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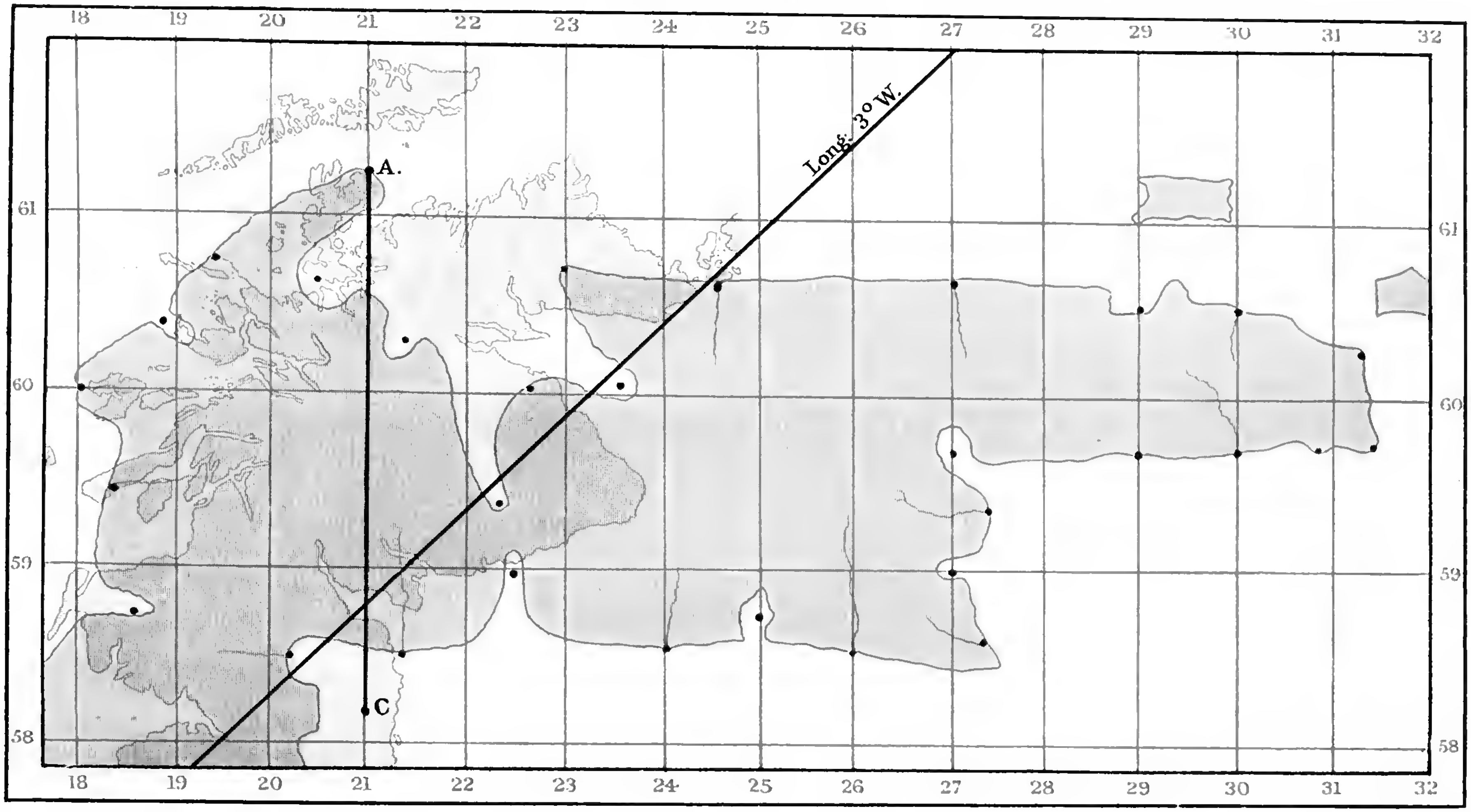
THE GEOGRAPHY

OF

PTOLEMY ELUCIDATED

PTOLEMY'S SCOTLAND WITH MODERN MAP IN SITU TO THE SAME SCALE.



[See Plate XVI. and Preface]

THE GEOGRAPHY

OF

PTOLEMY ELUCIDATED

BY

THOMAS GLAZEBROOK RYLANDS

F.S.A.; M.R.I.A.; M.R.A.S.; F.R.A.S.

Etc., Etc.

"Prove all things."—1 Thess. v. 21.

"Salvation is not by faith, but by verification."—Huxley.

Printed for the Author

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To

MY WIFE,

IN RECOGNITION OF THE CHEERFUL PATIENCE

WITH WHICH SHE WAS MY COMPANION

THROUGH THE LARGE INTERVALS AND THE SMALL HOURS WHEN I WAS

MOST SERIOUSLY AFFLICTED WITH WHAT MY FAMILY CALLED

"PTOLEMY ON THE BRAIN,"

This Volume

IS INSCRIBED WITH PROFOUND AFFECTION AND REGARD

BY HER HUSBAND,

THE AUTHOR.

AUTHOR'S PREFACE.

"If in the course of these inquiries I shall often find occasion to differ from those learned antiquaries who have engaged in this province before me, as I desire my sentiments may be no farther regarded than they appear to be supported by sufficient evidence, so I hope I need make no farther apology for such dissent."—*Brit. Rom.*, p. 355.

I LOOK upon a preface as an opportunity when an author may enter into somewhat confidential commune with his readers. This feeling has been expressed, I trust, in a way to enable those who peruse what follows, to approach the subject more nearly from my own point of view. Such has been my desire.

Although it is now fully forty-five years since my attention was first called to the Geography of Ptolemy, it must not be supposed that the whole of the interval has been occupied by this work. With the first futile effort to name our local river came the knowledge that the text of Ptolemy's work was still extant; but twenty-five years passed before a copy was met with for sale.

At the end of 1872, Mr. Quaritch advertised about a dozen editions; and led—or rather misled—by the Bibliotheca Spenceriana and Smith's Dictionary, I purchased two of them. These differed so egregiously, that a new difficulty was the result.

A long and careful bibliographical study was undertaken, during which very nearly every printed edition, and not a few of the manuscripts, in the libraries at home and abroad, including the Vatican, were examined, and a score or two of critical points selected, which, with ordinary care, determined the heredity of a copy. These extended over the text, the diagrams, and the maps. For study, most of the typical editions were placed upon my own shelves. The general result of this proceeding was, that the Greek text, except for the purpose of special verbal criticism, had little advantage over the Latin either in age or accuracy. Those that remain to us are largely taken from inferior texts, which were handled carelessly or ignorantly, and the "corrections," so called, are little more than the copying and perpetuation of readings and errors alike. So far as could be made out, we have no *editio* *princeps* worthy of the name. It was in the course of this study, after examining the two manuscript issues of Nicolaus de Donis, and the edition of 1482, that the conclusion was reached as to its value, and, as if by a "particular providence," a copy of it came into my possession within twenty-four hours.

It is not suggested that *any one* edition is a safe guide alone; but that, of all that have been examined, the edition of 1482 is, on the whole, the one which is most reliable. What the work wanted in the fifteenth century was not a mere copyist, nor a counter of codices, but an editor with some critical acumen: that man seems to have been Nicolaus de Donis; and it was cheering to find the appreciation in which his work had been held by the most careful and competent critics; indeed, they go further than I should be inclined to follow.[1]

It was after this that the work really began. The idea was—if it may be so described—to get to look over Ptolemy's shoulder while he was at work, and to investigate each succeeding point, until they could be combined into a consistent whole. This led to a rather large and wide amount of preliminary reading, which, however, fully repaid the time it cost.

The most fitting method of work seemed to be by diagrams. When a subject was selected, the preparatory lines were put in, but nothing was added until it had been thoroughly tested; thus some of these diagrams remained on the board for days, or even weeks, always in view.

Plates III. and VII. were among the most troublesome. The error of Alexandria in the former was, for a time, left as unaccountable, until it was reproduced and explained in Plate VI. The difficulty in Plate VII. was that, however much the data or calculation might be varied, there was constantly an outstanding error of either 5′ or 6′. When this was found to be the interval between Londinium and Greenwich (see Plate II.), it was welcomed as another verification.

Thus the work went on. I had no theory to support, and no object to serve. My interest in the subject was my only incentive.

In 1877 what I had to do was practically done; and the Paper which has been included as an Appendix in this volume was the first utterance I made. If the treatment there of the "authorities" should appear to be somewhat curt, let it be remembered that the time was short, that they had long been troublesome

[1] So far as I am aware no edition of the "Geography" has hitherto been printed in England, while more than seventy have been issued on the Continent. I have now good reason to believe that a photo-lithographic *fac-simile* of this Donis volume is likely to be published.

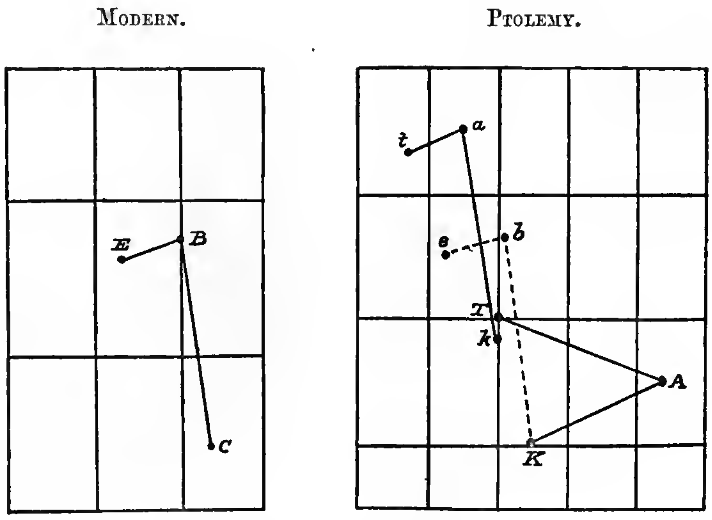
as an established obstruction, and that nothing could be done until they were put out of the way. They are a telling illustration of the carelessness or ignorance which obtained during the early years of this century, and of the pernicious practice of continuous copying without thought or examination.

Previous to this time arrangements were made for publication: but I care not to repeat or recall the treatment to which I was subjected; suffice it to say, that at last I recovered my Papers, and that they were laid aside in disgust. Years passed, and friends continued importunate in vain, until, in 1891, one of them added to his appeal that he would find me a competent young man to see the book through the press, and save me all the trouble. To this I yielded, and the present volume is the result.

After the final proofs of the following pages had been sent away, three points presented themselves as likely to be the better for a little further explanation. The frontispiece also was drawn.

I. Different Values of Errors.

Cases may probably be found where the error of a given station is stated to be 1° or more, while, on another page, the same error is given as only one or two minutes of arc. The two are consistent.



If K, A, and T be three stations of Ptolemy, and C, B, and E be their equivalents on a modern map, there are two modes of calculating their true positions and relations:—

1. They may be obtained from Ptolemy's Alexandria and his first meridian. In this case the result is *k*, *a*, and *t*, which are their true positions on his projection. *All* the errors are corrected.

2. In working with his sectional map of the British Isles, *K*, his nearest "fundamental" station, may be adopted as a fixed point from which to investigate its environment. Here only the *local* errors are concerned, and we get *K*, *b*, and *e*.

In the first case, the error of *A* is about 2°; in the second, $B - b$ is so small, that a much larger scale would be required to show it. The differences are simply the errors of *K*. The stations are Caturactonium, Albanus, and Trimontium.

II. West Coast of Scotland.

The purpose of the following pages is not controversial; criticism has been avoided. They are simply the record of my own investigations, which have been throughout a labour of love, and an honest search for truth.

Before finally dismissing the subject of Scotland, however, it will be instructive to see how this matter has been treated in the latest edition of the "Geographia" with which I am acquainted (Paris, 1883), especially as this inquiry has, for general purposes, its own peculiar interest.

The following is found on page 82 of that volume:—Novantum. "Est *Corsill Point* sive boreale promontorium Chersonesi, cujus isthmum efficiunt *Loch Ryan* (Rerigonius sinus Ptol.) et *Luce Bay*. Latitudo Britanniæ a Damnonio promontorio (12° 0′, 51° 30′) sive a *Lizard Point* usque ad Novantorum promontorium pertinens, secundum Marciani Codicem est stadiorum 3083, qui numerus etsi ad veram locorum distantiam proxime accedit, tamen corruptus est, mutandusque in 5083 tot enim stadia efficiunt 10° 10′ qui sec. Ptolemei tabulam inter promontoria ista intercedunt." Here the note ends without any attempt to reconcile its statements.

If 3083 stadia be very nearly true for *Corsill Point*—and the map of Ptolemy requires that this interval must be increased 2000 stadia, = 4° latitude, to reach his Novantum—it seems very obvious that *Corsill Point* cannot be Novantum. This difference is, indeed, the crucial question. Is the Novantum promontory of Ptolemy in Wigtown or in Skye? Plates XVI. and XIX. decided for Skye, but the above note includes new material; the arc is extended to the Lizard, the largest attainable, and it thus includes 9° 50′. Further, from Plate XIII. it will be seen that the Lizard has an abnormal error of "about 1° south." This will affect the results, and supply a new test of the discrimination of errors

The facts of the case are as follows:—

True latitudes, 57° 42′, 49° 57′; interval, 7° 45′.
Ptolemy gives, 61° 20′, 51° 30′; interval, 9° 50′.

From which 9° 50′ − $\frac{6}{5}$ 7° 45′ = 0° 32′, by which amount Ptolemy's interval is too large.

The particulars of the two stations were then calculated in the usual way:—

Latitudes.

—	Point of Aird.	Lizard Point.	—
L.	57° 42′	49° 57′	$L - L' = 7° 45'$.
A.	31° 10′	31° 10′	
$L - A$.	26° 32′	18° 47′	
$\frac{6}{5}(L - A)$.	31° 50′	22° 32′	$\frac{6}{5}(L - A) = 9° 18'$.
S.	5° 18′	3° 45′	
$L - 10'$.	57° 32′	49° 47′	
λ.	62° 50′	53° 32′	$\lambda - \lambda' = 9° 18'$.
π.	61° 20′	51° 30′	$\pi - \pi' = 9° 50'$.
ϵ.	− 1° 30′	− 2° 2′	$\epsilon - \epsilon' = - 0° 32'$.
S.	5° 18′	3° 45′	
η.	+ 3° 48′	+ 1° 43′	$\eta - \eta' = 2° 5'$.

As compared with *Land's End*, *Lizard* has $\epsilon = + 0° 52'$.
Land's End has $L = 50° 3'$, $S = 3° 47'$, $\lambda = 53° 40'$, $\pi = 52° 30'$, $\epsilon = 1° 10'$, $\eta = 2° 37'$.

Hence the true errors are found to be:—

Point of Aird, − 1° 30′; Lizard Point, − 2° 2′.

Here − 1° 30′ − 2° 2′ = + 0° 32′, as before.

This large error is not due to displacement of Novantum promontory, which differs only 0° 5′ from Londinium, and, when compared with St. Bee's Head and 61° 20′, showed only about one-half that amount of error.

But at Land's End we find $\epsilon = -1° 10'$; hence the errors are distributed thus :—

Interval from Point of Aird to Land's End = – 0° 20′.
Interval from Land's End to Lizard = + 0° 52′.

Leaving + 0° 32′, as before.

Finally, the errors of Land's End + Lizard Point = 1° 10′ + 0° 52′ = 2° 2′, which is the whole correction required at the latter station, while Novantum promontory, as Point of Aird, has only the normal error when calculated from Alexandria. The longitude is also correct (Plate XIX.). Corsill Point is not a station of Ptolemy, and, therefore, could not be introduced.

As the result, Novantum promontory *is* the Point of Aird.

III. The Frontispiece.

This map should be compared with Plate XVI., which was an attempt to produce *Ptolemy's Scotland corrected* by means of his own data. The main error seems to have been that the northern half of Scotland is made vertical; but the general distortion-angle was not then known. The line of true longitude, 3° 1′ W., was an after-addition.

In the explanation of the Plate referred to, instructions are given by which a more correct comparison may be made, the line *A C* of the frontispiece being the datum. If this line is made vertical, the modern Scotland will be inclined as it should have been in Ptolemy's Map. Ptolemy's scale is increased to $\frac{6}{5}$ to make the comparison more exact. The two maps are on the same scale.

The following notes seem desirable :—

1. In this northern half of the map the distortion-angle is consistently larger than the mean. *A C* is 46° 45′ from the true meridian.

2. Epidium promontory and Cape Wrath are both found in longitude 23°.

3. The easterly extension is greater here than in the 1482 map; but in this case "even" Nicolaus de Donis "*improved*" his master, and had to be corrected.

4. The reduction of the latitude by gnomon makes the distortion almost due East. The more southerly bearing of Tazalorum promontory has been explained by its changed error, &c.
5. Judged by their work generally, the Romans, whether marching or coasting, could have made no such error as is now shown. Practically, the whole area is outside the limits of the true coast, and it is larger than the whole of the true Scotland upon which they worked.

The "meteoroscopic" solution is found to be simple and consistent throughout; but a few lines may well be spared to trace the history of the popular opinion.

George (Mythos und Sage) defines a myth as "the creation of a *fact* out of an *idea*." If this be so, the notion that Ptolemy turned Scotland over 90° seems to be simply the addition of another myth to those which still retain vitality in our modern faiths.

Differences of opinion would arise as a truer outline of Scotland became known. There are traces of this in the earlier editions, where modern names are inserted in the list, but these need not detain us.

The *idea*, in a definite form, may be dated from Mercator. His attempt was to correct Ptolemy by the light of his own map-making knowledge, and he worked with a very free hand. His notes respecting Scotland will be found in his first edition of the maps (1578).

Bertius, in the *Theatrum* (1618), reprints these notes together with the map.

Horsley used Bertius as his authority.

Thus the pedigree is short and simple, and the *idea*, indorsed with the names of Mercator, Bertius, and Horsley, has since freely circulated as a *fact*.

The primitive and radical question, Is it turned? never suggested itself so far as can be ascertained. Horsley's map has been more or less exactly copied in every recent atlas that has come within my reach.

Under such circumstances all further work becomes apologetic—a confirmation or an explanation of the "*fact*."

The following will be sufficient:—

Bishop Nicolson (1702) admits that this general error of Ptolemy is very surprising and truly unaccountable, especially considering that his account of Scotland is reckoned in other respects just and true.

Professor M'Laurin conjectures that, as the Roman soldiers entered Scotland on the west side, and afterwards crossed to the east, they might mistake the length for the breadth.

Baron *Clerk* (1726) dismisses the case summarily with:—"I have always considered him (Ptolemy) amongst the most uncorrect of all ancient authors."

Let these suffice to show what may be called the *outside* opinion before the publication of Horsley's map. How far they include the turning is not quite evident.

All that remains is to state the short and simple process by which *it seems to be* became *it is*.

Mercator calls the most northerly of Ptolemy's Ebudæ (Hebrides) Maleos, and then simply says that it is now Mull! (Maleos insula nunc Mula). On this flimsy foundation—true only to his own mode—he carries up Mull about 4° of latitude, and, with a little accommodation, puts Corsill Point at the Point of Aird.

By this process Scotland is turned effectually.

Horsley followed his leader; but, as he had an authority to appeal to, he becomes somewhat more dogmatic, and, in addition, he gives us the map which he calls "A corrected map of Britain *according to Ptolemy*," which he claims as his own.

This rendered the whole thing permanent and palpable, and this it is which has been the foundation of our maps since.

In the very dawn of this investigation I wanted a reliable station north of Lancashire, and was led to the south-west of Scotland. Being then ignorant of what has just been written, and working with the extract from Bertius alone, I was naturally brought face to face with what has been called the primary and radical question, though without the idea of turning at all. The inquiry was therefore quite independent, and the result written on my first diagram is given in connexion with Plate XVI. Now the work is done, I am strongly tempted—and not without confidence—to repeat the question of Macduff—

"Stands Scotland where it did?"

In conclusion, I have to express my obligations to those who have helped to bring this opusculc to the birth. To my friend Mr. Scott I am largely indebted for the ability with which he mastered the text of the Geography, for the concise form in which he has arranged my scattered notes and papers, and for the way in which he has transmuted the memoranda on my diagrams into the Explanations of the Plates. Such work is always trying and troublesome, but Mr. Scott has been throughout zealous and persevering, and, so far as I have made out, what I wanted to say in the following pages is said.

To my son, Mr. W. Harry Rylands, F.S.A., I owe the production of the Plates. Labour and care have been given without stint, and the results, so far as I can see, are without blemish.

William Owen, Esq., F.R.I.B.A., has been "a friend indeed." When, at the last moment, I realized how desirable it was that the Frontispiece should form one of the series of Plates, an imbecile thumb disabled me. I could rule a line and dot in a station, but that was all. Mr. Owen very kindly gave me the use of one of his "hands," who completed the map. It is all that I could desire, and is even more instructive than I anticipated; it verifies several previous conclusions.

To the Printers and Lithographers my thanks are especially due for the way in which they have done their work. They are too well known to need praise from me.

Fair and honest criticism I enjoy, but anyone who has done me the kindness to read this Preface will understand me when I say, that if I had a work to do, I now consider it done.

T. G. R.

Highfields, Thelwall, near Warrington,
January 14th, 1893.

EDITOR'S PREFACE.

IN the preceding pages Mr. Rylands has recounted the growth of his intimacy with Ptolemy until our meeting, about two years ago, and the remaining events can best be told by the person upon whom the arrangements, culminating in publication, have devolved. At the time of our first interview I was powerfully impressed by the quality of the work that had lain dormant for so many years, and that was in imminent danger of being lost to the world—especially to English Antiquaries and Students of Ptolemy. Though completely ignorant of the text of the "Geographia," and, from my other studies, little inclined to make its acquaintance, I nevertheless promised to become the mouthpiece of Mr. Rylands' opinions.

At first I was overwhelmed by the range of the subject, and the immense number of data to be fused together into a readable whole. However, under kindly and patient guidance, and at times, perhaps, a little infected by the author's enthusiasm, I made a careful study of the materials submitted me, with copious mental notes upon the verbal commentaries with which they were accompanied.

These materials were of two kinds, a bulky pile of manuscript, consisting of many hundreds of pages, which formed the learned outline of a former editor's work. The learning was in its minuteness beyond my capacity, and altogether beyond the scope of my instructions, while the matter, though so large, had not yet reached Ptolemy! Besides many faults in arrangement, it was not difficult to see that, if the superstructure was to be commensurate with the foundation, it would grow to a veritable tower of Babel. Therefore, while using such portions as seemed necessary, I built the first chapter upon a simpler and much less ambitious plan, which I believe is more in accordance both with the aim of the author and the scope of the work. The second chapter is my own, under the guiding impulse of the criticism of Ptolemy advocated by Mr. Rylands throughout.

The materials for the remainder of the book were of a different order. As the study grew, Mr. Rylands was in the habit of drawing a diagram, and jotting down the results of minute and repeated study upon it. He showed me a carefully preserved portfolio of these, and it became necessary to choose which should be selected for reproduction. After a considerable amount of hesitation, nineteen were at first separated from the rest, and from these we decided to build the theory for the elucidation of Ptolemy's "Geography." The question of treatment was the great point of difficulty, as the intricacy and scholarship of the calculations disqualified them for a place in the body of the book. Moreover, as representing a chronological sequence, the question of classification was a matter of no little complexity. If, on the one hand, the diagrams were arranged as the various steps of rectification emerged, those who followed the argument must toil through the same difficulties that beset the whole original criticism. On the other hand, the tentative nature of the earlier diagrams made the ordinary methods of explanation hazardous in the extreme. The latter course was finally adopted, and the body of the work was completed by a popular summary of the main results of the diagrams, with a reference to the fuller explanations facing the Plates, to facilitate the complete solution of any question that may interest the reader. Afterwards I attacked the notes to the diagrams themselves, first arranging the various calculations as clearly as I could, sometimes omitting considerable quantities of superfluous matter, and sometimes developing a pencilled remark into another artery to the proof.

During the lengthy period the proofs have been sauntering through the press, various additions have been made. The interesting outlines of Plate X. were prepared for publication; also Plate XXI., with the Explanation as an Appendix extrinsic to the object of the work, but of considerable critical interest, besides representing a small portion of the careful work which characterized the diagrams as a whole.

More recently, in reading the proofs relating to Scotland, Mr. Rylands experienced a relapse of what he will pardon my calling the old Ptolemaic fever, and he again attacked the distortion of the northern portion of this island. The first result appears in copious additions incorporated in the Explanations. When the blank spaces there were exhausted, room was found for a Supplement, and during a further delay, necessitated by renewed revision, time was fortunately allowed for the composition of the happy criticisms embodied in the preceding Preface.

The full account of the growth of so many different elements before their union in the present volume will render clear the relations of Author and Editor in its production. I need scarcely any further disclaim part or lot in the rectifications involved, than by the mere statement that, when Mr. Rylands wrote "*finit feliciter*" below his last important diagram, his present editor was painfully learning how to hold a pen! My part, in truth, is far different. It is the sole merit of bringing to a birth thoughts that laboured for expression, the understanding of a peculiar method of inquiry and verification, a selection and consistent statement of principles that were clearly outlined in the author's mind, and only needed definite connexion to become tangible to the reader.

My functions as ambassador end with the writing of these pages, and I sincerely trust the results may be serviceable to others, as their acquisition has been advantageous to myself.

In Mr. Rylands I have invariably found a large-hearted patience in dealing with the irreverent treatment of his idol by an ignorant neophyte. From his work I trust I have learned habits of industry and accuracy, that are indispensible to the study that claims my more serious thoughts, and my deepest aspirations.

W. R. SCOTT.

19, Trinity College, Dublin,
January 16th, 1893.

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PTOLEMY ELUCIDATED.

CHAPTER I.

A BRIEF OUTLINE OF THE RISE AND PROGRESS OF GEOGRAPHICAL INQUIRY PRIOR TO THE TIME OF PTOLEMY.

§ 1.—Introductory.

In tracing the early progress of Geography it is necessary to remember that, like all other sciences, it arose from "small beginnings." When men began to move from place to place they naturally desired to tell the tale of their wanderings, both for their own satisfaction and for the information of others. Such accounts have now perished, but their results remain in the earliest records extant; hence, it is impossible to begin an investigation into the history of Geography at the true fountain-head, though it is in some instances possible to guess the nature of the source from the character of the resultant stream.

A further difficulty lies in the question as to where the science of Geography begins. A Greek would probably have answered when some discoverer first invented a method for mapping out the distance between certain places, which distance conversely could be ascertained directly from the map. In accordance with modern method, the science of Geography might be said to begin when the subject ceased to be dealt with mythically or dogmatically; when facts were collected and reduced to laws, while these laws again, or their prior facts, were connected with other laws—in the case of Geography with those of Mathematics and Astronomy. Perhaps it would be better to follow the latter principle of division, and consequently the history of Geography may be divided into two main stages—"Pre-Scientific" and Scientific Geography—though strictly speaking, the name of Geography should be applied to the latter alone. The first of these extends from the earliest period, and running through the poets, terminates about the time when the youthful science passed from the schools to men like Pytheas, who combined the accuracy of the astronomer with the intrepidity of a daring navigator, and from this time it grows by successive steps till we reach the full knowledge of Ptolemy.

Another point that may be worthy of note is one of general application. As has been said, the mind advances in "serpentine lines," and therefore every advance in time is not always an advance towards the truth; on the contrary, some thinkers seem, with singular perversity, to delight in reviving and defending opinions long routed by the victories of truth. One instance of this attitude of mind may be mentioned. The school of Philosophy, known as the Epicurean, wilfully denied the spherical form of the earth several centuries after it had become the common property of the Greek world. In a full history such retrogressions would have place, not so much from their intrinsic importance as from the influence they exercised over the minds of those who were brought face to face with them, and who were thus forced to reconsider carefully the main pillars upon which their scientific structure was based; but in the present Introductory Chapter, the aim of which is to show the progress of Geography in outline up to a certain time, these backward steps must be necessarily omitted.

§ 2.—Pre-Scientific Geography or Cosmography.

It is probable that Geography, like many other sciences, had its birth in the far East. Besides the analogy several other facts seem to point to the same conclusion, such as the ratio of a cubit to a mean degree of a great circle; while Greek and Roman writers explicitly affirm that a measurement of the earth was made by the Chaldeans.[1] Be this as it may, it is at least moderately certain that if there were any such geographical knowledge it was only known to the Greeks by tradition, and was rather assimilated unconsciously than classed as the basis of a science. In fact, if we can believe the testimony of Herodotus,[2] the astronomical theorems which the Greeks applied to the measurement of the earth were derived from the Babylonians, and yet so far were they from being aware of this indebtedness that they assigned a mythical origin to their measures, which were due to the same source; for instance, their foot was supposed to be the foot of Hercules. Therefore, when we come to the first written records—those of the Greeks—we find that whatever knowledge there may have been of the earth had already become inextricably involved with the myths of the popular religion, and as a

[1] See Achilles Tatius, "Isagoge ad Arati Phænomena," ch. 18. ("Petavii Uranologia," p. 137.)
[2] Herod, II., ch. 109.

result, the early information of the Greeks, and therefore the earliest information obtainable, must be treated as mythical from which true knowledge of the earth gradually grew.

This earliest discoverable information is found in the Greek poets where the knowledge of the time is clothed under the form of the wanderings of certain popular heroes—for instance, the wanderings of Ulysses. Thus the natural errors in the accounts of the early travellers were enormously multiplied by the vehicle by which they are known to us; for who would bind an epic down to mere stadia? Further, when men began really to investigate the different positions of different countries, and desired to ascertain the exact truth, they found it impossible to free themselves from mythic errors which they had assimilated almost with the air they breathed. Thus the early accounts of the world (or Cosmography) can all be distinguished by the presence of certain poetic fictions which are accepted as the real basis of knowledge. One instance will be sufficient to illustrate the cosmographical tendency, namely, the theory or fiction of a river Oceanus which flowed round the world, and which was the source of all smaller rivers. Accordingly, Æschylus makes Prometheus trace out the wanderings of Io to the Indus, following which she is to reach the delta of the Nile. Euthymenes, a voyager of the same age, sailing through the Pillars of Hercules and down the coast of Africa, found the *Nile* flowing out of the Outer Sea, giving as his reason for believing the ocean to be the source of the Nile that the water was sweet, and the animals were the same as those of the Nile. A somewhat similar instance of the influence of cosmographical ideas will be found in Herodotus' tale of the five adventurous youths (Book II. 31–34).

In fact, the influence of this myth of an all-embracing ocean may be traced through all antiquity from Homer, Hesiod, and all the poets, through Herodotus, Plato, Aristotle, and the various schools of Philosophy. Even the mathematical astronomers, scientific travellers, and geographers have not altogether shaken it off, as we find Pytheas censured for his freedom from it by the historians of Geography, Polybius and Strabo. Even in the midst of the geographical knowledge under the empire the mythical theories of cosmography manifest themselves not only among the poets such as Ovid, Virgil, and Lucan, but also with the philosophers, Seneca, Pliny, and Plutarch. They hold their ground under the declining Empire, and run through the middle ages—nay, they revive with the revival of learning—witness amid a vast number Munster's Cosmographia and the Nuremberg Chronicle.

§ 3.—The Germ of true Geography.

The first step towards scientific Geography was made when the information concerning the earth passed from the hands of the poets to men of general culture. Now, at this period, the great centres of culture were the schools of Philosophy, and so the great majority of contributors to the earliest Geography (as distinct from Cosmography) belonged to some of the first philosophic schools. To the Ionic sect Geography stands in special debt. Thales is named as the first to suggest the spherical figure of the earth, which he may have learnt from the Oriental astronomy with which we know he had been brought in contact. To his younger contemporary, Anaximander, we owe the first geographical tablet, or picture, or map. Anaximander's map acquired great celebrity, probably from the originality of the idea. Herodotus says (v. 49) that when the Ionians were planning their revolt from the Persian yoke, Aristagoras, the tyrant of Miletus, went to Sparta to ask assistance from Cleomenes; during the conference he produced a bronze plate on which was engraved a map of the whole world. This may have been either the original map or a copy.

Of the nature of the map itself we know little; but it must have been very crude, as there were no sufficient means of determining positions by observation or exact measurement, nor was there any definite datum line, much less anything corresponding to longitudes and latitudes to which to refer them. Greece was placed in the centre of the world, the lands round the Mediterranean were vaguely added, while distances were reckoned from travels and voyages, and the directions were guessed from the position of the sun and course of the winds.

Shortly after Anaximander's map appeared the treatise of Hecatæus, concerning which the most noticeable feature is the fact that it appeared *after* the first map—thereby exemplifying the general course of Greek Geography.[1]

Socrates appears to have had a map of the earth amongst the diagrams of his school,[2] while in the time of Plato and Aristotle maps had become an essential part of educational appliances, as may be seen from the will of Theophrastus.[3]

Plato and Aristotle began to apply general laws to the study of geographical phenomena. Their names are associated in Geography with the maintenance of the spherical figure of the earth. They do not however teach it as a new discovery, but rather as an accepted fact—indeed it began to be held some two centuries before their time. Aristotle held also the theory of celestial gravitation. He

[1] For information concerning Hecatæus, see Bunbury, ch. v., vol. i., p. 134, and fol.

