THE FATIGUE OF METALS

AUGUST WÖHLER

THE outstanding pioneer figure in the experimental study of the strength of materials under repeated stress, was born June 22, 1819, at Soltau, in the Kingdom of Hanover. He was the son of a schoolmaster, and obtained his schooling at a trade school in Hanover, where he had a scholarship. After his school days he had practical experience in railway construction in the Kingdom of Hanover, experience as a draftsman with the A. Borsig Locomotive Works in Berlin, and experience as an engine driver in Hanover.

He entered the Prussian Railway Service in 1847 and in 1852 was appointed a member of a commission to study causes of derailments and details of locomotive construction. This work led him to study axle failures in rolling stock.

In 1852 he succeeded in establishing in Berlin an experiment station for tests of metals under repeated stress. It was in this station that his great life work was done. This station was soon enlarged by the addition of equipment for general materials testing, became part of the Gewerbeakademie, and later a part of the Technical High School.

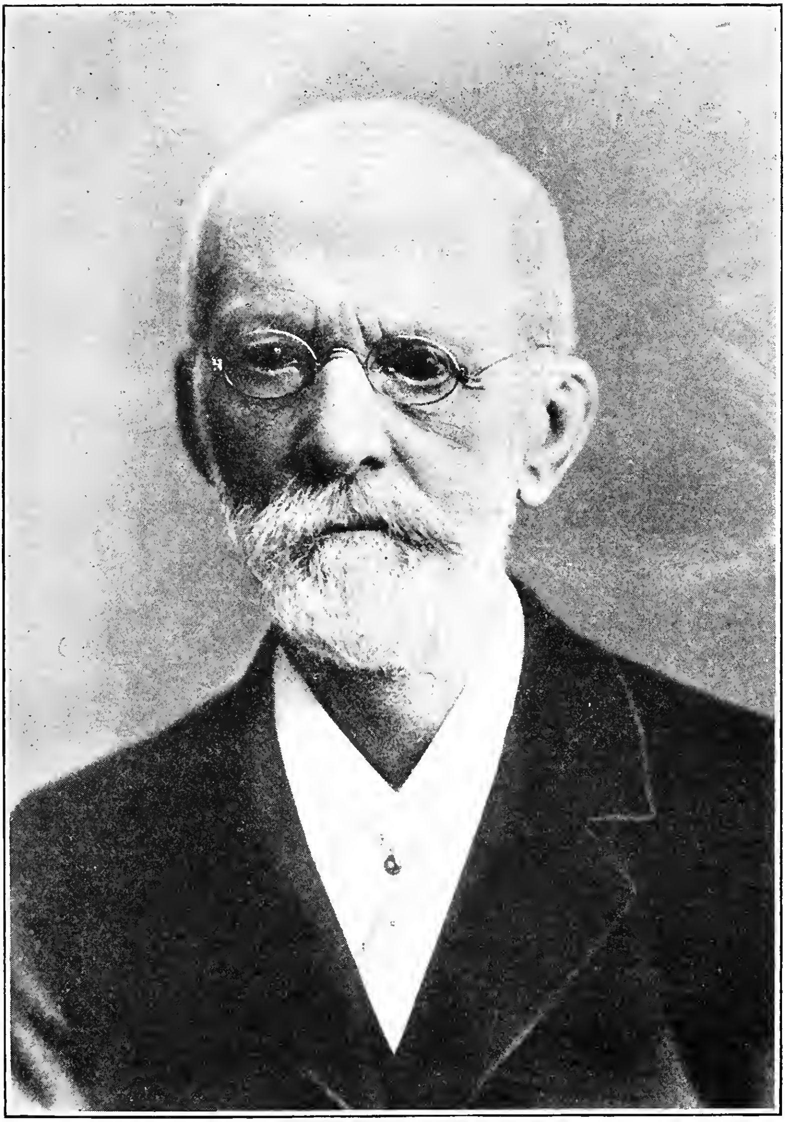
Wöhler's famous tests were made between the years 1852 and 1869. His machines are still preserved, and the commonest form of repeated-stress testing machine in use today, the rotating-beam machine is practically the same as the machine designed by Wöhler. In fact, the names "Wöhler machine" and "Wöhler test" are frequently used in connection with rotating-beam fatigue tests.

Wöhler did his work before the days of the metallurgical microscope, and his concepts are the stress-strain concepts of Weisbach and Rankine, but his critical value of stress, below which failure will not occur even after an indefinitely large number of cycles of stress, is the same value which today, under the names "endurance limit" and "fatigue limit," is regarded as the most significant index of the fatigue strength of a metal.

Wöhler became manager of the Berlin Railway Car Works in 1869, and in 1874 was made one of the general directors of the Alsatian State Railways. He retired from active professional life in 1889. During his later professional years he advocated impact tests for determining the acceptability of rails, axles, and tires. It was on his initiative that the Prussian government in 1876 issued an official classification of iron and steel.

In 1896 the Verein Deutscher Ingenieure awarded Wöhler their highest honor, the Grashof medal. In 1901 the Technical High School of Berlin-Charlottenberg conferred on him the degree of Doctor Ingenieur.

Wöhler died March 21, 1914, in the city of Hanover, a few months before his ninety-fifth birthday. His work endures.



August Wöhler
(*Courtesy of Dr. Ing. C. Matschloss, Verein Deutscher Ingenieure.*)

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THE FATIGUE OF METALS

WITH CHAPTERS ON THE FATIGUE OF WOOD AND OF CONCRETE

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To the many distinguished British investigators, who, especially during the present century, have been foremost in forwarding our knowledge of the fatigue phenomena of metals, this book is dedicated.

PREFACE

The growing use of high-speed machinery during the last quarter century has greatly increased the necessity for experimental knowledge of the behavior of materials, especially metals, under repeated stress. Failure of metal parts under repeated stress, "fatigue" failure as it is commonly called, is usually sudden, occurs without warning, and in many cases may be the cause of a major structural disaster.

One of the purposes of the authors in writing this book is to summarize the more important experimental facts concerning the strength of metals under repeated stress. Another purpose is to review briefly the more important of the current theories of fatigue of metals. A third purpose is to give a brief description of apparatus and methods used in making an experimental study of the fatigue of metals.

In compiling tables of results of tests of various materials, the guiding principle has been to make available to the reader, in compact and convenient form, typical results for various materials, giving in each case as complete a statement as possible of the chemical composition and heat treatment of each metal. This, it is believed, will enable the reader to form some judgment as to what may normally be expected from these metals. The results quoted are, for all the metals reported, obtained from test data of tests covering a sufficient number of repetitions of stress to render the results reliable.

It has been necessary to refer frequently to the use of the ordinary formulas of mechanics of materials, or to the elaborate mathematical methods of the theory of elasticity. For readers who may wish to refresh their memory of such formulas and analyses, reference is made to various standard texts on the mechanics of materials (such as Boyd, Seely, Merriman, Poorman, Maurer and Withey), and for those who have the time and the inclination to study the theory of elasticity, to Love's great work, "Mathematical Theory of Elasticity."

Frequent reference is also made to the metallographic study of metals and related matters. For those readers interested in these phases of the study of metals, it is suggested that "The Science of Metals" by Jeffries and Archer will be found an excellent book to be read in connection with this.

Although data concerning the fatigue strength of non-metallic materials are very few, two chapters have been given to a discussion of such data as are available for wood and for concrete.

The authors acknowledge the assistance of many friends and colleagues in the preparation of this book. Where photographs or drawings have been contributed, acknowledgement is made by a note. The authors have drawn very heavily on the published results of tests by H. J. Gough, of the British National Physical Laboratory, D. J. McAdam, Jr., of the U. S. Naval Engineering Experiment Station, R. R. Moore, of the McCook Aviation Field, Dayton, Ohio, T. M. Jasper, and others who have been coworkers at the Illinois Engineering Experiment Station, and the U. S. Forest Products Laboratory at Madison, Wis.

The authors wish to make special acknowledgement to Dr. D. J. McAdam, Jr., who read the manuscript of this book and made numerous valuable suggestions and constructive criticisms.

THE AUTHORS.

April, 1927.

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THE FATIGUE OF METALS

CHAPTER I

STRESS AND STRAIN IN METALS—THE ACCURACY OF THE ORDINARY CONCEPTS OF ELASTIC ACTION

Strain, Unit Strain.—Whenever a force is applied to any member of a machine or a structure, the shape of the member is altered. If the member has been properly designed to withstand the force, the change of shape is small, usually not visible to the unaided eye, and on the removal of the force, the member returns to its initial form as nearly as can be determined by any ordinary measurements. The change in any linear dimension of a member caused by the application of a force is called the "strain," or deformation, and the change per unit of linear dimension is called the "unit strain" or "unit deformation."[1]

Stress, Unit Stress.—If a machine part or structural member (Fig. 1) is acted on by forces P, P', there must be set up within the body internal forces (measured in pounds or kilograms) and called "stresses," which resist the tendency of the external forces to tear apart or to crush the member. Imagine the part of the body at one side of any section mn to be cut away; then to balance the force P (Fig. 1(b)) there must be stresses S. The summation of these stresses makes up the total stress on the section mn. If the stress over a small portion of the section be denoted by ΔS and the small area be denoted by ΔA, then for that

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[1] The definition used here is that common in American engineering texts on mechanics of materials; physicists use the term "strain" for what is defined above as unit strain (measured in inches per inch or millimeters per millimeter). No confusion between the two systems of units need arise if care is taken to keep in mind the units used for strain.

small area $\Delta S/\Delta A$ is the *intensity of stress,* or the *stress per unit area,* or more briefly, the *unit stress.*[1]

If the stress is uniformly distributed over the whole area of the cross-section *mn,* then the unit stress is P/A. Note that this is true only *if the stress is uniformly distributed.* In general, the unit stress will be different at different locations on the cross-section *mn.*

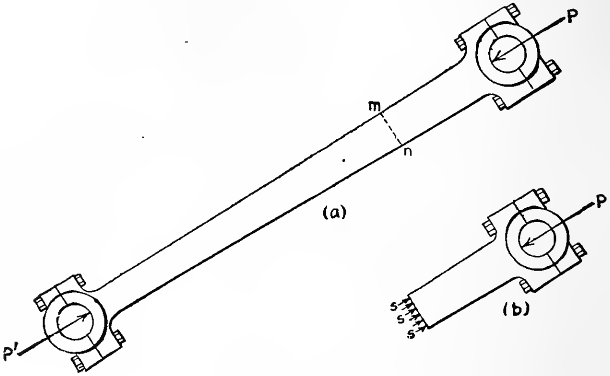

Fig. 1.—Machine part under stress.

Hooke's Law.—Under working conditions, for the materials commonly used to carry load in structures and machines, it is very nearly true that *stress is proportional to strain.* This statement is *Hooke's law,* and is named after the English physicist who first stated it. Under working loads Hooke's law agrees very closely with the observed general action of rolled and forged metals and of steel castings; it is a fairly close approximation for cast metals in general, for concrete, for brick, and for wood; it is a rough approximation for such materials as rubber, leather, rope, and textile fabrics. For rolled and forged metals and for steel castings there is a fairly well-defined limiting unit stress, known as the proportional limit (or the proportional elastic limit), above which Hooke's law does not hold.

[1] Physicists use the term "stress" for what is defined above as "unit stress."

Formulas for Computation of Stress and Strain.—For the ordinary computation of strain and stress in machine and structural parts and in test specimens of various metals, textbooks on the mechanics of materials give fairly simple formulas. These formulas have been put into their present form largely by Rankine, the famous Scotch engineer, and form the skeleton of what is frequently known as the Rankine mechanics of materials. Rankine mechanics of materials neglects several factors assumed to be of minor importance, notably, the strain which is at right angles to any tensile or compressive stress, and which always accompanies that stress. For anything approaching complete analysis of stress distribution in a machine or structural part, Rankine mechanics of materials becomes inadequate, and much more involved formulas become necessary. The elaborate system of mathematical analysis which attempts to take account of *all* stresses and strains in a structural or machine member is known as the mathematical theory of elasticity, although that name really includes the simple Rankine mechanics of materials. Sometimes the more elaborate system of stress analysis is known as the *Saint Venant* mechanics of materials, from the name of the distinguished French mathematician and physicist who is the outstanding figure in its development.

While there is not space in this book to show the derivation of the formulas for stress and strain in structural and machine parts, some of the commoner formulas are given for ready reference. All the formulas given are Rankine formulas commonly used by structural engineers and machine designers.

1. *Axial Loading.*—The stress (tension or compression) is assumed to be uniformly distributed over the cross-section of the piece loaded (rarely is this assumption an exact one), and the resulting unit stress is given by the formula

$$S = \frac{P}{A},$$

in which

S is the unit stress (pounds per square inch),
P is the axial load in pounds,
A is the area of the cross-section in square inches.

2. *Direct Shear.*—The stress on a rivet in a riveted joint is a good illustration of direct shear. Uniform distribution of stress over the cross-section of the piece is assumed, although this is nearly always a very rough approximation, and the formula for the unit stress is

$$S_s = \frac{P}{A},$$

in which

S_s is the shearing unit stress (pounds per square inch),
P is the shearing load in pounds,
A is the area of cross-section in square inches.

3. *Flexure.*—Under working loads on machine parts and structural members having cross-sections of symmetrical shape and loaded in a plane containing an axis of symmetry or at right angles to such an axis,[1] the stress is assumed to vary uniformly from a value of zero at a "neutral axis" passing through the centroid of a cross-section in a direction perpendicular to the plane of loading, to a maximum value at one side of the cross-section and to a minimum value at the other (maximum value and minimum have opposite signs). This assumption is a very close approximation for flexural members with a span not less than about ten times the depth. The maximum unit stress at the outside edge of the piece is given by the formula

$$S = \frac{Mc}{I},$$

in which

S is the unit stress in tension or compression (pounds per square inch),

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[1] For members with unsymmetrical cross-section (*e.g.*, a Z-bar) or for members not loaded in a plane of symmetry or at right angles thereto (*e.g.*, an angle bar loaded parallel to one leg), the formulas given here do not hold, and a much more elaborate analysis must be used. See Johnson, L. J., "An Analysis of General Flexure in a Straight Bar of Uniform Cross-section," *Trans. Am. Soc. Civil Eng.*, vol. 56, p. 169, 1906.

M is the bending moment at the section (inch-pounds),[1]
c is the distance in inches from the netural axis to the extreme edge of the piece (there are two values of c, one for tensile stress, one for compressive),
I is the moment of inertia of the cross-section in inches^4.[1]

4. *Torsion.*—Under working loads on round shafts and shafts whose cross-section is a hollow circle, the shearing stress is assumed to vary uniformly from zero at the axis of the shaft to a maximum value at the surface, and the shearing unit stress is given by the formula

$$S_s = \frac{Tr}{J},$$

in which

S_s is the shearing unit stress (pounds per square inch),
T is the twisting moment in inch-pounds (for a shaft transmitting H horsepower at N r. p. m., $T = 63{,}000\ H/N$),
J is the polar moment of inertia of the cross-section in inches.^{4} (For a solid circular shaft $J = 1.57\ r^4$; for a hollow circular shaft $J = 1.57(r^4 - r'^4)$ in which r is the outside radius, and r' is the inside radius),
r is the outer radius of the shaft in inches.

This formula does not apply to shafts of non-circular cross-section.

Assumptions Underlying the Mechanics of Materials.—In both the simple Rankine system of stress analysis and in the complicated Saint Venant system certain assumptions are made, among which are:

1. The material is homogeneous.
2. The material is isotropic (having equal elastic stiffness in all directions).
3. The material is capable of being subdivided to any desired extent if the elementary areas approach zero in magnitude, and the elastic properties of the elementary particles are assumed to remain unchanged.
4. Hooke's law holds.

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[1] Values of bending moment for various loadings and of moment of inertia for various cross-sections are given in texts on mechanics of materials and in engineering handbooks.

Under the metallographic microscope, metals are seen to be made up of mutually adhering crystalline grains with occasional "inclusions" of foreign matter. In general, pure metals and solid solutions with only one phase have only one kind of grain; but many structural metals, including steel, are made up of two or more kinds of crystalline grain, differing in strength. Moreover, by watching through a microscope the action of a metal under stress, it is seen that even in a pure metal the grains have planes of weakness, and that long before any general yielding of the metal has taken place, there is yielding in certain crystalline grains whose planes of weakness are unfavorably oriented to resist the stresses set up.

The above considerations make it evident that assumptions 1, 2, and 3, while they are true in a "statistical" way for a mass of metal comprising a considerable number of grains, are not true for individual grains, nor for an area so small as to include cross-sections of only a few crystalline grains. The exactness of Hooke's law has been discussed in a previous paragraph.

Why is it that two systems of stress analysis which depend on such inexact postulates have proved so reliable a guide for practical stress analysis? One answer is that the Rankine and the Saint Venant systems yield results which are "statistically" true for the more important stresses in structural members and machine parts. Just as it is possible to use mortuary statistics to predict the death rate of a community, so it is possible to use the common elastic formulas to predict the behavior under stress of a rather small *group* of crystalline grains. For example, in applying the common flexure formula to determine the maximum unit stress in an I-beam, the I-beam is pictured as divided into thin horizontal layers, and the average unit stress in the outer layer is what is determined by the formula. Just as it is impossible to use mortuary statistics to predict the death of individuals, so it is impossible to use the formulas of even the elaborate mathematical theory of elasticity to predict the failure under stress of a single crystalline grain of metal or the failure of a group of a few grains. Even if the complex differential equations involved could be solved, it is not at all likely that the use of the mathematical theory of elasticity would permit the accurate computation of unit stress at the root of a sharp V screw thread.

In general, this "statistical" correctness of the common methods of stress analysis makes them satisfactory for determining the *significant* unit stresses in structural parts under dead load, and in many machine parts. In this connection it may be noted that the *significant* unit stresses for ductile metals can be predicted with higher degree of accuracy than can the significant stresses in brittle materials. This statistical correctness also makes the mathematical theory of elasticity useful for indicating in a semiquantitative way localized unit stress around rivet holes, at the bottom of screw threads, and at other locations where the maximum unit stress developed affects only a minute area. The customary tacit assumption is that such localized stresses are not important under static load, especially for structural parts made of ductile metal. For example, a localized overstress around a rivet hole produces no noticeable general distortion in a water tank, nor does it interfere with the functioning of the structure. As will be more fully discussed in Chap. III, it must be recognized that under repeated stress these "negligible" localized stresses may become of prime importance, owing to the tendency to start a crack which spreads under successive cycles of stress.

Reliability of Stress Computations.—The ordinary (Rankine) formulas of mechanics of materials give a general idea of the principal significant stresses in a structure, but they do not give any idea of many localized stresses, frequently of high intensity, which may cause failure under repeated stress. These ordinary formulas may be used for computing stresses in repeated-stress specimens if those specimens are so designed as to be free from localized stress.

The more elaborate formulas of the mathematical theory of elasticity (Saint Venant's) afford a means of determining these localized stresses, when the differential equations can be solved;[1] but when the areas concerned are minute, the errors in underlying assumptions of homogeneity, isotropy, and indefinite divisibility of the material render the computed unit stresses inaccurate. It may be noted, however, that all test data available show that the use of the mathematical theory of elasticity gives results for localized stress which are on the safe side.

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[1] In some cases in which these differential equations cannot be solved, recourse may be had to mechanical means of solution, such as the examination of transparent specimens under polarized light (see Coker, in *Engineering* (*London*), p. 1, Jan. 6, 1911), and the soap-film method (see Griffith and Taylor in *Engineering* (*London*), p. 546, Dec. 21, 1917).

CHAPTER II

HISTORICAL SURVEY UP TO 1919—FUNDAMENTAL CONCEPTS

Introduction.—The three classes of stresses to which engineering materials are commonly subjected are static or steady stresses, repeated stresses, and impact stresses. It is possible, of course, to have repeated impact stresses, and these may be closely related to the simpler case of ordinary repeated stresses. An I-beam in the floor of an ordinary building is an example of a member subjected to steady stress, the axle of a moving railway car is an example of a member working under repeated stress, and the plunger of a steam hammer in operation is an example of a member subjected to impact stresses. To resist static stresses the "elastic limit," or in ductile metals the "yield point," of the material is the most important criterion.[1] To resist repeated stresses it will be shown that the "endurance limit" is the important criterion; while to resist impact stresses the modulus of resilience, or the capacity to absorb energy up to the elastic limit, is an important criterion, although the ability of some materials to withstand occasional extreme punishment without fracture is of great practical importance. The present discussion will relate especially to repeated stresses. These stresses may vary from zero to a maximum value, from a positive minimum to a positive maximum, or from a negative minimum to a positive maximum. The last case is usually spoken of as reversed stresses, and when the negative and positive stresses are numerically equal, the term "alternating

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[1] The term "elastic limit" is used here to designate the lowest unit stress at which there is observed appreciable inelastic action in a material. The value obtained for elastic limit depends on the delicacy of apparatus and methods used for detecting inelastic action.

stresses" is used. By the term "range of stress" is meant the algebraic difference between the maximum and minimum stress. By the term "endurance limit" is meant that unit stress which may be applied to a given material for an indefinitely large number of cycles without producing rupture.

The term "fatigue" has been applied to the phenomenon of fracture under repeated stresses, and while it is admittedly not a proper descriptive term, it has become so thoroughly established in the literature that it will be adhered to in this book. It will be shown that the term "progressive failure" is more precisely descriptive of the action of repeated stresses on a member.

"Crystallization" of Metals.—It has often been observed that when metals fracture due to repeated stresses, the fracture has a decidedly crystalline appearance. It used to be assumed that the metal had developed a crystalline structure due to the action of the repeated stresses, and even today this idea is quite commonly held. Many experiments have shown that this idea is quite incorrect. Metals are composed of crystals, and there is no change in their inherent crystallinity due to the action of repeated stresses. It will be shown that the action of repeated stresses is highly localized and that if a bar which has broken due to fatigue and shows a crystalline fracture is nicked at some point away from the fracture and broken by a single blow, it will be found that a crystalline fracture is again revealed. The idea of "crystallization" undoubtedly arose from the fact that many bars ruptured under fatigue showed a coarsely crystalline fracture due to overheating, defective chemical composition, or some maltreatment in fabrication. The bars broke in many cases because these defects made them particularly weak in resisting repeated stresses.

Work of Albert.—The earliest tests on the effect of repeated stresses of which the authors have seen any record are those of Albert,[1] made in Germany in 1829, on welded

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[1] *Stahl u. Eisen*, p. 437, 1896.

chain for mine hoists. In these tests a chain was held on a 12-ft. disk, one end of the chain carrying a load. By means of a crank coupling, the disk could be oscillated through an arc, thus subjecting the chain links to repeated bendings. The speed was 10 bends per minute, and tests of 100,000 bendings were recorded.

Work of Fairbairn.—One of the earliest recorded experiments on the effect of repeated stress is that of William Fairbairn[1] in 1864. He mentions previous experiments performed by Captains James and Galton in which bars were subjected to repeated loadings by means of cams. One cam produced considerable vibration in applying the load and the other released the load suddenly. By means of the first cam three cast-iron bars were tested, one being subjected to 10,000 bending repetitions at one-third of the statical breaking load without failure. The other two bars were subjected to one-half the statical breaking load and failed at 28,602 and 30,000 repetitions, respectively.

By means of the second cam, five cast-iron bars were subjected to deflections equal to those produced by one-third of the breaking load. Three bars bore 10,000 repetitions without failure, one failed at 51,538 repetitions, and one bore 100,000 repetitions without failure. Three bars subjected to deflections equal to those produced by one-half the breaking load failed at 490, 617 and 900 repetitions, respectively.

The first or vibratory cam next subjected a wrought-iron bar, 2 in. square in section and 9 ft. between supports, to a strain corresponding to five-ninths of the strain which permanently injured a similar bar. A permanent set of 0.015 in. was produced by 100,000 repetitions.

Some further tests were made on wrought-iron bars and a small box girder of boiler plate.

The conclusions drawn from these experiments regarding cast iron were that bars or girders were not safe when subjected to deflections caused by one-half the breaking load, that they were safe when the deflections were caused by

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[1] *Phil. Trans. Roy. Soc.*, p. 311, 1864.

one-third of the breaking load, and that these repeated deflections did not seem to have any injurious effects on the static properties of the metal.

With regard to the wrought-iron bars it was noticed that they showed a progressive increase in the deflections and a permanent set.

Fairbairn tested a wrought-iron girder 22 ft. long and 16 in. deep, made up of plates and angles. The web consisted of a plate ⅛ by 15¼ in.; the top flange of a plate ½ by 4 in. and two 2- by 2- by 5⁄16-in. angles, and the bottom flange of a plate ½ by 4 in. and two 2- by 2- by 3⁄16-in. angles. The load was applied to the beam by means of a lever and dead load, and this load was lifted off and again applied, causing more or less vibration.

The beam was subjected to 596,790 cycles of stress at one-fourth of the ultimate load, then to 403,210 more cycles at two-sevenths of the ultimate load, and then to 5,175 more cycles at two-fifths of the ultimate load, when the beam failed on the tension side near the middle of the span where the load was being applied. The beam was repaired and subjected to 3,150,000 cycles of stress at one-fourth of the ultimate load, and then to 313,000 more cycles at one-third of the ultimate load, when the beam failed on the tension side.

Fairbairn concluded from these experiments that wrought-iron girders when subjected to one-third of their ultimate load, or a unit stress of about 15,700 lb. per square inch in tension, are not safe, but that the unit stress of 11,000 lb. per square inch fixed as the maximum allowable unit stress by the Board of Trade was satisfactory.

Wöhler's Experiments.—The first comprehensive series of repeated-stress tests was that carried out by Wöhler[1] in repeated torsion, bending, and direct stress. These tests included torsion between zero and a maximum stress, torsion completely reversed, tension between various limits of minimum and maximum, and rotating bendings

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[1] *Zeit. für Bauwesen*, vols. 8, 10, 13, 16, and 20, 1860–1870; *Engineering* (*London*), p. 199, Mar. 24, 1871, and following issues.

in which the stresses were completely reversed. Wöhler designed very ingenious machines for stressing his specimens, but was forced to run his machines at slow speeds. Since his rotating bending machine had a speed of only 72 r. p. m., and since this was undoubtedly the highest speed used in any of his machines, it is not surprising that it was necessary for him to spend 12 years at the work. One of his rotating-beam specimens, for instance, was given 132,250,000 cycles of stress without producing fracture.

The materials used by Wöhler for test specimens have undergone such changes in manufacture and content that it is not deemed desirable to reproduce his numerical results. Only the conclusions to be drawn from Wöhler's experiments will be mentioned here, and they are as follows:

1. Wrought iron and steel will rupture at a unit stress not only less than the ultimate static strength of the material, but even less than the elastic limit, if the stress is repeated a sufficient number of times.
2. Within certain limits the range of stress rather than the maximum stress determines the number of cycles before rupture.
3. For a given minimum or maximum unit stress an increase of range of stress decreases the cycles necessary for rupture.
4. For a given minimum or maximum unit stress there appears to be a limiting range of stress which may be applied indefinitely without producing rupture.
5. As the maximum applied unit stress increases, this limiting range of stress decreases.

Wöhler also studied the effect of abrupt changes in cross-sections both under axial tension and under rotating bending, the effect of heat treatment, and the effect of the time element in applying stress.

It is interesting to note the conclusions regarding limiting stresses at which Wöhler arrived from a study of his results. He states that the unit stresses to which materials may be subjected indefinitely under various conditions of stress are those given in Table 1.

TABLE 1.—LIMITING REPEATED STRESSES (ENDURANCE LIMITS) DETERMINED BY WÖHLER

Material	Maximum unit stress, pounds per square inch	Minimum unit stress, pounds per square inch	Ratio of minimum to maximum unit stress (range ratio r)	Ratio of endurance limit to ultimate tensile strength (endurance ratio)
Bars subjected to cycles of bending or tension-compression				
Wrought iron	+17,100	−17,100	−1.0	0.36
	+35,300	0	0	0.74
	+47,100	+25,700	+0.54	0.99
Cast steel for axles	+30,000	−30,000	−1.0	0.29
	+51,400	0	0	0.49
	+85,600	+37,400	+0.44	0.82
Untempered cast steel for springs.	+53,500	0	0	
	+74,900	+26,800	+0.36	
	+85,600	+42,800	+0.50	
	+96,300	+64,200	+0.67	
Bars subjected to cycles of torsional stress				
Cast steel for axles	+23,500	−23,500	−1.0	
	+40,700	0	0	

He concluded that the safe unit stress for wrought iron under alternating stress might be 8,600 lb. per square inch; that under tension it might vary from 3,300 to 19,300 lb. per square inch; and that the range of stress was to be taken as constant if the minimum stress fell below 3,300 lb. per square inch.

For untempered cast steel the stress might be 12,800 lb. per square inch alternating, and from 11,500 to 35,000 lb. per square inch in one direction only.

For cast spring steel the stress might vary from 96,000 to 128,000 lb. per square inch.

Wöhler expressed the opinion that railway axles might occasionally be subjected to stresses equal to the endurance limit without serious damage, and that tempered spring steel might be subjected to three-fourths of the ultimate strength if the play of the spring was small compared with the total deflection.

The work done by Spangenberg[1] in Germany and by Baker[2] in England confirmed Wöhler's results.

Bauschinger's Researches.—The work done by Bauschinger[3] in studying the action of repeated stresses and allied matters is so important that it will be given in some detail. His conclusions were as follows:

1. A tension stress above the yield point[4] increases the yield point up to the applied stress, even if the bar is immediately retested. Upon release of load and lapse of time, the yield point increases even above the previous maximum applied stress. This increase is noticeable even after 1 day and may continue for weeks or even years.
2. A tension stress above the yield point reduces the elastic limit (determined by delicate measurements of deformation) often down to zero. Upon release of load and lapse of time, the elastic limit increases, reaches the applied stress after several days, and may rise above this stress after sufficient lapse of time.
3. A tension stress which lies above the elastic limit, but below the yield point, increases the elastic limit immediately, the more so the higher the initial stress. When the applied stress approaches the yield point, the elastic limit reaches a maximum, and is lowered when the yield point is exceeded.
4. As a rule, a stress above the yield point lowers the modulus of elasticity. Upon release of stress and lapse of

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[1] *Zeit. für Bauwesen*, 1874–1875.

[2] *Am. Soc. Mech. Eng.*, vol. 8, 1887.

[3] *Mitt. Mech.-Tech. Lab. Kgl. Tech. Hochs.*, Heft 13, *München*, 1886; *Dinglers polytech. Jour.*, Bd. 224; *Civil ingenieur*, 1881.

[4] The technical definition of "yield point" is: that unit stress at which a metal shows increase of deformation without additional stress. Practically, yield point means that unit stress at which inelastic action can be detected without the use of a delicate extensometer.

time, the modulus increases, and after several years is found to be considerably above the original value.

5. Severe jars, as by cold hammering, lower the elastic limit which has previously been raised by overstress and rest. If the hammering produces tension, the elastic limit decreases down to its original value, but otherwise it remains above it.

6. Heating followed by subsequent cooling will again reduce the elastic limit and yield point which have been increased by overstress and rest. For low-carbon steel the effect becomes noticeable at 350°C. if the cooling is rapid, and at 450°C. if the cooling has no effect on the two limits.

With wrought iron the effect is produced at about 400°C. both for rapid and slow cooling.

7. Rapid cooling lowers the elastic limit and yield point, especially the former, more effectively than slow cooling. Rapid cooling usually decreases the elastic limit to zero or nearly zero after heating up to 500°C., and surely by heating up to a cherry-red heat. This is true for wrought iron, low steel, and Bessemer steel. Slow cooling even after heating to a cherry-red heat does not produce such a great decrease.

8. A stress in tension (or compression) beyond the elastic limit reduces considerably the elastic limit in compression (or tension) the more the higher the applied stress is above the original elastic limit. Even relatively small transgressions of one elastic limit will reduce the opposite elastic limit to zero. A period of rest will not again increase the elastic limit, as is possible by loading in one direction above the yield point.

9. Gradually increasing alternating stress in tension and compression will not decrease the opposite elastic limit unless one of the original elastic limits is exceeded.

10. An elastic limit in tension (or compression) which has been lowered by previous stress above the elastic limit in compression (or tension) can again be increased by applying a gradually increasing alternating stress, but it can be increased only up to a value which is considerably below the original elastic limit.

11. Repeated stresses between zero and an upper limit which coincides with or is close to the elastic limit will not cause rupture. The elastic limit, however, must not previously have been raised artificially by tension or cold working, nor must there be any flaws in the material. In the latter respect homogeneous material like low-carbon steel is especially sensitive.

12. Repeated stresses between zero and an upper limit in tension which coincides with or lies slightly above the elastic limit will increase the elastic limit, and the more so the greater the number of repetitions, but not above a certain limiting value.

13. If by the previous action (as in conclusion 12) the elastic limit is increased above the applied stress, fracture will not take place; but if the applied stress is so high that the elastic limit cannot be augmented to this value, failure must take place after a limited number of repetitions.

14. Millions of repetitions of stress do not alter the structure of a material, nor do they reduce the ultimate static strength.

Bauschinger inferred from his experiments that a material with an artificially raised elastic limit would have its elastic limit reduced to a certain value by applying alternating stresses below this artificial limit, and that this new elastic limit would be the same as that obtained when an elastic limit which has been reduced is gradually increased by applying a slowly increasing alternating stress. This new limit Bauschinger called the "natural elastic limit," and he proposed the following principle:

15. If a material is to withstand an indefinite number of repetitions of alternating stress, then the applied stress must not exceed the natural elastic limit.[1]

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[1] Bauschinger, working before the day of the metallurgical microscope, very naturally emphasized "elastic limit" as a prime factor in determining fatigue strength. Later, the idea of fatigue failure as a spreading *fracture* became prominent. Practically, Bauschinger's "natural elastic limit" and the modern "fatigue limit," or "endurance limit," may be regarded as synonomous terms.

Slip Bands.—The work of Ewing and Rosenhain and Ewing and Humfrey did much to increase the available information as to the possible mechanism of fatigue failure. Ewing and Rosenhain[1] observed that when a metal is sufficiently stressed, the crystals of which the metal is composed yield by slipping on certain gliding planes within the crystal. This slipping has the effect of breaking up the polished surface of a grain into elevations and depressions in the nature of steps. Under vertical illumination the steps show as dark lines, which Ewing and Rosenhain called "slip bands." The appearance of these bands was straight in some metals, but in others the lines were wavy and tended to fork or branch. After severe straining there might be as many as four systems of intersecting slip bands on the surface of the same grain.

Ewing and Humfrey[2] carried on a similar study on slip bands when specimens were subjected to reversed bending stresses. Specimens were made of Swedish iron having an ultimate tensile strength of 52,800 lb. per square inch and a proportional elastic limit of about 29,100 lb. per square inch. A reversed stress of about 20,000 lb. per square inch produced slip lines on a few crystals after a few reversals. When the stress was comparatively high, many crystals were affected. With increase in the number of cycles of stress, additional slip lines appeared which had not been visible before, and the original ones showed a tendency to broaden. As the number of cycles increased, the broadening process continued, until some parts of the surface became covered by groups of dark markings. At this stage it was found that an actual crack had opened up along some of the broadened slip bands. The cracks were sometimes first seen on a single crystal, but they soon joined from crystal to crystal, until a continuous crack ran across the surface of the specimen, after which a few more cycles of stress produced fracture.

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[1] *Phil. Trans. Roy. Soc.*, vol. 193*A*, p. 352, 1899.

[2] *Phil. Trans. Roy. Soc.*, vol. 200*A*, p. 241, 1902.

The specimens showed no sign of damage when subjected to a stress of 11,200 lb. per square inch, but when the stress was increased to 15,700 lb. per square inch, signs of fatigue became visible after many cycles of applied stress. With a stress of 20,100 lb. per square inch the damage was so great that cracks were formed and the specimen failed. The presumption is that with a sufficient number of applied cycles the specimen would have failed at a stress of 15,700 lb. per square inch. It will be noted that this unit stress was only a little greater than half of the proportional elastic limit.

These experiments indicated that some crystals reach their limit of elasticity sooner than others, which is no doubt due to the fact that they are so oriented as to be in a favorable position to permit slip on their gliding planes. It is evident that a specimen built up in a complex manner of many crystals will have a distribution of stress from crystal to crystal which is by no means regular.

These experiments demonstrated that when a metal is subjected to alternating stress of sufficient magnitude certain crystals yield by slipping, as in other cases of non-elastic strain. Ewing and Humfrey are of the opinion that the surfaces on which slipping occurs continue to be planes of weakness and that the effect of repeated sliding and grinding results in the production of a burr, or rough and jagged irregular edge, suggesting the accumulation of débris. This repeated grinding tends to destroy the cohesion of the crystal on the surfaces of slip, and in certain cases actually develops into a crack. Once a crack is formed, it develops rapidly because of concentration of stress at the end of the crack. The tests show how a crack may develop to failure under the action of repeated stresses,[1] even in sound, flawless metal.

The experiments help to explain why a fatigue fracture shows no sign of local elongation, and why a specimen which

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[1] Recent researches, to be described later, have shown that slip may occur without fatigue failure, and that repeated stresses may have a beneficial as well as an injurious effect.

has been subjected to many reversals of stress shows no deterioration which can be detected by a tensile test. As long as cracks have not been formed, there is no reason to suppose that a tensile test would detect any deterioration.

Bairstow's Experiments.—A very excellent piece of work, which tends to throw much light on the behavior of metals under the action of repeated stress, is that of Bairstow.[1] He made tests on axle steel which had a yield point of 55,700 lb. per square inch and an ultimate strength of

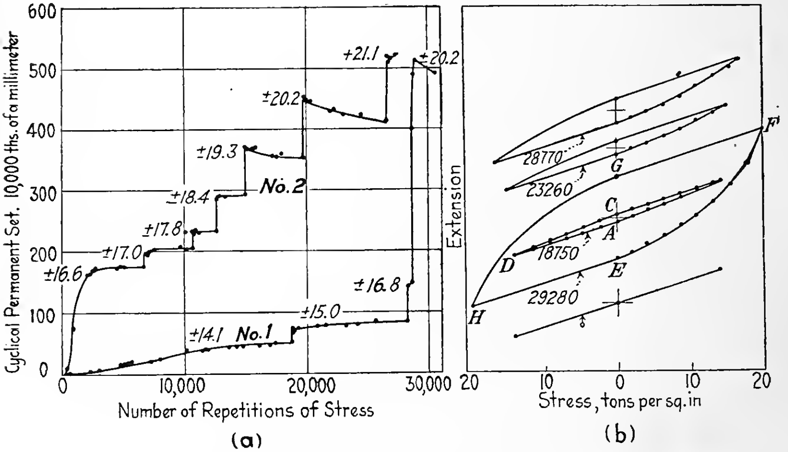

FIG. 2.—Set and mechanical hysteresis under repeated stress. (*Based on Bairstow, in Phil. Trans. Roy. Soc.*)

85,500 lb. per square inch. When a specimen was subjected to equal and opposite stresses of 31,600 lb. per square inch, the cycle of extensions was represented at first by the straight line shown in Fig. 2(*b*). As the number of cycles of stress increased, the straight line was changed into a loop, until after 18,750 cycles the width of the loop was about 11 per cent of the original elastic extension.

Although the extensometer measured to about 0.000004 in., evidently it could not detect the fact that the specimen was being subjected to stresses beyond its elastic limit, which repeated stresses made apparent by developing a hysteresis loop. Bairstow is of the opinion that at a slightly

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[1] *Phil. Trans. Roy. Soc.*, vol. 210*A*, p. 35, 1910.

smaller stress of about 29,000 lb. per square inch the specimen would have been perfectly elastic and that no number of cycles would have developed a loop.

The stress on this specimen was increased to 33,600 lb. per square inch, and an immediate increase in the width of the loop was produced. At a stress of 45,200 lb. per square inch and 29,280 cycles, the loop became very wide and had the shape shown by *EFGH* in Fig. 2(*b*). In all these loops the lines *FG* and *HE* were found to be parallel to the original elastic line. The portion *FG* was obtained when tension was reduced to zero, and *HE* when compression was reduced to zero. At the lower stresses the width of the loop tended to become constant as the number of cycles increased, and at higher stresses the width of the loop actually decreased with increase of cycles.

The observations were continued almost to the breaking point, and since the extensometer gave no warning of the deterioration which was going on, the actual damage must have been extremely local. Since Bairstow is of the opinion that the individual slips in the crystalline grains cannot have increased in extent due to repetitions of stress, this tends to give additional weight to the views of Ewing and Humfrey regarding the mechanism of fatigue failure.

Bairstow found that when the stresses were not completely reversed, there was developed a "permanent extension" due to the repeated stresses in addition to the width of the hysteresis loop, which width he called "cyclical permanent set." The "cyclical permanent set" is shown in Fig. 2(*a*).

A specimen was subjected to a maximum stress of 41,000 lb. per square inch in tension and a minimum stress of 18,600 lb. per square inch in compression. The width of the loop and the permanent extension gradually increased from zero with increase of repetitions, and when the "cyclical permanent set" became constant, the rate of "permanent extension" became small but not zero. Increase in range of stress produced a wider loop and great permanent extension.

Another specimen was subjected to a range of stress from zero to 52,000 lb. per square inch in tension. For the first 2,000 cycles the extensometer showed nothing, but shortly afterwards a slow yield took place and at the same time a hysteresis loop made its appearance. The width of the hysteresis loop reached a maximum value at about 7,000 cycles and then remained constant for another 8,000 cycles. During this time the rate of permanent extension was decreasing until it was very small.

When the range of stress for another specimen varied in tension from 16,800 to 63,600 lb. per square inch, permanent extension and a hysteresis loop occurred at the first load. It will be noted that in this case the maximum stress was greater than the yield point of the material.

The range of stress for a certain specimen was now made from 42,300 to 77,000 lb. per square inch in tension. It will be noted that this maximum stress was fairly near the ultimate static strength. A permanent extension occurred and a narrow hysteresis loop was formed, but this loop decreased in width until at 6,000 cycles it was practically a straight line. An increase of range did not increase the permanent extension nor form a loop, but with a still further increase of range a loop was formed and a slow extension commenced.

One significant observation made by Bairstow was that when the average unit stress of a given cycle of stress was tension, then an extension occurred during the adjustment of elastic limits to this cycle, and this extension was similar to the extension observed in an ordinary tensile test when the yield point is exceeded. This extension under repeated stress occurred even when the maximum unit stress applied was less than the static yield stress. The greater the extension, the greater was the amount by which the elastic limits were raised. There was no such extension when the stresses applied were equal and opposite.

The practical conclusion to be drawn from this phenomenon is that under conditions in which elongation could not be permitted, a unit stress somewhat less than the yield point would determine the upper limit of any cycle of stress in which the average unit stress was tension. It will be shown later that this also applies to shortening when the average unit stress of the cycle is compression.

Bairstow concludes from these experiments that iron and steel are capable of adjusting themselves to cyclical variations of stress after a sufficient number of cycles have been applied. When this adjustment is complete, the specimen is perfectly elastic throughout the cycle and fatigue does not occur, although slip may have occurred in the adjusting process. This adjustment to a given cycle is possible because the elastic limits are not fixed but can be raised or lowered by cycles of stress.

The amounts by which the elastic limits may be adjusted are limited, and if the range of stress is great enough, the specimen becomes and remains inelastic, and a certain amount of energy is expended in moving the portions of the crystals with respect to each other, and this is probably associated with the slip bands which Ewing and Humfrey found, and which gradually develop into cracks.[1]

Bairstow determined the elastic ranges of Swedish iron, axle steel of about 0.35 per cent carbon, and Bessemer steel of about 0.46 per cent carbon, by determining the range of stress which would not produce a hysteresis loop. He also found the safe ranges for the Swedish iron and the Bessemer steel by means of fatigue tests, using stresses completely reversed. These values checked each other fairly well.

Bairstow plotted range of stress as ordinates against minimum stress as abscissae for these elastic ranges. He also showed similar curves obtained from similar material by Wöhler's fatigue tests. These curves indicate that the elastic ranges found by Bairstow and the safe ranges found by Wöhler are identical. Incidentally it may be remarked that these curves indicate that for completely reversed

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[1] The exact connection between cracks and slip bands is not clear. It will be shown that there certainly may be slip bands without cracks, but whether there may be cracks without slip bands is not known.

stress the range is a maximum and that in general the range decreases as the minimum stress approaches the value of the maximum stress—in other words, as the ratio of minimum stress to maximum stress approaches unity.[1]

Other Work in Fatigue.—The formulas of Launhardt and Weyrauch and the diagrams of Goodman and J. B. Johnson will be discussed in Chap. VII.

In the United States a great amount of fatigue testing was carried on by Howard[2] at the Watertown Arsenal.

From 1896 to 1914 the British investigators of fatigue of metals were particularly active, and the slip and hysteresis of metals under repeated stress were given much study. Several investigators made series of repeated-stress tests, but no systematic series of long-time tests were found feasible, though the need for such tests was pointed out. Reynolds and Smith, Stanton and Bairstow, and J. H. Smith developed the inertia type of fatigue-testing machine. Kapp, Hopkinson, and Haigh developed the alternating-current magnet type of fatigue-testing machine. Stromeyer made studies of thermal effects produced by repeated stress, following lines suggested by Kelvin.

In 1900 Gilchrist[3] put forward a picture of fatigue failure which may be regarded as an early statement of the modern picture of this problem, and which emphasizes localized stress as a source of fatigue failure. In discussing Wöhler's results, Gilchrist sums up his views as follows:

> 1. The average stress in the bars broken in Wöhler's machines did not reach the statical breaking load.
>
> 2. The fracture was caused by the statical breaking limit being exceeded at one point only, from which, when once started, rupture spread, at first rapidly and then more slowly, sometimes continuing to complete separation of the two parts of the bar, but occasionally stopping short of complete rupture.[4]

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[1] Elastic failure seems to be associated with slip, and the connection between elastic failure and fatigue failure seems rather slight. Fatigue failure seems to start in actual tearing apart of particles of metal.

[2] "Tests of Metals," 1888–1895, 1900–1909.

[3] GILCHRIST, J., "Wöhler's Theories on Material under Repeated Stress," *The Engineer (London)*, vol. 90, p. 203, 1900.

[4] Modern experiments make it rather doubtful if the crack spreads at first rapidly and then slowly. Probably the reverse is true.

3. The raising of the stress at the point where the fracture commenced was due to an irregularity in the bar. This might be an irregularity or discontinuity in the metal, either on the surface or in the body of the bar.

4. A bar of uniform strength, whose surface was perfectly smooth, with no sharp corners in the longitudinal configuration, and with a perfectly homogeneous structure, would endure, without breaking, an indefinite number of repetitions of a stress varying between zero and a value near to the breaking strength.[1]

5. A bar similar to that under 4 could, under certain conditions, endure an indefinite number of repetitions of a load varying between tension and compression of equal values both beyond the ordinary primitive elastic limit.

In 1910 Basquin of Northwestern University presented an important paper before the American Society for Testing Materials.[2] In that paper he pointed out that for the test data available at that time the relation between stress and number of cycles of stress to cause fatigue failure might be expressed by the formula

$$S = KN^{-m}$$

or

$$\log S = \log K - m \log N,$$

in which

S is the maximum computed unit stress in the test specimen,
N is the number of cycles of stress required for fracture,
K and m are constants depending on the material and on the manner of making the test.

This formula fitted existing data fairly well, and where differences were noted, this formula was on the side of safety. It implied that there was no absolute endurance limit for actual materials. Basquin's formula has not been verified by later tests, but the discussion it caused was a powerful factor in developing research in the fatigue of metals, especially in the United States.

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[1] It may be pointed out that all machine parts made of any available material fall far short of the ideal conditions here pictured.

[2] "The Exponential Law of Endurance Tests," *Proc. Am. Soc. Testing Materials*, vol. 10, p. 625, 1910.

In 1914 it was evident that there was need of a large number of long-time tests to give more light on the question of the existence of an endurance limit for metals used in structural and machine parts. The development of the airplane and the exigencies of the World War accentuated this need, and several extensive series of investigations have been carried out. The British investigations have centered round the British National Physical Laboratory, where Gough and Hanson have done noteworthy work. In the United States three extensive investigations have been the source of much data, and are still in progress: (1) an investigation sponsored by the National Research Council, the Engineering Foundation, and several commercial firms, carried on at the University of Illinois under the direction of H. F. Moore and his associates, J. B. Kommers and T. M. Jasper; (2) an investigation carried on at the U. S. Naval Engineering Experiment Station under the direction of D. J. McAdam, Jr.; and (3) investigations carried on at McCook Aviation Field, Dayton, Ohio, under the direction of R. R. Moore.

This brief outline makes no pretense of giving anything like a complete list of the investigators who have made valuable contributions to the knowledge of the fatigue of metals. The types of machines used and the important results obtained will be discussed in succeeding chapters, and a rather complete bibliography will be found in Appendix A.

CHAPTER III

SLIP, OVERSTRAIN, AND HYSTERESIS

Slip under Static Stress.—The work of Ewing and Rosenhain on the behavior of metals under strain has been mentioned. They established the fact that the crystalline structure of metals is preserved even under severe plastic strain, which might be supposed to destroy crystalline structure. They concluded that the distinction formerly drawn between crystalline and non-crystalline metals was not justified.

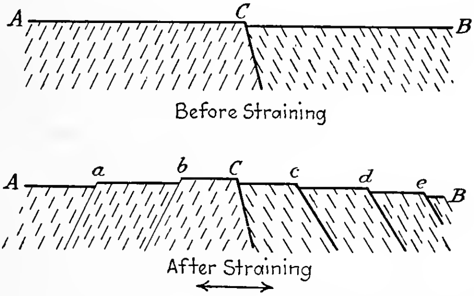

Fig. 3.—Intracrystalline slip in a metal. *(Based on Ewing and Rosenhain in Phil. Trans. Roy. Soc.)*

Figure 3 represents a section through the upper part of two adjacent grains, having cleavage planes as indicated by the dotted lines, AB being a portion of the polished surface and C being the junction between the two grains.

When the metal is strained beyond the elastic limit parallel to AB, for instance, yielding takes place by finite amounts of slip at a limited number of places as indicated at a, b, c, d, and e. This breaks up the polished surface into elevations and depressions in the nature of steps, which under vertical illumination show the "risers" of the steps as dark lines or bands. That this explanation was correct was proved by changing the incidence of the light, when the bright areas became dark and some of the dark lines became bright. Figure 4 shows a micrograph of a polished and etched surface of a steel sample which has been strained and which shows the slip bands running across the crystals.

Apparently these slip bands occur in all metals as soon as plastic deformation takes place, for they were found in many pure metals as well as in alloys. In the case of iron under tensile stress the bands appeared as soon as the yield point was exceeded. Slip bands were developed under all kinds of strain involving permanent deformation, and the more severe the straining the more slip bands were formed. The effect of a stress producing plastic strain is similar to that of a force overcoming the static friction between two surfaces. If the plastic strain takes place in this manner by slipping, then the orientation of the parts of one grain would remain uniform (except for the case of

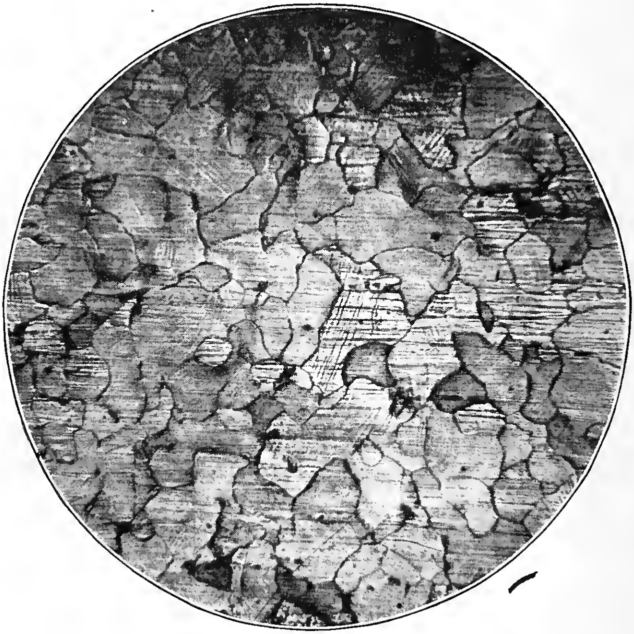

Fig. 4.—Slip bands in Armco iron. Magnification 75 ×. (*Micrograph by J. W. Harsch at the University of Illinois.*)

twinning)[1] no matter how much the outline of the grain might be changed by slips occurring within it. The crystalline structure of the metal would persist even after the most drastic strains.

The later researches of Rosenhain[2] showed that when a tensile stress is applied to nearly pure iron, the soft ferrite crystals of which iron is composed are elongated by the process of slipping. With continued elongation the limit of slip is reached and fracture takes place by tearing along the cleavage planes on which slip has been taking place.

An ordinary carbon steel contains ferrite and pearlite. Apparently the ferrite and pearlite are so closely interlocked that they are deformed about equally. The presence of fissures near the actual fracture appeared to show that for a while the pearlite was able to accommodate itself to the deformation of the ferrite; but it gradually reached its limit of deformation and then, being the stronger constituent, started tearing fissures in the ferrite.

Slip under Repeated Stress.—The work of Ewing and Humfrey on slip bands produced by repeated stresses has been mentioned in Chap. II. Stanton and Bairstow[3] later obtained similar results. The former investigators found that ordinary slip bands disappeared when the specimen was repolished and reetched,[4] but that when actual cracks had been formed, these remained visible. When an incipient crack had once formed across a certain set of crystals, the effect of further repetitions was confined mostly to this set of crystals, the other crystals changing very little. During reversals of small stresses, slip lines were generally found only in the central parts of crystals, not extending out to the boundaries.

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[1] Sometimes grains are formed in such a manner that they are structurally symmetrical with respect to a plane between them, one appearing to be the mirrored image of the other. These are called "twinned crystals," and twinning is of such a nature that if either part of the twin were revolved through an angle, the two parts would possess the same orientation.

[2] *Jour. Iron and Steel Inst.*, No. 2, p. 189, 1906.

[3] *Jour. Inst. Civil Eng.*, No. 4, p. 78, 1905–1906.

[4] Later metallographists have developed methods of etching which do not cover up all slip bands.

As is well known, one of the characteristic features of fractures due to repeated stresses is the fact that they take place suddenly, and, even with soft metals, show none of the local drawing out and necking down which are associated with ordinary tensile tests of ductile material. The development of slip bands and the formation of cracks explain why this type of fracture is to be expected under the action of repeated stresses.

Later Work on Slip Bands.—Gough and Hanson[1] report some interesting results on Armco iron, which has the surprising property, first reported for any metal by Moore and Kommers[2] in 1921, of having an endurance limit greater than the static yield point of the material. Gough and Hanson made careful metallographical examination of the stressed metal, both under static and alternating stress. When the stress was below the limit of proportionality in the static tests, they found no indications of strain, but for various stresses above the proportional elastic limit they found definite indications of plastic strain similar to the usual slip-band markings.

Under alternating stresses just below the limit of proportionality they found faint surface markings after 11,751,000 cycles. These were found on only a few crystals, the remaining crystals showing no effect of stress. When the stress was slightly above the proportional elastic limit, the markings were of the same character as before, but more crystals were affected.

When the stress was greater than the static yield point but less than the endurance limit, the appearance was different from that at the lower stresses. There now appeared dark areas on certain crystals similar to those found by Ewing and Humfrey. When these were examined under a magnification of 1,400 diameters, they appeared to consist of a series of roughly parallel lines which seemed to be identical with slip bands. It is interesting to note that this specimen withstood 40,000,000 cycles of the

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[1] *Proc. Roy. Soc.*, vol. 104*A*, p. 538, 1923.

[2] *Univ. Illinois Eng. Exp. Sta., Bull.* 124, 1921.

stress, which was only 900 lb. per square inch below the endurance limit. This seems conclusive proof that slip bands may form at stresses less than the endurance limit, and that the production of slip bands is not a criterion of ultimate failure by fatigue. When the stress was above the endurance limit, the markings were similar to those just described, but the dark areas were larger and more numerous.

Gough and Hanson are of the opinion that the dark areas consist of numerous slip bands rather than a few slip bands which have been widened by attrition. They believe that continuous slipping does not occur on the planes first formed. It will be noted that these conclusions are different from those of Ewing and Humfrey. Gough and Hanson found that in mild steel and copper, as well as in Armco iron, slip bands were formed at stresses less than the endurance limit. In this connection it may be stated that Moore and Kommers[1] found endurance limits higher than the proportional elastic limit for several steels; such cases have also been reported for non-ferrous metals by R. R. Moore,[2] Moore and Jasper,[3] D. J. McAdam, Jr.,[4] and Lessells.[5] McAdam has found that for all the pure metals and solid-solution metals which he tested in the fully annealed condition, the endurance limit is higher than the proof stress or Johnson's elastic limit. This was found to be true for both ferrous and non-ferrous metals. For annealed copper and annealed aluminum he found an endurance limit well above the highest value of stress which might be designated as the yield point of the metal.

Effect of Overstrain.—It is a well-known fact that metal which has been stressed beyond the yield point becomes temporarily inelastic, but that it recovers its elasticity by release of load and rest, and Muir[6] has shown that it may

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 124, 1921.

[2] *Proc. Am. Soc. Testing Materials*, p. 547, 1924.

[3] *Univ. Illinois Eng. Exp. Sta., Bull.* 152, 1925.

[4] *Amer. Soc. Steel Treating*, p. 59, 1925.

[5] *Proc. Am. Soc. Testing Materials*, 1924.

[6] *Phil. Trans. Roy. Soc.*, vol. 193*A*, p. 1, 1900.

have its recovery greatly accelerated by immersion in boiling water for a short time.

Beilby[1] was of the opinion that when the surface of a metal is polished, a thin layer of amorphous metal is produced, and he believes that when a metal deforms by slip, there are thin films of amorphous metal produced on the surfaces of slip. This film may be in a temporarily mobile condition. The hardening of metal due to overstrain is accounted for by the fact that these layers of amorphous metal harden.

Rosenhain[2] has extended this theory and believes that when a metal is subjected to mild deformation, there is formed on the surfaces of slip a thin layer of disturbed and temporarily mobile molecules, that these layers do not remain permanently amorphous, but become reabsorbed into the crystalline system from which they were formed. When, however, the deformation is more severe, the layers of amorphous material become too thick to be readily reabsorbed during the short time of temporary mobility. These layers, therefore, persist until the application of heat produces sufficient mobility to permit their reabsorption into the crystalline system.

The question may then be asked, why, if no amorphous material remains after slight straining, the properties of the metal are changed, as is well known to be the case. This is explained on two grounds, the first being that after slip takes place on the planes of easy slip, the conditions are no longer the same. These planes have lost their tendency to easy slip, and a greater force than before is needed to make them slip again, or else slip may take place on other planes which were slightly stronger than the first ones. This seems to be verified by the fact that when slip has produced slip bands, a somewhat greater strain not only produces new planes of slip but also deepens the old ones, and this even after elastic recovery has taken place.

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[1] *Jour. Brit. Inst. Metals*, No. 2, p. 5, 1911.

[2] *Jour. Brit. Iron and Steel Inst.*, No. 2, p. 189, 1906.

The second reason for the above view is that when slip has taken place on several planes,

> . . . the section of the original surface which was rectilinear to begin with will be stepped at every intersection with other surfaces of slip (see Fig. 5). Consequently further slipping on the original surface of easiest slip must come to an end as soon as slip in other planes has occurred; on further deformation the occurrence of slip is thus forced upon surfaces not initially favorably situated for its occurrence, so that increased force is required to bring it about.

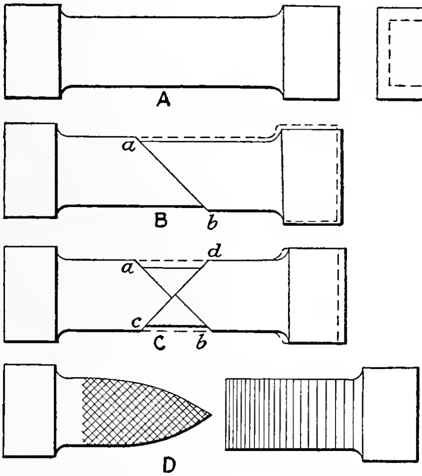

Fig. 5.—Progressive slip. (*Jeffries and Archer.*)

Jeffries and Archer[1] believe that Beilby's hypothesis, that there is a production of amorphous metal at all planes of slip and that slip can occur only once on each plane of slip, is not consistent with experimental facts. They believe that the greatest production of amorphous metal occurs at the crystal boundaries, and that the total amount formed is much less than has been assumed to be the case.

They sum up the causes for strain hardening as follows:[2]

> 1. Cold work produces a structure which simulates in many respects that of a very fine-grained metal.
>
> 2. Because of the manner of origin of the cold-worked structure, each grain fragment may have an orientation only slightly different

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[1] "The Science of Metals," p. 80.

[2] "The Science of Metals," p. 209.

from that of its neighbors, so that a large number of grain fragments may be so oriented as to be traversed by a single slip. Slip through such grain fragments is, however, interfered with by the disregistry at the fragment boundaries, and therefore the hardness is increased. In other words, the main cause of strain hardening is the slip interference resulting from the disregistry of slip planes at the boundaries of the grain fragments.

3. An additional cause of strain hardening is the disorganized layer of atoms at self-stopping slip planes and the additional amorphous metal generated at the old grain boundaries.

4. Since severe cold work tends to produce uniformity of orientation among the grain fragments, it is probable that there is a limit to the hardness attainable by cold work. Judging from the hardness of severely cold-worked iron and severely cold-worked aluminum, the maximum hardness attainable by cold work is much lower than that attainable by other methods, such as alloying and heat treating, and hence much lower than the hardness corresponding to the absolute cohesion of the metal.

Amorphous-cement Theory.—Bengough[1] and Rosenhain first suggested that the crystals of a metal were held together by an intergranular cement. Rosenhain and Ewen[2] have developed this idea further. The fact that in normal pure metals the intercrystalline boundaries are surfaces of special strength rather than weakness, and the further fact that a metal of fine-grained structure is stronger than a metal of coarse-grained structure, suggest the presence of a material which has a special strength.

The idea suggested is that the cement is of the same material as the metal itself and exists in an amorphous condition. When a molten metal cools, the last portions of the liquid are prevented from crystallizing in the regular crystalline system, and these portions retain the amorphous condition of the liquid and fill the microscopic spaces between the crystals in the body of the metal. This amorphous cement is essentially an undercooled liquid of great viscosity.

At ordinary temperatures fractures occur across the crystals of metal because of the strength of the cementing

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[1] *Jour. Brit. Inst. Metals*, No. 1, p. 123, 1912.

[2] *Jour. Brit. Insi. Metals*, No. 2, p. 149, 1912.

material; at higher temperatures fractures occur at the boundaries of crystals because of the greatly weakened condition of the cement.

Mechanical Hysteresis.—When a member is loaded in tension and then in compression, the stress-deformation curve may form a loop, as has already been pointed out in connection with Bairstow's work. This phenomenon is called mechanical hysteresis, from analogy with magnetic hysteresis. Bairstow expressed the opinion that when a material acts in a purely elastic manner, without the production of a loop, failure by fatigue would not occur. His experiments made plain the fact that static tests in which the material is carried through only a few cycles of stress and strain cannot be of much utility in the field of repeated stresses, for the reason that in some cases thousands of cycles of stress were necessary to produce a loop. Hysteresis in general is of interest, however, because of the light it may throw on the phenomenon of fatigue. It will be shown later that the production of a hysteresis loop is not necessarily a criterion of fatigue failure.

Ewing[1] made some tests on long metal wires of various kinds, loaded between two limits in tension below the elastic limit, and found evidences of hysteresis in all cases. He concluded that the work done in each of the cycles of stress had an obvious bearing on the conclusions of Wöhler regarding the deteriorating effect of repeated stresses.

Hopkinson and Williams[2] made some elastic-hysteresis experiments on an 0.18 per cent carbon steel. They took temperature readings with thermocouples accurate to about 0.05°C. and also measured the energy dissipated by elastic hysteresis under cyclical variation of stress at a speed of 7,200 cycles per minute. The results showed that the dissipation of energy increased about as the fourth power of the stress range, there being evidences of energy dissipation at a range of stress as low as about 24,600 lb. per square inch.

[1] *Brit. Assoc., Repts.*, p. 502, 1889.

[2] *Proc. Roy. Soc.*, vol. 87*A*, p. 502, 1912.

The stress difference at the maximum width of the hysteresis loop seemed to be somewhat greater in the static tests than in the high-speed tests. The result obtained on the maximum stress-width of the hysteresis loop was of the same order of magnitude as found previously by Ewing, and at a range of 38,000 lb. per square inch was about 0.59 per cent of the maximum unit stress applied. The maximum strain-width at the same range was about 0.000015 in. per inch. The results indicated that there was probably a decrease of hysteresis at speeds of 7,200 cycles per minute as compared with very low speeds, but that the difference could not be more than 30 per cent.

F. E. Rowett[1] carried on similar experiments, but he determined the area of the complete hysteresis loop more exactly than did Hopkinson and Williams. The experiments were made in torsion on thin tubes, and the high-speed experiments were made at about 4,200 cycles per minute. He found that the hysteresis was, probably within 5 per cent, the same at high speeds as at low speeds; and further that if the results of Hopkinson and Williams were calculated on the basis of the hysteresis-loop shape which he determined, their results were almost in exact agreement at high and low speeds.

The results indicated that for a hard-drawn tube of steel of about 0.17 per cent carbon, the hysteresis at all stress ranges was only about one-eighth of that for the same tube after annealing. For the annealed tube the hysteresis loss varied about as the cube of the stress range. The maximum stress-width of the hysteresis loop for the annealed tubes was about 3.5 per cent of the maximum unit stress applied at a range of 19,100 lb. per square inch. The unannealed tube at a range of 19,100 lb. per square inch gave a maximum stress-width of loop of 0.55 per cent of the maximum unit stress. It will be noted that the latter result is of the same order of magnitude as that of Hopkinson and Williams, but that the result for the annealed tube is very much greater.

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[1] *Proc. Roy. Soc.*, vol. 89*A*, p. 528, 1913–1914.

Guest and Lea[1] determined some torsion hysteresis loops on a mild-steel specimen about ⅞ in. in diameter and containing about 0.15 per cent carbon. A mirror device was used for measuring angle of twist and this permitted readings of 0.000000291 radian. Hysteresis loops were obtained at shearing unit stresses as low as ±1,500 lb. per square inch. With increase in range of stress larger loops were obtained. When a large loop had formed and the load was increased and decreased at any stress point on the loop, small hysteresis loops were obtained. At the lower stresses there was no perceptible "creep," by which is meant an increase of strain at constant stress with increase of time.

The effect of slight overstrain was also studied. In increasing the load for this test, creep was observed at a unit stress of 18,200 lb. per square inch, and a slight overstrain occurred at 22,800 lb. per square inch. After overstrain, the width of the loop for a range of ±9,100 lb. per square inch was as great as before overstrain at ±15,200 lb. per square inch.

The effect of a rest of 10 days on overstrained material was to reduce the width of the hysteresis loop about half, and the range of stress without creep was considerably increased by rest.

The effect of heating overstrained material to the temperature of boiling water for 1 hour was to decrease the width of the loop. This treatment was more effective than 18 days of rest. The effect of heating to 330°C. after overstrain was to reduce the width of the loop to about three-fourths of the width after the operation of boiling in water. After heating to 330°C., no creep took place up to the load which caused yielding.

Guest and Lea say:

> Since fatigue effects depend upon the gradual increase of the width of the hysteresis loop with repetition, it would appear that boiling and tempering at comparatively low temperatures remove initial strains, and thus considerably increase the resistance of the steel to repetition of stress.

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[1] *Proc. Roy. Soc.*, vol. 93*A*, p. 313, 1916–1917.

If hysteresis loops may be obtained in steel at stresses as low as ±1,500 lb. per square inch, as reported by Guest and Lea, then it is obvious that the production of hysteresis loops is not a criterion of final failure of the material in fatigue. Moore and Kommers[1] determined an endurance limit for Armco iron in torsion at ±12,500 lb. per square inch, and the material used by Guest and Lea would have an endurance limit at least as great as this. A stress as low as ±1,500 lb. per square inch would certainly not cause failure even after billions of repetitions. Since, on the other hand, Guest and Lea obtained yielding at 22,800 lb. per square inch, which is certainly above the endurance limit of this material, it seems clear that there is a certain range of stress below the endurance limit and within the ordinary static elastic limit of the material at which hysteresis loops are formed but which will not cause failure under repetition of stress. That a certain amount of movement, permanent change of position, and adjustment of metal particles may take place at stresses less than the endurance limit seems to be further conclusively demonstrated by the well-established fact that millions of repetitions of stress below the endurance limit improve the material, so that it is better able than before to withstand repetitions at higher stresses.

Gough and Hanson[2] report some very interesting results on hysteresis loops found in stressing Armco iron in reversed bending. For stresses less than the endurance limit, the width of the hysteresis loops usually increased at first, and then became constant or else actually diminished. When they applied a stress above the endurance limit, the loop steadily increased in width during 24,000 cycles. After a rest of 18 hr., followed by 12,000 cycles, the loop width had slightly increased, the effect of rest having been completely obliterated by the subsequent cycles of stress. The loop width remained constant during 111,200 more cycles. During a rest of 72 hr. followed by 50,000 cycles,

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 124.

