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W. Hoyle

Roots of Numerical Equations
1826

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EXTRACTION

OF THE

REAL ROOTS

OF

NUMERAL EQUATIONS

OF ALL

DENOMINATIONS.

BY WILLIAM HOYLE.

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1826.

INTRODUCTION.

THE following small treatise being chiefly intended for those who have already made some advance in the science of algebra, it will only be necessary to inform the general reader, that the extraction of the roots of an algebraic equation, and particularly the solution of the irreducible case in cubic equations, has been assiduously sought after during the last four centuries, by the most eminent mathematicians of Europe; amongst whom the following names (with regard to the subject of the present work) stand pre-eminent, viz. Scipio Ferreus, Nicholas Tartalea, Hieronymus Cardan, Lewis Ferrari, Raphael Bombelli, Vieta, Albert Girard, Harriot, Oughtred, Descartes, Sir Isaac Newton, Maclaurin, John and James Bernoulli, Fontaine, Euler, Waring, Simpson, Legendre, and Lagrange.

To the more advanced mathematician, who is already acquainted with the different methods employed by the above mentioned eminent persons, it will be necessary to give a demonstration of the system by which all the real roots (either positive or negative) of a numeral equation, containing only one unknown quantity, may be extracted.

In the following examples a number, as near the real root as possible, is assumed, which is placed in the quotient or root; the said number is then involved to within one power of the given equation, and the different powers multiplied by their proper coefficients, the products are then collected into one sum, due regard being had to the signs + or —, which sum being placed before the numeral (to which the equation is equal), may be supposed to act as a divisor in common arithmetic, the numeral as the dividend, and the root which has been involved as the quotient; multiply the divisor by the quotient and it will be the same as though the root had been involved to the proper powers of the given equation, as may be seen by the following; viz. $(x^4-2x^3+4x^2+5x+1)\times x=x^5-2x^4+4x^3+5x^2+x$; the product of the divisor and quotient is now subtracted from the dividend, the remainder will be the difference between an equation of the same order and coefficients, as the given equation, with the assumed number for its root; to the remainder bring down as many ciphers as there are units in the highest power of the given equation; another figure must be placed in the root, and managed according to the following formula.

The first figure in the root having had its powers extracted, may now be considered as known to be a part of the real root, and therefore may be represented by a, the second figure, which is not yet known to be a part of the real root, by x; the first figure standing to the left may be considered as a tens figure and the second a unit; from whence the present figures in the root may be represented by $a+x$; we must now involve this binomial to the highest power of the given equation; thus, let the given equation be of the fourth order, or $y^4+by^3-cy^2-dy=0$, and the fourth power of $a+x=a^4+4a^3x+6a^2x^2+4ax^3+x^4$, in which we may consider a^4 to have been already extracted, and as the sum will have to be multiplied by x at last, we shall have $4a^3+6a^2x+4ax^2+x^3$, but as a may be considered to be ten times the value of the figure by which it is represented, we have, in the following examples, begun with x^3, and retreated one figure tothe left, it being the same way that is practised in common numeral multiplication, in order to save the repetition of ciphers: now, as y^4 has only 1 for its co-efficient, the sum of $4a^3+6a^2x+4a^2+x^3$ may be now left to represent y^4. Then $\overline{a+x}|^3=a^3+3a^2x+3ax^2+x^3$ where the same allowances for a^3 and a lower power of x will have to be made as before, whence we shall have $3a^2+3ax+x^2$, which (as y^3 is multiplied by $+b$) must be multiplied by b, and then $3a^2b+3ab\,x+bx^2$ will represent $+by^3$.

Then $\overline{a+x}|^2=a^2+2ax+x^2$ where, proceeding as before, we have $-2ac-cx$ to represent $-cy^2$.

Then $a+x$ will be merely $-d$, to represent $-dy$, therefore we shall have

$$\begin{array}{l} 4a^3+6a^2x+4ax^2+x^3 \\ \quad 3a^2b+3abx+bx^2 \\ \qquad\quad -2ac-cx \\ \qquad\qquad\quad -d \end{array}$$

which, added together, will give the second divisor.

