EXPERIMENTAL PHYSICS FOR COLLEGES

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EXPERIMENTAL PHYSICS FOR COLLEGES

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PREFACE

This book is essentially a *text* on experimental physics for elementary students. The authors feel that students should be given more than a set of laboratory instructions for performing an experiment and have tried to bring out the connection which an experiment has with other experiments of a similar nature. In other words, students should look upon a laboratory physics experiment as part of the larger subject of "Experimental Physics." This book can be used as a laboratory text in conjunction with any general college physics text.

The chapters are so planned that experiments illustrating laws which are governed by the same or by a similar set of physical principles may be grouped together. Each chapter begins with a discussion of the general theory, followed by directions for each specific experiment. If any experiment requires more details as to the theory, these are given under the experiment heading.

The experiments are designed for *even front;* that is, enough sets of apparatus are utilized to enable a whole class to work in pairs on the same experiment. This makes it possible to use apparatus which is of a simpler and less expensive type. Occasionally, when experiments are inserted for completeness which by their nature require a more expensive piece of apparatus, a demonstration laboratory period may be substituted.

The authors' aim has been to make many of the electrical experiments inexpensive by the substitution of commercial rheostats, resistance units, etc., in place of the more expensive resistance boxes when the experiment does not warrant the accuracy of the latter. The apparatus mentioned is listed at the beginning of each experiment. Many pieces of apparatus mentioned in this text may be made cheaper and often better than those which can be bought, provided the facilities of a machine shop are available.

The experiments are designed for at least a full two-hour period of laboratory experimental work, but if one wishes, the period may be shortened by leaving out parts of the experiment or by taking fewer readings.

The book starts with a study of the *precision of measurements* so that the student may make a statement as to the accuracy of his work in every experiment performed. In the beginning, sample data are given and tabular forms suggested. Such details are made less definite, however, as the experiments progress. The purpose of this procedure is an attempt to teach the student to do his work in an orderly and systematic manner, first by example, and finally, by working out some orderly system of his own. The student is quite free to use his own ingenuity when obtaining his data in any way that he thinks will give him the best results. Every experiment is written in sufficient completeness, as to theory and laboratory instructions, so that the student may start his laboratory work immediately upon entering the room without instruction as to theory or procedure.

In order to make sure that the student will read the theory as well as the experiment before he enters the laboratory, it is well to require each student to solve, before the laboratory hour, the *problems* which have been placed at the end of each chapter for that experiment. It should be noted that these problems, which involve making statements concerning the theory as well as solving equations for numerical answers, cannot be done without some knowledge of the theory contained in the chapter.

In addition, at the end of every experiment there are *questions* concerning data, to be answered and passed in with the finished report. These questions, when answered and submitted, will help to make the student more careful in the taking of data and will give him a better understanding of his experiment as a whole. These questions will also serve, it is hoped, to suggest many other and perhaps more helpful ones to the instructor.

The authors take this opportunity to thank their fellow-members of the Physics Department at Washington Square College who have contributed valuable criticism and help in the choice and arrangement of material used in this text.

W. A. S.
L. B. H.

NEW YORK CITY
April, 1932

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CHAPTER I

INTRODUCTION

Physics, like the other sciences, is playing an ever-increasing part in shaping our environment and our mode of living. Were it not for the knowledge and application of the laws of Physics, one could not travel through space, ride under the sea, view the hitherto unknown celestial bodies, or examine the minutest forms of life and matter. Present-day labor-saving devices — the electric lights, radio, telegraphy, telephones, and the ocean cable — are often taken for granted and little thought is given to the years of scientific research which produced them. What is the fascination which makes great minds devote years of study to physical phenomena in order to present to the world wonders which will all too soon be labeled " necessities "? Is it not because there are always untold possibilities ahead — a story of surprises awaiting?

The Laboratory

Let us then consider the laboratory, which is the workshop of the scientist, as a place of interest where each one can perform for himself the experiments which brought joy to the heart of the man who first worked them out. Consider it a place where one can learn by actual practice to understand better the principles of and the care for mechanical devices which are used in everyday life. Remember that seeing and working with objective material and apparatus gives one the first-hand knowledge which is so valuable in rendering the descriptive more concrete, comprehensive, and usable, and that it initiates a better understanding of the formulation of laws, illustrates the working of a principle, concentrates attention to detail, and stimulates the exercise of deliberate judgment.

Success with an experiment comes only when all possible knowledge and underlying theory of the experiment have been obtained before starting. The teacher helps the student to acquire this in ways which he sees fit, and he points out certain delicate adjustments of apparatus and means of taking advantage of situations to obtain more accurate results, *but* it is up to the student to grasp and retain the lessons taught in the laboratory. Others cannot do this for him.

Conduct. The instructor expects his students to be honest and interested in their experiments, careful of University property, and methodical in replacing or leaving apparatus at the end of each class session. If the semester's work in the laboratory does not develop the habit of inquiry, the respect for seasoned opinions, the ability to handle apparatus carefully and intelligently, and the value of system and order in any work, then it has not accomplished its purpose.

Written reports. The instructor decides upon the particular method and form of the written report, but the following headings might be remembered as important in any write-up of an experiment: object, description, conventional drawing of apparatus, theory, data, conclusion, and discussion of errors. The treatment of each of these divisions will depend upon the wishes of the instructor. Avoid the use of personal pronouns in written reports.

UNITS

In laboratory work, generally speaking, results not given in definite units mean little or nothing. For instance, the numerical values of density depend upon the system of units used. Some results, such as specific gravity and specific heat, appear as abstract numbers because they are defined as ratios of numbers having the same units.

There are *three systems* of units in common use: the c. g. s. system (centimeter, gram, second), used almost exclusively by scientists; the f. p. s. system (foot, pound, second); and the gravitational system. The first two are called absolute systems because the derived units bear the simplest possible relation to the fundamental units of length, mass, and time. Thus, in the two absolute systems as stated, the unit of force (*i.e.*, dyne, poundal) is defined as that force which will give a unit mass, unit acceleration. In the gravitational system, the unit force (pound weight) is the force acting on a unit mass (pound) due to the attraction of the earth. Since this force varies, depending upon the location, it is seen that the pound weight is not an absolute unit of force. It is sufficiently accurate, however, for practically all engineering work.

As an example of the use of units, suppose that a train is observed to have a steady velocity of 36 kilometers per hour for thirty seconds. What distance was traversed in this time interval? The distance (s) traveled would be the velocity (v) times the time (t), *i.e.*,

$$s = vt.$$

The answer in centimeters is

$$s = \frac{36 \times 1000 \times 100}{60 \times 60} \times 30$$
$$= 3 \times 10^4 \text{ cm.}$$

It is better practice in general to convert all units, before using them in equations, to fundamental units, *i.e.*, $36 \frac{\text{km.}}{\text{hr.}} \approx$ (is equivalent to) $36 \times 1000 \times 100 \frac{\text{cm.}}{\text{hr.}}$ or $\frac{36 \times 1000 \times 100}{60 \times 60} \frac{\text{cm.}}{\text{sec.}}$, which in turn $=$ (equals to) $1000 \frac{\text{cm.}}{\text{sec.}}$. Notice that dimensionally the product, vt, gives $\frac{\text{cm.}}{\text{sec.}} \cdot \text{sec.}$ or cm. (The dot is often used to denote multiplication.) This is the same as the dimension found on the left-hand side of the equation (in other words, s).

Whenever large numbers are involved, it is better to express them in positive powers of ten. Thus, the number 237,000,000 may be expressed as 2.37×10^8. Likewise, all small decimal quantities may be expressed in negative powers of ten, *i.e.*, $0.0000000237 = 2.37 \times 10^{-8}$.

CHAPTER II

PRECISION OF MEASUREMENTS

Errors which enter into the results of a series of observations may be classified as personal, accidental, systematic, and instrumental errors. The magnitude of the personal errors depends upon the observer's experience and follows the law of chance unless there is a personal bias.

Personal bias. A common example of the personal bias of beginners is their tendency to give to a first reading of a series greater significance than the succeeding readings. A beginner asked to take three readings of the length of a table with a meter stick, to the nearest tenth millimeter, will bring back, generally, three readings all alike. Consider Figure 1, which represents a plan of the table top. Besides inaccuracies in the placing of the meter stick, it should be observed that the length must vary, depending upon whether the measurements were taken at A, B, or C.

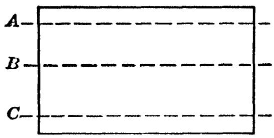


FIG. 1. — Measuring the length of a table top.

Personal errors. Personal errors include inaccurate settings, inaccurate estimations of a fractional division, insufficient development of one or more of the six senses, and parallax. Matching of color intensities, pitch of two sounds, and timing results are examples of measurements depending upon the use of sense organs. The error due to parallax will be discussed later (see page 17).

Accidental errors. Errors which are beyond the control of the observer are called "accidental." For instance, suppose galvanometer readings are to be taken every fifteen seconds. A sudden jar will cause the galvanometer to alter its readings from an otherwise good result. Other examples of "accidental" errors are found in fluctuations in magnetism, noise, temperature, pressure, electricity, wind, and the like.

Systematic errors. Such errors are characterized by their tendency to one direction only, *i.e.*, positive or negative. They may or may not be easily traced. For instance, if a meter stick is always used by taking measurements from one end, then this end may get worn and a constant error will occur in all measurements made with this meter stick when measuring from this end. If not detected, systematic errors may give a result far from the true result, and are, moreover, not accounted for in any calculations of the experimental error.

Instrumental errors. New instruments are usually calibrated to a certain degree of accuracy. If not, they should be calibrated in the laboratory or sent to some place, such as the U. S. Bureau of Standards for calibration. Once the required per cent accuracy is decided upon as necessary for the experiment, one should see that the apparatus used for any measurement has an accuracy a little better than that demanded for the tests. However, a calibrated instrument is of no value as a precision instrument unless operated under conditions similar to those used in its calibration. For instance, a steel tape calibrated for 20° C. would not be accurate for winter use unless its temperature coefficient is known. Furthermore, a calibrated instrument is reliable only with careful and intelligent use.

Arithmetical mean — most probable result. The accuracy of an experiment, then, depends upon a number of factors or conditions, many of which are not easily ascertained. In our discussion we shall assume that the apparatus is sufficiently accurate for the experiment and that there exists no personal bias nor any systematic error. This leaves us with certain personal and accidental errors which are subject to pure chance. With these limitations as to the nature of errors to be considered, our judgment tells us that the arithmetical mean of a number of observations will give us the most probable result. Actually, this represents the most probable reproducable result with the apparatus available rather than accuracy. Accuracy is better checked by other independent experiments using different methods of experimentation. Hence, such words as "per cent of error," "probable error," etc., refer to the mean result as found, since the true result is seldom known.

Calculation of Errors and Per Cent Errors

When an experimental result is to be obtained by direct measurement, we take one or more readings, depending upon the accuracy desired. Suppose a single measurement of length, less than a meter, is to be made with a meter stick. Let the measurement be 34.3$\bar{5}$ centimeters. All figures, except the last digit which has the dash over it, are measured digits. The last digit ($\bar{5}$) is doubtful. The most inexperienced observer could estimate probably to a half millimeter division. For the single reading above, we will designate as the error $\pm$ 0.05, the plus or minus sign ($\pm$) indicating that the reading may be too small or too large by this amount. Such estimations on a single reading are generally taken large enough to be called, with some reservations, a " maximum possible error." For this reason, the term " maximum possible error " is used by some authors. Our reading now becomes 34.35 $\pm$ 0.05 cm., which gives a per cent error of 0.15 $\left(i.e., \dfrac{0.05}{34.35} \times 100\right)$. If anything more than an approximate answer is required, more than one reading is essential. However, *when a single measurement only is taken, we shall designate all digits, including the first doubtful figure, as significant figures.*

When a number of readings of a given length are taken, we must deal with an average having a certain error. Calculations of the accuracy from such considerations is our present problem and is important since the result of our experiment has to represent an average of various readings and calculations. Consider, for instance, the following readings to have been taken for a given distance: 152.2$\bar{8}$, 152.3$\bar{6}$, 152.3$\bar{0}$, 152.3$\bar{2}$, and 152.3$\bar{8}$ cm. Notice that since the meter stick must be reset at least once for each measurement of length, the individual errors now are greater than the error in the single measurement of less than a meter, as illustrated in the previous paragraph. Having five readings for the same length, how shall we now find the number of significant figures? This is done by calculating the average (152.328 cm.) and the average error ($\pm$ 0.034 cm.). The **average error** is found by subtracting the mean from each reading, adding the errors without taking into account the signs, and dividing by the number of readings. In the above, the total error, neglecting signs, in five readings is $\pm$ 0.168, so that the

Reading	*Deviations*
152.2$\bar{8}$	− 0.048
152.3$\bar{6}$	+ .032
152.3$\bar{0}$	− .028
152.3$\bar{2}$	− .008
152.3$\bar{8}$	+ .052
5)761.64	5)$\pm$ 0.168
152.328	$\pm$ 0.0336

Length = 152.328 $\pm$ 0.034 cm.

average error is $\pm$ 0.034 cm. Now, in case of averages, we shall retain two doubtful figures in the result. Hence, *the number of significant figures where averages are considered will be designated as all digits including the first two doubtful figures.* This gives as the result of the above experiment a length of 152.328 $\pm$ 0.034 cm. The first doubtful figure is in the hundredths' place, so that the 2 is in doubt by 3.4 while the 8 is in doubt by 34.

The theory of least squares. The reason for retaining two doubtful figures in the case of averages is not given above, but may be justified if one studies the question of errors by means of the theory of least squares. The theory of least squares is based upon the probability curve. From this curve, we may find the "probable deviation," a technical term the numerical magnitude of which is such that the true error has equal chances of being larger or smaller. Now the "probable deviation" may be shown to be 0.6745 S, if S represents the root mean square error, which is the square root of the sum of the squares of the errors divided by the number of readings. In the previous example, where the length was found to be 152.328 $\pm$ 0.034,

$$S = \sqrt{\frac{(0.048)^2 + (0.032)^2 + (0.028)^2 + (0.008)^2 + (0.052)^2}{5}}$$
$$= \pm\ 0.037$$

and the probable deviation is

$$0.6745\, S = \pm\ 0.025 \text{ cm.},$$

which means that the true error is equally likely to be greater or smaller than $\pm$ 0.025. The theory of least squares, however, is based upon an infinite number of readings, so that the above value for the probable deviation error is only approximate. In general, the small number of readings taken in elementary work is not sufficient to conform very closely to the requirements of the theory. Moreover, to use the complete theory of least squares would lead us into detail which is beyond the scope of the present work. However, if the above example is representative, it may be inferred that the average error is a liberal estimate of the error of one's work. We shall use the average error in the estimation of accuracy of our future work in the laboratory.

It is often desirable to express the **error in per cent.** Thus for the measurement of the length referred to previously, the per cent of error is $\left(\frac{0.034}{152.328} \times 100\right) = 0.022\%$. It is to be observed that this is a measure of reproductibility and is not necessarily a true deviation from the "real" answer. For ordinary laboratory experiments, from three to five measurements are sufficient to establish a working average.

When a series of measurements is to be taken and it is desired to express the results graphically, one measurement is taken for each point on the graph. Thus in plotting deflections of a galvanometer, one reading is observed each on the "black" and "red" side for a particular value of current, and then a new value of the current is chosen. The curve itself will show qualitatively the errors and characteristics of the instrument.

Errors in indirect measurements. So far, we have assumed that the desired results depended only upon certain direct measurements. An extension of the above rules for finding errors will be necessary when the result is obtained by computation. Thus, to find the moment of inertia I from the equation:

$$I = I_0 + mh^2,$$

or, to find the surface of the sides of a cylinder from the equation:

$$S = 2\,\pi rl,$$

or, to find the area of a circle from the equation:

$$A = \pi r^2,$$

or, to find the value of g from the equation:

$$g = \frac{4\,\pi^2 l}{T^2},$$

one must perform the arithmetic calculations of addition, multiplication, squaring, and division respectively. It is assumed that each of the factors involved is an average obtained from several readings and that the error in each factor has been computed. Upon these assumptions, the following rules for errors in computing results are given:

Rule I. In *addition* or *subtraction*, retain as the error in the final result the largest numerical error found in any one of the quantities, *e.g.*:

Addition Distances in cm.	Per Cent Error
25.20 ± 0.23	0.91
5.312 ± 0.021	0.39
1.2534 ± 0.0025	0.20
31.76 ± 0.23 cm.	

Subtraction Distances in cm.	Per Cent Error
13.21 ± 0.022	0.17
7.315 ± 0.026	0.35
5.89 ± 0.026	

Thus, in the example of addition shown above, the result is 31.76 ± 0.23 cm., while in the example on subtraction, the result is 5.89 ± 0.026 cm.

Rule II. In *multiplication* or *division*, retain as the per cent of error the largest per cent of error found in any one of the terms. Thus the product, (12.57 ± 1.8%) × (1.325 ± 1.3%), is 16.66 ± 1.8%.

$$
\begin{array}{rll}
12.\bar{5}7 & \pm\ 0.23, & 1.8\% \\
1.3\bar{2}5 & \pm\ 0.018, & 1.3\% \\
\hline
\bar{6}\bar{2}\bar{8}\bar{5} & & \\
\bar{2}\bar{5}\bar{1}\bar{4} & & \\
37\bar{7}\bar{1} & & \\
12\bar{5}\bar{7} & & \\
\hline
16.\bar{6}\bar{5}\bar{5}\bar{2}\bar{5} & &
\end{array}
$$

Answer: 16.66 ± 1.8%.

The above rules will give a sufficient estimation of the accuracy of computed results for our purposes. These rules, in turn, give us certain hints as to the accuracy necessary in the various measurements. Thus with additions and subtractions the numerical error is important. This means that if the 25.20 cm. in the addition problem under Rule I had been measured with a meter stick, the other distances which were to be added to it could have been measured equally well with a meter stick. The accuracy of the other two figures (5.312 and 1.2534) show that they were measured with more accurate instruments. With multiplication and division, however, it is the per cent of error which is important. Hence, in this case, small quantities should be measured more accurately.

One may raise a question as to the reason for retaining four figures in the result of the multiplication. The reason for this will be seen by observing that in the multiplication all doubtful figures have a dash over them. The quantities to be multiplied show doubtful digits beginning with the third. Therefore, the number of significant figures in the result is four. When a number is squared, its per cent of error is multiplied by two, since the error in each factor would necessarily be in the same direction and of the same amount. Similarly in taking a square root, the per cent of error is divided by two.

When multiplying or dividing two factors, retain a number of digits in the result equal to the number of digits in the factor containing the smaller number of significant figures. In the multiplication shown, the number of significant figures was four in each factor. Therefore, the number of significant figures in the result is four. (In a product, such as $125\bar{7}\bar{2} \times 13\bar{4} = 1\overline{684648}$, the result may be expressed as 168×10^4.) This rule concerning significant figures is important especially where the result obtained is to be multiplied or divided by a third factor.

Slide rule. The number of significant figures in one's result indicates whether one could use a slide rule profitably. There are few calculations in the elementary laboratories which have more than three or four significant figures. Consequently, a slide rule is a very great time-saver in the laboratory.

Per cent of error from some standard value. Occasionally, the student is asked to check the value of a constant, say g, with his apparatus. If the accepted value for his location is $980.2\ \frac{\text{cm.}}{\text{sec}^2}$. and he gets an average value of $978.5\ \frac{\text{cm.}}{\text{sec}^2}$, the per cent of error from the accepted value is given as $\frac{980.2 - 978.5}{980.2} \times 100 = 0.17\%$.

Per cent of error from the mean. If two results are given with either one being equally probable, *i.e.*, $1\bar{2}\bar{8}\bar{1}$ and $1\bar{2}\bar{5}\bar{3}$, then the per cent of error from the mean is given as

$$\frac{1281 - 1253}{1281 + 1253} \times 100 = 1.1\%.$$

That is, take the difference, divide by the sum, and multiply by one hundred.

GRAPHICAL RESULTS

To show the relation between one variable and another, if any definite relation does exist it is often best to resort to plotting a curve. By this method, the mathematical relation existing between the two quantities may sometimes be determined. In any case, a graph represents pictorially and in concise form the nature of the results obtained. The curve should be drawn *smoothly* so as to fall on as many points as possible. The points which are noticeably in error should fall in approximate equal numbers on each side of the curve. When such a curve has been drawn, the magnitude of the errors is shown qualitatively by the distance which the points in error fall outside the smooth curve. If one wishes to find the value of the quantity for intermediate positions on the curve where readings were not actually taken, the curve is sufficiently accurate for that purpose. If the accuracy of the final result is desired, the per cent of error should be found by the methods outlined previously.

Whenever possible, the coördinates used for abscissae and ordinates should be so chosen from the experimental data that the plotted points will fall on a straight line within the limits of experimental error. A straight line is the only graph easily examined. If the graph is a curve, one might have considerable difficulty in determining whether it is a parabola, hyperbola, or even an ellipse, particularly if only a small portion is shown. A few illustrations of curve plotting are given.

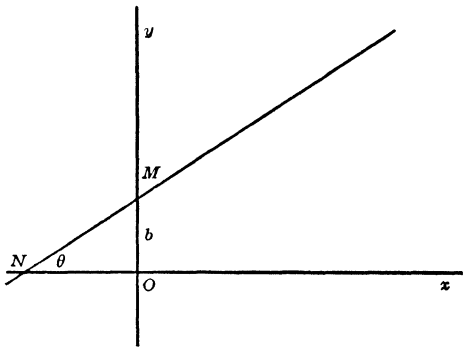


FIG. 2. — Graph of $y = ax + b$.

Example 1. Suppose that in a certain experiment it is found that y varies directly with x, as shown in the diagram. The graph is a straight line and any straight line can be represented mathematically by an equation of the form,

$$y = ax + b.$$

If this line cuts the x and y axes at N and M respectively, then it can easily be shown that the distance OM represents the value of b, and the slope of the line (*i.e.*, the tangent of the angle θ) is represented in the equation by a, and in the diagram by $\frac{OM}{NO}$ (Fig. 2).

Example 2. If we plot y against $\frac{1}{x}$, and obtain a straight line through the origin, it means that

$$y = c\left(\frac{1}{x}\right),$$

where c is a constant. If y represents the pressure and x the volume of a gas, then we have an experimental verification of Boyle's law ($pv = c$) if, when p is plotted against $\frac{1}{v}$, a straight-line graph results.

Example 3. If the electric current I is plotted against the heat-developed H, the curve is not a straight line but will be a straight line if I^2 is plotted against H. The known relation existing between these two quantities is

$$H = I^2\frac{R}{J},$$

when R and J are constants.

When one finds that the quantities which are plotted on the graph are not directly proportional to each other, and one suspects that one of the variables raised to some power is proportional to the other raised to some different power, then it is better to take the logarithms of both quantities before plotting them.

Thus, suppose the actual relation between x and y is

$$y^n = ax^m,$$

it being necessary to find the constants a, n, and m from the experimental data. When the logarithm of both sides is calculated, we have

$$n \log y = \log a + m \log x.$$

Now plotting $\log x$ against $\log y$ (Fig. 3), instead of x against y, we get a straight line; and from the intercepts on the $\log x$ and

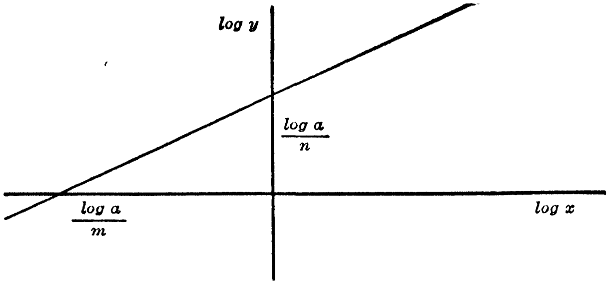


FIG. 3. — Graph of $n \log y = \log a + m \log x$.

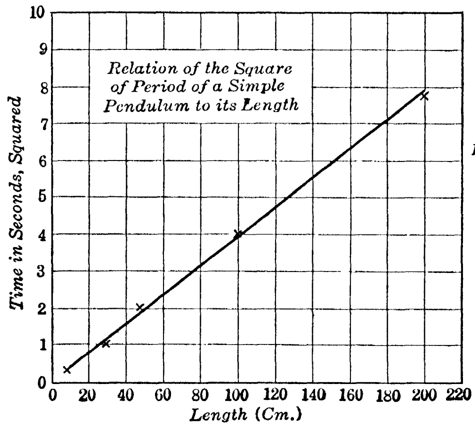
log y axes, we can find the constants log a, n, and m. In order to facilitate this procedure and save time in calculating the logarithms of the different values of x and y, it is possible to purchase log-log graph paper in which the axes are marked off in logarithmic units. In using this paper it is then only necessary to plot values of x and y directly, and if a relation of the above type exists, a straight line will result.

The following directions will be found helpful in plotting a useful curve:

1. Use coördinate paper with rulings of one millimeter on sheets of convenient size (say $8\frac{1}{2}$ by 11 inches).

2. Plot to such a scale that all significant figures will be used. The curve should be drawn to as large a scale as the sheet will allow.

3. Place a small dot (or cross) at every located point (Fig. 4).



Data:

Period (sec.)	Length (cm.)
0.6	10
1.0	25
1.4	50
2.0	100
2.8	200

FIG. 4.—An example of curve-plotting.

4. Every curve should have a title indicating which two quantities are plotted along the two axes.

5. Draw heavy lines for the coördinate axes and label the axes, the independent variable along the x axis and the dependent variable along the y axis.

6. The origin of the coördinate axes need not be shown on the curve.

7. If a relation exists between the quantities, draw, by means of a flexible rule, a smooth curve through the points. If all the points do not actually fall on a smooth graph, then draw the curve

so that approximately as many points will be on one side as on the other. A point that deviates from the curve markedly should be discarded as an accidental error.

PROBLEMS

1. Express the following numbers in powers of 10 and state the number of significant figures in each, assuming that they are correctly expressed: 0.0231, 10.32, 10306, 101.30.

2. Express the following results with the correct number of significant figures: 2.31 ± 0.023, 1325 ± 10, 123.5 ± 13, 1.25127 with accuracy to 0.5%.

3. Write the following physical constants in powers of 10, giving the proper number of significant figures:

Velocity of light = 29979600000 ± 400000 $\frac{\text{cm.}}{\text{sec.}}$

Mean radius of the earth = 637100000 ± 3500000 cm.

Mass of a hydrogen atom = 0.0000000000000000000000016617 gms. (error of 17 in 16617)

Mechanical equivalent of heat (15° C.) = 41852000 ± 6000 $\frac{\text{ergs}}{\text{cal.}}$

Gravitational constant (G) = 0.00000006664 (error of 2 in 6664).

4. What are the significant figures for the results of the following calculations:

(a) (1.372 ± 0.031) × (235 ± 11) × (0.1765 ± 0.0025)?

(b) (23.275 ± 0.015) + (101.3 ± 8.1) + (265.7 ± 1.5)?

What is the per cent of error in each of the above calculations?

5. Multiply 123 × 789 on the slide rule. Calculate your per cent of error from estimation of the error in reading the instrument. How does your resulting estimated per cent of error compare with the actual per cent of error? Repeat the above calculations for the following multiplication on the slide rule: 115 × 78 × 67.

6. The following readings of the time (in seconds) of 50 vibrations of a simple pendulum are found to be: 78.5, 78.1, 78.4, 78.2, 78.5. Calculate the average deviation and the per cent error in the above readings.

7. List the various kinds of errors that may arise in a physical measurement and discuss briefly the possibility of eliminating or calculating the error.

8. Five stones are dropped over a cliff in order to estimate the height of the cliff. The time (in seconds) noted in each case is 6.2, 6.8, 6.7, 6.7, and 6.3. Assume that $g = 32.19 \pm 0.01 \left(\frac{\text{ft.}}{\text{sec.}}\right)^2$. Calculate the height of the cliff from the formula

$$s = \tfrac{1}{2} gt^2,$$

where s is the height of the cliff in feet. Calculate the per cent of error in the answer.

9. Plot a curve showing the relation between the vapor pressure (or tension) of water (in cm. of mercury) and the temperature (from 0° C. to

103° C.). Look up the values of this pressure at various temperatures in a book of physical tables. Pressure should be plotted along the ordinate and temperature along the abscissa.

10. Perform the following calculation, using logarithms:

$$\frac{4324 \times (5692)^2 \times 0.0003468}{6824 \times .3021 \times 4568000}.$$

CHAPTER III

MEASUREMENT OF THE FUNDAMENTAL UNITS

The Measurement of Length

This is perhaps the most widely performed, as well as the simplest of the three quantities, length, mass, and time. In our everyday experiences, we frequently have to measure some length or other. Very seldom, however, do we stop to consider whether we are really performing the measurement with the best rule or apparatus for that particular purpose. Still less do we concern ourselves with the question of accuracy of measurement. The reason for this unconcern of ours is to be found in the fact that someone has already considered these points very carefully and the apparatus which we have used has been designed accordingly. The same is true when we have to measure mass and time, the other two fundamental quantities.

In the physical laboratory, we cannot be satisfied with such a superficial view, but must consider both the accuracy of our instruments and our measurements carefully. They become of much greater importance when a student of physics has devised some new and easier method of performing a certain measurement, which, according to his ideas, will give more accurate results. In such a case, a knowledge of the accuracy and reproducibility of his apparatus and of the precision of his result is necessary. The research worker in the field of physics or chemistry must be familiar with the different methods at his disposal, and with their relative accuracy, as well as the methods of calculating or estimating the errors of observations.

In these first three or four experiments, therefore, the student should try to discover for himself the precision that he has obtained with his apparatus; and in the future, when measuring a length, mass, or time interval, adapt the method to the precision required in the result.

The meter rule. The simplest way of measuring length is with the aid of an ordinary scale or rule. (A metric scale or " meter bar " is most frequently used in the laboratory.)

There are several errors that have to be considered when using such an instrument. The inherent accuracy in the instrument is limited by the fact that the lines on the scale have a finite thickness. *Observational errors* occur when estimating the fraction of the smallest division. When estimating the fraction of a millimeter on an ordinary meter rule, an uncertainty of 0.1 of a millimeter is about the limit of accuracy for all but the highest skilled observers. An error of equal or larger amount is probable in placing the object on the zero reading from which the measurement is made. Errors of this type can be minimized only by taking a sufficiently large number of readings. Another frequent source of error is the *error of parallax.* This error occurs when a scale of finite thickness is used and the eye is not always vertically above the scale and point being read, or even in the same relative position with regard to the scale. This will be more clearly seen in Figure 5. When the eye is placed at C and E, the distance AB will

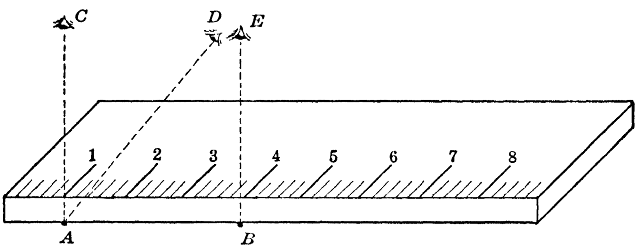


FIG. 5. — Error of parallax.

be read correctly. This will not be the case, however, if the eye should be placed at E and D. In practice, therefore, whenever it is possible, the rule should be placed in such a position — on edge in this case — so that the markings on the scale fall right on the points A and B. The nearer the division marks on the scale to the points being read, the smaller is the error due to parallax. In very many pieces of physical apparatus this error of parallax causes additional difficulties, and consequently a large number of ingenious artifices have been invented to overcome this error. Such an example is shown in Figure 6, which represents the method used very often with the better class of electrical voltmeters and ammeters. Since in these cases the pointer has to swing freely over the scale, the error of parallax may be present. It is overcome in this type of instrument by mounting a mirror underneath the pointer. When looking into the mirror, right underneath the pointer, an image of the latter will be seen. By adjusting the position of the eye above the pointer, this image can be made to

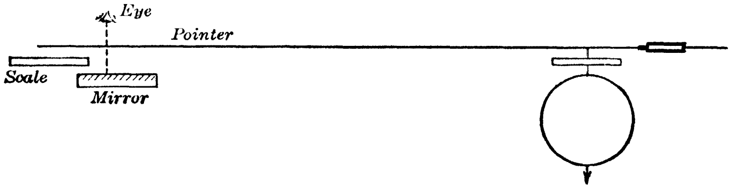


FIG. 6. — Arrangement of the points and scale in a voltmeter.

disappear underneath the pointer. When this is the case, the eye is vertically above and the reading on the scale, keeping the eye in this position, can then be taken accurately.

Very often the object or distance being measured cannot be placed along the side of the scale, or *vice versa*. Such cases as the inside or outside diameter of a vessel, the outside diameter of a sphere, etc., come into consideration here. The method used involves the use of transfer instruments whereby we transfer the original dimension to a pair of dividers, or " calipers " as they are called, and then measure the distance on a scale between the legs of the transfer caliper. Figure 7 shows both inside and outside calipers.

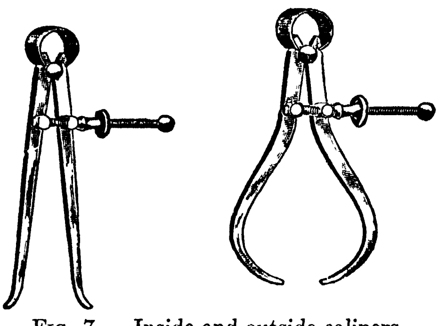


FIG. 7. — Inside and outside calipers.

The vernier scale. In taking a measurement with an ordinary meter scale, we try to estimate to tenths of a division. This requires a large amount of skill, and, even then, when the divisions on the scale are small, an error of one- or two-tenths is quite probable. A microscope will help in such cases.

A very ingenious device was discovered by P. Vernier (1580–1637) for the purpose of estimating this fraction of a division with great accuracy. The great advantage of his method is that we can measure to any fraction of a division, be it to tenths, twelfths, twenty-fifths, hundredths, etc.

The instrument consists of a scale — called the vernier scale — which can slide next to the ordinary scale. This vernier scale has divisions on it which may be either a little smaller or a little larger than the divisions of the ordinary main scale along which it slides. In most cases the divisions on the vernier are smaller than those on the main scale, and it is this case which we will consider here in detail. The vernier scale is marked off into n equal divisions, the one end being called zero and usually marked correspondingly or with an arrow. Placing this zero mark of the vernier on any main scale division, it will be seen that in the case under consideration n divisions of the vernier correspond to $n - 1$ divisions on the main scale. Hence each vernier division is $\frac{n-1}{n} = \left(1 - \frac{1}{n}\right)$ of a main scale division. Consequently the vernier division is $\frac{1}{n}$ shorter than a main scale division. This quantity $\frac{1}{n}$ of a main scale division is called the *least count* of the vernier and is usually expressed in centimeters or inches. Always determine the least count of a vernier before attempting to make a measurement.

Suppose next that we wish to make a measurement and find the zero of the vernier somewhere between two main scale divisions. From what has been said above about the least count, we can readily see that if the zero of the vernier is $\frac{1}{n}$ division (main scale) beyond the division line of the main scale (Fig. 8), then the first

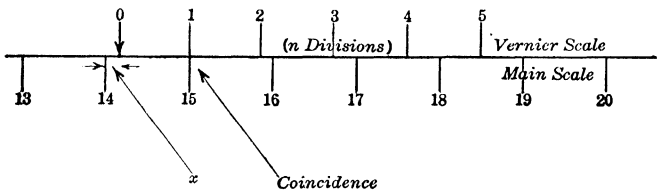


FIG. 8. — Reading 14.1 if $n = 10$.

vernier division should coincide with a division line on the main scale.

Similarly, if the zero of the vernier should be $\frac{2}{n}$ divisions to the right of the main scale division, we should expect the second line beyond the zero on the vernier to correspond with a main scale division. The reason for this, of course, is that the first vernier division reduces the discrepancy between vernier division and main division by $\frac{1}{n}$, and the second by $\frac{2}{n}$, therefore making them coincide on the second division of the vernier (see Fig. 9).

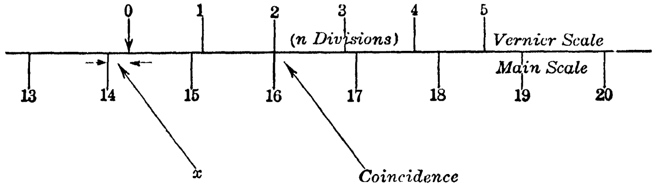


FIG. 9.—Reading 14.2 if $n = 10$.

In general, then, if we find the k^{th} division of the vernier coincides with a scale division, we will be able to make out that the part x, which we had to estimate with our eye previously, is exactly $\frac{k}{n}$ main scale divisions. *Always bear in mind that it is the zero mark on the vernier the position of which we are trying to locate as accurately as possible on the main scale.*

The procedure in reading an instrument having an attached vernier scale is as follows:

1. First determine the least count. This is usually done by moving the vernier along the main scale so that the zero mark on the vernier coincides with some division on the main scale Find out, by looking along the vernier, how many divisions on the vernier are necessary until a vernier and a main scale division coincide again. This enables one to determine the least count, as described above (*i.e.*, $\frac{1}{n}$ of a main scale division). Knowing the value of the main scale division one can express the least count in inches or centimeters.

2. Set the vernier on the instrument so as to measure the length of the required object and estimate approximately its length, by noting the position of the zero of the vernier. In Figure 9 this would be 14^+. . . .

3. Determine the fraction x by noting which vernier division coincides with a main scale division. In the above figure this would be the second. This gives for a final result, then, $14 \frac{2}{n}$ main scale divisions.

The least count is usually chosen as to give the required measurement in convenient and practical c. g. s. or f. p. s. units. Common arrangements on instruments are: in the c. g. s. system 0.01 cm. (Fig. 10), and in the f. p. s. system $\frac{1}{128}$ or $\frac{1}{1000}$ inch.

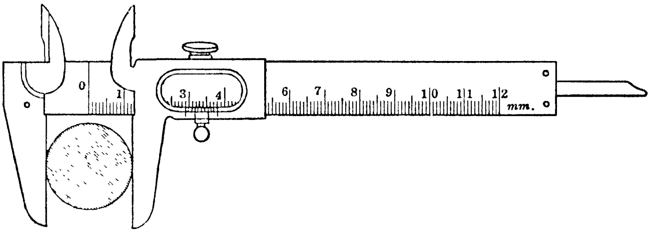


FIG. 10. — Vernier caliper, reading 2.89 cm.

In the accurate measurement of angles by means of a vernier the procedure is exactly the same. The verniers are usually arranged to read to a minute of arc (*i.e.*, $\frac{1}{60}$ degree). This is very often accomplished by using for the main scale division a unit of $\frac{1}{2}$ degree and placing thirty divisions on the vernier to correspond with twenty-nine divisions ($\frac{1}{2}$ degree) on the main scale. When this is the case, one should not forget part 2 of the above procedure. Be sure to see whether the fraction x is in the first or second half of the larger degree divisions. If it should fall in the second half, then we must add a half-degree to x in expressing our result in degrees.

A little practice will help considerably in understanding the above principles. A well-made vernier scale forms one of the most useful, and therefore most frequently used, adjuncts to physical apparatus having to do with the measurement of lengths or angles.

The micrometer caliper. The measurement of the size of small objects, or the comparison of lengths of objects that do not vary very much in size, can be done with a higher degree of precision by using a micrometer screw.

The instrument, which is shown schematically in Figure 11, is usually made in a more or less semi-circular form and has two so-called "jaws." One of the jaws is fixed and the other is movable. The movable one is made to advance a certain fixed distance for every revolution by having cut on it an accurate thread. The *pitch*, or distance that the movable jaw advances per revolution, is usually made to be 1 mm. or 0.5 mm. in the c. g. s. system and $\frac{1}{40}$ inch in the f. p. s. system. If now we attach a "head" on the end of the screw, which we divide up into a large number of equally spaced divisions, then we can measure the fraction of a

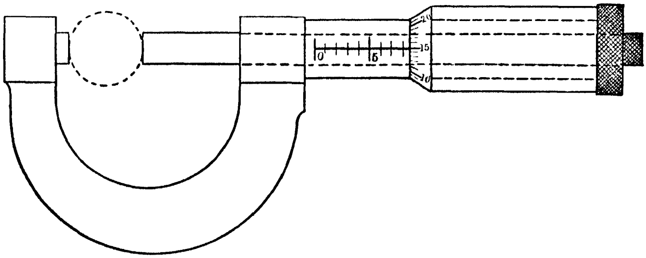


FIG. 11.—Micrometer caliper.

turn that the screw is advanced. In the case of a 0.5 mm. pitch screw with fifty divisions on the head, we can therefore measure to $\frac{1}{50}$ of $\frac{1}{2}$ mm. = $\frac{1}{100}$ mm. = 0.01 mm. In addition to the divisions on the screw head a horizontal scale is usually engraved along a fixed cylindrical barrel so that the whole number of turns will be indicated. Remember again that in this case the reading on the head, which goes up to fifty, may be in the first or second half of the millimeter, and if the latter is true, then 0.5 mm. must be added. When the pitch happens to be 1 mm. and there are 100 divisions on the head, then the last precaution mentioned is not necessary.

A good micrometer gauge or caliper has at the end of the movable jaw a friction or ratchet device which, when used, prevents too much pressure being applied to the jaws. This ratchet device serves a double purpose. First, it prevents the operator from applying too great a force to the jaws, thus damaging the thread and jaws and so making them useless for accurate measurements. Secondly, in many cases when too great a pressure is applied, the object being measured will be slightly deformed and an error introduced in the result.

The following *procedure* is suggested when using a micrometer screw gauge:

1. Study the *pitch* of the screw by turning the head through a certain number of counted revolutions and noting the movement on the horizontal scale. Observe how many divisions there are on the head and from this determine the amount that the screw

advances for a rotation of only one division on the head (this might be called the "least count" in this case). In the above example this amount is $\frac{1}{50}$ of $\frac{1}{2}$ mm. = 0.01 mm.

2. Study the *zero setting*. To test the zero, screw up the movable jaw until it just touches the fixed jaw and see whether the zero checks. If it does not, allowance must be made for a zero correction in future readings. In the better-made instruments the zero settings can be adjusted. This adjustment, however, should not be done by the student.

3. Insert the object between the jaws, using the *ratchet device* to insure just the right pressure, and take the readings.

EXPERIMENT 1

THE MEASUREMENT OF LENGTH

Part (a). *To measure the length of an object by means of a meter rule, vernier caliper, and micrometer screw, and to determine the probable error in each case.*

Part (b). *To become familiar with vernier scales as used in some physical instruments.*

Apparatus: *Part (a).* An object such as a cylinder (metal) about 2 cm. in length (or even a coin), a metric rule, a vernier caliper, a micrometer screw gauge.

Part (b). Instruments which have vernier scales attached, such as a barometer, spectrometer, Jolly balance, sextant, etc., placed about the laboratory.

Part (a). 1. *Determine the diameter of the coin using the metric rule.* Avoid errors of parallax. Take five readings of the diameter yourself, using various parts of the metric rule. Ask your partner to do the same. In each case estimate the fractions (tenths) of a millimeter. Record all your readings in tabular form.

From the data obtain the average diameter, the average deviation or "error." Express your result (the diameter) finally with its average error as well as the per cent of error.

2. *Determine the diameter of the coin using the vernier caliper.* Having studied carefully the least count, etc., determine first the zero correction. Next insert the object in the caliper, being careful not to force the jaws, and take five readings, turning the object so as to get an average value should the coin not be round. Ask your partner to take five more. Record all the readings in

tabular form. From the data determine the average diameter, the average error, and per cent of error. Pay strict attention to "significant figures."

3. *Determine the diameter using the micrometer screw.* After having studied the instrument, determine first the zero correction (*do not force the jaws*). Insert the object, turning it if necessary, and obtain all together ten readings. From the data determine as before the average diameter, average error, and per cent of error.

Part (b). Inspect the various verniers on the instruments and take a typical reading. Have the instructor check your reading. Keep a record of the least count on each instrument.

DATA

Part (a). 1. *Diameter of a coin using a metric rule.*

Readings (cm.)	Deviations
2.43	+ 0.011
2.45	+ 0.031
2.43	+ 0.011
2.42	+ 0.001
2.42	+ 0.001
2.41	− 0.009
2.40	− 0.019
2.40	− 0.019
2.42	+ 0.001
2.41	− 0.009
Total 24.19	0.112
Average 2.419	± 0.011

Hence the mean diameter of the coin = 2.419 ± 0.011 cm.

Error = 0.45%.

2. *Diameter of a coin using a vernier caliper.*

Readings (cm.)	Deviations
2.42	+ 0.005
2.42	+ 0.005
2.41	− 0.005
2.41	− 0.005
2.41	− 0.005
2.42	+ 0.005
2.42	+ 0.005
2.42	+ 0.005
2.41	− 0.005
2.41	− 0.005
Total 24.15	0.050
Average 2.415	± 0.005

Zero correction = 0.000 cm.
Least count = $\frac{1}{10}$ main scale.
Divisions = (0.1 × 0.1) = 0.01 cm.

Hence mean diameter of the coin (using vernier caliper) = 2.415 ± 0.005 cm.

Error = 0.21%.

3. *Diameter of a coin using a micrometer screw.*

READINGS (cm.)	DEVIATIONS
2.4133	+ 0.0009
2.4128	+ 0.0004
2.4122	− 0.0002
2.4120	− 0.0004
2.4123	− 0.0001
2.4123	− 0.0001
2.4120	− 0.0004
2.4113	− 0.0011
2.4130	+ 0.0006
2.4128	+ 0.0004
Total 24.1240	0.0046
Average 2.41240	± 0.00046

Zero correction = 0.0000 cm.
Pitch of screw = 0.5 mm.
Least count = 0.01 mm.
Hence the diameter of coin (using a micrometer screw) = 2.41240 ± 0.00046 cm.

Error = 0.017%.

Part (b).

Readings of setting on: *Least count*

1. Barometer c. g. s. . . . . .
   f. p. s. . . . . .
2. Spectrometer . . . .
3. Jolly balance . . . .
4. Sextant . . . .

QUESTIONS

(**a**). Which is the most accurate instrument of the three? How do the results obtained show this?

(**b**). State the number of significant figures in Part (a) 1, 2, and 3.

(**c**). Does the average for Part (a) 3 lie within outer limits of Part (a) 2? Give reasons for your answer.

(**d**). Find the sum of the three results of Part (a) and state the error in the result.

(**e**). State the least count as found in the instruments used in Part (b).

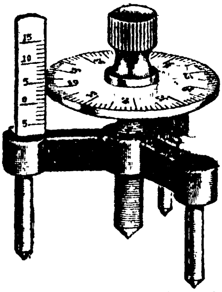
(**f**). Find the product of the three results of Part (a) and give the error of the result.

(**g**). The density of a coin can be found by dividing its mass by its volume. If the mass is given to 0.5%, which of the three instruments in Part (a) would you buy in order to measure the dimensions?

(**h**). Draw a sketch of a micrometer, reading to $\frac{1}{1000}$ inch, when the setting is 0.638 inch.

The spherometer. In certain indirect measurements of length, such as the radius of curvature of a lens, use is made of a spherometer (Fig. 12).

The principle of operation of this instrument is exactly the same as for a micrometer caliper, since the device consists of a movable micrometer screw attached to a head which is subdivided. The movable leg or micrometer screw is usually mounted vertically in a framework. This framework is supported on three legs placed at equal distances from each other. The movable leg is placed so as to be equidistant from the three fixed legs. Attached to the framework is a vertical scale. In using both the micrometer caliper and the spherometer remember that they should be treated carefully and never forced, because the whole accuracy of such an instrument depends upon the screw remaining accurate. This will not be the case when subjected to excessive strains and excessive wear.



Courtesy Central Scientific Company

FIG. 12. — Spherometer.

The *procedure* in using a spherometer is as follows:

1. First place the spherometer on a very flat and hard glass or metal surface and adjust the center leg until it touches the surface. When this has been done, the tips of all the four legs are in the same plane. This gives the zero reading.

2. Then the required thickness of a plate can be measured by placing the plate under the center leg only and measuring the amount which this leg has to be raised.

Difficulty will be experienced in determining exactly when the middle leg is just touching, unless the following or some similar method is used. Gently move one of the side legs back and forth while adjusting the center leg, and it will be found that as soon as the center leg becomes a little longer than the others, the instrument will rotate around this leg as a center. Perform the adjustment by having the center leg too short and then screwing it down until the spherometer just begins to turn on this middle leg and take the reading. Repeat the same adjustment but starting with the leg too long, and bring it back slowly until the instrument just does not rotate around this leg any more. Take a number of readings approaching the setting from both sides. With a little practice this method will give very accurate settings. Another way is to adjust the center leg so that the whole instrument will not just rock on this leg.

It will be found that this setting gives a reading somewhere in the neighborhood of the middle of the vertical scale. In many instruments the zero is consequently placed in the middle of the vertical scale. This leads to confusion in case we have to make a zero correction (which is almost always the case). It is a much better plan to call the lowest division mark on the vertical scale the zero of this scale, and then take the readings on the vertical scale with this point as zero.

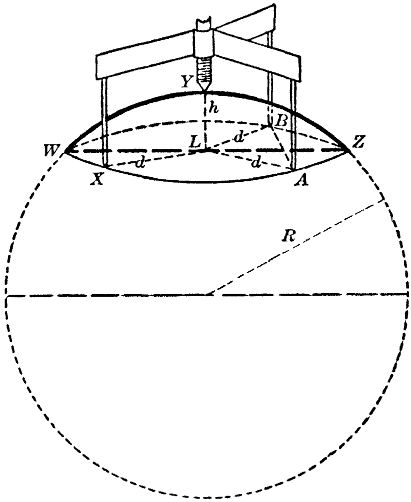


Fig. 13. — Spherometer placed on a spherical surface.

To Find the Radius of Curvature of a Spherical Surface

Very often it becomes necessary to find the radius of curvature of mirrors and lenses since this is an important property in determining their optical behavior. The spherometer adapts itself admirably for this purpose.

Let a portion of a lens, of which the radius of curvature R is desired, be represented by the spherical cap WYZ of Figure 13. This spherical cap is pictured as representing a portion of a sphere with the desired radius R. We wish to express R in terms of XL and YL, which distances can be measured.

From symmetry, $XL = BL = AL = d.$

And if $YL = h$

then we can write,[1]

$$h(2\,R - h) = d^2$$

or,

$$2\,Rh - h^2 = d^2$$

i.e.,

$$R = \frac{d^2 + h^2}{2\,h}.$$

[1] This follows from the theorem in geometry which states that when two chords of a circle intersect within the circle, then the product of two parts of one chord is equal to the product of the two parts of the other.

EXPERIMENT 2

THE SPHEROMETER

Part (a). *To measure the thickness of a small glass plate using a spherometer.*

Part (b). *To find the radius of curvature using a spherometer.*

Apparatus: A spherometer, a microscope slide, a flat surface (glass or metal about 10 × 10 cm.), a concave or convex lens or mirror surface, meter rule.

Part (a). First study the instrument, carefully determining the pitch, size of divisions on the vertical scale, and number of divisions on the head. This will enable you to calculate the amount of advance of the screw for a single rotation of the head:

Next study the zero setting by placing the instrument on the flat surface and adjusting the center leg as explained before. Make about five settings and tabulate your results.

Lastly, determine the height through which the center leg must be raised so as to touch the top surface of the microscope slide which is placed on the flat surface under the middle leg.

From your data calculate the thickness of the slide and state the error in your result.

DATA

Part (a).

Zero Readings (mm.)		Readings on the Plate (mm.)	
Readings (mm)	*Deviations*	*Readings (mm.)*	*Deviations*
20.139	− 0.003	23.046	− 0.0012
20.141	− 0.001	23.048	+ 0.0008
20.145	+ 0.003	23.047	− 0.0002
20.142	0.000	23.048	+ 0.0008
20.143	+ 0.001	23.047	− 0.0002
Total 100.710	0.008	115.236	0.0032
Average 20.1420	± 0.0016	23.0472	± 0.0006

Hence,

$$\text{Reading on glass} = 23.0472 \pm 0.0006 \text{ mm.}$$
$$\text{Zero of spherometer} = 20.1420 \pm 0.0016 \text{ mm.}$$
$$\text{Thickness of glass} = 2.9052 \pm 0.0016 \text{ mm.}$$
$$\text{Error} = 0.06\%.$$

Part (b). In order to find R (Fig. 13) it becomes necessary to measure d and h as accurately as possible.

Having first found the zero reading of the spherometer, by taking a reading of the instrument on a flat surface, place the spherometer on the curved surface and adjust, as explained before, until all four legs touch the surface and again take the reading. The difference between these two results gives the required distance h (Fig. 13).

Since the lengths XL, AL, and BL are not exactly the same in practice, we find the mean value of d by pressing the spherometer, when the four legs are in the same plane, not too heavily on the data sheet as in Figure 14. This gives the points A, B, X, and L, so that XL, AL, and BL may be measured with a meter rule and the average value for d obtained.

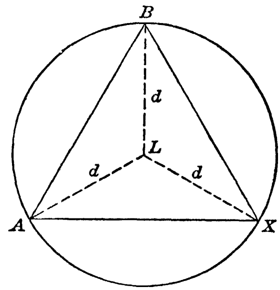


FIG. 14.—Measurement of d.

DATA

Part (b).

TO FIND h

Zero Setting: Readings (mm.)	Zero Setting: Deviations	Lens Setting: Readings (mm.)	Lens Setting: Deviations
4.967	+ 0.0032	7.809	+ 0.0084
4.966	+ 0.0022	7.814	+ 0.0034
4.964	+ 0.0002	7.817	+ 0.0004
4.960	− 0.0038	7.819	− 0 0024
4.962	− 0.0018	7.818	− 0.0014
Total 24.819	.0112	39.077	.0160
Average 4.9638	± 0.0026	7.8174	± 0.0032

TO FIND d

Distance	Reading (mm.)	Deviations
AL	23.1	+ 0.30
BL	22.0	− 0.80
XL	23.3	+ 0.50
Total	68.4	1.60
Mean "d"	22.80	± 0.53

Hence,

$$h = (7.8174 \pm 0.0032) - (4.9638 \pm 0.0026) = 2.8536 \pm 0.0032 \text{ mm. } (0.11\%)$$

$$h^2 = 8.145 \pm 0.22\%.$$

Hence,

$$d = 22.80 \pm 0.53 \text{ mm. } (2.3\%)$$

$$d^2 = 519.8 \pm 4.6\%.$$

Hence, $$R = \frac{d^2 + h^2}{2\,h} = 92.5 \pm 4.6\% \text{ mm.} = 92.5 \pm 4.3 \text{ mm.}$$

QUESTIONS

(a). Discuss the least count of the spherometer which you used.

(b). What would be the volume of a sphere having this radius and what might be the error in the volume (express in cc.)?

(c). Which affects the result (R) more, (1) the error in the measurement of h, or (2) the error in the value of d? Why?