[2] *Proc. Roy. Soc.*, vol. 104*A*, p. 538, 1923.

the width of the loop had diminished 46 per cent. After this, the stress cycles caused rapid increase in width of loop, and rupture took place after 250,000 more cycles.

It may be mentioned in passing that Gough and Hanson in careful static tests in tension found permanent sets at all stresses, indicating again that materials apparently do not behave in a perfectly elastic manner even at very low stresses.

Yield Stress and Yield Range.—J. H. Smith[1] made observations on the yielding of steel which led him to believe that his yield ranges and Wöhler's limiting ranges were identical. He determined his yield ranges as follows: A specimen was subjected to alternating stresses of zero mean stress, at about 1,000 cycles per minute, and an increment of steady tension was then added and the extensometer reading taken; the steady stress was then changed to the same amount in compression, and the reading again taken. This process was continued until a value of mean stress was reached for which on reversal of the steady stress the extensometer showed a yield. The maximum stress on the specimen when yielding occurred was not, in general, the ordinary static yield point as found in tension or compression tests; it might be greater or less than these values according to the range of stress employed. Yielding seemed to take place on the tension side if the mean stress was tension; on the compression side if the mean stress was compression; while if the mean stress was zero, the yield occurred in tension but was seldom obtained.

At this point it may be well to explain the synonymous terms steady stress, average stress, and mean stress. Each of these terms denotes the algebraic sum of the maximum stress and minimum stress divided by two. If the stress is ±10,000 lb. per square inch, then the range of stress is 20,000, and the mean stress is zero; if the maximum stress is 15,000 lb. per square inch tension and the minimum stress is 5,000 lb. per square inch compression, then the range of stress is 20,000 lb. per square inch and the mean

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[1] *Jour. Brit. Iron and Steel Inst.*, No. 2, p. 246, 1910.

stress is 5,000 lb. per square inch; if the maximum stress is 15,000 lb. per square inch tension and the minimum stress is 5,000 lb. per square inch tension, then the range of stress is 10,000 lb. per square inch and the mean stress is 10,000 lb. per square inch. The mean stress plus half the range of variable stress gives the maximum stress, while the mean stress minus half the range of variable stress gives the minimum stress.

In Smith's experiments the Wöhler limiting ranges were determined from tests most of which were at fewer cycles than 1,000,000, so that they can hardly be considered very reliable values. The experiments indicated that the yield ranges could be varied within wide limits when the mean stress was not zero, and that the Wöhler limiting range was not a fixed range even when the mean stress was zero. When a yield range was raised, the modulus of elasticity of the material appeared to be lowered.

One of the phenomena noted was that when specimens were subject to ranges so that the maximum stress was above the ordinary static yield point of the material, the specimens showed very perceptible changes of diameter. When the mean stress was compression, the diameter increased; while when the mean stress was tension, the diameter decreased. Furthermore, the same specimen could have its diameter first increased and then decreased. In one 0.63 per cent carbon steel in which the maximum stress did not exceed the static yield point, the change in diameter was not so marked as in the other cases, but still existed.

Stress-strain Loops.—Smith and Wedgwood[1] carried out further experiments with the object especially of studying the stress-strain loops formed under cyclical stress. They found that the static yield point of a material was not necessarily the upper limit of the yield range, the upper limit being in some cases greater and in some cases less. When the lower limit of the yield range was zero, then the upper limit was approximately equal to the ordinary static

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[1] *Jour. Brit. Iron and Steel Inst.*, No. 8, p. 365, 1915.

yield stress. This means that when the upper limit of the yield range was greater than the static yield point, then the lower limit of the range could not be of opposite sign.

The operations for getting the materials in the cyclic state were as follows: An alternating range known to be safe, with zero mean stress, was first applied. Mean stress was then applied in tension and the strains noted, after which the mean stress was gradually changed to the same amount in compression. When these operations were repeated a number of times, it was observed that the maximum strains settled down to definite values, which were repeated after each reversal of mean stress. These operations were repeated with increasing mean stress and in each case the strains settled down to fixed limits, until finally yielding occurred at the tension limit, at the compression limit, or at both limits. A material was considered to have been brought into the "cyclic" state when its yield stresses were equal in tension and compression. The speed of the alternating range was varied in different experiments, having values between about 500 and 1,000 cycles per minute.

When a yield range had been determined, the material could be brought back to a normal state by applying gradually diminishing mean-stress ranges. This normal state was not the primitive state, but a state in which there was an elastic range which was apparently the Bauschinger range.

When the material had been brought into the cyclic state, it was found that the stress-strain diagrams were complete loops as long as the stresses did not exceed the equal yield stresses mentioned above. If the range of stress was reduced, the loops diminished in size, and if the diminishing range of stress was kept between equal and opposite limits, the diminished loop became a straight line.

The first tests on loops were made with static loadings, and the shape of the loop was similar to that found by Bairstow and shown in Fig. 2. The loops for equal and opposite stresses were symmetrical with respect to the original elastic line of the stress-strain curve, and the straight-line portions of the loops were always parallel to the elastic straight line.

A rest of 24 hr. had no effect on the shape of the loop. A rest of 14 days had the effect of giving a smaller loop at first, but as the operations were continued, the loop became larger and larger, and Smith and Wedgwood believe that the material would finally have been brought into the condition of the original loop as obtained before the period of rest.

In one test, after a certain loop had been traced, the upper limit of stress was kept constant and the lower limit decreased. When the lower limit was decreased for successive loops, the loop diminished in width and finally became a straight line. The elastic line so determined was found to be of the same length for four different cases in which one limit was kept constant at a point on the original loop. This was true whether the upper limit was tension or compression. These loops with unequal stresses in tension and compression were not symmetrical with respect to the elastic straight line.

The unloading portion of a symmetrical loop, which was approximately a straight line down to zero stress, was investigated by unloading to zero and loading again, under which action a loop of narrow width was formed.

It may be of interest to note here that the original material had a static yield point in tension of 35,000 lb. per square inch, and the large symmetrical loop first traced had a maximum unit stress in tension and compression of 37,200 lb. per square inch. When the loop was successively traced with diminishing maximum and minimum stresses, the unit stress at which the loop became a straight line was ±22,400 lb. per square inch. This condition Smith and Wedgwood called the Bauschinger state.

After the static loops had been studied, tests were made in which a steady stress in tension or compression, plus an alternating stress, were obtained by means of revolving unbalanced masses, and the strains due to the alternating stresses were measured by a ray of light reflected from a concave mirror mounted on a double knife-edge. An illuminated line was used and not a spot of light, and to specify the strains completely, the length of the illuminated line and the change of position of its midpoint were noted.

The loops and diagrams which were obtained, therefore, represented the mean stress and the position of the midpoint of the illuminated line; that is, the deformation measured was that of the steady stress and did not include the deformation due to the alternating-stress cycle. It should be noted that the maximum and minimum stresses and the corresponding deformations for any one cycle of alternating stress were not recorded by this method. In this respect the tests are greatly different from those performed by Bairstow and also from the static-loop tests of Smith and Wedgwood, in which the strains of the specimen in going through any particular cycle of stress could be studied in detail.

It was found that loops traced in the manner described above were almost exactly the same as the static loops previously studied; but it must be borne in mind that while the static loops represented maximum stresses and corresponding maximum strains, the new loops represented mean stresses and corresponding strains.

The tables of values given for yield ranges show that apparently the static yield point cannot be greatly exceeded without producing a yield of the material, although the experiments did not determine how high the maximum stress might be increased with a small alternating range of stress before yield took place.

When a loop had been formed and the mean stress was gradually reduced by an increment for each succeeding loop, a condition was arrived at in which the mean stress-strain diagram did not plot as a loop but as a straight line. This condition was called by the authors the Bauschinger state, and represented a range from a certain stress in tension to an equal stress in compression. The authors took the mean-stress range represented by this straight line, added to it the alternating-stress range, and called the total range the Bauschinger range. The authors do not state in which way they consider that the Bauschinger range is related to the Wöhler limiting range.

It should be clearly understood that what the authors called the Bauschinger range is not related in any simple way to the Wöhler limiting range as commonly determined. In an ordinary fatigue test a maximum- and minimum-stress cycle is applied, and the steady stress, which may be zero or not, is kept constant throughout the test. Smith and Wedgwood, on the other hand, applied a constant alternating range of stress and then varied the mean stress, determining by diminishing loops a straight-line curve. What they called the Bauschinger range, determined as above described, does not help in answering the question as to what range of stress, applied in the ordinary way, *could be withstood without failure.*

The Bauschinger range as determined by Smith and Wedgwood in the static tests corresponds to the ordinary fatigue tests, and it is the opinion of the writers of this book that the Bauschinger range so determined is more likely to correspond to the Wöhler limiting range than the so-called yield range. The yield ranges produce loops and the Bauschinger ranges do not produce loops. It has already been pointed out, however, that the production of loops does not necessarily mean fatigue failure. Whether the Bauschinger ranges determined from the static tests, or the yield ranges, correspond to the Wöhler limiting range can be satisfactorily answered only by recourse to fatigue tests in which the endurance ranges are determined by long-time tests.

"Creep" Phenomena.—The work of Gough and Hanson in connection with slip bands and mechanical hysteresis has already been mentioned. They believe that failure under repeated stresses does not differ essentially from failure under static stresses. If a stress sufficiently high is applied, slip occurs on those crystals favorably oriented for easy slip, which results in local strain hardening. The amorphous metal formed on the plane of slip is hardened immediately on completion of slip and resistance of the metal to slip is strengthened on this plane. When the stress is reversed, slip takes place, but *not* on the original slip planes. If the process is repeated and the stress is not too great, the metal may become so strengthened that it will not fail under that range of stress. In other words, the metal can be cold worked by repeated stress just as it can be cold worked by static stress.

The amount of such overstraining is limited and ultimately a point is reached at which a crack is formed. Their experiments showed that the overstraining is localized in certain areas, and they believe that it is probably localized overstraining which causes a crack to be formed. This conception has been put forward by a number of investigators.

Gough and Hanson believe that when a metal is stressed to a certain value, plastic yielding and "creep" occur in certain unfavorably placed crystals. This will cause local redistribution of the internal stresses, which may cause an increase or decrease of stress in the immediate neighborhood. A local increase of stress acting on a suitably placed cleavage plane may cause further slip, inducing further redistribution. In certain ductile metals creep may cause sudden yielding at a particular load, and in others it may continue very gradually until it reaches a maximum. This creep will increase as the stress is increased until finally a stress is reached under which creep continues indefinitely and the metal fails.

Assuming that creep has ceased under a certain stress, the portions of the metal which have not suffered plastic deformation will be under higher stress than those which have slipped. When the stress is reduced, slip will occur in those portions which were previously free from slip. When the stress reaches zero, redistribution of stress continues in the so-called "elastic after-working." Creep and elastic after-working are two aspects of the same process, one being positive creep and the other negative creep.

Gough and Hanson refer to an experiment by Muir[1] (see Fig. 6), in which two specimens of the same steel were overstrained to the same extent, after which one was left at no load for 40 days and the other was left loaded at 55,000 lb. per square inch for the same length of time. In curve *A* the specimen was unloaded along *abc*, elastic after-working occurred at *cd*, the specimen was then loaded along *def*, and unloaded a second time along *a'b'c'*. In curve *B* the specimen was loaded along $\alpha\beta\gamma$, creep occurred for 3 min., the specimen was unloaded along $\delta\epsilon\zeta$, and loaded a second time along $\alpha'\beta'\gamma'$. If curve *A* is rotated through 180 deg., it will fit quite well on curve *B*. The *unloading*

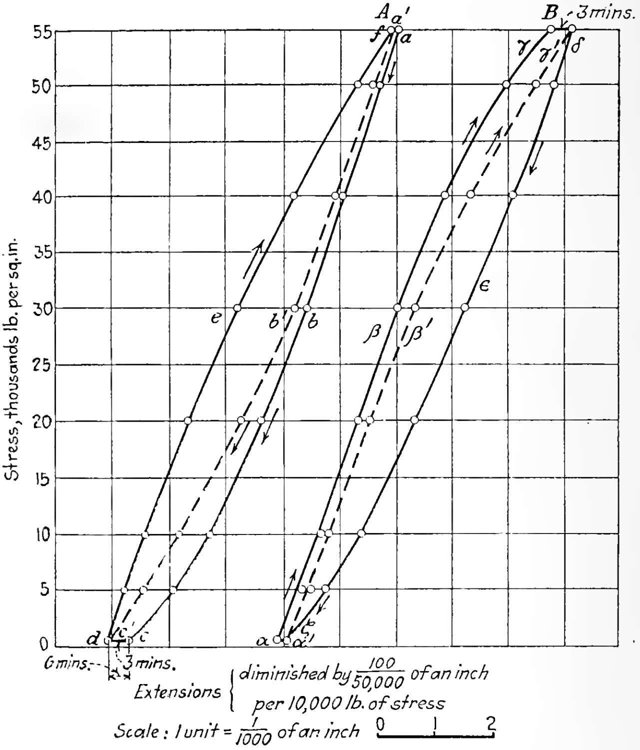

FIG. 6.—Cycles of loading and unloading for overstrained steel. (*Based on Muir in Phil. Trans. Roy. Soc.*)

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[1] *Phil. Trans. Roy. Soc.*, vol. 193*A*, p. 1, 1900.

part *ab* of curve *A* is straight and similar to the *loading* part $\alpha\beta$ of curve *B*, and also *bc* is similar to $\beta\gamma$. The *elastic after-working* for curve *A* is similar to the *creep* for curve *B*. Furthermore, the loop $defa'b'c'$, representing approximately cyclic conditions, is similar to the loop $\delta\epsilon\zeta\alpha'\beta'\gamma'$. In other words, this evidence indicates that the effects of loading and unloading are similar, but of opposite sign.

When metal is stressed within the fatigue range, the cyclic state is attained by plastic deformation and strain hardening. The cyclic state is attained when plastic strain ceases, and the metal can then withstand the cycle of stress indefinitely. When the stress is above the endurance limit, the slipping action is the same as that which occurs below that limit.

For cycles whose mean stress is not zero, the upper limit of stress can be applied safely only if the lower limit is *above* a certain stress. Since unloading causes plastic strain in a manner similar to loading, the process of unloading cannot be carried very far if the material is to withstand cycles of stress indefinitely.

Mason[1] found in torsion tests that the strain became greater when the speed of applying the cycles was reduced from 200 to 2 per minute, and became smaller again when the speed was again increased, the stress remaining the same throughout. This effect was absent when the strains were purely elastic. When the speed was reduced from 200 to 2 cycles per minute, the strain increased but immediately started to decrease toward a certain asymptotic value; while when the speed was increased to 200 cycles per minute, the strain decreased but immediately started to increase toward a certain asymptotic value.

This effect is explained by the writers of this book by the action of creep, which in turn is related to the time element. When the speed is 200 cycles per minute, there is not enough time for large strain, but in every succeeding cycle there is a readjustment of internal stress which is, of course, influenced by the previous history of stressing.

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[1] *Proc. Roy. Soc.*, vol. 92*A*, p. 373, 1915–1916.

When the speed is reduced to 2 cycles per minute, there is time for a greater strain, but in every succeeding cycle there is a readjustment of internal stress, which is again influenced by the history of stressing immediately preceding.

Mason considers as significant the hysteresis loops found by Bairstow, in which the unloading part of the loop is parallel to the original elastic curve. The writers of this book would explain this as follows: If when the plastic strain has occurred, the unloading is exactly similar to loading, except for sign, then the curves obtained by Bairstow and by Smith and Wedgewood are to be expected. Starting with the maximum stress in tension, the process of unloading begins. Some of the elements of metal have been stressed elastically, and they will return elastically to a lower stress. Some of the metal elements, however, were deformed plastically and hardened. These elements, being in a new state, presumably will act elastically. All the elements, therefore, for a certain range of unloading, can behave elastically. Soon a range of strain is reached which forces some of the crystals to slip plastically, and the stress-strain diagram becomes curved. It must be recalled that when these curves of Bairstow's were obtained, there had usually preceded the measurement of strain a considerable run at a constant stress, so that rather stable conditions of cyclic straining had been obtained.

Mason[1] performed the following experiment: Running at 200 cycles per minute, the strain range was 9.00 cm. on the deformation scale, while after stopping and immediately getting the range with dead weights, it was 11.65 cm., and the hysteresis loop was very far from being closed. Running at 2 cycles per minute, the range was 9.90 cm., and immediately after stopping, it was 10.26 cm., with dead weights, and the hysteresis loop was almost closed. The writers of this book explain this action as follows: The first change in strain was 2.65 cm. and was due to the great change in speed; the second change in strain was only 0.36 cm., because of the much smaller change in speed.

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[1] *Brit. Inst. Mech. Eng.*, 1917; *Engineering* (*London*), p. 211, Mar. 2, 1917.

The first range of strain at rest was indicated by 11.65 cm., and the second by 10.26 cm. In the first case the hysteresis loop was far from being closed, and the specimen was not adjusted to that range of strain. In the second case the range of 10.26 cm. was smaller and the hysteresis loop was almost closed, because there was so little difference between 2 cycles per minute and rest that little adjustment to this range of strain was necessary.

The well-known fact that metal which is stressed below its endurance limit is strengthened is in itself sufficient to show that even at these lower stresses there must be an action in the material which is not elastic. It is difficult to conceive how elastic action could strengthen the material, but it is easily understood how inelastic action could do this. The evidence of slip bands and hysteresis loops at stresses less than the endurance limit of the material is further evidence that a material has the power of adjusting itself to cycles of stress if these cycles of stress are within certain limits. That a process of strain hardening is going on under repeated stresses below the endurance limit is evidently quite as possible as it is under the action of static stresses above the yield point.

Creep at High Temperatures.—The phenomenon of creep at normal temperatures and also at higher temperatures has been studied by Lea and his collaborators. Budgen and Lea found[1] that a material had at a given temperature a "limiting creep stress," that is, a stress above which the material was progressively viscous. At ordinary temperatures specimens kept under observation for many weeks at stresses above the static yield point showed fairly steady creep for some hours, but the creep eventually ceased if the stress was below the ultimate strength. For each temperature, also, there seemed to be a stress below which creep ceased, but above which it was continuous.

Experiments were made on a 0.14 per cent carbon steel having a breaking strength of 68,500 lb. per square inch at 15°C., and 62,700 lb. per square inch at 400°C. At ordinary

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[1] *Brit. Assoc. Repts.*, 1924; *Engineering* (*London*), p. 500, Oct. 3, 1924.

temperatures the range of stress for 10,000,000 cycles was ±33,800 lb. per square inch, while at 400°C. it was ±39,200 lb. per square inch. When this material was tested statically to determine the limiting creep stress at 400°C., it was found to be slightly greater than 31,400 lb. per square inch. The range of stress, therefore, was greater than that which would cause continuous creep, and the half range was also greater.

This material was subjected (at 400°C.) to a maximum stress of 49,300 lb. per square inch and a minimum stress of 17,900 lb. per square inch. With this range of stress, which was equal to that which would cause continuous creep, millions of cycles of stress could be applied without producing failure. Lea is of the opinion that when the range of stress is above the limiting creep stress, fracture will probably occur ultimately. In some cases, however, 50,000,000 cycles of stress were withstood without fracture at such ranges.[1]

At ordinary temperatures under 20,000,000 cycles of equal and opposite stresses, the range of stress was about equal to the ultimate strength of the steel. Since the ultimate strength is the stress at which creep is continuous, it would seem that there may be a relation between range of stress and limiting creep stress.

Creep is apparently the criterion of slip, and persistent creep is evidence of the inability of the material to resist given shear stresses. Persistent creep implies an action in which time plays an important part.

Lea is of the opinion that if the range of stress is below that at which even for slowly applied loads there is no continuous creep, then the rate of application of stress is apparently of little consequence, and it is probable that an infinite number of cycles could be applied whatever the rate of application of stress. He has shown that the range of repetitions of stress can be raised more than 25 per cent by slowly increasing the range of stress during applied cycles.

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[1] French has reported in the *Proc. Am. Soc. Testing Materials* for 1925 and 1926 much more exhaustive studies of creep under high temperatures.

This seems to indicate that very small centers of possible creep can be healed by understraining. If, however, the stress first applied exceeds a certain amount, then the displacements are such as to prevent healing. Further, at ordinary temperatures the viscosity coefficient is small compared with what may be called the adhesive factor, and thus speed of application has not so important an effect as at high temperatures.

The limiting range of stress appears to be that range below which molecular slips can take place in the material, but after which new bonds may be established. This new bonding is materially helped by raising the temperature, and also by permitting slip to take place in very small increments during the application of cycles of stress. If the applied stress exceeds a certain amount, then the relative movement of the molecules is too great to permit rebonding, and molecular separation occurs which results in the formation of a fatigue crack and final failure.

Yielding in Static Tests and in Fatigue Tests.—As has been stated in Chap. II, Bairstow's experiments showed that for equal and opposite stresses steel did not show a permanent extension. Figure 7 shows the experimental results obtained by Bairstow when the mean stress of the cycle was not zero. In the figure the curve *OFEABC* is the curve obtained in an ordinary static tension test. Under a repeated stress equal to *OG* the first cycle did not show a measurable extension of the specimen, but continued applications of stress which was slightly greater than the safe range produced a slow yielding, represented by the line *GH*. When the adjustment of elastic limits was complete, there was no further extension beyond the point *H* due to continued applications of stress. The point *J* on the curve was obtained in a similar manner by repetitions of stress. For the stress *OE*, which is considered to represent the maximum non-destructive stress under completely reversed stress, no extension of the specimen occurred.

When the maximum stress of a cycle was above the yield point at *AB*, the extension was found to be due entirely to the maximum stress and was not influenced by the range of stress, which might be zero. Bairstow is of the opinion that extensions such as *GH* would probably be caused even by a range of stress which would not cause final failure.

For stresses below the yield point, therefore, iron and steel appear to be able to maintain an unstable condition for a considerable time under cyclical stress. The first application may not show an extension which is measurable, but this extension may increase thousands of times under a constant cycle of repeated stress.

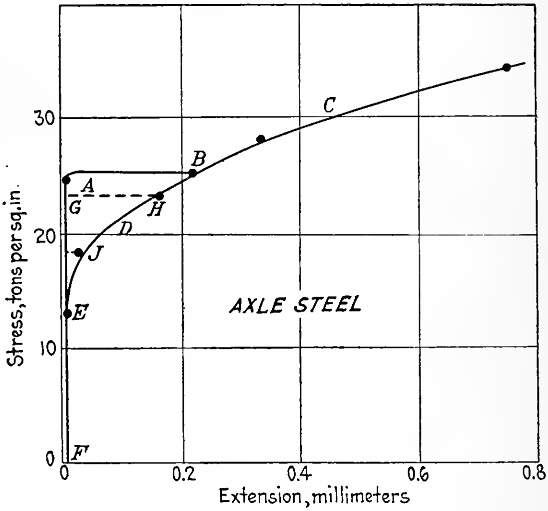

FIG. 7.—Permanent extension under cycles of stress. (*Based on Bairstow in Phil. Trans. Roy. Soc.*)

The experiments showed that the line *EJDHB* seemed to join smoothly with the static curve *BC*. The region *EABDE* seems, therefore, to be one which can be explored by repeated stresses, but about which no information can be obtained by a single application of stress as in a static test. This seems to reinforce again the contention that elastic limits and yield points obtained in a static test cannot be correlated with endurance limits.

It is well known that concrete and wood under dead load in long-duration tests yield gradually with lapse of time. It may be that this yielding phenomenon is similar to the yielding for iron and steel under repeated stress in the region *EABDE* in Fig. 7. Such yielding has not been shown to occur in iron and steel at normal temperatures below the static yield point.

Elasticity.—In an ordinary static tension test of steel the increments of unit stress and the corresponding increments of unit deformation are determined. In the usual test the value of unit stress is plotted as the ordinate and the value of unit deformation as the abscissa, and the limit of proportionality, or proportional elastic limit, is defined as the maximum unit stress at which the unit deformation remains proportional to the unit stress.

Sometimes a more tedious test is performed by going back to zero load after each increment of stress and determining whether there is any permanent set. The unit stress at which permanent set first appears is sometimes defined as the "true" elastic limit. The time consumed in making such a test is hardly justified, because extensometer measurements have shown that the determination of the stress at first permanent set is dependent on the precision and sensitiveness of the measurements. If the extensometer can detect very small deformations, then the "true" elastic limit is found at a comparatively low value. This evidence of inelastic action has been confirmed by sensitive thermal measurements both in static tests and in repeated-stress tests. It is evident, therefore, that the true elastic limit, obtained by a static test on virgin metal, can have little bearing on the phenomena of repeated stresses.

Bairstow found in repeated-stress tests of steel that a hysteresis loop was not developed in some cases until the specimen had been subjected to thousands of repetitions. On the other hand, tests of copper, a metal which has a curved stress-deformation curve, have shown that copper has a fairly well-defined endurance limit. It is evident, therefore, that neither initial apparent perfect elasticity, nor initial inelastic action, is a criterion of the behavior of materials under many applications of stress.

Perfect elasticity is sometimes defined as the quality which permits a material to be stressed and then to return to its original length without permanent set. Since a material may do this, however, and in the process form a loop which is closed at both ends, it does not seem that the above definition is admissible. Perfect elasticity might be defined as the quality which permits the stress-deformation curve under decreasing stress to coincide with the curve under increasing stress. Such perfect elasticity is evidently not common for engineering materials.

Bairstow was of the opinion that when the stresses were low enough so that the hysteresis-loop width was zero, then the specimen would not fail, but he also stated that the presence of a hysteresis loop was not necessarily a sign of failure. The experiments of Gough and Hanson and of Moore and Kommers have shown that perfect elasticity is not essential for indefinite endurance. The development of heat at stresses less than the endurance limit has confirmed this result.

Elastic Hysteresis.—The term "elastic hysteresis" is found in engineering literature and needs to be defined. If by elastic hysteresis is meant the action under repeated stress which may form a hysteresis loop, but which will not result in final failure, it is evident that the term may be used to describe a phenomenon which has been demonstrated by experiment. Elastic hysteresis is associated in fatigue tests with the attainment of stable conditions. A hysteresis loop may exist; but as long as its width does not increase under continued repetitions, the specimen will not fail. If the loop width does continue to increase, then the specimen will finally fail. A specimen of copper, therefore, might show an initial loop of considerable width, but if this loop reached and then maintained a constant width under continued repetitions, the specimen would not fail.

Concrete under repeated stress shows considerable initial permanent set, but if the specimen under test succeeds in reaching and maintaining a condition in which neither the deformation nor permanent set keeps on increasing, then apparently the specimen will not fail.

Temporary Effects.—These examples of the action of materials under fatigue illustrate the presence of temporary and transitory effects which tend to obscure the results which are of real importance. Bairstow found, for instance, that the width of the hysteresis loop at low, equal, and opposite stresses tended to become almost constant, but at higher stresses the width of the loop gradually decreased as the number of cycles was increased. Yet this decrease could not be looked upon as a sign that the specimen would not fail under these stresses, because the stress was known to be unsafe, and the effect was a temporary one which would not have continued indefinitely. Had the test been continued to failure, the width of the hysteresis loop would undoubtedly have decreased to some fairly constant value and then increased again until failure occurred.

This phenomenon of large initial hysteresis loops which gradually decrease is undoubtedly associated with the so-called "heat bursts" which have been observed by a number of experimenters. These heat bursts, as the name implies, cause a temporary rise in temperature, after a stress is first applied, followed by a subsequent fall in temperature. They indicate the transitory plastic strains which occur during the period when the specimen is adjusting itself to a particular cycle of stress. Hankins[1] has shown that if a specimen is subsequently tested at the same stress, heat bursts will not occur, the specimen being now adjusted to that particular cycle of stress.

Recovery under Repeated Stresses.—Another phenomenon which has effects which are sometimes permanent and sometimes temporary is the so-called "recovery," which may occur under the action of repeated stresses, and also that due to rest and mild heat treatment. The recovery which consists in a decrease in the width of the hysteresis loop after a stress is first applied is apparently

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[1] *Brit. Research Comm. Aero., Repts. and Mem.*, No. 789, 1921.

permanent, provided the stress is below the endurance limit. The metal seems to be cold worked due to the repeated stresses, making it stronger not only under subsequent static stresses but also under subsequent repeated stresses. If the stresses are above the endurance limit, however, and such recovery takes place, then the effect is only temporary, and continued repetition will again begin to increase the width of the hysteresis loop.

The effect of rest and mild heat treatment must be put into the class of temporary effects. Both metals and concrete show smaller deformations for the same stress after a period of rest, but experiments have shown that subsequent repeated stresses soon bring the deformations back to the value which they had before the rest period. Mild heat treatment seems to be similar in effect to a long period of rest, and as far as is known, it will produce only a temporary effect in decreasing deformations. It should, however, be noted that mild heat treatment may be very effective in relieving internal stresses, and such action must not be confused with the temporary effect which mild heat treatment may have on a specimen subsequent to a period of repeated stressing. Since the deformations after a period of rest or after mild heat treatment may be smaller for a time than just before the rest period, it is conceivable that the total number of repetitions before failure might be slightly increased by such treatment, but there is no evidence that the endurance limit is changed in any way.

In static tests the "healing" of overstrained metal by rest or mild heat treatment has been observed by a number of experimenters, and seems to be a well-established phenomenon. The strengthening of metal at ordinary temperatures by repeated stressing below the endurance limit also seems to be a well-established experimental fact.

Mason[1] made some tests to determine whether the temperature of boiling water would have an effect under cyclical stress similar to its effect under static stress. He found that a 0.12 per cent carbon steel seemed to be more resist-

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[1] *Advisory Comm. Aëro.*, vol. 2, p. 569, 1923–1924.

ant to alternating shear at 60 than at 212°F., and that for the same stress the range of strain was greater at 212 than at 60°F. This was just the opposite of what might have been expected. He concluded that either the healing which might be concurrent with cyclic stressing was less at 212 than at 60°F., or else if the healing was more pronounced at 212 than at 60°F., then evidently non-elastic strain was more easily produced at the higher temperature than at the lower.

On the other hand, as already mentioned in this chapter, Lea and Budgen found that on a 10,000,000-cycle basis a 0.14 per cent carbon steel had a higher endurance limit under reversed axial stress at 752 than at 59°F. There seemed to be no increase in endurance limit, however, until a temperature of 392°F. had been passed. For two other steels (chrome nickel) the endurance limit at normal temperatures was higher than at elevated temperatures. Lea and Budgen did not report the amounts of strain exhibited by specimens at the different temperatures.

Moore and Jasper[1] made static and fatigue tests on a normalized 0.49 per cent carbon steel, cyclops metal, a chrome-nickel steel with two heat treatments, and a heat-treated 1.02 per cent carbon steel. In the case of the 0.49 per cent carbon steel the endurance limit increased with the temperature up to about 900°F., and for one heat treatment of the chrome-nickel steel the endurance limit increased with the temperature up to about 500°F. For the other steels, the endurance limit decreased slightly with increase of temperature up to about 800 or 900°F. For all the steels tested the endurance limit fell off rapidly above 900°F.

Moore and Jasper determined the ultimate strength of these materials under tests lasting some hours, and they found that at high temperatures (about 1000°F.) the endurance limit at a speed of 1,500 cycles per minute approached, and in the cases of the chrome-nickel steel slightly exceeded, the ultimate tensile strength obtained from the tests lasting

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 152, p. 9, 1925.

some hours. This result indicates, of course, that at elevated temperatures the ultimate tensile strength falls off much more rapidly than the endurance limit does. Since, therefore, the material is weaker at the elevated temperatures and yet the ratio of endurance limit to ultimate strength is higher at these temperatures than at normal temperatures, the indication is that "healing" at elevated temperatures must be more effective than at ordinary temperatures.

Since the résumé of results given above as to the effect of heat on specimens subjected to cyclical stress shows more effective healing at some elevated temperatures but no effect at others, compared with the effect at normal temperatures, it is evident that at the present time no general conclusion can be drawn as to the healing effect at temperatures above the normal.

Bauschinger's Laws.—It may be of interest at this point to recall the evidence which has been presented to see whether or not it controverts the laws of Bauschinger given in Chap. II. The last sentence of the eighth law states that, after overstressing, a period of rest will not again increase the elastic limit for the opposite kind of stress, as is possible by loading in one direction only above the yield point. If it is assumed that subjecting the material to moderate heating has the same effect as a long period of rest, then there is some evidence to controvert this law. Moore and Kommers[1] made some tests on hot-rolled 0.18 per cent carbon steel which was cold stretched so that its diameter was reduced from 0.50 to 0.44 in. This material before cold stretching had an elastic limit of about 38,200, an ultimate strength of 61,500, and an endurance limit of ±28,000 lb. per square inch. After cold stretching the endurance limit was ±41,000 lb. per square inch. This material had been heated to 260°C. (500°F.) after cold stretching. This result would indicate that probably the elastic limit in compression which was reduced to zero by cold stretching must have been restored to something

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 124, 1921.

like 41,000 lb. per square inch, and possibly a period of rest would have had a similar effect, although the relative effectiveness of rest and mild heat treatment is somewhat uncertain.

It is believed that the remaining laws of Bauschinger have not been disproved by experiments made since they were formulated; but, on the other hand, a considerable body of evidence reinforcing these laws has been collected by various investigators.

Bauschinger's laws and the experimental evidence discussed in this chapter make it abundantly clear that the results obtained from ordinary static tests cannot be relied upon in drawing conclusions as to fatigue strength. It is hoped that the evidence thus far reviewed will help to point out some of the factors that are of importance in connection with fatigue strength.

CHAPTER IV

FRACTURE UNDER REPEATED STRESS

Introductory.—Although the microscope has shown that metals are not homogeneous in structure, not isotropic, and not capable of indefinite subdivision without change of properties, the theory of elasticity has such a commanding position and has proved so useful as a basis for design that the idea of perfect elastic material and of an absolute elastic limit below which no number of loadings can produce any structural damage in the material still persists. This explains, in part at least, the amount of attention paid to the phenomena of inelastic action—slip and mechanical hysteresis. In this study two facts have become apparent: (1) Before fatigue failure occurs, a crack develops in the metal, and (2) considerable slip may occur and considerable energy may be lost in mechanical hysteresis without starting a fatigue crack in some metals.

Recent tests have shown that for most metals the limiting stress for fatigue failure seems to be correlated with the ultimate tensile strength or the ultimate shearing strength rather than with any elastic limit. For some metals, especially for annealed pure metals, the fatigue limit is found above the elastic limit, and in some cases above the yield point; for some metals, especially for cold-drawn non-ferrous metals, the fatigue limit is found at a stress lower than that at which there is the first evidence of inelastic action.

Elastic failure of a machine part or of a test specimen involves a quite general slip throughout a considerable mass of metal; fatigue failure, on the other hand, may result from the spread of a crack at any cross-section. It is believed that a separate chapter may well be devoted to a study of the mechanism of the progressive fracture which constitutes a fatigue failure.

Limitations of Elastic Theory as Applied to Structural and Machine Parts.—The formulas of mechanics of materials have been and are of enormous use, but the assumptions on which they are founded are not strictly true. Materials, at least all ordinary structural materials, are not homogeneous and cannot be subdivided indefinitely without change of properties. This means that the ordinary formulas of mechanics of materials may be regarded as giving results "statistically" accurate, that is, accurate for the general behavior of a group of, say, a few thousand crystalline grains of metal, but not accurate for the behavior of the metal in any one grain. In considering *dead-load* strength of machine and structural parts made of *ductile metal,* such a "statistical" view is satisfactory. Unless a considerable mass of metal is deformed beyond the yield point, no serious structural damage is done. Around the rivet holes in an I-beam there may be dozens of minute areas stressed to the yield point, and no damage is done to the beam as a whole so long as the load is steady.

If, however, the material is brittle (for example, cast iron) then the case is different. The outstanding characteristic of brittle metal is its inability to adjust itself to local overstress without fracture. If an I-beam were made of cast iron, then highly localized stress around rivet holes probably would be a source of grave danger, even under dead load.

The case is still different for repeated loading, and under repetitions of loading, minute cracks tend to form at points of highly localized stress and to spread. This is true both for brittle and for ductile metal. The spreading of such a crack, like a minute hacksaw cut, gradually diminishes the area of sound metal remaining in any cross-section of a piece; and the end of a spreading crack is in itself a point of highly localized stress, so that there is a strong tendency for the crack to be self-perpetuating. *Under repeated load-*

ing, localized stress in a structural or machine part cannot be neglected even if parts are made of ductile metal.

Deformation and Slip from a Metallographic Viewpoint.—Taking now the viewpoint of the metallographist, three stages of deformation can be distinguished as metal is subjected to increasing static stress: elastic deformation, slip, and fracture.

Elastic deformation, as the machine designer and the structural engineer see it, consists in a very slight stretching, compressing, or sidewise shoving (shearing detrusion), and this slight deformation disappears if the stress is released. It has been noted that the engineer and the elastician think of stress in a metal as a regularly distributed internal force, but if metals be viewed through the metallographist's eyes, they are seen to be made up of irregular crystalline grains, and between grains and within grains unfavorably placed there must be many minute areas under very high stress. This is illustrated in a rather crude way in Fig. 8. If in addition to the irregularity of intergranular stresses there is considered the effect of non-metallic "inclusions" and of minute holes which are found in many metals, the possibilities of still higher localized stresses are evident. The stresses computed by the ordinary formulas

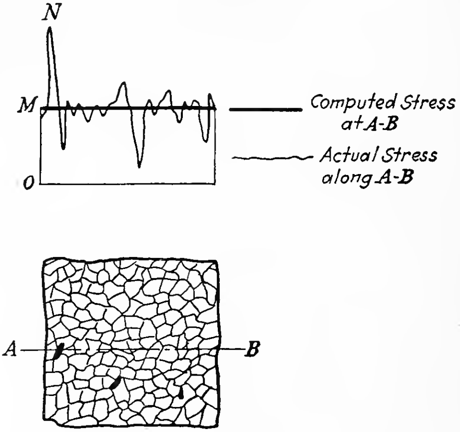

FIG. 8.—Nominal and actual stress in crystalline grained metal.

of mechanics of materials are much lower than the stresses developed over many minute areas in the metal; perhaps computed stresses are only a small fraction of the actual localized stresses existing in structural and machine parts.

In recent years the X-ray spectroscope has given a picture of the atoms in a crystalline grain of metal, held together by forces whose nature is as yet a mystery, and arranged in some regular geometric pattern with a border region at grain boundaries having a more or less irregular atomic arrangement. The regular pattern of atoms which is repeated to make up a crystal is known as the space lattice of a metal, and from the viewpoint of the student of atomic structure, elastic strain consists of a slight distortion of the space lattice, which distortion disappears when stress is released.

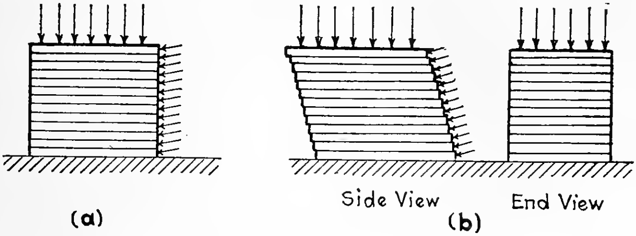

Fig. 9.—Diagram of action of slip.

As stress is increased in ductile materials, there comes about a state of affairs such that along certain planes of weakness in crystalline grains atomic bonds are broken. The divorced atoms slide over a few thousand other atoms, after which most of them find new partners and form new bonds with them. The remarkable thing is that the new bonds seem to be stronger than the old, after a brief period of restful adjustment. This action is known as "slip" and is shown under the metallographic microscope by a series of "slip lines" or "slip bands" such as are discussed in Chap. III and shown in Fig. 4.

Slip may be pictured as an action analogous to that shown by a pack of cards pressed together face to back and subjected to slightly oblique endwise pressure, an arrangement such as that shown in Fig. 9(*a*). Under a sufficiently heavy push the pack would take a position like that shown in Fig. 9(*b*). The end view of the pack is the end view of a stepped surface; it is the edgewise view of these steps that shows the slip lines through the microscope. If the cards are slipped repeatedly, the faces and the backs would become roughened and would offer increasing resistance to further slip.

To the metallographist a major significance of slip is the strengthening of planes of weakness within a crystalline grain. *If slip could be brought about with no other effects than the exchange and strengthening of atomic bonds*, it would be an entirely beneficial process so far as strength is concerned. In some cases, *e.g.*, cold-drawn steel, the process is actually somewhat beneficial to the strength.

The Progressive Course of Fracture.—It is not possible, however, for the process of slip to go on without there being some locations where atomic bonds are broken and new bonds are *not* formed; that is, minute, submicroscopic cracks are developed. The earliest metallographic picture of the mechanism of repeated stress of metals was a picture of cracks developing at slipping surfaces, growing to visibility under the microscope, and finally spreading to failure.[1] This is still a quite satisfactory picture, although two other pictures of the origin and spread of cracks have been recently presented—pictures which do not picture cracks as necessarily originating at slipped surfaces. These pictures will be shown in succeeding paragraphs.

If the strain on a metal is continually increased beyond the original strain where general slip takes place, actual fracture finally occurs. In the case of brittle materials such fracture occurs before slip becomes widespread enough to show a well-marked yield point. Under a single loading in tension the final fracture of a metal, either ductile or brittle, appears to take place simultaneously over the whole section of a piece of metal. That section seems to

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[1] EWING and HUMFREY, "Fracture of Metals under Repeated Alternations of Stress," *Phil. Trans. Roy. Soc.*, vol. 200*A*, p. 241, 1903.

act like the famous "one-horse shay," which went to pieces "all at once and nothing first, just as bubbles do when they burst." A careful study of the bursting of bubbles and of the failure of tension test pieces shows that in both cases the actual fracture is progressive, not instantaneous. If the fracture of a tension test piece is examined, there usually can be found evidence that the failure began at some definite region and spread rapidly across the piece. In some ductile metals fracture can be seen to progress across the test specimen (especially in the case of thin specimens). Fracture under a single loading is a very rapid progressive fracture. It may be safely stated that no experimenter has ever loaded a test piece of metal so carefully and so accurately that all the atomic bonds on a cross-section were broken at the same instant.

Under repeated loading a stress well below the ultimate tensile strength will start a fracture in metal which spreads, finally causing the failure of the entire cross-section of a piece. This spread under repeated loading is very much slower than the spread under a single increasing load. Thousands or even millions of cycles of stress may be required to develop the final failure of a machine part. Not infrequently the spreading crack can be detected before it has progressed to failure, and a disaster averted. This repeated-stress fracture spreads slowly like a minute hacksaw cut, but its rate of progress is accelerated, and just before fracture, it is almost as rapid as is the spread of fracture under a single increasing load. In fact, a typical fatigue failure usually shows two distinct zones: (1) a smooth surface where the crack has spread slowly and the walls of the crack are battered smooth by repeated opening and closing, and (2) a rough "crystalline" surface indicating the very much more sudden fracture of the core of the piece.

Figure 10 shows the fracture of a rotating-beam test specimen subjected to cycles of repeated flexure. Fracture started at the outer circumference, and a crack gradually spread inward. The walls of this crack were continually shoved against each other as the crack opened and closed under successive cycles of stress. The walls of this part of the crack were worn smooth, and occasionally little longitudinal breaks occurred, leaving steps in the surface which roughly resembled ripple marks left on sand by flowing water. When the cracks had spread to the inner circle shown in Fig. 10, the failure of the remaining metal progressed so rapidly that a rough "crystalline" surface was left, such a surface as is found when a steel specimen with a sharp shoulder is fractured under a single very heavy load.

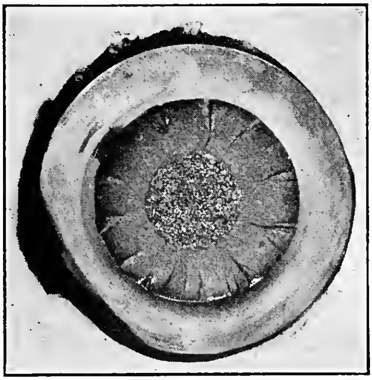
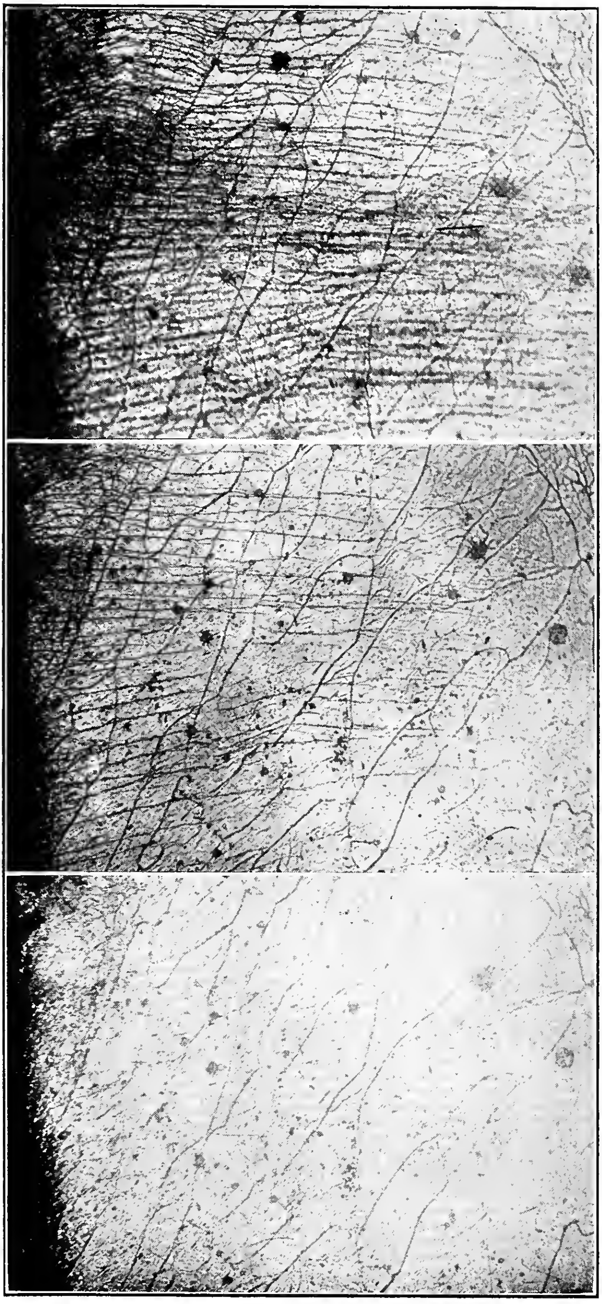


Fig. 10.—Fracture of rotating shaft under reversed bending.

Incipient Cracks.—Definite knowledge as to the nature of fatigue cracks in their initial stages is entirely lacking. Attempts to use the microscope to detect fatigue cracks in their very early stages have not met with much success. If, however, a specimen is notched so as to localize fractures at some definite cross-section, such fatigue cracks in large crystalline grains of metal can be detected quite early in the "life" of the piece, and they can be seen to multiply and to lengthen under successive cycles of stress. Figure 11 shows three views of a specimen of Armco iron subjected to violent reversals of flexure. The multiplication and lengthening of cracks is evident. It is, however, exceedingly difficult to detect fatigue cracks in small-crystalled metal, and it is exceedingly tedious to hunt for microscopic cracks over any considerable area of surface metal.

Figure 12 shows fatigue cracks in normalized 0.93 carbon steel, in brass, and in Armco iron. In Fig. 12(a) the crack is seen to traverse the ferrite of the steel, skirting the laminae of cementite. It seems as if the junction of ferrite

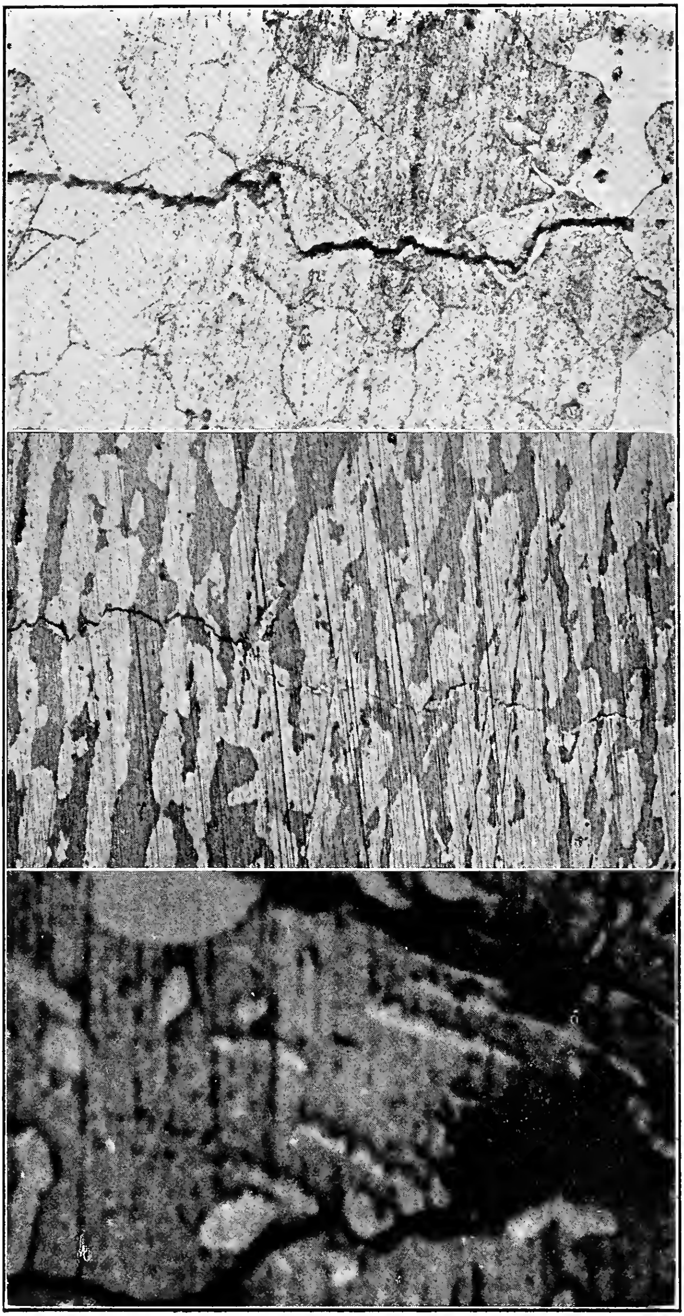


Unstressed

After 27 cycles of stress

After 424 cycles of stress

FIG. 11.—Development of cracks in Armco iron. Magnification 75×. (*Micrographs by H. R. Thomas at the University of Illinois.*)



(a)
0.93 per cent carbon steel
Magnification 5,000×

(b)
Brass
Magnification 275×

(c)
Armco iron, annealed
Magnification 275×

Fig. 12.—Fatigue cracks in metals. (*Micrographs by Julius Muller at the University of Illinois.*)

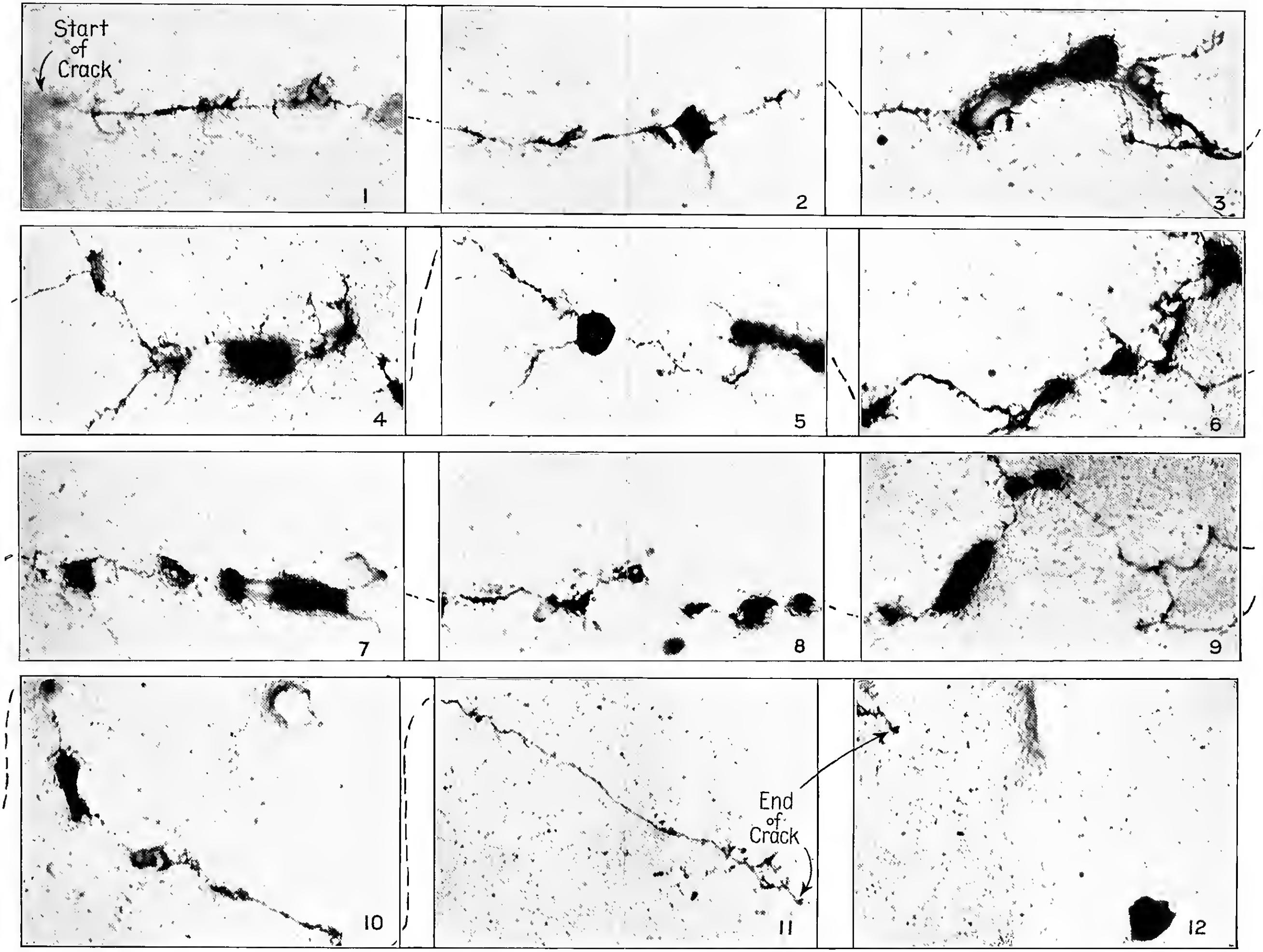


Fig. 12(*d*).—Course of fatigue crack in Armco iron. Magnification about 2,000 ×, magnification of original plates 3,560 ×. (*Micrographs by F. F. Lucas at the Bell Telephone Laboratories.*)

and cementite were the weak region in this steel. In Fig. 12(*b*) the brass is seen to be made up of two crystalline ingredients, but they seem to differ but little in strength, since the fatigue crack goes straight across both kinds of crystalline grains. A crack typical of fatigue cracks in large-grained pure metals is shown in Fig 12(*c*)—Armco iron. This crack is seen to cross grain boundaries, and to go out of a straight path to skirt "inclusions." There seems to be a tendency for fatigue cracks to pass through the boundary between an "inclusion" and the adjacent metal, as if the "weld" or the "cement" between inclusion and metal were a region of special weakness.

Figure 12(*d*) shows a fatigue crack in Armco iron. The magnification of the cut is 1,800 times, and at its narrowest visible part the crack is about 500 atoms wide—if the calculations of atom size by modern physicists are accepted. This remarkable micrograph by F. F. Lucas shows the crack seeking out some inclusions and avoiding others, and at once suggests the presence of a multitude of minute defects in the metal.

In spite of its difficulties, the microscopic study of fatigue cracks in metals offers a very promising field for the investigator. The question of initial stress in metals and of whether such regions of stress are sources of fatigue cracks also needs investigation. Some locations where there are no cracks may be under high internal stress, stress so high that but little additional stress is necessary to start a crack.

In structural members and machine parts it is sometimes possible to detect fatigue cracks before they have spread to fracture. Some railroads and street railways make a practice of inspecting axles of cars and locomotives at regular intervals to see if such small cracks can be detected. When this has been carefully done, there have been very few disasters due to fatigue fractures in axles. In some experiments now in progress at the University of Illinois, it has been found possible to detect cracks[1] in specimens 1⅛ in. in diameter cut from car axles when about one-half the "life" of the specimen has passed, unless the applied stress is very high.

Theoretical and Actual Strength of Metals.—The whole question of fracture in metals brings up the relation of theoretical cohesion and strength. From a determination of the latent heat of fusion and the latent heat of vaporization, physicists have computed the theoretical cohesion of atoms for many metals, and if cohesion in solids at ordinary temperatures is of the same order of magnitude as cohesion in melting solids and vaporizing liquids, then the tensile strength of most metals should be from fifteen to twenty times as great as it is found to be in ordinary tension tests.

The most obvious explanation of this great difference is that in the common metals the system of atomic bonds is far from perfect. It has already been noted that the elastician's picture of continuous, homogeneous material is not true for ordinary metals. If metals are considered from the viewpoint of the metallographist, a rough picture of the metals may be drawn by considering them as continuous but not homogeneous. If there are considered imperfect bonds between atoms, minute cracks, and severe internal stresses, which when slightly increased will produce cracks, a picture may be drawn from the engineer's viewpoint, a picture of metal which is homogeneous but not continuous. Neither of these pictures can claim to be complete. The metallographist's picture does not lend itself to mathematical computations of strength. The engineer's picture of metal which may be regarded as homogeneous but which has in it many small holes or many irregularities of outline, *does* lend itself to such computa-

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[1] A method in successful use consists of applying oil to the surface of the steel, rubbing off the free oil and then applying a coating of whiting and alcohol. When this coating is dry, the specimen is rotated under load. Oil which has penetrated the crack and was not removed when the surface was wiped is forced out, if the crack is on the compression side and discolors the whiting coating.

tion or, at least, to estimation. The mathematical theory of elasticity, and the mechanical means which can be used to solve some of its more complex equations can be employed to determine approximately the effect of these supposed discontinuities. With this apologia for using a method of analysis which is admittedly based on an incomplete picture, but which is believed to be useful, the writers of this book wish to present a discussion of two hypotheses of the mechanism of fatigue failure.

The Internal-flaw Hypothesis.—In an extremely valuable paper[1] the British physicist, A. A. Griffith, has developed a picture of the mechanism of the failure of materials under stress. The paper treats the subject both from the viewpoint of mathematical analysis of stress and strain, and from the experimental viewpoint.

Griffith found that the computed unit stresses at rupture existing at the ends of cracks in glass were of the order of 350,000 lb. per square inch, while the tensile strength of the glass, as determined by an ordinary tension test, was about 25,000 lb. per square inch. He also found, by drawing this glass out into very fine fibers and by a series of tensile tests of these fibers, in which the fragments of one test were in turn tested, that a tensile strength of 491,000 lb. per square inch was finally obtained. The above values approach in magnitude the theoretical cohesive strength for glass.

Griffith believes that the above-named results and also the great difference between the theoretical cohesion of solids and the actual values obtained in tension tests may be best explained by the hypothesis that in all solids there are, scattered throughout the mass of the solid, multitudes of minute discontinuities or flaws, whose ruling dimensions are large when compared with atomic dimensions and distances. He believes that the effective strength of engineering materials might be greatly increased, perhaps ten to twenty times, if such flaws could be eliminated.

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[1] "Phenomena of Rupture and Flow in Solids," *Phil. Trans. Roy. Soc.*, vol. 221*A*, p. 163, 1920.

Figure 12(*d*) supports the Griffith picture. Figure 13 is a cartoon of the Griffith idea. Metal is pictured as having in it a multitude of minute cracks—cracks, say, 0.0002 in. long and a few score atoms wide, cracks which cannot be detected by any present-day microscope. If such cracks exist, they must be very numerous and must be scattered throughout the metal, else the metal could not be produced with such dependable physical properties as is found to be the case. These minute cracks weaken the metal in two ways: (1) by diminishing the area of the cross-section, and (2) by causing very high localized stress at the ends of the crack. Attention must be called again to the limitations of the theory of elasticity and to the improbability that its formulas would apply with any high degree of accuracy to such small areas of metal as are involved in considering these cracks; however, the general qualitative conclusions of the theory of elasticity may be expected to furnish a useful guide for estimating the general effect of

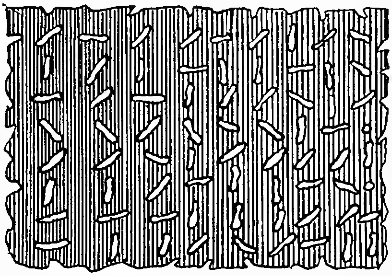

Fig. 13.—Diagram to illustrate the Griffith theory of the structure of metals.

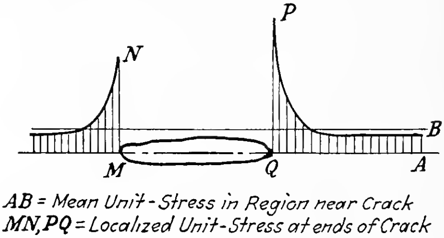

Fig. 14.—Stress intensification at the ends of a crack.

such cracks. From such general conclusions it would seem that if Fig. 14 represents such a small crack, the stress intensification at the ends is a function of the direction of the long axis of the crack with respect to the direction of the stress and a function of the sharpness of curvature at the end of the crack. It must be remembered that cracks have rounded ends. High-power micrographs, of which Fig. 15 is a sample, show this.

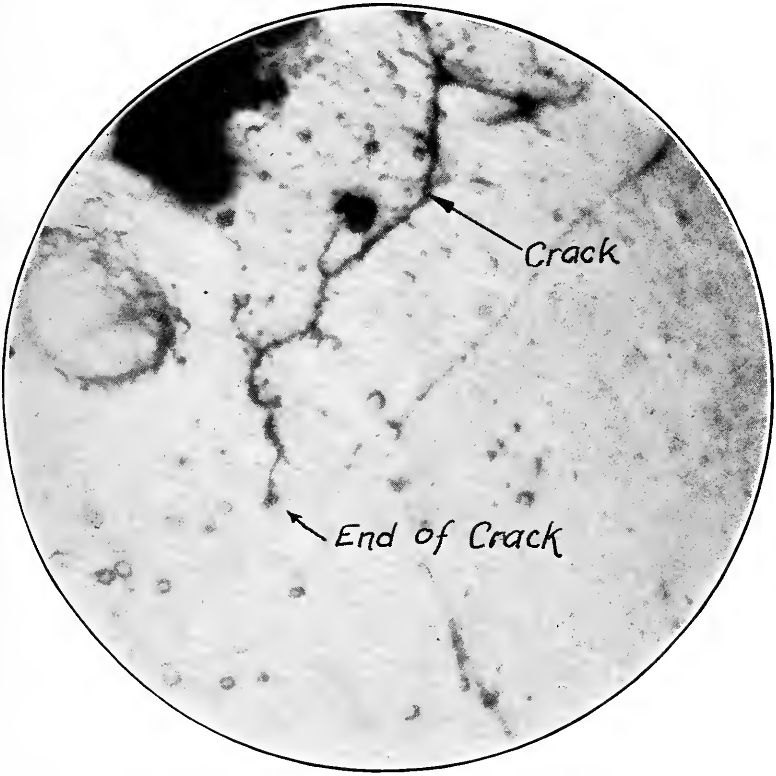

Fig. 15.—Fatigue crack in Armco iron. Magnification 3,560 ×. (*Micrograph by F. F. Lucas at the Bell Telephone Laboratories.*)

Griffith believes that the presence of such cracks may be explained if it is supposed that a change in volume occurs when the metal changes from the crystalline to the amorphous condition. Supposing a material contracts on decrystallizing, then a stress cycle, which causes repeated slipping in certain crystals, will produce amorphous material at the crystal boundaries. The volume of amorphous material will increase with repeated slipping, and if it fills less space than the crystalline material did, the material in the immediate neighborhood will be subjected to a tensile stress. When this tensile stress exceeds a certain critical value, a crack will form, and under further cycles of stress the crack will spread and final rupture will occur.

It may be noted here that considerable evidence is available to show that the effect of overstrain is to decrease the density of metals. If, then, amorphous material occupies a larger volume than the crystalline material, it is quite possible for this change in volume to produce both compressive and tensile stresses in the immediate neighborhood and thus again produce a crack. In the event of either decrease or increase of volume due to the formation of amorphous material, the damaging disturbance of internal structure takes place, not immediately at the end of an internal flaw, but some little distance away from it.

Beilby[1] has suggested that under alternating stresses a film of "hard-phase" material is formed on a surface of slip, and when the stress is reversed, slip occurs in the opposite direction, but not on the same plane as before, because the harder material is stronger. The second slip will occur on an adjacent plane, producing two hard layers with a soft layer between. If it be assumed that the production of the hard layers has produced a tension normal to the layers, then on further slipping it may be conceived that the hard layers thicken at the expense of the soft crystalline material. If this process is continued and the tension also, the crystalline material will be used up in thickening the hard sheets and an incipient crack will appear between them.

In considering this hypothesis in conjunction with the fact that overstrain seems to produce an increase in volume in the material, it is not clear why tension should be produced between the layers. It is conceivable, however,

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[1] *Proc. Roy. Soc.*, vol. 79*A*, p. 463, 1907.

how such an increase in volume of the hard layers could act as a wedge producing tension on some other microscopic portion of the body.

Whether or not either of the hypotheses mentioned represents what actually occurs, it is known that in fatigue some mechanism is at work which either produces a microscopic crack or else spreads a crack already existing in the virgin material. The action of the repeated stresses is such as to spread this crack until the member is so reduced in effective cross-section that complete failure results. This spreading of a crack, even in a ductile metal, explains the characteristic appearance of a fatigue fracture, which has the features usually associated with a brittle material, but none of the local elongation and reduction in cross-section which accompanies the ordinary tensile failure of a ductile material.

From the above pictures of failure an explanation is found for the fact that while scratches do have an appreciable effect on the fatigue strength of a metal, they do not have so serious an effect under the action of fatigue stresses as theoretical stress calculations would indicate. If fatigue failure is determined by the phenomena at the grain boundaries, then the strength is determined not so much by the stress range at the corner of a scratch as by the stress range at a distance of about one crystal layer away. Since theory indicates that the stress due to a scratch falls off very rapidly with distance from the corner of the scratch, this explains why fatigue tests show a smaller effect due to scratches and sudden changes of cross-section than would be expected from the calculation of maximum stresses based upon elastic theory.

It is, of course, well known that under the action of fatigue stresses failure occurs by the spreading of a crack. Griffith's theory of failure would indicate that in many cases the cracks are initially present in the material and that the action of the repeated stresses has the effect of spreading these cracks. Griffith's results show that the inherent local strength of a solid is many times greater than the average strength obtained in an ordinary tensile test, and it is clear, therefore, why a certain minimum stress is necessary to spread even those cracks which may exist initially in a body. The theory also suggests how a crack might be formed in material initially free from cracks, on the assumption that material in the amorphous state has a different volume from the same material in the crystalline state.

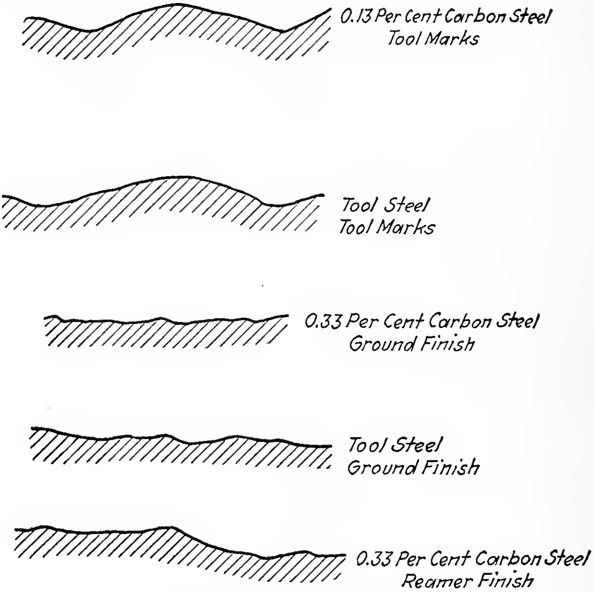

Fig. 16.—Surface irregularities of steel. Magnification 180 ×. (*Based on micrographs of sectioned gelatin casts obtained by W. Norman Thomas in tests for the British Aeronautical Research Committee.*)

The Surface-irregularity Hypothesis.—A second hypothesis for explaining the start of fatigue cracks is that such cracks start in the region of localized stress or in the adjacent region of structural damage, due to one of the many minute hills and hollows which are found even on the most carefully polished surfaces. Figure 16, which is based on the work of the British experimenter, W. Norman Thomas for the Aeronautical Research Committee,[1] shows actual surface irregularities magnified 180 times. Evidently

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[1] *Brit. Research Comm. Aero., Repts. and Mem.* 860, Vol. 2, p. 542, 1923–24.

there will be stress concentration at the bottom of the minute notches in the surface of the metal, and the magnitude of the stress concentration at the root of any notch depends on the depth of that notch and the sharpness of curvature at the root of the notch.

