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THE FLOW OF LIQUIDS IN PIPES

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THE FLOW OF LIQUIDS IN PIPES

BY

NORMAN SWINDIN

WITH NINETEEN ILLUSTRATIONS AND DIAGRAMS AND NINE TABLES

LONDON: BENN BROTHERS, LIMITED
8 BOUVERIE STREET, E.C.4
1922

CONTENTS

		PAGE
List of Symbols Adopted		6
CHAP.		
I	Introduction	7
II	Fundamental Researches on Viscous Flow	13
III	Kinematic and Absolute Viscosity	24
IV	Relation between Mean Velocity and Velocity at Axis of Pipes	30
V	The $\frac{mig}{v^2} - \frac{vd}{\nu}$ Diagram	33
VI	The Practical Application of the Kinematic Viscosity Equation	37
VII	Flow of Liquids in Channels	42
VIII	Other Pipe Line Losses	45
Summary		54
Appendix		58

SYMBOLS

η = co-efficient of absolute viscosity.

ρ = density gms. per cc. or lbs. per cu. ft. $\frac{\text{slugs}}{\text{ft}^3}$

r = radius of tube in cms. or feet.

h = head cms. or feet.

v = average velocity of flow in cc.'s or feet per second.

l = length of tube or pipe in cc.'s or feet.

g = acceleration of gravity 981 cm. or 32·2 ft. per sec.

ν = co-efficient of kinematic velocity.

m = hydraulic mean depth area/perimeter.

i = hydraulic gradient $\frac{h}{l}$.

d = diameter of tube or pipe = $2r$ cms. or feet.

c = constant in Chezy's formula $v = c\sqrt{mi}$.

R = resistance per unit of surface in absolute units.

n = co-efficient or roughness.

f = co-efficient in D'Arcy's formula, $h = f\,\frac{l}{d}\,\frac{v^2}{2g}$.

a = constant.

b = do.

P = $(1 + {\cdot}0336\text{T} + {\cdot}000221\text{T}^2)^{-1}$.

p = perimeter of a pipe or channel.

T = temperature °C.

THE FLOW OF LIQUIDS IN PIPES

I

INTRODUCTION

BEFORE any system of pumping or moving liquids can be designed and installed the magnitude of the resistances to flow in the conduits must first be accurately estimated. The chemical engineer here has a much more difficult task than his more fortunate brother the hydraulic engineer. The former deals with a much wider range of fluids and these under all manner of physical conditions, the latter confines himself to the flow of water and this only at ordinary temperatures. Thus the chemical engineer requires to know the general case of the law of resistance to flow of liquids, the hydraulic engineer considers only the particular case of water at ordinary temperatures. Moreover the chemical engineer by reason of the corrosive nature of many of the substances which he has to move is prevented from using mechanical pumping devices and is compelled to adopt such devices as the air lift, the design of which depends entirely on the accurate estimation of frictional resistances. It may be accepted as true that the tardy recognition of the merits of the air or gas lift is due, more than anything else, to the want of accurate methods of estimating and formulating the factors controlling resistances to liquid flow. The tendency of modern chemical engineering to develop continuous processes instead of batch or intermittent processes necessitates the use of careful and continuous methods of measurement of liquids and gases employed in the processes. Hence the laws of viscous flow must be accordingly determined.

This work is an attempt to provide the chemical engineer with a sound working scientific formula for the flow of liquids, of whatever character, under all possible conditions. This formula breaks new ground in that it departs entirely from the usual hydraulic formula, as it is based upon the researches on the two modes of motion of Osborne Reynolds, first published in 1883. Reynolds discovered the criterion of flow as a function of velocity, temperature and the quantity called kinematic viscosity. Unfortunately, however, there has not till recently been sufficient experimental data published to enable the ordinary engineer to use this criterion to obtain the correct value of a constant c in Chezy's formula:

$$v = c\sqrt{mi}.$$

Stanton working at the National Physical

Laboratory has carried out a complete series of experiments on the flow of air and the flow of gas through tubes of various sizes, and plotting his results according to a method suggested by Lord Rayleigh he has given us the complete relation between the forces necessary to overcome resistance to flow and the criterion $\frac{vd}{\nu}$. It may be recorded that the enormous literature which exists on hydraulic flow is solely concerned with purely surface friction of water in large conduits and rivers with here and there attempts to correct for viscosity in a purely empirical way.

The internal factors of resistance to flow have during the last fifty years been very carefully studied in connexion with the development of the American oil fields, and extraordinary means have been taken to reduce the internal resistance by means of rifled piping and the emulsifying of the oil with water. Unfortunately little attempt was made to incorporate these factors in a rational formula. However the Stanton curve upon which the following work is based has provided engineers of all classes with a complete scientific formula adequate enough to satisfy all practical requirements. The use of the Stanton formula would have been useless without the calibration and standardization of the viscosimeters to obtain readily the value of kinematic viscosity for any practical solution of which the constants were not known. Three such viscosimeters have been standardized, the Redwood, Engler and the Saybolt, and Chapter VI gives the formulæ and curves connecting the seconds flow with ν.

The factors which govern the rate of flow of a liquid moving through a pipe, fall into two main classes or groups. Group 1: the internal group; comprises the characteristics of the liquid itself, namely, density, which determines the energy available to produce flow; viscosity, the internal friction of the molecules of the liquid itself and which retards the flow; the inertias or the hydromechanical criteria which govern the formation of eddying streams, thus increasing the energy required to produce a certain mode of flow. Group 2: the external group; includes such factors as hydraulic mean depth, length, roughness of the surface of the pipe, and gravity head.

In the year 1883 Professor Osborne Reynolds read a paper on "An Experimental Investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels," in which the nature of the internal factors governing fluid resistance was first indicated. It is strange that it has taken almost forty years for this knowledge to filter down to ordinary hydraulic and mechanical engineering, for it is only within the last few years that any attempt has been made to incorporate these internal factors into a rational formula for the use of engineers.

Notwithstanding the basic character of D'Arcy's formula:

$$h = f\,\frac{l}{d}\,\frac{v^2}{2g}$$

and the many attempts to give a true value to the co-efficient f, the best thing that has yet appeared is the work of Barnes in a recently published book, *Hydraulic Flow Reviewed*, to devise a logarithmic formula which would fit the Mississippi River and a small earth channel $3\frac{5}{8}$ in. wide and 10 in. deep. This formula is:

$$v = 58{\cdot}4m^{\cdot 694}\,i^{\cdot 496},$$

where v = vel. ft. per sec.

m = hydraulic mean depth $\frac{d}{4}$ in feet.

i = hydraulic slope $\frac{h}{l}$.

Barnes' work is worth study, if only to show that logarithms may cover a multitude of sins and that pure empiricism can be dressed up to appear as the latest scientific novelty. He gives a host of formulæ based on the pattern: $v = \mathrm{K}m^{\alpha}i^{\beta}$, to suit all kinds of hydraulic flow, and when one sees the many attempts to give reasonable accuracy to value of f in D'Arcy's formula the pure hydraulic and mechanical engineer owes a great debt to Barnes for collecting a mass of very carefully conducted experimental results and for determining with wonderful accuracy the constants in his logarithmic though empirical formulæ. It may interest the reader of this pamphlet to notice some of these attempts to square D'Arcy with the truth. For instance, D'Aubusson, Prony, and Eytelwein make the co-efficient f depend on v only, thus: $\frac{1}{f^2} = a + \frac{b}{v}$.

Weisbach submits the following: $\frac{1}{f^2} = a + \frac{b}{\sqrt{v}}$.

Bazin makes f depend on d instead of v, thus:

$$\frac{1}{f^2} = a + \frac{b}{d}.$$

Knutter's co-efficient depends on hydraulic gradient and hydraulic mean depth, and he throws in a sub-co-efficient n for roughness.

For a clean iron pipe D'Arcy gives:

$$4f = \cdot 02\left(1 + \frac{1}{d}\right).$$

No wonder then Barnes rejected the above attempts to derive rational formulæ and took refuge in pure empiricism, but although his formulæ may be excellent for the case of water at ordinary temperature, they are not sufficiently general in application for use in chemical plant design.

II

FUNDAMENTAL RESEARCHES ON VISCOUS FLOW

In the paper already mentioned Reynolds set out to find an explanation for the two modes of fluid motion; one steady, stream-like motion, the other unsteady and sinuous. In the first case resistance varies directly as the velocity, while in the second resistance is as the square of velocity. Most of Reynolds' research work has a philosophical basis, and much of this paper is taken up with the study of Stokes' equations of fluid motion.

From the study of the equations of fluid motion Reynolds obtained his clue to critical velocity being dependent on—

$$\frac{P}{d} \text{ where } P \propto \frac{\text{viscosity}}{\text{density}},$$

but his work is memorable because of the ingenuity and beauty of his experimental work to determine the value of the critical velocity.