But in the addition of numerals it will have to be observed, that x is a decimal, and that therefore as many ciphers must be prefixed to the right of $-d$ as there are units within one in the highest power of the given equation; in the above case it will be $4-1=3$, the number of ciphers; and generally let n represent the number of decimals in the root and m the number of units in the highest power of the given equation, then the number of ciphers to be added will be

$$y^1=m-1\times n$$
$$y^2=m-2\times n$$
$$y^3=m-3\times n, \text{ \&c.}$$

which will be readily seen by adding together the root, square, cube, &c. of a decimal, as

$$\begin{array}{ll} .2 & .02 \\ .2^2=.04 & .02^2=.0004 \\ .2^3=.008 & .02^3=.000008 \\ .2^4=.0016 & .02^4=.00000016 \end{array}$$

where, to make an equal quantity of decimals in each line, it will be necessary to add 3 ciphers to .2 two to .04, 1 to .008, 6 to .02, 4 to .0004, and 2 to .000008.

When the second divisor has been multiplied by the last figure in the root and the product subtracted, to the remainder as many ciphers must be brought down as before, and every thing brought on in the same manner.

The two figures in the root will now be known to be a part of the real root, and therefore will be a, the figure next put in the root will be x.

After the first two or three divisors have been got it will be easily seen what the next quotient figure will be, as the significant figures, or those to the left in the divisors, will not then vary much.

This treatise would not have been published in its present form, if the author could have got it inserted in any of the periodical publications.

The solution of three different forms of the irreducible case in cubic equations was sent to the editor of the Mechanics' Magazine, London, on the 1st of August; its receipt was acknowledged on the 13th of August, but, as it has never yet appeared, what use the editor has made of it is not known. The solution of a cubic equation was sent to the editor of the Kaleidoscope, Liverpool, on the 12th of September, and appeared in it on the 4th of October; but the editor, in a note, November 1st, declined inserting, for the present, any thing more in mathematics.

Oldham, Dec. 1st, 1825.

[illegible]

The [illegible] [illegible] of the [illegible] [illegible] [illegible] August [illegible] [illegible]

[illegible], Dec. 1st, 18[illegible].

SOLUTION

OF

EQUATIONS,

CONTAINING ONLY ONE UNKNOWN QUANTITY.

Example I.

Given $x^3-15x^2+63x-50=0$.

$1\times 1=1$	1	$49)50(1.028=x$
$+63$	-15	49
64	-15	$357604)1000000$
-15		715208
49 1st divisor.		$35425744)284792000$
		283405952
		1386048

$2\times 2=\quad 4$	202
$3\times 10\times 2=\quad 60$	-15
$3\times 10\times 10=300$	-3030
30604	
$+630000$	
$+660604$	
-303000	
357604 2nd divisor.	

$8\times8=$	64	2048
$3\times102\times8=$	2448	—15
$3\times102\times102=$	31212	—30720
	3145744	
	+63000000	
	+66145744	
	—30720000	
	35425744	3rd divisor.

Solution 2. $x^3-15x^2+63x=50$

		9)50(6.576$=x$
		54
$6\times6=36$	6	—
+63	—15	—725)—4000
		—3625
99	—90	
—90		—49301)—375000
		—345107
9 1st divisor.		
		—4577004)—29893000
		—27462024
		—2430976

$5\times5=$	25	125
$3\times6\times5=$	90	—15
$3\times6\times6=$	108	
		—18750
	11725	+18025
	+6300	
		—725 2nd divisor.
	+18025	

```
        7×7=        49          1307
    3×65×7=       1365           —15
   3×65×65=12675                ————
           ———————          —1960500
           1281199          +1911199
           +630000          ————————
           ———————            —49301  3rd divisor.
          +1911199
          ——————————————————————————
        6×6=           36      13146
    3×657×6=        11826        —15
 3×657×657=1294947            ——————
           —————————      —197190000
           129612996      +192612996
          +  63000000     ——————————
           —————————        —4577004  4th divisor.
          +192612996
          ——————————————————————————
```

When $x=6.576$.

$$
\begin{aligned}
x^3 &= +284.371070976\\
-15x^2 &= -648.656640\\
+63x &= +414.288\\
&\;-\;50.\\
\hline
&\;+.002430976 \text{ the above remainder.}
\end{aligned}
$$

SOLUTION 3. $x^3-15x^2+63x=50$.

```
7×7=49          7         7)50(7.395=x.
   +63        —15           49
   ———        ———           ——
  +112       —105       189)1000
  —105                       567
  ————                       ———
     7  1st divisor.   44991)433000
                             404919
                             ——————
                     5172175)28081000
                             25860875
                             ————————
                              2220125
```

$3\times3=$	9	143
$3\times7\times3=$	63	-15
$3\times7\times7=$	147	-2145
	15339	
	$+$ 6300	
	21639	
	-21450	
	189 2nd divisor.	