(d). Would this method of finding R be of any value if the surface were not spherical? Give reasons for your answer.

The Measurement of Mass

There are two common methods by which the mass of an object may be determined. The one utilizes the principle of the lever, while the other is based upon the principle of elasticity.

The principle of the lever states, for our purpose, that, given a bar (called a lever), it is always possible to place it on some knife edge (or fulcrum) on which it may turn and be made to balance by placing weights properly on either side. With equal arm balances, such as the platform and fine balance, equilibrium will be established with an equal quantity of mass on each side. Hence, any standard series of masses may be used to determine the mass of an unknown with equal arm balances. With unequal arm balances, such as the fish scales and many standard scales which employ more than one lever, the masses used in the process of determining the unknown must be calibrated for the particular scale. In any case, the result of the determination of mass is not affected by changes in the force of gravity.

The principle of elasticity explains the behavior of elastic bodies under twisting, stretching, or compressional forces. Any body, which returns to its former position after a given distortion, is serviceable for a balance. The spring balance is the commonest and can be made quite sensitive. While it may be calibrated and used to measure mass, it actually records the gravitational attraction on the mass. For accurate work, it should be calibrated at every place used.

Two very useful kinds of balances sufficient for our purposes will be described: the one, a *platform balance* which will weigh objects up to one thousand grams with an accuracy of one-tenth of a gram; the other, a *fine balance* which will weigh objects up to one hundred grams with an accuracy of one ten-thousandth of a gram. Both of these balances are characterized by having pans supported by knife edges (called fulcrums) at either end of a lever arm and a third fulcrum at the center of the lever. The center fulcrum knife edge is raised slightly above the other two so that the balance arm will be in stable equilibrium.

The **platform balance** is used for quick weighings and is very useful where great accuracy is not necessary. The pans or platforms supporting the known and unknown weights are of heavy construction and are kept rigidly in an upright position by supporting rods under the balance (Fig. 15). The knife edges, which

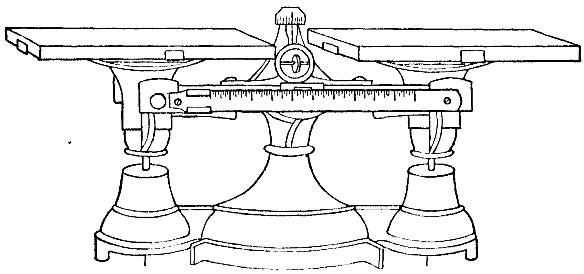


FIG. 15. — Platform balance.

constitute the fulcrum positions, are of steel construction and rest on hardened steel or agate plates. The pans are balanced at the factory by loading cups near the bottom of the balance under the platform. An adjustment for student use is found over the center fulcrum in the form of a threaded cylindrical nut which advances on a screw either to the left or to the right. A pointer travels in front of a scale marked in arbitrary divisions. The last ten grams of mass to be added to the load may be accomplished by a sliding weight placed on a graduated scale in front of the balance. This scale is graduated in tenth of a gram divisions. Because of this scale, the unknown is generally placed on the left while the known masses are placed on the right pan. Should the unknown be placed on the right-hand side, that mass indicated by the slider must be subtracted from the known masses placed on the left-hand pan.

The **fine balance** is used wherever an accurate determination of the mass is necessary. The ordinary fine balance will weigh to about one-tenth of a milligram. If much greater accuracy is desired, the buoyancy effect of the air, due to the volumes occupied by the known and unknown masses, must be taken into account.

The pans are suspended from agate planes on either end, which in turn are supported by knife edges. The central fulcrum knife edge is supported on an agate plane which is imbedded in a pillar. A pointer fastened to the lever arm swings in front of the pillar and in front of the scale, which is graduated in arbitrary divisions and located behind the lower end of the pointer. A small weight, clamped by a set screw, is fastened on the pointer and may be moved up or down so as to raise or lower the sensitivity. When the weight is raised on the pointer, the stability of the lever system is lessened, but the balance becomes more sensitive. When the balance is made very sensitive, a long period of swing of the lever system results and a longer time is required for the weighing. Hence the sensitivity of the balance for general use is adjusted to compromise between sensitivity and quick weighing. The zero position of the pointer on the scale can be adjusted by small threaded weights placed at the two ends of the lever arm. Every fine balance has some kind of a " rider " or chain system to make adjustments from 0.1 to 5 milligrams. Very often the rider will add or subtract from a load a maximum of 10 milligrams. The whole mechanism is placed in a glass case. When not in use, an arrestment mechanism is provided to release the agate planes from the knife edges or to raise the lever arm itself from its agate plane. This is accomplished by use of a screw head at the bottom of, and outside, the glass case of the balance. There is also a button at the left of this screw head, which may be pressed in to release the pans so that they will swing freely. When the button is pressed in and turned slightly, it will catch, so that the pans will be free without further pressure. A level is provided at the back of the balance, so as to indicate when the balance is horizontal. The balance itself is supported on three legs. The two front legs are threaded and adjustable in length.

Before attempting to weigh an object, especially when the balance is first set up, notice whether (1) the balance is level, (2) the knife edges are in position, and (3) the pointer is swinging so that the initial resting point is near the central division of the arbitrary scale. If the lever arm does not swing freely upon release by the screw head, the knife edges should be examined to see if they are properly seated when lifted by the screw head. If the pans are free to turn and the pointer does not have its zero position near the central division of the scale, an adjustment of one or both of the threaded units at each end of the lever arm should be made. All such adjustments are generally made by the instructor. With continued use, the student learns to make these adjustments himself at the direction of the instructor.

The knife edges are frequently found out of position, because of carelessness on the part of the student when weights (*i.e.*, standard masses) are added or taken off the balance. Weights should never be added or removed from the scale pan unless the balance arms are locked by the arrestment devices. Moreover, weights should always be lifted by tweezers, since the hand leaves grease marks, which increase the mass of the weight.

Suppose that the unknown is on the left pan and certain known masses are on the right pan, and we wish to see whether a balance exists. First, turn the screw head. Then if the pans do not tip, press in the button which frees the pans. If a balance does not exist, release the button so as to bring the pans to rest and then turn the screw head so as to lock the balance arms. The known masses are generally placed on the right-hand pan because of the fact that the rider is attached to the right-hand balance arm. Each scale must be studied by itself to learn its particular arrestment device.

Because of friction at the knife edges the exact *resting point* cannot be found by allowing the pointer to come to rest because it does not always come to rest at the same point. In a sensitive balance the time taken to come to rest is also inconveniently long. Hence a *method of swings* is used to overcome these difficulties. Turn the screw head so as to release the balance arms and then press in the button gently so as to give an initial swing of about 4 to 10 divisions. If the initial swing is not enough, wave the hand in front of one of the pans to give it the desired initial swing. Close the window so that air currents will not affect the to-and-fro motion of the pointer. Then take an odd number of consecutive readings of the extreme positions of the pointer.

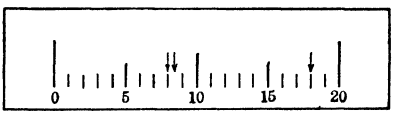


FIG. 16. — Pointer scale of a balance.

Assume the readings to have been 8, 18, 8.5 (Fig. 16). Now in order to calculate where the pointer would come to rest on the scale, first average the two left-hand readings. Then find the mean between this average reading and the reading on the right. In the example shown, the resting point would therefore be 13.12. This is so because the average left-hand reading is 8.25, and when the mean is found between this average and 18, the result becomes $\frac{18 + 8.25}{2}$, which equals 13.12.

The *sensitivity of a balance* is a very important constant in comparing balances and estimating the accuracy to be expected. It is defined as the mass which must be added to the scale pan in order to deflect the pointer one division. A little consideration will show that the sensitivity decreases somewhat, on account of friction, with the increase of the load on the balance arms.

Because of the large number of small standard masses (*i.e.*, weights) used in weighing with a fine balance, one tries to minimize the number of weights both for simplicity and to cut down the number of accumulative errors incident with each weight. The minimum number of masses will be used when we start with the weight next smaller to the one which overbalances the unknown mass and continue to add each time that weight which is next smaller than the one which overbalances the unknown. This process continues until the smallest weight is used or until the pointer stays on the scale without further addition of weights. Most fine balances have a rider or other mechanism to furnish readings of 5 milligrams or less.

With a large number of standard masses on the scale pan, there is a fair probability of error in adding these masses mentally. To avoid this it is good practice to write down the masses of the weights taken from the box by a study of the empty spaces in the box. Then check off the weights as they are taken from the scale pan and replaced in the box. This method is sure, saves one from doubt, and often also the necessity of repeating a whole experiment.

If the arms are found to be unequal, the correct mass of the unknown is found by placing it on each scale pan in turn and weighing. Suppose that, when the unknown mass is in the left-hand pan, the known mass in the right-hand pan is m_1, while when the unknown mass is in the right pan, the known mass in the left-hand pan is m_2, then the correct mass M is found by theory to be

$$M = \sqrt{m_1 m_2}.$$

EXPERIMENT 3

THE MEASUREMENT OF MASS

Part (a). *To measure the mass of an object with a platform balance.*

Part (b). *To find the sensitivity of a fine balance and to use this sensitivity in weighing an object with the greatest possible accuracy.*

Apparatus: *Part* (*a*). Platform balance, set of weights (10–500 grams), unknown mass.

Part (*b*). Fine balance, set of weights, unknown mass (*e.g.*, a coin), tweezers.

Part (**a**). Place the unknown first on the left-hand pan and weigh, then on the right-hand pan. Repeat three to five times, alternating the unknown weight from one pan to the other. Since the sliding weight in front of the scale is made for weighing when the known weights are on the right-hand pan, one should remember to subtract this weight when the known masses are placed on the left. From the data calculate the average mass for each side and its accompanying deviation. Find the per cent of error on each side, also the true weight.

DATA

Part (a).

UNKNOWN MASS IN GRAMS

	LEFT PAN		RIGHT PAN	
	Known Mass on Right Side	*Deviations*	*Known Mass on Left Side*	*Deviations*
	165.3	+ 0.10	164.6	− 0.17
	165.2	0.00	164.6	− 0.17
	165.0	− 0.20	164.9	+ 0.13
	165.3	+ 0.10	165.0	+ 0.23
Averages	165.20	± 0.10	164.77	± 0.17
Per cent of error		0.06%		0.10%

$$M = \sqrt{165.20 \times 164.77} = 164.98 \text{ grams.}$$

$$\text{Mass } (M) = 164.98 \text{ grams with } 0.05 \text{ per cent of error.}$$

Part (**b**). First determine the *zero resting point* of the balance when no load is on either pan. This should be found by the method of swings (as described in a previous section) by taking three consecutive readings of the position of the pointer on the scale (say two on the left and one on the right). From these three readings the zero resting point can be calculated. Repeat these three observations three times, obtaining an average value for this resting point. Record every reading on your data sheet.

Next place on the left pan the object to be weighed (having of course first clamped the balance arm) and on the right pan the standard weights until a balance point is found somewhere near the zero resting point as determined in the first part of this procedure. Determine this resting point now by taking as many readings for it as were taken for the zero resting point. Let us call the average value of this resting point the *load resting point 1*.

The next part of the procedure is for the purpose of finding the "sensitivity" at this load. This is done by adding (or subtracting) a small known weight to the others (say five milligrams) and redetermining the resting point with this additional little weight. Call the value thus found *load resting point 2*. By subtracting the two load resting points 1 and 2 we know how much effect the small weight had on the resting point, and consequently the sensitivity can be easily calculated since it is the weight necessary to move the pointer one division on the scale.

Now, using the sensitivity, *calculate the weight* of the object. To do this, it is to be noticed that what has to be calculated is the weight which would have to be added to (or subtracted from) the load at resting point 1, to bring it back to the zero resting point. *Knowing the sensitivity*, and the number of divisions we should like to have the pointer move to get it back to the zero resting point, we get, by multiplying these two quantities together, the weight which would have to be added (or subtracted) to bring the balance to the zero resting point.

Finally, if a rider is available, *check* this calculated value *by using the rider* on the balance arm.

Repeat the whole experiment of Part (b) by placing the weight on the left pan and the object on the right pan.

Calculations. *Unknown on left pan:* We see from the data that 0.005 gram moves the pointer $(12.65 - 11.88) = 0.77$ divisions.

$$\therefore \text{ Sensitivity } = \frac{0.005}{0.77} = 0.0065 \text{ gram/division.}$$

To bring the pointer back to the zero resting point from position 1, it would have to move $(12.65 - 10.43) = 2.22$ divisions; or, in other words, a mass of $(2.22 \times 0.0065) = 0.0143$ gram would have to be added to the standard masses.

Hence the final mass $= 6.200 + 0.0143 =$ **6.2143 grams.**

Unknown on right pan: Make calculations in a similar way and find the final mass for the unknown on the right pan.

DATA

Part (b).

UNKNOWN ON LEFT PAN

Description of Trial	Trials	Maximum Swing to			Average Left	Resting Point	Average R.P.
		Left	Right	Left			
Zero resting point	1	3.0	18.0	3.0	3.00	10.50	
	2	1.0	19.5	1.5	1.25	10.38	
	3	5.3	15.3	5.8	5.55	10.42	10.43
Resting point (1) with load of 6.200 gm.	1	6.0	19.0	7.0	6.50	12.75	
	2	9.0	14.0	9.5	9.25	11.63	
	3	9.0	18.0	9.3	9.15	13.57	12.65
Resting point (2) with load of 6.200 gm. + 0.005 gm.	1	4.0	19.5	4.5	4.25	11.88	
	2	9.0	14.0	9.5	9.25	11.63	
	3	7.5	16.5	8.0	7.75	12.13	11.88

UNKNOWN ON RIGHT PAN

Description of Trial	Trials	Maximum Swing to			Average Left	Resting Point	Average R.P.
		Left	Right	Left			
Zero resting point	1	1.3	19.0	2.0	1 65	10.32	
	2	2.0	18.6	2.5	2.25	10.42	
	3	5.9	15.1	6.0	5.95	10.62	10.45
Resting point (1) with load of 6.200 gm.	1	4.0	11 8	4.2	4.1	7.95	
	2	0.0	15.8	0.3	0.15	7.98	
	3	2.0	14.0	2.4	2.20	8.10	8.01
Resting point (2) with load of 6.200 gm. + 0.005 gm.	1	4.0	14.0	4.3	4.15	9.08	
	2	4.3	14.0	4.7	4.50	9.25	
	3	6.3	12.0	6.6	6.45	9.23	9.19

The true mass (allowing for inequalities in the balance arms) is found by:

$$M = \sqrt{m_1 m_2},$$

where m_1 and m_2 are the masses as found on the two sides.

QUESTIONS

(a). What is the average sensitivity of the fine balance for the particular load which was used? Express the result in milligrams per division.

(b). Calculate the per cent of error in finding the resting point 1, when the load is on the left pan.

(c). Calculate the per cent of error introduced in the true mass by inequalities in the lengths of the balance arms. Take the value as found for the load on the left pan.

(d). How many divisions would the pointer on the fine balance move if one milligram were added to the load?

THE MEASUREMENT OF TIME

The measurement of time is the most difficult operation in determining the three fundamental quantities: length, mass, and time. There are several reasons for this inherent difficulty.

The scientific unit of time is the *mean solar second* which was chosen, not for its convenience, but for lack of a better unit. This unit of time has to be determined astronomically. Having once established this unit, the next operation was to try to construct a mechanically oscillating system which would give exactly isochronous (*i.e.*, equal period) vibrations. No such system has yet been devised.

The greatest progress in this direction has been made within recent years at our large research laboratories by using vibrating crystals of quartz and recording their oscillations electrically. Fortunately for most of our scientific observations, such extreme accuracy is not demanded because other errors are much more important. Usually the procedure is to use a well-made clock and then find the correction factor for this clock by comparison with an astronomical clock.

Suppose for a moment that we did have a perfect clock, then how should we record or measure a certain interval of time exactly? It cannot be done! Personal errors, errors of lag and inertia come in and offset the accuracy of our clock. These errors — *i.e.*, errors which we make in starting and stopping a clock exactly at the beginning and end of an interval — are quite large and are difficult to determine.

Another error peculiar to a watch is that we cannot read a watch to parts of an oscillation of the flywheel. Every oscillation of the flywheel by means of a pawl actuates the seconds pointer and this pointer moves in jerks (*e.g.*, $\frac{1}{5}$ sec. jumps). This, then, limits the accuracy of reading such a watch to one-fifth of a second.

Assuming that the future will be able to produce oscillating systems with extremely constant oscillations, *the difficulties of recording intervals to accuracies even better than this become almost if not just as difficult a problem itself.* For most accurate work, therefore, the measurement of time is extremely difficult and requires the highest degree of skill and technique.

In this course we shall confine our attempts to measurements of periods with a stop watch of known accuracy. With practice this forms a fairly accurate, and certainly a comparatively simple, way of observing certain time intervals. However, even here, in order to obtain results which are to mean anything, certain rules must be remembered and strictly adhered to.

First, it is to be noticed that *the per cent of error depends upon the total time for which the oscillations or period is observed and not upon the number of swings.* Hence, if the period of swing is longer, it is not necessary to observe as many swings. If the same percentage of accuracy is required, the times of observing the swings in the two cases should be about the same. Of course lengthening the time of observation increases the accuracy.

Secondly, it is to be observed that it is extremely difficult, if not impossible, to estimate parts of a swing. For this reason *it becomes necessary to find the time of a certain whole number of swings, and not the number of swings in a certain time.*

Thirdly, in order not to count one swing more or less than the required number — a very common fault with beginners — the following procedure is recommended and should be practiced by a student for a few minutes before starting observations: Decide from which end of the swing you will start counting. Call this zero. Start counting backwards, from 3 say, until zero is reached, and when zero is reached, start the stop watch and keep on counting forward, finishing the measurement when the number designating the number of required swings is reached.

e.g., 3, 2, 1, 0, 1, 2, 3, ... 97, 98, 99, 100

↑ Start watch ↑ Stop watch

By always following this procedure, this too frequent source of error can be overcome.

EXPERIMENT 4

THE MEASUREMENT OF TIME

To find the period of a simple pendulum of different lengths and to record the results in the form of a graph.

Apparatus: A spherical pendulum bob of lead or brass about 2 to 2.5 cm. in diameter, flexible string 1.5 to 2 meters long, meter stick, mounting clamp, stop clock.

Start your experiment with the longest possible pendulum length, making sure that the pendulum is secured to the support (Fig. 17) in such a way that the string swings always with a definite and fixed length. *The amplitude of swing of the pendulum should not exceed 5° each side of the equilibrium position.*

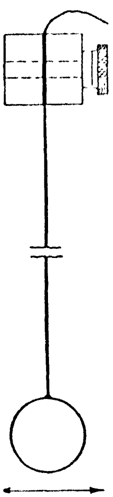


FIG. 17.—A simple pendulum.

Since the time available for this experiment is decidedly limited, we shall have to content ourselves with allowing the total period of each observation to be about one minute. Time these swings as accurately as possible with the stop watch available, remembering that the time for *one complete to-and-fro swing* of the pendulum is called T.

Record the time for two sets of oscillations for each length of the pendulum and let your partner do likewise for two more. Record the results in the tabular form as shown below. Calculate the per cent of error in your timing.

Measure the length of the pendulum from the point of the support to the center of the bob. Next make the length about one-half, one-fourth, one-eighth, and one-sixteenth of the original and carry out the same procedure for each of these lengths, timing the pendulum always for about one minute.

Calculate T, the time for one complete oscillation, as well as T^2. Then draw a graph showing the relation between (1) the length and T, and (2) the length and T^2. (Refer to the previous chapter for procedure in plotting graphs.) The two curves should be plotted on the same sheet with T and T^2 as ordinates.

DATA

(1)

Total Time (sec.)	Deviation
63.3	− 0.20
63.6	+ 0.10
63.5	0.00
63.6	+ 0.10
Total 254.0	0.40
Average 63.50	± 0.10
	0.15%

Length = 159.6 ± 0.2 cm.

No. of swings = 25.

$\therefore T$ = 2.540 sec. ± 0.15%.
T^2 = 6.452 sec^2. ± 0.30%.

(2)

Total Time (sec.)	Deviation
73.4	− 0.08
73.7	+ 0.22
73.5	+ 0.02
73.3	− 0.18
Total 293.9	0.50
Average 73.48	± 0.13
	0.18%

Length = 83.7 ± 0.2 cm.

No. of swings = 40.

$\therefore T$ = 1.837 sec. ± 0.18%.
T^2 = 3.375 sec^2. ± 0.36%.

(3)

Total Time (sec.)	Deviation
66.9	+ 0.10
66.7	− 0.10
66.8	0.00
66.8	0.00
Total 267.2	0.20
Average 66.80	± 0.05
	0.075%

Length = 44.3 ± 0.2 cm.

No. of swings = 50.

$\therefore T$ = 1.336 sec. ± 0.075%.
T^2 = 1.785 sec^2. ± 0.15%.

(4)

Total Time (sec.)	Deviation
66.1	+ 0.3
65.8	0.0
65.8	0.0
65.5	− 0.3
Total 263.2	0.6
Average 65.80	± 0.15
	0.23%

Length = 22.0 ± 0.2 cm.

No. of swings = 70.

$\therefore T$ = 0.9400 sec. ± 0.23%.
T^2 = 0.8836 sec^2. ± 0.46%.

(5)

TOTAL TIME (sec.)	DEVIATION
69.9	− 0.05
70.1	+ 0.15
70.0	+ 0.05
69.8	− 0.15
Total 279.80	± 0.40
Average 69.95	± 0.10
	0.14%

Length = 12.4 ± 0.2 cm.

No. of swings = 100.

$\therefore T = 0.6995$ sec. ± 0.14%.
$T^2 = 0.4893$ sec². ± 0.28%.

QUESTIONS

(a). How can you increase the accuracy of the value of T with the apparatus which you used?

(b). State the relation between the length (l) and T by examination of the graph.

(c). For what length does the experimental period have the largest deviation from the curve? Calculate the per cent of error in the period for this length.

(d). Calculate the length of a seconds pendulum (*i.e.*, period of 2 sec.) from each of the curves in your graph.

PROBLEMS

Experiment 1

1. Given that 1 inch = 2.54 cm., find a factor for converting miles into kilometers. Use this factor to convert 12 miles to kilometers.

2. A thin circular sheet of brass has a diameter of 50.00 cm. Find its area ($\pi = 3.142$). Assuming that the material has a thickness of 1 mm., find the weight of the sheet in grams. (Look up the density of brass in a book of physical tables.)

3. Define "least count" of a vernier. Define "error of parallax."

4. A scale is divided into sixteenths of an inch. It is required to read to $\frac{1}{128}$ of an inch by a suitable vernier. Calculate the number of divisions on the vernier and show by means of a diagram the position of the vernier on the main scale when reading $3\frac{35}{128}$ inches.

5. A micrometer gauge has 40 threads per inch. There are 25 divisions on the revolving head. To what accuracy will this gauge read?

Experiment 2

6. A spherometer has its outer legs at the three corners of an equilateral triangle of side 5 cm. If the center leg is at a distance of 4 mm. above the plane of the three corner legs when the instrument is placed on a lens surface, find the radius of curvature of the lens.

7. Through how many revolutions would a spherometer head have to be rotated if the center leg is to be raised 3.65 mm.? The pitch of the screw is $\frac{1}{2}$ mm. and there are 50 divisions on the head.

8. Plot a curve showing how R varies with h. Assume that $d = 5$ cm. and h has values varying from 1 to 3 mm.

Experiment 3

9. A balance has a zero resting point of 8.6. A load of 6.430 grams is placed on the left pan with the standard weights on the right. The resting point (1) is now found to be 7.1. On addition of a 10-milligram weight to the right pan the resting point is found to be 6.2. Calculate the sensitivity of the balance and also the weight of the unknown load.

10. Calculate the resting point, given the following consecutive readings on the scale: 4.2, 13.8, 4.7. Why is this method used for finding the resting point of the balance?

11. Define what is meant by the sensitivity of a balance. Finish the calculations and find the value of the sensitivity and the unknown mass when placed on the right-hand pan. Determine also the true mass. (Refer to data of Part (b), Exp. 3.)

Experiment 4

12. Given a swinging pendulum and a stop watch, what precautions would you take in finding the period of the pendulum with the greatest precision?

13. Plot two curves from the tabulated data on the pendulum (pages 41, 42) as follows: (1) length against time, (2) length against time squared (make the length the abscissa).

14. From the curves in Problem 13, show what the length of a seconds pendulum (*i.e.*, a pendulum having a period of two seconds) must be.

CHAPTER IV

STATICS

The branch of mechanics which deals with the equilibrium of a particle, or of a system of particles distributed at fixed distances relative to each other (*i.e.*, a rigid body), is termed *statics*.

Whenever a body is at rest, it does not necessarily mean that there are no forces acting on it. What more often occurs in practice is that there are forces acting on the body, but they act in such a way that they keep the body in equilibrium. Furthermore, should the particle or body be moving with a constant velocity (either linear or angular or both), then if it keeps on moving with the same velocity and in the same direction, we still speak of the system as being *in equilibrium*, and the same laws are true in this case as were true when the body or particle was being held stationary. In this chapter, we shall study experimentally some of the laws which govern the behavior of these forces which keep a system in equilibrium.

In studying the laws of Statics, it is natural, from what has been said above, to divide them into two groups, depending upon whether we are dealing with a *particle*, which has mass but no appreciable size, or a *rigid body*, which has both mass and size.

The Equilibrium Conditions for a Rigid Body

In this course we will simplify the general conditions by considering all the forces which act on the body to be in one plane. When this is the case, then we can show that if the body is to be in complete equilibrium, *both* the following two laws must be satisfied:

1. *The vector sum of all the external forces acting on the body must be zero.*

Another way of stating this same law is to say that the vector sum of all the components of the external forces in two directions must add up to zero.

2. *The sum of the moments of all the external forces acting on the body must be zero around any axis which we wish to choose perpendicular to the plane in which the forces act.*

In order to be able to apply these two Laws of Equilibrium, it is necessary to know how to deal with vectors, since forces fall under the general heading of vectors. The following section applies to vectors generally.

THE GEOMETRY OF VECTORS

Addition of vectors. In order to find the sum, or resultant vector, of the four vectors, proceed graphically as follows: Repre-

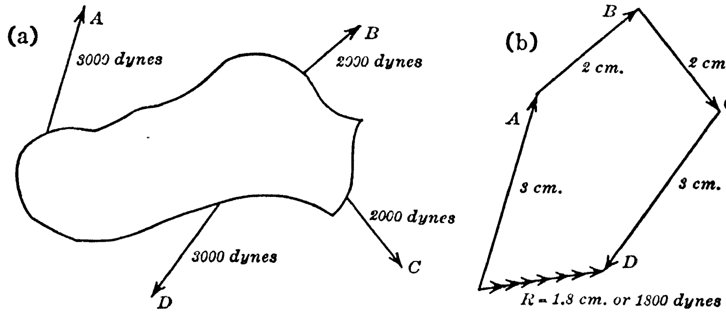


FIG. 18.

sent force A by an arrow of suitable length and proper direction; then, at the top of the arrow A, place the heel of the arrow which represents B, in magnitude and direction; at the tip of B, place the arrow representing C in direction and magnitude; and so on. Continue in this way until all the vectors have been drawn (in this case, four). Then finally draw an arrow from the heel of the first arrow to the point of the last arrow. This vector represents both in magnitude and direction the resultant or sum, shown as R in Figure 18 (b).

Resolution of vectors. In many cases in which we are dealing with a vector, the problem can be much simplified, by splitting a vector up into component parts or vectors, so that the sum of these component vectors together form the original vector. Then for the purposes of the problem in hand, we can neglect the original vector and deal only with the *component vectors* (the so-called "components"). Just as in algebra

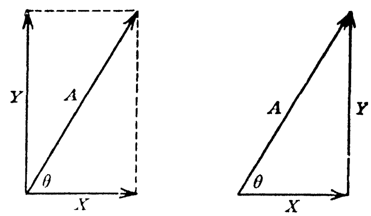


FIG. 19. — Resolution of a vector (A) into components.

we can split 5 up into 2 + 3, 2.5 + 2.5, 1 + 4, etc., and do so indefinitely, so we can find many components for a single vector. For practical purposes we usually *find the components of a vector in two directions at right angles to each other. E.g.*, in Figure 19 we see that the vector A can be split up into two components, X and Y, at right angles to each other, in such a way that $\overline{X} + \overline{Y} = \overline{A}$ (*i.e.*, when added as vectors). In this case we see that

$$X = A \cos \theta$$

and $$Y = A \cos (90 - \theta) = A \sin \theta.$$

Concurrent Forces

The equilibrium of a particle. This is the simplest case of equilibrium to consider. Here the forces all meet at a point, because the particle on which they act is negligible in size. Let us also assume, for the time being, negligible mass. *The law of equilibrium of a particle states that when a number of forces* A, B, C, D, *etc., act on a particle and keep it in equilibrium, the resultant force is zero;* or, in other words, when the lines representing the vectors A, B, C, D, etc., are placed end to end, they must form a closed figure so that no sum or resultant vector (R above) is possible.

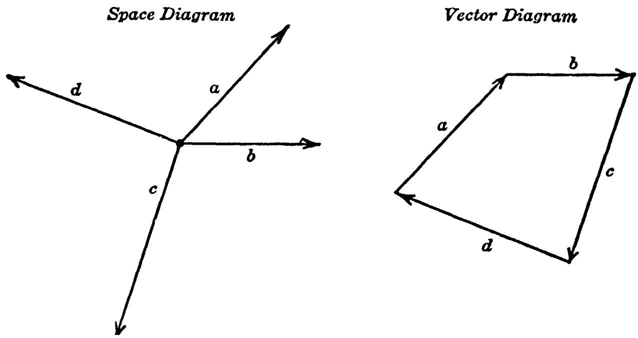


FIG. 20. — Four forces keeping a particle in equilibrium.

Example 1. Figure 20 represents four forces acting on a particle keeping it in equilibrium. All forces are known. The left diagram shows the arrangement of the forces in space, whereas the diagram to the right represents the vector diagram. Note that since equilibrium exists, the four vectors form the sides of a closed figure (the force-polygon).

Example 2. In this case there are three known forces keeping the particle in equilibrium. The vector diagram becomes a triangle, as shown in Figure 21.

Example 3. Figure 22 represents a particle in equilibrium acted upon by three forces, two of which are known. The problem is to find the unknown force x. There are two methods of procedure:

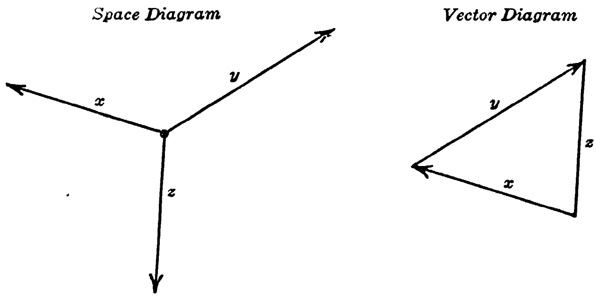


FIG. 21. — Three forces producing equilibrium.

Method 1. Draw the two known forces and complete the triangle with the third vector (since we have equilibrium).

Method 2. Parallelogram of force method. Draw the known vectors and complete a parallelogram with these two as sides. The diagonal represents the sum R (check these and see that it is the same as by the above method). Then R reversed is the unknown vector x, which will give equilibrium.

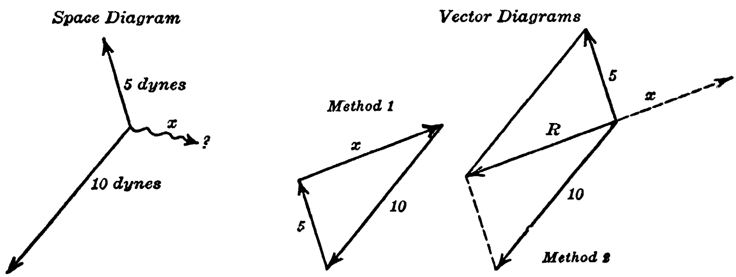


FIG. 22. — Graphical method for finding an unknown force.

Example 4. Another example is to be found by a consideration of the *inclined plane.* It is much easier to solve questions involving motion or equilibrium on an inclined plane, if we resolve all forces into components, either along the plane or perpendicular to its surface.

Suppose in Figure 23 (a) we have a particle being held in equilibrium on a smooth plane by a force P — a string, for example — acting up the plane. The remaining forces acting on this

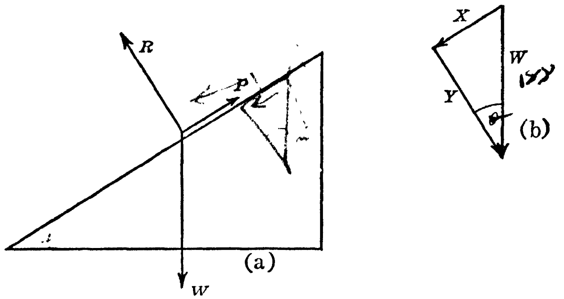


FIG. 23. — Equilibrium of a particle on an inclined plane.

particle are the thrust of the plane on the particle R, and the force of gravity W. Then we can resolve W into two components, X and Y, respectively parallel and perpendicular to the plane, *i.e.*, in the directions of R and P, such that

$$X = W \sin \theta,$$
$$Y = W \cos \theta.$$

Having done this, we need now only work with X and Y and can neglect W. For equilibrium we see that P must balance X, and R must balance Y, hence

$$P = W \sin \theta,$$
$$R = W \cos \theta.$$

The equilibrium of a rigid body. A rigid body has to have an additional test applied to it, to be sure it is in equilibrium. In the above cases of a particle, the law of equilibrium states that there can be no *translational* acceleration since the resultant force acting on the particle was zero. This same law still holds for a rigid body. This, however, is only half of the test for equilibrium, since a rigid body as a whole might still be in translational equilibrium while rotating around some axis. We need a further test to see if the body is in rotational equilibrium. This test consists in seeing whether it obeys the *Law of Moments. This law states that the sum of the moments around any axis must be zero.* Remember that the *moment of a force* is the product of the force, and the perpendicular drawn from the axis to the direction in which this force acts. A clockwise moment is usually called positive, and an anticlockwise moment, negative.

Friction. In dealing with equilibrium, we may frequently encounter the so-called *force of friction.* Practically this force can be of great help to us. For example, it is utilized in bringing an automobile to rest. At other times, however, we would like to eliminate this force. Its effect is to decrease the efficiency of machines. Although the whole story of these various types of frictional forces has not yet been told, the scientist has discovered many important facts relating to friction. Consider a body at rest on a rough surface, and let us try to slide it along the surface. The force which the plane will exert on the body will be in some unknown direction (not necessarily perpendicular to the surface). We can think of this force as having two components, (1) a thrust R normal to the surface, and (2) a frictional force F acting horizontally, which comes into play when we try to move the body (Fig. 24). This frictional force becomes larger and larger the more we push, and finally reaches a limiting value just before the body starts to slide. It is found experimentally that the limiting frictional force F depends upon the thrust R.

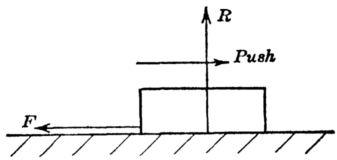


FIG. 24.—The force of friction.

The ratio $\frac{F}{R}$ is a constant and is called the coefficient of friction (μ).

In practice the force of friction is found to decrease slightly once the body has started moving, and then remains constant while the body is sliding along slowly without accelerating. This fact leads us to be more specific and define:

(1) *The coefficient of static friction,*

(2) *The coefficient of dynamic friction,*

depending upon whether we are dealing respectively with the frictional force necessary to start the body moving, or else with the force required to move it slowly at constant velocity.

EXPERIMENT 5

THE EQUILIBRIUM OF A PARTICLE

Part (a). *An experimental study of the law of equilibrium of a particle.*

Part (b). *Using the law of equilibrium, to find the mass of an unknown body.*

Apparatus: Force table (either horizontal or vertical type), four hangers having known masses, assorted slotted weights, thin string or heavy cotton thread, ruler, triangle, and an unknown mass.

Part (a). Draw a thin pencil line across a sheet of paper so as to divide the paper into halves, as shown in Figure 25. On the one half, plan to put the space diagram. This is simply a transfer to this half of the paper of the known forces and their directions from the actual conditions of the experiment. The other half can then be used to construct the vector-addition graphically. Now pin the paper by means of two or three thumb tacks to the force table in such a way that the center of one-half of the paper is approximately in the middle of the force board. Tie three pieces of string of approximately the same length together in a small knot, passing the other end over three pulleys arranged at the edges of the board. On these ends attach the hangers and add known weights until the knot comes to rest approximately at the center of the force table.

In order to obtain good results, two or three precautions have to be taken. First, see that the pulleys have as little friction as possible. Secondly, make sure that the strings are as close to the paper as possible without the knot touching the board. This point is usually taken care of in making the apparatus, but in some instances the heights of the pulleys above the plane of the board can be adjusted. Lastly, before taking any readings, see that the pulley groove and the string as it comes off the pulley are parallel. This necessitates having the pulley on a swivel.

Having made these adjustments, find the position of equilibrium of the particle (in this case the knot) by displacing the system slightly and noting the point to which it returns. If, on account of friction, the particle does not always return to the same point, displace the system several times, marking the points to which the particle returns and then set the knot in the center of this region.

The next problem is to transfer the directions of these forces to the paper. This is best done by making two small dots (as far apart as the paper will allow) with a sharp pencil immediately underneath each string. This has to be done with care so as not to displace the system and also to avoid errors of parallax. Having obtained these six points (in the case of three strings attached to the particle), remove the sheet of paper and join the pairs of points. Along these three directions write the corresponding masses that were attached to the strings. The accuracy of your results depends largely upon obtaining these directions exactly.

To construct the vector diagram. On the other half of the sheet, adjoining the force diagram, starting at C, draw a vector CD representing, on some suitable scale, the magnitude and direction of the known force along AB. In drawing this direction it is best to use a triangle and a straight edge as shown in Figure 25. This figure illustrates how a line can be drawn through C (namely, CD) accurately parallel to AB. The procedure is to place one edge of the triangle to coincide with the line AB. Then put a ruler or other straight edge along another side of the triangle, being careful to hold the triangle in place with one edge along AB. Now keep the ruler fixed and slide the triangle along the ruler until the side which was parallel to AB now passes through C. Then draw CD, which will be parallel to AB. From the end D, draw DE to represent the second force (along XY). Finally construct EF to represent the force along MN. Now if these forces had been represented correctly, then according to the *law of equilibrium of a particle, the third vector should finish exactly where the first vector began* (viz., at C).

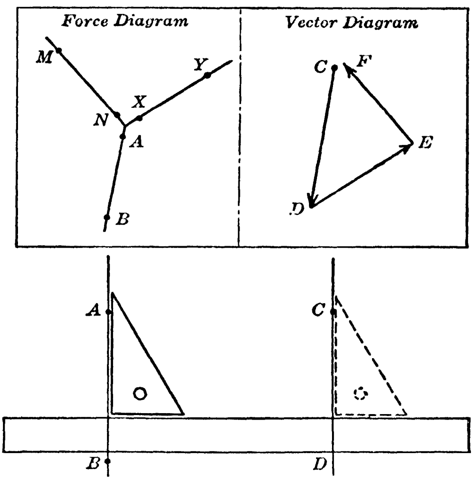


FIG. 25. — Constructing a triangle of forces.

Note the difference in your drawing between the points C and F, and from a measurement of the length of this difference calculate the error. Express the error in per cent (of the last force represented) and note also the error in direction. Note in all these constructions a fairly hard pencil with a sharp point must be used. Choose the scale of representation as large as the paper will allow.

Repeat the experiment, arranging the particle so that four known forces act and again find the error in your vector addition.

Part (b). Let the knot have three strings tied to it. Two known forces are applied at the ends of two of these strings, the third having some unknown force acting on it. The unknown force in this case is the force of gravity acting on an unknown mass.

Proceed as before, obtaining first a space diagram on one-half of another sheet of paper. Now, on the assumption that the Law of Equilibrium is true we should find that the three vectors representing these forces, when placed end to end, form a closed triangle if our drawing were done exactly and there were no friction. Hence, in drawing the vector diagram to scale, draw the two known forces end to end (using the triangle and ruler) and then join the finishing and starting points. This vector then represents the unknown force, and by finding the length of this vector determine the mass of the unknown body. Check by finding its mass on a balance and calculate your per cent of error. (Note that if we had drawn a parallelogram with the two known forces as sides, then the magnitude of the diagonal of this figure can be made to represent the magnitude of this unknown force. This is often referred to as the method of the parallelogram of the forces.)

QUESTIONS

(a). Having done this experiment, what is your conclusion about the law of equilibrium for a particle?

(b). How would you go about proving this law in terms of the components of the forces?

(c). Find graphically the magnitude of the components of all the forces in the direction AB for Part (a), when three forces act.

EXPERIMENT 6

THE INCLINED PLANE AND A DETERMINATION OF THE COEFFICIENT OF FRICTION

Part (a). *To find the unknown mass of a rolling block on the inclined plane.*

Part (b). *To find the coefficient of friction between a wooden block and a horizontal surface.*

Part (c). *Determination of the coefficient of friction by finding the limiting angle of repose.*

Apparatus: Inclined plane apparatus, rolling block, two sets of slotted weights (10–500 gm.), string, hanger, friction block, platform balance.

Part (a). Arrange the angle of the plane to be about 5° with the horizontal (Fig. 26) and determine the force necessary to hold the unknown rolling block in equilibrium. In order to overcome the effects of friction in finding this force, it is *necessary* to find the average between the force required to make the rolling block move *upward slowly* with constant velocity and the force necessary to make the same block move *downward* with the same velocity.

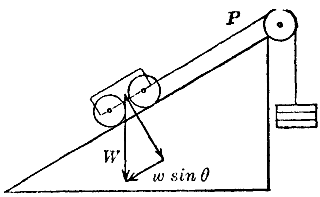


FIG. 26.

The force P pulling the rolling body should be kept parallel to the plane. This is done by adjusting the pulley accordingly. Obtain about six to eight readings, using different values of θ, and from these calculate the value of the mass and also the average error and the per cent of error. Check the mass by weighing on a balance.

P_1 FORCE UP	P_2 FORCE DOWN	P AVERAGE FORCE	θ	$\sin\theta$	$W = \frac{P}{\sin\theta}$

Part (b). The determination of the dynamic coefficient of friction consists in measuring the force of friction on a level surface. This force of friction is measured by applying the force P horizontally until the block just slides with a very slow and constant velocity. In the above apparatus make $\theta = 0$ and add just the correct weights until the block slides slowly (Fig. 27). This might necessitate interpolation if the slotted

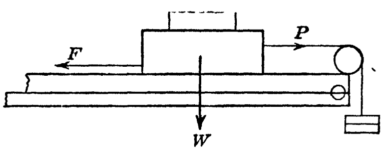


FIG. 27.

weights which are put on the hanger are not provided in small enough steps. Make three to five trials, adding weights to W, and redetermine P. Enter your results in tabular form. Calculate μ, the coefficient of friction, and state the accuracy of your result. Compare your result with the result given in a table of physical constants for these two kinds of surfaces.

Part (c). The coefficient of friction is found to be the tangent of the limiting angle of repose. The proof for this can be found in almost any physics text. The method consists in finding the limiting angle to which the plane can be raised so as to have the block slide down by itself slowly and with constant velocity. Make two or three trials of this angle for the block alone. Then try the effect of adding mass on top of the block, recording two readings of θ for each mass added.

Put your results in tabular form and calculate μ. Find the per cent of difference between the two values.

QUESTIONS

(a). Is this the dynamic or static coefficient of friction?

(b). Which of the two methods described under Part (b) and Part (c) gives better results for μ?

(c). In Part (b) how does the coefficient of friction vary with the weight of the body? Explain your answer.

(d). In Part (c) how does the coefficient of friction vary with the mass added to the body? Explain your results.

EXPERIMENT 7

AN EXPERIMENTAL STUDY OF THE LAWS OF EQUILIBRIUM OF A RIGID BODY

Part (a). *Determination of the center of gravity of a non-uniform bar.*

Part (b). *To find the mass of this bar being given a known mass (say 50 grams).*

Part (c). *To find two unknown masses being given a known mass.*

Apparatus: A meter bar which is non-uniformly leaded by little metal slugs built into the bar, a knife-edged clamp which can be fixed at any point along the bar, a vertical support for this knife-edge clamp, several movable clamps from which to hang the known or unknown masses, a known mass, and two unknown masses (Fig. 28).

Part (a). The position of the center of gravity of the bar can be simply determined by balancing the bar on a knife edge. The vertical plane through the knife edge passes through the center of gravity. Find the center of gravity by balancing the bar on all four sides.

Part (b). Balance the bar on a knife edge when a known mass of 50 grams is hung from some point along the bar. Having

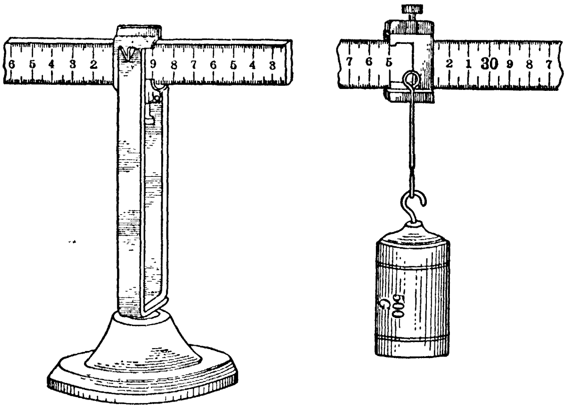


FIG. 28. — Arrangement for equilibrium of moments.

measured the distances from the fulcrum, write down the conditions of equilibrium and solve for W, the weight of the bar. (Note that the known weight of the hanger must be included.) For this calculation you may assume the position of the center of gravity as found in Part (a).

Repeat, using different positions for the 50-gram mass. Tabulate your results, and find an average value for the mass. Calculate the per cent of error in the mass. (If time allows, find the position of the center of gravity and the weight of the bar as described in Problem 9 at the end of this chapter.)

Part (c). Hang the two unknown masses X and Y from any two positions along the bar (*e.g.*, somewhere near the ends), and place the fulcrum so as to balance the system. Next add a known mass to either X or Y and readjust the distances for a balance. Measure the distances of all the forces from the fulcrum in either case (don't forget to consider W, the mass of the bar) and write down the conditions for equilibrium. Solve for X and Y. As a check find the masses X and Y on a balance.

Repeat this part of the experiment by adding the known mass to the other unknown.

QUESTIONS

(a). Why does Part (a) give us the center of gravity?

(b). Suppose the bar came to rest at 45°, where would the center of gravity be? Show by means of a sketch.

(c). Show clearly why the first condition for equilibrium is satisfied in each case.

PROBLEMS

Experiment 5

1. Find by graphical construction the vector sum (direction and magnitude) of the following vectors: 5 North, 4 West, 3 East, 6 East.

2. If the vectors in Problem 1 represent a number of forces acting on a particle, how will you determine whether the particle is in equilibrium? If it is not, then what force would be necessary to keep it in equilibrium?

3. Two forces of 10 pounds each act on a point. Find by graphical construction the resultant (or sum) of these two forces when the angle between them varies from 0° to 180°. Plot a curve showing the relation between the magnitude of the resultant and the angle between the forces.

Experiment 6

4. A body is moving up an incline of 35° with a velocity of 20 feet per second. What are the components of this velocity horizontally and vertically?

5. A body of mass 10 pounds is held in equilibrium on a smooth incline by a tension of 160 poundals acting upwards along the plane. Show that the angle of the plane must be 30° and also that the sum of the component forces, in any direction you wish to choose, must be zero.

6. Define "coefficient of friction," and distinguish between static and dynamic coefficients. Discuss practical conditions under which either one or the other is used.

7. Prove that the coefficient of friction is equal to the tangent of the limiting angle of repose.

8. A horizontal force of 9 pounds in weight can keep a load weighing 60 pounds in steady motion along a horizontal table. What is the coefficient of friction? Now if the table is tilted slightly, what is the minimum angle of tilt of the table for the body to slide down by itself without the 9-pound force?

Experiment 7

9. Without assuming the position of the center of gravity of the bar, how can you find both this position and the weight of the bar, when balancing the bar on a knife edge by means of the 50-gram mass twice?

10. A rod AB carries bodies weighing 5 pounds, 7 pounds, and 8 pounds, at distances of 2 inches, 8 inches, and 14 inches, respectively, from A. Neglect the weight of the rod and find the point at which the rod must be supported for equilibrium to be possible.

CHAPTER V

MACHINES

A machine is a device, or mechanism, which will transfer a force from one point of application to another for some useful advantage. The pulley system, the jack-screw, and the lever are everyday illustrations of machines. The pulley is commonly used to hoist heavy loads, such as steel girders, through great distances, while the jack-screw and lever are employed to lift massive objects through short distances. The primary object of these machines is the utilization of a great force by the application of a small force.

There are many cases, however, where a change in direction of the force is the primary object of a machine, but not a change in magnitude. Illustrations are supplied by a single pulley, reversing belts, and reversing gear wheels. More generally, however, the machine is arranged to change both the magnitude and direction. These characteristics are demonstrated by a study of the block and tackle, the differential pulley, the transmission gear, the wedge (*i.e.*, double inclined plane), and the wheel and axle.

Mechanical advantage and efficiency. While machines enable us to do work in an easier and more advantageous way, one should not be misled into thinking that the total work put into the machine (*i.e.*, input) to accomplish the task is any less than that obtained from the machine (*i.e.*, output). In fact, it is generally greater than that obtained from the machine because of frictional losses.

In sum, the useful advantage of a machine depends, theoretically, upon a knowledge of the resultant and applied forces, and consequently the frictional losses. *The ratio of the force exerted by the machine to the force applied by the operator is called the mechanical advantage*, while *the ratio of the work obtained from the machine (output) to that put into the machine (input) is called the efficiency.* To express the efficiency in per cent, the ratio is multiplied by 100. Notice that the mechanical advantage is a ratio of forces, while the efficiency is a ratio of work. The latter determines the frictional losses.

Two possibilities are to be considered in the development of the theory of machines, namely, (1) the *ideal case*, (2) the *actual case*.

The ideal case. The ideal case is thought of as a machine having no frictional losses. No such machines exist, but a few approach the ideal condition (such as, for example, the lever). The ideal case is useful to consider, however, since it gives us a limiting minimum force and is found to be related to the actual efficiency of the machine.

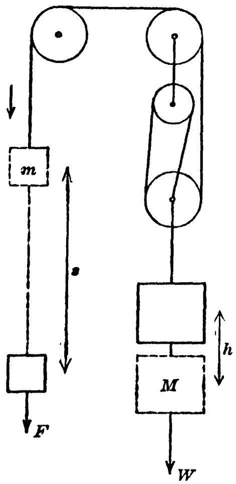


FIG. 29. — A pulley system — ideal case.

The ideal mechanical advantage (I. M. A.) is (Fig. 29) by definition

$$\text{I. M. A.} = \frac{W}{F},$$

and since, by the theory of conservation of energy,

$$Fs = Wh,$$

provided there are no frictional losses in the machine where s and h are the respective distances through which F and W operate, we have

$$\text{I. M. A.} = \frac{W}{F} = \frac{s}{h}.$$

It will be seen that, in the pulley system pictured in Figure 29 and Figure 30, when M goes up 1 cm., F will go down 3 cm. Hence the ideal mechanical advantage equals 3, since

$$\text{I. M. A.} = \frac{s}{h} = 3.$$

The ideal mechanical advantage may be figured from the dimensions of the machine as noted above. It should be noticed, however, that whatever is saved in force by the ideal machine is lost in distance through which the force operates, so that the work done is the same whether the machine is used or not. Consequently, the efficiency in the ideal case is

$$\text{Eff.} = \frac{Wh}{Fs} = 100\%.$$

The actual case. Frictional losses are always encountered in practice. To illustrate, Figure 30 represents a mass M being lifted where the frictional force is found to be of the magnitude f, while the ideal force (*i.e.*, force as found when there is no friction) is F. The frictional force f is found by experiment. By definition the mechanical advantage is

$$\text{M. A.} = \frac{W}{F + f}.$$

It is to be seen that in this case the work $[(F + f)s]$ put into the machine is greater than that (Wh) obtained from the machine, the difference being dissipated by friction inside the machine. The efficiency is given by:

$$\text{Eff.} = \frac{\text{output}}{\text{input}} = \frac{Wh}{(F + f)s},$$

and since $Wh = Fs$,

$$\text{Eff.} = \frac{Fs}{(F + f)s} = \frac{F}{F + f}.$$

Note that we may write

$$\frac{F}{F + f} = \frac{\dfrac{W}{F + f}}{\dfrac{W}{F}},$$

so that the efficiency may be written as

$$\text{Eff.} = \frac{\text{actual mechanical advantage}}{\text{ideal mechanical advantage}}.$$

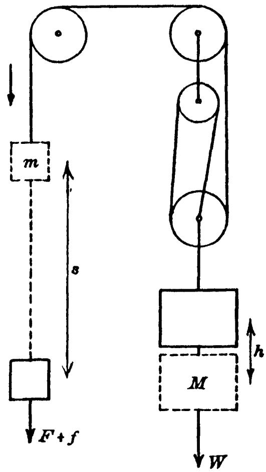


FIG. 30.—Pulley system—actual case.

When f is greater than F, the work to overcome the friction becomes greater than the output work of the machine. This gives us a self-locking machine, the efficiency of which is less than 50 per cent. The jack-screw, wedge, and the differential pulley are examples of this type. The large mechanical advantage usually associated with these machines makes them very useful for the transfer or dislodgment of very massive objects.

POWER

The rate at which work is done is called the power. Two machines which exert equal forces can have very different power ratings depending upon the rapidity with which the force moves and consequently the rate at which the machine does work.

The fundamental units of power will be:

(1) in the c. g. s. system = 1 erg per second.
(2) in the f. p. s. system = 1 foot-poundal per second.

In addition to these, two common units found in practice are:

1 ft.-lb./sec. = 32 ft. poundals/sec.
1 horse-power = 33,000 ft.-lbs./min.
1 watt = 10^7 ergs/sec.
= 1 joule/sec.

[*Note.* 1 H.P. = 746 watts.]

EXPERIMENT 8

PULLEYS

Part (a). *To find the actual mechanical advantage, efficiency, and force of friction in a pulley system.*
Part (b). *To calculate the horse-power output of the machine.*
Part (c). *To show graphically the effect of different loads on the machine.*

Apparatus: A mounted double and single pulley, a single pulley, strong twine, two sets of slotted weights (10–500 grams), two hangers, meter stick, and stop clock.

