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Mathematics for Common Schools

A

MANUAL FOR TEACHERS

INCLUDING

DEFINITIONS, PRINCIPLES, AND RULES
AND SOLUTIONS OF THE MORE
DIFFICULT PROBLEMS

BY

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HIGHER ARITHMETIC

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(Higher Arithmetic Manual.)

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MANUAL FOR TEACHERS

I

INTRODUCTORY

Plan and Scope of the Work. — In addition to the subjects generally included in the ordinary text-books in arithmetic, *Mathematics for Common Schools* contains such simple work in algebraic equations and constructive geometry as can be studied to advantage by pupils of the elementary schools.

The arithmetical portion is divided into thirteen chapters, each of which, except the first, contains the work of a term of five months. The following extracts from the table of contents will show the arrangement of topics:

First and Second Years

Chapter I. — Numbers of Three Figures. Addition and Subtraction.

Third Year

Chapters II. and III. — Numbers of Five Figures. Multipliers and Divisors of One Figure. Addition and Subtraction of Halves, of Fourths, of Thirds. Multiplication by Mixed Numbers. Pint, Quart, and Gallon; Ounce and Pound. Roman Notation.

Fourth Year

Chapters IV. and V. — Numbers of Six Figures. Multipliers and Divisors of Two or More Figures. Addition and Subtraction of Easy Fractions. Multiplication by Mixed Numbers. Simple Denominate Numbers. Roman Notation.

Fifth Year

Chapters VI. and VII. — Fractions. Decimals of Three Places. Bills. Denominate Numbers. Simple Measurements.

Sixth Year

Chapters VIII. and IX. — Decimals. Bills. Denominate Numbers. Surfaces and Volumes. Percentage and Interest.

Seventh Year

Chapters XI. and XII. — Percentage and Interest. Commercial and Bank Discount. Cause and Effect. Partnership. Bonds and Stocks. Exchange. Longitude and Time. Surfaces and Volumes.

Eighth Year

Chapters XIII. and XIV. — Partial Payments. Equation of Payments. Annual Interest. Metric System. Evolution and Involution. Surfaces and Volumes.

While all of the above topics are generally included in an eight years' course, it may be considered advisable to omit some of them, and to take up, instead, during the seventh and eighth years, the constructive geometry work of Chapter XVI. Among the topics that may be dropped without injury to the pupil are Bonds and Stocks, Exchange, Partial Payments, and Equation of Payments.

Grammar School Algebra. — Chapter X., consisting of a dozen pages, is devoted to the subject of easy equations of one unknown quantity, as a preliminary to the employment of the equation in so much of the subsequent work in arithmetic as is rendered more simple by this mode of treatment. To teachers desirous of dispensing with rules, sample solutions of type examples, etc., the algebraic method of solving the so-called "problems" in percentage, interest, discount, etc., is strongly recommended.

In Chapter XV., intended chiefly for schools having a nine years' course, the algebraic work is extended to cover simple equations containing two or more unknown quantities, and pure and affected quadratic equations of one unknown quantity.

No attempt has been made in these two chapters to treat algebra as a science; the aim has been to make grammar-school pupils acquainted, to some slight extent, with the great instrument of mathematical investigation, — the equation.

Constructive Geometry. — Progressive teachers will appreciate the importance of supplementing the concrete geometrical instruction now given in the drawing and mensuration work. Chapter XVI. contains a series of problems in construction so arranged as to enable pupils to obtain for themselves a working knowledge of all the most important facts of geometry. Applications of the facts thus ascertained, are made to the mensuration of surfaces and volumes, the calculation of heights and distances, etc. No attempt is made to anticipate the work of the high-school by teaching geometry as a science.

While the construction problems are brought together into a single chapter at the end of the book, it is not intended that instruction in geometry should be delayed until the preceding work is completed. Chapter XVI. should be commenced not later than the seventh year, and should be continued throughout the remainder of the grammar-school course. For the earlier years, suitable exercises in the mensuration of the surfaces of triangles and quadrilaterals, and of the volumes of right parallelopipedons have been incorporated with the arithmetic work.

II

GENERAL HINTS

Division of the Work. — The five chapters constituting Part I. of *Mathematics for Common Schools* should be completed by the end of the fourth school year. The remaining eight arithmetic chapters constitute half-yearly divisions for the second four years of school. Chapter I., with the additional oral work needed in the case of young pupils, will occupy about two years; the remaining four chapters should not take more than half a year each. When the Grube system is used, and the work of the first two years is exclusively oral, it will be possible, by omitting much of the easier portions of the first two chapters, to cover, during the third year, the ground contained in Chapters I., II., and III.

Additions and Omissions. — The teacher should freely supplement the work of the text-book when she finds it necessary to do so; and she should not hesitate to leave a topic that her pupils fully understand, even though they may not have worked all the examples given in connection therewith. A very large number of exercises is necessary for such pupils as can devote a half-year to the study of the matter furnished in each chapter. In the case of pupils of greater maturity, it will be possible to make more rapid progress by passing to the next topic as soon as the previous work is fairly well understood.

Oral and Written Work. — The heading "Slate Problems" is merely a general direction, and it should be disregarded by the teacher when the pupils are able to do the work "mentally." The use of the pencil should be demanded only so far as it may be required. It is a pedagogical mistake to insist that all of the pupils of a class should set down a number of figures that are not needed by the brighter ones. As an occasional exercise, it may be advisable to have scholars give all the work required to solve a problem, and to make a written explanation of each step in the solution; but it should be the teacher's aim to have the majority of the examples done with as great rapidity as is consistent with absolute correctness. It will be found that, as a rule, the quickest workers are the most accurate.

Many of the slate problems can be treated by some classes as "sight" examples, each pupil reading the question for himself from the book, and writing the answer at a given signal without putting down any of the work.

Use of Books. — It is generally recommended that books be placed in pupils' hands as early as the third school year. Since many children are unable at this stage to read with sufficient intelligence to understand the terms of a problem, this work should be done under the teacher's direction, the latter reading the questions while the pupils follow from their books. In later years, the problems should be solved by the pupils from the books with practically no assistance whatever from the teacher.

Conduct of the Recitation. — Many thoughtful educators consider it advisable to divide an arithmetic class into two sections, for some purposes, even where its members are nearly equal in attainments. The members of one division of such a class may work examples from their books while the others write the answers to oral problems given by the teacher, etc.

Where a class is thus taught in two divisions, the members of each should sit in alternate rows, extending from the front of the room to the rear. Seated in this way, a pupil is doing a different kind of work from those on the right and the left, and he would not have the temptation of a neighbor's slate to lead him to compare answers.

As an economy of time, explanations of new subjects might be given to the whole class; but much of the arithmetic work should be done in "sections," one of which is under the immediate direction of the teacher, the other being employed in "seat" work. In the case of pupils of the more advanced classes, "seat" work should consist largely of "problems" solved without assistance. Especial pains have been taken to so grade the problems as to have none beyond the capacity of the average pupil that is willing to try to understand its terms. It is not necessary that all the members of a division should work the same problems at a given time, nor the same number of problems, nor that a new topic should be postponed until all of the previous problems have been solved.

Whenever it is possible, all of the members of the division working under the teacher's immediate direction should take part in all the work done. In mental arithmetic, for instance, while only a few may be called upon for explanations, all of the pupils should write the answers to each question. The same is true of much of the sight work, the approximations, some of the special drills, etc.

Drills and Sight Work. — To secure reasonable rapidity, it is necessary to have regular systematic drills. They should be employed daily, if possible, in the earlier years, but should never last longer than five or ten minutes. Various kinds are suggested, such as sight addition drills, in Arts. 3, 11, 24, 26, etc.; subtraction, in Arts. 19, 50, 53, etc.; multiplication, in Arts. 71, 109, etc.; division, in Arts. 199, 202, etc.; counting by 2's, 3's, etc., in Art. 61; carrying, in Art. 53, etc. For the young pupil, those are the most valuable in which the figures are in his sight, and in the position they occupy in an example; see Arts. 3, 34, 164, etc.

Many teachers prepare cards, each of which contains one of the combinations taught in their respective grades. Showing one of these cards, the teacher requires an immediate answer from a pupil. If his reply is correct, a new card is shown to the next pupil, and so on. Other teachers write a number of combinations on the blackboard, and point to them at random, requiring prompt answers. When drills remain on the board for any considerable time, some children learn to know the results of a combination by its location on the board, so that frequent changes in the arrangement of the drills are, therefore, advisable. The drills in Arts. 111, 112, and 115 furnish a great deal of work with the occasional change of a single figure.

For the higher classes, each chapter contains appropriate drills, which are subsequently used in oral problems. It happens only too frequently that as children go forward in school they lose much of the readiness in oral and written work they possessed in the lower grades, owing to the neglect of their teachers to continue to require quick, accurate review work in the operations previously taught. These special drills follow the plan of the combinations of the earlier chapters, but gradually grow more difficult. They should first be used as sight exercises, either from the books or from the blackboard.

To secure valuable results from drill exercises, the utmost possible promptness in answers should be insisted upon.

Definitions, Principles, and Rules. — Young children should not memorize rules or definitions. They should learn to add by adding, after being first shown by the teacher how to perform the operation. Those not previously taught by the Grube method should be given no reason for "carrying." In teaching such children to write numbers of two or three figures, there is nothing gained by discussing the local value of the digits. During the earlier years, instruction in the art of arithmetic should be given with the least possible amount of science. While principles may be incidentally brought to the view of the children at times, there should be no cross-examination thereon. It may be shown, for instance, that subtraction is the reverse of addition, and that multiplication is a short method of combining equal

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numbers, etc.; but care should be taken in the case of pupils below about the fifth school year not to dwell long on this side of the instruction. By that time, pupils should be able to add, subtract, multiply, and divide whole numbers; to add and subtract simple mixed numbers, and to use a mixed number as a multiplier or a multiplicand; to solve easy problems, with small numbers, involving the foregoing operations and others containing the more commonly used denominate units. Whether or not they can explain the principles underlying the operations is of next to no importance, if they can do the work with reasonable accuracy and rapidity.

When decimal fractions are taken up, the principles of Arabic notation should be developed; and about the same time, or somewhat later, the principles upon which are founded the operations in the fundamental processes, can be briefly discussed.

Definitions should in all cases be made by the pupils, their mistakes being brought out by the teacher through appropriate questions, criticisms, etc. Systematic work under this head should be deferred until at least the seventh year.

The use of unnecessary rules in the higher grades is to be deprecated. When, for instance, a pupil understands that *per cent* means *hundredths*, that seven per cent means seven hundredths, it should not be necessary to tell him that 7 per cent of 143 is obtained by multiplying 143 by .07. It should be a fair assumption that his previous work in the multiplication of common and of decimal fractions has enabled him to see that 7 per cent of 143 is $\frac{7}{100}$ of 143 or $143 \times .07$, without information other than the meaning of the term "per cent."

When a pupil is able to calculate that 15 % of 120 is 18, he should be allowed to try to work out for himself, without a rule, the solution of this problem: 18 is what per cent of 120? or of this: 18 is 15 % of what number? These questions should present no more difficulty in the seventh year than the following examples in the fifth: (*a*) Find the cost of $\frac{3}{20}$ ton of hay at $12 per ton. (*b*) When hay is worth $12 per ton, what part of a ton can be bought for \$1.80? (*c*) If $\frac{3}{20}$ ton of hay costs \$1.80, what is the value of a ton?

When, however, it becomes necessary to assist pupils in the solution of problems of this class, it is more profitable to furnish them with a general method by the use of the equation, than with any special plan suited only to the type under immediate discussion.

In the supplement to the Manual will be found the usual definitions, principles, and rules, for the teacher to use in such a way as her experience shows to be best for her pupils. The rules given are based somewhat on the older methods, rather than on those recommended by the author. He would prefer to omit entirely those relating to percentage, interest, and the like as being unnecessary, but that they are called for by many successful teachers, who prefer to continue the use of methods which they have found to produce satisfactory results.

Language. — While the use of correct language should be insisted upon in all lessons, children should not be required in arithmetic to give all answers in "complete sentences." Especially in the drills, it is important that the results be expressed in the fewest possible words.

Analyses. — Sparing use of analyses is recommended for beginners. If a pupil solves a problem correctly, the natural inference should be that his method is correct, even if he be unable to state it in words. When a pupil gives the analysis of a problem, he should be permitted to express himself in his own way. Set forms should not be used under any circumstances.

Objective Illustrations. — The chief reason for the use of objects in the study of arithmetic is to enable pupils to work without them. While counters, weights and measures, diagrams, or the like are necessary at the beginning of some topics, it is important to discontinue their use as soon as the scholar is able to proceed without their aid.

Approximate Answers. — An important drill is furnished in the "approximations." (See Arts. 521, 669, 719, etc.) Pupils should be required in much of their written work to estimate the result before beginning to solve a problem with the pencil. Besides preventing an absurd answer, this practice will also have the effect of causing a pupil to see what processes are necessary. In too many instances, work is commenced upon a problem before the conditions are grasped by the youthful scholar; which will be less likely to occur in the case of one who has carefully "estimated" the answer. The pupil will frequently find, also, that he can obtain the correct result without using his pencil at all.

Indicating Operations. — It is a good practice to require pupils to indicate by signs all of the processes necessary to the solution of a problem, before performing any of the operations. This frequently enables a scholar to shorten his work by cancellation, etc. In the case of problems whose solution requires tedious processes, some teachers do not require their pupils to do more than to indicate the operations. It is to be feared that much of the lack of facility in adding, multiplying, etc., found in the pupils of the higher classes is due to this desire to make work pleasant. Instead of becoming more expert in the fundamental operations, scholars in their eighth year frequently add, subtract, multiply, and divide more slowly and less accurately than in their fourth year of school.

Paper *vs.* Slates. — To the use of slates may be traced very much of the poor work now done in arithmetic. A child that finds the sum of two or more numbers by drawing on his slate the number of strokes represented by each, and then counting the total, will have to adopt some other method if his work is done on material that does not permit the easy obliteration of the tell-tale marks. When the teacher has an opportunity to see the number of attempts made by some of her pupils to obtain the correct quotient figures in a long division example, she may realize the importance of such drills as will enable them to arrive more readily at the correct result.

The unnecessary work now done by many pupils will be very much lessened if they find themselves compelled to dispense with the "rubbing out" they have an opportunity to indulge in when slates are employed. The additional expense caused by the introduction of paper will almost inevitably lead to better results in arithmetic. The arrangement of the work will be looked after; pupils will not be required, nor will they be permitted, to waste material in writing out the operations that can be performed mentally; the least common denominator will be determined by inspection; problems will be shortened by the greater use of cancellation, etc., etc. Better writing of figures and neater arrangement of problems will be likely to accompany the use of material that will be kept by the teacher for the inspection of the school authorities. The endless writing of tables and the long, tedious examples now given to keep troublesome pupils from bothering a teacher that wishes to write up her records, will, to some extent, be discontinued when slates are no longer used.

XIII

NOTES ON CHAPTER TEN

The formal study of algebra belongs to the high-school; but some so-called arithmetical problems are so much simplified by the use of the equation that it is a mistake for a teacher not to avail herself of this means of lightening her pupils' burdens.

In beginning this part of her mathematical instruction, the teacher should not bewilder her scholars with definitions. The necessary terms should be employed as occasion requires, and without any explanation beyond that which is absolutely necessary.

849. Very young pupils can give answers to most of these questions; so that there will be no need, for the present, at least, of introducing a number of axioms to enable the scholar to obtain a result that he can reach without them.

850. Pupils will learn how to work these problems by working a number of them. They may need to be told that x stands for $1x$; and that, as a rule, only abstract numbers are used in the equations, the denomination — dollars, marbles, etc. — being supplied afterwards.

While the scholars should be required to furnish rather full solutions of the earlier problems, they should be permitted to shorten the work by degrees, writing only whatever may be necessary.

4. $x + 2x = 54.$

5. $x + 5x = 78.$

6. $7x + 5x = 156.$

7. $9x - 3x = 66.$

8. $x + 2x + 6x = 27000.$

9. $x + 5x = 72.$

10. $x + 2x + 3x = 54.$

11. $x + 6x = 42.$

12. $2x + 10x = 96$.

13. Let x = the fourth; then $4x$ = the third, $12x$ = the second, and $24x$ = the first.

$$x + 4x + 12x + 24x = 41.$$

14. x = the second, $2x$ = first, $9x$ = third.

15. $5x + 4x = 81$.

16. $24x = 456$.

17. $4x = 340$.

19. $3x + 4x = 175$.

20. Let x = each boy's share; $2x$ = each girl's share.

$$2x + 4x = 240.$$

21. x = number of days son worked; $2x$ = number father worked. $3x$ = son's earnings; $8x$ = father's earnings.

$$3x + 8x = 165.$$

22. x = number of dimes; $2x$ = number of nickels; $6x$ = number of cents.

$$(10 \times x) + (5 \times 2x) + (1 \times 6x) = 78,$$

or

$$10x + 10x + 6x = 78.$$

23. $15x - 12x = 75$.

24. $x + 4x + x + 4x = 250$.

25. Let x = cost of speller; then $3x$ = cost of reader.

26. Let x = smaller; then $5x$ = larger.

27. Let x = Susan's number; $2x$ = Mary's; $3x$ = Jane's.

851. 10 : $\frac{1}{3}x$ is the same as $\frac{x}{3}$.

852. Pupils already know that $\frac{3}{4}$ means $3 \div 4$, so that they can understand that $\frac{3x}{4}$ means $3x \div 4$, or $\frac{1}{4}$ of $3x$. When $\frac{1}{4}$ of something ($3x$) is 24, the whole thing ($3x$) must be 4 times 24, or 96; that is, when $\frac{3x}{4} = 24$, $3x = 96$.

When $\frac{2y}{3} = 24$, $2y = 24 \times 3$, or 72.

When $\frac{4z}{5} = 20$, $4z = 20 \times 5$, or 100.

From these examples can be formulated the rule for disposing of a fraction in one term of an equation, which is, to multiply both terms by the denominator of the fraction. In changing the first term of the equation, $\frac{3x}{4}=24$, to $3x$, it has been multiplied by 4, so that the second term must also be multiplied by 4.

853. In solving these examples by the algebraic method of "clearing of fractions," attention may be called to its similarity to the arithmetical method. To find the value of y in **2**, the pupil multiplies 8 by 5 and divides the product by 2; as an example in arithmetic, he would divide 8 by $\frac{2}{5}$, that is, he would multiply 8 by $\frac{5}{2}$; the only difference being that by the latter method he would cancel.

While $\frac{2y}{5}=8$ may be changed to $\frac{y}{5}=4$ by dividing both terms by 2, beginners are usually advised to begin by "clearing of fractions," short methods being deferred to a later stage.

854. **6** may be written $\frac{3x}{5}+\frac{5x}{7}=92$.

8. $2\frac{7}{8}x$ should be reduced to an improper fraction, making the equation, $\frac{23x}{8}=115$. Make similar changes in **12**, **14**, **18**, and **20**.

855. **2.** $x+\frac{5x}{2}=100$.

5. $\frac{x}{2}+\frac{x}{4}=\frac{267}{4}$; $2x+x=267$.

6. $\frac{3x}{4}-\frac{3x}{5}=15$.

9. Let $5x=$ numerator; $7x=$ denominator. $7x-5x=24$; $2x=24$; $x=12$. The numerator, $5x$, will be 5 times 12, or 60; the denominator will be 84; and the fraction, $\frac{60}{84}$. *Ans.*

10. Let $x=$ greater; $\frac{x}{7}=$ less.

$$x+\frac{x}{7}=480.$$

Clearing of fractions, $7x + x = 3360,$

$$8x = 3360,$$

$$x = 420, \text{ the greater number,}$$

$$\frac{x}{7} = 60, \text{ the less.}$$

Or, let $x =$ less; $7x =$ greater.

$$x + 7x = 480,$$

$$8x = 480,$$

$$x = 60, \text{ the less,}$$

$$7x = 420, \text{ the greater.}$$

The employment of the latter plan does away with fractions in the original equation.

11. $30x - x = 522$, or $x - \dfrac{x}{30} = 522.$

13. Let $x =$ number of plums; $4x =$ number of peaches. Then $2x$ will be cost of plums, and $12x$ the cost of the peaches.

$$2x + 12x = 70.$$

15. $$x - \frac{3x}{7} = 80.$$

17. $$x - \frac{3x}{8} - \frac{x}{4} = 24.$$

18. $$x + 1\tfrac{1}{2}x + (1\tfrac{1}{2}x \times 3\tfrac{1}{3}) = 15.$$

$$x + \frac{3x}{2} + 5x = 15.$$

19. Let $x =$ price per yard of the 48-yard piece; $2x =$ price per yard of the 36-yard piece; $48x$ will be the total cost of one, and $72x$, of the other.

$$48x + 72x = 240.$$

20. $160x + 120x = 840.$

856. The pupils should be permitted to give these answers without assistance.

In Art. 857 is explained what is meant by "transposing."

858. While these exercises are so simple that they can be worked without a pencil, they should be used to show the steps generally taken in more complicated equations. In **1**, for instance, the work should take the form here indicated, only a single step being taken at a time. In **19**, the first step is to clear the equation of fractions by multiplying by 6; the second step is to transpose the unknown quantities to the left side of the equation, and the known quantities to the right; the third step is to combine the unknown quantities into one, and to make a similar combination of the known quantities; the last step is to find the value of x.

$$\begin{aligned} x + 37 &= 56 \\ x &= 56 - 37 \\ x &= 19 \end{aligned}$$

$$\begin{aligned} 2x - \ \ 6 &= 16 + \frac{x}{2} - \frac{x}{3} \\ 12x - 36 &= 96 + 3x - 2x \\ 12x - 3x + 2x &= 96 + 36 \\ 11x &= 132 \\ x &= 12 \end{aligned}$$

After a little more familiarity with exercises of this kind, the pupil can take short cuts with less danger of mistakes; for the present, however, it will be safer to proceed in the slower way.

859. **5.** $x + (x + 75) + x + (x + 75) = 250.$

$$x + x + x + x = 250 - 75 - 75.$$

NOTE.—The parentheses used here are unnecessary. They are employed merely to show that $x + 75$ is one side of the field.

6. $x + (x + 8) = 86.$

7. $x + x + 318 = 2436.$

9. $x + x + 72 = 96.$

10. $x - \frac{x}{3} - \frac{x}{4} = 45.$

8. $x + \frac{x}{2} + 7 = 100.$

11. $x =$ one part; $2x - 6 =$ other part.

$$x + 2x - 6 = 45.$$

12. $x =$ John's money; $x + 5 =$ William's money.

$$3x + 15 + 5x = 103.$$

13. Let $x =$ price of a horse; $x - 80 =$ price of a cow;
$4x =$ cost of four horses; $3x - 240 =$ cost of three cows.

$$4x + 3x - 240 = 635,$$
$$7x = 635 + 240 = 875,$$
$$x = 125, \text{ price, in dollars, of a horse;}$$
$$x - 80 = 45, \text{ price, in dollars, of a cow.}$$

Other pupils may solve the problems in this way:

$$x = \text{price of a cow; } x + 80 = \text{price of a horse.}$$
$$3x + 4x + 320 = 635,$$
$$7x = 635 - 320 = 315,$$
$$x = 45, \text{ price, in dollars, of a cow;}$$
$$x + 80 = 125, \text{ price, in dollars, of a horse.}$$

14. $x =$ number of dimes; $x + 11 =$ number of five-cent pieces; $10x =$ value of dimes (in cents); $5x + 55 =$ value of five-cent pieces.

$$10x + 5x + 55 = 100.$$

15. $x =$ greater; $x - 48 =$ less.

$$x + x - 48 = 100.$$

Or, $x =$ less; $x + 48 =$ greater.

$$x + x + 48 = 100.$$

17. $x =$ share of the first;

$x + 2400 =$ share of the second;

$x + 2400 + 2400 =$ share of the third.

$x + x + 2400 + x + 2400 + 2400 = 18000.$

18. Let $x =$ less; $x + 33 =$ greater.

$$x + 33 - 3x = 11.$$

Bringing known quantities to the left side of the equation, and the unknown quantities to the right,

$$33 - 11 = 3x - x,$$
$$22 = 2x,$$
$$11 = x.$$

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Or,
$$x - 3x = 11 - 33,$$
$$-2x = -22.$$

Changing signs of both terms,
$$2x = 22,$$
$$x = 11.$$

This problem may also be worked in this way:
$$x = \text{less}; \; 3x + 11 = \text{greater}.$$
$$3x + 11 - x = 33.$$

19. $x =$ number of 5-cent stamps; $x + 15 =$ number of 2-cent stamps; $x + 30 =$ number of postal cards.
$$5x + 2x + 30 + x + 30 = 100.$$

20. $x =$ number of horses; $x + 17 =$ number of cows; $2x + 39 =$ number of sheep.
$$x + x + 17 + 2x + 39 = 88.$$

XIV

NOTES ON CHAPTER ELEVEN

With this chapter begins the regular work in percentage, and it is important that the pupils obtain, as soon as possible, a correct idea of what is meant by the term *per cent.*

Many of the various subdivisions of this topic found in some books, are taken up only incidentally, while others are omitted altogether, the aim being to give the pupils a foundation upon which they can subsequently build, rather than to scatter their energies over too great a diversity of subjects.

860. The reduction of a common fraction to a per cent, consists in changing the former to a decimal of two places. In reducing $\frac{1}{2}$ to a decimal, the result is .5, or 5 tenths; in changing $\frac{1}{2}$ to an equivalent per cent, the result is 50 per cent, or 50 hundredths. In reducing $\frac{1}{8}$ to a decimal, the answer is given in three places, .125, or 125 thousandths; in changing it to a per cent, the division is stopped at the second place, and the remainder written as a fraction, $12\frac{1}{2}$ per cent, or $12\frac{1}{2}$ hundredths.

The denominator of a per cent being always the same, 100, the comparative value of several per cents is known at sight. To compare $\frac{5}{8}$ and $\frac{3}{5}$ as common fractions, they must be changed to $\frac{25}{40}$ and $\frac{24}{40}$; if a further comparison is to be made between these and $\frac{7}{12}$, a new common denominator must be employed, and the fractions reduced to $\frac{75}{120}$, $\frac{72}{120}$, and $\frac{70}{120}$. Changing the fractions to decimals, 625 thousandths, 6 tenths, and $58\frac{1}{3}$ hundredths, simplifies the comparison; but it is still easier to determine their relative value when they are expressed as $62\frac{1}{2}$ per cent, 60 per cent, and $58\frac{1}{3}$ per cent.

The teacher must not be discouraged if the pupil fails to grasp at once the full meaning of percentage. Definitions will not help materially; much practice in working examples is necessary to give the knowledge desired.

863. Many children find it difficult to distinguish between $\frac{1}{2}\%$ and 50%. If the former is read in the business way, $\frac{1}{2}$ of one per cent, it may make the distinction plainer.

864. Per cents being generally given in two figures, scholars hesitate to give the correct answers: 300%, 250%, 125%, $1633\frac{1}{3}\%$, 420%, 910%.

865. While pupils will find $33\frac{1}{3}\%$ of 81 cows, by dividing 81 by 3, they should understand that they are really multiplying 81 by $33\frac{1}{3}$ hundredths, or 81 by $\frac{1}{3}$. In **4**, 6% of 150, or $\frac{6}{100}$ of 150, may be obtained by multiplying 150 by 6 and cutting off two ciphers; or by dividing 150 by 100, obtaining $1\frac{1}{2}$, and multiplying this quotient by 6; or by reducing 6% to $\frac{3}{50}$, and finding $\frac{3}{50}$ of 150. In **9**, the pupil should find 1% of \$640 and take one-half of the result.

The scholars should be permitted to use their own method of solving these problems, the different analyses given by the pupils furnishing their class-mates an opportunity to select a simpler method.

866. Although every pupil may not be able to determine at once the shortest way of calculating a given example, no one should be allowed to work **3**, by multiplying by $33\frac{1}{3}$. When the multiplication by $\frac{1}{3}$ has been performed, the answer has been obtained, except as to the location of the decimal point, and the waste of time in multiplying by 3, repeating this product, and adding three columns should not be tolerated. No fault should be found with the average pupil for failing to recognize in **1**, that $6\frac{2}{3}\%$ is $\frac{1}{15}$; or that in **12**, $3\frac{1}{3}\%$ is $\frac{1}{30}$. The general method should be to multiply by the figures given to represent the per cent, except in such cases as $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, 25%, $33\frac{1}{3}\%$, $37\frac{1}{2}\%$, 50%, and possibly a few others.

$$\begin{array}{r} \$1240 \\ \times .00\frac{1}{8} \\ \hline \$1.55 \end{array}$$

Where the given numbers are used, they should be made the multipliers and expressed as hundredths. Nothing is gained in **5**, by reducing the $\frac{1}{2}$ to a decimal; although in **13**, writing $5\frac{1}{2}\%$, .055, might make it easier for some.

$$\begin{array}{r} \$92.40 \\ \times .04\frac{1}{2} \\ \hline \end{array}$$

868. The rule generally given of finding the percentage, by multiplying by the rate expressed as hundredths, is here modified to the extent of using the common fraction to express hundredths, instead of the decimal, as being more in conformity with early algebraic methods.

The teacher that prefers to ascertain the base or the rate by the older arithmetical method, will omit **30–41**.

30. $\frac{x}{100} \times 65 = \frac{13x}{20}$. *Ans.*

31. $\frac{13x}{20} = 26$; etc.

32. $\frac{1}{4}$ of $x = \frac{x}{4}$. *Ans.*

33. $\frac{x}{4} = 42$; etc.

34. $x + \frac{x}{5}$. *Ans.*

35. $x + \frac{x}{5} = 132$; etc.

37. $x - \frac{x}{3} = 78$; etc.

38. $\frac{x}{100}$ of $\frac{2}{3} = \frac{x}{150}$. *Ans.*

39. $\frac{x}{150} = \frac{10}{3}$; etc.

40. $\frac{1}{800}$ of $x = \frac{x}{800}$. *Ans.*

41. $\frac{x}{800} = 23$; etc.

42. Let $x =$ rate. Then $\frac{x}{100}$ of $65 = 26$, or $\frac{x}{100} \times 65 = 26$.

In an equation containing quantities to be multiplied, the multiplication should be performed before the equation is cleared of fractions. This equation becomes $\frac{65x}{100} = 26$, or $\frac{13x}{20} = 26$, it being immaterial whether canceling be done or not.

43. After a little experience with this class of examples, the equation may be written at once, in the order in which the terms are given:

$$24 = \frac{18}{100} \text{ of } x, \text{ or } \frac{18x}{100} = 24.$$

44. $\frac{250}{100}$ of $x = 180$, or $\frac{5x}{2} = 180.$

45. $x + \frac{x}{4} = 85.$

46. $\frac{3}{5} = \frac{x}{100}$ of $\frac{4}{5}$, or $\frac{x}{125} = \frac{3}{5}.$

While the algebraic method is of no advantage to the bright scholar, it makes the employment of rules unnecessary in the case of the ordinary pupil.

48. $x \times \frac{5\frac{1}{2}}{100} \times 1 = \frac{11x}{200}.$

49. $\frac{11x}{200} = 44.$

50. $\frac{3}{5} \times \frac{2}{3}.$

51. $\frac{88x}{100} = 33.$

52. $\frac{1}{8}$ of $800 = 100$; $\frac{1}{8}\%$ of $800 = 1.$

53. $\$175 + \frac{1}{4}$ of $\$175.$

54. $2\frac{1}{2}\%$ of $x = 12.50$; that is, $\frac{x}{40} = 12.50.$

55. $6\frac{1}{4} \times .16.$

56. $3\frac{1}{3} = \frac{x}{100}$ of $\frac{2}{3}$; that is, $\frac{x}{150} = \frac{10}{3}.$

57. $\frac{x}{100}$ of $389.50 = 124.64$; $\frac{389.50x}{100} = 124.64$;

$389.5x = 12464$; $3895x = 124640.$

58. $\frac{95x}{100} = 174.04$; $95x = 17404.$

59. $x + \frac{16x}{100} = 1276.$

60. $984 = \frac{133\frac{1}{3}}{100}$ of x; that is, $\frac{4x}{3} = 984$.

62. $\frac{x}{4} = 386.75$.

65. $x =$ cost of oats; $x + \frac{2x}{100} = 1071$.

Divide the cost of the oats by 30¢ to find the number of bushels.

68. Assessed value $= \frac{2}{3}$ of \$48000 = \$32000. Taxes on 32 thousand dollars = \$18.50 × 32.

869. In giving answers to these and to all other exercises, no "guessing" should be allowed. The pupil should be permitted to obtain the correct result in his own way — that is, no inflexible rule should be given him to follow — but he should be able to get the answer, using the algebraic method if that seems to him the easiest, as it may be in some instances.

The examples are not arranged by "cases," so that each will have to be understood before it can be worked.

The careless pupil will probably give the wrong answer to **13**; saying 6, instead of 600; he will be likely too, in **14**, to use the larger number as a divisor, and to obtain $44\frac{4}{9}\%$ instead of 225%. These mistakes are less likely to occur if he uses equations — $3 = \frac{x}{200}$ and $\frac{9x}{100} = 20\frac{1}{4}$. Even those scholars that have solved in their arithmetic work of the lower grades, examples similar to **15**, will have new light thrown on their method by using the equation, $x + \frac{x}{4} = 20$. In mental work, however, the first term should be made $\frac{5x}{4}$, to reduce it in size, so that it can be more easily remembered. **24** is simplified by changing the fractions to whole numbers — $\frac{9}{12}$ is what per cent of $\frac{10}{12}$, 9 is what per cent of 10 — before beginning to calculate the rate. In **25**, $1\frac{1}{2}$ and $6\frac{2}{3}$ become $\frac{3}{2}$ and $\frac{20}{3}$, $\frac{9}{6}$ and $\frac{40}{6}$, 9 and 40.

870. **1–5** can be worked by the pupils without any explanation; **6–20** present more difficulty. The beginner in algebra desires to start at once with his x, without any preliminary calculations; and the usual method of treating these examples requires him first to ascertain the gain or the loss before commencing his equation. The formula employed in the first five examples is:

$$\text{Cost} \times \frac{\text{rate}}{100} = \text{gain or loss.}$$

When the pupil knows any two of these three terms, he can calculate the third; and **6–15** furnish data from which the necessary two items can be obtained. The pupil must, however, be careful in **11**, for instance, not to *subtract* the loss from the selling price to obtain the cost.

In the following equations, *cost* $\times \frac{x}{100}$ is made equal to the *gain* or the *loss*. No canceling has been done.

6. $\frac{600\,x}{100} = 18;\ 6\,x = 18.$

7. $\frac{1203\,x}{100} = 401.$

8. $\frac{86.20\,x}{100} = 12.93$, or $\frac{8620\,x}{100} = 1293.$

9. $\frac{908.40\,x}{100} = 181.68.$

10. $\frac{84\,x}{100} = 5.25;\ 84\,x = 525.$

11. $\frac{84\,x}{100} = 5.25.$

12. $\frac{125\,x}{100} = 25.$

13. $\frac{875\,x}{100} = 43.75;\ 875\,x = 4375.$

14. $\frac{934.56\,x}{100} = 116.82.$

15. $\frac{1012.50\,x}{100} = 168.75.$

In **16–20**, the cost is represented by x.

16. $x + \frac{x}{4} = 468.75.$

18. $x + \frac{x}{3} = 1646.08.$

17. $x - \frac{x}{5} = 73.84.$

19. $x - \frac{15\,x}{100} = 204.$

20. $x - \frac{4x}{100} = 66.30.$

21. Gain $= \frac{1}{5}$ of \$275.

22. $x\%$ of $60 = 15.$

23. $x + \frac{x}{5} = 960.$

24. $x\%$ of $32 = 16.$

25. $x\%$ of $175 = 25.$

26. $x\%$ of $200 = 25.$

27. $33\frac{1}{3}$¢ $+ \frac{1}{8}$ of $33\frac{1}{3}$¢.

28. $x - \frac{16x}{100} = 33.60.$

29. Gain $= 2\frac{1}{2}\%$ of \$8760.

30. $x - \frac{x}{10} - \frac{x}{4} - \frac{3x}{10} = 70.$

31. $6000 = x\%$ of 16000.

32. $6000 = x\%$ of 24000.

33. $1600 + 2\frac{7}{8}\%$ of 1600.

34. $4200 - 3\frac{1}{2}\%$ of 4200.

871. In **1**, the 30 cu. yd. are reduced to cubic feet by multiplying by 27. Instead of performing the different multiplications, they are merely indicated, so that work may be saved by canceling.

Although **2** should be a simple problem for a bright pupil, it is apt to prove puzzling unless an x is introduced. A pasteboard box may be used to represent the walls and the ceiling of a room, the sides and the top being then opened out to permit of its representation on the blackboard.

3. The area in square feet $= \frac{1}{2}$ of 132×110. This is reduced to acres by dividing by $9 \times 30\frac{1}{4} \times 160$.

$$\frac{132 \times 110 \times 4}{2 \times 9 \times 121 \times 160} = Ans.$$

4. Number of strips = 6 yd. ÷ 27 in. = 6 yd. ÷ $\frac{3}{4}$ yd. $= 6 \times \frac{4}{3}$.

7. The "development" of the fence will be represented by four adjoining rectangles, each marked 6 ft. high, the lengths being 25 ft., 100 ft., 25 ft., and 100 ft., respectively, the whole forming a rectangle 6 ft. × 250 ft.

8. A board's area in square feet $= 12 \times \frac{1}{2} = 6$. Dividing number of square feet in the fence by 6, gives the number of boards.

9. The cost of a square foot is obtained by dividing \$181.50 by $(160 \times 30\frac{1}{4} \times 9)$; this, multiplied by (300×200), gives the cost of the plot.

$$\frac{\$181.50 \times 4 \times 300 \times 200}{160 \times 121 \times 9}.$$

The amount received for the lots will be $\$160 \times 6$.

10. Number of cakes $= (320 \times 160) \div (4 \times 2)$.

11. Number of cubic feet $= 320 \times 160 \times 1\frac{1}{2}$.

12. $(320 \times 160 \times 1\frac{1}{2}) \div (15 \times 32)$.

13. Number of square feet originally $= 640 \times 440$. For building purposes, there will be four pieces, each measuring 300 ft. by 200 ft.

14. The difference between the above areas will represent the number of square feet in the streets.

872. Many of these exercises can be used for mental and sight work. For methods of solution, see Art. 870.

873. As a preliminary to the formal study of interest, the teacher will need to see that her pupils understand what is meant by the term. She can explain that a person borrowing money should pay for its use, just as a person who rents a house, etc.

874. In changing 4 mo. 10 da. to the fraction of a year, many teachers prefer to reduce the time to days and to write the result over 360, $\frac{130}{360}$, leaving the reduction to lowest terms for the subsequent cancellation. In the same way, 1 yr. 5 mo. 15 da. is changed to $(360 + 150 + 15)$ da., or 525 da. $= \frac{525}{360}$ yr. The reduction to days is done very rapidly.

875. 1. $\$750 \times \frac{6}{100} \times \frac{5}{2}$.

2. $\$84.75 \times \frac{4}{100} \times \frac{16}{60}$.

3. $\$308.25 \times \frac{5}{100} \times \frac{20}{360}$.

4. $\$464.75 \times \frac{6}{100} \times \frac{252}{360}$.

5. $\$360 \times \frac{5}{100} \times \frac{33}{360}$.

6. $\$94.43 \times \frac{7}{100} \times \frac{63}{360}$.

7. $\$400 \times \frac{9}{200} \times \frac{391}{360}$.

etc., etc.

877. The teacher should explain that a person that owes money, frequently gives a note as an acknowledgment of the debt, etc.

878. There is no general method applicable to these problems.

1. Interest for a year is \$12, or \$1 per month, which gives \$19 for 1 yr. 7 mo.

2. \$3.60 per year is 1¢ per day, 33¢ for 33 da.

3. \$6 per year, or \$9 for $1\frac{1}{2}$ yr.

4. \$6 per year, or \$15 in 2 yr. 6 mo.

5. If \$50 produces \$6 in 2 yr., it will produce \$3 in 1 yr.; rate, therefore, is 6%.

6. \$18 per year is \$1 for 20 da., or $\frac{1}{18}$ yr.

8. 4% per year = 1% for 90 da.; 1% of \$150 = \$1.50.

9. 5% per year = $\frac{1}{2}$% for 36 da.; $\frac{1}{2}$% of \$240 = $\frac{1}{2}$ of \$2.40.

11. \$1 is 100% of \$1; at 5% per year it will take 20 yr. to make 100%.

12. At 6% it will take $16\frac{2}{3}$ yr., or 16 yr. 8 mo., to make 100%.

13. Disregarding \$14.90, it will take 25 yr. at 4% to make 100%.

14. $\frac{1}{2}$% per month = 8% for 16 mo.; 8% of \$90 = \$7.20.

15. 5% for 360 da. = 1% for 72 da.

16. 360 da. ÷ $4\frac{1}{2}$ = 720 da. ÷ 9 = 80 da.

17. 5% for 1 yr. = 1% for 72 da.; 1% of \$75 = 75 cents.

18. 1% of \$63.	20. $\frac{1}{2}$% of \$840.	22. 1% of \$275.
19. 1% of \$570.	21. 1% of \$150.	23. 2% of \$360.

879. 1. 30 rd. 5 yd. 1 ft. = 511 ft.; 8 rd. 4 yd. 2 ft. = 146 ft.; $\frac{146}{511} = \frac{2}{7}$. *Ans.*

2. Number of feet deep = $(36 \times 5) \div (6 \times 4)$.

3. 3 mi. 96 rd. = 1056 rd.; 3 hr. 16 min. = $3\frac{4}{15}$ hr.; 1056 rd. $\times 3\frac{4}{15} = 1056$ rd. $\times \frac{49}{15} = \frac{17248}{5}$ rd. $= 3449\frac{3}{5}$ rd. = 10 mi. $249\frac{3}{5}$ rd. *Ans.*

4. $(\frac{3}{4}+\frac{7}{8}) \times (\frac{9}{2} \times \frac{8}{27}) \div (\frac{3}{7} \times \frac{84}{5}) = \frac{13}{8} \times \frac{9}{2} \times \frac{8}{27} \times \frac{7}{3} \times \frac{5}{84} = \frac{65}{216}$. *Ans.*

8. The first two figures express 1800; the second two, 5.4.

22. $48.37 ÷ $8\frac{1}{2}$.

880. 2. Provisions that will supply 450 men for 5 months will supply 5 times 450 men for 1 month, and will supply (5 times 450 men) ÷ 9 for 9 months, or 250 men. The number that must be discharged = 450 men − 250 men = 200 men. *Ans.*

15. $x - \frac{15x}{100} - \frac{20x}{100} = 19500.$

16. D bought $\frac{7}{8} \times \frac{4}{5} \times \frac{5}{9} \times \frac{6}{7}$ of the ship.

18. $100x + 50x = 340 \times 75.$

19. x = number distributed by each new man; $2x$ = number distributed by each experienced man.

$$16x + 32x = 36000.$$

20. $x - \frac{x}{4} = 1972.65.$

881. 10. 100 cents ÷ 1.13.

883. 3. $1.10 + 15% of $1.10.

4. $\frac{9876}{x} = 87 + \frac{45}{x}.$

$$9876 = 87x + 45.$$

5. 640 is what per cent of (640 + 560), etc.

6. 43 gal. 3 qt. 1 pt. = $43\frac{7}{8}$ gal.; $70.20 ÷ $43\frac{7}{8}$ = *Ans.*

7. $(48 \times 32) \div (16 \times \frac{1}{2}).$

8. 20 is what per cent of 160? 20 is what per cent of 180?

10. Selling price per bbl. $= \frac{3450}{600} = \frac{23}{4}$; $4\frac{1}{6}\% = \frac{25}{600} = \frac{1}{24}.$

Let x = cost per bbl. $x - \frac{x}{24} = \frac{23}{4}.$

11. $\frac{9.075x}{100} = 24.2.$

884. $425 + 99$ is 1 less than $425 + 100$; $425 + 999$ is 1 less than $425 + 1000$.

885. $565 - 99$ is 1 more than $565 - 100$; $1424 - 999$ is 1 more than $1424 - 1000$.

886. $24 \times 21 = (24 \times 20) + (24 \times 1)$. See Art. 786.

887. $16 \div .25 = 16 \div \frac{1}{4} = 16 \times 4$; $36 \div .75 = 36 \div \frac{3}{4} = 36 \times \frac{4}{3} = 12 \times 4$.

888. $7\frac{1}{2} \div \frac{3}{4} = \frac{15}{2} \div \frac{3}{4} = \frac{30}{4} \div \frac{3}{4} = 30 \div 3$. When the dividend is a whole number, it is frequently better to perform the division in the ordinary way: $63 \div 3\frac{1}{2} = 63 \div \frac{7}{2} = 63 \times \frac{2}{7} = 9 \times 2$.

889. **1.** $\$\frac{7}{8} \times 48$. **2.** $(48 \times \frac{3}{4})$ sq. yd. **3.** $(48 \div \frac{3}{4})$ yd. **7.** 9 into $83\frac{1}{4}$, 9 times and $2\frac{1}{4}$, or $\frac{9}{4}$, remaining; 9 into 9 quarters, $\frac{1}{4}$. *Ans.* $9\frac{1}{4}$. **10.** $\$1\frac{1}{4} \times 19$. **11.** $\$1\frac{7}{8} \times 120$. **12.** $(120 \div 1\frac{7}{8})$ yd. $= (120 \times \frac{8}{15})$ yd. $= (8 \times 8)$ yd. **14.** Dimensions of field $= 80$ rods $\times$ 80 rods. **16.** $95 \div 4\frac{3}{4} = 95 \div \frac{19}{4} = 95 \times \frac{4}{19} = 5 \times 4$. **17.** $4000 \div 2$. **19.** 3 T. will cost \$15; 480 lb. @ $\frac{1}{4}$¢ per lb. will cost \$1.20; total, \$16.20. **20.** For \$10, I can buy 2 tons; for 80¢, I can buy (80×4) lb., or 320 lb. **23.** $347 + 495 = (347 + 500) - 5$. **25.** One man can do $\frac{1}{24}$ in 1 day; the other can do $\frac{1}{48}$ in 1 day; both can do $\frac{1}{24} + \frac{1}{48}$ in 1 day, or $\frac{2}{48} + \frac{1}{48} = \frac{3}{48} = \frac{1}{16}$ in one day, thus requiring 16 days to do the whole work.

890. **1.** \$150, at 4%, for 3 years.

2. 12 cu. ft., at 60 lb. to cu. ft.

3. \$12 is 3% of what?

4. 250 is what per cent of 500?

5. 12 is what per cent of 4?

6. 20 M @ \$30 per M.

7. 4 bbl., 300 lb. each, @ 5¢.

8. $18 \div 4\frac{1}{2}$.

9. $84 \div 4$.

10. 75 @ \$79 (or \$80).

892. **4.** Find $\frac{1}{11}$ and annex cipher.

13. See Art. 791.

893. See Arts. 792, 716, 717, 714.

26. Multiply by 36, by subtracting 4 times the number from 40 times the number; multiply by 45, by subtracting 5 times the multiplicand from 50 times the multiplicand.

895. See Art. 563, p. 55.

897. **2.** $18x + 15x + 18x + 15x + (18 \times 15) = 930$.

6. Floor space $= (30 \times 24)$ sq. ft.
Air space $= (30 \times 24 \times 15)$ cu. ft.

7. $(30 \times 24) \div \frac{27}{36}$.

8. Reducing to yards: $[(10 \times 5) + (8 \times 5) + (10 \times 5) + (8 \times 5) + (10 \times 8)] \div 3$.

10. Commencing at lower right-hand corner: $(15 + 3 + 12 + 9 + 8 + 18 + 10 + 15 + 9 + 15)$ rd.

12. $[(22 \times 12) \times (14 \times 12) \times (9 \times 12)] \div 2150.4$.

15. Dimensions: 1000 yd., 2 yd., $3\frac{1}{2}$ yd.

16. Dimensions of pipe space: 1000 yd., $1\frac{1}{2}$ yd., $1\frac{1}{2}$ yd.

900. It may be necessary for the teacher to supplement the information given the pupils in connection with the demand notes in Art. 877. The present note is payable at a fixed time, and the place of payment is specified; but it does not bear interest. If, however, it is not paid at maturity it bears interest from that time at the legal rate.

While savings banks loan money only on good security, generally real estate, banks of deposit will advance money on a note, if the officers feel certain that it will be paid at maturity. When William Brown & Sons present the note for discount, they *endorse* it by writing their name on its back. This transfers the ownership of the note to any subsequent holder, and also makes the endorsers liable for the amount in case the maker fails to pay it at maturity. The discounting bank thus has two parties upon whom to depend for the money.

The sum charged by the bank for this service is the interest on the face of the note for the time it has to run. This sum is called the *discount*, as it is deducted from the sum named in the note; and the difference — called the *avails* or *proceeds* — is given to the owner of the note.

When the above note is due, it is sent to the Park National Bank for collection. If Thomas Tierney, or some other person, does not pay the money before the close of banking hours, the note is *protested;* that is, a notary public certifies that payment has not been made, and notifies the endorsers, William Brown & Sons, of their liability.

901. In states in which days of grace are no longer allowed, the pupils should not employ them even in calculating discount on notes made in places that still have days of grace. Two answers are given to each problem in discount, one including days of grace; the other, enclosed in parentheses, in which days of grace are not employed.

902. These exercises are nothing more than examples in interest, except that in some states, three days are to be added to the time mentioned.

Deducting the discount from the face of the note gives the proceeds.

903. As will be seen in Art. 906, the exact number of days is taken for periods less than a year.

904. Pupils should be led to see that banks are entitled to interest only for the number of days they have to wait for repayment. A failure to understand this, leads to frequent mistakes. Many careless scholars find the difference in time between the two dates named in the example, — from Feb. 27 to March 9, in 25, — disregarding entirely the time for which it is drawn.

Instead of explaining how to calculate the discount on the notes given in Art. 905, the teacher should permit the class to attempt to ascertain the result by themselves. In case of a failure to obtain the correct answer, a discussion of the matter will lead to a proper understanding of the principles involved.

906. 1900 is not a leap year. See Arithmetic, Art. 1303, Time Measure.

907. Find the total number of meters in the first twelve pieces, and ascertain their value at the price named. Do the same for the remaining four pieces. Reduce the total number of meters in the sixteen pieces to yards, by multiplying by 39.37 and dividing the product by 36.

908. See Arithmetic, Art. 758.

910. In $64\frac{1}{2} \times 11\frac{5}{7}$, the product by $\frac{4}{7}$ is found by multiplying the product of $\frac{1}{7}$, already ascertained, by 4.

912. 8. A yard is 36 in. If ribbon is 36 in. long, its width must be $(144 \div 36)$ in. to contain 144 sq. in.

14. Some pupils will say without reflecting, 200% — not seeing that the profit is equal to the cost, 100%.

15. 1% of $1500.

20. $10 =$ what per cent of $(40 + 10)$?

27. The remainder = 20%, or $\frac{1}{5} = \$2000$.

29. The thoughtful teacher must determine for herself just how much time she can afford to waste in giving the pupils a number of useless facts about taxes, brokerage, commissions, bonds, etc., etc. The time allotted to arithmetic should be spent chiefly in developing "power" in her scholars. If the latter can correctly apply mathematical principles in ordinary problems suited to their present experience, they will not find it difficult

in later life, after they understand the conditions, to solve such new problems as come up.

In this example, it will be sufficient to say that the "premium for insuring" means "cost of insuring."

913. 1. See Art. 685.

3. The pupils should attempt to frame the definitions asked.

5. (a) $\frac{7}{8}$ of $832 = Ans.$

(b) $832 = \frac{7}{8}$ of $x = \frac{7x}{8}$; etc.

6. $\frac{15x}{100} = 3750.$

8. The first boy gains $\frac{1}{4}$¢ on a 1¢ apple, or 25%; the second gains $\frac{1}{5}$¢, or 20%.

9. Sold (20×20) sq. rd. + 16 sq. rd.

15. The pupil should be able to state the rule.

19. $x + 2\frac{1}{2}x = 1050.$

21. $([(35 + 23 + 35 + 23) \times 13] + [35 \times 23]) \div 9.$

914. Use first as sight problems.

1. $\frac{2x}{3} - 10 = \frac{x}{4}.$

2. $x + \frac{5x}{7} = 24.$

3. $x + x + 5 = 31.$

4. If $\frac{2}{3}$ of A's money $= \frac{4}{5}$ of B's, A's money $= \frac{4}{5}$ of B's $\times \frac{3}{2}$ $= \frac{6}{5}$ B's. Let $x =$ B's, $\frac{6x}{5} =$ A's. $x + \frac{6x}{5} = 165$; $\frac{11x}{5} = 165$; dividing by 11, $\frac{x}{5} = 15$; $x = 75$. B's = \$75, A's = \$90.

8. After 2 days, there will be enough to feed 4 horses 4 days, or 1 horse 16 days, or 5 horses $3\frac{1}{5}$ days.

915. 7. An ounce avoirdupois contains 7000 gr. $\div$ 16; an ounce troy contains 480 gr.

8. 4 lb. 8 oz. avoirdupois $= 7000$ troy grains $\times 4\frac{1}{2}$.

9. The pure silver amounts to 192.9 gr. $\times .9$; divide by 480 to reduce to ounces, and multiply 75¢ by the quotient.

$$\frac{75¢ \times 192.9 \times .9}{480}.$$

11. Number of square feet $= [(16 + 14 + 16 + 14) \times 8] + 16 \times 14$.

NOTE. — When the bottom of a tank is covered with sheet lead, the side strips will be $\frac{1}{32}$ in. less than 8 ft. high, etc., but this small difference may be neglected in these four problems.

12. Multiply the number of square feet by $\frac{1}{32}$, and divide the product by 12. Cancel.

21. Assessed value, $\frac{3}{4}$ of \$24000, or \$18000. Taxes $= 1\frac{3}{4}\%$ of \$18000.

917. 8. Dividend $= 2\frac{1}{2}\%$ of (\$50 $\times$ 65).

918. 1. Area of surface to be papered: $[(16 + 14 + 16 + 14) \times 10] - 174$; divided by area of roll, 24 ft. by $1\frac{1}{2}$ ft.

4. When he sells 31 gills, the grocer charges for 32 gills, or $1\frac{1}{31}$ times the correct quantity, thereby charging for $\frac{1}{32}$ more than he gives. In 2 hhd. of 58 gal. 2 qt. 1 pt. each, there are $117\frac{1}{4}$ gal., the dishonest gain on which is $\frac{1}{31}$ of $117\frac{1}{4}$ gal. worth \$.80 $\times \frac{1}{31}$ of $117\frac{1}{4}$.

6. 30% of cost $(x) = 21$.

919. 1. (*a*) \$48.50 + (\$48.50 $\times \frac{6}{100} \times \frac{63}{360}$) = \$48.50 + .51 = \$49.01. *Ans.*

Omitting days of grace, \$48.50 + (\$48.50 $\times \frac{6}{100} \times \frac{60}{360}$) = \$48.50 + .48$\frac{1}{2}$ = \$48.98$\frac{1}{2}$, say \$48.99. *Ans.*

These examples should be worked with days of grace or without days of grace, but not in both ways. See Art. 901. Days of grace were not abolished in New York until January 1895.

(*b*) With days of grace, this note is due Dec. 17. Term Dec. 1 to Dec. 17 = 16 days. Discount on \$49.01 for 16 days at 6% $= \$49.01 \times \frac{6}{100} \times \frac{16}{360} = 13¢$. Proceeds = \$49.01 − .13 = \$48.88 *Ans.*

Omitting days of grace, the note is due Dec. 14. Term = 13 days. Discount on \$48.99 for 13 days at 6% $= \$48.99 \times \frac{6}{100} \times \frac{13}{360} = 11¢$. Proceeds = \$48.99 − .11 = \$48.88. *Ans.*

2. With days of grace, the amount due at maturity will be $\$175 + (\$175 \times \frac{6}{100} \times \frac{93}{360}) = \$175 + 2.71 + = \$177.71 +$. The term of discount = 93 days − 33 days = 60 days. Interest of \$177.71 for 60 days at 6% will be \$1.78 nearly. Proceeds = \$177.71 − 1.78 = \$175.93. More accurately, \$177.7125 − 1.7771 = \$175.9354 or \$175.94 −.

Without days of grace, the amount due at maturity will be $\$175 + (\$175 \times \frac{6}{100} \times \frac{90}{360}) = \$177.63 -$. Interest of this amount for 57 days = \$1.69 −. Proceeds = \$177.63 − 1.69 = \$175.94.

920. See Arithmetic, Arts. 821, 822.

921. **9.** 4) £183 14*s.* 8*d.*

13. Total number of days' work = 32 + 53 + 41 = 126. Value of 1 day's work = \$283.50 ÷ 126. Share of first man = (\$283.50 ÷ 126) × 32. Cancel.

14. Amount furnished, \$12000. The one furnishing \$3000, or $\frac{1}{4}$, is entitled to $\frac{1}{4}$ of \$1800; the second to $\frac{5}{12}$ of \$1800, etc.

15. After 10 days, there are rations for 1200 men for 30 days; which will last 1 man 30 days × 1200; and will last 1200 men + 300 men, (30 days × 1200) ÷ 1500 = *Ans.*

16. Train leaving B goes $1\frac{1}{2}$ times as fast as the other, so that meeting place will be $1\frac{1}{2}$ times as far from B as it is from A. If x represents distance from A, $1\frac{1}{2}x$ will represent distance from B, and $x + 1\frac{1}{2}x = 120$, or $x = 48$. Trains meet 48 mi. from A, or 72 mi. from B. The first train takes $\frac{48}{20}$ hr. to travel the distance, or 2 hr. 24 min.; second train requires the same time,

$\frac{72}{30}$ hr., or 2 hr. 24 min. Time of meeting $= 9$ A.M. $+ 2$ hr. 24 min. $= 24$ min. past 11.

Or, let $x =$ time required to reach meeting point; then $20x$ $=$ distance travelled by one train, and $30x =$ distance travelled by the other, and $20x + 30x =$ whole distance $= 120$, or $x = 2\frac{2}{5}$. Time is $2\frac{2}{5}$ hr., etc.

17. $x + 3x + 6x + 10x = 900.$

18. Let $x =$ share of third; $x + 75 =$ share of second; and $x + 75 + 48 =$ share of first.

$$x + x + 75 + x + 75 + 48 = 540.$$

19. If 4 men need 105 hr., one man would need 420 hr., and 6 men would need 70 hr., or $(70 \div 10)$ da.

20. Let $x =$ number of dozen bought, $15x =$ cost in cents; $x - 1\frac{1}{4} =$ number of dozen sold, 16 times $(x - 1\frac{1}{4}) = 16x - 20$ $=$ selling price; $16x - 20 = 15x$, or $x = 20$. He bought 20 dozen, or 240 eggs.

21. The interest on \$250 for 8 mo. is the same as that on \$1 for 8 mo. $\times$ 250, and on \$400 for (8 mo. $\times$ 250) $\div$ 400, or 5 mo.

22. Provisions for 3000 men would last $1\frac{1}{3}$ times as long as for 4000 men, or 18 wk. $\times 1\frac{1}{3} = 24$ wk. *Ans.*

Or, (18 wk. $\times$ 4000) $\div$ 3000.

24. Area of first plank in square feet, 20×1; of second, $24 \times x$. $24x = 20$.

$$x = \tfrac{20}{24}\ \textit{Ans.}\text{ in feet, or } (\tfrac{20}{24} \times 12)\text{ in.}$$

25. He can pay $\frac{2025}{3040}$ of his debts. Mr. Smith should receive $\$576 \times \frac{2025}{3040}$. Cancel.

922. **6.** See table, Arithmetic, Art. 795.

923. **10.** $23 \times 11 \times x = 2749.$

13. $48 \times 72 \times x = 2150.4 \times 40.$

$$x = \frac{2150.4 \times 40}{48 \times 72} = \textit{Ans.}\text{ in inches.}$$

22. 55 cts. $\times 6 \times 54 \times \frac{32}{36}$.

924. 1. The pupils should gradually become accustomed to business methods of obtaining results. In calculating the amount to be paid, a clerk writes the discount at once under the gross price. He takes $\frac{1}{20}$ by dividing by 2 and placing the first quotient figure one place to the left.

$$\begin{array}{r} \$583.40 \\ 29.17 \\ \hline \$554.23 \end{array}$$

2. In the first example, a discount of 5% is made for prompt payment; the discount here allowed is a reduction from what is called the "list" price. Catalogues are issued by some merchants on which the prices named are not the ones regularly charged, but are much larger so as to mislead persons that do not know the rate of discount allowed. Information as to this rate is communicated to customers, and varies from time to time owing to fluctuations in the market, the "list" price seldom being changed. The list price is sometimes called the "gross" price, the "net" price being the one actually paid.

A bill for the Roman candles would be made out as follows:

16 gross Roman Candles,	\$26.75,	\$428. –
	Less 60%,	256.80
	Net,	\$171.20

The product by .60 is written under the "gross" total, the first figure being written two places to the left.

The net cost can be directly obtained by multiplying \$428 by .40.

7. Two, three, and even more discounts are very frequent in business. An article catalogued at \$100 is sold, for instance, at \$70, and customers informed that the discount is 30%. A later reduction in price is accompanied by a notice that a further discount of 10% will be allowed. This does not signify 30% + 10%, or 40%, from the "list" price; it means that the regular price of \$70 is to be reduced \$7, making the new price \$63. A third discount of 5% means 5% of the last price, \$63; etc., etc. In writing these discounts, the per cent mark is written only after the last.

11. On a bill of \$100, 40 and 10% gives a "net" amount of \$60 − \$6, or \$54; 30 and 20% gives \$70 − \$14, or \$56; the former being better for the buyer by \$2.

12. \$100 less $33\frac{1}{3}$ and 10% = \$60. The discount is \$40 on \$100, or 40%. The net is 60%.

13. 100 − 40 = 60; 20% of 60 = 12; discount = 40% + 12% = 52%. *Ans.*

14. Let x = "list" price. After first discount of $\frac{1}{3}$ is deducted, there will remain $\frac{2x}{3}$. Deducting 10% of this, or $\frac{1}{10}$ of it, there will remain $\frac{9}{10}$ of it. $\frac{9}{10}$ of $\frac{2x}{3} = \frac{3x}{15} = 60$.

15. The first reduction is 100% − 20%, or 80% of the list; the second is 100% − 10%, or 90% of the former. 90% of 80% = $\frac{9}{10}$ of 80% = 72%. *Ans.*

16. 80% of 90% = 72%. *Ans.* The net price is the same for the same discounts in whatever order they are taken.

925. 3. $\frac{x}{100}$ of 5000 (cents) = 5 (cents); $50x = 5$; etc.

11. Value at par, \$50 × 96 = \$4800. Discount = \$4800 − \$4476 = \$324 = x% of \$4800, *i.e.*, $324 = 48x$; etc.

15. $\$500 \times \frac{8}{100} \times \frac{100}{365}$.

926. 3. 50% + [10% of (100% − 50%)] = 50% + 5% = 55%. *Ans.*

4. 30% + [30% of (100% − 30%)] = 30% + (30% of 70%) = 30% + 21% = 51%. *Ans.*

5. $\frac{9}{10}$ of gross price (x) = 729; $\frac{1}{10}x = 81$; $x = 810$.

927. 21. Cost per acre = \$40293 ÷ 396, at which price 112 acres were sold.

25. $40¢ \times \frac{30}{12} \times \frac{50}{12}$. Cancel.

26. Number of hours = $365 \div 4\frac{180}{320}$.

928. 2. $x \times \frac{5}{100} \times \frac{9}{4} = \frac{9x}{80} = 180$; $9x = 14400$; etc.

4. $4250 \times \frac{6}{100} \times x = 765$.

6. $2020 \times \frac{6}{100} \times x = 606$.

5. $x \times \frac{4}{100} \times 3 = 240$.

7. $6000 \times \frac{x}{100} \times \frac{5}{2} = 900$.

929. 1. The pupils should not be shown how to calculate these areas.

2. If any difficulty is experienced in finding the areas of these triangles, the pupils should be referred to **1**; after which they should be led to deduce the rule. Thus the area of the second triangle may be calculated from the figure in **1** by adding $\frac{1}{2}$ of (60×50) to $\frac{1}{2}$ of (60×50); that of the third by adding $\frac{1}{2}$ of (60×60) to $\frac{1}{2}$ of (60×40); and that of the fourth by adding $\frac{1}{2}$ of (60×70) to $\frac{1}{2}$ of (60×30). Each of these will be found equal to $\frac{1}{2}$ of (60×100).

3. The second rectangle is divided into three triangles, two of them right-angled. By deducting from the area of the rectangle the sum of the areas of the two right-angled triangles, they will obtain the area of the remaining triangle.

4. The area of each of these triangles can be ascertained by referring to the corresponding triangle of **3**. Let the scholars do this for themselves.

5. The areas of the oblique-angled triangles constituting the first and second quadrilaterals, can be calculated by the pupils that have benefited by the work in **4**. If they see that the area of each triangle of a parallelogram is equal to $\frac{1}{2}$ (base $\times$ altitude), the area of the latter is equal to base $\times$ altitude.

