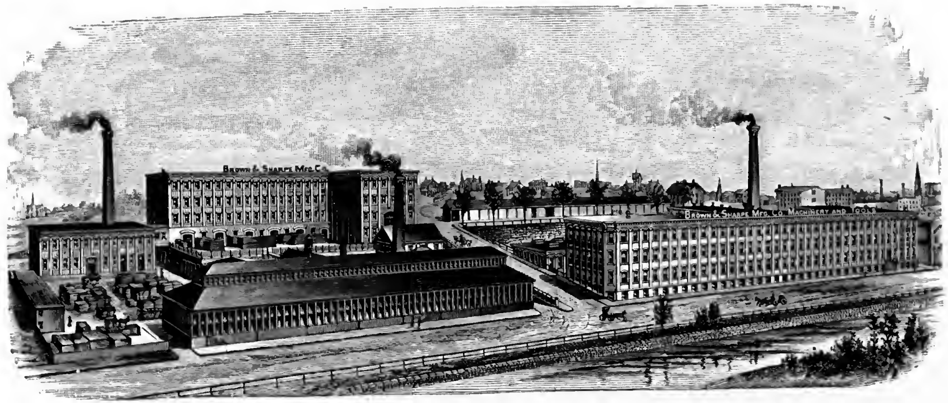
FORMULAS IN GEARING

BROWN & SHARPE MFG. CO.

Presented
to the Engineering Society
by the
Brown & Sharpe Mfg Co.

Jany 1895.



WORKS OF THE

BROWN & SHARPE MF'G. CO.,

Providence, R. I.

chnol. Dept

ERRATA.

P. 16. Formula "$2a = 2 \cos a$," should read $2a = 2s \cos a$.

P. 57. Example III. Instead of "we advance 8 teeth of our 147 tooth gear," should read—we advance 87 teeth.

P. 57. Example IV. Instead of "$x = 12 + \frac{6}{190}$" should read $x = 12 + \frac{60}{190}$ This would make it necessary to advance one additional tooth at a time of the change gear at 60 even intervals, which would not be desirable; but if other change gears were on hand, say with 88 or 95 teeth, better results would be obtained. If an 88 tooth gear were used we should advance one turn and 12 teeth at each indexing, and it would then be necessary to advance an additional tooth at only 8 intervals. If a 95 tooth gear were used the division would be exactly one turn and 13 teeth of the change gear with no correction to make.

P. 62. Fifth paragraph. Instead of "gear E (being fast on same shaft with E)," should read, on same shaft with D.

P. 62 & 63. D should be changed to E in all formulas under "Simple Gearing."

P. 65.

$$\text{Selecting} \begin{cases} E = 50 \\ G = 30 \\ H = 40 \end{cases}$$

Change "E = 50" to E = 74.

(OVER).

P. 66. Second paragraph should read,—Is E not divisible we find how many *turns* (V) of gear R are *made* to each full turn of the spindle. Dividing this number by 2 for double, by 3 for triple thread, etc., we advance R. so many *turns* and fractions of a turn, being careful to leave the spindle at rest.

The formula

$$V = \frac{E}{R}$$

for simple gearing, might be omitted as there would be no case when E was not divisible that the change could be made at R, and it is considered better practice to make the change at E.

The rules and formulas given on page 66 would be modified when the gear D is twice as large as the gear A (as explained in the fifth paragraph on page 62) and to provide for this it would be necessary to divide the results by 2 in each case.

November, 1893.

"FORMULAS IN GEARING."

BROWN & SHARPE MFG. CO.

PROVIDENCE, R. I.

FORMULAS

IN

GEARING.

WITH PRACTICAL SUGGESTIONS.

PROVIDENCE, R. I.
BROWN & SHARPE MANUFACTURING COMPANY.

1892.

PREFACE.

It is the aim, in the following pages, to condense as much as possible the solution of all problems in gearing which in the ordinary practice may be met with, to the exclusion of problems dealing with transmission of power and strength of gearing. The simplest and briefest being the symbolical expression, it has, whenever available, been resorted to. The mathematics employed are of a simple kind, and will present no difficulty to anyone familiar with ordinary Algebra and the elements of Trigonometry.

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FORMULAS IN GEARING.

CHAPTER I.

SYSTEMS OF GEARING.

(Figs. 1, 2.)

There are in common use two systems of gearing, viz.: the involute and the epicycloidal.

In the involute system the outlines of the working parts of a tooth are single curves, which may be traced by a point in a flexible, inextensible cord being unwound from a circular disk the circumference of which is called the *base circle*, the disk being concentric with the pitch circle of the gear.

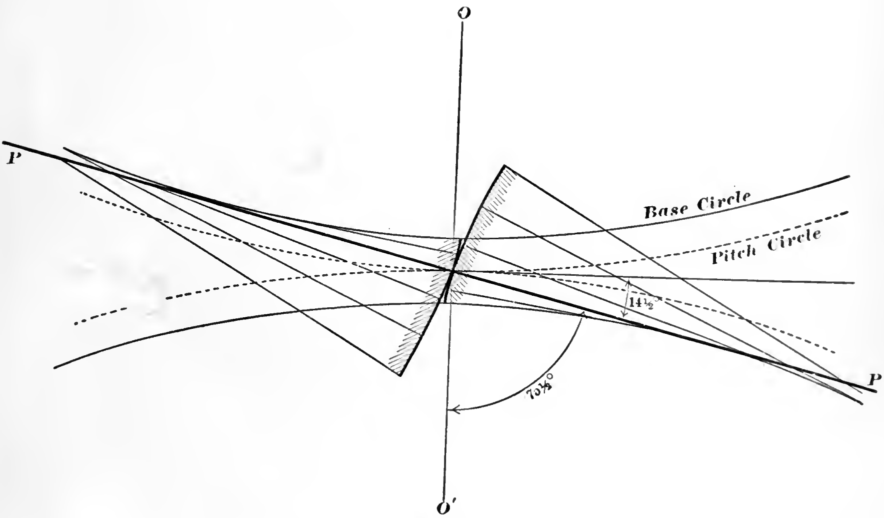


Fig. 1.

In *Fig. 1* the two base circles are represented as tangent to the line P P. This line (P P) is variously called "the line of pressure," "the line of contact," or "the line of action."

In our practice this is drawn so as to make with a normal to the center line (O O′) 14½°, or with the center line 75½°.

The rack of this system has teeth with straight sides, the two sides of a tooth making, together, an angle of 29° (twice 14½°).

This applies to gears having 30 teeth or more. For gears having less than 30 teeth special rules are followed, which are explained in our "Practical Treatise on Gearing."

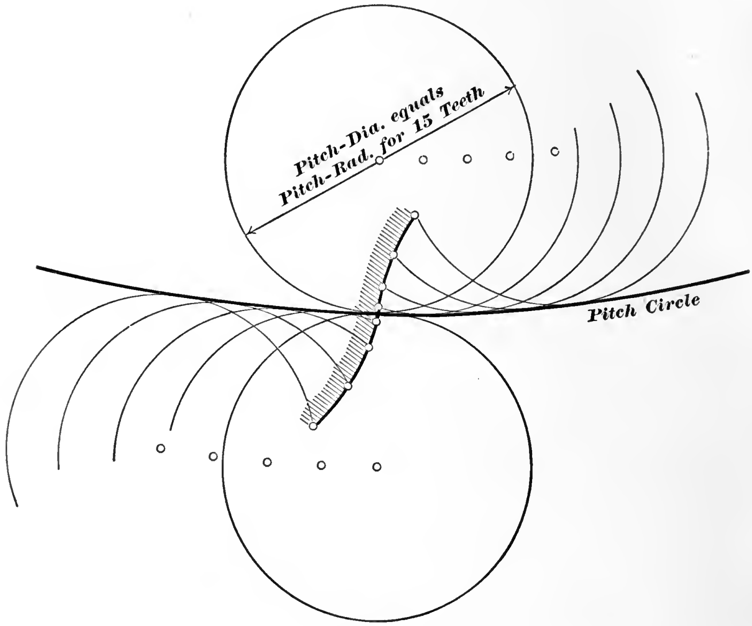


Fig. 2.

In epicycloidal, or double-curve teeth, the formation of the curve changes at the pitch circle. The outline of the faces of epicycloidal teeth may be traced by a point in a circle rolling on the *outside* of pitch circle of a gear, and the flanks by a point in a circle rolling on the *inside* of the pitch circle. The faces of one gear must be traced by the same circle that traces the flanks of the engaging gear.

In our practice the diameter of the rolling or describing circle is equal to the radius of a 15-tooth gear of the pitch required; this is the base of the system. The same describing circle being used for all gears of the same pitch.

The teeth of the rack of this system have double curves, which may be traced by the base circle rolling alternately on each side of the pitch line.

An advantage of the involute over the epicycloidal tooth is, that in action gears having involute teeth may be separated a little from their normal positions without interfering with the angular velocity, which is not possible in any other kind of tooth.

The obliquity of action is sometimes urged as an objection to involute teeth, but a full consideration of the subject will show that the importance of this has been greatly over-estimated.

The tooth dimensions for both the involute and epicycloidal gears may be calculated from the formulas in Chapter II.

CHAPTER II.

SPUR GEARING.

(Figs. 3, 4.)

Two spur gears in action are comparable to two corresponding plain rollers whose surfaces are in contact, these surfaces representing the pitch circles of the gears.

PITCH OF GEARS.

For convenience of expression the pitch of gears *may* be stated as follows:

Circular pitch is the distance from the center of one tooth to the center of the next tooth, measured on the pitch line.

Diametral pitch is the number of teeth in a gear per inch of pitch diameter. That is, a gear that has, say, six teeth for each inch in pitch diameter is six diametral pitch, or, as the expression is universally abbreviated, it is "six pitch." This is by far the most convenient way of expressing the relation of diameter to number of teeth.

Chordal pitch is a term but little employed. It is the distance from center to center of two adjacent teeth measured in a straight line.

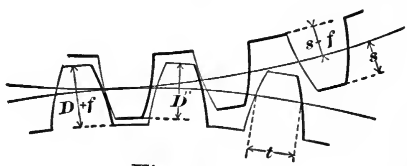


Fig. 3.

FORMULAS.

N = number of teeth.
s = addendum.
t = thickness of tooth on pitch line.
f = clearance at bottom of tooth.
D'' = working depth of tooth.
$D'' + f$ = whole depth of tooth.
d = pitch diameter.
d' = outside diameter.
P' = circular pitch.
P^c = chord pitch.
P = diametral pitch.
C = center distance.

$$P = \frac{N + 2}{d'}$$

$$P = \frac{\pi}{P'}$$

$$P' = \frac{\pi}{P}$$

$$s = \frac{1}{P} = \frac{P'}{\pi} = .3183\ P'$$

$$s = \frac{d}{N} = \frac{d'}{N + 2}$$

$$t = \frac{1}{2} P' = \frac{\pi}{2\ P}$$

$$f = \frac{1}{10} t$$

$$s + f = \frac{1}{P}\left(1 + \frac{\pi}{20}\right) = .3685\ P'$$

$$D'' = 2\ s$$

$$P^c = d \sin \frac{180^\circ}{N}$$

$$P' = d\pi \frac{\delta}{360^\circ} \quad \text{where } \sin \delta = \frac{P^c}{d}$$

$$d = \frac{N}{P}$$

$$d' = d + 2\ s$$

$$d = \frac{N\ P'}{\pi}$$

GEAR WHEELS.

TABLE OF TOOTH PARTS—CIRCULAR PITCH IN FIRST COLUMN.

Circular Pitch.	Threads or Teeth per inch Linear.	Diametral Pitch.	Thickness of Tooth on Pitch Line.	Addendum and $\frac{1''}{P}$	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.	Width of Thread-Tool at End.	Width of Thread at Top.
P'	$\frac{1''}{P'}$	P	t	s	D''	$s+f$	$D''+f$	$P'\times.31$	$P'\times.335$
2	$\frac{1}{2}$	1.5708	1.0000	.6366	1.2732	.7366	1.3732	.6200	.6700
$1\frac{7}{8}$	$\frac{8}{15}$	1.6755	.9375	.5968	1.1937	.6906	1.2874	.5813	.6281
$1\frac{3}{4}$	$\frac{4}{7}$	1.7952	.8750	.5570	1.1141	.6445	1.2016	.5425	.5863
$1\frac{5}{8}$	$\frac{8}{13}$	1.9333	.8125	.5173	1.0345	.5985	1.1158	.5038	.5444
$1\frac{1}{2}$	$\frac{2}{3}$	2.0944	.7500	.4775	.9549	.5525	1.0299	.4650	.5025
$1\frac{7}{16}$	$\frac{16}{23}$	2.1855	.7187	.4576	.9151	.5294	.9870	.4456	.4816
$1\frac{3}{8}$	$\frac{8}{11}$	2.2848	.6875	.4377	.8754	.5064	.9441	.4262	.4606
$1\frac{5}{16}$	$\frac{16}{21}$	2.3936	.6562	.4178	.8356	.4834	.9012	.4069	.4397
$1\frac{1}{4}$	$\frac{4}{5}$	2.5133	.6250	.3979	.7958	.4604	.8583	.3875	.4188
$1\frac{3}{16}$	$\frac{16}{19}$	2.6456	.5937	.3780	.7560	.4374	.8156	.3681	.3978
$1\frac{1}{8}$	$\frac{8}{9}$	2.7925	.5625	.3581	.7162	.4143	.7724	.3488	3769
$1\frac{1}{16}$	$\frac{16}{17}$	2.9568	.5312	.3382	.6764	.3913	.7295	.3294	.3559
1	1	3.1416	.5000	.3183	.6366	.3683	.6866	.3100	.3350
$\frac{15}{16}$	$1\frac{1}{15}$	3.3510	.4687	.2984	.5968	.3453	.6437	.2906	.3141
$\frac{7}{8}$	$1\frac{1}{7}$	3.5904	.4375	.2785	.5570	.3223	.6007	.2713	.2931
$\frac{13}{16}$	$1\frac{3}{13}$	3.8666	.4062	.2586	.5173	.2993	.5579	.2519	.2722
$\frac{3}{4}$	$1\frac{1}{3}$	4.1888	.3750	.2387	.4775	.2762	.5150	.2325	.2513
$\frac{11}{16}$	$1\frac{5}{11}$	4.5696	.3437	.2189	.4377	.2532	.4720	.2131	.2303
$\frac{2}{3}$	$1\frac{1}{2}$	4.7124	.3333	.2122	.4244	.2455	.4577	.2066	.2233

TABLE OF TOOTH PARTS.—*Continued.*

CIRCULAR PITCH IN FIRST COLUMN.

Circular Pitch.	Threads or Teeth per inch Linear.	Diametral Pitch.	Thickness of Tooth on Pitch Line.	Addendum and $\frac{1''}{P}$	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.	Width of Thread-Tool at End.	Width of Thread at Top.
P′	$\frac{1''}{P'}$	P	t	s	D″	$s+f$	D″$+f$.	P′×.31	P′×.335
$\frac{5}{8}$	$1\frac{3}{5}$	5.0265	.3125	.1989	.3979	.2301	.4291	.1938	.2094
$\frac{9}{16}$	$1\frac{7}{9}$	5.5851	.2812	.1790	.3581	.2071	.3862	.1744	.1884
$\frac{1}{2}$	2	6.2832	.2500	.1592	.3183	.1842	.3433	.1550	.1675
$\frac{7}{16}$	$2\frac{2}{7}$	7.1808	.2187	.1393	.2785	.1611	.3003	.1356	.1466
$\frac{2}{5}$	$2\frac{1}{2}$	7.8540	.2000	.1273	.2546	.1473	.2746	.1240	.1340
$\frac{3}{8}$	$2\frac{2}{3}$	8.3776	.1875	.1194	.2387	.1381	.2575	.1163	.1256
$\frac{1}{3}$	3	9.4248	.1666	.1061	.2122	.1228	.2289	.1033	.1117
$\frac{5}{16}$	$3\frac{1}{5}$	10.0531	.1562	.0995	.1989	.1151	.2146	.0969	.1047
$\frac{2}{7}$	$3\frac{1}{2}$	10.9956	.1429	.0909	.1819	.1052	.1962	.0886	.0957
$\frac{1}{4}$	4	12.5664	.1250	.0796	.1591	.0921	.1716	.0775	.0838
$\frac{2}{9}$	$4\frac{1}{2}$	14.1372	.1111	.0707	.1415	.0818	.1526	.0689	.0744
$\frac{1}{5}$	5	15.7080	.1000	.0637	.1273	.0737	.1373	.0620	.0670
$\frac{3}{16}$	$5\frac{1}{3}$	16.7552	.0937	.0597	.1194	.0690	.1287	.0581	.0628
$\frac{1}{6}$	6	18.8496	.0833	.0531	.1061	.0614	.1144	.0517	.0558
$\frac{1}{7}$	7	21.9911	.0714	.0455	.0910	.0526	.0981	.0443	.0479
$\frac{1}{8}$	8	25.1327	.0625	.0398	.0796	.0460	.0858	.0388	.0419
$\frac{1}{9}$	9	28.2743	.0555	.0354	.0707	.0409	.0763	.0344	.0372
$\frac{1}{10}$	10	31.4159	.0500	.0318	.0637	.0368	.0687	.0310	.0335
$\frac{1}{16}$	16	50.2655	.0312	.0199	.0398	.0230	.0429	.0194	.0209

GEAR WHEELS.

TABLE OF TOOTH PARTS—DIAMETRAL PITCH IN FIRST COLUMN.

Diametral Pitch.	Circular Pitch.	Thickness of Tooth on Pitch Line.	Addendum and $\frac{1''}{P}$	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.
P	P′	t	s	D′′	$s+f$.	D′′$+f$.
½	6.2832	3.1416	2.0000	4.0000	2.3142	4.3142
¾	4.1888	2.0944	1.3333	2.6666	1.5428	2.8761
1	3.1416	1.5708	1.0000	2.0000	1.1571	2.1571
1¼	2.5133	1.2566	.8000	1.6000	.9257	1.7257
1½	2.0944	1.0472	.6666	1.3333	.7714	1.4381
1¾	1.7952	.8976	.5714	1.1429	.6612	1.2326
2	1.5708	.7854	.5000	1.0000	.5785	1.0785
2¼	1.3963	.6981	.4444	.8888	.5143	.9587
2½	1.2566	.6283	.4000	.8000	.4628	.8628
2¾	1.1424	.5712	.3636	.7273	.4208	.7844
3	1.0472	.5236	.3333	.6666	.3857	.7190
3½	.8976	.4488	.2857	.5714	.3306	.6163
4	.7854	.3927	.2500	.5000	.2893	.5393
5	.6283	.3142	.2000	.4000	.2314	.4314
6	.5236	.2618	.1666	.3333	.1928	.3595
7	.4488	.2244	.1429	.2857	.1653	.3081
8	.3927	.1963	.1250	.2500	.1446	.2696
9	.3491	.1745	.1111	.2222	.1286	.2397
10	.3142	.1571	.1000	.2000	.1157	.2157
11	.2856	.1428	.0909	.1818	.1052	.1961
12	.2618	.1309	0833	.1666	.0964	.1798
13	.2417	.1208	.0769	.1538	.0890	.1659
14	.2244	.1122	.0714	.1429	.0826	.1541

TABLE OF TOOTH PARTS—*Continued.*

DIAMETRAL PITCH IN FIRST COLUMN.

Diametral Pitch.	Circular Pitch.	Thickness of Tooth on Pitch Line.	Addendum and $\frac{1''}{P}$	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.
P.	P′.	*t.*	*s.*	D″.	*s*+*f*.	D″+*f*.
15	.2094	.1047	.0666	.1333	.0771	.1438
16	.1963	.0982	.0625	.1250	.0723	.1348
17	.1848	.0924	.0588	.1176	.0681	.1269
18	.1745	.0873	.0555	.1111	.0643	.1198
19	.1653	.0827	.0526	.1053	.0609	.1135
20	.1571	.0785	.0500	.1000	.0579	.1079
22	.1428	.0714	.0455	.0909	.0526	.0980
24	.1309	.0654	.0417	.0833	.0482	.0898
26	.1208	.0604	.0385	.0769	.0445	.0829
28	.1122	.0561	.0357	.0714	.0413	.0770
30	.1047	.0524	.0333	.0666	.0386	.0719
32	.0982	.0491	.0312	.0625	.0362	.0674
34	.0924	.0462	.0294	.0588	.0340	.0634
36	.0873	.0436	.0278	.0555	.0321	.0599
38	.0827	.0413	.0263	.0526	.0304	.0568
40	.0785	.0393	.0250	.0500	.0289	.0539
42	.0748	.0374	.0238	.0476	.0275	.0514
44	.0714	.0357	.0227	.0455	.0263	.0490
46	.0683	.0341	.0217	.0435	.0252	.0469
48	.0654	.0327	.0208	.0417	.0241	.0449
50	.0628	.0314	.0200	.0400	.0231	.0431
56	.0561	.0280	.0178	.0357	.0207	.0385
60	.0524	.0262	.0166	.0333	.0193	.0360

Comparative Sizes of Gear Teeth. Involute.