Part (a). Set up the apparatus as shown in Figure 30. Place a standard mass (say 200 grams) in the pan meant for the load W. This mass, together with the hanger and movable pulley (neglect weight of string), constitutes the total load of weight W which the machine exerts. On the other pan, which is to supply the force applied to the machine, add a number of standard masses until this side of the pulley system *falls without acceleration.* To offset the effect of static friction, give the system a slight push to start it moving. These masses, together with the hanger, give the force $(F + f)$ necessary to raise the weight W. The two separate forces, the ideal force F and the force f opposite and equal

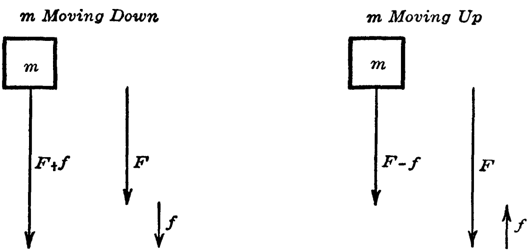


FIG. 31. — Vector representation of the forces.

to the force of friction, are pictured in Figure 31. Then take off masses on the side at which the force is applied until it will go up without acceleration. This force is $F - f$. Notice that the direction of f has changed. In this case the frictional force, which is acting equal and opposite to f, is directed downward. If the 10-grams mass is not sufficiently small for adjustments of mass to secure constant velocity for rise and fall, estimation of the correct mass should be made.

Repeat the experiment with standard masses of 400, 600, 800, 1000, and 1200 grams placed in succession on the weight pan. The following table will be found useful for recording the results.

W Including Hanger, Pulley	Force (gm. weight)			$\frac{Wh}{s}$	$(F+f) - F$	$\frac{W}{F+f}$	$\frac{F}{F+f}$
	Down $(F+f)$	*Up* $(F-f)$	F				

Calculate from the data the ideal force applied to the machine (*i.e.*, F), the frictional force $[f = (F + f) - F]$, the actual mechanical advantage $\left(\frac{W}{F + f}\right)$, and the efficiency $\left(\frac{F}{F + f}\right)$ for the various loads.

Part (b). Note with a stop clock the time it takes to raise the largest load with constant speed as far as the apparatus will allow. Measure the distance and calculate from this the horse-power output of the machine. This gives the horse-power output for this particular load and speed.

Part (c). (To be done at home.) Plot on a single coördinate paper the loads as abscissae against the efficiency, actual mechanical advantage, and friction respectively as ordinates. The curves are more easily observed if different-colored ink is used to trace each curve. The ordinates for each curve should be inked in corresponding colors. If colored ink is not available, the curves can be distinguished by use of broken lines, dots and dashes, following some systematic scheme, such as dot-dash, long dash-short dash, etc.

QUESTIONS

(a). How does the efficiency of the pulley system change with the load? Explain your results.

(b). How does the actual mechanical advantage change with the load? Explain your results.

(c). Express the power output (calculated) in terms of watts.

PROBLEMS

1. What is meant by the term " mechanical advantage "?

2. Give examples of the use of machines where the " mechanical advantage " is less than one.

3. Explain the use of the terms $(F + f)$ and $(F - f)$ as used in the theory.

4. Make two schematic drawings of pulley systems having a theoretical mechanical advantage of 5 and $\frac{1}{3}$ respectively.

5. Suppose that in a pulley system, as shown in Figure 30, $M = 600$ grams, while the mass m necessary to pull it up without acceleration is 230 grams. Find the following: (1) ideal force F, (2) frictional force due to the pulley system, (3) the actual mechanical advantage, (4) the efficiency. [*Note.* Assume that the ideal mechanical advantage is given by the number of cords supporting the mass of 600 grams.]

6. A painter is suspended to the side of a house by the pulley system in Figure 30. The upper block of the pulley system is fastened by a hook to the cornice of the building. The painter may fasten the free or hoisting end of the rope either to his staging or to the side of the building. If the weight of the man and pulley system is 600 lbs., examine the force exerted on the cornice of the building in each case.

CHAPTER VI

ELASTICITY

All bodies are deformed in some way by the application of a force, no matter how small that force may be. A perfectly elastic body will return to its original shape, or position, when the applied forces are removed. In any actual case, there is a limit to the magnitude of the force which may be applied, if the body is to return to its original state. This is called the *elastic limit.* A greater force would cause permanent distortion and finally fracture. Applications of the laws of elastic bodies may be seen in watch springs, automobile springs, spring balances, etc.

One of the most important properties of an elastic body is that, when it is bent, twisted, compressed, or stretched, *the ratio of the magnitude of the applied force to the deformation is constant.* In order that we may obtain the same constant for bodies, made of the same material, no matter what the dimensions of the object may be, we state that, within the elastic limit, the ratio of the stress to the strain is a constant e,

i.e.,
$$e = \frac{\text{stress}}{\text{strain}}.$$

(This is *Hooke's Law,* discovered by Robert Hooke in 1660.) The constant e is called the *modulus of elasticity.* Stress is measured by the magnitude of the force per unit area causing distortion. Actually, the term *stress* refers to the internal forces per unit area set up to oppose the external force. These forces are, however, equal and opposite for all cases which we shall consider.

The deformations may be, essentially, a change of shape, length, or volume. In practice, the most important deformation, resulting from a pulling or compressing force, is the change in length. For our purposes, then, we will consider only deformations which are essentially changes of length.

Stretching of Materials

Consider a wire of length l and of cross-section A (Fig. 32) to be stretched by a force F so that the new length is $(l + \Delta l)$. Application of Hooke's Law gives

$$\frac{\text{stress}}{\text{strain}} = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \text{constant.}$$

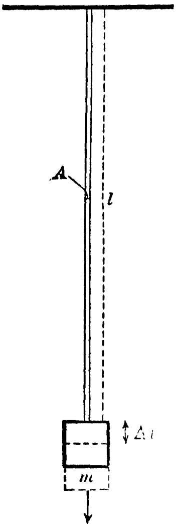


Fig. 32. — Wire stretched by a force F.

Instead of calling the constant e, we substitute for it the letter Y (and call this Young's modulus, in honor of Thomas Young, who first gave physical meaning to this constant), which expresses the relation between stress and strain.

Consequently,

$$(1) \qquad Y = \frac{Fl}{A\Delta l}.$$

Bending Beams

In engineering practice, considerably less material is needed if girders be placed so that the thin side is vertical, for bending is usually inversely proportional to the cube of the depth but only to the first power of thickness. The H type steel girder and the steel rail are examples of useful applications of the law of bending beams. Theory and experiment show that the bend B in a beam of rectangular cross-section is proportional to the force F and to the cube of its length l; and inversely proportional to its breadth b and to the cube of its depth d; that is:

$$B \propto \frac{Fl^3}{bd^3}$$

or

$$B = C\frac{Fl^3}{bd^3},$$

where C is a constant, depending upon the mode of support and the material of the rod. When the rod is supported by a fulcrum at either end and the force is applied midway between them, then

$$C = \frac{1}{4\,Y},$$

where Y represents Young's modulus, and

$$B = \frac{Fl^3}{4\,Ybd^3}.$$

Notice that bending is essentially a combination of stretching and compression. This explains why Young's modulus appears in our equation. The top half of the beam is compressed, while the lower half is stretched.

EXPERIMENT 9

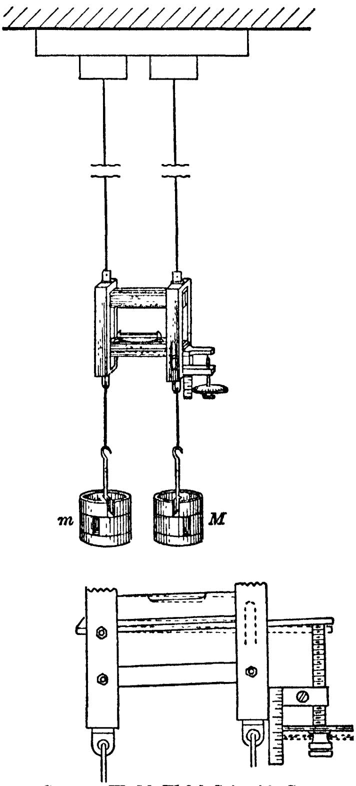
YOUNG'S MODULUS

A determination of Young's modulus for materials in the form of long wires.

Apparatus: Young's modulus apparatus, meter stick, weights (1–14 kg.).

The apparatus consists of two wires of equal length, each having one end fastened to a rigid support or beam in the ceiling, while the other end is attached to a rectangular frame, as shown in Figure 33. The frame is loaded under each wire with masses m and M. A level with micrometer screw attachment tells us how much the wire is stretched for a given load.

The mass m is used to keep the wire at a constant tension and, in particular, to eliminate any kinks in the wire. This mass m should be one or two kilograms, depending upon the size of the wire. The other mass M is variable and is used to vary the tension on the second wire.



Courtesy W. M. Welch Scientific Company

FIG. 33. — Young's modulus apparatus.

The procedure is to adjust the micrometer screw so that the bubble in the spirit level is in the center when a load M, of one or two kilograms, is applied. Then increase the value of M by two kilograms and bring the bubble to its zero position by adjustment of the micrometer screw. Repeat until 10 or 12 kilograms have been added. Now since Young's modulus is given by the equation

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{Fl}{A\Delta l},$$

we can find Y by substituting our measured values of F, A, Δl, and l in equation (1). In order to test Hooke's Law we shall plot the stress (*i.e.*, force per unit area) as ordinates and strain (*i.e.*, change of length per unit length) as abscissae. If Hooke's Law is valid, a straight line should result. Provided the elastic limit has not been exceeded, draw the best straight line through the plotted points. If the line does not pass through the origin, draw a parallel line that does. For large loads (*i.e.*, when the elastic limit is exceeded), the curve bends towards the horizontal.

Calculate Y from your straight line, which passes through the origin, by taking the ratio of any ordinate to the corresponding

Use C. G. S. Units

Total Force F (dynes)	Force per Unit Area $\frac{F}{A}$ (dynes per cm^2.)	Micrometer Readings (cm.)	Total Stretch Δl	Change in Length per Unit Length $\frac{\Delta l}{l}$

To Find Diameter of Wire (cm.)

Trials	*Diameter*
1	
2	
3	
4	
5	
Total Average diameter	

abscissa. Choose an ordinate of reasonably large value in order to minimize the error in reading your graph. The unit of Y will, of course, depend upon the units in which the stress is measured.

QUESTIONS

(a). What is the least count in centimeters for the micrometer screw in your experiment on Young's modulus?

(b). Which of the quantities that you measured in finding Young's modulus has the least per cent of error? If possible, give an estimate of the per cent of error in your result.

(c). Suppose you had made an error of one centimeter in measuring the length l, would the error be justified, considering the accuracy of the other measurements?

(d). If the micrometer screw which raises or lowers the level is not located immediately under the wire to be stretched, will any correction be necessary? Explain your answer.

EXPERIMENT 10

BENDING BEAMS

A determination of the coefficients of the factors in the equations representing the bend.

Apparatus: Bending beams apparatus, beam holder, weights (1–1000 grams), battery and sounder or electric light, meter rule.

The equation representing the bend B of a beam under conditions of the experiment have been given previously by the equation,

$$(2) \qquad B = \frac{CFl^3}{bd^3} = \frac{Fl^3}{4\,Ybd^3}.$$

We will now rewrite the above equation in the form,

$$(3) \qquad B = CF^h l^k b^m d^n,$$

the object being to obtain experimentally the values of the exponents h, k, m, and n. When these values are found experimentally equation (3) should be identical to equation (2). Our experiment, then, represents a method of establishing an equation experimentally.

By taking the logarithm of equation (3), one obtains

$$(4) \quad \log B = \log C + h \log F + k \log l + m \log b + n \log d.$$

Now, suppose that one wishes to find the value of h. First, find the bend B_1 when a force F_1 is placed on the hanger, and then, the bend B_2 when a force F_2 is employed. When the force F_1 is placed on the hanger, equation (4) becomes

(5) $\log B_1 = \log C + h \log F_1 + k \log l + m \log b + n \log d.$

Similarly, with F_2 on the hanger, equation (4) gives:

(6) $\log B_2 = \log C + h \log F_2 + k \log l + m \log b + n \log d.$

Solving equations (5) and (6) for h, gives

$$h = \frac{\log B_2 - \log B_1}{\log F_2 - \log F_1}, \tag{7}$$

which is the exponent of the force F. If equation (2) is correct, then the experimental value of h should be 1 (within experimental error).

Similarly, to find the value of k, one varies the length (l) of his beam only, keeping the force constant. If there are bends B_1' and B_2' corresponding to lengths l_1' and l_2', then it may be shown readily that

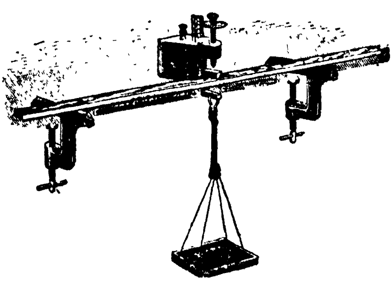
$$k = \frac{\log B_2' - \log B_1'}{\log l_2' - \log l_1'}. \tag{8}$$

The student may verify by methods similar to the above that

$$m = \frac{\log B_2'' - \log B_1''}{\log b_2'' - \log b_1''}$$

and

$$n = \frac{\log B_2''' - \log B_1'''}{\log d_2''' - \log d_1'''}.$$



Courtesy Central Scientific Company

FIG. 34.—Bending beams apparatus

However, the directions for the experimental procedure will be confined to the evaluation of h and k.

The apparatus is pictured in Figure 34. The beam is supported by two substantial knife edges. At the center of the bar a hanger is suspended from a metal stirrup. The metal stirrup is provided with a knife edge and binding post. The bend is measured by a micrometer screw which makes contact with the top of the knife edge. Contact is indicated by the closing of an electric circuit which operates a buzzer or lights a lamp. The micrometer screw should be examined before any measurements are taken to determine its pitch and value of each division on the milled disc in terms of centimeters. Then turn down the micrometer screw slowly in each case to make contact. There is a small difference in reading of the micrometer depending upon whether the micrometer is turned down to just make contact or turned up to just break contact. The micrometer should not be in contact with the bar when masses are being added or taken off the hanger. It is a good policy to have a zero load of 200 grams in the pan.

Suppose that we wish to find the value of h. Take the initial reading of the micrometer screw with the zero load of 200 grams. Now increase the force on the pan by adding a mass of 200 grams. Take the new reading. The difference between the two readings represents the bend B for a force of 200g dynes. Continue your experiment by reading the micrometer screw for each additional 200 grams mass added to the pan until the maximum safe load is obtained, say 1000 grams. Now, take the micrometer screw readings as the load is reduced by 200g dyne steps.

To calculate *h*. From your data obtained when the loads were increased by 200g dynes, calculate the mean value of h for the following combination of forces: 200g and 600g, 400g and 800g, 600g and 1000g. Likewise, find the mean value of h from the data obtained for the reduction of the loads in steps of 200g dynes.

To find *k*, clamp the knife edges near the extreme ends of the beam and take the micrometer reading of the 200g dyne zero load. Now add a force of 500g dynes to the load and take the new micrometer reading. Always keep the load exactly midway between the knife edges. You now have a bend B for a length of beam l due to a load of 500g dynes. Now move each knife edge in towards the center about 5 cm. and repeat the micrometer measurements for the zero load of 200g and the 500g additional load. Repeat the micrometer measurements two more times, moving each knife edge in towards the center 5 cm. each time. Remember that the measurement of the bend is made in each case by the addition of the constant load of 500g dynes to the zero load of 200g dynes. Obtain at least three values of k from your experiment.

To Find h

Relation between B and F	
Mass in Grams	*Micrometer Reading*
200	

To Find k

l (cm.)	*Zero Load* (200g) *Micrometer Reading*	*Micrometer Reading for an Additional Load of* 500g *Dynes*

Note. Measure the breadth b and depth d of the beam used to find the value of k. These values will be needed in Question (**b**) below.

QUESTIONS

(**a**). What is the least count in centimeters for the micrometer screw in your experiment on bending beams?

(**b**). From the data obtained in the calculation of exponent k, determine the value of Y for the maximum value of l.

(**c**). Draw a curve between the bends as ordinates and the corresponding lengths as abscissae from the data on the experimental determination of k. Repeat for the logarithms of the bends as ordinates and the logarithms of the corresponding lengths as abscissae (see page 12).

(**d**). Is either curve a straight line? What kind of curves would you expect if you plotted in a similar fashion the data obtained in the experimental determination of h?

PROBLEMS

Experiment 9

1. Assume that a wire could be stretched to twice its original length, and still remain within its elastic limit and that the change in its cross-section is negligible. What expression would you obtain for Young's modulus?

2. From an examination of the equation representing Young's modulus, what relation exists between the distorting force and stretch for any given wire?

3. What is the object of the initial load of two kilograms suspended from each wire before measurements are taken?

4. Are any corrections, due to the stretch of the initial load of two kilograms on each wire, to be made in the calculations for Young's modulus? State reasons for your answer.

5. A flat brass rod and a flat copper rod, each 4 meters long and 0.2 square centimeter in cross-section, are rigidly connected all along their lengths. If a mass of 25 kilograms is suspended from one end of the combined rods, what will be the resultant stretch of the bar and the restoring force exerted by each bar? Assume Young's modulus for brass to be $9.2 \times 10^{11} \frac{\text{dynes}}{\text{cm}^2.}$; for copper, $10 \times 10^{11} \frac{\text{dynes}}{\text{cm}^2.}$

6. What will be the length of a brass rod which stretches one-half the distance of a 5-meter steel rod when subjected to the same stretching force, assuming that the steel rod is one-half the diameter of the brass rod. If the stretching force is 15 kg. and the diameter of the steel rod is 0.5 mm., what will be the stretch in this rod? Young's modulus for steel is $22 \times 10^{11} \frac{\text{dynes}}{\text{cm}^2.}$ and for brass as given in Problem 5.

7. A 50-foot steel rod of diameter 3 inches changes in length 0.8 inch due to the difference between summer and winter temperatures. Calculate the mechanical force necessary to stretch the rod this amount.

8. Assuming Young's modulus for brass to be $13.4 \times 10^{6} \frac{\text{lbs.}}{\text{in}^2.}$, what diameter of brass wire will be necessary to sustain a load of 75 pounds? Express your result in terms of the B and S gauge as well as in centimeters.

9. From the results of Problem 8, calculate the minimum thickness of a brass wire that will be necessary to support a 150-pound weight.

Experiment 10

10. If the load in the experiment on bending beams is not applied at the center of the rod, what portion of the theory of the experiment might you expect to be altered?

11. Would you expect Young's modulus to appear in an expression representing the bending of a beam? Explain your answer.

12. A beam 2 meters long, 5 mm. wide, and 1.1 cm. deep is bent 8 mm. by a load F placed at its center. How long must a beam of similar material be which is 4 mm. wide and 5 mm. deep so as to bend 1 cm. with a load of magnitude $2\,F$ placed at its center?

13. Assume that a brass and a steel bar of equal lengths are rigidly connected along their lengths. If each is 1.8 meters in length and 0.6×0.6 cm^2. in cross-section, what will be the bend if the bars are supported at both ends by knife edges and have a load of 50 gms. suspended from their centers? Assume the combined rods are placed so that both the brass rod and the steel rod are in contact with the supports and that Young's modulus for each bar is the same as values given in Problems 5 and 6. What will be the weight supported by each bar?

CHAPTER VII

PERIODIC MOTIONS

From Hooke's Law we have seen (Chapter VI) that a distortion is proportional to the distorting force. Hence in Figure 35, if a force F displaces a mass m a distance x, against the restoring force due to the springs, then $F \propto x$, and our equation of motion becomes

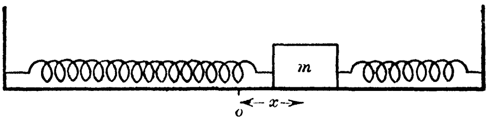


FIG. 35. — Simple harmonic motion on a frictionless table.

(1) $$F = kx,$$

where k is a constant to be determined. The oscillatory motion of the mass m is called simple harmonic motion. The equation tells us that *simple harmonic motion is a linear motion such that the magnitude of the restoring force is proportional to the displacement.* In addition, the restoring force and displacement are opposite in direction. This latter point is not indicated in the equation for simple harmonic motion and is omitted to add simplicity in the later development which may be accomplished without use of the negative sign for the displacement x. One of the most important characteristics of simple harmonic motion is its period, or the inverse, which is the frequency. Either of these could be obtained readily from equation (1) by the use of the calculus. Since the use of calculus is beyond the scope of the present work, we shall study the uniform speed of a point on the circumference of a circle, projected on any diameter. This projected motion is an example of simple harmonic motion.

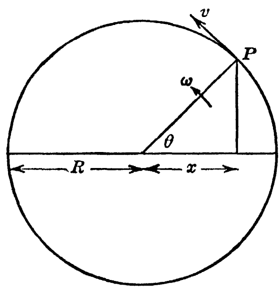


FIG. 36. — Circular motion of point P.

Consider the motion (Fig. 36) of a point P on the circumference of the circle of radius R and having a velocity v. We have from the figure that:

$$v = R\omega,$$

where ω is the angular velocity; also that

$$a = \frac{v^2}{R} = \omega^2 R.$$

The acceleration along the x axis is

$$a_x = a \cos\theta = \omega^2 R \cos\theta = \omega^2 x$$

and the force in the x direction is

$$F = ma_x = m\omega^2 x.$$

Since by definition,

$$\omega = \frac{2\pi}{T},$$

we have that:

(2) $$F = \frac{4\pi^2 m}{T^2}\cdot x.$$

This shows that the constant k for equation (1) has the value,

$$k = \frac{4\pi^2 m}{T^2}.$$

Our equation (2) gives us the expression

(3) $$T = 2\pi\sqrt{\frac{m}{\left(\frac{F}{x}\right)}}$$

for the period. It is the time interval taken by the object to pass any point (*e.g.*, the origin, Fig. 36) of reference two consecutive times in the same direction while making its to-and-fro excursions. In the case of a simple harmonic motion R is often called the amplitude.

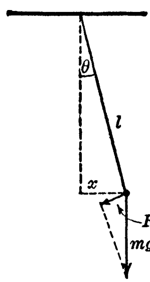


FIG. 37. — Simple pendulum.

The motion of a *simple pendulum* may be treated as simple harmonic if the oscillation occurs through a sufficiently small angle. Referring to Figure 37, if the initial angle θ is less than 10°, the oscillation of mass m will be approximately simple harmonic motion and the period of oscillation (a complete to-and-fro motion) will be a constant quantity.

In the figure a small but massive spherical ball is fastened to a cord of negligible weight. When the spherical ball is small, we may consider the length of the pendulum as the distance from the upper support to the center of the spherical mass. When the pendulum is displaced a slight distance, the restoring force F, due to gravity, is related to the displacement x by the equation

$$\frac{F}{mg} = \frac{x}{l}$$

and from equation (3), we obtain

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{4}$$

as the approximate period of a simple pendulum. The inexactness of the equation is due to the fact that x is not actually the displacement as required in simple harmonic motion. However, the error for $\theta = 10°$ is only of the order of 0.2% and correspondingly less for smaller angles. Hence, the pendulum may be made a fairly accurate instrument for the determination of the acceleration of gravity, provided the period (T) can be measured with accuracy.

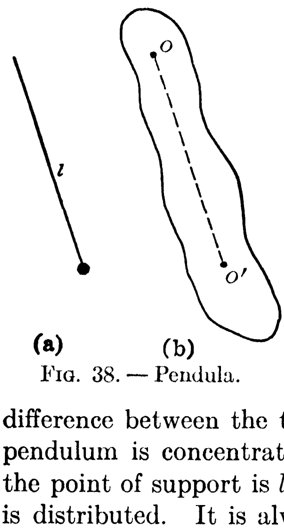


FIG. 38. — Pendula.

The more accurate measurements of g by the pendulum method, however, are usually accomplished with some form of a *compound pendulum*. In Figure 38 let b represent a compound pendulum and a, a simple pendulum. The essential difference between the two pendula is, that the mass in a simple pendulum is concentrated at a point the distance of which from the point of support is l, while the mass in a compound pendulum is distributed. It is always possible to find a simple pendulum of a length l which has the same period as a given compound pendulum.

Consider the simple and compound pendulum in Figure 38. The compound pendulum is assumed to have equal periods when suspended from points O and O'. These equal periods may be proved to be equal to the period of the simple pendulum whose length is the same as the distance between O and O'. The points O and O' of the compound pendulum have an important physical interpretation. If the compound pendulum is struck at O' by a force in the plane of the paper and at right angles to OO', it will oscillate freely about O without any force of the blow communicating itself to the point O. The point O' is called the center of percussion. Furthermore, if the compound pendulum is suspended from O', point O now becomes the center of percussion.

The *center of percussion* is often marked on baseball bats since if the ball is batted from this point there will be no force reaction (sting) on the hands. Of course, the center of percussion for the thin end of the bat is of no practical use.

In our experiment, then, the problem is to find the two points O and O' about which the two periods for the compound pendulum will be the same. When these two points are found, we shall know that the distance between O and O' represents the length of simple pendulum, having the same period as the compound pendulum suspended from O or O'. From this data the acceleration due to gravity may be calculated from the formula for a simple pendulum by using equation (4), or

$$g = 4\pi^2 \frac{l}{T^2}. \tag{5}$$

The above description of the ideas underlying the use of a compound pendulum to measure the acceleration due to gravity is substituted for the more formal proof which belongs to more advanced treatises.

Torsion Pendulum

Moment of inertia (I) is defined as the summation

$$I = \sum_{k=1}^{k=n} m_k r_k^2,$$

where there are n distinct particles. The mass in each case may be considered as concentrated at a point the distance of which from the center of rotation is r. The moments of inertia about any axis are usually found by the methods of calculus. Even by these methods the procedure is very difficult, and sometimes impossible when irregularly shaped bodies have to be considered. The moment of inertia of any body can, however, be readily found by experimental methods.

The student will probably have observed in his studies concerning the dynamics of rotation that this quantity I, which has been called the moment of inertia, plays about the same rôle there that mass does in the dynamics of translation.

Consequently, it can easily be shown that the general expression for the period of a simple harmonic motion of rotation has exactly the same form as the general expression for the period in the case of linear simple harmonic motion. For the linear case, $T = 2\pi\sqrt{\frac{mx}{F}}$, where x is the linear displacement produced by a force F, whereas for angular simple harmonic motion, $T = 2\pi\sqrt{\frac{I\theta}{L}}$, where θ is the angular displacement produced by a torque L, and I represents the moment of inertia of the rotating body around the axis of rotation.

A very convenient way of determining experimentally the moment of inertia of a body of any shape (around an axis of rotation) is to attach it to the end of a wire, clamped so that the wire hangs vertically and then allow it to perform rotational simple harmonic motion. The period of oscillation is carefully observed. Next a body of known moment of inertia is added and the period of the combination redetermined. From these two periods and the known moment of inertia of the body which was added, it is possible to calculate the unknown moment of inertia of the first original body.

For the first condition (unknown body only),

$$T_1 = 2\pi\sqrt{\frac{I\theta}{L}}, \tag{7}$$

where I is the unknown moment of inertia.

In the second case (unknown + known),

$$T_2 = 2\pi\sqrt{\frac{(I + I_1)\theta}{L}}, \tag{8}$$

where I_1 is the known moment of inertia.

Dividing (7) by (8), we get

$$\frac{T_1}{T_2} = \sqrt{\frac{I}{I + I_1}},$$

or

$$I = \frac{T_1^2}{T_2^2 - T_1^2} I_1,$$

from which I can be calculated.

In the apparatus used, the body added is in the form of a thin ring of mass m and average radius r, whose moment of inertia I around an axis through the center is known to be $I = mr^2$.

Note. If the ring cannot be assumed to be thin, but if instead it has a thickness which is appreciable compared to the radius, then the more exact formula should be used in calculating the moment of inertia of the ring, *viz.*, $I = \frac{1}{2} m(r_1^2 + r_2^2)$.

EXPERIMENT 11

CENTRIFUGAL FORCE

A verification of the law of force for uniform circular motion.

Apparatus: Any form of centrifugal force apparatus, spring balance.

The type of apparatus to be used is shown in the diagram (Fig. 39). A shaft AB can be made to rotate around its own axis. A crosspiece, having suspended from it a metal ball m, is rigidly attached to this shaft. When the shaft rotates, the ball will tend to fly outwards, but is prevented from doing this by the spring. When the proper speed is attained and kept constant, the wire, supporting the ball, will be vertical and its position will be marked by a fixed pointer P below. A string passing down the center of the spring acts as a safeguard in case the spring is stretched too much.

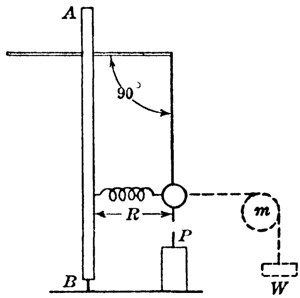


FIG. 39. — Centrifugal force apparatus.

Several **precautions** must be strictly observed, otherwise the ball, which has a considerable mass, might break off and cause injury to someone. With just a little common sense and care this is entirely unnecessary. The procedure should be somewhat as follows: Before starting, check the apparatus. See that the shaft AB is not too tight nor too lose in its bearings. Next see that the pointer P is in the correct place. Do this by stretching the spring (without rotation) and pulling the ball to the right until the wire hangs vertical. The pointer should then be under the projecting wire on the ball. Now gently start rotating the shaft, first by giving it a twist with the fingers and then keeping it in slow rotation by just running your finger along the shaft, helping it or retarding it as the case may be. Practice for about five minutes and see whether you can acquire the technique for holding the speed constant. You will note that the number of revolutions per second is not very large. Now practice keeping the speed such as to keep the ball exactly over the pointer. Let your partner do the same.

Having mastered the technique, you are ready to start your observations. We wish to show, within experimental error, that the centripetal force acting toward the center is correctly given by $F = \frac{mv^2}{R}$. Consequently the experimental measurements involve measuring, (1) the mass of the ball m; (2) the velocity v; (3) the radius of the circular motion R. The linear velocity of the ball is found by timing a certain number of revolutions with a stop watch when the ball is running with constant speed, the speed being the proper value to keep the two pointers over each other. Time the system for 50 or 100 turns. Make about three trials, recording each trial. Let your partner take three or four more. Average all these trials and calculate the average period for one revolution. Do not forget to calculate the error.

Next, to find R, it is necessary to measure only the distance from the center of the shaft to the pointer P. This can be done by measuring the distance from P to the shaft by means of a meter rule and then adding one-half the diameter of the shaft as measured with caliper.

Knowing now the time T for one revolution and the radius (in cms.), it is a simple matter to calculate the linear velocity v, since $\frac{2\pi R}{T} = v$. In this way find v and the per cent of error in the measured value of v. Finally, calculate F, and the per cent of error, from the formula for the centripetal force.

Check this value roughly, experimentally, by attaching a spring balance to the ball and pulling the latter out until the ball is again over the pointer P. Take the reading of the balance and express this force in dynes. Repeat two or more times in order to estimate the precision of this measurement. Calculate the percentage difference between the two values for the force.

QUESTIONS

(a). Which gives the more accurate method for finding F?

(b). Point out in your figure where and in which direction the centripetal and the centrifugal forces act.

(c). What would be the objection to having the pointer P at a distance from the axis of rotation not equal to that of the length of the arm c?

EXPERIMENT 12

THE PERIOD OF OSCILLATION AND FORCE CONSTANT OF A SPRING

Part (a). *To determine the force constant of the spring.*

Part (b). *To determine experimentally the period of oscillation of the system for several masses.*

Part (c). *To check on the experimental period found in (b), by calculation from the formula using the force constant found in (a).*

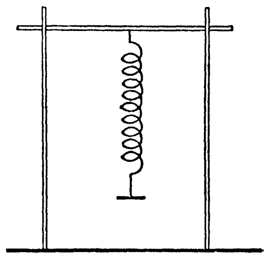


FIG. 40. — Simple harmonic motion apparatus.

Apparatus: Spiral spring mounted on rods and clamped rigidly to the table (Fig. 40), slotted weights (up to 1000 grams), hanger, stop clock, meter rule.

Part (a). Add to the hanger 100, 200, 300 grams, etc., up to 1000 grams. Place the meter rule vertically and take readings of a certain point on the hanger before and after each load. Record the stretch of the spring for every 100 grams added.

Tabulate your results as shown:

INITIAL MASS (grams)	MASS ADDED (grams)	INITIAL READING (cm.)	FINAL READING (cm.)	STRETCH (x) (per 100 grams)

Examine the successive values found for the stretch x, when additional loads of 100 grams are added. The value of x, of course, should be constant since Hooke's Law is assumed to hold. For the smaller loads it is quite possible that x is not constant. This, however, is due to negative tension in the spring which must be overcome before the spring really becomes stretched. In averaging your values to find x, take only those which show that beyond a certain point the negative tension has been overcome.

$$\text{Then the force constant} = \frac{F}{x} = \frac{100\text{ g.}}{\text{av. } x}\text{ dynes/(cm.).}$$

Part (b). Determine the periods of three masses experimentally by taking the average of one hundred oscillations for each mass. Each of the three masses should be in excess of that required to overcome any negative tension of the spring.

Mass of the spring =

m Includes Hanger and $\frac{1}{3}$ Weight of Spring	Time for 100 Oscillations	Period of Mass m		% Difference from the Mean
		By Theory Part (c)	By Exp.	

Part (c). Calculate from equation (3) the theoretical period for each of the three masses in Part (b), using the value of $\frac{F}{x}$ found in Part (a). The mass m must include the weight of the hanger as well as one-third the mass of the spring if the latter mass is appreciable compared to the other masses. (The reason for adding one-third the mass of the spring appears in more extended treatises.)

QUESTIONS

(a). In your experiment what would be the (1) maximum restoring force of the spring, (2) maximum acceleration of the mass, (3) maximum velocity of the mass if the amplitude of swing was 3 cm. and the mass on the spring was 500 gm.?

(b). What will be the error in the period of the smallest mass used in Part (b) if the mass of the spring is neglected?

EXPERIMENT 13

THE SIMPLE PENDULUM

To determine g, the acceleration due to gravity.

Apparatus: Same as in Experiment 4.

If the data for Experiment 4 are available, find the value of g from the curve or other observations made at that time, using the formula

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

If the previously taken data are not available, repeat the experiment.

QUESTIONS

(a). What must be the length of a simple pendulum in order that (1) the period may be one second? (2) The half period may be one second?

(b). In Experiment 12 you have found a curve showing the relation between T^2 and l. This should be a straight line passing through the origin. Prove that the slope of this line is equal to $\frac{4\pi^2}{g}$.

(c). Suppose that you had found experimentally that the value of this slope is 0.04, what is the value of g, the acceleration of gravity?

(d). What value did your results give for the slope of this line?

EXPERIMENT 14

KATER'S PENDULUM

To find the value of g.

Apparatus: Kater's reversible pendulum and support, meter stick, stop clock.

Kater's pendulum consists of a bar with masses attached so that one or all of them may be adjusted to several positions. It is constructed to have one or more suspension points from either end. The instrument as shown in Figure 41 is a straight steel bar weighing about $2\frac{1}{2}$ lb. with $\frac{3}{8}$-inch holes bored along the bar beginning one inch from either end and spacing two inches apart. They are slightly beveled and have a groove to engage the knife edge. The length of this pendulum bar is about $4\frac{1}{2}$ feet. The mass at the top consists of two discs, each weighing about $1\frac{3}{4}$ lb. and bolted to the bar by a bolt $\frac{3}{8}$ inch in diameter and a wing nut. This mass is kept in the same position throughout the experiment. The other mass, consisting of two discs near the bottom, is fastened to the bar in the same way as the one described above. The weight of each of these discs, however, is approximately 0.9 lb.

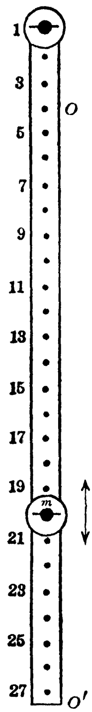


FIG. 41. — Kater's pendulum.

The method used in this experiment is to find the period from either support O and O' and to adjust the sliding mass m up or down until the periods from each end are equal. The approximate values of the periods for the preliminary work may be found by taking the average of ten complete oscillations. Suppose the periods to be about the same when the mass m is bolted through hole No. 20. To determine more accurately the period of oscillation for equal periods from each end, find the period about O and O' by taking the average of fifty complete oscillations with the mass m bolted to holes Nos. 18, 19, 20, 21, and 22. Plot the period about each point of support as an ordinate against the numbers representing the holes. The intersection of the two curves for the periods about O and O' will give the period of a simple pendulum the length of which is the distance between O and O'. The distance between the supports O and O' must be measured carefully.

PRELIMINARY READINGS

POSITION OF m	TIME FOR 10 VIB. ABOUT	
	O	O'

FINAL READINGS

POSITION OF m	TIME FOR 50 VIB. ABOUT	
	O	O'

	MEASURE OF DISTANCE BETWEEN O AND O' (l)
Average	

Value of T, the period, obtained from the curve $= g = \dfrac{4\pi^2 l}{T^2}$.

Per cent of error from accepted value =

QUESTIONS

(a). Estimate the error in your experiment on Kater's pendulum from the data obtained. Compare with the per cent of error from the accepted value.

(b). Let the period of a simple pendulum of length 99.6 cm. be 2.10 seconds. Find the distance between the two knife edges of a Kater's pendulum which has a period of 4.15 seconds at the same place.

EXPERIMENT 15

THE TORSION PENDULUM

To find the moment inertia of a disc.

Apparatus: Torsion pendulum with rod, table clamp and test tube holder, inertia ring, stop clock, meter stick.

The apparatus consists essentially of a disc rigidly attached to a wire of length l. The upper end of the wire is attached to a cylindrical block, which, in turn, is held rigidly by a clamp (Fig. 42). If the disc of mass M is given a small twist, say a half-turn, it will oscillate back and forth with a given period. If the ring of mass M is now placed on top of the disc so that the wire is at the center of the ring, the period will be longer. From these two values of the periods it is possible to obtain the moment of inertia of the disc of mass M. This, therefore, is *an experimental method for finding the moment of inertia of a disc. The method is applicable no matter what the shape of M.* In order to check the experimental determination, the body M is given a shape such that it is also possible to calculate, by methods of calculus, the moment of inertia. When M has the shape of a disc as shown in the figure, then the calculation by calculus gives I (disc around a central axis) $= \frac{1}{2} Mr^2$, where M is the mass of the disc and r its radius. Consequently in performing this experiment find the time, first without and then with the ring, of 25 or more complete angular oscillations. Make a number of observations of this same quantity, recording each one in a table or form shown below. By

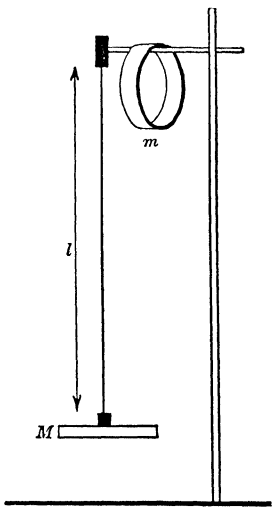


FIG. 42. — Torsion pendulum.

averaging each set of readings and then dividing by the number of oscillations, the period of a single oscillation can be calculated. Unless the mass of the disc and the mass of the ring are already given, it will now be necessary to weigh them on a balance.

Record also the radius of the disc and the inner and outer radius of the ring. These can all be found by averaging a number of readings taken with the vernier caliper. If the approximate formula is used for the ring (mr^2), then r must be the mean radius.

To Find the Period (Seconds)

Trial	Time for 25 Vibrations			
	$25\,T_1$	Deviations	$25\,T_2$	Deviations
1				
2				
3				
Total				
Average				

To Find the Radii (cm.)

	Diam. of Disc	Deviations	Diam. (Inside) Ring	Deviations	Diam. (Outside) Ring	Deviations
Total						
Average						
Radius						

Mass of the ring = grams
Mass of the disc = grams
Experimental value of T_1 = sec.
Experimental value of T_2 = sec.
also I_1 (ring) = gm.-cm.^{2}
hence I (disc) = gm.-cm.^{2}
also I (calc) = gm.-cm.^{2}

QUESTIONS

(**a**). Using the more exact formula for the moment of inertia of the ring, calculate the moment of inertia of the disc.

(**b**). Compare the two values for the moment of inertia of the disc as found by experiment (using the approximate and the exact values for the moment of inertia of the ring) and calculate the change in the error when the values are compared with the calculated value of the moment of inertia of the disc (*i.e.*, $I = \frac{1}{2} Mr^2$). 16300 dynes

1.42

PROBLEMS

Experiment 11

1. Suppose that, in Figure 39, the mass m revolves 50 times in 60 seconds. What will be the tension in the spring if $m = 500$ gm. and radius $R = 30$ cm.?

2. What would be the time T for one revolution in the question above if $m = 1000$ gm.?

3. What will be the time T for one revolution in Problem 1, if the tension of the spring and the mass m remain unchanged, but the radius $R = 60$ cm.?

Experiment 12 distance

4. Give a definition of simple harmonic motion.

5. What is the amplitude of a simple harmonic motion?

6. Is the period of oscillation in any way dependent on the amplitude of the oscillation?

7. If the period of your spring, with a mass m, should be obtained in another locality with a different value of g (the acceleration due to gravity), would you expect the period to be altered? Explain your answer.

8. If a mass of 50 grams stretches a given spring 1 cm., what would be the period of the spring if 200 grams were placed on the spring?

Experiment 13 greater force at mass

9. Is the oscillation of a simple pendulum an example of simple harmonic motion? Explain your answer.

10. How does the period of a simple pendulum vary with (1) the length, (2) the mass of the bob?

11. Would you expect the period of a simple pendulum to be greater or less at the top of a tall building when compared with the period at the base of the building?

12. A pendulum of length 99.5 cm. swings at a place where the acceleration due to gravity is 980.2 $\frac{\text{cm.}}{\text{sec.}^2}$. If it is intended to be a seconds pendulum (period of 2 seconds), what time is lost or gained per day? What should its length be?

Experiment 14

13. What is the meaning of the expression "center of percussion"?

14. When the period of a compound pendulum about O and O' (Fig. 38 b) is the same, what does this mean in terms of a simple pendulum?

15. What method is used in the experiment with Kater's pendulum to assure periods about two points O and O' to be equal?

Experiment 15

16. Does the period of a torsion pendulum vary with the angular displacement?

17. Calculate the moment of inertia of a disc around an axis through the center, given that the mass is 500 grams and the radius 10 cm.

18. Find the moment of inertia of a ring around a central axis if its mass is 500 grams and the inside and outside diameters are 12 and 14 cm. respectively.

19. Verify the equation $I = I_1\left(\frac{T_1^2}{T_2^2 - T_1^2}\right)$ for a torsion pendulum.

CHAPTER VIII

SPECIFIC GRAVITY AND DENSITY OF BODIES

The *density* of a body is defined as the mass per unit volume. Consequently, if d represents the density, v the volume, and m the mass, then

$$d = \frac{m}{v}.$$

In fundamental units the density is expressed in $\frac{\text{gm.}}{\text{cm.}^3}$ or $\frac{\text{lbs.}}{\text{ft.}^3}$. Thus in the English (f. p. s.) system the densities of water, mercury at 20° C., and gold (cast) are 62.4, 845.6, and 1204.6 $\frac{\text{lbs.}}{\text{ft.}^3}$ respectively, whereas in the metric (c. g. s.) system these densities become 1, 13.56, and 19.3 $\frac{\text{gm.}}{\text{cm.}^3}$.

It is very common, in the case of liquids, to compare the weight of a given volume to the weight of the same volume of some standard substance. The standard substance generally taken is water. *This ratio is called the specific gravity of the substance.* Specific gravity can also be expressed as the ratio of two densities, *i.e.*,

$$\text{Sp. gr.} = \frac{d_u}{d_k},$$

where d_u and d_k are the densities of the unknown and known substances respectively. Referring to the above figures for water and mercury, it will be seen that in either system of units the specific gravity of water is 1 and that of mercury is 13.56, if water is chosen as the standard.

Density is a concrete number with dimensions dependent on the system of units used; while *specific gravity* is an abstract number, and hence without dimensions. The value of the specific gravity of a liquid, therefore, does not depend on the system of units. Because of the fact that in the metric system a unit volume of water has a mass of 1 gram, the specific gravity and density in that system have equal numerical magnitudes.

EXPERIMENT 16

ARCHIMEDES' PRINCIPLE AND THE DENSITY OF SOLIDS

Part (a). *To find the density of solids heavier than water.*
Part (b). *To find the density of solids lighter than water.*
Part (c). *To measure the specific gravity of a liquid.*
Part (d). *To find the length of a tangle of wire.*

Apparatus: *Part* (*a*). Solids insoluble in water, platform balance, standard weights, thread.

Part (*b*). Solids lighter than water (*e.g.*, paraffin, wood, etc.), a sinker, balance as in Part (a).

Part (*c*). A liquid such as carbon tetrachloride, or glycerin, balance as in Part (a).

Part (*d*). Fine balance, platform to fit over left pan of fine balance, beaker (500 cc.), loosely tangled copper wire of approximately # 22 B & S gauge, micrometer screw, cleaning solution (see Part (d) of experiment).

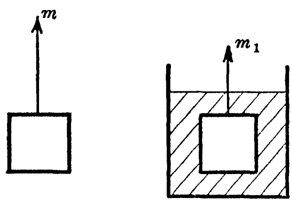


FIG. 43. — Solid denser than water.

Part (a). Weigh the solid in air and let its mass be m. Then weigh the solid (Fig. 43) in water. If the weight in water is m_1, its loss of weight in water is $m - m_1$. This represents the weight of water displaced, which numerically is equal to the volume of the block. Hence, the density is

$$d = \frac{m}{m - m_1}.$$

Part (b). Attach a sinker to the body of mass m, the density of which is desired. If the weight of the combined masses with sinker just immersed is m_1, and with both solids immersed is m_2, then the density of the mass m is (Fig. 44)

$$d = \frac{m}{m_1 - m_2}.$$

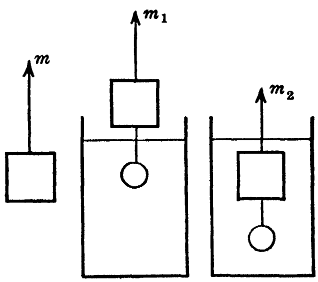


FIG. 44. — Solid less dense than water.

Part (c). Weigh a solid (insoluble in water or in the liquid whose specific gravity is desired) in air and call its mass m. Then weigh the solid immersed in water (Fig. 45), and then in the liquid

x. If the weight of the solid in water is m_1, and in the liquid is m_2, then the specific gravity of the liquid is

$$\text{Sp. gr.} = \frac{m - m_2}{m - m_1}.$$

This results (1) from Archimedes' principle, which states that the loss of weight of a body immersed in a fluid is equal to the weight of the fluid displaced; and (2) from the definition of specific gravity, which is the ratio of the masses of the two liquids when the volumes are the same.

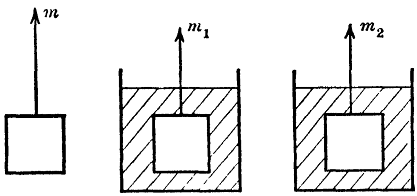


FIG. 45.—A liquid.

Part (d). In this experiment, one finds the volume of the wire by determining the loss of weight in water by means of the fine balance, and also the diameter d of the wire by means of a micrometer screw. Thus if l is the length of the wire, then the volume (v) of the wire is

$$v = \frac{\pi}{4} d^2 l, \quad \text{or} \quad l = \frac{4\,v}{\pi d^2}.$$

The loss of weight of the wire is found in the following manner: Clean the wire in an alkali solution made of sodium hydroxide (20 or more grams per liter) and rinse with clean water. Make a tiny hook at one end of the wire and attach to a hook directly under the left-hand knife edge of the balance arm. Then place a platform over the left-hand pan and arrange the apparatus so that the tangle is immersed in a beaker of water placed on this platform. Exercise great care in placing the platform and beaker of water over the scale pan so that no water is spilled on the balance. Carelessness in this operation ruins a balance and in this case is inexcusable. Weigh the tangled wire when immersed in the water. The difference between the two weights is numerically the volume. [*Note.* The length of the portion above water may be added to the resultant length under water, as found by calculation, later.]

QUESTIONS

(a). What effect would bubbles of air, gathered on the side of the solid when immersed in water, have on (1) the density of the solids in Part (a); (2) the specific gravity of the liquid in Part (c)?

(**b**). What kind of errors may be expected in Part (d) if the wire is tangled too tightly or if the wire is not clean?

(**c**). Discuss and approximate the accuracy of each result in Parts (a), (b), (c), and (d) of this experiment.

EXPERIMENT 17

THE DETERMINATION OF THE SPECIFIC GRAVITY OF LIQUIDS

Part (a). *By means of the specific gravity bottle.*
Part (b). *By means of the U-tube.*

Apparatus: *Part (a).* Fine balance and weights, specific gravity bottle, liquids, heated compressed air for drying out the specific gravity bottles.

Part (b). U-tube about one foot or more in length, liquids, meter stick, water, mercury, carbon tetrachloride, glycerin, etc.

Part (a). If the specific gravity bottle is not dry and clean, rinse with water and then dry by blowing heated compressed air into the bottle. It is then weighed on the fine balance. Fill the bottle with water and weigh again. Next, having emptied and dried the specific gravity bottle, it is again weighed when filled with the liquid the specific gravity of which is desired. The bottle should always be exactly full when at room temperature. If m, m_1, and m_2, in the order given, represent the mass of the empty bottle, the mass of the bottle when filled with water, and the mass of the bottle when filled with the liquid, the specific gravity is

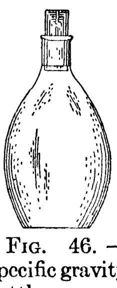


FIG. 46. — Specific gravity bottle.

$$\text{Sp. gr.} = \frac{m_2 - m}{m_1 - m}.$$

Note that there is a hole of capillary size along the axis of the ground stopper (Fig. 46). This hole is provided to allow excess liquid to escape from the bottle upon sealing with the stopper. The bottle should be wiped after closing with the stopper. *To prevent the formation of bubbles within the bottle,* insert the stopper in such a fashion as to have one side touching the neck of the bottle and the bottle tipped slightly.

Repeat your experiment one or more times and calculate the per cent of error. Compare the value found by experiment with the value as given in tables.

Part (b). Fill the U-tube about one third to one half full of the liquid, then add water to one side until the tube is filled to a desired height. In Figure 47, the shaded portion represents the liquid of which the specific gravity is desired, while the unshaded portion is water. The water column of length l_2 is balanced by the column of liquid of length l_1.

It may be shown that

$$l_2 d_2 = l_1 d_1$$

and, therefore, we have

$$\text{Sp. gr.} = \frac{d_1}{d_2} = \frac{l_2}{l_1}.$$

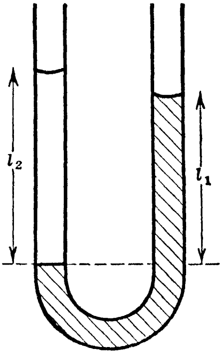


FIG. 47. — Specific gravity by U-tube method.

QUESTIONS

(**a**). What is the accepted specific gravity of the liquid used? State the source of information. What is the per cent of error of your experiment (use your average result) from this accepted value?

(**b**). Could you perform the experiment if some water appears on both sides of the tube?

(**c**). Compare the results of the specific gravity as found in Part (a) and Part (b). Which method gives more accurate results?

10.5 9.6

PROBLEMS

Experiment 1[illegible]

1. State all the differences you can think of between the definitions of specific gravity and density.

2. A spherical mass of cast gold weighing 100 grams is thought to have a hollow center. When weighed in water, it is found to weigh 90 grams. What is the volume of the inclosed air space? (Density of gold $= 19.3\ \frac{\text{gm.}}{\text{cm.}^3}$.)

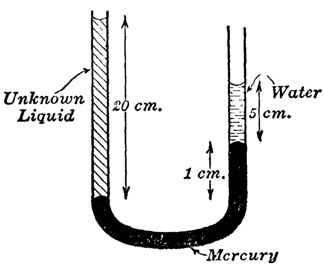


FIG. 48. — U-tube with liquids as required in Problem 8.

3. An alloy of silver and gold is made to have a density of $13\ \frac{\text{gm.}}{\text{cm.}^3}$ and a mass of 55 grams. What mass of silver was used? [*Note*. Take the densities of silver and gold to be $10.5\ \frac{\text{gm.}}{\text{cm.}^3}$ and $19.3\ \frac{\text{gm.}}{\text{cm.}^3}$ respectively.]

4. A glass receptacle contains a layer of mercury and then a layer of water. If a cast steel rectangle of density $7.6\ \frac{\text{gm.}}{\text{cm.}^3}$ is dropped into the dish, what portion of the depth of the rectangle will be above the mercury surface? $\left(\text{Mercury has a density of } 13.5\ \frac{\text{gm.}}{\text{cm.}^3}.\right)$

5. Do you think iron would sink to the bottom of the ocean?

Experiment 17

6. What advantage is obtained by the use of a specific gravity bottle with the kind of stopper described in Experiment 17, Part (a)?

7. What is the theory that leads to the conclusion $l_2d_2 = l_1d_1$? Would the theory be altered if the U-tube had a larger diameter on one side than on the other?

8. A U-tube contains liquids as shown in Figure 48. Assume that the density of water and mercury are known and calculate the density of the unknown liquid.

CHAPTER IX

EXPANSION OF SOLIDS, LIQUIDS, AND GASES

Most substances expand with an increase of temperature. A few, however, contract with temperature, while others change but little. The amount of expansion with heat seems to be related very closely with the grouping of the molecules and the forces between them. Thus with gases the motions of the molecules are wholly at random and the expansions are very uniform for many gases. In general, the expansions, with increase of temperature, are less marked in the order: gases, liquids, and solids.

By expansion, we may refer to the change in length, surface, or volume, due to a change in temperature. Thus with the laying of pipe lines, the building of bridges, or the construction of steel buildings, it is the change in length that is important; while in the construction of thermometer bulbs or the filling of liquid containers, the volume expansion is important. Surface expansions will not be considered.

The linear expansion of a solid is measured in terms of the change of length per unit length per unit rise of temperature. We call this the *linear coefficient of expansion* of a solid, and it is designated by the letter α. Thus, if Δl represents the total change produced in the length,

(1) $$\alpha = \frac{1}{l}\frac{\Delta l}{\Delta t}.$$

Here α is the average linear coefficient of expansion for the temperature interval Δt, and l is the length at the initial temperature. The average volume coefficient of expansion (β) for the temperature interval Δt is defined in a similar way, namely,

(2) $$\beta = \frac{1}{l}\frac{\Delta v}{\Delta t},$$

where v is the volume at the initial temperature.