The Russian physicist, Joffe, is inclined to feel that surface irregularities rather than internal flaws are the starting points for fatigue cracks, and he cites an interesting experiment in which a single crystal of a salt was ground to spherical shape and subjected first to thorough cooling in liquid air and then to sudden immersion in molten lead. Under such a change the surface of the sphere would be free from stress, but the interior would be under heavy tensile stress. Joffe calculated the stress set up to be nearly equal to the theoretical cohesion of the salt; yet after the test no evidence of any fracture external or internal was found.

The two hypotheses given above are not contradictory, but rather supplementary. For all we know, both internal flaws and surface irregularities may be effective agents in starting fatigue cracks. Moreover, the reasoning about stress concentration, production of amorphous metal at grain boundaries, splitting action due to "wedges" of strain hardened material, etc., is as applicable to the surface-irregularity picture as to the internal-flaw picture.

The Mechanism of Progressive Fracture.—Whether the origin of fatigue cracks is always at surface irregularities, or whether they may originate at internal submicroscopic flaws; whether they are always present, or whether they originate as the result of internal stress plus stress due to load; whether they always originate as the result of slip in a metal, or whether they may start without previous slip taking place; once they are started, they progress, sometimes to fracture and sometimes to a state of equilibrium without fracture. The following somewhat speculative picture of the progress of a crack is given in terms of a combination of the internal-flaw and surface-irregularity hypotheses.

In Fig. 17 the left-hand part of the figure refers to the internal-flaw picture, while the right-hand part represents the surface-irregularity picture. The reader is again reminded that cracks and notches have roughly rounded ends (see Fig. 15). Under increasing load at least three things happen: (1) The crack shown at *a* and the notch shown at *b* spread to the condition shown at *a′* and *b′*, respectively; (2) the curvature of the ends of the crack and the curvature at the bottom of the notch become less sharp, causing diminution of stress concentration and benefit to the metal; and (3) slip as well as fracture occur at the ends of the crack and at the bottom of the notch, probably causing diminution of internal stress, and tending to increase the strength of the material by cold working. Under a moderate increase of loading, equilibrium is reached, and the damage caused by the crack (or the notch) spreading slightly and causing a reduced cross-section is balanced by the diminished stress concentration at the ends of the crack or the notch, and furthermore by the tendency to strengthen the material at the points of highest

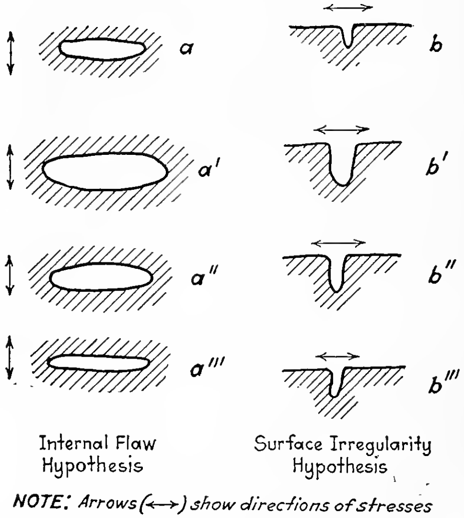

Fig. 17.—Diagram of growth of defects under repeated stress.

stress. Under a sufficiently great increase of load the spread of the defects overbalances the beneficial effects of lowered stress concentration; failure occurs, and its final progress is very rapid. If the metal has low ductility, the strengthening effect and the reduction of internal stress due to slip are slight, since brittle material slips very little before fracture.

To make this picture fit the case of repeated stress, a third part must be added. If a load is applied not sufficient to cause failure and that load is then released, the resulting state of affairs may be pictured as at a'' and b''. The crack a'' is longer than the initial crack a, and the notch b'' is deeper than the initial notch b. However, the curvature at the end of the crack and at the bottom of the notch may be either sharper or less sharp than at first. For example, if considerable slip has taken place at the end of a crack or a notch, cold working the metal there, there would be a tendency for the end to "stay open" on release of load, and the crack or notch would remain comparatively blunt ended. If, on the other hand, the crack spreads with but little slip accompanying its spread, on release of load there will be less resistance to the "closing up" of the end, and the crack or the notch will tend to be sharp ended. Stress concentration then may be either increased or diminished, and the damage done by succeeding loads may be either greater or less than that done by the first load, depending on the magnitude of the load and upon the changing nature of the material as the crack reaches different crystalline grains. It is, then, not difficult to picture how the repetition of a load smaller than that required to fracture the material at one application may fracture it under repeated applications.

If the load is not merely repeated but is *reversed,* the state of affairs may be pictured as at a''' and b'''. The reversed load might shorten the crack and the notch, although this is by no means certain. It would undoubtedly tend to make the curvature at the ends of the defects more sharp than that after release of load (a'' and

b'') and thus tend to increase stress concentration for the succeeding cycles of stress. It is then easy to see in a general way how cycles of *reversed* stress are more likely to spread a crack to failure than are cycles of one-direction stress of the same maximum value.

To the user of material, the significance of slip seems to lie in the location of some elastic limit or yield point which marks the practical limit of retention of original form by a machine part or a structural member. The significance of spreading fracture lies in the location of an endurance limit or fatigue limit, below which repetition or reversal of loading will not cause a crack to spread to failure. Both limits are most conveniently measured in terms of computed stress, that is, stress computed by the ordinary formulas of mechanics of materials, which, as has been pointed out, is really an average stress for a considerable number of crystalline grains of metal.

Explanation of the Discrepancy between Theoretical and Practical Effect of Holes, Scratches, Etc.—It is an observed experimental fact that holes, screw threads, notches, and other obvious defects in metals do not reduce the strength of metals under repeated stress as much as is indicated as probable by the theory of elasticity. In a foregoing paragraph reference has been made to an explanation of this fact on metallographic grounds, assuming the formation of amorphous material as the result of slip, and causing the damage to be done, not at the surface of the hole or scratch, but at some appreciable distance therefrom, perhaps at the next grain boundary. There is here offered an explanation based on considerations of stress and strain under spreading fracture. This explanation is believed to be neither more nor less fanciful than the explanation based on metallographic grounds, and is not at all contradictory to it.

Holes, scratches, nicks, screw threads, and other similar defects will be called "imposed" defects, while small internal flaws and irregularities of machined and rolled surfaces will be called "inherent" defects. First of all, imagine the case of ideal metal without any inherent defects, metal homogeneous and continuous. An imposed defect would then produce its full theoretical effect, as given by the mathematical theory of elasticity. For the case of an imposed defect consisting of a small circular hole, the localized stress at the edge of the hole would be about three times the average, and under a load of one-third the ultimate of the ideal metal itself, a crack would form, the stress concentration at its end would be high—higher than that due to the small hole—and failure would take place rapidly under repeated load.

Next, imagine a very defective metal in which the inherent defects are of the same order of magnitude as the imposed defects. For this metal an imposed defect would produce no weakening except that due to the actual area of metal removed. The inherent defects already have set up stress concentrations as bad as those set up by the imposed defect, and since they are of the same order of magnitude, the area of influence round the inherent defects is as great as the area of influence round the imposed defect. If there is imagined a specimen with a hundred small holes bored through it at points well scattered over the surface, the tensile strength under repeated load is not much lowered by boring one more hole.

In the third place, imagine metal in which the inherent defects are of a smaller order of magnitude than the imposed defect, which for purposes of illustration may be a small hole. The inherent defects are spread throughout the metal (or the inherent defects of surface are spread around the edges of the hole), and under load the stress-raising effects of inherent defects and of imposed defect are added. If, however, under this additive effect a crack starts and *spreads*, conditions change. The crack itself may be considered as of the same order of magnitude as the small inherent defects, and as it spreads, it soon begins to get out of the area of influence of the hole. This is crudely illustrated in Fig. 18, in which the dots represent inherent defects. Initially the stress at a is the sum of the stress S, the theoretical stress at the edge of the hole, and a very high stress Q, due to stress concentration at the inherent defect at a. If, however, the crack spreads to b, the stress due to concentration at the end of the crack may still be imagined to be of the order of magnitude of Q, but that due to the stress concentration at the hole will be not S but a smaller value S_b, since the stress falls away very rapidly as the distance from the hole is increased. Thus as a crack spreads, stress concentrations tend to become smaller. Hence when considering the spread of a crack to failure, an imposed defect may be imagined to *start* cracks as indicated by the theory of elasticity, but the imposed

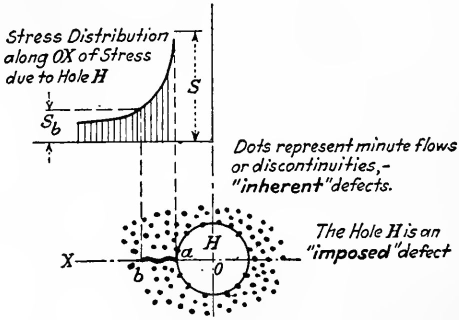

FIG. 18.—Diagram for stress as crack spreads from a large defect.

defect does not cause cracks to *spread to failure* as readily as the theory of elasticity would indicate.

It seems reasonable to picture metals with many very small inherent defects as approaching more closely to the assumed conditions of the theory of elasticity than do metals with fewer and larger inherent defects. Moreover, the size of widespread inherent defects is, in general, smaller for fine-grained than for coarse-grained metals. Thus it is reasonable to find fine-grained metals, such as heat-treated alloy steels, following more nearly the theory of elasticity, when they have imposed defects, than do coarser-grained metals; that is, the effect of holes, scratches, nicks, screw-threads, etc. might be expected to be relatively more serious on fine-grained alloy steels than on ordinary steels. This is found by experiment to be the case.

CHAPTER V

TESTING MACHINES AND SPECIMENS FOR FATIGUE TESTS OF METALS

Importance of Fatigue Tests and Testing Apparatus.—For any given metal the strength under repeated stress seems to be a function of the ultimate tensile strength and of the regularity of internal structure. The fatigue strength is best measured by an "endurance limit" or "fatigue limit," whose determination will be discussed in the next succeeding chapter. The ratio of endurance limit under cycles of reversed flexural stress to ultimate static tensile strength has been christened the "endurance ratio" by D. J. McAdam, Jr. The endurance ratio varies for different metals, and at least for a metal of unknown properties, direct experimentation under cycles of known stress seems to be the only way to determine fatigue strength satisfactorily. Hence, the apparatus, specimens, test methods, and methods of reducing test data for fatigue tests are of prime interest.

Types of Repeated-stress Testing Machines.—Testing machines for making fatigue tests under cycles of repeated or reversed stress may be classified according to the type of stress produced:

1. Machines for cycles of axial stress (tension-compression).
2. Machines for cycles of flexure.
3. Machines for cycles of torsion (shearing stress).

Another classification of testing machines for fatigue tests would divide them as follows:

a. Machines producing for each cycle a definite load or moment on the specimen, which remains constant throughout the test.

b. Machines producing for each cycle a definite deformation of the specimen, which remains constant throughout the test.

c. Machines in which both the load or moment and the deformation vary as the test proceeds.

In any repeated-stress testing machine, as in any ordinary "static" testing machine, there must be provided a mechanism for applying load or moment to the specimen, and a mechanism for measuring the load or moment applied to the specimen. The load-applying mechanism and the load-measuring mechanism may be combined in some machines.

In any repeated-stress testing machine there must be provided a counter for the number of cycles applied, and some device by which, when the specimen breaks, this counter automatically goes out of action. Frequently the device which throws the counter out of action acts to stop the testing machine itself.

Repeated-stress Testing Machines for Cycles of Axial Stress. 1. *Spring-type Machines.*—Figure 19 shows in diagram a typical axial-stress testing machine in which the cycles of load are applied by means of a crank and connecting-rod mechanism and in which the magnitude of load is measured by the deformation of a spring. The specimen *S* is directly attached to the heavy spring *G*. The end of the spring away from the specimen is given a reciprocating motion by means of the connecting rod *K*, which is actuated by the variable-throw crank *C*. The magnitude of tensile force or compressive force acting on the specimen is measured by the extension or the compres-

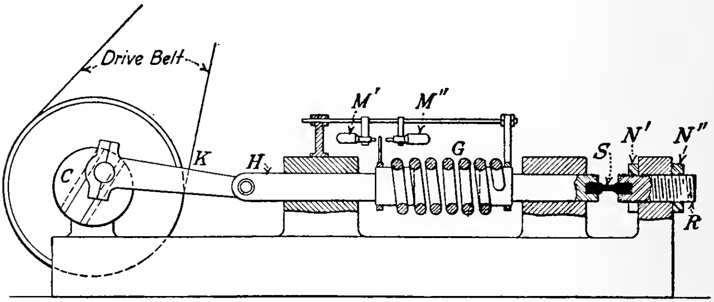

Fig. 19.—Diagram of axial-stress spring-type testing machine. (*Jasper.*)

sion of the spring G, and this extension or compression is measured by micrometers M' and M''. Varying range of load may be secured by adjusting the initial pressure on the spring G by means of the screw R and the nuts $N'N''$. If with the cross-head H at midstroke this initial pressure is zero, then the machine sets up cycles of completely reversed axial stress in the specimen; during a cycle, load changes from tension to compression of equal magnitude. If the spring is so adjusted that the pressure is zero at the end of a stroke, the machine sets up cycles of stress varying from zero to a maximum, cycles of tensile stress for zero adjustment at one end of the stroke, and cycles of compression for zero adjustment at the other. The particular machine shown in Fig. 19 was designed by T. M. Jasper at the University of Illinois.

In any repeated-stress testing machine of the spring type it is necessary to limit the minimum time of one cycle to a value well above that of the natural period of vibration of the spring, else there will be set up interfering waves of stress, and the measurement of stress will be very uncertain in accuracy. With a spiral spring such as that shown in Fig. 19 and a capacity of 4,000 lb. the maximum speed of the machine was found to be about 200 r. p. m. Using a flat spring of short natural period of vibration, spring-type repeated-stress testing machines have been successfully operated at speeds up to 1,000 cycles per minute.

The spring-type repeated-stress testing machine falls in class *c* (p. 84), since any deformation of specimen or loosening of grips during the test causes a reduced deformation of the spring and hence a falling off of the load. In using a spring-type testing machine to produce cycles of definite stress in the specimen, it is necessary to observe spring deformations at frequent intervals, especially during the early stages of the test, and to adjust the throw of the crank so as to keep the stress constant.

The axial-stress testing machines used by Wöhler in his classic tests[1] were of the spring type, and that fact limited

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[1] A very full account of Wöhler's tests including a description of his testing machines is found in *Engineering* (*London*), vol. 11, 1871.

the maximum speed to less than 100 cycles of stress per minute.[1]

2. *Inertia-type Machines.*—In this type of machine a mass of iron or other metal is given reciprocating motion in which the maximum and minimum accelerations are known. The alternating forces (+ and −) accompanying these accelerations are transmitted through the specimen.

Figure 20 shows in diagram an inertia-type machine used at the British National Physical Laboratory by Stanton and Bairstow. This machine has four reciprocating masses W_1, W_2, W_3, and W_4 attached to two pairs of opposed cranks C_1 and C_2, thus giving complete balance in both horizontal and vertical directions. The maximum acceleration of the reciprocating masses occurs at the ends of the strokes of the cross-heads, and the maximum force transmitted through the specimens S_1, S_2, S_3, S_4 is

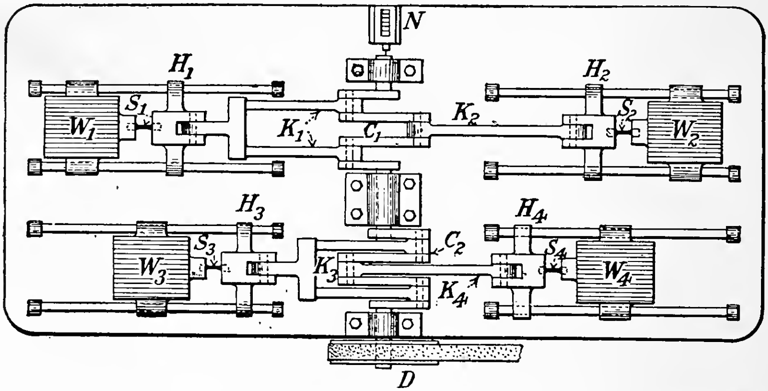

Fig. 20.—Diagram of axial-stress inertia-type testing machine. (*Stanton and Bairstow.*)

$$F = \pm \frac{W(2\pi n)^2 R}{g}\left(1 \pm \frac{R}{L}\right),$$

in which

F = the accelerating force at the end of a stroke in pounds,
W = the weight of the reciprocating mass in pounds,
n = the speed of rotation in revolutions per second,

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[1] For other descriptions of spring-type axial-stress machines, see *Univ. Illinois, Eng. Exp. Sta., Bull.* 142, p. 38.

R = the radius of the crank in feet,
g = the acceleration due to gravity = 32.2 ft. per second per second,
L = the length of the connecting rod in feet.

From the above equation it is evident that the machine sets up in the specimens cycles of partially reversed stress.

An advantage of the inertia type of repeated-stress testing machine is that it permits the use of high speeds. It will be noted, however, that the force applied to the specimen depends on the *square* of the speed of rotation. This necessitates a very close control of the speed of the line shaft or the motor which drives the machine. Ordinary sources of power rarely will give sufficiently constant speed for inertia-type machines, and rather elaborate speed-regulating devices are usually necessary.[1]

3. *Centrifugal-force-type Machines.*—This type of machine is really an inertia machine which, to set up cycles of stress, utilizes the centrifugal force of rotating unbalanced masses instead of the inertia of reciprocating masses. Figure 21 shows in diagram a machine of this type designed and used by J. H. Smith of Belfast, Ireland.[2] The specimen S is fastened at one end to the framework of the machine and at the other to the sliding cross-head C which slides freely in guides G. This cross-head carries a shaft on which are mounted disks D_1, D_2, on which are fastened eccentric weights W_1, W_2. The disks are driven through a universal joint U by a drive disk D_3, on which is mounted an eccentric weight W_3 which balances the combination W_1 W_2. Disk D_3 is driven by a shaft on which is a drive pulley or a connection to a motor. As the disks rotate, the centrifugal forces set up by the unbalanced weights W_1 and W_2 cause cycles of alternate tension and

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[1] See Stanton, T. E., "Alternating Stress Testing Machine at the National Physical Laboratory," *Engineering* (*London*), Feb. 17, 1905, for details of an inertia-type repeated-stress testing machine.

[2] See Smith, J. H., "Testing Machine for Reversals of Stress," *Engineering* (*London*), Mar. 10, 1905; and "Fatigue Testing Machine," *Engineering* (*London*), July 23, 1909.

compression in the specimen, the maximum tensile (or compressive) force being

$$F = \pm \frac{W(2\pi n)^2 R}{g},$$

in which

F = maximum tensile force and maximum compressive force in specimen in pounds,
W = combined weight of W_1 and W_2 in pounds,
n = number of revolutions per second,
R = radius to center of gravity of W_1 and W_2 in feet,
g = acceleration due to gravity = 32.2 ft. per second per second.

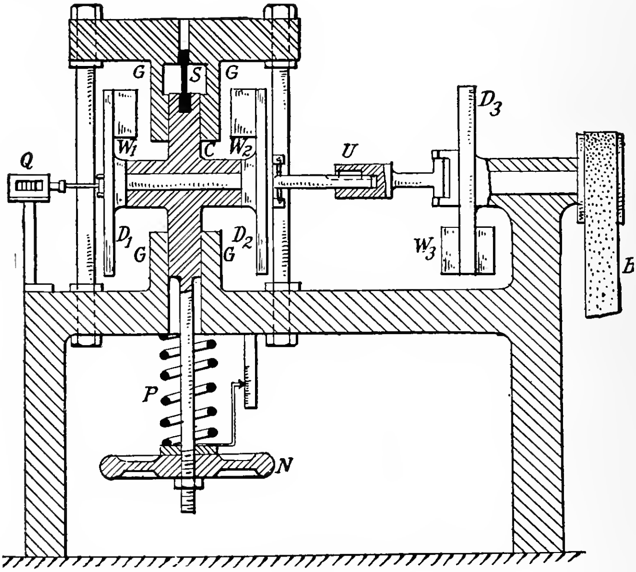

Fig. 21.—Diagram of axial-stress centrifugal-force-type testing machine. (*J. H. Smith.*)

If it is desired to set up cycles of stress not completely reversed, load is put on the specimen by tightening the nut N, thus putting a known steady load L on the specimen through the spring P. Then the cycle of load is from a value of $L + F$ to a value of $L - F$.

Centrifugal-force testing machines require very close speed regulation as do all inertia-type machines, the stress set up varying as the *square* of the speed of rotation.

In all inertia-type machines (including centrifugal-force machines) any deformation of specimen tends to increase the throw of the reciprocating masses or of the cross-head of the centrifugal force machine and hence tends to *increase* the range of stress developed in a cycle. This action is the reverse of that noted in the case of spring-type repeated-stress testing machines.

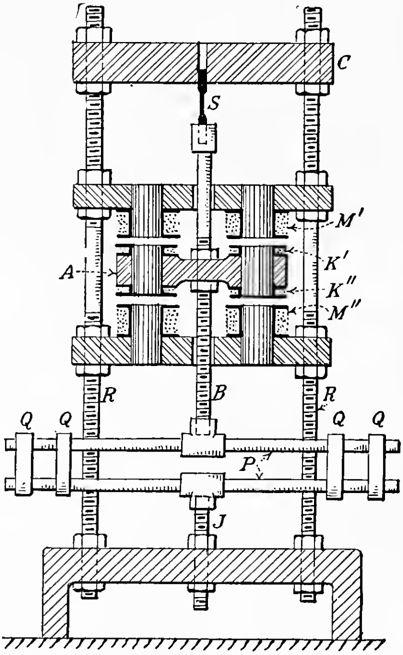

Fig. 22.—Diagram of axial-stress alternating-current magnet-type testing machine. (*Haigh.*)

4. *Alternating-current Magnet-type Machines.*—The general introduction of alternating-current electric circuits suggested to several investigators the use of alternating-current magnets as a means of setting up cycles of stress at a very rapid rate. Hopkinson[1] and Kapp[2] both devised such machines. A machine of this type designed by B. Parker Haigh of the Royal Naval Academy at Greenwich, England, has been developed commercially and is today, in spite of its very high cost, the most widely used fatigue-testing machine for repeated axial-stress tests. It is shown in diagram in Fig. 22. The specimen *S* is attached at one end to the framework of the machine and at the other to the armature *A*, which is placed between two magnets *M'* and *M''*. These magnets are energized by two-phase alternating current, one

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[1] *Proc. Roy. Soc.*, vol. 86*A*, November, 1911.

[2] *Zeit. Ver. deut. Ing.*, Aug. 26, 1911.

phase being connected to each magnet. Thus the specimen is alternately stretched and compressed by the action of the magnets. If the air gap between armature and pole pieces is the same above and below the armature, the current in both magnets is the same, and after setting up a specimen in the machine, the position of the armature is adjusted until this equality of current is established as shown by zero reading of a differentially wound ammeter connected to both phases.

A measure of the force exerted during each cycle of loading is obtained by the use of a voltmeter connected to the secondary coils K' K'' placed near the pole pieces of the magnets. The readings of the voltmeter are calibrated in terms of pounds pull or push by the use of a specimen with a mirror extensometer attached. The specimen in turn is standardized by means of a test in a static testing machine. It is assumed that the modulus of elasticity of the specimen is the same under cyclical loading as under static loading.

It is important that the machine be "tuned" so that there are introduced no unknown inertia forces. This tuning is accomplished by means of the spring P. With no specimen in place, the clamps Q are adjusted along the spring until the armature oscillates in unison with the magnetic pull. The magnetic pull does not vary greatly for slight variations in frequency of supply current. It is, however, important that the frequency of current be kept constant so that the vibrating parts of the machine are kept "in tune" with the magnet pulls, and unknown inertia stresses are avoided.

In this machine unequal deformation of the specimen in tension and in compression would move the armature nearer one pole piece than the other. It is usually necessary to watch the machine rather closely for the first hour of a test and, if necessary, adjust the position of armature and the amount of current supplied, so that the armature is kept midway between the pole pieces, and the forces developed during a cycle remain constant.

By adjusting the screw J, it is possible to superimpose a known steady load L (either tension or compression) upon the alternating load $\pm F$ set up by the magnetic pull, causing a range of load during a cycle from $L + F$ to $L - F$.

This machine is usually operated at a speed of 2,000 cycles of stress per minute, and a special generator is required to supply the low-frequency two-phase alternating current necessary. Properly adjusted and calibrated, this machine is capable of a high degree of precision. Its principal drawback is its high cost.

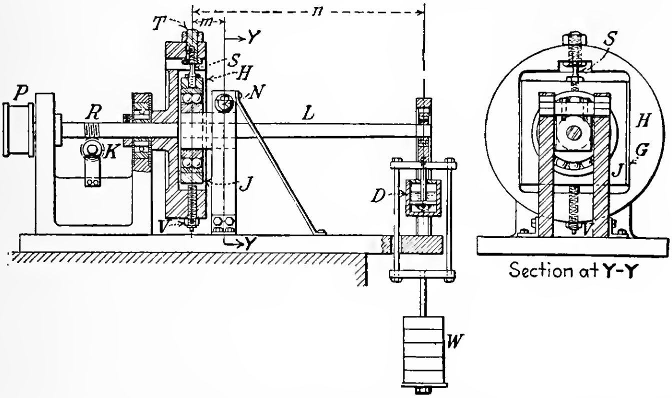

Fig. 23.—Diagram of axial-stress rotating-specimen-type testing machine. (*Jasper.*)

5. *Rotating-specimen-type Machines.*—Repeated-stress testing machines operated by the application and removal of dead weights have been used in a few special cases, but their speed of operation is very slow indeed. A machine in which the constant pull of dead weights causes cycles of alternating axial-stress in a rotating specimen is shown in Fig. 23.

This machine was designed by T. M. Jasper of the University of Illinois and used in the investigation of the fatigue of metals carried on at that institution. S is the specimen whose outer end is fastened to the block J, which can slide freely on guides G in the direction of the axis of the specimen. This block J is supported by the lever L, which rests on the knife-edge N. At the outer end of the lever is hung a weight W. In the position shown, the specimen is under compression, and the load on the specimen equals the weight $W \times n/m$. When the head H rotates 90 deg., the upward push of the lever acts directly against the guide G and produces no stress in the specimen. When the head rotates 180 deg. from the position shown, tensile stress is set up in the specimen. The machine gives cycles of completely reversed axial stress.

The machine is run at a speed of 1,000 r. p. m. A dash pot D minimizes surging of the weights. A revolution counter K driven by a worm R gives the number of cycles in a test. A screw V prevents excessive throw of the block J when a specimen breaks. A trigger device (not shown) is actuated by the drop of the outer end of the lever L when a specimen breaks, and this trigger device releases a spring which opens the switch on the motor driving the machine.

This machine is not very expensive and, carefully adjusted, is of a good degree of accuracy. It is necessary to renew the heavy main ball bearings occasionally, and this adds an appreciable item to the cost of upkeep of the machine. This machine comes under class *a* (p. 83). For each cycle there is applied a definite range of stress.

Repeated-stress Testing Machines for Cycles of Flexural Stress. 1. *Rotating-beam-type Machines.*—Probably 90 per cent of all repeated-stress tests made have been made on a type of testing machine in which a transverse load is applied to a rotating beam, either a cantilever beam or a beam supported at the ends In this type of machine one side of the rotating beam is under tensile stress and the opposite side is under compressive stress, and as the beam rotates, the stress on any longitudinal "fiber" changes from tension to compression. This type of machine is inexpensive, can be used at high speeds, and for stresses within the yield point of the metal tested produces stresses which can be accurately computed; furthermore, a definite range of stress is applied during each cycle.

Figure 24 shows the rotating cantilever machine used by Wöhler in his classic tests. S denotes the specimens, a pair to a machine. One end of each specimen is tapered as shown at T and the tapered end driven into a tapered hole in the axle C. The specimen is rotated by the drive pulley D, which is driven by a belt. At the outer end of each specimen is a gimbal G, and load is applied through a spring balance P. A counter (not shown) is attached either to a specimen or to the shaft driving the machine. From the records of Wöhler's tests it seems that this machine was run at a speed not exceeding 100 r. p. m.

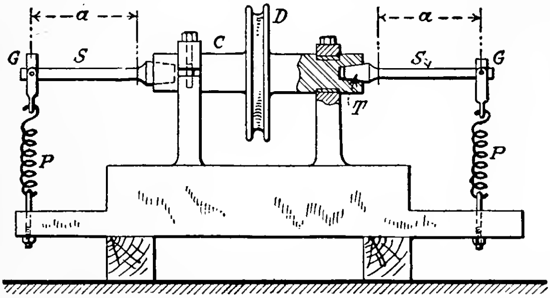

FIG. 24.—Diagram of rotating-cantilever-beam-type testing machine. (*Wöhler.*)

In a rotating-cantilever fatigue test the upper surface of the specimen is in tension, and the lower surface in compression. Cycles of reversed stress are set up in any "fiber" as the specimen rotates, and the magnitude of the maximum stress is

$$S = \frac{Mc}{I} \text{ and } M = Pa,$$

in which

M = bending moment at critical section of specimen in inch-pounds,
P = load hung on specimen (see Fig. 24),
a = distance from line of load to critical section of specimen in inches (a in Fig. 24),
I = moment of inertia of cross-section of specimen at critical section, for a circle $I = 0.049\ d^4$, in which d = diameter,

c = distance in inches from neutral axis to outer "fiber," for a circle c = one-half the diameter.

The above formula holds only within the proportional elastic limit of the metal. Up to the yield point of those metals which have a yield point, the above formula is of fairly satisfactory accuracy.

Later users of the cantilever machine have rather generally discarded the double specimen and have had one specimen to a machine. Figure 25 shows in diagram the cantilever machine used by McAdam at the U. S. Naval Engineering Experiment Station at Annapolis, Md. The special features of McAdam's machine are the shape of the specimen, which will be discussed in the section on specimens for repeated-stress tests, and the chuck for holding the specimen. This consists of a tapered split collet L to which is attached a hexagonal nut N''. Against a shoulder on this hexagonal nut N'' there bears a shoulder of the knurled nut N', and by means of the interaction of nuts N' and N'' the tapered collet may be forced into or removed from the tapered chuck H without the necessity of using a hammer to tighten or to loosen the specimen. At the outer end of the specimen is a gimbal G carrying the ball bearing B'', and from this gimbal are hung weights

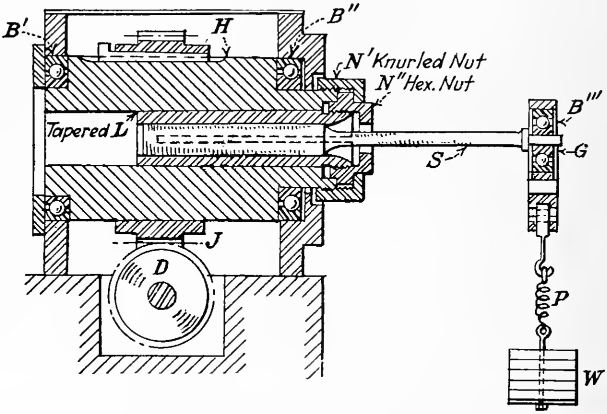

FIG. 25.—Diagram of rotating cantilever beam-type testing machine. (*McAdam.*)

W. The weights are attached to the gimbal through a spiral spring *P* which minimizes any tendency to surge.

The machine is driven by the spiral gear *J* and the drive gear *D*. A counter is provided to record the number of cycles of stress for each test. These machines are used in McAdam's laboratory in blocks of four machines driven by one motor.

In 1892 Sondericker[1] used a modification of the Wöhler rotating-beam machine in which the specimen was a rotating beam supported at the ends and loaded with two symmetrical loads. This type of machine has the advantage that between the symmetrical loads the bending moment is constant and the shear is zero; the action is pure flexure. Figure 26 shows a machine of this type following closely a design used by Farmer.[2] This machine has been widely used in recent fatigue tests of metals. The specimen *A* is driven through a leather flexible coupling *H* by the drive pulley *K*. The specimen is mounted in ball bearings *B*, *C*, *D*, and *E*. A tapered collet fastens the specimen in each bearing as is shown for bearing *D*. Gimbals attached

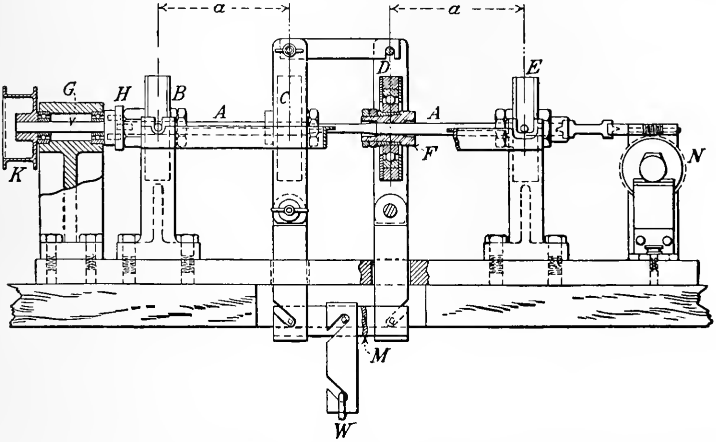

FIG. 26.—Diagram of rotating beam-type testing machine. (*Sondericker, Farmer.*)

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[1] "Some Repeated-stress Tests," *Tech. Quart.*, April, 1892.

[2] *Proc. Am. Soc. Testing Materials*, vol. 19, Pt. II, p. 709, 1919.

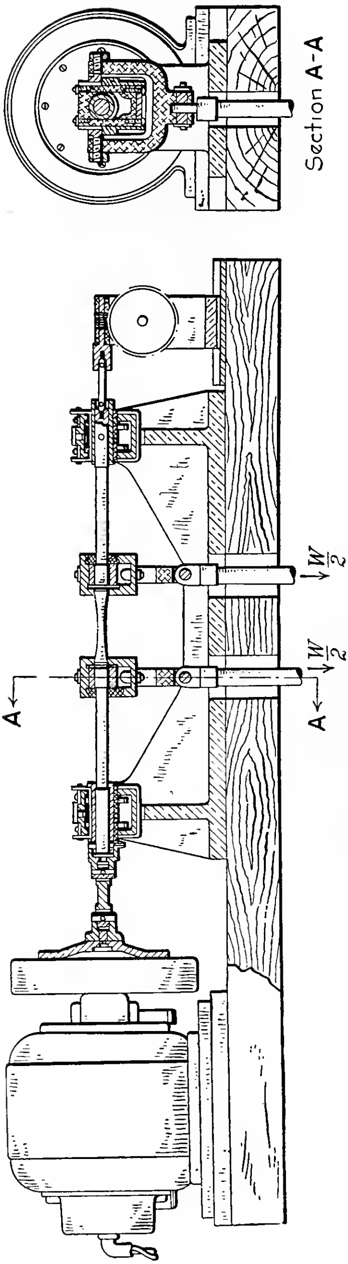

Fig. 27.—Diagram of rotating beam-type testing machine. (*R. R. Moore.*)

to the two central bearings *C* and *D* carry an equalizer bar *M*, and from the middle of this equalizer bar are suspended weights *W*, hanging from a short spiral spring not shown in Fig. 26. The specimen drives a counter *N*, and when the specimen breaks, the counter automatically stops. In addition there is provided a device (not shown) for throwing open the motor switch when a specimen breaks.

For tests of the stronger metals the type of machine shown in Fig. 26 serves very well indeed, but for tests of small specimens of weak metals, such as pure aluminum, straight bearings have been found to give smoother running than ball bearings. Figure 27 shows such a testing machine designed by R. R. Moore and used in the McCook Field (Dayton) laboratories of the U. S. Army Air Service. The machine is similar in its general action to the machine shown in Fig. 26, but in place of ball bearings straight ring-oiling bearings are used. It is possible to make the action of this machine so smooth

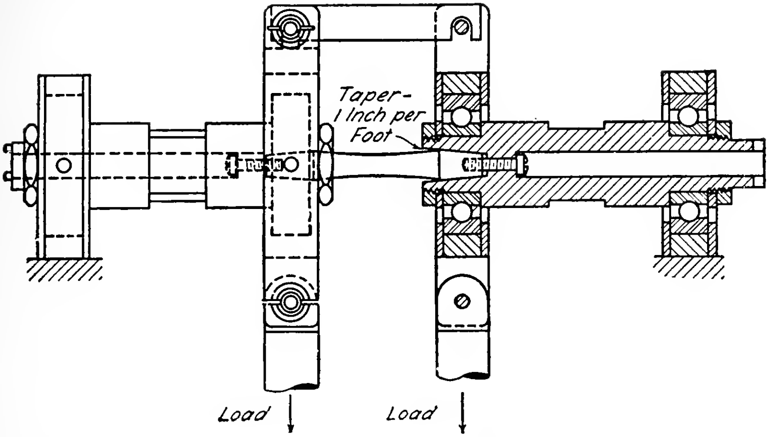

FIG. 28.—Holders for short specimen in rotating-beam-type testing machine. (*Ono.*)

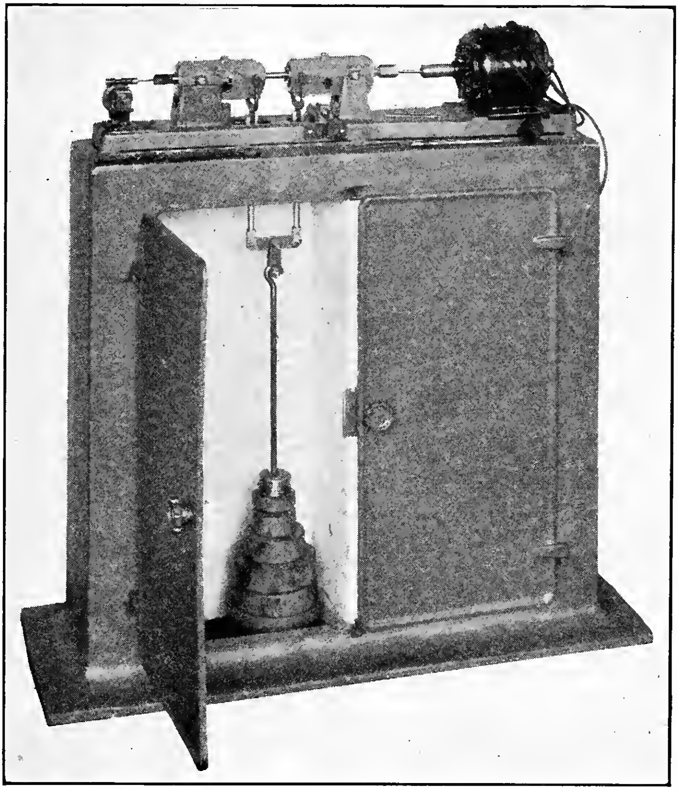

FIG. 29.—Rotating-beam-type testing machine. (*R. R. Moore.*)

that the vibration of the specimen is only a few ten-thousandths of an inch.

Figure 28 shows a device for using short specimens in a Sondericker-type machine. This special device was used by Prof. Ono of Kyushu Imperial University, Japan.[1] It has been applied to the type of machine shown in Fig. 26. Figure 29 is from a photograph of a very recent model of rotating-beam testing machine designed by R. R. Moore and combining the straight bearings shown in Fig. 27 with the chucks for using short specimens shown in Fig. 28.

In the Sondericker type of machine the computed unit stress at the critical section of the specimen is given by the flexure formula

$$S = \frac{Wa}{2I/c},$$

in which

S is the unit stress in the extreme fibers of the specimen, in pounds per square inch,

W is the total weight in pounds (hung at W in Fig. 26),

a is the distance in inches along the specimen from center of bearing E, Fig. 26, to center of bearing D (or from center of bearing B to center of bearing C),

I/c is the "section modulus" of the specimen in inches^3, for a specimen of circular cross-section $I/c = 0.0982 d^3$, when d is the diameter in inches of the specimen at the critical section.

The rotating-beam type of repeated-stress testing machine (including the rotating-cantilever type) is inexpensive, is practically independent of speed effect, and can readily be run up to speeds of 2,000 r. p. m.; also the value of bending moment set up can be computed accurately. It is the most generally useful type of repeated-stress testing machine.

2. *Rotating-spring-type Machines.*—In some repeated-stress tests it is desirable to have the specimen stationary so that it can be examined during the test. Figure 30

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[1] *Mem. Coll. Eng., Kyushu Imp. Univ.*, vol. 2, No. 2, 1921.

shows in diagram such a machine, designed by H. F. Moore, for tests under cycles of reversed flexure. One end of the specimen *S* is held rigid in the vise *V*, and the other end, which runs in the bearing *B*, is rotated in a small circle. Sidewise pressure, which can be adjusted by means of a screw, is brought on the bearing *B* by a calibrated indicator spring *I*. The compression of the spring, and hence the load and the bending moment, on the specimen, is measured by means of a strain gage spanning the gage holes *GG* shown near the ends of the spring. The rotating spring is carried in the cross-head *C*. Sidewise motion of the bearing *B* is prevented by placing the bearing in a slot, and excessive displacement of the bearing after the specimen breaks is prevented by the rod *R*. The cross-head *C* is driven by a shaft *H*, a pulley *P*, and a motor not shown. The number of revolutions of the shaft is measured by the counter *K* which is driven by a worm on the drive shaft.

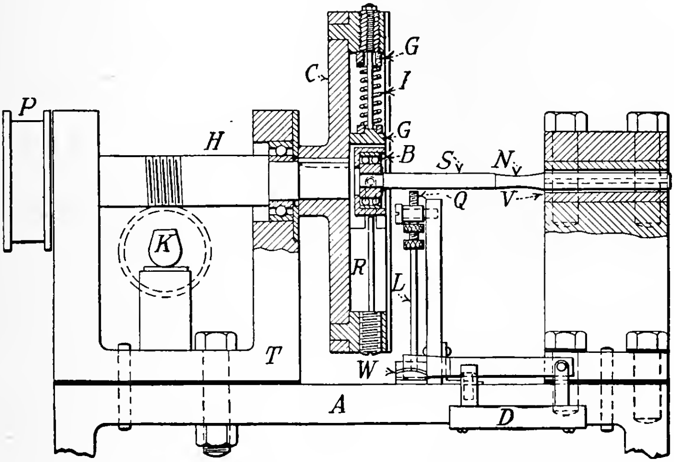

Fig. 30.—Diagram of reversed-flexure rotating spring-type testing machine. (*H. F. Moore.*)

When the specimen breaks, the broken end of the specimen hits a screw *Q* and kicks out the lever *L*. This releases the spring *W* which then opens the motor switch *D*, thus stopping the machine.

This machine sets up cycles of completely reversed flexural stress and can be run at a speed of 1,800 r. p. m. As in all spring-type machines, it is necessary to read the load-indicating device occasionally, especially during the earlier part of a test, and to adjust the load if found necessary.

3. *Spring-type Machines.*—Figure 31 shows a type of machine in which a specimen is subjected to cycles of flexural stress which are set up by the reciprocating action of a crank and connecting rod and whose magnitude is measured by the compression of calibrated springs. Power is furnished by a motor M (or from a line shaft). A crank C with adjustable throw is attached to a connecting rod R which bends the specimen S back and forth. The motion of the specimen is resisted by springs G acting through a bent lever A. The magnitude of bending moment applied to the specimen may be varied by changing the throw of the crank and is measured by the compression of the springs G. The magnitude of compression of springs is measured by the throw of the arm I, to the end of which is attached a pencil which records the throw on paper wrapped round the drum D, which is rotated by a worm-and-wheel drive from the main shaft of the machine. There is thus recorded on the paper a diagram whose width is a measure of magnitude of bending moment at the critical section of the specimen and whose length is a measure of the number of cycles of stress applied. The number of cycles is also indicated by a counter K. The stress in the specimen at the critical section is computed by the usual flexure formula (see p. 4). This type of machine may be used to produce either cycles of reversed flexural stress or, by varying the relative initial pressure on each of the springs G, to produce cycles of flexural stress with varying ratios of minimum stress to

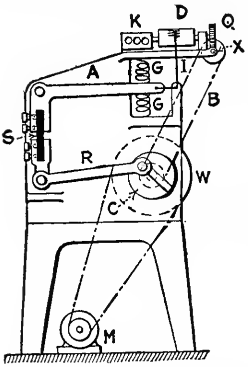

Fig. 31.—Diagram of repeated-flexure spring-type testing machine. (*Upton-Lewis.*)

maximum stress. The Upton-Lewis machine is the commonest example of this type of machine. It is made in two styles, one which runs at about 250 r. p. m., and the other which may be run at 1,000 r. p. m. if the springs used have a sufficiently short period of natural vibration.

Figure 32 shows a repeated-stress testing machine of the spring type designed for making flexure tests of thin, flat specimens. This machine is similar in principle to the machine described in the foregoing paragraph. The specimen *N* is fastened at one end to the calibrated flat spring *M*, and the other end of the specimen is vibrated back and forth by the connecting rod *K*, which is operated by the variable-throw crank *D*. If the throw of the crank is increased, the bending moment on the specimen is increased, and the deflection of *Q*, a mirror attached to the calibrated spring, is also increased, causing motion of a beam of light reflected from the lamp *L* to the screen *S*. There is provided an automatic trip which is operated by dropping of connecting rod *K* when the specimen breaks, and which opens the motor switch, thus stopping the machine. A counter is also provided which records the number of revolutions. This machine, developed by H. F. Moore of the University of Illinois, operates at 1,300 r. p. m. and has proved especially useful in testing

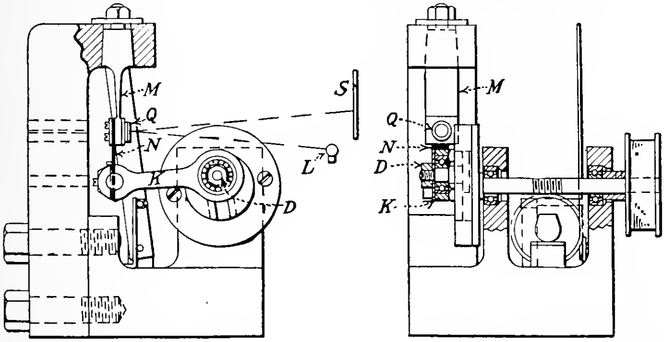

FIG. 32.—Repeated-flexure spring-type testing machine for thin flat specimens. (*H. F. Moore.*)

specimens from thin material and from locations very close to the surface of metal.

4. *Inertia-type and Magnetic-type Machines.*—The possibility of application of inertia stresses or of cyclical magnetic pulls to the construction of the repeated-stress testing machines for flexural stress is obvious. Machines of these types have been built[1] and used, but the convenience and simplicity of the rotating-beam type and the spring type have made these the common types of repeated-stress testing machine for flexure tests.

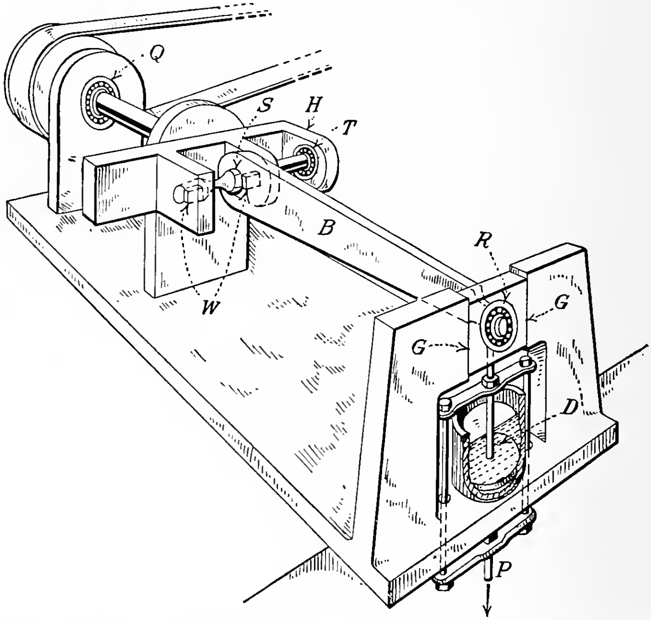

Fig. 33.—Diagram of reversed-torsion rotating-specimen-type testing machine. (*H. F. Moore.*)

Repeated-stress Testing Machines for Cycles of Torsion. 1. *Rotating-specimen-type Machines.*—The same general types of device for measuring the twisting moment (and resulting shearing stress) in specimens subjected to repeated torsion are used as in repeated-stress testing machines for cycles of flexure. Figure 33 shows a machine

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[1] Gough, "The Fatigue of Metals," pp. 32–34.

analogous in its action to the rotating-beam type of machine.

The specimen S is attached at one end to the rotating head H, and at the other end to a rotating flexible beam B. At the end of this beam a load P is applied by means of weights or a spring balance through the ball bearing R. When the axis of the specimen is horizontal and the specimen is on the left-hand side of the shaft (as shown in Fig. 33), the twisting moment on the specimen is counterclockwise; when the shaft of the machine has rotated 90 deg. from the position shown, there is no twisting moment on the specimen; when the shaft of the machine has rotated 180 deg., the twisting moment is clockwise. There is a complete reversal of torsional (shearing) stress during a rotation. Knowing the length of the flexible beam B, the pull P at the end of the beam, and the moment set up in the specimen by the weight of the beam, the torsional moment and the nominal shearing stress in the specimen can be computed. The beam B is made flexible, especially in one direction, to minimize vibration, and a dash pot D is also of service in this respect. The machine is operated at a speed of 1,000 r. p. m.

2. *Spring-type Repeated-torsion Machines.*—Figure 34 shows in diagram the Olsen-Foster machine for cycles of torsional stress. The specimen S is held at one end keyed in the vibrating arm A, and at the other it is keyed into the pivoted lever L. The arm A is vibrated by means of the adjustable pin which in turn is driven back and forth by sliding block B in the intermediate vibrating arm M. M is slotted and is driven by the crank pin Q which in turn is driven by the shaft and the drive pulley D (or a motor may be directly connected to the machine). By varying the position of the pin R up or down the arm A, the throw of the arm may be varied. The pivoted lever arm L bears against two calibrated springs P' and P'', and the compression of these springs measures the twisting moment transmitted by the specimen S. This compression is indicated and recorded by means of the arm H and the lever J

which carries a pencil at its outer end and records on the drum T a diagram whose width is a measure of the twisting moment on the specimen. The drum T is driven by the worm and wheel W which, in turn, is driven by a belt from the drive shaft of the machine. A counter K is also driven from the axle of the drum T. This machine is similar in its general action to the Upton-Lewis machine described on page 100, and the two machines are manufactured by the same firm. The Olsen-Foster machine usually is equipped with an attachment for making flexure tests. For torsion tests it can be run at speeds up to about 300 r. p. m.

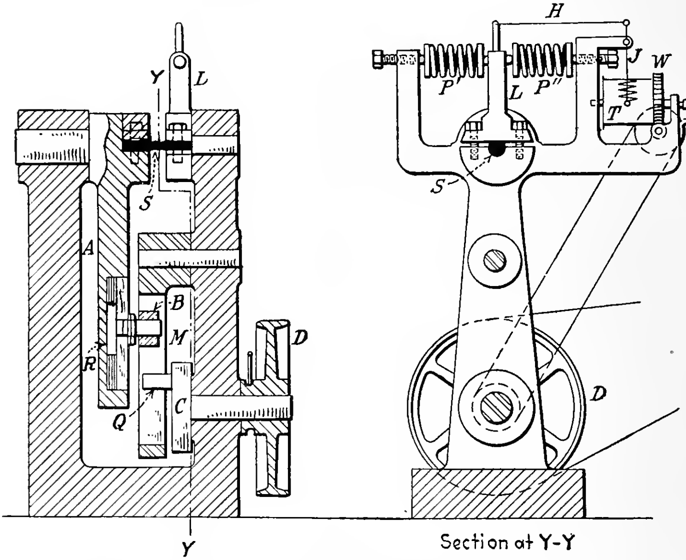

Fig. 34.—Diagram of repeated-torsion spring-type testing machine. (*Olsen-Foster.*)

3. *Inertia-type Machines.*—Fatigue-testing machines for cycles of torsional stress which use the inertia of a flywheel for producing the stress have been developed by Stromeyer[1]

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[1] Stromeyer, C. E., "The Determination of Fatigue Limits under Alternating Stress Conditions," *Proc. Roy. Soc.*, vol. 90, p. 411, 1914; McAdam, D. J., Jr., "A High-speed Alternating Torsion Testing Machine," *Proc. Am. Soc. Testing Materials*, vol. 20, Pt. II, p. 366, 1920.

and McAdam. Figure 35 shows in diagram the machine developed by McAdam. The specimen *S* is keyed at one end to a chuck in a shaft which is turned back and forth by the vibrating arm *A*, actuated by the connecting rod *K* and the variable-throw crank *C*. The drive pulley *D* is made with a heavy rim to give a flywheel effect. If the machine is equipped with direct motor drive, a flywheel is placed on the drive shaft. The right-hand end of the specimen *S* is keyed in a chuck which is a part of a shaft attached to the flywheel *J*, which is shown made up of several separate disks. To the shaft of flywheel *J* is attached a mirror which reflects a beam of light from the lamp *L* to the scale *Q*. When the machine is running, the oscillation of flywheel *J* causes a band of light to appear on the scale *Q*. The width of this band of light is a measure of the "throw" of the flywheel *J*; and knowing the speed of the machine, the throw of the crank *C*, and the dimensions of the rods *K* and *A*, the maximum angular acceleration of the flywheel *J* can be computed, and from this the maximum and minimum values of twisting moment on the specimen during a cycle of stress. This machine has been used at a speed of 2,100 r. p. m. As in all inertia-type machines it is very

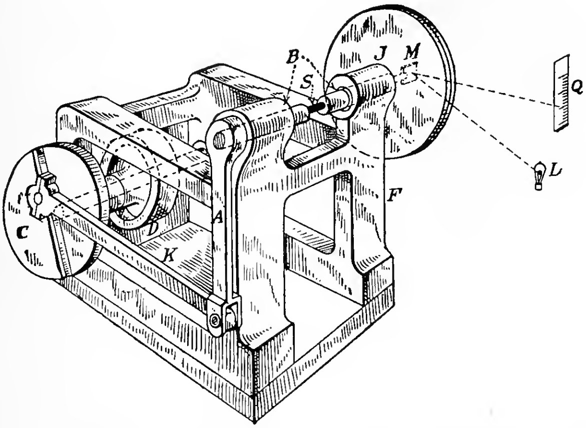

FIG. 35.—Diagram of reversed-torsion inertia-type testing machine. (*McAdam.*)

necessary that the speed control be very close, since the stress developed varies as the square of the speed.

Repeated-stress machines for torsion tests using alternating-current magnets (analogous to the Haigh machine for tension-compression tests) and machines using the action of centrifugal force have not been developed, but the application of either of these agencies could easily be made.

Repeated-stress Testing Machines for Tests under Combined Stresses.—Repeated-stress testing machines have been developed for tests under cycles of reversed flexure combined with steady tension, and for tests under cycles of combined flexure and torsion. Figure 36 shows a machine developed at the University of Illinois for tests under cycles of reversed flexure combined with steady tension. One end of the specimen S is held rigid in the vise V, and the other end, which runs in the bearing B, is rotated in a small circle. Sidewise pressure, which can be adjusted by means of a screw, is brought on the bearing B by a calibrated indicator spring I. The compression of the spring and hence the bending moment on the critical section of the specimen are measured by means of a strain gage

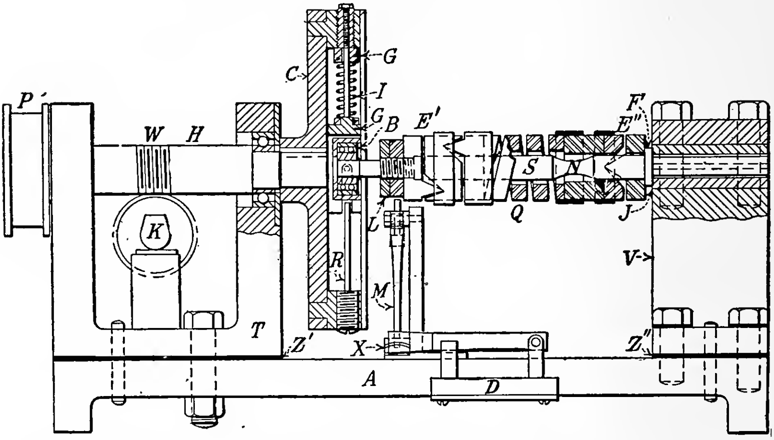

Fig. 36.—Diagram of testing machine for combined reversed flexure and steady tension. (*H. F. Moore.*)

spanning the holes *GG*. The rotating spring is carried by the cross-head *C*. The bearing *B* is placed in a radial slot in the cross-head. The method of driving the machine is evident from the figure. The steady-tension load on the specimen is set up by the action of the spiral spring *Q*, which is fitted at each end with a pair of crossed knife-edges *E′* and *E″*, so as to cause uniform tension in the specimen and at the same time not to interfere with the flexure of the specimen. There is a shoulder on the specimen at *F* which supports one end of the spring apparatus; the spring is compressed by tightening the nut *L*. As the specimen is bent, the fibers are subjected to a maximum stress, which is the sum of the direct tensile stress and the tensile stress due to bending, and then to a minimum stress which is the difference of the direct tensile stress and the compressive stress due to bending.

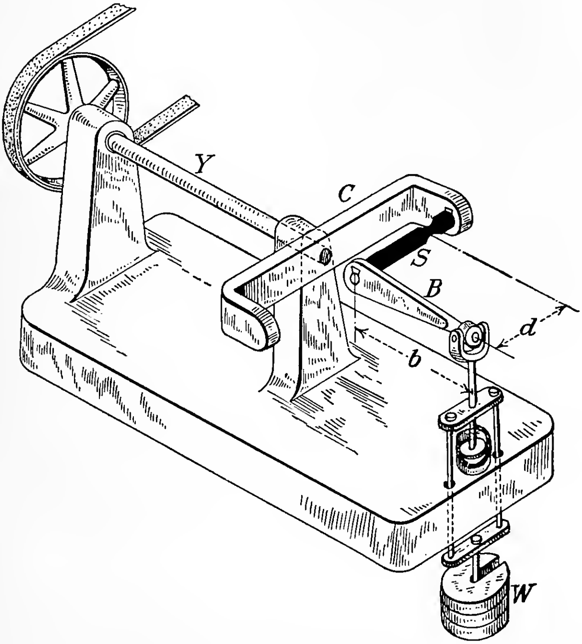

Fig. 37.—Diagram of testing machine for combined reversed flexure and reversed torsion. (*Stanton and Batson.*)

Figure 37 shows in diagram a machine used by Stanton and Batson[1] for tests under cycles of combined flexure and torsion. The machine is a combination of the Wöhler rotating-beam principle and the principle of the rotating-specimen torsion machine shown in Fig. 33. As a matter of fact, the machine shown in Fig. 33 was developed from the consideration of the work of Stanton and Batson.

In Fig. 37 S is the specimen which is located with its axis along a radial diameter of the rotating jaw C. One end of the specimen is rigidly fastened to the rotating jaw C. The arm B is attached to the free end of the specimen S, and its axis coincides with the axis of the drive shaft Y. The weight W is attached to the free end of the arm B, and as the jaw C rotates, the specimen is subjected to cycles of reversed bending moment varying from $+Wd$ to $-Wd$, and to cycles of torsional moment varying from $+Wb$ to $-Wb$. Evidently the deformation of the specimen S and arm B will cause the axis of B to get somewhat out of line with the axis of Y as the machine is operated, but by proper proportioning of parts and by the use of a dash pot attached to the rod carrying the weights W, the vibration resulting from this misalignment can be kept within workable limits. Stanton and Batson used a speed of 2,000 r. p. m. for this machine.

Ono[2] has used a rotating-beam machine for tests under cycles of reversed flexure combined with steady torsion, the steady torsion being set up by using the specimen as a drive shaft for transmitting power to an electrical absorption dynamometer.

Constant-deformation Fatigue-testing Machines.—Figure 38 shows a type of machine which has been used by Arnold and others for quick shop tests of resistance to repeated violent stress. No mechanism for measuring load or moment on the specimen is provided, but for each cycle the specimen is given a definite deformation—in

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[1] *Brit. Assoc. Repts.*, p. 288, 1916.

[2] "Fatigue of Steel under Combined Bending and Torsion," *Mem. Coll. Eng., Kyushu Imp., Univ.*, vol. 2, No. 2, 1921.

the machine shown in Fig. 38 a definite range of deflection. Specimens of various materials are subjected to this arbitrary deflection, and the value of the material is judged by the number of cycles withstood before fracture. Usually the deflection is such that the specimen is stressed well beyond the yield point of the metal.

It has so far proved impossible to correlate the results of tests in which length of endurance under definite cycles of deformation is regarded as the index of value of a metal with the results of tests in which the stress corresponding to indefinitely long endurance is regarded as the index of value. It is sometimes found that the use of short-time constant-deflection tests will arrange metals in a quite different order of merit from that found by the use of long-time tests at various stresses to determine the limiting stress for indefinitely long "life." The short-time tests seem to be more a measure of ductility or of toughness of metal than of its ability to resist millions of cycles of a given working stress.

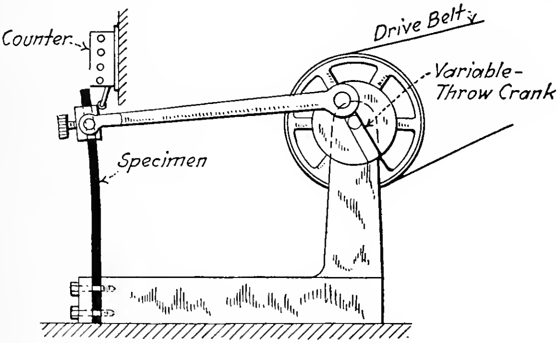

Fig. 38.—Diagram of constant-deformation repeated bending testing machine.

Repeated-impact Machines.—Testing machines have been used in which specimens have been subjected to flexural action produced by means of repeated blows of a swinging pendulum or of a falling weight. Usually there is a definite amount of energy in each blow, but it is, in general, impossible to translate this energy into terms of stress in the specimen, so that direct correlation of test results on stress-measuring fatigue-testing machines with test results on repeated-impact machines is, in general, impossible. Repeated-impact machines using some arbitrary amount of energy per blow and using the length of "life" under repeated blows have been used for acceptance tests of metal for automobile parts, because such tests were supposed to simulate service conditions. For such tests there must be used a specimen of a certain arbitrary size and shape, usually a notched specimen.

Perhaps the most widely used repeated-impact machine is one designed by Stanton.[1] In this machine the specimen is a simple beam in flexure. It is notched and is struck at the middle by the head of a small trip hammer which is driven by a motor. Between blows, the specimen is rotated 180 deg. around its axis, thus being subjected to reversed flexure. Various other investigators have developed machines utilizing this general idea.

Another form of repeated-impact flexure machine was designed by Gustafsson.[2] In this machine the specimen is a vertical cantilever held in a vise and struck by a pair of swinging pendulum hammers, first on one side and then on the other.

In repeated-impact tests, if the energy per blow is relatively high, the test results will arrange metals in an order of merit similar to that given by single-blow impact tests (Izod or Charpy tests). If the energy per blow is small, the order of merit will be similar to that given by repeated-stress tests, say on a rotating-beam machine. This has been shown by tests by Stanton, McAdam, and Lessells.[3] Repeated-impact tests which approach the conditions of ordinary repeated-stress tests are very time consuming indeed, since it is usually not feasible to use a speed of more

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[1] "Repeated Impact Testing Machine," *Engineering* (*London*), July 13, p. 33, 1906.

[2] Roos, J. O., "Some Static and Dynamic Endurance Tests," *Proc. Intern. Assoc. Testing Materials, Paper V2b*, 1912.

[3] Stanton and Bairstow, *Proc. Brit. Inst. Mech. Eng.*, November, p. 889, 1908; McAdam, *Proc. Am. Soc. Testing Materials*, vol. 23, Pt. II, p. 56, 1923; Lessells, *Proc. Am. Soc. Testing Materials*, vol. 24, Pt. II, p. 603, 1924.

than 100 blows per minute, whereas the repeated-stress test may be run at speeds up to 2,000 cycles per minute.

The interpretation of the results of repeated-impact tests is a matter of no small difficulty. No correlation with stress values for the material is possible. Moreover, the use of the number of blows to cause failure as an index of merit of the material means that results will show a great deal of "scatter," since a slight variation in the energy per blow makes a very considerable difference in the number of blows a specimen can withstand.

Specimens for Repeated-stress Tests.—The values taken for maximum stress in repeated-stress specimens are those given by the ordinary formulas of mechanics of materials. For the specimens ordinarily used these formulas are

For tension-compression specimens, $S = \frac{P}{A}$;

For flexure specimens, $S = \frac{Mc}{I}$;

For torsion specimens (circular), $S_s = \frac{Tr}{J}$.

In the above formulas

S = the maximum unit stress in pounds per square inch,

S_s = the maximum shearing unit stress in pounds per square inch,

P = the axial load, in pounds, on a tension-compression specimen,

M = the bending moment, in inch-pounds, at the critical section of the specimen at which the maximum unit stress is S,

T = the twisting moment, in inch-pounds, at the critical section of the specimen at which the maximum shearing unit stress is S_s.

I = the moment of inertia, in inches^4, of the area of the critical cross-section of the specimen (flexure),

J = the polar moment of inertial, in inches^4, of the area of the critical cross-section of the specimen (torsion),

$J = 0.098\ d^4$ for a circle of diameter d,

c = the distance, in inches, from neutral axis to outer fiber of flexure specimen,

r = radius of circular specimen in inches.

The foregoing formulas hold only within the proportional elastic limit of the material (for ductile metals they hold practically up to the yield point) and give values of unit stresses for a certain definite section of any given specimen. If specimens have sharp fillets, notches, screw threads, or other abrupt changes of form, maximum stresses may exist which are higher than those given by the nominal formula for the specimen. In the case of a test specimen for a "static" test, in which the load is increased gradually from zero to a maximum, the localized high stress at a notch or a sharp fillet has no marked effect on the ultimate tensile strength of a ductile metal on account of a general readjustment of stress distribution after the stress at such a point passes the yield point of the metal. In static-test specimens of *brittle* materials and in fatigue-test specimens of *all* materials, localized high stresses are of importance. In a fatigue specimen such localized high stress may be the source of a spreading crack which will cause final failure under repeated stress. For fatigue-test specimens, then, it is especially important that the specimen and the gripping devices be so designed that there are set up no appreciable localized stresses whose magnitude cannot be calculated.

Tension-compression Fatigue Specimens.—It is assumed by some engineers that fatigue tests using cycles of alternate axial tension and axial compression give more reliable results than do tests under cycles of flexural stress, because in tension-compression tests the stress can be computed even beyond the yield point of the metal. Such engineers usually neglect the fact that it is not at all easy to design specimens for repeated axial stress so that the load shall be a purely axial one without any flexure. In static tension-test specimens truly axial loading can be secured by the use of a long specimen and by using special spherical-seated grips. Figure 39 shows such a form of gripping device. This form of gripping device might be used in a repeated-tension test in which the load varied from minimum to maximum tension, but evidently could not be used in tests in which compressive stress was applied to the specimen. It is evident that if an additional bearing were used so that the device shown in Fig. 39 could be used to apply compression, the freedom of adjustment of the gripping device would be hampered. Moreover, a specimen to be subjected to cycles of alternate axial tension and compression cannot be long; else there is danger of buckling.

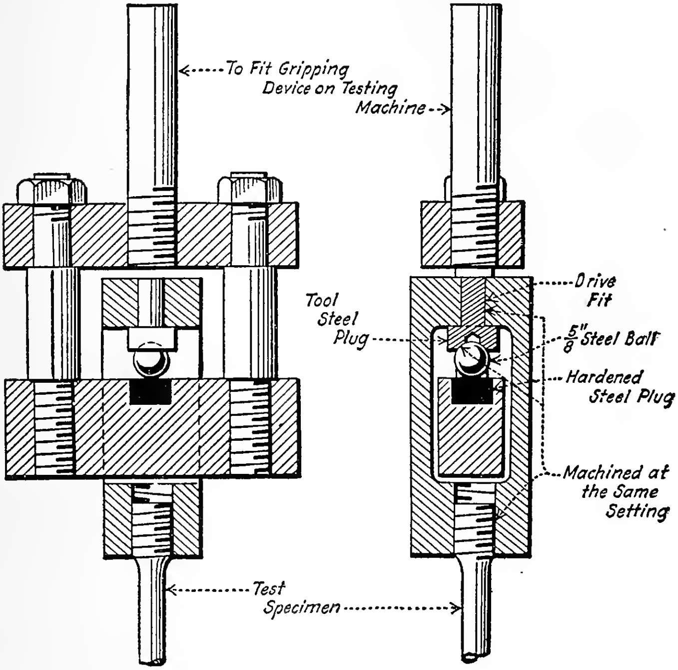

Fig. 39.—Shackles for insuring axial load on tension test specimen. (*Robertson.*)

Figure 40 shows the specimen used by Haigh in his axial-stress machine (see Fig. 22). To secure the necessary combination of rigidity of grip and true axial load, he uses a threaded-ended specimen and depends on careful machining and adjustment of the parts of the machine. The change in cross-section from the enlarged threaded ends to the middle is made gradually by means of a long taper. Great care must be taken to avoid a "stress-raising" notch at the junction of tapered portion and straight portion of the specimen.