$9\times9=$	81	1469
$3\times73\times9=$	1971	-15
$3\times73\times73=$	15987	-22035
	1618491	
	$+$ 630000	
	2248491	
	-2203500	
	44991 3rd divisor.	

$5\times5=$	25	14785
$3\times739\times5=$	11085	-15
$3\times739\times739=$	1638363	-221775
	163947175	
	$+$ 63000000	
	226947175	
	-221775000	
	5172175 4th divisor.	

When $x=7.395$.

$$
\begin{array}{rr}
x^3= & +404.403154875 \\
-15x^2= & -820.290375 \\
+63x= & +465.885 \\
\hline
 & +49.997779875 \\
+\text{ the above remainder } = & .002220125 \\
\hline
 & 50.0000000000
\end{array}
$$

$$
\text{The three values of } x \begin{cases} +1.028 \\ +6.576 \\ +7.395 \end{cases}
$$

$$
+14.999 \begin{cases} \text{second term with its} \\ \text{sign changed.} \end{cases}
$$

Ex. II. Given $x^4-8x^3+14x^2+4x=8$; or to get the negative root $x^4+8x^3+14x^2-4x=8$.

$7\times7\times7=343$ $\qquad$ $7\times7=49$

$$
\begin{array}{r}
49 \\
8 \\
\hline
392
\end{array}
$$

$$
\begin{array}{r}
7 \\
14 \\
\hline
9800 \\
3920 \\
343 \\
\hline
14063 \\
-4000 \\
\hline
10063
\end{array}
\text{ 1st divisor.}
$$

$$
\begin{array}{r}
10063)80000(.732 \\
70441 \\
\hline
29753947)95590000 \\
89261841 \\
\hline
30855146488)63281590000 \\
61710292976 \\
\hline
1571297024
\end{array}
$$

$-.732=x$

$3\times3\times3=27$ $\quad 3\times3=9$ $\quad 143$
$4\times7\times3\times3=252$ $\quad 3\times7\times3=63$ $\quad 14$
$6\times7\times7\times3=882$ $\quad 3\times7\times7=147$ $\quad$ ——
$4\times7\times7\times7=1372$ $\quad$ —— $\quad 2002$

—— $\quad 15339$
1462747 $\quad 8$
12271200 $\quad$ ——
20020000 $\quad 122712$
——
33753947
— 4000000
——
29753947 2nd divisor.

$2\times2\times2=8$ $\quad 2\times2=4$ $\quad 1462$
$4\times73\times2\times2=1168$ $\quad 3\times73\times2=438$ $\quad 14$
$6\times73\times73\times2=63948$ $\quad 3\times73\times73=15987$ $\quad$ ——
$4\times73\times73\times73=1556068$ $\quad$ —— $\quad 20468$

—— $\quad 1603084$
1562474488 $\quad 8$
12824672000 $\quad$ ——
20468000000 $\quad 12824672$
——
34855146488
— 4000000000
——
30855146488 3rd divisor.

SOLUTION 2. $x^4-8x^3+14x^2+4x=8.$

8 $\quad$ 4 $\quad$ 2 $\quad$ +8) 8(2.732$=x$
—8 $\quad$ 14 $\quad$ 16
— $\quad$ — $\quad$ —
—32 $\quad$ 28 $\quad$ —10657)—80000
8 $\quad$ —74599
28 $\quad$ ——
4 $\quad$ —16837253)—54010000
— $\quad$ —50511759
+8 1st divisor. $\quad$ ——
—17057525512)—34982410000
—34115051024
——
—867358976

$$\begin{array}{r}
7\times7\times7=\ \ 343 \\
4\times2\times7\times7=\ 392 \\
6\times2\times2\times7=168 \\
4\times2\times2\times2=32 \\
\hline
53063
\end{array}
\qquad
\begin{array}{r}
7\times7=\ \ 49 \\
3\times2\times7=\ 42 \\
3\times2\times2=12 \\
\hline
1669 \\
-8 \\
\hline
-133520 \\
+122863 \\
\hline
-10657
\end{array}\ \text{2nd divisor.}$$