It will be noticed that if we let l_0 and l_1 be the initial and final lengths, while t_0 and t_1 are the initial and final temperatures, we have from (1)

$$\alpha = \frac{l_1 - l_0}{l_0(t_1 - t_0)}$$

or

$$l_1 = l_0[1 + \alpha(t_1 - t_0)],$$

and if $t_0 = 0$,

(3) $$l_1 = l_0(1 + \alpha t_1).$$

Similarly,

$$v_1 = v_0[1 + \beta(t_1 - t_0)],$$

and when $t_0 = 0$,

(4) $$v_1 = v_0(1 + \beta t_1).$$

[*Note.* Many use the letter t instead of Δt to represent the change of temperature.]

Gases expand so markedly that the volume (or linear) coefficient of expansion will have different values, measurable in the laboratory, if different initial temperatures from which the expansion is measured are chosen. The initial temperature usually referred to is 0° C.

The volume coefficient of expansion of a gas at constant pressure with an initial volume (v_0) at 0° C. is defined as

$$\beta_p = \frac{v_1 - v_0}{v_0 t_1}, \quad \text{or} \quad v_1 = v_0(1 + \beta_p t_1).$$

The pressure coefficient of expansion at constant volume is

$$\beta_v = \frac{p_1 - p_0}{p_0 t_1}, \quad \text{or} \quad p_1 = p_0(1 + \beta_v t_1).$$

It may be shown that for a perfect gas

$$\beta_p = \beta_v = \frac{1}{T_0},$$

where $T_0 = 273$ (to a first approximation), the temperature on the absolute scale which corresponds to zero degrees on the Centigrade scale.

If we wish to study the expansion of gases where the initial state of the gas is not given at zero degrees Centigrade, it will be found more convenient, generally, to use the more general gas law,

$$pv = RT,$$

i.e.,

$$\frac{pv}{T} = \text{const.} = R.$$

In this equation, T is the temperature on the absolute scale and has the relation to temperatures t on the Centigrade scale that

$$T = 273 + t.$$

EXPERIMENT 18

THE AIR THERMOMETER

To find the value of β by use of the air thermometer.

Apparatus: Air thermometer, ice, large beaker, steam jacket, thermometer.

The apparatus (Fig. 49) consists of glass bulb with stem of capillary bore, connected to a flexible rubber tubing as shown. A straight glass tubing of about $\frac{1}{4}$ inch in diameter is connected to the other side of the rubber tubing. Clean mercury fills the space between the levels a and b.

Keeping the volume of air inclosed in the bulb constant, theory shows that the pressure in the bulb is related to the temperature of the bulb in the following way:

$$\beta_v = \frac{p_1 - p_0}{p_0 t_1}.$$

Consequently it becomes necessary to find the corresponding pressure in the bulb for two or more temperatures.

In the experiment, place the bulb in a mixture of chopped ice and water to obtain the pressure in the bulb for zero temperature. This pressure (p_0), if a is higher than b, will be:

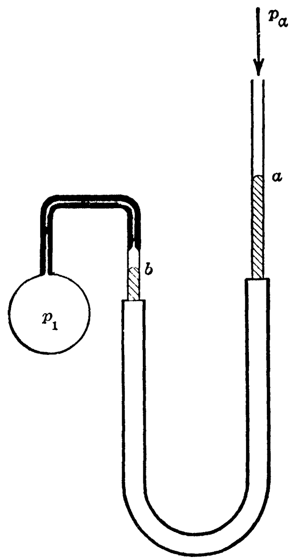


FIG. 49.—Air thermometer.

$$p_0 = p_a + (a - b) \text{ centimeters of Hg},$$

where p_a is the atmospheric pressure in centimeters of mercury (Hg), while $a - b$ is the difference in height of the two sides of the mercury column, measured in centimeters. The height of a and b may be measured from the table top if the apparatus contains no scale of its own. It may turn out that a is lower than b. In such a case the pressure in the bulb is less than atmospheric pressure; hence the difference $(b - a)$ is subtracted from p_a. The atmospheric pressure (p_a) is to be obtained from any good barometer.

Next insert the bulb in a steam jacket to get the pressure (p_1) for steam temperature. Remember that you are to adjust your apparatus so that the volume within the bulb is to remain constant. This means that the top of the mercury column at b must remain at the same position as before. To do this the other side of the mercury column must be raised. Again find the pressure, now at 100° C. From your results obtain a value for β. [*Note.* Before you take the bulb from the steam bath, lower the side which has the top of the mercury column marked with the letter a, so that mercury will not go into the bulb when it cools.]

Having surrounded the bulb with a water jacket, find the pressure corresponding to temperatures in the neighborhood of 25, 50, and 75 degrees Centigrade to determine whether the statement is true that the pressure increases linearly with the temperature. Plot your results with pressure as ordinate and temperature as abscissa. Extend your curve until it crosses the axis of the abscissa and interpret the intercepts. Enter your data in a tabular form as shown:

BAROMETER HEIGHT $(p_a) =$

TEMP.	HEIGHT OF Hg		$a-b$	$p_1 = p_a + (a-b)$	$\frac{p_1}{T}$
	a (Open)	b (Closed)			
Ice					
Room					
Medium					
Hot					
Steam					

$\beta_v =$

QUESTIONS

(a). What does your "curve" tell you as to the relation between the pressures and temperatures?

(b). At what temperature did your curve cross the axis of abscissa?

(c). What relation would you expect to find if you divided the pressure in the bulb by the absolute temperature for each of the four temperatures at which the pressures were measured? Explain your answer.

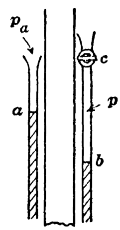
EXPERIMENT 19

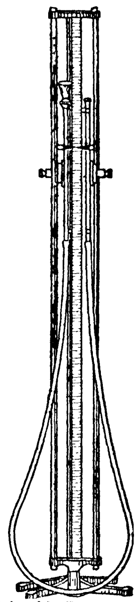
BOYLE'S LAW

To show that the product of the pressure and the volume of a gas (air) is a constant if the temperature is kept constant.

Apparatus: Any standard Boyle's law apparatus.

In the apparatus to be described (Fig. 50), the closed side of which the volume is to be determined is on the right, while an open end is provided on the left. There are two pieces of glass tubing of uniform bore connected by a flexible rubber tubing. The apparatus is filled with mercury to levels a and b, as shown. The volume of the inclosed space is changed by altering the height of the mercury column on the left side.





Courtesy Central Scientific Company

FIG. 50. — Boyle's law apparatus.

Since the tube is of uniform bore, Boyle's law,

$$p_1v_1 = p_2v_2,$$

may be written in the form,

$$p_1l_1 = p_2l_2,$$

where A is the cross-sectional area of the tube, and $v_1 = Al_1$ and $v_2 = Al_2$. The pressure p inside the inclosed gas (air) will be

$$p = p_a + (a - b)$$

centimeters of mercury where p_a is the pressure of the atmosphere. If we do not alter the height of the tube on the right, then the top of the closed tube (c) will remain at some constant reading on the scale, and the whole experiment may be done by reading different positions of a and b on the scale when the left-hand column is altered in height. Always allow the air in the closed tube to stand for a few minutes after changing the level to allow the air to acquire the temperature of the room.

Take several (8 to 12) readings by changing the height on the left-hand side and plot a curve between the reciprocal of the length ($l = c - b$) of the inclosed volume, as abscissae, against the pressure in centimeters of mercury as ordinates.

$p_a =$ cm. of Hg $c =$ cm.

Reading of Hg Levels		$a - b$ (cm.)	$p = p_a + (a - b)$	$l = (c - b)$	$\frac{1}{l}$	pl
a (cm.)	b (cm.)					

QUESTIONS

(**a**). In which way will a plot of $1/l$ against p prove Boyle's law?

(**b**). Plot l against p for your experiment. What is the shape of the curve?

(**c**). Why is it necessary to insure that the inclosed air is always at room temperature?

(**d**). Is the value of pl as found in the last column constant? If not, discuss possible reasons for the variation of this quantity.

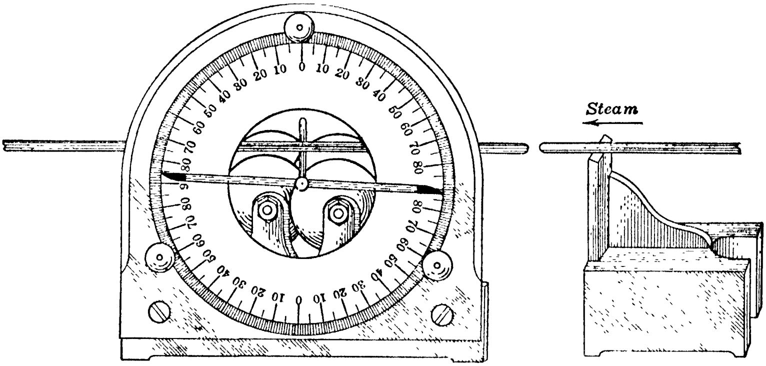
EXPERIMENT 20

LINEAR EXPANSION

To determine the linear coefficient of expansion of a rod.

Apparatus: Linear coefficient of expansion apparatus, thermometer, steam generator, bunsen burner with rubber tubing, meter stick, micrometer screw.

The apparatus (Fig. 51) consists of a hollow tube of copper or brass, resting at the one end, which is notched, on a pointed support; at the other, on a spindle which carries a pointer. Since the notched end is fixed, it follows that when the length changes, the pointer turns in front of a dial graduated in degrees. Record the temperature of the room near your apparatus, and assume it to be the initial temperature of your expansion rod. Take the initial reading of the dial. It is important not to disturb the apparatus after this initial dial reading is taken. Then pass steam, which has a temperature of 100° C. at normal pressure, through the hollow tubing. After a short interval of time the dial will come to rest at a new position, the reading of which should



Courtesy Central Scientific Company

FIG. 51. — Linear coefficient of expansion apparatus.

be taken. Let the angular change in degrees between the initial and final dial reading be called θ. Consequently the change in length (Δl) of the rod is

$$\Delta l = 2\,\pi r \frac{\theta}{360},$$

where r is the radius of the spindle. This radius is found by means of the micrometer screw by taking the average of several readings of the diameter and dividing by 2. The length l of the rod under consideration is, of course, the distance from the notch to the spindle.

The average linear coefficient of expansion of the rod for the temperature interval $\Delta t = t_1 - t_0$ is

$$\alpha = \frac{1}{l}\frac{\Delta l}{\Delta t}.$$

Repeat the experiment two or more times. Allow the rod to acquire room temperature before taking the initial reading.

QUESTIONS

(a). If the experiment is repeated only once, find the per cent of error from the mean. If repeated more than once, find the maximum per cent of error from the mean.

(b). Determine the per cent of error of your average result from the accepted value.

(c). Would you call the method for obtaining t_0, the initial temperature, very exact or scientific? Can you think of any other procedure that may have some merit?

PROBLEMS

Experiment 18

1. Why is the hydrogen gas thermometer called a "standard thermometer"?

2. Show that for a perfect gas, *i.e.*, a gas which satisfies the relation

$$R = \frac{p_0 v_0}{T_0} = \frac{p_t v_t}{T_0 + t}$$

the following is true:

$$\beta_p = \frac{v_1 - v_0}{v_0 t_1} = \frac{1}{T_0}; \qquad \beta_v = \frac{p_1 - p_0}{p_0 t_1} = \frac{1}{T_0}$$

3. Would carbon dioxide make a good "standard thermometer"?

4. Consult a handbook and record the values of the volume coefficients of expansion of five to ten representative gases, liquids, and solids. In which state do you find (1) the greatest variation of this coefficient and (2) the largest value for the volume coefficient of expansion?

5. Why is it necessary to specify some initial temperature from which to reckon the volume coefficient of expansion for gases while with solids the exact position of the initial temperature is not so important?

Experiment 19

6. Would Boyle's law hold for carbon dioxide at ordinary (20° to 25° C.) temperatures? Why?

7. A certain perfect barometer has a space of 5 cm. above the mercury level when the atmospheric pressure is normal. A little air is allowed to enter the barometer so that the level of mercury falls 3 cm. If the diameter of the barometer tube is 1 cm., what was the volume of air before it entered?

Experiment 20

8. With the pressure of the atmosphere as 74 cm. of Hg instead of normal (76 cm.), what would be the per cent of error in the calculated coefficient of expansion due to the temperature of steam being taken as 100° C.? Assume the initial temperature to be 22° C., the length of the rod 55 cm., and the expansion 0.5 mm.

9. How much will a steel bridge one mile in length expand between extreme temperatures of − 20° C. and + 40° C.?

CHAPTER X

MEASUREMENT OF HEAT AND CHANGE OF STATE

The science of calorimetry deals with the measurement of quantities of heat. Since the absolute heat energy in any body is a rather vague quantity and of course would be very difficult to measure, we confine ourselves in calorimetry to measurements of heat changes and transfers from one body to another. In any case, in our practical life it is only exchanges of heat and their effects that are of interest to us.

Heat measurements have always formed a very interesting part of practical physics, yet they are a very difficult part, however, when extreme accuracy is attempted. We find in this field scientists who have spent a whole lifetime in these researches and have won fame for the degree of skill and technique which they have developed. Such names as Joule, Rowland, and Regnault will never be forgotten, and the student would find the time well spent if he will read some of the records of the published works of these men.

Units Involved in Heat Measurements

The unit of heat in the c. g. s. system is the calorie, which is arbitrarily defined as the amount of heat necessary to raise the temperature of 1 gram of water 1° C., at some specific temperature. The unit varies somewhat, depending upon the temperature, but the variation is so small that for ordinary work the difference is negligible (less than 0.1% per degree change). We shall assume that the unit is the same for all the temperatures that we will use.

The specific heat or heat capacity (s) of a substance is defined as the amount of heat (measured in calories) necessary to raise the temperature of 1 gram of the substance through 1° C.

The water equivalent of a body or vessel (w) is the amount of water which would require the same amount of heat as the body, in order to raise the temperature through 1° C. Hence:

$$w = m \times s,$$

where m is the mass of the calorimeter and s is the specific heat of the metal of which it is made.

The latent heat of fusion (L) is defined as the amount of heat (in calories) necessary to change 1 gram of the substance from a solid to a liquid at the same temperature. ($L = 80$ calories per gram for water and the change takes place at 0° C.)

The latent heat of vaporization (l) is defined as the amount of heat (in calories) necessary to change 1 gram of the substance from a liquid to a vapor at the same temperature. ($l = 540$ calories per gram for water and the change takes place at 100° C.)

The following examples illustrate the use of these definitions.

1. The amount of heat necessary to change the temperature of $\frac{1}{2}$ kg. of water from 15° C. to 25° C. is $ms(t_2 - t_1)$ calories $= 500 \times 1 \times (25 - 15) = 5000$ calories.

2. The amount of heat necessary to raise $\frac{1}{2}$ kg. of iron through the same temperature interval would be

$$ms(t_2 - t_1) = 500 \times 0.115 \times (25 - 15) = 575 \text{ calories.}$$

3. The water equivalent of the iron block in example (2) is 57.5 grams because $m \times s = 500 \times 0.115 = 57.5$ grams.

4. The latent heat of fusion of lead is 5.4 calories per gram if the amount of heat necessary to change 500 grams of lead at 327° C. (the melting point) to liquid lead at 327° C. is 2700 calories.

5. The latent heat of vaporization of water is 540 calories per gram if 5400 calories are necessary to change 10 grams of water into steam at 10C° C.

Methods Used in Calorimetric Determinations

Although the methods are numerous, they can be conveniently grouped under two headings:

1. **Method of mixtures,** in which two or more systems having different temperatures are placed in contact in such a way that they interchange heat until all of them acquire the same temperature, at which time the interchange stops. Writing down, then, the fact that the *bodies at the higher temperatures give out an amount of heat equivalent to the heat absorbed by the bodies having a lower temperature,* we have usually sufficient data to find the unknown constant, be it the specific heat or latent heat.

2. **Methods employing steam or ice calorimeters** (*e.g.*, Black's and Bunsen's ice calorimeter or Jolly's steam calorimeter). The

method here used depends upon a knowledge of the latent heat of fusion or vaporization of water.

In the *ice calorimeter* the hot body, after having been raised to a known high temperature, is allowed to give off its heat, and in so doing melt part of a block of ice. The amount of ice melted is found experimentally and thus a knowledge of the amount of heat given off in cooling to 0° C. is obtained, enabling us to calculate the specific heat.

In the *steam calorimeter* (Jolly) the body at room temperature is suddenly surrounded by an atmosphere of steam. The amount of steam condensed on the body before it finally acquires the temperature of the steam is measured on a sensitive balance. This again gives us a knowledge of the amount of heat necessary to heat the body from its original temperature to the temperature of the steam. From this we calculate the specific heat.

The experiments to be described here will be confined to the method of mixtures. In this method the first essential is the calorimeter. This is usually a thin-walled copper vessel which is polished or nickel-plated both inside and outside. It is polished inside so as to reflect the radiant heat back into the vessel, and polished outside so as to be a poor emitter and absorber of heat.

Convection currents and radiation are minimized by surrounding the calorimeter with another metal vessel polished also inside and out and having as few contacts as possible with the inside calorimeter. Often a Dewar vessel can be used very effectively.

An important source of error in accurate calorimetric work is the error introduced by heat loss to the surroundings. Newton has introduced a method for calculating and measuring this heat loss and making correction to our calorimeter readings. The method of correction is very instructive and gives good results, but requires quite a little skill and practice. The student is referred, for a description of the method and procedure of correction, to other texts on practical physics.

In the experiments to be described here, we shall minimize this error as far as possible by arranging the experiment so that during half the period the calorimeter has a temperature above the surroundings, and gives off heat; whereas during the other half the calorimeter is below the temperature of the surroundings and absorbs heat from the surroundings. By arranging the experiment so that these two heats are approximately the same, this error is minimized.

EXPERIMENT 21

METHOD OF MIXTURES

A determination of the specific heat of a solid.

Apparatus: A solid block (*e.g.*, aluminum or copper) with attached thread for handling, calorimeter supported in an outer vessel (Fig. 52), a vessel in which the solid can be raised to 100° C., burner, tripod, balance, thermometer, and stirrer.

The method we shall use is the method of mixtures in which the solid, the specific heat of which we wish to determine, is first heated to 100° C. The hot solid is then quickly transferred to a calorimeter containing water at a known temperature. The temperature is observed continuously until the final temperature of the solid and water is reached. Then equate the heat given out by the solid in cooling to the final temperature and the heat absorbed by the water, and thus calculate the specific heat s.

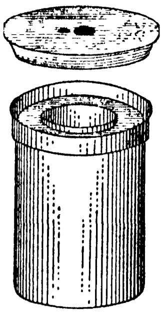


FIG. 52. — Calorimeter.

The first operation consists in finding the mass M of the unknown solid by weighing. Next suspend the solid in a vessel in which water can be boiled, and leave it in the boiling water or steam for at least fifteen minutes so as to be sure the whole solid has this same temperature throughout. This will be a temperature of 100° C. (unless correction has to be made for atmospheric pressure — see your instructor regarding this point). While the solid is heating, get the calorimeter ready. This includes finding the mass of the calorimeter (m_1) first, when empty, and then when half full of water. Let the mass of water in the calorimeter be m. When the object is immersed, the calorimeter should be about three-quarters full. The initial temperature of the water (t_0) should be from 3 to 5 degrees below room temperature.

When the solid has acquired uniform temperature of 100° C., transfer it quickly into the water, being careful to transfer as little condensed water along with the solid as possible, and also not to splash out any of the weighed water in the calorimeter. Note the temperature (t_0) of the water, when the solid was introduced, and record the highest temperature (t_f) attained by the water. The solid should be kept moving slowly, the student being careful never to have any part of the solid above the water. Record all the data with proper labeling on a data form.

Write down the expression for the heat given out by the solid in cooling from 100° C. to the final temperature (t_f). This is according to our definition $ms(100 - t_f)$. Equate this to the heat taken in by the water and water equivalent of the calorimeter, which is $(m + w) \times (t_f - t_0)$, where w = water equivalent of the calorimeter. (Note that the calorimeter is made of copper and has a specific heat s = 0.095 cal./gram.)

Solve for s, the specific heat of the unknown solid. Repeat the experiment if time allows.

[*Note.* The final temperature should be about as much above room temperature as the original temperature was below. If too large a difference exists, then error on account of radiation losses becomes important.]

QUESTIONS

(a). Why does a graph showing temperature-time relations help?

(b). Name possible sources of error in your experiment.

(c). Compare your result with results as given for the metal in a book of tables.

(d). In what way might you expect the water, which is transferred with the solid from the bath at 100° C. to the calorimeter, to effect the value of the specific heat as found in this experiment?

(e). Suggest a method by which the error referred to in question **(d)** may be overcome.

EXPERIMENT 22

LATENT HEAT OF FUSION

To find the latent heat of fusion of ice.

Apparatus: Double-walled calorimeter, thermometer, stirrer, blotting or filter paper, balance, ice.

In this experiment we shall again use the method of mixtures, by placing a piece of ice into a calorimeter containing water and noting the change in temperature produced when the ice has all melted. Equating then the total heats given out by the water and calorimeter and the heat absorbed by the ice, after equilibrium has been reached, enables us to find L.

The **procedure** is as follows: The mass m_1 of the calorimeter and stirrer must be found. The stirrer should be of special design so as to keep the ice below the surface (Fig. 53). Next, the calorimeter is half filled with water and weighed. The temperature of the water should be arranged to be about five or six degrees above the room temperature. Let the mass of the water be m grams. A piece of ice should be chosen which will conveniently go into the calorimeter and does not contain many corners or cracks that might have water clinging to them. The ice should be placed on a piece of blotting paper, dried, and *transferred as quickly as possible to the calorimeter. Handle the ice as little as possible,* so as to prevent melting after drying. Note the temperature of the water when the ice was introduced, and keeping the ice below the water surface and the water stirred, note the final lowest temperature (t_f) reached by the water. Finally weigh the calorimeter with its contents again and determine the mass M of the ice which must have been added.

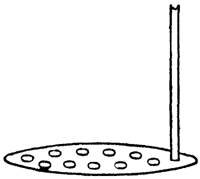


FIG. 53.—Stirrer.

Calculation to find L: The heat given out by the water and calorimeter in cooling from t_0 to t_f is given by $(m + w)(t_0 - t_f)$ calories, where w is the water equivalent of the calorimeter and stirrer.

The heat taken in consists of two parts: (1) heat taken in melting M grams of ice $= ML$ calories (see definition of L), and (2) heat necessary to change M grams of water from 0° C. to t_f° C. $= M(t_f - 0)$ calories. Hence the total heat taken in is $ML + Mt_f$.

Now since the heat given out is equal to the heat taken in, we have:

$$(a) \qquad ML + Mt_f = (m + w)(t_0 - t_f),$$

from which L can be found.

[*Note.* The student should not try to remember formulae such as (a) above; but should rather form the habit of following the reasoning in order to be able to reason through similar and allied cases of heat transfer.]

QUESTIONS

(a). Name possible sources of error in the above experiment and state whether the error would make L too large or too small.

(b). What should be the value of L? Calculate your per cent of error.

EXPERIMENT 23

LATENT HEAT OF VAPORIZATION

A determination of the latent heat of vaporization of water.

Apparatus: Calorimeters, thermometer, steam generator, steam trap.

The principle used in this experiment consists in passing steam into a known mass of water for a certain time. It will condense and so raise the temperature of the water. The amount of heat given to the water can be found if the temperature rise is known. Knowing the amount of steam condensed, we can calculate L, the latent heat of vaporization, which is also the heat necessary to condense 1 gram of steam at the boiling point.

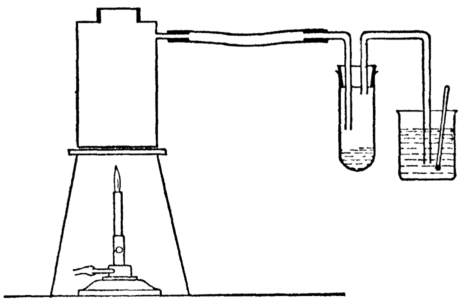


FIG. 54. — Latent heat of vaporization apparatus.

The apparatus used is shown in the photograph in Figure 54. The experimental procedure divides itself into several parts:

Part (a) consists in getting the calorimeter ready to receive the steam. Weigh the calorimeter and stirrer. Then the calorimeter should be filled about two-thirds full of water which has been cooled to about 5° C. by means of a piece of ice. Record the mass m_1 of calorimeter and stirrer, as well as the mass of water m.

Part (b). Prepare the steam generator. This should have a good stream of steam coming out before the steam is passed into the calorimeter. The steam should be passed through a so-called "steam-trap" (Fig. 54) which really catches any water, condensed on the way over, and prevents it from getting into the calorimeter. *It is most important that no condensed steam be allowed to get into the water in the calorimeter* if any accuracy is to be obtained. In order to help prevent this condensation before reaching the calorimeter, the tubing coming out of the steam-trap and going into the calorimeter should preferably be heat-insulated by lagging with cotton wool. *Any drops that might otherwise adhere to the glass nozzle should be shaken off before introducing the nozzle into the calorimeter.*

Part (c). Insert the nozzle into the water, recording the temperature of the water at the moment of introduction. Stir the water continuously while passing in steam. The nozzle of the steam generator should not be inserted very far. Any steam that escapes from the surface does not introduce an error because it does not condense. Keep on passing in steam until the temperature is approximately as much above room temperature as the water was below, when the steam was introduced initially. Then remove the steam nozzle from the calorimeter but keep on stirring, and record the maximum temperature reached. (The temperature will drop after a while on account of cooling toward room temperature.)

Part (d). Find the mass of the steam which has condensed by weighing the calorimeter and contents after the steam has condensed and subtracting the previous mass before introducing the steam. Let the mass of steam, so found, be M.

[*Note.* It is a good policy in recording the temperature of the water to read this every minute before and after introducing the steam and about every half-minute while the steam is being introduced. Then plot time against temperature and determine t_0 and t_f from your curve.]

Calculation. The water and calorimeter and stirrer start with a temperature t_0 and end with a temperature t_f. The total mass of water before introduction of the steam is the mass m of water plus the water equivalent of the calorimeter, viz., $m + w$. [*Note.* $w = m_1 s$, where s = the specific heat of the copper vessel = 0.089 cal./gram.] Hence the heat taken in by the water and calorimeter is

$$(m + w)(t_f - t_0).$$

Now the steam had to supply this heat. The heat given off by the steam consisted of two parts. One part was given off when the M grams of steam condensed. For every gram this amount is L, and hence for M grams this part will be ML calories. The other part consists of heat given out by the M grams of steam, which have now already condensed but are still at a temperature 100° C. and now cool until they reach the same temperature as the rest of the water, viz., t_f. The heat necessary for this part is, of course, $M(100 - t_f)$.

Hence the total heat given out by the steam is:

$$ML + M(100 - t_f).$$

Equating the heat given out to the heat taken in we have:

$$ML + M(100 - t_f) = (m + w)(t_f - t_0).$$

Solve this equation for L.

QUESTIONS

(a). Enumerate the various sources of error in your experiment and state whether the error would make L too large or too small.

(b). Do your calculations need correction for pressure of the atmosphere if not normal? How would you make such correction?

EXPERIMENT 24

THE MELTING POINT OF A SOLID

To determine the melting point of a solid by the method of cooling.

Apparatus: Pyrex test tube about 1 inch in diameter clamped to a stand, paraffin or acetamide, thermometer, burner, watch.

The method consists in heating the solid until it is molten. Then, after insertion of a thermometer into the liquid, the substance is allowed to cool slowly. The temperature is observed every minute. It will be found that on solidifying, the temperature remains constant for an appreciable time, until all the latent heat has been given off, after which the temperature falls again. The temperature at which this occurs is the solidifying (or melting) point.

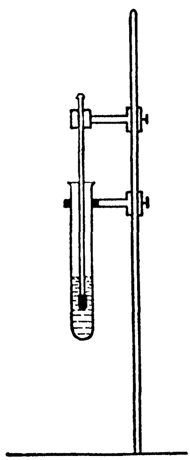


FIG. 55. — Melting point apparatus.

By applying heat gently, melt wax in the test tube until the vessel is about half full. A thermometer should be adjusted so that the bulb is nearly in the center of the liquid paraffin. A loosely fitting stopper will help in making this adjustment, or else the thermometer can be separately clamped. The test tube should be so placed that the thermometer can be conveniently read and so that no drafts or air currents might cause uneven cooling (Fig. 55). Record the time and temperature every minute. Have your partner plot these on a temperature (ordinate) — time (abscissa) curve, after having recorded the readings on the data sheet. The period of observation will be usually thirty minutes or more before the melting point has been well passed (in the case of paraffin). This time, of course, depends upon the substance and on conditions. Hence the advisability of plotting the curve as the experiment progresses. The substance should not be heated to a high temperature, but rather to a temperature just sufficient to melt it all. Do this slowly so as not to overheat the substance. Pick out from your curve the horizontal portion and thus find the melting point.

Check your result by looking up the value of this temperature in a book of physical tables. [*Note.* Before leaving, heat up the paraffin again so that it is just molten and remove the thermometer. Do not try to remove it in any other way, for fear of breaking it.]

An alternative and better way to measure the temperature is by use of a thermocouple, which has a very small heat capacity. The description of this instrument and its calibration can be seen from Experiment 39, later. If the thermocouple is used in this experiment, it is necessary to first calibrate it at two fixed temperatures, 0° C. and 100° C., and adjust the resistances so that the change in temperature from 0° C. to 100° C. will not cause too much deflection in the galvanometer. Having calibrated the thermocouple, the thermometer in the above experiment can be replaced by the one junction of the thermocouple. In this case, plot galvanometer deflections (ordinates) against time (abscissa). Then convert the galvanometer deflection for the melting point into temperature from the calibration.

QUESTIONS

(a). From your curve what can you say about the rate of fall of temperature before solidification and after?

(b). Find the rate of cooling (approximately) at the melting point.

(c). Why do we not find the melting point by performing a similar experiment when heating up the solid?

(d). Why does the temperature remain fairly constant during solidification?

PROBLEMS

General

1. Define: (*a*) the calorie; (*b*) specific heat; (*c*) water equivalent; (*d*) latent heat of fusion; (*e*) latent heat of vaporization.

2. Convert 1 pound — degree — Fahrenheit into calories.

Experiment 21

3. A copper calorimeter weights 80 grams and has a specific heat of 0.095 cal./gram. How many calories of heat are necessary to change its temperature from room temperature (15° C.) to 50° C.? What is the water equivalent of this vessel?

4. A piece of iron of mass 120 grams and temperature 80° C. is put into a calorimeter having 500 grams of water at 14° C. If the final temperature acquired is 16.5° C., find the specific heat of iron. (The water equivalent of the calorimeter = 50 grams.)

5. Can you devise some experimental method for finding the water equivalent of a vessel for which it would be difficult to calculate the water equivalent by the method used in this chapter?

6. Fifty grams of lead shot of specific heat 0.03 cal./gram and at a temperature of 100° C. are poured into a calorimeter containing a liquid of mass 100 grams. The calorimeter has a water equivalent of 20 grams. The initial temperature of the liquid was 15° C. and the final temperature 20° C. Find the specific heat of the liquid.

Experiment 22

7. How much heat is necessary to change 20 grams of ice at 0° C. into water at 30° C.?

8. What will be the fall in temperature if 40 grams of ice are put into a copper calorimeter of mass 50 grams and containing 200 grams of water at 20° C.? (Specific heat of copper = 0.095 cal./gram.)

9. Find the latent heat of fusion of ice, given that in a certain experiment a piece of dry ice of mass 50 grams was put into a calorimeter, having a water equivalent of 30 grams and containing 500 grams of water at 18° C. The final temperature was found to be 9.5° C.

Experiment 23

10. Find the final temperature if 20 grams of steam at 100° C. are introduced into a copper calorimeter of mass 100 grams containing 600 grams of water at 20° C. (Specific heat of copper = 0.095 cal./gram.)

11. Find the amount of heat necessary to heat 100 grams of ice from − 20° C. to steam at 120° C. (Specific heat of ice = 0.54 cal./gram and specific heat of steam = 0.42 cal./gram.)

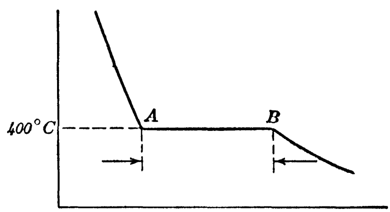


FIG. 56. — Cooling curve. (See Problem 12.)

Experiment 24

12. Find the latent heat of fusion of a substance which has a cooling curve as shown (Fig. 56). The temperature at which melting occurs is 400° C. The rate of fall of temperature at A and B is 10° per min. The specific heat of the substance is 0.1 cal./gram. The horizontal portion AB lasts for 25 minutes.

CHAPTER XI

SURFACE TENSION

When a molecule of a liquid gets near the surface, forces of attraction begin to exert a resultant force on this molecule. The direction of this resultant force is vertically downwards if the surface of the liquid is horizontal. The magnitude of this resultant downward force on the molecule increases as the molecule gets nearer the surface. (The reasons for this, based on the kinetic theory, will be found in most textbooks on Physics.)

Hence, as far as the molecule is concerned, the surface of the liquid behaves as if it had stretched over it a membrane, because a certain force is necessary to enable the molecule to break through.

To be more specific, we define the *surface tension of a liquid as the tension acting on both sides of an imaginary unit length in the surface.* In the c. g. s. system this tension is measured in dynes per cm. (Fig. 57).

Fig. 57. — Unit length of the surface of a liquid.

Another point which must be mentioned in connection with our definition is that, when we speak of the surface tension, we must specify the media on both sides of the surface. Usually when we refer to a water surface, we assume water below and air above. The value of T, *i.e.*, the surface tension, will be different if we have some other gas or liquid above the water.

Methods of Measurement

There are several methods that could be, and are, employed in measuring the surface tension of a liquid. These methods are based mostly on the effects produced by surface tension and are hence indirect methods. With practice and skill they lend themselves to accurate determinations of surface tension. In all surface tension measurements the main requirement for accurate and reproducible results is cleanliness. The slightest amount of foreign substance on the surface will cause large errors in the value of T. This is especially true for small amounts of grease. Hence the liquids or solids that are going to be in contact should not be touched with the fingers. Unless this precaution is observed, the value as found may vary by as much as 10 per cent or more.

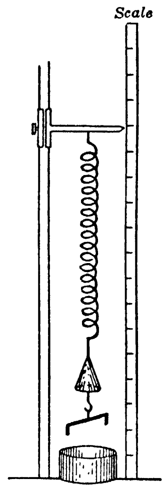


FIG. 58.—Jolly balance method.

Two methods will be described. The first method using the so-called *Jolly balance* is straight-forward in principle, but does not give such accurate results. The other method, making use of the effect of surface tension in producing so-called *capillary action*, will give very good results, although a large amount of skill and technique are required. It is included here, therefore, as an experiment designed to test and develop the student's technique in experimental procedure. If enough time were available, there is no reason why accurate results are not possible.

Direct determination of surface tension (Jolly balance). The Jolly balance consists essentially of a long spiral spring which hangs vertically and is fixed above to a crosspiece. This crosspiece can be moved up or down very slowly and the amount of motion measured by some suitable means. Such means are the observation of the motion of a certain point on the spring on a scale engraved on a mirror surface which remains stationary, or, by having a scale and vernier engraved on the telescope tube which moves up or down (Fig. 58).

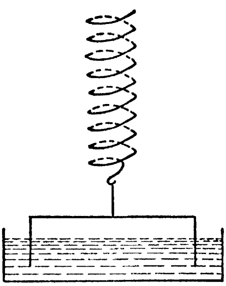


FIG. 59.—Frame in the liquid without film.

At the end of the spiral is attached a carefully cleaned platinum or aluminum frame. This frame is allowed to sink right into the liquid, and then the force necessary to pull this frame through the surface is measured in dynes. Let it be F dynes (Fig. 59).

Now the force F has to be applied against a film on each side of the frame. Hence this force has to be exerted against a length of film $2\,l$ (Fig. 60). Hence by definition $T = \dfrac{F}{2\,l}$ dynes per cm.

Effect of surface tension in capillarity. The forces of adhesion between glass and water molecules, being larger than the forces of cohesion between the water molecules themselves, cause the rising of the surface of water whenever it comes in contact with the glass. Although these forces are small, they can be readily shown to be appreciable if we make use of a capillary tube. The adhesive forces will make the water rise in the tube until these forces acting upwards just balance the force of gravity acting downwards on the column of water.

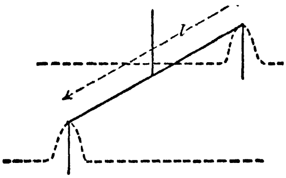


FIG. 60. — Frame in the liquid with film.

Of course the adhesive force A (Fig. 61) equals the force of surface tension acting all around the edge. Let the radius of the tube be r and let the surface of the liquid (*i.e.*, also the direction of T) make an angle θ with the surface of the tube. Then, by definition, T is the force acting on the surface on every centimeter and hence at the edge the total vertical force acting will be $T \cos \theta \times 2\pi r$ dynes. This is, therefore, the resultant force which the adhesive forces exert upwards.

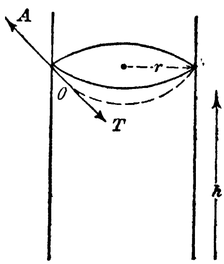


FIG. 61. — Capillarity.

The mass of liquid elevated in the capillary tube exerts a downward force Vdg, where V is the volume of liquid in the capillary and d is the density. Hence:

$$2\pi rT \cos \theta = Vdg.$$

If the liquid *wets* the surface, then the *angle of contact is zero.* The volume of liquid in the capillary tube will then consist of a column of area πr^2 and of height h (being the height from the free surface of the liquid in the vessel to the base of the meniscus) plus a volume of liquid which is the difference between a cylinder of height r and a hemisphere of radius r.

That is,

$$\begin{aligned} V &= \pi r^2 h + [\pi r^2 \cdot r - \tfrac{2}{3}\pi r^3] \\ &= \pi r^2 h + \tfrac{1}{3}\pi r^3 \\ &= \pi r^2(h + \tfrac{1}{3} r) = \pi r^2 l, \end{aligned}$$

where

$$l = h + \tfrac{1}{3} r.$$

Now for all cases in which the liquid wets the surface, the angle $\theta = 0$, and hence for these cases,

$$T = \frac{rldg}{2} \text{ dynes.}$$

It is often quite accurate enough to measure the distance h from the surface of the liquid to the bottom of the meniscus and, neglecting r, to call this distance l.

EXPERIMENT 25

THE JOLLY BALANCE

Determination of surface tension by means of a Jolly balance.

Apparatus: Jolly balance or similar spring balance that can be calibrated. A movable platform on which can be placed the glass dish containing the liquid. Several platinum frames with stems and hooks attached so that they can be easily attached to the end of the spring. A cleaning solution (20 grams of NaOH per liter of water) into which the frames can be dipped so as to remove grease, dirt, etc. A few small weights for calibration purposes (1 up to 10 grams), a meter bar, pair of tweezers, distilled water, soap solution. Other metallic frames, such as aluminum and nickel, may be used with proper cleaning solutions.

The experiment is divided into two parts as follows: (1) to calibrate the spring balance; (2) to measure the force due to the surface tension.

Part (a). To calibrate the spring balance, hang a very light pan on the bottom of the spring and make a note of the position of a certain convenient mark or pointer on the lower end of the spring and read the scale. (This reading can be in arbitrary spring balance units.) Next, place the known mass on the scale pan (say x grams). This of course will stretch the spring. The balance is restored by moving the upper support of the spring upwards until the point on the lower part of the spring, that was observed before, comes to rest in the same position. The amount of motion of the upper support is measured on the scale.

Having measured the elongation for x grams, it is a simple matter to calculate the force necessary to give a stretch or elongation of one scale division. Make several trials with the same mass and calculate the mean. Express the force in dynes per division elongation.

Part (b). To measure the force necessary to overcome surface tension, remove one of the frames by means of tweezers or forceps from the cleaning solution and rinse in a dish of clean water from the faucet. Care must be taken never to touch the frame with the fingers once it has been removed from the cleaning solution, nor must this frame be placed on a table, or allowed to come in contact with anything which will leave foreign substances on it. Hang this frame on the hook at the bottom of the spring and arrange a clean dish of clean water from the faucet on the movable platform, so that the legs of the frame hang centrally in the dish and are immersed for about half their length. Move the upper support down slowly until the whole frame is immersed. Then stretch the spring slowly until the frame is pulled out of the water. Note approximately how much the film can be stretched just before it breaks. (Sometimes an adjustable marker is provided which can be used to mark this point.) Now bring the legs of the frame back slowly into the liquid up to the position which the frame had when the film broke (being careful to see that there is no film formed on the frame). Read this position on the scale. The frame is now lowered into the liquid and pulled up again, this time with the film formed on the frame tending to keep the frame in the liquid. Raise the spring slowly until the film just breaks. Take the reading on the scale when the film just breaks. The difference gives the number of scale divisions that the spring is stretched due to the surface tension. Having previously determined the calibration of the spring, these scale divisions can be converted into units of force (F dynes). Measure the length (l) of the frame with a meter rule. From a knowledge of the force F, in dynes, and the length of the frame in cm., T, the surface tension can be calculated.

Repeat several times with the same frame, determining in each case the stretch of the spring. Take the average of these and determine the error in your experimental determination of the stretch. If time allows, repeat these observations, using another frame.

Record your results as follows (for one set of readings):

Length of frame, $l =$ cm.

No. of grams added in calibration = grams.

Calibration of Spring Balance			Determination of the Elongation Produced by the Film		
Zero Reading	*Reading after Addition of Grams*	*Elongation (1)*	*Zero Reading without Film*	*Final Reading When Film Breaks*	*Elongation (2)*

Average elongation (1) = ±
Per cent of error =
Average elongation (2) = ±
Per cent of error =
Hence one scale division = dynes.
$F =$ ± dynes,
and since $T = \frac{F}{2\,l'}$,
$\therefore\ T =$ ± dynes/cm.

Repeat the experiment, using another liquid.

QUESTIONS

(**a**). Suppose the rectangular frame were a horizontal circular wire of radius r, how would you determine the surface tension in this case?

(**b**). Will the surface tension be different if platinum or aluminum is used in making the frames?

(**c**). Between which media are you really finding the surface tension?

EXPERIMENT 26

SURFACE TENSION IN CAPILLARY TUBES

Determination of surface tension by the method of capillarity.

Apparatus: Small glass evaporating dish, transparent glass scale with millimeter divisions, a burner, glass tubing of about 2 or 3 mm.

diameter, a microscope with either a micrometer eye-piece or else a finely divided scale placed in the focal plane of the eye-piece.

The experiment is divided into *three parts* as follows:

(*a*). *Making the capillary tubes.*

(*b*). *Measuring the height of rise in the tubes.*

(*c*). *Measuring the diameter of the bore of these tubes.*

Part (a). The glass tubes which you are given have been well cleaned inside by rinsing with caustic soda solution, then water, then 10% nitric acid, and finally washed out with water again, and dried. Heat the glass about 5 cm. from the end until it is soft (rotating the tube during the process of heating). Then remove the heated glass tube from the flame and draw it out until a capillary tube about 1 mm. external diameter has been obtained. If the drawn-out section is long enough, cut or break it off into pieces about 15 cm. in length. Using the remaining sections of the original glass tube and holding by the drawn-out end, make some finer capillaries until about five have been obtained of varying sizes, each of length 10 to 15 cm. Handle the capillaries as little as possible.

Part (b). Fill the thin evaporating glass dish about half full of clean water from the faucet. It is understood of course that the glass dish itself has first been cleaned and is thoroughly free from grease or dirt. Stand the glass scale up in the water and next to it one of the fine capillaries. Usually the capillary will adhere to the scale if the latter has been wet a little. The water will rise in the tube. In order to be sure that the tube is wet inside (*i.e.*, $\theta = 0$) for the whole length of the capillary, lean the tube over, still keeping the lower end in the water, until the water fills the whole tube. Then tilt the capillary tube up again to see whether the water comes back to the same height as before. If this is not the case, reject this capillary and use another. If the capillary and water are both clean, the water will always rise to the same height.

Measure the height of the water column in the capillary, taking the measurement from the outside water surface to the bottom of the meniscus (the correction $\frac{1}{3}$ can be added here if the tube has a large radius, but in most cases this is negligible). Stick a small piece of gummed paper to the tube about 2 millimeters above the highest point of the column of water. Draw the capillary tube up about 5 millimeters and measure the height again, sticking a small piece of paper 2 millimeters above this latter height. If the tube is uniform, these two heights should agree. If there should be a large difference between these heights, use a tube of more uniform bore.

Repeat for two other sizes of capillary tubes. Be sure to mark them so as to know which tube was used in finding the measured elevations!

Part (c). By means of a fine file or a rough knife cut the capillaries off half-way between the gummed paper marks and mount in the V groove, prepared especially for this purpose, in front of the microscope. We wish to measure the diameter of these tubes. On looking into the eye-piece a scale will be seen. Adjust the distance of the microscope from the end of the capillary tube until the latter is in focus and on some convenient mark on the scale, and measure its diameter in micrometer scale divisions. To find out what these micrometer divisions mean, the micrometer which has the scale fixed in the eye-piece is moved back and forth until it is exactly focused now on a standard scale placed where the capillary tube was. In the eye-piece will be seen two scales, the original eye-piece scale and also the divisions on the standard scale. See how many microscope scale divisions are equal to a whole number of standard scale divisions. Knowing the size of the divisions of the standard scale, the size of the division on the micrometer scale can be calculated in centimeters. In this way the diameters of all the capillaries can be found.

Another method of finding the diameter, which gives greater accuracy, but requires considerably more technique and skill, is to introduce a thread of mercury into the capillary, measure the length of the thread, and find its mass on a balance. From this data, knowing the density of mercury, the diameter of the tube can be found. By this method the uniformity of the bore can also be tested.

Let r represent the radius to be determined, l the length of the mercury column, m the mass of the mercury column, and d the density of mercury.

Then
$$m = \pi r^2 l d,$$

and
$$r = \sqrt{\frac{m}{\pi l d}}.$$

In order to get a thread of mercury into the tube it is possible to insert the end of the tube into a small globule of mercury and suck a little into the tube. However, if this is not done carefully, the method is dangerous since mercury is a poison. A cheap rubber ball will serve the purpose just as well if a small hole is made in the ball and the capillary inserted (Fig. 62).

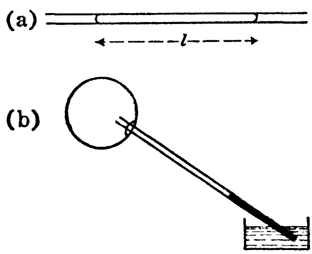


FIG. 62. — Drawing mercury into a capillary tube.

Calculate the value of the surface tension from the equation

$$T = \frac{rldg}{2}.$$

Record your results as follows:

Liquid under investigation,
Division on microscope scale = division on standard scale.
1 division on standard scale = cm.
∴ 1 division on microscope scale = cm.

Part (b) Part (c)

Capillary No.	Reading in Cm. at		Height of Column	Diameter in		Radius	T
	Outside Level	Meniscus		Micrometer Scale Div.	in Cm.		
1							
2							
3							

Find the average value of T. If time allows, repeat using a different liquid.

QUESTIONS

(a). Which of the above measurements, r or l, will cause the largest error in the result for T and what is approximately the per cent of error in each of the quantities measured?

(b). If the angle of contact is not zero, what additional test would have to be carried out?

(c). Why would the method of weighing (in order to find the diameter) be more accurate?

(d). Do your results show what relation exists between r and l?

(e). Would it be better, as far as accuracy is concerned, to take a very long tube of extremely fine bore or a short tube of fairly large bore?

(f). From your results, what is the per cent of error in r, in l, and in T?

PROBLEMS

Experiment 25

1. Define surface tension, angle of contact.

2. A rectangular frame made of platinum has one edge cut away and is so mounted that the two legs dip into a liquid. When the frame is immersed and pulled out again, it is found that a spring attached to the frame is stretched 2 cm. before the frame actually breaks through the surface. The length of the rectangular frame (*i.e.*, the length of the platinum wire immersed, neglecting the legs) is 8 cm. If the surface tension is 60 dynes/cm., how much elongation would a mass of 1.5 grams produce in the spring?

3. Find the surface tension of a liquid if a rectangular frame of length 9 cm. immersed in the liquid requires a stretch of 5 cm. in a spring. It is known that a mass of 3 grams will stretch the spring 20 cm.

Experiment 26

4. Prove the expression for capillary rise in a tube, namely, $l = \frac{2\,T\cos\theta}{rdg}$, giving full reasons for each step.

5. How would you prove experimentally the relation between:
(1) l and r?
(2) l and d?

6. The surface tensions of two liquids A and B are 35 dynes/cm. and 65 dynes/cm. respectively, whereas the angle of contact for glass and liquid A is 45° and for glass and B is 50°. Using the same capillary tube in both liquids, what is the ratio between the heights of the liquids A and B in this tube? (Given that the density of liquid A : density of liquid B = 4 : 5.)

7. Find the surface tension of a liquid which wets a capillary tube and rises in this tube of 3 mm. diameter to a height of 12 cm. The density of the liquid is 8 gm./cc.

CHAPTER XII

RELATIVE HUMIDITY

The determination of the quantity of water vapor in the atmosphere per unit volume is an important adjunct to the study of atmospheric conditions. The number of grams of water vapor per cubic meter is a common measure used.

The air is said to be saturated when it contains the maximum number of grams of water vapor per cubic meter without condensing. The maximum quantity of water vapor per unit volume increases with temperature, but not linearly.

The mass (in grams) of water vapor actually present in the atmosphere per unit volume is called the *absolute humidity*, while the temperature to which the air must be reduced to reach saturation is called the *dew point*. Below this point, the excess water above the amount necessary for saturation will precipitate on some solid or even in the air as mist, clouds, fogs, etc.

A term which is more frequently used to express the moisture content of the air is called the *relative humidity*. This is defined as the ratio of the mass of water vapor actually present per unit volume (absolute humidity) to the maximum mass of water vapor per unit volume that the atmosphere can hold at this temperature (saturated vapor). To express the relative humidity as per cent, multiply the above ratio by 100.

In finding the relative humidity, the procedure is first to find the dew-point temperature. *The number of grams of water vapor per cubic meter the air can hold at the dew point gives the absolute humidity.* This quantity can be found from tables. Then look in the handbook for the number of grams of moisture per cubic meter the air can hold at the present temperature of the air. The ratio of the absolute humidity to the vapor content, if the air were saturated at the present temperature, gives the relative humidity.

The relative humidity may also be found by use of a wet-bulb and a dry-bulb thermometer if a current of air (say, greater than 3 meters per second) is directed towards the thermometers so that the water will evaporate from the wet thermometer without saturating the surrounding air layer. The greater the moisture content of the air, the less the evaporation from the wet thermometer, and consequently the smaller is the cooling effect due to evaporation. This means that the difference between the temperature reading of the wet and dry bulb becomes less the higher the moisture content. When this method is used, tables are found in handbooks for calculating the results in terms of the relative humidity.

EXPERIMENT 27

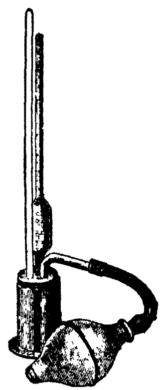
RELATIVE HUMIDITY

To find the relative humidity of the atmosphere by (a) the dew-point method; (b) the wet- and dry-bulb thermometer.

Apparatus: *Part (a).* Small nickel vessel, a medium-sized vessel, salt, and ice; or a nickel tube fitted with bulb and ether, thermometer.

Part (b). Wet- and dry-bulb thermometer or a sling psychrometer.

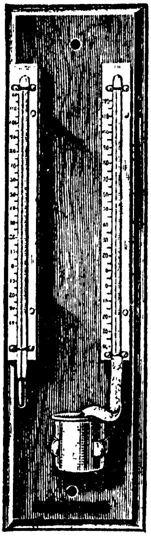
Part (a). The dew-point method. To determine the dew point with ice and salt, chop the ice into small portions and add slowly to water, which is to about one inch depth in the polished nickel can. Stir with a thermometer and take the reading of the thermometer when the dew first appears. Now add lukewarm water until the dew disappears and take the temperature reading again. The average of these two temperatures gives a fair estimate of the dew point and becomes more accurately located the smaller the difference between the two readings for the appearance and disappearance of the dew. Make several trials, recording all readings, and find the average. Water should be removed from time to time to keep the height about one inch. Do not handle the polished side of the can with the fingers or breathe on the can while the experiment is in progress.



Courtesy Central Scientific Company

FIG. 63.—Nickel tube with compression bulb.

When ether is used, partially fill the small nickel tube, fitted with a compression bulb (Fig. 63), with ether. The quantity of ether used depends upon the apparatus. The increased evaporation brought about by forcing air through the ether with the compression bulb cools the apparatus. The dew point is noted as in the previous salt and ice method, and the calculations for the relative humidity are made in a similar manner. The method is very useful for obtaining low temperatures. It is desirable not to inhale the ether more than necessary. Repeat the experiment out of doors.



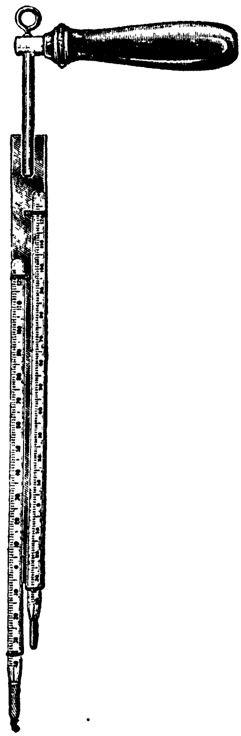
Courtesy Central Scientific Company

FIG. 64. — Wet- and dry-bulb thermometer.

Part (b). The wet- and dry-bulb thermometer method. If a wet- and dry-bulb thermometer is used (Fig. 64), see that the container holding the lower side of the wick is filled with distilled water and then start the fan, which should be at a distance of about two feet from the wet- and dry-bulb thermometers. When the temperatures of the thermometers cease to change further, take readings of both the wet- and dry-thermometers. Repeat two or more times, relocating the apparatus each time. Calculate your results from tables found in a handbook.

If a sling psychrometer is used (Fig. 65), the procedure is much the same, except that the instrument, consisting of a wet and a dry thermometer fastened to a metal frame, is whirled until both thermometers cease to change any in temperature. In this instrument, the wet-bulb thermometer is kept moist by means of a wet cheese cloth which is wrapped around the bulb. (Use distilled water for this purpose.)

Tabulate and record all your observations.



Courtesy Central Scientific Company

FIG. 65. — Sling psychrometer.

QUESTIONS

(**a**). When may ice, or ice and salt, be used in place of ether?

(**b**). What causes the appearance of dew on the side of the vessel?

(**c**). What would be the relative humidity out-of-doors today according to your experiment? (Assume the dew point out-of-doors the same as in the room where the experiment was performed.)