In one class of experiments water was drawn through glass tubes B, Fig. 1, of various diameters up to 2 in., immersed in a large tank C constructed with glass sides. On opening the cock A water flows through the experimental pipe B at velocities depending on the amount of the opening of the cock. A fine stream of water coloured with a dye is allowed to enter the trumpet end of pipe B from a vessel

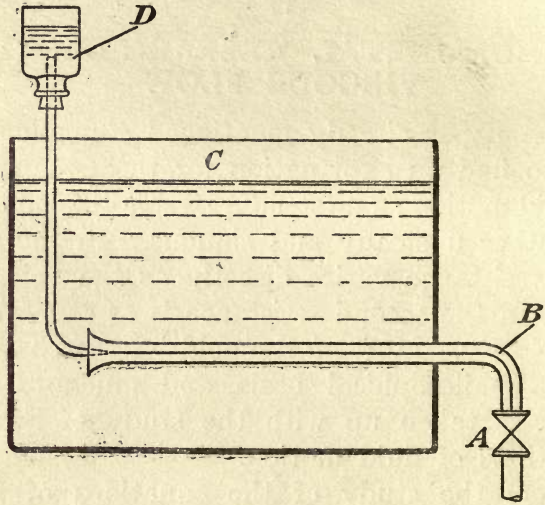

FIG. 1.

D. "When the velocities were sufficiently low, the streak of colour extended in a beautiful straight line through the tube" as in Fig. 2.

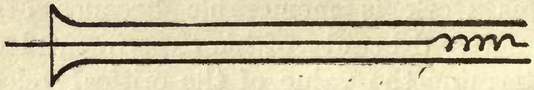

FIG. 2.

"If the water in the tank had not quite settled to rest at sufficiently low velocities the streak would shift about the tube, but there was no appearance of sinuosity.

"As the velocity was increased by small stages, at some point in the tube, always at a considerable distance from the trumpet or intake, the colour band would all at once mix up with the surrounding water and fill the rest of the tube with a mass of coloured water" as in Fig. 3.

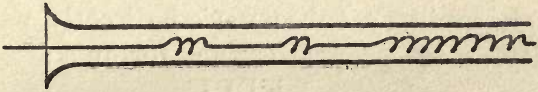

FIG. 3.

"On viewing the eddies by the light of an electric spark the mass of colour resolved itself into a mass of more or less distinct curls, showing eddies" as in Fig. 4.

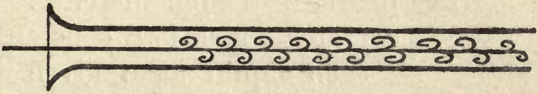

FIG. 4.

Any initial disturbance of the water tends to reduce the velocity at which the motion changes from steady to sinuous, and which is termed the critical velocity.

Reynolds in his paper mentions that when oil is poured on the surface of a pond, at one time the surface appears like a looking glass, and at other times the surface is rippled up and the reflection takes the form of grimacing images, like that of sheet glass. The clear impression corresponds to the stream-line motion; the rippled grimacing effect is due to the formation of eddies at the bounding surface of the oil and the water in the pond.

Poiseuille in connexion with some work on the flow of blood in animal tissues carried out very extensive researches to determine the rate of flow of liquids through small capillary tubes. He laid down a law that the flow through capillary tubes varied directly as the velocity. He proposed the formula:

$$V = \frac{KD^4H}{L}.$$

Where V = volume transpired in cc.
L = length of capillary.
D = diameter of capillary.
H = pressure in mm. mercury.
K = a factor which is a constant for each liquid at a given temperature.

This law governs the standardization of most of the modern viscosimeters.

Another experimenter in this field, D'Arcy, carried out a careful series of experiments in 1857 on the flow of water through cast-iron pipes having diameters ranging from ·5 in. to 20 in. His conclusions, which are set out in a memorable paper, are that the resistance to flow varied as the square of the velocity, and has left on record the general formula:

$$h = f\frac{lv^2}{d2g}$$

where f is a factor which, as already mentioned, depends upon many variables. How these two modes of motion are connected and at what point stream-line flow changes to eddying flow Reynolds was the earliest to show, first by observing the flow of coloured streams and second by investigating the law of resistance in the flow of liquid through pipes. If a force required to cause flow be represented as a slope of pressure, in other words, if the head h required be divided by the length l and

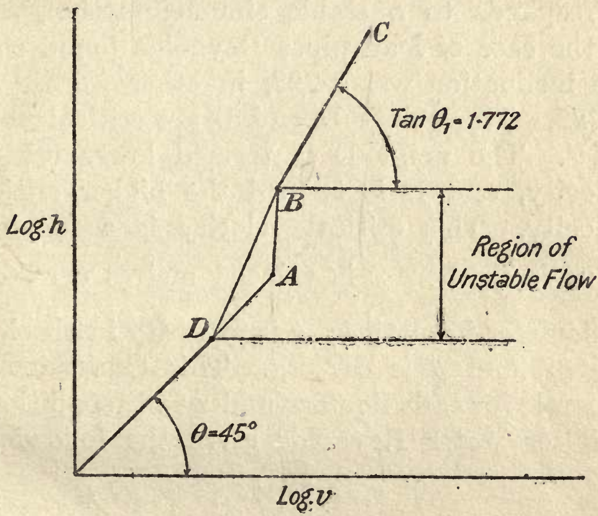

FIG. 5.

velocity v be considered to be the mean velocity of flow through the pipe, then if the log. of h be plotted against the log. of v, the position where streamline flow breaks down into sinuous flow is clearly indicated by the change in the angle of inclination of the curve. Referring to Fig. 5, the line from the origin to the point D makes an angle of 45° to the base, showing that the head h varies directly as v because tangent of the angle $\theta = 1$. From point D to B corresponds to a period of instability, and according to Reynolds' experiments the increase of head is three times that for stream-line flow. From point B onwards to C the flow is steady and corresponds to a stable sinuous motion, and in the case of lead pipes Reynolds found that the inclination was 1·722, in other words, $h \propto v^{1 \cdot 722}$. The points D and B are critical velocities. The point D is termed lower critical velocity, and B is termed the higher critical velocity. This critical velocity is a function of v, d, $\frac{\eta}{\rho}$, where v is velocity metres per sec.; d diameter in metres; η is viscosity in absolute units; and ρ is density. This expression is general for all liquids under all conditions, and for water Reynolds gives the following:

$$v_c = \frac{1}{b} \times \frac{P}{d},$$

where $P = (1 + \cdot 0336T + \cdot 000221T^2)^{-1}$
and T = temperature of water on °C.

The value of b is 43·79 for the lower critical velocity, and 278 for the high critical velocity.

The value of this formula, which may be termed the Reynolds' Criterion, is at last being appreciated by engineers in the development of a rational formula for liquid flow, which will apply to all liquids under all conditions. One point, however, must be noted—there has not been very much experimental data to obtain very accurate values for the constants and for a wide range of substances. However, Stanton (Reynolds' former Assistant) has completely verified the general law and supplied the engineer with a basis upon which to construct the new formula.

Reynolds, however, overlooked the importance of the function $\frac{vd}{\nu}$ as an argument or common denominator in fluid friction, but appears to have been the first to use logarithms in plotting his results. This is to be regretted, for it has set the fashion in exponential formulæ, of which Mr. Parry, the Engineer-in-Chief to the English Electric Co., tells me, one is announced once every month or oftener.

As already stated (page 10) the law of "similarity of conditions represented by $\frac{vl}{\nu}$ produces similarity of motion" was first made known by O. Reynolds in 1883. This law, though verified by many careful experiments, was really the outcome of mathematical insight which characterizes so much of Reynolds' work. His experimental data was then not sufficient to attract the attention of engineers who are satisfied with a simple formula accurate enough to guide them in practical work. However, the rapid development of the aeroplane has drawn attention of the scientific world to the need of a rational formula governing the resistance of planes through the air. Stanton and his assistants working on this subject at the National Physical Laboratory have carried out a classic series of experiments on the flow of air, water and oil through smooth drawn brass pipes of various diameters. The results put on a firm basis the above-mentioned law of similarity of fluid forming the modern theory of fluid friction more comprehensive in character than the older D'Arcy formula with its floating factor f, and excepting for the difficulty in comparing degrees of roughness for various sizes of pipes adequate enough to banish empiricism to the limbo of the past. The criterion which governs the law of fluid resistance is:

$$\frac{vl}{\nu}$$

where v = velocity

l = linear dimension of the system

ν = kinematic viscosity $= \frac{\eta}{\rho}$

where η = absolute viscosity

ρ = density

for a pipe d is substituted for l thus:

$$\frac{vd}{\nu}$$

and for a channel the perimeter p is the linear dimension: $\frac{vp}{\nu}$.

The new theory has been well summarized and adapted to the uses of the hydraulic engineer by Mr. E. Parry, Engineer-in-Chief to the English Electric Company, in a pamphlet on *A Theory of Fluid Friction and its Application to Hydraulics*, published in his Company's journal. Briefly, the new theory states that the resistance per unit area R is a function of the linear dimensions of the system, the velocity v, the viscosity η, and density ρ; if $\nu =$ ratio $\frac{\eta}{\rho}$, the kinematic viscosity, then $\frac{R}{\rho v^2} = \varphi \frac{(vl)}{\nu}$.