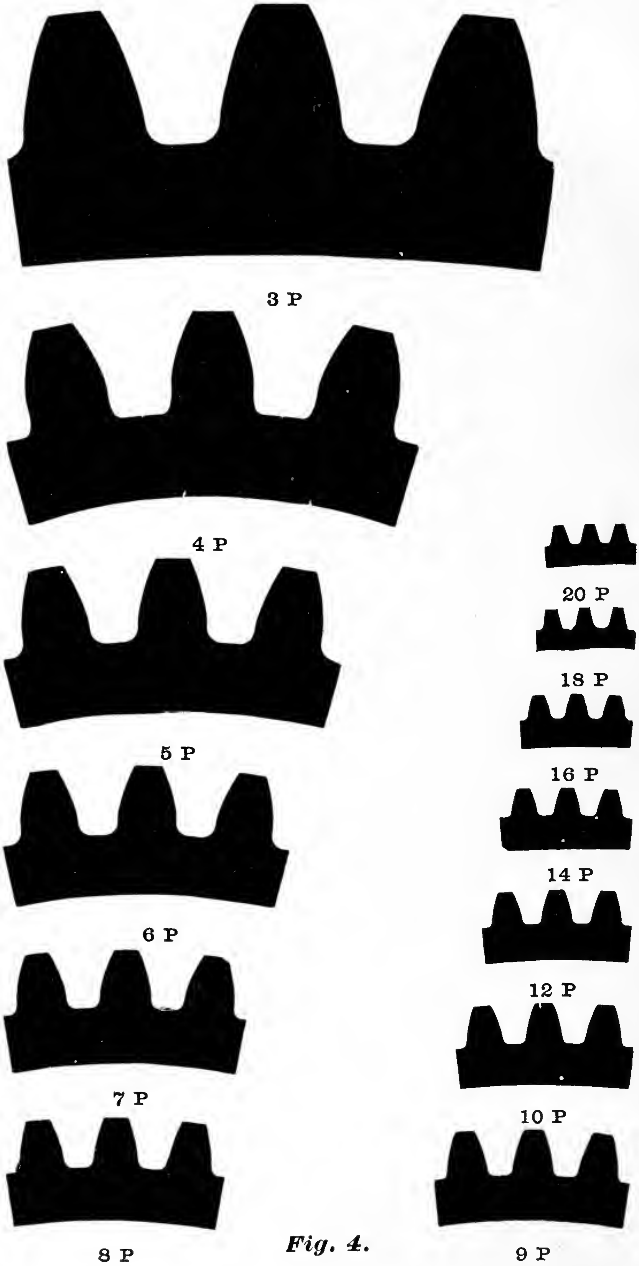
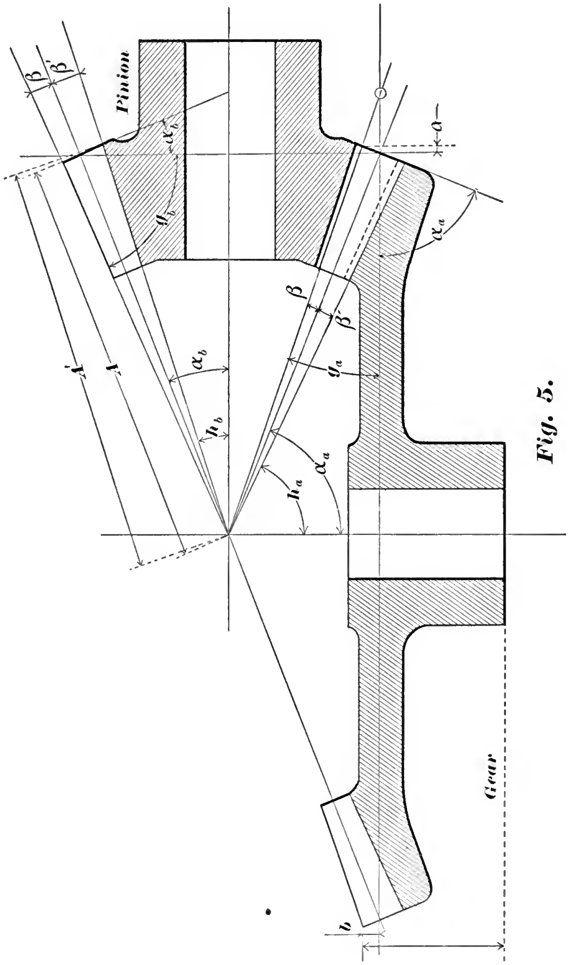


Fig. 4.

CHAPTER III.

BEVEL GEARS.—AXES AT RIGHT ANGLES.

(Fig. 5.)



FORMULAS.

$N_a =$, $N_b =$ } Number of teeth { gear. / pinion

$P =$ diametral pitch.

$P' =$ circular pitch.

$\alpha_a =$, $\alpha_b =$ } center angle = angle of edge or pitch angle { gear. / pinion.

$\beta =$ angle of top.

$\beta' =$ angle of bottom.

$g_a =$, $g_b =$ } angle of face { gear. / pinion.

$h_a =$, $h_b =$ } cutting angle { gear. / pinion.

$A =$ apex distance from pitch circle.

$A' =$ apex distance from large bottom of tooth.

$d =$ pitch diameter.

$d' =$ outside diameter.

$s =$ addendum.

$t =$ thickness of tooth at pitch line.

$f =$ clearance at bottom of tooth.

$D'' =$ working depth of tooth.

$D'' + f =$ whole depth of tooth.

$2a =$ diameter increment.

$b =$ distance from top of tooth to plane of pitch circle.

$F =$ width of face.

$$\tan \alpha_a = \frac{N_a}{N_b}; \quad \tan \alpha_b = \frac{N_b}{N_a};$$

$$\tan \beta = \frac{2 \sin \alpha}{N}; \text{ or } \quad \tan \beta = \frac{s}{A}.$$

$$\tan \beta' = \frac{\sin \alpha \left(2 + \frac{\pi}{10}\right)}{N} = \frac{2.314 \sin \alpha}{N}; \quad \tan \beta' = \frac{s + f}{A};$$

$$g_a = 90^\circ - (\alpha_a + \beta); \; g_b = 90^\circ - (\alpha_b + \beta)$$

$h = \alpha - \beta'$ (*See Note, page 52.*).

$$A = \sqrt{\left(\frac{N_a}{2P}\right)^2 + \left(\frac{N_b}{2P}\right)^2}$$

$$A = \frac{N}{2 P \sin \alpha}$$

$$A' = \frac{A}{\cos \beta'} \qquad A' = \frac{N}{2 P \sin \alpha \cos \beta'}$$

$$A = \frac{\frac{1}{2} d'}{\sin (\alpha + \beta)} \cos \beta$$

$$P = \frac{N}{2 A \sin \alpha}$$

$$d = \frac{N}{P} \text{ or } = \frac{N P'}{\pi} \qquad d' = d + 2a$$

$2a = 2s \cos \alpha$ (*See page 20.*)

$b = a \tan \alpha$ { a for gear $= b$ for pinion; a for pinion $= b$ for gear }

$$P = \frac{\pi}{P'} \qquad P' = \frac{\pi}{P}$$

$$s = \frac{1}{P} = \frac{P'}{\pi} = .3183 P' \qquad s = A \tan \beta$$

$$s + f = .3685 P' \qquad s + f = A \tan \beta'$$

$$s + f = \frac{1}{P}\left(1 + \frac{\pi}{20}\right) \qquad D'' = 2s$$

$$t = \frac{P'}{2} = \frac{\pi}{2P} \qquad f = \frac{1}{10} t$$

$$F = \frac{4}{P} + \frac{A}{7} \text{ or } = 2P' \text{ to } 3P'$$

NOTE.—Formulas containing notations without the designating letters a and b apply equally to either gear or pinion. If wanted for one or the other, the respective letters are simply attached.

BEVEL GEARS WITH AXES AT ANY ANGLE.

(Figs. 6, 7.)

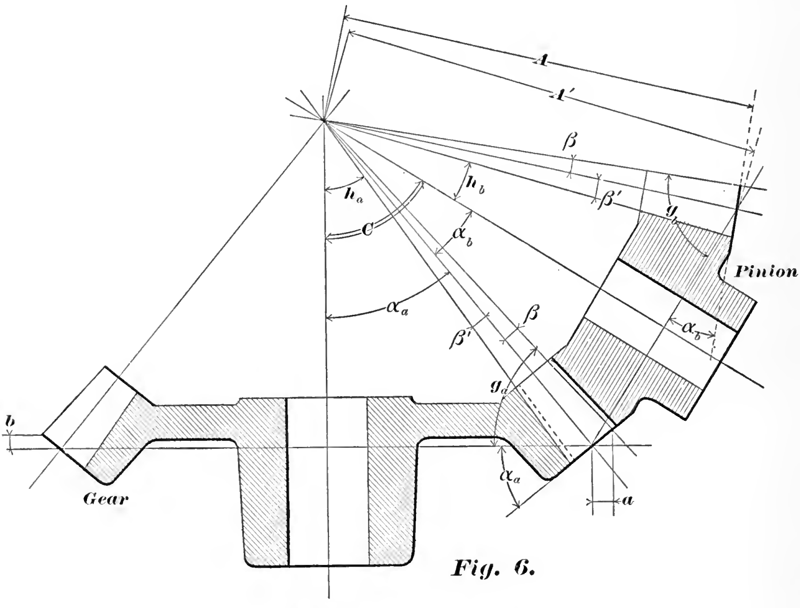


Fig. 6.

FORMULAS.

C = angle formed by axes of gears.

N_a = ⎫ number of teeth ⎧ gear.
N_b = ⎭ number of teeth ⎩ pinion.

P = diametral pitch.

P′ = circular pitch.

α_a = ⎫ angle of edge = pitch angle ⎧ gear.
α_b = ⎭ angle of edge = pitch angle ⎩ pinion.

β = angle of top.

β' = angle of bottom.

g_a = ⎫ angle of face ⎧ gear.
g_b = ⎭ angle of face ⎩ pinion.

h_a = ⎫ cutting angle ⎧ gear.
h_b = ⎭ cutting angle ⎩ pinion.

A = apex distance from pitch circle.

A′ = apex distance from large bottom of tooth.

d = pitch diameter.

d' = outside diameter.

$2a$ = diameter increment.

b = distance from top of tooth to plane of pitch circle.

NOTE.—The formulas for tooth parts as given on page 5 apply equally to these cases.

$$\tan \alpha_a = \frac{\sin C}{\frac{N_b}{N_a} + \cos C}; \text{ or } \cot \alpha_a = \frac{N_b}{N_a \sin C} + \cot C$$

$$\tan \alpha_b = \frac{\sin C}{\frac{N_a}{N_b} + \cos C}; \text{ or } \cot \alpha_b = \frac{N_a}{N_b \sin C} + \cot C$$

NOTE.—These formulas are correct only for values of C less than 90°. If C is greater than 90°, consult the following page.

$$\tan \beta = \frac{2 \sin \alpha}{N}; \quad \text{or } \tan \beta = \frac{s}{A};$$

$$\tan \beta' = \frac{\sin \alpha\left(2 + \frac{\pi}{10}\right)}{N} = \frac{2.314 \sin \alpha}{N}; \tan \beta' = \frac{s + f}{A};$$

$g_a = 90° - (\alpha_a + \beta)$; $g_b = 90° - (\alpha_b + \beta)$

$h = \alpha - \beta'$ (*See Note, page 52.*)

$$A = \frac{N}{2 P \sin \alpha}$$

$$A' = \frac{A}{\cos \beta'}$$

$d = \frac{N}{P}$ or $= \frac{N P'}{\pi}$ $d' = d + 2a$

$2a = 2 \cos \alpha$

a for gear $= b$ for pinion.

a for pinion $= b$ for gear.

NOTE.—See Foot Note on page 13.

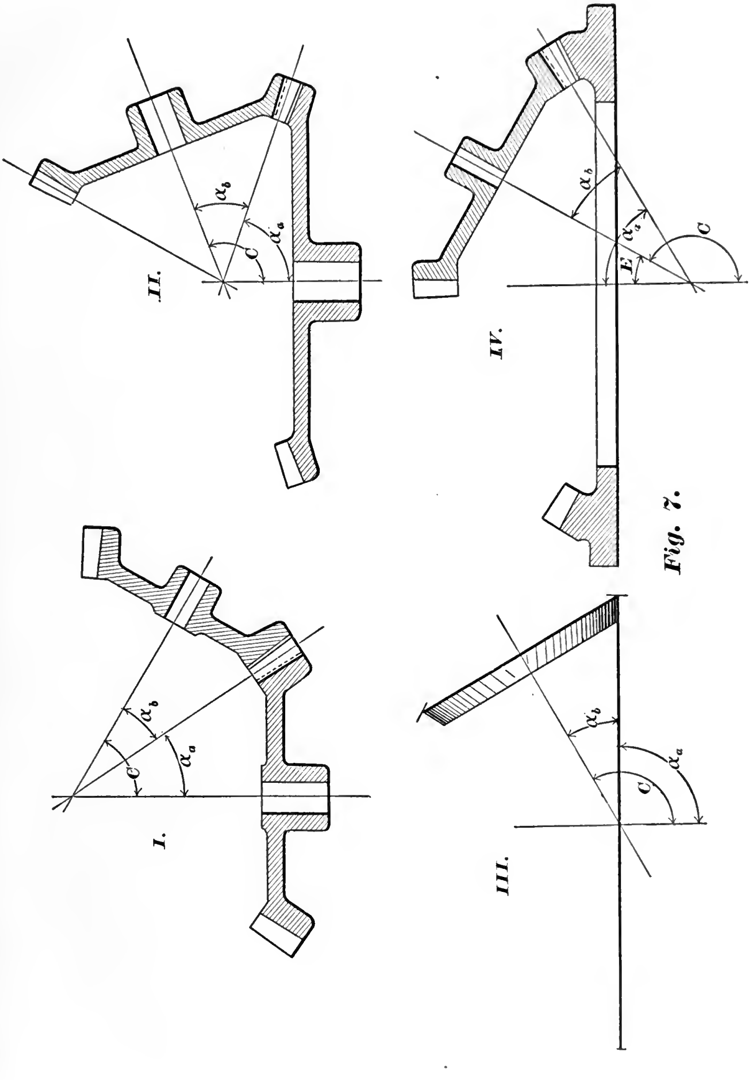


Fig. 7.

The formulas given for α_a and α_b (when C, N_a and N_b are known) undergo some modifications for values of C greater than 90°.

For bevel gears at any angle but 90° we may distinguish four cases ; C, N_a, N_b being given.

I. Case. See pages 14 and 16.

II. Case. C is greater than 90°.

$$\tan \alpha_a = \frac{\sin (180 - C)}{\frac{N_b}{N_a} - \cos (180 - C)} \; ; \quad \tan \alpha_b = \frac{\sin (180 - C}{\frac{N_a}{N_b} - \cos (180 - C)}$$

III. Case. $\alpha_a = 90°$; $\alpha_b = C - 90°$

IV. Case.

$$\tan \alpha_a = \frac{\sin E}{\cos E - \frac{N_b}{N_a}} \; ; \quad \tan \alpha_b = \frac{\sin E}{\frac{N_a}{N_b} - \cos E}$$

For an example to apply to Case III., the following condition must be fulfilled :

$$N_a \sin (C - 90°) = N_b$$

To distinguish whether a given example belongs to Case II. or case IV., we are guided by the following condition :

Is : $N_a \sin (C - 90°)$ { *smaller* than N_b, we have Case II. / *larger* than N_b, we have case IV.

UNDERCUT IN BEVEL GEARS.

By undercut in gears is understood a special formation of the tooth, which may be explained by saying that the elements of the tooth below the pitch line are nearer the center line of the tooth than those on the pitch line. Such a tooth outline is to be found only in gears with few teeth. In a pair of bevel gears where the pinion is low-numbered and the ratio high, we are apt to have undercut. For a pair of running gears this condition presents no objection. Should, however, these gears be intended as patterns to cast from, they would be found useless, from the fact that they would not draw out of the sand. We have stated on page 2 (see Fig. 1) that the base of our involute system is the 14½° pressure angle. If a pair of bevel gears with teeth constructed on this basis have undercut, we can nearly eliminate the undercut—and for the practical working this is quite sufficient—by taking as a basis for the construction of the tooth outline a pressure angle of 20°.

The question now is: When do we, and when do we not have undercut? Let there be:

N = number of teeth in gear.

n = number of teeth in pinion.

$$\frac{n\sqrt{N^2 + n^2}}{N} = p$$

where we have undercut for p less than 30.

This formula is strictly correct for epicycloidal gears only. It is, however, used as a safe and efficient approximation for the involute system.

DIAMETER INCREMENT.

2 *a*.

Rule.—The ratio being given or determined, to find the outside diameter divide figures given in table for large and small gear by pitch (P) and add quotient to pitch diameter.

RATIO.		GEARS. Large	GEARS. Small	RATIO.		GEARS. Large	GEARS. Small	RATIO.		GEARS. Large	GEARS. Small
1.00	1:1	1.41	1.41	1.65		1.05	1.70	4.40		.45	1.94
1.05		1.37	1.42	1.67	5:3	1.03	1.72	4.50	9:2	.44	1.95
1.07		1.36	1.43	1.70		1.01	1.73	4.60		.42	1.95
1.10		1.35	1.44	1.75	7:4	.99	1.74	4.80		.41	1.96
1.11	10:9	1.34	1.46	1.80	9:5	.97	1.75	5.00	5:1	.39	1.96
1.12		1.33	1 46	1.85		.95	1.76	5.20		.38	1.96
1.13	9:8	1 33	1.47	1.90		.93	1 77	5.40		.37	1.96
1.14	8:7	1.32	1.49	1 95		.91	1.78	5.60		.36	1.97
1.15		1.31	1.50	2 00	2:1	.89	1.79	5.80		.34	1.97
1.16		1.30	1.51	2 10		.87	1.80	6.00	6:1	.33	1.97
1.17	7:6	1.30	1.52	2.20		.84	1.81	6.20		.32	1.97
1.18		1.29	1.53	2 25	9:4	.82	1.82	6.40		.31	1.97
1.19		1.28	1.53	2.30		.80	1.83	6.60		.30	1 97
1.20	6:5	1.28	1.54	2.33	7:3	.78	1.84	6.80		.29	1.98
1.23		1.27	1.55	2.40		.76	1.85	7 00	7:1	.28	1.98
1.25	5:4	1.25	1.56	2.50	5:2	.75	1.86	7.20		.27	1.98
1.27		1.25	1.57	2.60		.73	1.86	7.40		.27	1 98
1.29	9:7	1.24	1.58	2.67	8:3	.71	1.87	7.60		.26	1 98
1.30		1.22	1.59	2.70		.69	1.87	7 80		.26	1.98
1.33	4:3	1.20	1.60	2.80		.67	1.88	8 00	8:1	.25	1.98
1.35		1.18	1.61	2.90		.65	1.89	8.20		.24	1.98
1 37		1.17	1.61	3.00	3:1	.63	1.91	8.40		.24	1.98
1.40	7:5	1.16	1.62	3.20		.60	1.92	8.60		.23	1.98
1.43	10:7	1.15	1.63	3.33		.58	1.92	8.80		.23	1.98
1.45		1.13	1.65	3.40		.56	1 92	9.00	9:1	.22	1.99
1.50	3:2	1.11	1.66	3.50	7:2	.54	1.93	9.20		.22	1.99
1.53		1.10	1.67	3.60		.52	1 93	9.40		.21	1.99
1 55		1.09	1.67	3.80		.50	1.94	9.60		.21	2.00
1.58		1.08	1.68	4.00	4:1	.49	1.94	9.80		.20	2.00
1.60	8:5	1.07	1.68	4.20		.47	1.94	10.00	10:1	.20	2.00

Note.—To be used only for bevel gears with axes at right angle.

TABLES FOR ANGLES OF EDGE AND ANGLES OF FACE.

The following three tables have been computed for the convenience in calculating datas for bevel gears with axes at right angle. They *do not hold* good for bevel gears with axes at any other angle.

To use the tables the number of teeth in gear and pinion must be known.

Having located the number of teeth in the gear on the horizontal line of figures at the top of the table, and the number of teeth in the pinion on the vertical line of figures on the left-hand side, we follow the two columns to the square formed by their intersections.

The two angles found in the same square are the respective angles for gear and pinion. The tables are so arranged that the angle belonging to the gear is always placed above the angle for the pinion.

TABLE 1

Angle of Edge.

