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GEOMETRY
PLANE TRIGONOMETRY
NATURAL TRIGONOMETRIC FUNCTIONS
LOGARITHMIC TRIGONOMETRIC FUNCTIONS

Published by
INTERNATIONAL TEXTBOOK COMPANY
SCRANTON, PA.

Printed in U. S. A.

PRESS OF
INTERNATIONAL TEXTBOOK COMPANY
SCRANTON, PA.
36F

95326

PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the nonessentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed.

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CONTENTS

NOTE—This volume is made up of a number of separate Sections, the page numbers of which usually begin with 1. To enable the reader to distinguish between the different Sections, each one is designated by a number preceded by a Section mark (§), which appears at the top of each page, opposite the page number. In this list of contents, the Section number is given following the title of the Section, and under each title appears a full synopsis of the subjects treated. This table of contents will enable the reader to find readily any topic covered.

GEOMETRY

(PART 1)

Serial 778A — Edition 1

PRELIMINARY DEFINITIONS

Note.—The study of Geometry is a process of systematic and orderly reasoning rather than a matter of memory. The student is advised to study the principles and propositions stated until he understands them thoroughly and sees their relation one to another, and, when a proposition is accompanied by an explanation in small type, to read over the explanation carefully one or more times, until he clearly understands the matter, following out the references to the figure when a figure is given. If he will do this he will find Geometry to be of great benefit and assistance to him in his subsequent studies. But he is not required to commit to memory the explanations or any part of the text except a few of the more important principles and propositions, such as those to which the Examination Questions relate.

1. Every material body possesses two general properties without regard to any other condition, namely: **form,** or **shape,** which is due to the relative positions of its parts; and **magnitude,** or **size,** which is due to the distance of its parts from one another.

The form and magnitude of a body can be described by the relative positions of *points*, *lines*, and *surfaces*.

2. A **point** has position without magnitude. A dot is commonly used to represent a point; but a dot, no matter how small, has length, breadth, and thickness, while a theoretical point has position only.

3. A **line** is the path of a point in motion; it has one dimension—length. Thus, if a point is moved from the position A, Fig. 1, to the position B, its path, or trace, is the line AB.

Fig. 1

Fig. 2

4. A **straight line, or right line,** Fig. 2, is a line that does not change its direction.

5. The **distance** between two points is the length of the straight line joining them.

Fig. 3

6. A **curved line,** Fig. 3, is a line that changes its direction at every point.

Fig. 4

7. A **broken line,** Fig. 4, is a line that changes its direction at only certain points. It is made up wholly of different straight lines.

The word *line*, when not qualified by any other word, is understood to mean a straight line.

8. A **surface** is the path of a line when moved in a direction other than its length. Thus, if a line is moved from the position *A B*, Fig. 5, to the position *C D*, the line describes the surface *A B D C*.

Fig. 5

9. A **flat surface, plane surface,** or simply a **plane,** is a surface such that a straight line between any two of its points lies wholly in the surface. If a straightedge is laid on a plane surface in any direction, every point of the straightedge will touch the surface.

10. A **figure** is any combination of points and lines. A figure that lies entirely in one plane is a **plane figure.**

In referring to a figure, a point is designated by a letter placed conveniently near it; thus, in Fig. 1, the left end of the line is referred to as the point *A*. The entire line is referred to as "the line *A B*," the letters *A* and *B* designating two points, usually the ends of the line. If a line is broken or curved, as many points are named as are considered necessary to designate the line.

11. Geometry is that branch of mathematics that treats of the construction and properties of figures.

12. To **produce** a line is to prolong it or to increase its length. A straight line can be prolonged or produced to any extent in either direction. Thus, in Fig. 6, the straight line *A B* is produced to the points *C* and *D*.

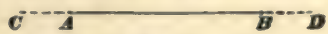


FIG. 6

13. To **bisect** any given magnitude is to divide it into two equal parts. Thus, the straight line *A B*, Fig. 7, is bisected at the point *C* if *A C* is equal to *C B*. When a given magnitude is bisected, each of the parts into which it is divided is one-half the given magnitude.

FIG. 7

STRAIGHT-LINE FIGURES

ANGLES AND PERPENDICULARS

14. An **angle**, Fig. 8, is the opening between two straight lines that meet in a point. The two straight lines are the **sides**, and the point where the lines meet is the **vertex**, of the angle. Thus, in Fig. 8, the straight lines *O A* and *O B* form an angle at the point *O*; the lines *O A* and *O B* are the sides of this angle, and the point *O* is its vertex.

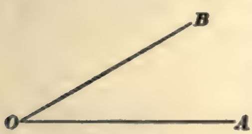


FIG. 8

An angle is usually referred to by naming a letter on each of its sides and a third letter at the vertex, the letter at the vertex being placed between the other two. Thus, the angle in Fig. 8 is called angle *A O B* or angle *B O A*.

An angle may also be designated by a letter placed between its sides near the vertex. Thus, the two angles *X C Y* and *Y C Z*, Fig. 9, may be referred to as the angles *A* and *B*, respectively.

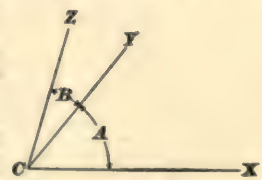


FIG. 9

An isolated angle, that is, an angle whose vertex is not the vertex of any other angle, may be designated by naming the letter at its vertex. For example, the angle in Fig. 8 may be called the angle *O*.

15. Two angles, as A and B, Fig. 9, having the same vertex and a common side $C\,Y$, are called **adjacent angles.**

16. Two angles are equal when one can be placed on the other so that they will coincide. Thus, in Fig. 10, the angles $A\,O\,B$ and $A'\,O'\,B'$ are equal, because $A'\,O'\,B'$ can be superimposed on $A\,O\,B$, so that with O' upon O and $A'\,O'$ along $A\,O$, $B'\,O'$ will take the direction of $B\,O$ and coincide with it.

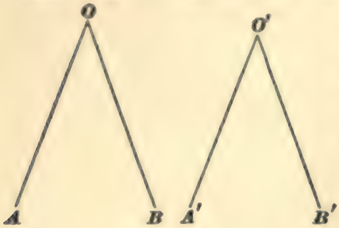


FIG. 10

17. Any angle may be thought of as being formed, or **generated,** by a line turning about the vertex as a pivot, from the position of one side to the position of the other. Thus, the angle $A\,O\,B$, Fig. 8, may be conceived as generated by a line turning about O from the position $O\,A$ to the position $O\,B$. The size of the angle does not depend on the length of the sides, which are supposed to be of indefinite length, but on the opening between the sides; or, what is the same thing, on the amount of turning necessary to bring one side to the position of the other.

18. If a straight line, as $A\,B$, Fig. 11, meets another straight line, as $C\,D$, so as to make with it two equal adjacent angles, each of these angles is a **right angle,** and the first line is said to be **perpendicular** to the second. The point where the first line meets the second is called the **foot** of the perpendicular. It is evident that all right angles are equal.

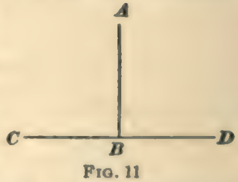


FIG. 11

19. A **horizontal line** is a line parallel to the horizon, or to the surface of still water.

20. A **vertical line** is a line perpendicular to a horizontal line, and having, therefore, the direction of a plumb-line. See Fig. 12.

FIG. 12

21. An **oblique angle** is any angle that is not a right angle. An **acute angle** is an oblique angle that is less than a right angle. An **obtuse angle** is an oblique angle that is greater than a right angle. In Fig. 13, BOC and AOC are oblique angles, BOC being an acute angle, and AOC an obtuse angle.

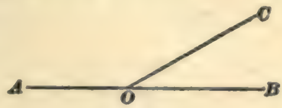


Fig. 13

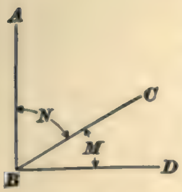


Fig. 14

22. Two angles are said to be **complementary** when their sum is equal to one right angle. Each of two complementary angles is called the **complement** of the other. Thus, in Fig. 14, in which AB is perpendicular to BD, the angles M and N are complementary, their sum being equal to the right angle ABD.

23. Two angles are said to be **supplementary** when their sum is equal to two right angles. Each of two supplementary angles is called the **supplement** of the other. In Fig. 15, AOD and DOB are supplementary angles, their sum being evidently equal to the sum of the two right angles POB and POA.

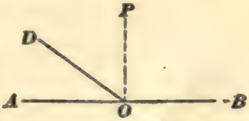


Fig. 15

It will be seen from this illustration that two adjacent angles whose non-common sides are in the same straight line are always supplementary. Conversely, if two adjacent angles are supplementary, their non-common sides are in the same straight line.

24. At a given point in a straight line, one perpendicular to the line and only one can be drawn.

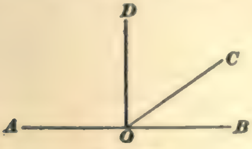


Fig. 16

Let O, Fig. 16, be the given point in the line OB. Suppose that with the point O fixed, the line OC starts from the position OB and revolves about O. In any position, as OC, it makes two angles with the line AB; one AOC, the other BOC. As OC revolves from the position OB to the position OA, the angle BOC will continually increase, and the angle *A O C* will continually decrease. There will therefore be one position, as *O D*, where the two angles are equal, and there can evidently be but one such position.

25. The sum of all the angles formed on the same side of a straight line about the same point in the line is equal to two right angles.

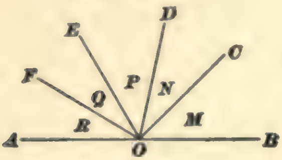


FIG. 17

In Fig. 17, the sum of the three angles *M*, *N*, and *P* is evidently equal to the angle *B O E*, and the sum of the angles *Q* and *R* is equal to the angle *E O A*. But, by Art. **23**, *B O E* + *E O A* is equal to two right angles. Hence, $M + N + P + Q + R$ = two right angles.

26. The sum of all the angles formed in the same plane about one point is equal to four right angles. Thus, in Fig. 18, $M + N + P + Q + R + S + T + U$ = four right angles.

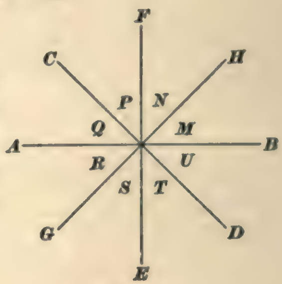


FIG. 18

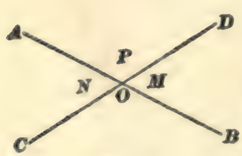


FIG. 19

27. When two lines, as *A B* and *C D*, Fig. 19, cut or cross each other, they are said to **intersect**. Their common point *O* is called their **point of intersection**, or simply their **intersection**.

28. Two intersecting straight lines determine four angles having a common vertex. Any one of these angles and the angle on the opposite side of both lines, as the angles *M* and *N*, Fig. 19, are called **vertical angles** with respect to each other. Vertical angles may also be defined as those having a common vertex and in which the sides of the one are the prolongations of the sides of the other.

Since *M* and *N* are each the supplement of *P*, they are equal to each other. Any angle is equal to its vertical angle.

29. If two straight lines intersect and one of the angles is a right angle, the other three angles are right angles, and the lines are perpendicular to each other.

30. Two oblique lines drawn from the same point in a perpendicular to a line, and cutting off on that line equal distances from the foot of the perpendicular, are equal.

Let PO and PQ, Fig. 20, be two oblique lines drawn from the point P in the perpendicular AB, and let BO and BQ be equal. Then, by turning the right side of the figure about AB, it will coincide with the left side; O will fall on Q, and PO will coincide with PQ. Hence, PO is equal to PQ.

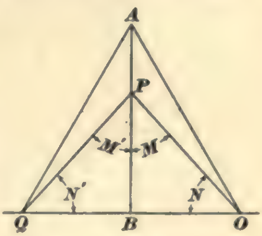


FIG. 20

31. Every point in the perpendicular at the middle point of a straight line is equally distant from the ends of the line. Thus, in Fig. 20, P, which may be any point in the perpendicular AB at the middle point B of OQ is equally distant from Q and O.

32. Two equal oblique lines drawn from the same point in the perpendicular to a straight line make equal angles with the straight line and with the perpendicular.

Since, when PBO, Fig. 20, is brought to coincide with PBQ, PO coincides with PQ and BO with BQ, the angle M = angle M', and angle N = angle N'.

33. A line that divides an angle into two equal angles is called the **bisector** of that angle. In Fig. 20, PB is the bisector of OPQ, since $M = M'$.

34. Two points, each of which is equally distant from the two extremities of a line, determine a perpendicular bisecting the line. Thus, in Fig. 20, A and P are two points equally distant from Q and O and determine the perpendicular bisecting the line OQ.

EXAMPLES FOR PRACTICE

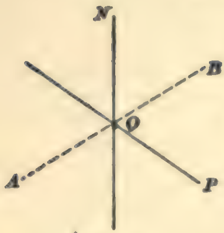


FIG. 21

1. Show that the bisectors of two vertical angles are in the same straight line.

SUGGESTION.—In Fig. 21, show that the sum of the angles on one side of the bisector *A B* of the angle *N O P* is equal to the sum of the angles on the other side.

2. Show that the bisectors of two supplementary adjacent angles are perpendicular to each other.

SUGGESTION.—In Fig. 22, show that the angle *E O F* is one-half of two right angles.

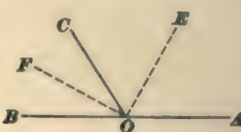


FIG. 22

PARALLELS

35. Parallel lines, Fig. 23, are straight lines that lie in the same plane and never meet, however far they are produced. Any two parallel lines have the same direction and are everywhere equally distant from each other.

FIG. 23

36. When two parallel lines, as *P Q* and *R S*, Fig. 24, are cut by a third line, as *X Y*, the cutting line *X Y* is called **a secant line** or **a transversal.**

The eight angles thus formed are named as follows: The angles *a*, *A*, *d*, and *D* are **exterior angles.** The angles *b*, *B*, *c*, and *C* are **interior angles.** The pairs of angles *a* and *d* or *A* and *D* are **alternate-exterior angles.** The pairs of angles *b* and *c* or *B* and *C* are **alternate-interior angles.** The pairs of angles *a* and *c*, *A* and *C*, *b* and *d*, or *B* and *D* are **exterior-interior** or **corresponding angles.**

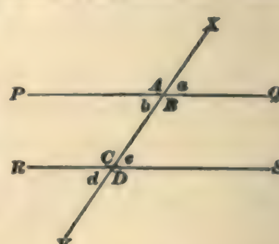


FIG. 24

37. When two parallel lines are cut by a transversal, the alternate-interior angles are equal.

Let CD and EF, Fig. 25, be the parallel lines and AB the transversal. The angles M and M' have their sides GD and HF parallel and AG and GH in the same line; hence, the turning in changing from the direction HF to the direction HG is equal to the turning in changing from the direction GD to the direction GA. That is, angle AGD, or M, is equal to the angle GHF, or M', Art. **17.** But angle M is equal to angle N, Art. **28**; therefore, angle N is equal to angle M'. In like manner, it can be shown that the angle DGH is equal to the angle GHE.

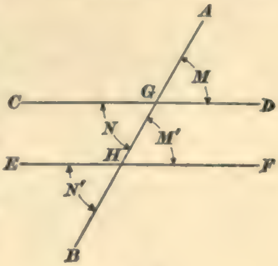


Fig. 25

38. It follows from the preceding article that the alternate-exterior angles are equal; also, the exterior-interior angles. Thus, in Fig. 24, we have $a = d$, $A = D$; $B = D$, $b = d$; $B = C$, $b = c$.

39. In Fig. 24, the angle a and the angle A are supplementary adjacent angles, and their sum is, therefore, equal to two right angles. From this, and from the principle stated in the preceding article, it follows that any angle in Fig. 24 marked by a capital letter and any angle marked by a small letter are together equal to two right angles.

The principles stated in this and in the two preceding articles may be summed up as follows: When two parallel lines are cut by an oblique transversal, the four obtuse angles are equal to one another; the four acute angles are equal to one another; and any of the obtuse angles is the supplement of any of the acute angles.

40. If a straight line is perpendicular to one of two parallel lines, it is perpendicular to the other also.

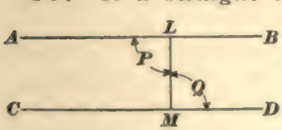


Fig. 26

In Fig. 26, AB and CD are parallel, and LM is drawn perpendicular to AB. Then, since the alternate-interior angles P and Q are equal, and since P is a right angle, Q must be a right angle also; that is, LM is perpendicular to CD.

41. The distance between two parallel lines is the length intercepted by the two parallels on any line perpendicular to them. Thus, *L M*, Fig. 26, is the distance between *A B* and *C D*.

42. If two straight lines *A B* and *C D*, Fig. 27, are cut by a third straight line *E F* so that the exterior-interior angles *M* and *N* are equal, the two straight lines are parallel.

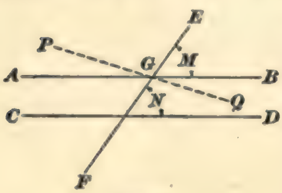


FIG. 27

If *A B* were not parallel to *C D*, we might draw through *G* a line *P Q* that was parallel to *C D*. But then the exterior-interior angles *N* and *E G Q* would be equal (Art. 38), which is obviously inconsistent with the supposition that *N* is equal to *M*.

43. If two lines, as *A B* and *C D*, Fig. 28, are parallel to a third line, as *E F*, they are parallel to each other.

Draw a transversal *G H*. Then, since *A B* is parallel to *E F*, the alternate-interior angles *M* and *N* are equal; and, since *C D* is parallel to *E F*, the alternate-interior angles *P* and *N* are equal. We have, therefore, *N* = *M*, *N* = *P*, and, consequently, *M* = *P*. As *M* and *P* are exterior-interior angles, it follows, from Art. **42**, that *A B* and *C D* are parallel.

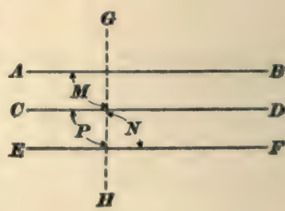


FIG. 28

44. Two angles whose sides are respectively parallel and lie in the same or opposite directions from their vertexes are equal.

In Fig. 29 (*a*), *B A* and *E D* are parallel and extend in the same direction; also, *B C* and *E F* are parallel and extend in the same direction from the vertexes. Let *O* be the point of intersection of the sides *B C* and *E D* produced. Then, since *B Q* and *E F* are parallel, the exterior-interior angles *E* and *M* are equal; and, since *B A* and *E G* are parallel, the exterior-interior angles *B* and *M* are equal. Therefore, the angles *B* and *E*, being each equal to *M*, are equal to each other.

In Fig. 29 (*b*), *B A* and *E F* are parallel and extend in opposite directions; also, *B C* and *E D* are parallel and extend in opposite directions from the vertexes. Producing *F E* and *D E*, we have, by the preceding case, *B* = *D' E F'*. As *D E F* and *D' E F'* ar vertical angles, they are equal, and, therefore, B, which is equal to $D'EF'$, is also equal to DEF.

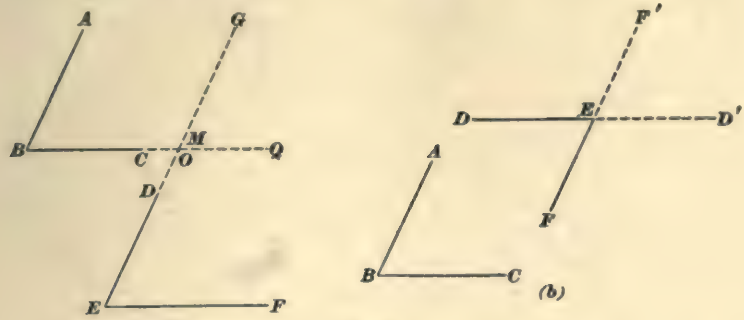


FIG. 29

45. If one side of an angle is parallel to one side of another angle, the two extending in the same direction from the vertexes, and if the other sides of the two angles are also parallel, but extend in opposite directions from the vertexes, the two angles are supplementary.

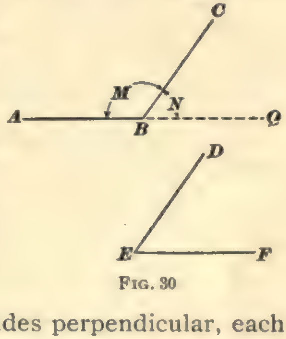


FIG. 30

In Fig. 30, BC and ED are parallel and extend in the same direction, while BA and EF are parallel and extend in opposite directions from the vertexes. Producing AB, we have, by Art. **44,** $N = E$. Now, $M + N =$ two right angles; therefore, $M + E =$ two right angles.

46. Two angles that have their sides perpendicular, each to each, are either equal or supplementary; they are equal if both are acute or both obtuse; and supplementary if one is acute and the other obtuse.

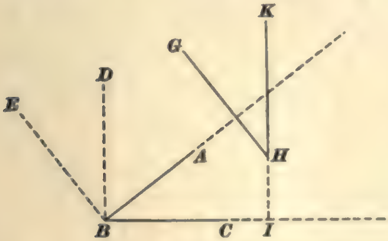


FIG. 31

In Fig. 31, let GH be perpendicular to AB, and KH perpendicular to BC. Draw BD parallel to KH, and BE parallel to GH. Then, by Art. **44,** DBE is equal to KHG. Since EBA and DBC are right angles, by taking DBA

from each of them *E B D* is seen to be equal to *A B C*. Hence, the acute angle *A B C* is equal to the acute angle *K H G*. Also, when one angle is the acute angle *A B C* and the other is the obtuse angle *G H I*, since *G H I* is the supplement of *K H G*, it must be the supplement of *A B C*.

POLYGONS

DEFINITIONS

47. A **polygon** is a portion of a plane bounded by straight lines. The boundary lines are the **sides** of the polygon. The angles formed by the sides are the **angles** of the polygon. The vertexes of the angles of the polygon are the **vertexes** of the polygon. The broken line that bounds it, or the whole distance around it, is the **perimeter** of the polygon. Thus, *A B C D E*, Fig. 32, is a polygon; the sides of this polygon are *A B*, *B C*, *C D*, *D E*, and *E A*; its angles are *A B C*, *B C D*, *C D E*, *D E A*, and *E A B*; and its vertexes are *A*, *B*, *C*, *D*, and *E*.

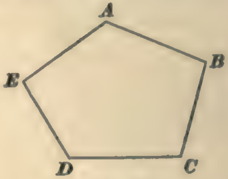


Fig. 32

48. The number of vertexes of a polygon is the same as the number of sides.

49. The least number of sides that a polygon can have is three, since two straight lines cannot enclose space.

50. Polygons are classified in various manners. One of these classifications is based on the number of sides. A polygon of three sides is a **triangle**; a polygon of four sides, a **quadrilateral**; a polygon of five sides, a **pentagon**; a polygon of six sides, a **hexagon**; a polygon of seven sides, a **heptagon**; a polygon of eight sides, an **octagon**; a polygon of nine sides, a **nonagon**; a polygon of ten sides, a **decagon**; a polygon of twelve sides, a **dodecagon**.

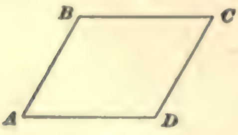


Fig. 33

51. An **equilateral polygon** is a polygon whose sides are all equal. Thus, in Fig. 33, $AB = BC = CD = DA$; hence, *A B C D* is an equilateral polygon.

52. An **equiangular polygon** is a polygon whose angles are all equal. Thus, in Fig. 34, angle A = angle B = angle D = angle C; hence, $ABDC$ is an equiangular polygon.

Fig. 34

53. A **regular polygon** is a polygon in which all the sides and all the angles are equal. Thus, in Fig. 35, $AB = BD = DC = CA$; and angle A = angle B = angle D = angle C; hence, $ABDC$ is a regular polygon. Some regular polygons are shown in Fig. 36.

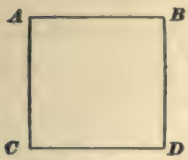


Fig. 35

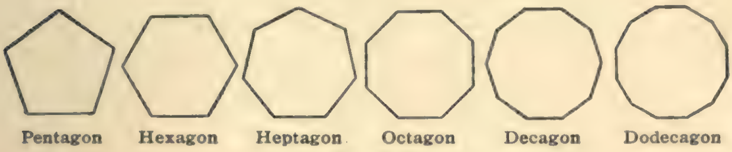


Fig. 36

54. A **reentrant angle** of a polygon is an angle whose sides if produced through the vertex will enter the surface bounded by the perimeter of the polygon. Thus, BCD, Fig. 37, is a reentrant angle.

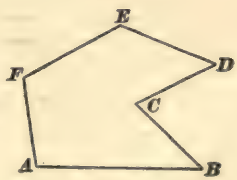


Fig. 37

TRIANGLES

55. Triangles are classified with regard to their sides into *scalene*, *isosceles*, and *equilateral* triangles.

Fig. 38

56. A **scalene triangle**, Fig. 38, is a triangle that has no two of its sides equal.

57. An **isosceles triangle**, Fig. 39, is a triangle that has two of its sides equal.

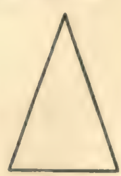
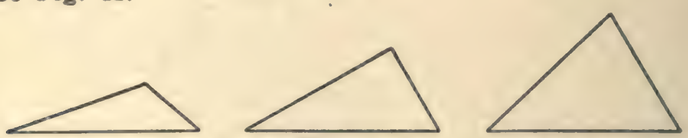


Fig. 39

58. An **equilateral triangle,** Fig. 40, is a triangle that has its three sides equal. An equilateral triangle is a particular kind of isosceles triangle. Thus, the triangle *A B C*, Fig. 40, may be regarded as an isosceles triangle whose equal sides are *A B* and *A C*, as an isosceles triangle whose equal sides are *B A* and *B C*, or as an isosceles triangle whose equal sides are *C A* and *C B*. All the statements made with regard to isosceles triangles are, therefore, true of equilateral triangles.

Fig. 40

59. Triangles are classified with regard to their angles into *right-angled*, *obtuse-angled*, and *acute-angled triangles*. See Fig. 41.


Obtuse-angled triangle Right-angled triangle Acute-angled triangle
Fig. 41

60. A **right-angled triangle,** or a **right triangle,** is a triangle having a right angle. The **hypotenuse** of a right triangle is the side opposite the right angle. The **legs** of a right triangle are the sides that include the right angle.

61. An **obtuse-angled triangle** is a triangle having an obtuse angle.

62. An **acute-angled triangle** is a triangle all the angles of which are acute.

63. An **oblique triangle** is a triangle that has no right angle. The class oblique triangles includes all obtuse-angled and acute-angled triangles.

64. An **equiangular triangle** is a triangle whose three angles are equal.

65. The **base** of a triangle is the side on which the triangle is supposed to stand. In a scalene triangle. any side may be considered as the base. In an isosceles triangle, the unequal side is usually, though not necessarily, taken as the base.

The angle opposite the base of a triangle is sometimes called the **vertical angle** of the triangle. In Figs. 42 and 43, AC is the base.

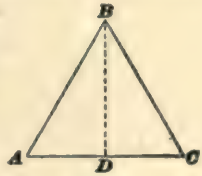


FIG. 42

66. The **altitude** of a triangle is the length of a line drawn from the vertex of the angle opposite the base perpendicular to the base. Thus, in Figs. 42 and 43, the length of BD is the altitude.

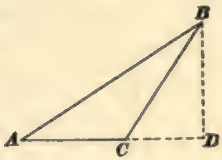


FIG. 43

67. An **exterior angle** of a triangle is an angle formed by a side and the prolongation of another side. Thus, in Figs. 43 and 44, the angle BCD, formed by the side BC and the prolongation of the side AC, is an exterior angle of the triangle ABC. The angle BCA is adjacent to the exterior angle BCD. The angles A and B are **opposite-interior angles** to the angle BCD.

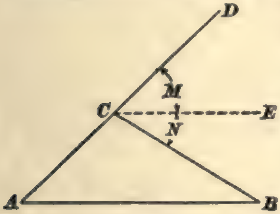


FIG. 44

68. In any triangle, an exterior angle is equal to the sum of the opposite-interior angles.

Let DCB, Fig. 44, be an exterior angle of the triangle ABC. Draw CE through C parallel to AB. Then, the angles M and A, being exterior-interior angles, are equal. Also, N and B, being alternate-interior angles, are equal. Hence, angle M plus angle N, that is, the exterior angle DCB, is equal to angle A plus angle B, or the sum of the opposite-interior angles.

69. The sum of the interior angles of a triangle is equal to two right angles.

In Fig. 44, the angles BCD and BCA, being supplementary adjacent angles, are together equal to two right angles. But, by the preceding article, the angle BCD is equal to the sum of the angles A and B. Hence, the sum of the three interior angles A, B, and BCA is equal to two right angles.

70. The following important propositions are immediate consequences of that stated in Art. **69:**

1. If two angles of a triangle are known, or if their sum is known, the third angle can be found by subtracting their sum from two right angles.

2. If two angles of a triangle are equal, respectively, to two angles of another triangle, the third angle of the first-mentioned triangle is equal to the third angle of the other triangle.

3. A triangle can have but one right angle, or one obtuse angle.

4. In any right triangle, the two acute angles are complementary.

5. Each angle of an equiangular triangle is equal to one-third of two right angles, or two-thirds of one right angle.

6. From a point without a line, only one perpendicular to the line can be drawn.

EXAMPLES FOR PRACTICE

1. If one acute angle of a right triangle is one-third of a right angle, what is the value of the other? Ans. Two-thirds of a right angle

2. If one angle of a triangle is one-half of a right angle, and another is five-sixths of a right angle, what is the third angle?

Ans. Two-thirds of a right angle

3. The exterior angle of a triangle is $1\frac{2}{5}$ right angles, and one of the opposite-interior angles is one-fourth of a right angle; what are the other angles of the triangle?

Ans. { Other opposite-interior angle $= \frac{23}{20} = 1.15$ right angles
Angle adjacent to exterior angle = three-fifths of a right angle

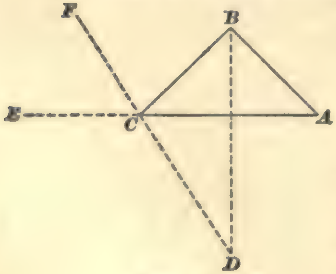


Fig. 45

4. Show that in the triangle *ABC*, Fig. 45, the bisector of the right angle *ABC* forms with the bisector of the exterior angle at *C* an angle that is equal to one-half of the angle *A*.

Suggestion.—Let *BD* be the bisector of *ABC* and *FD* the bisector of *BCE*. Then *BCF* is equal to *CBD* plus *CDB*, or *CDB* is equal to *BCF* minus *CBD*. Also, *ECB* is equal to *CBA* plus *A*, or *A* is equal to *ECB* minus *CBA*. Furthermore, *ECB* is equal to twice *BCF* and *CBA* is equal to twice *CBD*.

5. One angle of a triangle is one-half of a right angle: (*a*) What are the remaining two angles, if one is twice as large as the other? (*b*) What kind of triangle is this?

Ans. { (*a*) One-half of a right angle and one right angle
(*b*) An isosceles right triangle

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71. Two plane figures are **equal** when one can be placed on the other so that they will coincide in all their parts.

Thus, the triangles *A B C* and *A′ B′ C′*, Fig. 46, are equal, because if *A′ B′ C′* is imagined to be lifted off the paper, moved over and placed on *A B C*, the sides *A′ B′*, *B′ C′*, and *C′ A′* can be made to coincide

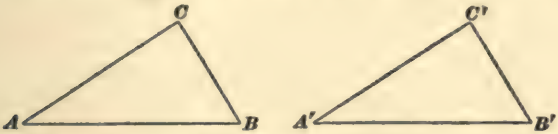

FIG. 46

with *A B*, *B C*, and *C A*, respectively, and the angles *A′*, *B′*, and *C′* to coincide with the angles *A*, *B*, and *C*. It is evident, from the figure, that if the vertexes of the two triangles coincide, the triangles will coincide throughout, and are, therefore, equal.

The polygons *A B C D E* and *A′ B′ C′ D′ E′*, Fig. 47, are equal, because *A′ B′ C′ D′ E′* can be imagined to be lifted, turned over, and placed on *A B C D E* so as to make the two polygons coincide in all their parts.

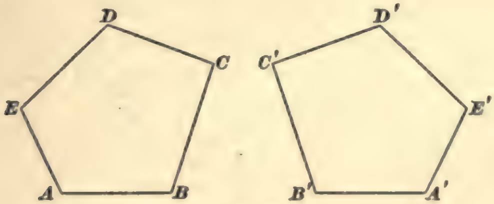

FIG. 47

72. Two triangles are equal when a side and two adjacent angles of one are equal to a side and two adjacent angles of the other.

Let *A′ B′*, Fig. 48, equal *A B*, the angle *A′* equal the angle *A*, and the angle *B′* equal the angle *B*. Now, if *A′ B′ C′* is placed on *A B C* so that *A′ B′* coincides with its equal *A B*, with *A′* on *A* and *B′* on *B*, *A′ C′* will take the direction *A C*; since the angle *A′* is equal to the angle *A*, and as *B′* is equal to *B*, *B′ C′* will take the direction *B C*.

Now, the point C' will fall somewhere on the line AC, and also somewhere on the line BC, and since two lines can intersect in only one point, C' must fall at the intersection of AC and BC, or at C. Hence, the vertexes of the triangles coincide and the triangles are equal.

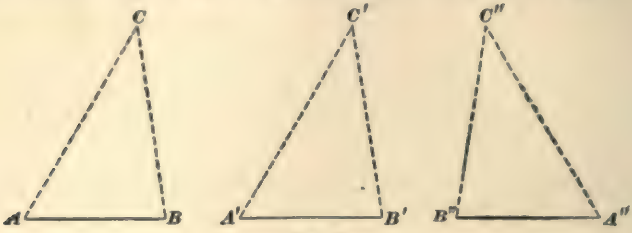

FIG. 48

The same reasoning applies to the triangles ABC and $A''B''C''$, in which $AB = A''B''$, and $A = A''$, $B = B''$; but the triangle $A''B''C''$ must be imagined to be lifted and turned over before it can be placed on ABC.

73. The following important principles are consequences of the preceding proposition:

1. Two triangles are equal when one side and any two angles of one are equal, respectively, to one side and the two similarly situated angles of the other.

2. Two right triangles are equal when one side and one acute angle of the one are equal, respectively, to one side and the similarly situated acute angle of the other.

74. Two triangles are equal when two sides and the included angle of one are equal to two sides and the included angle of the other.

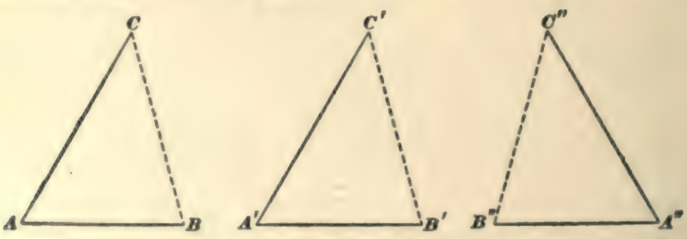

FIG. 49

In Fig. 49, $AB = A'B' = A''B''$, $AC = A'C' = A''C''$, and $A = A' = A''$. If $A'B'C'$ is placed on ABC, so that A' will coincide with A, and $A'B'$ with AB, the rest of the triangles will evidently coincide; for since $A' = A$, $A'C'$ will take the direction AC, and since $A'C' = AC$, C' will coincide with C. The same reasoning applies to $A''B''C''$, after the latter triangle has been turned over.

75. Two triangles are equal when the three sides of the one are equal, respectively, to the three sides of the other.

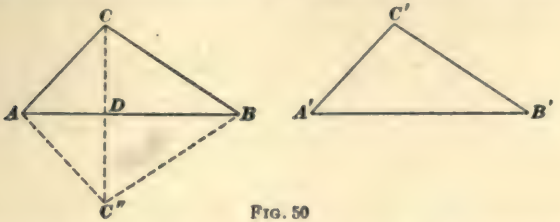

Fig. 50

In Fig. 50, let $A'B'$, $B'C'$, and $C'A'$ be equal, respectively, to AB, BC, and CA. Place $A'B'C'$ in the position ABC'', with its longest side $A'B'$ coinciding with AB, and C'' on the opposite side of AB from C; then join C and C''. Now, AC is equal to AC'', and BC is equal to BC''; hence, A and B determine a perpendicular to CC'' at its mid-point (Art. **34**). Then, by Art. **32**, the angle CAD is equal to the angle $C''AD$, or to $C'A'B'$, and by Art. **74** the triangles CAB and $C'A'B'$ are equal.

76. In an isosceles triangle, the angles opposite the equal sides are equal.

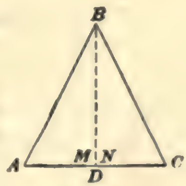

Fig. 51

Let ABC, Fig. 51, be an isosceles triangle in which $AB = BC$. Draw the bisector BD of the angle B. Then, by Art. **74**, the triangles ABD and CBD are equal. Therefore, $A = C$.

77. The equality of the triangles ABD and CBD, Fig. 51, gives $AD = DC$, and angle M = angle N = one right angle (since $M + N$ = two right angles). Hence,

1. The bisector of the vertical angle of an isosceles triangle bisects the base and is perpendicular to it.

2. Conversely, the perpendicular bisecting the base of an isosceles triangle passes through the vertex of the opposite angle and bisects that angle.

3. Also, the perpendicular drawn from the vertical angle of an isosceles triangle to the base, bisects both the base and the vertical angle.

78. If two angles of a triangle are equal, the sides opposite these two angles are equal, and the triangle is therefore isosceles.

In Fig. 51, let $A = C$. Draw BD perpendicular to AC. The right triangles BDC and BDA have the common side BD, and acute angle $A = C$. Therefore (Art. **73**), they are equal, and their hypotenuses BA and BC are equal.

79. It follows from Art. **76** that an equilateral triangle is also equiangular, and from the preceding article that an equiangular triangle is also equilateral.

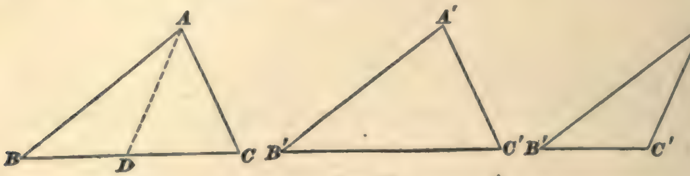

FIG. 52

80. If two sides of a triangle are equal, respectively, to two sides of another triangle, and the angle opposite one of these two sides in the first triangle is equal to the corresponding angle in the second triangle, the angles opposite the other two equal sides are either equal or supplementary.

In Fig. 52, let $A'C' = AC$, $A'B' = AB$, and the angle $B' = B$. Place $A'B'C'$ on ABC so that $A'B'$ coincides with AB. Then since $B' = B$, $B'C'$ will take the direction BC, and since $A'C'$ joins $B'C'$, C' must fall on BC, at either C or D. If C' falls at C, the triangles are equal and the angle $C' = C$; but if C' falls at D, ADB is the angle C', and ADB, the supplement of ADC, is the supplement of C, since, by Art. **76**, $ADC = C$.

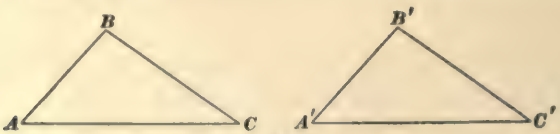

FIG. 53

81. If two triangles have two sides of the one equal to two sides of the other, and the angles opposite one pair of the equal sides are right angles or equal obtuse angles, the triangles are equal.

Since a triangle can have but one right or one obtuse angle, when the angles B and B', Fig. 53, are obtuse, the angle C' cannot be the supplement of C, hence C' must equal C.

82. Of two sides of a triangle, that is greater which is opposite the greater angle.

In the triangle ABC, Fig. 54, let the angle C be greater than the angle B. Draw CD, making with CB an angle BCD equal to the angle B. Then BCD is an isosceles triangle, and $CD = DB$. Therefore, $AD + DB$, or AB, is the same as $AD + DC$, which is evidently greater than AC.

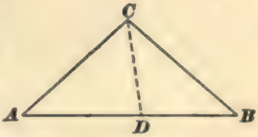

FIG. 54

83. Of two angles of a triangle, that is greater which is opposite the greater side.

Let A and B be two angles of a triangle, a the side opposite A, and b the side opposite B. Suppose that a is greater than b. If A were equal to B, the triangle would be isosceles, and $a = b$. If B were greater than A, then by the preceding article, b would be greater than a. Therefore, since B cannot be equal to or greater than A, it must be less, or A must be greater than B.

84. If from a point O, Fig. 55, without a line AB, a perpendicular OP to the line is drawn, and also two oblique lines OL and OL', the oblique line OL, whose foot L is farther from the foot P of the perpendicular, is the greater of the two oblique lines.

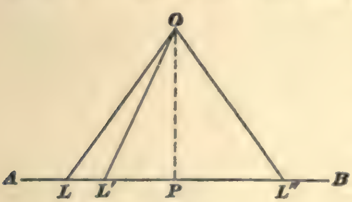

FIG. 55

Suppose the two oblique lines OL and OL' to be on the same side of the perpendicular. Since $OL'P$ is a right triangle and OPL' the right angle, the angle $OL'P$ is acute; also, the angle $OL'L$ is obtuse, since it is the supplement of $OL'P$. As the triangle OLL' can have but one obtuse angle, $OL'L$ is greater than OLP, and, therefore (Art. 82), OL is greater than OL'. If OL lies on the opposite side of the perpendicular from OL', as in the position OL'', and if $PL'' = PL$, which is greater than PL', then, by Art. **30**, $OL'' = OL$, which is greater than OL'.

85. If the hypotenuse, as AB, Fig. 56, and one leg, as BC, of a right triangle are equal, respectively, to the hypotenuse and one leg of another right triangle, as $A' B' C'$, the two triangles are equal.

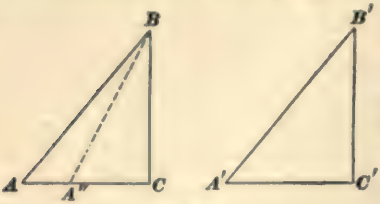

FIG. 56

Place $A' B' C'$ on $A B C$, so that $B' C'$ will coincide with its equal $B C$. Since $B' C' A'$ is a right angle, $C' A'$ will take the direction $C A$; and, since $B' A' = B A$, A' must fall on A; for if it fell to the right of A, as at A'', the hypotenuse $B A''$, or $B' A'$, would be less than $B A$ (Art. 84); and, if A' fell on the left of A, the hypotenuse $B' A'$ would be greater than $B A$.

EXAMPLES FOR PRACTICE

1. Show that, if two intersecting lines, as $A B$ and $D C$, Fig. 57, bisect each other, the lines $A C$ and $D B$ are parallel.

FIG. 57

2. If the value of the unequal or vertical angle of an isosceles triangle is two-fifths of a right angle, what is the value of each of the base angles? Ans. Four-fifths of a right angle

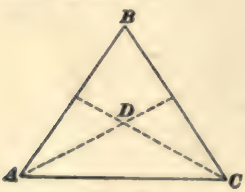

FIG. 58

3. Show that the bisectors of the base angles of an isosceles triangle form with the base an isosceles triangle; or that, $A D C$, Fig. 58, is an isosceles triangle.

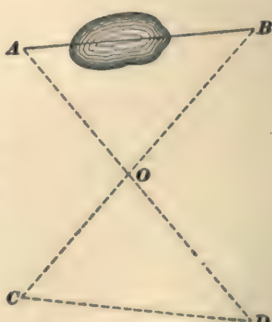

FIG. 59

4. Show that the length of the inaccessible line $A B$, Fig. 59, can be found by measuring $A O$ and $B O$, then making $O D = O B$ and $O C = O A$, and finally measuring $C D$.

QUADRILATERALS

86. There are three kinds of quadrilaterals: the *parallelogram*, the *trapezoid*, and the *trapezium*.

87. A **parallelogram** is a quadrilateral whose opposite sides are parallel. There are four kinds of parallelograms: the *rectangle*, the *square*, the *rhomboid*, and the *rhombus*.

88. A **rectangle**, Fig. 60, is a parallelogram whose angles are all right angles.

FIG. 60

FIG. 61

89. A **square**, Fig. 61, is a rectangle whose sides are equal.

90. A **rhomboid**, Fig. 62, is a quadrilateral whose opposite sides are parallel, and whose angles are not right angles.