For definitions of quadrilaterals see Art. 1265.

6. The area of the first is equal to the area of the rectangle, (50×60), plus the area of the triangle, $\frac{1}{2}$ of (50×60); or 4500 sq. ft.

The second is made up of a rectangle and of two triangles; its area is also 4500 sq. ft.

The pupils should be led to see that if, in the fourth, the upper left triangle were cut off and placed below, and if the lower right triangle were cut off and placed above, as indicated by the dotted lines, the resultant figure would be a rectangle 60×75.

Cutting off both triangles in the third, and placing them above, will make a 60×75 rectangle.

The area of each trapezoid is equal to $[\frac{1}{2} \text{ of } (50 + 100)] \times 60$.

7. The area of each of these quadrilaterals equals $\frac{1}{2}$ of $(30 \times 100) + \frac{1}{2}$ of (40×100), or $[\frac{1}{2} \text{ of } (30 + 40)] \times 100$.

The first three quadrilaterals are trapeziums. The last is a trapezoid. Which are the parallel sides?

8. A strip of paper of any uniform width may be used. Carefully cut a rectangle by making square corners with a card. Using the base of the rectangle as a measure, place two dots on the lower edge of the strip to mark the extremities of the base of a parallelogram equal in length to the base of the rectangle, and above these, at any convenient distance to the right or to the left, two others to mark the extremities of the opposite side of the parallelogram. Draw lines forming the right and left sides, and cut along these lines. That the parallelogram is equal in area to the rectangle, may be shown by carefully drawing a perpendicular at one corner; cutting off the triangle thus made, and placing it, in a reversed position, on the opposite side of the parallelogram.

9. See **6**, third and fourth trapezoid.

930. **2.** Four faces will measure 6 ft. by $4\frac{1}{2}$ ft. each, and two will measure $4\frac{1}{2}$ ft. by $4\frac{1}{2}$ ft. each.

4. Dimensions of floor, 57 ft. by 18 ft., or 19 yd. by 6 yd.

5. Volume in cubic feet, $4 \times 4 \times 12$. Multiply by 1000 to get the weight in ounces of an equal volume of water. Multiply by 2.8 to get weight of marble in ounces. Divide by 16×2000 to reduce to tons.

$$\frac{4 \times 4 \times 12 \times 1000 \times 2.8}{16 \times 2000}.$$

9. Outer dimensions, $14 \times 14 \times 14$, or 2744 cu. in. See if the same number of cubic inches of wood is obtained by calculating the volume of the wood in **6**—2 pieces, 12×12, 1 in. thick; 2 pieces, 12×14, 1 in. thick; 2 pieces, 14×14, 1 in. thick.

10. A cube of water 2 ft. long contains $(2 \times 2 \times 2)$ cu. ft., or 8 cu. ft. At 1000 oz. to a cubic foot, it weighs $\frac{1000}{16}$ lb. $\times 8$, or 500 lb. The cube of iron weighs 8 times as much as an equal volume of water.

A cube of iron 1 ft. long also weighs 8 times as much as a corresponding cube of water, or 8 times $\frac{1000}{16}$ lb. $= 500$ lb., or $\frac{1}{4}$ ton.

A 3 ft. cube of iron contains 27 cu. ft., weighing 8 times as much as a corresponding cube of water, or 216 times 1000 oz. $= 6\frac{3}{4}$ tons.

XV

NOTES ON CHAPTER TWELVE

931. While problems requiring the pupil to find the principal, the rate, or the time have very little "practical" value, they can be so readily taught by the algebraic method that the time spent upon them need not be very great. A pupil that is able to calculate one of a series of related items is benefited by being required to calculate the others, if he is not compelled to resort to a series of ill-understood rules in order to obtain the results.

Although there is no real difference between the algebraic method and the arithmetical one, a great number of scholars fail to obtain a thorough understanding of the latter. They can work a number of examples, following a model solution at the head of the lesson; but they fail to grasp the underlying principles. By the algebraic method, x is used to represent the number of years or dollars, or the rate, instead of the 1 year, \$1, or 1%, of the other; but this method seems to require the formulation of a number of rules, as against practically none in the case of the other.

After pupils have learned to work examples by the algebraic method, they can be encouraged to discontinue the use of the x; but they should not be taught both methods at one time.

933. Represent the required item by x. Simplify the first member before proceeding to solve the equation.

1. $2000 \times \frac{x}{100} \times 3 = 300.$

2. $1800 \times \frac{4}{100} \times x = 144.$

3. $x \times \frac{9}{200} \times \frac{8}{12} = 2.88.$

4. $38 + \dfrac{38 \times x \times 2}{100} = 40.28.$

Or, $38 \times \dfrac{x}{100} \times 2 = 40.28 - 38 = 2.28.$

5. $140 \times \frac{7}{200} \times \frac{105}{360} = x = Ans.$

6. $x + \dfrac{x \times 4 \times 5}{100 \times 2} = 39.60.$

7. $460 + \dfrac{460 \times 7 \times x}{200} = 484.15.$

8. $39.60 + \dfrac{39.60 \times 4 \times 585}{100 \times 360} = x = Ans.$

15. Principal $= \$97.57 - \$7.57 = \$90.$

$$90 \times \tfrac{4}{100} \times x = 7.57.$$

21. Let $x =$ principal.

$$x + \frac{x \times 4 \times 846}{100 \times 360} = 196.92.$$

The interest is then found by subtracting from \$196.92, the value obtained for x.

22. First find the principal (x).

25. See **15.**

934. The recommendation so frequently made, that all written work be preceded by oral exercises of the same character, should not be followed without some modifications. Oral work is necessarily accompanied by a number of devices that tend to simplify the task of handling numbers that are not seen; written work should follow general rules in order to be learned by a majority of the pupils, although later they may adopt some of the short-cuts of their oral exercises. Even the oral addition of two numbers of two figures each, is done in a manner different from the ordinary slate method, the operation in the former case being commenced generally with the tens' figures, and in the latter case with the units' figures. The reduction to a common denominator recommended in oral division of fractions, is seldom employed in slate work.

The average scholar is able to handle "mental" problems containing small numbers in a way that he cannot always explain, although he may endeavor to stultify himself by repeating a prescribed form of analysis. It is next to impossible, with the average teaching, to get the same pupil to work some varieties of "written" problems containing the same conditions.

In order to furnish a general method of treating some classes of examples, it has been thought best to commence with written work, leaving the mental exercises with their various devices until the former task is accomplished.

The accompanying exercises are so simple as not to need explanation by the teacher; but sufficient time should be given the pupil to work them out in his own way. They differ in this respect from the oral examples of a single operation containing larger figures, but which do not require any effort on the part of the scholar to determine which process is required.

1. Yearly interest is \$6; a year and a half will be needed to make the interest \$9.

2. The yearly interest is \$8, making the rate 4%.

3. Yearly interest is \$4, requiring a principal of \$100, at the given rate.

5. The pupils may remember (Art. 878, No. 15) that 5% for a year is 1% for 72 da.

6. 4% per year is 1% for 90 da.

11. 2 mo. 12 da. = 72 da. See **5.**

12. 1% for 80 da. is (360 ÷ 80) % for a year.

17. 2% for 6 mo.

18. \$3.60 per year is 1 cent per day.

20. See **18.**

935. First payment $=\frac{x}{3}$, leaving $\frac{2x}{3}$ remainder; second payment $=\frac{1}{2}$ of $\frac{2x}{3}=\frac{x}{3}$, leaving $\frac{x}{3}$ remainder; third payment $=\frac{3}{5}$ of $\frac{x}{3}=\frac{x}{5}$; last payment, $=\$2000$. The total cost of the house, $x=$ the sum of the payments, $\frac{x}{3}+\frac{x}{3}+\frac{x}{5}+2000$.

936. The books contain many methods of calculating interest, but it is questionable whether it is not time wasted in giving so much attention to this topic. The average person is required to do comparatively little work in this line; while those called upon to compute interest often, learn short methods of their own or use interest tables.

If a second method is to be taught at all, the one by aliquot parts is the most useful, as modifications of this method may be applied to other operations.

6. See Arithmetic, Art. 384.

937. A modification of the so-called "60-day method."

16. See Art. 901 as to days of grace.

938. 21. 10% gives 2 years' interest; then 1 yr. ($\frac{1}{2}$ of the foregoing); 6 mo.; 1 mo.; 18 da. ($\frac{1}{10}$ of 6 mo.).

942. 46. Term, 57 da. (54 da.).

47. Term, 92 da. (89 da.).

48. Term, 16 da. (13 da.).

49. Term, 34 da. (31 da.).

50. Term, 187 da. (184 da.).

943. 9. See Table, Arithmetic, Art. 1303.

944. 11. The net price of goods catalogued at x dollars, and sold at a discount of 20 and 10%, will be $\left(x-\frac{20x}{100}, \text{ or } \frac{80x}{100}\right)-\left(\frac{1}{10} \text{ of } \frac{80x}{100}\right)=\frac{80x}{100}-\frac{8x}{100}=\frac{72x}{100}$.

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13. If the selling price of the above is \$360, $\frac{72x}{100} = 360$; $72x = 36000$; $x = 500$. Catalogue price = \$500. *Ans.*

14. $750 - (\frac{1}{3} \text{ of } 750) = 500$;

$$500 - \left(\frac{x}{100} \text{ of } 500\right) = 500 - 5x = \text{net price.}$$

$$500 - 5x = 450.$$

Transposing, $-5x = -50.$

Changing signs of both terms, $5x = 50,$

$$x = 10.$$

945. 7. Let x = selling price of muslin.

$$(84 \times 40) + 105x = (84 \times 55) + (105 \times 20).$$

Another way: He loses 15¢ per yard on 84 yd., which is a loss of 15¢ × 84. This he must make up on 105 yd., which is (15¢ × 84) ÷ 105 on each yard, or 12¢. Selling price of muslin, 20¢ + 12¢, or 32¢. *Ans.*

8. $\frac{1}{4}$ of them brought \$120; $\frac{1}{3}$ of remainder, or $\frac{1}{4}$ of them, brought \$96; $\frac{1}{2}$ of remainder, or $\frac{1}{4}$ of them, brought \$40; remainder, or $\frac{1}{4}$ of them, brought \$30. Total amount received, \$286.

9. Proceeds of gas stock, \$25 × 165 = \$4125. Cost of lots, \$4125 − \$27 = \$4098. Number of square feet in lots, (32 × 115) + (30 × 105) = 3680 + 3150 = 6830. Value per square foot, \$4098 ÷ 6830 = \$0.60. *Ans.*

10. Two walls, each 16 × 14, and two others, each 12 × 14, contain (32 + 24) × 14, or (56 × 14) sq. ft. = 784 sq. ft. The ceiling contains (16 × 12) sq. ft. = 192 sq. ft. Adding this to the walls, makes a total of 976 sq. ft.

The deductions are (8 × 4) sq. ft. × 2, and (7 × 3) sq. ft. × 3, or 64 sq. ft. + 63 sq. ft. = 127 sq. ft. Number of square feet to be plastered = 976 − 127 = 849. Cost at $\frac{18}{9}$¢ per square foot = 2¢ × 849 = \$16.98. *Ans.*

946. **1.** A can do $\frac{1}{5}$ of the work in 1 hr., and B can do $\frac{1}{7}$ of it in 1 hr.; together they can do in 1 hr. $(\frac{1}{5}+\frac{1}{7})$ of the work, or $\frac{12}{35}$ of it; and to do the whole work it will take as many hours as $\frac{12}{35}$ is contained times in 1.

$$1 \div \tfrac{12}{35} = 1 \times \tfrac{35}{12} = \tfrac{35}{12} = 2\tfrac{11}{12}. \quad \textit{Ans. } 2\tfrac{11}{12} \text{ hours.}$$

2. Commission of $2\frac{1}{2}\% = \frac{1}{40}$ of amount collected $= \$1.60$. Amount collected $= \$1.60 \times 40 = \$64$. Amount remitted $= \$64 - \$1.60 = \$62.40$. *Ans.*

3. $\frac{1}{4}\%$ of $(\frac{3}{4}$ of $\$12000) = \frac{1}{4}\%$ of $\$9000 = \frac{1}{4}$ of $\$90 = \$22.50$. *Ans.*

NOTE. — It may be advisable to explain to the pupils that property is seldom insured for its full value, because it is not likely that a fire will completely destroy a building, and insurance companies reimburse the person insured, only to the extent of his loss.

4. $32 \times x = 6 \times 4$; $32x = 24$; $x = \frac{24}{32} = \frac{3}{4}$. *Ans.* $\frac{3}{4}$ yd. or 27 in.

5. 5% for 360 days $= 1\%$ for 72 days $= 2\%$ for 144 days. 2% of $\$87 =$ *Ans.*

6. 2% of $\$176$.

7. Let $x =$ commission; $40x =$ amount invested; $x + 40x = 41x = 8200$; $x = 200$. *Ans.* $200.

8. $500 is $\frac{1}{8}$ of cost, $4000.

9. Let $x =$ loss, or 20% of cost; $5x =$ cost; $5x - x = 4x =$ selling price.

x, the loss, is $\frac{1}{4}$ of selling price, $4x$.

10. Let $x =$ gain, which is 20%, or $\frac{1}{5}$, of the cost of the goods; $5x =$ cost; $5x + x$, or $6x$, $=$ selling price.

x, the gain, is $\frac{1}{6}$ of selling price, $6x$.

NOTE. — The amount of money given in these two examples, $1200, does not affect either result. It may be used or not, as the pupil prefers.

11. 3 men earn $\$72 \div 8$ in one day, or $3 per day each. 5 men earn $15 a day, or $165 in 11 days.

12. 3 quarters of the cost, or $\frac{3x}{4}$, $= 225$. Cost $= \$300$. By selling for \$325, there is a gain of \$25, or $\frac{1}{12}$ of the cost. $\frac{1}{12} = 8\frac{1}{3}\%$. *Ans.*

13. $2\frac{2}{3}$ yd., or $\frac{8}{3}$ yd. cost 40¢; 1 yd. costs $40¢ \div \frac{8}{3}$, or $\frac{3}{8}$ of $40¢ = 15¢$. 4 yd. 1 ft., or $4\frac{1}{3}$ yd., cost $15¢ \times 4\frac{1}{3}$.

947. The following is the solution without days of grace:

Let $x =$ face of the note.

Then, $$x \times \frac{6}{100} \times \frac{1}{12} = \frac{x}{200} = \text{discount};$$

$$x - \frac{x}{200} = \text{proceeds} = 1000.$$

$$200x - x = 200000,$$

$$199x = 200000,$$

$$x = \tfrac{200000}{199} = 1005.03.$$

Face of note $= \$1005.03$. *Ans.*

Proof. Face of note,		\$1005.03 –
Deduct 30 days' discount,	$\frac{1}{2}\%$,	5.03 –
Proceeds,		\$1000.00

949. **1.** When days of grace are omitted, the term of discount is 90 da.

10. Find the term, and add the number of days to March 15.

950. **2.** (*a*) 1 trillion, 500 billions, etc.

5. The first quarter of 1888 contained $(31 + 29 + 31)$ da., or 91 da. The man was employed 60 da., and unemployed 31 da. His \$3 additional paid the expenses of the working days. Deducting $\$2 \times 31$, or \$62, for the expenses of the other days, his net income $= \$350 - \$62 = \$288$. *Ans.*

6. Apr. 16, '79 to Mch. 19, '86,	$83\frac{1}{10}$	mo., @	\$8,	\$664.80
Mch. 19, '86 to Mch. 4, '87,	$11\frac{1}{2}$	" "	12,	138.00
Apr. 16, '79 to Sept. 1, '80,	$16\frac{1}{2}$	" "	2,	33.00
Apr. 16, '79 to Nov. 22, '82,	$43\frac{1}{5}$	" "	2,	86.40
			Ans.	\$922.20

10. Last quarter's salary = \$287 = 82% of previous quarter's salary $= \frac{82\,x}{100} = 287$; $x = 350$.

Salary of three quarters @ \$350 = \$1050; add last quarter's, \$287. Total for year, \$1337. *Ans.*

951. See Art. 784.

953. To find 19 times 91, subtract 91 from 20 times 91, or $1820 - 91$.

$$82 \times 19 = (82 \times 20) - (82 \times 1).$$
$$51 \times 29 = (51 \times 30) - 51.$$
$$27 \times 99 = 2700 - 27.$$

954. See Art. 706. $675 \div 37\frac{1}{2} = 6\frac{6}{8} \div \frac{3}{8} = 54 \div 3.$

955. $136 \times \frac{7}{8} = 136 - (\frac{1}{8} \text{ of } 136) = 136 - 17.$

$$290 \times \frac{9}{10} = 290 - 29.$$
$$64\tfrac{1}{2} \div 5 = 12\tfrac{4\frac{1}{2}}{5} = 12\tfrac{9}{10}.$$
$$22 \times 19\tfrac{1}{2} = (22 \times 20) - (22 \times \tfrac{1}{2}).$$
$$45 \times 9\tfrac{14}{15} = (45 \times 10) - (\tfrac{1}{15} \text{ of } 45).$$
$$160 \div 1\tfrac{1}{2} = 320 \div 3.$$
$$18\tfrac{4}{5} \div 1\tfrac{4}{5} = 94 \div 9 = 10\tfrac{4}{9}.$$

956. **3.** At 50¢ each, the cost would be \$8; at 1¢ apiece less, the cost is \$8.00 — \$.16 = *Ans.*

4. $(\frac{3}{4} \text{ of } 100 \text{ lb.}) \times 27 = 100 \text{ lb.} \times (\frac{3}{4} \text{ of } 27) = 100 \text{ lb.} \times \frac{81}{4} = 100 \text{ lb.} \times 20\frac{1}{4}.$

NOTE. — The 100 should not be used until the end; even then, $20\frac{1}{4}$ is changed to 2025 without thinking of multiplication, $\frac{1}{4}$ being considered 25, and annexed to 20. See Art. 649.

6. $900 \div 75 = (900 \div 100) \div (75 \div 100) = 9 \div \frac{3}{4}$.

7. See Art. 955.

9. $(10\frac{1}{2} \times 4) + \frac{1}{2}$ of $10\frac{1}{2} = 42 + 5\frac{1}{4} = 47\frac{1}{4}$.

12. $\frac{1}{2}$ of $(33 \times 42) = 33 \times 21$.

15. $\$16\frac{1}{4} \div \$1\frac{1}{4} = 65 \div 5$.

20. $16\frac{1}{2} \times 2\frac{1}{4} = (16\frac{1}{2} \times 2) + (\frac{1}{4}$ of $16\frac{1}{2})$.

957. 1. $\frac{35}{100}$ of $(27¢ \times 56 \times 37\frac{1}{2} \times \frac{30}{36})$.

2. $[(\$4.875 \times 17350) \div 196] + [\$4.9375 \times 122.75] + [\$.0825 \times 2240 \times 2\frac{1}{4}]$.

3. $x - \frac{7}{400}x = 49739.55\frac{3}{8}$.

12. Duty $= [\frac{35}{100}$ of $(55¢ \times 45 \times 38)] + [20¢ \times (45 \times 38 \times \frac{42}{36})]$.

958. In multiplying by 427, the first figure of the product by 42 (7×6) is placed under the 2; in multiplying by 832, the first figure of the product by 8 is placed under the 8, and the first figure of the product by 32 (8×4) is placed under the 2.

959. These exercises contain some examples worked by short methods explained in previous chapters. See Arts. 650, 714, 791, 792, and 891.

960. See Arithmetic, Art. 384.

964. 2. It won 17 games out of 30.

3. 1600%.

4. $\frac{8}{3}$ is what per cent of $\frac{16}{5}$? $\frac{1}{3}$ is what per cent of $\frac{2}{5}$? $\frac{5}{15}$ is what per cent of $\frac{6}{15}$? 5 is what per cent of 6? $\frac{5}{6} = 83\frac{1}{3}\%$.

6. The deduction of the first discount leaves 80% of the list price; the deduction from this of 10% of itself leaves 90% of 80%, or 72%.

7. One fills $\frac{1}{6}$ of tank in 1 hr., the other fills $\frac{1}{8}$ in 1 hr.; both together fill $\frac{1}{6} + \frac{1}{8}$ in 1 hr., or $\frac{4}{24} + \frac{3}{24}$, or $\frac{7}{24}$; to fill $\frac{24}{24}$ of tank, it will take 24 hr. $\div 7 = 3\frac{3}{7}$ hr.

8. 6% for 60 da. = 80¢; for 12 da., $\frac{1}{5}$ of 80¢ or 16¢; for 72 da., 80¢ + 16¢ = 96¢. Or, \$4.80 for year, and $\frac{1}{5}$ of \$4.80 for 72 da.

9. 16% = $\frac{4}{25}$; $420 = \frac{21x}{25}$; etc.

965. 5. Selling price = $\frac{3}{4}$ of \$1.50 = \$1.12$\frac{1}{2}$; gain = 22$\frac{1}{2}$¢ = $\frac{1}{4}$ of 90¢.

6. Selling price = \$9.60, a reduction of \$2.40 from marked price, or $\frac{1}{5}$ of \$12, or 20%.

7. The rug is sold for \$24. If this is $\frac{4}{5}$ of marked price, the latter is \$30.

8. See **7.**

NOTE. — It is not to be expected that all the pupils' work will be shortened to this extent, but the majority of the class should be able to give answers at sight to these four examples.

9. Find $\frac{1}{10}$ of £83 2*s.* 6*d.* by compound division; do not reduce to pence.

966. 4. Let x = profits first year; then $\frac{21x}{20}$ = profits second year; $x + \frac{21x}{20} = 6970$.

5. I wish to gain 15% of \$.96, or \$.14$\frac{2}{5}$, which makes my selling price \$.96 + \$.14$\frac{2}{5}$ = \$1.10$\frac{2}{5}$.

Let x = marked price.

$$x - \frac{15x}{100} = 1.104;\ \text{etc.}$$

Or, writing all the foregoing in one equation:

$$\frac{85x}{100} = \left(\frac{115}{100} \text{ of } .96\right),$$

$$85x = .96 \times 115,$$

$$x = \frac{.96 \times 115}{85}.$$

7. The average pupil will be able to obtain the meanings of these terms by inquiries of his parents, friends, etc.; and he will remember much longer what he learns in this way, than if he finds the answer in the text-book. As the penalties for taking usurious interest vary in the different states, the teacher should ascertain the law of her own state in this matter. See Art. 1306. A tax bill or a policy of insurance brought in by a pupil and described, will add to the interest. The teacher should not spend too much time upon details that have no relevancy in her section of the country. Poll taxes, for instance, should not be dwelt upon in cities in which they are not collected; etc.

9. The use of the hogshead as a measure of 63 gal. is fast becoming obsolete. The term "barrel," to indicate $31\frac{1}{2}$ gal., is occasionally used in giving the capacity of large tanks, etc. The U.S. authorities require prices to be stated in the currency of the country from which the articles are exported; but as this would make the problem more complicated, the text-books generally give prices in U.S. money. No allowance is now made for leakage, the quantity actually imported being ascertained by measuring.

10. A port of entry is a place in which there is a custom house, established by the government.

967. **4.** Any principal — \$150, \$575, or \$343.75 — will double itself at 5% in $(100 \div 5)$ yr.

12. The pupils will need to obtain a correct idea of the meaning of the word "premium" in this connection, as they will find it used differently when they come to the study of Bonds and Stocks. The premium is the amount paid to the company assuming the risk.

968. **11.** \$3500 is raised on property worth \$1750000; the rate is $\$3500 \div \$1750000 = 2$ mills on \$1. The man's property tax $= 2$ mills $\times\ 24000 = \$48$; adding to this 1 poll tax, at \$2, gives his total tax of \$50. *Ans.*

969. 2. Multiply the denominators of the first and the third; divide the numerators of the second and the fourth.

5. While pupils in lower grades may be permitted to reduce both amounts to pence, it is now time to use a shorter method. The sums given may be changed to $38\frac{3}{4}s.$ and $5\frac{2}{4}s.$, or $\frac{155}{4}s.$ and $\frac{22}{4}s.$

970. 1. See diagram, Arithmetic, Art. 897, Problem 2.

$$18x + 15x + 18x + 15x + (18 \times 15) = 63 \times 9,$$
$$66x + 270 = 567,$$
$$66x = 567 - 270 = 297,$$
$$x = 4\tfrac{1}{2}. \qquad \textit{Ans. } 4\tfrac{1}{2} \text{ ft.}$$

The arithmetical solution is apparent from the foregoing. The sides and the bottom contain 63 sq. yd., or 567 sq. ft. The bottom contains (18×15) sq. ft., or 270 sq. ft. The sides contain 567 sq. ft. $-$ 270 sq. ft. $=$ 297 sq. ft., which is the area of four rectangles, whose bases measure 18 ft., 15 ft., 18 ft., and 15 ft., respectively, the total being 66 ft. $297 \div 66$ gives $4\frac{1}{2}$ as the number of feet in depth.

2. There are 4840 sq. yd. in an acre. $(4840 \times 3) \div (\frac{1}{2} \text{ of } 242)$. Cancel.

3. The area of a trapezoid is found by multiplying one-half the sum of the parallel sides by the perpendicular distance between them. See diagrams, Arithmetic, Art. 929, Problems 6 and 9; and Art. 1265, Figs. 9 and 10.

4. The area $= \left(\dfrac{x + 100}{2}\right) \times 60 = 30x + 3000$. *Ans.*

$30x + 3000 = 5400$; $x = 80$. *Ans.* 80 yd.

5. $\left(\dfrac{80 + 120}{2}\right) \times x = 100x$. *Ans.*

$100x = 4000$; $x = 40$. *Ans.* 40 yd.

6. $\left(\frac{x+x+40}{2}\right)\times 60=(x+20)\times 60=60\,x+1200.$ *Ans.*

$60\,x+1200=6000$; $x=80$; $x+40=120$.

Ans. 80 yd. and 120 yd.

9. The number of square feet in the walls of a room $16\frac{1}{2}$ ft. long, $14\frac{3}{4}$ ft. wide, and $13\frac{1}{3}$ ft. high, may be obtained by adding the bases of the four sides, — $16\frac{1}{2}+14\frac{3}{4}+16\frac{1}{2}+14\frac{3}{4}=62\frac{1}{2}$, — and multiplying this by their common height, $13\frac{1}{3}$. Dividing by 9 gives the number of square yards. The operation of finding the cost may be indicated as follows:

$$\frac{10¢\times 125\times 40}{9\times 2\times 3}=\frac{25000}{27}¢=\$9.25\tfrac{25}{27}.\quad \textit{Ans. } \$9.26.$$

10. $(20\times 17\frac{1}{2})\div 2\frac{1}{2}$ gives the number of feet of carpet. Dividing this result by 3 gives the number of yards.

11. $41\frac{1}{4}$ lb. $\times(15\frac{3}{4}\times\frac{10}{12}\times\frac{1}{6})$. Cancel.

12. The "development" will be a modification of the one given in problem 20, Arithmetic, Art. 818. In drawing the development, the pupil should be required to approximate the proper proportions, and to place the faces in the proper order. It is not necessary to have the top and bottom faces in the positions shown in Art. 818.

The surface of the four vertical faces should be obtained in one operation, as in **9**; also the surface of the two horizontal faces:

$$[2\times(3\tfrac{3}{4}+2\tfrac{1}{2})\times 1\tfrac{1}{3}]+[2\times(3\tfrac{3}{4}\times 2\tfrac{1}{2})].$$

13. $(135\frac{1}{9}\div 12\frac{2}{3})$ ft.

14. $[(128\times 152\times 105)\div 2150.4]$ bu.

15. $[(77\times 45\times 54)\div 231]$ gal.

16. 40 acres $=(160\times 40)$ sq. rd. $=6400$ sq. rd. The dimensions are 80 rd. by 80 rd., making 320 rd. of fence necessary. There will be 640 posts, at 15¢ each; and 5 times 640 rails, or 3200 rails, at 10¢ each.

17. There will be 16 fields, 4 rows of 4 fields each. Five parallel fences, each a mile long, and five other parallel fences of the same length, and perpendicular to the first, will be required.

19. 1728 cu. in. of water weigh 1000 oz.; 1 cu. in. weighs $(1000 \div 16)$ lb. $\div$ 1728; 231 cu. in. weigh $[(1000 \times 231) \div (16 \times 1728)]$ lb.

21. Number of square feet $= (320 + 210 + 320 + 210) \times 6$. Divide by 9 to obtain square yards.

22. Area of outer rectangle in square feet $= 332 \times 222$; of inner rectangle $= (320 \times 210)$ sq. ft. Divide the difference by 9 to obtain square yards.

23. Area of outside plot $= (320 \times 210)$ sq. ft.; of inner plot $= (308 \times 198)$ sq. ft.

972. **1.** Each yard measured with the short yardstick contains $\frac{35}{36}$ yd.; the true length $= 25$ yd. $\times \frac{35}{36}$.

Or, each so-called yard is $\frac{1}{36}$ yd. short, and 25 yd. are $\frac{25}{36}$ yd. short; and the piece contains 25 yd. $- \frac{25}{36}$ yd. $= 24\frac{11}{36}$ yd. *Ans.*

4. 32 boys $=$ 20 men. If 15 men do the work in 12 da., 20 men do it in 12 da. $\times \frac{15}{20}$.

20. Change 6 lb. 14 oz. and 23 lb. 12 oz. to ounces, or to pounds and fractions.

976. **3.** A and B can mow $\frac{1}{7}$ of the field in 1 da., all three can mow $\frac{1}{5}$ of the field in 1 da. C mows $(\frac{1}{5} - \frac{1}{7})$ of the field in 1 da., or $\frac{2}{35}$ of it; in 5 da. he does $\frac{2}{35} \times 5$, or $\frac{2}{7}$ of the work, for which he should receive $\frac{2}{7}$ of \$25.

4.

$$\begin{aligned} 5 \text{ bu. @ } 80¢ &= 400¢. \\ 5 \text{ bu. @ } 60¢ &= 300¢. \\ x \text{ bu. @ } 30¢ &= 30x¢. \\ (x+10) \text{ bu. @ } 50¢ &= (30x + 700)¢, \\ 50x + 500 &= 30x + 700, \\ 20x &= 200, \\ x &= 10. \end{aligned}$$

Ans. 10 bu. of oats.

7. Total cost $= (65¢ \times 128) + 80¢$; quantity sold $= 128$ gal. $- 16$ gal. Selling price per gallon $= \frac{6}{5}$ of $[(65¢ \times 128) + 80¢] \div (128 - 16)$.

It is advisable to accustom children to understand that a gain of $\frac{1}{5}$ of cost makes the selling price $\frac{6}{5}$ of cost.

10. On sofas sold for \$1125 there was a loss of $\frac{1}{5}$, making the selling price $\frac{4}{5}$ of cost. On the remaining sofas, the selling price, \$1125, represents $\frac{6}{5}$ of cost.

4 fifths of cost = \$1125, selling price of 25 sofas.
1 fifth of cost = \$$281\frac{1}{4}$, loss on first lot.
6 fifths of cost = \$1125, selling price of remaining sofas.
1 fifth of cost = \$$187\frac{1}{2}$ = gain on second lot.

Loss on the transaction, \$281.25 – \$187.50.

The following is an algebraic solution without fractions:

Let $x =$ loss on first lot;
then $5x =$ cost of first lot.
$5x - x = 4x =$ selling price $= 1125$.
$x = 281\frac{1}{4} =$ loss in dollars.
Let $x =$ gain on second lot;
$5x =$ cost of second lot.
$5x + x = 6x =$ selling price $= 1125$.
$x = 187\frac{1}{2} =$ gain in dollars.
etc.

11. Let $x =$ cost per egg.

$18x =$ cost of 18 eggs.

$\frac{18x}{11} =$ selling price per egg.

Gain per egg $= \frac{18x}{11} - x = \frac{7x}{11}$; that is, on x cents I gain $\frac{7}{11}$ of x cents. A gain of $\frac{7}{11}$ of cost $= .63\frac{7}{11}$ of cost $= 63\frac{7}{11}\%$. *Ans.*

12. Let $x =$ marked price.

$$x - \frac{15\,x}{100} = 2.$$

14. $x = \frac{x}{3} + \frac{x}{4} + 5\frac{5}{6}.$

18. Let $x =$ cost of the horse.

$$x + \frac{x}{20} = \text{asking price} = \frac{21\,x}{20}.$$

He sold it at $\frac{19}{20}$ of asking price; *i.e.*, $\frac{19}{20}$ of $\frac{21\,x}{20}$, or $\frac{399\,x}{400}$.

$$\frac{399\,x}{400} = 275.$$

20. Length of field in rods $= (1600 + 146) \div 18$.

21. Let $10\,x$ represent cost, then loss $= x$; and selling price $= 9\,x = \$117$; $x = 13$; cost $= \$130$. A gain of 10% would be \$13, making the price at which he should have sold it to gain 10% $= \$130 + \$13 = \$143$. *Ans.*

22. Wife receives $\frac{2\,x}{3}$; son, $\frac{2}{3}$ of $\frac{x}{3}$, or $\frac{2\,x}{9}$; daughter, \$5000.

$$\frac{2\,x}{3} + \frac{2\,x}{9} + 5000 = x.$$

977. The ten problems of this section will call for no special treatment. An occasional problem of this kind has already been given, although, perhaps, with smaller numbers. After the pupils understand in **1**, that the joint capital, \$700, is the basis upon which the profits are distributed, they will have no difficulty in understanding that B is entitled to $\frac{300}{700}$ of \$182, and that C is entitled to $\frac{400}{700}$ of \$182. Cancellation should be employed. See Art. 1121, No. 5.

There is no need in this connection of discussing the subject of business partnerships that are continued for a year or longer. The division of profits in these cases is the subject of a special agreement, and is rarely made solely on the basis of the amounts invested. A yearly gain of \$4000 made in regular business by two partners, one of whom invested \$1000, and the other \$3000, might be divided in various equitable ways. The partner investing the larger sum, might first take out \$120 as interest on his excess of capital; and the remaining \$3880 might be equally divided, giving one of them \$1940, and the other \$2060. Another arrangement might permit each partner to withdraw a fixed sum for services, say \$1000, leaving \$2000 to be divided on the basis of 1 to 3. This would make the shares (\$1000 + \$500) and (\$1000 + \$1500), or \$1500 and \$2500.

10. Cases of this kind are found only in the books.

979. The cost of the first item is given. The second item contains 451 sq. ft.; the third contains ($4 \times 42 \times 7\frac{1}{6}$) sq. ft., or 1204 sq. ft.; the fourth contains ($3 \times 43 \times 7\frac{2}{3}$) sq. ft., or 989 sq. ft.; the total of the three items being 2644 sq. ft. The cost @ 10*d.* is

	£110 – 3 – 4
First item,	24 – 7 – 8
	£134 – 11
Less $2\frac{1}{2}\%$ ($\frac{1}{40}$),	3 – 7 – $3\frac{1}{4}$ +
Ans.	£131 – 3 – $8\frac{3}{4}$

The fraction of a penny in the discount is $\frac{3}{10}$, which is nearly 1 farthing, written $\frac{1}{4}d$.

980. German currency being a decimal one, the bill is computed in the ordinary way. The duty is found by reducing the marks to dollars by multiplying by \$.238, and taking 35% of the result.

981. Too much stress cannot be laid upon the importance of requiring children to estimate the probable answer to every "written" problem before placing a figure on paper. The mere drill on the "approximations" found in the text-book is of comparatively little value, unless it leads pupils to the employment of this device throughout all of their work. Besides preventing a pupil from making a very serious mistake in an ordinary computation, the habit of careful reading that is necessarily formed by the scholar that is not satisfied with a simple guess, will tend to make his methods simpler and more accurate. He will learn to apply to the apparently more difficult numbers of the written problem the processes employed in solving the comparatively simple "mental" questions.

While all of the pupils should not be expected to give the same answer to each of these examples, they should gradually approach more and more closely to the correct result.

1. 480 is what per cent of 960?
2. 52 bu. is about 3 times (17 bu. 37 lb.).
3. 500 cu. ft. ÷ 128 cu. ft. Less than 4 cords.
4. 120 cu. ft. ÷ about $1\frac{1}{4}$ cu. ft.
5. 1500 sq. rd. ÷ 160 sq. rd. Less than 10 acres.
6. A little less than 70×70.
7. $(6 \times 4 \times 5) \times$ about $7\frac{1}{2}$.
8. $64 \div \frac{1}{10}$, or $64.3 \div \frac{1}{10}$. More than 643.
9. About £200 @ $4.80 to £.
10. About 4 marks to $1. Over 400 marks.

982. The teacher that is allowed any discretion should omit all problems relating to Bonds and Stocks. The average grammar-school pupil cannot be made to understand the subject without the expenditure of more time and energy than should be given to a topic that he will learn by himself when he grows older if he gets the proper foundation.

If, however, the course of study requires that this topic be taken up, the teacher should aim to interest the pupils by making some local corporation the basis of the work. A certificate of stock should be obtained, or at least a copy made, which might be placed upon the blackboard.

The scholars should be led to see that large undertakings, such as the construction of a railroad, the building of water-works, and the like, require more money than any individual might have, or would care to risk. It then becomes necessary to interest a number of persons that will be willing to invest more or less money in the new enterprise. It is found, for instance, that $50000 will be needed to build and equip the street railroad mentioned in **1.** The projectors divide this amount into 500 shares, each of which represents a one-five-hundredth interest in the profits. It may happen sometimes that it is considered advisable to interest people of small means, and who are unwilling to take a $100 share. In these cases the original (par) value of the shares may be fixed as low as $10 each. When the par value is not given, it is understood to be $100.

In the distribution of profits, the owner of 10 shares is entitled to $\frac{10}{500}$ of $2000, or $40.

2. These profits are generally distributed annually, semi-annually, or quarterly. Before the time comes for "declaring" the dividend, the directors of the company meet and determine how much money shall be thus distributed. It may be considered advantageous to reserve a portion of the profits for the purchase of new cars, or for the extension of the road, etc.; so that the amount distributed at any time does not necessarily include all that has been gained. The dividend is generally announced as a per cent of the capital, which in this case is $50000; so that the semi-annual dividend is 4%, equal to 8% per year.

3. The owner of a $100 share, on which he receives $8 per year, is not likely to be willing to sell it for $100 if he can obtain only $4 per year interest on that sum of money deposited in a savings bank. The person desirous of obtaining stock after it is reasonably certain that the railroad is going to prove successful, will have to pay more than $100 per share. Mr. H. pays $150 per share, or 150% of the par value.

4. Mr. H. receives 4% dividend on \$3000, or \$120. From the savings bank he would obtain 2% on \$4500, or \$90.

5. The \$120 semi-annual dividend is $2\frac{2}{3}\%$ on the \$4500 invested, equivalent to $5\frac{1}{3}\%$ per year.

6. The words "per cent" are not used in stating the price of stock.

A \$100 share at $164\frac{1}{2}\%$ is worth $\$164\frac{1}{2}$; 30 shares are worth $\$164\frac{1}{2} \times 30$.

7. Assuming the par value of each to be \$100, the first pays \$6 per year on \$150, or 4%; the second pays \$7 per year on \$175, or 4%.

The par value of \$100 is assumed for convenience; a par value of \$50 would make the cost of a share of gas stock 150% of \$50, or \$75, and its annual dividend would be 6% of \$50, or \$3, the rate on the amount invested being 4%.

8. The buyer of stock at 125 wishes to receive 4% of \$125, or \$5. As the dividend is based on the par value (\$100), the rate must be 5% per year, or $2\frac{1}{2}\%$ semi-annually.

9. $[93\frac{7}{8}\% \text{ of } (\$50 \times 17)] + [102\frac{3}{4}\% \text{ of } (\$10 \times 143)]$.

10. A person that desires to buy stocks is not always likely to know where he can find any for sale; so he goes to a stock-broker, who makes a business of buying and selling stocks on commission. This commission is a small per cent of the par value, the charge of $\frac{1}{8}\%$ for buying or selling a share of the par value of \$100 being $12\frac{1}{2}$¢, whether the actual value of the stock be \$150 or \$50.

This broker receives, therefore, $\frac{1}{8}\%$ of

$$[(\$50 \times 17) + (\$10 \times 143)].$$

11. A bond is a note issued by a corporation, and is generally secured by a mortgage on its property. It is a much larger document than the note of an individual, and frequently contains at the bottom a number of "coupons," one for each half-year's interest, upon which is engraved the date when due and the sum payable. A 10 years' U. S. 4% coupon bond for \$1000 would have 20 coupons, each worth \$20 when due. At the expiration of each 6 months, the holder of the bond cuts off the proper coupon and presents it for payment. A "registered" bond contains no coupons, a check for the interest being mailed to the owner.

Although the railroad company in this example receives only \$95 for each \$100 bond, it promises to pay \$4 interest per year, and \$100 at the end of 20 years.

In considering the rate of interest received by the owner of such a bond, it is not customary to complicate the example too much by requiring the pupils to take into account the additional \$5 above the cost received when the bond is redeemed at the end of 20 years, although buyers of bonds include it in their calculations. For our purpose, at present, it will be sufficient to assume that the purchaser of one of these \$100 bonds receives \$4 on each \$95 invested, the rate being $400 \div 95$, or $4\frac{4}{19}\%$.

12. Omitting the question of redemption, at which time the purchaser for \$116.50 would receive only \$100, the rate is $400 \div 116\frac{1}{2}$, or $3\frac{101}{233}\%$.

The holder of a U. S. bond knows that the face value of the bond will be paid in full at maturity, and that the interest payments will be made on the dates when due; in the case of the bond of a railroad, or the like, there is always the possibility that something may occur to prevent the company from meeting its obligations.

13. The rate of income from stocks may vary at each dividend period, depending upon the amount of business done, etc.; the rate of income from bonds is fixed as stated on their face.

Bonds are redeemed at the time specified; there is no reason why a successful company should sell out and divide the proceeds among its stockholders. When, however, the property of a corporation is sold, the claims of bondholders and all other obligations must be satisfied before the stockholders receive anything.

14. The stock-broker's fee is called brokerage, or commission.

15. A cotton-mill obtains material through a cotton "factor"; property is purchased through a real estate agent; a grocer may buy butter, eggs, etc., from a commission merchant, the seller in each case remitting the amount received to the owner after deducting his fee, or commission.

16. The *base* in insurance is the sum for which the property is insured; in taxes, it is the assessed value of property; in brokerage, it is the par value of stocks or bonds; in commission, it is the sum for which goods are bought or sold; the principal is the base upon which interest is calculated; the face of the note is the base in bank discount; in commercial discount, the gross price is the base in the first instance, the base for each subsequent discount being the successive remainders left after the deduction of the previous discounts; in stocks and bonds, the base is the par value.

17. The assessed value of property is the value for purposes of taxation, and is fixed annually by officers chosen for this duty, generally called assessors.

18. $2\frac{1}{4}\%$ of assessed value $(x) = \$540$; assessed value $= \$24000$. This is $\frac{2}{3}$ of the actual value, or $66\frac{2}{3}\%$.

For various reasons, the assessed value is placed below the sum that would be realized by the sale of the property under favorable conditions; but care is taken that all property is assessed upon the same basis.

19. If all property were assessed at its actual value, the same amount of taxes would be produced by a lower rate. To obtain \$540 taxes on property assessed at \$36000, the rate would be $1\frac{1}{2}\%$.

21. 1674 ft. = 558 yd.; 558 yd. $\div 5\frac{1}{2}$ yd. = 1116 half-yd. $\div$ 11 half-yd., which gives 101 rd. and 5 half-yd. = 101 rd. $2\frac{1}{2}$ yd. = 101 rd. 2 yd. 1 ft. 6 in.

22. $\$8575 \div \$245 = 35 =$ number of shares. Quarterly dividend $= 2\frac{1}{2}\%$ of $(\$100 \times 35)$.

983. 27. Each figure of the product is written two places to the right of the corresponding figure of the multiplicand.

Principal,	$375.	
3%	11.25	
	$386.25	Amount $\frac{1}{2}$ yr.
3%	11.5875	
	$397.8375	Amount 1 yr.
3%	11.9351	
	$409.7726	Amount $1\frac{1}{2}$ yr.
3%	12.2931	
	$422.0657	Amount 2 yr.
1%	4.2206	
$\frac{1}{2}$%	2.1103	
	$428.3966	Amount 2 yr. 3 mo.
Principal,	375.	
	$53.3966	Interest 2 yr. 3 mo.

Ans. $428.40, amount; and $53.40, interest.

It is not necessary throughout the work to carry the multiplication beyond four places of decimals.

985. 6. After deducting the first 25%, or $\frac{1}{4}$, $\frac{3}{4}$ of list price remains; a second discount of $\frac{1}{4}$ of this remainder leaves $\frac{3}{4}$ of this remainder, or $\frac{3}{4}$ of $\frac{3}{4}$ of list price, or $\frac{9}{16}$ of list price $= 90¢$. List price $= 90¢ \div \frac{9}{16} = 90¢ \times \frac{16}{9} = \$1.60$.

7. All together can do $(\frac{1}{2} + \frac{1}{3} + \frac{1}{6})$ of the work in 1 hr.

8. Selling price, $\$60 = \frac{3}{4}$ of value of cow; $\frac{1}{4}$ of value $= \$60 \div 3 = \$20$, the loss.

9. Selling price, $\$60 = \frac{5}{4}$ of value of cow; $\frac{1}{4}$ of value $= \$60 \div 5 = \$12$, the gain.

986. 6. The wrong weights are $\frac{15\frac{1}{4}}{16}$, or $\frac{61}{64}$ of correct weights, so that the customer receives for $352, $\frac{61}{64}$ of this amount, the gain to the grocer being $\frac{3}{64}$ of \$352, or \$16.50. By selling $16\frac{1}{2}$ oz. to the pound, the grocer gives $\frac{33}{32}$ of the proper amount, his loss being $\frac{1}{32}$ of \$320, or \$10. The net gain is \$16.50 − \$10 = \$6.50. *Ans.*

7. Cost of alcohol, \$2.50 × 42 = \$105; 3 yr. interest on \$105 = \$18.90. Amount to be realized, \$105 + \$18.90 = \$123.90. Number of gallons to be sold, 42 − 7 = 35. Selling price per gallon, \$123.90 ÷ 35 = \$3.54.

8. £4500 = \$4.85 × 4500 = \$21825. Income on consols at 3% = \$654.75. Selling price of consols, \$21825 × .96; value of U. S. bonds, \$21825 × .96 ÷ 108. Canceling, we obtain \$19400; 6% of which gives the income on bonds = \$1164. Difference = \$1164 − \$654.75 = \$509.25. *Ans.*

9. Number of cubic feet = $9\frac{2}{3} \times 9\frac{1}{3} \times 6\frac{3}{4} = 609$; weighing 609000 oz.; etc.

988. **11.** Agent collected 80% of \$4500 = \$3600; on this, his commission at $7\frac{1}{2}$% is \$270; making the amount to be given me = \$3600 − \$270.

13. $x \times \frac{5}{100} \times \frac{3}{2}$, or $\frac{3x}{40} = 15.12$.

Find the discount for 17 da., the time from June 20 to July 7.

15. Ignoring the price — if he could buy 80 lb. with $81\frac{1}{2}$% of his money, he could buy with 100%, (80 lb. ÷ $81\frac{1}{2}$) × 100.

17. Let x = cost of each cow; $6x$ = cost of 6 cows; commission $= \frac{3}{100}$ of $6x = \frac{18x}{100}$; cost and commission $= 6x + \frac{18x}{100} = 525.30$.

18. The note is discounted 35 da. after it is made, so that it has (93 − 35) da. to run, or 58 da. [Without grace, 55 da.] The interest for a year is \$36, which is 10 cents a day, or \$5.80 for 58 days; etc.

19. If the selling price (regardless of its amount) is six-fifths of the cost, the gain is $\frac{1}{5}$ of cost, or 20%.

989. 6. The walls contain $[(20 + 15 + 20 + 15) \times 10]$ sq. ft.; the ceiling contains (20×15) sq. ft. Dividing by $\frac{10}{12}$, the width in feet of the paper, gives the number of feet of paper required, which is then reduced to yards.

991. 7. Dividend is $3\frac{1}{2}\%$ of $(\$9562.50 \div 1.27\frac{1}{2})$.

993. Mr. Smith wishes to pay Mr. Thompson the exact amount of his bill. A check on a Memphis bank for \$3475.86 would not be sufficient, as Mr. Thompson would have to pay a New York bank for collecting the check in Memphis. As the charge may not always be the same, Mr. Smith cannot know how much to add to the amount of his bill to cover this expense. From a banker that has an account in a New York bank, he can obtain a draft, payable in that city, for the exact amount, by giving the Memphis banker \$3475.86 + \$5.21, or \$3481.07.

Exchange is at a *premium* when the cost of a sight draft is greater than its face; it is at a *discount* when the cost of a sight draft is less than its face.

Bank Check.

Quogue, N.Y., Jan. 3, 1896. *No. 1492.*

SHINNECOCK NATIONAL BANK.

Pay to the order of *Lewis K. Thurlow,* *\$68$\frac{35}{100}$*

Sixty-eight $\frac{35}{100}$ *Dollars*

HOWELL & PENNIMAN.

An examination of the above check will show wherein it differs in form from the draft. A draft may be made payable at a future time, whereas a check is always payable on presentation.

8. $\frac{3}{40}\% = \frac{3}{4000}$; $x - \dfrac{3x}{4000} = 632.18$. **9.** $x + \dfrac{3x}{200} = 1000$.

10. $339.66 – (2% of $339.66) = sum remaining for the purchase of dry-goods, etc., and the commission. Dividing this by 1.02 gives the cost of the goods.

Another method of solving the foregoing is to indicate the money remaining as $339.66 × .98. Using 1.02 as a divisor, and canceling, gives the result.

$$\frac{339.66 \times 98}{102}.$$

994. **2.** Noon Monday to 6 P.M. Thursday = 78 hr. The loss in time = 35 sec. × 78 = 45 min. 30 sec. The time shown is 6 hr. – 45 min. 30 sec. = 5 hr. 14 min. 30 sec., or $14\frac{1}{2}$ min. past 5.

4. The number of rows, 2 ft. apart in a space 36 ft. – 4 ft., is $(32 \div 2) + 1 = 17$. The number of plants, 16 in. apart in a row 60 ft. – $2\frac{2}{3}$ ft., or $57\frac{1}{3}$ ft., in length, is $(57\frac{1}{3} \div 1\frac{1}{3}) + 1 = 44$. Total number of plants $= 44 \times 17 = 748$.

If the rows run crosswise there will be 29 of them, each containing 26 plants.

6. Number of revolutions = 14 mi. ÷ 13 ft. 4 in.

995. **8.** Length of a degree on the equator = 25000 mi. ÷ 360. 20° will measure (25000 ÷ 360) × 20. Cancel.

9. The circumference = 18 ft. × 3.1416. Divide by 360. Cancel.

10. The difference is 20°, and the distance will be about one-half that found in **8**.

The teacher should remember that the shortest distance between these two places is not measured on the parallel of 60°. The shortest distance between two points on a sphere is measured by the arc of a *great* circle joining the points, and the 20° are $\frac{1}{18}$ of a *small* circle.

11. 46° 22′ 30″ = $46\frac{3}{8}$°. The number of miles $= 69\frac{1}{4} \times 46\frac{3}{8}$.

12. The approximate length of the 45th parallel is 25000 mi. × .7071; the length of a degree on this parallel = (25000 mi.

$\times .7071) \div 360$; multiplying by $22\frac{1}{2}$ gives the required distance. Cancel.

996. Time drafts are so little used that it is scarcely worth while to spend much time on their study.

A sight draft being payable on presentation (except in those states allowing days of grace), there is no need of formal acceptance. Acceptance is necessary in the case of time drafts, as they are not payable until the specified time after this acceptance.

The acceptance of a draft makes the person or corporation accepting it liable to its owner for the amount, a draft being transferable by endorsement just as a check or a note.

997. In calculating the cost of a sight draft, days of grace — even when allowed — do not enter into the result, this being included in the rate charged. Time drafts are allowed days of grace, except in the states given in the Appendix, Art. 1305. The number of states in which days of grace are no longer allowed, increases yearly, there being no good reason for promising to pay in 60 da. when the intention of the signer is to take 63 da.

1000. 1. Although days of grace are not allowed in California, the pupils of other states should not be expected to know this. In states that grant days of grace, they should be allowed in every note or time draft, no matter where payable; while in the other states, pupils should be taught not to employ them in any case.

The premium on the draft $= \$1.75 \times .840 = \$1.47$. The interest (with days of grace) $= \$840 \times \frac{6}{100} \times \frac{93}{360} = \$13.02$. The cost of the draft $= \$840 + \$1.47 - \$13.02 = \$828.45$. *Ans.*

Or, without days of grace:

$\$840 \times \frac{6}{100} \times \frac{90}{360} = \$12.60$, the cost being $\$840 + \$1.47 - \$12.60 = \$828.87$. *Ans.*

Some teachers prefer to find the cost of a draft for \$1 at the given premium — in this case, \$1.00175; from which is deducted the interest on \$1 for 93 da., or \$.0155; making the cost of a

90-day draft for $1, $1.00175 − $.0155 = $.98625. Multiplying this by 840 gives $825.45, the cost of a draft for $840. This method is not so much shorter as to make it advisable to use it.

2. The discount $= \frac{1}{8}\%$ of $400 $= \frac{1}{800}$ of $400 = 50¢. The interest for 33 da. = $400 $\times \frac{6}{100} \times \frac{33}{360}$ = $2.20. Cost = $400 − $.50 − $2.20 = $397.30. *Ans.*

NOTE. — The word "interest" is used instead of "bank discount," to avoid the confusion arising from the use of "discount" with two meanings in the same example.

5. The six remaining examples should be omitted by pupils that do not use algebraic methods of solution. Scholars that have readily worked the first four examples will find no great difficulty in solving **5–10** by means of the equation.

The premium is $\$\frac{1}{4}$ per $1000, or $\frac{1}{4000}$ of the face of the draft, x. Premium $= \frac{x}{4000}$. The interest on x dollars for 63 da. at $6\% = x \times \frac{6}{100} \times \frac{63}{360} = \frac{21x}{2000}$. Adding the premium to the face of the draft, and deducting the interest, gives $x + \frac{x}{4000} - \frac{21x}{2000}$ as the cost of the draft. This may be changed to

$$\frac{4000x + x - 42x}{4000} = \frac{3959x}{4000}. \quad \textit{Ans.}$$

6. $\frac{1}{8}\%$ of $1200 = $1.50; interest for $(x+3)$ da. $= 1200 \times \frac{6}{100} \times \frac{x+3}{360} = \frac{x+3}{5}$; cost of the draft (with days of grace), $1200 - 1\frac{1}{2} - \frac{x+3}{5} = \frac{12000}{10} - \frac{15}{10} - \frac{2x}{10} - \frac{6}{10} = \frac{11979 - 2x}{10}$. *Ans.*

NOTE. — Unless the pupil has studied algebraic subtraction in Chap. XV., he may make a mistake in deducting $\frac{x+3}{5}$, by failing to change the sign of the second term. By writing $\frac{x+3}{5}$, $\frac{x}{5} + \frac{3}{5}$, he may see more clearly that the cost of the draft is $1200 - 1\frac{1}{2} - \frac{x}{5} - \frac{3}{5}$, etc. Without days of grace, cost $= 1200 - 1\frac{1}{2} - \frac{x}{5}$; etc.

1001. 1. $15° \times 3\frac{1}{3} = 50°$. *Ans.*

2. $(61 \div 15)$ hr. $= 4\frac{1}{15}$ hr. $= 4$ hr. 4 min. *Ans.*

3. Difference in time $= \frac{75}{15}$ hr. $= 5$ hr. London time $= 1$ P.M. $+ 5$ hr. $= 6$ P.M. *Ans.*

4. 2 P.M. $- 5$ hr. $= 9$ A.M. *Ans.*

5. Vienna is $\frac{25}{15}$ hr. later $= 1\frac{2}{3}$ hr. $= 1$ hr. 40 min. Time at Vienna 40 min. after 1 P.M., or 20 min. to 2 P.M. *Ans.*

6. 3 hr. 40 min. $= 3\frac{2}{3}$ hr. Difference in longitude $= 15° \times 3\frac{2}{3} = 55°$. *Ans.*

7. Difference in longitude $= 75° + 30° = 105°$. *Ans.*

8. Philadelphia time is $\frac{105}{15}$ hr. earlier, or 7 hr. 3 P.M. $-$ 7 hr. $= 8$ A.M. *Ans.*

9. Correct Washington time is $\frac{2}{15}$ hr., or 8 min. earlier than standard time.

10. A town in 84° west longitude is 6° east of 90°, so that its correct time is $\frac{6}{15}$ hr., or 24 min., later. Time, 12:24 P.M. *Ans.*

1002. 1. Longitude difference $= 15° \times 3\frac{44}{60}$. The pupil should see that $15 \times \frac{44}{60} = 44 \div 4 = 11$; so that $15° \times 3\frac{44}{60} = 45° + 11° = 56°$. *Ans.*

2. 15)37 hr. 18 min.
  2 hr. 29 min. 12 sec.

At 1 hr. to a degree, the difference in time would be 37 hr. 18 min.; as it requires 15° to make an hour's difference, dividing 37 hr. 18 min. by 15 gives the result.

Shorter methods should be deferred for the present. Using multiplication to obtain the difference in degrees, and division to obtain the difference in time, is more easily understood by beginners.

3. Time difference $= \frac{1}{15}$ of 87 hr. 35 min. earlier at Chicago, because it is west of Greenwich. Standard Chicago time is the time of 90°, or 90 hr. $\div 15 = 6$ hr. earlier than Greenwich. Standard time $= 1$ P.M. $- 6$ hr. $= 7$ A.M. *Ans.*

4. Vessel's time is $2\frac{1}{2}$ hr. earlier, showing that the vessel is $15° \times 2\frac{1}{2}$, or $37\frac{1}{2}°$ west of Greenwich. *Ans.* 37° 30′.

6. Time difference $= 1\frac{1}{2}$ hr. Longitude difference $= 15° \times 1\frac{1}{2} = 22\frac{1}{2}°$. The latter place, having the later time, is the more easterly; so that its longitude is $22\frac{1}{2}°$ east of 11° east, or 33° 30′ east. *Ans.*

7. 3 da. 12 hr. 17 min. $= 84\frac{17}{60}$ hr.; $3313.5 \div 84\frac{17}{60} =$ number of miles per hour.

11. $12\frac{1}{2}$ (ft.) $\times 3\frac{3}{4}$ (ft.) $\times x$ (ft.) $= 730\frac{810}{1728} = 730\frac{15}{32}$ (cu. ft.); $\frac{25}{2} \times \frac{15}{4} \times x = \frac{23375}{32}$; $x = (\frac{23375}{32} \div \frac{25}{2}) \div \frac{15}{4} = \frac{23375}{32} \times \frac{2}{25} \times \frac{4}{15} = \frac{187}{12} = 15\frac{7}{12}$. *Ans.* $15\frac{7}{12}$ ft. = 15 ft. 7 in.

12.

	48%	= 237 bu. 3 pk.	
	+ 4%	= 19 bu. 3 pk. 2 qt.	$\frac{1}{12}$ of 48%
Remainder,	52%	= 257 bu. 2 pk. 2 qt. *Ans.*	

14. Number of degrees $= (34 \times 24) \div 48.96$.

1003. **5.** 84 half-dollars − 84 cents = \$42.00 − \$.84.

9. After taking $\frac{1}{3}$, $\frac{2}{3}$ are left; when $\frac{1}{3}$ of the remainder is taken, $\frac{2}{3}$ of remainder are left, or $\frac{2}{3}$ of $\frac{2}{3} = \frac{4}{9} = 4$ gal.; etc.

1004. **5.** 12 men working 8 hr. daily build 90 rd. in 15 da.; 7 men working 10 hr. daily build 70 rd. in ? days.

$$\frac{15 \text{ da.} \times 12 \times 8 \times 70}{90 \times 7 \times 10}.$$

9. 72 (in.) × 48 (in.) × x (in.) = 2150.4 (cu. in.) × 75.

16. $x + (x + 15) + (x + 15 + 27) = 320$.

17. .64 bu. = 4 pk. × .64 = 2.56 pk.; .56 pk. = 8 qt. × .56 = 4.48 qt.; 3.64 bu. = 3 bu. 2 pk. 4.48 qt.; $\frac{9}{16}$ bu. = 4 pk. × $\frac{9}{16}$ = $\frac{9}{4}$ pk. = $2\frac{1}{4}$ pk. = 2 pk. 2 qt.; 3 bu. 2 pk. 4.48 qt. + 2 pk. 2 qt. + 1 bu. 3 pk. 6.52 qt. = 6 bu. 5 qt.; 10 bu. − 6 bu. 5 qt. = 3 bu. 3 pk. 3 qt. *Ans.*

18. Each step takes $7\frac{1}{2}$ in. + 10 in. = $17\frac{1}{2}$ in. = $\frac{17\frac{1}{2}}{36}$ yd. Cost = 90¢ × $\frac{17\frac{1}{2}}{36}$ × 18. Cancel.

24. $x + (x + 1211) = 9891$.

1005. Formerly, bills of exchange were issued to purchasers in sets of three bills, two of which were sent by different steamers to the foreign payee, who presented for payment or acceptance the one that reached him first. The third bill was retained by the purchaser, to be sent in case both of the others failed to reach their destination. At present, only two bills of a set are issued. The second will read as follows:

Exchange for £180 17*s.* 6*d.* NEW YORK, Dec. 14, 1895.

Sixty days after sight of this Second of Exchange (First unpaid) pay to the order of John W. Moran & Bro., One Hundred Eighty pounds sterling, seventeen shillings, six pence.

Value received, and charge the same to account of

To JAMES LENNON & Co.,
London.
No. 39.

PETER COMERFORD & SON.

1. No deduction for interest is made for the 60 da., the quotation giving the price per pound for 60-day bills.

The method given in the text-book is a form of the aliquot part method used in calculating interest.

1007. 3. Cost in dollars $= 1000 \div 5.1625$.

4. Cost in dollars $= 1874.35 \times .9525 \div 4$.

5. Number of marks $= 1000 \div (.955 \div 4) = 4000 \div .955$.

6. Number of francs $= 1637.5 \times 5.185$.

8.

	£437	5*s.*	10*d.*
Less 4%, or $\frac{1}{25}$,	17	9*s.*	10*d.*
	£419	16*s.*	

Cost $= \$4.885 \times 419\frac{16}{20} = \$4.885 \times 419.8$.

9. 18 pcs., 44 m. each, or 792 m.

@ fr. 25 =	fr. 19800	
Less $7\frac{1}{2}$%,	1485	fr. 18315
3 pcs. 50 m. each, or 150 m. @ fr. 20,	fr. 3000	
Less 5%,	150	2850
Packing charges,		60.50
		fr. 21225.50

Cost of bill $= 19\frac{1}{2}¢ \times 21225.5 = \$4138.97$. *Ans.*

10.

	M. 3598.60
Less 10%,	359.86
	M. 3238.74
Less 5%,	161.94
	M. 3076.80
Less $2\frac{1}{2}$%,	76.92
	M. 2999.88
Freight, 165 kilos @ M. 4.80,	792.00
	M. 3791.88

$95\frac{7}{8}¢ \times 3791.88 \div 4 = \$908.87$. *Ans.*

UNIVERSITY OF CALIFORNIA

XVI

NOTES ON CHAPTER THIRTEEN

1008. The word "endorsement" means something that is written on the back of a document. As applied to notes, checks, drafts, etc., it generally means the signature of the person in whose favor the note, check, etc., is made out, which is written on the back in order to transfer the ownership. If the payee of the following note sells it to William Simms, he writes his name on the back, as shown below.

Accotink, Va., March 4, 1897.

Four months after date, I promise to pay to the order of James McWilliams, Two Hundred Dollars, value received, at the Pohick National Bank.

$\$200\frac{00}{100}$.   Victor Struder.

(Endorsement in blank.)

James McWilliams

(Endorsement in full.)

Pay to the order of
William Simms
James McWilliams

The effect of the "endorsement in blank" is to make it payable to any holder; the "endorsement in full" transfers it to William Simms, who may transfer it to another by either kind of endorsement.

Besides transferring ownership in a note, the effect of an endorsement is generally to bind the signer to pay the note, in case of default by the maker or preceding endorsers. This liability is avoided if the endorser writes after his name the words, "without recourse."

The "endorsements" mentioned in this chapter are a record of the payments received by the holder of the note. This is usually kept on the back of the note, the date and the amount received being written in each instance.

1010. Although the maker of a note is generally supposed to pay the interest at the end of each year, the U. S. Courts, by whom this rule has been formulated, do not permit the collection of interest upon deferred payments of interest.

This rule is followed in all the states except Connecticut (see Art. 1307) for computing the amount due on notes that do not expressly provide for the payment of interest annually (Art. 1172). Connecticut pupils should learn only their own rule; in other states, no reference whatever to the Connecticut rule should be made.

See Art. 1307 for Connecticut-rule answers to the partial payment examples of this chapter.

1013. 6. Let $100x =$ cost of coal; $2x =$ commission.