	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27
12	73°41′ 16°19′	73°18′ 16°42′	72°54′ 17°6′	72°28′ 17°32′	72°2′ 17°58′	71°34′ 18°26′	71°5′ 18°55′	70°34′ 19°26′	70°1′ 19°59′	69°26′ 20°34′	68°50′ 21°10′	68°12′ 21°48′	67°31′ 22°29′	66°48′ 23°12′	66°2′ 23°58′
13	72°25′ 17°35′	71°59′ 18°1′	71°34′ 18°26′	71°7′ 18°53′	70°39′ 19°21′	70°9′ 19°51′	69°37′ 20°23′	69°5′ 20°55′	68°30′ 21°30′	67°53′ 22°7′	67°15′ 22°45′	66°34′ 23°26′	65°51′ 24°9′	65°6′ 24°54′	64°17′ 25°43′
14	71°9′ 18°51′	70°43′ 19°17′	70°15′ 19°45′	69°46′ 20°14′	69°16′ 20°44′	68°45′ 21°15′	68°12′ 21°48′	67°37′ 22°23′	67°0′ 23°0′	66°23′ 23°37′	65°42′ 24°18′	64°59′ 25°1′	64°14′ 25°46′	63°26′ 26°34′	62°36′ 27°24′
15	69°54′ 20°6′	69°26′ 20°34′	68°58′ 21°2′	68°28′ 21°32′	67°56′ 22°4′	67°23′ 22°37′	66°48′ 23°12′	66°12′ 23°48′	65°33′ 24°27′	64°53′ 25°7′	64°10′ 25°50′	63°26′ 26°34′	62°39′ 27°21′	61°49′ 28°11′	60°57′ 29°3′
16	68°41′ 21°19′	68°12′ 21°48′	67°42′ 22°18′	67°10′ 22°50′	66°37′ 23°23′	66°2′ 23°58′	65°26′ 24°34′	64°48′ 25°12′	64°8′ 25°52′	63°26′ 26°34′	62°42′ 27°18′	61°56′ 28°4′	61°7′ 28°53′	60°15′ 29°45′	59°21′ 30°39′
17	67°29′ 22°31′	66°58′ 23°2′	66°27′ 23°33′	65°54′ 24°6′	65°19′ 24°41′	64°43′ 25°17′	64°6′ 25°54′	63°26′ 26°34′	62°45′ 27°15′	62°1′ 27°59′	61°15′ 28°45′	60°28′ 29°32′	59°37′ 30°23′	58°44′ 31°16′	57°48′ 32°12′
18	66°18′ 23°42′	65°46′ 24°14′	65°14′ 24°46′	64°39′ 25°21′	64°4′ 25°56′	63°26′ 26°34′	62°47′ 27°13′	62°6′ 27°54′	61°23′ 28°37′	60°38′ 29°22′	59°51′ 30°9′	59°2′ 30°58′	58°10′ 31°50′	57°16′ 32°44′	56°19′ 33°41′
19	65°8′ 24°52′	64°36′ 25°24′	64°2′ 25°58′	63°26′ 26°34′	62°49′ 27°11′	62°10′ 27°50′	61°30′ 28°30′	60°48′ 29°12′	60°4′ 29°56′	59°18′ 30°42′	58°30′ 31°30′	57°39′ 32°21′	56°46′ 33°14′	55°51′ 34°9′	54°52′ 35°8′
20	64°0′ 26°0′	63°26′ 26°34′	62°51′ 27°9′	62°14′ 27°46′	61°37′ 28°23′	60°57′ 29°3′	60°15′ 29°45′	59°32′ 30°28′	58°47′ 31°13′	58°0′ 32°0′	57°10′ 32°50′	56°19′ 33°41′	55°24′ 34°36′	54°28′ 35°32′	53°28′ 36°32′
21	62°53′ 27°7′	62°18′ 27°42′	61°42′ 28°18′	61°4′ 28°56′	60°25′ 29°35′	59°45′ 30°15′	59°2′ 30°58′	58°18′ 31°42′	57°32′ 32°28′	56°43′ 33°17′	55°53′ 34°7′	55°0′ 35°0′	54°5′ 35°55′	53°7′ 36°53′	52°8′ 37°52′
22	61°47′ 28°13′	61°11′ 28°49′	60°34′ 29°26′	59°56′ 30°4′	59°15′ 30°45′	58°34′ 31°26′	57°51′ 32°9′	57°6′ 32°54′	56°19′ 33°41′	55°29′ 34°31′	54°38′ 35°22′	53°45′ 36°15′	52°49′ 37°11′	51°50′ 38°10′	50°49′ 39°11′
23	60°42′ 29°18′	60°6′ 29°54′	59°28′ 30°32′	58°49′ 31°11′	58°8′ 31°52′	57°25′ 32°35′	56°41′ 33°19′	55°55′ 34°5′	55°7′ 34°53′	54°18′ 35°42′	53°26′ 36°34′	52°31′ 37°29′	51°35′ 38°25′	50°36′ 39°24′	49°34′ 40°26′
24	59°39′ 30°21′	59°2′ 30°58′	58°23′ 31°37′	57°44′ 32°16′	57°2′ 32°58′	56°19′ 33°41′	55°33′ 34°27′	54°47′ 35°13′	53°58′ 36°2′	53°7′ 36°53′	52°15′ 37°45′	51°20′ 38°40′	50°23′ 39°37′	49°24′ 40°36′	48°22′ 41°38′
25	58°38′ 31°22′	58°0′ 32°0′	57°20′ 32°40′	56°40′ 33°20′	55°57′ 34°3′	55°13′ 34°47′	54°28′ 35°32′	53°40′ 36°20′	52°51′ 37°9′	52°0′ 38°0′	51°7′ 38°53′	50°12′ 39°48′	49°14′ 40°46′	48°14′ 41°46′	47°12′ 42°48′
26	57°37′ 32°23′	56°58′ 33°2′	56°19′ 33°41′	55°37′ 34°23′	54°54′ 35°6′	54°10′ 35°50′	53°24′ 36°36′	52°36′ 37°24′	51°46′ 38°14′	50°54′ 39°6′	50°1′ 39°59′	49°5′ 40°55′	48°7′ 41°53′	47°7′ 42°53′	46°5′ 43°56′
27	56°38′ 33°22′	55°59′ 34°1′	55°18′ 34°42′	54°36′ 35°24′	53°53′ 36°7′	53°7′ 36°53′	52°21′ 37°39′	51°33′ 38°27′	50°43′ 39°17′	49°51′ 40°9′	48°57′ 41°3′	48°0′ 42°0′	47°3′ 42°57′	46°2′ 43°58′	45°
28	55°40′ 34°20′	55°0′ 35°0′	54°19′ 35°41′	53°37′ 36°23′	52°53′ 37°7′	52°8′ 37°52′	51°20′ 38°40′	50°32′ 39°28′	49°41′ 40°19′	48°48′ 41°12′	47°55′ 42°5′	46°58′ 43°2′	46°0′ 44°0′	45°	
29	54°44′ 35°16′	54°3′ 35°57′	53°22′ 36°38′	52°39′ 37°21′	51°55′ 38°5′	51°9′ 38°51′	50°21′ 39°39′	49°32′ 40°28′	48°41′ 41°19′	47°49′ 42°11′	46°54′ 43°6′	45°58′ 44°2′	45°		
30	53°48′ 36°12′	53°7′ 36°53′	52°26′ 37°34′	51°42′ 38°18′	50°58′ 39°2′	50°12′ 39°48′	49°24′ 40°36′	48°35′ 41°25′	47°43′ 42°17′	46°51′ 43°9′	45°56′ 44°4′	45°			
31	52°54′ 37°6′	52°13′ 37°47′	51°31′ 38°29′	50°48′ 39°12′	50°2′ 39°58′	49°16′ 40°44′	48°28′ 41°32′	47°39′ 42°21′	46°47′ 43°13′	45°54′ 44°6′	45°				
32	52°2′ 37°58′	51°20′ 38°40′	50°38′ 39°22′	49°54′ 40°6′	49°9′ 40°51′	48°22′ 41°38′	47°34′ 42°26′	46°44′ 43°16′	45°53′ 44°7′	45°					
33	51°10′ 38°50′	50°29′ 39°31′	49°46′ 40°14′	49°2′ 40°58′	48°16′ 41°44′	47°29′ 42°21′	46°41′ 43°19′	45°51′ 44°9′	45°						
34	50°20′ 39°40′	49°38′ 40°22′	48°55′ 41°5′	48°11′ 41°49′	47°25′ 42°35′	46°38′ 43°22′	45°50′ 44°10′	45°							
35	49°31′ 40°29′	48°48′ 41°12′	48°5′ 41°55′	47°21′ 42°39′	46°35′ 43°25′	45°48′ 44°12′	45°								
36	48°43′ 41°17′	48°0′ 42°0′	47°17′ 42°43′	46°33′ 43°27′	45°47′ 44°13′	45°									
37	47°56′ 42°4′	47°14′ 42°46′	46°30′ 43°30′	45°46′ 44°14′	45°										
38	47°10′ 42°50′	46°28′ 43°32′	45°45′ 44°15′	45°											
39	46°26′ 43°34′	45°43′ 44°17′	45°												
40	45°42′ 44°18′	45°													
41	45°														

TABLE 1.—(*Continued.*)

ANGLE OF EDGE.

	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12
12	65° 14′ 24° 46′	64° 22′ 25° 38′	63° 26′ 26° 34′	62° 27′ 27° 33′	61° 23′ 28° 37′	60° 15′ 29° 45′	59° 2′ 30° 58′	57° 44′ 32° 16′	56° 19′ 33° 41′	54° 47′ 35° 13′	53° 7′ 36° 53′	51° 20′ 38° 40′	49° 24′ 40° 36′	47° 17′ 42° 43′	45°
13	63° 26′ 26° 34′	62° 31′ 27° 29′	61° 33′ 28° 27′	60° 31′ 29° 29′	59° 25′ 30° 35′	58° 14′ 31° 46′	56° 58′ 33° 2′	55° 37′ 34° 23′	54° 10′ 35° 50′	52° 36′ 37° 24′	50° 54′ 39° 6′	49° 5′ 40° 55′	47° 7′ 42° 53′	45°	
14	61° 42′ 28° 18′	60° 45′ 29° 15′	59° 45′ 30° 15′	58° 40′ 31° 20′	57° 32′ 32° 28′	56° 19′ 33° 41′	55° 0′ 35° 0′	53° 37′ 36° 23′	52° 8′ 37° 52′	50° 32′ 39° 28′	48° 48′ 41° 12′	46° 58′ 43° 2′	45°		
15	60° 1′ 29° 59′	59° 2′ 30° 58′	58° 0′ 32° 0′	56° 53′ 33° 7′	55° 43′ 34° 17′	54° 28′ 35° 32′	53° 7′ 36° 53′	51° 42′ 38° 18′	50° 12′ 39° 48′	48° 35′ 41° 25′	46° 51′ 43° 9′	45°			
16	58° 23′ 31° 37′	57° 23′ 32° 37′	56° 19′ 33° 41′	55° 11′ 34° 49′	53° 58′ 36° 2′	52° 42′ 37° 18′	51° 20′ 38° 40′	49° 54′ 40° 6′	48° 22′ 41° 38′	46° 44′ 43° 16′	45°				
17	56° 49′ 33° 11′	55° 47′ 34° 13′	54° 41′ 35° 19′	53° 32′ 36° 28′	52° 18′ 37° 42′	51° 0′ 39° 0′	49° 38′ 40° 22′	48° 11′ 41° 49′	46° 38′ 43° 22′	45°					
18	55° 18′ 34° 42′	54° 15′ 35° 45′	53° 7′ 36° 53′	51° 57′ 38° 3′	50° 43′ 39° 17′	49° 24′ 40° 36′	48° 0′ 42° 0′	46° 33′ 43° 27′	45°						
19	53° 51′ 36° 9′	52° 46′ 37° 14′	51° 38′ 38° 22′	50° 26′ 39° 34′	49° 11′ 40° 49′	47° 52′ 42° 8′	46° 28′ 43° 32′	45°							
20	52° 26′ 37° 34′	51° 20′ 38° 40′	50° 12′ 39° 48′	48° 59′ 41° 1′	47° 43′ 42° 17′	46° 24′ 43° 36′	45°								
21	51° 4′ 38° 56′	49° 58′ 40° 2′	48° 48′ 41° 12′	47° 36′ 42° 24′	46° 20′ 43° 40′	45°									
22	49° 46′ 40° 14′	48° 39′ 41° 21′	47° 29′ 42° 31′	46° 16′ 43° 44′	45°										
23	48° 30′ 41° 30′	47° 23′ 42° 37′	46° 13′ 43° 47′	45°											
24	47° 17′ 42° 43′	46° 10′ 43° 50′	45°												
25	46° 7′ 43° 53′	45°													
26	45°														

$$\tan \alpha_a = \frac{N_a}{N_b}$$

$$\tan \alpha_b = \frac{N_b}{N_a}$$

(*See page 13.*)

TABLE 2.

Angle of Edge.

	72	71	70	69	68	67	66	65	64	63	62	61	60	59	58	57
12	80° 33' 9° 27'	80° 26' 9° 36'	80° 16' 9° 44'	80° 8' 9° 52'	79° 58' 10° 1'	79° 51' 10° 9'	79° 42' 10° 18'	79° 32' 10° 28'	79° 23' 10° 37'	79° 13' 10° 47'	79° 3' 10° 57'	78° 52' 11° 8'	78° 41' 11° 19'	78° 30' 11° 30'	78° 19' 11° 41'	78° 7' 11° 53'
13	79° 46' 10° 14'	79° 37' 10° 23'	79° 29' 10° 31'	79° 20' 10° 40'	79° 11' 10° 49'	79° 1' 10° 59'	78° 51' 11° 9'	78° 41' 11° 19'	78° 31' 11° 29'	78° 20' 11° 40'	78° 9' 11° 51'	77° 58' 12° 2'	77° 46' 12° 14'	77° 34' 12° 26'	77° 22' 12° 38'	77° 9' 12° 51'
14	79° 0' 11° 0'	78° 51' 11° 9'	78° 41' 11° 19'	78° 32' 11° 28'	78° 22' 11° 38'	78° 11' 11° 49'	78° 1' 11° 59'	77° 51' 12° 9'	77° 40' 12° 20'	77° 28' 12° 32'	77° 17' 12° 43'	77° 5' 12° 55'	76° 52' 13° 8'	76° 39' 13° 21'	76° 26' 13° 34'	76° 12' 13° 48'
15	78° 14' 11° 44'	78° 4' 11° 56'	77° 54' 12° 6'	77° 44' 12° 16'	77° 34' 12° 26'	77° 23' 12° 37'	77° 12' 12° 48'	77° 0' 13° 0'	76° 48' 13° 12'	76° 36' 13° 24'	76° 24' 13° 36'	76° 11' 13° 49'	75° 58' 14° 2'	75° 44' 14° 16'	75° 30' 14° 30'	75° 15' 14° 45'
16	77° 28' 12° 32'	77° 18' 12° 42'	77° 7' 12° 53'	76° 57' 13° 3'	76° 45' 13° 15'	76° 34' 13° 26'	76° 22' 13° 38'	76° 10' 13° 50'	75° 58' 14° 2'	75° 45' 14° 15'	75° 32' 14° 28'	75° 18' 14° 42'	75° 4' 14° 56'	74° 49' 15° 11'	74° 35' 15° 25'	74° 19' 15° 41'
17	76° 43' 13° 17'	76° 32' 13° 28'	76° 21' 13° 39'	76° 10' 13° 50'	75° 58' 14° 2'	75° 46' 14° 14'	75° 33' 14° 27'	75° 21' 14° 39'	75° 8' 14° 52'	74° 54' 15° 6'	74° 40' 15° 20'	74° 25' 15° 35'	74° 11' 15° 49'	73° 56' 16° 4'	73° 40' 16° 20'	73° 24' 16° 36'
18	75° 58' 14° 2'	75° 46' 14° 14'	75° 35' 14° 25'	75° 23' 14° 37'	75° 10' 14° 50'	74° 58' 15° 2'	74° 45' 15° 15'	74° 31' 15° 29'	74° 17' 15° 43'	74° 3' 15° 57'	73° 49' 16° 11'	73° 33' 16° 27'	73° 18' 16° 42'	73° 2' 16° 58'	72° 45' 17° 15'	72° 29' 17° 31'
19	75° 13' 14° 47'	75° 1' 14° 59'	74° 49' 15° 11'	74° 36' 15° 24'	74° 23' 15° 37'	74° 10' 15° 50'	73° 56' 16° 4'	73° 42' 16° 18'	73° 28' 16° 32'	73° 13' 16° 47'	72° 58' 17° 2'	72° 42' 17° 18'	72° 20' 17° 34'	72° 9' 17° 51'	71° 52' 18° 8'	71° 34' 18° 26'
20	74° 29' 15° 31'	74° 16' 15° 44'	74° 3' 15° 57'	73° 50' 16° 10'	73° 37' 16° 23'	73° 23' 16° 42'	73° 9' 16° 51'	72° 54' 17° 6'	72° 39' 17° 21'	72° 23' 17° 37'	72° 7' 17° 53'	71° 51' 18° 9'	71° 34' 18° 26'	71° 16' 18° 44'	70° 59' 19° 1'	70° 40' 19° 20'
21	73° 45' 16° 15'	73° 32' 16° 28'	73° 18' 16° 42'	73° 4' 16° 56'	72° 50' 17° 10'	72° 36' 17° 24'	72° 21' 17° 39'	72° 6' 17° 54'	71° 50' 18° 10'	71° 34' 18° 26'	71° 17' 18° 43'	71° 0' 19° 0'	70° 43' 19° 17'	70° 24' 19° 36'	70° 6' 19° 54'	69° 46' 20° 24'
22	73° 1' 16° 59'	72° 47' 17° 13'	72° 33' 17° 27'	72° 19' 17° 41'	72° 4' 17° 56'	71° 49' 18° 11'	71° 34' 18° 26'	71° 18' 18° 42'	71° 2' 18° 58'	70° 45' 19° 15'	70° 28' 19° 34'	70° 10' 19° 50'	69° 52' 20° 8'	69° 33' 20° 27'	69° 13' 20° 47'	68° 54' 21° 6'
23	72° 17' 17° 43'	72° 3' 17° 57'	71° 49' 18° 11'	71° 34' 18° 26'	71° 19' 18° 41'	71° 3' 18° 57'	70° 47' 19° 13'	70° 30' 19° 30'	70° 14' 19° 46'	69° 57' 20° 3'	69° 39' 20° 21'	69° 20' 20° 40'	69° 2' 20° 58'	68° 42' 21° 18'	68° 22' 21° 38'	68° 2' 21° 58'
24	71° 34' 18° 26'	71° 19' 18° 41'	71° 5' 18° 55'	70° 49' 19° 11'	70° 34' 19° 26'	70° 17' 19° 43'	70° 1' 19° 59'	69° 44' 20° 16'	69° 26' 20° 34'	69° 9' 20° 51'	68° 50' 21° 10'	68° 31' 21° 29'	68° 12' 21° 48'	67° 52' 22° 8'	67° 31' 22° 29'	67° 10' 22° 50'
25	70° 51' 19° 9'	70° 36' 19° 24'	70° 21' 19° 39'	70° 5' 19° 55'	69° 49' 20° 11'	69° 32' 20° 28'	69° 15' 20° 45'	68° 57' 21° 3'	68° 40' 21° 20'	68° 21' 21° 39'	68° 3' 21° 57'	67° 43' 22° 17'	67° 23' 22° 37'	67° 2' 22° 58'	66° 41' 23° 19'	66° 19' 23° 41'
26	70° 9' 19° 51'	69° 53' 20° 7'	69° 37' 20° 23'	69° 21' 20° 39'	69° 4' 20° 56'	68° 48' 21° 12'	68° 30' 21° 30'	68° 12' 21° 48'	67° 54' 22° 6'	67° 34' 22° 26'	67° 15' 22° 45'	66° 55' 23° 5'	66° 34' 23° 26'	66° 13' 23° 47'	65° 51' 24° 9'	65° 29' 24° 31'
27	69° 27' 20° 33'	69° 10' 20° 50'	68° 54' 21° 6'	68° 38' 21° 22'	68° 20' 21° 40'	68° 3' 21° 57'	67° 45' 22° 15'	67° 26' 22° 34'	67° 8' 22° 52'	66° 48' 23° 12'	66° 28' 23° 32'	66° 7' 23° 53'	65° 46' 24° 14'	65° 25' 24° 35'	65° 2' 24° 58'	64° 39' 25° 21'
28	68° 46' 21° 15'	68° 29' 21° 31'	68° 12' 21° 48'	67° 55' 22° 5'	67° 37' 22° 23'	67° 19' 22° 41'	67° 1' 22° 59'	66° 42' 23° 18'	66° 22' 23° 38'	66° 2' 23° 58'	65° 42' 24° 18'	65° 21' 24° 39'	64° 59' 25° 1'	64° 37' 25° 23'	64° 14' 25° 46'	63° 50' 26° 10'
29	68° 4' 21° 56'	67° 47' 22° 13'	67° 30' 22° 30'	67° 12' 22° 48'	66° 54' 23° 6'	66° 36' 23° 24'	66° 17' 23° 43'	65° 57' 24° 3'	65° 37' 24° 23'	65° 16' 24° 44'	64° 55' 25° 5'	64° 34' 25° 26'	64° 12' 25° 48'	63° 50' 26° 10'	63° 26' 26° 34'	63° 2' 26° 58'
30	67° 23' 22° 37'	67° 6' 22° 54'	66° 48' 23° 12'	66° 30' 23° 30'	66° 12' 23° 48'	65° 52' 24° 8'	65° 33' 24° 27'	65° 14' 24° 46'	64° 53' 25° 7'	64° 32' 25° 28'	64° 10' 25° 50'	63° 49' 26° 11'	63° 26' 26° 34'	63° 3' 26° 57'	62° 39' 27° 21'	62° 14' 27° 46'
31	66° 42' 23° 18'	66° 25' 23° 35'	66° 6' 23° 54'	65° 48' 24° 12'	65° 29' 24° 31'	65° 10' 24° 50'	64° 50' 25° 10'	64° 30' 25° 30'	64° 9' 25° 51'	63° 48' 26° 12'	63° 26' 26° 34'	63° 3' 26° 57'	62° 40' 27° 20'	62° 18' 27° 42'	61° 53' 28° 7'	61° 28' 28° 32'
32	66° 2' 23° 58'	65° 44' 24° 16'	65° 26' 24° 34'	65° 7' 24° 53'	64° 48' 25° 12'	64° 28' 25° 32'	64° 8' 25° 52'	63° 47' 26° 13'	63° 26' 26° 34'	63° 4' 26° 56'	62° 42' 27° 18'	62° 19' 27° 41'	61° 56' 28° 4'	61° 32' 28° 28'	61° 7' 28° 53'	60° 41' 29° 19'
33	65° 23' 24° 37'	65° 4' 24° 56'	64° 45' 25° 15'	64° 26' 25° 34'	64° 7' 25° 53'	63° 47' 26° 13'	63° 26' 26° 34'	63° 5' 26° 55'	62° 43' 27° 17'	62° 21' 27° 39'	61° 58' 28° 2'	61° 35' 28° 25'	61° 11' 28° 49'	60° 47' 29° 13'	60° 21' 29° 39'	59° 56' 30° 4'
34	64° 43' 25° 17'	64° 25' 25° 35'	64° 5' 25° 55'	63° 46' 26° 14'	63° 26' 26° 34'	63° 5' 26° 55'	62° 45' 27° 15'	62° 23' 27° 37'	62° 1' 27° 59'	61° 38' 28° 22'	61° 15' 28° 45'	60° 52' 29° 8'	60° 28' 29° 32'	60° 3' 29° 57'	59° 37' 30° 23'	59° 11' 30° 49'
35	64° 5' 25° 55'	63° 45' 26° 15'	63° 26' 26° 34'	63° 6' 26° 54'	62° 46' 27° 14'	62° 25' 21° 35'	62° 4' 27° 56'	61° 42' 28° 18'	61° 19' 28° 41'	60° 57' 29° 3'	60° 33' 29° 27'	60° 9' 29° 51'	59° 45' 30° 15'	59° 19' 30° 41'	58° 53' 31° 7'	58° 27' 31° 33'
36	63° 26' 26° 34'	63° 7' 26° 53'	62° 47' 27° 13'	62° 27' 27° 33'	62° 6' 27° 54'	61° 45' 28° 15'	61° 23' 28° 37'	61° 1' 28° 59'	60° 38' 29° 22'	60° 15' 29° 45'	59° 51' 30° 9'	59° 27' 30° 33'	59° 2' 30° 58'	58° 37' 31° 23'	58° 10' 31° 50'	57° 43' 32° 17'
37	62° 48' 27° 12'	62° 28' 27° 32'	62° 8' 27° 52'	61° 48' 28° 12'	61° 27' 28° 33'	61° 5' 28° 55'	60° 44' 29° 16'	60° 21' 29° 39'	59° 58' 30° 2'	59° 35' 30° 25'	59° 10' 30° 50'	58° 46' 31° 14'	58° 20' 31° 40'	57° 54' 32° 6'	57° 28' 32° 32'	57° 1' 32° 59'
38	62° 11' 27° 49'	61° 51' 28° 9'	61° 30' 28° 30'	61° 9' 28° 51'	60° 48' 29° 12'	60° 26' 29° 34'	60° 4' 29° 56'	59° 41' 30° 19'	59° 18' 30° 42'	58° 54' 31° 6'	58° 30' 31° 30'	58° 5' 31° 55'	57° 39' 32° 21'	57° 13' 32° 47'	56° 46' 33° 14'	56° 19' 33° 41'
39	61° 33' 28° 27'	61° 13' 28° 47'	60° 53' 29° 7'	60° 31' 29° 29'	60° 10' 29° 50'	59° 48' 30° 12'	59° 25' 30° 35'	59° 2' 30° 58'	58° 39' 31° 21'	58° 14' 31° 46'	57° 50' 32° 10'	57° 24' 32° 36'	56° 58' 33° 2'	56° 32' 33° 28'	56° 6' 33° 54'	55° 37' 34° 23'
40	60° 57' 29° 3'	60° 56' 29° 4'	60° 15' 29° 45'	59° 53' 30° 7'	59° 32' 30° 28'	59° 10' 30° 50'	58° 47' 31° 13'	58° 24' 31° 36'	58° 0' 32° 0'	57° 35' 32° 25'	57° 10' 32° 50'	56° 44' 33° 16'	56° 19' 33° 41'	55° 52' 34° 8'	55° 24' 34° 35'	54° 57' 35° 3'
41	60° 26' 29° 40'	60° 0' 30° 0'	59° 39' 30° 21'	59° 17' 30° 43'	58° 55' 31° 5'	58° 32' 31° 28'	58° 9' 31° 51'	57° 45' 32° 15'	57° 21' 32° 39'	56° 57' 33° 3'	56° 32' 33° 28'	56° 6' 33° 54'	55° 39' 34° 21'	55° 12' 34° 48'	54° 44' 35° 16'	54° 16' 35° 44'
42	59° 45' 30° 15'	59° 24' 30° 36'	59° 3' 30° 57'	58° 40' 31° 20'	58° 18' 31° 42'	57° 55' 32° 5'	57° 32' 32° 28'	57° 8' 32° 52'	56° 43' 33° 17'	56° 19' 33° 41'	55° 53' 34° 7'	55° 27' 34° 33'	55° 0' 35° 0'	54° 33' 35° 27'	54° 5' 35° 55'	53° 37' 36° 23'