FIG. 62

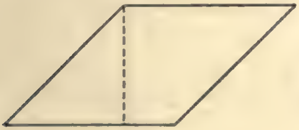

FIG. 63

91. A **rhombus**, Fig. 63, is a rhomboid having equal sides.

92. A **trapezoid**, Fig. 64, is a quadrilateral that has only two of its sides parallel.

FIG. 64

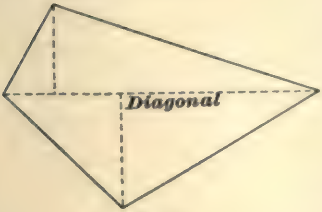

FIG. 65

93. A **trapezium**, Fig. 65, is a quadrilateral having no two sides parallel.

94. The **altitude** of a parallelogram, or of a trapezoid, is the length of the perpendicular distance between the parallel sides. See dotted line in Figs. 62, 63, and 64.

95. A **diagonal** of a quadrilateral is a straight line drawn from the vertex of any angle of the quadrilateral to the vertex of the angle opposite. A diagonal divides a quadrilateral into two triangles. See Figs. 60 and 65.

96. In a parallelogram, as $ABCD$, Fig. 66, the opposite sides and opposite angles are equal; that is, $AB = DC$, $AD = BC$, angle A = angle C, angle B = angle D.

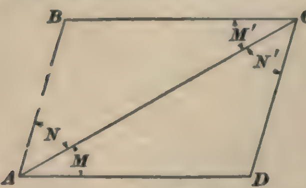

FIG. 66

Draw the diagonal AC. Then, angle M = angle M', and $N = N'$ (Art. **37**). The triangles ADC and ABC, having the common side AC and the adjacent angles M and N' equal, respectively, to M' and N, are equal (Art. **72**). Therefore, $AD = BC$, $AB = DC$, and angle B = angle D. Also, since $M = M'$ and $N = N'$, it follows that $M + N$, or BAD, is equal to $M' + N'$, or BCD.

97. The diagonal of a parallelogram divides the parallelogram into two equal triangles.

98. Parallel lines intercepted between parallel lines are equal. Thus, if the parallels AB and CD, Fig. 67, are cut by the parallels EF, GH, IJ, KL, we have, from Art. **96**, $MN = OP = QR = ST$.

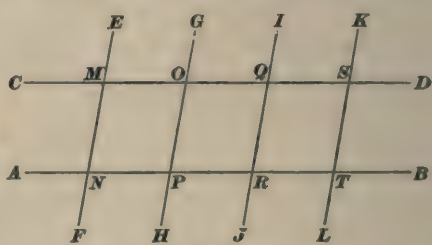

FIG. 67

99. The diagonals of a parallelogram, as AC and BD, Fig. 68, bisect each other; that is, denoting by O the point of intersection of the diagonals, $OA = OC$ and $OB = OD$.

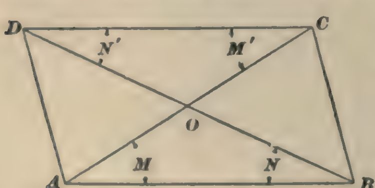

FIG. 68

In the triangles AOB and DOC, $AB = DC$ (Art. **96**), $M = M'$ and $N = N'$ (Art. **37**). Therefore, the triangles are equal (Art. **72**) and $OA = OC$, $OB = OD$.

EXAMPLES FOR PRACTICE

1. Show that if the diagonals of a quadrilateral bisect each other the figure is a parallelogram.

SUGGESTION.—In Fig. 68, assume that $OA = OC$, $OB = OD$. Then show that triangle BOC = triangle AOD, and triangle AOB = triangle DOC.

2. Show that the diagonals of a rectangle are equal.

SUGGESTION.—Show that in any rectangle $ABCD$ the triangle ABC = triangle ABD.

3. Show that if the opposite sides of a quadrilateral are equal, the figure is a parallelogram.

SUGGESTION.—Draw the diagonal. Then, by Art. **75**, the triangles formed are equal.

4. Show that if two sides of a quadrilateral are equal and parallel, the figure is a parallelogram.

5. Show that if one angle of a parallelogram is a right angle, the parallelogram is a rectangle.

ADDITIONAL PROPERTIES OF TRIANGLES

100. The bisectors of the three angles of a triangle meet in a point.

In the triangle ABC, Fig. 69, draw the bisectors of the angles A and B and let them meet at O. Join C and O, and draw the perpendiculars from O to the sides of the triangle. Then, in the right triangles BOF and BOE, BO is common and the angle OBF = angle OBE. Hence, by Art. **73**, these triangles are equal. Therefore, $OF = OE$. In a similar manner it can be shown that $OD = OE$. Therefore, $OD = OF$. The right triangles OFC and ODC, having $OD = OF$ and OC common, are equal (Art. **85**). Hence, angle OCF = angle OCD; that is, OC, which meets the bisectors AO and BO in O, is the bisector of the angle C.

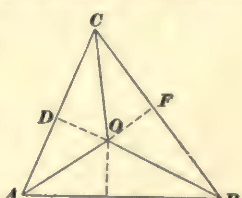

FIG. 69

101. Any point in the bisector of an angle is equally distant from the sides of the angle. For it has just been shown that, in Fig. 69, $OF = OE$.

102. The perpendiculars erected at the middle points of the three sides of a triangle meet in a point equally distant from the vertexes of the triangle.

In Fig. 70, draw the perpendiculars to CB and AC at their midpoints D and E, and let O be the point in which these perpendiculars meet. Now, O, being in OD, is equally distant from C and B (Art. **31**), that is, $OB = OC$: and being in OE, is equally distant from A and C; that is, $OA = OC$. From these two equalities it follows that $OB = OA$. Therefore, the perpendicular to AB at its middle point F passes through O. (Art. **77**).

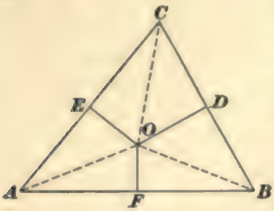

FIG. 70

103. If several parallel lines intercept equal distances on one transversal, they intercept equal distances on any other transversal.

In Fig. 71, let the parallels AE, BG, CI, DK intercept the equal distances AB, BC, and CD on the transversal PQ, and let RS be any other transversal. Draw EF, GH, IJ, parallel to AB. Then, by Art. **98**, $EF = AB$, $GH = BC$, $IJ = CD$. Hence, $EF = GH = IJ$. In the triangles EFG, GHI, and IJK, angle E = angle G = angle I (Art. **38**), and angle F = angle H = angle J (Art. **44**). Hence, by Art. **72**, these triangles are eoual, and, therefore, $EG = GI = IK$.

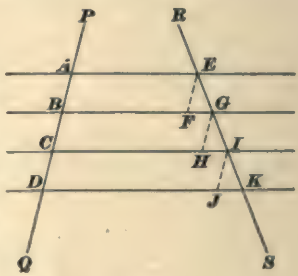

FIG. 71

104. A line parallel to one of the sides of a triangle and bisecting one of the other sides, bisects the third side also.

In Fig. 72, let DE bisect AB and be parallel to AC. Draw a line through B parallel to AC. Then since the three parallels intercept equal parts on AB, they intercept equal parts on BC; that is, $BE = EC$.

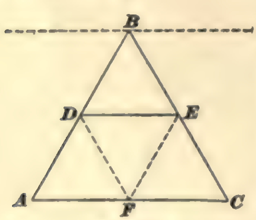

FIG. 72

105. A line joining the middle points of two sides of a triangle is parallel to the third side and equal to one-half of that third side.

In Fig. 72, let DE join D and E, the middle points of AB and BC. The first part of this proposition follows at once from the preceding article. Let F be the middle point of AC, and draw FE. This line is parallel to AB, and, therefore, $ADEF$ is a parallelogram. Consequently (Art. **96**), $DE = AF = \frac{1}{2} AC$.

106. The lines joining the middle points of the three sides of a triangle divide it into four equal triangles.

The diagonal DF, Fig. 72, divides the parallelogram $ADEF$ into two equal triangles AFD and DFE. Likewise, the diagonal EF divides $DECF$ into two equal triangles DFE and EFC; and the diagonal DE divides the parallelogram $BDFE$ into the two equal triangles DFE and BDE. Hence, triangle AFD = triangle DFE = triangle EFC = triangle BDE.

107. Any of the parallelograms $ADEF$, $FCED$, $DBEF$, Fig. 72, is equal to one-half the given triangle, since it contains two of the four equal triangles into which the given triangle is divided.

108. A line, as EF, Fig. 73, parallel to the bases AB and DC of a trapezoid and passing through the middle point E of one of the non-parallel sides, passes through the middle point of the other non-parallel side and is equal to one-half the sum of the parallel sides or bases.

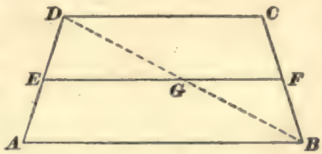

Fig. 73

Since the parallels AB, EF, and DC intercept equal parts on AD, they intercept equal parts on BC (Art. **103**); that is, $BF = FC$.

Draw BD, meeting EF in G. Then, by Art. **105**, in the triangle DCB, FG is one-half CD. Also, in the triangle ADB, GE is one-half BA. Hence, $FG + GE$, or FE, $= \frac{1}{2}(CD + BA)$.

109. The **medians** of a triangle are the lines drawn from the vertexes to the middle points of the opposite sides.

110. The medians of a triangle meet in a point whose distance from any vertex is two-thirds the length of the median from that vertex.

In Fig. 74, AD, BE, CF are the median lines of the triangle ABC; they meet at O, and $AO = \frac{2}{3} AD$, $BO = \frac{2}{3} BE$ and $CO = \frac{2}{3} CF$.

Let $A D$ and $C F$ meet at O. Join I and H, the mid-points of $C O$ and $A O$, respectively; also join D and F. Then, in the triangle $A O C$, $I H$ is parallel to $A C$ and equal to one-half $A C$ (Art. **105**). Also, in triangle $A B C$, $D F$ is parallel to $A C$ and equal to one-half $A C$. Hence, $I H$ and $D F$ are equal and parallel. It follows that the triangles $D O F$ and $H O I$ are equal, and that, therefore, $H O = O D$. But, by construction, $A H = H O$. Hence, $A H = H O = O D$, whence, $A O = \frac{2}{3} A D$. Similarly $C O = \frac{2}{3} C F$. That is, one median cuts off on the other median two-thirds of the distance from the vertex to the opposite side.

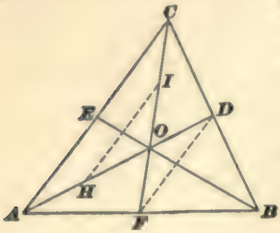

FIG. 74

POLYGONS IN GENERAL

111. Two polygons are equal when they can be divided into the same number of triangles equal each to each and similarly placed. Thus, the polygons shown at (*a*) and (*b*) in Fig. 75 are composed of the same number of triangles equal each to each and similarly placed, and it is evident that one polygon can be placed on the other so that they will coincide throughout; hence, they are equal.

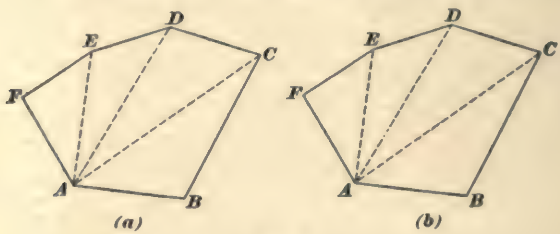

FIG. 75

112. An **exterior angle** of a polygon is an angle formed by any side and the prolongation of an adjacent side. In Fig. 76, the angles M and N are exterior angles of the polygon $A B C D E$.

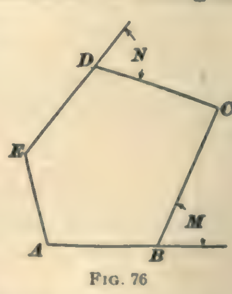

FIG. 76

113. A **diagonal** of a polygon is any line joining two vertexes not adjacent to the same side of the polygon. Thus, in Fig. 75, AC, AD, and AE are diagonals of the polygon $ABCDEF$.

114. The sum of the interior angles of any polygon is equal to two right angles multiplied by a number that is two less than the number of sides of the polygon.

Let (*a*), Fig. 75, be any polygon. Draw the diagonals from one vertex and thus divide the polygon into triangles. It is seen that the first triangle ABC and the last triangle AFE, each contains two sides of the polygon, while each of the other triangles contains but one side of the polygon. Thus, the number of triangles formed is two less than the number of the sides of the polygon. Hence (Art. **69**), the sum of the angles of the triangles, or of the polygon, is two right angles multiplied by a number that is two less than the number of sides of the polygon.

115. Let n = number of sides of a polygon;
S = sum of interior angles of the polygon, expressed in right angles.

Then, $$S = 2(n-2) = 2n - 4$$

If $n = 4$, then $S = 2 \times 4 - 4 = 4$ right angles; that is, the sum of the angles of a quadrilateral is equal to four right angles.

Example 1.—What is the value of one of the interior angles of an equiangular hexagon?

Solution.—The number of sides of a hexagon is six; hence, applying the formula, $S = 2 \times (6-2) = 8$ right angles, that is, the sum of the interior angles of a hexagon is equal to eight right angles. Since the hexagon is equiangular, one of the angles is equal to one-sixth of eight right angles, or $1\frac{1}{3}$ right angles. Ans.

Example 2.—If one of the interior angles of an equiangular polygon is equal to $1\frac{3}{7}$ right angles, what is the name of the polygon?

Solution.—If one of the interior angles is equal to $1\frac{3}{7}$ or $\frac{10}{7}$ right angles, their sum S is equal to $\frac{10}{7} \times n = \frac{10\,n}{7}$. But from the formula, $S = 2n - 4$. Therefore, $\frac{10\,n}{7} = 2n - 4$; whence, $n = 7$. A polygon of seven sides is a heptagon; therefore, the polygon is a heptagon. Ans.

EXAMPLES FOR PRACTICE

1. Show that if two angles of a quadrilateral are supplementary the other two angles are supplementary.

2. In a triangle $A B C$, the angle C is twice the angle B. Show that the line that bisects the angle C meets the line $A B$ at a point D so that $C D = B D$.

SUGGESTION.—Half the angle C = angle B. Then in the triangle $C D B$, angle $B C D$ = angle $C B D$.

3. What is the value of one of the interior angles of an equiangular octagon? Ans. $1\frac{1}{2}$ right angles

4. (*a*) What is the value of one of the interior angles of an equiangular quadrilateral? (*b*) What kind of quadrilateral is it?

Ans. { (*a*) One right angle; (*b*) Rectangle }

5. If one of the interior angles of an equiangular polygon is equal to $1\frac{5}{9}$ right angles, what is the name of the polygon? Ans. Nonagon

THE CIRCLE

DEFINITIONS AND GENERAL PROPERTIES

116. A **circle,** Fig. 77, is a plane figure bounded by a curved line every point of which is equally distant from a point within called the **center.**

FIG. 77

117. The **circumference** of a circle is the line that bounds the circle. The term *circle* is often used in the sense of *circumference.*

118. The **diameter** of a circle is a straight line drawn through the center and terminated at both ends by the circumference. Thus, $A E$, Fig. 78, is a diameter of the circle whose center is O.

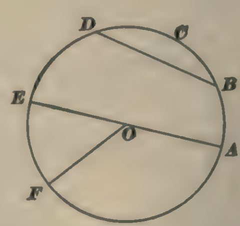

FIG. 78

119. The **radius** of a circle is any straight line drawn from the center to the circumference. The plural of *radius* is *radii*. Thus, OA, OE, and OF, Fig. 78, are radii of the circle whose center is O.

120. The distance from the center to the circumference is, by the definition of a circle, the same for all points in the same circle; hence, all radii are equal.

121. When any two radii, as OA and OE, Fig. 78, are in the same straight line, they form a diameter. Hence, the length of the diameter is twice the length of the radius.

122. An **arc** of a circle is any part of its circumference, as DCB, Fig. 78.

123. An arc equal to one-half the circumference is a **semi-circumference;** and an arc equal to one-fourth the circumference is a **quadrant.**

124. A **chord** is a straight line, as BD, Fig. 78, joining any two points in a circumference, or it is a line joining the extremities of an arc.

125. The longest chord that can be drawn in a circle is a chord that passes through the center and is, therefore, a diameter.

126. An arc of a circle is said to be **subtended** by its chord. Thus, the arc BCD, Fig. 78, is subtended by the chord BD.

Every chord in a circle subtends two arcs. Thus, BD subtends both the arcs BCD and $BAFED$.

When an arc and its chord are spoken of, the arc less than a semi-circumference is meant, unless the contrary is stated. The shorter arc is usually referred to by naming the letters at its extremities; thus, the arc BCD is called the arc BD.

127. A **segment** of a circle is a part of the circle enclosed by an arc and its chord. In Fig. 78, the part of the circle between the chord BD and the arc BD is a segment.

A segment equal to one-half the circle is a **semicircle.**

128. A **sector** of a circle is the space included between an arc and the two radii drawn to the extremities of the arc. In Fig. 78, the space included between the arc *F E* and the radii *O F* and *O E* is a sector.

129. Two circles are equal when the radius or diameter of one is equal to the radius or diameter of the other.

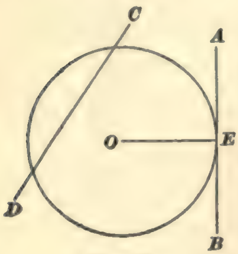

FIG. 79

130. A **tangent** to a circle is a line that touches the circumference in only one point. In Fig. 79, *A B* is tangent to the circle whose center is *O*. The point *E* at which the tangent touches the circumference is the **point of contact,** or **point of tangency.**

131. Two circles are tangent when they touch each other in one point only, as in Fig. 80. When two circles are tangent, they are tangent to the same straight line at the point of tangency.

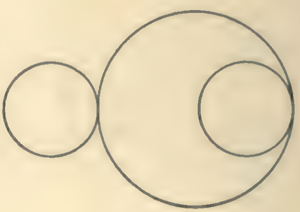

FIG. 80

132. A **secant,** as the term is used in geometry, is a line that intersects the circumference of a circle in two points. In Fig. 79, *C D* is a secant to the circle whose center is *O*.

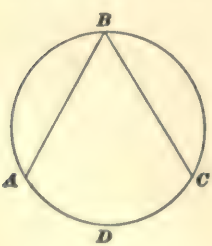

FIG. 81

133. An **inscribed angle** is an angle whose vertex lies on the circumference of a circle, and whose sides are chords. In Fig. 81, *A B C* is an inscribed angle.

134. **A central angle, or an angle at the center,** is an angle whose vertex is at the center of a circle and whose sides are radii. Thus, in Fig. 82, AOB is a central angle.

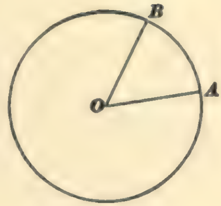

FIG. 82

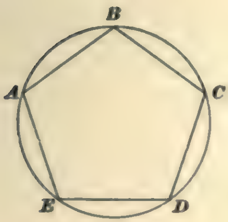

FIG. 83

135. An **inscribed polygon** is a polygon each of whose vertexes lies on the circumference of a circle, as in Fig. 83. The circle is said to be **circumscribed** about the polygon.

136. An **inscribed circle** is a circle whose circumference touches but does not intersect each of the sides of a polygon, as in Fig. 84. The polygon is said to be **circumscribed** about the circle.

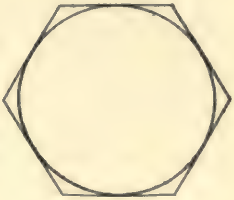
FIG. 84

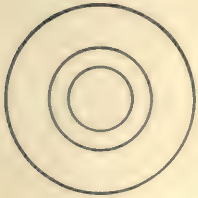
FIG. 85

137. **Concentric circles** are circles having the same center. See Fig. 85.

138. Every diameter of a circle bisects the circle and its circumference. Thus, in Fig. 86, both the arc and the portion of the circle on one side of the diameter AB are equal, respectively, to the arc and the portion of the circle on the other side.

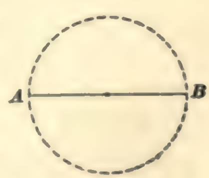

FIG 86

139. In the same circle, or equal circles, equal angles at the center intercept equal arcs on the circumference.

Let O and O', Fig. 87, be equal circles, and $A O B$ and $A' O' B'$ equal angles. Place the circle O' on O so that the point O' coincides with O and the line $O' B'$ takes the direction $O B$. Then, since $O B$ and $O' B'$ are equal, being radii of equal circles, B' will fall on B, and, since the angle O' is equal to the angle O, the line $O' A'$ will take the direction of $O A$, and, being equal to $O A$, its extremity A' will fall on A. Hence, the arcs $A B$ and $A' B'$ will coincide and are equal.

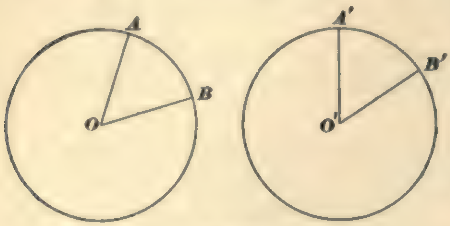

FIG. 87

140. In the same circle, or equal circles, equal arcs are intercepted by equal angles at the center.

Let O and O', Fig. 87, be equal circles, and $A B$ and $A' B'$ equal arcs. Place the circle O' on the circle O, with the points O' and A' on O and A, respectively. Then, since the arc $A' B'$ is equal to the arc $A B$, B' will fall on B. Then the angle O' is equal to the angle O, as the vertex and the sides of the angles coincide.

141. In the same circle or equal circles, equal chords subtend equal arcs.

Let $A B$ and $C D$, Fig. 88, be equal chords. Draw the radii $A O$, $B O$, $C O$, and $D O$, joining A, B, C and D to O. Then the triangles $A O B$ and $C O D$, having three sides of one equal to three sides of the other, are equal. Hence, the angle $A O B$ is equal to the angle $C O D$, and, therefore (Art. **139**), the arc $A B$ is equal to the arc $C D$.

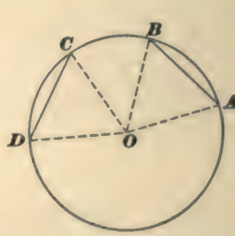

FIG. 88

142. In the same circle, or equal circles, equal arcs are subtended by equal chords.

143. A perpendicular from the center of a circle to a chord bisects the chord and the arc subtended by it.

Let OM, Fig. 89, be drawn from O perpendicular to the chord AB. Join O to A and B. The triangle AOB is isosceles, since the two sides OA and OB are radii of the same circle. Therefore (Art. **77**), $AM = MB$. Also, $AOM = MOB$ (Art. **77**); therefore (Art. **139**), arc AC = arc CB.

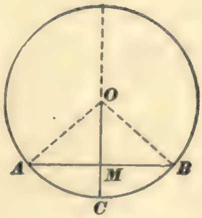

FIG. 89

144. The perpendicular erected at the middle of a chord passes through the center of the circle and bisects the arc subtended by the chord.

145. Through any three points not in a straight line a circumference can be passed.

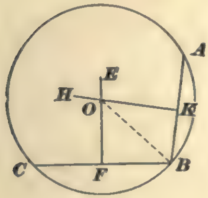

FIG. 90

Let A, B, and C, Fig. 90, be any three points. Draw AB and BC. At the middle point of AB draw KH perpendicular to AB; at the middle point of CB draw FE perpendicular to BC and meeting KH at O. As O is a point in the perpendiculars at the middle points of AB and BC, it is equally distant from A, B, and C. Therefore, a circle with O as center and OB as radius will pass through A, B, and C.

146. A straight line perpendicular to a radius at its extremity is tangent to the circle.

Let AB, Fig. 91, be perpendicular to OH at its extremity H. As OH is perpendicular to AB it is shorter than any other line, as OM, drawn from O to AB. Hence, M is without the circle, and any point in AB other than H is without the circle. Therefore, AB touches the circle in only the point H, and is, consequently, tangent to the circle.

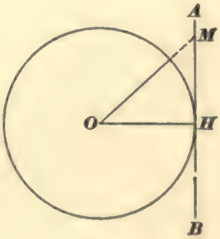

FIG. 91

147. A perpendicular to a tangent at the point of tangency passes through the center of the circle.

148. A tangent to a circle is perpendicular to the radius drawn to the point of tangency.

149. If two circles intersect, the line joining their centers bisects at right angles the line joining the points where the circles intersect.

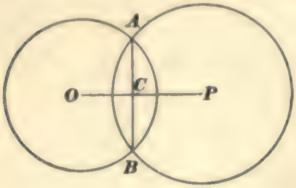

FIG. 92

Let the two circles whose centers are O and P, Fig. 92, intersect at A and B. The point P, being the center of a circle, is equally distant from A and B, points on the circumference. Similarly, O is equally distant from A and B. Hence, by Art. **34**, O and P determine the perpendicular bisecting AB.

150. The two tangents from a point to a circle are equal.

Let PA and PB, Fig. 93, be tangents from P to the circle whose center is O. Draw OA, OP, OB. Then the triangles POB and POA are right triangles (Art. **148**). In these triangles, PO is common and OA is equal to OB. Hence, the triangles are equal, and $PA = PB$.

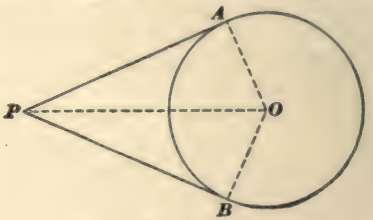

FIG. 93

151. The line joining an external point to the center of a circle bisects the angle made by the two tangents drawn from the point to the circle. Thus, the angle OPA, Fig. 93, is equal to the angle OPB.

EXAMPLES FOR PRACTICE

1. Show that the line joining the intersection of two tangents to the center of the circle bisects the chord joining the points of tangency.

2. Show that the bisector of the angle between two tangents passes through the center of the circle.

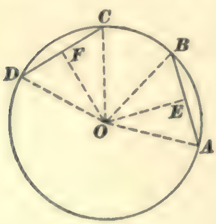

FIG. 94

3. Show that in the same circle, or equal circles, equal chords are equally distant from the center.

SUGGESTION.—Draw OE and OF, Fig. 94, perpendicular to the equal chords AB and CD. Then what is true of the triangles AEO and DOF?

4. Show that the tangents to a circle at the extremities of a diameter are parallel.

5. Show that in any circle a chord parallel to a tangent is bisected by the diameter drawn to the point of contact.

MEASUREMENT OF ANGLES

152. The **ratio** of one quantity to another of the same kind is the number of times that the first contains the second. When both quantities are represented by numbers, their ratio is the same as the quotient obtained by dividing one of the numbers by the other.

153. In the same circle, or equal circles, two central angles have the same ratio as their intercepted arcs; that is, in Fig. 95, angle AOB : angle COD = arc AB : arc CD.

Suppose the arc AB to be three-fifths of the arc CD. Divide AB into three equal parts, and CD into five equal parts, as shown, and join the points of division with the center.

Since $AB : CD = 3 : 5$, or $\frac{AB}{CD} = \frac{3}{5}$, it follows that one-third of AB is one-fifth of CD; that is, arc AE = arc DF, and, therefore, angle AOE = angle DOF. We have, therefore, angle $AOB = 3 \times$ angle AOE, angle $COD = 5 \times$ angle $DOF = 5 \times$ angle AOE; whence, $\frac{\text{angle } AOB}{\text{angle } COD} = \frac{3 \times \text{angle } AOE}{5 \times \text{angle } AOE} = \frac{3}{5} = \frac{\text{arc } AB}{\text{arc } CD}$.

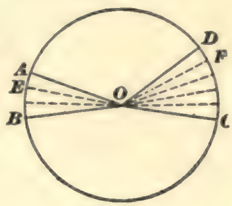

FIG. 95

154. Since the angle at the center and its intercepted arc increase and decrease in the same ratio, it is said that *an angle at the center is measured by its intercepted arc.*

155. The whole circumference of a circle is divided into 360 equal parts, called **degrees.** A degree is divided into 60 equal parts, called **minutes;** and a minute is divided into 60 equal parts, called **seconds.** Degrees, minutes, and seconds of arc are used as units for measuring circular arcs. Since the circumference of every circle contains 360 degrees, the length of a degree differs in different circles. Thus, if AOB, Fig. 96, is an angle of 1°, AB is an arc of 1° in the larger circle and CD is also and arc of 1° in the smaller concentric circle. A degree of the earth's equator is a little more than 69 miles

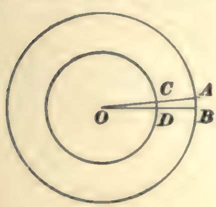

FIG. 96

long; and a degree of the circumference of a circle whose diameter is 360 inches is 3.1416 inches long.

Degrees, minutes, and seconds are indicated by °, ′, ″. Thus, 25° 3′ 10″ means 25 degrees, 3 minutes, and 10 seconds.

Since a right angle intercepts one quarter of a circumference, the number of degrees measuring it is $360 \div 4 = 90°$. The number of degrees measuring an angle equal to one-half of a right angle is $90° \div 2 = 45°$.

Usually, the magnitude of an angle is expressed by stating the number of degrees that it subtends. Thus, a right angle is referred to as an angle of 90°; one-third of a right angle, as an angle of 30°, etc.

156. An inscribed angle is measured by one-half the intercepted arc. Thus, in Fig. 97, the angle *A B C* is measured by one-half the arc *A D C*.

Draw the diameter *B O D* and the radii *O C* and *O A*. The angle *C O D*, the exterior angle of the triangle *O B C*, is equal to the angle *O B C* plus the angle *O C B*. But the angle *O C B* is equal to the angle *O B C*, as they are opposite the equal sides of an isosceles triangle. Hence, the angle *C O D*, which is measured by the arc *C D*, is equal to $2 \times OBC$. Therefore, *O B C* is measured by one-half the arc *C D*. Similarly, the angle *O B A* is measured by one-half the arc *A D*. Therefore, the angle *A B C* is measured by one-half the arc *A D* plus one-half the arc *D C*; that is, by one-half the arc *A C*.

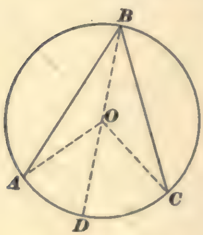

Fig. 97

157. In the same circle, or equal circles, equal arcs are intercepted by equal inscribed angles.

158. All angles inscribed in the same segment are equal.

159. Any angle inscribed in a semicircle is a right angle.

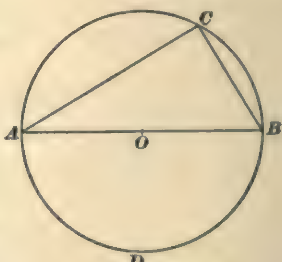

Fig. 98

The angle *A C B*, Fig. 98, is measured by one-half the arc *A D B*, which is a semi-circumference. As a semi-circumference contains 180°, the angle *A C B* is measured by one-half of 180°, or 90°, and is, therefore, a right angle.

160. The vertexes of all the angles of a given magnitude whose sides pass through two fixed points, lie on a circle that passes through the two fixed points and any one of the vertexes.

In Fig. 99, let APB be an angle of the given magnitude and A and B the fixed points. Through A, B, and P, pass a circle. Now, any angle, as AP_1B or AP_2B, whose sides pass through A and B and whose vertex lies on the arc APB is (Art. **158**) equal to the given angle APB.

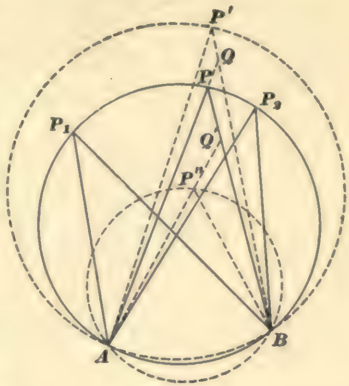

FIG. 99

Again, any angle, as $AP'B$, whose sides pass through A and B and whose vertex lies without the arc APB is less than the angle APB. For if AP is produced to meet BP' at Q, the angle APB being an exterior angle of the triangle BPQ, is equal to $PQB + PBQ$ and is therefore greater than PQB, and, as PQB is greater than $AP'B$ (since $PQB = AP'B + QAP'$), it follows that APB is greater than $AP'B$.

In like manner it can be shown that any angle, as $AP''B$, whose sides pass through A and B and whose vertex lies within the arc APB, is greater than the given angle APB.

161. An angle formed by a tangent, as TM, Fig. 100, and a chord, as TP, is measured by one-half the intercepted arc TEP.

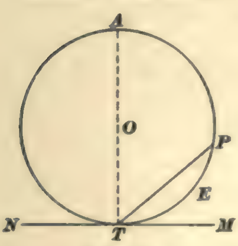

FIG. 100

Draw the diameter TOA. Then MTA is a right angle and is, therefore, measured by one-half the semi-circumference $TEPA$. The angle PTA is measured by one-half the arc PA. Hence, the angle MTP, equal to MTA minus PTA, is measured by one-half the difference between the semi-circumference and PA; that is, by one-half the arc TEP.

EXAMPLES FOR PRACTICE

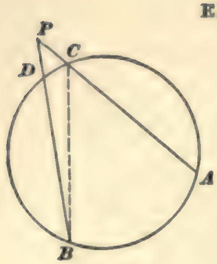

Fig. 101

1. Prove that the angle BPA, Fig. 101, formed by two secants intersecting without the circumference is measured by one-half the difference of the intercepted arcs AB and CD; that is, by $\frac{1}{2}(AB - CD)$.

SUGGESTION.—Join C and B. Then angle BCA is an exterior angle of the triangle BCP, and angle BPC is equal to angle ACB minus angle DBC.

2. Show that the angle APB, Fig. 102, formed by two tangents PT and PT' is measured by one-half the difference of the intercepted arcs TQT' and TRT'.

SUGGESTION.—Join T and T'. Then ATT' is an exterior angle of triangle $TT'P$, while ATT' and $PT'T$ are angles formed by a tangent and a chord.

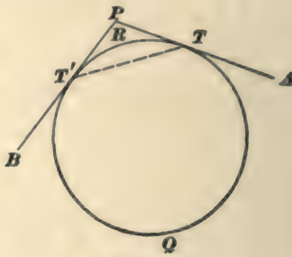

Fig. 102

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162. The **angle of intersection** of two tangents is the angle formed by one tangent with the prolongation of the other tangent. Thus, the angle APT', Fig. 103, is the angle of intersection of the two tangents TP and PT'.

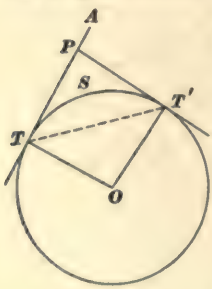

Fig. 103

163. The angle of intersection of two tangents is equal to the central angle whose sides pass through the points of tangency.

In Fig. 103, join TT'. Then, the angle APT' is equal to the sum of the equal angles PTT' and $PT'T$. But each of these angles is made by a tangent and a chord and is, therefore, measured by one-half of the arc TST'. Hence, the angle APT' is measured by the arc TST'. The central angle O is also measured by this arc; therefore, the angle O is equal to the angle APT'.

164. The opposite angles of an inscribed quadrilateral are supplementary; that is, their sum is equal to two right angles or 180°.

In Fig. 104, the angle B is measured by one-half the arc ADC, and the opposite angle D is measured by one-half the arc ABC. The sum of the arcs ADC and ABC is a circumference, or 360°. Hence, the sum of the angles ADC and ABC is measured by one-half of 360°, or 180°.

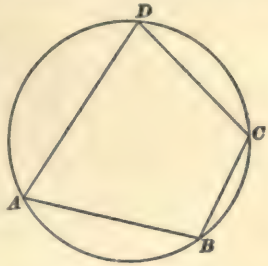

FIG. 104

165. If the opposite angles of a quadrilateral are supplementary, the quadrilateral can be inscribed in a circle.

EXAMPLE 1.—What is the number of degrees in each angle of an equilateral triangle?

SOLUTION.—The sum of the three angles of the triangle is two right angles, or 180°. Since the three angles are equal, each angle is one-third of 180°, or $\frac{180°}{3} = 60°$. Ans.

EXAMPLE 2.—The unequal angle of an isosceles triangle is 75° 32′ 10″; what is the magnitude of each of the equal angles?

SOLUTION.—Since the sum of the three angles is 180°, the sum of the two equal angles is 180° minus the other angle, or 180° − 75° 32′ 10″ = 104° 27′ 50″, and each of them is one-half of this sum, or (104° 27′ 50″) ÷ 2 = 52° 13′ 55″. Ans.

EXAMPLE 3.—The exterior angle of a triangle is 124° 3′ 40″, and one of the opposite-interior angles is 60°; find the other two angles of the triangle.

SOLUTION.—Let the given exterior angle be denoted by A, the given interior angle by B, the other opposite-interior angle by C, and the third angle of the triangle by A'. (Let the student draw the triangle and mark these angles.) Then, $A = B + C$; whence, $C = A - B$ = 124° 3′ 40″ − 60° = 64° 3′ 40″. Ans. Also, $A + A' = 180°$; whence, $A' = 180° - A$ = 180° − 124° 3′ 40″ = 55° 56′ 20″. Ans.

EXAMPLES FOR PRACTICE

1. Show that the only parallelogram that can be inscribed in a circle is a rectangle.

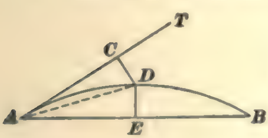

FIG. 105

2. Show that if from a point A, Fig. 105, on the arc of a circle a chord AB and a tangent AT are drawn, the perpendiculars DC and DE drawn to them from the middle point D of the subtended arc are equal.

3. The angle of intersection of two tangents is 100°; find the number of degrees in each angle formed by the tangents and the chord through the points of contact. Ans. 50°

4. One of the acute angles of a right triangle is 50°; what is the magnitude of the other acute angle? Ans. 40°

5. Each of the equal angles of an isosceles triangle is 45°; show that the triangle is right-angled.

6. Two angles of a triangle are 37° 41′ 36″ and 86° 51′ 2″; what is the value of the other angle? Ans. 55° 27′ 22″

GEOMETRY

Serial 778B (PART 2) Edition 1

PROPORTION

DEFINITIONS AND GENERAL PRINCIPLES

1. A **proportion** is an equality of ratios or of fractions. Thus, the fractions $\frac{4}{5}$ and $\frac{8}{10}$, being equal, form a proportion. In general, if $\frac{a}{b}$ is equal to $\frac{c}{d}$, these two ratios or fractions form a proportion, which may be written in any of the following forms: $\frac{a}{b} = \frac{c}{d}$, $a : b = c : d$, $a : b :: c : d$. When written in either of the last two forms, the proportion is read *a* is to *b* as *c* is to *d*.

2. Properties of Proportions.—The first and the fourth term of a proportion are called the **extremes**; the second and the third, the **means**. Thus, in the proportion $a : b = c : d$, the extremes are a and d, and the means, b and c.

3. If any four quantities are in proportion, the product of the extremes is equal to the product of the means. This principle follows at once from the definition of a proportion, as will be explained presently. If a, b, c, and d, are in proportion, then, by the definition,

$$\frac{a}{b} = \frac{c}{d} \qquad (1)$$

This equation may be treated the same as any other algebraic equation. Both members of the equation may be multiplied or divided by the same quantity, or the same quantity may be added to or subtracted from both members, and the proportion may thus be changed to a great number of forms without destroying the equality of the ratios. Different names are applied to these changes, some of the most common of which are given in the following articles.

In order to show that the product of the means is equal to the product of the extremes, multiply both members of equation (1) by bd to clear of fractions; the equation then becomes

$$ad = bc \qquad (2)$$

4. It is evident that if two fractions are equal, their reciprocals are also equal. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$; that is, if $a : b = c : d$, we have also $b : a = d : c$.

Taking the reciprocal of a fraction is called **inverting** the fraction. The operation of inverting the two fractions of a proportion is called **inversion.**

5. If both members of equation (2), Art. **3**, are divided by cd, there results

$$\frac{a}{c} = \frac{b}{d}, \text{ or } a : c = b : d$$

Or, if both members of equation (2) are divided by ba, the result is

$$\frac{d}{b} = \frac{c}{a}, \text{ or } d : b = c : a$$

Therefore, either the means or the extremes of a proportion can be interchanged. This operation is called **alternation.**

6. If 1 is added to each member of equation (1), Art. **3**, the equation becomes

$$\frac{a}{b} + 1 = \frac{c}{d} + 1$$

Reducing each member to an improper fraction,

$$\frac{a+b}{b} = \frac{c+d}{d}, \text{ or } a+b : b = c+d : d \qquad (1)$$

In a similar manner it can be shown that

$$a+b : a = c+d : c \qquad (2)$$

The proportions (1) and (2) are said to be derived from the original proportion by **composition.**

7. If 1 is subtracted from each member of equation (1), Art. **3**, the equation becomes

$$\frac{a}{b} - 1 = \frac{c}{d} - 1$$

Reducing each member to an improper fraction,

$$\frac{a-b}{b} = \frac{c-d}{d}, \text{ or } a-b:b = c-d:d \qquad (1)$$

In a similar manner it can be shown that

$$a-b:a = c-d:c \qquad (2)$$

The proportions (1) and (2) are said to be derived from the original proportion by **division.**

LINES DIVIDED PROPORTIONALLY

8. Two straight lines are **divided proportionally** when the corresponding segments or parts are in proportion; or when the ratio of the two segments of one is the same as the ratio of the two segments of the other. Thus, the lines AB and CD, Fig. 1, are divided proportionally in the points E and F if $AE:EB = CF:FD$.

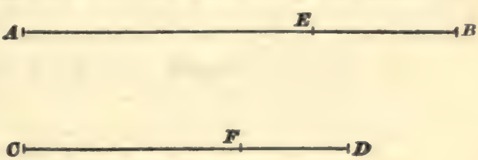

FIG. 1

9. A line parallel to one of the sides of a triangle divides the other two sides proportionally. Thus, in Fig. 2, where DE is parallel to BC, $AD:DB = AE:EC$.

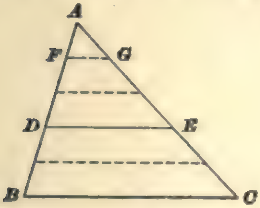

FIG. 2

Suppose that the ratio of AD to DB is as 3 to 2; that is, let $\frac{AD}{DB} = \frac{3}{2}$. Divide AB into five equal parts, and through the points of division draw lines parallel to BC These lines will intercept equal distances on AC (see *Geometry*, Part 1).

As the ratio of AD to DB is that of 3 to 2, AD will contain three. and DB will contain two, of the equal parts into which AB is divided.

Also, AE will contain three and EC two of the equal parts into which AC is divided; so that $AE = 3 \times AG$, and $EC = 2 \times AG$; whence

$$\frac{AE}{EC} = \frac{3 \times AG}{2 \times AG} = \frac{3}{2} = \frac{AD}{DB}$$

or,

$$AE : EC = AD : DB$$

10. Any two sides of a triangle are to each other as the segments into which they are divided by any line parallel to the third side. Thus, in Fig. 2, $AB : AC = AD : AE = DB : EC$.

From the preceding article, we have

$$AD : DB = AE : EC$$

whence (Art. 6),

$$AD + DB : DB = AE + EC : EC$$

that is,

$$AB : DB = AC : EC$$

and, interchanging the means (Art. 5),

$$AB : AC = DB : EC$$

In the same manner it may be shown that

$$AB : AC = AD : AE$$

11. If a line divides two sides of a triangle proportionally, it is parallel to the third side. Thus, if DE, Fig. 3, divides AB and AC so that $AD : DB = AE : EC$, then DE is parallel to BC.

If DE were not parallel to BC, a line DE' could be drawn through D parallel to BC. Then, by Art. 9, we should have $\frac{AD}{DB} = \frac{AE'}{E'C}$; whence, since we have assumed that $\frac{AD}{DB} = \frac{AE}{EC}$,

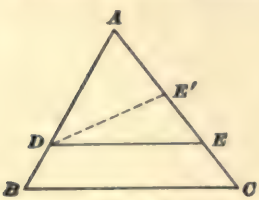

FIG. 3

$$\frac{AE}{EC} = \frac{AE'}{E'C}$$

By interchanging the means of this proportion, we obtain

$$\frac{AE}{AE'} = \frac{EC}{E'C}$$

This equality is evidently absurd, since AE is greater than AE', whereas EC is less than $E'C$ Therefore, no other line than DE can pass through D and be parallel to BC.

EXAMPLE 1.—Find the length of the line AB, Fig. 4, of which the end B is inaccessible.

SOLUTION.—There are several ways of solving this problem in practice. The one illustrated in the figure is as follows: Any convenient distance AC is measured and the angle C observed with a transit or compass. From C, a distance CE is measured, and at E an angle AED equal to C is turned off. The point D where the line of sight ED meets AB is marked, and the distances AD and AE are measured. Then, since AED equals C, the lines ED and CB are parallel (see *Geometry*, Part 1) and, therefore (Art. **9**),

$$AB : AC = AD : AE$$

whence (Art. **3**),

$$AB \times AE = AC \times AD$$

and, dividing by AE,

$$AB = \frac{AC \times AD}{AE}. \quad \text{Ans.}$$

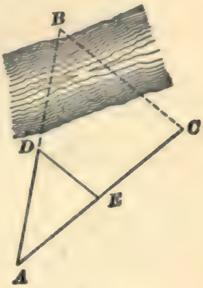

FIG. 4

EXAMPLE 2.—Divide a line AB, Fig. 1, of given length into two parts AE and EB whose ratio shall be the same as that of two given numbers m and n; that is, so that $AE : EB = m : n$.

