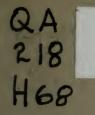
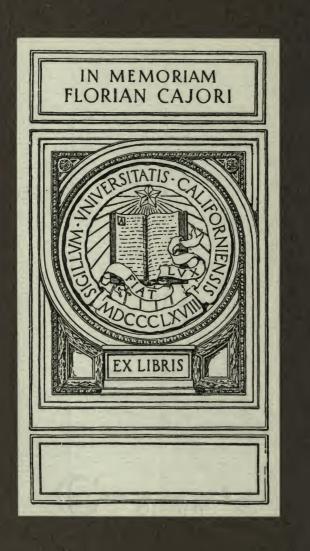
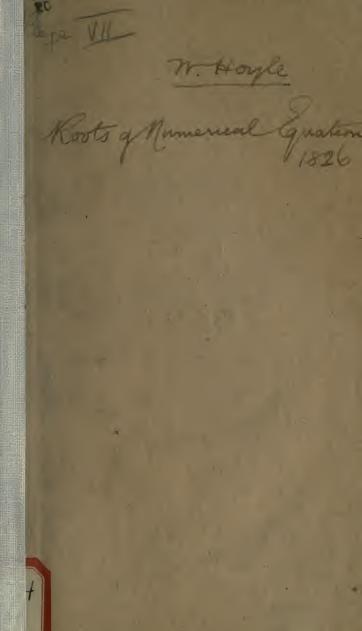
HOYLE

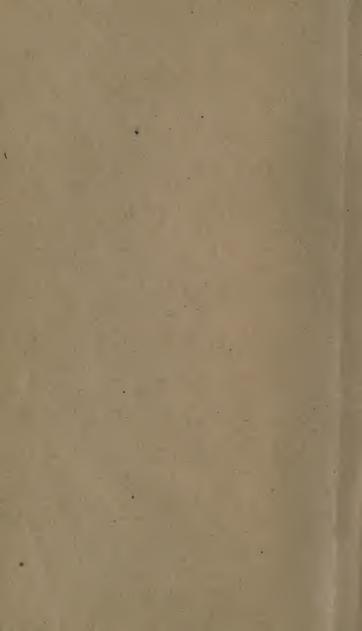




Q.A218 H68







EXTRACTION

2214

OF THE

REAL ROOTS

OF

NUMERAL EQUATIONS

OF ALL

DENOMINATIONS.

BY WILLIAM HOYLE.

LONDON:

PRINTED FOR ALL THE BOOKSELLERS, and

MAY BE HAD OF W. DEAN, MANCHESTER, AND J. WESTALL, ROCHDALE.

1826.

CAJORI

INTRODUCTION.

QA218

468

THE following small treatise being chiefly intended for those who have already made some advance in the science of algebra, it will only be necessary to inform the general reader, that the extraction of the roots of an algebraic equation, and particularly the solution of the irreducible case in cubic equations, has been assiduously sought after during the last four centuries, by the most eminent mathematicians of Europe; amongst whom the following names (with regard to the subject of the present work) stand preeminent, viz. Scipio Ferreus, Nicholas Tartalea, Hieronymus Cardan, Lewis Ferrari, Raphael Bombelli, Vieta, Albert Girard, Harriot, Oughtred, Descartes, Sir Isaac Newton, Maclaurin, John and James Bernoulli, Fontaine, Euler, Waring, Simpson, Legendre, and Lagrange.

To the more advanced mathematician, who is already acquainted with the different methods employed by the above mentioned eminent persons, it will be necessary to give a demonstration of the system by which all the real roots (either positive or negative) of a numeral equation, containing only one unknown quantity, may be extracted.