Figure 41 shows a form of tension-compression specimen used with marked success by Irwin.[1] The ends of the specimen are shouldered, and the reduction in cross-section from shoulder to midlength is made by turning down the specimen with a tool swung on the arc of a circle. Using this specimen with carefully adjusted grips in a Haigh machine, Irwin found substantial agreement between the fatigue strength of several kinds of metal as determined by a tension-compression test and the fatigue strengths of the same kinds of metal as determined by reversed-flexure tests.

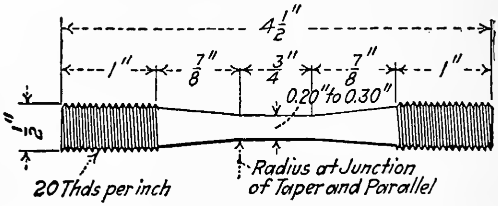

Fig. 40.—Specimen for repeated axial stress. (*Haigh.*)

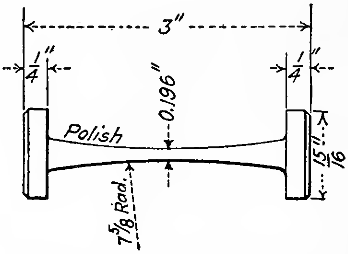

Fig. 41.—Specimen for repeated axial stress. (*Irwin.*)

Specimens for Repeated-stress Tests in Flexure. *Rotating-beam Specimens.*—If in a rotating-beam specimen for two symmetrical loads (see Fig. 26) no reduction of cross-section is made to localize the fracture, the localized stress under the bearings carrying the load introduces an uncertainty as to the actual value of maximum

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[1] *Proc. Am. Soc. Testing Materials,* vol. 25, Pt. II, p. 53, 1925.

stress. Hence, it is customary to reduce a portion of a specimen between the middle bearings so that it will be certain that the maximum stress occurs there. Figure 42(*a*) shows a specimen in which there is a reduced central portion consisting of a straight portion connected to the larger end portions by fillets. The flexure formula would neglect the localized stress at the junction of straight portion and fillet, and recent work by Timoshenko and Dietz[1] has shown that for the dimensions given in Fig. 42(*a*) the stress at the junction of straight portion and fillet would be theoretically about 1.35 times the stress at the middle of the straight portion.

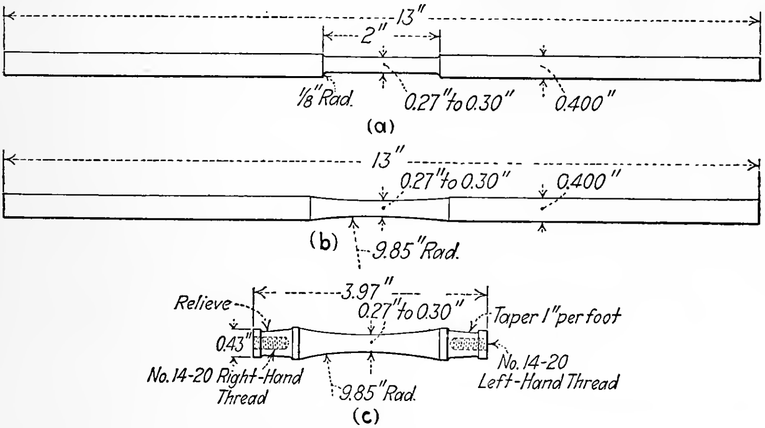

FIG. 42.—Rotating-beam specimens.

Figure 42(*b*) shows a form of rotating-beam specimen in which the reduced portion is formed by a lathe tool swung on the arc of a circle of radius much larger than the diameter of the specimen. The effect of stress concentration for this specimen is negligible. This form of specimen has the disadvantage that the maximum stress occurs only at one section and not along a length of the specimen. With the dimensions shown, however, the variation of maximum stress over the middle 0.1 in. is only 1 per cent.

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[1] "Stress Concentrations Produced by Holes and Fillets," *Trans. Am. Soc. Mech. Eng.*, vol. 47, p. 199, 1925.

It is sometimes inconvenient to use specimens as long as those shown in Fig. 42(*a*) and (*b*); Fig. 42(*c*) shows a short rotating-beam specimen designed to be used with holders such as those shown in Fig. 28.

Figure 43(*a*) shows a rotating-cantilever specimen as used by some experimenters. This form of specimen has the disadvantage of stress concentration at the fillet coming at the point of maximum stress, and also the disadvantage of having only one cross-section under maximum bending moment. Figure 43(*b*) shows a similar specimen except that it is drilled hollow so that the highly stressed outer fibers have very little backing of inner understressed fibers. In such a specimen the finish of the surface of the hole is of great importance. Figure 43(*c*) shows a rotating-cantilever specimen used by McAdam.[1] This specimen is tapered for a considerable portion of its length so that at the large end it is certain that the stress at the fillet is not the maximum stress in the specimen, and so that for about 1½ in. of length the maximum stress is very nearly constant.

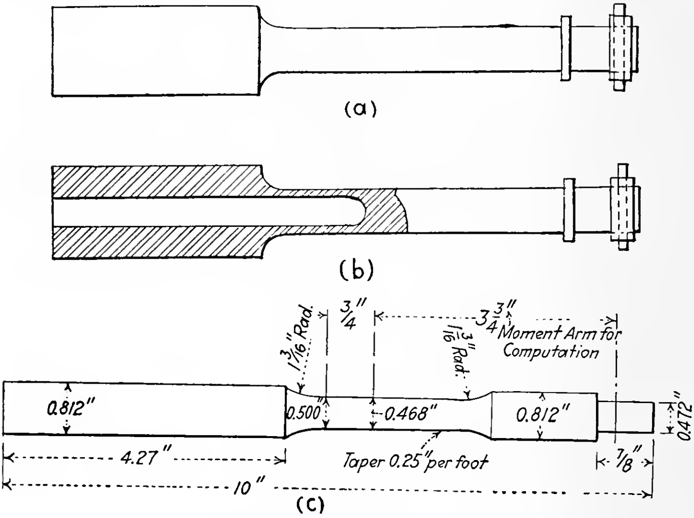

FIG. 43.—Rotating-cantilever-beam specimens.

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[1] *Proc. Am. Soc. Testing Materials*, vol. 23, Pt. II, p. 68, 1923.

Figure 44 shows a form of flexure specimen used at the University of Illinois for tests under a combination of reversed flexure and steady tension (see Fig. 36). This same form of specimen (with screw thread omitted) has been used for fatigue tests at elevated temperatures, in which case it is an advantage to have the maximum stress located within a short range of length of specimen so that it may be certain that temperature is measured at the point of maximum stress.

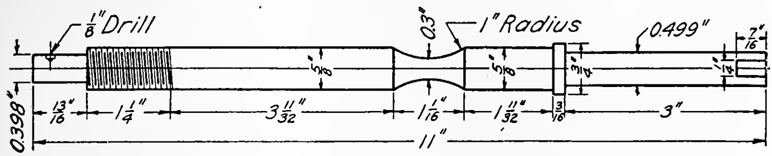

FIG. 44.—Specimen for combined reversed flexure and steady tension.

It may be noted here that specimens of the general form of Fig. 42(*b*) have been used by Gillett and Mack[1] in the Upton-Lewis machine (see Fig. 31).

Figure 45 shows a specimen used at the University of Illinois for tests of thin sheet metal under reversed flexure (see Fig. 32). Values for fatigue strength given by this specimen are usually lower than values given by rotating-beam specimens of the same metal. This is probably due to stress concentration at the fillets. For comparative

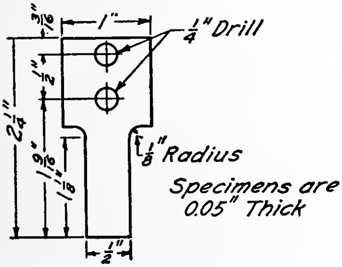

FIG. 45.—Specimen for fatigue tests of thin sheet metal.

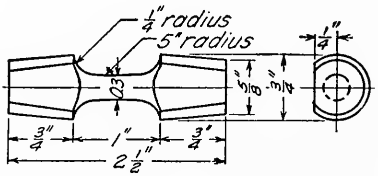

FIG. 46.—Specimen for fatigue test in torsion.

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[1] "Molybdenum, Cerium, and Related Alloy Steels," *Am. Chem. Soc, Monograph*, p. 259

results, however, the specimen shown in Fig. 45 is satisfactory.

Specimens for Repeated-stress Tests in Torsion.—The general design of repeated-stress specimens for torsion tests is similar in general character to the problem of repeated-stress specimens for flexure tests. Figure 46 shows a form of specimen quite commonly used for fatigue tests in torsion. The specimen is fastened into a chuck by a key which bears on the flattened portions of the tapered ends. This use of a flattened portion rather than the use of a keyway sunk in the specimen tends to lessen stress concentration at the grips.

Surface Finish for Repeated-stress Specimens.—A rough surface finish at the critical section of a fatigue specimen may reduce the fatigue strength by as much as 15 or 20 per cent; it is highly essential that a fatigue specimen be highly polished near its critical section. A good shop polish using No. 00 emery cloth is ordinarily sufficient.[1] For tension-compression and flexure specimens, circumferential scratches do more damage than longitudinal scratches, and where feasible it is slightly preferable to polish the specimens by rubbing with emery cloth in an axial direction, although this is usually rather inconvenient. For torsion specimens circumferential scratches do less harm than longitudinal, and polishing should be done by rubbing in a circumferential direction.

It is important to be sure that the polishing of specimens removes all tool marks or deep scratches near the critical section.

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[1] Experiments at the University of Illinois indicated that rouge polishing added fatigue strength only 1 per cent above that shown for specimens polished with No. 00 emery cloth. See *Univ. Illinois Eng. Exp. Sta., Bull.* 124, p. 108, 1921.

CHAPTER VI

CHARACTERISTIC RESULTS FOR FATIGUE TESTS

The S-N Diagram.—For determining the fatigue strength of metals from the results of repeated-stress tests—whether tension tests, flexure tests, torsion tests, tests under cycles of reversed stress, or tests under cycles of unidirectional stress of varying intensity—it is convenient to use diagrams in which values of computed unit stress are plotted as ordinates and values of number of cycles of stress for fracture are plotted as abscissae. Such diagrams are called stress-cycle diagrams by some experimenters and S-N diagrams by other experimenters (S for unit stress, N for number of cycles). The term S-N diagram will be used in this book.

Three methods of plotting S-N diagrams have been used: (1) plotting the values of both S and N to ordinary Cartesian coordinates, (2) plotting values of S to Cartesian coordinates and values of N to logarithmic coordinates (semilogarithmic plotting), and (3) plotting values of both S and N to logarithmic coordinates (logarithmic plotting). Figure 47 shows S-N diagrams for a number of steels plotted to both Cartesian and semilogarithmic coordinates. For all wrought ferrous metals tested, and for most non-ferrous metals the S-N diagrams become horizontal, as nearly as can be determined, for values of N ranging from 1,000,000 to 50,000,000. This seems to indicate a *fatigue limit* or an *endurance limit,* a unit stress below which the metal will withstand an indefinitely large number of cycles of stress without failure.

In investigations carried on in the United States, it has been customary to use either logarithmic or semilogarithmic plotting. The reason for this is twofold: (1) The use of a logarithmic scale for values of number of cycles makes it possible to plot on the same diagram both small and large values of N with the same percentage of accuracy; and (2) in a Cartesian diagram there is the danger that the general tendency towards curvature of the S-N diagram will lead the investigator to assume that an endurance limit has been reached at a comparatively low value of N when such is not the case. Figure 48, in which the data of tests on unannealed hot-rolled monel metal are plotted to Cartesian coordinates (upper), and to logarithmic coordinates (lower), illustrates this last-named point. The Cartesian diagram might lead the experimenter to report an endurance limit of 33,000 lb. per square inch, or if experiments had been carried only to values of N of 50,000,000, an endurance limit of 39,000 lb. per square inch might have been reported. The semilogarithmic and the logarithmic diagrams indicate that a well-marked endurance limit has not yet been determined.

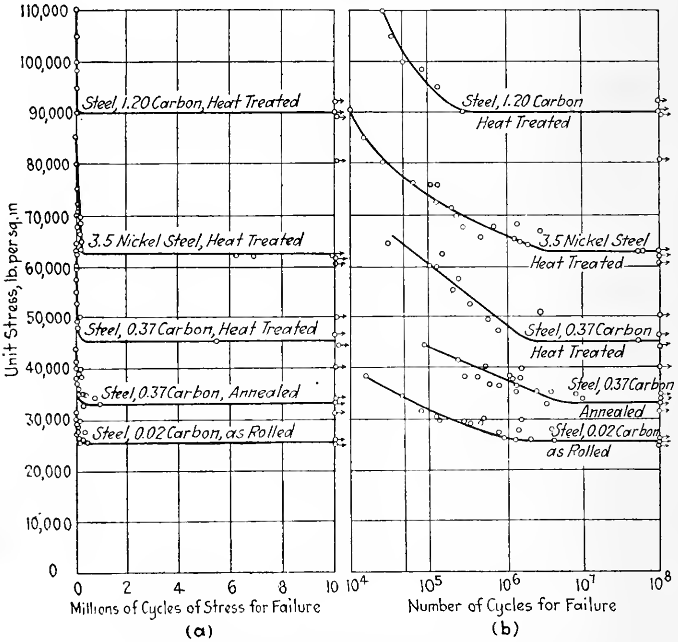

FIG. 47.—S-N diagrams—Cartesian and semilogarithmic.

In general, there does not seem to be any marked difference between the semilogarithmic and the logarithmic method of plotting as regards the determination of values for endurance limit. The criterion for reporting an endurance limit for a metal is that the *S-N* diagram shall become

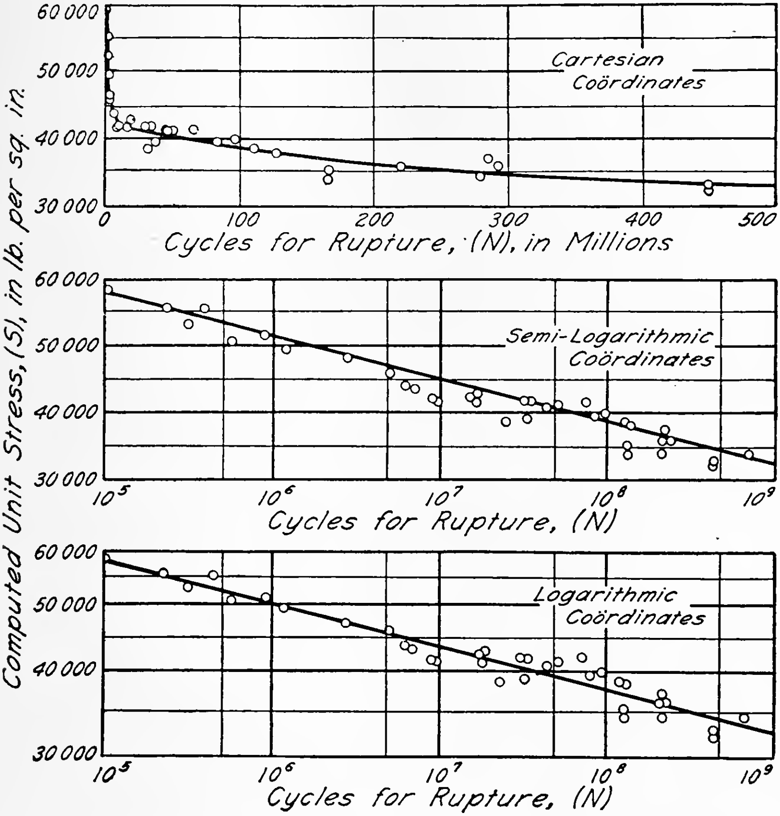

FIG. 48.—*S-N* diagram for special lot of monel metal hot-rolled without annealing—Cartesian, semilogarithmic, and logarithmic coordinates.

horizontal, or shall approach a horizontal line as an asymptote. Logarithmic *S-N* diagrams seem to show a "knee" (where the diagram approaches a horizontal line) more frequently than do semilogarithmic *S-N* diagrams. The choice between semi-logarithmic coordinates and logarithmic coordinates does not seem to be a matter of very deep significance. The authors, however, do recommend that either semilogarithmic or logarithmic plotting be used for S-N diagrams.

Typical S-N Diagrams for Various Metals.—Figures 49 to 52 give typical S-N diagrams for a number of metals.

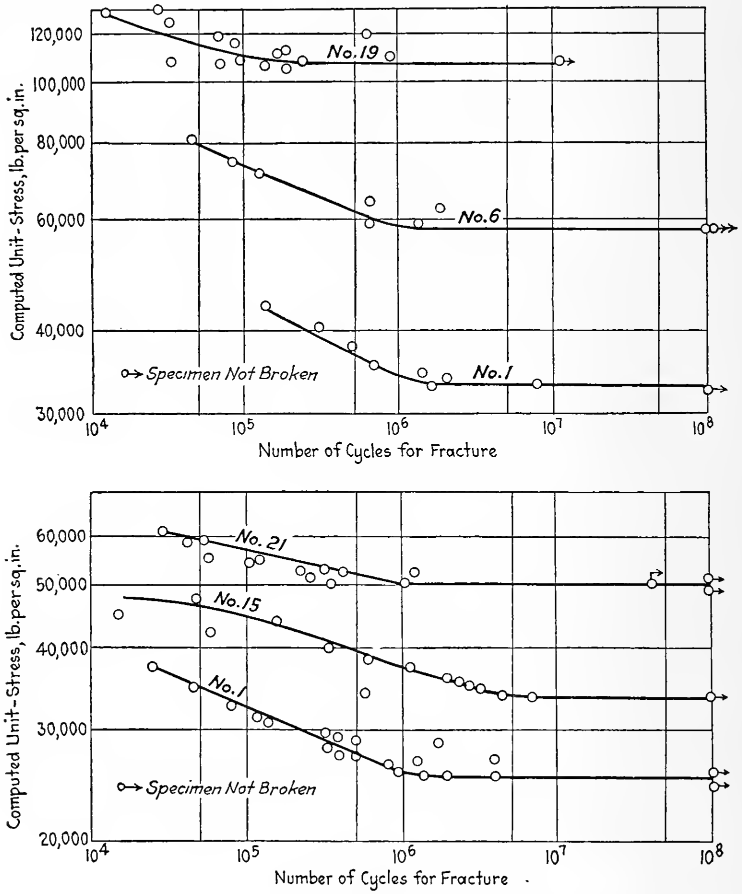

FIG. 49.—S-N diagrams for plain carbon steels. Upper, quenched; lower, not quenched.

Numbers on diagrams refer to numbers of steels in Tables 2B and 3B.

In a general way three kinds of S-N diagrams are shown: (1) diagrams such as those for the wrought ferrous metals, Figs. 49 to 50, and for certain non-ferrous metals (*e.g.*, No. 1, Fig. 52, light non-ferrous metals) with a well-marked horizontal portion; (2) diagrams, such as that shown for No. 3, Fig. 52 (lower part), and No. 9, Fig. 51, in which there is shown a distinct tendency for the diagram to become horizontal, and in which the diagram still has an appreciable downward slope at the greatest value of N observed;

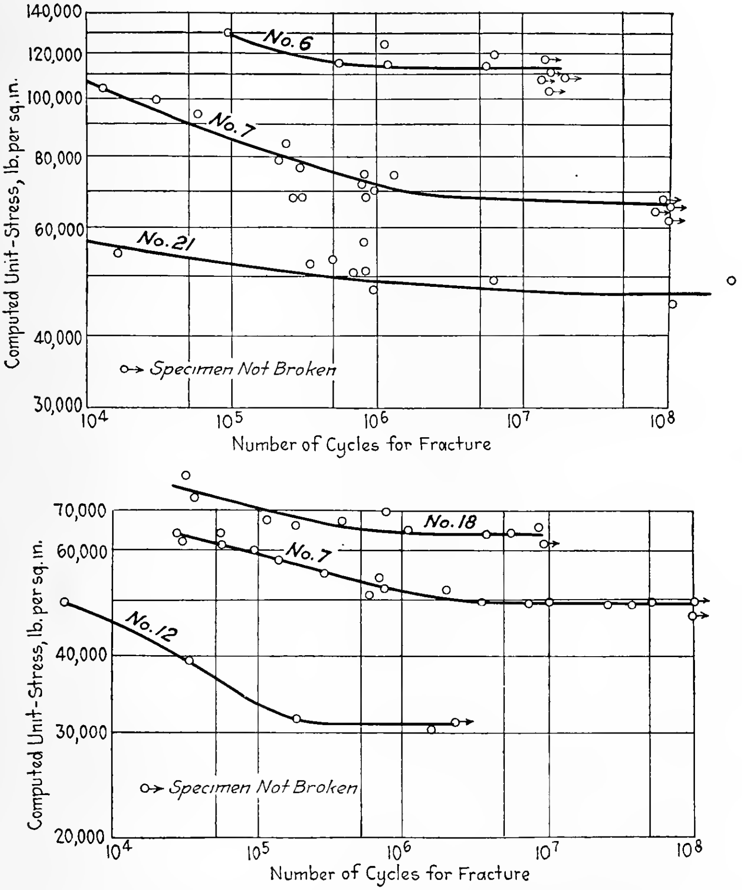

FIG. 50.—S-N diagrams for alloy steels. Upper, quenched; lower, not quenched. Numbers on diagrams refer to numbers of steels in Tables 4B and 5B.

and (3) diagrams, such as that shown for No. 2, Fig. 52 (lower), in which a straight-line relation (for logarithmic plotting) seems to hold between S and N even when tests are carried to several hundred millions of cycles of stress. It may be noted that S-N diagrams of this third type have not been found for any of the ferrous metals, and are unusual for non-ferrous metals.[1]

Evidence for the Existence of an Endurance Limit.—The endurance limit is evidently a very significant physical property for any metal to be used in structural or machine parts which in service are to be subjected to cycles of repeated stress. It seems fitting to examine the evidence for the existence of this limit. The results of long-time tests furnish three items of evidence for the existence of an endurance limit for wrought ferrous metals and for most non-ferrous metals.

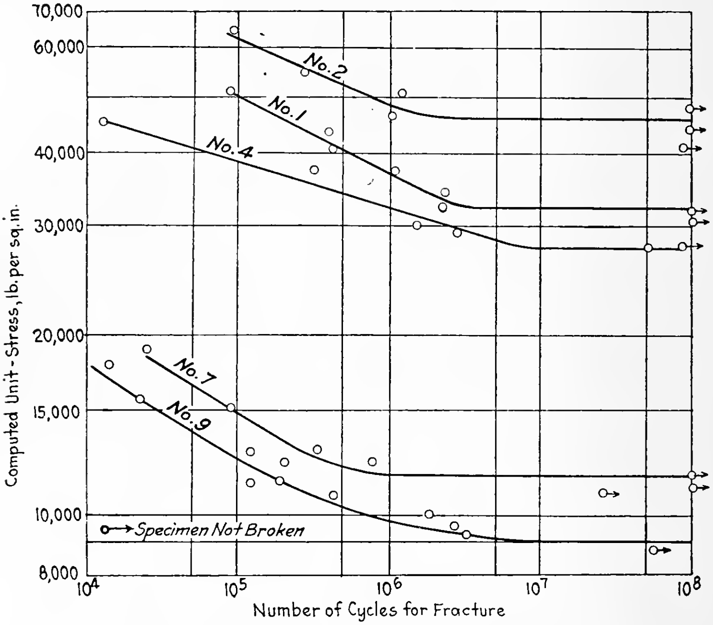

FIG. 51.—*S-N* diagrams for cast steels and cast irons. Numbers on diagrams refer to numbers of metals in Table 6*B*.

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[1] McAdam does not consider this third type a real type of *S-N* diagram. He holds that when such diagrams are obtained, it is because of corrosion-fatigue or some other influence not due to the nature of the metal. Probably if data were available for extending these "third-type" diagrams to a still greater number of cycles of stress, these "straight-line" diagrams would be found to bend to approach a horizontal asymptote. In any event, such diagrams are sometimes, though rarely, met with in making fatigue tests, especially if the tests are not carried to a very great number of cycles of stress.

1. For high values of N, the S-N diagrams, even when plotted to logarithmic coordinates, become horizontal, at least as nearly horizontal as can be determined by ordinary plotting. For all wrought ferrous metals tested, the horizontal part of the diagram is developed for values of N less than 10,000,000.

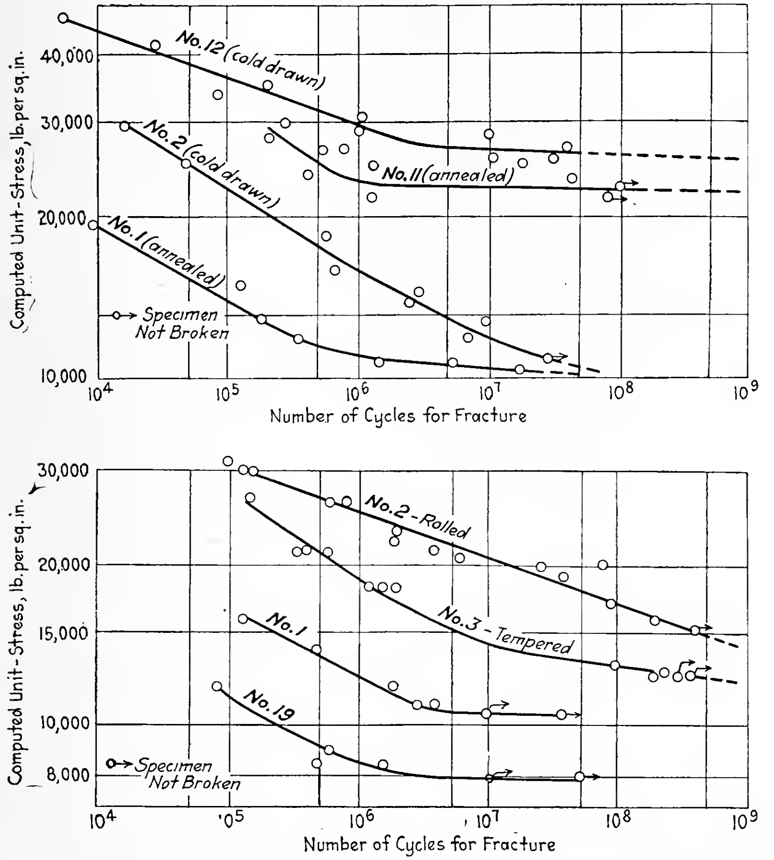

FIG. 52.—S-N diagrams for non-ferrous metals. Upper, heavy metals; lower, light metals.

Numbers on diagrams refer to numbers of metals in Tables 7B and 8B.

2. Specimens tested to millions of cycles of stress at or near the endurance limit, when retested under cycles of higher stress, have uniformly shown some gain in fatigue strength. Below the endurance limit the application of

repeated stress seems actually to improve the metal, rather than to injure it.

3. For wrought ferrous metals (and for some non-ferrous metals) at stresses near the endurance limit there can be noted a distinct rise in temperature. As is pointed out elsewhere, this is probably an indication of slip rather than of incipient fatigue failure, but for wrought ferrous metals slip usually precedes the formation of a fatigue crack. In a stress-temperature diagram there is near the endurance limit a fairly well-marked "knee"; below this "knee" the rise of temperature is very slight indeed, and the absence of continuing slip below the endurance limit as determined by long-time tests seems an indirect piece of evidence in favor of the existence of an endurance limit.

McAdam has given careful study to the form of the *S-N* diagram, giving particular attention to that part of the diagram corresponding to high computed stress in the specimen. He finds that under high computed stresses, especially for specimens not artificially cooled, there tends to be a reversal of curvature in the diagram, as indicated in the diagrams for steel No. 15, Fig. 49, and steel No. 12, Fig. 50. Of course, under such high computed stresses it is frequently the case that inelastic conditions prevail and the computed-stress value is purely nominal. Inelastic action, however, would tend to cover up such a reversal of curvature, and its persistence in diagrams is good evidence of the existence of such a tendency. Most of the diagrams in this book have not been carried to high enough stress values to show this tendency plainly.

It cannot, of course, be asserted dogmatically that for any metal there has been determined a limiting unit stress below which it is certain that the metal can withstand an infinite number of cycles of stress, but the authors believe that the data of long-time tests do show that for ferrous metals and for most (probably for all) non-ferrous metals the assumption of the existence of an endurance limit seems reasonably safe, and that such a limit in all probability exists.

Number of Cycles of Stress Necessary to Develop Endurance Limit.—In Figs. 49, 50, 51, and 52 are shown typical *S-N* diagrams for various ferrous and non-ferrous metals. They are plotted to logarithmic coordinates. From an examination of these diagrams and of other *S-N* diagrams, the following lengths of rotating-beam test have been found necessary to determine directly and accurately the endurance limit of a metal:

1. For wrought ferrous metals, from 500,000 cycles for very hard steel to 5,000,000 cycles for soft structural steel.
2. For cast steel and cast iron, not less than 10,000,000 cycles.
3. For non-ferrous metals the range of cycles necessary is very large, varying from less than 1,000,000 cycles for certain magnesium alloys to several hundred million cycles for some unusual lots of duralumin, and copper-nickel alloys. Usually 50,000,000 cycles are sufficient.

For certain lots of monel metal and duralumin, 500,000,000 cycles of stress failed to develop a well-marked endurance limit. Such results, however, are unusual and were obtained on unannealed, hot-rolled metal.

The test data plotted in Figs. 49 to 52 inclusive are from tests under cycles of reversed flexure, and from the data available it seems that under cycles of direct axial stress (tension-compression) the endurance limit is, in general, developed with a smaller number of cycles of stress.

In all metals tested it is found possible to make a close estimate of the endurance limit from tests run to not more than 10,000,000 cycles of stress. Even if the *S-N* diagram has not become horizontal, the curvature is usually sufficient to enable a close estimate to be made of the location of the horizontal line which the *S-N* diagram approaches,[1] or if the *S-N* diagram seems to be a straight line (a rare

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[1] For a systematic method of estimating the location of this horizontal line by extrapolation, see McAdam, "Stress-cycle Relationships and Corrosion Fatigue of Metals," *Proc. Am. Soc. Testing Materials*, Pt. II, p. 224, 1926.

TABLE 2A.—CHEMICAL COMPOSITION AND HEAT TREATMENT OF PLAIN CARBON STEELS, QUENCHED

Number	Material	Carbon, per cent	Chromium, per cent	Nickel, per cent	Silicon, per cent	Manganese, per cent	Phosphorus, per cent	Sulphur, per cent	Heat treatment
1	0.02 per cent Armco iron	0.02	....	....	0.02	0.03	0.005	0.042	Box anneal; heat to 1500°F., hold 15 min., quench in water
2	0.023 per cent C ingot iron	0.023	....	....	0.005	0.037	0.002	0.031	Heat to 1650°F., hold 1 hr., cool in water
3	0.24 per cent C	0.24	....	....	0.007	0.45	0.009	0.051	Heat to 1600°F., hold 1 hr., cool in oil, reheat to 900°F., hold 30 min., cool in furnace
4	0.25 per cent C	0.25	....	....	0.25	1.18	0.031	0.04	Not known
5	0.31 per cent C	0.31	....	....	0.16	0.47	0.013	0.03	Heat to 1525°F., quench in oil, anneal at 1200°F.
6	0.37 per cent C sorbitic	0.37	....	....	0.16	0.58	0.32	0.035	Heat to 1550°F., hold 15 min., quench in water, reheat to 1050°F., cool in air
7	0.37 per cent C sorbitic	0.37	....	....	0.16	0.58	0.032	0.035	Heat to 1550°F., hold 15 min., quench in water, reheat to 1050°F., cool in air
8	0.38 per cent C	0.38	....	....	0.04	0.57	0.033	0.048	Heat to 1550°F., hold 30 min., cool in oil, reheat to 800°F., hold 30 min., cool in furnace
9	0.38 per cent C	0.38	....	....	0.16	0.59	0.020	0.017	Not known
10	0.49 per cent C sorbitic	0.49	....	....	0.12	0.46	0.017	0.028	Normalize at 1700°F., heat to 1425°F., quench in water, reheat to 1200°F., cool in furnace
11	0.49 per cent C	0.49	....	....	0.12	0.46	0.017	0.029	Normalize at 1700°F., heat to 1450°F., hold 15 min., quench in oil
12	0.49 per cent C	0.49	....	....	0.12	0.46	0.017	0.029	Normalize at 1700°F., heat to 1450°F., hold 15 min., quench in oil, reheat to 1400°F., hold 15 min., cool in air
13	0.49 per cent C	0.49	....	....	0.18	0.63	0.011	0.036	Heat to 1600°F., quench in oil, anneal at 1350°F.

TABLE 2A.—(*Continued*)

14	0.52 per cent C sorbitic........	0.52	....	....	0.24	0.56	0.037	0.029	Normalize at 1550°F., heat to 1450°F., hold 15 min., quench in water, reheat to 1200°F., cool in air
15	0.93 per cent C sorbitic........	0.93	....	....	0.03	0.38	0.017	0.045	Normalize at 1600°F., heat to 1450°F., hold 15 min., quench in oil, reheat to 1200°F., hold 30 min., cool in air
16	0.93 per cent C troostitic........	0.93	....	....	0.03	0.38	0.017	0.045	Normalize at 1600°F., heat to 1450°F., hold 15 min., quench in oil, reheat to 850°F., hold 30 min., cool in air
17	1.02 per cent C...............	1.02	....	....	0.145	0.24	0.012	0.029	Heat to 1450°F., hold 30 min., cool in furnace, reheat to 1450°F., hold 30 min., quench in oil
18	1.20 per cent C sorbitic........	1.20	....	....	0.19	0.25	0.021	0.021	Anneal at 1460°F., heat to 1470°F., quench in oil, reheat to 860°F., hold 30 min., cool in air
19	1.20 per cent C...............	1.20	....	....	0.19	0.25	0.021	0.021	Heat to 1460°F., hold 20 min., cool in furnace, reheat to 1625°F., hold 15 min., quench in oil
20	1.20 per cent C...............	1.20	....	....	0.19	0.25	0.021	0.021	Heat to 1460°F., hold 20 min., cool in furnace, reheat to 1625°F., hold 15 min., quench in oil, reheat to 1900°F., hold 30 min., cool in air
21	0.42 per cent C...............	0.42	....	....	.....	.....	.....	.....	Normalize at 1580°F., quench in salt solution from 1560°F., draw at 210°F. for 1 hr.
22	0.42 per cent C...............	0.42	....	....	.....	.....	.....	.....	Normalize at 1580°F., quench in salt solution from 1440°F., draw at 590°F., for 30 min.
23	1.0 per cent C....	1.00	....	....	.....	.....	.....	.....	Normalize at 1520°F., quench in lime salt solution from 1470°F., draw at 570°F.

TABLE 2B.—MECHANICAL PROPERTIES UNDER STATIC LOADING AND FATIGUE LOADING OF PLAIN CARBON STEELS, QUENCHED

Number	Material	Proportional elastic limit, pounds per square inch	Yield point, pounds per square inch	Ultimate tensile strength, pounds per square inch	Elongation in 2 in., per cent	Reduction of area, per cent	Brinell number	Endurance limit, rotating beam, pounds per square inch	Investigator
1	0.02 per cent C Armco iron..........	35,000	36,300	50,000	36.3	76.0	109	33,000	Moore and Jasper
2	0.023 per cent C ingot iron...........	28,700	29,700	46,700	38.0	71.0	...	25,000	McAdam
3	0.24 per cent C.....................	47,500	48,500	67,500	35.6	68.2	...	29,500	McAdam
4	0.25 per cent C....................	43,900	43,900	75,900	36.2	66.7	...	35,500	McAdam
5	0.31 per cent C.....................	37,500	40,000	71,300	34.5	67.0	...	30,000	McAdam
6	0.37 per cent C sorbitic............	80,600	87,300	102,600	23.3	65.1	209	57,000	Moore and Kommers
7	0.37 per cent C sorbitic..............	61,500	63,100	94,200	25.0	63.0	...	45,000	Moore and Kommers
8	0.38 per cent C.....................	60,000	60,000	91,500	29.0	58.0	...	33,500	McAdam
9	0.38 per cent C.....................	37,000	38,000	77,300	28.0	53.1	...	30,500	McAdam
10	0.49 per cent sorbitic................	67,700	69,700	96,900	23.5	57.8	197	48,000	Moore and Kommers
11	0.49 per cent C.....................	72,000	80,900	126,500	12.5	52.2	228	65,000	Moore and Jasper
12	0.49 per cent C.....................	49,000	52,000	85,600	23.5	55.3	125	37,000	Moore and Jasper
13	0.49 per cent C.....................	53,800	56,400	101,700	25.5	52.5	...	41,400	McAdam
14	0.52 per cent C sorbitic..............	80,300	84,300	111,400	21.9	56.6	227	55,000	Moore and Kommers
15	0.93 per cent C sorbitic..............	60,300	67,600	115,000	23.0	39.6	227	56,000	Moore and Kommers
16	0.93 per cent C troostitic............	97,200	....	188,300	9.9	29.3	380	98,000	Moore and Kommers
17	1.02 per cent C.....................	109,000	....	200,400	9.3	22.5	415	105,000	Moore and Jasper
18	1.20 per cent C sorbitic..............	120,400	130,100	179,900	9.0	15.2	369	92,000	Moore and Kommers
19	1.20 per cent C.....................	137,500	....	220,000	0.8	2.4	444	105,000	Moore and Jasper
20	1.20 per cent C.....................	71,000	71,500	118,000	14.3	17.5	224	53,500	Moore and Jasper
21	0.42 per cent C.....................	36,000	....	230,500	1.8	5.1	578	81,000	Lessells
22	0.42 per cent C.....................	35,000	....	215,000	6.8	37.2	477	98,000	Lessells
23	1.0 per cent C......................	120,000	....	241,000	6.1	11.2	495	112,000	Lessells

case), that line can be extended to cover the probable "life" required.

TABLE 3*A*.—CHEMICAL COMPOSITION OF PLAIN STEELS, NOT QUENCHED

Number	Material	Carbon, per cent	Chromium, per cent	Nickel, per cent	Silicon, per cent	Manganese, per cent	Phosphorus, per cent	Sulphur, per cent
1	0.02 per cent C, as rolled (hot) Armco iron	0.02	.....	.....	0.02	0.03	0.005	0.042
2	0.023 per cent C, as rolled (hot) ingot iron	0.023	.....	.....	0.005	0.037	0.002	0.031
3	0.13 per cent C, as rolled hot...........	0.132	.....	.....	0.028	0.300	0.028	0.017
4	0.13 per cent C, annealed at 1350°F.....	0.13	.....	.....	0.17	0.56	0.008	0.047
5	0.14 per cent C, as rolled (hot).........	0.14						
6	0.18 per cent C, as rolled (hot).........	0.18	.....	.....	0.06	0.37	0.013	0.039
7	0.21 per cent C, as rolled (hot).........	0.21	0.017	0.206	0.08	0.82	0.060	0.080
8	0.27 per cent C, as rolled (hot).........	0.27	.....	.....	.....	1.06		
9	0.30 per cent C, annealed at 1200°F.....	0.30	.....	.....	0.17	0.52	0.010	0.051
10	0.32 per cent C, as rolled (hot).........	0.32	.....	.....	.....	0.31		
11	0.32 per cent C, as rolled (hot).........	0.32						
12	0.37 per cent C, normalized at 1495°F...	0.37	.....	.....	0.16	0.58	0.032	0.035
13	0.42 per cent C, annealed at 1560°F.....	0.42	.....	.....	0.19	0.60	0.010	0.038
14	0.48 per cent C, annealed at 1350°F.....	0.48	.....	.....	0.19	0.60	0.010	0.038
15	0.49 per cent C, normalized at 1700°F...	0.49	.....	.....	0.12	0.46	0.017	0.029
16	0.52 per cent C, normalized at 1550°F...	0.52	.....	.....	0.24	0.56	0.037	0.029
17	0.56 per cent C, annealed at 1470°F.....	0.56	.....	.....	0.08	0.55	0.023	0.035
18	0.77 per cent C, annealed at 1350°F.....	0.77	.....	.....	0.18	0.55	0.037	0.047
19	0.80 per cent C, annealed at 1470°F.....	0.80	.....	.....	0.12	0.51	0.029	0.036
20	0.93 per cent C, normalized at 1600°F., annealed at 1450°F..................	0.93	.....	.....	0.03	0.38	0.017	0.045
21	1.20 per cent C, normalized at 1460°F...	1.20	.....	.....	0.19	0.25	0.021	0.021

Values of Endurance Limit under Cycles of Reversed Flexure.—The commonest fatigue failure in machine or structural parts is a failure under repeated-flexure action, and the most convenient fatigue test to make is a fatigue test under reversed flexure. Accordingly there are given in Tables 2*B*, 3*B*, 4*B*, 5*B*, 6*B*, 7*B*, and 8*B* values of fatigue limit in reversed flexure. The values of fatigue limit under cycles of axial stress, of fatigue limit under cycles of shearing stress (torsion), and of fatigue limit under cycles of stress not completely reversed are discussed in subsequent paragraphs or in a subsequent chapter. In connection with Tables 2*B* to 8*B* inclusive, reference should be made to Tables 2*A* to 8*A* inclusive. In Tables 2*B* to 8*B* inclusive, in cases where the endurance limit was not clearly defined, a limit was determined by extending the *S-N* diagrams to a value of *N* of 1,000,000,000 cycles.

TABLE 3*B*.—MECHANICAL PROPERTIES UNDER STATIC LOADING AND FATIGUE LOADING OF PLAIN CARBON STEELS, NOT QUENCHED

Number	Material	Proportional elastic limit, pounds per square inch	Yield point, pounds per square inch	Ultimate tensile strength pounds per square inch	Elongation in 2 in., per cent	Reduction of area, per cent	Brinell number	Endurance limit, rotating beam, pounds per square inch	Investigator
1	0.02 per cent C, as rolled Armco iron (hot)	16,100	19,000	42,400	48.3	76.2	69	26,000	Moore and Kommers
2	0.023 per cent C, as rolled ingot iron (hot)	20,100	25,000	44,200	48.5	74.5	...	23,600	McAdam
3	0.13 per cent C, as rolled (hot)	......	53,600	59,100	22.8*	69.8	...	29,600	Haigh
4	0.13 per cent C, annealed at 1350°F.	26,200	26,200	54,500	43.5	70.5	...	26,000	McAdam
5	0.14 per cent C, as rolled (hot)	33,500	35,800	62,700	38.5	69.3	...	30,900	Lea
6	0.18 per cent C, as rolled (hot)	38,600	40,300	61,500	41.0	66.7	...	28,000	Moore and Kommers
7	0.21 per cent C, as rolled (hot)	39,900	40,500	70,700	35.3	62.9	...	33,400	McAdam
8	0.27 per cent C, as rolled (hot)	54,700	61,800	84,700	....	63.5	...	42,600	Rogers
9	0.30 per cent C, annealed at 1200°F.	35,000	36,900	69,900	34.5	50.1	...	30,000	McAdam
10	0.32 per cent C, as rolled (hot)	37,400	39,000	65,700	....	60.1	...	31,300	Rogers
11	0.32 per cent C, as rolled (hot)	36,700	38,100	77,200	35.5	65.5	...	31,400	Lea
12	0.37 per cent C, normalized at 1495°F.	34,500	34,900	71,900	29.4	53.5	132	33,000	Moore and Kommers
13	0.42 per cent C, annealed at 1560°F.	38,900	40,000	75,500	32.5	49.3	...	33,500	Kommers
14	0.48 per cent C, annealed at 1350°F.	38,800	38,800	81,400	31.3	53.5	...	35,000	McAdam
15	0.49 per cent C, normalized at 1700°F.	44,700	47,100	91,500	26.5	39.5	164	33,000	Moore and Jasper
16	0.52 per cent C, normalized at 1550°F.	45,500	47,600	98,000	24.4	41.7	193	42,000	Moore and Kommers
17	0.56 per cent C, annealed at 1470°F.	40,300	45,300	87,200	37.0	35.6	...	33,800	Kommers
18	0.77 per cent C, annealed at 1350°F.	46,300	47,500	111,100	17.3	27.3	...	39,000	McAdam
19	0.80 per cent C, annealed at 1470°F.	47,800	53,300	110,330	11.0	11.2	...	42,000	Kommers
20	0.93 per cent C, normalized at 1600°F., annealed at 1450°F.	28,000	33,400	84,100	24.8	37.2	162	30,500	Moore and Kommers
21	1.20 per cent C, normalized at 1460°F.	58,600	60,700	116,900	7.9	11.6	224	50,000	Moore and Kommers

Wrought iron tested by Moore and Jasper showed the following properties parallel to the direction of rolling: Prop'l Elas. Lim. 20,100 lb. per sq. in., Yield Point 24,000 lb. per sq. in., Ultimate Tensile Strength 46,900 lb. per sq. in., Elong. in 2 in. 34.9 per cent, Red'n Area 29 per cent, Brinell Number 105, Endurance Limit 23,000 lb. per sq. in.

*** In 8 inches.**

In using the values given in Tables 2B–8B inclusive it should be remembered that they have been obtained from tests of small specimens in which the heat treatment given penetrates the specimen very thoroughly. Values as high as these could not be expected for machine parts made of metal of the same nominal composition and under shop conditions, given the same nominal heat treatment as the specimens giving the values in the tables. This is especially true for large machine parts, such as car axles. In other words, a "factor of safety" is necessary when applying these test results to actual design problems.

Fatigue Strength under Cycles of Axial Stress (Tension Compression).—Repeated-stress tests of specimens under cycles of alternating tension and compression have proved decidedly difficult to carry out. There are two reasons for this difficulty. First, a slight deviation from true axial loading in a specimen causes serious flexural stresses. Repeated-stress specimens have to be held very rigidly during the progress of a fatigue test and there is so little opportunity for adjustment to true axial stress that appreciable eccentricity of unknown amount is very easily introduced. Second, it is very difficult to design tension-compression specimens so as to avoid high localized stress at shoulders. In static tension tests of ductile metals, this high localized stress has very little effect on the tensile strength, although with brittle materials it has a very appreciable effect, and static tensile tests of brittle materials are usually less satisfactory than flexural tests. Under repeated stress these high localized stresses start fatigue cracks even in ductile metals, and the actual maximum unit stress in a test under repeated axial stress is frequently indeterminate and appreciably higher than the nominal stress obtained by dividing load by minimum area of cross-section.

Various series of axial-stress tests have been made, and the reported ratio of endurance limit obtained to that obtained from reversed-flexure tests of the same material has ranged from 0.7 to 1.0. A recent series of tests under

TABLE 4A.—CHEMICAL COMPOSITION AND HEAT TREATMENT OF ALLOY STEELS, QUENCHED

Number	Material	Carbon, per cent	Chromium, per cent	Nickel, per cent	Vanadium, per cent	Molybdenum, per cent	Manganese, per cent	Silicon, per cent	Phosphorus, per cent	Sulphur, per cent	Heat treatment
1	Nickel steel	0.41	0.18	3.41	....	....	0.75	0.25	0.020	0.020	Heat to 1525°F., cool in furnace, heat to 1525°F., quench in oil, reheat to 1210°F., hold 2 hr., cool in furnace
2	Nickel steel	0.42	....	3.60	....	....	0.70	0.12	0.017	0.016	Heat to 1450°F., hold 30 min., cool in oil, reheat to 900°F., hold 30 min., cool in furnace
3	Nickel steel	0.42	....	3.60	....	....	0.70	0.12	0.017	0.016	Heat to 1450°F., hold 30 min., cool in oil, reheat to 1150°F., hold 30 min., cool in furnace
4	Nickel steel	0.42	....	3.60	....	....	0.70	0.12	0.017	0.016	Heat to 1450°F., hold 1 hr., cool in water, reheat to 900°F., hold 1 hr., cool in furnace
5	Nickel steel	0.42	....	3.60	....	....	0.70	0.12	0.017	0.016	Heat to 1450°F., hold 1 hr., cool in water, reheat to 1100°F., hold 1 hr., cool in furnace
6	Nickel steel	0.41	0.18	3.41	....	....	0.75	0.25	0.020	0.020	Heat to 1525°F., cool in furnace, heat to 1450°F., quench in oil, reheat to 400°F., cool in air
7	Chrome-nickel steel	0.24	0.87	3.33	....	....	0.15	0.37	0.019	0.025	Anneal, heat to 1525°F., quench in oil, reheat to 700°F., quench in oil
8	Chrome-nickel steel	0.24	0.87	3.33	....	....	0.15	0.37	0.019	0.020	Anneal, heat to 1525°F., hold 30 min., quench in oil, reheat to 1450°F., quench in oil, reheat to 1200°F., hold 1 hr., quench in water
9	Chrome-nickel steel	0.49	0.99	1.75	....	....	....	.....	.....	.....	Heat to 1550°F., hold 1 hr., cool in water, reheat to 900°F., hold 1 hr., cool in furnace
10	Chrome-nickel steel	0.49	0.99	1.75	....	....	....	.....	.....	.....	Heat to 1550°F., hold 1 hr., cool in water, reheat to 1110°F., hold 1 hr., cool in furnace
11	Chrome-vanadium steel	0.55	0.99	.....	0.19	....	....	.....	.....	.....	Heat to 1650°F., hold 1 hr., cool in water, reheat to 900°F., hold 1 hr., cool in furnace

12	Chrome-vanadium steel.....	0.55	0.99	.....	0.19	....	....	.....	.....	.....	Heat to 1650°F., hold 1 hr., cool in water, reheat to 1110°F., hold 1 hr., cool in furnace
13	Chrome-vanadium steel.....	0.24	1.16	0.173	0.23	....	0.30	0.29	0.046	0.044	Treated to produce a proportional elastic limit of about 60,000 lb. per square inch
14	Chrome-vanadium steel.....	0.53	1.46	.....	0.23	....	0.50	0.22	0.005	0.038	Treated to produce a proportional elastic limit of about 110,000 lb. per square inch
15	Chrome-molybdenum steel..	0.39	0.76	.....	....	0.18	0.49	0.195	0.032	0.035	Heat to 1600°F., hold 1 hr., cool in water, reheat to 900°F., hold 1 hr., cool in furnace
16	Chrome-molybdenum steel..	0.39	0.76	.....	....	0.18	0.49	0.195	0.032	0.035	Heat to 1600°F., hold 1 hr., cool in water, reheat to 1100°F., hold 1 hr., cool in furnace
17	Chrome-molybdenum steel..	0.31	0.85	.....	....	0.20	0.44	0.22	0.034	0.035	Heat to 1600°F., hold 1 hr., cool in water, reheat to 900°F., hold 1 hr., cool in furnace
18	Chrome-molybdenum steel..	0.31	0.85	.....	....	0.20	0.44	0.22	0.034	0.035	Heat to 1600°F., hold 1 hr., cool in water, reheat to 1100°F., hold 1 hr., cool in furnace
19	Chrome-molybdenum steel..	0.50	1.03	.....	....	0.19	0.48	0.24	0.030	0.039	Heat to 1600°F., hold 1 hr., cool in water, reheat to 900°F., hold 1 hr., cool in furnace
20	Chrome-molybdenum steel..	0.50	1.03	.....	....	0.19	0.48	0.24	0.030	0.039	Heat to 1600°F., hold 1 hr., cool in water, reheat to 1100°F., hold 1 hr., cool in furnace
21	Nickel-molybdenum steel....	0.41	....	1.72	....	0.13	0.45	0.20	0.037	0.020	Treated to produce a proportional elastic limit of about 60,000 lb. per square inch
22	Nickel-molybdenum steel....	0.42	....	1.68	....	0.12	0.47	0.25	0.038	0.020	Treated to produce a proportional elastic limit of about 110,000 lb. per square inch
23	Chrome-nickel molybdenum steel	0.41	0.86	2.39	....	0.75	....	.....	....	.....	Heat to 1500°F., quench in oil, reheat to 300°F., cool in air
24	Si-Mn spring steel..........	0.55	....	.....	....	....	0.65	1.97	0.024	0.037	Treated to produce a proportional elastic limit of about 60,000 lb. per square inch
25	Si-Mn spring steel..........	0.51	....	....	....	....	0.65	1.96	0.029	0.045	Treated to produce a proportional elastic limit of about 110,000 lb. per square inch

TABLE 4*B*.—MECHANICAL PROPERTIES UNDER STATIC LOADING AND FATIGUE LOADING OF ALLOY STEELS, QUENCHED

Number	Material	Proportional elastic limit, pounds per square inch	Yield point, pounds per square inch	Ultimate tensile strength, pounds per square inch	Elongation in 2 in., per cent	Reduction of area, per cent	Brinell number	Endurance limit, rotating beam, pounds per square inch	Investigator
1	Nickel steel	82,400	91,000	111,800	23.6	60.2	248	67,000	Moore and Kommers
2	Nickel steel	90,500	92.500	118,000	23.2	50.7	...	66,500	McAdam
3	Nickel steel	60,500	61,800	123,000	21.7	55.1	...	59,500	McAdam
4	Nickel steel	139,000	142,000	154,200	15.0	55.0	...	74,000	McAdam
5	Nickel steel	115,500	116,000	132,300	18.7	54.7	...	72,000	McAdam
6	Nickel steel	174,000	....	282,000	8.2	48.7	488	112,000	Moore and Jasper
7	Chrome-nickel steel	115,500	128,100	138,700	18.2	61.8	291	68,000	Moore and Kommers
8	Chrome-nickel steel	86,200	100,500	114,200	23.2	69.3	246	67,000	Moore and Kommers
9	Chrome-nickel steel	127,500	128,000	164,500	14.5	52.5	...	84,500	McAdam
10	Chrome-nickel steel	115,000	116,000	144,300	18.0	60.0	...	72,000	McAdam
11	Chrome-vanadium steel	164,400	165,000	201,000	13.3	53.5	...	94,500	McAdam
12	Chrome-vanadium steel	129,500	130,500	164,400	15.5	50.5	...	92,000	McAdam
13	Chrome-vanadium steel	47,500	49,200	79,600	30.7	57.2	...	36,100	McAdam
14	Chrome-vanadium steel	114,000	120,500	146,600	17.5	53.4	...	66,900	McAdam
15	Chrome-molybdenum steel	85,200	86,200	121,600	12.7	55.3	...	68,000	McAdam
16	Chrome-molybdenum steel	107,500	108,500	136,600	17.3	61.3	...	68,500	McAdam
17	Chrome-molybdenum steel	122,500	124,300	164,800	13.3	52.3	...	72,500	McAdam
18	Chrome-molybdenum steel	115,500	116,500	139,900	16.0	62.3	...	63,500	McAdam
19	Chrome-molybdenum steel	127,500	128,500	172,000	12.3	41.7	...	88,000	McAdam
20	Chrome-molybdenum steel	128,000	129,000	157,000	16.3	55.5	...	77,000	McAdam
21	Nickel-molybdenum steel	51,300	56,300	104,700	21.3	41.0	...	46,600	McAdam
22	Nickel-molybdenum steel	100,000	102,500	133,400	19.8	51.3	...	57,000	McAdam
23	Cr-Ni-Mo steel	170,000	....	312,000	11.2	26.5	530	135,000	Gillett and Mack
24	Si-Mn spring steel	57,500	60,000	111,800	24.5	40.3	...	49,100	McAdam
25	Si-Mn spring steel	100,000	101,000	157,500	16.5	40.0	...	62,000	McAdam

TABLE 5A.—CHEMICAL COMPOSITION OF ALLOY STEELS, NOT QUENCHED

Number	Material	Carbon, per cent	Chromium, per cent	Nickel, per cent	Vanadium, per cent	Molybdenum, per cent	Manganese, per cent	Silicon, per cent	Phosphorus, per cent	Sulphur, per cent	Copper, per cent	Heat treatment
1	Stainless iron	0.03	13.47	.....	0.27	....	....	....	0.040	0.01	....	Tested as received
2	Nickel steel	0.42	.....	3.60	....	....	0.70	0.12	0.017	0.016	....	Heat to 1450°F., hold 30 min., cool in furnace
3	Nickel steel	0.41	0.18	3.41	....	....	0.75	0.25	0.020	0.020	....	Heat to 1445°F., hold 1 hr., cool in furnace
4	Nickel steel	0.31	.....	3.35	....	....	0.64	0.13	0.026	0.028	....	Heat to 1475°F., hold 30 min., cool in air
5	Nickel steel	0.31	.....	3.35	....	....	0.64	0.13	0.026	0.028	....	Heat to 1475°F., hold 1 hr., cool in furnace
6	Chrome-nickel steel	0.49	0.99	1.75	....	....	....	....	.....	.....	....	Heat to 1550°F., hold 1 hr., cool in furnace
7	Chrome-nickel steel	0.24	0.87	3.33	....	....	0.37	0.15	0.019	0.025	....	Anneal, heat to 1525°F., hold 30 min., cool in air, reheat to 1450°F., hold 30 min., cool in furnace
8	Chrome-vanadium steel	0.55	0.99	.....	0.19	....	....	....	.....	.....	....	Heat to 1650°F., hold 1 hr., cool in furnace
9	Chrome-molybdenum steel	0.39	0.76	.....	....	0.18	0.49	0.195	0.034	0.035	....	Heat to 1600°F., hold 1 hr., cool in furnace
10	Chrome-molybdenum steel	0.31	0.85	.....	....	0.20	0.44	0.22	0.034	0.035	....	Heat to 1600°F., hold 1 hr., cool in furnace
11	Chrome-molybdenum steel	0.50	1.03	.....	....	0.19	0.48	0.24	0.030	0.039	....	Heat to 1600°F., hold 1 hr., cool in furnace
12	Corrosion-resistant steel	0.08	11.78	.....	....	....	0.07	0.11	0.017	0.007	0.05	Heat to 1800°F., hold 2 hr., cool in furnace
13	Corrosion-resistant steel	0.40	15.21	0.18	....	....	0.28	0.59	0.017	0.058	....	Tested as received (annealed by manufacturer)
14	Corrosion-resistant steel	0.61	15.81	0.24	....	....	0.39	0.03	0.018	0.035	1.10	Tested as received (annealed by manufacturer)
15	Corrosion-resistant steel	0.85	14.99	0.26	....	....	0.39	0.16	0.020	0.005	0.03	Tested as received (annealed by manufacturer)
16	Corrosion-resistant steel	0.24	5.42	22.90	....	....	0.80	1.65	0.010	0.027	0.78	Tested as received
17	Corrosion-resistant steel	0.45	8.38	28.20	....	....	0.49	1.39	0.012	0.022	0.67	Tested as received
18	Corrosion-resistant steel	0.38	15.95	15.88	....	....	0.71	2.36	0.020	0.048	0.08	Tested as received
19	Corrosion-resistant steel	0.80	19.01	12.19	....	....	0.86	1.70	0.017	0.045	0.05	Tested as received
20	Corrosion-resistant steel	0.39	17.71	25.27	....	....	0.66	1.44	0.018	0.030	0.12	Tested as received
21	Corrosion-resistant steel	0.70	17.32	25.81	....	....	0.72	3.08	0.018	0.060	0.05	Tested as received

TABLE 5*B*.—MECHANICAL PROPERTIES UNDER STATIC LOADING AND FATIGUE LOADING OF ALLOY STEELS, NOT QUENCHED

Number	Material	Proportional elastic limit, pounds per square inch	Yield point, pounds per square inch	Ultimate tensile strength, pounds per square inch	Elongation in 2 in., per cent	Reduction of area, per cent	Brinell number	Endurance limit, rotating beam, pounds per square inch	Investigator
1	Stainless iron	18,100	......	82,400	16.0	26.3	175	40,000	R. R. Moore
2	Nickel steel	69,500	70,300	113,600	22.0	43.5	...	51,100	McAdam
3	Nickel steel	60,800	64,600	101,600	25.5	52.2	194	54,000	Moore and Jasper
4	Nickel steel	49,500	50,000	91,100	29.5	55.0	...	42,500	McAdam
5	Nickel steel	50,500	53,500	103,900	22.5	44.5	...	49,500	McAdam
6	Chrome-nickel steel	55,500	56,500	117,800	20.0	44.5	...	50,000	McAdam
7	Chrome-nickel steel	56,700	59,800	87,300	32.9	67.1	167	49,000	Moore and Jasper
8	Chrome-vanadium steel	50,500	51,500	101,800	25.5	48.5	...	44,500	McAdam
9	Chrome-molybdenum steel	47,300	48,300	85,800	29.0	49.3	...	38,500	McAdam
10	Chrome-molybdenum steel	42,100	43,100	82,400	27.5	41.7	...	36,500	McAdam
11	Chrome-molybdenum steel	49,900	50,900	111,400	21.0	41.0	...	49,500	McAdam
12	Corrosion-resistant steel	20,000	......	63,300	40.0	72.0	...	31,000	McAdam
13	Corrosion-resistant steel	22,500	......	110,000	24.0	53.5	...	49,500	McAdam
14	Corrosion-resistant steel	25,000	......	99,800	28.0	48.5	...	54,500	McAdam
15	Corrosion-resistant steel	22,500	......	111,000	17.5	32.7	...	50,000	McAdam
16	Corrosion-resistant steel	25,000	......	96,000	33.0	59.8	...	50,000	McAdam
17	Corrosion-resistant steel	22,800	......	111,100	24.0	48.9	...	58,500	McAdam
18	Corrosion-resistant steel	35,000	......	130,000	24.5	37.5	...	64,000	McAdam
19	Corrosion-resistant steel	30,000	......	123,000	21.0	27.0	...	65,500	McAdam
20	Corrosion-resistant steel	30,000	......	118,300	21.5	33.0	...	54,000	McAdam
21	Corrosion-resistant steel	20,000	......	106,500	12.0	15.0	...	55,500	McAdam

TABLE 6A.—CHEMICAL COMPOSITION AND HEAT TREATMENT OF CAST STEEL AND CAST IRON

Number	Material	Graphitic carbon, per cent	Combined carbon, per cent	Manganese, per cent	Silicon, per cent	Phosphorus, per cent	Sulphur, per cent	Heat treatment
1	Cast steel	....	0.35	1.71	0.30	0.02	0.03	As cast from bottom of slug
2	Cast steel	....	0.35	1.71	0.30	0.02	0.03	Heat to 1650°F., cool in air, reheat to 1525°F, cool in air, reheat to 1000°F., cool in air
3	Cast steel	....	0.35	1.71	0.30	0.02	0.03	Heat to 1525°F., cool in air
4	Cast steel	....	0.25	0.68	0.32	0.01	0.03	As cast from bottom of slug
5	Cast steel	....	0.25	0.68	0.32	0.01	0.03	Heat to 1625°F., cool in air
6	Cast steel	....	0.25	0.68	0.32	0.01	0.03	Heat to 1625°F., cool in air, reheat to 1000°F., cool in air
7	Cast iron	2.72	0.84	0.32	1.42	0.75	0.06	From 6-in. pipe, cast by centrifugal process
8	Cast iron	2.76	0.68	0.62	1.10	0.51	0.09	From 12-in. cylinder, 1 in. thick, sand cast
9	Cast iron	2.78	0.57	0.59	1.10	0.51	0.09	From center of wall of 12-in. cylinder 3 in. thick, sand cast

TABLE 6*B*.—MECHANICAL PROPERTIES UNDER STATIC AND FATIGUE LOADING OF CAST STEEL AND CAST IRON

Number	Material	Proportional elastic limit, pounds per square inch	Yield point, pounds per square inch	Ultimate tensile strength, pounds per square inch	Elongation in 2 in., per cent	Reduction of area, per cent	Brinell number	Endurance limit, rotating beam, pounds per square inch	Investigator
1	Cast steel	39,000	......	80,800	2.1	4	179	32,000	H. F. Moore
2	Cast steel	59,600	......	103,600	22.5	48	188	45,000	H. F. Moore
3	Cast steel	47,500	......	107,600	13.7	23	201	48,000	H. F. Moore
4	Cast steel	23,600	26,700	67,200	22.0	33	119	27,000	H. F. Moore
5	Cast steel	42,800	43,600	76,600	30.5	51	136	35,000	H. F. Moore
6	Cast steel	41,300	42,700	76,100	31.7	56	133	33,000	H. F. Moore
7	Cast iron	......	......	26,200	....	..	162	12,000	H. F. Moore
8	Cast iron	......	......	31,600	....	..	148	11,000	H. F. Moore
9	Cast iron	......	......	25,300	....	..	132	9,000	H. F. Moore

reversed axial stress has been reported by Paul L. Irwin.[1] These tests were made on a Haigh testing machine in the laboratories of the Westinghouse Electric and Manufacturing Company. Extreme care was taken to insure axial load during the test and the specimen used was very carefully designed to avoid localized high stress. Irwin has tested in this way several kinds of steel and a few non-ferrous metals. Some of the metals tested had an endurance limit below the proportional elastic limit, and some had an endurance limit above the proportional elastic limit. He has found that endurance limits under axial stress had practically the same value as endurance limits in reversed flexure, obtained on a rotating-beam testing machine.

It seems desirable to point out that in machine parts and structural members subjected to repeated axial stress, localized stress at shoulders, screw threads, etc. is of very great importance, and is not usually computed or even estimated; moreover, very few machine or structural parts are subjected to pure axial atress without any flexural action. It is doubtful whether structural or machine parts which are subjected to reversed axial stress in engineering practice would be likely (because of the accompanying indeterminate bending stresses) to develop a nominal fatigue strength higher than about 70 per cent of the endurance limit given by tests of rotating-beam specimens. In the case of parts with U. S. Standard threads, it is doubtful whether the computed endurance limit at the root of thread (load divided by area at root of thread) would be higher than 25 per cent of the endurance limit determined by tests of carefully designed specimens under reversed flexure.

Fatigue Strength under Cycles of Shearing Stress.—In determining fatigue strength under cycles of shearing stress, the endurance limit under cycles of repeated torsion is taken as the index of fatigue strength. Test data for repeated-

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[1] Irwin, "Fatigue of Metals by Direct Stress," *Proc. Am. Soc. Testing Materials*, vol. 25, Pt. II, p. 53, 1925, and vol. 26, Pt. II, p. 218, 1926.