This equation is perfectly general and applies to all moving surfaces in any fluid, liquid or gaseous.

For pipes $\frac{R}{\rho v^2}$ becomes $\frac{mig}{v^2}$,

where $m =$ hydraulic mean depth
$i =$ hydraulic gradient
$g =$ acceleration of gravity

therefore $\frac{mig}{v^2} = \varphi \frac{vd}{\nu}$.

To apply this formula to channels the perimeter p is substituted for d, the diameter of pipe, thus: $\frac{mig}{v^2} = \varphi \frac{vp}{\nu}$.

The insertion of the kinematic viscosity function $\frac{\eta}{\rho}$ in the flow formula can only refer to the internal friction of the fluid stream and can therefore only extend to the surface of the conduit. The theory is true providing the state of the surfaces of the conduits are similar, that is to say a small smooth pipe may be as proportionately rough as a very large rough pipe. The next stage in the development of the theory is the finding of suitable factors of roughness. Parry has already made such an attempt. The factor kinematic viscosity $\frac{\eta}{\rho}$ then is the link which connects the flow of all liquids and gases through conduits. Lord Rayleigh first proposed to plot on a diagram values for the external resistances against those of the internal resistance, in other words, $\frac{R}{\rho v^2}$ as ordinates and $\frac{vl}{\nu}$ as abscissæ.

As R (c.g.s. units) is an inconvenient unit for the engineer, the equivalent for $\frac{R}{\rho v^2}$ for pipes $\frac{mi}{v^2}$ is adopted instead, which is obtained by dividing $\frac{R}{\rho v^2}$ by g.

Now before any values of these factors are given a word or two on units is advisable. One of the greatest difficulties the working engineer encounters is the great variety of units adopted in various countries and in the different professions. Throughout the world the unit for purely scientific work is the centimetre gram second. Stanton's researches as published by the National Physical Laboratory, London, are given in this unit. The ordinary hydraulic engineer uses the foot pound second, while the newly arisen chemical engineer, chiefly through the influence of the bench chemist who is not at home in the works with a foot-rule, is trying to adopt the metre kilogram second as a practical kind of c.g.s. system.

III

KINEMATIC AND ABSOLUTE VISCOSITY

VISCOSITY is a measure of the internal friction of a moving fluid. It is the force required to move a plane surface of unit area past another at unit distance at unit speed, the space between being filled with the fluid. There are various ways of measuring this quantity, but the usual method is to cause the fluids to flow at velocities below the critical value, i.e. without the formation of eddies, through small bore tubes of many diameters in length.

What happens when a liquid or a gas flows through a tube without eddying may be imagined as pulling out a telescope. The fluid is considered as made up of an infinite number of concentric cylinders moving axially along the tube. The outermost cylinder wets the wall of this tube and sticks there, the next cylinder slips past slowly and each successive cylinder a little faster than the last till the centre is reached where the velocity is greatest. The resistance to this slipping of the liquid cylinders is a measure of the viscosity. If the head required to cause the flow is carefully measured the value of the absolute viscosity is calculated from the expression $\eta = \dfrac{\rho g r^2 h}{8vl}$,

where η = co-efficient of absolute viscosity in dynes per sq. cm. ~~of~~ or poundals per sq. ft.

g = acceleration of gravity—981 cm. or 32·2 ft. per sec.

r = radius of tube in cm. or ft.

h = head in cm. or ft.

v = average velocity of flow in cm. or ft. per sec.

l = length of tube in cm. or ft.

The value of η in c.g.s. units must be divided by 14·9 to give the ~~practical~~ absolute English equivalent.

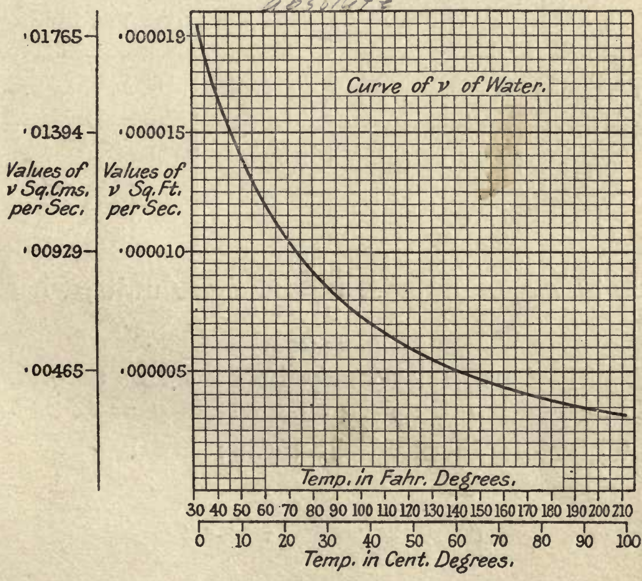

FIG. 6.

The kinematic viscosity ν or $\frac{\eta}{\rho}$ is the quantity measured by the ordinary viscosimeter of the Redwood, Engler or Saybolt type. The dimension of ν is $\frac{M}{LT} \times \frac{L^3}{M} = \frac{L^2}{T}$, the ratio $\frac{\text{c.g.s.}}{\text{ft. lb. sec.}}$ being $\frac{1}{30{\cdot}48^2} = \frac{1}{929{\cdot}03}$.

Table 1 in the Appendix gives the values of η, ρ, ν for water at temperatures from 0° C. to 100° C. in both sets of units.

These values are also given in Diagram Fig. 6.

CONVERSION FACTORS

η Absolute Viscosity		ν Kinematic Viscosity	
c.g.s. units	ft. lb. sec. units	c.g.s. units	ft. lb. seconds.
1	$\frac{1}{14{\cdot}9}$	1	$\frac{1}{929{\cdot}03}$

$\frac{R}{\rho v^2}$ in c.g.s. units $= \frac{mig}{v^2}$ in c.g.s. units, where $g = 981$.

$\frac{R}{\rho v^2}$ in foot lb. second units $= \frac{mig}{v^2}$ in foot lb. seconds, where $g = 32{\cdot}2$.

Therefore in practical units:

$$\frac{\frac{R}{\rho v^2}}{981} \text{ c.g.s.} = \frac{mi}{v^2} \text{ c.g.s.}$$

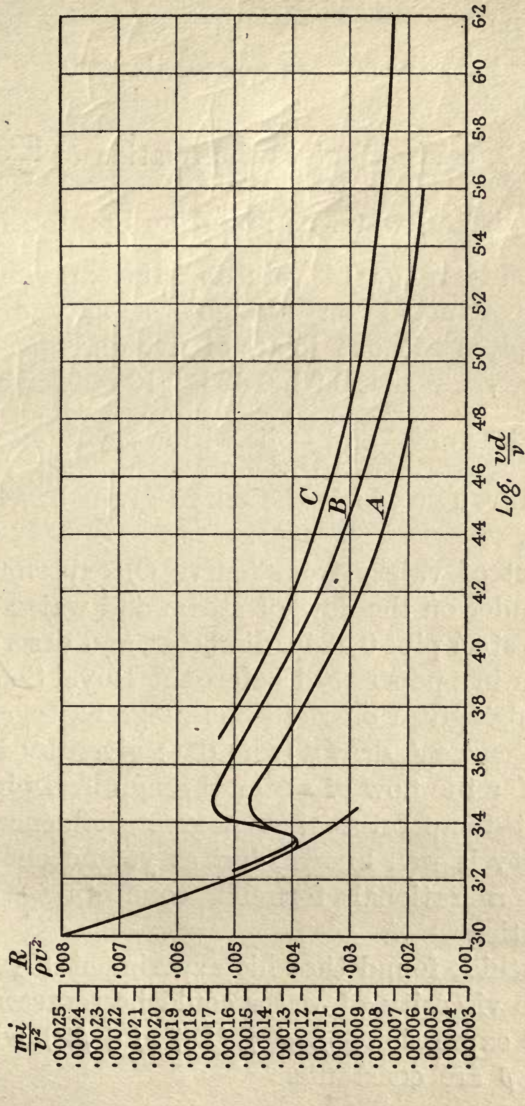
A
B
C
Log. $\frac{vd}{\nu}$
$\frac{R}{\rho v^2}$
$\frac{mi}{v^2}$
3·0
3·2
3·4
3·6
3·8
4·0
4·2
4·4
4·6
4·8
5·0
5·2
5·4
5·6
5·8
6·0
6·2
·001
·002
·003
·004
·005
·006
·007
·008
·00003
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FIG. 7.

$$\frac{R}{\frac{\rho v^2}{32\cdot 2}} \text{ ft. lb. sec.} = \frac{mi}{v^2} \text{ ft. lb. sec.}$$

Fig. 7, curve B, gives the relation of $\frac{mi}{v^2}$ and $\frac{vd}{\nu}$ for air and water as found by Stanton for a very wide range of values. His experiments were conducted on straight smooth drawn brass pipes without joints of five different diameters, viz. 0·144, 0·28, 0·493, 1·13, 4·96 inches. The values of $\frac{mi}{v^2}$ and $\frac{vd}{\nu}$ lie within a very narrow band through which the curve given has been drawn.