$102x =$ cost of coal $+$ commission $= 7650$.

1015. 1. The man expended 30% of 50% of $\frac{3}{5}$ of his money, or $\frac{3}{10} \times \frac{1}{2} \times \frac{3}{5}$ of it; which was $\frac{9}{100}$ of his money. $\frac{9}{100}x = 1\frac{1}{8} \times 728 = 819$; $x = 9100$ Two-fifths of \$9100 equals the balance in bank.

2. Cost per gallon, \$1.50; selling price, \$1.60; gain per gallon, 10¢ on 150¢, or $\frac{1}{15} = 6\frac{2}{3}\%$.

5. Let $10x =$ cost, then x will represent the loss in one case and the gain in another, making the selling prices $11x$ and $9x$.

$11x = 99$; $x = 9$, gain in dollars.

$9x = 99$; $x = 11$, loss in dollars, making the net loss \$2.

NOTE.—Some teachers, wishing to avoid fractions as far as possible in equations, assume x for loss or gain, making $10x = \text{cost}$; etc. Solutions of this kind are given occasionally as a suggestion to be followed or not, as may seem most desirable.

8. Arithmetic, Art. 924, **7** and **8.**

1016. As there is no such thing as "true discount," it is unprofitable to spend time upon it. Any problem involving finding the "present worth" can be solved by an intelligent pupil, from his previous work in interest.

1019. **2.** To $1\frac{1}{2}\%$ of $\frac{3}{4}$ of \$25000, add \$1.

3. Longitude difference $= 5° 59' 18''$. See Art. 1002, **2.**

10. Number of yards $= (5616 \div 1.04) \div 1\frac{1}{2}$. Art. 1013, **6.**

11. Term of discount 36 (33) days. Yearly interest is \$30, which is \$3 for 36 da., $\frac{1}{10}$ yr. \$500 − \$3 = proceeds, \$497. Without grace, the term is 33 days, the interest for which time is \$2.75. Proceeds, \$497.25.

13. Cost of an acre $= \$21.78 \times 5$. Cost per square foot $= (\$21.78 \times 5) \div (4840 \times 9)$, the divisor being the number of square feet in an acre. To gain 20% or $\frac{1}{5}$ of cost, the selling price must be $\frac{6}{5}$ of cost. Multiplying the foregoing by $\frac{6}{5}$, the selling price per foot will be

$$\frac{\$21.78 \times 5 \times 6}{4840 \times 9 \times 5}.$$

In getting the number of square feet in an acre, the pupil may use $160 \times 30\frac{1}{4} \times 9$, unless he remembers that there are 4840 sq. yd. in an acre.

14. Specific duty (duty by weight) $= \$\frac{1}{2} \times 700 \times 1\frac{1}{4}$. Advalorem duty $= 30\%$ of $\$1\frac{1}{4} \times 700$. The sum of the two gives the total duty.

1020. **7–9.** See Arithmetic, Art. 384.

1021. **4.** (50 ft. + 38 ft.) — (7 ft. + 2 ft.).

5. $\frac{9}{8}$ of cost of horse = \$90. Cost = \$80. A selling price of \$100 makes a gain of \$20 = $\frac{1}{4}$ of cost = 25%. *Ans.*

6. Loss $= \frac{1}{5}$ of cost $= 20\%$. *Ans.*

12. 100%. **15–16.** See Art. 878. **19.** $\frac{1}{4}\%$ of \$700. **20.** \$10 for 60 da. $+ \frac{1}{12}$ of \$10 for 5 da. **21.** 200%. **22.** $\frac{7}{20}$ of \$7. **26.** $4\frac{2}{10} \div \frac{3}{10} = 42 \div 3$.

28. $\frac{7x}{8} = \frac{7}{9}$ of $63 = 49$; $\frac{x}{8} = 7$; $x = 56$.

29. $\frac{2x}{3} = 8\frac{1}{3} = \frac{25}{3}$, $2x = 25$; $x = 12\frac{1}{2}$. *Ans.* 12 yr. 6 mo.

30. 5 qt. $= \frac{5}{32}$ pk.; $\frac{5}{8} = .625$; $.625 \div 4 = .156\frac{1}{4} = .15625$.

1022. **1.** $\$\frac{3}{4} \times 5\frac{1}{2}$ (yd.) $\times 4$ (yd.) $\div 1\frac{1}{2}$ (yd.).
$\$\frac{3}{4} \times \frac{11}{2} \times \frac{4}{1} \times \frac{2}{3}$. Cancel.

2. What sum in 4 years at 6% will amount to \$105.71? Arithmetic, Art. 1017.

$$x + (x \times \tfrac{6}{100} \times 4) = 105.71;\ x + \frac{24x}{100} = 105.71;$$

$$100x + 24x = 10571;\ 124x = 10571;\ x = 85.25.$$

\$85.25. *Ans.*

3. Term $= 27$ da. $+ 3$ da. $= \frac{1}{12}$ yr.

$$x - (x \times \tfrac{6}{100} \times \tfrac{1}{12}) = 95;\ x - \frac{x}{200} = 95;\ \text{etc.}$$

4. Problems of this kind may be solved without finding the cost; 18¢ per yard represents $\frac{9}{10}$ of cost; what price will represent $\frac{6}{5}$ or $\frac{12}{10}$ of cost? If $\frac{9}{10}$ of cost $=$ 18¢, $\frac{1}{10}$ will equal 2¢, and $\frac{12}{10}$ will equal 24¢. *Ans.*

5. $24\frac{1}{2}\% = \frac{49}{200}$; $\frac{49x}{200} = 1372$; $x = 5600$. The whole real estate was worth \$5600; the part remaining after the sale of \$1372 worth $=$ \$5600 $-$ \$1372 $=$ \$4228. \$14000 $+$ \$4228 $=$ \$18228. *Ans.*

6. 6 times (5×5) sq. ft.

7. Mr. Jones paid $\frac{3}{4}\%$ of $\frac{5}{6}$ of \$48000 $= \frac{\$48000}{1} \times \frac{5}{6} \times \frac{3}{400}$ $=$ \$300. The company loses \$40000 less the premium received, \$300.

8. This may be solved without finding the cost, although many pupils will prefer the more tedious way. $764.40 represents 91% of cost; what per cent does $894.60 represent?

$$\frac{91\% \times 89460}{76440} = \frac{213\%}{2} = 106\tfrac{1}{2}\%\text{; gain } 6\tfrac{1}{2}\%.$$

10. Cancel. $\$3\frac{7}{8} \times 25 \times 8 \times 8 \div 128$.

11. Total cost, $252.50; loss = $252.50 − $141.40 = $111.10, which is 44% of the total cost.

12. $[14\text{ (yd.)} + 5 + 14 + 5] \times 3$.

14. Profit $2 per ton, $\frac{1}{3}$ of cost. Cost = $\$198 \times 3$.

15. Art. 546. A bill is receipted by writing the words, "Received payment" at its foot, followed by the date and the name of the seller:

Recd. payment, Oct. 3, 1897,
Abraham & Straus,
per J. S.

If the money is received by a clerk, he writes his initials underneath, preceded by the word "per" or "by."

16. $\dfrac{8\frac{3}{4}}{x} = 9\frac{4}{5}$; $9\frac{4}{5}x = 8\frac{3}{4}$; etc.

17. Omit $4\frac{1}{2}$ bu. I gave away $\frac{1}{3}$ and $\frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$. The remainder $= \frac{4}{15} = .26\frac{2}{3} = 26\frac{2}{3}\%$.

21. $3 per yd. = 80% of cost; $? = 115%.

22. Number of gallons sold = (65 gal. × 60) − 80 gal. = 3820 gal. Selling price per gallon = $\frac{9}{8}$ of $\$1542 \div 3820$. Cancel.

23. $\$180 = \frac{4}{5}$ cost of one horse $= \frac{6}{5}$ cost of other.

24. Number of square yards in walls $= (6 + 4 + 6 + 4) \times 3$; in ceiling, 6×4. Number of cords $= (18 \times 12 \times 9) \div 128$.

25. Let x = less, $x + \frac{4}{5}$ = greater; $x + x + \frac{4}{5} = 4\frac{1}{5}$.

26. 16 cu. yd. is $x\%$ of $(10 \times 8 \times 2)$ cu. yd.?

27. 672 yd. @ $\$2\frac{1}{4} = \$1512$. Discount without grace $= \$1512 \times \frac{7}{100} \times \frac{1}{6} = \$17.64$. Profit is \$1 per yard less discount $= \$672 - \$17.64 = \$654.36$. *Ans.*

The discount for 3 days' additional (grace) $= \frac{1}{20}$ of $\$17.64 = \$.88$, making the profit 88¢ less than the above, or \$653.48. *Ans.*

28. $40¢ \times [(55 \times 600 \times 5\frac{1}{2}) \div 27]$. Cancel.

1023. **5.** The circumference of the wheel = distance traveled in 1 revolution = 1 mi. 94 rd. 2 yd. 1 ft. ÷ 526 = 6838 ft. ÷ 526 = 13 ft. = 4 yd. 1 ft.

6. The weight in pounds $= \frac{1000}{16} \times 9\frac{2}{3} \times 9\frac{1}{3} \times 6\frac{3}{4}$.

8. Rate of income received on 6% bonds $= 6 \div 1.18$; rate on $4\frac{1}{2}\%$ bonds $= 4\frac{1}{2} \div \frac{x}{100}$. The income being the same, and the same amount being invested, the rates must be equal; therefore, $\frac{600}{118} = \frac{450}{x}$; $600x = 118 \times 450 = 53100$; $x = 88\frac{1}{2}$. Price per \$100 = \$88.50.

9. The shrinkage being 1 lb. in 10, he must sell 9 lb. for the cost of 10 lb. to suffer no loss. 10 lb. cost \$1.80; by charging 20¢ per pound, he receives the cost. To gain 20%, he must sell for $\frac{6}{5}$ of 20¢, or 24¢ per pound. Since he loses 4% of the amount of sales, or $\frac{1}{25}$, he receives only $\frac{24}{25}$ of the price charged per pound. Therefore to receive 24¢, he must charge $24¢ \div \frac{24}{25} = 25¢$ per pound.

$$[(18¢ \div \tfrac{9}{10}) \times \tfrac{6}{5}] \div \tfrac{24}{25}.$$

1024. **1.** $12 \times x = 20 \times \frac{3}{4}$.

2. This may be solved by analysis, or the following method may be employed:

The solid contents of first beam in cubic feet $= 16 \times 2\frac{1}{4} \times \frac{2}{3}$; of the second $= x \times 3\frac{1}{4} \times \frac{15}{24}$. The second weighs $\frac{2028}{1280}$ times the first; its contents, therefore, $= \frac{2028}{1280}$ times the contents of the first.

$$x \times 3\tfrac{1}{4} \times \tfrac{15}{24} = 16 \times 2\tfrac{1}{4} \times \tfrac{2}{3} \times \tfrac{2028}{1280};$$
$$x = (16 \times 2\tfrac{1}{4} \times \tfrac{2}{3} \times \tfrac{2028}{1280}) \div (3\tfrac{1}{4} \times \tfrac{15}{24})$$
$$= 16 \times \tfrac{9}{4} \times \tfrac{2}{3} \times \tfrac{2028}{1280} \times \tfrac{4}{13} \times \tfrac{24}{15}. \quad \text{Cancel.}$$

3. The carpet costs 50¢ per foot. $\$\frac{1}{2} \times 22\frac{1}{2} \times 15\frac{3}{4} \div 2\frac{1}{4} = Ans.$ Or, changing all dimensions to yards: $\$1\frac{1}{2} \times 7\frac{1}{2} \times 5\frac{1}{4} \div \frac{3}{4} = Ans.$

4. As a sight example, some pupils may see that the width of the large box is double that of each small one, and its depth is three times that of each small one, so that with the same length as the small one, it would contain 2×3, or 6, small ones. A length twice as great — $8\frac{1}{3}$ ft. — is required to enable it to hold 12 boxes.

6. $[\frac{1}{2}$ of $(18\frac{1}{3} \times 11\frac{5}{6})]$ sq. ft.

8. $\dfrac{22 \times 14 \times 12 \times 1728}{2150.4}.$

Drop the decimal point in the denominator, and annex a cipher to one of the numbers in the numerator. Cancel.

9. $49 \times 44 \times 27 \div 231 =$ number of gallons. Cancel.

10. Solve at sight. 7 yd., 6 yd., 4 yd.

11. Number of gallons $= 5\frac{1}{2} \times 6 \times 7 \times 1728 \div 231$. Canceling, we obtain 1728 gal. One empties it in $(1728 \div 9)$ min. $= 192$ min.; the other in $(1728 \div 7)$ min. $= 246\frac{6}{7}$ min.; both in $(1728 \div 16)$ min. $= 108$ min.

12. The dimensions of the room are 6 yd. and 5 yd., and the carpet is $\frac{3}{4}$ yd. wide. 6 contains $\frac{3}{4}$ an exact number of times (8), so that if the carpet runs across the room it will take 8 strips each 5 yd. long. As $5 \div \frac{3}{4} = 6\frac{2}{3}$, to lay the carpet lengthwise would require 6 strips, and $\frac{2}{3}$ of a seventh strip, which would have to be cut.

Carpet 30 in. wide, $\frac{5}{6}$ yd., could be laid lengthwise without splitting the breadths, $5 \div \frac{5}{6}$, or 6, strips being needed, each 6 yd. long.

13. 36 yd. are needed to cover the floor; including $4\frac{1}{2}$ yd. cut off in matching the pattern, $40\frac{1}{2}$ yd. must be bought at 95¢. At 10¢ per yard, the sewing and laying should cost $10¢ \times 36 = \$3.60$, but the custom is to charge for the number of yards purchased, $40\frac{1}{2}$, making \$4.05; (5×6) sq. yd. or 30 sq. yd. @ 5¢ = \$1.50 for lining. Total cost, $\$38.47\frac{1}{2} + \$4.05 + \$1.50 = \$44.02\frac{1}{2}$, or \$44.03.

14. A strip $\frac{1}{2}$ of $(18 - 15)$ or $\frac{1}{2}$ of $(21 - 18)$ is left uncovered on each of the four sides, or $1\frac{1}{2}$ ft. The area of the uncovered space in square feet $= (21 \times 18) - (18 \times 15)$.

15. The number of square feet in the walls $= (18 + 24 + 18 + 24) \times 9$. The ceiling contains (18×24) sq. ft. Deduct 60 sq. ft. for two doors, 48 sq. ft. for two windows, 25 sq. ft. for the fireplace. The total number of feet around the four walls $= 18 + 24 + 18 + 24 = 84$ ft. Baseboard will not be required at the doors, 8 ft.; nor at the fireplace, 5 ft. — a deduction of 13 ft., making 71 running feet of baseboard, 1 ft. wide, containing, therefore, 71 sq. ft. The total deduction from the area of walls and ceiling, 1188 sq. ft., are 60 sq. ft. + 48 sq. ft. + 25 sq. ft. + 71 sq. ft. = 204 sq. ft., leaving 1188 sq. ft. − 204 sq. ft., or 984 sq. ft., to be plastered.

16. The first pile contains $(25 \times 20 \times 10)$ cu. ft. and costs \$1400. 1 cu. ft. cost $\$1400 \div (25 \times 20 \times 10)$. Multiplying by $(50 \times 40 \times 20)$, the number of cubic feet in the second pile, gives the cost:

$$\frac{\$1400 \times 50 \times 40 \times 20}{25 \times 20 \times 10}.$$

17. Pupils that endeavor to solve a problem without examining the conditions, will be likely to assume that this example resembles **16**. In the latter, the cost of the second pile is 8 times the cost of the first; in this one, the surface to be painted in the second room is 4 times that of the first room, making the cost \$56. As they may not be familiar enough with similar surfaces to know the ratio, they should find the surface of each.

1025. In dividing decimals, change the divisor to a whole number. See Arithmetic, Art. 663.

1026. **2.** $\overline{\text{XXV}} = 25000$.

5. The furlong is seldom used. 3.7082 mi. $\div$ 4 = .92705 mi., the length of one side. Multiplying by 8 to reduce to furlongs, we obtain 7.4164 fur. Change the decimal part, .4164 fur., to rods by multiplying by 40, obtaining 16.656 rd.; .656 rd. = 3.608 yd.; etc.; etc.

6. See Art. 986, **7.** That selling price, \$3.54, must be increased $\frac{1}{6}$, or \$.59, to gain $16\frac{2}{3}\%$. \$3.54 + \$.59 = \$4.13. *Ans.*

7. $\$\frac{5}{8} \times \frac{240 \times 38 \times 9}{27}$. Cancel.

10. The inventors of the expression "true discount" assume that interest is not payable in advance. They claim that a borrower that promises to pay \$380 at the end of 2 yr. 5 mo. should receive as a loan the principal that will amount to \$380 in that time.

$$\text{Let} \quad x = \text{principal.}$$

$$\text{Interest} = x \times \frac{6}{100} \times \frac{29}{12} = \frac{29x}{200}.$$

$$\text{Amount} = x + \frac{29x}{200} = 380;$$

$$200x + 29x = 229x = 76000;$$

$$x = 331.88 -.$$

The principal ("present worth") = \$331.88.

The "true discount" = \$380 − \$331.88 = \$48.12.

This "true discount" is the interest at 6% on \$331.88 for 2 yr. 5 mo.

The interest on \$380 at 6% for 2 yr. 5 mo. is \$55.10; the difference between the interest and the "true discount" being \$55.10 − \$48.12 = \$6.98. *Ans.*

1027. **1.** A gain of 25%, or $\frac{1}{4}$ of cost, makes the selling price \$9, equal $\frac{5}{4}$ of cost; $\frac{1}{4}$ of cost, the present gain, is \$9 $\div$ 5, or \$1.80. A gain of 50% would be \$1.80 × 2 = \$3.60.

N. B. — It is not necessary to find the cost.

2. \$30 in one case represents $\frac{5}{4}$ of cost, making the gain \$6; in the other case, \$30 represents $\frac{3}{4}$ of cost, making the loss \$10. Net loss \$4.

NOTE.—The thoughtful teacher will recollect that every member of the class does not "see through" an example in the same way, nor with equal rapidity. While quick work should be exacted where the question involves but a single arithmetical operation, time should be given, in problem work, to pupils that do not quickly grasp the conditions. Such as can dispense with unnecessary figures should be encouraged to do so as much as possible; but care should be taken not to injure others by requiring them to adopt a short method whose underlying principles they do not thoroughly understand. Each should, to a certain extent, be allowed to use his own mode of "analyzing" oral problems and of setting down his written ones; shorter ways, however, being presented from time to time in the oral and blackboard work of his brighter classmates. The scholar that reaches his results by a circuitous course will, by these models, be led to see the time saved by shorter methods, and he will probably try to master some of them.

3. Together they have $\$300 = x + 2x$.

6. \$40 in $3\frac{1}{3}$ yr. = \$12 per year. This is produced at 6% by \$200. *Ans.*

7. The interest of x dollars for 5 yr. at $6\% = \frac{3x}{10}$; $x + \frac{3x}{10}$, or $\frac{13x}{10} = 52$; $\frac{x}{10} = 4$; $x = 40$.

8. Yearly interest = \$3. To obtain \$18 interest will require 6 yr.

9. The interest is \$2, $\frac{1}{6}$ of principal in $3\frac{1}{3}$ yr.; in 1 yr. it is $\frac{1}{6} \times \frac{3}{10}$ of principal, or $\frac{1}{20} = 5\%$.

Or, the interest on \$12 @ 1% for a year is 12¢, or 40¢ for $3\frac{1}{3}$ yr.; to obtain \$2.00 interest, which is 5 times as much, the rate must be 5%.

10. I lose \$25, or $\frac{1}{3}$ of cost $= 33\frac{1}{3}\%$.

1028. **4.** $23\frac{1}{8} + (23\frac{1}{8} + 3\frac{3}{4}) + (23\frac{1}{8} + 3\frac{3}{4} + 3\frac{1}{8})$.

6. Let $3x$ represent the amount received by one; and $4x$ the amount received by the other. Then, $7x = 21.63$; $x = 3.09$; $4x = 12.36$, and $3x = 9.27$. *Ans.* \$9.27 and \$12.36.

7. $[(26 \times 12) \times (4 \times 12)] \div (8 \times 4)$.

8. $\dfrac{\$10.24 \times 2700 \times 890}{1500 \times 356}$. Cancel.

1044. In addition to the details usually given in a bill, an invoice shows the marks and the numbers placed upon each package shipped. In this invoice, the mark is given in the first column, and is the same on each case. The number of each case is written in the second column. Besides informing the receiver in what case a particular article is packed, it is required by the U. S. custom authorities. A certain percentage of cases in each invoice is examined, and their contents must agree in description, quantity, value, etc., with the invoice.

1500 yd.	@ $1\frac{7}{8}d.$,	£11 – 14 – $4\frac{1}{2}$
1500 "	" 2 "	12 – 10
3000 "	" $1\frac{7}{8}$ "	23 – 8 – 9
2889 "	" $2\frac{5\frac{1}{2}}{12}$ "	29 – 11 – $10\frac{1}{4}$
		£77 – 4 – $11\frac{3}{4}$
	Less $\frac{1}{40}$,	1 – 18 – $7\frac{1}{2}$
		£75 – 6 – $4\frac{1}{4}$

Value in U. S. money, \$366.53. Duty @ 50% = $\frac{1}{2}$ value = \$183.27, nearly.

English accountants employ to a great extent a method by aliquot parts called "practice." The cost of the fourth item is obtained as here shown. The cost at $1d.$ per yard, $2889d.$, is easily reduced to pounds, etc. It is repeated to obtain the cost @ $2d.$ $\frac{4}{12}d.$ is $\frac{1}{3}$ of $1d.$, $\frac{1}{12}d.$ is $\frac{1}{4}$ of $\frac{4}{12}d.$, and $\frac{\frac{1}{2}}{12}$ is $\frac{1}{2}$ of $\frac{1}{12}d.$ As the smallest denomination is the farthing, or $\frac{1}{4}$ of a penny, $\frac{1}{2}$ of $\frac{3}{4}d.$ is called 2 farthings, or $\frac{1}{2}d.$, instead of $\frac{3}{8}d.$

2889 @ 1 $d.$ =	£12 – 0 – 9
" 1 "	12 – 0 – 9
" $\frac{4}{12}$ "	4 – 0 – 3
" $\frac{1}{12}$ "	1 – 0 – $0\frac{3}{4}$
" $\frac{\frac{1}{2}}{12}$ "	10 – $0\frac{1}{2}$
	£29 – 11 – $10\frac{1}{4}$

A similar method may be used in reducing the above result to U. S. money. See Arithmetic, Art. 1005. The value of £50 is ascertained by taking $\frac{1}{2}$ of \$486.65, or \$243.325. £25 = $\frac{1}{2}$ of £50. $5s.$ = £$\frac{1}{4}$; etc.

In assessing duties, the government ignores fractions of a dollar in the cost less than $\frac{1}{2}$; over 50¢ is considered another dollar. The duty that would be collected upon the foregoing would be 50% of \$367, or \$183.50. Children should not, of course, be burdened with such details; their answer should be \$183.27.

Some zealous teachers fall into the mistake of endeavoring to make their pupils familiar with the methods of calculation peculiar to some callings. The time assigned to the study of arithmetic can be employed more profitably to the scholar by giving him the ability to handle ordinary problems with reasonable readiness, than by dissipating his energies in trying to make him understand a multitude of small matters that are entirely outside of his present experience. The average boy would make a better accountant if he did not hear of taxes, partnership, insurance, bonds, stock, brokerage, commissions, etc., during his school life, provided the time thus misspent were given to elementary algebra and constructive geometry, as well as to better work in what are considered the more elementary portions of arithmetic.

1046. The ratio of 3 to 6 is generally expressed as 1 to 2.

1049. 3. The ratio of the price of coffee to that of sugar is $\frac{5}{1}$, or 5 to 1.

4. A goes $1\frac{1}{4}$ times as fast as B. The ratio of A to B is $\frac{5}{4}$, or 5 to 4.

5. E's earnings are to D's as 4 to 3.

6. 3 to 4.

10. 30 to 3 = 10 to 1.

1050. 6. One wheel makes 70 revolutions per second; the other makes 90 per second.

7. The diameter = twice radius = 4 ft.

8. One goes 48 mi. per hour; the other goes the same distance in the same time.

9. Area of first = $6\frac{2}{3} \times 4\frac{1}{6}$; of second = $4\frac{2}{3} \times 2\frac{1}{12}$. Ratio = $\dfrac{4\frac{2}{3} \times 2\frac{1}{12}}{6\frac{2}{3} \times 4\frac{1}{6}}$. Reduce.

1051. 2. Cost of farm = \$75.50 × $156\frac{1}{2}$ × 124.6 ÷ 160 = \$9201.52. Interest for 1 yr. on one-half of the cost = \$230.04; \$4600.76 + \$230.04 = \$4830.80, amount of mortgage. Deducting \$500 then paid, gives \$4330,80, balance due.

4. Number of cubic feet of excavation $= 41\frac{1}{4} \times 8 \times 33 = 10890$. The inner dimensions of the cellar are $38\frac{1}{4} \times 30 \times 8$, deducting from the length and the breadth 3 ft., which is the thickness of two walls. The number of cubic feet in the cellar $= 9180$, leaving 10890 cu. ft. $-$ 9180 cu. ft. $=$ 1710 cu. ft. as the contents of the walls. The cost of the excavation is $\$\frac{1}{2} \times 10890 \div 27 = \$201\frac{2}{3}$; to lay the wall, costs $\$15 \times 17.1 = \$256.50$. Total cost, $\$201.66\frac{2}{3} + \$256.50 = \$458.16\frac{2}{3}$, or \$458.17. *Ans.*

NOTE. — One of the practices of builders in some sections is to take only the outside measure of walls in ascertaining the contents. The number of cubic feet in this case, according to their calculations, would be $(41\frac{1}{4} + 33 + 41\frac{1}{4} + 33) \times 8 \times 1\frac{1}{2} = 1782$ cu. ft., or 72 cu. ft. too many. By this method the four corners, measuring each $1\frac{1}{2} \times 1\frac{1}{2} \times 8$, or 72 cu. ft. in all, are included twice. As has already been said, children should be expected to obtain only the correct results, leaving later experience to furnish information as to local usages.

5. For \$107.25 there can be obtained, at $3\frac{1}{4}\%$, insurance amounting to $\$107.25 \div .0325 = \$3300$. If this is 80% of the value, the flour must have cost him $\$3300 \div \frac{4}{5} = \$4125$, which is $\$4125 \div 500$, per barrel, or \$8.25.

The algebraic method would be:

Let $x =$ the cost per barrel.

$$x \times 500 \times 80\% \times 3\tfrac{1}{4}\% = 107.25;$$

or,

$$500x \times \tfrac{4}{5} \times \tfrac{13}{400} = 13x = 107.25;$$

$$x = 8.25.$$

6. The bank discount is calculated on the sum due at maturity, which is \$1250 + interest from June 12 to Dec. 15 (12), 186 da. (183 da.), at 5% $=$ \$1250 + \$32.29 (\$31.77) $=$ \$1282.29 (\$1281.77). The discount for 30 da. (including grace) on \$1282.29 at 6% is \$6.41, making the proceeds \$1275.88. The discount on \$1281.77 for 27 da. (no grace) at 6% $=$ \$5.77, making the proceeds \$1276.

8. A furnished 5 men for 20 da. and 6 men for 15 da., which is the same as 100 men and 90 men for 1 da. B furnished the equivalent of 120 men + 180 men for 1 da. The money received should be divided on the basis of 190 men for A and 300 for B, 490 in all; and A should receive $\frac{190}{490}$ of the sum, and B $\frac{300}{490}$ of it.

$$\tfrac{190}{400} \text{ of } \$857.50 = \$332.50, \text{ A's share;}$$
$$\tfrac{300}{490} \text{ of } \$857.50 = \$525.00, \text{ B's share.}$$

9. Arrange the work so as to have the required term in the last place:

A ditch $(403 \times 3 \times 3)$ cu. ft. is dug in (62×13) hr. by 27 men.
A ditch $(750 \times 4 \times 3)$ cu. ft. is dug in (250×12) hr. by ? men.

If $(403 \times 3 \times 3)$ cu. ft. are dug in (62×13) hr. by 27 men,

1 cu. ft. will be dug in (62×13) hr. by $\dfrac{27 \text{ men}}{403 \times 3 \times 3}$;

1 cu. ft. will be dug in 1 hr. by $\dfrac{27 \text{ men} \times 62 \times 13}{403 \times 3 \times 3}$;

etc. See Arithmetic, Arts. 973, 974.

$$Ans. = \frac{27 \text{ men} \times 62 \times 13 \times 750 \times 4 \times 3}{403 \times 3 \times 3 \times 250 \times 12}. \quad \text{Cancel.}$$

10. Let $x =$ price per barrel; $\dfrac{23x}{4} =$ cost of flour. $2\frac{1}{2}\%$ of $\dfrac{23x}{4}$, or $\dfrac{1}{40}$ of $\dfrac{23x}{4}$, $= \dfrac{23x}{160} =$ commission.

Then $$\frac{23x}{4} + \frac{23x}{160} = 1508.80.$$

NOTE. — The words "after deducting his commission" mislead some pupils, who think that this requires them to begin work by deducting a commission of $\frac{1}{40}$ of the amount sent. If the purchasing agent did not receive money in advance, he would render his employer the following account:

To 256 bbl. flour @ $5.75,	$1472.00
Commission at $2\frac{1}{2}\%$,	36.80
	$1508.80

$2\frac{1}{2}\%$ on the amount sent, $1508.80, would be $2\frac{1}{2}\%$ of the cost of the flour, $1472, and $2\frac{1}{2}\%$ of the $36.80 commission.

A pupil that wishes to deduct the commission first, may work in this way:

Let $x =$ commission, then $40x =$ cost of flour;

$41x = 1508.80$; $x = 36.80$.

\$1508.80, amount sent, $-$ \$36.80, commission, $=$ \$1472 left for the purchase of the flour.

The commission to be deducted is $\frac{1}{41}$ of the sum sent.

1052. See Arithmetic, Arts. 936, 937.

1053. Pupils generally find it most convenient to change the time to days, leaving the reduction to lowest terms for the subsequent cancellation.

11. 2 yr. 11 mo. 18 da. $= 720$ da. $+ 330$ da. $+ 18$ da. $= \frac{1068}{360}$ yr.

$$\overset{2}{\cancel{240}} \times \frac{x}{100} \times \frac{\overset{356}{\cancel{1068}}}{\underset{\cancel{3}}{\cancel{360}}} = \frac{712x}{100} = \text{interest} = 32.04;$$

$$712x = 3204;\ x = 4\tfrac{1}{2}.$$

NOTE.—The teacher will notice that canceling 100 and 356 by dividing each by 4, and reducing the equation to $\frac{178x}{25} = 32.04$, is of no advantage.

The French method of solving an example like the foregoing, is to indicate all the work, and then to cancel:

(*a*) $$\frac{240 \times x \times 1068}{100 \times 360} = \frac{3204}{100};$$

(*b*) $$240 \times x \times 1068 = \frac{3204}{100} \times 100 \times 360;$$

(*c*) $$x = \frac{\overset{3}{\cancel{3204}} \times \cancel{100} \times \overset{3}{\cancel{360}}}{\cancel{100} \times \underset{2}{\cancel{240}} \times \cancel{1068}} = \frac{9}{2} = 4\tfrac{1}{2}.$$

The first member of equation (*a*) is "cleared of fractions" by multiplying the second number by the compound denominator, 100×360; see (*b*). The value of x is then found by dividing the second member by the compound coefficient of x, 240×1068; see (*c*). The result is then obtained by cancellation. In practice, the second equation (*b*) is not employed, the multiplication by 100×360, and the division by 240×1068, being indicated at the same time.

1054. See Art. 948.

In states that do not allow days of grace, there will be no difference in these examples between the words "term" and "time." The notes in **19**, **21**, and **25** are assumed to be discounted on the day they are drawn, which will make the "term" in states allowing days of grace, 3 days longer than the specified time. Teachers in other states are advised not to use "days of grace" in discount examples, no matter what is the date of the note or where it is made. Answers in which days of grace are not included, are enclosed in parentheses.

1055. The short methods previously studied should be employed. For **1**, **2**, **4**, see Arithmetic, Art. 891; **6** and **8**, see Art. 792; **10**, Art. 717; **12**, Art. 716; **14** and **16**, Art. 791; **17** and **18**, Art. 958; **19**, Art. 758; **20**, Art. 910.

3. $1648 \times 87\frac{1}{2} = \frac{7}{8}$ of 164800. See Arithmetic, Art. 891.

$$\begin{array}{l} 1648 \times 87\frac{1}{2} \\ \underline{\ \ 206 \text{ deduct } \frac{1}{8}} \\ 144200 \ \textit{Ans.} \end{array}$$

5. $2416 \times 875 = \frac{7}{8}$ of 2416000. **7.** $848 \times 125 = \frac{1}{8}$ of 848000. **9.** $= \frac{1}{4}$ of 79200. **13.** $= \frac{5}{8}$ of 157600. **15.** $= \frac{3}{8}$ of 1128000.

The pupils should be encouraged to employ these methods, where practicable. When too many figures are used in blackboard work, the teacher should call the attention of the class to the saving of time that is rendered possible by the employment of a shorter way.

1056. The use of diagrams will tend to simplify these problems for many pupils. The average scholar finds no difficulty in ascertaining the time difference when the difference in longitude is given, and *vice versa;* but the introduction of the other elements tends to confuse him.

First mark the meridian of 0°; then locate the two places, writing under each the longitude, as far as given; and above each, its time, as far as given. The next step is to find the time difference or longitude difference, writing it in the place designated; and from it to calculate the other, writing it in its place. The last step is to calculate the required time or longitude.

1. The difference in time is 1 hr. 24 min. The time at A is later; 1 hr. 24 min. is, therefore, deducted from 1:30 P.M., giving 12:6 P.M. as the time at B.

2. The longitude difference is 36°, which gives a time difference of 36 hr. ÷ 15 = 2 hr. 24 min., both of which should be written on the diagram. As the right-hand, or more easterly, place is later, the time at B is 12 M. − 2 hr. 24 min. = 9:36 A.M.

3. A look at the diagram shows the pupil which difference must be found, from which the other is to be calculated. The time difference in this problem is 55 min. 30 sec., making the longitude difference $55' 30'' \times 15 = 13° 52' 30''$. As B has the later time, it is the more easterly. A glance at the diagram, if correctly made, will show the pupil that he must deduct from the longitude of A, 156° 48′, the above difference, 13° 52′ 30″, to obtain the longitude of B, which is 142° 55′ 30″ west. Other rules than those already learned should not be given.

4. B is 52° 36′ east of 0°, or east longitude.

5. The time difference = 101 hr. ÷ 15 = 6 hr. 44 min. The time at A = 12 M. − 6 hr. 44 min. = 5:16 A.M.

1057. **6.** Long. diff. = 9°; time diff. = 36 min.; 9 A.M − 36 min. = 8:24 A.M.

7. Long. diff. = 25° 55′; time diff. = 1 hr. 43 min. 40 sec.; 1:45 P.M. — 1 hr. 43 min. 40 sec. = 12:1:20 P.M.

8. Time diff. = 55 min.; long. diff. = 13° 45′; 156° 48′ − 13° 45′ = 143° 3′ W.

1058. Find the square root of each term. After extracting the roots of **13–21**, reduce the resulting improper fractions to mixed numbers.

1059. **6.** Find the amount of $1030.05 at 4% for 2 yr. 9 mo. 12 da.

7. See Manual, Art. 1026, **10.**

1060. See notes on Special Drills of previous chapters.

1062. $36 \times 31 = (36 \times 30) + 36$. $36 \times 29 = (36 \times 30) - 36$. Art. 953.

1063. Use chiefly as "sight" work. $675 \div 75 = 6\frac{3}{4} \div \frac{3}{4} = 27 \div 3$; $225 \div 12\frac{1}{2} = 2\frac{1}{4} \times 8$; $150 \div 6\frac{1}{4} = 1\frac{1}{2} \times 16$; $825 \div 37\frac{1}{2} = 8\frac{2}{8} \div \frac{3}{8} = 66 \div 3$; $750 \div 62\frac{1}{2} = 7\frac{4}{8} \div \frac{5}{8} = 60 \div 5$.

1064. $315 \times 1\frac{14}{15} = (315 \times 2) - (\frac{1}{15}$ of $315)$; $32 \times 39\frac{7}{8} = (32 \times 40) - (\frac{1}{8}$ of $32)$; etc.

The square of $7\frac{1}{2}$ (Art. 1032) $= 7$ times $7 + (2 \times 7 \times \frac{1}{2})$, or 1 time $7 + \frac{1}{4} = 8$ times $7 + \frac{1}{4}$; $8\frac{1}{2} \times 8\frac{1}{2} = 9$ times $8 + \frac{1}{4}$. The square of 75 is found by affixing 25 to the product of 7 by 8, 5625; $85^2 = 7225$.

$18\frac{2}{5} \times 5\frac{1}{3} = 18\frac{2}{5} \times 5\ (92) + \frac{1}{3}$ of $18\frac{2}{5}\ (6\frac{2}{15}) = 98\frac{2}{15}$. See Arithmetic, Art. 758.

When the divisor is a whole number, $97\frac{1}{2} \div 3$, etc., do not reduce the dividend to an improper fraction; $\frac{1}{3}$ of $97\frac{1}{2} = 32$ with a remainder of $1\frac{1}{2}$, or $\frac{3}{2}$; $\frac{1}{3}$ of $\frac{3}{2} = \frac{1}{2}$. $19\frac{1}{5} \div 2\frac{2}{5} = \frac{96}{5} \div \frac{12}{5} = 96 \div 12$; etc.

1065. These problems are applications of the drills upon the preceding page. Their solution involves simply the ability to handle large numbers without a pencil, and does not require any mental effort in determining the nature of the operations required. It is of more advantage to the pupil to be able to obtain the results in examples of this kind than in the more complicated ones usually given in the higher grades, for which reason teachers should be careful not to omit them.

6. $[(30 \times 16) - (\frac{1}{2}$ of $16)]$ oz.

8. 5700 tenths $\div$ 19 tenths.

10. $[(12$ yr. $\times 9) + (10$ yr. $\times 6)] \div (9 + 6)$.

16. $77 \div 5\frac{1}{2} = 77 \div \frac{11}{2} = 77 \times \frac{2}{11} = 7 \times 2$.

17. $[(10\frac{2}{3} \times 6) + (\frac{1}{2}$ of $10\frac{2}{3})]$ sq. yd.

18. 31 doz. @ 15¢.

21. 100 marks = \$23.80; 50 marks = $\frac{1}{2}$ of \$23.80.

24. $1\frac{1}{4}$ thousand $\times 16 = 20000$.

25. $5600 \div 87\frac{1}{2} = 56 \div \frac{7}{8} = 56 \times \frac{8}{7} = 8 \times 8$.

1066. The value of "All others" should be written directly in its place. See Arithmetic, Art. 384. To obtain results with two decimal places that will give a total of exactly 100, it will be necessary to extend the division to three places, as here shown, and to increase by 1 the hundredths' figure of the four having the largest figures in the thousandths' place, rejecting the thousandths' figures of the others. The usual method of calling 5 thousandths or over 1 hundredth, does not always make the total correct. See **8**, in which .715 – is made .72, although the thousandths' figure is not quite 5.

1.	63.301 +	63.30
2.	13.143 –	13.14
3.	8.576 –	8.58
4.	3.826 +	3.83
5.	3.414 +	3.41
6.	2.094 –	2.09
7.	1.113 –	1.11
8.	.715 –	.72
9.	3.818 –	3.82
	100.000	100.00

1067. **7.** (*a*) $28.128 × $39\frac{1}{4}$ = $1104.02. A payment on Dec. 21 is entitled to a rebate of 11 days' interest on the foregoing, at $7\frac{3}{10}$%, or $2.46, making the amount actually paid = $1104.02 – $2.46 = $1101.56. If paid Jan. 15, the interest on $1104.02 at 9% for 45 da., $12.42, would be added, making a total of $1116.44.

1070. Permit the pupils to work out in their own way these preliminary examples. The sign : : is another form of the sign of equality. The rule for proportion is given in Arithmetic, Art. 1073.

1074. In oral problems, pupils should generally be permitted to use their own method. Nothing will be gained by requiring them to use proportion in the solution of these.

1076. **1.** 80% of 70¢. **2.** 90% of 60¢. **3.** 60% of 75¢. **8.** 90% of $66\frac{2}{3}$¢.

1077. **11.** The first discount is the given one, 30%; the second is 30% of the remaining 70%, or 21%; the total single discount = 30% + 21% = 51%. **12.** 20% + $\frac{1}{4}$ of (100 − 20)% = 20% + 20% = 40%. **13.** 25% + $\frac{1}{5}$ of (100 − 25)% = 25% + 15% = 40%. The results are the same in **12** and **13**, it making no difference which discount is taken first.

Another method of obtaining the single discount is to ascertain the net per cent, and to subtract this from 100%. Thus, in **11**, (100 − 30)% of (100 − 30) per cent = 70% of 70 per cent = 49 per cent. The single discount = (100 − 49)% = 51%.

1078. **21.** 30 and 20% = (70% of 80%) net = 56% net. 40 and 10% = (60% of 90%) net = 54% net. The latter, being the smaller price, is better for the buyer.

1079. **1.** 60% of 90% of \$250 = \$135. *Ans.* **2.** 95% of $(100 - x)\%$ of 800, or $\frac{95}{100}$ of $\frac{100 - x}{100}$ of $\frac{800}{1} = 684$; canceling, $\frac{3800 - 38x}{5} = 684$; $3800 - 38x = 3420$; $-38x = -380$; $38x = 380$; $x = 10$. *Ans.* 10%.

3. $66\frac{2}{3}\%$ of 90% of x, or $\frac{2}{3}$ of $\frac{9}{10}$ of $x = \frac{3x}{5} = 90$; $3x = 450$; $x = 150$. *Ans.* \$150.

4. 70% of $(100 - x)\%$ of $600 = 378$; $\frac{7}{10} \times \frac{100 - x}{100} \times \frac{600}{1} = \frac{2100 - 21x}{5} = 378$; $2100 - 21x = 1890$; $-21x = -210$; $21x = 210$; $x = 10$. *Ans.* 10%.

$$\begin{array}{lr} & 800 \\ 5\%, & \underline{40} \\ & 760 \\ x\%, & \underline{\frac{38x}{5}} \\ & 760 - \frac{38x}{5} \end{array}$$

NOTE. — Some pupils may prefer to use the longer method, deducting each discount in turn to obtain the net price, and making this equal to the given net price. Thus in **2** the operations would be shown as here indicated, and the equation would be $760 - \frac{38x}{5} = 684$, which, after clearing of fractions, reduces to the form given above, $3800 - 38x = 3420$. The first method is shorter and less likely to give rise to mistakes.

5. $x = 70\%$ of 20% of 16.

6. $\frac{1}{2}$ of $\frac{9}{10}$ of $x = 27$.

7. $\frac{4}{5}$ of $\frac{100 - x}{100}$ of $5 = 3.20$.

8. $\frac{7}{10}$ of $\frac{7}{10}$ of $x = 73.50$.

9. $x = \frac{3}{4}$ of $\frac{4}{5}$ of 200.

10. $\frac{1}{2}$ of $\frac{100 - x}{100}$ of $1.50 = .60$.

1084. 2. There are 5280 ft. in a mile. $5280 \times 176 \div 3520$ = number of minutes. Cancel.

3. $\frac{375}{10000}$ A. = \$9; 1 A. = \$9 $\div \frac{375}{10000}$ = \$9 $\times \frac{10000}{375}$; $\frac{3}{32}$ A. = \$9 $\times \frac{10000}{375} \times \frac{3}{32}$. Cancel.

4. Reduce both to pence.

5. A receives $\frac{2000}{7500}$ of \$576; etc.

6. (580 tiles $\times 6 \times 6) \div (4 \times 3)$. Cancel.

7. $\frac{x}{400} = 1500$.

10. 108 (in.) $\times 80 \times 77 \div 231$.

11. $.625 + .4375 + .75 + .09375 + 2.46 = 312.5\,x$.

12. $142.50 \div (5.25 - 4.95)$ = number of tons, 475. Total cost = \$4.95 $\times$ 475.

14. The distance traveled in 1 revolution is 13 ft., the circumference of the wheel.

15. Three hundred forty-nine thousandths; three hundred (units) and forty-nine thousandths; three hundred forty nine-thousandths; three hundred (units) and forty nine-thousandths.

In dictating such numbers, it would be necessary to be still more explicit.

16. 4 lb. 6 oz. 12 pwt. = 1092 pwt.; 7 lb. 9 oz. 12 pwt. = 1872 pwt. *Ans.* (£13 8*s.* 4*d.*) $\times 1872 \div 1092$ = (canceling) £13 8*s.* 4*d.* $\times \frac{12}{7}$. If preferred, £13 8*s.* 4*d.* may be reduced to pence.

17. Total amount realized = \$20000 + \$1500 + \$1000 = \$22500. A furnishes one-third of the \$15000 capital, and is entitled to $\frac{1}{3}$ of \$22500, or \$7500. Having drawn \$1500 already, he should receive \$6000 additional. B's share = \$15000 − \$1000 = \$14000.

18. The gain on an article bought for 80% of its value and sold for 120% of its value, is 40% of its value. 40% is $\frac{1}{2}$ of the cost, 80%, so that the gain is 50%.

The words "per cent" occur so frequently in the foregoing as to confuse some children. Calling the value $100x$, the cost is $80x$ and the selling price is $120x$, a gain of $40x$, which is $\frac{1}{2}$ of the cost, $80x$, or 50%.

19. $9 half-yearly, or $18 yearly, is the interest on the difference between $1200 and $750, or $450. The rate is 4%.

21. M does $\frac{1}{4}$ in a day; N, $\frac{1}{5}$; O, $\frac{1}{6}$; together, $\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$ in 1 da., or $\frac{15+12+10}{60}=\frac{37}{60}$ in 1 da. To do the whole work will require 1 da. $\div \frac{37}{60} = \frac{60}{37}$ da. $= 1\frac{23}{37}$ da.

The teacher should not neglect to have her pupils make an estimate of the answer of each problem before beginning work. They will find in doing this, that they can solve many of the problems without using the pencil. It will be found useful as an occasional class exercise to require each pupil to write on paper at a signal, his estimate of, say, ten problems similar to some of the foregoing. Anything that will cause a pupil to read carefully and understandingly before setting his pencil to paper, will by of great value to him in problem work. See Art. 981.

1086. **1.** Let $x =$ first cost of the goods. Then, $\frac{24x}{100} = 122.16$; $24x = 12216$; $x = 509$; cost $= \$509$. The number of yards purchased $= 32 \times 32 = 1024$; invoice price per yard $= \$509 \div 1024 = 49\frac{181}{256}$¢. Cost of the goods, including duties, etc., $= \$509 + \$122.16 + \$40.96 = \$672.12$; total cost per yard, $= \$672.12 \div 1024 = 65\frac{163}{256}$¢.

2. The amount paid $= \$12500 + 9$ months' interest on $\$3500 + 18$ months' interest on $\$2600 + 28$ months' interest on $2400.

3. At the end of 30 da., the provisions will last 1200 men 70 da. If half the men are then withdrawn, the remaining 600 will consume in 30 da. what 1200 men would use in 15 da., leaving 55 days' provisions for 1200 men. If the force is increased then by 900 men, there will be 1500 in the garrison. 55 days' provisions for 1200 men will last 1500 men $\frac{1200}{1500}$ of 55 da. $= \frac{4}{5}$ of 55 da. $=$ 44 da. Total time, 30 da. $+$ 30 da. $+$ 44 da. $=$ 104 da.

4. The ad valorem duty $= 20\%$ of $7¢ \times 267 \times 37$. The specific duty $= 5\frac{1}{2}¢ \times 267 \times 37 \times \frac{30}{36}$. The sum of both will be the entire duty.

5. $\frac{1}{5}$ of $\frac{3}{4}$ of $\frac{25}{8} = \frac{15}{32}$; $\frac{15}{32} \times \frac{1}{2}$ of $\frac{15}{32} \div \frac{1}{2}$.

6. $97\frac{1}{2}\%$ of amount $= \$762742.50$.

9. The tea cost 25¢ per lb. A gain of $15¢ = 60\%$.

1087. These exercises should be omitted if not required by the course of study.

	Face of Draft		+ Premium, or – Discount		– Interest	=	Cost of Draft
1.	100	$+$	.02			$=$	x
2.	x			$-$	$\frac{x}{1000}$	$=$	499.50
3.	1800	$+$	$\frac{9x}{5}$	$-$	18	$=$	1778.85
4.	x	$+$	$\frac{x}{400}$			$=$	701.75
5.	200	$-$	.20	$-$	1	$=$	x
6.	600	$+$	2.25	$-$	$36x$	$=$	598.95
7.	1000	$-$	.75	$-$	$60x$	$=$	999.25
8.	1200			$-$	$\frac{31x}{10}$	$=$	1178.30
9.	800	$+$	$\frac{4x}{5}$	$-$	3.20	$=$	796.80
10.	400	$+$	.80	$-$	$\frac{x}{10}$	$=$	400.30

2. Interest $= x \times \frac{6}{100} \times \frac{6}{360} = \frac{x}{1000}$.

3. The rate is x dollars per \$1000 premium; on \$1800, the premium $= \frac{1800}{1000} \times x = \frac{9x}{5}$. The interest on \$1800 for 60 days @ 6% = 1% of \$1800 = \$18. The equation becomes $\frac{9x}{5} + 1782 = 1778.85$; $\frac{9x}{5} = 1778.85 - 1782 = -3.15$; $9x = -15.75$; $x = -1.75$. The minus sign indicates that it is a discount. *Ans.* \$1.75 discount per \$1000.

6. Let x = time in years. The equation becomes $602.25 - 36x = 598.95$; $-36x = 598.95 - 602.25 = -3.30$; $36x = 3.30$; $x = \frac{3.3}{36} = \frac{33}{360}$. *Ans.* $\frac{33}{360}$ year = 33 days.

7. $-.60x = 0$; $x = 0$ years; *i.e.* sight.

8. The interest $= 1200 \times \frac{x}{100} \times \frac{93}{360} = \frac{31x}{10}$. The equation becomes $\frac{31x}{10} = 21.70$; $31x = 217$; $x = 7$. *Ans.* 7%.

9. Rate $= x$ per M. Premium $= \frac{800}{1000} \times x = \frac{4x}{5}$; $x = 0$. *Ans.* Par.

1088. **4.** 10 sq. ch. = 1 A. = 160 sq. rd. 1 sq. ch. = 16 sq. rd. 1 chain $= \sqrt{16}$ rods = 4 rods = $16\frac{1}{2}$ ft. × 4.

5. Each face contains 1350 sq. in. ÷ 6 = 225 sq. in. Length $= \sqrt{225}$ in. = 15 in.

6. Hypotenuse = 5 in.

7. Hypotenuse = $3\frac{1}{4}$ in.

1089. Another method of subdividing square C is here shown, Fig. 3. From each side of C, a right-angled triangle is cut, of the same dimensions as the original triangle m, leaving a small square X. In Fig. 4, is shown a rearrangement of these

FIG. 1.

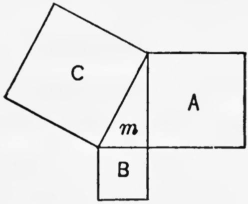

FIG. 2.

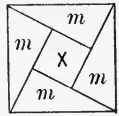

FIG. 3.

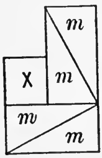

FIG. 4.

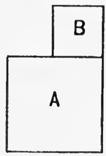

FIG. 5.

triangles, making a polygon equal in surface to the sum of the squares A and B, Fig. 5.

1091. **1.** To find $\frac{1}{40}$, divide by 4, placing each quotient figure one place to the right. Nothing should be written beyond what is shown in the Arithmetic. To find the interest, see Art. 983, **27**.

2. 1% each quarter. In dividing by 100, the dividend is repeated, with each figure two places to the right.

3. 3% each half year. Multiply by 3, placing each figure of the product two places to the right.

$$\begin{array}{rl}
\$1500. & \\
45. & \\
\hline
\$1545. & \tfrac{1}{2} \text{ year} \\
46.35 & \\
\hline
\$1591.35 & 1 \text{ year.}
\end{array}$$

1092. In some cities, the quotations of stocks give the price per share. A share whose par value is \$50 and which sells for \$48, is quoted in the New York papers as worth 96; *i.e.* 96%

of \$50. In a few places, the price is quoted as 48, meaning \$48 per share. In the examples given in the Arithmetic, the rate always means the per cent of the par value.

1. $(87\frac{3}{4}+\frac{1}{8})\%$ of (\$$10 \times 240$).

2. Brokerage $= \frac{1}{4}\%$ of (\$$100 \times 120$) = \$30. Value of stock = \$11460 − \$30 = \$11430; value per share = \$11430 ÷ 120 = \$95.25.

3. Let $x =$ par value per share. Then $(87\frac{1}{2}+\frac{1}{4})\%$ of $(x \times 150) = 5265$; $\frac{x \times 150 \times 87\frac{3}{4}}{100} = \frac{x \times 150 \times 351}{100 \times 4} = 5265.$

Using the cancellation method, $x = \frac{5265 \times 100 \times 4}{150 \times 351}.$

4. $\frac{1}{8}\%$ of \$27500.

5. $102\frac{1}{4}\%$ of (\$$25 \times 200$) = selling price. Deduct therefrom $\frac{1}{8}\%$ of (\$$25 \times 100$).

6. His income is 18% of the par value (\$$100 \times 60$) = \$1080. His investment is 450% of (\$$100 \times 60$) = \$27000.

\$1080 is 4% (*Ans.*) of \$27000.

Or, irrespective of number of shares, $18\% \div 4\frac{1}{2} = 4\%$. *Ans.*

7. The stock = \$$50 \times 4000$ = \$200000. The rate of dividend = \$20000 ÷ \$200000 = $\frac{1}{10}$ = 10%. *Ans.* 10% ÷ 1.75 = $5\frac{5}{7}\%$. *Ans.*

8. \$10000 interest must be paid to the bondholders, leaving (\$47500 − \$10000) \$37500 to be paid as dividends on \$1000000 of stock. $37500 \div 1000000 = .0375 = 3\frac{3}{4}\%$.

9. Let $x =$ brokerage. $\frac{168\frac{1}{2}+x}{100}$ of $25 \times 360 = 15176.25$; $9000\,(168\frac{1}{2}+x) = 1517625$; etc.

10. Let $x =$ amount of brokerage.

$$107\tfrac{3}{4}\% \text{ of } (100 \times 250) - x = 26875;$$
$$26937.50 - x = 26875;$$
$$x = 62.50; \text{ brokerage} = \$62.50. \textit{ Ans.} \quad \text{Etc.}$$

11. $\$35050 \div 1.75\frac{1}{4} = \$20000$ = par value of stock; $7\frac{1}{2}\%$ of $\$20000 = \$1500$. *Ans.*

12. Mr. Tower receives \$30 interest, and \$100, the face value of the bond, in six years, \$130 in all. His investment was \$104, on which he has received \$130 – \$104, or \$26 interest in 6 yr., or $\$4.33\frac{1}{3}$ per year. $4\frac{1}{3} \div 104 = .04\frac{1}{6} = 4\frac{1}{6}\%$. *Ans.*

1093. **1.** A corporation is an association of a number of persons legally empowered to transact business as a single individual. The charter specifies the name of the corporation, the amount of capital, the business it is authorized to carry on, the powers and privileges conferred, etc. The stock is the money invested in the business of the corporation; a share is one of the equal parts into which the stock is divided; a shareholder is the owner of one or more shares; a stockbroker is a person engaged in the business of buying and selling stocks on commission; a dividend is a *pro rata* division of profits among the stockholders; an assessment is a sum levied upon stockholders to meet some unexpected expenses, losses, etc.

2. Income from bonds = 6% of $\$125 \times 109 \div 1.09 = 6\%$ of $\$12500 = \$750$, an increase of \$68.75 over the previous income of \$681.25.

3. Sum loaned $= \$59.57 \div \frac{1}{6} = \$59.57 \times 6$.

4. In 3 yr. at 7%, \$1 will amount to \$1.21; *i.e.* a payment of \$1.21 in 3 yr. is considered equal to a cash payment of \$1. The "present worth" of the delayed payment $= \$4235 \div 1.21 = \$3500$, which is a loss of $\$3675 - \$3500 = \$175$. *Ans.*

5. The rate on one $= 5\% \div (.98\frac{1}{4} + .00\frac{1}{4}) = 5 \div .98\frac{1}{2}$; on the other $= 6\% \div 1.09$.

The cost of a \$100 5% bond is \$98.25 + 25¢ brokerage = \$98.50; the interest on the bond is 5% of \$100 = \$5; the rate is, therefore, $5\% \div .98\frac{1}{2}$; etc.

6. The time of the place being 1 hr. later than the time of the starting-point, the traveler is 15° east of the latter place.

8. As the note is stated to be due in 3 mo., which may be assumed to include days of grace where allowed, the date of its maturity is Sept. 20. The amount of the note for 92 da., at 9%, = $2455.20. The discount on this sum at 6% from Aug. 8 to Sept. 20, 43 da. = $17.60. Proceeds = $2455.20 − $17.60 = $2437.60. *Ans.*

10. $4800 less 63 days' interest, $50.40, = $4749.60.

($4797.58 less 60 days' interest, $47.98, = $4749.60.)

1097. **4.** Having found that √.10 = .316 +, the pupils should see that √.1 = .316 +.

In pointing off, begin at the units' place, pointing off two places to the right or the left.

7. 1.6 is made 1.60. See **10.**

1098. **7.** Find the amount of $467.50 at 6% from July 5, 1881, to Dec. 19, 1885.

8. Amount stolen = $650. The first sender is entitled to 150/650 of $523.25; the second, to 202/650 of $523.25; etc.

9. WASHINGTON, D.C., Dec. 1, 1896.

THE UNITED STATES

To JAMES RYAN, *Dr.*

		To Services, Nov. 19–30, 12 da.	45	65		
		" Traveling Expenses, 12 da., $4.00	48	00		
		" Stage Fare $12.50, St'mb't Fare $17.40, Telegraph $3.25	33	15		
					$126	80

10. The account for the first quarter, exclusive of box rents, is $124.96. On $50, the commission allowed = $50. On the remaining $74.96, the commission is 60% = $44.98. Total = $77 + $50 + $44.98 = $171.98.

Salary for the year = $171.98 + $194.01 + $174.08 + $167.87 = $707.94.

1099. $\text{Base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2$; $H = \sqrt{B^2 + P^2}$.

1. $225 + 64 = x^2$; $x^2 = 289$; $x = 17$, hypotenuse.

2. $1225 + x^2 = 1369$; $x^2 = 1369 - 1225 = 144$; $x = 12$, perpendicular.

3. $x^2 + 225 = 1521$; $x^2 = 1521 - 225 = 1296$; $x = 36$, base.

1100. **11.** Let x = perpendicular. Area $= \frac{1}{2}(200 + 160) \times x = 180\,x = 32400$; $x = 180$.

12. $\frac{1}{2}(20 + x) \times 15 = 225$; $\dfrac{300 + 15x}{2} = 225$; $300 + 15\,x = 450$; $15\,x = 450 - 300 = 150$; $x = 10$.

13. $\frac{1}{2}(x + x + 6) \times 10 = (x + 3) \times 10 = 10\,x + 30 = 150$; $10\,x = 150 - 30 = 120$; $x = 12$, $x + 6 = 18$. 12 rd. and 18 rd. *Ans.*

14. The base = 7 rd.; the altitude, or perpendicular, = 24 rd. Area $= \frac{1}{2}(24 \times 7)$ sq. rd.

15. Number of stones $= (84 \times 36) \div (6 \times 3)$. Cost $= \$1\frac{1}{4} \times (28 \times 12)$.

16. The angle of 90° indicates a right-angled triangle. Hypotenuse = 20 chains. Number of chains of fence $= 12 + 16 + 20 = 48$. 1 chain = 66 ft. = 4 rd. Number of rods $= 48 \times 4 = 192$.

17. A perpendicular let fall from the upper right corner would form a right-angled triangle, whose perpendicular is 40 ft., base 30 ft., making the hypotenuse $= \sqrt{1600 + 900}$ ft. = 50 ft., the fourth side.

The number of yards of fence $= (40 + 70 + 50 + 100) \times 5\frac{1}{2}$.

Area in acres $= [\frac{1}{2} \text{ of } (70 + 100) \times 40] \div 160$.

18. The diagonal is the hypotenuse of a right-angled triangle whose other sides measure, respectively, 90 yd. and 120 yd.

19. The diagonal $= \sqrt{1296 + 729}$ chains = 45 chains. The distance by the road = 27 chains + 36 chains = 63 chains. The saving is 18 chains, of 22 yd. each.

20. A 40-acre field contains 6400 sq. rd. Each side measures $\sqrt{6400}$ rd. $= 80$ rd. The diagonal $= \sqrt{80^2 + 80^2}$.

1101. **1.** The loss is $\frac{1}{6}$ of the cost, \$300.

2. If 3 boys solve 3 problems in 3 min., 1 boy will solve 1 problem in 3 min., and 6 boys will solve 6 problems in 3 min.

Or, 3 boys will solve 6 problems in 6 min., therefore 6 boys will solve 6 problems in 3 min.

3. The 3 boys eat 12 cakes, 4 each; for which the third boy pays 12¢, or 3¢ apiece. The boy that brought 7 cakes supplies 3; for which he should receive 9¢. The other boy furnishes 1, and is entitled to 3¢.

4. The cost of the article is 50¢; a sale for \$2.00 is a gain of \$1.50, or 300%.

6. A 6-ft. fence needs 2 posts, one at each end; a 12-ft. fence, 3 posts; a 30-ft. fence, 6 posts.

7. One half is profit, the other half is the cost. The profit equals the cost, and is 100%.

8. $\frac{1}{2}x \times \frac{3}{10}x = \frac{3}{20}x^2 = 60$; $3x^2 = 1200$; $x^2 = 400$; $x = 20$.

9. $\dfrac{3x}{400} = 90$.

10. The difference between 28 and 75, inclusive, $= (75 - 28) + 1$.

1102. **2.** The quantity of provisions for each man $= 2\frac{1}{4}$ lb. $\times 20 = 45$ lb. To last 24 da., the allowance should be 45 lb. $\div 24 = 1\frac{7}{8}$ lb. $= 1$ lb. 14 oz.

5. $80\frac{1}{2}$ min. $\times 112 \div 46$. Cancel.

6. See Arithmetic, Arts. 821, 822.

7. First beam contains $(66 \times 10 \times 8)$ cu. in. The second contains $(x \times 12 \times 12)$ cu. in. The contents of the latter $= \frac{3024}{924}$ times the contents of the former.

$$x \times 12 \times 12 = \frac{3024 \times 66 \times 10 \times 8}{924},$$

$$x = \frac{3024 \times 66 \times 10 \times 8}{924 \times 12 \times 12} = 120.$$

Ans. 120 in. = 10 ft.

9. The weight of the provisions = 3 lb. $\times 32 \times 45$. Dividing by 40 gives the daily allowance for the increased crew; dividing this by their number, $32 + 16$, gives the allowance of each.

$$\frac{3 \text{ lb.} \times 32 \times 45}{40 \times 48}.$$

12. The difference in deposits = \$450, which sum produces \$18 interest. The rate is 4%.

14. \$7500 at $x\%$ for $3\frac{1}{3}$ years produces \$1125 interest.

15. $\left(\frac{5}{100} \text{ of } \frac{x}{2}\right) - \left(\frac{4}{100} \text{ of } \frac{x}{2}\right) = 40$; $\frac{5x}{200} - \frac{4x}{200} = \frac{x}{200} = 40$; $x = 8000$.

16. $[\frac{5}{100} \text{ of } (x + 400)] - (\frac{4}{100} \text{ of } x) = 30$;

$$\frac{5x + 2000}{100} - \frac{4x}{100} = 30,$$

$$5x + 2000 - 4x = 3000,$$

$$x = 1000,$$

$$x + 400 = 1400.$$

Ans. \$1000 at 4%, \$1400 at 5%.

17. $\left(\frac{4}{100} \text{ of } \frac{4x}{5}\right) + \left(\frac{5}{100} \text{ of } \frac{x}{5}\right) = 2940.$

19. A receives $\frac{220}{800}$ of selling price; he invested, therefore, $\frac{220}{800}$ of \$600; etc.

20. F receives $\frac{7}{15}$. E's share is $\frac{2}{15}$ more than D's. If $\frac{2}{15} = \$90$, $\frac{1}{15} = \$45$; F's share, $\frac{7}{15}$, = \$315.