$$\begin{array}{r}
47 \\
14 \\
\hline
658
\end{array}
\qquad
\begin{array}{r}
4000 \\
65800 \\
53063 \\
\hline
+122863
\end{array}$$

$$\begin{array}{r}
3\times3\times3=\qquad 27 \\
4\times27\times3\times3=\quad 972 \\
6\times27\times27\times3=\ 13122 \\
4\times27\times27+27=78732 \\
\hline
80053947
\end{array}
\qquad
\begin{array}{r}
3\times3=\qquad 9 \\
3\times27\times3=\ \ 243 \\
3\times27\times27=2187 \\
\hline
221139 \\
-8 \\
\hline
-176911200 \\
+160073947 \\
\hline
\text{3rd divisor}\quad -16837253
\end{array}$$

$$\begin{array}{r}
543 \\
14 \\
\hline
7602
\end{array}
\qquad
\begin{array}{r}
4000000 \\
76020000 \\
80053947 \\
\hline
160073947
\end{array}$$

$2\times2\times2=$ 8
$4\times273\times2\times2=$ 4368
$6\times273\times273\times2=$ 894348
$4\times273\times273\times273=$81385668

81475146488

$2\times2=$ 4
$3\times273\times2=$ 1638
$3\times273\times273=$223587

22375084
—8

—179000672000
+161943146488

—17057525512 4th divisor.

5462	4000000000
14	76468000000
	81475146488
76468	161943146488

Solution 3. $x^4-8x^3+14x^2+4x=8.$

343	49	7	10223)80000(.763=x.
	—8	14	71561
	—392	9800	13201896)84390000
		4000	79211376
		343	
			13171136187)51786240000
		14143	39513408561
		—3920	
			12272831439
		10223	

$6\times6\times6=$	216	$6\times6=$	36	146
$4\times7\times6\times6=$	1008	$3\times7\times6=$	126	14
$6\times7\times7\times6=$	1764	$3\times7\times7=$	147	
$4\times7\times7\times7=$	1372			20440000
			15996	4000000
	1558696		—8	1558696
			—127968	25998696
				12796800
			2nd divisor	13201896

$3\times3\times3=$	27	$3\times3=$	9
$4\times76\times3\times3=$	2736	$3\times76\times3=$	684
$6\times76\times76\times3=$	103968	$3\times76\times76=$	17328
$4\times76\times76\times76=$	1755904		
			1739649
	1766328187		—8
			—13917192

1523	
14	
21322000000	
4000000000	
1766328187	
27088328187	
—13917192000	
13171136187	3rd divisor.

Solution 4. $x^4-8x^3+14x^2+4x=8$.

125	25	5	—1)8(5.236
	—8	14	—5
	—200	70	53288)130000
		4	106576
		125	
		—200	64626747)234240000
			193880241
	1st divisor	—1	
			66509117736)403597590000
			399054706416
			4542883584

$2\times2\times2=$ 8	$2\times2=$ 4	102
$4\times5\times2\times2=$ 80	$3\times5\times2=$ 30	14
$6\times5\times5\times2=$ 300	$3\times5\times5=$75	
$4\times5\times5\times5=$500		142800
	7804	4000
530808	—8	530808
	—62432	677608
		—624320
		2nd divisor 53288

$3\times3\times3=$ 27	$3\times3=$ 9
$4\times52\times3\times3=$ 1872	$3\times52\times3=$ 468
$6\times52\times52\times3=$ 48672	$3\times52\times52=$8112
$4\times52\times52\times52=$562432	
	815889
567317947	—8
	—6527112

1043
14

146020000
4000000
567317947

717337947
—652711200

64626747 3rd divisor.