TABLE 2.—(*Continued.*)

ANGLE OF EDGE.

	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42
12	77° 54′ 12° 6′	77° 42′ 12° 18′	77° 28′ 12° 32′	77° 15′ 12° 45′	77° 0′ 13° 0′	76° 46′ 13° 14′	76° 30′ 13° 30′	76° 14′ 13° 46′	75° 58′ 14° 2′	75° 41′ 14° 19′	75° 23′ 14° 37′	75° 4′ 14° 56′	74° 45′ 15° 15′	74° 25′ 15° 35′	74° 3′ 15° 57′
13	76° 56′ 13° 4′	76° 42′ 13° 18′	76° 28′ 13° 32′	76° 13′ 13° 47′	75° 58′ 14° 2′	75° 42′ 14° 18′	75° 26′ 14° 34′	75° 8′ 14° 52′	74° 51′ 15° 9′	74° 32′ 15° 28′	74° 13′ 15° 47′	73° 53′ 16° 7′	73° 32′ 16° 28′	73° 11′ 16° 49′	72° 48′ 17° 12′
14	75° 58′ 14° 2′	75° 43′ 14° 17′	75° 28′ 14° 32′	75° 12′ 14° 48′	74° 56′ 15° 4′	74° 39′ 15° 21′	74° 21′ 15° 39′	74° 3′ 15° 57′	73° 44′ 16° 16′	73° 25′ 16° 35′	73° 4′ 16° 56′	72° 43′ 17° 17′	72° 21′ 17° 39′	71° 58′ 18° 2′	71° 34′ 18° 26′
15	75° 0′ 15° 0′	74° 44′ 15° 16′	74° 29′ 15° 31′	74° 12′ 15° 48′	73° 55′ 16° 5′	73° 37′ 16° 23′	73° 18′ 16° 42′	72° 59′ 17° 1′	72° 39′ 17° 21′	72° 18′ 17° 42′	71° 56′ 18° 4′	71° 34′ 18° 26′	71° 10′ 18° 50′	70° 46′ 19° 14′	70° 21′ 19° 39′
16	74° 3′ 15° 57′	73° 47′ 16° 13′	73° 30′ 16° 36′	73° 12′ 16° 48′	72° 54′ 17° 6′	72° 35′ 17° 25′	72° 15′ 17° 45′	71° 55′ 18° 5′	71° 34′ 18° 26′	71° 12′ 18° 48′	70° 49′ 19° 11′	70° 26′ 19° 34′	70° 1′ 19° 59′	69° 35′ 20° 25′	69° 9′ 20° 51′
17	73° 7′ 16° 53′	72° 49′ 17° 11′	72° 31′ 17° 29′	72° 13′ 17° 47′	71° 54′ 18° 6′	71° 34′ 18° 26′	71° 13′ 18° 47′	70° 52′ 19° 8′	70° 30′ 19° 30′	70° 7′ 19° 53′	69° 43′ 20° 17′	69° 17′ 20° 43′	68° 52′ 21° 8′	68° 26′ 21° 34′	67° 58′ 22° 2′
18	72° 11′ 17° 49′	71° 53′ 18° 7′	71° 34′ 18° 26′	71° 15′ 18° 45′	70° 54′ 19° 6′	70° 33′ 19° 27′	70° 12′ 19° 48′	69° 50′ 20° 10′	69° 26′ 20° 34′	69° 3′ 20° 57′	68° 38′ 21° 22′	68° 12′ 21° 48′	67° 45′ 22° 15′	67° 17′ 22° 43′	66° 48′ 23° 12′
19	71° 15′ 18° 45′	70° 57′ 19° 1′	70° 37′ 19° 23′	70° 17′ 19° 43′	69° 56′ 20° 4′	69° 34′ 20° 26′	69° 12′ 20° 48′	68° 48′ 21° 12′	68° 25′ 21° 35′	67° 59′ 22° 1′	67° 34′ 22° 26′	67° 6′ 22° 54′	66° 38′ 23° 22′	66° 10′ 23° 50′	65° 39′ 24° 21′
20	70° 21′ 19° 39′	70° 1′ 19° 59′	69° 41′ 20° 19′	69° 19′ 20° 41′	68° 57′ 21° 3′	68° 35′ 21° 25′	68° 12′ 21° 48′	67° 48′ 22° 12′	67° 23′ 22° 37′	66° 57′ 23° 3′	66° 30′ 23° 30′	66° 2′ 23° 58′	65° 33′ 24° 27′	65° 3′ 24° 57′	64° 32′ 25° 28′
21	69° 26′ 20° 34′	69° 0′ 20° 94′	68° 45′ 21° 15′	68° 23′ 21° 37′	68° 0′ 22° 0′	67° 37′ 22° 23′	67° 13′ 22° 47′	66° 48′ 23° 12′	66° 22′ 23° 38′	65° 55′ 24° 5′	65° 28′ 24° 32′	64° 59′ 25° 1′	64° 29′ 25° 31′	63° 58′ 26° 2′	63° 26′ 26° 34′
22	68° 33′ 21° 27′	68° 12′ 21° 48′	67° 50′ 22° 10′	67° 27′ 22° 33′	67° 4′ 22° 56′	66° 40′ 23° 20′	66° 15′ 23° 45′	65° 49′ 24° 11′	65° 23′ 24° 37′	64° 55′ 25° 5′	64° 26′ 25° 34′	63° 57′ 26° 3′	63° 26′ 26° 34′	62° 54′ 27° 6′	62° 21′ 27° 39′
23	67° 41′ 22° 19′	67° 18′ 22° 42′	66° 55′ 23° 5′	66° 32′ 23° 28′	66° 8′ 23° 52′	65° 44′ 24° 16′	65° 18′ 24° 42′	64° 51′ 25° 9′	64° 24′ 25° 36′	63° 55′ 26° 5′	63° 26′ 26° 34′	62° 56′ 27° 4′	62° 24′ 27° 36′	61° 52′ 28° 8′	61° 18′ 28° 42′
24	66° 48′ 23° 12′	66° 26′ 23° 34′	66° 2′ 23° 58′	65° 38′ 24° 22′	65° 14′ 24° 46′	64° 48′ 25° 12′	64° 22′ 25° 38′	63° 54′ 26° 6′	63° 26′ 26° 34′	62° 57′ 27° 3′	62° 27′ 27° 33′	61° 56′ 28° 4′	61° 23′ 28° 37′	60° 50′ 29° 10′	60° 15′ 29° 45′
25	65° 57′ 24° 3′	65° 33′ 24° 27′	65° 9′ 24° 51′	64° 45′ 25° 15′	64° 20′ 25° 40′	63° 53′ 26° 7′	63° 26′ 26° 34′	62° 58′ 27° 2′	62° 29′ 27° 31′	61° 59′ 28° 1′	61° 29′ 28° 31′	60° 57′ 29° 3′	60° 24′ 29° 36′	59° 50′ 30° 10′	59° 14′ 30° 46′
26	65° 6′ 24° 54′	64° 42′ 25° 18′	64° 18′ 25° 42′	63° 52′ 26° 8′	63° 26′ 26° 34′	62° 59′ 27° 1′	62° 31′ 27° 29′	62° 3′ 27° 57′	61° 33′ 28° 27′	61° 3′ 28° 57′	60° 31′ 29° 29′	59° 59′ 30° 1′	59° 25′ 30° 35′	58° 50′ 31° 10′	58° 14′ 31° 46′
27	64° 16′ 25° 44′	63° 51′ 26° 9′	63° 26′ 26° 34′	63° 0′ 27° 0′	62° 34′ 27° 26′	62° 6′ 27° 54′	61° 38′ 28° 22′	61° 8′ 28° 52′	60° 38′ 29° 22′	60° 7′ 29° 53′	59° 35′ 30° 25′	59° 2′ 30° 58′	58° 28′ 31° 32′	57° 53′ 32° 7′	57° 16′ 32° 44′
28	63° 26′ 26° 34′	63° 1′ 26° 59′	62° 36′ 27° 24′	62° 9′ 27° 51′	61° 42′ 28° 18′	61° 14′ 28° 46′	60° 45′ 29° 15′	60° 15′ 29° 45′	59° 45′ 30° 15′	59° 13′ 30° 47′	58° 40′ 31° 20′	58° 7′ 31° 53′	57° 32′ 32° 28′	56° 56′ 33° 4′	56° 19′ 33° 41′
29	62° 37′ 27° 23′	62° 12′ 27° 48′	61° 45′ 28° 15′	61° 19′ 28° 41′	60° 51′ 29° 9′	60° 23′ 29° 37′	59° 53′ 30° 7′	59° 23′ 30° 37′	58° 52′ 31° 8′	58° 19′ 31° 41′	57° 46′ 32° 14′	57° 12′ 32° 48′	56° 37′ 33° 23′	56° 0′ 34° 0′	55° 23′ 34° 37′
30	61° 49′ 28° 11′	61° 23′ 28° 37′	60° 57′ 29° 3′	60° 29′ 29° 31′	60° 1′ 29° 59′	59° 32′ 30° 28′	59° 2′ 30° 58′	58° 32′ 31° 28′	58° 0′ 32° 0′	57° 27′ 32° 33′	56° 53′ 33° 7′	56° 19′ 33° 41′	55° 43′ 34° 17′	55° 5′ 34° 55′	54° 28′ 35° 32′
31	61° 2′ 28° 58′	60° 36′ 29° 24′	60° 6′ 29° 54′	59° 41′ 30° 19′	59° 12′ 30° 48′	58° 42′ 31° 18′	58° 12′ 31° 48′	57° 41′ 32° 19′	57° 8′ 32° 52′	56° 36′ 33° 24′	56° 1′ 33° 59′	55° 26′ 34° 34′	54° 50′ 35° 10′	54° 12′ 35° 48′	53° 34′ 36° 26′
32	60° 15′ 29° 45′	59° 48′ 30° 12′	59° 21′ 30° 39′	58° 52′ 31° 8′	58° 34′ 31° 26′	57° 54′ 32° 6′	57° 23′ 32° 37′	56° 52′ 33° 8′	56° 19′ 33° 41′	55° 45′ 34° 15′	55° 11′ 34° 49′	54° 35′ 35° 25′	53° 58′ 36° 2′	53° 21′ 36° 39′	52° 42′ 37° 18′
33	59° 29′ 30° 31′	59° 2′ 30° 58′	58° 34′ 31° 26′	58° 5′ 31° 55′	57° 36′ 32° 24′	57° 6′ 32° 54′	56° 34′ 33° 26′	56° 2′ 33° 58′	55° 30′ 34° 30′	54° 56′ 35° 4′	54° 21′ 35° 39′	53° 45′ 36° 15′	53° 8′ 36° 52′	52° 29′ 37° 31′	51° 50′ 38° 9′
34	58° 44′ 31° 16′	58° 16′ 31° 44′	57° 48′ 32° 12′	57° 19′ 32° 41′	56° 49′ 33° 11′	56° 19′ 33° 41′	55° 47′ 34° 13′	55° 15′ 34° 45′	54° 41′ 35° 19′	54° 7′ 35° 53′	53° 32′ 36° 28′	52° 52′ 37° 8′	52° 18′ 37° 42′	51° 40′ 38° 20′	51° 0′ 39° 0′
35	58° 0′ 32° 0′	57° 32′ 32° 28′	57° 3′ 32° 57′	56° 33′ 33° 27′	56° 3′ 33° 57′	55° 32′ 34° 28′	55° 0′ 35° 0′	54° 28′ 35° 32′	53° 54′ 36° 6′	53° 20′ 36° 40′	52° 44′ 37° 16′	52° 8′ 37° 52′	51° 30′ 38° 30′	50° 51′ 39° 9′	50° 12′ 39° 48′
36	57° 16′ 32° 44′	56° 48′ 33° 12′	56° 19′ 33° 41′	55° 49′ 34° 11′	55° 18′ 34° 42′	54° 47′ 35° 13′	54° 15′ 35° 45′	53° 42′ 36° 18′	53° 8′ 36° 52′	52° 33′ 37° 27′	51° 57′ 38° 3′	51° 20′ 38° 40′	50° 43′ 39° 17′	50° 4′ 39° 56′	49° 24′ 40° 36′
37	56° 32′ 33° 28′	56° 4′ 33° 56′	55° 35′ 34° 25′	55° 5′ 34° 55′	54° 34′ 35° 26′	54° 2′ 35° 58′	53° 30′ 36° 30′	52° 56′ 37° 4′	52° 23′ 37° 37′	51° 47′ 38° 13′	51° 12′ 38° 48′	50° 35′ 39° 25′	49° 56′ 40° 4′	49° 17′ 40° 43′	48° 37′ 41° 23′
38	55° 51′ 34° 9′	55° 21′ 34° 39′	54° 52′ 35° 8′	54° 23′ 35° 37′	53° 51′ 36° 9′	53° 18′ 36° 42′	52° 46′ 37° 14′	52° 12′ 37° 48′	51° 38′ 38° 22′	51° 3′ 38° 57′	50° 27′ 39° 33′	49° 49′ 40° 11′	49° 11′ 40° 49′	48° 32′ 41° 28′	47° 52′ 42° 8′
39	55° 9′ 34° 51′	54° 39′ 35° 21′	54° 10′ 35° 50′	53° 39′ 36° 21′	53° 7′ 36° 53′	52° 36′ 37° 24′	52° 3′ 37° 57′	51° 29′ 38° 31′	50° 54′ 39° 6′	50° 19′ 39° 41′	49° 42′ 40° 18′	49° 5′ 40° 55′	48° 27′ 41° 33′	47° 48′ 42° 12′	47° 7′ 42° 53′
40	54° 28′ 35° 32′	53° 58′ 36° 2′	53° 28′ 36° 32′	52° 58′ 37° 2′	52° 26′ 37° 34′	51° 54′ 38° 6′	51° 20′ 38° 40′	50° 46′ 39° 14′	50° 12′ 39° 48′	49° 36′ 40° 24′	48° 59′ 41° 1′	48° 22′ 41° 38′	47° 44′ 42° 16′	47° 5′ 42° 55′	46° 24′ 43° 36′
41	53° 48′ 36° 12′	53° 17′ 36° 43′	52° 48′ 37° 12′	52° 16′ 37° 44′	51° 45′ 38° 15′	51° 12′ 38° 48′	50° 39′ 39° 21′	50° 5′ 39° 55′	49° 30′ 40° 30′	48° 54′ 41° 6′	48° 17′ 41° 43′	47° 40′ 42° 20′	47° 1′ 42° 59′	46° 22′ 43° 38′	45° 41′ 44° 19′
42	53° 8′ 36° 52′	52° 38′ 37° 22′	52° 8′ 37° 52′	51° 36′ 38° 24′	51° 4′ 38° 56′	50° 32′ 39° 28′	49° 58′ 40° 2′	49° 24′ 40° 36′	48° 49′ 41° 11′	48° 13′ 41° 47′	47° 36′ 42° 24′	46° 59′ 43° 1′	46° 20′ 43° 40′	45° 40′ 44° 20′	45°

TABLE 3.

Angle of Face.

