

## THE LIBRARY <br> OF <br> THE UNIVERSITY OF CALIFORNIA LOS ANGELES

## 8



Digitized by the Internet Archive in 2007 with funding from
Microsoft Corporation

THE EYE
ITS REFRACTION AND DISEASES

The No.

## THE EYE

## ITS REFRACTION AND DISEASES

THE REFRACTION AND FUNCTIONAL TESTING OF THE EYE, COMPLETE IN ITSELF, IN TWENTY-EIGHT CHAPTERS WITH NUMEROUS EXPLANATORY CUTS<br>AND DIAGRAMS

## EDWARD E. GIBBONS, M.D.

## COPYRIGHT, 1904

By THE MACMILLAN COMPANY

Set up, electrotyped and printed January, 1904

PRESS Of
THE H1W EHA PHIMTING COMPANX

## PREFACE.

The author has attempted in the following pages to supply students of ophthalmology with the practicaloinformation needed upon the various subjects treated.

The deductions of the various formulæ used in optics have been simplified and inserted. It is customary to omit the mathematics of the subject from treatises of this kind, but the author feels that the student should be familiar with the physics involved for the proper understanding of the subject. The scope of the work precludes as frequent reference to authors as the writer would like. The author feels that the new material and diagrams the work contains justifies its publication, so offers no apology for adding one more to the numerous books upon the same subject.

## TABLE OF CONTENTS.

CHAPTER I
Light, its Propagation and Refraction. ..... I
CHAPTER II.
The Study of Rffraction by Prisms and Lenses. ..... 9
CHAPTER III.
Reflection and Image-Formation by Plane, Concave and Convex Mirrors ..... 47
CHAPTER IV.
The Gross Anatomy of and Physiology of the Eye and the Study of the Theory of Gauss ..... 56
CHAPTER V.
Visual Acuity and Accommodation. ..... 79
CHAPTER VI.
Mechanism of Accommodation and Optical Defects of the Eye. ..... 94
CHAPTER VII.
Ophthalmoscopy and Oblique Illumination. ..... 107
CHAPTER VIII.
The Appearance of the Normal Fundus Oculi ..... 133
CHAPTER IX.
The Study of the Field of Vision ..... 139

## CHAPTER X.

The Study of the Color Sense ..... 161
CHAPTER XI.
The Study of the Light Sense, After Images and Troxler Phe- nomenon ..... 171
CHAPTER XII.
Simulation of Blindness and Means of Detecting It. ..... 177
CHAPTER XIII.
Visual Impressions. ..... 183
CHAPTER XIV.
Entoptic Phenomena ..... 195
CHAPTER XV.
Movements of the Eyeballs. ..... 203
CHAPTER XVI.
The Study of the Laif of Listing. ..... 214
CHAPTER XVII.
Normal and Abnormal Refraction. Hyperopia and Myopia ..... 224
CHAPTER XVIII.
Abnormal Refraction (continued). Astigmatism. Anisometropia. ..... $24^{1}$
CHAPTER XIX.
Presbyopia ..... 260
CHAPTER XX.
Balance and Imbalance of the Extraocular Muscles. ..... 264
CHAPTER XXI.
Manner of Detecting and Correcting Errors of Refraction. ..... 268

CHAPTER XXII.
Optometers. Prisoptometry. Ridgeway's Chromatic Test. Hotz' Astigmatic Test ..... 308
CHAPTER XXIII.
Ophthalmoscopy in Measuring Errors of Refraction ..... 331
CHAPTER XXIV.
Retinoscopy ..... 344
CHAPTER XXV.
Ophthalmometry and Ophthalmophakometry ..... 379
CHAPTER XXVI.
Tests for Muscle Imbalance ..... 407
CHAPTER XXVII.
Aphakia and Post-Operative Refraction ..... 435
CHAPTER XXVIII.
Spectacle and Nose Glass Fitting and Neutralization of Lenses. ..... 443

## THE EYE, ITS REFRACTION AND DISEASES

## CHAPTER I

## LIGHT, ITS PROPAGATION AND REFRACTION

Light is that which renders the objects of the external world visible to us. Light emanates from all luminous and illuminated bodies, and passes in every direction in space. It travels at the rate of 198,000 miles a second, taking about eight minutes to reach us from the sun, the source of all light. Bodies are transparent when they transmit light to the extent of allowing vision through them; translucent when they allow light to pass through them, but cannot be seen through. Opaque bodies do not transmit light. Such are wood and stone. The intensity or brightness of light decreases as the distance of the source of light increases, in proportion to the square of the distance of the source of light. Thus at three feet away the brightness of a light is one ninth of what it is at one foot. There are two theories as to the nature and the manner of propagation of light. Either theory will explain all the phenomena of light concerned in the study of optics. The theories are:
$1^{\circ}$. The Corpuscular Theory, as considered by Newton, is that light emanates from all luminous bodies, and consists of very minute particles, too small and subtle to exhibit the properties of ordinary matter, and which travel in straight lines and produce the sensation of sight by passing into the eye, and striking upon the end-organ of the nerve of sight, the retina.
$2^{\circ}$. The Undulatory Theory, or Wave-theory of light. According to this theory, light is supposed to be propagated by the undulations of a subtle ethereal medium that pervades all space. The waves of
light spread from center to circumference, as the ripples spread in the water from the spot where a pebble has dropped. The waves of light do not, however, occupy only one plane as the ripples of water, but pass out from the source in all directions, in every plane. The cycle at the origin of the light is repeated in all its essentials in each surrounding particle. The theory that light travels in straight lines, or in a rectilinear course, is not at variance with the wave-theory of light, as at first may be supposed. The direction of the propagation of light is as if it traveled as the arrow flies. A chip of wood on the water may rise and fall with the waves but its course will be straight ahead with the current. Undulations of light are perpendicular to the course of its propagation, or ray. We know that light travels in straight lines, for many reasons. The one reason that will appeal to us all is our inability to see around corners. And if we look at an object through a long narrow tube the tube must be turned directly towards the object. Again, the shape of the shadows cast upon the ground by the sun proves that light has a straight path. The following experiment of classical interest is cited as a proof of the wave-theory of light. It is Young's experiment.
"Three parallel opaque screens are placed some few inches apart ; in the first is made a narrow slit, or opening with straight parallel edges $(S)$; in the second, there are made two slits parallel to that in the first screen ( $A$ and $B$ ), and quite close together. There is thus a narrow opaque portion between $A$ and $B$, and the two screens are so adjusted that when a source of light, as a candle, is placed in front of the first slit, the candle, the first slit, and the opaque portion between the two slits are in the same straight line. The two slits are now equally illuminated by the slit $S$, but there will not be uniform illumination over the third screen, which is receiving light from the slits $A$ and $B$. It may be observed that on this third screen there are bands, parallel to the slits. That is, strips of the screen are illumined while the screen in between the strips remains in dark-
ness. These bands will be at fairly equal distances apart. It may be necessary to use a microscope to see the bands clearly. The only satisfactory explanation of this phenomenon is as follow: the ether waves from the slit $S$ reach the two slits $A$ and $B$, thus making them two sources of waves, which are identical in all respects if proper precautions are observed. The two sets of waves that proceed from the slits $A$ and $B$ illuminate the third screen; but, to reach the same points on the screen the two trains of waves must go different distances in general."

The waves from one slit in the second screen will interfere with the waves from the other slit, that is the crest of one wave will fall into the trough of another, and the wave will be broken or neutralized. The disturbances in a wave, at a distance apart of half a wavelength, are exactly opposite to each other. The central portion of the third screen, that is the portion in line with the slit $S$ and the opaque strip between the slits $A$ and $B$, will be illuminated because the two trains of waves reinforce each other; but on each side of this bright portion there will be a series of points such as described above, where the difference in the path of the two trains is $l / 2(l$ being a wave-length, or the distance between one crest and the next) and there is darkness. It is obvious that the interference bands, as the stripes of shade on the third screen, between the light bands, are called, are not a phenomenon of light, but of waves.* Waves of light are either spherical or plane. Spherical when the wave-front is curved in outline, and plane when the wave-front is straight. If the spherical wave has its convexity in the direction of propagation of the light it is said to be positive, and to be negative if the convexity is in the opposite direction.

The arrow heads mark the direction of propagation of light. Fig. $A$ represents positive spherical waves, Fig. $B$, plane waves and Fig.

[^0]C, negative spherical waves. A ray of light is the direction of the propagation of the light. In Fig. $A$ the rays are diverging, in Fig. $B$ parallel, and in Fig. $C$ converging. A number of diverging rays are spoken of as a pencil of light. A number of parallel rays com-


A


B


C pose a beam of light. Waves of light are said to be positive to curved surfaces when the center of curvature of the surface and that of the waves are on opposite sides of the surface, and negative when on the same side.
$A$ is a curved surface of a lens. Waves $W$ are positive to it, and waves $W^{\prime}$, negative.

There are two remarkable laws of light, namely: Refraction and Reflection.

Refraction.-Refraction of light is the bending or alteration in the direction of a ray, when it passes obliquely from one medium, or substance, into another of different density. A wave of light passing through the air continues in the same direction until it meets with some obstruction to its progress at the surface of a denser medium, when its course is changed, its ray (course) being bent to-
 wards the perpendicular. On exit from the denser medium the ray is deviated from the perpendicular.

A Dioptric Surface is a surface that separates transparent or refracting media of different densities. A dioptric medium is any substance that can be seen through. From $\delta \iota a$, through, and ö $\pi \tau \epsilon \iota \nu$, to see.

The ray of light that impinges upon the surface of the refracting medium is the incident ray, and the ray at its exit from the dioptric medium, the refracted ray.

In figure, $A B C D$ is a piece of glass. Ray $I O$ is incident at the point $O . R R^{\prime}$ is normal to the surface at the same point. The
angle formed by the incident ray and the perpendicular to the surface at the point of incidence is called the angle of incidence; such is angle $I O R^{\prime}$.

If the ray did not change its course when it entered the refracting medium, it would continue on the path $O x$. It is, however, bent towards the perpendicular $R R^{\prime}$, and occupies a path along $O O^{\prime}$. The angle between the perpendicular and the refracted ray is the angle of refraction. If the ray $O O^{\prime}$ continued straight on as it emerged from the glass it would occupy the path $O^{\prime} x^{\prime}$, but on exit it is bent from the perpendicular to the surface at the point of exit, and takes the direction $O^{\prime} K$. At the surfaces of the medium refraction takes place. An angle of incidence and of refraction are formed at each surface of a dioptric medium. If the sides of the refracting medium are parallel, as are the sides $A C$ and $B D$, the ray on exit is parallel to the ray on entrance ; the ray is therefore simply displaced. If the incident ray is perpendicular to the refracting surface it passes through
 the medium unrefracted, as $L M$. The reason that light changes its course when it enters a medium of different density is because the medium of greater density offers more impediment to the progress of the light. The denser the medium the slower the light travels through it. Suppose that in the figure above the cross lines on the incident ray, which represent a train of plane light waves, be taken to represent successive columns of men, marching side by side. The men are instructed always to preserve the line. If they are marching in the direction of the arrow-head, those on the right of the line will enter the obstruction to their progress first. They will not be able to proceed so fast, and in order that the line may not be broken the men on the left of the column will wheel to the right. After all the men are in they progress with an equal
velocity. On exit from the impediment those on the right get out first, and are compelled to wheel to the left to preserve the straightness of the line. The relative resistance that a substance offers to the passage of light through it, as compared with the resistance offered by the air, is called its index of refraction. Air is taken as the standard, and its value to be one. The resistance of a substance as compared with that of a vacuum is the absolute index of refraction.

Snell's Law is that the sine of the angle of incidence is to the sine of the angle of refraction as the resistance of air is to the resistance offered by the refracting medium to the passage of light. In other words the index of refrac-
 tion of a substance is the sine of the angle of incidence divided by the sine of the angle of refraction. The law is illustrated in the following way:

Let $M$ be a refracting medium ; $a b c$ a train of plane waves of light, incident to the surface $M$. The wave-front $d$ would advance to the position of the line $a^{\prime} b^{\prime}$, if every portion of the wave impinged upon the surface $M$ at the same time. As the waves form an angle $I^{\prime}$ with the surface of the refracting medium, the retardation in the wave-front begins below and advances upwards. This causes the wave to assume the position $a^{\prime} b^{\prime \prime}$, after refraction, and the waves pass on as 1,2 and $3 . \angle I^{\prime}=$ angle of incidence, angles $I$ and $I^{\prime}$ are equal, being alternate. Angle $R=$ angle of refraction. $v$ and $v^{\prime}$ are radii of wave-fronts or rays, $v$ course without refraction, $v^{\prime}$, after refraction. Both $v$ and $v^{\prime}$ may be understood to mean the velocity of or resistance to the passage of the light in the medium $B$. In triangle $a^{\prime} b^{\prime} M, v / a^{\prime} M=$ sine of $I$, and in triangle $a^{\prime} b^{\prime \prime} M, v^{\prime} / a^{\prime} M=$ sine of $R$. As $a^{\prime} M$ is common to both, it may be dropped from each, then sine of $I=v$, and sine of $R=v^{\prime}$.

Ergo:

$$
\frac{\text { sine of } I}{\text { sine of } R}=\frac{v}{v^{\prime}} .
$$

Another way of illustrating Snell's Law is by the following diagram : $C D$ is the sine of the angle of incidence, and $M x$, the sine of the angle of refraction. An incident ray strikes the surface $S$. It forms the angle of incidence $D O C$ with the normal $P P^{\prime}$ erected at the point of incidence, with the surface $S$. The angle $M O x$ is the angle of refraction, formed by the refracted ray and the normal $P P^{\prime}$, erected at the point $O$ perpendicular to the surface $S$. Draw a circle around $O$ as a center. From $D$ and $M$ draw perpendiculars to the line $P P^{\prime}$. The sine of the angle $C O D$ is $C D / O D$, and the sine of the angle $M O x$ is $M x / M O$. Let $O D$ and $M O$, radii of the circle, be equal to I .
$O D$ and $M O$ can then be eliminated from each sine fraction. The
 index of refraction of the medium, whose surface is $S$, is then $C D / M x$. In this case $C D$ is taken to be equal to 4 , and $M x, 3$. Ergo: The index of refraction of the medium is $4 / 3$ or 1.33 times that of air. The indices of refraction of a few important substances are:

$$
\begin{aligned}
& \text { Crown Glass . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . } 5 \\
& \text { Flint Glass . . . . . . . . . . . . . . . . . . . . . . } \\
& \text { 1. } 3365 \\
& \text { Aqueous humor. . . . . . . . . . . . . . . . . } \\
& \text { I. } 3365+ \\
& \text { Cornea . . . . . . . . . . . . . . . . . . . . . . . } \\
& \text { 1.4371 } \\
& \text { Crystalline lens . . . . . . . . . . . . . . . . . . . } 1.3363 \\
& \text { Vitrous humor . . . . . . . . . . . . . . . . . . } 1.33
\end{aligned}
$$

Practical Demonstration of the Refraction of Light.-Place a coin in the bottom of an empty vessel, with opaque sides, so that it will just be hidden from your view by the side of vessel, as at $C$ in figure.

Fill the vessel with water and the coin will become visible from the same position. The rays of light that pass from the coin towards

the eye are bent at the surface of the water and pass into the eye, as is shown in figure. The coin then appears to be on the line $C^{\prime} K$.

Place a stick in a vessel of water. The stick appears bent at the surface of the water, causing its point to appear nearer the surface.


From the points $a, b$ and $c$ rays diverge. Some pass up as I, 2, and 3 ; others enter the eye $E$. When the vessel is filled with water the light that went to $E$ is refracted in the direction of $x$, and the light that went up as 1,2 , and 3 is refracted to the eye $E$. The end of the stick as well as each point in it beneath the surface of the water appears out of its true position. The end $C$, for instance, now appears at point $C^{\prime}$.

## CHAPTER II

PRISMS AND LENSES

A portion of glass or other transparent medium with plane (straight), but non-parallel sides is called a prism. Usually the sides of the prism incline towards each other and form an angle, the Refracting Angle of the prism, which is expressed in degrees. The prism may be cut in any shape, but still retains its properties so long as its sides are plane and non-parallel. If the sides are parallel, the medium is a plane. The apex of the prism includes the refracting angle, and the side opposite the apex is called the base. The position of the prism is described according to the direction of its base. Thus: Base up or down to the right or left. If the prism is before the eye and
 its base towards the nose, we speak of the base as in and as out when the base is next to the temple.

A ray of light entering a prism emerges deviated towards the base of the prism. The ray $R O$ would continue to the point $Q$ if refraction did not occur at the side $A B$ of the prism. The right ends of the wave-fronts $a, b$ and $c$ enter the prism first, and consequently the ray is bent towards the perpendicular to the surface at the point of incidence. The ray is therefore inclined towards the base of the prism. On exit the wave-fronts pass out of the prism first on the left end, and moving faster than those which are still within the prism, cause the ray to be inclined again towards the base of the prism. In the figure, I is the angle of incidence ; 2 , the angle of re-
fraction, and 3, the angle of deviation. Angles 3 and 2 are usually together equal to angle i. In weak prisms angle 3 is equal to one half of angle 2. If an object be looked at through a prism, it will appear to be displaced in space towards the thin edge or apex of the prism. The light appears to enter the eye from a point along the ray, as it emerges from the prism, the
 eye not taking cognizance of the fact that the light has twice undergone refraction at the surfaces of the prism. The eye then projects the object (imagines it to be) back along the course given to the ray at its last refraction. When a prism is held so that objects viewed through it are displaced the least, the prism is said to be in the position of minimum deviation. This occurs when the angle of incidence and the angle of emergence are equal. The angle through which a ray of light is bent in refraction is called the Angle of Deviation. The angle of deviation may be described as the angle between the direction along which an object really is and that in which it appears to be, when looked at through a prism. In prisms between one and ten degrees this angle is one half the refracting angle of the prism.

In the figure $A$ is the refracting angle of the prism, $I$ the incident ray, $E$ the ray on emergence, $i$ the incident and $e$ the angle of emergence, $D$ the angle of deviation, $D=i+e-A$, for $D=a^{\prime}+b^{\prime}$; and $A=180^{\circ}$ $-x=c+c^{\prime}$. The deviation is least when the angle of incidence $i$, is equal to the angle of emergence $e$ and
 the course of the ray is then symmetrical and we have $A=2 c$, and $D=2 i-2 c=2 i-A$.

By Snell's Law : $\sin i / \sin c=v / v^{\prime}$, in which $v / v^{\prime}$ is the index of refraction of the prism. $\operatorname{Sin} i=v / v^{\prime}(\sin c)$. We can replace the sines by the arcs if the latter are small. Ergo:

$$
i=v / v^{\prime}(c) \text {, and } D=2 v / v^{\prime}-A=\left(v / v^{\prime}-1\right) A .
$$

If the prism is made of glass, $v / v^{\prime}=\frac{3}{2}$ and $v / v^{\prime}-1=\frac{1}{2}$.
Therefore $D=\frac{1}{2} A$. The angle of deviation produced by a prism is then equal to one half of the angle of the prism. In stronger prisms the amount of deviation in comparison to the angle of the prism rapidly increases. The largest possible angle that an incident ray of light can make with the normal to a refracting surface, and still pass out of the medium, is called the Critical Angle. A ray of light can emerge from a piece of glass if it makes an incident angle equal to $41^{\circ} 48^{\prime}$, but if the angle of incidence is greater than this the ray is reflected back into the medium. This is the limit angle of refraction, as a ray cannot be bent more than that on entering a denser medium. The critical angle from water to air is $48^{\circ} 35^{\prime}$, and from glass to air $41^{\circ} 48^{\prime}$ (Ganot). The critical angle is designated as the angle of incidence that corresponds with the angle of refraction of $90^{\circ}$. Total refraction takes place when a ray passing through a dense medium meets the surface separating the denser medium from a rarer one. If the angle of incidence exceeds the critical angle all the light is reflected. This phenomenon is spoken of as Internal Reflection, or Total reflection. The light in a prism may suffer a number of reflections at the
 surfaces of the prism and then finally pass out. In this way a prism may give rise to multiple images.
$I$ represents a ray that makes an incident angle a little less than the critical angle and passes out of the medium nearly parallel to the surface. $I^{\prime}$ makes an angle larger than the critical and is reflected back into the medium, at the surface of the medium.

Dip a pencil into a glass of water, and we will see it mirrored from the surface of the water, if we look at the surface through the side of the glass, from below. The rays of light that pass from the points of the pencil in the water pass towards the surface where
they undergo reflection, back into the water and enter the eye of the observer. The surface of the water appears as a silvered, opaque surface, there being no ray that comes from above reaching the eye as all are refracted towards the bottom of the glass.

The most useful application of internal reflection is in the rectangular prism. Looking perpendicularly at one of the surfaces we see an object placed in front of the other face, formed by total reflection on the hypothenuse. The prism need not be rectangular ; a prism of 60 degrees gives the same results, but the three faces of the prism must be well polished.

This principle is made use of in certain optical instruments.

## METHODS OF NUMERATING PRISMS.

$1^{\circ}$. According to the refracting angle of the prism. Thus a prism that includes at its apex an angle of 5 degrees is called a $5^{\circ}$ prism. In prisms below ten degrees, the deviation of a ray passing through them is equal to half the angle of the prism, but for prisms above ten degrees, this does not apply, and therefore this scale of numeration is faulty.
$2^{\circ}$. Dr. Jackson proposed to number a prism according to the degrees of deviation, and to replace the degree mark by a small $d$, to avoid confusion, thus: Pr. Id, Pr. $2 d$, etc. The unit in this Deviation angle system is about double that of the Refracting-angle system.
$3^{\circ}$. Dennet's method has for its base an arc called a radian, whose length is equal to its radius of curvature. Such an arc has a length of $57.295^{\circ}$. A prism that causes a deviation in a ray of light one one-hundredth of this arc, is called a centrad, and denoted thus: Pr. $1^{\nabla}, 2^{\nabla}$ and so on. The amount of deviation of one centrad is $.57295^{\circ}$. The merit of this system is its uniformity, ten centrads having ten times the deviating power of one centrad.
$4^{\circ}$. The Prentice or Prism-diopter Method. The standard of this method is a prism that causes a deflection of 1 cm . in a ray of light at a distance of 1 m . By this method it is easy to ascertain the
amount of prismatic effect in a decentered lens, by multiplying the strength of the lens by the number of centimeters it is decentered. Thus: A 5-diopter lens decentered 1 cm . would cause a prismatic deviation of 5 pr . diopters or centrads since a prism-diopter and centrad are nearly equal. Their relation is shown in the figure (below). Prism diopters are denoted, Pr. $\mathrm{I}^{\Delta}$, and so on.
$5^{\circ}$. The Meter-angle System of Nagel. Its relation to the meterangle of convergence will be seen by referring to the latter subject. The meter angle is the angle made by the visual axis and the median plane when the eye fixes an object on that plane at 1 m . distant. The value of the angle depends of course upon the interocular distance, which must be conventionalized when used for the purpose of notation. An interocular distance of .o6 makes the (one) meter-angle equal to $3^{\Delta}$,
 being about the average interocular distance. The advantage of this system, as will be seen, is that it corresponds to the amount of convergence as well as the amount of accommodation for any given distance.

Light spreads, as we have seen, divergingly from its source; therefore all the rays of light that come to the eye are divergent, no matter from what distance they come. They are, however, so little divergent when they reach the eye from a distance of 6 m . or twenty feet, or beyond, that we in practice consider them parallel. The nearer the eye is to the source of light the more diverging are the rays of light that it receives, and vice versa. If a circular aperture one centimeter in diameter be made in an opaque disc, and a luminous point be placed at varying distances from it, for example at a distance of one meter and at ten meters, the rays of light coming from ten meters passing through the aperture will be less divergent than those that come from a meter. A cone of light will pass through the aperture in each case, but the shape of it will be different according to the distance of the source of light from the screen.

When the round hole one centimeter in diameter is at a distance of one meter from the source of light, the cone has a base one centimeter in diameter and the apex is situated in the luminous source at one meter's distance. The rays have diverged 1 cm . in traveling 100 cm . The metal disc cuts off all the rays having a greater divergence. If the cone of light passes through the aperture and falls upon a distant wall the cone will preserve the same proportions, that is its base will be $1 / 100$ of its altitude. If the point of light is at a very great distance, there will be no difference in the size of the luminous circle and the size of the aperture in the screen; the rays therefore have practically a parallel direction. Rays that enter the pupil of the eye from a distance of six meters have so little divergence that they may be considered parallel. The average size of the pupil is 4 mm . ; the divergence is therefore only $6 / 1000$. All rays, diverging more widely than this are excluded from the eye by the iris. If the aperture or pupil is large, then infinite distance is required to render rays parallel. The deviation of light as it passes through a prism depends upon several conditions : the wave-number of the incident light (the number of waves in a second), the value of the angle of incidence, the material of which the prism is composed, and the size of its refracting angle. The greater the wavenumber (smaller the waves) the more the refraction or deviation of the light. If light coming from a source that appears white to our eyes is made to pass through a prism, and thrown upon a screen, it will be seen that the area of illumination on the screen is not white but colored. The colors arrange themselves in a definite order and are together called the spectrum ; they are red, yellow, green and blue as well as intermediate shades. White light is considered as a mixture of the waves that produce these separate color sensations. The waves that produce the sensation of blue in our eyes are deviated more than those that produce red, in passing through the prism." The number of waves per second or " $n$," as it is denoted,

[^1]varies from 45 I million millions for red to 789 million millions for violet. A thermometer placed in the spectrum will register a higher temperature as it is moved from the violet towards the red. The heating effect of the sun's ray is due perhaps to the red rays. The red rays have the greatest velocity.

The function of a prism of breaking white light up into its constituent colors is called dispersion, or chromatic aberration. It is possible to make two prisms
 of different materials and refracting angles, so that when they are placed side by side, base of one opposite the apex of the other, the dispersion of light will be overcome and yet the refraction of the prism not destroyed. White light then in passing through such a prism will only be refracted. Such a prism is called an achromatic prism.

Light enters prism $A$, and is dispersed. Then entering prism $B$, the light is condensed, and passes out of the prism as parallel rays.

Generally the medium that has the greater index of refraction has also the greater dispersion, but the two are not proportional. Flint glass, for example, gives a dispersion nearly twice that of crown glass, while its index is 1.7, and that of crown glass 1.5. In the achromatic prism one portion is
 made of crown glass, and the other, whose angle is about half as large, is made of flint. A prism of flint glass produces a spectrum much longer than one of crown glass. Flint glass is so called because it was originally made from ground flint. It is the sort of glass used for optical instruments and table ware, and contains lead as an ingredient. Crown glass is the common window glass, and contains no lead. Its refractive power is less than that of flint glass.

In order that we may obtain a very clear spectrum we must make use of a very narrow slit, through which the light passes to the prism, and interpose a lens so that the rays of each color may be re-


Prism. à Vision direct united on the screen in a distinct image of the slit. The spectrum in reality is composed of a whole series of images of the slit. The length of the spectrum depends upon the size of the angle of the prism and the amount of its dispersion.

We can construct a series of prisms that possess little or no refraction, but considerable dispersion ; such are spoken of as prism, $\grave{d}$ vision directe, and are much used for spectroscopes.

A Maddox or doubling prism is an obtuse angle prism, and is to be considered as two prisms placed base to base. Such a prism has the function of causing an object seen through it to appear as two.

Lenses.-A lens is a transparent medium or sub-


Maddox Prism. stance bounded by at least one curved side. Lenses are divided into spherical and cylindrical lenses. Compound lenses are formed by the combination of a spherical and a cylindrical lens.

A spherical lens is one the surface of which is equally curved in different directions. It may be thought of as a section of a sphere. A cylindrical lens is curved in one direction, and in the direction at right angles thereto the surface is plane or straight. It may be considered as a surface section of a cylinder.

Spherical Lenses.

Convex, positive or + , converging or condensing lens. Are thicker in the center than on the edge.

Plano-convex. - One side plane or straight and the other convex. Most spherical spectacle lenses are of this variety.

Bi-convex.-Double-convex, both sides are convex.

Concavo-convex.-One surface concave and one convex. Convexity in excess of concavity. Positive meniscus or periscopic lens.

Spherical
Lenses. $\left\{\begin{array}{l}\text { Concave, negative or } \\ -, \text { diverging or dispers- } \\ \text { ing lens. Concave lenses } \\ \text { are thinner in the center. }\end{array}\right.$
Plano-concave.-One side plane, the other one concave. The weaker -. Spherical spectacle lenses are of this variety.

Bi-concave. - Both sides concave. Also called double-concave.

Convexo-concave.-One surface concave and the other convex, the concave surface in excess of the convex. Negative, or minus, meniscus, or periscopic lens.

Cylindrical lenses $\left\{\begin{array}{l}\text { Convex, } \\ \text { Concave }\end{array}\right.$

## Stherical Lenses.



Bi-convex-Plano-Curves.


Concavo-convex



Plano-concave. Bi-concave. Cylindrical Lenses-


Convex cylincrical Lens.


Concave cylindrical Lens.


Coquille. Mi-coqulile.

In the biconvex lens $L, E$ is its edge or equator, $S$ and $S^{\prime}$ its sides or surfaces; $P$ and $P^{\prime}$, the central points of these surfaces, called the anterior and posterior poles. Point $O$ is the geometrical center of the figure, and is called the optical center. $C$ and $C^{\prime}$ are the centers of curvatures of the surfaces of the lens. A line passing through the anterior pole, the optical center and the posterior pole
of the lens is called the principal axis. The principal axis likewise passes through the centers of curvature of the lens. This line is normal to the surfaces of the lens at
 its poles. Any other ray passing obliquely through the optical center of the lens is called a secondary axis. All rays of light that pass through the optical center of a lens pass out parallel to the path of incidence, but displaced as ray $A B^{\prime}$. Convex lenses act upon waves of light that pass through them as two prisms placed base to base. The rays (lines of propagation of the light) $A$ and $B$ enter the lens $S$. Kay $A$ strikes the upper prism and is refracted down; ray $B$ strikes the lower one and is refracted up. The two rays are therefore brought together at the point $F$. The point to which waves of light converge after passing through a convex lens is called a focus (real). If the waves are plane waves (parallel rays) they are brought to a focus at a point posterior to the lens, called the principal focus, and a vertical plane through this point, is the principal focal plane.


A Train of Plane Waves from a Distance Entering a Biconvex Lens.
The wave-fronts, $a, b, c, d$, progress with equal velocity throughout their length until they reach the obstruction offered by the lens.

The portion of the wave-front that enters the lens first is retarded in its progress before the peripheral portions. The waves are then caused to assume the form within the lens, as shown in the figure $(e, f, g, h)$. On emerging from the lens the waves are given a still greater curvature (as $i, j, k, l$ ), converging towards $F$, their focus.

The distance of the principal focal point from the optical center of the lens is called the focal distance or focal interval, and marks the strength of the lens. There are two systems in use for the numeration of lenses, namely, the inch and the metric system. The former is the older method of nomenclature. According to this system, a lens was numbered according to the length of its radius of curvature, assuming that its index of refraction was $1.5\left(\frac{3}{2}\right)$. A double spherical lens of such an index has a focal length equal to its radius of curvature. This results from the formula $F=R / 2\left(v / v^{\prime}-1\right)$, in which $F$ stands for the principal focus, $R$ for the radius of curvature, $v / v^{\prime}$ for the index of refraction of the glass and 1 the index of air. (See the deduction of this formula.) If $R=12, v / v^{\prime}=1.5$, the length of focus will be $F=12 / 2(1.5-1)=12$, that is, the focus is equal to the radius of curvature. Glass does not have the index of I.5, but, according to Nagel, varying from 1.52 to 1.55, and, according to Javal, r.54. If we substitute the last value in the formula, we have $F=12 / 2($ I. $54-1)=1$ I.I, that is, the focal length is less than 12 inches, nearly 11 inches. Again, the size of the inch varies according to its nationality ; thus the English inch is 25.3 mm .; the Austrian, 26.34 mm .; the Prussian, 26.15 mm . ; the French or Paris inch, 27.07 mm . Thus, the French and the English inch differ by $\frac{1}{12}$, there being about 37 French inches to 40 English inches. In the inch-system a lens with a focal distance of 1 inch is taken as the standard. The stronger the lens the shorter its focal distance and vice versa. A lens whose focal distance is 4 inches is $\frac{1}{4}$ as strong as the lens taken as the standard, and is called therefore a $\frac{1}{4}$ lens or a 4 -inch lens. Accordingly, the strength of a lens in the inch-system is expressed by a fraction, the numerator of which is I , the standard, and the denominator the focal length of the lens. Thus:
$\frac{1}{2}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}$, or a 2 -, 5 -, 6 - or 8 -inch lens. When the metric system of mensuration was introduced it was soon adopted by Monoyer in numbering lenses. In the metric or dioptric system a lens with a focal length of 1 m . is taken as the standard, and is called a I-diopter lens. A lens that has a focal interval of 20 cm . is five times stronger, and is called a 5 -D. lens $(100 \div 20=5 \mathrm{D}$.). Diopter is expressed by the letter D , and we combine with it the sign of the lens and its kind; thus, $a+2$ D. S. (diopter spherical), a -2 D. S., etc. Cylindrical lenses are numbered in the same way; thus, $\mathrm{a}-2 \mathrm{D}$. C. (diopter cylindrical). As a diopter is equal to about 40 of our inches (accurately, 39.37) it is easy to pass from one system to the other, by dividing the known strength of the lens in one system into 40. Thus: Convert the following lens strengths in the inch-system into their equivalents in the metric system : $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{8}$.
Ans. $\quad \frac{2) 40}{20 \mathrm{D} .}, \quad \frac{3 \lcm{40}}{13+\mathrm{D} .}, \quad \frac{5 \longdiv { 4 0 }}{8 \mathrm{D}}, \quad \frac{8) 40}{5 \mathrm{D} .}$.

Express the equivalents in the inch-system of the following: 2 D ., 5 D., 8 D.
Ans.
2) $\frac{40}{20 \mathrm{in} \text {. }}$,
5) $\frac{40}{8 \mathrm{in} .}$,
8) $\frac{40}{5 \mathrm{in}}$,
or $\frac{1}{20}-, \frac{1}{8}$ - and $\frac{1}{5}$-lens.
If the source of light is nearer the lens than 20 ft . or 6 m ., but beyond the principal focus, diverging rays (positive waves of light) strike the lens, and are brought to a
 focus posterior to the principal focus. This is a secondary focus, called also the posterior conjugate focus, the source of the light being the anterior conjugate focus. The conjugate foci bear the relation to each other that if light emanates from either it will be focused at the other. $P$ and $P^{\prime}$ are anterior and posterior conjugate foci of lens ; $F$ and $F$, anterior and posterior principal focal points.

If the light emanates from a position closer to the lens than the principal focus, it is not brought to a focus at all on emerging from the lens. The refracted rays are rendered relatively convergent only; that is they are less divergent after refraction than before.
$F$ is the principal focus of lens $L$; $a, b$ and $c$ are diverging rays of the incident waves. They are rendered less divergent only, taking the paths $a^{\prime}, b^{\prime}$ and $c^{\prime}$. Point $O$ on the same side of the lens as the
 source of light, and from which the rays of light appear to emanate after refraction, is called the virtual focus. The foci mentioned before have been real foci. Real foci are those produced by the actual crossing of rays of light after refraction and can be caught upon a screen. Virtual foci are points from which light seems to proceed after refraction. They cannot be caught upon a screen and have an existence to the eye of an observer only. Light that has been rendered convergent by a convex lens is rendered more so by a second convex lens. We have then the third law-that converging rays of light are brought to a focus,
 anterior to the principal focus.

Rays $A, B$ and $C$ have been rendered convergent by passing through a convex lens. Entering lens $L$ they are rendered more so and finally caused to gather about or to focus at the point $f . F$ is the point of the principal focus. All foci other than principal foci are called secondary. The following laws are very important in refraction work, and should be studied closely.

1. Parallel rays of light are brought together posterior to the lens at a point called the principal focus. Conversely : Light that emanates from the principal focus of a convex lens passes from the lens parallel or plane.
2. Diverging light is focused posterior to the principal focus, and the more diverging the further posterior. Conversely : Light eman-
 ating from a point further from a convex lens than the point of principal focus passes from the lens converging.
3. Converging rays of light are focused anterior to the principal focus. Conversely: Light emanating from a point nearer the lens than the principal focal point passes from the lens diverging.

Rays $A$ and $A^{\prime}$ are parallel and $F$ their focus.
Rays $B$ and $B^{\prime}$ are divergent and $f^{\prime}$ their focus.
Rays $C$ and $C^{\prime}$ are convergent and $f$ their focus.

## CONCAVE SPHERICAL LENSES.

Concave spherical lenses, from the way that they affect light passing through them, may be regarded as composed of two prisms, placed apex to apex at the center of the lens. Parallel light is rendered divergent by passing through a concave spherical lens. The point from which the light appears to emanate after refraction is the principal focus; it is situated on the same side of the lens as the source of light, as are all foci of concave lenses. Rays that are divergent on incidence are rendered more so by concave lenses, and rays that are convergent are rendered less so by passing through concave lenses. Convex lenses shorten focal distances and concave ones lengthen them. Rays $A$ and $B$, entering the lens, are each bent towards the base of the prism.
$L$ is a biconcave spherical lens ; $P$ and $P^{\prime}$, its anterior and posterior poles ; $O$, its optical center ; $S S^{\prime}$, its principal axis ; $C$ and $C^{\prime}$, the centers of curvature of its anterior and posterior surfaces respectively. Any ray that passes through the optical center but not through the poles of the lens is a secondary axis, as in convex lenses. $\quad R, R^{\prime}$ are rays or radii of the train of plane waves $a, b, c, d$, incident to the anterior lens surface. In passing through the lens they are rendered divergent, that is the waves are rendered positive, as $A, B, C$ and $D$. The refracted waves seem to have their origin at the point $C$, from which they appear to emanate. $C O$ is the focal interval of $L$. Concave lenses are numbered as convex ones are.


The strength of a lens depends upon the radius of its curvature, its thickness, and upon the index of refraction of the substance of which it is made. If the lens is made of ordinary glass its focal length nearly corresponds to its radius of curvature. In planospherical lenses the focal length is the diameter of the circle, of the circumference of which the curved side of the lens forms a part. If both sides of the lens are spherical and equally curved, the focal length is equal to the radius of a circle, of the circumference of which one side of the lens is an arc (if the index of refraction of the lens be considered I.5). If we represent the radius of curvature of the lens by $R$ and the index of the glass by $v / v^{\prime}$ (the ratio between the velocity of light in air and in the refracting medium), and taking the index of refraction of air to be 1 , we have the following formula
to ascertain the focal interval of a double convex lens, neglecting its thickness : $F=R / 2\left(v / v^{\prime}-1\right)$, and $F=R /\left(v / v^{\prime}-1\right)$, for plano-convex lenses.

The method of obtaining this formula is as follows: $L$ is a planoconvex spherical lens, $x$ being its plane surface. $R$ is an incident ray of light. It passes unrefracted to $O^{\prime}$, as it is perpendicular to the plane surface of the lens. $O O^{\prime}$ is the radius of curvature of the lens side, $y . \quad O^{\prime} R^{\prime}$ is the refracted ray, ergo : $C O^{\prime} R^{\prime}$ is the angle of deviation and $B O^{\prime} R^{\prime}$ the angle of refraction. Angle $R O^{\prime} O$ is the an-
 gle of incidence. Angles $R O^{\prime} O$ and $B O^{\prime} R^{\prime \prime}$ are proportional to their sines. $a a^{\prime}$ is the sine of angle $R O^{\prime} O$, and $b b^{\prime}$ the sine of angle $B O^{\prime} R^{\prime}$. Sides $a O^{\prime}$ and $O^{\prime} b$ being equal, $a a^{\prime}$ is inversely proportional to $b b^{\prime}$ as the index of re-
fraction of the lens is to that of the air. Represent the index of refraction of the lens by $v / v^{\prime}$ and that of the air as I , and we have $a a^{\prime}: b b^{\prime}:: 1: v / v^{\prime}$. Angles $R O^{\prime} O$ and $B O^{\prime} R^{\prime}$, being small, are proportional to their sines: $R O^{\prime} O: B O^{\prime} R^{\prime}:: a a^{\prime}: b b^{\prime}$, or substituting $R O^{\prime} O: B O^{\prime} R^{\prime}:: \mathrm{I}: v / v^{\prime}$, we have $v / v^{\prime}\left(R O^{\prime} O\right)=B O^{\prime} R^{\prime}$. If $R O^{\prime} O$ is I and $B O^{\prime} R^{\prime}$ is $v / v^{\prime}$, the angle of refraction $=v / v^{\prime}$ and the angle of incidence $=1 . \quad$ The angle of deviation $=$ angle of refraction - the angle of incidence $=v / v^{\prime}-1$.

In $\triangle S O O^{\prime}$ and $S O^{\prime} R^{\prime} ; \angle R O^{\prime} O$ and $O^{\prime} O S$ are equal and $\angle C O^{\prime} R^{\prime}$ and $O^{\prime} R^{\prime} S$ are equal. The side $S$ being common we have:

$$
O^{\prime} O S: O^{\prime} R^{\prime} S:: R^{\prime} S: S O . \quad O^{\prime} O S=1, O^{\prime} R^{\prime} S=i / v^{\prime}-1
$$

$O S=R$ (radius). $\quad S R^{\prime}=F$ (focal length). Ergo:

$$
\begin{gathered}
\mathrm{I}: v / v^{\prime}-\mathrm{I}:: F: R . \\
F=\frac{R}{v / v^{\prime}-1} .
\end{gathered}
$$

The focal length of a plano-convex lens is equal to its radius of curvature divided by the index of refraction of the lens - 1 .

In the double convex lens the lens is twice as strong and the focal interval half as long, therefore the formula is :

$$
F=\frac{R}{2\left(v / v^{\prime}-1\right)} .
$$

The deduction is not so simple if the lens has thickness. In the preceding formula it was supposed that the lens had no thickness; that a line joining its centers of curvature was its axis, and the optical center of the lens was the point it cut in passing through the lens. If the lens has thickness, we must suppose that the focus of - the first surface is the object for the second surface and take into account the distance between the surfaces.

Let us designate the radii of curvature of the two surfaces as $R_{1}$ (for the first) and $R_{2}$ (for the second).

Incident parallel rays to the first surface are refracted towards the posterior focus, the distance of which is

$$
-\frac{v / v^{\prime} r}{v / v^{\prime}-1} . *
$$

As this point is behind the posterior surface it is considered negative. In the formula $F_{1} / f_{1}+F_{2} / f_{2}=1, f_{1}$ is therefore equal to

$$
\begin{gathered}
-\frac{v / v^{\prime} R_{1}}{v / v^{\prime}-\mathrm{I}} \\
\frac{v / v^{\prime} R_{2}}{v / v^{\prime}-\mathrm{I}}
\end{gathered}
$$

and $F_{2}$ of

$$
\frac{R_{2}}{v / v^{\prime}-1} .
$$

## Ergo :

* When rays pass from a denser into a rarer medium we replace $v / v^{\prime}$ by $\frac{1}{v / v^{\prime}}$ in the formulæ.
or

$$
\begin{gathered}
-\frac{R_{2}}{R_{1}}+\frac{R_{2}}{\left(\frac{v}{v^{\prime}}-\mathrm{I}\right) f_{2}}=\mathrm{I} \\
\frac{R_{2}}{\left(\frac{v}{v^{\prime}}-\mathrm{I}\right) f_{2}}=\mathrm{I}+\frac{R_{2}}{R^{\prime}}=\frac{R_{1}+R_{2}}{R_{1}} \\
\frac{\mathrm{I}}{f_{2}}=\left(\frac{v}{v^{\prime}}-\mathrm{I}_{\mathrm{I}}\right) \frac{R_{1}+R_{2}}{R_{1} R_{2}}=\left(\frac{v}{v^{\prime}}-\mathrm{I}\right)\left(\frac{\mathrm{I}}{R_{1}}+\frac{\mathrm{I}}{R_{2}}\right) .
\end{gathered}
$$

The posterior focus is then deduced by the following formula:

$$
\frac{\mathrm{I}}{F}=\left(\frac{v}{v^{\prime}}-\mathrm{I}\right)\left(\frac{\mathrm{I}}{R_{1}}+\frac{\mathrm{I}}{R_{2}}\right) .
$$

This answers also for the anterior focus, save we must replace $R_{1}$ by $R_{2}$, and vice versa, which does not alter the expression however.

## Formation of images by spherical lenses.

If a convex spherical lens be placed between an illuminated object and a screen, and further from the screen than its principal focus, there will be formed on the screen an image of the object, varying in size according to the relative position of the object and the screen in regard to the lens. At times the image will be larger than the object, and at times smaller as the screen and the lens are moved closer together or further apart and their distance from the object
changed. The image will, however, always be upside down or inverted. If the object is at a distance from the lens greater than twice the focal length of the lens, the image will be found on the opposite side of the lens, smaller than the object, beyond the principal focus and inverted.

In figure let $L$ be a biconvex spherical lens; $A B$, the object ;
 $O C$, greater than $2 F$ (twice the focal interval). Whatever is true in regard to the two points, the extremities of the object, is true of all points in the object $A B$. Draw a ray from each extremity of the object parallel to the principal axis of the lens, Cc. These two rays will be focused at the point $F$ behind the lens, the principal or focus of parallel rays of light. Now draw a secondary axis to the lens from each end of the object, and continue the lines until they cross, behind the lens, the rays from the corresponding point in the object. The rays coming from the same point in the object then cross behind the lens, those from above cross below the principal axis of lens, and those from below cross above the principal axis. The points $a$ and $b$ of image are therefore corresponding points to $A$ and $B$ of the object. The image is therefore inverted and smaller.

$2^{\circ}$. The converse of the first proposition is: Object at a distance less than twice the focal distance of the lens but beyond the principal
focus. In this case the image is on the opposite side of the lens, from the object, inverted and real (it can be caught upon a screen), larger than object, and further from the lens than twice its focal length.
$F$ on each side of the lens is at a distance (from the lens) equal to the focal distance of the lens $L$.
${ }_{2} F$ is at twice the focal distance. Construct figure as for the previous proposition.
$3^{\circ}$. Object at a distance equal to twice the focal length of the lens. Image is on the opposite side of lens, inverted and of the same size as the object, and at the same distance from the lens.

$O$ is object and $i$ the image, each situated at twice the focal distance from the lens.
$4^{\circ}$. Object situated at the principal focus of the lens. There is in this case no image at all, as the rays from each point in the object
 emerge from the lens parallel, and therefore never cross to form foci. The ray from point $A$ of object, parallel to the primary axis of the lens $L$, passes through $F$, the principal focus, on the other side of the lens, but after refraction it and the ray from the same point forming a secondary axis to the lens are parallel. So are all other rays proceeding from a common point in the object.
$5^{\circ}$. If the object is nearer the lens than its principal focus, the image is virtual, erect, larger than the object and on the same side
of the lens as the object. The rays from the head of the arrow after refraction are still diverging. Entering the eye of the observer they appear to have started from the point $a$, the head of the image. The rays of light from the ends of object that are parallel to the

principal axis of the lens are still focused at the point $F$. Convex lenses used this way are often called magnifying lenses. A convex lens used as a magnifying lens is also called a simple microscope.

The magnifying power of any optical instrument is the ratio of the apparent size of the image to that of the object, both being at the distance of most distinct vision.

The magnification of a convex lens is greater : (i) When the object is placed just within the principal focal point of the lens. (2) The stronger the lens (the shorter the focal distance) the greater the magnification. (3) As the observer's distance of most distinct vision is greater.


In figure let $A B$ be the object and $A^{\prime} B^{\prime}$ its image formed at the distance of most distinct vision. $a^{\prime} b^{\prime}$ is the projection of $A B$ upon $A^{\prime} B^{\prime}$. The magnification equals $A^{\prime} O B^{\prime} \mid a^{\prime} O b^{\prime}$, or $A^{\prime} B^{\prime} / a^{\prime} b^{\prime}$, i. e.,
$A^{\prime} B^{\prime} \mid A B$. In the triangles $A^{\prime} O B^{\prime}$ and $A O B$ which are similar, $A^{\prime} B^{\prime}: A B:: D O: C O$.
$D O$ is the distance of most distinct vision, and $C O$ is nearly equal to $F O$, ergo: the magnification equals the ratio of the distance of the most distinct vision to the focal length of the lens. Let $x$ be the angular magnitude of the object seen by the naked eye, and $x^{\prime}$ the angular magnitude of the image whether real or virtual, then the magnification is $x \div x^{\prime}$. This rule applies to telescopes. The magnification spoken of here is linear magnification. Superficial mag. nification is equal to the square of the linear magnification of an object. The magnifying power of a combination of lenses is equal to the multiple of the magnifying power of each. Thus: If the objective of a microscope magnifies 20 times and the eyepiece ro times, the power of the microscope is equal to 200 .

The image formed on the retina of the eye is always smaller than the object and inverted, because the object is always situated at a distance from the eye greater than twice the focal length of the eyeball. Concave spherical lenses form erect, virtual images on the same side of the lens as the object and smaller than the object.

Draw lines $a$ and $b$ from the ends of the object $O$ through the optical center of the lens. Draw a ray from each end of the object to the lens, parallel to the principal axis $P P$, and call them $a^{\prime}$ and $b^{\prime}$. The rays $a^{\prime}$ and $b^{\prime}$ are rendered divergent by the lens, and take the course of $a^{\prime \prime}$ and $b^{\prime \prime}$ after refraction. Project the rays $a^{\prime \prime}$ and $b^{\prime \prime}$ backward and where they cross the rays $a$ and $b$ will be the corresponding end of the image $I$. The relative size of image and object are to each other as their distances from the lens.

In figure (on opposite page) let $L$ be a convex lens; $A B$, the object, denoted by $O ; a b$, the image denoted by $i ; O^{\prime} B$ equal to $D$;
and $O^{\prime} b$ equal to $d$. In the triangles $A O^{\prime} B$ and $b O^{\prime} a$, we have the following relation of sides.
$d: D:: i: O$ or $d / D=i / O$. If any three of the quantities in this proportion are known the other one is easily ascertained. Ascertain the distance of an image from the lens if the image and the object are of the same size, and the object at 100 cm . from the lens. If image and the object are of the same size
 $i / O=1$, and the proportion becomes $d / 100=1$, ergo : $d=100 \mathrm{~cm}$. Let $d=10 ; i=1$; and $O=200$. Find the distance of the object.

$$
\text { Іо } / D=\mathrm{I} / 200, D=2000
$$

If the focal distance of the lens is known, we have the following formula to ascertain the relative distances of image and object. In the above figure, let $B O^{\prime}=D, F^{\prime} O^{\prime}$ and $O^{\prime} F=F ; B D=a$ and $F b=b$.

In triangles $A B D$ and $D O^{\prime} E$ on one side and $G O^{\prime} F$ and $F b a$, on the other, we have $O / i=a / F=F / b$, or $a b=F^{2}$; which can also be written thus: $F / D+F / d=$ I or $1 / D+\mathrm{I} / d=\mathrm{I} / F$, or $1 / d=\mathrm{I} / F$ - I/ $D$. If the image and object are on the same side of the lensthat is, if the image is virtual as always the case in concave lenses, I / $D$ has a negative value, and the formula is written thus:

$$
\mathrm{I} / d=\mathrm{I} / F+\mathrm{I} D
$$

If the ratio between the size of the image and that of the object becomes larger when the position of the object in regard to the lens is altered, so does the image, and vice versa.

In both convex and concave lenses the image and the object move in the same direction when the position of either is changed, that is, when the object is moved towards the lens; in case of convex lenses, the image recedes further from the lens on the opposite side. In concave lenses, when the object is moved nearer the lens, the image
also approaches the lens on the same side. If the object is on the left of the lens in regard to the observer, when it is moved to the right, the image moves to the right in both cases. This is the reverse of what occurs in reflection.

## TO ASCERTAIN THE STRENGTH OF A LENS.

If the lens is a convex one an image of a distant object, one further off than twenty feet, may be focused on a screen and the distance of the lens from the screen measured. In case of concave lenses, we place a flame a great distance from the lens, so that the virtual image of the flame is formed at the principal focus of the lens. We find the position of a screen placed behind the lens, so that the diffusion or luminous circle of the lens has a diameter equal to twice the diameter of the lens. The distance of the lens from the screen equals the focal distance.

The reason why a concave lens is at a distance from a screen equal to its focal distance when the area of illumination on the screen

is just twice the diameter of the lens, is apparent from the diagram.

Let $L$ be a concave lens, $a b$ the extent of the refracting surface ; $R R^{\prime}$, incident rays emerging as $P P^{\prime} ; S$, the screen and $c d$ the diameter of area of illumination and $F$ principal focus. We are to prove that $F O=O O^{\prime}$, when $C D=2 a b$. $\quad C a^{\prime}=a O^{\prime}$, as $a O=a^{\prime} O^{\prime}$, ergo: $a a^{\prime}=F o$, being sides of equal angles, with equal opposite sides. Therefore $F O=O O^{\prime}$.

A convex lens is told from the fact that if an object is looked at through the lens held before the eye and the lens moved to the right the object will appear to move in the opposite direction, to the left and vice versa. The apparent motion of objects viewed through a concave lens when the lens is moved is in the direction that the lens is moved. The reason for these phenomena is that the lenses act like two prisms. As long as the light reaches the eye through the opti-
cal center of the lens it is not deviated, and its apparent corresponds with its true course; but when the lens is moved so as to dislocate the optical center to one side or the other the light reaching the eye from the object is deviated before entering the eye, causing it to appear to proceed from a false position in space. The figures above illustrate.


In the figure the two prisms of which the convex lens $L$ and the concave lens $L^{\prime}$ are composed are shown. The ray of light in each case is bent towards the base of the prism through which it passes, causing the object along $a$ to appear in the direction of $a^{\prime}$. The arrows indicate the direction the lenses have been moved before the eye. Furthermore, if a convex lens be made to approach the eye, the object viewed through it appears to recede, and vice versa. If the lens is concave the object and the lens appear to move in the same direction, that is, when the lens is brought nearer to the eye the object seen through it also approaches the eye. The apparent motion of an object looked at through a spherical lens as the lens is moved backward and forward before the eye is due to an illusion. When a convex lens is brought nearer to the eye the object seen through it appears to diminish in size as its retinal image decreases, which causes the object to appear to recede. The opposite is the case with concave lenses, that is, when they are brought nearer to the eye the image of the object increases, which causes the object to appear to approach. (See article on "Optical Illusions.")

One can determine the radius of curvature of a lens by applying the formula given under the head of mirrors. Knowing the radius and the focal distance, one can, from the following formula, ascertain the index of refraction of a lens :

$$
\mathrm{I} / F=\left(v / v^{\prime}-\mathrm{I}\right)\left(\mathrm{I} / R_{1}+\mathrm{I} / R_{2}\right)
$$

$R_{1}$ and $R_{2}$ being the radii of the two sides respectively.

Spherical lenses are not equally refractive throughout their surfaces. Positive spherical lenses are usually more refractive or stronger near the equator or edge than at the center, while concave lenses usually have the opposite defect, of being stronger at the center. This unequal refraction in different portions of a lens is called spherical aberration. It is said to be positive when it follows the rule in convex lenses, and negative when it follows the rule in concave lenses. A positive lens at times possesses negative aberration, and vice versa. If the aperture of the lens, that is, the angle formed at the principal focal point by drawing lines from the edge of the lens, does not exceed ten or twelve degrees the lens is free of the error of aberration. The degree of aberration increases as the square of the aperture and as the cube of the refracting power of a lens. It is also dependent upon the distance of the object and the form of the lens. A plano-convex lens possesses less aberration than a biconvex if the spherical side is turned towards the incident ray, and more if in the contrary direction. Spherical aberration in a lens may be overcome by placing before the lens a diaphragm with a central opening, thus excluding light from entering the lens save at the more central portions. The iris of the eye performs this function. The best form of lens is what is called a cros'sed or periscopic lens, in which the radius of the posterior surface is about six times that of the anterior. The refraction at 15 mm . from the center of several lenses is as follows: each has at the center a strength of 20 D .

Crossed lens,
21.ID.


$|$| Pl |
| :--- |

Plano-convex with plane surface in front, 23.6 D. 23.8 D.

Aberration may also be overcome to a degree by combining two lenses, situated at a little distance apart, to make up the required strength of lens. The aperture is also less, and it is not necessary to stop a great part of the lens, thus gaining in luminosity of the
image. The error of aberration increases, pari passu, towards the equator of the lens.

In the figure below, the rays $A, B$ and $C$ are brought to a focus on the principal axis at the points $a, b$ and $c$ respectively.


It will be noticed that the convex lens is stronger nearer the edge and the concave one nearer the center. The areas of illumination produced by the intersecting of refracted rays of light are spoken of as Caustics by refraction. There are produced by mirrors Caustics by reflection.

Experiments Demonstrating Spherical Aberration.-Take a plus 20 D.S. lens and upon it place a screen with four openings in it,

the one above the other, along the diameter of the lens. Make the apertures in the screen equidistant. From a distant luminous point
(a small circular opening in an opaque lamp shade) receive the images upon a screen held behind the lens. First placing the screen behind the principal focus of the lens, we have four luminous spots corresponding to the apertures, but inverted. The two central points reproduce the form of the source of light enlarged, but the peripheral ones are elongated in the horizontal direction, especially if the aberration is strong. By moving the screen still nearer, the two central points are blended into one. At this moment the screen is at the focal distance of the central part of the lens, but still beyond the peripheral foci. By altering the position of the screen behind the lens all the phases shown in the figure can be observed.

Where the rays from several holes in the screen on the lens cross, there will be produced only one point of illumination. Reference to the figure will explain. The vertical dotted lines represent the different positions of the screen; the dots on the lines, the number and form of the illuminated areas in the different positions of the screen.

All four images of a candle-flame produced through the openings I, 2, 3 and 4 in the screen over the lens can not be brought into good focus in any one position of the screen placed to receive them, behind the lens. The images produced through the central openings are well defined at a point further from the lens than those through the peripheral openings, showing the lens to be weaker nearer the center than on the edge. If we hold a strong convex lens nearer a screen than its principal focus, the image of a luminous point is a diffusion circle, the edge of the circle being brighter than the center, while if the screen is beyond the point of the principal focus the diffusion circle is brighter in the center. (A diffusion circle is the circular area of illumination on a screen produced by a point of light out of focus.) In figure above we see that the rays of light passing through the convex lens are condensed towards the border, between the lens and the focus and towards the center, beyond the focus. We can also gain some evidence of spherical aberration by the study of the shadows cast through lenses. Hold a convex lens of about twenty diopters in front of a screen beyond its principal
focus. Place a hat-pin against the lens. It will be noticed that the shadow of the pin cast through the lens on the screen is only straight when the pin coincides with the axis of the lens. Otherwise it will be curved with its convexity towards the center of the diffusion circle cast on the screen. If the lens is held closer to the screen than its principal focus, the shadows will have their concavities towards the center of the diffusion circle, but the curve is not so pronounced as before. By referring to the figure illustrating the error of aberration on the preceding page, it will be noticed that the circles of diffusion increase in widths towards the periphery, when the screen is beyond the principal focus, and diminish towards the periphery when the screen is nearer the lens than the principal focus.

No. I represents the lens divided into equal concentric circles, with the pin across its surface. No. 2 represents the circles of diffusion between the lens and its principal focus and as the zones become narrower towards the edge, $a^{\prime}$ is relatively nearer the center than $b^{\prime}$, which gives the shadow its particular curved form. No. 3 represents the circles of diffusion, and the shadows, when the lens is beyond its focus ( $a^{\prime}$ relatively further from the center than $b^{\prime}$ ).

Knowing the position of the concentric circles of the diffusion spots, it is easy to construct the form of the shadow, since the shadow of the point of the pin must be at the same angular distance from the center as the point of the pin itself. A lens with the reversed kind of aberration, that is an overcorrected lens, will show the reversed phenomenon, while an aplanatic one will not show any curving of the shadows. As all spherical lenses act upon light that passes through them as two prisms in the bending of the light, they likewise act as prisms in the dispersion of the light ; hence all lenses possess the error of chromatic aberration. This error or defect
is of small extent in lenses of the size that are used in refraction work, but in optical instruments where great accuracy and detail of images is needed it interferes very much. This evil can be overcome in a great measure by combining with the convex lens a concave lens having a different index of refraction, in the same manner as chromatic aberration is overcome in prisms. The convex lens is usually made of crown glass, and the concave one of flint glass. Such a lens is called an achromatic lens. Chromatic aberration is overcome to a great extent in the eye by the superficial layers of the crystalline lens having different indices of refraction, and also by the combination formed by the biconvex crystalline lens and the concavoconvex vitreous body, having an index of refraction differing from that of the lens. The human eye, however, is not entirely free of chromatic aberration. It has been variously estimated to be from I. 3 D. to as much as 3 D. Helmholtz gives as an average 1.8 D. The dispersion of the eye is a little greater than it would be if filled with water. By passing through a lens, the colored rays are separated, the violet rays being refracted to a greater extent than the red rays, and hence the focus of the former is nearer to the lens. This is the reason that the image formed by a convex lens is bordered with red inside the focus and with blue when beyond the focus. The image of an achromatic lens no longer presents red and blue borders, but there are often traces of green and purple. By combining several glasses of different kinds these colors can be made to disappear as shown by Zeiss.

The chromatism of the eye can be demonstrated by the experiment of Wollaston. A luminous point seen through a prism gives a linear spectrum, but it is observed that we cannot see distinctly the red and the violet end of the spectrum at one and the same time. If the eye is normal in regard to the way it acts upon the light entering it, or emmetropic as such a condition is spoken of, and the luminous point at a distance of twenty feet or more, the red end of the spectrum will appear as a narrow line, while the violet end will be drawn out into a wide band and will often appear to be divided.

If the observer goes nearer, taking care not to accommodate or adjust the eye for the violet extremity, he finds a place where his eye is adjusted for the violet end of the spectrum, while the red end is in turn diffuse. The observer can therefore determine his far-point (the distant point for which the eye is adjusted when at rest) for each end of the spectrum ; the difference gives the degree of chromatic aberration. Fraunhofer determined the distance at which he could see a spider's web suspended in red and in blue light, and finding that the distance differed, arrived at some very accurate results. The chromatic aberration of the eye increases as the pupil is dilated, so it is well to dilate the pupil when studying it. We could correct the chromatic aberration of the eye with a concave lens of flint, exactly as we correct the chromatic aberration of a convex lens made of crown glass. The dispersion of flint glass is about three times that of the eye. As the refracting power of the eye is about 60 D., a concave flint glass of about 20 D . would be needed to correct the aberration. A myope, then, of 20 D . would have at the same time his myopia and his chromatic aberration corrected. An emmetropic eye would need in addition to this a convex achromatic lens of 20 D. to remain emmetropic. Such glass has not been found to increase the visual acuity to any marked degree.

The eye free of spherical aberration sees a circle of light of uniform brightness, while if the center appears more luminous there is present aberration (positive). The edge will appear more luminous if the aberration is overcorrected, or negative, or if the luminous point is within the far-point. If one looks at a circle of light through a convex lens strong enough to render the eye myopic and to cause the luminous source to appear as a circle of diffusion, and holds a hat-pin in front of the eye, the shadow of the pin is seen in ordinary aberration, with its concavity towards the periphery. If the concavity is towards the center the aberration is overcorrected. Upon this principle Dr. Tscherning constructed the little instrument called Aberroscope. It consists of a plano-convex lens on the plane side of which
is a series of lines dividing the space into little squares. The instrument is held io or 20 cm . in front of the eye, to observe whether the lines appear curved or not. Young's optometer enables us to measure the amount of aberration of the eye directly. The optometer has the form of a little rule. On one side is drawn a fine white line on a black background. We look along this line through a lens of plus 1o D. In front of the lens moves a small horizontal rule in which are different groups of slits.

Placing the two slits that are in the middle of the slide in front of the lens, causes the white line on the rule to appear doubled, except where the line is seen distinctly. If the $E$. is not using his accommodation, the lines will appear to intersect at the
 far point of the objective lens or at 10 cm . from the eye. To measure the refraction the distance between the point where the lines intersect and the lens is measured in cm . and divided into 100 cm . to obtain the refraction in diopters. We can determine the near point of accommodation in the same manner ( $q . v_{0}$ ). The other groups of slits are used to measure the refraction in different parts of the pupillary area, or the square opening and the slide $b$, used. By lowering the rule " $b$ " more or less in front of the eye more or less of the center of the pupillary space is excluded from the visual act. The difference in the refraction between the central and the peripheral parts of the pupil gives the amount of spherical aberration resident in the eye. By rotating the instrument around its longitudinal axis the refraction can be measured in different meridians, and thus astigmatism detected. In this manner Thomas Young detected astigmatism in his own eyes.

Cylindrical lenses are either convex or concave. A convex cylindrical lens may be thought of as a vertical surface-section of a cylinder of glass, and a concave cylinder as a section of the mould in which the + cylinder was formed. A cylindrical that at right angles to its length and plane or lens like the cylinder is curved in one
direction, straight in the direction of its axis. A line passing through the middle of the lens parallel to its plane sides is called the axis of the cylinder.
$a b c d$ is a cylinder. aef-cgh, a surface section, or a convex cylindrical lens, and $i j l k-m n o p$, a concave cylindrical lens of the same strength.

Waves of light that enter the cylindrical lens parallel to its axis pass through unrefracted. Waves that enter at right angles to the axis are rendered convergent or divergent, according to the kind of cylinder it is, whether convex or concave. The refracting power of a cylindrical lens in the meridians, oblique to the axis, increases regularly from the vertical to the horizontal meridian.
$A B C D$ is a convex cylindrical lens; $a b c d$, a train of plane waves entering the lens parallel to the axis.
 They pass on through unchanged as $a^{\prime}, b^{\prime}, c^{\prime}$.

Waves $1,2,3$ are transverse, and on passing through the lens are
 rendered convergent to the point $R^{\prime}$. $R R^{\prime}$ is the line of propagation of both the vertical and horizontal waves of light. $S$ and $S^{\prime}$ parallel rays of crosswaves.
$A B C D E F$ is a concave cylinder: $a b c d$, a train of plane, vertical waves, passing through the lens unaltered.

The cross waves i, 2, 3 are rendered divergent on passing through the lens, as their rays will show, in the figure. $R R^{\prime}$ is a ray common to both the vertical and horizontal waves.

The focus of a cylindrical lens is a straight line, that is, every point of light is focused as a line. There can therefore be no image formation by cylindrical lenses. The focal line of a cylinder is always

parallel to the axis of the cylinder. Cylinders are numbered like spherical lenses according to the degree of curvature of their curved surfaces or according to their focal length. The position of a cylinder is described according to the angle at which its axis stands in regard to the horizontal. Thus: If the axis is vertical, we say the cylinder is at 90 degrees, and when the axis is horizontal ; at or i 80 degrees, and so on.
$A B C D$ is a convex cylinder, $O$ a point from which light proceeds. Lines $a$ and $d$ are rays of waves that proceed from the point $O$ and enter the lens parallel to its axis, and are not refracted but continue to diverge as $a^{\prime}, b^{\prime}$, and illuminate screen $S$ along the line $a^{\prime \prime} b^{\prime \prime}$. The dotted lines to the cross meridian of the lens represent waves that enter the lens at right angles to the axis. They are converged and come together at the point $O$ on the line $a^{\prime \prime} b^{\prime \prime}$. All light that enters the cylinder between these positions is focused as short lines running in the direction of the meridian that the light entered. These short oblique lines overlap and aid in building up the long line $a^{\prime \prime} b^{\prime \prime}$.

This figure represents the shape of the illumination on the screen, light entering the cylinder, in meridians oblique to its axis. The heavy lines or rays in each case are those nearest the observer, as he looks at the figure. $O^{\prime}$ is the focus of $O$, through the oblique meridians, but as these merid-

ians are feebler than the horizontal meridian, their focus lies behind the focus of the latter, so that while the focus of the point $O$ will be on the screen in the position represented, the screen will be illuminated in the direction of the oblique lines $a^{\prime}$ and $b^{\prime}$, through the oblique meridians $a$ and $b$.

If two or more spherical lenses are placed together the strength of the combination will be equal to the sum of the combined spheres.
 Thus a $+_{2},+_{3}$, and $+_{4}$ D. S. placed together, equals a lens of $+_{9}$ D.S.

A convex and a concave lens of the same strength neutralize each other, and such a combination acts upon light that passes through it as a plane glass. We use the sign $\bigcirc$ to express the fact that one lens is to be combined with another. Thus:- I D. S. (combined with) +2 D. S. $=+$ I D. S.

When a spherical and a cylindrical lens are combined, we call it a sphero-cylindrical combination. The strength of such a combination is equal to the strengths of the combined lenses in the direction at right angles to the axis of the cylinder, while the cylinder does not alter the strength of the sphere in the direction parallel to its axis. Sphero-cylindrical combinations in spectacle lenses are arranged as follows: If the combination is a positive one; and the cylinder is weaker than the sphere, it is ground on the side of the glass that is to be placed next to the eye, while the sphere is ground on the other side. If the cylinder is stronger than the sphere, the spherical side of the lens is placed next to the eye. If the combination is negative, the opposite method is pursued, thus : the spherical side is placed next to the eye whenever it is stronger than the combined cylinder, and vice versa. In the figures below, the number at the end of the dotted lines represents the strength of the sphero-cylinder combination in the direction of the axis of the cylinder, and the number at the
end of the continuous line, the strength of the combined lenses at right angles to the axis of the cylinder.

$$
\begin{aligned}
& +5 \mathrm{D} . \mathrm{S} . \frown+1 \mathrm{D} . \mathrm{Cyl} . \mathrm{Ax} \cdot 90^{\circ}= \\
& 2 \text { D. S. } \frown-\mathrm{I} \text { D. Cyl. Ax. } 180^{\circ}=+60 \\
& +4 \text { D. S. } \frown+3 \text { D. C. Ax. } 45^{\circ}=
\end{aligned}
$$

Never combine a convex sphere with a concave cylinder, or vice versa, if it is possible to avoid doing so, as such lenses are more difficult to grind accurately; the lens cannot be made as thin, and the combination possesses more aberration than when a sphere and cylinder of like signs are combined. Thus instead of ordering +3 D. S. - - I D. Cyl. Ax. $90^{\circ}$ its equivalent in terms of plus signs should be substituted.

$$
\begin{aligned}
& +_{3} \text { D.S. } \odot-1 \text { D. C. Ax. } 90^{\circ}=+2 \text { D.S. } \odot+1 \text { D. C. Ax. } 180^{\circ} \\
& +_{1} \text { D.S. } \odot 1_{1} \text { D. C. Ax. } 45^{\circ}=+1 \text { D. C. Ax. } 135^{\circ} .
\end{aligned}
$$

Toric Lenses.-A toric lens is one which has a cylindrical and spherical surface on the same side. It is a section of a tore. A tore is a large ring used at the base of a column. A hard rubber ring, such as is given to teething children, represents a tore. The surface into which a positive toric lens will fit is a negative toric lens. Torical lenses are more periscopic, freer of aberration than the usual sphero-cylindrical combination, giving a clearer and a flatter field of view. It is spherical aberration in a lens that causes the edges of objects looked at through them to appear blurred, elevated or curved while the middle of the object is well defined and vice
versa. The torical lens is at present little used and is expensive. It is best adapted to correcting the refraction after cataract extraction. If the spherical surface in a toric lens is stronger than the cylindrical, we speak of it as a sphero-toric lens, while if the cylindrical is the stronger - a cylindro-toric lens.
To convert a sphero-cylindrical lens into a sphero-toric, divide the greatest meridian in half for the sphere. Subtract this from each of the meridians in turn for the strength of the two toric curvatures. For example take: +8 D. S. $\bigcirc+2$ D. C. Ax. $90^{\circ}$.
If we divide the greatest meridian in half we have +5 D . as the power for the spherical surface of one side of the lens. Subtract 5 from 8 and +3 will be strength of one toric curve upon the other side of the lens. Subtract 5 from io and +5 D. will be the power of the other toric curve. When the cylindrical element is the stronger as in the following +2 D. S. $\bigcirc+4$ D. C. Ax. $90^{\circ}$, the strongest meridian is halved for one surface which in this case will be cylindrical ground upon one side of the lens; the other half will be the strength of one toric curve ground upon the other side of the lens, and the weaker meridian the other toric curve. The sphero-toric equivalent of the sphero-cylindrical combination taken as example ( $+8 \mathrm{D} . \mathrm{S}$. $\bigcirc+2$ D. C. Ax. $90^{\circ}$ ) is as follows: +5 D. S. $O$ toric +3 vertical and +5 horizontal. And the cylindro-toric: $(+2$ D. S. $\bigcirc+4$ D. C. Ax. $\left.90^{\circ}\right)+3$ D. C. Ax. $90^{\circ} \bigcirc$ toric +3 horizontal and +2 vertical.

One of the most useful apparatuses for the study of the laws of refraction and reflection is the one shown in the cut. It consists of a graduated brass circle mounted in a vertical plane upon a tripod. Two slides move around the circumference; on one of them there is a piece of ground glass, $P$, and on the other is an opaque screen, $S$, in the center of which there is a small aperture; fixed on the latter slide there is also a mirror, $M$, which can be more or less rotated but remains always in the plane at right angles to that of the graduated circle. Lastly, there is at the center of the circle a small stage $O$, upon which a mirror may be placed for studying reflection or replaced
by a hollow semicircular vessel, for the reception of liquids for the study of refraction. The following method of using the instrument is taken from Ganot's "Physics." To study refraction: The stage in the center of the graduated arc is replaced by a semicircular vessel filled with water until its surface is at the height of the center of the circle. The mirror $M$ is then so inclined that light passes through the central opening in the screen $S$, and falls upon the surface of the water at the center of the circle. The light suffers refrac-
 tion on entering the water, but passes straight out of the vessel below, as it is perpendicular to the curved side of the vessel. The arm carrying $P$ (a piece of ground glass) is then moved down until the refracted ray is received upon it at its central point. The sines of the angle of incidence and of the angle of refraction are read off from the arms $I$ and $R$, respectively, which move around the circle always at right angles to the vertical line that passes through the center of the circle. To study reflection, a mirror is placed at the center of the graduated circle, with its surface horizontal. Light is conveyed to it through the central opening in the opaque screen, $S$, and the amount of the angle of incidence and of the angle of reflection read off from the circumference of the circle after the arm carrying the screen $P$ has been placed in position to receive the reflected ray of light, upon the center of the screen $P$. In the above cut, $x c c^{\prime}=$ angle of incidence ; $c^{\prime} c y=$ angle of reflection ; $x^{\prime} c y^{\prime}=$ angle of refraction.

## CHAPTER III

## MIRRORS AND THE REFLECTION OF LIGHT

A mirror is any polished, smooth surface that separates two media of different densities, and which shows by reflection the image of an object presented to it. By the reflection of light is meant the sending back or returning of light when it meets a polished smooth surface. At the surfaces of all refracting media, light undergoes both refraction and reflection, each interfering with the distinctness of image formation by the other. It is impossible to make a refracting medium that will not have some power in reflecting light. For, if the surface of the medium be made rough or its polish be otherwise interfered with, the substance becomes nearly opaque from the irregular refraction and reflection on its surface. The substance of which a mirror is made must be opaque if we wish to obtain the best image by reflection. If the mirror is transparent, part of the luminosity of the image is lost by a portion of the light from the object passing through the mirror. As a rule mirrors are made of glass backed with amalgam, or of a highly polished metal. Mirrors are divided according to the shape of their reflecting surfaces into plane, concave and convex. The light that comes to a mirror is the incident light, and, after it is returned by the mirror, the reflected light. The angle of incidence is that formed between the normal or radius to the wave-front and that of the mirror, at the point of incidence ; the angle of reflection, as the angle between the normal of the reflected ray and that of the mirror. The two following laws are observed in the reflection of light :
(1) The angle of reflection is always equal to the angle of incidence, and (2) the incident and the reflected rays occupy the same plane. In the figure below $M$ is the face of a plane mirror ; $a, b, c, d$ are
plane waves, incident to the mirror ; $a^{\prime}, b^{\prime}, c^{\prime}$ and $d^{\prime}$, the same waves after reflection.

Angle $I N O$, angle of incidence ; $O N R$, angle of reflection. If the waves are parallel to the face of a plane mirror on incidence, the
 light is returned back along the same course. Under these conditions the value of the angle of incidence is zero, and the angle of reflection is likewise zero. A plane mirror does not alter the relation of rays of light that fall upon it. If they were parallel before incidence, they are so after reflection. If divergent or convergent, they are equally so after reflection. The action of the mirror is to change the direction of the light in reflection, so that it appears to start from a point behind the mirror, its virtual focus.

The image formed by a plane mirror is upright, of the same size as the object, virtual, on a line at right angles to the mirror from the object, and behind the mirror at a distance equal to the distance of the object in front of the mirror.

- $M$ is a plane mirror ; $O$, an object presented to it. Draw rays $a$ and $b$, from the head of the arrow to mirror. Erect perpendiculars at the points of incidence. Make the angles of reflection equal to the angles of incidence. Project the reflected rays back behind the mirror and where they come to a virtual focus is the point in the image corresponding to the head of the object (arrow). We can ascertain a point in the image corresponding to any point in the object by drawing a line per-
 pendicular to the mirror (extending its length, if necessary, in order to get the perpendicular line to strike it) from the given point in the object, and measuring off on this line posterior to the mirror a distance equal to the distance of the point in the object in front of the
mirror. When a mirror is rotated, that is when its normal (a line perpendicular to its surface) is made to incline in different directions, the reflected ray passes through twice the angle of rotation of the mirror.

Let $x$ be an incident ray to the mirror $M$, at the point $O$. Being perpendicular to the mirror, it is reflected back along the same path. Rotate
 $M$ to $M^{\prime}$. Normal $x$ moves to $y$, through angle $x o y$, the angle of rotation of the mirror. Ray $x$ passes off in the direction of $x^{\prime}$, making angle $x o y$ equal to angle $y o x^{\prime}$. While normal $x$ has moved through angle $x o y$, ray $x$ has moved through $x o x^{\prime}$, which is twice angle xoy. The image formed by a plane mirror moves in a
 direction opposite to the rotation of the mirror.
$I$ is the image of $O$, when mirror is in position $M$. When the face of mirror is inclined upwards, the image of $O$ descends to $I^{\prime}$, and vice versa.

Spherical mirrors are those whose surfaces are portions of hollow spheres. If the concave side is turned towards the object, the mirror is concave and if the convex side is next to object we have a convex mirror. In each case the center of the sphere, of which the mirror forms a part, is the center of curvature of the mirror. The central point on the surface of a mirror (spherical), is called its vertex. A line joining the center of curvature and the vertex is the principal axis. Any line passing through the center of curvature, but not through the vertex, is a secondary axis of the mirror. The radius of curvature is the radius of the sphere of which the mirror forms a part, a line drawn from the center of curvature to the mirror.

Mirror $M$ is concave to object $A$; $C$ is its center of curvature ; $C a$, its principal axis ; and, $C b$, a secondary axis. $M^{\prime}$ is convex to $A$,
and $C a^{\prime}$, its principal axis ; $C b^{\prime}$, a secondary axis. Spherical mirrors may be supposed to be made up of a number of plane mirrors, perpendicular to the radii of curvature of the mirror.

REFLECTION FROM AND IMAGE FORMATION BY CONCAVE MIRRORS.
When parallel light strikes a concave mirror, it is reflected from the mirror convergent, and is gathered to a point, called the principal focus of the mirror. The distance of this point from the mirror marks the strength of the mirror. If light emanates from the principal focus
 it is reflected from the mirror parallel. As in convex lenses the more divergent the rays of light that impinge upon the mirror, the further off is its focus. Divergent rays coming to a mirror from a near point are focused beyond the principal focus.

This secondary focus and the point from which the light emanates are to each other as conjugate foci.
$M$ is a concave mirror ; $R C$, its radius of curvature ; $a, b, c$, etc., a train of plane waves, advancing towards the mirror. They are reflected back as negative waves $a^{\prime}, b^{\prime}, c^{\prime}$, converging to the point $F$, the principal focus of the mirror. $F C$ is the focal length of the mirror.

Convex or positive waves emanate from the point $C$. They pass to mirror, upon which the ends of the rays impinge first. The waves after reflection are hence rendered
 converging to the point $C^{\prime}$, the conjugate focus of the point $C$. $F$ is the point of the principal focus. Concave mirrors form real images, inverted, and on the same side of the mirror as the object, so long as the object is further from the mirror than its principal focus. If the object is nearer the mirror than the principal focus the image is larger than the object, upright, virtual and behind the mirror.
$M$ is a concave mirror; $O$ an object, presented to it. To find the position of the image, draw rays $a$ and $c$ from the head of the object to the mirror. Ray $c$, passing through $C$, the center of curvature of the mirror, strikes the face of the mirror perpendicularly, and hence passes back along the course of incidence. Ray $a$, being parallel to the principal axis $b$ of the mirror, is reflected to the principal focus $F$. Where ray $a^{\prime}$ crosses the ray $c$, is formed the head of
 the image $I$, the tail of the image being on the principal axis.

## OBJECT NEARER THE MIRROR THAN THE PRINCIPAL FOCUS.

$F$ is the principal focus; $O$ is nearer the mirror than $F$. Draw from the head of the object ray $a$ parallel to the principal axis of the mirror ; it will be reflected through the point $F$. Draw ray $b$ diverging to mirror ; it will be reflected and focused at a point further from the mirror than the principal focus. Where reflected rays $a^{\prime}$ and $b^{\prime}$ cross will be found the head of the image $I$. As the foot of the object rests upon the principal axis, the foot of the image will do likewise. If the object is beyond the center of curvature of the mirror, the image is smaller than the object and is situated between the principal focus and center of curvature of the mirror. The converse is also true, that if the object is situated between the principal focus and the
 center of curvature, the image will be larger than the object and situated beyond the center of curvature. The image will be inverted in each case. (See figure above.) The light that emanates from the center of curvature of a concave mirror strikes one of the plane mirrors of which the spherical mirror is made up, and is reflected back along
the same path. There is consequently no image formed as image and object overlap each other. As in lenses, so in mirrors, the shorter the focus the stronger the mirror. The shorter the radius

of curvature, the smaller the image formed by the mirror. The focal interval of a concave mirror is equal to one half the radius of curvature, expressed :

$$
F=R / 2
$$

In the figure, $p$, is one of the plane mirrors of which the concave mirror $M$ is composed; $C d$, the radius of curvature of the mirror is normal to it ; ray $a$ is parallel to $P P^{\prime}$, the principal axis of the mirror. Angle $i$ and angle $r$ are equal, since the angle of incidence and the angle of reflection are equal. Angle $i=$ angle $x$, being alternate ; Therefore $C F=F d$, being sides of a triangle opposite equal angles. $F d=F P^{\prime}$, and therefore

$$
C F=F P^{\prime} \quad \text { or } \quad F P^{\prime}=C P^{\prime} / 2=R / 2, \quad \text { as } \quad C P^{\prime}=R
$$

The conjugate focus of any point at a greater distance than the principal focus is found according to the following formula:

$$
\mathrm{I} / f+\mathrm{r} / f^{\prime}=1 / F \quad \text { or } \quad \mathrm{I} / f^{\prime}=1 / F-\mathrm{I} / f,
$$

in which $F=$ distance of the principal focus from the mirror ; $f^{\prime}$, distance of conjugate focus or image from the mirror ; and $f$, the distance of the source of light or object from the mirror.

The linear dimensions of an object and its image are to each other as their distances from the mirror.


Let $B A$ be the object, and denote it by $O ; A^{\prime} B^{\prime}$, its image, and denote it by $i$. Let $A F=D$ and $A^{\prime} F=d$ and $F V=F$.

The triangles $B A F$ and $B^{\prime} F V$, on one side and $F V X$ and $A^{\prime} B^{\prime} F$ on the other, give the relations $O / i=D \mid F=F / d$, or $D d=F F$. $O / i=D / F$ can also be written $O / i=2 D / R . *$

Let $A V=f$ and $A^{\prime} V=f^{\prime}$. As $D=f-F$ and $d=f^{\prime}-F$, the formula, that of Newton, $D d=F F$, can be written thus:

$$
F / f+F / f^{\prime}=\mathrm{I} \quad \text { or } \quad \mathrm{I} / f+\mathrm{I} f^{\prime}=\mathrm{I} / F .
$$

or .
$\overline{\text { Distance of object from mirror }}+\frac{\mathrm{I}}{\text { Distance of image }}=\frac{\mathrm{I}}{\text { Focal interval }}$

$$
\begin{gathered}
O: i:: A F: A^{\prime} F \text { and } O: i:: A V: A^{\prime} V . \text { Let } A V=D \text {, and } \\
A^{\prime} V=d \text {, then } O / i=D / d .
\end{gathered}
$$

A mirror has a $20-\mathrm{cm}$. focus; light comes to it from a distance of 30 cm . How far off is the conjugate focus?

$$
1 / f^{\prime}=1 / F-1 / f=1 / 20-1 / 30=1 / 60, \quad f^{\prime}=60 \mathrm{~cm} .
$$

Convex Mirrors.-All rays of light impinging upon a convex mirror are rendered divergent. If they were divergent on incidence, they are more so after reflection. The foci and images of convex spherical mirrors are virtual, and back of the mirror. The images are smaller than the objects.

Let $C$ be the center of curvature of the convex mirror $M$; $A B$, an object ; $C C^{\prime}$, the principal axis of the mirror. To locate image draw ray $i$ parallel to the
 axis $C C^{\prime}$; it will be reflected off in the direction of $x$, having a virtual focus at $F$ behind the mirror. Draw $i^{\prime}$ so that it will pass through the center of curvature of the mirror. It will then strike the mirror at right angles and be reflected back along the same path. The reflected rays appear to originate from

* This formula is the one used in ophthalmometry.
the point $A^{\prime}$ behind the mirror, the point corresponding to the point $A$ of image. As in concave mirrors the focal interval is equal to one half of the radius of curvature. Conjugate focal distances are ascertained for convex mirrors by the same formula as for concave mirrors, save a minus sign is placed before $F$ and $f^{\prime}$, as both are virtual and have a negative value, being behind the mirror. The formula becomes then: $1 / f-\mathrm{I} / f^{\prime}=-\mathrm{I} / F$. Find the conjugate focus of a point at 60 cm . in front of a convex mirror of 20 cm . radius of curvature. $F=10, f=60$.
$\mathrm{I} / f-\mathrm{I} / f^{\prime}=-\mathrm{I} / F, \quad \mathrm{I} / f=-\mathrm{I} / \mathrm{Io}+\mathrm{I} / 60=-5 / 60 ; f=-12 \mathrm{~cm}$.
The conjugate focus is 12 cm ., behind the mirror.
From the following proportion it is easy to calculate the radius of curvature of a convex mirror, or any other of the unknown quantities in the proportion, if the remaining three quantities are given.


## $1 / 2$ radius of curvature $=$ Distance of object from the mirror . <br> Length of image

See page 53, formula $O / I=D / F$.
If one half the radius of curvature $=F$ (focal length of mirror) ; length of image, $I$; distance of object from mirror, $D$; length of object $O$; we have

$$
F=\frac{I \cdot D}{O} .
$$

Example: What is the focal length of a mirror, if an object 1 cm . in size, at a distance of 10 cm ., is reduced in the image to 1 mm .?

$$
F=\frac{I \cdot D}{O}=\frac{100 \mathrm{~mm} .}{10 \mathrm{~mm} .}=10 \mathrm{~mm} .
$$

Ascertain the size of an image of an object 1 cm . in size, at a distance of 10 cm . formed by a mirror of 10 mm . focus.

$$
\begin{gathered}
F=\frac{I \cdot D}{O}, \quad \text { Io } \mathrm{mm} . \\
\text { ergo }: \quad \frac{I \cdot 100 \mathrm{~mm} .}{10 \mathrm{~mm} .}=I \cdot 1 \mathrm{~mm} .
\end{gathered}
$$

What has been said in the preceding pages about spherical mirrors, applies only to mirrors of small apertures (see aperture of lenses), or to small portions immediately around the axis of the mirror if large. The phenomena of reflection are not so simple about the edge of a large mirror. Mirrors as lenses possess aberration. The peripheral rays of light are brought to a focus anterior to that of the central rays. Every re-
 flected ray cuts the one next to it, and their points of intersection form in space a curved surface which is called a caustic by reflection.