TABLE 7A.—CHEMICAL COMPOSITION AND HEAT TREATMENT OF HEAVY NON-FERROUS METALS

Number	Material	Silicon, per cent	Manganese, per cent	Nickel, per cent	Aluminum, per cent	Copper, per cent	Iron, per cent	Zinc, per cent	Tin, per cent	Carbon, per cent	Lead, per cent	Sulphur, per cent	Phosphorus, per cent	Heat treatment
1	Copper, annealed	....	....	....	....	99.89	....	......	....	....	....	....	....	Heat to 970°F., hold 30 min., cool in furnace
2	Copper, cold drawn after annealing	....	....	....	....	99.89	....	......	....	....	....	....	....	Reduced from ¾ to ½ in. in one draw
3	Copper, cold rolled	....	....	....	....	....	0.007	......	....	....	....	....	Trace	Tested as received
4	Copper, cold rolled (annealed)	....	....	....	....	....	0.007	......	....	....	....	....	Trace	Heat to 1200°F., hold 1 hr., cool in furnace
5	81-19 brass, cold drawn	....	....	....	....	81.00	0.05	19.06	....	....	0.01	....	....	Tested as received
6	81-19 brass, cold drawn and annealed	....	....	....	....	81.00	0.05	19.06	....	....	0.01	....	....	Heat to 1000°F., hold 1 hr., cool in furnace
7	70-30 brass, cold rolled	....	....	....	....	70.08	0.05	29.99	....	....	0.01	....	....	Tested as received
8	70-30 brass, cold rolled and annealed	....	....	....	....	70.08	0.05	29.99	....	....	0.01	....	....	Heat to 1200°F., hold 1 hr., cool in furnace
9	60-40 brass, annealed	....	....	....	....	60.25	0.02	39.61	....	....	0.02	....	....	Heat to 1020°F., hold 30 min., pickled and washed
10	Naval brass (61-38)	....	....	....	....	61.20	....	Remainder	0.43	....	0.10	....	....	Tested as received
11	95-5 bronze, annealed	....	....	....	....	94.96	....	......	4.89	....	....	....	....	Heat to 1290°F., hold 30 min., pickled and washed

TABLE 7A.—(*Continued*)

12	95-5 bronze cold drawn after anneal	....	....	....	....	94.96	....	......	4.89	....	....	....	....	Reduced from ¾ to ½ in. in one draw
13	95-5 bronze, cold rolled	....	....	....	....	95.57	0.09	......	4.05	....	0.01	....	0.39	Tested as received
14	95-5 bronze, cold rolled and annealed	....	....	....	....	95.57	0.09	......	4.05	....	0.01	....	0.39	Heat to 1200°F., hold 1 hr., cool in furnace
15	89-11 bronze, cold rolled	....	....	....	....	89.39	0.08	......	10.6	....	0.01	....	0.13	Tested as received
16	89-11 bronze, cold rolled and annealed	....	....	....	....	89.39	0.08	......	10.6	....	0.01	....	0.13	Heat to 1100°F., hold 1 hr., cool in furnace
17	Mn-bronze, cast	....	0.20	....	0.23	56.85	1.50	Remainder	0.32	....	....	....	....	Tested as received
18	Al-bronze, extruded	....	....	....	10.06	89.81	0.13	......	....	....	....	....	....	Heat to 1650°F., quench in water, reheat to 1150°F., hold 30 min., cool in furnace
19	Al-bronze cast	....	....	....	9.78	90.22	....	......	....	....	....	....	....	Tested as received
20	Al-bronze, cast and annealed	....	....	....	9.78	90.22	....	......	....	....	....	....	....	Heat to 1650°F., quench in water, reheat to 1200°F., hold 30 min., cool in furnace
21	Nickel, cold rolled	0.06	0.10	98.95	....	0.12	0.50	......	....	0.25	....	0.175	....	Tested as cast
22	Nickel, cold rolled (annealed)	0.06	0.10	98.95	....	0.12	0.50	......	....	0.25	....	0.175	....	Heat to 1600°F., hold 1 hr., cool in furnace
23	Nickel annealed	....	....	99.0	....	....	....	......	....	....	....	....	....	Annealed from 1600°F.
24	Monel metal, hot rolled	0.08	1.38	68.95	Trace	27.29	2.22	......	....	0.20	....	0.019	0.019	Hot rolled by manufacturers to 1⅛ in. round
25	Monel metal, cold rolled (annealed)	0.03	1.03	69.10	....	27.99	1.62	......	....	0.21	....	....	....	Heat to 1000°F., hold 3 hr., cool in air
26	Monel metal, cold rolled (annealed)	0.03	1.02	67.07	....	28.04	1.60	......	....	0.21	....	0.017	0.017	Heat to 800°F., hold 3 hr., cool in air

TABLE 7*B*.—MECHANICAL PROPERTIES UNDER STATIC AND FATIGUE LOADING OF HEAVY NON-FERROUS METALS

Number	Material	Proportional elastic limit, pounds per square inch	Ultimate tensile strength, pounds per square inch	Elongation in 2 in., per cent	Reduction of area, per cent	Brinell number	Endurance limit rotating beam, pounds per square inch	Investigator
1	Copper, annealed	3,170	32,400	56.4	71.4	47	10,000	Moore and Jasper
2	Copper, cold drawn after annealing	38,400	56,200	6.5	52.0	104	10,000	Moore and Jasper
3	Copper, cold rolled	10,000	52,000	13.0	50.0	...	16,000	McAdam
4	Copper, cold rolled (annealed)	......	32,000	57.5	71.8	...	10,500	McAdam
5	81-19 brass, cold drawn	21,000	76,500	16.0	64.8	...	23,000	McAdam
6	81-19 brass, cold drawn and annealed	7,500	44,000	64.3	80.0	...	17,500	McAdam
7	70-30 brass, cold rolled	......	73,200	20.0	46.3	...	17,500	McAdam
8	70-30 brass, cold rolled and annealed	......	45,000	72.3	66.0	...	15,000	McAdam
9	60-40 brass, annealed	15,600	54,200	56.0	61.0	72	22,000	Moore and Jasper
10	Naval brass (61-38)	33,400	68,200	27.0	53.0	135	21,100	R. R. Moore
11	95-5 bronze, annealed	12,800	45,700	66.9	83.5	74	23,000	Moore and Jasper
12	95-5 bronze, cold drawn after annealing	59,900	85,100	11.7	67.0	166	27,000	Moore and Jasper
13	95-5 bronze, cold rolled	23,000	55,800	47.5	79.0	...	22,500	McAdam
14	95-5 bronze, cold rolled and annealed	11,500	48,100	70.8	80.5	...	22,500	McAdam
15	89-11 bronze, cold rolled	38,000	82,800	37.8	63.0	...	27,000	McAdam
16	89-11 bronze, cold rolled and annealed	24,500	67,600	70.0	72.0	...	27,000	McAdam
17	Mn-bronze, cast	13,000	70,000	32.8	40.8	93	17,000	R. R. Moore
18	Al-bronze, extruded	16,900	77,500	35.5	34.0	128	34,000	R. R. Moore
19	Al-bronze, cast	5,100	59,300	20.0	28.1	96	22,000	R. R. Moore
20	Al-bronze, cast and annealed	24,900	77,800	14.0	19.0	192	26,000	R. R. Moore
21	Nickel, cold rolled	71,600	166,200	12.2	18.8	...	40,000	McAdam
22	Nickel, cold rolled (annealed)	......	70,400	47.8	54.8	...	25,500	McAdam
23	Nickel, annealed	11,300	69,900	45.1	72.0	90	28,000	Moore and Jasper
24	Monel metal, hot rolled	49,600	89,800	40.4	69.3	166	32,000	Moore and Jasper
25	Monel metal, cold rolled (annealed)	53,300	100,400	31.8	67.5	...	40,000	McAdam
26	Monel metal, cold rolled (annealed)	75,800	135,500	21.2	55.8	...	54,000	McAdam

TABLE 8A.—CHEMICAL COMPOSITION AND HEAT TREATMENT OF LIGHT NON-FERROUS METALS

Number	Material	Silicon, per cent	Manganese, per cent	Nickel, per cent	Aluminum, per cent	Magnesium, per cent	Copper, per cent	Iron, per cent	Zinc, per cent	Tin, per cent	Lead, per cent	Heat treatment
1	Aluminum, rolled	0.15	.....	...	Remainder	....	0.12	0.49	....	......	......	Tested as received
2	Duralumin, rolled	0.28	.....	...	Remainder	0.70	3.25	0.28	....	......	......	Heat to 950°F., quench in water
3	Duralumin, tempered	0.28	.....	...	Remainder	0.70	3.25	0.28	....	......	......	Heat to 950°F., hold 7 to 30 min., quench in water, reheat to 925°F., hold 30 min., quench in water
4	Duralumin, annealed	0.28	.....	...	Remainder	0.70	3.25	0.28	....	......	......	Heat to 950°F., hold 7 to 30 min., quench in water, reheat to 700°F., hold 20 min., cool in furnace
5	Duralumin, rolled	0.25	0.64	...	93.64	0.71	4.35	0.77	....	Less than 0.05	......	Tested as received
6	Duralumin	0.47	0.52	...	93.74	....	4.12	0.51	....	......	......	Annealed by manufacturer
7	Duralumin	0.34	0.62	...	Remainder	0.42	4.28	0.54	....	......	......	Tempered by manufacturer
8	Aluminum alloy	0.25	0.5	2.0	92.25	1.5	4.0	....	....	......	......	Tested as received
9	Aluminum-copper alloy	0.82	0.82	...	Remainder	....	4.55	0.56	....	......	......	Tempered by manufacturer
10	Mg-Al alloy, extruded	.....	.....	...	4.20	Remainder	.....	0.03	....	......	......	Tested as received
11	Mg-Al alloy, extruded	.....	0.26	...	4.40	Remainder	.....	0.03	....	......	......	Tested as received
12	Mg-Al alloy, extruded	.....	.....	...	6.70	Remainder	.....	0.04	....	......	......	Tested as received
13	Mg-Al alloy, extruded	.....	0.26	...	6.80	Remainder	.....	0.04	....	......	......	Tested as received
14	Mg-Al alloy, forged	0.023	.....	...	8.88	Remainder	0.026	0.041	....	......	......	Tested as received
15	Mg-Al alloy, cast	0.023	.....	...	8.68	Remainder	0.026	0.041	....	......	......	Tested as received
16	Al-Mg-Si alloy	0.56	0.006	...	Remainder	0.55	0.15	0.54	....	......	......	Tempered by manufacturer
17	Mg-Cu alloy, extruded	.....	.....	...	.....	Remainder	9.65	0.04	....	......	......	Tested as received
18	Electron metal	0.24	.....	...	.....	Remainder	0.41	0.25	4.38	......	......	Tested as received
19	Magnesium, extruded	0.02	.....	...	.....	Remainder	....	0.02	....	......	......	Tested as received

TABLE 8*B*.—MECHANICAL PROPERTIES UNDER STATIC AND FATIGUE LOADING OF LIGHT NON-FERROUS METALS

Number	Material	Proportional elastic limit, pounds per square inch	Ultimate tensile strength pounds per square inch	Elongation in 2 in., per cent	Reduction of area, per cent	Brinell number	Endurance limit rotating beam, pounds per square inch	Investigator
1	Aluminum, rolled	11,300	22,600	16.0	65.4	45	10,500	R. R. Moore
2	Duralumin, rolled	25,000	51,000	16.0	50.4	100	14,000	R. R. Moore
3	Duralumin, tempered	18,600	51,200	29.3	47.5	100	12,000	R. R. Moore
4	Duralumin, annealed	6,800	25,300	25.0	60.8	50	10,900	R. R. Moore
5	Duralumin, rolled	17,500	53,400	23.8	35.2	...	12,500	McAdam
6	Duralumin	14,200	29,500	19.4	46.8	...	11,000	McAdam
7	Duralumin	16,000	61,600	29.3	39.4	...	16,000	McAdam
8	Aluminum alloy	32,000 (Y. P.)	53,700	24.0	....	...	22,800	Gough
9	Aluminum-copper alloy	19,900	59,500	24.2	37.4	...	15,000	McAdam
10	Mg-Al alloy, extruded	8,100	35,200	21.7	28.3	52	12,000	R. R. Moore
11	Mg-Al alloy, extruded	13,500	39,000	15.5	30.8	58	15,000	R. R. Moore
12	Mg-Al alloy, extruded	12,000	41,300	16.0	20.3	61	13,000	R. R. Moore
13	Mg-Al alloy, extruded	15,300	44,400	13.8	16.9	65	15,000	R. R. Moore
14	Mg-Al alloy, forged	11,400	41,300	4.0	....	61	15,000	R. R. Moore
15	Mg-Al alloy, cast	6,900	28,000	4.0	....	61	12,500	R. R. Moore
16	Al-Mg-Si alloy	15,300	45,000	17.4	22.3	...	12,000	McAdam
17	Mg-Cu alloy, extruded	14,300	39,000	3.0	3.6	60	11,000	R. R. Moore
18	Electron metal	6,300	36,500	17.5	20.5	64	17,000	R. R. Moore
19	Magnesium, extruded	1,200	32,500	6.2	4.4	41	7,800	R. R. Moore

TABLE 9.—FATIGUE STRENGTH UNDER REVERSED SHEARING STRESS AND REVERSED FLEXURAL STRESS

Material	Endurance limit, pounds per square inch		Ratio endurance limit in torsion to endurance limit in flexure	Investigator
	Reversed torsion (shear)	Reversed flexure		
Plain carbon steels:				
Armco, annealed	12,700	26,000	0.49	Moore and Jasper
0.24 carbon, as rolled	14,000	25,500	0.55	McAdam
0.37 carbon, normalized	16,000	33,000	0.49	Moore and Jasper
sorbitic	32,500	57,000	0.57	
0.38 carbon, oil quench annealed	17,000	30,000	0.57	McAdam
normalized	17,500	32,000	0.55	
oil quench, drawn 1250°F	21,500	33,500	0.64	
oil quench, drawn 1000°F	16,500	33,500	0.48	
oil quench, drawn 800°F	20,500	33,500	0.61	
0.49 carbon, normalized	20,000	33,000	0.60	Moore and Jasper
sorbitic	28,000	48,000	0.58	
0.52 carbon, normalized	22,000	42,000	0.52	Moore and Jasper
sorbitic	31,500	55,000	0.57	
0.81 carbon, annealed	19,000	31,500	0.60	McAdam
0.93 carbon, annealed	16,300	30,500	0.53	Moore and Jasper
sorbitic	29,000	56,000	0.52	
troostitic	52,000	98,000	0.53	
1.20 carbon, normalized	24,500	50,000	0.49	Moore and Jasper
sorbitic	48,000	92,000	0.52	
Average for plain carbon steels			0.55	
Alloy steels:				
3.35 nickel, annealed	28,000	49,500	0.56	McAdam
oil quench, drawn 900°F	35,000	54,500	0.64	
oil quench, drawn 950°F	45,000	63,500	0.71	
oil quench, drawn 1150°F	37,500	54,000	0.69	
3.50 nickel, annealed	29,000	54,000	0.54	Moore and Jasper
oil quench, drawn 1100°F	36,000	64,000	0.56	
oil quench, drawn 1200°F	35,500	63,000	0.56	
3.60 nickel, annealed	22,500	51,000	0.44	McAdam
oil quench, drawn 900°F	38,000	66,500	0.57	
water quench, drawn 900°F	47,000	74,000	0.63	
water quench, drawn 1100°F	46,500	72,000	0.64	
Chrome-nickel, annealed	25,000	49,000	0.51	Moore and Jasper
oil quench, drawn 700°F	38,000	68,000	0.56	
oil quench, drawn 1200°F	33,000	66,000	0.50	
Average for alloy steels			0.58	
Non-ferrous metals:				
Nickel, cold-rolled	17,400	32,400	0.54	McAdam
Monel, hot-rolled	18,600	35,300	0.52	McAdam
Monel, cold-rolled	19,000	38,000	0.50	McAdam
Constantin-hot-rolled	15,800	34,500	0.46	McAdam
Copper-nickel-zinc alloy, hot-rolled	13,000	21,900	0.59	McAdam
Average for non-ferrous metals			0.52	

torsion tests are fewer than data for reversed-flexure tests. Table 9 gives the results of fatigue tests under cycles of reversed torsion. The tests quoted in Table 9 were made, some at the U. S. Naval Engineering Experiment Station and some at the University of Illinois. Other tests have been made at the British National Physical Laboratory, and Gough in his book, "The Fatigue of Metals," quotes the results of 49 series of fatigue tests which gave an average value for ratio of endurance limit for reversed shearing stress to endurance limit for reversed flexural stress of 0.56. An examination of Table 9 shows values of the above-named ratio ranging from 0.44 to 0.71 with an average of 0.55. Most of the values found lie between 0.49 and 0.60.

The endurance limit of metals in shear may then be regarded as having a value of about 55 per cent of the endurance limit in tension-compression. The general result of fatigue tests thus adds weight to other existing test data, which tends to show that the maximum-shear theory of the failure of metals is a safe approximation for ductile metals, but is not an exact statement of fact.[1]

Fatigue strength under cycles of torsion not completely reversed is discussed in Chap. VII.

Accelerated Tests for Fatigue Strength.—Fatigue tests to give data for determining endurance limit from an *S-N* diagram are very time consuming, and various attempts have been made to devise accelerated fatigue tests.

An accelerated fatigue test which is frequently proposed consists of comparative tests between specimens of different metals using a standard computed unit stress (or a standard deformation of specimen) for the series, and taking the length of "life" under this standard unit stress (or deformation) as an index of fatigue strength. In the opinion of the authors this test is a very unsatisfactory one. In the first place, a very slight accidental variation in stress or deformation makes a large change in the "life" of a specimen, and the results of such tests show a great deal of "scatter," so much as to render doubtful the quantitative value of the results. In the second place, if a number of metals are tested by this accelerated method and are then

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[1] TIMOSHENKO and LESSELLS, "Applied Elasticity," Chap. XVII, Westinghouse Technical Night School Press.

arranged in order of fatigue strength, the order of arrangement will depend on the severity of the standard stress used. A violent short test under high stress tends to emphasize the effect of ductility, while as the stress is lowered, the effect of strength is emphasized. The authors cannot recommend the use of this accelerated test.

An accelerated fatigue test which seems to have a limited field of usefulness is the rise-of-temperature test. In 1855 Lord Kelvin[1] called attention to the fact that a material subjected to elastic stress is cooled under tensile stress and heated under compressive stress, but that inelastic stress causes heat for either tension or compression. In 1913 C. E. Stromeyer devised and used an apparatus for determining fatigue limit by means of the heat generated under repeated stress. He used an inertia-type torsion testing machine in which a stream of water flowed over the specimen, and delicate mercury thermometers measured the temperature rise in the water. He did not, however, check his fatigue limits by means of long-time tests to destruction. In 1921 Putman and Harsch developed an apparatus for measuring the rise of temperature under repeated stress, using a delicate thermocouple to indicate rise of temperature. They found a good correlation between the endurance limit determined in this way and the endurance limit given by long-time tests, studying some 20 wrought ferrous metals. Gough developed a similar test at almost the same time.[2] The rise-of-temperature tests seems to give fairly reliable results for many wrought ferrous metals. It has not given uniformly reliable results for non-ferrous metals, especially for cold-worked non-ferrous metals.

Bauschinger in his classical work on the fatigue of metals always emphasized the idea of an elastic limit which was gradually acquired by a metal under repeated cycles of stress, which might be either higher or lower than the

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[1] *Quart. Math. Jour.*, 1855.

[2] References for further study of the rise-of-temperature test are: *Univ. Illinois Eng. Exp. Station, Bull.* 124, 1921; STROMEYER, C. E., *Mem. Chief Engineer, Manchester, England, Steam Users' Assoc.*, 1913; GOUGH, H. J., "The Fatigue of Metals," Chap. X.

elastic limit of the metal in its primitive state, and which he regarded as the fatigue limit of the metal. Gough[1] developed this general idea into an accelerated test for fatigue strength. In his apparatus the stretch of a tension specimen or the deflection of a flexure specimen was measured while a repeated-stress test was in progress. On a graph plotted with computed unit stress as ordinates and "running" stretch or deflection as abscissae, the endurance limit was located at the point of deviation of the graph from a straight line. Lessells also has used this accelerated test. This "running-deflection" test seems to be of the same order of reliability as the rise-of-temperature test. It does not give altogether trustworthy results for non-ferrous metals. It seems doubtful whether either the rise-of-temperature test or the running-deflection test would distinguish between effects due to sudden temporary slip (heat bursts) and effects due to the beginning of fatigue cracks.

In the opinion of the authors both the rise-of-temperature test and the running-deflection test give indications of the beginning of serious slip in the metal, and frequently, though not necessarily, they indicate the beginning of a fatigue crack, which seems for many metals to occur under about the same conditions which cause slip.

An interesting accelerated fatigue test has been used by McAdam. For the test an inertia type of testing machine is used, and the tendency for stress to increase as deformation increases (see p. 89) serves to cause the rapid spread of a fatigue crack. If the stress is below the endurance limit (with a number of cycles not sufficient to cause appreciable strengthening of the metal), the relation of stress (as shown by the amplitude of oscillation of flywheel, Fig. 35, p. 105) to strain (as shown by nominal amplitude of oscillation given by crank pin radius) remains constant. At the endurance limit this ratio does not remain constant during a

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[1] Gough, H. J., "Improvements in Methods of Fatigue Testing," *The Engineer, (London)*, p. 159, Aug. 12, 1921; also "The Fatigue of Metals," Chap. X.

run, but as the incipient fatigue cracks spread, the oscillation of the flywheel increases. The actual spread of the fatigue crack is accelerated, once it is started. McAdam has used this test for determining endurance limit under cycles of reversed torsion. Data are lacking on which to base an opinion of its general reliability, but it seems to be a promising test.[1]

It is the opinion of the writers of this book that for cases where it is not feasible to determine the endurance limit of a metal by a long-time test, the best accelerated test to use is a series of short-time tests to fracture, using varying values of stress, and estimating the location of the horizontal line which the *S-N* diagram presumably approaches (see p. 127, especially reference in footnote). Long-time tests running to several million cycles of stress, however, should always be made when at all feasible.

Effect of Rapidity of Application of Cycles of Stress.—Various experimenters have run fatigue tests at different speeds in attempts to determine whether or not the value of the endurance limit is affected by the rapidity of application of cycles of stress.[2] In general, it seems that for specimens of the size usually used in fatigue testing, which are free from high localized stress, the value of endurance limit is very little affected by variation of the speed of testing over a range from 200 cycles per minute to 5,000 cycles per minute. Recent tests in England by Jenkin indicate that, at speeds of 20,000 cycles of stress per minute and higher, the endurance limit is higher than for speeds up to 5,000 cycles per minute. It should again be noted that the studies of effect of rapidity of application of cycles of stress have all been made on specimens of steel reasonably free from flaws and inclusions and free from high localized stress.

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[1] For details of this test, see McAdam, "Accelerated Fatigue Tests and Some Endurance Properties of Metals," *Proc. Am. Soc. Test. Materials,* vol. 24, Pt. II, p. 454, 1924.

[2] For references to this subject see *Univ. Illinois Eng. Exp. Sta. Bull.* 124, p. 27, 1921 and *Bull.* 136, p. 58, 1923.

Effect of Cold Working of Metals on Fatigue Strength.—The outstanding effect of cold work on metals is to raise the elastic strength in the direction of the rolling. Commercial cold-drawing and cold-rolling processes markedly increase the elastic strength of steel, and have a still greater effect in increasing the elastic strength of non-ferrous metals. The effect of these processes is to increase the ultimate tensile strength of metals noticeably, but not to so great a degree as the elastic strength is increased.

Commerical cold-drawing or cold-rolling of steel seems to increase the fatigue strength to about the same degree as the ultimate tensile strength is increased. With the non-ferrous metals tested a variety of results has been obtained for the effect of cold-drawing on fatigue strength. Tests of cold-drawn brass and copper rods in which there had been brought about a reduction in area of 55 per cent in a single pass of the cold-drawing process showed no increase or only a slight increase in fatigue strength over the strength of the same metal hot rolled. Tests of nickel and of other non-ferrous metals subjected to a somewhat less drastic reduction than that mentioned above showed appreciable increases in fatigue strength over the same metal annealed. Heating cold-drawn non-ferrous metals to a temperature well below the critical range distinctly improves their fatigue strength.[1]

Cold working of metal seems to exert two opposing effects: (1) It elongates the crystalline grains in the direction of the cold-drawing or the cold-rolling and seems to reorient crystalline grains into more favorable positions for resisting slip and fracture; and (2) it tends to start new minute fractures, or at least to set up severe internal stresses in the metal so that fractures are likely to be started by a small additional applied stress. For some degree of severity of cold-rolling or cold-drawing, there is a maximum of net benefit, and for more severe cold working, the damage

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[1] See *Univ. Illinois Eng. Exp. Sta., Bull.* 124, p. 104, 1921 and *Bull.* 152, p. 56, 1925; also McAdam, D. J., Jr., article in *Trans. Am. Soc. Steel Treating*, December, 1925.

done increases more rapidly than does the benefit. This picture is in harmony with the fact often observed by metal workers that overdrawn metal is weakened. It seems probable that the internal strains set up play an important part in weakening metal, in view of the improvement in fatigue strength noted after slight heating of cold-worked non-ferrous metals. In this connection the prevention of "season cracking" in brass by slight heating is of interest.

It is also of interest to note that polished soft steel cold worked by direct tension in a testing machine and *not afterwards repolished* has its endurance limit lowered,[1] whereas cold-stretched soft steel polished after stretching has its endurance limit raised. In commercial cold-drawing and cold-rolling there is exerted a heavy lateral pressure on the steel as it is reduced in cross-section, and a smooth surface is produced. Possibly this explains the difference in results for commercial cold-drawn and cold-stretched steel. A further study of the internal stresses set up in cold working might throw further light on this subject.

Cold-drawing and cold-rolling may, then, be regarded as possible means of increasing the fatigue strength of steel, but their usefulness is limited by their tendency to set up severe internal strains in steel and, possibly, to cause minute cracks.

The Effect of Heat Treatment on Endurance Limit.—From what has been said regarding the correlation between endurance limit and ultimate strength, it is obvious that heat treatment of steels may greatly influence the magnitude of the endurance limit. Figure 53, taken from the results of the Illinois investigation on a 0.93 and a 1.20 per cent carbon steel, illustrates what may be expected. In the case of the 0.93 per cent carbon steel, the material in the relatively soft pearlitic condition had an endurance limit of 30,500 lb. per square inch. This was increased 84 per cent when the steel was given a sorbitic structure, and 221 per cent when it was given a troostitic structure,

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 136, p. 60, 1923 and *Bull.* 124, p. 104, 1921.

by heat treatment. The curves for the 1.20 per cent carbon steel show an increase in endurance limit from 50,000 to 92,000 per square inch, or 84 per cent.

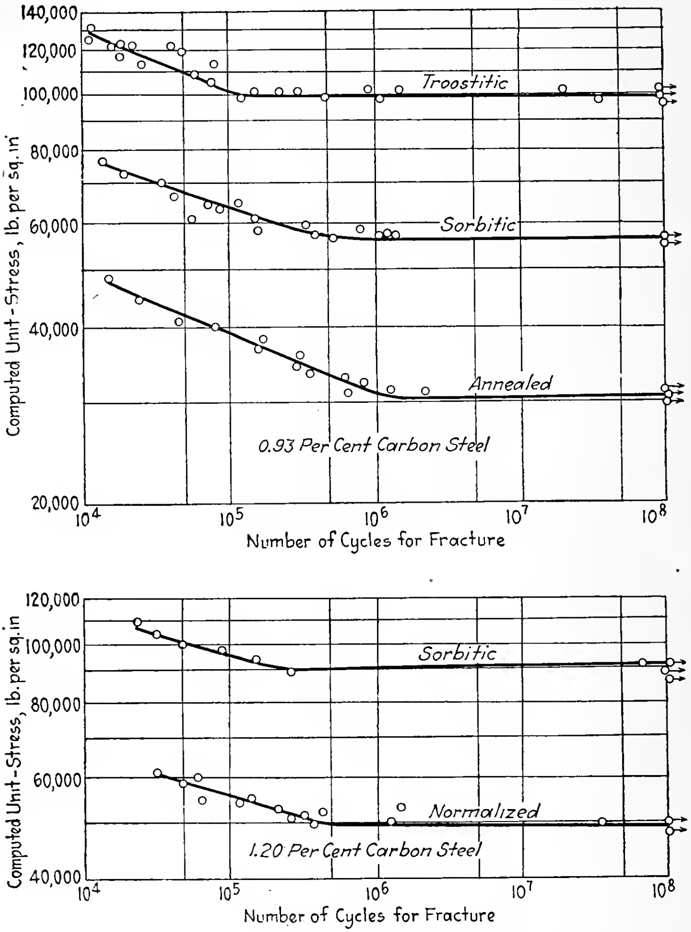

FIG. 53.—Effect of heat treatment on endurance limit.

The effect on endurance limit of various drawing temperatures is discussed in Chap. VIII.

Fatigue Strength of Steel at Elevated Temperatures.—Repeated-stress tests of steel at high temperatures have been made by Lea at Birmingham University, and by Moore and Jasper at the University of Illinois.[1] The testing machine used in the tests at Illinois was of the type shown in Fig. 30 (p. 99) with the addition of an electric furnace to heat the specimen. A thermocouple was attached to the specimen at the necked-down part, at the point of maximum stress, and this thermocouple was attached to a recording-controlling potentiometer, which maintained and recorded any desired temperature up to 1800°F. The fatigue tests were tests in reversed flexure, run at a speed of 1,500 r. p. m.

In connection with these fatigue tests static tests under high temperatures were also made. Some static tests were made in the ordinary manner, and other tests[2] were run, holding each increment of load until no increase of stretch (creep) could be observed after several minutes under load. These tests were called "prolonged and retarded" tests. Figures 54, 55, and 56 show graphically the results of tests at Illinois. It will be noted that, in general, the endurance limit does not fall off so rapidly under high temperature as does the tensile strength. It will be further noted that for some tests the endurance limit approaches in value the ultimate static strength given by a prolonged and retarded test. At temperatures so high that these two values become equal, fatigue failure ceases to be a matter of interest to the machine designer. In general, at elevated temperatures the ratio of fatigue strength to static ultimate strength is higher than it is for ordinary room temperatures. For the steels tested there does not seem to be much reduction of fatigue strength below 800°F., except for the Cyclops metal and for the 1.02 per cent carbon steel. Both these steels were heat-treated steels and the effect of fairly low temperatures

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[1] LEA, F. C., "The Effect of Low and High Temperatures on Materials," *Proc. Brit. Inst. Mech. Eng.*, Dec. 5, 1924, and *Univ. Illinois Eng. Exp. Sta., Bull.* 152, 1925.

[2] French, at the U. S. Bureau of Standards, has made much more prolonged static tests at high temperatures than these "prolonged and retarded tests." He has found that the ultimate tensile strength is still further reduced by further prolonging the time of test.

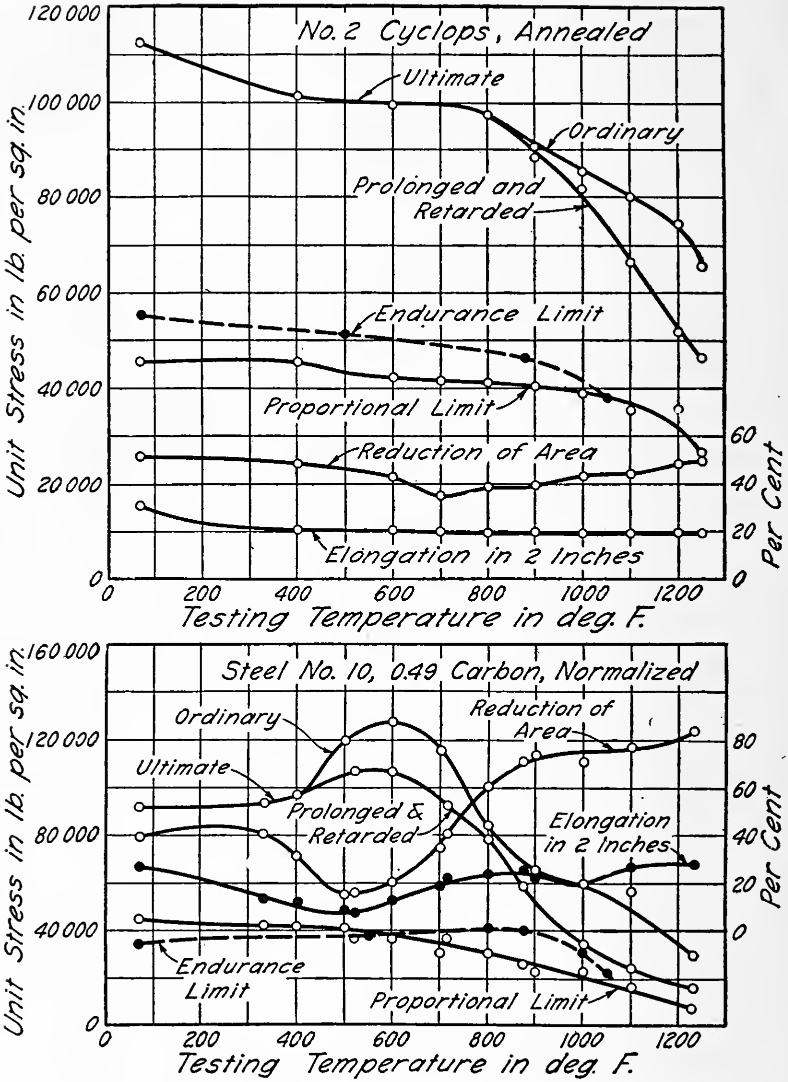

FIG. 54.—Effect of temperature on mechanical properties of metals.

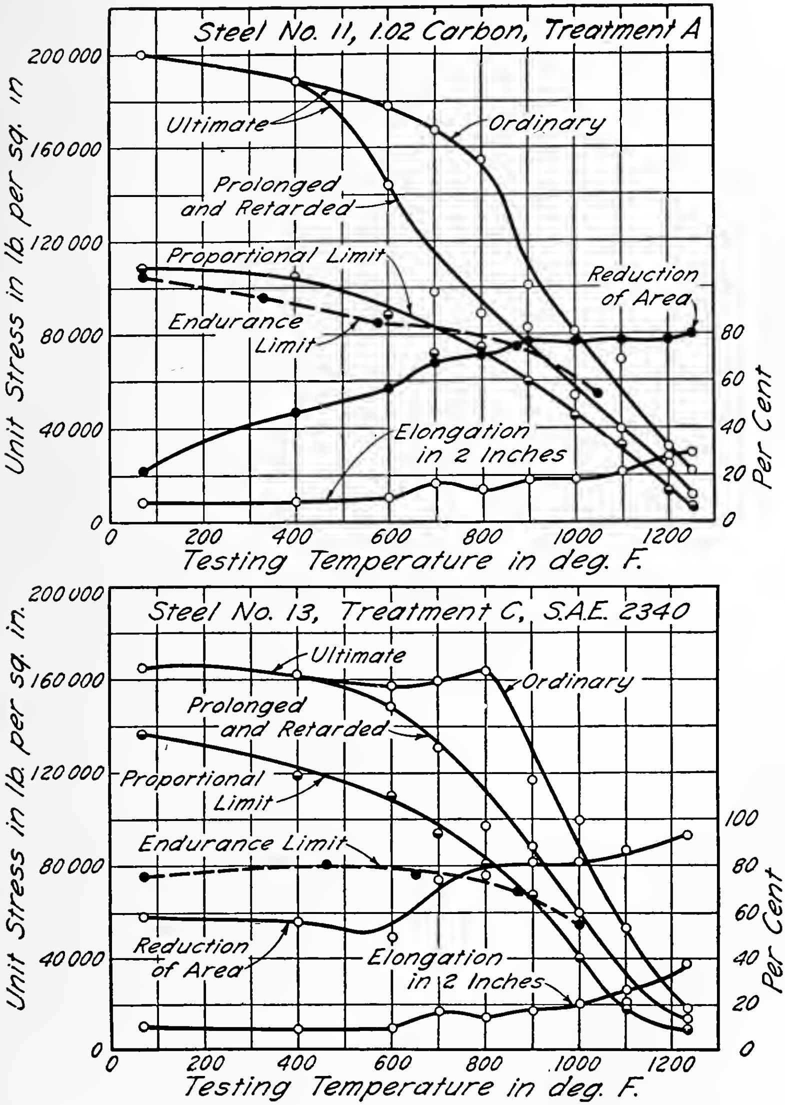

FIG. 55.—Effect of temperature on mechanical properties of metals.

would be to weaken the steels by "drawing," so that it is natural to find their endurance limits falling off with any increase of temperature above ordinary room conditions. Under high temperatures the ductility of steel is increased, and it seems reasonable to suppose that the ability of the steel to withstand plastic action without starting fatigue cracks is also increased.

A few fatigue tests on monel metal and on cast iron under elevated temperatures give indications that for these metals the fatigue strength is proportionately less reduced by high temperatures than is the case for rolled steel.

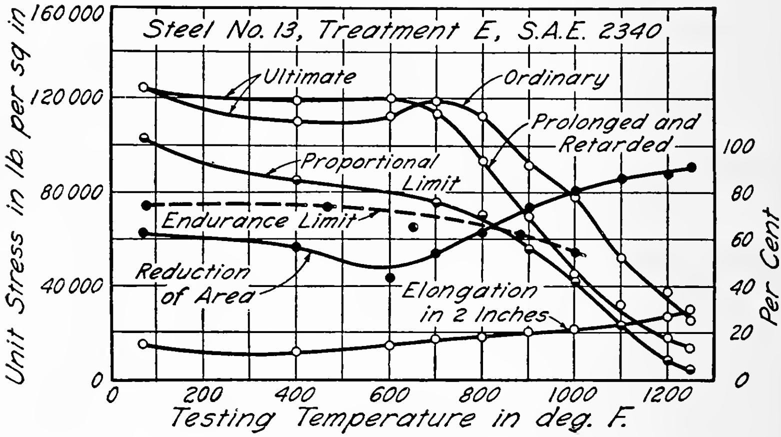

FIG. 56.—Effect of temperature on mechanical properties of metals.

Corrosion of steel and spreading fatigue cracks are mutually accelerative destructive factors. This action is discussed further in Chap. VIII on the effect of "stress-raisers" in metals.

The Effect of Case-carburizing on the Endurance Limit. Moore and Jasper[1] report the results of tests made by Muller on a 0.21 per cent steel and on Armco iron, after these materials had been carburized. The 0.21 per cent carbon steel was carburized in a gas-fired furnace at 1750°F. for 3 hr., producing a case thickness of 0.06 in. The Armco iron was carburized in an electric furnace at from 1650 to 1675°F.,

[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 152, p. 63, 1925.

for periods of 3½, 2½, and 2 hr., respectively. The depths of case resulting were 0.075, 0.025, and 0.015 in., respectively.

TABLE 10.—HEAT TREATMENTS USED WITH SPECIMENS OF CASE-CARBURIZED STEEL

Steel number	Designation	Heat treatment	Description
52	0.20 carbon steel..	*A*	Heat to 1600°F., hold 15 min., quench in oil, reheat to 1450°F., hold 15 min., quench in oil, reheat to 1200°F., hold 30 min., cool in air
		B	Heat to 1600°F., hold 15 min., quench in water, reheat to 1450°F., hold 15 min., quench in water, reheat to 1200°F., hold 30 min., cool in air
		C	Heat to 1600°F., hold 15 min., quench in oil, reheat to 1450°F., hold 15 min., quench in water, reheat to 1200°F., hold 30 min., cool in air
		D	Heat to 1600°F., hold 15 min., quench in water, reheat to 1200°F., hold 30 min., cool in air
9	0.02 carbon steel (Armco)........	*E*	Heat to 1600°F., hold 15 min., quench in oil, reheat to 1450°F., hold 15 min., quench in oil
		F	Heat to 1600°F., hold 15 min., quench in oil, reheat to 1450°F., hold 15 min., quench in oil, reheat to 1200°F., hold 30 min., cool in air
		G	Heat to 1450°F., hold 15 min., quench in oil
		H	After carburizing allow steel to cool in furnace

Table 10 shows the heat treatments to which the specimens were subjected after they had been carburized; Table 11 shows the results of the fatigue tests. Figures 57 and 58 show the *S-N* diagrams obtained from these tests.

Table 11 shows that the outside shell of high-carbon steel which is produced in the case-carburizing process is very effective in increasing the endurance limit, reaching a maximum of 162 per cent for Armco iron, heat treatment *E*. Treatments *A*, *G*, and *E* were particularly effective, while treatment *H*, which was practically an anneal, was apparently not at all effective.

TABLE 11.—RESULTS OF FATIGUE TESTS OF CASE-CARBURIZED STEEL SPECIMENS

All fatigue tests were made on a rotating-beam testing machine

Steel number	Designation	Case-carburizing treatment			Endurance limit, pounds per square inch	Increase of endurance limit over that of untreated steel, per cent
		Depth of case		Heat treatment		
		Inches	Percentage of diameter			
52	0.20 carbon steel..........	0	0	as received	33,000	0
		0.06	20	C	45,000	36
		0.06	20	B	48,000	45
		0.06	20	A	55,000	67
9	0.02 carbon steel (Armco)...	0	0	as received	26,000	0
		0.015	5.0	F	37,000	42
		0.015	5.0	E	44,000	69
		0.025	8.3	H	27,000	4
		0.025	8.3	G	56,000	115
		0.025	8.3	E	57,000	120
		0.075	25.0	F	50,000	92
		0.075	25.0	E	68,000	162

Figure 59 shows the relation between depth of case and endurance limit for Armco iron. The curves indicate that there is evidently a limit to the depth of case which is effective in increasing the endurance limit.

The results of tension tests on case-carburized specimens showed that, as might be expected, carburizing is less effective in increasing the strength of tension members, which have approximately uniform stress over the cross-section, than it is for flexure members, in which only the outer shell carries the high stresses.

Correlation of Fatigue Strength with Other Physical Properties.—It is not to be expected that there will be found any precise correlation between fatigue strength and any one other physical property of a metal. Probably elastic strength, ultimate strength, and ductility all have an effect on the fatigue strength. Fatigue failure, however, seems to be a progressive tearing apart or shearing apart of

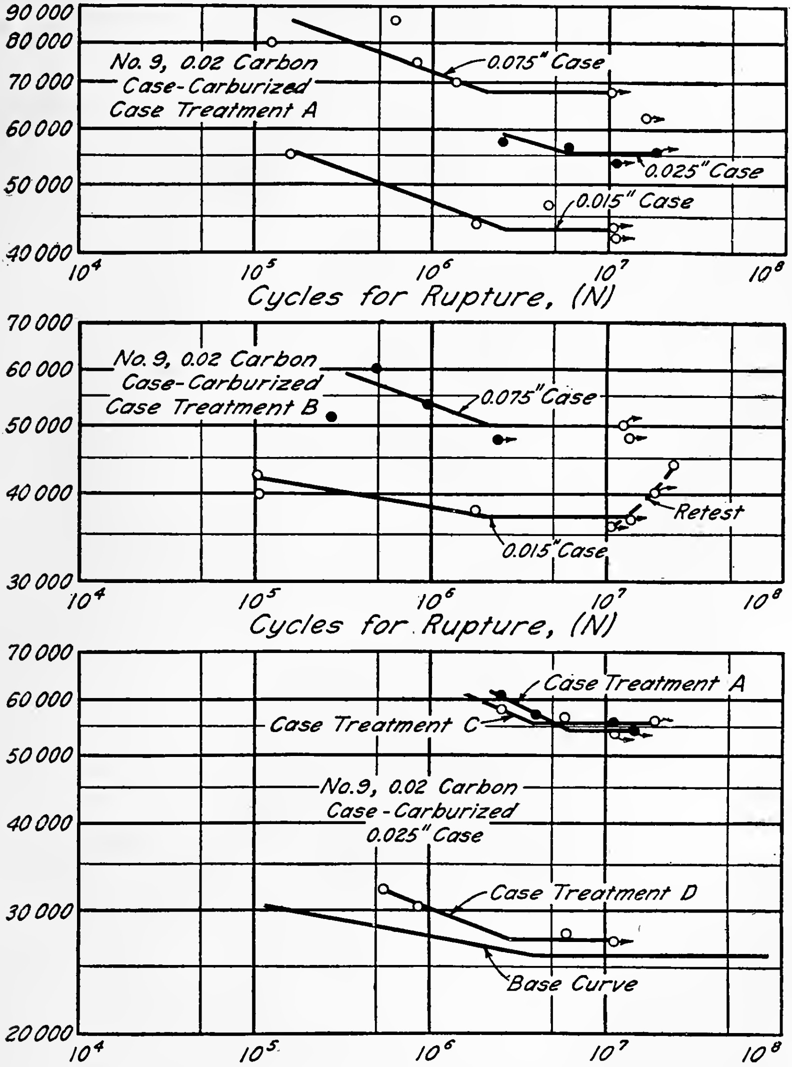

FIG. 57.—*S-N* diagrams for case-carburized Armco iron (0.02 carbon steel).

metal, and it is not surprising that there seems to be closer correlation between fatigue strength and ultimate tensile strength than there is between fatigue strength and any other one physical property. Figure 60 shows a correlation diagram between values of endurance limit and ultimate tensile strength for the metals listed in Tables 2*B* to 8*B* inclusive. For ferrous metals the comparatively narrow "scatter" of plotted points indicates a good degree of correlation, and as a rough approximation for obtaining the fatigue strength of a sound wrought ferrous metal for

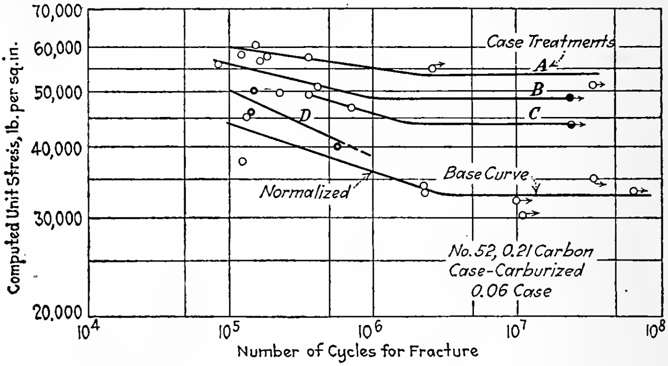

FIG. 58.—*S-N* diagrams for case-carburized steel (0.21 carbon).

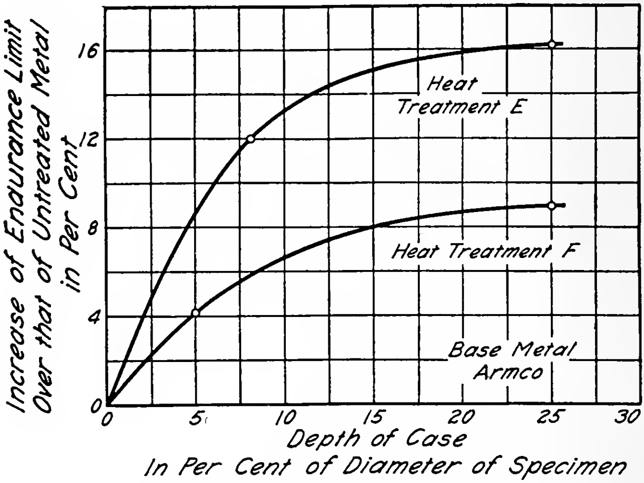

FIG. 59.—Effect of depth of case (Armco iron).

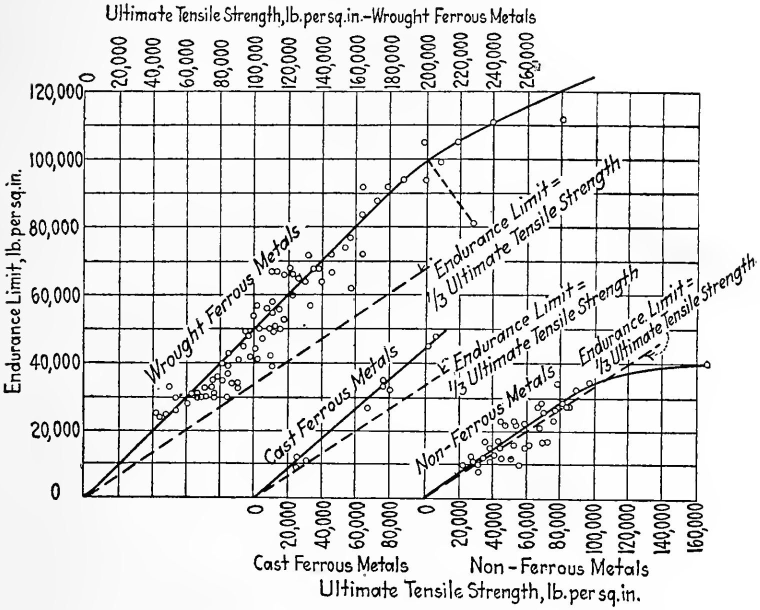

FIG. 60.—Correlation diagram, endurance limit and ultimate tensile strength.

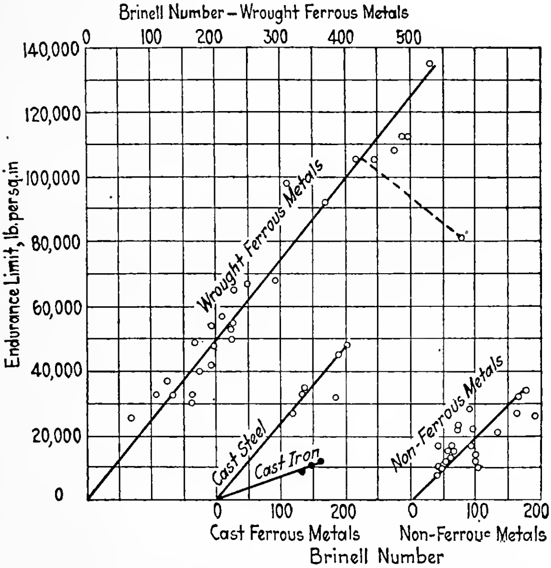

FIG. 61.—Correlation diagram, endurance limit and Brinell number.

which fatigue-test data are not available, the following formula may be used up to an ultimate tensile strength of about 200,000 lb. per square inch:

$$F.L. = 0.50\ T.S.,$$

in which $F.L.$ = estimated endurance limit of the wrought ferrous metal under cycles of reversed flexure (pounds per square inch),

$T.S.$ = ultimate tensile strength of the metal (pounds per square inch).

For cast ferrous metals the data are few, but such data as are available indicate that the following formula may be used to give a rough estimate for the endurance limit of cast steel free from blowholes and abnormal inclusions:

$$F.L. = 0.40 T.S.,$$

in which the symbols are the same as in the equation for wrought ferrous metals.

For non-ferrous metals, as might be expected, there is a wider range of ratio of endurance limit to ultimate tensile strength, the value of the ratio varying from 0.25 to 0.50. No general equation for non-ferrous metals can be given at the present time.

The Brinell number for wrought ferrous metals usually is proportional to the ultimate tensile strength. Figure 61 shows, as might be expected, a fairly good correlation between Brinell number and endurance limit for wrought ferrous metals. For purposes of estimation of fatigue strength of sound wrought ferrous metals, the following formula may be used:

$$F.L. = 250\ BHN,$$

in which $F.L.$ denotes the estimated endurance limit for cycles of reversed flexure (pounds per square inch),

BHN denotes the Brinell number.

For cast ferrous metals the data are too few, and for non-ferrous metals there is too much "scatter" of results to justify giving a formula correlating estimated endurance limit with Brinell number.

Figure 62 gives a correlation diagram for endurance limit and proportional elastic limit for the metals listed in Tables 2*B* to 8*B* inclusive. Here the correlation is distinctly poorer than that shown for wrought ferrous metals in Figs. 60 and 61. Since slip, which seems to be associated with elastic-limit phenomena, occurs before the start of fatigue fracture for most, if not all, metals, it would seem that the elastic strength of a metal is to be regarded as a factor in fatigue strength, but a minor one.

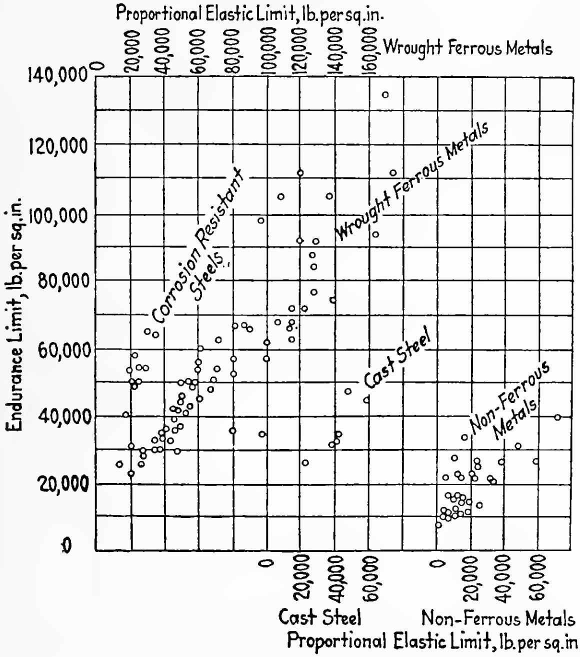

FIG. 62.—Correlation diagram, endurance limit and proportional elastic limit.

In Fig. 63 are plotted values of endurance limit against values of reduction of area of fractured tensile specimen—a value generally accepted as an index of ductility. No correlation is shown in this figure for wrought ferrous metals, cast steel, or for non-ferrous metals. Probably ductility is a major factor in determining resistance to a single impact (Charpy or Izod value) but a very minor factor in determining endurance limit. Ductility is, however, a valuable property of metals, and possibly it is a major factor in determining the effect on fatigue strength of regions of localized high stress.

Charpy impact values were not available for many of the metals listed in Tables 2*B* to 8*B* inclusive. In Fig. 64(*a*)

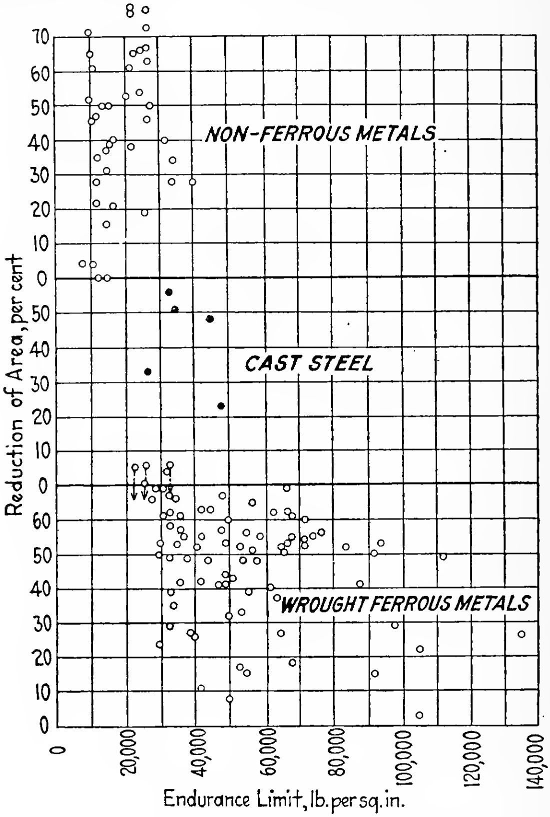

Fig. 63.—Correlation diagram, endurance limit and reduction of area.

is shown a diagram for some 40 steels tested at the University of Illinois, in which endurance limit is plotted against Charpy impact value. No correlation can be detected from a study of the diagram. Fig. 64(*b*) shows a diagram for the same steels in which endurance limit is plotted against the results of repeated-impact tests

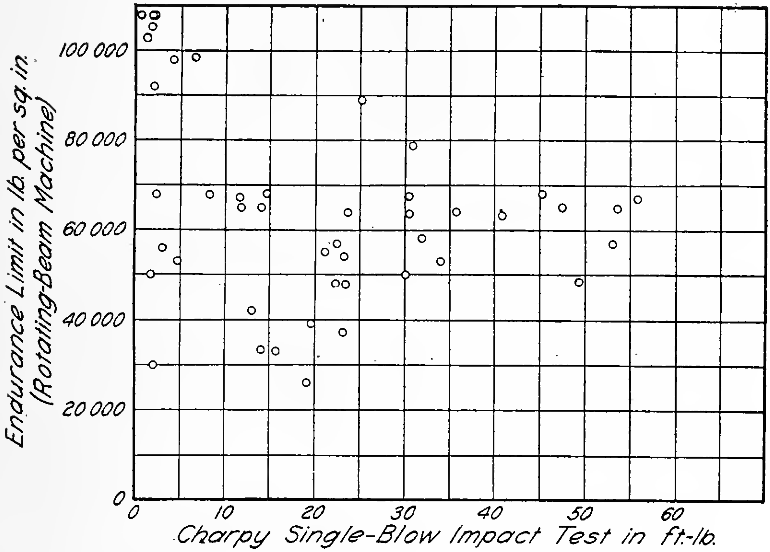

FIG. 64(*a*).—Correlation diagram, endurance limit and Charpy value.

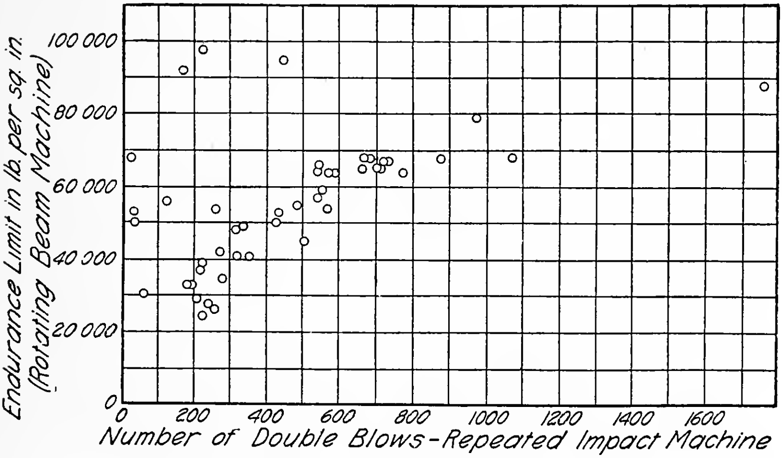

FIG. 64(*b*).—Correlation diagram, endurance limit and results of repeated-impact test.

on a special double-hammer machine. No correlation is shown.

"Scatter" of Fatigue-test Data and Its Significance.[1]—In fatigue tests of some metals the values determined lie very close to the line representing the *S-N* diagram, while in tests of other metals the *S-N* diagram is a line drawn through the estimated middle of a rather wide zone containing the plotted test results. Figure 65 shows two sets of data illustrating the above-mentioned distinction. In any laboratory in which the testing machines are kept in careful adjustment, and careful test methods are used, marked "scatter" of fatigue-test data for a metal may be interpreted as indicating non-uniformity of structure of the metal—dirty metal, metal with minute cracks in it, or badly segregated metal. If, for example, the metal contains many small particles of dirt, the strength of any test specimen depends to no small degree on the chance location of a piece of dirt near the critical section of the specimen. If a large number of specimens of dirty steel are tested, it may be expected that some would show normal strength and others low strength, depending on the chance

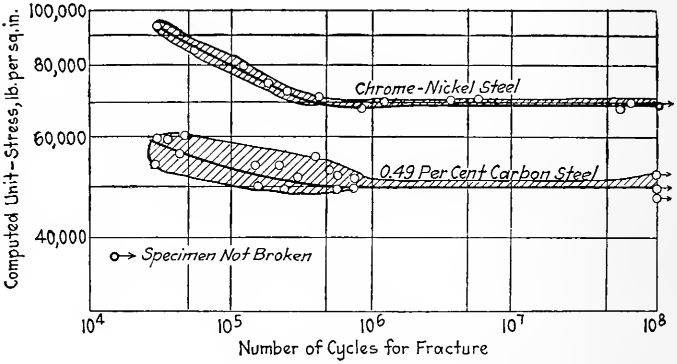

Fig. 65.—"Scatter" of data of fatigue tests.

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[1] So far as the writers have been able to ascertain, the term "scatter" as applied to the irregularity shown by plotted test data was coined by Prof. G. B. Upton of Cornell University.

location of pieces of dirt with reference to areas of high stress.

A quantitative measure of "scatter" would be the average deviation of a test result from the line representing mean results. The determination of this average deviation would involve a rather tedious use of the method of least squares. A qualitative idea of the amount of "scatter"

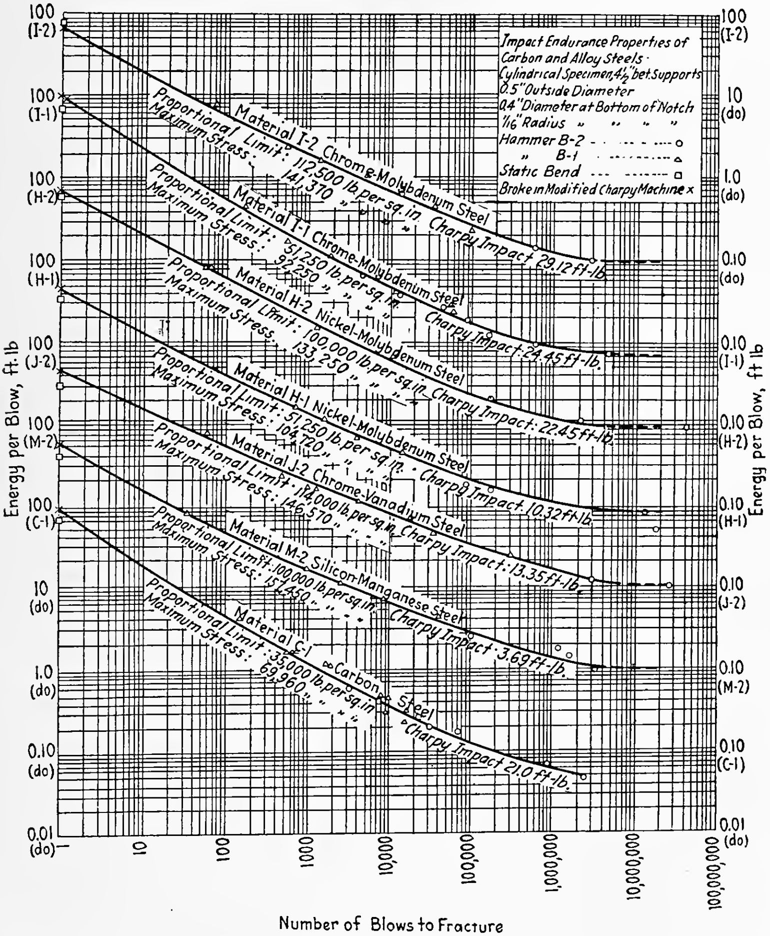

FIG. 66.—Results of impact-endurance tests—Specimen *A*. (*Based on McAdam in Proc. Am. Soc. Test. Materials.*)

may be obtained from the *area* covered by the plotted points, as indicated in Fig. 65.

Impact-endurance Tests.—Machine parts are sometimes subjected to repeated impacts, and it is of interest to inquire in what way the effect of repeated impacts is related to the result obtained from an ordinary endurance test, and also how it is related to the case of a specimen ruptured by a single blow.

McAdam[1] has made tests on various carbon and alloy steels, determining the ordinary rotating-beam endurance limit, an impact-endurance limit, the energy of rupture for a single blow in a modified Charpy machine, and the energy of rupture in slow bending.

Two types of specimen were used in the impact-endurance machine: type *A*, having a diameter of 0.5 in., diameter at bottom of notch 0.4 in., notch sides parallel, radius at bottom, $\frac{1}{16}$ in.; type *B*, diameter 0.75 inch, diameter at bottom of notch 0.6 inch, angle between sides of notch 60 deg., radius at bottom of notch, $\frac{1}{4}$ mm. The specimens were supported at the ends in the impact-endurance machine, as a simple beam, and subjected to reversed impacts by the drop of a hammer.

The purpose of the impact-endurance test was to determine the relation between the energy of blow and the number of blows necessary to cause fracture. By having various hammers and using various heights of drop, the number of blows necessary to cause fracture could be varied from about 500 to many millions.

In Fig. 66 are shown the results obtained with specimens of type *A*. On the left of the diagram are the results obtained from the slow-bending tests and from the single-blow test on a modified Charpy machine. It will be noted that the curves obtained are similar to the ones obtained in a rotating-beam test, in this case the horizontal asymptote being reached in the neighborhood of 10,000,000 impacts.

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[1] "Endurance Properties of Steel: Their Relation to Other Physical Properties and to Chemical Composition," *Proc. Amer. Soc. Testing Materials* vol. 23, p. 56, 1923.

In Table 12 are shown the rotating-beam endurance limits for 100,000,000 cycles and the impact-endurance limits for 10,000,000 cycles. As the last column in the table shows, the ratio of the two endurance limits is fairly constant.

TABLE 12.—IMPACT—ENDURANCE PROPERTIES OF CARBON STEELS AND OF ALLOY STEELS

Reported by D. J. McAdam, Jr., from the U. S. Naval Engineering Experiment Station, Annapolis, Md.

Steel	Charpy[1] value, foot-pounds	Impact-endurance limit, foot-pounds	Rotating-beam endurance limit,[2] pounds per square inch	Ratio of rotating-beam endurance limit to impact-endurance limit
I-2 chrome-molybdenum	61.5	0.100	67,500	670,000
I-1 chrome-molybdenum	92.6	0.075	47,000	630,000
H-2 nickel-molybdenum	68.0	0.100	57,000	570,000
H-1 nickel-molybdenum	46.3	0.085	46,500	550,000
J-2 chrome-vanadium	45.6	0.110	67,000	610,000
M-2 silico-manganese	56.4	0.098	62,000	630,000
C-1 carbon	93.2	0.048	30,000	620,000

[1] A modified Charpy test was used. Values given have relative significance.
[2] Rotating-cantilever type of testing machine used.

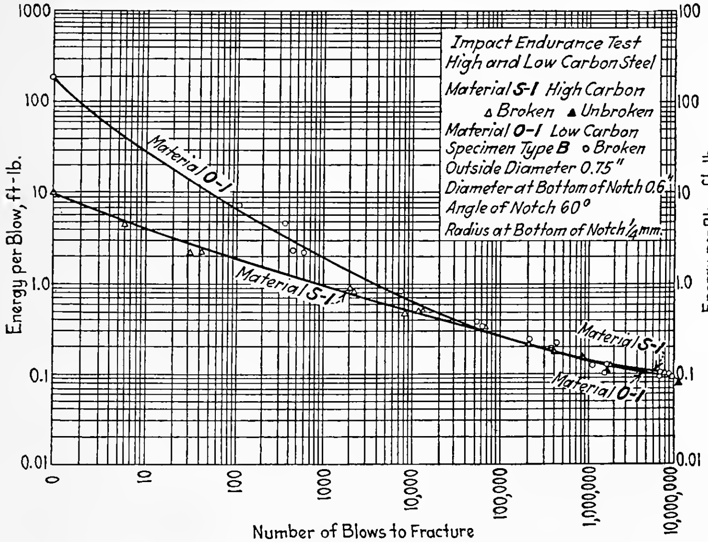

FIG. 67.—Results of impact-endurance tests—Specimen *B*. (*Based on McAdam in Proc. Am. Soc. Test. Materials.*)

McAdam says:

> Evidently, therefore, the ordinate at the extreme left of the impact-endurance graph depends on the impact properties, and the ordinates at the extreme right depend on the endurance properties of the metal. Between these two extremes the influence of the impact properties decreases and that of the endurance properties increases with increase in the number of blows necessary to cause fracture.

Figure 67 shows the results obtained on type *B* specimen on a high- and low-carbon steel. These tests were made because the type *A* specimen had a larger radius at the bottom of the notch than the standard impact specimen. At the left of the diagram in Fig. 67 the ordinates are approximately proportional to the Charpy values, and at the extreme right the ordinates are approximately proportional to the rotating-beam endurance limits for 10,000,000 cycles. The conclusions drawn from Fig. 66, therefore, apply also to this diagram.

CHAPTER VII

THE EFFECT OF RANGE OF STRESS ON FATIGUE STRENGTH

Range of Stress Defined.—While the most common laboratory tests of fatigue of metals are those in which the stress is completely reversed, yet in structural and machine parts there are numerous cases in which stress fluctuates between minimum and maximum limits which in some cases are the same in sign and in others opposite in sign. "Range of stress" is the algebraic difference between stress in the cycles to which a test specimen or a machine or structural part is subjected. This range of stress may be conveniently indicated by a numerical value which may be called the "range ratio," and which indicates the algebraic ratio of minimum stress to maximum stress during a cycle of stress. Thus a range ratio of -1.0 indicates that the ratio of minimum stress to maximum stress is -1.0, or in other words the stress is completely reversed during a cycle; a range ratio of 0 indicates that during a cycle the stress varies from 0 to maximum; a range of ratio of -0.25 indicates that during a cycle the stress varies from a maximum to a value of 25 per cent of the maximum but in the opposite direction.

If S_{max} denotes the maximum unit stress during a cycle,
S_{min} denotes the minimum unit stress during a cycle,
R denotes the range of stress during a cycle,
r denotes the range ratio for the cycle (r is never numerically greater than 1),
S_{av} denotes the mean stress during a cycle,

then $R = S_{\text{max}} - S_{\text{min}}$, remembering that S_{max} and S_{min} may be opposite in sign,

$r = S_{\text{min}}/S_{\text{max}}$, remembering that r may be either + or −, $S_{\text{av}} = \frac{1}{2}(S_{\text{max}} + S_{\text{min}})$, again remembering that S_{max} and S_{min} may be opposite in sign,

$$S_{\text{max}} = S_{\text{av}} + \frac{1}{2}R \text{ and } S_{\text{min}} = S_{\text{av}} - \frac{1}{2}R$$

Evidently the endurance limit for a metal will have different values for different ranges of stress. As indicated on page 23 the endurance limit for cycles of stress varying from zero to a maximum (range ratio 0) will be greater than the endurance limit for completely reversed stress (range ratio −1). For cycles of stress in which the stress varies between two limits of the same sign (range ratio +), the endurance limit is still higher. The developing of formulas to express the effect of range of stress on endurance limit has occupied the attention of a number of investigators, and some of the more important formulas will be discussed in this chapter.

Experimental data available for the study of the effect of range of stress are much fewer than could be wished. This is largely because of the fact that for every pair of points on a range diagram, that is, for every value of range ratio for a metal, it is necessary to make a number of tests sufficient to give data for an S-N diagram from which to determine the endurance limit for that particular range of stress. It is obvious that to get the data for determining a complete stress-range diagram requires a long time and many tests.

Gerber's Formula.—One of the early students of Wöhler's work was Gerber, who in 1872 proposed the following formula for effect of range of stress on endurance limit.[1]

$$S_{\text{max}} = \frac{R}{2} + \sqrt{S_u^2 - nS_uR},$$

in which S_{max} is the endurance limit for any given range of stress,

R is the range of stress during a cycle,

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[1] "Relation between the Superior and the Inferior Stresses of a Cycle of Limiting Stress," *Zeit. Bayerischen Arch. Ing.-Vereins*, 1874.

S_u is the static ultimate tensile strength of the metal,

n is an experimentally determined constant, usually between 1.33 and 2.00.

Gerber's formula with a proper value for n fits test results fairly well, but, as pointed out later by Barr,[1] a formula in terms of *range ratio* is a more convenient formula to use than one in terms of *range of stress.*

Formulas of Launhardt and Weyrauch.—Two early studies of effect of range of stress were those of Launhardt and of Weyrauch, who derived formulas from considerations based on the elastic action of metals and on the formulas of mechanics of materials, checked by a few experimental results. In 1873 Launhardt[2] suggested the following formula for cases in which the stress was not reversed (range ratio from 0 to +1.0)

$$S_{\max} = S_0 + r(S_u - S_0)$$

in which, S_0 is the endurance limit when $r = 0$, and the other symbols have the same values as on pages 173 and 174; r is always + for Launhardt's formula.