	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27
12	13° 37′ 70° 33′	13° 57′ 70° 33′	14° 18′ 70° 6′	14° 39′ 69° 33′	15° 1′ 69° 5′	15° 24′ 68° 32′	15° 49′ 67° 59′	16° 15′ 67° 23′	16° 43′ 66° 46′	17° 13′ 66° 5′	17° 43′ 65° 23′	18° 16′ 64° 38′	18° 51′ 63° 53′	18° 27′ 63° 3′	20° 5′ 62° 9′
13	14° 55′ 69° 45′	15° 17′ 69° 15′	15° 39′ 68° 47′	16° 1′ 68° 15′	16° 25′ 67° 43′	16° 51′ 67° 9′	17° 19′ 66° 33′	17° 46′ 65° 56′	18° 16′ 65° 16′	18° 48′ 64° 34′	19° 21′ 63° 51′	19° 57′ 63° 5′	20° 32′ 62° 14′	21° 11′ 61° 23′	21° 54′ 60° 28′
14	16° 13′ 68° 31′	16° 34′ 68° 0′	16° 59′ 67° 29′	17° 24′ 66° 56′	17° 50′ 66° 22′	18° 17′ 65° 47′	18° 45′ 65° 8′	19° 16′ 64° 30′	19° 48′ 63° 48′	20° 20′ 63° 6′	20° 56′ 62° 20′	21° 34′ 61° 32′	22° 13′ 60° 41′	22° 55′ 59° 47′	23° 38′ 58° 50′
15	17° 28′ 67° 16′	17° 53′ 66° 45′	18° 18′ 66° 14′	18° 44′ 65° 40′	19° 11′ 65° 3′	19° 40′ 64° 26′	20° 11′ 63° 49′	20° 44′ 63° 8′	21° 18′ 62° 24′	21° 53′ 61° 39′	22° 31′ 60° 51′	23° 10′ 60° 2′	23° 51′ 59° 9′	24° 35′ 58° 13′	25° 20′ 57° 14′
16	18° 42′ 66° 4′	19° 9′ 65° 33′	19° 35′ 64° 58′	20° 3′ 64° 28′	20° 32′ 63° 46′	21° 3′ 63° 7′	21° 36′ 62° 28′	22° 9′ 61° 45′	22° 40′ 61° 01′	23° 22′ 60° 14′	24° 1′ 59° 25′	24° 42′ 58° 34′	25° 26′ 57° 40′	26° 12′ 56° 42′	27° 1′ 55° 48′
17	19° 56′ 64° 54′	20° 24′ 64° 20′	20° 51′ 63° 46′	21° 21′ 63° 9′	21° 58′ 62° 31′	22° 24′ 61° 50′	22° 57′ 61° 9′	23° 33′ 60° 25′	24° 10′ 59° 40′	25° 50′ 58° 52′	25° 31′ 58° 1′	26° 14′ 57° 10′	27° 59′ 56° 18′	27° 47′ 55° 15′	28° 37′ 54° 18′
18	21° 9′ 63° 45′	21° 37′ 63° 9′	22° 6′ 62° 34′	22° 38′ 61° 56′	23° 8′ 61° 17′	23° 43′ 60° 35′	24° 18′ 59° 52′	24° 56′ 59° 8′	25° 34′ 58° 20′	26° 15′ 57° 31′	26° 57′ 56° 39′	27° 42′ 55° 46′	28° 29′ 54° 49′	29° 18′ 53° 50′	30° 9′ 52° 47′
19	22° 20′ 62° 36′	22° 49′ 62° 1′	23° 20′ 61° 24′	23° 52′ 60° 44′	24° 26′ 60° 4′	25° 1′ 59° 21′	25° 37′ 58° 37′	26° 15′ 57° 51′	26° 56′ 57° 4′	27° 30′ 56° 14′	28° 22′ 55° 22′	29° 8′ 54° 26′	29° 56′ 53° 28′	30° 43′ 52° 28′	31° 40′ 51° 24′
20	23° 30′ 61° 30′	24° 1′ 60° 53′	24° 32′ 60° 14′	25° 6′ 59° 34′	25° 40′ 58° 54′	26° 16′ 58° 10′	26° 56′ 57° 26′	27° 34′ 56° 39′	28° 15′ 55° 49′	28° 58′ 54° 58′	29° 44′ 54° 4′	30° 31′ 53° 8′	31° 21′ 52° 9′	32° 13′ 51° 9′	33° 8′ 50° 4′
21	24° 38′ 60° 25′	25° 10′ 59° 46′	25° 43′ 59° 7′	26° 18′ 58° 26′	26° 53′ 57° 43′	27° 30′ 57° 0′	28° 10′ 56° 14′	28° 50′ 55° 26′	29° 32′ 54° 36′	30° 17′ 53° 48′	31° 4′ 52° 50′	31° 52′ 51° 52′	32° 43′ 50° 53′	33° 36′ 49° 50′	34° 31′ 48° 47′
22	25° 46′ 59° 20′	26° 19′ 58° 41′	26° 53′ 58° 1′	27° 27′ 57° 19′	28° 5′ 56° 36′	28° 43′ 55° 51′	29° 22′ 55° 4′	30° 5′ 54° 17′	30° 48′ 53° 26′	31° 34′ 52° 32′	32° 22′ 51° 38′	33° 11′ 50° 41′	34° 3′ 49° 41′	34° 57′ 48° 37′	35° 54′ 47° 32′
23	26° 52′ 58° 16′	27° 26′ 57° 38′	28° 0′ 56° 56′	28° 36′ 56° 14′	29° 14′ 55° 30′	29° 58′ 54° 43′	35° 35′ 53° 57′	31° 18′ 53° 8′	32° 1′ 52° 16′	32° 48′ 51° 24′	33° 36′ 50° 28′	34° 27′ 49° 29′	35° 20′ 48° 30′	36° 15′ 47° 27′	37° 12′ 46° 20′
24	27° 57′ 57° 15′	28° 31′ 56° 35′	29° 7′ 55° 53′	29° 43′ 55° 11′	30° 22′ 54° 26′	31° 2′ 53° 40′	31° 46′ 52° 51′	32° 28′ 52° 2′	33° 14′ 51° 10′	34° 1′ 50° 6′	34° 50′ 49° 20′	35° 41′ 48° 22′	36° 35′ 47° 21′	37° 30′ 46° 18′	38° 28′ 45° 12′
25	28° 55′ 56° 15′	29° 34′ 55° 34′	30° 12′ 54° 52′	30° 49′ 54° 9′	31° 29′ 53° 23′	32° 10′ 52° 36′	32° 52′ 51° 48′	33° 37′ 50° 57′	34° 23′ 50° 5′	35° 11′ 49° 11′	36° 0′ 48° 14′	36° 52′ 47° 16′	37° 47′ 46° 15′	38° 43′ 45° 11′	39° 41′ 44° 5′
26	30° 1′ 55° 16′	30° 38′ 54° 34′	31° 14′ 53° 52′	31° 54′ 53° 8′	32° 34′ 52° 22′	33° 16′ 51° 35′	33° 58′ 50° 46′	34° 46′ 49° 56′	35° 31′ 49° 3′	36° 19′ 48° 7′	37° 10′ 47° 12′	38° 2′ 46° 12′	38° 56′ 45° 10′	39° 53′ 44° 7′	40° 52′ 43° 2′
27	31° 3′ 54° 19′	31° 39′ 53° 37′	32° 18′ 52° 54′	32° 57′ 52° 9′	33° 37′ 51° 23′	34° 20′ 50° 34′	35° 3′ 49° 45′	35° 49′ 48° 55′	36° 36′ 48° 2′	37° 25′ 47° 7′	38° 16′ 46° 10′	39° 10′ 45° 10′	40° 4′ 44° 10′	41° 1′ 43° 5′	42°
28	32° 2′ 53° 22′	32° 39′ 52° 39′	33° 18′ 51° 56′	33° 57′ 51° 11′	34° 39′ 50° 25′	35° 21′ 49° 37′	36° 7′ 48° 47′	36° 52′ 47° 56′	37° 40′ 47° 2′	38° 20′ 46° 6′	39° 21′ 45° 11′	40° 15′ 44° 11′	41° 9′ 43° 9′	42° 7′	
29	32° 59′ 52° 27′	33° 38′ 51° 44′	34° 17′ 51° 1′	34° 58′ 50° 16′	35° 39′ 49° 29′	36° 23′ 48° 41′	37° 8′ 47° 50′	37° 54′ 46° 58′	38° 42′ 46° 4′	39° 32′ 45° 10′	40° 24′ 44° 12′	41° 10′ 43° 14′	42° 13′		
30	33° 57′ 51° 38′	34° 36′ 50° 50′	35° 15′ 50° 7′	35° 56′ 49° 20′	36° 38′ 48° 34′	37° 21′ 47° 45′	38° 7′ 46° 55′	38° 53′ 46° 3′	39° 45′ 45° 9′	40° 32′ 44° 14′	41° 25′ 43° 17′	42° 16′			
31	34° 53′ 50° 41′	35° 31′ 49° 57′	36° 11′ 49° 13′	36° 52′ 48° 28′	37° 35′ 47° 39′	38° 20′ 46° 52′	39° 5′ 46° 1′	39° 52′ 45° 10′	40° 41′ 44° 15′	41° 32′ 43° 20′	42° 23′				
32	35° 46′ 49° 50′	36° 27′ 49° 7′	37° 6′ 48° 22′	37° 48′ 47° 36′	38° 31′ 46° 49′	39° 15′ 45° 59′	40° 1′ 45° 9′	40° 49′ 44° 17′	41° 38′ 43° 24′	42° 28′					
33	36° 39′ 48° 59′	37° 19′ 48° 17′	38° 0′ 47° 32′	38° 42′ 46° 46′	39° 26′ 45° 58′	40° 0′ 45° 8′	40° 56′ 44° 18′	41° 44′ 43° 26′	42° 33′						
34	37° 32′ 48° 12′	38° 11′ 47° 27′	38° 53′ 46° 43′	39° 35′ 45° 57′	40° 18′ 45° 8′	41° 4′ 44° 20′	41° 49′ 43° 29′	42° 37′							
35	38° 22′ 47° 24′	39° 3′ 46° 39′	39° 44′ 45° 54′	40° 26′ 45° 8′	41° 10′ 44° 20′	41° 55′ 43° 31′	42° 41′								
36	39° 11′ 46° 37′	39° 52′ 45° 52′	40° 34′ 45° 8′	41° 16′ 44° 21′	42° 0′ 43° 34′	42° 45′									
37	40° 0′ 45° 52′	40° 40′ 45° 8′	41° 22′ 44° 22′	42° 5′ 43° 37′	42° 48′										
38	40° 47′ 45° 7′	41° 28′ 44° 24′	42° 9′ 43° 39′	42° 52′											
39	41° 32′ 44° 24′	42° 14′ 43° 40′	42° 55′												
40	43° 18′ 43° 42′	42° 58′													
41	43° 2′														

TABLE 3.—(*Continued.*)

ANGLE OF FACE.

	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	
12	20° 46 61° 14	21° 31 60° 15	22° 18 59° 10	23° 8 58° 2	24° 3 56° 49	25° 2 55° 32	26° 3 54° 7	27° 11 52° 39	28° 25 51° 3	29° 43 49° 17	31° 11 47° 25	32° 44 45° 24	34° 26 43° 14	30° 16 40° 50	38° 17	
13	22° 37 59° 29	23° 26 58° 28	24° 15 57° 21	25° 9 56° 11	26° 6 54° 56	27° 8 53° 36	28° 14 52° 10	29° 25 50° 39	30° 42 49° 2	32° 4 47° 16	33° 34 45° 22	35° 10 43° 20	30° 55 41° 9	38° 48		
14	24° 25 57° 48	25° 16 56° 46	26° 8 55° 38	27° 5 54° 25	28° 4 53° 8	29° 9 51° 47	30° 20 50° 20	31° 33 48° 47	32° 52 47° 8	34° 18 45° 12	35° 50 43° 26	37° 28 41° 24	39° 15			
15	26° 11 56° 13	27° 3 55° 7	27° 58 53° 58	28° 58 52° 44	30° 0 51° 26	31° 6 50° 2	32° 19 48° 33	33° 36 47° 0	34° 66 45° 20	36° 23 43° 33	37° 57 41° 39	39° 38				
16	27° 52 54° 38	28° 45 53° 31	29° 43 52° 21	30° 44 51° 6	31° 60 49° 46	32° 58 48° 22	34° 12 46° 52	35° 31 45° 19	36° 54 43° 38	38° 23 41° 51	39° 57					
17	29° 30 53° 8	30° 26 52° 0	31° 26 50° 48	32° 28 49° 32	33° 35 48° 11	34° 47 46° 47	36° 0 45° 16	37° 21 43° 43	38° 45 42° 1	40° 15						
18	31° 5 51° 41	32° 2 50° 32	33° 4 49° 18	34° 8 48° 2	35° 15 46° 41	36° 28 45° 16	37° 45 43° 45	39° 5 42° 11	40° 31							
19	32° 36 50° 18	33° 36 49° 8	34° 38 47° 64	35° 48 46° 36	36° 53 45° 15	38° 6 43° 50	39° 24 42° 20	40° 45								
20	34° 5 48° 57	35° 6 47° 46	36° 8 46° 32	37° 16 45° 14	38° 26 43° 52	39° 39 42° 27	40° 67									
21	35° 31 47° 39	36° 32 46° 28	37° 37 15° 13	38° 44 43° 56	39° 54 42° 34	41° 8										
22	36° 52 46° 24	37° 65 45° 13	39° 0 43° 58	40° 8 42° 40	41° 19											
23	38° 12 45° 12	39° 15 44° 1	40° 20 42° 46	41° 28												
24	39° 29 44° 3	40° 32 42° 52	41° 38													
25	40° 43 42° 57	41° 46														
26	41° 63															

$$g_a = 90^\circ - (\alpha_a + \beta)$$
$$g_b = 90^\circ - (\alpha_b + \beta)$$

(*See page 13.*)

NATURAL SINE.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00291	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02326	.02617	.02908	.03199	.03489	88
2	.03489	.03780	.04071	.04361	.04652	.04943	.05233	87
3	.05233	.05524	.05814	.06104	.06395	.06685	.06975	86
4	.06975	.07265	.07555	.07845	.08135	.08425	.08715	85
5	.08715	.09005	.09295	.09584	.09874	.10163	.10452	84
6	.10452	.10742	.11031	.11320	.11609	.11898	.12186	83
7	.12186	.12475	.12764	.13052	.13341	.13629	.13917	82
8	.13917	.14205	.14493	.14780	.15068	.15356	.15643	81
9	.15643	15930	.16217	.16504	.16791	.17078	.17364	80
10	.17364	.17651	.17937	.18223	.18509	.18795	.19080	79
11	.19080	.19366	.19651	.19936	.20221	.20506	.20791	78
12	.20791	.21075	.21359	.21644	.21927	.22211	.22495	77
13	.22495	.22778	.23061	.23344	.23627	.23909	.24192	76
14	.24192	.24474	.24756	.25038	.25319	.25600	.25881	75
15	.25881	.26162	.26443	.26723	.27004	.27284	.27563	74
16	.27563	.27843	.28122	.28401	.28680	.28958	.29237	73
17	.29237	.29515	.29793	.30070	.30347	.30624	.30901	72
18	.30901	.31178	.31454	.31730	.32006	.32281	.32556	71
19	.32556	.32831	.33106	.33380	.33654	.33928	.34202	70
20	.34202	.34475	.34748	.35020	.35293	.35565	.35836	69
21	.35836	.36108	.36379	.36650	.36920	.37190	.37460	68
22	.37460	.37730	.37999	.38268	.38536	.38805	.39073	67
23	.39073	.39340	.39607	.39874	.40141	.40407	.40673	66
24	.40673	.40939	.41204	.41469	.41733	.41998	.42261	65
25	.42261	.42525	.42788	.43051	.43313	.43575	.43837	64
26	.43837	.44098	.44359	.44619	.44879	.45139	.45399	63
27	.45399	.45658	.45916	.46174	.46432	.46690	.46947	62
28	.46947	.47203	.47460	.47715	.47971	.48226	.48481	61
29	.48481	.48735	.48989	.49242	.49495	.49747	.50000	60
30	.50000	.50251	.50503	.50753	.51004	.51254	.51503	59
31	.51503	.51752	.52001	.52249	.52497	.52745	.52991	58
32	.52991	.53238	.53484	.53730	.53975	.54219	.54463	57
33	.54463	.54707	.54950	.55193	.55436	.55677	.55919	56
34	.55919	.56160	.56400	.56640	.56880	.57119	.57357	55
35	.57357	.57595	.57833	.58070	.58306	.58542	.58778	54
36	.58778	.59013	.59248	.59482	.59715	.59948	.60181	53
37	.60181	.60413	.60645	.60876	.61106	.61336	.61566	52
38	.61566	.61795	.62023	.62251	.62478	.62705	.62932	51
39	.62932	.63157	.63383	.63607	.63832	.64055	.64278	50
40	.64278	.64501	.64723	.64944	.65165	.65386	.65605	49
41	.65605	.65825	.66043	.66262	.66479	.66696	.66913	48
42	.66913	.67128	.67344	.67559	.67773	.67986	.68199	47
43	.68199	.68412	.68624	.68835	.69046	.69256	.69465	46
44	.69465	.69674	.69883	.70090	.70298	.70504	.70710	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

NATURAL COSINE.

NATURAL SINE.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	.70710	.70916	.71120	.71325	.71528	.71731	.71934	44
46	.71934	.72135	.72336	.72537	.72737	.72936	.73135	43
47	.73135	.73333	.73530	.73727	.73923	.74119	.74314	42
48	.74314	.74508	.74702	.74895	.75088	.75279	.75471	41
49	.75471	.75661	.75851	.76040	.76229	.76417	.76604	40
50	.76604	.76791	.76977	.77162	.77347	.77531	.77714	39
51	.77714	.77897	.78079	.78260	.78441	.78621	.78801	38
52	.78801	.78979	.79157	.79335	.79512	.79688	.79863	37
53	.79863	.80038	.80212	.80385	.80558	.80730	.80901	36
54	.80901	.81072	.81242	.81411	.81580	.81748	.81915	35
55	.81915	.82081	.82247	.82412	.82577	.82740	.82903	34
56	.82903	.83066	.83227	.83388	.83548	.83708	.83867	33
57	.83867	.84025	.84182	.84339	.84495	.84650	.84804	32
58	.84804	.84958	.85111	.85264	.85415	.85566	.85716	31
59	.85716	.85866	.86014	.86162	.86310	.86456	.86602	30
60	.86602	.86747	.86892	.87035	.87178	.87320	.87462	29
61	.87462	.87602	.87742	.87881	.88020	.88157	.88294	28
62	.88294	.88430	.88566	.88701	.88835	.88968	.89100	27
63	.89100	.89232	.89363	.89493	.89622	.89751	.89879	26
64	.89879	.90006	.90132	.90258	.90383	.90507	.90630	25
65	.90630	.90753	.90875	.90996	.91116	.91235	.91354	24
66	.91354	.91472	.91589	.91706	.91821	.91936	.92050	23
67	.92050	.92163	.92276	.92388	.92498	.92609	.92718	22
68	.92718	.92827	.92934	.93041	.93148	.93253	.93358	21
69	.93358	.93461	.93565	.93667	.93768	.93869	.93969	20
70	.93969	.94068	.94166	.94264	.94360	.94456	.94551	19
71	.94551	.94646	.94739	.94832	.94924	.95015	.95105	18
72	.95105	.95195	.95283	.95371	.95458	.95545	.95630	17
73	.95630	.95715	.95799	.95882	.95964	.96045	.96126	16
74	.96126	.96205	.96284	.96363	.96440	.96516	.96592	15
75	.96592	.96667	.96741	.96814	.96887	.96958	.97029	14
76	.97029	.97099	.97168	.97237	.97304	.97371	.97437	13
77	.97437	.97502	.97566	.97629	.97692	.97753	.97814	12
78	.97814	.97874	.97934	.97992	.98050	.98106	.98162	11
79	.98162	.98217	.98272	.98325	.98378	.98429	.98480	10
80	.98480	.98530	.98580	.98628	.98676	.98722	.98768	9
81	.98768	.98813	.98858	.98901	.98944	.98985	.99026	8
82	.99026	.99066	.09106	.99144	.99182	.99218	.99254	7
83	.99254	.99289	.99323	.99357	.99389	.99421	.99452	6
84	.99452	.99482	.99511	.99539	.99567	.99593	.99619	5
85	.99619	.99644	.99668	.99691	.99714	.99735	.99756	4
86	.99756	.99776	.99795	99813	.99830	.99847	.99863	3
87	.99863	.99877	.99891	.99904	.99917	.99928	.99939	2
88	.99939	.99948	.99957	.99965	.99972	.99979	.99984	1
89	.99984	.99989	.99993	.99996	.99998	.99999	1.0000	0
-	60′	50′	40′	30′	20′	10′	0′	Deg.

NATURAL COSINE.

NATURAL TANGENT.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00290	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02327	.02618	.02909	.03200	.03492	88
2	.03492	.03783	.04074	.04366	.04657	.04949	.05240	87
3	.05240	.05532	.05824	.06116	.06408	.06700	.06992	86
4	.06992	.07285	.07577	.07870	.08162	.08455	.08748	85
5	.08748	.09042	.09335	.09628	.09922	.10216	.10510	84
6	.10510	.10804	.11099	.11393	.11688	.11983	.12278	83
7	.12278	.12573	.12869	.13165	.13461	.13757	.14054	82
8	.14054	.14350	.14647	.14945	.15242	.15540	.15838	81
9	.15838	.16136	.16435	.16734	.17033	.17332	.17632	80
10	.17632	.17932	.18233	.18533	.18834	.19136	.19438	79
11	.19438	.19740	.20042	.20345	.20648	.20951	.21255	78
12	.21255	.21559	.21864	.22169	.22474	.22780	.23086	77
13	.23086	.23393	.23700	.24007	.24315	.24624	.24932	76
14	.24932	.25242	.25551	.25861	.26172	.26483	.26794	75
15	.26794	.27106	.27419	.27732	.28046	.28360	.28674	74
16	.28674	.28989	.29305	.29621	.29938	.30255	.30573	73
17	.30573	.30891	.31210	.31529	.31850	.32170	.32492	72
18	.32492	.32813	.33136	.33459	.33783	.34107	.34432	71
19	.34432	.34758	.35084	.35411	.35739	.36067	.36397	70
20	.36397	.36726	.37057	.37388	.37720	.38053	.38386	69
21	.38386	.38720	.39055	.39391	.39727	.40064	.40402	68
22	.40402	.40741	.41080	.41421	.41762	.42104	.42447	67
23	.42447	.42791	.43135	.43481	.43827	.44174	.44522	66
24	.44522	.44871	.45221	.45572	.45924	.46277	.46630	65
25	.46630	.46985	.47341	.47697	.48055	.48413	.48773	64
26	.48773	.49133	.49495	.49858	.50221	.50586	.50952	63
27	.50952	.51319	.51687	.52056	.52427	.52798	.53170	62
28	.53170	.53544	.53919	.54295	.54672	.55051	.55430	61
29	.55430	.55811	.56193	.56577	.56961	.57347	.57735	60
30	.57735	.58123	.58513	.58904	.59297	.59690	.60086	59
31	.60086	.60482	.60880	.61280	.61680	.62083	.62486	58
32	.62486	.62892	.63298	.63707	.64116	.64528	.64940	57
33	.64940	.65355	.65771	.66188	.66607	.67028	.67450	56
34	.67450	.67874	.68300	.68728	.69157	.69588	.70020	55
35	.70020	.70455	.70891	.71329	.71769	.72210	.72654	54
36	.72654	.73099	.73546	.73996	.74447	.74900	.75355	53
37	.75355	.75812	.76271	.76732	.77195	.77661	.78128	52
38	.78128	.78598	.79069	.79543	.80019	.80497	80978	51
39	.80978	.81461	.81946	.82433	.82923	.83415	.83910	50
40	.83910	.84406	.84906	.85408	.85912	.86419	.86928	49
41	.86928	.87440	.87955	.88472	.88992	.89515	.90040	48
42	.90040	.90568	.91099	.91633	.92169	.92709	.93251	47
43	.93251	.93796	.94345	.94896	.95450	.96008	.96568	46
44	.96568	.97132	.97699	.98269	.98843	.99419	1.0000	45
	60′	50′	40′	30	20′	10′	0′	Deg.

NATURAL COTANGENT.

NATURAL TANGENT.