The figure represents a number of parallel rays impinging upon the mirror. Those on the edge are focused nearest the mirror, and those nearest the center furthest from the mirror. The intersection of the reflected rays forms a curved area of illumination, which in section has the shape of the dotted lines.

## CHAPTER IV

## THE EYE, AND THE THEORY OF GAUSS

The human eyeball is more nearly a sphere than the eyeball of any other animal. It is enclosed within a bony socket ; protected anteriorly by the lids ; rests upon a fatty cushion ; held in place by fascia ; moved by six muscles; supplied by many vessels and nerves and provided with an apparatus to keep it moist - the lachrymal apparatus. The eyeball by outside measurements is on the average, antero-posteriorly, 24 mm .; transversely, 23.5 mm ., and vertically 23 mm .

The eyeball consists of three coats or tunics :
$I^{\circ}$. The external fibrous tunic, formed by the cornea and the sclera.
$2^{\circ}$. The middle or vascular tunic, called the uvea, formed by the iris, ciliary processes, and the chorioid.
$3^{\circ}$. The inner or nervous tunic, formed by the retina.
The posterior four fifths of the outer coat of the eye globe is formed by the opaque sclera, in shape conforming nearly to that of a sphere ; the anterior one fifth, by the transparent cornea, which resembles in its curve that of an ellipse. The cornea is not set upon the anterior portion of the sclera, after the manner of a watch-crystal upon the face of a watch, as is so often stated, but the junction of these segments is marked by a broad, shallow, annular groove, the sulcus scleræ. It is on account of this groove that the cornea seems to set upon the sclera. If it were not for this sulcus the curve of the sclera and that of the cornea would be a nearly continuous one. The middle coat is the nourishing tunic of the eyeball. It is formed by the iris in front with its central hole or pupil, by the chorioid behind, and by the ciliary processes between the two. The iris arises
just posterior to the junction of the cornea with the sclera. It rests by its pupillary border upon the crystalline lens which is immediately behind it. The crystalline lens is a bi-convex lens. It is surrounded by its capsule, and swung in its suspensory ligament arising from the ciliary processes. The inner coat lines the interior of the eyeball, posterior to the ciliary processes. The cavity enclosed by these tunics is divided by the lens and its suspensory ligament into two. The, posterior cavity is filled by the vitreous humor (the vitreous chamber), and the anterior cavity, by the aqueous humor. The latter is divided into two by the iris, namely : an anterior chamber, in front of the iris, and a posterior chamber, behind the iris and in front of the lens. Both of these chambers are filled with a watery lymph secreted by the ciliary processes and iris, the aqueous. The anterior and the posterior chambers communicate by means of the pupil. The dioptric media of the eye are the cornea, aqueous humor, crystalline lens and the vitreous humor, which acting upon light that enters the eyeball, bring it to a focus upon the retina. The stimulus is then carried to the brain, where the picture formed upon the retina is interpreted. The dioptric surfaces of the eyeball are four in number, namely: The anterior and posterior corneal surfaces and the anterior and posterior surfaces of the crystalline lens. The posterior corneal surface is usually omitted because the index of refraction of the cornea and of the aqueous are so nearly equal. This surface, however, exercises a definite influence over the refraction of light entering the eye, as its catoptric image is well marked. If the indices of refraction of the cornea and aqueous were identical there would be no image of reflection formed by the posterior surface of the cornea. The indices of the cornea and aqueous differ more than was formerly supposed. The eye in the arrangements of its parts and in the manner of its working resembles the photographer's camera or camera obscura, in which the eyelids represent the stop-shutter ; the iris the diaphragm ; the dioptric system of the eye the focusing apparatus, and the retina, the sensitized plate or film. In the camera the sensitive film or the lens is movable, in order that all objects, at
whatever distance, may be brought to an accurate focus on the film, inasmuch as the nearer the object to the lens the further behind the. lens is its image. In the human eye however the focusing is done by a change in shape, and consequently, a change in the strength of the crystalline lens. This act is called accommodation. It is only needed when the eye adjusts itself for a distance nearer than twenty feet, or six meters, as from beyond this distance the waves of light that reach the eye are so nearly plane that in practice they may be considered entirely so. Dr. Beer, in making a study of accommodation in the lower animals, finds that aquatic animals with highly developed eyes, as cephalopods and long fishes, have eyes that are normally adapted for near seeing. Such eyes undergo active accommodation for distant seeing. The round lens is brought nearer to the retina without changing its shape. The eyes of the terrestrial vertebrates are normally adjusted for distant seeing and undergo active accommodation for near objects.

In amphibia and snakes the unaltered lens is carried away from the retina ; in the rest there is a change in its curvature. In every class of animals except cuttle-fishes and birds there is a certain species whose eyes do not possess the power of accommodation.

Some of these nocturnal in their habits have pupils that contract to a linear form when exposed to light, forming a sort of accommodation. Cave-dwelling and subterranean animals have but poor vision and need no accommodation. No animal can see equally well in water and on the land without the use of accommodation. Aquatic animals are extremely myopic when on land, and terrestrial animals very hyperopic when in water. Animals that alter the accommodation by altering the distance of the lens from the retina, do not become presbyopic as do those that accommodate by changing the shape of the crystalline lens. (See Presbyopia.)

The normally refracting eye is called emmetropic ( $\tilde{\epsilon}^{\mu} \mu \mu \epsilon \tau \rho o s$, in due measure, and ${ }_{\omega}^{*} \psi$, eye), and is denoted by the letter $E$. The retina of such an eye lies at the principal focus of its dioptric system. This eye without accommodation (at rest) is adjusted for distant or plane
waves (parallel rays) of light. By the refraction of an eye we mean the state of its refraction when at rest, the relation of its retina to the principal focus of its dioptric system. If the eyeball is too short from before backward, so that the retina lies anterior to the principal focus of its dioptric system, the eye is far-sighted or hypermetropic, abbreviated to hyperopia, and denoted by the letter $H$ ( $\dot{v} \pi \epsilon \stackrel{\rho}{\rho}$, in excess of, and $\ddot{\omega} \psi$, sight). Such an eye is too little refractive. The opposite condition, where the eyeball is too long, so that the retina lies posterior to the principal focus of the dioptric media of the eye, constitutes myopia, denoted by $M$. The myopic eye is the near-sighted eye. It is so called from the habit that myopes have of squinting ( $\mu v ́ \epsilon \iota \nu$, to close the eyes, and $\ddot{\omega} \psi$, eye).

The myopic eye is over-refractive, in regard to the position of its retina.

In order that the rays of light, as they pass through the dioptric media of the eye, may be traced to the formation of images on the retina, one must know the curvature of the dioptric surfaces, their distances apart and their indices of refraction. As the two surfaces of the cornea in the pupillary area are practically parallel and as the indices of refraction of the cornea and of the aqueous humor are nearly equal, they are usually regarded as one refracting medium.

The refracting or dioptric surfaces of the eyeball then are three in number, namely: the anterior surface of the cornea, the anterior and the posterior surfaces of the crystalline lens. The radii of curvature of each of the above surfaces used for distant vision are : for the cornea, 7.829 mm .; for the anterior surface of the lens, 10 mm .; for the posterior surface of the lens, 6 mm . The indices of refraction of the several media are as follows :

Cornea and aqueous humor, 1.3365 (that of the cornea and aqueous being assumed to be the same);

Lens, I.437I;
Vitreous humor, I.3365.
The distance from the center of the cornea to the center of the anterior surface of the lens is 3.6 mm . The lens is 3.6 mm . thick
(on the average) and the distance of the center of the posterior surface of the lens to the retina 15 mm . Homocentric rays (rays from a point) enter the eye from the air with an index of refraction of 1.00025, into the cornea with an index of 1.3365 . In passing from the rarer to the denser medium, the light is converged somewhat, to the extent that it would be brought to a focus about 10 mm . behind the retina.

On entering the lens the light is further converged, so that it would come to a focus about 6.5 mm . behind the retina. The vitreous still further converges the rays to a focus upon the retina.

If the refracting media are not so thin that their thickness can be neglected nor so close together that their distances apart can be neglected, we find the position of the image by construction or by the rules given for locating the image by a spherical surface: we calculate in the first place, the image formed by the first dioptric surface; this image then serves as the object for the second surface and so on. We can thus follow the rays of light as they pass through the refracting media of the eyeball, but the conditions vary for every distance of the object from the eye, from which the light proceeds. To facilitate such calculations, schematic eyes have been devised. Gauss showed that every complicated dioptric system can, be reduced to, or replaced by, a single dioptric medium, composed of six cardinal

points and six cardinal planes perpendicular to a common axis, e. g., two focal points ; two principal points, and two nodal points. The cardinal planes are planes through the cardinal points at right angles to the common axis (focal, principal and nodal planes). There may be named four cardinal points for every dioptric surface. They are
" $n$," the center of curvature of the surface; " $v$, ," the vertex of the surface; $F^{\prime}$ and $F^{2}$, the first and second principal foci.

Properties of Cardinal Points. - The first focal point has the property that every ray of light that passes through it before refraction is parallel to the principal axis after refraction.

Rays that are parallel on incidence are focused at the second or posterior principal focus $F^{2}$. The second principal point is the image of the first principal point: that is, rays that pass through the first principal point pass through the second principal point after refraction. Planes through these points at right angles to the axis are the principal planes. The first principal plane is the conjugate of the second, or the second principal plane is the image of the first.


The second nodal point is the image of the first nodal point. Every ray that passes through the first nodal point prior to refraction passes through the second nodal point after refraction, and the rays before and after refraction are parallel to each other. The distance of the first focal point to the first principal plane is the anterior focal distance, and the distance of the second focal point from the second principal plane is the posterior focal distance. The distance of the first nodal point from the first focal point is equal to the distance between the second principal plane and the second focal point, or the second focal distance. The distance of the second nodal point from
the second or posterior focal point is equal to that between the anterior principal plane and the anterior focal point or the anterior focal distance. The distances of corresponding principal and nodal points from each other are equal then to the differences between the two focal distances. Also the distance between the two nodal points is equal to the distance between the two principal points. Lastly the focal distances are proportional to the refractive indices of the first and of the last dioptric media. The focal planes pass through the axis at the focal points. The theory of Gauss assumes that there is a centered system, that is that the optical centers of the refracting media are on a common axis. (A condition slightly departed from in the eye.)

From the properties of cardinal points, the position of an image in the last medium may be determined, and the course of the refracted ray in the last medium be known if its course in the first medium is given.

TO FIND IMAGE IN LAST MEDIUM OF AN OBJECT IN THE FIRST.
Let $O$ be an object, then $I$ is its image. Draw ray i parallel to the axis; it will pass through the posterior focal point $F_{2}$.


Draw the nodal ray 2 to the first nodal point $(N)$; it passes through the second unrefracted, simply displaced a little.

Draw ray 3 through the anterior focal point $\left(F_{1}\right)$. It is therefore parallel to the axis after refraction. Where these three rays cross after refraction will be a point in-the image corresponding to the point $A$ in the object.

TO FIND THE COURSE OF A REFRACTED RAY, IN A LAST MEDIUM, OF A GIVEN RAY IN THE FIRST MEDIUM.
Let $I$ be the incident ray. Continue it to the second principal plane parallel to the axis. From any point $(a)$ in the ray, draw lines I and 2. Line I parallel to axis will, after refraction, pass through the second point $F_{2}$. Line 2 is the nodal ray. Where 1 and 2 cross is the image of the point " $a$ ". $a^{\prime} b$ is the course of the ray in the last medium.

Let $V$ represent the in-
 dex of refraction of air which is taken as the standard and assumed to be equal to I , and $V^{\prime}$ the index of any other medium ; $F^{\prime}$ the anterior or first focal point ; $F^{\prime \prime}$, the posterior or second focal point
 and $f^{\prime}$ and $f^{\prime \prime}$ any other two focal points anterior and posterior respectively, and we have the most important formula in all geometrical optics. By it we can obtain the position of the first and second focal points, or these given ascertain the radius of the refracting surface.

$$
\begin{equation*}
V^{\prime}\left|f^{\prime \prime}-V\right| f^{\prime}=\frac{V^{\prime}-V}{r} \tag{I}
\end{equation*}
$$

In the formula $f^{\prime}$ and $f^{\prime \prime}$ are conjugate to $F^{\prime \prime}$ and $F^{\prime}$.
The deduction of the formula is as follows:
In the figure above let $a h$ represent the refracting surface ; $L P$, its axis; $n$, its center of curvature ; * $r$, its radius of curvature; $I R$ an incident ray of light that would pass to the point $P$, if it was not refracted. It is, however, bent to the point $J^{\prime}$. Angle $I=$ angle of incidence ; angle $R$, the angle of refraction.

* Center of $a h$ should be at $n$ instead of at $P^{\prime}$ as in figure.

In triangles reg and $\mathrm{re}^{\prime} g^{\prime}$,

$$
\frac{\operatorname{sine} I}{\operatorname{sine} d}=g / e(a) \quad \text { and } \quad \frac{\operatorname{sine} R}{\operatorname{sine} d}=g^{\prime} / e^{\prime}(b)
$$

Divide equation $a$ by $b$ to eliminate $d$, and we have

$$
\frac{\operatorname{sine} I}{\text { sine } R}=g e^{\prime} / g^{\prime} e .
$$

Letting the index of refraction of the first medium be $V$ and that of the second $V^{\prime}$ as mentioned above, and $V$ however accentuated be equal to i/ $V$, the equation of $\operatorname{Snell}, \sin I / \sin R=v / v^{\prime}$, becomes $\sin I V=\sin R V^{\prime}$.
$V^{\prime} \mid V=g e^{\prime} / g^{\prime} e(3)$. As $e$ is $f^{\prime}$, and $e^{\prime}$ is $f^{\prime \prime}$, the equation becomes $V^{\prime}\left|V=g f^{\prime \prime}\right| g^{\prime} f^{\prime}$. Giving $g^{\prime}$ its value of $f^{\prime}-r$, and $g^{\prime}$ its value of $f^{\prime \prime}-r$, in the figure above we have the following:

$$
V^{\prime} \left\lvert\, V=\frac{\left(f^{\prime}-r\right) f^{\prime \prime}}{\left(f^{\prime \prime}-r\right) f^{\prime}}\right.,
$$

which may be reduced to the following form :

$$
V^{\prime}\left|f^{\prime \prime}-V\right| f^{\prime}=\frac{V^{\prime}-V}{r} . \quad \text { Q. E. D. }
$$

To find $F^{\prime}$, the anterior principal focal point, give $f^{\prime \prime}$ in the formula the value of $\infty$, as the center of curvature of waves brought to a focus at $F^{\prime}$, lies at an infinite distance. i $/ f^{\prime \prime}\left(V^{\prime} / f^{\prime \prime}\right)=0$, and therefore disappears from the equation. We have then $f^{\prime}=V r \mid V^{\prime}-V$, for $F^{\prime}(4)$. To find the second or posterior principal focal point, or $F^{\prime \prime}$, proceed in the same way, giving $f$ the value of infinity. The equation then becomes $f^{\prime \prime}=V^{\prime} r \mid V^{\prime}-V$, for $F^{\prime \prime}$ (5).

Light enters a curved glass surface having a radius of 10 cm . Ascertain the positions of points $F^{\prime}$ and $F^{\prime \prime}$. The index of refraction of the glass equals 1.54, and that of air I. The surface $S$ is positive to waves emanating from the side of $F^{r}$ and negative to those pro-
ceeding from the side of $F^{\prime \prime}$, convex to ray $a$, and concave to ray $a^{\prime}$. Find $F^{\prime}$ and $F^{\prime \prime}$.

$$
f^{\prime \prime}=V^{\prime} r / V^{\prime}-V=I_{1} .54 \times 10 /{ }_{1} .54-1=15.4 / .54=28.5
$$

If the surface is concave to the incident rays $r=-10$,

$$
f^{\prime}=-V r / V^{\prime}-V=-28.5
$$

the - sign showing that the incident rays are negative to surface. If the surface is a mirror $V=-V^{\prime}$ and formula is written thus:

$$
\begin{gathered}
f=-V^{\prime} r / V^{\prime}-\left(-V^{\prime}\right) \\
f=-\frac{-(+) 1.54 \times 10}{\mathrm{I} .54+1.54}=\mathrm{I} 5.4 \mathrm{O} / 3.08=5
\end{gathered}
$$

We have the same value for $f^{\prime}$ and $f^{\prime \prime}$, showing that in spherical mirrors both concave and convex the focal interval is equal to half the radius of curvature. When the first and the second principal foci are known, a simple formula may be deduced to ascertain the position of the conjugate of any focal distance. Multiplying equation (1) by $r$ we have :

$$
r V^{\prime}\left|f^{\prime \prime}-r V\right| f^{\prime}=r\left(V^{\prime}-V\right) / r
$$

Dividing each numerator by $V^{\prime}-V$ :

$$
\frac{\frac{r V^{\prime}}{V^{\prime}-V}}{f^{\prime \prime}}-\frac{r V}{\bar{V}^{\prime}-\bar{V}} f^{\prime}=\mathrm{I}
$$

substitute for each numerator the values given in equations (4) and (5).

$$
F^{\prime \prime} \mid f^{\prime \prime}-F^{\prime} / f^{\prime}=\mathrm{I}
$$

Clearing of fractions and subtracting $F^{\prime} F^{\prime \prime}$ from each side we have $F^{\prime \prime} f^{\prime}+F^{\prime} f^{\prime \prime}-f^{\prime} f^{\prime \prime}-F^{\prime} F^{\prime \prime}=F^{\prime} F^{\prime \prime},\left(F^{\prime \prime}-f^{\prime \prime}\right)\left(f^{\prime}-F^{\prime}\right)=-F^{\prime \prime} F^{\prime}$,

Let $c$ and $c^{\prime}$ represent the distance between the principal foci and the conjugate on the same side, and we will have

$$
c c^{\prime}=F^{\prime} F^{\prime \prime}
$$

In the preceding formulæ $F^{\prime}, F^{\prime \prime}, f^{\prime}$ and $f^{\prime \prime}$ represent the positions of the first and second, principal and secondary foci.


Cardinal points fulfill the same function for a dioptric system that they do for a single dioptric surface. The focal points of a dioptric system are measured from the anterior and from the posterior principal points respectively. Methods of Finding the Cardinal Points of a Given System. - We draw an incident ray parallel to the axis and construct its course as shown on preceding page according to the Law of Descartes or by the formula deduced for refraction by spherical surfaces. The posterior focus is found. The incident and the emergent rays are then prolonged; their intersection is situated in the second principal plane, and a perpendicular let fall from this point to the axis, marks the second principal point. Repeating the same construction with a ray coming from the other side and parallel to the axis, we find after the same manner the anterior principal focus and the first prin cipal plane and point. Knowing these four points we can deduce the position of the nodal points, since the distance of the first nodal point from the anterior focus is equal to the distance of the second principal point to the posterior focus,
 etc.
$I_{1}$ and $I_{2}$ are incident rays parallel to the axis of the system S . $F_{2}$ is the focus of $I_{1} ; F_{1}$ the focus of $I_{2}$. Projecting the incident ray
and the emergent ray until they meet in each case gives us the points $a_{1}$ and $a_{2}$. These points lie in the first and second principal planes respectively. Perpendiculars dropped to the axis of the system from the points $a_{1}$ and $a_{2}$ locate the principal planes, and the principal points where the perpendiculars cut the axis of the system. Calculation. - Designate the two optic systems that we wish to combine by A and B ; their focal distances by $F_{1}^{\prime}, F_{2}^{\prime}$ (for system A) and by $F_{1}^{\prime \prime}$ and $F_{2}^{\prime \prime}$ (for system B), and the distance of the posterior focus of system A, behind the anterior focus of system B, by $d$. The cardinal points of the combined system can then be ascertained by means of the following formulæ:

$$
\begin{array}{ll}
y^{\prime}=\frac{F_{1}^{\prime} F_{2}^{\prime}}{d}, & F_{1}=\frac{F_{1}^{\prime} F_{2}^{\prime \prime}}{d}, \\
y_{2}=\frac{F_{1}^{\prime \prime} F_{2}^{\prime \prime}}{d}, & F_{2}=\frac{F_{2}^{\prime} F_{2}^{\prime \prime}}{d},
\end{array}
$$

in which $y^{\prime}$ indicates the distance of the focus of the combined system, behind the anterior focus of system A, and $y_{2}$, the distance of the posterior focus of the combined system in front of the posterior focus of system B. The deduction of these formulæ is as follows:


An incident ray $I . R$. parallel to the axis will pass after refraction by the system A , through its posterior focus, and after refraction by the system B, through the point $\phi$, the posterior focus of the combined surfaces. The prolongation of the incident ray meets the refracted ray at $D$, so that $h_{2}$ is the second principal plane of the compound system. According to the formula of Newton (q. v.) we have

$$
y_{2}=\frac{F_{1}^{\prime \prime} \cdot F_{2}^{\prime \prime}}{d} .
$$

The figure gives the following relations:

$$
\frac{a}{b}=\frac{F_{2}^{\prime}}{d+F_{1}^{\prime \prime}}=\frac{F_{2}}{y_{2}+F_{2^{\prime \prime}}^{\prime \prime}}
$$

or

$$
\begin{aligned}
F_{2} & =\frac{F_{2}^{\prime}\left(y_{2}+F_{2}^{\prime \prime}\right)}{d+F_{1}^{\prime \prime}} \\
& =\frac{F_{2}^{\prime}\left(\frac{F_{1}^{\prime \prime} F_{2}^{\prime \prime}}{d}+F_{2}^{\prime \prime}\right)}{d+F_{1}^{\prime \prime}} \\
& =\frac{F_{2}^{\prime}\left(F_{1}^{\prime \prime} F_{2}^{\prime \prime}+d F_{2}^{\prime \prime}\right)}{d\left(d+F_{1}^{\prime \prime}\right)} \cdot \\
& =\frac{F_{2}^{\prime} F_{2}^{\prime \prime}}{d}
\end{aligned}
$$

We find the value of $y^{\prime}$ and of $F^{\prime}$ by supposing the light to come from the other side. Knowing the focal distance and the position of the foci it is easy to calculate the other cardinal points. The figure above represents a negative system inasmuch as the posterior focus of $A$ is anterior to the anterior focus of $B . \quad F_{1}$ and $F_{2}$ as well as $y^{\prime}$ and $y_{2}$ are therefore negative. If $d=0$, parallel rays on incidence are parallel after refraction by the system. Such a system is called telescopic. The distance $d$ is called the interval, it determines the character of the combination.

As the focal distances are proportional to the indices of the first and last media, they should be equal if the first and last media are identical, which is true in all optical instruments. In such a case the distañce of the anterior focus from the first principal point is equal to its distance from the first nodal point ; in other words the first principal point coincides with the first nodal point, and the second principal point with the second nodal point. The nodal points and the optical center of thick lenses can be easily ascertained.
$L$ is a biconvex spherical lens; $C C^{\prime}$, its primary axis. Draw $C A$ and $C^{\prime} A^{\prime}$, radii of the respective surfaces of the lens, parallel to each other. $A A^{\prime}$ is the course of the ray $I$ within the lens. Where $A A^{\prime}$ intersects the axis $C C^{\prime}$ is the optical center of the lens. If the ray $I$ continued on without refraction it would meet the axis of the lens at the point $N$, the anterior nodal point, $N^{\prime}$ is the posterior nodal point and is located by projecting the refracted ray backward into the lens. The optical center is the image of $N$, in regard to the first surface, and of $N^{\prime}$ in regard to the second surface. In infinitely thin lenses the nodal points, principal points and optical center all coincide. If the refracting system is represented by a single surface both principal points coincide with the surface and the nodal points with the center.

## TO FIND THE CARDINAL POINTS OF THE CRYSTALLINE LENS.

Let us suppose that the lens has a thickness of 4 mm .. that the radius of curvature of the anterior surface is io mm ., and that of the posterior surface 6 mm . Let 1.33 be the index of refraction of the aqueous humor, and of the vitreous, and that of the lens r.06 in relation to these liquids. (That is I .437 I divided by I .33 ; the index of the lens divided by that of the aqueous.) In this case the system A
and system $B$ are represented by a single refracting surface. The focal distances of the system are :

$$
\left.\begin{array}{l}
F_{1}^{\prime}=\frac{V_{r}}{V^{\prime}-V}=\frac{10}{0.06}=167 \mathrm{~mm} . \\
F_{2}^{\prime}=\frac{V^{\prime} r}{V^{\prime}-V}=\frac{10 \times 1.06}{0.06}=177 \mathrm{~mm} . \\
F_{1}^{\prime \prime}=\frac{V_{r}}{V^{\prime}-V}=\frac{-6}{1 / 1.06-1}=106 \mathrm{~mm} . \\
F_{2}^{\prime \prime}=\frac{V^{\prime} r}{V^{\prime}-V}=\frac{-6 \times 1 / 1.06}{1 / 1.06-\mathrm{I}}=\frac{6}{0.06}=100 \mathrm{~mm} .
\end{array}\right\} \text { For A. }
$$

The interval $d$ is the distance of the posterior focus of the system A, from the anterior focus of the system B; the former is situated 177 mm . behind the anterior surface, the latter at 106 mm . in front of the posterior surface ; the thickness of the crystalline being 4 mm ., we have $d=177+106-4 \mathrm{~mm} .=279 \mathrm{~mm}$., and

$$
\begin{aligned}
& y^{\prime}=\frac{F_{1}^{\prime} F_{2}^{\prime}}{d}=\frac{167 \times 177}{279}=106 \mathrm{~mm} \\
& y^{\prime}=\frac{F_{1}^{\prime \prime} F_{2}^{\prime \prime}}{d}=\frac{106 \times 100}{279}=38 \mathrm{~mm} \\
& F_{1}=\frac{F_{1}^{\prime} F_{1}^{\prime \prime}}{d}=\frac{169 \times 106}{276}=63.4 \mathrm{~mm} \\
& F_{2}=\frac{F_{2}^{\prime} F_{2}^{\prime \prime}}{d}=\frac{177 \times 100}{279}=63.4 \mathrm{~mm}
\end{aligned}
$$

The anterior focus of the crystalline lens being situated 106 mm . behind the anterior focus of the first surface, which is at 167 mm ., its distance as far as that surface will be $167-106=61 \mathrm{~mm}$., and as the focal distance is 63.4 mm ., the first principal point of the crystalline lens will be placed 2.4 mm . behind the anterior surface. The
second principal point will be situated at an equal distance, at $100-38-63.4=-1.4 \mathrm{~mm}$., that is 1.4 mm . in front of the posterior surface. Both focal distances are equal as they always are when the surrounding media are alike. The refracting power of the crystalline lens would then be $1 / 63.4 \mathrm{~mm} .=\mathrm{I} 5.8 \mathrm{D}$.
the manner of combining the cornea with the crystalline lens.
Suppose the cornea to be a single refracting surface having a radius of 6 mm ., surrounded in front by air $(v=1)$ and behind by aqueous ( $v^{\prime}=1.33$ ). The distance of the anterior surface of the lens behind the anterior surface of the cornea is 3.6 mm .

In this case the cornea forms the system A. Its focal distances are:

$$
\begin{aligned}
& F_{1}^{\prime}=\frac{V r}{V^{\prime}-V}=24 \mathrm{~mm} . \\
& F_{2}^{\prime}=\frac{V^{\prime} r}{V^{\prime}-V}=32 \mathrm{~mm} .
\end{aligned}
$$

The principal points coincide with the surface. The focal distances of the system B are those found for the lens.

The interval $d$ is the distance from the posterior focus of the cornea to the anterior focus of the lens: $d=61 \mathrm{~mm} .+32-3.6=$ 89.4. With these data we compute the entire optical system of the eye:

$$
\begin{aligned}
& y^{\prime}=\frac{24 \times 32}{89.4}=8.6 \mathrm{~mm} \\
& y_{2}=\frac{63.4 \times 63.4}{89.4}=45 \mathrm{~mm} \\
& F_{1}=\frac{24 \times 63.4}{89.4}=17 \mathrm{~mm} \\
& F_{2}=\frac{32 \times 63.4}{89.4}=22.7 \mathrm{~mm}
\end{aligned}
$$

The anterior and posterior focal lengths of the eyeball are then, in diopters: $\mathrm{I} / \mathrm{I} 7 \mathrm{~mm} .=58.82 \mathrm{D}$., and $\mathrm{I} .3365 / 22.7 \mathrm{~mm} .=58.8 \mathrm{D}$., respectively.

The strength of a lens is $I / F$. It is the measure of curvature that it causes a wave of light to take that passes through it. Conver-

gence produced by a single refracting surface is greater on the side of the lesser index of refraction.

$$
F_{1} / F_{2}=V^{\prime} / V^{\prime \prime} .
$$

The strength of a system is the index of the last medium divided by the principal focal distance in that medium.

The Optic System and Constants of the Normal Human Eye.-The constants must be known before we can deduct the position of the cardinal points. The constants of the dioptric system of the eyeball, or the relation of the refracting surfaces to one another in regard to their distance apart, curvature, etc., after Tscherning are as follows :

Position of the anterior surface of the cornea.......... 0.
Position of the posterior surface of the cornea. ....... 1.15 mm .
Position of the anterior surface of crystalline lens.... 3.54 mm .
Position of the posterior surface of the lens.......... 7.60 mm .
Radius of the anterior surface of the cornea.......... $\quad 7.98 \mathrm{~mm}$.
Radius of the posterior surface of the cornea. . . . . . . . 6.22 mm .
Radius of the anterior surface of the crystalline lens... 10.20 mm .
Radius of the posterior surface of the lens............ 6.17 mm .
Index of refraction of the air.......................... I
Index of the cornea. . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1.337
Index of the aqueous humor. . . . . . . . . . . . . . . . . . . . . . 1.3365
Index of the lens (total index)........................ . . 1.42
Index of the vitreous humor.
1.3365

The optic system is as follows:
Of the cornea :
Position of the first principal point.................. - 0.13 mm .
OPTIC SYSTEM OF THE CRYSTALLINE LENS.
Position of the first nodal point. 5.96 mm .
Position of the second nodal point. 6.14 mm .
Focal distance of the lens 62.46 mm .
Refracting power ..... 16.01 D.
Combining these two systems we have the complete optic systemof the eyeball.
Position of the first principal point. ..... 1.54 mm .
Position of the second principal point ..... 1.86 mm .
Position of the first nodal point ..... 7.30 mm .
Position of the second nodal point. ..... 7.62 mm .
Position of the anterior focus ..... -15.59 mm .
Position of the posterior focus ..... 24.75 mm .
Anterior focal distance ..... ${ }^{1} 7.13 \mathrm{~mm}$.
Posterior focal distance ..... 22.89 mm .
Refracting power ..... 58.38 D.

It will be noticed that the cornea is two and one half times as refractive as the crystalline lens. The sum of the refracting power of the dioptric surfaces of the eyeball is not far from being equal to the refracting power of the eye, because the nodal points of the cornea and of the lens are very close together. The refracting power of the eye would be exactly equal to the sum of the refracting power of its
dioptric surfaces if the anterior principal point of the lens coincided with the posterior nodal point of the cornea.

In the formula :

$$
F_{1}=\frac{F_{1}^{\prime} F_{1}^{\prime \prime}}{d}
$$

$d$ would then equal $F_{1}^{\prime}+F_{1}^{\prime \prime}$ which gives
or $\mathrm{I} / F^{\prime}={ }_{\mathrm{I}} F_{1}^{\prime}+\mathrm{I} / F_{1^{\prime \prime}}$.

$$
F_{1}=\frac{F_{1} F_{1^{\prime \prime}}^{\prime}}{F_{1}^{\prime \prime}+F_{1}^{\prime}}
$$


The theory of Gauss assumes that the aperture of the optic system is small. In optical instruments an aperture of ten or twelve degrees is considered the limit in size, while in the eye, the pupil, being about 4 mm ., gives an aperture to the cornea of twenty degrees. We do not see the pupil in its real size or place, but a magnified image of it by refraction through the cornea. It is apparently moved forward and enlarged.

Its apparent place and size is easily determined. We can determine its apparent place by means of the following formula :

$$
\frac{F_{1}}{f_{1}}+\frac{F_{2}}{f_{2}}=1 .
$$

To find $f_{1}$, we give to the remaining quantities their values as deduced from the cornea of the simplified eye. $\quad\left(F_{1}=24 ; F_{2}=32\right.$, $f_{2}=3.6$, distance between cornea and iris.) $f_{1}$, is then found to be
equal to -3.04 mm . Suppose that the real size of the pupil is 4 mm ., its apparent size is 4.5 mm ., as found by the following formula:

$$
I / O=F / D
$$

$O, 4 \mathrm{~mm} . ; F, 32 \mathrm{~mm} . ; D, 3.6-32=-28.4 \mathrm{~mm}$. ; therefore

$$
I=\frac{4 \times 32}{28.4}=4.5 \mathrm{~mm} .
$$

The pupil appears therefore to be moved forward by about .5 mm ., and to be enlarged by about the same amount.

Tscherning applied the name of apparent iris and apparent pupil to these images of the real iris and pupil as seen through the cornea. If the iris and the pupil could be viewed by eye from behind the eyeball through the vitreous, it would appear .i mm. further back than it really is and enlarged about .2 mm . Rays coming from a point in the real pupil would proceed into the vitreous as if they came from a point in the crystalline image of the pupil. If the corneal and the crystalline images of the pupil be constructed, we would then know that a ray of light that passed through a certain point of the apparent corneal pupil, would after refraction by the lens pass through the same point in the crystalline apparent pupil. Light that enters the eye is limited by the apparent pupil, in its passage between the cornea and the lens, by the real pupil, and in the vitreous by the crystalline image of the pupil. There are analogous phenomena in optical instruments, wherever a diaphragm is between two lenses. Abbe proposes the names of pupil of entrance and pupil of exit for the images of the diaphragm. For further consideration of the subject the reader is referred to works of Helmholtz or Tscherning.

Accepting the theory of cardinal points, Donders, Listing and Helmholtz constructed schematic eyes. The data in the eye of Helmholtz are as follows:
Refraction index of air ..... I
Of cornea and aqueous humor ..... 1. 3365
Of the lens ..... 1.4371
Of the vitreous humor ..... 1. 3365
Radius of cornea. ..... 7.829 mm .
Of the anterior surface of the lens ..... 10 mm .
Of the posterior surface of the lens ..... 6 mm .
Distance of the apex of the cornea from the lens ..... 3.6 mm .
Thickness of the lens ..... 3.6 mm .

The position of the cardinal points in such a schematic eye are as follows :
$F_{1}$, first focal point is 13.745 mm . in front of the anterior surface of the cornea.
$F_{2}$, posterior focal point, 15.689 mm . back of the lens.
$H^{\prime}$, first principal point, 1.753 mm . back of the posterior surface of the cornea.
$H^{\prime \prime}$, posterior principal point, 2.106 mm . behind the cornea.
$N^{\prime}$, first nodal polnt, 6.968 mm . behind the apex of the cornea.
$N^{\prime \prime}$, second or posterior nodal point, 7.32 Imm . behind the apex of the cornea.

The anterior focal distance of this schematic eye is 15.494 mm ., and the posterior, 20.713 mm . When the eye is adjusted for near vision, the relation of these cardinal points is changed on account of the change in the curvature of the lens. Listing and Donders further simplified the schematic eye, by substituting for the refracting system a single refracting surface, bounded anteriorly by the air and posteriorly by the aqueous or vitreous humor as both have practically the same index of refraction. In this reduced eye the anterior principal and the anterior nodal points may be disregarded without introducing any error into the determination of the size of the retinal image. These points may be neglected as the distance separating them is so minute ( .39 mm .) . There is then in the reduced eye one principal and one nodal point, the latter being the center of curvature of a single refracting surface.

The dimensions of the reduced eye of Listing are as follows: From the anterior surface to the principal point, 2.106 mm .; to the nodal point, 7.321 mm . The anterior focal distance is 15.498 mm , 20.783 mm . The radius of curvature of the refracting surface is 5.215 mm ., the index of refraction is 1.3365 mm ., the same as that of the aqueous humor.

The reduced eye of Donders departed a little more from the conditions present in the natural eye. Its axial length is 20 mm .; the refracting surface has a curvature of 5 mm . radius ; the nodal point in consequence is 5 mm . behind the apex of the refracting surface; and 55 mm . in front of the retina. The index of the eye is that of the aqueous humor. The principal focal point of the normal human eye amounts to about 22 mm ., but calculations in regard to the size of retinal images, diffusion circles, etc., in the reduced schematic eye give results approximating closely those found for the real eye.

One often wishes to measure the size of a retinal image or of a lesion in the fundus of the eye.

To Ascertain the Size of the Retinal Image. - As has been shown the size of an image formed by a refracting surface is to the size of the object as the distance of the image from the nodal point is to the distance of the object from the nodal point. In the eye (reduced eye of Donders) the distance of the nodal point from the retina, is $\mathrm{I}_{5}$ mm ., and from the cornea 5 mm . If an object is situated at 2 m . distance the size of its retinal image is $15 / 2,000$ of the size of the object. The 5 mm ., the distance of the nodal point to the cornea, may be neglected in these calculations. The angle formed at the nodal point of the eye by lines drawn from the extremities of the object, is called the visual angle.

Angle $V$ is the visual angle ; $N$, the nodal point ; $O$, the object. This nodal point in the normal human eyeball lies a little posterior to the posterior pole of the crystalline lens.

Distance of $V$ to $N=7.321 \mathrm{~mm}$.; $V$ to $P, 3.6 \mathrm{~mm}$.; $P^{\prime}$ to $P, 3.6$ mm .; $V$ to $P^{\prime}, 7.2 \mathrm{~mm}$.; $P^{\prime}$ to $N$, .1 mm.; $N R=15.498 \mathrm{~mm}$. . 1 mm .
can be neglected, making the nodal point coincide with the posterior pole of the crystalline lens. The lens of the eye is often 4 mm .

thick, so under such a condition, the nodal point is dislocated .3 mm . anterior to the posterior pole of the crystalline lens.

## CHAPTER V

## VISUAL ACUITY AND ACCOMMODATION

The apparent size of an object depends upon the size of the angle of vision subtended by the object. The further from the eye an object is viewed the smaller does it appear, as the further away an object is the smaller the visual angle. Objects appear smaller to the hyperope and larger to the myope than to the emmetrope, as the area of retinal stimulation is smaller and larger respectively, than in the emmetropic eye. See figure.


Let $E$ be an eye; $H, E$ and $M$, the positions of the hyperopic, emmetropic and myopic retinas respectively. $N$ is the nodal point of the eye. $a$ and $b$ are two objects at different distances from the eye, each subtending the same visual angle, as lines drawn from the extremities of each form the same angle at $N$. If $a$ without change of size was moved to the position of $b$, it would appear smaller, as it would then subtend a smaller angle. The extent of the retinal impression in each is $x y, x^{\prime} y^{\prime}$ and $x^{\prime \prime} y^{\prime \prime}$, for $a$ or $b$, and for $a$ in the position $b, x z, x^{\prime} z^{\prime}$ and $x^{\prime \prime} z^{\prime \prime}$, respectively. The retinal image is embraced between the sides of the visual angle, that is between $a$ and b. An object to be visible to the unaided eye must subtend a visual angle of $\mathrm{I}^{\prime}$. If its shape is to be discerned it must subtend an angle of $5^{\prime}$ at the nodal point of the eye. This is called the limit visual angle. If an object is smaller than this its retinal image will not
embrace one percipient element of the retina. A distant star or a point of light is visible, even if it subtends a much smaller angle than $I^{\prime}$, but if two stars or points are to be discerned as two, they must be separate at least $60^{\prime \prime}$ or $\mathrm{I}^{\prime}$, otherwise the image of each will fall upon and influence the same percipient retinal element, and the stimulus will be carried to the brain as one. For two objects to be recognized separately, they must at least stimulate two retinal elements.

An object nearer than twenty feet cannot be seen clearly by the normal eye even if the object does subtend the proper size visual angle, without the use of accommodation. It is not only necessary that an object to be seen clearly should subtend a visual angle of $5^{\prime}$, but rays of light from each point in the object must be brought to an accurate focus upon the retina to form the corresponding point in the retinal picture.

If the object is at or beyond twenty feet from the eye, the light comes to the eye in practically parallel paths, and the focusing is accurate without the aid of accommodation, as the emmetropic eye is adjusted for parallel rays of light, its retina lying at the principal focus of its dioptric system. The angle that an object subtends at the greatest distance at which it is visible represents the maximum acute ness of vision. An object twice the size could be seen at twice the distance and vice versa. The size of an object denoting acuteness of vision is proportional to the distance.

Snellen devised a series of letters subtending an angle of $5^{\prime}$. The letters were formed of strokes in width one fifth the size of the entire letter, consequently each limb of the letter at the distance it was designed to be read subtended an angle of $\mathrm{I}^{\prime}$, while the whole letter subtended an angle of $5^{\prime}$. The openings or interspaces of the letter were likewise made to conform to this same standard. The relation of the size of a letter to the distance it should be read by the normal eye is twice the tangent of half the angle of $5^{\prime}=.001425$.

In the diagram let $N$ be the nodal point of an eye ; $A B$, an object at the distance $D$ from $N$. In either position of $A B$, the size of its
retinal image will be $i$. Let angle $V=5^{\prime}$, the limit visual angle. Draw line $N O$, and let it bisect the angle $V$ and the line $A B$.
$A O=O B$. In triangle $A N O, \tan V / 2=A O / N O . \quad A O=\tan$ $V / 2 \times D . \quad A O=A B / 2$, ergo : $A B=2 \tan V / 2 \times D$, or expressing the object as $O$ and the distance of it from the eye as $D$, the formula to ascertain the size of the object becomes :

$$
O=2 \tan 21 / 2^{\circ} \times D
$$

Twice the $\tan$ of $21 / 2^{\circ}=: 001425$, ergo: $O=.001425 D$.

The size of a letter to be seen
 at a given distance is then ascertained by multiplying .001425 by the distance expressed in millimeters. At a distance of one meter the size of a standard letter is 1.42 mm . (.001 $425 \times \mathrm{I}, 000 \mathrm{~mm}$.), and a letter to be read by a normal eye at six meters to express visual acuity must be (.001 $425 \times 6,000 \mathrm{~mm}$.) 8.5 mm . in size.

The size of the retinal image of a standard letter at six meters is ${ }^{15} / 6,000$ of 8.5 mm . or . 02124 mm . (The size of an object and its image are to each other as their respective distances from the lens.) Some people have better vision than that expressed by a $5^{\prime}$ standard, so letters subtending an angle of only $4^{\prime}$ at a given distance have been constructed. The retinal image of the limbs of such a letter is four fifths of the size of those of the $5^{\prime}$ standard letter, that is, .0034 mm . The perceptive elements of the central part of the retina vary in size from .0032 to .0036 mm ., showing a very close relation between the size of the cones of the retina and acute vision. There are many copies of test-letters to be had that do not conform closely to the $5^{\prime}$ standard. Perhaps the very best set of letters is that devised by Dr. Randall. The card has regular and practical intervals and closest practical adherence to the I-minute parts and interspaces.

Landolt has lately advocated the use of circles the thickness of Snellen's letters, each broken at some point in its contour by a space
equal to the thickness of the circle. The space in each case subtends an angle of one minute at the distance the character is designed to be read, and visual acuity is expressed by a fraction with the distance of the patient from the test-card for the numerator and the width of the gap in the circles of the line read for the denominator. The advantages claimed for these characters are that they can be made to conform to the one-minute standard; that they may

be used for those that read and illiterates alike, and that they are not easily memorized. In reading them the patient tells in each case in which direction the circle is open.

The black card with white letters is that devised by Dr. Gould. It is claimed that upon this card patients can usually read one or two lines better than upon the card with black letters, and that the black background is restful to the eyes of the patient, in consequence of which he does not so quickly tire. The writer has not found this the case. The white letters on the black background are certainly not as well seen as the black letters on the white card by artificial
light. An old card is better to use if one is compelled to use artificial light in refraction work, as the polish of the surface of a new card dazzles the eye of the patient and the white background appears to flow over into the letters, obscuring their edges. The white letters on the black card seem enlarged, as they make a very vivid impression on the retina and irradiation causes the letters to appear to flow over into the background. The letters, while appearing enlarged, are not read the more easily, as their edges are blurred by


3


4
the irradiation. (A white object on a black background appears larger than a black object of the same size on a white ground, due to irradiation.)

No. 3 is a test-card to be used for illiterates. The letter E is turned in different directions and is made according to the scale of $5^{\prime}$. The patient, instead of reading the letters, tells in which directions the strokes of the characters point, whether to the right or to the left, up or down. Card 4 is made in German for those who do not read English. There is also to be had a card with Hebrew characters. The picture cards intended for children too young to read are not reliable as it is almost impossible to make the pictures to conform with the usual standard of acute vision. If the examiner is
not familiar with the German and Hebrew characters he may hold in his hand a small tally card upon which the names of the characters are written in English and thus keep himself straight.

Dr. Williams has devised some test-cards to be used for testing the visual acuity of railroad employees, the characters being made to represent semaphores when placed in different positions, which correspond, when seen at a distance of twenty feet, to the apparent size of the semaphore arm when seen at a distance of 2,600 feet. The cards contain three different arrangements of letters for each of the standard distances, which avoids the difficulty caused by the patient memorizing the letters. The card shown in cut is designed to be seen at a distance of twenty feet.

The upper arm of semaphore is for the near track and the lower one for the off track. Where the arm forms a right angle with the
 pole, it implies that that section of track is blocked and when lowered that the road is clear.

Visual acuity is expressed by a fraction the numerator of which denotes the distance at which a certain letter or line of letters is read and the denominator the distance at which the letter or line should be read by the normal eye, or putting it another way: The numerator of the fraction denotes the distance of the patient from the test card, and the denominator the number of the line of letters read at that distance.

The lines on the test-card are numbered either above or at the side by figures denoting in feet or meters the distance at which the particular lines are designed to be read. If the refractionist has the space it is always better to test the eyes from a distance of 6 m . or $20^{\prime}$, as from that distance practically no accommodation is needed by the normal eye. If a closer distance has to be selected than 6 m ., the use of accommodation is stimulated which is apt to lead into error. If a patient is sitting at $20^{\prime}$ or 6 m ., and only reads the line designed to be read at a distance of $100^{\prime}$ or 30 m ., his vision is $V$ (from visus,
the Latin of sight) $=20 / 100$, or by the metric system $6 / 30$. If the vision is better than normal it is expressed by a fraction in the same way, save in such a case the numerator will be greater than the denominator. Thus : At $20^{\prime}$ a man sees that designed to be read only at $10^{\prime}$, his vision is then $20 /$ ro and so on. The fraction expressing the visual acuity is not reduced to its lowest terms, but left unreduced so that one seeing it may know at what distance the vision was tested. The Germans use $S$ instead of $V$ to denote the visual acuity (Sehsschärfe, meaning visual acuity).

In 1891, Guillery proposed to measure visual acuity by the distance at which a black spot on a white ground could be distinguished. He found that a black point seen under an angle of $50^{\prime \prime}$ corresponds to normal visual acuity. At five meters the point would have to be 1.2 mm . in diameter. The dots are numbered according to the size, no. 2 being twice the diameter of no. I, and so on. If a patient at a given distance saw no. 2, when he should have been able to see no. I, his vision is one half and so on. Javal has constructed a similar test using small squares so that the area of each square is always double that of the preceding square. The diagonal of one square is taken to be the side of the next succeeding square. If the side is 2 , the diagonal is

$$
\sqrt{2^{2}+2^{2}}=\sqrt{8} .
$$

Test-Type for Near Vision.-Those generally in use for this are Jaeger's or Snellen's. The latter are graduated that each should be read at the distance for which it is marked. The smallest should be seen at a distance of 50 cm ., and the largest at 4 m . ( 12 feet). These types are given to the patient and we note the smallest that he can read and also the nearest and furthest point from the eyes at which he can read it. The small types are chiefly used to test the accommodation, but they are also of service in testing for myopia or nearsightedness.

Dr. Ziegler's arrangement of Jaeger's near test-types is a good one. Each paragraph is numbered from I to 20, and the dioptric
equivalent is placed along the margins. A bar of notes for testing musicians and some symbols for the seamstress are appended.

The visual acuity falls off as soon as the object is moved away from the fovea, to one eighth or one tenth. To estimate peripheral visual acuity, Bjerrum repeats the perimetric examination, using smaller and smaller objects. He uses a distance of 2 m ., placing the patient in front of a large black curtain ; the objects used are small ivory discs of various sizes fixed on black rods i m. in length. It is said that in cases of optic nerve atrophy, the peripheral visual acuity is often diminished before the visual field has become curtailed. The limits of the visual field according to Bjerrum are as follows :

> With a 3 -mm. disc: 35 , outside ; 30, inside ; 30, below ; 25, above. With a 6-mm. disc : 50, outside ; 40, inside ; 40, below ; 35, above.

If we draw several dots on a sheet of paper and fix one of them for some time we will notice that now one and now another of the surrounding dots will disappear from view, to reappear after a little, generally if the eyes are moved or after winking. This is called Troxler's Phenomenon. The color of the ground as well as that of the spots plays no part. One must guard against an error in perimetric examination, due to this phenomenon. No explanation is offered as to the cause of this phenomenon.

Accommodation. - Accommodation is the change in the refraction of an eye, due to an increased convexity, and hence an increase in strength of the crystalline lens, due to the action of the ciliary muscle, whenever the eye looks from a distant to a near object. The normally refracting eye only needs accommodation when it fixes an object nearer than 20 feet or 6 m ., while the hyperopic eyeball needs it for distance as well if it is to receive well-focused retinal pictures. Accommodation does the same for an eye that the placing of a convex spherical lens in front of it would do, that is the increased convexity of the dioptric media shortens foci. The refraction of an eye when at rest, that is, without any effort on the part of the ciliary muscle, is spoken of as its static refraction, and the eye is adjusted so
that rays proceeding from a distant point, called its far-point, come to a focus upon the retina. When the eye is using all of its available accommodation it is adjusted for its near-point. The refraction of the eye when adjusted for its near-point is called the dynamic refraction. The entire amount (amplitude) of accommodation of an eye is employed as the eye passes from the far-point (denoted by R.) to its near-point.(denoted by pp. or by p.).

The far-point is spoken of as the punctum remotum, and the near point as the punctum proximum. The need of accommodation is at once apparent when we recall that the normal eye is adjusted when at rest for parallel rays of light only, and that from near objects the light proceeds from every point in the object in diverging paths, and that diverging rays of light are focused posterior to parallel rays of light,
 so that the image of near objects would be blurred, unless the focal length of the dioptric system of the eyeball is shortened.

In the figure $E$ represents an emmetropic or normally refracting eye. Rays I and 2 are parallel, coming to it from a distance of 20 feet or more, and are focused upon the retina without accommodation as the emmetropic eye is adjusted when at rest for focusing parallel rays (plane waves) of light. Rays 2 and 3 come to the eye divergent; as they emanate from a near point, they are not brought to a focus as the retina of the eye intercepts them ; if they continued they would come to a focus at the point $F^{\prime}$, because divergent rays of light are focused posterior to parallel rays. The circle of illumination on the retina embraced by the rays that emanate from the point $N$, is called a diffusion circle. In order that the point $N$ may be seen distinctly the focus of it must lie upon the retina of the eye. This is accomplished by the lens of the eye becoming stronger through accommodation, which shortens the focal distance of the eyeball, and brings $F^{\prime}$ up to $F$, or upon the retina.

It will be noticed that now the parallel rays of light have been brought to a focus anterior to the retina and crossing from a diffusion circle upon the latter. It is therefore evident that an eye cannot be adjusted at the same time for both parallel and divergent
 rays of light for distant and near seeing. It is not difficult to calculate the size of a diffusion circle on the retina. If the diameter of the pupil be designated by $p$ (the smaller the pupil the smaller the circle of diffusion); its distance from the retina by $a$; and the distance of the principal focus of the eyeball by $d$; we have the following formula to ascertain $x$, the diameter of the diffusion circle :

$$
x=p \frac{d}{d+a}
$$

In triangles $p \overline{F \bar{d}+a}$, and $x F d$ : sides $p$ and $x$ are to each other as $\overline{d+a}$ is to $d$.

$$
x / p=d / d+a \text {, or } x=d / \overline{d+a} \times p .
$$

The following is Listing's table of the size of diffusion circles in the emmetropic eye of objects at varying distances :

| Distance of Luminous Point, <br> Meters. | $\left.\begin{array}{c}\text { Distance of } \begin{array}{c}\text { Focus Behind the Retina, } \\ \text { Millimeters. }\end{array} \\ \hline \infty \\ 65\end{array}\right)$Diameter of Diffusion Circle, <br> Millimeter. |  |
| :---: | :---: | :---: |
| 25 | 0 | 0 |
| 12 | 0.005 | 0.0011 |
| 6 | 0.012 | 0.0027 |
| 3 | 0.025 | 0.0056 |
| 1.500 | 0.050 | 0.0112 |
| 0.750 | 0.100 | 0.0222 |
| 0.375 | 0.200 | 0.0443 |
| 0.188 | 0.40 | 0.0825 |
| 0.094 | 0.80 | 0.1616 |
| 0.88 | 1.60 | 0.3122 |

The amplitude or the amount of available accommodation is measured by the strongest convex spherical lens with which the
eye can see as well with as without when adjusted for the punctum proximum.

One begins with a weak convex sphere and places successively stronger and stronger lenses before the eye, until the vision begins to be blurred at the near point after having determined the position of the near-point by ascertaining the closest distance at which the finest discernible print can be read by the unaided eye. As successive strengths of lenses are added, the accommodation each time a stronger one is put on relaxes a little, allowing the glass lens to do the work for it, until all is relaxed. If the accommodation was not relaxed each time, the glass lens would spoil the vision by bringing the focus anterior to the retina, that is the eye would be made myopic. The amount of accommodation needed by the emmetropic eye for any given distance is equal to the lens whose focal length is that distance. Thus: For I m. distance, the E. needs i D. of accommodation ; or according to the inch system, as I m. equals $40^{\prime \prime}$, one fortieth accommodation.

Always divide the distance at which the object is seen, expressed in cm . into 100 cm . ( r m .) or the number of inches at which the object is seen into 40 , to ascertain the number of diopters of accommodation in use for that distance. This is evident by referring to the figure.

Let $L$ represent the lens of an emmetropic eye in accommodation, for the point $O$ at 1 m . distant
 from the eye. Portion $l^{\prime}$ is the increase of strength of lens caused by accommodation ; $l$, the strength of the lens when the accommodation is relaxed, or when the eye is at rest. 'The light from $O$ comes divergent to $L$. Portion $l$ ' renders them parallel and then portion $l$ focuses them upon the retina. The strength of the portion $l^{\prime}$ is I D., as rays of light emanating from a point at one meter's distance are rendered parallel by a i $D$. lens. How much accommodation is required by the emmetropic
eye to see distinctly an object ro cm. 50 cm ., 200 and 400 cm . away?
$100 / 10=10$ D., $100 / 50=2$ D., $100 / 200=.50$ D., $100 / 400=.25$ D.
The entire amount of or amplitude of accommodation is obtained by the following formula:

Amount of $=$ refraction of the eye at _refraction of the eye at accommodation $=$ the near-point $\quad$ the far-point.
or,

$$
A=p-R,
$$

or, as it is sometimes written

$$
A=D-S \text { (dynamic - static refraction of the eye). }
$$

In each of these equations the respective values of the quantities on the right-hand side are expressed in diopters. $R$ is the point for which the eye is focused when at rest. The emmetropic eye is focused when at rest for parallel rays of light. Parallel rays of light only come from a distance of infinity, therefore, the $R$ of the emmetropic eye lies at an infinite distance. The strength of a lens whose focal length is infinity is o D., that is it is no lens at all. How is it that parallel rays of light come to the eye only from a distance of infinity? When it was stated in a former section that when rays of light came to the eye from a distance of twenty feet or beyond, they may be regarded as parallel.

Strictly speaking, rays of light do come to the eye diverging from all distances short of infinity ; but from distances beyond $20^{\prime}$, the size of diffusion circles on the retina are so small that they may be disregarded, as they do not interfere materially with the sharpness of the retinal images. To obtain the value of $p$, Landolt's ophthal-dynometer is useful. A screen which is made to fit a candle has a number of perforations in it arranged in a row. Attached to the instrument there is a tape measure, marked in centimeters on one side and in the equivalent number of diopters on the other. The patient holds the instrument as close to his eyes as it is possible to see the per-
forations separately, and then the distance from the eye and the amount of accommodation in use for that distance is read off from the tape-line. The patient of course must exert all his effort to see as close by as possible in determining the value of $p$. The value of $p$ is then influenced to a certain extent by the will and intelligence of the patient. Theoretically the instillation of eserin sulphate would give the absolute position of the near-point, as the ciliary muscle is thrown into the strongest contraction by the drug. The value of $p$ thus obtained would, however, be too great for practical purposes. Very accurate results can be obtained by ascertaining the nearest point at which the finest readable print can be distinctly seen. Or, a number of black dots or small pin-holes can be made in a card, and the card brought closer and closer to the eyes until the dots or perforations are seen to fuse into one continuous line. In emmetropia then $A$ equals the dynamic refraction, inasmuch as $R$ has no value in the equation $A=p-R$.

If an eye emmetropic or made so by spectacles can read at ten centimeters the finest discernible print, the amount of accommodation is $\frac{10)}{10 \mathrm{IOD}}$. In myopia and in hyperopia, $R$ has its value in diopters. The myopic eye is not able to bring parallel rays to a focus on its retina, as the retina of the myopic eye lies posterior to the principal focus of the dioptric system of the eye. In order that such an eye may receive a clear retinal picture then, it is necessary that it should receive divergent rays of light, as divergent rays of light are focused posterior to parallel rays of light.

Inasmuch as divergent rays of light only come to the eye from a finite distance, $R$ has an equivalent finite value. The point situated in front of the myopic eye from which light emanating has the proper amount of divergence to be focused accurately upon the retina of the
eye, is the far-point of the myopic eye. The distance of this farpoint expressed in diopters is the value of $R$. The near-point is ascertained as before described. Suppose that $p$ is at 10 cm ., and $R$ at 1 m ., then $A=p-R=$ ıо $\mathrm{D} .-$ ı $\mathrm{D} .=9 \mathrm{D}$. accommodation. The near-point assumed for emmetropia and for myopia in the above cases is the same, that is 10 cm ., and from the formula one sees that the myope of I D. has to use one diopter less of accommodation for a given distance than does the emmetrope. If there is 3 D . of myopia, then 3 D . accommodation less are needed, and so on. In hyperopia the value of $R$ is negative and the formula for the amplitude of accommodation becomes: $A=p-(-R)=p+R$. The H. eye is adjusted when at rest for only converging rays of light. Such rays have only a virtual focus, a point behind the eyeball where they would come together if they were unrefracted as they passed through the dioptric media of the eyeball. Converging rays are found only in nature after they have passed through a convex lens. The strength of the lens that will impart to parallel rays of light entering it the proper amount of convergence so that they will be focused upon the retain of the hyperopic eye at rest is the value of $R$. Such a lens is spoken of as correcting the hyperopia. The sign of $R$ is negative as it lies behind the eyeball.


In the figure the hyperopic eyeball is focused for the converging rays $a^{\prime}$ and $b^{\prime}$, rendered so by the convex lens $L$, acting upon parallel rays $a$ and $b$. If $L$ is equal to 2 D ., $R$ is equal to -2 D . lying posterior to the eyeball at $-R$, where the rays $a^{\prime}$ and $b^{\prime}$ would meet if unrefracted by the dioptric media of the eyeball. If $p$ is assumed to be equal to 10 D . then, $A=10+2=12 \mathrm{D}$. The hyperopic eye,
then for a given distance needs the amount of accommodation of the emmetropic eye plus the amount of its hyperopia.

The range or region of accommodation is the space lying between the far-point and the near-point. The extent of the region of accommodation forms no estimate of the amount of work done by the ciliary muscle.

Compare region of accommodation ; the space between $R$ and $p$, with the amplitude $A$, in the above cases of E ., H. of 2 D ., and M. of 2 D. The normal near-point in emmetropia lies at $4-5$ inches,

or $10-12.5 \mathrm{~cm}$. from the eyes. It gradually recedes further and further from the eyes, until the age of seventy-five years, when it lies at an infinite distance.

Bull calls that region of accommodation lying between the puncta remota of the principal meridians of the astigmatic eyeball, the remote range of accommodation.

## CHAPTER VI

## ACCOMMODATION (CONTINUED)

Mechanism.-The eye could theoretically alter its focus or accommodate in any of the following ways: (a) By increase of corneal curvature ; (b) by increase of curvature of the crystalline lens; (c) by elongation of the eye-globe; (d) by advance of the crystalline lens ; (e) by the contraction of the pupil.

Each of these methods has had its adherents, theory $(d)$ is impossible from the anatomical arrangement of the crystalline lens system of the eyeball. In certain lower animals, however, the lens is drawn nearer the retina in focusing the eye for distant seeing.

Even if the crystalline lens could advance to the posterior surface of the cornea it would not suffice to explain any considerable amount of accommodation. Scheiner was the first to discover that looking at objects through a pinhole increased their distinctness, but the contraction of the pupil does not answer for any considerable amount of accommodation. Young was the first to discover that accommodation depended upon a change in the curvature of the lens of the eye. He disproved theories (a) and (c) by experiments upon his own eyes. Young decided that the cornea did not change during the accommodative act as he could notice no change in the size or shape of the corneal catoptric image, but a decided change in the image when he made only feeble pressure upon the periphery of the cornea, thus altering its contour. His most conclusive experiment consisted in putting his eye under water. He took a weak objective of a microscope which had nearly the same index of refraction as the cornea, filled the tube with water, and placed it before the eye also plunged into water. The effect of the cornea on the refraction of the eye was now eliminated as it was surrounded on both sides by a liquid with the same index of refraction. The cornea was replaced
by the lens of the objective. Under these conditions he noticed that the accommodation remained intact. He also disproved the fact that accommodation depended upon the elongation of the eyeball, by measurements made upon his own eye. Young had very prominent eyes, so he was enabled to perform the following experiment.

He turned his eye inwards as much as he could, and applied against its anterior surface a strong iron ring ; then he thrust the ring of a little key on the external side between the eyeball and the bone until the phosphene reached the fovea. Placed between the ring and the key the eyeball could not elongate during accommodation. If accommodation depended upon elongation of the globe, he should have found it abolished or see the phosphene due to pressure extend over a much larger surface. The accommodation remained unaltered and the size of the phosphene did not change. In Young's day the ciliary muscle had not been discovered, so he was unable to account for the change in shape of the crystalline lens. The crystalline lens rests in the fossa patellaris on the anterior surface of the vitreous body. It is enclosed within its capsule and held in place by a suspensory ligament, which passes from the ciliary processes over its anterior surface. (A small portion passes posteriorly, to fuse with the capsule of the lens.) The suspensory ligament is taut during the passive condition of the eye, and keeps the lens compressed beneath it. By the contraction of the ciliary muscle the pressure upon the lens is lessened, and it through its own elasticity becomes more convex, responding almost entirely upon its anterior surface. At the same time the lens becomes thicker, its diameter between the ciliary bodies becomes less. As a proof of the increased convexity of the crystalline lens during accommodation, the experiments of Hensen and Völckers may be cited. These experiments removed a part of the outer tunic of the eyeball lying over the ciliary muscle, in the cat, dog and monkey. They then introduced pins through the sclerotic, so that the point of one should rest upon the anterior surface of the lens about its center, and another upon the center of the posterior surface. Pins were also introduced into the chorioid. On electriz-
ing the ciliary muscle, the head of the pin resting upon the anterior surface of the lens moved backward, showing an advance of the anterior surface of the lens. The head of the pin resting upon the posterior of the lens moved forward-but to a much less degree than the head of the pin upon the anterior surface of the lens moved backward. The pins in the chorioid all moved backward, showing that the chorioid advanced during accommodation.

Coccius and Hjort in cases where part of the iris was absent, noticed that not only the chlorioid, but also the retina, advanced during accommodation. The theory that the iris played an important part in the act of accommodation was disproved in a case in Von Graefe's clinic where the iris was entirely removed without disturbing in the least the power of accommodation. The accommodation is also perfect in many cases of congenital absence of the iris. The ciliary muscle was discovered in 1846 by Bowmann and Brücke, working independently. It is divided into three portions, namely: A meridional, horizontal, or radiating portion (Brücke's muscle) ; a circular or annular (Müller's) and a diagonal or transitional portion. The circular fibers, or those of Müller, are the most important.

In eyes that accommodate a great deal they are excessively developed. The ciliary muscle lies upon the internal surface, of the sclera, and is bounded internally by the ciliary processes, sixty to seventy in number. The meridional portion of the ciliary muscle arises from the corneo-scleral junction, from the posterior wall of Schlemm's canal and from the adjacent sclera, and runs posteriorly to be attached to the anterior extremity of the chorioid coat. When it contracts it draws the chorioid forward, and through the vitreous being interposed, it renders the posterior surface of the crystalline lens stationary, so than the lens cannot recede during the act of accommodation. The circular fibers surround the orifice formed by the ciliary processes like a sphincter muscle in any other locality surrounds the orifice over which it presides. When they contract they lessen the circumference of the circle over which they have control, and thus slacken the suspensory ligament of the lens. The diagonal
fibers arise along with the horizontal ones and run diagonally backward and inward towards the center of the eyeball and are attached to the suspensory ligament, which is drawn forward off of the lens by their contraction.

According to some anatomists these latter fibers do not exist, claiming them to be transitions between the circular and the meridional. According to Heinrich Müller, the ciliary processes press upon the equator of the lens during accommodation, and squeezing upon it aid in its increased convexity. Accommodation begins to fail at the age of ten years and is nil at the age of seventy-five, due to a loss of elasticity of the lens, caused by a hardening or sclerosis of its fibers.

The gradual loss of elasticity and the consequent failure to respond to the contraction of the ciliary muscle causes the near-point to recede pari passu. When it has receded beyond the point of convenient near seeing presbyopia has set in. The normal nearpoint up to the age of forty years lies at $4-5 \mathrm{in}$. or $\mathrm{IO}-12.5 \mathrm{~cm}$. from the eyes. If it lies nearer than this, myopia is indicated ; and if further off, hyperopia, that is in individuals under the age of the advent of presbyopia. The accompanying table gives the range of accommodation at different ages according to Donders.

With the contraction of the ciliary
 muscle there is an associated contraction of the pupil, due partly to the rush of blood from the ciliary bodies into the iris, but caused chiefly by a stimulus from the third nerve, in the floor of the fourth ventricle, to the sphincter muscle of the iris. The eyes also converge (turn in towards the nose) during the act of accommodation.

By the bulging forward of the anterior surface of the lens the center of the anterior chamber is made shallower, while at its periphery, its depth is increased by the retraction of the root of the iris through contraction of the more deeply seated ciliary muscle fibers. By con-
traction of the pupil the iris is made more taut and the infiltration angle between the iris and the cornea opened.

The retraction of the periphery of the iris also aids in widening this space. The figure below shows the changes that take place in each eyeball during accommodation.

The right half of the figure represents a section of an eyeball with relaxed ciliary muscle ; the left half one, with contracted ciliary muscle. It will be noticed that accommodation causes the
 following changes: (i) Lens thicker, and equatorial diameter made less; the change in shape of lens being mostly on the anterior surface. The ciliary body coming in contact with the equator of the lens. (2) The center of the anterior chamber made shallower, and the periphery deeper. (3) The pupil is contracted (and in regard to both eyes, they are converged).

The theory of accommodation as outlined in the preceding paragraphs, and the one that is generally accepted is that of Von Helmholtz. Tscherning and Young dispute the theory that accommodation depends upon the relaxation of the suspensory ligament of the lens, due to con'traction of the ciliary muscle, and claim that the periphery of the lens is flattened by contraction of the ciliary muscle, and that the curve of the anterior surface of the lens assumes a hyperboloid form from the nearly spherical form that it has during a state of rest. The change in the shape of the lens in the pupillary area is seen by the change in the shape of and in the position of the images of Purkinje, which was discovered by Max Langenbeck in 1849. These images of Purkinje were described by the scientist whose name they bear at the beginning of the last century. They are catoptric, that is formed by reflection from the anterior surface of the cornea and from the anterior and posterior surfaces of the lens. (There are three other catoptric images seen under special conditions which will be described later.)

If a lighted candle be held in front of and a little to one side of an
eye with a dilated pupil, three images of the light are seen reflected from the eye, provided it has a crystalline lens. The brightest image is erect and is formed by reflection from the cornea. The largest but at the same time the faintest image is likewise erect, and is formed by the anterior surface of the lens. The third one is small, but bright and inverted,
 and formed by reflection from the posterior surface of the lens.

The conditions that are best suited to study this test are: That the room should be dark and that the light be placed about 30 degrees to one side of the eye and about 14 in. distant.

The observer should place himself on the opposite side of the eye and at an angle of 30 degrees with the visual axis of the eye. The images in the pupil will then bear the relation to each other shown in the cut no. i. If the eye now adjusts itself for a nearer object, without changing the direction of its gaze, the images will be seen to bear the relation shown in figure no. 2. The next figure explains how the change in the position of the catoptric images takes place during accommodation.

During the accommodative act the reflected image from the anterior surface of the crystalline lens has moved towards that from the anterior surface of the cornea.

Light falling upon the vertex of the cornea at $O$ forms an image that
appears in the position of $c$, when projected by the observer upon a horizontal plane $x y$. The relative position of the image $l$ from the vertex of the anterior crystalline surface is not altered as it lies in the horizontal plane. $L^{\prime}$ is the position of the image $O^{\prime \prime}$, formed at the vertex
 of the posterior crystalline surface. When the anterior surface of the crystalline lens bulges forward to the position of $a^{\prime \prime}$, its catoptric image moves to the point $L^{\prime \prime}$ in the horizontal plane. The change in the shape of the lens necessitated by accommodation according to the theory of Von Helmholtz, is shown in figure $a$, and the change that takes place according to the Tscherning theory, shown in figure $b$.
$S$ is the anterior surface of the lens in each case, and $S^{\prime}$ the posterior surface. The dotted lines represent the lens in accommodation. Tscherning believes that the inner portion of the ciliary muscle has its more fixed attachment posteriorly into the chorioid, which is rendered stationary by the tension of the vitreous, which is increased during accommodation due to a backward movement of the lens. The retraction of Müller's fibers which are not so circularly arranged as was formerly supposed, makes traction upon the suspensory ligament, and thus produces a change in the shape of the crystalline lens. The particular shape that the lens assumes depends in part upon the arrangement of the fibers. The deepening of the periphery of the anterior chamber is accounted for by the flattening of the edge of the lens during accommodation. There is often no change in the position of the catoptric images as described, but only a diminution in the size of the image formed by the anterior surface of the lens, showing that the surface has become more convex but has not advanced to any appreciable degree. To make out the change of size in the lenticular catoptric image during accommodation a small white and well-illuminated square should be used instead of the candle flame, as the object. The tension of the anterior chamber of the
eyeball diminishes during accommodation, as shown by Förster in 1864. He observed in patients with small keratoceles, that the protrusion diminished in size during accommodation, to swell up again as the accommodation was relaxed. The iris covers the periphery of the lens, so whichever theory is the correct one, the resulting increase of refraction in the pupillary area is the same.

In support of the Tscherning theory of accommodation is cited the fact that the amplitude of accommodation diminishes towards the edge of the pupil, as detected by the aberroscope. This instrument consists of a strong plano-convex lens, which on its plane side carries a micrometer in the form of little squares. We look at a distant luminous point through the lens, moving it to 10 to 20 cm . from the eye. The lens causes the light to assume the shape of a circle of light, and upon this diffusion circle most people see the lines in the field concave towards the periphery, due to the usual variety of spherical aberration of the crystalline lens, namely positive aberration.

But, on making an effort at accommodating, the forms of the shadows change; they now turn their concavity towards the center, indicating that the refraction increases towards the center of the lens, or that the edge of the lens is now weaker than the center; the aberration has been overcome by accommodation.

When an eye looks directly at an object the macula lutea of the retina is employed, and the eye is said to have fixed the object. This is central or direct vision, in contradistinction to vision performed with the peripheral portions of the retina, namely, indirect vision. When the eyes are viewing an object beyond twenty feet from them the visual axes are practically parallel. The visual axis is a line drawn from the macula lutea through the nodal point forward. When a near object is fixed the visual axes must deviate inwards, towards the nose, in order that each eye may receive its stimulus on its macula. This act is called convergence, and the angle through which the visual axis of each eye deviates, in fixing a near object, is the angle of convergence. The unit of measurement of convergence is the meter-angle (Nagle), that is the amount of convergence
that is necessary to fix an object at one meter's distance, in the median line. The value of one meter-angle is $x^{\circ} 50^{\prime}$ for each eye,
 or twice that amount ( $3^{\circ} 40^{\prime}$ ) for one eye if the object of fixation is directly in front of one eye.

The following is the manner of deduction if the object is in the median line:
$O O^{\prime}$ is the interocular distance; $O M$, one half of the interocular distance ; $C^{\prime} M$ is perpendicular to $O O^{\prime}$, so that for an object on it the convergence of each eye is equal. When the visual axes $J O$ and $J^{\prime} O^{\prime}$ are parallel the convergence is nil ; when however the visual line has deviated to $C^{\prime}, J O$ has moved to $O C^{\prime} \therefore J O C^{\prime}=$ angle of convergence.
$J O$ being parallel to $C^{\prime} M$, angle $J O C^{\prime}=$ angle $O C^{\prime} M$. In rt. $\Delta$ $O C^{\prime} M, O M / O C^{\prime}=$ sine $\angle O C^{\prime} M$. The value of $O C^{\prime} M$ depends upon the interocular distance. The average interocular distance is 64 mm ., making $O M=32 \mathrm{~mm} . \quad O C^{\prime}=1,000 \mathrm{~mm}$. ( I m. ); $O M / O C^{\prime}$ $=32 / 1,000=.032=$ the sine of one meter-angle, which corresponds to an angle of $\mathrm{I}^{\circ} 50^{\prime}$.

The amount of convergence for any distance is ascertained in degrees by multiplying the number of meter angles, necessary for that distance by $1^{\circ} 50^{\prime}$. The number of meter-angles of convergence needed for a given distance is the inverse of the distance, thus : for 2 m . there is needed one half meter-angle of convergence and for $1 / 2 \mathrm{~m}$., 2 meter-angles of convergence. The average convergent near-point denoted as c. p. $=2-3$ in. or $5-8 \mathrm{~cm}$., giving the normal limit of convergence equal to $20-12.5 \mathrm{~m} .4$ (meter-angles).

The meter-angle is closely related to the centrad ( $\nabla$ ), and is used to numerate prisms. The value of one meter-angle is easily ascertained by finding its relation to a centrad. The centrad is a prims that deviates a ray I/ ioo part of a radian, that is an arc whose length
is equal to its radius of curvature. It is then measured upon the tangent. The deviation of the meter-angle is measured on the sine (half the interocular distance). One meter-angle equals a deviation of 32 mm . as 32 mm . is half of the average interocular distance. For small angles the sine and the arc are almost equal. At one meter's distance the amount of deviation is equal to 3.2 centrads ( 32 in 1,000 or 3.2 in 100 , or $3.2 \nabla$ ). The nearer the fixation point; the greater the convergence necessary to maintain single binocular vision (seeing singly with the two eyes).

The accommodation and convergence are expressed in terms that indicate the same number of units for any given distance. Thus: an E. eye viewing an object at 33 cm . needs 3 D . Acc. $\left(\frac{33) \frac{100}{3 \mathrm{D} . \mathrm{Acc} .}}{}\right)$ and as 33 cm . is one third of a meter, the convergence necessary is 3 m. L. While using 5 D . Accommodation, 5 meter-angles of convergence are needed by the emmetrope, and so on.

The number of meter-angles that the eyes can call into use is the measure of the amount of convergence, the amplitude of convergence. It is measured from the far-point of convergence (C. R.) to the near point of convergence (c. p.). C. R. is the point for which the visual lines are directed when the convergence is relaxed to the utmost; c. p. the point for which the visual axes are directed when the convergence is at its utmost. The visual lines should be parallel when the eyes are adjusted for C. R. At times the visual lines actually diverge, causing an outward cast, and at other times there is convergence constituting an inward cast when the eyes are adjusted for C. R. Accommodation and convergence, as has been shown, are very closely associated, so that for every diopter of accommodation there is a corresponding meter-angle of convergence. The methods of measuring convergence will be described under the head of muscular inefficiencies.

This association is more or less a flexible one, however, so that accommodation and convergence are in a certain measure independent of each other. This varies in different individuals. The amount of accommodation that can be exercised without altering the amount
of convergence is called the relative accommodation, and the amount of convergence that can be brought into use without altering the amount of accommodation is the relative convergence. A patient is made to fix fine print at 33 cm . distance. Three diopters of accommodation are in use. Suppose that $p$ lies at Io cm .; there is then 7 D. of absolute accommodation in reserve ( $\mathrm{IOO} / \mathrm{IO}=10,10-3=$ 7 D.). The part of the relative accommodation in use when converging for a given distance is the negative portion. It is measured by the strongest convex spherical lens with which the patient can see as well as without at the given distance, the point for which the eyes are converged. As the spheres are added the accommodation is relaxed pari passu. The part of the relative
 accommodation in reserve is the positive portion. It is revealed by the strongest concave spherical lens with which the patient can see as well as without, at the distance for which the eyes are converged. As negative spheres are added the accommodation is increased pari passu until the whole amount is in use. The far-point of relative accommodation is denoted by $R_{1}$, and the near-point by $p_{1}$.

In the figure, $F . p$. is the fixation point at 33 cm . distance. While the eyes continue to converge for the point $F_{.}$. ., they can accommodate for a more distant point $R_{\mathrm{l}}$, or the nearer point $p_{1}$. If $R_{1}$ is I m. distant and $p_{1}, 20 \mathrm{~cm}$. distant, then :

$$
A_{1}=p_{1}-R_{1}=5 \mathrm{D} .-\mathrm{I} \mathrm{D} .=4 \mathrm{D} \text {. relative accommodation. }
$$

The print held at $F . p$. could then be read as well with a +2 D.S. as with no glass before the eye ; it could also be read as clearly with a -2 D.S. ( $\mathrm{r} 00 / 33 \mathrm{~cm} .-3$ D. accommodation, $100 / 20=5$ D. accommodation, $5-3 \mathrm{D}$. accommodation $=2 \mathrm{D}$.). With a minus lens because a concave lens stimulates accommodation.

The relative accommodation varies for each point of fixation. The amount of positive relative accommodation decreases as the fixation point approaches the eyes. Upon the relation of the segments of relative accommodation to each other depends the ability of the eyes to work without exhaustion, keeping up the necessary amount of accommodation and convergence. It is impossible to use the eyes for a distance at which the positive portion of the relative accommodation is not at least as great as the negative portion, otherwise fatigue soon results. At the distance of 33 cm ., the proper reading distance, the plus portion of the relative accommodation is about one and one half times greater than the minus portion, for which reason work can be carried on at this distance continuously without tiring. A lack of unity of accommodation and convergence is seen in the eye at the near point. The function of convergence being stronger than that of accommodation, the absolute near point is obtained at the sacrifice of single binocular vision, the convergence overacting and thus reinforcing the accommodation.

In hyperopia the amount of accommodation required is greater than in emmetropia, and the same tendency for the two functions to reinforce each other offers a stimulus to convergence which often results in internal squint.

Optical Defects. - The crystalline lens of the eye, as has been pointed out in a former chapter, possesses the errors of spherical and chromatic aberration, both of which defects interfere with the distinctness of the retinal images. In order that two objects side by side can be recognized as two, they must be separated at least by $60^{\prime \prime}$, so that the images of both do not fall upon the same retinal percipient element, as under these conditions two objects are not seen, but only one, the brain interpreting the stimulation of the same element twice, as one stimulation, from one object in space. If the retina was perfect two objects should be perceived even if their images did fall upon the same end element of the optic nerve. The dioptric system of the eyeball, furthermore, is a decentered system, that is, the center of the cornea is not in line with the center of the lens, nor the center
of the lens in line with the macula lutea. A line drawn from the center of the retina forward through the center of the lens is called the optic axis. This line does not cut the central point of the cornea.

A line from the point of fixation to the macula is called the visual axis; it cuts the optic axis at the nodal point, forming with it an angle called the angle beta.* The optic axis outside of the eyeball runs below and to the outer side of the visual axis, while within the eyeball it runs to the inner side of and above the visual axis. The visual axis as a rule pierces the cornea a little to its nasal side.

The corneal curve continued (that of anterior surface) forms an ellipse, the major axis of which, that is the corneal axis, runs to the outside of the visual axis and forms with it the angle called angle alpha. The angle alpha is usually equal to about five degrees, and is considered positive when the visual line lies to its inner side, and

negative when to its outer side. In E. and H. it is the rule to have the angle with a positive value, and when large it often causes the eye to have an apparent ontward cast. In M. it is negative and then causes, if large, an apparent inward cast.

The line of fixation is drawn from the object for which the eye is directed to the point of rotation, a point in the vitreous chamber 6 mm . behind the nodal point and 9 mm . in front of the retina. About this point the eyeball makes all of its movements as if upon a pivot. The angle formed by the fault of the optic axis and the line of fixation is the angle gamma, and the angle between the corneal axis and the line of fixation has been called the angle kappa (see figure).

[^2]
## CHAPTER VII

## OPHTHALMOSCOPY AND OBLIQUE ILLUMINATION

All objects that do not shine by their own light are visible to us by the light that is thrown off from them by reflection entering our eyes. The interior of the eyeball is illumined but the pupil appears black under ordinary conditions, as the light that is reflected back from the interior of the eye does not enter the eye of the observer.

The blackness of the pupil was formerly supposed to be due to the absorption of the light that entered the eye by the black pigment that coats the interior of the eyeball. In 1850 Brücke and Cumming discovered the means of making the pupil of the human eye luminous, and in the year 1851 Helmholtz invented the ophthalmoscope, the instrument that has revolutionized ophthalmology. Light passes out of an observed eye along paths close to those of entrance. The ophthalmoscope allows the observer to get his eye in a position relative to the observed eye, to receive some of the returned light from the latter, and as the observer's eye placed back of the sighthole in the mirror of the ophthalmoscope is practically at the source of light to the observed eye, the pupil of the latter appears illumined. Helmholtz first was able to see the interior of the eyeball by reflecting light into it from a plate of glass held obliquely before his own eye. The reflecting power of the glass was increased by placing several plates together. Plane amalgamated mirrors were soon used as reflectors, and finally after the suggestion of Reute, concave mirrors came into use for ophthalmoscopic mirrors. The concave mirror gives better illumination, as it gathers up or focuses the light that enters the eye. The ophthalmoscope is still to-day nothing but a reflector with a central peep-hole.

In the figure the light enters the eye, $E$, along the dotted lines, $s s^{\prime}$, illuminating the area $a b$ upon the retina. The light reflected back passes out of the eye along paths embraced by the lines $r r^{\prime}$, drawn through the nodal point $N$. The observer's eye $E^{\prime}$ receives no light from the eye $E$, so the pupil of $E$ remains unillumined to $E^{\prime}$. Rays $s$ and $r^{\prime}$, and $s^{\prime}$ and $r$,
 from points $a$ and $b$ respectively pass out of the eye parallel as the retina of the eye lies at the principal focus of its dioptric system, as the eye is emmetropic. If the observer would move close enough to receive some returned rays from the observed eye, his head would then intercept the entrance of light into the observed eye.

In the hyperopic eye one is able to see the pupil illuminated, especially if it is dilated. The reason is that the light passes out of the hyperopic eye in diverging paths and some falls into the eye of the observer near by.

If in the preceding figure the retina was moved to the position of $H$, the eyeball would be hyperopic. The light returning from the area of illumination ( $a^{\prime} b^{\prime}$ ) would be embraced by the rays $r^{\prime \prime}$ and $s^{\prime \prime}$, and as the observer's eye receives some of this returned light, the pupil of the observed eye would appear aglow. Rays of light from the same point in the fundus of the hyperopic eye leave the eyeball diverging as rays $s s^{\prime \prime}$ from the point $a^{\prime}$, and $s^{\prime} r^{\prime \prime}$ from the point $b^{\prime}$, because the retina of the hyperopic eye lies anterior to the principal focus of the dioptric system of the eyeball. The eyes of many animals are hyperopic, the reason in part that such eyes are seen aglow. The luminosity of their pupils is however enhanced by a peculiar reflecting medium in the chorioid coat of their eyes, called the tapetum.
(Some observers claim that the eyes of beasts and birds are naturally emmetropic and not hyperopic.) Human eyes that are devoid of lenses are intensely hyperopic and their pupils are therefore luminous.

This is seen after cataract extraction, especially if an iridectomy has been done. The luminosity of the pupils of the albino's eyes is explained in a different way. The light passes into the albino's eyes not only through the pupils but also through the non-pigmented iris, and even through the thin sclerotics which are peculiar to such eyes. Not only a limited area of fundus is illuminated at one time as in other eyes, but the whole fundus is flooded with light ; the rays of light from different parts of the fundus pass out of the pupil and pass in all directions in space, some of which enter the eye of the observer near by. That this is the correct explanation is proven by the fact that the pupil of the albino's eye looks black as soon as a screen with a hole in it the size of the normal pupil is placed before the eye. This excludes all light from the eyeball save that which enters through the pupil, making the eye of the albino like that. of a normal person in this respect.

Light from the original source of light at $L$ is reflected by the mirror $M$ (immediate source) into the observed eye, along the paths marked by the dotted lines. The returning
 rays pass out of the observed eye along the paths marked by continuous lines through the sight-hole of the ophthalmoscopic mirror into the eye of the observer. The observer is therefore enabled to see the interior of the observed eyeball. There are many kinds of ophthalmoscopes on the market. The simple ophthalmoscope is a mirror with a central opening or an area where the amalgam is wanting. Some have round mirrors, others oblong tilting mirrors. The round mirrors are preferable as the area of illumination produced by them is circular, while that from an oblong mirror is more or less drawn out into a streak, curtailing somewhat
one's view of the fundus (the area of illumination from a circular concave mirror is a circle if the mirror is out of focus, and especially if the source of light is at some distance from the mirror). Refraction ophthalmoscopes have in addition a number of lenses that can be revolved behind the sight-hole of the mirror and before the eye of the observer, so that the light returning from the observed eye can be given the direction necessary to bring it to an accurate focus upon the retina of the observer. The best refraction ophthalmoscope is the one with the largest number of lenses and the one in which any combination or strength of lens can be gotten with the least possible complication. Morton's ophthalmoscope is probably the best on the market, as the lenses are contained in an endless chain that revolves before the sight-hole of the mirror. The use of the refraction ophthalmoscope will be explained under the head of errors of refraction. The Loring ophthalmoscope is probably the most widely employed and is an excellent instrument, except for the correction of refraction errors. The following cuts show a few of the many varieties of ophthalmoscopes. The Meyrowitz model of the Morton ophthalmoscope is in all respects as good as the imported original model of Curry and Paxton, London.

