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Further Evidence of the Beta Stability and Tendency: An Application of a Variable Mean Response Regression Model

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Summary

A Variable Mean Response Regres ine
the stability and tendency of beta e R
model is a better model than the tra on
model in investigating the beta stab e
error criteria is also used to show
from the VMRR model are generally mo
future betas than those obtained fro
model.

I. Introduction

The stability of beta coefficients is of interest to both security and portfolio analysis. Levy [1971] has investigated short-term stationarity of beta coefficients; Blume [1971, 1975] and Klemkosky and Martin [1975] have studied the regression tendency of beta coefficients; and Fabozzi and Francis [1978] have conducted a random coefficient test on over 700 stocks. However, none of these studies has explicitly derived a model to allow both the regression tendency and the degree of uncertainty for a beta coefficient to be tested empirically. The main purpose of this paper is, therefore, to derive a variable mean response regression capital asset pricing model for testing simultaneously the stability and the regression tendency of beta coefficients.

Based upon either a linear or a quadratic time trend specification, it will be shown that the model presented in this paper allows for the co-existence of a random beta and a shifting beta.

In the second section, the problems of testing the mean-variance type of capital asset pricing model (CAPM) is discussed in accordance with the papers by Roll [1977, 1978]; the arbitrage pricing theory (APT) is reviewed in terms of the papers by Ross [1976, 1977, 1978]. In the third section, the model to be used in this study is developed; its relationship to previous studies in investigating the regression tendency and the stability of beta coefficients is explored; and the theoretical and empirical reasons for using the model are discussed. In the fourth section, the method for estimating the variable mean response regression market model is presented and asymptotic estimators are derived. In the fifth section, monthly data for 363 firms from January, 1965 to September, 1979 is used to test the stability of betas and the existence of regression tendency for beta coefficients in accordance with the model developed in this paper. Some of the empirical results are reported, and the implications of random and trend components on previous empirical risk decomposition analysis and the heteroscedasticity test of the CAPM are also analyzed. Finally, the results of the study are summarized in the last section.

II. Capital Asset Pricing Model and Arbitrage Pricing Theory[1]

Professor Roll [1977, 1978] has statistically and theoretically criticized the mean-variance type of CAPM developed by Sharpe [1964], Lintner [1965] and Mossin [1966]. Statistically, a researcher is generally unable to accurately measure the true market portfolio. Theoretically, there exist some tautological relationships in determining the efficient set and testing efficient market hypothesis. Statistical problems can be resolved by modern econometric methods. However, the tautological theoretical relationship is a much more serious problem than the statistical problem. As an alternative, Professor Roll suggests the APT theory developed by Ross [1976, 1977 and 1978]. The APT theory is basically a multi-index model developed under much less stringent assumptions than those of the CAPM. In the following section, the variable mean response regression model is defined. It will empirically be shown that the variable mean regression response methodology will be

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[1]This section is based upon one of the anonymous reviewer's constructive suggestions.

more applicable to the APT than either the fixed coefficient or the standard random coefficient regression model.

III. A Variable Mean Response Regression Capital Asset Pricing Model

Following Singh, Nagan, Choudhry, and Raj [SNCR, 1976], a variable mean response regression model for estimating the parameters of the capital asset pricing model developed by Sharpe [1964], Lintner [1965-a], and Mossin [1966] is defined as follows:

(1) $Y_{(t)} = \beta_{(t)}X_{(t)} + \varepsilon_{(t)}$,

where $Y_{(t)} = R_{jt} - R_{ft}$, $X_{(t)} = R_{ft}$,

R_{jt} = 1 + rates of return on jth asset,

R_{mt} = 1 + market rate of return,

R_{ft} = 1 + market rate of return,

$\varepsilon(t)$ = disturbance term, and

(2) $\beta_{(t)} = \overline{\beta} + \alpha_1 t + \alpha_2 t^2 + \eta_{(t)}$.