Deg.	0′	10′	20′	30′	40′	50′	60	
45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44
46	1.0355	1.0415	1.0476	1.0537	1.0599	1.0661	1.0723	43
47	1.0723	1.0786	1.0849	1.0913	1.0977	1.1041	1.1106	42
48	1.1106	1.1171	1.1236	1.1302	1.1369	1.1436	1.1503	41
49	1.1503	1.1571	1.1639	1.1708	1.1777	1.1847	1.1917	40
50	1.1917	1.1988	1.2059	1.2131	1.2203	1.2275	1.2349	39
51	1.2349	1.2422	1.2496	1.2571	1.2647	1.2723	1.2799	38
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37
53	1.3270	1.3351	1.3432	1.3514	1.3596	1.3680	1.3763	36
54	1.3763	1.3848	1.3933	1.4019	1.4106	1.4193	1.4281	35
55	1.4281	1.4370	1.4459	1.4550	1.4641	1.4733	1.4825	34
56	1.4825	1.4919	1.5013	1.5108	1.5204	1.5301	1.5398	33
57	1.5398	1.5497	1.5596	1.5696	1.5798	1.5900	1.6003	32
58	1.6003	1.6107	1.6212	1.6318	1.6425	1.6533	1.6642	31
59	1.6642	1.6753	1.6864	1.6976	1.7090	1.7204	1.7320	30
60	1.7320	1.7437	1.7555	1.7674	1.7795	1.7917	1.8040	29
61	1.8040	1.8164	1.8290	1.8417	1.8546	1.8676	1.8807	28
62	1.8807	1.8940	1.9074	1.9209	1.9347	1.9485	1.9626	27
63	1.9626	1.9768	1.9911	2.0056	2.0203	2.0352	2.0503	26
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25
65	2.1445	2.1609	2.1774	2.1943	2.2113	2.2285	2.2460	24
66	2.2460	2.2637	2.2816	2.2998	2.3182	2.3369	2.3558	23
67	2.3558	2.3750	2.3944	2.4142	2.4342	2.4545	2.4750	22
68	2.4750	2.4959	2.5171	2.5386	2.5604	2.5826	2.6050	21
69	2.6050	2.6279	2.6510	2.6746	2.6985	2.7228	2.7474	20
70	2.7474	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	19
71	2.9042	2.9318	2.9600	2.9886	3.0178	3.0474	3.0776	18
72	3.0776	3.1084	3.1397	3.1715	3.2040	3.2371	3.2708	17
73	3.2708	3.3052	3.3402	3.3759	3.4123	3.4495	3.4874	16
74	3.4874	3.5260	3.5655	3.6058	3.6470	3.6890	3.7320	15
75	3.7320	3.7759	3.8208	3.8667	3.9136	3.9616	4.0107	14
76	4.0107	4.0610	4.1125	4.1653	4.2193	4.2747	4.3314	13
77	4.3314	4.3896	4.4494	4.5107	4.5736	4.6382	4.7046	12
78	4.7046	4.7728	4.8430	4.9151	4.9894	5.0658	5.1445	11
79	5.1445	5.2256	5.3092	5.3955	5.4845	5.5763	5.6712	10
80	5.6712	5.7693	5.8708	5.9757	6.0844	6.1970	6.3137	9
81	6.3137	6.4348	6.5605	6.6911	6.8269	6.9682	7.1153	8
82	7.1153	7.2687	7.4287	7.5957	7.7703	7.9530	8.1443	7
83	8.1443	8.3449	8.5555	8.7768	9.0098	9.2553	9.5143	6
84	9.5143	9.7881	10.078	10.385	10.711	11.059	11.430	5
85	11.430	11.826	12.250	12.706	13.196	13.726	14.300	4
86	14.300	14.924	15.604	16.349	17.169	18.075	19.081	3
87	19.081	20.205	21.470	22.904	24.541	26.431	28.636	2
88	28.636	31.241	34.367	38.188	42.964	49.103	57.290	1
89	57.290	68.750	85.939	114.58	171.88	343.77	∞	0
	60′	50	40′	30′	20′	10′	0′	Deg.

NATURAL COTANGENT.

CHAPTER IV.

WORM AND WORM WHEEL.

(Fig. 8.)

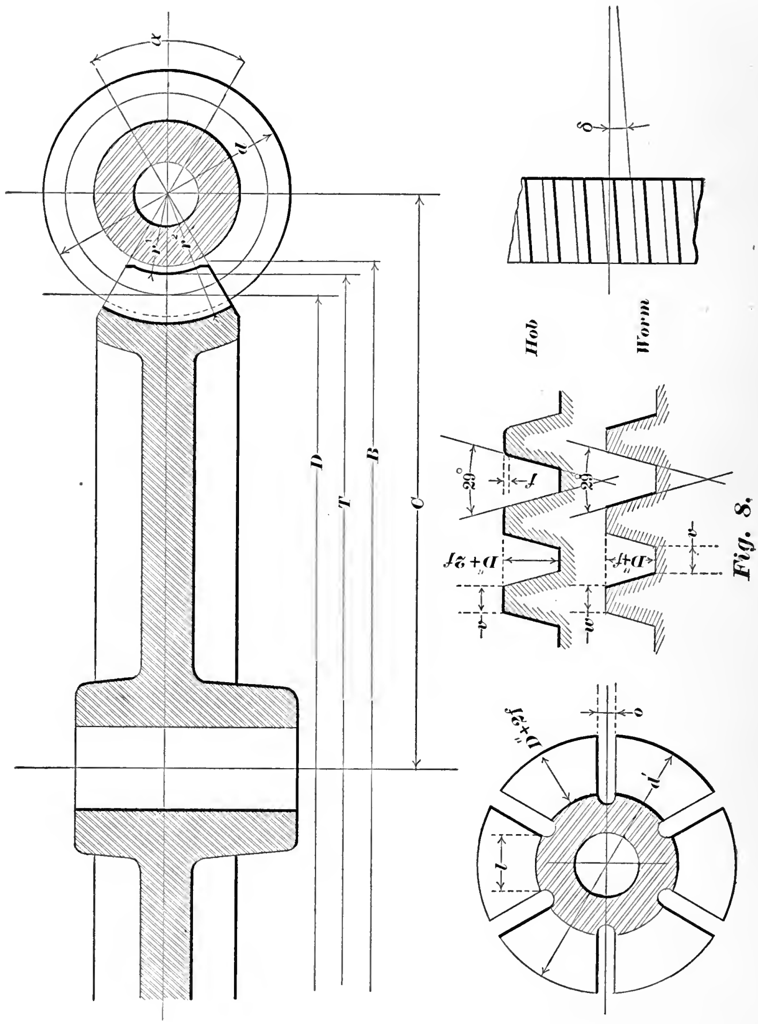


Fig. 8.

FORMULAS.

L = lead of worm.
N = number of teeth in gear.
m = threads per inch in worm.
d = diameter of worm.
d' = diameter of hob.
T = throat diameter.
B = blank diameter (to sharp corners).
C = distance between centers.
o = thickness of hob-slotting cutter.
l = width of bands at bottom.
b = pitch circumference of worm.
v = width of worm thread tool at end.
w = width of worm thread at tap.
P = diametral pitch.
P' = circular pitch.
s = addendum.
t = thickness of tooth at pitch line.
t^n = normal thickness of tooth.
f = clearance at bottom of tooth.
D'' = working depth of tooth.
$D'' + f$ = whole depth of tooth.
δ = angle of thread with axis.

If the lead is for single, double, triple, etc., thread, then

$$L = P',\ 2\ P',\ 3\ P', \text{ etc.}$$

$\alpha = 60^\circ$ to 90°

$L = \frac{1}{m}$

$P' = \frac{\pi T}{N + 2}$

$D = \frac{N P'}{\pi} = \frac{N}{P}$

$T = \frac{N}{P} + 2s$

$b = \pi (d - 2s)$

$\tan \delta = \frac{L}{b}$ { Practical only when width of wheel on wheel pitch circle is not more than ⅔ pitch diameter of worm.

$t^n = t \cos \delta$

$r^1 = \frac{d}{2} - 2s$

$r^2 = r^1 + D'' + f$

$C = \frac{D + d}{2} - s$

$B = T + 2\left(r^1 - r^1 \cos \frac{\alpha}{2}\right)$ A measurement of sketch is generally sufficient.

$o = \frac{.335\ P'}{2} + \frac{1}{8}''$

$l = D'' + 2f + \frac{1}{8}''$

$d' = d + 2f$

$v = .31\ P'$

$w = .335\ P'$

NOTE.—The notations and formulas referring to tooth parts, given on page 5 for spur gears, apply to worm wheels, and are here used.

NOTE.—Hob and worm should be marked, as per example:

4 threads per 1″ single .25 P′; .25 L.
2 threads per 1″ double .25 P′; .50 L.

UNDERCUT IN WORM WHEELS.

In worm wheels of less than 30 teeth the thread of the worm (being 29°) interferes with the flank of the gear tooth. Such a wheel finished with a hob will have its teeth undercut. To avoid this interference two methods may be employed.

First Method.—Make throat diameter of wheel

$$T = \cos^2 14\tfrac{1}{2}° \frac{N}{P} + 4s \qquad \text{or}$$

$$T = \frac{.937\ N}{P} + 4s$$

This formula increases the throat diameter, and consequently the center distance. The amount of the increase can be found by comparing this value of T with the one as obtained by formula on page 34. To keep the original center distance, the outside diameter of the worm must be reduced by the same amount the throat diameter is increased.

Second Method.—Without changing any of the dimensions we found by the formulas given on page 34, we can avoid the interference to be found in worm wheels of less than 30 teeth by simply increasing the angle of worm thread. We find the value of this angle by the following formula:

Let there be

2γ = angle of worm thread.
N = number of teeth in worm wheel.

$$\cos \gamma = \sqrt{1 - \frac{2}{N}}$$

From this formula we obtain the following values:

N	29	28	27	26	25	24	23	22	21	20
2γ	30¼	31	31½	32¼	32¾	33½	34¼	35	36	37

N	19	18	17	16	15	14	13	12
2γ	38	39	40	41½	42¾	44½	46¼	48

As this latter formula involves the making of new hobs in many cases, on account of change of angle, we prefer to reduce the diameter of worm as indicated by first method, if the distance of centers must be absolute.

CHAPTER V.

SPIRAL OR SCREW GEARING.

(Figs. 9, 10, 11.)

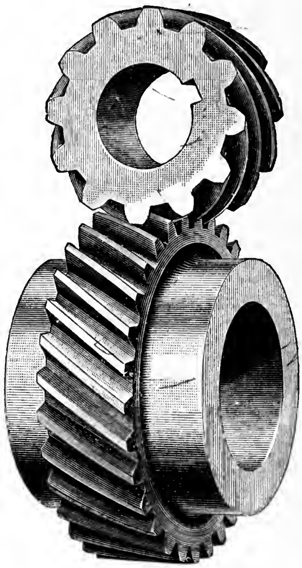


Fig. 9.

In spiral gearing the wheels have cylindrical pitch surfaces, but the teeth are not parallel to the axis. The line in which the pitch surface intersects the face of a tooth is part of a screw line, or helix, drawn at the pitch surface. A screw wheel may have one or any number of teeth. A one-toothed wheel corresponds to a one-threaded screw, a many-toothed wheel to a many-threaded screw. The axes may be placed at any angle.

Consider spiral gears with :

I. Axes parallel.

II. Axes at right angles.

III. Axes any angle.

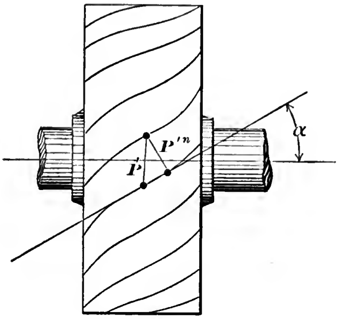


Fig. 10.

Let there be:

$N_a =$ } number of teeth in gears { a
$N_b =$ } { b

$C =$ center distance.
$P =$ diametral pitch
$P' =$ circular pitch.
$P^n =$ normal diametral pitch.
$P'^n =$ normal circular pitch.
$\gamma =$ angle of axes.
$L_1 =$ exact lead of spiral on pitch surface.
$L_2 =$ approximate lead of spiral on pitch surface.
$T =$ number of teeth marked on cutter to be used when teeth are to be cut on milling machine.
$D =$ pitch diameter.
$B =$ blank diameter.
$\alpha_a =$ } angle of teeth with axis
$\alpha_b =$ }
$t =$ thickness of tooth.
$s =$ addendum.
$D'' + f =$ whole depth of tooth.

NOTE.—Letters a and b occurring at bottom of notations refer to gears a and b.

I.—AXES PARALLEL.

Gears of this class are called twisted gears. The angle of teeth with axes in both gears must be equal and the spirals run in opposite directions. The angles are generally chosen small (seldom over 20°) to avoid excessive end thrust. End thrust may, however, be entirely avoided by combining two pairs of wheels with right and left-hand obliquity. Gears of this class are known as Herringbone gears. They are comparatively noiseless running at high speed.

II.—Axes at Right Angles.

Here we must always have:

1. The teeth of same hand spiral;
2. The normal pitches equal in both gears; and
3. The sum of the angles of teeth with axes $= 90°$.

Choosing Angle of Teeth with Axes.

1. If in a pair of gears the ratio of the number of teeth is equal to the direct ratio of the diameters, *i. e.*, if the number of teeth in the two gears are to each other as their pitch diameters, then the angles of the spirals will be 45° and 45°; for, this condition being fulfilled, the circular pitches of the two gears must be alike, which is only possible with angles of 45°. In such a combination either gear may be the driver.

2. If the ratio of the diameters determined upon is larger or smaller than the ratio of the number of teeth, then the angles are:

$$\tan \alpha_a = \frac{D_a\, N_b}{D_b\, N_a} \qquad \tan \alpha_b = \frac{D_b\, N_a}{D_a\, N_b}$$

In such gears the velocity ratio is measured by the number of teeth, and not by the diameters.

3. Given N_a, N_b and C:
If $P_a{}'$ is made $= P_b{}'$, then we have case "1" and

$$P' = \frac{\pi C}{\frac{1}{2}(N_a + N_b)}$$

But if $P_a{}'$ is assumed, then:

$$P_b{}' = \frac{C\,\pi - \frac{1}{2}\, N_a\, P_a{}'}{\frac{1}{2}\, N_b}$$

and

$$\tan \alpha_a = \frac{P_a{}'}{P_b{}'} \qquad \tan \alpha_b = \frac{P_b{}'}{P_a{}'}$$

The gear whose P′ or α is larger will be the driver, on account of the greater obliquity of the teeth.

4. Given N_a, N_b and C or D.
See case "7" under III., considering $\gamma = 90°$.

III.—Axis at any Angle (γ).

5. Given case "1," under II., then angles of spirals $= \frac{1}{2}\gamma$, for the same reason.

6. Analogous cases to "2" and "3," under II., may be worked out, when angles of axes $= \gamma$, but they have been

omitted, partly because the formulas are too cumbersome, and partly because they are to some extent covered by cases "5" and "7."

7. Given N_a, N_b and C, or one of the pitch diameters. We find the angles by a graphic method, which for all practical purposes is accurate enough; *r o* and *v o* are the axes of gears forming angle γ (see diagram, Fig. 11.) On these axes we lay off lines *o r* and *o v* representing the ratio of the number of teeth (velocity ratio), so that $N_a : N_b : : r s : s v$, and

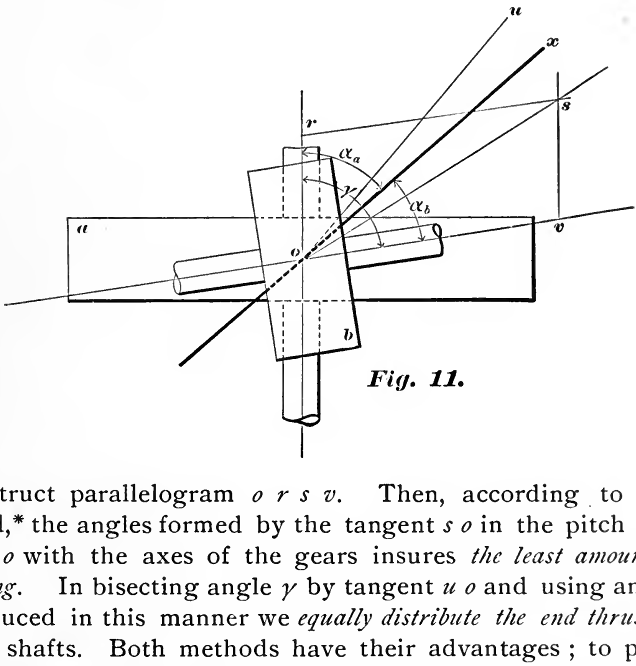


Fig. 11.

construct parallelogram *o r s v*. Then, according to McCord,* the angles formed by the tangent *s o* in the pitch contact *o* with the axes of the gears insures *the least amount of sliding*. In bisecting angle γ by tangent *u o* and using angles produced in this manner we *equally distribute the end thrust* on both shafts. Both methods have their advantages; to profit by both we select angles α_a and α_b, produced by tangent *o x*, bisecting angle *u o s*.

Thus we have when angles are found and C given,

$$P'^n = \frac{2\,C\,\pi \cos\alpha_a \cos\alpha_b}{N_a \cos\alpha_b + N_b \cos\alpha_a}$$

and when D_a given

$$P'^n = \frac{D_a\,\pi \cos\alpha_a}{N_a} \qquad \text{and}$$

$$D_b = \frac{P'^n\,N_b}{\pi \cos\alpha_b}$$

* McCord, Kinematics, page 278.

General Formulas.

$$\gamma = \alpha_a + \alpha_b$$

$$P_a{}'^n = P_b{}'^n$$

$$D = \frac{P' N}{\pi} \quad \text{or} = \frac{P'^n N}{\pi \cos \alpha}$$

$$B = D + 2s \quad \text{or} = D + \frac{2}{P^n}$$

$$P' = \frac{D \pi}{N} \quad \text{or} = \frac{P'^n}{\cos \alpha}$$

$$P'^n = P' \cos \alpha$$

$$P^n = \frac{\pi}{P'^n} \quad \text{(Pitch of cutter.)}$$

$$s = \frac{P'^n}{\pi} \quad \text{or} = \frac{1}{P^n}$$

$$t = \frac{P'^n}{2}$$

$$D'' + f = 2s + \frac{t}{10}$$

$$T = \frac{N}{\cos^2 \alpha} \quad \textit{(See Note 1.)}$$

$$L_1 = \frac{N P'}{\tan \alpha} \quad \text{or} = \frac{N \pi}{P \tan \alpha} \quad \text{or} = \frac{N P'^n}{\tan \alpha \cos \alpha}$$

$$L_2 = \frac{10 W G_2}{S G_1} \quad \textit{(See Note 2.)}$$

$$\begin{pmatrix} \cos 45^\circ = .70711 \\ \cos^2 45^\circ = .50 \end{pmatrix}$$

Note 1.—Cutters of regular involute system.

Use No. 1 cutter for T from	135 up.	No. 5 cutter for T from	21 to 25
" 2 " " " "	55 to 134	" 6 " " " "	17 to 20
" 3 " " " "	35 to 54	" 7 " " " "	14 to 16
" 4 " " " "	26 to 34	" 8 " " " "	12 to 13

Note 2.—Gears used on spiral head and bed for Brown & Sharpe milling machine:

W = number of teeth in gear on worm.
G_1 = " " 1st " stud.
G_2 = " " 2d " stud.
S = " " " screw.

Should a spiral head of different construction be used, the formula would not apply.

CHAPTER VI.

INTERNAL GEARING.

PART A.—INTERNAL SPUR GEARING.

(Figs. 12, 13, 14, 15, 16.)

A little consideration will show that a tooth of an internal or annular gear is the same as the space of a spur—external gear.

We prefer the epicycloidal form of tooth in this class of gearing to the involute form, for the reason that the difficulties in overcoming the interference of gear teeth in the involute system are considerable. Special constructions are required when the difference between the number of teeth in gear and pinion is small.

In using the system of epicycloidal form of tooth in which the gear of 15 teeth has radial flanks, this difference must be at least 15 teeth, if the teeth have both faces and flanks. Gears fulfilling this condition present no difficulties. Their pitch diameters are found as in regular spur gears, and the inside diameter is equal to the pitch diameter, less twice the addendum.

If, however, this difference is less than 15, say 6, or 2, or 1, then we may construct the tooth outline (based on the epicycloidal system) in two different ways.