[2] Aristophanes, Nubes, l. 206.

[3] Diog. Laert. v. 51. Eudoxus himself drew maps to illustrate his views.

supports his view by two main arguments :—When matter gravitates to a centre it must assume the spherical form, and all bodies on the earth gravitate to its centre, therefore it is a sphere. Secondly, the fact that the shadow of the earth upon the moon is always circular can only be explained by believing the earth to be itself a sphere. This sphere is at rest in the centre of the Universe, and round it the celestial bodies revolve. In size he made the earth about double its true diameter, but in reference to the enormous distance of the fixed stars it is "as nothing." After dividing the earth into zones the conception of Antipodes was reached—and was easily accepted if not explained—by reducing it under the already accepted fact of gravitation.

Archytas of Tarentum, the Pythagorean, a contemporary of Plato, and about a generation prior to Aristotle, is said to have had no small influence upon the geography of both. He was celebrated as a mathematician, and still more as the inventor of mechanical instruments. Horace (Odes, I., 28) describes him as the "measurer of sea and earth and the countless sands." It may seem absurd to collect his influence as a geographer from the verses of Horace were it not that the description corresponds almost literally with that of Aristotle, and the reference to the measuring of the countless sands seems to suggest the problem afterwards attempted by Archimedes of estimating the solid contents of the earth's sphere by the number of grains of sand which it would contain.

§ 4.—Pytheas.

(a) *His position and date.*—In Pytheas we meet Geography first treated as a special science and not as a branch of general education. He devoted alike the knowledge of an original mathematician and the bravery of a hardy traveller to the advancement of his chosen study. It is probably to this innovation that we may attribute the scorn with which his discoveries were greeted by the Greek historians of Geography. Besides his personal characteristics, the place of his birth pre-eminently fitted him to be the pioneer both of geographical discovery and its scientific systematization. Born in the flourishing colony of Massalia, which had defeated the fleets of Carthage, subdued the savage Celtic population, and formed alliances with Rome, he was placed in the centre of a circle of active commercial enterprise. How far the first extensive voyages of the Massalians were due to a desire to extend an already existing tin trade or else to compete with that already established by the Carthaginians it would be difficult to determine. Further, Alexander's expedition in the East would stimulate the Greeks in the West to efforts in maritime discovery. It has also been suggested that the second commercial treaty between Rome and Carthage would make the inhabitants of Massalia desire to strengthen the position of their city by the foundation of sister towns.

It was at this time that Pytheas lived. Unfortunately, his exact date is uncertain. It is probable that he lived between Eudoxus and Dicæarchus, as Hipparchus corrects the former from data supplied by Pytheas,[1] while the latter criticizes him unfavourably.[2] Moreover, his contributions to Astronomy and Geography would have been noticed by Aristotle had they been made before his time. Therefore, on the whole, the date of Pytheas may be fixed in the last quarter of the fourth century B.C., while his voyage may be associated with the date of the death of Aristotle, B.C. 322.

(b) *His astronomical and mathematical knowledge.*—The practical knowledge which Pytheas possessed of both these subjects enabled him to be of the greatest possible service in developing true Geography. Even his arch-accuser, Strabo, bears witness to his celebrity in this respect.[3] His observation respecting the stars round the pole, which Hipparchus adopted, has been confirmed by modern astronomy—the three stars mentioned being fixed as β Ursae Minoris and α and κ Draconis.[4] But there is another instance of his geographical and astronomical accuracy. He was the originator of the first observation by means of the gnomon on record, by means of which he fixed the position of his native city with the least possible amount of error.[5]

(c) *His Voyage.*—Besides observation and the comparison of records, Pytheas himself undertook an extensive voyage, which even to the present day forms the subject of much controversy. The reason is that none of his writings have been preserved, and such extracts as reach us or such accounts of his work as have been perpetuated come to us only after having passed through the distorting media of several other minds which were not always unprejudiced. Strabo, supported by Polybius, was especially hostile, since he held views which were contradicted by the reports of Pytheas, whom he considered led Eratosthenes astray.

The result of this voyage was the discovery of Britain to the Greeks, but regarding the details there is considerable doubt. Strabo traces Pytheas as the authority of Eratosthenes, whom he is criticizing,[6] from the Pillars of Hercules round the coast of Iberia and Gaul as far as the Promontory of the Osismii (Brittany)

[1] Hipparchus Comm. in Arati Phænom. i. 5. [2] Strabo, recensuit G. Kramer Berolini, 1844, ii.

[3] Strabo, vii., ch. 3, par. 295; ii. (ch. 4, § 1), par. 104.

[4] Delambre, "Astronomie Ancienne," vol. i. p. 110. Lelewel (Pytheas, p. 48) gives the first-named star as α, Ursae Minoris; also Professor Foster of the Berlin University.

[5] Mentioned three times by Strabo, pars. 63, 71, 115. [6] Strabo, i., pars. 63, 64.

and the Island of Uxisama (Ushant). After "some days' sail" from Celtica he reached Cantium, giving the length of the whole island of Britain as 20,000 stadia. The difficulty affecting Pytheas in this account lies in the expression "some days' sail." For how could the distance across the Straits of Dover occupy "some days"? The explanation lies in a fact which should be borne in mind in considering the similar difficulty in the case of Thule. The coast of Britain was quite new to Pytheas, and so it was but natural he should coast along it, and the time of the voyage might be spoken of (especially after filtering through several hands) as occupying some days!

From Cantium[1] he sailed up the east coast till he reached Thule, where "there was no longer either solid land nor sea nor air, but a sort of mixture of these . . . which could neither be travelled over nor sailed through. Returning he visited the whole ocean coast of Europe, from Gades to the Tanais." The perimeter of the island he gives as 40,000 stadia.[2]

As a possible confirmation of the voyage, the account given by Diodorus Siculus, of Britain, may be mentioned, which may have been in the main taken from Timæus, who again is connected with Pytheas.[3] Here, the form of the island is given as triangular, with three headlands, representing the three corners named respectively, Bolerium (Land's End), Cantium, and Orcas (Duncansby Head) "Of these sides the least is 7500 stadia long extending beside Europe, the second from the strait to the vertex is 15,000 stadia long, the remaining side 20,000, so that the whole circuit of the island is 42,500 stadia." Here, the near agreement of one of the sides with Strabo's account of the length of the island given by Pytheas, and also the similarity of the totals, are worthy of note.

(d) *The Thule of Pytheas.*—The last point of interest in connexion with Pytheas is his Thule, not only in vindication of his consistency, but still more from an independent ground of interest, because the Thule of Pytheas was the most northern point in all later maps, with the exception of Strabo's. The latter says[4]: "It is true that Pytheas the Massalian says that the furthest parts (of the habitable world) are those about Thule, the northernmost of the British Isles, at which

[1] Strabo, ii. 4, par. 104.

[2] These dimensions of Pytheas are so excessive that they *cannot* have been the results of his actual work; but the discordance may be explained so as to reconcile the discrepancy. In transmission the *day's sails* have been reckoned as *day-and-night's sails*, and these have been given in stadia. We know from Solinus that the technical value of a day's sail was changed, and so far as Thule is concerned the distance is reduced to one-third, and the same is equally true of the length of Britain. Further, Lelewel makes the distance from Cantium to Orcas 4821 stadia, which is not quite one-fourth of the number attributed to Pytheas.

[3] Diod. Siculus, v. 21, 22.

[4] Strabo, par. 114.

parts the summer tropical circle is the same as the Arctic (Circle). But from the other writers[1] I find no information, either that there is such an island as Thule nor whether the parts (of the world) are habitable up to the point where the summer tropic becomes the Arctic Circle. But I think the northern limit of the inhabited world is much further to the south."

Strabo also quotes a statement from Pytheas locating Thule "at six days' sail from Britain." Remembering the previous uncertainty connected with distance reckoned by "days' sail," this latter passage cannot be accepted as decisive, while in the former there are two statements which at least partially contradict each other. Thule is "the northernmost of the British Isles," and lies upon the Arctic Circle. But there is no island which was accessible to Pytheas that would answer this description; so the difficulty is which to accept? Certainly the acceptation of the latter view would be a serious blow to the accuracy of Pytheas as a geographer, and still more were we to take Pliny's explanation—that at Thule there is six months' day and six months' night.

But Pytheas was better informed. To take first the account of Pliny, one edition reads in the margin, "some copies have six days," and an Anglo-Saxon Manual of Astronomy actually reads six days—"Thule is the name of an island to the north of Britain, six days' voyage by sea, in which there is no night for six days at the summer solstice."[2]

But we can get even nearer the truth: it happens that the exact words of Pytheas have been recorded. "The barbarians showed us where the sun goes to his rest, for it happened about these parts that the night was only a little interval after the setting of the sun before it rose again."[3] The description Pytheas has given of the harvest, seen probably as he returned, would seem to tally with the time of the summer solstice, and therefore, upon the whole evidence, his Thule may be fixed as one of the Shetland Islands. Ptolemy, with much fuller evidence before him, and Marinus, place Thule in the position of the Shetlands.

§ 5.—Dicæarchus.

Dicæarchus was a native of Messana in Sicily, and survived in the year 296 B. C. He wrote a work entitled Γῆς περίοδος. He was one of the later pupils of Aristotle. His geographical work marks a new period in the history of

[1] Reading ἱστορῶ instead of ἱστορῶν or ἱστορεῖ, compare the old Latin version of Xylander, who reads (from a MS. now lost) "ab aliis nihil comperio."

[2] Ang.-Sax. Manual Astron., Ed. T. Wright; date about the 10th century.

[3] Gesenius, Elem. Astron. v. 22.

Geography—on the one side he is connected with the philosophic, while the specialising tendency of the teaching of Aristotle led him to devote his attention more especially to the study of the earth. By his discovery of a *base line* he may be named the father of ancient geodesy. From this basal line he measured the length of the habitable world, and to it he referred all positions. The discovery lay in the application to scientific purposes an already existing provision of nature. The known world was divided by the long and comparatively narrow basin of the Mediterranean, lying west and east between shores, roughly speaking, parallel. From its eastern point—the Gulf of Issus—the division was continued by the mountain chains which prolonged that of Taurus. This irregular *natural* line Dicæarchus reduced to a *mathematical* line, thereby discovering the longitudinal measurement of the habitable world, which may be named as the first step towards scientific Geography.

Though there is no direct evidence that Dicæarchus used his base line in remodelling the map of the world, there seems to be no other reason to account for his discovery, while it is known that he did actually construct maps of Greece. Therefore it may not be too much to conclude that he gave its final form to the "Ancient map" before it passed into the hands of Eratosthenes in the following century.

§ 6.—Eratosthenes.

(a) *His Life.*—Eratosthenes is closely connected with Dicæarchus; he supplemented, most materially, the "base line" of the latter, while further, we owe to him the first recorded scientific measurement of the circumference of the earth. Born in the year 276 B. C., he received his education at Athens, and was invited by Ptolemy III. (Euergetes) to preside over the great library of Alexandria. Here he was able still further to increase his remarkable acquaintance with the whole mass of literature extant, besides his observatory enabled him to pursue his researches in astronomy. He died at the age of eighty.

(b) *Measurement of the Earth.*—Before the time of Eratosthenes measurements of the earth had been hazarded by several of his predecessors. There is one due to Aristotle of 400,000 stadia, and another to Archimedes of 300,000 stadia; another, practically correct, has been attributed to Pytheas by calculation (by Lelewel), but without positive authority. The round numbers in which the measures already named have been expressed, show that they were the result of guesswork rather than any scientific process.

Eratosthenes starts from strictly astronomical data. The distance of the northern tropic from the Equator, determined by the vertical reflection of the sun in a well at Syene at the solstice, was the shrewd and simple method of this determination. He reckoned the distance from Syene to Alexandria as 5000 stadia, from which he estimated the circumference of the earth as 250,000 stadia; to this number he afterwards added 2000 stadia, probably to make it a multiple of the $\frac{1}{60}$th part of a great circle which he used instead of the modern division into 360ths or degrees. Thus the world of Eratosthenes was $\frac{1}{6}$th larger than the true earth.

(c) *His Map.*—It is easy to see how much this discovery enabled him to add to the bare base line of Dicæarchus. He was at once able to establish a fundamental meridian at right angles to the parallel which he owed to his predecessor. These two circles cut each other in the Island of Rhodes, which was considered due north of Alexandria. From this time forward the parallel of Rhodes occupied the same position in ancient Geography as the equator does in ours; in fact, the terrestial Equator was an unknown quantity, only to be determined from other known parallels by calculation. Eratosthenes proceeded to supplement the two measuring lines of his world by other parallels or meridians drawn through places whose position was supposed to be known. What we now call Latitude was recorded in *Climates*, of which there were originally seven, determined by the length of the longest and shortest days. The original number of seven was gradually increased as more exact knowledge was obtained. This method was constantly used up to the time of Ptolemy.

So much for the theory of the map—the map itself must have been very imperfect—in fact, Eratosthenes merely grasped the principle of a mathematical basis for Geography without attempting to carry it out in detail, as he did not believe that Geography was susceptible of any great degree of accuracy. For instance, though well knowing there was a difference of 400 stadia between the parallels of Rhodes and Athens, yet he drew his main parallel through both. Not only so, but he sometimes, when it suited his purpose, substituted for this dividing line a dividing belt of 3000 stadia in width! which shows how little he had grasped the true scope of his great discovery.

In connexion with Eratosthenes, it is perhaps worth while noticing his speculation concerning the possibility of circumnavigating the globe, where he comes curiously close to the actual distance to be traversed; but as being a mere speculation, it has less place in the history of Geography than the tales of M. Jules Verne in modern science.

§ 7.—Hipparchus.

(a) *His Position and Date.*—Hipparchus was one of the pioneers of discovery, without being himself a geographical discoverer. In his astronomical investigations he had solved the whole problem of Geography, but his solution was implicit, not explicit; consequently, it needed one of equal, if not greater genius, to grasp the true fertility of his principles. Therefore, it is not so astonishing as it might appear at first sight, that for three hundred years the discoveries of Hipparchus[1] lay hidden till Ptolemy had the genius to rediscover them after Strabo, Marinus, and others had passed them by. This being so, the genial and open manner in which Ptolemy acknowledges the long-forgotten claims of Hipparchus, gives us a transient glimpse of the frankness of his character, especially as he could so easily have appropriated the work of his predecessor without fear of detection—and yet we know but little of Hipparchus except through Ptolemy.

(b) *The surface of the Globe mathematically divided.*—Hipparchus as a mathematician at once made an important advance upon Eratosthenes' division of circles into sixtieths by substituting three hundred and sixtieths. He[2] then took a meridian so divided, and imagined circles drawn through each of these divisions parallel to the Equator, which thus correspond to our parallels of latitude. Upon these parallel circles he took stations from which might be determined the celestial phenomena dependent upon the latitude. Hipparchus desired to calculate the variations of these phenomena for all positions or stations at intervals of one degree—that is at intervals of 700 stadia, as Hipparchus accepted Eratosthenes' measurement of the earth which would be sufficiently accurate for his purpose. The use of the word calculate gives the essential point of distinction, as this work was purely astronomical and theoretical. If any proof of this were wanting beyond that which will be found in the mere nature of the case, it will be quite sufficient to mention that his system embraced the whole quadrant from the Equator to the Pole, much of which was outside the range of Geography.

With regard to longitudes Hipparchus had the true conception of their connexion with differences of times, as is shown by his proposal to determine them by means of eclipses—of which method Ptolemy proves the theoretical soundness, and at the same time shows the practical uncertainty in that age.[3]

[1] Hipparchus was alive 150 B.C., Ptolemy 150 A.D. Suidas places him from B.C. 160 to 145, without mentioning the dates of his birth or death. From Ptolemy's "Almagest" (v. p. 299) we learn that Hipparchus made an observation in the 197th year after the death of Alexander, that is in 126 B.C.

[2] See Strabo (ed. Kramer), Bk. ii., par. 131–2.

[3] Indeed the same remark applies to even late mediæval estimates. See the Supplement to Explanation of Plate XV., p. 77.

(c) *The application of these principles by Hipparchus.*—The object of Hipparchus being altogether astronomical, it is but natural to expect that his realization of these principles would be very imperfect. As Ptolemy says (Geogr. i. iv. § 1), "Hipparchus alone (among preceding writers) handed down to us the north polar elevation for *a few* cities among the great number laid down on the map, and that too of places lying on the same parallel." Further, how far theory prevailed with him over practice is shown by his remarkable error in the latitude of Byzantium. But most important of all he was not in the strict sense of the word a geographer, as Strabo plainly says, *οὐ γεωγράφουντι*. He did not make a map, neither original, nor did he rectify the ancient map, but rather endeavoured to apply that ancient map to his own astronomical purpose, and in so doing was led to criticize it as left by Eratosthenes.[1] His real service to geographical science was to suggest the idea of a scientific framework, at least with respect to the system of latitudes, which was carried out in practice by Ptolemy, but which formed no part of the work of Hipparchus to himself.

§ 8.—Recapitulatory.

Geography has now been traced from its origin with the poets through the philosophic schools till passing thence it became a science. The various steps in scientific Geography are sufficiently apparent; the travels of Pytheas astronomically verified: the base line or prime parallel of Dicæarchus, supplemented by the fundamental and other meridians of Eratosthenes: which again gave way to the more accurate determination of the length and breadth of the inhabited earth by Hipparchus, are all stages which pave the way for the man who could understand them, and accurately fit in stations consistently with the theory. Therefore, Ptolemy is the true follower of Hipparchus, and the names which are scattered through the intervening three centuries mark retreat rather than advance. However, a brief notice of these others may be added partly to keep up the connexion of the dates and partly also to show the great difficulties Ptolemy had to contend with, and the many paths which were offered to him—all leading from the truth.

§ 9.—From Hipparchus to Ptolemy.

(a) *Posidonius.*—Next after Hipparchus occurs the name of Posidonius, who lived in the first century, and resided chiefly at Rhodes. His name is connected with a new measurement of the earth; in fact the later calculation which he made

[1] See Strabo (ed. Kramer), Bk. ii., par. 93.

was accepted by Ptolemy. Eratosthenes having calculated from Alexandria to Rhodes, Posidonius likewise started first from his native city, and worked in the reverse direction from Rhodes to Alexandria. From observations such as the altitudes and disappearances of constellations or stars observed at each, he decided the distance to be 5000 stadia; he likewise over-estimated the arcual distance as $\frac{1}{48}$ of a great circle. Hence, multiplying the two together, he obtained the result 240,000 stadia as the whole circumference. Now it so happened that the errors in both calculations were upon the same side, namely, in excess, consequently they partly balance, and the result came nearer the truth than any other of the calculations upon record.

Unfortunately, in a second attempt he spoiled the harmony of his errors by correcting the nautical distance while the arcual distance remained the same. He now estimated the former at 3750 stadia, which gave him 180,000 stadia for the whole circumference. It is worth while remarking that he errs just as much in defect as Eratosthenes in excess—the globe of the latter is one-sixth too large, that of Posidonius one-sixth too small. Taking the true measure of a degree as 600 stadia, Eratosthenes took 700 st. and Posidonius 500 st. Perhaps, as coming last, or for some other reason we cannot now trace, this measure prevailed over that of Eratosthenes; and just as Hipparchus without question accepted the one, so Ptolemy equally without question accepted the other, each believing that, whether accurate or not, it would not make much difference for the observation of heavenly bodies from the several stations.

(b) *Strabo's Map.*—Strabo quite ignored the fertility of the principles of Hipparchus, and returned back to the older and less developed ideas. Accepting the earth as spherical and divided into zones, he took as the basis of his map the north temperate zone. Believing the frigid and torrid zone alike uninhabitable, he considered it an immediate inference to say that our habitable world lay in the North Temperate zone. Here it lay, like an island in the midst of the sea, occupying about half of the chosen zone in one hemisphere, and resembling in shape a military cloak or chlamys. The length he decides is about 70,000 stadia, and the breadth less than 30,000 stadia.

He next passes to the method of giving an actual visual picture of it which could be done by taking a sphere "like that of Crates," and dividing it into zones with the cloak-shaped inhabited world in the proper position. But considering how small a portion the inhabited world would cover, it would be necessary, he says, that the globe should be not less than ten feet in diameter. Therefore the student who cannot obtain one of such size had better delineate his map upon a plane tablet not less than seven feet long.

Here we reach the map of the world on a plane projection, and the infantile simplicity of this unscientific process—which indeed can scarce be called a projection—shows how much was left for Ptolemy to do. It is simply a division of the surface into irregular rectangles, determined by climates and distances, abandoning any attempt to preserve their proportionate intervals. For, he reasons, it will make little difference if instead of the circles, namely, both parallels and meridians, we use straight lines—those used instead of the parallels, parallel, and those used instead of the meridians perpendicular, since the imagination can easily transfer the form and magnitude seen on a plane surface to the curved and spherical surface.

This brief sketch of Strabo's map on a plane surface is quite sufficient to show how far he fell short of even the idea of a true projection. As a "descriptive geographer" his mind was possessed by the notion of a map as a picture, not so much for any scientific use in exhibiting true relations of distance, as for a framework on which to lay down the records of itinerary distances with an approximation to accuracy in the resultant positions.

(C) *Increase of data.*—The interval of about a century which separates the work of Strabo and Marinus is just the period at which the Roman Empire reached its widest extent and most undisputed power. The rule of the Cæsars, from Augustus to the Antonines, over the civilized world from Britain to Africa, and from the Rhine to the Euphrates, further the expeditions beyond these frontiers, and the vast extension of commercial enterprise prompted by the wealth and luxury of the Empire—all aided in the vast and rapid accumulation of information most serviceable to Geography. As examples there may be mentioned the exploration of the Nile to the Lakes by two centurions in the time of Nero, the expedition of Suetonius Paulinus into the heart of the Sahara; the regular voyage down the Red Sea along the coasts of Africa and Arabia, beyond the Straits into the Persian Gulf, and still further to India and the Eastern Archipelago, while simultaneously land travels were undertaken frequently by the merchants through the heart of Asia to the far East. The reign of Antoninus Pius, during which Ptolemy lived, was rendered remarkable in this connexion by the erection of the further wall in Britain and the despatching a Roman embassy to China.

This period is marked by the names of Pomponius Mela and Pliny, as well as certain important contributions to special Geography. These works, however, have little bearing upon the growth of Geography beyond being the literary expression of the rapidly accumulating mass of information. They, therefore, form a connecting link between the meagre details of earlier writers and the comprehensive knowledge displayed by Marinus.

(d) *Marinus.*—The name of Marinus has been saved to us by the careful manner in which Ptolemy acknowledges every instance in which he stands indebted to him. No one can read Ptolemy's account of Marinus (Geog., Bk. I., ch. 6) without recognizing the kindly spirit in which the latter geographer enumerates each of the excellencies of the earlier, which spirit is maintained all through the discussion. "Indeed," he goes on to say, "if we saw nothing wanting in the last edition, it would be enough to construct our map of the inhabited world from his alone of the commentaries." The defects pointed out are, that "even Marinus" has set down some things unworthy of belief, and secondly, that he did not exercise the required care about the plan of his map.

The meaning of the last assertion can be more fully seen if we consider what exactly Marinus did in connexion with map-making. He accepted Strabo's map, with its local right lines passing north and south and east and west, and upon this he ticked off new places as discovered—according to Ptolemy's account frequently changing their positions according to later information,[1] in fact, *he (Marinus) says himself*, in the last issue of his "Commentaries," that *he had not as yet reached the drawing of a map*.[2] He does not seem to have been aware that even if all his measurements were most accurately determined there will still have been a large amount of error inherent in the method of projection adopted by Strabo.

The relation of Marinus to Ptolemy may, therefore, be briefly characterized as follows:—Ptolemy, having established the mathematical basis of Geography,[3] chooses out Marinus as the commentator who has collected most data; these data he sifts most carefully, freeing them from internal improbabilities, establishing, where practical, a fixed standard for measuring the distance traversed by a day's march, or in a day's sail, correcting the few imperfect observations, and finally collating the accounts of Marinus with information which Ptolemy himself possessed. Then, and not till then, did Ptolemy adopt the corrected records.

[1] Geog., Bk. i., chs. 18, 19, 20, *passim*.

[2] The passage establishing this important fact in the history of ancient Geography has been hitherto entirely ignored. Critics have been misled by Ptolemy's mention of the "map of Marinus," and have consequently credited him with a mathematically constructed map of the greater part of the ancient world, which it was further supposed Ptolomy had appropriated. The following passage should tend to remove this misconception:—"*Τούτοις μὲν οὖν καὶ τοῖς τοιούτοις οὐκ ἐπέστησεν ὁ Μαρῖνος, ἤτοι διὰ τὸ πολύχουν καὶ κεχωρισεμένον τῶν συντάξεων, ἢ διὰ τὸ μὴ φθάσαι καὶ κατὰ τὴν τελευτάιαν ἔκδοσιν, ὡς αὐτός φησι, πίνακα καταγράψαι.*—Geog., Bk. i., ch. 17, § 1.

[3] This we shall see hereafter from the "Almagest," where all the mathematical principles necessary for geography are to be found, though Ptolemy, had not at that time dealt with the subject, or from the Geography itself, where these principles are briefly recapitulated *before* any mention is made of Marinus.

TABLE OF MEASUREMENTS OF THE EARTH.

Authority.	Stadia in Circumference.	Stadia in 1 Degree.
1. Aristotle, ...	400,000	1111$\frac{1}{9}$
2. Archimedes, ...	300,000	833$\frac{1}{3}$
3. Eratosthenes, ...	252,000	700
4. Hipparchus, (?)	275,000	763$\frac{8}{9}$
(Plin. ii. 12.)	*or* 276,000	766$\frac{2}{3}$
5. Posidonius (1), ...	240,000	666$\frac{2}{3}$
6. Posidonius (2) and Ptolemy, . .	180,000	500
True Measure,[1]	216,000	600

[1] Lelewel attempts to prove, by calculation, that this measure was used by Pytheas; but there is no positive authority for ascribing it to him, though he may well have approximated the truth by means of gnomon observations (see p. 6).

CHAPTER II.

PTOLEMY AND HIS GEOGRAPHY.

§ 1.—General Plan of his Work.

Ptolemy was not so much an author as a practical astronomer. The astronomy which he cultivated was not the theoretical science which has become accidentally associated with his name, but rather the practical observation of celestial phenomena. But as these vary according to the different positions of observers upon the earth's surface, Hipparchus had long before Ptolemy pointed out the necessity of so fixing positions on the earth, that the appearances of the heavens seen from each of them could be approximately recorded. The inquiry, thus indicated by Hipparchus, was actually carried out by Ptolemy, who undertook to construct an improved map of the world, in order that the positions thus determined might be used for astronomical observations. The construction of such a map for such ends was his real object—not the composition of a treatise on Geography—and the Geographia is simply an exposition of the principles upon which his map was made.

In the first book of the Geographia he explains how he made his map of the "habitabilis," and in the following books he gives us—not, as seems to be generally supposed, the list of stations or places from which he made his maps, but having first plotted, by a process of simple triangulation, the positions on his map—his lists were constructed from it. In other words, the list of places, with their longitudes and latitudes appended to the several maps, form an index to the maps, and not a table of the data from which they were constructed.

§ 2.—Its Astronomical Basis.

Turning to the Geographia itself we find that he refers us back to his mathematical treatise; and in the Almagest there is a cross reference to the Geographia, which was even then planned out.

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Having in the first two books of the Almagest explained fully the principles and method of the determination of positions upon the earth's surface, by means of angular measurements obtained from the celestial sphere, he concludes the second book with the following passage which completely links the Almagest to this contemplated work—the Geographia:—

"Now that we have finished the discussion of the angles, before completing the subject, we have still to investigate the principal positions of the notable cities, province by province, according to their longitude and latitude, with reference to the calculations observed at them. Inasmuch as such an exposition is of great importance in itself, and pertains to geographical science, we shall notice it by itself, following the accounts of those who have treated of the subject as far as possible. Further, we shall indicate how many degrees each of the cities is distant from the Equator, reckoned on the meridian drawn through it (*i. e.*, its latitude), and, moreover, how many degrees this meridian is distant from that drawn through Alexandria (*i.e.*, its longitude); for it is to this that we refer the times of the positions."[1]

We shall find that Ptolemy (Geo. ch. 2) expressly recapitulates the astronomical and mathematical data which he condenses from his larger work.

§ 3.—Its Consequent Limits.

Looking upon Geography as altogether ancillary to Astronomy, Ptolemy emphasizes this point in the opening chapter of the Geographia. He there defines it "an imitative delineation of that part of the earth comprehended within our knowledge as a whole, with its parts roughly (lit. generally) appended." It thus differs from Chorography, inasmuch as the latter selects the various regions and exhibits each separately by itself, copying all the details, even to the minutest, contained in that portion—such as harbours, villages, districts (or townships), the tributaries branching off from the main rivers, and other things like these. The proper object of Geography, on the other hand, is to exhibit the known earth in its unity and continuity, showing its actual condition and position, and the features belonging to it only in general outline, such as gulfs, the more important cities, nations, the more important rivers, the more remarkable things, each after its kind.

He goes on to further distinguish the two by saying that "Chorography deals

[1] "Almagest," Bk. ii. ch. 12.

with the part *only*, Geography with the *whole*, as the artist who copies an ear only or an eye only is to be distinguished from the artist who paints the whole figure. Further, the former is concerned with the kind (τὸ ποῖον), whereas the latter deals with magnitude (τὸ ποσόν), and thus aims at representing the proportion of distances, positions, and the general configurations by mere lines and the appended form."[1]

Lastly, Chorography having to do with depicting, has therefore "no need of mathematical science, but in Geography this plays the leading part."[2] For the Geographer "ought first to consider the form and magnitude of the whole earth; and, moreover, its position with regard to the surrounding heavens, in order that he may be able to say, with respect also to the part of it comprehended within our knowledge, both how great and of what kind it is, and, moreover, as to the several places in that part, under what parallels of the heavenly sphere they are situated.[3] From all which it will be in our power to determine both the length of the days and nights and which of the fixed stars become vertical and which of them always revolve above the earth or below it (*i. e.* those which never rise above the horizon of the place in question); and, in short, whatever is connected with an account of the *habitabilis*. All which forms the most sublime and beautiful study revealing to human understanding by the aid of mathematical science, on the one hand the nature of the heaven itself (inasmuch as it can be seen revolving round us), and, on the other, as regards the earth, showing by means of a sort of likeness that the real earth, which is very great and does not go round us, cannot be inspected by the same men either as a whole or in its several parts."

§ 4.—His Data and Pre-suppositions.

Having thus defined his meaning of the word Geography, he next proceeds to mention the data to be used. "I[4] may now ask the reader to accept what has

[1] Ptolemy's coast-line is always somewhat conventional, as he has but one consistent system for depicting promontories, gulfs, and other features. He knew nothing (other than by these differences) of the actual peculiarities of each station.

[2] It is possibly worth referring to Werner's ingenious note upon Chapter I., where he points out that Ptolemy has distinguished Geography from Chorography, according to each of the Aristotelian "Four Causes."

[3] It should be noted that in all works prior to Ptolemy *the lengths of days and nights determined the positions*, while with Ptolemy *the position* is the basis from which *to determine the lengths of days and nights.*

[4] Geogr. (ed. Müller), Bk. i., ch. 2.

been said as a rough sketch of the object to be set before anyone who proposes to do the work of a geographer, and the difference between him and a chorographer. Proposing, as we now do, to delineate or map out (καταγραψαι) the world as inhabited at our epoch, so that our map may tally as nearly as possible with the actual world, we consider it necessary to state, at the very outset, that our datum is the body of information obtained from travellers. Now this furnishes us with the greater part of our knowledge, which is derived from the accounts of those who traverse the countries in their several parts with most careful scrutiny. But both in such survey and the accounts of it part is 'geometric' and part 'meteoroscopic.' The 'geometric' method determines the positions of places *by base measurement* of their distances; the meteoroscopic, on the other hand, by *observations* taken by the astrolabe and gnomon.[1] The latter method is more satisfactory and accurate, while the other is more general and dependent upon the 'meteoroscopic.' For, first, suppose we would set down, according to the first method, to what part of the globe the distance between the required places is to be assigned, we must know not only the interval between them, but further towards what quarter of the Universe this interval points—for instance, whether to the North or the East, or the various directions sub-dividing the space between these. Now, it is impossible to determine anything of the kind accurately without the aid of the two instruments mentioned above; for by means of these at every place and every time, we can easily find the position of the meridian, and by means of the meridian the bearings of the distances already obtained.