$6\times6\times6=$ 216
$4\times523\times6\times6=$ 75312
$6\times523\times523\times6=$ 9847044
$4\times523\times523\times523=$572222668

573208125736

$6\times6=$ 36
$3\times523\times6=$ 9414
$3\times523\times523=$820587

82152876
—8

—657223008

10466
14

146524000000
4000000000
573208125736

723732125736
—657223008000

4th divisor 66509117736

$$\text{Four values of } x \begin{cases} -\ .732 & -\ 1571297024 \\ +2.732 & -\ 867358976 \\ +\ .763 & +12272831439 \\ +5.236 & +\ 4542883584 \end{cases}$$

$+7.999$ $\left\{\begin{array}{l}\text{second term with its} \\ \text{sign changed.}\end{array}\right.$

Ex. II. Given $x^4-12x^2+12x-3=0$ to find the four roots.

```
8      2        12                   —4)  3(2.858
     —12         8                         —8
      —        —24                        ———
     —24         —              11232)110000
               —4  1st divisor.          89856
                                        ———————
                             34388125)201440000
                                      171940625
                                      ─────────
                          364911101l2)294993750000
                                      291928880896
                                      ────────────
                                        3064869104
─────────────────────────────────

       8×8×8=  512                    48
     4×2×8×8=  512                   —12
     6×2×2×8=192                     ———
     4×2×2×2=32                     —576
              ───────
               56832
               12000
              —57600
              ───────
               11232  2nd divisor.
─────────────────────────────────

     5×5×5=        125                565
   4×28×5×5=      2800                —12
  6×28×28×5=  23520                   ———
4×28×28×28=87808                    —6780
              ─────────
              90188125
              12000000
             —67800000
              ─────────
              34388125  3rd divisor.
```

$8\times8\times8=$	512	5708
$4\times285\times8\times8=$	72960	—12
$6\times285\times285\times8=$	3898800	——
$4\times285\times285\times285=$	92596500	—68496
	————	
	92987110112	
	12000000000	
	—68496000000	
	————	
	36491110112 4th divisor.	

SOLUTION 2. $x^4-12x^2+12x-3=0.$

216	6	12000	
	—12	—7200	
	—	216	
	—72	———	
		5016 1st divisor.	

```
          5016)3.0000(.606=x.
               30096
               -----
 —1594953384)—9600000000
             —9569720304
             -----------
                30279696
```

$6\times6\times6=$	216	1206	12000000000
$4\times60\times6\times6=$	8640	—12	877046616
$6\times60\times60\times6=$	129600	——	—14472000000
$4\times60\times60\times60=$	864000	—14472	————
	————	2nd divisor	1594953384
	877046616		

SOLUTION 3. $x^4-12x^2+12x-3=0.$

64	4
	—12
	——
	—4800
	12000
	64
	——
	7264 1st divisor.

```
        7264)3.0000(.443=x.
             29056
             -----
     2217024)9440000
             8868096
             -------
  1748236667)5719040000
             5244710001
             ----------
              474329999
```

$4\times4\times4=$	64	84
$4\times4\times4\times4=$	256	—12
$6\times4\times4\times4=$	384	
$4\times4\times4\times4=$	256	1008

297024
—10080000
12000000

2217024 2nd divisor.

$3\times3\times3=$	27	883
$4\times44\times3\times3=$	1584	12
$6\times44\times44\times3=$	34848	
$4\times44\times44\times44=$	340736	10596

344236667
—10596000000
12000000000

1748236667 3rd divisor.

$$x^4-12x^2+12x-3=0,$$

or to get a —root $x^4-12x^2-12x-3=0$.

27 3 —21) 3(3.907 therefore —3.907 is a root.
—12 —63

—36 72249(660000
—12 650241
+27

1322315847 43)975900000000

—21 1st divisor. 925621093201

50278906799

$9\times9\times9=$	729	69
$4\times3\times9\times9=$	972	—12
$6\times3\times3\times9=$	486	——
$4\times3\times3\times3=$	108	—828
	——	
	167049	
	—82800	
	—12000	
	——	
	+72249 2nd divisor.	

$7\times7\times7=$	343	7807
$4\times390\times7\times7=$	76440	—12
$6\times390\times390\times7=$	6388200	——
$4\times390\times390\times390=$	237276000	—93684
	——	
	237915584743	
	—93684000000	
	—12000000000	
	——	
	132231584743 3d divisor.	

The four roots	2.858	+.003064869104
	.606	—.000030279696
	.443	+.000474329999
	—3.907	—.050278906799
	——	
	0	

Ex. IV. Given $x^5+6x^4-10x^3-112x^2-207x-110=0$.