21. The bank receives at the end of 63 days, the sum loaned, \$593.70, + \$6.30 interest. The problem is: At what rate will \$593.70 produce, in 63 days, \$6.30 interest?

$$593.70 \times \frac{x}{100} \times \frac{63}{360} = 6.30,$$

$$x = \frac{630 \times 100 \times 360}{59370 \times 63} = 6\tfrac{126}{1979}.$$

22. The two supply pipes fill $\frac{1}{2} + \frac{1}{3}$, or $\frac{5}{6}$, of tank in 1 hour. If all the pipes are set to work when the tank is full, the exhaust pipe takes off each hour $\frac{1}{6}$ of the tank more than the others supply. To empty the tank would, therefore, require 6 hours.

1103. **7.** The diagonal $= \sqrt{137\frac{1}{2}^2 + 137\frac{1}{2}^2}$.

9. Let $100x$ = value. Buying price $= 90x$; selling price $= 110x$; gain $= 20x$, which is $\frac{2}{9}$ of the cost, $90x$, or $22\frac{2}{9}\%$.

10. $420¢ \div (\frac{3}{4}¢ - \frac{2}{5}¢)$ = number.

1106. **1.** $\$500 \div \frac{5}{4}$ = cost of one portion; $500 \div \frac{5}{6}$ = cost of other portion.

4. If $\frac{2}{3}$ of farm is sold for $\frac{3}{4}$ cost, the selling price of the whole farm at the same rate would be $\frac{3}{4}$ cost $\div \frac{2}{3} = \frac{9}{8}$ cost, making the gain $\frac{1}{8}$ cost $= 12\frac{1}{2}\%$.

7. 10¢ per bu. of 60 lb. $= 16\frac{2}{3}¢$ per 100 lb., or $\frac{2}{3}¢$ higher than 16¢ per 100 lb., or $\frac{1}{24}$ of 16¢ higher, or $4\frac{1}{6}\%$.

8. See Art. 1084, **15**.

10. If $\frac{2}{5}$ of the selling price is profit, the cost must be $\frac{3}{5}$ of the selling price; the latter is therefore $\frac{5}{3}$ of the cost. This is a profit of $\frac{2}{3}$ of the cost, or $66\frac{2}{3}\%$. *Ans.*

Or, a profit of 2-fifths on a cost of 3-fifths is $\frac{2}{3}$ of the cost.

11. $x + 4\frac{1}{2}x = 60\frac{1}{2}$.

14. $1\frac{1}{2}\%$ of $\dfrac{2x}{3} = 150$.

17. See Arithmetic, Art. 915, **6–8**.

22. $(100 \div 5\frac{1}{2})$ years. The \$200 is unnecessary.

23. $(96 \times 48 \times 45) \div (231 \times 31\frac{1}{2})$. Cancel.

24. 20 acres $= 3200$ sq. rd. Each side measures $\sqrt{3200}$ rd. The diagonal = the square root of the sum of the squares of two equal sides. The square of each $= 3200$.

The diagonal $= \sqrt{3200 + 3200}$ rd. $= \sqrt{6400}$ rd. $= 80$ rd.

NOTE. — By constructing a square on the diagonal of a square, the pupils will see that the former will be twice as large as the latter; that is, that a square on the diagonal of the above will contain 6400 sq. rd., making each side 80 rd.

In the above example, the square root of 3200 should not be extracted.

1107. **6.** Find the proceeds of \$1572.50 for 81 da. (Omitting days of grace, the proceeds for 78 da. = \$1552.06.)

9. One costs $85\frac{1}{4}\%$ of \$1500; the other costs $102\frac{1}{4}\%$ of \$1300.

10. $(\$2562.50 \div 1.025) \div \$62.50 =$ number of acres.

1. It is frequently difficult, for various reasons, to measure the altitude of a triangular field. On this account, a method of determining the area when the lengths of the sides are given, is useful, even though the underlying principles be not understood. A pupil can satisfy himself as to its accuracy, by calculating the area of the right-angled triangle in **3**, and of the isosceles triangle in **5**.

2. The half sum $= \frac{1}{2}$ of $(35 + 84 + 91) = 105$. The remainders are: $(105 - 35)$ 70, $(105 - 84)$ 21, and $(105 - 91)$ 14. Area $= \sqrt{105 \times 70 \times 21 \times 14}$ sq. ft. $= 1470$ sq. ft.

3. This is a right-angled triangle, since $21^2 + 28^2 = 35^2$; its area, therefore, is $\frac{1}{2}$ of (21×28) sq. rd., or 294 sq. rd.

By the other method, the area $= \sqrt{42 \times 21 \times 14 \times 7}$ sq. rd., or 294 sq. rd.

4. The sides of one triangle measure 39, 52, and 65 rd. respectively. Its area $= \sqrt{78 \times 39 \times 26 \times 13}$ sq. rd. $= 1014$ sq. rd. The sides of the other are 25, 60, and 65 rd., respectively; and its area $= \sqrt{75 \times 50 \times 15 \times 10}$ sq. rd. $= 750$ sq. rd. The area of the quadrilateral $= 1014$ sq. rd. $+ 750$ sq. rd. $= 1764$ sq. rd.

Each of these triangles is right-angled, AC being their common hypotenuse. Their areas are $\frac{1}{2}$ of (39×52) sq. rd., and $\frac{1}{2}$ of (25×60) sq. rd., respectively.

5. Since the altitude AC, Fig. 2, Arithmetic, Art. 1263, of an isosceles triangle divides the base into two equal parts, $BC = 15$ yd. $AC^2 = AB^2 - BC^2 = 625 - 225 = 400$; $AC = 20$ yd. The area $= \frac{1}{2}$ of 600 sq. rd. $= 300$ sq. rd.

The area by the other method $= \sqrt{40 \times 10 \times 15 \times 15}$ sq. yd.

6. In the right-angled triangle ACB (Art. 1263, Fig. 2), $BC = \frac{1}{2}$ of 96 ft. $= 48$ ft; $AB = 64$ ft.; $AC = \sqrt{64^2 - 48^2}$ ft. $= \sqrt{4096 - 2304 = 1792}$ ft. The area $= \frac{1}{2}$ of (96×42.332) sq. ft.

7. $\frac{1}{2}$ sum $= 9$ ft. Area $= \sqrt{9 \times 3 \times 3 \times 3}$ sq. ft.

8. The base $= \sqrt{70^2 - 42^2}$ ft. $= 56$ ft. Area $= \frac{1}{2}$ (42×56) sq. ft.

9. $\sqrt{50^2 - 48^2} = 14$, the number of feet in one-half the base. See Fig. 2, Arithmetic, Art. 1263.

10. The common base of the two triangles will be one diagonal, 2 in. The altitude of a triangle will be one-half the other diagonal $= \sqrt{2^2 - 1^2}$ in. $= \sqrt{3}$ in. $= 1.732$ in. The diagonal $= 3.464$ in. The area of each triangle $= \frac{1}{2}$ (2×1.732) sq. in. $= 1.732$ sq. in. The area of the rhombus $= 3.464$ sq. in.

11. The scholars should be led to ascertain for themselves the approximate relation between the diameter of a circle and its circumference. Place a point marked on the circumference of a spool, or something similar, on a given point on the surface of a sheet of paper. Roll the spool until the point on the circumference again touches the surface of the paper. The distance between the two points on the paper will be equal to the circumference of the circle. Measure this distance very carefully, also the diameter of the circle, and ascertain the ratio.

12. See Art. 1274, **6–14**, for the reason for the rule for determining the area of a circle.

13. The $\frac{1}{2}$ circumference $= \frac{1}{2}$ $(2x \times 3.1416) = 3.1416x$; the $\frac{1}{2}$ diameter $= x$; the area $= 3.1416x \times x = 3.1416x^2$.

14. $3.1416x^2 = \text{area} = 314.16.$

$$x^2 = 100;\ x = 10.$$

15. Diameter $= 15.708$ ft. $\div 3.1416 = 5$ ft.

$$\text{Area} = [(\tfrac{1}{2} \text{ of } 15.708) \times (\tfrac{1}{2} \text{ of } 5)] \text{ sq. ft.}$$

1108. The balance, \$851.72, is found by adding the credits, and subtracting from \$2535.35 in one operation. See Art. 384.

1110. **1.**

Amount of \$500, July 25 to April 1, 250 da.,		\$520.83
Amount of \$100, Sept. 18 to April 1, 195 da.,	\$103.25	
" " \$200, Feb. 5, " " " 55 "	201.83	305.08
Due April 1, 1894,		\$215.75

3.

Amount of \$600 for 354 da.,		\$635.40
" " \$300 " 152 "	\$307.60	
" " \$200 " 57 "	201.90	509.50
Due at settlement,		\$125.90

5. In the debit column, place the interest for 329 da., \$46.06. The total of this column is written on the same line as the total of the credit column, and is \$886.06. The amount is also written as the total of credits. The total interest (\$28.38) on \$500 for 297 da., \$24.75, and on \$200 for 109 da., \$3.63, is written among the credits; and the cash payment is ascertained by writing in its place the sum necessary (\$157.68) to make the total, \$886.06. See Art. 384.

6. From the amount of \$725 for 234 da. + the amount of \$603 for 174 da., take the amount of \$600 for 183 da. + the amount of \$300 for 37 da.

1113. **3.** The field contains 160 sq. rd. $\times 7\frac{1}{2}$. Its length is $(160 \times 7\frac{1}{2} \div 30)$ rd. $= 40$ rd. The diagonal $= \sqrt{40^2 + 30^2}$ rd. $= 50$ rd.

4. The rate per cent that will produce \$36 interest in $1\frac{2}{3}$ yr. is $7\frac{1}{5}$. The interest on \$212.50 at $7\frac{1}{5}\%$ for the given time $=$ \$52.02. *Ans.*

Or, the problem may be solved as follows (Arithmetic, Art. 974):

Interest on \$300 for 20 mo. is \$36,
" " \$212.50 " $40\frac{4}{5}$ " " ?

$$\frac{\$36 \times 212.50 \times 40\frac{4}{5}}{300 \times 20} = \frac{\$36 \times 212.50 \times 204}{300 \times 20 \times 5}.$$

5. The date not being given, the number of days is taken as 120 + 3. The proceeds for 120 da. = \$490.

8. The interest on \$635 for 205 da. at 5% = \$18.08.

Amount = \$635 + \$18.08 = \$653.08.

XVII

NOTES ON CHAPTER FOURTEEN

1115. 1. The total interest on the given sums of money is equal to the interest of \$3000 for 1 mo. As the total sum is \$1000, the interest of \$3000 for 1 mo. equals the interest on \$1000 for 3 mo.

1116. 3. Since the time is required, the products may be expressed in years (or months or days), and their total divided by the total of the multipliers.

$$\begin{array}{llll}
2 & \text{yr.} \times & 600 = & 1200 \text{ yr.} \\
1\frac{1}{2} & \text{"} \times & 500 = & 750 \text{ "} \\
1 & \text{"} \times & 300 = & 300 \text{ "} \\
\frac{3}{4} & \text{"} \times & 400 = & 300 \text{ "} \\
\hline
? & \times & 1800 = & 2550
\end{array}$$

Ans. 2550 yr. $\div$ 1800 = 1 yr. 5 mo.

5. $[(15 \text{ da.} \times 200) + (30 \text{ da.} \times 300) + (45 \text{ da.} \times 400)] \div (200 + 300 + 400)$.

6. $[(1 \text{ mo.} \times 210) + (2 \text{ mo.} \times 210) + (3 \text{ mo.} \times 210) + (4 \text{ mo.} \times 210)] \div 840$.

Since the sum due at each period is the same, the 210 may be omitted; $(1 \text{ mo.} + 2 \text{ mo.} + 3 \text{ mo.} + 4 \text{ mo.}) \div 4$.

7. $[(2 \text{ mo.} \times 320) + (4 \text{ mo.} \times 160) + (5 \text{ mo.} \times 240) + (6 \text{ mo.} \times 240)] \div 960$.

Or, $(2 \text{ mo.} \times \frac{1}{3}) + (4 \text{ mo.} \times \frac{1}{6}) + (5 \text{ mo.} \times \frac{1}{4}) + (6 \text{ mo.} \times \frac{1}{4}) = 4\frac{1}{12}$ mo. *Ans.*

8. $(2 \text{ mo.} \times \frac{1}{10}) + (3 \text{ mo.} \times \frac{1}{5}) + (4 \text{ mo.} \times \frac{1}{6}) + (12 \text{ mo.} \times \frac{8}{15}) = 7\frac{13}{15}$ mo. = 7 mo. 26 da. *Ans.*

9. $(0 \text{ mo.} \times \frac{1}{3}) + (3 \text{ mo.} \times \frac{1}{6}) + (6 \text{ mo.} \times \frac{1}{6}) + (9 \text{ mo.} \times \frac{1}{3}) = 4\frac{1}{2}$ mo. *Ans.*

10. $[(0 \text{ da.} \times 300) + (30 \text{ da.} \times 800) + (60 \text{ da.} \times 1000)] \div (300 + 800 + 1000) = 40$ da. July $1 + 40$ da. $=$ Aug. 10. *Ans.*

1117. **11.** The total amount received $\div$ number of bushels sold $=$ average price.

The arrangement of the work may follow that given in Art. 1116, Problem 3.

12. The first puts in the equivalent of 180 cows for a week; the second, 120 cows for a week; the third, 180 cows for a week; a total of 480 cows for a week. The first should pay $\frac{180}{480}$ of \$84; the second, $\frac{120}{480}$ of \$84; the third, $\frac{180}{480}$ of \$84.

$$\begin{aligned} 15 \times 12 &= 180 \\ 20 \times 6 &= 120 \\ 18 \times 10 &= \underline{180} \end{aligned}$$

$$480 : 180 :: \$84 : x$$
$$480 : 120 :: \$84 : y$$
$$480 : 120 :: \$84 : z$$

Proportion is commonly employed in working examples of this kind. As the whole number for a week (480) is to the first man's number for a week (180) so is the whole rent (\$84) to the first man's share (x). The second proportion is used to ascertain the second man's share (y); and the third, to ascertain the third man's share (z).

13. A furnishes \$2000 for 2 yr. and \$1000 for 1 yr., which is the equivalent of \$5000 for a year. B furnishes the equivalent of \$6000 for a year. The total is \$11000 for a year, and the profits of \$1100 are distributed in the ratios of $\frac{5000}{11000}$ and $\frac{6000}{11000}$, as indicated by the proportions here given.

$$\begin{aligned} 2000 \times 2 &= 4000 \\ 1000 \times 1 &= \underline{1000} \quad 5000 \\ 3000 \times 2 &= \qquad\quad \underline{6000} \end{aligned}$$

$$11000 : 5000 :: \$1100 : \text{A}$$
$$11000 : 6000 :: \$1100 : \text{B}$$

A receives \$500 of the profits and his capital of \$3000, or \$3500 in all. B receives \$3600.

14.
$$\begin{aligned} 40x + 30(100 - x) &= 36 \times 100, \\ 40x + 3000 - 30x &= 3600, \\ 40x - 30x &= 3600 - 3000, \\ 10x &= 600, \\ x &= 60 = \text{number of bushels at } 40¢, \\ 100 - x &= 40 = \text{number of bushels at } 30¢. \end{aligned}$$

15. $(60 \times x) + (50 \times 80) = 52(x + 80).$
$$60x + 4000 = 52x + 4160; \text{ etc.};$$
that is, x bu. @ $60¢ + 80$ bu. @ $50¢ = (x + 80)$ bu. @ $52¢$.

16. A does $\frac{1}{20}$ in 1 da.; B does $\frac{1}{30}$ in 1 da.; both do $\frac{1}{20} + \frac{1}{30} = \frac{1}{12}$ in 1 da. They finish the work, therefore, in 12 da., and receive \$60. If A does $\frac{1}{20}$ in 1 da., in 12 da. he does $\frac{12}{20}$, and should receive $\frac{12}{20}$ of \$60 = \$36. B should receive \$24.

17. C's capital of \$4000 for $\frac{1}{2}$ yr. may be considered as \$2000 for a year. This, with \$4000 furnished by A and B, makes the capital \$6000. A takes $\frac{1500}{6000}$, or $\frac{1}{4}$ of the profits; etc. See **13.**

18.
$$\begin{aligned} &600 : 180 :: 180 \text{ tons} : \text{A's share}. \\ &600 : 105 :: 180 \text{ tons} : \text{B's share}. \\ &600 : 315 :: 180 \text{ tons} : \text{C's share}. \end{aligned}$$

19. Let x = number of quarts of water, then $40 - x$ = number of quarts of milk. 40 quarts of the adulterated article at $5¢$ per quart $= 200¢$. $(40 - x)$ quarts of milk at $6¢$ per quart $= (240 - 6x)¢$. x quarts of water cost nothing.
$$\begin{aligned} 200 &= 240 - 6x, \\ 6x &= 40, \\ x &= 6\tfrac{2}{3}. \end{aligned}$$
The can contains $6\frac{2}{3}$ qt. water and $33\frac{1}{3}$ qt. milk.

1120. 1. \$999 = rent for 1 yr. 10. mo. 6 da., or $22\frac{1}{5}$ mo. The rent for 1 mo. $= \$999 \div 22\frac{1}{5} = \$999 \times \frac{5}{111}$; for 12 mo. $= \$999 \times \frac{5}{111} \times 12$.

2. What per cent of 55 oz. is 121 oz.?
$$\frac{x}{100} \text{ of } 55 = 121;\ 55x = 12100;\ x = 220.$$

3. The field contains 1600 sq. rd.; each side measures 40 rd., or 220 yd.; etc.

4. A solves 20 per hour; B solves $15 \times \frac{60}{55}$, or $16\frac{4}{11}$, per hour; both solve $36\frac{4}{11}$ per hour. To solve 100 will require $(100 \div 36\frac{4}{11})$ hours.

6. Six faces, each containing (15×15) sq. in.

7. Cost 90¢. See Arithmetic, Art. 924, **7**. Selling price $= 90¢ \times 1.43\frac{1}{3}$.

8. March 4, 1861, + 4 yr. 1 mo. 11 da. = Apr. 15, 1865; Apr. 15, 1865, − 56 yr. 2 mo. 3 da. = Feb. 12, 1809. *Ans.*

9. The selling price, \$4.50 = 90% of cost; the latter is, therefore, \$5 per barrel. Loss per barrel, 50¢; on 50 bbl., \$25. A profit of 6% = 30¢ per barrel; the gain on 100 bbl. = \$30. Net gain = \$5. *Ans.*

10. 60% of $66\frac{2}{3}\% = 40\%$. If $\frac{40}{100}$ of a number = 810, the number $= 810 \times \frac{100}{40} = 2025$.

Note.—Observe the difference between this example and **8** of Art. 1101.

1121. **1.** Having used $\frac{7}{8}$ of $\frac{4}{7}$ box, the remainder $= \frac{1}{8}$ of $\frac{4}{7}$ box $= \frac{1}{14}$ box. If $\frac{1}{14}$ box $= \$\frac{17}{25}$, a box $= \$\frac{17}{25} \times 14 = 68¢ \times 14 =$ \$9.52. *Ans.*

3. The 40-ft. ladder forms the hypotenuse of two right-angled triangles; *CE* and *DE*, Arithmetic, Art. 1250, Problem **9**. *CA*, one perpendicular, measures 21 ft.; *DB*, the other, measures 33 ft. *AB* is the width of the street.

$$AE = \sqrt{40^2 - 21^2};\ EB = \sqrt{40^2 - 33^2};\ AE + EB = AB.$$

4. 16 oz. : 12 oz. :: \$28 : x.

5. A partnership is the association of two or more persons for the transaction of business on joint account. One advantage of a partnership is the employment of a larger capital, with a smaller percentage of expenses than would be the case were each member to establish a separate business. The firm obtains the combined business experience of its several members, and can utilize the services of each in the department in which he can best serve the interests of the firm; etc.

NOTE. — In the absence of an agreement to the contrary, each partner is entitled to an equal share of the profits, and is liable for an equal portion of the losses; in the examples given, however, the gains and the losses are distributed according to the amount invested by each and the length of time each one's capital remains in the business. See also Art. 977.

6. A, $\frac{16}{104}$ of \$13; B, $\frac{24}{104}$; C, $\frac{28}{104}$; D, $\frac{36}{104}$; etc.

7. The analysis of a mathematical problem or operation should be occasionally used as an exercise in composition, the same attention being given to penmanship, spelling, language, and arrangement as in other such exercises.

1122. **1.** Make *men* the last term; Art. 974, **8.**

If 1 be eaten in 35 da. at 24 oz. daily by 3600 men,
2 will be eaten in 45 da. at 14 oz. daily by ? men.

$$\frac{3600 \text{ men} \times 35 \times 24 \times 2}{45 \times 14} = 9600 \text{ men. } \textit{Ans.}$$

4. $\$3700 \div (1 - .075)$.

8. See Arithmetic Art. 1005, **2.** £500 $= \frac{1}{2}$ of \$4866.50. Take £250, £25, £5, 10*s.*, 5*s.*, 2*s.* 6*d.*, 1*s.* 3*d.*, 2*d.*

Or,

$$\begin{array}{r} 20\underline{)\$4.8665} \times 780 = \\ 12\underline{)\$\ \ .2433} \times \ \ 18 = \\ \$\ \ .0203 \times \ \ 11 = \end{array}$$

9. See Art. 1250, **8.** Calling the part remaining x, the part broken off will be $100 - x$. The latter is the hypotenuse of a right-angled triangle; x is the perpendicular; 40 is the base.

$$\begin{aligned} 40^2 + x^2 &= (100 - x)^2, \\ 1600 + x^2 &= 10000 - 200\,x + x^2, \\ 200\,x &= 10000 - 1600 = 8400, \\ x &= 42, \\ 100 - x &= 58. \end{aligned}$$

The length of the part broken off $= 58$ ft. *Ans.*

By assuming x as the length of the part broken off, the hypotenuse $= x$, and the perpendicular $= 100 - x$.

$$x^2 = 40^2 + (100 - x^2),$$
$$x^2 = 1600 + 10000 - 200\,x + x^2,$$
$$200\,x = 11600,$$
$$x = 58.$$

10. Number of feet in length $= 10 \div (\frac{15}{12} \times \frac{11}{12})$.

13. At $1\frac{1}{2}d.$ per pound, 2240 lb. are worth $3360d. =$ £14. Cost of 120 T. = £1680; the duty in English money is $\frac{1}{5}$ of £1680 = £336; freight = £30; etc.

15.

Dr. U.S. TREASURY DEPARTMENT. *Cr.*

1883.					1883.				
Jan.	3	To 2575 lb. Twine .12	309	—	Feb.	4	By Cash	175	—
Apr.	4	" 25 doz. Pens 25.—	625	—	Apr.	30	" Cash	350	—
May	7	" 645 reams Paper 2.—	1290	—	July	15	" Cash	700	—
July	9	" 45 doz. Ink 3.—	135	—	Nov.	5	" Cash	2300	—
Oct.	30	" 1000 M Envelopes 2.—	2000	—	Dec.	31	" Breakage	75	—
Dec.	5	" 8 doz. Inkstands 1.97	15	76	Dec.	31	" Shortage	60	—
Dec.	31	" Cartage	45	—	Dec.	31	" *Balance*	759	76
			4419	76				4419	76
1884.									
Jan.	1	To Balance	759	76					

The above represents the account as it stands upon the books of Samuel Adams. The debit column contains the amounts due from the Treasury Department, and the credit column contains the sums received, etc. The account is balanced by placing the footing of the debit column under each, and by writing in the credit side the words "By Balance" in red ink, followed by the sum necessary to make the total of the credit column equal to the total of the debit column. Red ink is used to show that the money has not been paid. The account is reopened by writing "To Balance" on the debit side, followed by the sum remaining due.

The statement rendered by Samuel Adams to the Treasury Department would be made out as follows:

WASHINGTON, D.C., Jan. 2, 1884.

U.S. TREASURY DEPARTMENT,

In Account with SAMUEL ADAMS.

1883.		— *Dr.* —				
Jan.	3	To Mdse., as per bill rendered	309	—		
Apr.	4	" " " " " "	625	—		
May	7	" " " " " "	1290	—		
July	9	" " " " " "	135	—		
Oct.	30	" " " " " "	2000	—		
Dec.	5	" " " " " "	15	76		
Dec.	31	" Allowance for Cartage	45	—	4419	76
1883.		— *Cr.* —				
Feb.	4	By Cash	175	—		
Apr.	30	" "	350	—		
July	15	" "	700	—		
Nov.	5	" "	2300	—		
Dec.	31	" Breakage 75.— Shortage 60.—	135	—	3660	—
		Balance due			759	76

The quantities and prices are omitted, as they have been given in the bills rendered at the time the articles were supplied.

For the form of this account as it would appear on the books of the Treasury Department, see Art. 1146, **24**.

1123. **1.** $5280 \div 15.7$.

The owner of a cyclometer should calculate the number of revolutions of a wheel necessary to move the index of the cyclometer over one of its smallest divisions. The circumference of a wheel, 26 in. in diameter, measures nearly $2\frac{1}{4}$ yd. Nine revolutions of the wheel should indicate a trifle over 20 yd. on the cyclometer; 8 revolutions should indicate a trifle over $\frac{1}{100}$ mile, 17.6 yd.

3. The premium $= \frac{3}{400}$ of $\frac{3}{4}$ of $\$6000 = \$33.75$. The loss will be the value of the uninsured one-quarter, $1500, and the above premium.

4. $\$1.40 \times (5\frac{1}{3} \times 5\frac{1}{2}) \times (8\frac{1}{2} \div 3)$.

$$\frac{\$1.40 \times 16 \times 11 \times 17}{3 \times 2 \times 2 \times 3}.$$

The rods are reduced to yards by multiplying by $5\frac{1}{2}$; the feet, by dividing by 3.

5. Let $x =$ the smaller number; $x + 1$ bu. 2 pk. 5 qt. $=$ the larger one.

$$x + x + 1 \text{ bu. } 2 \text{ pk. } 5 \text{ qt.} = 12 \text{ bu. } 1 \text{ pk. } 3 \text{ qt.},$$
$$2x = 12 \text{ bu. } 1 \text{ pk. } 3 \text{ qt.} - 1 \text{ bu. } 2 \text{ pk. } 5 \text{ qt} = 10 \text{ bu. } 2 \text{ pk. } 6 \text{ qt.},$$
$$x = 5 \text{ bu. } 1 \text{ pk. } 3 \text{ qt} = \text{the smaller number},$$
$$x + 1 \text{ bu. } 2 \text{ pk. } 5 \text{ qt.} = 7 \text{ bu.} = \text{the larger number.}$$

8. Each side $= 40$ rd. Area $= 1600$ sq. rd. $= 10$ A.

9. See Art. 1097. $\sqrt{.44'\,10}$.

10. 7000 gr. $\times .17\frac{11}{16} =$ the number of Troy grains. Reduce to pounds, ounces, pennyweights, etc.

11. At what rate (x) will \$324.61 in 2 yr. 7 mo. 13 da. produce (\$384.13 − \$324.61) interest.

$$324.61 \times \frac{x}{100} \times \frac{943}{360} = 59.52;$$

Canceling decimals, $30610723\,x = 214272000$;

$$x = 7 \text{ nearly.}$$

12. $\frac{7}{800}x = 140$.

13. The field contains 6400 sq. rd.; each side measures 80 rd.; the diagonal $= \sqrt{6400 + 6400}$ rd.

14. See Arithmetic, Art. 591.

15. One edge measures $\sqrt{256}$ ft. $= 16$ ft. Solid contents $= (16 \times 16 \times 16)$ cu. ft., or (256×16) cu. ft.

16. Cost and selling price.
Cost and profit (or loss).
Selling price and profit (or loss).

17. $92¢ \times 21643 \div 60$.

18. $\frac{1}{2}$ of $(22\frac{2}{3} \times 19\frac{3}{4})$ sq. ft.

19. See Arithmetic, Art. 924, **7**.

21. 14 da. in Oct. $+ 30 + 31 + 31 + 28 + 31 + 24$ in April exclusive of April 25 $= 189$ da. 9 tons will be needed.

22. 112 A. 96 sq. rd. $= 112.6$ A. Remainder $= 112.6$ A. $- 48.64$ A. $= 63.96$ A.; $63.96 = \dfrac{x}{100}$ of 112.6; $x = 6396 \div 112.6$ $= 63960 \div 1126 = 56\frac{452}{563}$. *Ans.* $56\frac{452}{563}\%$.

23. By 4-ft. wood is meant that the sticks are 4 ft. long. This makes the pile 4 ft. wide. Cancel.

25. See Art. 1117, **12**.

$1 = 18 \times 4 =$	72	$= 1008$ fourteenths.	Share	$= \frac{1008}{4905} = \frac{112}{545}$,
$2 = 12\frac{4}{7} \times 5 =$	$62\frac{6}{7}$	$= 880$ "	"	$= \frac{880}{4905} = \frac{176}{981}$,
$3 = 6\frac{1}{2} \times 11 =$	$71\frac{1}{2}$	$= 1001$ "	"	$= \frac{1001}{4905}$,
$4 = 16 \times 9 =$	144	$= 2016$ "	"	$= \frac{2016}{4905} = \frac{224}{545}$.
Total,	$349\frac{13}{14}$	$= 4905$ "		

26.
$$\tfrac{1}{2}(48 \times \text{perpendicular}) = 160 \text{ rd.} \times 13\tfrac{1}{2},$$
$$24 \times \text{perpendicular} = 160 \text{ rd.} \times 13\tfrac{1}{2},$$
$$\text{perpendicular} = \frac{160 \text{ rd.} \times 27}{24 \times 2} = 90 \text{ rd.}$$

Hypotenuse $= \sqrt{90^2 + 48^2}$ rd. $= 102$ rd. Length of fence $= 48$ rd. $+ 90$ rd. $+ 102$ rd.

1124. **2.** $\frac{1}{2}$ circumference $= 3.1416x$; $\frac{1}{2}$ diameter $= x$; area $= 3.1416x^2$.

NOTE. — The pupil should memorize the ratio between the circumference and the diameter, 3.1416. After learning from Art. 1274, **6–14**, that the area of a circle is equal to the product of the semi-circumference by the semi-diameter, and ascertaining from **2** that this is equal to the square of the radius multiplied by 3.1416, the latter rule can occasionally be employed. See **6**.

4. When the circumference is x, the diameter $= \dfrac{x}{3.1416}$. Then $\dfrac{x}{2} \times \dfrac{x}{3.1416 \times 2} = \dfrac{x^2}{12.5664} = .07958x^2$.

5. $18^2 \times 3.1416$.

6. $R^2 \times 3.1416 = 153.9384$; $R^2 = 153.9384 \div 3.1416 = 49$; $R = 7$. *Ans.* 7 yd.

7. Let $x =$ circumference; then, from **4**, $\frac{x^2}{12.5664} =$ area $= 198.95$; $x^2 = 198.95 \times 12.5664 = 2500.08528$; $x = 50.008$. *Ans.* 50 rd.

8. The square of the diagonal, x^2, = twice the square of a side. The square of a side is, therefore, $\frac{x^2}{2}$, which is the area of the square.

9. Let the pupil draw a square. On its diagonal, which may be called 150 rd., draw another square. Produce two sides of the smaller square so as to make diagonals of the larger one. An examination of the small square will show that its area is one-half that of the other, or $\frac{1}{2}$ of (150×150) sq. rd.

10–12. See **6**, **7**, and **1**, of Measurements, Art. 1107.

13. See **4**. 100 sq. ft. $\div$ 12.5664 = 7.958 sq. ft. *Ans.*

Area = circumference$^2 \times .07958$.

14. The altitude $= \sqrt{625 - 49} = 24$. Area $= (40 \times 24)$ sq. rd.

15. Find the perpendicular, $\sqrt{100^2 - 80^2}$; etc.

16. See **10** of Measurements, Art. 1107.

17. Calculate the altitude. Area $= \frac{1}{2}(60 + 130) \times$ altitude.

18. See **4**, Measurements, Art. 1107.

19. The altitude $= \sqrt{30^2 - 15^2}$.

20. Find the area as in **1**, Measurements, Art. 1107. Divide the area by one-half the base to obtain the altitude.

21. Area = 800 sq. rd.; area of the square constructed on its diagonal = 800 sq. rd. $\times$ 2 = 1600 sq. rd.; length of the diagonal $= \sqrt{1600}$ rd. = 40 rd. *Ans.*

22. See Measurements, Art. 1107, **7**, for the area of one of the six equal triangles.

23. ($6^2 \times 3.1416$) sq. in.

24. Its area is one-half the area of the square constructed on the diameter; that is, $\frac{1}{2}$ of 100 sq. ft.

25. The area of the sector is $\frac{1}{5}$ that of the circle. Area of circle $= 100 \times 3.1416$.

1125. **2.** See Supplement for the definitions.

3. \$6.75 is what per cent of \$2700?

6. The note is due April 13 (10), 1891; the term of discount is 102 (99) da.

7. Do not place the rate under the principal. See Arithmetic, Art. 983, **27**.

	\$261.
6%	15.66
	\$276.66

8. Make the divisor a whole number, $68702050000 \div 48665$. See Art. 1007, **7**.

9. To yield \$900 per annum, the bonds must have a face value of $\$900 \div .045 = \$20,000$. Their cost will be $\$20,000 \times 1.0525$.

10. See Art. 1051, **10**.

1126. **10.** $73\frac{1}{40}$ mi. $\div 2\frac{1}{6}$.

11. See Art. 1056.

12. $\$3 \times 4 \times 4 \div \frac{2}{3}$. Teachers should not require pupils to use pencils unnecessarily. See page 5.

16. $\dfrac{x}{25} + 68 + \dfrac{4x}{7} = x.$

18. See Arts. 546 and 1022, **15**.

19. ($\frac{7}{8}$ of $[(14 + 16 + 14 + 16) \times 8] \div 1\frac{1}{2}) \div 24$. Cancel. The perimeter of the room multiplied by the height gives the surface of the walls; $\frac{7}{8}$ of this gives the number of square feet remaining after the openings are deducted; dividing by $1\frac{1}{2}$, gives the number of feet of paper needed; dividing by 24, which is the number of feet in a roll, gives the number of rolls, or $11\frac{2}{3}$. As a part of a roll is not obtainable, 12 rolls must be purchased.

1127. **5.** Cellar contains $(10 \times 8 \times 2)$ cu. yd.

6. $(48 \times 32) \div (16 \times \frac{1}{3})$.

9. See Art. 1100, **19.**

1128. **7.** $16 : 20 :: x : 25$. See Arts. 1068–1073.

13. $\frac{4}{500}$ of $\frac{2}{3}$ of \$18000. Cancel.

14. $\$120 \div \frac{4}{500}$.

15. Rate, \$8 per \$1000.

1129. See notes on previous Special Drills.

1131. $44 \times 22 = (44 \times 20) + (44 \times 2)$; $44 \times 18 = (44 \times 20) - (44 \times 2)$. $26 \times 62 = (26 \times 60) + (26 \times 2)$; $26 \times 58 = (26 \times 60) - (26 \times 2)$.

1133. See Art. 1064. 49×49, Art. 1032.

1134. See Art. 1065.

5. 9 times 16 ft.

12. 175×12 hundredths $= 1\frac{3}{4} \times 12$.

14. $143 \div \frac{11}{5} = \frac{143}{11} \times 5 = 13 \times 5 = 65$.

15. 70% of 69 = 70% of 70 − 70% of 1.

17. $(29 \times 16) + 26 = 10$ more than (30×16).

20. $2 \times (87 + 49) = [(87 + 50) - 1] \times 2$.

21. Each brick contains $(\frac{2}{3} \times \frac{1}{3} \times \frac{1}{6})$ cu. ft. $= \frac{1}{27}$ cu. ft.

27. 30 da. after April 6 = May 6; 30 da. thereafter = June 5; 30 da. thereafter = July 5; adding days of grace = July 8.

28. Each side measures 2 yd. The surface of 1 face = 4 sq. yd.; of 6 faces = 24 sq. yd.

29. Without grace, the interest for 60 da. is 1% of \$100 = \$1; the proceeds = \$99.

For 3 additional days, the interest is $\frac{1}{20}$ of \$1, or 5¢; the proceeds = \$98.95.

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1135. See Art. 1044. The mark on each case is H. B. The numbers of the cases are 5453 and 5454. The goods are sent (consigned) to Messrs. Hamburger Bros., to be sold on commission.

$94\frac{1}{2}$ yd. at 2*s*. 3*d*.	£10	12*s*.	$7\frac{1}{2}$*d*.
$140\frac{3}{4}$ yd. at 1*s*. 9*d*.	12	6	$3\frac{3}{4}$
61 yd. at 3*s*. 0*d*.	9	3	0
348 yd. at 1*s*. 9*d*.	30	9	0
	£62	10	$11\frac{1}{4}$
Less $\frac{1}{40}$,	1	11	$3\frac{1}{4}$
	£60	19	8

The value in U.S. money = \$296.78. Ad valorem duty at 50% = \$148.39. Specific duty (295 lb. + 351 lb.) = 646 lb. @ 44¢ = \$284.24. The entire duty = \$432.63. *Ans.*

1137. Divide area by $\frac{1}{2}$ base. See Art. 1107, Measurements, **1** and **4**.

1139. **1.** A's equivalent = 72, (6 × 12); B's = 70, [(5 × 11) + (3 × 5)]. Total, 142. A pays $\frac{72}{142}$ of \$175; etc.

2. Total debts (\$750 + \$1125 + \$1245) = \$3120.

3120 : 750 :: \$1287 : A
3120 : 1125 :: \$1287 : B
3120 : 1245 :: \$1287 : C

3. K, 50 × 26; L, 60 × 26; M, 70 × 20; N, 90 × 22; etc.

4. Some persons prefer to employ smaller figures for the capital invested, by dividing each by the same number. A's can be taken as \$25; B's as \$40; and C's as \$50; or \$5, \$8, and \$10 may be used.

5 × 12 = 60
8 × 9 = 72
10 × 5 = 50

182 : 60 :: \$15000 : A's share.
182 : 72 :: \$15000 : B's share.
182 : 50 :: \$15000 : C's share.

5. Counting each ox as 3 sheep,

$$\begin{array}{ll}
60 \times 10 = 600 & \\
50 \times \ \ 8 = \underline{400} & 1000 \\
75 \times \ \ 8 = 600 & \\
30 \times \ \ 7 = \underline{210} & 810 \\
54 \times 10 = 540 & \\
10 \times 12 = \underline{120} & 660 \\
90 \times 12 = & \underline{1080} \\
& 3550 : 1000 :: \$152.50 : W \\
& 3550 : \ \ 810 :: \$152.50 : X \\
& 3550 : \ \ 660 :: \$152.50 : Y \\
& 3550 : 1080 :: \$152.50 : Z
\end{array}$$

6. 220 yd. were built in 11 da. by 18 men. 480 yd. will be built in 18 days by ? men.

$$\frac{18 \text{ men} \times 11 \times 480}{220 \times 18} = 24 \text{ men.} \quad 6 \text{ extra men. } \textit{Ans.}$$

7. See Art. 1122, **1.**

8. 14 men in $(8\frac{1}{4} \times 12)$ hr. mow 168 acres. 20 men in $(7\frac{4}{5} \times 11)$ hr. mow ? acres.

NOTE.—A thoughtless scholar will sometimes fail to see that the 15 min. should be joined to 8 hr.; he will, therefore, compare 8 hr. with 7 hr., and 15 min. with 48 min.

9. To do 4 times the work in $\frac{1}{5}$ of the time will take 20 times 12 men, or 240 men. (Omit 20 da.)

10. At the time they meet the sinking vessel, 60 men have provisions for 24 da.; these will last 72 persons 20 da.

$$72 : 60 :: 24 \text{ da.} : x$$

11. Omit the dimensions of the boards. If 76 are worth \$19.76, 50 are worth $\$19.76 \times 50 \div 76$.

1140. This table will furnish some practice in addition and division, and should not be passed over.

1141. The "developed" entire surface of a square prism is shown in Arithmetic, Art. 818, **20**. In drawing the development of the convex surface, the upper and lower squares, denoting the bases, will be omitted. The drawing should be done with reasonable care, to a scale of, say, $\frac{1}{2}$ inch to the inch.

In making a model of a solid, narrow strips for pasting should be added, as shown in Fig. 1. In the development of a triangular prism, the bases are usually drawn above and below the middle rectangle, but the pupils should learn that they may be placed in other positions, one of which is here shown. It will be noticed that the pasting flaps do not form rectangles, the sides being inclined at an acute angle to make neater work in the completed model.

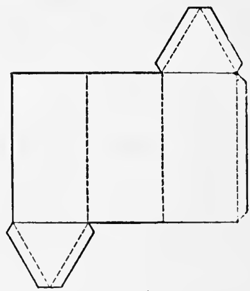


FIG. 1.

The shape and the arrangement of the gumming flaps for the bases of a cylinder are shown in Fig. 2. Interested pupils may be safely left to themselves to ascertain the length of the rectangle that is needed for the model of a given cylinder.

FIG. 2.

The scholars will learn more geometrical facts while constructing these models than they will obtain by memorizing many pages of definitions or listening to numerous "explanations."

5. The entire surface includes the convex surface and the surface of the bases.

8. If x represents one side of a cube, $x^2 =$ the surface of one face, and $6x^2 =$ the entire surface $= 216$ sq. in. *Ans.*

9. The convex surface $= 4x^2 = 144$ sq. in. *Ans.*

11. The perimeter $= (600 \div 15)$ ft.

Or, let $x =$ one side of base; the perimeter $= 4x$; the convex surface $= 4x \times 15 = 60x = 600$; $x = 10$; etc.

12. Let $x =$ the altitude. $(15+15+15+15) \times x + 15^2 + 15^2 = 1650$; $60x + 225 + 225 = 1650$; $60x = 1650 - 225 - 225 = 1200$; $x = 20$; the convex surface $= 60x = 1200$. 20 in.; 1200 sq. in. *Ans.*

13. Let $x =$ one side of base; $4x =$ the perimeter; $4x \times 15 = 60x =$ the convex surface $= 540$; $x = 9$. The entire surface $= 540$ sq. in. $+ 81$ sq. in $+ 81$ sq. in.

14. Circumference of base $= 3.1416$ ft. Convex surface $= (3.1416 \times 1)$ sq. ft. $= 3.1416$ sq. ft. Radius of base $= \frac{1}{2}$ ft.; area $= (\frac{1}{2} \times \frac{1}{2} \times 3.1416)$ sq. ft. $= .7854$ sq. ft. Entire surface $=$ 3.1416 sq. ft. $+ .7854$ sq. ft. $+ .7854$ sq. ft. $= 4.7124$ sq. ft. *Ans.*

15. See Arithmetic, Art. 1290. While pupils should be permitted to "develop" these solids in their own way, provided it be a correct one, they should be advised in making drawings for models to use a pattern that will require a minimum of pasting. While Fig. 3 would serve for the development of the entire surface, it would not answer as a pattern from which to construct a hollow pyramid.

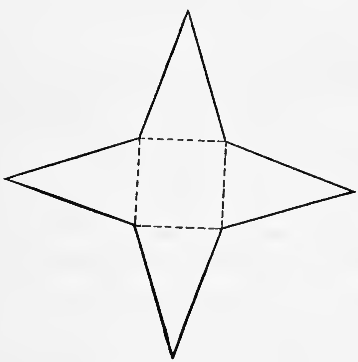


FIG. 3.

16. The convex surface of a pyramid is equal to the perimeter of the base $\times \frac{1}{2}$ the slant height. The slant height of a regular pyramid is the altitude of one of the equal triangles that constitute its convex surface.

18. One side of base $= \sqrt{144}$ in.

19. Either calculate the slant height, which is $\sqrt{2^2 - 1^2} = \sqrt{3}$; or employ the method given in **1**, of Measurements, Arithmetic, Art. 1107.

20. The developed convex surface of a cone is a sector, whose radius is the slant height of the cone and whose arc is equal in length to the circumference of the base of the cone.

The circumference of the base of the given cone = (4 times 3.1416) in. The circumference of the circle of which the sector forms a part, is (2 × 6 times 3.1416) in., or (12 times 3.1416) in.; the sector is, therefore, $\frac{1}{3}$ of the circle, and its arc measures $\frac{1}{3}$ of 360°, or 120°.

Any sector of 120° will form a hollow cone of the proper proportions.

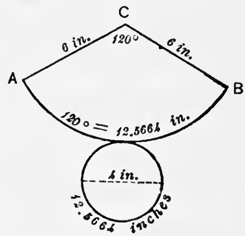


Fig. 4.

The base shown in Fig. 4 is not required by the terms of this problem; it is merely introduced here to show the development of the entire surface. As it is difficult to lay off the required number of inches for the arc AB, the pupil will appreciate the foregoing method of determining the number of degrees it should contain. The compasses or the protractor may be employed to construct an angle of 120° at C.

The convex surface of a cone is equal to the circumference of the base × $\frac{1}{2}$ slant height.

An examination of Fig. 4 will show the resemblance between the methods of calculating the surface of a sector and of a triangle. The area of a triangle = $\frac{1}{2}$ (base × altitude); that of a sector = $\frac{1}{2}$ (base × radius).

22. The altitude, one-half the base, and the slant height, form a right-angled triangle; and the lengths of the two first being 12 in. and 5 in., respectively, the length of the latter is $\sqrt{144 + 25}$ in., or 13 in. The convex surface = $\frac{1}{2}$ (10 × 3.1416 × 13).

23. The entire surface = [$\frac{1}{2}$ of (6 × 3.1416 × 10)] + (3^2 × 3.1416) = (30 × 3.1416) + (9 × 3.1416) = 39 × 3.1416.

Using π (pī) instead of 3.1416, the circumference of the base = 6π inches. The radius of the sector representing the development, is 10 in., and the circumference of the whole circle = 20π inches. As the arc of the sector must be 6π inches, it measures in degrees $\frac{6}{20}$ of 360°, or 108°.

24. The slant height will be $\frac{1}{2}$ of 6 in. The circumference of the base will equal the arc of the semicircle, 3π inches; its diameter will, therefore, be 3 in.

1142. **3.** Due Sept. (21) **24.** Term of discount from July $21 = 65$ (62) da.

4. \$600 yearly interest represents a principal of \$10000.

5. Length of one fence, $(20 + 20 + 20 + 20)$ rd.; of the other, $(40 + 10 + 40 + 10)$ rd.

6. The distance between the center of the first and of the last post $= 10$ ft. $\times (11 - 1) = 100$ ft. Adding $\frac{1}{2}$ of the diameter of each post, gives 100 ft. 6 in.; and an additional 3 in. at each end to fasten the wire, makes a total of 101 ft. of wire required for each length, or 303 ft. in all. *Ans.*

1143. **7.** Troy weight.

8. $43\frac{2}{5}$ mo. @ 25¢ per mo. 43 quarter dollars $=$ \$10.75; $\frac{2}{5}$ of 25¢ $=$ 10¢. Total \$10.85. *Ans.*

11. Without grace, 1% of \$400, or \$4. *Ans.* With grace, \$4 + $\frac{1}{20}$ of \$4, or \$4.20. *Ans.*

12. $\frac{1}{4}$ of cost $=$ \$2; etc.

13. The principal is unimportant. Time $= (100 \div 8)$ yr.

17. $(100 \div 6)$ yr. **23.** $\frac{1}{2}$% of \$1234.

24. 2% of \$1234.

25. The number of rings $=$ 60 pwt. $\div 2\frac{1}{2}$ pwt.

26. $2° 3' \times 15$. **27.** [Twice $(4 + 3) \times 10$] + [twice (4×3)].

28. \$1.12$\frac{1}{2}$ + $\frac{1}{4}$ of \$1.12$\frac{1}{2}$ = \$1.12$\frac{1}{2}$ + \$.28$\frac{1}{8}$.

30. Selling price $= \frac{3}{2}$ of cost $= \frac{1}{2}$¢; cost $= \frac{1}{2}$¢ $\div \frac{3}{2} = \frac{1}{3}$¢. *Ans.*

1144. **4.** 14% profit $=$ 7¢; cost $=$ 7¢ $\div .14 =$ 50¢; selling price $=$ 57¢. *Ans.*

7. $x + \dfrac{x}{40} + \dfrac{x}{200} = 7828$; etc.

17. Cost of 350 tons @ \$3.50 = \$1225. Selling price = $\$4.25 \times 350 \times \frac{2240}{2000}$.

NOTE.— The scholars should not use pencils to obtain answers to problems that can be solved at sight.

1145. See Art. 1290.

3. Area of base = 6 sq. in.; etc.

5. See Art. 1107, Measurements, **7.**

8. Changing given dimensions to inches, the number of gallons will be $36^2 \times 3.1416 \times 66 \div 231$.

9. 1 cu. ft. = $\frac{1000}{16}$ lb. Cubical contents = $(3^2 \times 3.1416 \times 5\frac{1}{2})$ cu. ft.

10, 11. Careful pupils will be much interested in ascertaining how closely their calculations as to the contents of the measure, agree with the number of cubic inches it is supposed to contain. There should be 231 cu. in. ÷ 8, in a quart. The cup used must be cylindrical. Tapering measures should be left until the frustum of a cone has been studied, Art. 1295. The paper box used for ice cream, a frustum of a pyramid, can also be employed at that time. Some of these measurements should be made out of school, and comparisons made as to the results obtained by different pupils and the methods employed by them to secure accuracy. A random measurement will not obtain the correct diameter of a quart measure.

After calculating the altitude of an equilateral triangle or the diagonal of a square, the pupil should draw the figure to a scale, measure the altitude or the diagonal, and compare the measured length with the length obtained by calculation.

Pupils should ascertain the weight of a cubic foot of water by weighing a quart of water, for instance, etc.

12. Measure the height to which the water rises in the box, etc.

If a solid heavier than water, is placed in a rectangular or a cylindrical vessel containing sufficient water to cover it, and the difference in the depth of the water before and after immersion is noted, the volume of the solid can be calculated. It will be equal to that of a solid whose base is the base of the vessel, and whose altitude is the difference in depth above mentioned.

If the water in a rectangular box, whose base measures $5\frac{1}{2}$ by 3 in., is raised $1\frac{7}{8}$ in. by the introduction of a piece of marble, the volume of the latter $= 5\frac{1}{2} \times 3 \times 1\frac{7}{8}$ cu. in.

This method is useful in determining the contents of a solid of irregular shape.

13. The radius of the base $= \frac{1}{2}$ of (25.1328 yd. $\div$ 3.1416) $= 4$ yd. Volume of cone $= (4^2 \times 3.1416 \times \frac{1}{3}$ of 18) cu. yd.

14. See Art. 1141, **22**.

15. The slant height of the pyramid $= \sqrt{24^2 + (\frac{1}{2} \text{ of } 14)^2}$. See Art. 1283, **13**.

1146. **6.** $[(10\frac{3}{8}¢ \times 1\frac{1}{4}) + (3\frac{7}{8}¢ \times 1\frac{3}{8})] \times 10840)$.

7. 1000 grams $\div$ 279 $=$ weight in grams of a 10-mark piece. Weight in Troy grains $= (1000 \div 279) \times 15.432349$. Dividing this result by $23\frac{22}{100}$ gives the number of U.S. gold dollars.

$$\frac{1000 \times 15 \odot 43.2349}{279 \times 23 \odot 22.}$$

NOTE. — 23.22 is changed to a whole number by removing the decimal point two places to the right, and a corresponding change is made in one of the numbers in the dividend.

11. Interest on

$$\begin{array}{rlrl} \$237453250 & @\ 3\ \% & = \$ & 7123597.50 \\ 250000000 & @\ 4\frac{1}{2}\% & = & 11250000. \\ \underline{737954700} & @\ 4\ \% & = & \underline{29518188.} \\ \$1225407950 & @\ x\ \% & = \$ & 47891785.50 \end{array}$$

$$4789178550 \div 1225407950 = 3.9083 +$$

The interest on the entire amount at $2\frac{1}{2}\%$ would be \$30635198.75, the saving being \$17256586.75. *Ans.*

12. \$100 worth of stock costs \$$85\frac{3}{4}$. The annual dividend is 5% of \$100, or \$5. This is $(5 \div .8575)$ per cent on the money invested.

13. $\$8930 \div 1.11\frac{5}{8}$.

14. See Art. 1122, **15.** The following shows the account as it stands on the books of the Interior Department. The items that appear as debits on Mr. Well's books, here appear as credits, and *vice versa*.

Dr. RICHARD WELLS. *Cr.*

1882					1882				
Jan.	31	To Cash	885	—	Jan.	1	By 645 bbl. Flour 9.45	6095	25
Feb.	5	" "	450	—	"	16	" 1912 bu. Oats .57	1089	84
Apr.	11	" "	615	35	Apr.	4	" 9231 lb. Bacon .09	830	79
May	30	" "	4162	15	May	3	" 8264 bu. Corn .74	6115	36
June	25	" 345 lb. Bacon .09	31	05	June	20	" 325 bbl. Pork 12.65	4111	25
"	25	" 35 bbl. Pork 12.65	442	75	"	30	" Cartage	65	—
"	30	" Penalty	75	—					
"	30	" Cash in full	11646	19					
			18307	49				18307	49

1148. Multiply the length in feet by the width in feet by the thickness in inches.

16. 3¢ (per ft.) $\times 15 \times 16 \times \frac{3}{4} \times 3$.

17. The floor contains $(36 \times 17\frac{1}{2})$ sq. ft., or 630 sq. ft. If 1-inch boards were used, 630 board feet would be required. The number of feet of $2\frac{1}{2}$-inch planks required $= 630$ ft. $\times 2\frac{1}{2}$.

18.
$$\begin{aligned}(150 \times 13 \times \tfrac{2}{3} \times 1) \text{ board ft.} &= 1300 \text{ board ft.}\\ (60 \times 14 \times \tfrac{3}{4} \times 2) \text{ `` `` } &= 1260 \text{ `` ``}\\ (40 \times 15 \times \tfrac{5}{12} \times 4) \text{ `` `` } &= \underline{1000} \text{ `` ``}\\ \text{Total, } & 3560 \text{ `` ``}\end{aligned}$$

the duty on which, at \$1 per M, is \$3.56. *Ans.*

19. The length of the fence $= 480$ ft. $+ 360$ ft. $+ 480$ ft. $+ 360$ ft. $= 1680$ ft. For a fence 4 boards high, (1680×4) running feet of boards will be needed, or 6720 running feet. If the boards are $\frac{1}{2}$ ft. wide and 1 in. thick, the number of board feet $= 6720 \times \frac{1}{2} \times 1 = 3360$ ft. Cost $= \$18 \times 3.36$.

20. The length of the fence $= (25 + 100 + 25 + 100)$ ft. Surface $= (250 \times 6)$ sq. ft. $= 1500$ sq. ft. As the boards are 1 in. thick, the number of board feet $= 1500$. Cost $= \$25 \times 1.5$

= $37.50. The number of posts = $250 \div 6\frac{1}{4} = 40$; cost, at 25¢ each, $10. The number of running feet of scantling, two strips, = $250 \times 2 = 500$; the number of board feet = $500 \times \frac{1}{4} \times 2 = 250$; cost, $18 × .25 = $4.50. Total cost, $37.50 + $10 + $4.50 = $52. *Ans.*

1149. 1. (*a*) A note made payable to the *order* of a certain person or to *bearer* is negotiable; in the former case, an endorsement is necessary to transfer its ownership. A note payable to *bearer* does not require endorsement. A note payable to Charles Naumann (without the words, "or order," or the like) is not transferable by endorsement. If Charles Naumann wishes to sell the note, he must *assign* his interest in it by another document.

NOTE. — The above is the general rule; in some states there are special laws bearing on the subject:

In Alabama and Kentucky, a note to be negotiable must be payable at a fixed place; in Indiana and Virginia, at a bank; in West Virginia, at a bank or public office. In Pennsylvania, it should contain the words "without defalcation"; in New Jersey, "without defalcation or discount"; in Missouri, "negotiable and payable without defalcation or discount."

(*b*) A person unable to write his name, makes his mark, as shown below, in the presence of a witness:

his
Witness: WILLIAM **X** DEVERS.
THEODORE H. FICKLIN. mark

3. In old deeds, the contents of a farm are given in acres (A.), roods (R.), and poles (P.), the rood being $\frac{1}{4}$ acre, and containing 40 poles, or square rods. In long measure, the word *pole* is occasionally employed instead of *rod.*

6. $2x = x + \frac{x}{2} + \frac{x}{3} + 18$; etc.

7. The distance between the ships is the hypotenuse of a right-angled triangle, whose other sides are 72 mi. and 128 mi., respectively.

8. The first capital $= \$3500 \div 1.40 = \$2500$; C put in $\frac{2100}{3500}$, or $\frac{3}{5}$ of $\$2500 = \$1500$; etc.

9. See Art. 1026, **10.**

10. See Supplement.

1150. **3.** $x^2 + x^2 = \text{hypotenuse}^2 = 100$. $x^2 = 50$, the area of the inscribed square. *Ans.*

4. Area of circle $= (5 \times 5 \times 3.1416)$ sq. in. $= 78.54$ sq. in.

5. Arc of $90° = \frac{1}{4}(10 \times 3.1416)$ in. $= 7.854$ in.; the chord $= \sqrt{50}$ in.

6. Arc of 90° in a circle whose radius is 10 in. $= 15.708$ in. Area of sector $= \frac{1}{2}$ of (15.708×10) sq. in. $= 78.54$ sq. in. *Ans.* Area of triangle $= \frac{1}{2}$ of (10×10) sq. in. $= 50$ sq. in.; area of segment $= 78.54$ sq. in. $- 50$ sq. in. $= 28.54$ sq. in. *Ans.*

8. $R^2 \times 3.1416$. *Ans.*

9. Area of outer circle in square yards $= (15^2 \times 3.1416) \div 9$; of inner circle $= (10^2 \times 3.1416) \div 9$.

10. (125×3.1416) sq. ft.

11. $[(6^2 - 3^2) \times 3.1416]$ sq. in. The radius of the outer circle is 6 in.; of the inner circle, 3 in.

12. $36 \times 3.1416 : 9 \times 3.1416 = 4 : 1$.

13. $[(30 \times 30) - (20 \times 20)]$ sq. ft.

14. Dividing the walk into four trapezoids, as in Fig. 5, the area of each will be $[\frac{1}{2}$ of $(30 + 20) \times 5]$ sq. ft. $= 125$ sq. ft. The broken line, XY, drawn midway between the parallel sides measures 25 ft.; the whole length of the center lines is 100 ft. The area of the walk is (100×5) sq. ft.

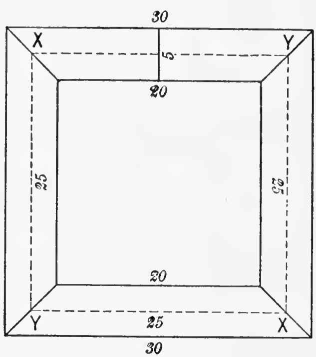


FIG. 5.

The area of the circular frame in **11** can be ascertained in the same way. The center of the frame is $4\frac{1}{2}$ in. from the center of the glass. The middle line of the frame $= (3.1416 \times 4\frac{1}{2} \times 2)$ in. $= 28.2744$ in. The area $= (28.2744 \times 3)$ sq. in.

15. The area of the first $= 240$ sq. ft.; of the second, 960 sq. ft.

1153. 18. The surface of the sphere $= (4\pi \times \frac{1}{4})$ sq. ft. $= 3.1416$ sq. ft.; the convex surface of the cylinder $= 3.1416$ sq. ft.

19. The entire surface $= 3.1416$ sq. ft. $+ (2 \times \frac{1}{4} \times 3.1416)$ sq. ft. $= 1\frac{1}{2}$ times 3.1416 sq. ft.

20. 4 times ($\frac{1}{2}$ circumference) $10 \times$ ($\frac{1}{2}$ diameter) $\frac{10}{3.1416} = \frac{400}{3.1416} =$ surface in square inches.

1154. 1. Rate per cent $= 18750 \div 12500 = 1\frac{1}{2}$. $1\frac{1}{2}\%$ of $\$6000 = \$90$.

3. The price of silver is now given by the ounce.

4. The interest on \$200 at $4\frac{1}{2}\% = \$9$. $\$9 \div .08 = \$112.50$. *Ans.*

5. $\$2100 + \$4400 + \$13000 + \$7200$ (90% of \$8000) $= \$26700 =$ total assets. Total liabilities $= \$1625 + \$5625 = \$7250$. $\$26700 - \$7250 = \$19450$. To this, add the amounts withdrawn, $\$850 + \$1075 = \$1925$, making the sum of the capital and profits \$21375. Of this, H is entitled to $\frac{1}{3}$, \$7125, less the amount withdrawn by him, \$1075, or \$6050.

1157. 9–11. Find the cube root of the numerator and of the denominator separately.

12–15. Reduce to an improper fraction before extracting the cube root; then reduce the root to a mixed number.

1159. 2. $[\frac{4}{3} \times 3.1416 \times (\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2})]$ cu. in. Cancel.

3. The volume of the first in cubic inches $= \frac{4}{3}\pi \times \frac{1}{8} = \frac{1}{6}\pi$. The volume of the second $= \frac{4}{3}\pi \times 1 = \frac{8}{6}\pi$.

4. The volume of the sphere $= .5236$ cu. in.; the volume of the cube $= 1$ cu. in.

6. The ball contains .5236 cu. ft. Its weight $= (.5236 \times 7.5 \times 1000 \div 16)$ lb.

1160. **1.** 40 tons were to be moved; there remain 22 tons to be moved in 3 da. In 6 da. 18 men moved 18 tons, so it requires 22 men to remove 22 tons in 6 da., or 44 men to remove 22 tons in 3 da. 44 men. *Ans.*

3. A diagonal on the floor measures $\sqrt{40^2 + 30^2}$ ft. $= 50$ ft. A diagonal on one wall measures $\sqrt{40^2 + 14^2}$ ft. $= 42.38 -$ ft.; on the other wall it measures $\sqrt{30^2 + 14^2}$ ft. $= 33.11 -$ ft.

4. Selling price, \$6500 + 15% of \$6500 = \$6500 + \$975 = \$7475.

As no date is given, the note may be assumed to be for 120 da., or 123 da. including grace. Proceeds of \$7475 for 123 da. = \$7321.76. Profits = \$7321.76 − \$6500 = \$821.76. *Ans.* Without grace, the proceeds would be \$3.74 more, making the profits \$825.50. *Ans.*

5. Let $x =$ face; $\frac{13x}{400} =$ premium; bank discount $= x \times \frac{33}{360} \times \frac{8}{100} = \frac{11x}{1500}$ (including grace).

$$x + \frac{13x}{400} - \frac{11x}{1500} = 4265,$$

$$6000x + 195x - 44x = 25590000,$$

$$6151x = 25590000,$$

$$x = 4160.30 -. \quad \$4160.30. \ \textit{Ans.}$$

Without grace, $x + \frac{13x}{400} - \frac{x}{150} = 4265$; etc.

7. Longitude difference $= 48°\ 24'$; time difference $= 48$ hr. 24 min. $\div 15 = 3$ hr. 13 min. 36 sec. As the more easterly place has the later time, the watch is fast. *Ans.*

The above result is based upon the assumption that the "sun time" of each place is used. The difference in the "standard" time of the two cities is 3 hr.

9. $\frac{1}{4}$ of $\frac{1}{3}$ of $\frac{3}{8}$ of $x = 6$; $\frac{x}{32} = 6$; $x = 192$.

10. $x + \frac{x}{5} = x - \frac{x}{8} + 1040$;

$$40x + 8x = 40x - 5x + 41600;$$
$$13x = 41600;\ x = 3200. \qquad \textit{Ans. } \$3200.$$

11. $2050 : x :: 41 : 69$. Cancel.

14. $\$1450 - \frac{1}{4}\%$ of $\$1450 - 63$ (60) days' interest on $\$1450$ at $5\% =$ cost of draft.

17. A can do $\frac{1}{12}$ in 1 da.; B can do $\frac{1}{20}$; both can do $\frac{2}{15}$ in 1 da., and can do the whole work in $7\frac{1}{2}$ da. *Ans.*

19. See Art. 1150, **4.**

1161. The definitions and principles called for throughout this chapter should be formulated, as far as possible, by the pupils, the latter being led through the teacher's questioning to see their mistakes, and to make the necessary corrections. If this preliminary work is done as it should be, the scholars will be ready to understand the definition finally given by the teacher. Too much time, however, should not be wasted on formal definitions, as they are of next to no help to a pupil in his mathematical work, and it is very unlikely that he will ever be called upon to use them in after life. See Supplement.

1. (*d*) A decimal fraction is frequently defined as one whose denominator is 10 or some power of 10. In this place, however, the expression is used as synonymous with "decimal." The rule asked for, refers to the method of "pointing off" the product.

7. If all three are opened, they will fill $(\frac{1}{10} + \frac{1}{6} - \frac{1}{5})$ or $\frac{1}{15}$ of the cistern in 1 hr. To fill the whole cistern will require 15 hr. *Ans.*

9. As they meet in 15 hr., they must approach each other at the rate of $(105 \div 15)$ miles per hour, or 7 mi. As one goes 3 mi. per hour, the other must travel 7 mi. $-$ 3 mi. $=$ 4 mi. per hour.

