverse implications of autocorrelation.

The solution proposed by Haugh (1976) and Pierce (1977) is to model the univariate series and then to analyze the relationship of the residuals. Pierce (1977, p. 14) specifies: "Intuitively, X_t causes Y_t only if <u>after</u> explaining whatever of Y_t that can be explained on the basis of its own past history, $\overline{Y}_t$, some more remains to be explained by X_s, $s < t$, i.e., by $\overline{X}_t$. This suggests relating X_t to that part of Y_t which cannot be explained by $\overline{Y}_t$. But this is exactly the innovation [meaning residual] v_t in the univariate time series model of Y_t....Similarly, to assess causality from Y_t to X_t we would whiten X_t, according to its univariate model."

<u>Modeling of the univariate series</u>. This first stage of the analysis consists of whitening (filtering the variable X_t in order to derive a residual u_t which is moving randomly) the series using the Box and Jenkins technique.

The general form of the model is described as follows:

$$\phi(B)\nabla^d X_t = \Theta(B)u_t$$

where: B is called a backward shift operator defined as $B^j X_t = X_{t-j}$,

$\phi(B)$ is a polynomial expression of B, of degree p, where

$$\phi(B) = 1 + \phi_1 B^1 + \phi_2 B^2 + \ldots + \phi_p B^p,$$

ϕ_j are the weights or parameters of the autoregressive AR(p) process,

∇ is a backward difference operator such that

$\nabla^d X_t = X_t - X_{t-d}$,

$\Theta(B)$ is a polynomial expression of B, of degree q, where

$\Theta(B) = 1 + \Theta_1 B^1 + \Theta_2 B^2 + \ldots + \Theta_q B^q$, and

Θ_i are the parameters of the moving average MA(q) process.

When a time series needs to be whitened by a combined use of an autoregressive process of order p, successive differencing of order d and a moving average process of order q, the series is said to follow a mixed Autoregressive Integrated Moving Average process of order (p, d, q), denoted ARIMA (p, d, q).

The whitening of the time series using the Box and Jenkins technique involves three steps: model selection, estimation, and diagnostic check. Model selection is designed to recognize the type of process exhibited by the series. This is done by looking at the estimated autocorrelation and partial autocorrelation functions for different lags of the series.

Estimation of the different parameters ϕ_j and Θ_i is then performed. The computer performs an iterative search using a least squares technique to explain the series.[5] Finally, a diagnostic check is performed using the residuals $\hat{u}_t$ of the series. If the residuals do not represent a white noise (random) sequence, the ARIMA model must be modified with a new hypothesis on the degrees of p, d and q.

For this section of the paper, the two series were expanded to 1968 through 1977, and each calendar year of data was examined individually rather than as a single series. The extension of both series

back to 1968 for this analysis gives us some opportunity to examine whether the relationship between CFI and S&P has changed over time in response to structural changes in the U.S. economy in the early 1970s caused by events which have produced significantly higher energy and food prices. The application of the Box-Jenkins technique on the daily S&P and CFI indexes provided models expressed in Table 5. All of the models contain autoregressive processes, and all but one are expressed in first differences. No model contains a moving average process.

The linear lead-lag relationship between the two series. Suppose the two series X_t and Y_t are described by the following models:

$$\hat{u}_t = F(B)X_t \tag{8}$$

$$\hat{v}_t = G(B)Y_t \tag{9}$$

The $\hat{u}_t$ and $\hat{v}_t$ are by definition constructed free from autocorrelation, so that the defects noted above in the use of correlation procedures on the original series should now be removed. Thus, following Haugh and Box (1977), the cross-correlation between the u's and v's defined at lag k as:

$$\rho_{uv}(k) = \frac{E(u_{t-k}, v_t)}{[E(u_t^2)E(v_t^2)]^{1/2}} \tag{10}$$

may be used to assess lead-lag relationship between X_t and Y_t. Some linear causal relationship of interest are shown in Table 6.

The u's and v's of Equation (10) are not observable. However, their estimates, $\hat{u}_t$ and $\hat{v}_t$ are fitted in Equations (8-9).

