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FIRST YEAR LECTURES IN DESCRIPTIVE GEOMETRY

J. Roy Cockburn, B.A.Sc.,

Descriptive Goemetry is the science by means of which solid objects may be represented upon a single plane surface and by means of which problems in solid geometry may be solved by graphical methods.

In the solution of problems by the methods of descriptive geometry all points and lines are represented by their projections upon plane or other surfaces.

Definitions

(1) A *plane* is a surface such that if any two points in the surface be joined by a straight line the line will be wholly in that surface.

(2) The *projection* of a point may be defined as follows:—If a straight line be drawn through a given point to intersect a given surface, the point where this line intersects the surface is the projection of the given point on the given surface and the straight line joining the point with its projection is called the projecting line of the point.

If the projection be upon a plane surface and the projecting line be perpendicular to that surface the projection is said to be *orthographic*. If the projecting line be inclined to the plane surface the projection is said to be *oblique*.

The projection of a given line is the line which contains the projection of every point in the given line and similarly the projection of any object may be found by obtaining the projection of every line and point in the object.

If all the projecting lines of any object are parallel, the projection is said to be a parallel projection and may be either orthographic or oblique.

If all the projecting lines of an object pass through some fixed point the projection is said to be conical or a conical projection.

Perspectives are the most familiar examples of conical projections. Many of the projections of the earth's surface used in map making also belong to this class.

(3) A *straight line* is said to be perpendicular to a plane if it is at right angles to every straight line in the plane drawn through its point of intersection with the plane.

(4) The *angle between* two planes is the angle between two lines one, in each of the planes and each passing through a point in the line of intersection of the two planes and perpendicular to the line of intersection.

PROPOSITIONS IN SOLID GEOMETRY

Proposition I

If two straight lines intersect they are contained in one plane, for, if a plane be passed through one of the lines it may be revolved about that line as an axis until it contains the other.

Proposition II

If two planes intersect, their line of intersection is a straight line. (*Fig. 1.*)

Let M and N be two planes which intersect.

If the two points A and B be taken in the line of intersection of M and N and joined by a straight line, the straight line will lie wholly in the plane M (def. of plane); and for the same reason it will lie wholly in plane N.

Therefore the straight line AB is the line of intersection of the two planes.

Proposition III

If a straight line be perpendicular to each of two intersecting straight lines at their point of intersection, it is perpendicular to their plane. (*Fig. 2.*)

Let AB be a straight line perpendicular to each of the straight lines BD and BF at their point of intersection B. Then the straight line AB must be perpendicular to the plane "M" containing BD and BF.

Construction:

Produce AB to C, making $BC = AB$. Join D and F and from any point on DF, such as E, draw the straight line BE to the point B. Join the points D, E and F with points A and C. Proof:

In triangles ABD and CBD,
side AB = side CB.
side DB is common,
and $ABD = CBD$.
$\therefore$ the side AD must = side DC.

Similarly

In triangles ABF and CBF
side AF may be proved = side CF

Again

side AD = side DC
side AF = side CF
side DF is common.

If triangle CDF be revolved about the side DF through a sufficient angle the sides DC and CF will fall on the sides AD and AF and the point C will coincide with the point A.

$\therefore$ line CE will coincide with and equal line AE.

Again

In triangles ABE and CBE
side AB = side CB
side BE is common
and side AE = side CE
$\therefore ABE = CBE$ and
$\therefore$ each = right angle.

$\therefore AB$ is at right angles to line BE and $\therefore$ to any straight line in plane M passing through point B.

It is therefore perpendicular to the plane M which contains the two straight lines BD and BF.

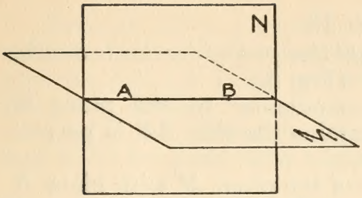

FIG. 1.

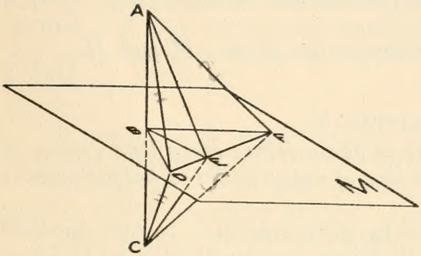

FIG. 2.

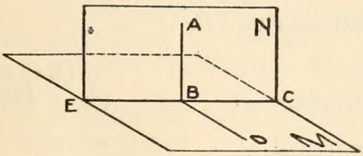

FIG. 3.

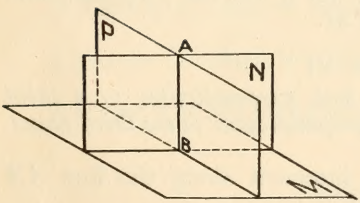

FIG. 4.

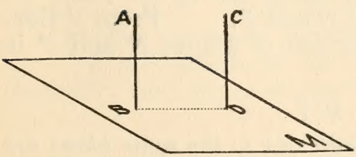

FIG. 5.

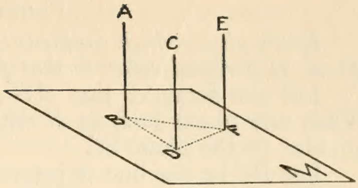

FIG. 6.

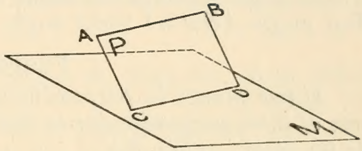

FIG. 7.

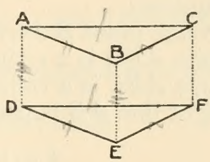

FIG. 8.

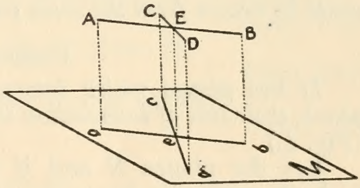

FIG. 9.

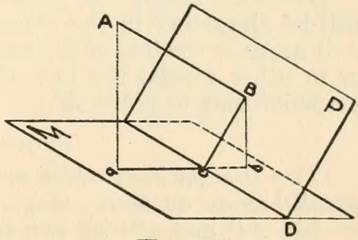

FIG. 10.

