EXACT ASTRONOMY.

A DYNAMICAL SOLUTION OF THE FUNDAMENTAL PROBLEMS

OF

Mathematical Astronomy.

BY

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TO

WALTER WILLIAM PALMER,

In recognition of the stimulant to persistence wrought by his kindness and intelligent appreciation, this work is respectfully dedicated

By his friend,

THE AUTHOR.

INTRODUCTORY.

Distance and Dimensions of the Sun.

FROM THE "SUN," BY PROFESSOR C. A. YOUNG.

The problem of finding the distance of the sun is one of the most important and difficult presented by astronomy. Its importance lies in this, that this distance—the radius of the earth's orbit—is the base line by means of which we measure every other celestial distance, excepting only that of the moon ; so that error in this base propagates itself in all directions, through all space, affecting with a corresponding proportion of falsehood every measured line, the distance of every star, the radius of every orbit, the diameter of every planet.

Our estimates of the masses of the heavenly bodies also depends upon a knowledge of the sun's distance from the earth. The quantity of matter in a star or planet is determined by calculations whose fundamental data include the distance between the investigated body, and some other body whose motion is controlled or modified by it; so that any error in it involves a more than three-fold error in the resulting mass. An uncertainty of one per cent. in the sun's distance implies an uncertainty of more than three per cent. in every celestial mass and every cosmical force.

Error in this fundamental element propagates itself in time also, as well as in space and mass. That is to say, our calculations of the mutual effects of the planets upon each other's motions depend upon an accurate knowledge of their masses and distances. By these calculations, were our data perfect, we could predict for all futurity, or reproduce for any given epoch of the past, the configurations of the planets and the conditions of their orb itsand many interesting problems in geology and natural history seem to require for their solution just such determinations of the form and position of the earth's orbit in by-gone ages.

Now, the slightest error in the data, though hardly affecting the result for epochs near the present, leads to uncertainty, which accumulates with extreme rapidity in the lapse of time ; so that even the present uncertainty of the sun's distance, small as it is, renders precarious all conclusions from such computations when the period is extended more than a few hundred thousand years from the present time. If, for instance, we should find as the result of calculation with the received data, that two millions of years ago the eccentricity of the earth's orbit was at a maximum, and the perihelion so placed that the sun was nearest during the northern winter (a condition of affairs which is thought would produce a glacial epoch in the southern hemisphere), it might easily happen that our results would be exactly contrary to the truth, and that the state of affairs indicated did not occur within half a million years of the specified date ; and all because in our calculation the sun's distance, or solar parallax by which it is measured, was assumed half of one per cent. too great or too small. In fact, this solar parallax enters into almost every kind of astronomical computations, from those which deal with stellar systems and the constitution of the universe, to those which have for their object nothing higher than the prediction of the moon's place as a means of finding the longitude at sea. Of course, it hardly need be said that its determination is the first step to any knowledge of the dimensions and constitution of the sun itself.

This parallax of the sun is simply the angular semi-diameter of the earth as seen from the sun; or, it may be defined in another way as the angle between the direction of the sun ideally observed from the center of the earth, and its actual direction as seen from a station where it is just rising above the horizon.

We know with great accuracy the dimensions of the earth. Its mean equatorial radius according to the latest and most reliable determination (agreeing, however, very closely with previous ones,) is 3962.720 English miles, and the error can hardly amount to more than 0.00001 of the whole—perhaps 200 feet one way or the other. Accordingly, if we know how large the earth looks from any point, or, to speak technically, if we know the parallax of the point, its distance can at once be found by a very easy calculation; it equals simply (206265 × the radius of the earth) ÷ (the parallax in seconds of arc).

Now, in the case of the sun it is very difficult to find the parallax with sufficient precision, on account of its smallness—it is less than 9″, almost certainly between 8″.8 and 8″.9. But this tenth of a second of doubtfulness is more than 0.01 of the whole, although it is no more than the angle subtended by a single hair at a distance of nearly 800 feet. If we call the parallax 8″.86, which is probably very near the truth, the distance of the sun will come out 92,254,000 miles, while a variation of $\frac{1}{20}$ of a second either way will change it nearly half a million miles.