SOLUTION.—Since $AE : EB = m : n$, we must have (Art. **6**),

$$\frac{AE + EB}{EB} = \frac{m+n}{n}, \text{ or, } \frac{AB}{EB} = \frac{m+n}{n},$$

whence, solving for EB,

$$EB = \frac{n \times AB}{m+n}. \quad \text{Ans.}$$

AE can be found in a similar manner, or by subtracting the value of EB from AB.

EXAMPLES FOR PRACTICE

1. If the measured distances in Fig. 4 are $AC = 100$ feet, $AE = 45.2$ feet, $AD = 48.36$ feet, what are the distances AB and DB?

   Ans. $\begin{cases} AB = 106.99 \text{ ft.} \\ DB = 58.63 \text{ ft.} \end{cases}$

2. If, in Fig. 3, $AD = 75$ feet, $DB = 16.25$ feet, and $AC = 80$ feet, find AE and EC.

   Ans. $\begin{cases} AE = 65.75 \text{ ft.} \\ EC = 14.25 \text{ ft.} \end{cases}$

3. If AB, Fig. 1, is equal to 125 feet, find the distances AE and EB so that the line will be divided at E in the ratio of 5 to 2.

   Ans. $\begin{cases} AE = 89.286 \text{ ft.} \\ EB = 35.714 \text{ ft.} \end{cases}$

12. If two lines, as AB and CD, Fig. 5, are cut by any number of parallel lines, as EM, GN, IO, etc., the corresponding intercepts are proportional; that is, $EG : GI = MN : NO$; $GI : IK = NO : OP$, or, by interchanging the means, $EG : MN = GI : NO = IK : OP$, etc.

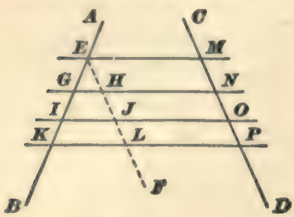

FIG. 5

Through E, draw EF parallel to CD. Then, $EH = MN$, $HJ = NO$, $JL = OP$. (See *Geometry*, Part 1.) Also, by Arts. **9** and **10**,

$$\frac{EK}{EL} = \frac{EG}{EH} = \frac{GI}{HJ} = \frac{IK}{JL}$$

that is,

$$\frac{EK}{MP} = \frac{EG}{MN} = \frac{GI}{NO} = \frac{IK}{OP}$$

or, $$EK : MP = EG : MN = GI : NO = IK : OP$$

13. In any triangle ABC, Fig. 6, the bisector BD of an angle divides the side opposite proportionally to the including sides; that is, $AB : BC = AD : DC$.

Draw CE parallel to BD and meeting AB produced in E. Then, in the triangle AEC, by Art. **9**,

$$AB : BE = AD : DC \qquad (1)$$

The angles DBC and M, being alternate-interior angles, are equal; that is, $M = \frac{1}{2}B$. The angles DBA and E, being exterior-interior angles, are equal; that is, $E = \frac{1}{2}B$. Therefore, $E = M$, and $BE = BC$. (See *Geometry*, Part 1.)

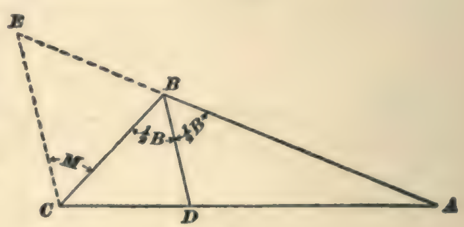

FIG. 6

Substituting, in equation (1), BC for its equal BE,

$$AB : BC = AD : DC$$

POLYGONS

SIMILAR POLYGONS

SIMILAR TRIANGLES

14. Similar polygons are those whose corresponding angles are equal and whose corresponding sides are proportional.

In order that two polygons may be similar, it is manifestly necessary that each angle of the one shall be equal to the corresponding angle of the other. But this is not sufficient; the corresponding sides must be proportional. For example,

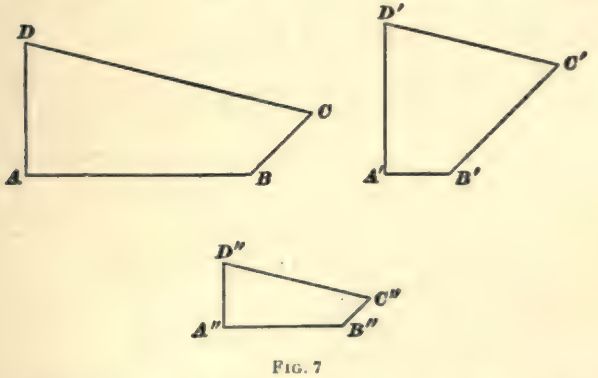

FIG. 7

the quadrilaterals *A B C D* and *A' B' C' D'*, Fig. 7, have their corresponding angles equal, but they are not similar, because their corresponding sides are not proportional. The quadrilaterals *A B C D* and *A'' B'' C'' D''* have their corresponding angles equal and their corresponding sides proportional, and are, therefore, similar.

The corresponding sides of similar polygons are called **homologous sides.**

15. Two triangles are similar when the angles of one are equal to the angles of the other.

In Fig. 8, let the angles of the triangle $A\,B\,C$ be equal, respectively, to those of the triangle $A'\,B'\,C'$. Place the triangle $A'\,B'\,C'$ upon $A\,B\,C$, so that the angle A' will coincide with its equal A. Then B' will fall along $A\,B$ and C' along $A\,C$, as at B'' and C'', respectively, and $B'\,C'$ will take the position $B''\,C''$. The angle B'', which is equal to B', is equal to B, and the angle C'', which is equal to C', is equal to C; hence, $B''\,C''$ is parallel to $B\,C$ (see *Geometry*, Part 1). Then, by Art. **10**,

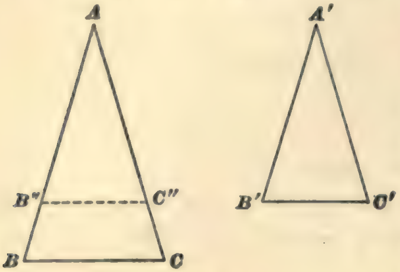

FIG. 8

$$A\,B : A\,B'' = A\,C : A\,C''$$

Substituting $A'\,B'$ and $A'\,C'$ for their respective equals $A\,B''$ and $A\,C''$,

$$A\,B : A'\,B' = A\,C : A'\,C'$$

In like manner, it can be proved that

$$A\,B : A'\,B' = B\,C : B'\,C'$$

Therefore, the triangles, having their angles equal and their corresponding sides proportional, are similar.

16. Two triangles are similar when two angles of the one are equal respectively to two angles of the other.

17. Two right triangles are similar when an acute angle of one is equal to an acute angle of the other.

18. A triangle is similar to any triangle formed by a line parallel to one of its sides and the segments it intercepts on the other two sides or the other two sides prolonged.

19. Two triangles are similar when the three sides of one are either parallel or perpendicular to the three sides of the other.

20. Two triangles are similar when their corresponding sides are proportional.

In Fig. 9, $A\,B : A'\,B' = A\,C : A'\,C' = B\,C : B'\,C'$

On AB, lay off AD equal to $A'B'$; on AC, lay off AE equal to $A'C'$, and join DE. Then, since $AB : AD = AC : AE$, DE is parallel to BC. Hence, by Art. **18**, triangles ABC and ADE are similar, and, consequently, triangles ABC and $A'B'C'$ are similar if it can be shown that $DE = B'C'$. Now,

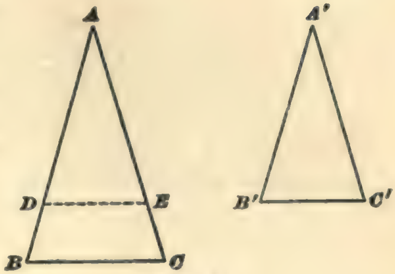

FIG. 9

$$AB : AD = BC : DE, \text{ or } AB : A'B' = BC : DE$$

But, $$AB : A'B' = BC : B'C'$$

The last two proportions are the same, term for term, excepting the last term; hence, DE is equal to $B'C'$, and the triangles ADE and $A'B'C'$ are equal. Therefore, the triangles ABC and $A'B'C'$ are similar.

21. Two triangles are similar when an angle of the one is equal to an angle of the other and the including sides are proportional.

22. In two similar triangles, corresponding altitudes have the same ratio as any two corresponding sides.

Let CD and $C'D'$, Fig. 10, be the corresponding altitudes of the two similar triangles ABC and $A'B'C'$. The right triangles ACD and $A'C'D'$, having angle A equal to the angle A', are similar; hence,

$$CD : C'D' = AC : A'C'$$

But, since the triangles ABC and $A'B'C'$ are similar,

$$AC : A'C' = AB : A'B' = BC : B'C'$$

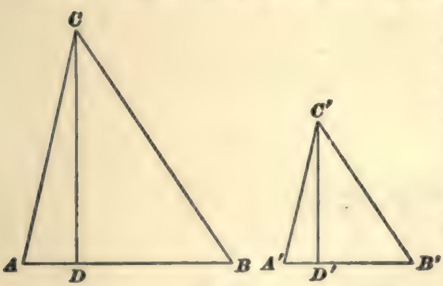

FIG. 10

Therefore,

$$CD : C'D' = AC : A'C' = AB : A'B' = BC : B'C'$$

23. As stated in Art. **14**, two polygons are similar when their corresponding angles are equal and their corresponding sides are proportional. It has now been shown that, in triangles, either of these conditions includes the other. This could have been expected from the fact that either the three angles or the three sides of a triangle fix its shape. This is not true of a polygon of more than three sides, as the angles can be changed without altering the sides, or the proportions of the sides can be changed without altering the angles.

EXAMPLE 1.—In the triangles ABC and $A'B'C'$, Fig. 11, angle A = angle A', angle B = angle B', and angle C = angle C', and the

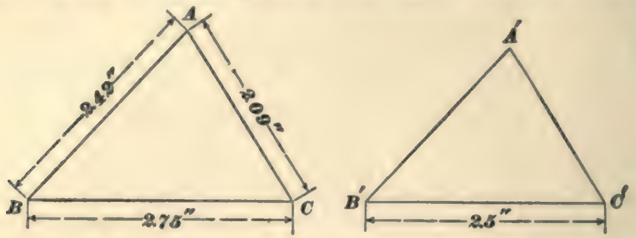

FIG. 11

sides BC, CA, AB, and $B'C'$ have the dimensions that are marked on them; find the lengths of the sides $C'A'$ and $A'B'$.

SOLUTION.—Since the two triangles are equiangular, they are similar, and hence the value of $A'B'$ and that of $A'C'$ are conveniently found as follows:

$$2.75 : 2.42 = 2.5 : A'B'$$

whence $$A'B' = \frac{2.42 \times 2.5}{2.75} = 2.2 \text{ in.} \quad \text{Ans.}$$

$$2.75 : 2.09 = 2.5 : A'C'$$

whence $$A'C' = \frac{2.09 \times 2.5}{2.75} = 1.9 \text{ in.} \quad \text{Ans.}$$

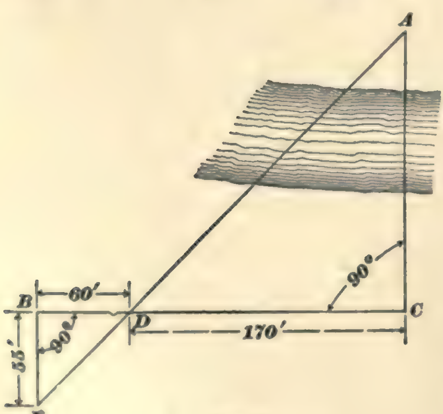

FIG. 12

EXAMPLE 2.—In Fig. 12, CD is perpendicular to AC and is 170 feet long; DB is 60 feet; and BE, perpendicular to BD, is 55 feet long; find the distance AC.

SOLUTION.—The right triangles ACD and DBE have the angles ADC and BDE equal; hence, they are similar, and

$$AC : CD = BE : BD,$$
or $$AC : 170 = 55 : 60$$

whence,

$$A\,C = \frac{170 \times 55}{60} = 155.83 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 3.—It is required to cut from a triangular plate $A\,B\,C$, Fig. 13, having the dimensions shown, a trapezoidal plate $B\,D\,E\,C$ whose upper base $D\,E$ shall be 6 inches; find the distances $A\,D$ and $A\,E$ that must be cut off.

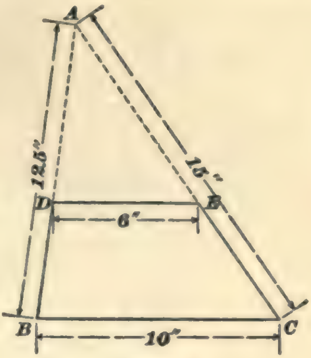

FIG. 13

SOLUTION.—The similar triangles $A\,D\,E$ and $A\,B\,C$ give,

$$\frac{A\,D}{A\,B} = \frac{D\,E}{B\,C};\ A\,D = \frac{A\,B \times D\,E}{B\,C} = \frac{12.5 \times 6}{10} = 7.5 \text{ in.} \quad \text{Ans.}$$

$$\frac{A\,E}{A\,C} = \frac{D\,E}{B\,C};\ A\,E = \frac{A\,C \times D\,E}{B\,C} = \frac{15 \times 6}{10} = 9 \text{ in.} \quad \text{Ans.}$$

EXAMPLE 4.—In order to measure the height $R\,H$ of a pier $A_1\,P\,Q\,R$, Fig. 14, whose base and top are, respectively, 22 feet and 12 feet square and whose sides all have the same inclination, a transit was set at a point O distant 250 feet from the side A_1 of the pier; that is, so that $O\,M_1 = 250$ feet. $A_1\,B_1$ was a rod on which the horizontal line of sight $O\,M_1$ intercepted a distance $A_1\,M_1 = 4.5$ feet. The same rod was

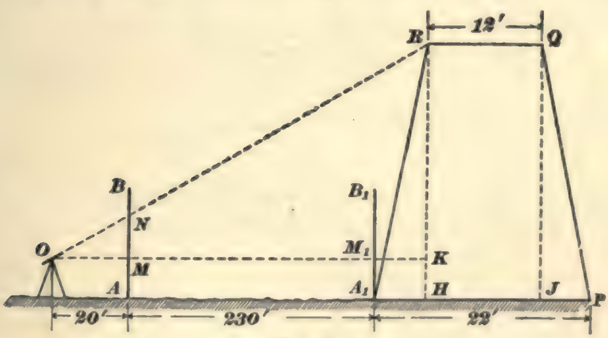

FIG. 14

held at a distance $O\,M = 20$ feet from the instrument, and the height $A\,M$ above the ground noted. Then the telescope of the transit was directed to the top R of the pier, and, with the rod still held at A, the height $A\,N$ was read on the rod. By subtracting $A\,M$ from $A\,N$, the distance $M\,N$ intercepted between the lines $O\,M_1$ and $O\,R$ was found to be 6 feet. What was the height $R\,H$?

SOLUTION.—The inclination of $A_1 R$ and that of PQ being equal we have, $A_1 H = JP$, and $HJ = RQ = 12$ ft. Now,

$$A_1 P = A_1 H + HJ + JP = 2 A_1 H + 12$$

whence, $$A_1 H = \frac{A_1 P - 12}{2} = \frac{22 - 12}{2} = 5 \text{ ft.}$$

The similar triangles OMN and OKR give

$$\frac{OM}{MN} = \frac{OK}{KR};\ KR = \frac{OK \times MN}{OM}$$
$$= \frac{(OM_1 + M_1 K) \times MN}{OM} = \frac{(OM_1 + A_1 H) \times MN}{OM}$$
$$= \frac{(250 + 5) \times 6}{20} = \frac{255 \times 6}{20} = 76.5 \text{ ft.}$$

Finally,

$RH = RK + KH = RK + M_1 A_1 = 76.5 + 4.5 = 81$ ft. Ans.

EXAMPLES FOR PRACTICE

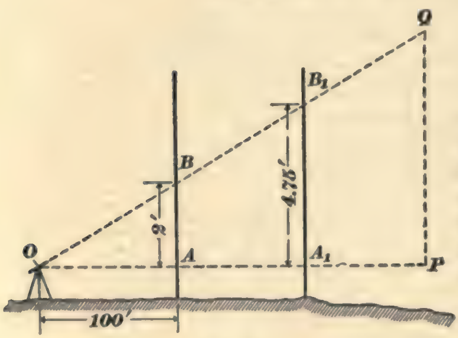

FIG. 15

1. In Fig. 15, the lines of sight OP and OQ of a transit intercept on a rod distances, $AB = 2$ feet and $A_1 B_1 = 4.75$ feet; if the distance OA is 100 feet, what is the distance OA_1? Ans. 237.5 ft.

2. In order to find the stress in the member BD, Fig. 16, by the method of moments, it is necessary to find the distance DO from D to

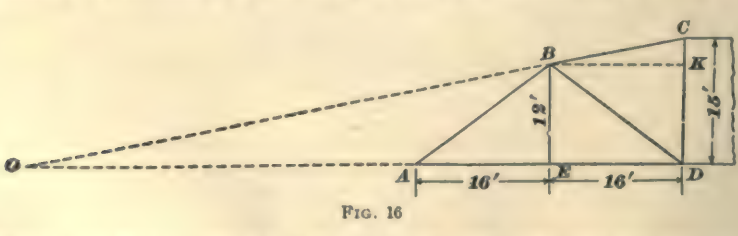

FIG. 16

the point of intersection O of DA and CB, both produced; the dimensions being as shown, what is that distance? Ans. $DO = 80$ ft.

3. $ABCD$, Fig. 17, is a trapezoid whose non-parallel sides produced meet at O; the line MN is parallel to the bases AD and BC; the dimensions of AD, BC, BM, and MA being as shown, find OB and MN.

Ans. $\begin{cases} OB = 315 \text{ ft.} \\ MN = 85.714 \text{ ft.} \end{cases}$

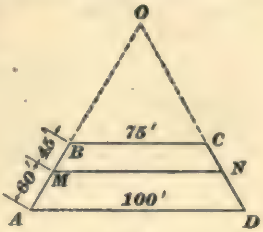

FIG. 17

4. In a triangle ABC, side $AB = 32$ feet, $BC = 34$ feet, and $AC = 48$ feet; if side $A'B'$ of a similar triangle $A'B'C'$ is 72 feet long, what are the lengths of the other two sides?

Ans. $\begin{cases} A'C' = 108 \text{ ft.} \\ B'C' = 76.5 \text{ ft.} \end{cases}$

5. The base of a right triangle is 24 inches, and its altitude 72 inches; at what distance from the top is the triangle 16 inches wide?

Ans. 48 in.

IMPORTANT CONSEQUENCES OF THE THEORY OF SIMILAR TRIANGLES

24. When the first of three quantities is to the second as the second is to the third, the three quantities are in **continued proportion**; the second is a **mean proportional** between the first and third; and the third is a **third proportional** to the first and second. Thus, if $a : b = b : c$, the three quantities a, b, and c are in continued proportion; b is a mean proportional between a and c; and c is a third proportional to a and b.

25. In a right triangle, as ABC, Fig. 18, the perpendicular CD drawn from the vertex of the right angle to the hypotenuse, divides the triangle into two triangles ACD and CDB that are similar to the whole triangle and to each other.

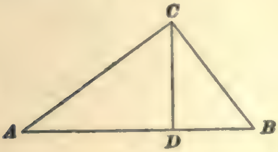

FIG. 18

The right triangles ABC and ACD are similar, by Art. **17**, as the angle A is common. Also, the triangles ABC and CBD, having angle B in common, are similar. Again, the triangles ACD and CBD, being each similar to ABC are similar to each other.

26. In a right triangle, the perpendicular to the hypotenuse from the vertex of the right angle is a mean proportional between the two parts or segments into which it divides the hypotenuse; that is, Fig. 18, $AD:CD = CD:DB$.

As the triangles BCD and ABC are similar, and the angle B is common, the angle BCD must equal the angle A, and similarly the angle ACD must equal the angle B. The triangles ACD and BCD are similar, hence the sides opposite equal angles are in proportion; that is,

$$\frac{AD \text{ (side opposite } ACD)}{CD \text{ (side opposite } B)} = \frac{CD \text{ (side opposite } A)}{DB \text{ (side opposite } BCD)}$$

Or, $$AD : CD = CD : DB$$

27. The side AC, Fig. 18, is a mean proportional between the whole hypotenuse and the segment AD on the same side of CD as the side AC; that is, $AB : AC = AC : AD$. Similarly, $AB : BC = BC : BD$.

The triangles ABC and ACD are similar, hence the sides opposite equal angles are proportional; that is,

$$\frac{AB \text{ (opposite right angle)}}{AC \text{ (opposite right angle)}} = \frac{AC \text{ (opposite } B)}{AD \text{ (opposite } ACD)}$$

Or, $$AB : AC = AC : AD$$

EXAMPLE 1.—In the right triangle ABC, Fig. 19, find the length of the perpendicular CD.

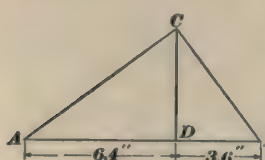

FIG. 19

SOLUTION.—The perpendicular is a mean proportional between the parts AD and DB into which it divides the hypotenuse; therefore,

$$6.4 : CD = CD : 3.6$$

whence, $$\overline{CD}^2 = 6.4 \times 3.6$$

and $CD = \sqrt{6.4 \times 3.6} = 4.8$ in. Ans.

EXAMPLE 2.—Find the length of the sides of the right triangle ABC, Fig. 20, in which CD is the perpendicular from the vertex of the right angle to the hypotenuse.

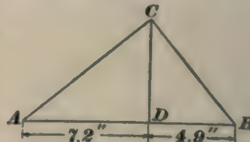

FIG. 20

SOLUTION.—The hypotenuse is 7.2 in. + 4.9 in. = 12.1 in. The side CB is a mean proportional between the hypotenuse AB and the part DB; therefore,

$$12.1 : CB = CB : 4.9$$

$$\overline{CB}^2 = 12.1 \times 4.9$$

$$CB = \sqrt{12.1 \times 4.9} = 7.7 \text{ in. Ans.}$$

The leg AC is a mean proportional between AB and AD; that is,

$$AB : AC = AC : AD$$
$$AC = \sqrt{AB \times AD}$$
$$= \sqrt{12.1 \times 7.2} = 9.34 \text{ in.} \quad \text{Ans.}$$

28. Since an angle inscribed in a semicircle is a right angle, it follows from Arts. **26** and **27**, that:

(*a*) A perpendicular CD, Fig. 21, drawn from any point on the circumference of a circle to a diameter AB, is a mean proportional between the segments into which it divides the diameter; that is,

$$AD : CD = CD : DB$$

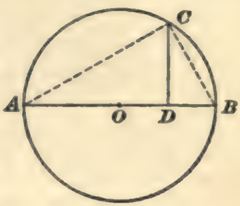

FIG. 21

(*b*) A chord CA drawn from a point in a circumference to the end of a diameter is a mean proportional between the whole diameter and the adjacent segment AD; that is,

$$AB : AC = AC : AD$$

29. If from a point without a circle, a tangent and a secant are drawn, the tangent is a mean proportional between the whole secant and the exterior segment; that is, in Fig. 22, $PB : PT = PT : PA$.

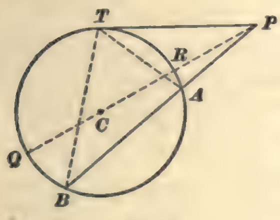

FIG. 22

In the triangles BPT and APT, the angle P is common. The angle B, an inscribed angle, and the angle PTA, an angle formed by a tangent and a chord, are equal, since each is measured by one-half the same arc AT. Hence, the triangles are similar by Art. **16**, and

$$\frac{PB \text{ (opposite angle } PTB)}{PT \text{ (opposite angle } PAT)} = \frac{PT \text{ (opposite angle } B)}{PA \text{ (opposite angle } PTA)}$$

$$\overline{PT}^2 = PB \times PA$$
$$PT = \sqrt{PB \times PA}$$

30. If from a point without a circle any two secants are drawn, the product of one secant and its external segment is equal to the product of the other secant and its external segment.

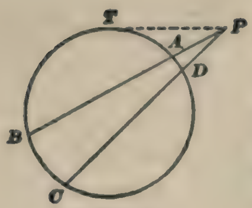

FIG. 23

In Fig. 23, PB and PC are secants. Draw the tangent PT. Then, from Art. **29**,

$$\overline{PT}^2 = PA \times PB$$

and $$\overline{PT}^2 = PC \times PD$$

hence, $$PA \times PB = PC \times PD$$

31. If any two chords be drawn through a point within a circle, the product of the segments of one is equal to the product of the segments of the other.

In Fig. 24, the angles D and B, being measured by one-half the arc AC, are equal. The angles BPC and DPA, being vertical angles, are equal. Hence, by Art. **16**, the triangles CBP and ADP are similar. Therefore,

$$\frac{AP}{CP} = \frac{PD}{PB}$$

and $$AP \times PB = CP \times PD$$

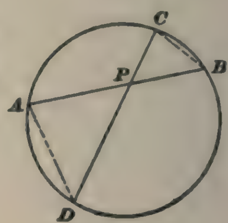

FIG. 24

EXAMPLES FOR PRACTICE

1. The perpendicular from the vertex of the right angle of a right triangle divides the hypotenuse into parts of 23.04 inches and 1.96 inches. Find: (*a*) the length of the perpendicular; (*b*) the length of the two sides of the triangle.

Ans. { (*a*) 6.72 in. (*b*) 24 in. and 7 in.

2. If, in Fig. 22, the distance CP of the point P from the center of the circle is 65 feet, and the radius CR is 25 feet, what is the length of the tangent PT? Ans. 60 ft.

3. The chord of the arc of a segment is 14 inches long and the height of the segment is 2 inches; what is the radius? Ans. $13\frac{1}{4}$ in.

OTHER SIMILAR POLYGONS

32. Two polygons are similar when they are composed of the same number of triangles similar each to each and similarly placed.

Thus, in Fig. 25, the polygons $ABCDE$ and $A'B'C'D'E'$ are composed of the same number of similar triangles similarly placed.

Since the triangle $A E D$ is similar to the triangle $A' E' D'$, angle E = angle E' and angle $A D E$ = angle $A' D' E'$. Also, in the similar triangles $A D C$ and $A' D' C'$, angle $A D C$ = angle $A' D' C'$. Hence, the sum of the angles $A D E$ and $A D C$, or the angle $E D C$, is equal to the sum of the angles $A' D' E'$ and $A' D' C'$, or the angle $E' D' C'$. In like manner, angle DCB = angle $D' C' B'$,

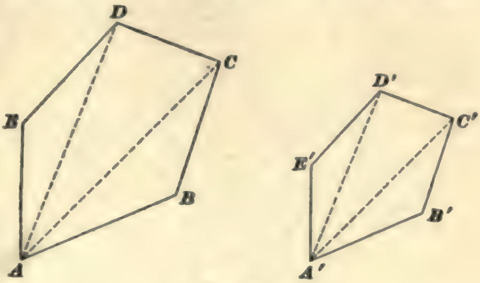

FIG. 25

angle B = angle B', and angle $B A E$ = angle $B' A' E'$. Since the triangles are similar,

$$E D : E' D' = A D : A' D' \text{ and } A D : A' D' = D C : D' C'$$

hence,

$$E D : E' D' = D C : D' C'$$

In like manner,

$$D C : D' C' = C B : C' B' = B A : B' A' = A E : A' E'$$

Therefore, as the angles of the one polygon are equal to the corresponding angles of the other and the sides of the one polygon are proportional to the sides of the other, the polygons are similar.

33. Two similar polygons can be divided into the same number of similar triangles similarly placed.

34. The perimeters of two similar polygons are in the same ratio as any two homologous sides.

In Fig. 25, let P be the perimeter of the polygon $A B C D E$, and P' the perimeter of the polygon $A' B' C' D' E'$. Since the polygons are similar

$$\frac{A E}{A' E'} = \frac{E D}{E' D'} = \frac{D C}{D' C'} = \frac{C B}{C' B'} = \frac{B A}{B' A'} \qquad (1)$$

Let each of these equal ratios be denoted by R; that is, let

$$\frac{A E}{A' E'} = R, \frac{E D}{E' D'} = R, \frac{D C}{D' C'} = R, \frac{C B}{C' B'} = R, \frac{B A}{B' A'} = R.$$

From these equations we obtain,

$$A E = R \times A' E', E D = R \times E' D', D C = R \times D' C',$$
$$C B = R \times C' B', B A = R \times B' A'$$

Adding the sides of these equalities,

$$A E + E D + D C + C B + B A$$
$$= R \times A' E' + R \times E' D' + R \times D' C' + R \times C' B' + R \times B' A'$$
$$= R (A' E' + E' D' + D' C' + C' B' + B' A')$$

whence $$\frac{AE+ED+DC+CB+BA}{A'E'+E'D'+D'C'+C'B'+B'A'}=R$$

But $$R=\frac{AE}{A'E'}=\frac{ED}{E'D'}=\frac{DC}{D'C'}, \text{ etc.};$$

therefore, $$\frac{P}{P'}=\frac{AE}{A'E'}=\frac{ED}{E'D'}=\frac{DC}{D'C'}, \text{ etc.}$$

35. Equation (1) of the preceding article is a series of equal ratios, of which the numerators are the antecedents and the denominators the consequents. The general truth was shown in that article, that in a series of equal ratios the sum of the antecedents is to the sum of the consequents as any antecedent is to its consequent.

AREAS OF POLYGONS

36. Definitions.—The **area** of a surface is the superficial space included within its boundary lines. Area is expressed by the ratio of the surface to a surface of fixed value chosen as a unit and called the **unit of area.**

37. A square whose side is equal in length to the unit of length is usually taken as the unit of area, and its area is called the **square unit.** For example, if the unit of length is 1 inch, the unit of area, or square inch, is the square whose sides measure 1 inch, and the area of any surface is expressed by the number of square inches that the surface contains. If the unit of length were 1 foot, the unit of area would measure 1 foot on each side, and the area of the surface would be expressed in square feet. Square inch and square foot are abbreviated to sq. in. and sq. ft., respectively, and are often indicated by the symbols □″ and □′.

38. Two surfaces are **equivalent** when their areas are equal.

39. Comparison of the Areas of Two Rectangles. The areas of two rectangles $ABCD$ and $A'B'C'D'$, Fig. 26, having equal altitudes are to each other as their bases; that is, area $ABCD$: area $A'B'C'D' = AB : A'B'$.

Suppose that $A'B'$ is four-fifths of AB, or that $AB : A'B' = 5 : 4$. Divide AB into five equal parts AE, EF, etc., and $A'B'$ into four equal parts $A'E'$, $E'F'$, etc. It is evident that $A'E' = AE$, for, since AB is to $A'B'$ in the ratio of 5 to 4, any quantity, as AE, that is contained five times in AB must be contained four times in $A'B'$. Through the points of division E, F, E', F', etc., draw perpendiculars

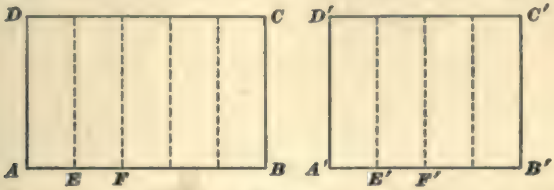

FIG. 26

to AB and $A'B'$. Each large rectangle is thus divided into small rectangles, all of which are equal. As $ABCD$ contains five, and $A'B'C'D'$ contains four, of the small rectangles, the ratio of the two large rectangles is that of 5 to 4, which is also the ratio of their bases.

40. Since any of the sides of a rectangle can be considered as the base, it follows that the area of two rectangles having equal bases are to each other as their altitudes.

41. The areas of any two rectangles are to each other as the products of their bases by their altitudes.

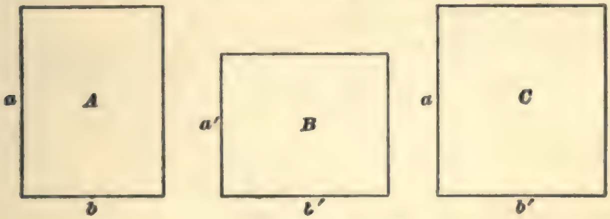

FIG. 27

Let A and B, Fig. 27, be two rectangles whose altitudes are a and a' and whose bases are b and b', respectively. Construct a rectangle C with an altitude a and a base b'. Then, by Arts. **39** and **40**,

$$A : C = b : b' \qquad (1)$$

and

$$C : B = a : a' \qquad (2)$$

Multiplying equation (1) by equation (2),

$$AC : BC = ab : a'b' \qquad (3)$$

Dividing the terms of the first member of equation (3) by C,

$$A : B = ab : a'b'$$

42. Area of a Rectangle.—The area of a rectangle is equal to its base multiplied by its altitude; that is, in Fig. 28,

$$A = b\,h.$$

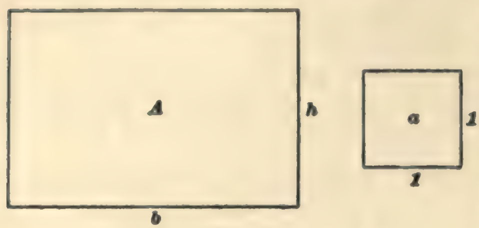

FIG. 28

Construct a unit square a. Then (Art. 41),

$$A : a = h \times b : 1 \times 1;$$

or, $$\frac{A}{a} = \frac{h \times b}{1 \times 1}$$

But a is a unit square, and its area is therefore equal to 1; hence,

$$A = b\,h$$

43. Area of a Triangle.—The area of a right triangle is equal to one-half the product of the two legs of the triangle; that is, in Fig. 29, area $A\,B\,C = \frac{1}{2}\,a\,b$.

For the triangle $A\,B\,C$ is one-half the rectangle $A\,B\,C\,D$ and the area of the latter is $a\,b$.

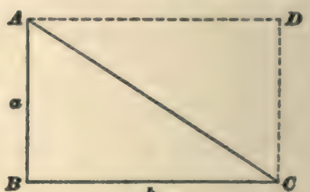

FIG. 29

44. The area of any triangle is equal to one-half the product of its base and altitude.

In Fig. 30 (a), let $A\,C$ be the base and $B\,H$ the altitude of the triangle $A\,B\,C$. The area $A\,B\,C$ is equal to the sum of the right triangles $A\,H\,B$ and $C\,H\,B$, which, by the last article, is

$$\tfrac{1}{2}\,B\,H \times A\,H + \tfrac{1}{2}\,B\,H \times H\,C = \tfrac{1}{2}\,B\,H \times (A\,H + H\,C) = \tfrac{1}{2}\,B\,H \times A\,C$$

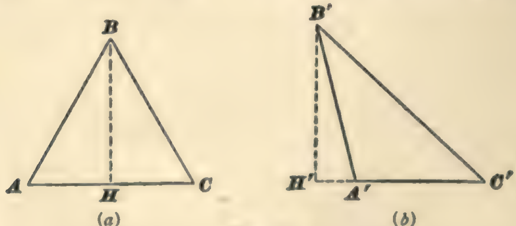

FIG. 30

In Fig. 30 (b), the area $A'\,B'\,C'$ is the difference between the areas of the right triangles $B'\,H'\,C'$ and $B'\,H'\,A'$; that is,

$$\tfrac{1}{2}\,B'\,H' \times H'\,C' - \tfrac{1}{2}\,B'\,H' \times H'\,A' = \tfrac{1}{2}\,B'\,H' \times (H'\,C' - H'\,A')$$
$$= \tfrac{1}{2}\,B'\,H' \times A'\,C'$$

Let b be the base, h the altitude, and A the area of any triangle; then,

$$A = \frac{1}{2} b h$$

45. Two triangles having the same base are to each other as their altitudes, and two triangles having the same altitude are to each other as their bases.

46. Two triangles having the same base and the same altitude are equivalent.

It should be borne in mind that any side of a triangle can be taken as the base, the altitude being the perpendicular to that side from the opposite vertex.

47. To find the area of a triangle from the lengths of its three sides, apply the following:

Rule.—*From half the sum of the three sides subtract each side separately; multiply together the half sum and the three remainders and extract the square root of the product.*

Let a, b, and c be the three sides of a triangle, and A the area; let

$$s = \frac{1}{2}(a + b + c)$$

Then $$A = \sqrt{s(s-a)(s-b)(s-c)}$$

The geometrical proof of this rule is very laborious, and will not be given here. A proof will be found in *Trigonometry*.

Example.—What is the area of a triangle having two sides 19.8 feet long, and one side 28 feet long?

Solution.—It is immaterial which side is called a, b, or c. $s = \frac{a+b+c}{2} = \frac{28 + 19.8 + 19.8}{2} = 33.8$; taking b and c as the short sides, $s - a = 33.8 - 28 = 5.8$, and $s - b$ and $s - c$ are each $33.8 - 19.8 = 14$. Then, applying the formula

$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{33.8 \times 5.8 \times 14 \times 14} = 196$ sq. ft., nearly. Ans.

48. A triangle equivalent to any given polygon may be constructed as follows:

Let $ABCDEF$, Fig. 31, be the given polygon. Produce any of the sides, as AF, in both directions, as indicated by XY. This line will be referred to as the base. Starting from one of the ends of AF, as A, draw a diagonal AC forming a triangle with AB and BC. Draw BB_1 parallel to CA, meeting the base at B_1, and join C to B_1.

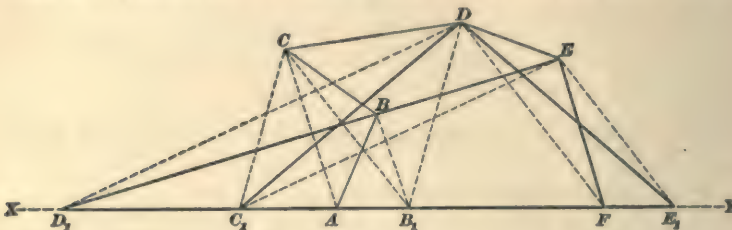

FIG. 31

The polygon B_1CDEF has one side less than the given polygon, and is equivalent to it. For

$$ABCDEF = B_1BCDEF + \text{triangle } B_1BA$$
$$B_1CDEF = B_1BCDEF + \text{triangle } B_1BC$$

The two triangles B_1BA and B_1BC are equivalent, for they have the common base B_1B, and their altitudes, being each equal to the distance between the parallels AC and B_1B, are equal. Proceeding with the polygon B_1CDEF as with the original polygon, draw the diagonal B_1D, forming a triangle with B_1C and CD. Draw CC_1 parallel to DB_1, and join D and C_1. It can be shown as before that the polygon C_1DEF is equivalent to B_1CDEF, and, therefore, to the original polygon. Finally, draw the diagonal C_1E, and DD_1 parallel to it, meeting the base at D_1. Then will the triangle D_1EF be the required triangle equivalent to the given polygon.

In practice, it is more convenient, as well as more accurate, to reduce about one-half of the polygon on one side of A and the rest on the other side of F. Thus, having reduced the polygon to the quadrilateral C_1DEF, the diagonal FD is drawn from F; EE_1 is drawn through E parallel to DF, and E_1 joined to D. This gives C_1DE_1 as the required triangle.

49. Area of a Parallelogram.—The area of a parallelogram is equal to its base multiplied by its altitude; that is, in Fig. 32, area $ABCD = AD \times MN$.

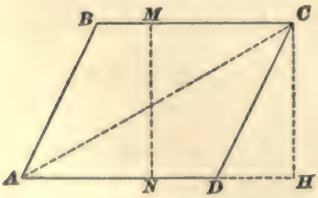

FIG. 32

For $ABCD$ is equal to the sum of the equal triangles ABC and ADC, or to twice either of them, as ADC; that is, $ABCD = 2 \times \frac{1}{2} AD \times CH = AD \times CH = AD \times MN$.

Example 1.—What is the cost of paving a street 1,800 feet long and 36 feet wide with asphalt, the price being $2 per square yard?

Solution.—The surface to be covered is a rectangle whose sides are 36 ft. and 1,800 ft., or 12 yd. and 600 yd., and whose area is, therefore, $12 \times 600 = 7{,}200$ sq. yd. The cost of paving is, then, $2 \times 7{,}200 = \$14{,}400$. Ans.

Example 2.—One side of a triangular plot of land is 125 feet long and the perpendicular distance from the opposite vertex to this side is 174.24 feet; it is desired to find a side of a rectangle that has the same area as the triangle and one side 75 feet long.

Solution.—The area of the triangle is $\frac{1}{2} \times 125 \times 174.24 = 10{,}890$ sq. ft. Then the other side of the rectangle is $10{,}890 \div 75 = 145.2$ ft. Ans.

Example 3.—Divide a triangular plot of land into any number of equal parts by lines from a vertex to the opposite side.

Solution.—Divide the side opposite the vertex through which the lines are to be run into the required number of equal parts and run lines from the vertex of the triangle to the points of division. Then, since the triangles thus formed have equal bases and their vertexes in the same point, they are equivalent. Ans.

Example 4.—Divide a given triangle into parts proportional to any given numbers by lines run through a vertex.

Solution.—Let the given triangle be ABC, Fig. 33, and let it be required to divide it into parts proportional to 3, 4, and 5, by lines drawn from the vertex A.

The base BC is divided into parts proportional to the numbers 3, 4, and 5, by dividing it into $3 + 4 + 5 = 12$ equal parts, and then marking the third and the seventh points of division. From the points thus marked, lines are run to the vertex A. Then, by Art. **45**,

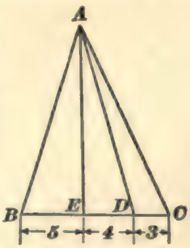

Fig. 33

$$CAD : ADE = 3 : 4$$

and, $$ADE : ABE = 4 : 5 \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the area of a square whose side is 5 feet 9 inches. Ans. 33.062 sq. ft

2. Find the area of a rhombus whose length is 12.5 feet, and whose height is 9.25 feet. Ans. 115.62 sq. ft.

3. One side of a room is 16 feet long; if the floor contains 240 square feet, what is the length of the other side? Ans. 15 ft.

4. In a trapezium two not adjacent sides are 16 and 14 inches, respectively. A diagonal divides the trapezium into two triangles whose altitudes from their vertexes to the given sides as bases are 17 inches and 3 inches, respectively; what is the area of the trapezium? Ans. 157 sq. in.

5. The base BC of a triangle is 150 chains and the perpendicular from the opposite vertex A to BC is 45 chains; it is desired to divide the triangle into two parts equal in area by a line from A to BC; how far from B is D, the intersection of this line with BC? Ans. 75 ch.

6. From the mid-point E of the side AB of a parallelogram $ABCD$, lines are drawn to the vertexes D and C and to the mid-point of the side CD; show that these lines divide the parallelogram into four triangles that are equal in area.

7. Find the area of a triangle whose three sides are 13, 14, and 15 feet. Ans. 84 sq. ft.

8. Find the area of a right triangle whose hypotenuse is 50 feet and one of whose legs is 40 feet. Ans. 600 sq. ft.

---

50. Area of a Trapezoid.—The area of a trapezoid is equal to one-half the sum of the parallel sides multiplied by the altitude; that is, in Fig. 34, area of trapezoid $ABCD = \frac{1}{2}(AB + DC) \times MN$.

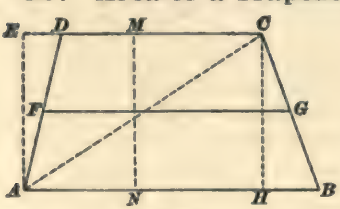

FIG. 34

The area of the trapezoid is equal to the sum of the areas of the two triangles ABC and ADC; hence,

$$\begin{aligned} ABCD &= \tfrac{1}{2} AB \times CH + \tfrac{1}{2} DC \times AE \\ &= \tfrac{1}{2} AB \times MN + \tfrac{1}{2} DC \times MN \\ &= \tfrac{1}{2}(AB + DC) \times MN \end{aligned}$$

Let b_1 = length of lower base;

b_2 = length of upper base;

h = altitude.

Then, the area A of the trapezoid $ABCD$ is

$$A = \frac{1}{2}(b_1 + b_2)h$$

51. Since the median line FG, Fig. 34, joining the mid-points of the non-parallel sides is equal to $\frac{1}{2}(AB + DC)$, the area of a trapezoid is equal to the product of the median line by the altitude.

EXAMPLE.—Divide a plot of ground in the form of a trapezoid into any number of equal parts by lines intersecting the two bases.