Proposition IV

Every plane which contains a straight line perpendicular to another plane is perpendicular to that plane. (*Fig. 3.*)

Let the straight line AB be perpendicular to the plane M. Then any plane such as N which contains the line AB is perpendicular to the plane M.

Let BC be the line of intersection of the plane M with plane N.

Through the point B where line AB intersects the plane M draw the straight line BD at right angles to BC.

The straight line AB is perpendicular to line BC. Def. 3
and BD is perpendicular to BC. Const.
∴ the angle ABD is the angle between the planes M and N.
But angle $ABD = 1$ right angle. Def. 3

Proposition V

If two planes be perpendicular to one another every line drawn in one of them perpendicular to their line of intersection is perpendicular to the other. (*Fig. 3.*)

Let the two planes M and N be perpendicular to one another and let AB be a line in plane N drawn perpendicular to the line of intersection BC.

Construction:

Through point B draw BD perpendicualr to BC.
The angle ABD is a right angle.
The angle ABC is a right angle. Def. 4
∴ straight line AB is perpendiculat to plane M. Prop. III

Collary—Proposition V

If through any point in M a straight line be drawn perpendicular to the plane N it will intersect it in BC; for if not, two perpendiculars could be drawn from the same point in M.

Proposition VI

If two planes which intersect be both perpendicular to a third plane, their line of intersection is also perpendicular to the third plane. (*Fig. 4.*)

Let the planes N and P which intersect along the line AB be both perpendicular to the plane M.

The point B is a point common to all three planes.

Then a straight line perpendicular to M through the point B will lie altogether in the plane N and altogether in the plane P.
∴ it must be the line of intersection of N and P, Prop. V Cor.
or in other words, the line of intersection of planes N and P is perpendicular to plane M.

Proposition VII

Two straight lines which are perpendicular to the same plane are parallel to one another. (*Fig. 5.*)

Let AB and CD be two straight lines, each perpendicular to the plane M. They are therefore parallel.

Because AB is perpendicular to the plane M, the plane ABD must be perpendicular to plane M. Prop. IV and the line CD being perpendicular to the plane M must lie in the plane ABD. Prop. V Cor.

$\therefore$ AB and CD are in the same plane and as the angles ABD and CDB are each right angles, AB and CD are parallel.

Proposition VIII

If one of two parallel straight lines is perpendicular to a plane the other is also perpendicular to that plane. (*Fig. 5.*)

Let AB and CD be two parallel straight lines and one of them AB perpendicular to the plane M. Then CD must also be perpendicular to plane M.

Proof:

Because AB is perpendicular to plane M, then the plane ABD must be perpendicular to the plane M. Prop. IV

But CD is in the plane, ABD being parallel to the line AB, and as ABD is a right angle $\therefore$ CDB must be a right angle and $\therefore$ line CD must be perpendicular to the plane M. Prop. V

Proposition IX

If two straight lines are each parallel to a third straight line they are parallel to one another. (*Fig. 6.*)

AB and CD are two straight lines each parallel to the straight line EF.

AB and CD must then be parallel to each other.

Construction:

Through some point in EF pass a plane M perpendicular to EF.

AB being parallel to EF must be perpendicular to plane M. Prop. VIII

Likewise CD must be perpendicular to plane M. Prop. VIII

AB and CD are therefore parallel to one another. Prop. VII

Proposition X

A plane which contains one of two parallel straight lines is parallel to the other. (*Fig. 7.*)

AB and CD are two parallel straight lines and M is a plane containing CD. Plane M must then be parallel to the line AB.

AB and CD being parallel are in the same plane. $ABCD$.

$\therefore$ if AB intersects the plane M it must intersect it along the line CD; but, this is impossible, as AB is parallel to CD.

Proposition XI

A straight line which is parallel to a plane is also parallel to the line of intersection with that plane of any plane containing the line (*Fig. 7.*)

Let the straight line AB be parallel to the plane M. AB is then parallel to CD, the line of intersection with M of any plane containing AB, such as P.

If AB be not parallel to CD, the line of intersection of M and P, it will meet CD; but, as CD is wholly contained in plane M, AB would meet M at the same point; but, this is impossible; therefore AB and CD are parallel.

Corollary 1.—Proposition XI

If two straight lines which meet be each parallel to a plane their plane must be parallel to the plane first mentioned.

Proposition XII

If two planes which are parallel to each other be intersected by a third plane the lines of intersection must be parallel. Proof.

If the two lines of intersection be not parallel they will meet, since they are in the same plane; but, the planes would then meet in the same point which is impossible.

Proposition XIII

Planes to which the same straight line is perpendicular are parallel. Proof:

If the two planes be not parallel they will meet in a straight line and from any point in this line lines could be drawn, one in each plane, to intersect the given line, and these lines would each be perpendicular to the given line.

But, this is impossible: therefore, the given planes cannot meet and therefore are parallel.

Proposition XIV

If two intersecting straight lines be parallel respectively to two other intersecting straight lines situated in a different plane from the first two, the first two and the second two contain equal angles and their planes are parallel. (*Fig. 8.*)

AB and BC are parallel respectively to DE and EF.

Then the angle ABC = the angle DEF and plane ABC is parallel to plane DEF.

Construction:

Cut off AB and DE equal and BC and EF equal.

Join AD, BE, CF, AC and DF.

Proof:

Because AB and DE are equal and parallel, then AD and BE must be equal and parallel.

Because BC and EF are equal and parallel, then BE and CF must be equal and parallel, and $\therefore$ AD and CF must be equal and parallel. Prop. IX

Then because AD and CF are equal and parallel, AC and DF must be equal and parallel.

In triangles ABC and DEF
side AB = side DE
side BC = side EF
side AC = side DF $\therefore$ angle ABC = angle DEF.