When a surveyor has to find the distance of an inaccessible object he lays off a convenient base line, and from its extremities observes the directions of the object, considering himself very unfortunate if he cannot get a base whose length is at least $\frac{1}{10}$ of the distance to be measured. But the whole diameter of the earth is less than $\frac{1}{11000}$ of the distance of the sun, and the astronomer is in the predicament of a surveyor, who, having to measure the distance of an object ten miles off, finds himself restricted to a base of less than five feet, and herein lies the difficulty of the problem.

Of course, it would be hopeless to attempt this problem by direct observations, such as answer perfectly in the case of the moon, whose distance is only thirty times the earth's diameter. In her case, observations taken from stations widely separated in latitude, like Berlin and the Cape of Good Hope, or Washington and Santiago, determine her parallax and distance with very satisfactory precision; (very unsatisfactory, rather, since the error is very nearly half of one per cent.—AUTHOR) but if observations of the same accuracy could be made upon the sun, (which is not the case, since its heat disturbs the adjustments of an instrument) they would only show the parallax to be somewhere between 8″ and 10″,—its distance between 126,000,000 and 82,000,000 miles.

Astronomers, therefore, have been driven to employ indirect methods, based on various principles.

On the Method of Zadig.

FROM A LECTURE BY HUXLEY.

It is a usual and a commendable practice to preface the discussion of the views of a philosophic thinker by some account of the man and of the circumstances which shaped his life and colored his way of looking at things; but, though Zadig is cited in one of the most important chapters of Cuvier's greatest work, little is known about him, and that little might, perhaps, be better authenticated than it is.

It is said that he lived at Babylon in the time of King Moabdar; but the name of Moabdar does not appear in the list of Babylonian sovereigns brought to light by the patience and the industry of the decipherers of cuneiform inscriptions in these later years; nor indeed am I aware that there is any other authority for his existence than that of the biographer of Zadig, one Arouet de Voltaire, among whose most conspicuous merits strict historical accuracy is perhaps hardly to be reckoned.

Happily Zadig is in the position of a great many other philosophers. What he was like when he was in the flesh, indeed whether he existed at all, are matters of no great consequence. What we care about in a light is that it shows the way, not whether it is lamp or candle, tallow or wax. Our only real interest in Zadig, lies in the conceptions of which he is the putative father; and his biographer has stated these with so much clearness and vivacious illustration, that we need hardly feel a pang even if critical research should prove King Moabdar and the rest of the story to be unhistorical, and reduce Zadig himself to the shadowy condition of a solar myth. Voltaire tells us that, disenchanted with life by sundry domestic misadventures, Zadig withdrew from the turmoil of Babylon to a secluded retreat on the banks of the Euphrates, where he beguiled his solitude by the study of nature. The manifold wonders of the world of life had a peculiar attraction for the lonely student; incessant and patient observation of the planets and animals about him sharpened his naturally good powers of observation and of reasoning; until it length he acquired a sagacity which enabled him to perceive endless minute differences among objects which, to the untutored eye, appear absolutely alike.

It might have been expected that this enlargement of the powers of the mind and of its store of natural knowledge could tend to nothing but the increase of a man's own welfare and the good of his fellow men. But Zadig was fated to experience the vanity of such expectations.

One day walking near a little wood, he saw, hastening that way, one of the queen's chief eunuchs, followed by a troop of officials who appeared to be in the greatest anxiety, running hither and thither like men distraught, in search of some lost treasure.

"Young man," cried the eunuch, "have you seen the queen's dog ?" Zadig answered modestly, "A bitch, I think, not a dog." "Quite right," replied the eunuch ; and Zadig continued : "A very small spaniel who has lately had puppies ; she limps with the left fore leg, and has very long ears." "Ah ! you have seen her, then ?" said the breathless eunuch. "No," answered Zadig, "I have not seen her ; and I really was not aware that the queen possessed a spaniel."

By an odd coincidence, at the very same time, the handsomest horse in the king's stables broke away from his groom in the Babylonian plains. The grand huntsman and all his staff were seeking the horse with as much anxiety as the eunuch and his people, the spaniel; and the grand huntsman asked Zadig if he had not seen the king's horse go that way. "A first-rate galloper, small hoofed, five feet high ; tail three feet and a half long ; cheek pieces of the bit of twenty-three carat gold ; shoes silver," said Zadig.