SOLUTION.—Divide each of the bases into the same number of equal parts into which the trapezoid is to be divided and run lines through the corresponding points of division. The trapezoids thus formed have equal bases and the same altitude and are, therefore, equal in area. Ans.

EXAMPLES FOR PRACTICE

1. The parallel sides of a trapezoid are 321.51 and 214.24 feet, and the perpendicular distance between them is 171.16 feet; what is the area of the trapezoid? Ans. 45,849 sq. ft.

2. Find the area of a trapezoid whose parallel sides are 20.5 and 12.25 chains, the perpendicular distance between them being 10.75 chains. Ans. 17.603 A.

3. The parallel sides of a trapezoidal plot of ground are 400 feet and 360 feet long; the distance between the parallel sides is 100 feet. It is desired to divide this plot into five lots by lines intersecting the parallel sides; what will be the length of the front and the rear of one of the lots? Ans. 80 ft. and 72 ft.

4. How many square feet are there in a board 12 feet long, 18 inches wide at one end, and 12 inches wide at the other end? Ans. 15 sq. ft.

---

52. Area of Any Polygon.—The area of any polygon can be found by dividing the polygon into triangles, determining the area of each triangle, and adding the results.

53. Comparison of the Areas of Similar Polygons. The areas of two similar triangles are to each other as the squares of their homologous sides.

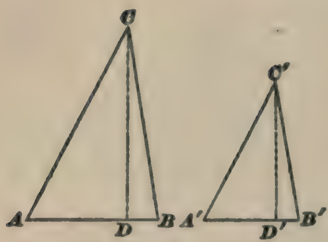

FIG. 35

In Fig. 35,

Area $ABC = \frac{1}{2} AB \times CD$ (1)

Area $A'B'C' = \frac{1}{2} A'B' \times C'D'$ (2)

Dividing equation (1) by equation (2), $\frac{ABC}{A'B'C'} = \frac{AB}{A'B'} \times \frac{CD}{C'D'}$ (3)

but, by Art. **22**, $\frac{CD}{C'D'} = \frac{AB}{A'B'}$;

hence, substituting in (3)

$$\frac{AB}{A'B'} \text{ for } \frac{CD}{C'D'},\ \frac{ABC}{A'B'C'} = \frac{AB}{A'B'} \times \frac{AB}{A'B'} = \frac{\overline{AB}^2}{\overline{A'B'}^2}$$

that is, $ABC : A'B'C' = \overline{AB}^2 : \overline{A'B'}^2$

54. The areas of two similar triangles are to each other as the squares of any two homologous lines.

55. The areas of two similar polygons are to each other as the squares of their homologous lines.

By Art. **33**, two similar polygons can be divided into the same number of similar triangles. The sums of these triangles will, by Art. **35**, be to each other as any triangle of one polygon is to the corresponding triangle of the other. But these triangles are to each other as the squares of any two homologous lines. Hence, the sum of the triangles, or the polygons, are to each other as the squares of any two homologous lines.

EXAMPLE 1.—Divide a given triangle by a line parallel to the base into parts such that the given triangle shall be to the triangle cut off as $m : n$.

SOLUTION.—Let ABC, Fig. 36, be the given triangle, and ADE be the triangle cut off so that $ABC : ADE = m : n$. By Art. **18**, ADE and ABC are similar; hence, by Art. **53**,

$$ABC : ADE = \overline{AB}^2 : \overline{AD}^2$$

But by the conditions of the problem,

$$ABC : ADE = m : n$$

Therefore, $\overline{AB}^2 : \overline{AD}^2 = m : n$;

whence, $AD = AB\sqrt{\frac{n}{m}}$. Ans.

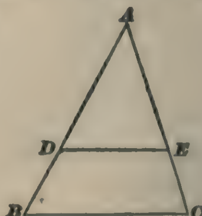

FIG. 36

When the triangle ABC is to be divided into two equal parts,

$$AD = AB\sqrt{\frac{1}{2}} = .70711\,AB$$

Example 2.—Let the length AB of example 1 be 32 chains and the area of ABC be 25.6 acres; what is the length of AD, if it is desired to make the triangle ADE contain 15 acres?

Solution.—The area ABC is to be to the area of ADE as 25.6 : 15; hence, $m : n = 25.6 : 15$.

Then, $$AD = 32\sqrt{\frac{15}{25.6}} = 24.495 \text{ ch.} \quad \text{Ans.}$$

Example 3.—Divide a given triangle ABC, by lines parallel to the base, into n equal parts.

Solution.—Let ABC, Fig. 37, be the triangle, and DE, FG, HI, etc., divide it into n equal parts. Then ADE is one part, AFG is two parts, and so on. Hence,

$$ABC : ADE = n : 1$$
$$ABC : AFG = n : 2; \text{ etc.}$$

Then, by example 1,

$$AD = AB\sqrt{\frac{1}{n}};\ AF = AB\sqrt{\frac{2}{n}};$$

$$AH = AB\sqrt{\frac{3}{n}};\ \text{etc.} \quad \text{Ans.}$$

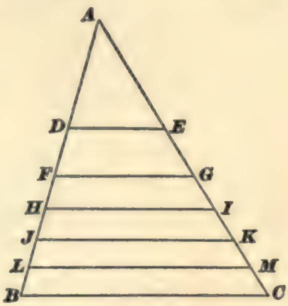

Fig. 37

Example 4.—Two triangles ABC and $A'B'C'$ are similar. The sides of the triangle ABC are: $AB = 10$ inches, $BC = 21$ inches, $AC = 17$ inches, and in the triangle $A'B'C'$ the side $B'C' = 42$ inches; what is the area of the triangle $A'B'C'$?

Solution.—In the triangle ABC, $s = \frac{10 + 17 + 21}{2} = 24$. Then $s - a = 3$, $s - b = 7$, $s - c = 14$, and the area is $\sqrt{24 \times 3 \times 7 \times 14}$ $= 84$ sq. in. By the principle of Art. **53**,

$$\text{area of } A'B'C' : \text{area of } ABC = \overline{B'C'}^2 : \overline{BC}^2,$$

that is, $$\text{area of } A'B'C' : 84 = 42^2 : 21^2$$

But $$42^2 : 21^2 = 4 : 1$$

hence, $$\text{area of } A'B'C' : 84 = 4 : 1$$

whence, $$\text{area of } A'B'C' = 4 \times 84 = 336 \text{ sq. in.} \quad \textbf{Ans.}$$

EXAMPLES FOR PRACTICE

1. Suppose that the sides of the triangle $A'B'C'$ in example 4 of Art. **55** are $A'B' = 20$ inches, $B'C' = 42$ inches, and $C'A' = 34$ inches; show that the answer that is given to the example is correct.

2. The triangles ABC and $A'B'C'$ are similar; being given $BC = 13$ inches, $CA = 14$ inches, $AB = 15$ inches, and $B'C' = 19.5$ inches; find the area of the triangle $A'B'C'$. Ans. 189 sq. in.

3. Let $A B$, one side of a triangle $A B C$, be 60 chains long, and let it be required to divide, by lines parallel to $B C$, the triangle $A B C$ into five equal parts. (*a*) What are the lengths of the lines $A D$, $A F$, $A H$, and $A T$? (*b*) Let the area of $A B C$ be 120 acres; by means of Art. **53**, prove your results.

Ans. $\begin{cases} A D = 26 \text{ ch. } 83.3 \text{ l.} \\ A F = 37 \text{ ch. } 94.7 \text{ l.} \\ A H = 46 \text{ ch. } 47.6 \text{ l.} \\ A T = 53 \text{ ch. } 66.6 \text{ l.} \end{cases}$

4. Find the lengths of $A D$ and $A F$ when the triangle of example 3 is divided into three parts, whose areas shall be proportional to the numbers 3, 4, and 5.

Ans. $\begin{cases} A D = 30 \text{ ch.} \\ A F = 45 \text{ ch. } 82.6 \text{ l.} \end{cases}$

Hint.—This is the same as if the triangle were divided into 3 + 4 + 5 equal parts and $A D E$ contained three, and $A F G$, seven of these equal parts.

---

56. The Theorem of Pythagoras.—In any right triangle, the square described on the hypotenuse is equivalent to the sum of the squares described on the other two sides.

Let $A B C$, Fig. 38, be a right triangle. Draw an equal triangle in the position $C B' C'$, so that $C B'$ will be in the prolongation of $B C$. Construct the squares $A B D E$ and $B' C' F D$ on $A B$ and $B' C'$, respectively. Since $M + N_1$ $(= M + N)$ is a right angle, $A C C'$ is also a right angle. Produce $E F$ to A', making $F A' = B A = D E$. Then, since $E F$ is the difference between $D F$ and $D E$, or $B C$ and $A B$, $E A' = B C$. Draw $A A'$ and $C' A'$. Each of the right triangles T_2 and T_3 is equal to T, since their legs are respectively equal. The quadrilateral $A C C' A'$, having all its sides equal and a right angle C, is a square—the square on the hypotenuse $A C$. This square is equal to the shaded figure plus the sum of the triangles T_2 and T_3; or to the shaded figure plus twice the triangle T. The sum of the squares $A B D E$ and $B' C' F D$ is equal to the shaded figure plus the sum of the triangles T and T_1, or to the shaded figure plus twice the triangle T. Therefore, square $A C C' A'$ = square $A B D E$ + square $B' C' F D$.

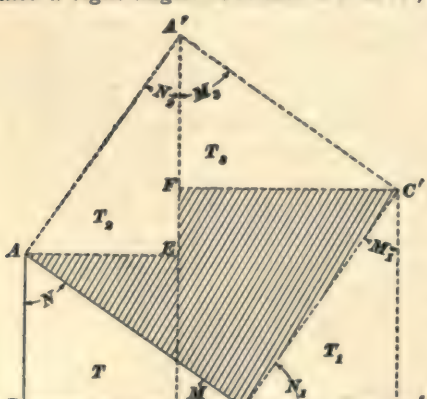

Fig. 38

A particular case of the proposition just proved is shown in Fig. 39.

Let c be the hypotenuse, and a and b the other two sides of any right triangle.

Then,

$$c^2 = a^2 + b^2 \qquad (1)$$

$$c = \sqrt{a^2 + b^2} \qquad (2)$$

$$a = \sqrt{c^2 - b^2} \qquad (3)$$

Formula **3** may be written

$$a = \sqrt{(c - b)(c + b)} \qquad (4)$$

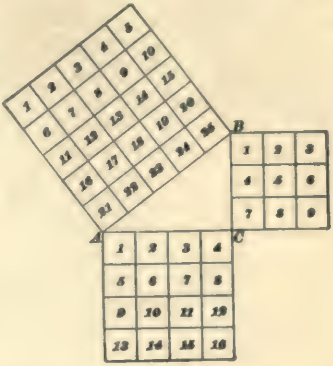

FIG. 39

EXAMPLE 1.—If $AB = 3$ inches and $BC = 4$ inches, what is the length of the hypotenuse AC, Fig. 38?

SOLUTION.—

$$AC = \sqrt{\overline{AB}^2 + \overline{BC}^2}$$
$$= \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ in. Ans.}$$

EXAMPLE 2.—The side given is 3 inches ($= b$, say), the hypotenuse is 5 inches ($= c$); what is the length of the other side?

SOLUTION.—Applying formula **4**, Art. **56**,

$$a = \sqrt{(5 - 3)(5 + 3)} = \sqrt{16} = 4 \text{ in. Ans.}$$

Also, $a = \sqrt{c^2 - b^2} = \sqrt{5^2 - 3^2} = 4$ in. Ans.

EXAMPLE 3.—If, from a church steeple that is 150 feet high, a rope is to be attached at the top and to a stake in the ground 85 feet from its foot (the ground being supposed to be level), what must be the length of the rope?

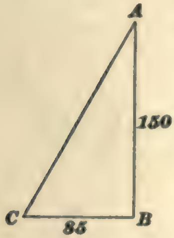

FIG. 40

SOLUTION.—In Fig. 40, AB represents the steeple 150 ft. high; C, a stake 85 ft. from the foot of the steeple; and AC, the rope. Here we have a triangle right-angled at B, of which AC is the hypotenuse. The square of $AC = 85^2 + 150^2 = 7{,}225 + 22{,}500 = 29{,}725$. Therefore,

$$AC = \sqrt{29{,}725} = 172.4 \text{ ft., nearly. Ans.}$$

EXAMPLE 4.—Referring to Fig. 16, it is required to find the length of the post AB and that of the member BC.

SOLUTION.—Draw BK parallel to ED. Then, $BK = ED = 16$ ft. and $CK = CD - DK = CD - EB = 15 - 12 = 3$ ft. The right triangles AEB and BCK give

$$AB = \sqrt{\overline{AE}^2 + \overline{EB}^2} = \sqrt{16^2 + 12^2} = \sqrt{400} = 20 \text{ ft. Ans.}$$

$$BC = \sqrt{\overline{BK}^2 + \overline{CK}^2} = \sqrt{16^2 + 3^2} = \sqrt{265} = 16.279 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1. If the two sides about the right angle in a right triangle are 52 and 39 feet long, how long is the hypotenuse? Ans. 65 ft.

2. A ladder 65 feet long reaches to the top of a house when its foot is 25 feet from the house; how high is the house, supposing the ground to be level? Ans. 60 ft.

3. The shortest distance from a point to a line is 25 inches; the distances from this point to the extremities of the line are 54 inches and 40 inches, respectively; what is the length of the line? Ans. 79.08 in.

4. Show that the diagonal of a square is equal to the side multiplied by $\sqrt{2}$.

REGULAR POLYGONS

57. A **regular** polygon is a polygon that has equal sides and equal angles, that is, it is equilateral and equiangular.

58. A circle can be circumscribed about any regular polygon.

Take any three vertexes of the regular polygon $ABCDE$, Fig. 41, as the vertexes A, B, C, and pass a circle through them. Let O be the center of this circle. Join O to A, B, C, D, and E. The polygon being equiangular, the angle ABC = angle BCD. The angles OCB and OBC, being opposite equal sides OC and OB of the triangle OBC, are equal. Hence,

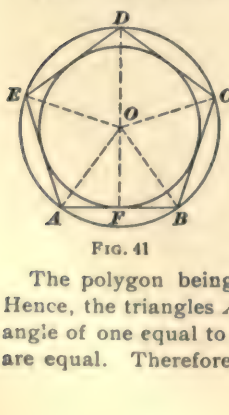

FIG. 41

$$ABC - OBC = BCD - OCB$$

or, $$ABO = OCD$$

The polygon being equilateral, the sides AB and CD are equal. Hence, the triangles AOB and OCD, having two sides and included angle of one equal to two sides and included angle of the other equal, are equal. Therefore, $OD = OA$, and a circle passing through A, *B*, and *C* must pass through *D*. In like manner, it can be shown that the circle passes through *E*.

59. A circle can be inscribed in any regular polygon.

In Fig. 41, *O A*, *O B*, *O C*, *O D*, and *O E*, being radii of the circumscribed circle, are equal and divide the polygon into equal isosceles triangles that have a common vertex *O*. The altitudes of these equal triangles are equal, hence the perpendicular distances, as *O F*, from *O* to each of the sides are the same. Therefore, a circle drawn with *O* as center and a radius equal to *O F* will be inscribed in the regular polygon.

60. The **center** of a regular polygon is the common center of the circumscribed and the inscribed circle.

61. The **radius** of a regular polygon is the radius of the circumscribed circle, as *O A*, Fig. 41.

62. The **apothem** of a regular polygon is the radius of the inscribed circle, as *O F*, Fig. 41.

63. The **angle at the center** of a regular polygon is the angle included by the radii drawn to the extremities of any side.

64. The angle at the center of any regular polygon is equal to four right angles, or 360°, divided by the number of the sides.

65. If n is the number of sides of a regular polygon, the sum of its interior angles is $2(n-2)$ right angles (see *Geometry*, Part 1), or, $90° \times 2(n-2) = 180° \times (n-2)$, and, since all the angles are equal, each angle is equal to $\frac{180° \times (n-2)}{n} = 180° - \frac{360°}{n}$. Since this value depends only on the number of sides, all regular polygons of the same number of sides have the same angles.

66. Regular polygons of the same number of sides are similar; their perimeters are to each other as any two homologous lines, and their areas are to each other as the squares of any two homologous lines.

67. The area of a regular polygon is equal to one-half the product of the perimeter and the apothem.

Let l be the side MN of a regular polygon, Fig. 42, n the number of sides, $p(= n\,l)$ the perimeter, $a(= OF)$ the apothem, and A the area. As A is equal to the sum of n triangles, each equal to MON, we have,

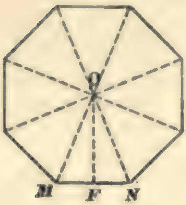

FIG. 42

$$A = (\tfrac{1}{2}\,MN \times OF) \times n = \tfrac{1}{2}\,l\,a \times n = \tfrac{1}{2}\,n\,l \times a,$$

or,

$$A = \tfrac{1}{2}\,p\,a$$

EXAMPLE.—Find the area of a regular pentagon whose side is 25 feet and apothem is 17.2 feet.

SOLUTION.—The figure is a pentagon, hence it has five sides. The perimeter is 5×25 and the area is $\frac{5 \times 25 \times 17.2}{2}$ = 1,075 sq. ft. Ans.

68. The areas of regular polygons each of whose sides is equal to 1 are given in the following table:

TABLE I

AREAS OF REGULAR POLYGONS

Name	Number of Sides	Area When Side = 1	Name	Number of Sides	Area When Side = 1
Triangle .	3	.4330	Octagon .	8	4.8284
Square . .	4	1.0000	Nonagon .	9	6.1818
Pentagon .	5	1.7205	Decagon .	10	7.6942
Hexagon' .	6	2.5981	Undecagon	11	9.3656
Heptagon .	7	3.6339	Dodecagon	12	11.1960

From the principle of Art. **55**, the following rule is derived:

Rule.—*To find the area of any regular polygon, square the length of a side and multiply by the area of the similar polygon whose side is equal to the unit of length.*

Let A = area; l = length of side of required polygon; a = area of similar polygon whose side is 1; then, by Art. **55**,

$$A : a = l^2 : 1^2$$

whence,

$$A = a\,l^2$$

EXAMPLE.—The side of a regular octagon is 3 inches, find its area.

SOLUTION.—From the table, the area of a regular octagon whose side is 1 in. is 4.8284 sq. in. Hence, the area of the octagon whose side is 3 in. is 4.8284×3^2 = 43.456 sq. in. Ans.

69. If the vertexes of a regular inscribed polygon are joined to the middle points of the arcs subtended by the sides of the polygon, the joining lines form a regular inscribed polygon of double the number of sides. Thus, the octagon *A F B G*, etc., Fig. 43, is formed by joining the middle points of the arcs subtended by the sides of the square *A B C D*.

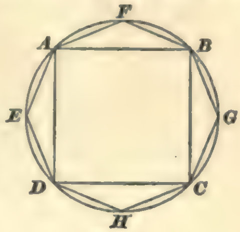

FIG. 43

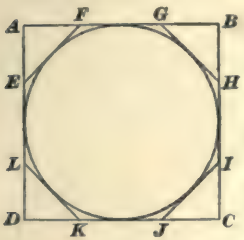

FIG. 44

70. If tangents are drawn at the middle points of the arcs between adjacent points of contact of the sides of a regular circumscribed polygon, a regular circumscribed polygon of double the number of sides is formed. Thus, in Fig. 44, the octagon *E F G H*, etc., is formed by drawing tangents at the middle points of the arcs between adjacent points of contact of the sides of the circumscribed square *A B C D*.

CIRCULAR MEASUREMENTS

THE CIRCLE

LENGTH OF ANY ARC

71. If any two circles are taken, and two regular polygons of the same number of sides are inscribed in them, the perimeters of these polygons are to each other as the radii of the circles (Art. 66). This relation holds whatever the number of sides of the polygon. Now, it is evident that, as this number increases, the perimeters of the two polygons approach the circumferences of their respective circles. We may, therefore, consider these circumferences as extreme cases of the perimeters of regular polygons, in which the number of sides is increased indefinitely; whence we conclude that the circumferences, also, are to each other as their radii.

If c and c' are the circumferences of any two circles, and r and r' their respective radii, we may write,

$$c : c' = r : r'$$

whence,

$$c : r = c' : r'$$

or,

$$\frac{c}{r} = \frac{c'}{r'}$$

Dividing both numbers by 2, and denoting the diameters by d and d',

$$\frac{c}{2r} = \frac{c'}{2r'}$$

that is,

$$\frac{c}{d} = \frac{c'}{d'}$$

As c and c' are any two circumferences, it is seen that the ratio obtained by dividing any circumference by its diameter is the same for all circumferences. This ratio is usually denoted by the Greek letter π (pronounced *pi*). We have, therefore, for any circle,

$$\frac{c}{d} = \pi$$

whence, $$c = \pi d = 2\pi r$$

72. The quantity π can be determined by elementary geometrical methods, which may be found in treatises on geometry; but these methods are very laborious. A much better method is afforded by the theory of series, which is treated in works on trigonometry and the differential calculus. It is found that π cannot be expressed as an exact fraction, either decimal or vulgar. Its value can, however, be calculated to any desired degree of approximation. The following value is approximate to fifteen decimal places:

$$\pi = 3.141592653589793+$$

For nearly all practical purposes, 3.1416 is a sufficiently close value. This value is used very generally, and will be used in this Course, unless otherwise stated. The student should commit it to memory. A value that is often used in rough calculations is $\frac{22}{7}$; it can be used when no more than three significant figures are required in the result.

73. The length of an arc, when the number of degrees in the arc and the radius of the circle are given, may be found as follows:

The length of the arc is evidently the same part of the length of the circumference ($2\pi r$) as the number of degrees in the arc is of the number of degrees in the whole circumference, or 360°. Thus, if n is the number of degrees in the arc, and l is its length, we shall have,

$$\frac{2\pi r}{l} = \frac{360}{n}$$

whence, $$l = \frac{\pi r n}{180}$$

In applying this formula, minutes and seconds should be expressed as fractions of a degree.

EXAMPLE 1.—Find the length of a rope that will go around a wheel or drum 7.5 feet in diameter.

SOLUTION.—The required length is equal to the length c of the circumference of the wheel or drum. Here $d = 7.5$ ft., and, taking $\pi = 3.1416$, we have, by formula of Art. **71**,

$$c = 3.1416 \times 7.5 = 23.562 \text{ ft. Ans.}$$

Using $\frac{22}{7}$ for π, the result, to three significant figures, is

$$c = \frac{22}{7} \times 7.5 = 23.6 \text{ ft. Ans.}$$

EXAMPLE 2.—Find the diameter of a circular race track 1 mile in length.

SOLUTION.—Here c is given (= 1 mi. = 5,280 ft.) and the quantity required is d. From the formula $c = \pi d$, we get

$$d = \frac{c}{\pi} = \frac{5{,}280}{3.1416} = 1{,}680.7 \text{ ft. Ans.}$$

EXAMPLE 3.—What is the length of a railroad circular curve having a radius of 1,540 feet and subtending an angle at the center equal to 26° 35′?

SOLUTION.—To apply formula of Art. **73**, we have $r = 1{,}540$ ft., $n = 26\frac{35}{60}° = 26.583°$, nearly. Therefore,

$$l = \frac{3.1416 \times 1{,}540 \times 26.583}{180} = 714.50 \text{ ft. Ans.}$$

74. When only the chord AB, Fig. 45, of an arc and the height, or "rise," CD of the segment are known, the following approximate method gives good results. AC, the chord of half the arc, has the value

$$AC = \sqrt{\overline{AD}^2 + \overline{CD}^2} = \sqrt{\left(\frac{AB}{2}\right)^2 + \overline{CD}^2}$$

Then, to find the length of the arc:

Rule.—*From eight times the chord of half the arc, subtract the chord of the whole arc and divide the remainder by 3.*

That is,

$$\text{arc } ACB = \frac{8 \times AC - AB}{3}$$

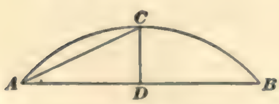

FIG. 45

Let c = chord of whole arc;
h = height of segment;
l = length of arc.

Then, $$AC = \sqrt{\frac{c^2}{4} + h^2} = \frac{1}{2}\sqrt{c^2 + 4h^2}$$

and $$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3}$$

This formula gives the length of an arc less than one-sixth of the circumference correct to four figures, and it gives the length of an arc less than one-third of the circumference correct to three figures.

EXAMPLE.—Find the length of the arc ACB, Fig. 46.

SOLUTION.—In this example, $c = 72$, $h = 8$. Therefore,

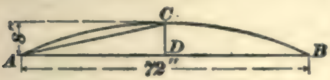

FIG. 46

$$l = \frac{4\sqrt{72^2 + 4 \times 8^2} - 72}{3} = 74.34 \text{ in.}$$ Ans.

75. For very flat arcs, that is, when $\frac{h}{c}$ is very small (say not greater than .1), the following approximate formula may be used, the notation being the same as in the preceding article:

$$l = c + \frac{8h^2}{3c}$$

EXAMPLE 1.—Find the length of the arc AB, Fig. 46.

SOLUTION.—

$$l = 72 + \frac{8 \times 8^2}{3 \times 72} = 72 + 2.37 = 74.37. \text{ Ans.}$$

This is not a very close approximation, because the ratio $\frac{h}{c}\left(= \frac{8}{72} = \frac{1}{9}\right)$ is not very small; however, the approximate value thus found would be close enough for most practical purposes.

EXAMPLE 2.—The chord of a railroad curve is 675 feet long, and the rise (or, "middle ordinate," as the rise is called in railroad work) is 40 feet; what is the length of the curve?

SOLUTION.—Here $c = 675$, $h = 40$, and therefore

$$l = 675 + \frac{8 \times 40^2}{3 \times 675} = 675 + 6.32 = 681.32 \text{ ft. Ans.}$$

76. Circular Measure of an Angle.—The following equation follows from the formula of Art. **73**:

$$\frac{l}{r} = \frac{\pi n}{180} = \frac{\pi}{180} \times n$$

If we assume the radius to be 1, then

$$l = \frac{\pi}{180} \times n \qquad (1)$$

This equation gives the length of the arc that the angle subtends on a circle whose radius is equal to unity. The length of such arc is called the **circular measure** of the angle, and the angle is often referred to by stating that measure. Thus, an angle of 1.34, circular measure, means an angle that subtends an arc of length 1.34 on a circle whose radius is 1. An angle expressed in circular measure is also said to be expressed **in radians.**

If in equation 1 we make $n = 180°$, we obtain, for the circular measure of 180°, $l = \pi$, that is, 180° is equivalent to π radians. Likewise, 90° is equivalent to $\frac{\pi}{2}$ radians, etc.

EXAMPLES FOR PRACTICE

1. Find the distance around the outside of a waterwheel whose outside diameter is 22 feet 8 inches. Ans. 71.21 ft.

2. The wheel of a carriage is observed to turn 375 times in going from a certain place to another; the diameter of the wheel is 3.5 feet; what is the distance between the two places? Ans. 4,123.4 ft.

3. A circular column measures 45.5 inches around the outside; what is its diameter? Ans. 14.483 in.

4. A belt covers an arc of 50° on a pulley whose diameter is 5 feet; what length of the belt is in contact with the pulley? Ans. 2.1817 ft.

5. How long will it take a train to move over a curve subtending an angle of 100°, the radius of the curve being 1,800 feet, and the train going at the rate of 20 miles an hour? Ans. 1.79 min.

6. The length of arc of a circle is equal to the radius; find the number of degrees in the arc. Ans. 57.3° = 57° 18′, nearly

7. The chord of a railroad curve is 600 feet long and the middle ordinate is 80 feet; what is the length of the curve? Ans. 628 ft.

AREAS BOUNDED BY CIRCULAR ARCS

77. The area of a circle is equal to one-half the product of its circumference and radius (Art. **67**). This at once follows by considering the circle as an extreme case of a regular polygon.

Let A = area of circle;
c = circumference of circle;
r = radius of circle.

Then, $A = \frac{1}{2} c r$

or, since $c = 2 \pi r$,

$$A = \frac{1}{2} 2 \pi r \times r$$

or, simplifying,

$$A = \pi r^2 = 3.1416\, r^2 \qquad (1)$$

Writing $\frac{d}{2}$ for r, we obtain for the area in terms of the diameter,

$$A = \frac{\pi d^2}{4} = .7854\, d^2 \qquad (2)$$

These formulas serve likewise to find r or d when A is given. Since $2 \pi r = c$, we have

$$r = \frac{c}{2 \pi}, \text{ and } \pi r^2 = \pi \left(\frac{c}{2 \pi}\right)^2 = \frac{c^2}{4 \pi}$$

that is, $A = \frac{c^2}{4 \pi}$ (3)

This formula gives the area of a circle when its circumference is known.

Example 1.—The steam pressure on a piston is 75 pounds per square inch, and the diameter of the piston is 15 inches; what is the pressure on the whole surface of the piston?

Solution.—The required pressure is evidently seventy-five times the number of square inches in the surface of the piston, or seventy-five times the area A of the piston. Here $d = 15$ in., and formula **2** gives

$$A = .7854 \times 15^2$$

whence the total pressure is

$$75 \times .7854 \times 15^2 = 13{,}254 \text{ lb. Ans.}$$

Example 2.—The distance around a circular park is 2.75 miles; what is the area of the park, in acres?

Solution.—Here c is given equal to 2.75 mi. = (2.75×80) ch. Therefore, the area of the park, in square chains, is (formula **3**)

$$\frac{(2.75 \times 80)^2}{4 \times 3.1416}$$

The area, in acres, is one-tenth of this, or

$$\frac{1}{10} \times \frac{(2.75 \times 80)^2}{4 \times 3.1416} = \frac{220^2}{125.664} = 385.15 \text{ A. Ans.}$$

Example 3.—What must be the diameter of a circular sewer pipe that its cross-section may be 12.75 square feet?

Solution.—Solving formula **2** for d,

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{12.75}{.7854}} = 4.03 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1 The cable of a suspension bridge measures 40 inches around its circumference; find: (*a*) the diameter *d* of the cable; (*b*) the area *A* of the cross-section.

Ans. $\begin{cases} (a)\ d = 12.732 \text{ in.} \\ (b)\ A = 127.32 \text{ sq. in} \end{cases}$

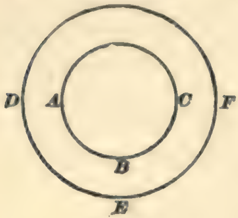

FIG. 47

2. Find a formula for the area *A* of the space enclosed between two circles *A B C* and *D E F*, Fig. 47, the diameter of the outer circle being *D*, and that of the inner circle *d*.

Ans. $\begin{cases} A = \frac{\pi}{4}(D^2 - d^2) \\ A = \frac{\pi}{4}(D + d)(D - d) \end{cases}$

3. What must be the inner diameter of a circular chimney, that its inner cross-section may be 14 square feet? Ans. 4.2221 ft

4. The diameter of a circular airway of a mine is 10 feet; find: (*a*) the circumference *c*; (*b*) the area *A* of the cross-section.

Ans. $\begin{cases} (a)\ c = 31.416 \text{ ft.} \\ (b)\ A = 78.54 \text{ sq. ft.} \end{cases}$

---

78. A sector is the same part of a circle as its arc is of the circumference.

Let A = area of circle;
A' = area of sector;
n = number of degrees in arc of sector.

Then, $$A' : A = n : 360$$

whence, $$A' = \frac{n A}{360} = \frac{\pi r^2 n}{360}$$

EXAMPLE.—The angle of a sector of a circle is 75°; the diameter of the circle is 12 inches; what is the area of the sector?

SOLUTION.—The area A of the circle is $12^2 \times .7854$ sq. in. Then the area of the sector is

$$\frac{n A}{360} = \frac{75 \times 12^2 \times .7854}{360} = 23.562 \text{ sq. in.} \quad \text{Ans.}$$

79. The area of a sector is equal to one-half the product of its base by the radius of the circle.

$$A' = \tfrac{1}{2} r l$$

If l is the length of the arc, or **base**, of a sector, we have (Art. **73**):

$$l = \frac{\pi r n}{180}$$

whence, $$n = \frac{180\, l}{\pi r}$$

This value of n substituted in formula of Art. **78** gives

$$A' = \frac{\pi r^2}{360} \times \frac{180\, l}{\pi r}$$

or, reducing, $$A' = \tfrac{1}{2} r l$$

EXAMPLE.—If the radius of an arc is 5 feet and the length of the arc is 4 feet, what is the area of the sector?

SOLUTION.—By formula of Art. **79**,

$$A' = \frac{l r}{2} = \frac{4 \times 5}{2} = 10 \text{ sq. ft.} \quad \text{Ans.}$$

80. The area of a segment, as ADB, Fig. 48, is evidently equal to the area of the sector $AOBD$ minus the area of the triangle AOB.

EXAMPLE 1.—The diameter of a circle is 10 inches, and the chord of the arc of a segment is 7 inches; what is the area of the segment?

SOLUTION.—In Fig. 48, let $AB = 7$ in. and the diameter $= 10$ in. Then, $OB = 5$ in., and $CB = 3.5$ in. Hence, $OC = \sqrt{5^2 - 3.5^2} = 3.57$ in., and $CD = 5 - 3.57 = 1.43$ in. Then, by formula of Art. **74**,

$$\text{arc } ADB = \frac{4\sqrt{7^2 + 4 \times 1.43^2} - 7}{3} = 7.75 \text{ in.}$$

Hence, area of sector $AOBD = \frac{1}{2} \times 5 \times 7.75 = 19.38$ sq. in. The area of the triangle $AOB = \frac{1}{2} \times 3.57 \times 7 = 12.50$ sq. in. Therefore, the area of the segment is $19.38 - 12.50 = 6.88$ sq. in. Ans.

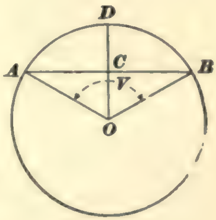

FIG. 48

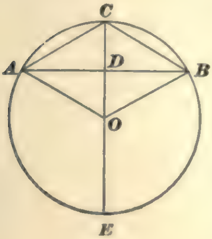

FIG. 49

EXAMPLE 2.—The chord of the arc of a segment is 79 inches and the height of the segment is 20 inches; find the area of the segment.

SOLUTION.—Let $ACBE$, Fig. 49, be the circle; let $AB = 79$ in. and $CD = 20$ in. Then, $AD = \frac{1}{2} \times 79$ in. $= 39.5$ in. By Art. **28**,

$$CD : AD = AD : DE$$

or, $$20 : 39.5 = 39.5 : DE$$

whence, $$DE = 78.01$$

Hence, the diameter = 20 + 78.01 = 98.01 in., and the radius = 49. Then the arc $ACB = \frac{4\sqrt{79^2 + 4 \times 20^2} - 79}{3} = 91.7$ in. Hence, the area of sector $AOBC = 91.7 \times \frac{1}{2} \times 49 = 2{,}246.65$ sq. in. The area of the triangle $AOB = \frac{1}{2} \times 79 \times 29 = 1{,}145.5$ sq. in. Therefore, the area of the segment = 2,246.65 − 1,145.50 = 1,101.15 sq. in. Ans.

THE ELLIPSE

81. An **ellipse** is a plane figure bounded by a curved line such that the sum of the distances of any point on that line from two fixed points within is always equal to the length of the line passing through the fixed points and terminating at both ends in the curved line.

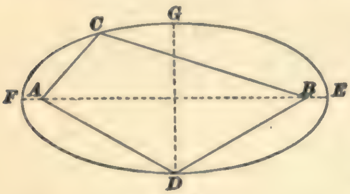

FIG. 50

In Fig. 50, the fixed points are A and B, and if C and D are any two points on the curve, $AC + CB = AD + DB = FE$. The two fixed points are the **foci**. The line FE through the foci is the **transverse**, or **major, axis.** The line GD, which is the perpendicular bisector of FE, is the **conjugate**, or **minor, axis.** The foci may be located from G or D as a center by striking arcs with a radius equal to one-half FE.

82. There is no simple and exact method of finding the periphery (perimeter) of an ellipse. The following formula gives values very nearly exact:

Let C = periphery;
a = half the major axis;
b = half the minor axis;
$D = \frac{a - b}{a + b}$.

Then, $$C = \pi(a + b)\frac{64 - 3D^4}{64 - 16D^2}$$

EXAMPLE.—What is the periphery of an ellipse whose axes are 10 inches and 4 inches long?

SOLUTION.—In this example, $a = 5$, $b = 2$, $D = \frac{5-2}{5+2} = \frac{3}{7}$.

Then, $$C = 3.1416(5+2)\frac{64 - 3(\frac{3}{7})^4}{64 - 16(\frac{3}{7})^2} = 23.013$$

Therefore, the periphery is 23.013 in. Ans.

83. The area of an ellipse is equal to the product of its two semiaxes multiplied by π.

Let a = half the major axis;
b = half the minor axis;
A = area.

Then, $$A = \pi a b = 3.1416\, a b$$

EXAMPLE.—What is the area of an ellipse whose axes are 10 inches and 6 inches?

SOLUTION.—Here, $a = \frac{1}{2} \times 10 = 5$, $b = \frac{1}{2} \times 6 = 3$.
Then, $$A = 3.1416 \times 5 \times 3 = 47.124$$
Therefore, the area is 47.12 sq. in. Ans.

EXAMPLES FOR PRACTICE

1. The number of degrees in the angle formed by drawing radii from the center of a circle to the extremities of an arc of the circle is 84; the diameter of the circle is 17 inches; what is the area of the sector? Ans. 52.96 sq. in.

2. Given the chord of the arc of a segment equal to 24 inches, and the height of the segment equal to 6.5 inches, find: (*a*) the diameter of the circle; (*b*) the area of the segment. Ans. { (*a*) 28.7 in. (*b*) 109.5 sq. in. }

3. (*a*) What is the perimeter of an ellipse whose axes are 15 inches and 9 inches? (*b*) What is the area? Ans. { (*a*) 38.29 in. (*b*) 106.03 sq. in. }

4. The base of a sector is 24 inches and the diameter of the circle is 54 inches; what is the area of the sector? Ans. 324 sq. in.

THE MENSURATION OF SOLIDS

84. A **solid**, or body, has three dimensions: length, breadth, and thickness.

85. The **entire area** of a solid is the area of the whole outside of the solid.

The **convex area** of a solid having one or two flat ends is the same as the entire surface, except that the areas of the ends or bases are not included.

86. The **volume** of a solid is expressed by the number of times that it will contain another volume, called the unit of volume. Instead of the word *volume*, the expression **cubical contents** is frequently used.

THE PRISM AND CYLINDER

87. A **prism** is a solid whose ends are equal polygons in parallel planes, and whose sides are parallelograms.

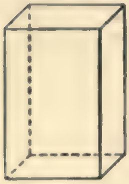

FIG. 51

88. A **parallelopipedon**, Fig. 51, is a prism whose bases (ends) are parallelograms.

89. A **cube**, Fig. 52, is a parallelopipedon whose faces and ends are squares.

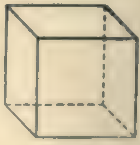

FIG. 52

90. The cube whose edges are equal to the unit of length is taken as the unit of volume when finding the volume of a solid.

Thus, if the unit of length is 1 inch, the unit of volume will be the cube each of whose edges measures 1 inch, or 1 cubic inch; and the number of cubic inches the solid contains will be its volume. If the unit of length is 1 foot, the unit of volume will be 1 cubic foot, etc. Cubic inch, cubic foot, and cubic yard are abbreviated to cu. in., cu. ft., and cu. yd., respectively.

91. Prisms take their names from their bases. Thus, a *triangular prism* is one whose bases are triangles; a *pentagonal prism* is one whose bases are pentagons, etc.

92. A **cylinder,** Fig. 53, is a round body of uniform diameter with circles for its ends.

FIG. 53

93. A **right prism,** or **right cylinder,** is one whose center line (axis) is perpendicular to its bases.

94. The **altitude** of a prism or cylinder is the perpendicular distance between its two ends.

95. To find the convex area of any right prism, or right cylinder:

Rule.—*Multiply the perimeter of the base by the altitude.*

Let p = perimeter of base;
h = altitude;
c = convex area.

Then, $$c = p\,h$$

EXAMPLE 1.—What is the convex area of a right prism whose base is a square, one side of which is 9 inches, and whose altitude is 16 inches?

SOLUTION.— $9 \times 4 = 36$ in., the perimeter of the base. Applying formula of Art. **95**,

$c = 36 \times 16 = 576$ sq. in., the convex area. Ans.

To find the entire area, add the areas of the two ends to the convex area.

EXAMPLE 2.—What is the entire area of the parallelopipedon mentioned in the last question?

SOLUTION.—The area of one end is $9^2 = 81$ sq. in. $81 \times 2 = 162$ sq. in., is the area of both ends. $576 + 162 = 738$ sq. in., the entire area of the parallelopipedon. Ans.

EXAMPLE 3.—What is the entire area of a right cylinder whose base is 16 inches in diameter, and whose altitude is 24 inches?

SOLUTION.— $16 \times 3.1416 = 50.27$ in., or the perimeter (circumference) of the base. $50.27 \times 24 = 1{,}206.48$ sq. in., the convex area.

$16^2 \times .7854 \times 2 = 402.12$ sq. in., the area of the ends.

$1{,}206.48 + 402.12 = 1{,}608.6$ sq. in., the entire area. Ans.

96. To find the volume of a prism, or cylinder:

Rule.—*The volume of any prism or cylinder is equal to the area of the base multiplied by the altitude.*

Let A = area of base;
h = altitude;
V = volume.

Then, $$V = A\,h$$

If the given prism is a cube, the three dimensions are all equal, and the volume equals the cube of one of the edges. Hence, if the volume is given, the length of an edge is found by extracting the cube root.

If the volume and the area of the base are given, the altitude is $h = \frac{V}{A}$. If the cylinder or prism is hollow, the volume is equal to the area of the ring or base multiplied by the altitude.

EXAMPLE 1.—What is the volume of a rectangular prism whose base is 6 inches by 4 inches, and whose altitude is 12 inches?

SOLUTION.—The base of a rectangular prism is a rectangle; hence, $6 \times 4 = 24$ sq. in., the area of the base. Applying formula of Art. **96**,

$V = 24 \times 12 = 288$ cu. in., or the volume. Ans.

EXAMPLE 2.—What is the volume of a cube whose edge is 9 inches?

SOLUTION.— $9^3 = 9 \times 9 \times 9 = 729$ cu. in., the volume. Ans.

EXAMPLE 3.—What is the volume of a cylinder whose base is 7 inches in diameter, and whose altitude is 11 inches?

SOLUTION.— $7^2 \times .7854 = 38.48$ sq. in., the area of the base. Applying formula of Art. **96**,

$V = 38.48 \times 11 = 423.28$ cu. in., the volume. Ans.

THE PYRAMID AND CONE

97. A **pyramid,** Fig. 54, is a solid whose base is a polygon, and whose sides are triangles uniting at a common point, called the **vertex.** If the base is a regular polygon, and the sides have the same inclination to the base, the pyramid is a **regular pyramid.**

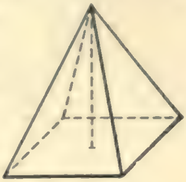

FIG. 54

98. A **cone,** Fig. 55, is a solid whose base is a circle, and whose convex surface tapers uniformly to a point called the **vertex.**

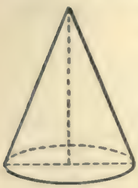

FIG. 55

99. The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

100. The **slant height** of a regular pyramid is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a cone is a straight line drawn from the vertex to the circumference of the base, and lying on the surface of the cone.

101. To find the convex area of a regular pyramid or a cone:

Rule.—*The convex area of a regular pyramid or of a cone is equal to the perimeter of the base multiplied by one-half the slant height.*

Let p = perimeter;
s = slant height;
c = convex area.

Then, $$c = \frac{ps}{2}$$

EXAMPLE 1.—What is the convex area of a regular pentagonal pyramid, if each side of the base measures 6 inches and the slant height measures 14 inches?

SOLUTION.—The base of the pentagonal pyramid is a pentagon, and consequently it has five sides. $6 \times 5 = 30$ in., or the perimeter of the base. Applying formula of Art. **101,**

$$c = \frac{p\,s}{2} = \frac{30 \times 14}{2} = 210 \text{ sq. in., the convex area. Ans.}$$

EXAMPLE 2.—What is the entire area of a cone whose altitude is 15 inches, and whose base is 16 inches in diameter?

SOLUTION.—The slant height of the cone is the hypotenuse of a right triangle whose legs are the radius of the base and altitude of the cone, respectively. Therefore, the slant height is equal to $\sqrt{15^2 + 8^2} = 17$ in. (Art. **56**). The perimeter of the base is $16 \times 3.1416 = 50.2656$ in. Applying formula of Art. **101**,

$$c = \frac{50.2656 \times 17}{2} = 427.26 \text{ sq. in.}$$

The area of the base is $16^2 \times .7854 = 201.06$ sq. in. The entire area is, therefore, $427.26 + 201.06 = 628.32$ sq. in. Ans.