"But, secondly, suppose all this were granted, such a measurement by stadia does not make the apprehension of the truth quite definite. For we rarely meet with perfectly straight ways, since many deviations occur both in itinerary distances on land and in a ship's course on sea, therefore to find the straight line required we must subtract from the whole amount of the account the probable amount of deviation in the case of travels by land, while in sea voyages we must allow for the variable force of the winds. Further, suppose the net distance between places on the line of march were accurately determined, even this does not give the proportion to the whole circumference of the earth nor the position with regard to the Equator and the Poles.

"But the measurement by observation of celestial phenomena determines accurately each of these points. It shows, moreover, the magnitude of the arcs,

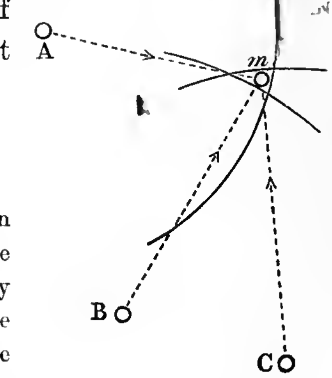
[1] Under the head of observations were classed astronomical observations, especially lunar eclipses, variations of climate, natural productions. Ptolemy's strong preference for this class of data is evident from the astronomical purpose of his work.

which the parallel and meridian circles intercept upon each other. By this I mean the parallels cut off the arcs of the meridian between themselves and the Equator, while the meridians cut off those arcs which lie between them, both on the parallels and the Equator. Further, we learn what arc the two places intercept from the great circle of the earth drawn through them. Now this method does not require any reference to stadial reckoning for showing the proportion of the parts of the earth or for the whole plan of the delineation (or map). For it is sufficient, if we assume the circumference of the earth as consisting of any number of parts whatsoever, to show the several distances as occupying so many of those parts described upon a great circle of the earth. On the other hand, when we come to reduce the whole circumference or any of its parts to certain and known intervals we cannot do without the measurement by stadia. And for this one reason alone was it necessary to fit in some one of the straight roads with an equal arc of a great circle; and so, by determining both the proportion of this (arc) to the whole circle by observation, as well as the length of the road in stadia by measurement, we can find the number of stadia in the whole circumference. For we start from the hypothesis as already established by mathematical science that the continuous surface of land is, on the whole, spherical. Moreover, this sphere has the same centre as the sphere of the heavens, and therefore the sections cut off by each of the planes drawn through that centre is a great circle (in the one case the celestial, in the other of the terrestrial sphere), and the angles in that plane meeting at the centre subtend similar arcs of the two circles. Therefore, from all these considerations it follows that, though we can obtain the true value in stadia of distances on the earth (if only they are straight) by measurement, we cannot obtain the *ratio* of such distances to the *whole circumference* from such measurements, but only from the similar arc of the celestial sphere. But it is possible to find the ratio of this arc to the whole circumference, and the ratio of the similar arc on the earth to its great circle is the same."

§ 5.—Manipulation of the Data.

Having given a decided preference, theoretically, to the astronomical ("meteoroscopic") class of data, when Ptolemy comes to deal with them practically he is met by a difficulty. He had too few of them to serve his purpose. As he says (ch. iv.): "These things then being so, if only travellers in the several countries had happened to have used such observations, we should have had the means of delineating the inhabited world in a form beyond dispute." "But, as a matter of fact," he continues, "Hipparchus is the only person who has given polar elevation of some few cities,[1] while some of his successors have recorded 'the positions lying opposite to one another,' that is, those approximately under the same meridian. Moreover, distances (especially east and west) have been inaccurately reported, owing to the want of sufficient astronomical knowledge, and also to the neglect of the observation of lunar eclipses."[2] This being so there remained no other course but to set down certain fundamental points, whose position Ptolemy believed was accurately determined, and *starting from these* the remaining positions were consecutively put in according to stadial measures or other evidence. He thus himself explains this process: "It would then be reasonable that a person undertaking to make a map according to such data should first lay down in his delineation, as foundations (*κάθαπερ θεμελίους*), the points derived from the more accurate observations; and next, he should *fit into these* the information derived from other sources until the relative positions of the latter to one another are found to preserve, with their relative position to the fundamental points, as near an agreement as possible with the more acc[illegible] reports of travellers."[3] [illegible]hese.

Ptolemy next proceeds to fix the position of some of the most importa[illegible] the these fundamental points[4] from south to north and from west to east. In so d[illegible]lace he shows his intimate acquaintance with preceding measurements, and after[illegible]s of consideration gives much weight to the commentaries of Marinus, whose data, however, he does not accept without the most careful investigation.[5]



[1] See passage quoted, p. 12.

[2] This passage is quoted, *infra*, p. 26.

[3] The meaning is as follows:—Having first inserted on his projection the stations accurately determined (A, B, C), he then added the subordinate stations (for instance *m*), according to their distances (see arcs in figure), by a simple process of triangulation. This mode of construction is the only one at once consistent with the facts and with Ptolemy's description. It will be noticed that the subordinate station, *m*, does not exactly coincide with any one of the given distances; but Ptolemy expressly allowed for this by placing it midway between such arcs as he could obtain, believing the error would be in no case greater than 0° 5′.

[4] These fundamental stations are separately given in the Geographia, but their purpose has been obscured by the editors.

[5] See *ante* Marinus.

§ 6.—Projection.

Having ascertained such fundamental points as he considered necessary, it is obvious that the last step is to project the "habitabilis" upon a plane surface. At the end of the first book Ptolemy describes two projections, the first of which is of a conical, the second of a globular type. After describing his conical projection, he says (Bk. I., ch. 24, § 9) that the plan of the map might be made "both truer and more symmetrical" if a second projection be adopted. He then describes a projection of the globular type, and concludes by saying (§ 22) that the globular is preferable, and he will use it both here and elsewhere, though both are given on account of the difficulty in drawing the latter. It is needless to enter into a description of the projections at present, as that adopted by Ptolemy is fully discussed in the following chapter.[1] It is a curious fact, which should not be omitted, that near the end of Book VII., there will be found instructions for drawing the map of the earth within the armillary sphere. Though this, at first sight, may appear to have been intended as a projection, there is no pretence that Ptolemy ever used it, and it was probably intended for decorative purposes, of which we see so many examples in ancient mural paintings. For the present work its most important purpose is the exhibition of the ignorant carelessness of the editors, of which it is a remarkable example.[2]

It is worth noting that in some MSS. and early editions the conical projection is used, but there is reason to believe that these maps represent the work of Agathodæmon.

Having described the plan of projection adopted, Ptolemy's work was ended, and the remaining books of the Geographia are merely a series of indices to his sectional maps; so it may be said that to Ptolemy we owe the first *Atlas*, as his work would be named in the present day.

[1] See Explanations of Plates IV. and V.

[2] See Note upon the corrupt passage in Book vii., ch. 6, dealing with the problem of describing the Earth within the Armillary Sphere, being an Explanation of Plate XXI., pp. 81–83.

CHAPTER III.

CAN PTOLEMY'S ERRORS BE DETERMINED?

§ 1.—Introductory.

We have seen how Ptolemy, the Alexandrian astronomer, set to work to map the whole world as then known. We have seen further what his data were, and how from these he proceeded to project his map and fit in upon it the distances as far as his knowledge served him. Now all this being known and admitted, let us take, as an illustration or example, his map of the habitabilis. Upon reference to it we find that his degrees, both of latitude and longitude, differ from ours; and, moreover, his places are apparently in different positions; for example, he sometimes maps a continent where we have islands scattered up and down an ocean.

One of three courses is forced upon us. We may either give up Ptolemy's Geography altogether and say that his positions either did not exist or else cannot be verified: or we may endeavour to fit in some of the positions, from similarity of nomenclature or some such guide: or, finally, we may search for some general principle or principles of rectification which, if true, must necessarily enable us to verify any position of which the latitude and longitude is given, with the same position upon a modern map. Both of the former courses have already been adopted. With regard to the first it may be remarked that Ptolemy's position has been peculiar. From a place of veneration he has been deposed, and has fallen rather into oblivion and contempt. At present all that need be said is, that so far the case has been prejudged. If Ptolemy was venerated there must have been some reason more or less true for such veneration, and to depose him rightly involves the whole refutation of the older opinion. This has not as yet been satisfactorily accomplished. According to the second method it must be observed, that were such identification complete Ptolemy's latitudes and longitudes will still remain over baffling systematic reduction.

There remains, therefore, only the third method, viz. a general principle of elucidation which, if true, will provide not only the principle itself, but as a result a formula from which any of Ptolemy's positions may be with close approximation located upon a modern map. This then is the problem which has hitherto baffled all previous students of Ptolemy, and which, if satisfactorily decided, will raise him from subordinate to sovereign rank.

§ 2.—Where Ptolemy's Errors are most likely to be found.

In marking out places on his map Ptolemy used two classes of data—the one determined by measurement, the other by observation. It is obvious that both are liable to serious error. Further, the former (measurement) admits of sub-division, namely, whether as measured by land or sea. In the next place it must be remembered that Ptolemy looked out with an astronomer's gaze over his habitabilis. But even an astronomer is not independent of measurement, and it is just possible that Ptolemy may have had imperfect data to build upon. Finally, before locating his stations, it was necessary to make a *projection* or plan of the earth, and here there is a further possibility of error. Consequently *internal* (as contradistinguished from *external*, *e.g.* misinterpretations of data supplied, misreading and consequent errors in the text, &c.) sources of error may be classified as follows:—

Errors, . . .
- (*a*) Of measurement
  - (*a*) by land.
  - (*b*) by sea.
- (*b*) Of observation.
- (*c*) Of scale, *i.e.* in identifying a distance known by stadia with apparently the same distance as measured by scale.
- (*d*) Of projection.

§ 3.—Errors of Measurement by Land and Sea.

It is obvious that when distances were measured in the primitive method employed in the time of Ptolemy, by the length of time necessary to traverse a given distance by land or sea, that errors were likely to occur. Further, such errors, it is obvious, admit of no *general principle* of elucidation, because there is no ratio between the variation of the distance and the variation of the time.

One point, however, is worthy of note. As might be expected, distances measured by land are much more reliable than those measured by sea—indeed the consensus of opinion of a body of soldiers in motion as to the number of stadia traversed may be accepted as nearly approximating the truth.

§ 4.—Errors of Observation.

Ptolemy believed observations to be a more certain and accurate datum than "mere" measurement;[1] hence he gives them an invariable preference, and the chief observations were—

(*a*) Gnomon for latitude.

(*b*) Lunar eclipses for longitude.

[1] See p. 20, "Geographia," Bk. i., ch. 4.

Ptolemy lays special stress upon these latter, because he believed they would give the true interval distinct from all error except such as might be contained in the observation itself. But when we consider the imperfection of the measures of time existing in Ptolemy's age, it is manifest that there is a possibility of error, especially at outlying stations, which were ill-provided with the requisite instruments. As a matter of fact, where Ptolemy himself mentions an observation of this nature, viz. that an eclipse has been observed "at Arbela in the fifth hour, and at Carthage in the second" (Bk. I., ch. iv.), there is an actual error of 11°. After such a striking example of Ptolemy's preference for observation, and that too in one of the districts with which he was most familiar, it is scarcely necessary to search for examples. Fortunately for the permanent worth of his work, such errors can seldom be detected, but it may not be out of place to mention a very probable case, which will account for the distortion of Scotland (see Explanation of Plates XV., XVI.), and another in the position of Rome, which (so to speak) breaks the back of the Italian peninsula (see Explanation Plate VIII.).

It may be remarked then that the paucity of errors of both classes stands forth prominently as an indication of the exceeding care and accuracy of Ptolemy's work, while again it makes possible the discovery of a general principle for the correction of those that remain.

§ 5.—Errors of Scale or Interval.

Suppose now we return to Ptolemy's map of the *habitabilis* with the errors so far discovered, can we say we have come any nearer the possibility of locating the actual position of some required station? No, assuredly we cannot. If we could say, "this is where Ptolemy's station ought to be," our present corrections might enable us to explain *why it was found slightly displaced.* Further than this at the present stage we could not go.

Now, to take the third possible source of error, namely, in the "scale," or fitting the degree of a geometrical circle to an actual distance upon the surface of the earth. Ptolemy dismisses this point briefly by saying, with apparent regret (Bk. I., ch. ii.), that it was necessary to introduce the uncertainty of measurement by stadia in order to fit in either the whole or parts of the perimeter with certain and known intervals. But when he came to fit in 1° with his "known and perfectly straight road,"[1] he fell into an error. He calculated a degree as equal to 500 stadia,

[1] "Geographia" (ed. Müller), Bk. i. ch. 2

whereas it is really equal to 600 stadia; hence Ptolemy's degree is only $\frac{5}{6}$th of the true degree (see Explanation of Plate I.).

But here a most important consideration must not escape our notice. It must be remembered that stadial measures form only one branch of the total data; hence it is absolutely indispensable to make a reservation in the case of intervals obtained by the other class of data—*i. e.* observations. Therefore it is only stations whose position has been obtained by measurement that are subject to scale error.

The result may be exemplified as follows, if *S* be a station determined by measurement, and *O* be obtained by observation, it follows that *S* will be chargeable with errors of both measurement and projection, while *O* would be charged only with the error of the observation itself.

Here, then, we begin to feel firm ground. If Ptolemy has made his degree contain $\frac{1}{6}$th too little, it is plain that the degrees will be $\frac{1}{6}$th too many; therefore when allowance is made for the errors already explained, if we deduct the interpolated $\frac{1}{6}$th, or, in other words, multiply by $\frac{5}{6}$th, Ptolemy's degrees should become the true. But here an extraordinary result appears. In some small sectional maps, the Ptolemaic Latitudes and Longitudes, when reduced by $\frac{1}{6}$th, are true within his own limits of error, yet outside these the correction is not sufficient, there is still a differentia which needs explanation, and which must be determined to complete the solution of the problem. What is it?

§ 6.—Error of Projection.

Now, to take a specific case. If we take the two stations, Alexandria, and Londinium—the former[1] being the starting point from which Ptolemy fixed his first and fundamental measures, and the latter being that from which we fix our own—if, then, we take these two stations and reduce Ptolemy's longitude to allow for the scale error (§ 5), there still remains a residuum of error unexplained (see Plate II.). And yet these are two points most likely to be accurate. Not only so, but upon the return journey from London to Alexandria (see Plate III.) there actually is an error of 2° 26′ to the west in the position of the latter. Is it possible that Ptolemy could have made a mistake in the position of his own starting point. Plainly not, and yet the error remains! Upon reference to the Plates it will be seen that there is a fundamental error in the plan of his projection which demands a further contraction of his sphere.

[1] See passage quoted from "Almagest" (Bk. ii. ch. 12), p. 18.

The next step is plainly to project Ptolemy's sphere accurately according to his instructions. In attempting this it was found that there was a constant difference in the process of construction between the editions prior to, and those following, 1525 (Pirkheimer). Circumstances led to the conviction that an additional error was involved, and that a correction had inadvertently been made, at some time not long previous to the edition of Pirkheimer already referred to. The preface of this book called attention to the previous work of John Werner, published in 1514. Further, we know that Pirkheimer worked from Werner, and that the next popular edition—that in Latin by Bertius—was based upon Pirkheimer. So far as can be ascertained, it was in Werner's work that the error was first corrected—but merely as a typographical fault.

In the instructions given (in ch. 24, bk. i.) we are told how to obtain the centre from which to draw the parallels of latitude (see Explanation Plate IV.). The length of the radius is there obtained from the Equator, but in all the older editions it had been measured off from the southern limit of the "habitabilis" (see Plate IV.—fig. 1 being the correct, fig. 2 the incorrect, radius). The result is, therefore, a further reduction of the scale of Ptolemy's sphere, since he mapped the larger chart on the smaller sphere, or, in other words, graduated the smaller with the scale of the larger. Having proved the existence of the projection error the next step was to obtain its actual value. Turning back to Ptolemy's instructions in the first instance for making his projection, and taking the proportion of the true length of the radius to that actually used, we find the exact value of the error in projection to be ·9172. Then (from Explanation Plate I.), multiplying by the value of scale error, we get a formula for the reduction of Ptolemaic to true degrees—for both errors combined ·7632; or conversely for the reduction of modern to Ptolemaic 1·3103 (see Explanation Plate V.).

Hence we have now reached the answer to our original problem, and have obtained a general formula for the rectification of Ptolemy's positions with the exception of such incidental errors as depend solely on the limits of his information.

Proceeding a step farther, let us use these data not only for the interval between London and Alexandria, but also for the whole half of the "habitabilis" from the western limit to Ptolemy's central meridian (see Explanation Plate VI.). Here upon an extended field, from several distinct considerations, exactly the same error (2° 26′ too far west) appears in the position of Alexandria, which may be regarded as strong confirmation of the accuracy of the whole criticism involved.

For still further verification let the formula be tested for the whole 180° of

Ptolemy's projection (see Explanation Plate VII.). For a specific starting-point, it was found advisable to adopt Cambericum, the central mouth of the Ganges, the position of which could be determined with practical accuracy—the error was found to be constant, and so the exact length of the habitabilis could be obtained. Not to enter into details, it is sufficient to mention here that the eastern limit was fixed from five distinct calculations as 114° 29′ east of Greenwich. The conclusion then is completed by the indication of the true position of Ptolemy's limits and salient stations upon a modern map (see Explanation Plate VIII.).

It will be found from the diagram that applying Ptolemy's rectified longitudes and latitudes to a modern map the Fortunate Islands, as the ancient Hesperides, include both the Canaries and Cape Verdes. As a matter of fact, Ptolemy finding it impossible to discriminate between them, gave them the longitude of the one and the latitude of the other.

The error arising from insufficient data begins to appear at the Indus, and acquires the larger dimensions of 10° 50′ at the Ganges. This is due to the discussion of the day, concerning the coast of India, whether it ran north and south or east and west. The data of Ptolemy led him to conclude that it ran east and west, and hence the increase of his error between the Indus and the Ganges. The same fact explains the enlargement of Taprobana (Ceylon) to obtain the known width of the straits. Taking Ptolemy's 180°, which has been proved equal to 114° 29′ E., his Thyne Metropole becomes a little S.W. of the centre of Borneo.[1] Further confirmation will be found in the fact that Cattigara agrees with "Kottawaringin" on the S.W. coast of the same island.

The limits for some of the more important fundamental stations being fixed, there remains nothing but the final completion and further verification of the formula of elucidation, namely, to arrange upon a modern map of the world the latitudes and longitudes of Ptolemy, subject only to errors of measurement. The needful instructions are given in the Explanations of Plates VII., VIII., and the whole of these corrections may be applied from Plate IX.

[1] Extensive ancient ruins have been recorded as existing a little S.W. of the centre of Borneo, but no further particulars are given. They are apparently in the position where Thyne Metropole should be found. Other evidences of early eastern civilization have been found in the same region. (For information relative to the latter we are much indebted to Herr S. W. Tromp, Resident of the Western Division of Dutch Borneo, Correspondent of the Royal Academy of Sciences and of the Royal Dutch Geographical Society, both in Amsterdam.)

CHAPTER IV.

PTOLEMY'S ERRORS IN BRITAIN.

§ 1.—The Origin of the whole Inquiry.

Though the present Chapter is the last, the investigations which it records originally led to the whole inquiry. As far back as the year 1847 the author was asked by an antiquarian friend: "*Is the Belisama of Ptolemy the Mersey or the Ribble?*" The question had long been of great local interest, and had often been discussed, but no satisfactory reply seemed forthcoming. Without even a knowledge of the existence of the text of the "Geographia," the question was resolutely faced, and it appeared that Ptolemy's river was about Formby Point, just half way between the two rivers already mentioned.

Upon reading Henry's "History of Great Britain," and afterwards Horsley's "Britannia Romana," new light came with the British portion of Ptolemy's table. But this discovery necessitated the extension of the inquiry to, at least, a considerable portion of the west coast of England. Even this area was found too contracted, and London was taken as probably a fixed point. Still the same baffling contradictions, for the error at this well-known station seemed greater than at the coast! A solution was demanded from Ptolemy's own residence and primary station at Alexandria, which after minute inquiry was proved to be placed in error by the amount already recorded. The next step was to project Ptolemy's map of the world in strict accordance with his instructions. This led to the discovery of the "Projection Error." When this solution was found to be valid for the whole *habitabilis*, it was plain that at last the vexed question was solved.

As these results gradually grew up, a few stray hints of the progress of the work were given, in two papers, to the members of the Historic Society of Lancashire and Cheshire.[1] One of these seemed of sufficient importance to be reprinted as an Appendix to the present volume. The address had never been written, and no report could be taken at the meeting; but the Secretary of the Society afterwards drew up an epitome, which was published amongst the transactions, and which is now given with few alterations.

[1] "Ptolemy's Geography of the Coast from Carnarvon to Cumberland": Transactions of the Historic Society of Lancashire and Cheshire, vol. xxx., p. 81 (1878); and "The Map-History of the Coast from the Dee to the Dudan," by T. Glazebrook Rylands, F.S.A., F.L.S., F.G.S., Liverpool, vol. xxxi. (1879).

The summary of the errors which follows is a generalized statement of many independent inquiries, which generalization is made easier by a kind of progress in the errors themselves. This enables us to follow almost without break the natural course of the coast-line, beginning at the original starting-point, the much-disputed Belisama.

§ 2.—Errors of Measurement and Scale.

Though these two errors have been previously classed as distinct in dealing with a practical case, it is better to treat them together. But before commencing the investigation care must be taken that the materials are judiciously selected. Regarding maps, this may well be illustrated by a series of typical tracings of that part of the coast from Ituna to Tisobius, which will be first dealt with (see Explanation of Plate X.).[1] Starting, then, from Ituna (the Solway), and descending the west coast, it will be found that allowing for both these errors we are enabled to identify Ptolemy's stations. The much-disputed Belisama "can only be the Mersey" (see Plate XI.).[2] Seteia is the Dee (see Plate XII.). But now arises the question, where is Ianganorum Prom.? It is allowed on all hands to be Brachypwll, but in every edition of Ptolemy's maps the promontory of Carnarvonshire is cut off.[3] Ptolemy records the names as they occurred from north to south, and we find the Tisobius, which is the next station S. of Ianganorum Prom., occurs to the north of the latter. Now, an editor having no knowledge of the return of the coast upon itself, would endeavour to alter the latitudes and longitudes to suit the position of the names in the tables. This is confirmed by the readings of different editions. Consequently the Tisobius can be identified with Traeth Mawr, "and the Promontory of Carnarvonshire has been cut off, not by Ptolemy, but by his editors." Ptolemy knew nothing of Anglesea. He had no station between the Point of Carnarvonshire and the Dee. It may be noted in passing that this latter point affords an instance of a source of error previously called external, but which for obvious reasons has not been tabulated.

The coast south of Ianganorum Promontory requires no correction till we reach Bolerium Promontory. Here we see a striking example of the comparative

[1] The coasts according to Donis and Buckinck give the earliest form of the maps of Ptolemy, and of Agathodæmon. It is worth noting the change in the position of the Tisobius. (See Explanation Plate X.)

[2] This section of the coast was explicitly discussed, station by station, by Mr. Rylands, in a Paper read before the Historic Society of Lancashire and Cheshire: see Transactions of this Society, vol. xxx. See Appendix.

[3] Trans. Hist. Soc. of Lancashire and Cheshire, xxx., p. 90.

amount of the two sub-classes of measurement—the superior accuracy of data obtained from land marches as compared with those obtained from sea voyages. The whole coast line has been obtained from the latter source; the errors vary from 1° in longitude between Land's End and the Tamar to $\frac{1}{2}°$ between the Tamar and Southampton (see Explanation Plate XIII.). Further, we have here a discrepancy in *position* as well as a discrepancy in accuracy. For towns situated on the banks of rivers (the former being obtained necessarily from the land, the latter from the sea) differ so far that the town is generally placed $\frac{1}{2}°$ westward from its river. Still even here one is forced to pay a compliment to Ptolemy's accuracy; for the mere fact of giving us such a map of Cornwall, which by rectification of the error of 1° can be reduced to the comparatively accurate form as given in figure 2, speaks volumes for the care with which he sifted the confused accounts of sailors who were even then verging on the dread unknown of the Western Ocean.

Passing along the south coast then many stations will be found which, upon reference to the Plate admit of easy identification. Upon the east coast, bearing the rectification of the scale error in mind, it becomes easy to recognize the chief stations (see Explanation Plate XIV.). Thus the Iamasius is the Thames, the Vedra the Tees, and the Albanus the Tweed. The most noticeable point here is that Novus Portus is not New Haven, as the name would suggest, but Dungeness, and that Nucantium must be placed upon the mainland not upon Thanet, as has hitherto been the prevailing opinion.

§ 3.—Errors of Observation.

Referring again to the east coast, at the Vedra it suddenly breaks away from its parallelism to the modern. Now here we have an example of another class of error, namely, that of observation; and the only explanation of which the distortion of Scotland, which has so long puzzled students of Ptolemy, seems to be susceptible, is by the supposition of an observation of a lunar eclipse, for if such observation were recorded Ptolemy was bound, according to what he has expressly stated, to accept it in preference to the stadial measurements. It will be seen upon reference to the Plate that the error in such a case is only 15 minutes and 4 seconds (see Explanation Plate XV.). This error of his longitude at Duncansby is largely confirmed by the greatly reduced latitude, which seems to point distinctly to the fact that a gnomon observation also was taken there. But his latitude as calculated, is reduced very nearly 4°, bringing it within half a degree of the truth.

To realize fully the complete distortion of Scotland, it will be well to refer to Plate No. 16, where the part coloured red represents Ptolemy's error, and the part coloured blue the true Scotland which he has missed, while the part where the colours interlap is common to the two. It may be noticed in this Plate that the exigencies of his scale have necessitated the reduction of the *length* of the true Scotland from N. to S., but for purposes of the present illustration this fact is of no importance—indeed the true northern half of Scotland was really beyond the limits of Ptolemy's area (see Plate IX.).

Following Ptolemy's Scotland it will be seen that on the west coast it returns back upon the true, until from the "Point of Aird," Skye, the *general direction* of the coast line is substantially accurate.[1]

We have now completed the circle of Britain, and all errors but those in some way dependent upon projection have been exemplified.

§ 4.—Error dependent upon Projection.

In speaking of Ptolemaic stations in Britain exemplifying the errors previously mentioned, it may have been noticed that we have spoken with a considerable amount of reserve. As a matter of fact no other course was open, as there remains another error to be pointed out rather than exemplified. In the previous account in ch. 3 of Ptolemy's *general* errors, it was not possible to introduce it, as it is a special error connected solely with the sectional map. In so far as it depends upon projection, it may be regarded as a corollary from, or modification of, the general projection error.

We can easily trace its progress from incidental remarks of Ptolemy himself.[2] After his express adoption of the globular type of projection, he again refers incidentally to the matter in Bk. ii., ch. 1, inasmuch as in his sectional maps he adopts the value of the latitudes and longitudes at their centres, and the fact that he employs it—lines both vertical and horizontal—in such areas, he considers, would produce no error greater than that involved in the data themselves. But Ptolemy, no more than any other man, could lay down on a plane so large a portion of the sphere without distortion; hence, having corrected his scale error and the additional reduction already referred to, there still remained to be investigated the amount of this special distortion. In the case of a modern atlas this, which was a source of error to Ptolemy, may be almost entirely avoided,

[1] See Supplement to Plate XV., p. 77.

[2] See p. 20 and Geography, Bk. i., ch. 24, § 29.

but it must be remembered that in distinction from our methods he simply adopted on his sectional maps the numbers he had obtained in his map of the world.

In investigating this point it seems to be very desirable, as it will be sufficient to confine our inquiry to this Tabula I., Europæ, which contains the British Isles. This distortion will be most easily realized by contrasting rather than comparing the two areas taken from his corrected Projection, as shown in Plate No. 17. The first attempt to measure this error was to rule sheets of paper with lines to the scales of latitude and longitude, and then to insert the needful stations according to modern determination and as given in Ptolemy's lists. Here it was found the Ptolemy errors were always + the latitudes being all too high. For example his apparent error in the case of Londinium was 3° 29′ too far north. This view of the variation of his error round the coast of the island was so instructive that a second sheet was prepared, using both scales coincident at Alexandria. In this case the errors were reversed, and those now obtained were *too low*, and the proportion of the errors became inverse. To illustrate this, the smallest + error in the first sheet was at Peterhead, which becomes the largest in the second sheet; and in the case of London the error became 1° 31′ too low. This difference of errors may be elucidated by including a diagram of much later date, which also includes the final results, and which was made to investigate this point. In it these errors became + 3° 29′ and – 1° 25′ (see Plate XVIII.).

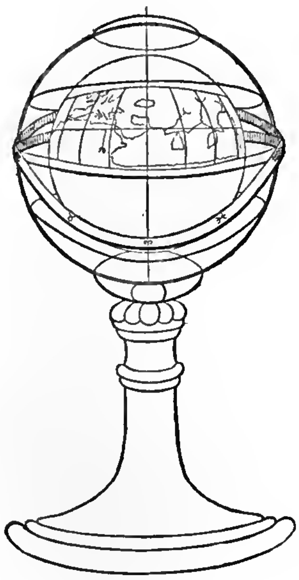
The errors in longitude were then treated in the same way. They were minus by a gradually decreasing error from Land's End to near Berwick-on-Tweed. From this point northwards they become plus until Peterhead was reached, where the error reaches a maximum.

Further, Londinium appeared to be nearly 4° too far west. These facts necessarily led to a further investigation. The first step was to enlarge and divide into degrees the two areas shown in Plate XVII., Distortion of Areas. It was found that when made consistent at the south-west corner of the map they were also coincident at Thule.

Between these two points, from the intersection of each degree of the two areas, a very slightly curved line of no distortion (bearing 44° 15′ east of north) was drawn between the north and south limits of the map, and the errors were found to be proportionate to the distance of the stations from this line (see Explanation of Plate XIX.). This line was afterwards found to be the *true* meridian of Thule according to Ptolemy's projection. The line of no distortion in *longitude* is here also determined, and the actual displacement of the stations in both directions may be obtained.

Other diagrams were made to test these results, but ultimately it was seen that the facts would come out more forcibly if they were viewed in profile, and taking the line of "no distortion" as a basis, the diagram of "Caturactonium and London" (Plate XX.) was drawn. In this plate the error due to the transference of positions from the general map of the habitabilis to the special sectional map will be seen worked out in detail.

But Caturactonium and Londinium were two of Ptolemy's fundamental stations in England, and as the positions of the majority of the other stations were put in from them by a process of plotting, when this special error of projection has been allowed for no further correction beyond those already mentioned is necessary. It is hoped that any further explanation the reader may desire will be found in the explanations of the plates, and that too in a more convenient form than if embodied in the text.



The ΚΟΣΜΟΣ.

GENERAL FORMULA OF REDUCTION.

(*Example*—Thule.)

I.—Latitude.

Ptolemy to True.

		° ′	
Thule		63 0	
Alexandria		31 0	
$T - A$		32 0	
SP	−	7 35	
λ	=	24 25	
Lat. Shetlands − Alexandria		28 50	true.
ϵ	= −	4 25	true.
SP	=	7 35	
η	= +	3 10	

True to Ptolemy.

		° ′	
Shetlands L	=	60 0	
Alexandria	=	31 10	
		28 50	
$\frac{1}{SP}$	+	8 57	
λ	=	37 47	
Lat. Thule − Alexandria		32 0	Ptol.
ϵ	= +	5 47	Ptol.
$\frac{1}{SP}$	=	8 57	
η	= −	3 10	

II.—Longitude.

Ptolemy to True.

		° ′	
Alexandria		60 30	
Thule		33 0	
$A - T$		27 30	
SP	−	6 30	
λ	=	21 0	
Long. Alexandria to Shetlands	=	31 15	true.
ϵ	−	10 15	true.
SP	−	6 30	
η	= +	3 45	(Pl. XV.)

True to Ptolemy.

		° ′	
Alexandria to Shetlands		31 10	
Greenwich to London		5	
		31 15	
$\frac{1}{SP}$	+	9 41	
λ		40 56	
Long. Alexandria to Thule		27 30	Ptol.
ϵ	+	13 26	Ptol.
$\frac{1}{SP}$		9 41	
η	−	3 45	

Note.—*Latitude.*—On the map, $L - L' = 3° 0'$, but the above includes the error of Alexandria, 0° 10′.

Longitude.—Duncansby to Shetlands = 1° 46′; Tarvedume to Thule = 1° 40′. Here, also, $\epsilon = 10° 15'$ against the 10° 21′ on Plate XV.

III.—Positions on a Modern Map.

Latitude.—True, 28° 50′ + 31° 10′ = 60° 0′ N.
Ptolemy, 32° 0′ + 31° 0′ = 63° 0′ N.

Longitude.—True, Alexandria to Shetlands = 31° 10′.
,, to Greenwich = 29° 55′.
True longitude = 1° 15′ W.

With so large a difference, the determination of *Ptolemy's* longitude on a modern map is not quite as simple.

Although it would be strictly true to say 33° − 20° = 13°, this would be of no practical use. His error must be corrected and verified.