"Which way did he go ? Where is he ?" cried the grand huntsman.

"I have not seen anything of the horse, and I never heard of him before," replied Zadig.

The grand huntsman and the chief eunuch made sure that Zadig had stolen both the king's horse and the queen's spaniel; so they haled him before the high Court of Desterham, which at once condemned him to the knout and transportation for life to Siberia. But the sentence was hardly pronounced when the lost horse and spaniel were found. So the judges were under the painful necessity of reconsidering their decision, but they fined Zadig four hundred ounces of gold for saying that he had seen that which he had not seen.

The first thing was to pay the fine; afterwards Zadig was permitted to open his defense to the Court, which he did in the following terms:

"Stars of justice, abysses of knowledge, mirrors of truth, whose gravity is as that of lead, whose inflexibility is as that of iron, who rival the diamond in clearness, and possess no little affinity with gold:— Since I am permitted to address your august assembly, I swear by Ormuzd that I have never seen the respectable lady dog of the queen nor beheld the sacrosanct horse of the king of kings.

"This is what happened: I was taking a walk toward the little wood near which I subsequently had the honor to meet the venerable chief eunuch and the most illustrious grand huntsman. I noticed the track of an animal in the sand, and it was easy to see that it was that of a small dog. Long, faint streaks upon the little elevations of sand between foot marks convinced me that it was a she dog with pendent dugs, showing that she must have had puppies not many days since. Other scrapings of the sand, which always lay close to the marks of the fore paws, indicated that she had very long ears; and as the imprint of one foot was always fainter than those of the other three, I judged that the lady dog of our august queen was, if I may venture to say so, a little lame.

"With respect to the horse of the king of kings, permit me to observe that, wandering through the paths which traverse the wood, I noticed the marks of horseshoes. They were all equidistant. 'Ah,' said I, 'this is a famous galloper.' In a narrow alley, only seven feet wide, the dust upon the trunks of the trees was a little disturbed at three feet and a half from the middle of the path. 'This horse,' said I to myself, 'had a tail three feet and a half long, and, lashing it from one side to the other, he has swept away the dust.' Branches of trees met overhead at the height of five feet, and under them I saw newly fallen leaves; so I knew the horse had brushed some of the branches and was therefore five feet high. As to his bit, it must have been of twenty-three carat gold, for he had rubbed it against a stone which turned out to be a touchstone, with the properties of which I am familiar by experiment. Lastly, by the marks which his shoes left upon pebbles of another kind, I was led to think that his shoes were of fine silver."

All the judges admired Zadig's profound and subtle discernment; and the fame of it reached even the king and the queen. From the ante-rooms to the presence-chamber, Zadig's name was in everybody's mouth; and, although many of the magi were of the opinion that he ought to be burned as a sorcerer, the king commanded that the four hundred ounces of gold which he had been fined should be restored to him. So the officers of the court went in state with the four hundred ounces; only they retained three hundred and ninety-eight for legal expenses, and their servants expected fees.

Those who are interested in learning more of the fateful history of Zadig must turn to the original; we are dealing with him only as a philosopher, and this brief excerpt suffices for the exemplification of the nature of his conclusions and of the method by which he arrived at them.

PREFACE.

These few pages effect the exhaustive solution of the fundamental problems of mathematical astronomy, by the application, after the manner of Zadig, of the elementary mathematics only to accurately known data.

The remark in the introduction to Herschel's "Outlines of Astronomy" that (together with the story of Zadig) suggested to my mind the mode of attacking the great problem of the sun's distance, best expresses the thought I wish to enunciate,—"the pearls of analytical research are invariably strung on the central thread of common sense."

The substitution of dynamical for statical treatment of the problem, has revolutionized the oldest of the sciences, and dissipated the intellectual fog in which abstruse methods and deference to authority have so long enshrouded it.

Human nature remains the same through all ages; and my experience of the reception accorded truths newly discovered by a layman, by the guild of scientists, demonstrates that Voltaire's caustic satire is as applicable to the scientific pretension, arrogance and dishonesty of to-day as to that of his own time.