Once the white noise residuals are obtained for each original time series, statistical tests of the significance of the calculated

Table 5. Results of Box-Jenkins Analysis on S&P and CFI Indexes, 1968-1977

Year	Index	AR Process[a]	Differences	MA Process	Coefficients[b]		
1968	CFI	1	0	0	.99		
	S&P	1,3	1	0	.24	.16	
1969	CFI	1	1	0	-.18		
	S&P	1,2	1	0	.38	-.11	
1970	CFI	1	1	0	-.20		
	S&P	1,2,6	1	0	.38	-.08	-.12
1971	CFI	1	1	0	-.03		
	S&P	1	1	0	.26		
1972	CFI	1,2,4	1	0	.26	-.09	.20
	S&P	1	1	0	.29		
1973	CFI	1,2,4	1	0	.20	-.18	.16
	S&P	1,2,5	1	0	.26	-.11	-.12
1974	CFI	1,2,4	1	0	.01	-.14	.16
	S&P	1,2	1	0	.31	-.09	
1975	CFI	1	1	0	.03		
	S&P	1,2,5	1	0	.28	-.16	-.00
1976	CFI	1	1	0	-.06		
	S&P	1	1	0	.14		
1977	CFI	1	1	0	.12		
	S&P	1,6	1	0	.20	-.15	

[a]The numbers indicate the specific autoregressive processes in the model.

[b]The coefficients correspond to the specific autoregressive element in the model.

Table 6. Conditions on Cross-correlations of Whitened Series for Causality Patterns

Relationship	Cross-correlations at lag k
1. X leads Y	$\rho_{u,v}(k) \neq 0$ for some $k > 0$
2. Y leads X	$\rho_{u,v}(k) \neq 0$ for some $k < 0$
3. X and Y are instantly related	$\rho_{u,v}(0) \neq 0$
4. Feedback between X and Y	$\rho_{u,v}(k) \neq 0$ for some $k > 0$ and for some $k < 0$
5. Y does not lead X	$\rho_{u,v}(k) = 0$ for all $k < 0$
6. X does not lead Y	$\rho_{u,v}(k) = 0$ for all $k > 0$
7. X leads Y, no feedback from Y to X	$\rho_{u,v}(k) \neq 0$ for some $k > 0$ and $\rho_{u,v}(k) = 0$ for all $k < 0$
8. X and Y are related instantly but in no other way	$\rho_{u,v}(k) = 0$ for all $k \neq 0$ and $\rho_{u,v}(0) \neq 0$
9. X and Y are independent	$\rho_{u,v}(k) = 0$ for all k

Source: Pierce (1977).

cross-correlations between the $\hat{u}$'s and $\hat{v}$'s, denoted as the $r_{\hat{u}\hat{v}}(k)$'s, may be used to infer the lead-lag relationship between X_t and Y_t. If X_t and Y_t are independent, the $r_{\hat{u}\hat{v}}(k)$'s are asymptotically, independently, and normally distributed with zero mean and variance N^{-1}, where N is the sample size.

As discussed in Pierce (1977), the hypothesis that X_t and Y_t are independent may be rejected at significant level α if:

$$Q_{2m+1} = N \sum_{k=-m}^{+m} (r_{\hat{u}\hat{v}}(k))^2 > X^2_{\alpha,2m+1}$$

where $X^2_{\alpha,2m+1}$ is the upper α percentage point of the chi-square distribution with 2m+1 degrees of freedom; and m is chosen so as to include all $\rho_{\hat{u}\hat{v}}(k)$'s expected to differ from zero. The contention that X_t leads Y_t is suggested at significant level α if:

$$Q_m = N \sum_{k=1}^{m} (r_{\hat{u}\hat{v}}(k))^2 > X^2_{\alpha,m}.$$

Similarly, Y_t leads X_t may be asserted at α if:

$$Q_{\bar{m}} = N \sum_{k=-1}^{-m} (r_{\hat{u}\hat{v}}(k))^2 > X^2_{\alpha,m}.$$

The significance of an individual $r_{\hat{u}\hat{v}}(k)$ may be determined by comparison to its standard error, $N^{-1/2}$. The convention is to judge an $r_{\hat{u}\hat{v}}(k)$ significant if it is at least twice as large as its standard error (theoretically $\pm$ 2 $(N-k)^{-1/2}$).

Table 7 summarizes the results obtained from the daily data 1968-77. To test the dependence between the two indexes CFI and S&P, we look at three different lags: 5 days, 3 days, and 1 day. We compute respectively Q_{11}, Q_7 and Q_3, the Q_{2m+1} statistics related to 5, 3 and 1 days of lag.

Then we compare the Q statistics with the value of $X^2_{\alpha,2m+1}$, with m = 5, 3 and 1 (degrees of freedom) and α = 95% or 90% of confidence. The notation is as follows:

(i) if $Q_i > X^2_{\alpha,i}$, then S&P and CFI are dependent; the notation is + in Table 7;

(ii) if $Q_i = X^2_{\alpha,i}$, then we suppose S&P and CFI are dependent; the notation is $\pm$ in Table 7;

(iii) if $Q_i < X^2_{\alpha,i}$, then S&P and CFI are independent; the notation is - in Table 7.