There are several previous results which will apply:—His Londinium is in long. 20°; on Plate III., the error of Alexandria is 2° 26′ W.; on Plate IV., London, as measured from Alexandria, is 1° 45′ E.; and on Plate VIII., the general error of Londinium is 1° 19′ E.

From which 33° 0′ − 13° 26′ = 19° 34′
Londinium = 20° 0′

∴ Ptolemy's longitude = 0° 26′ W.

Then − 0° 26′ − 1° 45′ = + 1° 19′, as on Plate VIII.; and 1° 19′ + 2° 26′ = 3° 45′, the error found above. Thus the true longitude being 1° 15′ W., and Ptolemy's error being 0° 26′ W., his Thule, on a modern map, should be placed in longitude 1° 41′ W.

The fixing of the longitude of Thule, while it completes the foregoing instructions, is otherwise interesting after reading of Iceland and Norway.

There are two readings in the text:—

$\bar{\lambda}\gamma'$ (30° 20′) and $\bar{\lambda}\gamma$ (33° 0′).

The former of these became fashionable after the publication of the Servetus Edition of 1533. So far it has not been found earlier.

The above calculations were based upon the idea that Pytheas landed about Sumburgh Head; but he describes Thule as an *Island*. If he went round it the error would differ little from that at Tarvedume; but the data on this point are not forthcoming.

PLATE I.

PTOLEMY'S ERRORS AND INTERVALS.

EXPLANATION OF PLATE I. (1)

(See Chap. III., § 5.)

Ptolemy accepted 500 stadia = 1° of a great circle of the earth.

but 600 (2) stadia = 1° of a great circle of the earth.

Therefore, Ptolemy's sphere is $\frac{1}{6}$th too small.

If Ptolemy wished to map out certain positions this scale error would appear differently, according as his data were derived from stadial measurements or from observation (see ch. II., § 3).

(*a*) Suppose the positions *A*, *B*, *C* are derived from measurement. Now, let the interval from *A* to *B* be 6000 stadia, and from *B* to *C* 18,000 stadia. It is evident that the *interval* remains unaffected by scale error, but Ptolemy would map them as 12° and 36° on *his* scale, instead of 10° and 30°, respectively, on the true.

(*b*) Suppose the same positions fixed by angular measurement, we should find them at *A*, *B*, *C*, when the *number of degrees* in the interval is right, but the interval itself is wrong—thus *A B* = 10°, but only 5000 stadia.

(*c*) One station may be determined by one method, the other by the other method, and so may be mapped as *B C* or *B C*.

But we find few examples where data derived from observation conflict with the ordinary stadial measurements (see ch. II., § 4, and ch. III., § 4); therefore, bearing this and what has been said above in mind, the following formula eliminates the scale error from Ptolemy's longitudes and latitudes. (From data supplied by Lt.-Gen. James as to the exact value of 1°, and measurements ascertaining as nearly as possible the length of a stadium—where *S* = Scale Error.)

$$\left.\begin{array}{l} S = 0{\cdot}832 \\ \dfrac{1}{S} = 1{\cdot}202 \end{array}\right\}$$

NOTE.—To determine the effect of these differing conditions (*a*, *b*, and *c*) upon the position of places contained in the Geography—

Let λ and λ' be two positions of known distance from *A*, but mapped by Ptolemy at π and π'.

Let ϵ, ϵ' = his *actual* errors.

η, η' = his apparent errors of MEASURED distances.

ω, ω' = his apparent errors of observed distances.

Then $\omega, \omega' = \pm (S + \epsilon)$ that is, + in longitudes, – in latitudes.

I. Both stations by measurement,

$$\pi - \pi' = (\lambda - \lambda') + (\epsilon - \epsilon').$$

II. Both stations by observation,

$$\pi - \pi' = (\lambda - \lambda') - (\omega - \omega').$$

III. π by observation, π' by measurement,

$$\pi - \pi' = (\lambda - \lambda') + (\omega - \epsilon').$$

IV. π by measurement, π' by observation,

$$\pi - \pi' = (\lambda - \lambda') - (\omega' - \epsilon).$$

Here $\pi - \pi' = 20°$ varies from $\pi - \pi' = 30°$ to $\pi - \pi' = 14°$.

(1) Originally it was intended to use red ink and Greek letters to distinguish Ptolemy, but this was not carried out consistently in the sequel.

(2) The round number 600 is given here as sufficient for the purpose; the exact value of the error will be found below (see p. 88).

PTOLEMY'S ERRORS AND INTERVALS.

FOR STATIONS NORTH AND WEST OF ALEXANDRIA.

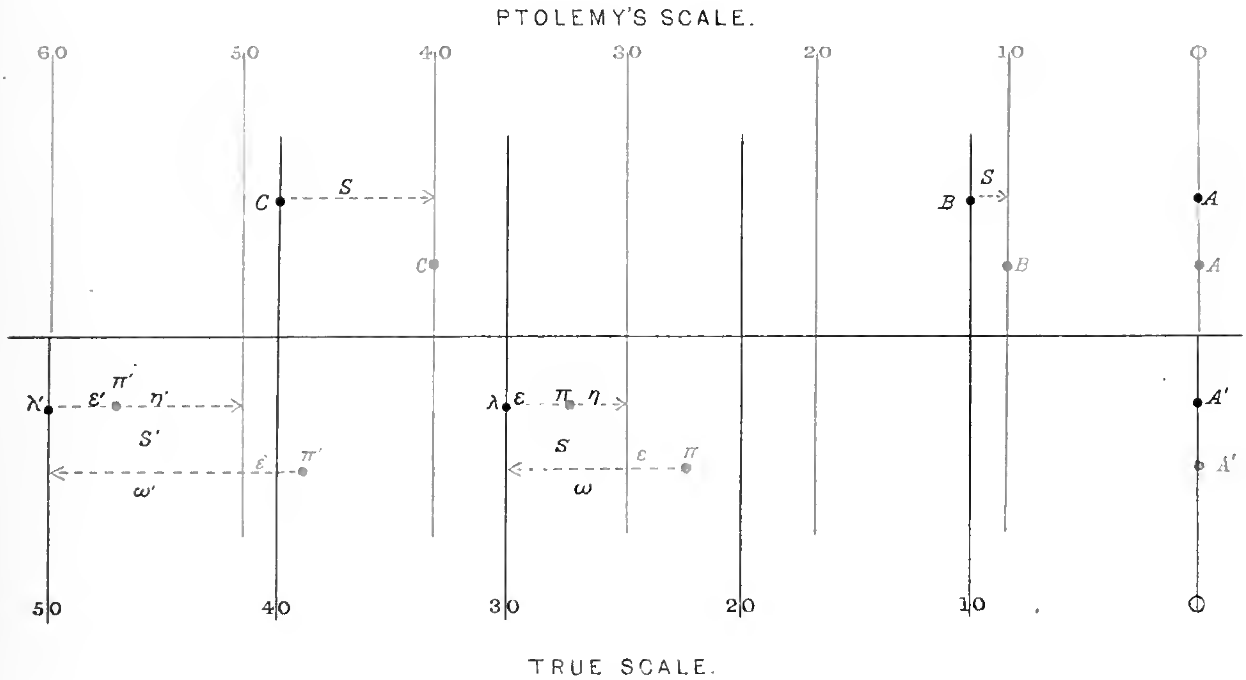


PLATE II.

ALEXANDRIA (TRUE) TO LONDINIUM.

EXPLANATION OF PLATE II.

(Alexandria (true) to Londinium.)

Let TT and CC be two concentric circles, such that TT represents the true and CC the World of Ptolemy as subject to scale error (see Explanation of Plate I.).

Now, if the true position of Alexandria be marked at A and that of London at L, their positions, as measured on Ptolemy's scale, should be at a and η respectively, since $a\eta = AL$.

But if L be found at π, how is $\eta\pi$ to be accounted for?

If the radius ZV (also known) were accidentally used for ZC, and the circle VV be described; since $\delta m = AL$, L would be mapped at m.

But $\delta Zm = aZ\pi$, then if V were unconsciously graduated with the scale of C, L would be mapped at π.

If Ptolemy made such an error, the values of the radii can be supplied from his work, thus—

$$\frac{ZC}{ZT} \text{ call this } S \text{ (as before)} = 0{\cdot}832, \quad \text{(see Explanation of Plate I.)}$$

$$\frac{ZV}{ZC} \text{ call this } P = 0{\cdot}917,$$

$$\frac{ZV}{ZT} \text{ call this } SP = 0{\cdot}763.$$

Now $AL = 30°$; $\therefore\ \delta m = 30° \dfrac{1}{SP} = 39°\ 19' = a\pi.$

But Ptolemy gives $a\pi = 40°$; $\therefore\ 40° - 39°\ 19' = 0°\ 41'$, which would be his actual error upon this supposition. (I.)

Now, to turn to Ptolemy's numbers and distances alone,

$$a\pi = 30° \frac{1}{SP} = 39°\ 19',$$

$$a\eta = 30° \frac{1}{S} = 36°\ 3'.$$

Therefore (by subtraction) $\pi\eta = \text{excess} = 3°\ 16'.$ (II.)

Then Ptolemy's longitude of Alexandria $= \phi a = 60°\ 30'$, which is made up as follows —

$\pi a = 40°$ and $\phi\pi = 20°\ 30'$

And	$\phi a = 60°\ 30' \cdot S$		$= 50°\ 20'$
	$\phi\pi = 20°\ 30' \cdot S = 17°\ 4'$		
	$\eta a \cdot S$	$= 30°$	$= 47°\ 4'$
Therefore	$\eta\pi$	$=$ excess	$= 3°\ 16'$ (II.)

Using $\eta\pi = 3°\ 16'$ to determine *actual* error on map.

On map	$a\pi = 40°$
	$a\eta = 36°\ 3'$
Excess *on map*	$= 3°\ 57'$
But Projection Error $= \eta\pi$	$= 3°\ 16'$ (II.)
Therefore Actual Error	$= 0°\ 41'$ as before. (I.)

REMARK.—Lastly, to compare Ptolemy's longitudes from A with the modern from Greenwich, mark the latter at G, and make

$$o\gamma = LG = 6' \text{(Ptol.)} = 5' \text{(true)}.$$

PROJECTION ERRORS.

ALEXANDRIA (TRUE) TO LONDINIUM.

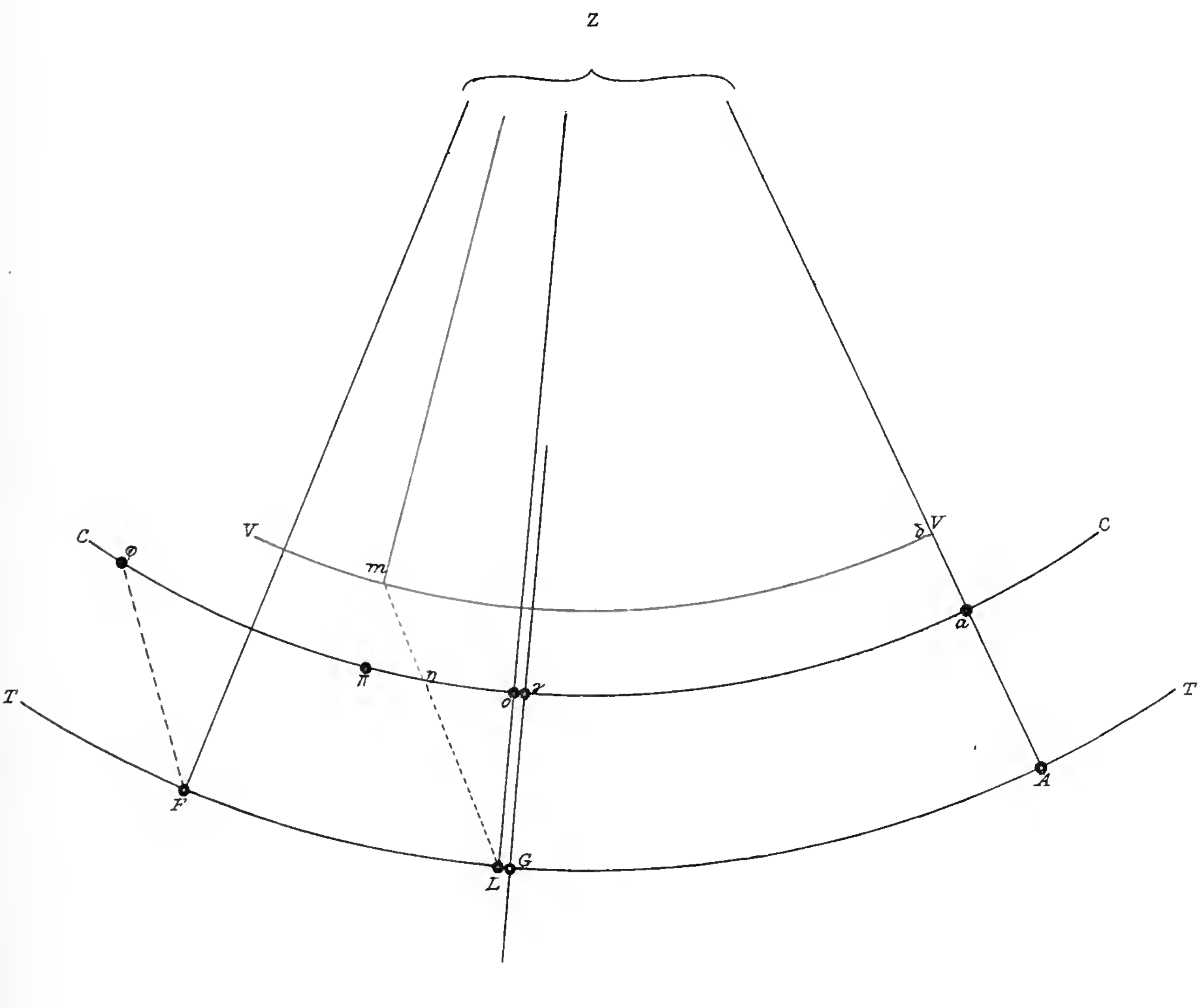


PLATE III.

LONGITUDE OF LONDINIUM, LONDON (TRUE) TO ALEXANDRIA (PTOL.).

EXPLANATION OF PLATE III.

(London (true) to Alexandria (Ptol.), *i.e.* the reverse of Plate II.)

If A be Alexandria, and L Londinium on the true world, construct CC Ptolemy's *calculated*, and VV his *virtual*, projections as before.

Then $AL = \alpha\lambda = \delta\tau$, being the true projection of AL on each scale (in figures) $AL = 30°$; $\alpha\lambda = 30° \cdot S = 36° \ 3'$; $\delta\tau = 30° \cdot SP = 39° \ 19'$.

But V was graduated with the scale of C, therefore produce $Z\tau$ to m.

Now LA should have been mapped ma; but Ptolemy maps it πa.

Therefore actual error $= \pi a - ma = 40° - 39° \ 19' = 0° \ 41'$. (I.) as in Plate II.

CALCULATION OF LONDINIUM.

LA			=	30° 0′
S	=	λo	=	6° 3′
Ptolemy's interval	=	λa	=	36° 3′
Interval on projection	=	πa	=	40° 0′
Therefore apparent error	=	$\pi\lambda$	=	3° 57′
But P is developed from 90°	=	$\mu\lambda$	=	5° 42′
Therefore Error on Projection	=	$\mu\pi$	=	1° 45′.

RELATIONS OF m.

Make $EL = m\lambda = \epsilon o = \eta\tau$; then $EA = ma = \eta\delta$,

but $\eta\omega = \mu o = S + P$ in terms of C.

That is, $a\mu = 30° + 6° \ 3' + 5° \ 42'$	=	41° 45′	
but $a\pi$	=	40° 0′	
Therefore $\pi\mu$ (by subtraction)	=	1° 45′	as above.
Then $\mu m = \mu a - ma = 41° \ 45' - 39° \ 19'$	=	2° 26′	(III.)
Therefore πm (by subtraction)	=	0° 41′	(I.) Plate II.

REMARK I.—While S is developed from *Alexandria*,
P originates from the central meridian 90° of Ptolemy's sphere.

REMARK II.—In Ptolemy's Tab. I. Europæ, he makes πa 40° 30′, in which case μa becomes 41° 49′; $\mu m = 2° \ 30'$; $\pi m = 1° \ 11'$, and $\pi\mu = 1° \ 19'$. For the present purpose the change is of no importance, but in working with Ptolemy's *sectional maps* there is an important difference. These maps are simply drawn to scale ($\frac{4}{5}$), therefore P disappears, η being brought to λ and m to ϵ, while A coincides with a and L with λ; in this case λ represents both L and η, thus giving $\lambda o = S$, while $\eta = S - \epsilon$, and $\epsilon = S - \eta$. (See Plate I.)

NOTE.—η on V is the true projection of E on T. The excess on C from mo becomes μo, and μm is the actual displacement of $A = 2° \ 26'$. (III.) *cf.* Plate VI. A. Thus Ptolemy's Alexandria is 2° 26′ too far west upon his projection.

LONGITUDE OF LONDINIUM.

LONDON (TRUE) TO ALEXANDRIA (PTOL:)

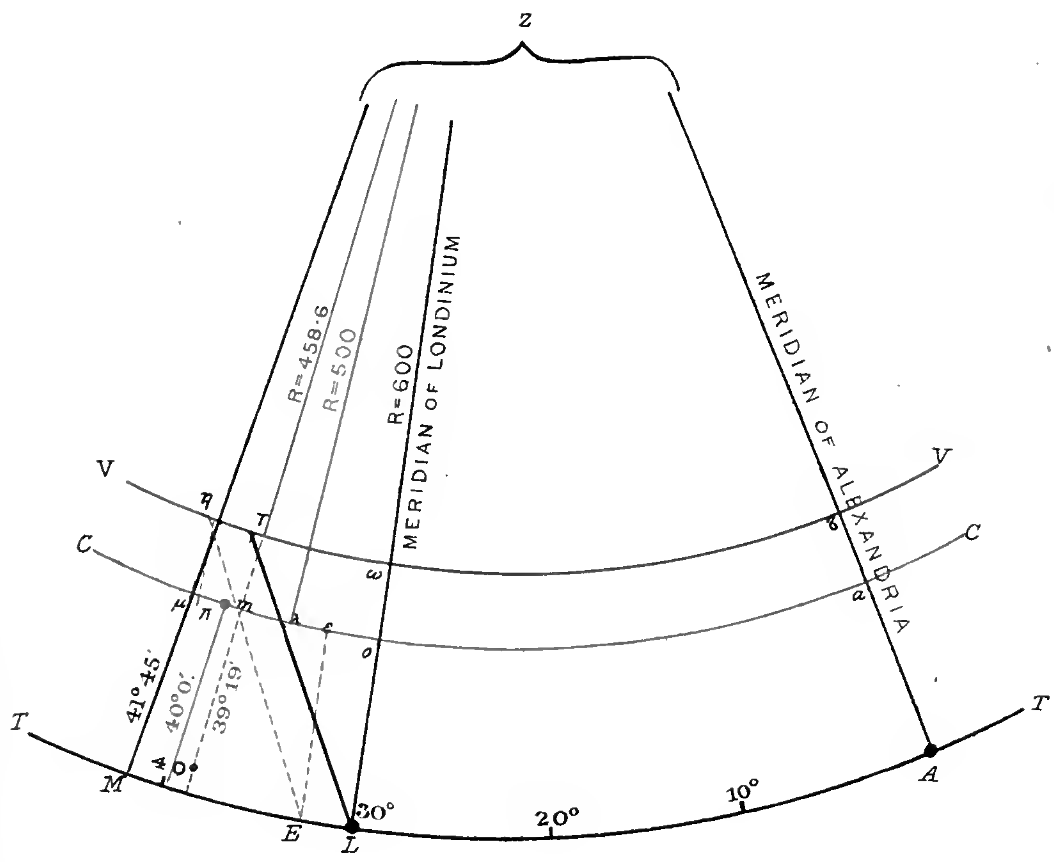


PLATE IV.

PROJECTION ERROR.

EXPLANATION OF PLATE IV.

The projection error originated as follows. In the latter portion of Bk. i. ch. 24, Ptolemy describes his "globular projection."

The first step (see fig. 1) is to determine the centre from which the parallels are to be drawn—$\alpha\beta\gamma\delta$ being a meridian of the earth passing through the equinoxes $\beta\delta$. Divide $\epsilon\gamma$ into 90 degrees, or parts; of which make $\epsilon\zeta$ 23° 50′; then it is required to draw the arc $\beta\zeta\delta$ (the Equator), $\beta\epsilon\delta$ being the ecliptic. ϵ is very nearly his middle latitude, and is therefore used as the centre of the map. The construction employed is sufficiently evident in the figure—the other parallels are to be described from the centre η, at distances determined by $\epsilon\gamma = 90°$.

The next step (see fig. 2) is to construct the projection. *Retaining the scale of fig.* 1, construct $\alpha\beta\gamma\delta$, a right-angled parallelogram, in which $\alpha\beta = \gamma\delta = 180°$, and $\alpha\gamma = \beta\delta = 90°$, and let it be equally divided by $\epsilon\zeta$.

The instructions then are—Divide a line $= \epsilon\zeta$, the central meridian, into 90°, from it set off $\zeta\eta = 16°$ 25′, $\eta\theta = 23°$ 50′, and $\eta\kappa = 63$. Then η is a point on the Equator; through θ is to be drawn the Tropic or Parallel of Syene, through κ the Parallel of Thule, and through ζ the southern limit of the "habitabilis."

So far all is well, and the texts agree; but we are next told to extend the "line of these instructions"—in other copies to produce the line $\zeta\epsilon$ to 181° 50′ at λ, and then from centre λ, with distances κ, θ, ζ, to describe the arcs $\pi\kappa\rho$, $\xi\theta o$, and $\mu\eta\nu$.

Now, if Ptolemy wrote this—as it appears he did, for ζ was plainly at the bottom of the figure, and $\mu\eta$ is afterwards mentioned as an arc of the Equator—no one before Werner constructed the projection without including the error.

Hence we find two forms of correction: first, $\mu\eta\nu$ for $\mu\zeta\nu$, and secondly (Ptolemy certainly intended the arc $\sigma\zeta\chi$), μ and ν are removed to M and N, leaving the Equator with no mark but η, and $\mu\zeta\nu$ becomes correct. But then $\mu\eta$ is no longer an arc of the Equator, and so, in these texts, the $\mu\eta$ drops out, and finally, Pirkheimer, apparently translating from the paraphrase of Werner, omits this mention of the Equator entirely; in this he is copied by Bertius, whose Latin is thus not a translation of his Greek. But Werner also corrected $\zeta\epsilon$ to $\eta\epsilon$, and this correction too was adopted by Bertius, so that the original confusion and error entirely disappeared in that popular edition, which has since been so generally used.

But ζ, in fig. 2, is no longer a point on the Equator, as it was in fig. 1, and the 181° 50′ should have been measured from η. It is to be feared that this slip was due to Ptolemy himself. By this mistake the radius becomes too short and the projection too small by $\frac{\eta\lambda}{\zeta\lambda} = \frac{181°\ 50'}{198°\ 15'} = P$.

Note.—The confusion of ζ in fig. 2 with ζ in fig. 1 was easy, and it should, perhaps, be mentioned that $\mu\zeta\nu$ was the correct notation in his "conical projection," described in the same chapter. Whatever be the explanation, the rectification of this error rectifies his whole projection.

PROJECTION ERROR.

Fig. 1. Fig 2.

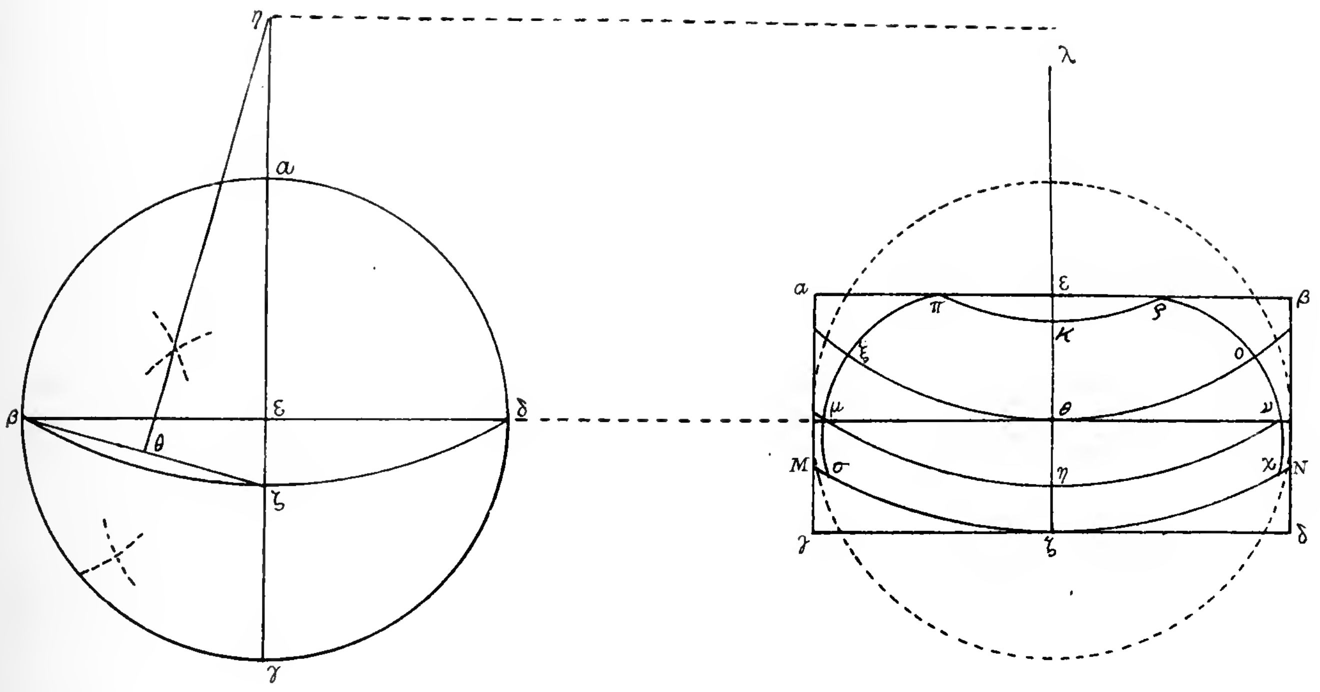


PLATE V.

PROJECTION ERROR.

(Continued.)

EXPLANATION OF PLATE V.

These figures show the relations of Ptolemy's *intended* and his *virtual* projection.

They may be regarded as portions of two spheres one within the other, having radii about 11 and 12 respectively, coincident in the plane of the Equator and in contact at the point *O*.

In the larger figure, drawn to scale, the black lines show the construction of the intended chart, while the red give the virtual one.

Ptolemy's *actual projection* was neither one nor the other of these. By working from the centre of the red sphere, and then using the scale of the black, he really *mapped the larger chart upon the smaller sphere*, or, in other words, graduated the small chart with the scale of the large one.

The smaller figures, *not to scale*, may represent a vertical and horizontal section through the point *O*.

The actual numbers for the reduction of Ptolemy's intervals to the true intervals, and *vice versâ*, are as follows:—

$$\text{Projection Error} = P = \frac{181{\cdot}833}{198{\cdot}250} = 0{\cdot}9172$$

$$\frac{1}{P} = 1{\cdot}0902$$

$$\text{Scale Error from Explanation of Plate I.} = S = 0{\cdot}8320$$

$$\frac{1}{S} = 1{\cdot}2019$$

$$\text{P Combined with Scale Error} = SP = 0{\cdot}7632$$

$$\frac{1}{SP} = 1{\cdot}3103$$

Note.—After this calculation had been fully determined, the value of the Greek foot was more carefully ascertained. This value is—

1 Greek foot = 1·01146 English = 12·13752 inches;

∴ 1° = 600 st. = 364125·6 English feet.

Though this correct determination has come to light, it does not in the least detract from the value of the original calculation, which must be taken as high praise to the accuracy of Mr. Rylands. For,

by the old unity of measurement, Ptolemy's $180^\circ \times S \cdot P = 137^\circ\ 22'\ 23''$

by the new unity of measurement, Ptolemy's $180^\circ \times S \cdot P = 137^\circ\ 23'\ 40''$

Difference, $0^\circ\ 1'\ 18''$

which is less than 1½ miles (1·48) in the whole length of Ptolemy's world [Ed.].

PROJECTION ERROR

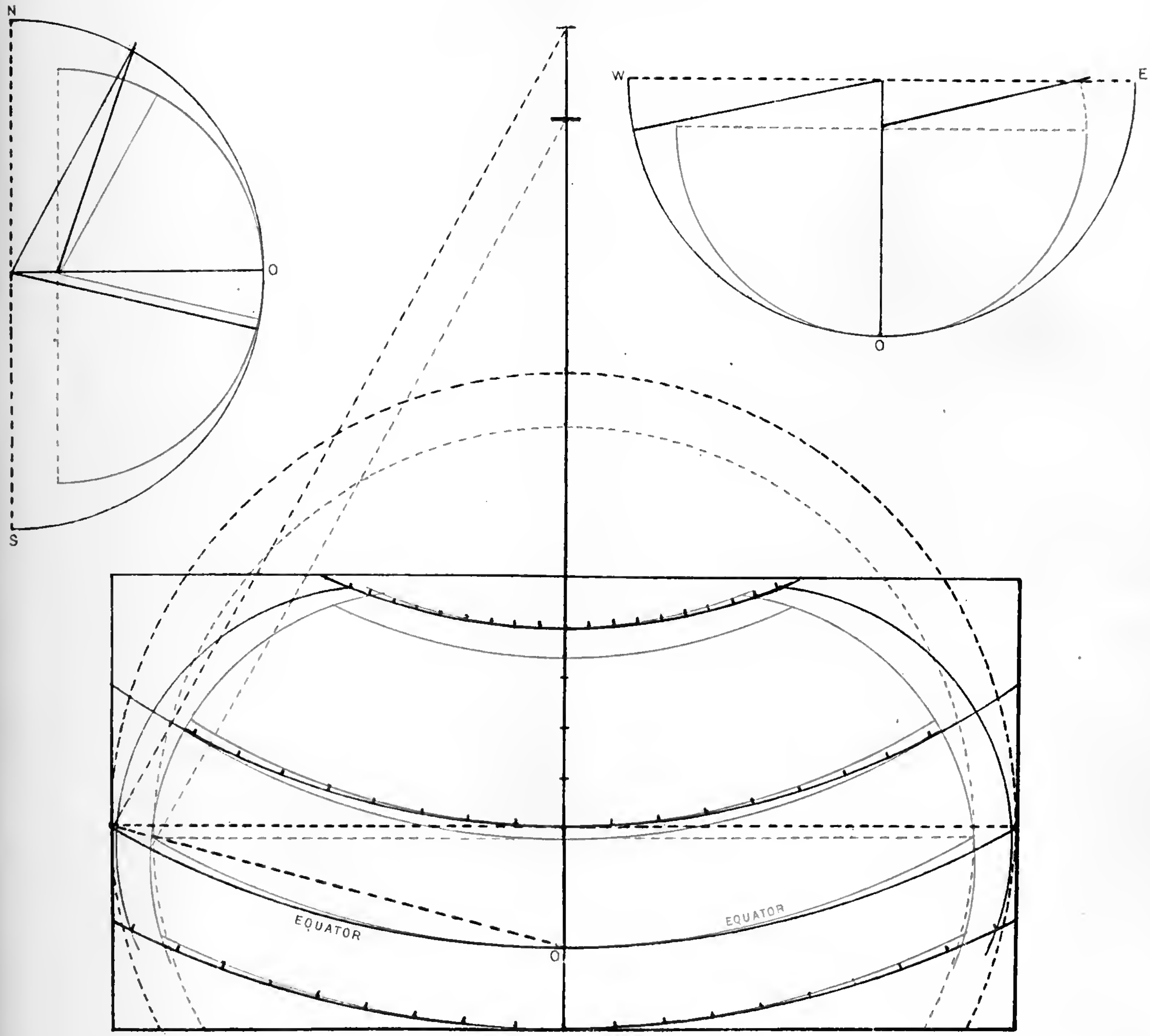


PLATE VI.

TRUE EQUIVALENT OF PTOLEMY'S FIRST MERIDIAN.

EXPLANATION OF PLATE VI.

Let fn and FN be two lines of such proportionate lengths that $\frac{FN}{fn} = P$, and let a and A be any two points in them so placed that $\frac{fa}{FA} = \frac{an}{AN}$, then $FA = fa \cdot P$, and $an = AN \cdot \frac{1}{P}$; for $\frac{fn}{FN} = \frac{fa}{FA} = \frac{an}{AN} = \frac{1}{P}$. Also $an - AN = an - an \cdot P$. If a be made coincident with A, since the ratio is unchanged, excepting as to position, the relations remain the same. But let λ be any other point in fn, then $\lambda n = AN \cdot \frac{1}{P} + \lambda a$, while $f\lambda = FA \cdot \frac{1}{P} - \lambda a$. Further, if λ be made coincident with A to obtain an, knowing FA and λa, $an = fn - (f\lambda + \lambda a) = [FN - (FA + \lambda a)] \cdot \frac{1}{P}$, while $fa = FA \frac{1}{P} + \lambda a$.