A set of values from (curve C) experiments by Lander on the flow of steam and water in a drawn steel pipe 0·42 in. diameter, and described by him in a paper read before the Royal Society in 1916, is given on the same diagram, together with a curve A drawn from data given by Reynolds for the flow of water through lead pipes.

That temperature has a great influence on viscosity is well known, but as yet no one has found a rational formula connecting these quantities.

Poiseuille found that his experimental results for the viscosity of water could be represented by the expression: $\eta_t = \eta_o (1 + \alpha t + \beta t^2)$, where α and β are constants.

Meyers' formula:

$$\eta_t = \eta_o (1 + at)$$

is too simple and only applies over a limited range.

Slottes comes nearer with a formula of the type:

$\eta_t = \eta_o (1 + \beta t)^n$, where β and n are constants.

Graetz proposed:

$\eta_t = \dfrac{A(\theta - t)}{t - t_1}$, in which θ is the critical temperature and t_1 is a temperature below the melting point, where the viscosity is infinite. Thorpe and Rogers, reviewing the various formulæ above, select Slottes' as being the most accurate. Slottes' formula, when expanded on the assumption that the temperature coefficient is small, simplifies to:

$$\eta = \frac{A}{1 + at + \beta t^2}.$$

Constants in the foregoing formulæ for many pure substances and solutions are given in Castell-Evans' tables.

In practice for liquids of unknown composition it is preferable to use one of the standard viscosimeters and deduce η from the curves shown on Fig. 9. Care must be taken to determine accurately the temperature. The flow of water is increased about $1\frac{1}{2}$ times from 0° C. to 100° C. boiling point.

IV

RELATION BETWEEN MEAN VELOCITY AND VELOCITY AT AXIS OF PIPES

In the case of stream-line flow of a viscous fluid through a pipe the ratio of the mean velocity over the section to the velocity at the centre is 0·5. Here both theory and experiment are in agreement. The value of this ratio for eddying or turbulent flow depends on the function $\frac{vd}{\nu}$. Stanton's careful experiments

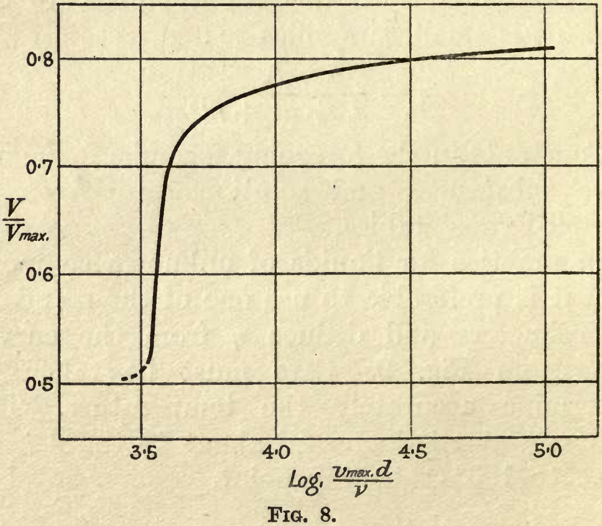


FIG. 8.

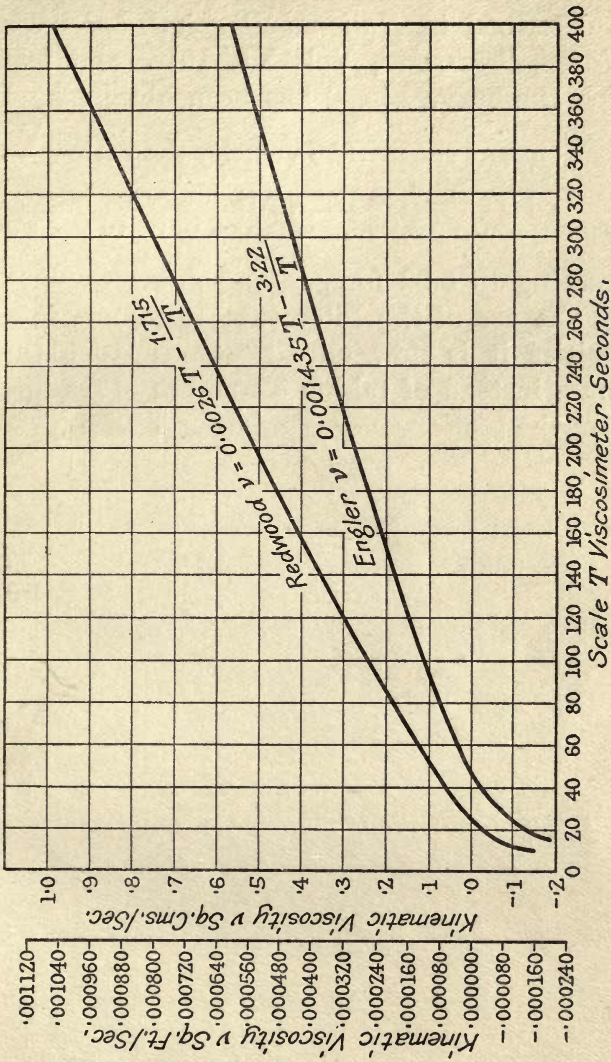

Redwood $\nu = 0.0026\,T - \frac{1.715}{T}$
Engler $\nu = 0.001435\,T - \frac{3.22}{T}$
Scale T Viscosimeter Seconds.
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400
Kinematic Viscosity ν Sq.Cms./Sec.
1·0 ·9 ·8 ·7 ·6 ·5 ·4 ·3 ·2 ·1 0 −·1 −·2
Kinematic Viscosity ν Sq.Ft./Sec.
·001120 ·001040 ·000960 ·000880 ·000800 ·000720 ·000640 ·000560 ·000480 ·000400 ·000320 ·000240 ·000160 ·000080 ·000000 −·000080 −·000160 −·000240

Fig. 9.

are described in *National Physical Laboratory Collected Researches*, Vol. XI, 1914, and from which the curve Fig. 8 has been obtained. It is seen that at the critical velocity the ratio $\frac{V}{V_{max.}}$ is log. $\frac{vd}{\nu}$ 3·5 or 2,050, rising at once to 0·7 and increasing to 0·82 for eddying flow.

In the use of the Pitot tube for ascertaining velocities it is now only necessary to obtain V max. instead of taking a number of readings across the section and calculating V mean.

V

THE $\dfrac{mig}{v^2} - \dfrac{vd}{\nu}$ DIAGRAM

Owing to the enormous range of values of $\dfrac{vd}{\nu}$ (1–1,000,000) and of $\dfrac{mig}{v^2}$ (·001–1·0), the relation of these quantities is plotted logarithmically, Fig. 10. The curve between values of $\dfrac{vd}{\nu}$ from 1 to 2,050 is a line sloping downward at an angle of 45° to the horizontal and refers to stream-line flow where the head h varies directly as the velocity v.

The equation of this portion of the curve is:

$$\frac{mi}{v^2} = 8\frac{\nu}{vdg} \text{ or } \frac{R}{\rho v^2} = 8\frac{\nu}{vdg}.$$

The point B indicates the lower critical point where:

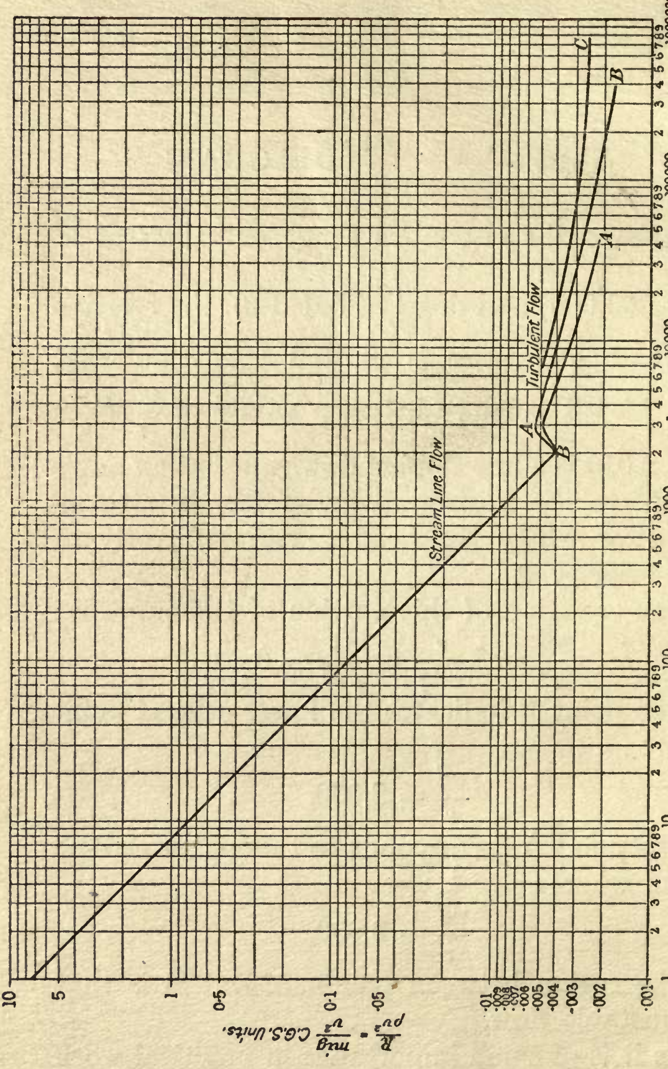
$$\frac{vd}{\nu} = 2,050.$$

The point A corresponds to the higher critical value where:

$$\frac{vd}{\nu} = 2,900.$$

The region between points A and B is that of unstable flow.