In the following examples a number, as near the real root as possible, is assumed, which is placed in the quotient or root; the said number is then involved to within one power of the given equation, and the different powers multiplied by their proper coefficients, the products are then collected into one sum, due regard being had to the signs + or -, which sum being placed before the numeral (to which the equation is equal), may be supposed to act as a divisor in common arithmetic, the numeral as the dividend, and the root which has been involved as the quotient; multiply the divisor by the quotient and it will be the same as though the root had been involved to the proper powers of the given equation, as may be seen by the following; viz. $(x^4 - 2x^3 +$ $4x^2+5x+1$ $\times x=x^5-2x^4+4x^3+5x^2+x$; the product of the divisor and quotient is now subtracted from the dividend. the remainder will be the difference between an equation of the same order and coefficients, as the given equation, with the assumed number for its root; to the remainder bring down as many ciphers as there are units in the highest power of the given equation; another figure must be placed in the root, and managed according to the following formula.

The first figure in the root having had its powers extracted, may now be considered as known to be a part of the real root, and therefore may be represented by a, the second figure, which is not yet known to be a part of the real root, by x; the first figure standing to the left may be considered as a tens figure and the second a unit; from whence the present figures in the root may be represented by a+x; we must now involve this binomial to the highest power of the

given equation ; thus, let the given equation be of the fourth order, or $y^4 + by^3 - cy^2 - dy = 0$, and the fourth power of α $+x=a^{4}+4a^{3}x+6a^{2}x^{2}+4ax^{3}+x^{4}$, in which we may consider a^4 to have been already extracted, and as the sum will have to be multiplied by x at last, we shall have $4a^3 + 6a^2x$ $+4ax^2+x^3$, but as a may be considered to be ten times the value of the figure by which it is represented, we have, in the following examples, begun with x^3 , and retreated one figure tothe left, it being the same way that is practised in common numeral multiplication, in order to save the repetition of ciphers: now, as y^4 has only 1 for its co-efficient, the sum of $4a^3+6a^2x+4a^2+x^3$ may be now left to represent y^4 . Then $\overline{a+x}^{3} = a^{3} + 3a^{2}x + 3ax^{2} + x^{3}$ where the same allowances for a^3 and a lower power of x will have to be made as before, whence we shall have $3a^2 + 3ax + x^2$, which (as y^* is multiplied by +b) must be multiplied by b, and then $3a^2b+$ $3ab x + bx^2$ will represent $+ by^3$.

Then $\overline{a+x_i}^2 = a^2 + 2ax + x^2$ where, proceeding as before, we have -2ac-cx to represent $-cy^2$.

Then a+x will be merely -d, to represent -dy, therefore we shall have $4a^3+6a^2x+4ax^2+x^3$ $3a^2b+3abx+bx^2$

-2ac-cx

-d

May do a mill of 352

which, added together, will give the second divisor.

But in the addition of numerals it will have to be observed, that x is a decimal, and that therefore as many ciphers must be prefixed to the right of -d as there are units

V

within one in the highest power of the given equation; in the above case it will be 4-1=3, the number of ciphers; and generally let *n* represent the number of decimals in the root and *m* the number of units in the highest power of the given equation, then the number of ciphers to be added will be $u^{1}=m-1 \times n$

$$y^{2} = m - 2 \times n$$

$$y^{3} = m - 3 \times n. \&c.$$

which will be readily seen by adding together the root, square, cube, &c. of a decimal, as .2 .02

$.2^2 = .04$.02* = .0004
.23=.008	.023=.000008
. 24- 0016	024- 00000014

where, to make an equal quantity of decimals in each line, it will be necessary to add 3 ciphers to .2 two to .04, 1 to .008, 6 to .02, 4 to .0004, and 2 to .000008.

When the second divisor has been multiplied by the last figure in the root and the product subtracted, to the remainder as many ciphers must be brought down as before, and every thing brought on in the same manner.

The two figures in the root will now be known to be a part of the real root, and therefore will be a, the figure next put in the root will be x.

After the first two or three divisors have been got it will be easily seen what the next quotient figure will be, as the significant figures, or those to the left in the divisors, will not then vary much.

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This treatise would not have been published in its present form, if the author could have got it inserted in any of the periodical publications.

The solution of three different forms of the irreducible case in cubic equations was sent to the editor of the Mechanics' Magazine, London, on the 1st of August; its receipt was acknowledged on the 13th of August, but, as it has never yet appeared, what use the editor has made of it is not known. The solution of a cubic equation was sent to the editor of the Kaleidoscope, Liverpool, on the 12th of September, and appeared in it on the 4th of October; but the editor, in a note, November 1st, declined inserting, for the present, any thing more in mathematics.