(d). What would the relative humidity, according to the wet- and dry-bulb thermometer experiment, have been if the temperature of the dry thermometer had been found to be 10° C. higher than that actually found?

(e). What is the probable accuracy of Part (a)? Part (b)? Compare the actual results to see if the figures and probable accuracy are compatible.

PROBLEMS

1. Does the maximum amount of vapor the air will hold (*i.e.*, saturated vapor) increase linearly with temperature? Plot the mass per cubic meter of saturated vapor as ordinates against temperature as abscissae between the temperatures of $-20°$ to $35°$ Centigrade with intervals of 5 degrees between each reading. Consult some handbook for the mass of saturated vapor per cubic meter.

2. Is it necessary that the atmosphere next to the earth be saturated in order to have rain?

3. Given that the absolute humidity of the air at noon is 13.5 $\left(\frac{\text{gm.}}{\text{m.}^3}\right)$, what is the relative humidity for 22° C.? Suppose the temperature near the ground dropped to 12° C. that night, speculate on what may happen.

4. Which has the lower reading, the wet- or dry-bulb thermometer?

5. Under what conditions will the wet- and dry-bulb thermometer show large differences? Why?

6. Why must the air be circulated about the wet- and dry-bulb thermometer by a fan or by other methods?

CHAPTER XIII

ELECTRIC AND MAGNETIC FIELDS

All materials are composed of atoms or combinations of atoms (molecules). The atoms are made up of electrons and protons. The electron is found to be a negative charge of electricity, probably with electrical inertia only, while the proton is the positively charged hydrogen nucleus. All the mass seems to be associated with the nucleus and the positive charge has, up to the present, never been identified separately from the nucleus.

The electron is very mobile. Certain of the electrons are easily removed from some substances by friction. On the other hand, positive charges, being associated with the mass of the nucleus, which is heavy compared to the electron, appear to be fairly immobile so far as experiments have shown.

Electric Properties of Materials

Consider a hard rubber rod which is rubbed with fur, or a glass rod which is rubbed with silk. The hard rubber takes on a negative charge because the fur loses electrons easier than hard rubber does in the process of rubbing. The glass takes on a positive charge because it loses more electrons than it accumulates. The above substances are known as dielectrics (non-conductors) because a charge will not flow from one point to another.

All substances may be charged by friction as above. If conductors (*i.e.*, metals) are to be charged, however, they must be insulated so that the charge will not leak off as fast as it is generated by friction. Ebonite, hard rubber, sulphur, and dry air are examples of good insulators. Moist air, however, is a much better conductor than dry air and slowly conducts charges off metallic surfaces. Many experiments in electrostatics are partial or complete failures in damp weather. Moreover, the air becomes more conducting in the presence of condenser discharges, radioactive compounds, lighted matches, etc., which serve to ionize the air.

The fundamental law governing the force between charges is *Coulomb's Law.* This law states that for similar charges in air or vacuum the force is one of repulsion and of amount

$$F = \frac{q_1 q_2}{d^2} \text{ dynes,}$$

where q_1 and q_2 represent the electric charges and d their distance apart. For unlike charges the force is one of attraction.

In speaking about charges it has become customary to refer to the charge on an ebonite (or hard rubber) rod, when rubbed with fur, as the negative charge. Consequently once this arbitrary designation has been established, all other charges can be classified as positive or negative, depending upon whether they attract or repel charges on the hard rubber.

The region around an electric charge or system of charges is called an electric field. When charges or uncharged bodies are placed in this electric field, forces due to this field act on them. It is necessary to define and measure what is usually termed the *field intensity* or *strength of the electric field.* There are two ways in which an electric field E can be defined and measured. In the final analysis the two methods can be shown to be the same.

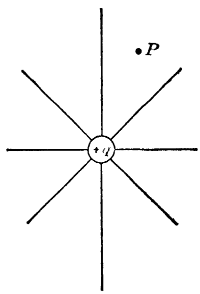


FIG. 66. — Electric field.

First, the field intensity at a point P (Fig. 66) is defined as the force which would act on a unit charge placed at the point in question. In order to find the strength of the electric field at all points in a certain region it would be necessary to put the unit charge at all these points and measure the force acting on it.

Secondly, the field intensity at any point P can also be defined in terms of the space rate at which a quantity, called the potential V, varies $\left(E_x = -\frac{\Delta V}{\Delta x}\right)$. From this definition it can be seen that a knowledge of the potential at all points in the field enables us to find the field intensity. This method is similar to the method used in geographical maps of representing the country by lines having equal elevations. These topographical maps show right away whether the country is fairly level or mountainous and enable us to calculate the grade or steepness (corresponding to field intensity in the electrical case).

In the electric field, lines of force are often drawn. These are lines which show at all points through which they pass the direction of the electric field. They also represent the direction in which a unit charge will travel if allowed to do so on account of the repulsion.

We will next analyze the effects produced by some electric charges. Consider, for instance, the electrophorus (Fig. 67 and Experiment 28), the dielectric plate of which has been charged negatively by whipping with catskin or flannel. The metallic disc will have a positive charge induced on the under side when in " contact " (Fig. 67 a) with the dielectric. A negative charge is induced on the upper side. The effect of the contact points of the disc along the dielectric is negligible, for these points are relatively few, and besides, charges in a dielectric are not conducted from point to point. The induced negative charge is allowed to flow to ground by touching the top of the metallic disc with the hand (Fig. 67 b). Remove the hand and the metallic disc has left a positive charge (Fig. 67 c). This is called *charging by induction.* Referring again to Figure 67 b, we will call the potential of the metallic disc v_2 a " ground " potential. If the disc is raised slightly, as in Figure 67 c, the " lines " are stretched and work is required to raise the disc. Moreover, it may be shown that these " lines " tend to separate from each other. That is, there is a " pressure," or force, at right angles to the lines tending to split the lines off from their present terminals. The higher the plate is raised, the more work it takes and the greater the number of lines split off so that a large number of lines (Fig. 67 d) will now terminate on other surrounding objects and the charge on the disc itself becomes less bound to the dielectric. This means that the potential of the disc is now raised to a value v_1. The difference in potential $(v_1 - v_2)$ is measured by the work

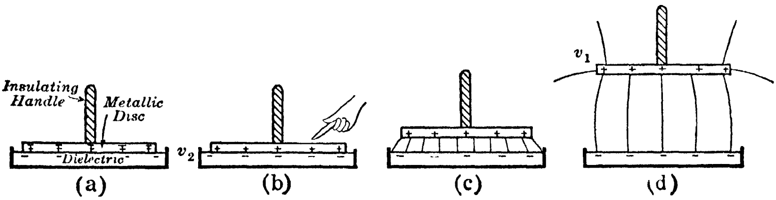


FIG. 67. — Metallic disc charged by induction.

done in raising the disc with a charge q from a potential v_2 to a potential v_1, *i.e.*,

$$w = q(v_1 - v_2),$$

where w is expressed in ergs if q and $(v_1 - v_2)$ are measured in electrostatic units.

By similar reasoning it may be shown that a body will hold a greater charge (*i.e.*, its capacity increased) for a given potential if another conducting body, usually well grounded, is brought near to it. This can be shown with the electroscope which has a metallic disc attached to it and charged as shown in Figure 68. The leaves diverge an amount depending upon the charge given to the system. Now bring a grounded conductor such as a metallic disc (Fig. 69) into the vicinity of the electroscope. The leaves of the instrument come closer together, indicating that the potential is lowered. This means that the charge which the electroscope arrangement could hold, for a given potential, is increased.

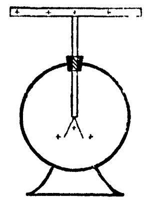


FIG. 68. — Charged electroscope.

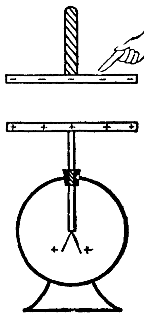


FIG. 69. — Grounded metallic disc near charged electroscope.

MAGNETIC PROPERTIES OF MATERIALS

The magnetic properties of lodestone have been known for centuries. The Greeks called it *magnetite* (Fe_3O_4). It has the property of picking up iron filings and small pieces of iron. Later compasses were discovered and the magnetic nature of the earth was observed. Magnets have a north-seeking (N) and a south-seeking (S) pole. Like poles repel, while unlike poles attract.

It is now known that an electric charge in motion produces a magnetic field. The orbital motion of the electrons about the nucleus of the atoms goes a long way toward explaining the magnetic properties of substances.

The space surrounding a magnet is called the *magnetic field.* We can draw in this field *lines of force* which, in reality, are directions along which a positive (N) pole would travel if placed in the field. These lines of force may be located readily by means of iron filings or a small compass. The magnetic field around a magnet is shown in Figure 70. This field will be distorted because of the effect of the earth's magnetic field. In fact, the earth's field itself may be distorted at any given position in a building because of the nearness of steel girders, steam pipes, and other iron or steel structural work.

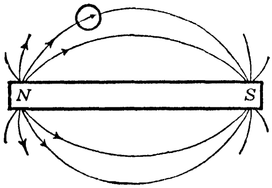


FIG. 70. — Magnetic field surrounding a bar magnet.

In Figure 71, one effect of interest due to the earth's field is to cause two points (P_1 and P_2) to have no magnetic field in any direction. A compass needle placed at these points would have no tendency to turn in any particular direction. *These are called neutral points.* If the horizontal intensity of the earth's field is found to be 0.2 gauss, it means that the field due to the magnet itself at P_1 and P_2 is also 0.2 gauss with a direction opposite that of the magnetic field due to the earth. This type of experiment gives us one way of determining the pole strength of a magnet, knowing the strength of the earth's field.

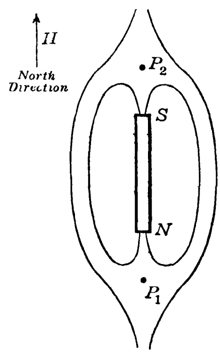


FIG. 71. — Effect of the earth's field.

The pole strength of a magnet is measured in terms of a unit magnetic pole. This unit is based on Coulomb's experimental law which states that if m_1 and m_2 represent the strength of two "isolated" poles, the force (F) of repulsion (if poles are alike in sign) will be

$$F = \frac{1}{\mu}\frac{m_1 m_2}{d^2},$$

where d is the distance between them and μ is the permeability. The permeability for air is approximately unity. From Coulomb's law of force between two poles, we define *unit pole* as that pole which, when placed a distance of 1 cm. in a vacuum from an equal and like pole, will repel it with a force of one dyne. The expression $\frac{m}{\mu d^2}$ is known as the *field intensity* at a point P which is at a distance d from an isolated pole of strength m, and is the force per unit pole at P due to a single magnetic pole of strength m.

If there is more than one pole to be considered, the resultant field intensity at a point is the vector sum of the separate field intensities which are calculated from the known strength of each pole. The field intensity is usually denoted by the letter H.

EXPERIMENT 28

ELECTRIC CHARGES

Methods of obtaining and detecting electric charges.

Apparatus: Ebonite, sealing wax, glass rod, fur, flannel, silk, pith ball, two aluminum-covered pith balls, condenser, electroscope with metallic disc attachment, electrophorus, electrostatic voltmeter (one is sufficient), stand with horizontal rod for suspending pith balls.

The *electroscope* (Fig. 72) is an instrument to detect (and measure) electric charges. It consists essentially of an insulated metal rod with very thin gold, or aluminum, leaves attached to the lower end. The rod is suspended in a metal container with glass windows by means of the insulating plug, which may be made of ebonite (or better, of amber or sulphur).

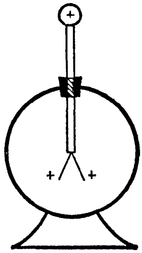


FIG. 72. — Electroscope.

The *electrostatic voltmeter* is a form of calibrated electroscope.

The *electrophorus* (Fig. 73), an instrument designed to obtain large charges, consists of a dielectric, usually inclosed in a metal dish, and a metallic disc with an insulating handle. The procedure in obtaining a charge on the metallic disc is as follows: The ebonite is charged negatively by whipping or rubbing with flannel or catskin. Bring the metallic disc down on the charged ebonite. The disc touches the ebonite in relatively few places, so that the metallic disc becomes charged by induction, positively on the lower side and negatively on the upper side. Touch the upper side with the hand for an instant. The negative charge becomes grounded. Remove the metallic disc. It is now charged positively. We say that the plate was charged positively by induction. All charging by induction is carried out in the above manner. If the metallic plate had been in contact at all points with the charged dielectric, the former would have become charged negatively by conduction.

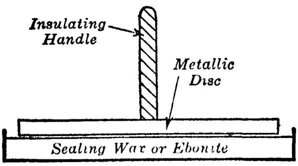


FIG. 73. — Electrophorus.

Procedure. For the most part, the data are to be recorded by transferring the diagrams to your data sheet, filled in with the correct sign of charge (*i.e.*, + for positive and − for negative) and any other additional drawings to make the data sheet more complete. The experiment is to be done in parts as provided below.

Part 1. Charge the ebonite rod with fur. Bring the ebonite rod near bits of paper. The paper becomes charged by induction

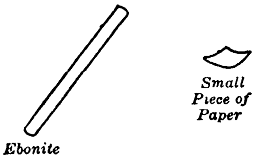


FIG. 74. — Refer to Part 1 of the experiment.

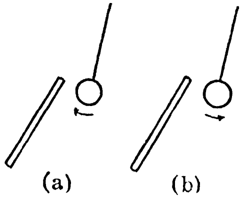


FIG. 75. — Refer to Part 2 of the experiment.

and is drawn towards the rod. Indicate the nature of the charges on the bits of paper and rod (Fig. 74).

Part 2. A charged ebonite rod is brought near a suspended aluminum pith ball (Fig. 75 a) and attracts it. After hitting the ebonite, it flies away (Fig. 75 b). Fill in the proper signs of electric charges for Figure 75 a and b.

Part 3. Charge the electroscope negatively by conduction. Record nature of the electrification on figures as in Figure 76 a, b, and *c*. Note that the position of the leaves of the electroscope as well as the nature of the charges in Figures 76 a, b, are left to the student to supply. Write in the kind of rod used and indicate its charge.

Part 4. Charge the electroscope negatively by induction. Referring to Figure 77, record the results as above in Part 3.

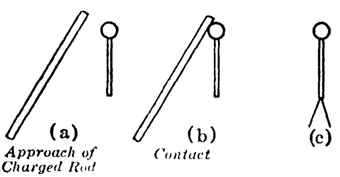


FIG. 76. — Refer to Part 3 of the experiment.

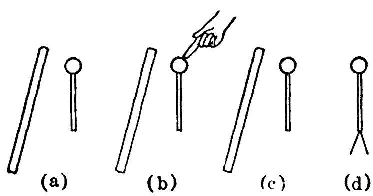


FIG. 77. — Refer to Part 4 of the experiment.

Part 5. Charge the electroscope and place a lighted match near it. What happens?

Part 6. Place the metallic disc attachment on the electroscope and charge the system negatively. Bring the metallic disc with insulated handle near the charged electroscope with and without grounding the former (Fig. 69). What do you observe in each case? Draw diagrams and explain, in your report.

Part 7. Charge the knob of the Leyden jar (condenser) by means of the charged metallic disc of the electrophorus, repeating until the condenser has acquired a considerable change. Bring the finger near the knob.

Part 8. Discharge the charged metallic disc of the electrophorus once on the electrostatic voltmeter. Record the voltage obtained.

QUESTIONS

(a). How could you tell whether an unknown charge was positive or negative by means of an electroscope?

(b). Explain what happened in Part 5 of the experiment.

(c). What is the cause of the result observed in Part 6?

(d). Why is the magnitude of voltage (see Part 8), which you have produced by means of the electrophorus, usually dangerous but not so in this experiment?

EXPERIMENT 29

MAGNETIC FIELDS AND POLES

Part (a). *To determine the field about a magnet for a given direction of the poles.*

Part (b). *To find the strength of the poles from the position of the neutral points.*

Apparatus: Two bar magnets, two small magnetic compasses, two drawing boards, thumb tacks, large sheet of paper (about 12 × 15 inches for 4-inch magnets), meter rule.

It is desirable that each person should perform this experiment by himself if the apparatus is available.

Part (a). Determine, by means of a small compass needle, which is the north-south magnetic direction of the earth, for the particular location of your experiment. Place the long edge of the data sheet parallel to this direction and then tack the paper down upon a drawing board or table top. Determine, next, which is the north pole of your bar magnet and place it parallel to the previously determined direction of the compass needle (consequently also parallel to the edge of the paper), so that the north pole of the bar magnet is in the same direction as the north pole of the compass needle (Fig. 78). When determining the north-south magnetic field of the earth by means of the compass needle, keep the bar magnet as far away as possible.

Draw an outline of the magnet on the paper and also construct a line AA' bisecting the magnet at right angles. It can be shown

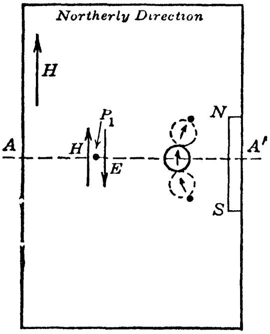


FIG. 78. — Plotting lines of force.

that the neutral point P_1 will lie on this line AA'. It must satisfy the condition that the earth's field H must be just equal in magnitude to the field E of the magnet. Consequently the resultant field at P_1 is zero. This point P_1 can be quite accurately located by moving the small compass needle back and forth along AA'.

In order to plot some lines of force, place the small compass on AA' about one inch away from the magnet. Make dots with a sharp pencil at both ends of the compass needle. Now move the small compass so that the last dot falls at the one end of the needle and make another dot at the other end. Continue in this way, following up from the previous dot, until the magnet or edge of the paper is reached. Having drawn a line of force through these points, plot other lines by starting at points two, three, four, five, and six inches away from the magnet. A non-magnetic (*e.g.*, wooden) pencil should be used for this work.

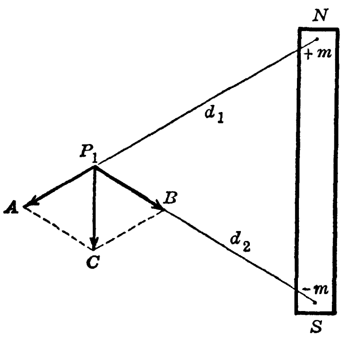


FIG. 79. — Measurement of pole strength.

Part (b). To find the pole strength, consider the neutral point (P_1) to have been found (Fig. 79). If there are no secondary poles, the distance from P_1 to the poles $+ m$ and $- m$ is the same. The position of each pole is found by noting where the lines of force converge. The field intensity (H_1) due to $+ m$ is

$$H_1 = \frac{m}{d_1^2}$$

in the direction P_1A, and the field intensity due to $- m$ is

$$H_2 = \frac{m_2}{d_2^2}$$

in the direction P_1B.

Now the magnitude of H_1 is equal to the magnitude of H_2. The distance d_1 (or d_2) can be measured, so that the magnitude of m may be calculated if H_1 can be found.

To find H_1, construct a parallelogram as in Figure 79, where P_1C is drawn so that 1 cm. represents one-tenth dyne of force. Look up in a handbook the horizontal intensity for the earth's magnetic field for your locality. If it is 0.2 dyne per unit pole, then make P_1C two centimeters in length. From P_1 continue the line drawn from $+ m$ to P_1, and from C draw a line parallel to the one from $- m$ to P_1. These two lines interesect at A. The line AP_1 represents the magnitude of H_1, each centimeter of length representing one-tenth dyne. Calculate the pole strength from the relation,

$$m = H_1 d_1^2.$$

QUESTIONS

(a). In the above arrangement where should another neutral point be found?

(b). In what way would secondary poles affect your result for the pole strength? Draw a magnet with secondary poles.

(c). What does P_1C represent, the resultant field due to the magnet or the field due to the earth?

(d). What is the resultant field intensity at point P_1 in Part (b) (Fig. 79)?

PROBLEMS

Experiment 28

1. State two definitions for field intensity at a point.

2. What kind of charges are actually transferred, as far as we know, when two dissimilar substances are rubbed together?

3. What is meant by the terms charging a body by (1) conduction, (2) induction?

4. What is meant by an electric field?

5. State Coulomb's law in words and by formula.

6. What is meant by (1) difference in potential, (2) field intensity?

7. What is the field intensity at a distance of 15 cm. from a charge of 450 e. s. u.?

8. Two pith balls, each with the same charge q, and each weighing one-twentieth of a gram, are suspended from the same point by strings of 100 cm. in length. Find the magnitude of the charge on each if they are separated by a distance of 6 cm.

Experiment 29

9. If magnetism is a molecular property of iron, how would you expect heat or hammering (in general) to affect the piece of steel which has been magnetized?

10. Can magnetic poles be isolated?

11. What would be the magnetic field intensity at a point on the longitudinal axis of a bar magnet at a distance of 15 cm. from one end? The poles are at the very end of the bar which has a length of 5 cm. Consider the strength of each pole to be 50 electromagnetic units.

12. Suppose the poles of the last problem were at the very ends of a magnet of length 50 cm. What would be the error introduced if the far pole were neglected in the calculation of the field intensity?

CHAPTER XIV

THE MEASUREMENT OF CURRENT — GALVANOMETERS

Whenever a charge flows through a conductor we speak of an *electric current* and define it as the rate of flow of charge past any cross-section of the conductor. Hence,

$$i = \frac{\Delta q}{\Delta t},$$

where Δq is the charge and Δt the time taken for this charge to flow past. If the current is constant, we can simply write $i = \frac{q}{t}$.

The analogy between electric current and rate of flow of water in a pipe (in, say, cubic feet per second) is seen to be a very close one. In the case of water some driving force is necessary before the water flows. This is usually supplied by a pump or by having a difference in level (potential energy) between the two ends of the pipe. The same is true in the electrical flow of current. A potential difference e is necessary before the current flows.

Now it is found that the potential difference existing between the two ends of a conductor is directly proportional to the current flowing through this conductor, all other conditions remaining the same. Hence we can state that

$$e \propto i.$$

The proportionality sign can be replaced by a sign of equality if we put in a constant and write:

$$(1) \qquad e = ri,$$

where r is now a constant for the conductor and called its resistance. This equation was first stated by G. S. Ohm and is known as *Ohm's Law*.

In the practical system of units we measure the potential difference in *volts*, the current in *amperes*, and the resistance in *ohms*. The relation between these and the fundamental units (e. m. u.) will be found in college physics texts and need not be given here.

[*Note.* In these chapters we will use capital letters to signify quantities measured in practical units (*e.g.*, I in amperes, E in volts, etc.) and small letters for the corresponding absolute units.]

An electric current can be measured by any one of the effects which it produces. There are three of these that lend themselves readily to observation and measurement. They are the *heating*, *chemical*, and *magnetic* effects. All three are used, since we know the laws governing the relation between current and any one of these effects. The only question then is simply which of them shall we use under the particular condition of convenience, accuracy, speed, portability, etc. We shall discuss the measurement of current under three headings, with particular reference to the laws of the effects and the apparatus.

I. Chemical Effect

The laws underlying the relation between current and amount of chemical action have been fully stated by Faraday (see Chapter XVII. This is really the way in which we should measure an electric current, since the international ampere was defined, at the Conference on Electrical Units and Standards held in London in 1908, in terms of the amount of substance deposited chemically by an electric current in a certain time. *Unit current* is that unvarying current which, when passed through a silver nitrate solution in water, in accordance with the specifications attached to this resolution, deposits silver at the rate of 0.00111800 gram per second.

The apparatus is called a silver coulometer. It consists simply of a platinum vessel or crucible containing the silver nitrate solution and having immersed in it a disc of pure silver. Many precautions are necessary when great accuracy is desired. Although the method is extremely difficult, long, and tedious, it has the advantage that it will enable us to reproduce accurately or measure a certain current, since the measurement finally becomes one of measurement of mass, which can be determined anywhere. The various bureaus of standardization still have to use this method. Legally this represents the way current should be measured.

For ordinary commercial and practical work, when such great accuracy is unnecessary, this method of measurement is out of the question since the apparatus is not portable. It is messy, time-consuming, and requires an expert to carry out the experiment.

II. The Heating Effect

An electric current flowing through a conductor heats the material according to the law:

$$W = JH = I^2Rt \text{ joules.}$$

Stated in other words: The heat produced per second is directly proportional to the square of the current. Hence by the heating effect, we can measure an electric current if we can find a convenient way of measuring the amount of heat produced. Several ways of doing this immediately suggest themselves, such as measuring the change in length of a wire produced by the heating, or else by allowing the wire to heat up a calorimeter with water and measure the rise in temperature, etc. The method most commonly used at the present consists in attaching a thermocouple junction to the wire and so measuring the heat produced in the wire by means of the e. m. f. produced by the thermocouple (Fig. 80). Such current-measuring instruments are known as thermo-galvanometers and have a very useful field of application.

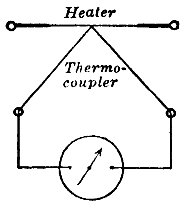


Fig. 80. — Thermo-galvanometer

The thermo-galvanometer is used extensively in measuring alternating currents, *i.e.*, currents that reverse their direction of flow periodically. The reason for their use in this connection is because the most common forms of current-measuring instruments (using the magnetic effect — see below) will not measure currents that reverse their direction of flow. It becomes a difficult problem to measure such alternating currents, especially when the frequency of the alternation becomes higher and higher. This heating effect furnishes about the only satisfactory means available at present for measuring frequencies of 10,000 alternations per second and higher — usually referred to as radio-frequencies — a field of current measurement becoming more and more important.

On account of heat losses to the air, wires, etc., it becomes difficult to calculate the current from a knowledge of the temperature of the wire, etc., and hence these instruments are usually calibrated by comparison with other ammeters or galvanometers.

III. Magnetic Effect

This is by far the most common effect that is used in steady current measurement. Every current flowing in a wire shows a magnetic field around it, and consequently a measure of the magnetic field gives us also a measure of the current.

There are two ways of measuring a magnetic field produced by a current: (1) by noting its effect on a magnet, or (2) by noting the effect of another magnetic field on the conductor carrying the current, when this other magnetic field is placed in the neighborhood of the current to be measured. For these two methods, two types of instruments have been developed to a remarkable (perhaps limiting) degree of sensitiveness to current. These two types are:

1. The moving-magnet type of galvanometer.
2. The moving-coil (or D'Arsonval) form of galvanometer.

1. The moving-magnet galvanometer or tangent galvanometer. This apparatus consists essentially of a vertical coil of wire which carries the current. At the center of the coil a small compass needle is mounted. In order to enable deflection of this small compass needle to be observed, a long, thin aluminum pointer is attached to this needle, usually at right angles, and allowed to swing over a graduated circular scale divided into degrees. When great sensitiveness is desired, the needle is suspended by a very thin quartz fiber and has a small mirror attached, so that a beam of light reflected from the mirror can be used to replace a pointer.

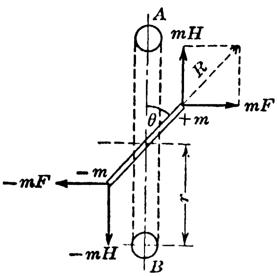


FIG. 81. — Tangent galvanometer.

Suppose in Figure 81 AB represents a cross-section of the coil which has been arranged so that its plane is in the direction of the earth's field H. Then when a current i flows through the coil, the magnetic field at the center can be shown to be $F = \frac{2\pi ni}{r}$, where n represents the number of turns and r the radius.

If now a compass needle is placed at the center, each pole of the little magnet will be acted on by the resultant of these two forces F and H, and hence will rotate until its direction is in line with this resultant R. Therefore, if θ is the angle through which the compass needle turns when the current is passed through the coil, we see that

$$\tan\theta = \frac{F}{H} = \frac{2\pi ni}{rH},$$

and solving for i, we get

$$i = \frac{rH}{2\pi n}\tan\theta = K\tan\theta,$$

where i is measured in absolute units.

Since r, H, and n are all constants which are known or can be found, we can see that $i \propto \tan\theta$.

This is the reason for the name, tangent galvanometer.

In order to increase the sensitiveness we want i to be as small as possible for a certain value of θ (or $\tan\theta$). This is done by making r and H as small as practical and n as large as possible. A limit is reached, however, for two reasons: first, increasing n, the number of turns, means an increasing resistance and hence a smaller current; and secondly, if H is made very weak, stray magnetic fields will often cause spurious results.

2. The moving coil or D'Arsonval galvanometer. The conductor carrying the current to be measured is put in the form of a coil and either suspended by means of a thin wire or strip, or else mounted between delicately constructed jeweled bearings so as to enable the coil to rotate. The magnetic field is supplied by a permanent magnet. The interaction of the two fields produces the rotation. With a carefully designed galvanometer of this type, the current can be made to give deflections proportional to this current. These instruments can be made very sensitive and are not subject to as many of the difficulties in their use as is the moving-magnet type.

The following table gives some idea of the sensitiveness of the various types:

Type	Approximate Resistance	Current Sensitivity	Voltage Sensitivity	Period
Moving magnet (tangent galvanometer)	$1 \longrightarrow 4000$ ohms	1×10^{-9} amps. to 3×10^{-11} amps.	1×10^{-9} volts to 1×10^{-7} volts	about 6 seconds
Moving coil (D'Arsonval)	$10 \longrightarrow 2500$ ohms	1×10^{-7} amps. 1×10^{-11} amps.	$.2 \times 10^{-6}$ volts to $.5 \times 10^{-7}$ volts	2 to 22 seconds

In this table the sensitivities are all stated in terms of a standard deflection, and the following definitions are used:

1. **A standard deflection** is chosen as the deflection of 1 millimeter on a scale 50 cm. away from the galvanometer when viewed through a telescope, the latter being also placed 50 centimeter distant from the galvanometer.

2. The **current sensitivity** (figure of merit) is the current necessary (usually given in amperes) to give a standard deflection.

3. The **megohm sensitivity** is the number of millions of ohms (megohms) that must be placed in series with a galvanometer, when one volt is applied, to give a standard deflection.

4. The **voltage sensitivity** is the potential difference (usually given in volts) which must be applied to the instrument directly to give a standard deflection.

5. The **period** is defined as the time for a complete vibration.

Example: Suppose that the current sensitivity, or figure of merit, is 10^{-9} amperes per standard deflection, then the megohm sensitivity would very approximately be

$$\left(\frac{1}{10^{-9}}\right)\frac{1}{10^6} = 1000 \text{ megohms},$$

if the galvanometer resistance is neglected. Suppose that the same galvanometer has a resistance of 100 ohms, then its voltage sensitivity would be $10^{-9} \times 100 = 10^{-7}$ volts per standard deflection.

General precautions. Whenever a circuit is being wired up, it should become a habit always to connect the battery or source of supply into the circuit last. Even then, the battery should not be connected unless the circuit has been carefully checked. This precaution cannot be too strongly emphasized because much valuable apparatus can be ruined if this precaution is not observed. For the same reason, when disconnecting your apparatus, always disconnect the source of supply first.

Another point of importance should be carefully observed: When using a resistance box, see that sufficient plugs have been removed in the high resistance range so that the circuit has a high resistance. Never make connections to resistance boxes and apply the battery or source until proper plugs have been removed from the resistance box. In general, if the resistances are all as large as possible, you are on the safe side and it is a simple matter to reduce the resistance to the desired value. Place the spare resistance box plugs flat on the top of the box so that they will not become dirty or misplaced.

EXPERIMENT 30

THE TANGENT GALVANOMETER

A measure of the earth's horizontal intensity using a tangent galvanometer.

Apparatus: A tangent galvanometer, about 3 feet of twisted leads, a reversing switch, a low-range resistance box (or rheostat), an ammeter to read a current of 0.1 or 0.2 ampere, dry cell.

Instead of using a tangent galvanometer to measure the value of a current, we shall use it to measure the strength of the earth's field, H (*i.e.*, the horizontal component). To find H accurately in practice involves a long and difficult experimentation. We can very easily get a fair value for H by the following procedure:

The arrangement of apparatus will be seen from Figure 82. It consists of a tangent galvanometer T through which we can pass a known current, as read on the ammeter A.

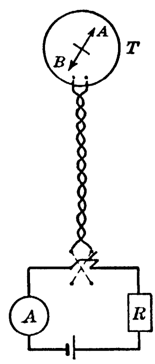


FIG. 82. — Determination of H.

Knowing the current and the constants of the galvanometer, we can calculate H, using the relation between current through the galvanometer and the tangent of the angle of deflection, as described above.

There are, however, several sources of error which have to be avoided, or else allowed for. Since we are going to measure H, the value of the magnetic field due to the earth at a certain region, it is important not to introduce any extraneous magnetic fields. It is for this reason that the ammeter should be kept as far as possible from the tangent galvanometer, and the leads through which the current flows should be close together.

The next part of the procedure is to set up the tangent galvanometer. Remember that the earth's field is being measured in the region in which the little magnetic needle is located. The long aluminum pointer is attached to the little magnet simply to facilitate reading the deflection. Before passing the current we must arrange to have the plane of the coil in line or parallel with the earth's field, so that the magnetic field produced later by the current will be at right angles to the magnetic field of the earth.

After being sure the needle swings freely, turn the coil until the plane of the latter is in line with the needle. A small straight edge placed above and in line with the needle on the compass-box will help to line up needle and coil. When this has been done, leave the coil now in this position and turn the compass-box scale until the reading of the pointer on the scale is zero.

We are now ready to pass a current. Before closing the switch see that a fairly large resistance is in the circuit and the ammeter is connected into the circuit with proper terminals, using the 1.5 ampere range. The ammeter should be connected (+ of ammeter to + of battery) right next to the battery as shown in Figure 82. Close the switch and note the ammeter current and galvanometer deflection. Adjust R until the deflection is between 30° and 60°. Note that throwing the switch the other way gives an opposite deflection.

Finally, we wish to obtain the necessary data for finding H. Between readings do not change the galvanometer adjustment, although the current can be varied so as to give about four or five different angles between 30° and 60°. Always measure the angle by reading both sides of the pointer. Record your data as follows:

No. of turns =
Average radius =

Location	Zero Reading		Abs. Current (ab.-amp.)	Current Direct		Current Reversed		Deflection		Av. Defl. (θ)	H (Calc.)
	End A	*End B*		*End A*	*End B*	*End A*	*End B*	*End A*	*End B*		
1. Three feet from south wall and two feet from east wall										Mean	=
2.											

Find the value of H in as many different localities as time allows.

QUESTIONS

(**a**). If the current is doubled, does the angle of deflection become twice as large?

(**b**). Do the angles measured by the end A and the end B of the needle agree? If not, give reasons.

(**c**). Give the various ways in which you could increase the sensitivity of the tangent galvanometer in practice.

EXPERIMENT 31

SENSITIVITY OF D'ARSONVAL GALVANOMETER

A determination of the current sensitivity, megohm sensitivity, and voltage sensitivity of a moving coil galvonometer.

Apparatus: D'Arsonval galvanometer, two high-range resistance boxes, one low-range resistance box, one key, dry cell, voltmeter (0–3 volt range).

Theory. There are many circuits that may be designed to measure the sensitivity of a galvanometer. They are all so arranged that the current passed through the galvanometer is of the proper magnitude so as not to burn out the galvanometer by being excessive in amount, but rather to give a reasonable deflection.

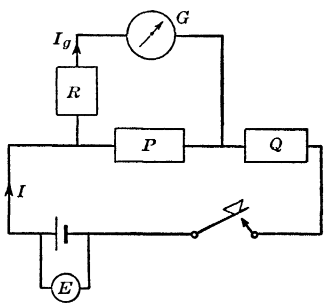


Fig. 83. — Sensitivity of a galvanometer.

Referring to Figure 83, it will be seen that the current through the galvanometer can be made small by arranging P to be very small and R and Q very large in resistance. Let I_g = current through the galvanometer and I = main current through the battery. Then $I_g = I \left(\dfrac{P}{P + R + G}\right)$ (see next chapter, page 155), where G is the galvanometer resistance. Since $I = \dfrac{E}{Q}$ approximately, where E is the voltmeter reading, we get

$$I_g = \frac{EP}{Q(R + G)},$$

approximately.

To adjust a galvanometer telescope and focus on the scale. Before passing a current through the galvanometer, it is necessary to get the cross-hairs and scale in focus. Look through the telescope and you will observe a *cross-hair*. This will be seen distinctly and sharply if the eye-piece end (the end you look through) is moved slightly in or out. Next, without looking through the telescope, you will find, if the scale is at all illuminated, that you can see an image in the galvanometer mirror of this scale when you get your eye at the proper level. Move your eye up or down, next to the telescope, until you see this image when looking into the mirror. Having spotted this image, see that the telescope is at the same height as your eye. (Use the raising or lowering screw on the telescope arm for this purpose.) Now if you should look through the telescope and it is pointing in the proper direction and is at the correct height, you will probably see the image of the scale faintly or blurred. Bring this image finally into sharp focus by the adjustment on the telescope tube (not the eye-piece). When final adjustment is obtained, both cross-hairs and scale should be in clear or sharp focus and no parallax should exist between them. Having made these adjustments, try not to jar the instrument any more. If you experience difficulty, do not hesitate to ask the instructor for help in making these adjustments.

To obtain data for calculating the sensitivity. Having connected the apparatus as shown in Figure 83, be sure that the resistance in boxes R and Q is as high as possible and the resistance in P is very low (say one ohm or less), before closing the key. The first time you close the key, this should be done with extreme care. Just press the key down for an instant. If there are any wrong connections, this fact will show up in such a brief deflection, and furthermore, such a procedure might save some valuable apparatus. It is always a safe policy to call the instructor over to your desk and have him check your connections. Having closed the key after everything is connected up correctly, you will probably find that the deflection produced, on looking through the telescope, is very small. To obtain a larger deflection reduce the value of R. In this experiment we would like to get a deflection right up to the end of the scale, namely, 24 or 24.5 cm. If reducing R does not produce the required deflection, then reduce Q in steps, being careful, however, not to make Q less than 50 ohms. If this manipulation still does not give the required result, then use a larger value of P.

Knowing that the deflection of 25 cm. can be obtained, adjust

R and Q until the steady deflections to the right or to the left are obtained approximately at the following points: 4, 8, 12, 16, 20, 24 cm. Read the voltmeter and deflection when the key is depressed and the deflection is steady, stating on which side your deflection was taken.

Record your data as follows:

Galvanometer resistance = ohms
Scale distance = cm.

P (Ohms)	R (Ohms)	Q (Ohms)	Deflection (mm.)	E (Volts)	Sensitivity (Current) amp./mm.	Sensitivity (Megohm) megohms/mm.	Sensitivity (Voltage) volts/mm.

[*Note.* The calculation of sensitivity should be for a standard deflection.]

Plot a curve with current sensitivity as ordinate and deflection as abscissa. Draw a smooth curve and do not join the plotted points.

QUESTIONS

(a). Can you think of any reason why it is better to use a constant value of P through all the measurements?

(b). With the approximations made in the derivation of the formula for finding I_g, is it better to have P large or small? Explain.

(c). Why is the sensitivity not constant for all amounts of deflection?

(d). Over what range of the galvanometer which you used can you assume linearity between current and deflection?

(e). Explain the shape of the curve by a study of the nature of the magnetic field in which the coil turns.

The Construction of Voltmeters and Ammeters

A galvanometer, when combined with the proper resistances, can be used to measure currents of any magnitude. The currents to be measured we shall assume to be larger than the current which the galvanometer, by itself, can carry without danger of damage. By a proper choice of resistances also, the same galvanometer can be made to indicate voltage or potential difference across its terminals. Of course it must be realized that a galvanometer really gives a deflection because a current is flowing, and in a well-designed instrument this deflection is proportional to the current. But if we let this current flow through a resistance, we can calculate the potential difference across this resistance, the value of the potential drop being proportional to the current if the resistance remains constant. *In this sense a galvanometer can be made to serve as a voltmeter.*

The ammeter. When a current of large magnitude has to be measured, then some provision must be made in an ammeter so that most of the current will be deflected through a branch circuit, and *only the proper amount passes through the galvanometer.*

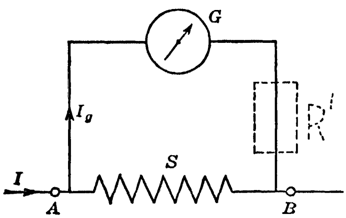


FIG. 84.—Arrangement of galvanometer and shunt to form an ammeter.

This is accomplished, as shown in Figure 83, by connecting a very low resistance shunt S across the galvanometer. The resistance R', shown dotted, is not really necessary but is inserted quite frequently to aid in making adjustments when calibrating the deflections of the galvanometer. This is done by comparison with the readings of a standard ammeter connected in the main circuit.

The relation between main current I and galvanometer current I_g is easily shown to be (see Chapter XV):

$$I_g = I\left(\frac{S}{S+G}\right).$$

In practice, if we wish to measure a fairly large current, then the ratio $\frac{I_g}{I}$ is a very small fraction, perhaps of the order $\frac{1}{10^4}\cdot$ We see from the above equation that the value of the term in the brackets must be of the same order of magnitude. This can only be accomplished by making the resistance of S a very small fraction of the resistance of the galvanometer. In the case under consideration, if the galvanometer had a resistance of 100 ohms, S would have a value of approximately 0.01 ohm, so that $\frac{S}{S+G}$ would be of the order $\frac{1}{10^4}\cdot$

Since the currents to be measured mostly in practice are quite large, a less sensitive galvanometer can be used. This means that the coil does not have to be suspended by a very fine wire, but can be mounted more ruggedly in conical jewel bearings, thus making the instrument more portable. *Note that the resistance of an ammeter* (AB) *is very low* (*less than that of the shunt resistance*).

The commercial ammeter, shown in Figure 85, is of the multiple-range type. The binding post at the extreme right is marked +, meaning that this side should be connected to the positive (+) pole of the battery. If the approximate magnitude of the current to be measured is not known, connect the negative side of your circuit to the negative binding post of the ammeter showing the maximum range, which is 15 amperes in the diagram. If you find that the current is less than 1.5 amperes, change your negative terminal connection from the binding post marked 15 amperes to the one marked 1.5 amperes. The same precaution should be taken in using voltmeters.

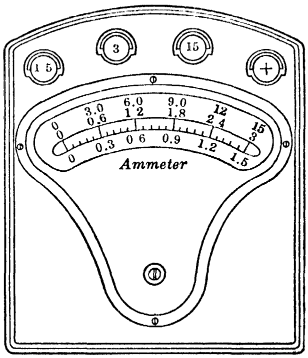


FIG. 85. — Commerical ammeter.

The voltmeter. The use of a galvanometer for indicating potential difference is very easily arranged. As a matter of fact, it is a very simple matter if we know the resistance of the galvanometer and the sensitivity, since $E = I_g R_g$, where R_g = the resistance of the galvanometer and I_g = the current through the galvanometer, and hence E is proportional to the deflection. This is only possible, however, when the voltages to be measured are very small. In general, it becomes necessary to insert a resistance R, in series with the galvanometer, as shown in Figure 86, when larger voltages have to be measured. Let us suppose, for example, that the potential difference existing across the galvanometer when carrying its maximum allowable current is 0.01 volt and we wish to measure a potential difference across AB of 100 volts. This means that we will have to insert a resistance at R so that most of the potential fall occurs across R (namely, 99.99 volts) and only

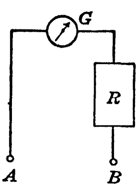


FIG. 86. — Arrangement of a voltmeter.

0.01 volt across G. If the current I_g under these conditions is say 0.001 amp., then R would have to have a value given by $I_gR = 99.99$, or $R = 99{,}900$ ohms. In this way, then, by applying various potentials at AB, the deflections of the galvanometer are proportional to the potential difference between A and B. (Full scale above = 100 volts between A and B.) *Note that a voltmeter has a very high resistance.*

EXPERIMENT 32

THE CONSTRUCTION OF VOLTMETERS AND AMMETERS AND THEIR CALIBRATION

Part (a). *To construct and calibrate an ammeter to read from 0 to 1.5 amperes.*

Part (b). *To construct and calibrate a voltmeter to read from 0 to 1.5 volts.*

Apparatus: Galvanometer (or milliammeter), a suitable low-resistance shunt (*i.e.*, low-value resistance box will do nicely), a variable high resistance or rheostat, standard ammeter (0 to 1.5 amps.), standard voltmeter (0 to 1.5 volts), 2–4–6 volt storage battery, about 15-ohm rheostat to carry 1.5 amps., a 150-ohm rheostat.

Part (a). The apparatus is connected as shown in Figure 87. The standard ammeter is shown at I and the home-made ammeter to be calibrated by the apparatus, consisting of resistances S, R', and galvanometer G. All the apparatus connected between A and B represents the home-made ammeter. The variable rheostat marked 11 Ω is meant for varying the current flowing through the main circuits. Be sure to see that all the 11 ohms are in the circuit to start with.

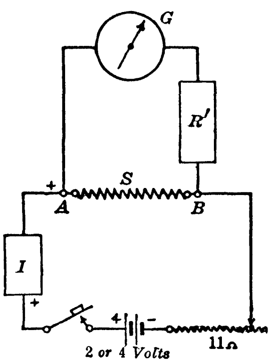


FIG. 87. — Calibration of an ammeter.

Before depressing the key, or connecting the battery into the circuit, have the instructor come to your desk and check your connections. The clips which connect on the battery enable you to obtain either two or four volts from the battery. Having made R' and the rheostat (11 Ω) as large as possible, close the key only for an instant. Note whether the direction of deflection is correct.

If the ammeter should read backwards, interchange the wires leading to this instrument. The deflections should now be on the scale of both instruments. The current can be varied by adjusting the value of the 11 ohm rheostat. Use care in decreasing this resistance, so that not more than 1.5 amperes pass through the ammeter I.

Adjust next the value of R' so that when the standard ammeter reads 1.5 amperes, then the galvanometer reading will be a maximum also. Make this adjustment accurately. Now reduce the current in the main circuit and take readings on both instruments for about ten points on the scale (to do this you will have to work out the main current value of a scale division on the galvanometer). In taking these readings set the standard ammeter as exactly as possible on a division line and read the galvanometer (estimating to tenths of a division). Tabulate your results.

Part (b). The adjoining Figure 88 shows the connections. Above the dotted line we have our home-made voltmeter, consisting of galvanometer and a variable high resistance R'. Be sure that R' is a very high resistance. We shall calibrate this voltmeter by comparison with a standard voltmeter V. The 150 Ω rheostat serves as a potential divider (Fig. 95 in the next chapter). By varying the position of the slider the potential difference applied to the two voltmeters can be varied from 0 to 2 volts, of which of course we will only need as much as 1.5 volts. Set the standard voltmeter V on 1.5 volts and adjust R' so that G reads full scale. Calibrate as before for various points on the scale, tabulating your results.

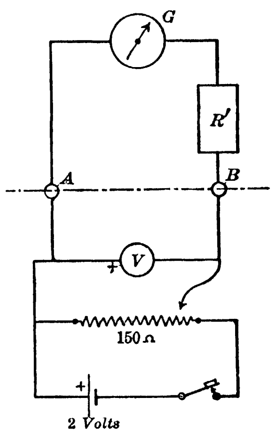


FIG. 88. — Calibration of a voltmeter.

QUESTIONS

(a). Draw correction curves for both the ammeter and voltmeter after having found the deviation at each point. If the reading is too large, call the deviation +, if too small, −. Then plot deviations (as ordinates) against divisions on G as abscissae.

(b). Discuss these curves as to possible methods of correction.

(c). If the voltmeter V in Figure 88 were removed, would the home-made voltmeter still give the same reading? Why?

PROBLEMS

General

1. A current flows for five hours through a copper sulphate solution. If, during this time, 300 coulombs have passed through the solution, what was the average value of the current (1) in amperes, (2) in absolute electromagnetic units of current?

2. A resistance of 50 ohms has a current of 3 amperes flowing through it. Find the potential difference across the terminals. How much e. m. f. would be necessary to furnish this current if the battery has a resistance of 0.5 ohm?

3. If the current in the above problem also passed through a silver coulometer, how much silver would be deposited in two hours?

4. How many calories of heat are produced in Problem 2 in two hours?

5. If a resistance of 50 ohms is connected across a 110-volt line, how much heat will be generated in one hour?

Experiment 30

6. (1) Is there any theoretical limit to the amount of current that can be measured with a tangent galvanometer? Explain. (2) What are the practical limitations of this method in measuring such large currents?

7. How many turns of wire are there in the coil of a tangent galvanometer of radius 8 cm. if a current of 0.4 ampere gives a deflection of 45° in the earth's field of strength 0.2 gauss?

8. Find the value of the horizontal component of the earth's magnetic field, being given that a tangent galvanometer of 200 turns and an average radius 10 cm. gives a deflection of 55°, when a current of 0.01 ampere flows through the coil.

Experiment 31

9. A galvanometer of 500 ohms resistance has a current sensitivity of 2×10^{-9} amperes per mm. What is the megohm sensitivity? What is the voltage sensitivity?

10. A battery of e. m. f. 1.5 volts furnishes current to two resistances of 2 ohms and 16,000 ohms in series. Across the 2 ohm resistance is connected a galvanometer of resistance 200 ohms. The deflection produced is 10 cm. Calculate the current through the galvanometer and the current sensitivity.

11. A galvanometer has a resistance of 20 ohms, and requires a current of 40 milliamperes (0.04 amp.) to give full-scale deflection. What is the potential difference across the galvanometer when giving full-scale deflection?

Experiment 32

12. The galvanometer of Problem 11 has a shunt connected across its terminals. What must be the value of this shunt if the whole is to be used as an ammeter for measuring a current of 50 amperes maximum value?

13. What resistance must be placed in series with the galvanometer of Problem 11 so that now the instrument forms a voltmeter of range 0–50 volts?

14. A galvanometer of resistance 50 ohms and sensitivity 10^{-6} amperes per division deflection has a resistance R, connected in series with it. Across this combination of galvanometer and resistance is connected a resistance of one ohm. What must be the value of R so that the whole arrangement forms an ammeter of maximum range 1.5 amperes if there are fifteen divisions on the scale?

CHAPTER XV

THE MEASUREMENT OF RESISTANCE

We have seen in the previous chapter that the constant which gives the relationship between the current produced in a certain circuit when an e. m. f. is applied is called its resistance. This is usually expressed by the equation $E = RI$, which is referred to as *Ohm's Law.*

This law was extended by Kirchhoff to take into account the continuity of the currents and the potential difference relations in any kind of complex arrangement of conductors that may arise in practice. These relations are put in the form of two laws, known as *Kirchhoff's Laws.*

Kirchhoff's First Law: The algebraic sum of all the currents meeting at a point is zero.

Kirchhoff's Second Law: In any closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the applied electromotive force in that circuit.

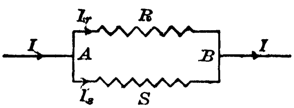


FIG. 89.—Currents flowing through resistances connected in parallel.

As an example of the use of these two laws, consider a case which occurs often in practical work. Let us find the current through one branch when two resistances are connected in parallel into a circuit having a current flowing in it.

This case is shown in Figure 89.

Call the currents flowing into a point $+$, and the currents flowing out $-$. Then for the point A we have (using the first law)

$$+ I - I_s - I_r = 0$$

or

(1) $$I = I_s + I_r.$$

Now use the second law and apply it to the closed circuit $ASBR$. There is no applied e. m. f. in this circuit. Let us go around the circuit in an anti-clockwise direction, starting at B. Then the second law gives

$$+ RI_r - SI_s = 0,$$

and hence

$$I_r = \frac{SI_s}{R}.$$

Substituting this value of I_r in (1), we get

$$I = I_s\left(1 + \frac{S}{R}\right) = I_s\left(\frac{R + S}{R}\right),$$

or finally,

(2) $$I_s = I\left(\frac{R}{R + S}\right).$$

This result is used so often that it is well to remember it as follows: *When a current enters two branches connected in parallel, the current through one branch (say* S*) equals the main current* (I) *multiplied by the resistance of the other branch* (R) *and divided by the total resistance of both branches* (R + S).

These two laws of Kirchhoff are very general laws and enable us to solve practically any case that may arise. A very frequent application consists in finding the current in a branch of a complicated network of wires when a certain e. m. f. is applied to the network. Suppose we have a simple circuit with several resistances, say R_1, R_2, R_3, etc., in series. Then by use of these laws we can show that as far as the current is concerned these resistances behave as if we had a resistance in the circuit of value R, where R equals $R_1 + R_2 + R_3$, etc. When these resistances are connected in parallel, then it can be shown that the equivalent resistance R has a value such that

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots.$$

EXPERIMENT 33

CURRENT FLOW IN NETWORKS

To analyze the currents and voltages existing in a complex network of resistances and to find the values of these resistances by the voltmeter-ammeter method.

Apparatus: A source of supply (for example, 110 volts D. C.); a board on which are mounted the resistances as shown in Figure 90 (screw sockets with Ward Leonard mounted resistances to screw into these sockets will do nicely); sockets for insertion of plugs ("parallel prong" and "polarized" prong types); single throw, single-pole knife switches; a plug and cord to fit parallel-prong type socket; a plug and cord to fit "polarized" type socket; ammeter (0–1.5 amps. range); voltmeter (0–150 volts).

[*Note.* The resistances are chosen here so that a 110-volt line can be used in conjunction with an ammeter of range 0–1.5 amps. With suitable resistance changes, a 6-volt battery can be used with the same ammeter and a voltmeter reading from 0 to 6 volts.]

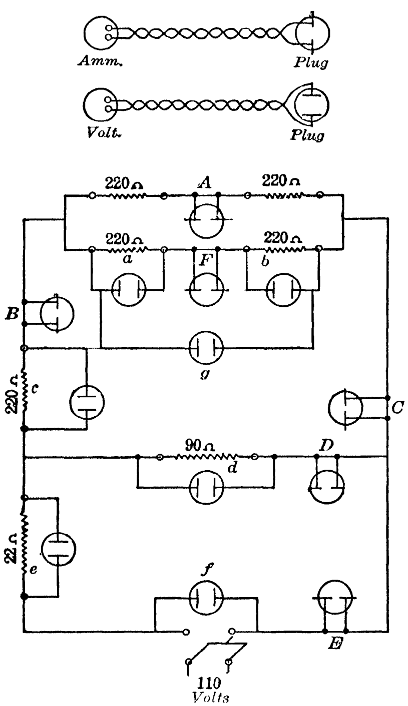


FIG. 90. — Network of resistances.

Before connecting the network to the 110-volt line, be sure to analyze and check up the connections in the wiring. Draw a schematic diagram of the circuit as arranged on the board and mark on your diagram the values of the resistances if they are given. Note the sockets *a*, *b*, etc., are connected *across* the resistances and are hence meant for measuring the potential drops across these resistances by means of a voltmeter. Therefore be sure to see that the proper plug is connected to the voltmeter. This should be the plug which has the two prongs parallel, one under the other. The other plug in which the prongs are at right angles to each other is the one that is connected to the ammeter. Note that the sockets into which the ammeter plug fits are connected across knife switches. Opening up a knife switch allows the current to flow through the ammeter.