102. To find the volume of any pyramid or cone:

Rule.—*The volume of any pyramid or cone equals the area of the base multiplied by one-third of the altitude.*

Let A = area of base;
h = altitude;
V = volume.

Then,
$$V = \frac{A\,h}{3}$$

EXAMPLE 1.—What is the volume of a triangular pyramid, each edge of whose base measures 6 inches, and whose altitude is 8 inches?

SOLUTION.—The base is an equilateral triangle; hence, applying the rule of Art. **68**, the area is $6^2 \times .433 = 15.59$ sq. in. Applying formula of Art. **102**,

$$V = \frac{A\,h}{3} = \frac{15.59 \times 8}{3} = 41.57 \text{ cu. in. Ans.}$$

EXAMPLE 2.—What is the volume of a cone whose altitude is 18 inches, and whose base is 14 inches in diameter?

SOLUTION.— $14^2 \times .7854 = 153.94$ sq. in., the area of the base. Applying formula of Art. **102**,

$$V = \frac{A\,h}{3} = \frac{153.94 \times 18}{3} = 923.64 \text{ cu. in., the volume. Ans.}$$

103. It has been stated that the volume of a cone or a pyramid is equal to one-third the product of the area of the base multiplied by the altitude. Similarly, the volume of any solid whose base is a plane figure and which tapers to a point like a cone or a pyramid is equal to one-third of the product of its base and altitude.

EXAMPLE.—Find the volume of an elliptical cone, whose base is an ellipse with diameters 8 inches and 6 inches, and the altitude is 7.5 inches.

SOLUTION.—The area of the ellipse at the base is $3.1416 \times 4 \times 3$ The volume is equal to one-third the product of the area of the base and altitude; that is,

$$V = \frac{1}{3} \times 3.1416 \times 4 \times 3 \times 7.5 = 94.248$$

Hence, the volume is 94.248 cu. in. Ans.

EXAMPLES FOR PRACTICE

1. Find the volume of a triangular pyramid of which the altitude is 4 inches and the base is an equilateral triangle having each side 3 inches long. Ans. 5.2 cu. in.

2. Find the weight of a steel bar 16 feet long and 2 inches in diameter, the weight of steel being taken as .28 pound per cubic inch. Ans. 168.89 lb.

3. What is the entire area of a hexagonal prism 12 inches long, each side of the base being 1 inch long? Ans. 77.196 sq. in.

4. (*a*) Find the convex area of a cone whose altitude is 12 inches, and the circumference of whose base is 31.416 inches. (*b*) Find the volume of the cone. Ans. { (*a*) 204.2 sq. in. (*b*) 314.16 cu. in

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THE FRUSTUM OF A PYRAMID OR A CONE

104. If a pyramid is cut by a plane parallel to the base, as in Fig. 56, so as to form two parts, the lower part is called a **frustum** of the pyramid.

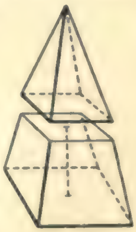
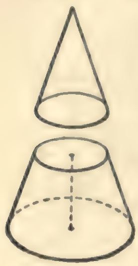


FIG. 56

105. If a cone is cut in a similar manner, as in Fig. 57, the lower part is called a **frustum of the cone**.

106. The upper end of a frustum of a pyramid or cone is called the **upper base**, and the lower end the **lower base**. The **altitude** of a frustum is the perpendicular distance between the bases.

107. To find the convex area of a frustum of a regular pyramid or of a cone:



Rule.—*The convex area of a frustum of a regular pyramid or of a cone equals one-half the sum of the perimeters of its bases multiplied by the slant height of the frustum.*

Let p = perimeter of lower base;
p' = perimeter of upper base;
s = slant height;
c = convex area.

FIG. 57

Then,
$$c = \left(\frac{p + p'}{2}\right)s$$

EXAMPLE 1.—Given the frustum of a triangular pyramid in which each side of the lower base measures 10 inches, each side of the upper base measures 6 inches, and whose slant height is 9 inches; find the convex area.

SOLUTION.— 10 in. × 3 = 30 in., the perimeter of the lower base. 6 in. × 3 = 18 in., the perimeter of the upper base. Applying formula of Art. **107**,

$c = \left(\frac{p + p'}{2}\right)s = \frac{30 + 18}{2} \times 9 = 216$ sq. in., the convex area. Ans.

EXAMPLE 2.—If the diameters of the two bases of a frustum of a cone are 12 inches and 8 inches, respectively, and the slant height is 12 inches, what is the entire area of the frustum?

SOLUTION.— $\frac{(12 \times 3.1416) + (8 \times 3.1416)}{2} \times 12 = 376.99$ sq. in., the convex area.

$8^2 \times .7854 = 50.27$ sq. in.
$12^2 \times .7854 = 113.1$ sq. in.

113.1 + 50.27 = 163.37 sq. in., the area of the two ends. 376.99 + 163.37 = 540.36 sq. in., the entire area of the frustum. Ans.

108. To find the volume of the frustum of a pyramid or a cone:

Rule.—*Add the areas of the upper base, the lower base, and the square root of the product of the areas of the two bases; multiply this sum by one-third of the altitude.*

Let A = area of lower base;
a = area of upper base;
h = altitude;
V = volume.

Then, $$V = (A + a + \sqrt{A\,a})\frac{h}{3}$$

EXAMPLE 1.—Given a frustum of a hexagonal pyramid in which each edge of the lower base measures 8 inches, and each edge of the upper base measures 5 inches, and whose altitude is 14 inches, what is its volume?

SOLUTION.—A hexagonal pyramid is one whose base is a regular hexagon, as shown in Fig. 58. Hence, applying formula of Art. **68**,

$$A = 8^2 \times 2.5981 = 166.28 \text{ sq. in.}$$

In a similar way, the area of the upper base is found to be 64.95 sq. in. Then, applying formula of Art. **108**,

$$V = (166.28 + 64.95 + \sqrt{166.28 \times 64.95})\frac{14}{3}$$

$$= 335.15 \times \frac{14}{3} = 1{,}564.03 \text{ cu. in., the volume. Ans.}$$

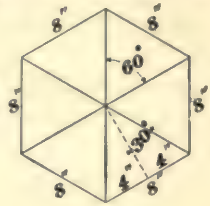


FIG. 58

EXAMPLE 2.—What is the volume of a frustum of a cone whose upper base is 8 inches in diameter, whose lower base is 12 inches in diameter, and whose altitude is 15 inches?

SOLUTION.—The area of the upper base is $8^2 \times .7854 = 50.27$ sq. in. The area of the lower base is $12^2 \times .7854 = 113.1$ sq. in., nearly. The square root of their product is $\sqrt{50.27 \times 113.1} = 75.4$.

Then, $$V = (50.27 + 113.1 + 75.4)\frac{15}{3}$$

$$= 238.77 \times \frac{15}{3} = 1{,}193.85 \text{ cu. in., the volume. Ans.}$$

THE WEDGE

109. A **wedge,** as here considered, is a solid whose base is a rectangle, two of whose opposite faces are parallel triangles, and two are parallelograms whose intersection is called the **edge** of the wedge. A wedge may therefore be defined as a triangular prism having one rectangular face, called the **base.** In Fig. 59, $ABCD$ is the base and EF the edge of the wedge.

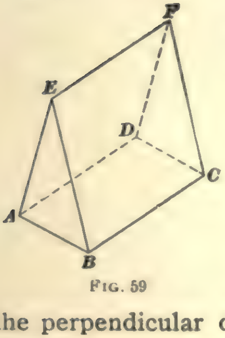


FIG. 59

110. The **altitude** of a wedge is the perpendicular distance between the base and the opposite edge.

111. To find the volume of a wedge:

Rule.—*The volume of any wedge is equal to the area of the base multiplied by one-half the altitude.*

Let A = area of base;
h = altitude;
V = volume.

Then,
$$V = \frac{A\,h}{2}$$

EXAMPLE.—What is the volume of a wedge whose base is a rectangle 6 feet long and 4 feet wide, and whose altitude is 10 feet?

SOLUTION.—The area of the base is $4 \times 6 = 24$ sq. ft. Applying formula of Art. **111**,

$$V = \frac{24 \times 10}{2} = 120 \text{ cu. ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Steel weighs .28 pound per cubic inch; find the weight of a steel wedge whose base is a rectangle 3 inches by $1\frac{1}{2}$ inches and whose altitude is 8 inches. Ans. 5.04 lb.

2. Find the volume of the frustum of a square pyramid of which the larger base is 15 inches square, the smaller base, 14 inches square, and the altitude, 3 inches. Ans. 631 cu. in.

3. A round tank is 8 feet in diameter at the top (inside) and 10 feet at the bottom; if the tank is 12 feet deep, how many gallons will it hold, there being 231 cubic inches in a gallon? Ans. 5,734.2 gal.

4. (*a*) What is the convex area of the frustum of a square pyramid whose altitude is 16 inches, one side of whose lower base is 28 inches long, and of the upper base 10 inches? (*b*) What is the volume of the frustum.

Ans. { (*a*) 1,395.18 sq. in.
(*b*) 6,208 cu. in.

THE SPHERE

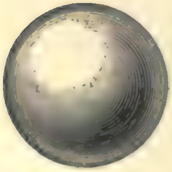


FIG. 60

112. A **sphere**, Fig. 60, is a solid bounded by a uniformly curved surface every point of which is equally distant from a point within, called the **center.**

The word **ball** is commonly used instead of sphere.

113. To find the area of the surface of a sphere

Rule.—*The area of the surface of a sphere equals the square of the diameter multiplied by* π.

Let S = surface;
d = diameter.

Then, $$S = \pi d^2$$

EXAMPLE.—What is the area of the surface of a sphere whose diameter is 14 inches?

SOLUTION.—Applying formula of Art. **113**, $S = 3.1416 \times 14^2 = 3.1416 \times 14 \times 14 = 615.75$ sq. in., the area. Ans.

114. To find the volume of a sphere:

Rule.—*The volume of a sphere equals the cube of the diameter multiplied by* $\frac{\pi}{6}$.

Let V = volume;
d = diameter.

Then, $$V = \frac{\pi}{6}d^3 = .5236d^3$$

EXAMPLE.—What is the weight of a lead cannon ball 12 inches in diameter, a cubic inch of lead weighing .41 pound?

SOLUTION.—Applying formula of Art. **114**, $V = .5236 \times 12 \times 12 \times 12 = 904.78$ cu. in., the volume of the ball.

$$904.78 \times .41 = 370.96 \text{ lb. Ans.}$$

The volume of a spherical shell, or hollow sphere, is equal to the difference in volume between two spheres having, respectively, the outer and the inner diameter of the shell.

115. To find the diameter of a sphere of known volume:

Rule.—*Divide the volume by .5236 and extract the cube root of the quotient. The result is the diameter.*

$$d = \sqrt[3]{\frac{V}{.5236}} = 1.2407\sqrt[3]{V}$$

EXAMPLE.—The volume of a sphere is 96.1 cubic inches; what is its diameter?

SOLUTION.—Applying formula of Art. **115**,

$$d = \sqrt[3]{\frac{V}{.5236}} = \sqrt[3]{\frac{96.1}{.5236}} = 1.2407\sqrt[3]{96.1} = 5.68 \text{ in. Ans.}$$

116. If any solid is cut into two parts by a plane, the surface of either part exposed by the removal of the other part is called a **plane section** of the solid.

Plane sections are divided into three classes: longitudinal sections, cross-sections, and right sections. A **longitudinal section** is any plane section taken lengthwise through the solid. Any other plane section is called a **cross-section.** If the surface exposed by taking a plane section of a solid is perpendicular to the center line of the solid, the section is called a **right section.** The surface exposed by any longitudinal section of a cylinder is a rectangle. The surface exposed by a right section of a cube is a square; of a cylinder or a cone, a circle. An oblique cross-section of a cylinder is an ellipse.

THE CYLINDRICAL RING

117. A **cylindrical ring** is a solid that may be generated by a circle revolving about an external axis in its plane.

118. To find the convex area of a cylindrical ring:

Rule.—*Multiply the circumference of an imaginary cross-section on the line A B, Fig. 61, by the length of the center line D.*

EXAMPLE.—A piece of round iron rod is bent into circular form to make a ring for a chain; if the outside diameter of the ring is 12 inches and the inside diameter is 8 inches, what is its convex area?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters, $\frac{12+8}{2} = 10$, and $10 \times 3.1416 = 31.416$ in., the length of the center line. The radius of the inside circle is 4 in., of the outside circle 6 in.; therefore, the diameter of the cross-section on the line *A B* is 2 in. Then, $2 \times 3.1416 = 6.2832$ in., and $6.2832 \times 31.416 = 197.4$ sq. in., or the convex area. Ans.

119. To find the volume of a cylindrical ring:

Rule.—*The volume will be the same as that of a cylinder whose altitude equals the length of the dotted center line D.*

Fig. 61, and whose base is the same as a cross-section of the ring on the line A B, drawn from the center O. Hence, to find the volume of a cylindrical ring, multiply the area of an imaginary cross-section on a line A B, by the length of the center line D.

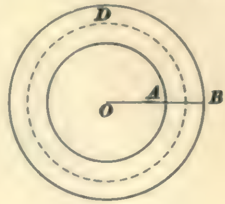


FIG. 61

EXAMPLE.—What is the volume of a cylindrical ring whose outside diameter is 12 inches, and whose inside diameter is 8 inches?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters, $\frac{12+8}{2} = 10$. $10 \times 3.1416 = 31.416$ in., the length of the center line. The radius of the outside circle is 6 in., of the inside circle, 4 in.; therefore, the diameter of the cross-section on the line *A B* is 2 in. Then, $2^2 \times .7854 = 3.1416$ sq. in., the area of the imaginary cross-section; and $3.1416 \times 31.416 = 98.7$ cu. in., the volume. Ans.

EXAMPLES FOR PRACTICE

1. (*a*) What is the area of the surface of a sphere 30 inches in diameter? (*b*) What is the volume of the sphere?

Ans. { (*a*) 2,827.44 sq. in.
(*b*) 14,137.2 cu. in.

2. (*a*) What is the convex area of a cylindrical ring, the outside diameter of the ring being 10 inches and the inside diameter $7\frac{1}{2}$ inches? (*b*) What is the volume of the ring?

Ans. { (*a*) 107.95 sq. in.
(*b*) 33.734 cu. in.

3. The volume of a sphere is 606.132 cubic inches; what is the convex area of a cone whose slant height is 10 inches, and the diameter of whose base is the same as the diameter of the sphere?

Ans. 164.934 sq. in.

THE PRISMOID

120. A **prismoid** is a solid whose two bases are any polygons in parallel planes, and whose lateral faces may be divided into triangles and trapezoids by lines joining the vertexes of one base with those of the other. Thus, the solid shown in Fig. 62 is a prismoid; its bases are the pentagon *A B C D E* and the quadrilateral *F G H I*, which lie in parallel planes; and its faces are the triangle GBC and the trapezoids $GCDH$, $HDEI$, $IEAF$, and $FABG$.

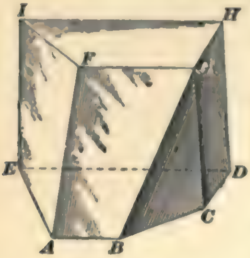


Fig. 62

121. The **altitude** of a prismoid is the perpendicular distance between the bases or parallel faces.

122. The parallel faces or bases of a prismoid are commonly called its **end sections.**

A prismoid is also defined as a solid having two parallel end faces, and composed of any combination of prisms, wedges, and pyramids, whose common altitude is the perpendicular distance between the parallel faces.

123. The **middle section** of a prismoid is the polygon formed by a plane, parallel to the bases, and cutting the prismoid at equal distances from the two bases or end sections. Thus, polygon $PQRS$ is the middle section of the prismoid shown in Fig. 63.

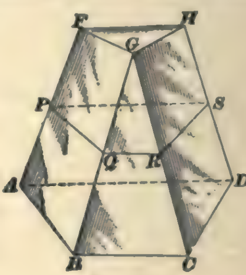


Fig. 63

124. Any dimension of the middle section of a prismoid may be taken equal to one-half the sum of the corresponding dimensions of the two end sections or bases. Thus, in Fig. 63, $PQ = \frac{1}{2}(AB + FG)$, $QR = \frac{1}{2}BC$, $RS = \frac{1}{2}(GH + CD)$, and $SP = \frac{1}{2}(HF + DA)$.

125. The area of the middle section of a prismoid may be measured directly, or calculated from its dimensions as determined from the dimensions of the end sections. It is not, in general, equal to one-half the sum of the areas of the bases.

The area of the middle section of a prism is the same as the area of either base; the area of the middle section of a wedge is equal to one-half the area of the base; the area of the middle section of a pyramid is equal to one-fourth the area of the base.

126. To find the volume of a prismoid:

Rule.—*Multiply the sum of the areas of the two end sections plus four times the area of the middle section by one-sixth the altitude.*

Let A = area of one base or end section;
A' = area of opposite base or end section;
M = area of middle section;
h = altitude;
V = volume of prismoid.

Then, $$V = \frac{h}{6}(A + A' + 4\,M)$$

This formula for finding the volume of a prismoid is known as the **prismoidal formula.** It is theoretically exact for determining the volumes of those solids to which it applies.

The derivation of this formula is as follows:

A prismoid can always be divided into elementary parts that will be prisms, wedges, and pyramids. From formula of Art. **96,** the volume of a prism is $V = A\,h$; from formula of Art. **111,** the volume of a wedge is $V = \frac{A\,h}{2}$; and from formula of Art. **102,** the volume of a pyramid is $V = \frac{A\,h}{3}$. If these expressions are reduced to a common denominator, there will result,

For a prism, $$V = \frac{6\,A\,h}{6} \qquad (1)$$

For a wedge, $$V = \frac{3\,A\,h}{6} \qquad (2)$$

For a pyramid, $$V = \frac{2\,A\,h}{6} \qquad (3)$$

Since any prism is of uniform cross-section throughout its length, every section will have the same area A, and equation (1) may be written

$$V = \frac{6\,A\,h}{6} = \frac{h}{6}(A + A' + 4\,M)$$

For a wedge, evidently $A' = 0$, and $M = \frac{1}{2}A$. Hence, equation (2) may be written

$$V = \frac{3\,A\,h}{6} = \frac{h}{6}(A + 0 + 2\,A) = \frac{h}{6}(A + A' + 4\,M)$$

For a pyramid, $A' = 0$, and $M = \frac{1}{4}A$. Hence, equation (3) may be written

$$V = \frac{2\,A\,h}{6} = \frac{h}{6}(A + 0 + A) = \frac{h}{6}(A + A' + 4\,M)$$

Each of these formulas is the same as the formula given in this article; which shows that the latter formula applies correctly to the volume of a prism, pyramid, or wedge, and since it applies to each, it applies also to their sum, or the volume of a prismoid.

EXAMPLE.—Find the volume of the prismoid shown in Fig. 64, whose altitude is 14 inches.

SOLUTION.—Let PQR be the middle section. Then,

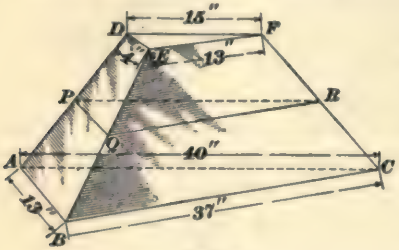


FIG. 64

$PQ = \frac{1}{2}(AB + DE) = \frac{1}{2}(13 + 4) = 8.5$ in.
$QR = \frac{1}{2}(BC + EF) = \frac{1}{2}(37 + 13) = 25$ in.
$RP = \frac{1}{2}(AC + DF) = \frac{1}{2}(40 + 15) = 27.5$ in.

The areas of the triangles are calculated by formula of Art. **47**, which gives the area of $ABC = 240$ sq. in., area of $DEF = 24$ sq. in., and area of $PQR = 105.2$ sq. in., nearly. Hence,

$$V = \frac{14}{6} \times (240 + 24 + 4 \times 105.2) = 1{,}597.9 \text{ cu. in., nearly.} \quad \text{Ans.}$$

127. A familiar example of a prismoid is a railway cutting where the roadway is a horizontal plane, the side slopes are inclined planes, and the original surface of the ground is more or less inclined and irregular.

For calculating the volume of cuts and fills the prismoidal formula, though theoretically exact, gives results that are only approximate, on account of the inequalities of the surface of the ground. The nearer to each other the cross-sections are taken, the more accurate will be the result.

EXAMPLE 1.—Find, by the prismoidal formula, the volume of the frustum of a square pyramid of which the larger base is 2.5 feet square, the smaller base is 1 foot square, and the altitude is 16 feet.

SOLUTION.—The area of the larger base is $2.5 \times 2.5 = 6.25$ sq. ft.; the area of the smaller base is $1 \times 1 = 1$ sq. ft. The middle section is a square whose side is one-half the sum of the side of the upper and lower base; that is, $\frac{1}{2} \times (2.5 + 1) = 1.75$ ft. The area of the middle section is $1.75^2 = 3.0625$ sq. ft. Applying formula of Art. **126**, the volume of the frustum is

$$\frac{1}{6} \times 16 \times (6.25 + 1 + 4 \times 3.0625) = 52 \text{ cu. ft.} \quad \text{Ans.}$$

EXAMPLE 2.—In a railway cutting 200 feet long, the following are the areas, in square feet, of the cross-sections taken every 50 feet, namely: 2,700, 2,619, 2,556, 2,484, 2,610. What is its volume?

SOLUTION.—The volume between the first and the third cross-section is, by formula of Art. **126**,

$$V = \frac{100}{6}(2{,}700 + 2{,}556 + 4 \times 2{,}619) = 262{,}200 \text{ cu. ft.}$$

The volume between the third and the fifth section is

$$V = \frac{100}{6}(2{,}556 + 2{,}610 + 4 \times 2{,}484) = 251{,}700 \text{ cu. ft.}$$

The volume of the cutting is the sum of the volumes of the two prismoids, which is 513,900 cu. ft. = 19,033 cu. yd. Ans.

128. Average End Areas.—In practice, the volume of cuts and fills is often calculated by what is known as the **method by average end areas**, or simply as the **end area method**. By this method, the volume of the solid is found by multiplying one-half the sum of the two end areas by the distance between the two sections. Thus, let

A = area of one cross-section;
A' = area of next cross-section;
h = perpendicular distance between sections;
V = volume.

Then,
$$V = \frac{h}{2}(A + A')$$

Results obtained by this formula are approximate and slightly larger than those given by the prismoidal formula. On account of its simplicity, the average end area formula is much used in practical earth-work calculations. The inequalities of the surface of the ground make it impossible to find the exact volume of a cut or fill, however accurate may be the formula applied.

EXAMPLE.—The areas of two cross-sections of a fill 50 feet apart are 2,700 and 2,619 square feet respectively; find the volume of the section, in cubic yards.

SOLUTION.—In this case, A = 2,700; A' = 2,619; and h = 50; then

$$V = \frac{50}{2}(2{,}700 + 2{,}619) = 132{,}975$$

Hence, the volume is 132,975 cu. ft. = 4,925 cu. yd. Ans.

EXAMPLES FOR PRACTICE

1. Find the volume of a right prismoid whose bases are rectangles that measure 10 inches by 8 inches and 8 inches by 6 inches, and whose height is 40 inches. Ans. 2,533.3 cu. in.

2. A railway cutting is 800 feet in length; the areas, in square yards, of cross-sections taken every 100 feet are: 237, 220, 204, 187, 171, 186, 204, 210, 220. Find the number of cubic yards in the cutting: (*a*) by the prismoidal formula; (*b*) by average end areas.

Ans. { (*a*) 53,633 cu. yd.; (*b*) 53,683 cu. yd. }

3. Find, by the prismoidal formula, the volume of a frustum of a hexagonal pyramid, each side of the lower base being 12 inches; of the upper base, 8 inches; and the altitude being 12 inches.

Ans. 3,159.3 cu. in.

4. Find, by the prismoidal formula, the volume of a wedge whose base is a rectangle 15 feet in length and 9 feet in width, and whose altitude is 12 feet. Ans. 810 cu. ft.

PLANE TRIGONOMETRY

(PART 1)

Serial 779A　　　　Edition 1

THE TRIGONOMETRIC FUNCTIONS

DEFINITIONS

1. Trigonometric Functions and Trigonometry Defined.—Let A, Fig. 1, be any acute angle; AM and AN, its sides; BC, a perpendicular drawn to the side AN from any point on the side AM; and $B'C'$, a perpendicular drawn to the side AM from any point on the side AN. In the right triangle ABC, one of the vertexes of which is the vertex of the angle A, the hypotenuse AB will be referred to as *the hypotenuse;* the perpendicular BC, opposite the vertex of the angle A, as *the side opposite;* and the leg AC, containing the vertex of the angle A, as *the side adjacent.* Likewise, in the right triangle $AB'C'$, the hypotenuse is AB'; the side opposite is $B'C'$; and the side adjacent, or the leg containing the vertex of the angle A, is AC'. It should be borne in mind that these terms are used in connection with, or with reference to, the angle A.

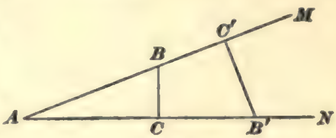


FIG. 1

The two right triangles ABC and $AB'C'$, having the acute angle A in common, are similar. Therefore,

$$\frac{AB}{AC} = \frac{AB'}{AC'}, \quad \frac{BC}{AB} = \frac{B'C'}{AB'}, \quad \frac{BC}{AC} = \frac{B'C'}{AC'}$$

It will be observed that, from whichever side the perpendicular is drawn, and whatever the point from which it is drawn, the ratio of the hypotenuse to the side adjacent remains unchanged, or is **constant.** The same is true of the ratio of the side opposite to the side adjacent, and, in general, of the ratio of any two of the three lines—hypotenuse, side adjacent, and side opposite. Evidently, these ratios are different for different angles. Thus, if A is 45°, both acute angles B and B' are also 45°; the triangles $A\,B\,C$ and $A\,B'\,C'$ are isosceles; and therefore

$$\frac{B\,C}{A\,C} = \frac{B'\,C'}{A\,C'} = 1$$

If A is greater than 45°, $B\,C$ is greater than $A\,C$, and the ratio $\frac{B\,C}{A\,C}$, having its numerator greater than its denominator, is greater than 1.

Confining ourselves to the ratio $\frac{B\,C}{A\,C}$ of the side opposite to the side adjacent, it is seen that the value of this ratio depends on the magnitude of the angle, and may, therefore, be used for the determination of the angle. Thus, it has just been shown that when the angle is 45° the ratio is equal to 1; hence, if in the solution of a problem it is found that the two legs of a right triangle are equal, or that their ratio is 1, it can be at once concluded that each of the acute angles is 45°.

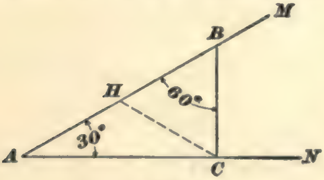


FIG. 2

Consider now an angle A, Fig. 2, of 30°. The right triangle $A\,B\,C$ having been constructed, $B\,C$ is the side opposite and $A\,C$ the side adjacent. If H is the middle point of the hypotenuse, the line $H\,C$ is equal to $A\,H$, or $\frac{A\,B}{2}$; for, if a semicircle is described on $A\,B$ as a diameter, with $H\,A$ as a radius, that semicircle must pass through C, since the angle $A\,C\,B$ is a right angle. Now, $H\,C$ being equal to $H\,B$, the angle $H\,C\,B$ is equal to B, or 60°; and, as the sum of the three angles of the triangle $B\,H\,C$ is 180°, the angle $B\,H\,C$ must be 60°. The triangle $H\,B\,C$ being equiangular, it is also equilateral, and therefore $B\,C = B\,H = \frac{A\,B}{2}$, and the ratio of the side opposite to the hypotenuse is $\frac{BC}{AB} = \frac{\frac{1}{2}AB}{AB} = \frac{1}{2}$. Suppose, now, that in dealing with a right triangle the hypotenuse is found, by measurement, to be 1,500 feet and one of the sides 750 feet. Since the ratio of 750 to 1,500 is $\frac{1}{2}$, we at once conclude that the angle opposite the 750-foot side is 30°, and the other angle of the triangle, 60°.

These illustrations give a general idea of the practical value and use of the ratios under consideration. These ratios are determined for each angle, by methods that will be again referred to further on, and collected together in a table, from which the angle corresponding to any given ratio can be determined. Thus, if in a certain angle the ratio of the opposite side to the hypotenuse is $\frac{1}{2}$, this ratio is looked for in the table, where it is found as that belonging to 30°. In this manner, the value of the angle is determined from the ratio in question, that ratio being obtained from the measured lengths of certain lines.

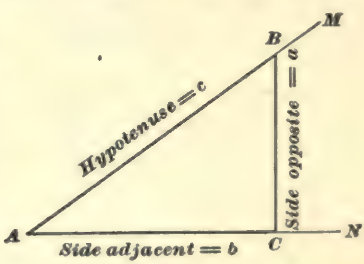


FIG. 3

2. The ratios considered in the preceding article are called **trigonometric functions** of the angle A. In the triangle ABC, Fig. 3, two ratios are obtained by dividing any of three sides by each of the other two. Hence, there are six trigonometric functions of the angle A. This is true of any angle, since A is here used to represent any angle whatever. These functions have very important and useful properties, which make them exceedingly valuable for the solution of geometrical problems by computation.

3. Trigonometry is that branch of mathematics that treats of the properties of trigonometric functions and of their application to the solution of triangles.

4. The Sine and the Tangent.—Two of the most important of the trigonometric functions are the ratio of the side opposite to the hypotenuse, and that of the side opposite to the side adjacent; that is, $\frac{a}{c}$ and $\frac{a}{b}$, Fig. 3. They are called, respectively, the **sine** of A and the **tangent** of A. The words *sine* and *tangent* are abbreviated to *sin* and *tan*, respectively, and the expressions *sin A*, *tan A*, are for brevity read *sine A*, *tangent A*, instead of *sine of A*, and *tangent of A*. We have, then,

$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{a}{c} \qquad (1)$$

$$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{a}{b} \qquad (2)$$

If these formulas are fixed in the mind, little difficulty will be experienced in remembering the others that will be given. It should be noticed that the side opposite is the numerator in both ratios. The occurrence of the letter *a* in both the words *adjacent* and *tangent* will help one to remember which of the two fractions represents the tangent and which the sine.

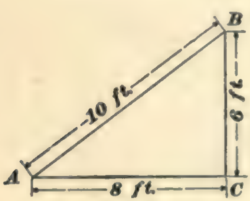


FIG. 4

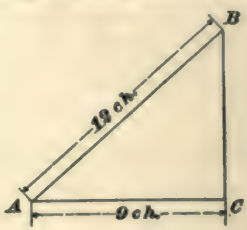


FIG. 5

EXAMPLE 1.—In the right triangle $A B C$, Fig. 4, the lengths of the sides are shown; find the sine and the tangent of A.

SOLUTION.—In this case, the hypotenuse $A B = 10$; the side adjacent, $A C = 8$; side opposite, $B C = 6$. These values in formulas 1 and 2 give

$$\sin A = \frac{6}{10} = .6. \quad \text{Ans.}$$

$$\tan A = \frac{6}{8} = .75. \quad \text{Ans.}$$

EXAMPLE 2.—In the right triangle $A B C$, Fig. 5, the hypotenuse is 12 chains, and the side $A C$ is 9 chains; find: (*a*) the sine and the tangent of A; (*b*) the sine and the tangent of B.

Solution.—(*a*) For the angle A, we have

$$\text{hypotenuse} \quad AB = 12$$
$$\text{side adjacent, } AC = 9$$
$$\text{side opposite, } BC = \sqrt{\overline{AB}^2 - \overline{AC}^2} = \sqrt{12^2 - 9^2} = 7.9372$$

Substituting in formulas **1** and **2**,

$$\sin A = \frac{BC}{AB} = \frac{7.9372}{12} = .66143. \quad \text{Ans.}$$
$$\tan A = \frac{BC}{AC} = \frac{7.9372}{9} = .88191. \quad \text{Ans.}$$

(*b*) For angle B, we have

$$\text{hypotenuse} \quad BA = 12$$
$$\text{side opposite, } AC = 9$$
$$\text{side adjacent, } BC = 7.9372$$

Therefore,
$$\sin B = \frac{AC}{AB} = \frac{9}{12} = .75. \quad \text{Ans.}$$
$$\tan B = \frac{AC}{BC} = \frac{9}{7.9372} = 1.1339. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. In a right triangle ABC (make a sketch of this triangle), A and B are the two acute angles; the hypotenuse = 40 feet; side opposite B = 15 feet; find: (*a*) sin A and tan A; (*b*) sin B and tan B.

Ans. $\begin{cases} (a) \sin A = .92703, \tan A = 2.47207 \\ (b) \sin B = .37500, \tan B = .40452 \end{cases}$

2. From a point on one side of an angle M, a perpendicular is drawn on the other side; it is found that this perpendicular is 12.5 inches long, and that it meets the other side at a distance of 7.75 inches from the vertex; find the sine and the tangent of the angle M. (Make a sketch of this triangle.)

Ans. $\begin{cases} \sin M = .84988 \\ \tan M = 1.61290 \end{cases}$

3. From a point on one side of an angle A distant 10 inches from the vertex, a perpendicular is drawn on the other side; the distance from the vertex to the foot of the perpendicular is 6 inches; find sin A and tan A.

Ans. $\begin{cases} \sin A = .80000 \\ \tan A = 1.33333 \end{cases}$

4. The two acute angles of a right triangle are P and Q; the side opposite P is 150 feet, and that opposite Q is 225 feet; find: (*a*) sin P and tan P; (*b*) sin Q and tan Q.

Ans. $\begin{cases} (a) \sin P = .55469, \tan P = .66667 \\ (b) \sin Q = .83204, \tan Q = 1.50000 \end{cases}$

5. The Cosine and Cotangent.—The **cosine** and **cotangent** of an angle are, respectively, the sine and the tangent of the complement of the angle. The words *cosine* and *cotangent* are abbreviated to *cos* and *cot*, respectively, and the expressions *cos A*, *cot A* are read *cosine A*, *cotangent A*. Denoting any angle by A, its complement is $90° - A$; therefore, according to the definitions just given,

$$\cos A = \sin (90° - A) \qquad (1)$$
$$\cot A = \tan (90° - A) \qquad (2)$$

Since the complement of $90° - A$ is A, it also follows that

$$\cos (90° - A) = \sin A \qquad (3)$$
$$\cot (90° - A) = \tan A \qquad (4)$$

With reference to the angle B, Fig. 3, BC is the side adjacent and AC the side opposite. Therefore, by formulas **1** and **2**, Art. **4**,

$$\sin B = \frac{b}{c}, \tan B = \frac{b}{a}$$

and therefore, since A is the complement of B,

$$\cos A = \sin B = \frac{b}{c}$$

$$\cot A = \tan B = \frac{b}{a}$$

or, again referring to the angle A, which is the angle under consideration,

$$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} \qquad (5)$$

$$\cot A = \frac{\text{side adjacent}}{\text{side opposite}} \qquad (6)$$

The student will, after some practice, become familiar with these formulas. Whenever he forgets them, he should refer to the definitions of the cosine and cotangent, which will at once enable him to write down the formulas, provided that he remembers those for the sine and the tangent.

6. The Secant and Cosecant.—The **secant** of an angle is the reciprocal of the cosine of the angle; that is, 1 divided by the cosine.

The word *secant* is abbreviated to *sec*. According to the definition, we have

$$\sec A = \frac{1}{\cos A} \qquad (1)$$

It follows that

$$\cos A = \frac{1}{\sec A} \qquad (2)$$

7. The **cosecant** of an angle is the secant of the complement of the angle. The abbreviations *cosec* and *csc* are used for *cosecant*. According to the definition, we have

$$\csc A = \sec (90° - A) \qquad (1)$$

Since A is the complement of $90° - A$, we have also

$$\csc (90° - A) = \sec A \qquad (2)$$

By means of formula **1**, Art. **6**, this relation may be written

$$\csc A = \sec (90° - A) = \frac{1}{\cos (90° - A)}$$

or, since $\cos (90° - A) = \sin A$ (formula **3**, Art. **5**),

$$\csc A = \frac{1}{\sin A} \qquad (3)$$

Therefore, the cosecant of an angle may also be defined as the reciprocal of the sine. Notice very particularly that

secant = reciprocal of co*sine*
co*secant* = reciprocal of *sine*

From formula **3** above follows

$$\sin A = \frac{1}{\csc A} \qquad (4)$$

8. Cofunctions and Complementary Functions. The functions cosine, cotangent, and cosecant are sometimes called **cofunctions** of the angle considered; while the sine, tangent, and secant are called **fundamental functions**. As has been explained, the cofunctions of an angle are the corresponding fundamental functions of the complement of the angle. Thus, the cosine of A is the sine of $90° - A$; the cotangent of A is the tangent of $90° - A$; etc.

A fundamental function and its corresponding cofunction are called **complementary functions** of each other. The sine, for example, is the complementary function of the cosine; and the cosine is the complementary function of the sine.

EXAMPLE 1.—Find: (*a*) the cosine of the angle A, Fig. 5; (*b*) the cotangent; (*c*) the secant; (*d*) the cosecant.

SOLUTION.—(*a*) The cosine of A is equal to the sine of B, or

$$\frac{A\,C}{A\,B} = \frac{9}{12} = .75. \quad \text{Ans.}$$

(*b*) The cotangent of A is equal to the tangent of B, or (see example 2, Art. 4)

$$\frac{A\,C}{B\,C} = \frac{9}{7.9372} = 1.1339. \quad \text{Ans.}$$

(*c*) The secant of A is 1 divided by cos A, or

$$1 \div \frac{9}{12} = \frac{12}{9} = 1.33333. \quad \text{Ans.}$$

(*d*) The cosecant of A is 1 divided by sin A, or

$$1 \div \frac{B\,C}{A\,B} = \frac{A\,B}{B\,C} = \frac{12}{7.9372} = 1.51187. \quad \text{Ans.}$$

EXAMPLE 2.—Find the functions of 30°.

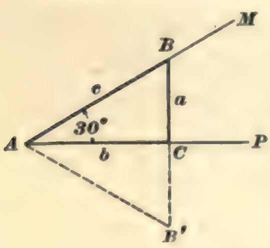


FIG. 6

SOLUTION.—Let the angle $M\,A\,P$, Fig. 6, be 30°. Draw $B\,C$ perpendicular to $A\,P$, produce it to B', making $C\,B' = C\,B$, and draw $A\,B'$. The triangle $B\,A\,B'$ thus formed is isosceles, and angle $C\,A\,B' = C\,A\,B = 30°$. Therefore, $B\,A\,B' = 30° + 30° = 60°$. Also, angle $B = 90° - 30° = 60°$; and angle $B' =$ angle $B = 60°$. As the three angles of $A\,B\,B'$ are equal, the sides are also equal, and $c = B\,B' = 2\,a$. Now, the figure gives,

$$b = \sqrt{c^2 - a^2} = \sqrt{(2\,a)^2 - a^2} = \sqrt{3\,a^2} = a\sqrt{3}$$

Bearing these values in mind, we have

$$\sin 30° = \frac{a}{c} = \frac{a}{2\,a} = \frac{1}{2}. \quad \text{Ans.}$$

$$\tan 30° = \frac{a}{b} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}. \quad \text{Ans.}$$

$$\cos 30° = \frac{b}{c} = \frac{a\sqrt{3}}{2\,a} = \frac{\sqrt{3}}{2}. \quad \text{Ans.}$$

$$\cot 30° = \frac{b}{a} = \frac{a\sqrt{3}}{a} = \sqrt{3}. \quad \text{Ans.}$$

$$\sec 30° = \frac{1}{\cos 30°} = 1 \div \frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}. \quad \text{Ans.}$$

$$\csc 30° = \frac{1}{\sin 30°} = 1 \div \frac{1}{2} = 2. \quad \text{Ans.}$$

NOTE.—It is only in a few cases that the values of the trigonometric functions of an angle can be derived by elementary principles, as above. The general method for determining the functions of any angle is comparatively complicated, and is beyond the scope of this work. The trigonometric functions of any angle can be obtained from a table, as will be presently explained.

EXAMPLES FOR PRACTICE

1. The acute angles of a right triangle are B and C; the side opposite B is 1,200 feet; and that opposite C is 1,500 feet; find the fundamental functions of B, and from them the cofunctions of C.

$$\text{Ans.} \begin{cases} \sin B = .62471, \tan B = .8, \sec B = 1.2806 \\ \cos C = .62471, \cot C = .8, \csc C = 1.2806 \end{cases}$$

2. From example 2, Art. 8, derive the functions of 60° ($= 90° - 30°$).

$$\text{Ans.} \begin{cases} \sin 60° = \frac{\sqrt{3}}{2}, \tan 60° = \sqrt{3}, \cos 60° = \frac{1}{2} \\ \cot 60° = \frac{\sqrt{3}}{3}, \sec 60° = 2, \csc 60° = \frac{2}{3}\sqrt{3} \end{cases}$$

3. Given $\sin A = \frac{2}{3}$ and $\cos B = \frac{4}{5}$, find $\csc A$ and $\sec B$.

$$\text{Ans.} \begin{cases} \csc A = 1.5 \\ \sec B = 1.25 \end{cases}$$

4. Find the trigonometric functions of 45°. (Notice that here the side opposite is equal to the side adjacent. Denote the hypotenuse by c, and express the other two sides in terms of c.)

$$\text{Ans.} \begin{cases} \sin 45° = \cos 45° = \frac{1}{2}\sqrt{2} \\ \tan 45° = \cot 45° = 1 \\ \sec 45° = \csc 45° = \sqrt{2} \end{cases}$$

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9. The Versed Sine and Coversed Sine.—The **versed sine** (*vers*) of an angle is 1 minus the cosine; and the **coversed sine** (*covers*) is 1 minus the sine.

$$\text{vers } A = 1 - \cos A \qquad (1)$$
$$\text{covers } A = 1 - \sin A \qquad (2)$$

These two functions are not much used, except in railroad work.

10. Summing Up.—The foregoing definitions are summed up in the table given below, which contains the expressions for the functions of the angle A, Fig. 7, in terms of the hypotenuse c, the side opposite, a, and the side adjacent, b.

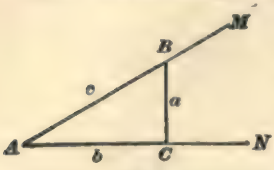

FIG. 7

TABLE I

Function	sin	tan	cos	cot	sec	csc	vers	covers
Value . . .	$\frac{a}{c}$	$\frac{a}{b}$	$\frac{b}{c}$	$\frac{b}{a}$	$\frac{c}{b}$	$\frac{c}{a}$	$1-\frac{b}{c}$	$1-\frac{a}{c}$

The ratios $\frac{c}{b}$ and $\frac{c}{a}$ for the secant and cosecant are obtained from the formulas $\sec A = 1 \div \cos A = 1 \div \frac{b}{c} = \frac{c}{b}$, $\csc A = 1 \div \sin A = 1 \div \frac{a}{c} = \frac{c}{a}$.

11. Representation of the Trigonometric Functions by Lines.—Let A, Fig. 8, be any angle. From its

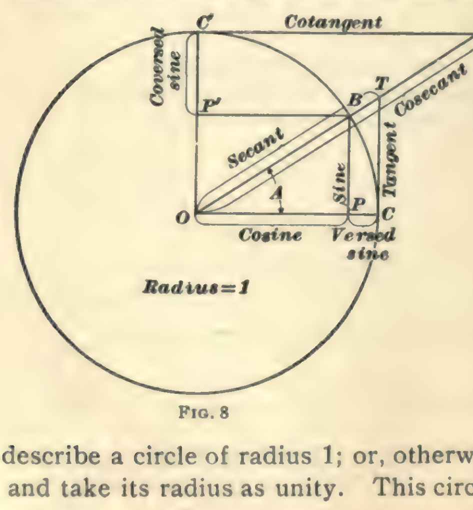

FIG. 8

vertex O, describe a circle of radius 1; or, otherwise, describe any circle and take its radius as unity. This circle intersects the sides of the angle at B and C. Draw the tangent CT, meeting OB produced at T; the radius OC' perpendicular to OC; the lines BP and BP' perpendicular to OC and OC', respectively; and the tangent $C'T'$, meeting OB produced at T'.

Since the angle A is measured by the arc CB, the trigonometric functions of the angle are said to be likewise the trigonometric functions of the arc. It is, for instance, immaterial whether we say that 1 is the tangent of an angle of 45° or of an arc of 45°.