The red glare that fills the pupil of the eye when viewed with the ophthalmoscope is called the chorioidal reflex. It is the reflection back from the vascular chorioidal coat of the eye. With the ophthalmoscope the interior of the eyeball can be studied in detail. Many diseases on the interior of the eye that formerly lay hidden from view now lie exposed as upon the surface. In the interior of the eyeball nerves and blood-vessels can be studied which are nowhere else in the body exposed to view except by dissection. The eye-ground is frequently an index to troubles located in distant organs of the body, as in the kidney per example, and in many grave conditions the use of the ophthalmoscope gives the most important diagnostic signs. There are two methods employed to see the interior of the eye, namely, the direct and indirect methods of ophthalmoscopy. The indirect method is the more easily mastered, and as a whole more

I. Loring's Ophthalmoscope, with 7 lenses, set in revolving disc, with round or tiltng mirror.
2. Loring's Complete Ophthalmoscope, with 3 rows of figures, tilting mirror and covered disc.

This instrument consists of a full disc and a quadrant of a disc, as shown in the cut. The quadrant rotates immediately over the disc and around the same center, and contains 4 lenses, -.50-16 and $+.50+16$. When not in use the quadrant is beneath its cover, and the instrument then represents a simple ophthalmoscope with 16 lenses, the series running with an interval of I D., and extending from I to 7 plus and from I to 8 minus. If the higher numbers are desired they are obtained by combination with those of the quadrant. These progress regularly up to 16 D ., every dioptric being marked upon the disc ; above this up to +23 D . and -24 D . we simply have to add the glass which comes beneath the 16 D. , turning always in the same direction.
useful, except in the estimation of refraction errors and in studying small details of the fundus. By the indirect method more of the fundus is seen at one time, although not as much enlarged as when seen by the direct method. To study a lesion under magnification, then, the direct method is preferable.

It is well for the beginner in ophthalmoscopy to have the pupil of the eye to be examined expanded by a mydriatic, as then more light can enter the eye and consequently the picture of the fundus will be seen more clearly, and furthermore, the reflection of the light from

3. Loring's Ophthalmoscope.
4. Loring's Post-Graduate Ophthalmoscope, with round or tilting mirror.

Consists of two superimposed dics, by means of which 31 combinations can be made. The lower disc contains 15 convex lenses, from .50 to 17.50 D., giving the convex series; in the upper disc, which serves also as a cover, is a concave 18 D . lens, which when brought into position before the mirror hole gives a series of 16 concave lenses from $\cdot 50$ to 18 D .

(Full size cut of Dr. Knauer's Ophthalmoscope.)
Fig. A, back of instrument, showing mirror and slide on front in dotted lines.
Fig. B, same, with cover removed, showing interior mechanism.

## 5. Knauer's Ophthalmoscope.

The Knauer Ophthalmoscope is an instrument of novel design and its main features are the great range of foci, its compactness, ease of manipulation and simplicity of construction. It contains two discs holding lenses of different foci, so arranged that whatever focus may be produced by their combination, is plainly indicated in the little window situated in the center of the back of the Ophthalmoscope, where both the number and the sign of the lenses in si.u appear.

A special point in the construction of the instrument is that all the gear-work is completely covered.
The lower disc is operated by the small driving wheel, and this disc again acts automatically upon the upper disc in such a manner that the consecutive combinations are effected in intervals of one diopter from +1 D. to +20 D ., and from - I D. to -39 D . There is besides, situated behind the tilting mirror, a slide containing $\mathrm{a}+.5 \mathrm{D}$. lens, which can be brought in instant combination with any of the lenses in the series by an easy and natural movement of the index finger, thus giving a regular interval oi. 5 D . from +.5 to +20.5 D., and from -.5 D. to -39 D., making in all 119 combinations.

For ne puipose of rapidly changing the lenses, the upper disc can be revolved indepencently by neans ci two small projecting pins, a slight turn of the same causing a change of io numbers in the series ; if, for instance, the - 38 D . were before the opening, a simple turn of the upper disc would change it rapidly to $-28 \mathrm{D} .,-18 \mathrm{D}$., or -8 D ., thus obviating the turning of the small driving wheel until the lower focus is reached in single numbers.

The small driving wheel that operates the discs is so placed that the finger of the operator is not brought between the Ophthalmoscope and the face of the patient.
6. Payne's Ophthalmoscope, with tilting mirror.

Consists of two superimposed discs, each holding respectively 17 convex and concave lenses, ranging from 0.25 D. to 20 D . The rotation of the discs is accomplished by means of two small cogged driving wheels.


Roth's improved Ophthalmoscope with automatic quadrant and index and tilting mirror.
In the construction of this Ophthalmoscope the general plan and dimensions recommended by Loring have been adhered to, and the object of the improvement is mainly to overcome the difficulty experienced in obtaining and reading the higher combinations, which formerly necessitated the removal of the instrument from the eye to bring the quadrant lenses before the sight hole, thus interrupting the examination. Dr. Roth has completely overcome this difficulty by his invention of the Automatic Quadrant and

Registering Index, which greatly simplifies the manipulation of the instrument, inasmuch as the combination of the higher numbers is effected automatically by a continuous revolution of the disc, which also moves a pointer which indicates the number of each lens or combination of lenses plainly upon a dial.

In addition to the lenses contained in the disc and quadrant, $\mathrm{a}+.50 \mathrm{D}$. lens is mounted in a slide attached to the front of the instrument, directly behind the tilting mirror, in such a manner that it can instantly be brought into combination with any of the lenses contained in the series by an easy and

natural movement of the index finger, thus giving a regular interval of .50 D . throughout the entire series from +.50 D . to +23.50 D ., and from -.50 D. to -24 D ., making in all 95 distinct combinations.

Fig. 7 shows the exact size of the Ophthalmoscope with the index finger pointing to 0 . The convex series, which are shown in white figures on the instrument, occupy the inner circle, while the concave numbers, which are marked in red, run in the reverse direction toward the outer margin of the dial.

Fig. 8. Same with small driving wheel.
Morton's ophthalmoscope consists essentially of twenty nine separate lenses, inclosed in an endless groove and propelled by a strong driving wheel. In addition to the lenses just mentioned are four others, set in a separate disc, and so placed that they can be instantly put in front of or removed away from the sight hole without rotating the whole series of convex or concave lenses. At the same time that the driving wheel propels the lenses it rotates a disc on which at a certain aperture is indicated the lens presented at the sight hole.

On the front of the instrument is an arrangement similar to the nosepiece of a microscope, revolving on a central pivot and carrying three mirrors-one plane and one concave mirror of 10 -inch focus at one end and a small concave mirror of 3 -inch focus at the other. The first two, which are set back to back in one mounting and are reversible, are for indirect examination and retinoscopy.

The advantages claimed for this Ophthalmoscope are, briefly:

1. A continuous series of single lenses, sufficient for all ordinary purposes.
2. The provision of a few separate, easily adjustable, lenses for extraordinary cases.
3. The lens in the sight hole is always shown on the indicating disc (except in the rare cases where one of the extra lenses just mentioned is used).
4. The numbers of the lenses and their relative positions being fully exposed on an indicating disc, the direction in which this latter has to be rotated to bring any particular lens to the sight hole is at once made manifest.
5. There is only one driving wheel.
6. A pupilmeter, which is set in the face of the driving wheel.
7. The provision of two mirrors revolving on a central pivot, so that either can be at once brought into position.
8. The width of the instrument is only $11 / 4$ inches, while the driving wheel, being 3 inches below the sight hole, is unimpeded in its action by contact with the face of observer or patient.
9. Lastly, the instrument balances well in the hand, is light and packs into a small compass.
the cornea of the observed eye is not so annoying. The room should be dark and the source of light shaded. The latter is not necessary, however, but if the source of light is a circular opening in an opaque shade covering the flame, the beginner is not so much confused by the image of the flame or source of light in the eye to be observed.

If the source of light is not shaded it is well to have it at some distance from the mirror, as then the area of illumination from the concave mirror is much out of focus and hence more circular in outline. The most convenient source of light is an Argand gas-burner. The Welsbach light is too bright and very trying upon the eyes of the patient ; it is also not easy to estimate a slight change in the color of the fundus of the eye, as in incipient inflammatory changes, when the
source of light is so powerful. Indeed, one often lessens the illumination of the eye by using a plane mirror when searching for small lesions.

The Indirect Method. - The patient and the examiner sit opposite one another and about arm's length from each other. Their eyes should be at about the same height, and the source of light back of the patient and upon his left hand. Holding the handle of the ophthalmoscope horizontally, and with the edge of the mirror against the side of the nose the examiner reflects the light into the eye to be examined. The pupil is seen at once to become luminous, but no details of the interior of the eye are seen unless there be present an error of refraction, either myopia or hyperopia, in the observed eye. Holding the reflection steady in the eye, a convex spherical lens of about 2 to 3 ( 20 D . to I $_{3} \mathrm{D}$.) inch focal distance is held before the eye at about its focal length. This lens is spoken of as the objective

lens. It should be held between the thumb and index finger of the left hand and the hand steadied by resting the little finger against the brow or forehead of the patient. No attempt should be made to look through the objective lens into the eye, as nothing will be seen; but the attention should be directed to a point just in front of the objective lens, that is in the air between yourself and the patient, as it is here that a real image of the fundus of the eye is formed. See figure on preceding page for explanation.

The continuous lines are rays of illumination to the eyeball $A$; dotted lines, the returning rays of light from the eyeball $A$, forming the image $a$ of the fundus $I$ through the objective lens $L$. The broken lines represent the light from the image $a$ to the fundus of eye $B ; a^{\prime}$, image on retina of eyeball $B$ of the image $a$. The image formed by the indirect method of ophthalmoscopy is inverted. Whatever is above in the image is below in the eyeball, and that to the right in the image is upon the left in the eyeball, and vice versa. Accommodation or an equivalent convex lens is needed to see the image of the fundus of an eye by the indirect method. The image has a contrary motion also; that is, if the observer moves upward, the image moves downward ; if the observer moves to the right, the image moves to the left and so on. The upper part of the image must be viewed if it is desired to see the lower part of the eye-field, and the right side of the image if the portion of the eye-field to the left is to be examined.

Heine describes the following method of examining one's own eye by the indirect method. An ordinary hand mirror is held between the light and the left eye, so as to shade it ; then with an ophthalmoscopic mirror the light is reflected into the left eye by aid of the hand mirror, illuminating the region of the fundus to the temporal side of the fovea. Now, by holding a convex lens of I3 D. before the left eye the inverted image of the fundus is seen. By converging the left eye strongly the entrance of the optic nerve may be examined in the inverted image.

Direct Method of Ophthalmoscopy. - (Also called the method of the upright or erect image.) By this method the observer approaches quite close to the patient and looks directly into the eyeball without the interposition of any lens. To examine the right eye of the patient the observer uses his right eye, and vice versa, so that the noses will not interfere with getting as close as is necessary to the patient's eye. The light should be placed behind and on the side of the patient's head next to the eye under examination, about on a level with his ear, so that it will illuminate his temple, leaving his eye and his face
in darkness. Holding the instrument in the right hand, if the right eye is to be examined, the observer throws the light into the patient's eye from a distance of several inches. He then approaches closer, all the time keeping the light in the eye until the ophthalmoscopic mirror is almost in contact with the patient's eye. The observer must not attempt to look directly into the eye, as with the use of accommodation nothing of the interior of the eyeball will be seen, unless the observer's or the observed eye is hyperopic. Any refraction error of the observer should be corrected by the wearing of proper spectacles, and then no confusion will arise from its presence. One should stare vacantly into the eyeball as if trying to see an object at some distance behind the patient's head. The accommodation may be encouraged to relax by forcibly elevating the upper eyelids. The effect of this over accommodation may be seen thus: If, while reading, the upper lids be elevated forcibly, the printed matter will fade out of legibility. The patient should gaze vacantly into space and avoid looking into the light as reflected from the mirror, otherwise the pupil will contract to its utmost, interfering greatly with the test. If it is not convenient to place the patient under the effect of a mydriatic it is a good plan to have him look at some large letters hung at a distance of twenty feet across the room which is made semi-dark, so as to encourage the accommodation to relax. The examination of the cornea and of the lens of the eye are made by placing a magnifying lens back of the sight-hole of the ophthalmoscope.

A 20 D. S. lens is a convenient strength to use. In order to make the lens magnify a lesion of the cornea it is necessary to view the cornea from a point closer than the focal point of the lens. The focal interval of a 20 D . lens is $5 \mathrm{~cm} .(100 \div 20=5 \mathrm{~cm}$.), therefore the examiner must draw closer than 5 cm . to the eye to be examined. Any opacity of the media of the eyeball looks black by reflected light, no matter what its color may be. (At times the opacity is dense enough to be seen by the light that it reflects, and then it is seen in its true color. Hemorrhage into the vitreous, cataracts and dense
scars of the cornea are often seen in their true color by reflected light.)


Let $O$ be an opacity of any sort in one of the media of the eyeball. As no light can enter the eye through the opacity, rays $a$ and $b$ being excluded, there can be no returned light from the eye, and the pupil in that locality, in consequence, looks black. After examining the cornea and the lens with the 20 D. S. lens back of the sight-hole of the ophthalmoscopic mirror the motility of the iris should be tested, whether it reacts evenly and promptly under the influence of light thrown into the eye from different directions. The vitreous is examined by throwing light into the eye from a distance of eighteen to twenty inches while the eyeball is moved up, down, in and out. Opacities in the vitreous, neoplasms and a detached retina are seen in the erect image, because they are within the range of accommodation of the observer. If he views them at a closer distance he must place a convex lens behind the sight-hole of the ophthalmoscope, to accommodate or converge the rays of light for him as they enter his eye; otherwise nothing but an indistinct image will result. It is always better to use a convex spherical lens than accommodation in ophthalmoscopic examinations, for if one accommodates at times and relaxes accommodation at other times he will not acquire as good a control over his ciliary muscle as if he gets into the habit always of viewing the interior of an eye with relaxed accommodation. And, unless accommodation is completely relaxed one cannot get any reliable information in regard to the refraction error of the eye under examination. To place or locate properly an opacity in the transparent media of the eye, the parallactic test is employed.

If the opacity is a faint one the plane mirror is used to the best advantage. It is necessary at times to ascertain whether the opacity is movable or fixed. If movable it will be seen to float about in the eyeball when the latter is moved up, down, in and out, while the
light is kept steady in the eye; the observer watching the pupil. If the opacity is immobile its situation in a plane according to the test referred to above may be ascertained by the observer moving his head from side to side while the eye under examination fixes an object in any given direction (preferably one straight in front). All opacities in the media of the eye are either anterior or posterior to the plane of the iris. If the opacity is anterior to the plane of the iris, that is, in the cornea, it will appear to move in a direction opposite to that in which the observer moves his head, or is as we say against the movement of the head.

If the opacity is in the plane of the iris, that is, at the anterior pole of the crystalline lens, it apparently does not alter its position when the head is moved. If in the vitreous, however, behind the plane of the iris, it will appear to move in the same direction as that in which the head of the observer moves, and will finally disappear behind the iris on the same side. The illustration of this parallactic displacement is simple. Take two pencils and hold them in front of the eyes, the one behind the other and a few inches apart. First let the posterior pencil represent the plane of the iris, and the anterior one an opacity in the cornea, anterior to this plane. Holding the pencils still, move the head from side to side. The anterior pencil appears to move in the opposite direction from the head. Now let the posterior pencil represent the opacity and the anterior one the iris. This time when the head is moved from side to side the posterior pencil appears to move with the head.

Let $a, b, c, d$ and $e$ be opacities in the cornea, at the anterior pole of the lens, center of lens, posterior pole of lens and in the vitreous respectively. An observer at $O$ sees them all as occupying the center of the pupil. As he moves his eye down to $O^{\prime}$, capacity $a$ appears to move up, that is, it is projected upon the plane of the iris on the opposite side of the pupil. $b$, occupying the center of the pupil and in the plane of the iris, does not appear to change its position. $c, d$ and $e$ appear to move with the head of the observer, and finally get out of sight behind the iris on the
same side of the pupil. At position $O^{\prime}, e$ has gotten out of sight behind the iris, and at $O^{\prime \prime}, c, d$ and $e$ are out of sight. The quicker an opacity gets out of sight behind the iris, moving with the head of the observer, the further back in the vitreous is it located. The examiner sees the same portion of the eye-ground when he moves his head to the right or to the left by the indirect as he does by the direct method of ophthalmoscopy. Starting with the entrance of the optic nerve in view with the direct method, the observer moves his head to the right. He thus brings into view a portion of the retina to the left of the optic nerve. The nerve has moved out of the field to the right of the observer. The image moves with the observer. In the indirect method, with the image of the nerve in view, when the observer moves his head to the right, he sees the same portion of the retina that he saw when using the direct method. Being to the left of the nerve its image appears to the right of the nerve (as everything is inverted by the indirect method). The nerve thus appears to have moved towards the left, or in a contrary way to teh observer's head.

The ophthalmoscopic field is larger in the indirect examination of the fundus than in the direct ; that is, the amount of retina or fundus that can be seen at once is greater by the indirect method. The following diagrams explain the construction of the ophthalmoscopic fields according to Helmholtz.

## FIELD BY THE DIRECT METHOD.

The field is constructed by drawing lines from the center of the pupil of the observer's eye to the borders of the pupil of the observed eye, and then continuing them as if they were rays, through
the dioptric media of the eyeball. .Of course, being divergent when they enter the eyeball, they will not be brought to a focus until they

come to the point $C^{\prime}$. As will be seen the field or the extent of the fundus illuminated is greater in the hyperopic and less in the myopic eye than in the emmetropic, if the observing eye is beyond the principal focal point of the eye which is nearly always the case.* As it is the pupil that limits the field of view, we can enlarge the field by instilling a mydriatic into the observed eye.

This deduction is an example of inverse reasoning that is so often used in optics. We imagine that the pupil of the observer's eyeball is luminous and we see how much of the fundus of the observed eyeball it can illumine. The result is the same on account of the reversibility of the process. The field, as shown in the figure above, is really a little smaller than the real field - as to be accurate we would not construct $C^{\prime}$ the image of the one point, $C$ of the pupil of the observing eye, but the image of the entire pupil, or rather of the sight-hole of the ophthalmoscope, which would bring points now lying upon the edge of the field, as constructed above well into the field.

## FIELD BY THE INDIRECT METHOD.

The field by the indirect method is constructed as follows : So that the field may be as large as it is possible to make it by the indirect method, the objective lens should be held at a distance from the observed eye equal to its focal distance. The field then occupies the entire objective lens, and the iris disappears from view. We construct the field as before by supposing the center of the pupil of the

[^3]observing eye to be luminous, and ascertaining what part of the fundus of the patient's eye it could illuminate. In the figure above

it has been assumed that the center of the pupil, $c$, coincides with the nodal point of the observed eye. The rays then that enter the observed eye suffer no refraction. It will be noticed that the field, $a b$, does not depend upon the size of the pupil of the observed eye. The field is only limited by the borders of the objective lens. It is therefore better to use a large lens.

If the lens be moved nearer or further away from the eye it will be noticed that the field becomes limited by the iris of the observed eye. If the pupil is large it is easier to hold the lens just in the proper place for the iris to disappear. By the indirect method details of the fundus are magnified about four times, while by the direct method they are magnified about fourteen times, (provided no projection of the image behind the eyeball takes place). The amount of magnification by either method above that stated depends upon the distance at which the image of the fundus is projected behind the eye. The following figure illustrates the manner in which the fundus of an eye is seen, and also what is meant by projecting the image.
$a^{\prime \prime}$ is the image in the fundus of the observing eye of the object in the fundus of the observed eye. According to the laws of lenses, an image and the object are embraced between secondary axes of the lens, that is between lines passing diagonally through the optical center of the lens. In the above case the image and the object are embraced by the lines $r$ and $r^{\prime}$, secondary axes to the observing eye as they pass through its nodal point. (The nodal point of the eyeball is to the eye what an optical center is to a lens.)

For a given sized image, $a^{\prime \prime}$, on the observer's retina the object may then be the size of $a^{\prime}, b$ or $c$, according to the amount of projection. With both eyes open the picture of the fundus as seen with the ophthalmoscope can be projected upon a screen placed back of the patient's head just as an image in a microscope can be projected upon a piece of paper laid beside the instrument upon the table. Both eyes should always be kept open in using any optical instrument, as the pressure of the forcibly closed lids of the eye not in use, acts deleteriously upon the cornea, interfering with its nutrition and ofttimes giving rise to alteration in its curvature, and consequent


The Lens of each Eye is omitted for Simplicity of Figure
deficient vision. It is always the eye unused that troubles one when he is in the habit of closing one eye. Again if two eyes are not used much magnification of the fundus is missed. Landolt determined for accuracy in calculating the size of fundal lesions, that the fundus of the eye is magnified twenty times when projected behind the eyeball 30 cm . To facilitate measurements and to make a record of the same, Dr. Flavel Tiffany has devised a tablet which is checked off into squares by parallel lines, 5 mm . apart. There is a tablet for each eye. The tablet is placed at 30 cm . behind the patient and on the side of the eye to be examined, fastened to the back of his chair or held by a holder upon a table by his side. At first it is difficult to project the image of the fundus of the eye under
examination upon the card, but a little practice will soon bring it out. When examining either eye it helps to have the patient look a little in, towards the nose, and for the observer to view the fundus from within outwards. At first the image is seen projected upon the cheek of the patient, but on having him look out a bit, the image will shift from the cheek to the tablet or card. With the card it is easy at once to determine the size of an object in the fundus of the eye, and the distances apart of different portions of the retina or of diseased areas in the fundus. Thus: If a lesion is seen to cover a space of 1 cm . upon the card when projected upon it by the observer, it has .5 mm . actual size, and so on.

The numerical value given to ophthalmoscopic magnification upon the preceding page is obtained by comparing the retinal image formed in the observing eye
 with the retinal image that the eye would have of the same object if placed free in the air at the usual working distance of the observer. In the following calculation let us suppose that the eye of the observer, as well as that of the patient, is emmetropic. For the direct method the deduction is as follows:
$A^{\prime} B^{\prime}$ is the image in the fundus of the observer's eye of the object $A B$, in the fundus of the observed eye. Let $A B$ be equal to $O$, and $A^{\prime} B^{\prime}$ to $I$, being included between parallel lines.

In each eyeball a ray is drawn from either extremity of the object or image parallel to the optic axis. Therefore, after refraction these rays will pass through the anterior focal point of the eyeball. As both eyes are considered emmetropic, their anterior focal distances are equal, that is, $O O^{\prime}=O^{\prime} I$. In the triangles $C O^{\prime} D$ and $C^{\prime} O^{\prime} D^{\prime}$

$$
\begin{equation*}
\frac{I}{O}=\frac{F_{1}}{F_{1}}={ }_{\mathrm{I}} \tag{I}
\end{equation*}
$$

or $I=O$. The fundus of the observed eye forms in the observing
eye an image equal to itself. By placing the fundus of the eye in the air at a working distance of 20 cm . the retinal image $F^{\prime}$ of the object $O$ is easily found by the formula $O / I^{\prime}=D / d=200 / d$. (This formula means that the size of the object is to the size of its image as the distance of the object from the anterior focal point of the eye is to the distance of image from the anterior focal point.)
$d=F_{1}$, the anterior focal distance-
(2)

$$
\therefore O / I^{\prime}=\frac{200 \mathrm{~mm}}{F_{1}}
$$

Multiplying two by one we obtain the amount of magnification -

$$
G=\frac{I}{I^{\prime}}=\frac{200}{F_{1}}=\mathrm{I}_{3}+
$$

The formula of magnification for any distance is

$$
G=\frac{D}{F_{1}}
$$

The amount of magnification of the image by the indirect method, if we use an objective lens of about 13 D., is four to five times.

$A B$ and $A^{\prime} B^{\prime}$-Object; $C D$.Image.
$A_{1}$ - Ant. Focal Point of Eyeball.
OC-Focal Length of Objective Lens.
The rays of light from the head of the object $A B$ in the fundus of the observed eyeball pass out of the eyeball parallel as the eye is considered emmetropic. (The rays start from the retina which lies at the principal focus of the dioptric system of the eye.) One of the returning rays, the one parallel to the optic axis, passes through the
anterior focal point of the eyeball, and the other one without refraction through the optical center of the objective lens. In triangles $A^{\prime} B^{\prime} F^{\prime}$ and $O C D$, we have this relation :

$$
\begin{aligned}
\frac{C D}{A^{\prime} B^{\prime}} & =\frac{O C}{B^{\prime} F^{\prime}}, \text { or } \frac{I}{O}=\frac{O C}{F_{1}} \\
\frac{I}{O} & =\frac{77 \mathrm{~mm} .}{15 \mathrm{~mm} .}=5+.
\end{aligned}
$$

If a weaker objective lens is used the image is situated at a greater distance from the lens, and is at the same time larger. The observer must then use more accommodation to see it, move further back or use a convex spherical lens back of the sight-hole of his ophthalmoscope. If a convex lens is used in the ophthalmoscope, and the observer approaches the image of observed eye closer than the focal length of the lens in use, the image will be seen much magnified. (A convex spherical lens, when held nearer to an object than the principal focus of the lens, magnifies the object.) If the observed eye is myopic, the image of the fundus is smaller than in the E., and if hyperopic, larger. In emmetropia the image of the fundus of the observed eyeball is formed at the principal focus of the objective lens as the rays of light from the object pass into the lens parallel. If the eye is myopic the light returns from it in converging paths, and entering the lens is focused anterior to the principal focus, hence the image is smaller, while in hyperopia the light passes from the eyeball divergent and is focused by the objective lens posterior to its focal point, making the image larger than that of the emmetropic eye.

After cataract extraction, when the eye has been rendered very hyperopic by the removal of its lens, the magnification of the fundus is extreme, and its image is formed at a great distance from the objective lens. To see this image the observer must move back or use a stronger objective lens, say an 18 D. S.

Auto-ophthalmoscopy. - The examination of one's own eye-ground is of limited utility. The method is especially useful to those who
are studying fundus details, without any model save their own eye. The simplest method of seeing the interior of one's own eyeball is that of Coccius.* A plane mirror with a large sight-hole is held in front of the eye so that the light will shine into the eye through the central hole of the mirror. The emergent rays of light are caught by the margins of the opening in the mirror and reflected back into the eye upon the macula lutea. The image of the fundus will be projected by the observer's eye upon the background behind the source of light. This is not a mere suggestion as are the images of Purkinje, but can be seen and reproduced in good detail.

Light $a$ entering the eye through sight-hole of mirror falls upon optic papilla and is
 reflected back along path $b$; impinging upon mirror, it again enters the eye along path $c$. The macula then sees an image of the nervehead behind mirror.

Ophthalmoscopy by Direct Sunlight (Jackson). - For ordinary ophthalmoscopic examination, direct sunlight is manifestly unfit, but for certain diagnostic purposes it is superior to light from any other source. Its intensity gives it a greater penetrating power than any other form of light when the media are hazy or partially opaque. There is little or no danger to the eye from concentrating the intense light and heat of the sun upon the eye. The accurate focusing of the light upon the retina is prevented and even a slight haziness of the media renders impossible such a concentration of the light as experience has shown to be injurious. The pupil must be dilated or unresponsive to the light thrown into the eye. It is not necessary to have a dark room. The patient sits with his back to the sunshine

[^4]which enters a window, or which is reflected into the room by a mirror outside of the window.

To examine the anterior segment of the eye, the mirror is held so that the light is focused a little anterior to the eye. To study the vitreous or the fundus the mirror is held so that the light is focused somewhere in the vitreous chamber. The important point is to avoid concentrating the light upon the skin of the face or upon the coats of the eyeball. At times better results are obtained by not looking through the hole in the mirror but a little to one side of it. Or it may be best to illuminate the eye, by concentrating the light (not focusing it) upon the sclera and then view the interior of the eye through the pupil. Examination of the eye by means of direct sunlight as outlined is of value in cataracts and in cases with cloudy cornea and in diagnosing the presence of tumors within the eyeball, etc.

As an auxiliary in diagnosing lesions of the fundus of the eye, Neuschuler has introduced ophthalmo-chromoscopy. He accomplishes this by placing a series of glasses of different colors behind the sight-hole in the ophthalmoscope. In the application of his method he makes use of a well-known law that when an image of a color is viewed through colored media the corresponding color of the interposed media will always appear pale. Red, green and blue were the only colors he found necessary to employ. This method gives very valuable results when it is desired to produce an exact picture of the fundus. It may also be used in retinoscopy as well as in the examination of the throat and ear.

Henocque employs spectroscopy in studying the eye. By means of his ophthalmo-spectroscope he claims to obtain readily the different spectral colors from the tissues of the eye.

Reflections are very annoying to the beginner in ophthalmoscopy. The one that annoys most is the reflection of the light from the patient's cornea. Its perception is less when one is nearer the eye and one can usually look to one side of it or through its edge, but it is always present to disturb the view of the fundus details in a measure.

Almost as annoying is the reflection of light from the surfaces of the objective lens. These however can be shifted from the field of view by tilting the lens up or down a little. There are other reflections seen in the fundus of the eye which will be described under the appearances of the fundus.

Examination of the Eye by Oblique Illumination or Transmitted Light.-Oblique illumination of the eye is performed by focusing light upon it by means of a strong convex spherical lens held close to the eye, and between it and the source of light. Oblique illumination is useful in searching for small foreign bodies in the cornea, conjunctiva or under the lids, and in examining small lesions of the cornea. By holding the lens a little closer to the eye the light can be concentrated upon the iris or upon the crystalline lens and these structures examined under good illumination. Opacities in the anterior portion of the vitreous humor or a detached retina as well as growths that


Binocular Magnifying Lens. present themselves in that region may be studied by means of oblique illumination. A second convex lens may be used to magnify the lesion revealed by oblique illumination, being held closer to the eye than its focal distance.

Jackson recently devised a binocular magnifying lens made of two lenses ground in one piece, with which the apparent relation of the parts of the eye is not altered as it is when viewed with one eye only.

The binocular magnifier has since been mounted with prisms in tubes, to which the name of stereoscopic loupe is given. The instrument is held before the eyes by a spring or rubber band about the head.

In detecting the presence of slight opacities in the media of the eyeball a plane mirror is very useful. The light is reflected into the eye from a distance of about one foot, the room being perfectly dark and with the source of light about six inches behind the patient's head. Any opacity of the media is then seen to interrupt the homogeneous
red glare of the pupil, and the appearance of irregular astigmatism described under the head of retinoscopy may be seen, that is the pupil appears to be broken up into areas of light and shade, that move some with and some against the direction of rotation of the ophthalmoscopic mirror. (See section on Retinoscopy.)

Oblique illumination by direct sunlight will frequently enable one to discern growths within the eyeball when the media are cloudy. Great care is needed not to bring the light to an accurate focus upon the cornea. The room should be darkened and the sunlight reflected in by a mirror held outside the window.

## CHAPTER VIII

## OPHTHALMOSCOPIC APPEARANCES OF THE FUNDUS OF THE EyE

The color of the normal eye-ground varies from a light pink in the albino to almost a slate color in the black-skin races. The lighter the complexion of the individual, the lighter the color of the fundus. The shade of the fundus is due to the amount of pigment in the stroma of the chorioid. The overlying retina is transparent in normal conditions and does not affect the color of the eye-ground. The epithelium of the retina can at times be recognized as a finely granular surface, by the direct method, especially if the refraction is perfectly corrected. In examining the fundus there are two regions of especial importance, namely: the entrance of the optic nerve and the macular region of the retina. Each of these regions should be examined separately for pathological changes. To bring the optic disc or papilla, as the nerve head is called, into view (into line of vision), when using the indirect method of ophthalmoscopy, the eye under examination must be turned towards the nose about fifteen degrees, as the optic nerve enters the eyeball about fifteen degrees to the nasal side of the center of the retina. As the front of the eyeball moves in towards the nose the posterior portion moves out, bringing the nerve head in line of sight for the examiner's eye. If the examiner holds the ophthalmoscope before the right eye, with the handle of the instrument horizontal, the right eye of the patient will be adducted to the proper amount if he looks at the uplifted little finger of the examiner's hand holding the ophthalmoscope. To examine the nerve of the left eye, the patient fixes the opposite ear of the examiner. It makes little difference whether the patient simply turns his eyes in the required direction or turns his head, so that he fixes the proper point. Both eyes should be trained to use the ophthal-
moscope. One can tell whether the optic nerve is in line of vision or not before putting up the objective lens.

It will be noticed that when the light is thrown into the eye that the pupil immediately shuts up, and the fundus is poorly illuminated, evidenced by the dark color of the reflection from the pupil. When the patient directs his eye in the proper direction for the light to fall upon the entrance of the optic nerve, the illumination becomes brighter and the pupil of the eye dilates, because the nerve head is of a lighter hue than the surrounding fundus and because the nerve head is a blind spot, so that light falling upon it does not stimulate the pupil to contract. The optic disc is more or less circular in outline and of a light pink color, and from it radiate and ramify the blood-vessels of the retina. It varies from 1.5 to 1.7 mm . in diameter. It is more or less sharply defined from the surrounding darker colored chorioid, by a more or less well-defined black border, called the chorioidal ring. It is formed by a heaping up of the chorioidal pigment at the nerve entrance. This ring may be entirely absent or only marked in part of the circumference of the nerve. Within the pigment ring is most frequently a well-defined white ring, called the scleral ring. The latter was formerly supposed to be the white sclera showing through, the chorioid being deficient in that locality; but now we know it to be the nerve sheath or perineurium, formed of connective tissue.

At times the scleral ring is entirely absent. There is usually seen within the confines of the nerve and rarely extending to its margin a depression whiter than the rest of the papilla. This is the physiological pit or excavation. It lies to the nasal side of the center of the papilla and frequently its nasal edge is steep and overhanging, while the pit becomes shallower and finally reaches the level of the rest of the nerve in the opposite direction. The pit is formed by the rapid arching of the optic nerve fibers as they divide to supply the different parts of the retina. In the bottom of the pit is exposed to view the connective tissue that supports the nerve, the lamina cribrosa, which imparts the white color to the depth of the pit and the in-
terstices between the fibers of the lamina gives the pit a stippled appearance.

Through a larger opening between the fibers of the lamina, called porus opticus, enters the retinal artery. The artery is accompanied by its vein, lying to its temporal side. The vessels frequently show a bend in their course as they emerge from the pit. Climbing up the steep side of the pit they suddenly come to view after bending over the edge of it. The artery is of a lighter color than the vein, and smaller in diameter. Both have a light streak running along the middle of their walls. This is a reflex streak, formed by the reflection of light from the anterior convex surface of the column of blood within the vessels. The arteries show this streak more plainly than the veins, due to the lighter color of their contents.

Loring's explanation of the reflex streak seen along the course of the retinal vessels is that the light is condensed in passing through the column of blood in them and then reflected back by the underlying tissue. Noyes concurs in this opinion, as he noted a case in which there was a hemorrhage beneath a retinal vessel that did not interfere with the continuity of the vessels but abolished its reflex streak. And, again that in cases in which a retinal vessel was seen to cross an area of fatty degeneration in the retina that the reflex streak was intensified by the glistening surface beneath the vessel. Dimmer thinks that the reflex in the veins is due to a reflection from the surface of the column of blood within them, but that in the arteries it is from the axial part of the blood stream. This reflex streak differentiates the retinal vessels from the flat ribbon-like vessels of the chorioid, which are also told by their free anastomosis.

At times artery and artery or vein and vein anastomose upon the surface of the papilla or after leaving it. The arteries of the retina are end arteries like those of the brain and kidney. The artery and vein often cross and recross each other, and if the vessel walls are not thickened by a pathological change taking place in them the one vessel will not obscure the other from view as it passes over it.

Pulsation of the retinal veins is not infrequent, especially if they are large. Schön concluded from the study of one case that the pulsation in the vein is communicated from the artery, as the artery and vein lie side by side in the optic nerve. Pulsation of the arteries of the disc occurs in cases of increased intraocular tension, and in diseases of the heart especially in aortic stenosis and inefficiency. At times there is a small arterial twig seen to enter the eyeball through the medulla of the optic nerve. Such branches are usually derived from the posterior or short ciliary arteries that pierce the sclera around the optic nerve to supply the chorioid coat. They are called aberrant or cilio-retinal arteries Cilio-retinal arteries are oftenest seen in eyes with myopic crescents, and are destined to nourish the tissue about the macula lutea.

The retinal vessels run chiefly upwards and downwards, branching in an arborescent fashion. At the entrance of the optic nerve the following points are to be considered: A pink medullary portion of the nerve, a white physiological pit, vessels radiating from the pit, white scleral ring, black chorioidal ring.

There are many variations, within physiological limits, in the form and appearance of the optic papilla. Its border may be well defined with a chorioidal or scleral ring or with both. Either ring may be marked only in part of the circumference of the nerve, or entirely absent. The edge of the nerve may be rather illy defined, denoted simply by an abrupt change in the color of the fundus. There may be no physiological pit, and the vessels not infrequently spring from about the center of the medullary portion of the disc. The disc is to be considered normal if it is of a clean, clear, pink hue and if its edge is to be readily discerned, and if the vessels upon its face are clear and well-defined. The macula lutea, or area of most distinct vision, is the most important part of the retina. It is about 2 mm . in diameter and situated near the posterior pole of the eyeball. It is difficult to see, as the pupil shrinks to its smallest when the macular region of the fundus is illuminated, and the corneal reflex is most annoying when the eye is in position for viewing the macula. The macula
lutea or yellow spot was so named from the yellow color that it presented in dissection. The yellow color is now believed to be a postmortem change. The position of the macula is marked in the living eye by a darker red spot in the fundus with a shifting reflex. The foveal reflex is an inverted image formed by the concave surface of the excavation or central pit of the macula, and has the shape of the illuminated area of the ophthalmoscope. The reflex is annular if the sight-hole of the ophthalmoscope is exactly centered with the pupil of the observed eye, elliptical or comet-shaped if the sight-hole and pupil are decentered. Around the macula the retina grows gradually thinner. This sloping of the retina gives back a reflection in the form of a halo or ring of smoke surrounding the macula.

This ring is complete only when the sight-hole of the mirror and the pupil are centered. This reflex is seldom well seen by the direct method. The ring is usually horizontally oval, and slightly larger than the optic disc in diameter, and one or two disc diameters from its lower margin. These reflections are best seen with a concave mirror, and they change in shape and distinctness as the focusing of the light upon the retina is altered by shifting the objective lens backward and forward. This fact will enable the beginner to differentiate them from infiltrations into the retina, for which they are often taken. These reflections become dim with old age. There is not infrequently seen, especially in dark-colored eyes and in those of children, a shimmering or watered silk appearance along the course of the retinal vessels.

These reflections change with each motion of the mirror. They move against the rotation of the mirror and thus show that they are reflections from concave surfaces, where the contour of the vessels pass into the retinal level. There is a reflex of the same nature at times seen parallel to the nasal side of the papilla The macular region of the retina has very few vessels while its central portion has none at all, by which it may be recognized with the ophthalmoscope. Pathological changes are frequent in this locality. The peripheral portions of the retina have no especial peculiarities. The periphery
is at times the seat of the earliest pathological changes, as in retinitis pigmentosa. In the periphery of the fundus, the chorioid is often thinner than elsewhere, exposing to view its deeper layer of vessels, which are seen running as flat ribbon-like vessels of a dark red color and frequently anastomosing. There is not infrequently a heaping up of the chorioidal pigment in the periphery likewise, giving it a mottled appearance.

In fair eyes the vessels of the chorioid are seen as darker colored streams enclosing lighter colored areas, while an abundance of pigment in the chorioid causes the vessels to appear lighter in color than the intervascular tissue, especially if the retinal pigment epithelium is thin. Such a fundus is called a tesselated fundus. A thick retinal pigment gives the fundus a more homogeneous appearance. In some eyes the beginnings of the venæ vorticosæ are recognizable.

Differences in the level of the fundus are made apparent by the indirect method by parallactic displacement. The convex lens (objective lens) is moved a little up or down during the examination. If the parts of the fundus in the field occupy the same plane they do not alter their relative position to each other as the objective lens is moved. If to the contrary there exists a difference of level between them we will see them approach and recede from each other, as the lens is moved. By the direct method of ophthalmoscopy we determine the exact difference in level of two portions of the fundus, by ascertaining the amount of refraction error produced by the elevation of excavation. The former gives rise to hyperopia and the latter to myopia.

## CHAPTER IX

## FUNCTIONAL TESTING. THE FIELD OF VISION

Inasmuch as functional testing is subjective entirely, we are dependent upon the intelligence and good will of our patients for a correct test. In children and mentally deficient individuals the facts derived from functional testing are frequently misleading. When we look at an object we can recognize its form, its color and its brightness. The first is called the form sense ; the second, the color-sense ; and the third the light-sense. These three faculties are resident in the retina throughout its entire extent, but to a varying degree, and diminish from the center to the periphery. Central vision is that performed by means of the macula lutea, and when we look directly at an object we employ direct or central vision, and the eye is said to have fixed the object. We employ central vision for all useful seeing, as when we read, work or what not. It is in regard to the central vision that we test and correct refraction errors, and estimate the amount of accommodation and visual acuity. Peripheral or indirect vision is that performed by other portions of the retina. Indirect vision is poorer for form, color and light the further from the center vision is performed. The extent of indirect vision for form or space is called the field of vision. If we get such poorly defined perceptions by indirect vision, of what use is it at all?

A person devoid of peripheral vision is in the same condition as one looking through a long narrow tube. He can read the finest print straight ahead but is not in a condition to go about alone, as he would stumble and fall over everything in his path. Peripheral vision is of value in enabling us properly to place objects in space, and thus placing them, avoid falling over them or avoid colliding with them. This faculty of assigning to objects their proper position in space is called orientation. While we are walking along, a stone in our path
forms its image on the periphery of our retinas, our attention is attracted to it thereby, and we then turn our eyes directly towards it. Our central vision then gives us a clear conception of the nature of the obstacle, and we then avoid it. The same thing is done when moving objects approach us from the side. Their images are first formed upon the periphery of the retina, which awakens our attention to their approach. If it were not so, we would not be able to go about in a crowded street without being run over by some vehicle. Exner has shown that the peripheral portion of the retina has quite a high degree of sensitiveness for the projection of motion in an object, so that our attention is attracted most certainly to it.

We refer an object in space along a line which we imagine drawn from the retinal image through the nodal point to the outside. The object and its retinal image are then diametrically opposite. The upper part of the retina is stimulated by an object below the level of the eyes when the eyes are directed straight ahead and vice versa.


In the figure, let $N$ be the nodal point of the eyeball $A B, A^{\prime} B^{\prime}$ and $A^{\prime \prime} B^{\prime \prime}$, the objects of attention. A line drawn from the head of each object through the nodal point of the eye will stimulate the retina at the point $a$, and likewise the tail of each image will stimulate the retina at the point $b$, in the fundus of the eye. The lines connecting different points of the object with like points in the image are called visual axes. It will be seen from the figure that the image $a b$, corresponds to either object, $A B, A^{\prime} B^{\prime}$ or $A^{\prime \prime} B^{\prime \prime}$. The distance of the object before the eyes is however estimated subjectively by the amount of accommodation needed to bring it to a sharp focus upon the retinx, and by the amount of convergence necessary to see it singly with the two eyes. Whenever the retina is stimulated, the stimulation is projected into space along lines as shown above. This is called projecting an object into space. By virtue of this fact we see the objects in the external world arranged in their proper relation in regard to one another (objective orientation).

But, to get a correct idea of the arrangement of objects in space, we must have a knowledge of the relation that they bear to our own position, in space (subjective orientation). The latter depends upon us having a correct idea of the position of our bodies in space, and the position that our eyes occupy in our heads. The former is given to us by the sense of equilibrium, through the internal ear and the cerebellum, and the latter by the muscular sense resident in the extraocular muscles themselves, which informs us how our eyes are turned in our orbits. By means of objective and subjective orientation we can assign to an object that is presented to our view, its true position in space. As a rule we see with both eyes at once, but the two images, one upon each retina, are fused, $i$. e., the stimulation in each eye is conveyed to the brain and interpreted as one image corresponding to one object in space so long as corresponding parts of the two retinas are stimulated, the macula lutea in each or the same distance to the right or left, above or below the macula in each eye.

According to the law of projection, when the same part of each retina is stimulated, the object of stimulation is located by both eyes at the same point in the outer world, and hence the object is seen as one with both eyes, or there is then single binocular vision.

In stereoscopic vision, or the estimation of solidity of an object, the portion of the retina stimulated in one eye does not correspond with that stimulated in the other eye, as will be seen later on, although there is fusion of the retinal images, giving perspective to the object. If a prism be placed before one eye so that the light that enters it is deviated from its proper path the eye will place the object in a faulty direction in space. This is called false orientation. If one eye is turned out of the line of vision for the object of attention, as in cases of crossed eyes, the same thing happens. Each eye projecting its image in a different direction in space causes the two eyes to see a single object as two. There is binocular diplopia, or double binocular vision. Diplopia exists only in cases of rather recent squints, for, after the squint has existed for a while the patient learns not to take account of the image formed in the deviating eye, as diplopia is very
annoying. There is suppression of the retinal image in the squinting eye, as we speak of it.

## EXAMINATION OF THE FIELD OF VISION.

The examination must be made for each eye separately. The eye under test is kept fixed upon a point straight ahead, and the fellow eye bandaged. While the eye maintains central fixation, an object is moved from the periphery over the field of view, or the reverse, that is, beginning at the point of fixation the object is moved towards the periphery in different directions. The simplest but at the same time a crude way to estimate the extent of indirect vision is for the physician to place himself directly in front of the patient, and at a short distance from him, and then move his uplifted finger or a small white ball upon a black rod, gradually from the periphery towards the center and midway between himself and the patient. The examiner closes the eye that is opposite the bandaged one of the patient. The patient is instructed to say so as soon as he sees the hand or the object approaching. Taking it for granted that the field of the examiner is normal this will occur at the time the physician himself first sees the object approaching, if the limits of the patient's field are normal.

The hand is moved from either side and from above and below, and the point at which the physician and the patient see the hand for the first time compared. Small defects in the visual field are not recognized by this method. It is the only way that we can test patients with deficient visual acuity, and in cataract cases, where the vision is still better than the perception of light only, but when smaller test objects can no longer be distinguished. The field is to be gotten more accurately upon a blackboard, but with considerable inconvenience. We place the person in front of a blackboard at a distance of 25 cm ., which distance must be maintained throughout the test. Directly in front of the patient's eye we make a mark on the board, which he is to fix during the examination. A stick of white chalk is now gradually brought from the edge of the board
towards the center, and the point at which it first comes into view is. marked. By thus marking the limits of the vision in every direction. and connecting the points by straight lines we outline the extent of indirect vision. The size of the field as depicted upon the board is of course dependent upon the distance of the patient from the board. If we divide the board up into squares of 1 cm . each, and have the patient at exactly 25 cm . from the point of fixation, the value in degrees of arc of the squares on the plane surface may be ascertained from the following table:

| 2.2 cm. | 5 degrees. | 17.5 cm. | 35 degrees. |
| ---: | :--- | :--- | :--- |
| 4.4 | IO | 2 I | 40 |
| 6.7 | I 5 | I 5 | 45 |
| $9 . \mathrm{I}$ | 20 | 30 | 50 |
| 11.7 | 25 | 36.7 | 55 |
| 14.4 | 30 | 33.3 | 60 |

This method is a very imperfect one, due to the fact that it is impossible to project a hollow sphere upon a plane. Equal distances upon the plane correspond to unequal distances upon the retina. The second evil is that the whole field cannot find a place upon a plane. The normal field extends on the temporal side to ninety degrees and even beyond; therefore, the temporal limits of the field can never be projected upon a plane. The figure illustrates each of these points.

In the figure the eye $E$ is represented as fixing the point $F$. Points $a, b$ and $c$ are equidistant along the horizontal plane $L L^{\prime}$. The dotted lines $a a^{\prime \prime}$ and $b b^{\prime \prime}$ are drawn through the nodal point of the eye without any refraction, as they enter the dioptric media of the eyeball ; points $a^{\prime \prime}$ and $b^{\prime \prime}$ would then be the areas of stimulation from the objects $a$ and $b$ respectively, but refraction takes place in rays from $a$ and $b$, as they enter the eyeball; ergo: point $a^{\prime \prime}$ is shifted to point $a^{\prime}$, and point $b^{\prime \prime}$ to $b^{\prime}$, the true positions of the retinal images of the objects $a$ and $b$. Intervals $F a, a b$ and $b c$ are equal, but the intervals corresponding to them upon the retina are unequal, i.e., $F^{\prime} a^{\prime}$,
$a^{\prime} b^{\prime}$ and $b^{\prime} c^{\prime}$ are unequal, diminishing towards the periphery. Point $d$ at a right angle with the visual axis $F F^{\prime}$, can stimulate the retina as the light from it undergoes a sharp bending as it enters the diop-

tric media of the eye, and so the light passes into the eyeball through the pupil. As $L L^{\prime}$ and od are parallel, point $d$ can have no place upon the plane $L L^{\prime}$, therefore the whole field of vision cannot be projected upon a plane.

There is only one way to properly represent the visual field, and that is to project it upon a hollow sphere (Aubert). .The instrument employed thus to take the field of vision is called a perimeter. There are many different types on the market, but all are constructed so that the field is projected upon an arc, the section of a hollow sphere, that can be rotated in different meridians. In most of the instruments the test object is moved along the arc by the hand of the operator, and when it first comes into view, its position is marked upon a chart which comes for the purpose. The latest improved instruments have a system of gearing that enables the operator to move the test object along the arc of the instrument by operating a small milled wheel, at the back of the perimeter. In order to save time there is also an arrangement to punch or mark the chart in the proper place, when the patient first sees the approaching object. Several models of perimeters are appended.

Dana's pocket perimeter is a very convenient one for the oculist who has to carry his instrument to the bedside of an invalid or sick patient, or for one who visits many institutions. This instrument consists of a metal semi-circular protractor, graduated from central o to 90 degrees in both directions. The movable bar which carries the white and colored test objects rotates on a pivot over the surface of the graduated scale. To obtain the field in the horizontal direction, the scale is placed di-
 rectly under the eye, resting the instrument against the cheek bone, and the extent of the field is denoted by the position of the bar upon the scale.

The vertical meridian of the field is taken by resting the instrument against the temple and reading in the same manner. The instrument can be easily carried in the vest pocket, by sliding the extension rod into the hollow handle, the colored discs being held by a small clip upon the reverse side of the scale. The model that is perhaps the most universally employed is that shown in the cut below, of which there are various makes.

This instrument combines the most practical points in the Landolt and Priestly Smith perimeters. It has a broad hard-rubber arc with a sliding object carrier after the pattern of Landolt and the registering attachment of the Priestly Smith instrument. It has an adjustable double chin rest sliding upon an upright bar, the end of which carries the cheek rest. The chart is fixed to a hard-rubber disc attached to the back of the instrument and revolving with it. A stationary scale, mounted upon an upright arm, is graduated to correspond to the divisions upon the arc, and is placed immediately back of the disc holding the chart. By means of this scale the exact position of the object upon the arc, and the meridian of the arc itself is marked by a
single puncture. To avoid orientation in the mind of the examiner the scale back of the chart is graduated in two colors, red and white; the degrees on the arc are likewise in red and white. A simple observation of the color and degree on the arc at which the object stands only is necessary to prick the chart in the proper place. There is an opening through the center of the spindle of the arc to allow the ex-


Meyrowitz Perimeter.
aminer to watch the eye of the patient, so that central fixation is maintained throughout the test. The color discs show colors of white, red, green, blue and yellow in squares of 5 mm ., and are carried on a handle 45 cm . long. The two instruments below are self-recording. Dr. Skeel's is the more complicated and more apt to get out
of order. The chart is fixed upon the platform, as shown in the cut. The knob on the back of the disc is rotated, drawing the object along the arc until it comes into the field of view, the lever at the bottom
 of chart platform is then pressed and the chart is marked in the proper place. In the Hardy instrument the chart is fixed upon the disc that projects from the instrument and the carrier moved along the arc by rotating a milled head. The chart is marked 3 in the proper place by pressing the chart up against a needle-point.

One should have an adjustable table for the perimeter so that it can be placed quickly at the proper height for the patient.
The Extent of the Field


Hardy Perimeter.
the temporal side, that is in the temporal field, even if they lie a little
posterior to a plane passing through the pupil. This is on account of the strong refraction that the light from such a point undergoes at the surface of the cornea, enabling it still to enter the pupil. The field is much less extensive to the inside and to the inside and below. The cause of this is that the nose and the cheek intercept vision in these directions, but if the head be rotated so that these obstacles are removed, the field will even then be found not to extend as far to the nasal side as to the temporal. This is because the retina does not extend as far to the temporal side as to the nasal side of the eyeball, for it is the nasal retina that sees objects in the temporal field and vice versa. And as Landolt pointed out, the temporal retina is not as acute as the nasal portion, as it is not used so much. The limits of the field of vision for white in round numbers are: Above, $50^{\circ}$; nasally, $60^{\circ}$; below, $70^{\circ}$; temporally, $90^{\circ}$. The color visual fields are less extensive than that for white. Blue is recognized at the greatest distance from the center, then yellow, orange, red, green and violet are recognized at decreasing distances from the center in
 the order named. The blue field is five degrees within the white one; the red ten degrees within the blue, and the green ten degrees within the red field.

In the figure the limits of the white area represent the extent of the field of vision for white. The small circle near the center of each chart represents the position of the physiological blind spot, i. e., the head of the optic nerve. The dotted line in the left-hand chart is the extent of the field for blue ; the stroke and dot line, that for red, and the cross line, the field for green. The extent of the field for white and for colors varies very much in different individuals within physiological limits. If either field is curtailed ten degrees or more
within the limits given it is to be considered abnormal. As we ordinarily measure the visual field, as Baas pointed out, we measure the relative visual field in contradistinction to the absolute field of vision. The relative visual field records the limits of vision for an object of a definite size, while the absolute field is the limit of the field of vision without regard to the size of the object. The figures given above represent the extent of indirect vision taken with a test object 1.5 cm. square.

Pathological alterations in the field of vision consist in its curtailment. This may be concentric (contracted equally in all directions) with the retention of good direct (central) vision. When the field is much encroached upon the individual has lost the faculty of orientation. At times the contraction is more decided on one side of the field than on the other, as in glaucoma, in which the nasal field suffers the greatest shrinkage. When the contraction is in the shape of a triangle with its apex at the center of the field, it is spoken of as a sector-shaped contraction. The entire one half of the field may be wanting, constituting the hemianopic field. Blind spots within the limits of the visual field are called scotomata. One of these exists in the normal eye at the entrance of the optic nerve. It is known as Mariotte's blind spot. It lies to the temporal side of the point of fixation about fifteen degrees. The scotomata that occur as the result of disease are known according to their locality as central or peripheral. A central scotoma involves the macula lutea. The patient with such a blind spot can no longer see fine print or do fine work, but is perfectly able to get safely about alone, just the reverse of one who has a contracted field, with retention of normal central vision. At times there exists an annular scotoma around the point of fixation, leaving the macula intact. Von Graefe was the first one to show the importance of taking the field of vision in eye practice. He showed that many intraocular diseases had their peculiar forms of alteration in the visual field. Concentric contraction with retention of normal or good central vision is met with especially in cases of retinitis pigmentosa, less frequently in glaucoma and in atrophy
of the optic nerve. In most cases of marked peripheral contraction of the visual field central vision is greatly reduced. We find sectorshaped contraction especially in cases of occlusion of the retinal artery and in optic nerve atrophy. In glaucoma marked contraction of the nasal field is of frequent enough occurrence to be of some pathognomonic importance. Scotomata are most frequently to be met with in diseases of the fundus with circumscribed lesions, therefore in retinitis disseminata. The blind spots in chorioidal diseases correspond to the lesions seen in the fundus of the eye with the ophthalmoscope. So long as these spots are peripheral the vision does not suffer to any great extent, but if numerous the field assumes a sieve-like character. If the cause of the scotoma is not apparent in the fundus of the eye it must be looked for in the optic nerve. Scotomata are divided into positive and negative scotomata. By the first is meant a spot in the field in which objects are not distinctly visible, a cloud apparently being interposed between the eye and the object. If the cloud is faint and does not entirely obscure the object we have a relative positive scotoma; while on the other hand if the cloud is dense enough entirely to obscure the object of attention from view, we call it an absolute positive scotoma.

Positive scotomata are produced by opacities in the media of the eyeball which throw their shadows upon the retina, or retinal exudates or hemorrhages into the retina anterior to the layer of percipient elements. As these lesions cast shadows they are perceived as dark spots. If the opacities are in the vitreous they are most likely to be mobile, giving rise to motile scotomata. Fixed scotomata are caused by opacities in the cornea, lens or retina. These scotomata are best made apparent by having the patient gaze upon a uniformly bright surface, as a sheet of white paper. They will be more apparent, if slight, when the illumination is reduced. The patient may be instructed to outline the spot seen upon the paper with a pencil, from which the extent of the diseased area can be estimated. If the eye of the patient is at 25 cm . from the paper, and the projection of the scotoma as outlined by him upon the paper is 1 cm . in diameter, the
lesion in the fundus causing the blind spot is .06 mm . in diameter. The size of an object and its image are to each other as their respective distances from the lens.

$$
I / O=d / D, \quad i / \mathrm{ro}^{2}=\mathrm{I}_{5} / 250 \mathrm{~mm} ., i=.06 \mathrm{~mm} .
$$

A negative scotoma is simply a break in the field, an area in which nothing is distinctly seen, but nothing appears to be interposed. If the object looks pale but can still be seen, there exists a relative negative scotoma, while if the object is not discernible in that part of the field we have an absolute negative scotoma. All the perception of light is wanting within the confines of an absolute negative scotoma. Negative scotomata are only discovered by taking the field of vision, with small and especially with colored objects, for as the visual acuity diminishes, the ability to distinguish colors disappears before the ability to distinguish form or outlines of the object.

This is best illustrated in cases of poisoning from alcohol and tobacco. Such a case examined with a white test object seems to have a normal visual field, while if a small red or green colored object be used, there is found a central negative scotoma, in which location red or green is not recognized as such. The examination of the field for colors is of great practical importance and too infrequently done. It not infrequently happens that the colored fields show curtailment some time before any change is noted in the form of the white field. Rapid falling off of the color fields is associated with disease of the optic nerve and soon leads to blindness. The charts used for recording the field of vision do not represent the part of the retina of the eye under test stimulated, but the part or location of external space in which the test object has become visible; so that in marking the chart when the object is seen on the temporal side of the eye, a dot is placed upon the chart on the side marked temporal. If the chart be held in front of the patient there can no mistake be made, as then all objects seen on the right of the eye will be marked on the right and those on the left on the left side of the chart in regard to the patient. The binocular field of vision, or that area in which an
object is simultaneously visible to the two eyes, is more or less circular in outline. It extends above to the

abed-
Binocular Field for White upper limits of the monocular field and to the extent of the field of each eye below, and laterally from the fixation point to about sixty degrees. The fixation point is the center of this area and the physiological blind spot of each eye on the corresponding side of the fixation point. The binocular field of vision is not the space in which an object is visible to ether eye, but that in which the object is visible to both eyes at once. The temporal field of each eye extends thirty degrees beyond the limits of the binocular field.

The Technique of Taking a Field of Vision. -Seat the patient in front of the perimeter so that his chin will rest upon the chin-rest comfortably, with the cheek pressed against the upright piece that springs from the side of the chin-rest. The curved top of the cheekrest should fit into the depression under the malar bone. The head is directed straight forward and the eye not under test is bandaged, or closed by a card held in front of it by the patient, care being taken that the card or bandage does not rise above the bridge of the nose to occlude the vision in the lower nasal field. The eye under the test fixes the white dot at the center of the arc of the perimeter, and throughout the test the eye remains constantly fixed upon this dot. The test-object is then moved up from the periphery and the point at which it comes into view is noted upon the chart. The arc of the instrument, which up to this time has been in a vertical position, is turned fifteen degrees the one way or the other, and the point at which the object is first seen in this new meridian is marked. The arc each time is turned through fifteen degrees, and the limits of the field noted in each meridian of rotation. While the arc is in the same position, two points may be taken, one above and the other below, or to the right or left as the case may be, and thus some time saved. The dots upon the chart are now connected by straight lines
and the field thus outlined. The color fields are now taken in the same way, usually blue, red and green only being selected. Small test-objects are used when hunting for blind spots. The operator should wear black gloves during perimetric examination, so that his white hands and cuffs do not attract the eye of the patient away from the point of fixation. It is not necessary to do this in the Hardy and Skeel instruments as the test-object is moved along the arc by operating a small milled head behind a disc that hides the hand of the operator from view. If one desires to study from day to day the change in shape and size of a scotoma, it can be outlined upon a larger scale than is usually don'e by placing the patient's eye further from the fixation point. The distance at which the field is taken from day to day should be constant. Fifty centimeters is a convenient distance to employ. If the projection of the lesion occupies five degrees of arc when taken in the usual manner it will appear twice as big if the eye is at twice the distance from the point of fixation.

The numbering of meridians has not yet been universally agreed upon. Priestly Smith and others assume that the top of the vertical meridian is the starting point and number in either direction 180 degrees, denoting the temporal field as plus and the nasal as negative. According to Förster we should begin at the left, and go around 360 degrees, 90 degrees being at the top. We must always remember that the temporal fields correspond to the crossed fibers in the optic tracts and the nasal fields to the uncrossed.

Measurements of the blind spot are made by using a small bright test object. It is increased in cases of neuritis and myopia. It varies in size when normal from $4^{\circ}$ to $7^{\circ} 30^{\prime}$.

One of the reasons given for the reduction of visual acuity outside the macula lutea is that the images falling upon the periphery of the retina are distorted by the rays of light entering the eyeball at very oblique angles. Flick has shown, however, that the eye is nearly periscopic on account of the laminated structure of the crystalline lens neutralizing this distortion. That the eye is not exactly peri-
scopic may be seen in emmetropic eyes, which are hyperopic in very oblique axes. This is best seen by performing retinoscopy at a decided angle with the visual axes.

The field of vision is altered from a disturbance located anywhere in the visual paths. The following divisions of the visual apparatus are made: (I) The orbital portion (consisting of retina, optic nerve and chorioid); (2) the optic nerves ; (3) the optic chiasm; (4) the optic tracts ; (5) the primary optical centers ; (6) the intra-cerebral tracts leading to the cortical centers ; (7) the visual centers in the cortex of the brain.

The manner in which a diseased condition involving any of these structures influences the field of vision will be considered. Inasmuch as the outer layers of the retina are almost entirely nourished by the chorioid any disease in the latter interferes with the welfare of the former, as the contiguous layers of the retina soon become involved. In affections of the outer layers of the retina scotomata soon appear in patches when the nourishment of the delicate structure is altered by a diseased process, while in diseases of the inner layers of the retina (hemorrhages, sclerosis of vessels, degenerations, etc.) there is no characteristic change in the visual field, although there is present a certain amount of amblyopia. At the first scotomata can often only be outlined by aid of diminished light. In order to render as clear as possible the symptomatology of diseases of the optic disc and inner layers of the retina (nerve fiber layers of the retina) in contradistinction to that of the outer retinal layers (epithelial nerve layers), as well as of the chorioid (as affected in chorioido-retinitis) Wilbrand compiled the following table:

General Symptomatology.
Diseases of the Optic Nerve, Optic Papilla, Diseases of the Outer Layers of the Retina and Innermost Layers of the Retina.

Dark objects on a white ground Torpor retinæ. picked out as well almost as by healthy eye.

## Field of Vision by Diminished Light.

The eye acts as a sound one and shows the same faults as by ordinary daylight illumination.

By diminished light the field of vision shows either concentric contraction zonular defects, or central scotomata, while by ordinary light no defects can be found, or the small defects that are obtained are exaggerated by diminished light.

Forms of the Defects of the Field of Vision.
Mostly concentric limitation with Large irregular defects with zones, sector-shaped reëntering angles. The and islands (visus reticulus). Circular field of color perception is diminished scotomata. as compared with that for the perception of white. Central scotomata with diminution of the field on one side are present.

## Nature of Scotomata.

Central and peripheral negative scotomata which are not recognizable but in which objects are not ordinarily spots. recognized.

## Central Visual Acuity.

Usually better by diminished illumination. Nyctalopia.

Usually less by diminished illumination. Hemeralopia.

## Perception of Color.

Typical disappearance of color from the field; green goes first, then red and lastly blue. Zones of dulled perception of color (color-blindness) to absolute loss of power of perception of color. In partial optic atrophy, the fields of white and color approximate towards the point of the disease.

In ordinary daylight concentrically limited field boundaries. In full light the colors appear just as they do to the normal eye by diminished light. In the disturbed portion of the field green appears bluish, yellow appears reddish and violet appears as gray.

Metamorphopsia (retinal).
Not present.
Commonly present.

The Field of Vision in Diseases Affecting the Optic Nerve.-Both inflammatory and atrophic changes in the nerve give rise to alteration in the field of vision. The disturbances in vision in inflammatory affections of the nerve are due to destruction of conduction caused by the swelling and increase of the connective tissue septa pressing upon the nerve-fibers, and partly by the disturbance in nutrition.

In simple optic atrophy the changes in vision are caused by evenly spreading loss of power in the individual nerve fibers, of the whole cross-section of the nerve. It is progressive and always ends in blindness. In neuritic atrophy the visual changes depend upon the duration and intensity of the inflammatory reaction, and the permanency of the disturbances in nutrition, to which the individual bundles of the optic nerve are exposed. In secondary descending atrophy the amount of change is commensurate with the loss of conductivity of the nerve fibers produced by disease of their ganglion cells. In diseases of the optic nerve and especially in simple atrophy the diminution of the field for white and for color bear a definite relation to each other, which is of the greatest importance for diagnosis and prognosis. Defects first appear in those portions of the field whose corresponding retinal areas have in the normal physiological condition the weaker visual acuity. The early defects are then found towards the periphery of the field.

If a sector-shaped area of the optic nerve is attacked more intensely, there will be found a similar shaped defect projected into the field of white. The field for blue will follow the same conformation. If the limits of the colored field approach close to the limits of the white field it proves that the eccentric acuteness of vision remains near normal. If the defect divides both the color and the white fields so that the limits of the two fields coincide, we have a condition known as partial atrophy, which indicates a stationary disease, and favorable prognosis. The relation of central acuteness of vision to the defects in the visual field can not be regarded as a constant one. Jacobson and Wilbrand have arranged the following rules:

If central vision falls off at the same time with eccentric vision, it is a sign that the disease has attacked the whole tract. If central vision is diminished and the field lessened by sector-shaped defects which cut into the field from the periphery, it is a sign that the whole tract is affected, those parts related to the sector-shaped defects being the more disturbed. If central vision is much diminished while the periphery of the visual field is not much disturbed, a central scotoma relative or absolute is to be thought of. The following is the anatomical condition of the optic nerves as given by Henschen.


The macular bundle lies ventro-laterally in the papilla and also immediately behind it. At the latter place it forms a keystone-shaped sector with its base turned towards the pial sheath and its apex towards the central vessels. See figures. Further back this bundle

is half-moon-shaped ; still further back it takes the form of an upright oval, and approaches nearer the axis of the optic nerve. In the foramen it assumes an axial position. In front of the chiasm it assumes the form of a horizontal oval.

The macular bundle contains both crossed and uncrossed fibers. In the papilla the crossed fibers lie ventrically, and the uncrossed ones more eccentrically, being in proximity to the other uncrossed fibers. The spreading of the fibers
 over the retina is as shown in the cut below. Further back the macular bundle becomes drawn together towards the center, as shown in the second figure. The dorsal half of these fibers goes to the dorsal half of the retina, and the ventral ones to the ventral half. The eccentrically distributed uncrossed bundle is divided in the anterior division of the optic nerve into two fascicles, a dorso-lateral uncrossed dorsal part and a ventro-lateral uncrossed ventral part. In the lamina cribrosa these fibers are separated by the macular bundle. Behind the entrance of the central vessels the fascicles approach one another and form a united half-moon-shaped bundle, which includes the lateral periphery and lies somewhat ventro-laterally. The crossed peripheral distributed bundle forms a closed cord in the whole of the optic nerve. In the papilla it is situated dorsomesially, and retains this position until it passes the chiasm.


Field of Vision in Diseases Affecting the Chiasm. - The disturbance of vision that is pathognomonic of disturbances in conduction at the chiasm is temporal hemianopsia in its various manifestations.

It is observed only in organic lesions and does not occur as a functional disease. By hemianopsia (hemiablepsia) is meant, complete or partial loss of sight affecting one half of the field of vision.
The term temporal or nasal hemianopsia has to do with the portion of the field affected and not with the portion of retina involved, thus temporal hemianopsia means loss of sight in the temporal field. The eye affected with hemianopsia is said to be hemianoptic (hemiopic).*

The figure above represents the hemianoptic fields, with the scotomata reaching up to and involving half of the fixation point. At times the line of separation between the halves of the field of vision in temporal hemianopsia does not always lie vertically and may be situated beyond the point of fixation, forming the so-called overshot field of rision. The latter could only occur in cases
 in which the maculæ were provided with a double set of nerves, one set from each hemisphere. The arrangement of the optic fibers in the chiasm is shown in the figure.

Pressure from an exudate or tumor involving the upper or lower half of the chiasm may give rise to a binocular inferior or superior hemianopsia.

The Field of Vision in Diseases of the Optic Tracts.-The pathognomonic disturbance in the visual field in diseases involving the tracts is homonymous hemianopsia, also called lateral hemianopsia. In this condition the left or the right field in each eye is destroyed. There is some difference in regard to the line of separation of the two portions of the field in different cases. In some the line of separation cuts through the point of fixation. In another type the line

[^5]of separation is carried past the fixation point to the advantage of the remaining halves of the field of vision. Lateral hemianopsia is caused by organic diseases only and is absolute, that is within the blind portions of the fields nothing at all is seen. The hemianoptic fields may be incomplete, that is a scotoma may occupy the corresponding part of the field of each eye.


LEFT HOMONYMOUS HEMIANOPSIA CAUSED BY INVOLVEMENT OF THE RIGHT OPTIC TRACT.
Hemianopsia in Lesions of the Primary Optical Centers.-By the primary optical centers is meant the corpus geniculatum, the corpus quadrigeminum, and the optic thalamus. A diseased focus affecting the primary optic centers gives rise to homonymous hemianopsia, and hemiplegia and hemiæsthesia of the same side from common involvement of the neighboring fibers that pass through the internal capsule of the pedunculus cerebri. If the posterior end of the tract is involved the hemianoptic pupillary reaction is observed.

If the lesion is placed more centrally, that is posterior to the thalamus, the pupils will react when the light is thrown upon the blind half of either retina, and there is homonymous hemianopsia perhaps only for colors, or there is complete blindness. In some cases central vision is retained if the maculæ have a double innervation.

## CHAPTER X

## THE COLOR-SENSE

The ability of the eye to discern different colors is called the color sense. As has been seen, a ray of white light when passed through a prism is broken up into spectral colors: red, orange, yellow, green, blue, indigo (?) and violet. These colors are called simple colors because they cannot be further divided. They likewise compose the visible spectrum, the part of the entire spectrum that can be perceived by the unaided eye. Objectively the various colors consist of rapid vibrations of the ether, from about 400 millions of millions per second for red, to about 760 millions of millions per second for violet. At each end of the visible spectrum there are waves proceeding of such a refrangibility that the colors are not perceived by the eye. These are the ultra-red and ultra-violet rays. The reason given for their invisibility is that the waves are of such a length that they are absorbed by the media of the eye, and therefore never reach the retina to stimulate it.*

The ultra-violet rays can be perceived by receiving them upon a photographer's plate, or upon a fluorescent screen, or by eliminating all other light by painting the screen with a fluorescent substance, as sulphate of quinine or fluorescein. These substances struck by the ultra-violet rays send back visible rays usually bluish or greenish. With the proper precautions the ultra-violet rays may be seen of a grayish color, perhaps as the retina is itself fluorescent. It is the ultra-violet rays that are abundant in winter and in high altitudes that are supposed to be the cause of snow-blindness. It is possible to mix or blend color sensations in the eye by stimulating the same area of the retina by different colors at the same time or in rapid

[^6]succession. The following table shows the results of such experiments performed by Von Helmholtz :

|  | Violet. | Indigo | Cyan-blue. | Blue Green. | Green. | Green Yellow. | Yellow. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Red. | Purple. | Dark rose. | White rose. | White. | White yellow. | Gold yellow. | Orange. |
| Orange. | Dark rose. | White rose. | White. | White yellow. | Yellow. | Yellow. |  |
| Yellow. | White rose. | White. | White green. | White yellow. | Green yellow. |  |  |
| Green blue. | White. | Whitegreen. | White green. | Green. |  |  |  |
| Green. | White blue. | Water green. | Bluish green. |  |  | - |  |
| Bluishgreen. Cyan-blue | Water blue Indigo. | Water blue. |  |  |  |  |  |

These are mixed colors. As will be seen only two new colors can be produced, namely : white and purple, the other mixed colors having their equivalent in the spectrum. White and purple have no objective equivalent in a simple number of ether vibrations. Any two colors that produce white when mixed together are called complementary colors. Such are red and green-blue, golden yellow and blue, green and purple. The mixture of all the spectral colors of course produces white. Different results are obtained by mixing colored pigments, however. On the painter's palette yellow and blue produce green, but in the eye white. The reason for this is that the colors of nature are mixtures of simple colors, as can be seen by spectroscopic analysis, or by mixing spectral colors. There are three primary qualities resident in all colors: hue (ton), purity or tint (saturation), and brightness or luminosity (intensitè). The first gives the name to the color, red, blue or what not. The second depends upon the admixture of white in all colors, save those of the spectrum. The third quality depends upon the objective intensity of the light and the sensibility of the observer's retina. As pointed out on a preceding page the ability to see the different colors diminishes towards the periphery of the retina, blue being seen the furthest from the center.

Theories of Color Perception.-The theory of Helmholtz originated by Thomas Young, assumes that there is in the retina three different kinds of end-organs, each of which is loaded with its own
photo-chemical substance, which is decomposed by a certain number of ether vibrations and thus excites the optic nerve.

One group is loaded with a red sensitive substance, and is affected mainly by the red end of the spectrum; another group of end-organs is loaded with a green sensitive substance, and answers to the green portion of the spectrum, while the third group is provided with a blue sensitive substance, chiefly decomposed by the blue-violet end of the spectrum.

All three end-organs are present in different parts of the retina, and are connected by special nerve fibers with special parts of the brain, in the cells of which is stored the memory of the sensation of red, green or blue. All other color sensations arise from these primary sensations. If a color mainly excites the red or green substance, we term it red or green respectively. The equal and simultaneous stimulation of the red and green carrying end-organs gives rise to a sensation of yellow, that of red and blue to the sensation of purple, and that of blue and green to that of blue-green. The simultaneous stimulation of all three substances of a certain area to the same degree gives the impression of white. Complementary colors, according to this theory, are those that excite all three colorsubstances at the same time and to the same degree. Color-blindness is supposed to be caused by two of the color-substances being or becoming similar, or equal to each other or that one or more of them is absent. Thomas Young supposed that each spectral color stimulated one or perhaps two of the color end-organs in varying degrees, while Helmholtz supposed that all spectral colors stimulated all three end-organs at once, but to different degrees.

The theory of Hering was developed about four years later, in 1874. It assumes that the process of restitution in a nerve element is capable of exciting a sensation. There are accordingly three visual substances in the retina, namely: a white-black, a red-green and a yellow-blue visual substance. A destructive process in the white-black substance, not only occasioned by white light, but as well by any simple or mixed color, produces the sensation of white,
while the process of assimilation or restitution, gives the sensation of black.

Red light similarly produces a destruction in the red-green substance and the sensation of red while its reconstruction by a green light causes the sensation of green. The sensation of yellow has its cause in the decomposition of the yellow-blue substance by yellow light, while blue light causes a reconstruction and the sensation of blue. Simultaneous processes of dissimilation and assimilation in the same substance antagonize each other and cause the sensation of no color, white being the result, by decomposition of the whiteblack substance. Color blindness is explained by this theory by assuming that either the red-green or the yellow-blue substance in the retina is absent.

Among the more recent theories of color perception may be mentioned that of Ebbinghaus, who supposes that there exists in the cones a green substance, the decomposition of which would produce the sensation of red and green, while the visual purple by its decomposition would produce the sensation of yellow and blue. Perinaud supposes that the stimulation of the rods produces a sensation of non-colored light, while stimulation of the cones produces all possible sensations, the sensation of white and that of color. The retina would then have two systems sensitive to light, one monochromatic and the other trichromatic. Von Kries agrees with the views of Perinaud. Arthur König believes that the decomposition of the retinal purple into yellow produces a weak sensation of gray, which causes any color when it is sufficiently weak.

Further decomposition produces the sensation of blue. The perception of the two other principal colors, namely green and red, is affected through the pigment cells, while the cones are considered to be dioptric agents intended to concentrate the light upon the epithelial layer of the retina.
H. Müller measured the distance of the retinal vessels from the sensitive layer of the retina by means of the parallax of the vessels seen entoptically. In collaboration with Zumft he made these experi-
ments with spectral light. He found that the distance gradually increased as he approached towards the red end of the spectrum. The layer sensitive to red light would then be posterior to that sensitive to blue. These experiments have been repeated by others without success.

Congenital color-blindness is called daltonism (achromatopsia) from the English physicist (Dalton) who first described the anomaly, he being himself color-blind. There is a form of acquired color-blindness due to poisoning by tobacco and alcohol. The commonest form of daltonism is red-blindness; according to the Hering theory, red-green-blindness. It occurs oftener in males than in females, about three to four per cent. of the former being affected. Perhaps women are less often color-blind from undergoing a sort of training in busying themselves with colors, as in making dresses, fancy articles and so on-the taste for these things being inherited.

Color-blindness does not cause any inconvenience to its possessor further than that it unfits him for certain callings in life, as that of a painter, dyer, tailor or for the railroad or marine service. Those employed in railroad or marine service should have a good color sense as the signals used in each are the colors that are most apt to be confounded by the color-blind ; they are red and green. Employees should also be tested once a year for acquired color-blindness, if they are users of alcohol or tobacco. The next most frequent form of congenital color-blindness is yellow-blue-blindness, also called violetblindness.

It seems strange that the two colors which are most often confused should be used in a service where the lives of hundreds of people are concerned. The colors in use are, however, those that have the highest degree of luminosity, and, therefore, can be seen at the greatest distance off. White light, having the greatest penetrability, is likewise used for signaling. A white light seen through a bank of haze or fog often appears red, as the red rays have the longest wave-length of the spectrum and are, therefore, best able to penetrate the obstruction offered by the foggy medium. They are most
conspicuous at the end of their journey. In the neighborhood of large towns and cities the sun almost always sets as a red ball of fire, as above them usually floats a bank of haze. As the sun sinks to rest, the green rays are first absorbed, then the yellow and lastly the orange and the red, the latter often the only one to get through. (The blue of the vault of the heavens is said to be due to the reflection and scattering of the blue rays of light by the myriads of atoms of dust that pervade all space. The blue rays, having a short wavelength, are unable to pass through). A person born blind to certain colors can often name properly one of those colors when it is presented to him. To him colors are recognized simply by their luminosity or valence (intensitè). Since early childhood the color-blind individual has been hearing people say that leaves of trees are green and that cherries are red and so on. The association of the color and the object is mentally fixed and every time in adult life that he sees a cherry he thinks of red and looking at the trees and grass he calls them green. He is apt to become confused, however, if red and green differing from the shades found in nature be given him. To the absolutely color-blind a landscape looks like an etching, made of varying shades of gray. Or, if blind only for certain colors, those colors will be wanting in his conception of the picture, each being replaced by a gray of the same intensitè as the color itself. The color-blind engineer may get along all right in clear weather, when there is no fog or mist, and be able to keep separate in his mind the difference between the red and green signal lights. But, if a fog comes up to obscure the signal somewhat, then the luminosity of the light being altered, he has nothing by which to judge of the color, and an accident happens through the failure on the part of the engineer to appreciate the difference between red and green lights. The demonstration of color-blindness requires accurate and painstaking testing. Many of those who are aware of their defect try to conceal it from the examiner, especially if a good position is dependent upon a normal color sense. We must look out then for all sorts of tricks, when testing men for the railroad or marine service.

Frequently by practice the man has made himself adept in the methods of testing commonly used for the detection of color-blindness. On the other hand, persons with poor color appreciation (dyschromatopsia) and education may be considered color-blind if they are simply asked to name colors placed before them. We never test the color sense by setting different colors before the patient and ask him to name them, for the color-blind individual will often by a little attention give the right answer and the uneducated will just as often give the wrong answer. The test is best conducted by setting before the patient those colors which experience has shown are most apt to be confused, and then a large collection of colored worsteds are given him from which he is to select those skeins that match the confusion color. If then colors quite dissimilar are placed together, for instance gray, red and green, color-blindness exists. It is possible to tell what kind of color-blindness the individual has according to the colors he confuses. Thus, if a red skein is given him to match and he places besides it other colors of the same shade, it is evident that he is blind to red, and if he fails properly to match the green he is blind to green and so on. This test is the Seebeck or Holmgren test.

The next most commonly employed test is that with the pseudoisochromatic diagrams of Stilling. These consist of squares composed of different colors, so arranged that they form figures or letters. The colors of the test have been selected by a color-blind painter so that they will correspond to the confusion colors of the color-blind.

To the latter then all the squares appear to be of the same shade and the letters or figures are not seen. The spectroscope is indispensable for the scientific estimation of color-blindness. We can find out what colors in the spectrum are missing and by showing the patient an isolated portion of the spectrum make him tell both by naming and by matching the color how the spectrum looks to him. For the quantitative color sense, the tests of Donders, Weber and Wollfberg are used. Small discs of colored paper upon a background of black satin serve for test objects.

If the color sense is normal colored discs of different sizes must be recognized at given distances. The distance at which the disc is discernible varies for different colors. The stronger the color sense

Dr. Williams' Lantern for Testing the Color Sense of Employees in the Railway or Marine Service who use Colored Signals by Night.


On the front of this lantern are two movable discs, the lower has five green and five red glasses of different shades, one yellow, one blue and one white glass. The upper disc has London smoke glass of three different shades and some clear openings. By rotating the discs by hand any of the colors can be shown, either alone or in combination with the smoke glass. Between the discs is a movable diaphragm which regulates the size of the colored area shown. Inside the lantern are two pieces of ground glass which can be placed before the light when desired, giving the effect of fog. The lamp is arranged with two sep-

arate burners, and by having two openings in the front of the lantern it is possible to show two colors at the same time in contrast, or by moving the diaphragm to have only one color shown. In this way red and green, two reds or two greens of different shades, red and white, etc., can be shown at the same time, or any one color can be shown by itself. In connection with each glass there is an illuminated number or letter which can be seen by the person making the examination, but which is screened from the person tested. By this means a record can be kept of the color shown, and the name given to it. This lantern makes a very useful addition to the Worsted Test and will detect some cases of defective color perception, especially where it is confined to a small central area in the retina, which are not discovered by the Worsted Test.
of the individual the further can he recognize the given color and vice versa. The intensity of the color sense for the color in question is the distance at which it begins to be recognized. Dr. Oliver has
devised a convenient apparatus for measuring the color sense at a given distance. He found that red requires 2.65 mm . of square surface to be recognized at a distance of 5 m .; yellow a slightly larger area, and violet 22.75 mm . of square surface. Instead of colored papers we may use colored lights, and thus more closely approximate the conditions present in railroad and marine service. These tests are called lantern tests. The lantern shown in the cut above is a sample and probably the best lantern on the market for testing the color sense of employees in railroad and marine service.

One other test will be mentioned, that of Meyer. If a border of gray paper is placed upon red paper, it appears to have the complementary color, that is green. This is especially the case when the whole is covered with a piece of tissue paper. A color-blind person who does not recognize the color of the background fails to tell the color of the border as well. Daltonism (dichromasia) is incurable. Acquired color-blindness is caused by affections of the light-perceiving apparatus.

There seems to be a special color center in the occipital cortex, and affections of this as well as affections of the optic tract, nerve or retina destroy the appreciation of color, leaving the form or space sense intact. There is evidence that this color center is capable of perceiving six separate and independent colors. There is a tendency of late to accept this central color-sense theory to the exclusion of the retinal theories.

Diseases of the optic nerve are the most frequent causes of acquired color-blindness. Such is never absent when the vision is much reduced. The loss of the color sense sets in gradually and not for all the colors at once. First of all the perception of green and red is extinguished, then that for yellow, and lastly that for blue. That color for which the least extent of the retina is sensitive disappears first and that for which most of the retina is sensitive persists the longest. The perception of color remains intact when the vision is reduced from other causes than implication of the light-perceiving apparatus.

No definite results can be obtained in infants under six months of age as to the condition of color perception. After this age there is usually a reaction obtained for red, orange and yellow, that is the infant will reach after and seem pleased at the sight of ribbons of these colors. Between the ages of ten and twelve months there is an equal reaction for all colors. Tests for color preference show preference for red between the seventh and the twenty-fourth month. Blue preference begins in 33 per cent. between the ages of two and three years, and increases to 93 per cent. in the thirteenth year. A typical choice without preference begins with 44 per cent. between the ages of three and four years and decreases to 7 per cent. at the thirteenth year.

## CHAPTER XI

## THE LIGHT SENSE

Two persons that have the same visual acuity in good illumination are often not able to read equally well when the illumination is reduced. These two people have the same space sense, but their retinæ are differently affected by brightness or difference of illumination. The one who can read the better when the light is reduced has the better light sense.

The light sense is tested in several ways. We can determine the smallest amount of illumination by which an object can be seen (minimum stimulus) or the smallest difference of brightness that can be appreciated (minimum of difference). In practice when we only wish to know whether the light sense is reduced or not, as patients with certain eye diseases see better in feeble illumination, while others read better in a bright light, we can gradually lessen the amount of light that falls upon the test card, and note whether the patient sees poorer in proportion as the light is reduced than we do, that is whether his vision falls off faster than ours. The light-sense is not always diminished when the vision is disturbed, but the vision may be very poor and the light sense very good and vice versa. When a patient sees better by day or by a bright light, he has what is termed hemeralopia, and when he sees better when the illumination is feeble, we call it nyctalopia. These two terms are often confused, the latter being at times used to indicate night-blindness. The first term, hemeralopia, is derived from the two Greek words: $\dot{\eta} \mu \epsilon \dot{\epsilon} \rho a$, day, and $\ddot{\omega} \psi \iota \varsigma$, sight, that is day sight. The etymology of the latter term, nyctalopia, is: $\nu v \xi$, night, $\begin{gathered}\text { ü } \\ \text { cs, sight, that is night sight. }\end{gathered}$

The scientific method of testing the light sense is with one of the various photometers, which gives the minimum stimulus. The instrument usually employed is that of Forster. It consists of a box,
light-tight and blackened on the inside. In one corner of the box there is a normal candle - one burning 120 grs. in one hour - made of spermaceti. The candle illumines the interior of the box through the window $W$, the size of which can


Förster's Рнотомeter. be altered by means of the screw $S$. The window is covered with oiled paper so that the light emitted will be perfectly uniform. Upon the opposite side of the interior of the box is hung a card made of alternating black and white stripes, to be used as the test object.

The patient after remaining in the dark for a while, in order that his eyes may become adapted to the darkness, looks into the apparatus with the window $W$ closed, and the test card unillumined in consequence. Then the light is allowed to enter the box slowly through the window, until the stripes upon the card can be seen. The size of the opening needed for this purpose is a measure of the light sense of the individual under examination.

One day Fechner noticed a scarcely perceptible difference between the brightness of two clouds, and that this difference persisted on looking through a smoked glass. This observation caused him to formulate the following law: The smallest difference of perceptible illumination is a constant fraction (about one per cent.) of the total illumination.