As it is of small importance in practical work



Stream Line Flow
Turbulent Flow
A
B
C
$\frac{vd}{\nu}$
$\frac{R}{\rho v^2} = \frac{m i g}{v^2}$ C.G.S. Units.

FIG. 10.

its significance need not here be discussed. Gibson, in his work on hydraulics, explains this point of unstable flow at length. From the point A onwards the curve sweeps gently downwards to parallelism at infinity. No portion of this curve is straight, so that the logarithmic gentlemen may rest their souls in silence. No logarithmic formula has any basis in reality and is pure empiricism. The work of Reynolds and Froude extends over too narrow a region for the errors of the index ratio to be seen.

The curve from point A onwards corresponds to sinuous or turbulent flow on which the main hydraulic flow formulæ are built. It will be seen later that the flow of many chemical liquids is stream line. Thus, if strong sulphuric acid be taken as example, we have $\eta = \cdot 2658$ in absolute units at 20° C.

$$\rho = 1{\cdot}84$$

$$\frac{\eta}{\rho} = \nu = \frac{\cdot 2658}{1{\cdot}84} = \cdot 1442$$

$$d = 5 \text{ cm. (about 2 in.)}$$

then critical velocity is reached when:

$$\frac{vd}{\nu} = 2{,}050$$

$$v = \frac{2{,}050 \times \cdot 1442}{5} = 59 \text{ cm. per sec. (about}$$

2 ft. per second).

This rate of flow is equivalent to about 16 gallons per minute. It is obviously an advantage to keep the flow below the critical velocity because the head required varies only as velocity, and not as velocity squared as in the usual hydraulic flow.

That portion of the curve relating to turbulent flow is plotted to a larger and different scale in Fig. 7. The actual values of $\frac{mi}{v^2}$ in foot pound second units and $\frac{R}{\rho v^2}$ in c.g.s. units are plotted as abscissæ and log. $\frac{vd}{\nu}$ as ordinates.

The lower curve A gives the flow through lead pipes from Reynolds' experiments on water; the middle curve B Stanton's results on the flow of water and air through commercially obtained smooth solid drawn brass pipe; while the upper curve C gives Lander's figures on the flow of steam and air through solid drawn steel pipes. Parry in his pamphlet has plotted a chart giving the flow of water through large steel and cast-iron mains of about 48 in. diameter, from which an idea of the factor of roughness can be obtained. As already stated the position of the turbulent-flow curve on the $\frac{mi}{v^2} - \frac{vd}{\nu}$ graph is a criterion of the surface roughness of the pipe or conduit only. The resistance offered by cast iron (asphalted) is much the same as that of Stanton's brass pipes, while riveted steel pipes seem to offer a much greater resistance.

VI

THE PRACTICAL APPLICATION OF THE KINEMATIC VISCOSITY EQUATION

THE chemical engineer has this advantage over the hydraulic engineer that he has at command the resources of a modern chemical laboratory. If Parry can urge the use of this formula for water, how much more should the chemical engineer seize upon this new method of expressing resistance to fluid flow. On the other hand, it can be stated that water has been studied almost *ad nauseam*, while many of the fluids required to be dealt with by the chemical engineer are almost unknown outside a special industry. In certain industries, however, knowledge of viscosity is essential and in them convenient types of viscosimeters have been developed. The petroleum industry possesses two standard types of viscosimeters from which the kinematic viscosity is easily derived, one designed by Sir Boverton Redwood, the English standard ; the other the Saybolt, the adopted standard of the United States. The German type of the instrument, the Engler, is also in use. In the dope, varnish, and similar industries, other types more on the line of the scientific apparatus for determining the absolute viscosity are in use.

From what has already been said it is an easy matter to ascertain the flow of any liquid if its absolute viscosity and density are known. All published tables of physical constants, of which a selection is given in the Appendix, give such data for many substances in the pure and diluted form, but in the process of manufacture it is essential to be able to measure the flow of liquids of unknown composition. In such cases the kinematic viscosity can be obtained by determining the time of outflow for any of the three viscosimeters just mentioned. The relation between kinematic viscosity ν and T time of outflow is for the various viscosimeters as follows (in c.g.s. units):

Redwood $\quad \nu = 0{\cdot}0026 \quad T - \dfrac{1{\cdot}715}{T}$

Saybolt $\quad \nu = 0{\cdot}0022 \quad T - \dfrac{1{\cdot}8}{T}$

Engler $\quad (a)\ \nu = 0{\cdot}00147 \quad T - \dfrac{3{\cdot}74}{T}$

$\quad (b)\ \nu = 0{\cdot}001435 \quad T - \dfrac{3{\cdot}22}{T}$

Nos. 1 and 4 of these equations are plotted in Fig. 10, from which it can be seen that for time of outflow greater than 100 seconds the kinematic viscosity varies directly as viscosimeter seconds T. Below 100 seconds ν falls more rapidly than T, till at 40 seconds the value of ν descends to impossible figures. It may be taken that the Redwood and Saybolt formulæ do not apply below 40 T and Engler below 60 T.

It is now possible to give examples showing the application of the new relation of flow in actual practice.

It is required to determine the head necessary to cause flow as follows:

Liquid . . .	Concentrated sulphuric acid
Pipe . . .	Lead 2 in. dia. ($m = \frac{1}{24}$ ft.)
Temperature . .	20° C.
Kinematic viscosity from Table No. 4	·00013 sq. ft. per sec.
Flow . . .	3 ft. per sec., or 25 gals. per min.

Now $$\frac{vd}{\nu} = 3 \times \frac{1}{6} \times \frac{1}{\cdot 00013}$$
$$= 3{,}850$$
$$\log. 3{,}850 = 3{\cdot}5855$$

From the curve Fig. 7 or 10 the value of $\frac{mi}{v^2}$ corresponding to $\frac{vd}{\nu}$ of log. 3·5855 is ·00016

$$\frac{mi}{v^2} = \cdot 00016$$
$$\therefore i = \frac{\cdot 00016 \times 9}{\frac{1}{24}}$$
$$= \cdot 0352,\text{ about 3 ft. 6 in. per 100 ft.}$$

As a check the calculation may be done in c.g.s. units as follows:

$$d = 2 \text{ in.} = 5 \text{ cm.}$$

$$m = \frac{d}{4} \quad = \frac{5}{4}$$

$$\nu = \cdot 121 \text{ sq. cms. per sec.}$$

$$v = 90 \text{ cm. per sec.}$$

Now $$\frac{vd}{\nu} = \frac{90 \times 5}{\cdot 121}$$

$$= 3{,}800 \text{ approx.}$$

$$\log. 3{,}800 = 3 \cdot 5798$$

From curve Fig. 7 or 10

$$\frac{mi}{v^2} = \cdot 0052 \times g$$

$$= \cdot 000005$$

$$\therefore i = \cdot 032$$

An American engineer, discussing Stanton's researches, prefers to use the criterion $\frac{vd}{\nu}$ to obtain the correct value of f in what he terms the Fanning formula:

$$h = f \frac{lv^2}{d2g}$$

known on this side as D'Arcy's formula. This he does by graphing f in the above formula against $\frac{vd}{\nu}$, which of course gives a curve which is a function of Stanton's $\frac{R}{\rho v^2} - \frac{vd}{\nu}$ curve. The first method seems to be simpler as most of the labour involved is in determining the value of $\frac{vd}{\nu}$. Having obtained $\frac{vd}{\nu}$ the corresponding value of $\frac{mi}{v^2}$ is read from the curve.

VII

FLOW OF LIQUIDS IN CHANNELS

It has already been stated that if p be the wetted perimeter then $\frac{vp}{\nu} = \varphi \frac{mi}{v^2}$ or $\frac{R}{\rho v^2}$.