MYRON HUTCHINSON.

San Francisco, October, 1889.

EXACT ASTRONOMY.

A Dynamical Resolution of the Solar Parallax.

The conception of the solar horizontal parallax postulates a circular orbit described with the radius R, equal to the mean distance between the centers of terrestrial and lunar revolution, measured by an arc x of the said great circle; equal in length to the earth's equatorial radius. If x be taken in seconds, so must R be, and since the ratio of the sidereal year in seconds T and R is constant, and that $T = \frac{PI \times T}{3 \times 6^3 \times 10^3} \times R$, it follows that factoring R factors T also. Wherefore, the numerical expression of the mean radius of the real elliptical orbit, by the ratio $\frac{R}{x}$, compels the expression of the time of its description by the ratio $\frac{T}{x}$. Now, conceiving resistance to the revolution of a planetary mass without volume, at the distance 1 or x, eliminated by compression of the sun's volume, it would revolve in x seconds. The proportion $x^2 : x^3 :: T^2 : R^3$ is consequently explicit in Kepler's third law.

Whence: $x = \frac{R^3}{T^2} = 8''.8115507443113$.

Also: $\frac{T^2}{x^2} \div \frac{R^3}{x^3} = \frac{T^2 x}{R^3} = 1$.

Multiplying the equation by $\frac{R}{x}$, $\frac{R}{x} = \frac{T^2}{R^2} = (\frac{T}{R})^2 = (152.-99822531136870)^2 = 23408.4569484283500$.

A Dynamical Resolution of the Equatorial Radius.

The universal law of gravitation prescribes the multiplication of the moon's sidereal period, t, by the square root of $1 + m$, the sum of the relative masses of earth and moon, the division of the product by the periodic time of a hypothetical satellite, conceived to revolve around the earth at the distance 1, and the involution of the quotient to the fractional index $\frac{2}{3}$ for the moon's mean distance from the center of her motion. Conceiving resistance eliminated by compression of the earth's volume, the periodic time in seconds, of such satellite, is the product of three factors, to wit: the terrestrial orbital arc x, the square of the solar day in hours, d^2, and the reciprocal of $(1+m)^{\frac{1}{6}}$; $\frac{t(1+m)^{\frac{1}{2}}}{xd^2 \div (1+m)^{\frac{1}{6}}} = \frac{t(1+m)^{\frac{2}{3}}}{xd^2} = \frac{2360591s.5(1+m)^{\frac{2}{3}}}{5075.4532287233088} = 465.0996460061535(1+m)^{\frac{2}{3}}$, involved to the index $\frac{2}{3}$, equals $60.029400472 2072(1+m)^{\frac{4}{9}}$. The simple ratio of the distances of sun and moon: $23408.4569484 2835 \div 60.0294004722072(1+m)^{\frac{4}{9}} = \frac{389.9498706349091}{(1+m)^{\frac{4}{9}}}$. All that precedes is rigorously demonstrated by the absolute identity of the sesquiplicate ratio of the astronomical periodic times of earth and moon, and the simple ratio of their mean distances: $\frac{T}{x} \div \frac{t(1+m)^{\frac{1}{2}}}{xd^2} = \frac{Td^2}{t(1+m)^{\frac{2}{3}}} = 7700.3979709322854 \div (1+m)^{\frac{2}{3}}$. The cube root of the square of same equals $389.9498706349084 \div (1+m)^{\frac{4}{9}}$. The difference, $0.0000000000007 \times 60.0294 \times 20926084 \times 12 = 0.01$ inch, is the necessary consequence of incommensurability. With Herschel's value of the sidereal year, which is $0^s.3$ greater, the above discrepancy of 0.01 inch is increased to 33.83 miles. Consequently, no variation whatever from the values employed in the computation is possible; which is admittedly the highest order of proof. I will assume $\frac{1}{80}$ for the moon's mass, and then demonstrate that it cannot be either more or less: $(1+m)^{\frac{1}{6}} = (1.0125)^{\frac{1}{6}} = 1.0020725$.