Again:

Because AB is parallel to DE, it is parallel to the plane DEF. Prop. X

Similarly BC is parallel to plane DEF.
$\therefore$ plane ABC is parallel to the plane DEF. Prop. XI Cor. 1

Proposition XV

If two straight lines be at right angles to one another their orthographic projections on a plane parallel to either one of them are also at right angles. (*Fig. 9.*)

Let AB and CD be two straight lines at right angles to one another, and M a plane parallel to AB.

The orthographic projections of AB and CD, viz., ab and cd are also at right angles.

Proof:

AE is parallel to ae — Prop. XI
and ae is perpendicular to eE
$\therefore$ AE is perpendicular to eE — Prop. III
and $\therefore$ perpendicular to plane eED, and
$\therefore$ ae is perpendicular to plane eED — Prop. VIII
and $\therefore$ to line ed.

Proposition XVI

If the projections of two straight lines be at right angles to one another and one of the lines be parallel to the plane of projection, the two lines are at right angles. (*Fig. 9.*)

Let AB and CD be two straight lines of which the projections ab and cd are at right angles and the line AB is parallel to the plane of projection M.

AE and ae are parallel — Prop. XI
and ae is perpendicular to the projecting plane Eed. — Prop. III
$\therefore$ AE is perpendicular to the plane eED
and $\therefore$ to ED. — Prop. VIII

Proposition XVII

If a straight line be perpendicular to a plane the orthographic projection of the line on another plane is perpendicular to the line of intersection of the two planes. (*Fig. 10.*)

Let AB be a straight line perpendicular to the plane P, and M a plane intersecting P along line CD.

ab is the orthographic projection of AB on the plane M.

ab is $\therefore$ perpendicular to CD.

Proof:

As AB is perpendicular to P, the plane $ABca$ is perpendicular to P

and plane $ABca$ is perpendicular to M.

$\therefore$ plane $ABca$ is perpendicular to CD — Prop. VI

$\therefore$ CD is perpendicular to ac. — Def. III

PROBLEMS RELATIVE TO POINTS, STRAIGHT LINES AND PLANES

In general, points and lines are represented by their projections upon two imaginary planes: one plane, called the *vertical plane of projection*, being chosen in a vertical position; and the other plane, called the *horizontal plane of projection*, being chosen in a horizontal position.

In the representation of points and lines by their projections upon these planes it is found convenient to consider that the projections have been made and then one of the planes revolved about the line of intersection of the two planes until both planes coincide. In this way the projections of the points and lines can be represented upon a single plane surface.

The two imaginary planes divide space into four parts or quadrants and these quadrants are usually called the first, second, third and fourth, being numbered as shown in Fig. 11.

The point of sight or point of observation is considered to be in the first quadrant.

If the horizontal plane be revolved as indicated by the arrow heads until it coincides with the vertical plane of projection, a point such as "A" in the first quadrant would have its vertical projection a^v above the ground line (intersection of the two planes) and its horizontal projection a below the ground line.

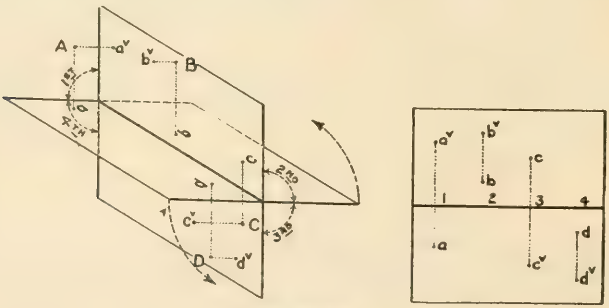

FIG. 11.

The points B, C and D in the other quadrants will be represented as shown.

Definitions

The vertical trace of a straight line is the point where it intersects the vertical plane of projection.

The horizontal trace of a straight line is the point where it intersects the horizontal plane of projection.

The vertical trace of a plane is its line of intersection with the vertical plane of projection.

The horizontal trace of a plane is its line of intersection with the horizontal plane of projection.

Problem I

Given the projections of a straight line, to find its horizontal and vertical traces. (*Fig. 12.*)

Let ab and a^vb^v be the horizontal and vertical projections of the straight line AB.

Produce the vertical projection a^vb^v to intersect the ground line in the point c^v. c^v is then the vertical projection of the point C where the line AB intersects the horizontal plane and the horizontal projection of this point must be at the intersection of the horizontal projection of AB, viz., ab, with the line c^1C drawn perpendicular to the ground line.

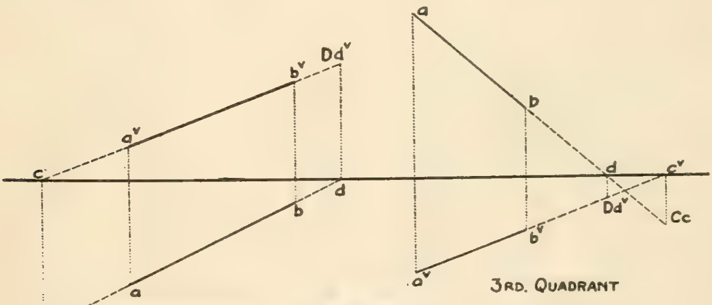

Fig. 12.

As the point C is a point on the horizontal plane, its horizontal projection coincides with the point itself. Therefore, the point C or c is the horizontal trace of AB.

In the same way the vertical trace D may be found.

Problem II

Given the points in which a straight line pierces the planes of projection, to find its projections. (*Fig. 12.*)

Let the points C and D be the points where the line pierces the horizontal and vertical planes respectively.

The horizontal projection of the point C is the point C itself.

Its vertical projection is the point c^v found by drawing line cc^1 to intersect the ground line at the point c^v.

Likewise D is the vertical projection of D and d the horizontal projection of D.

Join the horizontal projection of C to the horizontal projection of D by the line Cd. Cd is then the horizontal projection of the required line, and similarly c^vD is its vertical projection.

Problem III

To find the length of a straight line whose projections are given and also to find the angle which it makes with the planes of projection (Fig. 13.)

Let AB be the given line, given by its horizontal projection ab and its vertical projection a^vb^v.