Before applying the voltage have your apparatus checked by the instructor. In your report tabulate all your readings, having read them as accurately as you can estimate them on the instruments. This experiment is a test of the accuracy and judgment that you are capable of exercising in reading a pointer-type instrument. The following tests are to be carried out:

1. Measure the potential drops across *a*, *b*, *c*, *d*, *e*, *f*, *g*. Use the 150-volt scale (*i.e.*, binding posts marked + and 150).

2. Measure the currents at the various points *A*, *B*, *C*, *D*, *E*, *F*.

Calculate from your data the following:

1. The values of the various resistances (using Ohm's law, neglect the effect of the voltmeter and ammeter resistances — see next section on measurement of resistance).

2. Choose three points and show that Kirchhoff's first law is satisfied. (Give experimental error.)

3. Choose two closed circuits and from the above readings show that Kirchhoff's second law is satisfied. (Again calculate your experimental error.)

4. Having found from (1) the values of all the resistances, calculate the resistance of the whole circuit. (Do this by taking the various branches part by part and combining their resistances in proper fashion until the whole circuit has been replaced by a single resistance.)

5. Knowing the voltage applied to the circuit and the total resistance, calculate the current and compare with current as read at *E*.

QUESTIONS

(a). What effect does the resistance of the ammeter have in the various branches? How could you make correction for its effect and still calculate the unknown resistance?

(b). What is the similar effect and correction necessary in using the voltmeter?

(c). Do your experimental results check Kirchhoff's two laws within what you estimate to be the experimental error? If not, what explanation do you have?

Methods Used in the Measurement of Resistance

In practical work, current and potential differences are usually measured directly with instruments made for the purpose. These instruments, we have seen, are called voltmeters and ammeters and their indications give us the result directly. For most commercial work these readings have the necessary accuracy. Other and even more complicated methods must be resorted to if the accuracy so obtained is not sufficient.

The measurement of resistance, however, is not quite so simple and direct. We shall discuss here only the measurement of resistances of such magnitude as are commonly found in practice. These values might range from 0.01 ohm to 100,000 ohms. For values beyond these two limits, that is, larger than 100,000 ohms and less than 0.01 ohm, the accurate methods are considerably more complicated, require an exceptional amount of skill, and, in the case of extremely high resistances, require a technique all their own.

The simplest way o measuring a resistance is the *volt-ammeter method.* The principle of the method consists simply in measuring the current through the resistance, and the potential difference existing at the terminals. Then applying Ohm's law in the form: $R = \frac{E}{I}$, we find the resistance. The difficulty comes when we measure E and I. Using a voltmeter, as in Figure 91 (a), we note that the voltmeter reads the P. D. across R, but the ammeter reads the current through both R and V. We want the current only through R. However, the current through V in Figure 91 (a) becomes relatively less important the larger the resistance of V relatiye to R. Hence, assuming we are given a voltmeter V having a fixed resistance (say 100 ohms per volt), the smaller the value of R the more accurate our result will be.

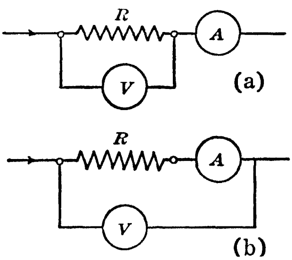


FIG. 91. — Volt-ammeter method of measuring resistance.

If we connect the voltmeter and ammeter as shown in Figure 91 (b), we find that the ammeter reads the correct current through R, but the voltmeter now measures the potential difference, not only across R, but across A also. In this case, therefore, the smaller the potential drop across the ammeter A relative to the P. D. across R, the more accurate our result. Hence R should have a relatively high value compared to the resistance of the ammeter. Usually of course the ammeter has a fixed resistance of fairly low value (say 0.01 or less), and hence the method is quite accurate for resistances from 10 ohms up.

It can be shown [1] that the dividing line between methods (a) or (b) occurs at a value $R = \sqrt{R_v R_a}$, where R_v = resistance of the voltmeter and R_a = resistance of ammeter. Hence for resistances

[1] A. W. Smith, *Electrical Measurements*, page 15.

less than this value, use the method shown in Figure 91 (a), and for resistances larger than this, the method of Figure 91 (b). With caution, therefore, the volt-ammeter method can be made to give fairly accurate results. Its main advantage of course is its simplicity.

The Wheatstone bridge. The method used for all accurate measurement of the resistance, in the range under discussion, is the *Wheatstone bridge method.* The method finds its application in various forms of apparatus having entirely different appearances. The principle of the balanced Wheatstone bridge network, however, can be traced in each one. Consider the arrangement of resistances as shown in Figure 92. The current entering at a splits into two parts, one part going along ad and the other along ab. The branch db is connected to the circuit at d and b. If a current is to flow through db, then there must exist a potential difference between d and b, because a current does not flow unless a potential difference exists. If, then, we say that there is no current flowing between d and b (a fact which we can easily determine by noting whether the galvanometer in this circuit shows deflection), we can reason conversely and say that no potential difference exists between the points d and b. This does not mean that the potentials of the points d and b are zero — they will not be — it simply means that they are the same.

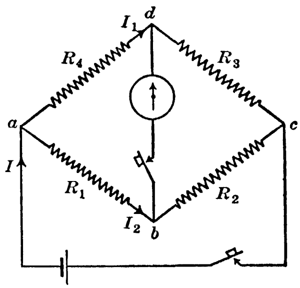


FIG. 92. — Wheatstone bridge network.

Consider then the two paths for the main battery current, viz., adc and abc. The potential of a is common to both, and we have seen that b and d have the same potential, hence the potential drop from a to d must equal the potential drop from a to b. Hence,

$$R_4I_1 = R_1I_2. \tag{3}$$

And similarly,

$$R_3I_1 = R_2I_2. \tag{4}$$

This, of course, is only true if no current flows from d to b or, in other words, if the bridge is balanced. Eliminating I_2 and I_1 from equations (3) and (4), we get,

$$\frac{R_1}{R_2} = \frac{R_4}{R_3}.$$

Now if we had one resistance unknown (say $R = X$), and knew the other three values, then we could find the unknown X by the relation $R_4 = X = R_3\left(\frac{R_1}{R_2}\right)$.

Usually the battery and galvanometer are interchangeable in their relative positions, but if the values of the resistances are very different, say R_1 and R_4 very large, and R_2 and R_3 very small, then the galvanometer should be connected between the points joining the two higher and the two lower resistances.

Only two common forms of apparatus will be described, although the student should from this description be able to use any other form, the similarity in their use being extremely close.

The slide wire bridge (meter bridge). Essentially this form consists of a uniform wire exactly one meter in length stretched along or above a meter rule. The ends of this wire are soldered to heavy copper or brass pieces shown shaded in the diagram of Figure 93.

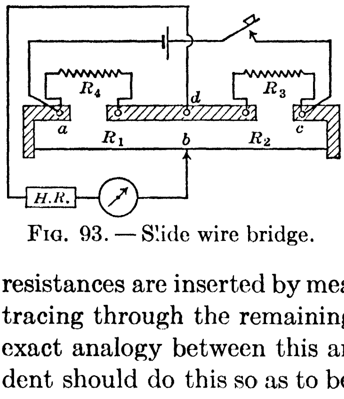


FIG. 93. — Slide wire bridge.

A slider which can make contact on the wire is so arranged that its position can be read on the scale. The two parts of the wire form two arms of the bridge. The other two resistances are inserted by means of binding posts at R_3 and R_4. By tracing through the remaining connections, the student will see the exact analogy between this and the Wheatstone bridge. The student should do this so as to be able to write down the conditions for the balance. In using such a bridge, the unknown is inserted at, say R_4, a variab e standard resistance box being connected in at R_3. Balance is then obtained by moving the slider until the galvanometer reads zero. Then assuming that the wire has constant resistance per unit length,

$$R_4 = X = R_3\left(\frac{R_1}{R_2}\right) = R_3\frac{l_1}{l_2},$$

where l_1 and l_2 are the distances between slider and the two ends of the wire.

Although this form of apparatus is very simple to understand and use, there are several possibilities of a considerable error being introduced when accuracy is required. First, we have neglected the effects of the end pieces to which the wire has been soldered. These should be included in the balance condition. The bar-resistance from d to R_3 and R_4 has also been neglected. Further, the wires may not be soldered exactly at the 0 and 100 division marks on the scale. (This is very difficult to accomplish in practice.)

A good way to overcome or minimize these errors is to proceed as follows: Place the unknown at the R_4 position and at R_3 insert a standard variable resistance box. Make a guess at the value of the unknown resistance and put that value which you have guessed into the standard resistance R_3. Move the slider near one end of the wire. Depress first the key in the battery circuit and while this key is down make the contact with the wire only for a short time — just long enough to see which way the galvanometer is going to swing. Next move the slider to the other end, and if the connections are correct, the galvanometer should now deflect in the opposite direction. If it does not, check up your connection. Now move the slider along until the deflection is neither one way nor the other. From the readings on the scale, calculate approximately the value of the unknown. Unless your guess has been very good, the chances are that the balance point is not near the center (*i.e.*, the 50 cm. mark). Having calculated the approximate value of the unknown, change the value of the resistance standard (R_3) so as to have approximately this value, and repeat, finding the balance point, this time very exactly. It should now be somewhere near the middle of the wire. Next, leaving the leads unchanged, interchange the positions of R_3 and the unknown resistance, being careful not to change the value of R_3, and find the balance point again as accurately as you can. Then it can be shown quite easily that if l_1 and l'_1 are the readings (in cm.) on the scale for the direct and reversed positions, then

$$\frac{R_4 \text{ or } X}{R_3} = \frac{100.0 + (l_1 - l'_1)}{100.0 - (l_1 - l'_1)}.$$

This procedure, although a little longer, eliminates many of the errors that arise on account of contacts and corrections. This method, however, is only effective if the balance points are arranged to be near the center of the wire.

Another error which is likely to creep in is found in the fact that the wire may not have been uniform originally, or else may have lost its uniformity by improper use. The only possible remedy is to replace the wire or else calibrate it. The method for doing this is beyond the scope of this book. The student should take special precautions not to spoil the uniformity of the wire by depressing the key too hard on the wire and producing kinks, which of course will ruin the wire as far as accurate results are concerned.

RESISTIVITY OR SPECIFIC RESISTANCE

The property of a substance to conduct electricity is called the *electrical conductivity.* When designing electrical instruments and apparatus, it is essential that the designer have an accurate and complete knowledge of the conductivity of the various materials at his command. In all this work the property or quantity to which reference is made is the so-called *resistivity* or *specific resistance.*

This quantity is arrived at by the following considerations: It is found experimentally that if the temperature of a substance is kept constant, then the resistance of a piece of the material depends upon its dimensions. The dimensions which affect the resistance are the length and cross-sectional area. We find, for example, that the resistance varies directly with the length of the specimen and inversely as the cross-sectional area.

Hence,
$$R \propto \frac{l}{A},$$
or
$$R = \rho\frac{l}{A},$$
where ρ is the proportionality constant which takes into account the kind of the material; ρ is called the *resistivity* of the substance and $\frac{1}{\rho}$ is called the *conductivity.*

[Note that $\rho = R$, if $A = 1$ and $l = 1$, which means that the specific resistance (resistivity) is numerically equal to the resistance of a conductor 1 cm. in length and 1 sq. cm. in cross-sectional area.]

A measurement of ρ, as we see from the above equation, involves a measurement of the total resistance (in ohms) of the sample, the length (in cm.) and the cross-sectional area (in sq. cm.).

Thus far we have assumed that the current has the same density per sq. cm. over the cross-sectional area, or, in other words, is uniform. In alternating currents this assumption is not valid, the current density becoming more non-uniform as the frequency becomes higher.

Dependence of Resistance on Temperature

Since the electrical conductivity, and consequently the resistance, vary with temperature (usually increasing with the temperature), one of the important quantities in electrical measurements is the *temperature coefficient of resistance.* In most cases if the resistance is plotted for corresponding temperatures and a graph drawn (Fig. 94), a smooth curve will be found to result which is not far from a straight line, especially in the useful range of temperatures.

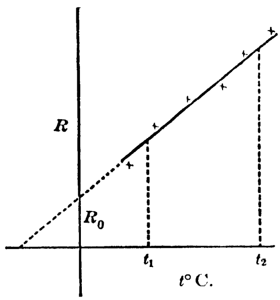


Fig. 94. — Variation of resistance with temperature.

In order to distinguish between the different behavior of materials in regard to this change in resistance we specify the temperature coefficient of resistance (α) which is defined by the relation,

$$R_t = R_0(1 + \alpha t),$$

where

R_t = Resistance at $t°$ C.
R_0 = Resistance at 0° C.
t = Change in temperature from 0° C. to $t°$ C.

Determining experimentally the values for R_t, R_0, and t, we can solve for α.

Should we not be able to find the value of the resistance at 0° C., we can still find the constant α, knowing the values of the resistances at two other temperatures t_1 and t_2, as follows:

(5) for temperature t_1 we have $R_1 = R_0(1 + \alpha t_1)$,
(6) for temperature t_2 we have $R_2 = R_0(1 + \alpha t_2)$.

Dividing equation (5) by (6), we have

$$\frac{R_1}{R_2} = \frac{1 + \alpha t_1}{1 + \alpha t_2},$$

whence

$$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}.$$

Resistance Standards

In the case of resistance standards, where permanence and accuracy are paramount, great precaution must be exercised to see that the principles to be mentioned are carefully considered and taken into account.

It is important that the resistances used have a small temperature effect, *i.e.*, the temperature coefficient of resistance be very small. The resistance wire used should also have a high resistivity or specific resistance so as to economize in bulk and cost.

It is found that various alloys of copper-nickel, etc., can be made to have high resistivity. It has been a difficult matter, however, to find alloys which have at the same time a low temperature coefficient. For example, German silver, which is an alloy of copper, zinc, and nickel, has high resistivity but an objectionable high temperature coefficient. The most satisfactory material at present for use in the construction of resistance standards is *manganin*, an alloy of copper, nickel, and manganese. This material has a very low temperature coefficient and high resistivity. It has, however, another very important property which has not been mentioned before — its thermo-electric e. m. f. between it and copper or brass is very small. When we connect in a resistance box with binding posts into a circuit, we do not wish to introduce bothersome thermal electromotive forces which would have to be allowed for. On account of this, such materials as German silver and constantan are not desirable for use in electrical measuring circuits.

Finally, great care and ingenuity must be exercised in the mounting of resistances. The calibration should be permanent. Temperature must not distort the forms on which the wires are wound so as to strain the material. Humidity and dust should not affect the insulation between coils, and, in the case of highest precision, provision must be made to keep the coils at a constant known temperature.

The student is referred to other more advanced texts on electrical measurements [1] for a detailed description of the methods used in mounting resistance coils. The methods used in varying the resistances are of two types: plug or dial. In the use of such resistance standards the student should not employ brute force

[1] See F. A. Laws. *Electrical Measurements*.

on the plugs or dials. Insert or remove plugs with a slight twisting motion, pressing down gently while so doing.

Current Limiting Rheostats

In very many cases it is necessary to vary and control a current. The nature of the variation required determines the type of resistance to choose for the purpose. As long as the resistance produces the required variations, we are not interested in its actual value. For this purpose it is unnecessary to use a carefully calibrated variable standard. Resistances designed for this purpose are called *rheostats.*

In general we can divide rheostats into two classes: slide wire types and carbon compression types. The first kind is used when the range of variation is to be large and, for practical purposes, it is satisfactory if the resistance change occurs in very small jumps. Slide wire rheostats are used nowadays almost exclusively. When large amounts of current have to be controlled and finer continuous adjustment is necessary, a carbon rheostat is used. The resistance change can be made practically infinitesimal by changing the pressure on the blocks slightly.

A word of caution should be given here to the student on the use of rheostats, especially when used in power supply lines for controlling current. The resistance and carrying capacity of the rheostat are usually marked on it and should always be known. The voltage being known on which the rheostat is to be used, divide this by the resistance of the rheostat, and so calculate the minimum current that will be produced when the rheostat is connected on this voltage. The current so found should always be well within the current-carrying capacity as marked on the rheostat. Most of the variable rheostats have connections as shown in Figure 95. Having three binding posts and a slider enables the rheostat to be used as a potential divider (very commonly but erroneously called a potentiometer). When the instrument is connected into a circuit at A and C only, we have a variable rheostat. When connected to a source of potential as shown by the dotted lines, then between A and C we have a variable potential difference, variable from zero to the full potential difference of the battery.

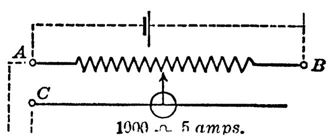


Fig. 95.—Rheostat.

EXPERIMENT 34

MEASUREMENT OF RESISTANCE WITH A SLIDE WIRE BRIDGE

Part (a). *To measure several resistances with a slide wire bridge.*
Part (b). *To check as accurately as possible the laws of combination of resistances.*
Part (c). *To measure the resistance of a galvanometer.*

Apparatus: A slide wire meter bridge, a standard plug or dial resistance box, galvanometer of medium sensitivity, a good dry cell, a single contact key, fairly heavy cotton-covered copper wire for connections, a high resistance rheostat.

Part (a). Connect up the slide wire bridge as shown in Figure 93, putting in one of the unknowns at R_4. Find first the value of the unknown by the simple relation $R_4 = X = R_3 \frac{l_1}{l_2}$. Make several trials by changing the value of R_3 slightly. Record all data in tabular form and find the average value of X together with its per cent of error. The final setting should be somewhere near the center of the slide wire. Measure in the same way all the other unknown resistances.

Next repeat the measurement using the direct and reversing method outlined above and obtain again several trials by changing R_3 slightly. For each valued R_3 calculate X and find the average value of X together with the per cent of error. Do the same for the other resistances given. Record all data and put it in tabular form. Label your data carefully so as not to get the ratio upside down when interchanging R_3 and X.

Part (b). Connect the resistances that you have measured, and measure in series by the simple method the resistance of the combination. Find the total resistance by calculation from Part (a) and also the per cent of error between calculated and experimental value. Repeat for the parallel connection. Compare the errors found in the calculated and experimental result.

Part (c). By means of a slight modification of the Wheatstone bridge the slide wire bridge can be used to measure the resistance of a galvanometer. The connections are as shown in Figure 96. A second galvanometer is not necessary to tell when a balance is obtained.

Note that in this case a high resistance (H. R.) is put in the battery circuit. Since a current is flowing through the galvanometer all the time, even when a balance is obtained, the value of H. R. should be kept such as to limit the current through the galvanometer G to a safe value. Balance is shown by the fact that when the key K_2 is depressed, no change is observed in the deflection of the galvanometer (k_1, of course, being closed first). The procedure in obtaining a balance is the same as before. Obtain other settings by changing the value of R_3 (still keeping balance point near the center of slide wire). Calculate the galvanometer resistance by the ordinary Wheatstone bridge formula. Average all values and so obtain value of the galvanometer resistance and the experimental error. Tabulate all your results.

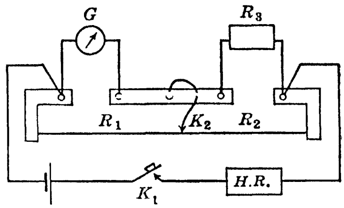


FIG. 96. — Measurement of galvanometer resistance.

QUESTIONS

(a). Do your results show that the method of interchanging X and R_3 gives better results for the value of the unknown?

(b). In Part (b) is the error as calculated from the single resistances the same as the error as found for the combination measurement?

(c). Approximately what was the accuracy of setting of the slider on the wire (*i.e.*, how much movement could occur before a change would be noticed on the galvanometer)?

(d). Explain why the Wheatstone bridge formula holds in Part (c). What is the difference between this and an ordinary bridge?

EXPERIMENT 35

TEMPERATURE COEFFICIENT

To determine the temperature coefficient of resistance of copper wire.

Apparatus: Temperature coefficient apparatus, thermometer, Wheatstone bridge, burner.

As can be seen in Figure 97, the apparatus consists of a coil of wire which can be immersed in a beaker of oil, preferably an oil which is not inflammable (olive oil will do very well). This beaker and coil is surrounded by another outer water jacket, the temperature of which can be raised or lowered by heating the water.

The resistance is found by means of a dial Wheatstone bridge.

In using such a bridge the student should first study the apparatus carefully, making frequent reference to the circuit as explained in Figure 92. In this figure it will be seen that two keys are needed: a key in the battery circuit, and a key in the galvanometer circuit. On the dial bridge these keys are mounted and marked, *BA*(ttery), and *GA*(lvanometer). Always close the battery key before depressing the galvanometer key. The latter key should be closed last, and then only for an instant, to show the direction of deflection.

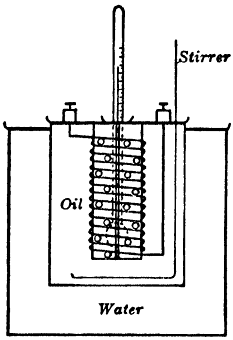


FIG. 97. — Temperature coefficient of resistance apparatus.

In the theory of the Wheatstone bridge it was shown that when a balance occurs, $R_4 = R_3\left(\frac{R_1}{R_2}\right)$. Now in order to find R_4, it is only necessary to know the value of R_3 and the ratio between R_1 and R_2 and not the actual values of R_1 and R_2. Hence in most assembled bridge networks, provision is made to determine and vary their ratio, rather than their actual resistances. A dial is provided which performs this known ratio variation (marked 1,000, 100, 10, 1, 0.1, 0.01, 0.001). The remaining dials form the variable resistance R_3, which is variable in steps of units, tens, hundreds, and thousands.

When the experiment is in progress, there is no time to spare, and measurements of the resistance have to be taken quickly and accurately. Having become familiar with the parts of the bridge, the student should next practice the following procedure and manipulation. The galvanometer should be connected to the binding posts marked *GA*(lvanometer). The battery (a dry cell) is connected to the proper terminals, and the resistance to be tested at the binding posts marked X.

Since the student usually has no idea of the value of the resistance under test, it is first necessary to find the approximate value of X. Set the ratio dial on the ratio marked 1. Put a large value in R_3 (say 5,000) and note the direction of galvanometer swing when the galvanometer key is depressed momentarily. Then put in a small value (say 1 ohm) and again note deflection. If the deflection is in the other direction, then we can say that X lies between these two values (1 ohm and 5,000 ohms). Now by changing

R_3, narrow down the limits until you have a fair idea of the approximate value of X (say between 8 and 9 ohms). Since the resistance is to be measured as accurately as possible, it is necessary to use all four dials (*i.e.*, measure to four significant figures) if this be possible. In the case mentioned, the value of R_3 would be made 8,000 and the ratio 0.001, because $X = 8{,}000 \times 0.001 = 8$ ohms approximately. Then by using the hundreds, tens, and units dial, the balance can be found accurately (interpolating the last figure if necessary). Suppose the value of R_3 so found is 8,235. The value of X will consequently be $8{,}235 \times 0.001 = 8.235$ ohms. Always remember to arrange your ratio such that R_3 can be made as high as possible. This means a greater number of significant figures and greater accuracy.

Having become familiar with the method of measuring a resistance, the student should now be ready to start his experiment. *Stir the oil well* by means of the stirrer provided so as to be sure of a uniform temperature throughout. The first measurement of the resistance should be made at room temperature. Close watch should be kept on the thermometer to see whether the temperature is settling down and becoming steady. Never take a reading while the temperature is changing rapidly or the apparatus is being heated up to another temperature. Control the heat supplied by the flame until the temperature has settled down about 9 or 10 degrees higher. When the thermometer shows that the temperature is almost steady, then be prepared to measure the resistance and *record the temperature and resistance exactly when a balance is obtained.* Then raise through another 9 or 10 degrees and so take readings up to the boiling point of the water. [*Note.* If the student has become familiar and expert at measuring the resistance, the variations in temperature of the oil and coil can be followed much more closely by observing the change in resistance by the drift of the galvanometer. When the drift is very small or zero, then the temperature is practically constant. A thermometer has a very appreciable lag which should always be considered.] Tabulate all results, showing values for R_3, ratio $\frac{R_1}{R_2}$, temperature and the calculated value of X.

If time permits, take measurements on cooling. Plot results on graph paper as described in the section on temperature coefficient of resistance. In doing this, do not join the plotted points, but draw the best straight line representing the data and make your calculations from this line. Check your result with the value given in a book of tables.

QUESTIONS

(a). Why not join individual points and call this the graph of the resistance *versus* temperature?

(b). Calculate from your results on your curve the value of R_0 (the resistance at 0° C.).

(c). Estimate the accuracy of your resistance measurement.

(d). Estimate the accuracy of your temperature measurement.

(e). Estimate the accuracy of your calculated α.

(f). Would you expect points on the curve for heating to fall on the same curve as for cooling? Give reasons for your answer.

EXPERIMENT 36

THE SPECIFIC RESISTANCE OF MATERIALS IN THE FORM OF WIRES

The use of a dial Wheatstone bridge for measuring a fairly low resistance such as occurs in finding the resistivity of materials.

Apparatus: Dial form of Wheatstone bridge, dry cell ($1\frac{1}{2}$ volts), galvanometer, known length of wire mounted on a board with three mercury cups.

In finding the specific resistance of a substance, we have seen that three measurements are necessary: total resistance, length, and cross-sectional area. Of these, the most difficult to measure with accuracy are the cross-sectional area and the resistance. In order that the non-uniformity of the wire is relatively small and also to aid in measuring the cross-section, it is preferable to have the diameter of the wire as large as possible. This means, however, that for a given length the resistance will be very low and great precautions must be taken in measuring such a low resistance accurately. Contact resistances must be reduced to a minimum and even the values of the lead resistances should be known.

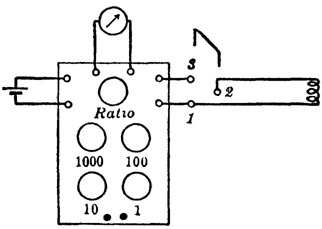


FIG. 98. — Specific resistance apparatus.

In the above form of apparatus (Fig. 98), contacts are made in mercury cups because this gives a uniform and low contact resistance. Thermal e. m. f.'s are to a great extent eliminated. The procedure in finding the resistance of the wire of known length is to connect by a copper strap cups 2 and 3 and measure the resistance (say R_L). This includes the unknown resistance plus leads, etc. Then take out the strap between 2 and 3 and place it in 1 and 3, thus omitting the long wire. Measure this resistance (say R_T). Then the required resistance is $R_L - R_T$.

When measuring the resistances the same method should be followed in finding a balance as in Experiment 35 (refer to the experiment for details in manipulating the dial bridge). Measure, in several places if possible, the diameter of the wire by means of a micrometer screw (calculating the error). Obtain the length of the wire and calculate the resistivity. Tabulate all data and compare your results with tables.

Leave the mercury-coated copper strap in the mercury cups when the apparatus is not in use.

QUESTIONS

(a). Compare the errors in the various measurements that you had to make in finding ρ.

(b). Explain fully why ρ is the same for all sizes or dimensions of wire made of the same material.

(c). What would be the effect of change of temperature on ρ?

Experiment 33

PROBLEMS

1. A battery has an internal resistance of 0.05 ohm and an electromotive force (*i.e.*, the potential difference between the terminals when no current is drawn from the battery) of 1.45 volts. A radio tube operating on 1.1 volts and requiring a current of 0.25 amp. is to be operated with this battery. How much resistance must be put in the circuit (usually done with a so-called rheostat) so that the tube will just be operating under the required condition? What is the resistance of the radio tube?

2. Given two parallel circuits as shown in Figure 99. Calculate the current through each branch, using first Kirchhoff's laws and secondly the rule as given in this chapter for two parallel circuits. Calculate also the total resistance.

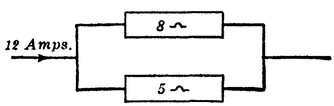


FIG. 99.

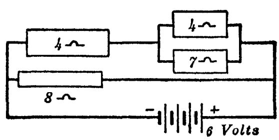


FIG. 100.

3. In the network shown in Figure 100, calculate the total resistance and also the current which the battery has to furnish.

4. Apply Kirchhoff's laws to Figure 101 and find the current flowing through each 7-ohm resistance.

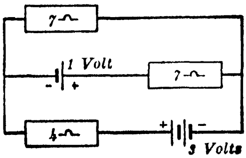


FIG. 101.

5. Calculate the total resistance of the circuit shown in Figure 100.

Experiment 34

6. Write down the similarities between the slide wire bridge and the Wheatstone bridge. State all the errors that come into a slide wire bridge measurement and give methods of overcoming some of these.

7. How much resistance would have to be placed in parallel with two parallel resistances of 80 ohms and 60 ohms, so that the total resistance be 30 ohms?

Experiment 35

8. Calculate the resistance of a coil of copper wire at 90° C. which has a resistance of 20 ohms at 0° C. (Look up temperature coefficient of resistance of copper in a handbook.)

9. Given that a certain wire has a resistance of 200 ohms at 20° C. and 208 ohms at 65° C., find the temperature coefficient of resistance.

10. Define or explain the meaning of the term "temperature coefficient of resistance." What is the importance of a knowledge of this constant from a practical standpoint?

11. From the discussion in this chapter on the change of resistance with temperature, does a method suggest itself to you for measuring temperatures?

Experiment 36

12. Calculate the resistance of 40 miles of number 20 B and S gauge copper wire (look up size of this wire and resistivity in a book of tables).

CHAPTER XVI

THE MEASUREMENT OF POTENTIAL DIFFERENCE BY THE POTENTIOMETER METHOD

We have seen in a previous chapter (page 149) that a voltmeter will measure the potential difference across its terminals. As a matter of fact this formed a very quick and convenient way of performing the measurement. This method, however, has two drawbacks which in many types of measurement preclude this method of measuring potential difference. These two are, first, very limited accuracy, and secondly, the fact that the insertion of the voltmeter changes the conditions in the original circuit (as to current and potential difference) so that it is not possible by this method to measure the original potential difference.

In such cases, where a voltmeter cannot be used, and in cases when great accuracy is needed, especially in the case of thermocouple e. m. f.'s, the potentiometer will give excellent results.

The potentiometer uses essentially a null method in its measurement of potential differences, meaning by this that when a balance obtains, the instrument gives its reading, and *no current is drawn from the source of potential difference to be measured.* Hence the instrument measures the original potential difference.

In practical work, for example in power stations, the temperature is measured by having a thermocouple in the furnace connected by leads to the switchboard in front of the operator. The e. m. f.'s produced are measured with a millivoltmeter, or a potentiometer when greater accuracy is needed. In many cases a continuous record of the temperature is kept on a recording instrument which does the balancing of the potentiometer, and recording of the e. m. f. or temperature automatically.

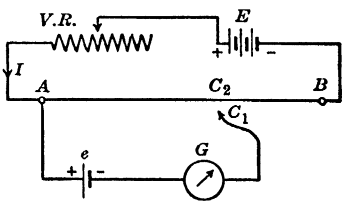


FIG. 102. — Potentiometer circuit.

The theory underlying the operation of a potentiometer can be studied by reference to Figure 102. This figure represents, schematically, the wiring of a simple potentiometer. A main battery E furnishes the source of e. m. f. for the current I which flows through a variable resistance $V.R.$ and a uniform slide wire AB. The current I can be varied by changing $V.R.$

Now when a current flows through the slide wire from A to B a potential drop will exist from A to B (let us say about 2 volts). From A to the middle of AB, *i.e.*, about C_2, the potential difference (drop) will be just one-half the value from AB (hence about 1 volt). Let us next connect in at A another cell, e, so that the positive pole of the battery e is connected to A. In series with this cell we place a galvanometer G and then connect this end to the contact maker or the slider. Suppose the contact, however, is not yet made, *i.e.*, C_1 is not yet connected to a point C_2 on the slide wire. Now if the cell, e, has an e. m. f. of 1 volt, then of course the potential drop from A to C, even before making the contact between C_1 and C_2, must be 1 volt. But we have seen that when the current I flows in the main circuit, the potential drop from A to C_2 is 1 volt, hence if the drop from A to C_1, and from A to C_2 — in each case 1 volt — and the potential A is common to both circuits, then C_1 and C_2 must have the same potential even before the contact is made. Now since no current existed in the circuit from A to C_1 (*i.e.*, through the galvanometer circuit) before making the C_1C_2 contact, no current will flow after making the contact because C_1 and C_2 had the same potential. If we had made contact at some other point on the wire, then the potentials at the point of contact would not have been the same and a current would have passed through the galvanometer, in one direction when C_1 is to the left of C_2, and in the other direction when C_1 is to the right of C_2.

Making use of this principle, suppose the slide wire, which we shall assume to be a meter in length and marked off in millimeters, had across it a known potential difference of 2 volts, and suppose that the battery e had an e. m. f. that was unknown. Then in order to find the unknown e. m. f., the procedure would be to move the slider along the wire until the point on the wire is found at which C_1 and C_2 have the same potential, which fact, of course, will be manifested by zero deflection of the galvanometer. Suppose that the reading on the scale was 37.6 cm. The e. m. f. therefore of the cell, e, will be $\frac{37.5}{100} \times 2$ volts $= 0.750$ volt. Using this method, then, we see that it becomes possible to calibrate the potentiometer to be direct-reading. In actual practice a means is always provided on the instrument for making this adjustment.

The next question that naturally suggests itself is: How did we know what the fall of potential across the slide wire originally was? This leads us up to the question of the use of standard cells.

A standard cell, as the name suggests, is a source specially designed to give a very steady and constant e. m. f. The development of such a cell has required years of experimental research with only the purest of chemicals. As a result, however, of this intensive study, using certain prescribed chemicals and standard methods of preparation, it is now possible to write down what the e. m. f. will be, for a given temperature, to approximately 1 part in 100,000.

For a *Weston normal cell* the e. m. f. at a temperature $t°$ C. is given by:

$$E_t = 1.01830 - 0.0000406(t - 20° \text{ C.}) - 0.00000095(t - 20° \text{ C.}) \\ + 0.00000001(t - 20° \text{ C.}).$$

Such a cell has for its electrodes mercury and cadmium, the solution being mercurous sulphate. The student should refer to his textbook for a detailed description of standard cells.

Other standard cells have been constructed, two of which are known as the *Eppley cell* and the *Weston unsaturated cell.* They differ from the normal cell described above in that they have an e. m. f. which is not quite as reproducible, when constructed over and over again, but they have the advantage that their e. m. f. does not vary considerably with change in temperature, as is the case with the normal cell. They are usually calibrated, after being constructed, by comparison with a Weston normal cell and then used with this calibration.

As with all chemical sources of e. m. f., standard cells have the disadvantage that they *polarize.* However, if small currents are drawn from the cells, the polarization is negligible, and even if it should occur, the cell will, when allowed to stand, rectify itself again. Remember any appreciable current drawn from the standard cell will ruin its use as a standard source of e. m. f. Never use a standard cell to furnish current. *Even connecting a voltmeter across a standard cell will spoil it,* the current used by the voltmeter being too large. No current larger than 0.0001 ampere should ever be drawn from the cell, and even then only for an instant.

The importance of having only very small currents go through the standard cell is taken care of in a potentiometer, by having a number of high resistances which can be placed in series with the standard cell in the galvanometer circuit. When one has no idea as to where the balance point occurs on the wire, a very high resistance is put into the standard cell circuit so that if the two potential drops are not the same, and the standard cell tends to furnish current, then this current will be cut down to a very small value on account of this high resistance. As the balance point is approached, the resistance can be cut out, but not before.

EXPERIMENT 37

THE POTENTIOMETER

Part (a). *Method of calibration by the use of a standard cell.*
Part (b). *Measurement of the e. m. f. of several cells.*
Part (c). *To measure the internal resistance of a cell.*

Apparatus: Slide wire mounted along a meter rule with the necessary binding posts, variable rheostat (V. R. say 15 ohms), a 4 or 6 volt storage battery, double-pole double-throw switch, single-pole knife switch, single-pole double-throw switch, a fixed high resistance (say 1000 ohms), a galvanometer (portable type is satisfactory), standard cell e (standardized by the instructor by comparison with a Weston or Eppley cell), Daniell cell, a variable resistance box p (total, 1000 ohms), several cells of various types and ages.

The purpose underlying this experiment is to acquaint the student with the technique used in standard cell and potentiometer methods. The first part of the experiment consists in calibrating the main circuit (namely E, $V.R.$, A, B, in Figure 103), so that we will have a known potential drop from A to B. This is done by using a standard cell whose e. m. f. we know. For the purpose of this experiment, obtain from your instructor a new dry cell which has been calibrated by him against a Weston or Eppley standard.

Note in this connection that the potentiometer which the instructor uses is substantially the same as yours except that the wire, instead of being stretched out straight, is wound into a circle. Ask to have this potentiometer explained to you. Write down the e. m. f. of the dry cell measured by him, and from then on treat this dry cell of known e. m. f. as your standard; *i.e.*, do not ever let it furnish any appreciable current.

Next connect your apparatus as shown in Figure 103. In doing so leave one wire off one terminal of each battery until your hook-up has been checked by the instructor. Be careful to see that your polarity is correct — the high potential or positive side of all batteries should connect to *A* when the switches are closed. Trace this through in your circuit.

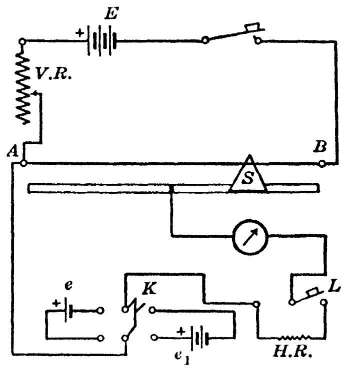


FIG. 103. — Measurement of the e. m. f. with a potentiometer.

Parts (a), (b). After your circuit has been checked by the instructor, start closing the circuits as follows:

The variable resistance is set for maximum resistance and the main battery circuit is closed. Next, see that the contact-maker *S* is not connected (remove entirely) and throw the switch connected to the *H. R.* resistance to the proper side (to the right in the diagram) such that the current will have to flow through this resistance. Now close the switch *K* so that *e* is thrown into the circuit.

Before connecting in the contact-maker *S*, note carefully what the purpose of the experiment is in Part (a). We want to know the potential drop across the wire. This we wish to make 2 volts across the 100 cm. of wire. Knowing the e. m. f. of the standard, *e*, figure out at which point on the slide wire a balance should occur and set the contact-maker at this point. The galvanometer will deflect (*precaution:* be sure to have the *H. R.* in the circuit), showing that the main current is not of the proper value to give 2 volts across the wire. Adjust this current by means of *V. R.* until the galvanometer reads zero. Now throw over the switch *L*, cutting *H. R.* out of the circuit, and make a finer adjustment of the main current. If an exact balance is now obtained, then you have adjusted the main current so that the drop across *AB* is exactly 2 volts. In further measurements in Part (b) do not change this main current except in one case only, namely, if when you return to the setting for the standard cell, *e*, you should find that the main current has changed slightly and needs readjustment.

Part (c). In order to measure the internal resistance of a cell it is necessary to measure some potential differences. On the right side of the switch K is connected a Daniell cell or a dry cell as shown in Figure 104. If you should have to move the Daniell cell, be careful not to mix any more than necessary the two solutions of copper sulphate and zinc sulphate. A variable resistance standard P (0 to 1000 ohms) is connected to the switch K. Start by having infinite resistance across the cell (*i.e.*, P on open circuit). Be sure also that the positive pole (copper) is connected to the correct side of the switch. If this is not done, a balance will never be obtained.

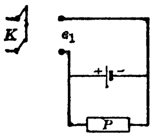


FIG. 104. — Internal resistance of a cell.

Next see that the resistance $H. R.$ is in the galvanometer circuit. Find the balance point on the slide wire, first approximately, then accurately, by cutting out $H. R.$ From the location of the balance point on the meter scale calculate the potential difference of the unknown Daniell cell, in this case when no current is drawn from the cell.

Make sure of the proper balance point by approaching it from both sides of the wire. Make a note also of the amount of movement possible of the slider before a difference can be detected in the balance of the galvanometer.

Repeat the measurement of the P. D. of the cell, furnishing currents for the following resistances which are connected across it: 1000, 500, 200, 100, 50, 30, 20, 10, 5 ohms.

Also take the measurement again of the e. m. f. of the cell on the open circuit.

For accurate work it is necessary from time to time to throw in the standard cell e, set the slider to the value for the standard, and see whether the circuit is still balanced. If it is not, a readjustment of $V. R.$ is necessary to bring the main current back to its proper value, assuming a drop of 2 volts across the wire.

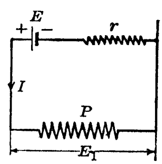


FIG. 105. — Relation between e. m. f. and P. D. of a cell.

From the above data calculate the internal resistance r of the cell. In Figure 105, P represents the external resistance and E is the e. m. f. of the cell, E_1 being the potential difference measured. From this figure we see that

$$E_1 = IP = \left(\frac{E}{P + r}\right) P.$$

Hence solve for r.

DATA

Part (a).

E. M. F. of main battery =

E. M. F. of standard cell as obtained from the instructor =

Calculated setting of slider for standard cell balance =

Calibration of wire: 1 mm. =

Accuracy of setting (in mm.) =

Part (b).

E. M. F. of cell marked A =

E. M. F. of cell marked B =

E. M. F. of cell marked C =

Part (c).

p	E_1	r	Accuracy of Setting	I
∞	E =		0.5 mm.	
1000				
500				

Average r = ± ohms

Plot the values of r as ordinates against I as abscissa.

[*Note.* When you are finished with the experiment, see that the Daniell cell when not in use always has a resistance of approximately 40 ohms across it. This is necessary to prevent the two solutions from mixing.]

QUESTIONS

(a). What is the maximum current that this particular Daniell cell can furnish?

(b). Suppose you had to have more current, what would you do if you only had Daniell cells at your disposal?

(c). What effect would a slide wire of ten times the length have on the accuracy of your measurement? What would have been the calibration in this case?

(d). Suppose the resistance of the wire were 4 ohms, what is the current in the main circuit when the current has been properly adjusted?

PROBLEMS

Experiment 37

1. A cell of e. m. f. (*i.e.*, when not furnishing current) 1.52 volts is connected to a voltmeter which only reads 1.48 volts. If the voltmeter has a resistance of 200 ohms, what is the internal resistance of the cell?

2. Make a list of the important precautions that must be taken in using a potentiometer and classify them under the headings (1) to take care of the apparatus, (2) to care for the standard cell, (3) to obtain accurate results.

3. How can a potentiometer be made direct-reading in volts on a slide wire?

4. You are given a slide wire 100 cm. long and of resistance 5 ohms. This is to be made into a direct-reading potentiometer of 1 mm. = 1 millivolt. If the main battery is a storage battery of negligible resistance and e. m. f. 4 volts, work out the details of the necessary resistances to give the proper calibration in the main circuit. Does the galvanometer circuit affect the calibration?

5. Explain fully the advantages of a potentiometer in measuring potential differences.

CHAPTER XVII

LAWS OF ELECTROLYSIS

When an electric current is sent through a chemical compound in solution or a fused chemical, certain chemical changes take place. We call this breaking down of the compound by the electrical current *electrolysis.*

The current is generally led into the solution by means of metal plates called *electrodes.* If a metal is deposited at one of the electrodes, the process is called *electroplating.* We shall confine our discussion to the simple case of electro-deposition of copper from an acid copper sulphate solution, because this example of electro-deposition will be sufficient to illustrate the application of the laws of electrolysis.

Consider a solution of copper sulphate, through which a current is passed, as pictured in Figure 106. The electrode at which the current enters the solution is known as the anode, while the other electrode is called the cathode. The copper sulphate, which is ionized because of its solution in water, begins to travel towards the electrodes as soon as the current is turned on. The negative SO_4 ions move towards the positive electrode pole, while the copper ions are attracted to the negative electrode. The solution of copper sulphate will continue to dissociate as long as energy is supplied to keep the ions in motion so that the electrodes will be kept in a charged condition. The ions which reach the respective electrodes lose their charge, the copper atoms are deposited at the cathode, and the SO_4 ion reacts with the anode to produce more copper sulphate.

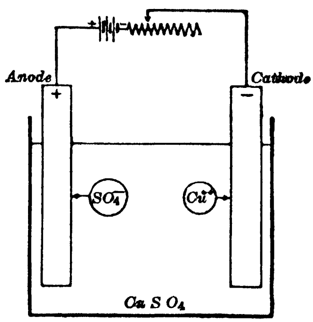


FIG. 106. — Electrolytic cell.

The reactions at the electrodes are not always as simple as described. We have chosen copper sulphate because of the ease with which the laws of electrolysis, first stated by Faraday (1853), may be applied to this solution. *Faraday's laws may be summarized briefly as follows:*

1. *The mass* (M) *of material deposited is proportional to the quantity* (Q) *of electricity which flows through the solution.*
2. *The mass* (M) *of material deposited for different substances is proportional to the chemical equivalent* $\left(\frac{\mathrm{m}}{\mathrm{v}}\right)$, *where* m *and* v *stand for the atomic weight and valence respectively.*

Hence the mass deposited is proportional to the product of the two conditions as stated in Faraday's Laws.

$$M \propto \frac{m}{v} Q.$$

That is,

$$M = \frac{1}{F}\frac{m}{v} Q = \frac{1}{F}\frac{m}{v} It,$$

where I is the current (in amperes) flowing for a time t secs. The constant of proportionality (F) is known as the Faraday, and $Z = \frac{1}{F}\frac{m}{v}$ is called the electrochemical equivalent, *i.e.*,

$$M = ZIt \text{ grams},$$

where Z can be defined as the number of grams of substance liberated by one coulomb of electricity.

There are some practical difficulties in testing these laws because of secondary reactions which may occur at or near the electrodes. Thus, if a neutral solution of copper sulphate is used to test the laws by weighing the copper deposited, one will find that the deposit may be brown due to formation of a copper compound at the cathode. This is caused by the alkaline condition at the anode due to the reducing action of copper when being precipitated out. This difficulty at the cathode is remedied by making the solution acid.

A satisfactory solution for plating copper on copper, as done in our experiment, is the following:

$CuSO_4 \cdot 5\,H_2O$ . . .	200 – 250 gm./liter
H_2SO_4 (conc.) . . .	50 – 80 gm./liter

Every object to be plated must first be cleaned. For our purposes, polishing the copper strip with clean emery paper will be sufficient. After cleaning, the fingers should not touch the portion to be plated. Should a *cleaning solution* be found necessary, the following alkali bath is very useful, especially when heated:

Sodium carbonate . . .	60 gm./liter
Sodium hydroxide . . .	15 gm./liter

or

Sodium carbonate crystals	
($Na_2CO_3 \cdot 10\,H_2O$) . .	165 gm./liter
Sodium hydroxide . . .	15 gm./liter

The object should be dipped into the alkali bath, then into water, and finally into very weak H_2SO_4 solution. The last dipping in the sulphuric acid solution is desirable when the object is to be plated in an acid copper sulphate solution.

EXPERIMENT 38

THE COULOMETER

The calibration of an ammeter by means of the copper coulometer.

Apparatus: Copper coulometer, ammeter, resistance unit of about 15 ohms and 1.5 amperes capacity, knife switch, fine balance.

The apparatus is assembled as shown in Figure 107. A low voltage supply of about 4 to 6 volts is connected to an adjustable resistance which, in turn, is connected in series with the copper sulphate solution, ammeter, and knife switch. The copper sulphate solution is made up as described in the theory.

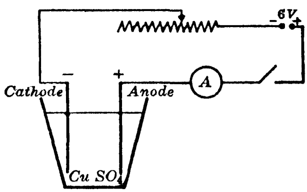


Fig. 107.—Copper coulometer.

Before starting your weighing in this experiment, the connections should be inspected and current turned on to see if the ammeter is connected so that the current will flow in the proper direction. It is also necessary to adjust the rheostat to see that an excessive current does not pass through the solution to "burn" the deposit. Adjust the rheostat so as not to deposit copper at a rate in excess of 5 amp./dm^2. A current density of 3 to 4 amp./dm^2. will give a rapid deposit and yet be well within the safe limit.

Now throw the switch off and remove the cathode (strip of copper to be plated). Rinse, dry, clean by polishing with emery paper, then weigh accurately on the fine balance. Do not touch the portion to be plated with the hands after cleaning. Replace the weighed cathode and deposit copper on it for one-half to three-fourths hour. Take readings on the ammeter every two minutes if the fluxations are small, or else keep the rheostat adjusted so as to keep the current constant. Record the total time during which the copper is being deposited. After the copper has been deposited, clean with several rinsings of water, and dry. The drying process is hastened considerably by pressing filter paper against the surfaces of the deposit, but avoid any rubbing. Handle the deposit as little as possible with the hands before weighing. Weigh the plate. The increase in weight will be the mass of copper deposited. The current, as found by the copper coulometer, will be given by

$$I = \frac{M}{Zt},$$

where $Z = 0.0003294$ gm./coulomb for copper (valence 2). Calculate the current from this equation. This method of determining the average current is very useful where the current, over a long period, varies by small unknown amounts.

Average all the readings of the ammeter and so find the average current recorded by the ammeter. Note the difference in per cent of your calculated result from the value as given by the ammeter.

QUESTIONS

(a). What data would be necessary to calibrate your ammeter completely?

(b). Examine the edges of the plate on which you deposited the copper. How does it compare in color with the center of the plate?

(c). If all the copper sulphate is not removed from the cathode in the rinsing just before the final weighing, will the current found experimentally by deposition be too large or too small?

(d). Suppose all the copper deposited did not adhere to the cathode due to a dirty portion of the surface, how would this affect the calculated value of the current?

(e). Calculate the approximate errors in your work to see if the percentage difference between the two results, one for the calculated current and the other for the average ammeter reading, come within experimental error.

PROBLEMS

1. What is meant by (1) chemical equivalent, (2) electrochemical equivalent?

2. If a copper sulphate and a nickel sulphate solution are connected in series with a source of current and it is found that 5 grams of copper have been deposited from the copper sulphate solution, how much nickel would be deposited from the nickel solution? (Consult handbooks for electrochemical equivalents.)

3. If nickel is to be deposited on a rectangular block of copper of dimensions $12 \times 30 \times 50$ cm^3. at the rate of 2 amperes per square decimeter, how long will it take to deposit 300 grams?

CHAPTER XVIII

THERMOCOUPLES

Electromotive forces, in general, exist at the surfaces of separation of two dissimilar conductors. The surfaces of separation may be liquid to liquid, metal to metal, or metal to liquid.

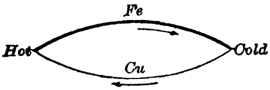


FIG. 108. — Seebeck effect.

If two dissimilar metals such as iron and copper are joined as in Figure 108, with the one junction heated and the other kept cool, a current will flow from the copper to iron at the hot junction and from iron to copper at the cold junction. This is known as the *Seebeck* effect, discovered in 1822.

The inverse effect was discovered by *Peltier* (1834) and is as follows: If a current is caused to flow as indicated in Figure 109, the junction at which the current flows from copper to iron becomes cooled (heat absorbed), and the junction where the current flows from iron to copper becomes hot (heat evolved). At the cold junction, work is done upon the electric current. That is, the current is made to flow from a lower to a higher potential. At the hot junction, heat is evolved since the current flows from a higher to lower potential and expends electrical energy. The electrical energy necessary to force the current through the two junctions is the difference between the heat absorbed at the cold junction and the heat evolved at the hot junction.

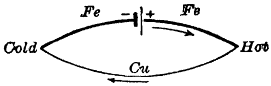


FIG. 109. — Peltier effect.

In the Seebeck effect (Fig. 108), heat is absorbed at the hot junction and transformed into electrical energy by the thermocouple. At the same time this junction would cool due to the fact that current is flowing (Peltier effect). At the cold junction, the current flowing from a higher to a lower potential causes a decrease of electrical energy, and therefore a heating effect. Hence the cold junction would heat up if not kept cool by some agent (generally a mixture of ice and water).

The resultant e. m. f. might be considered as due to the sum of the Peltier effects at the junctions if it were not for another effect called the Thomson effect, which alters the e. m. f.'s of the thermocouple. Therefore the thermoelectric force, as found in the Seebeck effect, will be considered as due to the combined Peltier e. m. f.'s at the junctions and the Thomson e. m. f.'s along the conductors. The total e. m. f. produced is not constant but depends upon the difference in temperature and rises to some maximum, then returns to zero (and even reverses) at higher temperatures. For smaller temperature differences and lower temperatures, the curve appears approximately straight.

Thermocouples are widely used for measurements of temperature by means of the e. m. f.'s developed. They may be made extremely sensitive to minute temperature differences. They often have an advantage over mercury thermometers in that they have a small heat capacity and absorb but little heat from the source whose temperature is to be measured.

EXPERIMENT 39

THE THERMOCOUPLE

The electromotive force of a thermocouple as a function of the temperature of the hot junction.

Apparatus: Base metal thermocouple, galvanometer, high series resistance for the galvanometer, thermometer, ice, steam generator, two small containers for chopped ice and water mixture.

The thermocouple consists of a constantan and a nickel-chromium wire. The couple is fastened to a bakelite panel. The wire runs in grooves on the back to binding posts at the right end (Fig. 110). One wire leading from each junction is insulated from the other by sections of porcelain tubing. The bakelite panel has a brass " collar " and clamping screw at the back with a hole of sufficient size to allow clamping on a standard rod. This provision enables one to suspend the apparatus at any given height. *Adjust the height so that it will not be necessary to bend the thermocouple wires.* Figure 110 shows a high resistance R in series with the galvanometer G.

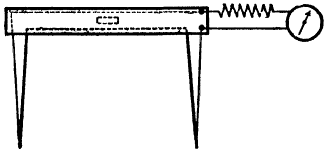


Fig. 110. — Thermocouple.

Adjust your galvanometer (with $R = 0$) to its zero reading by placing both the hot and the cold junction of the thermocouple in beakers containing ice (chopped) with just enough water to cover. This mixture will then be at 0° C. If the galvanometer does not come to rest on the zero, adjust to zero (see instructor), or take the reading. Then place the cold junction in the chopped ice and the hot junction in live steam. Adjust the resistance R so that the deflection will be between 200 and 250 millimeters (*i.e.*, not off scale). After this adjustment of the resistance box, record the steam temperature and the corresponding scale deflection. Then determine the temperature and scale deflections for intervals of about 10° decrease in temperature until the difference between the cold and the hot junction is less than 10° Centigrade. Now place a low flame under the beaker containing the hot junction and take the temperature and scale deflections for intervals of about 10° increase in temperature until the boiling point is reached. When changing the temperature, always bear in mind that the thermometer and couple do not change in temperature at the same rate. This is because of their different heat capacities. Always give the thermometer a chance to become steady before readings.