Let $P = \frac{181 \cdot 831}{198 \cdot 250}$, and let $\lambda a = 0° \ 5'$.

Now, the ratio of the Ptolemaic to the true scale being known, if a sufficient interval can be found unaffected by Ptolemy's errors, the expansion of the one scale ought to produce the other.

In the western quadrant of his projection, Alexandria is placed 60° 30′ east of his first meridian. As to *longitude* at least, the Fortunate Islands are the Canaries, and Ptolemy's group includes Ferro. Now the centre of Ferro is almost exactly 18° W. of Greenwich. Let it be taken as his first meridian; it is almost certainly true, and can at least be used in all cases alike.

Then λ being coincident with A, $\left.\begin{matrix} fn = 90° \ 0' \\ fa = 60° \ 30' \end{matrix}\right\}$ are given to find an.

A.—True position of Alexandria verified.

First, $90° \cdot SP$	=	68° 41′	
and $FA + \lambda a = 18° + 29° \ 55' + 0° \ 5'$	=	48° 0′	
Therefore (subtracting) $AN - \lambda a$	=	20° 41′	
but $20° \ 41' \frac{1}{SP}$	=	27° 4′	
Further, Ptolemy gives the interval = 90° − 60° 30′	=	29° 30′	
Therefore (subtracting) Alexandria is misplaced on projection by $29° \ 30' - 29° \ 30' \cdot P$	=	2° 26′	(III.)

B.—Verification of $FA + \lambda a = 48°$ as used above.

Ptolemy makes $an = 90° - 60° \ 30' = 29° \ 30'$.

Although 48° gives the true value of an, $48° \frac{1}{SP}$ should produce $fa + (a\lambda - aA) = 48° \cdot \frac{1}{SP} = 62° \ 51'$, and $\lambda a - aA = 2° \ 26' - 5' = 2° \ 21'$. Therefore $62° \ 51' - 2° \ 21' = 60° \ 30' = fa$ (as above).

C.—Londinium.

Londinium is said to be $2\frac{2}{3}$ hours = 40° west of Alexandria, 60° 30′ − 40° = 20° 30′.

Greenwich is 29° 55′ west of Alexandria (true).

Then $60° \ 30' \cdot SP$	=	46° 10′
Less 29° 55′	=	29° 55′
Therefore longitude west of Greenwich of Ptolemy's first meridian	=	16° 15′

Now 18° − 16° 15′ = 1° 45′ (II.), while error from Alexandria is 2° 26′ (III.).

In this last case, however, Ptolemy's Alexandria and the true Greenwich have been used. What is his error between Londinium and Alexandria?

London (Ptolemy's Londinium) 30° from Alexandria (true); $30 \cdot \frac{1}{SP}$	=	39° 19′
Ptolemy's interval	=	40° 0′
Therefore the error	=	0° 41′ (I.) as in Plate II.

Consequently, Ptolemy's Alexandria is 2° 26′ (III.) too far west.

The true Londinium is 0° 41′ (I.) too far east.

And 2° 26′ − 0° 41′ = 1° 45′ (II.) as before.

D.—Ptolemy's First Meridian verified.

Ptolemy's Fortunate Islands are placed 1° east of his first meridian.

Therefore longitude of Ptolemy's first meridian (from above)	=	16° 15′ W.
Less $1° \cdot SP$	=	0° 46′
Therefore (by subtraction) true longitude of Ptolemy's Fortunate Islands	=	15° 29′ W.,

which is almost exactly true of the *Grand Canary*, and differs but little from centre of the group.

Reckoned from a, therefore, f is − 2° 26′ (III.); from Londinium it is − 1° 45′ (II.). Therefore 16° 15′ + 1° 45′ = 18° W., which *proves* the position of Ptolemy's first meridian to be 18° W. exactly.

PROJECTION ERROR.

TRUE EQUIVALENT OF PTOLEMY'S FIRST MERIDIAN.

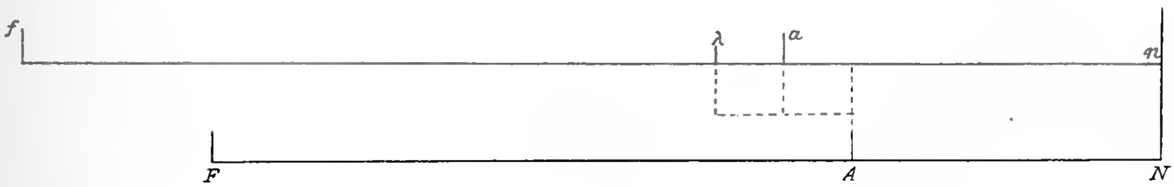


PLATE VII.

TRUE EQUIVALENT OF 180° (PTOL.).

EXPLANATION OF PLATE VII.

(True equivalent of 180° of Ptolemy.)

Make $\frac{fo}{FO} = \frac{fa}{FA} = \frac{an}{AN} = P$. From A set off $A\omega = ao$ and $A\phi = af$; then $\phi f = \omega o = Aa$.

Again, make $F'O' = \frac{21}{20} f\omega$ and $F'N' = fn = no = N'\omega'$. From ω' set off $\omega' n' = N'O'$ and $N'F' = nf$, then $f'F' = n'N' = \omega' o'$; also $f\omega = \phi o - 2Aa$.

Lastly, make $a'n' = an$ and $A'N' = AN$; then $a'A' = N'n' - Aa$.

It remains now only to determine how far the scale SP will satisfy the whole 180° of Ptolemy.

A.—180°, including errors.

180° SP	=	137° 22′
Interval from Ptolemy's western limit to Greenwich	=	16° 15′
Therefore interval to Ptolemy's eastern limit from Greenwich	=	121° 7′ E.

B.—180°, using the Ganges.

The most easterly position on Ptolemy's projection which, up to this point, can be adopted without dispute is the Ganges. The position of *Cambericum* (the middle one of Ptolemy's ostia) may be fixed with sufficient accuracy.

Error at Cambericum	=	10° 50′
Less error at F (= 1° 45′ and λa = 0° 5′)	=	1° 50′
Net error at Cambericum	=	9° 0′
Longitude of 180° from A	=	121° 7′
Therefore equivalent of 180°, if error as at Ganges,	=	112° 7′ E.

C.—Reverse of B.

If the error is the same at 180° as at Ganges, the inverse calculation should produce 180°, or the difference will be the difference of their errors.

Equivalent from B	=	112° 7′
Alexandria (true) east of Greenwich	=	29° 55′
(Therefore subtracting) 180° east of Alexandria (true)	=	82° 12′ E.
But 82° 12′ $\frac{1}{SP}$	=	107° 42′
Add (FA = 60° 30′ – London to Greenwich = 0° 6′)	=	60° 24′
Therefore equivalent of 180°	=	168° 6′
Less	=	180° 0′
Therefore error at 180° = $f'F' = \omega' o'$	=	11° 54′
and 11° 54′ . SP = 9° 5′		

D.—180°, less net errors and 90° – 90° P.

By Ordnance Survey data 180 S.			=	149° 47′
West of Greenwich	=	18° 0′		
Error at Greenwich (= 41′ + 5′ = 46′) + net error at 180° (= 9° 5′)	=	9° 51′		
and 90° – 90° P.	=	7° 27′		
(Adding) west of Greenwich, errors and 90° – 90° P.	=	35° 18′	=	35° 18′
Therefore true equivalent of 180° (east of Greenwich)			=	114° 29′ E. (IV.)

E.—(180° – gross errors) × SP

				180° 0′
West of Greenwich	=	18° 0′		
Londinium to Greenwich	=	0° 6′		
Error at 180° from C	=	11° 54′		
Gross errors	=	30° 0′	=	30° 0′
Subtracting (180° – gross errors)			=	150° 0′ E.
150° SP = true equivalent of 180°			=	114° 29′ E. (IV. as before.)

F.—180°, independently of Cambericum.

Ptolemy reckoned his distances from Alexandria (see p. 34). Therefore, if his projection be superinduced upon a modern map, and the two made coincident at Alexandria, Ptolemy's would be – 2° 26′ 30″ at 0° = ϕf and + 2° 26′ 30″ at 180° = ωo. The actual interval of the true map covered by it, and therefore included in it, having regard to this error, would be reduced by (2° 26′ 30″) × 2 = 4° 53′.

Then	180° · SP	=	137° 22′
	Less 18° 0′ + 4° 53′	=	22° 53′
	True equivalent therefore	=	114° 29′ E. (IV. as before.)

Lastly, the same result IV. can be deduced from A and B.

From A—121° 7′ – (1° 45′ + 4° 53′)	=	114° 29′ E. (IV.)
,, B—112° 7′ + (2° 26′ 30″ – 5′)	=	114° 28′ 30″.

Now the true error at 180° = $\omega' o'$ = 9° 5′ being known, A and a may be neglected. On a modern map graduate 18° W. to 114° 29′ E. into 180°, and these lines should represent the true positions of Ptolemy's meridians upon the true earth. Any departure from them will be his absolute errors of longitude from all causes.

Parallels of latitude on scale SP (making Alexandria 31° N.) will similarly give his absolute errors of latitude. The latitude of Alexandria was known. The error, according to Lelewel, was – 0° 14′ 17″, and it was thus placed upon the map. In plotting from it, the effect of P upon Londinium was reduced practically to about + 1° 40′ 0″, but we have found ϵ (see Plate XVIII.) – 1° 25′. (Subtracting), therefore the outstanding error is + 0° 14′ 48″, which almost exactly corrects Ptolemy's error of Alexandria, so that London was placed with almost absolute accuracy.

PROJECTION ERROR.

TRUE EQUIVALENT OF 180° PTOLEMY.

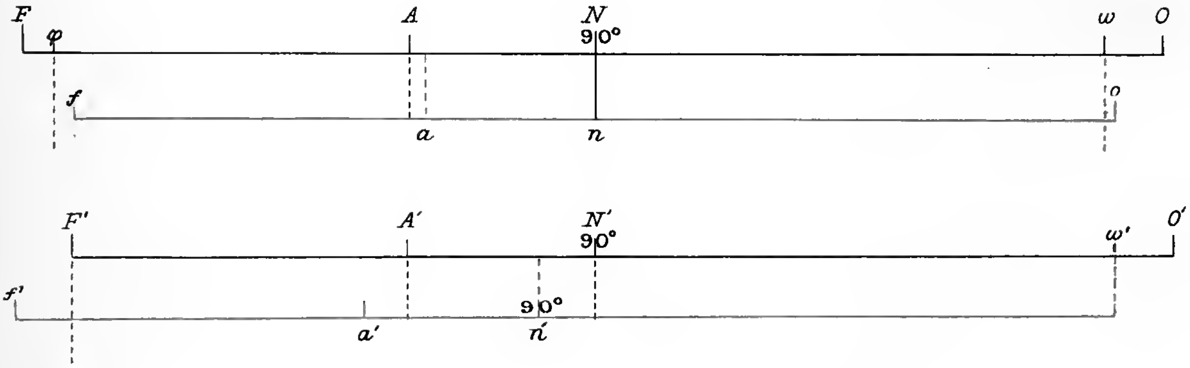


PLATE VIII.

PTOLEMY'S LONGITUDES AND LIMITS.

EXPLANATION OF PLATE VIII.

This Plate represents a map graduated, as described in the Explanation of Plate VII. The black lines attached to the red or Ptolemaic stations indicate the amount of error dependent upon *Projection*, while the intervening spaces between these lines and the black stations show the errors of *measurement*.

Longitude 90° of Ptolemy is the line of no distortion in longitude, and the arrows indicate the direction of his errors.

The following positions seem to call for special note.

The *Fortunate Islands* include the Hesperides, that is, both the Canaries and the Cape Verdes—they have the longitude of the one and the latitude of the other.

The errors at *Rome* are interesting in connexion with the distortion of Italy. These errors are probably due to Astronomical observation.

The error eastward begins at *Indus*, and is doubled (= 10° 50′) at *Ganges*. It was a matter of discussion whether the coast of India ran north and south or east and west, and Ptolemy adopted the latter conclusion, hence the breadth is extended, and the peninsula almost disappears.

Iabadium, which was noted for its metals, becomes Banca, *not Java*, which was far beyond his limits.

Calculation puts *Thyne Metrop.* in longitude 114° 29′, which is a little S.-W. of the centre of Borneo.

Cattigara agrees almost exactly with *Kottawaringin*, on the S.-W. coast of the same island.

Zabe is exactly *Cambodia*, and Ptolemy's coast line justifies the result.

Fretum Herculaneum is traditional.

The other stations are very correct: but *Ceylon* was enlarged to form the known straits. The Coast of India was believed to run east and west, hence the large error at the *Ganges* and the enlargement of Ceylon.

London is 1° 19′ E. It is desirable to note this on account of more local inquiries.

Ptolemy's "habitabilis," as will be seen, extends from 18° W. to 114° 29′ E.

PTOLEMY'S LONGITUDES AND LIMITS

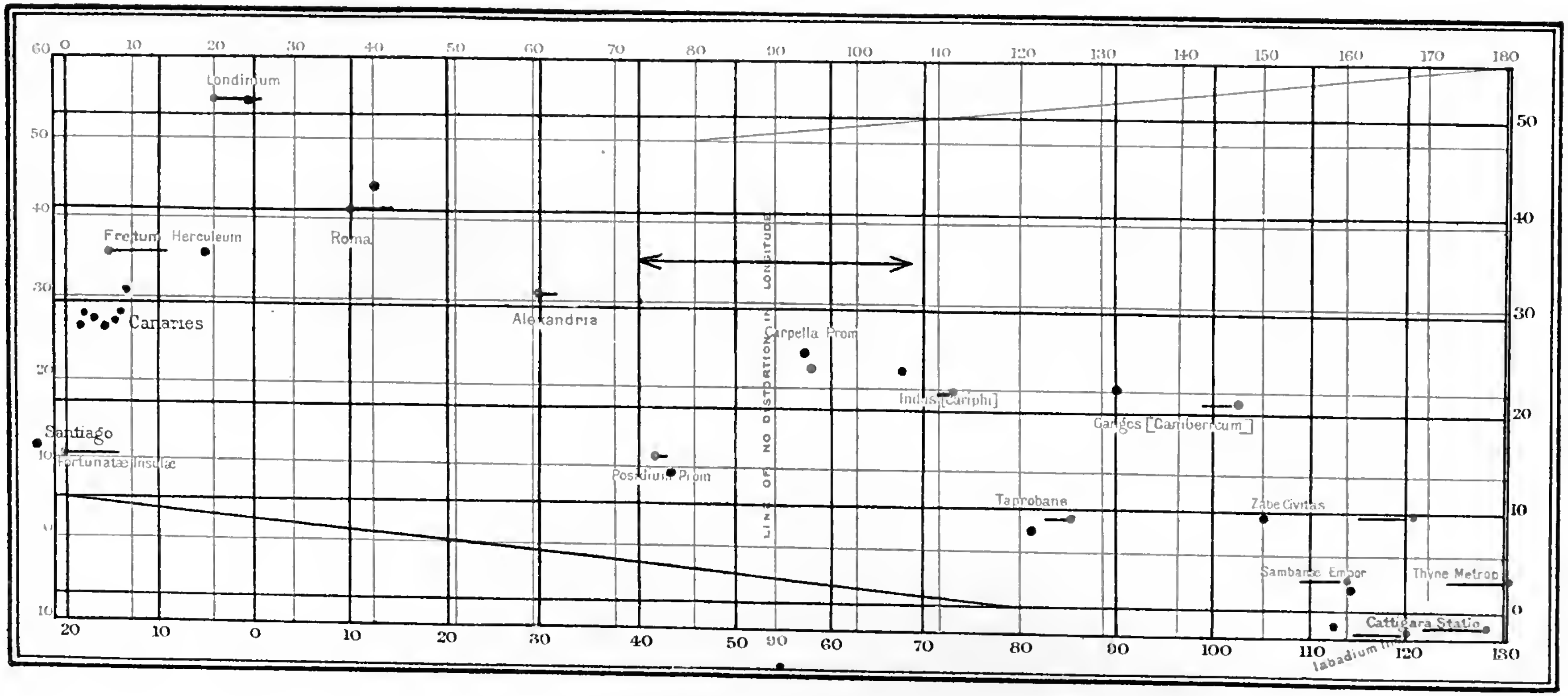


PLATE IX.

TRUE AREA OF PTOLEMY'S PROJECTION.

EXPLANATION OF PLATE IX.

On a Mercator's Chart of the World draw *longitudes* 18° W. and 114° 29′ E. of Greenwich. Divide this interval into 180°.

Through *Alexandria* draw the latitude 31° N., and from it, by a Table of meridional parts, set off, to the scale of the longitudes obtained, the Equator, and thence the other parallels from 63° N. to 16° 25′ S.

The included area may then be cut out, mounted, and graduated. The larger the scale of the chart the better.

The differences between Ptolemy's positions and those obtained from a chart thus graduated are the total errors resulting from all causes whatsoever. Thus longitude 20° passes through the Bristol Channel instead of London, or Londinium was placed 4° too far west, while 60° 30′ is fully 5° west of Alexandria.

But such a chart has a further and very much more important value. On comparing it with Ptolemy's Map of the World, it will be seen that the central meridian 90° of the *chart* coincides almost exactly with 80° (Ptolemy)—really with his 79° 51′ = *C*.

If then a scale *ACB*, = 180° of the chart, be so divided that *AC* = 90° 9′, and *CB* = 89° 51′, and *AC* be divided into 80°, while *CB* is made 100°, the two halves of the chart, with regard to this scale, ought to agree with Ptolemy's map, or rather as we have now, for the first time, a true map measured by his actual scale, the outstanding errors are reduced to a minimum—it might almost be said to those of measurement alone. These, in the case of land marches especially, must be small; and those which resulted from the longer sea voyages are mainly included in the construction.

The scale of longitudes being once obtained, for all future purposes those according to the scale *A*, *C*, *B*, need only be inserted in the map.

Compare now the dotted lines with Ptolemy's map, remembering that actual measures were made from the *true* Alexandria, but the graduation from the Alexandria of *Ptolemy*, the traditional extension of the Mediterranean, the discussion as to the coast of India, that the Caspian Sea includes also the Sea of Aral, the omission of Sumatra, the shortening of the Malay Peninsula (about 7°), and the consequent change in the latitudes of Zabe, Cambodia, Cattigara (Borneo), and Iabadium Ins. (Banca), and lastly, in special reference to local inquiries, that the north of Scotland is cut off (see Plate XV., and Supplement, p. 77).

This Plate completes the general investigation; for it would involve almost endless calculations to rectify *all* the positions on Ptolemy's maps; but the general principles have now been stated by which any portion of such work may be done. In this volume these corrections have only been applied to the district which first gave rise to the whole inquiry—and any other locality may be similarly treated.

TRUE AREA OF PTOLEMY'S PROJECTION.

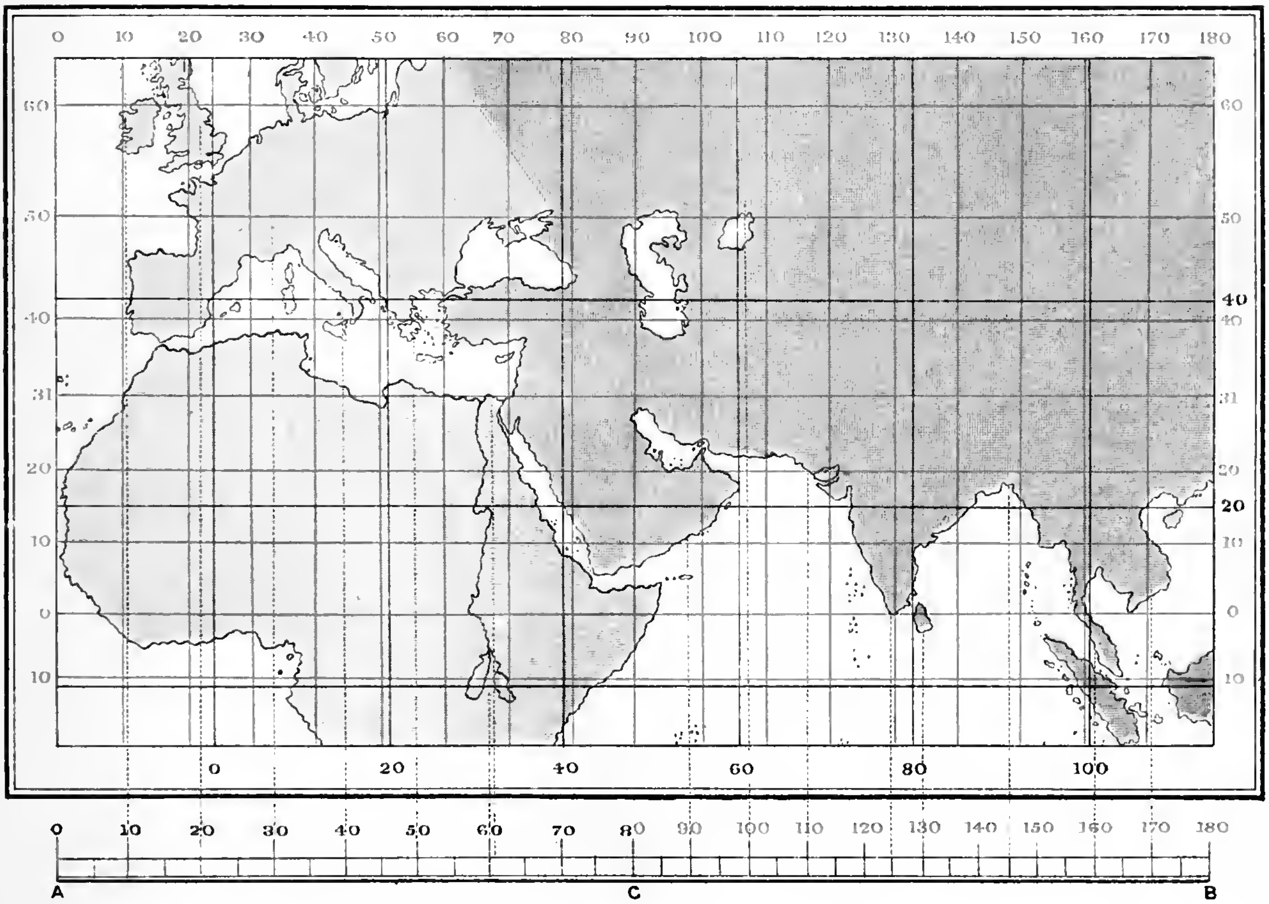


PLATE X.

ITUNA TO TISOBIUS.

EXPLANATION OF PLATE X.

(See Chap. IV.)

Plate X. is designed to show how much care is needed in selecting a map of Ptolemy, so as to form just judgment of the details of his work. The coast-line, according to eight editions, extending from Ituna to Tisobius (the Solway to Traethmawr), is given to exhibit the grotesque and absurd changes which have been made. These maps extend from 1460 to 1730, and are fairly typical of the various editions. They begin with the map of Nicholaus Donis, who, because he defined the terms of Ptolemy and adhered to the definitions he had adopted, has produced a coast-line which may be taken confidently to represent Ptolemy's conclusions more truly than any of the others. The Pirkheimer Map (1513–1541) was largely influenced by Nicholaus Donis. It will be noted that in Donis, Buckinck,[1] and Pirkheimer, the Tisobius follows Ianganorum Prom. Much study of both MSS. and text, as well as the maps of these and other editions, led to the conclusion that Donis, in the edition of 1482, was most to be relied upon.[2] It is needless to say that it is by no means free from error, but it suffers less from careful criticism than any of the others; nor was Nicholaus de Donis an unlikely man to do such good work. The following account is from the *Res Literaria Ordinis S. Benedicti* of G. Zugelbauer:—

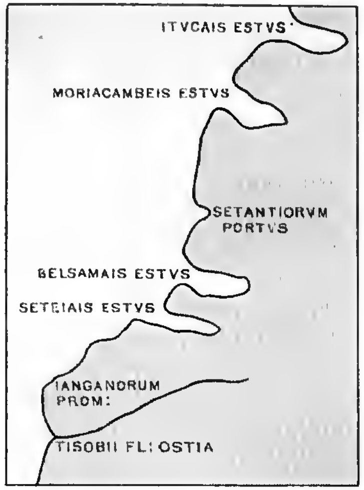
"Alius Nicholaus . . . cognomento *de Donis* aeternâ nominis famâ sub medium seculi xv. in Germaniâ claruit. Erat is monachus Richarbachensis, congregationis novae Bursfeldensis, divinarum scriptarum non ignarus, et Graecè atque Latinè insigniter doctus, Philosophus, Mathematicus, et Cosmographus nulli suo tempore secundus. Cum suá igitur vigilantiâ Ptolemaei tabulas cosmographicas multis ante seculis deperditas reperisset, sagacitate suá ita eas instauravit, et admirando opere cum picturis et novis tabulis elegantissime ordinatoris diligenter conexit, ut novi operis potius compositor quam reparator existimitur. Quod in septum partis seu libros divisum, Paulo II. Pontifici Maximo dedicavit."

[1] It is believed that the coast-lines of Donis and Buckinck represent the earliest forms we possess of the maps of Ptolemy and of Agathodæmon.

[2] The Donis map may be contrasted with that in the Mount Athos MS., the original of which is in the British Museum.

ITUNA TO TISOBIUS, PTOLEMY.

C. 1460-1486.

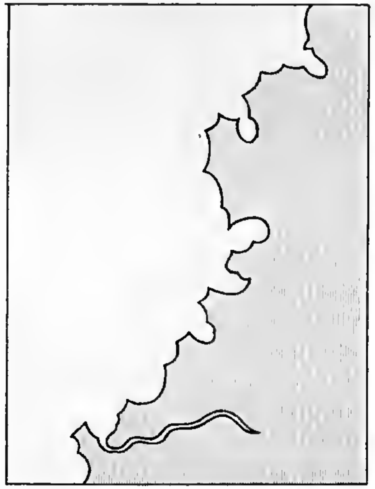


N. DONIS.

1478-1490.

A. BUCKINCK.

C. 1480.

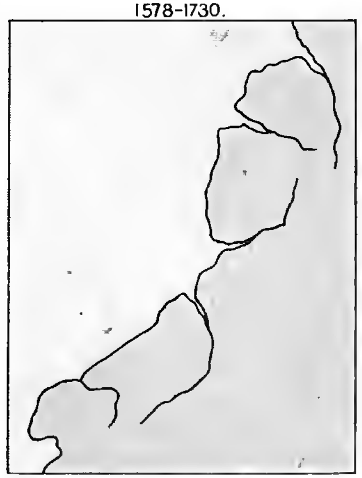


BERLINGHIERI.

1511.

SYLVANUS

1578-1730.



MERCATOR

1513-1541.

PIRKHEYMER

1540-1545.

MUNSTER.

1548-1598

GASTALDO.

PLATE XI.

PTOLEMY'S ACTUAL BELISAMA ERROR.

EXPLANATION OF PLATE XI.

The Coast from Carnarvon to Cumberland.

[Stations are numbered 1 to 5 from north to south; B is St. Bee's Head; A is Point of Ayr.]

Ptolemy's intervals calculated $= \lambda - \lambda' + (\eta - \eta')$.

Data. For scale, stations 4. – 5. = 1° latitude, 2° longitude.

For coast-line, 5. A and B from Ordnance Maps.

Longitude.

Between 5. and 4. $= 1° 49' + A$ to 4. $= 0° 7' = 1° 56'$: therefore error $-$ 0° 4'

or 4 is placed nearly 3 miles too far west.

Latitudes.

Between 5. and 4. $= 0° 41' + 0° 26' = 1° 7'$; therefore error + 0° 7

or 4 placed nearly 8 miles too far south.

Between A' and B' $= (59° 11' - 57° 49') + 0° 31' = 1° 53'$ = 113′

120

But intervals 5. to 4. and AB overlap; therefore (subtracting) $A4$ = 1′

therefore $A'B'$ = 119′

Therefore total interval 5. B. $= (1° 6' - 0° 1') + (1° 54' - 0° 1')$ = 2° 58′

that is, 5. B. = (5. 4. – 1) + (AB – 1′), where 4 represents distortion of 4.

This map wss originally drawn for the investigation of the two positions of 5 and 5.2. given in different editions of the Geographia. On noticing the scale error $\left(\frac{A'B'}{A'\pi}\right)$ it was discovered that it could be used for the investigation of the Belisama error. But positions marked ● were put in without reference to it.

The Belisama.

It was found that $\frac{A'B'}{A'\pi} = \frac{119}{119 - 21} = \frac{600}{500}$.

Therefore $A'\pi$ of Ptolemy = $A'B'$ modern, and the errors must balance. (See Scale in margin.)

Now $\epsilon o = \eta\omega$ or $\epsilon\eta = o\omega$;

therefore $\epsilon\eta$ is mapped on the scale of AB, *not* on the scale of AB.

On the same scale $\pi\epsilon$ should be $\frac{6}{5}$ $B'o$.

By Ordnance Map $B'o = 31'$, and $\frac{6}{5}$ 31′ = 37′

but $\pi\epsilon$ = 19′ = $A'\eta$

therefore (subtracting) the actual error = 18′

This is compounded of – 6′ and + 12′ = 4. 4. and ϵo.

Let the error between A' and η be 18′ – 6′ = 12′, and that between ϵ and π becomes – 12′ + 6′

then $\epsilon\pi$ = 19′ + 18′ = 37 (as before).

Thus the errors do balance, and the whole coast is consistent from A to B. B. 1. must be 18′ too small, and 4. to 3. is 12′ too large.

The Belisama can only be the Mersey. (See Plate XII.)

PTOLEMY'S ACTUAL BELISAMA ERROR

Data: For Scale 4.-5.-1° Lat. 2° Long. For Coast Lines 5. A & B from Ordnance Maps.

PLATE XI.

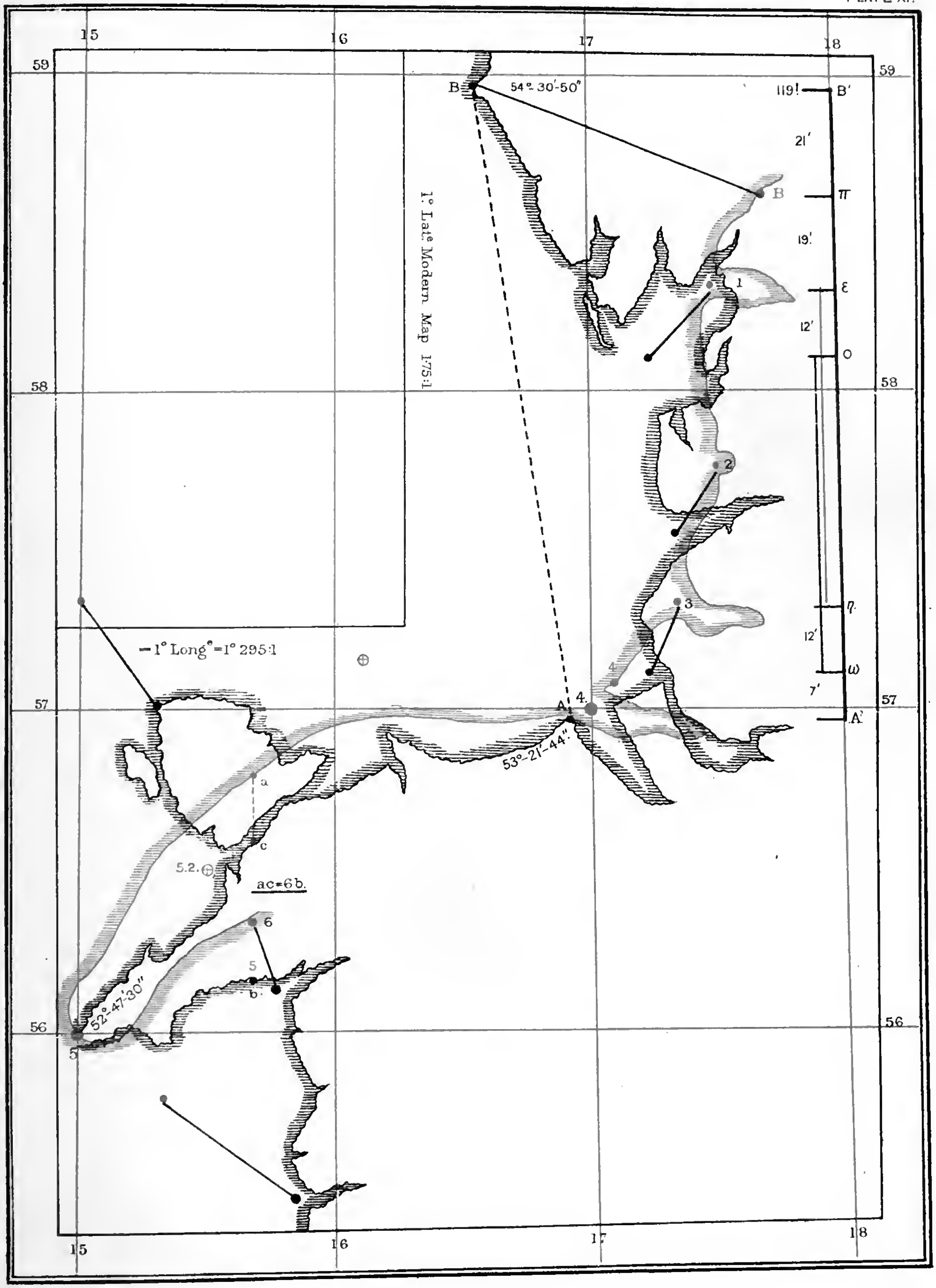


PLATE XII.