If the straight line AB be revolved about a straight line through "A" and perpendicular to the vertical plane until it becomes

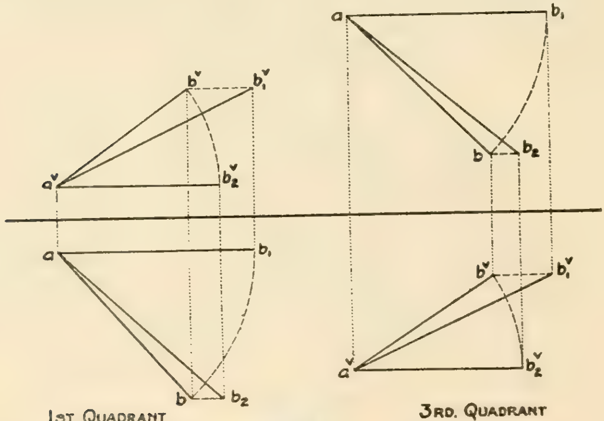

Fig. 13.

parallel to the horizontal plane, its horizontal projection ab_2 will be equal to the line in length, and the inclination of the projection ab_2 to the ground line will equal the angle between the line and the vertical plane.

During this revolution the horizontal projection of B moves from b to b_2 along a line parallel to the ground line.

The vertical projection of point B moves around the arc of a circle, whose centre is a^v, from point b^v to point b^v_2, *i.e.*, until $a^vb^v_2$ becomes parallel to the G. L.

The line ab_2 is equal in length to the line AB in space.

The angle ab_2b equals the angle between the line AB and the vertical plane of projection.

In the same way the angle $b_2^v a^v b_1^v$ equals the angle which the line makes with the horizontal plane of projection.

Problem IV

Given the length of a line, the angles which the line makes with the planes of projection and the projections of one extremity of the line, to find the projections of the line. (*Fig. 13.*)

Let the given line be AB and the projections of one extremity be the points a and a^v.

First, find the projections of the line when it is parallel to the vertical plane and making the required angle with the horizontal plane as follows:—

Through point a^v draw line $a^v b_1^v$ inclined to the ground line at an angle equal to required angle between AB and "H" plane, and make $a^v b_1^v$ equal to the required line AB.

Through point a draw line ab_1 parallel to the ground line.

To find point b_1 draw through b_1^v the line $b_1^v b_1$ perpendicular to the ground line and to intersect ab_1 in the point b_1 ab_1 and $a^v b_1^v$ are the horizontal and vertical projections of the line AB when it is parallel to the vertical plane and making the required angle with the horizontal plane.

Next find in the same way the horizontal and vertical projections ab_2 and $a^v b_2^v$ of AB when it makes the required angle with the vertical plane and is parallel to the horizontal plane.

Again: As long as line AB is inclined at the same angle to the vertical plane and point A remains stationary, the vertical projection AB must be constant in length and the distance of the horizontal projection of B (viz., b) from the ground line must be constant.

Also, as long as the line AB makes the same angle with the horizontal plane of projection, the length of its horizontal projection must be constant and if point A be stationary, then the distance of the vertical projection of B from the ground line must be constant.

Therefore, to find vertical projection of point B, draw line $b_1^v b^v$ parallel to the ground line and with centre a^1 and radius $a^x b_2^v$ describe arc $b_2^v b^v$ to intersect line $b_1^v b^v$ at b^v. b^v is required vertical projection.

In the same way, b, the horizontal projection of B, may be found.

$a^v b^v$ is required vertical projection.

ab the required horizontal projection.

Problem V

To find the traces of a plane containing three given points. (*Fig. 14.*)

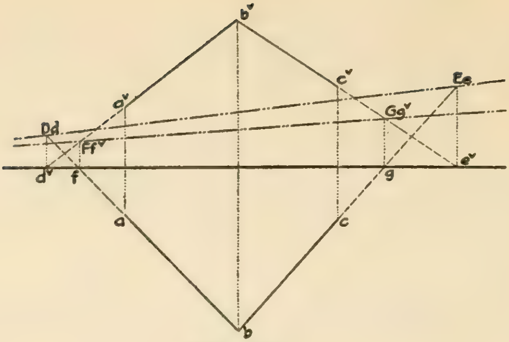

Fig. 14.

Note—The vertical trace of the plane is its intersection with the vertical plane.

The horizontal trace is its intersection with the horizontal plane.

Let *ABC* be the three given points, given by their projections $a^v b^v c^v$ and *abc*.

A plane passing through these three points must contain a straight line joining any two of them and the points where such a line intersects the planes of projection must be points on the required traces.

Find the vertical trace of the line joining *A* and *B*, viz., *F*. Prob. I

Also the vertical trace of the line joining *B* and *C*, viz., *G*. Prob. I

Join *FG*, obtaining the vertical trace of the plane.

Problem VI

To find the angle between two given straight lines which intersect. (*Fig. 15.*)

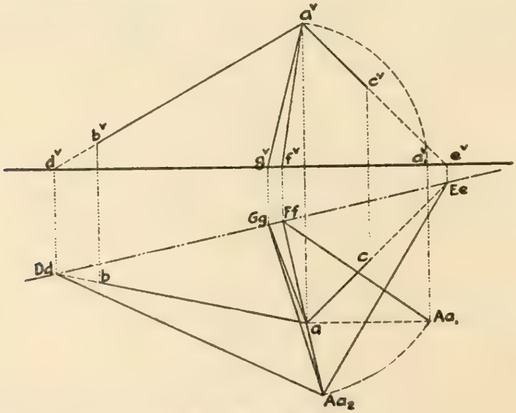

Fig. 15.

Let AB and AC be the two lines given by their projections ab, a^vb^v and ac, a^vc^v and intersecting at point A.

To find the angle between them.

Find the horizontal trace of the plane containing these lines. Prob. V

The three lines AB, AC and the trace DE just found form a triangle.

Revolve this triangle about the line DE until it coincides with the horizontal plane.

To do this, find the projection of the line AF from point A, perpendicular to the line DE and intersecting it at point F. The horizontal projection of AF is the line aF drawn perpendicular to DE. Its vertical projection is the line a^vf^v, f^v being point on ground line and on perpendicular Ff^v to the ground line.