In 1876 Weyrauch[3] suggested the following formula for cases when the stress was wholly or partly reversed (r is negative):

$$S_{\max} = S_0 + r(S_0 - S_{-1})$$

in which S_{-1} is the endurance limit for complete reversal of stress ($r = -1$) and the remaining notation is the same as in Launhardt's formula.

Figure 68 shows their two formulas plotted in a diagram. It is inconvenient to have to determine both S_0 and S_{-1} for a metal, and a consideration of the combined Launhardt-Weyrauch diagram led J. B. Johnson to suggest a combined straight-line diagram which is equivalent to the Goodman diagram discussed in a succeeding paragraph.

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[1] KIMBALL and BARR, "Elements of Machine Design," p. 86, New York.

[2] *Zeit. des Arch. Ing.-Vereins*, Hanover, 1873.

[3] *Proc. Brit. Inst. Civil Eng.*, vol. 63, p. 275, 1880–1881.

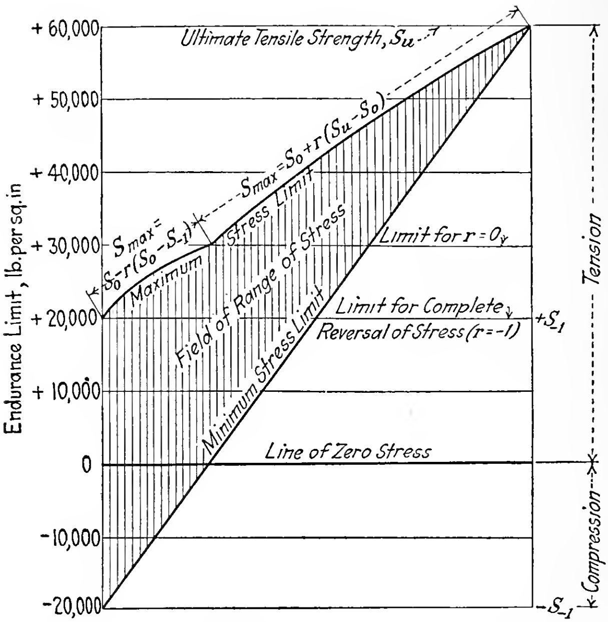

FIG. 68.—Launhardt-Weyrauch diagram for range of stress.

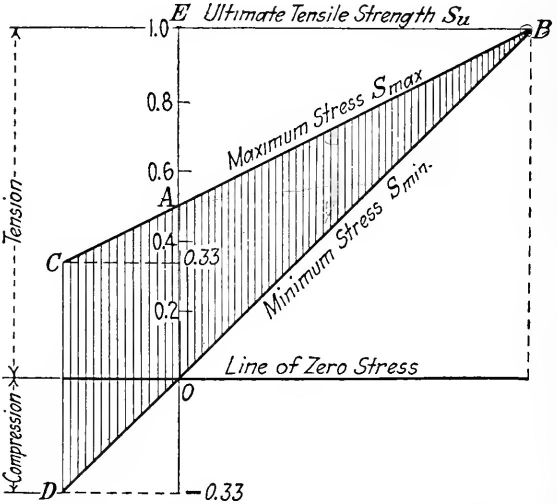

FIG. 69.—Goodman diagram for range of stress.

The Goodman Diagram.—Figure 69 is a stress-range diagram proposed by Goodman[1] and covering the field of both the Launhardt and the Weyrauch formulas. The ordinate of the line *EB* represents the static ultimate tensile strength of the metal. The minimum stresses are plotted along a 45-deg. line *DOB*. The horizontal line through *O* is the line of zero stress, tensile stress being plotted above the line and compressive stress below it. According to the "dynamic theory" of suddenly applied loads, the minimum or dead-load stress plus *twice* the live-load stress equals the static ultimate strength; and the maximum applied stress should fall on a line *CAB*, such that the point *A* is five-tenths of the ultimate static strength. Goodman plotted endurance limits obtained by various investigators after the material had been subjected to over 4,000,000 cycles of stress, and found that these experimental points fell fairly well on the straight line *CAB*.

As the diagram shows, when the minimum stress is zero, then the maximum stress for indefinite endurance should be five-tenths of the ultimate static strength. When the stress is completely reversed at *CD*, then the plus and minus stresses should be one-third of the ultimate stress. Presumably a diagram similar to the one shown in Fig. 69 would hold when the stress above the zero line is compression and that below it is tension. Experimental data, however, are very meager for these combinations of compressive stress.

According to the Goodman diagram the range of stress (algebraic difference between maximum and minimum) is greater for reversed stresses, and decreases as the maximum stress is increased above one-third of the ultimate strength, being actually zero when the maximum stress coincides with the ultimate. In other words, as the maximum stress is increased, the minimum stress must also be increased algebraically in order that the material may be stressed indefinitely without fracture.

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[1] "Mechanics Applied to Engineering," 9th ed., p. 634. See also Fidler, "Practical Treatise on Bridge Construction."

Criticism of the Goodman Diagram.—The statement has been made that the Goodman diagram does not always give values which are accurate, and that sometimes it gives values which are not safe.[1] The Goodman diagram and the Johnson-Goodman formula (see p. 179) imply that the endurance limit of a metal under cycles of completely reversed stress is one-third of the static ultimate tensile strength. An examination of Fig. 60 (p. 163) shows that for ferrous metals, both wrought and cast, the endurance limit is higher than this value, for non-ferrous metals this ratio of one-third averages results fairly well, but several values of endurance limit fall below it. So for completely reversed stress the Goodman diagram and the Johnson-Goodman formula seem to give results which are on the safe side for ferrous metals, but which are not on the safe side for a considerable number of non-ferrous metals. The writers of this book have examined with especial care all the recorded results which they could find for metals tested under varying ranges of stress. The list included results of experiments by Wöhler, Bauschinger, Haigh, Smith, Smith and Wedgwood, and Moore and Jasper. The discussion will be limited to the two important cases in which the stress is completely reversed, and in which the minimum stress is zero. The authors find that in only two out of twenty-four series of tests were *both* endurance limits respectively less than 0.33 and 0.50 of the ultimate tensile strength as called for by the Goodman diagram. For one case one endurance limit was 0.25 and the other 0.47 of the ultimate, while for the second case the values were 0.29 and 0.493, respectively. In one other series of tests the endurance limit for minimum-stress zero was less than 0.50 of the ultimate, and this series gave a value of 0.44. For this last case no information was given for completely reversed stresses. In five cases out of the twenty-four series of tests the endurance limit for completely reversed stress was less than one-third of the ultimate tensile strength, but in all these cases the endurance limit

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[1] GOUGH, "Fatigue of Metals," p. 72.

for minimum-stress zero ($r = 0$) was equal to or greater than one-half the ultimate tensile strength. These five cases gave values for ratio of endurance limit for completely reversed stress to ultimate tensile strength of 0.270, 0.314, 0.299, 0.302, and 0.263, respectively. The results cited in this paragraph were from Wöhler's tests and from Bauschinger's tests, and four of the five results cited in the sentence before this were from Bauschinger's experiments on axial stress, for which it is admittedly difficult to avoid the setting up of unknown bending stresses.

Another series of experiments which has been cited[1] as an argument against the Goodman diagram, is a series by J. H. Smith[2] on plain carbon and nickel-alloy steels. In these experiments the endurance limits obtained by Smith's "yield" experiments, were checked by actual repeated-stress tests. Most of these tests, however, were carried out to less than 1,000,000 cycles, so that the endurance limits would in general be too high. Gough calculated the value of the range of stress when the minimum stress is zero, and compared these values with the experimental values. He found by this method that five cases out of nineteen gave computed values which were greater than the experimental values, the worst case being 7 per cent in error.

The Johnson-Goodman Formula.—Working independently of Goodman, J. B. Johnson "straightened out" the Launhardt-Weyrauch diagram (Fig. 62) and obtained a diagram like the Goodman diagram.[3] Johnson developed a formula which could be used in place of the diagram, and this formula was afterwards simplified by Barr.[4] The formula is

$$S_{\max} = \frac{0.5S_u}{1 - 0.5r}$$

in which the notation is that given on page 173. It is

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[1] GOUGH, "Fatigue of Metals," p. 78.

[2] *Jour. Brit. Iron and Steel Inst.*, No. 2, p. 246, 1910.

[3] JOHNSON, "Materials of Construction," 5th ed., p. 781.

[4] *Sibley Jour. Eng.*, December, 1901.

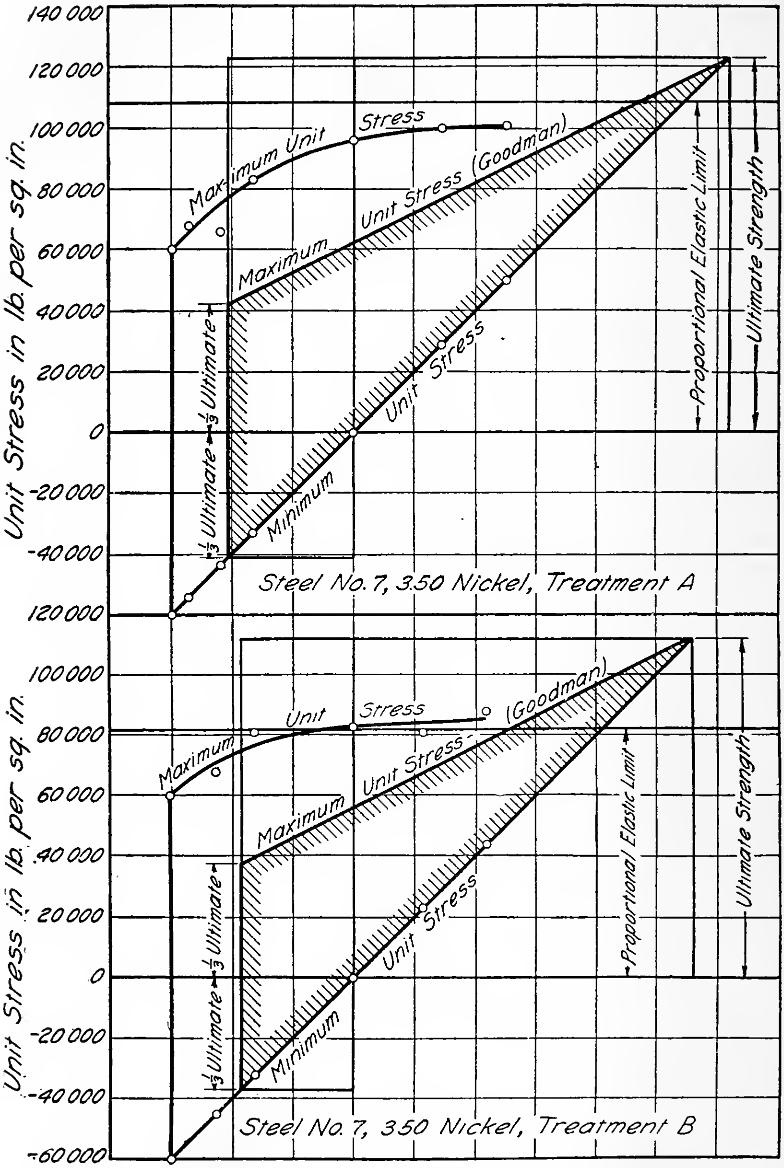

FIG. 70.—Comparison of experimental results with Goodman's diagram for 3.5 per cent nickel-steel (*A* and *B*). (*Bull.* 136, *Univ. of Ill. Eng. Expt. Sta.*)

again called to mind that r, the range ratio, is positive if the stress limits of a cycle are both tension or both

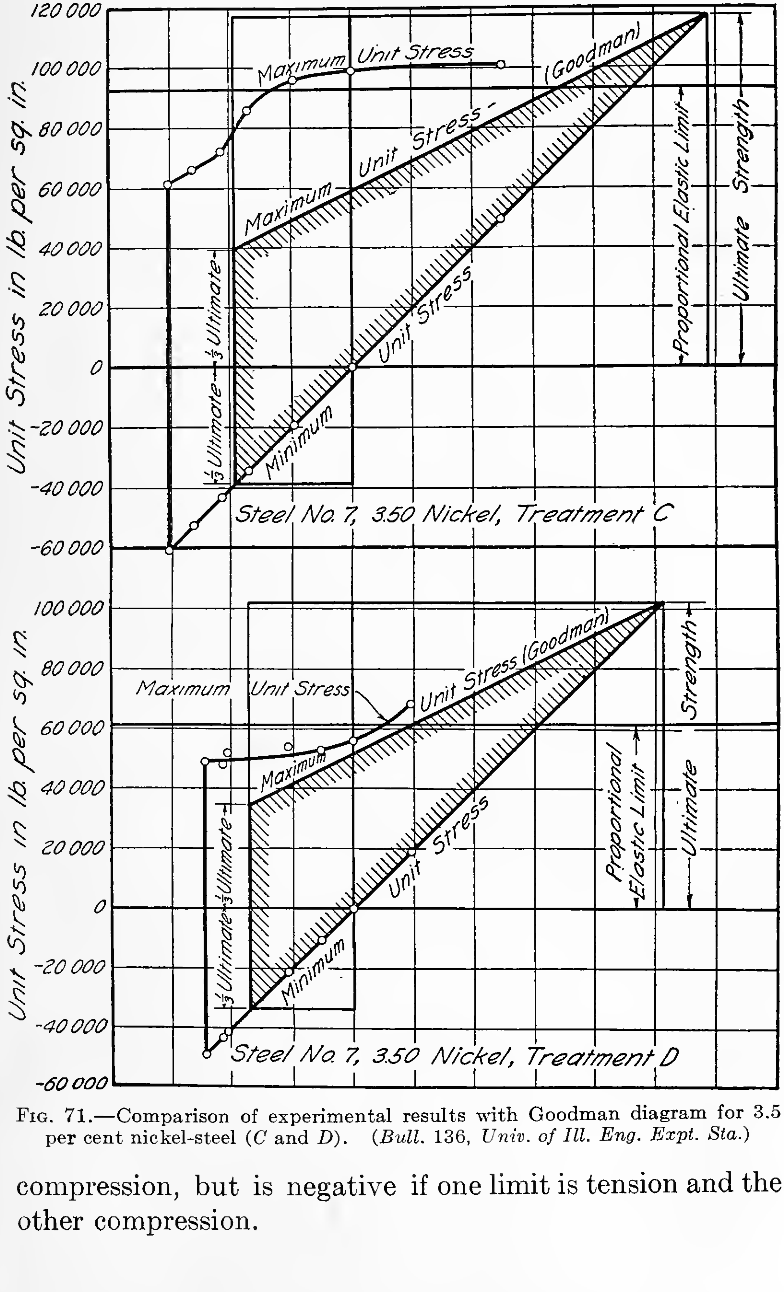

FIG. 71.—Comparison of experimental results with Goodman diagram for 3.5 per cent nickel-steel (*C* and *D*). (*Bull.* 136, *Univ. of Ill. Eng. Expt. Sta.*)

compression, but is negative if one limit is tension and the other compression.

The Goodman diagram not only implies that $S_{-1}: S_u = \frac{1}{3}$, but also that $S_0 : S_{-1} = 1.5$ (see p. 175 for notation). The writers of this book wish to examine this second implication of the Goodman diagram in the light of the available test data for fatigue tests with varying range of stress. Examining the test data referred to in the foregoing paragraphs, it is seen that, taking first the results for tests made

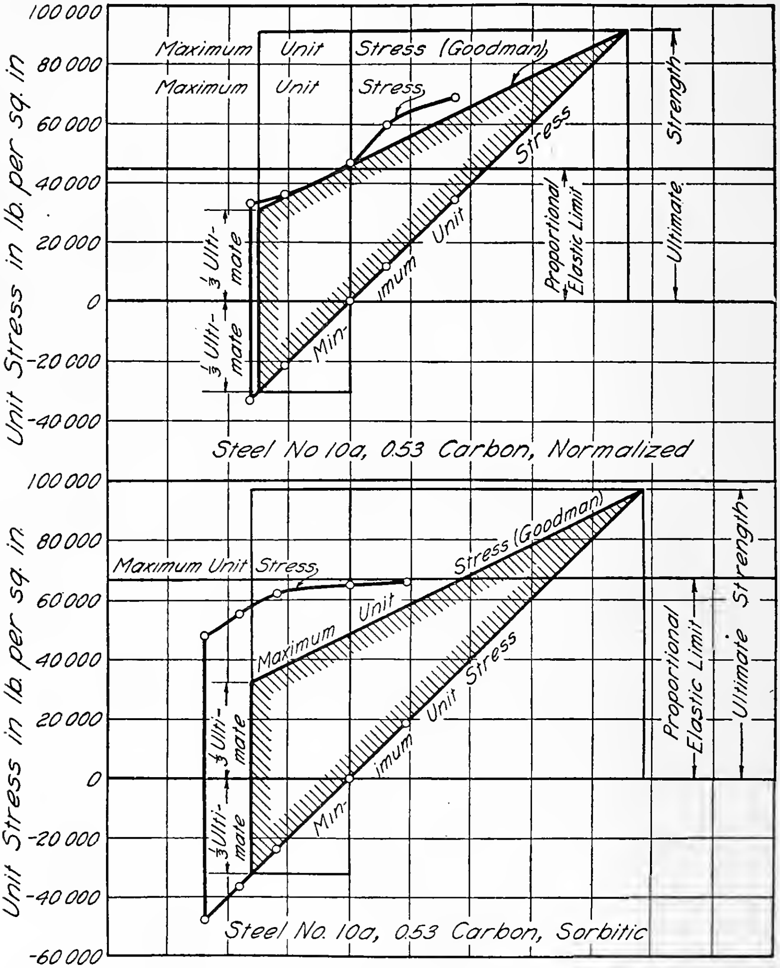

FIG. 72.—Comparison of experimental results with Goodman diagram for 0.53 per cent carbon steel. (*Bull.* 136, *Univ. of Ill. Eng. Expt. Sta.*)

by applying repeated stress to destruction (omitting for the moment the tests by Smith and Wedgwood), only two series show a ratio of $S_0:S_{-1}$ less than 1.5. One of these is a series of tests of nickel steel by Moore and Jasper which shows a ratio of 1.47,[1] and the other is a series of tests by J. H. Smith in which the value of the ratio is 1.44, but in which S_0 was 85 per cent of the ultimate tensile strength, probably above the elastic limit of the material, so that failure was elastic failure rather than fatigue failure.

Figures 70, 71, and 72 show results obtained at the University of Illinois[2] on a number of different steels tested under various ranges of stress. It will be noted that in a number of these steels the maximum unit stress was above the proportional elastic limit of the material, and in no case was the maximum unit stress less than that given by the Goodman diagram. In view of the fact, however, that Bairstow found that when the mean stress is tension, the specimen suffers a permanent extension, it is probable that it would be unwise in any case to use a stress range in which the maximum stress exceeded the elastic limit. For the same reason, when the Goodman diagram is used, it seems desirable not to use a maximum stress which exceeds the elastic limit of the material. It must always be remembered that elastic failure is usually as important a consideration as is fatigue failure.[3]

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[1] Moore and Jasper report some other tests with values of ratio S_0: S_{-1} less than 1.50, but in all cases the stress at S_0 was above the proportional elastic limit of the metal, and failure was probably elastic failure rather than fatigue failure.

[2] *Univ. Illinois Eng. Exp. Sta., Bull.* 136, pp. 67–69, 1923.

[3] As noted on p. 188, results for repeated torsion tests indicate that there is only a slight difference in range of stress for various range ratios for cycles of shearing (torsional) stress. McAdam is of the opinion that Figs. 70, 71, and 72, showing results for flexure tests, support the idea that for cycles of tensile-compressive stress the endurance range is practically constant within the elastic range. In the figures named, he suggests that the line for maximum stress may be drawn parallel to the line for minimum stress up to values slightly above the proportional elastic limit, and that beyond that portion the maximum-stress line should be drawn horizontal (see footnote, p. 190).

Considering now the experiments by J. H. Smith in which endurance limit was determined by "yield" method, and to which reference is made on page 179, the writers of this book would like to present a comparison of experimental values and computed values, the computed values of S_0 being obtained by multiplying the experimental values of S_{-1} by the factor 1.5. Since there is not for all metals a constant ratio $S_{-1}:S_u$, this method tends to correct errors due to individual characteristics by reducing or increasing computed values of S_0 when the value of S_{-1} happens to be below or above normal. This computation gives the values for Smith's results shown in Table 13.

TABLE 13.—EXPERIMENTAL AND COMPUTED VALUES OF S_0 THE ENDURANCE LIMIT FOR CYCLES OF STRESS VARYING FROM ZERO TO A MAXIMUM

Based on tests made by J. H. Smith

Series number	Experimental value of S_0, tons per square inch	Computed value of S_0, tons per square inch	Error based on experimental value, per cent
1	18.5	17.8	− 4
2	19.4	18.9	− 3
3	19.2	21.6	+13
4	18.6	19.7	+ 6
5	20.6	20.3	− 1
6	21.2	20.6	− 3
7	21.7	21.7	0
8	22.4	22.2	− 1
9	23.6	23.2	− 2
10	19.3	19.3	0
11	25.6	25.5	0
12	22.3	21.4	− 4
13	27.1	26.6	− 2
14	23.2	20.3	−13
15	21.2	21.5	+ 1
16	23.2	21.8	− 6
17	27.4	28.4	+ 4
18	28.0	29.4	+ 5
19	22.0	25.7	+17

As already explained, the experimental method used by Smith would tend to give values of S_{-1} which are too large.

Since the computed values of S_0 are 1.5 times S_{-1}, these values would also tend to be too large. Table 13 shows six computed values which are higher than the experimental, and only two of these are seriously in error. It is interesting to note that the average error in the table is +0.4 per cent.

It is the opinion of the writers of this book that the criticism of the Goodman diagram and of the Johnson-Goodman formula is justified in so far as it is a criticism of the assumption that for *all* metals the ratio $S_{-1}:S_u$ has the value $\frac{1}{3}$. It is believed that the implication that the ratio $S_0:S_{-1}$ has a value of 1.5 for all metals is a reasonably safe assumption so far as available test data show.

Modified Johnson-Goodman Formula.—The writers of this book wish to suggest a formula for effect of range of stress, a formula which has the general form of the Johnson-Goodman formula, but which is not based on any assumed ratio of $S_{-1}:S_u$, but rather on an experimentally determined value of S_{-1} for each metal. The value of 1.5 for the ratio $S_0:S_{-1}$ is retained, and the proposed formula is

$$S_{\max} = \frac{1.5S_{-1}}{1 - 0.5r} \text{ or } \frac{S_{\max}}{S_{-1}} = \frac{3}{2 - r}$$

in which the notation is that given on page 173.[1] If $r = 0$, $\frac{S_{\max}}{S_{-1}} = 1.5$, the Goodman ratio.

In Fig. 73 this equation is plotted as a dotted line, and there are also plotted test results from the University of Illinois and from the work of Haigh at the Greenwich Royal Naval Academy. This formula seems to fit the experimental data available fairly well, and where it differs from such data, it usually gives results which are on the side of safety. It is to be noted that in the modified Johnson-Goodman formula S_{-1} is an *experimentally deter-*

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[1] This modified Johnson-Goodman formula was used by Moore, Kommers, and Jasper as the basis of the curve shown in Fig. 21 of the paper on "Fatigue of Metals" presented before the American Society for Testing Materials in 1922, and the formula is given in Moore's "Textbook of the Materials of Engineering," 3d ed., p. 52.

mined endurance limit—an endurance limit which may be determined by a series of rotating-beam tests. It should be further noted that neither the modified Johnson-Goodman nor any other formula justifies the consideration of unit stresses which are high enough to cause static failure—either elastic failure or rupture.

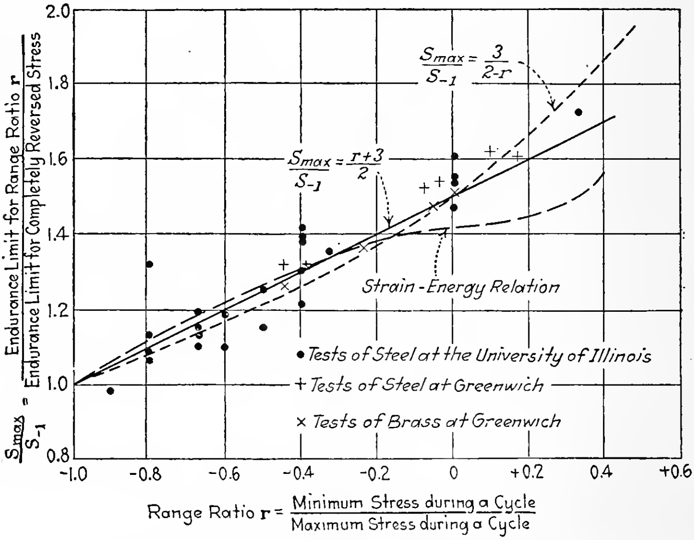

FIG. 73.—Diagrams for modified Johnson-Goodman formula, Howell formula, and strain-energy relation for effect of range of stress.

The Howell "Straight-line" Formula.—Howell[1] has suggested an empirical "straight-line" formula based on test results obtained at the University of Illinois. The Howell formula is

$$\frac{S_{\max}}{S_{-1}} = \frac{r+3}{2}$$

in which the notation is that given on pages 173 and 175. The graph of the Howell formula has been plotted in Fig. 73, and it is seen to agree fairly well with test data for values of r from -1.0 to 0. For this range of values of r the modified Johnson-Goodman formula gives slightly

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 136, p. 89, 1923.

"safer" values than does the Howell formula. For values of r above 0, the Howell formula is the more conservative.

The Strain-energy Hypothesis.—In 1919 Haigh[1] suggested that the strain energy absorbed within the elastic limit might be of more general application as a criterion of failure than the hypotheses of Saint Venant, Rankine, or Guest.

In 1923 Jasper[2] applied this method to the case of repeated stresses and suggested that the limiting energy per unit volume per cycle of stress might be found to be the same for the cases of reversed stresses and those not reversed. For the case in which the maximum stress is of opposite sign to the minimum stress, he derived the formula

$$\frac{S_{\max}}{S_{-1}} = \frac{2}{1 + r^2}$$

For the case in which the maximum and minimum stresses are of the same sign, he derived the formula

$$\frac{S_{\max}}{S_{-1}} = \frac{2}{1 - r^2}$$

in which the notation is that given on page 173.

In Fig. 73 the graph of the strain-energy relation is plotted. The modified Johnson-Goodman formula, the Howell "straight-line" formula, and the strain-energy relation all fit the experimental data fairly well for values of r from -1.0 to 0. In this range the modified Johnson-Goodman formula is slightly more conservative than either of the others. Beyond this range, that is, for positive values of r, the strain-energy relation gives values of $S_{\max}$ lower than the other two formulas, and also lower than test results. Test data, however, are very few for tests with positive values of r, and the value of endurance limit in most cases exceeds the elastic limit, with the result that for structural and machine parts subjected to cycles of stress varying between a maximum and a minimum of the same sign, the danger of elastic failure is usually greater than the danger of fatigue failure.

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[1] *Brit. Assoc. Repts.*, p. 486, 1919.

[2] *Phil. Mag.*, p. 609, 1923.

The writers of this book believe that in the present state of knowledge of fatigue of metals the modified Johnson-Goodman formula is a safe and a convenient formula to use in the design of structural and fatigue parts subjected to cycles of tensile-compressive stress. This formula should be regarded, however, as a tentative empirical formula which may be modified or superseded as the result of further experimental investigation.

Range of Stress in Torsion.—Moore and Jasper[1] have made some torsion tests on six series of steels in various conditions of heat treatment, including 0.49 per cent carbon, 1.20 per cent carbon, and 3.5 per cent nickel-alloy steels. The ratio of the endurance limit for various values of r to the endurance limit for complete reversal varied from 1.08 to 2.

The cases in which the stress was completely reversed and in which the minimum stress was zero are of particular interest here. The ratio $S_0:S_{-1}$ varied from 1.85 to 2. According to the Goodman diagram this ratio should be 1.5.

It is desirable to translate these results in terms of range of stress. According to Goodman's diagram the range of stress for the case in which the minimum stress is zero should be 0.75 of the range for complete reversal. The above results show that the range of stress, for minimum-stress zero, was either the same as for completely reversed stress or in the worst case 0.93 of that range. This indicates, therefore, that for the case of torsion the range of stress is much more nearly constant than it is for the case of bending stress and axial stress.

Similar results have been obtained by McAdam[2] at the U. S. Naval Engineering Experiment Station at Annapolis in eight series of tests including plain carbon steels and alloy steels. He found that in the worst case the difference in range of stress was 10 per cent, and averaged about 5 per cent.

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 142, p. 72, 1924.

[2] *Proc. Am. Soc. Testing Materials*, vol. 24, Pt. II, p. 574, 1924.

It is interesting to note that McAdam found in his experiments with stress of the same sign that there seemed to be a maximum stress beyond which the upper limit of stress could not be moved without a corresponding increase in the minimum stress, and therefore a decrease in the range of stress. He found this "endurance yield point" to be in the neighborhood of the elastic limit and yield point. This fact seems to indicate that in torsion tests, also, the upper limit of any range of stress should not exceed the elastic limit of the material.

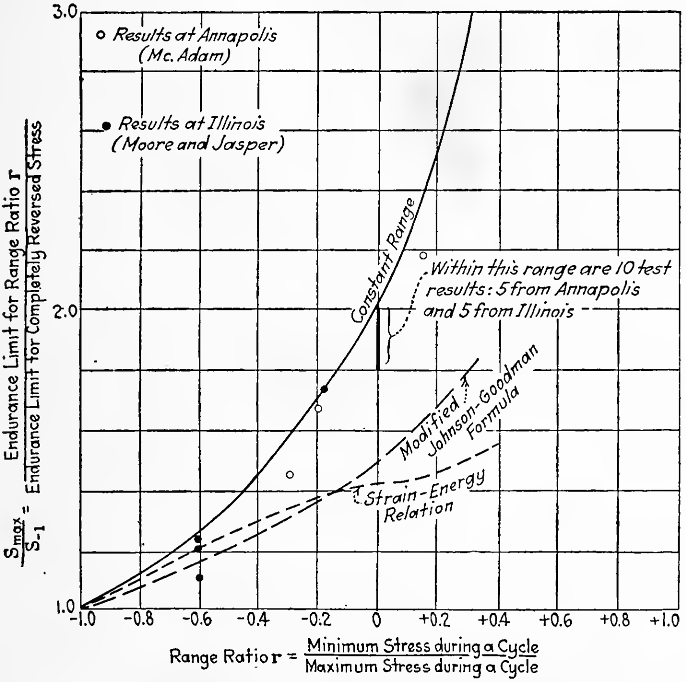

FIG. 74.—Diagrams for modified Johnson-Goodman formula, strain-energy relation, and constant-range relation for effect of range of stress in torsion. (*Shear.*)

Figure 74 shows the results of the tests at Annapolis and the tests at Illinois.[1] The graphs indicate that for cycles of shearing stress the constant-range relation fits

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[1] Omitting the cases for which the stress was above the elastic limit of the metal.

experimental results better than the strain-energy relation, or than the modified Johnson-Goodman formula.

From these results it may be concluded that for cycles of torsional (shearing) stress the assumption of a constant-range relation involves no serious error, at least for stresses below the proportional elastic limit of the metal. It is also evident, as pointed out by McAdam, that for torsion stresses a steel of high elastic ratio as well as high tensile strength is desirable for machinery parts to resist fatigue.

On the basis of the constant range hypothesis the following formula may be used:

$$S_{\max} = 2S_{-1} + S_{\min} \text{ or } S_{\max} = \frac{2S_{-1}}{1 - r}.$$

Here S_{-1} denotes the endurance limit under cycles of completely reversed stress, and $S_{\min}$ and r are minus if the stress is wholly or partly reversed. When S_{-1} is known, $S_{\max}$ may be calculated if $S_{\min}$ or r (the range ratio) is known.[1]

Formulas Involving Number of Cycles.—Before it was definitely established that metals had an endurance limit, a number of formulas were developed which attempted to show the relation between the maximum stress at which a material would fail and the corresponding number of cycles for rupture. One developed by Moore and Seely[2] may be cited as an example:

$$S_{\max} = \frac{B}{(1 - r)N^{1/8}}.$$

Here B was a constant depending upon the kind of material, r the ratio of minimum to maximum stress, and N the number of cycles of stress necessary to produce rupture. The formula was based upon the assumption that the S-N diagram when plotted on log paper was an inclined straight

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[1] McAdam holds that this formula, involving a constant range relation, may be used for cycles of flexural stress or of compression-tension, with the *limitation* that $S_{\max}$ must never be considered to be higher than the elastic limit of the metal.

[2] *Proc. Amer. Soc. Testing Materials*, vol. 15, p. 438, 1915, and vol. 16, p. 470, 1916. (The 1916 paper corrects a numerical error in the 1915 paper.)

line which extended to any value of N, however large. That assumption is now known to be wrong, and the formulas have no value for stresses below the endurance limit.

It does not seem at the present time that there is any great need for a formula involving the number of cycles for rupture. Even when the number of cycles to which a structural or machine part is to be subjected is definitely known (which is rarely), it does not seem probable that a stress could be chosen with the degree of precision which is here contemplated. It would seem that if the endurance limit for a particular material is known, there is available all that is necessary for the designer. At this point of departure nothing can take the place of engineering judgment in determining in a particular case what factor of safety is to be allowed for such contingencies as unexpected loads, danger to life in case of failure of a part, and the many other factors which particular conditions bring up for consideration.

If at any time a general diagram should be desirable showing the numbers of cycles of stress which can be withstood under various conditions, the authors wish to suggest one which may prove useful as a basis for a rough estimation. The form of the diagram was suggested by a diagram published by Stromeyer,[1] but the authors wish to apply it to a modified Goodman diagram, based on an experimentally determined S-N diagram for any given metal under cycles of reversed stress and the modified Johnson-Goodman formula. Figure 75 shows such an endurance diagram for 1.02 per cent carbon steel, oil quenched from 1450°F. The S-N diagram for reversed stress showed the endurance limit developed at 500,000 cycles, a "life" of 100,000 cycles for a stress range of ±115,000 lb. per square inch, a "life" of 10,000 cycles for a stress range of ±130,000 lb. per square inch, and a "life" of 1,000 cycles for a stress range of ±150,000 lb. per square inch. To construct the diagram, locate the pairs of points AC, DE, FG, and HK from the S-N diagram for reversed stress for the metal.

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[1] *Proc. S. Wales Inst. Eng.*, 1922.

Locate *B* at a height corresponding to the ultimate tensile strength of the metal, and at any convenient distance to the right of line *KOH*. Draw the straight lines *CB*, *EB*, *GB*, and *KB*. Draw the curved lines *AB* by the use of the modified Johnson-Goodman formula, making $QP = 1.5\,OA$. Draw the curved lines *DB*, *FB*, and *HB*, making the vertical spacing from the line *AB* the same for any given abscissa as the corresponding spacings for the lines *CB*, *EB*, *GB*, and *KB*.

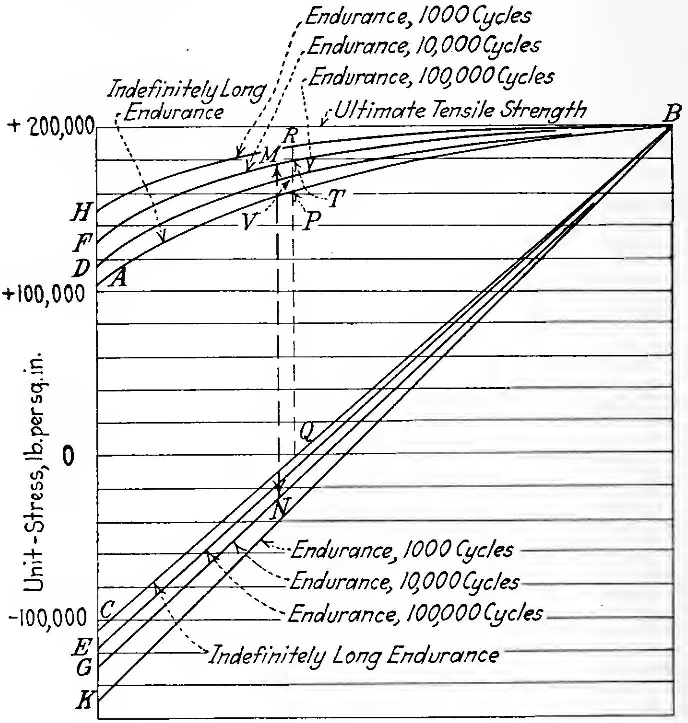

FIG. 75.—Diagram for estimating length of endurance under repeated stress (1.02 per cent carbon steel, oil quenched).

In Fig. 75 the lines *APB* and *CQB* determine the stress ranges for indefinitely long endurance; the lines *DVB* and *EB* determine the stress ranges for an endurance of 100,000 cycles; the lines *FTB* and *GB* determine the stress ranges for an endurance of 10,000 cycles; and the lines *HRB* and

KB determine the stress ranges for endurance of 1,000 cycles.

As an example of the use of such a diagram, suppose that a machine part made of this steel will be satisfactory if it lasts for 10,000 cycles of stress, and that a maximum stress is to be 175,000 lb. per square inch; it is desired to determine the range of stress to which the part may be subjected. From the line *FTB*, where it crosses the ordinate for 175,000 lb. per square inch at *M*, drop a vertical line *MN* to the line *GB*. The intersection of *MN* and *GB* is found at −25,000 lb. per square inch, so that the range of stress will be from 25,000 lb. per square inch in one direction to 175,000 lb. per square inch in the other.

This method is, of course, a rough graphical method rather than a careful analytic method, but in view of the great variation found in length of endurance for any given stress, such a rough graphical method is believed to be as precise a method as the circumstances justify.

The Effect of Steady Torsion on the Range of Stress in Reversed Flexure.—Shafts transmitting power are frequently subjected to a combination of cycles of reversed flexure together with a constant twisting stress. Recently Lea[1] has made fatigue tests on specimens of three kinds of steel subjected to varying combinations of reversed flexure and steady torsion. The three steels tested were a chrome-nickel steel, a 0.14 per cent carbon steel, and a 0.32 per cent carbon steel.

Lea found that so long as the shearing stress due to the steady torsion was below a critical value, no marked effect on the endurance limit was noticeable. Above this limiting value the endurance limit was markedly lowered. His tests are not sufficient in number to determine definitely this limiting value of shearing stress, but it seems to be nearly equal to the endurance limit for reversed flexure. For example, in his tests of 0.14 per cent carbon steel, the

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[1] Oxford Meeting, *Brit. Assoc. Advancement Sci.*, 1926; also *Engineering (London)*, Aug. 20, 1926. In discussion, Ono reported results confirming Lea's results, in a general way.

endurance limit for reversed flexure with no steady shearing stress was about 37,000 lb. per square inch. For reversed flexure together with a steady shearing stress of 33,800 lb. per square inch the endurance limit was about 39,200 lb. per square inch (an actual slight increase), and for reversed flexure combined with a steady shearing stress of 43,500 lb. per square inch the endurance limit was about 32,500, a distinct decrease.

CHAPTER VIII

"STRESS RAISERS" AND THEIR EFFECT ON FATIGUE STRENGTH—STRESS AND CORROSION

Effect of Internal Flaws.—Gillett and Mack[1] coined the term "stress raisers" to denote internal flaws, abrupt changes in cross-sections, and other factors which tend to cause local increase in stress not taken into account by the ordinary formulas of mechanics.

The work of Griffith in showing the effect of scratches in glass in producing high local stresses has been mentioned in Chap. IV. He showed that the computed local stresses in glass were at least ten times as high as the ordinary ultimate strength. He then carried his argument a step further by proving that it was possible to have stresses in glass of this high order of magnitude. He did this with very thin fibers of glass. He believes that the weakness of ordinary solids is due to discontinuities and flaws whose ruling dimensions are large compared with molecular distances.

If these extremely minute flaws assumed by Griffith really exist, they are of a smaller order of magnitude than the inclusions, dirt, minute cracks, blow holes, etc., which can be detected in unsound steel either by the unaided eye or by the microscope. These minute flaws, if they exist, must be very generally and very uniformly distributed throughout the mass of a piece of metal, since the test strength of sound metal is found to be uniform and reliable. It is again noted that instead of assuming these minute flaws, it is possible to visualize a picture of the mechanics of fatigue failure either on the hypothesis that in metal there are high internal stresses which make possible the start of cracks with slight additional imposed stress, or on

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[1] *Proc. Amer. Soc. Testing Materials*, vol. 24, Pt. II, p. 476, 1924.

the hypothesis that minute surface irregularities are the starting points of fatigue cracks.

Consideration will now be given to the effect of such minute flaws as can be seen in steel, either by the unaided eye or through the microscope.

Gillett and Mack have done a considerable amount of work in studying the effect of non-metallic inclusions and other inhomogeneities. One series of tests which they carried out was with steels containing cerium.[1] These steels were always dirty, that is, full of non-metallic inclusions, and gave, on the average, lower results for fatigue strength than would be expected from the tensile-test results and the usual relation of endurance limit to tensile strength. The results on these steels also showed a wider "scatter" of results than the other steels.

While they found that the greater amount of evidence indicated that clean steels gave better results than dirty steels, yet often the opposite appeared to be the case. They came to the conclusion that it is practically impossible to polish the surface of a fractured specimen and show the actual starting point of failure. They are of the opinion that polishing removes some of the material and thus destroys the evidence, so that metallographic examination merely shows the condition more or less remote from the actual point where failure began.

They say:

> Examination of successive surfaces showed that the distribution of non-metallic inclusions is so extremely non-uniform that unless tedious study of many surfaces indicates that the specimen is uniformly very clean or very dirty, it is quite impossible to say that the steel was clean or dirty at the actual point of fracture.

Gillett and Mack further point out that because the volume of metal subjected to the maximum unit stress in an ordinary fatigue test is very small, it is quite possible for the element of chance to play an important part in determining whether this small volume is clean or dirty.

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[1] "Molybdenum, Cerium, and Related Alloy Steels," Chap. VIII, especially p. 158, The Chemical Catalog Company, New York.

They point out that an inclusion which is some distance away from the point of maximum stress need not necessarily cause failure, and that a flaw or longitudinal scratch which lies parallel to the direction of stress does not markedly increase the local stress. They examined some specimens of normalized 0.52 per cent carbon steel which had been received from H. F. Moore. This material had an endurance limit of 42,000 lb. per square inch. A specimen was run at 40,400 lb. per square inch for 100,000,000 cycles without failure. This specimen had a large inclusion, lying in the longitudinal direction, with its tip 0.01 in. below the surface and about 0.1 in. away from the point of maximum stress. Evidently this flaw did not cause a local stress greater than the endurance limit. Another specimen of the same steel failed after 3,500,000 cycles at a unit stress of 40,400 lb. per square inch. This specimen, under examination, showed some deep circumferential scratches and a finish decidedly poorer than the unbroken specimen.

Another pair of Moore's specimens of a 3.5 per cent nickel steel seemed to indicate that inclusions caused failure in one case, and circumferential scratches caused failure in the second case.

A pair of specimens of molybdenum-nickel steel sent to Gillett and Mack by McAdam seemed to indicate that one specimen failed at a low stress and after comparatively few cycles because of inclusions and poorer surface finish than a second specimen which ran at a higher stress for 250,000,000 cycles before failure.

Gillett and Mack conclude that inclusions appear to act as local stress raisers, and that when they are so shaped, so oriented, and so placed with respect to the direction of stress application as to produce a local stress higher than the endurance limit, they may start fatigue failure even though the nominal computed stress is below the endurance limit of the material.

Moore and Jasper[1] report having made some tests on "dirty" steel. They also report erratic results, some

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[1] *Univ. Illinois, Eng. Exp. Sta. Bull.* 142, p. 65, 1924.

specimens giving fatigue results as high as those for clean steel, while other specimens gave low results.

McAdam[1] found that for crank-shaft and propeller-shaft material the endurance limits were usually lower for specimens taken in a transverse direction than in a longitudinal direction, although the tensile results did not differ greatly. He thinks this is probably due to the unfavorable orientation of inclusions in the transverse specimens.

Lea[2] speaks of examining bolts taken from couplings and connecting rods that had broken in service. These bolts revealed no weakness in tensile tests, but microscopic, and even naked-eye examination, often revealed slag inclusions or planes of separation, at which cracks undoubtedly started which led to failure. It is his opinion that it is a mistake to use wrought-iron bolts in such cases.

In a discussion of materials used in aircraft construction Aitchison[3] says that one of the most potent groups of imperfections in metals is the one including slag, non-metallic inclusions, and the like. He points out that the effect of these imperfections on ductility and toughness is much the same as their effect on fatigue strength.

Effect of Abrupt Changes in Cross-section.—The effect of external cracks, scratches, notches, and other abrupt changes in cross-section is attested by the results of a number of different experimenters. The results of Moore and Kommers[4] may be cited as an example of the effect of abrupt changes in cross-section. Figure 76 shows the five different kinds of specimens which they used. Figure 77 shows the results of the rotating-beam tests, the upper part of the figure giving results on a heat-treated 0.49 per cent carbon steel in the sorbitic condition and the lower part on Armco iron. The endurance limit for the specimen with the 9.85-in. radius was about 49,000 lb. per square inch. With a ¼-in. radius the endurance limit was reduced 8 per

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[1] *Proc. Amer. Soc. Testing Materials*, vol. 23, Pt. II, p. 100, 1923.

[2] *Proc. Inst. Civil Eng.*, 1923; *Engineering* (*London*), vol. 115, p. 253.

[3] *Engineering* (*London*), p. 90, Jan. 18, 1924.

[4] *Univ. Illinois Eng. Exp. Sta.*, *Bull.* 124, p. 20, 1921.

cent, with square shoulders it was reduced 51 per cent, and with a V-notch it was reduced 60 per cent. With the Armco iron the percentage of reduction was not quite so great.

Stanton and Bairstow[1] found that specimens with Whitworth screw threads, and also those with square shoulders plus a small fillet, suffered a reduction in endurance strength of about 30 per cent for hard steel, for soft steel, and for wrought iron, while specimens with square shoulders suffered a reduction of about 50 per cent for hard steels and from 25 to 45 per cent for mild steels and wrought iron. Eden, Rose, and Cunningham[2] found that a sharp V-notch reduced the endurance strength of bright-drawn mild

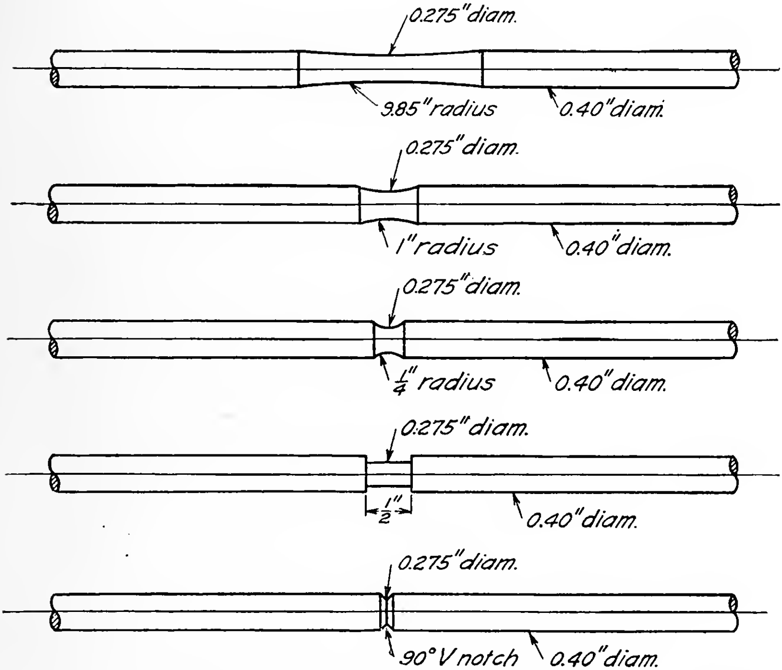

FIG. 76.—Specimens for study of effect of shape on endurance limit. (*Bull.* 124, *Univ. of Ill. Eng. Expt. Sta.*)

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[1] *Proc. Brit. Inst. Civil Eng.*, vol. 4, p. 78, 1905–1906.

[2] *Proc. Brit. Inst. Mech. Eng.*, vols. 3 and 4, p. 839, 1911.

steel about 25 per cent. Square shoulders reduced the strength of both hard and soft steels by 40 per cent, while keyways at flange couplings reduced the strength of steel by 50 per cent, and the strength of wrought iron by 23 per cent. Wöhler[1] found in some tests on axle steel

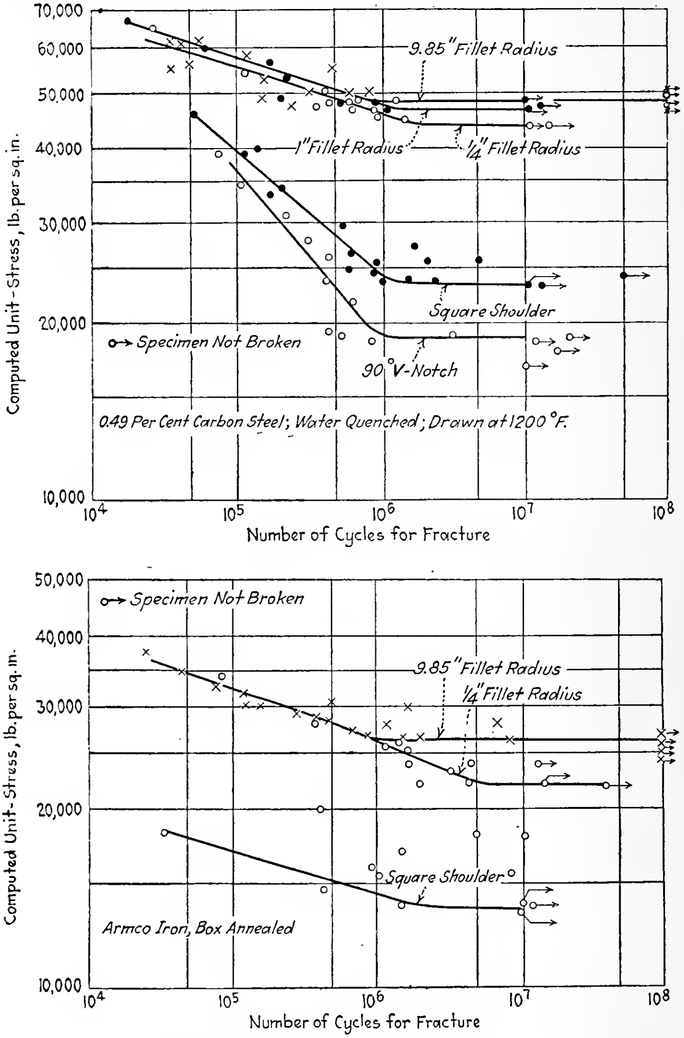

FIG. 77.—*S-N* diagrams for specimens of different shapes. (*Bull.* 124, *Univ. of Ill. Eng. Expt. Sta.*)

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[1] *Engineering* (*London*), vol. 11, 1871.

stressed from zero to a maximum in repeated tension, that specimens with square shoulders as compared with specimens having well-rounded shoulders, had their strength reduced about 37 per cent. On rotating-beam specimens of wrought iron the reduction of strength due to square shoulders ranged from 11 to 22 per cent.

R. R. Moore[1] has found that a single circumferential groove around a rotating-beam specimen reduces the fatigue strength much more than does a length of thread cut with the same tool as the groove. His results were confirmed by H. F. Moore. From this it is judged that a thread with nuts taking up most of its length would weaken a rod of metal in fatigue more than would the thread alone, and that the fatigue strength of the rod might be somewhat increased by cutting a longer thread on it.

All these results indicate the importance of avoiding abrupt changes of section in members of machines which are to be subjected to repeated stresses. Whenever a change of section is necessary, generous fillets should be provided at all shoulders.

Effect of Surface Finish.—Moore and Kommers[2] studied also the effect of surface finish on endurance strength. They used five degrees of smoothness: (1) a high polish in which after using Nos. 0 and 00 emery cloth, the specimens were polished with emery papers Nos. 1, 0, and 000, and finally with rouge and broadcloth, a microscope with a magnification of 100 diameters being used to make sure that all scratches were removed; (2) their standard finish, using Nos. 0 and 00 emery cloth; (3) a ground finish, obtained with a grinding wheel; (4) a smooth-turned finish using a lathe tool; and (5) a rough-turned finish, using a lathe tool. These tests were made on a heat-treated 0.49 per cent carbon steel in the sorbitic condition, and a few tests also on Armco iron.

Figure 78 shows the results of these tests. For the 0.49 per cent carbon steel the rough-turned specimens, which

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[1] *Proc. Am. Soc. Testing Materials,* vol. 26, Pt. II, p. 255, 1926.

[2] *Univ. Illinois Eng. Exp. Sta., Bull.* 124, p. 108, 1921.

were the weakest, had their endurance limit reduced about 18 per cent below that of the rouge finished. For the Armco iron the turned specimens had their endurance limit reduced from 8 per cent to 12 per cent compared with specimens of standard finish.

Eden, Rose, and Cunningham[1] found that polished specimens of mild steel which had their surfaces scratched with an ordinary sewing needle suffered an appreciable reduction in fatigue strength. Specimens of Bessemer steel with a turned surface showed a fatigue strength about 18 per cent lower than specimens of the same material which had been turned and polished. Sondericker[2] found that a rotating-beam specimen of soft steel with a groove 0.003 in.

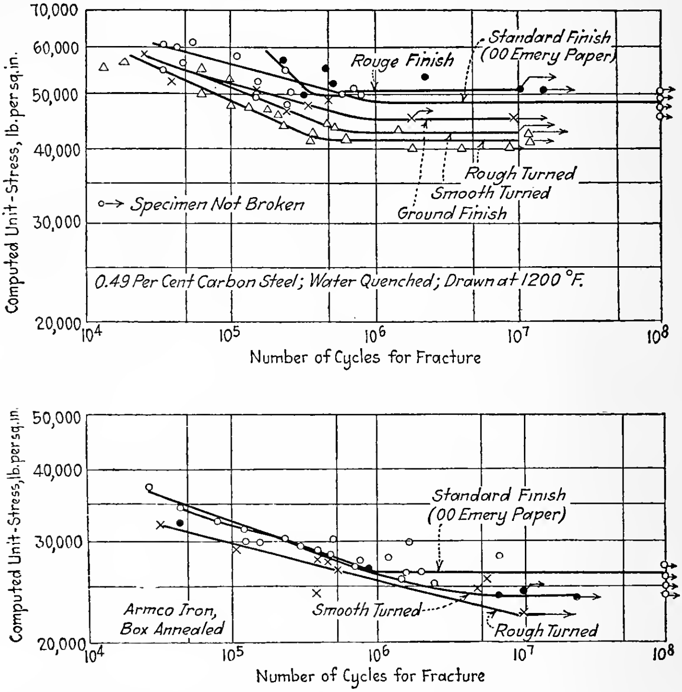

FIG. 78.—*S-N* diagrams showing effect of surface finish on endurance limit. (*Bull.* 124, *Univ. of Ill. Eng. Expt. Sta.*)

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[1] *Proc. Brit. Inst. Mech. Eng.*, vols. 3 and 4, p. 839, 1911.

[2] *Tech. Quart.* (*Boston*), March, 1899.

deep, cut with a diamond point, had its fatigue strength reduced by 40 per cent. In some tests in which annealed, cold-rolled steel was stressed in reversed bending beyond the yield point, Kommers[1] found that specimens which had been turned in a lathe and specimens which had been turned and then filed had their life reduced 30 per cent and 18 per cent, respectively, as compared with specimens which had been turned, filed, and polished.

TABLE 14.—EFFECT ON FATIGUE STRENGTH OF VARIOUS WORKSHOP FINISHES

Results obtained by W. Norman Thomas of the staff of the British Aeronautical Research Committee

Finish of Surface	Maximum Reduction in Fatigue Strength from Polished Surface, Per Cent[1]
Turned	12
Coarse file	18 to 20
Bastard file	14
Smooth file	7.5
Coarse emery (No. 3)	6
No. 1 emery	4
No. O or FF emery	2 to 3
Fine carborundum	2 to 3
Fine ground finish	4
Accidental scratches (maximum found)	16

[1] These values were estimated by determining the ratio for the various scratches of depth of scratch to radius of curvature at the bottom of the scratch. Values of depth and radius were determined by making gelatine casts of the surface of the metal, slicing the casts with a microtome, and then magnifying the outline of the slice by means of a projection apparatus. From these values it was possible to make an estimate of the probable effect of scratches on fatigue strength on the basis of fatigue tests on specimens scored with 72-deg. V-grooves with various ratios of depth to radius of curvature at the bottom of the scratch.

W. Norman Thomas[2] of the staff of the British Aeronautical Research Committee has made an extensive study of the effect of scratches and grooves resulting from various workshop processes. The materials used were tool steel, a 0.33 per cent carbon steel, a 0.13 per cent carbon steel, aluminum, and cast iron. Table 14 gives values which are a rough indication of the maximum effect on fatigue strength

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[1] *Proc. Intern. Assoc. Testing Materials*, art. V4*a*, 1912.

[2] *Engineering* (*London*), p. 449, Oct. 12, 1923.

to be expected from various finishes. Attention is also called to Fig. 16 (p. 76).

In order to determine whether the effect of the size of scratches would be appreciable, Thomas made some additional tests in which the depth of the scratches ranged from 0.0051 to 0.0448 in. instead of the maximum value of 0.00244 in. in the previous experiments. These grooves showed reductions in strength varying from 32 to 55 per cent, considerably larger than the reduction shown for the smaller scratches.

These results by various investigators all indicate that the surface finish of a machine member subjected to fatigue may have an appreciable effect on the fatigue strength. A poor surface finish may lower the fatigue strength of a metal by as much as 15 to 20 per cent. The results of Moore and Kommers indicate that fine grinding would probably be a satisfactory commercial finish.

Effect of Internal Stress.—The presence of internal stress in a metal will be such as to increase the resultant maximum stress above the computed stress, when the applied stress is of the same kind as the internal stress. The result will be an apparent reduction of the endurance limit as computed on the basis of the applied load. Experiments on quenched and tempered steels seem to indicate that the quenching operation introduces internal stresses which may be relieved to a considerable extent by heating, even below the critical temperature for the metal.

Table 15 gives some results obtained by Aitchison[1] on a 0.30 per cent carbon steel containing 0.56 per cent molybdenum, 4.30 per cent nickel, and 1.44 per cent chromium, air hardened from 1480°F.

These results show that the drawing temperature of 390°F. decreased the ultimate strength but actually increased the endurance limit. As the drawing temperature was increased, the ratio of endurance limit to ultimate strength did not change greatly, but the endurance limit decreased with decrease of ultimate strength. There is

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[1] "Engineering Steels," p. 209, 1921.

evidently a particular drawing temperature which relieves the internal stress considerably and produces the greatest absolute value of endurance limit.

TABLE 15.—EFFECT OF "DRAW" ON STATIC AND FATIGUE PROPERTIES

Drawing temperature, degrees Fahrenheit	Proportional elastic limit, pounds per square inch	Yield point, pounds per square inch	Ultimate tensile strength, pounds per square inch	Elongation in 2 in., per cent	Reduction of area, per cent	Endurance limit, pounds per square inch	Ratio of endurance limit to ultimate tensile strength
Values quoted from Aitchison for a nickel-chromium air-hardened steel							
None	45,000	176,000	244,000	11	36.5	102,000	0.418
390	81,000	173,000	227,000	12.5	41.5	115,000	0.507
750	119,000	179,000	220,000	10	36	106,000	0.482
930	105,000	159,000	185,000	15	46.5	93,000	0.502
1,110	91,500	141,000	157,000	17.5	55	79,500	0.506
Values quoted from Moore and Jasper for a 0.49 carbon oil-quenched steel							
None	72,000	80,900	126,500	12.5	52	65,000	0.513
600	73,300	80,800	126,800	11.5	52	68,000	0.536
800	75,800	78,800	121,800	11.5	51	64,000	0.526

Whyte[1] found that the endurance limit for a nickel-chromium steel rose for drawing temperatures up to 750°F. and then started to fall.

Moore and Jasper[2] report the results shown in the second part of Table 15 for a 0.49 per cent carbon steel quenched in oil from 1450°F. Here again the drawing temperature of 600°F. had the effect of relieving the internal stresses and actually increasing the endurance limit.

Gillett and Mack[3] report that a 0.31 per cent nickel-chromium-molybdenum steel, quenched in oil at 1500°F. and drawn at a temperature of 980°F. had an endurance limit of 100,000 lb. per square inch. This endurance limit was increased to 112,000 lb. per square inch by raising the specimens to a temperature of 1110°F. and cooling slowly. Another similar steel with 0.44 per cent carbon was quenched

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[1] *Proc.* (*British*) *Inst. Automotive Eng.*, vol. 15, p. 512, 1921.

[2] *Univ. Illinois Eng. Exp. Sta.*, *Bull.* 136, 1923.

[3] *Proc. Amer. Soc. Testing Materials*, vol. 24, Pt. II, p. 476, 1924.

at 1460°F. in oil and drawn at 980°F. This steel had its endurance limit increased from 87,000 to 123,000 lb. per square inch by 2¼ hours more of heating at 980°F. This additional heating produced no decrease in hardness. Several other steels are cited to show the same beneficial effect on endurance limit of more prolonged drawing.

Moore and Kommers[1] report a result on a 0.24 per cent carbon chrome-nickel steel which illustrates the effects of double heat treatment. The steel was first quenched in oil at 1525°F., reheated to 700°F., and again quenched in oil. This treatment gave an ultimate strength of 138,700 and an endurance limit of 68,000 lb. per square inch. Specimens of this steel were also quenched in oil at 1525°F., but then reheated to 1450°F., quenched in oil, and next reheated to 1200°F., held for 1 hour, and quenched in water. This treatment gave an ultimate strength of 114,200 and an endurance limit of 67,000 lb. per square inch. In other words, the second method gave a much decreased ultimate strength with an endurance limit almost equal to that of the first method. Undoubtedly, the internal stresses set up by the first method were much greater than by the second method.

These results indicate that the influence of quenching temperature, drawing temperature, and especially length of draw, are very important; and that it appears to be possible to reduce greatly the internal stresses and thereby increase the endurance limit by proper procedure in heat treatment. Very often this is accompanied by increase in ductility and no appreciable decrease in ultimate strength.

Discrepancies between Experiment and Theory.—The mathematical investigations of Suyehiro,[2] Inglis,[3] and Griffith[4] and the static experiments of Coker and Scoble[5] and others have shown that the effect of holes, scratches,

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 124, 1921.

[2] *Engineering (London)*, p. 280, Sept. 1, 1911.

[3] *Trans. Brit. Inst. Naval Arch.*, Pt. I, p. 219, 1913.

[4] *Brit. Advisory Comm. Aero. Rept. and Mem.*, No. 1275*T*, December, 1916.

[5] *Trans. Brit. Inst. Naval Arch.*, Pt. I, p. 207, 1913, and other papers.

and discontinuities in general is to produce high local stress. On the other hand, experiments in fatigue have shown that the endurance limit of specimens which have been provided with holes and scratches does not show the reduction in value which would be predicted by the mathematical theory of elasticity or by the static experiments.

An interesting series of experiments relating to this question are those of Thomas[1] (see also p. 203). His results showed that the increase of stress due to scratches, provided the variation in depth is small, depends approximately on the ratio d/ρ, in which d is the depth of the scratch and ρ is the radius of curvature of its extremity.

Now Inglis[2] has shown by an elaborate mathematical analysis that, for the case of a flat plate notched at one edge, the stress at the bottom of the notch is approximately

$$S_i = S\left(1 + 2\sqrt{\frac{d}{\rho}}\right),$$

in which S_i = the unit stress at the bottom of notch,
S = the mean stress in the plate,
d = the depth of the notch,
ρ = the radius of curvature at the extremity of the notch.

The assumption is made that the elastic limit of the material is not exceeded.

Furthermore, A. A. Griffith has shown by means of his soap-film[3] apparatus that the stresses in a shaft due to a twisting moment are greater at the bottom of a V-shaped groove than at the surface of a similar unscratched shaft, according to the values shown in Table 16. Griffith also showed by mathematical analysis[4] that when a shaft was

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[1] *Engineering (London)*, p. 449, Oct. 12, 1923.

[2] *Trans. Brit. Inst. Naval Arch.*, Pt. I, p. 219, 1913.

[3] This ingenious device is described in *Engineering (London)*, p. 546, Dec. 21, 1917, and in the *Proc. Brit. Inst. Mech. Eng.*, October–December, p. 755, 1917.

[4] "The Effect of Surface Scratches on the Strength of Shafts and Other Members," *Brit. Advisory Comm. Repts. and Mem.*, No. 1275*T*, December, 1918.

TABLE 16.—THEORETICAL STRESS CONCENTRATIONS AT THE BOTTOM OF LONGITUDINAL V-GROOVES IN SHAFTS UNDER TORSION[1]

Angle of V notch, degrees	Ratio of computed maximum stress at bottom of groove to the surface stress in an unscratched shaft				
	Values of $\frac{d}{\rho}$				
	½	1	3	5	9
0	1.85	2.01	2.66	3.23	4.54
60	1.84	2.00	2.54	3.06	3.99
90	1.81	1.95	2.40	2.64	3.12
120	1.66	1.75	1.95	2.06	2.13

[1] The values in this table were obtained by the use of the Griffith and Taylor soap-film method for determining theoretical stress.

d = depth of groove in inches.

ρ = radius of curvature at bottom of groove in inches.

subjected to a bending (as contrasted with twisting) moment, the ratio of the increased maximum tensile stress to the original maximum tensile stress could be obtained by multiplying the values in Table 16 by the factor

$$\frac{1 + 2\frac{d}{\rho}}{1 + \frac{d}{\rho}}.$$

This is true for grooves perpendicular to the direction of stress (that is, circumferential). The ratio was less for grooves in other directions.

Returning to the consideration of Thomas' experimental study of the structural damage done by stress concentrations at V-grooves, it is to be noted that he judged the amount of this structural damage by the results of fatigue tests, using a 0.33 per cent carbon steel. Two different grooves were used, one made by a lathe tool with an angle of about 72 deg. and the other made by a diamond—a shallow groove having an angle of about 120 deg. The size and shape of the grooves were determined accurately by making gelatine casts, slicing these casts with a microtome, and then magnifying the slices with a projection apparatus. The fatigue tests were made on a rotating-beam testing machine (see also Fig. 16).

If S_i denotes the intensified stress at the root of the groove, S denotes the nominal stress in the bar, as computed by the common flexure formula, and if the ratio of increase of intensified stress to nominal stress be assumed to be proportional to $\sqrt{d}$,[1] there results

$$\frac{S_i - S}{S} = c\sqrt{\frac{d}{\rho}}, \text{ or } c = \frac{S_i - S}{S}\sqrt{\frac{\rho}{d}},$$

in which c is a constant to be determined from the results of the tests.

Table 17.—Comparison of Theoretical Stress Concentrations at the Roots of V-grooves with the Effective Stress Concentrations as Shown by Fatigue Tests

Results obtained by W. Norman Thomas of the staff of the British Aeronautical Research Committee

Item	I Values of d/ρ	II Theoretical semi-elliptical groove, Inglis formula	III Soap-film experiments, 72-deg. V, Table 16	IV Results of fatigue tests, 72-deg. V (small)
1. Values of C....................	..	2.0	1.75	0.15
2. Nominal stress at fracture, tons per square inch..................	1	6.1	6.1	15.8[a]
	4	3.7	4.0	14.0[a]
	7	2.9	3.2	13.0[a]
3. Approximate decrease in strength, per cent......................	1	66.7	66.7	13.5
	4	80.0	78.0	23.5
	7	84.0	82.5	29.0
4. Ratio of maximum stress to nominal stress = S_1/S..............	1	3.0	3.0	1.16
	4	5.0	4.5	1.31
	7	6.3	5.7	1.42

[a] Endurance limits.

Table 17 shows some of the theoretical and experimental results obtained by Thomas. It was found that an

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[1] This assumption implies the use of the Inglis formula (p. 207) rather than the Griffith formula (p. 208). For the range covered, however, the values given by the two formulas differ only slightly.

unscratched specimen had an endurance limit (nominal stress at fracture) of 18.3 tons per square inch, and it was assumed that at the endurance limit for a scratched specimen the unit stress was also 18.3 tons per square inch. When $c = 2$ and $d/\rho = 1$, then from the Inglis formula (p. 207) there results

$$\frac{S_i - S}{S} = 2\sqrt{1} \text{ or } S_i = 3S.$$

The theoretical ratio of maximum stress to nominal stress is therefore 3, and this value is recorded in column II of Table 17 over against item 4 for the value $d/\rho = 1$. One-third of 18.3 tons per square inch is 6.1 tons per square inch, and this value is recorded in column II over against item 2 from $d/\rho = 1$. The values in column III of Table 17 are obtained from Table 16, using a similar procedure. Column IV gives the experimental results for endurance limit over against item 2, and 18.3 tons per square inch divided by these values gives the values in column IV over against item 4.