First Method.—To explain this method better, let us suppose the case as in Fig. 12, in which the difference between gear and pinion is more than 15 teeth. Here the point o of the describing circle B (the diameter of which in the best practice of the present day is equal to the pitch radius of a 15 tooth gear, of the same pitch as the gears in question) generates the cycloid o, o^1, o^2, o^3, etc., when rolling on pitch circle L L of gear, forming the face of tooth ; and when rolling on the outside of L L the flank of the tooth. In like manner is the face and flank of the pinion tooth produced by B rolling outside and inside of E E (pitch circle of pinion). A little study of Fig. 12 (in which the face and flank of a gear tooth are produced) will show the describing circle B divided into 12

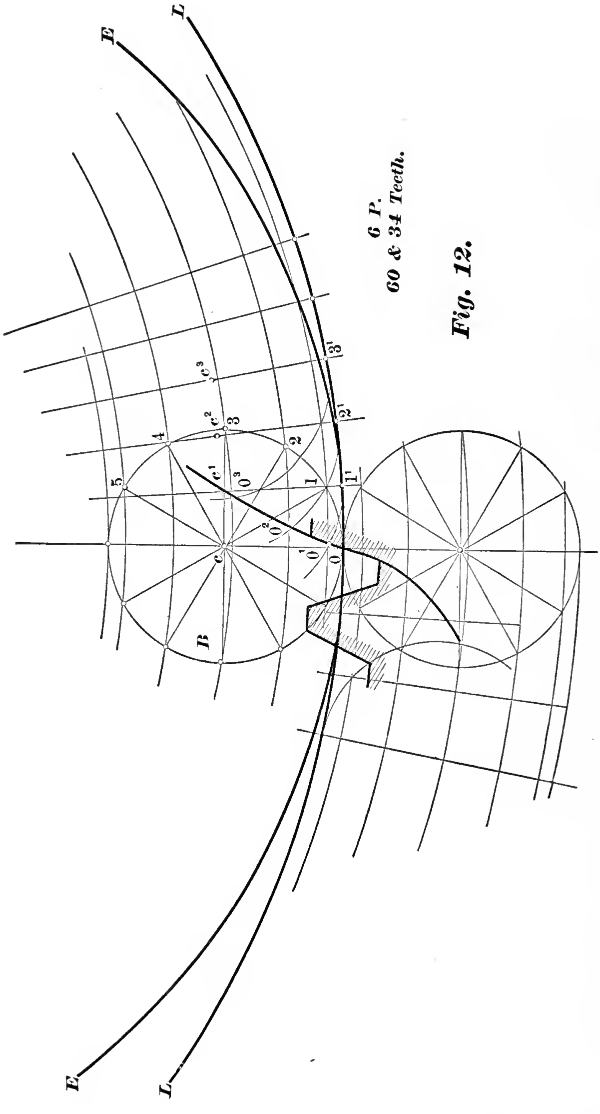


Fig. 12.

equal parts and circles laid through these points (1, 2, 3, etc.), concentric with L L. We now lay off on L L the distances 0–1, 1–2, 2–3, etc., of the circumference of B, and obtain points

1^1, 2^1, 3^1, etc. [Ordinarily it is sufficient to use the chord.] It will now readily be seen that B in rolling on L L will successively come in contact with 1^1, 2^1, 3^1, etc., c meanwhile moving to c^1, c^2, c^3, etc. (points on radii through 1^1, 2^1, 3^1, etc.), and the generating point o advancing to o^1, o^2, o^3, etc., being the intersections of B with c^1, c^2, c^3, etc., as centers and the circles laid through 1, 2, 3, etc. Points o, o^1, o^2, o^3, etc., connected with a curve give the face of the tooth; in like manner the flank is obtained.

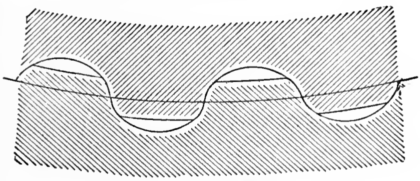
In this manner the form of tooth is obtained, when the difference of teeth in gear and pinion is less than 15, with the exception that the diameter of describing circle B

$$= \tfrac{1}{2}\left(\frac{N - n}{P}\right)$$

where P = diametral pitch, N and n number of teeth in gears.

The distances of the tooth above and below the pitch line as well as the thickness t are determined as in regular spur gears by the pitch, except when the difference in gear and pinion is very small, where we obtain a short tooth, as in Figs. 13 and 14. In such a case the height of tooth is arbitrary and only conditioned by the curve. In internal gears it is best to allow more clearance at bottom of tooth than in ordinary spur gears.

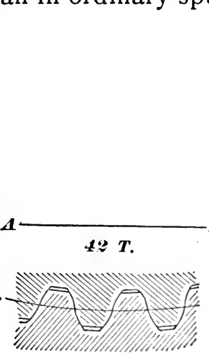
P′=1″
29 Teeth



30 Teeth
A B

Fig. 13.

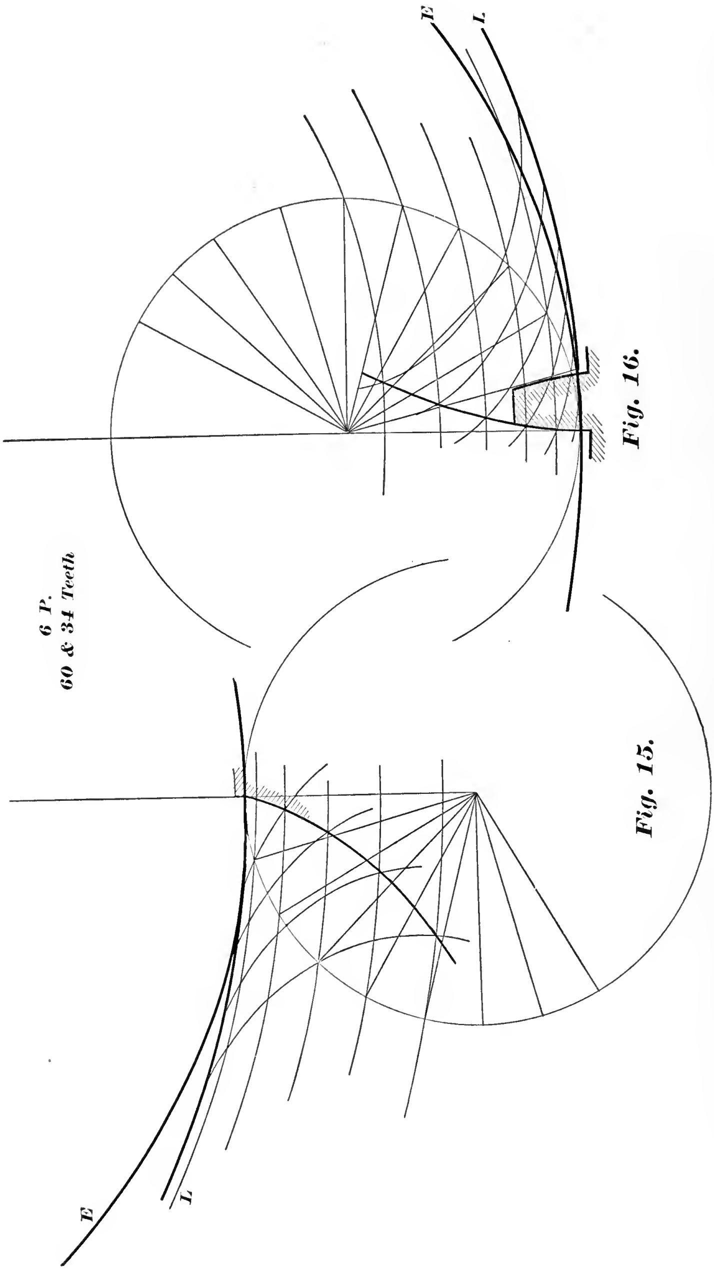
A B
42 T.
8 P.



46 T.

Fig. 14.

In a construction of this kind it is suggested to draw the tooth outline many times full size and reduce by photography. An equally multiplied line A B will help in reducing.



6 P.
60 & 34 Teeth

Fig. 16.

Fig. 15.

Second Method.—The difference between gear and pinion being very small, it is sometimes desirable to obtain a smooth action by avoiding what is termed the "friction of approaching action."* This is done, the *pinion driving*, by giving gear only flanks, Fig. 15, and the *gear driving*, by giving gear only faces, Fig. 16. In both these cases we have but one describing circle, whose diameter is equal to the difference of the two pitch diameters. The construction of the curve is precisely the same as described under A. The describing circle has been divided into 24 parts simply for the sake of greater accuracy.

PART B.—INTERNAL BEVEL GEARS.

(Fig. 17.)

The pitch surfaces of bevel gears are cones whose apexes are at a common point, rolling upon each other. The tooth forms for any given pair of bevel gears are the same as for a pair of spur gears (of same pitch) whose pitch radii are equal to the respective apex distances of the normal cones (*i. e.*, cones whose elements are perpendicular upon the elements of the bevel gear pitch cones). (Compare Fig 19, page 50.)

The same is true of internal bevel gears, with the modification that here one of the pitch cones rolls inside of the other. The spur gears to whose tooth forms the forms of the bevel gear teeth correspond, resolve themselves into internal spur gears (Fig. 17). The problem is now to be solved as indicated in the first part of this chapter.

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* McCord, Kinematics, pages 107, 108.

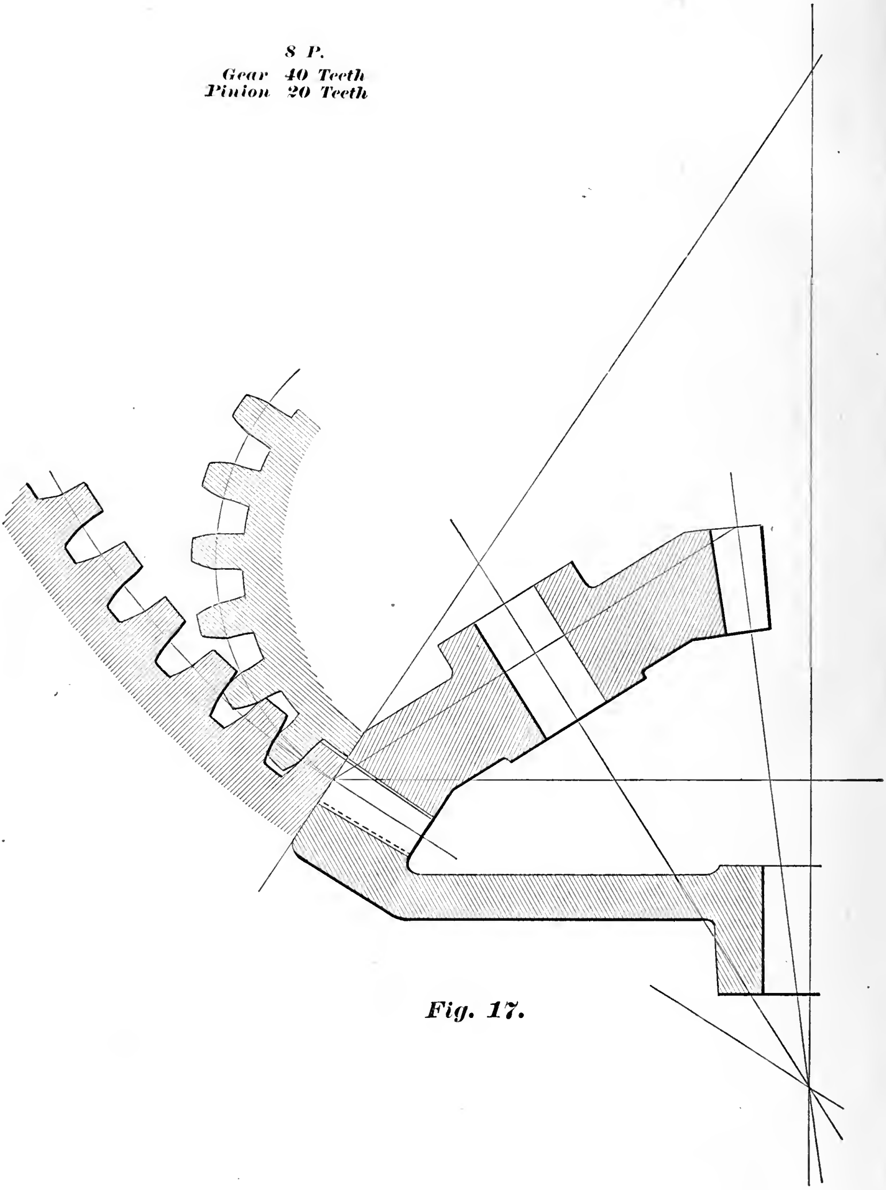


Fig. 17.

CHAPTER VII.

GEAR PATTERNS.

(Fig. 18.)

To place in bevel gears the best iron where it belongs, the tooth side of the pattern should always be in the nowel, no matter of what shape the hubs are.

Hubs, if short, may be left solid on web ; if long they should be made loose. A long hub should go on a tapering arbor, to prevent tipping in the sand. 1° taper for draft on hubs when loose, and 3° when solid is considered sufficient.

Coreprints as a rule are made separate, partly to allow the pattern to be turned on an arbor, partly for convenience, should it be desirable to use different sizes.

Put rap- and draw-holes as near to center as possible. Referring to Fig. 18, make L = D for D from ¾″ to 1½″, or even more, should hubs be very long. Otherwise if D is more than 1½″ leave L = 1½″.

Iron pattern before using should be marked, rusted and waxed.

Shrinkage—For cast-iron, ⅛″ per foot.
For brass, $\frac{3}{16}$″ "

Cast-iron gears, especially arm gears, do not shrink ⅛″ per foot. In making iron patterns the following suggestions have been found useful :

Up to 12″	diameter	allow	*no* shrink.		
From 12″ to 18″	"	"	⅓	regular	shrink.
" 18″ to 24″	"	"	½	"	"
" 24″ to 48″	"	"	⅔	"	"
Above 48″	"	"	.10″	"	"

for cast-iron.

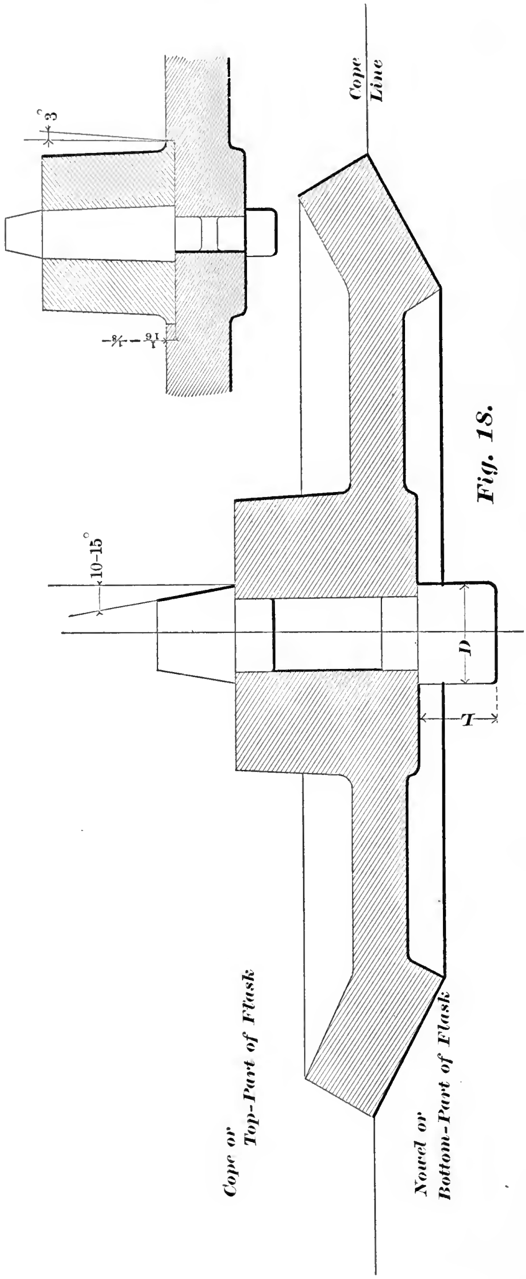

Cope Line
3°
10-15°
D
L
Cope or Top-Part of Flask
Nowel or Bottom-Part of Flask

Fig. 18.

If in gears the teeth are to be cast, the tooth thickness *t* in the pattern is made smaller than called for by the pitch, to avoid binding of the teeth when cast. No definite rule can be given, as the practice varies on this point. For the different diametral pitches we would advise making *t* smaller by an amount expressed in inches, as given in the following table :

Diam. Pitch.	Amount *t* is Smaller.	Diam. Pitch.	Amount *t* is Smaller.
16	.010″	5	.020″
12	012″	4	.022″
10	.014″	3	.026″
8	.016″	2	.030″
6	.018″	1	.040″

CHAPTER VIII.

DIMENSIONS AND FORM FOR BEVEL GEAR CUTTERS.

(Fig. 19.)

The data needed to determine the form and thickness of a bevel gear cutter are the following:

P = pitch.
N = number of teeth in large gear.
n = number of teeth in small gear.
F = length of face of tooth, measured on pitch line.

After having laid out a diagram of the pitch cones $a\ b\ c$ and $a\ b\ f$, and laid off the width of face, the problem resolves itself into two parts:

PART I.—DETERMINE PROPER CURVE FOR CUTTER.

It will be remembered that in the involute system of cutters (the only one used for bevel gears that are cut with rotary cutter), a set of eight different cutters is made for each pitch, numbering from No. 1 to No. 8, and cutting from a rack to 12 teeth. Each number represents the form of a cutter suitable to cut the indicated number of teeth. For instance, No. 4 cutter (No. 4 curve) will cut 26 to 34 teeth. In order to find the curve to be used for gear and pinion we simply construct the normal pitch cones by erecting the perpendicular $p\ q$ through b, Fig. 19. We now measure the lines $b\ q$ and $b\ p$, and taking them as radii, multiplying each by 2 and P we obtain a number of teeth for which cutters of proper curves may be selected. From example we have:

Gear: $b\ q = 9\frac{3}{4}''$; $2 \times P \times 9.75 = 97$ T No. 2 curve.
Pinion: $b\ p = 3\frac{1}{2}''$; $2 \times P \times 3.5 = 35$ T No. 3 curve.

The eight cutters which are made in the involute system for each pitch are as follows:

No. 1 will cut wheels from 135 teeth to a rack.
" 2 " " " 55 " " 134 teeth.
" 3 " " " 35 " " 54 "
" 4 " " " 26 " " 34 "
" 5 " " " 21 " " 25 "
" 6 " " " 17 " " 20 "
" 7 " " " 14 " " 16 "
" 8 " " " 12 " " 13 "

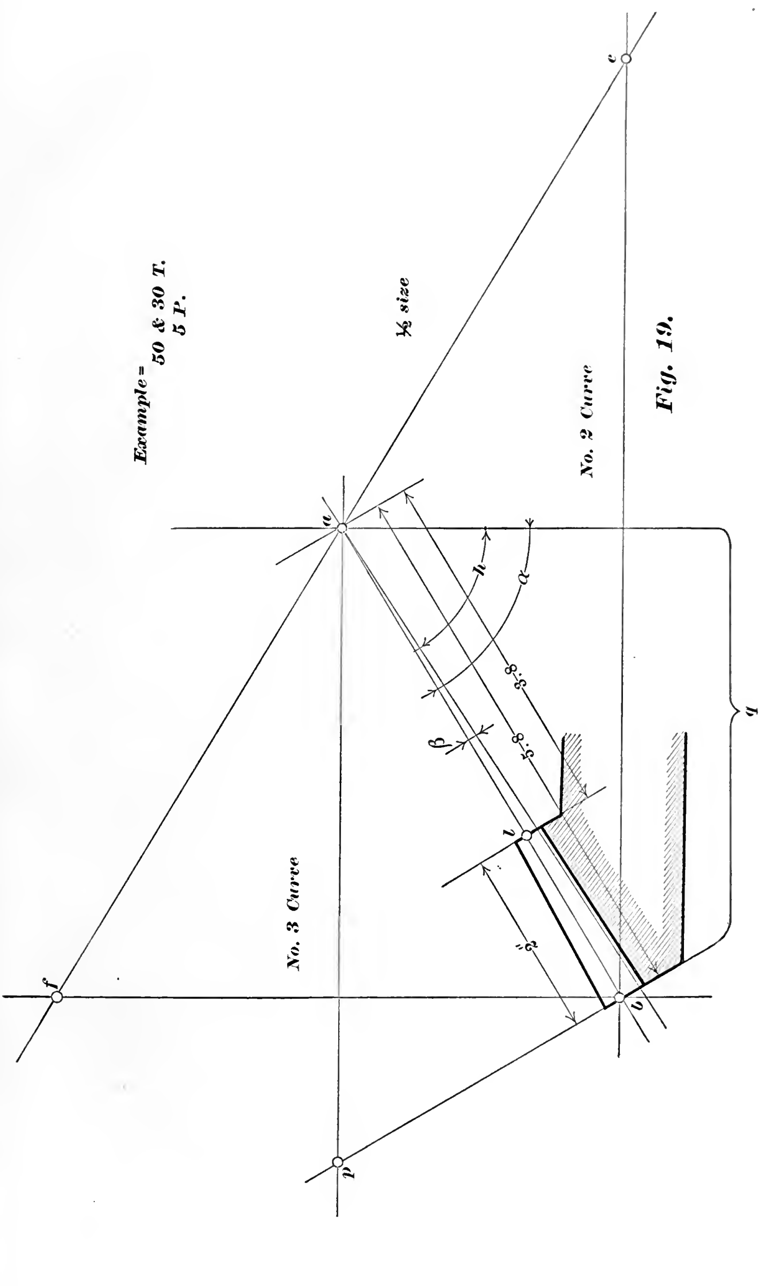

Example = 50 & 30 T.
5 P.
½ size
No. 2 Curve
No. 3 Curve
a
b
c
d
f
l
h
q
α
β
3·8
5·8
2·

Fig. 19.

Part II.—Determine Thickness of Cutter.

It is very evident that a bevel gear cutter cannot be thicker than the width of the space at small end of tooth ; the practice is to make cutter .005″ thinner. Theoretically the cutting angle (h) is equal to pitch angle less angle of bottom (or $h = \alpha - \beta'$). Practically, however, better results are obtained by making $h = \alpha - \beta$ (substituting angle of top for angle of bottom), and in calculating the depth at small end, to add the full clearance (f) to the obtained working depth, giving equal amount of clearance at large and small end. This is done to obtain a tooth thinner at the top and more curved. As the small end of tooth determines the thickness of cutter, we shall have to find the tooth part values at small end. From the diagram it will be seen that the values at large end are to those at small end as their respective apex distances ($a\ b$ and $a\ l$). The numerical values of these can be taken from the diagram and the quotient of the larger in the smaller is the constant wherewith to multiply the tooth values at large end, to obtain those at small end. In our example we find :

$$\frac{a\,l = 3.8}{a\,b = 5.8} = .655 = \text{constant}$$

For 5 P we have :

$t = .3141$	$t' = .2057$
$s = .2000$	$s' = .1310$
$f = .0314$	$f = .0314$
$s + f = .2314$	$s' + f = .1624$
$D'' + f = .4314.$	$s' = .1310$
	$D''' + f = .2934$

From the foregoing it is evident that a spur gear cutter could not be used, since a bevel gear cutter must be thinner.

If in gears of more than 30 teeth the faces are proportionately long, we select a cutter whose curve corresponds to the midway section of the tooth. The curve of the cutter is found by the method explained in Part I. of this Chapter.

CHAPTER IX.