$4^4=256$ 64 16
6 —10
+384 —160
+256
—160 4
—448 —112
—207 —448
1st div—175

—175) 110(4.464=x
—700
16676496)81000000
66705984
222439918096)1429401600000
1334639508576
23102222205747456)9476209142400000
9240888822989824
235320319410176

4×4×4×4= 256
5×4×4×4×4= 1280
10×4×4×4×4= 2560
10×4×4×4×4= 2560
5×4×4×4×4=1280
15629056

4×4×4= 64
4×4×4×4= 256
6×4×4×4= 384
4×4×4×4=256
297024
+6
17821440
15629056
33450496
—16774000
2nd divisor 16676496

4×4= 16
3×4×4= 48
3×4×4=48
5296
—10
—52960

84
—112
—9408000
—2070000
—5296000
—16774000

$$
\begin{array}{rr}
6\times6\times6\times6= & 1296 \\
5\times44\times6\times6\times6= & 47520 \\
10\times44\times44\times6\times6= & 696960 \\
10\times44\times44\times44\times6= & 5111040 \\
5\times44\times44\times44\times44= & 18740480 \\
\hline
 & 192586012496
\end{array}
$$

$$
\begin{array}{rr}
6\times6\times6= & 216 \\
4\times44\times6\times6= & 6336 \\
6\times44\times44\times6= & 69696 \\
4\times44\times44\times44= & 340736 \\
\hline
 & 347769176 \\
 & +6 \\
\hline
 & 208661505600 \\
 & 192586012496 \\
\hline
 & 401247518096 \\
 & -178807600000 \\
\hline
\text{3rd divisor} & 222439918096
\end{array}
$$

$$
\begin{array}{rr}
6\times6= & 36 \\
3\times44\times6= & 792 \\
3\times44\times44= & 5808 \\
\hline
 & 588756 \\
 & -10 \\
\hline
 & -58875600000 \\
 & -99232000000 \\
 & -20700000000 \\
\hline
 & -178807600000
\end{array}
\qquad
\begin{array}{r}
886 \\
-112 \\
\hline
-99232
\end{array}
$$

$$
\begin{array}{r}
4\times4\times4\times4= \quad 256 \\
5\times446\times4\times4\times4= \quad 142720 \\
10\times446\times446\times4\times4= \quad 31826560 \\
10\times446\times446\times446\times4= \quad 3548661440 \\
5\times446\times446\times446\times446=197837875280 \\
\hline
1981930598323456
\end{array}
$$

$$
\begin{array}{r}
4\times4\times4= \quad 64 \\
4\times446\times4\times4= \quad 28544 \\
6\times446\times446\times4= \quad 4773984 \\
4\times446\times446\times446=354866144 \\
\hline
355343827904 \\
+6 \\
\hline
2132062967424000 \\
1981930598323456 \\
\hline
4113993565747456 \\
-1803771360000000 \\
\hline
\text{4th divisor } 2310222205747456
\end{array}
$$

$$
\begin{array}{r}
4\times4= \quad 16 \\
3\times446\times4= \quad 5352 \\
3\times446\times446=596748 \\
\hline
59728336 \\
-10 \\
\hline
-597283360000000 \\
-999488000000000 \\
-207000000000000 \\
\hline
-1803771360000000
\end{array}
\qquad
\begin{array}{r}
8924 \\
-112 \\
\hline
999488
\end{array}
$$

$x^5+6x^4-10x^3-112x^2-207x-110=0$, or to find a — root $6x^4-x^5+10x^3-112x^2+207x-110=0$; on trying this, —2 will be found to be a root.

Dividing the equation by $x+2$ we have $x^4+4x^3-18x^2-76x-55=0$; or to find a — root $x^4-4x^3-18x^2+76x-55=0$, whence —1 is evidently a root; and again dividing this equation by $x+1$, we have $x^3+3x^2-21x-55=0$: or to find a — root $3x^2-x^3+21x-55=0$; by inspection —5 is a root, and again dividing, we have $x^2-2x-11=0$, we now have four roots. The original equation will only admit of one more, and by adding the roots together we find it must be —, therefore $x^2+2x-11=0$.

2	44	486	4924
2	20	200	2000
4	64	686	6924

The five roots $\begin{cases} +4.464 \\ -2 \\ -1 \\ -5 \\ -2.464 \end{cases}$

$\overline{\quad 6 \quad}$

```
4)11(2.464   therefore —2.464
  8            is a root.
  —
64)300
   256
   ———
686)4400
    4116
    ————
6924)28400
     27696
     —————
       704
```