This is a general law of perception as it applies to the other senses. If a string must have a length of 105 mm . before we can tell by observation that it is longer than one that has a length of 100 mm ., we find that another must be at least 210 mm . in length before we can tell that it is longer than one of 200 mm . This proportion holds no matter how long the string is. It is also with sound and the estimate of the difference between two weights. Bouguer observed the fact upon which Fechner based his law some time previously. He also described the following experiment to determine the ratio between the smallest difference of perceptible illumination and the total illumination.
$A$ and $B$ are two lights placed at different distances in front of screen $S S^{\prime} . O$ is a stick so placed that it casts two shadows upon $S S^{\prime}: a$ from $A$, and $b$ from $B$. The shadow $b$ is illuminated and thus rendered fainter by $A$, as is $a$ by $B$. By moving the light $B$ further away the shadow $b$ becomes feebler and at last disappears when the light $B$ is ten times further from the screen than $A$. The same thing occurs with lights of different intensities, that the moment when
 the shadow of one fades away, that light is ten times further off than the other one. The amount of illumination is proportional to the intensity of the luminous source, and inversely proportional to the square of the distance. Suppose that $B$ is 200 cm . from the screen, and $A$ at 20 cm . and $l$ their intensity. $A$ gives to the screen a lumination of $1 / 20^{2}$, and $B$ of $\mathrm{I} / 200^{2}$, and the shadow $b$ receives $\mathrm{I} / 2 \mathrm{O}^{2}$ illumination. The difference between the illumination of the shadow and the screen is

$$
\left(1 / 20^{2}+1 / 200^{2}\right)-1 / 20^{2}=1 / 200^{2}
$$

(The intensity of light varies inversely as the square of the distance.)
The ratio between this difference and the illumination of the screen is

$$
\frac{\frac{l}{200^{2}}}{\frac{l}{20^{2}}+\frac{l}{200^{2}}}=\frac{l}{101} \text { or } 1 / 100 \text {; }
$$

since the measurements were purposely taken in round numbers to simplify the problem, but are not very exact.

If the intensity of the lights is doubled, then :

$$
\frac{\frac{2 l}{200^{2}}}{\frac{2 l}{2 \mathrm{O}^{2}}+\frac{2 l}{200^{2}}}=\frac{\mathrm{I}}{\mathrm{IOI}}
$$

Many phenomena daily observed are explained by the law of Fech ${ }^{-}$ ner. We read by artificial light as well as we do by daylight, although the illumination in the daytime is enormously greater than by gaslight, because the light reflected by the white paper and the black letters remains the same. This law is only true for medium degrees of illumination. If the illumination is very feeble the relative difference must be much more. If the gaslight is much lowered, we can not see any longer to read, although the illumination of the print and the paper bears the same ratio. It is possible that this difference is due to what is called the retina's own light, designating the feeble glow which may be perceived in a dark room, which is due to internal causes, perhaps the friction of the blood in the vessels against the sensitive layer of the retina, and perhaps also certain processes in the brain. If this light is added to that of the printed sheet, the difference of brightness between the letters and the paper-may fall below the limit of Fechner. The law also ceases to act when the light is very strong - the reason why we cannot see the sun's spots with the naked eye on account of dazzling, but very well with a smoked glass.

The illumination of a fair day is the most favorable amount of light to verify Fechner's Law. The acuity of the light-sense is expressed by the inverse of the fraction of Fechner. If it be one one-hundredth then the luminous sense is 100 , and if by greatly diminishing the illumination the fraction rises to one fiftieth, we say that the acuity is 50 and so on.

The degree of illumination that forms the lowest limit of visibility is called the threshold. According to Aubert, the weakest light that the eye can distinguish, or the threshold of the normal eye, is a sheet
of white paper illumined by a candle placed at a distance of 200-250 m . For very feeble illumination the macula lutea is less sensitive than the parts of the retina that immediately surround it. By fixing a point a little to one side of the macula, we better distinguish the brightness, which differs only slightly from that of the background, as when we try to see very dim stars, for example. Parinaud says that this is because the macula is not able to adapt itself like the rest of the retina, as the cones in that locality contain no visual purple, which is considered the organ of adaptation of the retina. The inferiority of the macula may be due, as has been suggested, to the yellow pigmentation of the macula absorbing a part of the blue rays that play such an important part in feeble illumination. The period of adaptation and the time it takes for the visual purple to reproduce itself are about alike, namely, twenty minutes.

After-Images. - After-images are the visual impressions that persist after the eyes are closed or turned away from the object. If the after-image has a color complementary to that of the object, it is spoken of as a negative after-image, while if it is of the same color as the object itself, it is called a positive after-image. An example of the first kind is obtained by gazing for a few moments at a colored surface upon a white background. Soon the object begins to change its color and to become surrounded by a border of its complementary color.

The part of the retina where the image of the colored object is formed is fatigued for the color in question. If the look is transferred to a sheet of white paper, we perceive an after-image tinged with the complementary color of the object. This fact enhances the colorperception theory of Hering. To obtain a positive after-image, close the eyes and cover them with the hands to exclude all light. Remain in this position until all previous impressions have faded from the retina. Then remove the hands and open the eyes for a second without changing the direction of the gaze, shut them immediately and cover them again. If the experiment is successful we will then see a very clear after-image of the external object. The less illu-
mined parts of the image first disappear, while the more illumined parts change color, becoming bluish, violet, orange and so forth. An afterimage may last as long as a fifth of a second, as when one moves a piece of burning wood around, giving the appearance of the trail of fire familiar to all.

The Troxler Phenomenon. - If we draw several dots on a piece of white paper and fix one of them for some time we will see first one and then another of the surrounding dots fade from view, to reappear after a little especially at the moment of winking or of making a slight movement of the eye. This phenomenon was described at the beginning of the last century by Troxler. It has been studied of late by Dr. Holth. The color of the ground upon which the dots are drawn plays no part and when the dots fade from the view the background is seen in place of the dots. The gap in the field is then filled in after the manner of the blind spot of Mariotte. If we fix a square on a chess-board, we will often notice that now one and now another of the surrounding squares will disappear, to reappear after a little. Luminous objects can be made to disappear in a like manner according to Dr. Holth. The same phenomenon occurs in slowly moving bodies, so some care must be taken on this account in taking a field of vision, if we wish to perform it with precision.

The explanation of this curious phenomenon has not yet been advanced.

## CHAPTER XII

## MALIGNERING OR SIMULATION OF BLINDNESS

At times patients make an attempt to fool the physician into believing that. they are blind or that their vision is very defective. They are often led to do this in order that the doctor may testify to their unfitness for military service, or that they may get damages from some railroad company upon whose cars they met with some accident, and having nothing to show for the damage done, simulate blindness. It happens that sailors who want to desert their ships will, when they touch land, often complain of poor sight, and antering some hospital feign blindness until their ship has cleared port. We are ofttimes led to believe that the patient is a faker from the surroundings of the case.

If, on examination, the fundi oculorum are normal in appearance and especially if the pupil of the supposed blind eye is of the same size as its fellow, and reacts, that is, contracts, when light is thrown into the eye, the chances are that the eye sees. All blind eyes have dilated pupils, unless the cause of blindness is to be found posterior to the primary optical centers, namely; the optic thalami, the corpora geniculata, and the corpora quadrigemina. Any lesion anterior to these parts in the path of the optic fibers will abolish the contraction of the pupil to the influence of light. To prove that the sphincter of the pupil is not paralyzed, being the cause of the mydriasis; light is thrown into the sound eye, and if the pupil of the supposed blind eye is not adherent to the lens capsule behind it will contract. It is most likely that the eye has some vision if the pupil reacts to exposure to light. It is rarely that one simulates bilateral blindness. It is often very difficult to detect malignering. Many tests have been devised for its detection, and a knowledge of them all is not superfluous, as when one fails the other may be tried.

Tests for Simulated Blindness. - The patient is asked to look at his own hand held in front of him with the good eye closed. A blind man can do this through the muscular sense, that is, he can properly direct his sightless visual organ, but a faker knowing that his good eye is closed will most likely allow the fellow eye to wander here and there in search for the hand (Schmidt-Rimpler). A lighted candle is held before the good eye and then slowly moved towards the supposed blind eye. The fraud is detected if the candle is seen after it is concealed from the good eye by the bridge of the nose (Cuignet). We give the patient some reading matter and ask him to read. While he is reading we hold a pencil vertically between the book and his eyes, in the median line. If he continues to read uninterruptedly it is proof that both eyes are being used, while if the pencil obscures the print from him he is presumably using only one eye. This test is not very reliable, for the patient may claim that he cannot see the moment the pencil is placed in front of the book (Cuignet).

A strong convex sphere is placed before the sound eye. The lens makes the eye artificially myopic. If a 5 D.S. is used the eye is rendered that much myopic if it was emmetropic before. Such an eye cannot read further off than its far-point, which in this case lies at ( $100 \div 5=20 \mathrm{~cm}$.) 20 cm . distance. Some print is' then taken and held at or closer than twenty centimeters, to show the patient that his good eye is not screened. As he reads the print is withdrawn further and further from the eyes. If he continues to read after the book has receded beyond the far-point of the sound eye, he is reading with his supposed blind eye.

We make a show of occupying ourselves with the good eye only, telling the patient that inasmuch as he has only one good eye left, it is wise to examine it to see whether it needs glasses or not. that it is not well to put all the strain upon the unaided eye. Some distant test type is given him to read. While he reads a strong convex spherical lens is slipped in front of the good eye. If the patient does not immediately discover that he cannot see so well he is not blind in the eye before which there is no glass. One must
watch that both eyes are always kept open. Frequently by this method one can ascertain the exact amount of vision in the supposed blind eye.

The patient is told to look at a candle flame at twenty feet distance. Before the acknowledged seeing eye a Maddox doubling prism is placed, with the intersecting line bisecting the pupil. Two candles are seen with the eye before which is placed the prism. We say to the patient: You see two candles? He says that he does. The Maddox prism is then slipped up or down so that the eye only sees through the upper or lower half of it. If on inquiry the patient still sees two candles, both eyes are seeing, one candle seen by each eye. Instead of a Maddox doubling prism, a simple prism may be used. , It should be placed before the sound eye with its base bisecting the pupil, so that monocular diplopia is produced. The test then proceeds as in the former case. If a reading test is used for this examination, we can compel the person under examination to read sometimes the upper and sometimes the lower of the two paragraphs or lines and thus ascertain the visual acuity of each eye separately without the patient being aware of it (Alfred Graefe).

Snellen has constructed letters that are alternately red and green. To read them a pair of spectacles with a red glass in one side and a green glass in the other is placed upon the patient. Through the red glass the red letters alone can be read and through the green glass the green letters. This test presupposes that the letters are made upon a black background, for red letters upon a white ground are invisible through a red glass, and the green ones upon a white ground, invisible through a green glass. If the patient has the red glass before the blind eye and reads some of the red letters his fraud is detected. A recent and a very complete test for simulated blindness is that shown in the cut on next page.

## DR. PERCY FRIDENBERG'S TEST FOR SIMULATING BLINDNESS.

This instrument was devised by Dr. Percy Fridenberg, to reflect the image of a test card in such a way that it can be seen by only
one eye at a time, and a quantitative demonstration of vision made without the subject of examination obtaining any clue as to which eye is being tested.

The mirror is mounted on a horizontal arm in such a way as to permit of varying its distance from the test card, and of presenting it alternately to either eye by revolving the bearing through an arc of $180^{\circ}$. The lateral tilt of the mirror can be changed at will, and is indicated by a pointer on a horizontal scale. When the pointer is

at $90^{\circ}$, the plane of the mirror is at right angles to the line of vision of the eye on the corresponding side, and this eye sees its own image. The test card on this side, however, is not normal to the mirror, and its reflection is seen only by the opposite eye, which the subject presumes to be unconcerned in the visual act, as it does not appear in the mirror.

A slight tilting of the mirror to the temporal side, bringing the pointer to $95^{\circ}$ or $100^{\circ}$, is sufficient to reverse the optical conditions so that the test card is seen only by the eye on the same side. By switching the mirror over to the opposite side of the arm, a similar double test can be applied, so that in all eight variations are rapidly obtained, as follows :


The mirror can be adjusted laterally to correspond exactly with the inter-pupillary distance, and correcting glasses inserted in the trial frame, if necessary.

The test is simple, rapid and exact, gives no clue to the simulant, and can be demonstrated without theoretical explanations to the members of a commission or jury.

The stereoscope may also be used for detecting simulated blindness. Half pictures are used. For example on one side of the test card is the picture of a horse, while on the other is that of a jockey. The diaphragm of the stereoscope prohibits the right eye from seeing the picture before the left eye and vice versa. If one eye alone sees, either the horse or the rider will be seen, while if both eyes see, the patient will at once say when questioned that he sees a horse with a man upon his back. Care must be taken that the patient does not close one eye.

Hering's test of stereoscopic vision is not practical, as many have quite a good estimation of depth or perspective with one eye alone, and others with two eyes can with difficuity interpret the test properly. Hering's test is that at the end of a long tube or box little pith balls are dropped first anterior and then posterior to a screen made of threads or fine wire. The patient with his eye at the other end of the tube is asked to tell each time a ball is dropped whether it passes in front of or behind the screen. When the malignerer is pretending that both eyes are blind it is very difficult to trip him up.

If the man complains of pain in the eyes at the same time, one may instill cocain a few times into the eyes, and if the man is a faker he will not acknowledge that the anæsthetic has relieved his pain in the least. The following illustrates such a case well. An electrician
while repairing bumpers in a power house, claimed that one blew out, causing an intense flash of light which he received directly into the eyes. He was much frightened, and at the same time received a little shock of electricity. For a day he was unable to see, and light was very painful to the eyes. When I saw him all vestige of trouble had disappeared, but he had made up his mind that he would sue the railroad company, so he came groping into my office. His eyes in all particulars presented a healthy appearance. He claimed that the light annoyed him so that he could not keep the eyes open. I instilled some cocain solution (four per cent.) and after several instillations, the pain was not relieved in the least. I gave him some cocain solution to take home, telling him to use it every two hours. He returned and said that the medicine had not done him a bit of good, although his pupils were now widely dilated from the drug. The pupils were active at his first visit. After some persuasion his vision began to rise rapidly. At times the fraud may be detected by making a quick motion as if to hit the patient in the eye, when he will flinch, and bat the eyes. A few days in bed on low diet, pilocarpine sweats, or electric cautery, or a very strong faradic current applied frequently to the spine will bring the patient to terms.

## CHAPTER XIII

## VISUAL IMPRESSIONS

Since the retinal image of an external object bears a definite relation to the object in regard to its size, position and form, it would be expected that the sensation produced would correspond to the sensory impulse, originating in the formation of the retinal image. We should expect that our mental condition resulting from looking at an object, would correspond exactly to the retinal image of the object. This is not the case. There arise certain discrepancies between our perception and the retinal image, some having their origin in the retina, some in the brain, and others being of such a nature that it is difficult to say where the discrepancy is introduced. Such discrepancies are called optical illusions, among which may be mentioned the phenomena of irradiation and contrast.

Irradiation.-A white spot on a dark background looks larger than a black spot on a white background. This is to be especially noticed when the object is somewhat out of focus, and is to be partly explained by the formation of diffusion circles, which encroach in each case from the white upon the dark, but besides this any sensation arising from a stimulated area of the retina gives the impression of a larger object in the field of vision when the rest of the retina and visual apparatus are at rest than when they are simultaneously excited. The retinal or central visual structures seem to be thrown into action sympathetically at the same time

Contrast.-If a white strip of paper or a white paper figure be placed upon a black background, the edges of the figure will appear whiter than the central portions. The middle of the figure may look so dark that it will appear to be shaded. This occurs even when the object is well in focus. The apparent greater whiteness of the borders of the figure is due to the contrast between the border and the black background.

In the figure it will be noticed that at the intersections of the white lines there appears a shadow with illy defined borders. When the attention is directed to one of these spots, it disappears, while the other persists.

If a small piece of gray paper be placed upon a sheet of green paper and both covered with a piece of tissue paper, the gray will appear to be of a pink color, the complementary color to green. If white paper is used instead of gray this effect of contrast is absent. The contrast will disappear if a broad
 black line be drawn around the small piece of gray paper, so as to isolate it from the ground color. If a book be placcd vertically upon a sheet of white paper and illumined on one side by the sun and on the other side by a candle, two shadows will be produced. The one cast by the sun is illumined by the yellow light of the candle, and the one cast by the candle will be illumined by the white light from the sun. The first appears yellow, the latter however appears, not white, but blue, a color complementary to that of the candle light that surrounds it. If the candle is moved the blue light disappears. If some part of the area illumined by the candle is looked at through a tube blackened on the inside, the blue color disappears because there can be no contrast, but if the edge of the area is looked at through the tube the blue color reappears, as then there is a contrast formed between it and the white paper around.

Our judgments of a color depend a great deal upon the color of the surrounding medium, or upon simultaneous contrast, as in the two experiments mentioned above. The phenomenon of contrast is due to a false conception of white. A piece of paper that is white by daylight is still considered white by us when illumined by the yellow light of a candle or by the red light of a coal fire. Javal pointed
out that we consider all objects as white that reflect or return the most light, no matter what the color of the light may be. If the real color of the paper differs much from its color by daylight, which fact we unconsciously recollect; it seems white to us with a faint colored tone. Through a red glass, when the eye receives only red rays of light, a piece of white paper appears reddish white. We may regard the zero of our color sensations diplaced and with it the entire scale. If the yellowish-white shadow that illuminated the screen in the experiment with the book, mentioned above, appears white to us it is not strange that we should consider the white light that illuminates the other shadow as blue, that is less yellow than the screen.

The colors of after-images are due to what is known as successive contrast, and are explained by the retina becoming fatigued for the colors in question, and by regeneration giving rise to the complementary color. (See article upon After-images.)

The Physiological Blind Spot and the Filling Up of It in the Visual Field.-The optic nerve entrance is blind; it is even devoid of light perception. As has been seen it lies to the nasal side of the center of the retina about fifteen degrees, and a little above it. 'The presence of this blind spot (Mariotte's) can be easily demonstrated by the following plan. Upon a card make two dots several inches apart. Take the card in the right hand, and keeping the left eye closed look steadily at the left-hand dot as you cause the card to approach the eye. In a certain position according to the distance of the dots apart, the right-hand dot on the card will disappear from view. It will appear again if the card or the eye is moved a little. The image of the dot upon which the eye is fixed falls upon the macula, and that of the other dot falls upon the optic nerve entrance.

Notwithstanding this, we are not conscious of any gap in our visual field. Some of the printed matter, when we are reading, must throw its image upon the blind optic papilla, but no gap is noticed. We would not expect to see a black area or patch, for black is the sensation produced by the absence of light from structures that are sensitive to light. There must be a visual organ to see black. Visual
organs (rods and cones) are absent at the optic nerve entrance, and it is in no way affected by the light that falls upon it. The reason no gap is perceived in the field is that we refer the sensation produced by two points lying upon opposite margins of the disc, as coming from two points lying very close together, as we have no indication of the amount of space that separates them. The existence of the blind spot is of little importance, inasmuch as it is out of the way of direct vision, and the image of an object could never fall upon the blind spot of the two eyes at one time, so the one eye is able to fill the deficiency of the other in this regard. Again, images that are formed on the periphery of the retina as far removed from the center as the optic disc are seen so indistinctly that if a gap in the field was present at that locality it would most likely not be noticed. We may have a visual sensation in the entire absence of light. Any stimulation or irritation of the retina or optic nerve may give rise to the sensation of light. Pressure upon the eyeball gives rise to the appearance of colored rings of light, known as phosphenes. A blow upon the eye causes a flash of light. The optic nerve answers to a stimulus of any kind by causing the sensation of light and not pain. Complex and coherent visual images may arise in the brain without any corresponding objective cause. These phantoms or ocular spectra have a realness quite as striking as those of ordinary visual perceptions. They are seen with the eyes open and closed. The phantoms seen in a case of delirium tremens are a good example of ocular spectra.

Appreciation of the Apparent Size and Distance of an Object.With the eye alone we can only estimate the apparent size and distance of an object. We can tell what part of the field it occupies, and by comparing the visual angles of two objects estimate their relative sizes. The real size of an object is determined by other means. Our perception of the apparent size of an object may be so modified that it cannot be relied upon. The moon for instance looks to be of different sizes to different individuals. Any ocular deception (as has been noted) is called an optical illusion. Thus, let a
line $A B$ be divided into two equal parts, $A C$ and $C B$. Subdivide portion $A C$ into equal parts by distinct marks. It will now be noticed that the portion subdivided appears the longer, that is, $A C$ appears longer than $B C$.

If two squares $A$ and $B$ of equal dimensions are marked with stripes running cross-wise in the one and vertically in the other, the former will appear higher and the latter broader than it really is.

In order to observe this phenomenon cover square $A$ and it will be noticed that square $B$ appears to be broader than long, while $A$ appears taller than broad; when $B$ is cov-
 ered. The moon looks larger on the horizon, as then it can be easily compared with the size of terrestrial bodies. A short man and a tall one side by side causes the former to appear shorter and the latter taller than he really is by contrast. On the other hand, absence of comparison may lead us to suppose that an object is larger than it is. A man in a fog looks larger than he is. Seeing him indistinctly we imagine that he is further off than he is; subconsciously connecting the size of the visual angle he subtends with the indistinctness with which we see him our conclusion is drawn that he must be a very large man.

On the other hand, distant objects in a very clear atmosphere are judged to be nearer than they really are, as they are seen so distinctly. Distances upon the water are very deceptive, appearing shorter than they are, as there are no intervening objects. Our previous experience is the most potent factor in our estimations of size and distance ; our visual perceptions with our visual judgments.

Visual Fudgments. - Binocular vision or the seeing with two eyes is of use to us inasmuch as the one eye can fill up the insufficiencies of the other. Over and above the filling up of the blind spot one eye supplies that part of the visual field that is lacking to the other. The great use of binocular vision is however to inform us of the size, shape and distance of objects from the eyes and their distance from each other.

Fudgment of Distance and Size. - By the association of visual sensations with those of touch and handling of objects, and with the sensation derived from the movements of the eyeballs necessary to make any such part of the field as corresponds to a particular object distinct, we are led to form judgments which gradually become fixed in our sensoria. Even with one eye we can to a certain extent form some judgment, not only as to the position of an object in a plane at right angles to our visual axis, but also as to its distance along the visual axis. If the object is near we must accommodate for it, to render it distinct ; if far away, we must relax our accommodation to make it clear to us. The muscular sense of this effort through the ciliary muscle enables us to judge somewhat of the distance of the object. We judge of the distance separating two objects by the amount of innervation necessary for the ocular muscles to turn the eyes from the one object to the other. Looking directly at the object $O$, another object $I$ at its side casts its image upon the peripheral parts of our retinas. The distance separating the images of the external objects regulates the amount of innervation sent to the eyes, in order that they may be promptly and exactly directed from the one object to the other. That the amount of innervation needed to accomplish this is the real estimate of the separation of the two objects there is no doubt.

If a patient with a paralysis of the right external rectus muscle closes his left eye and is told to look at an object to the right, he supplies the necessary will power, but the innervation to the right external rectus muscle is at fault so the eyeball does not move. The patient thinks that it has, and there results a false projection and orientation. If he is told to move his finger rapidly to the right and touch the object, he most often moves the finger too far, as he judges of the distance of the object to the right from the amount of innervation sent to the paralytic muscle plus that caused by false orientation.

In the figure the right external rectus muscle is considered paralytic. $O$ is an object to the right of the eyes. In the left eye the
image of $O$ falls upon the macula as the eye is able to turn towards it but the right eye does not move, so the image of $O$ falls to the left of the macula. $M$ is the macula in each; $N$, the nodal point.


The individual supposes that the right eyeball has turned towards the object, and inasmuch as the nasal retina is influenced, that the object must lie further to the right than it does. $O^{\prime}$ is the apparent position of the object $O$ to the eye $R$, for if when the eye is turned towards the object $O$, an object that forms its image at $M^{\prime}$ must be at $O^{\prime}$.

The close association of touch and sight in forming judgments of distances and size is illustrated in those born blind but who have afterwards received their sight through an operation. Such individuals are at first unable to properly estimate distances. If a foot-rule is shown it will be imagined to be much longer or shorter than it really is, to the surprise of the individual when he is allowed to handle it. Likewise they cannot tell the shape of an object, for example whether it is spherical or cuboidal. These facts are fixed in the mind of the growing child through the association of touch and sight.

Knowing the narrow range of accommodation and the slight mus-
cular effort that it entails, one can easily see that all monocular judgments are subjected to much error. The person who tries to thread a needle without using both eyes knows how difficult it is to estimate distances properly with only one eye. When the object is near and both eyes are being used, we converge the visual axes of the two eyes towards the object ; and when distant, we bring them to parallelism. This contraction of the extraocular muscles gives us a sense that aids the muscular sense of accommodation, to estimate properly the distance of an object. If by any means the amount of accommodation needed to bring an object into focus is lessened, the object appears to have receded, and if more accommodation is used than one is accustomed to use for a given distance, the object looked at appears nearer than it is. The same is the case with the convergent effort, that is the more convergence is needed to bring the object clearly into view before the two eyes, the nearer the object is judged to be. These facts are well illustrated in the stereoscope, where a picture at very short range is looked at through convex glasses and prisms so that no accommodation or convergence is required. The pictures therefore appear to be at a great distance from the eyes. The judgment of the size of an object is closely connected with that of distance. Our perceptions gained exclusively from the field of vision go no further than the apparent size of an object. The real size of an object is only rightly conjectured from the apparent size of an object when its distance from the eyes is taken into account.

Thus: When there appears in our field of vision the form of a man ; knowing the ordinary size of a man, we infer from his apparent size the distance of the man from us. If, on the other hand, we have an estimate of the distance at which the man is through the presence of intervening objects or from experience, we judge of his real size from the apparent size he has at that distance. An image upon a screen when gradually enlarged appears to approach, inasmuch as all approaching objects subtend progressively larger and larger visual angles as they draw nearer. We have subconsciously connected the apparent increase in size of an object with its approach
since early childhood, and therefore any object that is increasing gradually in size appears to be moving forward and vice versa.

Fudgment of Perspective or Depth in an Object, or Stereoscopic Vision. - When we look at a square all parts of it are at the same distance from us ; all parts are equally well focused whether we look at it with one or two eyes. When, on the other hand, we behold a cube, we realize that all its parts are not at one distance from us, as we are compelled to accommodate for successively different portions, to bring them into view. Perhaps an adjustment of the eyes to this side or that is necessary. From this effort on the part of the eyes to see different parts of the body clearly, we form the idea of solidity in the cube, or that of perspective or depth, that is, that all parts of the object do not lie in the same plane. Our idea of solidity is much more correct when both eyes are used, just as in the case of the estimation of the distance of an object, the muscular sense of the eyeball aiding the accommodative sense. We are much assisted by the arrangement of the light and shade in an object when judging of its shape, as a depression may appear to be an elevation by the proper arrangement of light and shade as well as by the linear perspective of the object. For single binocular vision it has been noted that an object in space must form its image upon the corresponding parts of the two retinas. This is not true in stereoscopic vision, however. The above is only true of objects or portions of objects occupying one plane, and not those having solidity or perspective. Each eye projects out into space the area of retinal stimulation and places the object on a line that passes through its nodal point. This line connects the object and its retinal image. If the projected images join end to end and side to side, a solid or stereoscopic picture is built up. The image of any solid body in the right eye cannot be exactly like that that falls upon the retina of the left eye, though both are combined into a single visual perception. The right eye sees a little more on the right of the object than the left eye and vice versa. - The truncated pyramid $P$, when looked at in the median line in front of the eyes, forms a retinal image the shape of $R$ in the right
eye, and in the left eye the retinal image is the shape of $L$. Both together after projection make the appearance of $P$. It may be supposed that the judgment of solidity which arises when two dissimilar images are thus combined in one perception was due to the fact that all parts of the two images cannot fall upon corresponding parts of


L


P


R
the two retinas at the same time, and that therefore the combination of the two needs some movement of the eyes. Thus, if figure $L$ is placed upon $R$, the two bases will coincide when the apices do not and vice versa; hence when the bases fall upon any particular part, the apices will not be combined in a single image, and there must be a slight but rapid movement of the eyes, that they may be combined. That no such movement is necessary is proven by the fact that when a solid body is illumined by an electric spark too quick to allow of the movement of the eyes, the solidity is easily recognized. The fusion of images falling upon non-corresponding parts of the two retinas is the operation of the cerebrum, resulting in an ocular judgment. If the images of two surfaces, one black and the other white be made to fall upon corresponding parts of the two retinas, the perception is not always a fusion of the two colors, that is a gray, but a sensation is produced similar to that when a polished surface is looked at, that is the surface appears brilliant. The reason of this is because when we look at a polished surface, more light enters the one eye than the other according to the inclination of the mirror ; hence we associate an unequal stimulation of the two retinas with a polished surface. When two colors of different hue are made to stimulate the same area of retina in the two eyes, the resulting color is not the fusion of the two, as when both
fall upon the same area of a single retina, but first the one color and then the other is seen, immediate tints being passed through. The change of the color is frequent. This phenomenon is spoken of as the struggle of the two fields.

The change from one color to the other may arise from the difficulty of accommodating for the two at the same time. If the two eyes, one of which is regarding a red object and the other a blue one, be accommodated for red rays, the red will overlap the blue, and the sensation produced will be red and vice versa. For, as the rays of the spectrum are of different refrangibility, the amount of accommodation needed for a certain color is different from that needed for another color. The blue end of the spectrum needs less accommodation to be seen than the red end, as the blue rays are refracted more in passing through the dioptric media of the eyeball.

The Impressions of the Two Maculas are Projected Towards the Same Place in Space. - When both eyes are properly directed, each towards the object, this is not surprising, but the fact is the same when the two eyes do not fix the same object, as is seen in stereoscopic exercises. The following experiment of Tscherning illustrates this fact. It is necessary to be able to squint in order to perform the experiment. One can cause himself to squint externally, by pinching up a piece of skin near the outer canthus of the eye, and then trying to look in the opposite direction. The conjunctiva is drawn tightly across the eyeball by pinching up the skin and the eyeball prohibited from moving inwards towards the nose properly. One eye is closed and with the other one a lighted candle is looked at for a few minutes, long enough to produce an after-image. We then fix a certain point with the eye that was closed while an attempt is made to squint. The after-image in the squinting eye will always place itself upon the point of fixation, no matter how much the visual lines may converge or diverge. If we behold a near object with both eyes, a more distant one appears doubled ; as two, and vice versa. If we hold one finger behind another at a distance of two feet and then accommodate first for the one finger and then for the other, this
fact is made evident, that the finger out of focus appears doubled. This phenomenon is called physiological binocular diplopia, and was first described by Alhazon. The suppression of the image in one eye plays an important part in binocular vision. It is this fact that makes physiological binocular diplopia unobserved unless the attention is called to it. According to Dr. Javal it is the image that occupies the smallest amount of retinal surface that disappears. In the majority of cases it is the image of the less developed eye, the one that is used less frequently, that is suppressed.

Physiological diplopia manifests itself because the different position of the two eyes is not mentally taken into account. We cannot tell without some examination which image belongs to the right and which to the left eye. Every visual impression from one or both eyes is referred to a single visual center, which corresponds to the right or to the left eye in most cases, but in ideal binocular vision should correspond to a point midway between the two eyes. We should see objects in space in their true position by having regard of this center of reference.

## CHAPTER XIV

## ENTOPTIC PHENOMENA

Objects on the interior of the eyeball may, when light enters the pupil, cast shadows upon the sensitive layer of the retina, and thus be perceived. Listing called the examination of objects upon the interior of our own eyes entoptic observation. If a clear sky is looked at through a pinhole in a card held near the anterior focal point of the eyeball, that is about 15 mm . anterior to the cornea, or if a light at the distance of about 5 m . be viewed through a strong convex lens (spherical), held at two or three inches in front of the eye, a bright disc of light will be seen limited by the shadow of the border of the pupil, upon which certain things are visible.

The traces of the lids upon the cornea may be seen by half closing the eyes. These horizontal lines caused by the wrinkling of the epithelium of the cornea remain an instant after the pressure has ceased, and if the pressure is prolonged it leads to an irritable condition called tarsal asthenopia. According to George Bull, tarsal asthenopia is often made very pronounced by reading while lying upon the back. By rubbing the eye the luminous area appears speckled, due to irregularities of the cornea, which soon disappear. The tears and drops of mucus and particles of dust can be seen to move across the cornea from below upwards. Certain long striæ are seen running from above downwards, on winking the lids. These are caused by tears near the lid borders assuming the form of a prism with a concave surface ; the eyelashes are also distinctly seen.

In using the microscope all have noticed that at times the lashes seem to get in the way of seeing. The crystalline lens or some of its portions may be seen if the opening used is very small, the light being homocentric. In the normal lens the radiating star-figure and certain round objects like hyaline globules can be observed. These
are seen best in later life. The beginning of cataract in one's own eye can be seen in this manner (Donders). By closing and opening the eye the action of the pupil to light is well observed. When the eye is directed towards the sky or any surface of uniform brightness, as a sheet of white paper upon which the sun is shining, the field is seen to be filled with little bright bodies moving with considerable rapidity, and in more or less regularly curved lines. The uniformity of their movements suggests that they are confined to certain definite channels. These bodies are called muscæ volitantes; they frequently cause alarm to the observer when seen for the first time. They are exaggerated in neurasthenia and are seen among its earlier symptoms.

These bodies are supposed by some to be floating cells and fibers in the vitreous humor. They have no pathological significance. They can be studied to a better advantage if the observer looks through a piece of cobalt blue glass. These bodies are also called Bowditch's bodies. Dr. Willets gives a number of reasons which seem conclusive to him as well as to others that these bodies are the white corpuscles of the corneal circulation. Gould suggests that all positive entoptic sensations be called phoses and all negative ones aphoses.

The latter authority is convinced that the bodies (phoses) described and named after Bowditch, and the so-called spontaneous phosphenes of the retina are due to a reflection from the corpuscles of the retinal capillaries varying in appearance according to the method of observation, illumination, etc.

The retinal vessels may be seen by several methods: $(a)$ If in a darkened room, a lighted candle is held to the side of and a little in front of the eye, while the observer looks straight ahead into the darkness; the retinal vessels will come into view as dark lines on a yellowish background, and they appear to move whenever the candle is moved. The black lines running the course of the vessels are the shadows of the retinal vessels thrown upon the percipient layer of the retina, lying posterior to the vessels. (b) If a stenopaic opening held between the eye and the sky is kept in motion, the vessels are
distinctly seen, even the smallest around the macula. (c) If a strong light is focused upon the sclera as far back as possible and moved from side to side, the same phenomena occur.

The figure explains the manner in which the retinal vessels are seen when the light is focused upon the sclera. $L$ and $L^{\prime}$ are two positions of the convex lens; $l$ and $l^{\prime}$, the shadows cast by the vessel $V$.

Müller measured the movement of the retinal vessels projected upon a surface at a known distance, and the movement of the light on the sclera
 which produced the excursion, and calculated the distance that the sensitive layer of the retina must lie behind the retinal vessels. His results coincide very closely with the actual distance between the vessels and the rods and cones. Konig and Zumft claim that different colors are perceived at different levels of the retina, and apply this principle to color-vision. They say that violet is perceived by the most anterior and red by the most posterior sensitive layer of the retina. We do not perceive the shadows cast by the retinal vessels upon the rods and cones under ordinary conditions, as the brain has learned not to regard them, to suppress their images. When light is caused, however, to enter the eyeball obliquely the shadows are cast upon portions not accustomed to the condition and therefore are seen. Or, according to Müller, the light enters the eye; illuminates the retina over a certain area, which by reflection

illuminates another portion in which the shadows are perceived.

The shadows cast by the retinal vessels can also be perceived if light be focused upon the anterior portion of the sclera with a convex lens as noted above. If the object of entoptic observation is behind the plane of the iris it will move in the direction of the visual axis ; contrary to it ; if anterior to the plane of the iris.

Light enters the eyeball along the path $x$. Striking the fundus at the point $F$, it is reflected towards $V$. The vessel $V$ in the retina consequently casts its shadow upon the sensitive layer of the retina behind it at $V^{\prime}$.

In the figure $A$, let $N$ be the nodal point of the eyeball ; $O$ and $O^{\prime}$ two opacities, one anterior and the other posterior to the plane of the iris. Light entering the eye from the candle, throws a shadow of each upon the retina at $O^{\prime \prime}$. The eye therefore projects each of them in the direction of the source of


A


B light. In figure $B$, the eyeball is represented turned up. The opacity anterior to the iris, that is $O$, now throws its shadow upon the fundus at $O^{\prime \prime \prime}$, which the brain supposes corresponds to an object along a line drawn from the point of stimulation through the nodal point forward, that is direction $O^{\prime \prime \prime} x$, so that when the eye has moved up the object appears to have descended. The corpuscle behind the iris throws its shadow upon the fundus at point $O^{\prime \prime}$, which is projected out into space along line $O^{\prime \prime} x^{\prime}$. The opacity appears to have moved in the same direction as the eye.
Entoptic observation may be used to detect slight movements of the eyeball which would be difficult if not impossible to detect by any other means. For this purpose Dr. Tscherning constructed an instrument which he called the entoptoscope. It consists of a circular piece of a hollow sphere made of brass. Across the concavity of the cap are stretched two strings, as cords of the arc ; at the point where the strings cross is made a stenopaic opening in the brass cap. The whole is mounted upon an upright and attached to a mouth piece to be held between the teeth. When this instrument is held
between the teeth and we look towards the sky we see an entoptic field containing the image of the crossed threads much magnified. A certain point in the cross is selected for a fixation point. The position of the cross is invariably dependent upon that of the head, if therefore we observe a displacement of the cross in the entoptic field it is due to a displacement of the eyeball as a whole. It can be proven with this instrument that the eyeball is elevated a little when the eye is closed or when we wink and depressed somewhat when the eye is opened widely. When the head is tilted a little to one side the eyeball is found to undergo a slight displacement in the direction of its weight.

These phenomena are made more striking when eserine is instilled, as then the field is much smaller. The displacement of the cross will then reach as much as one third of the entire field. In order to determine the distance of an object of entoptic observation from the retina, Brewster used two luminous points. Two circles of diffusion are then seen which partly overlap, and each object within the eye produces two shadows. The distance between the two shadows of the same object and the diameter of the free part of one of the circles of diffusion forms a ratio that is equal to that between the distance of the object from the retina and that of the pupil from the retina.

Let $A$ and $B$ be two luminous points in the anterior focal plane of eyeball ;
 $o$, the object ; $p$, the center of pupil ; $s$ and $s^{\prime}$ the shadows of $o$, cast by $A$ and $B$, and $c$ and $c^{\prime}$ the centers of the circles of diffusion. Since $A$ and $B$ are at the anterior focus of eye, line $p c$ is parallel to os and $p c^{\prime}$ parallel to $o s^{\prime}$. $\Delta c p c^{\prime}$ and sos are then similar.
$\therefore \frac{s s^{\prime}}{c c^{\prime}}=\frac{o s}{p c}$. Figure 2 shows: $c c^{\prime}=D E=R$ (radius of circle) $+a$.

To take these measurements we look at a piece of white paper well illumined, through two stenopaic openings, and notice the place where the shadows are projected as well as the borders of the circles (Donders). Duncan made the measurements by comparing the entoptic phenomenon with a scale viewed by the other eye. If the object of observation is more or less transparent, it will not cast a definite shadow but being more or less refracting thạn the surrounding medium, will look brighter in the center or on the edge. If the body is more refracting than the surrounding medium on account of its shape, or having a different index, it will appear lighter in the center, while if less refracting the image will be dark with a light border. This difference of refraction is well marked in case of the star-figure of the lens in people with incipient cataracts ; in some the strix appear dark and in others light. Another phenomenon due to the influence of the light that passes into the eye through the sclera is observed when we place ourselves near a window, so that one eye is illumined while the other one is in the shade. After a while it will be noticed that the white objects seen with the illuminated eye assume a greenish tint, while they appear red to the fellow eye, when first one and then the other eye is closed. The light that passes through the sclera and the chorioid is colored red by the blood in the latter tunic, and the retina becoming fatigued for red objects appears tinted with the complementary color, green ; the other eye sees red by contrast.

When we read in sunlight we at times see the letters colored vividly red. The light that passes through the membranes of the eyeball is added to that which passes through the pupil. It is not sufficiently great to change the color of the white paper illuminated by the sun, but does change the color of the black letters that contain very little white light.

If we look at a very fine luminous point we see it surrounded by a number of fine colored radiations, which are known under the name of ciliary corona. Its extent varies with the intensity of the light looked at. The cause of this, in all probability, is to be found in the fibrous structure of the crystalline lens. Besides the ciliary corona
most folks see around the entire luminous source a vivid-colored or diffraction ring, red outside and blue inside. The diameter of the ring is about three degrees. A larger ring appears to every eye when the pupil is dilated, as pointed out by Druault \& Solomonsohn. It presents the colors in the same order as the smaller ring. Quite near the source of light there are several small well-defined black rings to be seen, due to diffraction by the edge of the pupil. The smaller colored ring is probably due to the epithelium of the cornea, and is analogous to the rings seen when we look through a piece of glass covered with a thin layer of lycopodium powder. Druault found that the epithelium on the anterior surface of the cornea could be removed without disturbing the colored rings, but that they disappeared when Descemet's endothelium was interfered with. He observed these facts by looking through the cornea of a dead eye. The larger colored circle seen about the point of light is due to the fibrous structure of the crystalline lens acting as a grating. Druault also produced this phenomenon with a dead crystalline lens. Glaucomatous patients see rings of a similar nature but they are larger, ten to eleven degrees in diameter. As the diameter of the rings are inversely proportional to the size of the object producing them, the glaucomatous halo must be produced by the cells of the endothelium which are much smaller than the cells of the epithelium of the cornea (Schiotz).

Dr. Tscherning recently described a sort of entoptic phenomenon, which is observed under the following circumstances. We surround a lamp with a transparent shade made of some layers of colored tissue paper for example. We place ourselves at a few meters' distance and interpose an opaque screen, in which there is made a vertical slit. The screen should be at about 30 cm . in front of the eye. Keeping the left eye closed we fix a point on the screen near the right border of the slit. We begin by holding the head so that the eye will be in darkness, and then move the head so that the eye enters the luminous pencil that passes through the slit, while maintaining fixation at the same spot. Immediately the phenomenon will
be seen to appear under the form of two arcs, feebly luminous but bright, going from the slit towards the blind spot, curving about the point of fixation. The arcs soon disappear and the space between them becomes filled by a bluish light, and then the whole disappears to reappear on the least motion of the eyeball. The form of the arc resembles the course of the nerve fibers between the macula and papilla, and its appearance resembles that of phosphorescent bodies, by the bluish color and by the impression that it gives of being feeble and bright at the same time.

## CHAPTER XV

## MOVEMENTS OF THE EYEBALLS

The eyeball in its movements simulates very closely a ball-andsocket joint. It is capable of being moved in many directions by six muscles that are attached to it. All movements of the eyeball take place about a point in the vitreous lying about 1.7 mm . posterior to the center of the globe. This point is called the center of rotation, and coincides with the center of curvature of the sclera, lying 10 mm . anterior to its posterior surface.

Donders and Dojer determined the position of the center of rotation in the following manner: They first measured the diameter of the cornea by means of the ophthalmometer, and then placed a hair stretched vertically across a ring in front of the cornea. The angular size of the lateral movements of the eyeball needed to bring the hair successively over the inner and the outer edge of the cornea was measured.

$$
p=x \text { tangent of angle } A C D,
$$

 from which the value of $x$ is calculated. Adding to this value the height of the cornea, we find the distance of the center of rotation from the vertex of the cornea, namely, 12 mm .

Through this point of rotation pass three lines or axes about which the eyeball makes all its simple movements. The primary axes are the antero-posterior, horizontal and vertical. The antero-posterior axis joins the center of rotation with the object looked at, and nearly coincides with the visual axis. It is the line of fixation. About this axis the eyeball makes its rotary movements or torsion, as it is termed. At right angles to this line and joining the centers of rota-
tion of the two eyes, is the horizontal or transverse axis, around which the movements of elevation and depression take place (sursumduction, or -version, and deorsumduction or -version). At right angles to both of these lines is the vertical axis around which the eyeball makes its lateral movements, that is, adduction, towards the nose, and abduction, towards the temple.

The six extra-ocular muscles can be divided into three pairs, each pair having a common axis around which it moves or rotates the eyeball. Only the common axis of the laterally-acting muscles, that of the internal and external recti, coincides with one of the three axes just described, that is, with the vertical axis. The other two pairs of muscles have their own axes of action and must be analyzed with regard to all three ocular axes, each of the four muscles producing elevation, depression, abduction or adduction. The superior and inferior recti muscles rotate the eyeball around an axis that
 passes horizontally through the center of rotation and running posteriorly cuts the median plane of the body at an angle of about 70 degrees. The obliques rotate the eyeball around a horizontal axis that passes through the center of rotation and cuts the median plane of the body anterior to the eyes, at an angle of about 35 degrees. Each of these axes is at right angles to the direction of the insertion of their respective muscles.

A reference to the figure will explain the axes of rotation just described; the verticalaxis being at right angles to the plane of the paper, cannot be shown in the figure.

The continuous lines in the figure drawn as diameters represent the primary ocular axes, with the exception of the vertical one ; the dotted lines the axes of rotation, or secondary axes.

Each muscle moves the eyeball as follows, reference being had to the direction that the cornea is made to face: The internal rectus simply draws the eyeball in towards the nose; the external rectus, outwards towards the temple. The superior rectus turns the cornea upwards, slightly inwards, and twists the upper end of the vertical meridian of the cornea inwards, the eyeball turning upon its anteroposterior axis (mesial torsion). The inferior rectus moves the eyeball downwards and inwards, and tilts the lower end of the vertical meridian of the cornea inwards, throwing the upper end outwards (lateral torsion). In regard to torsion the superior and inferior recti oppose each other. The superior oblique turns the cornea downwards, outwards, and produces mesial torsion ; the inferior oblique directs the eyeball upwards, outwards, and produces lateral torsion. The superior recti and the inferior obliques, and the inferior recti and superior obliques work together to maintain the vertical position of the vertical meridian of the cornea in vertical movements of the eyeball. The table gives the actions of the extraocular muscles at a glance.

Adduction by: superior, inferior and interior recti. The adducting action of the vertical recti increases in proportion as the eyeball is adducted.

Abduction: exterior recti and superior and inferior obliques. The abducting action of the obliques increases in proportion as the eyeball is abducted.

Sursumduction: superior recti and inferior obliques. Deorsumduction: inferior recti and superior obliques. Depression and elevation are effected mainly by the obliques when the eye is in adduction and by the recti when the eye is in abduction.

Inwards and upwards: interior and superior recti and inferior obliques.

Inwards and downwards: interior and inferior recti and superior obliques.

Outwards and upwards: exterior and superior recti and inferior obliques.

Outwards and downwards: exterior and inferior recti and superior obliques.

While this table is theoretically true, it is probable that all six extra-ocular muscles are concerned each time the eyeball makes any movement. The axis around which the eyeball turns is therefore always different from the three described above (primary axes), and perhaps no ocular movement is as simple as would be indicated from the above table. When the two eyes have their visual axes parallel and lying in the horizontal plane, the eyes are said to be in the primary position.

The primary position is more exactly defined as that from which the eyeball can make both vertical and horizontal movements without affecting the vertical meridian of the cornea.

All other positions of the eyeball are secondary. Under normal conditions both eyes move together in associate or conjugate movements, of which there are three forms, as follows: Both eyes moving in the same direction, as turning to the right (dextroversion), to the left (sinistroversion), turning up (bilateral sursumversion), or down (bilateral deorsumversion). (2) The movement of convergence, and (3) the movement of the eyes back to parallelism, or even divergence as in certain stereoscopic exercises, from convergence. There are two other associated muscular movements which may be mentioned, namely, that between the superior recti and the obliques in preventing torsion, in oblique directions of the gaze, and that between the sphincter of the pupil and the ciliary muscle in accommodation. The eyes are able to rotate upwards through 33 degrees of arc; inwards, 48 ; downwards, 50 ; outwards, 53 degrees, according to Dr. Stevens. According to Landolt, the eyeball is able to move through about 45 degrees of arc in every direction. If in any one direction the excursion of the eyeball is less than 30 , it may be considered pathological.

The field of fixation is not by any means a fixed quantity. The excursions of the eyeball are obtained most accurately by using Stevens' tropometer, illustrated and explained below.

It consists essentially of a telescope in which the inverted image of the examined eye is found at the eyepiece, where, either as an aërial image or as an image upon the ground glass, its movements

can be accurately observed. A graduated scale in the eyepiece permits every movement of rotation, in any direction, to be exactly measured.

Description of the Scale, Fig. B. - The long line between and at right angles to the shorter lines divides two similarly graduated scales running in different directions; the larger circle represents the outer border of the cornea, the edges of which are in contact with the two strong lines; the interval between each pair of short lines of the scale is ten degrees of arc, commencing at the strong line in each case. If now the head of the person examined is held
firmly in the primary position and the eye caused to rotate strongly in a given direction, the arc through which the border of the cornea passes may be accurately read upon the scale. In the figure B the curved dotted line represents a new position of the border of the cornea.

Suppose that the person examined has been directed to look strongly upward, then the cornea has moved down the scale and reaches the point in this example of $40^{\circ}$, that being the measure of this rotation. By means of the small lever the scale can be placed 'horizontally, vertically or obliquely, and by means of the two graduations measurements in opposite directions can be made.
Directions for Use of Tropometer. - If it is desired to determine the upward rotation the border of the cornea is made to coincide with the strong line which appears in the upper part of the scale at the right hand.

The adjustment is made by means of the milled head at the side of the standard. As the eye rotates up, the image moves down. In determining the downward rotation the strong line at the lower lefthand side of the scale is taken as the point of departure.

For lateral rotation the scale is turned to the horizontal position and the corresponding strong lines used as before.

In order to adjust the upper border of the cornea on the line, it will generally be necessary for the examiner to place the left hand upon the forehead of the patient and make gentle traction of the upper eyelid by the thumb. This application of the hand to the forehead is advisable in all measurements, as by this means the
examiner is able to detect even a slight movement of the head, which would vitiate any measure of the rotation. In adjusting the head to the head-rest, the teeth should be closed and the line of the upper lip just below the nose should be in a vertical line below the glabella or ridge just above the root of the nose.

The eye cannot fix the same point for even a little while without being annoyed by the formation of after-images or without the phenomenon of Troxler interfering. The eyes are therefore in perpetual motion which is made by jerks. They fix a point, make a movement; fix another point, and so forth. During reading the eyes move by jerks, four or five movements for a line of an ordinary book, and a greater movement when the eye is shifted from one line to another. Lamar constructed a little instrument, that he supported against the upper eyelid, and which was connected with the ears of the observer by rubber tubes. Each movement of the eyeball could then be heard and the number of movements easily counted. It has been shown that the shorter the printed line within certain limits, the less fatiguing is it upon the eyes.

The length of the line in the ordinary newspaper column is about the length that fatigues the eyes the least. It is impossible to cause a movement to be made with one eye without the other one moving also, in an associated movement. The following simple experiment however would seem to indicate the contrary. Suppose that the two eyes are fixing a point $a$, and we place an object $b$ in the visual line of the right eye. If we ask the person to fix the object $b$, the left eye is directed towards this point while the right eye remains apparently motionless. But, if we observe closely we will see that the right eye makes really two slight changes of position, for instead of receiving no innervation, as one would think, its muscles receive two, one that would cause it to make an associated movement to the right and another that would cause it to make a movement of convergence to the left ; the two innervations neutralize each other so that the eyeball remains apparently motionless. It was Hering that first described this experiment.

When the eyelids are closed, especially forcibly, the eyeballs are rolled up and out. This phenomenon is called Bell's phenomenon. It was until recently supposed to be due to a connection between the facial and the oculo-motor nerves, but histological dissection does not warrant any such supposition. Lately there has been advanced what is called the pressure-reflex theory, which is: That the cornea to escape the pressure of the stiff tarsal plate of the lids, when they are closed, rolls up behind the softer portions of the lids in obedience to a reflex started in the fifth nerves of the cornea, and carried to the eye muscles through the third nerve. (During sleep the pupils are contracted and possibly the ciliary muscles also.)

The field of fixation is the extent of space in which the eye can successively fix an object that is gradually moved in different directions before it. We begin by placing the object directly in front of the eyeball, with the eye in the primary position, and move the object from the center towards the periphery. The field of fixation is conveniently taken upon the perimeter, and the result recorded upon one of the charts used for recording the field of vision. The extent of the field of fixation is a good estimate of the ability on the part of the eye to rotate in different directions. 'The extent of the field taken with the perimeter should coincide with the results given by the tropometer.

To take the field of fixation the patient is placed as for taking the field of vision. Small printed matter is used for the test object. The patient's head is rendered stationary by having him grasp a wooden bar that extends cross-wise in front of the chin rest between the teeth, or the hand of the operator may be placed upon the head of the patient so as to perceive any motion of the head. The fine printed matter is then moved along the arc of the instrument from the center towards the periphery and the patient told to follow it with the eye (the other one being bandaged). The patient continues to turn his eye as the print moves toward the periphery and the moment when he has turned his eye in that direction to the utmost
the print passes out of view, that is, it becomes illegible as the macula can no longer be brought to bear upon it.

The furthest point at which the patient is able to read the print without moving the head expresses the limit of rotation of the eyeball in that direction. This is done for different meridians and the points on the chart thus obtained connected by a straight or curved line. If the vision is too poor to take the field of fixation with printed matter a lighted taper may be used as the object of fixation. As it is moved along the arc the eye follows it.

When the eyeball has rotated in any one direction to its fullest extent the catoptric image of the candle from the cornea becomes eccentric, no longer seen at the center of the cornea, as it was so long as the eye was directed at the light.

In the figure, $t, t^{\prime}$ and $t^{\prime \prime}$ represent three positions of the test object along the perimetric arc. As the object is moved from $t$ to $t^{\prime}$, the cornea is depressed, causing the macula to ascend from $m$ to $m^{\prime}$, so that the image of $t^{\prime}$ still falls upon it, but when the object reaches the point $t^{\prime \prime}$ the eyeball can no longer tollow it, so the image of $t^{\prime \prime}$ now falls off of the macula and the print of the test object is no longer legible.

A rough but sufficiently accurate test of the motility of the eyeball for practical purposes is to have the patient follow with the eyes the finger of the observer as it is moved about in front of them. One should notice whether both eyes move together, and whether in elevation and depression of the eyeballs, the lids accompany the excursions of the latter.

In lateral movements of the eyeball, that is, adduction and in abduction, linear mensuration of the excursions of the eyeball is applicable. The patient is first made to look straight ahead at an object at a distance of twenty feet in the horizontal plane with the eyes and in the median line of the face. An ink dot is then made upon the lower lid below the outer edge of the cornea (with the eyeball in this primary position), and below the outer angle of the lids, and the distance between them measured (distance $b d$ in the figure below). The eye then looks out to its fullest extent and a dot is then made below the outer edge of the cornea in the new position of the eyeball as before (distance $c d$ in the figure). The patient next looks in towards the nose as far as possible, and another dot
 made on the lower lid below the outer edge of the cornea (distance $a d$ in the figure).

Suppose that the first value is 8 mm ., and the second one I mm., then the amount of abduction equals 7 mm . Suppose that the distance from the outer edge of the cornea to the outer angle of the eye, when the eye is adducted is 18 mm ., the amount of adduction is then 10 mm . The measurement of the excursions of the eyeball is important in order to determine the degree of paralysis of extra-ocular muscles, and the effect of treatment; the prognosis of squint operations and so on.

The binocular field of fixation, or the extent of space in which there is single binocular fixation, is called the horopter or, as Dr. Savage suggests, the monoscopter ( $\mu$ óvos, single; $\sigma \kappa o ́ \pi \epsilon \omega \nu$, seeing). The monoscopter occupies different planes and is a surface of revolution formed by revolving a circular plane upon its chord, a line joining the centers of rotation of the two eyes.

The limits of the monoscopter in all directions is not less than thirty-three degrees. If binocular diplopia occurs uniformly when the eyes have been carried less than thirty degrees from the primary position, it is distinctly pathological. By reference to the cut it will
be seen that the three points chosen for the construction of the circle are: $b$, the point of fixation of the two eyes; $c$ and $c^{\prime}$, the centers of retinal curvature of the left and right eye respectively. The line of single vision is in the plane of the visual axis. Points situated anywhere on this line capable of sending light into the two eyes, will be seen singly by them. Such are the points $\alpha$ and $d$, the furthest points in the extreme periphery that can send light to the two eyes, and therefore can be seen singly. The monoscopter is formed by the revolution of the circular plane ( $a b c d$ ) upon its
 chord $c c^{\prime}$. Such a surface is a combination of a concave sphere and a concave cylinder (a section of a concave tore). All angles formed between the visual lines in the monoscopter are equal to each other, therefore there is the same inclination of the visual lines to each other throughout the surface.

## CHAPTER XVI

THE LAW OF LISTING

Each time that the look returns to the same point, the eye reassumes the same position (Donders). If we gaze upon a colored ribbon (red for instance), stretched horizontally, long enough to give rise to an after-image, and then project the latter upon the wall, keeping the head motionless, the projected after-image assumes the same position each time that particular spot is fixed, although not always horizontal. This position is determined by the law of Listing, according to which law the eye may be brought from the primary to any secondary position by a rotation around an axis that is perpendicular to the two successive directions of the visual line. Or in other words, as the eye passes from the primary to a secondary position, the angle of torsion of the eye is the same in the secondary position as if the eye had rotated about a line perpendicular to the first and second positions of the line of fixation.

The axes of Listing are all contained in a plane that is perpendicular to the primary position and pass through the center of rotation of the eyeball. This plane is invariably connected with the head as the primary position is that, when the eyes are directed straight ahead, giving to the head the position that seems most natural. It may be necessary for the individual to lean the head a little backward or forward in order to make the primary position horizontal. To demonstrate the law of Listing we place ourselves at a distance of one or two meters from the wall upon which is placed a fixation mark on level with the eyes. The position of the head is rendered secure. A rectangular cross is placed upon the wall with its arms horizontal and vertical. The cross is made to contrast with the color of the wall so that a plain after-image can be obtained after gazing at it for a while. We then move the head a little forward or backward
or to the one side or the other, until a position is obtained that on moving the look along the prolongation of each of the arms of the cross, the after-image of this arm glides all the time along itself. We observe that there exists only one position of the head in which this is possible. In every other position of the head the after-image of the cross turns partially around during the displacement of the look.

Suppose that we fix a point to the side of the cross on the same horizontal line. Since the meridian that was horizontal when fixing $A$ is also horizontal when fixing $B$, it is clear that the look may be brought from $A$ to $B$, by a motion around a vertical axis, that is around an axis that is perpendicular to the two positions of the visual line. It is the same for displacement in the vertical direction; to prove that it is true likewise for oblique displacements of the gaze we tilt the cross.

It is then easy to prove that the after-image of one of the arms of the cross glides all the time along its prolongation, when the look follows this prolongation, and that the eye in consequence turns around an axis perpendicular to this meridian, and thus the law of Listing is verified. In fixing the point $C$, figure I, we notice that the

vertical arm in the cross of the after-image is no longer vertical ; it has undergone a rotation, the upper extremity has gone to the right. This fact is simply in consequence of the law of Listing. The meridian that was vertical when fixing $A$ cannot remain vertical when the eye turns around an axis that is at right angles to the direction $A C$. In Fig. 2 the cross is tilted to show better how the after-image
glides along the prolongation of its arms. Donders attributed this phenomenon to a rotary movement of the eyeball around the visual axis. The displacement of the horizontal arm of the cross in the contrary direction is due to the fact that the projection of the after-image is upon a plane not perpendicular to the visual line. If we project the image upon the concave surface of a hollow hemisphere, at the center of which is placed the eye, the cross seems to have undergone a complete rotation to the right. In these experiments the position of the two eyes is exactly the same: we can cover one eye or the other without changing the position of the after-image.

The law of Listing merely defines the position of the eye in repose and it is probable that the eyeball never makes a movement about the axes of Listing, nor that the eyeball always follows the same path in passing from one point to another. As Tscherning suggests, the best way probably to study the manner of movements of the eye would be to quickly bring the look from one point to another, leaving the eye all the while exposed to a rather intense light. The after-image would then be in the form of a line that permits some conclusion to be drawn in regard to the direction of the movement of the eye. Meissner's method seems to verify the law of Listing in a very exact manner, as follows: If we hold a plumb line in front of the wall and fix an object nearer to the eyes than the line, we see the latter in double images (homonymous). We should expect to see two parallel vertical lines, but the two lines seen converge above. If we fix a point behind the line, the line again appears double but the two diverge above. If we look at a rectangular cross, its arms being vertical and horizontal with the right eye the upper right and lower left angles appear larger than the other two angles while the reverse takes place for the left eye. Since to the right eye a vertical line appears to slant to the left there must be a leaning to the right of the vertical meridian of the retina that seems vertical. The direction of this line may be determined in the following manner: Upon a circular piece of cardboard draw a diameter; place the disc upon a vertical wall; now let the observer try to turn the disc so
that the diameter will be vertical, using one eye at a time. With the right eye the upper extremity of the line will be placed several degrees too far to the right, and with the left eye too far to the left.

This fact will be observed if a plumb line be held so that it will pass through the center of the disc. Another method of determining the angle formed between the apparently vertical diameters of the retinæ is that of Volkmann. Place upon the wall two small discs, so that the distance between their centers will be equal to the distance between the pupils of the observer. Upon each disc is to be drawn a radius. The right eye now observes the right-hand disc and vice versa. The same experiment may be performed by using the stereoscope, as suggested by Javal. One of the radii is placed vertically and an attempt made to place the other one so that the two will form a single continuous straight line. It will be found that it is necessary that the lines should form an angle of about two degrees, in order that they may appear straight. There is a slight difference in the direction of the apparent and the real horizontal meridians of the two retinas as well, but to a much less degree. It is probable that these phenomena are due to the more important part played by the downward look in our every-day life, as when reading, walking, etc. If, while performing the experiment of Meissner, we draw the lower extremity of the plumb line towards us, we observe that the lines become parallel when the plumb line has a position at right angles to the visual lines.

Volkmann found that each eye in converging for a distance of 30 cm . made a rotatory movement of one degree (mesial torsion), which it would not have made if the visual lines were parallel in taking the same position, so that the law of Listing does not hold when the visual lines are not parallel. Place two candles one meter from each other, and look at them from a distance of one or two meters, getting the eyes as nearly in the primary position as possible. We then try to converge as if to fuse the candles. We then observe that the candles appear slightly inclined towards each other. The
inclination increases as we approach the candles towards each other and may reach as much as fifteen degrees.

Heuck taught that the eyeball underwent a rotation around the visual axes when the head was inclined to the one side or the other, in order that the vertical meridians of the retinas may always remain vertical. This rotatory motion does exist but to a slight extent, as Javal showed. If, however, no rotation took place around the line of vision when the head was inclined to the one side or the other, the axes of astigmatic spectacles would continue to coincide with the axes of the wearer's astigmatism, but this is not the case, as any one wearing astigmatic spectacles may demonstrate. With the head erect all the lines on the astigmatic dial may appear alike, but will appear unequal if the head is tilted to one side or the other. Dr. Stevens has lately devised an instrument that he calls the clinoscope, designed to further the study of the declinations of the vertical meridians of the retinæ, and to make more accurate such studies as well as to measure with a degree of certainty the extent of torsion that the eyeball undergoes in various movements. See the following cut and the explanation:


The clinoscope consists of two cylindrical tubes, each about three centimeters in diameter and fifty centimeters in length. The tubes are mounted on a brass platform, which holds them firmly in the same
horizontal plane, at a distance of 6.5 centimeters between the centers at a fixed point. The attachment to the platform permits the tubes to be adjusted in parallelism, in convergence or in divergence in the plane of the platform. The platform is attached by a movable joint to the upright standard, so that the instrument may be given any desired dip; a scale and pointer indicate, as in the case of the clinometer, the dip with respect to the horizon.

The tubes are caused to rotate upon their longitudinal axes by means of thumb screws, as seen in the figure, and the pointer and scale above the tubes mark the rotation with accuracy.

At the proximal end of each tube is a clip, in which, if desired, the observer may insert a glass for the correction of his refraction or any glass from the trial case. At the distal end is another clip and provision for maintaining precise position of the diagrams to be used in the investigation. These diagrams are haploscopic figures, calculated to aid in the various experiments which may be made. These may be varied according to the wish of the investigator. Below are repre-


Fig. 1.
sented three pairs of these diagrams. Figure I represents two pins, one to be seen by each eye. As the trained observer looks into the tubes, the two tubes and the two pins blend as one, and the head of the long pin now appears in the middle. An adjustment of the tubes by rotation on the long axis will show the true direction of the meridian, vertical or horizontal, according to the position of the diagrams, of each eye upon the scale above the tubes. For one who desires to test the accuracy of Helmholtz's theory of the leaning of the vertical meridian upward and outward, while the horizontal meridian remains
practically horizontal, Helmholtz's diagram (figure 2), familiar to students of physiological optics, may be used. The engraver has rep-


Fig. 2.
resented these diagrams as quadrilateral. They are in fact circular, like those of figure 1 .

For testing the ability of the eyes to rotate upon the antero-posterior axis (torsion), a straight line running across each disc (figure 3)


Fig. 3.
is the most useful figure. The lines may be placed vertically or horizontally. It will be found that the rotating ability is much greater when the lines are vertical.

The clinoscope is primarily an instrument for physiological research, but as a practical instrument it has been found of much value in determining the declination of the meridians in paralysis of the eye muscles, and in anomalous adjustments of the eyes in respect to the horizontal visual plane, and those who may be interested in determining the power of torsion, or who hope to increase the torsional ability by exercise, will find the clinoscope a correct measure in the first case, and an efficient aid in the second. The instrument, as
now constructed, is mounted on a base similar to the tropometer, with a head piece, which permits the easy and perfect adjustment of the patient to the desired position.

Dr. Savage questions the truth of Listing's law and brings the following arguments to bear in support of his unbelief. That the threatened torsion in rotations of the eyeball in oblique directions does not occur, being prevented by action of the obliques, is evident from the following experiment. Says he: An astigmatic with his correcting cylinders may be asked to look at an object up and to the right at the maximum obliquity. If torsion has occurred, revolving the axes of the cylinders in the direction taken by the best meridian would improve the vision, while as a matter of fact it renders the vision worse.

Again there are persons who have small corneal scars or marks upon the iris near the horizontal meridian, in whom we should be able to detect torsion of the eyeball as they look up and to one side. In fact these marks, if in the horizontal meridian at the beginning, remain so no matter what be the direction of the gaze. If physiological torsion did take place we would be deprived under certain conditions of the power to judge of verticality and horizontality. If the obliques did not prevent torsioning when we looked up and to the right or down and to the left 45 degrees, we would see a vertical line inclining $9^{\circ} 44^{\prime}$. An oblique position of the eyes does not deprive us of the idea of verticality, as Savage shows our idea of verticality depends upon the fact that the vertical axes of the eyes are parallel with the vertical plane of the head. The greatest argument in favor of Listing's law is the experiment of after-images described upon a foregoing page.

Such leaning of after-images as shown would be expected, for the stimulus of an after-image is not great enough to call into full action the fifth conjugate innervation, or that between the recti and the obliques. In passing to an oblique secondary position then according to many the eye revolves first about the vertical axis and then about the horizontal axis.

The following table is taken from Savage, which is equivalent to that of Maddox. It expresses the value of torsion, when the inclination of the axis is $45^{\circ}$, for various degrees of rotation if the obliques did not act.

| Angle of Rotation. | $5^{\circ}$ | $10^{\circ}$ | $15^{\circ}$ | $20^{\circ}$ | $25^{\circ}$ | $30^{\circ}$ | $35^{\circ}$ | $0^{\circ}$ | $45^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Torsi | 61/2' | $26^{\prime}$ | $1^{\circ}$ | $14^{0} 7^{\prime}$ | $24^{\circ} 9^{\prime}$ | $4^{\circ} 6^{\prime}$ | $5^{\circ} 40^{\prime}$ | $7^{\circ} 33^{\prime}$ | $9^{\circ} 44^{\prime}$ |

The use of the Maddox rod for testing the ability of the eyeball to undergo torsional movements was first pointed out by Maddox himself, but has been used by very few for this purpose. The Maddox rod is admirably adapted for determining the degree of torsion in cases of paralysis of the vertically acting muscles of the eyeballs. It also serves to determine in any given case whether lines are truly vertical and horizontal or not, or whether two intersecting lines are at right angles to each other or not, and we can ascertain how far these estimates are affected by the application of glasses.

If the notions of the patient in regard to the direction and relations of horizontal lines are much affected by glasses we may expect that they will cause for a while at least some amount of annoying distortion in objects seen through them. Cylindrical lenses frequently produce such distortion, even when the lenses are otherwise satisfactory. In order that the rod of Maddox may be readily applied to the solution of these problems Duane has constructed an instrument which he called a clinometer. It consists of two multiple Maddox rods mounted so as to revolve freely in a frame. Each frame is provided with a spring catch and is made to slide along the horizontal arm of the Stevens phorometer. The rod for the left eye has a ruby glass backing to distinguish the images as seen by the two eyes. The sides of the frames away from the patient's eyes bear a graduated arc and two indices, so that when the index marked $V$ is at $O$, and the arm of the phorometer perfectly horizontal, the line of light seen through the rod is perfectly vertical, and when the index $H$ is at $O$, the light is horizontal. The rods are revolved by means of two small handles.

The patient is seated so as to face a small but brilliant light on the other side of the room at a distance of about twenty feet, the room being perfectly dark. The patient is then directed to look at the light with the right eye through the right Maddox rod and the latter rotated until he says that the streak of light seen through it is perfectly vertical. The other eye should be screened. If when the patient declares the line to be vertical the index stands at zero or very near to it we know that his vertical meridian is truly vertical, or on the contrary if he should say the line is vertical when the index points.a number of degrees to the one side or the other of the zero mark we know that his vertical meridian is rotated that amount to the right or to the left as the case may be.

The test is now repeated with the left eye alone, the right one being covered, and finally with both eyes open, and the rods turned until the two lines of light are coincident or at least parallel. The same test is now made for horizontal lines. To estimate the accuracy with which the patient estimates right angles we turn one rod so that the streak is vertical and the other one so that the streak is horizontal, the true amount of deviation from the vertical or the horizontal to make the lines appear at right angles being read from the graduation. The eyes should occupy as nearly the primary position as possible during the test. The instrument shows that judgments are on a whole as accurate for horizontal lines as for vertical ones, and this about equal in the two eyes.

To measure the torsional ability, each rod with its line of light vertical is placed before its respective eye. The patient then sees a blended red and white line. One rod is then rotated until he just begins to see the lines diverge. The amount of rotation of the rod read from the scale then indicates the ability of the eyes to undergo torsion.

## CHAPTER XVII

## NORMAL AND ABNORMAL REFRACTION

By normal refraction we mean emmetropia, or the condition in which the retina lies at the principal focus of the dioptric system of the eyeball. Such an eye when at rest or in repose is focused for distant objects that send to it parallel rays (plane waves) of light. Any departure from this ideal constitutes an error of refraction, or ametropia, of which there are three varieties, namely, hyperopia, myopia and astigmatism. Presbyopia, or the failure of accommodation incident to age, is not included as it is a physiological condition that comes to all eyes, emmetropic and ametropic alike, and is not concerned with the focusing of the eye for distant objects, or the focusing of parallel rays of light. Errors of refraction lead to imperfect vision or eye-strain or to both. If the error is small it may be overcome by muscular effort on the part of the eyeball (if it is of such a nature that accommodation can correct the error), while if the defect is great, effort on the part of the eye being of no avail, eyestrain does not so readily ensue, as the eyes seem to discover the uselessness of the task and make no attempt to correct the trouble through muscular strain. On the other hand, part of an error may be overcome, while beyond this some remains to render the vision indistinct. Emmetropia is rarely found, although it is the ideal condition or state of refraction. The deviations of the eyeball from the normal are insignificant as compared with the deviations of other portions of the body, but sufficient to cause error and consequent inconvenience to many.

Eye-strain results from the efforts of the eye to prevent indistinctness of vision. It may be present in normal eyes working under unfavorable conditions, through poor illumination, or what not, or by excessive use. It may be caused by the excessive use of accommodation from
looking at too small objects or caused by hyperopia or astigmatism. It may also result from strain put upon the extra-ocular muscles, as from strain of convergence from viewing near work at too close a range. Strain may result from an effort to keep the eyes properly directed, as when the normal relation between the accommodation and the convergence is disturbed, as is caused by hyperopia or by myopia, or by lack of balance between opposing extra-ocular muscles (muscular inefficiency). In such cases the pain felt in and about the eyeball is due to a muscular fatigue to the greatest extent, also to the exhaustion of the nerve centers in the effort to overcome the error or to appreciate and properly interpret the blurred retinal pictures. The ciliary muscle cannot in itself give rise to an ache or pain due to fatigue as it is an unstriped muscle and such do not have any high degree of irritability (sensibility), but strain from excessive accommodation gives rise to reflex pain which radiates in different directions along the branches of the fifth nerve.

Eye-strain is more manifest when the health of the individual is below par, and is frequently unnoticed until during convalescence from an exhausting disease. Eye-strain makes itself evident through various affections of the eye, such as relaxation of accommodation, with the consequent blurring of the reading matter during near work, or by spasm of the ciliary muscle causing the eye to be rendered myopic with consequent blurring of distant seeing especially. It occasions inflammations of the lids, cornea and conjunctiva, and less frequently changes in the chorioid and retina, as well as in the crystalline lens. Opacities of the lens and vitreous body are at times caused by eye-strain. At times it manifests itself in the decrease of hyperopia or in the increase of myopia, if the latter originally existed. Headache is very frequent and is caused most likely by nerve exhaustion. It is most often frontal, extending to the temples and in some cases to the occiput and nape of the neck. Vertical headaches are among the rarer manifestations of refraction errors. Perhaps eye-strain may express itself in the form of a hemicrania.

The pain in the head is frequently found dependent upon the use of the eyes, but may be constant, ofttimes comes on arising in the morning, and may wear away as the day progresses, but in many the pain appears only towards evening. Nausea, vomiting, palpitation of the heart, despondency, neurasthenia and hysteria are frequently caused by eye-strain as well as twitchings of the face- or eye-muscles. Astigmatic errors especially and the kind with oblique axes frequently give rise to tinnitus aurium and vertigo, simulating very closely an incipient internal ear trouble. The connections between the eye and the ear will be pointed out in a subsequent chapter.

Hypermetropia, also hyperopia, hypermetropy and hyperpresbyopia ( $\dot{v} \pi \dot{\epsilon} \rho$, over; $\mu \epsilon ́ \tau \rho o \nu$, measure ; $\ddot{\omega} \psi$, eye), denoted by H., is an error of refraction caused by the retina being situated anterior to the principal focal point of the dioptric system of the eyeball. As parallel rays of light are not yet come to a focus when the retina of the eye intercepts them there is formed upon the latter an area of illumination the shape of the pupil, and varying in size according to the amount of ametropia. This area of illumination being circular in most instances is spoken of as a circle of diffusion. For every point in the object there is formed a circle of diffusion, and by the overlapping of these circles a blurred image of the object is built up. The unaided hyperopic eye has no point of clear vision. When at rest it is adjusted for converging rays of light. Such do not naturally exist, but the accommodation can render parallel rays from a distance so, and bring the focus of them upon the retina, and thus
 the eye is given sharp distant vision. A convex spherical spectacle lens does the same thing as increased convexity of the crystalline lens of the eyeball and therefore corrects the hyperopia.
Point $P$ is the principal focus of the parallel rays $A$ and $A^{\prime} ; r$, the retina lying anterior to the principal focal point of the dioptric system. In figure 1 the accommodation is represented as shortening focal in-
terval and bringing $P$ up to $P^{\prime}$. In figure 2, convex spherical lens $L$ takes the place of the accommodation. The rays $A$ and $A^{\prime}$ are rendered convergent by it, and then by the aid of the dioptric media of the eyeball are brought to a focus upon the retina at the point $P^{\prime}$. If one diopter of accommodation or a one-diopter
 convex spherical lens is needed to bring parallel rays of light to a focus upon the retina of an eye we may say there exists one diopter of hyperopia. One diopter of H . is equivalent to an actual shortening of the eyeball of .32 mm . Beginners are often confused to see how a lens with a focal length of one meter that is a I-D. lens can correct an error that depends upon a shortening of the eyeball of only .32 mm ., the distance between the retina and the principal focus of the dioptric system of the eyeball. The actual antero-posterior diameter of the emmetropic eyeball equals 24.2 mm ., which represents a refraction of 4 I .3 D . ( $1,000 \div 24.2=4 \mathrm{I} \cdot 3$ ), supposing that the dioptric system of the eyeball to be replaced by a single refracting surface, in the position of the cornea and the globe filled with air. A shortening of . 32 mm . of the eyeball makes its diameter 23.88 mm . $(24.2-.32 \mathrm{~mm}$. $=23.88)$ which represents the focal length of a $42+\mathrm{D}$. lens, one diopter more than the natural strength of the dioptric media of the eyeball to be supplied by a convex spherical lens ( $100 \mathrm{~cm} . \div 2.38=42+$ D.).

If a 2-D. lens is needed to bring the focus of parallel rays upon the retina the error is equal to 2 D., and so on. For any given distance the hyperope has to accommodate more than the emmetrope, that is, the same amount as the emmetrope plus the amount of the hyperopia. This causes the hyperopic eye to suffer earlier than the emmetropic eye from the loss of accommodation due to increasing age of the patient. The hyperopic eye for the same reason fatigues sooner than the emmetropic eye, other things being equal. It is deprived of the periods of rest that the emmetropic eye enjoys, that is, when adjusted for a distance. We have $H$. then causing eye-strain
or blurred vision or both. H., in the largest number of cases, is due to an antero-posterior shortening of the eyeball, axial hyperopia. This is most often caused by a diminution of the eyeball in all its diameters and not alone by a flattening from before backwards. Other less frequent causes of hyperopia are flattening of the curves of the cornea or of the lens, called curvature hyperopia, and a low index of refraction of the crystalline lens, or its removal from the eyeball, index hyperopia. To the latter class belongs the H . produced by cataract operations, which is spoken of as aphakial hyperopia (H. from absence of the lens).

The portion of the figure in each case above the horizontal line $A B$ represents the emmetropic eyeball in section, while that below the error of refraction produced by the several causes respectively.


In figure I the hyperopic eye's retina intercepts the light at $P^{\prime}$ before it comes to a focus; in figure 2 the lens is omitted for sake of simplicity; the cornea being too flat, the focal point is thrown back to point $H$, which lies behind the retina; in figure 3 the index of the lens is too little and the light is not bent enough in passing through the lens to be focused upon the retina, but would come to a focus at the point $H$ if the retina was not interposed; in the last figure, the light as it enters the eyeball is only refracted by the cornea, aqueous and vitreous, and is therefore not brought soon enough to a focus, so the retina receives a diffusion circle instead of a focus. Hyperopia is a congenital defect and is never acquired save by the loss of the crystalline lens. It is present in nearly all eyes to a certain extent at birth. Most observers believe that the amount of hyperopia is overcome in a great measure as the child grows.

Dr. Randall says that the eyeball increases its length as the child grows, from about 16 mm ., and decreases its refraction pari passu, so that not more than a diopter of hyperopia is outgrown. As the lens grows it increases in width, faster than in thickness, which causes its curves to flatten with a consequent diminution of its refraction.

There are several varieties of hyperopia recognized in practice, based upon the amount of accommodation that is used. The amount of error that remains after the accommodation is exerted to its utmost is called the absolute; it is the amount of hyperopia that the accommodation can not cover up or overcome.

A certain amount of hyperopia is covered by the action of the ciliary muscle and revealed by the relaxation of accommodation when a plus spherical lens is placed before the eye. This amount of hyperopia that can be covered up or not by will is spoken of as the facultative hyperopia. There is still a certain amount of error that remains covered after the strongest convex spherical lens is placed before the eye with which the patient maintains the same distant vision as without and which remains hidden by the action of the ciliary muscle. From early childhood the ciliary muscle has been active and the habit can not at once be entirely given up. This portion habitually covered by the accommodation is called the latent hyperopia. The latent hyperopia is never entirely revealed in folks under 60 years of age unless a cycloplegic be instilled into the eyes, to paralyze the ciliary muscles. The absolute and facultative make up the manifest hyperopia or the amount of error that can be ascertained without the use of drugs to abrogate the function of the ciliary muscle. The manifest and the latent together make up the total hyperopia. The scheme on following page illustrates.

The more active the accommodation is, and the lower the degree of hyperopia, the less the amount of absolute hyperopia. In the young the amount of absolute is less than in the old, as in the latter the accommodation is enfeebled. More and more of the latent becomes manifest, and manifest absolute as accommodation fails incident to increasing years.
Hyperopia $\left\{\begin{array}{l}\left.\begin{array}{l}\text { Absolute, the amount exceeding the power of } \\ \text { accommodation. } \\ \text { Facultative, amount corrected or not accord- } \\ \text { ing to conditions. } \\ \text { Voluntary. } \\ \text { Latent, amount habitually covered by accom- } \\ \text { modation. } \\ \text { Involuntary. }\end{array}\right\} \begin{array}{c}\text { Manifest } \\ \text { and }\end{array} \begin{array}{c}\text { Latent equals } \\ \text { total hyperopia. }\end{array}\end{array}\right.$

The proportion between the amount of latent and manifest hyperopia is not a fixed one, but varies in different individuals, and from day to day or from minute to minute in the same eyeball. The higher degrees of hyperopia alone give rise to trouble in childhood as the lower amounts can be overcome by the act of accommodation. The earliest symptom arising in many cases in children is cross-eyes. One eye, usually the one that has the higher error and the poorer vision, turns in towards the nose (internal squint). As the accommodative effort in hyperopia is necessarily great to keep objects well in focus, some of the nerve influence that is sent to the ciliary muscle overflows, so to speak, and causes contraction of those muscles that are closely associated with the ciliary muscle, the internal recti. The squint may only be apparent when there is most strain upon the accommodation, as when using the eyes for near work. It may be present at all times or only upon occasions. Internal squint due to hyperopia is most apt to occur before the sixth year. It is associated with high but not with the highest degrees of refraction error. It occurs when the hyperopia can be corrected by straining the accommodation. In the very high degrees of hyperopia the vision is found below par when the error is properly corrected by convex spherical lenses, caused by a faulty development of the eyeball and especially of the retina. The deficient vision which can not be made perfect by accommodation or glasses causes the child to hold his reading matter very close to the eyes, so that enlarged retinal images will compensate for diminution in distinctness.

Treatment. - No drugs or means of any sort will cause the eyeball to increase its antero-posterior diameter and to become emmetropic, or to do away with any of the causes that make hyperopia. The wearing of convex spherical lenses in spectacle or nose-glass frames is the only way to put the hyperopic eyeball in the same position for working as the emmetropic eye. The eye is said to have been rendered emmetropic when its correcting lens is adjusted. The wearing of glasses is not indicated because there is merely hyperopia present, but from the evidences of eye-strain alone. The rule is to correct the total hyperopia, unless there is a weakness of convergence, in which case a full correction of the hyperopia would exaggerate the muscular anomaly by taking off the extra nerve influence that enables the interni to do their work, received through the effort on the part of the accommodation to overcome the hyperopia

Glasses should be worn only for near work if the eyes do not trouble one at other times, if they annoy whether occupied with near-seeing or not, the glasses are to be worn all the time. Many hyperopes learn how to look at distant objects with relaxed accommodation and are therefore comfortable without their glasses. The manner of determining the proper glasses for hyperopia will be described later.

Myopia (from $\mu \dot{v} \epsilon \nu \nu$, to close, and $\ddot{\omega} \psi$, eye, so called from the manner myopes have of partially closing the eyes to increase their vision). -Also called brachymetropia, short-sightedness or near-sightedness. It is denoted by M.

Myopia is the error of refraction that results from the retina lying behind the principal focus of the dioptric system of the eyeball. Such an eyeball can not by any means bring parallel rays of light from a distance to a distinct focus upon its retina. It is adjusted for divergent rays of light that emanate from a point at a comparatively short distance. As parallel rays of light come to a focus anterior to the retina there is formed a diffusion circle upon the latter by the crossing of the rays of light. Parallel rays of light are brought to a focus upon the retina by passing them through a concave spherical
lens placed before the eye. The concave lens gives to the light the proper divergence, as if they emanated from a near point, for which the eyeball is adjusted in a state of rest. Concave spherical lenses then correct myopia. In the figure $M$ represents a myopic eyeball
 bringing rays $a$ and $a^{\prime}$ to a focus in front of the retina. The concave lens $L$ interposed diverges the rays to $b, b^{\prime}$, and thus throws the focus back upon the retina. (Divergent rays of light are focused posterior to parallel rays.)

The rays $a$ and $a^{\prime}$ have the same divergence after passing through lens as if they emanated from the point $C$.

An abnormally long antero-posterior axis of the eyeball or too great a curvature of its refracting media may be inherited. But the great mass of myopic eyes are pathological and acquired. They are prone to take on distinct and characteristic lesions, that may be due to the myopia, but which on the other hand cause an increase in the latter. The sclera of the eyeball is distended normally by an intraocular pressure equal to $25-30 \mathrm{~mm}$. of mercury. This outward push preserves the shape of the eyeball. The normal eye does not yield before this pressure by the bulging of its walls. An inherited weakness of the eye-coats, an acute disease, or a diathesis lowers the resistance of the sclera to withstand this intra-ocular pressure. Furthermore there is started by eye-strain or by nutritive deficiency, an inflammatory action in the ocular tunics which softens them and causes them to bulge. This distention mostly always occurs at the posterior pole of the eye-globe, causing myopia or the increase of it, if it was already present, by the elongation of the antero-posterior axis.

After such distention begins anything that favors the rise of intraocular tension or lowers the nutrition, tends to cause an increase in the trouble. The most important factors in the increase of myopia are faulty methods of using the eyes during school life. The myopic
eyeball is a long one and does not readily turn in the orbit. An excessive effort at convergence, due to holding the work too close to the eyes, causes pressure upon the eyeballs. It is squeezed as it were between the internal and external recti muscles. The interni contract to maintain single binocular fixation at the reading distance, and the externi are wrapped tightly around the globe. To maintain the proper amount of convergence is quite a task for the myopes of higher degrees, as the reading matter or work must be held very close to the eyes in order that there may be received distinct retinal pictures. Studying in a stooped posture, causing a congestion of the neck, head and eyeballs is deleterious.

Bad hygienic surroundings and insufficient light are to be blamed for a great deal of increase in myopia during early life. Most all cases of myopia progress to a certain extent. They may finally become stationary due to an increase in the resisting power of the eye-tunics. There are some cases of myopia that continue to progress, until convergence is made too difficult to be sustained, when the more defective eye is allowed to deviate. Often there is simply a loss of binocular fixation for near objects. The power of convergence soon diminishes ; the internal recti become weaker from nonuse and finally the eyeball is turned or pulled towards the temple by the stronger external rectus. This external or divergent squint may be intermittent, or remittent at first. After the squint becomes permanent there is no more desire on the part of the eyes to converge and consequently the myopia is most apt to become stationary. In a few cases, however, the sclera has become so thin by this time that it continues to yield before the intra-ocular tension and eventually blindness results. To these cases the term of malignant myopia is given.

After the sclera has become attenuated the intra-ocular pressure, and the tension of the ocular muscles upon the eyeball during sleep undoubtedly aid in the increase of the myopia. The eyeballs are rolled up and out during sleep - causing the globe to be more or less squeezed upon by the superior and inferior muscles. The
transverse diameter of the globe is thus increased, which causes the chorioid to be dragged upon and to develop inflammation adjacent to the side of the optic nerve. Sleeping in a light room would further increase the trouble, inasmuch as the eyes would be rolled up to a greater extent in their endeavor to rid themselves of the light.