18. A's capital at end of year is \$8000, B's is \$10000, C's, \$3000: total, \$21000. Profits = \$18000 + \$12000 − \$21000 = \$9000.

$$\begin{array}{llrr}
\text{A,} & 10000 \times \frac{1}{2}\ (\text{yr.}) & = 5000 & \\
 & 8000 \times \frac{1}{2} & = \underline{4000} & 9000 \\
\text{B,} & 6000 \times \frac{1}{2} & = 3000 & \\
 & 10000 \times \frac{1}{2} & = \underline{5000} & 8000 \\
\text{C,} & & & \underline{3000}
\end{array}$$

$$\begin{array}{l}
20000 : 9000 :: 9000 : 4050 \\
20000 : 9000 :: 8000 : 3600 \\
20000 : 9000 :: 3000 : 1350
\end{array}$$

A is entitled to his capital at the end of the year, \$8000, and \$4050 profits, or \$12050. As he receives \$12000 worth of goods, his cash receipts are \$50. B receives \$10000 + \$3600 = \$13600. C receives \$3000 + \$1350 = \$4350.

19. $\frac{x}{5} + \frac{1}{4}$ of $\frac{4x}{5} + 20 + \frac{2x}{5} = x;$

$$\frac{x}{5} + \frac{x}{5} + \frac{2x}{5} + 20 = x; \text{ etc.}$$

20. He sold 800 bbl. at \$7.50, which amounted to \$6000. The cost was \$7200 + \$312 + \$350 = \$7862. His profits being \$138, he must have received \$7862 + \$138 = \$8000. As the flour realized only \$6000, he must have received \$2000 from the railroad company.

28. 16% of the person's money = 20% of \$160000; etc.

29. 4 men do $\frac{1}{3}$ of the work in 60 hr.; to do the remainder, they would need 120 hr.; and 1 man would require 480 hr. There are 8 da. of 10 hr., or 80 hr., in which to finish it; 6 men, therefore, will be needed to complete it, or 2 men additional.

30. Cost = \$100 + \$5 + \$50 + \$20 = \$175. Selling price, \$300 less 10% (\$270) − \$20 = \$250. Profit, \$75.

If the commission merchant had refunded \$20 before making returns, his commission would have been 10% of \$280, or \$28; and the gain would have been \$77.

36. (*b*) See Arithmetic, Art. 668.

38.

$$\begin{array}{llr}
\text{A,} & 5000 \times \frac{1}{2} = 2500 & \\
 & 3000 \times \frac{1}{2} = \underline{1500} & 4000 \\
\text{B,} & & 6000 \\
\text{C,} & 4000 \times \frac{1}{2} = 2000 & \\
 & 12000 \times \frac{1}{2} = \underline{6000} & \underline{8000} \\
 & & 18 : 4 :: \$6000 : \text{A, etc.}
\end{array}$$

39. Proceeds of \$12000 for 93 da. = \$11814. Amount of \$10000 for 6 mo. at 6% = \$10300. Sum remaining = \$11814 − \$10300 = \$1514.

Without grace, the proceeds of 90-days note = \$11820; sum remaining = \$1520.

40. Cost, \$40 each. 10 were sold @ \$44 each = \$440; 10 @ \$46 each = \$460; 15 were sold for \$100: total \$1000. To obtain \$900 for remaining 15, he must charge \$60 each.

1164. **1.** $(30+30+31+30+31+31+30+31+30+17)$ da. = 291 da. Exact interest $= \$400 \times \frac{1}{20} \times \frac{291}{365}$.

$$\begin{array}{rcrcrl}
0 \text{ da.} & \times & 630 & = & & 0 \text{ da.} \\
243 \text{ “} & \times & 820 & = & 199260 & \text{“} \\
274 \text{ “} & \times & \underline{950} & = & \underline{260300} & \text{“} \\
 & & 2400 & &)\underline{459560} & \text{da.} \\
 & & & & 191+ & \text{da.}
\end{array}$$

4. Business men generally find the number of days' credit to which each item is entitled. \$820 is due March 5, or 243 da. after July 5; \$920 is due April 5, or 274 da. after July 5. The equated time is 191 da. after July 5, or Jan. 12. *Ans.*

Using months instead of days, the average term of credit is found to be 6 mo. 9 da., nearly, making the equated time Jan. 14.

5.

$$7500 : \text{A} :: 1200 : 250$$
$$7500 : \text{B} :: 1200 : 950.$$

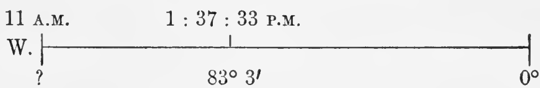
6. Bank stock pays $(25 \times 7\frac{1}{2} \div 85)$ per cent interest semi-annually; the railroad stock pays $(25 \times 3 \div 31)$ per cent interest semi-annually.

9. The bases are equilateral triangles, each side of which measures 5 in. Art. 1107, **7**, Measurements.

1165. **1.** See Supplement.
Quantity is anything that can be measured.
See Arithmetic, Art. 1072.

2. $\frac{2}{3}$ of $\frac{11}{12} \times \frac{81}{1} \times \frac{2}{11} \times \frac{3}{2} \times \frac{3}{11} \times \frac{1}{6} \times \frac{11}{2}$. Cancel.

3. Time difference = 2 hr. 37 min. 33 sec.



Longitude difference = ?

The longitude difference = 2° 37′ 33″ × 15 = 39° 23′ 15″. Longitude of San Francisco = 83° 3′ + 39° 23′ 15″ = 122° 26′ 15″ west.

5. $x + 2x + 5x = 11480$.

6. The equated time for the payment of \$600 is $[(200 \times 1) + (400 \times 2)]$ yr. ÷ 600 = $1\frac{2}{3}$ yr. The present worth of \$600 due in $1\frac{2}{3}$ yr. = \$600 ÷ 1.1 = \$545.45 +.

Another way is to calculate the present worth of each, and to add the results: (\$200 ÷ 1.06) + (\$400 ÷ 1.12) = \$188.68 + \$357.14 = \$545.82. *Ans.*

The latter is the more consistent way inasmuch as it employs the "present worth" method throughout. The first solution uses the "present worth" method to calculate the value at date, of \$600 whose equated time has been found by the "interest" (bank discount) method.

7. Selling price = \$120 + 15% of \$120 = \$138. Asking price = \$138 + \$12 = \$150. I threw off \$12 from \$150, or 8%.

8. Arithmetic, Art. 1250, **8.** $AD = 39$, $AC = 52$, $DC = \sqrt{52^2 + 39^2} = 65 = BC$. Height of tree = 52 ft. + 65 ft. = 117 ft.

9. $\sqrt[3]{10.125000} = Ans.$

10. If $\frac{4}{5}$ gain $= \frac{2}{15}$ cost, the gain $= \frac{2}{15}$ cost $\times \frac{5}{4} = \frac{1}{6}$ cost $=$ $16\frac{2}{3}$ per cent.

1166. 4. $[1 \div (\frac{1}{20} + \frac{1}{15})]$ weeks.

5. $x + x + 1\frac{3}{4} = 7\frac{1}{2}$.

15. B's gain of \$1400 is $\frac{7}{12}$ of total gain; $\frac{1}{12}$ of total $=$ \$200; A's gain, $\frac{5}{12}$ of total $=$ \$1000.

1167. 4. Let $x =$ cost per barrel.

$$75\% \text{ of } 500\,x \times .02\tfrac{1}{5} = 80.85.$$

5. A does $\frac{1}{27}$ in 1 da.; B does $\frac{1}{15}$ in 1 da. A does $\frac{12}{27}$ of the work, or $\frac{4}{9}$, and B does $\frac{5}{15}$ of it, or $\frac{3}{9}$, leaving $\frac{2}{9}$ to be done by C in 4 da. To do the whole work C would require 4 da. $\div \frac{2}{9}$ $=$ 18 da.

6. A ditch 20 yd. $\times$ 18 in. $\times$ 4 ft. is dug in (3×10) hours by 72 men. A ditch 30 yd. $\times$ 27 in. $\times$ 5 ft. is dug in (9×15) hours by ? men.

$$\frac{72 \text{ men} \times 3 \times 10 \times 30 \times 27 \times 5}{20 \times 18 \times 4 \times 9 \times 15} = Ans.$$

7. £2400 income is produced at 3% by bonds whose face value is £80000. Their cost $=$ £80000 $\times .94\frac{3}{8} =$ £75500 $=$ \$367433$\frac{1}{3}$, 12% of which $=$ \$44092.00.

$$\$4.86\tfrac{2}{3} \times (2400 \div .03) \times 94\tfrac{3}{8} \times .12.$$

8. $x - \dfrac{40\,x}{100} = 30 + 30\% \text{ of } 30 = 39.$

$$100\,x - 40\,x = 3900;\ \text{etc.}$$

10. $[(15 + 10 + 15 + 10) \times 9\frac{3}{4}] + (15 \times 10) =$ number of square feet in the walls and ceiling $= 637\frac{1}{2}$ sq. ft. $= 70\frac{5}{6}$ sq. yd. The cost $= 21d. \times \frac{425}{6} = 1487\frac{1}{2}d.$; etc.

1168. 1. $\frac{2}{3} + \frac{3}{4} = 1\frac{5}{12} = 1.41666 +$; $\frac{1}{2} + \frac{1}{5} + \frac{5}{7} = 1\frac{29}{70} =$ $1.41429 -$.

$.7409375 \div 237100 = .007409375 \div 2371$. See Arithmetic, Art. 668.

2. $$\frac{1\frac{1}{4}}{1\frac{1}{12}} \text{ of } \frac{2}{3} = \left(\frac{5}{4} \div \frac{13}{12}\right) \times \frac{2}{3} = \frac{5}{4} \times \frac{12}{13} \times \frac{2}{3} = \frac{10}{13}.$$

$$\frac{2\frac{1}{2} - 1\frac{5}{6}}{\frac{1}{4} + 1\frac{5}{6}} = \frac{2}{3} \div 2\tfrac{1}{12} = \frac{2}{3} \div \frac{25}{12} = \frac{2}{3} \times \frac{12}{25} = \frac{8}{25}.$$

$$8\tfrac{1}{5} \div 7\tfrac{4}{5} = \tfrac{41}{5} \div \tfrac{39}{5} = 41 \div 39 = \tfrac{41}{39}.$$

$$\frac{10}{13} + \frac{8}{25} - \frac{41}{39} = \frac{750 + 312 - 1025}{975} = \frac{37}{975};$$

$$\frac{37}{975} \div x = \frac{2}{3}; \quad \frac{37}{975x} = \frac{2}{3}.$$

Clearing of fractions, $37 = 650x$; $x = \frac{37}{650}$. *Ans.*

$$\tfrac{32}{9} \text{ of } \tfrac{8}{21} \text{ of } \tfrac{3}{5} \text{ of } \tfrac{7}{12} \times \tfrac{17}{6} \times \tfrac{11}{14} \times \tfrac{21}{4}.$$

$$x = \text{smaller number}; \; x + \tfrac{7}{18} = \text{larger}.$$

$x + x + \frac{7}{18} = \frac{113}{126}$; $126x + 126x + 49 = 113$; $252x = 113 - 49 = 64$; $x = \frac{64}{252} = \frac{16}{63}$; $x + \frac{7}{18} = \frac{16}{63} + \frac{7}{18} = \frac{32}{126} + \frac{49}{126} = \frac{81}{126} = \frac{9}{14}$. *Ans.* $\frac{16}{63}$ and $\frac{9}{14}$.

3. $16s.\ 4\frac{1}{3}d. = 196\frac{1}{3}d.$ *Ans.* $= £(\frac{5}{9} \text{ of } 196\frac{1}{3}) \div 240$.

$\frac{5}{8}$ mile $= 1000$ meters; 1 mi. $= 1000$ m. $\div \frac{5}{8} = 1600$ m. 17 mi. $= 1600$ m. $\times 17 = 27200$ m.; 6 furlongs $= 200$ m. $\times 6 = 1200$ m.; $82\frac{1}{2}$ yd. $= \dfrac{1600 \text{ m.}}{1760} \times \dfrac{165}{2} = 75$ m.

$$27200 \text{ m.} + 1200 \text{ m.} + 75 \text{ m.} = 28475 \text{ m. } \textit{Ans.}$$

27 yd. 2 ft. 9 in. $= 1005$ in.; 17 yd. 1 ft. 11 in. $= 635$ in.

(635×1) sq. in. cost \$25.40; 1 sq. in. $=$ \$2540 $\div 635$; $(1005 \times \frac{7}{8})$ sq. in. $=$ (\$25.40 $\div 635) \times 1005 \times \frac{7}{8} =$ \$35.17$\frac{1}{2}$.

4. Let $x =$ B's money; $x + 17.50 =$ A's.

$$\frac{2x}{5} = \frac{x + 17.50}{3}.$$

A's rate is $\frac{17}{18}$ of B's; B's time is $\frac{17}{18}$ of A's. To run the whole distance, A needs 34 min. $\div \frac{17}{18} = 36$ min. If he runs $2\frac{1}{3}$ miles in $16\frac{4}{5}$ min., in 1 min. he runs $2\frac{1}{3}$ mi. $\div 16\frac{4}{5}$, and in 36 min. he runs $(2\frac{1}{3}$ min. $\div 16\frac{4}{5}) \times 36 = \frac{7}{3}$ mi. $\times \frac{5}{84} \times \frac{36}{1} = 5$ mi. *Ans.*

5. $x - 1\frac{7}{8}\%$ of $x = 96084$; $x - \frac{15x}{800} = 96084$; $800x - 15x = 76867200$; $785x = 76867200$; $x = 97920$.

6. For the information of the teacher, the following method is given:

$$\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{2-2\sqrt{2}+1}{2-1} = \frac{3-2\sqrt{2}}{1} = 3-2\sqrt{2}.$$

We learn in algebra that the sum of two numbers $(x+y)$ multiplied by their difference $(x-y)$ gives the difference of their squares (x^2-y^2). If the sum of $\sqrt{2}$ and 1 be multiplied by their difference $(\sqrt{2}-1)$, we obtain the difference of their squares $(2-1)$. Multiplying the numerator also by $\sqrt{2}-1$, we retain the equality and obtain a divisor that has no decimals. See Art. 1169, **3**.

It is not expected that this method should be given to the pupils.

7. For 42 da., 50 men were at work. To do the same work, 30 men would have required 70 da., or $(70+40)$ da. to do the whole work. 110 da. $-$ 84 da. $=$ number of days the contractor would have been behindhand.

8. The number of square feet in the wall $= (23\frac{2}{3} + 15\frac{5}{6} + 23\frac{2}{3} + 15\frac{5}{6}) \times 11\frac{3}{4} = 928\frac{1}{4}$ sq. ft. The two windows contain (19×5) sq. ft. $= 95$ sq. ft.; the fireplace contains $(4\frac{1}{2} \times 6)$ sq. ft. $= 27$ sq. ft.; the door contains $(7\frac{1}{2} \times 3\frac{1}{2})$ sq. ft. $= 26\frac{1}{4}$ sq. ft.; a total of 95 sq. ft. $+$ 27 sq. ft. $+ 26\frac{1}{4}$ sq. ft. $= 148\frac{1}{4}$ sq. ft. There remain to be papered $928\frac{1}{4}$ sq. ft. $- 148\frac{1}{4}$ sq. ft. $= 780$ sq. ft. $= \frac{780}{9}$ sq. yd. A roll of paper contains $12 \times \frac{26}{36}$ sq. yd.; the number of rolls will be, therefore, $\frac{780}{9} \div \left(12 \times \frac{26}{36}\right) = \frac{780 \times 1 \times 36}{9 \times 12 \times 26}$; and its cost, $\frac{\$4.08 \times 780 \times 1 \times 36}{9 \times 12 \times 26} = \$40.80$. *Ans.*

9. $x+\frac{7x}{9}=336.$

10. The "present worth" of \$365 due in 30 da. = \$365 ÷ 1.005 = \$363.18 + = the cost of the horse in cash. The "present worth" of the selling price = \$435 ÷ 1.02 = \$426.47 +. Gain = \$426.47 − \$363.18 = \$63.29, which is 17.43% of the cost.

If the seller has the note for \$435 discounted at a bank, he will receive in cash \$435 − \$8.70 = \$426.30. If he uses this money to buy the note he has given, he should pay, at bank rates, \$365 − \$1.82½ = \$363.17½. The profit would be \$426.30 − \$363.17½ = \$63.12½, which is 17.38% of the cost.

11. £57 1*s.* 8*d.* = 13700*d.*; £2 11*s.* 4½*d.* = 616½*d.*

$$13700\times\frac{15}{200}\times x=\text{interest}=\frac{2055x}{2}=616\tfrac{1}{2},$$

$$2055\,x=1233,$$

$$x=\tfrac{1233}{2055};\ \tfrac{1233}{2055}\text{ yr.}=7\text{ mo. }6\text{ da. } \textit{Ans.}$$

13. A man that does only $\frac{5}{6}$ of a day's work, does 14 da. less work in 84 da. than the average. The contractor therefore loses, in 84 da. on three men, 14 da. + 12 da. + $9\frac{1}{3}$ da. = $35\frac{1}{3}$ da. He gains on two others $10\frac{1}{2}$ da. + $8\frac{2}{5}$ da. = $18\frac{9}{10}$ da. The net loss = $35\frac{1}{3}$ da. − $18\frac{9}{10}$ da. = $16\frac{13}{30}$ da. The extra 17 men have to do the equivalent of $16\frac{13}{30}$ days' work; each has, therefore, to do $16\frac{13}{30}$ days' work ÷ 17 = $\frac{29}{30}$ of a day's work, or $\frac{1}{30}$ less than the average.

14. Making no allowance for waste, etc., two strips, each 260 ft. long, will be needed for two sides; two strips, each (93 − 3 − 3) ft., or 87 ft. long, will be needed for the other two, or 520 ft. + 174 ft. = 694 ft. = $231\frac{1}{3}$ running yards, 1 yd. wide, making $231\frac{1}{3}$ sq. yd. Cost at 90¢ = \$208.20.

The surface to be carpeted = (260 − 5) ft. by (93 − 5) ft. = 85 yd. × $29\frac{1}{3}$ yd. Cost = \$2.09 × 85 × $29\frac{1}{3}$ ÷ $\frac{38}{36}$ = \$4936.80. Total, \$4936.80 + \$208.20 = \$5145. *Ans.*

15. Since the meeting-place is twice as far from A as from B, the first man goes twice as fast as the other; the latter, therefore, walks $2\frac{1}{2}$ mi. per hour. If x is the distance between A and B, the first will require $\frac{x}{5}$ hr., and the second $\frac{x}{2\frac{1}{2}}$ hr. $= \frac{2x}{5}$ hr.

$$\frac{2x}{5} - \frac{x}{5} = 1;\ x = 5. \quad \textit{Ans.}\ 5 \text{ mi.}$$

16. Let $x =$ number of miles between A and C. Then $x - 15$ = distance between B and C.

$\frac{x-15}{15} =$ time required for first train to run from B to C; $\frac{x}{25} =$ time required for second train to run from A to C. As the latter train leaves 3 hr. later and arrives one-half hour later, the running time of the first is $2\frac{1}{2}$ hr. longer.

$$\frac{x-15}{15} = \frac{x}{25} + \frac{5}{2};$$

$$10x - 150 = 6x + 375;$$

$$4x = 525;\ x = 131\tfrac{1}{4}. \quad \textit{Ans.}\ 131\tfrac{1}{4} \text{ mi.}$$

1169. **2.** The width of the road $= (60 \div 16\frac{1}{2})$ rd.; its area $= (104 \times 60 \div 16\frac{1}{2})$ sq. rd. $= [104 \times (60 \div 16\frac{1}{2}) \div 160]$ acres.

$$\text{Its cost} = \frac{\$154 \times 104 \times 60 \times 2}{33 \times 160} = \$364.$$

$$\text{The cost of grading} = \frac{\$200 \times 104}{320} = \$65.$$

$$\text{The cost of fencing} = \$\tfrac{1}{2} \times 104 \times 5\tfrac{1}{2} = \$286.$$

3. See Art. 1168, **6**.

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{5+2\sqrt{15}+3}{5-3}$$

$$= \frac{8+2\sqrt{15}}{2} = 4 + \sqrt{15}.$$

$$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{5-2\sqrt{15}+3}{5-3}$$

$$=\frac{8-2\sqrt{15}}{2}=4-\sqrt{15}.$$

$$4+\sqrt{15}-(4-\sqrt{15})=2\sqrt{15}=\sqrt{60}.$$

4. A cubic centimeter $=.3937^3$ cu. in. $=(.3937^3 \div 1728)$ cu. ft. Its weight $=$ weight of 1 gram $=(.3937^3 \times 1000 \div 1728)$ oz. $=[(.3937^3 \times 1000) \div (1728 \times 16)]$ lb. A kilogram $=[(1000 \times .3937^3 \times 1000) \div (1728 \times 16)]$ lb. The weight of the anchor in kilograms $=6500$ lb. $\div$ the number of pounds in a kilogram; or,

$$\frac{6500 \times 1728 \times 16}{1000 \times .3937 \times .3937 \times .3937 \times 1000}.$$

5. $x-(x \times \frac{8}{100} \times \frac{63}{360})=1500;$

$$x-\frac{7x}{500}=1500;\ 500x-7x=750000;$$

$$493x=750000;\ x=1521.30-. \quad \textit{Ans.}\ \$1521.30.$$

The proceeds of the new note $+\$200$ must pay the note of $\$2000$; the proceeds must therefore be $\$1800$.

$$x-(x \times \tfrac{8}{100} \times \tfrac{93}{360})=1800.$$

1170. 2. $(80.005-.013) \div 88.$

5. Let $x=$ number of cents received by Thomas. Then $\frac{3x}{5}$ $=$ number received by Henry, and $\frac{6x}{25}=$ number received by Richard.

$$x+\frac{3x}{5}+\frac{6x}{25}=4.14;$$

$$25x+15x+6x=103.50;$$

$$46x=103.50;\ x=2.25.$$

6. 272 liquid quarts $=231$ cu. in. $\times 272 \div 4$. A dry quart $=2150.4$ cu. in. $\div 32$.

Number of dry quarts $=\dfrac{231 \times 272 \times 32}{4 \times 2150.4}=\dfrac{231 \times 272 \times 320}{4 \times 21504}.$

7. The tub holds $12\frac{1}{2}$ qt. $\times 4\frac{1}{3} = 54\frac{1}{6}$ qt. Both pipes discharge $12\frac{1}{2}$ qt. $+ 83$ qt. $= 95\frac{1}{2}$ qt. The time required to fill it $= (54\frac{1}{6} \div 95\frac{1}{2})$ min.

8. The number of hours that must elapse before all will again be together at the starting point, is the least common multiple of $\frac{5}{36}, \frac{2}{9}, \frac{35}{99}$. The least common multiple of the numerators is 70. The smallest fraction that will contain the above fractions an exact number of times must have 70 for its numerator, and for the denominator the largest number that will divide 36, 9, and 99 without a remainder; *i.e.* the greatest common divisor of these numbers. The G.C.D. is 9, and the fraction is $\frac{70}{9}$. In $\frac{70}{9}$ hr., therefore, A, B, and C will be at the starting point. A will have walked around the circle $(\frac{70}{9} \div \frac{5}{36})$ 56 times; B, $(\frac{70}{9} \div \frac{2}{9})$ 35 times; C, $(\frac{70}{9} \div \frac{35}{99})$ 22 times.

NOTE.—The scholars will readily understand that the fraction which is the least common multiple of $\frac{5}{36}$, $\frac{2}{9}$, and $\frac{35}{99}$, should have 70 for its numerator. The following may make clear to them why 9 should be the denominator:

$$\frac{70}{x} \div \frac{5}{36}, \text{ or } \frac{2}{9}, \text{ or } \frac{35}{99} \text{ should be a whole number;}$$

i.e.
$$\frac{70}{x} \times \frac{36}{5}, \text{ or } \frac{9}{2}, \text{ or } \frac{99}{35} \text{ should be a whole number.}$$

An examination of the second line, in which the divisors are inverted, will show that 70 contains the three denominators, 5, 2, and 35, an exact number of times; the numerators, 36, 9, and 99, should contain x an exact number of times; x, therefore, must be a divisor of these numbers, etc.

1172. This work may be slightly abbreviated by combining the interest on the annual interest into one item of 6 years' interest instead of the three separate ones of 3 years' interest, 2 years' interest, and 1 year's interest. In beginning a new topic, however, pupils should not be confused by short methods.

2. $1200 + $300 + (4 + 3 + 2 + 1) years' interest on $60.

For partial payments on notes bearing annual interest, see Art. 1308. The special rules for New Hampshire and Vermont will be found in Arts. 1309 and 1310.

In states in which the collection of annual interest is not allowed, the teacher should omit this topic.

1173. In the older states, time should not be spent upon this topic.

1176. 1. $\frac{1}{4}$ of $\frac{1}{2}$ a section $= \frac{1}{8}$ of 640 A.

2. $\frac{1}{2}$ of $\frac{1}{4}$ section measures 80 rd. by 160 rd.

3. A line from the southwest corner of Sec. 1, to the northeast corner of Sec. 30 (see township diagram on the opposite page), is the hypotenuse of a right-angled triangle whose perpendicular, the eastern boundaries of Secs. 11, 14, and 23, is 3 mi. long; and whose base, the southern boundaries of Secs. 20, 21, 22, and 23, is 4 mi. long.

5. The number of rods of fence $= 80 + 160 + 80 + 160 = 480$. The number of feet $= 16\frac{1}{2} \times 480 = 7920$. A fence 4 boards high requires 7920 ft. $\times$ 4, or 31680, running feet of boards. If the latter are $\frac{1}{2}$ ft. wide, the number of board feet $= 31680 \times \frac{1}{2} = 15840$.

1184. 2. $26.50 \times .85$. 4. Multiply 135 by 69, and point off two places in the product. 5. Find the base. 6. 8.50 francs $\times (10 \times 1 \times 3.25)$. 7. Each dimension can be expressed in decimeters, $105 \times 80 \times 65$, whose product is the number of liters; or the product of the dimensions in meters — $10.5 \times 8 \times 6.5$ — may be multiplied by 1000. 8. 0*l*.75 means .75*l*., the denomination in France being generally written before the decimal. 9. 1.25 marks $\times [(68 \div 10) \times 36]$; *i.e.* $1\frac{1}{4}$ marks $\times 6.8 \times 36$. 10. The number of liters $= 50 \times 40 \times 30 = 60000$; 92% of this number gives the weight in kilograms (kilos).

1186. These problems are given for practice in obtaining the approximate values of the metric units in terms of our weights and measures. The use of 39.37 in. makes the work too tedious.

13. 4 in. by 4 in. by 4 in. A quart $= \frac{231}{4}$ cu. in.

14. A hectoliter = 100 liters = 6400 cu. in. = (6400 ÷ 2150.4) bu. 6400 cu. in. = (6400 ÷ 231) gal.

15. A liter of water, 64 cu. in., weighs a kilo. 1 cu. in. of water = $\frac{1000}{1728}$ oz.; 64 cu. in. = [(64000 ÷ 1728) ÷ 16] lb. = 4000 lb. ÷ 1728.

16. 400000000 in. = $\frac{1}{4}$ circumference. See Arithmetic, Art. 1177.

17. A square meter = (40 × 40) sq. in.

18. An are = (400 × 400) sq. in. A hectare = [(400 × 400) × 100] sq. in.

19. Hectometer = 100 meters = 4000 in.

20. A stere = (40 × 40 × 40) cu. in. = (64000 ÷ 1728) cu. ft.

21. 1000 grams weigh (4000 lb. ÷ 1728); 1 gram weighs 4 lb. ÷ 1728 = 28000 grains ÷ 1728.

22. A kilometer = 40000 in.; a mile = 63360 in.; a mile = (6.336 ÷ 4) Km.

1197. The average pupil should be permitted to use a pencil for his first solution of these problems.

1. Let x = the value of the second suit. Since \$12 and the overcoat = $2x$, the overcoat = $2x - 12$. The second suit (x) and the overcoat ($2x - 12$) = three times the first suit (36).

$$x + 2x - 12 = 36;\ \text{etc.}$$

The second suit is worth \$16; the overcoat, \$20. *Ans.*

2. $x - 22 + \frac{x - 22}{4} = \frac{x}{3}$; etc.

The arithmetical analysis might assume some such form as this: The remainder + $\frac{1}{4}$ of the remainder, or $\frac{5}{4}$ of the remainder = $\frac{1}{3}$ of original sum. The remainder = $\frac{1}{3}$ of original sum × $\frac{4}{5} = \frac{4}{15}$ of original sum. The sum lost is $1 - \frac{4}{15}$, or $\frac{11}{15}$ of original sum. As this is \$22, the original sum = \$22 × $\frac{15}{11}$ = \$30. *Ans.*

3. Let $x =$ time past noon; $x + 12 =$ time past midnight. $x = \frac{x+12}{5}$; $5x = x + 12$; $4x = 12$; $x = 3$. The time is 3 hr. past noon, or 3 P.M. *Ans.*

4. At 3 o'clock, the hour hand is 15 minute spaces in advance. To be only 5 spaces behind, the minute hand must gain 10 spaces. While the minute hand goes 1 space, the hour hand goes $\frac{1}{12}$ space; so that each minute, the minute hand gains $\frac{11}{12}$ space. To gain the 10 spaces necessary, the minute hand must travel $(10 \div \frac{11}{12})$ minutes $= 120$ min. $\div 11 = 10\frac{10}{11}$ min. The time is $10\frac{10}{11}$ min. past 3.

5. $\frac{1}{5}$ A $= \frac{4}{5}$ B; A $= 4$ B; 5 B $= 30$. B's age $= 6$ yr.; A's age $= 24$ yr. *Ans.*

6. A takes \$15 less than $\frac{3}{4}$ of the profits. If his capital is \$30 less than $\frac{3}{4}$ of the whole, the latter must be double the profits, or \$1440. A's capital $=$ \$525 $\times 2 =$ \$1050; B's $=$ \$390. *Ans.*

Or, A takes $\frac{525}{720}$, or $\frac{35}{48}$, of the profits; he owns, therefore, $\frac{35}{48}$ of the capital. If $\frac{35}{48}$ of the capital $+$ \$30 $= \frac{3}{4}$, or $\frac{36}{48}$, of the capital, $\frac{1}{48}$ of the capital $=$ \$30; etc.

7. Let $x =$ the number of sheep; $\frac{80}{x} =$ cost of each; $x - 5$ $=$ number remaining; $\frac{2x-10}{3} =$ number sold; $\frac{2x-10}{3} \times \frac{80}{x} =$ $\frac{160x - 800}{3x} =$ sum received $= 40$.

$160x - 800 = 120x$; $40x = 800$; $x = 20$. *Ans.* 20 sheep.

Or, if he received \$40 for $\frac{2}{3}$ of the remainder, he would have received \$60 for the remaining sheep. \$60 being $\frac{3}{4}$ of \$80, $\frac{3}{4}$ of the sheep remained, and $\frac{1}{4}$ of them died, or 5 sheep. The whole number was, therefore, 20 sheep.

8. Let $x =$ A's age; then $x + 10 =$ B's age;

$$\frac{x}{2} = \frac{x+10}{3};\ \text{etc.}$$

1198. 3.

$$
\begin{array}{r}
7\,\underline{)\ 19 \text{ mi. } 180 \text{ rd. } 2 \text{ yd. } 0 \text{ ft. } 9 \text{ in.}} \\
2 \text{ mi. } 254 \text{ rd. } 1 \text{ yd. } 2 \text{ ft. } 8\tfrac{1}{7} \text{ in.} \\
\underline{\times 12\phantom{\text{ in.}}} \\
33 \text{ mi. } 172 \text{ rd. } 0 \text{ yd. } 2 \text{ ft. } 1\tfrac{5}{7} \text{ in.}
\end{array}
$$

4. 18 hr. 24 min. 12 sec. $\div$ 15 = 1 hr. 13 min. $36\frac{4}{5}$ sec.

1 hr. 45 min. time difference $= 1° \, 45' \times 15 = 26° \, 15'$ difference in longitude. As the place has the later time, it is more easterly.

7. Calling it 1 in. thick, the number of board feet $= 16 \times \frac{3}{4} = 12$. $\$40 \times .012 = 48¢$. *Ans.*

$\$40 \times (16 \times \frac{3}{4} \times 2\frac{1}{2}) \div 1000 = \$1.20$. *Ans.*

9. [96 (in.) $\times$ 90 (in.) $\times$ 48 (in.)] $\div$ 2150.4.

10. If the strips run lengthwise, their number will be 8 yd. $\div \frac{3}{4}$ yd. $= 10\frac{2}{3}$. The number purchased must be 11, each 9 yd long, or 99 running yards of carpet.

XVIII

NOTES ON CHAPTER FIFTEEN

While the work contained in this chapter is intended more particularly for use in such schools as extend their instruction beyond the eighth year of the elementary course, it can profitably replace some of the less useful arithmetical topics taught during the eighth school year.

1199. These exercises should be taken up without any preliminary explanations. Their previous work in simple equations has so familiarized the pupils with the use of letters to express numbers, etc., that they need no assistance in the first ten examples. The necessary technical terms should be employed as occasion requires, and their meanings should be made clear; but exhaustive treatment of the different operations should be left for the study of the science of algebra in the high school.

1200. The explanation of the meaning xy, abc, etc., may be deferred until Art. 1238. For the present, the use of the word *coefficient* may be limited to simple numerical ones, as given in the text-book. The teacher should not yet explain that in the expression $5xy$, $5x$ may be considered the coefficient of y; nor that in $9abc$, $9a$ may be considered the coefficient of bc, and $9ab$ the coefficient of c.

1204. So far, the pupils have been required to add only single columns containing the same letters. When the signs are alike throughout, as in Art. 1199, they have found the sum of the coefficients, annexed the letter or letters, and prefixed the common sign. When the signs are unlike, the difference between the sums of the coefficients of the positive and of the negative terms is written, preceded by the sign of the greater sum.

It will scarcely be necessary to state to pupils that algebraic expressions containing dissimilar terms are added by placing the plus sign between them; thus the sum of $4ab$ and $3ac$, for instance, is written $4ab + 3ac$.

1205. The expressions employed in the preceding exercises are called *monomials*, or algebraic expressions of *one* term. Those of more than one term are called *polynomials*.

A polynomial of two terms is called a *binomial;* one of three terms, a *trinomial.*

6. The scholar will readily see that in the addition of polynomials, each column should contain similar terms; *i.e.* terms containing the same letters. That the letters should also be affected by the same exponents, need not be told him for the present.

1207. From some of the preceding examples, may be seen the use of the plus and of the minus sign to indicate direction north and south, and east and west; past and future time, etc.

In **2**, is required the difference between $-10°$ and $+90°$. In **6**, there is asked the sum of $+40¢$ and $-50¢$. Calling the distance north of the starting point $+50$ miles, in **8**, and the distance south -70 miles, the required location will be $(+50$ miles$) + (-70$ miles$) = -20$ miles, or 20 miles south.

6 and **8** are problems in algebraic addition; **10**, like **2**, is a problem in subtraction. The results of a man's transactions during a month are ascertained by subtracting the value of his possessions at the beginning of the month from their value at the end. In **10**, a man is worth $-\$250$ on Feb. 1; deducting from this $+\$150$, which represents his condition on Jan. 1, we obtain $-\$400$. The operation may be indicated thus: $(-\$250) - (+\$150) = -\$400$, the minus sign in the result indicating a loss.

The algebraic analyses of these problems, if asked at all, should not be required until the pupils have solved them in their own way. The main object of teaching subtraction at this stage, is to enable the scholars to understand the reasons for the change of signs that accompanies the removal of a parenthesis preceded by a minus sign. See Art. 1210.

1210. Considering (*a*) as an example in subtraction, it may be made one in addition (*b*) by changing the signs of the subtrahend. It may then be written $84 - 49 + 25$.

$$(a)\quad \begin{array}{rr} \text{From} & 84 \\ \text{Take} & \underline{49 - 25} \end{array}$$

$$(b)\quad \begin{array}{r} 84 \\ \underline{-49 + 25} \end{array}$$

1211. The pupil will readily ascertain that a parenthesis preceded by a plus sign may be removed without any alteration being required in the signs of the quantities enclosed within it. $57 + (33 - 16) = 74$, may be written $57 + 33 - 16 = 74$. In **2**, the signs of the quantities within a parenthesis must be changed. The first number within the parenthesis, being without a sign, is positive; it therefore becomes negative when the parenthesis is taken away. The equation then becomes $92 - 63 - 25 = 4$. In **5**, the multiplier 4 affects only the quantity within the parenthesis. The brackets heretofore used, have been omitted in **5** and **6**, to make these arithmetical equations resemble more closely the algebraic equations of Art. 1213. **5** becomes $75 + 60 - 40 = 95$; **6** becomes $75 - 60 + 40 = 55$.

$$\begin{array}{rr} \text{From} & +92 \\ \text{Take} & \underline{+63 + 25} \end{array}$$

$$\begin{array}{l} 92 \\ \underline{-63 - 25} \\ 92 - 63 - 25 \end{array}$$

1213. It has not been considered necessary to give any previous practice in multiplying simple algebraic polynomials by an ordinary number. The average pupil will readily understand that 6 times two x = twelve x.

1. $12x - 30 = 5x + 12.$

$$12x - 5x = 12 + 30;\ 7x = 42;\ x = 6.$$

Proof. $6\,(12 - 5) = 30 + 12$; *i.e.*, 6 times $7 = 42$.

2. $7x + 14 = 3x + 50$; etc.

3. $15 + 5x + 16 = 61$; etc.

4. $48 - 3x = 52 - 4x$; etc.

7. $2x - 2 - 4x + 38 = 3x - 9$; etc.

8. $12x - 30 - 5x = 12$; etc.

9. $5x - 12x + 30 = -12$; etc.

10. $11 - 3x + 10x = 38$; etc.

1215. 12. $3x - 3 - 2x + 4 = 12.$

13. $6x - 6 - 4x + 8 - 3x + 9 + 24 = 0.$

15. $14x - 16 - 18x - 36 = 12x + 15 - 6x - 12.$

Transposing, $14x - 18x - 12x + 6x = 15 - 12 + 16 + 36.$

Combining, $-10x = 55.$

Changing the signs of both members,

$$10x = -55.$$

Or, $x = -5\frac{1}{2}.$

17. $\frac{39}{4} - \frac{5x}{4} + \frac{x}{2} = \frac{3x}{8} + \frac{15x}{4}$; etc.

18. $2x = 3 + \frac{9x}{4} - 5 - \frac{2x}{5} + \frac{13}{5}.$

19. $\frac{3x}{4} + 9 = 2x + \frac{3x}{5} - \frac{x}{2}.$

22. $x - 20 = \frac{4x}{7} + 60.$

1216. 1. $\frac{3\frac{2}{3}x - 7}{16} = 3.$

Multiplying by 16, $3\frac{2}{3}x - 7 = 48$; or $\frac{11x}{3} - 7 = 48.$

Clearing of fractions, $11x - 21 = 144$; etc.

2. $\frac{3x}{8} = x - 60$; etc.

3. $x+\frac{x}{3}+x+\frac{x}{4}+x+\frac{x}{5}+x+\frac{x}{6}=99.$

5. $x-\frac{x}{6}-\frac{x}{8}+\frac{x}{3}=x+4.$

6. $x+12=$ son's present age; $2x+12=$ father's present age.

$$x+12+2x+12=138;$$
$$3x=138-24=114;$$
$$x=38, \text{ the son's age 12 yr. ago};$$
$$2x=76, \text{ the father's age 12 yr. ago}.$$

The present age of the son is 50 yr. $(x+12)$; the present age of the father is 88 yr. $(2x+12)$. *Ans.*

7. $(80+x)=2\frac{1}{2}(60-x)$; etc.

8. $2(11+x)=25+x.$

9. Let $x=$ the number of gallons originally in the cask. $\frac{x}{4}=$ amount drawn off, leaving $\frac{3x}{4}$ in the cask. $60-\frac{3x}{4}=$ number of gallons required to fill the cask.

$$24=60-\frac{3x}{4};$$
$$96=240-3x; \text{ etc.}$$

10. $\frac{x}{3}-40=104.$

11. $\frac{x+430}{x}=4+\frac{76}{x}.$

Clearing of fractions, $x+430=4x+76.$
Transposing, $x-4x=76-430.$
Combining, $-3x=-354.$
Changing the signs of both members,

$$3x=354.$$

Or, $x=118$, the smaller number;

$$x+430=548, \text{ the larger number.}$$

12. Let $x =$ the number of \$2 bills.

Then $29 - x =$ the number of \$5 bills.

$$2x + 5(29 - x) = 103;\ \text{etc.}$$

13. $4(x + 4) = x + 34.$

14. $(x + 3) \times \dfrac{180}{x} = 225.$

Multiplying, $$\frac{180x + 540}{x} = 225.$$

Clearing of fractions, $180x + 540 = 225x$; etc.

15. $33(x + 1) = 40x + 12.$

16. The numbers are x and $x + 17$.

Then $$x + x + 17 = 47;\ \text{etc.}$$

17. Let $x =$ number of years.

The mother's age will then be $41 + x$, and the son's $5 + x$.

$$5 + x = \tfrac{1}{3}(41 + x) = \frac{41 + x}{3};$$

$$15 + 3x = 41 + x;\ \text{etc.}$$

1218. 1. Substituting the value of $8x$, the equation becomes

$$16 + 7y = 44;\ \text{etc.}$$

2. $9 + 5z = 34$; $5z = 25$; $z = 5$. *Ans.*

1219. 11.

$$\begin{array}{ll} & 3x + 14y = 78 \\ & \underline{2x + 14y = 66} \\ \text{Subtracting,} & x \qquad\quad\ = 12 \end{array}$$

Substituting this value of x in the first equation,

$$36 + 14y = 78;\ \text{etc.}$$

14. Multiplying the first equation by 2, we have $2x + 2y = 30$. Subtract the new equation from the second one, $2x + 3y = 38$, thus finding the value of y. Substitute this value in either of the original equations.

17. Multiplying the first equation by 3, and the second by 2, we have $6x+9y=120$ and $6x+4y=70$.

18. Multiply the first by 2, and the second by 7.

19. Multiply the first by 9, and the second by 5.

20. Multiply the first by 8, and the second by 3.

1221. **21.** If we add, we have $2x=22$. Or, subtracting, $2y=14$. By adding, y is eliminated; by subtracting, x is eliminated. From this example, pupils should see that either addition or subtraction may be employed to eliminate one of the unknown quantities; and that either of the two may be eliminated, as may be found convenient.

27. $5x-6y=5$,
$3x-5y=-4$.

28. $3x+5y=-8$,
$2x-y=12$.

29. $-10x+y=-1$, (1)
$-5x+y=9$. (2)

Subtract (1) from (2).

30. $3x+8y=204$,
$10x+5y=160$.

31. $3x+2y=252$,
$7x+5y=609$.

34. $3x+7=15y-20$; $3x-15y=-27$. (1)
$7x-6=10y+6$; $7x-10y=12$. (2)

Multiply (1) by 7 and (2) by 3, to eliminate x; or (1) by 2 and (2) by 3, to eliminate y.

35. $51x+44y=804$, (1)
$45x-32y=72$. (2)

Multiply (1) by 8, and (2) by 11. Add.

37. $2x-22-2y+18=6$,
$15x+135=32y-96$.

The pupils should be taught to indicate the common denominator of $y-3$ and 15, as $15(y-3)$, which contains $y-3$, 15 times; and 15, $(y-3)$ times.

39. $2x+5y+3=18x-24y-12$; $-16x+29y=-15$.
$4x-7y+5=5x-10y+10$; $-x+3y=5$.

1222. 1. $x + y = 37$; $2x + 3y = 96$. This problem can also be solved by the use of one unknown quantity, by calling the numbers x and $37 - x$. The equation becomes $2x + 3(37 - x) = 96$; or, $2x + 111 - 3x = 96$.

2. Using one unknown quantity, the numbers are x and $x + 28$. $5x - 2(x + 28) = 197$.

3. $5x + 3y = 37$; $6x - 5y = 10$.

4. $x + y = 65$; $x - y = 19$.
By one unknown quantity, $x + (x + 19) = 65$.

5. $x + y = 32$; $2x + 5y = 103$. See Art. 1216, **12**.

6. $x + y = 25$; $7x + 5y = 145$.

7. $10x + 4y = 38$; $6x + 7y = 32$.

8. $5x + 3y = 375$ (cents); $8x + y = 505$ (cents).

9. $125(x) + 45(4x) + 10(8x) + 5(\frac{1}{2}x) = 1550$.

11. $x + y = 19$; $y + 10x - (x + 10y) = 45$.

12. $13x = 5y$; $x + y = 126$.

13. $15x = 8y$; $x - y = -147$. $\frac{8}{15}$ being a *proper* fraction, any equivalent fraction must have a denominator exceeding the numerator.

1223. 4. Eliminate z by comparing the first and the second equation, multiplying the latter by 5. Multiply the second by 3 and compare with the third equation, eliminating z.

5. First eliminate y.

6. $x + x + y = 42$; $3x + 3y - x + y = 96$.

7. $15x - 25y + 30 = 4x + 2y$; $11x - 27y = -30$.
$96 - 3x + 6y = 6x + 4y$; $-9x + 2y = -96$.

9. $15x - 9 - 9x + 57 = 24 - 6y + 2x$,
$16x + 8y - 18x + 14 = 12y + 36 - 4x - 5y$.

Transposing and combining, $4x + 6y = -24$; $2x + y = 22$. Eliminate x by dividing the first equation by 2; etc.

1224. 1. $x + \frac{x}{30} + \frac{x}{50} = 31600.$

2. Let $x =$ number of B's chestnuts; $x + 18 =$ number of A's chestnuts.

$$x + 18 + 4 = 4(x + 4).$$

3. $x + y = 8$; $23x + 17y = 166.$

4. $x + y = 55$; $x + z = 62$; $y + z = 83.$

Comparing the first two, we get $y - z = -7$; adding this to the third eliminates z; etc.

6. $x = \frac{x}{5} + \frac{16x}{45} + \frac{8x}{45} + 24.$

7. $x - \frac{3x}{20} = 510.$ 8. $x - \frac{x}{50} = 147.$

10. $\frac{3x}{7} = \frac{x}{5} + 16.$

11. Let $x =$ value of the clothes. $x + 280 =$ yearly wages. $\frac{1}{2}(x + 280) =$ wages for 6 months $= x + 130.$

Clearing of fractions, $x + 280 = 2x + 260$; etc.

1228. 14.

$$\begin{array}{rr} & 3x + 6 \\ & 2x - 3 \\ \hline 2x(3x+6) & 6x^2 + 12x \quad\quad \\ -3(3x+6) & -9x - 18 \\ \hline & 6x^2 + 3x - 18 \quad \textit{Ans.} \end{array}$$

1236. 5. $5x^2 + 85 - 3x^2 + 63 = 198$; etc.

7. $x^2 + 2x + 1 - x^2 = 49$; etc.

8. $4y^2 + 20 - 6y^2 + 54 = 24$; etc.

9. Clearing of fractions, Art. 1221, **37**, we have $(z + 7)(z - 9) = (z - 5)(z - 3).$

Performing the multiplication indicated,

$$z^2 - 2z - 63 = z^2 - 8z + 15;$$
$$6z = 78; \; z = 13. \; \textit{Ans.}$$

10. $20x(x+1)=30x(x-1)$; etc. Divide by x.

13. $(x+4)(x+4)=8x+80$; etc.

15. $6x^2+36=5x^2+72$.

16. $x^2-6x+9-(x^2-10x+25)=12$; removing the parenthesis, $x^2-6x+9-x^2+10x-25=12$; etc.

18. The common denominator is $36x$. Clearing of fractions,

$$9x^2+144=4x^2+324; \text{ etc.}$$

19. $(x+7)(x-9)=(x-3)(x-5)$.

20. $(y-9)(y+7)=(y-3)(y-5)$.

1237. **1.** Let $x=$ the breadth; $2x=$ the length. The area $=x\times 2x=2x^2=1800$; etc.

2. Let $x=$ the length of one edge. The area of one face $=x^2$; that of six faces is $6x^2$, and is equal to 96 sq. in.

$$6x^2=96; \text{ etc.}$$

3. Let $x=$ one number; $\frac{4x}{5}=$ the other.

$$x\times\frac{4x}{5}=\frac{4x^2}{5}=\text{their product}=80; \text{ etc.}$$

4. $\frac{x}{3}\times\frac{2x}{5}=\frac{2x^2}{15}=270$; etc.

5. $\frac{30x}{100}\times\frac{40x}{100}=\frac{3x^2}{25}=300$; etc.

6. 40% of a number $(x)=\frac{2x}{5}$; 30% of $\frac{2x}{5}=\frac{3}{10}$ of $\frac{2x}{5}=\frac{3x}{25}$. $\frac{3x}{25}=300$; etc.

7. Let $x=$ the length of the perpendicular; $\frac{3x}{4}=$ the length of the base. The area $=\frac{1}{2}\left(x\times\frac{3x}{4}\right)=\frac{3x^2}{8}$.

$$\frac{3x^2}{8}=96;\ 3x^2=768;\ x^2=256;\ x=\pm 16.$$

Neglecting the negative result, the perpendicular measures

16 rd., and the base 12 rd. The hypotenuse $= \sqrt{16^2 + 12^2}$ rd. $= 20$ rd.

8. $x^2 + \left(\frac{3x}{4}\right)^2 = 15^2$; $x^2 + \frac{9x^2}{16} = 225$; $16x^2 + 9x^2 = 3600$; etc.

9. $(x+9)^2 = x^2 + 15^2$; $x^2 + 18x + 81 = x^2 + 225$; etc.

10. $(x+1)^2 - x^2 = 49$; etc.

1238. The pupils should be informed that the product of the numbers represented by two letters is represented by writing the letters together; thus a times b is written ab, m times n is written mn, just as 3 times x is written $3x$.

1242. 1. $x^2 + 6x + 9$.

2. $x^2 - 12x + 36$.

3. $x^2 - 8x + 16$.

6. $x^2 + 2x + 1$.

7. $x^2 - 4x + 4$.

8. $x^2 - 10x + 25$.

1244. 1. $x^2 + 6x + 9 = 49$; $x + 3 = \pm 7$. *Ans.*

2. $x^2 - 12x + 36 = 64$; $x - 6 = \pm 8$. *Ans.*

5. $x^2 + 18x + 81 = 19 + 81 = 100$; $x + 9 = \pm 10$. *Ans.*

6. $x^2 + 2x + 1 = 24 + 1 = 25$; $x + 1 = \pm 5$. *Ans.*

7. $x^2 - 14x + 49 = 15 + 49$; $x - 7 = \pm 8$. *Ans.*

1246. 1. $x^2 - 6x + 9 = 7 + 9$; $x - 3 = \pm 4$; etc.

2. $x^2 - 12x + 36 = 108 + 36$; etc.

3. $x^2 + 2x + 1 = 48 + 1$; etc.

1247. The first member is made a complete square by adding the square of $\frac{1}{2}$ of the coefficient of x.

1248. 1. $x^2 + x + \frac{1}{4} = 12 + \frac{1}{4} = \frac{49}{4}$.

$x + \frac{1}{2} = \pm \frac{7}{2}$; $x = \frac{6}{2}$, or $-\frac{8}{2} = 3$ or -4. *Ans.*

2. $x^2 - 3x + \frac{9}{4} = 10 + \frac{9}{4} = \frac{49}{4}$;

$x - \frac{3}{2} = \pm \frac{7}{2}$; etc.

3. $x^2 + 5x + (\frac{5}{2})^2 = -4 + (\frac{5}{2})^2$; etc.

1249. 1. $x^2 - x = 6$; $x^2 - x + \frac{1}{4} = 6 + \frac{1}{4} = \frac{25}{4}$; etc.

1250. 1. $12x - x^2 = 32$.
Changing the signs, and rearranging,

$$x^2 - 12x = -32.$$

Completing the square, $x^2 - 12x + 36 = -32 + 36 = 4$.
Extracting the square root, $x - 6 = \pm 2$.
Transposing, $x = +2 + 6 = 8$; or $x = -2 + 6 = 4$.

$$12 - x = 12 - 8 = 4; \text{ or } 12 - 4 = 8.$$

8 and 4, or 4 and 8. *Ans.*

2. $x^2 + 50x + 625 = 2400 + 625 = 3025$.

$$x + 25 = \pm 55;$$
$$x = 30 \text{ or } -80.$$

Neglecting the negative result, the altitude is 30 ft. *Ans.*

3. $x^2 + 225 + 30x + x^2 = 5625$.
Transposing, $2x^2 + 30x = 5625 - 225 = 5400$.
Dividing both members by 2, $x^2 + 15x = 2700$.
Completing the square, $x^2 + 15x + (\frac{15}{2})^2 = 2700 + (\frac{15}{2})^2$; etc.

4. Perpendicular $= \sqrt{\frac{25}{16}x^2 - x^2} = \sqrt{\frac{9}{16}x^2} = \frac{3}{4}x$.

Area $= \dfrac{1}{2}\left(x \times \dfrac{3x}{4}\right) = \dfrac{3x^2}{8} = 150.$

Clearing of fractions, $3x^2 = 1200$,

$$x^2 = 400,$$
$$x = 20.$$

Base $= 20$ yd.; hypotenuse $= 20$ yd. $\times 1\frac{1}{4} = 25$ yd. *Ans.*

5. The convex surface $= 6(x + x + x + x) = 24x$; the surface of the two bases $= 2x^2$; the entire surface $= 2x^2 + 24x = 170$; $x^2 + 12x = 85$; etc.

6. The area of the walk = area of outside rectangle − area of inner rectangle.

$$(40+2x)(50+2x)-40\times 50=784;$$
$$2000+180x+4x^2-2000=784;$$
$$4x^2+180x=784;$$
$$x^2+45x=196; \text{ etc.}$$

7. 12 acres = 1920 sq. rd.

$$\frac{15x^2}{8}=1920;\ 15x^2=15360;\ x^2=1024;\ x=\pm 32.$$

Base = 32 rd.; perpendicular = 32 rd. $\times 1\frac{7}{8}$ = 60 rd.; hypotenuse = $\sqrt{32^2+60^2}$ rd. = 68 rd. Diagonal = 68 rd. *Ans.*

8. $x^2+30^2=(50-x)^2.$

$$x^2+900=2500-100x+x^2;$$
$$100x=1600;\ x=16.$$

AC = 16 ft., CB, the part broken off = 50 ft. − 16 ft. = 34 ft. *Ans.*

Or, making $BC=x$, $AC=50-x$.

Then, $(50-x)^2+30^2=x^2.$

$$2500-100x+x^2+900=x^2$$
$$-100x=-3400;$$
$x=34$, the length in feet of the part broken off.

9. $60^2+(58-x)^2=56^2+x^2;$

$$3600+3364-116x+x^2=3136+x^2;$$
$$x^2-x^2-116x=3136-3600-3364$$
$$-116x=-3828,\text{ or } 116x=3828;$$
$$x=33=AE.$$

The length of the ladder in feet $=\sqrt{56^2+33^2}=\sqrt{3136+1089}$ $=\sqrt{4225}$. 65 ft. *Ans.*

10. From ABD, the square of $BD=13^2-(15-x)^2$. From BCD, the square of $BD=4^2-x^2$. Therefore

$$13^2-(15-x)^2=4^2-x^2;$$

or, $$169-(225-30x+x^2)=16-x^2.$$

Removing the parenthesis, $169 - 225 + 30x - x^2 = 16 - x^2$.

Transposing and combining, $30x = 16 - 169 + 225 = 72$;

$$x = 2\tfrac{2}{5}.$$

$$BD = \sqrt{BC - CD} = \sqrt{4^2 - 2\tfrac{2}{5}^2} = \sqrt{16 - \tfrac{144}{25}} = \tfrac{16}{5} = 3\tfrac{1}{5}.$$

Altitude $= 3\tfrac{1}{5}$ ft. *Ans.*

11. $AF = \sqrt{\overline{AB}^2 - \overline{BF}^2} = \sqrt{1156 - 256} = \sqrt{900} = 30$;

$FC = \sqrt{\overline{BC}^2 - \overline{BF}^2} = \sqrt{400 - 256} = \sqrt{144} = 12$;

$AC = AF + FC = 30 + 12 = 42$. 42 ft. *Ans.*

Let $AE = x$; $EC = 42 - x$;

$$\overline{ED}^2 = \overline{AD}^2 - \overline{AE}^2 = 26^2 - x^2 = 676 - x^2;$$

$$\overline{ED}^2 = \overline{DC}^2 - \overline{EC}^2 = 40^2 - (42 - x)^2 = 1600 - (1764 - 84x + x^2).$$

Therefore $676 - x^2 = 1600 - 1764 + 84x - x^2$

$$-84x = 1600 - 1764 - 676 = -840;$$

$$x = 10.$$

$$ED = \sqrt{26^2 - 10^2} = \sqrt{576} = 24;$$

or, $= \sqrt{40^2 - 32^2} = \sqrt{576} = 24$. 24 ft. *Ans.*

XIX

NOTES ON CHAPTER SIXTEEN

The geometry work contained in this chapter should be commenced not later than the seventh year of school, and should be continued throughout the remainder of the grammar-school course.

1251. No formal definitions of *lines*, *angles*, etc., should be given at the beginning. After drawing angles of various sizes and with lines of different lengths, the pupils will be able to understand that "an angle is the difference in direction of two straight lines that meet in one point, or that would meet if produced."

1255. The semi-circular protractor is better than the common rectangular one for beginners, as they see more clearly by using the former that an angle is measured by the arc of a circle. Two protractors are printed on a fly-leaf in the back of the text-book, for the use of such pupils as cannot procure others. Protractors made of stout manilla paper can be obtained from the Milton Bradley Co., New York, at one cent each in quantities. A large protractor is needed for blackboard use. This can be made of pasteboard; or wooden ones can be bought of the Keuffel & Esser Co., New York.

Many scholars that are able to measure an angle one of whose sides is horizontal, Fig. 1, find it difficult at first to ascertain the number of degrees in an angle formed by two oblique lines, Figs. 2 and 3. They should be permitted to discover the method for themselves. All that is necessary, is to place the center (A) of the base of the protractor on the vertex of the angle, Figs. 1–3, and the edge of the protractor on one of the sides, the other side cutting the circumference. In Figs. 2 and 3, the number of degrees in the angle XAZ is determined by the number of degrees

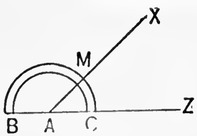


FIG. 1.

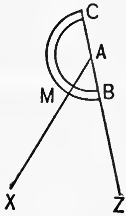


FIG. 2.

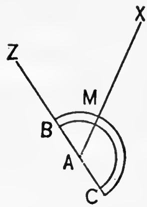


FIG. 3.

in the arc BM, and the upper row of figures is used, having the zero mark at B. In Fig. 1, the number of degrees in the angle is measured by the arc CM, which requires the use of the lower row of figures.

1256. The average class will find the 100 exercises to Art. 1269, inclusive, sufficient for the first year's work. This will give three per week, and leave some time for review. Pupils should work the exercises at home without any preliminary discussion in class. After the exercises are brought in, they should be done on the blackboard, at which time the mistakes made can be pointed out. While first-class drawing cannot be expected from the instruments used by school-children, the teacher should exact the best work possible under the circumstances. A hard pencil, kept sharp, is necessary to secure the requisite fineness of line.

1. In drawing an angle, commence at the vertex.

This exercise is given to remove the impression sometimes formed, that the size of an angle depends upon the length of the lines, instead of their greater or less difference in direction.

In this and all other exercises, the pupils should be encouraged to commence occasionally with an oblique line. No two results should be exactly alike. If two pupils compare notes, it should be for the purpose of producing a different drawing. One pupil's angle may have its vertex at the right, another at the left; one vertex may be above, another below; etc. The better the teaching, the greater will be the variety of results in exercises that permit of variety.

3. It is expected that the pupils will see for themselves that each arc will contain $\frac{1}{4}$ of 360°.

4. Using the ruler, draw the first line of any convenient length and in any direction. Placing *A* of the protractor at either end, mark off 45°, being careful to use the proper row of figures. Remove the protractor; place the ruler so that its edge just touches the end of the line and the 45° point, and draw the second line. This latter should not be of the same length as the first, unless for some good reason; so that pupils will not consider that the lines forming an angle should be equally long.

Write the number of degrees in each angle.

5. The teacher should not inform the pupils in advance how many degrees they will find in the second angle. They should measure it for themselves, using the protractor.

In drawing these angles, the figure in the book should not be followed. The second line should be drawn to the left in some cases; the lower angle may be made 60°; etc.

When two lines meet to form two angles, it is not at all necessary that the point of meeting should be at the *center* of one line.

1257. Pupils should be taught that horizontal lines are lines parallel to the surface of still water. Floating straws are horizontal, and may point in any direction. A spirit level is used by the carpenter to determine whether or not a beam, for instance, is horizontal. A vertical line is one that has the direction of a plumb line, which is used by a mason to ascertain if a wall is perpendicular.

In drawings, however, lines that will be horizontal when the paper is placed upon the wall, are called horizontal lines; and lines that will be vertical when the paper is placed upon the wall, are called vertical lines.

6. The perpendicular need not be drawn to the center of either of the others, nor need it always be drawn above. The teacher should encourage variety.

8. The pupils should draw these lines, and mark in each angle the number of degrees it contains. Encourage the greatest possible variety in the size of the angles and the direction of the lines.

9. While pupils may be able by this time to give the result without drawing the angles and measuring the second one, the teacher should not fail to give them the necessary practice in constructing angles of a given number of degrees, and in measuring the contents of others.

Many scholars make as ridiculous mistakes in the measurement of angles as they do in their work in numbers, frequently reading the wrong figures, and figures from the wrong row — marking an angle of 45°, for instance, 135°; etc. They should learn to "approximate" the size of an angle, as well as to "estimate" the probable answer to an arithmetical problem. An acute angle should not be marked as containing over 90°; etc.

10. Having learned by observation that the sum of two adjacent angles is 180°, the pupils should now discover that the sum of any number of angles formed on one side of a straight line is 180°. When they have learned this from drawing the first exercise, they may be permitted to calculate the result in the other two, especially as the protractors are not marked for fractions of a degree.

The first exercise should show the same variety in the work of the different pupils as has been recommended for previous work.

12. In constructing a square, the protractor is used to erect a perpendicular at each corner. These perpendiculars are made

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equal to each other and to the original line. A fourth line is drawn. The accuracy of the work may be tested by measuring, with the protractor, the two upper angles.

The base lines used by different pupils should be of different lengths. The pupils should be permitted, also, to construct the square in their own way. Some may erect a perpendicular at one end of the given line, and at the extremity of the second line erect another perpendicular. Some pupils may not measure the first line, drawing the second and third lines lightly of indefinite length, and using compasses to make them equal to the first. In this case, the light lines should not be erased; but the square should be marked off by heavier lines.

It is a good practice to have the pupils give a written description of their method of working one of these exercises, which should be accepted as a regular composition. The language should be correct; the proper technical terms should be employed; and there should be sufficient detail to enable any one not familiar with the work to understand just how it was done.

13. Pupils should be permitted to learn for themselves from this exercise and from **14**, that vertical, or opposite, angles are equal.

17. After drawing the required lines, the scholar should mark in each angle its contents in degrees.

19. This exercise should enable the pupil to see that the sum of all of the angles formed about a point will be 360°.

20. The teacher should not give unnecessary assistance. If the scholars have a few days in which to work out an exercise, they should find no difficulty in managing this.

The word "adjacent" in geometry is applied to each of the two angles formed by one straight line meeting another. In **19**, the two lower angles are adjacent; but none of the upper three angles is adjacent to any other, because three straight lines are used to construct two angles in each case. No two angles in **20** are adjacent, and no two are vertical.

21. There are no adjacent angles. They are all vertical, because the lines forming each are produced to form an opposite angle.

22. The pupil is not yet ready to do this in the geometrical way. The teacher should be satisfied if he adds 65° and 25°, and uses the protractor to make an angle of 90°; etc.

24. If the pupil examines a clock, he will see that the number of degrees between 12 and 1 is $\frac{1}{12}$ of 360°, or 30°. He has learned already that the length of the sides has nothing to do with the magnitude of the angle.

25. The minute hand goes 90° in a quarter of an hour. The hour hand goes 30° in an hour; 15° in $\frac{1}{2}$ hr.; $7\frac{1}{2}$° in $\frac{1}{4}$ hr.

To ascertain the angle at 12:15, the pupils should draw a clock face, locating the hands properly. Some will place the hour hand at 12, forgetting that it has gone $7\frac{1}{2}$° in $\frac{1}{4}$ hour; and will give the answer as 90° instead of the correct one of $90° - 7\frac{1}{2}°$, or 82° 30′. At 6:30, the minute hand is at 6, and the hour hand is half way between 6 and 7, or $\frac{1}{2}$ of $30° = 15°$. At 8:20, the minute hand is at 4, and the hour hand $\frac{1}{3}$ of the way between 8 and 9 — the number of hour spaces being $4\frac{1}{3}$, corresponding in degrees to $30° \times 4\frac{1}{3} = 130°$.

Pupils should understand that while the angle at 4 o'clock is $30° \times 4$, or 120°, and while the angle at 5 o'clock is $30° \times 5$, or 150°, the angle at 7 o'clock is not $30° \times 7$, or 210°. By making a drawing, they will see that in the last case the angle should be measured on the left, which will make it $30° \times 5$, or 150°.