Consequently, the arc in terms of radius that the hypothetical satellite (under the conditions specified) would describe in one second: $\frac{6.2831853071794 \times 1.0020725}{5075.4532287233088} = 0.001240521 1761$. An arc so small cannot be sensibly different from its tangent. Hence, $(1+\text{curv.})^2 - 1^2 = \tan.^2 = \text{arc}^2 = 0.0000015388927 88352$.

Completing the square and extracting root:

$$\text{Curv.}^2 + 2\ \text{curv.} + 1 = 1.000001538892788352.$$

$$\text{Curv.} + 1 = 1.0000007694462.$$

Curvature $= 0.0000007694462$ is identical with the fall of a weight (all resistance being eliminated) in one second at sea level on the equator; because its deflection in equal times from the tangent to its path is the same, whether dropped from rest or projected laterally with any velocity. The acceleration of gravity at London has been determined with the required accuracy: 32.1928306 feet per second.

Gravity is as the square of the number of vibrations by the same pendulum in equal times.

A pendulum that beats seconds at London loses 136 beats in 24 hours at sea level on the equator. On the equator gravity is diminished $\frac{1}{289}$ part of itself by the momentum of the earth's rotation. All resistance eliminated, the fall of a weight in 1 second: $\frac{32.1928306}{2} \times \left(\frac{86264}{86400}\right)^2 \times \frac{289}{288} = 16.1014958005563$ feet.

Length of the equatorial radius: $16.1014958005563 \div 0.0000007694462 = 20926084$ feet.

The Figure of the Earth Dynamically Determined.

Pendulum observations have demonstrated the regular increase of gravity with increase of latitude; equal to $\frac{1}{194}$ part of equatorial gravity at the pole.

The distance a weight would there fall in 1 second: $\frac{32.1928306}{2} \times \left(\frac{86264}{86400}\right)^2 \times \frac{195}{194} = 16.128491495012 5$ feet.

The earth's oblateness, concluded from the geodetical measurement of meridional arcs, is about $\frac{1}{299}$ part of the equatorial radius. I will now demonstrate that the terrestrial radii are in the inverse duplicate ratio of gravity at their intersections with sea level, computing by this rule: length of the polar radius: $\left(\frac{16.1014958005563}{16.1284914950125}\right)^2 \times 20926084 = 20$-856091 feet.

Oblateness: $\frac{69993}{20926084} = \frac{1}{298.974}$.

The identity of results demonstrates the rule, which certainly, simply and economically determines any radius that intersects land. The tangential force being as the square of the distance from the axis of rotation, is as the square of the latitude cosine. Corrected for rotation, the fall of a weight at London: $\frac{289}{289-(0.62229)^2} \times 16.0964153 = 16.$-118012 5916476 feet in one second.

Radius: $\left(\frac{16.1014958005563}{16.1180125916476}\right)^2 \times 20{,}926{,}084 = 20{,}883{,}218$ feet.

A pendulum that beats seconds at London gains 79 beats in 24 hours at Spitzbergen, in latitude 79° 50′. Correcting for rotation, the fall of a weight: $\frac{289}{289-(0.17651)^2} \times \left(\frac{86479}{86400}\right)^2 \times 16.0964153 = 16.1276029752894$ feet in one second.

Radius: $\left(\frac{16.1014958005563}{16.1276029752894}\right)^2 \times 20926084 = 20858389$ feet.

The correction for altitude is equally certain and easy, and I feel confident that a weighing apparatus that would accurately determine the intensity of gravity at sea is not beyond the ingenuity of American inventors.

With this adjunct to the pendulum, the exact figure of the earth would be quickly determined.

Distance, Mass and Dimensions of the Sun.

Distance between the centers of terrestrial and lunar revolution: $\frac{23408.45694842835 \times 20926084}{5280} = 92774116.75$ miles.

With the kilometer, 3280.8992 feet for unit = 1493027-69.32 kilometers.