In the figure constructed as just explained, the trigonometric functions of the angle A, or of the arc CB, may be represented by lines, as marked. For, in the right triangle OPB, in which BP, OP, and OB are, respectively, the side opposite, the side adjacent, and the hypotenuse, we have

$$\sin A = \frac{BP}{OB}, \cos A = \frac{OP}{OB}$$

or, since $OB = 1$,

$$\sin A = \frac{BP}{1} = BP, \cos A = \frac{OP}{1} = OP$$

In the triangle OCT, in which CT and OC are, respectively, the side opposite and the side adjacent, and OT is the hypotenuse,

$$\tan A = \frac{CT}{OC} = \frac{CT}{1} = CT$$

$$\sec A = \frac{OT}{OC} = \frac{OT}{1} = OT$$

By the same reasoning, it can be shown that $C'T'$ and OT' are, respectively, the tangent and the secant of the angle $C'OT'$, or the cotangent and the cosecant of A, since $C'OT'$ is the complement of A.

Let the student verify that, according to the definitions of the versed sine and coversed sine, these functions are represented by PC and $P'C'$, respectively.

RELATIONS AMONG THE FUNCTIONS OF AN ANGLE

12. Method of Marking a Triangle.—The triangle *A B C*, Fig. 7, has the angles marked by the capital letters *A*, *B*, and *C* and the sides opposite these angles marked by the small letters *a*, *b*, and *c*, respectively. This method of marking a triangle is very useful and convenient, as it points out at once the relative position of the sides and the angles. In a right triangle, the right angle is usually designated by *C*. In the figures that follow, when only the angles are marked, the sides opposite are taken as marked by the small letters corresponding to the capital letters that mark the angles.

13. Relation Between Tangent and Cotangent. In Fig. 7,

$$\tan A = \frac{a}{b}, \cot A = \frac{b}{a}$$

Multiplying these equations together gives

$$\tan A \times \cot A = \frac{a}{b} \times \frac{b}{a} = 1$$

whence,

$$\cot A = \frac{1}{\tan A}$$

$$\tan A = \frac{1}{\cot A}$$

That is, the tangent and cotangent are each the reciprocal of the other. This is a very important relation, and should be committed to memory, together with those given in the two articles following.

14. Tangent and Cotangent in Terms of Sine and Cosine.—In Fig. 7,

$$\sin A = \frac{a}{c}, \cos A = \frac{b}{c}$$

Dividing these equations member by member gives

$$\frac{\sin A}{\cos A} = \frac{a}{c} \div \frac{b}{c} = \frac{a}{b}$$

that is, since $\frac{a}{b} = \tan A$,

$$\tan A = \frac{\sin A}{\cos A} \qquad (1)$$

Also, because the cotangent is the reciprocal of the tangent,

$$\cot A = \frac{\cos A}{\sin A} \qquad (2)$$

15. Relations Between the Squares of Certain Functions.—A power of a trigonometric function is indicated by writing the exponent immediately after the abbreviation used for the function. Thus, the square of the sine of A, or of $\sin A$, is written $\sin^2 A$, and read *sine square A*. Similarly, the cube of $\tan A$ is written $\tan^3 A$, and read *tangent cube A*, etc.

In the right triangle ABC, Fig. 7, we have

$$a^2 + b^2 = c^2$$

Dividing both members of this equality by c^2 gives

$$\frac{a^2}{c^2} + \frac{b^2}{c_2} = 1$$

that is, $$\sin^2 A + \cos^2 A = 1 \qquad (1)$$

Again, dividing both members of the equation $c^2 = a^2 + b^2$ by b^2,

$$\frac{c^2}{b^2} = \frac{a^2}{b^2} + 1 = 1 + \frac{a^2}{b^2}$$

that is, $$\sec^2 A = 1 + \tan^2 A \qquad (2)$$

Similarly, if both members of the equation $c^2 = a^2 + b^2$ are divided by a^2,

$$\frac{c^2}{a^2} = 1 + \frac{b^2}{a^2}$$

that is, $$\csc^2 A = 1 + \cot^2 A \qquad (3)$$

16. To Express Any Function in Terms of Any Other Function.—In the triangle ABC, Fig. 7, we have

$$a^2 + b^2 = c^2 \qquad (1)$$

Dividing both members of this equation by c^2 gives

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1 \qquad (2)$$

From these two equations, any of the six ratios $\frac{a}{b}, \frac{a}{c}, \frac{b}{a}, \frac{b}{c},$

$\frac{c}{a}$, $\frac{c}{b}$ can be found when one of them is given. If, for instance, $\frac{a}{c}$ is given, $\frac{c}{a}$ is obtained by dividing 1 by $\frac{a}{c}$; $\frac{b}{c}$, by solving equation (2) for $\frac{b^2}{c^2}$ and taking the square root; $\frac{c}{b}$, by taking the reciprocal of the value just found for $\frac{b}{c}$. To find $\frac{a}{b}$, divide both members of equation (1) by b^2, which gives

$$\frac{a^2}{b^2} + 1 = \frac{c^2}{b^2}$$

whence, multiplying through by $\frac{b^2}{c^2}$,

$$\frac{b^2}{c^2}\left(\frac{a^2}{b^2} + 1\right) = \frac{c^2}{b^2} \times \frac{b^2}{c^2} = 1$$

and hence, dividing through by $\frac{a^2}{b^2} + 1$,

$$\frac{b^2}{c^2} = \frac{1}{\frac{a^2}{b^2} + 1}$$

Substituting this value of $\frac{b^2}{c^2}$ in equation (2) and solving for $\frac{a}{b}$, the latter ratio is obtained in terms of $\frac{a}{c}$.

Table II gives the relation between any two functions of any angle A.

TABLE II

RELATIONS BETWEEN THE FUNCTIONS OF AN ANGLE

In Terms of	$\sin A$	$\cos A$	$\tan A$	$\cot A$	$\sec A$	$\csc A$
$\sin A =$	$\sin A$	$\sqrt{1-\cos^2 A}$	$\frac{\tan A}{\sqrt{1+\tan^2 A}}$	$\frac{1}{\sqrt{1+\cot^2 A}}$	$\frac{\sqrt{\sec^2 A-1}}{\sec A}$	$\frac{1}{\csc A}$
$\cos A =$	$\sqrt{1-\sin^2 A}$	$\cos A$	$\frac{1}{\sqrt{1+\tan^2 A}}$	$\frac{\cot A}{\sqrt{1+\cot^2 A}}$	$\frac{1}{\sec A}$	$\frac{\sqrt{\csc^2 A-1}}{\csc A}$
$\tan A =$	$\frac{\sin A}{\sqrt{1-\sin^2 A}}$	$\frac{\sqrt{1-\cos^2 A}}{\cos A}$	$\tan A$	$\frac{1}{\cot A}$	$\sqrt{\sec^2 A-1}$	$\frac{1}{\sqrt{\csc^2 A-1}}$
$\cot A =$	$\frac{\sqrt{1-\sin^2 A}}{\sin A}$	$\frac{\cos A}{\sqrt{1-\cos^2 A}}$	$\frac{1}{\tan A}$	$\cot A$	$\frac{1}{\sqrt{\sec^2 A-1}}$	$\sqrt{\csc^2 A-1}$
$\sec A =$	$\frac{1}{\sqrt{1-\sin^2 A}}$	$\frac{1}{\cos A}$	$\sqrt{1+\tan^2 A}$	$\frac{\sqrt{1+\cot^2 A}}{\cot A}$	$\sec A$	$\frac{\csc A}{\sqrt{\csc^2 A-1}}$
$\csc A =$	$\frac{1}{\sin A}$	$\frac{1}{\sqrt{1-\cos^2 A}}$	$\frac{\sqrt{1+\tan^2 A}}{\tan A}$	$\sqrt{1+\cot^2 A}$	$\frac{\sec A}{\sqrt{\sec^2 A-1}}$	$\csc A$

TRIGONOMETRIC TABLES

TABLES OF NATURAL FUNCTIONS

17. To facilitate calculations, tables of the trigonometric functions are used. The tables give values for the sines, cosines, tangents, and cotangents of angles from 0° to 90°. The values of the secant and cosecant are not generally given in tables; they are obtained by dividing 1 by the cosine and the sine, respectively, according to formula **1**, Art. **6**, and formula **3**, Art. **7**.

There are two kinds of trigonometric tables; namely, the table of *natural functions* and the table of *logarithmic functions*. The table of natural functions gives the actual values of the functions, while the table of logarithmic functions gives the logarithms of the functions. It may be remarked that, except in making a table, the values of the functions are never calculated directly because the process is so long and laborious that it would require considerable time to calculate even the value of one function of an angle; nor is there a simple method of calculating the angle corresponding to a given function.

NOTE.—In all that follows, the number of seconds by which an angle exceeds a whole number of degrees and minutes will be referred to as *the odd seconds*, or *the number of odd seconds*, or simply *the number of seconds* in the angle; while the expression *total number of seconds* will be applied to the number obtained by reducing the degrees and minutes to seconds, and adding the odd seconds. Thus, the odd seconds, or, for shortness, the seconds in 34° 36′ 16″ are 16; while the total number of seconds is the number of seconds in 34°, plus the number of seconds in 36′, plus 16; that is, $34 \times 60 \times 60 + (36 \times 60) + 16 = 124,576$. A similar notation will be used with regard to minutes. The explanations that follow refer to the *Trigonometric Tables* used with this Course.

18. To Find the Natural Functions of an Angle Less Than 45° and Containing No Odd Seconds.—The required function is found in the double column marked at the top with the given number of degrees, in the subdivision of that column headed by the name of the given function, and horizontally opposite the number in the left-hand column (marked ′) that expresses the number of odd minutes in the angle. When the function considered is a sine or a cosine, it is taken from the table headed Natural Sines and Cosines; when a tangent or cotangent, from the table headed Natural Tangents and Cotangents.

EXAMPLE.—Find the natural functions of an angle of 37° 23′.

SOLUTION.—On page 30 of the table headed Natural Sines and Cosines, the double column headed 37° is found. Looking in the left-hand minute column for 23 (number of odd minutes in the given angle), and glancing along the horizontal row to the right of 23, the number .60714 is found in the single column marked Sine under 37°; and the number .79459 is found in the column marked Cosine. Therefore,

$$\sin 37° 23' = .60714. \text{ Ans.}$$
$$\cos 37° 23' = .79459. \text{ Ans.}$$

The tangent and cotangent are taken in a similar manner from the table headed Natural Tangents and Cotangents, page 39. The results are:

$$\tan 37° 23' = .76410. \text{ Ans.}$$
$$\cot 37° 23' = 1.30873. \text{ Ans.}$$

EXAMPLES FOR PRACTICE

Verify the following values:

(*a*) $\sin 39° 55' = .64167$; $\cos 39° 55' = .76698$; $\tan 39° 55' = .83662$; $\cot 39° 55' = 1.19528$.

(*b*) $\tan 16° 32' = .29685$; $\cos 16° 32' = .95865$; $\sec 16° 32' = 1.04313$; $\csc 16° 32' = 3.51407$.

(*c*) $\cot 43° 2' = 1.07112$; $\csc 43° 2' = 1.46537$; $\tan 43° 2' = .93360$; $\cos 43° 2' = .73096$.

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19. To Find the Natural Functions of an Angle Greater Than 45° and Containing No Odd Seconds. The required function is found in the double column marked at the bottom with the given number of degrees, in the subdivision of that column having at the bottom the name of the given function, and horizontally opposite the number in the right-hand column (marked ′) that expresses the odd minutes in the angle. It will be observed that the number of degrees at the bottom of the pages decrease as the pages increase, and that the number of minutes in the right-hand column increase from bottom to top.

EXAMPLE.—Find the functions of 53° 43′.

SOLUTION.—The double column marked 53° at the bottom is found on page 30 of Natural Sines and Cosines. Looking along the horizontal row determined by the number 43 in the right-hand minute column, the number .80610 is found in the single column marked Sine at the bottom, and the number .59178 in the single column marked Cosine at the bottom, these two columns forming the double column marked 53° at the bottom. Therefore,

sin 53° 43′ = .80610. Ans.
cos 53° 43′ = .59178. Ans.

The tangent and cotangent are similarly taken from page 39 of Natural Tangents and Cotangents. The results are:

tan 53° 43′ = 1.36217. Ans.
cot 53° 43′ = .73413. Ans.

EXAMPLES FOR PRACTICE

Verify the following values:

(*a*) sin 67° 45′ = .92554; cos 67° 45′ = .37865; tan 67° 45′ = 2.44433; cot 67° 45′ = .40911.

(*b*) cot 74° 3′ = .28580; csc 74° 3′ = 1.04004; sin 74° 3′ = .96150.

(*c*) cos 48° 9′ = .66718; cot 48° 9′ = .89567; csc 48° 9′ = 1.34248.

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20. To Find the Natural Functions of an Angle Containing Odd Seconds.—The method of solving this problem by means of the table is founded on the following principle, which applies within the limits of approximation with which the table is constructed:

If several angles are taken within an interval not greater than 1′; that is, so that the difference between the greatest and the smallest shall not exceed 1′, the ratio of the difference between any two of these angles to the difference between any other two is the same as the ratio obtained by dividing the difference between the values of any trigonometric function for the first pair of angles, by the difference between the values of the same function for the second pair of angles. For instance, if the angles 43° 46′ 32″, 43° 46′ 34″, 43° 46′ 40″, and 43° 47′ are taken between 43° 46′ and 43° 47′, then

$$\frac{43° 47' - 43° 46' 40''}{43° 46' 34'' - 43° 46' 32''} = \frac{\sin 43° 47' - \sin 43° 46' 40''}{\sin 43° 46' 34'' - \sin 43° 46' 32''}$$

In general, if A, B, C, D are any angles within an interval of 1′, then

$$\frac{A-B}{C-D} = \frac{\sin A - \sin B}{\sin C - \sin D} = \frac{\cos A - \cos B}{\cos C - \cos D}$$
$$= \frac{\tan A - \tan B}{\tan C - \tan D} = \frac{\cot A - \cot B}{\cot C - \cot D}$$

Similarly,

$$\frac{A-B}{B-C} = \frac{\sin A - \sin B}{\sin B - \sin C} = \frac{\cos A - \cos B}{\cos B - \cos C}, \text{ etc.}$$

Let A be the number of degrees and minutes in any angle, and s the number of odd seconds. Then the angle, which will be represented by $A + s''$, lies between A and $A + 1'$ or between A and $A + 60''$. For instance, if the angle is 25° 15′ 37″, it lies between 25° 15′, which is represented by A, and 25° 16′, which is 25° 15′ + 1′, or $A + 1'$, or $A + 60''$. In this case s represents 37″. From the principle stated above we have,

$$\frac{(A + 60'') - A}{(A + s'') - A} = \frac{\sin (A + 60'') - \sin A}{\sin (A + s'') - \sin A}$$

or,
$$\frac{60}{s} = \frac{\sin (A + 1') - \sin A}{\sin (A + s'') - \sin A}$$

whence, solving this equation for $\sin (A + s'')$,

$$\sin (A + s'') = \sin A + [\sin (A + 1') - \sin A] \frac{s}{60} \qquad (1)$$

Similarly,

$$\tan (A + s'') = \tan A + [\tan (A + 1') - \tan A] \frac{s}{60} \qquad (2)$$

For the cosine, we have

$$\cos (A + s'') = \cos A + [\cos (A + 1') - \cos A] \frac{s}{60}$$

but, since the cosine of an angle decreases as the angle increases, $\cos A$ is greater than $\cos (A + 1')$, and therefore it is better to write the formula thus,

$$\cos (A + s'') = \cos A - [\cos A - \cos (A + 1')] \frac{s}{60} \qquad (3)$$

Similarly,

$$\cot (A + s'') = \cot A - [\cot A - \cot (A + 1')] \frac{s}{60} \qquad (4)$$

The functions of A and $A + 1'$ can be readily taken from the table, as explained in the preceding articles, and from them the functions of $A + s''$ are determined by the formulas just given, or by the following rule, which states in words what the formulas express in symbols:

Rule.—*Find, in the table, the sine, cosine, tangent, or cotangent corresponding to the degrees and minutes in the angle.*

For the seconds, find the difference between this value and the value of the sine, cosine, tangent, or cotangent of an angle 1 minute greater; multiply this difference by a fraction whose numerator is the number of seconds in the given angle and whose denominator is 60.

If the sine or tangent is sought, add this correction to the value first found; if the cosine or cotangent is sought, subtract the correction.

EXAMPLE.—Find: (*a*) the sine of 56° 43′ 17″; (*b*) the cosine; (*c*) the tangent; and (*d*) the cotangent.

SOLUTION.—(*a*) Here $A = 56° 43'$, $s = 17$, $A + 1' = 56° 44'$.

$$\sin(A + 1') = \sin 56° 44' = .83613$$
$$\sin A = \sin 56° 43' = .83597$$
$$\text{Difference} = .00016$$
$$\times \frac{17}{60}$$
$$.00005, \text{ nearly}$$

Adding this product to sin A, we have

$$\sin 56° 43' 17'' = .83597 + .00005 = .83602. \quad \text{Ans.}$$

(*b*)
$$\cos A = \cos 56° 43' = .54878$$
$$\cos(A + 1') = \cos 56° 44' = .54854$$
$$\text{Difference} = .00024$$
$$\times \frac{17}{60}$$
$$.00007, \text{ nearly}$$

Subtracting this product from cos A, we have

$$\cos 56° 43' 17'' = .54878 - .00007 = .54871. \quad \text{Ans.}$$

(*c*)
$$\tan(A + 1') = \tan 56° 44' = 1.52429$$
$$\tan A = \tan 56° 43' = 1.52332$$
$$\text{Difference} = .00097$$
$$\times \frac{17}{60}$$
$$.00027, \text{ nearly}$$

Adding this product to tan A, we have

$$\tan 56° 43' 17'' = 1.52332 + .00027 = 1.52359. \quad \textbf{Ans.}$$

(*d*)

$$\begin{aligned} \cot A &= \cot 56° 43' = .65646 \\ \cot (A + 1') &= \cot 56° 44' = .65604 \\ \text{Difference} &= .00042 \\ &\times \tfrac{17}{60} \\ &\overline{.00012}, \text{ nearly} \end{aligned}$$

Subtracting this product from cot A, we have

$$\cot 56° 43' 17'' = .65646 - .00012 = .65634. \quad \textbf{Ans.}$$

EXAMPLES FOR PRACTICE

Verify the following values:

(*a*) sin 18° 54′ 45″ = .32412; tan 18° 54′ 45″ = .34262.

(*b*) cos 34° 17′ 18″ = .82621; cot 34° 17′ 18″ = 1.46659.

(*c*) sin 72° 26′ 20″ = .95340; cot 72° 26′ 20″ = .31647.

(*d*) cos 65° 6′ 9″ = .42100; tan 65° 6′ 9″ = 2.15457.

(*e*) sin 80° 0′ 3″ = .98481; cot 80° 0′ 3″ = .17631.

(*f*) tan 14° 14′ 14″ = .25373; cos 14° 14′ 14″ = .96928.

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21. To Find the Angle Corresponding to a Given Function, When the Function Is in the Table.—This case does not present any difficulty. Having found the given function in the table, the degrees in the angle are taken from the top or the bottom, and the minutes from the left- or the right-hand column, according as the name of the function is at the top or at the bottom of the page.

EXAMPLE 1.—The sine of an angle is .47486; what is the angle?

SOLUTION.—Glancing down the columns marked Sine in the table of Natural Sines and Cosines, .47486 is found (on page 28) in the column headed 28°. The number of minutes, 21, is found in the left-hand minute column, horizontally opposite .47486. Therefore, .47486 = sin 28° 21′. Ans.

EXAMPLE 2.—Find the angle whose cosine is .27032.

SOLUTION.—Looking in the columns marked Cosine at the top of the page, the given cosine is not found; hence, the angle is greater than 45°. Consequently, looking in the columns marked Cosine at the bottom of the page, .27032 is found (on page 26) in the double column marked 74° at the bottom, and in the horizontal row beginning with 19 in the right-hand minute column. Therefore, the angle whose cosine is .27032 is 74° 19′; or, .27032 = cos 74° 19′. Ans.

EXAMPLE 3.—Find the angle whose tangent is 2.15925.

SOLUTION.—On searching the table of Natural Tangents, the given tangent is found to belong to an angle greater than 45°, so that it must be looked for in the column marked Tangent at the bottom. It is found in the column having 65° at the bottom and opposite 9′ in the right-hand minute column. Therefore, 2.15925 = tan 65° 9′. Ans.

EXAMPLE 4.—Find the angle whose cotangent is .43412.

SOLUTION.—From the table of Natural Cotangents, it is found that this value is less than the cotangent of 45°, so it must be found in the column marked Cotangent at the bottom. Looking there, it is found in the column having 66° at the bottom, and opposite 32′, in the right-hand column of minutes. Therefore, the angle whose cotangent is .43412 is 66° 32′, or .43412 = cot 66° 32′. Ans.

EXAMPLES FOR PRACTICE

1. Find the angle whose sine is .47486. Ans. 28° 21′
2. Find the angle whose cosine is .74353. Ans. 41° 58′
3. Find the angle whose tangent is 2.06247. Ans. 64° 8′
4. Find the angle whose cotangent is 1.20665. Ans. 39° 39′
5. Find the angle whose sine is .76903. Ans. 50° 16′
6. Find the angle whose tangent is 9.93101. Ans. 84° 15′

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22. To Find the Angle Corresponding to a Given Function, When the Function Is Not in the Table. Since the table includes the functions of all angles containing no odd seconds, a function not found in the table must correspond to an angle having odd seconds. Let the odd seconds that are to be determined be denoted by s, and the degrees and minutes by A, as in Art. **20.** Now, two consecutive functions including the given function can always be found in the table; that is, two consecutive functions of which one is greater and the other less than the given function. The required angle must, therefore, lie between the two angles corresponding to these two consecutive functions, and its number of degrees and minutes, A, is the number of degrees and minutes in the smaller of the two angles. The larger angle is $A + 1'$, or $A + 60''$, while the required angle is $A + s''$. Having determined A, it only remains to determine the number of odd seconds, or s. This is done by means of the following formulas, obtained by solving for s the formulas found in Art. **20**.

If the given function is a sine or tangent,

$$s = \frac{\sin(A + s'') - \sin A}{\sin(A + 1') - \sin A} \times 60 \qquad (1)$$

$$s = \frac{\tan(A + s'') - \tan A}{\tan(A + 1') - \tan A} \times 60 \qquad (2)$$

If the given function is a cosine or cotangent,

$$s = \frac{\cos A - \cos(A + s'')}{\cos A - \cos(A + 1')} \times 60 \qquad (3)$$

$$s = \frac{\cot A - \cot(A + s'')}{\cot A - \cot(A + 1')} \times 60 \qquad (4)$$

Observe that, although $A + s''$ is not known, its sine, cosine, etc., as the case may be, is known, or given. Thus, if the problem is to find the angle whose cotangent is .97888, we have $\cot(A + s'') = .97888$.

The foregoing formulas lead to the following general rule for finding the angle corresponding to a given function:

Rule.—*Find the difference of the two numbers in the table between which the given function lies, and use that difference as the denominator of a fraction.*

Find the difference between the function belonging to the smaller angle and the given function, and use that difference as the numerator of the fraction mentioned above. Multiply this fraction by 60. The result will be the number of seconds to be added to the smaller angle in order to obtain the required angle.

EXAMPLE 1.—Find the angle whose sine is .57698.

SOLUTION.—Looking in the table of Natural Sines, in the columns marked Sine, it is found that the given sine lies between .57691 (= sin 35° 14′) and .57715(= sin 35° 15′). The difference between them is $.57715 - .57691 = .00024$. The difference between the sine of the smaller angle, or .57691, and the given sine, or .57698, is $.57698 - .57691 = .00007$. Then, $\frac{.00007}{.00024} \times 60 = \frac{7}{24} \times 60 = 18''$, nearly, and the required angle is 35° 14′ 18″; or, .57698 = sin 35° 14′ 18″. Ans.

NOTE.—In practice, only the significant figures of the differences forming the terms of the function are used, the decimal point being dispensed with. Thus, $.57715 - .57691 = 24$, it being understood that this means 24 units of the fifth decimal order, or .00024.

EXAMPLE 2.—Find the angle whose cosine is .27052.

SOLUTION.—Looking in the table of Cosines, the given cosine is found to belong to a greater angle than 45° and therefore it must be looked for in the columns marked Cosine at the bottom of the page. It is found between the numbers .27060(= cos 74° 18′) and .27032(= cos 74° 19′). The difference between the two numbers is .27060 − .27032 = 28 units of the fifth order. The cosine of the smaller angle, or 74° 18′, is .27060, and the difference between this and the given cosine is .27060 − .27052 = 8 units of the fifth order. Hence, $\frac{8}{28} \times 60 = 17''$; and, therefore, .27052 = cos 74° 18′ 17″. Ans.

EXAMPLE 3.—Find the angle whose tangent is 2.15841.

SOLUTION.— 2.15841 falls between 2.15760(= tan 65° 08′) and 2.15925 (= tan 65° 9′). The difference between these numbers is 2.15925 − 2.15760 = 165 units of the fifth order; 2.15841 − 2.15760 = 81 units of the fifth order. Hence, $\frac{81}{165} \times 60 = 30''$, nearly, and therefore 2.15841 = tan 65° 8′ 30″. Ans.

EXAMPLE 4.—Find the angle whose cotangent is 1.26342.

SOLUTION.— 1.26342 falls between 1.26395(= cot 38° 21′) and 1.26319 (= cot 38° 22′). The difference between these numbers is 1.26395 − 1.26319 = .00076. Also, 1.26395 − 1.26342 = .00053. $\frac{53}{76} \times 60 = 42''$, and therefore 1.26342 = cot 38° 21′ 42″. Ans.

EXAMPLES FOR PRACTICE

1. Find: (*a*) the sine of 48° 17′; (*b*) the cosine; (*c*) the tangent.

Ans. (*a*) .74644; (*b*) .66545; (*c*) 1.12172

2. Find: (*a*) the sine of 13° 11′ 6″; (*b*) the cosine; (*c*) the tangent.

Ans. (*a*) .22810; (*b*) .97364; (*c*) .23427

3. Find: (*a*) the sine of 72° 0′ 2″; (*b*) the cosine; (*c*) the tangent.

Ans. (*a*) .95106; (*b*) .30901; (*c*) 3.07778

4. (*a*) Of what angle is .26489 the sine? (*b*) Of what angle is it the cosine?

Ans. (*a*) 15° 21′ 37″; (*b*) 74° 38′ 23″

5. (*a*) Of what angle is .688 the sine? (*b*) Of what angle is it the cosine? (*c*) Of what angle is it the tangent?

Ans. (*a*) 43° 28′ 20″; (*b*) 46° 31′ 40″; (*c*) 34° 31′ 40″

TABLE OF LOGARITHMIC FUNCTIONS

23. The student is already familiar with the use of the table of logarithms of numbers. As stated in Art. **17**, a **table of logarithmic functions** is a table containing the logarithms of the natural functions, these logarithms being, for convenience, called **logarithmic functions.** Thus, the logarithm of the sine of an angle is referred to as the **logarithmic sine** of the angle.

The connection between the tables can be seen from the following:

From table of natural functions, cot 44° . . . . . = 1.03553
From table of logarithms, log 1.03553 . . . . . = .01516
From table of logarithmic functions, log cot 44° = .01516

Few tables give the logarithmic secants and cosecants. These logarithmic functions may be obtained from the relations,

$$\sec A = \frac{1}{\cos A}, \csc A = \frac{1}{\sin A}$$

which give,

$$\log \sec A = -\log \cos A, \log \csc A = -\log \sin A$$

That is, instead of adding the logarithmic secant or cosecant, the logarithmic cosine or sine, respectively, may be subtracted. Likewise, instead of subtracting the logarithmic secant, the logarithmic cosine may be added, and instead of subtracting the logarithmic cosecant, the logarithmic sine may be added.

24. Description of the Table.—The table of logarithmic functions contains for every minute the logarithms, to five decimal places, of the trigonometric sines, cosines, tangents, and cotangents of angles from 0° to 90°. From 0° to 45°, the degrees are placed at the top of the page and the minutes in the column headed ′ on the left. From 45° to 90°, the degrees are at the bottom of the page, the minutes in the last whole column at the right, and the name of the trigonometric function is placed at the bottom of the column.

This arrangement is similar to that in the table of natural functions. It will be observed that the numbers of degrees at the top of the pages increase in the order of the pages from 0° to 44°, while those at the bottom decrease from 89° to 44°.

The general description of the table will be better understood by referring to one of its pages. Take, for instance, the page marked 11° at the top and 78° at the bottom. The first column on the left (marked ′) contains the natural numbers from 1 to 60. These numbers represent minutes. Horizontally opposite to these numbers, and in the columns marked at the top log sin, log tan, etc., are printed the logarithmic functions, each function being in the same horizontal line as the number of minutes by which the corresponding angle exceeds 11°. Thus, the logarithmic tangent of 11° 39′, which is $\bar{1}.31425$, is found in the column marked log tan at the top, and in the same horizontal line as the number 39 in the left-hand column. Similarly, the number $\bar{1}.99072$, being in the column marked at the top log cos, and in the same horizontal line as 48 in the left-hand column, is the logarithmic cosine of 11° 48′. In some tables, several mantissas are printed under and to the right of the same characteristic, and are understood to belong with that characteristic. Thus, in the logarithm just considered, only the mantissa .99072 is printed, the characteristic being the same as the first one found above that mantissa.

The last column but one (marked ′ at the bottom) contains the natural numbers from 1 to 60, increasing from bottom to top. It will be observed that any angle determined by the number of degrees at the bottom (78 in this case) and any number of minutes in the right-hand minute column, is the complement of the angle determined by the number of degrees at the top (11 in this case) and the number of minutes in the left-hand minute column, horizontally opposite the number of minutes in the right-hand minute column. Thus, the number 18 in the right-hand minute column is horizontally opposite the number 42 in the left-hand column, and we have, $78° 18' + 11° 42' = 90°$. Therefore, since the fundamental functions of an angle are equal to the cofunctions of its complement,

$$\sin 11° 42' = \cos 78° 18'$$
$$\cot 11° 42' = \tan 78° 18', \text{ etc.}$$
and $$\log \sin 11° 42' = \log \cos 78° 18', \text{ etc.}$$

For this reason, the notation log tan is written at the bottom of the column headed log cot, to indicate that the logarithms in this column are the logarithmic tangents of angles whose number of degrees is the number (78 in this case) at the bottom of the page, and whose number of minutes is opposite those logarithms in the right-hand minute column. Similarly, the columns marked log sin, log tan, and log cos at the top are marked, respectively, log cos, log cot, and log sin at the bottom.

25. After the column marked log sin there is a column marked d. This column contains the differences, expressed in units of the fifth decimal order, between the consecutive logarithmic sines given in the sine column. Thus, referring to the page headed 11°, the first number in the d-column following the sine column is 65; it will be observed that this number is opposite the space between the logarithmic sines $\bar{1}.28125$ and $\bar{1}.28060$, and is the difference, in units of the fifth decimal order, or expressed in hundred thousandths, between these two logarithmic sines. These differences are called **tabular differences.** Similar differences are printed in the column marked d after the cosine column, and in the column marked c. d. between the tangent and the cotangent column. The notation c. d. means *common difference*, as the differences between the successive logarithmic tangents are the same as those between the corresponding cotangents. although obtained by reversing the order in which the functions are subtracted; that is to say, $\log \tan A - \log \tan B = \log \cot B - \log \cot A$.

The tabular differences for the cosines are not given in the first ten pages, both for want of space and because they are so small that they can be readily determined by mental subtraction.

The use of the tabular differences, the use and contents of the column marked p. p. in all pages but the first three, and the peculiarities and applications of these first three pages of the table will be explained further on.

26. To Find the Logarithmic Functions of an Angle Having No Odd Seconds.

Rule.—*For an angle less than 45°, look for the degrees at the top of the page and for the minutes in the column (marked ′) at the left of the page on which the number of degrees is found. Then look across the page along the horizontal row containing the given number of minutes, into the column headed by the name of the function whose logarithm is required. The desired logarithm is found in this row and column.*

For an angle between 45° and 90°, find the degrees at the bottom of the page and the minutes in the column (marked ′) at the right of the page. Then look across the page, along the horizontal row containing the given number of minutes, into the column marked at the bottom with the name of the function whose logarithm is to be found. The row and column thus determined contain the desired logarithm.

Example 1.—Find the logarithmic sine and the logarithmic tangent of 15° 24′.

Solution.—On the page marked 15° at the top, in the column headed log sin, and in the same horizontal row with 24, the number $\bar{1}.42416$ is found; and in the column headed log tan, the number $\bar{1}.44004$ is found. Hence,

$$\log \sin 15° 24' = \bar{1}.42416. \quad \text{Ans.}$$
$$\log \tan 15° 24' = \bar{1}.44004. \quad \text{Ans.}$$

Example 2.—Find the logarithmic tangent and cosine of 73° 10′.

Solution.—As 73 is greater than 45, it is found at the bottom of the page. Looking for the number of minutes (10′) in the right-hand minute column, and following the horizontal row determined by this number into the column marked log tan at the bottom, the number .51920 is found. Likewise, the number $\bar{1}.46178$ is found in the column marked log cos at the bottom, and horizontally opposite the number 10 in the right-hand minute column. Therefore,

$$\log \tan 73° 10' = .51920. \quad \text{Ans.}$$
$$\log \cos 73° 10' = \bar{1}.46178. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find: (*a*) the logarithmic cosine of 36° 58′; (*b*) the logarithmic tangent.
Ans. $\begin{cases} (a) & \bar{1}.90254 \\ (b) & \bar{1}.87659 \end{cases}$

2. Find: (*a*) the logarithmic tangent of 23° 39′; (*b*) the logarithmic cotangent.
Ans. $\begin{cases} (a) & \bar{1}.64140 \\ (b) & .35860 \end{cases}$

3. Find: (*a*) the logarithmic sine of 79° 45′; (*b*) the logarithmic cosine.
Ans. $\begin{cases} (a) & \bar{1}.99301 \\ (b) & \bar{1}.25028 \end{cases}$

4. Find: (*a*) the logarithmic tangent of 46° 59′; (*b*) the logarithmic cotangent.
Ans. $\begin{cases} (a) & .03009 \\ (b) & \bar{1}.96991 \end{cases}$

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27. To Find the Logarithmic Functions of an Angle Containing an Odd Number of Seconds.—Let the number of degrees and minutes in an angle any of whose logarithmic functions is required be denoted by A, and the number of odd seconds by s. Thus, if the angle is 37° 43′ 19″, A will equal 37° 43′, and s will equal 19″; also, $A + 1'$, or $A + 60''$, will equal 37° 43′ + 1′, or 37° 44′. (See Art. **20.**) Since the table gives the logarithmic functions of any angle containing no odd seconds, the logarithmic functions of A and $A + 1'$ may be readily found, as explained in the last article. Let these logarithmic functions be denoted by l and l', respectively, and the required logarithmic function by L. In the general theory of logarithms, treated in advanced works on mathematics, it is shown that if two consecutive angles (as 37° 43′ and 37° 44′) are taken from the table, the difference between any logarithmic function of the greater and the same logarithmic function of the smaller angle is to the difference between the same logarithmic function of any intermediate angle (as 37° 43′ 19″) and the same function of the smaller angle, as the difference between the greater and the smaller angle is to the difference between the intermediate and the smaller angle. If the notation $F(A)$, read *function of A*, is employed to denote any logarithmic function of an angle A, we have, writing $A + 60''$ instead of $A + 1'$,

$$\frac{F(A+60'')-F(A)}{F(A+s)-F(A)}=\frac{(A+60'')-A}{(A+s)-A}=\frac{60}{s}$$

that is,
$$\frac{l'-l}{L-l}=\frac{60}{s}$$

whence,
$$L-l=(l'-l)\frac{s}{60}$$

and
$$L=l+(l'-l)\frac{s}{60}$$

The difference between l' and l, being the difference between two consecutive logarithmic functions, may be taken from the column of tabular differences in the table. (See Art. **25.**) Denoting the tabular difference $l'-l$ by D, the preceding equation becomes

$$L=l+D\times\frac{s}{60}$$

It should be observed that, since the sine and the tangent increase with the angle, while the cosine and cotangent decrease as the angle increases, $l'-l$ is positive or negative according as the functions considered are fundamental functions (sine, tangent) or cofunctions (cosine, cotangent). In the latter case, D in the formula should be treated as negative; that is, the product $D\times\frac{s}{60}$ should be subtracted from l

It should also be borne in mind that the tabular difference D is expressed in units of the fifth order of decimals, or hundred thousandths. Thus, if the number of seconds s is 15, and the tabular difference is 36, the quantity to be added to l is $.00036\times\frac{15}{60}=.00009$.

If $l=\bar{1}.59812$, the work is arranged as follows:

$$l=\bar{1}.59812$$
$$D\times\frac{s}{60}=\qquad 9$$
$$L=\bar{1}.59821$$

When, as in this case, the product $D\times\frac{s}{60}$ is small, it can readily be added or subtracted mentally. Only the significant figures of D (those given in the d-column) are used, it being understood that the result expresses units of the fifth order of decimals. Thus, instead of writing $D = .00036$, and $D \times \frac{s}{60} = .00036 \times \frac{s}{60}$, the following abbreviated notation is used: $D = 36$; $D \times \frac{s}{60} = 36 \times \frac{s}{60}$, the latter product expressing decimal units of the fifth order, or hundred thousandths.

The foregoing formula indicates the process by which the logarithmic functions of an angle containing odd seconds are obtained. It may be stated in words as follows:

Rule.—*Drop the seconds, and find the logarithmic function of the remaining angle. Find the tabular difference between this logarithmic function and the same function of the angle next higher in the table. Multiply this tabular difference by the number of seconds in the angle and divide the product by 60. Add this result to or subtract it from the logarithm found, according as the logarithm to be determined is that of a fundamental function or that of a cofunction. The result thus obtained is the required logarithmic function.*

EXAMPLE 1.—Find: (*a*) the logarithmic sine of 15° 40′ 32″; (*b*) the logarithmic cosine.

SOLUTION.—(*a*) Dropping the seconds, 15° 40′ is obtained, whose logarithmic sine, found as in Art. **25**, is $\bar{1}.43143$; that is, $l = \bar{1}.43143$. Opposite the space between this logarithm and the following, and in the column marked d, is found the tabular difference 45($= D$). Applying the formula given in Art. **27**,

$$L = \bar{1}.43143 + .00045 \times \tfrac{32}{60}$$

$$l = \bar{1}.43143$$

$$D \times \frac{s}{60} = 45 \times \frac{32}{60} = \qquad 24$$

$$L = \bar{1}.43167$$

that is, log sin 15° 40′ 32″ = $\bar{1}.43167$. Ans.

In practice, it is not necessary to write all the figures of l before adding the correction $D \times \frac{s}{60}$. Having found the value of l in the table, one places and keeps the finger on that value and calculates the correction $D \times \frac{s}{60}$. In the majority of cases, this correction can be added mentally to l. Thus, in the example just explained, the correction is 24, which, being mentally added to the number 43 formed by the last two figures of l, gives 67 as the last two figures of L. The other figures of L are the same as those of l.

(b) The logarithmic cosine of 15° 40′ is $\bar{1}.98356 (= l)$. Horizontally opposite the space between this logarithm and the following, the tabular difference $4 (= D)$ is found in the column marked d on the right of the cosine column. As the function under consideration is a cofunction, the correction $D \times \frac{s}{60}$ must be subtracted for l. We have, then,

$$l = \bar{1}.98356$$

$$D \times \frac{s}{60} = 4 \times \frac{32}{60} = \qquad 2, \text{ to the nearest unit}$$

$$L = \bar{1}.98354$$

Therefore, $\log \cos 15° 40' 32'' = \bar{1}.98354$. Ans.

In practice, the correction 2 would be subtracted mentally, without previously writing the value of l.

EXAMPLE 2.—Find the logarithmic tangent of 63° 39′ 27″.

SOLUTION.—Dropping the seconds, and referring to the page marked 63° at the bottom, the logarithmic tangent of 63° 39′ is found to be $.30512 (= l)$. Since in this case the angles increase from bottom to top, the tabular difference to be used is that horizontally opposite the space between the logarithm just taken and the one immediately above it in the column (that is, .30543). This difference is 31, printed in the column marked c. d. on the left of the cotangent column. We have, therefore,

$$l = .30512$$

$$\frac{s}{60} \times D = \frac{27}{60} \times 31 = \qquad 14, \text{ to the nearest unit}$$

$$L = .30526$$

Therefore, $\log \tan 63° 39' 27'' = .30526$. Ans.

EXAMPLE 3.—Find the logarithmic cotangent of 54° 8′ 9″.

SOLUTION.—Dropping the seconds, the value of l is found to be $\bar{1}.85913$. The tabular difference in the c. d. column and horizontally opposite the space between this logarithm and the one immediately above it is 26. As the cotangent is a cofunction, the correction $\frac{s}{60} \times D$ is to be subtracted from l. Then,

$$l = \bar{1}.85913$$

$$\frac{s}{60} \times D = \frac{9}{60} \times 26 = \qquad 4$$

$$L = \bar{1}.85909$$

Therefore, $\log \cot 54° 8' 9'' = \bar{1}.85909$. Ans.

EXAMPLES FOR PRACTICE

1. Find the logarithmic sine, tangent, and cosine of 33° 21′ 46″.

$$\text{Ans.}\begin{cases}\log\sin = \bar{1}.74032\\ \log\tan = \bar{1}.81852\\ \log\cos = \bar{1}.92179\end{cases}$$

2. Find the logarithmic sine and cotangent of 23° 3′ 17″.

$$\text{Ans.}\begin{cases}\log\sin = \bar{1}.59286\\ \log\cot = .37100\end{cases}$$

3. Find the logarithmic tangent and cosine of 49° 12′ 12″.

$$\text{Ans.}\begin{cases}\log\tan = .06395\\ \log\cos = \bar{1}.81516\end{cases}$$

4. Find the logarithmic sine, tangent, and cosine of 72° 52′ 49″.

$$\text{Ans.}\begin{cases}\log\sin = \bar{1}.98031\\ \log\tan = .51143\\ \log\cos = \bar{1}.46890\end{cases}$$

5. Find the logarithmic sine and cotangent of 81° 38′ 28″.

$$\text{Ans.}\begin{cases}\log\sin = \bar{1}.99536\\ \log\cot = \bar{1}.16712\end{cases}$$

6. Find the logarithmic tangent and cosine of 65° 0′ 47″.

$$\text{Ans.}\begin{cases}\log\tan = .33159\\ \log\cos = \bar{1}.62574\end{cases}$$

7. Find the logarithmic secant and cosecant of 59° 0′ 9″.

$$\text{Ans.}\begin{cases}\log\sec = .28819\\ \log\csc = .06692\end{cases}$$

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28. Use of the Column of Proportional Parts.—The method described in the preceding article can be applied to any table of logarithmic functions. Some tables, however, among them the table furnished with this Course, contain a column giving the products of the tabular differences by the fractions $\frac{6}{60}$, $\frac{7}{60}$, $\frac{8}{60}$, $\frac{9}{60}$, $\frac{10}{60}$, $\frac{20}{60}$, $\frac{30}{60}$, $\frac{40}{60}$, and $\frac{50}{60}$. These products are called **proportional parts,** and are given in the right-hand column (marked p. p. at the top) of each page, beginning with 3°. The tabular differences are here printed in heavy figures. Under each tabular difference are given the products of it by $\frac{6}{60}$, $\frac{7}{60}$, etc., the number of sixtieths being printed horizontally opposite the product, on the left of a vertical line. Thus, referring to the right-hand column of the page marked 13° at the top, the numbers 54, 53, 52, printed in heavy type, are tabular differences. The number 27, directly under 54, and horizontally opposite the number 30 on the left of the vertical line, is the product of 54 by $\frac{30}{60}$. Likewise, 17.3, found under 52, and horizontally opposite 20, is the product of 52 by $\frac{20}{60}$. The proportional parts for 1, 2, 3, 4, 5 are obtained from those for 10, 20, 30, etc., by moving the decimal point one place to the left. Thus, the proportional part for 20, under the tabular difference 52, is 17.3, as just explained. The proportional part for 2, that is, the product of 52 by $\frac{2}{60}$, is 1.73.