ERROR AT SETEIA ESTUARY.

EXPLANATION OF PLATE XII.

Fig. 1.—Positions (to Scale).

The Calculations are as follows :—

		Point of Ayr.	Brachy-pwll.	
True latitude	L	53° 22′	52° 48′	$L - L' = 34'$
True latitude of Alexandria	a	31° 10′	31° 10′	
	$L - a$	22° 12′	21° 38′	
	$\frac{6}{5}(L - a)$	26° 39′	25° 58′	
Scale Variation	S	+ 4° 27′	+ 4° 20′	
And Ptolemy's Alexandria = $L - 10'$		53° 12′	52° 38′	$\lambda - \lambda'$
Ptolemy's latitude calculated	λ	57° 39′	56° 58′	41′
	π	57° 0	56° 0′	$\epsilon - \epsilon'$
Ptolemy's actual error	ϵ	− 0° 39′	− 0° 58′	19′
	S	4° 27′	4° 20′	$\eta - \eta'$
Ptolemy's apparent error		+ 3° 48′	+ 3° 22′	26′

From which $(\lambda - \lambda') + (\epsilon - \epsilon') = 60' = \pi - \pi'$

but $(\lambda - \lambda') + (\eta - \eta') = 67'$

Seteia Estuary is placed 1′ N. of Point of Ayr. Therefore, for Seteia, we have

$$(\lambda - \lambda') + (\epsilon - \epsilon') = 42' + 18' = 60' = \pi - \pi'$$

and $$(\lambda - \lambda') + (\eta - \eta') = 42' + 25' = 67'$$

Now Ptolemy gives the interval $\pi - \pi' = 60'$; therefore the position of these two stations is determined, and a comparative scale has been obtained.

Ianganorum Prom. is Brachy-pwll if Seteia is the Dee, which identifications have never been questioned by editors.

Fig. 2.—Intervals.

Here Ianganorum Prom. is made identical in position with Brachy-pwll without errors; in other words, $5 = L' = \lambda' = \pi'$. From this point the intervals given are $L - L'$, $\lambda - \lambda'$, and $\pi - \pi'$ to scale.

In this case λ is not coincident with L, but is 7′ above it $= (\eta - \eta') - (\epsilon - \epsilon')$.

The comparison here includes the apparent errors, and therefore though the scale of the parallels in the previous Plate is correct for Seteia and places north, they are practically 7′ too low: latitude 57° is placed at L instead of λ. Had this been otherwise, the errors would have been 12′ + 7′ = 19′, which is the actual error found both in Ptolemy and Plate XI.

Fig. 3.—The Error.

In the above, as in Plate XI., the amount and position of the error is disguised. But ϵ is mapped at E, not at o; and on the Plate scale AB, we have

$$AE = 18' + 7' = 25'$$

$$EB = 18' - 7' = 11'$$

therefore difference $= + 14'$ for amount of error.

In Plate XI., stations 1, 2, and 3 appear to be too high by this amount. This is only apparent, however, and due to the process.

The direction and position of the error is established by the following independent results of other calculations :—

Places	λ	L	ϵ
St. Bee's Head	59° 4′	54° 33′	− 24′
Belisama (Mersey)	57° 44′	53° 27′	− 24′
Seteia (Dee)	57° 39′	53° 22′	− 39′

Plainly the coast from St. Bee's Head is correct. At the Mersey, $\epsilon = - 24'$, and the Dee is placed 15′ too low : therefore $\epsilon = - 39'$, instead of − 37′, as found from the map. The difference of 1′ is in Plate XI. The estuary stations are slightly uncertain.

ERROR AT SETEIA ÆST.

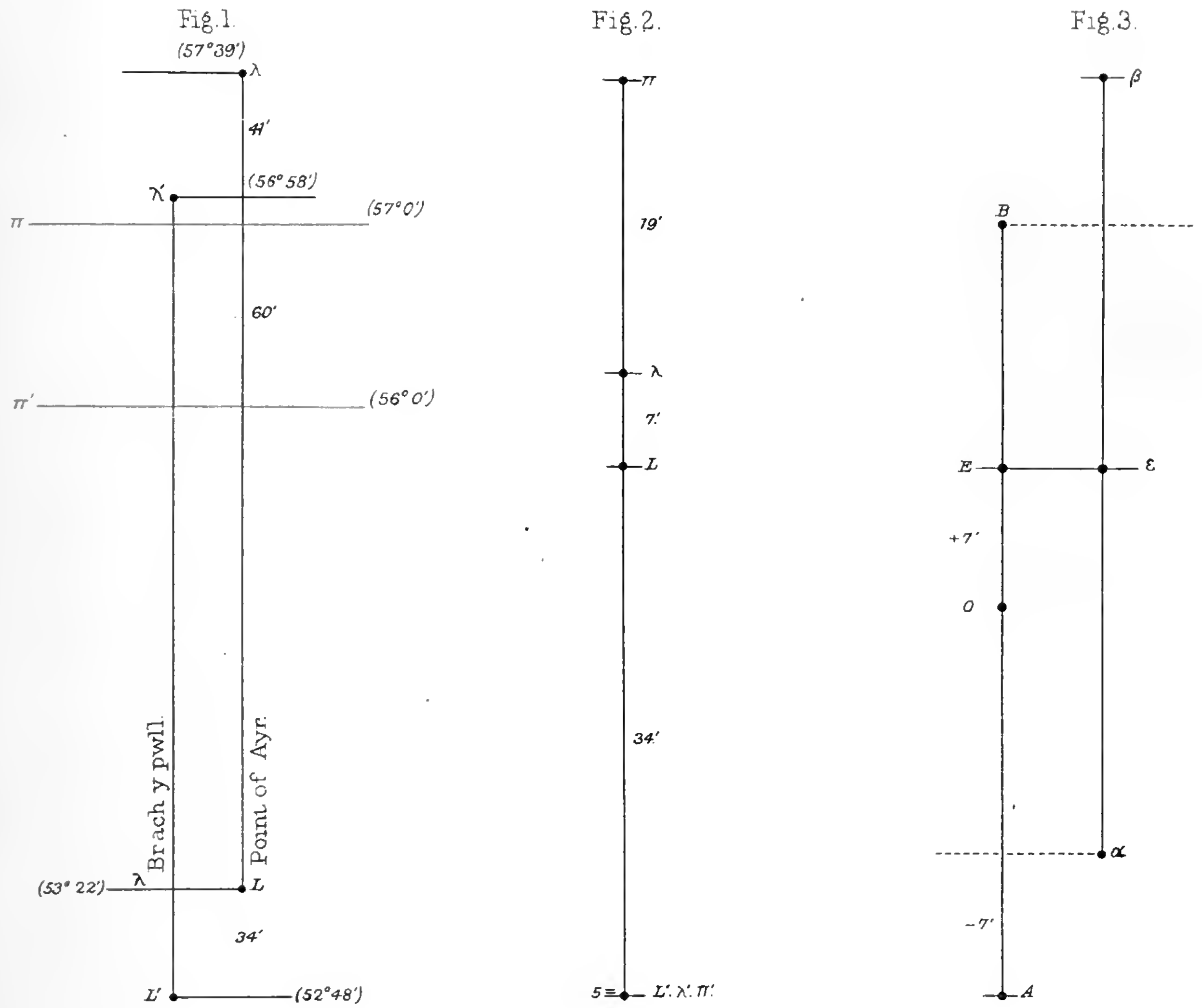


PLATE XIII.

STATIONS ON THE SOUTH COAST.

EXPLANATION OF PLATE XIII.

The scale is Ptolemy's. The *dotted lines* rising from it mark the positions of the places, the names of which are attached. All these are calculated from *Londinium of the map.*

The *horizontal lines* show the amount of correction required to bring them to the coast stations of the map.

From Southampton to the Tamar these corrections are uniformly about half a degree; and these errors would disappear if the *Londinium of the projection* were made the datum. The stations, however, being investigated as on the map (not as on the projection), the errors of the former are shown.

There is no pretence that the coast stations were fixed by itinerary measurements; they were the result of a coast survey. Inland stations, determined by measure from the Londinium of the map, would originally be inserted about half a degree westward of the rivers upon which they stood; for instance, Dunium, Iscalis, and Tamarus, which have been moved eastward as the easiest mode of placing them right. Uzela, if so changed, would be about *V*, and thus would be on a branch of the river so named.

The real error commences westward of the Tamar, and increases rapidly.

Calculation shows an error (probably in Ptolemy's data) of about 1° in the latitude of *Damnonium Prom.* If this be corrected, having regard to the conditions, some such outline as fig. 2 is the result. Damnonium Prom., being removed from *B* to *C*, becomes the Lizard. If *Kenion* depended upon *B*, it will now be removed from *D* to *E*, and will show the normal error of half a degree. The whole error then is confined to the points *A* and *C*. Voliba is a coast station, but undescribed: if a river mouth, it is slightly in error, but is consistently placed for the Fowey, as between the Tamarus and Kenion of the map.

The error of the *Promontories* is interesting. In the investigation of the longitude of Londinium (Plate III.), and afterwards in the projection error (Plates II.–VII.), it was proved that, on the projection, Alexandria is given longitude 60° 30′ instead of 58° 4′, *i. e.* is misplaced 2° 26′. All points, therefore, which have been determined from Alexandria, if correctly placed, will have longitudes relatively 2° 26′ greater than those which were independently placed upon the projection. In the case of London, as already pointed out, the error of 2° 26′ was reduced to 1° 45′ by an error of 0° 41′. Now, as determined from Alexandria and from Londinium, the errors of *A* and *C* are as follows:—

From Alexandria in 60° 30′	*A* – 2° 20′	*C* – 2° 30′
,, Londinium in 20° 30′	*A* – 1° 39′	*C* – 1° 49′
,, Alexandria in 58° 4′	*A* – 0° 6′	*C* + 0° 4′

It should be specially noticed in this Plate that we reach a point where the Alexandrian measures have interfered with the positions, *B* and *C* having much enlarged errors in the contrary direction (westward instead of eastward). If the error of Alexandria be corrected—that is, if Alexandria be read 58° 4′, instead of 60° 30′—this error disappears. In other words, *B* and *C* are true to the projection, while the other stations are referred to Alexandria as his primary datum, but called 60° 30′.

NOTE.—There are two early readings of Isca Fl., 17° and 17° 40′. There are also two for Iscalis, 16° and 16° 40′. But copies which have the river at 17° 40′ have the station at 16°, and *vice versa*. The suggestion is, that some early editor has, by mistake, removed Isca eastward, instead of Iscalis. In this case, we should have Isca at *a*, and it would be the Axe at *e*.

In many copies a station, or stations, will be found thus—

Isca, ...	17° 30′
Leg. II., Augusta, . . .	17° 0′

If there were really two stations, then Isca would be on the Axe at *a*, and Leg. II. on the Exe; and as a result, we reach eventually *four* "Iscas"—the Axe, the Exe, the Axe, and the Usk.

STATIONS ON THE SOUTH COAST.

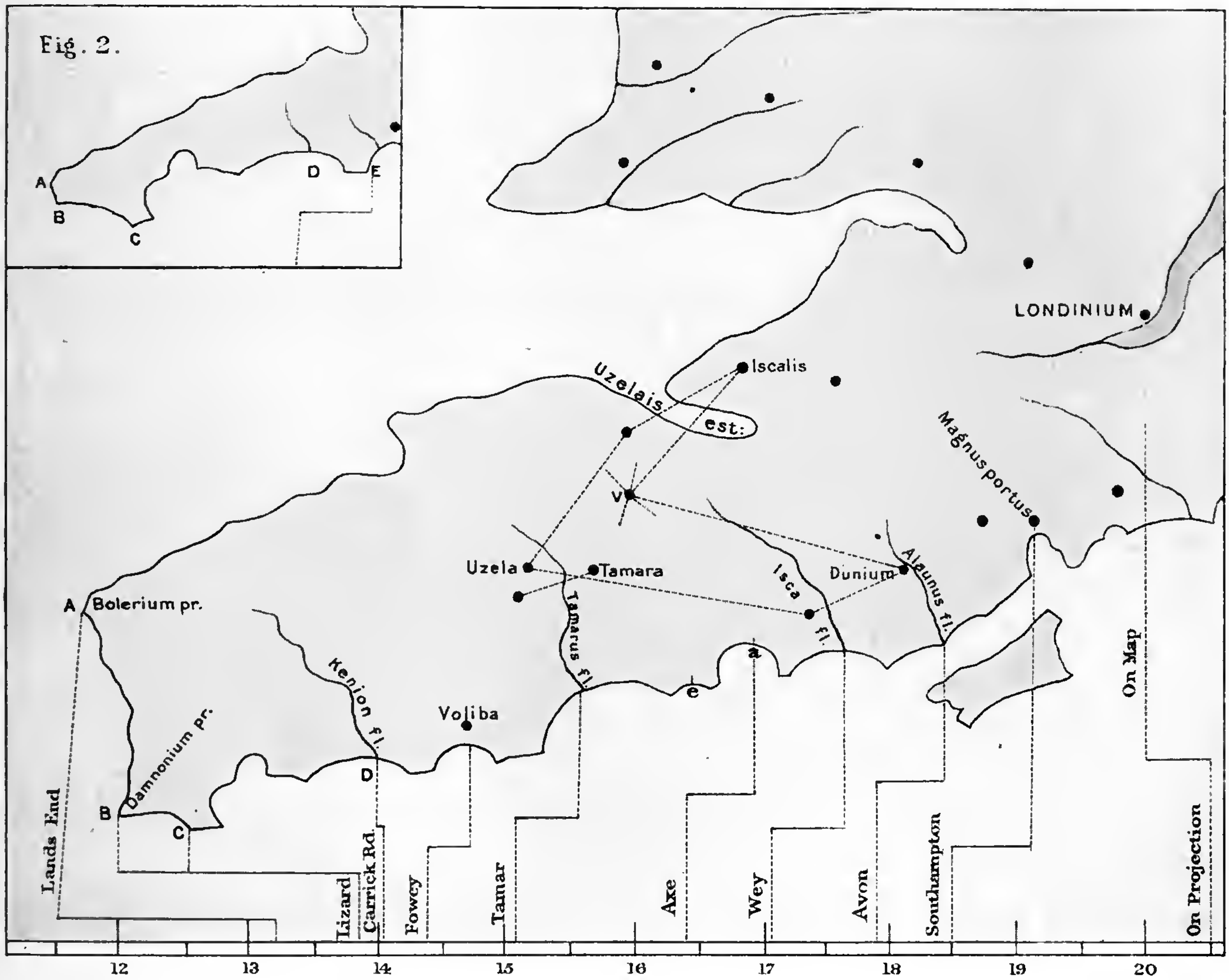


PLATE XIV.

STATIONS ON THE EAST COAST.

EXPLANATION OF PLATE XIV.

Between *Catterick* and *Berwick* 1° 10′ was omitted by Ptolemy in transferring his fundamental points or stations (see Plate XX.) to the map of Albion. This is the interval *AA*.

Caturactonium was originally 57° 30′, and *Londinium* 54°. These were changed to 58° and 54° 30′, which represent the true positions. The coast stations *near each* were corrected, but the others were left the half degree too low. See *BB*, *CC*, *DD*, &c.

The most remarkable correction in the series is Nucantium Prom., which has been made the North Foreland, overlooking the fact that Ptolemy's station is on the mainland, and not on Thanet.

Lastly, Novus Portus is *not* New Haven, as the name might lead us to infer. With these corrections, the coast is consistent. The stations from North to South are as follows:—

Bogderiais Est.	Forth River.	
Albani Fl. ost.	Tweed River (see also Plate XX.).	
Vedræ Fl. ost.	Tees River.	
Dunus Sinus.	Filey Bay.	
Gravantuicorum Port. sin.	Bridlington Bay.	
Ocelum Prom.	Spurn Head.	
Abi Fl. ost.	Humber River.	
Megaris Est.	The Wash.	
Gariennis Fl. ost.	Ouse River.	N.B.—There is no station between
Sidumanis Fl. ost.	Stour River.	the Ouse and the Stour !
Iamasiais Est.	Thames River.	
Toliatis Ins.	Sheppey.	
Convennos Ins.	Thanet.	
Nucantium Prom.	South Foreland.	
Novus Portus.	Dungeness.	
Trisanconis Fl. ost.	Cuckmere River.	

Note.—The map employed for the black outline was afterwards found to be slightly faulty. The Wash, 52° 55′, and the Stour, 51° 55′, are both too high. Further, the true latitude of Beachy Head is 50° 44′, not 50° 50′.

STATIONS ON THE EAST COAST.

Maps on the same Scale proved data Catterick and London.

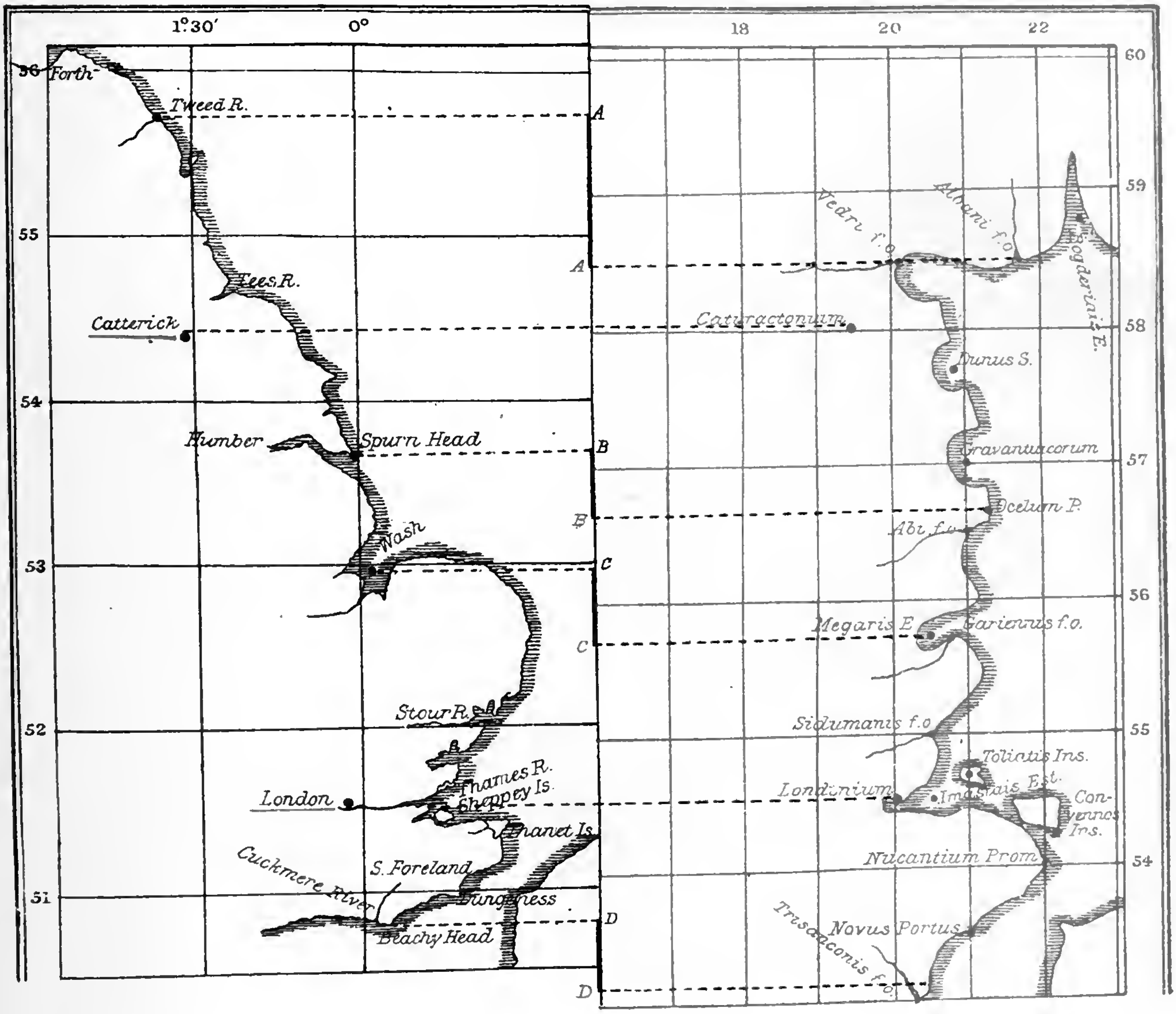


PLATE XV.

DISTORTION OF DUNCANSBY HEAD.

EXPLANATION OF PLATE XV.

It has been proved that *intervals* fixed by measurement are affected by errors of measurement only; thus, AD would be mapped $a\delta$, and $aC\delta = \frac{5}{6} ACD$.

But where a station has been determined by astronomical observation, the displacement will amount to the errors of observation plus the scale variation; thus (with an error $= DH$), AD would be mapped $ao = a\delta - (\delta\epsilon + \epsilon o)$.

Therefore observed stations should have their apparent errors increased by the amount due to scale variation.

Again, as such stations would be mapped by, and in accordance with, the observations, the above result should be exact and consistent.

Consequently, in the case of an error due to observation, it may be determined when the true angular distance is known—the scale variation being uniformly $\frac{5}{6}$.

Apply these considerations to Duncansby Head (D). If that station had been accurately determined by measurement, it would be mapped at δ, or if by observation, at ϵ; and any deviation from these positions will be the amount of error of measurement or of observation. It is in fact mapped at o.

The question then is: Was the point o determined by measurement or by observation?

I.—According to Measurement.

In latitude 58° 40′, 1° longitude	= 36 miles.	
Cape Wrath to Duncansby Head	= 2°	= 72 miles.
Ptolemy gives the interval, 8° 20′	= $\frac{5}{6}$th 300 miles	= 250 miles.

Hence the error would be 250 : 72, or very nearly $3\frac{1}{2}$: 1. Therefore it is plain the interval has not been determined by measurement.

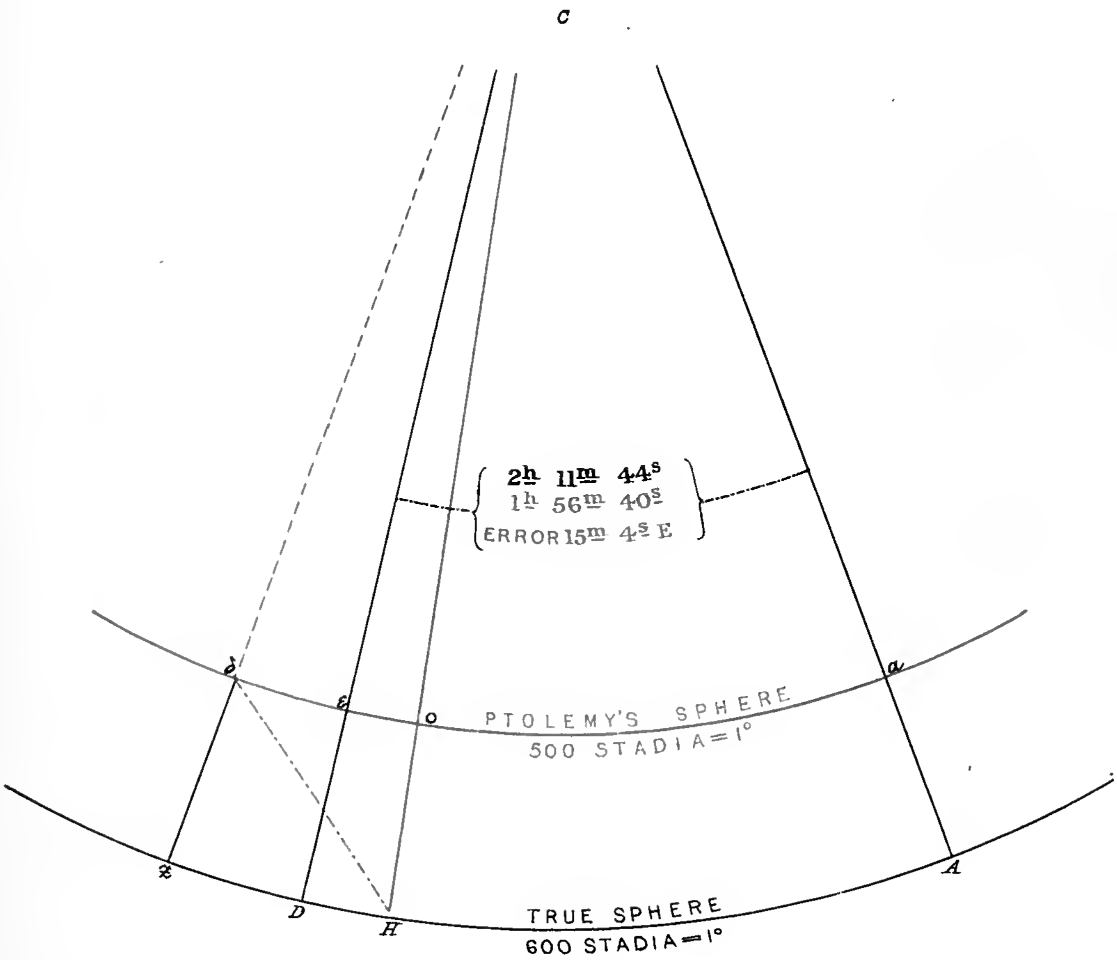
II.—According to Astronomical Observation.

True longitude, ACD	=	29° 55′ + 3° 1′	=	32° 56′
Less Ptolemy's longitude, aCo	=	60° 30′ − 31° 20′	=	29° 10′
Error of Observation, DH		. . . . .	=	3° 46′
Scale variation, S	=	$\frac{1}{5}$th 32° 56′	=	6° 35′
Therefore (by addition) total apparent error, δo			=	10° 21′
Or Ptolemy gives the longitude			=	31° 20′
Longitude calculated	=	60° 30′ − (32° 56′ × $\frac{6}{5}$)	=	20° 59′
Therefore Ptolemy's actual error			=	10° 21′ (as before).

In this case the error of observation—supposing it all real—when reduced to time, amounts only to 15 minutes 4 seconds. This must be considered small in the case of an eclipse of the moon, when the appliances of the time are taken into account, especially as there is an error of 44 minutes = 11° in that which Ptolemy himself records (see p. 26), and which he used in preference to itinerary measures. Lastly, had there been such an observation recorded, Ptolemy would certainly have used it (see pp. 37, 41, 47; and Geog., bk. i., ch. 4).

Unless Ptolemy gratuitously reduced his latitude of this station 4°, a gnomon observation is the only explanation of the change, and is strongly confirmatory of an astronomical basis of observation for latitude also. The reduction of 4° leaves only a probable margin of error. *(See Supplement to this Plate, p. 77.)*

LONGITUDES. DISTORTION OF DUNCANSBY HEAD.



D. Duncansby Head. *A.* Alexandria.

PLATE XVI

SCOTLAND, WITH NORMAL DISTORTION.

EXPLANATION OF PLATE XVI.

Having been compelled to reject the usual idea (Note 1) that Scotland had been turned over, "making the length to be almost directly east and west" ("Britannia Romana," p. 361), this Plate was an early attempt to explain the distortion simply by proportionate corrections. The "Meteoroscopic solution" did not present itself until a year later, when it was worked out as in Plate XV.

The corrections were here called normal, because they were made consistent with many general considerations, but they are by no means exact or sufficient—x is too far west, hence the blue area north of the Moray Firth and Loch Torridon is too large, Aberdeen and Elgin too small, and $E\ C$ is still too long; in short, it was necessarily a compromise, the fairest practicable with the facts then at hand.

Its claim to a place in the present series is to show pictorially the character of the distortion, and in its measure to confirm Plate XV. It detected the errors, but it did not explain them.

A more correct comparison may be obtained as follows:—

On a modern map of Scotland, very close to its centre, a point will be found about latitude 56° 47′, where a degree of $\frac{\text{latitude}}{\text{longitude}} = \frac{20}{11}$. As these are also the Ptolemaic proportions, the map is, so far, true for both, and the needful scale has been obtained.

Then from Point of Aird draw a line through the bend of the Tweed at Coldstream. This is Ptolemy's longitude 21°; and making the latitude of Point of Aird 61° 20′, the other parallels and meridians may be ruled in according to scale, any departure of Ptolemy's positions from these lines is the amount of his errors from all causes. His coast stations may then be added and his map drawn. (See Frontispiece and Preface.)

The inland stations are almost entirely itinerary, and the area bounded by the coast has, as it were, slid under them, without effect upon their positions. Thus Trimontium, as the Eildon Hills (but the name refers to a Station, not to the Hills), seems to be placed somewhere in Lanark; while, when the due correction is applied, its errors differ only 2′ or 3′ from those at Caturactonium. In any general inquiry it might be adopted as a fundamental station.

The portion coloured *red* in the Plate is Ptolemaic, and Ptolemaic only.

The portion coloured *blue* aimed at showing the Ptolemaic Scotland after the corrections.

Where the colours overlap, these two likewise overlap.

While the east coast was much distorted the west remained unaffected, thus:—

True Latitude of D Calculated. (Note 2.)

1. From St. Bee's Head.

$$L = L' - \tfrac{1}{6}(\pi - \pi') + \epsilon = 57^\circ\ 32'\ 32''$$

2. From $\pi = 61^\circ\ 20'$

$$L = \tfrac{5}{6}(\pi - a) + a + \epsilon = 57^\circ\ 42'\ 20''$$

Mean, $= 57^\circ\ 37'\ 26''$

Latitude, Point of Aird, Skye, $= 57^\circ\ 40'\ 0''$

Therefore error $= 0^\circ\ 2'\ 34''$

Hence Novantum Prom. is the Point of Aird. (As to longitude, see Plate XIX.)

Note 1.—The conclusion arrived at was—Horsley's solution amounts to this: Ptolemy, who mapped the whole length of the Island with an error of less than 40 miles, in mapping a distance of little more than 40 miles made an error nearly equal to the whole length of the Island.

Note 2.—The D of this Plate should not be confounded with the D of Plate XV.

SCOTLAND WITH NORMAL DISTORTION.

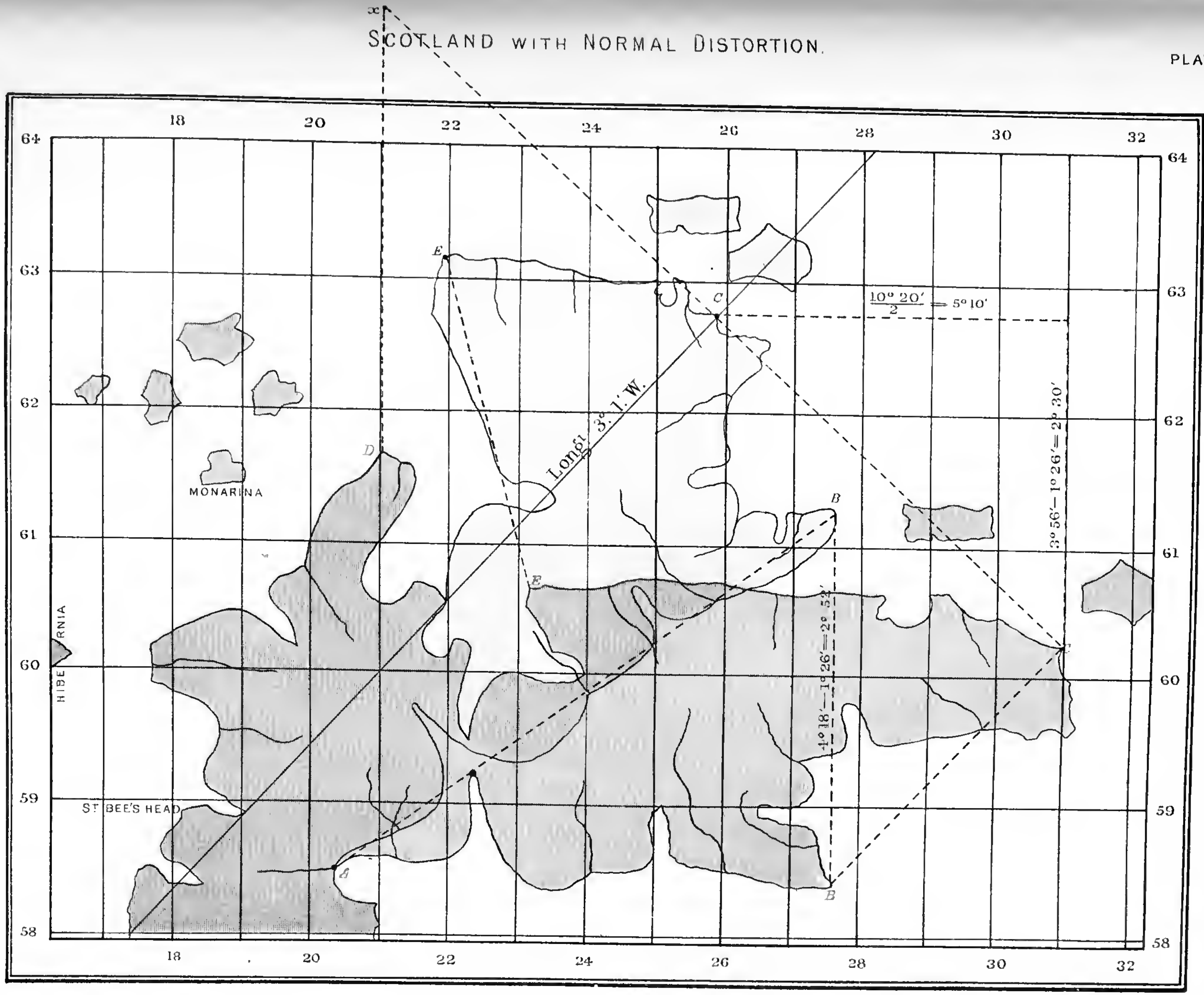


PLATE XVII.