Oldham, Dec. 1st, 1825.

and have I have all products I have a such a strat will all To which all of for the could go all on a set the many on and second and there is helder you

SOLUTION

EQUATIONS,

CONTAINING ONLY ONE UNKNOWN QUANTITY.

EXAMPLE I.

Given $x^3 - 15x^2 + 63x - 50 = 0$.

$1 \times 1 = 1$ +63	1 —15	49)50(1.028—x 49
64 	-15	357604)1000000 715208
49	lst divisor.	35425744)284792000

1386048

$\begin{array}{rrrrr} 2 \times 2 = & 4 \\ 3 \times 10 \times 2 = & 60 \end{array}$	202
- 3×10×10=300	-3030
30604 +630000	
+660604 -303000	
357604	2nd divisor.

	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
+6300 +6614 -3072 	5744
Solution 2. $x^2 - 1$	$5x^2 + 63x = 50$
and and all and all and	9)50(6.576 = x) 54
6×6=36 6	-121- 134
+63 -15 -	-725)-4000 -3625
99 -90	
	-49301)—375000
The state of the state of the	-345107
9 1st divisor.	577004)-29893000
240885	-27462024
	2430976
20 <u>0</u> 70]	= X3 N0 = X01 X0
$5 \times 5 = 25$	125
$3 \times 6 \times 5 = 90$	-15
$3 \times 6 \times 6 = 108$	10750
11725	-18750 +18025
+6300	W/H HISTORY
	-725 2nd divisor.
+18025	10575.6

$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$
Solution 3. $x^3 - 15x^2 + 63x = 50$.
$7 \times 7 = 49$ 7 7)50(7.395 = x. +63 -15 49
- 10 - 10 - 10 - 10 - 10 - 10 - 10 - 10
+112 -105 189)1000 -105 567
7 1st divisor. 44991)433000 404919
5172175)28081000 25860875
2220125

$3 \times 3 = 9$	143
$3 \times 7 \times 3 = 63$	-15
$3 \times 7 \times 7 = 147$	1710
	-2145
15339	
+ 6300	
21639	- CATTER &
21450	



9×9= 81	1469
$3 \times 73 \times 9 = 1971$	-15
3×73×73=15987	
	-22035
1618491	
+ 630000	
1 00000	
2248491	
-2203500	
44001 9	Brd divisor.
44991	JIU UIVISOF.

$5 \times 5 =$ $3 \times 739 \times 5 =$ $3 \times 739 \times 739 =$	= 11085		$ \begin{array}{r} 14785 \\ -15 \\ 221775 \end{array} $
+	163947175 - 63000000	and the set	221110
There are a	226947175 -221775000		
X 11077	5172175	4th divisor	10

When x=7.395. $x^3 = +404.403154875$ $-15x^2 = -820.290375$ +63x = +465.885+49.997779875+ the above remainder = .002220125 50.0000000000 The three values of $x \begin{cases} +1.028 \\ +6.576 \\ +7.395 \end{cases}$ +14.999 {second term with its sign changed. Ex. II. Given $x^4 - 8x^3 + 14x^2 + 4x = 8$; or to get the negative root $x^4 + 8x^3 + 14x^2 - 4x = 8$. $7 \times 7 \times 7 = 343$ $7 \times 7 = 49$ 8 392 7 10063)80000(.732 14 70441 29753947)95590000 9800 3920 89261841 343 30855146488)63281590000 14063 61710292976 -40001571297024 10063 1st divisor.