In the first three pages of the logarithmic table, no proportional parts are given, the use of these pages being different from that of the others. In pages 45, 46, and 47, not all the tabular differences are given in the p. p. column, owing to want of space; but the proportional part for any tabular difference is easily obtained by means of the proportional parts for digits given at the bottom of the p. p. column. Referring, for example, to page 45, the tabular difference 215, which is found in the c. d. column, does not appear in the p. p. column. If we wish to find the product of 215 by $\frac{30}{60}$, we look in the p. p. column for the tabular difference next lower than 215, which is 212. Horizontally opposite 30, and under 212, we find 106; that is, $212 \times \frac{30}{60} = 106$. As $215 = 212 + 3$, we must add to the product just found (106), the product of $3 \times \frac{30}{60}$. This is taken from the column headed 3 near the bottom of the p. p. column: there we find 1.5 horizontally opposite 30; that is, $3 \times \frac{30}{60} = 1.5$. Therefore, $215 \times \frac{30}{60} = 106 + 1.5 = 107.5$. The addition of these two products can usually be effected mentally.

The correction $D \times \frac{s}{60}$ to be applied to l in order to find L (formula of Art. **27**) is found from the table of proportional parts as follows:

Rule.—*Having found the tabular difference D, look for this difference in the column of proportional parts. If this difference is found in that column and the number of seconds is a digit greater than 5 or a digit followed by a cipher, look for it on the left of the vertical line under D; the correction is then found horizontally opposite this number, and directly under D. If the* *number of seconds is a digit less than 6, add a cipher, find the proportional part corresponding to the resulting number, and move the decimal point one place to the left. If the number of seconds consists of two significant digits (as 39), find the correction for the first digit followed by a cipher, and that for the second digit, and add the two corrections. (Thus, if the number of seconds is 43, the correction is found by adding the corrections for 40 and 3.)*

If the tabular difference D is not found in the p. p. column (which may happen only on pages 45 to 47), take, as just explained, the proportional part corresponding to the next lower tabular difference found in the p. p. column; then, from the digit columns found at the bottom of the p. p. column, find the proportional part corresponding to the difference between D and the tabular difference just used. Add the two proportional parts thus found.

EXAMPLE 1.—Find: (*a*) the logarithmic tangent of 22° 17′ 8″; (*b*) the logarithmic cosine.

SOLUTION.—(*a*) Dropping the seconds, we find log tan 22° 17′ $= \bar{1}.61256 (= l)$; $D = 36$. Turning to the column of proportional parts, 36 is found in heavy type near the top of the page. Following the horizontal row that begins with 8 (number of seconds) at the left of the vertical line under 36, we find in that row, and directly under 36, the correction 4.8, which may be called 5, as there are no other numbers to be combined with it. Therefore,

$$l = \bar{1}.61256$$
$$\frac{s}{60} \times D = \text{p. p.} = \qquad 5$$
$$L = \bar{1}.61261$$

That is, log tan 22° 17′ 8″ $= \bar{1}.61261$. Ans.

(*b*) $l = \log \cos 22° 17' = \bar{1}.96629$; $D = 5$. Looking for the column headed 5 among the proportional parts, the correction .7 (or say 1) is found directly under 5 and horizontally opposite 8. Therefore,

$$l = \bar{1}.96629$$
$$\frac{s}{60} \times D = \text{p. p.} = \qquad 1$$
$$L = \bar{1}.96628$$

That is, log cos 22° 17′ 8″ $= \bar{1}.96628$. Ans.

EXAMPLE 2.—Find the logarithmic sine of 3° 18′ 9″.

SOLUTION.— $l = \sin 3° 18' = \bar{2}.76015$; $D = 219$. The difference 219 is not found in the p. p. column; the tabular difference in the p. p. column next lower is 216. Under 216, and horizontally opposite 9, is found 32.4. The difference between 219 and 216 is 3. Looking for 3 in the digit columns at the bottom of the p. p. column, .5 is found under 3, and horizontally opposite 9. Therefore, $219 \times \frac{9}{60} = 32.4 + .5 = 33$, nearly.

$$l = \bar{2}.76015$$
$$219 \times \frac{9}{60} = \quad 33$$
$$L = \bar{2}.76048$$

That is, $\log 3° \, 18' \, 9'' = \bar{2}.76048$. Ans.

EXAMPLE 3.—Find: (*a*) the logarithmic tangent of 53° 47′ 04″; (*b*) the logarithmic cosine.

SOLUTION.—(*a*) $l = \log \tan 53° \, 47' = .13529$; $D = 26$; the proportional part for 40, under D, that is, under 26, is 17.3; the proportional part for 4 is $\frac{17.3}{10}$, or 2, nearly.

$$l = .13529$$
$$26 \times \frac{4}{60} = \quad 2$$
$$L = .13531$$

That is, $\log \tan 53° \, 47' \, 4'' = .13531$. Ans.

(*b*) $l = \log \cos 53° \, 47' = \bar{1}.77147$; $D = 17$. The number horizontally opposite 40, in the column headed 17 among the proportional parts, is 11.3; the proportional part for 4 is, therefore, $\frac{11.3}{10} = 1$, nearly.

$$l = \bar{1}.77147$$
$$17 \times \frac{4}{60} = \quad 1$$
$$L = \bar{1}.77146$$

That is, $\log \cos 53° \, 47' \, 4'' = \bar{1}.77146$. Ans.

EXAMPLE 4.—To find the logarithmic cotangent of 72° 35′ 47″.

SOLUTION.— $l = \log \cot 72° \, 35' = \bar{1}.49652$; $D = 45$. Looking among the proportional parts for the column headed 45, the correction for 40 is found to be 30, and that for 7 is found to be 5.3. Therefore,

$$l = \bar{1}.49652$$
$$\text{p. p. for } 40 = 30.0$$
$$\text{p. p. for } 7 = 5.3$$
$$\text{p. p. for } 47 = \qquad 35$$
$$L = \bar{1}.49617$$

That is, $\log \cot 72° \, 35' \, 47'' = \bar{1}.49617$. Ans.

In practice, it would not be necessary to write down the corrections 30 and 5.3, which would be added mentally. The same remark applies to all similar cases.

EXAMPLES FOR PRACTICE

1. Find the logarithmic sine and cotangent of 9° 39′ 17″.

$$\text{Ans.}\begin{cases}\log\sin = \bar{1}.22456\\ \log\cot = .76924\end{cases}$$

2. Find the logarithmic sine, tangent, and cosine of 39° 8′ 52″.

$$\text{Ans.}\begin{cases}\log\sin = \bar{1}.80025\\ \log\tan = \bar{1}.91065\\ \log\cos = \bar{1}.88959\end{cases}$$

3. Find the logarithmic cotangent and cosecant of 80° 3′ 46″.

$$\text{Ans.}\begin{cases}\log\cot = \bar{1}.24352\\ \log\csc = .00657\end{cases}$$

4. Find the logarithmic sine, secant, and tangent of 49° 0′ 54″.

$$\text{Ans.}\begin{cases}\log\sin = \bar{1}.87788\\ \log\sec = .18319\\ \log\tan = .06197\end{cases}$$

5. Find the logarithmic tangent and cosine of 4° 2′ 4″.

$$\text{Ans.}\begin{cases}\log\tan = \bar{2}.84838\\ \log\cos = \bar{1}.99892\end{cases}$$

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29. To Find the Angle Corresponding to Any Logarithmic Function When the Given Function Is Found in the Table.—In this case, the angle, which contains no odd seconds, is found as follows:

Rule.—*Find the given logarithm in the column marked by the name of the function whose logarithm is given. Then, if the name of the given function is at the top of the column, the number of degrees in the angle is that at the top of the page, and the number of minutes is horizontally opposite the logarithm, in the left-hand minute column. If the name of the function is at the foot of the column, the number of degrees in the angle is that at the foot of the page, and the number of minutes is in the right-hand minute column, horizontally opposite the given logarithm.*

In searching the table for a given logarithm, it should be borne in mind that the logarithmic sines and tangents increase, and the cosines and cotangents decrease, from 0° to 90°. Therefore, in the columns marked log sin and log tan at the top, the logarithms increase, and in the columns headed log cos and log cot the logarithms decrease, from the first to the last page. The sines and tangents continue to increase, and the cosines and cotangents to decrease, from the last page to the first, in the columns marked with the names of these functions, respectively, at the bottom. Thus, the last page contains, in the column headed log sin, the logarithmic sines of the angles between 44° and 45°. The sines are continued in the column marked log sin at the bottom, which contains the logarithmic sines of the angles between 45° and 46°; the preceding page contains the sines of angles between 46° and 47°, etc. Here the logarithmic sines increase from bottom to top, and in the inverse order of the pages.

When looking for a given logarithmic sine, open the table at random. Glance at both of the sine columns, that is, the column marked log sin at the top and the column marked log sin at the bottom, and compare the logarithms in them with the given logarithm. If the given logarithm is less than those found in the column marked log sin at the top, said given logarithm must be in that column, but in a preceding page. If the given logarithm is greater than those in the column marked log sin at the bottom, said given logarithm must be in that column, but in a preceding page. If neither of these is the case, the given logarithm must be in a subsequent page. Turn a few pages forwards or backwards, as the case may be, and repeat the operation. The comparison of the two columns, however, is not usually necessary after the first three figures of the given logarithm have been found in one of them, as that logarithm is then found in that column, and can be readily seen among the logarithms beginning with those three figures.

Proceed exactly in the same manner when the given function is a cosine; that is, treat the cosine as though it were a sine; but, having found the given logarithm, treat it as that of a cosine and take the angle accordingly.

As the tangents of angles less than 45° are less than 1, their logarithmic tangents have negative characteristics, and as the tangents of angles greater than 45° are greater than 1, their logarithmic tangents have positive characteristics. Therefore, a logarithmic tangent should be looked for in the column marked log tan at the top or at the bottom, according as its characteristic is negative or positive. For a logarithmic cotangent, the rule should be reversed.

EXAMPLE 1.—Find the angle whose logarithmic sine is $\bar{1}.57669$.

SOLUTION.—Opening the table at random, say at the page marked 36° at the top, it is at once seen that the logarithms in the column marked log sin at the top are greater than the given logarithm. This logarithm must, therefore, be in that column, but in a preceding page. Turning the pages backwards, a few at a time, the given logarithm is found on page 64, among those logarithms whose first three figures are $\bar{1}.57$. As the name of the function is at the head of the column, the number of degrees (22) is taken from the top of the page, and that of minutes (10) from the left-hand minute column. Therefore, the angle whose logarithmic sine is $\bar{1}.57669$ is 22° 10′, or $\bar{1}.57669 = \log \sin 22° 10'$.

Suppose that the table had first been opened at page 56. Since the given logarithm is greater than those in the column marked log sin at the top and less than those in the column marked log sin at the bottom (or log cos at the top), the given logarithm is to be found in a subsequent page. Suppose also that, turning the pages forwards, a few at a time, we come to page 63, and find the first three figures ($\bar{1}.57$) of the given logarithm in the column marked log sin at the top. Then, without consulting the other column, we follow the former column to the bottom, and into the next page, where we find the given logarithm, and take the corresponding angle as before.

EXAMPLE 2.—To find the angle whose logarithmic sine is $\bar{1}.89810$.

SOLUTION.—Open the table at random, say at page 73. Since the given logarithm is greater than those in the column marked log sin at the top, and less than those in the column marked log sin at the bottom, it must be found in a subsequent page. Suppose that we turn next to page 85. We see at once that the given logarithm is greater than those in the column headed log sin, and also than those in the column marked log sin at the bottom. Therefore, it must be in the latter column in some preceding page. Turning the pages backwards, we find the first three figures ($\bar{1}.89$) of the given logarithm on page 79, and among the logarithms to which these three figures are common, we find $\bar{1}.89810$. As this is a logarithmic sine, and the name sine is at the bottom of the column, the degrees in the corresponding angle are taken from the bottom of the page, and the minutes from the right-hand minute column. Therefore, 52° 16′ is the angle whose logarithmic sine is $\bar{1}.89810$; that is, $\bar{1}.89810 = \log \sin 52° 16'$. Ans.

EXAMPLE 3.—Find the angle whose logarithmic cosine is $\bar{1}.86924$.

SOLUTION.—Treating this as though it were a logarithmic sine, it is found, as explained above, on page 84, in the column marked log sin at the bottom. Since the name cosine is at the top of the column, the required angle is 42° 16′. That is, $\bar{1}.86924$ = log cos 42° 16′. Ans.

Example 4.—Find the angle whose logarithmic cotangent is .15639.

Solution.—As the characteristic is positive, the logarithm should be looked for in the column marked log cot at the top. After looking in a few pages, the first three figures (0.15) of the logarithm are found on page 76, and among them is found the given logarithm. The name of the function being at the head of the column, the degrees in the angle are taken from the top of the page, and the minutes from the left-hand minute column. Therefore, .15639 = log cot 34° 54′. Ans.

EXAMPLES FOR PRACTICE

1. Find the angle whose logarithmic sine is $\bar{1}.57885$. Ans. 22° 17′
2. Find the angle whose logarithmic sine is $\bar{1}.66731$. Ans. 27° 42′
3. Find the angle whose logarithmic sine is $\bar{2}.93740$. Ans. 4° 58′
4. Find the angle whose logarithmic sine is $\bar{1}.98345$. Ans. 74° 17′
5. Find the angle whose logarithmic cosine is $\bar{1}.92086$. Ans. 33° 33
6. Find the angle whose logarithmic cosine is $\bar{1}.57232$. Ans. 68° 4
7. Find the angle whose logarithmic cosine is $\bar{1}.84949$. Ans. 45° 0′
8. Find the angle whose logarithmic tangent is $\bar{1}.97649$. Ans. 43° 27′
9. Find the angle whose logarithmic cotangent is $\bar{2}.89274$. Ans. 85° 32′
10. Find the angle whose logarithmic tangent is .67377. Ans. 78° 2′
11. Find the angle whose logarithmic cotangent is .35517. Ans. 23° 49′
12. Find the angle whose logarithmic tangent is 1.28060. Ans. 87° 0′

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30. To Find the Angle Corresponding to a Given Logarithmic Function When the Function Is Not in the Table.—*Without the Use of Proportional Parts.*—From the formula given in Art. 27, the following may be obtained:

$$s = \frac{(L - l) \times 60}{D} = \frac{(L - l) \times 60}{l' - l}$$

Therefore, if the function L is given and it is found to lie between the consecutive logarithms l and l', the corresponding angle $A + s$ is that corresponding to l increased by the number of seconds determined by the formula just given. It will be remembered (see Art. **27**) that l and l' are, respectively, the logarithmic functions of two angles (A and $A + 1'$) differing by one minute. If the function is a fundamental function (sine or tangent) l' is greater than l; and since L lies between l and l', L is also greater than l; therefore, both $L - l$ and $l' - l$ are positive. If the function is a cofunction, l is greater than l', and also greater than L; therefore, both $L - l$ and $l' - l$ are negative, and $\frac{L - l}{l' - l}$ is positive. In such case, however, it is better to write this fraction in the form $\frac{l - L}{l - l'}$.

From the formula and the explanations just given, the following rule is derived for finding the angle corresponding to any given logarithmic function:

Rule.—*Find in the table the two consecutive logarithmic functions between which the given function lies. The degrees and minutes in the smaller of the angles corresponding to these two functions are the degrees and minutes in the required angle.*

Find the difference between the given function and that of the smaller angle; multiply that difference by 60, and divide the product by the tabular difference between the two functions in the table. The result will be the number of odd seconds in the required angle.

As the tabular difference is expressed in units of the fifth decimal order, the difference $L - l$ should be likewise expressed. Thus, if $L = \bar{1}.25198$, and $l = \bar{1}.25168$, the difference $L - l$ will be called 30.

Example 1.—Find the angle whose logarithmic sine is $\bar{1}.47867$ ($= L$).

Solution.—The first three figures of the given logarithm are always found in the table, and this makes it easy to determine the functions between which the given logarithm lies. Searching the sine columns of the table, it is found that $\bar{1}.47867$ lies between $\bar{1}.47854$ ($= l$) and

$\bar{1}.47894 (= l')$ on page 59. The smaller of the two angles corresponding to these two logarithms is $17° 31' (= A)$. Now, $L - l = 13$, $l' - l$ (tabular difference taken from table) $= 40$. Therefore,

$$s = \frac{13 \times 60}{40} = 19.5'', \text{ or, say, } 20''$$

and $$A + s = 17° 31' + 20'' = 17° 31' 20''$$

that is, $$\bar{1}.47867 = \log \sin 17° 31' 20''. \text{ Ans.}$$

EXAMPLE 2.—Find the angle whose logarithmic tangent is .27743 $(= L)$.

SOLUTION.—As the characteristic is positive, the logarithms between which L lies should be looked for in the column marked log tan at the bottom. These two logarithms are $.27738 (= l)$ and $.27769 (= l')$. The smaller angle corresponds to .27738, and is $62° 10' (= A)$. Also,

$$L - l = 5, \; l' - l (= D) = 31$$

$$A + s = A + \frac{5 \times 60}{31} = 62° 10' + 10'', \text{ nearly}, = 62° 10' 10''$$

that is, $$.27743 = \log \tan 62° 10' 10''. \text{ Ans.}$$

EXAMPLE 3.—Find the angle whose logarithmic cotangent is $\bar{1}.85899 (= L)$.

SOLUTION.— L is found to lie between $\bar{1}.85887 (= l')$ and $\bar{1}.85913 (= l)$. It will be noticed that here l is the greater, and l' the smaller of the two logarithms. Angle corresponding to $l = 54° 8' (= A)$.

$$l = \bar{1}.85913$$
$$L = \bar{1}.85899$$
$$l - L = 14; \; l - l' = 26$$

$$A + s = 54° 8' + \frac{14 \times 60}{26} = 54° 8' 32'', \text{ nearly}$$

that is, $$\bar{1}.85899 = \log \cot 54° 8' 32''. \text{ Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the angle whose logarithmic sine is $\bar{1}.45566$.
   Ans. 16° 35′ 27″

2. Find the angle whose logarithmic tangent is $\bar{1}.33471$.
   Ans. 12° 11′ 44″

3. Find the angle whose logarithmic sine is $\bar{1}.89798$.
   Ans. 52° 14′ 42″

4. Find the angle whose logarithmic cosine is $\bar{1}.67412$.
   Ans. 61° 49′ 23″

5. Find the angle whose logarithmic cosine is $\bar{1}.92386$.
   Ans. 32° 56′ 45″

6. Find the angle whose logarithmic cotangent is .54139.
Ans. 16° 2′ 20″

7. Find the angle whose logarithmic tangent is $\bar{1}.86712$.
Ans. 36° 22′ 7″

8. Find the angle whose logarithmic cosine is $\bar{1}.99785$.
Ans. 5° 42′ 0″

9. Find the angle whose logarithmic cotangent is $\bar{1}.12345$.
Ans. 82° 25′ 52″

---

31. *With the Use of Proportional Parts.*—Having found the degrees and minutes in the angle as in the preceding case, the number s of odd seconds may be conveniently found from the column of proportional parts. In order to facilitate the explanations that follow, the proportional parts corresponding to the tabular difference 105 are here copied from page 48 of the table. It will, therefore, be assumed that the value of D is 105, and, for what is said below, the student should refer to these proportional parts. Such being the case, the formula given at the beginning of the preceding article may be written,

	105
6	10.5
7	12.3
8	14.0
9	15.8
10	17.5
20	35.0
30	52.5
40	70.0
50	87.5

$$s = \frac{(L - l) \times 60}{105}$$

The value $L - l$, which is the difference between the given logarithm and the logarithm of the degrees and minutes (A) in the required angle, is readily determined, as already explained. It is only necessary to repeat that, if the function is a cofunction, $l - L$ should be used instead of $L - l$. Since the numbers on the right of the vertical line are the products of $\frac{105}{60}$ by the numbers on the left, it follows that the numbers on the left are the products of those on the right by $\frac{60}{105}$. Thus, $52.5 = \frac{105}{60} \times 30$, and $30 = 52.5 \times \frac{60}{105} = \frac{52.5 \times 60}{105}$. Therefore, if $L - l$ is found among the numbers directly under 105, the value of s is the number on the left of the vertical column horizontally opposite $L - l$. For example, if $L - l = 35$, then $s = 20''$. If $L - l = 16$, then

$s = 9''$, the number 9 being opposite 15.8, which, to the nearest unit, may be called 16.

It will be remembered that the proportional parts opposite 10, 20, 30, 40, 50, when divided by 10 (that is, when the period is moved one place to the left), give the products of $\frac{105}{60}$ by 1, 2, 3, 4, and 5. From those parts we may, therefore, find by inspection the products of $\frac{105}{60}$ by all the digits from 1 to 9; and, in what follows, we shall proceed as if the products 1.75, 3.50, 5.25, 7.00, 8.75 of $\frac{105}{60}$ by 1, 2, 3, 4, and 5 were actually printed in the table opposite those digits; that is, it will be assumed that the proportional parts run in this order: 1.75, 3.50, 5.25, 7.00, 8.75, 10.5, 12.3, 14.0, etc., up to 87.5, the corresponding numbers on the left being, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50. The proportional parts 1.75, 3.50, 5.25, 7.00, 8.75 will be referred to as proportional parts found in the table, corresponding to 1, 2, 3, 4, and 5 seconds, respectively.

This being understood, the number s of odd seconds in the angle is determined as follows:

Rule.—*Find l, l', $L - l$,* and *$l' - l$ (= tabular difference, or D), as before. Look for the tabular difference D in the column of proportional parts. Look for $L - l$ in the column of proportional parts directly under D. If $L - l$ is found there, the number horizontally opposite it on the left of the vertical line is the required number of seconds s. If $L - l$ is not found under D, take the proportional part next lower, which call p. Find the difference between $L - l$ and p, and look among the proportional parts under D for this difference, or the part nearest to it, whether higher or lower. Call this part p'. Add the numbers horizontally opposite p and p' on the left of the vertical line. The result will be the required number of seconds s.*

EXAMPLE 1.—Find the angle whose logarithmic tangent is $\bar{1}.42822 (= L)$.

SOLUTION.— $l = \bar{1}.42805$, $A = 15° 0'$, $L - l = 17$, $D = 51$. Looking in the column marked p. p. for 51, the number $17 (= L - l)$ is found under it, horizontally opposite the number 20 on the left of the vertical column. Therefore, $s = 20''$, and

$$\bar{1}.42822 = \log \tan 15° 0' 20''. \quad \textbf{Ans.}$$

Example 2.—Find the angle whose logarithmic cosine is $\bar{1}.52783 (= L)$.

Solution.— $l = \bar{1}.52811$, $A = 70° 17'$, $l - L = 28$, $D = 36$. The proportional part under 36 next lower than 28 is 24; $28 - 24 = 4$; the proportional part nearest 4 is 4.2; the number horizontally opposite 24 is 40; and the number horizontally opposite 4.2 is 7; hence, $s = 40 + 7 = 47''$, and therefore

$$\bar{1}.52783 = \log \cos 70° 17' 47''. \quad \text{Ans.}$$

Example 3.—Find the angle whose logarithmic sine is $\bar{1}.66191 (L)$.

Solution.— $l = \bar{1}.66173$; $A = 27° 19'$; $L - l = 18$; $D = 24$. Looking in the p. p. column for 24, the proportional part next lower than 18 is $16 (= p)$, horizontally opposite which is 40. $18 - p = 18 - 16 = 2$. This difference is found among the proportional parts in the table (since it is the same as 20 with the decimal point moved one place to the left), and corresponds to $5'' \left(= \frac{50}{10}\right)$. Therefore, $s = 40 + 5 = 45''$, and

$$\bar{1}.66191 = \log \sin 27° 19' 45''. \quad \text{Ans.}$$

Example 4.—Find the angle whose logarithmic cotangent is $\bar{1}.00375 (= L)$.

Solution.— $l = \bar{1}.00427$; $A = 84° 14'$; $l - L = 52$; $D = 126$. The proportional part under 126 next lower than 52 is $42 (= p)$, which corresponds to $20''$; $52 - 42 = 10$. The proportional part nearest to 10 is $10.50 \left(= \frac{105.0}{10}\right)$, which corresponds to $5'' \left(= \frac{50}{10}\right)$. Therefore, $s = 20'' + 5'' = 25''$, and

$$\bar{1}.00375 = \log \cot 84° 14' 25''. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the angle whose logarithmic sine is $\bar{1}.78988$. Ans. 38° 3′ 20″

2. Find the angle whose logarithmic tangent is $\bar{1}.78540$. Ans. 31° 23′ 15″

3. Find the angle whose logarithmic sine is $\bar{1}.77777$. Ans. 36° 49′ 56″

4. Find the angle whose logarithmic cosine is $\bar{1}.87341$. Ans. 41° 39′ 21″

5. Find the angle whose logarithmic cotangent is .31789. Ans. 25° 41′ 9″

6. Find the angle whose logarithmic cosine is $\bar{1}.34567$. Ans. 77° 11′ 38″

7. Find the angle whose logarithmic cotangent is $\bar{1}.00381$.

Ans. 84° 14′ 22″

8. Find the angle whose logarithmic tangent is 1.00300.

Ans. 84° 19′ 42″

9. Find the angle whose logarithmic sine is $\bar{2}.99001$.

Ans. 5° 36′ 30″

---

32. Tabular Values Increased by 10.—To avoid calculating with negative characteristics, they may be made positive by increasing them by 10. Thus, log sin 27° may be given as 9.65705 instead of $\bar{1}.65705$. The true logarithm is, therefore, $9.65705-10$; the -10 is usually not written, but is implied. In many books this method is used for the logarithms of trigonometric functions. In applying such logarithms to the solution of a problem, the characteristic in the final result must be corrected to agree with the conditions of the problem.

GENERAL PRINCIPLE OF INTERPOLATION

33. It has been explained in some of the preceding articles how to determine the natural or the logarithmic functions of any angle containing an odd number of seconds, and therefore, not found in the table; also, how to find the angle corresponding to a given function, when that function is not in the table but lies between two values given in the table. The operation by which such intermediate values are determined from a table is called **interpolation.** The values that are actually given in the table are called **tabular values.** For example, in the table of logarithmic functions already described are found all angles that lie between 0° and 90° and contain no odd seconds, and also the logarithmic sines, cosines, etc. of such angles; those are all tabular values. Angles containing odd seconds are not in the table, nor are their logarithmic functions. Both these angles and their functions are intermediate values, and it is in connection with them that interpolation is used.

34. The general principle of interpolation, to be explained presently, is of the utmost importance, and of great value to the engineer, whose work requires the frequent use of tables of various kinds. That principle, although only approximately true, applies to nearly all tables with which the engineer has to deal, and the student should endeavor to make himself thoroughly familiar with it.

Let a table be constructed on the general type shown on the margin, the left-hand column containing values of a quantity X, and the right-hand column corresponding values of some quantity whose values depend on the values of X. Thus, the values of X may be the natural numbers 1, 2, 3, 4, etc., and the corresponding values of F may be the logarithms or the square roots of those numbers; or the values of X may be angles, and those of F may be sines, cosines, etc., either natural or logarithmic. So far as the principle of interpolation is concerned, it is immaterial what kind of quantity is represented by X, and what kind of quantity is tabulated under F. It should be stated, however, that the principle applies only to tables in which the differences between consecutive values of X and the differences between the corresponding values of F do not vary very rapidly.

X	F
—	—
—	—
—	—
x_1	f_1
x	f
x_2	f_2
—	—

Let x_1 and x_2, as shown in the above general form, be two consecutive values of X given in the table, and f_1 and f_2 the corresponding values of F. Let x be a value of X lying between x_1 and x_2, and f the corresponding value of F. Neither x nor f is in the table, but one of them is given, and the problem is to find the other by interpolation. For instance, if the table is one of natural tangents in which the angles increase by whole minutes, x_1 and x_2 may be, respectively, 31° 42′ and 31° 43′, and f_1 and f_2 their corresponding tangents; while x may be any angle between 31° 42′ and 31° 43′, and f its tangent. Either x may be given to find f; or f may be given to find x.

The quantity by which the tabular value x_1 must be algebraically increased in order to obtain x will be called the **increment** of x_1, and denoted by $i(x_1)$, read *increment of* x_1 (mathematicians use the notation Δx_1, read *delta* x_1). We have, then,

$$x = x_1 + i(x_1) \qquad (1)$$

Using a similar notation for f_1,

$$f = f_1 + i(f_1) \qquad (2)$$

If x is given, $i(x_1)$ may be assumed as given, since $i(x_1) = x - x_1$. Then $i(f_1)$ is determined by interpolation, as explained below, and f is found from formula 2. Similarly, if f is given, $i(f_1)$ is likewise given, and x is found by interpolation.

The difference, as $x_2 - x_1$, of two consecutive values of X, will be called the **interval** of X; and that between two consecutive values of F, the interval of F. The notation $I(x_1)$, read *interval of* x_1, will be used to denote the interval $x_2 - x_1$. Similarly, $I(f_1)$ will denote the interval $f_2 - f_1$.

The principle of interpolation is this: *The increments $i(x_1)$ and $i(f_1)$ are to each other as the corresponding intervals $I(x_1)$ and $I(f_1)$; or, algebraically,*

$$\frac{i(x_1)}{i(f_1)} = \frac{I(x_1)}{I(f_1)} \qquad (3)$$

This formula is very easily remembered on account of its symmetry. The following, derived from it, serve, respectively, to find $i(f_1)$ when x is given, and $i(x_1)$ when f is given:

$$i(f_1) = I(f_1) \times \frac{i(x_1)}{I(x_1)} \qquad (4)$$

$$i(x_1) = I(x_1) \times \frac{i(f_1)}{I(f_1)} \qquad (5)$$

The last two formulas may be stated in the form of a general principle, as follows: *Either increment is equal to the corresponding interval multiplied by the ratio of the other increment to the other interval.* It is easy to remember what the numerator of this ratio is, by noticing that the ratio is always less than 1, and that, since the increment is always less than the interval, the former must be the numerator and the latter the denominator. It should be noted that $i(x_1)$, $i(f_1)$, $I(x_1)$, and $I(f_1)$ may be expressed in any convenient units, it being understood that $i(f_1)$, as determined from formula **4**, is in the same units as $I(f_1)$; and that $i(x_1)$, as determined from formula **5**, is in the same units as $I(x_1)$. Thus, if the values of f_2 and f_1 in the table are, respectively, 4.3476 and 4.3463, then, $I(f_1) = f_2 - f_1 = .0013$, or, if one ten-thousandth is taken as the unit, we may write $I(f_1) = 13$. The value of $i(f_1)$, determined from formula **4**, must be understood to express ten-thousandths. For instance, if $\frac{i(x_1)}{I(x_1)} = .3$, then, $i(f_1) = 13 \times .3 = 3.9$ (ten-thousandths) = 4 (ten-thousandths), nearly.

The value of f is then found thus,

$$\begin{aligned} f_1 &= 4.3463 \\ i(f_1) &= \underline{\qquad\ 4} \\ f &= 4.3467 \end{aligned}$$

Usually, the correction $i(f_1)$ can be added to f_1 mentally, in order to find f.

EXAMPLE 1.—Find the logarithm of 57,846 by means of a five-place table giving the logarithms of numbers consisting of four figures.

SOLUTION.—Only the mantissas will be considered, since the characteristics are determined by inspection. The given number lies between 57,840($=x_1$) and 57,850($= x_2$), whose logarithms are, respectively, .76223($= f_1$) and .76230($= f_2$). We have, therefore, expressing $f_2 - f_1$, or $I(f_1)$, in units of the fifth order

$$\begin{aligned} x &= 57846 & f_2 &= .76230 \\ x_1 &= 57840 & f_1 &= .76223 \\ i(x_1) &= \quad\ 6 & I(f_1) &= \qquad 7 \end{aligned}$$

$$I(x_1) = x_2 - x_1 = 10$$

Then (formula 4),

$$i(f_1) = 7 \times \frac{6}{10} = 4.2 = 4, \text{ nearly}$$

and

$$f = \begin{cases} f_1 \\ + i(f_1) \end{cases} = \begin{cases} .76223 \\ +4 \end{cases} = .76227. \text{ Ans.}$$

EXAMPLE 2.—Find, by means of a five-place table, the number the mantissa of whose logarithm is .47693.

SOLUTION.—Here $f(= .47693)$ lies between the tabular values $.47683(= f_1)$ and $.47698(= f_2)$, which are, respectively, the logarithms of $29{,}980(= x_1)$ and $29{,}990(= x_2)$. We have, then,

$$\begin{aligned} f_2 &= .47698 \\ f &= .47693 \\ f_1 &= .47683 \\ I(f_1) = f_2 - f_1 &= \quad 15 \\ i(f_1) = f - f_1 &= \quad 10 \end{aligned} \qquad \begin{aligned} x_2 &= 29{,}990 \\ x_1 &= 29{,}980 \\ I(x_1) &= \quad 10 \end{aligned}$$

Then (formula 5),

$$i(x_1) = 10 \times \frac{10}{15} = 7, \text{ nearly}$$

and $$x = x_1 + i(x_1) = 29{,}980 + 7 = 29{,}987. \quad \text{Ans.}$$

This gives the significant figures of the number. The decimal point should be placed according to the characteristic of the given logarithm.

EXAMPLE 3.—Find the angle whose natural tangent is $.56781(= f)$ by means of a table giving the natural tangents of angles varying by minutes.

SOLUTION.—Here f is found to lie between $.56769(= \tan 29° 35' = f_1)$ and $.56808(= \tan 29° 36' = f_2)$. Expressing $x_2 - x_1$, or $I(x_1)$, in seconds, we have

$$\begin{aligned} x_2 &= 29° 36' \\ x_1 &= 29° 35' \\ I(x_1) &= \quad 60'' \end{aligned} \qquad \begin{aligned} f_2 &= .56808 \\ f &= .56781 \\ f_1 &= .56769 \\ I(f_1) &= \quad 39 \\ i(f_1) &= \quad 12 \end{aligned}$$

Then (formula 5),

$$i(x_1) = 60'' \times \frac{12}{39} = 18'', \text{ nearly}$$

and $$x = x_1 + i(x_1) = 29° 35' 18''. \quad \text{Ans.}$$

EXAMPLE 4.—In Searles' field book is given a table of lengths of arcs for different degrees of curvature. Part of it is as follows (lengths in feet):

Degree of Curve $(=X)$	Length of Arc for One Station $(=F)$
10° 10′	100.131
10° 20′	100.136
10° 30′	100.140

Find the length of the arc between two stations for a 10° 26′ curve

SOLUTION.—Here we have, $x = 10° 26'$, which lies between $10° 20'$ $(= x_1)$ and $10° 30 (= x_2)$. Expressing $I(x_1)$ and $i(x_1)$ in minutes, and $I(f_1)$ and $i(f_1)$ in thousandths, we have $I(x_1) = 10$, $i(x_1) = 6$, $I(f_1) = 140 - 136 = 4$.

Therefore (formula 4),

$$i(f_1) = 4 \times \frac{6}{10} = 2, \text{ nearly}$$

and $$f = f_1 + i(f_1) = \begin{Bmatrix} 100.136 \\ +2 \end{Bmatrix} = 100.138. \quad \text{Ans.}$$

In all simple cases like this the operations can be performed mentally and very rapidly.

EXAMPLES FOR PRACTICE

1. From the following table, find, by interpolation, the cube root of 347.3 and that of 349.7.

Number	Cube Root
347	7.0271
348	7.0338
349	7.0406
350	7.0473

Ans. $\begin{cases} 7.0291 \\ 7.0453 \end{cases}$

2. Find, from the following table, the diameter of a circle whose circumference is 63.57318.

Diameter	Circumference
20.1	63.14601
20.2	63.46017
20.3	63.77433

Ans. 20.236

SOLUTION OF RIGHT TRIANGLES

35. Fundamental Equations.—Let $A\ B\ C$, Fig. 9, be a right triangle, in which A, B, and C are the angles and a, b, and c are the lengths of the sides, c being the hypotenuse. Since A and B are complementary angles, we have

$$\sin A = \cos B \qquad \tan A = \cot B$$
$$\cos A = \sin B \qquad \cot A = \tan B$$

Also, from the definitions of the trigonometric functions, $\sin A = \frac{a}{c}$, $\tan A = \frac{a}{b}$, $\cos B = \sin A = \frac{a}{c}$, $\cot A = \frac{b}{a}$; whence, expressing the value of a from each of these equations,

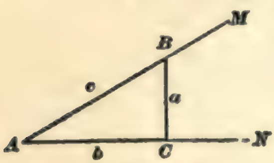

Fig. 9

$$a = c \sin A \qquad (1)$$
$$a = b \tan A \qquad (2)$$
$$a = c \cos B \qquad (3)$$
$$a = b \cot B \qquad (4)$$

From formulas **1** and **3**, the following values are found for c:

$$c = \frac{a}{\sin A} = a \csc A \qquad (5)$$

$$c = \frac{a}{\cos B} = a \sec B \qquad (6)$$

Finally, from geometry,

$$c^2 = a^2 + b^2 \qquad (7)$$

Of the trigonometric formulas just given, it is only necessary to commit to memory formulas **1** and **2**, as the others are immediate consequences of these. These two formulas may be stated in words thus:

Either leg of a right triangle is equal to the hypotenuse multiplied by the sine, or to the other leg multiplied by the tangent, of the opposite angle.

It should be observed that, since a is either leg whose opposite angle is A, and adjacent angle B, the letters a and b may be interchanged in the preceding formulas, provided that A and B are likewise interchanged. Thus, by interchanging a and b, A and B in formulas **1** and **5**, we obtain,

$$b = c \sin B, c = \frac{b}{\sin B} = b \csc B$$

36. Solution of a Right Triangle.—In general, when some of the parts of a triangle are given, the process of determining the others is called **solving the triangle,** or the **solution of the triangle.** The latter expression is applied also to the triangle determined in accordance with the given data.

In order to solve a right triangle, two parts, one at least of which should be a side, must be known in addition to the right angle. The two parts may be either (1) one side and one of the acute angles, or (2) two sides.

37. Case I.—*Given a Side and an Acute Angle.* The other acute angle is found from the relation $A + B = 90°$, and the other two sides by means of formulas **1** to **7**, Art. **35**, as illustrated by the following examples:

EXAMPLE 1.—In Fig. 10, the length of the hypotenuse $A B$ of the right triangle $A C B$, right-angled at C, is 24 feet, and the angle A is 29° 31′; find the sides $A C$ and $B C$, and the angle B.

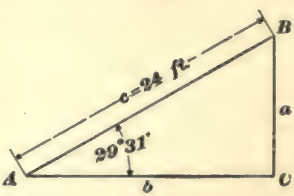

FIG. 10

NOTE.—When working examples of this kind, make a sketch and mark the known parts, as shown in the figure.

SOLUTION WITHOUT LOGARITHMS.— $B = 90° - A = 90° - 29° 31' = 60° 29'$. By formula **3**, Art. **35**, interchanging a and b, and A and B,

$b = c \cos A = 24 \cos 29° 31' = 24 \times .87021 = 20.89$ ft., nearly.

By formula **1**, Art **35**,

$a = 24 \sin 29° 31' = 24 \times .49268 = 11.82$ ft., nearly.

$$\text{Ans.}\begin{cases} B = 60° 29' \\ A\ C = 20.89 \text{ ft.} \\ B\ C = 11.82 \text{ ft.} \end{cases}$$

Solution by Logarithms.—By formulas **3** and **1**, Art. **35**,

$$b = 24 \cos 29° 31' \quad (1)$$
$$a = 24 \sin 29° 31' \quad (2)$$

Logarithms for (1)	Logarithms for (2)
$\log 24 = 1.38021$	$\log 24 = 1.38021$
$\log \cos 29° 31' = \bar{1}.93963$	$\log \sin 29° 31' = \bar{1}.69256$
$\log b = 1.31984$	$\log a = 1.07277$
$b = 20.89$	$a = 11.82$

In working examples of this kind, the two logarithmic functions should be taken from the table at the same time. It saves time and space to arrange the operations as follows:

$$\begin{aligned} \log a &= 1.07277;\ a = 11.82 \\ \log \sin 29° 31' &= \bar{1}.69256 \\ \log 24 &= 1.38021 \\ \log \cos 29° 31' &= \bar{1}.93963 \\ \log b &= 1.31984;\ b = 20.89. \quad \text{Ans.} \end{aligned}$$

The logarithm of 24 is written first, and then the logarithms of the sine and cosine, one over, the other under, log 24, the addition being performed upwards in one case and downwards in the other.

Example 2.—One leg of a right triangle *A C B*, Fig. 11, is 37 feet 7 inches long; the angle opposite is 25° 33′ 7″; what are the lengths of the hypotenuse and the side adjacent, and what is the other angle?

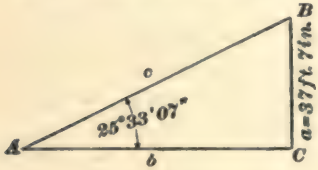

Fig. 11

Solution Without Logarithms. $B = 90° - 25° 33' 07'' = 64° 26' 53''$. Reducing 37 ft. 7 in. to ft., we have, $a = 37.583$ ft., nearly.

By formula **5**, Art. **35**,

$$c = \frac{37.583}{\sin 25° 33' 07''} = \frac{37.583}{.43133} = 87.133 \text{ ft., nearly.}$$

By formula **4**, Art. **35**, interchanging a and b, and A and B,

$$b = a \cot A = 37.583 \times 2.09166 = 78.611 \text{ ft., nearly.}$$

$$\text{Ans.} \begin{cases} B = 64° 26' 53'' \\ A\,C = 78.611 \\ A\,B = 87.133 \text{ ft} \end{cases}$$

Solution by Logarithms.—As before,

$$c = \frac{37.583}{\sin 25° 33' 7''}$$

Also,

$$b = 37.583 \cot 25° 33' 7''$$

$$\begin{aligned} \log b &= 1.89548;\ b = A\,C = 78.611 \text{ ft.} \\ \log \cot 25° 33' 7'' &= .32049 \\ \log 37.583 &= 1.57499 \\ \log \sin 25° 33' 7'' &= \bar{1}.63481 \\ \log c &= 1.94018;\ c = A\,B = 87.132 \text{ ft.} \quad \text{Ans.} \end{aligned}$$

It is to be noted that the value of AB given by logarithms is different in the fifth figure from the result given by natural functions. This is due to the fact that in using five-place tables the results can be depended on to be correct to only four figures, and to have a very close approximation to the fifth figure.

NOTE.—In the majority of cases, the solution by logarithms is far more expeditious than the solution by natural functions. The student is strongly advised to form the habit of solving all trigonometric problems by means of logarithms and the logarithmic functions, whenever these functions can be used.

38. Case II.—*Given Two Sides.* If the given sides are the two legs a and b, A is found from formula **2**, Art. **35**, and B, from the relation $A + B = 90°$. To find c, formula **7**, Art. **35**, may be used; but, unless a and b are convenient numbers to square, it is preferable to determine c by formula **5**, Art. **35**, after having determined A.

If the given sides are the hypotenuse c and one leg, say a, the angle A is found by formula **1**, Art. **35**, B from the relation $A + B = 90°$, and b from either formula **4**, or formula **7**, Art. **35**. The latter gives

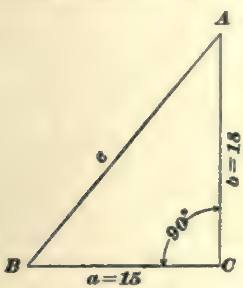

FIG. 12

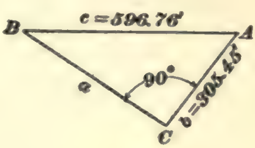

FIG. 13

$$b = \sqrt{c^2 - a^2}$$

Unless c and a are convenient numbers to square, the quantity under the radical should be replaced by the product $(c + a)\ (c - a)$, and then

$$\log b = \tfrac{1}{2}\ [\log\ (c + a) + \log\ (c - a]$$

from which b can be readily determined.

EXAMPLE 1.—Given a and b as shown in Fig. 12, to find A, B, and c

SOLUTION.—Formula **2**, Art. **35**,

$$\tan A = \frac{a}{b} = \frac{15}{18} = \frac{5}{6} = .83333$$

$$A = 39°\ 48'\ 20''$$

$$B = 90° - 39°\ 48'\ 20'' = 50°\ 11'\ 40''$$

Formula **5**, Art. **35**,

$$c = \frac{15}{\sin A} = \frac{15}{\sin 39^\circ\ 48'\ 20''}$$

$$\log 15 = 1.17609$$
$$\log \sin 39^\circ\ 48'\ 20'' = \bar{1}.80630$$
$$\log c = 1.36979;\ c = 23.431$$

Otherwise,

$$c = \sqrt{15^2 + 18^2} = \sqrt{(3 \times 5)^2 + (3 \times 6)^2} = 3\sqrt{5^2 + 6^2} = 3\sqrt{61} = 23.431.$$

Ans.

EXAMPLE 2.—The hypotenuse c and the leg b having the values shown in Fig. 13, find the acute angles and the leg a.