DISTORTION OF AREAS BY PROJECTION.

EXPLANATION OF PLATE XVII.

The object of this Plate is to show the extent of the secondary projection error pointed out in Ch. IV., § 4.

This Plate is a portion of Ptolemy's projection corrected, in which his central meridian (90°) bisects the area *ABCD*, which area shows the smallest amount of distortion between Ptolemy's parallels, 50° and 63°. The second area to the extreme west, also marked *ABCD*, is the distorted area, including the British Isles. Having compared this distorted area with the same area bisected by 90°, Ptolemy's rectangular Tab. 1 Europæ may be reproduced by drawing *AC′BD′*. If we bisect *AB* and *CD*, and draw the line *mo*, we get what would be the central meridian in Ptolemy's map, which, it will be seen, does not coincide with the meridian 20° of the sphere, and it is from this difference that the westerly error = 0° 30′ is found in the position of Londinium on the map—in short, the meridian 20° on his map is drawn at longitude 20° 30′ on his sphere. After the insertion of the fundamental stations, the others were plainly *plotted* in from them, and in the main they include the error. This fact explains why the stations on the south coast derived from Londinium should show the same amount of error in the contrary direction (see Plate XIII.). It was from the same cause that the latitude has an error of – 0° 30′ (see Explanations of Plates XIV. and XX.).

Ptolemy's Projection—Distortion of Areas

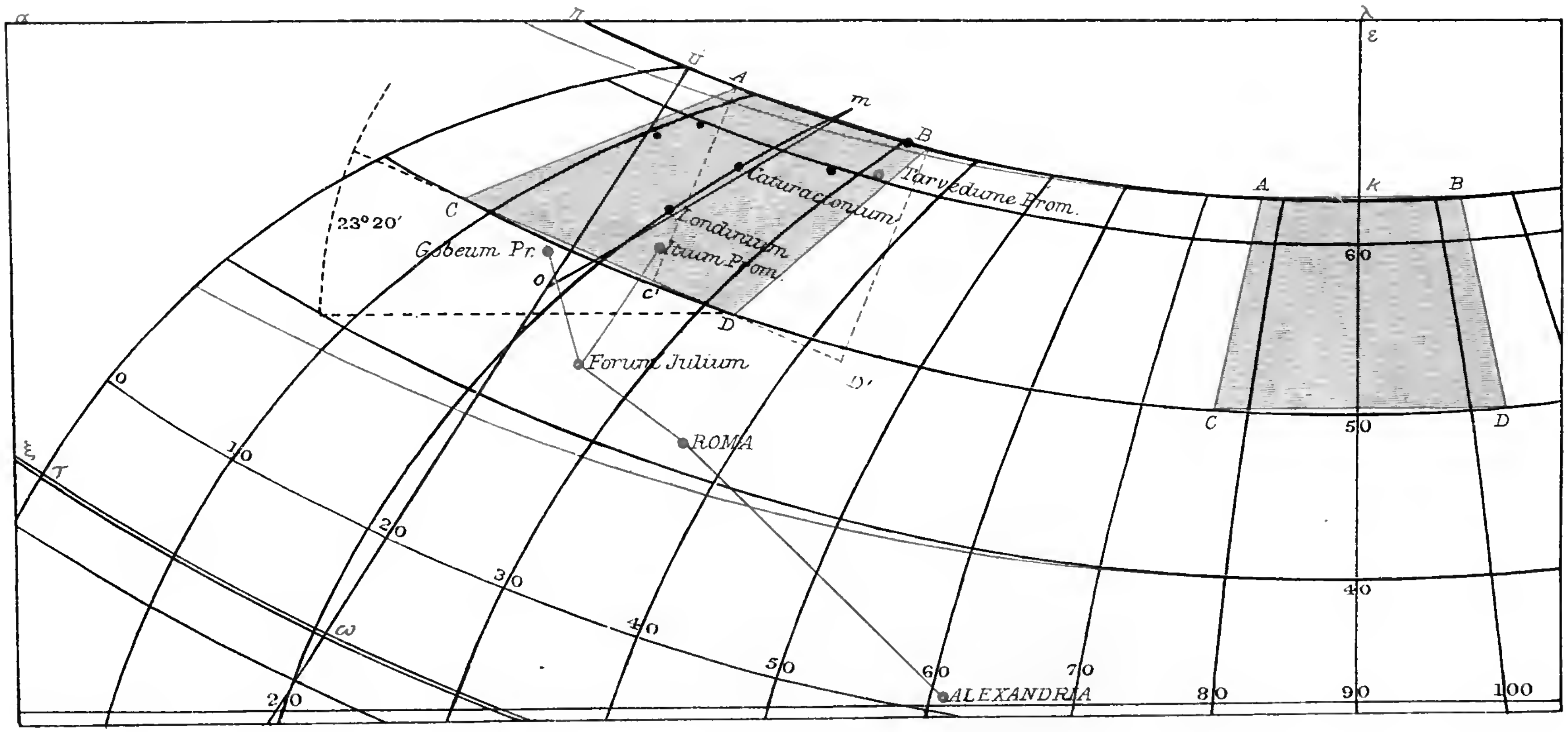


PLATE XVIII.

LATITUDE OF LONDINIUM.

EXPLANATION OF PLATE XVIII.

By including $\zeta\zeta$ in $\eta\zeta$, Ptolemy shortened his intended radius of the parallels in his projection by $\eta\eta$. Hence (as proved in Plate V.) his projection error $= \frac{\eta\zeta}{\eta\zeta} = \frac{181 \cdot 83}{198 \cdot 15}$, the effect being an alteration in his *latitudes* of almost exactly $\frac{1}{11}\lambda$.

Thus $TD'\chi$ is the true projection of latitude 54°, as obtained by a cone cutting the sphere at χ and χ' equidistant from A, which is Ptolemy's $\epsilon\zeta$. The intervals on this parallel will be to those on the Equator as $\frac{\cos}{\tan}$ 54° : 1. Then, by removing η to $H = \Delta D'$, we obtain $D.D'$, Ptolemy's distorted projection, and $\left(\text{by making } D'C = \frac{54^\circ}{11}\right)$ his calculated parallel. The intersection of these by $H\pi$ gives $\delta\pi$, in which $\lambda\pi = \epsilon$ and $\delta\lambda = \eta$, in accordance with the calculation.

Calculation of Londinium.

True latitude of London	=	51° 31′
True latitude of Alexandria	=	31° 10′
Therefore true latitude of London from Alexandria	=	20° 21′
but $S = \frac{20^\circ\ 21'}{5} = 4^\circ\ 4'$ + Ptolemy's Alexandria = 31°	=	35° 4′
Therefore (adding) λ	=	55° 25′
and Londinium π	=	54° 0′
Therefore (subtracting), $\epsilon = \lambda\pi$	= −	1° 25′
but $P = \frac{54^\circ}{11}$	=	4° 54′
Therefore $\eta = \delta\lambda$	= +	3° 29′

See Explanation of Plate XX., where it is shown that Ptolemy's Londinium should have been in latitude 55° 10′ and not 54°, from which 55° 10′ − 54° = 1° 10′, or η + 1° 10′ = + 3° 39′. But this excludes the correction of Alexandria = − 0° 10′, so finally we have on the projection, as above, η = + 3° 29′, and ϵ = − 1° 25′.

Note.—This Plate should be compared with Plate IV., to which it is an interesting supplement.

LATITUDE OF LONDINIUM.

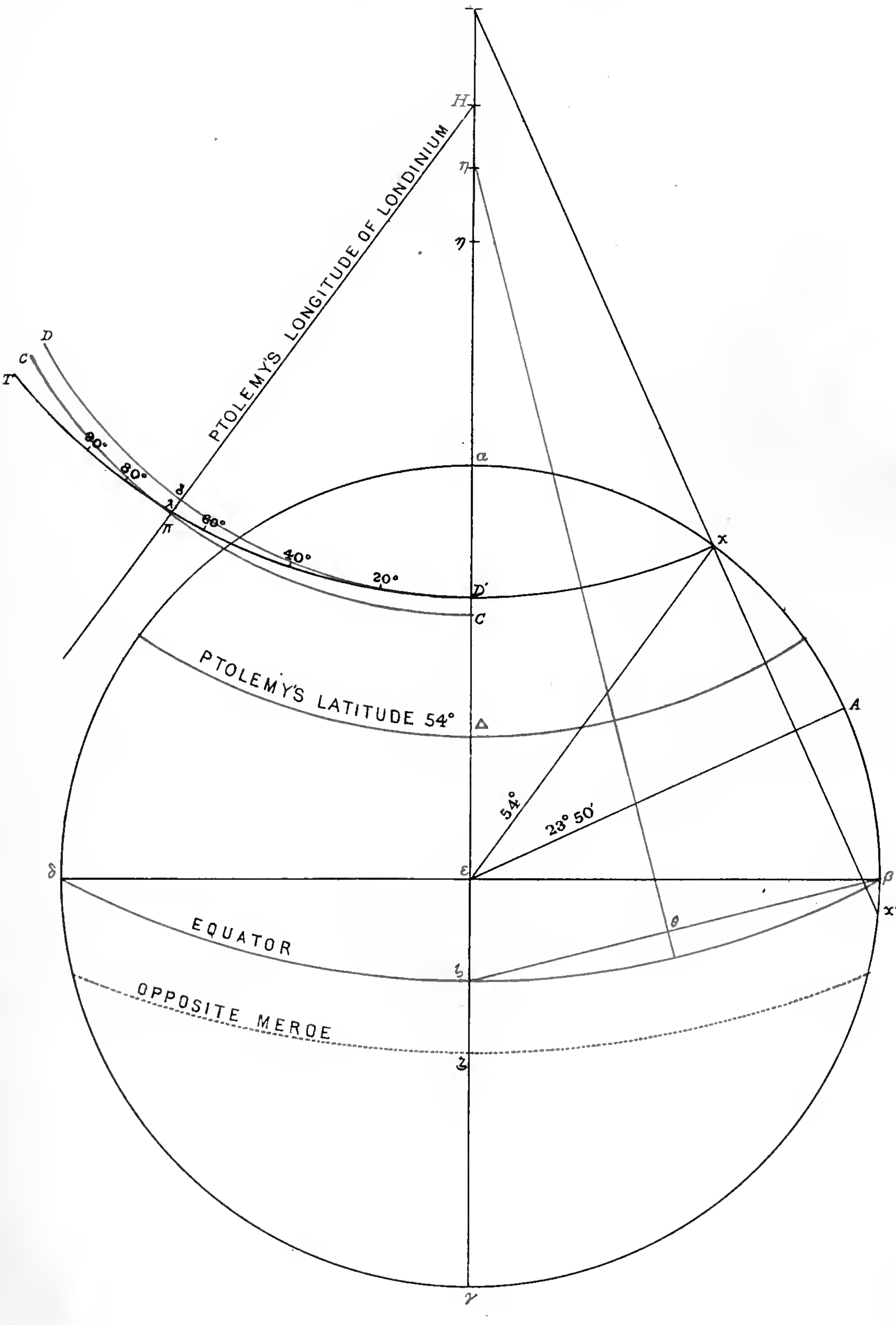


PLATE XIX.

DISTORTION BY PROJECTION.

EXPLANATION OF PLATE XIX.

The meridian 3° 1′ W. was traced from *Duneansby Head* to *Brean Down* (7° 19′), the lowest point determinable upon Ptolemy's map. A line was then drawn from *Tarvedume* to that point, and it very nearly agreed with the distortion angle (see p. 34). Further, since Dublin Bay has the same latitude as the Point of Ayr, a perpendicular to the line, mentioned above, was drawn, and *passed through both these stations*—therein agreeing with Ptolemy's map.

But Tarvedume being a fixed point, had any position other than *Brean Down* been selected, it is plain that the distortion angle, &c., could not have been obtained.

Now, why should Brean Down yield these results?

In addition to actual displacements, Ptolemy's systematic errors were in his *scale* and in his *projection*. Therefore, the former being allowed for, the positions assigned to his stations ought to coincide with his projection.

The exceptions will be—(*a*) *Positions astronomically determined*, which will differ from the projection by the errors of observation. Tarvedume correctly observed would have been placed at *T*. (*b*) After the fundamental stations or points had been fixed, subordinate stations were put in from them. The latter would be affected by the results of detailed survey.

Now, in Ptolemy's map the meridian of Tarvedume passes from *S* to Brean Down, and by *R*. *C*. and *N*. to *O*. The explanation is somewhat as follows:—

From *S* to Brean Down. Though the survey was imperfect, the distortion angle is less by 22°.

,, Brean Down to *R* (Ribble). Country being known, *error is nil.*

,, *R* to *C* (Carlisle). Country little known; angle is same as *NO*.

,, *C* to *N*. Gap caused by error of Tarvedume – error total.

,, *N* to *O*. Angle really true for Tarvedume.

The following points should be noted: If *Tarvedume* has an additional error of 3° 46′ east, it should be a little to the east of the distorted meridian of London. In the above projection it is *exactly* on that meridian. The scale of the meridians is slightly too small.

Secondly, it seems clear, also, that the red line from *O*, and not the black one from *T*, has controlled the distortion. For the red line drawn from Tarvedume to Brean Down, with its perpendicular, divides Elbana and Manapia, and passes through the point of Ayr—agreeing with Ptolemy's map.

Lastly, the east of Scotland, England, Ireland, and the Island Scitis have been governed by *Tarvedume*, while Dumna and the rest of the Orcades, with the north-west of Scotland, are true to the smaller angle. Here, again, we have Novantum Prom. agreeing with Point of Aird, Skye (see Plate XVI.).

Hence we learn from the above considerations that Ptolemy's *normal angular distortion* for the *British Isles* is 44° 15′ E., that this distortion has been transferred from his sphere to his map (Plate XVII.), and that it has been changed only under certain circumstances which can be explained.

DISTORTION BY PROJECTION.

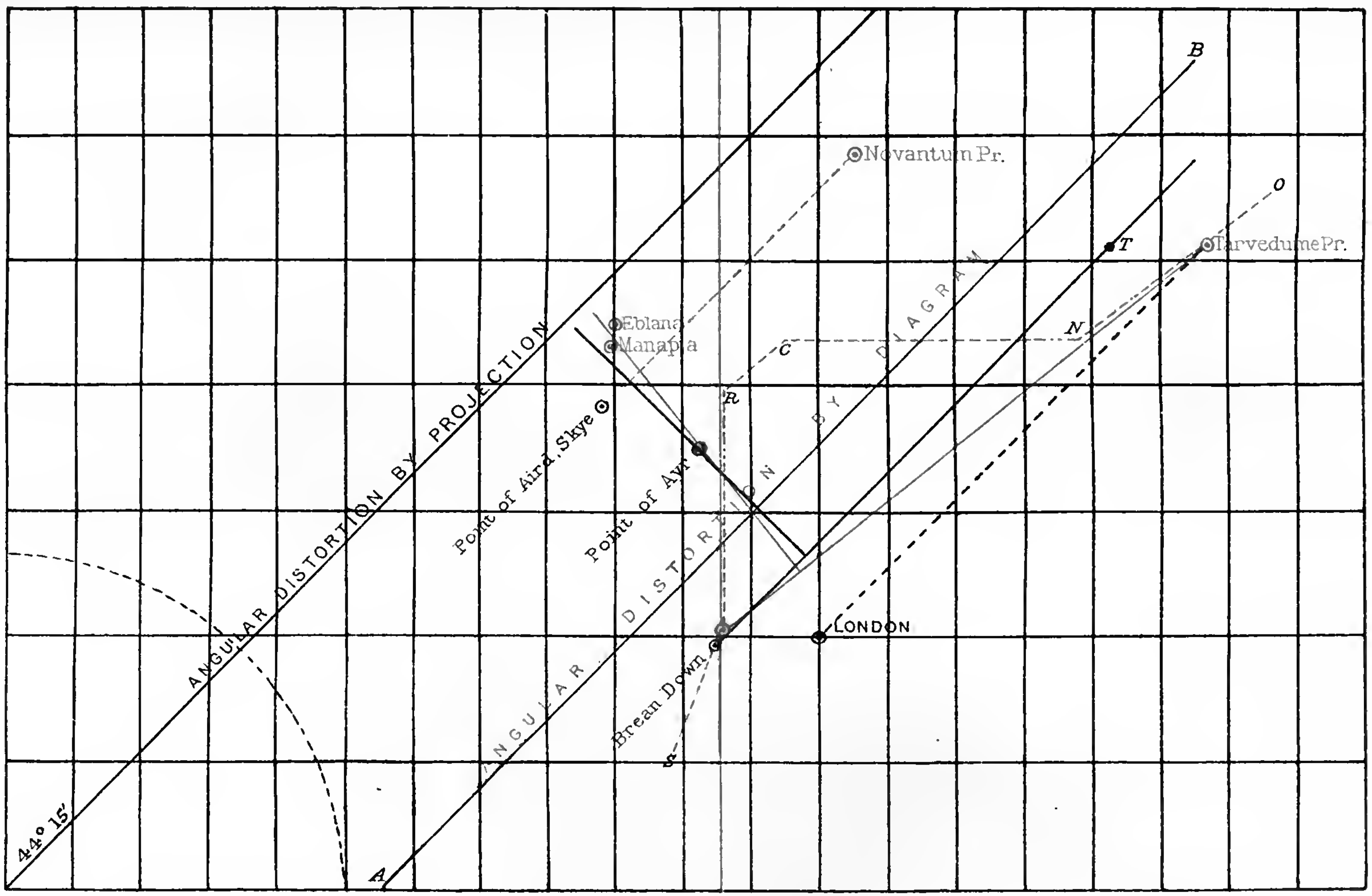


PLATE XX.

CATURACTONIUM AND LONDINIUM.

EXPLANATION OF PLATE XX.

Let *AB* represent the line of "no distortion," in latitude 58°. Then, with radii proportionate to the ***true*** and to the ***distorted*** latitudes (1 : 1·35), describe *CDE*, and *CDE* as arcs of meridian 20°. In these arcs ***radii from A*** will cut ***coincident*** parallels, while ***horizontal lines*** mark ***equivalent parallels.*** Thus, *E* is coincident with *E*, and equivalent to *D*.

Now, dealing with Caturactonium and Londinium, a difficulty of variant readings meets us.

A.—The earliest readings were—Caturactonium, 57° 30′; Londinium, 54°. The former is indicated by the position of the name in Ed. 1482; the latter is still the reading of all texts in Bk. ii. Then $\alpha\beta$ in the map = 4° 12′; whereas it should be 3° 30′—that is, the interval might be either $\alpha\beta$ or $\alpha\beta$. If Ptolemy had changed from the former to the latter, and thus made the positions correct, the latitudes become 58° and 54° 30′ (see Note 1).

B.—But the maps and Bk. ii. place 54° at *E*, the coincident of *E*; and Londinium, instead of being at 54°, is really too low by 1° 10′ (55° 10′ – 54° 0′), according both to construction and calculation. Ptolemy uses *E* for *E*, calling it 54°, instead of 55° 10′, its equivalent. By this course, not only is *CE* reduced to *CD*, but as *CC* becomes changed in the map from 57° 30′ to 58° 12′, the 1720 stadia are reduced to 1147.

It is very clear that under the above conditions one of two things must happen. Either the interval $\alpha\beta$ in the map will be made too small by $\frac{1147}{1720}$; or, if actual itinerary measures were used to determine the interval, it would commence at *D* and extend to *A*—that is, the compression must be looked for further north than *D*.

Now, whether $\alpha\beta$ or $\alpha\beta$ be adopted, the interval is correct. It is 1720 st. = 3° 30′ (Ptol.); therefore the missing 1° 10′ must be looked for north of *C*.

Assuming Albani Fl. Ost. to be the Tweed at Berwick, as the next probable station north, Ptolemy gives it lat. 58° 30′, which is 0° 30′ north of Caturactonium; but the true interval is very much larger, being in fact 1° 23′. Knowing the true positions of both, the true error may be calculated (see Note 2); and it appears that the compression sought for is found exactly in the interval under consideration. Further, Albani Fl. Ost. *is* the Tweed. (See Plate XIV.)

Calculation of Errors and Intervals.

True latitude Catterick	=	54° 23′	
,, ,, London	=	51° 31′	
Interval	=	2° 52′	
but $\frac{6}{5}$ (2° 52′)	=	3° 26′ 24″	
Ptolemy gives it	=	3° 30′ 0″	
difference	=	0° 3′ 36″	
Or actual error for *London*	=	1° 35′	
,, ,, Catterick,	=	1° 32′	
difference	=	0° 3′	or interval measured to 3 miles.

For London on the Map, $S = \frac{1}{5}(51° 31' - 31° 10') = 4° 4'$

$\eta = \lambda - L = (54° 0' - 51° 31') = 2° 29'$

$\epsilon = \eta - S = 2° 29' - 4° 4' = -1° 35'$

1720 stadia = 3° 30′ (Ptol.).

Note 1.—

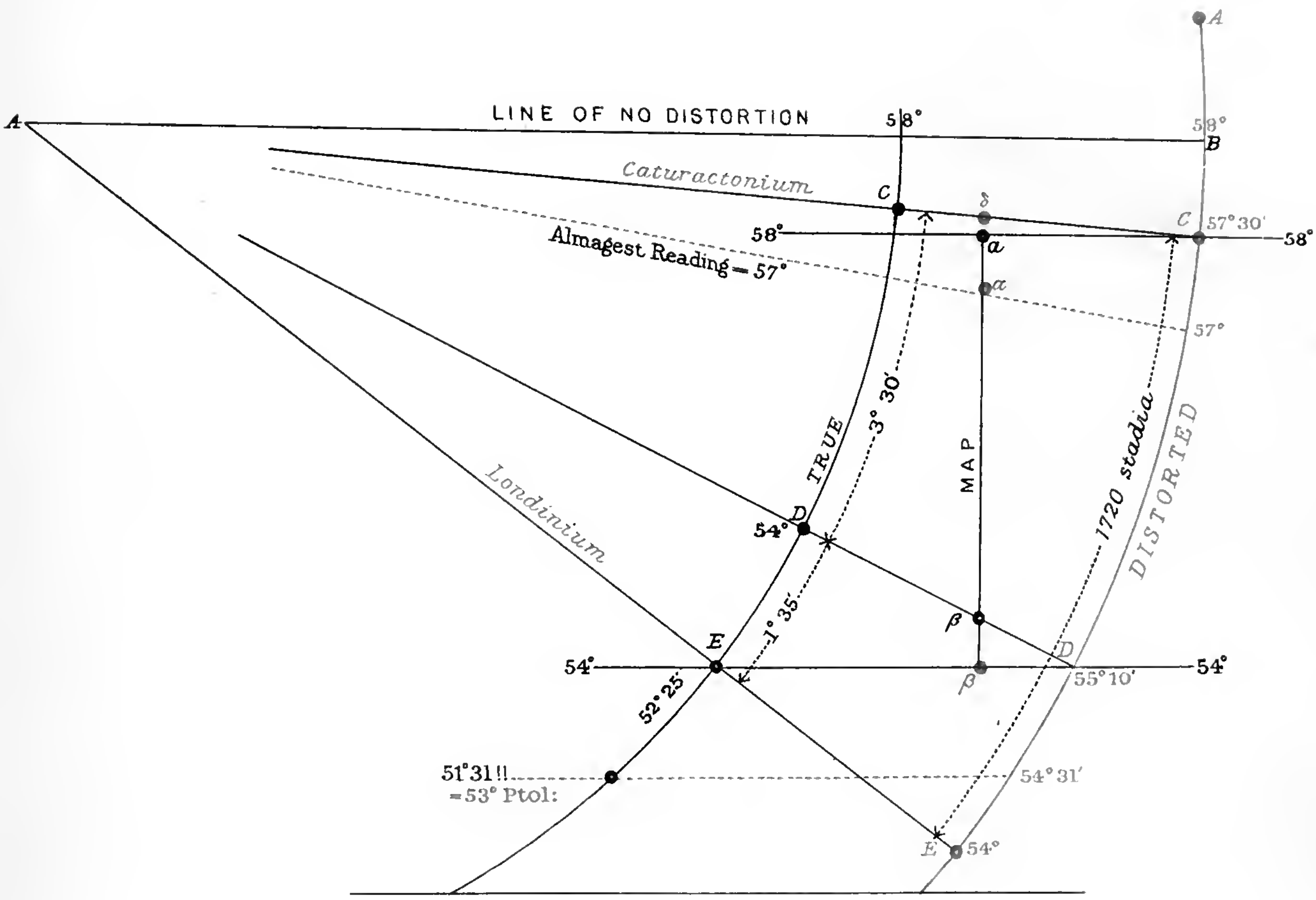
True latitude of London	=	51° 31′
S	=	+ 4° 4′
		55° 35′
P	=	– 2° 39′
DE = ε as above	=	+ 1° 35′
therefore Ptolemy's latitude calculated (seen above to be equivalent of 51° 31′)	=	54° 31′

Note 2.—

True latitude of Berwick	=	55° 46′
,, ,, Catterick	=	54° 23′
difference	=	1° 23′
but $\frac{6}{5}$ 1° 23′	=	1° 40′
Less Ptolemy's interva	=	0° 30′
Error = 55° 10′ – 54° 0′	=	1° 10′

Thus *AC* = *DE* = 1° 10′, as in *B*.

CATVRACTONIUM AND LONDINIUM.



SUPPLEMENT TO EXPLANATION OF PLATE XV.

THE following remarks may tend to elucidate what has been said in the Explanation of Plate XV.:—

1. METEOROSCOPIC OBSERVATIONS.

For the measurement of Longitude by means of eclipses the interval between Nuremberg and Rome seems to have been a favourite exercise. The following are a fair selection from some of the results:—

Regiomontanus makes it . .	9°	0′	Error,	7°	36′
Werner (1497), . . .	8°	0′	,,	6°	36′
Moletius (1562), . . .	2°	35′	,,	1°	11′
Maginus (1597), . . .	2°	20′	,,	0°	56′

So that Ptolemy's 11° compares favourably with even very late mediæval times, and the error found upon Plate XV. is less than the mean of the four given above.

While this subject was under consideration, an observation for Latitude and one (a single phase) for Longitude were made under conditions open to anyone. Indeed, the gnomon was a common surveyor's staff, not an inch in diameter, and the shadow was very faint and 25 ft. long. The Eclipse too was observed from a station while waiting for a train. Even under these conditions the Latitude was given true to less than three miles, and the Longitude was only two seconds wrong. In this latter case, however, the *time was known*, and the second station, Greenwich, was without sensible error. Such observations, therefore, even in the rough, may give good results.

Although we know that Ptolemy would have used the "meteoroscopic" results, it must be remembered that so large an extension of the north coast of Scotland would, of necessity, require commensurate changes elsewhere. Such changes must be looked for on what became his south coast, and they should be found in his map.

In *Longitude* Epidium prom. (Cape Wrath) is true to Novantum, so that the error is confined to the *true* north coast; but in *Latitude* it follows Tarvedume, hence the interval between Novantum and Epidium is considerably distorted.

Now Ptolemy reckoned the interval from Albanus fl. to Tazalorum prom. as *Longitude:* hence the length of his coast-line was nearly doubled (correctly $\frac{20}{11}$).

Between Moray Firth and Duncansby Head the same rate of expansion was continued, but it is a little disguised by his longitude error.

It may be mentioned that strong confirmation is given us in the inverse case—from Peterhead to Moray Firth—where the interval was treated as *Latitude*, and the distance is reduced to nearly one-half.

Lastly, on maps simply reduced to the same scale, without any corrections, direct lines were drawn between the extreme stations—their ratio being $\frac{20}{10 \cdot 51}$.

From Ptolemy's point of view the two coasts were harmonized by the above extension, and no difficulty appeared.

They are of consistent length as measured from Tarvedume to Novantum and Caturactonium, the limiting stations unaffected by the change.

Intermediate stations should fall into their places, naturally being subject to the same conditions.

2. Latitudes—Coast of Scotland.

D being Duncansby Head, *P*, Peterhead, *T*, the Tweed.

With a scale expanding in the ratio of $\frac{6}{5}$, Ptolemy's Latitude of *D* would be 64° 0′, but his limit was fixed at Thule in Latitude 63°, and space would still be wanting for the "six days sail" of Pytheas.

Under these circumstances either a violent compression or a large overlap appears to be unavoidable—neither was admissible. But if a gnomon observation changed the Latitude of *D* to 60° 15′ the margin would be ample, and the excess would be reduced to 1° 46′. This error was continued to *P*, where, as will be seen, it is really reduced to 1° 6′, and the 1° 6′ at *P* is the 1° 10′ found at *T* in Plate XX. For

$$(D - T) - 2° 52' = P - 1° 6' = T - 1° 10'; \text{ or } P - 1° 6' = T - 1° 10'.$$

Though this final form of the compromise is found at *P*, it was concealed from Ptolemy and unknown to him.

At *T* the expanded Latitude is + 2° 53′ and 2° 53′ − 1° 6′ = 1° 47′, thus at both stations η becomes + 1° 47′, and the discordance is eliminated by making the latitude of *P* and *T* the same—

$$2 \times (1° 47') + 0° 10' = 3° 44' \text{ which was the true error at } D.$$

3. The Calculation of Errors and Intervals.

—	Duncansby, *D.*	Peterhead, *P.*	Tweed, *T.*
	° ′	° ′	° ′
True latitude *L*, ...	58 39	57 30	55 47
True latitude of Alexandria, *A*, ...	31 10	31 10	31 10
$L - A$, ...	27 29	26 20	24 37
$\frac{6}{5}(L - A)$, ...	33 0	31 36	29 32
S, ...	5 31	5 16	4 55
For Ptolemy's Alexandria = $L - 0°\ 10'$ . .	58 29	57 20	55 37
Calculated λ, ...	64 0	62 36	60 32
On map π, ...	60 15	58 26	58 30
True error ϵ, ...	− 3 45	− 4 10	− 2 2
S, ...	5 31	5 16	4 55
Apparent error, η.	+ 1 46	+ 1 6	+ 2 53

4. Intervals from Tweed.

—	$D - T.$	$P - T.$
	° ′	° ′
$L - L'$, ...	2 52	1 43
$\pi - \pi'$, ...	1 45	+ 0 4
$\epsilon - \epsilon'$, ...	− 1 43	− 2 8
$\eta - \eta'$, ...	− 1 7	+ 1 47

Under $D - T$ we find $L - L = 2°\ 52'$,
and at T $\eta = 2°\ 53'$,
while, for D and P $\eta + \eta' = 2°\ 52'$.

Further, with $L - L' = 2°\ 52'$, $D - T$ has $\epsilon - \epsilon' = -\ 1°\ 43'$,
while with $L - L' = 1°\ 43'$, $P - T$ has $\eta - \eta' = +\ 1°\ 47'$,
with $\pi - \pi' = -\ 0°\ 4'$.

The explanation of this last difference is

$$\tfrac{6}{5}\,(1^\circ\ 43') = 2^\circ\ 4', \quad \text{while} \quad \epsilon - \epsilon' = 2^\circ\ 8',$$

or $+\ 0^\circ\ 4'$, by which amount T is mapped north of P.

Thus, the error was cancelled at P, but as the result, there could be no difference of latitude between P and T. The $-\ 1^\circ\ 43'$ was more than covered by $+\ 1^\circ\ 47'$, so that, excepting $\pi - \pi'$, they would appear to agree. The eclipse observation supplied the space in which to insert the intervening stations. It was in this way that the coast was compelled to trend "almost exactly east and west," though it can afford no support whatever to Horsley's idea of a turned Scotland.