NOTE. — Angles of 180°, 210°, etc., may be left for more advanced work.

1260. From **26** should be learned that two lines perpendicular to a third line are parallel to each other; and from **27**, that two lines running in the same direction and making the same angle with a third line, are parallel to each other.

29. DE will be drawn parallel to BC by means of the protractor, an angle of 58° being made at the intersection of AB and

DE. Six of the twelve angles will contain 58° each; the remaining six will each measure $180° - 58° = 122°$. The pupils should be permitted to ascertain this for themselves.

The card suggested in the note is to be used in schools in which small wooden triangles are not obtainable.

32. To draw from *P*, a line parallel to the oblique line *AB*, place the perpendicular of the triangle on the line *AB*, Fig. 4, and place the ruler *XY* against the base of the triangle. Holding the ruler in position, slide the triangle along it until the perpendicular passes through the point *P*. A line *PE* drawn along this side of the triangle will be parallel to *AB*.

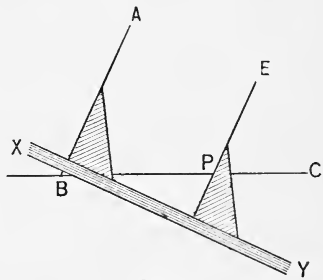


FIG. 4.

Time should be given the pupils to discover this or a similar method of drawing by means of a ruler and a triangle a line parallel to another line. The method may be made the subject of a composition.

33. *QR* and *UV* are drawn parallel by means of the ruler and the triangle. They may lie in any position, care only being taken to cut them by a line making angles of 50° and 130° with one of them. Three of the remaining six angles will measure 50° each; and the others, 130° each.

1261. **35.** If the work is done as it should be, the angle at *C* will measure 80°.

The line *AB* does not need to be horizontal; nor should all the pupils draw *AB* of the same length.

36. The third angle will measure 60°.

37. There are 68° in *a*, and 57° in *c*. In *b*, there are $180° - (68° + 57°)$, or 55°. In *d*, which is vertical to *b*, there are 55°.

38. The angle *e* should measure 28°, and *f* 120°. There are 180° in $e\ (28°) + g + f\ (120°)$.

39. $PRQ = 180° - (70° + 60°) = 50°$. $PRS = 180° - 50° = 130°$. PRS is therefore equal to the sum of the angles P and Q.

40. 180°. *Ans.*

41. $180° - (36° + 65°)$.

45. In measuring the side of a triangle, use the smallest fraction marked on the ruler. When the ruler in use has the denominations of the metric system on one face, that face should be used, and the length of the line given in millimeters. This will not require any teaching of the metric system beyond showing pupils how to read their rulers, and it will do away with the need of using fractions.

46. The length of each side should be marked. If two of them are not found exactly equal, the construction is faulty.

47. The three sides should be equal.

50. Each of the oblique angles will contain 45°.

52. Angles 2 and 3, 90° each; 4, 50°; 5 and 6, 40° each.

53. Angles 1 and 4, $67\frac{1}{2}°$ each; 2 and 3, 90° each; 5 and 6, $22\frac{1}{2}°$ each.

54. The angle p contains 120°; m, 30°; n, 30°.

1266. **56.** Construct this parallelogram by drawing two lines of the given lengths, meeting at an angle of 60°. By means of the ruler and the triangle, construct the other two sides.

If the work is properly done, these two sides will measure 2 in. and 3 in., respectively; and the remaining angles will measure 120°, 60°, and 120°, respectively. From this exercise, the pupils should learn that the opposite sides and the opposite angles of a parallelogram are equal, and that the sum of the four angles is 360°. The angles being oblique, the figure is a rhomboid.

The work of the scholars should show the variety suggested in previous exercises. It is not essential that the longer of the two given sides should be taken as the base, nor that the base should be parallel to the lower edge of the paper.

57. Different pupils will construct this trapezoid in different ways. Some, seeing that one angle is a right angle, will use the triangle to draw the second side, 3 in.; and at the extremity of this side, will draw, by the same means, an indefinite perpendicular line to form the third side, which is parallel to the base. The fourth side is drawn to make an angle of 60° with the base.

The remaining angles will measure 90° and 120°, respectively; and the sides will measure 5 in., 3 in., nearly $3\frac{1}{4}$ in., and nearly $3\frac{1}{2}$ in.

58. The triangle cut off will form a rhombus when opened out, unless the base and the perpendicular are equal. In this case, the paper, when opened, will form a square.

Making one angle of the triangle 30° (or 40°) will give a rhombus containing 60° (or 80°).

60. When the three sides of a triangle are equal, its three angles are equal; but the rhombus has four equal sides without having equal angles.

61. A triangle that contains three equal angles, has its sides equal; but an oblong has four angles of 90° each, with unequal sides.

62. To construct the rhomboid, draw a base of $2\frac{1}{2}$ in. Two inches above, draw a parallel line $2\frac{1}{2}$ in. long, with the extremities of the latter on the right or the left of the extremities of the base. If the two remaining sides are exactly equal to each other, the work is correctly done.

It is not necessary to draw the altitude, though a broken line may be used. While different methods may be employed, the use of an incorrect one should not be permitted. The work done on the board by a pupil, should be criticised by the class if it be faulty; or a better way may be suggested, if the one employed by the pupil at the board require too much time or unnecessary work.

The lengths of the two remaining sides should be measured, and marked on the papers. Those of different pupils should be different, there being no limit except the size of the paper: they must, however, be longer than 2 in. each; and they should not be just $2\frac{1}{2}$ in., which would make the figure a rhombus.

63. If the line that shows the altitude of the rhomboid can be drawn within the figure, a right-angled triangle can be cut from one side and transferred to the other, making the figure a rectangle. Arithmetic, Art. 929, **5**, last parallelogram.

In the case of a rhomboid whose altitude does not fall

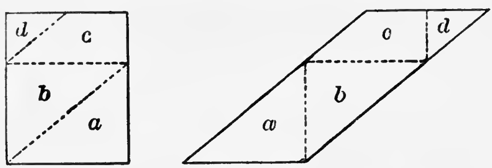


FIG. 5.

within the figure, several cuts will be necessary to change it into a rectangle. See Fig. 5.

65. The areas will be equal, because each rhomboid is equal in area to a rectangle 3 in. by 2 in.

66. The three rhomboids in Fig. 6 have their respective sides equal each to each, but their angles are unequal; hence their altitudes and their areas are unequal.

To construct these rhomboids, draw base lines of three inches,

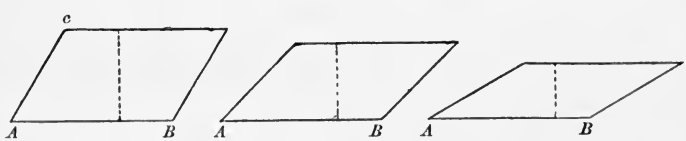


FIG. 6.

and inclined to each, at any angle except one of 90°, a two-inch line. Use the ruler and the triangle to complete the figures.

68. As the altitudes are not given, the areas of the figures drawn by different pupils should vary. The protractor or the triangle should be used in drawing the altitudes, which should then be carefully measured.

69. See **62**, making the upper side $2\frac{1}{2}$ in. The fourth trapezoid in Arithmetic, Art. 929, **6**, shows the manner of determining the dimensions of the required rectangle.

1267. **78.** The diameters of these circles should be such as not to admit of the use of the protractor in drawing them. To ascertain the extremities of an arc of 120°, two lines are drawn meeting in the center of the circle at an angle of 120°. The portion of the circumference intercepted by these lines will constitute an arc of 120°. The remainder of this circumference will form an arc of 240°.

79. The pupil should be permitted to make the first attempts in his own way. He will doubtless soon discover that the distance between the points of his compasses in drawing the circle, is the length of the chord required, and that by placing one point of the compasses on any portion of the circumference of the circle just drawn, the other point will indicate on the circumference the other extremity of the chord.

To measure the length of the arc in degrees, draw radii to its extremities, and use the protractor to determine the angle made by these radii, which may be produced, if necessary. If the work is properly done, the angle should measure 60°, which is the length of the arc.

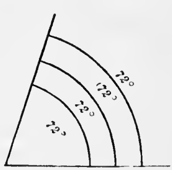


FIG. 7.

80. Draw two light lines meeting at an angle of 72°. Using the vertex of the angle as a center, and any radius, draw an arc between the lines. This arc will measure 72°. Darken the lines from the arc to the vertex to show the radii of the circle of which the arc is a part.

81, 82. Figs. 8 and 9 show sectors of 90° and 270°, respectively; Figs. 10 and 11, segments of 120° and 240°, respectively.

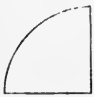


Fig. 8.

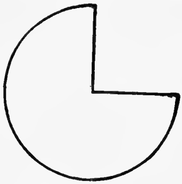


Fig. 9.


Fig. 10.

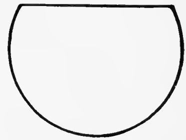


Fig. 11.

1268. **86.** A diameter will divide the circle into two equal parts. A second diameter perpendicular to the first, drawn by means of the protractor or the triangle, will divide the circle into four equal parts.

To divide the circumference into four equal parts, it will not be necessary to draw the diameters. When he has the ruler in the proper place to draw the first diameter, the pupil needs to mark only the two points where the ruler cuts the circumference. The third point can be indicated when the triangle is placed at the center of the circle and against the ruler; etc.

While the scholars may be permitted in the beginning of this work to draw a number of unnecessary lines, and while it may be an advantage to even require it, they should gradually learn to make as few lines as possible. The construction lines that are employed, should be drawn very lightly and should not be erased. Other lines should be made more conspicuous. Careful pupils may be allowed to use ink for this purpose.

1269. It is not intended that the methods here suggested should be communicated in advance to pupils. Each should be allowed to try the problem in his own way. The discussion of the method employed afterwards on the blackboard, will suggest other and possibly better modes of procedure.

88. To inscribe a regular pentagon in a circle, it will be necessary to divide the circumference into five arcs of 72° each. The protractor should be used to obtain the first arc; the remaining ones can be set off by the compasses, the first being used as a measure. Careful work should be exacted by the teacher.

89. Many pupils will have learned in their drawing lessons the regular method of inscribing a hexagon in a circle. Those unfamiliar with this way, should not be shown it until after they have constructed the hexagon by means of the protractor.

It is a pedagogical mistake to suggest "short-cuts" to pupils before they thoroughly understand a general method. For this reason, the teacher should permit the members of her class to use the protractor in the construction of the inscribed triangle, leaving it to themselves to discover a simpler way. She should encourage, also, the employment of a variety of methods even if some of them are not very direct. The experiments made by pupils to discover a new mode of constructing a polygon, will help them in their geometrical study.

The chord of 60° being equal in length to the radius (**79**), the shortest method of inscribing a hexagon is to apply the radius as a chord six times. Two of these divisions of the circumference will make arcs of 120°, the chords of three of these forming the sides of an inscribed equilateral triangle.

90. Each of the six angles at the center contains 60°. Since the two sides AC and AB, enclosing any central angle, are radii of the circle, and therefore equal to each other, the angles oppo-

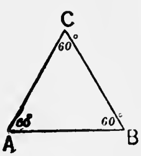


FIG. 12.

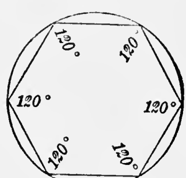


FIG. 13.

site those sides are equal; that is, angle $A =$ angle B. The angle at C being 60°, $A + B = 180° - 60° = 120°$, and $A = B = 60°$. Angles 1 and 2 (see Arithmetic), therefore, measure 60°

each, and the whole angle contained between two adjoining sides of the hexagon, measures 120°. After determining by this method the number of degrees in each angle of a regular hexagon, the pupils should be required to construct one, and to mark in each angle its contents in degrees, as in Fig. 13, verifying the result by using the protractor.

91. A careless scholar, measuring the number of degrees in each angle of a regular pentagon (Arithmetic, Art. 1268), will sometimes read from the wrong row of numbers on the protractor, getting the result 72°, instead of the correct one of 108°. As there are 72° in each division of the circumscribing circle, he will have no doubt of the correctness of his answer, unless he has been trained to estimate the size of an angle. In this case, he will see that each angle of a regular pentagon is obtuse, and, therefore, greater than 90°.

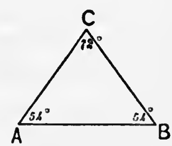


Fig. 14.

One method of calculating the number of degrees, is to divide the pentagon into five equal triangles, one of which is shown in Fig. 14. The angle at C is 72°. The sides CA and CB, being radii, are equal; which makes equal angles at A and B, each of which is $\frac{1}{2}$ of (180° − 72°), or 54°. Each of these angles is the half of one of the angles of the pentagon, so that these latter angles measure 108° each.

92. A circumscribed square touches the circle at four points, each side constituting a *tangent.* A tangent being perpendicular to the radius drawn to the point of contact, the square may be constructed by drawing perpendiculars to two diameters intersecting at right angles, using the triangle or the protractor for the purpose. The ingenious pupil will discover other ways; drawing, for instance, at each extremity of the two diameters, a line parallel to the intersecting diameter, by means of the ruler and the triangle; etc. No method should be permitted that merely approximates accuracy, such as determining that a line is parallel or perpendicular by the eye alone. The average class will contain many members intelligent enough to pass upon the correctness of a given method, and they should be called upon to give reasons for any criticisms they may have to offer.

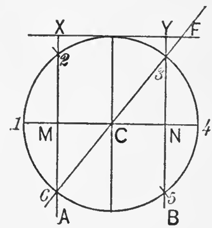


FIG. 15.

In circumscribing some polygons, a hexagon for instance, many pupils prefer locating points X and Y, instead of using the triangle and the ruler to draw a tangent XF. After dividing the circumference (Fig. 15) into six equal parts at 1, 2, 3, 4, 5, and 6, they draw a diameter from 1 to 4. Through 2 and 6 they draw a secant XA, making MX and MA each equal to the radius. Through 3 and 5 a second secant is drawn, and NY and NB are also made equal to the radius. A line drawn through X and Y will form one side of the circumscribed hexagon. One extremity of this side can be determined by producing the diameter $3C6$ to F, and the other by a line through $2C5$.

NOTE. — A *secant* is a line that cuts a circle at two points; a *tangent* is a line that touches a circle at one point.

93. The smallest number of triangles into which a pentagon can be divided, is three (Fig. 16). The three angles of each triangle contain 180°, making the sum of angles 1–9, 540°. Since $1 = A$, $2 + 4 = B$, $6 + 7 = C$, $8 = D$; $3 + 5 = E$, the sum of the five equal angles (A, B, C, D, and E) of a regular pentagon $= 540°$, and each equals 108°. This is the result that was found in **91** by another method.

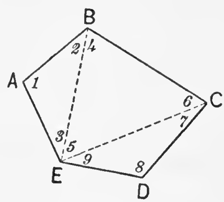


FIG. 16.

94. The hexagon is divisible, as above, into four triangles, containing $180° \times 4$, or 720°; making each angle $720° \div 6$, or 120°.

95. A quadrilateral is divisible into 2 triangles; a pentagon, into 3; a hexagon, into 4; a heptagon, into 5; an octagon, into 6, being 2 triangles less in each case than the number of sides in the polygon.

96. The number of degrees in each angle of a regular octagon $= [180° \times (8-2)] \div 8$.

97. At each end of the 2-inch line, draw a 2-inch line at an angle of 108°. At the farther extremity of each of those lines draw a line at an angle of 108°. These last lines meet at an angle of 108° if the work is correctly done, and are each two inches long to the point of their intersection.

98. A line drawn to each extremity of the base at an angle of 60° will form an equilateral triangle. Use angles of 90° for the square, 108° for the pentagon, 120° for the hexagon, nearly 129° for the heptagon, 135° for the octagon, 140° for the nonagon, etc.

The first line should be placed near the center of the bottom of the paper to give room for the successive polygons. See Fig. 17. Bright pupils will calculate the necessary angles and continue to construct polygons as far as the space will permit.

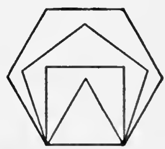


FIG. 17.

99. By drawing the diameters AB and XY (Fig. 18), the inscribed square will be divided into four triangles, while the circumscribed square contains eight, being double the area of the inscribed square.

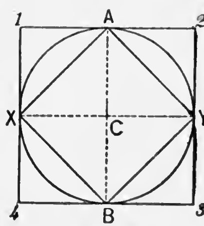


FIG. 18.

1270. These problems are given to enable the pupils to learn how to bisect lines, erect perpendiculars, construct angles, etc., by means of the ruler and the compasses, and incidentally to learn a number of geometrical facts. The use of other instruments is unnecessary, and should, therefore, not be tolerated.

1. The distance between the centers $= 1\frac{1}{2}$ in. $+ 1$ in.

2. $1\frac{1}{2}$ in. $- 1$ in.

4. The line XY joining the two points of intersection of the equal circles (Fig. 19), bisects the line AB connecting the centers. The radii AX, BX, AY, BY are each 2 inches long.

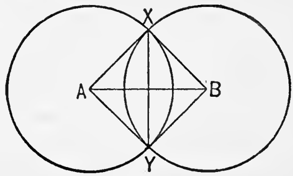


FIG. 19.

5. The previous exercise should lead the pupils to see the steps necessary to the construction of the required triangle. The 3-inch base AB is first drawn. The next requirement is to find a point X (Fig. 19) 2 inches from A and from B. A circle of 2-inches radius with B as a center, will contain every point that is 2 inches from B. A similar circle with A as a center, contains every point 2 inches from A. The intersections, X and Y, are each 2 inches from both points A and B. Using either one as a vertex, draw lines to A and B, forming the required triangle.

Authorities differ somewhat as to the advisability of requiring pupils to employ circles rather than arcs in geometrical problems. While it may be better at first to use circles, the point to be finally reached is the employment of the shortest lines possible. This should not be inconsistent with the acquirement of geometrical knowledge. A scholar should know after a very few exercises that each point of an arc is 2 inches from the center, as well as he does when he draws the entire circle.

In his later construction of an isosceles triangle, the pupil should know that the vertex is above (or below) the center of the base. For this reason the first arc need not be a very long one. The second should be still shorter.

6. Fig. 19 will suggest the necessary steps. Using each end of the $3\frac{1}{2}$-inch base as a center, and with a radius of 4 inches, draw two circles. Draw lines corresponding to XA and XB for the required triangle. Placing the ruler on X and Y will give the perpendicular, which should not extend below the base.

If arcs are used, the upper intersection determines the position of the vertex. A lower intersection is used to determine the direction of the perpendicular. The pupil will gradually learn that while a definite radius, 4 inches, is required to locate the vertex of the triangle, intersecting arcs, each of 3 inches or 5 inches or any other radius, will serve to locate the second point, used with the vertex to determine the direction of the perpendicular. The point of their intersection may be above the base or below it, according to convenience. A point below secures greater accuracy, by being probably at a greater distance from the vertex than one above is likely to be.

7. This is a variation of **6**, but without directions as to length of sides. If the perpendicular is correctly drawn, it will bisect the base. See Exercise 48, Art. 1263. The compasses should be employed to determine the equality or inequality of the segments of the base.

8. The same procedure is required as in **7**, except that the sides of the triangle are not drawn. The bisecting line should be extremely short.

9. With a 2-inch radius, draw intersecting arcs, using as centers the two extremities of a 2-inch line.

11. Either side may be used as the base; and different pupils should use a different base, although the greatest number will probably take the longest side.

Using the 2-inch side as a base, draw from one end, as a center, an arc with a radius of 1 inch; and from the other, an arc with a radius of $1\frac{1}{2}$ inches. The intersection of the arcs will be the vertex of the required triangle.

With the 3-inch line as a base, the radius of the arcs will be 2 inches and $2\frac{1}{2}$ inches, respectively.

Besides employing different bases, the pupils should use oblique lines and vertical lines as bases, and the vertex in some instances should be below the base; etc.

12. A scholar should be permitted to discover for himself that the intersection of the 2-inch arcs will be at the center of the 4-inch base. After endeavoring, also, to make a triangle whose sides shall measure 1, 2, and 3 inches, respectively, he will understand that the third side of a triangle must be shorter than the combined lengths of the other two.

13. If the pupil, in drawing arcs to locate the bisecting line, employs the radius used in drawing the circle, he will discover that one intersection will take place at the center of the circle. This will lead him to see that only one set of intersecting arcs is necessary — the one beyond the circle, the center of the circle serving as a second point.

The teacher should be in no hurry to inform the pupil using two sets of arcs, that a single set will be sufficient; knowledge that comes in some other way than merely by direct telling, is apt to last longer.

14. Before dividing a sector (Figs. 8 and 9) into two equal parts, some scholars may consider it necessary to draw the chord. Permit them to do so at first.

15. The pupils have already learned that a line drawn from the vertex of an isosceles triangle to the center of the base, is perpendicular to the base. The method employed in bisecting a line is practically to consider it the common base of two isosceles triangles having their vertices on opposite sides of the base, although the equal sides of the triangles are not drawn. This bisecting line is perpendicular to the assumed base. The pupils have probably discovered that the perpendicular line bisecting the chord in **13**, would, if produced, pass through the center of the circle. From this previous knowledge, they will doubtless be able to answer the last question of **15**.

The scholars that have drawn the chord in dividing the sector in **14** into two equal parts, will be likely to see that the radius CX (Fig. 20) is perpendicular to the bisected chord AB.

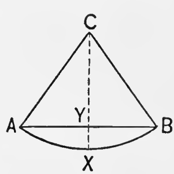


Fig. 20.

16 shows that a perpendicular that bisects a chord, AB (Fig. 20), also bisects the arc AXB.

It will readily be seen by the pupils that an arc AXB can be bisected without drawing the chord AB.

17. This problem is the same as **8**. The drawing, however, should show a longer line, and one that does not cut the given line, the requirement being that the perpendicular be drawn *to* the latter. A perpendicular drawn to a horizontal or to an oblique line from below it, would be correct; although it might not be accepted as satisfactory if the more common wording of the problem were employed: *Erect* a perpendicular at the middle point of a line.

18 requires no explanation.

19. Bisecting one of the four divisions of a circumference gives the dividing point between two one-eighths. A ruler placed upon this point and on the center of the circle will indicate the location of another. The distance between two points can be ascertained by the compasses, which can then be used to locate the remaining two dividing points.

The lines used to divide a circumference should not be too long.

The thoughtless pupil sometimes fails to see, when he employs his compasses to measure the distance between two points on a circumference, that he is measuring the chord of an arc, and not the arc itself. It may be necessary for the teacher to explain this in connection with **25**.

22. Unless the pupils have learned in their drawing work the method of erecting a perpendicular at the end of a line, some of them may experience a little difficulty in solving this problem. One way of drawing the circumscribed square is to construct on AY (Fig. 18) an isosceles triangle $A2Y$ equal to ACY. Produce $2A$ to 1, making $A1$ equal to $2A$. Produce $2Y$ to 3 in the same manner. Lines from 1 through X, and 3 through B, will intersect at 4, which completes the square.

24. The object of **23** and **24** is to lead the pupils to see again that the side of an inscribed hexagon is equal in length to the radius of the circle.

26. The four triangles will together constitute a circumscribed equilateral triangle.

27. Draw lightly an arc less than 90° in length. After cutting off 60°, darken this portion. No radii should be drawn.

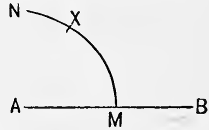


Fig. 21.

To construct an angle of 60°, draw an indefinite line AB (Fig. 21). With any convenient radius, as AM, draw an arc MN. With M as a center, and using the same radius, cut the arc at X. This makes MX an arc of 60°. From A draw a line through X.

28. An arc of 30° is constructed by the bisection of an arc of 60°.

To construct an angle of 30°, draw AB (Fig. 21) and the arc MN; and cut off MX, as in **27**. With the same, or any other convenient radius, and using M and X as centers, draw two arcs intersecting on the right of MX. A line drawn from A through this intersecting point will make with AB an angle of 30°.

29. To construct an angle equal to $60° + 30°$ (or $30° + 60°$), bisect the arc XM (Fig. 21), and from the bisecting point, which is 30° from M, lay off an additional 60°. A line drawn from A through this point will make with AB an angle of 90°.

30. One method of erecting a perpendicular at the end of a line is given in **29**. A somewhat similar method consists in prolonging the arc, and marking off at Q and R two divisions of 60° each. Using these two points as centers, draw arcs intersecting at F. From D draw a line through F.

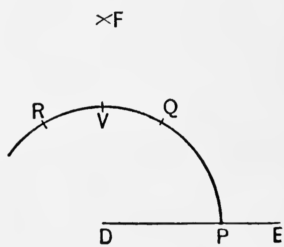


Fig. 22.

The line DF bisects the arc QR, which makes the angle $FDE = 60° (PQ) + 30° (QV) = 90°$.

31. The bisection of VP gives an angle of 45°. Do not draw DF.

The recommendation so often made as to getting a variety of drawings from the various members of the class, applies to these problems. The erection of a perpendicular at the end of a vertical line should call forth at least four different results. The perpendiculars may be erected at either end, and may run to the right or to the left. In constructing an angle of 45°, a greater variety is possible. The first line may be horizontal, vertical, or oblique; the second line may start from either end; and it may extend above or below, to the right or to the left; the lines forming the angles need not be of the same length on all papers, nor need both of them be of the same length on any one paper.

$22\frac{1}{2}° = \frac{1}{2}$ of $45°$; $135° = 90° + 45°$; $15° = \frac{1}{2}$ of $30°$; $75° = 60° + 15°$.

33. The preceding problem should give a hint as to the method of solving the present one. In **32**, a circle was drawn with A as a center and AX as a radius. This circle cut XY (Fig. 23) in M. To erect a perpendicular at A, the center of the circle, the extremities X and M of the diameter XM were used as centers to draw arcs intersecting at J. A perpendicular was then drawn from A through J.

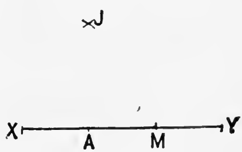


FIG. 23.

This problem does not furnish a circle, nor is one necessary. The line XY is given, and the point A at which the perpendicular is to be erected. With A as a center, and a radius equal to AX, cut off AM. This gives us the diameter of the circle employed in the preceding problem.

It will be noticed that a second set of intersecting arcs on the opposite side of XM is not needed, the point A answering instead. This problem amounts to the bisection of an arc of 180°, of which only the chord XM is drawn, A being the center of the circle.

34. The first two divisions of this problem are reviews of parts of **17** and **30**. The erection of a perpendicular at a point between the end and the center, is a variation of the preceding problem. Let RS (Fig. 24) be the required line and A the point. With A as a center, and any convenient radius, lay off the points X and M, which will be equidistant from A. Using X and M as centers, draw arcs intersecting at J. Draw the required perpendicular from A through J.

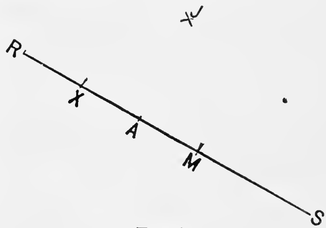


FIG. 24.

In problem **33** the point M was located at a distance from A, equal to AR; and while greater accuracy is obtained by having the points M and X as far apart as possible, the present method is suggested to show that the only essential requirement as to their location is to have them equidistant from A.

35. The base need not be a horizontal line.

36. Proceed as in the construction of a right-angled triangle, base 2 inches, perpendicular 2 inches; but do not draw the hypotenuse. With B and H as centers (Fig. 25), and a radius of 2 inches (BP or PH), draw arcs intersecting at O. BO and HO will form the remaining sides.

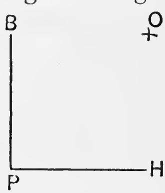


FIG. 25.

To construct the rectangle, PH (Fig. 25) should be made 3 inches long, BP measuring 2 inches. With B as a center, and a radius of 3 inches, draw an arc. Intersect this by a second arc drawn with H as a center, and a radius of 2 inches. From the point of intersection, draw lines to B and H.

Some scholars may prefer to erect a perpendicular at each end of the base, etc.

38. Draw a base line, WV (Fig. 26), 3 inches long. At any convenient point, M, erect a perpendicular, MP, $2\frac{1}{2}$ inches long.

At its extremity P, draw the perpendicular PO. With W as a center and a radius of 3 inches, locate the point X on the line PO; WX will be the second side of the rhombus. On PO draw XN 3 inches; and connect NV. This last line should measure 3 inches if the work is properly done.

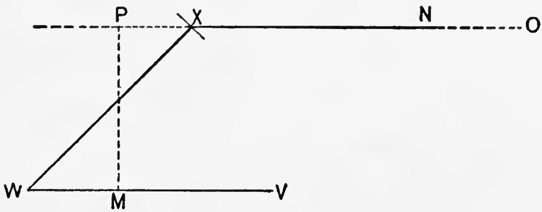


FIG. 26.

To construct a 3-inch rhombus containing an angle of 60°, draw WV; at W construct an angle of 60°, and make WX 3 inches. At X or at V, draw a 3-inch line making an angle of 120° with XW or VW, etc.; or, using X and V as centers, and with a radius of 3 inches, draw arcs intersecting at N; draw NX and NV.

39. Proceed as in the first problems of **38**; but make WV and XN each 4 inches long.

40. See BPH (Fig. 25); draw BH.

Some pupils, preferring to commence with a horizontal base, calculate the number of degrees in each angle at the base, $\frac{1}{2}(180° - 90°)$, or 45°. To each end of a base line of any length (Fig. 27), they draw a line making with the base an angle of 45°.

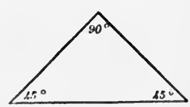


FIG. 27.

If one angle is 120°, the angles at the base will be 30° each; if one is 135°, the angles at the base will measure $22\frac{1}{2}°$ each.

The method shown in Fig. 27 requires the construction of two angles; and is, therefore, not so direct as drawing two equal lines meeting at the given angle.

41. Erect a 3-inch perpendicular at the middle of a 3-inch line.

A perpendicular, AB (Fig. 28), bisects the angle at A of the equilateral triangle. Erect a 3-inch perpendicular at B, on an indefinite line CD; at A, construct two angles of 30° each, as shown in the figure; draw AX and AY, forming the required equilateral triangle AXY. Test the work by measuring XY.

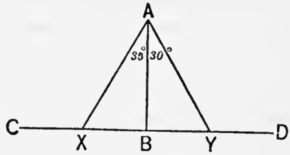


Fig. 28.

42. To construct a scalene triangle of the required dimensions, erect the 3-inch perpendicular at a point not in the center of the 3-inch base, nor at either extremity.

To construct the obtuse-angled triangle, produce the 3-inch base by a dotted line, and erect the 3-inch altitude at some point outside of the base.

43. This problem may puzzle the pupils at first. If the triangle were isosceles, XYM, for instance, there would be no difficulty. The solution of the problem consists practically in making an isosceles triangle, although the line XM is not drawn. With X as a center (Fig. 29), and XY as a radius, the point M is located (without drawing the arc shown in the figure). Using M and Y as centers, and with any convenient radius, arcs are drawn intersecting at R. XR gives the direction of the altitude.

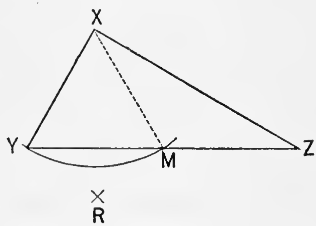


Fig. 29.

44. The given triangle in this case will be XMZ (Fig. 29). Produce ZM indefinitely towards Y, and with XM as a radius, cut the base at Y; etc. XY is, of course, not drawn; MY should be a light line or a broken line; etc.

45. This is the same problem in another form. Let YZ (Fig. 29) be the given line, and X the given point, from which a perpendicular is to be let fall upon YZ. In **43–44**, the arc described with X as a center, passes through one extremity of the line. The pupils should see that this is not essential, and that it would be very inconvenient in the case of a very long line. All that is necessary is to intersect the line YZ at two points sufficiently far apart to secure accuracy.

47. Each angle will measure 60°; the arc on which it stands measures 120°; an angle at the circumference, therefore, is measured by one-half the arc intercepted by its sides.

Apply this to the regular pentagon (Arithmetic, Art. 1268). The angle at the top of the figure stands on an arc containing $72° + 72° + 72°$, or 216° (made up of three arcs); it contains, therefore, $\frac{1}{2}$ of 216°, or 108°. See Fig. 30.

An angle of a square stands on an arc of 180°; its contents are, therefore, $\frac{1}{2}$ of 180°, or 90°; etc.

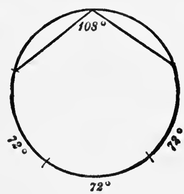


Fig. 30.

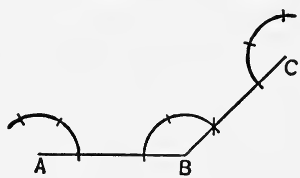


Fig. 31.

48. The number of degrees in each angle of a regular hexagon is 120° (**94**, Art. 1269). From each end of the given line AB (Fig. 31) draw a line equal in length to AB, and making with it an angle of 120°; etc.

While this is not the best way to construct a regular hexagon, it gives the pupils an opportunity to try the general method on a polygon of six sides.

49. For the construction of an octagon, the angles at A, B, C, etc., should measure 135° (Problem **31**).

By a later problem, the pupils will learn how to find the center of the circumscribing circle after two sides of the octagon are drawn; for the present, however, they should be required to repeat the construction of the angle of 135° at the required number of corners. The work should be as accurate as possible, considering the necessary imperfections of the tools employed.

50. A right angle whose vertex is at the circumference, subtends an arc of 180°, or a semi-circumference.

51. By means of a ruler, placed on the center of the circle, mark points A and B (Fig. 32) the extremities of a diameter (not drawn). The arc AMB is one-half of the circumference, or 180°; and the angles X, Y, and Z, whose sides XA and XB, YA and YB, ZA and ZB, subtend this arc, are each measured by one-half of it, and contain, therefore, $\frac{1}{2}$ of 180°, or 90°.

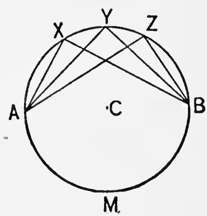


Fig. 32.

52. The hypotenuse of an inscribed right-angled triangle is always the diameter of the circle.

In Fig. 32 draw the diameter ACB (or any other). Bisect the arc $AXYZB$, which gives the vertex of the required triangle. Connect this point with the points A and B.

Note. — It may be necessary to have the scholars understand that all the vertices of an inscribed polygon must lie in the circumference; an inscribed triangle, therefore, has three vertices in the circumference.

53. The diameter of a circle having a radius of 2 inches, will be the diagonal of the inscribed square. The location of the remaining two corners is easily determined.

The square may also be constructed by drawing 4-inch diagonals bisecting each other at right angles; etc.

The rhombus will consist of two 3-inch equilateral triangles having a common base.

54. Every angle being on the circumference, each will be measured by one-half the intercepted arc, and the sum of the three angles will contain one-half the number of the degrees contained in the three arcs; *i.e.*, $\frac{1}{2}$ of 360°.

55. See the rhombus in **53.**

56. The two triangles will be equal in all respects, as will those in **57.**

58. From **56** and **57** the pupils should learn that two triangles are equal in every respect (a) when two sides and the included angle of one are equal to two sides and the included angle of the other, each to each; and (b) when two angles and the included side of one are equal to two angles and the included side of the other, each to each. From **59**, they should learn that when two triangles have the three sides of the one equal to three sides of the other, each to each, they are equal throughout. From the present problem, they should learn that any number of triangles can be constructed having their corresponding angles, equal each to each, but whose corresponding sides are unequal, each to each.

60. Impossible, because the sum of three angles of 75° each is greater than 180°.

61. The radius of each circle must be one-half of the hypotenuse, or $1\frac{1}{2}$ in. See Problem **52.**

Any inscribed triangle having one side the diameter of the circle will answer the conditions of the first case.

The 2-inch base of the second triangle is obtained by taking A as a center (Fig. 32), and with a radius of 2 inches, drawing an arc at, say, Y; AY will be the required base, YB the perpendicular, and AB the hypotenuse.

Using a radius of $1\frac{1}{2}$ inches, as above, will give a perpendicular, AX, of $1\frac{1}{2}$ inches, etc.

NOTE.—The term *base* does not necessarily imply a horizontal line; nor *perpendicular*, a vertical one.

63. To construct an angle at Y (Fig. 33) equal to the angle at X, take X as a center and any convenient radius, and draw the arc Aa; then take Y as a center and the same radius, and draw the arc Cc. The angle X is measured by the arc AB, the length of whose chord is determined by means of the compasses. Lay off the same on *Cc*, making the arc *CD* equal to the arc *AB*. An angle formed by drawing a line from *Y* through *D*

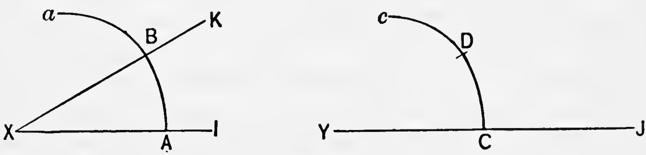


FIG. 33.

will be equal to the angle at *X*, as each angle is measured by an arc of the same number of degrees. See note to Problem **19**, Art. 1270.

64. Draw the first line at any angle; and by the preceding problem, draw the two intermediate lines, making, with the horizontal line, angles equal to that made by the first line. The pupil will have no difficulty with the oblique line at the other extremity of the horizontal line.

65. Proceed as in the second part of **64**, using a horizontal 3-inch line for the diagonal, drawing from one end a $2\frac{1}{2}$-inch oblique line running upward at any angle, and from the other end a similar line running downward at the same angle.

The lines divide the 3-inch diagonal into 5 equal parts, each part measuring $\frac{3}{5}$ inch.

66. The scholars should gradually learn that it is unnecessary to measure the oblique lines, *Km* and *Ln* (Fig. 34). In setting off the divisions *K*1, 1 2, etc., *La*, *ab*, etc., it will be found unnecessary to use a definite measure; all that is required is that they be equal. This equality is obtained by opening the compasses slightly and marking off the divisions with them. There is no need of completing the

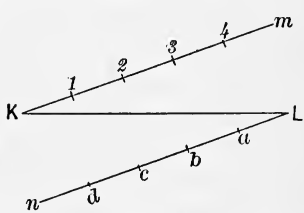


FIG. 34.

rhomboid, nor of drawing the lines $4a$, $3b$, $2c$, etc. Place the ruler on 4 and on a, and use a short line on KL to mark off one division; etc.

It is expected that the teacher will not give this method at the outset.

To draw a line exactly $\frac{3}{5}$ of an inch in length, a construction line, 3 inches long, KL (Fig. 34), is drawn lightly, also Km and Ln of indefinite length. Only the first division is required on Km. Placing the ruler on 1 and d, mark off on KL $\frac{3}{5}$ inch, and darken this portion

One-seventh of a 5-inch line $= \frac{5}{7}$ inch.

67. When the base of a right-angled triangle measures 3 inches, and the perpendicular measures 2 inches, the hypotenuse will measure $\sqrt{3^2+2^2} = \sqrt{9+4} = \sqrt{13}$ inches.

68. A line $\sqrt{13}$ inches long is drawn by constructing a right-angled triangle having a 3-inch base and a 2-inch perpendicular. The hypotenuse is the required line.

A base of 2 inches and a perpendicular of 1 inch give a hypotenuse of $\sqrt{5}$ inches.

A hypotenuse of 4 inches and another side of 3 inches give a remaining side of $\sqrt{7}$ inches. For the construction of this triangle, see Problem **61.**

69. The side opposite the angle of 30° is one-half as long as the side opposite the angle of 90°.

71. The chord of an arc of 60° is 2 inches long; the chord of an arc of 180° is the diameter, and is 4 inches long. The chord of an arc of 300° is 2 inches long.

72. The perpendicular "erected" at one end of the chord should be turned downwards if the chord is drawn above the center of the circle.

The perpendicular and the diameter meet at the circumference. See Problem **61.**

73. The triangle will retain its shape, because only one triangle can be drawn with sides of a given length. Problem **59.** The rectangle will not retain its shape, because an indefinite variety of parallelograms can be constructed having their corresponding sides equal, each to each. See Exercise **66**, Art. 1266.

77. From Problem **15**, the pupils have learned that a perpendicular bisecting a chord, passes through the center. If there are two lines passing through the center, the latter must be located at their intersection.

78. To inscribe a circle in any triangle, draw lines bisecting the three angles. See problem **28.** The intersection of these three lines will be the center of the circle.

In practice, only two lines are drawn; but the third serves to test the accuracy of a pupil's work.

79. The sides of an inscribed triangle are chords of the circumscribing circle. In Problem **77**, we have learned that the intersection of two perpendiculars that bisect chords, locates the center of the circle. With this center, and with a radius equal to the distance from the center to the vertex of any angle of the triangle, draw the circle.

80. The larger the circle the better for beginners. The average scholar may consider it necessary to draw two adjacent chords, but every purpose will be served by cutting the circumference by three short lines to mark the boundaries of two adjoining arcs. The bisection of these arcs by perpendiculars will locate the center of the circle.

82. Use a cup to draw the arc. Divide it at random into two parts. Bisect each, as above.

83. The altitude is 2 in. Problem **70.**

84. Experienced draughtsmen save time by changing the radius as infrequently as possible. To construct a square on a 3-inch line AB (Fig. 35), take a radius of 3 inches, and with A as a center draw the arc Bm. With B as a center, mark off $Bn = 60°$. Bisect Bn,

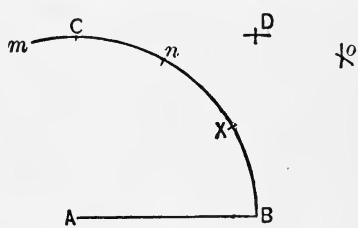


Fig. 35.

using the same radius to obtain the arcs intersecting at o, with B and n as centers. $BX = 30°$; place the compasses on X, and cut the first arc at C, which makes XC 60°, and BC 90°. C is the third corner of the square. Using B and C as centers, and with a radius of 3 inches, draw arcs intersecting at D, the fourth corner of the square.

86. We have found in Problem **26**, that constructing an equilateral triangle on each side of an inscribed equilateral triangle gives a circumscribed equilateral triangle. Locate m, n, and o, 120° apart. With each as a center, and with a radius equal to mo, draw arcs intersecting at A, B, and C. Do not draw the triangle mno, shown in the figure.

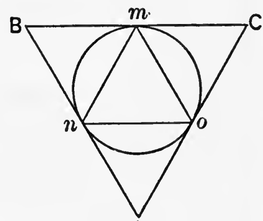


Fig. 36.

87. See Problem **41**, second part.

89. See Arithmetic, Art. 1089 (Fig. 2).

90. Construct a right-angled triangle having a base and a perpendicular measuring 2 inches and 3 inches, respectively. The hypotenuse will be the side of the required square.

Do not construct squares on the other two sides.

91. The hypotenuse is 3 inches, etc. See Problem **68**.

92. The square constructed on the hypotenuse of a right-angled triangle whose base and perpendicular measure 3 inches each, will be double the area of a 3-inch square.

When the 3-inch square is constructed, measure the length of the diagonal (without drawing it); and on a line of this length as a base, construct the required square.

93. Four. See Fig. 36.

94. Nine; sixteen; twenty-five.

95. 1, $1\frac{1}{2}$, 2 inches.

96. 3, $4\frac{1}{2}$, 6 inches.

97. The radius of the second circle is 2 inches.

98. A square described on the base of an isosceles right-angled triangle will be one-half the size of the square described on the hypotenuse. $A = a + a = 2a$; $a = \frac{1}{2}A$. The area of an equilateral triangle whose base is YZ, is one-half that of one whose base is XY. The area of a circle whose radius is YZ (or XZ), is one-half that of a circle whose radius is XY.

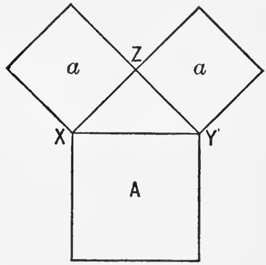


FIG. 37.

NOTE.—If the square A in Fig. 37 is drawn above XY instead of below it, the ratio of the large square to that of each small one will be more apparent.

Construct an isosceles right-angled triangle having a hypotenuse of 2 inches (Problem **52**). The base of this triangle will be the radius of the required circle.

The area of a circle of 2 inches radius $= 2^2 \times \pi = 4\pi$.

The radius of the other circle $= \sqrt{2}$; its area is $(\sqrt{2})^2 \times \pi = 2\pi$, which is one-half of 4π.

99. The side of the required triangle measures $\sqrt{2}$ inches. See **98.**

100. Cab (Fig. 38) is one-sixth of an inscribed regular hexagon, and CAB is one-sixth of a circumscribed regular hexagon. Calling the radius of the circle 2 inches, the altitude CX of the large triangle is 2 inches. Since ab is 2 inches, $ax = 1$ inch; $Ca = 2$ inches. In the right-angled triangle Cax, $\overline{Cx}^2 + \overline{ax}^2 = \overline{Ca}^2$; $\overline{Cx}^2 + 1 = 4$; $\overline{Cx}^2 = 4 - 1 = 3$; $Cx = \sqrt{3}$. The areas of the two triangles will be proportional to the squares of their respective altitudes. $\overline{Cx}^2 = 3$ and $\overline{CX}^2 = 4$. That is, the area of the larger triangle is $1\frac{1}{3}$ times the area of the smaller; hence the area of the circumscribed hexagon is $1\frac{1}{3}$ times the area of the inscribed hexagon.

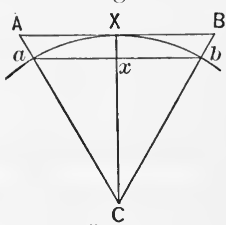


FIG. 38.

The area of the circumscribed square is double the area of the inscribed square; and the area of the circumscribed equilateral triangle is four times the area of the inscribed equilateral triangle.

1271. **3.** At the middle point X of the perpendicular of the right-angled triangle (Fig. 39), cut Xm parallel to the base. Re-arranging the two parts gives a rectangle whose dimensions are 4 inches and $1\frac{1}{2}$ inches. Figs. 40 and 41 show how the

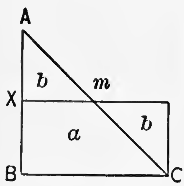


Fig. 39.

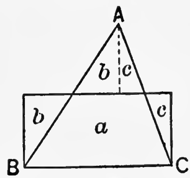


Fig. 40.

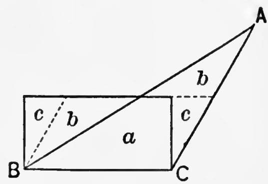


Fig. 41.

other two triangles are divided to make rectangles of the same dimensions.

4–6. See **56–59**, Art. 1270.

7. The two triangles have two sides of one, AC and BC, equal to two sides of the other, CE and CD; and the angle ACB equal to its opposite angle DCE; hence the third side, DE, of one triangle is equal to the third side, AB, of the other.

9. The area of the larger triangle is four times that of the smaller.

11. Another method of dividing a line into equal parts. See Problem **66**, Art. 1270.

1273. Much interest is added to this work by employing the method here given, in calculating heights and distances in the neighborhood of the school. A comparison between the results obtained by calculations and those obtained by actual measurements, will be useful in teaching the pupils the necessity of great accuracy in their preliminary work.

2. The hypotenuse of each triangle represents a ray of light from the sun, etc.

3. $VT = 110$ ft.

4. $CD : DE :: CB : BA$; $50 : 90 :: 1200 : BA$; $BA = 2160$ ft. The width of the river $= 2160$ ft. $- 100$ ft. $= 2060$ ft. *Ans.*

5. $CD = CG - DG = 6$ ft. $- 4$ ft. $= 2$ ft.; $EH = 177$ ft. $+ 3$ ft. $= 180$ ft; $AH = 120$ ft.; $AB = 120$ ft. $+ 4$ ft. $= 124$ ft.

6. The two triangles are similar, because the angles of one are equal to the angles of the other. Angle $D =$ angle $A = 90°$. The two angles at C are vertical angles, and, therefore, equal to each other. The remaining angle B must be equal to the remaining angle E, because the sum of the angles in each triangle is the same.

$$CD : DE :: AC : AB;$$
$$3.25 : \quad 5 \quad :: (12 - 3.25) : AB.$$

7. $10\frac{1}{2} : (10\frac{1}{2} + 195 + 15) :: (12 - 4\frac{1}{2}) : hf$. $hf = 157\frac{1}{2}$ ft.; $fi = 157\frac{1}{2}$ ft. $+ 4\frac{1}{2}$ ft. $= 162$ ft. *Ans.*

8. $RP : RN :: PQ : MN$; $6 : (120 + 6) :: 4 : MN$. $MN =$ 84 ft. *Ans.*

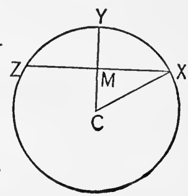
9. The tree is 3 ft. $\times$ 36, or 108 ft. high. *Ans.* $AC = AB$, because angle $C =$ angle $B = 45°$.

10. $AC = AB$. A line CB makes an angle of 45° with AC, angle $A = 90°$; angle $ABC = 45°$.

NOTE. — In the succeeding problems, the triangle and the protractor may be employed when necessary.

1274. **1.** The circumference of the circle, or $360° = 4$ in. $\times 3.1416$. The arc of 60° is $\frac{1}{6}$ of the circumference; the arc of $120° = \frac{1}{3}$ of it; etc.

2. Draw the circle and the various chords. The chord of 60° $=$ chord of $300° =$ radius $= 2$ inches; the chord of $180° =$ diameter $= 4$ inches.



If the pupils draw the chord of 120°, ZX (Fig. 42), they will find that it bisects the radius YC, making MC 1 inch. $CX = 2$ inches; therefore $MX = \sqrt{4 - 1} = \sqrt{3} = 1.732 +$; $ZX = 1.732 +$ in. $\times 2 = 3.464 +$ in. $=$ chord of $120° =$ chord of 240°.

Diagrams should be employed, unless the pupils can do satisfactory work without them. Since the arc of 180° is three times as long as the arc of 60°, the more careless members of the class may jump to the conclusion that the chord of 180° is three times as long as the chord of 60°. This mistake cannot be made if the circle is constructed, and the chords are drawn and measured.

5. See Exercise **98**, Art. 1269.

6. Each side of the hexagon measures 1 inch; the perimeter $= 1$ inch $\times 6 = 6$ inches. *Ans.*

Circumference $= 2$ in. $\times 3.1416 = 6.2832$ inches. *Ans.*

7. The pupil should use the ruler to ascertain the length of the apothem, which is about $\frac{7}{8}$ in. It can be calculated as follows: —

Cb (Fig. 38) is the radius $= 1$ inch; $bx =$ one-half the side of the inscribed hexagon $= \frac{1}{2}$ inch; $Cx =$ apothem. $\overline{Cx}^2 = \overline{Cb}^2 - \overline{bx}^2 = 1 - \frac{1}{4} = .75$; $Cx = \sqrt{.75} = .866+$, or nearly $\frac{7}{8}$.

8. The base of each triangle measures one inch, so that AB (Fig. 43) $= 3$ in. AX (apothem) $= \frac{7}{8}$ in. nearly.

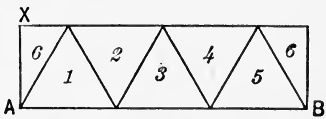


Fig. 43.

9. The base of each triangle measures about $\frac{3}{4}$ in. so that the base of the rectangle (the half perimeter) will be nearly 3 inches and the apothem about $\frac{15}{16}$ in. Area about $3 \times \frac{15}{16}$ sq. in. $=$ about $2\frac{13}{16}$ sq. in.

10. $AB \times AX = 3 \times \frac{7}{8} = 2\frac{5}{8}$. *Ans.* $2\frac{5}{8}$ sq. in.

11. See **9.**

12. The perimeter of a 16-sided polygon will be greater than that of an octagon. The 16-sided polygon will have the greater apothem.

13. As the number of sides increases, the perimeter approaches more and more closely the circumference, 6.2832 inches; and the apothem approaches the radius, 1 inch.

14. The base of the rectangle will be 3.1416 inches, one-half the perimeter (circumference); the apothem will be 1 inch (radius). Area $= 3.1416$ sq. in. *Ans.*

15. One-half the circumference $= 2 \times 3.1416 = 6.2832$, multiplied by the radius, 2, gives answer in square inches, 12.5664 sq. in.

16. 78.54 sq. in. *Ans.*

17. 3.1416 sq. in. *Ans.*

18. 314.16 sq. in. $\div 6 =$ *Ans.*

19. Subtract from the area of the outer circle, 113.0976 sq. in., the area of the inner circle, 28.2744 sq. in. *Ans.* 84.8232 sq. in.

1282. *Right* prisms, cylinders, etc., are meant when the word *oblique* is not used.

1. See Arithmetic, Art. 818, Problem **20.** The upper squares need not be drawn, as only the convex surface is required.

2. Three rectangles and two triangles. See **1.**

3. Three rectangles, each 3 inches high, bases 1, $1\frac{1}{2}$, and 2 inches, respectively.

5. A hollow paper cylinder, without bases, can be opened out into a rectangle whose base is the circumference of the base of the cylinder, and whose height is the altitude of the cylinder.

6. The slant height of a square pyramid is the distance from the apex to the center of one side of the base. This problem requires the pupil to draw, side by side, four isosceles triangles, the base of each being 2 inches and the altitude 3 inches. After constructing the first (Fig. 44), by erecting a 3-inch perpendicular at the center of a 2-inch base, he should use B'' and A as centers, and radii equal to $B''B'$ and AB'' to locate B'''. On AB' construct a third triangle, using A and B' as centers and the radii previously given. Upon one side of this triangle construct a fourth.

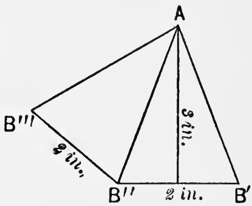


FIG. 44.

The pupils can discover for themselves the method of drawing geometrically the required convex surface.

The altitude should be carefully measured. It will be equal in length to the perpendicular of a right-angled triangle whose hypotenuse is the slant height of a pyramid, 3 inches; and whose base is the distance from the center of one edge of the base of the pyramid (the foot of the slant height) to the foot of the altitude, 1 inch. *Ans.* $\sqrt{8}$ inches $= 2.83$ inches nearly $=$ about $2\frac{13}{16}$ inches.

7. The area of each convex face of a regular pyramid is found by multiplying one side of the base by one-half the slant height; therefore, the area of all the faces forming the convex surface, is obtained by multiplying the sum of all the sides of the base, that is, the perimeter of the base, by one-half the slant height.

8. The pupil requires the slant height in order to proceed as in **6**; and he should obtain it by drawing it rather than by calculating it. He should be able to see that the altitude is a line drawn from the vertex to the center of the base, and that its foot is 1 inch distant from the foot of the altitude. Constructing a right-angled triangle whose base is 1 inch, and whose perpendicular is 3 inches, will give a hypotenuse equal to the required slant height.

Some scholars will bring in a prism whose slant height is 3 inches. The mistake should be pointed out, but not the mode of correcting it; and a pyramid of the required dimensions should be insisted upon.

1283. When the diameter of the base of a cone is 2 inches, the arc *BDC*, which forms the circumference when folded, measures 2π inches, or 6.2832 inches.

NOTE. — The Greek letter π (pi) represents 3.1416.

9. The semi-circumference of paper $= 3\pi$ inches, which is the circumference of base of cone. The diameter of base of cone $= 3\pi$ inches $\div \pi = 3$ inches. The radius of base $= 1\frac{1}{2}$ inches.

The slant height = radius of paper = 3 inches = diameter of base of cone = twice radius of base of cone.

10. The area of any sector of a circle = radius $\times \frac{1}{2}$ length of arc. The arc when folded becomes the circumference of base, and the radius becomes the slant height; so that convex surface $= \frac{1}{2}$ circumference $\times$ slant height = circumference $\times \frac{1}{2}$ slant height.

11. Length of arc of $90° = \frac{1}{4}$ circumference $= \frac{1}{4}$ of 6π inches $= 1\frac{1}{2}\pi$ inches. This is the circumference of the base of the cone, which makes the diameter $= 1\frac{1}{2}\pi$ inches $\div \pi = 1\frac{1}{2}$ inches. The slant height is 3 inches.

Length of arc of $60° = \frac{1}{6}$ circumference $= \frac{1}{6}$ of 6π inches $= \pi$ inches. The diameter of the base of the cone $= \pi$ inches $\div \pi$ $= 1$ inch; slant height, 3 inches.

12. The circumference of the base of the cone $= 3\pi$. This equals the length of the arc of the required sector. If the slant height is 5 inches, the circumference of which the sector is a part $= 10\pi$. The arc of the sector is, therefore, $\frac{3}{10}$ of the circumference; and its length is $\frac{3}{10}$ of $360° = 108°$.

13. XY represents the base of the pyramid; and AC, its altitude. The slant height of two faces, AM (Fig. 45), is the hypotenuse of a right-angled triangle, base $1\frac{1}{2}$ in., perpendicular 4 in. Using this as the perpendicular of a new triangle, with a base MY, gives as the hypotenuse AY (Figs. 45 and 46), one of the edges.

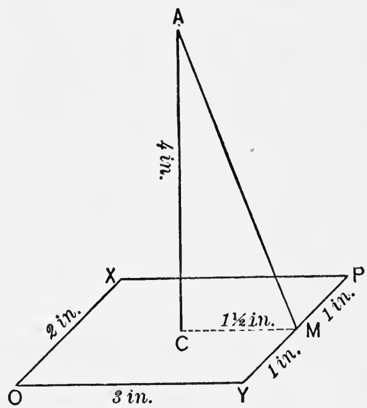


FIG. 45.

To draw the development, take AY as a radius, and draw an arc. On this lay off successive chords of 3 in., 2 in., 3 in., and 2 in., connecting the extremity of each with the center A. These four triangles

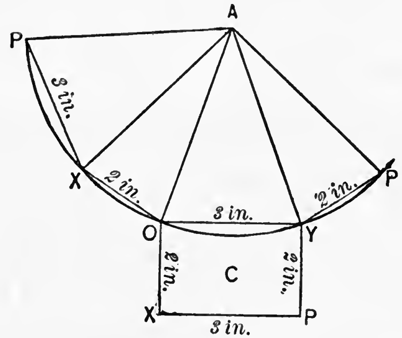


FIG. 46.

constitute the convex faces of the pyramid. On one of them, construct a rectangle 3 inches by 2 inches for the base.

14. In this pyramid, AC (Fig. 45) measures 12 inches, CM measures $\frac{1}{2}$ of 18 inches, or 9 inches, making AM, one slant height, $=\sqrt{144+81}$ inches $=15$ inches. The other slant height, drawn to the center of OY, $=\sqrt{144+25}$ inches $=13$ inches.

The pupils should be encouraged to construct stout paper pyramids and cones of required dimensions, making the necessary calculations themselves. The previous fourteen problems will present no difficulty whatever to pupils that are interested. As examples in dry calculation, they may prove somewhat tiresome. Many scholars will, of themselves, construct models of solids much more complicated than the foregoing.

1284. If there are no solids at hand, the pupils should construct a supply of paper ones, or make them of modeling clay, turnips, etc. Drawings are not sufficient for effective instruction.

15. The short method of ascertaining the convex surface should not be given until **18**. Each of the convex faces is a trapezoid, whose parallel sides measure 4 inches and 8 inches, respectively, the altitude being 10 inches.

16. Two inches apart, draw two parallel lines, *AB* and *CD* (Fig. 47), measuring 1 inch and 2 inches, respectively, a 2-inch perpendicular, *XY*, connecting the middle point of each. Draw *AC* and *BD*, and produce them until they meet in *M*. With this as a center, and *MC* as a radius, draw an arc; on which three other chords equal to *CD* are laid off, as in Fig. 46. With *M* as a center and a radius *MA*, lay off another arc, on which chords are laid off equal to *AB*; etc. See Arithmetic, Art. 1296.

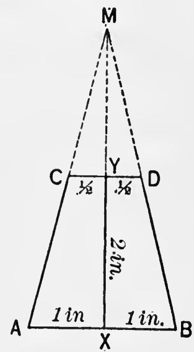


Fig. 47.

17. The entire surface = convex surface + 4 sq. ft. + 9 sq. ft.

18. The pupils can readily understand this rule, using the frustums of problems **15** and **17** as illustrations.

19. The pupil should first locate the apex of the cone. This he can do by following the method shown in **16**; making *AB* (Fig. 47) $2\frac{1}{2}$ inches; *CD*, $1\frac{1}{2}$ inches; and *AC*, 2 inches. This makes *MA* 5 inches, the slant height of the whole cone. The circumference of the base of the cone $= 2\frac{1}{2}\pi$. The slant height, 5 inches, is the radius of the required sector, its circumference being 10π. $2\frac{1}{2}\pi$, the length of the arc of the sector, being one-fourth of 10π, shows that the required sector is a quadrant.

20. The number of square inches in the convex surface = [circumference (perimeter) of upper base (9×3.1416) + circumference of lower base $(6 \times 3.1416)] \times \frac{1}{2}$ slant height (2). Adding to this, the area of the bottom (3^2) 9 sq. in. $\times$ 3.1416 (Art. 1124, **2**) gives the number of square inches of material required.

22. Circumference of upper base $= 6 \times 3.1416$
Circumference of lower base $= 10 \times 3.1416$

One-half sum $= 8 \times 3.1416$

Multiplying by slant height gives 48×3.1416
Add to this the area of the upper base, 9×3.1416
And the area of the lower base, 25×3.1416

Total in square yards, 82×3.1416

or $([(3+5) \times 6] + 9 + 25) \times 3.1416$.

23.

$$EA : EC :: 6 : 8$$
$$x : x + 9 :: 6 : 8$$
$$8x = 6x + 54$$
$$2x = 54$$
$$x = 27.$$

Ans. 27 ft.

The slant height of the whole cone $= 27$ ft. $+ 9$ ft. $= 36$ ft.

24. The convex surface of the whole cone $= \frac{1}{2}\,(8 \times 3.1416 \times 36)$ sq. ft.; of the part cut off $= \frac{1}{2}\,(6 \times 3.1416 \times 27)$ sq. ft.

1287. A sphere (a croquet ball, for instance) and a hemisphere should be used to illustrate these problems. On the plane face of the latter can be drawn the lines *AD*, *FG*, *HI*, *CI*, etc.; while on the curved face can be drawn *HYI*, *FXG*, etc.

25. $\frac{1}{6}$ of 25,000 miles.

26. $IH =$ chord of $60°$ of the great circle $=$ radius of the great circle $= 4000$ miles. $IB = \frac{1}{2}$ of $IH = 2000$ miles.

27. The diameter, *HI*, of the small circle is $\frac{1}{2}$ diameter *FG* of the great circle; the circumference $HYI = \frac{1}{2}$ of 25,000 miles, or 12,500 miles.

28. The length of a degree of longitude on the 60th parallel is about one-half of the length of a degree on the equator. (See Art. 995, Problem **10.**)

29. On the plane face of the hemisphere suggested above (Art. 1287), draw diameters AD and FG at right angles; and 45° from G, a chord NM parallel to FG. (This chord will not *bisect* AC.) As $MCG = 45°$, $WCM = 45°$; and the triangle WCM is a right-angled isosceles triangle, and $\overline{WM}^2 = \frac{1}{2}\overline{CM}^2$; $WM = .7071 CM$. (See Art. 995, Problem 12.) If CG measures 4000 miles, $CM = 4000$ miles, and
$WM = \sqrt{8,000,000} = 2828.4$ miles.

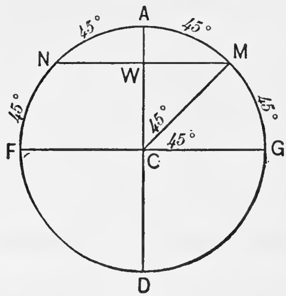


FIG. 48.

1289. The pupils have already learned that the volume of a rectangular prism is equal to the area of the base multiplied by its altitude; these problems are intended to show that the same is true of all prisms and of the cylinder (6).

1292. 8. The volume of the frustum is obtained by deducting the volume of the part cut off from the volume of the whole pyramid (Problem 7). The rule is given later.

10. Fig. 49 gives the method of calculating the slant height. The illustration shows a section of the frustum formed by passing a plane through the center of the frustum perpendicular to the base. The center of the upper base of the frustum of a right pyramid is directly above the center of the lower base, so that a perpendicular let fall from A will fall on CD at a point X, 5 in. from C. In the right-angled triangle AXC, AX = altitude = 12 in. $\overline{AC}^2 = \overline{AX}^2 + \overline{XC}^2 = 144 + 25 = 169$; $AC = \sqrt{169} = 13$.

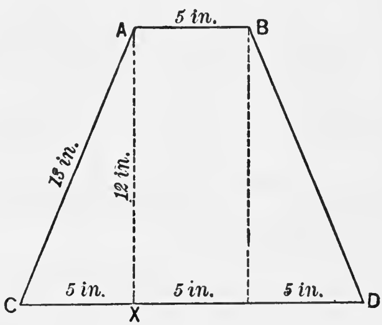


FIG. 49.