Taking the earth's weight as 1, it follows from the law of gravitation, that the mass of the sun is the inverse duplicate ratio of the computed periodic times of the supposed earth and satellite, both revolving at the distance 1 :

$$\left(\frac{x\,d^2}{x\,(1+m)\,\frac{1}{6}}\right)^2 = \frac{d^4}{(1+m)^{\frac{1}{3}}} = \frac{331776}{1.0041493} = 330405.$$

The sun and earth both revolve in a year about their common center of gravity; distant from the sun's center; $92774117 \div 330405 = 280.8$ miles.

The earth and moon both also revolve about the outer end of the earth's radius-vector eccentric to her center: $239230 \div 80 = 2990$ miles. But being always in the direction of the moon from the center, its mean position, with reference to the sun's distance, is at the center. The sun's mean apparent diameter, $1923''.6$, as observed at Greenwich Observatory in latitude $51°29'$, depends on the observer's distance from the sun's center, which is less by the cosine of the latitude than to the earth's center.

Wherefore, the actual diameter of the sun:

$$\frac{92774117+281-2468}{206264''.806247} \times 1923''.6 = 865{,}180 \text{ miles.}$$

His volume: $\left(\frac{432590}{3963.2735}\right)^3 = (109.15)^3 = 1{,}300{,}383$ earths.

Distance and Diameter of the Moon.

$60.0294004722 \times 1.00553639 \times 3963.2735 = 239230.11$ miles is her mean distance from the center of her motion. Since her mean position is in the plane of the equator, her mean distance from the center of the earth is 2,990 miles greater. By reason of proximity, her mean apparent diameter, $1854''$, measured at Greenwich, varies slightly with the latitude of the observer's station. Consequently, her mean distance from the earth's surface is the square root of the the sum of the squares of the sine of the observer's latitude and the distance from its intersection with the plane of the equator to the moon's center. Hence, her mean distance from Greenwich in lat. $51^\circ\ 29'$: $[(239230 + 2990 - 2468)^2 + (3101)^2]^{\frac{1}{2}} = 239{,}782$ miles.

Her actual diameter:

$$239782 \times 1854'' \div 206264.''8 = 2155.2 \text{ miles.}$$

A Crucial Test

Of the foregoing determinations is applied by the observed duration, 118 seconds, of the total phase of the solar eclipse of Jan. 1st, 1889, at Willows, California, in latitude $39\frac{1}{2}^\circ$. It is obvious that the diameter of the moon and that of her shadow parallel to its motion at intersection with the earth are sections of an equilateral triangle standing on the sun's diameter. It is equally obvious that the duration of the observer's immersion in the moon's shadow, when she is on his meridian, is the diameter, perpendicular to her direction, of the shadow's intersection with the earth, divided by the rate of its motion. Because of the great eccentricity of the moon's orbit, her motion is far from uniform, and the velocity of the earth's rotation is also greatest at the equator. But, in consequence of the divergence from parallelism of the planes of the two motions towards the east, and of the earth's convexity, the velocity of the shadow's intersection therewith is the difference of the moon's mean velocity and the equatorial velocity of the earth's rotation:

$$\frac{239230 \times 6.2831853}{2360591s.5} - \frac{3963.2735 \times 6.2831853}{86400s} = 0.348541 \text{ mile per second.}$$

Diameter of the shadow's intersection with the earth perpendicular to the moon:

$$118 \times 0.348541 = 41.127838 \text{ miles.}$$

Taking the apparent semi-diameters of the sun and moon, computed for date of the eclipse, as given in the *Nautical Almanac*, the observer's distance from the sun's center: $92774117 \times 962'' \div 976''.2 = 91{,}424{,}606$ miles.

Now, let x equal the length of segment of the moon's shadow cut off by the earth, and we have from similar triangles the proportion:

$$x : 41.127838 :: x + 91424606 : 865180.$$
$$865138.872162x = 3760096384.781828.$$
$$x = 4346.23 \text{ miles.}$$

Also, 41.127838 : 4346.23 :: 2155.2 : moon's shadow.

Total length of shadow = 227,753 miles.

The observer's distance from the moon:

$$227753 - 4346 = 223407 \text{ miles.}$$

Her mean distance from the center of her motion:

$$[(223407 + \text{cosine } 39\tfrac{1}{2}^\circ, 3058)^2 - (\text{sine } 39\tfrac{1}{2}^\circ, 2521)^2]^{\frac{1}{2}} \times \frac{992''.4}{927''} = 239,226 \text{ miles.}$$

The deficiency of 4 miles may be credited to error of observation.