$3 \times 3 \times 3 = 27$ $3 \times 3 = 9$ 143
$4 \times 7 \times 3 \times 3 = 252 3 \times 7 \times 3 = 63 14$
$6 \times 7 \times 7 \times 3 = 882 \qquad 3 \times 7 \times 7 = 147 \qquad$
$4 \times 7 \times 7 \times 7 = 1372 \qquad 2002$
15339
1462747 8
12271200
20020000 122712
33753947
- 4000000
29753947 2nd divisor.
$2 \times 2 \times 2 = 8$ $2 \times 2 = 4$ 1462
$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$6 \times 73 \times 73 \times 2 = 63948 \ 3 \times 73 \times 73 = 15987 =$
4×73×73×73=1556068 20468
1603084
1562474488 8
12824672000
20468000000 12824672
34855146488
- 400000000
MEDIXIX MEDIXIX
30855146488 3rd divisor.
Solution 2. $x^4 - 8x^3 + 14x^2 + 4x = 8$.
8 4 2 +8) $8(2.732 = x)$
200 - 11 Mar - 61
-32 28 -10657) -80000
8 -74599
28
4 —16837253)—54010000
+8 1st divisor.
-17057525512) - 34982410000
-34115051024
an of Proceedings of the Proceed

$7 \times 7 \times 7 = 343$ $4 \times 2 \times 7 \times 7 = 392$ $6 \times 2 \times 2 \times 7 = 168$	$7 \times 7 = 49$ $3 \times 2 \times 7 = 42$ $3 \times 2 \times 2 = 12$
$4 \times 2 \times 2 \times 2 = 32$ 53063	1669 —8
The Party	-133520 +122863
ARALYCA	-10657 2nd divisor.
47 14 658	4000 65800 53063
. 000	+122863

	3X	3=	9
$3 \times$	27×	3=	243
3×2	7×2	27=21	87

221139 —8

80053947

27

972

-176911200+160073947

3rd divisor -16837253

4000000 76020000 80053947

160073947

800

 $3 \times 3 \times 3 =$

 $6 \times 27 \times 27 \times 3 = 13122$ $4 \times 27 \times 27 + 27 = 78732$

 $4 \times 27 \times 3 \times 3 =$

114.014

543

7602

		273×2	
			$\times 2 = 894348$ 273=81385668
		-	81475146488
		3×273>	$\begin{array}{cccc} & & 4 \\ \hline & & & \\ \hline \hline & & & \\ \hline \hline & & & \\ \hline \hline \\ \hline & & & \\ \hline \hline & & & \\ \hline \hline \\ \hline & & & \\ \hline \hline \hline \\ \hline \hline \hline \\ \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \hline \\ \hline \hline$
			22375084
			8
			-179000672000
			+161943146488
			-17057525512 4th divisor.
	5462		400000000
	14		76468000000
	76468		81475146488
	10400		161943146488
	Solution 3.	x4-	$-8x^3+14x^2+4x=8$.
343	49 —8	7 14	$\begin{array}{c} 10223 \\ 80000 (.763 = x. \\ 71561 \end{array}$
	-392	9800	13201896)84390000
		4000	79211376
		343	10171106107171706040000
		14143	13171136187)51786240000 39513408561
	7	-3920	
	TRUET	10223	12272831439

$6 \times 6 \times 6 = 216$	$6 \times 6 = 36$	146
$4 \times 7 \times 6 \times 6 = 1008$	$3 \times 7 \times 6 = 126$	14
$6 \times 7 \times 7 \times 6 = 1764$	$3 \times 7 \times 7 = 147$	
$4 \times 7 \times 7 \times 7 = 1372$		20440000
Base and a state of the state o	15996	4000000
1558696	-8	1558696
	-127968	25998696
		12796800
	0 11 11 1	10201000

2nd divisor 13201896

-13917192

;1523 14

21322000000 4000000000 1766328187

27088328187 -13917192000

13171136187 3rd divisor. C

	SOLUTION	1. x ⁴ -	$8x^3 + 14x^2 + 4x = 8$.
125	25	5	-1)8(5.236
	-8	14	-5 1
10.1	-200	70	53288)130000
	-200	70 4	106576
	in ta	125	
		-200	64626747)234240000
			193880241
	lst divi	sor —1	
			66509117736)403597590000

399054706416

4542883584

$2 \times 2 \times 2 = 8$	$2 \times 2 = 4$	102
$4 \times 5 \times 2 \times 2 = 80$	$3 \times 5 \times 2 = 30$	14
$6 \times 5 \times 5 \times 2 = 300$	$3 \times 5 \times 5 = 75$	1.1 - 1.7
$4 \times 5 \times 5 \times 5 = 500$	No	142800
	7804	4000
530808	-8	530808
	· · · · · · · · · · · · · · · · · · ·	
	-62432	677608
		-624320