Find the true length of this line AF, viz., A_1F. Prob. III

Through point F draw line FA_2 equal A_1F and perpendicular to DE. The point A_2 gives the revolved position of point A.

Join A_2 with points D and E.

The lines A_2D and A_2E are the positions of the lines AD and AE when revolved to coincide with the horizontal plane.

The angle DA_2E shows the true size of the angle between the lines AB and AC.

Problem VII

To find the projections of a line bisecting the angle between two given straight lines which intersect. (*Fig. 15.*)

Let AB and AC be the two given lines intersecting at the point A.

Find the true size of the angle BAC, viz., DA_2E in the same way as in problem VI, by the method of revolving into the horizontal plane.

While in revolved position, bisect the angle DA_2E by the line A_2G intersecting line ED at point G.

Find vertical projection of point G, viz., g^v.

Join G with a: line aG is horizontal projection of required line.

Join a^vg^v and line a^vg^v is the vertical projection of required line.

Problem VIII

To find the projections of the line of intersection of two oblique planes. (*Fig. 16.*)

Let LM be the vertical trace and MN the horizontal trace of one of the planes and LP the vertical and PN the horizontal trace of the other plane.

The vertical traces of the two planes intersect at point L.

The horizontal traces of the two planes intersect at point N.

L and N are therefore, points on the required line of intersection and as the line of intersection is a straight line, it must be the line joining points L and N.

Find projections of points L and N.

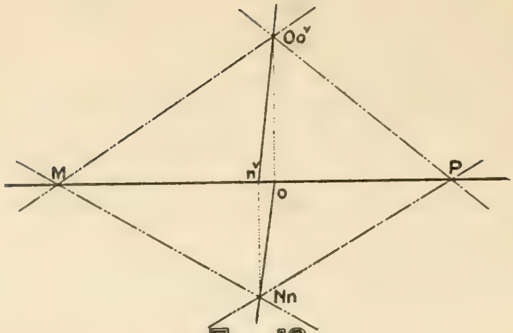

Fig. 16.

l^v is vertical projection and l the horizontal projection of point L.

n^v is vertical and n the horizontal projection of point N.

Join $l^v n^v$ obtaining the vertical projection of required line of intersection. ln is the horizontal projection.

Problem IX

To find the projection of the line of intersection of two planes, both of which are parallel to the ground line. (*Fig. 17.*)

Let AB be the vertical trace and CD the horizontal trace of one of the planes and WX the vertical trace and YZ the horizontal trace of the other.

Note—The planes being parallel to the ground line, must have their traces parallel to the ground line. Prob. XI

Pass an auxiliary plane through both of the given planes. One perpendicular to the two planes will be most convenient.

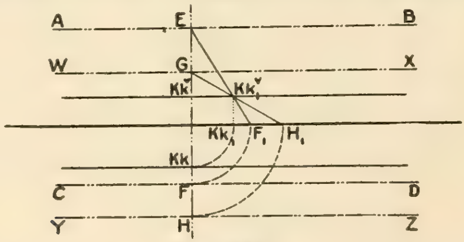

Fig. 17.

Such a plane is the plane $EFGH$, having both traces perpendicular to the ground line.

The vertical trace AB is intersected in the point E by the vertical trace EG of the auxiliary plane and the horizontal trace CD in the point F by the horizontal trace FH.

The line joining the points E and F is the line of intersection of the plane $ABCD$ by the auxiliary plane, and similarly, the line GH is the line of intersection of the plane $WXYZ$ by the auxiliary plane.

If the auxiliary plane be revolved through an angle of 90° about its vertical trace *EG*, it will coincide with the vertical plane and the lines of intersection *EF* and *GH* will in that position be in the vertical plane, and their point of intersection (*K*) will be shown in position K_1.

Revolve the auxiliary plane back to the original position. finding k^v and k, its vertical and horizontal projections. *K* being common to the three planes, *ABCD*, *WXYZ* an *dEFGH* must be on the line of intersection of *ABCD* and *WXYZ*.

Problem X

To find the projection of the point of intersection of a given straight line and a given plane. (Fig. 18.)

Let *LM* be the vertical trace of the given plane and *MN* the horizontal trace, and *AB* be the line given by its vertical projection $a^v b^v$ and its horizontal projection *ab*.

Find the traces of the plane containing the line *AB* and its horizontal projection.

OP and *PQ* are the traces.

Find the line of intersection of the plane *LMN* by the plane

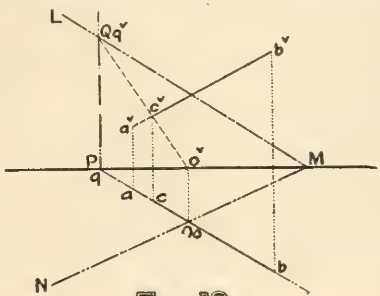

Fig. 18.

OPQ. Such a line is *OQ*. Its vertical projection is o^1q^1 and its horizontal projection *oq*.

The vertical projection $o^v q^v$ intersects the vertical projection a^1b^1 at the point c^1, this the vertical projection of the point where the line *AB* intersects the line *OQ* and ∴ where it intersects the plane *LMN*.

Find the point *c* the horizontal projection of point c^1 or *c* on line *AB*.

Problem XI

To find the projection of a line passing through a given point and perpendicular to a given plane and to find the distance from the point to the plane. (Fig. 19.)

Let *LM* be the vertical trace and *MN* the horizontal trace of the given plane, and a^v the vertical projection and *a* the horizontal projection of the given point.

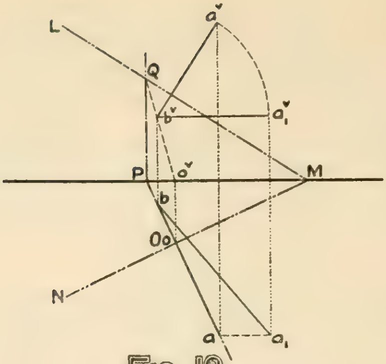

Fig. 19.