Myopia reaches a much higher degree than hyperopia, and high myopia constitutes a large number of all cases of myopia. Myopia of 20 D. is as common as hyperopia of half that amount. Myopia is designated as low when it is less than 2.50 D., in which case some accommodation is used for near work. Moderate myopia, from 2.50 to 5 D., in which near work can be done without accommodation ; and high myopia, from 5 to 10 D., in which near work is performed at the far-point. Very high myopia is that of 10 to 20 D. or more, and is usually associated with extensive inflammatory changes in the eye tunics. It is often detected by the child being unable to see the board in school unless he takes a front seat. If the myopia does not exceed 3 D . the child may not hold the reading matter closer to the eyes than is proper ; when, however, the error is over 3 D., the book must be held closer than 33 cm ., the proper reading distance. The patient may notice spots in his field of vision due to opacities in the vitreous humor. The pupil of the myopic eye is rather larger than in the emmetropic because less often exercised by contraction in the act of accommodation and convergence.

Myopes often have a rather vacant and stupid stare as not seeing well, they are unable to respond to the change of expression of the countenances of others. They show a distinct inclination for reading and pursuits that do not require good distant vision. With the ophthalmoscope there are seen many intra-ocular changes that are indicative of the cause and of the increase of the myopia. The most frequent are alterations in the chorioid coat. The change may consist simply in congestion or edema, causing reddening, blurring of details in spots, and lighter areas. This condition is spoken of as a woolly, patchy or fluffy chorioid. The pigment in parts of the fundus may be reduced, while in other parts it may be heaped up - moth-
eaten fundus. There are frequently present patches of chorioidal atrophy or active inflammation in the latter.

The characteristic change in the fundus of the myopic eyeball, however, is the myopic crescent. This consists of a crescenticshaped area of chorioidal atrophy adjacent to and upon the temporal side of the optic papilla. The crescent arises from the stretching of the eyeball about its posterior pole. By the yielding of the sclera, the delicate chorioid coat is pulled upon and readily takes on an inflammatory action, which in turn gives rise to atrophy. There may be several crescents present, the one to the outside of the other. The one adjacent to the nerve will consist of a spot of complete atrophy, the next one of partial atrophy and the most external, a spot of active chorioiditis. If the edge of the crescent is well and sharply defined, the myopia at that time is at a standstill ; if the edge is blurred, on the other hand, active inflammation is going on, indicative of an increase in the elongation of the eye-globe. Sometimes the crescent is of a more or less triangular form and then it is called a conus. Both the crescent and the conus are called posterior staphylomata, or the crescent or conus of Scarpa. The optic papilla is tilted by the recedence of the sclera and is therefore seen obliquely, causing an apparent diminution in its width, appearing as a narrow oval. Late in the course of high myopia, cataracts and vitreous opacities develop and not infrequently detachment of the retina follows.

We recognize the following varieties of myopia: Axial myopia, so-called because the antero-posterior axis is too long as compared with the focal length of the dioptric system of the eyeball. It is seldom congenital ; begins as eye-strain ; increases, or if it becomes stationary in early life, it may be outgrown by the slow growth of and consequent flattening of the crystalline lens. Axial myopia may begin in middle or in old age as a symptom of diabetes. Curvature myopia may begin at any time ; the cornea becomes distended in a form of a cone or globe, due to weakness of its fibers brought about by poor nutrition. This is myopia from conical cornea. Index my-
opia comes in old age as a precursor of cataract, due to swelling of
 the crystalline lens, also called second sight, because when it sets in the patient no longer needs his convex lenses for reading and for seeing near by, but is able to throw them aside. It is usually complicated with astigmatism against the rule. Lastly, spasmodic myopia, caused by a spasmodic contraction of the ciliary muscle. The lens is rendered abnormally convex and then the light from a distance is brought to a focus anterior to the retina. This spasm is most apt to occur in convergent or accommodative inefficiency, and in low degrees of hyperopia, although it occurs in emmetropia, and less frequently in myopia, increasing the latter error. It is occasionally unilateral, and then affects the eye that is used the most. Spasm of accommodation often renders latent a considerable amount of hyperopia. In the opposite figures, the portions above the horizontal line represent sections of emmetropia, while those below the line represent the respective errors.

Treatment. - Distant vision is rendered distinct by the use of concave spherical lenses, and if the error is of considerable amount, above 3 D., one is enabled by the use of concave lenses to remove printed matter to the proper reading distance. The progress of myopia will be checked permanently if near work is avoided. If on the increase in a school child, the child should be kept from school until all inflammatory reaction in the eyeball has subsided and then the most careful hygiene of the eyes preserved. Correcting lenses in the vast majority of cases render myopia stationary. Accommo-
dation and convergence are both accused of being the cause of the increase in myopia. Accommodation is far more strained in hyperopic eyes, but such eyes seldom show any tendency to elongate. On the other hand hyperopia is an obstacle to the straining of convergence while myopia favors it, as the accommodation near point in myopia lies very close to the eyes. Again the myopia does not cease to progress when the accommodation is reduced to a minimum, or when it is rendered unnecessary for near vision, but it often ceases when convergence is not needed, as when single binocular fixation is lost.

Von Graefe was the first to advocate the undercorrection of myopia as is still practiced by many. He noticed with the ophthalmoscope that there developed a pulsation of the retinal veins during accommodation and therefore said if accommodation gave rise to increased intra-ocular tension it was a bad thing for myopia. Pulsation of the retinal veins occurs however in many eyes under the full effect of atropine, so the rise of tension must be due to the pressure of the extra-ocular muscles upon the eyeball. That this is the correct explanation is proven by the fact that no pulsation is observable during accommodation in cases of paralysis of the extra-ocular muscles.

The myope should then have glasses that prohibit excessive convergence. If binocular fixation for near objects is lost it is not advisable to try to restore it, as convergence would then again become active in increasing the error of refraction. Myopes should wear their glasses constantly, so that the requirements made on accommodation will act as a check upon convergence. As vision is rendered more distinct at the proper reading distance from the eyes, by aid of the glasses, printed matter will be held further off, and thus convergence relieved, also, the myope not being able to accommodate well will hold his reading matter as far from the eyes as is consistent with good vision, to use as little accommodation as possible. Some advise only a part correction of the error with the idea that accommodation may react injuriously upon the eyes. Too weak lenses however often prove a menace to the welfare of the eye. Looking
obliquely through their edges gives better distant vision, by increasing their spherical effect. The patient soon discovers this fact, and avails himself of it.

Looking obliquely through a lens also gives it the effect of a cylinder, which subjects the eye to a strain similar to that caused by uncorrected astigmatism ; a strain that is illy borne, as it is constant in its operation. Partial correction for near work is deleterious, as then objects are held closer than is safe. In myopia give the full correcting lenses as often as is possible, for constant use.

Certain exceptions to this rule exist. If presbyopia has set in weaker concave lenses must be given for near work. The same is the case if there is weakness of accommodation from other causes. Presbyopia sets in late in cases of non- or under-corrected myopias, as there is no need of the usual amount of accommodation, and its loss is therefore not noticed until far advanced. A full correction should be given for distance and a lens a diopter or so weaker for near seeing, if the accommodation is below par; until the ciliary muscle becomes stronger from proper use. The myope must further learn to hold all near work as far from the eyes as possible. Good light and good large print should be demanded always. The patient should always sit with his or her back to the light, and never with it shining in the eyes while employed at near work. Prolonged near work is to be avoided. One should take frequent resting spells, to allow relaxation of convergence. Reading in a stooped or recumbent posture should not be allowed, as both favor ocular congestion. If there is any chorioiditis present, atropine should be instilled and complete rest from all near work taken. Correcting lenses should be changed as the myopia increases or diminishes.

Fukula advised the removal of the crystalline lens in cases of very high degrees of myopia. Cases of $18-20$ D. or over are alone suited for operation as about that much hyperopia is made by the removal of the lens, causing more nearly emmetropia. Of course the eyeball is then devoid of accommodation, but in no worse condition than an eye that has been operated upon for cataract. In cases where cor-
recting lenses cannot be comfortably worn or where with them the patient has not vision good enough to carry on his avocation in life is this measure alone adopted. Experience has shown that the operation does not put a stop to the increase in the myopia, as was claimed by Fukula. The operation is not contraindicated by the presence of inflammation in the fundus of the eyeball, over which it exercises no influence, for good or bad. The operation is best performed by needling the lens, and in people over thirty years of age combined with a subsequent extraction of the opaque lens substance. All are not agreed as to the advisability of running the necessary risk of operation and of a subsequent destructive inflammation for the amount of benefit derived in the majority of cases.

By the operation the visual acuity is improved by the increase in size of retinal images, by the nodal point being brought up to the apex of the cornea and inasmuch as the concave lenses correcting the error likewise considerably diminished the size of the retinal images.

TABLE OF AXIAL HYPEROPIA. (LANDOLT.)

| Degree of Hyperopia. | Amount of Shortening. | Total Length of Axis. | Degree of Hyperopia. | Amount of Shortening. | Total Length of Axis. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.00 mm . | 22.824 mm. | 8.0 | 2.28 mm . | 20.54 mm . |
| 0.5 | 0.16 | 22.67 | 8.5 | 2.41 | 20.41 |
| 1.0 | 0.31 | 22.51 | 9.0 | 2.53 | 20.29 |
| 1.5 | 0.47 | 22.35 | 9.5 | 2.66 | 20.16 |
| 2.0 | 0.62 | 22.20 | 10.0 | 2.78 | 20.04 |
| 2.5 | 0.77 | 22.05 | 10.5 | 2.90 | 19.92 |
| 3.0 | 0.92 | 21.90 | 11 | 3.02 | 19.80 |
| 3.5 | 1.06 | 21.76 | 12 | 3.25 | 19.57 |
| 4.0 | 1. 21 | 21.61 | 13 | 3.49 | 19.35 |
| 4.5 | I. 35 | 21.47 | 14 | 3.69 | 19.13 |
| 5.0 | 1. 50 | 21.32 | 15 | 3.91 | $18.91$ |
| 5.5 | 1. 62 | 21.20 | 16 | 4.11 | 18.71 |
| 6.0 | 1.76 | $21.06$ | 17 | $4 \cdot 32$ | $18.50$ |
| 6.5 | 1.90 | $20.92$ | 18 | 4.52 | 18.30 |
| 7.0 | 2.03 | 20.80 | 19 | 4.71 | I8.11 |
| 7.5 | 2. 16 | 20.66 | 20 | 4.90 | 17.92 |

It will be seen from the table that the shortening in axial length of the eyeball to make I D. of error is .3 mm . up to 7 D . beyond which the shortening necessary to make i $D$. of hyperopia is less thav .3 mm .

TABLE OF AXIAL MYOPIA.

| Degree of Myopia. | Amount of Lengthening. | Total Length of Axis. | Degree of Myopia. | Amount of Lengthening. | Total Length of Axis |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.00 mm . | 22.824 mm . | 8.0 | 2.93 mm . | 25.75 mm . |
| 0.5 | 0.16 | 22.98 | 8.5 | 3.14 | 25.96 |
| 1.0 | 0.32 | 23.14 | 9.0 | 3.55 | 26.17 |
| 1.5 | 0.49 | 23.31 | 9.5 | 3.58 | 26.40 |
| 2.0 | 0.66 | 23.48 | 10.0 | 3.80 | 26.62 |
| 2.5 | o. 83 | 23.65 | 10.5 | 4.03 | 26.85 |
| 3.0 | 1.01 | 23.83 | II | 4.26 | 27.08 |
| 3.5 | 1.19 | 24.01 | 12 | 4.73 | 27.55 |
| 4.0 | 1.37 | 24.19 | 13 | 5.23 | 28.05 |
| 4.5 | I. 55 | 24.37 | 14 | 5.74 | 28.56 |
| 5.0 | 1.74 | 24.56 | 15 | 6.28 | 29.10 |
| 5.5 | 1.93 | 24.75 | 16 | 6.83 | 29.65 |
| 6.0 | 2.13 | 24.95 | 17 | 7.41 | 30.23 |
| 6.5 | 2.32 | 25.14 | 18 | 8.03 | 30.85 |
| 7.0 | 2.52 | 25.34 | 19 | 8.65 | 31.47 |
| $7 \cdot 5$ | 2.73 | 25.55 | 20 | 9.31 | 32.13 |

Unlike hyperopia the necessary increase in the axis of the eyeball to make one additional diopter becomes larger as the myopia increases. A certain amount of shortening of the eyeball does not give rise to the same degree of hyperopia as the same extent of lengthening of the eyeball does to myopia, on account of the relative distances of the principal points being different in the two cases.

## CHAPTER XVIII

## ABNORMAL REFRACTICN (continued). ASTIGMATISM

Astigmatism, dencted As. (from $a$, not, and $\sigma \tau i \not \gamma \mu a$, a point), also called astigmism, is the error of refraction due to an inequality of the curvatures of the dioptric surfaces of the eyeball, in different meridians. The error is so named because the refraction being different in different meridians of the astigmatic eye, a point of light is not focused upon the retina as a point of light, but as a line or in the form of an ellipse. In an astigmatic eyeball, then, the form of the retinal illumination from a point of light is, according to the position of the retina, one of the following :


This can be demonstrated by taking a sphero-cylindrical combination and moving it backward and forward before a screen upon which is thrown the focus of a luminous point or small luminous circle. As the position of the screen is altered, the area of illumination upon the screen will be seen to change its shape, gradually passing through the series of figures as shown above. If the cylindrical lens in the combination is tilted so that its axis is inclined the lines and ellipses will likewise be inclined either in the same direction or in the opposite direction according to the position of the screen.
The astigmatic defect is either resident in the cornea, lens or retina. When the location of the error is not designated it is assumed to be located in the cornea, but in the vast majority of cases it resides in the lens as well. The astigmatic cornea is well illustrated by a circular section taken from the edge of a watch,
curved in one direction and more curved in the direction at right angles thereto. Regular astigmatism is a curvature ametropia in which the different meridians are equally curved throughout their course, and in which the meridians of the greatest and the least refraction are at right angles to each other. Irregular astigmatism is that in which either of the above conditions of regular astigmatism is departed from, or produced by an oblique position of the screen, the retina. This latter occurs in cases of myopia, where the macula is not at the summit of the posterior staphyloma, but upon its side, and operates to render the vision imperfect in certain cases of myopia. Irregular astigmatism is also produced by an oblique position of the crystalline lens of the eyeball, known as astigmatism by incidence.

The meridians of the greatest and the least curvatures in astigmatism are called the principal or chief meridians. If they are vertical and horizontal, the astigmatism is styled as vertical or horizontal, according to the direction that the meridian of the greatest ametropia takes. If the principal meridians are oblique or inclined to the vertical and horizontal, the astigmatism is classed as oblique. The cornea on its anterior surface is normally slightly more curved from above downward than from side to side (the radii of curvature differing by about. i mm.). In the majority of cases the anterior or the posterior surface of the lens or both compensate for this inequality, by having their weakest curves in direction at right angles to the strongest curve of the cornea, and vice versa. As this inequality in the corneal meridians is a rule in eyes, an exaggeration of it gives rise to what is termed astigmatism with the rule (direct astigmatism); while if the cornea is most bulged or curved from side to side, we have astigmatism against the rule (indirect astigmatism). The figure illustrates the manner in which light is refracted on entering an astigmatic eyeball. The lens of the eyeball in the diagram is omitted for the sake of simplicity.

Parallel rays of light $A, A^{\prime}$ from a distant point pass into the eyeball through the more bulged vertical meridian of the cornea, which
with the aid of the lens and the vitreous brings them to a focus upon the retina, and therefore the vertical meridian of the eye is called emmetropic. Parallel rays $B, B^{\prime}$ from the same point pass to the eyeball through the cross, flatter meridian of the cornea. The refraction being feebler in this meridian the focus lies behind the retina at the point $H^{\prime}$. The cross meridian of the eyeball is therefore called hyperopic. If one of the principal meridians is emmetropic, the astigmatism is spoken of as simple; if both principal meridians are hyperopic but the one more so than the other, as compound hyperopic ; if both are myopic, but to different degrees, as compound myopic. While if one principal meridian is hyperopic, and the other one

myopic, the astigmatism is called mixed. The form of illumination at the point $E^{\prime}$, in the preceding figure, where all the rays from a common external point come to a focus, is that of a horizontal line, while that at the point $H^{\prime}$, where they again come together, is a vertical line. These lines are called the focal lines of the astigmatism. The distance between the first and the second focal lines is the interval of Sturm. The length of this interval represents the amount of astigmatism. The smaller it is the less is the astigmatism, and vice versa.

The first or the anterior focal line is formed by refraction through the greatest refracting meridian of the eyeball and is at right angles to the direction of the latter. The second focal line is formed by the least refracting meridian and is at right angles to that meridian. The
diffusion spots are everywhere elliptical except at one point between the focal lines, where the spot is circular in outline. The rays of light that pass through the principal meridians meet the axis at their respective focal points, but the light that passes into the eye through secondary axes does not meet the axis of the dioptric system of the eyeball. The following method of calculating the length of the focal lines is according to Tscherning. The length of these lines is proportional to their distances from the lens.

In the figure let $P$ be the diameter of the lens; $F^{\prime}$, the first focal

$P^{\prime}-$ Horizontal Focal Line, $P^{2}=$ Vertical Fucal Líne
line ; $F^{\prime \prime}$, the second focal line ; $P^{\prime}$ and $P^{2}$, the lengths of the two focal lines respectively. Then in the two triangles, we have

$$
P^{\prime} \left\lvert\, P=\frac{F^{\prime \prime}-F^{\prime}}{F^{\prime \prime}}\right., \text { and } P^{2} / P=\frac{F^{\prime \prime}-F^{\prime}}{F^{\prime}} ;
$$

by dividing $P^{\prime} / P^{2}=F^{\prime} / F^{\prime \prime}$.
The circle of diffusion is at $a$, where the diameters are equal. It divides the interfocal distance into two parts proportional to the focal distances. Designating the diameter of this circle by $a$, and the two parts of the interfocal distance by $x$ and $y$ we have:

$$
\begin{aligned}
& a / P^{\prime}=y / x+y, \text { and } a / P^{2}=x / x+y, \text { and by dividing } \\
& y / x=P^{2} / P^{\prime}=F^{\prime \prime} \mid F^{\prime} .
\end{aligned}
$$

All other diffusion spots are ellipses, of which it is not difficult to calculate the axes. If the screen is placed at the distance $b$, from the
second focal line we see that the axes $c$ and $d$ are found by the following equations :

$$
\frac{c}{P}=\frac{b-\left(F^{\prime \prime}-F^{\prime}\right)}{F^{\prime}} \text { and } \frac{d}{P}=\frac{b}{F^{\prime \prime}}
$$

equations which give as the relation between the axes:

$$
\frac{c}{d}=\frac{b-\left(F^{\prime \prime}-F^{\prime}\right)}{F^{\prime}} \times \frac{F^{\prime \prime}}{b}
$$

Knowing the axes we can find the ellipse by construction. Wa make a circle with half the length of $d$ (previous figure) for its radius, and draw within two diameters, a vertical one and a horizontal one. The points $A^{\prime}$ and $E^{\prime}$ are then marked so that $O A^{\prime}=c / 2 . \quad B D$ and $A^{\prime} E^{\prime}$ are then the two axes of the ellipse.

We can find any point $G^{\prime}$, of the ellipse, by letting fall a perpendicular $G H$ to the long axis so that

$$
G^{\prime} H \mid G H=c / d .
$$

This same construction can be used
 to find the course of rays that do not lie in the principal planes. If the dioptric system were spherical and of the power of the meridian of the least refraction, we would get a circle of diffusion of the diameter $B D$. This circle would only be a diminished image of the lens and it would be easy to determine the position of the point $K$, through which the ray would pass. We then can find the point $K^{\prime}$ by diminishing the distance of $K$ from the long axis in the proportion of $c / d$.
Lines that run in different directions are seen unequally distinct by the astigmatic eye. A line is distinct or not according to whether its edges are well focused or not. The distinctness of vertical lines depends upon the accuracy of focusing through the horizontal meridian of the eye, and so with any line, it is distinctly seen if the meridian
of the eyeball at right angles to its direction forms a clear retinal picture of it.
In the figure let $a, b$ and $c$ represent three horizontal meridians of the cornea $C ; d$, a vertical meridian of same. In the vertical line $L$ there are three points, the one above the other, and what is true of


The transposition in refraction causing a point above in the line to appear below in its image, etc., has been omitted for simplicity. these three points is true of the whole line, as the line is built up of the apposition of the points that compose it.

Point I sends rays of light to meridian $a$, and is focused at the point $\mathrm{I}^{\prime}$, behind; lights from points 2 and 3 pass through meridians $b$ and $c$ respectively and are focused at points $2^{\prime}$ and $3^{\prime}$. The image of the line $L$ is then built up by juxtaposition of the images of the points in the line $L .5$ and 6 are two rays that enter the cornea through the vertical meridian $d$, which is more curved than the horizontal meridians and therefore has a shorter focal length, and brings the rays to a focus at the point $O$, before the screen is reached. The rays diverge again from the point $O$ and form an area of.illumination upon the screen the shape of a vertical line. By the overlapping of the short lines, the line $L^{\prime}$ is built up. It is evident, therefore, that as long as the cross focusing of points in line $L$ is accurate, the vertical line $L^{\prime}$ is distinct. The only blurring of the line will be at its ends, where the rays of light from the end points of $L$ pass through the vertical meridians of the cornea. An ocular demonstration of this fact is as follows: Look at crossed lines through a strong convex or concave cylindrical lens, say one of 2 D., so as to have the test decisive; and it will be seen that the lines at right angles to the direction of the axis of the cylinder before the eye are seen the better. The refraction of the eyeball is not changed by the cylinder in the direction of its axis, and therefore the cross lines
are seen well as the focusing of them through the vertical meridian is not interfered with when the axis of the cylinder is held vertically. The same fact is well proven by photography. Focus a can crossed lines and then place a cylindrical lens with its axis vertical across the lens of the instrument, and then take the picture of the lines. It will be seen that the horizontal lines are distinctly taken while the vertical ones are very indistinct.
diagrams illustrating the focusING IN DIFFERENT VARIETIES of astigmatism.
$V$ focusing through the vertical meridian of the eye, and $H$ that through the horizontal meridian. The rays are represented parallel as they enter the eye. Each one save that of compound myopic astigmatism is a case with the rule, the vertical meridian of the cornea being more bulged than the horizontal one.

Astigmatism may be acquired from the habit of closing one eye while looking through an optical instrument with the other one. The pressure of the tightly closed lids upon the cornea finally leads to altera-
 tion in its nutrition and curves. There is usually remaining a high degree of astigmatism against the rule after cataract extraction operations, caused by the healing of the transverse wound in the cornea, and consequent flattening of its vertical curve. The axis as
well as the amount of astigmatism may undergo a change in the course of time, and especially in those with a rheumatic or gouty diathesis, as pointed out by Dr. Lautenbach. The action of the extra-ocular muscles, too, perhaps has an effect over the nature and amount of the corneal astigmatism (see discussion of errors of refraction caused by the extra-ocular muscles, at end of chapter).

By the elongation of the eyeball due to eye-strain, a hyperopic meridian would of course first become emmetropic and finally myopic, transforming the case of simple hyperopic astigmatism into one of simple myopic astigmatism.

Symptoms of Astigmatism.-Lines that run in the direction of one of the principal meridians of the eye can alone be seen distinctly, in the majority of cases, although in some, by an effort of accommodation, all the lines in the radiating star-figure used for detecting astigmatism may be made to appear equal in distinctness. There exists a very characteristic diminution of visual acuity in astigmatism. Some letters are more blurred than others of the same size, due to the direction of their strokes. Curved letters are furthermore not seen as distinctly as straight-line ones. The indistinctness of print produced by astigmatism is, however, not more than half as great as that produced by hyperopia or myopia of equal amount, when the eyes are not using their accommodation. Some have claimed that the astigmatic eyeball endeavors to correct its anomaly by unequal contraction of the ciliary muscle in different meridians, causing the lens to become more convex in one meridian than in another, or possibly by the tilting of the crystalline lens, giving rise to an astigmatism of incidence, of the opposite nature to the inherent error. It is not demonstrable that either of these phenomena ever occurs. It is more likely that the astigmatic eye covers up its error by a rapid change in the accommodation, so as to bring a sharp focus of the object through each meridian, upon the retina, in rapid succession, so that these impressions may add in a single mental impression.

The theory of sectional accommodation was first advocated by Martin, for the relief and correction of astigmatism. The ciliary
muscle according to this theory is supposed to act in two opposite sections, while the balance of the circular muscle remains quiet or only in slight action. The parts of the muscle in greatest action are supposed to coincide with the corneal meridian of least curvature, the lenticular astigmatism neutralizing the effect of the corneal error. If this is true the ciliary muscle is the only sphincter muscle in the body that is able to contract in sections. It is most likely that sectional accommodation does not occur as the ciliary muscle is not innervated in sections by different nerves, but the same nerve from one nucleus or center presides over the action of the entire muscle. If sectional accommodation does occur it is of no avail in myopic astigmatism; as it could do nothing but convert the case into one of simple myopia, in which case the vision would be rendered poorer. Eye-strain only exists when by it the vision can be rendered better, and when this cannot be accomplished, the strain is not instituted.

This effort on the part of the ciliary muscle leads to all the symptoms of eye-strain. High degrees of myopic astigmatism as myopia causes squinting or partial closure of the eyelids to increase the visual acuity. This as in myopia may give rise to an irritable condition, with secondary disturbances in the cornea. We not infrequently have internal strabismus associated with hyperopic astigmatism, and external strabismus with myopic astigmatism. Oblique astigmatism causes a strain upon the oblique muscles of the eyeball, because the focal lines are inclined and occupy oblique meridians upon the retina. The eyes would then see everything slanting in space unless there was a rotation, so that the usually vertical meridians of the retinæ were brought to coincide with the meridians of stimulation, or a physiological compensation for the obliquity of the retinal images. In cases where glasses have never been worn undoubtedly the oblique meridians of the retina answer for objects vertical in space, as it is rare that such a case sees vertical objects, slanting in space, but, when the correcting cylinders are worn, and the retinal images are rendered more upright thereby, the patients will often complain for a varying length of time that the glasses cause a slanting of objects,
which will, if notice be taken, be found to occur in the direction opposite to the inclination of the axis of the correcting cylinders, in hyperopic astigmatism and in the same direction in myopic astigmatism. (For further consideration of the action of the oblique muscles in oblique astigmatism see chapter upon Muscular Errors.)

It is believed by some that the tension of the extra-ocular muscles upon the eye-globe impart to it its characteristic refraction and is capable of causing a decided change in its refraction. The sclera being more or less yielding and surrounded by the four recti muscles, its shape is said to depend somewhat upon the tension of these muscles upon the eye-globe. It is claimed that undue tension of all four recti muscles, by drawing the eye-globe back against the cushion of fat behind it, shortens its antero-posterior diameter, giving rise to hyperopia, and that anomalous tension in the pair of vertically-acting muscles aided by the weight of or sinking down of the vitreous humor which does not entirely fill the globe, gives rise to change of form of the cornea resulting in astigmatism against the rule, while on the other hand overaction of the vertical-acting muscles causes astigmatism with the rule, and anomalous tension of the oblique muscles, ublique astigmatism. Shulin says that instead of internal squint being caused by hyperopia it is more likely that the hyperopia is caused to increase by the internal squint. The above is entirely hypothetical. If it were true we should find a decrease in the amount of ametropia, after the instillation of atropia which causes a disappearance of the squint, but this is not the case, or again there is observed no alteration in the refraction of the eye after operations upon the eye muscles. The tension of the extra-ocular muscles may play some part in the ocular refraction, especially astigmatism, but so far no alteration of the curves of the cornea has been noted after the action of the muscles has been abrogated by paralysis or operations. The effect of muscular tension upon the production and increase of myopia has already been pointed out.

Treatment of Astigmatism.-Astigmatism is corrected by the use of cylindrical lenses, convex or concave as the case may be. In
compound astigmatism, which is nothing more than hyperopia or myopia with astigmatism added, there is needed a spherical lens to correct the hyperopic or myopic error and a cylindrical lens combined with it for the astigmatism. Such a combination of lenses is spoken of as a sphero-cylindrical combination. The lens that corrects the astigmatic error causes all parallel rays that enter the eye, no matter through what meridian, to be focused upon the retina.

Any astigmatic case may be corrected by one of three combinations of lenses. Suppose for example that we have a compound hyperopic case in which the vertical meridian of the eye is hyperopic to 4 D., and the horizontal meridian hyperopic to i D. (astigmatism against the rule). The amount of astigmatism in this case is $4^{-\cdot} I=3$ D.) 3 D. This amount of astigmatism can be corrected by $a+3 \mathrm{D}$. cylinder with its axis at $180^{\circ}$, that is with the active plane or curved side of the cylinder placed vertically. There still remains uncorrected I D. of hyperopia, which needs the addition of a I D. S. The first method then is the following:

$$
+1 \text { D.S. }+3 \text { D. cylindrical axis } 180^{\circ} .
$$

Secondly, the case may be corrected by a concave cylinder with its acting plane crosswise (axis vertical), lengthening the focal interval in that meridian, thus causing the same amount of error in both the vertical and horizontal meridians of the eye. There would then be left 4 D . of hyperopia to be corrected by the addition of a 4 D . S. lens. Such a combination would be written thus: $+_{4}$ D. S. -3 D. cylindrical axis $90^{\circ}$.

Lastly, the ametropia may be corrected by the employment of crossed cylinders, that is by two cylinders with their axes at right angles to each other. Thus: $+_{4} \mathrm{D}$. cylindrical axis $180^{\circ},+1 \mathrm{D}$. cylindrical axis $90^{\circ}$.

Of these three combinations it will be seen that the first has the least or the weakest curved surfaces. It is most free from spherical aberration and the lenses can be ground much thinner. Such a combination with the cylinder and the sphere of like signs, either
plus or minus, is the combination that should-always be employed when possible. This is always possible save in cases of mixed astigmatism. (Any combination in which the cylinder is of less strength than the sphere and of opposite sign can be reduced to simpler terms or like signs.) Crossed cylinders should always be avoided as they cannot be ground with the same degree of accuracy as their equivalents. Astigmatics should as a rule wear their glasses all the time. If after a while the patient learns to look in the distance with relaxed accommodation, as the emmetrope does, he may do without them for distance, if so doing causes him no headache or eye-pain. Myopic astigmatism as a rule gives rise to no strain for distant seeing, as the myopic astigmatic eye can not aid itself by accommodation, therefore glasses for distance may be dispensed with more readily than in cases of hyperopic astigmatism.

Irregzular Astigmatism.-Irregular astigmatism can not be corrected by the use of lenses. It is caused by corneal disease giving rise to a distortion of the corneal surface, either through ulceration or by an alteration of its curves, as in conical cornea, or the closely allied form of globular cornea. Cataract by altering the refraction of the lens in different meridians, and in different parts of the same meridian, gives rise to irregular astigmatism. Obliquity of the crystalline is another cause, whether congenital or caused by a partial dislocation of the lens.

Obliquity of the crystalline lens causes what is called astigmatism by incidence. Place a spherical lens at some distance from a luminous point and catch the image of this point upon a screen. If the lens is rotated around a vertical axis, the screen ceases to be at the point ; it must be moved nearer the lens and at the same time it is seen that the pencil is astigmatic. The horizontal focal line is at a greater distance from the lens than the vertical focal line.

The refraction has increased in both meridians, but more in that which contains the axis of the lens. The focal lines are not distinct, especially if a small diaphragm is not used. They are rather diffusion spots drawn out in one direction. The pencil has one true
focal line, however, which is horizontal, if the lens is rotated around a vertical axis. We can find it by rotating the screen around a vertical axis, but in a direction the reverse of that of the lens.

A pencil reflected or refracted obliquely by a spherical surface is also astigmatic by incidence ; the same is true of refraction obliquely through plane surfaces. All mirrors have this defect due to the thickness of the glass in front of the coating.


Let $a b c d$ be an incident beam of light parallel to the primary axis of a refracting spherical surface. Suppose that the beam is cylindrical, so that $a b$ is the diameter of a small round portion, representing the aperture of the surface. $A b$ is one of the principal meridians of the surface and the meridian at right angles to $a b$, the other. On account of spherical aberration, the ray $a F^{\prime}$ is refracted more and reaches the axis of the surface sooner than does $b F^{\prime}$. The first focal line which is perpendicular to the plane of the paper, is at $F^{\prime}$. On the other hand the rays must all reach the axis of the surface at the second focal line, $F^{\prime \prime} F^{\prime \prime \prime}$.

Astigmatism of the human eyeball was discovered by Thomas Young in the year 18or. He proved the defect in his own eye by means of his optometer and also by observing the forms of circles of diffusion formed by a luminous point. He furthermore proved that the astigmatism in his own eye was not resident in the cornea, as the amount was not altered by immersing his eye under water, and substituting a spherical lens for the cornea. He thought that his astigmatism was due to an obliquity of the crystalline lens and remarks
that the error could be corrected by placing a spherical lens obliquely in front of the eye.

Airy, an astronomer and professor in Cambridge, was the first to correct astigmatism by the use of cylindrical lenses. Donders was the first to have cylindricals put in the test cases. A luminous point was at first used to detect the principal meridians and then a stenopaic slit used, and the refraction measured separately in the two meridians; later Javal introduced the method of using the star-figure and the cylinders in applying the test and correcting astigmatism.

The thing to do with cases of irregular astigmatism is to study them carefully, and correct the regular part of the ametropia, and thus bring vision up as much as possible. It may be supposed that if the meridians of the greatest and the least refraction of an eye with irregular astigmatism were not at right angles to each other that the case may be corrected by the use of crossed cylindrical lenses with their axes inclined towards each other, but this is not the case, for the meridians of the greatest and the least refraction are always at right angles to each other no matter what the deviation of the axes of two combined cylinders. If they are of equal strength and of same sign, the meridian of the greatest refraction equally divides the angle between the active planes of the two cylinders, and the meridian of the least refraction equally divides the angle between the axes of the cylinders. If the combined cylinders are unequal but of the same sign, the meridian of the greatest refraction is deviated towards the active plane of the stronger cylinder, and the meridian of the least refraction towards the axial plane of the same cylinder. If the cylinders are equal in strength but of contrary signs, the meridian of the greatest positive refraction equally divides the angles between the active plane of the convex and the axial plane of the concave cylinder and the plane of the greatest negative refraction equally divides the angle between the active plane of the concave and the axial plane of the convex cylinder. If the convex cylinder in the combination is the stronger, the meridian of the greatest positive refraction is deviated towards the active plane of the convex cylinder,
while if the concave cylinder is the stronger, the meridian of the greatest negative refractive will be deviated towards the active plane of the concave cylinder of the combination.

There are no short rules for ascertaining the sphero-cylindrical equivalent of crossed cylindricals, with varying inclination of their axes. It is just as well to neutralize the combination with trial lenses, holding the cylinders so that they do not slip from their proper positions in regard to each other. A slight slipping will make it impossible to properly neutralize. Opticians are in the habit of cementing the two crossed cylindrical lenses together, before trying to neutralize, when searching for the equivalent sphero-cylindrical lens.

In some cases where lenses fail to improve the vision at all or to a slight amount only, stenopaic spectacles are of great service. Stenopaic spectacles are those in which the lenses are replaced by oqaque discs, perforated by a slit running in the preferred direction (nearest emmetropic), or more commonly perforated by a pinhole, through which the patient sees. The slit excludes all meridians of the eyeball from the visual act save that which is parallel to its course. A lens may be combined to render the eye emmetropic in the latter meridian. The pinhole acts by excluding from the eye all light that would enter save through a definite small part of the cornea that has little astigmatism on account of its size, and furthermore the small opening limits the size of diffusion circles and thus renders more accurate the focusing of all external objects. A lens, convex or concave as the case may be, placed before the opening may improve the vision by further diminishing the size of diffusion areas. Stenopaic spectacles are objected to upon the ground that they limit the field of vision, but it often happens that a man or woman cannot see well enough to make a living without them. The curtailment of the field is lessened and one is enabled to see to the side without turning the head so much, by employing discs that are perforated in several places, instead of only at the center. The several perforations should, be far enough apart so that no two of them fall within the pupillary
area at the same time. The stenopaic spectacles shown in the cut are those designed by Dr. Heilbron, and are claimed to be of great service in cases of very high myopia, in which the vision can not be materially improved by the use of concave spherical lenses. The perforations are too close together and as a rule, they are of little value. The vision is not materially improved by them.


The star figure used in detecting astigmatism affords information only on the astigmatism that can be corrected by cylindricals, but the form under which a luminous point is seen (the manner of detecting regular astigmatism in the beginning) furnishes fuller information. The opening in the light shade or source of light'should be .2-. 3 mm . in diameter. There is no optical defect that does not have its especial figure caused by distortion of the luminous point. No two eyes see a luminous point exactly alike. The same individual sees it different with the two eyes unless they are exactly equal in refraction. The form of light area upon the retina being so varied in different eyes and at different distances renders it practically impossible to properly interpret the figure, but they can be analyzed to a certain extent. The following are the rules of analysis taken from Tscherning :

1. We can always decide whether a part of the figure is formed by crossed rays or not, by covering a part of the pupil. If it is the homonymous part of the figure which disappears, this part is formed
by rays which have already crossed the axis before reaching the retina, or if it is the heteronymous part of the figure which disappears, the rays have not yet crossed the axis.
2. If the luminous point is beyond the punctum remotum, and if the observer notices a concentric brightness on a part of the diffusion spot, this part corresponds to a less refracting part than in the remainder of the pupil, for the focus of this part is nearer the retina and its rays are therefore less dispersed.
3. If within the focus, the figures are elongated in one direction, and in the same direction beyond the focus, the eye is more refract ing in this direction.
4. The aberroscopic phenomenon always tells us in what direction the refraction increases or diminishes, starting from the center of the pupil, and finally the optometer of Young permits a very exact study of these phenomena. For further consideration of this subject of luminous point and irregular astigmatism the reader is referred to Tscherning's "Physiological Optics."

As yet we know of no way of correcting irregular astigmatism by glasses. The method of Sulzer of applying glasses in contact with the cornea would correct most of the error if resident in the cornea, but the cornea will not tolerate such abuse. Other contact glasses have been made with rims to be supported upon the sclera, but they likewise seem to cause much annoyance.

Anisometropia.-Anisometropia means an unequal refraction in the two eyes. It is rather the rule in eyes. Frequently it is of a different variety as well as of degree in the two eyes, thus, myopia in the one and hyperopia in the other. The importance of this difference in refraction depends entirely upon the degree and not upon the difference in the kind of ametropia in the two eyes. We follow the general rule to correct each eye and to prescribe for it its proper lens unless the anisometropia is too great. If the difference in the refraction of the two eyes is over four or five diopters, it is seldom that the patient can wear with any degree of comfort the correcting lens of each eye.

The correcting lenses, besides altering the position of the foci, affect the size of the retinal images, so that if these images are not of nearly the same size the brain cannot fuse the images into a single ocular perception, but there will exist a diplopia, caused by the superimposition of the projected retinal image of one eye upon that of the other. Bearing this fact in mind the refractionist should always place the spectacle lenses as near to the anterior focal point of the eye as possible so that the size of the retinal images will be influenced as little as possible by the lenses, in high degrees of anisometropia. If the optical center of the correcting lens coincides with the anterior focal point of the eyeball, the retinal image is always the same size no matter what the ametropia may be, for the rays from the extremi-
 ties of the object pass into the eyeball without deviation and are parallel after refraction by the dioptric system of the eye, so that the retinal image is always the same size no matter what the distance of retina (figure).
If the correcting lens is placed in front of the anterior focus of the eyeball the retinal image of the myopic eye is smaller and that of the hyperopic eye larger than that of the emmetropic, which is seen by constructing a figure analogous to the one above. First construct the image formed by the lens and then connect the extremities of the image with the anterior focus of the eyeball. Patients often claim that concave glasses diminish objects; this may be due to the glass being placed anterior to the anterior focus of the eyeball or to the fact that objects seen more distinctly appear smaller, because there are no longer any diffusion circles formed. If the glass is too strong the patient uses his accommodation and then the anterior focus of the eyeball approaches the cornea and the images become smaller for that reason. However a very few individuals prefer to have the full amount of their anisometropia corrected when high because it is impossible to maintain the correcting lenses at the anterior focal point of the eyeball.

Anisometropia in the young is very liable to give rise to squint. In some cases one can correct one eye for distant seeing and the other one for near seeing. Under such conditions a person soon learns to use the eyes alternately. If both eyes cannot be used together after their respective correcting lenses are adjusted, it is the rule to correct the better seeing eye, allowing the worse eye its full correction of astigmatism, with a sphere equal to the spherical correction of the other eye. To decide whether one sees as well with the proper correction before each eye as he does when the whole amount of the ametropia is corrected the stereoscope may be used. If the patient fuses well with the correcting lenses on, he will have little trouble to adapt his eyes to them. Another way is to alternately cover the poor eye, and ask the patient whether he sees distant and near printed matter better when the eye is closed or not. Anisometropia that is caused by the unequal change in refraction of the two eyes as in cases of progressing myopia, is more troublesome than any other variety. The anisometropia caused by illy fitting spectacles or nose-glasses is likewise very annoying.

Schulin finds that as a rule in cases of anisometropia the right eye has less hyperopia or more myopia than the left one. As most people are right-handed, and have a tendency to use the right eye more than the left one, one becomes impressed with the idea that anisometropia has some bearing upon the use of the eyes. The high degree of anisometropia produced by the extraction of the crystalline lens from one eyeball should not be corrected by spectacles, as the eye without its lens receives the larger retinal images and it is rarely that an individual can fuse images differing so much in size. As the anterior focal point is dislocated about I cm. further anterior by removal of crystalline lens it is impossible to wear the correcting lens at the anterior focus of the aphakic eyeball. The distant vision, especially, will be found to be much better with the aphakic eye uncorrected. The patient will prefer monocular clear vision to binocular blurred vision, and perhaps diplopia.

## CHAPTER XIX

## PRESIBYOPIA

Presbyopia. - The power of accommodation decreases with increasing years. This diminution is manifested by the gradual recession of the near point of accommodation. The decrease in the power of accommodation can not be referred to loss of strength of the ciliary muscle, due to senile changes, as the power of accommodation begins to wane in childhood, when other muscles are gaining in strength. The cause of the failure of accommodation lies in the gradual loss of elasticity of the fibers of the crystalline lens, due to a loss of water or process of sclerosis, that begins centrally and extends gradually to the surface of the lens. The lens becomes harder and harder and does not respond to the relaxation of pressure upon it, through the suspensory ligament being slackened by the contraction of the ciliary muscle, by bulging forward and becoming more convex. This loss of accommodation does not become troublesome until the near-point has receded beyond the point of convenient near-seeing.

The man who holds small objects close to the eyes is the one that will notice failure of accommodation first, for example a watchmaker or typesetter. As the loss of accommodation is a gradual process, the time when presbyopia sets in must be arbitrarily established.

The amount of accommodation diminishes in a very regular manner with age, so much so that after the twenty-fifth year it is possible to determine the age of the individual within a few years according to the amount of accommodation he possesses.

Donders assumed that presbyopia begins when the near-point recedes beyond 22 cm . (Jaeger says 25 cm .) when the amplitude of accommodation is equal to 4.50 D . $(100 / 22=4.50 \mathrm{D}$.). This occurs
in the majority of eyes at the fortieth year. After this age reading is laborious and glasses are needed for near work.

If an individual reads fine print without glasses when over fifty years of age he must be myopic if the pupil is of ordinary size.

Presbyopia comes to all eyes, setting in sooner in the hyperope than in the myope, as the former needs more accommodation than the emmetrope for a given distance and is therefore quicker to appreciate any loss of it, while the myope needing less accommodation than the emmetrope is not so soon hampered by the failure of it. People with presbyopia avoid reading fine print and hold their reading matter at a great distance from the eyes. They often find that they can read better if they sit with their faces towards the light. The reason for this is that the light shining into the eyes contracts the pupils, and as the size of the diffusion circles is thus lessened, there is less blurring of the retinal picture. The physiological diminution of the pupil incident to old age in the same way partially corrects presbyopia.

The vision of the presbyope can be made materially better by reading through a pinhole in a card held before the eye, by lessening the size of diffusion circles. The presbyopic patient does not experience pain in the use of the eyes as does the hyperope. He will notice that he can read very well for a short time, but that the print then becomes blurred. If the eyes are closed or rested for a short while the sight again comes up.

This is explained by the fatigue and consequent relaxation of the ciliary muscle in its effort to influence the sclerosed lens. Presbyopia demands the use of convex spherical lenses for near work. They must be of the strength to make the near point of accommodation occupy the place demanded by the particular kind of work the patient wishes to do with his eyes. The manner in which convex spherical lenses correct presbyopia is shown in the figure.

Rays $r$ and $r^{\prime}$ are parallel and are focused without accommodation upon the retina. The lens is omitted for simplicity without in any degree altering the conditions. Suppose that the eyeball did not
alter its refraction, that is, the crystalline lens did not change its shape as the eye looked at point $O$. The rays from the near point $O$ would then not be focused, but form a diffusion area upon the

retina and the point $O$ would be seen indistinctly. The conjugate focal point of $O$ is behind the retina at $O^{\prime}$. The convex lens $L$ causes the light from point $O$ to emerge from it in plane waves, for which the eyeball is adjusted when at rest, and therefore the point $O$ by aid of the lens $L$ is seen distinctly. In the figure the eye is represented as using no accommodation at all when it directs itself to the near-by point, and therefore the lens $L$ has a focal interval equal to the distance of the point $O$ from its optical center. If the eyeball is capable of some accommodation, the strength of $L$ must be decreased by that amount. So first in correcting presbyopia we always find out how much accommodative power the eye has left. Always correct the error of refraction before testing for presbyopia.

Presbyopia and hyperopia should not be confounded because in each the near point of accommodation is further off than in emmetropia.

Ciliary or accommodative asthenia also simulates presbyopia. It occurs however in folks under forty years of age. It is due to inherent weakness of the ciliary muscle, or occasioned by exhausting diseases, or by paralysis of the ciliary muscle. It is quite the rule in the higher grades of myopia to find an inherent weakness of the ciliary muscle. Ciliary asthenia is corrected in the same manner as presbyopia with convex spheres. Exercise of the ciliary muscle with the internal administration of strychnine will develop it if not paretic in origin. This is especially true in myopia. Exercise is carried on
by reading for awhile daily with a pair of weak concave spheres which in cases of myopia should be slightly stronger than those needed for reading. Paralysis of the ciliary muscle will be considered under the head of paralysis of the ocular muscles.

With the recession of the near point there is a series of recurrent weaknesses of the extra-ocular muscles. This is best seen in monocular adduction, and during convergence. The weakness of the muscles is not corrected by the glasses that correct the error of focusing. For this reason it at times happens that weak prisms are needed (bases in) in addition to the convex spheres for the correction of presbyopia.

## CHAPTER XX

## MUSCULAR INEFFICIENCY

Balance of the Extra-ocular Muscles.-Under normal conditions when the eyes are in the primary position, there is no tendency to depart therefrom. When adjusted for an object at a distance of twenty feet or beyond, the eyes should be passively directed. There should be no action or contraction of any extra-ocular muscle, to maintain the eyeballs in their proper direction. All antagonistic muscles should be at rest and equally balanced in regard to their tension upon the eyeballs. This ideal condition of the extra-ocular muscles is spoken of as orthophoria (ỏ $\rho \theta$ ós, straight, and $\phi \epsilon ́ \rho \epsilon \iota \nu$, to bear or sustain). Any tendency on the part of one eye or both to turn away from the object of attention, being held in the right position only by an excessive amount of innervation to the weaker of the opposing muscles, constitutes heterophoria ( $\epsilon \tau \epsilon \rho o s$, different, and $\phi \epsilon \rho \epsilon \iota \nu$, a bearing or tending in some other way than the normal). It is also spoken of as muscle imbalance, muscular inefficiency or insufficiency, or muscular asthenopia. 'The two eyes are kept properly directed by what is known as the guiding sensation, which is nothing more than the effort on the part of the eyes to have single binocular vision. The abhorrence of diplopia is so great that if one eye actually deviates from the object of attention the brain soon learns not to take account of the image formed in the deviating eye, or as we say suppresses the image of the deviating eye (seen in cases of squint). There may be a tendency to deviate on the part of one or both eyes, but if each eye is a good seeing one the guiding sensation will cause innervation to go to the right muscles so that the eyes will be properly directed, inasmuch as seeing with both eyes is preferable when both are good, as then the one helps the other.

Muscular inefficiency may be described as a preponderance of strength of one muscle or a set of muscles over their antagonists within the limits of single binocular vision. If the difference in strength is too great to be habitually overcome by nerve innervation, the stronger muscles gain power over the eyes and squint results. Von Graefe described muscle imbalance as latent squint or strabismus (dynamic squint) ; imbalance of the extra-ocular muscles is rather common. It is at times caused by refraction errors and disappears when the latter are corrected. Such are called pseudo-inefficiencies. The lack of balance may be due to the weakness of one muscle (asthenic heterophoria) or to the excessive strength of its antagonist (sthenic heterophoria). If binocular single vision ceases to exist through the loss of sight of one eye, or if binocular diplopia is produced by some artificial means, the guiding sensation ceases to act, and each eyeball rotates in the direction that its strongest acting muscle pulls it. Under these conditions the strain upon the nervous system to keep the eyes properly directed is useless, as monocular vision is all that can be obtained, so the guiding sensation ceases to operate, and thus reserves nervous energy for other functions.

If there is a tendency for the visual axes to deviate inwards, that is in convergence, we speak of it as esophoria (from $\stackrel{\ddot{\epsilon}}{\epsilon} \sigma \omega$, in, and $\phi \epsilon \in \rho \iota \nu$, to bear or tend), and when the eyes tend to deviate outwards, exophoria (from $\epsilon \xi$, out, and $\phi \epsilon \rho \epsilon \iota \nu$, to bear or tend). If either eye tends to deviate upwards, or its fellow downwards, we call it right or left hyperphoria ( $\dot{v} \pi \grave{\epsilon} \rho$, above, and $\phi \epsilon ́ \rho \epsilon \iota \nu$ ), according to which eyeball, the right or left, has the tendency to elevate. If both optic axes tend to rise above the median plane, we have anaphoria ( $\dot{\alpha} \nu \alpha$, upward, and $\phi \epsilon ́ \rho \epsilon \iota \nu)$, and if there is a tending of both eyes to be depressed below the median plane, cataphoria ( $\kappa \alpha \alpha^{\alpha}$, down, and $\phi \epsilon ́ \rho \epsilon \iota \nu)$. Each eye always turns from the weak muscle towards the strong one; esophoria therefore means a weakness of the external recti, and exophoria of the interni. Cyclophoria (кv́клоs, circle, and $\phi \epsilon \rho \epsilon \iota \nu)$ is the term employed when the eyeballs tend to undergo torsion, or when the oblique muscles are insufficient to keep the vertical
meridians of the eyes parallel with the median plane of the head. It is spoken of as plus when the vertical axes of the eyes have a tendency from the median plane of the head, and as minus when they have a tendency towards the median plane of the head in conformity with the terms plus and minus torsion, used by Maddox.

The lack of muscular equilibrium may be evident when the eyes are adjusted for both near and distant seeing, or only when adjusted for the one or the other. There are several possibilities in regard to the nature of heterophoria. There is no one theory that will hold for all cases. One view is that there is a congenital feebleness of one muscle as compared with its opposing muscle; the weakness being either due to the fewness of its fibers or to the manner of its insertion to the eyeball, the weak muscle being abnormally far from the corneo-scleral junction, or again to the lack of sufficient innervation. Some deny that heterophoria is ever congenital ; that the growth of the muscle has been normal, but that some irritation in or about the eye, or in some distant organ of the body, excites a spasm, tonic in its nature, in one of a pair of muscles, thus destroying their harmonious action. A few cases may be due to eccentricity of the maculæ lutea, that is, that the maculæ do not occupy corresponding places in the two retinæ.

A congenital displacement of one macula in or out will give rise to eso- or exophoria. This latter theory includes those cases produced by a faulty position of the eyes in the orbits. If one orbit is higher or lower than the other one, a hyperphoria will be the result. The last theory is that of Dr. Savage. In many cases there is esophoria for distance and exophoria for near seeing at variance with all the theories. There is a condition called reversed heterophoria. This is where the eyes have a tendency to turn in the direction of the inherently weaker muscles. The weak muscles, in their attempt to do their work of maintaining the eyes in the proper positions, have become thrown into a spasm. Thus there may be a true exophoria (an actual preponderance of strength of the external recti), yet so great is the effort on the part of the nerve centers to overcome its
manifestation that an excess of nerve impulse is sent to the interni, throwing them into a spasmodic contraction which not only conceals the exophoria, but actually carries the eyes so far inwards that the tests reveal an esophoria.

## CHAPTER XXI

## DETECTION AND CORRECTION OF ERRORS OF REFRACTION

It is essential to keep an accurate record of each ophthalmic patient, so that any change from time to time may be accurately noted, and in case the patient loses his glasses, they can then be supplied without a reëxamination. Records are best kept, according to the author's opinion, in a book. After taking the history as to how the patient complains and so on, and after examining the eyes by oblique illumination and with the ophthalmoscope for any diseased conditions (which should always be done), one is prepared to examine the refraction. The fundus of the eye should in all cases be carefully examined, even if vision is found to be perfect, as it not infrequently happens that the vision is not disturbed when there is a rather advanced lesion of the fundus. It is well, therefore, to get into the habit of examining the fundi of the eyes before proceeding to refract them. In all cases where a diseased condition is found, the refraction error should be corrected if there is any reason to suppose whatever from the nature of the trouble or from the presence of recurring attacks or what not, that the condition depends upon eyestrain.
Quite a few folks apply to have their eyes tested for the relief of poor vision, when there is an intra-ocular lesion that is interfering with the seeing. Therefore, opticians should not be allowed by law to test eyes for glasses, as any diseased condition of the eyes would naturally be overlooked by them. Some of the more conscientious of them will refer a patient to an oculist if he finds that he cannot bring the vision up to normal with his glasses, but they are few. It is necessary in the majority of cases to have the eyes under the effect of a cycloplegic to intelligently test and properly correct the error of refraction. It frequently happens that accommodation ren-
ders an error latent, or covers it up, and the trouble is not revealed without the use of a cycloplegic. Hyperopia is made to appear less or even emmetropia, or again converted into myopia under the action of the ciliary muscle. The eyes should in most cases be tested while under the effect of a cycloplegic, and then retested after the effect of the drug has worn off, and the proper allowance and change in the strength of glasses made for the return of accommodation. This is especially necessary for the beginner and in the more complicated cases of astigmatism, as mixed and compound oblique.

Again -with mydriases the patient may use some other part of the cornea than the visual zone, so that when the pupil returns to its normal size the glasses are unsatisfactory.

Subjective methods depending upon the good will and intelligence of the patient are of course inferior to objective methods of examination. The latter, however, should always be confirmed by the former whenever possible, as there-is a physiological side to vision, and not entirely of an optical nature.
For example there is a certain small number of cases of objective astigmatism that are not comfortable when the total amount of the astigmatic error is corrected, there being present an unusual amount of physiological astigmatism, which when corrected disturbs the physiological balance or the new conditions imposed upon the eyes by fully correcting the ametropia annoy. How to tell these cases beforehand I am not certain. It is said that physiological astigmatism does not disturb vision to the same extent as pathologic. Physiological astigmatism is certainly overestimated by the foreign ophthalmologists. It seems to me that it should always be placed as low as or less than .50 D. In a series of cases of astigmatism tabulated several years ago by Dr. Julian J. Chisolm, embracing about two thousand cases in all, it was found that a .25 D. and a .50 D. of error seemed to give rise to the most annoyance, other than faulty vision. The difference in the strength of the cylinder accepted under the fogging test (see Fogging Test), and that required to correct the astigmatism after the convex sphere has been removed in many cases is
due probably to physiological astigmatism. In the majority of cases glasses do not prove satisfactory at once, but there is a varying period of adaptation, lasting from a few days to as much as several weeks. The patient should be informed of this fact so that he does not suppose that the glasses are at fault, if he does not see as well at first with them on as he does with them off; which will happen in cases of hyperopia, or if his eye-strain does not immediately disappear. The quickest way to become accustomed to glasses is to wear them constantly from the first. The objection often raised to the use of cycloplegics that it takes time, for while the effect of the drug lasts the patient is compelled to do without the use of his eyes, is overcome in a great measure by the use of the following solution :

| Homatropin hydrobromate, <br> Cocain muriate, <br> Water,āā gr. i. <br> dr. ii. |
| :--- |

Two or three drops are instilled into the eyes every ten minutes for a period of two hours previous to examination. The effect of this solution mostly wears off in twelve hours, and always in twentyfour hours. At times however this solution is not strong enough to completely relax the ciliary muscle, and a solution of atropine gr. 4 to oz. i, has to be used for several days. Below is found a table showing the various strengths of the several mydriatics and cycloplegics in use, and the duration of their action upon the ciliary muscle and so forth.

TABLE OF MYDRIATICS.

|  | Relative Power <br> Name of Drug, and the Salt <br> Commonly Used. | Strength of <br> Solution <br> in <br> Same <br> Solution. | Time when <br> Solution <br> Sroduces | Period of <br> Maximum <br> Effect. | Effect <br> Effect. | Complete <br> Decline. |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Recovery. |  |  |  |  |  |  |

Euphthalmin is a new mydriatic claimed by many to be devoid of any effect upon the ciliary muscle. The salt used is the muriate, in ${ }_{2}-5$ per cent. solution. A 5 per cent. solution will dilate the pupil fully in from 20 to 30 minutes. Hinshelwood says a 5 per cent. solution produces a paresis of accommodation that lasts about two hours. All mydriatics affect to some extent the ciliary muscle, cocain and euphthalmin to the least extent. When a drug has put the accommodation in abeyance, the pupil is dilated fully, unless held by adhesions to the crystalline lens, and the patient is unable to read or to see small objects close by. With the rod optometer he will be unable to see the finest print save at the ten-inch point. Often when the eye is fully under the effect of the medicament, there is noticeable a circumcorneal injection due to the blood being pushed out of the iris into the circumcorneal loops of blood-vessels. This is particularly the case after the use of homatropin.

## SUBJECTIVE TESTING.

The Trial Case.--The case of test lenses represented below contains 35 pairs of convex and concave spherical lenses ranging from .125 D. to 20 D., and 23 pairs of convex and concave cylindrical lenses ranging from .125 D. to 6 D., thirteen prisms, from. 5 to 20 degrees, four plain colored glasses; one white glass; one half-ground glass; two metal discs, with stenopaic slit ; one stenopaic disc with hole; one solid metal disc, and two pairs of trial frames, one with revolving cells.

The usual arrangement of the contents of the trial case is as shown in the cut. The convex lenses are to be found in the right half of the case and the concave in the left half. Upon the partition separating the lenses of each pair is marked the strength of the glass in the adjoining space. The numerations are usually in both the inch and dioptric system ; the former to the right, and the equivalent number of diopters in the left-hand column.

The General Plan of Procedure. - It is well to have some definite order in testing eyes for glasses, and then nothing will be overlooked.

The following routine is that used by the author: (1) Estimation of vision ; (2) employment of oblique iilumination ; (3) ophthalmoscopic examination ; (4) optometry ; (5) ophthalmometry ; (6) muscle balance tests ; (7) retinoscopy without cycloplegic, as an aid for subjective test with trial case ; (8) confirmation subjectively with trial case. If vision is not brought to normal, or if there is any disagreement between the objective and the subjective tests, a cycloplegic is pre-

scribed and on a subsequent day, the shadow test employed and confirmed by the trial case. For the methods of applying these tests, consult the articles upon them, respectively.

The vision of both eyes together and then of each eye separately is to be tested. There are many that will not see better than 20/30 with either eye, but will have $20 / 20$ or better with both eyes open. Placing the opaque disc before one eye seems to interfere with the
vision of the uncovered eye. If this fact had not been taken into account, time would be wasted in endeavoring to render the vision in the eye under test normal. The near point of distinct vision should then be ascertained (with or without the rod optometer).

If the distant vision is found imperfect place a stenopaic disc with a hole before the eye, and if the imperfection in vision is due to an error of refraction, the vision will be improved. If the poor vision is due to a diseased condition or congenital defect the pinhole disc will not improve it and therefore glasses would be useless as far as the immediate improvement of visual acuity is concerned.

Amblyopia is the term used to denote poor vision without any discernible cause with the ophthalmoscope. It is the rule in the higher errors of refraction to find one eye, usually the one with the higher error, amblyopic. This is at times due to faulty development, but also ensues from non-use (amblyopia exanopsia). Congenital amblyopias have vision of $20 / 200$ or less as a rule, while the acquired cases have $20 / 100$ or better with the refraction correction. Acquired amblyopia is usually associated with squint and is occasioned by the suppression of the retinal image in the squinting eye to avoid diplopia. In cases where the eyes are straight, the image of one eye may be suppressed so that the effort for binocular vision is not made - there being present an anomaly of extra-ocular muscles which renders this difficult. All cases of amblyopias should be corrected accurately by some objective method, and the proper glass worn before the eye, as most of them improve in seeing after awhile especially if the vision is 20/100 or better, or if any improvement is at first noted with the glasses. Some have suggested that atropine be instilled into the good eye at intervals and the patient made to depend upon the faulty eye. This may do in children but few adults are willing or able to be so afflicted.
In astigmatism of high degrees there is often present a peculiar sort of amblyopia or inability on the part of the eye to see lines drawn in certain directions. It will be referred to under the head of Astigmatism.

In regular ametropias, the pinhole disc usually brings the vision up to about the same point as correcting lenses, if the test-card is well illuminated. In irregular astigmatism, the vision is often made much better by aid of the hole than can be hoped for by the aid of lenses. In astigmatism the visual acuity through the hole is not quite
 as good as in cases of hyperopia or myopia, as notwithstanding the smail area of cornea exposed by the hole, there is still enough inequality in curvature to distort the retinal images to a degree. The higher the ametropia the larger the diffusion circles upon the retina and the poorer the vision. The stenopaic opening lessens the size of the diffusion circles, and in cases of astigmatism renders them more circular, and thus raises the visual acuity.
$a, b, c, d$ is a beam of light, that on entering the eyeball $A$ would be brought to a focus at the point $F$, forming a diffusion circle the size of $x$ in diameter. By placing the pinhole disc $D$ in front of the eye, all rays save those passing through the central opening $O$ are excluded from the eye-
 ball. All entering rays must then fall within the limits of $c, b$, and coming to a focus at the point $F$, a diffusion circle is formed upon the retina the size of $y$. A small opening also magnifies to some extent an object looked at through it.

In both figures the crystalline lens is omitted for simplicity. Let $A$ be the object, and $a$ its retinal image, formed by lines drawn through the nodal point $N$ from extremities of object. $D$ is a screen and $O$ a central opening. With the screen interposed the light that
enters the eyeball diverging from object has its immediate origin at $O$. The size of image through $O$ is $1-2$, larger than $a$. Rays $r r^{\prime}$ and $s s^{\prime}$, being divergent, are not brought to a focus, until the points $V^{\prime} V$ are reached.

As has been said it is better to refract at a distance of twenty feet or more, as then the accommodation does not so readily enter in to complicate matters. Just as good work can be done at less distance if allowance is made for the shorter distance in prescribing. Many have not twenty feet at their disposal, and must refract at less. There are test-cards constructed to be used at a distance of twelve feet or four meters, a very convenient distance.

The glass that is correct for seeing at four meters is of course .25 D. too strong if it is a convex glass and . 25 D . too weak if concave,

for greater distances than four meters. Divide the distance at which the refraction work is done into one meter or 100 cm ., and ascertain the amount of accommodation employed at that distance. Thus, at 4 m . distance .25 D . accommodation is used ( $1 / 4=.25 \mathrm{D}$.). To overcome the necessity of having to add or subtract from every correction, when refracting at less distance than twenty feet (at twenty feet the amount of accommodation used is so little that it is ignored), Dr. Förster has devised some reversed test-types, subtending the
correct angle, which when hung above the head of the patient are seen properly in a mirror hung in front. The distance of the patient
 from the mirror plus that from the testtype to the mirror is made equal to 20 feet, to give the correct test for distant vision as the patient looks into the mirror. These cards are shown in previous cut. The type is also made of translucent material so that a light placed behind them will illuminate them if daylight is not available.

Test-type should be hung at the height of the patient's eyes. The most convenient method is to have them so arranged that one or the other can be shown without the necessity of leaving one's position to hang another card up, and as the patient soon learns the cards by heart, it is well to have those out of use out of sight of the patient, and only exposed as needed. This is accomplished by having the cards enclosed within a wall cabinet and lowered before the patient's eyes as needed by means of strings. The strings are passed through screw eyes, and attached to a panel beside the examiner. To avoid the necessity of looking at the cards to see if the patient is reading properly, one has a number of tally cards, placed upon the panel, under the strings that operate the respective testtype. The accompanying cut shows the ophthalmic cabinet.

The objection to the ophthalmic cabinet is that all the cards must be of the same size, which is not always possible with different style cards. One may arrange the cards in the same manner without the
cabinet, and use any size card. If hung close enough together the cards will slide smoothly upon one another. There are a number of good test-card brackets and stands to be had ; several of the best are shown in the cuts below.

Many cards may be hung one over the other. The cards are always in place. There is no taking down or hanging up. The cards do not become soiled, bent or broken. When it is desired to

change cards, all that is necessary is to turn the upper card on the rod toward the left. The other cut shows a portable stand consisting of an upright and heavy iron base carrying an adjustable gas bracket with Argand burner and mirror reflector in combination, with an additional arm to which is fastened a reversible frame containing two cards of test-letters.

If an individual has normal or nearly normal vision, myopia may be excluded, for the myopic eye can not help itself, by accommodating (increased convexity of the crystalline lens), the greater refraction only exaggerating the error.

EVIDENCES GAINED FROM THE VISUAL ACUITY.

| Distant Vision. | Emmetropia <br> $20 / 20$ or better. | Hyperopia may be <br> $20 / 20$ by accom- <br> modation but <br> often less. | Myopia, can not be <br> $20 / 20$. | Astigmatism, if not <br> over I D., may <br> be 20/20 by ac- <br> commodation. |
| :--- | :--- | :--- | :--- | :--- |
| Near Vision. | Snellen, 1, 5 in. <br> to 20 in. <br> largen, No. I or but not <br> close up. | Snellen, I or 2, but <br> not as far off as <br> distance noted. | Larger print only. |  |

To exclude the effect of astigmatism over visual acuity lines running in the direction of most distinct vision may be used. We can exclude all influence of refractive errors by placing a stenopaic opening before the eye under test. With the pinhole the vision is not quite as good as with the refraction correction, as it excludes a great deal of light. When using it the card should be very well illumined. In testing the vision of diseased eyes which have at the same time errors of refraction this method is of utility as we wish to know how much poor vision is due to the disease and how much to the error of refraction.

Following the routine method no matter whether the vision is below par or not, weak convex spherical lenses are placed in the trial frame before the patient's eyes. If hyperopia exists the patient will see just as well with the lens on as with it off, or as we say vision is maintained with the convex lens. This would not be possible unless the eye relaxed some accommodation and allowed the glass lens to take its place, and the fact that the eye was using accommodation for distant objects proves the presence of hyperopia.

The test is, not does the patient see better, but does he maintain the same vision with the glass. If the error is high he may even see better with the lens on, as will always happen if his unaided vision is below 20/20. The case in hand is not under the effect of a cycloplegic. Under a cycloplegic the convex lens either makes vision better or worse. If the visual acuity is the same in each eye and keratometry shows little or no astigmatism one may begin by placing the same strength lens before each eye, and then the strength increased until the strongest that allows the patient to see as well
or renders the vision best is ascertained. The idea is to get the patient to relax all the accommodation possible, and thus to reveal the greatest amount of error. At times this is accomplished better by placing before the eyes at first lenses that are too strong to see clearly with and then gradually diminishing them, until the strongest pair is ascertained that allows of the best vision. A shade should now be placed before one eye and the lens before the other altered and made stronger or weaker, if need be to make the vision the best possible ; the strongest lens
 within these limits of course being selected, and so with the other eye. The lenses thus as-


2 certained are to be prescribed, if the patient shows no evidence of astigmatism. The latter will most likely exist if in default of any intra-ocular lesion the vision is still below par with the proper spherical lenses. The tests for astigmatism should always be applied however as there may be as much as 1 D. of astigmatism present and the vision normal. As will be recalled,
the astigmatic eye can not focus equally well in all directions or meridians at once; it happens therefore that such an eye can not see lines drawn in different directions with equal distinctness. The line that runs at right angles to the meridian that has the sharpest focusing will be seen the best. The patient with the convex spherical lenses before his eyes, that correct the hyperopia, is then made to look at a chart made after the manner of either one shown in the cuts.


3 Green's Improved.

4. Dr. Jno. Green's.

The chart most commonly used is Dr. Green's, style no. 4. Number 1 or 2 is however the most convenient, as the meridians are marked every fifteen degrees apart. The meridians upon the card are numbered around from the left to the right as the hands of the watch move, from o degrees at nine o'clock, to 180 degrees at three o'clock. Both ends of the horizontal line are frequently marked 180 , as in charts 1 and 2. Besides the astigmatic dials there are several other tests for astigmatism, one of the best is Dr. Pray's astigmatic letters. As will be seen by referring to the figure, these letters are made of stripes running in a different direction in each letter. In using card no. 3 the patient should notice whether the white sectors appear equally distinct in every direction, and if he can trace them equally well towards the center in all directions. In
using Dr. Hill's card, the striped line is rotated so as to occupy different meridians, the patient watching the line as it rotates, and noticing whether the cross lines appear more distinct in one direction of the line than in another. If on card no. 4 the cross lines are seen the best, the line on Hill's card will be seen best when vertical, as then in each case the stripes are crosswise. In using Pray's card the individual notices if all the letters are black and distinct alike.


Pray's Letters.


Hill's Card.

Upon the same principle there is made a card with a number of circles striped in different directions in each. The cut below shows a recent astigmatic card. The fact that all the lines or letters look alike is no evidence that there is no astigmatism present in an eye not under the action of a cycloplegic. Even if all the lines do appear alike, further attempt should be made to render manifest any latent astigmatism. This is accomplished by Javal's or Bull's fogging method. A convex spherical lens strong enough to perceptibly blur the vision is placed in the trial frame, in front of the correcting lenses. The tendency on the part of the eye is to relax any accommodation that may be in use, in attempt to render the vision better. (Blurred retinal pictures are always a stimulus for the action or relaxation of the ciliary muscle, adjustment taking place until the
best vision is obtained.) The fogging lens (spherical lens) should not be strong enough to blur out of sight the lines upon the astigmatic dial. If with this lens on, the lines all look equally blurred, the lens should be made weaker or stronger, and the lines again looked at. If they all appear alike still, there is most likely no astigmatism present.

The strength of the fogging lens should be changed for it is possible that there may be astigmatism present, but which is rendered mixed astigmatism by the convex lens, being of a strength that brings the focus of the least hyperopic meridian now upon the retina


Verhóeff's Astigmatic Dials.
by aid of the lens that corrects the hyperopia anterior to the retina, while it brings the focus of the most hyperopic meridian behind the retina forward to the same extent. The amount of hyperopia and myopia being equal in the mixed astigmatism thus produced the size of the diffusion circles for all meridians of the eyeball will be the same, and therefore all the lines upon the astigmatic cards appear alike. The fogging lens must be greater than the amount of astigmatism, therefore. If all the lines do not look alike either with or without Bull's method, there remains an error of astigmatism to be corrected. It is well to turn the chart around especially when ex-
amining children to see if the same meridian is always selected irrespective of the position of the chart, or whether the patient simply follows the same line as it is moved through different meridians. If the latter is done, the answers given can not be relied upon and it is better to at once instil a cycloplegic and examine the case objectively. Any existing astigmatism after the hyperopia has been corrected by the strongest convex spherical lens, with which. distant vision is maintained, or when overcorrected as
 is done in the fogging method, is converted into myopic astigmatism.

The figure represents a compound hyperopic astigmatic eye, in which the horizontal meridian of the cornea $H$ is the most hyperopic. $A$ is the principal focal point of the vertical meridian, and $b$ that
 of the horizontal meridian. The convex spherical lens $L$ brings both focal points forward, causing the most hyperopic to be brought upon the retina and the least hyperopic, anterior to it, or perhaps both focal points are brought anterior to the retina. This myopic astigmatism is corrected by a concave cylindrical lens with its axis parallel to the meridian of the eyeball that is not to be influenced, or what is the same thing at right angles to the line upon the dial that is seen the best.

This will be dealt with more fully under the head of correction of astigmatism. Of course the sphero-cylindrical combination must be reduced to one with like.signs for sphere and cylinder, a plus sphere combined with a plus cylinder, for the reasons noted upon page 25 I . It is impossible to ascertain the full amount of latent hyperopia and astigmatism without the use of a cycloplegic. The latent hyperopia may not give rise to trouble, but latent astigmatism seldom fails to
annoy, so it is much better if there is any reason to suppose from the contradictory evidence gained through subjective examination that there is latent astigmatism to instil a cycloplegic and examine the case objectively.

It may happen that during the subjective test the patient will accept the weakest convex sphere, that is the one that corrects the least ametropic meridian of the eyeball. Under these conditions a concave cylinder exaggerates the astigmatism, which is made evident by the fact that all the lines upon the astigmatic chart look more uniform without the lens
 than they do with it on. A convex cylinder is needed then to correct the astigmatism (see figures).

Each of the cases represented show astigmatism with the rule, that is the most hyperopic meridian is crosswise. In figure I the lens $L$ corrects the meridian of the least hyperopic, causing the principal focus of the vertical meridian to lie upon the retina. The horizontal lines upon the astigmatic card are then seen the best as they are focused through the vertical meridian of the eye. In figure II the lens $L$ corrects the most hyperopic meridian, the other principal meridian, bringing the focus of the cross meridian upon the retina, but at the same time carrying the focus of the vertical meridian in front of the retina, giving rise to a myopic astigmatism. The vertical lines are now seen the best inasmuch as their focus lies upon the retina. In case no. I, a convex cylinder, axis vertical, corrects the astigmatism, while in case no. 2 a concave cylinder with its axis at 180 , or horizontal, is needed. As a rule then use a corvex cylinder to clear up vertical lines and a concave
one to clear up horizontal lines. This rule applies to astigmatism with the rule, the reverse being true in astigmatism against the rule, that is, a concave cylinder for clearing cross-lines and a convex cylinder for clearing up vertical lines.

The cylinders are placed before the eye with their axes at right angles to the lines upon the astigmatic dial that are seen the most distinctly. (The same thing as placing the axis of the cylinder across or parallel to the nearest emmetropic meridian of the eyeball.) When the accommodation is in abeyance through the use of a mydriatic, the aim is to ascertain the lens that gives the best vision. From this lens or combination must be deducted . 25 D. for the return of tone of the ciliary muscle, and in addition the amount of accommodation that would be needed for the distance at which the refraction work is done. The tone of the ciliary muscle is really a partial contraction, which is relaxed by use of the cycloplegic and regained after the latter wears off. If there is a weakness of the internal recti muscles, a further deduction must be made after the manner described. The greater the inefficiency of the interni the more is to be deducted and vice versa.

The younger the individual the greater the amount of latent error, so more hyperopia has to be deducted from the total correction in the young than in those of more advanced years. The only exact way to determine how much to deduct to allow for the return of accommodation is to reëxamine the patient after the effect of the cycloplegic has worn off. A certain amount of experience in refraction work enables one to tell how much to deduct in the majority of cases.

If the patient refuses convex spherical lenses, that is if the vision is rendered worse by them, hyperopia is excluded. Concave spherical lenses are then placed before the eyes until the weakest lens is found that allows of the best distant vision. Both eyes may be tested together, and after the lenses are found that give the best vision the lens before each eye modified to suit it, being made a little stronger or weaker as the case may be, or one eye at a time
may be tested. In myopia we select the weakest lens with which the patient can see the best, as the patient can see just as well with one a little too strong by using his accommodation. Under these conditions the myopia is said to be overcorrected and the patient is left hyperopic, with all the eye-strain that hyperopia entails. Whenever myopes are allowed to select glasses for themselves they most always select them too strong.

Figure I is a myopic eyeball, $A$ its posterior principal focus. By aid of concave lens $L$, the parallel rays $a a^{\prime}$ are rendered divergent and
 therefore come to a focus behind the retina, at the point $A^{\prime}$, as the lens is too strong, but by the aid of accommodation as is shown in figure II, the focus $A^{\prime}$ can be brought forward to the point $R$ upon the retina, and the eye secure sharp distant vision. For this reason, that accommodation can overcome a concave lens that is too strong, concave lenses are never placed before an eye in the beginning of the test as the eye will of course see as well with the glass, and by the fact that the letters are made smaller by the concave lenses, and therefore blacker, the patient may think that the lens has increased his visual acuity. After the proper spherical lens has been decided upon, one proceeds to test for astigmatism. In myopia the fogging method is applied by placing before the eye a concave spherical lens that is too weak and which therefore leaves the vision much blurred. Under these conditions, if the lens before the eye corrects the least myopic meridian of the eyeball, a concave cylinder will be needed in addition, to correct the astigmatism, applied at right angles to the line seen the best upon the astigmatic chart, while if the concave sphere before the eye corrects the most myopic meridian, at the same
time rendering the least myopic, hyperopic, a convex cylindrical lens will be required to make all the lines appear uniform. The resulting sphero-cylindrical combination must then be reduced to one of like signs. In hyperopia and myopia when the spherical lens before the eye is too strong a cylinder of opposite sign will be needed to render the astigmatic dial uniform. In the higher degrees of hyperopia and myopia the vision is frequently below par when the ametropia is properly corrected. After the lenses that give the best distant vision in myopia have been selected, the near vision should be tested with them on, and the strength of them decreased, if need be to give the best near vision at the proper reading distance.

If the myopia is of high degree the patient cannot read with his distant glasses, even after a deduction of three diopters to allow for accommodation being nil, on account of concave lenses diminishing the size of the retinal images. In such a case the concave lenses must be reduced in strength and the patient allowed to bring the print closer to the eyes, until the best vision is obtained at the greatest distance. If both eyes are now being used, denoted by convergence for this point, or by the stereoscope, prisms should be added with bases towards the nose, so that the eyes while regarding a close point will need no more convergence than if they were being used for a distance of 33 cm . Thus, suppose the greatest distance at which the patient can read is 20 cm . For that distance five meterangles of convergence are needed (see measurement of convergence) while only three meter-angles are needed for a distance of 33 cm . The patient must therefore have supplied a prism of two meterangles before each eye.

It may be found that the myope cannot read comfortably at 33 cm . on account of the convergence being weak with the accommodation. In such a case prisms with bases in are to be used in addition for reading. If the myope has lost single binocular fixation for near objects however it is not wise to try to restore it for reasons already stated.

If a cycloplegic is used there is not the same danger of overcor-
recting myopia as there is without it. The fogging method is only applied to detect astigmatism when the eyes are not under the effect of an agent that paralyzes the accommodation muscle, and is only intended to render the.ciliary muscle quiet during the test, and at the same time to reveal as much latent error as possible.

Astigmatism. - The astigmatic individual frequently shows a tendency to confuse curved letters more than others. In astigmatism one eye should always be tested at a time. If the visual acuity is fair, and positive spherical lenses are refused, making the vision worse, simple hyperopic astigmatism is suspected. The patient is then asked to look at one of the astigmatic test-cards, with the lines drawn in different directions, and to note whether all the lines appear uniform. If the accommodation is active they may, even if there is a considerable degree of astigmatism present, by the eye accommodating first for one line and then the other in quick succession, giving the retina a succession of distinct images. To obviate this as much as is possible the patient should be told to view the central white space upon the card, and then all the lines will be seen at the same time and an impression as to their uniformity or not gotten. After the patient has decided that the lines appear the plainer in a given direction, a card with lines drawn in that direction and at right angles thereto is placed before him. If he views these crossed lines with his accommodation in abeyance, the lines that run at right angles to the emmetropic meridian of the eye will be seen the plainer, but if accommodation is used, those that are parallel to the emmetropic meridian will appear the plainer. This fact must ever be borne in mind, otherwise a case of hyperopic astigmatism will often be corrected for one of myopic astigmatism (see figures).

Both of these figures represent simple hyperopic astigmatism with the rule. No. I is represented as viewing crossed lines, with relaxed accommodation. Line 1 is focused through meridian I of the eyeball, and line 2, through meridian 2. The horizontal lines are seen the better as their focus is upon the retina. No. II represents the eye as looking at crossed lines, using accom-
modation. The increased convexity of the crystalline lens brings both foci forward. The vertical lines are now seen the better as their focus is upon the retina. If the astigmatism is against the rule, that is the flatter curve of the cornea being vertical, the vertical lines will be seen the better when the accommodation is in abeyance, and vice versa. Simple hyperopic astigmatism is corrected by a plus cylinder with its axis parallel to the emmetropic meridian of the eye, that is perpendicular to the best line seen, if the accommodation is relaxed and parallel to it if accommodation is being used. As most cases of astigmatism are those with the rule, it may be said that usually a positive cylinder, with its axis vertical or at 90 degrees is needed to correct hyperopic astigmatism.

A very common error of some opticians is to correct the pseudomyopic astigmatism produced by accommodation as shown in opposite figure as one of true myopic astigmatism, with a concave cylindrical lens, placed at right angles to the best seen line upon the chart. The myopic cylinder lengthens the myopic focus (2), and causes it to lie upon the retina with focus no. I. The astigmatism is corrected and now all the lines upon the astigmatic dial look uniform, but the eye is left hyperopic, using its accommodation to focus distant objects as it was the action of the ciliary muscle that transformed the case into one of myopic astigmatism. Always follow this rule : If the eye is not under the effect of a cycloplegic, no matter what line upon the card appears the best, try first with a plus cylinder to cor-
rect the astigmatism, placing it before the eye with its axis parallel to the best line seen. If the case is one of hyperopic astigmatism in which accommodation is being used, it will be relaxed and the curve of the cylinder crosswise before the eye will shorten the crossfocusing of the eyeball and bring the focus of the vertical lines upon the retina, and all the lines will appear uniform, of the same shade, and at the same time the vision will be rendered better.

If, on the other hand, the case is one of simple hyperopic astigmatism with relaxed accommodation a plus cylinder placed before the eye with its axis parallel to the best line seen, will render the horizontal focusing of the eye myopic, bringing it in front of the retina. Vertical and horizontal lines will be made to appear alike, by the blurring of the horizontal or previously better line, but the visual acuity will not be improved. Then the cylinder is to be turned until the axis is at right angles to the best line upon the card. A plus spherical lens may be accepted by the astigmatic eye using accommodation, simply allowing the glass to do what the accommodation did, that is, to transform the case into one of pseudo-myopic astigmatism. A concave cylinder at right angles to the best line is then needed to make the lines appear all alike, and to render the vision normal. In cases of simple hyperopic astigmatism the strength of cylinder needed will be found to be equal to the strength of the sphere already on, and the equivalent lens will be a plus cylinder of the same strength as the concave, but with its axis at right angles to the latter, and which should be substituted for the combination of plus sphere and concave cylinder.

The same method is followed in simple hyperopic astigmatism-if the eye was not accommodating for the lines upon the card, but a convex sphere was accepted. When the eye is not under a cycloplegic, it is whether the eye sees as well with a convex sphere as with none and not whether it sees better. In ascertaining the kind of cylinder and inclination of axis needed, $\mathrm{a}+.50 \mathrm{D}$. is a convenient strength to use. If all the lines should look uniform to the patient under examination a convex sphere is placed before the eyes to en-
courage the relaxation of accommodation, fogging the lines upon the astigmatic card. Any existing astigmatism is then converted into myopic astigmatism and corrected after the manner of the latter. If a convex spherical lens is first accepted, before astigmatism is corrected, diminish its strength or remove it, if the vision is not $20 / 20$ or better with it, as has been noted the sphere like the accommodation has caused pseudo-myopic astigmatism, which in turn was transformed into myopia by the application of the convex cylinder applied according to the rule when the eye is not at rest, that is parallel to the best seen line. In the higher degrees of simple hyperopic astigmatism the axis of the correcting cylinder is always placed before the eyes at right angles to the best seen line whether the eyes are under a cycloplegic or not, as the accommodation does not endeavor as in the low degrees of error to render it latent. If the astigmatism is over several diopters the ciliary muscle never converts the hyperopic case into a myopic one.

Now and again the patient will not accept the cylindrical lens that corrected the astigmatism when the eyes were under the effect of a cycloplegic, when the latter wears away. The brain seems to have become accustomed to interpret distorted.images from early childhood, and does not immediately appreciate the benefit of perfectly formed retinal pictures. For the same reason in some a cylinder fails to improve the vision to any marked extent even when the error is high. The rule is thought to correct all the astigmatism. Such cases as those just mentioned will finally accept the proper strength cylinder, if the glass is worn while the effects of the cycloplegic is wearing off, or if the eye be led up to the proper strength, by beginning with weaker cylinders of the same kind and gradually increasing their strength at intervals of several days or weeks as the case may be, until the proper one is worn.

There are infrequent cases that appear to be more comfortable without the astigmatic correction. The majority of these however are cases in which the proper correction has not been obtained and as a rule cases that have only been examined subjectively without
the use of any agent to relax the ciliary muscle. There are though a few cases of compound hyperopic astigmatism, cases that are more comfortable without the correcting cylinders, wearing simply spheres to correct their hyperopia. There is no way to tell this beforehand. Correct the full error of astigmatism and look after muscular anomalies and then if the eyes are not comfortable the cylinders should be reduced or omitted, especially if they do not in a marked degree improve the visual acuity. To be certain that one has ascertained the meridians of the least and the greatest refraction of the eye without the use of a cycloplegic is impossible ; the lines that have their focus upon the retina are seen the best, and these may be in the meridians corresponding to the principal meridians of the eye or to some other, according to the amount of accommodation being used during the testing. However this makes little difference in most cases. To be reasonably certain that the patient is not guessing or that the accommodation is not interfering much with the test, it is well to have the patient pick out the most blurred as well as the best seen lines upon the card, as the two must always be at right angles to each other, unless there is present irregular astigmatism, and this would be noted by pathological changes in the cornea scars or change in its shape, which could be detected by oblique illumination, by incipient cataract or obliquity of the crystalline lens, detected by the ophthalmoscope.

In the practical application of the test for astigmatism, one begins with a weak plus cylinder, placed before the eye according to the rules laid down, and if the lines are rendered more alike by it, it is of the right sign and inclination ; successively stronger and stronger cylinders are then placed before the eye until the one is found that makes all the lines appear exactly alike. If one is chosen that causes the previously blurred line to appear the better, it is too strong and is said to have overcorrected the astigmatism. The next weaker cylinder is then chosen. If two successive strengths are found, the one under and the other overcorrecting the astigmatism, the former is to be chosen. In simple hyperopic or myopic astig-
matism the formerly blurred line is never made the better by a cylindrical lens that overcorrects the astigmatism. The lines at right angles to one another become more and more alike until the correcting cylinder is applied, when they are exactly alike. If a cylinder too strong is now put before the eye the lines that appeared the better without any lens again begin to be seen the plainer. It is therefore necessary to increase the strength of the cylinder gradually in correcting astigmatism, or otherwise one may overcorrect the case, and from the appearances of the lines suppose that the proper strength cylinder has not yet been arrived at. The fact that a cylindrical lens reverses the lines, that is causes the opposite lines to appear the better, is proof that the astigmatism is compound, that is, that the hyperopia or myopia has not been corrected.


Such cases should have a cycloplegic instilled.
In figure I, light rays 1,2 from cross lines pass into the eyeball
through the vertical meridian, and are focused upon the retina; ergo: they are seen plainer than the vertical lines, whose focus is behind the retina at $x$.