2nd divisor 53288

$3 \times 3 \times 3 =$	27	$3 \times 3 =$	9
$4 \times 52 \times 3 \times 3 =$	1872	$3 \times 52 \times 3 =$	468
$6 \times 52 \times 52 \times 3 =$	48672	$3 \times 52 \times 52 = 8$	112
$4 \times 52 \times 52 \times 52 = 50$	62432	-	
		8	15889
5	67317947		

-6527112

10

-

1043 14	A server all and
146020000 4000000 567317947	
717337947 —652711200	
64626747	3rd divisor.
$ \begin{array}{r} 6 \times 6 \times 6 = \\ 4 \times 523 \times 6 \times 6 = \\ 6 \times 523 \times 523 \times 6 = \\ 4 \times 523 \times 523 \times 523 = 572 \end{array} $	216 75312 9847044 222668
573	208125736
$6 \times 6 = 36$ $3 \times 523 \times 6 = 9414$ $3 \times 523 \times 523 = 820587$	10466
82152876 8	146524000000 4000000000 573208125736
-657223008	$723732125736 \\ -657223008000$
4th d	ivisor 66509117736
four values of x $+2.732$ $ +3.763$ $+15.236$ $+15.236$	1571297024 867358976 2272831439 4542883584
+7.999 {second sign	term with its changed.

F

Ex. 11. Given $x^4 - 12x^2 + 12x - 3 =$ roots.	O to find the four
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	
-4 1st divisor.	89856
34388125)201440000 171940625
3649111011	2)294993750000 291928880896
	3064869104
And and a state of the second state of the sec	10 - X F
8×8×8= 512	48
$4 \times 2 \times 8 \times 8 = 512$ $6 \times 2 \times 2 \times 8 = 192$	12
$4 \times 2 \times 2 \times 2 = 32$	-576
56832 12000 57600	
11232 2nd divis	sor.
$5 \times 5 \times 5 = 125$ $4 \times 28 \times 5 \times 5 = 2800$	565 —12
$6 \times 28 \times 28 \times 5 = 23520$ $4 \times 28 \times 28 \times 28 = 87808$	
West and a second secon	1.1
90188125 12000000	
-67800000	
34388125 3rd div	isor.
Street as ord un.	

$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$
92987110112 1200000000 68496000000 36491110112 4th divisor.
Solution 2. $x^4 - 12x^2 + 12x - 3 = 0.$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$
30279696
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
Solution 3. $x^4 - 12x^2 + 12x - 3 = 0$.
$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
7264 1st divisor. 5244710001
474329999

D

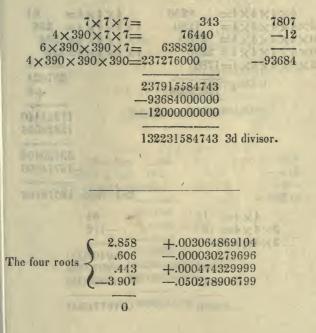
$4 \times 4 \times 4 = 64 = 84$
$4 \times 4 \times 4 \times 4 = 256 \qquad -12$
$6 \times 4 \times 4 \times 4 = 384$
$4 \times 4 \times 4 \times 4 = 256$ 1008
. 297024
12000000
2217024 2nd divisor.
And the state of the state of the state of the
$3 \times 3 \times 3 = 27$ 883
$4 \times 44 \times 3 \times 3 = 1584 \qquad 12$
$6 \times 44 \times 44 \times 3 = 34848 =$
344236667
-10596000000
1200000000
1748236667 3rd divisor.
is the state of the state of the
$x^4 - 12x^2 + 12x - 3 = 0$,
or to get a —root $x^4 - 12x^2 - 12x - 3 = 0$.
$\begin{array}{cccccccccccccccccccccccccccccccccccc$
-12 -63 (is a root.
-36 72249(660000 -12 650241
-12 650241 $+27$ $$
1~.