To find the vertical projection of a line through point A and perpendicular to the plane LMN, draw the line $a^v b^v$ through a^v perpendicular to LM. Prop. XVII

In the same way, find the horizontal projection ab.

To find the distance to the plane LMN from the point A, find the projections (b^v and b) of the point B where the line AB intersects the plane LMN.

Find the distance from the point A to the point B and the distance AB is the required distance. Prob. X
Prob. III

Problem XII

To project a given line orthographically on a given plane and to find the position of the projection relatively to one of the traces of the plane. (Fig. 20.)

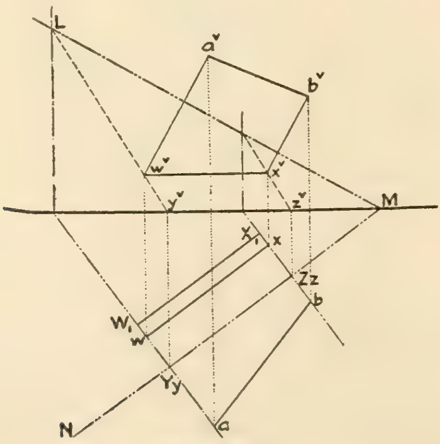

Fig. 20.

Let LM be the vertical trace and MN the horizontal trace of the given plane, and $a^v b^v$ the vertical projection and ab the horizontal projection of the given line AB.

To find the orthographic projection of the line AB on the plane LMN it is necessary to find the orthographic projection of the two extremities A and B.

Through the point A draw the line AW perpendicular to the plane LMN and find the point where AW intersects the plane LMN (or) this plane. Prob. X

The point W shown by its projections w^v and w is the orthographic projection of the point A on the plane LMN. Similarly the point X given by its projection x^v and x, is the orthographic projection of the point B on the plane LMN. $w^v x^v$ is the vertical projection and wx, the horizontal projection of the required line. The second part of this problem may be solved by revolving the plane LMN about its horizontal trace into the horizontal plane.

PROBLEM XIII

Through a given point to pass a plane perpendicular to a given line. (*Fig. 21.*)

Let X be the point given by its projections x^v and x, and AB be the line given by its projections $a^v b^v$ and ab.

A plane perpendicular to the line AB must have its vertical trace perpendicular to the vertical projection of the line AB and its horizontal trace perpendicular to the horizontal projection of the line AB. Prop. XVII

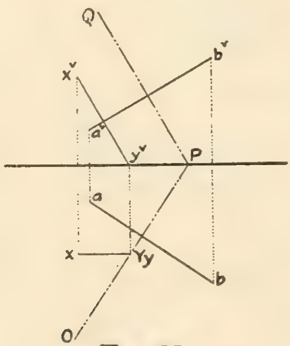

FIG. 21.

Through point X draw the line XY parallel to the vertical trace of the required plane. Its vertical projection is $x^v y^v$ perpendicular to $a^v b^v$ and its horizontal projection xy is parallel to the ground line.

Find the point Y where this line intersects the horizontal plane. As the line XY is parallel to the vertical trace of the required plane and it passes through the point X it must be contained in the required plane. Y is therefore a point on the horizontal trace of the required plane.

Problem XIV

Through a given point to pass a plane parallel to two given straight lines. (*Fig. 22.*)

Let X be the point given by its projections x^v and x, and AB and CD be the given lines, a^vb^v and ab being the projections of one, and c^vd^v and cd the projections of the other.

Through the point X draw a line parallel to AB, e.e., through x^v draw a line o^vm^v parallel to a^vb^v and through x draw om parallel to ab obtaining projections of the required line.

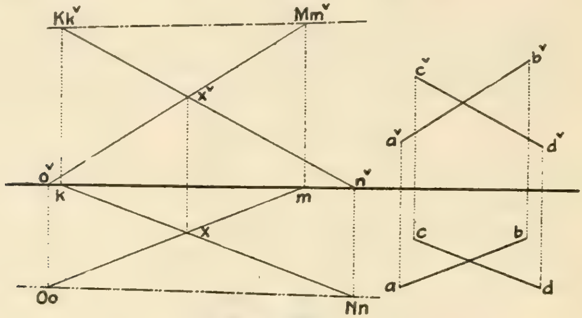

Fig. 22.

Through X draw a line parallel to CD: l^vn^v being the vertical and ln the horizontal projection.

Find the traces of a plane containing these two lines LN and MO. Prob. V

LM is the vertical trace of plane and ON the horizontal trace of the required plane.

Problem XV

To pass a plane through a straight line parallel to another straight line. (*Fig. 23.*)

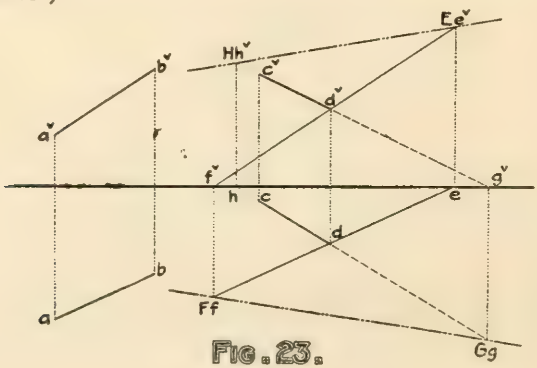

Fig. 23.

Let AB and CD be the given lines. It is required to pass a plane through CD parallel to AB.

Through the point D on CD draw the line EF parallel to AB.

Find the traces of a plane containing CD and EF.

Prob. V

Problem XVI

To find the shortest distance from a given point to a given straight line. (Fig. 24.)

Let the point X be given by its projections x^v and x and the straight line AB given by its projections a^vb^v and ab.

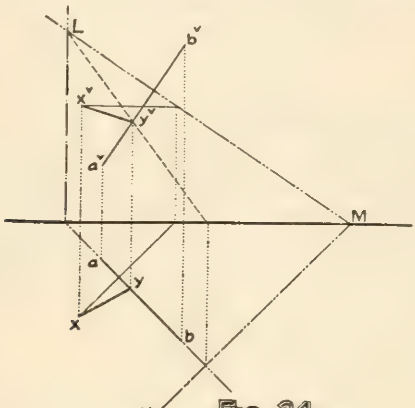

Fig. 24.