Figure II is a case of compound hyperopic astigmatism. The focus through the vertical meridian of the eyeball is nearest accurate, therefore cross-lines are seen the clearer by the eye.

Figure III represents simple hyperopic astigmatism overcorrected by a plus cylinder. The cylinder brings the focus through the cross meridian anterior to the retina. The horizontal lines still have their focus upon the retina and are therefore seen the better, as in case no. I, where the astigmatism was uncorrected.

Figure IV. Compound hyperopic astigmatism overcorrected by a plus cylinder. The focus $b$ through the vertical meridian is not altered, but the cylinder has brought the focus $a$ through the cross meridian of the eyeball upon the retina, and therefore the vertical lines are seen the better. The astigmatism was corrected when $a$ was brought to $b$, but is now overcorrected.

There are some that are unable to discern any difference in the distinctness of the lines upon the astigmatic dial even when a great deal of objective astigmatism is present. Their eyes have always been accustomed to see things in a distorted manner and any difference in the distinctness of the lines upon the dial is considered normal. One can often bring to the attention of such a case the fact that all the lines do not look alike by placing a strong plus cylindrical lens before the eye. The lines that run at right angles to the cylinder are seen the best, the axis of the cylinder is then changed. After the difference in the distinctness of the lines has been noted several times the cylinder is removed, when the astigmatic patient will still notice a difference in the lines, caused by his error of refraction. Now and then one will say that the lines on the astigmatic dial are seen better on one side of the center of the card than on the other. This is mostly imaginative, but in a few cases is caused by astigmatism by incidence, that is obliquity of the crystalline lens of the eye, or by an oblique position of the screen or retina as in eyes
with posterior staphylomata. This may be corrected by tilting the lenses before the eye. Astigmatism that can not be ascertained by the usual methods can be revealed by placing a weak convex cylinder in the trial frame and rotating it through 180 degrees. If the vision is found to be better in one position of the cylinder than in another, astigmatism exists. In order that the patient may notice whether he sees better in certain positions of the cylinder, he should put his attention upon several letters only in the lowest line upon the test-card that can be read. The cylinder should be rotated through 180 degrees several times to make sure that the patient actually does see better when the axis of the cylinder has a certain inclination or position. Folks over fifty frequently have astigmatism against the rule, which is only discoverable by the increase of visual acuity gained by placing a plus cylinder at $180^{\circ}$ or minus cylinder at $90^{\circ}$ before the eye.

If a positive cylinder seems to blur the vision, whatever the direction its axis occupies, then a minus cylinder is placed before the eye and rotated in the same manner. A . 25 D. C. is the most convenient one to use for this purpose. If a plus or minus .25 D. C. renders the vision better, with the axis in a certain position, then a .50 D. C. is substituted, and so on. The cylinder that allows of the best vision is the one chosen as the measure of the error. After plus cylinders have been tried and found to make the lines upon the astigmatic dial more unlike or rendering them alike, spoil the visual acuity, the case is considered to be one of myopic astigmatism. If plus cylinders make the lines more unlike, a minus cylinder is placed in the frame, always with its axis at right angles to the best seen line, whether the eye is under a cycloplegic or not, and the proper strength selected to render the lines uniform. If the eye is not under a mydriatic, there is no way to tell, save by trial, whether the case is hyperopia or myopia ; but supposing it to be the former and using accommodation a plus cylinder is placed before the eye with its axis paralfel to the best seen line. If all the lines are made alike, but the vision is still below par, then a minus sphere is placed in front of the
cylindrical lens, the weakest being chosen that allows of the best vision. In simple myopic astigmatism plus spheres will be promptly refused as making the vision worse, as they convert the case into one of compound myopic astigmatism, by bringing the focal line that is upon the retina anterior to it . When all the lines upon the astigmatic card are seen more unlike or alike but less distinctly with a plus cylinder, there is myopic astigmatism present. In hyperopic astigmatism the lines that are seen best upon the astigmatic chart with the fogging glass on are at right angles to those that are seen the best without the lens, while in myopic astigmatism the same lines appear the best with and without the fogging lens.

Mixed Astigmatism is made evident by the fact that neither plus nor minus spheres improve the vision to any extent, although the pinhole test shows that the poor vision is due to ametropia, by increasing the visual acuity. Plus spheres correct the meridian of hyperopia and exaggerate the meridian of myopia, and minus spheres correct the meridian of myopia and exaggerate the meridian of hyperopia. If the hyperopia and the myopia are equal in amount in the principal meridians all the lines upon the astigmatic dial are seen alike, as the diffusion circles in different meridians will be of equal size. The lines will however appear unequally distinct, when a convex or a concave spherical lens is placed before the eye. The sphere lessens the ametropia in one principal meridian and increases it in the other.

If there is a convex sphere on, the lines that are seen the best are seen through the meridian of hyperopia and vice versa. After the best seen line has been chosen with the sphere on, say it is a I D., the card with two sets of lines, drawn at right angles to each other, is placed before the patient so that one of the sets of lines occupies the meridian of the best vision. The lens in front of the eye is now made weaker or stronger, until the one is found with which the lines in that meridian are seen the best, care being taken to choose the weakest concave or the strongest convex, as the case may be. The attention is now placed upon the lines at right angles to these, or a
card upon which is a single set of lines may be used and the lines made now to occupy the opposite direction. The lens (sphere) that renders the lines best in this position is now ascertained. If one meridian needs a plus, the other needs a minus and vice versa. Suppose for an example, that when a convex spherical lens is placed before the eye that the lines that occupy the meridian of 60 degrees, that is those that run between $X I$ and $V$ upon the dial, are seen the best of all. The meridian of the eye at right angles to that, that is 150 degrees, is the meridian of hyperopia if the eye is not under a cycloplegic, or the meridian of least ametropia if the eye is under a cycloplegic. Lines are then placed so that they occupy meridian of 60 degrees, and the strongest convex sphere with which they are seen the best selected. Say it is plus 3 D. S. Then the card is turned so that the lines occupy the meridian between $I I$ and VIII, that is 150 degrees. Say that a minus 3 D. S. now renders them most distinct. The astigmatism is equal to the difference in the strength of the two lenses, or 6 D ., and the correction is the following :

$$
-3 \text { D. S. } \bigcirc+6 \text { D. Cyl. ac. } 60^{\circ}
$$

or

$$
+3 \text { D. S. } \asymp-6 \text { D. Cyl. ac. } 150^{\circ} .
$$

The astigmatism is known to be of the mixed variety from the fact that the sphere and cylinder in the combination are of unlike signs, and the cylinder of greater strength than the sphere. The axis of the combined cylinder in each case is placed in the meridian that is corrected by the sphere, and the strength of the cylinder combined with the sphere is equal to the sum of the refraction in diopters of the two principal meridians of the eye.

If there is a difference in the lines noticed on first looking at the astigmatic card, the examiner selects a weak positive spherical lens, and placing it before the eye under test asks the patient if the lines seen best without it on are made better with it or if they are at least. just as good. If the lines are seen just as well with the convex
sphere it argues that the meridian at right angles to these lines is hyperopic; the examiner then proceeds as in the former case. It is in this kind of an error (mixed astigmatism) that a cycloplegic is most needed to obtain accurate results. Accommodation causes the hyperopia to be lessened and the myopia to be increased to the same degree, not altering therefore the amount of the astigmatism, but leads one astray in the estimation of the amount of ametropia in each principal meridian, as accommodation may be used while refracting one principal meridian and in abeyance while refracting the other. After the effect of the cycloplegic has worn away it will be found that the eye requires an equal decrease in the strength of the plus lens and increase in the concave to give it the best vision, and to render the lines upon the astigmatic dial uniform.

The same rules that govern the amount to be deducted in hyperopia apply whenever the hyperopia is in excess of the myopia and the correction is the same as in myopia whenever the myopia is in excess of the hyperopia. " To allow for the return of accommodation in mixed astigmatism, if the combined sphere is positive, one simply deducts the required amount from it, which of course increases the relative refraction of the combined cylinder, while if the combined sphere has a minus sign, the sphere is made stronger, which lessens the refraction of the combined cylinder.

In the foregoing pages it was advised to ascertain the spherical correction first and then to test for astigmatism. At times, however, especially in mixed astigmatism and compound astigmatism of higher degrees, one can do better by correcting the astigmatism first and then adding the proper sphere. By retinoscopy beforehand the sort of astigmatism is discovered and if of the mixed variety the direction of the meridians of hyperopia and myopia located, a cylindrical lens of the strength indicated by keratometry is then placed in the trial frame and modified until the lines upon the astigmatic dial are rendered uniform or vision made best, then spheres are added and vision tested. Each time the sphere is changed the strength of the cylinder and the inclination of its axis is altered until the best vision
is obtained. Sometimes the following plan works well: A sphere may be dropped into the trial frame and then the cylinder of approximate strength added. The strength of the sphere and then that of the cylinder is changed until the vision is rendered best. The visual acuity test in astigmatism should always take precedence over the other tests, for it often happens that the vision is not as good with the cylinder that makes all the lines in the astigmatic dial uniform as with one weaker or stronger.

The stenopaic slit is at times useful in the estimation of astigmatic errors of refraction, and especially so in mixed astigmatism. A few facts about the stenopaic slit in general, as proven by photography. The emmetrope sees lines drawn in different directions perfectly uniform, with the stenopaic slit before his eye, and his vision is practically the same as without it, the only difference being that the slit emits less light, and for that reason, the vision may be slightly impaired by it. If a camera is put in accurate focus for an astigmatic dial, and then a stenopaic slit placed before the objective, and a picture taken of the lines it will be seen that all are equally well-defined, no matter what the direction of the slit. If the camera be put out of focus now and a negative made of the lines, those that occupied the meridian upon the card parallel to the slit are seen to be in best focus. The ametrope therefore with the stenopaic slit sees the lines parallel to the slit the best, because the areas of diffusion upon the retina are in vertical lines (when the slit is vertical before the eye), and these vertical diffusion areas overlapping build up a well-defined vertical line upon the retina, while the cross lines are seen much blurred, because their image upon the retina is a broad blurred line, formed by the apposition of the vertical diffusion areas. Each point in the horizontal lines is focused as a vertical line. It follows that an astigmatic eye will see best and all the lines most alike when the stenopaic slit occupies the meridian of least ametropia. The manner of using the stenopaic slit is as follows: Place the disc with slit in the trial frame before the eye, and slowly rotate it through i8o degrees while the patient puts his attention upon a certain line of test-type,
and notices whether he sees better in any particular position of the slit before the eye. If so astigmatism exists and the direction of the slit when the vision is the best marks the meridian of the least ametropia, the other principal meridian being at right angles to it.

If in the preferred position the vision is $20 / 20$ or better, and a convex sphere is refused, or if the eye is under atropine, there is emmetropia in that meridian. The disc is now rotated so that the slit will occupy a position at right angles to the former one, and the vision ascertained. If the vision is less acute than before there exists astigmatism. A concave or convex spherical lens is now sought that will render the vision normal in this meridian. The difference in the strengths of the two lenses needed to make the vision normal in the two meridians is the measure of the amount of the astigmatism. If in one meridian the vision is normal and positive spherical lenses are refused, the astigmatism is of the simple variety, while in compound astigmatism the vision will be below par in all directions of the slit, better in some however than in others, and improved by concave or convex spherical lenses as the case may be compound myopic or compound hyperopic astigmatism. Mixed astigmatism is evidenced by the fact that a concave lens is needed in one meridian and a convex one in the meridian at right angles thereto to render the vision perfect.

In all cases of astigmatism of whatever variety after the correction has been applied according to the rules laid down, it is well to alter the strength of the sphere, then that of the cylinder, to see whether the vision can be made still better, especially if one has failed to bring it to normal by aid of the glasses. The cylinder should be made a little stronger then a little weaker to see if the vision is improved, by holding in front of it a weak cylinder of opposite sign, then one of the same sign, the axis of each being held parallel to the axis of the combined cylinder before the eye. Then the strength of the sphere is tested by holding in front of the combination before the eye a weak convex and then a weak concave spherical lens. To alter the strength of the sphere and of the cylinder in the correction
at the same time is often desirable. This is best done with crossed trial cylinders. These are two cylinders of equal strength but of opposite sign ground on one glass with their axes at right angles to each other. The most convenient strengths are made of . 25 D. Cyls. and of .50 D. Cyls. These combinations are denoted thus :

$$
+.25 \text { D. C. } \perp-.25 \text { D. C. and }+.50 \text { D. C. } \perp-.50 \text { D. C. }
$$

and are equivalent to

$$
+.25 \text { D. S. } \frown-.50 \text { D. C. and }+.50 \text { D. S. } \frown-1.00 \text { D. C. }
$$

The most convenient way to have the crossed cylinders is mounted upon long handles so that they can be put on and off without the hand obscuring the vision. The crossed cylinder is placed before the eye with one of its axes parallel to that of the sphero-cylindrical combination, and vision noted and then turned so as to cause the opposite axis to occupy the same position. If the vision is made better in either direction of the crossed cylinder a change is made in the correction before the eye accordingly, the sphere in the combination being changed to the same extent as the cylinder. If the one is made stronger the other is made weaker or vice versa. Thus: Suppose that before the eye there is the following combination:
+2 D.S. $\bigcirc .50$ D. C. Ax. $90^{\circ}$, and the patient saw better when the .50 D. crossed cylinder was placed before the above combination, with the axis of the concave cylinder parallel to that of the cylinder in the sphero-cylindrical combination. The lens needed then is :
(1) +2 D.S. +.50 D. C. Ax. $180^{\circ}$. If the vision is improved by rotating the crossed cylinder, until the axis of the positive cylinder is parallel to that of the combined cylinder, then the glass needed is:
$(2)+1.50$ D. S. $\subseteq+1.50$ D. C. Ax. $90^{\circ}$, and so on.
Each time that the sphere or the cylinder before the eye is changed, the cylinder should be rotated a little this way or that to
ascertain in what position of its axis the vision is the best, that is if the eyes are not under a cycloplegic - for without it, the principal meridians vary according to the amount of accommodation used depending to a certain extent upon the strength of the plus $S$. before the eye.

In high degrees of astigmatism there is often present what the

( +2 D. S. $\subseteq+.50$
D. C. Ax. $90^{\circ}$ )

Place III upon II with axis of - cylinder at $90^{\circ}$ and the resulting refraction


Crossed Cyl.
If III is turned, so that axis of + cylinder lies vertically, we have :

v author chooses to denominate meridional amblyopia, that is a deficient vision or anæsthesia of the retina corresponding to the meridian of greatest ametropia. In consequence it is impossible to get the eye to see lines at right angles to this meridian with any degree of distinctness, notwithstanding the fact that the astigmatism may be properly corrected. Such cases should be corrected objectively and the weakest cylinder with which the vision is best worn.

Correction of Presbyopia. The convex spherical lens that enables the presbyope to read with comfort would be too strong for him to work with at arm's length, if he was a cabinet maker, and vice versa. In prescribing for the presbyope one should always ascertain at what distance he wishes to use his eyes and the glasses given accordingly. If we know how much accommodation the patient has and how much he needs for a given distance it is not difficult to ascertain
what he lacks, the amount to be supplied by spectacles. The amount of accommodation needed by the emmetrope, for any given distance, is ascertained by dividing the distance expressed in centimeters, into 100 centimeters, or one diopter. This gives the number of diopters of accommodation in use for that distance, as explained upon a previous page.

If working in the inch system, the distance in inches is expressed in a fraction with one as the numerator. Thus: How much accommodation is used when reading at a distance of fifty centimeters and at twenty inches? Ans. $100 / 50=2 \mathrm{D}$. accommodation; $1 / 20$ accommodation. Any refraction error should always be corrected before the eyes are tested for the presence of presbyopia. The presbyopic correction is added to the distant correction for the nearseeing glasses. An emmetropic bookkeeper needs glasses for near work. He is fifty years old. He wishes to use his glasses at 50 cm . distance, at his desk. Of late print has been blurred at that distance. The nearest point at which he can read with comfort is at i m., and at that distance he reads the type that is designed to be read at that distance. (This shows that he is emmetropic.) What glasses does this patient need? How much accommodation does he require for the distance at which he works? $100 / 50=2 \mathrm{D}$. accommodation needed. How much accommodation has he left? 100/100 $=\mathrm{I}$ D. accommodation left. $\quad 2-\mathrm{I}=1$ D. accommodation lost, and needed to be supplied by lenses.

We would give this patient a 1.25 D. S. lens, inasmuch as a i D. S. allows him to read at 50 cm . distance only when he is using all his available accommodation, and under these conditions he would soon tire, as all of his accommodative strength is being put forth at each moment. A little stronger lens allows him to hold a little accommodation in reserve, and as the ciliary muscle is not constantly exerted to its utmost the eyes do not so soon tire. A patient may come complaining that his eyes soon become tired when using them for near things. He reads all day long. The proper reading distance is at 33 centimeters, and at that distance 3 D . accommoda-
tion are needed. The patient's near-point of accommodation is 33 cm . which is only attained however by straining the accommodation, as is proven by the fact that the patient can not read a bit closer than 33 cm . Presbyopia has been troubling him since his near point has receded beyond 22 cm . (Donder's presbyopic point). What glasses does this man need to make him comfortable? The amount of accommodation used for 22 cm . distance is a trifle less than 4.50 D. The patient's near point lies at 33 cm . representing 3 D . accommodation. The patient has therefore lost since presbyopia began 1.50 D. accommodation, which must be supplied by convex spherical lenses in spectacles. Presbyopia then equals $4.50-\mathrm{I} / p$, in which $p$ $=$ near point of accommodation expressed in the metric system. If the eyes have been under the effect of a mydriatic for the estimation of the refraction error, one must wait until the accommodation has fully returned before attempting to estimate the degree of presbyopia. It is a bad policy to follow a table and to correct the presbyopia according to the age of the patient, for then paresis of accommodation will certainly be overlooked if it exists, and not a few people are less presbyopic than their age would indicate.


The rod optometer is a very useful instrument for detecting and measuring the amount of presbyopia. It consists of a two-foot rule, numerated in inches or centimeters, or both. Moving along the rule
there is a carrier for test-type, and at one end is a clip holding a plus spherical lens of 4 D . Through this convex spherical lens the patient should be able to at least read fine print at 12.5 cm . after the refraction error has been corrected. At 12.5 cm . there is needed, without any lens before the eye, 8 D . accommodation. The convex lens of the optometer does the work of 4 D . accommodation, and the eyeball supplies the other 4 D . If the inch system is used the amount of accommodation represented by 12.5 cm . or 5 inches is $\frac{1}{5}$. If a person does not read as close as 5 inches after the error of refraction is corrected, ciliary asthenia or presbyopia exists; the latter if the patient has passed the age of forty.

With the rod-optometer, subtract from what the patient should do, what he can do, to obtain the presbyopic correction. Thus, a man reads at 8 inches and he should read the same type at 5 inches. What glass does he require? Subtract the lesser fraction from the greater, $\frac{1}{5}-\frac{1}{8}=\frac{3}{40}$. According to the dioptric system, patient should be able to read at 12.5 cm . and can read only at the more distant point of 20 cm . What lens does he need? 12.5 cm . is equivalent to 8 D . accommodation ( $\frac{100}{12 i}=8 \mathrm{D}$.), and 20 cm . to 5 D . accommodation $\left(\frac{100}{20}=5\right.$ D. $) ; \cdot 3^{\prime}$ D. S. is the lens needed therefore. The rod-optometer can also be used when the eyes are under the effect of a cycloplegic, to estimate the refraction error, which will be spoken of later. The following plan, that of Dr. De Schweinitz, is to be pursued in cases of myopic astigmatism with presbyopia. "A patient with simple myopic astigmatism reads best with a plus cylinder of the same strength, its axis being reversed." Thus, a patient whose distant correction is -2 D . Cyl. Ax. $180^{\circ}$, will be comfortable with $\mathrm{a}+2$ D. Cyl. Ax. $90^{\circ}$ for reading. The convex cylinder with the myopic astigmatism produces a myopia of 2 D ., in all meridians, so he prefers this lens to the concave cylinder that makes him accommodate for reading. Simple myopic astigmatism may be utilized to ascertain the reading glass in all patients past their thirty-fifth year of age, provided the degree is not too high. A convex cylinder of the same strength as the concave one correcting the astigmatism,
with its axis reversed, is all that is required for the reading lens. If the degree of myopia thus produced is too great for comfortable reading a concave spherical lens may be added to the convex cylinder reducing the myopia the requisite amount. Thus an astigmatism corrected by a -4 D. C. Ax. $180^{\circ}$ would probably require a -1.50 D. S. $\bigcirc+4$ D. C. Ax. $90^{\circ}$.

If the degree of the astigmatism is unequal in the two eyes, a spherical lens is required to equalize the refraction. For example: R. E. -5 D. C. Ax. $180^{\circ}$; L. E. -3 D. C. Ax. $180^{\circ}$, needs -2 D. added to the right eye to make its refraction equal to that of the left one. R. E. -2 D. S. $\subseteq+5$ D. C. Ax. $90^{\circ}$; L. E. +3 D. C. Ax. $90^{\circ}$.

When in cases of compound myopic astigmatism, the myopia equals several diopters, the reading lens is secured by a sufficient decrease in the spherical lens of the combination, without changing the cylinder. When in the lower forms of compound myopic astigmatism it is desired to increase the refraction one or more diopters the procedure is somewhat different. Thus if the combination is -.50 D. S. -1 D. C. Ax. $180^{\circ}$, and the spherical lens is dropped, there is .50 D . gained. By substituting for the concave cylinder, with its axis reversed, an additional 1 D. is secured. $+_{1}$ D. C. Ax. $90^{\circ}$, in this case is the same as plus 1.50 D . S. added to the combination. If still more refractive power is desirable, say 2 D . in all, +.50 D. S. $\subseteq+1$ D. C. Ax. $90^{\circ}$, gives the additional amount.

There will usually appear in accommodation about 4-5 degrees of exophoria. This should be taken into account in the adjustment of reading glasses; for if the exophoria in accommodation is abolished, the patient is given a practical esophoria. The proper glasses having been selected, place them in an adjustable trial-frame, with the optical centers at the height of the pupils, then adjust the phorometer for exophoria of five degrees, and bring the slide of the instrument within two inches of the eyes of the patient. Hold a card on which there is a dot upon a vertical line, in front of the instrument at the usual reading distance. Now by the screw of the adjustable trial frame, by altering the pupillary distance, bring the dots plumb and the vertical lines to coincide. Measure the distance between the optical centers of the lenses, when the dots are seen exactly the one under the other, which is the distance at which the centers of the permanent reading glasses should stand.

## CHAPTER XXII

## OPTOMETERS

There are many appliances that have been brought forward from time to time to take the place of the trial case of lenses, spoken of as optometers, or refractometers.


Javal-Bull's Optometer. The advantage claimed for the majority of them is that they do away with the necessity of changing the lenses so frequently before the patient's eyes, during the test, and that more accommodation is relaxed as the strength of the lens is gradually altered, and not by jumps as we do when we replace a .25 D. by a .50 D . and so on in the trial frame. The instrument shown in the accompanying, cut is that of Javal \& Bull.

It consists of two superimposed discs, one carrying spherical and the other cylindrical lenses; each disc containing fourteen concave and fourteen convex lenses, running from $a+.50$ to $a+8$ D., and from -.50 to -10 D. spherical and the same in cylindrical lenses. There is in addition on the reverse of the disc, a clip carrying a plus and minus io D., by which the spherical lens series may be increased 308
accordingly, and the different combinations are easily read on either side of the discs. The instrument is so arranged that either eye can be tested with equal facility. The lenses are brought before the eye by turning the large screw-heads seen below the discs, and the cylinders are rotated upon their axes by a rack-and-pinion movement seen at the center of the discs, which operates upon a cog-wheel arrangement, and the axes are indicated by a pointer upon a scale. The whole is mounted upon a stand four feet high, with a heavy iron base, and can be raised or lowered to any height. The lenses can be changed very rapidly before the eye, thus encouraging the ciliary muscle to relax.

The fogging method can be employed to advantage. A strong

convex or a suitable concave lens is placed before the sight-hole, accordingly, and diminished or increased by slow gradations in strength, until the best vision is ascertained. One does not need an optometer of this kind and a trial-case both in the office, and as the latter is portable it is in the whole more useful, as it is often necessary to take your lenses to the patient when the patient can not come to you.

The next cut represents De Zeng's optometer or refractometer, based upon the fogging method. It consists as shown in the cut of a tubular casing about $2 \frac{1}{8}$ inches in diameter and $8 \frac{1}{2}$ inches long. It is mounted upon the bracket and pivoted to an upright pillar which is supported below upon an oxidized iron base. The front end of the tube is left open, while at the other end there is a negative eye-piece consisting of a biconcave sphere of 20 D . focus. The head of the instrument is provided at its circumference with a circular rack, rotated by means of a milled head. The degree of rotation is indicated by a scale with which coöperates an index secured to the upper side of the tube. This scale and index denote at any point of rotation of the head the axis of the cylindrical lens that may be exposed at the aperture of the eye-piece. Pivoted within the head there are two revolving discs or lens carriers having within them apertures in which are arranged the concave cylindrical lenses with their axes at right angles to the radii of the discs containing them. These discs bear the figures upon their front surfaces indicating the power of the lens that is exposed at the eye-piece. The cylindrical lenses contained in the disc are of a negative quantity, and by combining the lenses in the two discs any strength of lens may be obtained. Sliding within the main tube of the instrument is an inner one constituting the lens carrier, having in its rear end an achromatic convex lens being open at the other end. This tube is graduated upon its upper surface in diopters and fractions thereof, and the scale so formed indicates myopia as the refractive error, and the concave spheres for its correction. The range of the scale is from zero to nine diopters, inclusive, and is subdivided. The graduated circular dial on the side of the cut denotes myopia and hyperopia or concave and convex spheres, respectively, ranging from 0 to 18 for the convex and that of the concave numbers from $\circ$ to -1.50 inclusive, the continuation of the latter being upon the tube as has been noted.

All the positive effects obtained with the instrument are indicated by the red figures upon the circular dial, and the minus, in white
upon the circular dial, and upon the tube. There is an adjustable indicator on the upper side of the instrument, that has a white line on its under surface which coöperates with radiating white lines on a small range scale, of from three to six meters inclusive. The instrument may be arranged to perform accurately at either a three-, four-, five- or a six-meter range. Attached to the right side of the tube is a small finder, composed of a small tube having a convex lens at one end ; a plane mirror set at an angle at the rear end and a ground-glass screen at its side. Upon the ground-glass finder are fixed two intersecting lines at the intersection of which the image of the test-card will appear when the instrument is properly directed towards the test-type. Through the relative adjustment of the achromatic objective and the stationary negative eye-piece, all the spherical equivalents from +18 to -9 D . inclusive are obtained at the eye-piece, and are recorded upon the revolving and sliding scales. Owing to the negative scale being limited to 9 D., there are two auxiliary lenses accompanying the instrument, a -10 and $\mathrm{a}-20$ D., which may be placed over the eye-piece when required, thus raising the negative scale. By reason of its construction, the instrument has a magnification of two and a third diameters, and in consequence the test-type to use with the instrument are reduced to three-sevenths of the size of Snellen's distant test letters, so that the visual acuity may be readily estimated with the instrument.

Method of Testing. - The best method of testing with the refractometer is what is known as the fogging method. This consists in overcorrecting the hyperopic eye with a plus sphere and the undercorrecting of myopia with a concave sphere, of such a strength as to render all the lines and letters upon the test-cards blurred, in order that the accommodation may be relaxed and any hidden error brought to light and measured with the manifest. It is claimed for the instrument that it develops more latent error than any other instrument of the kind, not aided by the use of a cycloplegic, due to the fact that the illumination is better than in any other method,
and that the large volume of light stimulates the ciliary muscle to relax. Experience does not substantiate this statement. (A flood of light rather stimulates the accommodation.) First set up the instrument in line with the test-card and with range for which it is already adjusted. Next see that it is set at zero for both spheres and cylinders, that no focusing may exist when the test is begun, and further that the test-cards appear in the center of the visual field of the refractometer, using the crossed hairs in the finder to determine this.

Now adjust the eye-piece to cover one eye, while the brow of the other touches the eye-piece throughout the test. Turn the focusing adjustment either to the right or to the left until the test-card is seen the most distinctly, from which point turn it towards higher numbers among the red graduations or lower ones among the white, as the case may require, until all the radiating lines constituting the fan at the top of the card are completely fogged. Then turn the focusing adjustment slowly back to the point at which the card was seen the best, requesting the observer to name the line or lines in the fan which may first begin to clear up, that the presence or absence of astigmatism may be detected and its meridians located.

When all the lines in the fan clear uniformly the registration of the highest red or the lowest white as the case may require, with which the smallest letters on the test-card are seen distinctly, will indicate the nature of and the amount of defect present, and the kind and power of glass to be prescribed for its correction, it being simple hyperopia if the numbers are red and simple myopia if they are white. Should one or more of the lines in the fan clear before the others, the point at which they are first seen distinctly should be noted and the blurred lines rendered equally distinct by the employment of the cylindrical lenses as follows: Rotate the head of the instrument to a point where the axis register stands opposite to a graduation thereon agreeing with the meridian of the blurred lines, bring the cylinders to the eye-piece, beginning with the front discs containing the weakest numbers. Revolve it to the right or to the left until by the use
of one disc or the other, or both, a cylindrical power is obtained which will render the fan uniform to the eye. After securing a uniformity in the radiating lines as described, throw them all back into the fog, that on bringing them out again any under- or over-correction may be noted. If under-corrected, the same lines that came out first will come out first again, while if over-corrected, the lines at right angles to those first appearing to clear, will clear up first. If any difference exists, correct it after the manner described, before proceeding.

The power and axis of the cylinder thus obtained should be verified, before completing the astigmatic test, as follows: Direct the attention of the observer to the letters on the card beneath the fan, while a change in the strength and axis of the cylinder before the eye is made and note which cylinder and at what inclination produces the clearest definition of the smallest letters that can be read upon the card and is most acceptable to the eye. Always prescribe the weakest cylinder that will correct the difference in the distinctness of the lines, and gives the best vision. The figures engraved upon the faces of the discs containing the cylinders indicate the power of the lenses exposed at the eye-piece. When the click of the springstop tells the proper position, they stand directly opposite the $180-$ degree mark on the axis register directly beneath them. Should the lenses in both discs be exposed at the same time, their power should be combined and the equivalent expressed as one lens. In simple hyperopia always obtain the highest plus and in myopia the weakest minus that allows of the best vision. The results obtained by the use of this instrument are equally good but in no sense better than those obtained by use of the trial case, save by the latter method a certain amount of time is lost in changing the lenses before the eyes.

Mention should be made of the other optometers that have been devised, those of Badal, Sous, Coccius, Donders, Mile, Scheiner, and Young. All of them have this defect, that they stimulate the use of accommodation, rendering the myopia too high, as they operate at such a short range. They have not superseded the use of the trial
case for this reason. There are a few optometers founded on the use of a single biconvex lens. By displacing an object in relation to this lens, the image of it can be formed at any distance, and we can thus find the place where the object appears distinct. Such were the optometers of Donders, Sous and Coccius. Graefe constructed an optometer of a Galilean telescope; myopes have to shorten their opera glasses to see distinctly through them. By providing the opera glass with a scale it therefore may be used as an optometer, so may also the telescope.

The optometer of Badal is composed of a single convex lens, the focus of which coincides with the anterior nodal point of the eyeball. The position of the eye is made secure by an eye rest. Along the rod on the other side of the lens there is movable a diminished copy of the Snellen's chart. The retinal image of the chart always remains the same size no matter what the position of the chart in regard to the lens, according to the following figure, but the patient can not see the letters equally well at different distances from the lens as the light proceeds to the lens in more or less diverging paths according to the position of the test card along the optometer rod. This will be explained more fully later on. The visual acuity is measured by placing the test-card at the principal focus of the lens of the instrument, whence the rays pass through the lens parallel, and enter the eye of the observer as if proceeding from a distance of twenty feet or more.

The Rod Optometer (Modification of the Instrument of Badal).Much information that aids in the fitting of glasses can be obtained by means of the rod optometer. By its use the refraction of the eye and the glass needed to correct it are readily ascertained. It consists of a two-foot rule, provided with a clip at one end for a lens and a slide movable along the rule, for test-type. As a rule a io-inch or 4 D. S. lens is placed in the clip, and a diminished Snellen's card used for the test-type. By moving the printed matter along the bar the 4 D. lens exercises more or less influence over it according to its position. If the object is nearer the lens than its principal focus the light
passes through the lens in diverging paths. The eye behind the lens then must either be myopic or exert its accommodation to bring such rays to a focus upon its retina. On the other hand if the object is


The object $O$ in position $O, O^{\prime}$ or $O^{\prime \prime}$ is embraced by parallel rays $a a^{\prime}$, which passing through the lens $L$ come to a focus at $N$, and diverging include the image $i$ of $O$ upon the retina. $a$ and $b$ are rays emanating from the principal focal point of $L$ and, therefore, pass out of it parallel.
beyond the point of the principal focus of the eye-piece, rays emanate from the lens in converging paths, for which the hyperopic eye is alone focused. As the card is moved from the one-inch mark near the lens to the 24 -inch mark at the end of the rule, there has been placed before the eye, lens strengths ranging from -36 D . S. to +2.50 D. S., and if the patient can see to read with any glass within these limits there will be some position along the rod where he can


Figure II. represents $L$ placed so that its principal focus coincides with the anterior focal point of the eyeball. $O, O^{\prime}$ and $O^{\prime \prime}$ are different positions of the object and $i$ its retinal image.
at least read some of the test letters, and at the same time the degree of improvement expected from the use of glasses ascertained from the smallest size type the patient reads. If no reading point is found add four to eight more diopters to the eye-piece by placing the additional lenses in a trial frame. If no reading point is now found, there is no lens that will improve the vision, and the eye should be exam-
ined for lesions. In high degrees of hyperopia, the additional lens may be necessary, then after the correction is ascertained it should be added to or taken from the lens in the trial frame according to whether it is positive or negative.

More information can be gotten from the rod optometer in a short while than from any other one source alone, and the more one uses it, the more helpful does he find it. There are two points from which we calculate when using the instrument ; they are the five- and the ten-inch point. We reckon from the first, when ascertaining the glass needed for reading purposes, and from the latter for the refraction correction. In testing poor seeing eyes from cloudy mediæ or fundus lesions with the rod optometer we can often ascertain a reading glass which gives good near vision, but fails to improve the distant vision. The patient selects an abnormally near point for reading so that enlarged retinal images compensate for loss in distinctness. A small astigmatic fan is used to detect the presence of astigmatism. The following deductions will be in the inch system, inasmuch as it is customary to have the optometer bar graduated in inches. The eyeball is focused when at rest for parallel rays of light. It therefore sees the smallest test letters when they are at the ten-inch point, or at the principal focal point of the eye-piece. The range of accommodation of the eye before the advent of presbyopia is from the tento the five-inch mark.
Young and supple eyes can often see closer than five inches, showing an abundance of accommodation. If an eye to see the test card requires that it be closer than ten inches there is myopia present, and if it can see the letters further off than ten inches hyperopia exists. In astigmatism the range of accommodation is found to be much shortened. Suppose that the patient can see the small type the best at fifteen inches from the lens, then he needs $1 / 10-1 / 15=$ I/30 plus lens for distant seeing. The same patient can also see the print at eight inches, but no closer; he needs then for reading $1 / 5-1 / 8=1 / 13$ plus lens. Suppose that in another case the most distant point of distinct vision is eight inches, and the nearest point
is five inches, there is then indicated myopia and the lenses needed for distant and near use are respectively: $1 / 8-1 / 10=1 / 40$ minus spherical lens, and $1 / 5-1 / 5=0$ D.S.; in other words presbyopia has set in and no lens is needed for reading.

When testing for near-seeing glasses find the closest point at which the patient can read comfortably and deduct this from $1 / 5$. For distant glasses ascertain the most distant point at which the patient can see to read, and deduct this value expressed in a fraction with one as the numerator from $\mathrm{I} / \mathrm{ro}$ or vice versa, according to which value is the major fraction. The degree of ametropia may also be estimated in the following manner in the metric system. The greatest distance of distinct vision minus the focal length of the lens at the eye-piece divided by the common multiple of these numbers, or by dividing the distance by which the type is read in centimeters into 100 cm . to ascertain the refraction for that point, and subtracting this from the strength of the eye-piece. Thus, suppose that the patient reads best at $40 \mathrm{~cm} .40 \mathrm{~cm} .=2.50 \mathrm{D}$. of refraction; the strength of the lens of the instrument is 4 D.; $4^{-2.50}=1.50$ D. of hyperopia. Or, suppose that the point of the most distinct vision is at 10 cm . 10 cm . represents io D. of refraction ; го $-4=6 \mathrm{D}$., the amount of myopia.

After the regular ametropia has been corrected, one tests for astigmatism, especially if the range of accommodation is still shortened, or if the patient has been unable to read the finest print at any place along the rod. The small astigmatic fan is placed in the carrier and brought from a distance at which the lines are invisible towards the eye, moving the carrier along slowly. If the lines in different meridians come into view at different distances there is astigmatism present. The set of lines that first come clearly into view mark one principal meridian, that of least refraction. The card is then started close to the lens and moved slowly away, and the lines that first clear up noted. These two sets of lines should be at right angles to each other in cases of regular astigmatism, and if they are not it indicates
that the patient is accommodating, and that the test cannot be relied upon without the use of a cycloplegic.

In cases of simple hyperopic astigmatism the lines occupying one principal meridian are seen the best at the ten-inch point, and those at right angles to them best beyond that point. In simple myopic astigmatism one set of lines is seen best at the ten-inch and the others nearer the lens than this point. In compound hyperopic astigmatism the lines occupying each meridian are seen at least just as well if not better beyond the ten-inch point, and in compound myopic astigmatism all the lines must be drawn closer to the lens than its principal focus to be discerned clearly. In mixed astigmatism one set of lines is seen best beyond the principal focus of the lens, and those at right angles to them best closer to the lens than its principal focus.

After the principal meridians have been located, a card with two sets of lines, conforming to the direction of the principal meridians, is placed in the carrier. Suppose that one set of lines in a case of compound hyperopic astigmatism is seen best at twelve inches, which represents an error of $1 / 60$, and the set at right angles to them at fifteen inches, which represents a refraction of $1 / 30$. The amount of astigmatism is equal to the difference between the refraction of the principal meridians or $1 / 60$. A I / 60 plus sphere is now placed before the eye, correcting the meridian of least ametropia and a $/ 60$ cylinder combined with it, with its axis at right angles to the lines seen best when the carrier is at the ten-inch point. In a case of astigmatism with the rule (vertical meridian the least hyperopic) the cross lines will be seen the better at the twelve-inch point, the correcting cylinder would therefore be placed at ninety degrees at right angles to the better seen lines. The amount of astigmatism is ascertained in the same manner in compound myopic astigmatism. Thus, suppose one set of lines is seen best at five inches and the other set at eight inches, the refraction of the first point is $1 / 10(1 / 5-1 / 10)$, and of the second $1 / 40(1 / 8-1 / 10)$, which equals an astigmatism of $3 / 40$ or $1 / 13$. A $1 / 40$ concave sphere is now placed before the eye and the lines moved to the ten-inch mark. In a case of astigma-
tism with the rule the horizontal lines will now be seen the better, and the astigmatism is corrected by placing a $/ 13$ concave cylinder before the eye with its axis at right angles to the lines seen the better. In simple astigmatism the refraction of only one meridian is needed, thus:

Suppose that in a case of simple hyperopic astigmatism one set of lines is seen best at ten inches, representing emmetropia, and the other set at right angles thereto, at twenty inches, representing a refraction error of $\mathrm{I} / 20$, the case is corrected by a $1 / 20$ cylinder axis at right angles to the lines seen best with the test card at ten inches.

If in a case of simple myopic astigmatism one set of lines is seen best at eight inches, the astigmatism is corrected by a $1 / 40$ concave cylinder, axis at right angles to the lines seen the better with the card at ten inches.

Suppose that in a case of mixed astigmatism one set of lines comes into prominence at twenty inches, and the other set at eight inches, how is one to proceed ? When the card is at twenty inches it represents $\mathrm{I} / 20$ plus refraction and when at eight inches $\mathrm{I} / 40$ minus refraction. A plus or minus sphere either can be placed before the eye, the card brought to the ten-inch mark and a contrageneric cylinder of the strength of the two principal meridians combined added to the sphere with its axis at right angles to the better seen lines, which will be at right angles to the meridian corrected by the sphere.

After correcting astigmatism, the test card (astigmatic fan) is moved off until all the lines fade from sight and it is then brought slowly towards the eye. If all the lines come into prominence at the same place along the rod, the astigmatism is properly corrected. If the lines that primarily were seen the best come into prominence first, the astigmatism is undercorrected, while if those at right angles to the ones seen first without any lens before the eye clear up first now, the astigmatism is overcorrected. The strength of the cylindrical lens should be modified until one is ascertained that causes lines drawn in all meridians to come into distinctness at the same time. The distant and near points of seeing should then be ascer-
tained to see if the amount of hyperopia, myopia or presbyopia is properly corrected, using a card of small printed matter.

The Experiment of Scheiner. - This consists in having the patient look at a distant candle flame through a disc or card with two minute perforations close enough together, that light passing through each will enter the pupil of the eye. The light is placed at twenty feet from the observer. Light passes into the eyeball through each of the perforations, and if the eyeball is emmetropic, the light will meet in a common focus upon the retina and a single candle flame will be perceived.
$A$ and $b$ are parallel rays of light coming from some point in the distant candle flame. They are brought to a focus upon the retina, so
 with all the points in the flame and therefore a single flame is seen. If the eye is ametropic the rays coming through the two openings will not have a common focus, but will stimulate two separate areas upon the retina and give rise to the perception of two objects. If the eye is hyperopic, the rays from the two perforations will reach the retina before meeting in a focus, and two points of stimulation are formed, and two candle flames seen. Each point of the retina stimulated projects its point to the opposite side in space, so that the two flames are crossed, that

is the one caused by the right-hand hole in the disc is seen to the left and vice versa. A red glass or piece of celluloid may be placed behind the right-hand hole, coloring the left-hand image, so that
the examiner will have no difficulty in telling the nature of the diplopia.
$H$ is a hyperopic eyeball with Scheiner's disc before it; $a$ and $b$ course of rays to eyeball ; $F$, point of their principal focus; $r, r^{\prime}$, areas of retinal stimulation ; $N$, the nodal point and $c, d$, the direction of projection into space. The opposite condition of affairs pertains in myopia. The flame is seen double but its images are not

crossed, the red one being on the right side of the yellow one. Again if the disc be moved to the right, thus excluding the right-hand hole from sending light into the eyeball, the right-hand image will disappear in myopia and the left-hand one in hyperopia.

The lettering and explanation in figure above are the same as for the figure on page 320 . The strongest convex or the weakest concave spherical lens that unites the two images of the flame into one is the measure of the ametropia. Astigmatism is detected by placing the disc before the eye so that there will be produced vertical diplopia and then measuring the ametropia in the vertical meridian of eyeball. The holes in the disc must occupy the vertical direction. Or the disc may be slowly rotated while the patient watches the candle, after the ametropia of one meridian is corrected. If there is no astigmatism present, the flame appears as one throughout rotation of disc. To make the test still more accurate, Dr. Thompson arranged two small gas flames along a graduated bar, so that they could be made to approach or recede from one another. Viewing the flames through the double perforations of Scheiner's disc, four flames are seen. One flame is then made to approach the other by
sliding it along the bar, until three flames only are seen by the coinciding of the two middle images. The distance in centimeters separating the flames is now read off from the graduated bar bearing them, and the refraction of the eye deduced in diopters, by dividing the number of centimeters separating the two flames into 100 cm ., or the number of diopters of refraction may be read off directly from the bar. The arrangement of the ametrometer is shown in the cut. If the two middle images of the flame cannot be made to fuse with the bar of the instrument horizontal it is inclined until they do, and


AMETROMETER.
the degree of inclination denoted on the scale $E$ by the pointer $F$, giving the axis of the astigmatism. The refraction can be directly measured in different meridians of the eye by rotating $R$ through different meridians. This test may at times be of service but is not to be relied upon to the exclusion of others, as the personal equation enters largely into the test, rendering it more or less unreliable. Scheiner's test can be nicely demonstrated by looking at two pins placed one behind the other, through the double perforation of the disc. It will be noticed that if the eye is adjusted for the anterior
pin, that the posterior one will appear double and vice versa, showing that the pin out of focus is seen double, so in ametropia.

The optometer of Young is based upon the experiment of Scheiner. It consists of a rule upon one face of which is drawn a white line upon a black ground. The patient looks along this line through $a+i o D . S$ lens. In front of the lens moves a small horizontal rule in which there are numerous slits arranged in groups. The two parallel slits act like the openings in the experiment of Scheiner. Each point of the line looked at appears double, except that which is in focus; an emmetrope, not using his accommodation must see two lines that intersect at his punctum remotum, his artificial farpoint at 10 cm . from the eye, the principal focal point of the lens of the instrument. To determine the refraction of an eye, a small cursor is placed at the point at which the lines are seen to intersect. A dioptric scale placed along the side of the rule permits the refraction to be read off directly. The near-point is determined in the same manner. The other groups of slits are used for measuring the refraction in different parts of the pupil, as explained in the chapter upon spherical aberration. To test for astigmatism the instrument is rotated around a horizontal axis, so that the slits at the eye-piece occupy different meridians. The inexperienced observer almost always calls his accommodation into action when using the instrument. As the distances are so small, therefore, better results are obtained by using a weaker lens at the eye-piece, say 4 D . as has been noted in the modification of Badal's instrument.

Prisoptometry. - The prisoptometer consists of a revolving double prism set in a disc upon which there is a scale of $180^{\circ}$ subdivided into eighteen equal parts of ten degrees each and an indicator to denote the position of the prisms. In the center of the disc in which the prisms are set is an opening through which the patient views the test object. The test object is a white disc four inches in diameter. This disc is doubled by the instrument so that when it is placed at a distance of sixteen feet, two discs are seen side by side, appearing with their edges just in contact to the emmetropic eye. If the eye
is myopic the discs seem to overlap each other, while if hyperopia is present they are separated from each other. The weakest concave sphere in myopia or the strongest convex sphere in hyperopia which
 causes the edges of the discs to appear in contact, is the measure of the amount of the ametropia present.

When the prisms in the instrument are revolved one disc appears to roll around the other one. If there is no astigmatism present, the edges of the two discs will remain in contact throughout the revolution. If astigmatism is present they will separate or overlap in some meridian. The meridians in which the discs are nearest together and furthest apart indicate the principal meridians of the eye. The scale and pointer upon the instrument enables one to read the inclination of these meridians. A cylinder of proper sign is then selected that will in the most ametropic meridian cause the discs to become in contact. The prisms are then again rotated to see if the discs remain tangentially to one another throughout the revolution. If they do not the astigmatism is not simple, but there is still needed a minus or a plus sphere as the case may be. This instrument needs the assistance of the patient to give accurate results, and any test that relies upon the good will and intelligence of the patient is an inferior one. The principle underlying the test is shown in the figure on page 325 .

Rays $a, b$ and $c$ come from the upper edge, center and lower edge of the test circle $C$, and pass through the upper portion of the double prism. They are bent down by the prism and entering the eyeball
stimulate areas i, 2 and 3 respectively. The light that emanates from the circle and passes through the lower portion of the prism is bent up, and stimulates the areas 4,5 and 6 respectively. As will be seen, the areas of stimulation overlap each other in the myopic eyeball ; are just in contact in the emmetropic and are separated in the hyperopic eyeball.

Kinescopy. - Shreiner's experiment is made the basis of a new test for measuring ametropia by Dr. Holth, which he calls kinescopy. Dr. Holth causes the rays of light to enter the eye through a narrow aperture before the pupil. An apparent movement of the object looked at occurs when the position of the aperture is shifted except when the light is perfectly focused upon the retina. In myopia the apparent movement of the object is in the direction the aperture is moved and in hyperopia against it. For astigmatism the movement is most pronounced in the meridian of greatest ametropia. The amount of ametropia is measured by lenses placed before the eye which prevent any apparent movement of the object as the aperture is shifted. In astigmatism the error is measured in both principal meridians. The difficulty in the practical test was to get rapid movements of the opening within the limits of the pupil. Dr. Holth invented an instrument which he called the kinescope in which a slit one or two millimeters wide or
 an opening one or two millimeters in diameter is given a rapid move-
ment limited to three millimeters before the pupil. The object looked at is either a white circle or a narrow white band upon a black background. As Holth mentions, there is a direct parallel between his test and retinoscopy. Kinescopy may be regarded as a sort of subjective retinoscopy. This method is found of especial value in eyes with extremely poor vision from haziness of media or what not: It is perfectly applicable in cases of nystagmus when retinoscopy becomes so difficult, and when the vision is too poor to make the test with trial case of importance.

Ridgeway's Chromatic Test. - This test is based upon the phenomenon of chromatic aberration. A spherical lens decomposes white light that passes through it very much in the
 same way as does a prism; into its constituent parts. It will be recalled that the red rays are the least deviated from their path, and the violet or blue rays the most refracted. If white light is intercepted by a piece of cobalt blue glass, only the red and blue rays pass through, the other spectral colors being absorbed within the glass. Ridgeway's chromatic glass is shown in the cut. The central portion of the glass is thicker, making its absorbing properties greater, being composed of several layers of blue glass separated by white glass. This glass is placed before the eye to be tested. Through it the patient looks at a circle of light -a small hole in an opaque screen -at a distance of twenty feet, placed directly in front of his eye. Only the red and the blue rays pass through the glass into the eye. The blue light undergoes more refraction as it passes through the dioptric media than the red, and is, therefore, brought earlier to a focus. If the eye is emmetropic the red and the blue rays cross each other upon the retina. The circle of white light, therefore, appears purple, a mixture of the red and blue light. If the diffusion circle of red is smaller than that of the blue, the light will seem to have a red center and a
blue fringe, while if the blue focusing is more accurate upon the retina, the opposite condition will pertain, that is, the circle of light will have a blue center and a red border. In hyperopia the center is blue, and in myopia the center is red, with a border of the other color. The figure explains.

The size of the colored areas depends upon the amount of the ametropia. Thus, it will be seen by referring to the figure that the size of the colored areas depends upon the position of the retina. The amount of ametropia is determined by the lens that will correct the aberration, making the circle of light appear a homo-

geneous color. This is a very delicate test, but cannot be relied upon, save in the simpler errors, as the test is too complicated to give very definite information in astigmatic errors. It would not be so if the patient was always able to tell the examiner just how the light appears to him, but the ability to do this varies very much with different individuals. In astigmatism what is known as a chromoscope is used. The chromoscope consists of a card about two feet square, with a dial divided into degrees. In the center of the dial
there is a circular opening three eighths of an inch in diameter, also a rotating ring having an indicator or pointer through the center of the opening and extending across it. The indicator turns with the ring. See figure.

The card is placed at twenty feet in front of the patient so that the light from an Argand burner shines through the opening. The eye not under test is covered and before
 the other one is placed Ridgeway's glass. If the light through the opening appears circular, no astigmatism exists, but if the light appears to be drawn out into an ellipse, astigmatism is present. In simple hyperopic astigmatism the circle of light is elongated, having a blue center and a red border, or is in the form of a cross figure, the blue extending out from each side. The hand of the chromoscope is then rotated to the longest axis of the light area and one principal meridian read off from the dial. The strongest convex or the weakest concave cylinder that will render the figure of the light circular gives the amount of the astigmatism. In simple myopic astigmatism the elongated figure has a red center with blue extremities, the longest axis indicating the meridian of the astigmatism. See figure for the appearance of the light in different forms of astigmatism.

Dr. Hotz's Astigmatism Test. - This test depends upon the apparent shape of a circular area of light to the astigmatic eye. The drawing on next page illustrates Dr. Hotz's astigmometer.

The instrument consists of a hard-rubber plate to which is attached a rotatory disc of seven centimeters diameter. In the disc are two small circular apertures, four millimeters in diameter, and upon its edge a small arrowhead. The centers of the apertures and the head of the arrow lie in the same radius. When the disc is rotated the pointer always indicates upon the scale the meridian in which the two apertures lie. Each hole is covered by a thin piece of celluloid
upon its posterior side to give a uniform illumination. The diffused daylight from a window or the light of an Argand burner may be used. (The two apertures may be replaced by two circular black spots upon a white ground, giving about as good results.) The bracket should be fastened to the wall so that the two apertures are about on a level with the eye of the patient. The patient should sit at fifteen or twenty feet distance from the instrument and the latter


Hotz's Astigmometer.
should be turned full face towards him with the indicator at ninety degrees. The spherical lens that improves the vision should be before the eye under the test for astigmatism. The patient must look steadily at the two holes in the disc to ascertain what shape they seem to have. If the holes appear oblong or distorted in any manner astigmatism is present. The direction of the elongation marks out one of the two principal meridians of the eyeball. If the elongation of the perforations is in an oblique direction, the exact
degree of inclination can be ascertained by turning the disc to the right or to the left until the long axes of the two oblong holes are seen to be continuous, or lie in the same radius; the arrow will then indicate the obliquity of the meridian. Astigmatism does not always produce an oblong distortion of the holes. They may appear like half moons, or diamond-shaped, or round holes with a light line through them ; sometimes each hole appears as two, in which case the position of the secondary hole indicates the direction of the astigmatic elongation. Dr. Hotz claims for the instrument that it will detect astigmatism in 92.5 per cent. of cases. The writer's experience has not substantiated this claim. As an adjunct it is a very useful test.

## CHAPTER XXIII

## OPHTHALMOSCOPY IN MEASURING REFRACTION ERRORS

Objective Methods of Estimating Errors of Refraction.-In the objective examination of the refraction of an eye we employ ophthalmoscopy, retinoscopy and ophthalmometry. The latter is of utility only in astigmatic errors as commonly applied, consisting in the mensuration of the corneal curves.

Ophthalmoscopy. - The estimation of the sort of refraction of an eye with the ophthalmoscope may be done either by the direct or by the indirect method. The former is the one usually employed and is upon the whole the most reliable and accurate. In either case the accommodation of the patient must be in abeyance through the use of a cycloplegic and that of the observer voluntarily relaxed. The quickest way to learn to relax one's accommodation is to forcibly raise the upper eyelids and try to stare at the eye under examination as if you were trying to look through it at some distant point behind the patient's head. Both eyes must always be kept open, as it is nearly impossible to completely relax one's accommodation if one eye is closed. In closing the eye there is a tendency to action on the part of the ciliary muscle ; as the two eyes work together it is impossible to accommodate with the one and to relax accommodation with the other one.

Information Gained by Use of the Mirror Alone.-The observer must have his own ametropia corrected by spectacles if any exists. He then seats himself opposite to and in front of the patient at a distance of two or three feet. The patient must look to the left eye of the observer's head when the left eye is under observation, and to the right when the right eye is examined. The light is now reflected into the eye and keeping the fundus well illumined the observer moves his head from side to side.
I. If nothing more than a red reflex is seen, or at most a blurred image of some portion of the fundus vessels or optic nerve, the eye is emmetropic or very slightly myopic. In the medium and higher degrees of ametropia details of the fundus are easily observed.
2. If the details of the fundus appear to move in the same direction as the observer's head the eye under examination is hyperopic.
3. If the vessels are visible in one meridian only, and move with the observer's head, there is hyperopia in one meridian only, in that at right angles to the one in which the vessels are visible. There is then present simple hyperopic astigmatism.
4. If the image of the disc or vessels appears to move in a direction opposite to that in which the head of the observer is moved, there is myopia.
5. If the details are seen in one meridian only, and they appear to move against the head, there is present simple myopic astigmatism ; the myopic meridian being at right angles to that in which the vessels are visible.
6. If the fundus details are seen moving in one direction in one meridian and in another direction in a meridian at right angles thereto, there exists mixed astigmatism. This error is, however, very difficult to diagnose by the movement of vessels or fundus details.
7. If the details of the fundus, instead of moving regularly and evenly across the pupillary area, move slowly across the center of the pupil, and rapidly and irregularly at the periphery, giving the appearance of rotating bent spokes of a wheel, there exists irregular astigmatism.

These tests are not adapted for the estimation of the amount of refraction error present, but are of diagnostic import only. The accommodation of the observer must be active to properly discern the movement of the fundus details. Rays of light from any point of an illuminated fundus pass out of the eye in a cone of light, in which the rays have a certain relation according to the kind of refraction present. In emmetropic the rays pass out of the eyeball in parallel paths, as the retina, the immediate source of light, lies at
the principal focus of the emmetropic eye. The light leaves the hyperopic eye in diverging paths as the retina lies anterior to the principal focus of the dioptric system of the eyeball. And, in myopia the light passes out in converging paths, as the retina lies posterior to the principal focus.

The figure represents an emmetropic eye ; 1, 2 some object in its fundus ; $N$, its nodal point. The light passes out of the eyeball in cylinders of parallel rays, which diverging soon leave a space between

them, as $S$, in which there are no rays from either of the points i and 2. If the observer's head is in this space, it is evident that he can not get an image of the points 1 and 2. He will, however, receive parallel rays from some luminous point of the object, or perhaps two cylinders of rays may run so close together that on entering the eye of the observer may form an image of the portion of the object included by them. Such an image is rarely obtained, because it requires absolute suspension of accommodation, which is rarely met with.

In hyperopia the rays pass out of the eyeball in diverging paths, from every point in the fundus, as rays $a, b$ and $c$. The cones of light composed of them entering the eye of the observer appear to emanate from the points $I^{\prime}$ and $2^{\prime}$, behind the eyeball, the extremities of the virtual image of the object $\mathrm{r}, 2$. The image is seen upright. All motion of the fundus details is judged according to the direction of apparent displacement in regard to the plane of the iris, the latter being stationary and seen at the same time. Thus, if a finger of each hand be held behind one another, before the eye, and the head
moved from side to side it will be seen that the front finger appears to move against the movement of the head in regard to the rear finger while the rear finger moves with the head in regard to the front finger. The image of the fundus in hyperopia is posterior to

the iris, and when the head is moved from side to side the image apparently moves with the head of the observer in regard to the iris. In myopia the image of the fundus is formed at the far-point of the eye, where the returning rays converge and cross. The image being in front of the iris in this case causes it to move in a direction contrary to the movement of the head of the observer. See figure.


In simple astigmatism we do not see any vessels that run at right angles to the emmetropic meridian, because waves of light from their transverse planes pass out of the eyeball in diverging cylinders of light, just as in emmetropia, the same explanation holding for both, whereas waves from the transverse planes of vessels running at right angles to the myopic or hyperopic meridian produce in the
former case a real inverted image and in the latter a virtual erect image of the vessels.

There follows then the same rule of movement that pertains in simple hyperopia or myopia. In cases of common myopic astigmatism if the observer uses his accommodation, vessels at right angles to the most myopic meridian will be distinctly seen much closer to the patient's eye than those in the opposite direction. Rays from the transverse planes of the former converge and form an inverted image much sooner than do those from the latter. In compound hyperopic astigmatism we find that for the vessels at right angles to the most hyperopic meridian more accommodation is required than for those in the opposite direction from a given distance.

A greater extent of the former than of the latter vessels is however at the same time visible. Rays from the former are more divergent, and the cones of light take longer to separate. In mixed astigmatism images of the vessels may be seen sometimes inverted and sometimes erect according to the meridian through which the rays emerge, and varying with the observer's accommodation. The details are more visible than in emmetropia but are nevertheless very indefinite (Morton). The manner of estimating the amount of refraction error by the direct method will now be considered. The patient and the physician seat themselves as for the direct examination of the fundus of the eye. The accommodation of the patient must be relaxed by the use of a cycloplegic, the pupil being expanded by the same drug. The accommodation of the observer is voluntarily relaxed. His refraction error must be corrected by proper spectacle lenses or by revolving back of the sight-hole of the ophthalmoscopic mirror the appropriate lens strength. The former is the better method, as then no deductions need be made from the reading of the ophthalmoscope, after the completion of the test to ascertain the amount of refraction error present in the eye under observation. With no lens behind the sight-hole of the mirror the examiner throws the light into the patient's eye, and then moves closer and closer towards the eye until the ophthalmoscopic mirror is nearly in contact with the cornea. If
the observer is exerting no accommodation there will be seen nothing save a very blurred image of the fundus or only a red glare, unless the eye under examination is emmetropic, in which case the details of the fundus will be distinctly seen. In emmetropia the vessels and the disc can not be seen furthermore, if the accommodation of the observer is exerted, or if a plus or minus lens is revolved behind the sight hole of the mirror.

The dotted lines in the figure represent the path of light from the mirror, illuminating the fundus of the eyeball $E$, over the area $a b ; O$ is an object within that area from which light emanates along the continuous lines. The rays from the upper end of object $O$ pass out of the eye parallel as $x x^{\prime}$, and are focused at the $O^{\prime}$ below in the

eye $E^{\prime}$. Rays $y^{\prime} y^{\prime \prime}$ emanate from lower portion of object and are focused in $E^{\prime}$ above. As the retina of $E$ lies at the principal focus of the dioptric system, the light passes out in parallel paths and as $E^{\prime}$ is emmetropic they are focused accurately upon the retina of $E^{\prime}$ without the need of accommodation. The observer's eye therefore sees distinctly the details within the eye $E$. Any lens back of the mirror would of course spoil the image, as the rays passing through it would have a different relation imparted to them and they would no longer be focused upon the retina of $E^{\prime}$.

Test for Myopia. - In myopia no details of the fundus of the observed eyeball can be seen with or without accommodation. It is necessary to place a concave spherical lens behind the sight-hole of the mirror in order that a view of the fundus may be gotten. The returning rays in $M$ are convergent and such rays are focused anterior to the retina of the normal eye. Accommodation on the part of the observer still further shortens the focus. A concave lens causes the rays to diverge and when they are rendered parallel by the proper lens strength, the observer sees distinctly the details of the fundus of the observed eye, as the former is adjusted for parallel rays of light. The minus lens back of the ophthalmoscope renders

the eye under test a little more hyperopic each time its strength is increased, until the amount of hyperopia produced is equal to the amount of the myopia originally present.

The ophthalmoscope should be held against the side of the nose with the mirror perfectly vertical and the lenses are revolved into place by turning the small milled wheel at the side of the instrument with the index finger, turning it to the light to get plus lenses in and to the left for the minus lenses. The instrument should be so constructed that one may know what lens is in place without taking the instrument down from the eye. Morton's improved ophthalmoscope is the most convenient. The figure above shows how minus lenses correct myopia and indicate the amount of error.

Rays $a$ and $b$ proceed from eye $M$, in converging paths. Minus
lens $L$ renders them parallel as $c, d$; they are then brought to a focus upon the retina of the eye $E$.

In hyperopia the details of the observed eyeball may be seen clearly only when the observer uses his accommodation or when he revolves a convex sphere behind the sight-hole of his ophthalmoscope. The diverging paths of light as they pass out of the observed eye into the observer's, are rendered parallel by accommodation, but if the ciliary muscle is at rest come to a focus posterior to the retina and only a blurred picture of the details of the fundus of the observed eye is gotten. See figure below.

Rays $a, b$ and $a^{\prime} b^{\prime}$ leave the hyperopic eyeball from points $x$ and $y$ of the fundus, respectively, in diverging paths. They are rendered

parallel by the convex lens, and are then focused by the dioptric media of the eye $E$, upon its retina, giving it a clear picture of the fundus of $H$.

Astigmatism can be diagnosed from the apparent shape of the optic papilla, which instead of round appears more or less oval. In the erect image (direct method) the long axis of the oval corresponds with the meridian of the greatest refraction, that is, in myopia the meridian of greatest myopia and in hyperopia the meridian nearest emmetropia (least hyperopia). As a rule the long axis of the oval is vertical. In the inverted image the direction of the oval is at right angles to the meridian of the greatest refraction, provided the objective lens is nearer the eye than its own focal length. Astigmatism
is furthermore suspected when an undilating retinal vessel is seen clear and blurred in parts. This condition is not to be confused with an effusion into the retina. It is differentiated by the fact that a concave or a convex lens back of the sight-hole of the ophthalmoscope will bring into distinctness the blurred portions of the vessel, which would not be the case if an exudate was present. The appearance of a retinal vessel in astigmatism may well be imitated by looking at a wavy line through a strong cylindrical lens. The quantitative test of astigmatism by the direct method is to find the lens that will clear up the vessel in different directions. "The small vessels about the macular region are the best ones to focus upon. The lens that brings the vessel running crosswise into clear view denotes the refraction of the eye at right angles to the course of the vessel, and the difference in the strengths of lenses needed to clear up a vessel in its vertical and horizontal portions, or in any other two directions at right angles to each other, represents the amount of the astigmatism. Thus a vessel that runs up and down is seen best with a plus 2 D. S., and when it runs crosswise, best with a plus i D. S. The amount of astigmatism is $2-I=I D$., and the correcion is written as follows :

$$
+ \text { I D.S. } \bigcirc+\text { I D. C. Ax. } 90^{\circ} .
$$

The above is a case of compound hyperopic astigmatism. If minus lenses are needed for both meridians, the case is one of compound myopic astigmatism, and mixed when a minus lens is needed for one and a plus for the other principal meridian. The mensuration of the refraction error is not difficult in simple hyperopia or myopia, but in astigmatism and especially if the principal meridians are oblique, the test is very difficult and very few are masters of it enough to rely upon it. In astigmatism it is very difficult to measure closer than I D. Abroad this method of correcting refraction errors is used a great deal, but in this country where we care for accurate results in refraction work, and correct even as low as . 12 D., ophthalmoscopic correction is not considered of very great importance by the refrac-
tionist. By ophthalmoscopy the principal meridians are denoted by the direction of the best and poorest seen vessels in the fundus of the eye, without any lens, or with the weakest one that clears up the vessels running in any given direction.

Indirect Method.-After ascertaining the direction that the vessels of the fundus of the eye seem to move, when the observer moves his head from side to side, the objective lens is held before the eye. If the objective lens be made to approach and to recede from the eye, it will be noticed that in the emmetropic eye there is no change in the size of or shape of the optic disc. If on moving the lens backward and forward there is a change in the appearance of the disc, we are dealing with ametropia. In hyperopia it will be seen that on withdrawing the lens from the eye under observation that the image of the disc becomes smaller (small or short eyeball, smaller image) and in myopia that the image apparently increases in size (large eye, larger image). The explanation of this is found in the fact that the relative sizes of image and object are to each other as their distances from the lens. To find the distance of an image from the lens we have the following formula: $1 / d=1 / f-1 / D$, in which $d$ is the distance of the image from the lens, $f$ the focal length of the lens, and $D$ the distance of the object from the lens. Let $f=4 \mathrm{~cm}$. In emmetropia, the light issuing from the fundus of the eye passes out in parallel paths, as if emanating from an object situated at an infinite distance. It is of this supposed object that we get an image of the disc by means of the objective lens. In emmetropia it matters not where the lens is held, the parallel rays go to form the image at the principal focus of the lens, the relative distances of the image and the object from the lens remain constant, therefore the size of the image does not change as the lens is withdrawn. In hyperopia rays emerge from the eyeball as if emanating from an object at a certain distance behind the eye. It is of this virtual object that we get an image with the convex lens held close to the patient's eye. Suppose the lens to be held at 6 cm . from the object, the distance of the image from the lens is according to formula $1 / d=1 / 4-1 / 6=1 / 12$, or $d=12 \mathrm{~cm}$.

The ratio between the distance of the image and object is: $d / D$ $=12 / 6=2$.

If we now withdraw the lens to 12 cm ., the distance of the image from the lens will be: $1 / d=1 / 4-1 / 12=1 / 6$, or $d=6 \mathrm{~cm}$. ; the ratio in this case is $d / D=6 / 12=1 / 2$. The ratio of the distance of the image from the lens as compared with that of the object from the lens being greater in the first case than in the second, so is the size of the image. In myopia rays emerge from the disk converging, and form at a certain distance in front of the eye a real inverted image of the disc. This image is to be regarded as the object of which we get an image when the objective lens is held close to the patient's eye. In this case the object and the image are on the same side of the lens. The object being on the opposite side of the lens from which the rays proceed, it is customary to make i/ $D$ negative, so that the same formula may apply in each case. The formula then becomes

$$
\mathrm{I} / d=1 / f-(-1 / D)=1 / f+1 / D .
$$

If the lens is placed 12 cm . nearer the eye where the image would be formed we have the image formed at 3 cm . from the lens.

$$
1 / d=1 / 4+1 / 12=4 / 12=1 / 3, d=3 \mathrm{~cm} \text {. Ratio } d / D=1 / 4 .
$$

If we now withdraw the lens, that is towards the object, but from the eye until it is 6 cm . from the former, the distance of the image will be nearly two centimeters, given a ratio of $1 / 3$. As the ratio is greater in the second case, so is the image. This explanation holds good so long as the objective lens is not withdrawn beyond the far point of the eye plus its own focal length ; for hyperopia so long as the focal power of the lens is greater than the degree of hyperopia. The latter is always the case as the objective lens is usually of a three-inch or less focal distance, save in acquired hyperopia after the removal of the crystalline lens. In very high degrees of myopia, however, if the lens be further from the eye than the aerrial inverted image of the fundus, plus its own focal length, the erect image of the fundus is formed between the lens and the observer, the variations in the size of which
are subjected to the same rules as those for the inverted image in hyperopia (Morton).

The Quantitative Test of Refraction by the Indirect Method of Ophthalmoscopy. - If we know the distance of the image of the fundus of an eye from the objective lens, and the focal length of the latter, we can without difficulty deduce the refraction of the eye. The method of measuring the refraction of an eye by the indirect method of ophthalmoscopy is that of Schmidt-Rimpler. The method has never become very popular. The observer makes himself artificially myopic by placing a convex lens back of the sight-hole of the ophthalmoscope. A convex 5 D. S. is the one generally used, which makes the eye of the observer just $5 \mathrm{D} . \mathrm{M}$. if he has an emmetropic eye. Such an eye cannot see anything distinctly further off than the distance of its own far-point, which in this case lies at 20 cm . in front of the eye. The observer then throws the light into the eye under test. An objective lens of known strength, preferably a io D., as that strength makes the deduction easier, is held before the eye.

From the usual distance at which the fundus of an eye is examined the artificially myopic observer does not see anything but a red glare from the chorioid. As he moves his head closer to the patient the image of the fundus comes clearly into view, when he is at a distance from it equal to the distance of his far-point, that is at 20 cm . from the image. Knowing the distance of his far-point and the distance of his eye from the objective lens, which latter is indicated upon a tape line that the observer holds stretched between the ophthalmoscope and the objective lens, he can ascertain the position of the image in regard to the objective lens. The image of the fundus is situated at a greater or a less distance from the objective lens according to the state of refraction. If we use a convex objective lens of io D. the inverted image of the emmetropic eye will lie at ten centimeters from the lens, that is at its principal focus, as rays of light pass out of the emmetropic eye parallel. The image of the fundus of the hyperopic eye will be further away and that of the myopic eyeball nearer the lens than 10 cm . Each centimeter nearer
the lens adds one diopter of myopia, and each centimeter further off I D. of hyperopia. For example, if the image is situated at 6 cm . from the lens there is 4 D . of myopia ( $10-6=4 \mathrm{D}$.); if 20 cm . away $20-10=10$ D. of hyeropia and so on.

The Application of the Test.-The observer is at 30 cm . from the objective lens, the furthest point at which he can distinctly see the image of the fundus. What is the refraction of the eye? 30 cm . (distance of observer from lens) -20 cm . (distance of image from observer's eye) $=10 \mathrm{~cm}$. (distance of image from objective lens). As 10 cm . is the focal distance of the lens in use, the eye under test is emmetropic, inasmuch as the image of the fundus is formed at the principal focus of the objective lens. This method is not practical in astigmatic errors and is less accurate than the direct method. For this method, as well as for the direct, the accommodation of the observer's eye, as well as that of the patient, must be in abeyance.

## CHAPTER XXIV

## RETINOSCOPY

Retinoscopy, skiascopy or the shadow test, is the most accurate and the most reliable test at our disposal for detecting and estimating the amount of refraction error of an eye. It is independent of the good will and intelligence of the patient. It does not rely upon the answers of the patient as does the subjective test with the trial case, and the accuracy of the test is not influenced by the presence of an error of refraction in the observer, so long as he can see well enough to discern the movement of light and shade in the pupil of the observed eye. This method of examining ocular refraction was described by Cuignet in 1873 and called by him keratoscopy. Parent especially developed the method and was the first who gave the correct explanation of the test. By retinoscopy we measure the amount of myopia resident in the eye or that produced by a convex spherical lens placed before the eye, by ascertaining the position of its farpoint. Light is reflected into the eye under examination from a mirror ; the illuminated area in the eye is seen to move as the mirror is tilted, and according to the direction of this movemont, which varies with the position of the observer in regard to the far-point of the eye, the latter is located.

If the observer is nearer to the observed eye than its far-point, the movement of light in the eye is in one direction, while if he is further away than the far-point the movement is in the opposite direction. The test may be performed with either a concave or plane mirror. There are many kinds of retinoscopes on the market. The sighthole of the mirror should be very small, not exceeding 1.5 mm . in diameter, and made by scratching the amalgam off from the back of the mirror in preference to boring a hole through the mirror. If the sight-hole is bored through the glass, one is annoyed by reflection
from the sides of the bore, unless they are kept well blackened. The metal backing to the mirror should be very thin, so that there is no room for the collection of dust. The mirror should be provided with an arrangement so that its reflecting surface may be diminished or increased, as a different size mirror is often needed in different stages of the test. The one shown in the cut is that devised by Dr.


Jackson ; it is as cheap as any and at the same time the most convenient one to be had. In figure no. I only a small central area of the mirror is exposed, while in figure no. 2, the entire mirror.
If the far-point of the eye under examination lies at the principal focus of the convex lens before it, the eye is emmetropic. The light is reflected into the eye, the fundus is illumined, the returning rays pass out of the eyeball into the lens before it parallel and converging are brought to a focus at the principal focus of the lens. This point is the far-point of the eye rendered myopic by the convex lens before it. Therefore emmetropia exists if the far-point of the eye rendered myopic lies at a distance from the lens equal to its own focal distance. The fact that the returning rays of light from the eyeball are focused at the principal focus of the lens is evidence that they entered the lens parallel, emitted thus from the eye.
$a$ and $b$ are rays passing out of eyeball ; $F$, focus of $a^{\prime} b^{\prime}$ through $L$. $d e=1 \mathrm{~m}$. The strength of $L$ is known to be 1 D . Therefore $a^{\prime} b^{\prime}$ are parallel on entering $L$. In this case the eye is rendered I

D. M. Hyperopia is denoted if it takes a stronger lens than a plus I D. to render the eye under observation I D. myopic.

Light emanates from the illuminated fundus of the eye $H$, in diverging paths as $a b$. Portion $l^{\prime}$ of the convex spherical lens $L$, renders the rays parallel, and therefore corrects the hyperopia, portion $l$ brings the rays rendered parallel to a focus at a meter's distance. The strength of $l$ is then known, being i D.; the strength of

$L$ is known, having been selected by the examiner-in this case it is supposed to be a +4 D. S. lens. The amount of hyperopia is then:

$$
4-1 \mathrm{D} .=3 \mathrm{D} .
$$

The far-point of the myopic eyeball in retinoscopy is called the point of reversal. It is likewise the external conjugate focus of the dioptric system of the eyeball. The point of reversal is so called, because when the observer is nearer the eye than this point the
illuminated area in the eye appears to move in one direction and when beyond this point the movement is the reverse, as the skiascope is turned about its axis. The catoptric image of the light is called the immediate source of light to the eye under observation in contra-distinction to the original source of light to the mirror. The light that illumines the eye is the reflected image of the original source of light. The original source of light should be shaded, and the room made perfectly dark to exclude all extraneous light. The most convenient light-shade is that of Dr. Thorington shown in the cut. It consists of an iris diaphragm attached to a blackened asbestos chimney. The size of the opening in the shade is regulated by a small lever that projects from the side of the diaphragm. An Argand burner is the most convenient source of light. It should be fastened to an upright or adjustable wall bracket, so that the height of the light can be altered,
 and drawn closer or pushed further off as the observer desires.
In the great majority of cases under fifty years of age it is necessary to employ a cycloplegic, especially in astigmatism, as it is impossible to accurately measure a quantity that is apt to change from time to time during the test. To apply retinoscopy with the greatest ease furthermore needs a pupil moderately dilated. Like other methods it will not give as accurate results if the pupil is very narrow, and on account of aberration and irregular astigmatism that usually exists near the margin of the lens and the cornea, very wide dilatation of the pupil introduces difficulties. The pupil should never be less than 4 mm . in diameter. If this is the case in the elderly cocaine may be used as the mydriatic.

If a mydriatic is not used for some reason or other (as in eyes in which one fears an attack of glaucoma) the accommodation is rendered fixed, and relaxed to a certain extent by having the patient look at large letters hung at a distance of twenty feet, in a partially
darkened room, especially if the eye not under the test that sees the letters is fogged by a strong convex lens. The patient should look a little to one side of the mirror so that the pupil will be as large as possible.

Description of the Test with a Plane Mirror. - Let us begin with an emmetropic eye. The examiner reflects the light into the eye and turning the mirror from side to side (about a vertical axis) notices that the light area in the eye moves with the rotation of the mirror, and with the light on the face of the patient, that is when the mirror is turned to the right the light moves to the right in the eye as well as upon the face of the patient. The apparent movement of the light in the eye, that is as it appears to the observer, is the same as the real movement of light in the eye under observation, that is as it really moves across the fundus of the eye, and would appear to an observer if he were behind the eye and looking through the sclera and chorioid. The reason that the light moves with the rotation of the mirror is as follows: When the mirror is tilted to the left the light is reflected to the left and vice versa.

In the figure ( 1 ) is the image of the original source of light when the plane mirror is in position (I) ; the light entering the eye as if it originated at image (i), illuminates the fundus over the area (I). When the mirror is rotated or turned up to position (2), in the direction of the arrow, the immediate source of light descends to (2), illuminating the fundus now at (2). The light in the eye moves in the same direction as the mirror and appears to the observer to move in the direction it does, as the rays that return from the emmetropic eye never change their relation to each other, remaining parallel, and those that emerge from above remain above until they enter the eye of the observer. In emmetropia, then, the light appears to move in the direction of rotation of the mirror or with the light upon the face at all distances from the eye. The same is true in hyperopia, as in it the rays returning never cross or change their relation to each other, so that those above become below or vice versa, because the light leaves the hyperopic eye in diverging paths. The observer will
notice that the movement of the light area in the eye is always erect, that is the apparent motion is the same as the real motion of the light, at all distances from the eye. If the observer moves backward and forward, keeping the light in the eye as he rotates his mirror, he will notice that the movement of the light is ever with the rotation of the mirror. This occurs in emmetropia and hyperopia.

With the ophthalmoscopic mirror alone at a distance of several feet from the eye, a view of the fundus is obtained in an erect image, so

by retinoscopy when the light in the eye really moves down, up or how not, its apparent motion is the same as the real motion because the observer is getting an upright image of the fundus.
In myopia the same rule as to the part of the fundus of the eye illumined holds, that is the real movement of light in the eye is with the rotation of the mirror. The apparent movement is not the same at all distances from the eye however as in emmetropia and hyperopia. The rays of light emerging from the myopic eyeball converge, and crossing in front of the eye at the point of reversal they change their
relation to each other, so that those that come from above enter the eye of the observer below and those that came from below, enter the eye of the observer above, giving him the impression that they
 originated above in the eye under observation. When the apparent movement of the light in the eye is opposite to the real movement it is said to be an inverted movement. In myopia so long as the observer is nearer the eye than the point at which the emitted rays cross, the apparent and the real movement of light are the same (with the mirror), but when the point of crossing (farpoint of eye) is nearer the eye than the observer, the motion of light in the observed eye is inverted, appearing to move contrary to the direction of rotation of the mirror, that is, when the mirror is turned to the right the light seems to move to the left in the eye, and vice versa. The explanation is according to the following figure (after Jackson).

In the figure $C$ and $D$ are the external conjugates of $A$ and $B$, areas in the fundus of the eye. $\quad N$ is a point nearer the eyeball than these reversal points and $N^{\prime}$ one further away. If the eye of the observer is at $N$, the light that comes to it from point $B$ in the lower portion of the observed eyeball is that which passes out of the pupil of the observed eye below and is turned up.

The observer places the point $B$ in its true position, not taking into account the refraction of the light as it passes from the observed eye to his own, he projects the point $B$ as lying along a straight line at point $b$. He then sees what is below, as below. The light that comes to his eye from the point $A$ is thought to lie along the line $N a ; A$ that is above is then seen above. If the observer now moves
to the point $N^{\prime}$ beyond the points of reversal, the conditions have changed. The ray that comes to his eye now from $A$ above is the one that passes through the lower part of the pupil of the observed eye and is bent up; $A$ is then supposed to lie along a straight line at $a^{\prime}$, that is while $A$ is really above it appears to be below. During the test then when the light in the eye moves up it appears to move down. By this fact one knows that the point of reversal lies between him and the patient, or as we say the movement has become reversed. In myopia there is formed at the far-point of the eyeball an inverted real image of the fundus, which can be seen by the observer. As it is inverted so the movement of light in the eye during retinoscopy appears inverted seeming to be below when really above and vice versa. (In retinoscopy we do not view the image of the fundus, but movement upon the fundus.)

The rapidity with which the light appears to move across the pupil in retinoscopy depends upon the rapidity of movement of the light area upon the retina and upon the magnification of the latter. The rapidity of the real movement of the light upon the retina depends upon the rate of movement of the mirror and the distance of the mirror from the eye under examination, upon the distance of the original source of light from the mirror and upon the distance of the retina from the nodal point of the eye. The rate of movement of the mirror and the distance of the original source of light from the mirror determine the rate of movement of the immediate source of light, being quicker the faster the mirror is rotated and the nearer the original source of light to the mirror. The excursion that the immediate source of light makes is limited by the width of the mirror, and the extent of the movement of the immediate source of light upon the retina depends upon the relative distances of the mirror and the retina from the nodal point of the eye under test.

On account of the relative distances of the retina from the nodal point the extent of the movement of light upon the retina, other things being equal, is least in highest hyperopia and greatest in highest myopia. Practically the rate of movement of the light area
on the retina depends more upon the extent to which the real movement of light is magnified than to the actual rate of the real movement. The retina as it is viewed from different distances is seen under different degrees of magnification. When the eye of the observer is at the point of reversal all the rays from a single point in the fundus converge to the nodal point of the observer's eye, so that every point of illumination upon the retina appears to occupy the whole pupillary area. The retinal illumination therefore is indefinitely magnified.

It is disputed by some that the apparent rate of movement upon the retina depends upon the magnification of the movement when close to the point of reversal. They say that the apparent enlargement of the reflex is due to the increased diffusion upon the retina of the observer and not upon that of the observed, such diffusion reaching its maximum when the light from the observed eye comes to a focus just posterior to the cornea of the observer's eye, and the rate of movement depends upon the nearness of the reversal point to the eye of the observer. That the explanation of magnification accounts for both the diffusion and quick movement at the reversal point is proven by study with a convex lens and dot upon a card, as pointed out in the following pages. Diffusion upon the retina of the observer's eye renders the image of the light indistinct proportionately throughout the test, as can be demonstrated by rendering oneself artificially myopic with a convex lens placed back of the sight-hole of the retinoscope, but does not make the movement of light one bit quicker. The pupil of the observed eye and the area of retinal illumination appear enlarged through the convex lens, but they bear the same relation in size to each other as they do without the convex lens.

The nearness of the point of reversal to the nodal point of the observer's eye is however partly the cause of the increased rapidity of motion in the observed eye, as the point of reversal is reached. A near object moving with the same speed as a distant one appears to cross the field more rapidly.

As the eye of the observer departs from the point of reversal, it receives rays of light from an increasing area of retina, and the retina is therefore seen less magnified. See figure from Jackson.
$A$ is supposed to be the point of reversal of the eyeball $M$. At $A$ the observer receives all the light from the point $a$, and this point therefore appears to occupy the entire pupil. If however the observer places his eye at the point $B$ from which rays would be focused at $b$ behind the retina, he will see in the pupil all the retina included between the points $m$ and $n$, the extent of the circle of dif-

fusion if the light emanated at $B$. Or if the observer's eye was placed at the point $C$ from which rays would be focused at $c$, he will be able to perceive the portion of retina included between the broken lines, the area upon which would be formed a circle of diffusion if the light emanated from the point $C$. It follows then that the nearer the observer's eye is to the point of reversal the more the retina and the real movement of light upon it are magnified, and that in consequence the swifter the apparent movement across the pupil. Hence the rate of the apparent movement of light in the pupil is quicker the nearer one is to the point of reversal.

The Form of the Retinal Light Area. - The real form of the light area on the retina except in certain conditions in astigmatic eyes is a more or less blurred retinal image of the original source of light, partaking of its form, with either a plane or a concave mirror. The reflection from a circular plane mirror when thrown upon a screen is seen to be circular, no matter what is the shape of the original source of light. The same is true of a concave mirror unless the screen is
placed at the focus of the latter, then an inverted image of the original source of light is formed upon the screen. But after the reflected rays from either mirror pass through the dioptric system of the eyeball there is formed a more or less blurred image of the original source of light upon the fundus of the eyeball. When a circular opening is used for the source of light, the area of retinal illumination


Form of Light Area Near the Point of Reversal.


Form of Light Area in High Ametropia.
is circular. When the immediate source of light (catoptric image from the mirror) occupies the point of reversal, the external conjugate focus of the retina, the focusing is most accurate and the area of illumination small, circular and well defined, but as the immediate source departs from this point, the focusing becomes more and more diffused the higher the ametropia, and the edge of the light area approaches a straight line, due to the overlapping of diffusion areas of the shape of the pupil of the observed eye but its edge never becomes straight save in cases of astigmatism. In no case does the observer see a distinct image-for his eye is focused for the pupillary plane of the observed eye, while the image that he observes in myopia is in front of, and in hyperopia behind this plane. The image is therefore seen vaguely, each point in it being represented by a diffusion circle, which as always corresponds to the pupil of the observer. The theory as outlined is that of Parent.

The explanation given to retinoscopy by Leroy, which is widely accepted throughout Germany, is in agreement with that of Parent. The illuminated area upon the retina is bordered by a shadow, upon which many place their attention during the performance of retinos-
copy. This shadow is caused by no light entering the eye of the observer from that portion of the pupil of the observed eye, or in other words the iris of the observer as Leroy suggests produces the shadow. See the figure below.
$x$ represents an illuminated area of the retina, from which starts a luminous cone. As the eye is supposed to be myopic, or rendered

so by a convex sphere, the light from $x$ converges to point $y$. The observer sees luminous only that part of the pupil which sends rays to it. The lower part of the observed pupil therefore appears unillumined as the light passing through it is intercepted by the iris of the observer. The curved form of the border of the illumination in the observed eye is not explained, however, by the form of the pupil of the observer. The form of the pupil of the observer plays no part, as the phenomena do not change if the observer looks through a stenopaic slit or a triangular aperture placed in front of his pupil. The form of each diffusion circle of the light area, shaped by the iris of the observer, has no influence upon the form of the border of the light area upon the retina of the observed eye, the distinctness of its image only being altered.

The brilliancy of the light in the pupil depends upon the brightness of the original source of light, and upon the extent to which the retina is magnified. The more nearly the light is focused upon the retina the brighter the illumination in the pupil. The luminosity is diminished as the point of reversal is reached by the increasing magnification of the retina, which causes the light from a smaller part of the retina to occupy the whole pupil. The brightest reflex is obtained not at the point of reversal but about i $D$. therefrom.