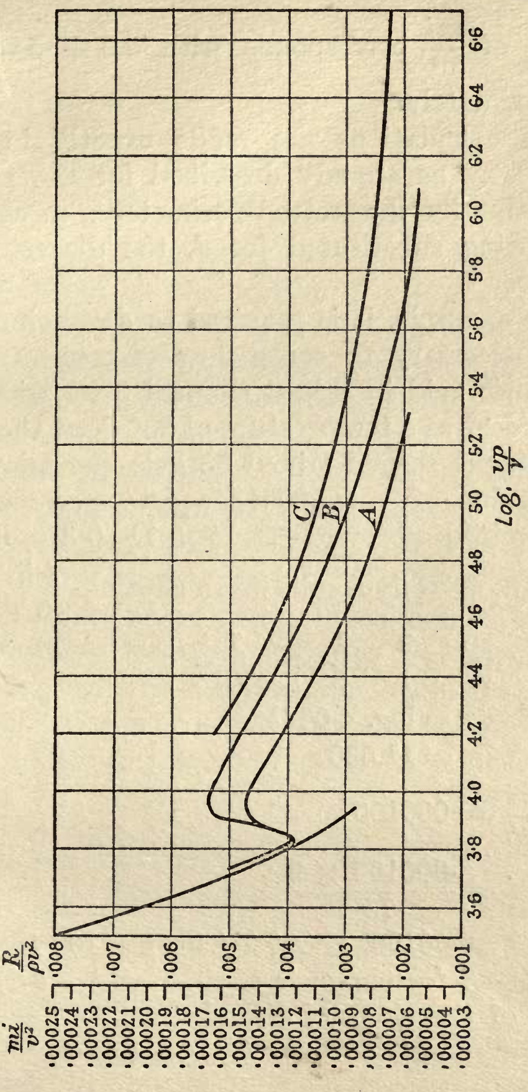
This equation enables us to determine the flow of liquid in channels. In Fig. 11 are graphed curves of this function for channels of varying degrees of roughness. Thus:

Curve A	Lead pipe	Reynolds
,, B	Smooth drawn brass pipes	Stanton.
,, C	Drawn steel pipes	Lander.

It is not, however, necessary to draw a separate graph for the resistance to flow in channels because the value of p is a simple ratio of d the diameter of a pipe:

$$p = \pi d. \qquad \therefore \frac{vp}{\nu} = \pi \frac{vd}{\nu}$$

or $\log. \frac{vp}{\nu} = \log. \frac{vd}{\nu} + \log. \pi = \log. \frac{vd}{\nu} + 0{\cdot}5$ approx. thus: referring to graph Fig. 7 curve B the value of $\frac{mi}{v^2}$ corresponding with $\log. \frac{vd}{\nu} = 5{\cdot}0$ is $\cdot 00007$, while on the $\frac{vp}{\nu}$ chart (Fig. 11) this



$\frac{R}{\rho v^2}$
.008
.007
.006
.005
.004
.003
.002
.001
$\frac{mi}{v^2}$
.00025
.00024
.00023
.00022
.00021
.00020
.00019
.00018
.00017
.00016
.00015
.00014
.00013
.00012
.00011
.00010
.00009
.00008
.00007
.00006
.00005
.00004
.00003
3·6
3·8
4·0
4·2
4·4
4·6
4·8
5·0
5·2
5·4
5·6
5·8
6·0
6·2
6·4
6·6
C
B
A
$Log, \frac{vp}{v}$

FIG. 11.

value of $\frac{mi}{v^2}$ corresponds with $5{\cdot}0 + {\cdot}5 = 5{\cdot}5$ approximately.

The calculations are made exactly in the same way as already described for the pipes, the value for the wetted perimeter p in cm. or feet being substituted for d, the diameter of a pipe.

For example, it is required to determine the head necessary to cause flow of concentrated sulphuric acid in a lead channel 3 in. wide by $1\frac{1}{2}$ in. deep at a temperature of 20° C. at the rate of 3 ft. per second (about 35 gals. per minute). The value of ν is ·00013 sq. ft. per second, $p =$ wetted perimeter $= 1\frac{1}{2} + 1\frac{1}{2} + 3 = 6$ in. $= {\cdot}5$ ft., $m =$ hydraulic mean depth $= \frac{4{\cdot}5}{6} = {\cdot}75$.

$$\text{Now } \frac{vp}{\nu} = \frac{3}{1} \times \frac{1}{2} \times \frac{1}{{\cdot}00013}$$
$$= \text{log. } 4{\cdot}0621$$
$$= 11{,}530$$

$$\frac{mi}{v^2} = {\cdot}000165$$

$$i = \frac{{\cdot}000165 \times 9}{{\cdot}75}$$
$$= {\cdot}00198, \text{ about } 2{\cdot}4 \text{ in. per 100 ft.}$$

Note.—i for channels is $\sin\theta$ where θ is the angle of the slope of the channel.

VIII

OTHER PIPE LINE LOSSES

THE foregoing chapters deal with resistance to flow offered by the surface of a straight pipe. We have now to consider the losses of bends, valves, elbows, deviation from the straight, pipe junctions, sudden enlargements and contractions in the pipe area.

In ordinary hydraulics all losses are assumed to be proportional to v^2 because we suppose water to flow in the eddying stream form. If the flow is stream line it is as reasonable to assume that losses are proportional to v only. Accordingly therefore it must be known first whether the flow is stream line or sinuous, which has already been explained, and then to take the ordinary hydraulic figures for sinuous flow losses and correct for stream-line flow in the ratio $\frac{v}{v^2}$. This may not be scientific in the real sense, but then these resistances are of such varied character that approximations can only be attempted. In an ordinary chemical works it can be accepted that pipe lines are seldom straight and that lead pipes are apt to be walked on and knocked, bulged, dented and otherwise misshaped, that often the "other pipe-line losses" comprise most of the resistance. Numerous examples could be cited to show the extent of "other pipe-line losses" in the average chemical works. One must suffice here. The writer in investigating pipe flow in one of the large tar works in the country stumbled across a case of a ram pump working at 52 lb. pressure pumping water to the Glover acid coolers which stood on a platform not 12 ft. from the ground. Theoretically 12 ft. is equal to $5\frac{1}{2}$ lb. per sq. in. pressure. Double this for pipe friction in the usual chemical way—that is, 11 lb. per sq. in.—so that it can be reckoned that 41 lb. per sq. in. was wasted. Though fuel may be cheap in such works the cost of the extra power used would be many times that of the interest on a larger and properly designed pipe line.

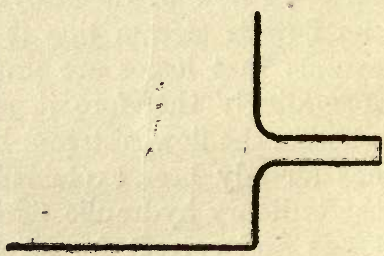


FIG. 12.

Professor Gibson, of Manchester, has kindly permitted me to extract and adapt from his book, *Hydraulics and its Application*, the following factors.

Losses at Entrance to a Pipe Line.—These depend on the form of the entrance. For bell mouthpiece, as in Fig. 12, loss of head is about $\cdot 05 \frac{v^2}{2g}$ or $\cdot 05 \frac{v}{2g}$ for stream-line flow. For re-entrant mouthpiece as in Fig. 13, the loss is

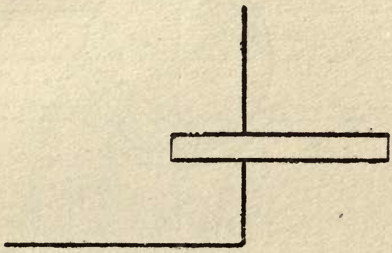


FIG. 13.

$\frac{v^2}{2g}$ or $\frac{v}{2g}$.

For the commonest case where pipe opens flush with side or bottom of tank or reservoir, loss of head is about $\cdot 47 \frac{v^2}{2g}$ or $\cdot 47 \frac{v}{2g}$ for stream line (Fig. 14).

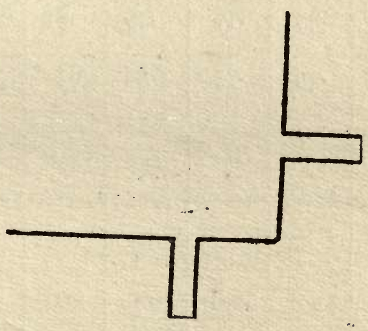


FIG. 14.