In equation (2), $\overline{\beta}$ represents the constant component of the beta coefficient; t represents the time trend; $\eta_{(t)}$ is the random stock associated with $\beta_{(t)}$; and α_1 and α_2 are parameters associated with the time trend. The estimated variance of $\eta_{(t)}$, σ_{η}^2, can be used to measure the degree of uncertainty for $\beta_{(t)}$; and the signs and magnitudes of the estimated values for α_1 and α_2 will test for the existence of beta tendency. It should also be noted that the specification of equation (2) can be used to make better estimates of the beta coefficient and the random risk.

The specification of equation (2) can be considered as a general case of previous research. There are three reasons why this is so. First, if the estimated values for α_1, α_2, and σ_η^2 are not significantly different from zero, then the fixed coefficient ordinary least squares (OLS) estimator will be an acceptable method for estimating the betas. This is because equation (2) can be reduced to $\beta_{(t)} = \overline{\beta}$. Secondly, if the estimated value of σ_η^2 is significantly different from zero while the trend components are not, the random coefficient model developed by Theil and Mennes [1959], and Hildreth and Houcks [1968] could be used to analyze the degree of uncertainty and to estimate beta. Fabozzi and Francis [1978] have utilized this approach to investigate the stability of betas. And thirdly, if the estimated values for α_1, α_2, and σ_η^2 are all statistically different from zero, this may indicate that beta coefficients are moving randomly around a trend line. As a result, the SNCR's variable mean response regression model could be used to estimate explicitly the constant component, the trend component, and the random component of beta coefficients. These three components can be used to describe the arbitrage process of capital asset pricing determination. This has not been considered in previous studies.

Further, the necessity of using a variable mean response regression model instead of fixed coefficient model to estimate the beta coefficients can be justified on both empirical and theoretical grounds. Empirically, Cohen and Pogue [1967] and Lee and Lloyd [1977] have found that the multi-index model can generally be applied to improve the explanatory power of the market model. If a single index model is adopted instead of a multi-index model, the necessity of using the variable mean response regression model can be justified by the impact of omitted variables.[2] This is consistent with Hildreth and Houck's argument that existence of population variances associated with the regression coefficients can be explained by the impact of omitted variables. Cooley and Prescott [1973] have also argued that sequential parameter variation may arise because of structural change, mis-specifications, and the problem of aggregation. Theoretically, Merton [1973] has shown that the investment opportunity set generally shifts over time unless the interest rate is constant over time. Black [1976] has argued that shocks in the capital market should be regarded as random fluctuations in the beta coefficient of a dynamic capital asset pricing model. These arguments, obviously, strongly support the hypothesis advanced in this paper.

IV. Estimation Methodology

To test equation (2), SNCR's study proposed alternative estimation methods. In this study, the Hildreth-Houck method is used. Substituting equation (2) into equation (1), the following is obtained:

$$(3)\quad Y_{(t)} = \bar{\beta}X_{(t)} + \alpha_1 X^*_{(t)} + \alpha_2 X^{*2}_{(t)} + w_{(t)},$$

where

$$(4)\quad X^*_{(t)} = tX_{(t)} \text{ and } X^{*2}_{(t)} = t^2 X_{(t)}, \text{ and}$$

$$(5)\quad w_{(t)} = \eta_{(t)} X_{(t)} + \varepsilon_{(t)}.$$

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[2] Turnbull [1977] has theoretically shown how a firm's systematic risk can be affected by both the firm-related variables and by general economic variables.