The results show a positive lead-lag relationship between CFI and S&P for three years: 1969, 1970 and 1972. CFI and S&P are independent for each of other years. These results are consistent regardless of the number of lags.

To determine which series is leading the other one for the three years 1969, 1970 and 1972, we compute Q_m and $Q_{\overline{m}}$ for the lag 3 days. The results are in Table 8. They show the following:

1969: S&P is leading CFI (1 day),

1970: S&P and CFI are instantaneously related within one day,

1972: S&P is leading CFI (1 day).

These results show that S&P had a tendency to lead CFI prior to 1973, but the tendency was not strong. From 1973 and on, there is no relationship between the two series. This possibly indicates that the

Table 7. Test Results of Univariate Cross-Correlation Analysis[a]

	Q_{11} 11=(2x5)+1	$X^2_{95,11}$ (19.7)	$X^2_{90,11}$ (17.3)	Q_7 7=(2x3)+1	$X^2_{95,7}$ (14.0)	$X^2_{90,7}$ (12.0)	Q_3 3=(2x1)+1	$X^2_{95,3}$ (7.8)	$X^2_{90,3}$ (6.2)
1968	7.1	-	-	2.1	-	-	1.3	-	-
1969	15.6	-	-	14.2	+	+	7.5	±	+
1970	19.3	±	+	13.4	±	+	7.2	±	+
1971	8.6	-	-	3.6	-	-	1.6	-	-
1972	22.6	+	+	16.8	+	+	10.2	+	+
1973	15.9	-	-	9.4	-	-	4.0	-	-
1974	13.6	-	-	9.1	-	-	1.2	-	-
1975	9.2	-	-	3.3	-	-	1.6	-	-
1976	12.3	-	-	5.8	-	-	3.8	-	-
1977	9.7	-	-	4.3	-	-	1.4	-	-

[a]See text for explanation of notation.

Table 8. Tests to Determine Direction of Causality[a]

	Q_3 (S&P-CFI)	$X^2_{95,3}$ (7.8)	$X^2_{90,3}$ (6.2)	$Q_{\overline{3}}$ (CFI-S&P)	$X^2_{95,3}$ (7.8)	$X^2_{90,3}$ (6.2)
1969	12.5	+	+	1.2	-	-
1970	5.4	-	-	1.4	-	-
1972	7.8	+	+	1.7	-	-

[a]See text for explanation of notation.

structural change in the early 1970s has affected the relationship between S&P and CFI.

V. Implications and Conclusions

This study has carefully investigated the historical relationships between S&P and CFI. The first three moments of the distributions of both indices indicates that CFI has outperformed S&P, regardless of investment horizon. Regression analysis revealed that virtually no relationship exists between the rates of return of the two series. However, the first three moments of the distributions are generally not independent of horizon. Finally, a parametric time-series technique was used to investigate further the lead-lag relationships between the two series and these results confirmed the regression results in that the two series are independent of each other, at least for the most recent years. S&P was found to lead CFI by one day in 1969 and 1972 while the two were instantaneously related in 1970. Data for 1973 through 1977 show complete independence, regardless of evaluation technique.

Thus, inclusion of commodity futures contracts in an equity portfolio has a strong opportunity to reduce the risks and enhance the performance of the portfolio. The futures contracts will not only provide diversification which reduces overall risks, but the commodity contracts may well outperform the stock investments to generate higher returns, and the contracts contain positive skewness. Also, the longer the commodity futures contracts are held, the better their performance, as long as the trader is on the "right" side of the market.

Some of our research results, especially that commodity futures contracts outperform stocks, are consistent with Bodie and Rosansky

(1980). However, our methodology looks at the relationship between the markets in much more depth and detail and concerns itself with investment horizon. Incidently, the time-series technique used here could be employed to reexamine intertemporal differences in systematic stock price movements as investigated by Francis (1975) and others. Nevertheless, this paper provides new information about the relationship between the commodity futures market index and Standard and Poor's 500 index, and the overwhelming evidence of independence between the two series in recent years, confirmed by two completely separate techniques of analysis, should be of interest to security analysts and portfolio managers as they plan their investment strategies.

Footnotes

[1]One other study, Bodie and Rosansky (1980), has compared rates of return on commodity futures contracts to those earned on stocks and bonds, but their study examines individual contracts with quarterly data from 1950 to 1976, and does not provide the diversity of tests employed here.