50278906799

 $9 \times 9 \times 9 = 729$ $4 \times 3 \times 9 \times 9 = 972$ $6 \times 3 \times 3 \times 9 = 486$ $4 \times 3 \times 3 \times 3 = 108$

167049 ---82800 ---12000

+72249 2nd divisor.



Ex. IV. Given x ⁵ -	$+6x^{4}-10x^{3}-$	$-112x^2 - 207x - 110 = 0$	•
4 ⁴ =256 64 16 6 -10	—175) $110(4.464 = x)$ -700	
+384 - 160 +256	166764	96)81000000 66705984	
-207	2224399180	96)1429401600000 1334639508576	
448 lst div_175 2	31022220574	7456)9476209142400000)
		9240888822989824	
		235320319410176	;
$4 \times 4 \times 4 \times 4 = 5 \times 4 \times 4 \times 4 \times 4 = 4$	256	$4 \times 4 \times 4 = 64$ $4 \times 4 \times 4 \times 4 = 256$	
$10 \times 4 \times 4 \times 4 \times 4 =$	2560	$6 \times 4 \times 4 \times 4 = 384$	
$10 \times 4 \times 4 \times 4 \times 4 =$		$4 \times 4 \times 4 \times 4 = 256$	
$5 \times 4 \times 4 \times 4 \times 4 = 1$	1280	005004	
10 D On 1	5629056	297024 +6	
100	1002000		
		17821440	
	14 T 12 T 11	15629056	
		33450496	
		-16774000	
		2nd divisor 16676496	
$4 \times 4 =$	16	84	
$3 \times 4 \times 4 =$		-112	
$3 \times 4 \times 4 = 4$	18		
F	5296	-2070000	
	-10	-5296000	
-	0000	10****	
	960		

192586012496

 $\begin{array}{rrrr} 6 \times 6 \times 6 = & 216 \\ 4 \times 44 \times 6 \times 6 = & 6336 \\ 6 \times 44 \times 44 \times 6 = & 69696 \\ 4 \times 44 \times 44 \times 44 = 340736 \end{array}$

347769176 +6

208661505600 192586012496

401247518096

3rd divisor 222439918096

886 -112 -99232

 $6 \times 6 = 36$ $3 \times 44 \times 6 = 792$ $3 \times 44 \times 44 = 5808$

-178807600000

 $\begin{array}{rrrr} 4 \times 4 \times 4 \times 4 = & 256 \\ 5 \times 446 \times 4 \times 4 \times 4 = & 142720 \\ 10 \times 446 \times 446 \times 446 \times 4 = & 31826560 \\ 10 \times 446 \times 446 \times 446 \times 4 = & 3548661440 \\ 5 \times 446 \times 446 \times 446 = 197837875280 \end{array}$

1981930598323456

> 355343827904 +6

2132062967424000 1981930598323456

4th divisor 2310222205747456

 $\begin{array}{rrrr} 4X4 = & 16\\ 3X446X4 = & 5352\\ 3X446X446 = 596748 \end{array}$

-1803771360000000

 $x^5+6x^4-10x^3-112x^2-207x-110=0$, or to find a ---root $6x^4-x^5+10x^3-112x^2+207x-110=0$; on trying this, -2 will be found to be a root.

Dividing the equation by x+2 we have $x^4+4x^3-18x^2-76x-55=0$; or to find a — root $x^4-4x^3-18x^2+76x-55=0$, whence —1 is evidently a root; and again dividing this equation by x+1, we have $x^3+3x^2-21x-55=0$: or to find a — root $3x^2-x^3+21x-55=0$; by inspection --5 is a root, and again dividing, we have $x^2-2x-11=0$, we now have four roots. The original equation will only admit of one more, and by adding the roots together we find it must be —, therefore $x^2+2x-11=0$.

$\frac{2}{2}$		486 200 686	4924 2000 6924
9-	-4.464 -2	4)11(2.464 8	therefore —2.464 is a root.
The five roots	-1 -5 -2.464	64)300 256	
	6	686)4400 4116	
		6924)28400 27696	
		704	









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