DIRECTIONS FOR CUTTING BEVEL GEARS WITH ROTARY CUTTER.

(Fig. 20.)

In order to obtain good results, the gear blanks must be of the right size and form. The following sizes for each end of the tooth must be given the workman :

Total depth of tooth.
Thickness of tooth at pitch line.
Height of tooth above pitch line.

These sizes are obtained as explained in Chapter VIII.

The workman must further know the cutting angle (see (formula on page 13 and compare Chapter VIII.), and be provided with the proper tools with which to measure teeth, etc.

In cutting a gear on a universal milling machine the operations and adjustments of the machine are as follows :

1. Set spiral bed to zero line.

2. Set cutter central with spiral head spindle.

3. Set spiral head to the proper cutting angle.

4. Set the index on head for the number of teeth to be cut, leaving the sector on the straight or numbered row of holes, and set the pointer (or in some machines the dial) on cross-feed screw of milling machine to zero line.

5. As a matter of precaution, mark the depth to be cut for large and small end of tooth on their respective places.

6. Cut two or three teeth in blank to conform with these marks in depth. The teeth will now be too thick on both their pitch circles.

7. Set the cutter off the center by moving the saddle to or from the frame of the machine by means of the cross-feed screw, measuring the advance on dial of same. The saddle must not be moved further than what to good judgment

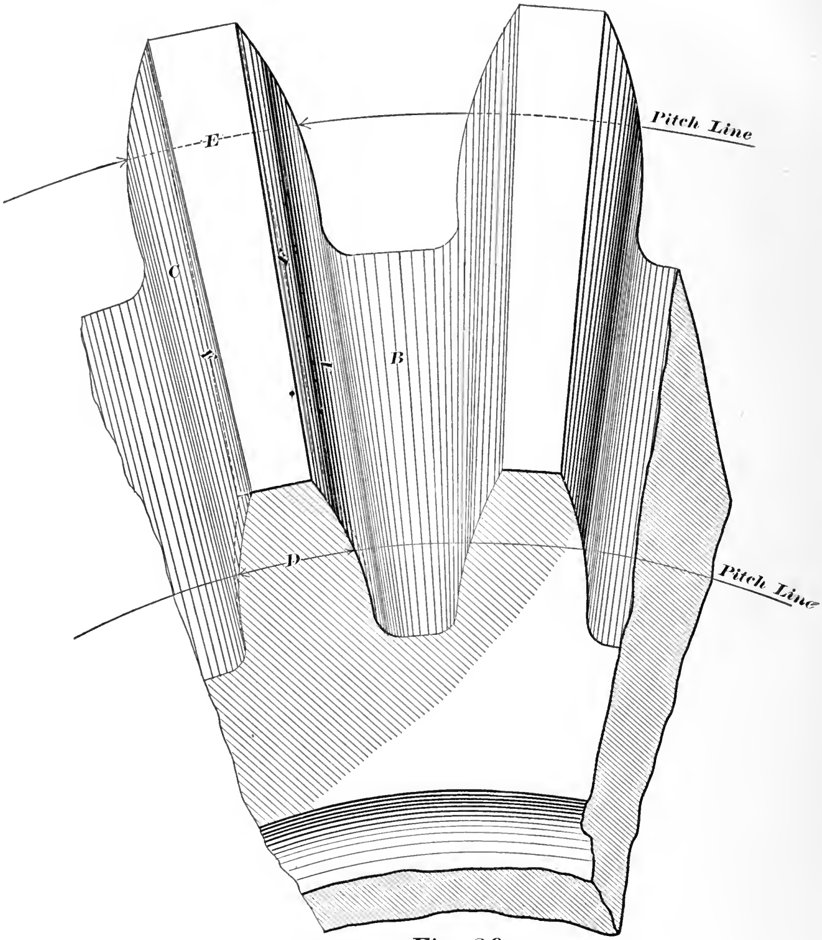


Fig. 20.

appears as not excessive; at the same time bearing in mind that an equal amount of stock is to be taken off each side of tooth.

8. Rotate the gear in the opposite direction from which the saddle is moved off the center, and trim the sides of teeth (A) (Fig. 20.)

9. Then move the saddle the same distance on the opposite side of center and rotate the gear an equal amount in the opposite direction and trim the other sides of teeth (C).

10. If the teeth are still too thick at large end E, move the saddle further off the center and repeat the operation, bearing in mind that the gear must be rotated and the saddle moved an equal amount each way from their respective zero settings.

It is generally necessary to file the sides of teeth above the pitch line more or less on the small ends of teeth, as indicated by dotted lines F F. This applies to pinions of less than 30 teeth.

For gears of coarser pitch than 5 diametral it is best to make one cut around before attempting to obtain the tooth thickness.

The formulas for obtaining the dimensions and angles of gear blanks are given in Chapter III.

CHAPTER X.

THE INDEXING OF ANY WHOLE OR FRACTIONAL NUMBER.

(Fig. 21.)

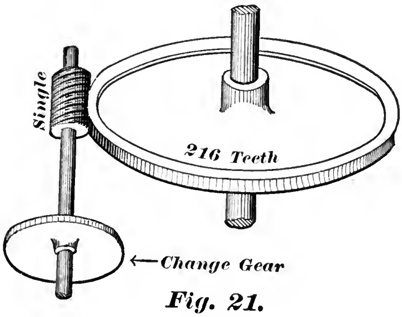


Fig. 21.

In indexing on a machine the question simply is: How many divisions of the machine index have to be advanced to advance a unit division of the number required. To which is the

$$\text{answer} = \frac{\text{divisions of machine index}}{\text{number to be indexed}}$$

Suppose the number of divisions in index wheel of machine to be 216.

EXAMPLE I.—Index 72.

Answer: $\frac{216}{72} = 3$ (3 turns of worm).

EXAMPLE II.—Index 123.

$$\frac{216}{123} = 1 + \frac{93}{123}$$

If now we should put on worm shaft a change gear having 123 teeth, give the worm shaft, Fig. 21, one turn, and in addition thereto advance 93 teeth of the change gear (to give the fractional turn), we would have indexed correctly one unit of the given number, and so solved the problem. Should we not have change gear 123 we may try those on hand. The question then is: How many teeth (χ) of the gear on hand (for instance 82) must we advance to obtain a result equal to the one when advancing 93 teeth of the 123 tooth gear? We have:

$$\frac{93}{123} = \frac{\chi}{82} \text{ where } \chi = 62$$

EXAMPLE III.—Index 365, change gear 147.

$$\frac{216}{365} = \frac{\chi}{147} \text{ where } \chi = 87 - \frac{3}{365}$$

Here 147 is the change gear on hand. In indexing for a unit of 365 we advance 8 teeth of our 147 tooth gear. It is evident that in so doing we advance too fast and will have indexed three teeth of our change gear too many when the circle is completed. To avoid having this error show in its total amount between the last and the first division, we can distribute the error by dropping one tooth at a time at three even intervals.

EXAMPLE IV.—Index 190.

$$\frac{216}{190} = 1 + \frac{26}{190} \qquad \text{Change gear on hand 90 T}$$

$$\frac{26}{190} = \frac{\chi}{90} \text{ where } \chi = 12 + \frac{60}{190}$$

To distribute the error in this case we advance one additional tooth at a time of the change gear at six even intervals.

EXAMPLE V.—Index 117.3913.

$$\frac{216}{117.3913} = 1 + \frac{986087}{1173913}$$

This example is in nowise different from the preceding ones, except that the fraction is expressed in large numbers. This fraction we can reduce to lower approximate values, which for practical purposes are accurate enough. This is done by the method of continued fractions. [For an explanation of this method we refer to our "Practical Treatise on Gearing."]

$$\frac{986c87}{1173913}$$

```
986087) 1173913 (1
        986087
        187826) 986087 (5
                939130
                 46957) 187826 (3
                        140871
                         46955) 46957 (1
                                46955
                                    2) 46955 (23477
                                       46954
                                           1) 2 (2
                                              2
                                              0
```

$$\frac{986087}{1173913} = \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{23477 + \cfrac{1}{2}}}}}}$$

1	5	$c = 3$	1	23477	2
$a = 1$	$b = 5$	$d = 16$	21	493033	986087
$a^1 = 1$	$b^1 = 6$	$d^1 = 19$	25	586944	1173913

NOTE.—Find the first two fractions by reduction $\frac{1}{1} = \frac{1}{1}$ and $\frac{1}{1 + \frac{1}{5}} = \frac{5}{6}$; the others are then found by the rule $\begin{cases} b\,c + a = d \\ b^1\,c + a^1 = d^1 \end{cases}$

The fraction $\frac{21}{25}$ is a good approximation; putting therefore a change gear of 25 teeth on worm shaft, we advance (beside the one full turn) 21 teeth to index our unit.

Of course, in using any but the correct fraction we have an error every time we index a division; so that when indexed around the whole circle, we have multiplied this error by the number of divisions.

In the present example this error is evidently equal to the difference between the correct and the approximate fraction used. Reducing both common fractions to decimal fractions we have:

$$\frac{986087}{1173913} = .84000006$$

$$\frac{21}{25} = \frac{.84000000}{.00000006} = \text{error in each division.}$$

$.00000006 \cdot 117.3913 = .00000703348$ total error in complete circle. This error is expressed in parts of a unit division. (To find this error expressed in inches, multiply it by the distance between two divisions, measured on the circle.) In this case the approximate fraction being smaller than the correct one, in indexing the whole circle we fall short .00000703348 of a division.

EXAMPLE VI.—Index 15.708

$$\frac{216}{15.708} = 13 + \frac{11796}{15708}$$

$$\frac{11796}{15708} = \frac{983}{1309}$$

```
983) 1309 (1
      983
      326) 983 (3
           978
             5) 326 (65
                30
                 26
                 25
                  1) 5 (5
                     5
                     0
```

$$\frac{983}{1309} = \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{65 + \cfrac{1}{5}}}}$$

1	3	65	5
$\frac{1}{1}$	$\frac{3}{4}$	$\frac{196}{261}$	$\frac{983}{1309}$

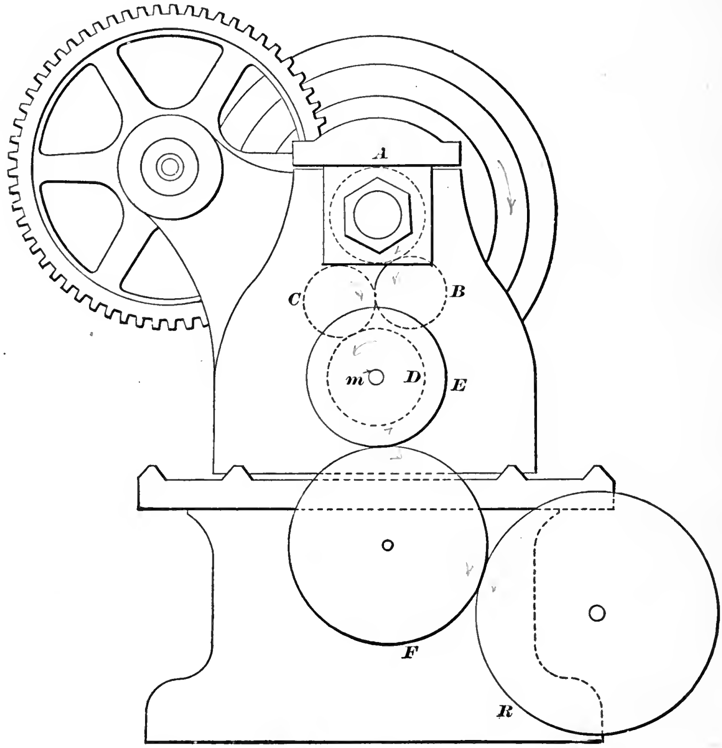
In using the approximation $\frac{196}{261}$ the error for each division (found as above) will be .000002917, for the whole circle .0000458. In this case, the approximation being larger than the correct fraction, we overreach the circle by the error.

CHAPTER XI.

THE GEARING OF LATHES FOR SCREW CUTTING.

(Figs. 22, 23.)

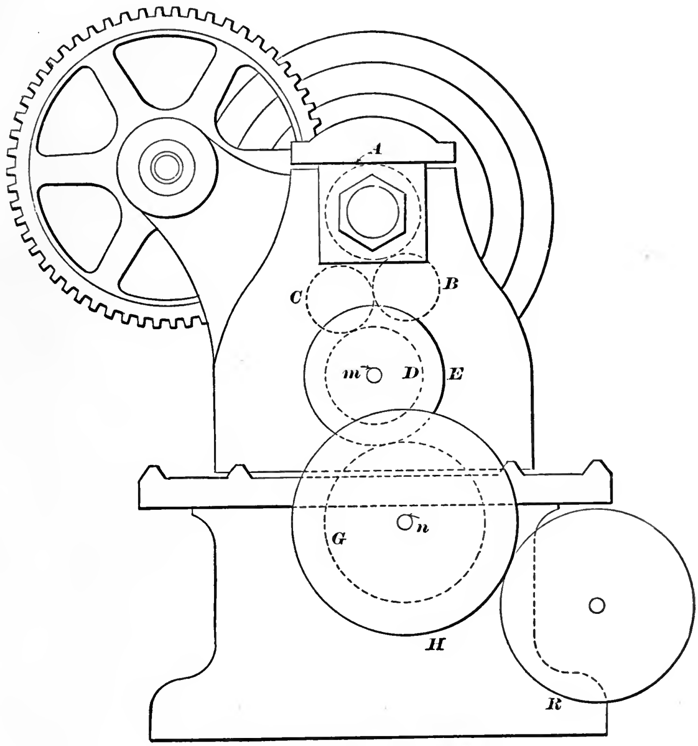
The problem of cutting a screw on a lathe resolves itself into connecting the lathe spindle with the lead screw by a train of gears in such a manner that the carriage (which is actuated by



Simple Gearing.
Fig. 22.

the lead screw) advances just one inch, or some definite distance, while the lathe spindle makes a number of revolutions equal to the number of threads to be cut per inch.

The lead screw has, with the exception of a very few cases, always a single thread, and to advance the carriage one inch it therefore makes a number of revolutions equal to its number



Compound Gearing.
Fig. 23.

of threads per inch. Should the lead screw have double thread, it will, to accomplish the same result, make a number of revolutions equal to half its number of threads per inch. It follows that we must know in the first place the number of threads per inch on lead screw.

It ought to be clearly understood that one or more intermediate gears, which simply transmit the motion received from one gear to another, in no wise alter the ultimate ratio of a train of gearing. An even number of intermediate gears simply change the direction of rotation, an odd number do not alter it.

The gearing of a lathe to solve a problem in screw cutting can be accomplished by

A. Simple gearing.
B. Compound gearing.

Referring to the diagrams, Figs. 22 and 23, we have in Fig. 22 a case of simple, and in Fig. 23 a case of compound gearing.

In simple gearing the motion from gear E is transmitted either directly to gear R on lead screw or through the intermediate F. In compound gearing the motion of E is transmitted through two gears (G and H) keyed together, revolving on the same stud *n*, by which we can change the velocity ratio of the motion while transmitting it from E to R. With these four variables E, G, H, R, we are enabled to have a wider range of changes than in simple gearing.

B and C, being intermediate gears, are not to be considered. If, as is generally the case, gear A equals gear D, we disregard them both, simply remembering that gear E (being fast on same shaft with E) makes as many revolutions as the spindle. Sometimes gear D is twice as large as gear A, then, still considering gear E as making as many revolutions as the spindle, we deal with the lead screw as having twice as many threads per inch as it measures.

SIMPLE GEARING.

Let there be: the number of teeth in the different gears expressed by their respective letters, as per Fig. 22, and

s = threads per inch to be cut,
L = threads per inch on lead screw ; then

1. $$\frac{s}{L} = \frac{R}{D}$$

If now one of the two gears D and R is selected, the other will be:

$$R = \frac{s\,D}{L}\ ; \quad D = \frac{L\,R}{s}$$

2. The two gears may be found by making

$$\left.\begin{aligned} R &= p\,s \\ D &= p\,L \end{aligned}\right\}\text{ where } p \text{ may be any number.}$$

3. The above holds good when a fractional thread is to be cut, but if the fraction is expressed in large numbers, as, for instance, $s = 2.833$ ($2\frac{833}{1000}$), we first reduce this fraction ($\frac{833}{1000}$) to lower approximate values by the process of continued fraction (see pages 57 and 58).

```
833) 1000 (1
      833
      ---
      167) 833 (4
           668
           ---
           165) 167 (1
                165
                ---
                  2) 165 (82
                     16
                     --
                      5
                      4
                      -
                      1) 2 (2
                         2
                         -
                         0
```

1	4	1	82	2
$\frac{1}{1}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{414}{497}$	$\frac{833}{1000}$

$$\frac{5}{6} = .833 \text{ (nearly) and } s = 2\frac{5}{6}$$

If in this case $L = 4$, and we select $D = 48$, then, since

$$R = \frac{s\,D}{L} \qquad R = 34$$

COMPOUND GEARING.

4. In a lathe geared compound for cutting a screw the product of the drivers (E and H, Fig. 23) multiplied by the number of threads to be cut must equal the product of the driven (G and R) multiplied by the number of threads on lead screw. This is expressed by

$$E\,.\,H\,.\,s = G\,.\,R\,.\,L \text{ or } \frac{E\,.\,H\,.\,s}{G\,.\,R\,.\,L} = 1$$

If three of the gears E, H, G, R have been selected, the fourth one would be either

$$E = \frac{G\ R\ L}{H\ s} \quad \text{or}$$

$$H = \frac{G\ R\ L}{E\ s} \quad \text{or}$$

$$G = \frac{E\ H\ s}{R\ L} \quad \text{or}$$

$$R = \frac{E\ H\ s}{G\ L}$$

$$s = \frac{R\ G\ L}{E\ H} = L\left(\frac{R\,.\,G}{L\,.\,E\,.\,H}\right)$$

If a fractional thread is to be cut, as under "3," we reduce the fraction to lower approximate values.

Example.—Gear for 5.2327 threads per inch, lead screw is 6 threads.

$$.2327 = \frac{2327}{10000}$$

```
2327) 10000 (4
      9308
       692) 2327 (3
            2076
             251) 692 (2
                  502
                  190) 251 (1
                       190
                        61) 190 (3
                            183
                              7) 61 (8
                                 56
                                  5) 7 (1
                                     5
                                     2) 5 (2
                                        4
                                        1) 2 (2
                                           2
                                           0
```

4	3	2	1	3	8	1	2	2
$\frac{1}{4}$	$\frac{3}{13}$	$\frac{7}{30}$	$\frac{10}{43}$	$\frac{37}{159}$	$\frac{306}{1315}$	$\frac{343}{1474}$	$\frac{992}{4263}$	$\frac{2327}{10000}$

$$\frac{10}{43} = .2327 \text{ (nearly) and } 5.2327 = 5\frac{10}{43}$$

Selecting E = 43, H = 52, R = 50, and

$$G = \frac{E\,.\,H\,.\,s}{R\,.\,L} \text{ we have } G = \frac{43\,\cdot\,52\,\cdot\,5\frac{10}{43}}{50\,.\,6} = 39.$$

5. The examples so far given all deal with single thread. The pitch of a screw is the distance from center of one thread to the center of the next. The lead of a screw is the advance for each complete revolution. In a single thread screw the pitch is equal to the lead, while in a double thread screw the pitch is equal to one-half the lead ; in a triple thread screw equal to one-third the lead, etc.

If we have to gear a lathe for a many-threaded screw (double, triple, quadruple, etc.), we simply ascertain the lead, and deal with the lead as we would with the pitch in a single thread screw, *i. e.*, we divide one inch by it, to obtain the number of threads for which we have to gear our lathe.

EXAMPLE.—Gear for double thread screw, lead = .4654. Number of threads per inch to be geared for is :

$$\frac{1}{\text{Lead}} = \frac{1}{.4654} = 2.1487$$

Lead screw is four threads per inch.

As in previous examples, we reduce the fraction $.1487 = \frac{1487}{10000}$ to lower approximate values by the process of continued fraction.

From the different values received in the usual way we select :

$$\frac{11}{74} = .1487 \text{ (nearly) and } 2.1487 = 2\tfrac{11}{74}$$

We have therefore :

$$s = 2\tfrac{11}{74}$$

$$L = 4$$

Selecting $\begin{cases} E = 30 \\ G = 30 \\ H = 40 \end{cases}$ 74

$$R = \frac{E\,.\,H\,.\,s}{G\,.\,L} = \frac{74\,.\,40\,.\,2\tfrac{11}{74}}{30\,.\,4} = 53$$

NOTE.—In using any but the original fraction we commit an error. This error can be found by reducing the approximate fraction used to a decimal fraction, and comparing it with the original fraction. In the above example the original fraction is

$$\begin{array}{l} .1487 \text{ and} \\ \tfrac{11}{74} = .14864 \\ \hline \text{Error} = .00006 \text{ inch in lead.} \end{array}$$

In cutting a multiple screw, after having cut one thread, the question arises how to move the thread tool the correct amount for cutting the next thread.

In cutting double, triple, etc., threads, if in simple or compound gearing the number of teeth in gear E is divisible by 2, 3, etc., we so divide the teeth ; then leaving the carriage at rest we bring gear E out of mesh and move it forward one division, whereby the spindle will assume the correct position.

Is E not divisible we find how many teeth (V) of gear R are advanced to each full turn of the spindle. Dividing this number by 2 for double, by 3 for triple thread, etc., we advance R so many teeth, being careful to leave the spindle at rest.

We have for simple gearing :

$$V = \frac{E}{R}$$

for compound gearing :

$$V = \frac{E \, . \, H}{G \, . \, R}$$

If in simple gearing both E and R are not divisible, one remedy would be to gear the lathe compound ; or the face-plate may be accurately divided in two, three or more slots, and all that is then necessary is to move the dog from one slot to another, the carriage remaining stationary.