A Sophistical Assumption Refuted.

Professor E. S. Holden, of the Lick Observatory, and a number of College Professors of Mathematics have asserted that my resolution of the solar parallax is fallacious, because time and distance cannot be measured by the same unit; and that the expansion or contraction of the earth would not alter its periodic time. The sophistry of the first objection is apparent, since every astronomer assumes that the only possible unit of the earth's mean distance is an arc of a great circle of the celestial sphere, which must become the unit of the time in which the whole circumference is described, when taken as the unit of circumference. The second objection is fallacious, puerile and absurd because, although true of abstract time, the numerical expression of the periodic time is the ratio of the periods of revolution and rotation. The subdivisions of the latter are arbitrary and their number constant. By the laws of mechanics, expansion would retard, contraction accelerate, the velocity of rotation. Consequently, the day which is the measure of the tangential force would vary by the expansion or contraction of its subdivisions, as the square root of the equatorial radius; and the sidereal year would vary in the same proportion, inversely. Hence, the cube of the constant R, divided by the square of the varying T, would always express the solar parallax,—their duplicate ratio the mean orbital radius, without change of abstract time or distance. The whole assumption of my critics is sophistical and devoid of honesty.

The Velocity of Light.

The progressive motion of light is demonstrated by the annual apparent oscillation of stars in the ecliptic, which becomes an oval of decreasing eccentricity with increase of the star's declination. The center of this apparent motion is the star's real place, and can arise only from the projection on the vault of the celestial sphere of the terrestial orbit seen in perspective and, consequently, foreshortened to its major axis by the apparent motion of a star in the plane of the real orbit and perpendicular to its major axis. All other stars in the ecliptic trace, of course, the diameter that is perpendicular to each. The longest diameter of the oval traced by stars having latitude is, of course, that diameter of the orbit parallel thereto.

The assumption of the text-books that all the stars in the plane of the orbit exhibit the maximum amount of aberration, and that a star exactly at the pole would apparently describe a circle, is manifestly erroneous. The phenomenon of aberration results directly from the earth's orbital velocity, being a sensible fraction of the velocity of light, which projects the orbit on the sky, at a distance $R \times \frac{V}{v}$ from the eye, R representing the semi-diameter, V and v the velocities.

Hence, the aberration of stars in opposition when the earth is at equinox and solstice, gives an independent measure of orbital eccentricity that is, perhaps, more reliable than the sun's disc. The time in which the earth describes the maximum arc of aberration or reduced semi-major axis and light describes the real semi-major axis :

$$31558149^{s}.3 \times 20''.445 \div 1296000'' = 497^{s}.84441546.$$

The velocity of light : $92774116.75 \div 497.84441546 =$ 186352 miles per second :

$$\frac{23408.45694842835}{497.84441546} \times \frac{20926084}{3280.8982} = 299898.5 \text{ kilometers per second.}$$

Charlatanry Unmasked.

About three years ago the scientific journals, led by *Nature*, executed a grand fanfare in glorification of the alleged independent experimental determination of the velocity of light by Professor Simon Newcomb, at the U. S. Naval Observatory and by Professor Michelson, at Cleveland.

My determination of the solar parallax had been submitted to Professor E. S. Holden several months before.

With the value of the equatorial radius accepted by American astronomers : 3962.720 miles (*vide* Professor C. A. Young's *The Sun*), and mine of the solar parallax, the velocity of light becomes :

$$\frac{23408.45694842835}{497.84441546} \times \frac{20923161.6}{3280.8992} = 299856.5 \text{ kilometers per second ;}$$

which is the exact mean of 299860 and 299853, the determinations in question.

The allowance of 30 kilometers, plus or minus, conceded for possible error, has proved inadequate to the correction of the actual error of the value adopted for the equatorial radius. The combination of this velocity with Nyren's determination of aberration for the sun's distance, which is expanded 200,000 miles thereby, is a very clumsy device for masking the fraud.