The reason for using quadratic functional form of time trend is that the quadratic form is used by most researchers and it also allows us to test the regression tendency. To estimate equation (3), the Hildreth-Houck two-step estimator as extended by SNCR is applied. For simplicity, equation (3) can be expressed in matrix notation as:

(6) $Y = Z\beta + w,$

where Y is a Tx1 vector of observations on the dependent variable Y_t; and

(7) $Z = [X\ X^*],$

X and X* are Tx1 and Tx2 matrices of regressors, $X_{(t)}$ and $[X^*_{(t)} X^{*2}_{(t)}]$ respectively; and

(8) $\beta' = [\bar{\beta}'\alpha'],$

where $\bar{\beta}$ and α are column vectors such that $\alpha' = [\alpha_1 \alpha_2]$ is a row vector. The distribution of w is assumed to be:

(9.1) $E(w) = 0$, and

(9.2) $E(w'w) = E(X\eta\eta'X') + E(\varepsilon\varepsilon')$

$= X\Delta X' + \sigma^2_\varepsilon I$

$= \Omega.$

To estimate β, the following is first obtained:

(10) $\hat{w} = Mw,$

where M is a symmetric, idempotent matrix such that:

$$(11) \qquad M = I - Z(Z'Z)^{-1}Z.$$

Next, OLS is applied to:

$$(12) \qquad \dot{\hat{w}} = \dot{M}\dot{X} + u = G\Delta + u,$$

where $\dot{\hat{w}}$, $\dot{M}$, and $\dot{X}$ are the vector and matrices of the squared elements of $\hat{w}$, M and X, respectively; and u is a vector of random error. The Δ estimator is thus:

$$(13) \qquad \hat{\Delta} = (G'G)^{-1}G'\dot{\hat{w}}.$$

With Δ estimated, Ω can be constructed following equation (9.2). Finally, the generalized least squares estimator for β can be written as:

$$(14) \qquad \hat{\beta} = (Z'\hat{\Omega}^{-1}Z)^{-1}Z'\hat{\Omega}^{-1}Y,$$

with the variance-covariance matrix:

$$(15) \qquad Var(\hat{\beta}) = (Z'\hat{\Omega}^{-1}Z)^{-1}.$$

V. Empirical Results and Their Implications

The monthly rates of return for 363 New York Stock Exchange (NYSE) companies from January, 1965 to September, 1979 are selected for this study in accordance with the model developed in the previous section. Both stock dividends and splits are adjusted for proper rates of return. The Standard and Poor (S&P) composite index is employed to calculate the monthly market rate of return. The monthly treasury bill rate is used to proxy the risk-free rates of return. To compare the results obtained from the variable mean response regression CAPM with those obtained from the traditional and fixed coefficient CAPM, the OLS method is first applied to estimate the traditional fixed coefficient betas (β_j) and the unsystematic risk. Secondly, the model of equation (3) without the term $\alpha_2 t^2 X_{(t)}$ is used to estimate $\bar{\beta}_j$, α_1, σ_η^2, and σ_ε^2.[3] The generalized least squares (GLS) estimated constant component of systematic risk ($\hat{\bar{\beta}}$) and the GLS estimated α_1 are calculated as defined in equation (14); and the asymptotic variance estimators associated with $\hat{\bar{\beta}}$ and $\hat{\alpha}_1$ are calculated as defined in equation (15). The least squares estimators for both σ_η^2 and σ_ε^2 are found in equation (9.2).[4]

To measure the degree of uncertainty, an index of uncertainty (IOU) is defined following the coefficient of variation concept as:

$$(16) \qquad IOU_j = \hat{\sigma}_{\eta j} / |\hat{\bar{\beta}}_j|,$$

where $\hat{\sigma}_\eta$ is the estimated population standard deviation associated with the beta coefficient for the jth firm. $\hat{\bar{\beta}}_j$ is the estimated constant component of the beta coefficient for the jth firm.[5] This index

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[3] $t^2 X_{(t)}$ was added to test the convergence of beta tendency. However, due to the high degree of multicollinearity, this variable was dropped in the final model.

[4] There exists about twenty percent of stocks with negative estimated σ_η^2. As Fabozzi and Francis [1978] pointed out, the restricted GLS employing quadratic programming can be used to deal with this kind of problem.