The shortest distance from x to AB is evidenlty the length of a perpendicular from X to AB.

Through X pass a plane LMN perpendicular to line AB.

Prob. XIII

Find the point Y where line AB intersects plane LMN.

Prob. X

The line joining the points X and Y being in a plane which is perpendicular to line AB must, therefore, be perpendicular to AB.

The length of the line XY is the required distance. This may be found by Problem III.

Problem XVII

To find the angle between a given straight line and a given plane. (Fig. 25.)

Def.—The angle between a straight line and a plane is the angle between the line and its orthographic projection on the plane.

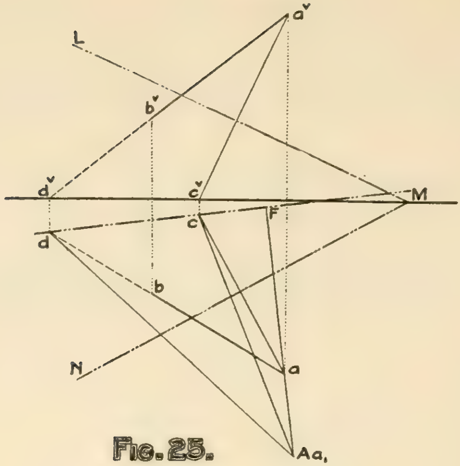

FIG. 25.

If a line be drawn from a point on the given line perpendicular to the given plane, the angle which it makes with the given line is the complement of the required angle.

Let *LMN* be the given plane and *AB* be the given straight line.

Through the point *A*, draw the line *AC* perpendicular to the plane *LMN*. Find the angle between *AB* and *AC*.

Prob. VI

90°—This angle is the required angle.

PROBLEM XVIII

To find the angle between two given planes. (*Fig. 26.*)

Let *LM* be the vertical trace and *MN* the horizontal trace of one plane and *LP* the vertical trace and *PN* the horizontal trace of the other plane.

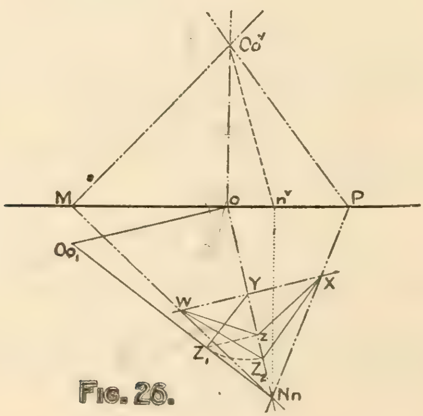

FIG. 26.

The angle between two planes is the angle between two lines, one in each plane, perpendicular to the line of intersection.

Therefore, if a plane be passed through the two planes so that it is at right angles to their line of intersection the two lines of intersection thus found are at right angles to the line of intersection on the planes LMN and LPN. The angle between these two lines is the required angle.

WX is the horizontal trace of the auxiliary plane which is taken at right angles to LN, the line of intersection of the two planes.

In order to obtain the horizontal projection of the point Z in which the line LN is intersected by the auxiliary plane WXZ revolve the line LN and the auxiliary plane WXZ, about the horizontal projection of LN, until the line takes the position L_1N on the horizontal plane, the auxiliary plane becoming perpendicular to the horizontal plane of projection, YZ being the horizontal trace in the revolved position.

Z_1 is the point where the plane WXZ intersects the line L_1N.

Revolve the plane and line back to the original position, obtaining z, the horizontal projection of the required point.

Join W and X to z, obtaining the projections of the lines cut from the planes LMN and LPN by the auxiliary plane.

Revolve the auxiliary plane WXZ about its horizontal trace, until it coincides with the horizontal plane of projection. The point Z falls at Z_2 YZ_2 being equal to ZY_1.

The angle WZ_2X equals the angle between the two planes.

Problem XIX

Either trace of a plane being given and the angle which the plane makes with the corresponding plane of projection, to find the other trace. (Fig. 27.)

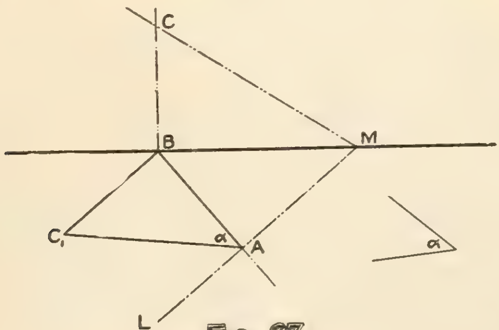

FIG. 27.

Let LM be the horizontal trace of the plane and a the angle which the plane makes with the horizontal plane of projection.

At any point A in line LM draw AB perpendicular to LM and in the horizontal plane. It intersects the ground line at the point B.

Draw BC_1 perpendicular to AB and in horizontal plane.

Make angle $BAC_1 = a$.

Revolve plane of triangle ABC_1 through 90° about AB or until BC_1 coincides with he vertical plane. C_1 will fall at C. BC being equal to BC_1 join CM and the line CM is the required vertical trace.

Problem XX

To find the shortest line that can be drawn terminated in two given straight lines. (*Fig. 28.*)

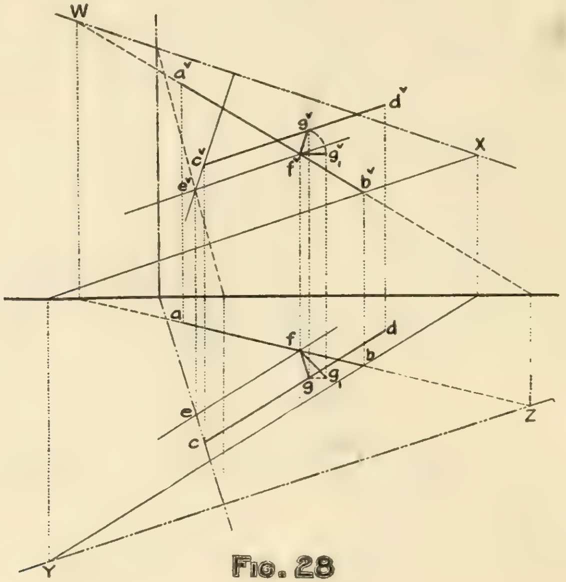

Fig. 28

Let AB and CD be the two given lines.