It is evident from the table that, theoretically, the ratio of maximum stress to nominal stress varied from 3 to 6.3, while, experimentally, the ratio varied from 1.16 to 1.42. While the theoretical decrease in strength varied from 67 to 84 per cent, the experimental values varied from 14 to 29 per cent. This makes plain the fact that the effect of grooves and scratches *is not so serious as the mathematical theory of elasticity would indicate.* This conclusion is borne out by the results of a number of other investigators and of tests on stress concentration at small holes as well as at the root of grooves.

Moore and Jasper[1] investigated the effect of small holes, 0.055 in. in diameter, on the endurance limit of a number of different metals. They used both round and flat specimens in reversed bending. Their results are given in Table 18.

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 152, p. 25, 1925.

TABLE 18.—EFFECT OF A SMALL HOLE ON THE FATIGUE STRENGTH OF A SPECIMEN

Results obtained by Moore and Jasper in the Joint Investigation of the Fatigue of Metals at the University of Illinois

Steel	II Rotating beam (Farmer specimen)	III Rotating beam (Farmer specimen) with hole	IV Flat specimen without hole	V Flat specimen with hole	VI Column III : Column II Per Cent	VII Column V : Column II Per Cent	VIII Column IV : Column II Per Cent	IX Column V : Column IV Per Cent
1.20 carbon, normalized	50,000	......	40,700	40,000	....	80.0	81.5	98.3
Cyclops metal:								
as received	55,000	33,600	......	......	61.1			
annealed	55,000	36,000	......	......	65.5			
0.52 carbon:								
normalized	42,000	......	34,000	32,000	....	76.1	81.0	94.1
troostitic	70,000	36,000	......	......	51.5			
0.37 carbon, normalized	33,000	......	32,000	27,000	....	81.8	97.0	84.4
Chrome nickel:								
oil quench, drawn at 700°F	68,000	......	54,000	50,000	....	73.6	79.4	92.6
oil quench, drawn at 1200°F	65,000	......	44,000	42,500	....	65.4	67.7	96.6
oil quench, drawn at 1200°F	67,000	......	46,000	38,000	....	56.7	68.6	82.6
3.5 nickel:								
oil quench, drawn at 1100°F	64,000	......	51,000	49,000	....	76.5	79.7	96.0
annealed	54,000	......	49,000	39,000	....	72.2	90.7	79.5
0.02 carbon (Armco)	26,000	13,000	24,000	20,000	50.0	77.0	92.3	83.3
0.49 carbon:								
normalized	33,000	......	29,000	26,000	....	78.7	87.9	89.7
sorbitic	48,000	26,000	......	......	54.2			
Average	......	......	......	......	56.5	73.8	82.6	89.7

The seventh column in the table gives the ratio of apparent endurance limit given in the fifth column for flat specimens with holes to that given in the second column for rotating-beam specimens without holes. According to the theory of elasticity, this ratio would be 0.333. The test results, however, give values ranging from 0.567 to 0.818, indicating that under fatigue loading a small discontinuity is not nearly so serious in its weakening effect as is indicated by the formulas of the theory of elasticity. It is to be noted that the weakening effect of a small hole is, in general, more marked in the alloy steels tested than in the carbon steels.

The eighth column in Table 18 shows that the ratio between apparent endurance limit for flat specimens with ⅛ inch fillets and that for rotating-beam specimens without holes varies from 0.677 to 0.970. Professor Coker has shown that under static load within the elastic range this ratio should be 0.696.[1] The test results show a value approximately equal to this for only two materials, both heat-treated alloy steels.

Timoshenko and Dietz[2] have made an experimental study and a theoretical investigation of the stress concentration around holes and fillets, and its effect on fatigue strength. They find that stress concentrations lower endurance limit less than the amount indicated by the mathematical theory of elasticity, and that specimens of chrome-nickel steel were more damaged by stress concentration than specimens of carbon steel.

Wilson and Haigh[3] found that perforated thin plates under repeated axial stress did not fail under such low stresses as might be predicted by the mathematical theory of elasticity.

R. R. Moore[4] found that for six non-ferrous metals and one steel the reduction in endurance limit caused by a notch ranged from 25 to 45 per cent for the non-ferrous metals, and 58 per cent for the steel. The mathematical computation of stress indicated that the endurance limit would be reduced 78 per cent.

All these results reinforce the conclusion that the destructive effect of stress concentration cannot be neglected in

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[1] *Brit. Assoc. Advancement Sci., Rept.,* 1924.

[2] *Trans. Am. Soc. Mech. Eng.,* vol. 47, p. 199, 1925.

[3] *Brit. Assoc. Advancement Sci. Repts.,* p. 368, 1923.

[4] *Proc. Amer. Soc. Testing Materials,* vol. 24, Pt. II, p. 547, 1924; and vol. 26, Pt. II, 1926.

parts subjected to repeated stress, but it is not so great as that indicated by the mathematical theory of elasticity, and the investigations of Moore and Jasper, of Timoshenko and Dietz, and of R. R. Moore also indicate that stress concentration produces different degrees of damage for different metals. Tempered alloy steels seem to be more damaged by stress concentration than other metals studied.

The explanation for these results offered on the basis of Griffith's theory is that metals have in them minute cracks and flaws, so that when experimental scratches are so small as to be comparable with the cracks already existing, their effect will be small, and that the theoretical reduction in strength will be approached only when the grooves are large.

Another explanation for the observed results is based on the fact that experiments seem to show that redistribution of stress by slipping seems possible even within the endurance limit of a material. The tests of Gough and Hanson[1] in which they found slip lines in Armco iron below the endurance limit seem to bear out this statement. The results of Moore and Kommers[2] on Armco iron, in which the yield point was below the endurance limit, of Hankins[3] on nickel, and of R. R. Moore[4] on seven different nonferrous metals, in which the proportional elastic limit was below the endurance limit, all indicate that slip and hence cold working can occur below the endurance limit of a material. The results of repeated stresses would, therefore, be in the nature of repeated cold work, which not only permits redistribution of stress but strengthens the material against fatigue.

Attention is called to the discussion in Chap. IV (p. 75) of this discrepancy between the theory of elasticity and results of fatigue tests of metals.

Effect of Corrosion on Fatigue Strength. 1. *Corrosion of Unstressed Metal.*—Corrosion roughens the surface of metal

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[1] *Proc. Roy. Soc.*, vol. 104*A*, p. 538, 1923.

[2] *Univ. Illinois Eng. Exp. Sta.*, *Bull.* 124, p. 98, 1921.

[3] *Brit. Advisory Comm. Aero.*, *Repts.*, vol. 2, p. 414, 1922–1923.

[4] *Proc. Amer. Soc. Testing Materials*, vol. 24, Pt. II, p. 547, 1924.

and causes many minute pits and grooves which act as "stress raisers." The tests of McAdam and R. R. Moore[1] on the fatigue strength of steel, corroded previous to testing for fatigue strength, show a reduction of fatigue strength of steel varying from 1 to 12 per cent. Corrosion of unstressed metal seems to have a purely mechanical effect and is comparable, as to injury caused to a poor surface finish.

2. *Effect of Simultaneous Corrosion and Repeated Stress.*—In 1917 Haigh[2] reported fatigue results of some brasses in contact with strong corrosive agents. He observed that when corrosion was simultaneous with the stress application the stress cycle graph under some conditions was slightly lowered. When the corrosion was prior to the stress application, the stress-cycle graph was not lowered. He made no tests on steel.

In 1926 McAdam[3] reported test results for a large number of ferrous metals subjected to the simultaneous action of a stream of fresh water and of cycles of reversed flexural stress. He found that for constant corrosion intensity there is a definite fatigue limit and that this limit is usually below (sometimes much below) the ordinary endurance limit. To this phenomenon he gave the name "corrosion-fatigue," and to the fatigue limit obtained under such conditions he gave the name "corrosion-fatigue limit." Using specimens like that shown in Fig. 43(*c*) he found the following reductions of fatigue strength of specimen subjected to corrosion-fatigue as compared with specimens subjected to reversed flexure alone.

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[1] *Proc. Am. Soc. Testing Materials*, vol. 26, Pt. II, 1926.

[2] *Jour.* (*British*) *Inst. Metals*, Sept., 1917; see also *Engineering* (*London*), Sept. 21, p. 315, 1917.

[3] *Proc. Am. Soc. Testing. Materials*, vol. 26, Part 2, p. 224, 1926. *Trans. Am. Soc. for Steel Treating*, 1926.

	Per Cent
3.5 per cent nickel steel:	
quenched and drawn	64
annealed	41
0.49 per cent carbon steel:	
quenched and drawn	62
annealed	32
0.36 per cent carbon steel:	
quenched and drawn	63
annealed	26
0.24 per cent carbon steel:	
annealed	41
0.11 per cent carbon steel:	
annealed	36
Ingot iron (average value)	23
High chromium-nickel steel (average value)	16
Stainless iron (average value)	29

Corrosion-fatigue tests with salt water as the corroding agent showed markedly greater reductions of fatigue strength than those listed above.

The corrosion-fatigue limit is surprisingly little affected by heat-treatment or chemical composition, except as such heat-treatment or composition affects corrosion-resistance. For "stainless" (high chromium) and other corrosion-resistant steels the corrosion-fatigue limit is higher than for carbon steels and other alloy steels.[1]

When corrosion and repeated stress act *together* there is, in addition to ordinary mechanical stress, an action which, following a suggestion by McAdam, may be called "chemical stress." By means of micrographs of the surface, McAdam has shown that fatigue cracks start from spots corroded so slightly that the corrosion can scarcely be detected except by examination with a microscope. Once corrosion and stress together start a fatigue crack, it apparently spreads much as does an ordinary fatigue crack.

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[1] An interesting question, as yet unanswered, is, "What connection, if any, exists between the corrosion-fatigue results reported by McAdam and the 'caustic embrittlement' of boiler plate under combined steady stress and corrosion?" This is discussed by Parr and Straub. See *Proc. Am. Soc. Testing Materials*, vol. 26, Pt. II, 1926.

An interesting problem in corrosion fatigue requiring experimental study is the comparative effects of corrosion fatigue on small specimens and on large pieces.

Corrosion of unstressed steel seems to be a rather minor factor in reducing its fatigue strength. Corrosion and fatigue acting *simultaneously* seem to constitute a factor of major importance, one which must be given careful consideration by the machine designer and the structural engineer.

Significance of Ductility.—Moore and Kommers[1] have pointed out that it is unlikely that ductility, as represented by the percentage of elongation and the percentage of reduction of area, will have much direct influence on the fatigue strength. Ductility is based upon the action of a bar as a whole, and in ductile materials is dependent upon the final necking down after the ultimate has been reached, while in fatigue failures there is no necking down. Furthermore, fatigue failures are extremely localized and involve only a small portion of the bar. A study of fatigue results shows no correlation between ductility and endurance limit (see Fig. 63, p. 166). The authors mentioned above cite the case of a 0.93 per cent carbon troostitic steel which had low elongation and reduction of area, but a high endurance limit and a high ratio of endurance limit to proportional elastic limit.

Ductility, therefore, does not influence the endurance limit directly, but the authors wish to emphasize the fact that ductility is for other reasons one of the most valuable properties of metals. Its influence on toughness is particularly important, toughness being defined as the quality of a material which permits it to absorb large amounts of energy without shattering failure. This quality is dependent on the two factors of strength and ductility.

A bar of brittle material which has local concentration of stress, due perhaps to an abrupt change in cross-section, would be almost sure to fail when subjected to a shock. If the material is ductile and tough, however, these quali-

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 124, 1921.

ties will permit permanent deformation to take place without actual failure. Such permanent deformation at a point of high local stress produces redistribution of stress and thus relieves the situation at the local point.

Moore and Kommers[1] have shown that a stress which is a considerable percentage above the endurance limit may be applied from 1,000 to 5,000 times without greatly influencing the fatigue strength under subsequent application of lower stresses. Their results on a heat-treated 0.49 per cent carbon steel in the sorbitic condition showed that a stress 10 and 20 per cent above the endurance limit applied 5,000 times, a 29 per cent overstress applied 1,000 times, and a 38 per cent overstress applied 100 times, did not appreciably reduce the endurance limit as subsequently determined. However, an overstress of 35 per cent applied 1,000 times reduced the endurance limit 4 per cent, while an overstress of 29 per cent applied 5,000 times reduced the endurance limit about 11 per cent.

A heat-treated 1.20 per cent carbon steel in the sorbitic condition whose original endurance limit was 50,000 lb. per sq. in. was subjected to 20 per cent overstress for 5,000 and 10,000 cycles, respectively. The endurance was reduced 12 per cent and 14 per cent, respectively. Comparing the result of 20 per cent overstress applied 5,000 times in the case of the 0.49 per cent carbon steel, whose Brinell hardness was 197, and the 1.20 per cent carbon steel, whose Brinell hardness was 369, it is seen that the harder steel was much more influenced by the overstress than the softer steel. It should be noted here that the absolute value of the overstress was practically the same in the two cases, because the endurance limits did not differ greatly.

Moore and Jasper[2] report some results on the effect of overstrain of a different type. They applied a heavy axial tensile load twenty times, producing stresses greater than the original endurance limit by various percentages. They found that the endurance limit was not affected appreci-

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 124, p. 112, 1921.

[2] *Univ. Illinois Eng. Exp. Sta., Bull.* 136, p. 60, 1923.

ably until the maximum stress applied approached the static proportional elastic limit, which was about 41 per cent above the original endurance limit. For stresses near or above the proportional limit the endurance limit was decreased from 18 to 22 per cent. The specimens were not polished after being overstressed. There appeared to be little difference in the results whether the overstressed specimens were tested immediately, were immersed in boiling water before testing, or rested three months before testing.

Moore and Jasper[1] did some further work on the effect of overstress in reducing the endurance limit. The results are shown in Table 19. In these tests the amount of axial overstress ranged from 15 to 80 per cent and in all but one case was applied twenty times. It will be noted that in all cases except one the effect of overstress was to reduce the subsequent endurance limit below the original value by amounts ranging from 3 to 23 per cent.

The tests on the annealed specimens of 0.49 per cent carbon steel require some explanation. The annealed *A* specimens were annealed at 1500°F. and then polished; and they gave an endurance limit of 32,000 lb. per square inch. The *B* specimens were annealed as were the *A* specimens, then given an overstress of 80 per cent applied twenty times, and repolished, with the result that the endurance limit was 31,000 lb. per square inch. The *C* specimens were annealed as were the *A* specimens and polished, reannealed and repolished, giving an endurance limit of 33,000 lb. per square inch. The *D* specimens were annealed as were the *A* specimens, then given an overstress of 80 per cent applied twenty times, reannealed, and repolished, giving an endurance limit of 30,000 lb. per square inch. The *E* specimens were annealed as were the *A* specimens, then given an overstress of 40 per cent applied twenty times, reannealed and repolished, giving an endurance limit of 29,300 lb. per square inch.

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 142, p. 32.

TABLE 19.—EFFECT OF OVERSTRESS AND OF SUBSEQUENT TREATMENT OF STEEL ON THE ENDURANCE LIMIT

Results obtained by Moore and Jasper in the Joint Investigation of the Fatigue of Metals at the University of Illinois

Steel	Axial stress applied before fatigue test of specimens		Excess of axial stress above original endurance limit, per cent	For specimens subjected to fatigue test immediately after axial stress		For specimens placed in boiling water and rested about 10 hr. after axial stress		For specimens rested 3 months after axial stress	
	Unit stress, pounds per square inch	Number of applications		Endurance limit, pounds per square inch	Reduction from original endurance limit, per cent	Endurance limit, pounds per square inch	Reduction from original endurance limit, per cent	Endurance limit, pounds per square inch	Reduction from original endurance limit, per cent
0.49 carbon, sorbitic.....	none	0	0	48,000	0				
	55,200	20	15	45,000	6.2	46,000	4.2	47,000	2.1
	62,400	20	30	48,000	0	47,000	2.1	47,000	2.1
	67,200	20	40	40,000	16.7	40,000[a]	16.7	39,000	18.8
	72,000	20	50	37,000	22.9	40,000	16.7	39,000	18.8
	81,600	20	70	37,000	22.9	37,000	22.9	39,000	18.8
	81,600	20	70	40,000	16.7	Specimens polished after axial stress			
0.49 C, annealed *A*......	none	0	0	32,000	0	Specimens not subjected to axial stress			
0.49 C, annealed *B*......	57,600	20	80	31,000	3.1	Specimens polished after axial stress			
0.49 C, annealed *C*.......	none	0	0	33,000	0	Specimens not subjected to axial stress			
0.49 C, annealed *D*......	57,600	20	80	30,000	6.2	Specimens polished after axial stress			
0.49 C, annealed *E*......	44,800	20	40	29,300	8.4	Specimens polished after axial stress			
1.20 C, treatment *AA*....	none	0	0	85,000	0	Specimens not subjected to axial stress			
1.20 C, treatment *AA*....	151,000	1	78	65,000	23.6	Specimens not polished after axial stress			

[a] Estimated from tests of at least four specimens.

For specimens *A* and *B* the overstress reduced the endurance limit 3.1 per cent. For specimens *C* and *D* the overstress reduced the endurance limit 6.2 per cent, even though the specimens were annealed and polished after the overstrain. For specimens *C* and *E* the overstress reduced the endurance limit 8.4 per cent, even though the specimens were annealed and polished after the overstrain. The beneficial effect of reannealing and repolishing on endurance limit seemed negligible.

The 0.49 per cent carbon steel in the annealed condition is evidently better able to withstand overstress than in the sorbitic condition. The sorbitic steel had its endurance limit reduced 22.9 per cent by 70 per cent overstress, while the annealed steel had its endurance limit reduced less than 10 per cent by 80 per cent overstress. Apparently the inherent ability of the metal for "healing" scars due to slip, is diminished by any heat treatment which raises the strength and lowers the ductility.

One of the valuable characteristics of materials which are ductile and tough comes into play both under static and repeated stress, this characteristic being the one which permits the material to deform under an unexpected, high stress. Such permanent deformation without actual failure gives a warning of impending failure which a brittle material cannot give.

The Effect of Understressing.—Moore and Jasper[1] investigated the effect of subjecting specimens to reversed bending stresses at or near the endurance limit. They retested 118 specimens which had received at least 10,000,000 and in some cases 100,000,000 cycles of stress without failure, the specimens being subjected to stresses which were increased by small increments until the specimens failed. In some cases, the unit stress at fracture was 25 per cent above the original endurance limit. Figure 79 shows the results obtained on some of the ferrous metals which had originally been subjected to 100,000,000 cycles of stress without failure. The figure shows two different groups of

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[1] *Univ. Illinois Eng. Exp. Sta., Bull.* 142, p. 27, 1924.

metals; those in the upper group show a very marked increase in fatigue strength, while those in the lower group show comparatively little increase in strength. It is evident, therefore, that all metals are not equally susceptible to increase in strength due to understressing. Those metals which have had their strength materially increased by heat

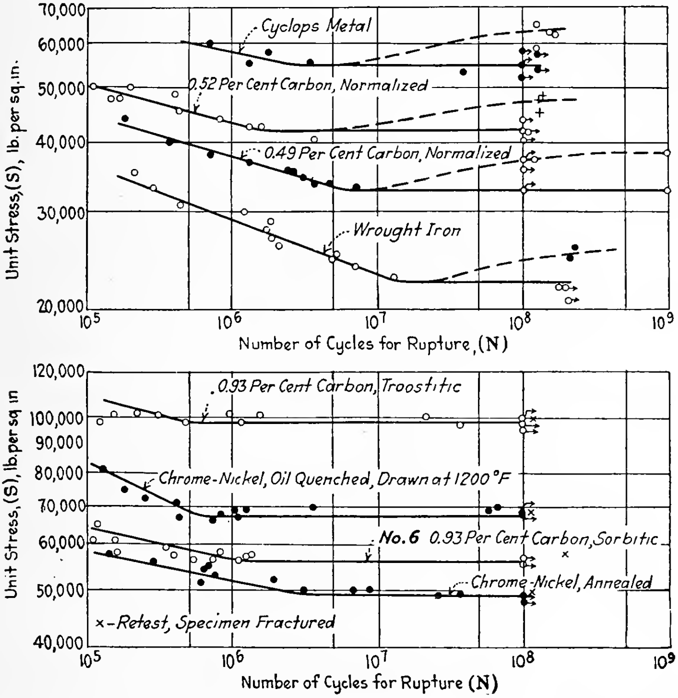

FIG. 79.—Effect of "understressing" on endurance limit. (*Bull.* 142, *Univ. of Ill. Eng. Expt. Sta.*)

treatment apparently do not have their strength increased much by understressing.

Bauschinger's conclusion given as No. 12 in Chap. II says:

> Repeated stresses between zero and an upper limit in tension which coincides with or lies slightly above the elastic limit will increase the elastic limit, and the more so the greater number of repetitions, but not above a certain limiting value.

The experiments of Moore and Jasper indicate that for reversed stresses an analogous increase of endurance limit may be expected in steels susceptible to cold work when the stresses are at or slightly below the original endurance limit.[1]

It is of course well known that certain kinds of cold work increase the static and fatigue strength of metals, and it is conceived that repeated stressing tends to produce repeated cold work over minute areas. There is evidently the possibility of a localized rearrangement of particles which were disadvantageously placed initially; thus considerable strengthening is produced at critical locations.

Another series of tests on understressing was carried out by Moore and Jasper on a 1.20 and a 0.49 per cent carbon steel, both in the sorbitic condition, and also on the 0.49 per cent carbon steel in the annealed condition. In these tests the specimens were first subjected to a small number of cycles of stress above the original endurance limit, then subjected to 10,000,000 or more cycles at or near the original endurance limit, and finally tested for endurance limit in the usual way. Table 20 shows the results of these tests.

As pointed out previously, the overstressing would tend to reduce the endurance limit while the understressing would tend to increase it. Table 20 shows that in every case there is some restoring action due to the understressing. In some cases the effect of overstressing seems to be entirely overcome by the effect of understressing.

Tests were also made to determine the effect on static properties of reversed axial stresses. Specimens were subjected to 10,000,000 cycles of reversed axial stress at or near the endurance limit, and subsequently tested in static tension. Table 21 shows the results of these tests. All the steels except one show an increase in static ultimate strength due to the understressing, and most of the steels

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[1] Recent tests by N. P. Inglis at the University of Illinois indicate that the fatigue strength of *cast iron* may be materially increased by understressing. This interesting result on a brittle metal indicates the need of further study of understressing.

TABLE 20.—SUMMARY OF RESULTS FOR STEEL SUBJECTED TO STRESS ABOVE THE ORIGINAL ENDURANCE LIMIT BUT NOT TESTED TO FAILURE; THEN SUBJECTED TO 10,000,000 OR MORE CYCLES OF STRESS AT OR NEAR THE ORIGINAL ENDURANCE LIMIT OF THE METAL

Results obtained by Moore and Jasper in the Joint Investigation of the Fatigue of Metals at the University of Illinois.

In considering the values given in this table three endurance limits must be kept in mind: (1) the original endurance limit of the metal, (2) the endurance limit of the metal after overstress, and (3) the endurance limit of the metal after overstress (not carried to failure) followed by cycles of stress at or near the endurance limit. Endurance limit (3) is greater than endurance limit (2), as is shown in this table, but endurance limit (3) is, however, less than endurance limit (1).

Specimen	Total cycles of stress	Increase over endurance limit after overstress (2) of endurance limit after subsequent understress (3), per cent		Amount of overstress applied	
		Greater than	Less than	Number of cycles	Excess over original endurance, limit per cent
Steel No. 1, 1.20 carbon, sorbitic					
1F0C	46,536,000	5.0	7.5	5,000	20
1F52G	42,615,900	18.8	21.4	5,000	20
1F39F	43,065,800	4.1	6.5	5,000	20
1F39D	80,686,600	13.5	16.1	5,000	20
1F39A	63,974,800	20.7	22.6	10,000	20
1F26B	66,153,200	19.9	24.4	10,000	20
Steel No. 10, 0.49 carbon, sorbitic					
10B26D	13,594,600	0	1.0	100	38
10B169B	11,395,600	0	1.0	100	38
10C13B	36,453,200	12.7	15.8	100	38
10D104D	22,157,400	0.7	3.3	1,000	35
10G143A	10,646,200	0.8	2.9	1,000	29
10F26A	38,656,400	15.1	18.6	5,000	29
10F13C	142,769,300	23.2	26.8	5,000	29
10C104C	11,496,500	0	2.1	5,000	20
10B65D	26,447,900	6.9	10.1	5,000	10
10G26D	13,443,600	2.5	4.8		
10K143B	116,649,100	6.0	7.8	20 axial	15[a]
10K143X	31,045,100	2.0	4.0	20 axial	15[b]
10N13C	138,780,600	18.1	21.3	20 axial	15[c]
10K52B	104,506,800	0	2.5	20 axial	30[a]
10K156N	37,416,200	7.4	9.8	20 axial	30[b]
10K69A	115,870,600	2.1	4.7	20 axial	30[c]
10K25A	123,751,900	6.4	8.5	20 axial	30[c]
10L39C	106,154,200	4.4	6.8	20 axial	30[d]
10N0C	166,415,100	41.5	45.2	20 axial	40[a]
10K78B	78,951,800	26.2	28.8	20 axial	40[b]
10K156B	36,717,900	17.3	19.5	20 axial	40[b]
10L182C	128,534,700	10.0	12.1	20 axial	50[a]
10K78	102,378,900	36.0	39.0	20 axial	50[b]
10K156	57,604,300	22.4	24.8	20 axial	50[b]

TABLE 20.—(*Continued*)

Specimen	Total cycles of stress	Increase over endurance limit after overstress (2) of endurance limit after subsequent understress (3), per cent		Amount of overstress applied	
		Greater than	Less than	Number of cycles	Excess over original endurance limit per cent
		Steel No. 10, 0.49 carbon, sorbitic			
10K0C	103,670,300	0	1.0	20 axial	50[c]
10L65A	122,138,600	4.0	6.3	20 axial	50[c]
10K91B	123,804,400	15.6	19.0	20 axial	50[d]
10M143A	138,875,300	17.1	20.2	1 axial	60[a]
10M143B	86,184,100	20.2	23.2	1 axial	60[a]
10M127B	86,089,400	13.9	15.9	1 axial	60[a]
10M78B	52,016,900	17.1	20.2	1 axial	60[a]
10M65B	40,580,700	0	1.9	10 axial	60[a]
10M130A	81,138,500	17.0	20.0	10 axial	60[a]
10M169	131,945,300	11.7	15.1	20 axial	60[a]
10K91A	121,695,900	7.1	10.2	20 axial	60[d]
10L0A	139,603,700	14.6	17.1	20 axial	60[d]
10K169B	136,771,700	18.1	21.6	20 axial	70[a]
10N13B	140,016,200	14.6	17.2	20 axial	70[c]
10K117A	127,660,900	9.7	12.4	20 axial	70[c]
10K13B	108,764,000	0	23.8	20 axial	70[c]
10N0A	77,084,000	15.1	18.1	20 axial	70[c]
10K104C	120,925,900	6.7	9.0	20 axial	70[d]
10N52A	125,299,000	9.5	11.7	20 axial	70[e]
10N52D	147,039,500	21.2	24.0	20 axial	70[c]
		Steel No. 10, 0.49 carbon, annealed			
10V117C	107,500,100	10.3	14.0	20 axial	80[e]
10V117A	112,388,800	9.7	13.2	20 axial	80[e]
10V78B	137,581,300	16.1	20.6	20 axial	80[e]
10U117D	121,017,300	20.1	23.6	20 axial	40[e]

[a] Specimen tested immediately after overstress.

[b] Specimen rested from 3 to 15 days after overstress.

[c] Specimen boiled in water for 1 hr., cooled and tested immediately.

[d] Specimen rested 3 months after overstress before testing.

[e] Specimen polished after overstress and tested immediately.

NOTE: Specimen 10G26D, 0.49 carbon, sorbitic was subjected to 2,000,000 cycles of stress 10 per cent below the original endurance limit of the metal, and its endurance limit after these cycles of understress was raised more than 2.5 per cent and less than 4.8 per cent.

show a decrease in percentage of reduction in area. It will be noted that this effect of increase of ultimate strength and decrease of reduction area is precisely the effect which static cold work produces, and indicates, therefore, that the understressing is in the nature of repeated cold work.

TABLE 21.—EFFECT ON THE STATIC STRENGTH AND DUCTILITY OF 10,000,000 CYCLES OF REVERSED AXIAL STRESS AT OR NEAR THE ORIGINAL ENDURANCE LIMIT

Results obtained by Moore and Jasper in the Joint Investigation of the Fatigue of Metals at the University of Illinois

Metal	Values before application of reversed axial stress		Values after application of reversed axial stress		Change in values due to reversed axial stress	
	Ultimate tensile strength, pounds per square inch	Reduction of area, per cent	Ultimate tensile strength, pounds per square inch	Reduction of area, per cent	Ultimate tensile strength, per cent	Reduction of area, per cent
0.37 carbon steel, normalized	71,900	53.5	85,300	40.2	+18.7	−24.8
(same) as rolled, transverse specimen	71,600	43.9	83,300	23.3	+16.3	−47.0
(same) as rolled, longitudinal specimen	72,900	51.0	88,500	47.3	+21.5	− 7.3
(same) as rolled, sorbitic specimen	104,200	41.5	106,800	40.1	+ 4.4	− 3.4
0.93 carbon steel, sorbitic	115,000	39.6	116,800	44.0	+ 1.6	+11.1
3.5 nickel steel:						
oil quenched, drawn at 1200°F	111,800	60.2	116,200	55.8	+ 3.9	− 7.3
oil quenched, drawn at 1100°F	117,500	60.1	123,200	62.1	+ 4.9	+ 3.3
annealed	101,600	52.2	104,000	48.9	+ 2.3	− 6.3
0.49 carbon steel, normalized	91,500	39.5	105,500	29.2	+15.3	−26.0
Wrought iron as rolled:						
transverse specimens	34,400	5.1	33,300	7.0	− 3.2	+37.2
longitudinal specimens	46,900	29.3	52,000	31.6	+10.9	+ 7.9

CHAPTER IX

FATIGUE FAILURE UNDER SERVICE CONDITIONS

Laboratory and Service Conditions.—In a laboratory test to determine the endurance limit of a metal, specimens are subjected to cycles of known stress. It is obviously a prime essential in laboratory work that tests be performed under known conditions. It then becomes very necessary that the machine designer keep in mind the fact that in machine parts and structural members the range of stress developed is not constant, and the magnitude of the stresses is not known with any high degree of accuracy. Freight-car axles are subjected to occasional high stresses of very uncertain magnitude, due to flat wheels, lateral flange pressure at frogs, and bad rail joints. There are very few stress-carrying joints in boiler plates which are not subjected to occasional very high localized stress at the edges of rivet holes. The steering knuckle of a motor car is subjected to frequent repetitions of rather violent stress, due to shocks transmitted from rough pavement. These stresses are, however, quite impossible of computation.

The endurance limit determined by laboratory tests is a very useful index of the strength of a metal under cycles of normal stress. It is a value which should be known by the designer before he designs machine parts to be made of the metal, but it is by no means the only physical property to be considered. No one physical property of a metal is sufficient to enable a designer to design a machine part or a structural member so that it will be both safe and economical.

It has been previously noted (see p. 165) that there seems to be little correlation between endurance limit and any index of elastic strength, such as the yield point or the elastic limit (however determined). Some metals show endurance limits under reversed stress above the elastic limit of the metal as received. If, however, parts in service were subjected to widely distributed[1] stresses above the elastic limit, failure would be likely to occur, not a fatigue failure, but a failure by plastic distortion of sufficient magnitude to impair the working of the machine or the integrity of form of the structure.

Effect of Occasional Overstress.—As noted in the foregoing paragraph, one factor affecting the serviceability of metal in machine parts is the effect of occasional overload—load which causes stresses above the yield point or above the endurance limit of the metal. It is to be noted that such stresses *change* the metal. A stress beyond the yield point improves the elastic strength after a period of rest, and a stress beyond the endurance limit starts minute fatigue cracks in the metal. Both experience and laboratory tests indicate that once a fatigue crack is started it will spread under subsequent cycles of stress somewhat below the original endurance limit of the metal. How much below the original endurance limit the subsequent stress must be to be incapable of spreading the existing crack is a matter of uncertainty.[2]

Metals seem to vary markedly in their ability to withstand occasional overstress without developing disastrous fatigue cracks. In general, ductile metals are superior to brittle metals in this respect, but ductility is not the only factor involved. Certain ductile alloy steels seem to be highly susceptible to damage by occasional overstress. Occasional overstress in machine parts is frequently applied very rapidly, and the ability of the part to absorb the

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[1] The term "widely distributed stress" is used to exclude the action of localized stress. The stresses computed by the ordinary (Rankine) formulas of mechanics of materials may be regarded as widely distributed stresses.

[2] The statement that overstress lowers the stress at which fatigue cracks will continue to spread seems to be contradicted by the raised fatigue strength observed in cold-drawn and cold-rolled iron and steel. It should be noted, however, that drawing through dies, or passing between rolls, causes lateral compression during cold working, and leaves a very smooth surface. Cold working by simple overstress tends to roughen the surface.

energy of the cycles of overstress is in some cases as important as the ability to withstand high unit stress.

The "life" of a machine part or of a structural member may be considered as made up of two parts: (1) cycles of normal working stress, which cover (say) 99 per cent of the "life" of the part, and (2) cycles of abnormally high stress, which cover (say) 1 per cent of the "life." To insure satisfactory service under (1), it is necessary that the working stresses shall be well below both the endurance limit and the yield point (if any exists) of the metal. To make probable the satisfactory service under (2), it is necessary that the metal shall be tough, so that the damage done by the occasional periods of high stress will not start disastrous fatigue cracks.

Warnings of Impending Fatigue Failure.—For parts made of ductile metals fatigue failures are likely to be more disastrous than are dead-load failures in machine and structural parts. Dead-load (static) failures of ductile metal are usually failures by plastic yielding unless the member is long enough to collapse by buckling; such failures usually occur without causing serious injury to the structure as a whole. For example, under an accidental overload, a steel crane-hook may be badly distorted without causing it to let go its load.

For all kinds of loads on brittle materials and for repeated loads on ductile metals, failure, if it occurs, usually means a shattering fracture without much warning. In some cases, however, careful systematic inspection will show signs of approaching fatigue failure. For example, in wire ropes bent around sheave wheels, approaching failure may usually be detected by the snapping of individual wires at the surface of the rope.

In car axles it is frequently possible to detect incipient fatigue failures by careful periodic inspection.[1] Experi-

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[1] An effective method of detecting incipient cracks is as follows: Oil the surface of the axle over the portion where cracks are expected to develop. Wipe off the oil on the surface. Then coat the surface with a wash of whiting and wood alcohol. This soon dries and then, if the axle is rotated and struck smartly with a hammer, the oil which has penetrated into any

ments in the Fatigue of Metals Laboratory of the University of Illinois indicate that in axle steel such cracks can be detected at about 50 per cent of the "life" of the axle for stresses slightly above the endurance limit. Figure 80 shows a crack in axle steel at an early stage of development. For higher stresses the chance of detection is, of course,

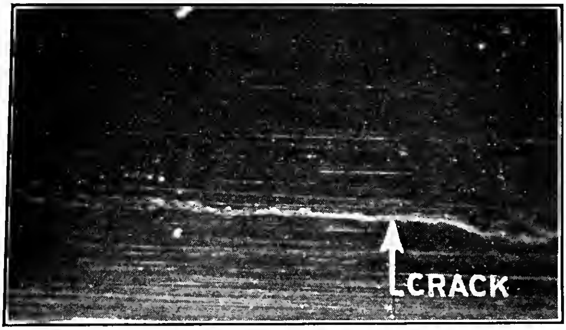

Fig. 80.—Fatigue crack in car-axle steel.

The specimen had been coated with whiting for the oil-whiting test and some of the whiting had worked into the crack.

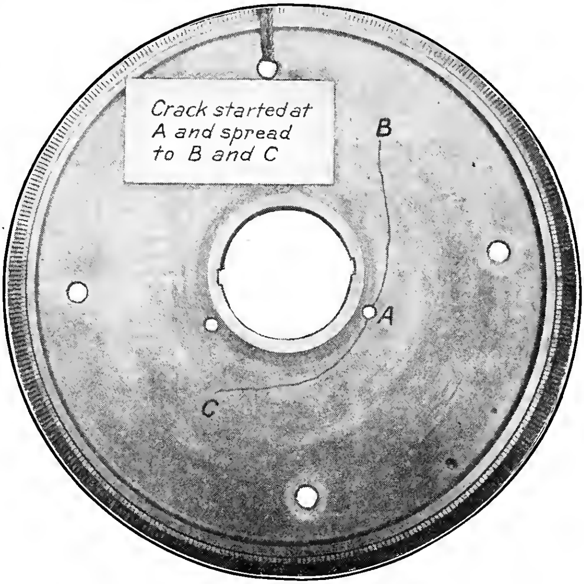

Fig. 81.—Fatigue crack in steam turbine disc.

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little cracks will be forced out, discoloring the whiting coating. By this means, cracks invisible to the unaided eye may be located.

less, but in general, the higher the stress the more rarely is it developed, and the fewer the number of cycles at any one period of such high stressing. Fatigue cracks have been detected in some steam-turbine disks before the cracks had spread to failure (see Fig. 81), although in other cases disastrous failures have occurred before any cracks were detected. In machine parts the commonest form of fatigue failure seems to be one in which a crack or cracks are started during short and infrequent periods of overstress and then spread slowly under normal loads or occasional periods of loads slightly above normal. For such failures the chance of detecting cracks before a disaster occurs is fairly high if periodical inspections can be made.

Probably examination under a high-power microscope would detect a fatigue crack at an earlier stage than the whiting-and-oil method outlined in the footnote on page 228, but the search over any considerable area of metal for a microscopic crack would involve so much labor in polishing surfaces and in traversing them with the microscope that the method would rarely be feasible in practice.

Another method which has been used with success in detecting small cracks in iron and steel pieces is that of H. S. Rawdon of the U. S. Bureau of Standards.[1] In this method the piece to be examined is polished, magnetized, and covered with a wash of finely divided iron ("iron mud" from cast-iron lapping disks) suspended in kerosene. The gathering of iron particles shows the location of cracks.

Typical Fatigue Fractures.—Whether a failure in a machine part is due to the progressive spreading of a fatigue crack or to some other cause can frequently be told by an examination of the fracture. Figure 82 shows three fatigue fractures which are typical of shafting failures under reversed flexure, such as occur in car axles. These particular failures were failures of laboratory specimens loaded as rotating-cantilever beams. In each failure two parts can be clearly distinguished: (1) a relatively smooth surface which marks the spread of the fatigue crack and which has

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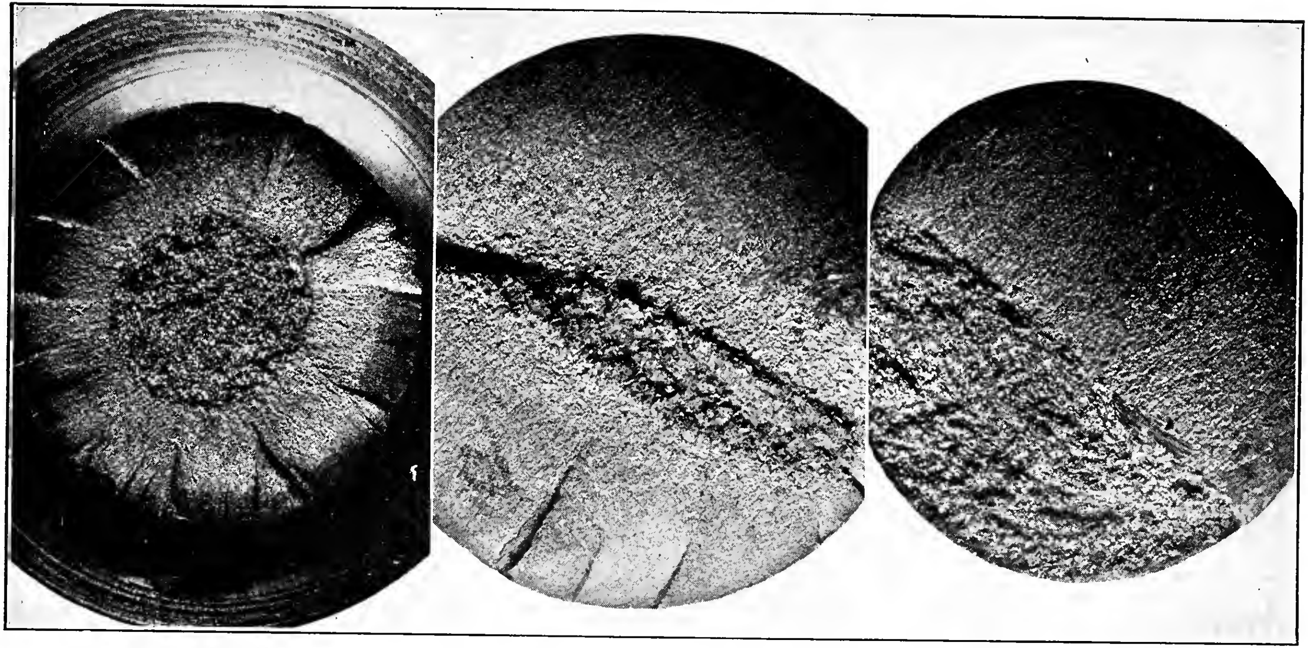
[1] *U. S. Bur. Standards, Tech. Paper*, 156.



(a) (b) (c)

FIG. 82.—Typical fatigue failures of shafting under reversed flexure.

been battered smooth by the repeated opening and closing of the crack, and (2) a rough "crystalline" surface which represents the final sudden failure of the small area of sound metal not reached by the fatigue crack when final failure occurred.

Figure 82(*a*) shows a fatigue crack which started all around the circumference and spread very evenly inward, leaving the remaining sound metal almost circular in shape. On the surface of the fatigue crack will be noted a number of radial lines, marking the edges of axial "steps" in the surface and bearing some resemblance to ripple marks left by flowing water on sand or clay. Such "ripple marks" are frequently found on the surface of fatigue cracks.

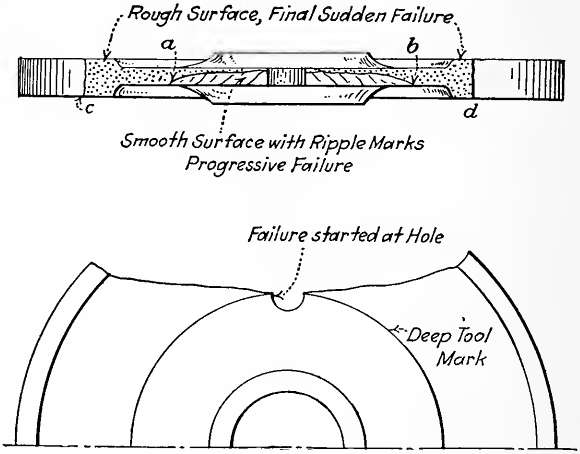


FIG. 83.—Diagram of typical fatigue failure caused by axial vibration in a thin disc.

Figure 82(*b*) shows a fracture in which fatigue cracks started at opposite sides of the shaft and spread toward a diametral line, leaving the remaining area of sound metal in the approximate shape of an elongated ellipse. Figure 82(*c*) shows a fracture in which a fatigue crack started at one side only of the shaft and spread inward, leaving the remaining sound metal in the shape of a segment of a circle.

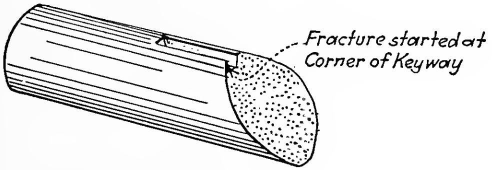
Figure 83 is a sketch showing a typical fracture due to axial vibration in a thin steel disk rotating at a high speed.

The axial vibration caused reversals of radial stress in the metal and the high speed of rotation set up steady tensile radial stresses. The fatigue fracture started at the intersection of a deep tool mark and a hole and spread over the length *ab*. At that stage the remaining sound metal *ac, bd* was so reduced in area that the steady centrifugal force caused a sudden tensile failure. The portion of the fracture *ab* showed a smooth surface with "ripple marks" like the surface shown in the outer ring of Fig. 82(*a*). The parts *ac* and *bd* of the fracture and the upper edge of the center portion showed a rough "crystalline" surface like the surface of the fractured core shown in Fig. 82(*a*).

Fatigue Fracture in Torsion
Transverse Shear

Fatigue Fracture in Torsion
Transverse and Longitudinal Shear

(a)



Fatigue Fracture in Torsion
Tension on inclined Section

(b)

FIG. 84.—Typical fatigue fractures in torsion.

Figure 84 shows typical fatigue fractures under repetitions of torsional stress. Figure 84 (*a*) is a sketch showing the development of longitudinal and also circumferential shearing fractures. It should be borne in mind that the longitudinal shearing unit stress in a shaft subjected to torsion is as great as is the transverse shearing unit stress. Figure 84(*b*) shows a failure not by shearing under torsion but by tension along an inclined plane. On a plane making 45 deg. with the axis of a shaft the extreme tensile unit stress is as great as is the extreme shearing unit stress on a section at right angles to the axis or the extreme shearing unit stress parallel to the axis of the shaft. Such "spiral" fractures under torsion as that shown in Fig. 84(*b*) are characteristic of rather hard, brittle metals, though under repeated stress such fractures sometimes occur in fairly ductile shafting steel. Figure 85 is from a photograph of a shaft which failed in service under cycles of torsional stress.

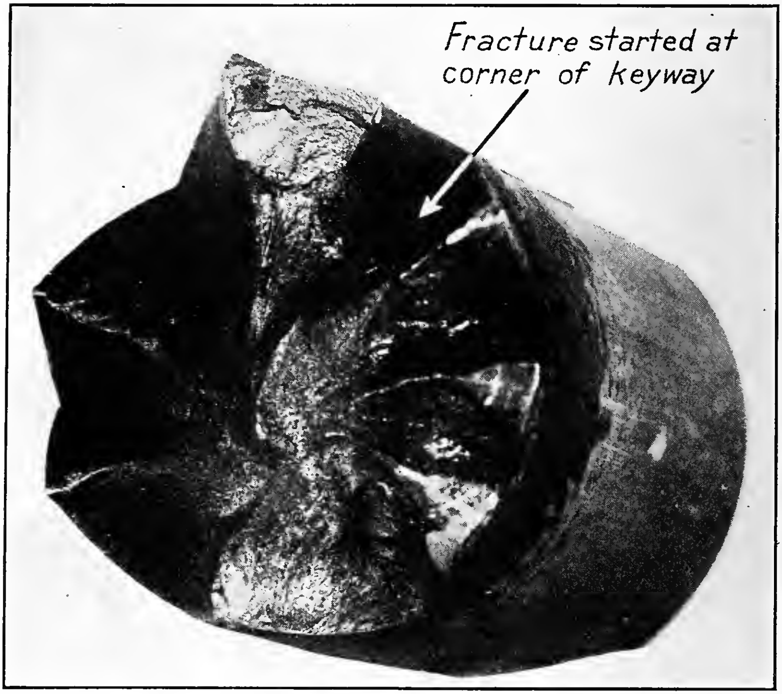


Fig. 85.—Fatigue fracture of shaft in service under repeated torsion.

The peculiar star-shaped fracture indicates a series of inclined tensile-stress failures. The failure started at a corner of the keyway, which is a point of high stress concentration.

Figure 86 shows a fatigue fracture of a bolt at the root of the screw thread under repeated axial loading. The rather irregular distribution of smooth fatigue-crack surfaces and rough final-failure surfaces is in marked contrast with the regular distribution of those surfaces shown in

Fig. 82 for specimens which failed under cycles of flexural stress.

Typical Fatigue Failures in Service. 1. *Structural Members in Bridges and Buildings.*—Fatigue failures are very rare in structural members of bridges and buildings. Such members are, for the most part, subjected in service to rather narrow ranges of stress, and very few such members are subjected to reversals of stress. Certain members in lift bridges are, however, subjected to partial stress reversal in service. Most structural members in bridges have rivet holes in them and at the edges of rivet holes there are high stress concentrations; probably under normal load the

Fig. 86.—Fatigue fracture of bolt under repeated axial tension.

localized unit stress at the edges of rivet holes is frequently as high as the yield point of the metal. In a few cases in practice fatigue cracks have developed at the edges of rivet holes and have spread into the member. In all cases with which the writers are familiar, such fatigue cracks were detected before a disastrous failure had occurred and the parts affected were replaced or patched.

2. *Boiler Plates.*—Fatigue cracks sometimes develop in boiler plates usually extending from one rivet hole toward the next. Both localized stress and corrosion effects are most marked at the edges of rivet holes, and corrosion and localized stress are mutually accelerative. Usually before a disastrous failure occurs, a crack can be detected by the leakage of steam or water through it; but there have been cases in which the combined effect of corrosion and stress caused a sudden tearing of plate through a large number of rivet holes at once, and a disastrous explosion followed, although there had been no leakage detected.

3. *Car Axles.*—Fatigue cracks sometimes develop in railway car axles. Such cracks practically always occur near fillets, where the localized stress is higher than the value computed by the ordinary formulas of mechanics of materials. Probably fatigue cracks begin under the occasional high stresses to which all car axles are occasionally subjected, high stresses caused by flat wheels, wheel flange pressure against "tight" frogs, bad joints in rails, etc. Once started, such cracks will spread under stresses lower than those necessary to cause the first fatigue cracks. Some railroads scrap axles after a certain mileage, while some street railroads take out axles after a certain mileage (usually about 100,000 miles), take off the wheels, and make a careful search for fatigue cracks. If no cracks are found, the axles are put back into service; if cracks are found the axles are scrapped. In detecting fatigue cracks in axles the process described in the footnote on page 228 is used. In view of the comparatively rare occurrence of periods of overload and rough service, the method of periodic inspection of axles for cracks seems to be a fairly effective pre-

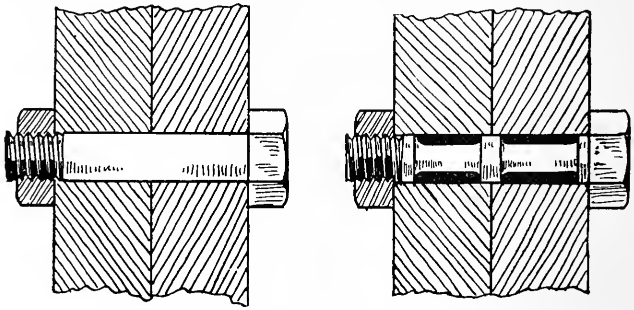
caution against disastrous fatigue failures in service. Axles cannot be inspected for cracks while in service, and when fatigue failures of axles occur, they occur suddenly, frequently with disastrous results.

4. *Automobile Axles.*—Automobile driving axles are not infrequently subjected to severe repeated stress, both torsional and bending. There is no opportunity for careful inspection of axles in service, and when fatigue failure occurs, there is no warning. Fatigue failure generally starts at the edge of a keyway, at a deep tool mark, or at a rough spot on the surface of the axle. In most cars the breaking of an axle does not usually cause a wreck, and a broken axle can easily be replaced.

5. *Automobile Steering Knuckles.*—Steering knuckles are subjected to occasional sudden, severe loads, usually not reversed. Fatigue cracks cannot well be detected in service, and a failure is frequently the cause of a serious accident. The only precaution available against fatigue failure seems to lie in the choice of the metal for the knuckle and in the careful design to minimize localized stress in the knuckle.

6. *Bolts and Studs.*—Bolts and studs have high stress concentrations at the roots of the threads, stress concentrations reaching probable values as high as four times the average stress on the section at the root of a thread. There is very little chance for inspection, and when fatigue failure occurs there is no warning.

Bolts and studs are frequently subjected to load very rapidly applied—shocks and blows, for example. Under rapidly applied load, an important criterion of strength is the ability of the bolt or stud to absorb *energy* without fracturing. This ability is somewhat different from the ability to carry load without fracture, and both ductility and strength contribute to the ability to resist energy loading. When it is feasible, the reduction of area of the shank of a bolt to a size slightly smaller than the section at the root of the threads (as shown in Fig. 87(*b*)) increases the energy-absorbing capacity of the bolt. In Fig. 87(*a*) the energy of a sudden load will be absorbed almost entirely in the very short sections of metal at the roots of the threads, and this metal cannot stretch sufficiently to absorb the energy of a heavy shock without fracture or, at least, the starting of a crack. In Fig. 87(*b*) the reduced shank will stretch appreciably, as well as the metal at the root of the threads. As stretch is one factor in energy absorption, the stress will be less for a bolt such as that shown in Fig. 87(*b*) than for a bolt like that shown in Fig. 87(*a*). Many years ago John Sweet stopped the frequent failures which occurred in the connecting-rod bolts of the "straight-line" steam engine by changing the design of the bolts from that shown in Fig. 87(*a*) to that shown in Fig. 87(*b*).



(*a*) (*b*)

Fig. 87.—Two designs for bolt to resist energy loading in tension.

7. *Springs.*—Springs are usually designed to give large elastic deformations and to absorb energy. The capacity for elastic-energy absorption varies as the square of the stress, and hence the prime requisite for the material is a high elastic strength. Springs are sometimes fitted with stops to prevent overstress, and are usually made of hard, brittle steel. The failure of a spring rarely causes a serious accident. Fatigue failures are not uncommon and occur without warning; usually the fatigue crack is started by a period of unusually high stress, and the crack spreads gradually with final failure often occurring under normal load. Careful lubrication of leaf springs increases the deformation under any given load, and reduces stress concentration due to wear at bearing points.

8. *Railroad Rails.*—Railroad rails are subjected to partial reversal of high stress in service. Normally, the head of the rail is worn out by traffic; this factor necessitates the use of rather hard steel not very high in ductility. The passage of loaded wheels over the rail cold rolls the steel in the head. This one-sided cold rolling in all probability sets up heavy stresses in the interior of the rail head. In exceptional cases the combination of high stress due to heavy wheel loads and high internal stress due to cold rolling of the surface of the rail start a "shattered zone" or a "transverse fissure," apparently from a focal point in the interior of the rail head. Figure 88 shows a rail fractured by a progressive failure starting from a transverse fissure *ABC*, which apparently started at a focal, minute, area *O*.

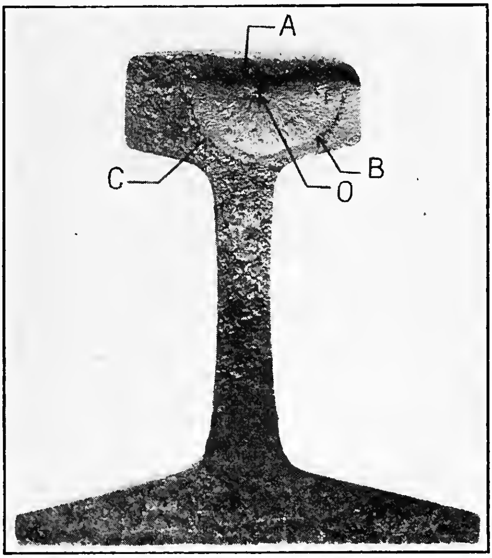


Fig. 88.—Fatigue fracture of rail, started at a transverse fissure.

The whole subject of transverse fissures in rails is a very fertile field for debate; metallurgists and engineers are divided in opinion as to whether abnormal rolling-mill conditions which produce poor steel or severe service conditions should bear the chief blame for their existence. Transverse fissures usually develop much more frequently in heats of steel from certain rolling mills and seem to develop from some defect which is initially in the rail and which acts as the nucleus of a fatigue failure under the high stresses set up by heavy wheel loads in service. As noted above as a matter of experience, transverse fissure failures in rails are not at all common. When they do occur, they occur without warning.

9. *Rotating Disks.*—Thin rotating disks have critical speeds at which severe axial vibration ("fluttering") is likely to occur, with consequent cycles of reversed flexural stress in a radial direction. Under such conditions a fatigue crack may be started. Such disks may be subjected to many thousand severe vibrations before there occurs a chance for inspection; hence there may occur a disastrous fatigue failure before a crack is detected. Figure 83 shows such a failure and Fig. 81 shows a disk in which a crack was detected before it had spread to failure. Most of such failures start at a deep tool mark, a hole, or other point of high localized stress.

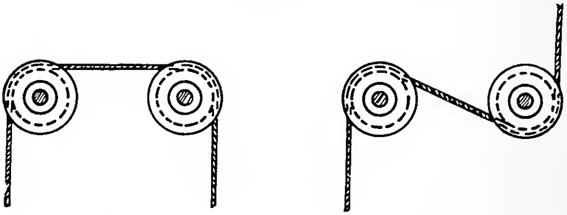

(*a*) (*b*)

Fig. 89.—Wire rope bent round sheaves.

The available means of safeguarding disks against this fatigue failure are: (1) careful surface finish, (2) avoiding of holes at regions of possible high stress, and (3) the designing of the disk so that the running speed of the machine will not approach closely the critical speed causing axial vibration in the disk.

10. *Wire Ropes Bent around Sheaves.*—The wires in wire ropes running over sheaves are subjected to cycles of flexural stress of a magnitude depending on the size of the individual wires and the diameter of the sheaves. Figure

89(*a*) shows a rope bent around sheaves in which there would be repetition of stress but not reversal. Figure 89(*b*) shows a rope bent around sheaves in which there would be reversal of stress. Usually, before complete fatigue of the rope occurs, individual wires snap, and as the wires on the outside of the rope are subjected to wear as well as flexural and tensile stress, the outside wires usually snap first. Hence the failure can be detected before it causes a serious accident. For wire ropes inspection should be frequent. The rope should be replaced when wires begin to break.

Summary of General Principles of Design of Members Subjected to Repeated Stress.—Members subjected to repeated stress should be designed so that the normal working stress will be well below the endurance limit of the metal for the range of stress imposed in service. In addition to this precaution the designer should so shape a machine part as to reduce localized stress to a minimum, and he should calculate or estimate the magnitude of localized stress when feasible. This means that he must avoid as far as possible sharp corners and notches in the outline of the parts, must provide generous fillets at shoulders, must avoid holes in regions of high stress (or must allow for a stress concentration of about twice the nominal computed stress if holes cannot be avoided), and should avoid using screw threads to transmit repeated stress, as far as is feasible.

In choosing metal for such parts, the designer must consider not only its strength, but also its ability to withstand occasional overstress without starting a progressive fatigue crack.[1] The designer must consider how serious would be the results of fatigue fracture of any part and use lower stresses, and should devise safeguards to minimize damage in the cases of a member whose failure would cause a disaster, and in the case of a member which cannot be frequently or readily inspected for incipient fatigue cracks.

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[1] Usually this means choosing as ductile a metal as is consistent with the necessary strength.

TABLE 22.—ESTIMATED SERVICE REQUIRED OF VARIOUS MACHINE AND STRUCTURAL PARTS SUBJECTED TO REPEATED STRESS[1]

Part of machine or structure	Range ratio[1] r	Approximate number of cycles of normal stress, millions	Approximate number of cycles of overstress	Remarks
Railroad bridge, chord members	+0.10 to +0.50	2	10,000	Range ratio higher for long span bridges than for short
Elevated railroad structure, floor beams	0	40	400,000	Overstress due to flat wheels and other poor conditions in rolling stock
Railroad rail, locomotive wheel loads	−0.25	0.5	10,000	Overstress due to unbalance in wheels, connecting rods, etc.
Car axles	−1.0	200	1,000,000	Overstress due to flat wheels, tight frogs, poor track conditions
Crank shafts, steam engine	0	1,000	1,000,000	Overstress due to hard starting conditions
Crank shafts and connecting rods:				
stationary gas engine	0	1,000	1,000,000	Overstress due to hard starting conditions
automobile	0	120	1,500,000	Overstress due to road conditions
airplane engine	0	18	0	Driven at full power practically all the time
Steam engine piston rods and connecting rods	−1.0	1,000	1,000,000	Overstress due to hard starting conditions
Line shafting in shops	0	400	4,000,000	Overstress due to sudden power requirements
Steam turbine shafts, bending stresses	−1.0	15,000	10,000,000	Overstress due to sudden power requirements
Steam turbine blades	0	100,000	100,000,000	Overstress due to sudden power requirements

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[1] The values in this table are to be regarded as rough estimates rather than as precise values.

Estimated Number of Cycles of Stress for Various Machine and Structural Parts.—Table 22 gives values of the estimated number of cycles of stress which must be withstood in the normal "lifetime" of various machine parts and structural members. The values given are to be regarded as rough estimates, giving the "order of magnitude" of the number of cycles of stress rather than anything even approaching a precise number. The values given for probable range of stress are also to be regarded as estimates rather than precise values.

CHAPTER X

FATIGUE OF WOOD

Fatigue Failure of Wood in Service.—Structural and machine parts subjected to repeated stress are rarely made of wood. Bridge timber, electric-wire poles, and floor beams in mill buildings are occasionally subjected to "vibration" which is equivalent to a reversed flexural stress superimposed on a steady stress, so that the net range of stress is narrow. On account of its light weight, wood has been widely used in airplane construction, and in this service it is subject to a considerable amount of repeated stress. Fatigue test data for wood are very scarce, and only a few tentative conclusions can be drawn as to the fatigue strength of wood.

Vibration Tests.—It will be of interest to study some of the results of repeated-stress tests which have been obtained at the Forest Products Laboratory at Madison, Wis. These results have not been published hitherto.

Some of the first tests which were made were suggested by the fact that certain members of airplanes were subject to vibration. It can be shown[1] that the frequency of vibration of a simple beam is given by the formula

$$F = 30.8\frac{EI}{Wl^3},$$

in which F = frequency of complete vibrations per second,
E = the modulus of elasticity in static bending, in pounds per square inch,
I = moment of inertia of the cross-section, in inches^4,
W = weight of the stick between centers, in pounds,
l = length between centers, in inches.

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[1] See Morley, "Strength of Materials," p. 406.

For the case of a vibrating-cantilever beam the formula is

$$F = 11.04\frac{EI}{Wl^3}.$$

The experiments consisted in determining the experimental constants for simple and cantilever beams, which are given as 30.8 and 11.04 in the above formulas.

The apparatus consisted of a vibrating beam having a brass stylus which traced a record on a rotating drum. An electric tuning fork traced a record on the same drum, thus making it possible to obtain the frequency. Since the other quantities in the formula could also be determined, it was possible to compute the constant. This constant was found to have an average value of 31.2 for simple beams, and 10.5 for cantilever beams. There was comparatively little variation in the constant for the various cross-sections and lengths of beam used.

The dimensions of the beams used in the experiments varied from ½ to ⅞ by 35 in. long, to ¾ by ¾ by 70 in. long for the simple beams, and from ½ by ⅞ by 24 in. long to 1 by 2 by 42 in. long for the cantilever beams. The species tested were red gum, yellow birch, yellow pine, white pine, hard maple, black walnut, Douglas fir, and Sitka spruce, a total of 128 specimens being tested.

Damping of Vibrations.—Some experiments were also made on the damping of vibrations in wooden beams. Since the specimens were fastened to a heavy concrete column, it was thought that the energy loss due to vibrations imparted to the column must have been very small. The energy loss due to air friction was also found to be small, so that most of the energy loss must be largely due to mechanical hysteresis. The tests indicated that there was a difference in damping of vibrations in the various species of wood, and that this damping was independent of the modulus of elasticity of the wood.

Effect of Vibration on Strength and Stiffness.—The next series of tests was made to determine the effect of vibrations on the strength and stiffness of relatively long Sitka spruce specimens. The dimensions used were $\frac{3}{8}$ by 2 by 52 in. long, and $1\frac{3}{16}$ by 2 by 78 in. long. About one-half of the tests were carried out on matched pairs of test pieces, one specimen being vibrated and one not vibrated, a total of 44 specimens being tested.

The number of vibrations used in the tests was 900 per minute. Then using the formula $F = 31.2\frac{EI}{Wl^3}$, and knowing E, I, and the weight per cubic foot of the beam, and having adopted a length for the beam, it was possible to compute the depth of cross-section required. Assuming a value of 7,200 lb. per square inch for the elastic limit for air-dry Sitka spruce, it was possible to calculate the amplitude of vibration for the specimen from the ordinary deflection formula, in order to make sure that the elastic limit of the material would not be exceeded. The amplitude of all test pieces was then kept at about one-half of that computed from the elastic-limit stress.

Specimens were vibrated and then tested statically, the time of vibration varying between 15 min. and 96 hr., representing 13,500 and 5,184,000 cycles of stress, respectively. The modulus of elasticity was not greatly affected by the vibration, although, in general, there was a reduction in modulus due to the vibration which varied from 1.5 to 10.5 per cent. This reduction seemed to be as great after 1 hr. of vibration as after 16 hr. While the conclusion was drawn that the change in modulus might perhaps be due to changes in moisture content and temperature of the specimen due to vibration, yet the results obtained on concrete (see Chap. XI) would lead one to suppose that such reduction of modulus of elasticity due to repeated stressing might well be expected.

It may be noted here that weight and moisture determinations were made during the tests, and in most cases practically no difference in weight could be determined, indicating that, in general, variations in moisture content were negligible.

The effect of vibration on the elastic limit and modulus of rupture could not be detected from the results, the values being about the same for the specimens which had been vibrated and those which had not been vibrated.

Fatigue Tests of Wood.—Another series of tests which was carried out at the Forest Products Laboratory consisted in making rotating-beam tests on wooden specimens. Specimens 2 in. square were gripped in a lathe chuck, and the projecting portion was then turned down to a diameter of $\frac{5}{8}$ in. A generous fillet joined this portion to the fixed end of the specimen. At the free end of the specimen a brass ferrule was attached, and through this the specimen was loaded by means of a lignum-vitae roller. Forty-five specimens each of kiln-dried Sitka spruce, kiln-dried Douglas fir, and green southern white oak were tested in fatigue, and five specimens of each species were tested in static bending. Some air-dried specimens of Douglas fir were also tested. The static bending tests were made on specimens held and turned in the lathe and loaded in a manner exactly like the fatigue specimens. Half the tests in static bending were made with the plane tangent to the annual rings in a vertical position, and half with the plane tangent to the annual rings in a horizontal position, the load in all cases being applied vertically. The speed used in the fatigue tests was 2,880 cycles per minute.

Table 23 shows the results obtained from the tests. Figure 90 shows the *S-N* diagrams plotted from the results of these tests.

The fatigue tests were not carried out to a sufficient number of cycles to make the determination of an endurance limit certain, but the indications are that the endurance limit of wood can be determined at a much smaller number of cycles than is the case with metals. In this respect wood resembles concrete (see Chap. XI). In all cases tests were carried out at least to 300,000 cycles. The curves show that when the applied unit stress is one-third of the

TABLE 23.—RESULTS OF STATIC TESTS AND OF FATIGUE TESTS OF WOOD

Test results obtained at the U. S. Forest Products Laboratory, Madison Wis.

Kind of wood	Moisture content, per cent	Static modulus of rupture, pounds per square inch	Estimated endurance limit (rotating-beam test), pounds per square inch	Ratio of endurance limit to modulus of rupture
Sitka spruce, kiln dried	13.8	12,100	3,200	0.27
Southern white oak, green	above fiber saturation point	10,600	3,200	0.30
Douglas fir, kiln dried	14.3	15,000	4,000	0.27
Douglas fir	23.8	12,800	3,900	0.31

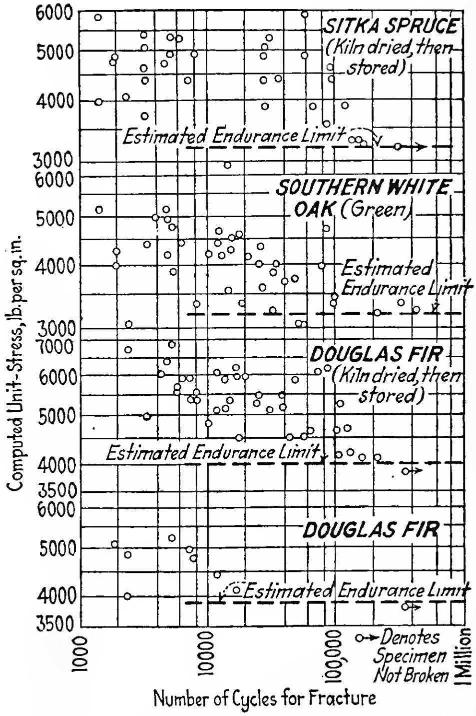

FIG. 90.—*S-N* diagrams for fatigue tests of wood. (*U. S. Forest Products Lab.*)

static modulus of rupture[1] or a little larger, failure may be expected to take place quite rapidly. The indications for endurance limit are in the neighborhood of 0.25 of the static modulus of rupture. It is of interest to note that the kiln-dried Sitka spruce and Douglas fir show a lower ratio of endurance limit to modulus of rupture than the other two woods which had a much greater percentage of moisture.

Tests Made at the National Physical Laboratory.[2]—Stanton reports some tests made on spruce wood which was to be used for airplane wing spars. This material had an ultimate tensile strength of 6,800 lb. per square inch, and the fatigue specimens were tested in rotating bending. The specimens were selected so as to have similar distribution and thickness of the annual rings.