[5] This index has been used by Kau and Lee [1977] to measure the degree of stability of the density gradient for an urban structure study.

provides a criteria for security analysts to decide whether the historical beta coefficient of a firm is an acceptable predictor for the future beta of the firm. The mean and standard deviation of the estimated IOU for 363 firms are 1.209 and 1.671 respectively. It is obvious that the estimated population standard deviation associated with the beta coefficient ($\hat{\sigma}_\eta$) can also be used to formulate the interval estimate for a firm's beta coefficient. An index of uncertainty can also be an indicator of the marginal benefit of utilizing the multi-index CAPM. As the index approaches zero, the implication is that $\bar{\beta}_j$ is an efficient population estimate and requires no additional data to estimate the multi-index CAPM. According to the estimates derived in this study, sixty-five percent of the sample firms have $\hat{\sigma}_\eta$ values which are significantly different from zero. This suggests that the residuals associated with these firms are not homoscedastic. One hundred five firms (29%) have an estimated α_1 value that is significantly different from zero. This indicates that a significant portion of the sample firms have shifting betas during the period from 1965 to 1975. This result, in addition to the significant σ_η^2 values, could well indicate that a variable mean response regression market model better describes the beta coefficient for some firms than does the constant beta and/or pure random beta models. From the signs of $\hat{\bar{\beta}}_j$ and $\hat{\alpha}_1$, it is found that there exists some regression tendency as established earlier by Blume [1975]. This argument is based upon the fact that the estimated value of α_1 is a factor for adjusting the beta coefficient toward unity. In other words, the sign of $\hat{\alpha}_1$ is generally positive when the magnitude of of the estimated beta is less than one, and negative if the magnitude of beta is greater than one. Hence the sign, magnitude, and degree of significance of $\hat{\alpha}_1$ provides information for security analysts to improve their forcasting of future beta coefficients.

To investigate the possible success of beta predictions of variable mean response regression model relative to the traditional fixed coefficient regression model, the mean squared error (MSE) technique developed by Mincer and Zarnowitz [1969] as indicated in equation (17) will be used to do empirical analysis.

$$\text{(17)} \qquad MSE = \underbrace{(\bar{\beta}_{t+1}-\bar{\beta}_t)^2}_{\text{bias}} + \underbrace{(1-b_1)^2 S_{\beta t}^2}_{\text{inefficiency}} + \underbrace{(1-R^2_{\beta t+1,\beta t})S^2_{\beta+1}}_{\text{random error}}$$

where $\bar{\beta}_{t+1}$ and $\bar{\beta}_t$ are the means of all beta in period t+1 and t, respectively. b_1 is the slope coefficient of $\hat{\beta}_{t+1}$ regressed on $\hat{\beta}_t$, $S^2_{\beta+1}$ and $S^2_{\beta t}$ are the sample variance of $\hat{\beta}_{t+1}$ and $\hat{\beta}_t$, respectively. $R^2_{\beta+1,\beta t}$ is the coefficient of determination for regression of $\hat{\beta}_{t+1}$ or $\hat{\beta}_t$. Note that Klemkosky and Martin [1975] have used this kind of technique to analyze the performance of alternative beta forcasting technique.

To estimate the MSE as indicated in equation (17), data from January, 1965 to March, 1975 are used to estimate the related information of period t and the data from April, 1975 - September, 1979 are used to estimate the related information in period t+1. The results associated with equation (17) are listed in Table 1. It is found that the MSE of forcasting obtained from the time-varying coefficient beta estimates is smaller than that obtained from constant coefficient beta estimates. The reduction of MSE is essentially due to the reduction of

Table 1

MSE of Constant Coefficient Betas
and Time-Varying Betas

	Constant Coefficient	Time-Varying Coefficient
$\overline{\beta}_{t+1}$	1.01325	.99928
$\overline{\beta}_{t}$	.94682	.96648
b_1	.30780	.42850
$S_{\beta t}$	.1355	.18913
$S_{\beta t+1}$	.2396	.2448
$R^2_{\beta t+1,\ \beta t}$	.03030	.10957
bias	.00441	.00108
In efficiency	.00880	.01168
random error	.05570	.05338
MSE	.06892	.06613

bias. Note that the inefficiency from time-varying coefficient estimate is larger than that for constant coefficient estimates. In sum, it seems reasonable to conclude that the main contribution of time-varying beta estimates in forcasting betas is to reduce the bias. Besides this kind of advantages, the time-varying coefficient can also be used to improve the risk-return trade-off test as shown in the latter portion of this section.