Obtain from the instructor the sensitivity of the galvanometer. Knowing the sensitivity of the galvanometer, the resistance of the galvanometer, and R, we can calculate the e. m. f. of the thermocouple (*i.e.*, product of the sensitivity, deflection, and total resistance). The resistance of the thermocouple itself may be neglected in comparison to other resistances. Plot on the same graph paper a curve for each of the two sets of the data of e. m. f.'s obtained, one for the temperature of the hot junction, decreasing and the other for the temperature of the hot junction increasing. The e. m. f.'s are to be plotted as ordinates and the temperature as abscissae. The average of these two curves should be taken as the correct curve.

DATA

Initial reading of the galvanometer =

R =

Resistance of the galvanometer =

Temperature of the cold junction =

Sensitivity of the galvanometer =

TEMPERATURE DECREASING

TEMP.	DEFL.	CURRENT THROUGH GALVA-NOMETER	E. M. F. OF THERMO-COUPLE
Steam			

TEMPERATURE INCREASING

TEMP.	DEFL.	CURRENT THROUGH GALVA-NOMETER	E. M O THEI COU

QUESTIONS

(**a**). Are the plotted curves linear?

(**b**). Do your two curves coincide within the experimental error of experiment? If not, how would you explain the non-coincidence?

(**c**). If the e. m. f. of the thermocouple is a function of temperature how would your results be altered if the temperature of the cold junction room temperature (say 25° C.) instead of 0° C.?

(**d**). Plot from your average curve as obtained in the experiment anc curve, as follows: Obtain from your curve the increase in e. m. f. (ΔE) 0° to 10° and divide by 10 (Δt = 10). Let this ratio $\left(\frac{\Delta E}{\Delta t}\right)$ be an ordi which corresponds to $\frac{\Delta E}{\Delta t}$, the average temperature (t) of 5° C. Find ΔE the temperature interval 10° to 20°, from which we may find $\frac{\Delta E}{\Delta t}$ for the ave temperature t of 15° C. Continue until the upper limit is reached. [I The ratio $\frac{\Delta E}{\Delta t}$ is known as the thermoelectric power.] What is the natu the curve obtained? Do you think it will cross the axis of abscissae? V is the physical significance if it crosses the axis of abscissae somewhere?

PROBLEMS

1. Explain the Seebeck effect on the assumption that the pressu " free " electrons at the junction of the two metals (Fig. 108) becomes gr in one metal than in the other with increase of temperature.

2. What is the (1) Peltier effect, (2) Thomson effect?

3. What is the order of magnitude of the e. m. f. developed by va thermo-junctions? (See book of physical tables.)

CHAPTER XIX

THE MEASUREMENT OF CHARGE

The fundamental or absolute unit of charge is defined from Coulomb's law. This law states that when we are given two electric charges q_1 and q_2, then the force exerted by one on the other is directly proportional to the magnitude of either charge and inversely proportional to the square of the distance separating them. Expressed in symbols we can therefore write:

$$F = \frac{q_1 q_2}{d^2}. \tag{1}$$

The only other factor which we have to consider in evaluating this force, F, is the medium separating the two charges q_1 and q_2. Experiment shows that this force is different in amount, depending upon the medium.

Usually we express this by saying that we put into equation (1) a constant $\left(\frac{1}{k}\right)$, which takes into account the medium and so makes the law perfectly general. Hence,

$$F = \frac{1}{k}\frac{q_1 q_2}{d^2}, \tag{2}$$

where k is called the *dielectric constant* of the medium.

Next, the question of units must concern us. We note that, in equation (2), the units for F and d are of course our fundamental units of force and distance, viz., the dyne and the centimeter in the c. g. s. system. Now since we do not have any units for charge q we can use Coulomb's law, to give us our unit for charge as follows: studying equation (2) mathematically, we see that, if $q_1 = q_2$, and $F = 1$, $k = 1$, and $d = 1$, then $q_1 = q_2 = 1$, and we have a unit charge. Put into physical terms, we have a unit charge (e. s. u.) when we take two similar point charges (magnitude and sign), place them a distance apart of 1 cm. in vacuum (by definition this makes $k = 1$) and find that the force between them is 1 dyne.

Now in measuring a charge, the first question to consider is:

Is the charge at rest, or in uniform motion (constant velocity), or does its velocity change? The method to be adopted in each case is different.

In the first case, in which the charge to be measured is at rest (such as in electrostatics) we use some form of electroscope or electrometer. The method used by Coulomb, which consists in finding the amount of twist in a wire when charges are placed at the ends of an insulated crosspiece attached to the end of the wire and the unknown charge placed near one of these, is theoretically the most direct, but practically offers many experimental difficulties and objections. At the present time the method used for qualitative work, where great accuracy is not necessary, involves the use of the leaf electroscope, in which the deflection depends upon the amount of charge. The instrument of course must be calibrated. For accurate work, a modified form of torsion balance, known as a quadrant electrometer, in which the unknown charge is made to attract an oppositely charged suspended system, is used.

Now in the second case in which the charges are in uniform motion we have a steady electric current. The methods used in finding the strength of electric currents have already been discussed in the chapter dealing with the measurement of electric currents (see Chapter XIV). If the currents are very small, such as ionization currents in the air, then we must use electrometer methods, by finding the rate at which charge accumulates on, or leaks off, an insulated system.

In the last case, in which the currents are variable, the measurement of charge resolves itself mostly into measuring the total charge which has passed in a certain small interval of time. Some important cases of common occurrence come in for consideration here; usually when a condenser charges up to a certain potential through the application of a potential difference a definite amount of charge flows into this condenser. This charge flows in rapidly in the beginning and more slowly towards the end, the whole process being over in a fraction of a second. A somewhat similar process occurs when the discharge takes place.

When the condenser is fully charged, we have:

$$Q = CV,$$

where Q = total charge, C = capacity, V = potential difference. In practical units these are measured in coulombs, farads, and volts respectively. Of course at any instant the current flowing is defined as the rate of flow of charge, viz.,

$$i = \frac{\Delta q}{\Delta t}.$$

Any electrical system which is capable of holding or storing an electrical charge when a potential difference is applied to the system is called a *condenser*. *The amount of charge it can hold for a certain fixed potential difference* (*one unit*) *is called its capacity.*

The unit in which we measure capacity is the "farad," or more commonly the "microfarad" (10^{-6} farads), in the practical system of units. In the absolute system of units (e. s. u.) we use a unit called the "centimeter."

A condenser will have a capacity of 1 farad when a potential difference of 1 volt will store in it a charge of 1 coulomb. A condenser will have a capacity of 1 centimeter if a unit charge (e. s. u.) will raise its potential by 1 unit (e. s. u.). The relation between the farad and the cm. is that

$$9 \times 10^{11} \text{ cm.} \approx 1 \text{ farad.}$$

or

$$9 \times 10^{5} \text{ cm.} \approx 1 \text{ microfarad.}$$

or

$$0.9 \text{ cm.} \approx 1 \text{ micro microfarad.}$$

FIG. 111. — Condensers connected in series.

In practice, condensers mostly consist of two parallel metal plates separated by a dielectric, such as, for example, air, mica, waxed paper, etc. If several of these are connected in series as shown in Figure 111, then it can easily be shown that the total capacity is less than the capacity of even the smallest condenser, in fact:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

If the condensers are connected in parallel or multiple as shown in Figure 112, then the capacities may be shown to give a total capacity

$$C = C_1 + C_2 + C_3.$$

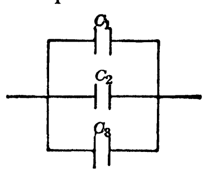


FIG. 112. — Condensers connected in parallel.

In the above discussion of condensers we are interested mainly in the resultant charge required. Another very important class of electrical measurement of charges which flow for a very short time occurs when dealing with induced currents. Suppose, for example, we have a closed coil of wire through which we pass, very rapidly, one pole of a bar magnet. A current is induced in the coil, the time for which the current flows being perhaps of the order of $\frac{1}{100}$ sec. It is difficult to find out what the current was at every instant. But it is relatively simple to find out, experimentally, how much total charge has passed. Then knowing how much charge has passed, we can calculate the strength of the magnet. Methods of this type are therefore most important in connection with measuring the strength of magnets or magnetic fields and also measuring the inductance of coils.

In all these measurements, of inductance and capacity, it becomes necessary to measure a certain quantity of charge, independent of the time for which this charge flows. The instrument which will measure charges of this nature is called a *ballistic galvanometer.*

The construction of a ballistic galvanometer is exactly similar to the D'Arsonval galvanometer, except that we apply an additional condition to a D'Arsonval galvanometer. This condition is, that all the charge flows through the galvanometer before the coil has moved appreciably from its equilibrium position. The coil then deflects after that, due to the impulse or " kick " which it received initially. It can be shown theoretically that the deflection then is directly proportional to the impulse, and consequently also to the total charge that went through the coil in the beginning. In order to satisfy this necessary condition of use, the current must flow for an extremely short time. If this cannot be assumed, then we must give the coil a very large moment of inertia and consequently a very long period of swing. The coil is usually made very broad so as to help in satisfying this condition. Since a ballistic galvanometer therefore gives deflections proportional to the total charge that has passed through it, we can use it as an instrument for comparison of total charges.

Suppose, for example, we wish to compare two capacities C_1 and C_2. We shall apply the same potential difference to each one and so charge them up. The total charge each will acquire will be different because their capacities are different, but the potential difference in this case will be the same. Hence we can write

$$Q_1 = C_1 V$$

for condenser C_1, and

$$Q_2 = C_2 V$$

for condenser C_2. Now if the deflections produced, when these two charges are allowed to pass rapidly through a ballistic galvanometer, are d_1 and d_2 then,

$$Q_1 \propto d_1, \text{ or } Q_1 = kd_1,$$

and

$$Q_2 \propto d_2, \text{ or } Q_2 = kd_2,$$

where k is a constant of the particular galvanometer.

Hence we see:

$$\frac{Q_1}{Q_2} = \frac{d_1}{d_2} = \frac{C_1}{C_2},$$

which shows that the ratio between the capacities is simply the ratio between the deflections produced.

We note in the above case that the constant of the galvanometer, namely, k, does not enter when using a ballistic galvanometer for purposes of comparison. If we wanted to know the magnitude of the charge (*i.e.*, Q_1), then we would have to know k. Note further that k is a constant whose value for different galvanometers would give us some idea of the sensitivity of the galvanometer in terms of the total charge which went through. To be more specific, we define the charge or micro-coulomb sensitivity (k) as the charge in micro-coulombs [*Note.* 1 micro-coulomb = $\frac{1}{10^6}$ coulombs] necessary to give a standard deflection (see page 142). Once we know the value of k, therefore, we can find Q very simply from the relation $Q = kd$, since we observe the deflection.

Calibration of a ballistic galvanometer when measuring a magnetic field or flux. When a ballistic galvanometer is connected to a coil of wire (Fig. 113), the conditions are not quite the same as when connected to a condenser. In the latter case the resistance in the galvanometer circuit may be considered to be infinite and hence it makes little difference which condenser we connect to the galvanometer. When connecting a coil, however, to the galvanometer, the resistance of the circuit of course depends upon the coil, and in most cases this is quite low.

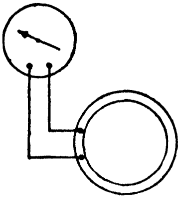


FIG. 113.—Measurement of flux.

Suppose now we produce a magnetic field near the coil, then some lines of force (N, say) will pass through the coil, and in doing so they will induce an e. m. f. in the coil, of amount, $e = -\frac{\Delta N}{\Delta t}$, for each turn of the coil. This e. m. f. will start a motion of charges through the wire and consequently an electric current will flow for a short time. The amount of current which will flow will of course depend upon the resistance of the circuit, consisting of the external coil and the galvanometer coil. The larger each one of these resistances the less the current. The current now having passed, the coil starts deflecting. As soon as it starts deflecting, however, the induced current set up by the rotating coil, according to Lenz's law, is such as to oppose further rotation. The smaller the external resistance the larger the induced current and so the less the deflection. When we calibrate a ballistic galvanometer in terms of flux cut and deflection produced, it will be seen that we do so *for a particular circuit*, consisting, in the above case, of an external coil and a galvanometer coil. Thus, by allowing the flux to cut or pass through the coil, we can measure the amount of flux cut.

We shall discuss one example of the use of the ballistic galvanometer in measuring flux. The problem is to study the nature of the induced e. m. f. in a coil of wire which makes one complete revolution in a magnetic field. This is done, with the apparatus provided, by allowing the coil to move rapidly through a very small angle (θ) say in Figure 114 and then measuring the total charge produced, which, in this case, is proportional to the average induced e. m. f. during this small interval. By measuring the amount of induced e. m. f. for all these small steps we can plot the amount of e. m. f. induced as the coil rotates in the magnetic field with reference to the coil position. It will be seen that the e. m. f. induced is a so-called "alternating" e. m. f., reversing its direction every 180° of rotation.

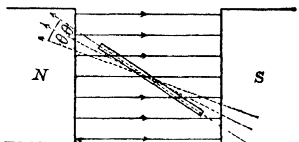


FIG. 114. — Rotating coil in a magnetic field.

EXPERIMENT 40

BALLISTIC GALVANOMETER

Part (a). *To measure the charge-sensitivity of a galvanometer.*
Part (b). *To compare capacities with a ballistic galvanometer.*
Part (c). *To check the laws of series and parallel connections of condensers.*

Apparatus: Ballistic galvanometer, short-circuit key, standard condenser, two or three unknown condensers, voltmeter (0–3 volts), charge-discharge key, dry cell.

Part (a). The apparatus is connected as shown in Figure 115. G is the ballistic galvanometer; k_1 is a key which may be found useful between measurements for bringing the galvanometer to rest. Key k_2 is a two-way single-pole switch. The dry cell is connected at e with a voltmeter across it to measure the potential difference which the battery furnishes for charging the standard condenser. [*Note.* Be very careful to see that your connections at the key k_2 are correct. It would be better not to connect in the battery, e, until the instructor has checked your circuit. Failure to do this might burn out the galvanometer coil — *so be over-cautious.*] The connections of the unknown condenser block are shown in Figure 116.

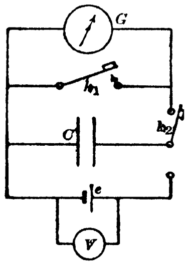


FIG. 115. — Measurement of capacity.

In reading a deflection the procedure is to throw k_2 on the battery side for just a few moments, and then quickly throw it to the galvanometer side of the switch. What is required is the farthest deflection on the scale. If you did not get the first one, repeat the charging and try again. Record all your trials. The zero from which the deflection occurs does not have to be the zero of the scale so long as the zero position is recorded and is steady. The deflection is the difference between the farthest deflection and the zero reading. Record both these for every trial. Average the deflections to find the average deflection. Record also the value of capacity used, as well as the P. D. of the battery and calculate the micro-coulomb sensitivity.

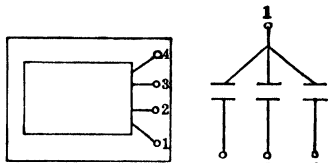


FIG. 116. — Condenser block.

Part (b). Use the largest value of the capacity as standard and assume that the others are unknown. Find their capacity by inserting them at C in Figure 115, and record the deflections they produce.

Part (c). Connect as many of the condensers as you can in parallel and measure their total capacity by comparison with the largest individual one assumed to be standard. Compare the total capacity as measured with the sum of the measured individual capacities.

Repeat this last part for as many condensers connected in series as possible.

DATA

Part (a).

Value of Capacity (C)	Voltmeter Reading (V)	Zero Reading	Max. Reading	Deflection (d)	Deviation	Remarks
Average V		Total				
		Mean		±		

Per cent of error =

Hence: Capacity .... farads (C)
P. D. .... ± .... volts (V)
Deflection .... ± .... millimeters

and since

Charge $C \times V$ coulombs
∴ Charge .... ± .... coulombs
Sensitivity .... ± .... coulombs/mm.
.... ± .... micro-coulombs/mm.

Part (b).

Value of Capacity Used	Zero Reading	Max. Reading	Deflections	Deviations	Average Deflection with Deviation
Standard (1–4)					
½ microfarad					

Part (b). — *Continued.*

Value of Capacity Used	Zero Reading	Max. Reading	Deflections	Deviations	Average Deflection with Deviation
1–3					
1–2					

Hence the calculated values of capacity from the relation $\frac{C_1}{C_2} = \frac{d_1}{d_2}$ give the following results:

Capacity	*Value*			
1–3	....	±	....	microfarads
1–2	....	±	....	microfarads

Part (c). Prepare your own data forms.

QUESTIONS

(**a**). Make a list of the various sources of experimental error, in finding the sensitivity of the ballistic galvanometer.

(**b**). With the cell provided in your experiment, calculate from your results obtained in Part (b) approximately the smallest and largest capacity that you could measure with your galvanometer.

(**c**). Do the results you obtained for series and parallel connections check the laws for such connection within experimental error? Discuss this point with reference to your experimental errors.

(**d**). Suppose you had to measure an unknown capacity by this method, but with the galvanometer you used in this experiment the deflection was too small to measure. How could you go about it?

EXPERIMENT 41

LENZ'S LAW

Part (a). *To compare the strength of two magnets by means of a ballistic galvanometer.*

Part (b). *To demonstrate Lenz's law.*

Part (c). *To show how the mutual induction falls off with distance between two coils.*

Part (d). *To find the nature of the induced e. m. f. (wave-form) when a coil is rotated in a magnetic field (model dynamo).*

Apparatus: Ballistic galvanometer, two bar magnets about 8 inches long, a fixed coil of about 500 turns, movable coil of about the same size, a dry cell, single contact key, model dynamo.

This experiment is designed to show how a ballistic galvanometer can be used for many purposes in which we wish to investigate either induced e. m. f.'s produced by coil cutting a known magnetic field or else to measure magnetic fields by the use of a coil which is allowed to move in this magnetic field. The three parts of this experiment are independent of each other, and consequently if the apparatus for any of these parts is not available, the remaining parts can still be worked.

The **theory** for this experiment may be summarized by noting that from the equations $q = i\Delta t$ and $e = ri$ (page 137), and $e = -\dfrac{\Delta N}{\Delta t}$ (page 194), one may obtain the equation

$$(1) \qquad q = -\frac{\Delta N}{r}.$$

If the number of turns on the coil is n and if k is the sensitivity of the ballistic galvanometer, then the deflection (d) of the galvanometer will be, since $q = kd$,

$$(2) \qquad d = -\frac{n\Delta N}{kr}.$$

This equation is true if the time of flow of charge is short compared to the period of the galvanometer.

Part (a). The galvanometer and fixed coil are connected together as shown in Figure 117. When the magnetic field inside the coil is allowed to change, then an induced e. m. f. is produced while the change takes place. Note what happens when the N pole of a magnet is thrust quickly into the coil, say from left to right. Measure the deflection produced. Several trials will be necessary to obtain this deflection. Record them all to find the average deflection and error. Repeat for the other magnet

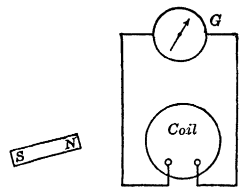


FIG. 117. — Lenz's law.

and thus compare their pole strengths. Construct your own data form.

[*Note.* In obtaining the deflection it is necessary to read and record the zero reading every time before observing the deflection, the reason being that the galvanometer is usually overdamped when a low-resistance coil is connected to it. This means that the galvanometer takes an enormously long time to come back to its original position, due, of course, to induced e. m. f.'s being created when the coil tries to come back, these induced e. m. f.'s opposing this motion. Since we cannot wait for the galvanometer to come back to the same zero, we ordinarily use the position from which it starts its deflection as zero. When using the galvanometer this way, of course, the student should wait long enough so that the zero does not drift very much. There are two other ways of overcoming this effect of overdamping or lag settling back to zero. One way is to open up the circuit somewhere, long enough for the galvanometer to swing back, and then close it as before. A better way sometimes is to induce a small e. m. f. in such a direction as to bring it back to zero. This is done very simply with the aid of a small magnet. The student should try any one or all of these methods in obtaining a zero.]

Part (b). With the same set-up as in the previous part, perform the following tests and explain your results, using diagrams.

1. Push an N pole of a magnet into the coil quickly and note the magnitude and direction of the deflection (the pole only is pushed through and not the whole magnet).

2. Pull out the N pole of the magnet rapidly (after having inserted it as in (1)) and observe the direction and magnitude of swing.

3. Push in the S pole — observe as before and show diagrammatically the direction of flow of current.

4. Pull out the S pole — and explain as in (3).

5. Having inserted the N pole as in (1), pull it out with various speeds, noting the deflection in each case. Record and explain your results.

6. Pull the whole magnet (both poles) through very rapidly and observe.

7. Connect another coil which can be placed close to the previously used coil and insert in series with this new coil a dry cell and a contact key. Close the key, keep it closed, and then open

the key, noting in each case the deflection. Draw a diagram to show in which direction the current flows in each case. Explain in terms of Lenz's law.

Part (c). Using the apparatus as shown in Figure 118, note the deflection in the galvanometer when A is placed at several distances from coil B. Start with A as close to B as possible, close the key and take a reading for the deflection produced when the circuit A is opened (it is not necessary to record the deflection when circuit A is closed). Take the mean of several trials. Increasing the distance between A and B, record the distance and deflection together with their mean. Do this for about ten distances until the readings are too small to observe with accuracy. Since this distance is unknown, it would be a wise policy to separate them as far as is necessary to get about a 2 or 3 mm. deflection, then divide the distance into approximately ten parts and set the coils at these calculated distances. Record your results in tabular form and then plot on a graph the deflection (or induced e. m. f.) as ordinate and distance as abscissa.

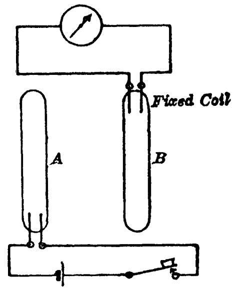


Fig. 118. — Induction apparatus.

Part (d). Connect the model dynamo to the ballistic galvanometer. Note the mechanism for rotating the coil through 10° at a time. The spring tension should be kept always about the same. Arrange to start your readings from the position in which the plane of the rotating coil is at right angles to the magnetic field produced by the poles of the magnets. Record your ballistic deflection every 10°, for a complete revolution (360°). Plot your results on a graph with degrees as abscissa and e. m. f. or deflection as ordinate. Draw a smooth curve through as many of the points as possible.

QUESTIONS

(**a**). What further information would you need in this experiment to calculate the e. m. f. developed at any instant when an N pole is thrust into the coil which is connected to the galvanometer?

(**b**). Does the induced e. m. f. (which of course depends upon the magnetic field) in Part (c) fall off linearly with distance? Explain.

(**c**). If the coil in Part (d) is rotated continuously, what would be the nature of the e. m. f. produced by this coil?

PROBLEMS

Experiment 40

1. Given that two similar charges, each of magnitude 200 e. s. u., when separated in a medium by a distance of 10 cm. are repelled with a force of 50 dynes, find the dielectric constant of the medium, and then from a book of physical tables find approximately the material of which the medium consists.

2. Calculate the amount of charge which flows into a condenser of capacity 4 microfarads when charged to a potential difference of 150 volts. Express your results in coulombs.

3. Find the capacity (in farads, microfarads, micro-microfarads, and centimeters) if a potential difference of 1.5 volts produces a charge in the condenser of three micro-coulombs.

4. Calculate the current in Problem 2 if the condenser could charge up uniformly in $\frac{1}{1000}$ second.

5. Given three condensers of capacity 0.25, 1, and 3 microfarads, find the total capacity when connected (1) in parallel and (2) in series.

6. A condenser of capacity two microfarads is connected to a dry cell giving 1.55 volts. When discharged through a ballistic galvanometer, the deflection is 18 cm. Find the micro-coulomb sensitivity. If another condenser is charged from the same battery and produces 4 cm. deflection, find the capacity of the second condenser as well as the amount of charge which this second condenser had.

7. Distinguish carefully between the use of a ballistic galvanometer and a constant current galvanometer, pointing out their similarities and differences in construction and use. Under what conditions could an ordinary D'Arsonval galvanometer be used for both purposes?

Experiment 41

8. A coil having 250 turns is connected to a ballistic galvanometer. It is known that 1000 lines of force are made to pass through this coil in $\frac{1}{2000}$ sec. Find the e. m. f. induced in the coil.

9. Draw diagrams to illustrate Lenz's law and current flow in the following cases:

(*a*) a North pole is thrust into a coil.
(*b*) a South pole is withdrawn.
(*c*) a current is made to flow in one circuit and induces a current in the second.
(*d*) a current is broken in the first circuit.

10. Derive an expression for the e. m. f. induced in a coil when it rotates in a uniform horizontal magnetic field.

CHAPTER XX

VIBRATING SYSTEMS — SOUND

Waves are a means of transfer of energy from one point to another. This transfer, when occurring in material media, is brought about by some elastic property of the media.

Thus, when a wave travels along a rope as shown in Figure 119, the elastic property is determined by the tension. In sound waves, however, it is determined by the bulk modulus. These two types of waves have one important difference, as to propagation characteristics, namely: in rope waves the vibrations of all portions of the rope are at right angles to the direction of propagation of the wave, while in sound waves the vibrations of the medium are back and forth in the direction of propagation of the wave. The former is representative of a class called *transverse waves* because the vibrations are perpendicular to the direction of motion. The latter type are representative of *longitudinal waves* because the vibrations are parallel to the direction of propagation.

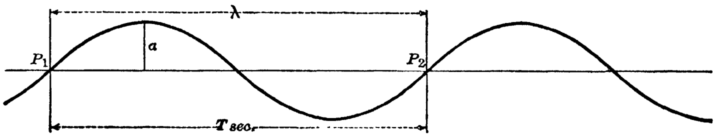


FIG. 119. — Wave along a rope.

Consider now a long train of sine waves, proceeding in one direction, known commonly; as progressive waves. Such a series of waves might exist, for example, along a rope (Fig. 119) of very great length. Figure 119 represents a portion of the series of waves traveling to the right with neither the beginning nor end of the train shown.

The distance from a point P_1 in the vibrating rope to another point P_2, the displacement and direction of vibration of which is the same as that at P_1, is called a wave length (λ). The time it takes a wave to travel the distance λ is called the period (T) of the wave, so that the velocity is, by definition,

$$v = \frac{\lambda}{T} = n\lambda,$$

where n is the frequency with which the waves pass a given point per second. Again, the maximum displacement of the segments of the rope from their neutral positions is called the amplitude (a).

While transverse waves are easily represented as in Figure 119, sound waves may be represented by such a simple diagram only if we represent the forward longitudinal displacements upwards on the y-axis when the waves are considered as proceeding to the right on the x-axis. Reference to Figure 120 shows one method

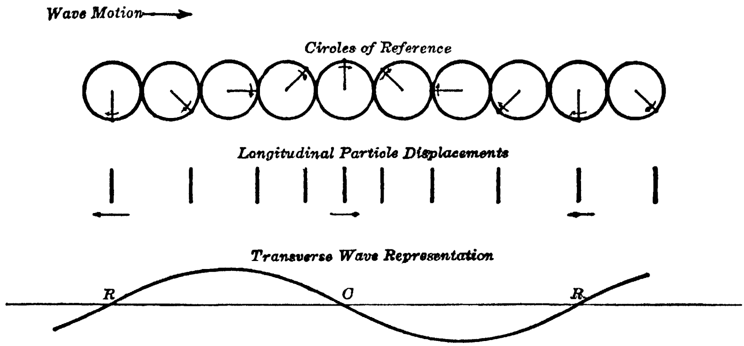


FIG. 120. — Wave representation.

of using this transverse representation of sound waves. At the points where the particles pass through their equilibrium positions, we have alternately condensations and rarefactions. At the condensation, the particles are moving with the wave, while at a rarefaction the particles are moving in the opposite direction. It should be noted, however, that both the condensation and the rarefaction as such are moving to the right each with velocity v, and what the ear hears depends upon the magnitude (*i.e.*, pressure changes) and frequency of the condensations and rarefactions, rather than upon the velocity of the gas particles. In Figure 120, the circles are constructed so that a point in each circle is in constant motion. The point in each succeeding circle represents an earlier moment of time. The series of straight lines below the circles represent the corresponding simple harmonic motions (*i.e.*, longitudinal wave) and the curve is the transverse representation of the longitudinal wave.

Consider now a rope of finite length with one end fastened to a wall (Fig. 121) and the other end P subjected to a vertical simple harmonic force. This simple harmonic force may be produced by attaching the string to one prong of a vibrating tuning fork. This is an example of forced oscillation, and if the tension of the rope is properly adjusted, stationary waves will be produced as shown in Figure 121 (b). When this happens, the frequency of the fork and string will be found to be the same, a special case of forced vibrations, called resonance. These stationary waves have *places where there are no motions of the rope, called nodes* (N), *and*

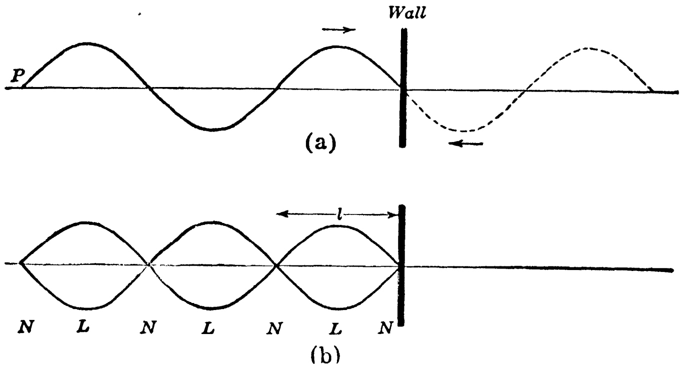


FIG. 121. — Reflection of waves.

places of maximum displacements of the rope, called loops (L) *or antinodes.* The presence of these nodes and loops may be explained if we consider a wave coming in from behind the wall with a displacement equal in magnitude but opposite in direction, as shown in Figure 121 (a). If you draw the resultant amplitude for such a reflected wave and the approaching wave, at intervals of one-eighth period over eight such intervals, for example, you will find that the transverse displacements of the rope at a given point will assume at the proper time intervals all values between the extreme displacement of each point as shown in Figure 121 (b) and zero displacement, for all points along the rope. If we call the distance from one node to the next, l, then from the definition of wave length, we have that $\lambda = 2\,l$, also that $v = n\lambda$, or

$$n = \frac{v}{2\,l}.$$

It is known that the velocity of a transverse wave along a flexible stretched wire, having a tension T (measured in dynes), is

$$v = \sqrt{\frac{T}{\sigma}},$$

where σ is the mass per unit length. Hence, the frequency n of the string represented in Figure 121 (b) will be

$$n = \frac{1}{2\,l}\sqrt{\frac{T}{\sigma}}. \tag{1}$$

If it is a sound wave that is being considered, then in order to have standing waves produced we must confine the waves, in some manner, such as in a hollow tube the diameter of which is small compared to a wave length. We shall use as our applied, simple harmonic force, a tuning fork. Hit the tuning fork with a rubber hammer (or tap the tuning fork against a rubber block or cork stopper) and hold it over a hollow tube as shown in Figure 122. If the tube is of the proper length, standing waves will be produced in the air column. Figure 122 (a), (b), (c) represents waves set up in three tubes of lengths d, $3\ d$, $5\ d$, etc., when the same fork is used, where d is the distance from node to loop.

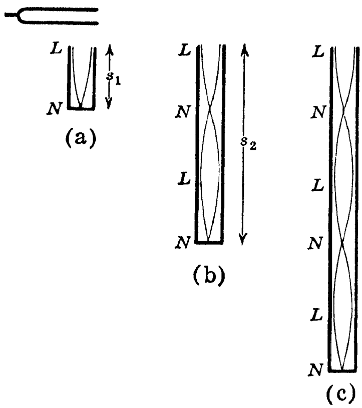


FIG. 122. — Resonance in an air column.

In order to tell when these standing waves are set up, listen for the reënforcement of the sound of the tuning fork, which is due to the resonant vibration of the air column. The frequency of the vibrating air column will be the same as that of the fork. This is a well-known example of resonance in the study of sound waves. The closed end represents the place of greatest change in pressure, yet here there is no motion of the molecules if the walls are rigid, and thus represents a node. The open end of such a tube will be found to be a place of greatest change in velocity of the air molecules with but little (if any) change in pressure because the medium is less confined here of all positions which have anything to do with the vibrating air column. Hence the open end represents an antinode, or a loop. With these restrictions in mind as to nodes and loops, the representation of nodes and loops in Figure 122 (a), (b), and (c) should be clear.

Unfortunately, the loop L is not located exactly at the open end of the tube but is a little beyond. It takes a short distance for the equalization of the pressures to take place. This additional distance is a function of the radius r and is usually between $0.6\,r$ and $0.8\,r$. One way to find the correction factor c is to find the resonance length as in Figure 122 (a), (b), with a tube of adjustable length. Call the length of the pipe for first resonance s_1, and the length for second resonance s_2. Then $(c + s_1)$ will be a quarter-wave length or

$$\lambda = 4(c + s_1), \tag{2}$$

also

$$\lambda = 2(s_2 - s_1), \tag{3}$$

so that, eliminating λ from the above equations, we have the correction factor

$$c = \frac{s_2 - 3\,s_1}{2}. \tag{4}$$

While the wave length of a fork of unknown frequency may be determined by use of equation (2), assuming a value for c, the determination of the wave length by the use of equation (3) is more accurate because the correction factor does not enter. In any case the frequency of the fork is determined from the equation $v = n\lambda$, where the velocity at $t°$ C. is

$$v = 331.7 + 0.6\,t \text{ meters per second.}$$

EXPERIMENT 42

STATIONARY WAVES IN A STRING

To determine the pitch of a tuning fork by means of stationary waves set up in a string.

Apparatus: Electrically driven tuning fork, flexible twine (fish cord), hanger to hold lead shot, platform balance and weights, meter stick.

The apparatus consists (Fig. 123) of an electrically driven tuning fork mounted on a board so that it may be secured vertically to rods and clamps or to an " arm " which may extend from the wall. One end of a string is fastened to one prong of the fork and a hanger (made from an aluminum tea ball) is fastened to the other end of the string.

The procedure is to start the fork into vibration and pour shot into the basket until stationary waves of one or more loops are formed. Find the distance l between two nodes. This is best done by taking an average of all the nodes, remembering that the node formed at or near the vibrating fork is not usually very definitely defined. It is better to omit this node when possible. Then weigh the basket with shot to find the tension, $T = mg$. The frequency n of the fork will be

FIG. 123. — Standing waves in a string.

$$n = \frac{1}{2\,l}\sqrt{\frac{T}{\sigma}},$$

where σ is the mass per unit length. Obtain this constant from your instructor, or weigh the string and divide the mass obtained by the length.

Repeat two or more times, obtaining each time a different number of segments (*i.e.*, nodes and loops) by changing the load on the string.

Frequency marked on the tuning fork =

No. of Segments	Length of Segments (l) 1st	2nd	3rd	etc.		Average l	m	T	$\frac{1}{2\,l}\sqrt{\frac{T}{\sigma}}$	
										$\sigma =$

QUESTIONS

(a). Did you find any marked variation in the length of the segments between successive nodes? Should there be any?

(b). Calculate the probable accuracy of your work.

(c). Assuming that your result is correct, calculate the per cent of error from the frequency as marked on the fork.

EXPERIMENT 43

THE SONOMETER

Part (a). *To test the relation between frequency and length.*

Part (b). *To test the relation between frequency and mass per unit length.*

Part (c). *To find the frequency of a given tuning fork by means of the sonometer.*

Apparatus: Sonometer equipped with a steel and a brass wire of different diameters, two tuning forks (say of frequencies 256 v. p. s. and 384 v. p. s.), about 12 kg. of standard masses (4 two-kg. masses, 2 one-kg. masses, 4 one-half-kg. masses), hanger.

The sonometer (Fig. 124) consists of a hollow resonance box on which are mounted two wires (one shown here). The tension is determined by the known weights and the length of the string is controlled by a movable bridge.

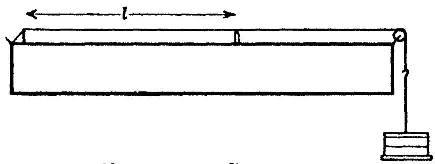


FIG. 124. — Sonometer.

[*Caution:* The twisted wire at the anchorages becomes weakened with use. Consequently one should keep the eyes as far from any direct line of the stretched wire as possible.]

Note that the distance between two nodes (l), as used in equation (1), now becomes the distance between the fixed and movable bridges when the wire vibrates in its fundamental mode.

Part (a). Place about 6 kg. in the hanger to which the steel wire is attached. Adjust the movable bridge until the frequency of the wire of length l when plucked is the same as that of the one fork. Record the length of the wire and the frequency marked on the fork. Repeat using the other fork. Next carry out the same procedure for the brass wire. From the theory [equation (1)] it will be seen that nl should be a constant if the same wire and tension are used.

There are two convenient methods for telling when the frequencies of the fork and wire are the same. The one method is to adjust the frequencies until no beats are heard. Beats are heard when the frequencies are close together and disappear either when the frequencies are the same or far apart. The other method is to place a tiny paper rider on the wire at the center between the fixed and movable bridge. Now start the fork into vibration and place the tip firmly on the top of the resonance board. When the frequencies are the same, the rider will jump off, due to resonance of the string with the fork.

Some tuning forks are so constructed that but little vibration is communicated to the stem of the fork. In this case, place the stem of the vibrating fork near or on the movable bridge.

Part (b). To test the relation between the frequency and the mass per unit length directly, a number of forks of different frequencies would be required. If we make use of the information gained in Part (a), namely, that $n \propto \frac{1}{l}$, then if $n \propto \frac{1}{\sqrt{\sigma}}$ also, the effect of both factors would be that $n \propto \frac{1}{l\sqrt{\sigma}}$ or

$$l\sqrt{\sigma} = \text{constant},$$

if n and T are constant.

Place 6 kg. on each hanger and adjust the bridges so that each wire will vibrate with the same frequency as one of the forks. Record the lengths necessary to give the same frequency. Be sure to have the tension on each wire the same. Obtain the values of σ from your instructor or else weigh a known length of wire on a balance.

Part (c). Adjust the tension and length of the wire so that it will be in tune with the fork. Record the length, tension, and mass per unit length and calculate the frequency from equation (1). Repeat by changing the tension and length. If time allows, repeat once more by using the other wire. Construct your own data form for Part (c).

DATA

Part (a).

Wire	m (kg.)	n	l	nl	Average	Per Cent of Difference from Mean
Steel	6 + hanger	256				
	6 + hanger	384				
Brass						

Part (b).

WIRE	m (kg.)	n	l	σ	$l\sqrt{\sigma}$	AVERAGE	PER CENT OF DIFFERENCE FROM MEAN
Steel	6 + hanger	256					
Brass	6 + hanger	256					
Steel	6 + hanger	384					
Brass	6 + hanger	384					

QUESTIONS

(a). Calculate the value of $\frac{1}{2}\sqrt{\frac{T}{\sigma}}$ from the data obtained in Part (a) and compare with your value of the constant nl for the steel wire.

(b). Calculate the value of $\frac{\sqrt{T}}{2n}$ from the data obtained in Part (b), and compare with your value of the constant $l\sqrt{\sigma}$ for the steel wire.

(c). What is the per cent of difference between the calculated and marked value of the frequency in Part (c)?

EXPERIMENT 44

THE RESONANCE TUBE

To find the frequency of a tuning fork by resonance and to determine the end correction for the resonance tube.

Apparatus: Two tuning forks (frequencies 512 and 768), resonance apparatus with resonance tube 22 to 24 inches in length, cork stopper or rubber mat.

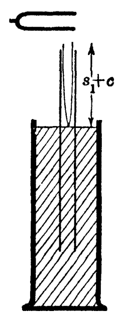


FIG. 125.—Resonance tube.

The apparatus consists of a tall jar filled with water. A glass tube about 1 to $1\frac{1}{2}$ inches in diameter is placed in the jar. The length is adjusted until resonance is obtained when the fork is struck against a cork stopper or other soft material and held over the tubing as shown in Figure 125. A rubber band around the glass tubing placed at the water line when resonance occurs aids materially in obtaining more exact measurements. Assume the length of the tubing is s_1, the corrected length is $(s_1 + c)$. Now find the second resonance point and call the length of the tube s_2. By means of equation (3) calculate the wave length λ. Using the known velocity of sound find the frequency of the fork.

The end correction c is determined by equation (4),

$$c = \frac{s_2 - 3\, s_1}{2}.$$

Repeat for the other fork.

DATA

$$v = 331.7 + 0.6\, t\, \frac{\text{meters}}{\text{sec.}}$$

FREQUENCY OF FORK (Manufacturer)	s_1	s_2	$n = \frac{v\ (\text{cm./sec.})}{\lambda\ (\text{cm.})}$	$\frac{s_2 - 3\, s_1}{2}$

Radius of the tube =

QUESTIONS

(**a**). Calculate the per cent of deviation of your result from the manufacturer's value for each fork.

(**b**). Does the correction factor c appear to be essentially constant for different frequencies with your tube?

(**c**). Assuming that the correction factor is a linear function of the radius of the resonance tube, let $kr = c$, where r is the radius of the tube and k is the constant factor which is to be found. Calculate the value of k. How close does your result agree with the theory?

EXPERIMENT 45

THE VELOCITY OF SOUND

To determine the velocity of sound in a metal by the Kundt's tube method.

Apparatus: Kundt's tube apparatus, lycopodium powder, resin, chamois, meter stick.

The Kundt's tube apparatus (Fig. 126) consists of a hollow glass tubing closed at one end (right side) with a disc which is fastened to a movable rod a. A steel or other metallic rod b is clamped to the other end (left) at its center. It also has a light metallic disc at the end which is inside the glass tubing, but this light disc does not touch the sides of the tubing. The tubing should contain some lycopodium powder or cork dust sprinkled the whole length.

The procedure is to stroke the portion of the metallic rod b to the left of the center with chamois which has powdered rosin

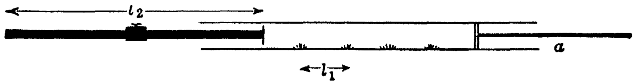


FIG. 126. — Kundt's tube.

sprinkled on it. A little practice will enable you to obtain the fundamental tone of the rod. Move the rod a back and forth until the powdered dust particles within the tubing form sharp heaps. Measure the distance between a number of these dust heaps and divide by the number of segments included within your measure. This will give the average distance l_1 between segments. These dust heaps represent nodes, the places of no motion. Hence l_1 is a half-wave length of frequency n, which is the tone being emitted by the metallic rod. If the velocity of sound in air is v_1, then

$$v_1 = n\lambda_1 = n(2\,l_1),$$

and if we call the velocity of sound in the metallic rod v_2, then

$$v_2 = n\lambda_2 = n(2\,l_2),$$

since the two ends of the rod are loops if the rod is secured at the center. Therefore

$$\frac{v_2}{v_1} = \frac{l_2}{l_1}$$

or

$$v_2 = v_1\frac{l_2}{l_1}.$$

The value of v_1 (velocity of sound in air) for the temperature of the room is found from the theory as given earlier in the chapter. Repeat for two other positions of the rod a.

QUESTIONS

(a). Calculate the probable experimental error. What is the maximum deviation from the mean result in per cent?

(b). Look up in tables the values of the bulk modulus of elasticity and density of the metallic rod b used in your experiment, and calculate the velocity of longitudinal waves in the rod by the equation, $v = \sqrt{\frac{E}{d}}$.

(c). What differences might you expect in the frequency pattern of the metallic rod if not secured at the center?

PROBLEMS

Experiment 42

1. Show by graphical construction the production of standing waves Choose a sine wave and take the time interval between the waves as $\frac{T}{8}$ sec.

2. If the distance between the loops of a vibrating cord, as shown in Figure 123, is shortened by one-half, what must be the change in tension?

3. A cord of length 40 cm. and of mass 0.025 gm. is loaded as in Figure 123, so that it vibrates in one segment when actuated by a tuning fork of 60 vibrations per second. What is the tension (in dynes) in the string?

Experiment 43

4. Two wires, held at the same tension, have a mass per unit length ratio of $\frac{4}{5}$. What will be the ratio of lengths to give the same frequency?

5. Assume you had two wires, each supposedly of the same mass per unit cross-section but which were not homogeneous in reality so that σ varies slightly from point to point in the following fashion: one wire a is a little thinner at the center of the string than the average, while the other wire b is a little thicker at the center than the average. These wires are tuned to unison and then the movable bridge (Fig. 124) is adjusted so that the wires are half the original length. The frequencies will be very approximately twice the original values but beats will be heard. Explain briefly why. Which wire will have the higher frequency?

Experiment 44

6. Referring to Figure 122 (c), which overtone (*i.e.*, 1st, 2nd, etc.) is represented by the drawing of the nodes and loops as shown? What is the frequency relation between this overtone and the fundamental for this tube?

7. A closed organ pipe of 150 cm. length is tuned correctly when the temperature is 20° C. What will be the change in frequency if the temperature rises to 30° C.? What would be the change in frequency for an organ pipe of 5 cm. length under the above conditions?

Experiment 45

8. Suppose that the distance l_1 between two adjacent nodes in the Kundt's tube apparatus, for a given metal rod, was 5 cm. If the metal rod clamped at its center was brass, determine its length. [*Note.* Look up in a handbook the velocity of a longitudinal wave in brass.] Assume that air was the gas medium in the tube.

9. Suppose that the tube above had been filled with hydrogen, what would have been the distance between adjacent nodes? [*Note.* Look up the velocity of sound through hydrogen.]

CHAPTER XXI

PHOTOMETRY

Photometry is the science which deals with the measurement of the intensity of light sources and illumination produced on absorbing and reflecting surfaces.

These measurements are usually relative and are made by comparison with some standard source such as the German standard Hefner lamp, which is a lamp burning amyl acetate at a certain definite rate, with a definite height of flame and definite conditions of air pressure, humidity, etc. This Hefner standard of light intensity is found to be 90 per cent of the international candle. The United States Bureau of Standards has standardized certain incandescent lamps for routine testing. The maintenance of standards is no easy task.

The instruments used to compare the intensities of light sources are called photometers. The Bunsen and Lummer-Brodhun photometers are representative of the simpler type, while the flicker and integrating photometers are examples of the more specialized forms.

The theory of our experiment is based on the fact that the light is assumed to be radiating from a " point " source out upon an expanding spherical surface. Hence, the luminous flux falling on one square foot of a surface gets less when the surface is further removed from the source — in other words, the illumination of the surface gets less. Now it is a *general law, in connection with radiation of energy from a point source, that the energy passing normally through 1 sq. cm. falls off inversely as the square of the distance from the source and hence the illumination produced on a screen by a point source of light falls off inversely as the square of the distance between source and screen.*

If we should place two point sources of light consecutively in front of a screen and each produced the same illumination, as observed by the eye, then we would conclude that the two sources have the same brightness or intensity. On the other hand, if one were four times as intense, then according to the above it would need to be placed at twice the distance from the screen.

The student should note that when we are dealing with the point source of light, we are interested in its intensity, which is measured in *candle power* by reference to the standard international candle. If we are dealing with the illumination of a surface on which this light falls, then we measure the illumination in *foot-candles*. This unit of illumination is, by definition, the illumination produced on a surface at a distance of one foot from the standard international candle. If the intensity of the source remains the same, then it is the illumination that falls off as the square of the distance.

In Figure 127, suppose we wish to compare the intensities of the two sources I_1 and I_2 (units of candle power). We place the two sources so as to illuminate the screens at B_1 and B_2. The illumination produced can be observed visually. Now if I_1 is the brighter source, then it will have to be placed further from the screen than I_2, the diminution of illumination of the screen B_1 falling off according to the inverse square law. Suppose that when I_1 and I_2 are in the positions shown, and the illumination of B_1 and B_2 are the same, we can then write

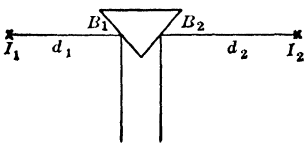


FIG. 127. — Comparison of intensities of sources.

$$\frac{I_1}{I_2} = \frac{d_1^2}{d_2^2}.$$

Knowing the intensity of I_1 (in c. p.) and measuring d_1 and d_2, we can find the intensity of I_2. This forms the basis of the photometry of sources of light.

EXPERIMENT 46

THE PHOTOMETER

Variations of luminous efficiency of a source of light with voltage.

Apparatus: Photometer, standard incandescent lamp, two incandescent lamps of unknown candle power, rheostat, ammeter, voltmeter.

The apparatus (Fig. 128) is a modified form of a Bunsen photometer. You will find the apparatus connected as shown in

Figure 128. Check the circuit. The device c, used for comparing the sources, consists of two thin rectangular pieces of paraffin separated by a strip of tinfoil (or aluminum foil) and mounted on a bench between two lamps. In our experiment, the lamps are kept at a fixed distance from each other, usually 100 cm. With the switch open, insert the plug in the 110-volt circuit. The standard lamp of candle power I_1 will light. Now adjust the rheostat, acting as a potentiometer device, so that little or no voltage will be across the lamp whose candle power (I_2) is to be determined. After the instructor has checked the wiring, close the switch to see if the polarity through the voltmeter and ammeter is correct. If not, withdraw the plug and reverse. Then adjust the rheostat for maximum voltage through the lamp.

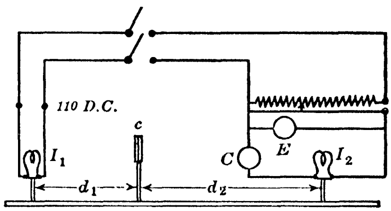


FIG. 128. — Photometer.

For our purpose we shall take the candle power per watt as a measure of the relative light efficiency of lamps because greater efficiency is associated with greater candle power per unit power consumption. Hence, with the rheostat set for maximum voltage through the lamp, the procedure is to adjust the comparing device c so that it is illuminated equally on both sides of the aluminum foil when viewed from the side (*i.e.*, perpendicular to the plane of the aluminum foil). When this is done, the illumination on both sides of c from the two sources is the same. Record the distances d_1 and d_2.

Then assuming the inverse square law and point sources, we have

$$\frac{I_1}{I_2} = \frac{d_1^2}{d_2^2}.$$

Reduce the voltage, approximately, in steps of 10 volts, keeping the distance between the lamps constant, and repeat the above measurement in every case until the low luminosity or difference in color of the lights make further measurements useless. The uncertainty of matching light intensities due to color differences may be minimized by averaging the extreme distances in either direction for which you are sure that no intensity match exists.

Repeat, using the other lamp.

Data for one lamp:

E (Volts)	C (Amps.)	EC (Watts)	d_1	I (Calc.)	CANDLE POWER PER WATT

$I_1 =$

$d_1 + d_2 =$

QUESTIONS

(a). Draw a graph with the candle power per watt as ordinates against E as abscissae.

(b). Calculate from your data or graph (Question a) the per cent decrease of the efficiency from its value at the maximum voltage, for voltages of 10, 20, and 30 less than the maximum value.

(c). At what voltage would you operate the lamp for greatest economy?

PROBLEMS

1. One electric lamp of 26 candle power is separated from another of 50 candle power by 200 cm. At what distance, or distances, on a line joining the two lamps will the illumination from the two lamps be the same? Calculate the distance from the lamp of smaller candle power. Calculate also the illumination in foot-candles at this point of balance.

2. Two sources of light, each of 5 candle-power intensity, are placed at distances of 2 feet and 4 feet respectively on the same side of the paraffin block. At what distance must a source of intensity 10 candle power be placed on the other side so as to produce equal illumination on both sides of the paraffin block?

3. What is meant by "candle power" and what is meant by "foot-candle"? What does each measure? If the intensity of illumination produced on a screen 30 meters from an arc lamp is equivalent to that produced by a standard Hefner lamp at 1 meter, what is the candle power of the arc lamp?

CHAPTER XXII

REFLECTION AND REFRACTION AT PLANE SURFACES

When a beam of light strikes a surface of different density, we notice that some of the light is reflected and the remainder transmitted through the separating boundary. The relative amounts depend upon the optical conditions at the surface. In this chapter, we are interested only in the paths taken by a reflected or a transmitted beam of light in an isotropic medium.

Reflection of Light at a Plane Surface

Consider the reflection of light, coming from a source S, and striking a plane mirror as shown in Figure 129. Let SO and OR represent the paths of the incident and reflected ray respectively, and let ON be the normal to the plane at the point of incidence. The following two laws may be proved experimentally:

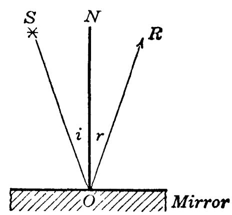


Fig. 129. — Angles of incidence and reflection.

1. *When a ray of light strikes the surface of the mirror at any point* O, *the angle of incidence* (i) *is equal to the angle of reflection* (r).
2. *The plane determined by the normal and the reflected ray coincides with the plane determined by the normal and the incident ray.*

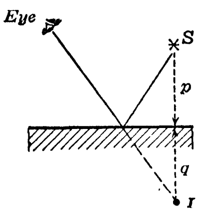


Fig. 130. — Image of a point source in a plane mirror.

Applying these two laws to all the rays coming from an object and being reflected at the surface leads to the important result that the image I of an object S will appear as far behind the mirror as the object is in front of the mirror. That is, referring to Figure 130:

$$q = p.$$

Very frequently it becomes necessary to locate the position of an image. The method of parallax is useful in such cases. Place a pin, which may be seen over the top of the mirror, at the point I where you think the image is located. Now move your eye back and forth in a direction perpendicular to the plane defined by the pin and a line connecting the image and source. This is called locating an image by parallax. *Parallax* is defined as the apparent angular separation of two objects due to a real displacement of the observer. Thus in Figure 131 (a) there is parallax when the eye is moved from the point A to the point B. In Figure 131 (b) there is negligible parallax, which vanishes entirely when the pin and image coincide.

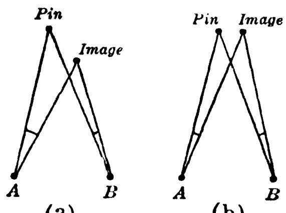


Fig. 131. — Parallax.

It will be seen in Fig. 131 (a) that the angular separation of the images at point A is greater than at point B. There is no such difference in the angles at points A and B in Fig. 131 (b), and hence in this case the parallax is not noticeable.

Refraction of Light at a Plane Surface

The laws of refraction of light from flat surfaces may be stated briefly as follows:

1. *The ratio of the velocity of light* (v_1) *in the first medium to the velocity of light* (v_2) *in the second medium is a constant* (see Fig. 132).

2. *The plane of the refracted angle coincides with the plane of the incident angle.*

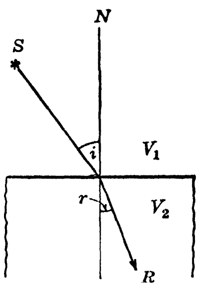


Fig. 132. — Refraction at a plane surface.