Through the line AB pass a plane parallel to the line CD.

Prob. XV

WX is the vertical trace and YZ the horizontal trace.

Project the line CD on the plane $WXYZ$ (Prob. XII), EF is the required projection intersecting line AB at the point F, of which f and f^1 are the horizontal and vertical projections. Through F draw line FG perpendicular to $WXYZ$ to intersect line CD in point G. Prob. XIII. The length of the line FG is the required length, and is shown at fg_1.

AXOMETRIC PROJECTION

If in a solid object three principal directions or axes be assumed each one perpendicular to the other two and the object be projected orthographically on a plane which is inclined to these axes the orthographic projection thus formed is called an Axometric or Axonometric Projection.

The method of obtaining the Axometric Projection is to assume a sufficient number of straight lines parallel to the axes and then find the projections of these lines.

In the case of a rectangular solid such as a cube or prism it is only necessary to find the projections of the edges when they are placed parallel to the assumed axes.

In Fig. 29 let aB, aC, and aD be the orthographic projections of three lines AB, AC, and AD each one of which is at right angles to the other two, and BC, CD and DB the intersections of the plane of projection with the planes ABC, ACD, and ADB respectively.

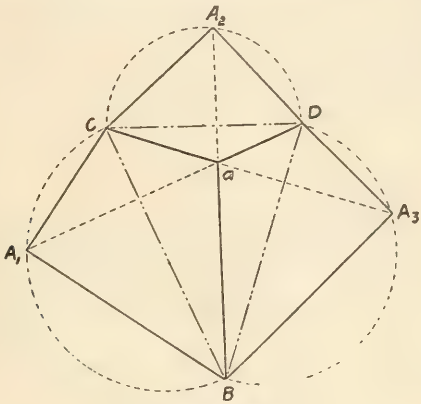

FIG. 29.

If the plane ABC be revolved about BC to coincide with the plane of projection A will fall at A_1.

As the angle BA_1C is a right angle the arc BA_1C must be a semi circle. The line DaA_1 is perpendicular to BC_1.

Prop. XVII

A_1B is the true length of the line AB.

aB is its projected length.

The ratio of the length aB to the length A_1B is therefore the ratio of the length of the projection of any line parallel to AB to the length of the line.

The corresponding ratios for lines parallel to AC and AD are found in a similar manner.

The practical method of making an axonometric projection is as follows:

First construct a scale as in Fig. 29. The angles at a may be any convenient size.

Secondly assume straight lines in the object each one of which can be placed parallel to one of the assumed axes. The projections of the lines will be parallel to their respective axes and the lengths of their projections can be found from the scale.

ISOMETRIC PROJECTION

If the three principal directions or axes in an axometric projection be all inclined at the same angle to the plane of projection the projection is called an Isometric Projection.

In order to prove certain properties of the Isometric Projection, take as an example the projection of a cube on a plane which makes equal angles with all its edges. Such a plane would be at right angles to a diagonal of the cube.

Let AB (Fig. 30) be a diagonal of a cube as shown above and AC, AD and AE, three edges meeting at the point A.

In Fig 30 the ratio of the line BD to the line DE equals the ratio of the diagonal of a face of a cube to the diagonal of a cube, *i.e.*, it is equal to the ratio between the length of the isometric projection of a line to the actual length of the line

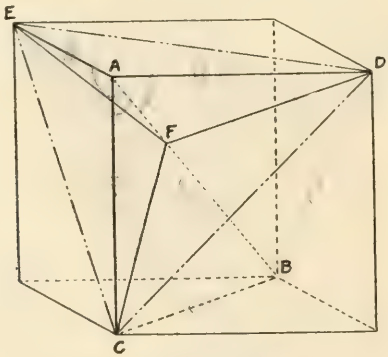

FIG. 30.

In the three triangles ABC, ABD and ABE

The sides AC, AD and AE are all equal, and sides BC BD and BE are all equal and side AB is common. ∴ angles BAC, BAD and BAE are all equal.

If perpendiculars be dropped from the three points C, D and E on the line AB, they will all fall at a common point F, and a plane containing these three lines will be perpendicular to the line AB.

In triangles, ACF, ADF and AEF,

The sides AC, AD and AE are all equal.

Angle FAC = angle FAD = angle FAE and

side AF is common,

∴ angle AFC = angle ADF = angle AEF

and side $FC = FD = FE$

The orthographic projection of AC on plane through C, D and E is line CF,

of AD is line DF,

of AE is line EF

In triangles FCD, FDE and FEC

sides FC, FD and FE are all equal.

and sides CD, DE and EC are all equal

$\therefore$ angles CFD, DFE and EFC are all equal and each $= 120°$.

$\therefore$ the projections of AC, AD and AE on the plane CDE are all equal and make angles of 120° with each other, and the projections of these lines on the plane CDE are their isometric projections because plane CDE makes equal angles with all three lines.

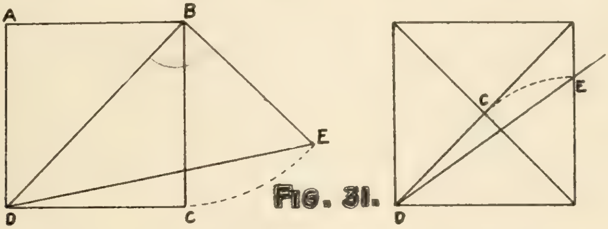

FIG. 31.

To determine the relation beween the lengths of the projections of the sides AC, AD and AE and the length of the lines themselves.

In the similar triangles BAC and AFC

angle $AFC =$ angle $ACB = 1$ right angle

$\therefore AC : CF = AB : BC$

i.e., $AC : CF =$ diagonal of cube: diagonal of face of cube.