Finally, the findings of this paper have other implications for previous research. Following equation (3), the total risk of each individual security can be decomposed as:

$$\text{(18)} \qquad Var(Y_t) = [(\bar{\beta}^2 + \alpha_1^2 t^2 + 2\bar{\beta}\alpha_1 t)Var(X_t)] + [\sigma_\eta^2 X_t^2 + \sigma_\varepsilon^2].$$

If the estimates for both α_1 and σ_η^2 approach zero, equation (17) can be reduced to:

$$\text{(19)} \qquad Var(Y_t) = \bar{\beta}^2 \; Var(X_t) + \sigma_\varepsilon^2.$$

Equation (18) indicates that total risk can be decomposed into systematic and unsystematic risk by the OLS regression method as discussed by Francis [1979] and others.[6] However, this result does not hold unless β is a deterministic variable. As the beta coefficient is stochastic, the OLS estimate of nonsystematic risk actually has two components, i.e., $\sigma_\eta^2 X_t + \sigma_\varepsilon^2$. Therefore, the partition of systematic risk and unsystematic risk is not possible since the random risk is confounded with "noise"

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[6]Fabozzi and Francis (1978) have similar arguments. However, their decomposition is only a special case of our decomposition as indicated in equation (18).

from shifting betas. Both Lintner [1956] and Douglas [1969] found that the rates of return on individual stocks are strongly correlated with random risk and this is contrary to the capital asset pricing theory. Subsequently, Miller and Scholes [197] carefully reexamined this issue and still failed to provide a satisfactory explanation.

This issue is reexamined in the present study by running two tests. First, the OLS residual variance is used as a measure of random risk, and a simple linear regression is run with the monthly rates of return for each firm as the dependent variable. The results are consistent with Lintner and Douglas's finding that rates of return are strongly correlated with random risk. However, if the "pure residual" variance σ_ε^2 is utilized in place of the OLS residual variance, it is found that the relationship between rates of return and random risk is not statistically significant. This finding could imply that both Lintner and Douglas's result may be due to the problem of using the fixed beta coefficient to decompose the total risk while, in fact, beta is not a constant.

V. Summary and Concluding Remarks

This paper has pointed out some of the problems of previous research in the investigation of beta stability and beta tendency. None of the previous studies explicitly derived a model that allows for coexistence of both beta instability and beta tendency. With the application of the variable mean response regression model, beta can be decomposed into constant, trend, and random components. The model developed in this study can therefore be considered as a general case of the previous studies. Monthly data for 363 firms was used in this paper to test beta stability and beta tendency. The empirical results revealed that sixty-five percent of the sample firms had significant σ_η values, which indicates that more than half of the firms had an unstable beta over the ten year period. In addition, 105 firms (29%) had significant α_1 values which implies that a significant portion of the sample firms had shifting betas. The sign and magnitude of α_1 is consistent with Blume's [1975] finding that there exists some regression tendency of beta coefficients over time.

The MSE of forecasting obtained from the model developed in this paper is smaller than that obtained from the traditional fixed coefficient beta estimates. This study also reexamined the Lintner and Douglas "paradox." After the elimination of both trend and random components of beta risk, the residual variance was found not to be correlated with the security return. This suggests that the risk partition method that has been used by previous studies may have a fundamental flaw, i.e., random risk can be confounded with "noise" from random betas. To improve estimates of beta risk and/or random risk, the model present in this study is a viable alternative. The causes of beta tendency, however, are still unknown and future research in this area is needed.

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