The sums expended by the different civilized governments in futile efforts to exactly determine the sun's distance by triangulation to the celestial bodies and by geodetical measurements, aggregate many millions of dollars. The problems of the solar parallax and of the dimensions and figure of the earth, are exhaustively solved by these few equations.

I have now arrived at the conviction that the solar horizontal parallax, as defined by the astronomers, is not comprehended by the average reader.

Its dynamical resolution is the key to the exact determination of all the numerical relations of the solar system and is, in fact, the foundation-stone of the entire structure of mathematical astronomy. The name discourages the non-mathematical reader, but the subject itself is simple and within his mental reach, when accurately described in simple terms.

Although the earth's orbit is not exactly a circle, it is exactly equal to the circumference of a circle whose radius equals the earth's mean distance from the center of her motion. Now, the circumference of every circle contains 1,296,000″, and the segment thereof which is equal to its radius contains 206264″.806247102477. The plane of the earth's equator is always nearly coincident with the plane of her orbit—exactly so when she is at the vernal or autumnal equinox. At those times her equatorial radius occupies a small segment of the above described great circle that contains an unknown number x of seconds. Said arc is the thing named by the astronomers solar horizontal parallax.

It is now obvious that the ratio $\frac{206264.''8+}{x''}$ is the ratio of the mean orbital radius and equatorial radius of the earth.

We remain ignorant of the sun's actual distance till this ratio is multiplied by the ascertained length of the earth's equatorial radius.

The so-called wonderful difficulty and complexity of the problem of the sun's distance have been enlarged upon by all the astronomers, but I have shown most conclusively that the problem is an extremely simple one and is easily solved by elementary mathematics.

Since the astronomers are themselves lost in the fog raised by their *conception* of solar parallax, which, rightly conceived, is the key to the exact determination of all astronomical magnitudes, whether of distance, mass or volume, I will endeavor to make the thing so styled clear to the minds of my non-mathematical readers, by detaching it from all ideas of parallax. The earth's orbit is an ellipse of small eccentricity, but is exactly equal to a circle the radius of which is the earth's distance from the center of her motion when she is at the vernal or autumnal equinox; she is then at her mean distance.

At those times, the plane of her equator exactly coincides with the plane of her orbit, and her equatorial semidiameter is concentric and coterminous with a very small segment of the aforesaid great circle, since so small an arc is not sensibly different from its chord. This minute arc containing the unknown number x seconds, being taken as the unit of the radial arc is made thereby to measure the whole circumference, and therefore to measure the time of its description, or the sidereal year, containing 31558149.3 time seconds, equal to 1296000″ × 24.3504238425926. It is now self-evident that these arcs of the said great circle are in the same ratio as the earth's mean-orbital and equatorial radii. The sesquiplicate ratio of the time and distance units must be the same as that of the wholes; whence, and from

Kepler's third law: $\frac{x^3}{x^2} = x = \frac{R^3}{T^2} = 8''.8115507443113$, $\frac{R}{X} = \frac{206264''.806247102477}{8''.8115507443113} = 23408.45694842835.$

Also, $\frac{R}{x} = (\frac{T}{R})^2 = (152.9982253113687)^2 = 23408.4569-4842835.$

We remain ignorant of the sun's actual distance till this ratio is multiplied by the ascertained length of the earth's equatorial radius, to the exact determination of which the arc x is the key.

Parallax is a distant object's apparent displacement, resulting from the observer's translation to another point of view, transverse to the direction of the object. Applied to the arc x, the term is fallacious and misleading, because implying its resolution by triangulation, which is the essential principle of all the methods hitherto employed. The diverse results obtained are inevitably vitiated by instrumental and atmospheric instability.

It is now patent that the annoying complexity of the problem, so much bemoaned by the astronomers, is of their own weaving and entirely foreign to its wonderful natural beauty and simplicity. Finally, astronomical treatises are devoted *ad nauseam* to glorification of the science, to the apotheosis of its founders, and to mutual back-scratching of its votaries.

The deduction of solar parallax and of the earth's dimensions from the periodic times of earth and moon, and from the acceleration of terrestrial gravity, is what is meant by the dynamical resolution of the problems, and is wholly original with the author. These data are positively known; all instrumental determinations being admittedly inaccurate.