Practical Application of the Test. - The eye to be tested must have its accommodation at rest and pupil dilated by a cycloplegic, and the room made very dark. The light should be back of the patient, at the height of his eye and on the same side of his head as the eye to be tested. The opening in the shade that screens the light should be large at first and made smaller as one approaches the


Gruenig's Set of Hand Skiascopes. point of reversal ; the mirror also should be small when near the reversal point. The ordinary trial frame and test case may be used for retinoscopy, but it is much more convenient and conducive to good work to be provided with some sort of a refractometer, by which one is enabled to revolve before the eye lenses in quick succession without leaving one's position. To leave your seat every time a change in the lens before the eye is needed is tiresome and takes time. The refractometer of Lambert is as good if not better than any on the market. The Cross retinoskiameter, and the Meyrowitz improved refractometer are among the best. Many use the hand skiascope-the patient holds the slide in front of the eye and moves it up or down as the examiner dictates. It is hardly ever held properly and not nearly as convenient as an instrument that can be operated by the observer. The Lambert instrument consists of two superimposed metal discs,
eleven inches in diameter, one of which contains nine convex lenses ranging from +I . to +9 D., and ten concave lenses from -I . to - roD., while the other carries both the convex and concave fractional and io D . lenses. On the reverse side of the instrument is an arm,

carrying an eye-piece, which can be swung to either side of the disc, according to which eye is to be tested. Attached to this arm is a graduated cell in which the cylindrical trial lenses can be placed at
any desired angle, The combination of all foci is obtained in the same manner as in the Loring ophthalmoscope, and the graduated cell permits the rotation of the cylinders to all axes. The lenses are one and one half inches in diameter, and the instrument can be used for refraction work as well as retinoscopy. Attached to the eye-piece is an extra cell which will hold either a solid disc to cover the other eye in retinoscopy or a trial lens in testing both eyes for refraction.

Both discs can be revolved either independently or together, with


Appearances of Source of Light with Ridgeway's Glass.
one hand, at the will of the operator, when at a distance of one meter from the patient, by means of a gear movement operated by a rod and hollow tube. The rod, which turns within the hollow tube, operates the main disc ; the auxiliary disc is rotated by means of the tube. A quarter turn brings a lens, by consecutive increase or decrease, before the eye-piece.

The elbow joint, which supports the lens discs, rotates on the up-
right standard in such a manner that the lenses can be placed before either eye of the patient without any change of position.

The figures are in large black and white letters, and can readily be seen in the dark-room.

The Meyrowitz refractometer, one of the newer instruments upon the market, consists of a highly polished base to which are fixed two uprights which carry a number of discs, each disc containing twentyone lenses, ranging from 0.25 D . to 7 D . in both positive and minus lenses. In making the test the chin of the patient rests in an ad


The Cross Retinoskiameter.
justable chin-piece which is raised or lowered by a milled head, while the forehead is placed against the head-rest which connects the two uprights holding the discs. The pupillary distance is obtained by moving the two discs in a lateral position.

The discs are rotated by two rods having milled heads, the length of these rods gives the required distance of one meter between the lens discs and the eye of the operator. The objection to all instruments of this kind is that the lenses are at too great a distance from the eyes of the patient.

The mechanism is so arranged that a distinct click is noticed as each lens comes before the eye, and the lenses are changed with rapidity without disturbing the patient in any way.

Two of the discs contain plus lenses, and two contain minus lenses, the disc not in use being accommodated in two drawers in the base of the instrument.

The Cross retinoskiameter consists of two 7 D. convex and two 7 D. concave cylindrical lenses mounted in cells, with their axes at right angles to each other, each lens being inclined slightly on its axis from the perpendicular. The two concave lenses are stationary, while the two convex ones are movable, their cells sliding on rods and being controlled by a double cord forty inches in length. The pointers on the side of each tube show the kind and the focal strength of the lenses. The tubes revolve, changing the axes of the
 cylinders. The cells are operated either backward or forward by a simple devise. By the construction of the instrument the pupil of the observed eye is magnified several times, thus facilitating the test. A point about the instrument upon which stress is laid is that the lens strength before the eye is changed gradually and not by jumps as we do usually when we replace a . 25 by a .50 D . lens and so on, thus causing more acccommodation to be relaxed, but as it is necessary to use a mydriatic in retinoscopy, as in other methods of testing refraction to be sure that the eyes are at rest, this is of no especial importance, unless one wishes to test without the aid of a cycloplegic.

The Lambert refractometer is to be preferred. The instrument is placed before the patient with the eye to be tested behind the sighthole, and the other eye protected by a shield held in a clip attached to the eye-piece. The handle of the instrument, which is detachable, is put in place. The examiner takes a position of one meter or
slightly more from the patient, and reflects the light into the eye. He then sees the pupil filled with a red glare, and on rotating the mirror a movement in this light area is noticed. Some advise that the attention be placed upon the edge of the light or bordering shadow, but it is much easier to watch the movement of the light itself. The retinoscope is supported by resting the edge of the mirror against the side of the nose, and the handle of the instrument is held horizontally. Rotation can then be performed in any meridian with the greatest ease. On rotating the mirror from side to side the observer will note that the light in the eye moves in the same meridian unless oblique astigmatism is present. In astigmatism if rotation of the mirror takes place in a meridian forming an angle with one of the principal meridians of the eye, the light area in the eye moves in a meridian oblique to that in which the mirror is rotated.

This oblique movement is more decided in the higher forms of astigmatism, and therefore in mixed astigmatism as a rule. If the principal meridians are 15 degrees or so from the vertical and horizontal the light will be seen to move vertically when the mirror is turned from side to side, and obliquely from above downwardsfrom the upper nasal quadrant to the lower temporal, or upper temporal to lower nasal or vice versa, if the principal meridians are in the neighborhood of 45 and 135 degrees.

If the light moves in the same meridian in the eye as it does upon the face of the patient, there is no astigmatism present or the rotation of the mirror is across one principal meridian. If astigmatism is present, rotating the mirror across a meridian forming an angle with that of the primary rotation, will cause the light in the eye to move obliquely. Unless the error is over .50 D., this oblique movement can not be detected. If the light in the eye appears to move in the same direction as that in which the mirror is rotated, the observer moves further back, and if the movement continues erect, there is present either emmetropia or hyperopia in that meridian, but myopia exists, if as he recedes from the eye the movement becomes very indistinct and then inverted, that is against the rotation of the mirror or the
light upon the face of the patient. If the movement of the light in the observed eye is erect at all distances one differentiates between emmetropia and hyperopia in the following manner: Seated at a distance of 1 m ., the observer rotates before the eye a I D. S. lens, which if there is emmetropia present will impart to the eye i D. of myopia, and as the far point of the eye would then be anterior to the eye of the observer, the movement of the light in the pupil would now be inverted, that is against the rotation of the mirror, or in a direction opposite to the movement of the light upon the face of the patient. 'The observer now rotates the mirror in different directions, and if the movement of the light in the eye is always parallel to the axis of rotation of the mirror, the shape of the light always circular, inverted in all meridians, and not inverted by a weaker lens in any meridian, the test is concluded with emmetropia as the diagnosis.

In a case of hyperopia the movement of light and shade in the pupil is seen to be with the mirror, and it continues to be so with a plus one diopter lens before the eye. Suppose that the present case is one of 3 D . of hyperopia. The movement of light will continue erect as successive strengths of lenses are added until a plus 4 D . S. is reached, when the movement becomes inverted, being now against the rotation of the mirror, moving to the right when the mirror was turned to the left and so on. The eye is rendered myopic by the plus 4 D . S. lens and the point of reversal is brought to lie just in front of the eye of the observer. If it takes a plus 4 D . lens to make an eye one diopter myopic there must have been 3 D . of original hyperopia ( $4-\mathrm{I}=3$ D.).

Suppose that it takes only $a+.50 \mathrm{D}$. lens to reverse the movement of light in the eye, what is the error? Inasmuch as it takes a plus lens to reverse the movement we subtract 1 D. from the strength of the lens before the eye when the movement has become reversed, to obtain the original refraction of the eye, which would be $+.50 \mathrm{D} .-$ $. I=-.50 \mathrm{D}$. or .50 D . of myopia. If the eye under test has over one diopter of myopia the movement of the light will be found in-
verted without any lens before the eye at the distance of one meter and twenty centimeters, as the observer seated at that distance would be behind the point of reversal for 1 D. of myopia. Myopia less than i D. is detected as above, or as the observer moves further from the eye, the movement will become inverted just as soon as he passes beyond the far-point of the eye.

If when beginning the test in myopia the light in the eye is seen to move against the mirror, at the usual distance of 1 meter minus spherical lenses are added in increasing strengths until the weakest one is found that causes the movement of light to be erect, that is causes the light to move with the rotation of the mirror as it does in hyperopia and in emmetropia. There still remains i D. of myopia after the reversal of light occurs, the lens before the eye under the test only partially correcting its error. Then, in an eye, if it takes $a-2 \mathrm{D}$. lens to correct all of the myopia save i D. there must originally have existed 3 D . of myopia. So to obtain the amount of myopia we add one to the lens that is needed to reverse the movement of light in the eye, or to have the same rule apply as in hyperopia we say that we subtract - I D. from the lens of reversal, which is the same thing as adding one $(2-(-1)=3$ D. $)$.

Take for example a myopia of 6 D . At first glance the light moves against the rotation of the mirror as it does when - $1,-2$, -3 and -4 D. S. lenses are placed before the eye. When a -5 D. is placed before the eye the movement becomes erect, as the farpoint of the eye or the point of reversal has been brought to lie posterior to the observer's nodal point. The same lens causes the movement in all meridians to be erect, unless astigmatism is present.

Instead of the observer taking his position before the observed eye and adding lenses until the point of reversal has been brought to his eye, he may change the lenses less often and move backwards and forwards until he finds the place where the direction of the movement of light changes, marking the location of the point of reversal. The distance of this point in cm . divided into 100 cm . will give the amount of myopia. Thus, suppose that the movement of light is
erect nearer to an eye than 50 cm . and further from the eye than this point, inverted. The reversal then takes place in the vicinity of 50 cm . from the observed eye. This error is equal to 2 D . of myopia ( $100 \div 50=2$ D.). If the error is high, shown by the poor illumination in the pupil of the observed eye and by the slow movement of the light, as the mirror is rotated, a minus lens of approximate strength is placed before the eye, so as to remove the point of reversal further off, in order that its position may be more accurately determined.

The far-point of io D . of myopia lies anterior to the eye 10 cm . $(100 / 10=10)$ and that of 9 D., i 1 cm . anterior to the eye, a centimeter only separating the far-points of errors differing by a diopter, but let each case be nearly corrected, then the distance between the far-points increases. Suppose that the amount uncorrected equals a . 50 D. in one instance and 1.50 D. of myopia in another, then the farpoints lie at 2 m . and 66 cm . respectively. To accurately ascertain the distance of the point of reversal from the observed eye, the patient holds one end of a centimeter tape line at the outer angle of the eye, the observer holding the other end in the hand that holds the retinoscope. If the end of the line is weighted, it will not be necessary to take up any slack as the observer approaches the patient if the tape is held loosely in the hand. The furthest point at which the erect movement is seen is noted, and the nearest point at which the inverted movement is perceived is noted. The distance between these points is halved and the amount added to the greatest distance at which the erect movement was observed, and divided into 100 cm .

In hyperopia, instead of gradually changing the strength of the lens until the point of reversal is reached, one may by trial obtain a lens that over-corrects the case, leaving it myopic. The amount of myopia is then measured by finding the position of the far-point after the manner described. The amount of myopia is then subtracted from the lens before the eye, to obtain the amount of original hyperopia. Thus, suppose that we have a case of 4 D. H. A + 6 D. lens is placed before the eye by guess, the movement of light in the eye
is seen to be inverted at about a meter's distance. The observer then moves backward and forward as he rotates his mirror and at about 50 cm . from the eye he notices that the movement of the light changes. The amount of myopia then equals 2 D . $(\mathrm{roo} / 50=2 \mathrm{D}$.). 6 D . (lens before the eye) -2 D . (amount of myopia produced) $=$ 4 D. (amount of original hyperopia).

The method of maintaining one's position and adding lenses to the eye to bring the point of reversal to one meter's distance is rather more accurate than the method of changing one's position in search of the point of reversal. By retinoscopy it is possible to measure . 12 D of error but much practice is necessary to do that well, especially in astigmatism. Whenever there is any doubt as to which lens reverses the movement, select the weaker and then there is no danger of over-correction.

The astigmatic eye, myopic or rendered so by the addition of a positive spherical lens, has many points of reversal, but we are alone concerned with the points of reversal of the principal meridians. There are then two practical points of reversal in the astigmatic eye. The figure below shows a myopic astigmatic eye, with the two points of reversal, one for each principal meridian.
$M$ is the far-point of the eye through the meridian $m$, and $N$ the far-point through the meridian $n$. If the eye of the observer is nearer one far-point than the other, the retina is seen more magnified in the meridian of that far-point,
 each point of illumination upon the retina appearing to occupy the whole pupillary area. This with the unequal focusing of the light upon the retina in the two principal meridians of the eye causes the light to be drawn out into a band or ribbon running across the pupil, bounded by the linear shadow of Bowman.

As taught by Jackson and others, the band appearance is most
distinct when the eye of the observer is at the point of reversal for one principal meridian and the immediate source of light at the point of reversal for the other meridian. By reference to the figure above it will be seen that when the eye of the observer is at point $N$, the retina will be most magnified in meridian $n$, causing each point of light upon the retina to occupy the whole pupil, causing the illumination to appear in the form of a band in the vertical meridian. If the source of light is at $M$ the conjugate of the retina in the horizontal meridian, the focusing of the light crosswise through the meridian $m$ will be accurate, adding to the distinctness of the band. Then, with a plane mirror, after the strongest lens has been found that reverses the movement of light in all meridians of the eye, the observer moves closer to the eye until he finds the point of reversal for the most myopic meridian (with lens on) by noting the place where the movement becomes erect for that meridian. His eye is now at one point of reversal, the other lying behind him. Inasmuch as the reflected image from a plane mirror always lies behind the mirror the original source of light must be pushed further away (it having been close to the mirror up to this point), so as to throw the immediate source of light back to the point of reversal of the least myopic meridian of the eyeball. In myopia, after the lens is found that reverses the movement in all meridians, the observer must move backward to get his eye at the point of reversal for the most


Band of Light in Astigmatism. myopic meridian. The band of light when using the plane mirror then occupies the meridian of the greatest refraction (most myopic).

If a concave mirror is used the observer locates the point of reversal of the least myopic meridian and then draws the original source of light towards the mirror. This throws the immediate source of light nearer the eye, inasmuch as the catoptric image from the concave mirror lies in front of the mirror. The band is then best defined the moment the light occupies the
point of reversal of the meridian of the greatest refraction. With the concave mirror then, the band occupies the meridian of the least refraction. Jackson says that with either mirror the band can not be made to appear to occupy both meridians, but in this he is mistaken. The chief factor in the development of the band appearance is the fact that in astigmatism the retina appears magnified most in the meridian that has its far-point at the observer's eye.

This band appearance of light in the eye is characteristic of astigmatism. The direction of the band of light marks the principal meridians, if the strongest or the weakest spherical lens that reverses the movement of the light in all, or in any one direction is before the eye. After the band appearance is developed in astigmatism denoting the direction of the principal meridians, the refraction is measured in each of these meridians as in simple hyperopia or myopia. To ascertain with accuracy the direction that the ribbon or the band of light occupies a metallic disc with a central opening is placed before the eye (axonometer), in the lens holder. Across the face of the disc is painted a broad white line, bisecting the opening at its center.

After the band is developed, the disc is rotated until the line across its face and the light band in the pupil are continuous. The inclination of the line is then read from the graduations upon the trial frame that holds the disc. To get this appearance, so characteristic of astigmatism, the source of light should be shaded. A circular opening in the shade of 5 mm . is the proper size for nicety of testing, especially when working near the point of reversal. A larger opening emits more light, but then the observer is annoyed by the presence of any spherical aberration in the eye under observation.

The smaller the area of retina illumined, the easier is it to confine one's attention to the movement of the light in the visual zone. When the area of illumination is large it passes into the eye both through the center and the equator of the lens. The equator of the lens being weaker or stronger than the center causes the light in the pupil to appear brighter in the center or upon the edge of the pupil, according to which portion has its point of reversal nearest to the
observer's eye. When near the point of reversal for the visual zone the light in the center of the pupil and that in the periphery of the pupil appear to be separated by a shadow to which the name of the paracentral shadow has been given. Bitzos was the first to describe this shadow. The following is the explanation of this phenomenon. Let it be supposed that in an emmetropic eyeball the peripheral parts of the pupil are so refractive that there is actually myopia. Rays emanating from the fundus would then have the direction as shown in the cut.

Rays I and 2 are parallel and enter the observing eye. Rays 3 and 4 through the myopic edge of the pupil come to a conjugate focus at $A$, and diverge ; ray 4 enters the observer's pupil while ray 3 is intercepted by the observer's iris. The observer therefore sees the portion of the pupil of the observed eye dark in place correspond-

ing to ray 3. This paracentral shadow is nothing else than the manifestation of spherical aberration. To ascertain with accuracy which movement of light belongs to the visual zone when near the point of reversal is difficult. Frequently the movement of light upon the edge and that in the center of the pupil are in reverse directions. This annoyance is done away with by placing before the eye an opaque screen with a central opening the size of the undilated pupil $(4 \mathrm{~mm}$.$) . The light is cut off from the periphery of the pupil by the$ screen. The test is not rendered any easier thereby, however, because it is more difficult to tell the direction of the movement of light when its excursion is so small, and more difficult to keep the light in the pupil, the excursions of the mirror being necessarily short. There is danger also of measuring through the edge of the cornea if the patient tilts his head.

This phenomenon of spherical aberration is frequently taken for the appearance so often presented in cases of irregular mixed astigmatism, what is termed the scissor movement of the light. The scissor movement is that on rotating the mirror, two areas or bands of light are seen to approach and to recede from each other, but never passing, each area of light only moving partly across the pupillary space, and not the crossing of two differently moving areas of light in the pupil, as is seen in cases of aberration. The scissor movement occurs only in cases of irregular mixed astigmatism. In compound irregular astigmatism the two areas or bands of light move together either with or against the rotation of the mirror as the case may be, but separated by a shadow. The typical scissor movement is seldom seen except in cases of ectasia corneæ, or in subluxation of the crystalline lens. The band appearance of light in the observed eye is often seen without any lens before the eye on first throwing the light into the eye, especially if the plane mirror is used.

In simple hyperopic astigmatism of no amount is there to be seen at first a well-defined band of light in the pupil. A slight flattening of the circular light area may be noticed in the meridian of the great-


The Bands in Compound Irregular Astigmatism Moving down Together.


Scissors Movement ; Bands Separating on Rotating Mirror.


The Bands in Compound Irregular Astigmatism Moving Up Together as Mirror is Rotated.
est refraction. When plus spherical lenses are added, and the point of reversal of the previously emmetropic meridian is approximated, a band appears running across the previously emmetropic meridian. In compound astigmatism of no degree is there to be seen the band appearance, until a plus or minus sphere is placed before the eye that reverses the movement in the least ametropic meridian ; the band then
appears running across the meridian of least ametropia. In mixed astigmatism of all degrees a band of light is seen running across the meridian of myopia. There is always a more apparent movement of the light from side to side than from above downward, as the excursion of the light from side to side is not interfered with by the edges of the lids. This is especially true in cases of astigmatism with the rule, as it is more difficult to discern movement in the direction of the length of the band of light than it is at right angles to it. To obviate this difficulty, a cylinder of the proper strength to reverse the movement at right angles to the band should be placed before the eye with its axis over the meridian to be measured, then spheres added until the point of reversal is reached for that meridian.

The refraction correction of the eye in the two principal meridians is represented by the strength of the cylinder and the sphere before the eye minus one, respectively.

Unless the astigmatic error is over .50 D. it is difficult to develop a band of light in the pupil, or the apparent oblique movement of light when rotation of the mirror takes place in a meridian other than one of the principal ones. The astigmatism is then detected by the movement becoming inverted in one meridian before it does in the others. So after the lens is found that apparentlyreverses the movement in all meridians, the observer should move a little closer and note whether the movement becomes erect in all directions at the same point and then moving a little further back notice whether the movement becomes inverted for all the meridians at once.

The use of a stenopaic slit opening in the shade enclosing the original source of light for retinoscopy has been advocated. The slit is placed vertically when the horizontal meridian of the eye is measured and horizontally when the vertical meridian is refracted, but there is nothing to be gained by this method.

In estimating the error in the chief meridians, the rotation of the retinoscope is kept parallel with and at right angles to the band of light. This is done by keeping the axonometer before the eye and causing the light reflected upon the face and into the eye to travel
along the painted line upon the face of the disc. After one meridian is measured the disc is turned so that the direction of the line corresponds to the other principal meridian, which is measured in like manner. We measure the refraction of the eye by retinoscopy in the meridians of rotation of the mirror. The cross meridian is measured by rotating the mirror crosswise and vice versa. One usually begins the test by turning the retinoscope from side to side and then from above down, to form an idea of the sort of refraction of the eye. It at times happens that on rotating the mirror from side to side the light in the eye is seen to move in an oblique direction, as has been said. By changing the axis of rotation of the mirror, while the light is kept moving in the eye, a meridian is at last found in which the light moves in the eye in the same meridian as it does upon the face of the patient ; the approximate angle of the movement of the light in the eye is then noted and the perforated disc placed before the eye, and the band of light developed to definitely ascertain the inclination of the principal meridians.

Instead of using an axonometer the meridian of the rotation of the mirror may be fixed by some device. Such an appliance has been invented by Dr. Fuller, of Chicago. It consists of an opaque disc containing a mirror set axially in a ring on which are marked degrees. Rotation of the disc in the rim changes the axis of the mirror without altering its plane. The outer rim is attached to a standard which can be fastened to the edge of a table, but it may be held in the hand. If the mirror is laid aside and taken up again it will have the same axis of rotation as before.

The apparent movement of light in the observed eye oblique to the movement of the light upon the face of the patient as reflected from the mirror, in cases of oblique astigmatism, when the mirror is rotated from side to side or from above downwards, or in any case of astigmatism when the rotation of the mirror is not across one principal meridian, is an optical illusion. In astigmatism the form of the retinal illumination is in the form of an ellipse which, as it sweeps across the retina, gives the impression that it is moving obliquely if
its motion forms an angle with one of its axes, when its real motion is horizontal. This fact is demonstrable by drawing an ellipse with oblique axes upon a card, and viewing it through a hole in another card, while it is moved from side to side. The ellipse appears to move obliquely whenever its motion is at an angle to one of its axes.

In mixed astigmatism the movement of the light is inverted in one direction while it is erect in the meridian at right angles thereto. This likewise occurs during the correction of compound astigmatism when the lens before the eye undercorrects one principal meridian and overcorrects the other. In cases where the principal meridians are vertical and horizontal there is no especial need to develop the band of light, as the case can be just as readily corrected by measuring the refraction in the two principal meridians after the manner of dealing with a simple hyperopic or myopic case, but in astigmatic eyes with oblique axes the band must be developed to ascertain the inclination of the axes.

In regular astigmatism the refraction is the same in different parts of the pupil in any given meridian, though differing in different meridians. In case's of spherical aberration and irregular astigmatism the refraction differs in different parts of the pupil in the same meridian. All eyes present variations of this kind which form an obstacle to the measurement of refraction by retinoscopy or by any other method. Inasmuch as the refraction of the eye with irregular astigmatism differs in different parts of the same meridian, there are many points of reversal, and the observer's eye being nearer some than others causes the pupil to appear broken up into light and shade without any degree of regularity, in the place of the homogeneous red reflex seen in other eyes by reflected light. On rotating the mirror there is seen a motion of the small light and dark areas, some
against and others with the mirror. The appearance of the eye with irregular astigmatism caused by corneal disease is shown in figure no. r, and in figure no. 2 the appearance presented by incipient cataract.

In figure no. 2 the darker areas are opacities in the lens, the lighter ones shadows produced by irregular astigmatism, showing the change in the lens preceding senile cataract. At times the same appearance is seen in the eyes of young people, as a congenital defect, and perhaps not increasing in years. If the difference of refraction in different parts of the pupil is slight, the observer will not notice the difference in the illumination and movement until he gets his eye close to the point of reversal. The only way to deal with irregular astigmatism by retinoscopy is to understand the optical principles involved in the test, and to apply them as far as possible in each case.

In cases of irregular astigmatism one attempts to ascertain the principal light area, that is the one that crosses the visual zone as the mirror is rotated, and the refraction corrected thereby. It is a great disadvantage to have the pupil of the irregular astigmatic eye much dilated, as the movement of light and shade in the pupil is rendered more confusing. All in all retinoscopy in irregular astigmatism is anything but satisfactory.

The variation in the refraction of the crystalline lens does not proceed regularly from the center to the equator, but the central area is comparatively uniform over a considerable extent, and towards the margin the change in refraction becomes increasingly more marked. The behavior of the light in the center of the pupil is what concerns us in retinoscopy, as it is this area that is used in the visual act, the periphery of the cornea


As mirror is turned from side to side light area is seen to sweep around the center of the pupil. normal condition, and only exposed by dilating the pupil. In keratoconus and lenticonus, on rotating the mirror, the light is seen to sweep around or revolve about the center of the pupil, in somewhat the same manner as the spokes of a wheel
about the hub, and besides there is usually the appearance of irregular astigmatism.

The concave mirror gives a feebler illumination than does the plane mirror and is therefore not as well adapted to the test as a rule. In irregular astigmatism we can often get better results by use of the concave mirror at a closer distance than one meter. If the test is performed at 50 cm . distance we deduct 2 D . $(100 \div 50=2)$ from the plus lens or add 2 D . to the minus lens that reverses the movement. With the concave mirror the movement of light in the eye in all conditions is just the reverse of what it is with the plane mirrorthat is against the rotation of the mirror in emmetropia, hyperopia and myopia less than one diopter at one meter's distance.

With the plane mirror and the original source of light back of the patient - with no lens before the eye or the weakest one that reverses the movement of light in any meridian, the band appearance of light is seen to be across the meridian of emmetropia in simple hyperopic astigmatism ; myopia in simple myopic astigmatism ; least hyperopia in compound hyperopic astigmatism ; least myopia in compound myopic astigmatism ; meridian of myopia in mixed astigmatism.
If the lens before the eye is strong enough to reverse the movement of light in all directions the band appearance, though not so well defined, runs across the meridians at right angles to those above.

The final test in astigmatism is to bring the two points of reversal together at the distance of one meter. To do this the cylinder that corrects the astigmatism is placed before the eye together with the convex sphere which will bring the point of reversal to the desired distance. With these lenses before the eye the test is again applied. The observer now moves closer to and further from the point of reversal and inspects the movement of the light in all meridians of the eye. If reversal for all meridians occurs at the same distance from the eye the cylinder is correct in strength and in position of its axis. If, however, the movement of light seems to cease in one meridian and to continue in a meridian at right angles thereto, it is evident that the cylinder chosen does not correct the astigmatism. If the
remaining astigmatism has the same principal meridians as those already ascertained, the direction of the axis of the cylinder is correct, but its strength is not right. Whether the strength needs to be increased or diminished will appear from the fact that the more myopic meridian continues to be more myopic, or that what was before less myopic has become the more myopic meridian. If the principal meridians with the cylinder before the eye do not correspond with those found primarily, the inclination of the axis of the cylinder is incorrect. If the cylinder is of the right strength or too weak its axis needs to be turned towards the axis of a similar cylinder which would correct the remaining astigmatism. If the cylinder before the eye is too strong, its axis needs to be turned towards the axis of a cylinder of opposite kind that would correct the remaining astigmatism.

Measurement of Accommodation. - Retinoscopy affords the only objective method at our disposal for the measurement of range and amplitude of accommodation. This is important in cases of suspected paralysis of the ciliary muscle in children, and all for whom the subjective tests can not be relied upon, and in eyes with imperfect vision.

The patient directs his gaze at some large letters hung upon the wall opposite in a partially darkened room, in such a position that the visual axis of the eye under the examination shall pass as close as possible to the eye of the observer. The refraction is then measured and a lens placed before the eye that will bring the point of reversal for all meridians at a distance of a meter or a little less. The finger or the point of a pencil is then held at about the near point of convergence, and in the visual line so that the direction of the visual axis shall not be changed during the test. The refraction is again measured with the eye adjusted for the near-point and the increase in myopia equals the amount of accommodation effort. The observer can tell whether the patient is exerting all his accommodation and convergent effort by watching the other eye.

The results of retinoscopy should be confirmed by the use of the trial case whenever possible. If the lines on the astigmatic dial are not all alike the cylinder of the sphero-cylindrical combination before
the eye should be made stronger or weaker as the case may be until they are. The sphere of the combination should then be altered until the best vision is obtained. If there is any doubt about the inclination of the combined cylinder it should be changed as the patient looks at the test-type and the position of the best vision decided upon.

To Study the Test.-To properly study the test the student should begin with known conditions of refraction, and study the movement of the light in the eye from known distances. He should study carefully the erect and the inverted movements from within and beyond the point of reversal. Only by making himself familiar with the varying appearances of the light and shade in the pupil can he ever hope to become efficient in the application of retinoscopy to the measurement of refraction errors. The best way to do this is to provide himself with a retinoscopic artificial eye. The cut below illustrates the Thorington retinoscopic eye. Accompanying the in-
 strument are several lenses, for the study of aberration and irregular astigmatism in its various manifestations.

To aid in understanding. the optical principles involved in the test, one may take a strong convex lens, and a piece of cardboard with an arrow upon it. Let the lens represent the dioptric system of the eyeball and the arrow upon the card the retina and light area upon it. The card is held a little further from the lens than its focal distance and the arrow viewed through the lens from varying distances. Nearer the lens an erect image of the arrow will be seen, blurred of course and further
off an inverted image, and between the two points the point of reversal, a place where no distinct image at all can be gotten. The movement of the light upon the retina may be gotten by moving the card a little from side to side. The appearances presented in astigmatism may be illustrated by using a round dot for the object, and combining with the spherical lens a cylindrical lens. The enlargement of the dot as the point of reversal is approached and its diminution as that point is departed from together with its increased distinctness, are to be noted. The combination of dot and lens will also beautifully demonstrate the phenomenon of aberration, with the central and peripheral areas of different movement, the one an erect and the other an inverted image.

In using the artificial eye any amount of regular ametropia may be made by shortening or lengthening the eye according to the scale upon the telescopic tube. Astigmatism is made by placing a cylinder in the clip in front of the eye. It should always be remembered that hyperopic astigmatism is made by the use of a concave cylindrical lens and vice versa unless the student will be puzzled by the appearances presented.

For the practice of retinoscopy, the media of the eye under examination must be comparatively transparent. A slight but diffused clouding of cornea - incipient cataracts and opacities of the vitreous body - interfere greatly and often render the test impossible even when the vision is not greatly reduced. In such cases the trial case, rod optometer and ophthalmometer have to be relied upon.

Reference to the figure on page 350 shows that it makes no difference what the refraction of the eye of the observer is, as the rays i, 2 and 3, 4 entering the eye at $N$ and $N^{\prime}$ respectively, stimulate the same portion of the fundus of the eye whether there is hyperopia, emmetropia or myopia. An inherent
 refraction error of the observer therefore could not reverse the move-
ment of light in the observed eye. The same is true of artificial myopia produced in the observer by placing a convex lens of any strength before the eye. No matter what the strength of the plus lens the relation of the rays entering the eye is not changed. But this is not the case in artificially produced hyperopia, for when a minus lens of about 5 D . is placed before the eye of the observer, the entering rays are diverged so that rays 2 and 4 stimulate the upper part of the fundus of the eye at $N$ and $N^{\prime}$. respectively.

The concave lens diverges the rays I and 2 , to $\mathrm{I}^{\prime}$ and $2^{\prime}$, which stimulate the upper and the lower part of the fundus respectively, thus reversing the apparent movement of the light in the eye under examination. If the convex lens before the eye was strong enough to bring rays 1 and 2 to a focus anterior to the cornea, and the eye refractive enough to then transpose the rays as they impinged upon the retina, the movement would be inverted by the convex lens; but
 this is impossible, for suppose that the lens was at 6 mm . from the cornea and the focus of the rays 4 mm . anterior to it, the eye to bring these rays together upon the fundus would need to be 250 D . refractive ( $\mathrm{r}, 000 / 4=250$ D.), and of a greater amount than this to cross them. Such ocular refraction does not exist. I and 2 are focused at $F$ by $L$.

If the eye was 250 D . refractive, the light diverging from point $F$ would be focused at $b$ and at $O$ if still more refracted, transposing the relation of the rays.

## CHAPTER XXV

## OPHTHALMOMETRY AND OPHTHALMOPHAKOMETRY

All the light that enters a lens from an object does not aid in the formation of its image, but some undergoes reflection at the surfaces of the lens, either passing out again and becoming lost, or after being reflected by the posterior surface of the lens to the anterior, passes out of the lens behind and interferes with the distinctness of the image. The incident rays are then divided into three portions : the useful or image-forming rays, the lost or those that pass out anteriorly, and the harmful rays that interfere with the distinctness of image, passing out of the lens posteriorly (see figure).

The harmful rays may enter the eye as it is observing the image formed by the useful rays, causing annoyance because it does not contribute to the formation of the image. In a simple lens about 8 per cent. of light is lost by reflection and much more in complicated apparatuses. The harmful light represents only
 about one five-hundredth of the incident light. In ophthalmometry about 33 per cent. of light is lost by reflection. The eye loses less light than any other optical instrument, only about 2 per cent. The useful light forms the retinal image; the lost light forms four images of reflection from the dioptric surfaces of the eye, and the harmful light a series of false images of which one only is visible. The images formed by the lost light are called the images of Purkinje. They are, as said, four in number, one from each surface of the cornea and each surface of the lens. They were discovered by the scientist whose name they
bear, but the second one, that from the posterior surface of the cornea, was lost sight of until Tscherning again called attention to it.

The Manner of Observing the Images of Purkinje. - The brightest image is formed by the anterior surface of the cornea, and is easily observed. To see the second image, that formed by the posterior surface of the cornea, one places himself a little in front of and to one side of the eye, which he examines with a magnifying glass without focusing the light upon the eye. Examining the corneal image of the flame we see as it approaches the edge of the cornea that it is accompanied by a smaller image situated near it. The smaller image is situated between the larger one and the pupil which indicates that the posterior surface of the cornea is more curved than the anterior. If two lamps are used, one on each side and the distance between them considered as the object, we shall see that the distance separating the smaller images is less than that separating the larger images, indicating that the curvature of the posterior surface of the cornea is greater than that of the anterior. At the center of the cornea the smaller image is not visible as it is hidden behind the larger one. The image formed by the anterior surface of the crystalline lens always preserves a more or less diffused appearance, due to the fact that the index of refraction of the lens varies in its superficial layers. To properly observe it, the observed eye should look so as to bisect the angle between the light and the examiner. After having observed it the light may be concentrated upon the eye, the image is thereby magnified and fills the entire pupil. The fourth image is seen under the same conditions as the preceding one and offers little difficulty. This image being inverted moves in a direction contrary to the others. To make a more accurate study of these images the ophthalmophakometer of Tscherning may be used (which see).

All the reflected rays that emerge from the eye to form the four catoptric images, with the exception of those of the first image, meet surfaces which again reflect a part of the light. This light is very feeble for most of the surfaces. It is only at the anterior surface of the
cornea that there is reflected enough light to be visible. Thus there are two more images formed ; a fifth, formed by a first reflection from the anterior surface of the crystalline lens and a second from the anterior surface of the cornea, and the sixth, due to a first reflection from the posterior surface of the lens and a second reflection from the anterior surface of the cornea. These images are entirely subjective, as the rays forming them return towards the retina. The fifth image is furthermore entirely theoretical. It ought to be situated near the posterior surface of the crystalline lens, but no trace of it can be seen. The sixth image is as a rule easy to observe. In a semi-darkened room a distant object is fixed, while one moves a lighted candle from side to side, in front of and a little to one side of the eye. The candle is made to approach and recede from the visual line but never reaching it. There is then seen projected on the opposite side of the visual line a blurred inverted image of the flame. If the refractive index of the superficial layers of the crystalline lens had been higher the image would be brighter and hence more annoying. The brightness of this image is really about $1 / 40,-$ ooo of that of the useful image. By the study of the images of Purkinje we can locate the internal refracting surfaces of the eye. These images have no function so far as the eye is concerned, but are of importance only in physiological optics. Their study constitutes ophthalmometry, by which term we mean the mensuration of the surfaces of the dioptric media of the eye, and unless otherwise indicated keratometry or the mensuration of the anterior surface of the cornea is implied.
The curvature of the posterior surface of the cornea does not enter as a causative factor in astigmatism to any great extent as the index of the cornea and that of the aqueous humor behind are practically the same as it is equally curved as a rule in all directions. There are then practically but three surfaces that may give rise to astigmatism, namely, the anterior surface of the cornea and the anterior and posterior surfaces of the crystalline lens. We measure and compare the curvature of these surfaces in different meridians by aid of their
catoptric images. Any inequality of the curvature of the anterior surface of the cornea in different meridians may be recognized by Placido's disc. This instrument consists of a polished disc of aluminium, about eight inches in diameter, upon which are painted concentric

circles in black enamel. The instrument is held about ten inches from the eye and the light reflected from it upon the cornea. Distortion of the reflected images of the circles indicates astigmatism. In regular astigmatism the circles appear as ovals, the long axes corresponding to the meridian of least refraction, that is the greatest hyperopic or the least myopic meridian, for the flatter the curve of a mirror, the longer the radius of curvature, and the larger the reflected image.

If the sight-hole of the keratoscope is not at the same height as the pupil of the observed eye, and if the disc is not held parallel to the iris of the eye under examination, the reflected images will appear oval and astigmatism thus simulated. Claiborne's hand ophthalmometer is a modification of the Placido disc, having in addition radiating lines as seen in the figure indicating the preferred positions in hyperopic and in myopic astigmatism, and a revolving double


The Javal-Schiötz Ophthalmometer.
carrier fastened to the back of the disc, which holds a strong magnifying lens and a short-focus retinoscopic mirror. The radiating lines enable the examiner to readily tell the direction of the elongation of the reflected circles. The lens back of the disc magnifies and sharpens the image. Both of these instruments are of little value, as a slight tilting of the disc causes the appearance of astigmatism. In
irregular astigmatism when it was impossible to intelligently use the ophthalmometer of Javal and Schiotz on account of the great distortion of the reflected images of the mires, they are of utility in deriving some information as to the meridians of the greatest and the least refraction.

The ophthalmometer of Javal and Schiotz enables one to diagnose and to measure with accuracy the corneal astigmatism.

The improved ophthalmometer consists of a telescope mounted upon an upright column, with a rack-and-pinion movement enabling the operator to easily adjust the height of the instrument. The column rests upon a japanned base, upon which it moves backward and forward, by rack-and-pinion movement, in focusing the instrument. At the base of the upright there is a rotating joint for the lateral adjustment of the instrument. The large steel disc seen in the cut is to protect the eyes of the observer from the light.

The telescope consists of an adjustable eye-piece $D$, and an achromatic objective of a compound character. The objective carries between its lenses, $A$ and $C$, a Wollaston birefringent prism $B$, made of two pieces of quartz cut at opposite axes, in such a manner as to double the image reflected by the cornea. At the point $x$, the conjugate focus of the objective lens, a crossed spider web is placed, upon which the ocular is focused, and upon which the image is centered. The large metallic disc in the older instruments had graduations upon its anterior face from $O$ to 180 degrees, to indicate the axes of any existing astigmatism. The figures were painted reversed so that they were read correctly when reflected by the cornea.

There was also upon the disc a series of concentric circles, so arranged that when reflected by the cornea they appeared equidistant and showed the size of the corneal image. These graduations are replaced in the newer models by a small-graduated dial attached to the telescope behind the large disc. In front of the telescope and attached to it there is an arc graduated upon its outer edge in half centimeters, to determine the size of the reflected image, and upon the inner edge from $5^{-13}$, to indicate the radius of curvature of the cornea in the meridian in which the arc stands. Upon this arc travel two targets or mires, moved along the arc by means of a milled head upon the rear of the large disc. One mire consists of eight white steps and is called the step mire. The other one is of the form of a parallelogram, six centimeters long and three wide. Through each there runs a black line, which appears continuous in the reflected image. The two pointers at the side of the mires indicate the position of the axis of the correcting cylinder, in the primary position in hyperopic astigmatism and in the secondary position, the axis of the correcting cylinder for myopic astigmatism. Upon the back of the large disc there is a small scale to correspond with the reading upon the arc, which is indicated by a white line upon the foot of the step mire. Day light, gas, or electric light may be used to illuminate the mires. In either case the cornea must be in the shade, as any light falling upon it interferes with the distinctness of the images of the mires.

Attached to the swinging eye-shield upon the head rest of the instrument is an artificial glass cornea of about the curvature of the average cornea. It is very useful in testing the accuracy of the ophthalmometer and for practice in the use of the instrument. Any amount of astigmatism may be made by placing the proper cylinder in the graduated cell below the artificial cornea, and any inclination given to the axis of the cylinder. As the lens and the cornea do not occupy the same focal plane, the reading of the instrument differs from the focus of the lens according to the following scale:

| Focus of Trial Lens. | Result with Artificial Cornea. | Focus of Trial Lens. | Result with Artificial Cornea. |
| :---: | :---: | :---: | :---: |
| . 25 | . 12 | 2.50 | 1.75 |
| . 50 | . 25 | 2.75 | 2.00 |
| . 75 | . 50 | 3.00 | 2.00 |
| 1.00 | . 62 | 3.50 | 2.50 |
| 1.25 | . 75 | 4.00 | 3.00 |
| 1.50 | 1.00 | 4.50 | 3.25 |
| 1.75 | 1.25 | 5.00 | 3.50 |
| 2.00 | 1.50 | 5.50 | 4.00 |
| 2.25 | 1.75 | 6.00 | 4.50 |

Manner of Using the Ophthalmometer. - The examiner should first adjust the telescope by looking through it and turning the eyepiece, until the crossed hairs are in perfect focus, then rotate the telescope, so that the arc bearing the mires is horizontal, told by the pointers upon dial being at o and 90 . Adjust the head of the patient by means of the chin rest so that the eyes are upon the level with a painted line on the side of the head-piece, with the forehead pressed against the top of the head-piece. See that the eyes are horizontal by sighting through the slit in the disc above the telescope. Cover one eye of the patient with the swinging eye-shield attached to the head rest, and sight the telescope on the eye to be examined by looking along the upper side of the barrel of the instrument. When once pointed to the cornea adjust the instrument by faising or lowering, and focus by moving it backward and forward until the images of the mires reflected from the cornea come clearly into view. The corneal reflection of the mires is doubled by the instrument, and of the four mires seen in the field, only two are to be taken into account, they are the steps and parallelogram that appear in close proximity in the center of the field. The images of these are made to fall upon the point of intersection of the crossed hairs in the ocular, by manipulating the telescope. The mires are moved along the arc by means of the milled head on the back of the large dial, until the two middle images are just in contact by their edges, and the telescope is then rotated until the bisecting line through each forms a continuous line if it is not already so. This is now what is termed the primary position. The telescope should not be turned
further to the right or to the left than 45 degress in finding the primary position, or astigmatism with the rule becomes confounded with that against the rule and vice versa. In regular astigmatism the lines always become continuous within 45 degrees to the right or to the left of o. The axis of the primary position, which is the axis of one principal meridian is noted upon the disc behind the upright of the instrument according to the position of the hand that bears the plus sign.
The appearance of the reflection in the primary position is shown in the cut.


After the primary position is ascertained the telescope is rotated through 90 degrees to the secondary position by grasping the barrel of the instrument behind the upright and turning it towards the left hand, against the direction that the hands of a watch move. If there is no corneal astigmatism present the mires will remain in contact throughout the revolution, but if astigmatism is present the images of the middle two mires will be seen to separate or overlap. The parallelogram mire appears to overlap the step mire, as seen by the increased whiteness or milkiness of the steps overlapped. If the astigmatism is regular the line through each mire will be continuous, and the mires of their proper shape, while if the lines through the middle of each are not continuous in the secondary position, or if the mires appear much distorted in the reflection, there is irregular corneal astigmatism present. If the mires overlap as the telescope is turned to the secondary position, there is astigmatism with the rule, while if they

separate there is astigmatism against the rule, but there is no way to tell whether this astigmatism is hyperopic or myopic. The figure opposite shows four diopters of astigmatism with the rule, each step overlapped denoting one diopter.

If the mires separate on rotating to the secondary position, they should be brought into contact again, and the telescope turned back to the primary position, when they will be seen to overlap, and the amount of overlapping will indicate the amount of astigmatism against the rule.

The amount of astigmatism can also be read off on the graduated scale on the back of the large dial with graduations corresponding to those on the arc, as follows: See that the brass pointer with knob (secondary pointer) on back of dial covers the lower pointer (primary pointer) exactly, so that the pin on the lower pointer fits into the hole of the secondary pointer (see figure $C$ ). (This is done by rotating the nurled head until both pointers are brought in line.) Bring the mires to the primary position.

Turn the telescope 90 degrees to the secondary position, when in case of astigmatism the images will overlap; lift the secondary pointer so that it will stand at right angles to the dial,


Fig. C.
and move the mires until they again just touch as in the primary position, lay the secondary pointer against the scale, and the difference between the two pointers will then indicate the amount of astigmatism in diopters. To ascertain the amount of refraction of the anterior surface of the cornea in diopters, one must add 20 to the graduations of diopters upon the arc or upon the small dial upon the back of the large disc. This is necessary because in all the older instruments the


Fig. D. left-hand mire was stationary and clamped at twenty degrees upon


Fig. E. the arc, and the mensuration has not been changed in the newer instruments. The reason that the images of the mires overlap in astigmatism with the rule is as follows:

The mires are imaged by the meridian of the cornea in which the arc carrying the mires stands. In astigmatism with the rule it will be recalled that the cross curve of the cornea is flatter-that is its radius of curvature is longer than the vertical curve. The larger the radius of curvature of a convex mirror which the cornea represents, the larger the catoptric image and vice versa.

The mires may be supposed to be the extremities of an object. They recede from each other in case the object increases in size and approach each other in case the object grows smaller. The doubling prism in the instrument enables us to tell whether the image has increased or diminished in size. The ends of the doubled image are brought in contact ; the right-hand end of the left image being on the right of the left-hand end of the right image, as will be seen by reference to the figure of the mires upon the preceding page. If the curve of the cornea from above down is sharper than it is from side to side, the catoptric image of the mires will decrease in size as the telescope is rotated, which causes the step and the parallelogram in each case to approach each other. This causes the middle two mires, one of each half of the double image, to overlap. The reading of the instrument is recorded as follows: If the astigmatism is with the rule, and the secondary position at 120 degrees, and the amount of astigmatism I D., it is denoted thus:-1 D. C. ax. 30, or +1 D. C. ax. 120 .

The proper sign belonging to each degree is denoted by the signs upon the pointers on the telescope moving around the disc, behind the upright of the instrument. One pointer bears a minus and the other one a plus sign. In some cases it is impossible to bring into a continuous line the middle line of the two mires on account of irregular astigmatism from scars of the cornea or from conical cornea. Not infrequently the ophthalmometer indicates that the principal meridians are not at right angles to each other, for example the reading may be: 3 D. C. ax. 80, or 3 D. C. ax. 180 . In such cases if there is hyperopia the axis of the correcting cylinder should be placed at 80 degrees and if there is myopia, it should be at 180 degrees. The objective astigmatism as measured with the ophthalmometer is not always equal to the subjective ascertained by means of the trial lenses, on account of the same kind or of a reverse form of astigmatism resident in the curvatures of the crystalline lens coming in to influence the result. To get as near as possible to the true amount of astigmatism subtract from .50 to .75 D. from the corneal astigmatism when it is according to the rule and add the same
amount to the reading of the ophthalmometer when the astigmatism is against the rule. The difference between the corneal and the subjective astigmatism was first noticed by Donders and Knapp, and they attributed it to a lens astigmatism. This was hypothetical, as no one has endeavored to measure the curves of the crystalline lens save Tscherning. He proposes the name supplementary astigmatism for that which influences the corneal astigmatism. The part which it plays is as follows according to most observers :
I. If there is no ophthalmometric astigmatism there is usually to be found a small amount of subjective astigmatism against the rule.
2. If the reading is against the rule, the subjective astigmatism is usually against the rule and greater in amount.
3. If the ophthalmometric astigmatism is with the rule and of a value between 1 and 3 D . the subjective astigmatism generally differs only slightly from it.
4. If the ophthalmometer gives an astigmatism greater than 3 D., the subjective astigmatism is also with the rule and frequently greater in amount.

Javal tried to express the relation between ophthalmometric astigmatism and subjective astigmatism by the following formula:
$A s_{t}=k+p . A s_{c}$, in which $k$ and $p$ are two constants, $k=.50 \mathrm{D}$. against the rule and $p=\mathrm{I} .25$. Accordingly we have the following relations :

|  | Against the Rule. | With the Rule. |
| :---: | :---: | :---: |
|  | c $2-1$ |  |

Astigmatism, subjective $\quad 3-1.75-.5-.75-2-3.25-4.5-5.75-7 \quad$ "
This formula is entirely empiric, as the supplementary astigmatism depends upon so many factors, that it is impossible to express the relation in a formula.

Among the factors may be mentioned :
I. Deformity of the internal surfaces, as that of the posterior surface of the cornea, which according to measurements made by Tscherning was frequently found to be more curved from above down than from side to side. This deformity causes astigmatism
against the rule, inasmuch as the posterior corneal surface acts like a minus lens ( 4.73 D .). The same defect in the anterior surface of the cornea gives rise to astigmatism with the rule. In the few cases measured, the anterior surface of the crystalline lens possessed astigmatism with the rule and the posterior surface astigmatism against the rule.
2. Obliquity of the crystalline lens may give rise to a small amount-about . 50 D - as any obliquity is nearly compensated for by the special structure of the lens, as pointed out by Hermann.
3. Sectional accommodation or astigmatic accommodation may overcorrect the corneal astigmatism, if such a thing as astigmatic accommodation exists, which is very doubtful.
4. The influence of the distance of the correcting cylinder from the eye must not be overlooked, on account of which concave lenses are weaker and convex ones stronger than the total astigmatism. That is a concave cylinder of greater strength and a convex cylinder of less strength is needed to correct the subjective astigmatism than that indicated by ophthalmometry. Certain observers have found that astigmatism with the rule often exceeds that found with the ophthalmometer. This is due to the fact that they use concave cylinders perhaps in their tests.
5. The most important factor that influences supplementary astigmatism, perhaps, is irregularity of astigmatism in different zones of the cornea. This exists in nearly all eyes, and is the cause of some of the indecision on the part of patients in selecting the proper cylinder when tested subjectively. Furthermore, the axes of the objective and subjective astigmatism frequently fail to correspond, as the axes of the astigmatism of the internal surfaces are oblique to each other or to the axes of the anterior surface of the cornea. The effect is then that of crossed cylindrical lenses with oblique axes. While measuring with the ophthalmometer the patient should look directly into the telescope, unless one wishes to measure the peripheral portions of the cornea. The refraction of each corneal meridian may be read off from the small dial on the posterior surface of the large
disc. The small dial is graduated on its edge in half diopters and within in millimeters of radius of curvature, a pointer indicating upon the inner circle the radius of the anterior surface of the cornea. Both mires should be movable, as in the Meyrowitz model of ophthalmometer, as then the reflection of each is obtained at the same distance from the center of the cornea, and each mire equally distant from the cornea, which is not the case if only one mire is movable, as in the old models, as the instrument must be twisted to get the reflection of each mire at the same distance from the center of the cornea, giving rise to inaccuracies. Reflection from the periphery of the cornea shadows the presence of astigmatism that does not enter into the visual act save in a number of cases with widely dilated pupils. The plane of the iris should also be parallel to that of the mires, and, as this is not always possible, the reading of the instrument is subjected to error.

All in all the ophthalmometer is disappointing and should not be relied upon to the exclusion of the trial case. In about 50 per cent. only does it give the proper amount of astigmatism and in about 75 per cent. the true axis of the subjective astigmatism, in the writer's experience. After the proper deductions are made it will be found of a decided help in the majority of cases, especially in ascertaining the axes of the astigmatism. Some have ventured the opinion that one radius of curvature belongs to emmetropia and that by taking the radius into account or the distance it was necessary to have the telescope to bring the images of the mires into focus would give a clue as to the amount of ametropia. This is a mistake, for ametropias of curvature are rare, for as Javal has said a mouse and an elephant may each be emmetropic, but their corneal radii must be very different. The radius of curvature of cornea varies from 7 to 8.5 mm . in emmetropes, the average being 7.8 mm . The radius is greater in persons of tall stature and with large cranial circumference. Hyperopia and myopia from altered corneal curvature does not exist except in microphthalmic eyes and those with applanatic corneæ, keratoconus or keratoglobus. Even in high anisometropia save in
cases of astigmatism it is very seldom that we find any difference in the radii of the two corneæ.

The basis of ophthalmometry is the following formula:

$$
F=\frac{I \times D}{O}
$$

in which $F$ is the focal length of the mirror, $I$ its catoptric image, $D$ the distance of the object from the mirror, and $O$ the size of the object in linear dimensions. If we then know the size of the object and the distance of it from the reflecting surface, and can measure the size of the reflected image, the strength of the reflecting surface can be easily ascertained. The size of the image may be obtained by means of a micrometer scale placed in the ocular of the instrument, but as it is nearly impossible to keep the eye perfectly quiet, it is very difficult to compare the image with this scale. Young and later Helmholtz borrowed a method of measuring, already in use in astronomical calculations, and apapplied it to ophthalmometry, that is the method of doubling (Deboulement). Suppose that we wish to measure the distance between two points, $a$ and $b$. By means of an instrument we see $a$ and $b$ doubled, then instead of two points we see four.

$a a^{\prime}=b b^{\prime}$, and let $a a^{\prime}=Y$, and $a b=X$.
If we vary the amount of doubling until $a^{\prime}$ and $b$ coincide, that is when $X=Y$, we have obtained the distance between $a$ and $b$, if we know the amount of doubling. When $a^{\prime}$ and $b$ touch we say that we have obtained contact. The following methods may be employed to double the images :
I. A Maddox doubling prism may be used.
2. A plate of glass with parallel sides placed before each half of the objective, oblique to the axis of the telescope, will double the image and render the images clearer than when a prism is used.
3. By sawing the objective in half and displacing one half laterally.
4. By removing a vertical piece from the middle of the objective, and cementing the two remaining portions together.
5. The best method however is that of Wollaston, employing doubly refracting crystals made of quartz. The two prisms are placed together so that they form a piece with parallel sides. One prism is cut with its axis parallel to the axis of the crystal and the other one with its axis at right angles to the axis of the crystal. Each ray of light that passes through the prism is divided into two, deviation of each being symmetrical to the incident ray.

By expressing the radius of curvature of the cornea in millimeters we can obtain the refraction of the cornea in diopters from the formula:

$$
F=\frac{v / v^{\prime}-\mathrm{I}}{R} .
$$

Taking the index of refraction of the cornea to be 1.3375 (Tscherning)

$$
F=\frac{.3375}{R} \text {, and } D=\frac{.3375}{R} \times \frac{1000}{\mathrm{I}}=\frac{337.5}{R} \text {, and } R=\frac{337 \cdot 5}{D} \text {. }
$$

From this formula the following table is computed :

| Refraction. | Radius of Curvature. |
| :---: | :---: |
| 50 D. | 6.75 mm. |
| 49 | 6.89 |
| 48 | 7.03 |
| 47 | 7.18 |
| 46 | 7.34 |

Refraction. 45 D.
44
43
42
41

Radius of Curvature.
7.50 mm .
7.67
7.85
8.04
8.23

In the formula

$$
F=\frac{I \times D}{O},
$$

the formula expressing the relation between the size of an image formed by a lens to that of the object, substitute the value of $R$ just found.

$$
\begin{gathered}
F=R / 2 \\
R / 2=I D / O, \text { or } O=\frac{2 I D \cdot(D)}{337 \cdot 5} .
\end{gathered}
$$

At the moment of contact of the two middle images the amount of doubling is equal to the size of the image. Let $a$ represent the linear length of one degree. If this length must equal one diopter, the object which corresponds to the image $I$ must have the size $(D) a$, ergo :

$$
\text { (D) } a=\frac{2 I D(D)}{337.5}, a=\frac{2 D I}{337.5^{\circ}} \text {. }
$$

As $a$ must be one degree long we have :

$$
\begin{gathered}
1^{\circ} / 360^{\circ}=a / 2 \pi D \\
a=\frac{2 \pi D}{360^{\circ}}=\frac{2 D I}{337.5}, I=\pi \frac{337.5}{360^{\circ}}=2.94 \mathrm{~mm} .
\end{gathered}
$$

In order that one diopter may correspond with one degree of arc, the doubling of the prism in the instrument must equal 2.94 mm ., so it has been made.

Ophthalmometry is especially useful in the higher degrees of astigmatism, and in oblique cases, and in those who are unable for some reason to aid in the subjective examination with the trial case. After cataract extraction, it would be expected that the ophthalmometer would give most valuable information, as the anterior surface of the cornea is practically the only one that gives rise to astigmatism in the aphakic eyeball, but the agreement between the subjective and the ophthalmometric astigmatism after cataract extraction is frequently less than in the normal eye, due partly to the distance of the correcting glasses from the eye and partly to the irregularity of the cornea that remains after the operation.

The advantage of the ophthalmometer of Javal and Schiotz over all others lies in the readiness with which one can ascertain the prin-
cipal meridians by means of difference of level. When the mires are in the principal meridians their images are on the same level, and the middle line through each continuous, but outside the principal meridians the images are in different planes and the middle lines of the mires not continuous in the images. The greater the astigmatism the more pronounced is this fact.

The difference between images produced by a spherical and an astigmatic cornea may be illustrated by drawing a circle upon a piece of paper with two diameters at right angles to each other, but oblique, and viewing it through a convex spherical lens and a sphero-cylindrical combination. Through the convex lens which is held at some distance from the eye the image of the circle is seen to conform exactly with the object, the diameters appearing to be at right angles to each other. Through the sphero-cylindrical combination the circle appears drawn out into an ellipse in the direction of the axis of the combination.

The diameters of the ellipse do not appear any longer to be at right angles but form obtuse angles with each other above and below. If the sphero-cylindrical combination is turned so that the axis of the cylinder coincides with one of the diameters of the circle, the diameters appear at right angles although the circle appears drawn out into an ellipse. In the first instance the image (of the mires) is in the plane of the object (mires) and as the doubling of the ophthalmometer is in the meridian of the mires, there is no difference of level-the diameters of the circle becoming obtuse to each other when looked at through the astigmatic glass shows that the reflection from an astigmatic cornea (image of the mires) is not in the same plane as the object (mires) and as the doubling takes place in this meridian, it follows that on obtaining contact the images of the mires are not upon the same level.

The Ophthalmophakometer. -The ophthalmophakometer of Tscherning, as shown in the cut, on next page, consists of a small telescope supported upon a stand, and an arc of a radius of 86 cm . movable about the axis of the telescope, and graduated so that the zero mark
coincides with the axis of the telescope. Upon the arc move several cursors which carry electric lamps. Each lamp is enclosed within a tube, closed in front by a plano-convex lens, which concentrates the

light upon the observed eye. The cursor $A$ carries one lamp of six volts while cursor $B$ carries two smaller lamps upon a rod. The rod $C$ with the bright face serves as a fixation object.

Measurememt of the Angle Alpha (angle between the corneal axis and the visual axis). -If, as is often the case, the major corneal axis coincides with the optic axis, the angle alpha is included between the visual and the optic axes, but if the corneal axis and the optic axis do not coincide, then the angle between the visual axis and the optic axis is called angle beta.

The arc of the ophthalmophakometer is placed horizontally and the cursor $B$ at the zero graduation of the arc, so that its two lamps
are in the same vertical plane as the objective of the telescope. The patient looks towards the latter place. It is clear that if the dioptric surfaces were centered around the visual axis, we would see six images of reflection in the same vertical line. The images formed by the posterior corneal surface are not visible in this experiment.

As a matter of fact, the six images are never seen in the same vertical plane. We always see, as in the figure below, on the right the images from the anterior surface of the lens on one side, those

from the posterior surface of the lens on the other, and those from the anterior surface of the cornea in the middle.

The bright ball on the cursor $C$ should then be fixed, and the cursor moved until the images seen in the eye are made to occupy the same vertical plane, as nearly as possible, for it is not possible to get them exactly in the same plane; two pairs of them will come is, but the third remains outside. This takes place because the axis of the crystalline lens does not pass through the center of the cornea, usually a little above the center of the cornea in decided defects of this kind. The optic axis now lies in the vertical plane passing through the objective of the tele-
 scope and the angular displacement of the fixation mark along the arc indicates how much the visual line deviates from the optic axis, usually from $4^{\circ}$ to $7^{\circ}$.

The arc is now placed vertically, so that the two lamps are in a horizontal plane. The observed eye then fixes the cursor $C$, and the operator moves it until the images
of reflection are seen in line. The angle at which the cursor $C$ stands marks the vertical deviation of the visual line, $2^{\circ}$ to $3^{\circ}$ downwards. The figure shows the usual position of the six images of reflection, when the eye fixes the objective of the telescope, the lamps being in the horizontal plane. To measure the radii of the internal refracting surface of the eye ball we must first determine the distance of the refracting surface from the summit of the cornea, or the position of the surface, and the position of the center of the refracting surface.

We can measure the radii of the surfaces directly, as will be explained, but it must be remembered that all sizes that we measure are apparent sizes only, and that to find the real sizes certain deductions are necessary according to rules later laid down. To make this deduction it is necessary to know the position of the surfaces, which is necessary also that we may be able to combine the surfaces with one another, so that we may calculate the entire system.

## DETERMINATION OF CENTERS OF INTERNAL SURFACES

Take the anterior surface of the crystalline lens as an example, and suppose that we make the measurements in the horizontal direction. The pupil should be well dilated. Place the arc of the instrument horizontally, and the carrier $A$ as far as possible from the telescope. The lamp must be sufficiently brilliant that its reflection from the surface to be measured may be quite visible. The fixation mark is then carried to a place at which the optic axis will bisect the angle between the telescope and the carrier $A$. It is necessary, therefore, to have previously calculated the value of angle $\beta$. We then move cursor $B$, the lamps of which should be feeble, so that their corneal reflections are alone visible, until the crystalline image of $A$ is in line with the corneal images of $B$. We now possess the elements necessary to calculate the distance of the anterior surface of the crystalline lens from the summit of the cornea.
$S^{1}$ represents the anterior corneal surface and $C^{\prime}$ its center; $S^{2}$, anterior crystalline lens surface, and $C^{2}$, its center; $C C^{2}$, the optic axis of the eyeball, and $R$ the radius of curvature of the cornea.

In the figure below, let $c$ represent half the angular distance between the telescope and the cursor $A$, and the angle $d$ half the angular distance of $B$ from the telescope. Suppose that we know

the radius of the cornea which is measured previously. In triangle $O^{2} C^{1} P$ :

$$
O^{2} C^{1}=\frac{\sin d}{\sin c} R
$$

and for the distance sought

$$
G^{1} O^{2}=R-O^{2} C^{1}=R\left(\mathrm{I}-\frac{\sin d}{\sin c}\right)=R \frac{\sin c-\sin d}{\sin c}
$$

and

$$
O^{2} C^{2}=C^{1} C^{2}+O^{2} C^{1}=\left(\frac{\sin a}{\sin b}+\frac{\sin c}{\sin d}\right) R
$$

Unless great exactness is desired, the sines may be replaced by the arcs.

Example.- Let the radius of the cornea be 7.98 mm .; the distance of $A$ from the telescope nasally, 28 degrees, and the distance of $B$ i 6.8 degrees nasally. We will have

$$
O^{1} O^{2}=7.98\left(1-\frac{\sin 8.4^{\circ}}{\sin 14^{\circ}}\right)=3.16 \mathrm{~mm}
$$

The apparent depth of the anterior chamber would therefore be 3.16 mm ., from which we can find the true value or depth, 3.73 mm ., by placing in the formula: $F_{1} / f_{1}+F_{2} / f_{2}=1$, the values $F_{1}=23.64$; $F_{2}=31.61$, and $f_{1}=-3.16$.

## determination of radil of internal surfaces

We place $A$ above the telescope, and move $C$ with the fixation mark as far as possible from the telescope, but so that the image does not disappear behind the iris ; $B$ is then moved until the corneal

image of its two lamps are on the same vertical line as the lens image of $A$. The axis of the telescope is now perpendicular to the surface of the crystalline lens. In the figure, in triangle $\mathrm{C}^{\prime} \mathrm{C}^{2} \mathrm{O}$,

$$
\begin{gathered}
C^{1} C^{2}: O C^{1}:: \sin b: \sin a, \\
C^{\prime} C^{2}=\frac{\sin b}{\sin a} R, \\
O C^{2}=R+C^{1} C^{2}=R\left(1+\frac{\sin b}{\sin a}\right)=R\left(\frac{\sin a+\sin b}{\sin a}\right) .
\end{gathered}
$$

Example.-Let $a=5.1^{\circ}$; the distance of $B$ from the telescope $12.4^{\circ}$ temporally and $C 9.9^{\circ}$ nasally.

$$
\text { Then } 7.98\left(\mathrm{I}+\frac{\sin 6.2^{\circ}}{\sin 4.8^{\circ}}\right)=18.28 \mathrm{~mm} \text {. }
$$

The apparent radius would be $18.28-3.16 \mathrm{~mm}$. (apparent depth of anterior chamber) $=15.12 \mathrm{~mm}$.

Considering that we have obtained the apparent values with reference to the refraction of the cornea, we must in the formula $F_{1} / f_{1}+$ $F_{2} / f_{2}=1$, put $F_{1}=23.64 ; F_{2}=31.61$, and $f_{1}=-18.28$, giving $f_{2}=$ 13.78, the position of the real center and the radius of the real surface $13.78-3.73$ (depth of anterior chamber) $=10.05 \mathrm{~mm}$.

The posterior lens surface is located and measured in the same manner as the anterior surface, but owing to the fact that it lies very near the nodal point of the eyeball the apparent position differs little from the real position. In regard to the anterior lens surface the true position or surface is about 5 mm . behind the apparent surface, and the radius of curvature of the apparent surface is about 15 mm . instead of 10 mm .

Measurement of the Posterior Corneal Surface. -The image of the more powerful lamp $A$ is viewed as formed by the posterior surface

of the cornea ; the images of the two smaller lamps $B$, reflected from the anterior surface of the cornea are brought into line with the image of $A$, by sliding their carrier along the arc. The eye to be measured is directed at the center of the objective of the telescope, and the cursor $A$ is moved along the arc until the fainter image $a^{2}$ is seen clearly defined at a little distance from the brighter image $a^{\prime}$.

The two lamps $B$ are now lighted and the carrier moved along the arc on the same side, until the two images reflected from the anterior corneal surface are seen in line with the image $a^{2}$. The lamps are then moved to the symmetrical position $A^{\prime}, B^{\prime}$, and the same pro-

cedure repeated. Referring to the figure it will be seen that $a^{\prime}{ }_{2} a^{2}$, represents both the length of image of $A A^{\prime}$, reflected from the posterior surface of the cornea, and of $B B^{\prime}$, reflected from the anterior surface of the cornea. The difference between $O^{\prime}$, the center of curvature of the anterior surface of the cornea, and $O^{2}$, the center of curvature of the posterior surface from the middle of the arc is so small that we may without sensible error assume the common value of I for both, which by the construction of the instrument is 86 cm .

The formula for ascertaining the radius of curvature of a convex mirror is :
$\frac{\text { Half the radius of curvature }}{\text { Length of the image }}=\frac{\text { Distance of the object }}{\text { Length of the object }}$, or $R / 2=\frac{I D}{O}$,

Designating the half of the radius of curvature of the anterior surface by $x$ and that of the posterior surface by $x^{\prime}$, we have for the two surfaces -

$$
\frac{x}{a_{2}^{\prime} a^{2}}=\frac{1}{B B^{\prime}}, \text { and } \frac{x^{\prime}}{a_{2}^{\prime} a^{2}}=\frac{1}{A A^{\prime}}
$$

Dividing equation I by 2 , member by member, we have:

$$
x / x^{\prime}=A A^{\prime} \mid B B^{\prime} \text { and } x^{\prime}=\frac{B B^{\prime}}{A A^{\prime}} .
$$

The value of $x$ is found with the ophthalmometer and $A A^{\prime}$ and $B B^{\prime}$, by the scale upon the arc $C$.

The average measurements give the following result, $x^{\prime}=0.77 x$, which corresponds to about 6 mm ., radius of curvature for the posterior corneal surface, or about 2 mm . less than for the anterior surface. By reason of the index of refraction of the aqueous differing little from that of the cornea, the refraction of the posterior corneal surface is very little, equaling in effect a negative lens of -4.73 D . S . The relation between the radii of the anterior and the posterior surface of the cornea is the same everywhere, the posterior surface undergoing a flattening at the periphery analogous to the anterior surface.

The relation of the lights in the pupil when measuring the posterior surface of the cornea is shown in cut on page 403.

The Direct Determination of Radii.-The distance separating the reflected images of the lamps may be supposed to be the length of the image and the distance between the lamps the length of the object. The ratio between the distances separating the two images of a lamp placed at corresponding points on opposite sides of the telescope, is equal to the ratio between the apparent radii of the reflecting surfaces, according to the formula $R / 2=I D / O$, as $O$ and $D$ are the same for all the surfaces. The measurements are accurate if we make use of two cursors similar to $A$ and two similar to $B . A$ is placed so that the reflection from the anterior surface of the crystalline lens is clearly visible. $B$, whose lamps are feeble, is now placed so that the corneal reflection is in plane with one of the images formed by the lens. The distance between the cursors $A$ is considered the object for the anterior surface of the crystalline lens and that separating $B$, as the object for the anterior surface of the cornea. The
radii are inversely proportional to the objects as the images are alike. Knowing the radius of curvature of the anterior corneal surface by previous measurement, we calculate the apparent radius of the anterior surface of the crystalline lens. To determine astigmatism the measurements must be repeated in the vertical meridian. An exactness of more than a half of a millimeter can not be guaranteed in any of these measurements. An error of $1 / 2 \mathrm{~mm}$. in measuring the anterior corneal surface corresponds to about 3 D . whilst the same error in measuring the anterior lens surface corresponds to $x / 3$ D., but as to the thickness of the lens, which is only 4 mm ., such an error is of great importance. The practical result obtained by measurement of the internal dioptric surfaces does not pay for the trouble, for each surface must be measured in at least two meridians, and as each surface calls for two measurements (of the radius and of its center), twelve measurements are necessary, and then one would have to deduce the real values to ascertain the astigmatism of each surface and finally to combine these astigmatisms with that of the anterior surface of the cornea. This would become more complicated if the principal meridians of the astigmatic surfaces did not coincide. It is also likewise impossible to measure the radius of curvature of the posterior surface of the cornea in the center, as the reflected image from the posterior surface of the cornea is not visible at the center of the cornea.

## CHAPTER XXVI

## TESTS FOR HETEROPHORIA

All the tests for the detection of inefficiency of the extra-ocular muscles depend upon the fact that when single binocular vision is interfered with, the guiding sensation ceases to operate, and both eyes rotate in the direction in which the strongest acting muscle or muscles pull them. All the tests depend upon destroying single binocular vision. The simplest test is that known as Von Graefe's screen test. The patient looks at a candle flame at 20 feet distance, or better a round dot upon a square piece of cardboard, with both eyes open. The examiner then places a card in front of one or the other eye, excluding it from the visual act. If there is balance of the extra-ocular muscles the eye behind the card will remain perfectly directed towards the object of observation, but if binocular vision is ordinarily maintained only by exerting an effort, the eye back of the card will be pulled in the direction of the strongest muscle. This fact will be noted by a movement on the part of the eyeball to get back into the proper direction when uncovered. This latter movement is called a redress movement. The redress is always towards the weakest muscle. Instead of covering and uncovering one eye, first one and then the other may be covered in quick succession, but not too quickly for the eye, as it is uncovered to adjust itself before the screen is again passed over it. In case there is imbalance of the extrinsic muscles, each eye will make a redress movement when it is uncovered. At the same time the patient will notice a shifting of the object looked at, as the screen is passed from in front of one eye to the other. This movement as observed by the patient has been called by Dr. Duane the parallax test. The examiner observes the eye as it changes from exclusion to fixation, and neutralizes the movement with prisms, placed with their bases in the direction of the
redress, until no more movement can be seen by the examiner when the eye fixes. The observation of the patient is then brought to bear, and stronger prisms used until the patient can no longer detect any movement in the object, when either eye is uncovered.
The extent of movement ascertained by the foregoing tests is called the deviation in exclusion. The same tests are used to detect any deficiency of convergence. After the refraction error is corrected a card with a dot upon it is held three inches from the eyes, and on uncovering a movement of the eyeball will take place toward the nose if convergence is below par, and the patient will notice that the dot appears to move as first one eye and then the other is screened.

By repeating the screen test with each eye alternately it can be told whether there is habitual binocular fixation, an alternating fixation or a squint present. If, when the screen is removed, the eye uncovered alone moves, there is heterophoria, and not squint, present. If both eyes move, or, in spite of there being an evident deviation, both eyes remain fixed, there is a squint. If in the latter case, when the left eye is uncovered, the eyes behave in the same way as they do when the right eye is uncovered (both alike moving or standing still) the squint is alternating. Lastly, if when one eye is uncovered both eyes move, and when the other eye is screened and uncovered, both eyes remain steady, the squint is permanent and belongs to that eye. The other tests for heterophoria depend upon the production of diplopia.

A strong convex or concave cylinder is placed before one eye, and the patient looks at a distant candle flame. The cylinder causes the flame to appear drawn out into a long streak of light, which is so different from the natural appearance of the flame as seen by the other eye that the brain can not fuse the two retinal images, especially if a red glass is placed before one eye, and immediately the guiding sensation that keeps the two eyes properly directed ceases to act, and the eye whose vision is interfered with rotates in the direction that the strongest acting muscle pulls it. The axis of the cylinder is placed crosswise before the eye when one wishes to test
the laterally-acting muscles. The streak of light then appears to run vertically, in the neighborhood of the candle flame seen by the other eye. The streak and the flame are kept better before the mind if before one eye ; say the one with the cylinder before it, is placed a red glass to color the image seen by that eye. If both eyes are directed at the flame of the candle, its image will fall in each upon corresponding parts of the two retinas, that is, in the same vertical meridian in each. The streak of light will then appear to run directly through or over the flame of the candle, each eye projecting its image to the same point in space. If the externi or the interni are acting too strongly, the images in the two eyes fall upon non-corresponding points of the retinas, and the eyes project their images to different points in space, the red streak of light appearing to the right or to the left of the flame. The eye that sees the flame (the one whose vision is not interfered with), maintains its fixation, keeping the image of the flame upon its macula, while the fellow eye deviates. If the cylindrical lens is placed before the other eye it will be seen that the streak of light is at the same distance on the other side of the flame. This is not always the case, for in not a few more inefficiency will be demonstrable in the muscles of one eye than in those of its fellow.

The image of each eye is always deviated towards the weaker-acting muscle. Suppose that the cylinder is before the right eye, and that the streak is seen to the right of the flame. The streak is to the right of the flame, the muscle to the right side of the right eye is the external rectus, which is the weak muscle. The flame seen by the other eye is to the left of the streak of light. The muscle to the left of the left eye is the external rectus, which is the weak one. The supposed condition is then one of weak external recti or esophoria, a tendency on the part of the eyes to turn in.

In the figure the left eye is represented as having the cylinder before it, and as there is a lack of balance between opposing muscles the eye turns in, as binocular single vision is impossible. The line of fixation of left eye is in the direction $x$. The light passing into
each eye does not impinge upon the same portion of each retina. It stimulates the macula in the right eye and internal to the macula in the left eye, causing the left eye to project its image to the left, the right eye's image being to the right. The left eye projects its image

in the direction that $x^{\prime}$ would occupy if $x$ was brought to occupy the position of $x^{\prime}$. This relation of the streak of light and flame is seen in cases of esophoria. The relations of light and streak in the different named heterophorias is shown in cut above.

To test the vertically-acting muscles the cylindical glass is placed with its axis vertical, causing the streak of light to run crosswise, which appears to run through the center of the flame if there is balance of the vertical recti. If there is imbalance each eye's image is deviated towards the weaker muscle of the corresponding eye.

With cylinder before the left eye:

R. E.
R. Hyper
R. E.
L. E.


Maddox Groove.


Aiken's Phoroscope.


Maddox Rod.

If the cylinder was before the other eye the above forms of diplopia would indicate just the opposite condition of the muscles. Instead of a cylinder being used to distort the image seen by one eye, Maddox's rod or groove is used as a rule. Maddox's rod consists of a piece of a glass rod set back of a stenopaic slit, in an opaque disc. The improved Maddox rod (Aiken's phoroscope) consists of a number of small rods of glass set side by side, and backed by a piece of red glass to color the image seen through them. The rods of glass act like strong cylinders to draw the candle flame out into a long streak of light.

Maddox's groove upon red glass acts as a strong concave cylinder in distorting the image of the light, but the multiple rods are preferable as the streak seen through them is much longer and better defined. Maddox has devised another test, known as the Maddox prism test Maddox's prism is a doubling prism made of two six-degree prisms
base to base. The prism causes all objects seen through it to appear doubled, two candle flames are then seen by the eye before which the prism is placed, if the base line of the prism intersects the visual zone-and the other eye
 sees its object between the two, both eyes together seeing three images of the flame. When the flames are arranged vertically, that is when the prism is placed with its apices above and below before the eye, one flame should be directly under the other, if there is balance between the laterallyacting recti muscles, otherwise each eye's image is deviated in the direction of its weak muscle. The prism is now turned so that the three lights are side by side, and if there is balance in action between the vertically-acting recti the three flames will lie in the same horizontal plane.

The test as depicted in the figure above shows balance of the vertically-acting muscles. This test is a good one inasmuch as inefficiencies of several mucles may be detected at the same time, according to the position of the middle flame, in relation to the other two. See figures on next page, remembering always that the image is deviated towards the weaker muscle of the corresponding eye.

A test for insufficiency of the extra-ocular muscles can be made by producing diplopia, by placing before one or both eyes a prism stronger than the eyes can overcome (a six-degree prism is a convenient strength). Such a prism is placed before the right eye-say with its base down. The image seen by the right eye is then above, displaced in the direction of the apex of the prism and that seen by the left eye below. The two images of the flame should be directly
over one another if there is a balance between the laterally-acting muscles. In testing the vertically-acting muscles, a prism strong enough to produce diplopia is placed before one eye with its apex out. To measure the amount of weakness of a muscle, one places successive strengths of prisms, beginning with the weaker, before one or the other eye with the base of the prism over the weaker muscle, until one is ascertained that establishes balance of the extra-ocular-muscles. Half of the strength of this prism then represents the amount of error in each eye. There is no more convenient apparatus for measuring the amount of error, as well as the extent of adduction and abduction than Risley's rotating prism.

It consists of two fifteen-degree prisms, which are revolved in op-


Risley's Prism. posite directions by a milled head screw, thus furnishing a prism the strength of which can be increased from zero to thirty degrees. It is made the diameter of a trial glass and can be placed before the eye in the trial frame. All of these tests are faulty inasmuch as the objects seen are thrown out of their relation to each other if the head of the observer is inclined to the side, as the axis of the cylinder or rod must be absolutely horizontal and vertical to obtain accurate results (must coincide with the vertical meridians of the retinæ). To obviate this possible error certain instruments called phorometers have been devised. The same methods
of testing are applied but the test apparatus is held before the eyes independent of the head of the patient. Within certain limits the eyes rotate by action of the obliques so that the vertical meridians of the retinas remain vertical, when the head is inclined to the side, and the rod or what not being always vertical before the eye no discrepancies can enter into the calculations. The most widely used phorometer is that devised by Dr. Stevens, shown in the cut below ; $E$ is a small spirit level upon a horizontal arm carrying a prism holder ;

$F$ is leveling screw and $G$ is clamp to keep the arm horizontal ; $a$ is a card at the end of a rod for the near-test of inefficiency.

The prism holder or slide contains two cells, in each of which rotates a disc, each disc carrying a prism of five degrees. Each disc is furnished with a border of teeth or cogs. A small gear wheel
placed between the discs communicates movement from one disc to the other. Around the outer part of the border of each cell there is a narrow band, on which is marked a scale of degrees, increasing from the center each way from zero to eight degrees, the numbers representing the refracting angle of a prism. To the outer side of this scale there is an equivalent scale of prism diopters. The prism

holder is placed upon the arm of the phorometer and the arm rendered absolutely horizontal by means of the small spirit level attached to it. The side of the slide upon which the scales are marked is placed away from the patient. The edge marked $R . H$. and $L . H$. will then be before the patient's right eye and that marked Es, and Ex. before his left. To determine hyperphoria bring the handle of the instrument to a vertical position. The pointer will then be at zero. The slide should be about six inches in front of the patient's eyes. The patient now looks through the two prisms at a card hung at a distance of twenty feet, across the room, upon which is drawn a dot and crossed lines through it, thus:


The patient sees double images of this cross, side by side. The horizontal line is made long enough so that when doubled the two horizontal lines butt end to end, enabling the patient to readily tell whether one dot is higher than the other or not. If one image is higher than the other we ascertain to which eye that image belongs. Thus the prisms are now before the eyes with their bases in, therefore, the images are homonymous, that is the image belonging to the right eye is on the right and that of the left eye on the left. If the right image is higher it indicates that the right eye has a weak superior rectus muscle, and the trouble is called left hyperphoria and vice versa. The right image is to be brought down to the level of the other one, when the scale upon the outer edge of the disc will indicate the amount of weakness.

The apex of the right prism must then be pulled down, by pulling the handle to the patient's right hand. By making the rotation slowly more inefficiency will be revealed than if the rotation is made quickly. To ascertain the amount of esophoria and exophoria bring the handle to the horizontal position. The images of the cross will now be seen, the one above the other, the prisms being before the eyes, the one with the apex up and the other with the apex down. The vertical lines through the two images of the cross should now be continuous if there is balance between the laterally-acting muscles. If one dot is not exactly over the other the amount of error is ascertained by rotating the handle until they are exactly over each other. The lower of the two crosses belongs to the right eye and if it is to the right there is esophoria, and if to the left exophoria. The handle is then moved so that the pointer will move in the direction of the esophoria or exophoria as the case may require. Trial prisms may be graduated after the same manner to work in a trial frame, as the prisms in the slide of the Stevens phorometer. When a prism is rotated before the eye, the object looked at through the prism appears to describe a circle following the apex of the prism. The circle that the object appears to describe is a projection or a smaller circle upon the eye's retina described by the rays of light that pass
through the prism, and are refracted towards the base of the prism. If no bending of the light took place as the light passed through the prism, no such appearance would take place by the rotation of the prism. Usually a four- or six-degree prism is graduated for use in the trial frame.

The following applies to a graduation of a six-degree prism. Divide a horizontal line into twelve equal parts. With this line as a diameter construct a circle; from each of the subdivisions of the line erect a perpendicular ; from the intersections of the perpendiculars with the circumference of the circle draw radii. Lay the prism to be marked upon the figure thus constructed, with its axis coinciding with the central radius, and with its base upon the diameter of the circle. The prism is now marked with a glass cutter over each radius drawn. The intervals correspond to one half degrees. The prism is now placed in the trial frame with its base down, and if diplopia is not developed another prism is placed before the other eye with its base in the opposite direction. The graduated prism is then rotated to the right or to the left until one light is seen directly over the other one. The amount of lateral deviation is then read off by counting the number of graduations between the 90 -degree mark upon the trial frame and the axis of the prism. In testing for vertical deviations, the prisms are turned so that the two lights are seen side by side, that is with bases in.

The amount of hyperphoria is then read off by counting the number of graduations between the 180 -degree mark upon the trial frame and the axis of the prism. Dr. Stevens has devised another test, which is easy of application and one in which the position of the patient's head has no influence. A convex lens say about a ${ }_{13} \mathrm{D}$. S. is covered except at its optical center, where a circular opening of 3 mm . or less acts as a stenopaic opening, preventing the adjustment of the lens as a prism and sharply circumscribing the light area as seen through the opening. A flame looked at through this opening appears as a large bright circle of light, in the center of which is seen the natural flame when the other eye is opened, if orthophoria
exist. The disc of light may be colored red to contrast with the color of the flame. If the candle flame is below the center of the disc of light and seen by the left eye there is present right hyperphoria ; if to the right of the center of the disc there is exophoria and if to the right and below, a combination of the two exists, i.e., right hyper-exophoria.



Orthophoria.


Heterophoria.

A convenient method of testing the extra-ocular muscles is as follows: A piece of black cardboard about two feet square, with a central round opening and divided by equidistant horizontal and vertical lines, is hung before a window or argand burner so that the light shines through the central opening. A Maddox rod is placed before one eye of the patient. If there is present imbalance of the muscles the streak of light seen by the eye before which the Maddox rod is placed, is seen to run to one side or the other of the central opening as seen by the other eye. The amount of error can be read off directly by noting which line upon the card the streak of
light overlaps. The lines upon the card should be separated I cm . for every meter's distance at which the test is to be employed. Thus for 6 m . distance the lines should be 6 cm . apart. Maddox's oneprism test and Prentice's test also enable one to read off directly the amount of exophoria or esophoria.

The former test consists of a series of figures beginning at zero and extending to the right in black and to the left in red figures. A prism of about six degrees is used to throw the vision of the right eye say below that of the left. The arrow which divides the two sets of figures then points to the one indicating the amount of deviation. The Prentice test is about the same save the chart is composed of a series of vertical lines, numbered from the middle to the right in black and to the left in red characters. The Wilson and Lewis phorometers combine many of the above tests in the one instrument. The test of the balance of the muscles when the eyes are adjusted for the usual reading point should also be made. In the ideal condition of the extra-ocular muscles there will appear near exophoria of from four to six degrees. A card with a dot and crossed lines upon it is held at the reading distance and the phorometer set for four degrees of exophoria. The patient now views the doubled image of the dot and lines which should in the normal condition be directly over each other. The Maddox doubling prism is also of value in testing the near balance. Any sort of a deviation may be recognized by its aid. The eye not behind the prism is the one considered under the test. A card upon which there is a dot and horizontal line drawn through it is held at the reading distance. With a doubling prism before one eye there appears three dots and three horizontal lines. If there is balance of the muscles the three lines are parallel and the three dots beneath each other. If the middle dot seen by the right eye is too far to the left it shows exophoria, and if too far to the right esophoria. Nearer the upper dot than the lower one bespeaks left hyperphoria, and if nearer the lower dot right hyperphoria. Being above and to the right indicates left hyperphoria-esophoria and so on.

In oblique astigmatism, according to Dr. Savage, there is frequently a weakness of the oblique muscles. In oblique astigmatism the retinal images of upright objects are oblique, as proven by photography, and for the eye to recognize the object as upright in space the image must fall upon a vertical retinal meridian. The function of the obliques in oblique astigmatism is to rotate the eyeballs so that this occurs. In oblique astigmatism of any kind when the meridians of the greatest curvature diverge above there is a necessity for action on the part of the superior oblique muscles in order to prevent diplopia. This action begins early in childhood and continues until the case is corrected or until single binocular vision is lost.

If the meridians of the greatest curvature converge above, the images of all objects in the two eyes are so displaced that the inferior obliques are called into action to prevent diplopia. In oblique astigmatism without an exception, the retinal image is displaced towards the meridian of greatest curvature, which in hyperopic astigmatism is the best curvature, and meridian of myopia in myopic and mixed astigmatism. When the astigmatic meridians are vertical and horizontal the ciliary muscles are alone called upon to overcome the error. Weakness of the superior obliques is more annoying than that of the inferior, as the former are called into action every time the eyes are turned down as in the act of reading. They are often unequal to their task and give rise to symptoms of asthenopia. The inefficiency of the oblique muscles is detected by the following method: With a Maddox prism before the right eye the left one is considered under the test. The patient looks at a dot with a horizontal line drawn through it held at the reading distance. If the obliques are normal in their action, the three lines seen by the two eyes will be parallel, while if either oblique muscle is under-acting the middle line of the three will be seen to dip towards the upper or the lower line at one end or the other according to the muscle involved. If the right ends of the middle and the bottom lines converge, while the left ends diverge, the superior oblique of the left eye is shown to be in a state of under-action.