Loss due to Cocks in Cylindrical Pipe.—Referring to Fig. 15:

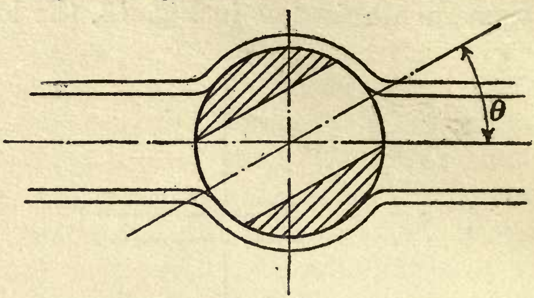


FIG. 15.

a = sectional area through valve plug
A = area of pipe
θ = angle through which valve is turned
F = constant in formula:

$$\text{Loss} = F\frac{v^2}{2g} \text{ or } F\frac{v}{2g}$$

	5°	10°	15°	20°	25°	30°	35°
$\frac{a}{A}$	·93	·85	·77	·69	·61	·54	·46
F	·05	·29	·75	1·56	3·1	5·47	9·68
	40°	45°	50°	55°	60°	65°	82°
$\frac{a}{A}$	·39	·32	·25	·19	·14	·09	Valve
F	17·3	31·2	52·6	106	206	486	Closed

Loss at Bends, Elbows and Tees.—

Referring to Fig. 16:

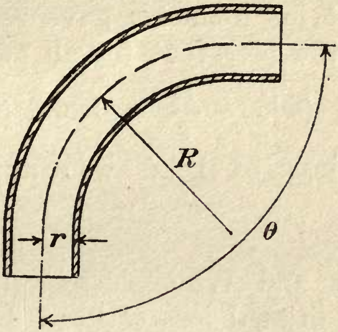


Fig. 16.

If R be the radius of the bend
r „ „ „ pipe
θ „ angle „ bend
then the following values for F in the formula $F\frac{v^2}{2g}$ or $F\frac{v}{2g}$ are given by the authorities cited.

Weisbach:

Loss of head due to bend $= \frac{h}{B} = F\frac{v^2}{2g} = \frac{\theta}{180}$ in ft.

and F for circular pipes $= \cdot 131 + 1\cdot 847 \left(\frac{r}{R}\right)^{\frac{1}{2}}$.

For sharp bends and elbows Weisbach gives:

$$\text{Loss} = F\frac{v^2}{2g} \text{ ft.}$$

$$\text{and } F = \cdot 946 \sin \frac{2\theta}{2}\ 2\cdot 05\ \sin \frac{4\theta}{2}.$$

Gibson states that the above formulæ give low results.

Brightmore experimenting on cast-iron pipes 3 in. and 4 in. diameter gave results which pointed to the fact that for all curves of the best radius (where $\frac{R}{r}$ is from 5 to 15) the additional loss of head was approximately $\cdot 3 \frac{v^2}{2g}$.

Alexander and Williams' experiments show that the additional loss of head due to bends of 90° of radius $R = 5r$ is equal to that offered by a straight pipe of length $3{\cdot}38l$ where l is the length of the curved portion of the bend. Other authorities state that the resistance of easy right-angled bend is equal to that of a straight pipe 10 to 15 diameters long. The best radius of a bend is $\frac{R}{r} = 30$, but if this large radius is impracticable then adopt values of R between 5 and 10. Values of R less than 5 or between 10 and 20 should be avoided.

Loss of Head in Commercial Screwed Pipe Elbows and Tees.—Gibson in his work, *Hydraulics and its Application*, has tabulated carefully the experiments of Schroder, Dalby, Bain, Davis, from which the following average values of resistance are calculated:

Name of fitting	Velocity in pipe ft. per sec.	Length of straight pipe giving equal loss of head	Value of F in formula $h = F\frac{v^2}{2g}$ (Mean)
		in pipe diameters	
Elbow:			
$\frac{3}{4}$ in. in black mall. (old)	2 to 10	23	0·76
$\frac{3}{4}$ in. in galvd. mall. (new)	2 to 10	16	0·53
1 in. in black mall. (old)	2 to 10	23	0·70
1 in. in C.I. (old) .	2 to 10	31	0·95
2 in. in M.I. . .	2 to 10	34	0·72
2 in. in C.I. . .	2 to 10	63	1·34
3 in. in C.I. . .	1 to 25	25	0·54
4 in. in C.I. . .	1 to 25	27	0·60
6 in. in C.I. . .	3 to 16	29	0·50
Tees:			
2 in. in C.I. A .	2 to 10	85	1·9
2 in. in C.I. B .	2 to 10	70	1·5
3 in. in C.I. B .	1 to 25	115	2·45
3 in. in C.I. A .	1 to 25	67	1·43
4 in. in C.I. B .	1 to 25	65	1·41
4 in. in C.I. A .	1 to 25	54	1·24

A refers to tee piece fixed as in Fig. 17.

B refers to tee piece fixed as in Fig. 18.

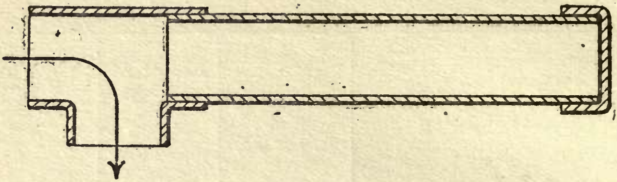


FIG. 17.

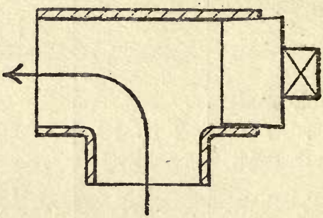


FIG. 18.

Losses at Sudden Changes in Section of the Pipe.—

If a = area of small pipe
A = area of large pipe
V = velocity in large pipe
v = velocity in small pipe

$$m = \frac{A}{a}$$

then loss of head due to sudden enlargement =

$$h = \frac{V^2}{2g}(m-1)^2 = \frac{V^2}{2g}\left(1 - \frac{1}{m}\right)^2$$

For pipe diameters ranging from 0·5 in. to 6 in. and values of m from 2·25 to 10·96 the loss expressed as a percentage of $\frac{(v-V)^2}{2g}$ is about 100. In other words, the factor F in formula $h = F\frac{v^2}{2g}$ is unity.

Losses at Joints.—For purely chemical work involving lead, glass and earthenware pipe lines care must be taken in assessing fair values for the various joints. The lead burners joint in lead, for instance, varies in smoothness according to the degree of skill of the workman. There is always a length of pipe containing burrs, restrictions which are bound to set up eddies and so increase resistance to flow. In glass pipe lines, as for example in air lifts for circulating liquors in acid-absorbing towers, there are many types of joints, and it does not follow that the joint offering the least resistance to flow is the one best to be used.

Tightness in the case of corrosive fluids has first consideration. One of the best joints for glass pipes is constructed of an ordinary earthenware packing ring, into which the ends of the glass pipe are inserted, the joint being made with a stout rubber bung as in Fig. 19.

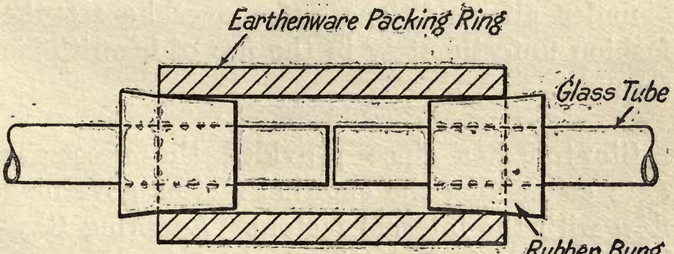


FIG. 19.

SUMMARY

Modern chemical engineering demands a general formula for viscous flow; the usual D'Arcy formula and the many exponential formulæ are either wrong or of restricted application.

The factors governing viscous flow fall into two classes: internal and external, the former in the case of chemical fluids being by far the most important.

Reynolds' researches revealed the two modes of fluid motion—stream-line flow and turbulent flow. He discovered the criterion $\frac{vd}{\nu}$, but failed to see it as an argument of general application to the flow of all fluids—liquids or gases—under all conditions. Moreover he set the fashion unfortunately in the use of logarithmic formulæ.

Stanton has since provided the necessary data connecting the external factors governing flow with the internal factors. Adopting Rayleigh's method of plotting he has given a curve

$\frac{R}{\rho v^2} = \varphi \frac{vd}{\nu}$ for flow over a very large range of air, water and oil through brass pipes.

Where the absolute viscosity η and density ρ of any liquid are known, the ratio $\frac{\eta}{\rho} = \nu$ is at once determined.

For other liquids kinematic viscosity ν can be obtained by determining T, the flow in seconds, of one of the three standard viscosimeters—Redwood, Engler and Saybolt, and using the appropriate formula.

The influence of temperature on viscosity is enormous, and in consequence also on the rate of flow. Therefore it is important that ν be carefully determined at the temperature of flow.

The $\frac{mig}{v^2} - \frac{vd}{\nu}$ diagrams show clearly the point of critical velocity: $\frac{vd}{\nu} = 2{,}050$ or $2{,}900$ in c.g.s. units. By designing pipe flow in the stream-line region much power is saved. For stream-line flow head varies directly as the velocity, while for turbulent flow head varies as $v^{1\cdot7 \text{ to } 2}$. Moreover, surface of pipe has no effect in the stream-line region, but in the turbulent flow region the nature of the surface is evident by the position of the curves, A, B, C, for lead, brass and steel respectively.

Stanton has shown that ratio of mean velocity to velocity of central filament is a function of $\frac{vd}{\nu}$. For stream-line flow the ratio $\frac{V}{V_{max.}}$ is 0·5, and for turbulent flow $\frac{V}{V_{max.}}$ is ·7 rising gradually to ·82.