It may be shown that the angles i and r are related to v_1 and v_2 by the equation,

$$\frac{v_1}{v_2} = \frac{\sin i}{\sin r} = \text{constant} = \mu_{1,2},$$

where $\mu_{1,2}$ is the relative index of refraction for these two substances. The subscripts 1, 2, indicate that the light is considered as proceeding from medium of velocity v_1 to medium of velocity v_2. If the velocity of light in a vacuum is c, then the absolute index of refraction of light entering the first medium is $\mu = c/v_1$, while that of the light entering the second is $\mu = c/v_2$. In an experiment we usually determine the index of refraction by measuring any angle of incidence i and the corresponding angle of refraction r, and calculate the ratio $\frac{\sin i}{\sin r}$, which by definition is μ.

EXPERIMENT 47

THE BEHAVIOR OF LIGHT AT A PLANE SURFACE

A study of the laws of reflection and refraction of light at plane surfaces.

Part (a). *To show that the angle of incidence* (i) *is equal to the angle of reflection* (r), *and that the image and object are at equal distances from the mirror.*

Part (b). *To show that for light passing from one medium to another,*

$$\frac{\sin i}{\sin r} = \text{const.} = \mu.$$

Part (c). *To obtain the index of refraction of a prism by measurement of the prism angle and the angle of minimum deviation.*

Apparatus: Mounted mirror, four pins about 2 to 4 inches in length, one rectangular piece of plate glass with two opposite sides polished, glass prism, ruler, protractor, drawing board and thumb tacks.

Part (a). Tack a sheet of paper to the drawing board. In the approximate center of the sheet draw one thin line XX' and locate the reflecting side of the mirror exactly along this edge. Place two pins S_1 and S_2 (Fig. 133) in front of the mirror. Insert two more pins R_1 and R_2 in line with the two images of S_1 and S_2 as seen in the mirror. Remove the mirror and at the point (O) of intersection of S_1S_2 with R_1R_2 erect a normal to the surface (whose trace is XX'). Measure i and r with a protractor. Repeat this experiment two or more times, using different values of angle i each time. Find the per cent of error from the mean of i and r for each experiment.

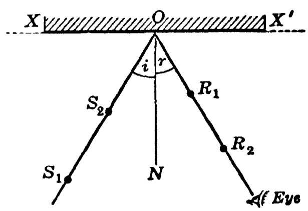


FIG. 133.—Reflection from a plane mirror.

Find the distance q of the image from the mirror by using the parallax method described above and repeat two or more times, varying the distance p of the object from the mirror. Find the per cent of error from the mean of p and q for each experiment.

Angles		Per Cent Error from Mean
i	r	

Distances		Per Cent Error from Mean
p	q	

Part (b). Place the glass plate (Fig. 134) on your data sheet and outline the edges $(XX'Y'Y)$ with a pencil. Place a pin at each of the points marked P and O. Now with the eye brought close to the paper, sight through the glass so as to bring the pins O and P in line with each other. Place another pin at some point D along the path. Remove the glass and connect the points PO and OD with straight lines. The lines may be extended to any distance out from O. Choose some point along OD, say E, and determine the distance OE. Then make OE' on OP equal to OE. From E and E' erect perpendiculars to the normal NN' drawn through O. By definition of the index of refraction of light from glass to air we have Figure 134.

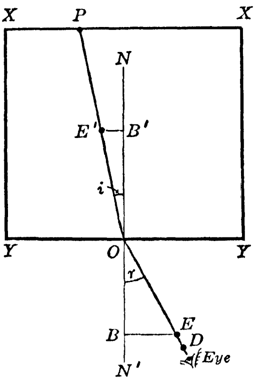


Fig. 134. — Refraction through rectangular plate of glass.

$$\mu_{ga} = \frac{\sin i}{\sin r} = \frac{E'B'}{EB}.$$

Measure the angles i and r, as well as the distances $E'B'$ and EB. Calculate μ_{ga} by means of the two ratios $\frac{\sin i}{\sin r}$ and $\frac{E'B'}{EB}\cdot$ Repeat your experiment two or more times, using different values of the incident angle i.

Angles		Distance		$\frac{\sin i}{\sin r}$	$\frac{E'B'}{EB}$
i	r	$E'B'$	EB		

Average Value of	
$\frac{\sin i}{\sin r}$	$\frac{E'B'}{EB}$

Average value of

$$\frac{1}{\frac{E'B'}{EB}} = \mu_{ag} =$$

Average value of

$$\frac{1}{\frac{\sin i}{\sin r}} = \mu_{ag} =$$

Part (c). Place two pins A and B in your data sheet as in Figure 135. With your eye close to the sheet, look through the prism towards the pins AB. When you get these pins in line, rotate the prism back and forth, around an axis at the apex of the angle A perpendicular to the paper until the angle D becomes a minimum. You can tell when D is a minimum by noting the extreme excursion (towards the line OF) of the image of AB through the prism. The rotation desired may be conveniently accomplished by causing a slight pressure at the apex of the angle A. When the minimum angle (D) is found, place pins at each of two points, such as C and E. Draw the line AB, extending it to F; also, the line CE, extending it to O'. The angle D, shown in the figure by $EO'F$, is also the angle of minimum deviation. Measure this angle with a protractor. The index of refraction from air to glass may be shown to be

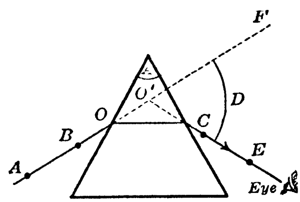


FIG. 135.—Refraction through a glass prism.

$$\mu_{ag} = \frac{\sin\left(\frac{A + D}{2}\right)}{\sin \frac{A}{2}}.$$

Repeat two or more times. Calculate the average index of refraction, also the per cent of error.

QUESTIONS

(a). In Part (b) of your experiment, what was the least count of (1) your protractor, (2) your measuring rule?

(b). Which of the two ratios $\left(i.e., \frac{\sin i}{\sin r} \text{ and } \frac{E'B'}{EB}\right)$ did you find the more accurate for determining the value of μ_{ga} in Part (b)? State the reasons for your answer.

(c). Do the errors in Part (a) and Part (b) of your experiment appear to be influenced in any way by the magnitude of the angle i?

PROBLEMS

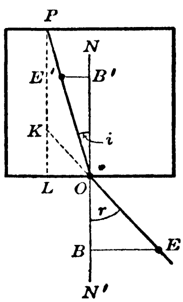


FIG. 136.

1. Prove that the index of refraction (μ_{ga}) from glass to air is given by the relation

$$\mu_{ga} = \frac{E'B'}{EB}.$$

(See Fig. 134.)

2. If a perpendicular PL (Fig. 136) is dropped from P and the line EO extended to K, prove that

$$\mu_{ga} = \frac{OK}{OP}.$$

3. Prove, if D is the angle of minimum deviation for a glass prism and A is the angle of the prism, that the index of refraction from air to glass is given by the equation,

$$\mu_{ga} = \frac{\sin \dfrac{A + D}{2}}{\sin \dfrac{A}{2}}.$$

4. Show that if A is very small in the above problem, say 5°, we may write the deviation D as

$$D = A(\mu_{ag} - 1).$$

CHAPTER XXIII

REFLECTION AND REFRACTION AT CURVED SURFACES

The laws of reflection and refraction of a narrow beam or ray of light as given in the last chapter for plane surfaces are applicable to curved surfaces if we apply these laws to each individual point of the curved surface where the light impinges. Consider a tangent plane to be drawn at every point where the light strikes. Then the laws of reflection and refraction apply to this tangent plane. Since every such plane will be inclined at an angle to all others, the reflected or refracted rays from an object will be spread out or brought together in some manner depending upon the curvature of the surface.

In dealing with light, reflecting at or passing through curved surfaces, it is convenient often to speak of the *wave front* which is perpendicular to the direction of the ray of light. The wave front of any beam of light coming from a very distant source is, for all practical purposes, plane. Another way of stating this same fact is by saying that the radius of curvature of the wave front is infinite.

MIRRORS

Consider a plane wave front proceeding towards a curved mirror (Fig. 137). The latter can be thought of as being a small portion of a sphere with its center at C and of radius r. The center of the mirror-surface is called the pole (Fig. 137), and the line drawn from the pole perpendicular to the spherical surface at that point is called the principal axis. The beam

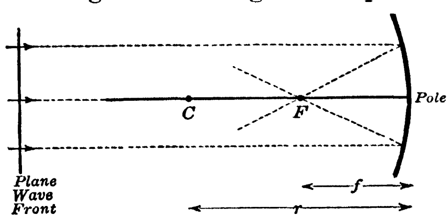


FIG. 137.—Concave mirror with source at infinite distance.

will strike the mirror and be reflected such that, when the laws of reflection are applied to each point on the mirror, the new wave front is found to be spherical and converging towards one point F on the principal axis called the principal focus. The distance f from the pole to the principal focus is called the focal distance.

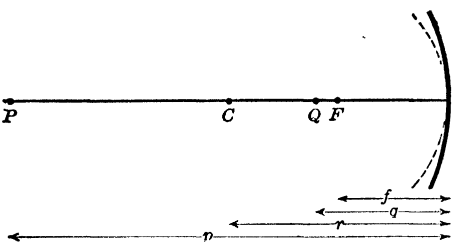


FIG. 138. — Concave mirror with source at finite distance.

If the source P, the light of which is being reflected, is at a finite distance p from the mirror (Fig. 138), the reflected wave front (the dotted line) will converge or focus at some point Q on the principal axis at a distance q from the pole. The relation between p, q, and r is known to be

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{r}. \tag{1}$$

By definition, the focal length of a mirror is the distance from the pole to the point at which light will focus when coming from an infinite distance ($p = \infty$). Hence in the above equation, when $p = \infty$, $q = f$, or $\frac{2}{r} = \frac{1}{f}$. That is, we may write the above equation for mirrors as,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}, \tag{2}$$

if the focal length is given instead of the radius of curvature. The proof of this equation can be found in almost any textbook on college physics.

While the above equation (2) was written for concave mirrors (*i.e.*, center of curvature towards source), it holds for all types of spherical mirrors. To make it applicable for all cases, note first that all distances (p, q, r) are positive for a real image formed by the concave mirror and that the object, image, and center of curvature are on the same side of the pole. When the image and object are on opposite sides of the pole, the distance of the image from the pole is negative. Likewise the radius of curvature (or focal length) is negative if the center of curvature and object are on opposite sides of the pole.

While the use of the wave-front method is convenient for developing mirror and lens formulae, it is not a convenient method for obtaining the more exact locations and relative sizes of images. The location and sizes of images is usually carried out by the ray method. Figure 139 represents the geometrical construction, using rays, for finding the image. This image we found in Figure 138 by the wave method. In this case, however, the relative size O of the object to the image I can be observed by referring to Figure 139.

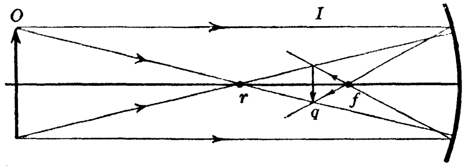


Fig. 139. — Construction of images in concave mirror.

A convenient set of conventions as to the location of images is given as follows: (1) Draw from any convenient point in the object a ray parallel to the principal axis. This ray, by definition, will reflect and pass on through the principal focus; (2) draw a ray starting from the same point in the object, through the center of curvature. This ray will be reflected back along its same path. The image of the given point in the object will be found where these lines meet. Usually the two end points of the object are sufficient for the location of the whole image. In fact, if the image is considered perpendicular to the principal axis and half of the object is above the principal axis and the other half below, then the image is readily located by drawing the two lines mentioned above from one point only.

It will be noticed that the image in Figure 139 is inverted. This always happens when the two rays of light actually meet after reflection from the mirror. We call such an image, which is formed by the actual crossing of the rays, a real image. Images formed by rays which appear to cross, but actually do not, are called virtual images. They are always erect.

It may be shown that the ratio of the magnitude of the image to the object is given by the relation.

$$\frac{I}{O} = \frac{q}{p} = M,$$

where M is generally called the magnification.

The following general diagrams on the next page may be found helpful in the construction of images formed by mirrors in specific cases:

CONCAVE MIRRORS

Example 1. If $p > r$,

then $$r > q > f,$$

and $$\frac{I}{O} < 1. \qquad \text{(Fig. 140)}$$

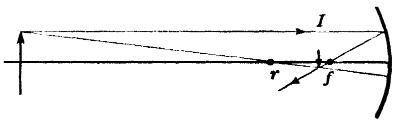


Fig. 140.

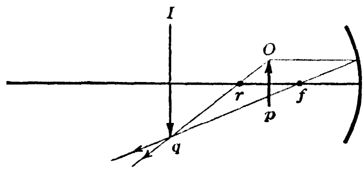


Fig. 141.

Example 2. If $r > p > f$,

then $$q > r,$$

$$\frac{I}{O} > 1. \qquad \text{(Fig. 141)}$$

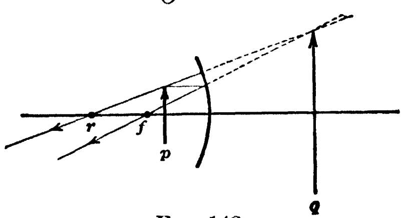


Fig. 142.

Example 3. If $$p < f,$$
then q is on the opposite side of the mirror and is negative numerically. (Fig. 142)

CONVEX MIRRORS

Example 4. If $$\infty > p > 0,$$

then $$q < f.$$

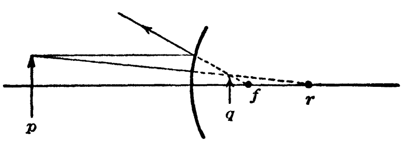


Fig. 143.

For these mirrors, the numerical magnitudes of r, f, and q are negative. (Fig. 143)

Lenses

Now, consider a plane wave advancing towards a double convex lens (Fig. 144) as shown in the diagram. Upon passing through the lens, it will come to a focus at a point on the principal axis called the principal focus, which is at a distance f from the center of the lens. The direction of the ray of the advancing wave is parallel to the principal axis, which is defined as a line, joining the two centers of curvature (C_1 and C_2) of the spherical surfaces of the lenses.

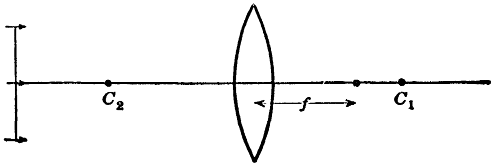


FIG. 144. — Double convex lens with source at an infinite distance.

If we consider a wave front (see Fig. 145) starting from a source P at a distance p from the center of the lens, it will, upon passing through the lens, refract (dotted curve) so as to converge to a point Q which is a distance of q units from the center of the lens. The relation between p, q, and f is known to be

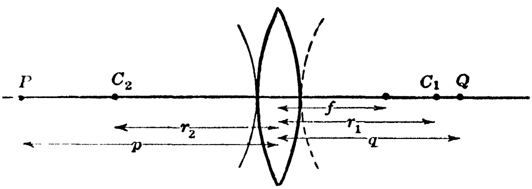


FIG. 145. — Double convex lens with source at finite distance.

(3)
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f},$$
where

(4)
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right).$$

This formula holds not only for converging lenses, which are thick at the center and thin at the edges, but also for diverging lenses, which are thin at the center and thick at the edges. A common form of a diverging lens is the double concave lens shown in Figure 146. As in the case of mirrors, if any of the distances (*i.e.*, q, f, r_1, r_2) are measured in a direction compared to the source, opposite to that shown in Figure 145, the sign becomes negative. Thus the focal length of a convex lens is positive, while that of a double concave lens is negative.


FIG. 146. — Double concave lens.

While the formula for a lens is readily obtained from the curvature of the wave fronts, the geometrical construction of images is, again, more readily obtained by use of the ray, or geometrical method. We will redraw Figure 145, using the ray method. Our new figure (Fig. 147) shows the object, of size O, at a distance p from the center of the lens. The method of locating the image of size I is as follows: (1) Draw from some point in the object a ray parallel to the principal axis. It will pass through the focal point; (2) draw another straight line, starting from the same point O, through the optical center of the lens and continue this line until it crosses the other ray which was refracted. If the object is perpendicular to the principal axis and symmetrical with it, the location and size of the image can be drawn without further image construction.

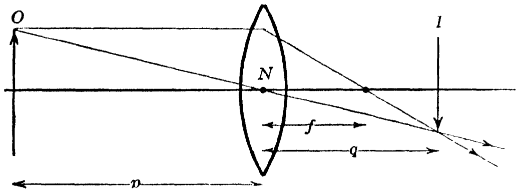


FIG. 147.—Construction of images in a convex lens.

The optical center N is the point through which the light passes from one side of the lens to the other without change of direction. There is a lateral displacement but this is negligible for thin lenses.

Just as in the case of mirrors, real images are inverted and the refracted rays actually intersect. Virtual images are erect and the refracted rays only appear to intersect.

It may be shown that the ratio of the magnitude of the image to the object is

$$M = \frac{I}{O} = \frac{q}{p}.$$

The following general examples are given for reference:

CONVERGING LENSES

Example 1. If $\infty > p > 2f,$

then $f < q < 2f,$

and $\frac{I}{O} < 1.$ (Fig. 148)

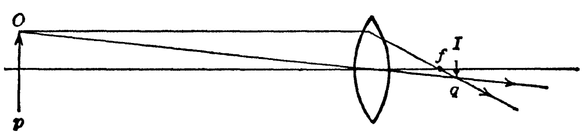


FIG. 148.

Example 2. If $2f > p > f,$

then $2f < q < \infty,$

and $\frac{I}{O} > 1.$ (Fig. 149)

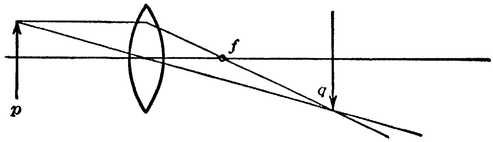


FIG. 149.

Example 3. If $f > p > O,$

then $\infty > q > O,$

and q is negative and the image virtual. (Fig. 150)

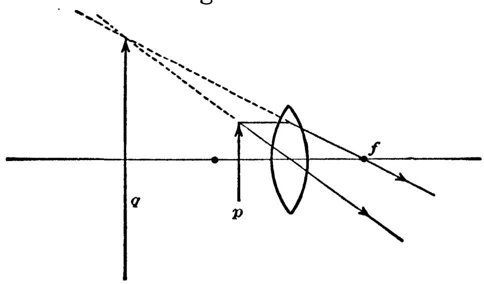


FIG. 150.

DIVERGING LENSES

Example 4. If $\infty > p > O,$

then $f > q > O,$ (Fig. 151)

where both f and q have negative values and the image is virtual.

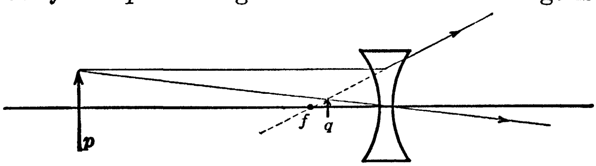


FIG. 151.

EXPERIMENT 48

THE BEHAVIOR OF LIGHT AT A CURVED SURFACE

A study of the reflection and refraction at curved surfaces.

Part (a). *To determine the radius of curvature of a concave spherical mirror by means of parallax.*

Part (b). *To locate, in general, any image by the method of parallax.*
Part (c). *To determine the focal length of a double convex lens.*

Apparatus. Mounted concave mirror, two mounted pins about four inches in length, ruler, optical bench, convex lens, illuminated object, screen.

Part (a). From the equation for mirrors, it is seen that, for a concave mirror, when the object is at the center of curvature, the image will be formed at the same place. Hence, place a pin in a movable block and slide it towards the concave mirror until a position is reached such that there is no observable parallax between the pin and the inverted image of the pin as observed in the mirror. The distance from the pin to the center of the mirror gives the radius of curvature of the mirror. Repeat the adjustment two or more times. Record all data.

Part (b). Place a pin in front of your concave mirror so that its distance p in front of the mirror is greater than its radius of curvature r (Fig. 152). Locate the image of this pin by means of parallax. That is, place another pin where you think the image of the object O should be. Move your head back and forth. The image is located where this latter pin and image do not displace relative to each other upon to-and-fro motion of the head. Measure the distance p of the object, and the distance q of the image from the mirror and calculate r from the equation

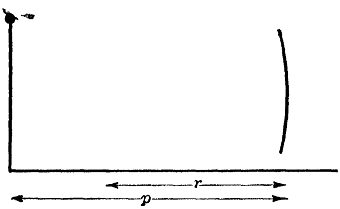


FIG. 152. — Concave mirror.

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{r}.$$

Repeat for two or more positions of the object O, and find the average value of r. What is the value of f?

Part (c). For this experiment, we have a source of light S, an illuminated object O, say a copper grid, and a screen on which the image I can be formed. These are all mounted on an optical bench (Fig. 153). When the copper grid is illuminated, an image will form on the screen with the lens in some position such as (1) in the figure, provided the distance d between O and I is such that $d > 4f$. This image will be larger than the object. Now move the lens to some position such as (2), when another image.

but this time smaller than the object, forms. This second image formation is to be expected, because of the symmetrical relation between p and q in the lens formula. We also see that position (2) of the lens is the same as would be obtained if the illuminated object and screen were interchanged. Because of this symmetry, our measurements consist of finding the distance d and the distance a between the two positions of the lens. Then, for purposes of calculation, if we consider the lens in position (1), we have that

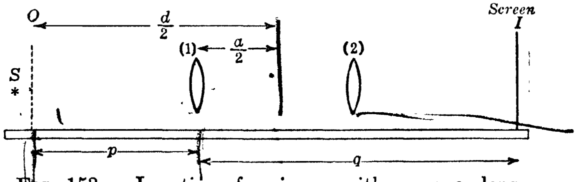


FIG. 153. — Location of an image with a convex lens.

$$p = \tfrac{1}{2}(d - a),$$

and

$$q = \tfrac{1}{2}(d + a),$$

and thus obtain for the focal length f,

$$f = \frac{d^2 - a^2}{4\,d}.$$

Repeat two or more times with different values for d.

The above gives a very accurate method of finding the focal length. If a distant illuminated object is available, compare the focal lengths obtained directly with the above experimental method.

If the distance d is very much greater than $4\,f$, then the image formed in the one case will be too large to focus sharply, and in the other case too small to be seen.

QUESTIONS

(a). Did you notice whether there was a tendency for the image of the pin to bend at the edges of the concave mirror? To what is this due if such an effect exists?

(b). What is the per cent of error from the mean for your average values of r, the radius of curvature, as obtained each in Part (a) and in Part (b) of your experiment?

(c). Represent by a diagram, drawn to scale, the distance of the image and object from the mirror and the relative sizes of the image and object for any one experiment of Part (b). Indicate plainly which set of data you used.

(d). What happens in Part (c), (1) if $d = 4\,f$, (2) if $d < 4\,f$?

PROBLEMS

1. Show by use of the formula for spherical mirrors that one would expect the image to appear at a distance behind a plane mirror equal to the distance of the object in front of the mirror.

2. When an object is placed 10 cm. in front of a concave mirror, the image of that object appears, by the method of parallax, to be 15 cm. behind the mirror. Is the image real or virtual, erect or inverted? What is the radius of curvature of the concave mirror? Find also the position of the image graphically.

3. Prove for a double convex lens, the equation

$$\frac{1}{p} + \frac{1}{q} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right),$$

by the wave-front method (*i.e.*, the sagitta method).

4. Show that for Part (b) of Experiment 48,

$$f = \frac{d^2 - a^2}{4\,d}.$$

5. In Example 3 on page 231, let $p = 10$ cm. and $f = 15$ cm. How far is the image from the lens? Draw a diagram showing the image, object, and lens position and construct the rays showing how the image is formed.

6. Suppose that the radii of curvature of the two spherical surfaces of a given double convex lens were 12 cm. each. What would be the index of refraction of the lens if its focal length was 20 cm.?

CHAPTER XXIV

THE TELESCOPE AND MICROSCOPE

In order to see distant objects in more detail, we either bring the object nearer to the eye or go nearer to the object. That is, we increase the angle subtended at the eye by the object. Usually the eye, when normally adjusted and at ease, just brings to a focus objects at infinity on the retina or receiving surface of the eye ball. When we wish to see objects that are closer to the eye, the self-focusing lens in the eye has to be made more convex so as to bring the image again on the retina. This is done automatically by certain muscles attached to this lens. The *limit of accommodation* (*i.e.*, distinct vision) is usually about 25 cm. and we shall use this figure in our calculations. Consequently, if we wish to see objects in still greater detail, a convex lens may be placed in front of the eye in order to make the effective focal length of the combination much less. This serves to decrease the distance of the object for distinct vision, so that it may be brought much nearer and thus subtend a much greater angle.

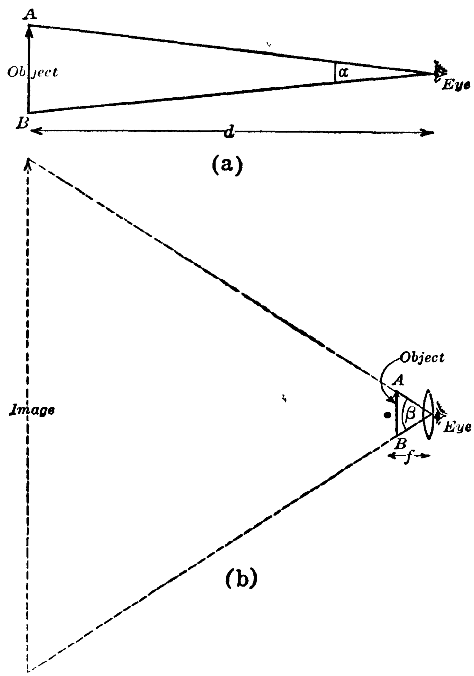


FIG. 154. — Magnification by a convex lens.

The effect of a single convex lens is illustrated in Figure 154, a and b. In Figure 154 (a), an object of length AB is seen at the limiting distance (d) of distinct vision (*i.e.*, 25 cm.). The angle subtended at the eye by AB is α. Now place a lens of focal length, f, between the object and eye as shown in Figure 154 (b). Adjust the object so that the image will appear at the limit of distinct vision. If the lens is close to the eye so that the angular opening at the eye is essentially the same as that at the lens, then the new angle subtended by AB is β. *Hence we may define visual magnification* M *as the ratio of the angular opening at the eye when aided to the angular opening at the eye when unaided, i.e.,*

$$M = \frac{\beta}{\alpha}. \tag{1}$$

It is often convenient to work with linear magnification, especially when the instrument to be considered has a number of lenses, each producing a given magnification. Thus, if I is the length of the image produced by the object whose height is O, then the magnification M, as defined before, becomes in this case

$$M = \frac{I}{O} = \frac{q}{p}, \tag{2}$$

where q and p represent the image distance and object distance respectively from the lens.

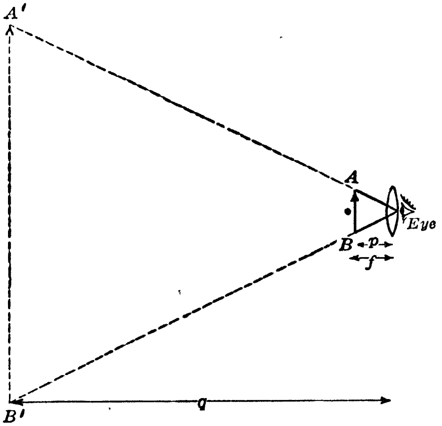


FIG. 155. — Magnification by a convex lens.

Thus, in Figure 155, the magnification produced by a single double convex lens is

$$M = \frac{A'B'}{AB} = \frac{q}{p},$$

or

$$M = \frac{d}{f} + 1,$$

where $q = d$, the limit of distinct vision.

The astronomical telescope (Fig. 156) consists, essentially, of an objective lens (L_1) of focal length F and an eyepiece (L_2) of focal length f. Since the distance of the object AB is usually great compared to the length of the telescope, the angle α is essentially the same as that subtended by AB with the unaided eye. Hence, the magnification of the image $A''B''$ over the object AB is given by

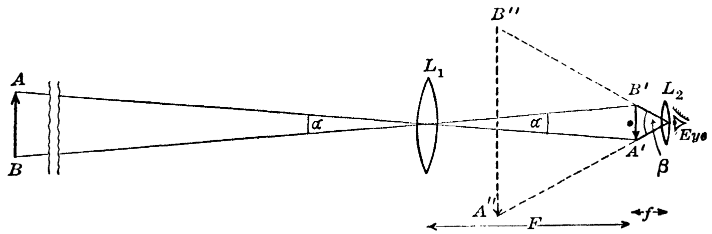


FIG. 156. — Astronomical telescope.

$$M = \frac{\beta}{\alpha} = \frac{\frac{A'B'}{f}}{\frac{A'B'}{F}}.$$

approximately, or

$$M = \frac{F}{f}.$$

That is, the magnification depends upon the relative magnitudes of the focal lengths of the objective lens and the eyepiece. The eyepiece consists very often of a combination of lenses. This does not change the magnification, as given above, since f now represents the effective focal length of the combination.

The compound microscope (Fig. 157) consists essentially of an objective lens (L_1) of very short focal length (f_1) and an eyepiece (L_2), also of short focal length (f_2). While in practice these lenses are made up of combinations to eliminate the various defects of a single lens, we shall treat them as simple lenses for purposes of obtaining an approximate magnifying power.

In order to calculate the approximate magnification of the microscope, we will first find the magnification of each lens by itself and then multiply the two separate magnifications to obtain the resulting magnification of the instrument as a whole. The object (Fig. 157) of length AB is placed just beyond the focal length (f_1) of the objective. A real image of length $A'B'$ will be formed at a distance q_1 from the objective. Hence the linear magnification M_1, due to the objective, is given by the expression,

$$M_1 = \frac{A'B'}{AB} = \frac{q_1}{p_1} = \frac{q_1}{f_1} - 1$$

or, approximately,

$$M_1 = \frac{q_1}{f_1}.$$

The eyepiece is focused so that a virtual image ($A''B''$) of the real image ($A'B'$) is seen at the distance (d) of distinct vision. Hence

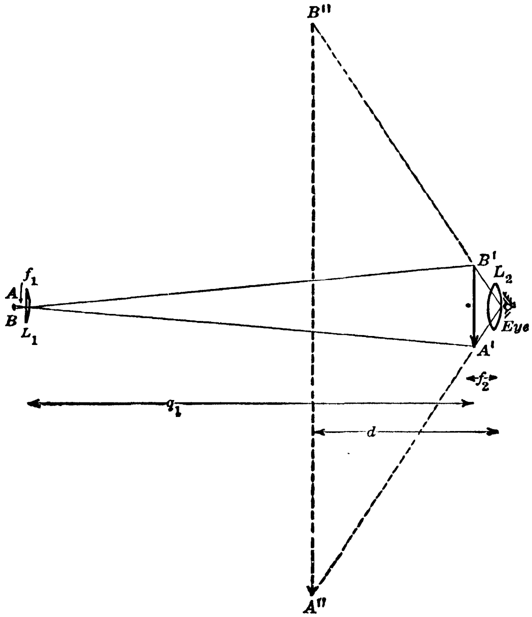


FIG. 157.—Compound microscope.

the approximate magnification (M_2) due to the eyepiece is seen by equation (3) to be

$$M_2 = \frac{A''B''}{A'B'} = \frac{d}{f_2},$$

and the magnification M of the microscope is

$$M = M_1 M_2 = \frac{q_1 d}{f_1 f_2}.$$

Since the focal lengths f_1 and f_2 are very short, the distance q_1 is essentially L, the distance between the lenses. Hence we may write the magnification of the microscope as

$$M = \frac{Ld}{f_1 f_2}.$$

EXPERIMENT 49

THE TELESCOPE

Part (a). *Magnification produced by a telescope.*
Part (b). *Magnification produced by a microscope.*

Apparatus: A galvanometer telescope, two convex lenses of very short focal length mounted for a microscope, meter stick.

Part (a). The galvanometer telescope contains an eyepiece and cross-hair arranged to slide in another tube containing the objective lens. Separate the two tubes and obtain the focal length of the eyepiece and objective by forming the image of a distant object or electric light source on a piece of white paper or cardboard. The distance from the lens to the image is the focal length. Repeat this measurement three or four times, recording all readings. The object used in finding the focal length is assumed to be far enough away so that all rays coming from it to the lens are parallel.

To measure the magnifying power of the telescope, mark off on the blackboard, or on some cardboard placed several feet away, a convenient scale. View this scale through the telescope with one eye, and directly with the other. Note the number of divisions seen by direct vision at the board between any two divisions as seen through the telescope with the other eye. The number of divisions found with the eye unaided between any two divisions as found by the other eye looking through the telescope is the magnification (*i.e.*, ratio of apparent to real size). Make several measurements estimating parts of divisions, if necessary. Always record all measurements on your data sheet. Then calculate the mean value and error. Compare your result with the result as found from the equation deduced above; *i.e.*,

$$M = \frac{F}{f}.$$

Part (b). We shall construct the optical system of a simple microscope by means of lenses which can easily be set up on an optical bench. Find the focal length of each lens as in Part (a). Again repeat the measurements several times for each lens.

The magnification is found in a manner similar to that in Part (a), with the exception that in this case the lines are ruled close together, and placed at a distance from the objective a little greater than its focal length. Make several trials and check your result with the theoretical formula,

$$M = \frac{Ld}{f_1 f_2},$$

where $d = 25$ cm.

QUESTIONS

(a). Did you notice whether the image through the telescope was erect or inverted?

(b). How would you have proceeded to make a telescope the image of which is inverted compared to the one you observed?

(c). Did you notice any imperfections in your instrument? If so, describe them in a few words.

(d). Did you notice whether your image was erect or inverted?

PROBLEMS

1. What will be the approximate magnification of a reading glass which has a focal length of 4 cm.? (Assume 25 cm. as the limit of distinct vision in these problems.)

2. Given that the objective and the eyepiece of a small telescope have focal lengths of 96 cm. and 3 cm. respectively. When viewing a distant object, what will be: (*a*) the magnification? (*b*) the distance between the objective and eyepiece?

3. Suppose that a building 200 feet high is viewed through the telescope of Problem 2 at a distance of five miles. What is the size of the image due to the objective? What are the magnitudes of the angles α and β in radians?

4. Suppose that a real image due to the objective of a compound microscope is formed at a distance of 30 cm. from the objective and is then magnified by the eyepiece of focal length 1.5 cm. What is the magnification? (Assume that the focal length of the objective is 1 cm.) How far is the object from the objective?

CHAPTER XXV

THE DIFFRACTION GRATING

A physicist or a chemist in trying to discover new laws and facts in connection with the properties and behavior of matter finds very often that the eye is very limited in its scope. Of course this is no serious criticism when one considers what wonderful mechanisms and optical instruments our eyes really are. We marvel more and more when we try to extend the scope and vision of our eyes by building optical instruments based on the physical laws and facts familiar to us. By means of a telescope we can extend the limit of vision of the eye into larger distances, and by means of a microscope we can extend our limit of vision into smaller dimensions. In either case, however, the result achieved is insignificant compared with the fact that our eyes can see at all. Just imagine how interesting it would be if our eyes could see dimensions of the size of molecules of matter.

The diffraction grating is just another optical device, or part of an optical instrument, designed to aid our eyes in "seeing" smaller dimensions. The distances which interest us here are the wave lengths of light waves. By means of a diffraction grating wave lengths of the order of 0.00005 cm. or even smaller can be measured with great accuracy. Very often a grating is used for the same purpose for which a prism would be used; namely, the breaking up of a complex light beam into its constituent colors or wave lengths in order to identify the nature of the source emitting the light beam.

A grating consists essentially of a number of very narrow and evenly spaced slits. The width of a slit or line is of the order $\frac{1}{10000}$ of an inch. The difficulties of constructing or ruling such a grating are quite large and there are in existence only a very few machines capable of ruling such fine lines with accuracy. Depending upon whether the lines are ruled on a glass surface or a metal surface, the light either passes through, or else is reflected from, the surface. The former is known as a transmission grating, the latter as a reflection grating.

For a detailed description the student should refer to a text. A few equations necessary to clear up the experimental procedure will be derived here. Let AB (Fig. 158) represent a cross-section of **a transmission grating with light falling normally on the surface from the left.** The light, after emerging from the slits, produces little wavelets according to Huyghen's principle which in turn produce a wave front. It is easily seen that a wave front travels in direction Ⓢ. If necessary, this wave front can be converged by means of a lens to form a real image at the focus of the lens. There are, however, other directions in which wave fronts may be formed. In a direction designated by ① in the diagram it would be possible to have reinforcement of the light waves from the various successive slits, if the difference in path is just equal to one wave length of light (λ). Along this direction ① then, it is possible to get another image produced. This is called the first order image. It is easily seen from Figure 158 that $\lambda = d \sin \theta$ where d is the distance between two slits or lines, and θ = angle between the direct image Ⓢ and the first order image ①. Similarly if the difference in path is $n\lambda$, we get the nth order image and hence in general we have,

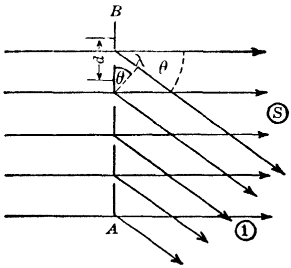


FIG. 158. — Transmission grating.

$$n\lambda = d \sin \theta. \tag{A}$$

In practice it is found that the higher orders get so faint as to be invisible. The case treated above assumes that all the light coming through the slits is of one wave length. This is never the case. Consequently, the image as seen really consists of several images close together, but coming together in slightly different directions. We usually say the light beam is spread out into a spectrum.

Let us suppose then that the problem is to find the wave length of a certain color. It becomes necessary only to measure θ in Equation A, since the distance d is usually known from the maker of the grating.

Method of minimum deviation.[1] There is another way of using a grating to measure a wave length. The reason for the necessity

[1] See L. W. Taylor's *College Manual of Optics*, page 57.

of any change or refinement in the previously outlined method is caused by the experimental difficulty, for it requires very elaborate adjustments in order to be quite sure that the beam of light coming from the left falls normally on the plane. These adjustments for accurate work are, of course, necessary and are usually studied in a more advanced course.

Using the idea of minimum deviation (compare with the case for a prism in Chapter XXII), let us suppose that light (Fig. 159) is coming in parallel rays from the left in direction S, making an angle i with the normal to the grating. The angle δ is the deviation, *i.e.*, the angle between this direction and the direction in which the first order image will be formed. If i' is the angle between the diffraction image ① and the normal, it will easily be seen from the above figure that

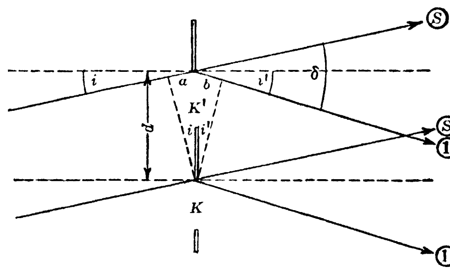


FIG. 159. — Transmission grating.

$$\delta = i + i'.$$

Now, as before, using Huyghen's principle, it follows that if the difference in path between the two beams, from the corresponding parts of the consecutive slits K and K', is $n\lambda$, then we will have the various order of spectra formed in directions ① ② *etc.*, when n has the values 1, 2, 3, $\cdots$.

Hence $\quad a + b = n\lambda$ for reinforcement.

But $\quad a = d \sin i, \quad$ and $\quad b = d \sin i'$.

$$\therefore \; n\lambda = d(\sin i + \sin i')$$

by using the theorem in trigonometry:

$$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}.$$

We can therefore write in this case,

$$n\lambda = 2\,d \sin \frac{i + i'}{2} \cos \frac{i - i'}{2}.$$

Now in order to find the minimum value of $\delta = i + i'$, we solve for this quantity, viz.,

(B) $$\sin\frac{i+i'}{2} = \frac{n\lambda}{2}\,\frac{1}{\cos\frac{i-i'}{2}}.$$

Now δ will have a minimum value when equation (B) is a minimum. This will be the case when $\cos\frac{i-i'}{2}$ has its maximum value since n and λ remain constant. The maximum value for $\cos\frac{i-i'}{2} = 1$, and this occurs when $\frac{i-i'}{2} = 0$. Consequently $i = i' = \frac{\delta}{2}$ is the condition for minimum deviation.

If we, therefore, use this condition of minimum deviation, then general equation (A) becomes

(C) $$n\lambda = 2\,d \sin\frac{\delta}{2}.$$

It is consequently not necessary, in performing the experiment, to arrange the grating normal to the beam since we can measure the position of minimum deviation. The position is easily found, by slightly rotating the grating around a vertical axis, when it will be seen that the first order image will move to an extreme position and return. This extreme position is the necessary position. Thus having measured δ, it is only necessary to apply the equation (C).

Finally, another method for finding the angle of deviation is to set the grating by eye as nearly normal as possible to the incident beam. Then, find the position of the first order image to the left of the direct image and the first order image to the right. Having read the angular scale readings for both those positions, find the angle by subtraction. This will be twice the angle of deviation θ. Then apply equation (A), namely, $n\lambda = d\sin\theta$, since we are not here dealing with the case for minimum deviation.

[*Note.* The student should verify for himself that this angle will be double the angle of deviation — assume that incidence is not normal.]

The wave length of a light ray is very often stated in "angstrom units,"

$$10^8 \text{ A. U.} \approx 1 \text{ cm.}$$

Light Sources

The grating, as we have seen, will spread out the light coming from a source into its characteristic spectrum. For a detailed description of the various types of spectra the student should refer to his text. In the laboratory the three main types of spectra, *i.e.*, continuous, absorption, and line spectra, should be produced and compared.

A continuous spectrum is obtained from sunlight, or, if this is not possible, from an incandescent tungsten lamp, placed before a slit and its light focused on the slit, the latter acting as source.

A line spectrum can be produced by a bunsen burner having in the flame various salts of such elements as sodium, potassium, etc. A mercury arc placed in a corner of the room and having its light focused on the slit serves admirably. A neon- or hydrogen-filled glass tube with sealed-in electrodes, which has an electrical discharge through it, will furnish further spectra for purposes of comparison or calibration.

An absorption spectrum is perhaps best typified by the famous absorption lines (Fraunhofer lines) in the sun spectrum. Another example consists in a source of continuous spectrum (say an electric lamp) of which the light has passed through various absorbing dye solutions or filters.

EXPERIMENT 50

THE WAVE LENGTH OF LIGHT

To measure the wave length of light using a diffraction grating mounted on a grating table.

Apparatus: Grating table, grating, slit, sodium burner, meter rule.

The arrangement is as shown in Figure 160. A source of light giving out monochromatic rays (*e.g.*, a sodium burner) is placed at S. A slit of width about 1 or 2 mm. is placed in front of S. A grating is placed at G normal to the light coming from S. On placing the eye in front of the grating and looking along directions ① I_1 and ② I_2, several images I_1 and I_2, etc., of the slit will be seen. The eye serves the purpose here of focusing the beam emerging from the grating on the retina of the eye.

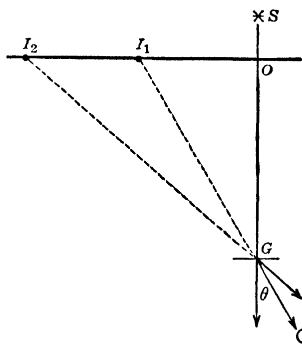


FIG. 160. — Grating table.

These images correspond to the first and second order images as described in this chapter. The angle θ is the angle of deviation of the beam of light coming from S.

In this experiment the **data** should be taken with the following points in mind:

1. To get a measurement on as many orders of images as is possible.

2. To measure the angle θ, or else the sin θ, and calculate the wave length in cm. and angstrom units for each order. [*Note.* Sine $\theta = \frac{OI_1}{I_1G}$.]

3. Place G at various distances from S. Repeat all observations under (1) and (2).

4. If another source is available, repeat all three parts, (1), (2), and (3).

[*Note.* The student should learn to make out a complete table for data before taking any readings — obtain the value of d from the instructor and calculate the wave length from equation (A).]

QUESTIONS

(**a**). What effect would you notice in the readings if the grating had ten times as many lines per inch?

(**b**). What fact did you observe about the intensity of the higher orders of the spectrum?

(**c**). What is the effect of a shorter wave length on the angle θ?

EXPERIMENT 51

A STUDY OF SPECTRA

To study various types of spectra and to measure the wave length of some lines in line spectra.

Apparatus: Grating spectrometer, several light sources, grating.

The instrument used in this experiment is called a spectrometer. A diagram of the essential parts is shown in Figure 161. The purpose of the collimator is to make a parallel beam out of the light coming from the slit S which now acts as a source. The function of the telescope is to converge the parallel beam after having passed through the grating to a point. The eyepiece assists in observing this image.

Important. *Don't touch the spectrometer until the following instructions have been carefully studied.* Before touching or turning any adjustment screw, be sure to learn what the screw is for. The spectrometer, as you receive it, will have had several adjustments requiring much time and skill already made. Do not upset these adjustments by turning a wrong screw. This is particularly true of the telescope and collimator which have already been focused for you. Let the instructor tell you which controls you have to manipulate. Before starting the experiments, be sure that you can read the scale and vernier.

The procedure in performing this experiment should be as follows:

1. Note that the telescope arm is controlled by two thumb-screws, a clamping screw, and a slow-motion screw. *Never force these screws* — a slight pressure is sufficient. Note that the clamping screw must be screwed in before the slow-motion screw has an effect. Practice reading the angle on the scale for various settings of the telescope.

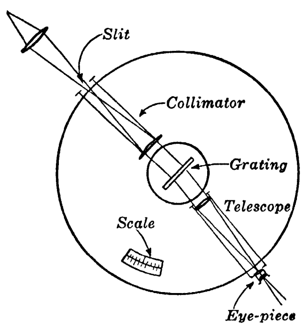


FIG. 161. — Grating spectrometer.

2. Note the clamping screw and slow-motion device on the center grating table.

3. Note the slit end of the collimator — a knurled screw around the end enables one to adjust the slit width.

4. A source (*e.g.*, sodium flame or incandescent bulb) is set up so as to form an image of the source on the slit, either by means of a lens, as shown in Figure 161 or by means of a concave mirror, as shown in Figure 162, large enough to cover the slit opening. If a concave mirror is used, the mirror should be placed in line with the axis of the collimator.

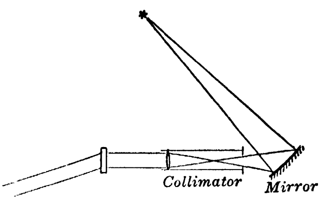


FIG. 162. — Mirror for focusing light in collimator.

5. See that the slit is partly open, turn the grating until it is approximately at right angles to the collimator and clamp.

6. Locate approximately with the eye the direct image (exactly in line with the collimator).

7. Set the telescope in this position and locate the cross-hair, which is observed when looking through the telescope (using the slow motion) exactly at the middle of the image. For this purpose the slit must be narrowed down so that this setting can be made accurately. Take a reading on the scale.

8. Now unclamp the telescope and move it slowly around to the left. Make a note of what you observe. Repeat to the right of the initial position and record all observations. Note also what the effect is of widening or narrowing the slits.

9. In order to find the position of minimum deviation, set the telescope on the first image to the left. Now loosen the clamping screw on the grating table and turn the grating slowly in one direction, noting the movement of the image through the telescope. The position of minimum deviation is thus easily found and the cross-hairs set on this position. Then clamp telescope and grating table.

10. Record the setting as read on the scale. Now go back and check the direct image setting. To do this unclamp only the telescope (leaving grating set) and get a reading for the direct image. The difference between the two readings gives the angle of minimum deviation.

Having mastered the technique of the instrument, take the readings necessary to fill in the table below.

The observations found in (7) and (8) are used to calculate the wave length from equation (A). The position of minimum deviation, as found from (9) and (10), requires, of course, the use of formula (C) in the calculation of the wave length λ.

DATA

Part (a). Grating has .... lines per inch, $\therefore$ d ...... cm.

Method of double angle:

Source Used	Particular Line Measured	Value as Given in Tables (A.U.)	Reading of Direct Image (a)	Reading for First Order Image Left (b)	Reading for First Order Image Right (c)	Double Angle of Deviation $d = (b) - (c)$	Angle of Deviation $\theta = \frac{d}{2}$	λ Using (A) in A. U.	Remarks
Sodium burner	Yellow								
Copper arc									

Method of minimum deviation:

Source Used	Particular Line Measured	Value as Given in Tables	Reading of Direct Image	Reading for First Order Image	Reading for Second Order Image	λ in A. U. Using C Calculated from 1st Order	λ in A. U. Calculated from 2nd Order	Average λ in A. U.	Remarks
Sodium burner	Yellow								

Part (b). Investigate the types of spectrum obtained from incandescent bodies such as a tungsten wire in a bulb — or if possible use the sun as a source. Note how many orders are visible and write down any further observations as to similarities and dissimilarities with other types of spectra in Part (a).

Part (c). Place various filters or absorbers before the source and note the effect produced on the spectrum. Record any observations that you may make.

QUESTIONS

(a). Classify the spectra as found in Parts (a), (b), and (c) under the headings: continuous, absorption, or line spectra.

(b). In each case how many orders of spectra were visible?

(c). Explain why the higher orders get weaker and weaker.

(d). How could the angle of minimum deviation be increased for a particular order of a spectrum?

(e). Do your results under Part (a) show whether the double angle or the angle of minimum deviation method is better?

PROBLEMS

Experiment 50

1. Distinguish between a continuous spectrum, absorption spectrum, and line spectrum, both as regards appearance and also with respect to their physical interpretation. (See any textbook on " types of spectra.")

2. Prove that for a diffraction grating, $n\lambda = d \sin \theta$, where θ is the angle of deviation for a beam incident normal to the grating.

3. What must be the wave length of a monochromatic beam of light if it falls normally on a grating and after having passed through the grating forms an image of the slit 10 cm. distant from the direct or central image on a screen? The screen is 50 cm. away and the grating has 10,000 lines per inch, the screen being parallel to the grating.

4. At what distance from the central image will the third order image be formed in the grating and screen of Problem 3?

Experiment 51

5. Prove that the second method, described in the theory for finding the angle of deviation if the grating is not at right angles to the beam, gives the correct deviation if we read the positions of the first order images on each side of the central image and then divide this total angle by two.

6. Prove that for the position of minimum deviation

$$n\lambda = 2\,d \sin \frac{\delta}{2}.$$

7. Find the wave length of a monochromatic source in centimeters as well as angstrom units, given that the angle of minimum deviation for the second order spectrum is 60° when the grating has 10,000 lines per cm.

8. Write down the ranges of wave lengths in centimeters and in A. U. for the regions of the spectrum usually designated by ultraviolet, visible, infra-red. (Look up the wave lengths in a textbook.)

APPENDIX

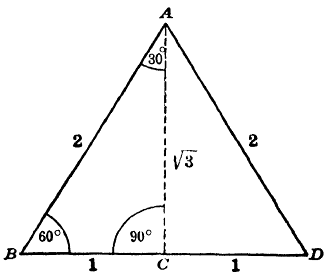
The Trigonometric Functions for 30°, 45°, and 60°

In any right-angled triangle:

The sine of an angle $= \dfrac{\text{side opposite}}{\text{hypotenuse}}.$

The cosine of an angle $= \dfrac{\text{side adjacent}}{\text{hypotenuse}}.$

The tangent of an angle $= \dfrac{\text{side opposite}}{\text{side adjacent}}.$



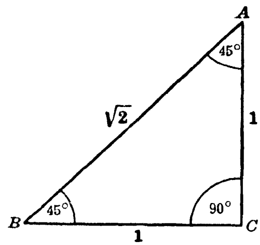


Fig. 163.

Function	0°	30°	45°	60°	90°
Sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Miscellaneous

The circumference of a circle (radius r) $= 2\pi r$.
The area of a circle (radius r) $= \pi r^2$.
The area of the surface of a sphere (radius r) $= 4\pi r^2$.
The volume of a sphere (radius r) $= \frac{4}{3}\pi r^3$.

1 mile = 5280 feet	1 kilometer = 1000 meters
1 foot = 12 inches	1 meter = 100 centimeters
1 inch = 2.54 centimeters	1 centimeter = 10 millimeters
1 meter = 39.37 inches	1 pound = 453.6 grams
1 liter = 61.0 cubic inches	1 kilogram = 2.205 pounds

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	16	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	4	5	7	9	10	12	14	16
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	12	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	5	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	7	8	9	11	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	2	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	4	5	6	7	8	9	11
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	6	7	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	5	6	7	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	3	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
N	0	1	2	3	4	5	6	7	8	9	1	2	2	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	1	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	3	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	5	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	3	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	3	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	3	3	4	4	5	6
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	3	3	4	4	5	6
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	4	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	1	1	2	2	3	3	4	4	5
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	3	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Degrees	Sine	Cosine	Tangent	Degrees	Sine	Cosine	Tangent
0	000	1.000	000	46	719	695	1.03
1	017	1.000	017	47	731	682	1.07
2	035	999	035	48	743	669	1.11
3	052	999	052	49	755	656	1.15
4	070	998	070	50	766	643	1.19
5	087	996	087	51	777	629	1 23
6	104	994	105	52	788	616	1.28
7	122	992	123	53	799	602	1.33
8	139	990	140	54	809	588	1.38
9	156	988	158	55	819	574	1.43
10	174	985	176	56	829	559	1.48
11	191	982	194	57	839	545	1.54
12	208	978	213	58	848	530	1.60
13	225	974	231	59	857	515	1.66
14	242	970	249	60	866	500	1.73
15	259	966	268	61	875	485	1.80
16	276	961	287	62	883	469	1.88
17	292	956	306	63	891	454	1.96
18	309	951	325	64	899	438	2.05
19	326	945	344	65	906	423	2.14
20	342	940	364	66	913	407	2.25
21	358	934	384	67	920	391	2.36
22	375	927	404	68	927	375	2.47
23	391	920	424	69	934	358	2.60
24	407	913	445	70	940	342	2.75
25	423	906	466	71	945	326	2.90
26	438	899	488	72	951	309	3.08
27	454	891	509	73	956	292	3.27
28	469	883	532	74	961	276	3.49
29	485	875	554	75	966	259	3.72
30	500	866	577	76	970	242	4.01
31	515	857	601	77	974	225	4.33
32	530	848	625	78	978	208	4.70
33	545	839	649	79	982	191	5.14
34	559	829	674	80	985	174	5.67
35	574	819	700	81	988	156	6.31
36	588	809	726	82	990	139	7.11
37	612	799	754	83	992	122	8.14
38	616	788	781	84	994	104	9.51
39	629	777	810	85	996	087	11.4
40	643	766	839	86	998	070	14.3
41	656	755	869	87	999	052	19.1
42	669	743	900	88	999	035	28.6
43	682	731	932	89	1.000	017	57.3
44	695	719	966	90	1.000	000	∞
45	707	707	1.000				

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(Boldface numbers refer to Experiments.)