Figure II represents the relation of the lines when the left inferior oblique muscle is too weak.

The same appearances if the right eye was under the test would indicate exactly the opposite condition of affairs, that is in the first case, inefficiency of the inferior oblique and in the second, inefficiency of the superior oblique muscle. If there is perfect equilibrium the three lines will be parallel as in III.

The sort of diplopia caused by oblique astigmatism, and the manner in which rotation of the eyeballs by the oblique muscles corrects it, is demonstrated in the following way:

Place a-3 D. C. before each eye in a trial
 frame. The concave cylinder creates 3 D . of hyperopic astigmatism in each eye. Place the axis of the cylinder before the left eye at 90 degrees and that of the right eye at 135 degrees. Place an opaque disc in front of the right eye and a doubling prism in front of the left. A horizontal arrow, head to the left, drawn upon a card is now looked at through the prism. Two arrows parallel to each other are seen. The opaque disc should now be removed from before the right eye. A third arrow then appears between the other two but not parallel with them. On removing the doubling prism two arrows are at once seen, and in a few seconds they begin to open and shut like a pair of scissors, and finally they merge into one. This occurs when the eyes are under the effect of atropine so that the correction of the diplopia can not be due to ciliary muscle action. The only way that this phenomenon can be explained is that the obliques rotate the eyeballs until the two meridians of retinal stimulation are rendered horizontal.

Any test for inefficiency of the extra-ocular muscles is not to be relied upon for several hours after a mydriatic or cycloplegic has been instilled. An inefficiency caused by a refraction error is called pseudo. The difference between the apparent weakness of muscle before the use of a cycloplegic (manifest pseudo-inefficiency), and
that discoverable after its use is called the latent pseudo-inefficiency, caused by the latent refraction error and corrected by the correction of the latter. According to the
 experiments of Guita and Bardelli it would seem that the instillation of cocaine renders manifest any latent muscular inefficiency. The manner in which this agent accomplishes this result is not clear.

Before adjusting glasses the amount of prism adduction, abduction, sursumduction and deorsumduction should be ascertained. For it will at times happen that when the other tests show a balance of the muscles that the adduction or the abduction is found below par. To test the strength of a muscle begin with a prism that the eye can overcome. The patient regards a distant candle flame and the prism is placed before the eye with its apex over the muscle to be tested or called into action. Successively stronger prisms are now placed before the eye in the same manner, until the strongest one is ascertained which allows of single binocular vision. By placing successively stronger prisms before the eye the muscle behind the apex of the prism is gradually led to do more work by a gradual increase in the amount of inner-
vation sent to it-just as any other muscle in the body. Either a Risley prism or a prism pile is the most convenient appliance for testing the amount of prism rotation. The Noyes prism pile is shown in the cut on page 422 .

This set is made up of three prism piles, two of which are arranged so as to furnish, in alternate numbers, all degrees from $1 / 2^{\circ}$ to $22^{\circ}$. The third consists of two superimposed prisms of $5^{\circ}$ and $10^{\circ}$ respectively, which combined produce a prism of $15^{\circ}$ in the center.

The power of adduction is tested by placing the apex of the prism in, of abduction by placing the apex of prism out; the power of elevation, by placing the apex up, and the power of depression, by placing apex of prism down. It is said that a normal external rectus muscle should overcome a prism of 8-10 degrees and an internal rectus one of $40-50$ degrees, and the vertical recti prisms of ${ }_{2-5}$ degrees. The writer is convinced that the amount of adduction and abduction given above is too high, adduction seldom reaching 6 degrees and abduction hardly ever over 30 .

If there is a paresis of the abducens, say, of the one eye, the prism adduction of that eye will be much less than that of the fellow eye, but it will now and then happen in other cases that the adducting power for instance will be found to be a few degrees greater in one eye than in the other, caused perhaps by the one eye having a congenitally inherently stronger muscle than its fellow, which under the same amount of innervation is able to do more work. In such cases the treatment of the heterophoria should be unequally divided between the two eyes.

A good converging ability may be absent even if there is a marked deviation of the visual lines inward, so there may be excessive converging power, when exophoria of a high degree is manifest. The fact of a deficient converging power with esophoria or vice versa is indicative of anomalous tension in the vertically-acting muscles, in the form of a hyperphoria or a tendency for both visual lines to rise above or to fall below the horizontal plane, namely: anaphoria or cataphoria. The absence of converging power should warn one against
the assumption that he is dealing with a simple case of esophoria although the latter is shown by the tests. Similar caution should be exercised in dealing with cases of apparent exophoria with excessive degrees of convergence. Anaphoria and cataphoria are only diagnosticated by taking the fields of rotation or fixation, or by measuring the amount of prism rotation above and below the horizontal plane. In anaphoria the whole field will be found to be moved upwards and the superior recti stronger than the inferior, and in cataphoria the field of fixation is dislocated downwards, and the inferior recti stronger than the superior. Those with anaphoria usually look straight ahead from under the brows, that is with chin thrown down and those with cataphoria look straight ahead, with head thrown well back during their unconscious moments.

There is likewise a characteristic position of the head in cases of hyperphoria, that is, the head is carried on the side, tilted towards one or the other shoulder. The cause of the tilting of the head is a complicating cyclophoria so often found in cases of hyperphoria. If the complication is a positive cyclophoria the head is tilted towards the cyclophoric side.

Inefficiency of convergence is of more frequent occurrence than is generally supposed and is found chiefly in those with a great interpupillary distance. The eyes should be able to converge upon a point at 3 in . or 7.6 cm . distance, in inefficiency of convergence the c. p. may be removed to six to ten inches. The prism adduction may or may not be found to be below par, and there may be exo phoria for distance or muscle balance, but it is the rule to find an abnormal amount of exophoria for the reading distance over $4^{\circ}$ to $6^{\circ}$. In some cases the inefficiency of convergence is so great that there is actually a paralysis of convergence, while each eye can adduct to its normal amount in associated movements with the fellow eye, there is no response to an impulse of convergence. The patient may overcome the effect of the error by reading with one eye closed, thus doing away with the need of convergence.

Treatment. - The treatment of pseudo-inefficiency is to correct the error of refraction. Otherwise to develop the weak muscle or muscles by exercise, or to relieve them of their work by prisms in form of spectacles worn with the base of the prism over the weak muscle to do a partial tenotomy on the stronger muscle, or to strengthen the weaker one by shortening or advancement, or by some similar operation.

Allen recommends that an attempt be made to convert the phoria into a squint by daily increasing the strength of the prisms used, their bases being placed over the weak muscles, and then to relieve the squint or cast by operation. If there appears an esophoria in hyperopia, correct the total hyperopia save . 25 D . to allow for the return of the ciliary muscle tone, while if hyperopia is associated with exophoria correct little or none of the error, but try to develop the internal recti muscles. The same rules apply in cases of hyperopic astigmatism. Myopia and myopic astigmatism, on the other hand, is to be totally corrected when associated with exophoria and only partially so when associated with esophoria. The rationale of this is the association between convergence and accommodation. The greater the call upon accommodation the greater the power of convergence, and vice versa.

The extra-ocular, like other muscles, can be developed by exercise. The wearing of a prism with its apex over the weak muscle, the strength of the prism being a little less than that required to produce diplopia, or to cause the eye to overcome successive strengths of prisms, a change to a stronger prism being made as soon as the patient overcomes the diplopia, is inferior to rhythmic exercise. Contraction and relaxation short of fatigue is the sort of exercise that develops a muscle anywhere in the body. The internal recti muscles respond to exercise much more perceptibly than do the externi. There are some cases in all classes of heterophoria that will resist this form of treatment. Only low degrees of lateral heterophoria, that is less than six degrees, can be converted into orthophoria by exercise. The higher degrees can only be overcome by converting
them into squints by exercise of the stronger muscles and then operating, or by a slight tenotomy. There may not in a given case be established balance of the muscles by exercise of the weaker one but the patient is rendered comfortable, all the error that was incompatible with comfortable use of the eyes being corrected. Exophoria or inefficiency of the internal recti may first be taken for study. There are two methods to be employed by either of which the muscles may be strengthened. The patient looks at a point of a pencil or a small lighted taper held in the hand at arm's length in the median line of the face. While regarding the object it is made to approach the eyes slowly, to within five inches of the latter. Holding it there a few seconds, the patient closes his eyes for a moment to allow the muscles to relax and then repeats the same procedure, and so on until the eyes begin to feel weary. These sittings are repeated two or more times daily for weeks or months. The best time to exercise is in the morning when the eyes are fresh from sleep. The second method is by the use of prisms, bases out. The strength of the prisms used is from one to eight degrees. . One should be placed before each eye. The treatment begins with the weaker prisms, and the stronger ones brought into use as the case develops. The prisms are worn in the form of spectacles. With the prism's before the eyes the patient regards the flame of a candle at twenty feet distance, and then raises the spectacles from his eyes, continuing to look at the candle flame. The guiding sensation quickly causes the eyes to rotate back to the primary position, when the lenses are elevated, and so the procedure is repeated.

In cases where exercise fails to develop the power of convergence or when the patient can not be treated for some reason by that method prisms may be applied, bases in, for relief of the asthenopia. The strength of the prisms needed is ascertained in the following manner. A card with a small dot upon it, is moved closer and closer towards the eyes of the patient, in the median line, until the limit of convergence upon the dot is reached. This fact is made evident by the dot appearing double, or by one eye of the patient ceasing to fix the dot
and wandering towards the temple. The distance of C. P. is then measured in centimeters from the eye, and divided into 100 cm . to ascertain the number of meter angles of convergence in use. Suppose that the patient ceases to converge at 10 cm . There is then in use 10 m . angles of convergence. The normal function equals 12.5 meter angles of convergence. The patient then lacks 2.5 m . a. to be supplied by prisms. Now a meter angle for the average pupillary distance equals $\mathrm{I}^{\circ} 5 \mathrm{O}^{\prime}$ (for each eye), therefore $2.5 \mathrm{~m} . \mathrm{a} .=4.35$ which is practically equal to a $4^{\circ}$ prism, before each eye with the base in. The prisms are to be worn for near work only. Some folks are unable to wear any but the very weakest prisms on account of the distortion produced by them and inability to properly judge distances for the first few days. Under these conditions the correction is as follows: With the refraction correction on have the patient put his attention upon a dot upon a card held at the distance for which he wishes to use his eyes. Beginning with the weakest prisms, successive strengths are placed before the eyes with the bases in until the weakest is ascertained with which there is no redress on the part of the eye or deviation in exclusion.

There is but one method to develop the externi and that is by the use of prisms. The prisms used must not exceed four degrees in strength, and are worn before the eyes with their bases in, and as with the interni a distant candle flame is viewed with the glasses on and then they are raised until the guiding sensation rotates the eyes back into the primary position, which has occurred as soon as single binocular vision has been restored, and so on. Hyperphoria is likewise only susceptible to exercise with prisms. The strength of the prisms should vary from one fourth to two degrees. The base of the prism should be down before the eye with the weak superior rectus and the base of the other up before the fellow eye, that is the base of the prism before the eye with the hyperphoria should be up. When there is a general weakness of the ocular muscles unaccompanied by a general debility of the muscular system of the body, there will usually appear esophoria for distance and exophoria for near, and
neither muscle can overcome anything like the strength of prism that it should. To relieve either the esophoria for distance or the exophoria for near would exaggerate the other condition, that is the esophoria or the exophoria as the case may be. Both the externi and the interni should be exercised as they are in cases of uncomplicated esophoria or exophoria, a different time of the day being selected for the exercise of the antagonizing muscles. General tonics also are to be administered.

Cyclophoria or inefficiency of the obliques is to be corrected, according to Savage, by the following method: The treatment is by means of cylindrical lenses so placed as to lead the guiding sensation to cause contraction of the weak muscle. A plus 1.50 D. C. is the most useful for this purpose. One is placed before each eye, and if the superior oblique is the weak muscle the axis of the cylinder is placed in the lower temporal quadrant, near the vertical at first, say fifteen degrees therefrom, and the inclination of the axis increased as the case improves. The axes of the cylinders are never carried beyond 55 degrees, however. The frames are raised and lowered before the eyes every five seconds or so during the exercise. In inefficiency of the inferior obliques, the axes of the cylinders are placed in the upper temporal quadrant, near the vertical, and the inclination of the axes increased as the case improves. Steele's rule in cases of oblique hyperopic astigmatism is: In those cases where the axes of the proper cylinders for the two eyes diverge above, place the cylinders, correcting the astigmatism, at the points which will give the axes the greatest divergence, and at the same time allow of the best vision. In those cases in which the axes of the correcting cylinders converge above, place the cylinders so that their axes will have the greatest amount of convergence consistent with the best visual acuity. There is such a condition as lateral orthophoria, in which neither the internal nor the external recti are sufficiently strong.

Again the vertically-acting muscles may be in balance, but in weakness. On the other hand, the balance may be one in strength. Likewise we may have imbalance in weakness or in strength. We
speak of the imbalance in weakness as asthenic heterophoria and of the imbalance in strength as sthenic heterophoria. Asthenic heterophoria is differentiated by the fact that neither of the antagonizing muscles can overcome the usual strength of prism. An operation may be performed for sthenic imbalance in the form of a tenotomy, but for the relief of the asthenic form an advancement operation, if any, is to be done. The weaker muscle is to be brought up to the stronger one, and then an attempt made to develop both muscles by certain exercises. Thus, if the lateral muscles are too weak the patient is to perform what is called the wall-to-wall evercise. That is : With the head stationary the patient is to look alternately from one side wall of the room to the other, resting now and then, and stopping the moment he feels fatigued. If the vertically-acting muscles are at fault, the ceiling-floor exercise is used to develop them. The patient, with fixed head, first looks at the ceiling and then at the floor.

## TREATMENT OF SPASMODIC ORTHOPHORIA AND REVERSED

## HETEROPHORIA.

As explained in a former chapter, spasmodic orthophoria is a condition in which there is apparent balance between the extra-ocular muscles, due to a partial spasm of the weaker antagonists, while reversed heterophoria is that condition where there appears an esophoria, for instance, due to a spasm of the internal recti, when in reality there is a preponderance of inherent strength in the external recti muscles. In all cases where there is evidence of great drain of nervous force, and when correction of all latent refraction errors, and apparent muscular errors fails to give relief of symptoms, one should think of reversed heterophoria or spasmodic orthophoria. Exophoria is the most common muscular anomaly found in spasmodic orthophoria, and may be suspected when there is distant muscle balance with inefficiency of convergence. This condition can be differentiated from true inefficiency of converging power in the following manner:

Place upon the patient prisms of two to four degrees, or a strength just short of diplopia, bases in, and allow him to sit awhile in the
office with them on. If there is a concealed heterophoria, he will, after awhile, be able to overcome stronger prisms, and then a slightly stronger pair is given him to wear home and to keep on constantly. If in a few days he feels much better, one may be reasonably sure of the diagnosis. If the wearing of prisms, bases in, in this manner aggravates the conditions, the diagnosis is at fault. It is undoubtedly a fact that myopia is often caused to increase by the excessive tension caused by a concealed or reversed heterophoria upon the eyeball.

Reversed heterophoria should be thought of in cases of hyperopia with exophoria and myopia with esophoria and the treatment outlined for spasmodic orthophoria applied, placing the bases of the prisms over the weaker-acting muscles. After all the muscular error has become manifest, or as much of it as can be developed an operation is performed (tenotomy or advancement as outlined) to establish balance. Slight errors are corrected by exercise. A safe line to draw between operative and non-operative cases is that pointed out by Dr. Savage, which is: If diplopia is caused by placing a red glass before one eye, as it will in cases that threaten to pass into squints, one should operate, and if not, not. Next to concealed exophoria in frequency comes concealed hyperphoria. A hyperphoria may be wholly latent to diffusion tests, but can be uncovered in the following manner: With both eyes as fully corrected as far as the plus refraction is concerned, place before the right eye a two-degree prism base down and if the patient fuses increase the strength of prism, until diplopia occurs, then record the result. Remove the prism and allow the right superior and the left inferior recti to relax from the strain under which they have been placed. After a while repeat the same, placing the base of the prism up before the same eye. If the eyes will fuse under stronger prisms when the base of the prism is down before the right eye, there is right hyperphoria present, and vice versa. The subsequent wearing of higher prisms will demonstrate the amount. Spasmodic orthophoria and reversed hyperphoria can only be determined by the effect produced by wearing of prisms.

Esophoria is seldom latent or reversed, but often spurious, caused by hyperopia (pseudo-esophoria).

Imbalance of the extra-ocular muscles not within the limits of single binocular vision will be dealt with under the head of strabismus. In all the cases considered single binocular vision is not only possible but habitual. A full correction of heterophoria should always be made if possible, but gradually, that is the strength of the prisms worn increased every several weeks until the total error is corrected. The more sensitive the patient the weaker must be the first pair of prisms worn and the slighter their change in strength each time.
Before leaving the subject of muscular inefficiencies, a few words must be said about the stereoscope as a means of detecting and correcting heterophorias. If the half pictures of the stereoscope are at the focal distance of the eye-piece, and the refraction error of the patient corrected, they will be fused if they are separated a distance equal to the pupillary interval. If the pictures are separated a distance equal to the pupillary distance, the visual lines of the two eyes need only to be parallel to bring the image of each half picture upon symmetrical portions of each retina so that when the images are projected they give the appearance of a whole stereoscopic picture, with its characteristic appearance of solidity or depth. The fused stereoscopic picture appears to lie midway between the two half pictures. The prisms in the oculars of the instruments with their bases out cause the image from each half picture to fall upon the temporal portion of the retina of the corresponding eye. Each eye therefore projects its image nasally. The stereoscope designed by Dr. Derby enables the observer to secure a displacement of the half pictures in both the lateral and vertical senses. The background of the half pictures is a white surface upon which vertical and horizontal lines are engraved, being one centimeter apart in each case. The vertical lines are numbered right and left from the middle of the card. Experience has shown that the measure of displacement of a half picture 1 cm . equals a prism rotation of three degrees. In front of
the graduated background two small travelers move. Each, like the half of the ordinary trial frame, is semicircular and graduated from o to 180 degrees. In these travelers the half pictures are placed, the vertical axis of the pictures being at 90 degrees for an ordinary anomaly of the muscles. The half pictures may be moved either in a lateral or vertical sense, and their position determined by the graduated background. The distance between the eye-piece and the pic-

tures can be altered from 12 to 16 cm ., the normal distance being 14 cm ., the focal length of the convex prisms of the oculars. The lenses are 7 D. S., combined with five-degree prisms, bases out. The distance between the lenses of the eye-piece may be altered to suit varying pupillary distances. When there is dynamic equilibrium the center of each half picture will be ordinarily 3.5 cm . from the middle line (that is for p. d. of 70 mm .), and centered on the same horizontal
line. If one or the other half picture has to be moved this way or that in order that the two may be fused there is indicated a weakness of the muscle of the eye on the same side from which the half picture has to be moved. Thus suppose that the pictures have to be moved closer together than 70 mm . before the observer is able to fuse them. We would then know that the externi were under-acting. Every centimeter nearer together or further apart the half pictures are placed causes the eyeballs to undergo three prism-degrees of rotation to keep the pictures fused. By placing the pictures nearer together than the pupillary distance the internal recti are caused to contract, the muscles relaxing again when the eyes are closed or when the pictures are again separated the distance of the pupils apart. To call the external recti into action the pictures must be moved apart. To develop any muscle with the stereoscope the half picture is moved from a position directly in front of the eye towards the muscle to be developed and after keeping it at the extreme point of fusion for a while it is move back towards the primary position in which the visual axes were parallel. Usually we wish to strengthen corresponding muscles of the two eyes and then both half pictures are moved, thus, for developing the externi, both half pictures are moved outwards until fusion is no longer possible. When for reason the ocular muscles fail to respond to exercise, or when the patient cannot afford to do without the use of his eyes while they growing stronger, prisms are adjusted in trial frames in such a manner that they take the strain off of the weak muscles. The eyes are relieved by the prisms of maintaining the proper position for single binocular vision ; each eye rotating in a position of rest behind the prism, thus relieving the system of much nervous strain. The prisms together, one for each eye, must be slightly weaker than is needed to render the case orthophoric for a very pronounced change is not well borne. Prisms stronger than four degrees can seldom be worn for any length of time with any degree of comfort, as objects seen through them appear so distorted that the confusion arising therefrom is greater than the discomfort for which they are adjusted. The bases of the prisms must
be placed over the muscles to be relieved. If after a while the prisms cease to give relief, it will be found that at that time there is more manifest heterophoria, or perhaps an actual squint has developed. In the latter case a first class tenotomy is done and in the former a stronger pair of prisms adjusted. The hair-splitting operations (partial tenotomies) done for the relief of slight heterophorias are to be condemned, as they seldom establish balance and if too much cutting is done the case is reversed, that is an esophoria converted into an exophoria or vice versa, and the condition is as bad if not worse than before. The other operations upon the muscles such as tendon tucking or advancement are likewise too uncertain to be of much value in phoria cases except those that threaten to pass into squints. It is much better to convert the case into a squint, as outlined above, if such a thing will come to pass, and if not the constant wearing of prisms or exercise will relieve the symptoms of asthenopia.

## CHAPTER XXVII

## APHAKIA

Aphakia means the absence of the crystalline lens of the eye. It is rarely congenital, being caused, as a rule, by the removal of the lens for cataract, either by solution or by extraction. The eyeball in consequence is rendered about II to I3 D. hyperopic. Any preexisting hyperopia is not necessarily increased by this amount, nor is the original amount of myopia always diminished to the same extent. The exact amount of H. in the aphakic eyeball may be calculated by the following formula:

$$
F^{\prime}\left|f^{\prime}+F^{\prime \prime}\right| f^{\prime \prime}=1
$$

From the values of the simplified eye we have, $F^{\prime}=24$ (anterior focal distance of cornea), $F^{\prime \prime}=32$ (posterior focal distance of cornea), and $f^{\prime \prime}=24.7$ (posterior focal distance of eye), giving the value of $f^{\prime}=-8 \mathrm{I} .2$. The far-point is then situated 8 I .2 mm . behind the apex of the cornea. This ametropia is corrected by a lens of 96 mm . focal length, that is, a 10.4 D . lens, placed at I 5 mm . in front of the cornea. To ascertain the correcting glass for aphakics we proceed as in other conditions of ametropia; we can not find the refraction of the eyeball simply by diminishing the ante-operative refraction by ir D . The following table showing the relation of the ante- and post-operative refraction is that of Dr. Stradfelt.

| Before operation. <br> After operation. | $\begin{gathered} \mathrm{H} . \\ 7 \\ 7 . \\ \mathrm{H} 5 \end{gathered}$ | $\begin{gathered} \text { H. } \\ 5 \\ \mathrm{H} . \\ \mathrm{I} 3.8 \end{gathered}$ | $\begin{gathered} \mathrm{H} . \\ 3 \\ \mathrm{H} . \\ 12.5 \end{gathered}$ | $\begin{gathered} \text { H. } \\ \text { I } \\ \text { H. } \\ \text { II. } 3 \end{gathered}$ | $\begin{gathered} \text { E. } \\ \text { H. } \\ \text { IO. } 6 \end{gathered}$ | $\begin{gathered} \text { M. } \\ \text { I } \\ \text { H. } \\ \text { IO. I } \end{gathered}$ | $\begin{gathered} \text { M. } \\ 3 \\ \text { H. } \\ 8.9 \end{gathered}$ | $\begin{gathered} \text { M. } \\ 5 \\ \text { H. } \\ 7.8 \end{gathered}$ | $\begin{gathered} \mathrm{M} . \\ 7 \\ \mathrm{H} \\ 6.6 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Before operation. <br> After operation. | $\begin{gathered} \text { M. } \\ 9 \\ \text { H. } \\ 5.5 \end{gathered}$ | $\begin{aligned} & \text { M. } \\ & \text { II } \\ & \text { H. } \\ & 4.4 \end{aligned}$ | $\begin{aligned} & \text { M. } \\ & \text { I3 } \\ & \mathrm{H} . \\ & 3.4 \end{aligned}$ | $\begin{aligned} & \mathrm{M} . \\ & \text { I5 } \\ & \mathrm{H} . \\ & 2.3 \end{aligned}$ | $\begin{aligned} & \text { M. } \\ & \text { I7 } \\ & \mathrm{H} \\ & \mathrm{I} .3 \end{aligned}$ | $\begin{aligned} & \text { M. } \\ & \text { I9 } \\ & \mathrm{H} . \\ & \mathrm{o.2} \end{aligned}$ | $\begin{gathered} \mathrm{M} . \\ 2 \mathrm{I} \\ \mathrm{M} . \\ \mathrm{o} .8 \end{gathered}$ | $\begin{aligned} & \mathrm{M} . \\ & 23 \\ & \mathrm{M} . \\ & \mathrm{I} .8 \end{aligned}$ | $\begin{gathered} \mathrm{M} . \\ 25 \\ \mathrm{M} . \\ 2.7 \end{gathered}$ |

In general after the extraction of the crystalline lens of the eye, one divides by two the number of diopters of the correcting glass of the complete eye, and when concave subtracts it from II, and when convex adds it to II $D$. to ascertain the amount of refraction of the aphakic eyeball.

Let us call the correcting lens of the complete eye $L$, and that of the resulting aphakia after the loss of the crystalline lens $L^{\prime}$, we will then have

$$
\begin{aligned}
& L=2\left(L^{\prime}-\mathrm{i} \mathrm{I}\right) ; \text { or } L=2 L^{\prime}-22, \\
& L^{\prime}=\frac{22+L}{2} ; \text { or } L^{\prime}=\mathrm{I} \mathrm{I}+L / 2 .
\end{aligned}
$$

The length of an eyeball is ascertained by the formula

$$
C^{\prime \prime}=\frac{F^{\prime} \times F^{\prime \prime}}{C^{\prime}} \text {, derived from } C^{\prime} C^{\prime \prime}=F^{\prime} F^{\prime \prime}
$$

$C^{\prime}=$ difference between the first conjugate and the first principal focal points $=f^{\prime}-F^{\prime}$.
$f^{\prime}=$ distance of punctum remotum from the first principal point of the eye. Accepting the measurements of the schematic eye of Helmholtz in the following deductions, if the correcting lens is placed 13 mm . in front of the cornea, it is situated at the anterior focus of the eye.
$f^{\prime}-F^{\prime}=$ the focal distance of the correcting lens.
$F^{\prime}=15.5 \mathrm{~mm} . ; F^{\prime \prime}-20.7 \mathrm{~mm}$.
$F^{\prime} \times F^{\prime \prime}=32 \mathrm{Imm}$. If the correcting glass is one diopter, then

$$
C^{\prime}=1,000 \mathrm{~mm} . \text { and } C^{\prime \prime}=\frac{F^{\prime} \times F^{\prime \prime}}{C^{\prime}}=\frac{32 \mathrm{I}}{1,000}=.321 \mathrm{~mm} . *
$$

That is if the correcting lens is placed at the anterior focus of the complete eye, each diopter corresponds to a difference in axial length of .321 mm .

* Measurements of Tscherning give $: \frac{353}{1000}=.353$.

There is a source of error in the examination of aphakics as Dimmer first pointed out, due to the fact that when the lenses in the trial cases are combined the result is not the same as when ground into one as supplied by the optician. The higher strengths of spherical lenses in the trial cases are biconvex, while the optician grinds the sphero-cylindrical combination, with the spherical correction on one side of the glass and the cylindrical upon the other, which is placed next to the eye whenever the cylinder of the combination is weaker than the sphere if convex and away from the eye if concave. The strongest convex or the weakest concave side of the lens is always placed away from the eye. The optical center of a biconvex lens is situated in the center of the lens, while that of a plano-convex lens is situated at the apex of the curved side. It follows that the spherical effect of a sphero-cylindrical combination is little greater than that of a biconvex lens, having the same focal distance, the posterior focus being situated a little nearer the lens in the former case. The strong lenses of the trial cases should be made plano-convex, to overcome this possible error. The distance at which the sphero-cylindrical lens is placed in front of the eye exercises an influence over its strength, as Ostwalt first pointed out.

Suppose that the eye is corrected by +8 D. S. +2 D. C. ax. $90^{\circ}$ placed at 15 mm . in front of the cornea. The focal distances of such a glass are : $1,000 / 8=125 \mathrm{~mm}$. and $\mathrm{I}, 000 / \mathrm{IO}=\mathrm{I} 00 \mathrm{~mm}$. The far-point in the first meridian is $125-15 \mathrm{~mm} .=110 \mathrm{~mm}$., and in the other, $100-\mathrm{r}_{5}=85 \mathrm{~mm}$.

$$
1,000 / 110=9+D ., \text { and } 1,000 / 8.5=11.7 D .
$$

The amount of astigmatism is then II.7 D. -9 D. $=2.7$ D., instead of 2 D. This makes little difference in the subjective examination of the refraction, as the lenses from the trial case are situated at about the same place as those the patient will wear in spectacles, but this is not the case with ophthalmometry. The amount of error
indicated by the ophthalmometer is higher than the lens that the patient selects signifies. In simple cylindrics the same influence makes itself felt to a less degree.

The absence of the crystalline lens of the eye is detected by the absence of two of the images of Purkinje. The aphakic eyeball is practically devoid of all accommodation, and a +3 D.S. should then be added to the distant correction for reading. $\mathrm{A}+4 \mathrm{D} . \mathrm{S}$. lens added to the distant correction will frequently afford better new vision than $a+3 \mathrm{D}$. as the print is seen more magnified. It is not advisable to add any stronger lens than this, as then the reading point will lie too close for comfortable use of the eyes. If the patient wishes to see smaller objects than this affords, at closer range, he may get the advantage of a stronger lens by moving his spectacles further down upon his nose. Now and then a case will be seen that has a certain amount of accommodation left, due perhaps to a bulging forward and an altered form of the anterior surface of the vitreous, brought about by action of the ciliary muscle. Accommodation would then be favored by a small round pupil, and would be in reverse relation to the thickness of the posterior capsule, and take place only when there is a marked difference in the index of the aqueous and vitreous, as is often the case. Schneller claims that the accommodation in aphakic eyeballs is due to elongation of the globe, due to the pressure of the extra-ocular muscles when the eyes converge. If this was the case, the eyeball would need to undergo a lengthening of 2.7 mm . for the reading distance. Sadtler pointed out that such a distention is impossible from the nature of the sclera. If the accommodation was due to an alteration in the curvature of cornea it would call for a diminution of .5 mm . in fixing an object at 30 cm . Keratoscopic examination shows that the cornea does not alter its curvature.

If the pupil is very small and round and the posterior capsule thin, there is enough accommodation at times preserved, so that the patient can use the same lens for distant and near seeing, as was seen recently in a case that came under the writer's notice.

Inasmuch as the optician always places the cylindrical surface of a compound lens next to the eye of the patient, whenever the cylinder is weaker than the combined sphere, the examiner in testing should place the cylinder behind the sphere in the trial frame, especially in correcting aphakia, and in the higher degrees of ametropia. The sphero-toric lens possesses many advantages over its sphero-cylindrical equivalent in the correction of astigmatism, and especially in aphakia. As described in a former section a toric lens is a section of a tore, which is represented by a watch, with its sharp curve in one direction and long curve in the other, upon its edge. Take a slice off the side of a similarly shaped piece of glass, and we have a toric lens. Besides the serious error that results from the optician substituting a lens, which involves a change in the position of the nodal points, sphero-cylindricals give rise to unpleasant phenomena due to internal reflection taking place between the surfaces of the lens. The patient is compelled to look, as it were, through a self-luminous medium, the reason why most folks can not see as well in feeble and by artificial light with their glasses as without them. Frequently the test lenses will give a patient normal vision, while the sphero-cylindrical equivalent supplied by the optician does not allow of more than two thirds or three fourths of the normal, on account of the high degree of aberration and internal reflection in such lenses. It occurred to Mr. Prentice that the internal reflection of the lens could be much lessened by constructing the lens with less curve upon its anterior surface; therefore a lens having a toric surface upon one side and a spherical surface upon the other was suggested. Such lenses, although expensive, give comfort, and are extremely light in comparison with their sphero-cylindrical equivalents, as they can be made much thinner.

In aphakia in which the cornea represents the only refracting surface, the length of the eye is identical with the posterior conjugate focal distance $f^{\prime \prime}$, therefore we have

$$
f^{\prime \prime}=l \pm C^{\prime \prime}
$$

According to Helmholtz we put $l$ the length of the normal eye $=23.8 \mathrm{~mm}$.


We must now calculate the anterior conjugate focal distance corresponding to $f^{\prime \prime}$, which we do by the following formula:

$$
f^{\prime}=\frac{f^{\prime \prime} \times F^{\prime}}{f^{\prime \prime}-F^{\prime \prime}}
$$

which is derived from

$$
F^{\prime}\left|f^{\prime}+F^{\prime \prime}\right| f^{\prime \prime}=\mathrm{I} .
$$

$F^{\prime}=23.26 \mathrm{~mm}$., the anterior principal focal interval of the aphakic eye. $F^{\prime \prime}=31 \mathrm{~mm}$., the posterior principal focal distance of the aphakic eye. If the correcting glass is placed 13 mm . in front of the cornea then its focal length must be

$$
f^{\prime} \pm 13 \mathrm{~mm}
$$

In the reduced eye in which the cornea is the only refracting surface, an axial length of $2_{3}+\mathrm{mm}$. represents 43.4 D . of refraction. Taking the strength of the lens of the eye to be ir D ., we reduce the refracting power of the eye to the same extent, for the removal of the lens. This would leave the eye $43-$ ir $\mathrm{D} .=32$ D. refractive, which corresponds to a focal interval of $31+\mathrm{mm}$.

To Calculate the Refraction of the Complete Eye from the Known Refraction of an Aphakic Eye.- $f^{\prime \prime}$ is the quantity sought, $f^{\prime}, F^{\prime}$ and $F^{\prime \prime}$ being known. If an 8 D . S. lens is needed at 13 mm . in front of
the cornea to correct the ametropia produced by removal of the crystalline lens, what was the original refraction of the eye?

$$
f^{\prime \prime}=\frac{f^{\prime} \times F^{\prime \prime}}{f^{\prime}-F^{\prime}}
$$


$\left\{\begin{array}{l}R=\text { Punctum Remotum } \\ \phi^{\prime}=\text { Ant. Focal Point } \\ \phi^{\prime \prime}-\text { Post ut "t } \\ H=\text { First Principal Poınt } \\ F^{\prime}=\text { Ant. Focal Distance } \\ F^{\prime \prime}-\text { Post .. }\end{array}\right.$
$f^{\prime}$ is negative in value as the first conjugate focus lies behind the eyeball, the refraction being hyperopic, at a distance of $125-13 \mathrm{~mm}$. $=\mathrm{I}$ I 2 mm .

$$
\left(\frac{8) 1000}{125 \mathrm{~mm} .}\right)
$$

Therefore :

$$
f^{\prime \prime}=\frac{f^{\prime} \times F^{\prime \prime}}{f^{\prime}+F^{\prime}}=\frac{112 \times 31}{112+23.26}=25.6 \mathrm{~mm} .
$$

Inasmuch as the eye was longer than the normal one ( 23.8 mm .) , the refraction was originally myopic, and inasmuch as each 3 D . of refraction corresponds to a difference in axial length of I mm., the difference between the length of this eye and the normal eye indicates that the eye with its own lens needed in addition one of -5.3 D. S. To simplify this for practical use, let us consider the formula

$$
C^{\prime \prime}=\frac{F^{\prime} \times F^{\prime \prime}}{C^{\prime}}
$$

in which $C^{\prime \prime}=$ the difference in length between the normal and the
ametropic eye, and $C^{\prime}=$ the focal interval of the lens placed at the anterior principal focal point of the eye. Its value is

$$
C^{\prime}=\frac{\mathrm{I} m}{C^{\prime}}
$$

If we make $C^{\prime \prime}=$ I mm., $C^{\prime}=F^{\prime} \times F^{\prime \prime}$. In the complete eye $F^{\prime} \times F^{\prime \prime}=321 \mathrm{~mm}$. Therefore

$$
1 / C^{\prime}=1,000 / 321 \mathrm{~mm} .=\text { about } 3 \mathrm{D} .
$$

In the aphakic eyeball, $F^{\prime} \times F^{\prime \prime}=72 \mathrm{Imm}$. Therefore

$$
\mathrm{I} / \mathrm{C}^{\prime}=\frac{\mathrm{I}, 000}{72 \mathrm{I}}=\text { about } \mathrm{I} .4 \mathrm{D} .
$$

One millimeter's difference in length of the normal eye equals about 3 D . difference of refraction, while in the aphakic eyeball only a difference of 1.5 D . A difference of $\mathrm{I} D$. in the correcting glass in aphakia indicates double the difference of length that it does in the complete eye, or a i D. lens has twice the influence over the complete eye that it does over the aphakic eye. The above values are not exact since slightly different results are obtained according to the measurements accepted for the normal complete eye.

The aphakial eyeball does not appreciate the difference in strength between two lenses unless it is equal to one diopter, because, as pointed out, a one-diopter lens has twice the influence over the refraction of the complete that it has over the refraction of the aphakial eyeball. After cataract extraction, as well as after iridectomies and other operations upon the cornea, there is a flattening of the corneal curve at right angles to the direction of the corneal section. After cataract operation this gives rise to several diopters of astigmatism against the rule, usually corrected by a one, two or three plus cylindrical lens with its axis transverse. For several months after the operation, the amount of astigmatism undergoes a change due to the contraction of the wound cicatrix at first, and finally to the cornea establishing its rotundity to a certain degree. In a few cases we find a high degree of corneal astigmatism with the rule after a cataract operation.

## CHAPTER XXVIII

## spectacle, nose-glass fitting, measuring lenses, etc.

To accurately fit a pair of spectacles, the following measurements must be taken. The distance between the pupils, the distance between the temples ; the distance from the tips of the lashes to behind the ear on a line with the epitragus; the height of the nose ; the
 tical centers of the spectacle lenses must be this distance apart.

Maddox suggests that the interaxial distance instead of the interpupillary distance be taken to get the lenses properly centered before the eyes, for there are a few cases that have decidedly eccen.
trically placed pupils and others have abnormally large angles alpha. The distance between corresponding parts of the reflected image of a window or other object from the cornea of each eye is measured, or, better still, light is thrown into the eyes from an ophthalmoscopic mirror, the central hole of which the patient fixes. The distance between the catoptric images is then measured.

If the lenses are not properly centered they have a prismatic effect. At times we desire to decenter the lenses in their rims, in cases of heterophoria, to relieve the strain upon the weak muscles. If the lenses are convex and decentered inwards the effect is that of prisms with bases in, each of a strength (in prism-diopters) equal to the strength of the lens multiplied by the number of centimeters it is decentered. If convex spectacle lenses are decentered outwards the effect is that of a pair of prisms with bases out. It is just the reverse with concave lenses. Vertical decentration of lenses also produces a prismatic effect. In the majority of cases the distance between the temples is regulated by the pupillary distance, but some cases require an unusual width between the temples, owing to an extreme fullness of the patient's face. It is desirable to take this measurement separately so that the temple pieces do not cut into the sides of the face. This distance may be taken by aid of a rule, or, better, take a spectacle frame that is comfortable and measure the distances between the temple pieces one inch behind the lenses.

The height of the nose is necessary to determine the depth of the


Fig. 1. nose-piece of the spectacles to be furnished. It is obtained by drawing an imaginary line across the face on level with the pupils, and measuring from the point where it intersects the bridge of the nose to the crest or root of the nose (Fig. I). This determines the height of the nosepiece for spectacles for distance or constant wear. It brings the lenses well up in front of the eyes (Fig. 2). The centers of the read-
ing lenses should be a bit nearer together and slightly lower than for distant use. The distance between centers is made $1 / 16$ to $1 / 8$ inch less and the saddle $1 / 16$ to $1 / 8$ inch deeper (Fig. 3).

Reading glasses should be tilted out at the top, as shown in Fig. 3, so that their plane is parallel to that of the reading matter. Looking obliquely through the lenses if they are set vertically before the eyes imparts to them a cylindrical effect, ren-


Fig. 2.


Fig. 3. dering the glasses less comfortable than they should be and giving rise perhaps to eye-strain.

The cylindrical effect produced by looking obliquely through a spherical lens is demonstrated by viewing a square through a strong convex or concave sphere held slantingly before the eye. The square no longer appears square but as a parallelogram, longer than it is broad, or vice versa, according to the position of the lens. Lenses that are worn constantly should be slightly tilted out at the top but to a less degree than those used only for near vision. The width of the nose is taken at two points, namely, at the crest and at the base. These measurements may be taken by sighting the nose over a graduated rule or by means of a pair of calipers.

Good results are obtained by shaping a piece of lead wire across the bridge of the nose, and then measuring the wire curve. The depth of the nose regulates the distance of the bridge of the spectacles in front of or behind the plane of the lenses, and is according to the prominence of the patient's nose, or eyes or length of eyelashes. This measurement may be taken by holding a piece of cardboard as near to the eyes as the sweep of the lashes will permit, and then measuring the distance between the card and the crest of the nose. The contour of the nose is best fitted with a saddle bridge which has superseded all other kinds of spectacle bridges. By lengthening or
shortening the shanks of the saddle bridge, the lenses are made to occupy the desired position in front of the eyes. When the eyes are prominent and the nose flat, the saddle is ordered back of the frame, and when the nose is prominent and the eyes receding, the bridge is ordered in front of frame. It is spoken of as being on line when the

edge of the bridge lies in the plane of the lenses. The two figures show the saddle in front of and back of frame.

By far the best method of measuring for spectacle frames is to have a number of assorted trial spectacle frames. From one, the set of the bridge may then be ascertained, and from another the width of temples and so on. By putting the measurements thus gotten together a well-fitting pair of spectacles is obtained. When a trial frame is found to fit the patient, its measurements may be ascertained by aid of Well's frame measure, or by aid of the rule shown in the cut on next page. The latter answers all purposes.

On one side of the rule the measurements are taken in inches, and on the other in millimeters, the frame to be measured is laid down upon the rule with the right lens coinciding with one of the ellipses at the end of rule, according to the size of the glass. The pupillary distance is then noted at the point where the nasal edge of the other lens is in contact with the rule. The width of the saddle at base is obtained by placing it upon the tapering end of the rule, and the
temple distance ascertained by placing the rule between the temple pieces and noting the distance upon the rule at the place marked "temple distance."

The instrument shown on next page is designed to facilitate taking the various measurements of spectacle frames. The scale is made from spring tempered steel, the graduations engraved in the best manner, and the metal parts finely nickel plated.

All measurements of any style frame can be readily determined either in millimeters or inches, including height, inclination and base of nose-piece, pupillary distance, etc., and by the graduation on the reverse side of the largest scale the distance between temples, length of temples, etc., can be obtained. The instrument can also be used as a pupillometer.

If it is desired that the one pair of spectacles answer for both near and distant seeing, when the patient requires a different strength for each, the reading glass is added to the distance one in the form of a segment or lenticular. Such lenses are called bifocals. The split bifocal is made of two separately ground lenses set edge to edge. This is the oldest form of bifocal and is called the Franklin lens. The cemented bifocal is made by cementing upon the distant lens a piece from the edge of the lens needed in addition for near seeing. The solid or whole bifocal, which is little used on account of its prismatic effect, is made by grinding off of the upper third of the lens that is proper for near seeing sufficient to make it the proper distant

correction. There are a few people that never can get accustomed to lenticulars. They are continually trying to see the floor in going up and down stairs or what not through the strong lower part of the glass or confused by seeing the edge of the lenticular, or on account of an anaphoria are unable to properly direct their eyes downward.


Such must have two pairs of glasses, one for distance and the other for near work. Instead of the two pairs, necessitating the taking of one pair off and putting the other on every time the patient wants to read or sew, a pair of fronts, or spectacles without the temple pieces, but small hooks to fasten over the distant correction may be used. Fronts most always give satisfaction but the lenses of the spectacles over which they are worn soon become scratched from their use. Half fronts, those cut away above, may be used if the person needs to glance up frequently from his work to look at a distance.

For nose-glasses there are only three measurements needed, namely, the pupillary distance and the width of the nose at crest
and base. The latter two measurements are necessary to ascertain how far apart the clips or guards must be so that when the glasses are upon the nose the lenses will stand horizontally. For those with astigmatism, especially of a high degree, it is better to prescribe spectacles unless the person is careful to note when the glasses fail to set horizontally and reports to have them straightened up. With the introduction of offset clips many of the difficulties of fitting noseglasses were overcome. Figure i shows the old form of regular clip,


Fig. 1.
which is not now used save for reading glasses with simple spheres. Figure 2 shows the offset clip.


Fig. 2.
The best way to get measurements between the clips above (nose at crest) and below (nose at base) is to place upon the patient's nose
a pair of offset clip nose-glasses, placing them in the proper position upon the nose - that is where they seem to hold firmly and feel comfortable, and then mark with ink and pen a horizontal line across each lens, bisecting the pupils. The glasses are then taken off and held so that the two horizontal lines will form a continuous line and the distance between the clips above and below measured with the rule. One pair of old nose-glasses will do for all measurements between clips (see figure). When the glasses are taken off the dis-

tances $c$ and $d$ are measured, while lines $S$ are held horizontally.
The proper P. D. is obtained by
 altering the size of the lenses, or by means of extended posts. Posts are made in graduated lengths from o to 4 , the length increasing one six-
 teenth of an inch with each number. They are especially useful when the distance from the crest of the nose is greater on one side than on the other, making it necessary for the center of one lens to be further from the nose than the other. Whenever possible, order post No. I and alter the size of lens to correspond as a more satisfactory fit is obtained when the glasses set
close to the side of the nose, being more or less top-heavy when extended posts are used.

If a receding bridge of nose is accompanied by long lashes, we use the set-back post, or a clip with an extra long shank. The set-back post accomplishes for eye-glasses what the saddle bridge does for spectacles. The set-back posts carry the lenses one eighth inch further forward, from the usual position, on plane with the lenses. The posts can be reversed, carrying the lenses backward for cases with prominent noses and receding eyes. They may also be combined with the extended posts.


If the patient has a very prominent brow, it is advisable to have the spring of the nose-glasses tilted forward as shown in the cut. This form of spring is known as the


Grecian or Tilted Spring. Grecian spring. If, on account of the shape of the patient's nose, the lenses in the usual position set too high up, or if the glasses are to be used only for reading, the lenses can be lowered by altering the position of the posts on the frame or lenses at the point of their attachment. The figure below shows the regular manner of attaching the posts on the center line and the so-called C. D. No. 2 position, in which the posts are set one eighth of an inch above it. By combination of extended posts, set-backs, tilted spring, C. D. No. 2 position, and so forth, with malleable pivot clips, which can easily be bent to the proper shape, almost any one can be fitted with a pair of eye-glasses.
The oculist should always examine his patient's glasses to see if they are as ordered. The fit should be carefully looked after and then the lenses examined. A convex spherical lens is recognized by
the fact that when an object is looked at through the lens and the lens moved from side to side or backward and forward, the object appears to move in a direction contrary to that in which the lens is moved. The same sort of motion occurs no matter in which direction the lens is moved, whether up and down or crosswise, proving that the lens is a spherical one. A concave spherical lens causes objects seen through it to appear to move with the lens, that is, to the right when the lens is moved to the right, and vice versa. See page 32 for explanation. To ascertain the strength of the sphere,

take a spherical lens of the opposite sign from the trial case, of about the supposed strength of the spectacle lens, and, holding the two together, move them from side to side and from above downward. If objects seen through them still appear to move, the one lens does not neutralize the other, and then a weaker or a stronger lens is selected from the trial case until one is ascertained which, when combined with the spectacle lens, allows of no motion in objects viewed through them. The strength of the lens that neutralizes the spectacle lens is, of course, known, as it was taken from the trial case. The spectacle lens is of the same strength but of opposite sign (contrageneric).

Strong contrageneric spherical lenses, that is those over 9 D., fail to neutralize. The power of a lens depends upon its index, its curves and its thickness. The thickness of the lens may be neglected in concave spherical lenses between .25 and 20 D . as their centers can be made of infinite and equal thinness. In convex lenses there is
however an increasing thickness as the strength of the lens grows greater. It is evident that the radius of curvature of a lens of a given strength must be different when the thickness is considered than when it is not. Two lenses of the same curvature but of different thickness do not have the same refractive power. The lens surfacemeasure (shown in cut on page 457) is unreliable in convex lenses over 8 D . as the thickness of the lens constantly enters into the calculation of its strength. The only way to measure strong convex lenses is with the screen and graduated bar. When we prescribe convex lenses stronger than 8 D . we should remember that if they are measured by neutralization with concave lenses they are always weaker than the strength of the concave lens would indicate. The convex lenses in the trial cases are not exactly what they are numbered. Every lens should be ground as thin as it can be made.

A cylindrical lens is known by the fact that objects looked at through the lens appear to move unequally when the lens is moved in a different direction. When the cylinder is moved in the direction of its axis there is no apparent movement of objects seen through it, and when moved in a direction at right angles to that there is decided motion either with or against the movement of the lens according to

its sign. If the cylinder is held so that its axis is oblique then objects appear to move in an oblique manner when the lens is moved vertically or horizontally. If a straight line, the edge of the door or win-
dow frame or what not be looked at through a cylindrical lens with an oblique axis, the part of the line seen through the cylinder will be inclined to the right or to the left according to the position of the axis of the cylinder while the line as seen above through and below the cylinder will appear continuous if the lens has a vertical or horizontal axis.

The inclination of the axis of the cylindrical spectacle lens must first of all be ascertained. This can be done with a protractor. The protractor is a circle divided into degrees; at the center is a figure the shape of a spectacle lens, upon which the lens to be tested is laid. An ink line is first drawn across the lens coinciding with its axis, and then laid upon the protractor. The angle at which the ink line now stands denotes the position of the axis of the cylinder in the spectacle or nose-glass frames. The lens is laid upon the protractor with the side that goes next to the eye of the wearer up. Inasmuch as a line looked at through a cylinder with oblique axis appears broken and can be made to appear continuous by either rotating the cylinder to the right or to the left, so that the axis is either parallel to or at right angles to the line looked at, some definite plane must be followed in marking the axis of a cylinder.

When the upper end of a line seen though a cylindrical lens is tilted to the right, rotate the lens to the right until the line is continuous, then mark the lens parallel to the observed line. This line will be parallel to the axis of the cylinder if it is convex and at right angles to it if concave. When the portion of the observed line seen through the cylinder tilts to the left, rotate the lens to the left, until the line is continuous, and then mark lens. This line is parallel to the axis of the lens if convex, and at right angles to it if concave. After the axis of the lens has been determined hold the lens so that the axis is vertical, and then move up and down and crosswise. If the lens is a plain cylinder there will be motion in objects when the lens is moved up and down, and the motion crosswise will indicate the sign of the cylinder. Then select a contrageneric cylinder from the trial case of lenses, until one is found that neutralizes motion in ob-
jects seen through the lens when the latter is moved from side to side.
The axis of a cylindrical lens can also be readily determined in the following ingenious manner: From the center of the draftman's protractor attach a thread with sealing wax, weight the end of the thread with a bullet. Place the spectacles upon the upper edge of the protractor with the lenses occupying the position they do upon the face. The center of the lens to be measured should be over the insertion of the thread. A vertical line is then sighted through the lens and the latter with the protractor rotated until it appears continuous, rotating to the right if the upper end of line tilts to right and to the left if the upper end tilts to the left. The weighted thread will indicate upon the circumference of the protractor the degree of inclination of the axis of the lens if convex and 90 degrees from the axis if the lens is concave.

The axis of a cylinder alone or in combination may be located in the following manner: Place the glasses in a trial frame, with the lenses in the position that they would occupy upon the face. Sight a straight line through the cylinder, and then turn the cylinder with the trial frame so that the line seen through it appears upright. Now place a contrageneric cylinder from the trial case in front of the spectacle lens, of proper strength to neutralize the latter. The inclination of the selected cylinder is then read from the graduations of the trial frame, which is also the inclination of the axis of the cylinder in the spectacle lens.

A sphero-cylindrical combination is recognized by the fact that when the lens is neutralized in one meridian there is still motion of objects seen through it when it is moved at right angles to the former direction. We first ascertain the position of the combined cylinder as outlined above, then we ascertain the nature of the combination, whether it is positive or negative, by moving the lens parallel to and in a direction at right angles to its axis. A contrageneric sphere is then selected that will neutralize motion of objects in direction of the axis of the lens. The strength of this sphere indicates the strength of the sphere in the combination. A cylinder is then found that will
in addition neutralize the motion of objects in a direction at right angles to the axis of the combination spectacle lens, or a stronger sphere may be selected the second time, and then the difference between the strength of the two spheres indicates the amount of astigmatism.

The lenses unless otherwise desired should be properly centered in the frames. To locate the optical center of a lens, view a straight line through the lens and move the lens to the right or to the left until the portion of the line seen through the lens is continuous with

945. that seen above and below the lens. Draw an ink line across lens over the line seen through it. Turn the lens, and make a second line upon it in the same manner at right angles to the first. The two diameters thus constructed will cross at the optical center of the lens. Or one may view crossed lines through the lens and move it one way or the other until both arms of the cross seen through the lens are continuous with the portion of the arms seen above, below and to either side of the lens.

Prisms or prismatic combinations are recognized by the fact that an object looked at,through them appears to describe a circle when the lens is rotated about its center. Objects also seen through them appear displaced in space in the direction of the apex of the prism. The strength of the prism may be ascertained by neutralizing with a prism from the trial case, or the Dennet prism scale may be used, which is designed for the measurement of prisms by any of the units thus far proposed. It gives the tangent of angular displacement at the distance designated for each unit. The prism should be held in the position of minimum deviation. The scale is hung vertically upon the wall, and the prism so held before one eye that its apex intersects the pupil, giving rise to a monocular diplopia. The origin of the scale is then seen doubled, the distance separating
the doubled images of the prism upon the scale denoting the strength of the prism tested according to one of the several units of measurement depending upon the distance of the observer from the scale.

If the scale is regarded with one eye, and the prism to be tested is placed before the other one as is usually done, producing a binocular diplopia, any muscular inefficiency of the observer influences the correctness of the test. Within certain limits one can measure centrads and prism-diopters with equal accuracy. To do this it is necessary to stand at five meters. To measure meter-angles the left-hand numbers are to be used, and one must move seven centimeters towards the scale for every millimeter that the pupillary distance exceeds 60 mm ., and ten centimeters further away from the scale for every millimeter that it falls short of that number. To measure deviation in degrees it is necessary to hold the prism three quarters of a meter further back, and the scale reading is then divided by two. To measure refracting angles it is necessary to be at 5.28 m . In none of the cases will the error be more than the width of a line. To get the prismatic power of a strong sphero-cylinder, cover it with a diaphragm, having a central aperture of about 5 mm . and then produce in one eye diplopia or doubling of the scale by the quick passage of the lens back and forth before the eye. Lenses over 5 D. may be accurately measured in the following manner. To the origin of the scale a very small reflecting surface or a small gas jet is fixed. On the lens, as it is passed in front of the eye as above directed, a pin is so held that its point is at the geometrical center of the lens. In the midst of the diffusion
 caused by the point of light will be seen the image of the pin, and always at the same distance from the
origin, which is easily measured. If the observer wears glasses he should remove them as turning the head and looking obliquely through the glasses often vitiates the experiment. Instead of the pin an ink dot may be made upon the lens. With two such lenses, one minus and the other plus of at least 5 D., all spectacle lenses may be examined. There are a number of appliances upon the market for the measurement of lenses. The one commonly employed is shown in the cut.

The lens is pressed upon the three points $a, b$ and $c$, and the hand indicates the refracting power. It can be used for spheres, cylinders and compound lenses. To measure the refracting angle a prism may be placed between the legs of an ordinary pair of dividers, so that its legs fit closely along its two sides in the base-apex direction. If the frame of the glass interferes or its edge is cut too thin, a small coin placed upon one or both sides of the prism will obviate this difficulty. The number of degrees are then read off by placing the dividers upon a protractor the radius of which is equal to the length of the legs of the dividers. If the prism is combined with a lens, the dividers must be made to embrace it at its geometrical center.

Table Showing the Deviation Produced by the Refracting Angles Series, by Centrads, by Prism-Diopters, and by Meter-Angles.

|  |  |  |  |
| :---: | :---: | :---: | :---: |
| $\mathrm{I}^{\circ}=0^{\circ} 32^{\prime} \mathrm{co}^{\prime \prime}$ | $\mathrm{I}^{\mathrm{V}}=\mathrm{O}^{\circ} 34^{\prime} 22^{\prime \prime}$ | $\mathrm{I}^{\Delta}=0^{\circ} 344^{\prime} 22^{\prime \prime}$ | $\mathrm{m}=\mathrm{m}^{\circ} \mathrm{r}^{\circ} \mathbf{3}^{\prime} 6^{\prime \prime}$ |
| $2=1450$ | $\mathbf{2}=1845$ | $2=1$ $=1$ | $2=32612$ |
| $3=13720$ | $3=1437$ | $3=143$ | $3=5 \quad 918$ |
| $4=2120$ | $4=21730$ | $4=217$ | $4=65224$ |
| $5=2428$ | $5=25153$ | $5=252$ | $5=8305$ |
| $6=31450$ | $6=32615$ | $6=326$ | For p.d. of .06 |
| $7=34720$ | $7=4038$ | $7=4$ |  |
| $8=420 \quad 2$ $9=45140$ | $8=43310$ $9=5923$ | $8=434$ $9=512$ | $\begin{aligned} & \mathbf{1}=\mathbf{1} 50 \\ & \mathbf{2}=34043 \end{aligned}$ |
| $10=52340$ | $10=54346$ | $10=543$ | $3=53041$ |
| $11=55820$ | $\mathrm{II}=6188$ | $11=617$ | $4=72123$ |
| $12=632$ | $12=65331$ | $12=651$ | $5=9123$ |
| $13=7450$ | $13=92653$ | $13=724$ | For p.d. of . 064 |
| $14=738$ | $14=8$ 1 16 | $14=758$ |  |
| $15=81132$ | $15=83539$ | $15=832$ |  |

One often wishes to know with exactness the amount of prismatic effect caused by decentering a lens a given amount. The relations between decentering and prismatic action are easily obtained from the properties of lenses. Every ray passing through a lens makes an angle with the central ray passing through the lens after refraction, which is the amount of deviation produced by the lens at that point. The tangent of this angle (between central and refracted ray) is the amount of decentering of the lens, the focal distance of the lens being the radius. Hence if $D$ represent the strength of the lens in diopters, and $y$ the amount of decentering, $f$ the focal distance (equal to I $/ D$ ), and $d$ the amount of deviation, we have

$$
y / f=\tan d, \text { or } D y=\tan d
$$

Multiplying both sides of this equation by 100 to reduce to prism diopters, $100 D y=d^{\Delta}$. The same formula applies to centrads, that is $100 D y=d^{\nabla}$. For degrees reduce by the coefficient .57295, $57 D y=d^{\circ}$. For meter-angles divide by $3,19 D y=d^{\mathrm{m}}$.

At times the strength of two prisms combined with their axes not parallel is required. The problem is solved by spherical trigonometry, where each side of the triangle is an arc, equal in degrees to the deviation, and where one of the angles-that between the two prisms-is given. The formula is

$$
\begin{equation*}
\cos a=\cos b \cos c-\sin b \sin c \cos A \tag{I}
\end{equation*}
$$

in which the small letters represent the sides and the large ones angles. The following example will illustrate. What is the result of putting before the eye two prisms, the one being at $0^{\circ}$ and the other at $35^{\circ}$ on the trial frame, supposing that $b=9^{d}$, $c=5^{\text {d. }}$, and $a$ the strength of
 the resulting prism? $\operatorname{Cos} a=.9877 \times .9962-.1564 \times .0871 \times .8192$ $=.97174$, which is the cosine of the resulting prism, $13^{\circ} 24^{\prime}$. The customary method is to consider this question one in simple
trigonometry, using the formula for the solution of oblique triangles, changing the sign of the cosine as before because the angle between the two prisms becomes the angle in the triangle to be solved. The formula is then

$$
\begin{equation*}
a^{2}=b^{2}+c^{2}-2 b c \cos A \tag{2}
\end{equation*}
$$

When the angle $A$ between the two prisms is 90 degrees the last term in the equation becomes 0 . The problem is then reduced to the proposition of Pythagoras, namely - that the resultant prism is the square root of the sum of the squares of the other two prisms.

The strength of the prisms being given, either of the angles can be found by the formula

$$
\begin{equation*}
\cos B=\frac{a^{2}}{2 a c}+\frac{b^{2}}{2 a c}-\frac{c^{2}}{2 a c} \tag{3}
\end{equation*}
$$

or when $A$ is a right angle,

$$
\tan B=c / b
$$

If one prism is placed before each eye, the sign of the last term of the equation No. 2 is to be changed from minus to plus, and the last term of equation No. 3, from plus to minus, or add $180^{\circ}$ to the position of the prism in the trial frame and the formulæ are used as they are written above.

Suppose we wish to know how to insert a lens in the frame, without taking the trouble to focus or measure it. It will be recalled that the strongest convex or the weakest concave curve is to be placed in the frame away from the eye. If we know which surface of the lens goes from the eye the inclination of the combined cylinder will be right. To decide which is the plane, curved or cylindrical surface of the lens we view the reflection of some distant object from its surfaces. Suppose the lens is a I D. C. ax. 30. On one side of the lens we can easily get the reflection of some distant object ; we can turn the lens with either its short or long diameter at right angles to the eye and still the distinctness of the reflection will not be interfered with. But, if we turn the other side of the lens up, and use it
as a reflector, we will find that there is only one direction in which we can obtain a distinct reflection, and as we turn the lens the image becomes distorted. Therefore this surface is the cylindrical one, and as this surface in this case is to go out or away from the eye we screw the lens in the frame so that the long axis of the lens is horizontal and if the lens is ground correctly the axis of the cylinder will have the proper inclination. It makes no difference which edge goes towards the nose or temple side, so long as the cylindrical surface is kept away from the eye. Again take as example a lens of $+_{1}$ D.S. $w /+1$ D.C. ax. 90. On sighting across this lens as before we will find that on one surface we get a more or less distorted image, which is equally so no matter which way the lens is turned. This then is the spherical surface. If we now look on the opposite side of the lens we will find that there is only one position in which we can obtain a distinct image, and as the lens is rotated it becomes much distorted, hence this is the cylindrical side which in this case goes towards the eye. Suppose again that we have a lens of - $4 w /-1$ ax. $165^{\circ}$.
We will find as before that one side gives a more or less altered or distorted image in all directions, and that the other surface gives a distinct reflection in one direction and a distorted one in every other. As the strongest concave side goes towards the eye, the cylinder will go out.

## APPENDIX.

The following formulæ are those commonly employed in optics. The deduction of each will be found in the text of the book.
$\frac{\sin \angle I}{\sin \angle R}=\frac{V}{V^{\prime}}$. (Snell's Law) (page 7)

$$
\begin{gather*}
\angle D=\frac{\angle A}{2} .  \tag{II}\\
F=\frac{R}{\bar{V}} .  \tag{25}\\
F=\frac{R}{2\left(\frac{V}{V^{\prime}}-\mathrm{I}\right) .}  \tag{25}\\
\frac{\mathrm{I}}{F}=\left(\frac{V}{V^{\prime}}-\mathrm{I}\right)\left(\frac{\mathrm{I}}{R^{\prime}}+\frac{\mathrm{I}}{R^{2}}\right) .  \tag{26}\\
\frac{d}{D}=\frac{I}{O} .  \tag{30}\\
\frac{\mathrm{I}}{d}=\frac{\mathrm{I}}{F}-\frac{\mathrm{I}}{\bar{D}}(\text { for convex }) . \\
\frac{\mathrm{I}}{\mathrm{I}^{\prime}} \text { (for concave). }  \tag{3I}\\
\frac{F}{\mathrm{I}^{\prime}}=\frac{R}{2} .  \tag{52}\\
\frac{\mathrm{I}}{f}+\frac{\mathrm{I}}{f^{\prime}}=\frac{\mathrm{I}}{F^{\prime}} .  \tag{52}\\
\frac{\mathrm{I}}{f^{\prime}}=\frac{\mathrm{I}}{F}-\frac{\mathrm{I}}{F^{\prime}} .  \tag{53}\\
\frac{O}{I}=\frac{D}{F}=\frac{F}{d} . \tag{53}
\end{gather*}
$$

$D d=F F$ (Newton's Formula).

$$
\begin{gather*}
\bar{f}+\frac{F}{f^{\prime}}=\mathrm{I} .  \tag{53}\\
\frac{\mathbf{1}}{f}+\frac{\mathbf{1}}{f^{\prime}}=\frac{\mathbf{1}}{F} .  \tag{53}\\
\frac{1}{f}-\frac{\mathrm{I}}{f^{\prime}}=-\frac{\mathbf{1}}{F} .  \tag{53}\\
F=\frac{I \cdot D}{O} .  \tag{54}\\
\frac{V^{\prime}}{f^{\prime}}-\frac{V}{f^{\prime \prime}}=\frac{V^{\prime}-V}{R} .  \tag{64}\\
\frac{F_{2}}{f_{2}}-\frac{F_{1}}{f_{1}}=\mathrm{I} .  \tag{65}\\
C C^{2}=F^{\prime} F^{\prime \prime} .  \tag{66}\\
y^{\prime}=\frac{F_{1}^{\prime} F_{2}^{\prime}}{d} .  \tag{67}\\
y_{2}=\frac{F_{1}^{\prime \prime} F_{2}^{\prime \prime}}{d} .  \tag{67}\\
F^{1}=\frac{F_{1}^{\prime} F_{2}^{\prime}}{d} .  \tag{67}\\
F_{2}=\frac{F_{1}^{\prime \prime} F_{2}^{\prime \prime}}{d} .  \tag{67}\\
F^{\prime} / F_{2}=V^{\prime} / V^{\prime \prime} .
\end{gather*}
$$

## APPENDIX.

$$
\begin{array}{cr|c}
O=2 \tan 2.5^{\circ} \cdot D . & (8 \mathrm{I}) & \frac{c}{d}=\frac{b-\left(F^{\prime \prime}-F^{\prime}\right)}{F^{\prime}} \times \frac{F^{\prime \prime}}{b} .(245) \\
x=p \frac{d}{d+a} . & (88) & A s_{t}=k+p \cdot A s_{c_{0}} \\
A=p-R=D-S . & (90) & R=\frac{337.5}{D} . \\
A_{1}=p_{1}-R_{1} . & (104) & (395) \\
G=\frac{D}{F_{1}} . & (127) & C^{\prime \prime}=\frac{F^{\prime} \times f^{\prime \prime}}{C^{\prime}} . \\
y / x=p^{\prime \prime} \left\lvert\, p^{\prime}=\frac{F^{\prime \prime}}{F^{\prime}} .\right. & (244) & f^{\prime \prime}=l \pm C^{\prime \prime} .
\end{array}
$$

,

## INDEX.

Abduction, 205, 423
Aberration,
Chromatic, $\mathrm{I}_{5}$ Of eye, 38,39
Test, 326
Spherical, 34, 35
Aberroscope, 40, 101
Accommodation, 58, 86, 260, 275
Amplitude of, 87, 89
Exophoria in, 306
Helmholtz theory of, 98
Influence of age on, 304
Mechanism of, 94
Relative, 104, 105
Tscherning's theory of, 98, 100
Achromatopsia, 165
Adaptation, period of, 270
Adduction, 205, 423
Amblyopia, 273
Exanopsia, 273
Acquired, 273
Congenital, 273
Meridional, 302
Ametrometer, Thompson's, 322
Ametropia, 324
Anaphoria, 265, 424
Test for, 424
Angle,
Alpha, ro6, 398
Beta, 106
Gamma, 106
Kappa, 106
Critical, II

Angle,
Of deviation, 10
Of incidence, 5
Of refraction, 5 .
Of reflection, 47, 48
Refracting, II
Meter, 102
Visual, 77, 79
Limit visual, 79
Anisometropia, 257
Aphoses, 197
Aphakia, 435
Apparatus, divisions of visual, 154
Artery,
Retinal, ${ }_{35}$
Cilio-retinal, 136
Asthenia, ciliary, 262
Treatment of, 262, 263
Astigmatism, 241, 253, 288
Against the rule (indirect), 242, 295
Band appearance in, $365,366,367$, 369
By incidence, 242, 252
Causes of, 241
Compound, 242
Correction of irregular, 255
Diffusion in, 244
Focal lines of, 243, 244
Horizontal, 242
Irregular, 242, 252
Latent, 283
Manner of detecting, 254

Astigmatism, Mixed, 296
Mixed, in retinoscopy, 365
Oblique, $24^{2}$
Ophthalmometric, $39^{1}$
Principal meridians of, 2
Physiological, 269
Regular, $24^{2}$
Retinoscopy in, 365
Simple, 242
Stenopaic slit in, 299
Supplementary, 391
Symptoms of, 248, 249
Tests for, 280, 287, 292
To exclude effect of, 278
Treatment of, 250, 251
Varieties of, 247, 256, 257
Vertical, 242
Astigmometer, Hotz', 329, 372
Axis or axes,
Corneal, 106
Listing's, 214, 215
Optic, 106
Primary ocular, 203
Secondary ocular, 205
Visual, 106
Axonometer, 367
Bands, interference, 3
Of light in astigmatism, 365,366 , 367, 369
Blind spot,
Mariotte's, 149
Filling in of, 185
Demonstration of, 185
Bodies, Bowditch's, 196
Cabinet, ophthalmic, 276
Cards, Astigmatic, 279
Green's, 280
Hill's, 28 I
Pray's, 28I

Carus, Verhoff's, 282
Cataphoria, 265, 424
Centers, primary optical, 160
Center, visual, 194
Of rotation, 203
Chromoscope, 328
Circle of diffusion, $36,83,90,226,261$, ${ }^{2} 74$
Size of, 88
Listing's table of size of, 88
Clinometer, 222
Clinoscope, 218
Color-blindness, 165
Colors,
Complementary, 162,163
Hue, 162
Intensitè, 162
Tint, 162
Mixing of, in the eye, 161, 162
Colored pigment, $\mathrm{I}_{3}$
Preference for, 170
Theories of perception, $162,16_{3}, 164$
Contrast, 183
Simultaneous, r84
Successive, 185
Convergence, IoI
Amplitude of, ro3
In myopia, 287
Manner of estimating, 102, 103
Relative, 104
Unit of measure of, IoI
Cross-eyes, 203
Crossed cylinders, 3 오
Cyclophoria, 265, 428
Cycloplegics, 458
Daltonism, 165
Seebeck or Holmgren test for, 167
Stilling's test for, 167
Test with spectroscope, 167

Daltonism, Lantern test for; 168
Meyer's test for, 169
Donder's, 167
Weber's, ${ }^{1} 67$
Wollfberg's, 167
Deorsumduction, 205, 423
Dichromasia, 169
Diplopia, $\mathrm{I}_{4} \mathrm{I}$
Physiological, 194
Disc, Stenopaic, 273
Use of pin-hole, 274
Placido's, 382
Optic, 134
Displacement, parallactic, 121, 138
Dyschromatopsia, ${ }^{167}$
Emmetropia, 58, 224
Entoscope, 199
Esophoria, 265, 410, 413
Euphthalmin, 271
Examination, routine of, 272
Exophoria, 265, 410, 413
Experiment of Young, 2
Of Wollaston, $3^{8}$
Of Fechner, 173
Of Hering, 209
Of Javal, 217
Of Meissner, 216
Of Scheiner, 320
Of Völcker, 95
Of Volkmann, 217
Eye, schematic, 75
Of Helmholtz, 76
Donder's reduced, 77
Listing's reduced, 77
Skiascopic, ${ }_{37} 6$
Eyeball, cardinal points of, 105, 106
Catoptric images of, 99
Centers of surfaces of, 400
Dioptric surfaces of, 59

Eyeball, Gross anatomy of, 56, 57
Measurements of, 56
Movements of, 203
Optical defects of, 105, 106
System of, 71, 72, 73
Radii of surfaces of, 402
Refraction of, 59 unics of, 56
Eye ground, color of, I 33
Appearance of, 134, I 35
Eye Strain, 224, 225
Causes of, 224, 225
Symptoms of, 225
Field, hemianopic, 149
Ophthalmoscopic, 122, 123
Struggle of the two, 193
Fixation, binocular field of, 212, 213
Field of, 210, 211
Line of, 106
Focus or foci,
Of concave mirrors, $5^{\circ}$
Of convex mirrors, 53
Of cylindrical lenses, 41
Principal focus of convex lens, 20, 22
Real, 21
Secondary, 20, 21
Virtual, 2 I
Fundus, tessellated, $\mathrm{I}_{3} 8$
Color of, 133
Appearance of, 134, I35
Hemeralopia, 171
Hemiablepsia, 159
Hemianopia, 159
Hemianopsia, temporal, 159
In lesions of optical centers, 160
Inferior, 159
Superior, 159 .
Lateral (homonymous), 159

Heterophoria, 264, 406, 4 II
Asthenic, 265
Causes of, 266
Reversed, 265, 429
Steven's test for, 418
Sthenic, 265
Hyperphoria, 265
Hyperopia, 59, 226, 278
Absolute, 229
Consequences of, 226, 227, 230
Correction of, 226, 278, 285
Facultative, 229
Latent, 229
Manifest, 229
Total, 229
Treatment of, 231
Table of axial, 239
Varieties of, 228
Illumination, oblique, $\mathrm{I}_{3} \mathrm{I}$
By direct sunlight, $13{ }^{2}$
Illusions, optical, 183, 187
Images, after, 175
By concave spheres, 30
By convex spheres, 26, 27, 28, 29,30
By concave mirrors, 50, 51, 52
By convex mirrors, 53
By plane mirrors, 48
Of Purkinje, 98, 379, 380
Projection of in ophthalmoscopy, 124, 125
Suppression of retinal, 142
Impressions, visual, 183
Index, refraction,
Of crown glass, 7
Of flint glass, 7
Of aqueous humor, 7
Of cornea, 7
Of crystalline lens, 7
Of vitreous humor, 7

Index, of water, 7
Interval,
Astigmatic, of Sturm, 243
Iris, apparent, 75
Irradiation, 183
Kinescopy, 325
Keratoscopy, 344
Law, Fechner's, 172
Listing's, 214, 216
Snell's, 6, 7
Lamina cribrosa, 134
Lens, achromatic, $3^{8}$
Aperture of, 34
Concave spherical, $=2$
Cylindrical, 16, 40, 453
Cylindro-toric, 45
Definition of, $\mathbf{x} 6$
Focal plane of, 17
Interval of, 13
Focus of convex, 17
Numeration of, 18, 23
Optical center of, 17
Periscopic, 34
Poles of, 17
Principal axis of, 17
Secondary axes of, 17
Focus of, 20
Spherical, 16
Sphero-cylindrical, 43, 254, 455
Sphero-toric, 45
Toric, 44
To ascertain strength of, $3^{2}$
Letters, Snellen's, 80
Standard, 8I
Test, 81
Lens measure, 457
Light, I
Beam of, 4

Light, Corpuscular theory of, I
Electro-magnetic theory of, 3
Intensity of, I
Immediate source of, 348
Number of waves of, 14,15
Original source of, 348
Refraction of, 4, 47
Ray of, 4
Pencil of, 4
Propagation of, 2
Spectrum of, 14, I 6
Undulatory theory of, I
Velocity of, 1
Light-shade, Thorington's, 347
Macula lutea, 1 $_{3} 6,137$
Projection of, 193
Malignering, 177
Tests to detect, 178 , 179,180 , 181
Measure, Frame, 448
Medium dioptric, 4
Maddox groove, 4 II
Rod, 4 I
Mirror, definition of, 47
Concave, 49
Conjugate foci, 53. 54
Convex, 49, 53
Focal interval of, $5^{2}$
Plane, 48
Principal focus of, 50
Axis of, 49
Radius of curvature of, 49
Spherical, 49
Vertex of, 49
Monoscopter, 212
Muscle or muscles,
Action of extra-ocular, 205
Balance of extra-ocular, 264
Imbalance of extra-ocular, 264
Brücke's, 96

Muscle or muscles, Müller's, 96
Inefficiency of extra-ocular, 264
Insufficiency of extra-ocular, 264
Of oblique, 42 I
Myopia, 59, 231
Cause of, 232, 233
Correction of, 232, 285
Malignant, 233
Symptoms of, 234, 235
Table of axial, 240
Treatment of, 236, 237
Varieties of, 235
Nerve, anatomy of optic, 157
Nose-glass fitting, 448
Object, apparent distance of, $\mathbf{1 8 6}$
Size of, 186
Opacities in dioptric media, $\mathrm{I}_{3} 1$
Ophthalmometer, hand, 382
Of Javal and Schiötz, 383
Manner of using, 386
Ophthalmometry, 379
Basis of, 394
Primary position in, 386
Secondary position in, 387
Ophthalmophakometer, 397
Ophthalmo-dynamometer, 90, 9 I
Ophthalmoscope, 107
Invention of, 107
Knauer's, II3
Loring's, III
Morton's, IIO, II5
Refraction, 109
Roth's, II4
Simple, 109
Ophthalmoscopy, direct, 110, II2, II8, 331
In hyperopia, 333
In myopia, 337

Opthalmoscopy,Indirect, 1 IO, 112 , 340343
Auto-, 118, I28, 129
By sunlight, 129
Magnification by, 124, 126, 127
Ophthalmo-cromoscopy, 130
Ophthalmo-spectroscope, ${ }^{1} \mathrm{C}$
Optometers, 308
Javal-Bull's, 308
Coccius's, 3 I4
DeZeng's, 309
Donder's, 314
Rod, 304, 314
Sous's, 3 I4
Young's, 40
Orthophoria, 264
Spasmodic, 429
Orientation, $\mathbf{I}_{39}$
False, 141
Objective, $\mathbf{I} 40$
Subjective, 14 I
Perimeter, 144
Dana's, 143
Meyrowitz's, 146
Skeel's, 147
Hardy's, 147
Phoses, 196
Phenomenon, Bell's, 210
Troxler's, 176, 209
Phenomena, entoptic, 195-202
Phorometer, Stevens's, 414
Phoroscope, Aiken's, 4 II
Phosphenes, 186
Photometer, Förster's, 172
Planes, cardinal, 60
Points, cardinal, 60
Properties of cardinal, 6I
Method of locating cardinal, 66
Of crystalline lens, 69, 70,71

Points, Of schematic eye, 77, 78
Point, Donder's presbyopic point, 304
Of reversal, 351, 365
Porus opticus, I35
Position, primary, 206
Secondary, 206
Prisoptometry, 323
Prism,
Achromatic, 15
Apex of, 9
Base of, 9
Definition of, 9
Dispersion of, $\mathrm{I}_{5}$
Position of, 9
Minimum deviation, of, 10
Numeration of, 12, 13
Refraction through, 9
Angle of, 9
Risley's, 413
Maddox, 16
Presbyopia, 97, 224, 260
Correction of, 262, 302
DeSchweinitz's method in, 305
Symptoms, 261
Projection of object, 140
Pupil, apparent size of, 74
Apparent position of, 75
Apparent, 75
Of entrance and exit, 75
Pupillometer, 443
Puncture proximum (p), 87
In E. H. \& M., 90-93
Remotum (R), 87
In E. H. \& M., 90-93
Rays of light, 4
Harmful, 379
Homocentric, 60
Incident, 4
Lost, 379

Rays of light, Refracted, 4
Reflected, 47
Useful, 379
Ultra, 16 I
Redress, movement of, 407
Reflections in ophthalmoscopy, I30, 13I
Internal, I I
Total, II
Reflex, foveal, 137
Macular, 137
Chorioidal, iro
Refraction, normal, 224
Absolute index of, 6
Caustics by, 35, 55
Demonstration of, 7,8
Dynamic, 87
Error of, 224
Explanation of, 5, 6
Index of, 6
Limit angle of, II
Static, 86
Total, 1 I
To exclude error of, 278
To deduce, of complete eye from. that of aphakic eye, 440
Refractometer, Lambert's, 357
Retinoscope, 345
Fuller's, $37{ }^{1}$
Retino-skiameter (Cross), 359, 360
Retinoscopy, 344
Apparent movement in, 348
Description of, 348
Form of light area in, 353
Magnification of retina in, 35 I
Mixed astigmatism in, 372
Measurement of accommodation by, 375
Real movement of light in, 348
Leroy's explanation of, 354
Paracentral shadow in, 368

Retinoscopy, Practical application of, $35^{6}$
Spherical aberration in, 369
Ridgeway's test, 327
Rotation, centers of, 203
Axes of, 205
Scale, prism, 456
Simulated blindness,
See malignering
Scotomata, 149
Absolute; 149
Annular, 149
Central, 149
Fixed, 149
Motile, 149
Negative, 149, 15 I
Peripheral, 149
Positive, 149, $\mathrm{I}_{5} \mathrm{I}$
Relative,
Sensation guiding, 264
Sense, color, I39, 161
Form, 139
Light, ${ }^{39}$ 3, 171
Acuity of, 174
Shadow Test, 334
Skiascopy, 344
Skiascopes, 356
Spectacle fitting, 443
Spectra, ocular, 186
Spot, Mariotte's blind, 149
Measurement of blind, $\mathrm{I}_{53}$
Size of, 53
Stenopaic slit, 299
Manner of using, 299
Streak, reflex, 135
Cause of, $\mathbf{I} 35$
Surface, dioptric, 4
Sursumduction, 205, 423
Squint, 265, 273
Dynamic, 265

Squint, Latent, 265
Caused by myopia, 265
Table of cycloplegics, 270
Prismatic deviation, 458
Torsion, 222
Test, fogging, 269, 282, 283, 296
Von Graefe's screen, 407, 408
Parallax, 407
Test, Cards,
Brackets for, 277
Förster's, 275
Guillery's, 85
Jaeger's, 85
Javal's, 85
Snellen's, 85
Randall's, 81
Reversed, 275
Landolt's, 8 i
Williams', 84
Ziegler's, 85
For near vision, 85
Threshold, the, 174
Theory of parent, the, 354
Trial case, 271
Plan of Procedure with, 27 r

INDEX.
Tropometer, 207
Directions for using, 208
Vein, retinal, 136
Vision, acuteness of, 80
Absolute field of, 149
Binocular field of, 153
Central, I 39
Curtailment of field of, 149
Estimation of field of, 86
Extent of field of, 147
Evidences gained from field of, 278
Field of, $\mathbf{1 4 2}^{2}, \mathrm{r}_{5}$
In optic nerve diseases, ${ }^{5} 5$, 157
Chiasm diseases, 158 , 159 Tract diseases, 159
Diseases of optical centers, 160
Manner of expressing acuity of, 82 , 84
Overshot field of, 159
Peripheral, 139
Relative field of, 149
Stereoscopic, 14I, I9I

## THE REFRACTION OF THE EYE

INCLUDING A COMPLETE TREATISE ON OPHTHALMOMETRY; A CLINICAL TEXT-BOOK FOR STUDENTS AND PRACTITIONERS

By A. EDWARD DAVIS, A.M., M.D.

## Cloth. 8vo. \$3.00 net. With 119 Engravings, 97 of which are original

The author outlines a routine method of examination to be followed in every case. Each step of the examination that is necessary to be made in fitting a patient with glasses is described in detail. With the use of the ophthalmometer to detect the corneal astigmatism, and by following this routine method of examination, spasm of accommodation, if present, can, in the great majority of cases, be overcome, and if not present, the liability of causing it avoided. Thus the use of a mydriatic is rendered unnecessary, except on rare occasions-in not more than one per cent. of all cases of errors of refraction.

The entire subject of the refraction of the eye is treated in this volume. A feature of the book is a report in full of one hundred and fifty clinical cases, illustrating practical points in the fitting of glasses and in the use of the ophthalmometer. Many diagrams are used to show the focus of the principal meridians of the eyes, so that the merest tyro must understand them.

The most complete and detailed description of the ophthalmometer, together with concise and definite rules for its use, are given. These rules contain the best practical directions for using the instrument accurately, and by their aid alone the careful student will learn to use the instrument correctly.

# THE OPHTHALMIC PATIENT 

A MANUAL. OF THERAPEUTICS AND NURSING IN EYE DISEASE

By PERCY FRIDENBERG, M.D.<br>Ophthalmic Surgeon to the Randall's Island and Infants' Hospital, Assistant Surgeon New York<br>Eye and Ear Infirmary

Cloth. 16 mo . \$1.00 net
" The author aims to explain the principles and to describe the various procedures and appliances of ophthalmic nursing, the technique of operative assistance, and the nature and use of ocular remedies, as exemplified in private practice as well as in the established routine of well-equipped institutions. The book is intended to serve as a practical guide to physicians, students, and nurses who lack special training in the care of ophthalmic cases, as well as to supplement the invaluable routine of the ward and the training-school with theoretical instruction. The author has thought it advisable to lay most stress on actual nursing, and to treat of the topics of pathology, symptomatology, and diagnosis only in so far as it was necessary to elucidate his own theme, for this volume is in no way a treatise on diseases of the Eye." -From the preface.

# DEFECTIVE EYESIGHT 

By D. B. ST. JOHN ROOSA, M.D., LL.D.<br>Cloth. 16 mo . \$1.00 net

This treatise takes up all conditions requiring the use of glasses, and indicates in the most careful manner the indications and rules for describing them. It is well known that the author is a conservative in regard to the value of glasses, believing that there is a limitation to their use, and that they ought not to be prescribed unless of positive value. No pains have been spared to make the manual a complete guide to the practitioner who wishes to understand and practice the rules for the prescription of lenses for the improvement of impaired sight. The book may also be interesting to educated men in all departments of life, who desire to be informed as to advances that have been made in this interesting subject, one which concerns such a large proportion of the human race.

# HANDBOOK OF OPTICS FOR STUDENTS OF OPHTHALMOLOGY 

By WILLIAM NORWOOD SUTER, B.A., M.D.

Cloth. 16 mo . $\$ 1.00$ net

"Simplicity has been sought so far as this is not incompatible with thoroughness. But the demonstrations, some of which may appear formidable to the student, require no knowledge of mathematics beyond that of simple algebraic equations and the elementary truths of geometry. For those who may not be familiar with the trigonometrical ratios, a brief synopsis has been furnished in an appendix.' ${ }^{\text {'-From the preface. }}$

# UNIVERSITY OF CALIFORNIA LIBRARY 

## Los Angeles

This book is DUE on the last date stamped below.



A 000414497


[^0]:    * Light, electro-magnetic and sound waves are similar save in the number of vibrations per second. If the waves are comparatively few in number they are perceived as sound by the ear, if of greater frequency they traverse space as electro-magnetic waves, while if of still greater frequency they call forth sight. In many particulars light and electro-magnetic waves behave alike, which has given rise to the so-called electro-magnetic theory of light.

[^1]:    * The relocity of the red rays is the greatest ; they are therefore retarded the least in passing through the dioptric medium, and in consequence bent least from their course.

[^2]:    * Often called angle alpha.

[^3]:    * If the observer's eye is at the anterior principal focus of the observed eye the fields will be equal in $E, H$ and $m$, as the rays including them will be parallel.

[^4]:    * The manner of performing autoöphthalmoscopy by the indirect method has been described on previous page.

[^5]:    * The terms hemiopia and hemianopsia are often used synonymously, but really hemiopia signifies loss of perceptive power of one half of the retina, while hemianopsia means obscuration of one half of the visual field.

[^6]:    * Only those waves whose wave-length is between .00036 and .00075 can be seen.