To determine head for a given flow through a pipe:

(1) Determine ν for known liquids ratio $\frac{\eta}{\rho}$.
for unknown liquids by use of standard viscosimeters and curves, Fig. 9.

(2) Fix v and d and calculate $\frac{vd}{\nu}$.

(3) From diagram Figs. 10 and 7 find value of $\frac{mi}{v^2}$ corresponding to value $\frac{vd}{\nu}$ as determined in (2).

(4) The hydraulic slope i then is:

$$i = x\left(\frac{vd}{\nu}\right)\frac{v^2}{m}.$$

Flow through channels can be determined

from the curves of $\frac{mig}{v^2} - \frac{vd}{\nu}$ for pipes by substituting for d in criterion $\frac{vd}{\nu}$ the factor p—the wetted perimeter thus:

$$p = \pi d \quad \therefore \quad \frac{vp}{\nu} = \frac{\pi vd}{\nu}.$$

Other pipe-line losses are given in the form of a factor F in the formula $h = \mathrm{F}\frac{v^2}{2g}$ for turbulent flow and $h = \mathrm{F}\frac{v}{2g}$ for stream-line flow. These losses in a chemical works pipe are often greater than that due to purely pipe friction.

APPENDIX

Tables of viscosity η with values of kinematic viscosity in sq. cms. per sec. and in sq. feet per sec. derived therefrom.

Dimensions of η is $\frac{M}{LT}$ conversion factor 14·85.

" " ν " $\frac{L^2}{T}$ " " 929·03

TABLE 1

Water

According to Hoskin, and Kaye and Laby.

Temp. ° C.	η c.g.s. Units.	ρ Grams per cubic cm.	$\nu = \frac{\eta}{\rho}$ Sq. cms. per sec.	ν Sq. ft. per sec.
0	0·01793	0·99987	0·01793	0·00001931
5	·01522	·99998	·01522	·00001639
10	·013105	·99973	·01311	·00001411
15	·01142	·99911	·01143	·00001230
20	·011006	·99823	·01008	·00001085
30	·00800	·99567	·00803	·00000865
40	·00657	·99220	·00662	·00000713
50	·00550	·98810	·00557	·00000599
60	·00469	·98320	·00477	·00000514
70	·00406	·97780	·00415	·00000447
80	·00356	·97180	·00366	·00000394
90	·00316	·96530	·00327	·00000352
100	·00284	·95840	·00296	·00000319

TABLE 2

Densities and Viscosities of Glycerol Solutions at 20° C. (68°) according to Archbutt and Deeley and Gerlach.

Per cent. Glycerol.	ρ Density in grams per cubic cm.	η Viscosity in c.g.s. Units.	ν Kinematic viscosity. c.g.s. Units.
5	1·0098	0·01181	0·01170
10	1·0217	·01364	·01335
15	1·0337	·01580	·01529
20	1·0461	·01846	·01765
25	1·0590	·02176	·02055
30	1·0720	·02585	·02411
35	1·0855	·03115	·02870
40	1·0989	·03791	·03450
45	1·1124	·04692	·04218
50	1·1258	·05908	·05248
55	1·1393	·07664	·06727
60	1·1528	·1031	·08943
65	1·1662	·1451	·1244
70	1·1797	·2149	·1822
75	1·1932	·3371	·2825
80	1·2066	·5534	·4586
85	1·2201	1·025	·8401
90	1·2335	2·076	1·683
95	1·2465	4·801	3·852

TABLE 3

Density and Viscosity of Castor Oil according to Kahlbaum and Raber.

Temp. in ° C.	ρ Density in grams per cubic cm.	η Viscosity in c.g.s. Units. dynes/sq cm	ν Kinematic viscosity. c.g.s. Units.
5	0·9707	37·60	38·74
6	·9700	34·475	35·54
7	·9693	31·56	32·56
8	·9686	28·90	29·84
9	·9679	26·45	27·33
10	·9672	24·18	25·00
11	·9665	22·075	22·84
12	·9659	20·075	20·78
13	·9652	18·25	18·91
14	·9645	16·61	17·22
15	·9638	15·14	15·71
16	·9631	13·805	14·33
17	·9624	12·65	13·14
18	·9617	11·625	12·09
19	·9610	10·71	11·15
20	·9603	9·86	10·27
21	·9596	9·06	9·44
22	·9589	8·34	8·70
23	·9583	7·67	8·00
24	·9576	7·06	7·37
25	·9569	6·51	6·80
26	·9562	6·04	6·32
27	·9555	5·61	5·87
28	·9548	5·21	5·46
29	·9541	4·85	5·08
30	·9534	4·51	4·73
31	·9527	4·21	4·42
32	·9520	3·94	4·14
33	·9513	3·65	3·84
34	·9506	3·40	3·58
35	·9499	3·16	3·33
36	·9492	2·94	3·10
37	·9485	2·74	2·89
38	·9478	2·58	2·72
39	·9471	2·44	2·58
40	·9464	2·31	2·44

TABLE 4

Densities and Viscosities of Sulphuric Acid.

Temp. °C.	Strength.	η Viscosity. c.g.s. Units.	ρ Density. c.g.s.	ν Kinematic viscosity. c.g.s.
11·2	96 per cent.	0·31953	1·84	0·17366
20·0	96 ,,	·21929	1·84	·11891
15·0	23·5 ,,	·022144	1·17	·018927
25	23·5 ,,	·017234	1·16	·014832
45	23·5 ,,	·011598	1·149	·010094
15	15·5 ,,	·017156	1·135	·015115
25	15·5 ,,	·013532	1·130	·011975
45	15·5 ,,	·008981	1·113	·0080692
15	7·87 ,,	·01404	1·067	·013159
25	7·87 ,,	·01100	1·063	·010345
45	7·87 ,,	·007523	1·055	·0071337

Temp. °C.	Grams H SO4 per litre.	η c.g.s.	ρ c.g.s.	ν c.g.s.
20	33·7	0·010850	1·02	0·010896
20	59·0	·011228	1·04	·01077
20	114·0	·012354	1·08	·011439
20	228·3	·015353	1·14	·013467
20	458·4	·023685	1·26	·0188
20	748·3	·040686	1·42	·028652
20	922·6	·062068	1·52	·040834
20	1240·4	·144730	1·68	·086149
20	1839·6	·221496	1·84	·12037

TABLE 5

Hydrochloric Acid

Temp. °C.	Strength.	η Viscosity. c.g.s.	ρ Density. c.g.s.	ν Kinematic viscosity. c.g.s.
20	30 per cent.	0·017764	1·155	0·015393
15	23 ,,	·01657	1·12	·014786
25	23 ,,	·01388	1·115	·01245
45	23 ,,	·01017	1·106	·0091954
15	16·1 ,,	·01443	1·08	·013373
25	16·1 ,,	·01201	1·075	·011183
45	16·1 ,,	·00868	1·066	·0081297
15	8·14 ,,	·01281	1·04	·012317
25	8·14 ,,	·01046	1·035	·010126
45	8·14 ,,	·007236	1·025	·0070595
25	Normal	·01290	1·018	

TABLE 6

Acetone

Temp. °C.	η c.g.s.	ρ c.g.s.	ν c.g.s.	
10	0·00415			
15	·004015			
20	·003794			
25	·003762			
30	·003636			
40	·0033745			
50	·0031215			

TABLE 7

Benzol

Temp. °C.	c.g.s.	ρ c.g.s.	ν c.g.s.	
0	0·00902			
10	·00746			
12	·00739			
16·5	·00688			
20	·00645			
30	·00561			
40	·00492			
50	·00433			
60	·00389			
70	·00351			

TABLE 8

Nitric Acid

Temp. °C.	Strength.	η c.g.s.	ρ c.g.s.	ν c.g.s.
0	100 per cent.	0·02273	1·53	0·014856
10	100 ,,	·01770	1·51	·01171
20	100 ,,	·01003	1·50	·00669
15	23·3 ,,	·0145	1·138	·012742
25	23·3 ,,	·01181	1·132	·01042
45	23·3 ,,	·008346	1·12	·00743
15	18·2 ,,	·01255	1·11	·011307
25	18·2 ,,	·010338	1·105	·00936
45	18·2 ,,	·007341	1·095	·0067
15	8·37 ,,	·011984	1·046	
25	8·37 ,,	·009894	1·041	
45	8·37 ,,	·006783	1·036	
25	Normal	·012896	1·035	

TABLE 9

MERCURY. KAYE AND LABY

Temp. ° C.	η c.g.s. Units.	ρ Grams per c.g.s. Unit.	ν c.g.s. Units.
20° C.	·0186	13·6450	0·001362
0	·0169	13·5955	·0012313
20	·0156	13·5462	·0011512
50	·0141	13·4729	·0010465
100	·0122	13·3518	·00091372
200	·0101	13·115	·00077011
300	·0093	12·881	·000722

Printed in Great Britain by Butler & Tanner, *Frome and London*