XX

NOTES ON THE APPENDIX

1306. A person that contracts to receive a rate of interest greater than is permitted by law, is liable to a penalty, except in Connecticut. In Delaware, Minnesota, etc., the penalty is the forfeiture of the contract; in New York, the forfeiture of the contract, $1000 fine, and 6 months' imprisonment; in Indiana, Kansas, Kentucky, Maryland, Michigan, Mississippi, Ohio, Pennsylvania, Tennessee, Vermont, Virginia, and West Virginia, the forfeiture of the excess of interest; in North Carolina, the forfeiture of double the amount of interest; in Georgia and New Hampshire, the forfeiture of three times the excess of interest; in Alabama, Delaware, Florida, Illinois, Iowa, Louisiana, Missouri, Nebraska, New Jersey, South Carolina, Texas, and Wisconsin, the forfeiture of all interest; in Arkansas and Oregon, the forfeiture of principal and interest.

1307. 3.

Amount of $1000, June 1, 1896, to June 1, 1897	$1,060.00
Amount of $150, Sept. 16, 1896, to June 1, 1897, 8½ mo.	156.37
Due June 1, 1897	$903.63
Interest to settlement, Apr. 16, 1898, 10½ mo.	47.44
Amount Apr. 16, 1898	$951.07
Amount of $50, Sept. 16, 1897, to Apr. 16, 1898, 7 mo.	51.75
Balance due	$899.32

4.

Amount of $500, July 25, 1896, to Apr. 1, 1897, 8 mo. 6 da.		$520.50
Amount of $100, Sept. 18, 1896, to Apr. 1, 1897, 6 mo. 13 da.	$103.22	
Amount of $200, Feb. 5, 1897, to Apr. 1, 1897, 1 mo. 26 da.	201.86	305.08
Balance due		$215.42

5. Amount of $870.50, Jan. 2, 1894, to March 18, 1896, 2 yr. 2 mo. 16 da.			$985.99
Less payments of $35 and $200			235.
New principal			$750.99
Interest March 18, 1896, to Jan. 2, 1897, 9 mo. 14 da.			35.54
Amount			$786.53
Amount of $250, Aug. 24, 1896, to Jan. 2, 1897, 4 mo. 8 da.			255.33
Due at maturity			$531.20

Below will be found answers to the partial payments examples of Chapters XIII and XIV, according to the Connecticut rule.

NOTE. — Although the legal rate in Connecticut is 6%, the rates given in the examples should be used.

Art. 1008. $70.51. Art. 1009. **1.** $224.22. **2.** $261.21. **3.** $1278.15. Art. 1011. **4.** $771.24. **5.** $899.32 (see Art. 1307, **3**). Art. 1013. **5.** $1088.31. Art. 1015. **7.** $700.50. Art. 1023. **7.** $1232.26. Art. 1051. **7.** $77.07. Art. 1090. **3.** $649.13. **4.** $224.64. Art. 1107. **2.** $1089.64.

The time in each of the preceding examples was found by compound subtraction, taking 30 days to each month. In the examples in Art. 1110, the Connecticut rule has been followed. In these, the time is taken in days. (See Art. 1111.)

1308. 7. Principal		$700.00
Interest to June 15, 1897, 2 yr.		84.00
1 year's interest on $42, unpaid interest		2.52
Amount June 15, 1897		$786.52
Amount of $20, Nov. 15, 1895, to June 15, 1897	$21.90	
Amount of $80, Feb. 15, 1897, to June 15, 1897	81.60	103.50
New principal June 15, 1897		$683.02
Interest to Oct. 15, 1899, 2 yr. 4 mo.		95.62
Interest for 1 yr. 4 mo. on $40.98, unpaid interest		3.28
Interest for 4 mo. on $40.98, unpaid interest		.82
Amount Oct. 15, 1899		$782.74
Amount of $15, Sept. 15, 1898, to Oct. 15, 1899		15.98
Due Oct. 15, 1899		$766.76

NOTE. — If four places of decimals are used, the answer to **6** is $367.6442, or $367.64 +; to **7**, $766.7658, or $766.77 −.

The teacher that wishes other examples of this kind can use **9** and **10** of Art. 1309, and **12**, **14**, and **15** of Art. 1310, the answers to which are as follows: Art. 1309. **9.** $1734. **10.** $738.29. Art. 1310. **12.** $1901.18. **14.** $3432.20. **15.** $1010.95.

1309. Pupils in New Hampshire should not be taught the preceding method, but should be confined to the rule laid down by the courts of their own state.

An examination of **8** will show the manner of ascertaining the balance. The interest for 1 year is $36. As no interest is due except that which accrued during the year, and as the sum paid is less than the interest due, no interest is allowed on the payment of $30, made during the year. This payment being $6 less than the interest due at the end of the year, the interest on $6 is added to the interest on the principal at the end of the next year, making a total of $642.36 then due less the *amount* of $100 for 11 months. Interest is allowed on the $100 payment because it is in excess of the interest then due.

9. Principal		$2500.00
Annual interest due May 1, 1897	$150.00	
Payment (no interest) Oct. 1, 1896	100.00	
Balance of interest	$ 50.00	
Annual interest due May 1, 1898	150.00	
Interest on balance of interest, 1 yr.	3.00	203.00
Amount May 1, 1898		$2703.00
Amount of $1000, June 1, 1897, to May 1, 1898		1055.00
New principal May 1, 1898		$1648.00
Interest on $1648 to May 1, 1899	$98.88	
Payment (no interest) Nov. 1, 1898	50.00	
Balance of interest	$48.88	
Interest on $1648 May 1 to Oct. 1	41.20	
Interest on balance of interest, 5 mo.	1.22	91.30
Due Oct. 1, 1899		$1739.30

10. Principal		$1000.00
Annual interest due Jan. 3, 1896	$60.00	
Payment (no interest) June 1, 1895	10.00	
Balance of interest	$50.00	

Amounts brought forward		$ 50.00	$ 1000.00
Annual interest due Jan. 3, 1897		60.00	
Interest on balance of interest, 1 yr.		3.00	
Interest due Jan. 3, 1897		$ 113.00	
Amount of payment March 14, 1896, 9 mo. 19 da.		10.48	
Balance of interest		$ 102.52	
Annual interest due Jan. 3, 1898		60.00	
Interest on balance of interest, 1 yr.		6.15	
Annual interest due Jan. 3, 1899		60.00	
Interest for 1 year on $ 102.52 and $ 60		9.75	238.42
Amount Jan. 3, 1899			$ 1238.42
Amount of $ 500, Sept. 30, 1898, to Jan. 3, 1899			507.75
New principal Jan. 3, 1899			$ 730.67
Interest on $ 730.67 to March 11, 2 mo. 8 da.			8.28
Due March 11, 1899			$ 738.95

NOTE. — The *amount* of the payment of March 14 canceled the $ 3 interest on interest and $ 7.48 additional, leaving $ 102.52 interest still unpaid Jan. 3, 1897 on which two years' interest is taken to Jan. 3, 1899. Two years' interest on the principal is also taken, and one year's interest on the annual interest due Jan. 3, 1898 and unpaid. It will be noticed that no interest is taken on the interest upon interest, $ 3 and $ 6.15. See **14** of Art. 1310, as calculated by the N.H. rule: —

Principal			$ 3000.00
Interest to March 17, 1900, 4 yr.		$ 720.00	
Interest on $ 180 (3 + 2 + 1) yr.	$ 64.80		
Amount of $ 20, 10 mo.	21.00		
Unpaid interest on interest	$ 43.80		
Interest to March 17, 1903, 3 yr.		540.00	
Interest on $ 180 (3 + 3 + 3 + 3 + 2 + 1) yr.	162.00	205.80	
Total interest due March 17, 1903		$ 1465.80	
Amount of $ 1000, 6 mo.		1030.00	435.80
Due March 17, 1903			$ 3435.80

NOTE. — Interest is not taken on $ 43.80.

As additional examples, the teacher may use **7** of Art. 1308, and **12**, **14**, and **15** of Art. 1310. The answers by the N.H. method are **7**, $ 767.60; **12**, $ 1901.27; **14**, $ 3435.80; **15**, $ 1011.40.

1310. Vermont pupils should omit Arts. 1307, 1308, and 1309.

12. Principal		$ 2500.00
Annual interest due June 1, 1897		309.00
Amount June 1, 1897		$ 2809.00
Amount of $ 100 to June 1, 1897	$ 104.00	
Payment June 1, 1897	1000.00	1104.00
New principal		$ 1705.00
Annual interest to June 1, 1899	$ 210.74	
Amount of $ 50, 7 mo.	51.75	
Unpaid interest	$ 158.99	
Interest on $ 1705 to settlement	34.10	
Interest on $ 158.99 to settlement	3.18	196.27
Due Oct. 1, 1899		$ 1901.27

13. Principal Jan. 3, 1895		$ 1000.00
Interest due Jan. 3, 1896	$ 60.00	
Amount of $ 10, 7 mo. 2 da.	10.35	
Balance of interest	$ 49.65	
Interest due Jan. 3, 1897	60.00	
1 year's interest on $ 49.65	2.98	
Total interest	$ 112.63	
Amount of $ 10, 9 mo. 19 da.	10.48	
Balance of interest	$ 102.15	
Interest on $ 1000 for 2 years	120.00	
2 years' interest on $ 102.15	12.26	
1 year's interest on $ 60	3.60	238.01
Amount Jan. 3, 1899		$ 1238.01
Amount of $ 500, Sept. 30, 1898, to Jan 3, 1899		507.75
New principal Jan. 3, 1899		$ 730.26
Interest to March 11, 1899		8.28
Due March 11, 1899		$ 738.54

14. Principal March 17, 1896			$ 3000.00
Interest to March 17, 1900, 4 yr.		$ 720.00	
Interest on yearly interest (3 + 2 + 1) yr.	$ 64.80		
Amount of $ 20, 10 mo.	21.00		
Unpaid interest on yearly interest	$ 43.80		

Amounts brought forward	$43.80	$720.00	$3000.00
3 years' interest to March 17, 1903		540.00	
Interest on yearly interest, 3 yr. on $720			
(2 + 1) yr. on $180	162.00	205.80	
Total interest due March 17, 1903		$1465.80	
Amount of $1000 to March 17, 1903		1030.00	435.80
Due March 17, 1903			$3435.80

15. Principal Feb. 25, 1893		$1200.00
Annual interest to Feb. 25, 1897, 4 yr.		313.92
Amount Feb. 25, 1897		$1513.92
Amount of $400, 11 mo. 29 da.		423.93
New principal Feb. 25, 1897		$1089.99
Annual interest on $1089.99, 2 yr.	$134.72	
Amount of $10, 8 mo. 14 da.	10.42	
Unpaid interest Feb. 25, 1899	$124.30	
Annual interest on $1089.99, 2 yr.	134.72	
Interest for 2 yr. on $124.30	14.91	273.93
Amount Feb. 25, 1901		$1363.92
Amount of $400, 4 mo. 29 da.		409.93
New principal Feb. 25, 1901		$953.99
Annual interest on $953.99, 1 yr. 1 da.		57.41
		$1011.40

As additional examples, **7** and **9** may be used. The answer to **7** by the Vermont rule is $766.75; to **9**, $1733.73.

1311. 1. $[(\$8500 + \$600) \times .0155] + \$2 = \$141.05$. *Ans.*

2. The amount to be raised on property $= \$2500 - (\$2 \times 150) = \$2200$. Rate on $\$1 = \$2200 \div 275000 = \$.008$, or 8 mills. 8 mills on $1, or $\frac{4}{5}\%$. *Ans.*

4. Mr. Simmons' grand list $= \$95 + \$2 = \$97$. Tax $= \$2.45 \times 97 = \$237.65$. *Ans.*

5. $1500 + 2x =$ grand list. The grand list multiplied by the rate gives the amount to be raised; $(1500 + 2x) \times 2 = 3600$; $3000 + 4x = 3600$; $4x = 600$; $x = 150$. 150 taxable polls. *Ans.*

6. Mr. Hallock's grand list $= \$120 + \$6 = \$126$. Since his taxes are \$252, the rate $= \$252 \div 126 = \$2$.

Let $x =$ appraised value of property.

$$\frac{x}{100} + 400 = \text{grand list.}$$

$$2\left(\frac{x}{100} + 400\right) = \text{total levy} = 6800.$$

$$\frac{2x}{100} + 800 = 6800.$$

$$2x + 80000 = 680000.$$

$$x = 300000.$$

Appraised value of property is \$300000. *Ans.*

SUPPLEMENT

DEFINITIONS, PRINCIPLES, AND RULES

A **Unit** is a single thing.

A **Number** is a unit or a collection of units.

The **Unit of a Number** is one of that number.

Like Numbers are those that express units of the same kind.

Unlike Numbers are those that express units of different kinds.

A **Concrete Number** is one in which the unit is named.

An **Abstract Number** is one in which the unit is not named.

Notation is expressing numbers by characters.

Arabic Notation is expressing numbers by figures.

Roman Notation is expressing numbers by letters.

Numeration is reading numbers expressed by characters.

The **Place of a Figure** is its position in a number.

A figure standing alone, or in the first place at the right of other figures, expresses *ones*, or *units of the first order*.

A figure in the second place expresses *tens*, or *units of the second order*.

A figure in the third place expresses *hundreds*, or *units of the third order;* and so on.

A **Period** is a group of three orders of units, counting from right to left.

RULE FOR NOTATION. — *Begin at the left, and write the hundreds, tens, and units of each period in succession, filling vacant places and periods with ciphers.*

Rule for Numeration. — *Beginning at the right, separate the number into periods.*

Beginning at the left, read the numbers in each period, giving the name of each period except the last.

ADDITION

Addition is finding a number equal to two or more given numbers.

Addends are the numbers added.

The **Sum,** or **Amount,** is the number obtained by addition.

Principle. — *Only like numbers, and units of the same order can be added.*

Rule. — *Write the numbers so that units of the same order shall be in the same column.*

Beginning at the right, add each column separately, and write the sum, if less than ten, under the column added.

When the sum of any column exceeds nine, write the units only, and add the ten or tens to the next column.

Write the entire sum of the last column.

SUBTRACTION

Subtraction is finding the difference between two numbers.

The **Subtrahend** is the number subtracted.

The **Minuend** is the number from which the subtrahend is taken.

The **Remainder,** or **Difference,** is the number left after subtracting one number from another.

Principles. — *Only like numbers and units of the same order can be subtracted.*

The sum of the difference and the subtrahend must equal the minuend.

Rules. — I. *Write the subtrahend under the minuend, placing units of the same order in the same column.*

Beginning at the right, find the number that must be added to the first figure of the subtrahend to produce the figure in the corresponding order of the minuend, and write it below. Proceed in this way until the difference is found.

If any figure in the subtrahend is greater than the corresponding figure in the minuend, find the number that must be added to the former to produce the latter increased by ten; then add one to the next order of the subtrahend and proceed as before.

II. *Beginning at the units' column, subtract each figure of the subtrahend from the corresponding figure of the minuend and write the remainder below.*

If any figure of the subtrahend is greater than the corresponding figure in the minuend, add ten to the latter and subtract; then, (a) *add one to the next order of the subtrahend and proceed as before; or,* (b) *subtract one from the next order of the minuend and proceed as before.*

MULTIPLICATION

Multiplication is taking one number as many times as there are units in another number.

The **Multiplicand** is the number taken or multiplied.

The **Multiplier** is the number that shows how many times the multiplicand is taken.

The **Product** is the result obtained by multiplication.

Principles. — *The multiplier must be an abstract number.*

The multiplicand and the product are like numbers.

The product is the same in whatever order the numbers are multiplied.

Rule. — *Write the multiplier under the multiplicand, placing units of the same order in the same column.*

Beginning at the right, multiply the multiplicand by the number of units in each order of the multiplier in succession. Write the *figure of the lowest order in each partial product under the figure of the multiplier that produces it. Add the partial products.*

To multiply by 10, 100, 1000, etc.

RULE. — *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier.*

DIVISION

Division is finding how many times one number is contained in another, or finding one of the equal parts of a number.

The **Dividend** is the number divided.

The **Divisor** is the number contained in the dividend.

The **Quotient** is the result obtained by division.

PRINCIPLES. — *When the divisor and the dividend are like numbers, the quotient is an abstract number.*

When the divisor is an abstract number, the dividend and the quotient are like numbers.

The product of the divisor and the quotient, plus the remainder, if any, is equal to the dividend.

RULE. — *Write the divisor at the left of the dividend with a line between them.*

Find how many times the divisor is contained in the fewest figures on the left of the dividend, and write the result over the last figure of the partial dividend. Multiply the divisor by this quotient figure, and write the product under the figures divided. Subtract the product from the partial dividend used, and to the remainder annex the next figure of the dividend for a new dividend.

Divide as before until all the figures of the dividend have been used.

If any partial dividend will not contain the divisor, write a cipher in the quotient, and annex the next figure of the dividend.

If there is a remainder after the last division, write it after the quotient with the divisor underneath.

FACTORING

An **Exact Divisor** of a number is a number that will divide it without a remainder.

An **Odd Number** is one that cannot be exactly divided by two.

An **Even Number** is one that can be exactly divided by two.

The **Factors** of a number are the numbers that multiplied together produce that number.

A **Prime Number** is a number that has no factors.

A **Composite Number** is a number that has factors.

A **Prime Factor** is a prime number used as a factor.

A **Composite Factor** is a composite number used as a factor.

Factoring is separating a number into its factors.

To find the Prime Factors of a Number.

RULE. — *Divide the number by any prime factor. Divide the quotient, if composite, in like manner; and so continue until a prime quotient is found. The several divisors and the last quotient will be the prime factors.*

CANCELLATION

Cancellation is rejecting equal factors from dividend and divisor.

PRINCIPLE. — *Dividing dividend and divisor by the same number does not affect the quotient.*

GREATEST COMMON DIVISOR

A **Common Factor** (divisor or measure) is a number that is a factor of each of two or more numbers.

A **Common Prime Factor** is a prime number that is a factor of each of two or more numbers.

The **Greatest Common Factor** (divisor or measure) is the largest number that is a factor of each of two or more numbers.

Numbers are prime to each other when they have no common factor.

The greatest common divisor of two or more numbers is the product of their common prime factors.

Principles. — *A common divisor of two numbers is a divisor of their sum, and also of their difference.*

A divisor of a number is a divisor of every multiple of that number; and a common divisor of two or more numbers is a divisor of any of their multiples.

To find the Common Prime Factors of Two or More Numbers.

Rule. — *Divide the numbers by any common prime factors, and the quotients in like manner, until they have no common factor; the several divisors are the common prime factors.*

To find the Greatest Common Divisor of Numbers that are Easily Factored.

Rule. — *Separate the numbers into their prime factors; the product of those that are common is the greatest common divisor.*

To find the Greatest Common Divisor of Numbers that are not Easily Factored.

Rule. — *Divide the greater number by the less; then divide the last divisor by the last remainder, continuing until there is no remainder. The last divisor is the greatest common divisor.*

If there are more than two numbers, find the greatest common divisor of two of them; then of that divisor and another of the numbers until all of the numbers have been used. The last divisor is the greatest common divisor.

LEAST COMMON MULTIPLE

A **Multiple** of a number is a number that exactly contains that number.

A **Common Multiple** of two or more numbers is a number that is a multiple of each of them.

The **Least Common Multiple** of two or more numbers is the smallest number that is a common multiple of them.

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PRINCIPLES. — *A multiple of a number contains all the prime factors of that number.*

A common multiple of two or more numbers contains each of the prime factors of those numbers.

The Least Common Multiple of two or more numbers contains only the prime factors of each of the numbers.

To find the Least Common Multiple of Two or More Numbers.

RULE. — *Divide by any prime number that is an exact divisor of two or more of the numbers, and write the quotients and undivided numbers below. Divide these numbers in like manner, continuing until no two of the remaining numbers have a common factor. The product of the divisors and remaining numbers is the least common multiple.*

FRACTIONS

A **Fraction** is one or more of the equal parts of anything.

The **Unit of a Fraction** is the number or thing that is divided into equal parts.

A **Fractional Unit** is one of the equal parts into which the number or thing is divided.

The **Terms of a Fraction** are its numerator and its denominator.

The **Denominator** of a fraction shows into how many parts the unit is divided.

The **Numerator** of a fraction shows how many of the parts are taken.

A fraction indicates division; the numerator being the dividend and the denominator the divisor.

The **Value of a Fraction** is the quotient of the numerator divided by the denominator.

Fractions are divided into two classes — **Common** and **Decimal.**

A **Common Fraction** is one in which the unit is divided into any number of equal parts.

A common fraction is expressed by writing the numerator above the denominator with a dividing line between.

Common fractions consist of three principal classes — **Simple, Compound,** and **Complex.**

A **Simple Fraction** is one whose terms are whole numbers.

A **Proper Fraction** is a simple fraction whose numerator is less than its denominator.

An **Improper Fraction** is a simple fraction whose numerator equals or exceeds its denominator.

A **Compound Fraction** is a fraction of a fraction.

A **Complex Fraction** is one having a fraction in its numerator, or in its denominator, or in both.

A **Mixed Number** is a whole number and a fraction written together.

The **Reciprocal of a Number** is one divided by that number.

The **Reciprocal of a Fraction** is one divided by the fraction, or the fraction inverted.

PRINCIPLES. — *Multiplying the numerator or dividing the denominator multiplies the fraction.*

Dividing the numerator or multiplying the denominator divides the fraction.

Multiplying or dividing both terms of a fraction by the same number does not alter the value of the fraction.

Reduction of fractions is changing their terms without altering their value.

To reduce a Fraction to Higher Terms.

RULE. — *Multiply both numerator and denominator by the same number.*

To reduce a Fraction to its Lowest Terms.

RULE. — *Divide both terms of the fraction by their greatest common divisor.*

A fraction is in its lowest terms when the numerator and the denominator are prime to each other.

To reduce a Mixed Number to an Improper Fraction.

RULE. — *Multiply the whole number by the denominator; to the product add the numerator; and place the sum over the denominator.*

To reduce an Improper Fraction to a Whole or to a Mixed Number.

RULE. — *Divide the numerator by the denominator.*

A **Common Denominator** is a denominator common to two or more fractions.

The **Least Common Denominator** is the smallest denominator common to two or more fractions.

To reduce Fractions to their Least Common Denominator.

RULE. — *Find the least common multiple of all the denominators for the least common denominator. Divide this multiple by the denominator of each fraction, and multiply the numerator by the quotient.*

ADDITION OF FRACTIONS

PRINCIPLE. — *Only like fractions can be added.*

RULE. — *Reduce the fractions, if necessary, to a common denominator, and over it write the sum of the numerators.*

If there are mixed numbers, add the fractions and the whole numbers separately, and unite the results.

SUBTRACTION OF FRACTIONS

PRINCIPLE. — *Only like fractions can be subtracted.*

RULE. — *Reduce the fractions, if necessary, to a common denominator, and over it write the difference between the numerators.*

If there are mixed numbers subtract the fractions and the whole numbers separately, and unite the results.

MULTIPLICATION OF FRACTIONS

RULE. — *Reduce whole and mixed numbers to improper fractions; cancel the factors common to numerators and denominators, and write the product of the remaining factors in the numerators over the product of the remaining factors in the denominators.*

DIVISION OF FRACTIONS

RULES. — I. *Reduce whole and mixed numbers to improper fractions. Reduce the fractions to a common denominator. Divide the numerator of the dividend by the numerator of the divisor.*

II. *Invert the divisor and proceed as in multiplication of fractions.*

To reduce a Complex Fraction to a Simple One.

RULES. — I. *Multiply the numerator of the complex fraction by its denominator inverted.*

II. *Multiply both terms by the least common multiple of the denominators.*

DECIMALS

A **Decimal Fraction** is one in which the unit is divided into tenths, hundredths, thousandths, etc.

A **Decimal** is a decimal fraction whose denomination is indicated by the number of places at the right of the decimal point.

The **Decimal Point** is the mark used to locate units.

A **Mixed Decimal** is a whole number and a decimal written together.

A **Complex Decimal** is a decimal with a common fraction written at its right.

To write Decimals.

RULE. — *Write the numerator; and from the right, point off as many decimal places as there are ciphers in the denominator, prefixing ciphers, if necessary, to make the required number.*

To read Decimals.

RULE. — *Read the numerator, and give the name of the right-hand order.*

PRINCIPLES. — *Prefixing ciphers to a decimal diminishes its value.*

Removing ciphers from the left of a decimal increases its value.

Annexing ciphers to a decimal or removing ciphers from its right does not alter its value.

To reduce a Decimal to a Common Fraction.

RULE. — *Write the figures of the decimal for the numerator, and* 1, *with as many ciphers as there are places in the decimal, for the denominator, and reduce the fraction to its lowest terms.*

To reduce a Common Fraction to a Decimal.

RULE. — *Annex decimal ciphers to the numerator, and divide it by the denominator.*

To reduce Decimals to a Common Denominator.

RULE. — *Make their decimal places equal by annexing ciphers.*

ADDITION AND SUBTRACTION OF DECIMALS

Decimals are added and subtracted the same as whole numbers.

MULTIPLICATION OF DECIMALS

RULE. — *Multiply as in whole numbers, and from the right of the product, point off as many decimal places as there are decimal places in both factors.*

DIVISION OF DECIMALS

RULE. — *Make the divisor a whole number by removing the decimal point, and make a corresponding change in the dividend. Divide as in whole numbers, and place the decimal point in the quotient under (or over) the new decimal point in the dividend.*

ACCOUNTS AND BILLS

A **Debtor** is a person who owes another.

A **Creditor** is a person to whom a debt is due.

An **Account** is a record of debits and credits between persons doing business.

The **Balance** of an account is the difference between the debit and credit sides.

A **Bill** is a written statement of an account.

An **Invoice** is a written statement of items, sent with merchandise.

A **Receipt** is a written acknowledgment of the payment of part or all of a debt.

A bill is receipted when the words, "Received Payment," are written at the bottom, signed by the creditor, or by some person duly authorized.

DENOMINATE NUMBERS

A **Measure** is a standard established by law or custom, by which distance, capacity, surface, time, or weight is determined.

A **Denominate Unit** is a unit of measure.

A **Denominate Number** is a denominate unit or a collection of denominate units.

A **Simple Denominate Number** consists of denominate units of one kind.

A **Compound Denominate Number** consists of denominate units of two or more kinds.

A **Denominate Fraction** is a fraction of a denominate number.

A denominate fraction may be either **common** or **decimal.**

Reduction of denominate numbers is changing them from one denomination to another without altering their value.

Reduction Descending is changing a denominate number to one of a lower denomination.

RULE.—*Multiply the highest denomination by the number required to reduce it to the next lower denomination, and to the product add the units of that lower denomination, if any. Proceed in this manner until the required denomination is reached.*

Reduction Ascending is changing a denominate number to one of a higher denomination.

RULE.—*Divide the given denomination successively by the numbers that will reduce it to the required denomination. To this quotient annex the several remainders.*

To find the Time between Dates.

RULE.— *When the time is less than one year, find the exact number of days; if greater than one year, find the time by compound subtraction, taking* 30 *days to the month.*

PERCENTAGE

Per Cent means hundredths.

Percentage is computing by hundredths.

The elements involved in percentage are the **Base, Rate, Percentage, Amount,** and **Difference.**

The **Base** is the number of which a number of hundredths is taken.

The **Rate** indicates the number of hundredths to be taken.

The **Percentage** is one or more hundredths of the base.

The **Amount** is the base increased by the percentage.

The **Difference** is the base diminished by the percentage.

To find the Percentage when the Base and Rate are Given.

RULE.— *Multiply the base by the rate expressed as hundredths.*

To find the Rate when the Percentage and Base are Given.

RULE.— *Divide the percentage by the base.*

To find the Base when the Percentage and Rate are Given.

RULE.—*Divide the percentage by the rate expressed as hundredths.*

To find the Base when the Amount and Rate are Given.

RULE.—*Divide the amount by* 1 + *the rate expressed as hundredths.*

To find the Base when the Difference and Rate are Given.

RULE. — *Divide the difference by* 1 — *the rate expressed as hundredths.*

PROFIT AND LOSS

Profit or **Loss** is the difference between the buying and selling prices.

In Profit and Loss,

The buying price, or cost, is the *base.*

The rate per cent profit or loss is the *rate.*

The profit or loss is the *percentage.*

The selling price is the *amount* or *difference*, according as it is more or less than the buying price.

COMMERCIAL DISCOUNT

Commercial Discount is a percentage deducted from the list price of goods, the face of a bill, etc.

The **Net Price** of goods is the sum received for them.

In Commercial Discount,

The list price, or
The face of the bill } is the *base.*

The rate per cent discount is the *rate.*

The discount is the *percentage.*

The list price diminished by the discount is the *difference.*

In successive discounts, the first discount is made from the list price or the face of the bill; the second discount, from the list price or face of the bill diminished by the first discount; and so on.

COMMISSION

Commission is a percentage allowed an agent for his services.

A **Commission Agent** is one who transacts business on commission.

A **Consignment** is the merchandise forwarded to a commission agent.

The **Consignor** is the person who sends the merchandise.

The **Consignee** is the person to whom the merchandise is sent.

The **Net Proceeds** is the sum remaining after all charges have been deducted.

In buying, the commission is a percentage of the *buying price;* in selling, a percentage of the *selling price;* in collecting, a percentage of the *sum collected;* hence:

The sum invested, or
The sum collected } is the *base.*

The rate per cent commission is the *rate.*

The commission is the *percentage.*

The sum invested increased by the commission is the *amount.*

The sum collected diminished by the commission is the *difference.*

INSURANCE

Insurance is a contract of indemnity.

Insurance is of three kinds — **Fire, Marine,** and **Life.**

Fire Insurance is indemnity against loss of property by fire.

Marine Insurance is indemnity against loss of property by the casualities of navigation.

Life Insurance is indemnity against loss of life.

The **Insurance Policy** is the contract setting forth the liability of the insurer.

The **Policy Face** is the amount of insurance.

The **Premium** is the price paid for insurance.

The **Insurer,** or **Underwriter,** is the company issuing the policy.

The **Insured** is the person for whose benefit the policy is issued.

In Insurance,

The policy face is the *base.*

The rate per cent premium is the *rate.*

The premium is the *percentage.*

TAXES

A **Tax** is a sum of money levied on persons or property for public purposes.

A **Personal,** or **Poll Tax,** is a tax on the person.

A **Property Tax** is a tax of a certain per cent on the assessed value of property.

Property may be either personal or real.

Personal Property consists of such things as are movable.

Real Property is that which is fixed, or immovable.

In Taxes,

The assessed value is the *base.*

The rate of taxation is the *rate.*

The tax is the *percentage.*

DUTIES

Duties are taxes on imported goods.

Duties are either **Specific** or **Ad Valorem.**

A **Specific Duty** is a tax on goods without regard to cost.

An **Ad Valorem** duty is a tax of a certain per cent on the cost of goods.

In Ad Valorem Duties,

The cost of the goods is the *base.*

The rate per cent duty is the *rate.*

The ad valorem duty is the *percentage.*

INTEREST

Interest is the sum paid for the use of money.

The **Principal** is the sum loaned.

The **Amount** is the sum of the principal and interest.

The **Rate of Interest** is the rate per cent for one year.

The **Legal Rate** is the rate fixed by law.

Usury is interest at a higher rate than that fixed by law.

Simple Interest is interest on the principal only.

To find the Interest when the Principal, Time, and Rate are Given.

RULE. — *Multiply the principal by the rate expressed as hundredths, and this product by the time expressed in years.*

To find the Time when the Principal, Interest, and Rate are Given.

RULE. — *Divide the given interest by the interest for one year.*

To find the Rate when the Principal, Interest, and Time are Given.

RULE. — *Divide the given interest by the interest at one per cent.*

To find the Principal when the Interest, Rate, and Time are Given.

RULE. — *Divide the given interest by the interest on* $1.

To find the Principal when the Amount and Time and Rate are Given.

RULE. — *Divide the given amount by the amount of* $1.

INTEREST BY ALIQUOT PARTS.

To find the Interest for Years, Months, and Days.

RULE. — *Find the interest for one year and take this as many times as there are years.*

Take the greatest number of the given months that equals an aliquot part of a year and find the interest for this time. Take aliquot parts of this for the remaining months.

In the same manner find the interest for the days.

The sum of these interests will be the interest required.

To find the Interest when the Time is Less than a Year.

RULE. — *Find the interest for the time in months or days that will gain one per cent of the principal.*

Find by aliquot parts, as in the first rule, the interest for the remaining time.

The sum of these interests will be the interest required.

Interest by Six Per Cent Method.

To find the Interest at 6%.

Rule.—**For Years:** *Multiply the principal by the rate expressed as hundredths, and that product by the number of years.*

For Months: *Move the decimal point two places to the left, and multiply by one-half the number of months.*

For Days: *Move the decimal point three places to the left, and multiply by one-sixth the number of days.*

To find the interest at any other rate per cent, divide the interest at 6% by 6, and multiply the quotient by the given rate.

To find Exact Interest.

Rule.—*Multiply the principal by the rate expressed as hundredths, and that product by the time expressed in years of* 365 *days.*

ANNUAL INTEREST

Annual Interest is interest payable annually. If not paid when due, annual interest draws simple interest.

To find the Amount Due on a Note with Annual Interest, when the Interest has not been Paid Annually.

Rule.—*Find the interest on the principal for the entire time, and on each annual interest for the time it remained unpaid. The sum of the principal and all the interest is the amount due.*

COMPOUND INTEREST

Compound Interest is interest on the principal and on the unpaid interest, which is added to the principal at regular intervals. The interest may be compounded annually, semi-annually, quarterly, etc., according to agreement.

To find Compound Interest.

Rule.—*Find the amount of the given principal for the first period. Considering this as a new principal, find the amount of* *it for the next period, continuing in this manner for the given time.*

Find the difference between the last amount and the given principal, which will be the compound interest.

PARTIAL PAYMENTS

Partial Payments are part payments of a note or debt. Each payment is recorded on the back of the note or the written obligation.

UNITED STATES RULE.—*Find the amount of the principal to the time when the payment or the sum of two or more payments equals or exceeds the interest.*

From this amount deduct the payment or sum of payments.

Use the balance then due as a new principal, and proceed as before.

MERCHANTS' RULE.—*Find the amount of an interest-bearing note at the time of settlement.*

Find the amount of each credit from its time of payment to the time of settlement; subtract their sum from the amount of the principal.

BANK DISCOUNT

Bank Discount is a percentage retained by a bank for advancing money on a note before it is due.

The **Sum Discounted** is the face of the note, or if interest-bearing, the amount of the note at maturity.

The **Term of Discount** is the number of days from the day of discount to the day of maturity.

The **Bank Discount** is the interest on the sum discounted for the term of discount.

The **Proceeds** of a note is the sum discounted less the bank discount.

Problems in bank discount are calculated as problems in interest.

In Bank Discount,

The sum discounted is the *principal.*

The rate of discount is the *rate of interest.*

The term of discount is the *time.*

The bank discount is the *proceeds.*

EXCHANGE

Exchange is making payments at a distance by means of drafts or bills of exchange.

Domestic Exchange is exchange between places in the same country.

Foreign Exchange is exchange between different countries.

Exchange is *at par* when a draft, or bill, sells for its face value; *at a premium* when it sells for more than its face value; *at a discount* when it sells for less.

The cost of a sight draft is the face of the draft increased by the premium, or diminished by the discount.

The cost of a time draft is the face of the draft increased by the premium, or diminished by the discount, and this result diminished by the bank discount.

To find the Cost of a Draft.

RULE.—*Find the cost of* \$1 *of the draft; multiply this by the face of the draft.*

To find the Face of a Draft.

RULE.—*Divide the cost of the draft by the cost of* \$1 *of the draft.*

EQUATION OF PAYMENTS

Equation of Payments is a method of ascertaining at what time several debts due at different times may be settled by a single payment.

The **Equated Time** of payment is the time when the several debts may be equitably settled by one payment.

The **Term of Credit** is the time the debt has to run before it becomes due.

The Average Term of Credit is the time the debts due at different times have to run, before they may be equitably settled by one payment.

To find the Equated Time of Payment when the Terms of Credit begin at the Same Date.

RULE. — *Multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient will be the average term of credit.*

Add the average term of credit to the date of the debts, and the result will be the equated time of payment.

To find the Equated Time when the Terms of Credit begin at Different Dates.

RULE. — *Find the date at which each debt becomes due. Select the earliest date as a standard.*

Multiply each debt by the number of days between the standard date and the date when the debt becomes due, and divide the sum of the products by the sum of the debts. The quotient will be the average term of credit from the standard date.

Add the average term of credit to the standard date, and the result will be the equated time of payment.

RATIO

Ratio is the relation one number bears to another of the same kind.

The **Terms** of the ratio are the numbers compared.

The **Antecedent** is the first term.

The **Consequent** is the second term.

The antecedent and consequent form a *couplet.*

PRINCIPLES. — See Fractions.

PROPORTION

A **Proportion** is formed by two equal ratios.

The **Extremes** of a proportion are the first and last terms.

The **Means** of a proportion are the second and third terms.

PRINCIPLES. — *The product of the means is equal to the product of the extremes.*

Either mean equals the product of the extremes divided by the other mean.

Either extreme equals the product of the means divided by the other extreme.

RULE FOR PROPORTION. — *Represent the required term by x.*

Arrange the terms so that the required term and the similar known term may form one couplet, the remaining terms the other.

If the required term is in the extremes, divide the product of the means by the given extreme.

If the required term is in the means, divide the product of the extremes by the given mean.

PARTNERSHIP

Partnership is an association of two or more persons for business purposes.

The **Partners** are the persons associated.

The **Capital** is that which is invested in the business.

The **Assets** are the partnership property.

The **Liabilities** are the partnership debts.

To find the Profit, or Loss, of Each Partner when the Capital of Each is Employed for the Same Period of Time.

RULE. — *Find the part of the entire profit, or loss, that each partner's capital is of the entire capital.*

To find the Profit, or Loss, of Each Partner when the Capital of Each is Employed for Different Periods of Time.

RULE. — *Find each partner's capital for one month, by multiplying the amount he invests by the number of months it is employed; then find the part of the entire profit, or loss, that each partner's capital for one month is of the entire capital for one month.*

INVOLUTION

A **Power** of a number is the product obtained by using that number a certain number of times as a factor.

The **First Power** of a number is the number itself.

The **Second Power** of a number, or the **Square**, is the product of a number taken twice as a factor.

The **Third Power** of a number, or the **Cube**, is the product of a number taken three times as a factor.

An **Exponent** is a small figure written a little to the right of the upper part of a number to indicate the power.

Involution is finding any power of a number.

To find the Power of a Number.

RULE.— *Take the number as a factor as many times as there are units in the exponent.*

EVOLUTION

A **Root** is one of the equal factors of a number.

The **Square Root** of a number is one of its two equal factors.

The **Cube Root** of a number is one of its three equal factors.

Evolution is finding any root of a number.

Evolution may be indicated in two ways: by the *Radical Sign*, $\sqrt{\ }$, or by a *fractional exponent.*

The **Index** of a root is a small figure placed a little to the left of the upper part of the radical sign, to indicate what root is to be found. In expressing square root, the index is omitted.

In the fractional exponent, the numerator indicates the power to which the number is to be raised; the denominator indicates the root to be taken of the number thus raised.

To find the Square Root of a Number.

RULE.—*Point off in periods of two figures, commencing at units. Find the greatest square in the first period and place the root in the quotient. Subtract this square from the first period, and bring down the next period.*

Multiply the quotient figure by two, and use it as a trial divisor. Place the second figure in the quotient, and annex it also to the trial divisor. Then multiply the figures in the trial divisor by the second quotient figure, and subtract.

Bring down the next period, and proceed as before until the square root is found.

To find the Square Root of a Fraction.

RULE. — *Reduce the fraction to its simplest form, and find the square root of each term separately.*

To find the Cube Root of a Number.

RULE. — *Point off in periods of three figures each, beginning at units.*

Find the greatest cube in the first period and place the root in the quotient. Subtract this cube from the first period, and bring down the next period.

Multiply the square of the first quotient figure by three and annex two ciphers for a trial divisor. Place the second figure in the quotient. Then, to the trial divisor add three times the product of the first and second figures, also the square of the second. Multiply this sum by the second figure and subtract.

Bring down the next period, and proceed as before until the cube root is found.

To find the Cube Root of a Fraction.

RULE. — *Reduce the fraction to its simplest form, and find the cube root of each term separately.*

STOCKS AND BONDS.

Capital Stock is the money or property employed by a corporation in its business.

A **Share** is one of the equal divisions of capital stock.

The **Stockholders** are the owners of the capital stock.

The **Par Value** of stock is the face value.

The **Market Value** of stock is the sum for which it may be sold.

Stock is at a *premium* when the market value is above the par value; at a *discount*, when below par.

Bonds are interest-bearing notes issued by a government or a corporation.

A **Dividend** is a percentage apportioned among the stockholders.

A **Stock Broker** is a person who deals in stocks.

Brokerage is a percentage allowed a stock broker for his services.

In Stocks and Bonds,

The par value is the *base.*

The rate per cent premium, or discount, is the *rate.*

The premium, discount, or dividend is the *percentage.*

The market value is the *amount,* or *difference.*

NOTES, DRAFTS, AND CHECKS.

A **Promissory Note** is a written promise to pay a specified sum on demand, or at a specified time.

The **Face** of a note is the sum named in the note.

The **Maker** is the person who signs it.

The **Payee** is the person to whom the sum specified is to be paid.

The **Indorser** is the person who signs his name on the back of the note, thus becoming liable for its payment in case of default of the maker.

An **Interest-bearing Note** is one payable with interest.

If the words "with interest" are omitted, interest cannot be collected until after maturity.

A **Demand Note** is one payable when demand of payment is made.

A **Time Note** is one payable at a specified time.

A **Joint Note** is one signed by two or more persons who jointly promise to pay.

A **Joint and Several Note** is one signed by two or more persons who jointly and severally promise to pay.

In a joint note, each person is liable for the whole amount, but they must all be sued together. In the joint and several note, each is liable for the whole amount, and may be sued separately.

A **Negotiable Note** is one that may be transferred or sold. It contains the words "or bearer," or "or order."

A **Non-negotiable Note** is one not payable to the bearer, nor to the payee's order.

The **Maturity** of a note is the day on which it legally falls due.

A **Draft**, or **Bill of Exchange**, is a written order directing the payment of a specified sum of money.

The **Face** of a draft is the sum named in it.

The **Drawer** is the person who signs the draft.

The **Drawee** is the person ordered to pay the sum specified.

The **Payee** is the person to whom the sum specified is to be paid.

A **Sight Draft** is one payable when presented.

A **Time Draft** is one payable at a specified time.

An **Acceptance** of a time draft is an agreement by the drawee to pay the draft at maturity, which he signifies by writing across the face of the draft the word "accepted" with the date and his name.

A **Check** is an order on a bank or banker to pay a specified sum of money.

ANSWERS. — PART III.

Page 448.
2. 20 and 80.
3. $ 2000; $ 4000; $ 12,000.
4. 18 girls; 36 boys.
5. 13 and 65.
6. 13.

Page 449.
7. 11.
8. $ 3000; $ 6000; $ 18,000.
9. 12 and 60.
10. 9 marbles; 18 marbles; 27 marbles.
11. 36 years; 6 years.
12. 8.
13. 1; 4; 12; 24.
14. 30; 15; 135.
15. 9 pounds.
16. 19 rods.
17. 85 feet.

Page 450.
18. Son, $ 40; daughter, $ 80.
19. 25 days.
20. Girl, $ 80; boy, $ 40.
21. Father, 30 da.; son, 15 da.
22. 3 dimes, 6 nickels, 18 cents.
23. 25 yards.
24. 25 rods; 100 rods.
25. Speller, 15 ¢.; reader, 45 ¢.
26. 60 and 12.
27. 18 nuts; 9 nuts; 27 nuts.

Page 452.
1. 24.
2. 24.
3. 42.
4. 84.
5. 24.
6. 70.
7. 72.
8. 40.
9. 360.
10. 160.
11. 18.
12. 18.
13. 8.
14. 16.
15. 12.
16. 20.
17. 900.
18. 60.
19. 60.
20. 32.

Page 453.
21. 36.
22. 222.
23. 180.
24. 72.
25. 320.
26. 7.

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1. 15 and 75.
2. $28\frac{4}{7}$; $71\frac{3}{7}$.
3. $ 816.
4. $ 180.
5. 89.
6. 100.
7. 40; 15.
8. $\frac{28}{32}$.
9. $\frac{60}{84}$.

Page 454.
10. 60; 420.
11. 540; 18.
12. 9.
13. 20 peaches; 5 plums.
14. $ 200; $ 600; $ 700.
15. $ 60; $ 140.
16. $ 300.
17. 64 marbles.
18. $ 2; $ 3; $ 10.
19. $ 4; $ 2.
20. 3 horses; 12 cows.

Page 455.
1. 19.
2. 22.
3. 47.
4. 14.
5. 9.
6. 10.
7. 6.
8. 33.

Page 456.
9. 27.
10. 3.
11. 28.
12. 96.
13. 144.
14. 18.
15. 24.
16. 6.
17. 32.
18. 18.
19. 12.
20. 20.

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2. 15.
3. 9.

4. 15 marbles; 33 marbles.

Page 457.

5. 25 ft.; 100 ft.
6. 39 acres; 47 acres.
7. 1059 votes; 1377 votes.
8. 62 years.
9. 84; 12.
10. $108.
11. 17; 28.
12. $16; $11.
13. Cows, $45; horses, $125.
14. 3 dimes; 14 half dimes.
15. 74 and 26.
16. 21 boys; 33 girls.

Page 458.

17. $3600; $6000; $8400.
18. 44; 11.
19. 5 five-cent stamps; 20 two-cent stamps; 35 postal cards.
20. 8 horses; 25 cows; 55 sheep.

Page 460.

1. $6.34.
2. $4.50.
3. $9.60.
4. $1.55.
5. $4.158.
6. $415.80.
7. $10.32.
8. $1255.80.
9. $9.16.
10. $3.68.
11. $1.60.
12. $1.58.
13. 1.62\frac{1}{4}$.
14. $326.80.
15. 3 cents.
16. $13.09.
17. $19.98.
18. $1.17.
19. $3.16.
20. $7.50.
21. 3.21\frac{3}{4}$.
22. $11.32.
23. $63.04.
24. $19.95.
25. $550.
26. $44.40.
27. $323.40.
28. $4.65.

Page 461.

30. $\frac{13x}{20}$.
31. 40%.
32. $\frac{x}{4}$.
33. 168.
34. $\frac{6x}{5}$.
35. 110.
36. $\frac{2x}{3}$.
37. 117.
38. $\frac{x}{150}$.
39. 500%.
40. $\frac{x}{800}$.
41. 18,400.
42. 40%.
43. 133$\frac{1}{3}$.
44. 72.
45. 68.
46. 75%.

Page 462.

47. 83$\frac{3}{4}$%.
48. $\frac{11x}{200}$.
49. $800.
50. $\frac{2}{5}$.
51. 37$\frac{1}{2}$%.
52. 99.
53. $218.75.
54. $500.
55. 1.
56. 500%.
57. 32%.
58. $183.20.
59. 1100.
60. 738.
61. 10,800.
62. $1547.
63. $764.80.

Page 463.

64. $4120.
65. $170.
66. 3500 bu.
67. $1440.
68. $592.

Page 464.

1. Selling price, $2157.40.
2. Selling price, $29.40.
3. Selling price, $1181.25.
4. Selling price, $831.25.
5. Selling price, $1051.38.

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6. 3%.
7. 33$\frac{1}{3}$%.
8. 15%.
9. 20%.
10. 6$\frac{1}{4}$%.
11. 6$\frac{1}{4}$%.
12. 20%.
13. 5%.
14. 12$\frac{1}{2}$%.
15. 16$\frac{2}{3}$%.
16. Cost, $375.
17. Cost, $92.30.
18. Cost, $1234.56.
19. Cost, $240.
20. Cost, $63.75.
21. $55.
22. 25%.
23. $800.

Page 466.

24. 50%.
25. 14$\frac{2}{7}$%.
26. 12$\frac{1}{2}$%.
27. 37$\frac{1}{2}$ cents.
28. $40.
29. $219.
30. $200.
31. 37$\frac{1}{2}$%
32. 25%.
33. $1646.
34. $4053.
35. $113.75+

Page 467.

1. 3 feet.

2. 18 feet.
3. $\frac{1}{6}$ acre.
4. 1430 yards.
5. 8 strips.

Page 468.

6. 3600 sheets.
7. 250 ft. 1500 sq. ft.
8. 250 boards.
9. $ 710.
10. 6400 cakes.
11. 76,800 cu. ft.; 2208 tons.
12. 160 feet.
13. 281,600 sq. ft.; 240,-000 sq. ft.
14. $ $2773\frac{1}{3}$.
15. 96 lots.
16. 164 sq. yd.

Page 469.

1. $11\frac{1}{9}$ %.
2. 10 %.
3. $11\frac{1}{9}$ %.
4. $ 26.40.
5. $ 2.80.
6. $ $1.22\frac{1}{2}$.
7. 45 %.
8. 1350 %.
9. $4\frac{1}{2}$ %.
10. $\frac{9}{20}$ %.
11. 25 %.
12. $33\frac{1}{3}$ %.
13. $16\frac{2}{3}$ %.
14. $ 75.
15. $21\frac{67}{73}$ %.
16. $ 35.
17. $ $71.34\frac{3}{4}$.
18. $ 19.20.
19. $33\frac{1}{3}$ %.
20. 12 %.
21. $ 1.92.
22. $ 24.24.
23. $ 39.20.
24. $ 1.57.

Page 470.

25. $ 18.33.
26. $119\frac{1}{21}$ %.
27. $ 200.
28. $ 893.20.
29. $ 1.
30. 2 %.
31. $ 18.
32. $ 626.05.
33. $ 1.57.
34. $ 68.18.
35. $9\frac{39}{229}$ %.
36. $14\frac{2}{5}$ %.
37. $18\frac{26}{93}$ %.
38. $ 27.20.
39. 15 cents.
40. $ 6300.
41. $ 871.83.
42. $ 309.14.
43. 8 %.
44. $7\frac{9}{13}$ %.
45. $ 21.
46. $ 677.25.
47. $ 7692.
48. $ 2.36.
49. $7\frac{1}{2}$ %.
50. $ 21.55.

Page 471.

1. $ 112.50.
2. $90\frac{2}{5}$ cents.
3. $85\frac{5}{8}$ cents.
4. $ 19.52–.
5. $ 1.65.
6. $ 1.16–.
7. $ 19.55.
8. $ 2.94.
9. $ $1.11\frac{1}{9}$.
10. $ $67.72\frac{1}{2}$.
11. $ 49.77+.
12. $ $235.75\frac{1}{2}$.

Page 472.

13. $ 18.88.
14. $ 7.80.
15. $ 18.74–.
16. $ $1050.80\frac{1}{4}$.
17. $ $1035.73\frac{1}{3}$.
18. $ 33.73.–.
19. $ 49.04–.
20. $ $154.87\frac{1}{2}$.
21. $ 884.53+.
22. $ 6191.20.
23. $ 2841.81–.
24. $ 2344.50+.
25. $ 161.00+.
26. $ 835.31–.
27. $ 886.17–.
28. $ 411.65+.

Page 473.

29. $ 1550.21–.
30. $ 118.14–.

Page 474.

18. 63 cents.
19. $ 5.70.
20. $ 4.20.
21. $ 1.50.
22. $ 2.75.
23. $ 7.20.

———

1. $\frac{2}{7}$.
2. $7\frac{1}{2}$ feet.
3. 10 mi. $249\frac{3}{5}$ rd.
4. $\frac{65}{216}$.
5. 16 %.
6. $ 7.35.
7. $\frac{10}{23}$.
8. $33\frac{1}{3}$ times.
9. $\frac{47}{400}$ cent.

Page 475.

10. 1.15; .000625; .0040625; $1\frac{3}{20}$; $\frac{1}{1600}$; $\frac{13}{3200}$.
11. $9\frac{3}{8}$ %.
13. 15 times.
14. $ 530.
15. .0223125 mile.
16. 2.432013984.
17. $3\frac{3}{4}$ %.
18. $ 2.64–.
19. $ 3999.24.
20. 162 days.
21. $16\frac{2}{3}$ %.
22. $ 5.69+.
23. $ 753.
24. $ 11.28–.
25. $\frac{9}{10}$; $\frac{85}{104}$.

Page 476.

1. 3613 flags; $1\frac{41}{72}$ yards remaining.
2. 200 men.
3. 317 acres; $ $14,325.59\frac{1}{4}$; $ $45.19\frac{145}{1268}$.
4. $ 288.75.
5. 7,002,079.-003129.
6. 482.9638599.
7. 2.0635+.
8. $ 10.38–.

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9. $56\frac{28}{47}$ %; $13\frac{14}{47}$ %; $30\frac{5}{47}$ %.
10. \$.06703+. \$.04648–; \$.0055; \$.00675; \$.00475; \$.0225; \$.005359+.
12. 8272.08512.
13. 8.522.
14. 3.7857+.
15. 30,000 men.
16. $\frac{1}{3}$.
17. 15 miles.

Page 478.

18. 170 bushels.
19. 1500 letters; 750 letters.
20. \$2630.20.

---

1. 1,163,117,-683.002129.
2. 69,092.-80236843.
3. 176.303–; 2.0247–.
4. \$66.45–.

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5. \$42.96–.
6. 2, 3, etc. 18 G. C. D.
7. 25 %.
8. 400 %.
9. $6\frac{13}{20}$.
10. $88\frac{56}{113}$ cents.
11. 7.625.
12. $1\frac{1}{23}$.
13. $\frac{673}{900}$.
14. $7\frac{7}{8}$.

Page 480.

1. $2\frac{10}{1311}$.
2. 82.
3. \$$1.26\frac{1}{2}$.
4. 113.
5. $53\frac{1}{3}$ %; $46\frac{2}{3}$ %.
6. \$1.60.
7. 192 planks.
8. $12\frac{1}{2}$ %; $11\frac{1}{9}$ %.
9. \$17.76+.
10. \$6.
11. $266\frac{2}{3}$ %.

Page 484.

1. 165.
2. 252.
3. 4048.
4. 910.
5. $2594\frac{2}{5}$.
6. $5646\frac{2}{3}$.
7. $1977\frac{3}{7}$.
8. $6373\frac{1}{2}$.
9. $5067\frac{9}{10}$.
10. $8852\frac{1}{4}$.
11. 46,018.
12. 79,520.
13. $25,554\frac{3}{8}$.
14. 106,908.
15. 65,471.
16. $65,635\frac{1}{2}$.
17. 86,855.
18. $27,702\frac{1}{6}$.
19. 37,411.
20. 26,969.
21. 41,382.
22. 58,975.
23. 899,100.
24. 426,000.
25. 16,800.
26. 1,172,880.
27. 4290.
28. 71,400.
29. 67,716.
30. $293,249\frac{2}{3}$.

Page 485.

"All other articles," \$73,327,274; \$72,122,469; Increase, \$26,-976,455.

Page 486.

1. $6975\frac{42349}{87659}$.
2. $6126\frac{41169}{45678}$.
3. $11,202\frac{30471}{34567}$.
4. $11,699\frac{33039}{68439}$.
5. $786\frac{272899}{576258}$.
6. $9809\frac{46939}{71685}$.
7. $49,167\frac{13938}{18789}$.
8. $9631\frac{59744}{59764}$.
9. $7288\frac{162890}{169375}$.
10. $6462\frac{188202}{483729}$.

---

1. 630 boards; 140 posts.
2. 10 feet.
3. \$1.50.

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4. 4840 sq. yd.; about 70 yards.
5. 400 rods.
6. 18 sq. ft.; 270 cu. ft.
7. $106\frac{2}{3}$ yards.
8. $86\frac{2}{3}$ bundles.
9. 810 sq. yd.
10. 114 rods.

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11. 300 panes.
12. 20,736 gal.
13. 72 cords.
14. 13,440 bu.
15. 7000 cu. yd.
16. 2250 cu. yd.

---

1. \$313.31–.
2. \$136.50.
3. \$500.
4. 8 %.
6. \$410.16–.

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7. 29.623– feet.
8. \$$30.87\frac{1}{2}$.
9. $12\frac{1}{4}$ tons.
10. 3.

---

1. \$.41+; (\$$.37\frac{1}{2}$).
2. \$.55+; (\$.46–).
3. \$2.89+; (\$2.75+).
4. \$.37–; (\$.32).
5. \$6.89–; (\$6.72).

Page 490.

6. \$$173.59\frac{1}{2}$ (\$173.70).
7. \$36.23+ (\$36.25+).

8. $791.42+; ($791.89−).
9. 176.74\frac{1}{2}$; ($176.85).
10. 985.41\frac{2}{3}$; ($986).
11. $3.10; ($2.80).
12. $2.60; ($2.48).
13. $5.64; ($5.46).
14. $1.74−; ($1.69+).
15. $5.38−; ($5.29+).
16. $2.52; ($2.40).
17. $5.58; ($5.40).

Page 491.

18. Proceeds, $87.34−; ($87.38+).
19. Proceeds, $122.66+; ($122.75−).
20. Proceeds, $502.05−; ($502.34+).
21. Proceeds, $71.65−; ($71.68+).
22. Proceeds, $232.99−; ($233.10+).
23. Proceeds, $95.58+; ($95.63−).
24. Proceeds, $162.65+; ($162.76+).
25. Proceeds, $81.91+; ($81.95+).

---

1. $24.28.
2. $10.15+.
3. $2.52+.
4. $367.42−;
5. $10.99+.
6. $2.13.
7. $11.23+.
8. $12.78+.

Page 492.

674.7+ yards.
$613.98+.

Page 493.

1. 371$\frac{7}{8}$.
2. 530$\frac{1}{3}$.
3. 129$\frac{1}{15}$.
4. 127$\frac{25}{48}$.
5. 517$\frac{1}{3}$.
6. 289$\frac{9}{16}$.
7. 233$\frac{38}{49}$.
8. 260$\frac{35}{64}$.
9. 734$\frac{13}{24}$.
10. 1078$\frac{1}{8}$.
11. 85$\frac{1}{4}$.
12. 89$\frac{5}{6}$.
13. 264$\frac{16}{35}$.
14. 361$\frac{13}{27}$.
15. 337$\frac{17}{40}$.
16. 674$\frac{11}{16}$.
17. 81$\frac{7}{12}$.
18. 401$\frac{1}{24}$.
19. 421$\frac{37}{50}$.
20. 434$\frac{1}{8}$.

Page 495.

1. $\frac{1}{32}$; .01020201; 10.01.

Page 496.

2. $4.90.
3. $115.54−.
4. 5.37\frac{1}{2}$; $224 07−.
5. $728; 950\frac{6}{7}$.
6. $25,000.
7. 20 %.
8. The first; 5 % more.
9. 1$\frac{2}{5}$ acres.
10. .0006216.
11. $1200; $4608; 25 %.
12. 6\frac{2}{7}$.

Page 497.

13. $55.91−.
14. 4$\frac{7}{30}$; 6$\frac{22}{63}$ days; $85,333$\frac{1}{3}$.
15. $173,668; $201.880618.
17. $1453.76.
18. 4.23+ times; 64.68+ inhabitants; 6.07− inhabitants; $15,124,032.
19. 300 sheep.
20. $8572.20.

Page 498.

21. 257 sq. yd.
22. 6$\frac{1}{2}$ %; $287.50.
23. 52 weeks.
24. 33,750 qt.; 2250 qt. average.
25. $153.75.
26. $33,519.20.
27. $85.
28. 70 feet.
29. $187.36.
30. 9$\frac{1}{11}$ %; 8$\frac{1}{3}$ %; 100 %.

Page 499.

1. $46.80.
2. $18.40.
3. 17 lb. 11 oz. 5 pwt. 19 gr.

Page 500.

4. 23,220 gr. gold; 2322 gr. silver; 258 gr. copper.
5. 24 spoons.
6. $2800.
7. $\frac{144}{175}$; 109$\frac{5}{7}$ %.
8. $105.
9. 27+ cents.
10. 23$\frac{3}{11}$ gal.
11. 704 sq. ft.
12. 1$\frac{5}{6}$ cu. ft.
13. 1317$\frac{17}{24}$ lb.

14. $13,405\frac{1}{11}$ gal.; 1440 bu.
15. 180 %.
16. $82\frac{2}{7}$ %.
17. $ 309.42.

Page 501.
18. 300 pounds.
19. A, $43\frac{1}{2}$ %; B, 36 %; C, $20\frac{1}{2}$ %.
20. 672 yards.
21. $ 315.

Page 502.
1. 101.5901+.
2. $71\frac{1}{2}$.
3. 72.
4. 390.
5. 3 yards.
6. $ 4.
7. $20\frac{5}{6}$ %.
8. $ 81.25.

---

1. $11\frac{5}{6}$ rolls.
2. 1.5548–.
3. $5333\frac{1}{3}$ bu.
4. $ $3.02\frac{18}{31}$.

Page 503.
5. $ $60.62\frac{1}{2}$.
6. 70 cents.
7. $11\frac{593}{2120}$ cts.
8. 17 spoons.
9. $48\frac{3}{34}$ bu.

---

1. $ 49.01– ; ($ 48.99–) ; $ 48.88– ; ($ 48.88–).
2. $ 175.94– ; ($ 175.94–).
3. $ 350.55+ ; ($ 350.55+).

Page 504.
4. $ 846.26– ; ($ 845.77+).
5. $ 724.85+ ; ($ 724.69–).

---

1. 38 rd. 3 yd. 11 in.
2. 113 rd. 3 yd. 1 ft. 6 in.
3. 175 rd. 1 yd. 1 ft. 6 in.
4. £ 16 13 s. 4 d.
5. 21,090 d.
6. £ 26 5 s.
7. $ 27.37+.
8. £ 108 4 s. 6 d.
9. £ 45 18 s. 8 d.
10. $773\frac{7}{16}$ oz.
11. $ 175.

Page 505.
12. $ 123.
13. $ 72 ; $ 119.25 ; $ 92.25.
14. $ 450 ; $ 750 ; $ 600.
15. 24 days.
16. $2\frac{2}{5}$ hours ; 48 miles from A.
17. $ 45 ; $ 135 ; $ 270 ; $ 450.
18. $ 237 ; $ 189 ; $ 114.
19. 7 days.
20. 240 eggs.
21. 5 months.
22. 24 weeks.

Page 506.
23. $ $37,033.13\frac{1}{2}$.
24. 10 inches.
25. $ 360.

---

1. 125,422,928,-368.01.
6. 12 T. 6 cwt. 2 qr. 13 lb.

Page 507.
7. 300.
8. $11\frac{2}{3}$ days.
9. $3416\frac{16}{25}$ lb.
10. £ 3068 15 s. 10 d.

---

4. 302.
6. $11\frac{1}{20}$.
7. $9\frac{3}{8}$.
9. 1250.

Page 508.
10. $10\frac{219}{253}$.
11. $37\frac{1}{3}$ pieces.
12. 43,200 min.
13. $21\frac{8}{9}$ inches.
14. 362.16 mi.
15. $ 6.72–.
17. $ 4761.90+.
18. $ $8.28\frac{3}{4}$.
19. $ 870.48.
20. A, $ 50 ; B, $ 90 ; C, $ 110.
21. $ 380.
22. $ 158.40.

Page 509.
1. $ 554.23.
2. $ 171.20.
3. $ $1782.67\frac{1}{2}$.
4. $ 158.40.
5. $ 28.50.
6. $ $60.56\frac{1}{4}$.
7. $ 50.85.
8. $ 13.
9. $ 7.45.

Page 510.
10. $ 27.84–.
11. 40 and 10 % ; $ 2 difference.
12. $ 60 ; 40 % discount ; 60 % net.
13. 52 %.
14. $ 100.
15. 72 %.
16. 72 %.

---

1. $ 81.
2. 75 %.
3. $\frac{1}{10}$ %.
4. 627.5.
5. .0075 ; $\frac{3}{400}$.
6. $77\frac{7}{9}$ %.
7. $ $2.63\frac{1}{5}$.
8. $ 24.
9. 600.

Page 511.

10. $3575.
11. $6\frac{3}{4}$%.
12. $65.
13. $101.50.
14. $375.
15. $10.96–.
16. $23.14–.

Page 512.

1. $19.
2. 35.
3. $2624\frac{2}{3}$ yards.
4. $27.20 gain.
5. $20.
6. 1767.5.
7. $5.49–.
8. $35\frac{5}{9}$%.
9. $1431.27.

Page 513.

10. $30.
11. $4\frac{5}{31}$.
12. .00007865.
13. 108.86– bu.
14. 2 bu. 1 pk. 4 qt.
15. $7425.
16. 25%.
17. $49.31+.
18. $594.50; ($594.80).
19. 25%; $32\frac{1}{7}$%; $42\frac{6}{7}$%.
20. $$1.56\frac{1}{4}$.
21. Loss, $177.
22. $3.60.
23. $.01\frac{2}{9}$.

Page 514.

24. 15,203.
25. $$4.16\frac{2}{3}$.
26. 80 hours.
27. 250 cu. ft.
28. $$7.12\frac{1}{4}$.
29. $18.
30. $77.95, ($79).