When stresses of ±2,510 lb. per square inch were applied, the specimen began to show cracks at 6,000,000 cycles and failed after 16,860,000 cycles. With stresses of ±1,970 lb. per square inch cracks developed at 16,250,000 cycles and failure set in after 16,860,000 cycles. With stresses of ±1,625 lb. per square inch, no cracks were visible even after 125,700,000 cycles. Evidently, therefore, the endurance limit of this material lay between 1,600 and 1,970 lb. per square inch, and Stanton concluded that ±1,875 lb. per square inch would be below the endurance limit. The endurance limit was therefore about 25 per cent of the ultimate tensile strength given above.

Repeated-impact Tests.—The Forest Products Laboratory also made some tests on the effect of repeated impacts on Douglas fir specimens. The machine used for these tests dropped a heavy hammer (about 500 lb.) repeatedly through a distance of 0.02 in. This action produced a stress in the specimen which was a little greater than the elastic limit in static bending. After specimens had

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[1] The modulus of rupture is a value obtained by dividing bending moment at fracture by the value I/c for a flexure specimen. Modulus of rupture is measured in pounds per square inch and serves as a comparative measure of static flexural strength.

[2] *Engineering* (*London*), p. 605, June 23, 1916.

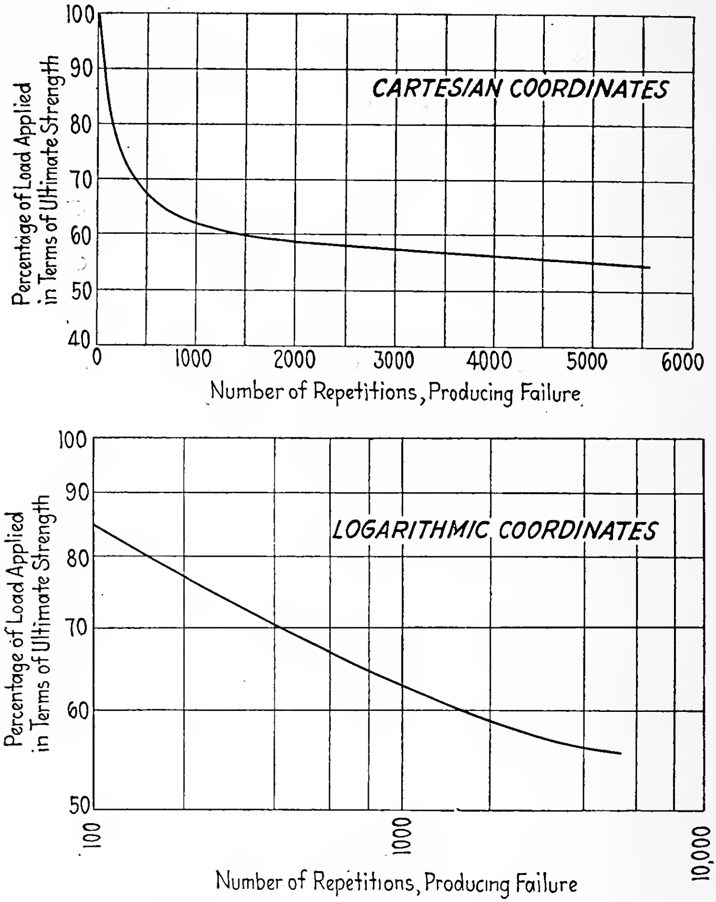
been subjected to this repeated-impact test, in some cases to as many as 8,000 impacts, they were tested in static bending. When these results were compared with results on similar specimens which had not been subjected to impact, it was found that the repeated impact had produced no significant change in the properties of the wood.

CHAPTER XI

FATIGUE OF CEMENT AND CONCRETE

Fatigue of Concrete in Service.—Concrete in service is most commonly subject to steady loading. Reinforced-concrete bridges and concrete arches are subjected to loads varying from dead load to dead load plus live load, cycles of stress not involving reversal. Concrete highway slabs are subjected to repeated load varying from practically zero to a maximum. The significant endurance limit for concrete is the endurance limit for cycles of stress varying from zero to a maximum.

Limitations of Experimental Study of the Fatigue of Concrete.—Fatigue tests of concrete must cover only a short time or must be made on concrete several months old; else the results will be affected by the natural gain in strength with age. Concrete test specimens must be of considerable size; else their strength is determined largely by the strength and location of a few large pieces of gravel or stone. Large-size specimens require testing machines of high capacity, of much higher loading capacity than the testing machines used for fatigue tests of metals. The machines used have been either single-lever machines or ordinary "static" testing machines designed to be automatically operated between definite limits of load. In either case the speed of testing was slow, and judged by its performance under static load tests, the strength of concrete is markedly affected by the speed of testing. As a result of these conditions, the test data available for determining the fatigue strength of concrete are very meager, and long-time test data are almost entirely lacking. Most of the tests involve cycles of stress varying from a small compressive stress to a large compressive stress. Some estimates of the fatigue strength of concrete are made in this chapter, and these estimates are based largely on extrapolation of the available test data, assuming a general similarity of behavior under test between concrete and metals.



(*a*) Upper (*b*) Lower

FIG. 91.—*S-N* diagrams for neat cement cubes under repeated compression. (*Basdeon Van Ornum in Trans. A. S. C. E.*)

Fatigue Tests of Cement.—Van Ornum[1] reports some fatigue tests at Washington University, St. Louis, made on 92 2- by 2-in. cubes of neat cement. The static ultimate compressive strength was determined, and then various

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[1] *Trans. Amer. Soc. Civil Eng.*, p. 443, 1903.

specimens were subjected to cycles of stress with a range from practically zero to a maximum. The maximum unit stress varied from 55 to 95 per cent of the ultimate compressive strength. The tests were made on blocks 4 weeks old. Figure 91(*a*) shows the results which were obtained. While the tests were not carried out to a sufficient number of cycles to determine the endurance limit of the material accurately, yet the indications are that the endurance limit for cycles of stress ranging from zero to a maximum, ($r = 0$) would probably be found to be about 50 per cent of the ultimate static strength. Figure 91(*b*) shows Van Ornum's diagram to a logarithmic scale.

The similarity of this curve to the *S-N* diagram for metals is obvious, and suggests that cement subjected to repeated stresses behaves in a manner which is similar to the behavior of metals under similar conditions. Furthermore, the indications are that the material has an endurance limit which has a relation to the ultimate static strength of the material.

Fatigue Tests of Concrete.—In the investigation mentioned above, similar tests were also carried out on 18 concrete cubes 7 by 7 in. in cross-section, subjected to a range of stress from practically zero to a maximum. The results indicated that concrete also obeyed the same general law shown in Fig. 91, breaking under repeated stresses which were much less than the ultimate static strength.

Van Ornum,[1] ~~in a~~ later research, made tests on both concrete compression blocks and reinforced-concrete beams under cycles of stress ranging from a small value to a maximum. Crushed limestone was used in a 1:3:5 mix, and the loads were applied by means of an oil-pressure piston at the rate of from 2 to 4 per minute.

The tests on the compression blocks were made at ages of 1 month and 1 year. The number of repetitions before rupture varied from 1 to 83,000, and the maximum stresses used in the fatigue tests were various percentages of the average compressive strength as determined from static

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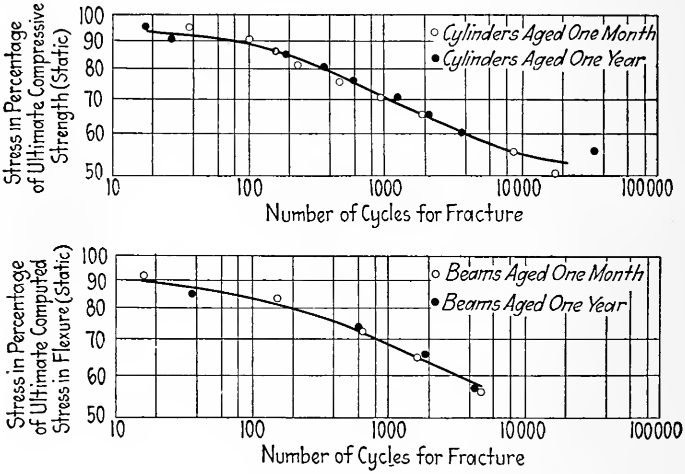
[1] *Trans. Amer. Soc. Civil Eng.*, p. 294, 1907.

tests of similar blocks. Three specimens loaded at 55 per cent of the ultimate static strength withstood over 40,000 cycles without failure. The present authors have plotted these results to logarithmic coordinates in Fig. 92(*a*). There is some evidence of a break in the curve for the cylinders aged one year and tested at 55 per cent of the ultimate static strength, indicating an endurance limit at this value of stress.

The tests of the reinforced-concrete beams were made at ages of 1 month, 6 months, and 1 year. The number of repetitions before rupture varied from 1 to 14,000, the stresses being again various percentages of the load causing failure in static tests. The results are plotted in Fig. 92 (*b*) to a logarithmic scale. There is no evidence of a break in the curve, and presumably the endurance limit of the material had not been determined.



(*a*) Upper (*b*) Lower

FIG. 92.—*S-N* diagrams for concrete cylinders and beams under repeated stress. (*Based on Van Ornum in Trans. A. S. C. E.*)

The failure of the beams began with tension cracks, then usually (but not always) diagonal tension cracks, and finally a compressive failure at the top of the beam near the loading point. These indications began with minute amounts which gradually increased until failure occurred.

It was found that in the large majority of cases the gradual and progressive destruction of the bond between the steel and the concrete had an important influence upon the results.

Van Ornum concluded that the stress-cycle curve became horizontal at about 50 per cent of the static ultimate strength. The present authors feel that the tests were not carried out to a sufficient number of cycles to make this conclusion certain. Evidence which will be mentioned later, however, indicates that the material can adjust itself to a certain cycle of stress, and presumably withstand this stress indefinitely.

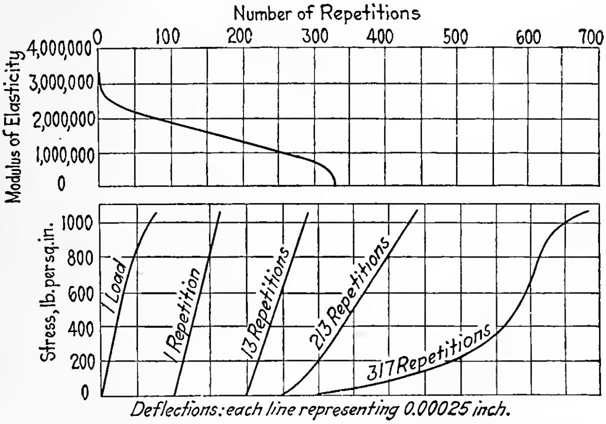


FIG. 93.—Elastic behavior of concrete cylinders under repeated compression. (*Based on Van Ornum in Trans. A. S. C. E.*)

Elastic and Inelastic Behavior of Concrete under Repetition.—The above research by Van Ornum included some very interesting observations on the elastic behavior of the compression cylinders subjected to repeated stresses. Compressometers with gage lengths of 8 in. were used to measure the compressive strains. It was found that the stress-deformation curve for the first loading was a straight line for the lower stresses, becoming convex upward for the higher stresses, as shown in Fig. 93. For the second loading the stress-deformation curve was practically a straight line up to the maximum stress applied, this straight line being parallel to the straight portion of the curve for the first loading. It was found that this first stage under repeated loading might continue for a considerable number of repetitions. The second stage of the test was characterized by a gradual decrease in slope of the straight line. This stage did not continue long for cases in which the number of cycles required for rupture was small. The third stage began relatively near the failure point, and was characterized by increase of deformation, which produced a curve which was concave upward. Finally, the fourth stage added to this curve a portion which was convex upward for the higher stresses, the deformations continuing to increase until failure occurred. These various types of deformation curves are shown in Fig. 93 (*cf.* Fig. 101).

The modulus of elasticity of the blocks was computed, based on the maximum stress and the corresponding deformation. The curve for modulus showed an increase for the second loading, a decrease during the next few applications to its original value, and then a gradual straight-line decrease during the greater portion of the test, terminating in a downward curve as the failure point was approached. This is shown in the upper part of Fig. 93.

A few specimens were also observed when subjected to maximum unit stresses which were too small to cause final failure. In these tests, as shown in Fig. 94, the first two stages of the test were the same as before, but the third and fourth were absent. For the case of the test data shown graphically in Fig. 94, the specimen was subjected to 30,000 cycles at a stress of 1,000 lb. per square inch without failure. The modulus curve decreased to a constant value equal to about two-thirds of its original value, after about 12,000 cycles. The stress-deformation curves, after the modulus of elasticity had become constant, were parallel straight lines.

Another phenomenon which was observed during these tests was that a permanent set occurred in all specimens during the first few loadings. For specimens which finally failed, the permanent set became comparatively small during the second stage, but again increased during the third and fourth stages. For specimens which presumably would have withstood an indefinitely large number of cycles, the evidence of permanent set rapidly disappeared.[1]

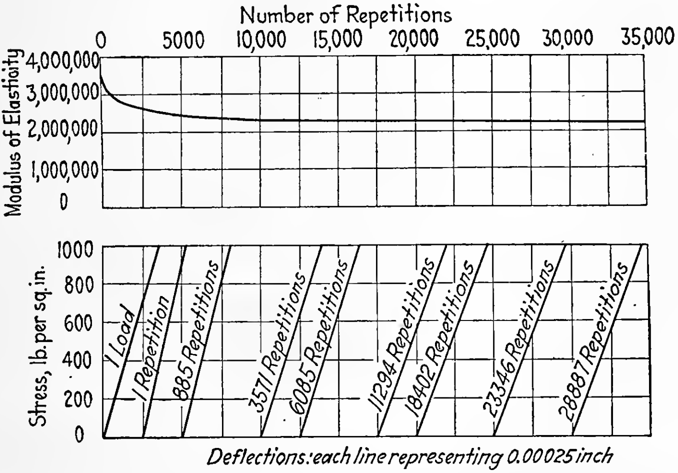


FIG. 94.—Elastic behavior of concrete under repeated compression below endurance limit. (*Based on Van Ornum in Trans. A. S. C. E.*)

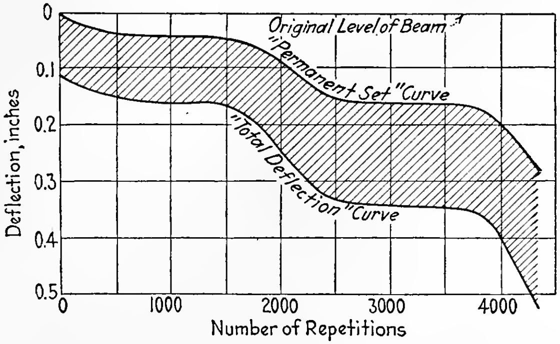


FIG. 95.—Elastic behavior of reinforced concrete beam under repeated flexure. (*Based on Van Ornum in Trans. A. S. C. E.*)

Beams under Repeated Loading.—Observations similar to the above were also made on reinforced-concrete beams,

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[1] In the opinion of the authors of this book this statement applies only after the adjustment to a certain cycle of stress is completed.

the deflections of the beam being read when the maximum repetition load was on and also when it was off. These results are shown in Fig. 95, which shows the total deflection curve and the permanent-set curve. The shaded area represents the range of deflections throughout the test.

The figure indicates five stages during the progress of the test. The first stage represents initial adjustment of the beam to the applied stress, accompanied by the development of small tension cracks in the beam. The second stage is characterized by a very slight downward trend in the curve, indicating fairly stable conditions. The third stage shows a rather rapid downward slope in the curve, accompanied by the breaking of the bond between the steel and concrete, and the enlargement of tension and diagonal cracks. The fourth stage is again nearly horizontal on the curve, indicating fairly stable conditions. The fifth stage shows a rapid downward trend in the curve, accompanied by the failure of the beam in compression and ending in complete failure.

It was found that when a beam was subjected to stresses which presumably could have been withstood indefinitely, only the first two stages shown in Fig. 95 were developed. The curves became horizontal, and the vertical distance between the two curves remained constant with increase in the number of cycles.

These results are especially interesting in indicating that concrete, under stresses which will not produce final failure, apparently acts just as metals do under similar conditions. Concrete, like metals, evidently has the power to adjust itself to cycles of stress, when the maximum unit stress is within a certain limit. These experiments show that after such adjustment the behavior of the material is elastic, and that the modulus of elasticity reaches and maintains a constant value. The similarity in behavior between concrete and the metals is obvious, and tends to reinforce the conclusion that the endurance limit of a material is a definite physical property.

Tests of Bond between Concrete and Steel.—Van Ornum also made tests on the effect of fatigue on the bond between steel and concrete. He used beams 4 by 4 in. in cross-section, and 15 in. long, with a ⅝-in. square plain steel bar placed with its center 1 inch from the tension side of the beam. The specimen was clamped on a fairly rigid frame. A machine was devised which had two metal struts, having a reciprocating motion. On the front end of these struts was a metal cross-head which was faced with 1½ in. of oak. By means of this device, the specimen was subjected to blows at the rate of 150 per minute, thus being subjected to impact, bending, and vibration. The tests were made when the concrete was 1 month old.

To obtain comparative results, 18 specimens had the steel rods pulled from the concrete in static tests without preliminary fatigue treatment. Under these conditions the average bond strength was 150 lb. per square inch, and the frictional resistance after the bond was broken was 100 lb. per square inch.

Thirty specimens were subjected to fatigue treatment, the average number of blows being 50,000. After this treatment the steel was pulled from the concrete and developed an initial average bond strength of 125 lb. per square inch, and a subsequent average frictional resistance of 90 lb. per square inch. The bond strength had therefore been reduced 17 per cent, and the frictional resistance 10 per cent, due to the repeated blows.

Withey[1] also made some tests on the effect of fatigue on the bond between steel and concrete in reinforced-concrete beams. In these tests the steel was imbedded at the ends of the beam and exposed near the middle of the beam. This made it possible to attach extensometers to the exposed rods to determine the deformation in the steel and the load on the rods. The beams rested on end supports and had the load applied at two symmetrical points on the top of the beam. The repeated loads were applied at one of the supports by means of a cylinder and piston, at the rate of

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[1] *Univ. Wisconsin, Bull.* 321, 1909.

100 repetitions per minute. The concrete for the repeated-stress tests was a 1:2.2:4.4 mixture, having crushed limestone for the coarse aggregate. The tests were made on concrete 1 month old.

From static tests the conclusion was drawn that the maximum bond strength for plain rods less than ¾ in. in diameter would be about 250 lb. per square inch and for rods of larger size about 200 lb. per square inch. The static bond for rusted rods was considerably greater than for plain rods, and was about twice as great for corrugated rods as for plain rods.

Under repeated loading the results showed that about 50 to 60 per cent of the static ultimate bond stress could be repeated many times on plain rods without failure. Rusted rods showed better bond strength than plain rods. The tests on corrugated rods indicated that about 60 to 70 per cent of the static bond stress could be repeated many times without failure. The number of repetitions used in these tests varied from 1,000 to 104,000 for the different beams, and in quite a few cases the tests were not carried out to destruction. No attempt was made to determine the bond stress which could be withstood indefinitely without failure.

Beam Tests by Berry.—Berry[1] made some tests at the University of Pennsylvania on reinforced-concrete beams 8 by 11 in. in cross-section, using a span of 12 ft. A mix of 1:2:4 was used, the age of the beams when tested was 4 weeks, and the rate of load application was 30 per minute. Compression cylinders showed that the concrete had an ultimate strength of 1,630 lb. per square inch at the age of 6 weeks.

Four beams were subjected to repeated stresses, and three of these had duplicates which were subjected to an ordinary static test. The tensile stresses in the steel varied from 14,300 to 18,300 lb. per square inch for the different beams, and the compressive stresses in the concrete varied from 628 to 940 lb. per square inch. After receiving from

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[1] *Proc. Amer. Soc. Testing Materials*, vol. 8, p. 454, 1908.

200,000 to 1,100,000 cycles of stress, the beams were subjected to increasing stresses until failure occurred. The results showed that the maximum deflection at failure and the ultimate load for the two beams of each set were much the same. The results also indicated that the hundreds of thousands of cycles of stress which were applied did not have any marked effect on the static strength and the deflection at static failure of the beams tested.

In the fatigue tests the deflection of the beam increased with the number of repetitions. The elastic deflection for any constant load remained nearly constant, but the permanent set increased. At least one-third of the set present after from 300,000 to 1,000,000 repetitions occurred during the first 10,000 cycles, and a very considerable part of the set occurred during the first few cycles. Berry says:

> While it is evident that the rate of increase in the set is relatively very large for the first few applications of the load, there is nothing to indicate that for a working load the set would cease to become greater.

On the basis of the tests performed by other investigators it seems clear that if the maximum stress applied in fatigue is sufficiently low, the permanent set will cease to increase and the beam apparently will withstand the stresses indefinitely.

Berry drew the following conclusions from his tests:

> 1. That the ultimate static strength of a reinforced-concrete beam is not materially affected by 1,000,000 repetitions of high working stresses.
> 2. That the maximum deflection is not affected.
> 3. That hair-line cracks become visible for such loads at intervals of 6 to 8 in., and grow deeper as the number of repetitions is increased; but that for 1,000,000 repetitions no crack extended beyond the neutral axis.
> 4. That the bond between the steel and the concrete is not appreciably affected, as shown by the difficulty with which the steel was removed in breaking up the beams.
> 5. That the position of the neutral axis is not changed by repetitions of load.
> 6. That the greater part of the set in the deformation in the plane of the steel occurs in the first few thousand applications of the load.

7. That the set in the deformation on the compressive side of the beam is also relatively large for the first few thousand repetitions, and that it increases with the stress applied and the number of repetitions.

Compression Tests by Williams.—Williams[1] made some repeated-stress tests on concrete cylinders, using a 1:2:4 mix, and testing at the age of 28 days and 6 weeks. The number of repetitions applied was small (less than 75), and his results indicated that the permanent set increased somewhat with increase of repetitions, and that the modulus of elasticity increased also. The modulus of elasticity increased only slightly, and since the concrete was only 6 weeks old or less, it is probable that the increase in strength with age affected the results. The present authors do not consider this evidence sufficient to controvert the findings of Van Ornum that the modulus of elasticity decreases with increase in number of repetitions. It will be recalled that this decrease to some constant value occurred even when the stress was low enough so that failure by fatigue did not occur.

Beam Tests by Bureau of Standards.—Slater[2] and his associates at the Bureau of Standards carried out some tests which were different from any of those mentioned hitherto, in that they were tests of double-reinforced-concrete beams subjected to reversed stresses. This work was done in order to get information which would be of use in the construction of concrete ships. Five beams 6 by 8 in. in cross-section were tested on a span of 8 ft. One of the beams had an I-shaped cross-section and web reinforcement in addition to the longitudinal rods. Four beams were made from a 1:⅔:1⅓ mix, and the fifth was made from a 1:¾:1½ mix. The strength of the compression cylinders made from the same concrete varied between 4,200 and 6,200 lb. per square inch. The beams were from 2 to 5 months old when the tests were begun, and the rate of application of load was 17 cycles per minute. The load

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[1] *Proc. Amer. Soc. Testing Materials*, vol. 20, Pt. II, p. 235, 1920.

[2] *U. S. Bur. Standards, Tech. Paper* 182, 1920.

was applied and released by lowering and raising weights acting at the ends of levers.

Beam 5*A*1 had a measured maximum stress in the steel which varied between 5,400 compression to 21,600 lb. per square inch tension. The maximum compressive stress in the concrete was 1,565 lb. per square inch. This beam failed after 709,041 cycles of stress by rupture of the steel in fatigue. During the first 1,000 cycles, the width of the tension cracks increased to about 0.02 in., the steel deformation changed slightly, and the beam deflections changed markedly. After the first 1,000 cycles the crack widths, deformations, and deflections remained practically constant for about half the "life" of the beam, and finally the deflection downward and the crack widths increased gradually until failure occurred.

Beam 5*B*1 had a measured maximum stress in the steel which varied between zero and 22,800 lb. per square inch tension, and a maximum compressive stress in the concrete of 1,210 lb. per square inch. The beam failed after 59,377 cycles by rupture of the steel in fatigue. During the first 300 cycles the deflections increased rapidly, and during the first 7,000 cycles the crack widths did the same. After that, the deflections and crack widths remained practically constant until failure was imminent.

Beam 5*C*1 had a measured maximum stress in the steel which varied between zero and 11,000 lb. per square inch tension, and a maximum compressive stress in the concrete of 1,425 lb. per square inch. These stresses represented the working stresses used in concrete ship design. The beam did not fail, and the test was discontinued after 2,008,000 cycles. The downward deflection, downward permanent set, and crack width at the bottom (where the steel stress was a maximum) increased gradually for about 400,000 cycles, and then remained practically constant. When the test was discontinued, the beam showed no indications of approaching failure.

Beam 5*F*1 had the I-shaped section and the web reinforcement, and was designed to give a computed unit stress of 175 lb. per square inch in the web. The measured maximum stress in the steel varied between 7,000 compressive and 11,700 lb. per square inch tensile unit stress, while the computed stress in the concrete was 1,641 lb. per square inch. The beam failed after 544,448 cycles by rupture of the steel in fatigue. During the early stages of the test the deflections increased rapidly with a gradual increase of crack widths. At 3,600 cycles a horizontal crack appeared at the top edge of the web. As the test continued, small diagonal cracks developed downward from the horizontal crack toward the center of the web. From 3,600 cycles until about three fourths of the life of the specimen, deflections and crack widths were practically constant, but unit deformations varied considerably.

Beam 5*L*1 had a measured maximum stress in the steel varying between 4,000 compressive and 18,000 lb. tensile unit stress, and a maximum compressive stress in the concrete of 2,080 lb. per square inch. The beam failed after 451,821 cycles by rupture of the steel in fatigue. It had been designed to develop a bond stress of 161 lb. per square inch. During the first 70,000 cycles there was a gradual increase in deflections, crack widths, and permanent set. The downward permanent set, deflection, and bottom crack widths increased gradually throughout the test. It is interesting to note that up to about 7,000 cycles the slip of the steel bars was less than 0.001 in., that is, less than the amount which tests of bond between steel and concrete have shown to be a criterion of safe condition. Due to repeated stress, however, this slip had been finally increased to 0.06 in., and failure seemed imminent due to this cause, when the steel failed in tension.

The ultimate tensile strength of the steel used in these tests varied between 53,000 and 57,000 lb. per square inch. None of the four beams which failed by fatigue of the steel developed an endurance limit which might have been expected, considering the ultimate static strength and the maximum repeated stress in the steel. Three of the four beams had steel which was high in phosphorus, and two out of the four failed where gage holes had been drilled in the steel. While these factors may have contributed to low fatigue strength, it would seem that another cause had a larger influence. This was the fact that in all four beams large cracks extended entirely cross the section of the beam at the places where the tension failure of the steel occurred. It seems likely that a sharp local bending of the steel may have been induced by the presence of these large cracks. In any case these tests indicate that when steel is subjected to large stresses in reinforced-concrete beams, developing large cracks in the concrete, then the fatigue strength of the steel will not be so great as is normally to be expected.

Beam Tests by Clemmer.[1]—Investigation of concrete-pavement failures in Illinois suggested that the failures were due to the effect of repeated stresses, and since the standard design in Illinois was based on the theory that the corners were the weakest part of the slab, and acted as cantilever beams, it was decided to devise a fatigue apparatus to produce similar conditions. Furthermore, since most of the loads were applied to roadways by means of rubber tires, it was decided to stress the specimens in the same way. This work was carried out by the Illinois Division of Highways.

The apparatus for these tests was arranged so that concrete beams radiated from a central support like the spokes from the hub of a wheel, the beams being rigidly held at the center and being free at the outer end. Concrete blocks were placed between the ends of each pair of beams thus completing a circular track upon which the loading device could travel. The track was made as level as possible to prevent impact.

The loading device was constructed by using the rear wheels and axle of a Ford automobile, the span between the wheels being increased to 7 ft. By means of a vertical housing attached to the differential casing, and a horizontal pulley attached to this vertical housing, the wheels could be driven in a circle on the track which had been prepared. Weight boxes placed on the horizontal housing near the

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[1] *Proc. Amer. Soc. Testing Materials*, vol. 22, Pt. II, p. 408, 1922.

wheels permitted various loads to be applied to the specimens. The rate of application of load was 40 per minute.

The first series of tests consisted of 15 plain concrete beams, 6 in. square and 36 in. long, of a 1:2:3½ mix. Two of the beams were tested under static load to determine their modulus of rupture, and the fatigue tests were then carried out on 7 beams using a stress equal to 50 per cent of the modulus of rupture. After 1,130,976 applications of load no failures had resulted.

The same beams were then subjected to 60 per cent of their modulus of rupture (as determined after the beams had failed). When a beam failed, a new beam was put in its place. Under the stress 7 beams failed under applications ranging from 16,782 to 199,836 in number. The average age of the specimens which failed was 174 days.

Some of the original beams which had been subjected to 1,130,976 cycles at 50 per cent of the modulus of rupture, and to 409,655 cycles at 61 per cent, were now stressed to 70 per cent of the modulus of rupture. Under this stress all the remaining specimens failed.

It was noted that beams which had resisted 1,000,000 cycles of stress, presumably less than the endurance limit, resisted more applications than new specimens which had not been so stressed. Furthermore, tests showed that the moduli of rupture of understressed specimens were somewhat higher than those of beams which had not been subjected to repeated stress. This strengthening effect of understressing is a well-established phenomenon in the fatigue testing of metals, and this evidence of a similar phenomenon in concrete is of interest.

In a second series of tests a 1:3:5 mix was used. When these beams were subjected to 70 per cent of their modulus of rupture, all but one failed at 1,525 cycles or less. This single beam was then broken statically, giving a modulus of rupture of 772 lb. per square inch. The unstressed end of the beam gave a modulus of rupture of 808 lb. per square inch, so that presumably the beam would have failed under more applications.

A third series of beams was made with a 1:3 mortar, these also being subjected to 70 per cent of their modulus of rupture. These beams all failed at 5,280 cycles or less.

A fourth series of tests was made to determine the relation between the increase in strength of concrete with age and the decrease in strength due to fatigue. Three sets of beams were made of 1:2:3½ concrete, each set consisting of 15 beams. In all cases the beams of this series were placed in the machine at the age of 30 days.

The beams of the first set, under a stress of 62 per cent of the modulus of rupture, all failed at 7,000 cycles or less. The beams of the second set, under a stress of 51 per cent of the modulus of rupture, all failed at 33,000 cycles or less.

The beams of the third set were stressed to 48 per cent of the modulus of rupture, and 2,000,000 cycles applied without a single failure. A stress of 50 per cent of the modulus of rupture, *at the attained age of 90 days,* was then applied 512,000 times without a single failure. A stress of 51 per cent of the modulus of rupture *at the then attained age of 120 days* was then applied 441,000 times without a single failure. A stress of 54 per cent of the modulus of rupture was then applied (at the age of 129 days), and all the beams failed after 700,000 cycles or less.

The deflections and the permanent sets of the beams were measured with a strain gage and with an Ames dial. The curves indicated that there was a deviation from the straight-line relation of load and deformation or of load and deflection at about 50 per cent of the modulus of rupture.

Figure 96 shows typical curves of deflections and recovery during the progress of a fatigue test. These curves indicate that as the fatigue test proceeds, the deflections increase and the recovery decreases. The curves are so drawn that the ordinates between the two curves indicate permanent set, and it is apparent that the permanent set increases with the number of applications. It should, however, be pointed out that this was the case for beams which were stressed above the endurance limit, so that they finally failed. It will be recalled that Van Ornum's tests show that when the stresses are below the endurance limit, the permanent set would not continue to increase.

Clemmer drew the following conclusions:

1. Concrete beams will fail under a number of repetitions of loads which produce stress equal to or greater than a certain percentage of that required to cause transverse failure when tested under one application of load.

2. Loads which produce stress less than a certain percentage of the modulus of rupture as determined in the testing machine will not cause failure on repetitions, but rather, as indicated in the first and fourth series, the strength of the specimens would actually be increased by this condition of load if the load is near (but below) this critical percentage.

3. For cycles of stress ranging from zero to a maximum ($r = 0$), the critical percentage or limit of endurance of the concrete specimens was between 51 and 54 per cent of the modulus of rupture as determined from one application of load.

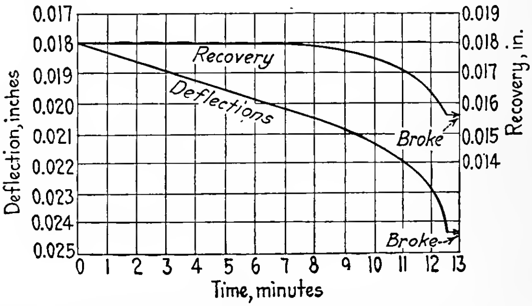

FIG. 96.—Deflection and recovery during test of concrete beam under repeated flexure. (*Based on Clemmer in Proc. Am. Soc. Test. Materials.*)

4. For the same percentage of ultimate strength, a considerably less number of applications of load is required to cause failure in the 1:3:5 mix specimens than in the 1:2:3½ mix specimens.

5. Stresses below the limit of endurance do not cause permanent deformation in the specimen.

6. For stresses beyond the limit of endurance, the number of repetitions of load required to produce failure decreases with increase of percentage of stress.

In connection with this same research of the Illinois Division of Highways, Older[1] reported that test beams subjected to a unit stress of about 50 per cent of the modu-

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[1] *Trans. Amer. Soc. Civil Eng.*, p. 1180, 1924.

lus of rupture had withstood 5,000,000 repetitions of stress without failure.

Concrete slabs under actual traffic conditions were studied. A corner break on a slab 4 in. thick, under a 3,500-lb. wheel load, indicated that at the corners the slab was subjected to stresses equal to or exceeding the critical value of 50 per cent of the modulus of rupture. An increase of wheel load of 1,000 lb. would therefore increase the stresses to about 78 per cent of the modulus of rupture, under which conditions rapid failure might be expected to follow. This actually proved to be the case, since many corners were broken and the progressive destruction was quite rapid. This same phenomenon had been noted on Illinois highways in service, which had withstood normal traffic for a number of years, and then began to give way under increased highway loadings.

It may be of interest to note here that the Illinois highway tests have led to the following formula for computing the depth of the slab:

$$d = \sqrt{\frac{3W}{S}},$$

in which W = the maximum wheel load, in pounds,
S = modulus of rupture of the concrete, in pounds per square inch,
d = depth of the concrete slab, in inches.

When S is the modulus of rupture, W would represent the breaking load. It is recommended that for design purposes S be taken equal to 50 per cent or less of the modulus of rupture of the concrete. The various other recommendations on slab design may be found in the original paper.

Compression Tests by Probst.—Some interesting experiments by Probst[1] were made on concrete compression specimens 7 by 7 cm. in cross-section and 28 cm. long. Two specimens were stressed with a maximum stress of 1,848 lb. per square inch, but one had a minimum stress of

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[1] *Festshrift zur Hundertjahr feier Tech. Hochs., Karlsruhe,* 1925.

114 and the other 1,422 lb. per square inch. The maximum stress was about 70 per cent of the static ultimate compressive strength. The first specimen withstood 341,000 cycles before failure, and the second 1,500,000 cycles without failure.

These specimens were also measured for deformation, a Martens mirror apparatus being used on a gage length of 20 cm. Unit deformations could be read directly to 0.00001 and by estimation to 0.000001 cm. per centimeter.

In one case a specimen had a static ultimate strength of 2,104 lb. per square inch, and was stressed with a minimum stress of 114 and a maximum stress of 789 lb. per square inch, the maximum stress being, therefore, about 38 per cent of the ultimate static strength. The loadings were repeated at the rate of 60 per minute. With increase in number of cycles the elastic deformations increased at first faster than the permanent sets, but later this was reversed. The elastic deformations finally reached a constant value and somewhat later the permanent sets also. (It will be understood that the elastic deformation plus the permanent set equals the total deformation at any stress.) After 453,000 cycles the test was discontinued because both the elastic and permanent deformations had reached a stable condition. The modulus of elasticity had decreased during the progress of the test, from an initial value of 3,590,000 lb. per square inch to a value of 2,810,000 lb. per square inch at the end of 453,000 cycles.

After this test the specimen was subjected to increasing stresses, beginning at 341 lb. per square inch, and the same stress was repeated to determine whether stable conditions of deformation had been reached. Up to a stress 1,000 lb. per square inch, or 50 per cent of the ultimate static strength determined after failure, there was no permanent deformation, and the modulus of elasticity had become constant. At a stress of 1,068 lb. per square inch a unit permanent set of 0.000005 was obtained, the elastic deformations were stable after seven applications, but the permanent sets were not. At a stress of 1,166 lb. per square inch and 10 repetitions, neither the elastic nor the permanent deformations were stable, and the permanent deformation had reached a unit value of 0.000014. This condition existed also at 1,220 lb. per square inch with a unit permanent deformation of 0.000022, and failure took place at 2,030 lb. per square inch. The increase of permanent deformation had been accompanied by decrease of modulus of elasticity. The specimen had been made elastic by repeated stresses, even beyond the original maximum value of 789 lb. per square inch. This increase of elasticity by means of repeated stresses is similar to that found by Bauschinger for metals.

Virgin specimens were then tested for elastic behavior, and the average of three results is given in Table 24.

TABLE 24.—TESTS OF CONCRETE COMPRESSION SPECIMENS FOR ELASTIC PROPERTIES

Stress, pounds per square inch	Elastic Unit deformations, inch per inch	Permanent unit deformation, inch per inch	Modulus of elasticity, pounds per square inch	Number of cycles required for attainment of stable conditions	Remarks
341	0.000081	0.000003	4,220,000	6	
489	0.000120	0.000006	4,070,000	10	
628	0.000158	0.00011	3,980,000	10	Stable for elastic but not for permanent deformation
789	0.000222	0.000020	3,570,000	10	Not stable
940	0.000295	0.000031	3,180,000	10	Not stable
1,010	0.000332	0.000039	3,040,000	10	Not stable
1,068	0.000365	0.000047	2,920,000	10	Not stable
1,166	0.000428	0.000057	2,720,000	10	Not stable
2,104	Broke				

[1] Specimens not stressed previous to tests. Each value is the average of three test results. Results obtained by Probst at the Karlsruhe Technical High School.

These results show that at a unit stress of 789 lb. per square inch the sum of the elastic and permanent deformations is greater for the specimen subjected to repeated stress than for the virgin specimens, the former being 0.000339 and the latter 0.000242. With increase of stress the deformations of the virgin specimens surpassed the deformations of the fatigue specimen, and that more rapidly for the elastic than for the permanent deformations. By permanent deformations for the fatigued specimen is meant the permanent deformation which occurred *after* stable conditions had been established by 453,000 cycles of stress at 789 lb. per square inch.

When the logarithm of unit stress was plotted against the logarithm of unit deformation, for elastic deformations, straight lines resulted both for the fatigued and for the virgin specimens. When the logarithm of unit deformation was plotted against the logarithm of cycles, the curve showed a sharp break and became parallel to the cycles axis, in a manner similar to the break in the *S-N* curve for ferrous metals.

These results reinforce the conclusions arrived at by Van Ornum. At a unit stress of 789 lb. per square inch, for instance, the fatigued specimen reached a constant value of modulus of elasticity which was about 79 per cent of the value at the same stress for the virgin specimens. This ratio undoubtedly would have been nearer the two-thirds value found by Van Ornum if the maximum stress in the fatigue test had been 50 per cent of the static ultimate instead of only 38 per cent. These results indicate, as did Van Ornum's, that concrete can adjust itself to a condition of repeated stress, and presumably can withstand the stress indefinitely if it does not exceed a certain maximum value.

Beam Tests at Purdue University.—Hatt[1] has reported tests made at Purdue University on beams 4 by 4 in. in cross-section, 30 in. long, and fabricated with a 1:2 mortar. In order to minimize the effect of increase in strength due to age, the tests were made on specimens which were over 6 months old. The specimens were subjected to bending stresses, and Berry strain gages on each side of the specimens measured the tensile and the compressive deformations over a gage length of 10 in. The fatigue testing machine applied the load in such a way as to produce reversals of stress on the two sides of the beam, which

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[1] *Proc. Highway Research Board, Nat. Research Council*, p. 47, 1925; also *Purdue Univ. Bull.* 24, p. 46, 1925.

was placed in a vertical position. The tests to determine the static properties were made in the same machine in which the fatigue tests were carried out. In the fatigue tests the rate of application of load was 10 per minute.

It was found that no definite endurance limit could be determined for mortar at early ages, because the increase in strength due to age might produce a greater effect than the fatigue action. For beams only 28 days old the endurance limit under cycles of reversed stress was found to be as low as 40 per cent of the static breaking load.

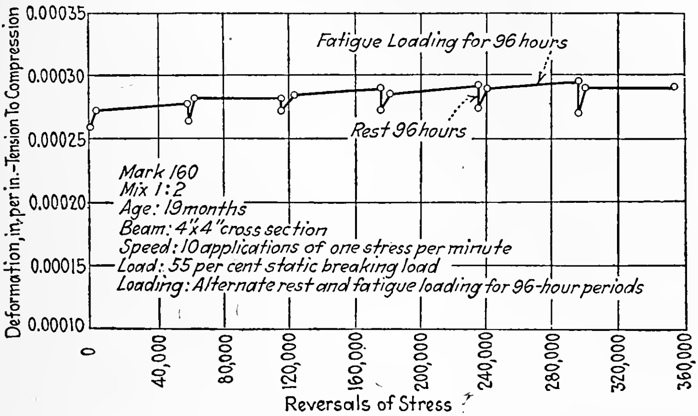

FIG. 97.—Progressive deformation for concrete beam under reversed flexure. (*Based on Hatt in Bull. 24, Purdue Univ.*)

Figure 97 indicates the action of a beam in fatigue in those cases in which final failure occurred. The deformation increased with increase of reversals of stress until final failure occurred. Rupture occurred first on the outer fibers where the deformation was a maximum, after which failure was progressive toward the center of the beam, sometimes requiring a considerable additional number of reversals to make failure complete.

Figure 97 also indicates the recovery that occurred during periods of rest, in this case periods of 96 hr. This decrease of unit deformation after a period of rest was also observed after 16 and 42 hr. It will be noted that this stiffening effect due to rest is only temporary, and that the value of deformation, existing before rest, is again attained after a certain number of stress reversals. This stiffening phenomenon was noted at stresses above and also below the endurance limit of the material.

In one case it became necessary to stop the machine for 5 weeks, and the specimens all showed a recovery in deformation. One specimen which had been stressed to 55 per cent of the breaking strength, and which was believed to have been on the point of failure, showed the greatest recovery. Under further stressing it gave a curve of slightly increasing and then decreasing deformation, indi-

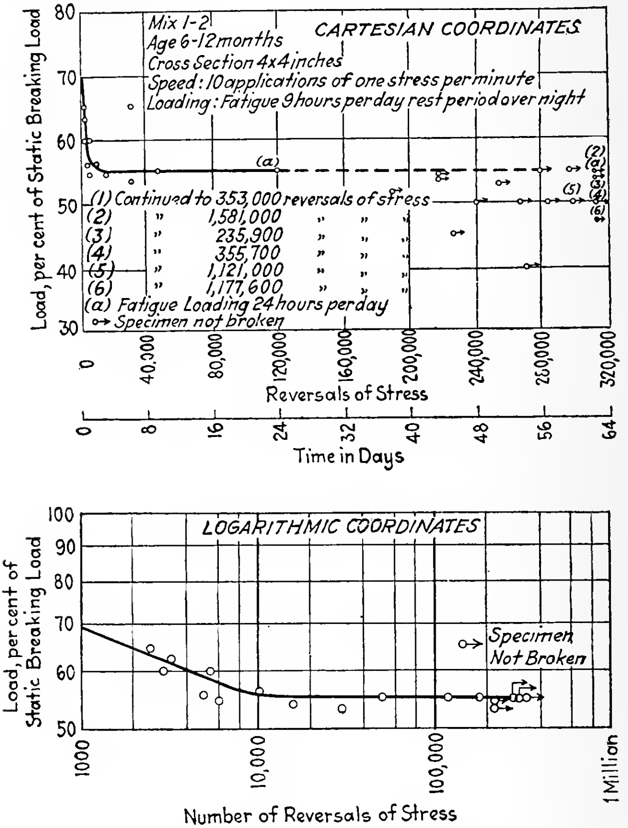

FIG. 98.—*S-N* diagram for concrete beam under reversed flexure. (*Based on Hatt in Bull.* 24, *Purdue Univ.*)

cating apparently complete recovery from the previous overstressing. Since these beams were 5 months old, it was thought that the increase of strength due to increase of age was of minor importance.

Figure 98 shows a typical *S-N* diagram. Figure 98(*a*) is plotted to Cartesian coordinates, and Fig. 98(*b*) to logarithmic coordinates. It should be noted that if the stress is below the endurance limit, the specimen will not fail, but if it is stressed only slightly above the endurance limit, a comparatively few reversals of stress will cause failure. It seems probable, from these tests and various others which have been described in this chapter, that the endurance limit of mortar and concrete can be determined at a much smaller number of cycles than is necessary in the case of metals.

It will be recalled that the tests of Van Ornum on compression specimens and those of Clemmer on beams were carried out between a minimum stress near zero and a maximum stress of the same sign. They reported some endurance limits lying between 50 and 55 per cent of the static breaking strength. The Purdue tests employed completely reversed stresses from tension to compression, and the reported endurance limit lies between 50 and 55 per cent of the static breaking strength. While the materials experimented on are not strictly comparable, yet the indication is that for these materials the range of stress for completely reversed stress may be approximately twice as great as for the case when the range lies between zero and an upper limit.

The Purdue results showed that when a beam was being subjected to a stress which would ultimately cause failure, then the plastic set at zero load might be fairly large. For stresses which could be endured a long time without failure, the plastic set was small and, after a period of rest, was practically zero. The magnitude of this plastic set seemed to be dependent upon the age of the mortar, being inversely proportional to the age.

Another phenomenon which was observed was the strengthening effect of repeated stresses which were less than the endurance limit. Understressing strengthened the beams so that a later stress greater than the original endurance limit could be withstood without failure. This effect, therefore, seems to be a characteristic one for all materials which have been subjected to fatigue stresses.

Hatt states the following conclusions drawn from the Purdue tests:

1. (*a*) For 28-day tests: No definite endurance limit between 40 and 60 per cent of that static load required to break the beam under a single application can be assigned to mortar of this age.

(*b*) For 4-month test: The load at the endurance limit is approximately 50 to 55 per cent of the static breaking load.

(*c*) For tests over 6 months: The load at the endurance limit is 54 to 55 per cent of the static breaking load.

2. The endurance limit does not seem to differ materially for beams under continuous fatigue loading from that for beams under fatigue loading with short rest periods.

3. The number of reversals of stress necessary to cause failure decrease in a proportion to the respective increase of the percentage of static load above the endurance limit.

4. Stresses above the endurance limit cause continual progressive deformation.

5. Stresses below the endurance limit may cause progressive deformation for short periods with a tendency to become constant or to decrease with continued loading.

6. The endurance limit may be raised by repeatedly stressing below 55 per cent of the static breaking load.

7. The amount of recovery in deformation seems to depend somewhat upon the length of rest period.

8. Plastic set in fatigue is more pronounced in mortar of early age. A sufficient rest period may reduce the plastic set to zero.

Tests by Mehmel.—The investigation of Probst (see p. 269) was supplemented by important investigations by Mehmel.[1] He used compression specimens 7 by 7 cm. in cross-section and 28 cm. long subjected to stresses from a lower limit near zero to an upper limit. The concrete mix was a 1:6 gravel mix, and the water-cement ratio was 0.63.

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[1] *Mitt. Inst. Beton Eisenbeton an der Tech. Hochs., Karlsruhe*, 1926.

Careful measurements of deformation were made on a 20-cm. gage length with Martens' mirror apparatus. The repeated-stress testing was done at a rate of 60 cycles per minute, and the average age of the specimens was about 1 year.

Tests Which Did Not Cause Failure.—A certain specimen was loaded in compression with a minimum stress of 114 and a maximum of 704 lb. per square inch, the upper limit being 29.5 per cent of the ultimate static strength determined from similar specimens. At various intervals, readings of deformation were taken so that stress-deformation and deformation-cycles graphs could be drawn.

The original stress-deformation curve was convex upward, and even after 10 cycles of stress the deformation had been increased. The deformation-cycles graph with increase of cycles became an inclined straight line, and the increase of deformation was then so slow that many cycles of stress were necessary to make the increase apparent. This was true for the elastic as well as the permanent deformations. The elastic deformation, corresponding to the maximum stress, was 0.000158 at 10 cycles, increased to 0.000178 at 150,000 cycles, and then remained constant up to 610,000 cycles. The permanent set was 0.00001 at 10 cycles, increased to 0.000086 at 400,000 cycles, and then remained constant up to 610,000 cycles. The elastic deformation had therefore increased 12.6 per cent, and the permanent set 760 per cent. The ratio of permanent set to elastic deformation was 0.0633 at 10 cycles, and 0.477 at 400,000 cycles.

The repeated loadings were discontinued after about 610,000 cycles, and the specimen was subjected to a static test. The stable condition which had been reached by the deformations was indicated by the value of modulus of elasticity which was constant not only up to the maximum repeated stress of 704 lb. per square inch, but even up to 803 lb. per square inch. At 917 lb. per square inch a permanent set was developed, and at the same time the modulus of elasticity decreased. Above the value of 704 lb. per square inch the stress-deformation curve was steeper than for similar specimens which had not been subjected to repeated stress. The specimen had evidently been strengthened by the repeated stressing even for stresses higher than that used in the fatigue test. Continuing the static test to destruction, it was found that the ultimate strength was practically the same as for specimens which had not been subjected to repeated stresses.

Two other specimens were subjected to repeated compression with the same minimum stress of 114 lb. per square inch as before, but with maximum stresses of 790 and 1,138 lb. per square inch, respectively; which stresses were 37.5 and 47 per cent, respectively, of the ultimate static strength.

For the specimen stressed to an upper limit of 790 lb. per square inch, stable conditions were reached at 169,000 cycles for the elastic deformations, and at 260,000 cycles for the permanent deformations. The ratio of permanent to elastic deformation changed from 0.0128 to 0.208 in going from 10 to 260,000 cycles, and the elastic deformation increased about 20 per cent.

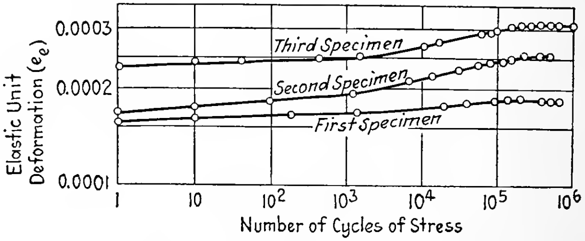

FIG. 99.—Progressive elastic deformation of concrete specimen under repeated compression. (*Mehmel, at Karlsruhe.*)

For the specimen stressed to an upper limit of 1,138 lb. per square inch, stable conditions were reached at 200,000 cycles for the elastic deformations, and at 400,000 cycles for the permanent deformations. The ratio of permanent to elastic deformation changed from 0.10 to 0.667 in going from 10 to 200,000 cycles, and the elastic deformation increased 24 per cent.

Figure 99 shows the logarithmic plot for these three specimens, using unit elastic deformation as ordinates and number of cycles as abscissae. Figure 100 shows both the elastic deformations and the permanent deformations plotted against number of cycles, using Cartesian coordinates. In both curves it is evident that when stable conditions are reached, the deformation line approaches a horizontal line as asymptote.

For the three specimens above mentioned, which were stressed to 29.5, 37.5, and 47 per cent, respectively, of the static ultimate strength, the elastic deformation increased during the period of repeated stressing 13, 20, and 24 per cent, respectively, starting with a base of 10 cycles.

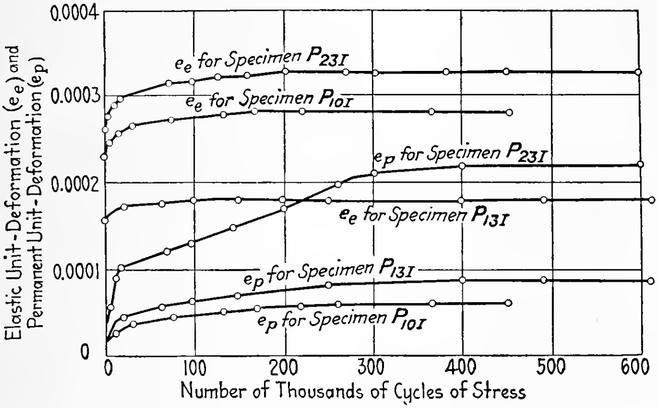

FIG. 100.—Progressive elastic and permanent deformation of a concrete specimen under repeated compression. (*Mehmel.*)

A study of the relation between unit stress and unit deformation under repeated stressing, showed that the modulus of elasticity is not merely a function of unit stress, but is dependent on the method of applying the stress, the number of times the stress is applied, the previous history of stressing, and other factors. When a specimen has been repeatedly stressed many times, it is possible for it to become elastic within the limits of stress to which it has been subjected, and within these limits Hooke's law is valid. This condition may be reached for all stresses from zero up to the endurance limit.

In this connection the authors of this book wish to point out the decrease in modulus of elasticity which the repeated stressing produced. Two specimens which were stressed in a static test to 29.5 per cent of the ultimate strength, had a secant modulus of elasticity based on elastic deformation, which varied from about 4,840,000 to 4,420,000 lb. per square inch, showing that the stress-deformation curve was not a straight line. A similar specimen after many cycles of repeated stress had a modulus of 3,960,000 lb. per square inch, a value about 85 per cent of the above.

Similarly, two specimens subjected to 47 per cent of the ultimate in a static test showed a modulus of elasticity varying from 4,730,000 to 4,280,000 lb. per square inch. A similar specimen after many cycles of repeated stress had a modulus of elasticity of 3,480,000 lb. per square inch, a value of 77 per cent of the above.

This last specimen reached a stable condition for deformations after 400,000 cycles. After 450,000 cycles had been applied, the specimen rested for 36 hours, and during this time the permanent set diminished from 0.000022 to 0.0000179; but the elastic deformation was not influenced. At 600,000 cycles the previous stable condition had again almost been reached. Evidently, therefore, the change in permanent deformation was temporary; but it seems questionable whether there is ever established a condition of stability in the strict sense of the word.

Tests Which Caused Failure.—In order to study the behavior of concrete under repeated stress which would finally cause failure, a specimen in Mehmel's tests was stressed from 114 to 1,990 lb. per square inch, or about 80 per cent of the static ultimate strength. The first period of stressing consisted of 1,470 cycles. The first few loadings gave a stress-deformation curve convex upward, but even after 20 cycles this had become concave upward; and as the number of cycles increased, this concavity upward increased, the curve being steeper at the higher values of stress. This effect is shown in Fig. 101 in which the stress-deformation curves are drawn after various numbers of cycles of stress had been applied. In other words, for the first loading the increment of deformation for a small increment of stress was greater at the higher stresses than at the lower, but after repeated loading this condition was reversed and the increment of deformation was greater for the lower stresses than for the higher. During this period of stressing the elastic deformation increased about 31 per cent, and the permanent deformation increased about 244 per cent.

During the second period of stressing of this specimen the maximum stress was decreased to 1,138 lb. per square inch, a reduction of 43 per cent. The shape of the stress-deformation curve remained the same as in the first period of stressing, and after about 800,000 cycles a condition of stability was reached by the elastic deformation but not by the permanent deformation. A previous specimen had been stressed to the same maximum stress as in this case, and it seemed that the stress was below the endurance limit; but in the present case the first period of stressing to 80 per cent of the ultimate static strength had evidently so weakened the specimen that even 1,500,000 cycles of lower stress were not sufficient to produce stable conditions in both the permanent and the elastic deformations.

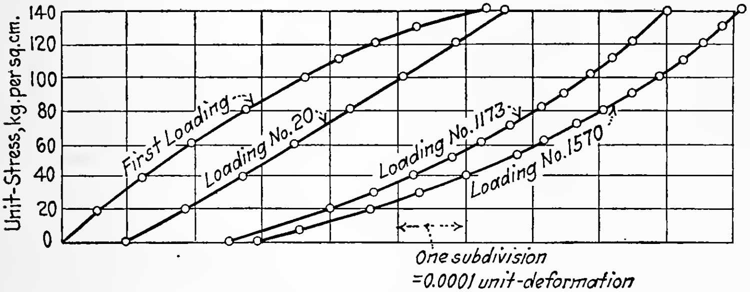

Fig. 101.—Stress-strain diagrams for concrete specimen after cycles of compression. (*Mehmel.*)

After the second period of stressing of about 1,490,000 cycles, the maximum stress was again increased to 1,990 lb. per square inch. The character of the stress-deformation curve remained the same, but both the elastic and permanent deformations increased. The third period of stressing consisted of 1,460 cycles.

In the fourth period of stressing, the maximum stress was again decreased to 1,138 lb. per square inch. After 86,000 cycles, occurred a period of rest of 12 days, during which time the total permanent deformation decreased about 14 per cent. The specimen was so near failure, that this rest period did not affect results to any extent.

During the fifth period of stressing the maximum unit stress was again increased to 1,990 lb. per square inch. After only a few loadings the deformations increased rapidly and in 588 cycles after the rest period the specimen failed. Failure was preceded by a soft crackling noise which gave sufficient warning so that the mirror apparatus could be removed before complete destruction occurred.

A specimen was next stressed to a maximum of 1,848 lb. per square inch, or about 70 per cent of the ultimate static strength. The action in this case was much like that in the previous specimen, similar stress-deformation curves being determined which were concave upward. The test showed very clearly that for an increment of stress at the lower limit of the stress cycle the elastic deformation increased from the beginning to the end of the test. For the same increment of stress at the upper limit of the stress cycle the elastic deformation decreased slightly at first with increase of cycles, and then increased very slightly during the test, and slightly more just before failure. For the purposes of this computation Mehmel employed a range of stress from zero to 426 and from 1,422 to 1,848 lb. per square inch. The specimen finally failed after about 341,000 cycles.

The character of the fracture of concrete failing under repeated stress was found to be similar to that observed in static tests.

The next test was one in which the maxium and minimum stresses were high but in which the range of stress was low. The maximum stress was 1,848 and the minimum 1,422 lb. per square inch, thus stressing the specimen from 54 to 71 per cent of the ultimate static strength. The deformation measurements soon showed that failure need not be feared, and, indeed, the specimen withstood 1,500,000 cycles without failure. The specimen was then tested to failure under a static load, and it was found that the repeated stresses had reduced the static ultimate strength about 18 per cent. The curves showed that the permanent deformation had not reached a stable condition at the end of the fatigue test, and it seems probable (to the writers of this book) that failure would have occurred under continued cycles.

In order to determine the ratio of endurance limit to static ultimate strength, various other tests were carried out beside those already mentioned. Two specimens were tested with a maximum stress of about 60 per cent of the static ultimate. After withstanding 1,500,000 cycles without failure, these specimens were broken under static load, and it was found that their ultimate static strength had been reduced by 8.3 and 10.3 per cent, respectively.

Mehmel concluded that as long as the maximum unit stress remained within a certain limit, it was possible for the deformations to attain a condition of stability. When stability was attained and when, furthermore, the ultimate static strength had not been decreased by the repeated stresses, it was concluded that the maximum stress lay below the endurance limit. Under these conditions the specimen subjected to repeated stresses showed a linear relation between unit stress and unit deformation up to the applied maximum stress and even higher.

When the maximum unit stress was increased, the stress-deformation curve which at first was convex upward, became a straight line, and next became concave upward. Using a constant increment of stress of about 300 or 400 lb. per square inch, it was found that at the maximum stress the total elastic deformation corresponding to this stress decreased with increased cycles, while for the same increment of stress at the minimum stress of the stress cycle the elastic deformation increased, so that the ratio $e_{\max}/e_{\min}$ became less than unity. Up to rupture this ratio became smaller for increased number of cycles. Figure 102 shows the variation in this ratio for a certain specimen which failed under fatigue.[1]

Bauschinger and Bach had previously found in tests of various natural rocks that the stress-deformation curve was concave upward, either from the origin or after a certain unit stress was reached. Bach subjected specimens of marble to a small number of repetitions of loading, and found that after the fifth loading the deformations for a constant increment of stress were larger for the lower stresses and smaller for the upper stresses than they had been for the first loading.

Since the aggregate for concrete has the same elastic properties that natural rocks have, and since neat cement is known to give a stress-deformation curve which is convex upward, Mehmel concluded that concrete gets its elastic character from the cement content. Under repeated stresses, however, when these are sufficiently high, the elastic properties of the aggregate come into play.

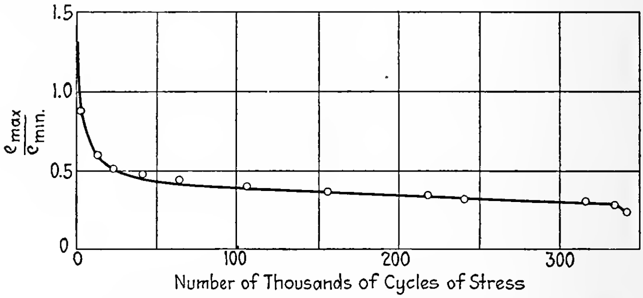

Fig. 102.—Progressive variation of the ratio e_{max}/e_{min}. (*Mehmel.*)

The cement, therefore, is the first of the component ingredients of concrete which becomes fatigued; it takes on less and less of the elastic stress energy, and becomes plastic and acts merely as a binder, so that the curve of elastic deformations is determined mostly or entirely by the aggregate.

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[1] In this figure the e_{max} refers to the elastic deformation corresponding to the increment of stress from 1,422 to 1,848 lb. per square inch, and the e_{min} refers to the elastic deformation corresponding to the same increment of stress from 0 to 426 lb. per square inch.

Mehmel concludes, therefore, that the shape of the deformation diagram is a criterion of the fatigue of concrete. A weakening of the material shows itself by the reversal of the stress-deformation curve from convex upward to concave upward, and the weakening is greater according to the amount of this reversal. This may be expressed by saying that a weakening of the concrete has occurred when the ratio of $e_{\max}/e_{\min}$ (meaning the same as on p. 284) becomes smaller than unity, and is greater according as this ratio becomes smaller. The ratio $e_{\max}/e_{\min} = 1$ may be looked upon as the critical value of the effect of repeated stresses on concrete. Concrete will fail under repeated stresses when the cement is so affected that it loses its ability to store up stress energy. The tests show that concrete can probably withstand a certain small number of repeated stresses above the endurance limit without becoming fatigued, provided that the ratio of $e_{\max}/e_{\min}$ remains equal to unity.

Mehmel concluded from his tests that the determination of the fatigue of concrete is measured much better by the progress of the curve of elastic deformations than by the curve of permanent deformations. He concluded also that the endurance limit of concrete subjected to stresses from zero to a maximum value lay between 47 and 60 per cent of the ultimate static compressive strength.

Some measurements were made of the increase of temperature of concrete specimens subjected to repeated stresses. These measurements were made with thermocouples, and the test showed that the temperature rise with increasing cycles of stress gave a curve similar to the curve of permanent deformation as determined with the mirror apparatus. Mehmel concludes that the thermal measurements may prove useful for the determination of deformation changes shortly before the failure of a specimen, when the mirror apparatus would have to be removed.

Summary of Conclusions.—The following is a summary of the conclusions which Mehmel arrived at from the results of his investigations:

1. The actions in a stress-deformation diagram of concrete, which at first are not reversible, may become reversible through repeated loadings, but only so long as a certain critical stress is not exceeded.

2. After repeated stressing the total deformation, the elastic deformation, and the permanent deformation all increase. The increase in the permanent set is considerably greater than the increase in elastic deformation, so that the ratio e_p/e_e (in which e_p = permanent and e_e = elastic deformation) increases.

3. A stable condition is reached first for the elastic and considerably later for the permanent deformation.

4. The attainment of stability is earlier or later depending on the absolute and relative increase in the permanent and elastic deformation, and the increase in the quotient e_p/e_e is greater the higher the upper limit of stress.

5. The conception of stability is limited in the case of concrete, a completely stable condition being apparently not attainable. Elastic after-working occurred even after 100,000 cycles had not changed the deformations. The stress-deformation curve was subject to change within a given strip covering the stress-deformation line, but this change affected only the permanent deformation.

6. A comparatively small number of cycles have the effect of changing the original curved stress-deformation curve to a straight line. The further cycles necesssary to produce a stable condition do not influence the form but only the position of the stress-elastic-deformation curve. This curve, with increase of cycles, undergoes a turning about the origin of coordinates.

7. The repeated cycles (within the critical stress) produce a strengthening effect on the material above the upper limit of the applied stress. The effect is only local, for the ultimate static strength is not changed thereby. The elastic properties of the concrete are changed, in a manner analogous to steel under repeated stresses or under cold working, so that the material is more favorably situated to withstand loads "gradually" applied. In relation to impact loading

two opposing influences may be noted: a decrease in the capacity for plastic deformation, and a decrease in the modulus of elasticity.

8. When the applied stress exceeds a certain critical value, the specimen becomes fatigued sooner or later. The cement is the first ingredient of the concrete to become fatigued. The stress-elastic-deformation curve becomes concave upward, so that the ratio $e_{\text{max}}/e_{\text{min}}$ (meaning the same as on p. 284) becomes less than unity. A stress-deformation curve to which this criterion applies is called the fatigue curve (see Fig. 101). The degree to which concrete has been fatigued is measured by the smallness of the ratio $e_{\text{max}}/e_{\text{min}}$. A condition of stability is not reached, and the deformations increase up to the time of failure. The increase in the elastic deformations is such that the ratio $e_{\text{max}}/e_{\text{min}}$ decreases and the weakening of the material proceeds in the lower stress increments.

9. The character of the curve between stress and elastic deformation is the criterion of fatigue, and not the permanent deformation.

10. All typical results of repeated stressing occur at a slower rate according to the intensity of the unit stress. The intensity is dependent on the upper and lower limits of stress and on the rapidity of application of cycles, in such manner that the intensity increases with an increase in the upper limit of stress and with an increase of the time interval per cycle. (The latter may not be true for extremely rapid applications of stress.)

11. The endurance limit for stresses with the lower limit zero, determined by this investigation, is identical with the critical stress mentioned above. It lies between 47 and 60 per cent of the ultimate static compressive strength. The factor of safety under repeated stresses is therefore between one-half and three-fifths of the factor of safety for steady loads.

12. It is possible to measure the thermal effects which result from stress-deformation phenomena in concrete.

This method seems to be adapted to the study of the behavior of the specimen shortly before failure.

Summary on Fatigue of Concrete.—The results on the fatigue of concrete which have been reviewed in this chapter indicate that concrete under repeated stresses is very similar in its behavior to metals under the same conditions of stress.

While some of the conclusions which follow must be looked upon as tentative until verified by further experiments, yet the indications are such that the following summary of the important phenomena may be regarded as fairly well established:

1. Concrete will fail under repeated loads at unit stresses which are much less than the ultimate static strength.

2. When the unit stress to which concrete is subjected in fatigue is decreased, the number of cycles for rupture is increased.

3. For concrete, as for metals, as the maximum limiting stress in a cycle is increased, the minimum stress must be increased algebraically if failure is not to occur. No formulas for the effect of range of stress have been developed for concrete.

4. While tests of many millions of cycles of stress have not been carried out on concrete, yet the indications are that its endurance limit for cycles of stress ranging from zero to a maximum ($r = 0$), is about 50 to 55 per cent of the static ultimate strength, both for compression cylinders and for beams.

5. Even when the cycle of stress (from zero to a maximum) is less than the endurance limit, a permanent set occurs during the first few loadings. If, however, the cycle of stress is such that the permanent set reaches and maintains a constant value, then the indications are that failure will not occur.

6. For stresses below the endurance limit concrete seems to be able to adjust itself to the imposed cycles of stress, the stress-deformation curve becomes a straight line, and the stress can be withstood indefinitely.

7. In the above process of adjustment to a cycle of stress the modulus of elasticity also reaches and maintains a constant value.

8. Stresses above the endurance limit cause progressive deformation and final failure.

9. Periods of rest seem to have only a temporary effect on the recovery from deformation, and do not seem to change the endurance limit.

10. Stressing concrete below the endurance limit increases its strength, just as is the case with metals.

11. In order that the effect of increase of strength with age shall not seriously affect the factors being investigated in fatigue tests of concrete, it is necessary that tests be made on concrete which has an age of 6 months or greater.

APPENDIX A

BIBLIOGRAPHY

In the preparation of this bibliography the authors acknowledge their indebtedness to the bibliography reported in 1913 by Mason in the *Reports of the British Association for the Advancement of Science,* and to that by Mailänder reported in *Stahl und Eisen* of May 22, 1924.

This bibliography has been made inclusive rather than selective, although the authors realize that, in all probability, they have overlooked many important contributions. The reader is advised to pay especial attention to the dates of the articles enumerated, if he wishes to look up the results of the more modern investigations.

This bibliography is divided into three sections: Metals, Wood, and Concrete.

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