But even if this distortion be accurately corrected, the Scottish coast stations are very far from settled. On the North and East coasts, where we are still somewhat dependent upon contour and distances, there need be little trouble, but the West coast seems to set all criticism at defiance.

Ptolemy had but four stations between the Solway and the Point of Aird to construct a coast-line that would require, at least, ten times the number, and of these four, three were river mouths and the fourth an estuary.[1]

If a sheet of paper be ruled with rectangles 20 × 11, and the six stations inserted in position, the bare materials at Ptolemy's command will be seen without any attempt to connect them by a coast-line, the only data for the formation of which consisted of the words "River Mouth" and "Estuary."

By calculation, the four positions given seem to apply—but rather widely—to the Clyde, Loch Andail, Lorn, and the Passage of Coll, or the Sound of Sleat. It is a question whether these resulted from a coast-survey at all; inland inquiries on the subject seem, at least, as likely when the old is contrasted with a modern map.[2] This is disappointing, but the same thing has occurred in our own day on coasts as little known.

[1] Indeed we have here a *terra-incognita* in the N. W. almost as complete as that which bounds the S. E. of the *habitabilis*.

[2] It is possible that the west coast stations may be connected with Philemon's account of Ireland.—"Geog.," Bk. i., ch. 11.

THE EARTH WITHIN THE ARMILLARY SPHERE.

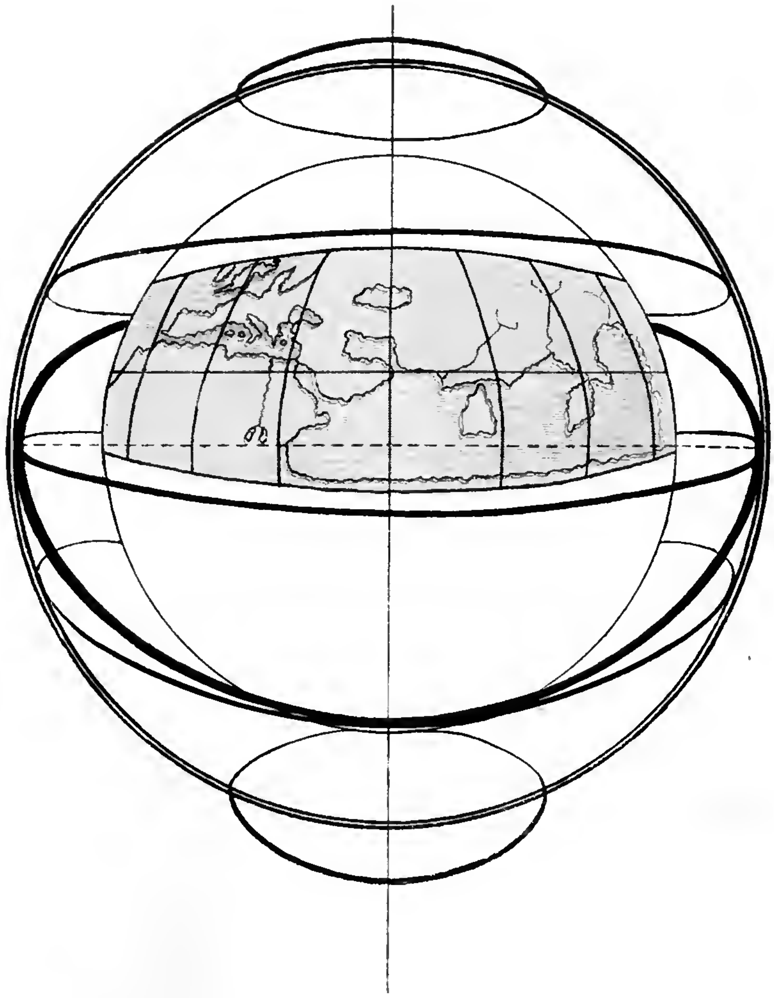


PLATE XXI.

(Note to Page 23.)

THE EARTH WITHIN THE ARMILLARY SPHERE.

NOTE

UPON THE CORRUPT PASSAGE IN BOOK VII. CH. VI., DEALING WITH THE PROBLEM OF DESCRIBING THE EARTH WITHIN THE ARMILLARY SPHERE, BEING AN EXPLANATION OF PLATE XXI.

The above subject seems to justify the following additional note upon the decorative picture of the "earth within the armillary sphere," which occurs in the closing chapters of the "Geographia." As has already (p. 23) been said, the discussion is interesting, moreover, as providing a striking example of the errors and their consequent troubles which are due to the ignorant carelessness of the editors. In reading any one of the editions it is quite impossible to gain a grasp of the meaning or terminology of the original; and it will, therefore, be more convenient to follow what must have been the evolution of the construction in the mind of Ptolemy, returning afterwards to a brief discussion of the readings.

First, it is necessary to remember that his purpose is to place the terrestrial within the celestial sphere, so as to show the whole of the *habitabilis* at one glance between two of the rings which were fixed upon the sphere of the heavens, the Equator and the northern Tropic.

Having procured a representation of the celestial sphere, let a vertical line be drawn representing its diameter; and, having found the centre (ε)[1] let the radius between the centre and the North Pole (α) be bisected (at ο); and, further, let a perpendicular fall from the end of the parallel in the heavenly sphere which passes over Syene (ζ) (meeting the diameter in a point called ι). Since this line is the co-sine of the angle intercepted between the Tropic and the Equator, the ratio of the sine to the radius is equal to that of the line joining the base of the perpendicular with the centre (ε), or $\frac{4}{10}$, or the ratio of the former line to half the radius (that is ει to εο) is 4 to 5.—(I.)

Again, as a second solution, the ratio of half the radius (εο) is to 4 very nearly[2] as "that same 4" is to 3.—(II.)

[1] These letters are inserted in brackets to explain what follows.

[2] εγγιστα.

Therefore, the radius of the celestial sphere will be $\frac{4}{3}$ the radius of the terrestrial, and if such a point be found in the celestial diameter the circumference of the earth may be drawn, and the important parallels filled in at the usual distances from the Equator.

Finally, the position for the eye of the observer is ascertained by joining the points where the Tropic and the Equator cut the diameter and the circumference of the celestial sphere respectively; also, a line passing through the terrestrial parallel of Syene. If these lines be produced, their point of intersection is the one required.

Now, to show the complete corruption of the texts, it will be sufficient to note down the true solution given above, and compare it with those of various editions. Using the symbols it is:—

$$\left.\begin{array}{c} \epsilon\iota : \epsilon o \\ 4 : 5 \end{array}\right\} \text{I. See above.}$$

$$\left.\begin{array}{c} \frac{1}{2}\,\epsilon\alpha : 4 \\ 4 : 3 \end{array}\right\} \text{II. See above.}$$

	1482.	1511.	1525.	1542.	1597.	1618 (Gk.).	1618 (Lat.).	Pal. MS.
I.[1]	$\epsilon\iota : \epsilon o$ 4 : 5	$\beta\zeta : \beta\alpha$ 4 : 15	$\beta\zeta : \beta\alpha$ 4 : 15	$\beta\zeta : \beta\alpha$ 4 : 15	$\beta\zeta : \beta\alpha$ 4 : 15	$\beta\zeta : \beta\alpha$ 4 : 15	$\beta\zeta : \beta\alpha$ 4 : 15	— —
II.	$\frac{1}{2}\,\epsilon o : \epsilon\alpha$ 4 : 15	$\epsilon\iota : \epsilon\alpha$ 4 : 20	$\frac{1}{2}\,\epsilon\iota : \epsilon\alpha$ 4 : 20	$\frac{1}{2}\,\epsilon\iota : \epsilon\alpha$ 4 : 20	$\frac{1}{2}\,\epsilon\alpha : \epsilon\iota$ 4 : 3	$\frac{1}{2}\,\epsilon\alpha : \epsilon\iota$ 4 : 3	$\frac{1}{2}\,\epsilon\alpha : \epsilon\iota$ 4 : 20	$\frac{1}{2}\,\epsilon\iota : \epsilon\alpha$ 4 : 3

It is almost amusing to read Pirckheimer's lament on this subject in his preface :—

"Porro quantum temporis et laboris, in sola armillari cum orbe sphæra mihi insumptum sit, nemo novit, nisi qui et ipse in similibus periculum fecit, ibi enim non solum numeri sed et literarum notæ linearum deductiones, omnia denique adeo depravata ac inversa erant, ut penitus inter se dissiderent ac pugnarent, planeque conjectore aliquo indigerent. Verum, ni fallor, ita restituta sunt singula, ut non minus authoris menti, quam rationibus quadrent mathematicis."

Yet he left the matter no better than he found it.

[1] It will be seen that I. is given correctly only in the edition of 1482.

APPENDIX.

A MORE DETAILED DISCUSSION OF PTOLEMY'S COAST-LINE FROM CARNARVON TO CUMBERLAND, BEING THE SUBSTANCE OF A LECTURE DELIVERED BY MR. RYLANDS BEFORE THE "HISTORIC SOCIETY OF LANCASHIRE AND CHESHIRE," ON DECEMBER 13TH, 1877, AND ORIGINALLY PUBLISHED IN THE TRANSACTIONS OF THAT SOCIETY, THIRD SERIES, VOL. VI., PAGES 81–92.[1]

I RECEIVED a short time ago a request from Dr. Hume, that I would give you what he was good enough to call "a chip from my workshop." I at once took hammer in hand, and broke off one for the purpose. It is a little larger than I intended: so that it will allow me time for but these few words by way of introduction.

The piece of coast which forms the subject of my Paper has these advantages:—First, it is one well known to ourselves, and has been the subject of considerable discussion; and this, too, in connexion with the geography of Ptolemy. At the same time, because its position is in the extreme N.W. of Ptolemy's *habitabilis*, and marked neither by special information nor peculiar interest, it affords, perhaps, the best test for us of his accuracy as a geographer.

Before I begin, let me ask you, in what follows, to make a broad distinction between *facts* and *opinions*. I shall be satisfied if you will weigh my facts with all the care you can command; and if I am led to express opinions, I will cheerfully allow you to take them for what they are worth.

One objection I have met with more than once, in dealing with this portion of

[1] This lecture was originally delivered in connexion with the portion of the coast which gave rise to the whole inquiry. It is here reprinted, as showing the detailed investigation of a specific portion of Ptolemy's work; but it must be remembered that any want of completeness of treatment is due to the exclusion of the more general theorem, now (it is to be hoped) fully demonstrated in the preceding pages. Moreover, the Paper now printed suffers from the fact that it is a mere epitome of that originally delivered. Mr. Rylands gave his original lecture from a few brief notes, which contained only quotations and figures.—[ED.]

Ptolemy's geography. It is this:—Is it worth while to re-open this question, when the best authorities have so long decided it? My reply is,—Let us see: a brief retrospect of what has been already done will clear the ground for all of us. It will be sufficient as an illustration to take a single station of Ptolemy, and I have selected his *Belisama*.

Let us see what the authorities teach us.

In such inquiries, every Englishman at least begins with Camden.[1] He makes the *Belisama* the Ribble, or, as he would write it, the "Rhe-bell," and he sees in that word the remains—as somebody expresses it—of the words River-Belisama. I do not know what you will think of such an etymology. With me, I confess, it has no value whatever.

Our next authority is John Horsley. He says the *Belisama*, "from its situation, must be the Mersey." Here we get the first geographical element; but Horsley's "must be" (as he gives us no reason for the statement) is of little more value than Camden's etymology. Indeed it may be sufficient to oppose to it one by your townsman, Mr. Picton, who says[2]—"By no mode of calculation can the situation of the Mersey and the description of the geographer be made to agree." I am very willing to admit that Mr. Picton meant, by no mode of calculation with which he was acquainted. But it might have been as well if he had said so.

We must not, however, leave Mr. Horsley; for though I have the most profound respect for his work and the manner in which he did it, yet, as I believe much of the confusion which exists has been due to a passage in his "Roman Antiquities of Britain," I must ask you to allow me to quote it. He says:—

> "As for the degrees of longitude, what I would most wish for is, to know with exactness and certainty *what space or number of miles he allowed to a degree in the several parts of Britain.* One would think that the *common well-known property of the sphere*, that at 60° latitude, the space answering to a degree of latitude, or of the great circle, is double the space of a degree of longitude, *could not escape Ptolemy's notice. And this would adjust the proportion of one to the other.* If a degree of longitude in any part of Britain be, according to Ptolemy, 40 miles (as some affirm), it must be in the south of England, where the latitude is least. *Nor must we here allow them the usual length of the English computed miles.* A degree of latitude, or a degree in the great circle, seems to me, according to Ptolemy, *to be near enough our usual reckoning*, 60 *computed miles*."—Page 361.

Let me now call your attention to those passages in the extract, which, for the purposes of distinction, I have printed in italics. The first is, he would like to know "what space or number of miles," &c. It is quite evident from this passage, that though good John Horsley had copied out with sufficient accuracy

[1] "Britannia." Ed. 1594, p. 582.

[2] Liverpool Literary and Philosophical Society, February 19th, 1849.

Ptolemy's description of Britain, he had not otherwise read the book; because Ptolemy allows the same number of miles to a degree of longitude in every part of Britain.

The next is, "one would think that the common property of the sphere," &c. This passage not only confirms the statement already made, but shows how a wrong has been done to Ptolemy, who has been classed by the late Mr. De Morgan (than whom there was no better judge) with "the two other great leaders —Aristotle and Euclid."

Next we read, "nor must we here allow," &c. Here he refers to Halley, in the "Phil. Trans." (No. 193), but does not give the quotation. Halley's paper says 26 English miles = $28\frac{1}{2}$ Roman.

We are now prepared to see the error which has resulted from the concluding words of the quotation. "A degree of latitude," &c. This has been read to mean, by writers even to the present time, that a degree of latitude, both as held by Ptolemy and by moderns, is 60 English miles. What Horsley plainly meant to say was, that a degree according to Ptolemy must be taken at about 60 computed miles of our usual reckoning, which is not *very* far from the truth; while the modern mean degree of a great circle is a little over 69 miles. It is needless to state the error and confusion resulting inevitably from such a mistake as this. Mr. Horsley may be right as regards the *Belisama*, but we have nothing except his bare statement on the subject.

We next come to Mr. Whitaker, the historian of Manchester, with whom Horsley's book was a manual in constant requisition. He says that the *Belisama* is the Mersey, but Mr. Whitaker not only makes a degree of latitude 60 English miles (both Ptolemy and modern), but in latitude 57 deg. he makes a degree of longitude 60 miles also! He says in his "History of Manchester":[1] "and coming 20 miles to the N. (*i. e.* 20 min. of a degree of latitude), and 30 miles to the E. (*i. e.* 30 min. of a degree of longitude)"; thus he reckons both degrees alike, and both wrong.

Next we come to Dr. Whitaker, the historian of Whalley, of whom the editor of the recent edition of his work says, "his book exhibits all the carelessness of genius," or, as I should be inclined to read it, all the genius of carelessness. Dr. Whitaker not only adopts without remark the errors of his predecessor, but adds others of his own; thus making confusion worse confounded. Let us have his own words:—"But if we stretch from the mouth of the Dee, 20 miles northward, according to the geographer's directions, we shall find ourselves out at sea

[1] "History of Manchester," B. 1, c. 5.

indeed, but in a latitude exactly corresponding to the mouth of the Ribble. And turning thence at a right angle to the E. for 30 miles, we shall stretch a little further inland than Mr. Whitaker's supposed station (which, however, was certainly not *Setantiorum Portus*), near the Neb of the Neze."[1] Thus we are to go 20 miles N. and 30 miles E.; but the peculiar genius of Dr. Whitaker inverts these numbers. He actually goes 30 miles N. instead of 20, and 20 miles E. instead of 30. By this means he makes the *Belisama* the Ribble! Need we go further? This passage from Dr. Whitaker has been allowed to pass current from the commencement of the present century until this moment without challenge; and almost all who have since written may be said to have endorsed the dictum of Dr. Ormerod, who says: "the *Belisama* is the Ribble, and Dr. Whitaker has abundantly proved it." We need not be surprised that the atlases of the Library of Useful Knowledge, Mr. Pearson and Dr. William Smith—and we must add the map in the Monumenta Historica Britannica—make the *Belisama* the Ribble.

Local writers, who might have known better, have followed "the authorities," perhaps naturally. It is almost amusing to see with what marvellous facility they raise or depress the Hundred of Wirral to suit their purposes.

Dr. Black, of Manchester, following Dr. Ormerod—who, by the way, has since retracted the statement relied upon by him—makes the Mersey fall into the Dee; so that "the *Seteia Aestuarium* will include the mouths of both rivers, having an island at their confluence, somewhat similar to what is faintly depicted in Ptolemy's map."[2] Now, I am not aware of any edition of "Ptolemy's Geography" existing in this country which I have not examined; and I do not hesitate to say that in not one of them is there any trace of an island—even "faintly depicted"—at the mouth of the Dee. In the edition of 1511, there is a very distinct island at the mouth of the *Belisama*, possibly the Burbo Bank; but this edition is unsupported by any other. The whole volume gives evidence of large changes made to support a theory, and is affected throughout by the knowledge of the time.

The late Rev. Mr. Massey, of Chester,[3] on the faith of the remainder of a bridge found in the neighbourhood of Birkenhead, not only depresses the valley of the Ellesmere Canal, after the manner of Dr. Black, but raises the N.W. portion of Wirral, so as entirely to close the mouth of the Mersey! And, in confirmation of this conclusion, he quotes Speed's Map of the Invasions of England, 1627 (in which the rivers are all plainly exaggerated), to show that at that

[1] "History of Whalley," ed. 1818, p. 6.
[2] "Memoirs of Literary and Philosophical Society, Manchester," vol. vii. (1846), p. 387.
[3] "Journal of Chester Society," vol. i., p. 76.

time there was a much broader waterway between the Mersey and the Dee than even the impossible rivulet found in the older maps, and which probably owes its origin and its existence to the imperfect investigations of Leland and others. And he did this, although the map of Cheshire, in the same volume, showed that the Flooker's Brook at Chester was at that time no larger than it is at present. Mr. Massey's conclusion involves, as we may see, a very considerable change of level in the face of Wirral since Roman times. I mention this, more especially because the same conclusion forms the basis of the conjectures of most of those who have recently written upon this subject.

But, as the result of the very careful investigation of Dr. Hume, in his "Ancient Meols," 1863, p. 22, we find that the principal Roman remains are found upon the large forest-bed, which is only two feet below the basis of the present sandhills, and these remains cannot have risen to a higher stratum with time.

I know not at present any other writer to whom I need refer. Sufficient has been said to enable me to ask you—If these are not your authorities, who are? I know no other. If they are, have they "decided the matter"?

And now, gentlemen, having cleared the ground, we will begin to build.

In order to lay the foundation of what follows, I shall ask you to accept two numbers; but I must frankly warn you, at the same time, that if you do accept these numbers as correct, the whole case is in my hands. I mean this so far as concerns that portion of coast which is the subject of the present Paper.

First, we require the true value of a degree of a mean great circle of the earth. In order to obtain this, I wrote to the late Sir Henry James, of the Ordnance Survey, and his reply was, 1 deg. = 69·0556 miles. I ask you to accept that.

In the next place, we require to know the value of the degree of the great circle according to Ptolemy. Those who have read the first book of the "Geography" will admit, without hesitation, that his degree consisted of 500 stadia. We require then the value of a stadium. After the examination of all the measurements within my reach, I adopted—1 stadium = 608·016 English feet There was some trifling uncertainty as to the exact value of a stadium; but as between those best ascertained the difference was so small, that for our present purpose, in order that I may secure your assent, you are at liberty to select which of them you please. I shall use throughout the value which I have just given.

Then we add, from the above figures, 500 stadia = 304,008·0 feet; and five-sixths of a true degree of the great circle = 303,844·4 feet. The difference, therefore, in using Ptolemy's scale as five-sixths of the true one, leaves an error of 54 yards in 69 miles, or about one mile in 33 degrees![1]

With these figures we are in a position to construct and to discuss Ptolemy's coast included in the title of the present Paper. The only requisites are that our scale shall be in accordance with Ptolemy's instructions, and that each geographical feature shall be strictly defined. What I mean is that in the present case the latitudes and longitudes shall be parallelograms, having their sides in the proportion of 11 to 20; and that, *losing sight entirely of our own knowledge of the coast*, each geographical feature shall be drawn in strict accordance with a predetermined plan. Thus, for example, the word "æstuarium" shall be represented by the same outline in each case.

In the map before us (Plate I.), the word "estus" occurs four times; and on reference to the map it is so represented, the form being almost the same. The same remark applies to other terms. I have adopted the edition of 1482, because I believe that it represents Ptolemy more faithfully than any other edition.[2]

Under these conditions, as Ptolemy gives the latitude and longitude of all his stations, the construction of one of his maps is little more than a mere mechanical act. The accompanying one (Plate II.) is a sheet prepared in this way, as described, the included stations being numbered from north to south. The direction of the coast-line here is left in no doubt, and the outline may be readily drawn in, attention being paid to the definition of the terms.

Ianganorum Prom.,[3] with which I will commence (marked No. 7), is allowed on all hands to be Brachypwll, at the extreme of Carnarvonshire; but in every edition of Ptolemy with which I am acquainted the promontory of Carnarvonshire is cut off. Ptolemy himself tells us that in such a coast the names are recorded as they occur from north to south; but if we imagine some early editor constructing the map as we propose to do, he would find the positions laid down occurring 5, 6, **8**, **7**, 9, and not in their natural order. He, having no idea whatever of the coast, and knowing Ptolemy's mode of proceeding, would, I beg to assume, believe there was an error. But mind, an assumption is utterly worthless until it is proved. Suspecting an error, there are at least three distinct courses which might be taken to correct it. The number at fault is No. 8. (The Tisobius.)

(*a*) The first course might be—it is only a river mouth, it makes no difference in the coast, it may be left out altogether.

[1] *Cf.* Explanation of Plate I.

[2] See also Explanation of Plate X.

[3] From this point the Paper was illustrated at large by means of maps and tracings prepared for future publication.

(*b*) The second method would be to invert the order of the names in Ptolemy's text, and write No. 8 below No. 7.

(*c*) There still remains a third course. The more careful editor would argue that the order of the names could scarcely be wrong; but that the numbers in Ptolemy's tables were frequently in error; and he would so alter the readings of No. 7, as to put it in a possible position. For example, add half a degree to the latitude and half a degree to the longitude.

Now, it happens that all these various readings occur in editions of Ptolemy in my possession. It is perfectly clear from this that some confusion has early existed in the text: and I am free to admit that we have here a totally corrupt passage. But, without breaking off at this point, as has been too much the custom with modern critics, do you not see that the man who altered the order of the names would keep the numbers right, while he who altered the numbers would preserve the original order of the names? Further, there is no doubt as to the station intended by No. 7. The question then remains, Ought *Ianganorum Prom.* to read 15 long. and 56 lat. or $15\frac{1}{2}$ long. and $56\frac{1}{2}$ lat.? Calculation shows that, as determined from London, the latitude ought to be 56 deg., the error being only 2 or 3 minutes of a degree; and that the long. ought to be 15 degrees, the error being about 7 minutes. It is plain, therefore, that the order of the stations on the map we are proposing to draw, is correct, and that No. 8, the *Tisobius* of Ptolemy, follows No. 7. In short, that the coast returns upon itself, as we know that it does. The *Tisobius*, however, is no longer the Conway, but the Traeth Mawr. The promontory of Carnarvonshire has been cut off, not by Ptolemy, but by his editors. Thus, then, we have two of Ptolemy's stations satisfactorily determined, viz. *Ianganorum Prom.*, which is Brachypwll, and *Tisobius*, which is Traeth Mawr.

We next proceed to Seteia (the Dee). Ptolemy places the *Seteia aest.* in long. 17 and lat. 57. Thus it differs from *Ianganorum Prom.* by 1 deg. lat. and 2 deg. long. Adopting these differences on a true outline of the coast, it was found that stations between *Belisama aest.* and *Morecambe aest.* were mapped in on the true scale of 600 stadia, and not of 500 to a degree of a great circle.

In previous work the point marked *B* (Plate I.), was found to represent St. Bees' Head; and it was found also that the space included between *Seteia* and the point *B* inclosed only, and almost exactly, 500 stadia to 1 deg. It is necessary, therefore, that as between the two the errors must balance.

Calculation shows at once the error at each particular station, and gives the following results:—

The error at *B* and at *Belisama aest.* amounts to 24 min. of a degree south, while the error at *Seteia* is 39 min. of a degree south. The result therefore is, that the space in Ptolemy's map between *B* and *Morecambe* is 15 min. of a degree too small, while that between *Belisama* and *Seteia* is 15 min. too great. Thus the errors do balance, and *Seteia* is found 15 min. of a degree too low. It will be seen that those writers who have commenced the examination with *Seteia* as a known station or point determined, have assumed it to be a station without error; while, in fact, the increased width of Wirral in Ptolemy's map is due entirely to the fact that the *Seteia* is placed a quarter of a degree too low.

Seteia being thus determined, the stations follow with sufficient correctness, until we pass *Morecambe*. *Belisama* can only be the Mersey,[1] and *Setantiorum Portus* the Ribble.

But what has become of Anglesea? The common suggestion is that it is *Mona;* but this can hardly be. In the first place, Ptolemy describes Mona as an island off the east coast of Ireland: this cannot be Anglesea. In the next place, he maps it, in the language of Cæsar, "*medio cursu*," half way between England and Ireland. Further, by calculation of lat. (whether from London or Brachypwll), Ptolemy's Mona is the Isle of Man pure and simple. The error as calculated from the centre of the Isle of Man to Brachypwll is but 3 min. of a degree. The true difference of long. between the two islands is so small, that calculation is of little avail.

But the question still remains, What has become of Anglesea? I may be told that the coast from Brachypwll to *Seteia* must have been explored, and that the island could not be overlooked. This may be quite true; but referring to Ptolemy's map of our east coast, a similar question may be asked respecting Norfolk or Suffolk. Plainly he had no station between the Stour and the Wash, and his only course was to outline almost directly the coast-line between. Now, he had, in the materials he used, no recorded station between Brachypwll and *Seteia*, and, as the result, not Anglesea alone but a considerable piece of mainland is omitted by his line drawn from one of these stations to the other. Ah! but someone says, What about Tacitus? True, Tacitus mentions Anglesea; but you know he was blowing his father-in-law's trumpet; he was praising Agricola, not writing geography. This is by no means the only case in which I have found the geography of Tacitus at fault. He had to carry the Roman soldiers to an island on the west coast of Britain; and, with artistic instinct, he applied to it the name of the only island in the channel with which he seems to have been acquainted. And this was the Mona of Cæsar. In his day Ptolemy had not written.

[1] *Cf.* the more complete determinations in the Explanation of Plate XI.

There remains now only the *Ituna*, which is admitted to be the Solway. The interval between this station and *Morecambe* is only about one-third of the true distance; and if time allowed, although it is beyond the limits of my paper, I would gladly explain how this diminution is to be accounted for. Suffice it to say that mapping actual measures at the rate of 600 stadia on a map of the scale of 500, Ptolemy was too fast approaching the limit that was fixed by *Thule* in lat. 63. At *Ituna* he had reached nearly the 60th degree, and the remaining space was insufficient. The explanation is briefly as follows:—

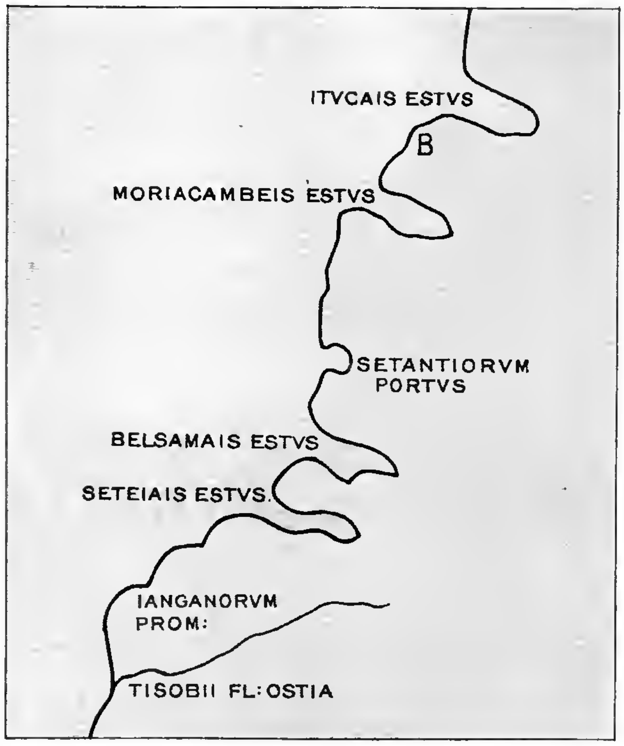
Up to *Morecambe* the stations were determined from Londinium, but northward of that, at *Ituna*, Caturactonium, his second principal station with a different error, was his new point of departure. Calculation and construction alike show that, with this explanation, his position of *Ituna* is almost exactly correct.[1]

Thus I have, I hope, made as intelligible as the time at my command would allow, the true interpretation of Ptolemy's "Geography,"—so far, at least, as this portion of our coast is concerned. In my attempt to do this, I have left out of consideration altogether certain other corrections which are required to elucidate a more extended inquiry into Ptolemy's work. And I wish you distinctly to understand, that although the scales of 600 and 500 stadia have been sufficient for our purpose this evening, they will not alone satisfy a more general inquiry.

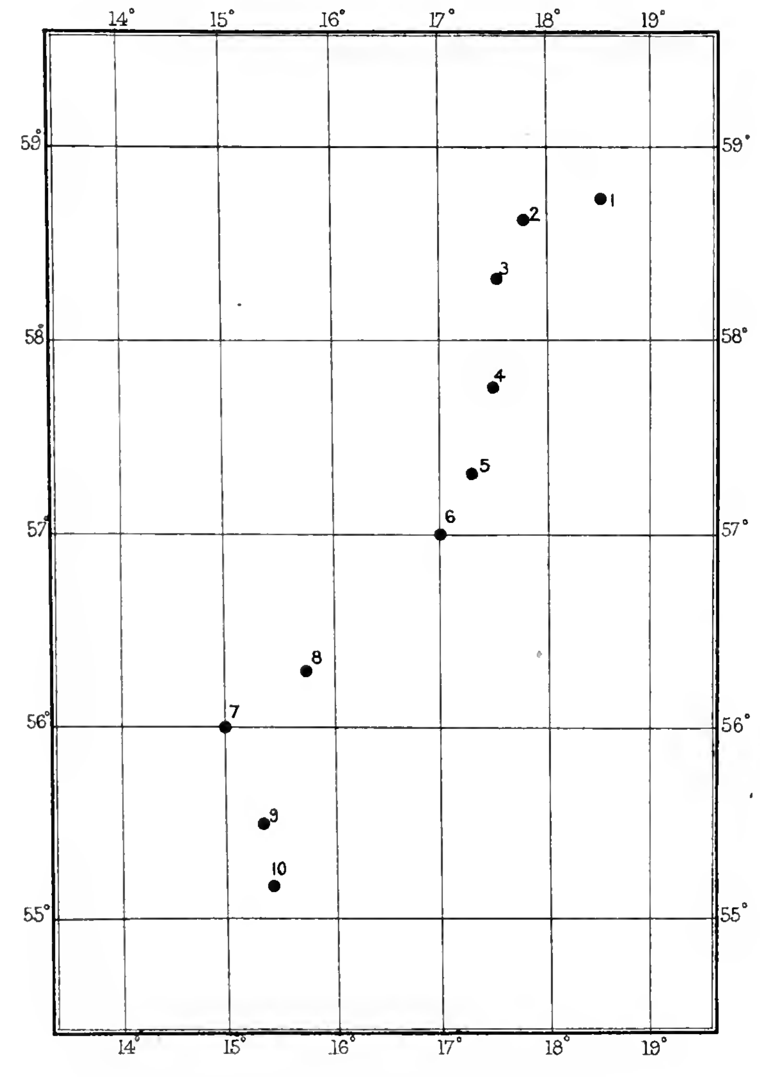
I am glad to be able to state, as the result of more extended investigation, of which you will remember this is merely a "chip," somewhat rudely broken off, that I have succeeded in ascertaining the true interpretation of Ptolemy's mode of working, and that my results enable me to hope that I have arrived at the true solution of the whole question. I have been, and am still, preparing a treatise on the subject for publication; and for whatever is imperfect to-night, you must allow me to refer you to that work when it is complete.

[1] Explicitly discussed in Explanation of Plate XX.

Ptolemy's Coast Line from Carnarvon to Cumberland.
A Tracing from the Edition "Ulmæ, 1482."



PTOLEMY'S COAST STATIONS CARNARVON TO CUMBERLAND.



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