---

1. $\frac{9x}{80}$.
2. $1600.
3. $255x$.
4. 3%.
5. $2000.

Page 515.

6. 5 years.
7. $150x$.
8. 6%.
9. $1875+\frac{375x}{32}$.
10. 682.90+ $\frac{6829x}{3200}$.
11. $\frac{21x}{2000}$; $\left(\frac{x}{100}\right)$.
12. $\frac{11x}{30}$; $\left(\frac{x}{3}\right)$.
13. $\frac{x+3}{20}$; $\left(\frac{x}{20}\right)$.
14. $\frac{41x}{60}$; $\left(\frac{2x}{3}\right)$.
15. $\frac{5997-x}{10}$; $\left(600-\frac{x}{10}\right)$.
16. $\frac{477x}{400}$.
17. $\frac{1969x}{2000}$; $\left(\frac{197x}{200}\right)$.

Page 517.

1. $121\frac{1}{2}$ cu. ft.
2. $162.

Page 518.

3. $126.
4. $85.50.
5. 16.8 tons.
6. 2 pieces, 12×12; 2 pieces, 12×14; 2 pieces, 14×14.
7. $44\frac{12}{54}$ lb.
8. 5280 lb.
9. Outside dimensions, $14\times14\times14$; 2744 cu. in. wood and marble; 1728 cu. in. marble; 1016 cu. in. wood.
10. 8 times; $\frac{1}{4}$ ton; $6\frac{3}{4}$ tons.

Page 520.

1. 5%.
2. 2 years.
3. $96.
4. 3%.
5. $1.43–.
6. $36.
7. 1 yr. 6 mo.
8. $42.17+.
9. $5000.
10. 6%.
11. 1 yr. 9 m 2 da.
12. $144.
13. $$83.26\frac{1}{2}$.
14. 3%.
15. 2 yr. 1 mo. 7 da.
16. $80.
17. $2181.99–.
18. $72.

Page 521.

19. 5 mo. 23 da.
20. 6%.
21. $16.92.
22. $402.22.
23. 37 days.
24. 5%.
25. 11 mo. 29 da.

Page 522.

1. 570,073,438,098.53.

Page 523.

6. .37875.
7. $70.20.
8. $281.25.
9. $15,000.
10. $67.71–.

Page 524.

2. $41.99+.
3. $951.13+.
4. $112.74–.

5. $119.43–.
6. $13.91–.

Page 525.
7. $147.19–.
8. $8.10–.
9. $52.33–.
10. $1005.50.
12. $967.78.
13. $21.79–.
14. $16.53–.
15. $1274.21–.

Page 526.
16. ($860.50–).
17. $24.74–; ($23.94).
18. $761.06–; ($761.45).
19. $43.99–.
20. $786.39–.
21. $65.40.
22. $625.03+.
23. $98.49+.
24. $993.27+.
25. $61.68–.
26. $252.37+; ($252.52–).
27. $13.09–; ($12.87½).
28. $486.10–; ($486.43–).
29. $2.33+; ($1.94+).
30. $989.67–. ($990).
31. $1938.43–.
32. $8.06+.

Page 527.
33. $1473.52–.
34. $6.55+.
35. $278.16–.
36. $1.23–.
37. $196.64+.
38. $1.10.
39. $389.60.
40. $6.11+.
41. $4.09+.
42. $56.32–.
43. $16.72–.
44. $95.43–.
45. $15.71+.
46. $594.30; ($594.60).
47. $10.73+; ($10.38+).
48. $793.73+; ($794.13+).

Page 528.
49. 45 cents; (nothing).
50. $968.83+; ($970).

---

1. 470,952.

Page 529.
5. 8614.20375.
6. $2\frac{4}{7}$.
7. $110.85+ gain.
8. $16.25.
9. 45 T. 5 cwt. 2 qr.
10. $43.33⅓.

Page 530.
4. $196.17.
5. $600.01+.
6. $1506.12.
7. 4269.22+ francs.
8. $1563.55–.
9. $1563.55–.
10. $1547.37.
11. $\frac{18x}{25}$.

Page 531.
12. $400 - 4x$.
13. $500.
14. 10 %.
15. Same.

---

1. 688,965,549,-176.65.

Page 532.
6. .0019.
7. 32 cents.
8. $46 gain.
9. 60 cents.
10. $16.98.

Page 535.
1. $106.33+; ($106.38).
2. 7 %; ($7\frac{7}{30}$ %).
3. 24 days.
4. $1200.
5. $4.68; ($3.90).
6. 5 %.
7. 72 days.
8. $1200. ($1260).
9. $304.26–. ($304.05+).
10. Apr. 8, 1894.
11. $853.27+; ($853.71–).
12. $56.62½; ($57.50).

Page 536.
3. $2\frac{4}{15}$.
4. 375.
5. $288.
6. $922.20.
7. $573.47½.
8. $1700.
9. $1400; $1200.
10. $1337.

Page 539.
1. $165.37½.
2. $1453.42–.
3. $50,625.50.

Page 540.
4. $893,615,929.
7. 284,106,409,-352.02.
8. 45.29– %.
9. $223,852.8835.
10. $107\frac{2}{3}$ cents.

Page 541.
11. $20,000.
12. $728.17½.
15. Profits, $4414.10.
16. $\frac{3}{4}$; $\frac{1}{8}$; $\frac{3}{16}$; $\frac{2}{5}$.

Page 542.
17. $4752.
18. 37½ %; 25 %; 12½ %; 25 %.
19. $81.90.

20. \$299.88–.

---

1. 1,287,400.
2. 3,370,185.
3. 598,969.
4. 2,883,736.
5. 816,669.
6. 5,127,460.
7. 2,455,038.
8. 42,327,198.
9. 2,513,420.
10. 22,944,747.

Page 543.

11. 857,712.
12. 6,482,112.
13. 1,230,828.
14. 921,776.
15. 3,460,704.
16. 5,888,304.
17. 1,460,025.
18. 10,563,960.
19. 3,911,322.
20. 2,982,840.
21. 714,186.
22. 3,277,719.
23. 456,375.
24. 174,600.
25. 362,250.
26. 104,787$\frac{1}{2}$.
27. 128,550.
28. 625,975.
29. 213,966$\frac{2}{3}$.
30. 413,866$\frac{2}{3}$.
31. 8650$\frac{1}{2}$.
32. 10,757$\frac{2}{3}$.
33. 21,873$\frac{1}{5}$.
34. 24,292$\frac{1}{2}$.
35. 485,072.
36. 167,276.
37. 24,418$\frac{1}{2}$.
38. 24,130$\frac{2}{5}$.
39. 651,329$\frac{5}{11}$.
40. 1,932,560.
41. \$277,133.11.
42. \$60,887.10.
43. \$48,554.08.

Page 544.

44. £7 5 s. 9 d.
45. 11 yd. 1 ft. 11 in.
46. 12 bu. 2 pk. 5 qt.
47. 11.76+.
48. 13.72+.
49. 14.41+.
50. 13.34+.
51. 19.05–.
52. 22.30–.

---

1. 79.98 %.
2. 5.02 %.
3. 4.28 %.
4. 3.83 %.
5. 3.81 %.
6. 2.44 %.
7. .54 %.
8. .10 %.
Total, \$872,270,283.

Page 545.

1. 237.49 %.
2. 234.60 %.
3. 563.36 %.

Page 546.

4. 0.04 %.
5. 25 %.
6. 20 %.
7. \$24; \$30.
8. \$3.
9. £73 3 s.
10. 6$\frac{2}{3}$ days.

---

2. \$58.93–.
3. 6 %.
4. \$3400; \$3570.

Page 547.

5. \$1.30–.
6. \$10,000.
8. \$264.25.
9. \$135.40–.

Page 548.

1. \$107.65+.
2. 6 yr. 6 mo.
3. 7$\frac{1}{2}$ %.
4. \$750.
5. \$882.
6. \$97.
7. \$6; \$144.
8. 33$\frac{1}{3}$ %.
9. \$8960.
10. \$360.

Page 549.

11. \$50.
12. 20.
13. \$61.87$\frac{1}{2}$.
14. \$1150.
15. $\frac{5}{49}$.

---

1. 4$\frac{29}{80}$.
3. $\frac{5}{11}$.
4. 16$\frac{23}{36}$.
5. $\frac{22}{155}$.
6. 8$\frac{12}{35}$.
7. 73; 240.
8. 4$\frac{35}{48}$.
9. $\frac{9}{11}$; $\frac{8}{13}$.
10. Largest, $\frac{4}{9}$; smallest, $\frac{2}{3}$ of $\frac{5}{8}$.

Page 550.

11. 2$\frac{8}{99}$.
12. 2 da. 15 hr. 50 min. 35 sec.
14. 11$\frac{79}{147}$.
15. 1$\frac{1}{12}$.
16. $\frac{37}{80}$.
17. 180.
18. 3$\frac{3}{4}$.
19. $\frac{26}{119}$.
20. 36$\frac{2}{3}$.
21. 1$\frac{2}{13}$.
22. 3 hr. 22 min.
23. 85$\frac{1}{3}$ rods.
24. 1$\frac{31}{36}$.
25. 15$\frac{3}{8}$.

---

1. 4$\frac{1}{2}$ feet.
2. 120 yards.

Page 551.

3. 8750 sq. yd.
4. $3000 + 30x$; 80 yards.
5. $100x$; 40 yards.
6. $60x + 1200$; 80 yd. and 120 yd.
7. 10,000 sq. yd.
8. 1920 flagstones.
9. \$9.26–.
10. 46$\frac{2}{3}$ yards.

11. $90\frac{15}{64}$ lb.
12. $35\frac{5}{12}$ sq. ft.

Page 552.
13. 10 ft. 8 in.
14. 950 bushels.
15. 810 gallons.
16. $416.
17. 10 miles.
18. 615 cords.
19. $8\frac{409}{1152}$ lb.
20. $27\frac{709}{4608}$ lb.
21. $706\frac{2}{3}$ sq. yd.
22. $722\frac{2}{3}$ sq. yd.

Page 553.
23. $690\frac{2}{3}$ sq. yd.

---

1. $24\frac{11}{36}$ yards.
2. 28 cents.

Page 554.
3. 84 days.
4. 9 days.
5. $49.
6. 180 bushels.

Page 555.
8. 60 bushels.
9. 720 pounds.
10. 4600 sheets.
11. $312.
12. 8 da. 4 hr.
13. $28.80.
14. 96 rods.
15. 50 days.
16. $45.
17. 120 men.

Page 556.
18. 360 men.
19. 10 hours.
20. $1.20.

---

2. 329.
3. $$7\frac{1}{7}$.
4. 10 bushels.
5. $4\frac{2}{45}$.
6. 12 feet.
7. 90 cents.

Page 557.
8. 1680.
9. 85,800.
10. Loss, $93.75.
11. $63\frac{7}{11}$ %.
12. $2.35+.
13. 99.99.
14. 14 feet.
15. $69.05–.
16. 1050 acres.
17. $9792\frac{2}{9}$ lb.
18. $275.69–.

Page 558.
20. $873; $184;
21. $143.
22. Estate, $45,000; son's share, $10,000.
23. $10.86+.
24. $6\frac{4}{9}$ days.

---

1. $78; $104.
2. $36; $17; $48.
3. $25; $20.
4. $16.20; $13.80.
5. 45 lb. gold; $4\frac{1}{2}$ lb. silver; $\frac{1}{2}$ lb. copper.

Page 559.
6. Saltpeter, 54 lb.; sulphur, $7\frac{1}{5}$ lb; charcoal, $10\frac{4}{5}$ lb.
7. $20.25; $18.63; $15.12.
8. 864 bales 540 bales; 396 bales.
9. $12; $20;
10. $60; $72.

---

2. 326.9843.
3. 6408.
5. 4.2633; 1.405712.

Page 560.
6. 16.5393.
9. 28.165; 305.36721612.
10. £ .3375.
12. Sum, 8.08690625.
14. .019104141.
15. .05; .03965.
16. 1093.3524
17. 2.8.
18. .215625 m.
19. 12 da. 20 hr. 31 min. 12 sec.
20. 5934.47– meters.
21. 6 quarts.
22. .8125 pound
23. .25 rod.
24. 100 links.
25. .1 acre.

Page 561.
£ 131 3 s. $8\frac{3}{4}$– d. 368.90 marks; $30.73–.

Page 562.
1. $40.
2. 8 %.
3. $150; 150 %

Page 563.
4. $30.
5. $5\frac{1}{3}$ %.
6. $435.
7. Same.
8. 5 %; $2\frac{1}{2}$ %.
9. $2267.26+.
10. $2.85.
11. $4\frac{4}{19}$ %.
12. $3\frac{101}{233}$ %.

Page 564.
17. $$459.37\frac{1}{2}$.
18. $24,000; $66\frac{2}{3}$ %.
19. $1\frac{1}{2}$ %.
20. $18,000.
21. 101 rd. 2 yd. 1 ft. 6 in.
22. 35 shares; $87.50.

23. 37 rd. 4 yd. 11 in.
24. \$106.12.

Page 565.
25. 229 rd. 4 yd. 2 ft. 6 in.
27. \$428.40–; \$53.40–.
28. \$119.10+.
29. \$149.14+.

Page 566.
30. 51 cents.

---

1. 15 %.
2. \$400.
3. \$102.50.
4. \$2.70.
5. \$40.
6. \$1.60.
7. 1 hour.
8. \$20.
9. \$12.

Page 567.
10. \$20 loss.

---

4. 1.
5. 962 feet.
6. \$6.50 gain.
7. \$3.54.

Page 568.
8. \$509.25.
9. 19 T. 62 lb. 8 oz.
10. 24 cents.

Page 570.
1. \$45.
2. $10\frac{7}{8}$ acres.
3. \$$452.32\frac{1}{2}$.
4. \$4.50.
5. \$1.50; \$5.40; 20 %;
6. \$6000.
7. \$$3333\frac{1}{3}$.

Page 571.
8. \$212.51–; \$439.79–.
9. \$5250; 18 %.
10. \$402.50; 35 %.
11. \$3330; \$4.25.
12. \$9.30.
13. \$201.60; \$125.93–; \$722.40+.
14. $2\frac{7}{9}$ pounds.
15. $98\frac{26}{163}$ lb.
16. \$71.25.

Page 572.
17. \$85.
18. \$594.20; (\$594.50).
19. 20 %.
20. \$$11{,}356.01\frac{1}{2}$
21. 700.
22. \$38.59+.
23. 12.
24. \$$1006.13\frac{1}{2}$.
25. 63 sq. ft.; $10\frac{16}{25}$ rods.
26. \$57,600.
27. \$$2.37\frac{1}{2}$.
28. \$387.36.

Page 573.
29. \$1000; 100 %.
30. \$5487.

---

2. 688,450.
3. $3\frac{23}{63}$; $18\frac{26}{27}$.
4. .140625; 88.088.
5. 5.820068; 1000.

Page 574.
6. 400 yards.
7. \$$2\frac{2}{5}$.
8. \$10,500.
9. 7400 inhabitants.
10. \$10,500.

Page 575.
1. (*a*) 1575.355671; (*b*) .028376604.
2. 49,999.74999.
3. (*a*) 73; (*b*) 2016.
4. \$72.
5. \$53.95+.
6. \$160; \$140; \$240.
7. \$262.50.

Page 576.
1. \$3481.07+.
2. \$$1874.77\frac{1}{2}$.
3. \$2461.76+.
4. \$1000.50+.
5. \$1632.

Page 577.
6. \$1845.90+.
7. \$946.04–.
8. \$632.65–.
9. \$985.22+.
10. \$326.34.

---

1. 9 hr. 45 min.
2. $14\frac{1}{2}$ minutes past 5 P.M.
3. 4 ft. 6 in.

Page 578.
4. 748 plants; 754 plants.
5. 20 da. 6 hr. 40 min.
6. 5544 revolutions; 1320 revolutions.
7. 28 mi. $130\frac{10}{11}$ rd.
8. $1388\frac{8}{9}$ mi.
9. .15708 foot.
10. $694\frac{4}{9}$ miles.
11. $3211\frac{15}{32}$ mi.

Page 579.
12. $1104\frac{27}{32}$ mi.

---

1. \$828.45. (\$828.87).
2. \$397.30; (\$397.50).

3. $554.40; ($554.68).

Page 580.

4. $625.33+.
5. $\frac{3959x}{4000}$; $\left(\frac{3961x}{4000}\right)$.
6. $\frac{11979-2x}{10}$; $\left(\frac{11985-2x}{10}\right)$.
7. $\frac{7956+8x}{5}$; $\left(1592+\frac{8x}{5}\right)$.
8. $1200; ($1199.39+).
9. 30 days; (33 days).
10. $1.50 discount per $1000. ($2 discount per $1000).

Page 581.

1. 56°.
2. 2 hr. 29 min. 12 sec.
3. 5 hr. 50 min. 20 sec.; 7 A.M.
4. 37° 30′.
5. 1 hr. 14 min. 52 sec.

Page 582.

6. 33° 30′ east longitude.
7. $39\frac{1587}{5037}$ miles per hour.
8. 1800 lb.
9. $1000.80.
10. 8 yd. 6 in.
11. 15 ft. 7 in.
12. 257 bu. 2 pk. 2 qt.
13. 20 spoons.
14. 16° 40′.

Page 583.

1. $67.46; $168.65; $236.11.
2. 1 lb. 3 oz.
3. .93.
4. 1 yr. 4 mo. 26 da. nearly.
5. 16 days.
7. $\frac{19}{253}$.

Page 584.

8. $630.76−.
9. $46\frac{2}{3}$ inches.
10. .0005207.
11. $\frac{50}{121}$ acre.
12. 30 min. 50 sec.
13. $225.67−.
14. Dec. (10) 13, 1888.
15. Dec. 29, 1892.
16. A, $107\frac{2}{3}$; B, $92\frac{2}{3}$; C, $119\frac{2}{3}$.
17. 3 bu. 3 pk. 3 qt.
18. $$7.87\frac{1}{2}$.
19. $19\frac{893}{1225}$.
20. $\frac{1}{80}$; $\frac{1}{16}$; $\frac{61}{900}$; $\frac{253}{40000}$.
21. 1.299609375.
23. $109\frac{13}{23}$ feet.

Page 585.

24. A, 4340 votes; B, 5551 votes.
25. $133.32−.

---

1. $880.86+.
2. $1229.01−.

Page 586.

3. $193.70+.
4. $446.33−.
5. 4188.48+ marks.
6. 8490.44− francs.
7. £307 7 s. $6\frac{1}{2}$ d. nearly.
8. $2050.72+.
9. $4138.97+.

Page 587.

10. $908.87−.

Page 589.

1. $224.46+.
2. $261.19−.

Page 590.

4. $772.37−.
5. $899.91+.

---

3. 12.
4. $2\frac{11}{15}$.

Page 591.

6. $150; $7500.
7. 61 %.
8. 15,544,041.45 francs.

---

1. $3640.
2. $4; $6\frac{2}{3}$ %.

Page 592.

5. $2 loss.
7. $701.53−.
8. $13.65.
9. $8\frac{1}{5}$ %.
10. $$200.02\frac{1}{2}$.

---

1. $137.61+.

Page 593.

2. $12.39−.

---

1. $8500.
2. $282.25.
3. 23 min. $53\frac{1}{5}$ sec.
4. $17.82.
5. 11 feet.
6. 12 rods.
7. $56.
8. $1\frac{1}{4}$ cents.
9. $300.
10. 3600 yards.

Page 594.

11. $497; ($497.25).
12. 5%. 13. $\frac{3}{10}$ ct.
14. $700.
15. $375.10−.

---

1. $1,181,021.50.
2. $1,037,124.44.
3. $214,854.74.
4. £14 7 s. 2 d.
5. 25 bu. 1 pk. 4 qt.
6. 23 rd. 2 yd. 1 ft. 6 in.

Page 595.

7. $249,981.53.
8. $318,808.78.
9. $202,722.44.

Page 597.

1. $11.
2. $85.25.
3. $95.48–.
4. 24 cents.
5. $18,228.
6. 150 sq. ft.
7. $39,700.
8. $6\frac{1}{2}$%.
9. 138 feet.

Page 598.

10. 48.43\frac{3}{4}$.
11. 44%.
12. 33.12\frac{1}{2}$.
13. 114 sq. yd.
14. $594.
15. $2090.25.
16. $\frac{25}{28}$.
17. $26\frac{2}{3}$%.
18. $363.
19. 6400 lb.; $11.40.
20. $870.62–; ($870.14–).
21. 4.31\frac{1}{4}$.
22. $45\frac{315}{764}$ ¢.
23. $150; $225.

Page 599.

24. $33.60; $15\frac{3}{16}$ cords.
25. $2\frac{1}{2}$; $1\frac{7}{10}$.
26. 10%.
27. $653.48–; ($654.36).
28. 2688\frac{8}{9}$.
29. $3682.19–; ($3684.37$\frac{1}{2}$).
30. $4940.28.

Page 600.

3. 135.62.
4. $12\frac{30}{161}$.
5. 13 feet.
6. 16 T. 19 cwt. 3 qr. $10\frac{1}{2}$ lb.
7. $1235.21–.
8. $88\frac{1}{2}$.
9. 25 cents.
10. $16.48–.

Page 601.

1. $1\frac{1}{4}$ yards.
2. 18.72 feet.
3. $78.75.
4. 8 ft. 4 in.
5. $14,910.75.
6. $108\frac{17}{36}$ sq. ft.
7. 44.17875 sq. ft.
8. 2970 bu.
9. 252 gallons.
10. 146 sq. yd.
11. 3 hr. 12 min.; 4 hr. $6\frac{6}{7}$ min.; 1 hr. 48 min.;

Page 602.

13. $40\frac{1}{2}$ yards; 44.02\frac{1}{2}$.
14. $1\frac{1}{2}$ ft.; 108 sq. ft.
15. 984 sq. ft.; 71 ft.
16. $11,200.
17. $56.

Page 603.

1. .00007722.
2. 1.485135.
3. .450522.
4. 7.70904.
5. .0712111.
6. .0018393.
7. .63672.
8. .0374715.
9. .8220672.
10. .00004768.

———

1. .68515625.
2. 589.84.
3. 153.6.
4. 3265.
5. 50.
6. .064.
7. .00002375.
8. .0115.
9. 79,000.
10. 219.32.

———

3. 4.120275.

Page 604.

4. $1\frac{1}{15}$.
5. 7 fur. 16 rd. 3 yd. 1 ft. 9.888 in.
6. $4.13.
7. $1900.
8. 3 T. 7 cwt. 2 qr. 17 lb. $4\frac{1}{9}$ oz.
9. $15,000.
10. $6.98–.

Page 605.

2. 1,345,595.
3. $3\frac{3}{20}$.

Page 606.

4. 80 miles.
5. .49184.
6. $9.27; $12.36.
7. 468 bricks.
8. $46.08.
9. 55 feet.
10. $5700.
11. $1700.
12. $77.14.
13. $44.48.
14. $215.36+.
15. $747.44–.

Page 609.

1. 14.
2. 16.
3. 18.
4. 24.
5. 26.
6. 36.
7. 35.
8. 42.
9. 44.
10. 51.

11. 53.
12. 54.
13. 61.
14. 63.
15. 72.
16. 75.
17. 83.
18. 84.
19. 91.
20. 95.

---

1. 7568.
2. 5107.
3. 6008.
4. $6285\frac{122064}{607432}$.
5. 6098.
6. 3007.
7. 98,640.
8. 75,064.
9. 70,921.
10. 78,905.

Page 610.

1. $ 183.27–.
2. £150 8s. 4d.

Page 612.

1. $\frac{123}{167}$.
2. $\frac{4}{3}$.
3. $\frac{83}{414}$.
4. $\frac{123}{107}$.
5. $\frac{525}{556}$.
6. $\frac{7}{9}$.

Page 613.

7. $\frac{1250}{3927}$.
8. Equal.
9. $\frac{7}{20}$.
10. $\frac{29}{2}$.

---

1. $ 389.61–.
2. $ 4330.80+.
3. 4 lb. tea, 27 lb. coffee.
4. $ $458.16\frac{2}{3}$.
5. $ 8.25.

Page 614.

6. Dec. (12) 15; (27) 30 da.; $ 1275.88+; ($ 1276.00+).
7. $ 77.61–.
8. $ 332.50; $ 525.
9. 18 men.
10. 256 barrels.

---

1. $ 2167.09–.
2. $ 543.16–.
3. $ 1335.23+.
4. $ 911.66–.

Page 615.

5. $ 784.54–.
6. $ 724.03+.
7. $ 1586.88–.
8. $ 1008.10+.
9. $ 2607.48–.
10. $ 718.00–.
11. $4\frac{1}{2}$ %.
12. $ 600.
13. 4 yr. 6 mo. 9 da.
14. 2 yr. 8 mo. 1 da.
15. $ 600.
16. 4 %.
17. 63 days.
18. $ 400.
19. $ 295.35. ($ 295.50).
20. 7 %.
21. $ 4.10; ($ 4).
22. $ 2750.
23. 90 da.
24. 18 da.
25. $ 475. ($ 474.76–).

Page 616.

1. 567.
2. 915.
3. 144,200.
4. 25,758.
5. 2,114,000.
6. 86,526.
7. 106,000.
8. 374,625.
9. 19,800.
10. 7312.
11. 466,000.
12. 50,133.
13. 98,500.
14. 4180.
15. 423,000.
16. 40,096.
17. 419,904.
18. 310,050.
19. $230\frac{5}{27}$.
20. $227\frac{1}{2}$.

---

1. 12 : 6 P.M.

Page 617.

2. 9 : 36 A.M.
3. 142° 55′ 30″ West.
4. 52° 36′ East.
5. 5 : 16 A.M.

Page 618.

6. 8 : 24 A.M.
7. 12 : 1 : 20 P.M.
8. 143° 3′ West.
9. 82° 40′ West.
10. 11 : 33 : 12 A.M.
11. 3 : 37 : 20 P.M.
12. 88° 4′ East.
13. 32° 34′ East.
14. 2 : 45 : 42 A.M.
15. 23° West.

---

3. $\frac{13}{21}$.
6. $\frac{15}{32}$.
7. $\frac{18}{47}$.
8. $\frac{23}{62}$.
9. $\frac{77}{95}$.
10. $\frac{89}{93}$.
11. $\frac{48}{65}$.
12. $\frac{55}{71}$.
13. $3\frac{1}{2}$.
14. $3\frac{1}{3}$.
15. $1\frac{5}{7}$.
16. $2\frac{2}{15}$.
17. $2\frac{11}{18}$.
18. $3\frac{6}{25}$.
19. $12\frac{1}{2}$.
20. $16\frac{1}{4}$.
21. $10\frac{1}{8}$.

Page 619.

1. 666,862,394,588.21.
5. $7\frac{13}{16}$.
6. $ 1144.73–.
7. $ 5166.69+; $ 283.31–.

Page 622.

Table.

1. 63.30 %.
2. 13.14 %.

3. 8.58 %.
4. 3.83 %.
5. 3.41 %.
6. 2.09 %.
7. 1.11 %.
8. .72 %.
9. 3.82 %.
All others, $ 26,632,801.

Page 623.

1. 63,805,573.34.
2. $ 869,109.89.
3. 524,658,551,-760.
4. $986\frac{2535}{7428}$.
7. (*a*) $1104.02+.
(*b*) $1101.56+; $1116.44+.

Page 624.

8. $ 9862.33+.
9. $\frac{1}{7}$.
10. $ 72.06.

---

1. 80.
2. 4.
3. $17\frac{1}{2}$.
4. 28.
5. 5.
6. 7.
7. 36 cents.
8. 70 cents.
9. 10 bottles.
10. 4 men.

Page 625.

3. 12.
4. 8.
5. 70.
6. $15\frac{157}{162}$.
7. $\frac{35}{72}$.
8. $\frac{1}{7}$.
9. $1\frac{3}{5}$.
10. $\frac{9}{32}$.
11. 1.
12. $1\frac{43}{45}$.
13. 6.

Page 626.

2. $ 625.92.
3. 3968 revolutions.
4. 27 kilometers.

Page 627.

5. 770 meters; 42 kilos.
6. $ 504.
7. $8\frac{1}{2}$ days.
8. $ 82.50.
9. 14 yards.
10. $ 2.45.

Page 628.

21. 40 and 10 %.
22. 50 and 10 %.
23. 30 and 10 %.
24. 30 and 5 %.
25. 30 and 15 %.
26. 50 and 10 %.
27. 20 and 50 %.
28. 40 and 5 %.
29. 60 and 10 %.
30. 40 and 15 %.

---

1. $ 135.
2. 10 %.
3. $ 150.
4. 10 %.
5. $ 8.96.

Page 629.

6. 60 cents.
7. 20 %.
8. $ 150.
9. $ 120.
10. 20 %.

---

1. 254.
2. 27.1.
3. 1.37.
4. 26.8.
5. 3.76.
6. .838.
7. 16.27.
8. .4876.
9. .306.
10. .069.

Page 630.

1. $ 6.24.
2. 4 hr. 24 min.
3. $ 22.50.
4. 150 barrels.
5. $ 153.60; $ 192; $ 230.40.
6. 1740 tiles.
7. $ 600,000.
8. $4\frac{1}{4}$ %.
9. 5.0004.
10. 2880 gal.
11. .013972.
12. $ 2351.25.

Page 631.

13. $\frac{47}{378}$.
14. 1 mi. 95 rd. 1 yd. 6 in.
16. £ 23.
17. $ 6000; $ 14,000.
18. 50 %.
19. 4 %.
20. $ 372.
21. $1\frac{23}{37}$ days.

---

1. $7\frac{311}{952}$.
2. $\frac{163}{315}$.

Page 632.

3. $22\frac{7}{144}$.
4. $\frac{4}{9}$; 4 s. 6 d.
5. $13\frac{83}{144}$.
6. $\frac{3}{800}$ acre; 218 lb. 12 oz.
7. $\frac{8}{15}$; $\frac{17}{83}$.
8. $\frac{1}{8}$.
9. $42\frac{65}{99}$.
10. 1.
11. 10.
12. £ $\frac{3}{8}$; $\frac{3}{10}$ da.
13. 8.
14. 13 s. $3\frac{11}{21}$ d.
15. $\frac{881}{1247}$.
16. $3\frac{3}{28}$.
17. 370.
18. $ 520.
19. $ 918.75.
20. £ 1060 18 s. 9 d.

Page 633.

1. $49\frac{181}{256}$ cents; $65\frac{163}{256}$ cents.
2. $ 13,227.50.
3. 104 days.
4. $ 591.09+.
5. $\frac{225}{1024}$.
6. $ 782,300.
7. $ 100.
8. $ 1005.50.
9. 60 %.

Page 634.

1. $100.02.
2. $500.
3. $1.75 discount per $1000.
4. $700.
5. $198.80.
6. 33 days.
7. Sight.
8. 7 %.
9. Par.
10. 5 %.

---

1. 23 inches.
2. 1 ft. 11 in.
3. 120 rods; 2640 yards.

Page 635.

4. 66 feet; 7.92 inches.
5. 15 inches.
8. $12\frac{1}{2}$ inches.
9. 24 inches.
10. 5 feet.

Page 636.

1. $8.40; ($8).
2. $629.20.
3. $650.39–.
4. $225.16+.
5. $343.61–. ($343.43+).
6. $484.75; ($485).
7. $60.63+.
8. $2.

Page 637.

10. 5 acres; $2\frac{3}{31}$ days.

---

1. $327.05+.
2. $154.44+.
3. $291.08–.
4. $1276.28+.
5. $874.75+.

Page 638.

1. $2109.
2. $95.25.
3. $40.
4. $34.37½.
5. $5106.25.
6. $1080; 4 %.
7. 10 %; $5\frac{5}{7}$ %.
8. $3\frac{3}{4}$ %.
9. $\frac{1}{8}$ %.

Page 639.

10. $62.50; $\frac{1}{4}$ %.

---

2. Increased, $68.75.
3. $357.42.
4. $175.
5. 6's.
6. East, 15°.
7. 7 A.M.
8. $2437.60+.
9. $4800; ($4797.58–).

Page 640.

1. 2.646–.
2. 3.742–.
3. 6.164+.
4. 8.602+.
5. 18.708+.
6. 27.532–.
7. 28.408–.
8. 37.202+.
9. 43.290–.
10. 63.245–.

---

1. .316+.
2. .632+.
3. .949–.
4. .316+.
5. .632+.
6. .949–.
7. 1.265–.
8. 1.586+.
9. 1.897+.
13. 2.214–.
14. 2.530–.
15. 2.846–.
16. 3.162+.
17. .348–.
18. .379+.
19. .411+.
20. .443–.

---

3. 16,203.03.

Page 641.

4. 172,030.
5. 9 %.
6. $1160.32+.
7. $592.48–.
8. $120.75; $162.61; $62.79; $83.72; $93.38.

Page 642.

9. $126.80.
10. $171.98; $194.01; $174.08; $167.87.

Page 643.

1. 17.
2. 12.
3. 36.
4. 29.
5. 28.
6. 33.
7. 73.
8. 48.
9. 16.
10. 113.
11. 180 yards.
12. 10 rods.
13. 12 rd.; 18 rd.
14. $\frac{21}{40}$ acre.

Page 644.

15. $420.
16. 192 rods.
17. 50 rods; 1430 yd; $21\frac{1}{4}$ acres.
18. 150 yards.
19. 396 yards.
20. 113.14– rd

---

1. $16\frac{2}{3}$ %.
2. 3 minutes.
3. 9 ¢; 3 ¢.

Page 645.

4. 300 %.
5. 7 × 10, etc.
6. $1.50.
7. 100 %.
8. 20.

9. 12,000.
10. 48 cases.

---

1. £45 16 s.
2. 1 lb. 14 oz.
3. 10 men.
4. 44 yards.
5. 3 hr. 16 min.
6. 319 rd. 4 yd. 1 ft. 6 in.

Page 646.

7. 10 feet.
8. 4 acres; 16 acres.
9. $2\frac{1}{4}$ pounds.
10. 432 pounds.
11. 6 days.
12. 4 %.
13. $4\frac{3}{10}$ %.
14. $4\frac{1}{2}$ %.
15. $8000.
16. $1000; $1400.

Page 647.

17. $70,000.
18. 68 rd. 3 yd. 4 in.
19. $165; $210; $225.
20. $315.
21. $6\frac{126}{1979}$ %; 6 %.
22. 6 hours.

---

1. $337.68+.
2. 1.5625.
3. (*a*) $2\frac{19}{105}$. (*b*) $2\frac{7}{48}$. (*c*) 180. (*d*) $37\frac{1}{2}$. (*e*) Impossible.

Page 648.

4. $24.
5. 1 mi. 7 fur. 27 rd. 3 yd. 2 ft. $11\frac{1}{2}$ in.
6. L.C.M. 360.
7. 194.45+ ft.
8. $6500.
9. $22\frac{2}{9}$ %.
10. 1200 peaches.

---

1. $4.45–.
2. $3.60+.

Page 649.

3. $5.76.
4. $4.38.
5. $1.17+.
6. $16.72–.
7. $11.94.
8. $51.40.
9. $76.80.
10. $92.38+.

---

1. $1000.
2. $189.92–
3. $31,000.
4. $12\frac{1}{2}$ %.
5. .7525 miles.
6. 10 mo. 17 da. nearly.
7. (10¢ per bu.); $4\frac{1}{6}$ %.

Page 650.

9. $1.72–.
10. $66\frac{2}{3}$ %.
11. 11.
12. 10,000.
13. $\frac{13}{21}$; $\frac{17}{23}$.
14. $15,000.
15. 896 pounds.
16. $472\frac{11}{112}$.
17. $$28.84\frac{1}{2}$.
19. $637\frac{1}{5}$.
20. $$166.58\frac{1}{3}$.
21. 6000 copies.
22. $18\frac{2}{11}$ years.
23. $28\frac{268}{539}$ bbl.
24. 80 rods.

Page 651.

2. $1091.66+.
3. $128\frac{4}{7}$ %.
4. $34,312.50.
5. $86.44–.
6. $1551.27+; ($1552.06–).
7. $6000.

Page 652.

8. (*a*) $33,000; (*b*) $13\frac{1}{2}$ %.
9. N. Y. & N. E. $50.50.
10. 40 acres.

---

2. 1470 sq. ft.
3. 294 sq. rd.
4. 1764 sq. rd.
5. 300 sq. yd.
6. 42.332+ ft.; 2031.94– sq. ft.

Page 653.

7. 15.588+ sq. ft.
8. 1176 sq. ft.
9. 672 sq. ft.
10. 3.464+ sq. in. Diagonals, 2 in. and 3.464+ in.
11. 21.008+ feet.
12. 50.2656 sq. in.
13. $3.1416\,x^2$.
14. 10 inches.
15. 19.635 sq. ft.

Page 654.

1. $215.75.
2. $414.33+.
3. $125.90.
4. $104.55.

Page 655.

5. $157.68+.
6. $453.61+.

---

1. 84 sq. ft.
2. 234 sq. yd.
3. 264 sq. rd.
4. 84 sq. in.
5. 990 sq. ft.
6. 900 sq. ft.
7. 420 sq. yd.

Page 656.

8. 330 sq. rd.
9. 744 sq. rd.
10. 240 sq. ch.

---

1. $215\frac{499}{715}$.

2. 1.975.
3. $100.
4. $52.02.
5. $489.75.
6. 93 lb. $9\frac{3}{5}$ oz.
7. $4\frac{661}{722}$.
8. $653.08–.
10. 2, 5, 7, 13, 23.

Page 657.

1. 3 months.

Page 658.

2. 9 months.
3. 1 yr. 5 mo.
4. $3\frac{4}{9}$ months.

Page 659.

5. $28\frac{1}{3}$ days.
6. $2\frac{1}{2}$ months.
7. $4\frac{1}{12}$ months.
8. 7 mo. 26 da.
9. $4\frac{1}{2}$ months.
10. August 10.
11. $94\frac{3}{4}$ cents.
12. $31.50; $21; $31.50.
13. A, $3500; B, $3600.

Page 660.

14. 60 bu.; 40 bu.
15. 20 bushels.
16. A, $36; B, $24.
17. A, $875; B, $1458.33+; C, $1166.67–.
18. A, 54 tons; B, $31\frac{1}{2}$ tons; C, $94\frac{1}{2}$ tons.
19. $6\frac{2}{3}$ quarts.

---

1. 186,441.

Page 661.

2. $34,538,549\frac{6}{7}$.
3. $82,739\frac{101}{789}$.

---

1. $540.
2. 220 %.
3. $440.
4. $2\frac{3}{4}$ hours.
5. $1440.
6. 1350 sq. in.

Page 662.

7. $1.29.
8. Feb. 12, 1809.
9. $5 gain.
10. 2025.

---

1. $9.52.
2. 1.464375.
3. 56.65– feet.
4. $21.
6. A, $2; B, $3; C, $3.50; D, $4.50.

Page 663.

8. $\frac{84}{96}$; ($\frac{7}{8}$).
9. 4725 lb.

---

1. 9600 men.
2. $37.75–.
3. $9.84+.
4. $4000.
5. 2.16603.
6. 7.95– %.
7. 5 yr. 5 mo. 20 da.
8. $3800.47+.
9. 58 feet.

Page 664.

10. $8\frac{8}{11}$ feet.
11. 7 %.
12. 1 yr. 10 mo.
13. $9956.86–.
14. $4\frac{5}{6}$.
15. $759.76.

Page 665.

1. $336\frac{48}{157}$.
2. $17\frac{5}{11}$ rods.
3. $1533.75.
4. $116.36–.
5. 7 bu; 5 bu. 1 pk. 3 qt.
8. 10 acres.
9. .66+.
10. 21 lb. 5 oz. 18 pwt. $20\frac{1}{2}$ gr.
11. 7 %.
12. $16,000.
13. 113.14– rd.

Page 666.

14. $\frac{17}{31}$.
15. 4096 cu. ft.
17. $331.86–.
18. $223\frac{5}{8}$ sq. ft.
19. $28.01+.
20. $5\frac{17}{75}$ tons.
21. $40.50.
22. 56.8+ %.
23. $27.84+.
24. 16– cents.
25. $\frac{112}{545}$; $\frac{176}{981}$; $\frac{1001}{4905}$; $\frac{224}{545}$.
26. 240 rods.

Page 667.

1. $6.2832x$.
2. $3.1416x^2$.
3. $.7854x^2$.
4. $.07958x^2$.
5. 1017.8784 sq. ft.
6. 7 yards.
7. 50 rods.
8. $\frac{x^2}{2}$.
9. 11,250 sq. rd.
10. 2688 sq. m.
11. 62.35+ sq. ft.
12. 1469.69– sq. yd.
13. 7.958 sq. ft.
14. 960 sq. rd.

Page 668.

15. 4800 sq. yd.
16. 541.27– sq. rd.
17. 3800 sq. rd.
18. 234 sq. rd.
19. 779.42+ sq. yd.
20. 36 feet.
21. 40 rods.
22. 93.53+ sq. in.
23. 113.10– sq. in.
24. 50 sq. ft.

Page 669.
25. 62.832 sq. in.

---

1. 18,990.59+.
3. $\frac{1}{4}$%.
4. $49.02.
5. $21.

Page 670.
6. $220.54–; ($220.67–).
7. $310.85+.
8. £1,411,734 6 s. 1 d.
9. $21,050.
10. $1200.

---

1. $1400.
2. $8.63–.
3. $1917.
4. 284 days.
5. 357.
6. 143.
7. .8.
8. 14 feet.

Page 671.
9. 3 mi. 207 rd. 1 yd. 1 ft. 6 in.
10. 33 mi. 225$\frac{3}{13}$ rd.
11. 1 hr. 2 min. 52 sec. P.M.
12. $72.
13. .6 week.
14. $34.26+.
15. 42$\frac{2}{3}$ yards.
16. 175 sheep.
18. $425.51.
19. 11$\frac{2}{3}$ rolls.
1. $6224.35.
2. 2 yr. 4 mo. 3$\frac{3}{4}$ da.

Page 672.
3. $1441.94+.
4. $150; $270.
5. 10%.
6. 288 boards.
7. 49 rods.
8. $1540.
9. 14 rods.
10. $50,000.

---

1. $69.85–.

Page 673.
2. $12,500.
3. 3%.
7. 20.
10. 14 weeks.
11. 21 men.
12. 240 miles.
13. $96.
14. $15,000.
15. $128.

Page 677.
$432.63–.

Page 678.
1. 306 sq. ft.; 12 ft.
2. 126 sq. yd.; 12 yd.
3. 1110 sq. rd.; 15 rd.
4. 210 sq. ch.; 15 ch.
5. 600 sq. in.; 16 in.
6. 210.
7. 1260.
8. 594.
9. 3366.
10. 3060.

---

1. $88.73+; $86.27–.
2. 309.37\frac{1}{2}$; 464.06\frac{1}{4}$; 513.56\frac{1}{4}$.
3. 27\frac{1}{12}$; 32\frac{1}{2}$; 29\frac{1}{6}$; 41\frac{1}{4}$.

Page 679.
4. $4945.05+; $5934.07–; $4120.88–.
5. $42.96–; $34.80–; $28.35+; $46.39+.
6. 6 men.
7. 31$\frac{3}{7}$ days.
8. 208 acres.
9. 240 men.
10. 20 days; 24 days.

Page 680.
11. $13.
12. $226.80.
13. 1363$\frac{1}{5}$ lb.
14. 12$\frac{1}{2}$ days.

Table.
1883, $25.923.
1884, $26.254.
1885, $28.972.
1886, $26.306.
1887, $27.543.
1888, $27.307.
1889, $29.499.
1890, $25.772.
1891, $25.680.
1892, $27.801.

Page 681.
1. 96 sq. in.
2. 72 sq. in.
3. 144 sq. in.
5. 75.3984 sq. in.
7. 294 sq. in.; 6 x^2.
8. 6 inches.
9. 216 sq. in.

Page 682.
10. 128 sq. in.
11. 10 feet.
12. 1200 sq. in., 20 in.
13. 702 sq. in.
14. 4.7124 sq. ft.
15. 48 sq. in.
17. 64 sq. in.
18. 576 sq. in.
19. 6.93+ sq. in.
20. 37.6992 sq. in.
21. 122.5224 sq. in.

Page 683.
22. 13 inches.
23. 122.5224 sq. in.

---

1. 130,548 sessions.
2. 11 poems.

3. $237.40;
($237.52).
4. $10,000.

Page 684.

5. $40.
6. 303 feet.

Page 686.

1. $10.34+;
$505.38–.
2. $78,133$\frac{1}{3}$.
3. 68$\frac{1}{2}$%.
4. 57 cents.
5. 4 lots.
6. $55.15–.
7. $7600.
8. 285 acres ;
$213.75
commission.
9. $425.52+;
$443;
($446.26+).
$900;
($899.54+).
10. $1394.05.

Page 687.

11. 4$\frac{31}{36}$ tons.
12. 21$\frac{1}{8}$ acres.
13. .56; .0012.
14. 144 yards.
15. 26$\frac{11}{14}$ cents.
16. 1$\frac{79}{121}$ cents.
17. $441.
18. $12,204.
19. 27 sq. yd.
20. 7$\frac{1}{7}$%.

Page 688.

21. 395,999.-
922186;
$$\frac{21}{40}$; $\frac{3}{2}$.
22. 24$\frac{4}{9}$ yards.
23. $2022.22+;
$371.44–;
(371.62\frac{1}{2}$).
24. 20%.
25. $1280.42+;
($1279.54+).
26. $8,575,875.
27. $54,368.52$\frac{1}{2}$.
28. 3960 inches.
29. 5$\frac{5}{9}$% gain;
5$\frac{5}{19}$% loss.
30. 67,750 acres.

Page 689.

1. 18 cu. ft.
2. 400 cu. in.
3. 60 cu. in.
4. 82$\frac{1}{3}$ cu. yd.
5. 83.14– cu. ft.
6. 30.56– cu. ft.
7. 169.65– cu. meters.
8. 1163.29– gal.
9. 9719.325 lb.
10. 57$\frac{3}{4}$ cu. in.

Page 690.

12. 3$\frac{1}{2}$ inches.
13. 301.59+ cu. yd.
14. 13 inches.
15. 700 sq. in.;
896 sq. in.;
1568 cu. in.

———

1. $1,468,380,-
830.
2. 1,001,101.
3. 202,100,001.-
00006.
6. 1983.38\frac{1}{8}$.

Page 691.

7. $2.382+.
8. 10 T. 17 cwt.
3 qr. 8 oz.
9. $51,000;
$1260;
$49,740.
10. $353,369,-
654.14;
94.84– %.
11. $47,891,-
785.50;
3.9083+ %.
12. 5$\frac{285}{343}$%.

Page 692.

13. $8000.
14. Due, $11,-
646.19.

Page 693.

1. 16 (board) ft.
2. 7 (board) ft.
3. 8 (board) ft.
4. 14 (board) ft.
5. 4 (board) ft.
6. 18 (board) ft.
7. 7 (board) ft.
8. 24 (board) ft.
9. 14 (board) ft.
10. 16 (board) ft.
11. 40$\frac{1}{2}$ (board) ft.
12. 27 (board) ft.
13. 30 (board) ft.
14. 15 (board) ft.
15. 9 (board) ft.
16. $16.20.
17. 1575 (board) ft.
18. $3.56.

Page 694.

19. $60.48.
20. $52.

———

2. 3$\frac{67}{75}$.
3. 6 lots.
4. 5889\frac{1}{3}$.
5. 36 days.
6. 108.
7. 146.86+ mi.
8. B, $1000;
C, $1500.

Page 695.

9. $5.94–.

———

1. 682.32 sq. ft.
2. 117.81 sq. in.
3. 50 sq. in.
4. 28.54 sq. in.
5. 7.854 in;
7.071+ in.
6. 78.54 sq. in.;
28.54 sq. in.
7. 3.1416 sq. in.;
12.5664 sq. in.

8. 1 to 9; area $=R^2\times 3.1416$.

Page 696.

9. $43.6\frac{1}{3}$ sq. yd.
10. 392.7 sq. ft.
11. 84.8232 sq. in.
12. 3 : 1.
13. 500 sq. ft.
14. 500 sq. ft.
15. 1 : 4.

Page 698.

16. 12.5664 sq. in.
17. 31 cents.
18. Equal.
19. 2 : 3.
20. 127.328 sq. in.

---

1. $1\frac{1}{2}$ %; $90.
2. $88,922.4231.

Page 699.

3. $13,173.60.
4. $$112\frac{1}{2}$.
5. $6050.

Page 700.

1. 13.
2. 21.
3. 32.
4. 41.
5. 53.
6. 62.
7. 75.
8. 82.
9. $\frac{8}{9}$.
10. $\frac{11}{14}$.
11. $\frac{20}{23}$.
12. 1.5.
13. $1\frac{1}{8}$.
14. $1\frac{1}{11}$.
15. $1\frac{7}{9}$.

Page 701.

1. 113.0976 cu. in.
2. 14.1372 cu. in.
3. 1 : 8.
4. .5236 : 1.

Page 702.

5. 113.0976 in.; 4071.5136 sq. in.; 24,429.0816 cu. in.
6. 245 lb. 7 oz.
7. 2 : 3.
8. .4764; about $\frac{1}{2}$.
9. .2146; about $\frac{4}{5}$.

---

1. 44 men.
2. A, $100; B, $120; C, $120.
3. 7.62+ ft.; (16.89+ ft.).
4. $821.76+; ($825.50).

Page 703.

5. $4160.30–; ($4157.60–).
6. 452.3904 sq. in.; 904.7808 cu. in.
7. 3 hr. 13 min. 36 sec. fast.
8. 185.61– sq. ft.
9. 192 shares.
10. $3200.
11. 3450 copies.
12. A, $375; B, $150; C, $100.
13. $1059.35+; ($1059.39–).

Page 704.

14. $1434.29+.
15. 2 : 58 : 48 P.M.
16. $19\frac{13}{98}$ days.
17. $7\frac{1}{2}$ days.
18. $13,000.
19. 2013.7824 sq. in.
20. 452.3904 cu. in.

---

2. (*a*) $1\frac{79}{112}$; (*b*) $\frac{5}{16}$.

Page 705.

6. $17,728.53–.
7. 15 hours.
8. $73.80.
9. 4 mi. per hr.
10. 10 hours.

Page 706.

12. (*a*) 8; (*b*) $1\frac{7}{8}$; (*c*) .625.
13. (*a*) 2 rd. 1 yd.
14. (*a*) .001, .0001024, 32.004; (*b*) .003672; (*c*) 1600.
16. (*a*) $\frac{3}{7}$; (*c*) .125 acre.
17. 25 %.
18. A, $50; B, $13,600; C, $4350.

Page 707.

19. 100 miles.
20. $2000.
22. (*a*) $\frac{1}{3}$; (*c*) $\frac{11}{32}$.
23. (*a*) $\frac{16}{21}$; (*b*) .00375; (*c*) .16.

Page 708.

25. (*b*) 4000; (*c*) 13.163; (*d*) 1.706.
27. $$33\frac{1}{3}$.
28. $200,000.
29. 2 men.
30. $75.

Page 709.

32. (*a*) 7; (*c*) $1\frac{11}{30}$.
33. (*a*) $\frac{7}{10}$; (*b*) 22 rd. 4 yd. 2 ft. $1\frac{5}{7}$ in.; (*c*) .0625.
35. (*b*) 20,007.253; (*c*) .00003.
36. (*a*) .00091; (*b*) 00006.
37. 8 men.

38. $1333\frac{1}{3}$; $2000; $2666\frac{2}{3}$.

Page 710.

39. $1514.
40. $60.

---

1. 113.
2. 124.
3. 155.
4. 341.
5. 2.35.
6. 4.06.

Page 711.

1. $109; $15.95–.
2. $8\frac{1}{3}$%; 15 years.
3. $600; 90 (87) days.
4. January 12.
5. $950; $5937.50.
6. $2\frac{7}{31}$%; $2\frac{13}{31}$%.
7. 60 feet; 231.
8. $2\frac{1}{2}$ feet; 263.8944.
9. 171.65+ sq. in.

Page 712.

10. 351.8592 sq. in.; 502.656 cu. in.

---

2. $3\frac{3}{8}$.
3. 122° 26′ 15″ West.
4. .00005.
5. A, $2870; B, $7175; C, $1435.
6. $545.82+.
7. 8%.
8. 117 feet.
9. 2.16+.
10. $16\frac{2}{3}$%.

Page 714.

1. $44.83\frac{1}{2}$.
2. $\frac{39}{40}$.
3. $25.62\frac{1}{2}$.
4. $9.80.
5. 18 days.
6. 45 men.
7. $44,092.
8. $65.
9. $246.36+; ($246.15+).

Page 715.

10. £6 3 s. $11\frac{1}{2}$ d.

---

1. 17,000; .0002938. 1st, greatest; 2d, least. .000003125.
2. $\frac{37}{650}$; $5\frac{73}{135}$; $\frac{9}{14}$, $\frac{16}{63}$.
3. £.4545–; 28,475 m.; $35.17\frac{1}{2}$.
4. A, $105; B, $87.50. 5 miles.
5. $5600; $97,920.

Page 716.

6. 30,013; .1716–.
7. 26 days.
8. $40.80.
9. $189; $147.
10. 17.43–%.
11. 7 mo. 6 da.
12. The latter.
13. $\frac{1}{30}$ less.

Page 717.

14. $5145.
15. 5 miles.
16. $131\frac{1}{4}$ miles.

---

1. .023825+; $18\frac{3}{4}$%; $\frac{2063}{9900}$.
2. $715.
3. 18.000002+; 7.745967–.

Page 718.

4. 2945 kilos nearly.
5. $1521.30–, ($1520.27+); $1837.99–, ($1836.73+).

---

1. Latter, $\frac{1}{72}$ greater.
2. .909.
3. $101.86+.
4. 23,048,771 sq. in.
5. Thos., $2.25; Henry, $1.35; Richard, $0.54.
6. $233\frac{3}{4}$ qt. dry measure.
7. $\frac{325}{573}$ minute.

Page 719.

8. A, 56 times; B, 35 times; C, 22 times.

---

1. $756.96.
2. $1530.
3. $821.57+.
4. $1178.46.

Page 721.

1. $150.
2. $360.
3. 5 miles.
4. 162 miles.
5. 15,840 feet.
6. 760 acres.

Page 722.

1. 989.95 francs.
2. 22.525 sq. m.
3. 176.715 sq. m.
4. 93.15 ares.
5. 1029 ares.

Page 723.

6. 276.25 francs.
7. 546,000 liters.
8. 296 bottles.
9. 306 marks.
10. 55,200 kilos.
11. .62137– mi.

12. 15,748 feet.
13. 64 cu. in.; $1\frac{25}{231}$ qt.
14. $2\frac{41}{42}$ bushels; $27\frac{163}{231}$ gal.

Page 724.

15. $2\frac{17}{54}$ pounds.
16. 25,2525.25+ miles.
17. $1\frac{19}{81}$ sq. yd.
18. $2\frac{5398}{9801}$ acres.
19. $20\frac{20}{99}$ rods.
20. $37\frac{1}{27}$ cu. ft.
21. $16\frac{11}{54}$ grains.
22. 1.584 Km.

Page 725.

23. \$19.82–.
24. 289.56– mi.
25. \$24.80–.

Page 726.

1. 101 lb. 7 oz. 12 pwt. 12 gr.
3. 33 mi. 172 rd. 2 ft. $1\frac{5}{7}$ in.

Page 727.

4. 1 hr. 13 min. $36\frac{4}{5}$ sec.; 26° 15′ east.
5. \$96 gain.
6. 18.8496 feet; 28.2744 sq. ft.; 12.5664 cu. in.
7. 48 ¢; \$1.20.
8. 40 acres.
9. $192\frac{6}{7}$ bushels; 231 cu. in.
10. 99 yards.

Page 729.

3. $-6\,xy$.
4. $11\,abc$.
5. $-3\,xyz$.
6. $x+6$.
7. $-10\,a-8\,x$.
8. $-a+5\,b-4\,c$.
9. $1\frac{1}{4}\,x-1$.
10. $3\frac{1}{3}\,x+38\frac{1}{2}$.

Page 731.

9. $2\,x+4$.
10. $8\,x+1$.
11. $-4\,x+9$.
12. $-2\,x+20$.
13. $4\,x+5$.
14. $-x-3$.
15. $2\,x+3\,a+4$.
16. $15\,y-z-5\,b$.
17. $-2\,d+e+f$.

Page 732.

1. 6.
2. 9.
3. 6.
4. 4.
5. 9.

Page 733.

7. 9.
8. 6.
9. 6.
10. $3\frac{6}{7}$.

Page 734.

11. 11.
12. 11.
13. 35.
14. 9.
15. $-5\frac{1}{2}$.
16. 2.
17. 2.
18. 4.
19. $6\frac{2}{3}$.
20. 120.
21. 120.
22. $186\frac{2}{3}$.
23. 12.
24. $\frac{1}{2}$.
25. 12.

Page 735.

1. 15.
2. 96.
3. 20 years.
4. \$1600.
5. 96 marbles.
6. Father, 88 years; son, 50 years.
7. 20 cents.
8. 3 years.

Page 736.

9. 48 gallons.
10. 432.
11. 118; 548.
12. 14 two-dollar bills; 15 five-dollar bills.
13. 6 years; 36 years.
14. 12; 15.
15. \$3, son; \$4, father.
16. 32; 15.
17. 13 years.

---

1. 10 cents.

Page 737.

2. 8.
3. 50 cents.
4. 2 pigs.
5. 40 years.
6. 4 cents.
7. 12 yards.
8. 24 cents.
9. 6.

---

1. 4.
2. 5.
3. 12.
4. 7.
5. 24.

Page 738.

6. 4.
7. 6.
8. 9.
9. $8\frac{1}{3}$.
10. 50.
11. Coat, \$12; vest, \$3.
12. 3.
13. 8.
14. $x=7, y=8$.
15. $x=9, y=6$.
16. $x=3, y=4$.
17. $x=5, y=10$.

Page 739.

18. $x=6, y=8$.
19. $x=1, y=1$.
20. $x=1\frac{1}{2}, y=2\frac{1}{2}$.
21. $x=11, y=7$.

22. $x=2, y=3$.
23. $x=12, y=3$.
24. $x=1, y=2$.
25. $x=3, y=4$.
26. $x=\frac{1}{3}, y=\frac{1}{2}$.
27. $x=7, y=5$.
28. $x=4, y=-4$.
29. $x=2, y=19$.

Page 740.

30. $x=4, y=24$.
31. $x=42, y=63$.
32. $x=5, y=7$.
33. $x=64{,}000$, $y=36{,}000$.
34. $x=6, y=3$.
35. $x=8, y=9$.
36. $x=11, y=15$.
37. $x=23, y=18$.
38. $x=13, y=9$.
39. $x=10, y=5$.

---

1. 15 and 22.
2. 47 and 19.
3. 5 and 4.
4. 23 and 42.

Page 741.

5. 19 two-dollar bills; 13 five-dollar bills.
6. 10 pigs; 15 sheep.
7. Oranges, 3 ¢; peaches, 2 ¢.
8. Tea, 60 ¢; coffee, 25 ¢.
9. 4 horses, 16 cows, 32 sheep, 2 pigs.
10. Raisins, 12 ¢; cheese, 17 ¢.
11. 12 and 7.
12. $\frac{35}{91}$.
13. $\frac{168}{315}$.
14. 40 pounds.

Page 742.

15. 40 pounds green tea; 60 pounds black tea.

Page 743.

2. $x=2, y=3, z=5$.
3. $x=7, y=13, z=1$.
4. $x=12, y=31, z=19$.
5. $x=10, y=13, z=16$.

Page 744.

6. $x=12, y=18$.
7. $x=12, y=6$.
8. $x=6, y=5$.
9. $x=19\frac{1}{2}$, $y=-17$.

---

1. $30,000.
2. A, 20 chestnuts; B, 2 chestnuts.
3. $5; $3.
4. 17, 38, and 45.

Page 745.

5. 21 and 32.
6. $18; $32; $16.
7. $600.
8. $150.
9. $66\frac{1}{3}$ acres; $38\frac{5}{6}$ acres; $25\frac{1}{12}$ acres.
10. 70 yards.
11. $20.

Page 746.

1. $x^2+7x+10$.
2. $x^2+17x+72$.
3. $2x^2+9x+10$.
4. $2x^2+26x+72$.
5. $3x^2+22x+7$.
6. $4x^2+4x+1$.
7. $x^2+6x-27$.
8. x^2+x-42.
9. x^2-25.

Page 748.

1. ± 7.
2. ± 5.
3. ± 5.
4. ± 5.
5. ± 5.
6. 24.
7. 24.
8. ± 5.
9. 13.
10. 5.
11. ± 7.
12. ± 1.
13. ± 8.
14. ± 4.
15. ± 6.
16. 7.
17. 13.
18. ± 6.
19. 13.
20. 13.

---

1. 60 rods; 30 rods.
2. 4 inches.
3. 10 and 8
4. 45.
5. 50.

Page 749.

6. 2500.
7. 12 rods; 20 rods.
8. 12 yards.
9. 8 feet.
10. 24 and 25.

Page 751.

1. $x+3=\pm 7$.
2. $x-6=\pm 8$.
3. $x-4=\pm 6$.
4. $x-8=\pm 5$.
5. $x+9=\pm 10$.
6. $x+1=\pm 5$.
7. $x-7=\pm 8$.
8. $x-11=\pm 12$.
9. $x+7=\pm 10$.
10. $x-11=\pm 13$.

---

1. 7 or -1.
2. 18 or -6.
3. 6 or -8.
4. 5 or -23.
5. 13 or 1.
6. 10.
7. 5 or -25.
8. 2 or -28.
9. 24.
10. 24 or -16.
11. 3 or 1.
12. 5 or -35.

13. 1 or − 29.
14. 4 or − 26.
15. 8.
16. 2 or − 38.

Page 752.
1. 3 or − 4.
2. 5 or − 2.
3. − 1 or − 4.
4. 8 or − 1.
5. − 4 or − 5.
6. 7 or 4.
7. − 6 or − 7.
8. 19 or − 4.
9. 18 or − 1.
10. − 18 or − 1.

---

1. 3, − 2.
2. 4, − 5.
3. 3, − 7.
4. 8, 2.
5. 9, − 7.
6. 3, − 6.
7. 5, 4.
8. 1, − 8.
9. 2, − 9.
10. 4, 1.

Page 753.
1. 4 and 8.
2. 80 feet.
3. 60 yards.
4. 25 yards.
5. 5 feet.
6. 4 feet.
7. 68 rods.

Page 754.
8. 34 feet.
9. 65 feet.
10. $3\frac{1}{5}$ feet.

11. $AC = 42$ ft. $ED = 24$ ft.

Page 784.
9. 1 : 4.
10. 90°, 5 in.; $1\frac{1}{2}$ in., 2 in., $2\frac{1}{2}$ in.; 37°, 53°, 90°.

Page 785.
11. $\frac{23}{72}$ in.

---

1. 60 feet.
2. 48 feet.

Page 786.
3. $109\frac{1}{11}$ feet.
4. 2160 feet; 2060 feet.
5. 124 feet.

Page 787.
6. $13\frac{6}{13}$ chains.
7. 162 feet.
8. 84 feet.

Page 788.
9. 108 feet
10. 119 yards.

---

1. 2.0944 in.; 4.1888 in.; 6.2832 in.; 8.3776 in.; 10.4720 in.
2. 2 inches; 3.464+ in.; 4 inches; 3.464+ in.; 2 inches.

3. 35°; 40°; 105°; 105°.

Page 793.
10. 14.1372 sq. in.
11. $1\frac{1}{2}$ in., 3 in.; 1 in., 3 in.

Page 794.
12. 108°.
14. Two, 15 in.; two, 13 in.; 384 sq. in.
15. 240 sq. in.
17. 40 sq. ft.

Page 795.
20. 122.5224 sq. in.
21. 175 sq. in.
22. 257.6112 sq. in.
23. 27 feet.

Page 796.
24. 452.3904 sq. ft. 254.4696 sq. ft.
25. $4166\frac{2}{3}$ mi.
26. 2000 miles.
27. 12,500 miles.
28. 1 : 2.
29. 2828.4+ mi.

Page 797.
30. 113.0976 sq. in.
31. 56.5488 sq. in.; 28.2744 sq. in.; 84.8232 sq. in.

---

1. 72 cu. in.
2. 36 cu. in.
3. 36 cu. in.

Page 798.
4. 36 cu. in.; 124.71− cu. in.
5. 690 cu. in.
6. 62.882 cu. in.

Page 799.
7. 144 cu. in.; 18 cu. in.
8. 126 cu. in.
9. 1350 cu. in.; 50 cu. in.
10. 1300 cu. in.; 520 sq. in.; 770 sq. in.

Page 800.
11. 1300 cu. in.
12. $65\frac{1}{3}$ cu. ft.
13. 2232 cu. in.

Page 801.
14. 1927.3716 cu. in.
15. 929.9136 cu. in.; 4.0256− gal.

Page 803.
16. 3053.6352 cu. in.
17. 381.7044 cu. in.; 190.8522 cu. in.

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