[2]Mathematically, one would expect the geometric rates of return to vary proportionately with horizon, as is the case for the CFI in Table 1. However, that is not the case in Table 2 for S&P because of instability at the end of the sample period and varying ending observations. For example, for a time series of 11 observations the average geometric return for a 1-day horizon is $(-\log P_1 + \log P_{11})/10$, for a 2-day horizon it is $(-\log P_1 + \log P_{11})/5$, while for a 4-day horizon it is $(-\log P_1 + \log P_9)/2$. Note that in the 4-day horizon case the last observation differs from the 1- and 2-day horizons, and that two observations are lost. Thus, in our sample of 1505 observations, the ending observation is different for most horizons, and between the 21- and 22-day horizons it can vary as much as one-half month. In empirical application it is difficult to estimate returns over several horizons without losing observations, and the actual returns will not coincide with theoretical expectations if the time series shows instability at the end.

[3]See Folger and Radcliff (1974), McEnally (1974), Kraus and Litzenberger (1976) and Lee (1977) for detail.

[4]The results show that 14 out of the 22 beta coefficients are, in fact, negative. Negative beta coefficients can be used to cancel other positive betas. Therefore, the negative beta is not a systematic risk in terms of the portfolio diversification process (Ben-Horim and Levy, 1980).

[5]The least squares technique is appropriate as long as the model has no moving average parameters.

REFERENCES

Beedles, W. L. "On the Asymmetry of Market Returns," Journal of Financial and Quantitative Analysis. 14(1979): 653-60.

Ben-Horim, M. and H. Levy. "Total Risk, Diversifiable Risk and Non-Diversifiable Risk: A Pedagogic Note," Journal of Financial and Quantitative Analysis. 15(1980): 289-97.

Bodie, Z. and V. Rosansky. "Risk and Return in Commodity Futures," Financial Analyst Journal. 36(May-June 1980): 27-39.

Brenner, M. "On the Stability of the Distribution of the Market Component in Stock Price Changes," Journal of Financial and Quantitative Analysis. 9(1974): 945-61.

Cheng, P. L. and M. K. Deets. "Systematic Risk and the Horizon Problem," Journal of Financial and Quantitative Analysis. 8(1973): 299-316.

Fogler, H. K. and R. C. Radcliff. "A Note on Measurement of Skewness," Journal of Financial and Quantitative Analysis. 9(1974): 485-9.

Francis, J. C. "Intertemporal Differences in Systematic Stock Price Movements," Journal of Financial and Quantitative Analysis. 10 (1975): 205-17.

Granger, C. W. J. and P. Newbold. "Spurious Regressions in Econometrics," Journal of Econometrics. 2 (1974): 111-20.

Haugh, L. D. "Checking the Independence of Two Covariance-Stationary Time Series: A Univariate Residual Cross Correlation Approach," Journal of the American Statistical Association. 71 (1976): 378-85.

Haugh, L. D. and G. E. P. Box. "Identification of Dynamic Regression (Distributed Lag) Models Connecting Two Time Series," Journal of the American Statistical Association. 72(1977): 121-130.

Kraus, A. and R. H. Litzenberger. "Skewness Preference and the Valuation of Risky Assets," Journal of Finance. 31(1976): 1084-1100.

Lee, C. F. "Functional Form, Skewness Effect, and the Risk-Return Relationship," Journal of Financial and Quantitative Analysis. 12 (1977): 55-72.

Lee, C. F. "Investment Horizon and the Functional Form of the Capital Asset Pricing Model," Review of Economics and Statistics. 58(1976): 356-63.

Lee, C. F. and K. Morimune. "Time Aggregation, Coefficient of Determination and Systematic Risk of the Market Model," The Financial Review. (1978): 36-47.

Levhari, D. and H. Levy. "The Capital Asset Pricing Model and the Investment Horizon," Review of Economics and Statistics. 59(1977): 92-104.

McEnally, R. W. "A Note on the Return Behavior of High Risk Common Stock," Journal of Finance. 29(1974): 199-202.

Pierce, D. A. "Relationships--and the Lack Thereof--Between Economic Time Series, with Special References to Money and Interest Rates," Journal of the American Statistical Association. 72(1977): 11-22.

Sharpe, W. "A Simplified Model for Portfolio Analysis," Management Science. (1963): 277-93.

Snedecor, G. W. and W. G. Cochran. Statistical Methods, 5th edition. Ames: Iowa State University Press, 1956.

M/E/202-2