SOLUTION.—By formula **3**, Art. **35**, interchanging a and b, A and B,

$$\cos A = \frac{b}{c} = \frac{305.45}{596.76}$$

$\log 305.45 = 2.48494$
$\log 596.76 = 2.77580$
$\log \cos A = \bar{1}.70914;\ A = 59^\circ\ 12'\ 46''$

$90^\circ = 89^\circ\ 59'\ 60''$
$A = 59^\circ\ 12'\ 46''$
$B = 30^\circ\ 47'\ 14''$

Formula **2**, Art. **35**,

$$a = 305.45 \tan 59^\circ\ 12'\ 46''$$
$$\log 305.45 = 2.48494$$
$$\log \tan 59^\circ\ 12'\ 46'' = .22489$$
$$\log a = 2.70983;\ a = 512.66$$

Otherwise, $a = \sqrt{c^2 - b^2} = \sqrt{(c+b)(c-b)}$

$c + b = 902.21$
$c = 596.76$
$b = 305.45$
$c - b = 291.31$

$\log (c + b) = 2.95531$
$\log (c - b) = 2.46435$
$2)5.41966$
$\log a = 2.70983;\ a = 512.66$

Ans. $\begin{cases} A = 59^\circ\ 12'\ 46'' \\ B = 30^\circ\ 47'\ 14'' \\ a = 512.66 \text{ ft.} \end{cases}$

EXAMPLES FOR PRACTICE

1. In a right triangle ACB, right-angled at C (let the student make a sketch), the hypotenuse AB = 40 inches and angle $A = 28^\circ$ $14'\ 14''$; solve the triangle.

Ans. $\begin{cases} \text{Angle } B = 61^\circ\ 45'\ 46'' \\ AC = 35.239 \text{ in.} \\ BC = 18.925 \text{ in.} \end{cases}$

2. In a right triangle ACB, right-angled at C, the side BC = 10 feet 4 inches; if angle $A = 26^\circ\ 59'\ 6''$, what are the other parts?

Ans. $\begin{cases} \text{Angle } B = 63^\circ\ 0'\ 54'' \\ AB = 22 \text{ ft. } 9\frac{1}{4} \text{ in., nearly} \\ AC = 20 \text{ ft. } 3\frac{1}{4} \text{ in., nearly} \end{cases}$

3. In a right triangle ACB, the hypotenuse $AB = 60$ feet and the side $AC = 22$ feet; solve the triangle.

$$\text{Ans.}\begin{cases}\text{Angle } A = 68° \ 29' \ 22'' \\ \text{Angle } B = 21° \ 30' \ 38'' \\ BC = 55.821 \text{ ft.}\end{cases}$$

4. In a right triangle ACB, right-angled at C, side $AC = .364$ foot and side $BC = .216$ foot; solve the triangle.

$$\text{Ans.}\begin{cases}\text{Angle } A = 30° \ 41' \ 6'' \\ \text{Angle } B = 59° \ 18' \ 54'' \\ AB = .423 \text{ ft.}\end{cases}$$

PRACTICAL EXAMPLES

39. When an object is viewed by an observer, the object may be either above or below a horizontal plane passing through the observer's eye. The angle made with this plane by the line of sight, that is, by the line from the observer's eye to the object, is called an **angle of elevation** if the object is above that plane; an **angle of depression** if the object is below that plane. The object is said to be seen at an angle of elevation or at an angle of depression according as it is above or below the plane in question. For example, a lighthouse is seen from a ship at sea at an angle of elevation, while the ship is seen from the lighthouse at an angle of depression.

EXAMPLE 1.—The angle of elevation of the top of a vertical cliff, CB, Fig. 14, at a point 100 feet from its base, is 36° 50′; find the height of the cliff.

SOLUTION.—By formula **2**, Art. **35**, required height $= a = 100 \times \tan 36° \ 50' = 100 \times .74900 = 74.9$ ft. Ans.

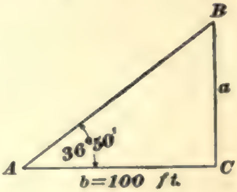

FIG. 14

EXAMPLE 2.—A statue is placed on the top of a column. At a point on the ground 130 feet from the base of the column, the angle of elevation of the top of the statue and that of the column are 43° 38′ and 40° 58′, respectively; find the height of the statue and column. (Let the student make a sketch.)

SOLUTION.—Let h = height of column;

h' = height of column and statue.

Then, $\tan 40° \ 58' = \frac{h}{130}$

Whence, $h = 130 \times \tan 40° \ 58' = 112.875$

Also, $\tan 43° \ 38' = \frac{h'}{130}$

Whence, $h' = 130 \times \tan 43° 38' = 123.942$

Therefore, the height of the column is 112.875 ft. Ans.

The height of the statue is $123.942 - 112.875 = 11.07$ ft. Ans.

EXAMPLE 3.—The top and bottom of a lighthouse LT, Fig. 15, located on a hill MN, are observed from a ship S with a sextant. It is found that the angles of elevation of T and L are, respectively, 6° 27′ and 5° 15′. If the height of the lighthouse is 128 feet, and the surface of the sea is assumed to be plane, what are: (a) the height $h(= CL)$ of the hill above sea level? (b) the horizontal distance $d(= SC)$ of the ship from the lighthouse?

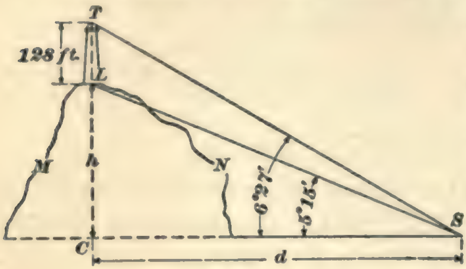

FIG. 15

SOLUTION.—(a) In the right triangles LCS and TCS, we have

$$d = h \cot 5° 15', \text{ and } d = (h + 128) \cot 6° 27'$$

Equating the two values of d,

$$h \cot 5° 15' = (h + 128) \cot 6° 27'$$

whence,

$$h = \frac{128 \cot 6° 27'}{\cot 5° 15' - \cot 6° 27'} = \frac{128 \times 8.84551}{10.8829 - 8.84551} = 555.72. \quad \text{Ans.}$$

(b) From (a),

$$d = h \cot 5° 15' = 555.72 \cot 5° 15' = 6{,}047.8 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 4.—In Fig. 16, $P_1 T_1$ is the track of a railroad that curves into a circular arc $T_1 M T_2$ at T_1. The chord $T_1 T_2$ of the whole arc is found, by measurement, to be 764.7 feet, and the chord $T_1 M$ of half the arc, 393.2 feet. Find: (a) the external angle I between $P_2 T_2$ and $P_1 T_1$ produced; (b) the radius $r(= CT_1)$ of the curve $T_1 M T_2$.

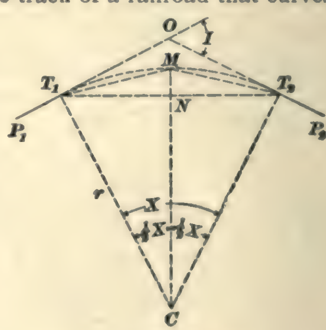

FIG. 16

SOLUTION.—(a) Draw CT_1, CM, and CT_2, as shown. Since $P_1 O$ and $P_2 O$ are tangent to the circle, the angles $OT_1 C$ and $OT_2 C$ are right angles; and as the sum of the angles in the quadrilateral $OT_1 CT_2$ is four right angles, we must have $X + T_1 O T_2 = 2$ right angles $= 180°$; we have also, $I + T_1 O T_2 = 180°$; therefore, $I = X$. The line CM bisects both the angle X and the chord $T_1 T_2$. As the angle MT_1T_2 is measured by one-half the arc $M T_2$, it is equal to one-half of $M C T_2$, or to $\frac{1}{4} X$. The right triangle $M T_1 N$ gives

$$\cos \frac{1}{4} X (= \cos M T_1 N) = \frac{T_1 N}{T_1 M} = \frac{\frac{1}{2} T_1 T_2}{T_1 M} = \frac{382.35}{393.2}$$

whence, by either logarithms or natural functions (logarithms are far preferable in this case),

$$\frac{1}{4} X = 13° 29' 20''; I (= X) = 4 \times 13° 29' 20'' = 53° 57' 20''. \quad \text{Ans.}$$

(*b*) In the right triangles $C T_1 N$,

$$r (= C T_1) = \frac{T_1 N}{\sin \frac{1}{2} X} = \frac{382.35}{\sin 26° 58' 40''} = 842.83 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 5.—Fig. 17 is a cross-section of a dam, the dimensions being as shown. The batter of the face $A B$ is 30 in 100, or .3. Find: (*a*) the width $w_1 (= A B)$ of the face; (*b*) the batter of the back $C D$; (*c*) the width $w_2 (= C D)$ of the back.

NOTE.—By the **batter** of one of the sides of an inclined wall is meant the rate at which that side deviates from the vertical. Thus, in Fig. 17, the side $B A$ deviates from the vertical by the amount $M N$ in the vertical distance $B N$, or by the amount $A P$ in the distance $B P$. Either of the ratios $\frac{M N}{B N}$ or $\frac{A P}{B P}$ expresses the batter of the wall. A batter of 30 in 100 is the same as $\frac{30}{100}$, or .3. It will be noticed that the batter is equal to the tangent of the inclination of the side of the wall to the vertical.

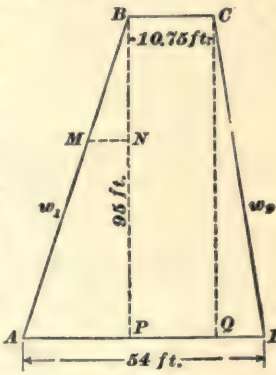

FIG. 17

SOLUTION.—(*a*) As just explained, tan $A B P$ = batter = .3; whence $A B P = 16° 41' 58''$. The triangle $A B P$ gives,

$$w_1 = \frac{B P}{\cos A B P} = \frac{95}{\cos 16° 41' 58''} = 99.182 \text{ ft.} \quad \text{Ans.}$$

(*b*) The triangle $A P B$ gives,

$$A P = P B \tan A B P = 95 \times .3 = 28.5 \text{ ft.}$$

From the figure,

$$Q D = A D - A P - P Q = 54 - 28.5 - 10.75 = 14.75 \text{ ft.}$$

The triangle $C Q D$ gives,

$$\text{batter of } C D = \tan Q C D = \frac{Q D}{Q C} = \frac{14.75}{95} = .15526$$

or, say, 15.5 in 100; also, $Q C D = 8° 49' 31''$. Ans.

$$(c) \quad w_2 = \frac{C Q}{\cos Q C D} = \frac{95}{\cos 8° 49' 31''} = 96.139 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 6.—Fig. 18 represents a derrick; the dimensions being as shown, determine: (*a*) the inclination A of the boom $Q R$ to the vertical; (*b*) the inclination M of the rod $P R$ to the vertical; (*c*) the point U at which the guy rope $P U$ must be tied, that it may make an angle of 60° with the horizontal; (*d*) the length $P U$ of the guy rope.

SOLUTION.—(*a*) The triangle RQS gives,

$$\sin A = \frac{QS}{QR} = \frac{28}{42} = \frac{2}{3}; \text{ whence, } A = 41° 48' 38''. \quad \text{Ans.}$$

(*b*) The same triangle gives,

$$RS = \sqrt{(42 + 28)(42 - 28)} = \sqrt{70 \times 14} = 31.305$$

The triangle PTR gives,

$$RT = RS - ST = RS - QP = 31.305 - 11.5 = 19.805 \text{ ft.}$$

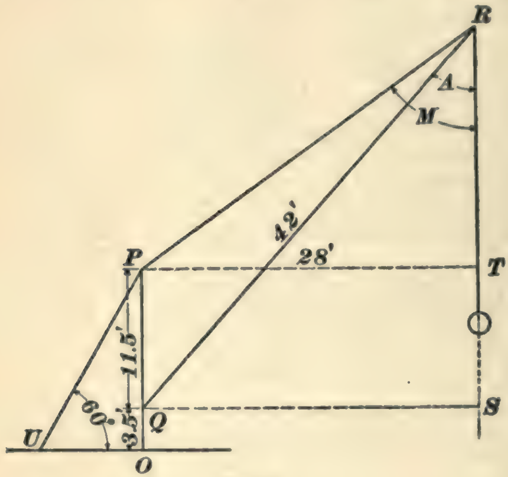

FIG. 18

$$\tan M = \frac{PT}{RT} = \frac{28}{19.805}; \text{ whence, } M = 54° 43' 38''. \quad \text{Ans.}$$

(*c*) In the triangle POU,

$$OU = OP \cot 60° = 15 \cot 60° = 8.660 \text{ ft.} \quad \text{Ans.}$$

(*d*) In the same triangle,

$$PU = \frac{OP}{\sin 60°} = \frac{15}{\sin 60°} = 17.320 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. In order to determine the distance CB, Fig. 19, across an intervening stream, a line CA, at right angles to CB, was measured; the angle CAB was also measured, and found to be 50° 16′. If CA = 100 feet, what is the distance CB? Ans. CB = 120.31 ft.

2. A ship was observed from the top of a lighthouse under an angle of depression of 50°; if the top of the lighthouse is 250 feet above sea level, what was the horizontal distance of the ship from the lighthouse? Ans. 209.78 ft.

3. From two points P_1, P_2, Fig. 20, assumed to be on the same horizontal line, the angles of elevation of the top O of a column were found to be as shown. If $P_1 P_2 = 300$ feet, and the points P_1 and P_2 are 9 feet higher than the base of the column, find: (*a*) the height h ($= OH$) of the column; (*b*) the horizontal distance d from P_1 to the axis of the column.

Ans. (*a*) $h = 366.77$ ft.; (*b*) $d = 292.31$ ft.

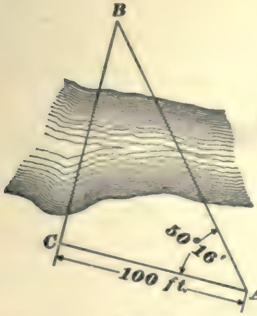

FIG. 19

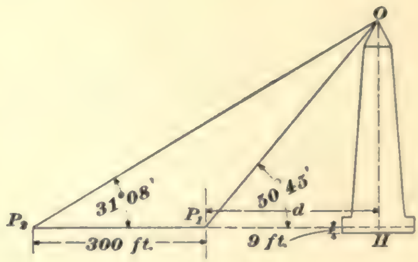

FIG. 20

4. A water pipe has a grade of 5.5 per cent. (that is, the pipe drops or rises 5.5 feet in every hundred feet measured horizontally); find: (*a*) the inclination of the pipe to the horizontal; (*b*) the length of pipe required for a horizontal distance of 2,764 feet.

Ans. (*a*) 3° 8′ 54″; (*b*) 2,768.2 ft.

5. The face $A B$, Fig. 17, and back $C D$ of a dam 80 feet high are to have a batter of 26 and 12 in 100, respectively; if the base $A D$ is to be 45 feet wide, find: (*a*) the angles A and D at the base; (*b*) the width $B C$ of the top.

Ans. (*a*) $A = 75° 25′ 33″$, $D = 83° 9′ 26″$; (*b*) $B C = 14.6$ ft.

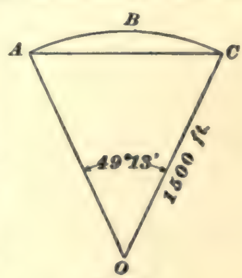

FIG. 21

6. Show that the base of an isosceles triangle is equal to twice one of the equal sides multiplied by the sine of one-half the vertical angle (angle opposite base).

7. A railroad curve $A B C$, Fig. 21, radius 1,500 feet, subtends a central angle of 49° 13′. (*a*) Find the length of the chord $A C$. (*b*) What will be the error in taking the length of the chord for the length of the arc? (Determine the latter length by the rules of geometry).

Ans. (*a*) $A C = 1,249.2$ ft.; (*b*) 39.3 ft.

PLANE TRIGONOMETRY

(PART 2)

Serial 779B — Edition 1

LOGARITHMIC FUNCTIONS OF SMALL ANGLES

1. Angles less than 3° are of comparatively rare occurrence in practice. When, however, they do occur, and they contain odd seconds, their logarithmic sines, tangents, and cotangents cannot be accurately determined by the general formulas and rules given in *Plane Trigonometry*, Part 1. These functions are found from a special table, which covers the first three pages of the general table of logarithmic functions furnished with this Course. These pages differ from the others in several respects, namely:

(*a*) The column of seconds on the left, marked ″ at the top, gives the total number of seconds in all angles between 0° and 3°, at intervals of 1 minute. Thus, on page 43, the number 6,360 in the column of seconds is horizontally opposite 46 in the minute column, and is, therefore, the total number of seconds in 1° 46′.

(*b*) The column headed S T, between the sine and the tangent column, contains the values of log tan A − log A'', and log sin A − log A'' for all values of A between 0° and 3°, varying from minute to minute; A'' is the total number of seconds in the angle A. The first four figures of these differences are common to the tangent and the sine and are printed near the head of the column; the other two figures are printed under S for the sine and under T for the tangent. The two figures corresponding to any angle are horizontally opposite the total number of seconds in the angle, this total number of seconds being given in the left-hand column. Thus, for 1° 45′ (= 6,300″), the value of *S*, or of log sin 1° 45′ − log 6,300, is $\bar{6}.68551$; and the value of *T*, or of log tan 1° 45′ − log 6,300, is $\bar{6}.68571$.

(*c*) Next to the cotangent column, there is a column marked C, containing the values of $-T$. The first four figures of these values are common to all angles between 0° and 3°, and are printed but once; the other two are printed horizontally opposite the number of seconds in the corresponding angles. Thus, for 1° 51′ (= 6,660″), the value of *C* is 5.31427. The values of *S*, *T*, and *C* will here be referred to as **corrections.**

2. To Find the Logarithmic Sine or Tangent of an Angle Between 0° and 3°.—If there are no odd seconds in the angle, the logarithm may be at once taken from the table, as in *Plane Trigonometry*, Part 1. Here it will be assumed that the angle contains a number of odd seconds. Let the angle be denoted by *A*, and the total number of seconds in it by *A″*; that is, let *A″* be the angle reduced to seconds. (See Art. **1.**)

Rule.—*Open the table at the page headed by the number of degrees in the given angle. Look in the minute column for the number of minutes nearest* (*whether greater or less*) *to the number of odd minutes and seconds in the given angle.* (*Thus, if the given angle is 2° 36′ 40″, look for 2° 37′; if the given angle is 2° 36′ 21″, look for 2° 36′.*) *Take from the column headed* S T *the correction horizontally opposite the number of minutes found as just described, using the correction under* S *for the sine, and that under* T *for the tangent. Look in the column of seconds at the left of the page for the number horizontally opposite the number of minutes in the given angle, and to it add the number of odd seconds in that angle. The result will be the total number of seconds* (*A″*) *in the given angle. Find the logarithm of this number of seconds from the table of logarithms of numbers. Add to this logarithm the correction found as above. The result will be the required logarithmic sine or tangent, according to the correction used.*

Example 1.—To find the logarithmic sine of 1° 3′ 45″ (= A).

Solution.—Opening the table at page 43 (headed 1°), we look for 4′ in the minute column, since 3′ 45″ is nearer to 4′ than to 3′. Horizontally opposite 4, and in the column headed S T, the sine correction $\bar{6}.68555$ (= S) is found. We now look in the minute column for the number of minutes (3) in the given angle; horizontally opposite it in the left-hand column is the number 3,780, number of seconds in 1° 3′; adding 45″, we obtain 3,825 (= A'') for the total number of seconds in the given angle.

$$\log A'' = \log 3{,}825 = 3.58263$$
$$S = \bar{6}.68555$$
$$\log \sin A = \bar{2}.26818$$

that is, $\log \sin 1° \, 3' \, 45'' = \bar{2}.26818$. Ans.

Example 2.—To find the logarithmic tangent of 2° 36′ 17″.

Solution.—On page 44, the correction for the tangent, opposite 36′, is $\bar{6}.68587$ (= T). Number of seconds opposite 36′ in the left-hand column, 9,360; $A'' = 9{,}360 + 17 = 9{,}377$.

$$\log 9{,}377 = 3.97206$$
$$T = \bar{6}.68587$$
$$\log \tan 2° \, 36' \, 17'' = \bar{2}.65793. \quad \text{Ans.}$$

3. To Find the Logarithmic Cotangent of an Angle Between 0° and 3°.

Rule.—*Find C, A'', and log A'' exactly as in the last article, C being taken from the correction column next to the cotangent column. Subtract log A'' from C. The result will be the required logarithmic cotangent.*

Example.—To find the logarithmic cotangent of 1° 52′ 37″.

Solution.—On page 43, the correction under C, and horizontally opposite 53′, is 5.31427; $A'' = 6{,}720 + 37 = 6{,}757$.

$$C = 5.31427$$
$$\log A'' = \log 6{,}757 = 3.82975$$
$$C - \log A'' = 1.48452$$

that is, $\log \cot 1° \, 52' \, 37'' = 1.48452$. Ans.

4. To Find the Logarithmic Tangent, Cosine, or Cotangent of an Angle Between 87° and 90°.—These functions also are to be taken from the first three pages of the table of logarithmic functions. The simplest way to proceed is to subtract the angle from 90° and look for the corresponding complementary function as explained in Arts. **2** and **3.** Thus, log cos 88° 55′ 38″ is obtained by looking for log sin (90° − 88° 55′ 38″) = log sin 1° 4′ 22″.

EXAMPLES FOR PRACTICE

1. Find the logarithmic sine of 1° 6′ 19″. Ans. $\bar{2}.28532$
2. Find the logarithmic sine of 0° 2′ 41″. Ans. $\bar{4}.89240$
3. Find the logarithmic tangent of 2° 56′ 57″. Ans. $\bar{2}.71196$
4. Find the logarithmic cotangent of 1° 30′ 18″. Ans. 1.58049
5. Find the logarithmic cosine of 88° 50′ 49″. Ans. $\bar{2}.30370$
6. Find the logarithmic tangent of 89° 3′ 9″. Ans. 1.78151
7. Find the logarithmic cotangent of 88° 0′ 25″. Ans. $\bar{2}.54157$

---

5. To Find the Angle Corresponding to a Given Logarithmic Function, When the Function Lies Between Two of the Functions in the First Three Pages of the Table.—I. *Sine and Tangent.*—As explained in Art. **1,** log sin $A = S + \log A''$; therefore,

$$\log A'' = \log \sin A - S \qquad (1)$$

Likewise, when log tan A is given,

$$\log A'' = \log \tan A - T \qquad (2)$$

From these formulas is derived the following

Rule.—*Find in the table the logarithm nearest to the given one. Take the correction horizontally opposite this logarithm, and subtract it from the given logarithm. The result will be the logarithm of the total number of seconds* (A'') *in the given angle. Find the number corresponding to this logarithm, and reduce it to degrees, minutes, and seconds.*

It is here assumed that the given function lies between two functions in the column marked log sin or log tan, as the case may be, at the top. If the names of the functions are at the bottom, the sine should be treated as in *Plane*

Trigonometry, Part 1; the tangent should be treated as if it were a cotangent, according to the directions to be given presently, and when the angle corresponding to that cotangent is found, it should be subtracted from 90°.

II. *Cotangent.*—Since log cot $A = C - \log A''$ (Art. **3**), we have

$$\log A'' = C - \log \cot A \qquad \textbf{(3)}$$

From this formula is derived the following

Rule.—*Find in the table the logarithmic function nearest the given cotangent. Take from the* C *column the correction horizontally opposite the logarithm just found, and from it subtract the given logarithmic cotangent. The result will be the logarithm of the total number of seconds in the angle.*

Here, as before, it is assumed that the given cotangent lies between two of those marked log cot at the top. If it lies between two logarithms in the column marked log cot at the bottom, it should be treated as if it were a tangent, and having found the angle corresponding to this tangent, it should be subtracted from 90° to obtain the required angle.

III. *Cosine.*

Rule.—*If the given cosine lies between two of those in the column headed log cos, apply the general rule given in Plane Trigonometry, Part 1. If it lies between two of the logarithms in the column marked log cos at the bottom, treat it as if it were a sine, find the angle corresponding to that sine as above, and subtract the result from 90°.*

EXAMPLE 1.—To find the angle whose logarithmic tangent is $\bar{2}.32803$.

SOLUTION.—The logarithmic tangent nearest to $\bar{2}.32803$ is $\bar{2}.32711$, found in the column headed log tan on page 43. The T correction horizontally opposite $\bar{2}.32711$ is $\bar{6}.68564$.

$$\begin{aligned} \log \tan A &= \bar{2}.32803 \\ T &= \underline{\bar{6}.68564} \\ \log A'' &= 3.64239 \end{aligned}$$

From the table of logarithms of numbers,

$$A'' = 4{,}389'' = 1^\circ\ 13'\ 9''. \quad \text{Ans.}$$

EXAMPLE 2.—To find the angle whose logarithmic cotangent is 2.49567.

SOLUTION.—The nearest logarithmic cotangent found in the table is 2.49488. The number opposite this logarithm in the C column is 5.31442.

$$C = 5.31442$$
$$\log \cot A = 2.49567$$
$$\log A'' = 2.81875;$$
$$A'' = 659'' = 0°\ 10'\ 59''. \quad \text{Ans.}$$

NOTE.—Angles are here given to the nearest whole second.

EXAMPLE 3.—To find the angle whose logarithmic cosine is $\bar{2}.63723$.

SOLUTION.—The nearest logarithm, $\bar{2}.63678$, is found on page 44, in the column headed log sin. The given function is, therefore, to be treated as if it were a logarithmic sine, and the angle A, corresponding to this sine is to be subtracted from 90° to obtain the required angle A. The correction horizontally opposite $\bar{2}.63678$, in the S column, is $\bar{6}.68544$.

$$\log \sin A_1 = \bar{2}.63723$$
$$S = \bar{6}.68544$$
$$\log A_1'' = 3.95179;$$
$$A_1 = 8{,}949'' = 2°\ 29'\ 9''$$
$$A = 90° - 2°\ 29'\ 9'' = 87°\ 30'\ 51''. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

Verify the following values:

(a) $\bar{2}.17645 = \log \sin 0°\ 51'\ 37''$ (e) $\bar{2}.48790 = \log \cot 88°\ 14'\ 19''$

(b) $\bar{3}.94316 = \log \sin 0°\ 30'\ 10''$ (f) $2.47608 = \log \cot 0°\ 11'\ 29''$

(c) $\bar{2}.65783 = \log \cos 87°\ 23'\ 36''$ (g) $1.31009 = \log \tan 87°\ 11'\ 48''$

(d) $\bar{2}.58349 = \log \tan 2°\ 11'\ 41''$ (h) $\bar{3}.95377 = \log \cos 89°\ 29'\ 6''$

6. Use of the Column of Seconds for Obtaining the Angle Corresponding to a Given Function.—In order to avoid confusing the student by too many rules, the reduction of A'' to degrees, minutes, and seconds was, in the preceding articles, effected by the ordinary rules of arithmetic, without any reference to the table. The following is a more expeditious method:

Let the given function lie between the functions of two consecutive angles, A_1 and $A_1 + 1'$. Then, the degrees and minutes in the required angle are those in A_1, and may be at once written down. The number in the column of seconds on the left, horizontally opposite the number of minutes in A_1, gives the total number of seconds in A_1. Denoting that number by A_1'' and the number of odd seconds in the required angle by s, we have

$$s = A'' - A_1''$$

EXAMPLE.—To find the angle whose logarithmic tangent is $\overline{2}.30217$.

SOLUTION.—The given function lies between $\overline{2}.29629$ and $\overline{2}.30263$. The angle corresponding to the first of these two functions is 1° 8′ ($= A_1$); $A_1'' = 4{,}080''$.

$$\log \tan A = \overline{2}.30217$$
$$T = \overline{6}.68563$$
$$\log A'' = 3.61654;\ A'' = 4{,}136$$
$$s = A'' - A_1'' = 4{,}136 - 4{,}080 = 56''$$
$$A = A_1 + s = 1°\ 8'\ 56''. \quad \text{Ans.}$$

The subtraction $A'' - A_1''$ can usually be effected mentally.

EXAMPLES FOR PRACTICE

Apply the method just described to the Examples for Practice given after Art. 4.

GENERAL TRIGONOMETRIC FORMULAS

ANGLES AND THEIR TRIGONOMETRIC FUNCTIONS

7. Angle of Any Magnitude.—In trigonometry, an angle is considered as being generated by a straight line turning about one of its ends, which is the vertex of the angle. In this motion, any point in the turning line describes a circular arc, whose number of degrees is the measure of the angle. The turning line is called the **generating line.** The position that this line occupies before it begins to turn, and from which arcs are measured, is called the **initial line,** or the **initial position** of the generating line; and the position it occupies after turning through a certain angle

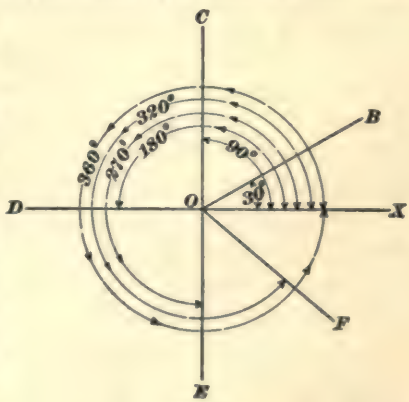

FIG. 1

is called the **final position.** In Fig. 1, for example, the initial position of the generating line is OX. The turning is supposed to take place about the point O and in a direction opposite to that in which the hands of a clock move. When the line turns from the position OX to the final positions OB, OC, OD, OE, OF, it generates angles of 30°, 90°, 180°, 270°, 320°, respectively, as indicated on the figure. If the line makes a complete turn, so that its final position coincides with its initial position OX, the angle generated is 360°.

8. Positive and Negative Angles.—When an angle is described by a line turning in a direction contrary to that in which the hands of a watch move, the angle is considered **positive;** if described in the opposite direction, it is considered **negative.** Referring to Fig. 2, the angle XOB, whose supplement is A, may be regarded as having been described in any of the following manners:

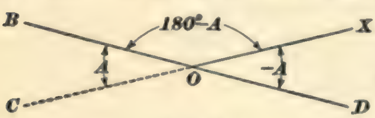

FIG. 2

(*a*) By turning the generating line about O from the position OX in the positive direction through $(180 - A)$ degrees to the position OB.

(*b*) By turning the generating line about O in a positive direction through an angle of 180°, when it will be in the position OC, and then turning it back from OC in the negative direction through the angle $-A$ (negative, because turned in the negative direction) into the position OB.

(*c*) By turning the generating line about O in the negative direction through the angle $-A$, into the position OD, and then turning it back in the positive direction through 180° into the position OB.

It is to be noticed that, however the angle $(180° - A)$ may be regarded as described, the resulting angle XOB is the same.

9. Quadrants.—Let OX, Fig. 3, be the initial position of the generating line, and OM_1, OM_2, OM_3, OM_4 final positions, determining, respectively, the angles A_1, A_2, A_3, A_4, all measured from OX upwards and toward the left. Producing XO and drawing through O a perpendicular YY' to OX, the plane of the figure is divided into four right angles, called **quadrants.** Taking them in order, following the direction in which positive angles are reckoned, they are distinguished as follows: XOY is the **first quadrant;** YOX', the **second quadrant;** $X'OY'$, the **third quadrant;** and $Y'OX$, the **fourth quadrant.**

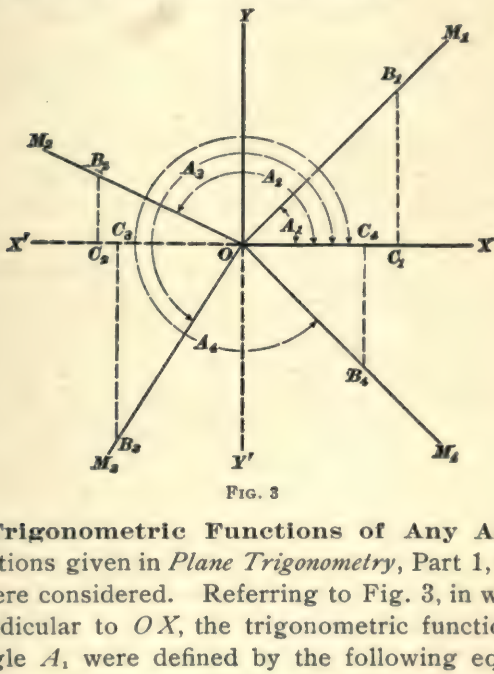

FIG. 3

10. Trigonometric Functions of Any Angle.—In the definitions given in *Plane Trigonometry*, Part 1, only acute angles were considered. Referring to Fig. 3, in which $B_1 C_1$ is perpendicular to OX, the trigonometric functions of the acute angle A_1 were defined by the following equations:

$$\sin A_1 = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{B_1 C_1}{O B_1} \qquad \tan A_1 = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{B_1 C_1}{O C_1}$$
$$\cos A_1 = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{O C_1}{O B_1} \qquad \cot A_1 = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{O C_1}{B_1 C_1}$$
$$\sec A_1 = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{O B_1}{O C_1} \qquad \csc A_1 = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{O B_1}{B_1 C_1}$$

These formulas serve as the definitions of the trigonometric functions of any angle; that is, the **sine** of any angle is the ratio of the side opposite to the hypotenuse; the **tangent** is the ratio of the side opposite to the side adjacent, etc. But, in order that these definitions may be correct, it is necessary to apply to them some algebraic rules relating to signs.

In Fig. 3, the hypotenuse used for the determination of the functions of A_1 is any portion OB_1 of the side OM_1, which is the final position of the generating line. From B_1, a perpendicular B_1C_1 is drawn on the initial line OX, thus determining the right triangle OB_1C_1. The length of the perpendicular B_1C_1, which is the side opposite the vertex of the angle, is the distance of B_1 above the initial line OX, and the length of the adjacent side OC_1 is the distance of the point B_1 to the right of the vertex, measured along the initial line; or, what is the same thing, OC_1 is the distance of B_1C_1 from the vertex, measured toward the right.

Consider now the angle XOM_2, or A_2, in which the final position OM_2 of the generating line lies in the second quadrant. As before, the hypotenuse to be used in the definitions of the trigonometric functions of A_2 is any portion OB_2 of the side OM_2, which is the final position of the generating line. As before, also, a perpendicular from B_2 is drawn on the initial line OX; but, in this case, the perpendicular falls on OX produced. In the right triangle OB_2C_2, the perpendicular B_2C_2 is the side opposite the vertex of the angle A_2, and OC_2 is the side adjacent. It should be noted very particularly that the terms *side opposite* and *side adjacent* are used to describe the positions of the legs of the right triangle with reference to the *vertex* of the angle considered, not to the angle itself. Thus, B_2C_2 is not opposite the angle A_2, but opposite the vertex O of that angle. The length of the side opposite, B_2C_2, measures the distance of B_2 above the initial line; and the length of OC_2, or the side adjacent, measures the distance of the opposite side B_2C_2 to the left of the vertex; or, in the language of algebra, it may be said that $-OC_2$ is the distance of B_2C_2 to the *right* of O.

Having defined the cosine of any angle as the ratio of the side adjacent to the hypotenuse, and the side adjacent as the distance of the side opposite from the vertex, measured toward the right of the vertex, it is necessary, when the side opposite is to the left of the vertex, to consider its distance from the vertex, or the side adjacent, as negative. This is in accordance with the general principle of algebra, that, if distances counted in one direction are treated as positive, distances in the opposite direction must be treated as negative. In the triangle $O B_2 C_2$, therefore, $O C_2$ should be treated as negative, and therefore, the cosine of A_2 is $\frac{-O C_2}{O B_2}$.

Considering now the angle A_3, the hypotenuse is, as above, any portion $O B_3$ of the side $O M_3$, which is the final position of the generating line. From B_3, the perpendicular $B_3 C_3$ on the initial line (produced) is drawn, and thus a right triangle is determined, in which $B_3 C_3$ is the side opposite, and $O C_3$ the side adjacent. As previously explained, $O C_3$ should be treated as negative. The opposite side $B_3 C_3$, which is the distance of B_3 below the initial line, should also be treated as negative; for if distances above the initial line are treated as positive, those below the initial line must be treated as negative.

Finally, in the angle A_4, which terminates in the fourth quadrant, $O C_4$, the side adjacent, is positive, while $B_4 C_4$, the side opposite, is negative.

The foregoing explanations may be summed up as follows: The side opposite is positive or negative according as the hypotenuse is above or below the initial line. The side adjacent is positive or negative according as it extends toward the right or toward the left of the vertex. The hypotenuse is always positive.

11. Algebraic Signs of the Functions.—Referring again to Fig. 3, it will be observed that, for any angle, as A_1, terminating in the first quadrant, both the side adjacent and the side opposite, or $O C_1$ and $B_1 C_1$, are positive, and therefore all the functions are positive; for any angle, as A_2, terminating in the second quadrant, the side adjacent, or $O C_2$, is negative, and the side opposite, or $B_2 C_2$, is positive. Therefore,

$$\sin A_2 = \frac{+B_2C_2}{+OB_2}, \text{ positive} \qquad \tan A_2 = \frac{+B_2C_2}{-OC_2}, \text{ negative}$$

$$\cos A_2 = \frac{-OC_2}{+OB_2}, \text{ negative} \qquad \sec A_2 = \frac{+OB_2}{-OC_2}, \text{ negative}$$

$$\csc A_2 = \frac{+OB_2}{+B_2C_2}, \text{ positive}$$

The signs of the functions of angles terminating in the third and in the fourth quadrant are similarly determined. The results are tabulated below.

TABLE I

Function	Quadrant			
	First	Second	Third	Fourth
	Sign of Function			
Sine	+	+	−	−
Cosine	+	−	−	+
Tangent	+	−	+	−
Cotangent	+	−	+	−
Secant	+	−	−	+
Cosecant	+	+	−	−

12. Trigonometric Functions of 0° and 90°.—In the right triangles ACB, Fig. 4, the hypotenuse AB may be taken to have any value whatever. It is evident that BC, the side opposite, decreases as the angle CAB decreases, and becomes zero when the angle becomes zero; and that BC coincides with the hypotenuse AB when the angle CAB is 90°. Again, AC, the adjacent side, increases as the angle decreases, and is equal to the hypotenuse AB when the angle CAB is 0°. Also, AC becomes zero when CAB is 90°. Now, from the definitions of the trigonometric functions,

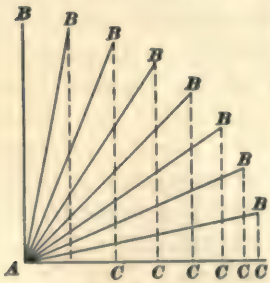

FIG. 4

$$\sin CAB = \frac{\text{side opposite}}{\text{hypotenuse}}, \text{ whence} \begin{cases} \sin \ \ 0^\circ = \dfrac{0}{AB} = 0 \\ \sin 90^\circ = \dfrac{AB}{AB} = 1 \end{cases}$$

$$\cos CAB = \frac{\text{side adjacent}}{\text{hypotenuse}}, \text{ whence} \begin{cases} \cos \ \ 0^\circ = \dfrac{AB}{AB} = 1 \\ \cos 90^\circ = \dfrac{0}{AB} = 0 \end{cases}$$

In like manner,

$$\tan 0^\circ = \frac{0}{AC} = 0 \qquad \tan 90^\circ = \frac{BC}{0} = \infty$$

$$\cot 0^\circ = \frac{AC}{0} = \infty \qquad \cot 90^\circ = \frac{0}{CB} = 0$$

NOTE.—The cotangent of CAB is equal to $\frac{CA}{CB}$. Now, as the angle decreases, the side CB becomes less and less, and it is evident that, as the denominator of a fraction becomes less and less, the numerator remaining the same, the value of the fraction increases. As the denominator decreases indefinitely, the value of the fraction increases indefinitely, and when the value of the fraction exceeds any known quantity, however great, it is said to be infinite. The sign ∞ is used to express an infinite number.

13. Functions of $(180° - A)$.—Let XOM, Fig. 5, be any angle, and A $(= MOX')$ its supplement. Draw OM' making with OX an angle equal to A, as shown. Take any part OB of OM for the hypotenuse, and draw BC perpendicular to OX produced; draw BB' parallel to OX, and $B'C'$ perpendicular to OX. Then, $BC = B'C'$; $OB = OB'$; $OC = -OC'$ (Art. **10**); and, by the definitions of the functions,

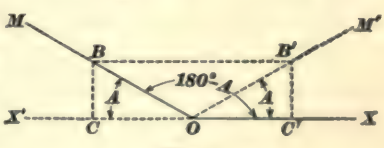

FIG. 5

$$\sin XOM = \frac{BC}{OB} = \frac{B'C'}{OB'} = \sin A$$

$$\cos XOM = \frac{OC}{OB} = \frac{-OC'}{OB'} = -\cos A$$

that is,

$$\sin (180^\circ - A) = \sin A \qquad (1)$$

$$\cos (180^\circ - A) = -\cos A \qquad (2)$$

Similarly, $\tan(180° - A) = -\tan A$ (3)

$$\cot(180° - A) = -\cot A \qquad (4)$$

These relations are especially useful for finding the logarithmic functions of angles greater than 90°, since these functions are arithmetically equal to those of the supplements of the angles; that is, when signs are disregarded, any function of an angle and that of its supplement are equal. For example, $\sin 105° = \sin(180° - 105°) = \sin 75°$; $\cos 105° = -\cos(180° - 105°) = -\cos 75°$.

14. Functions of $(90° + A)$.—By formula **1** of Art. **13**,

$$\sin(90° + A) = \sin[180° - (90 + A)] = \sin(90° - A)$$

or, since $\sin(90° - A) = \cos A$,

$$\sin(90° + A) = \cos A \qquad (1)$$

The following formulas may be derived in a similar manner:

$$\tan(90° + A) = -\cot A \qquad (2)$$

$$\cos(90° + A) = -\sin A \qquad (3)$$

$$\cot(90° + A) = -\tan A \qquad (4)$$

15. Functions of Negative Angles.—The complement of an angle is the algebraic difference between the angle and 90°. If the angle is greater than 90°, its complement is negative. Thus, the complement of 95° is $90° - 95° = -5°$. The cofunctions of an angle are the corresponding fundamental functions of its complement, whether that complement be positive or negative. Thus, $\cos 85° = \sin(90° - 85°) = \sin 5°$; $\cos 95° = \sin(90° - 95°) = \sin(-5°)$. Similarly, $\sin 95° = \cos(90° - 95°) = \cos(-5°)$. It is, therefore, necessary to know how to determine the functions of negative angles.

If $90° + A$ is any angle, its complement is $90° - (90° + A) = -A$; and, therefore,

$$\cos(90° + A) = \sin(-A), \quad \cot(90° + A) = \tan(-A)$$
$$\sin(90° + A) = \cos(-A), \quad \tan(90° + A) = \cot(-A)$$

whence, replacing the values of cos $(90° + A)$, cot $(90° + A)$, etc. from the preceding article,

$$\sin(-A) = -\sin A \qquad (1)$$

$$\tan(-A) = -\tan A \qquad (2)$$

$$\cos(-A) = \cos A \qquad (3)$$

$$\cot(-A) = -\cot A \qquad (4)$$

ADDITION OF ANGLES

16. To Express the Sine or Cosine of the Sum or Difference of Two Angles in Terms of the Sine and Cosine of the Angles.—The following formulas are fundamental; being of frequent occurrence, they are very important, and should be committed to memory:

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \qquad (1)$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \qquad (2)$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B \qquad (3)$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B \qquad (4)$$

NOTE.—The derivation of these formulas is given in the Appendix at the end of this Section, under the Roman numeral I. That Appendix contains this and a few other demonstrations that are comparatively laborious and may be found irksome by some. They are not essential to the understanding of the formulas, and the student is not required to learn them. He is, however, advised to peruse them carefully, as they are good exercises in the handling and transforming of both algebraic and trigonometric expressions.

These formulas are not used, as they seem to imply, to determine the sine or the cosine of the sum or difference of two angles, when the sine and cosine of those angles are given. They can be used for this purpose, but there would be no advantage in so doing. Their main value consists in their application to transforming complicated trigonometric expressions into simpler ones. The student will often have occasion to employ them in this manner. In order that he may have an idea of this application of the formulas, two examples are given here.

EXAMPLE 1.—To determine the angle A from the relation

$$\frac{\sin(A+28°)}{\sin A} = .95$$

SOLUTION.—Applying formula **1**, we have

$$\frac{\sin(A+28°)}{\sin A} = \frac{\sin A \cos 28° + \cos A \sin 28°}{\sin A}$$

$$= \frac{\sin A \cos 28°}{\sin A} + \frac{\cos A \sin 28°}{\sin A} = \cos 28° + \cot A \sin 28°$$

replacing $\frac{\cos A}{\sin A}$ by its equal $\cot A$ (see *Plane Trigonometry*, Part 1). Substituting this value of the quotient $\frac{\sin(A+28°)}{\sin A}$ in the given equation, we have,

$$\cos 28° + \cot A \sin 28° = .95$$

whence $\cot A = \frac{.95 - \cos 28°}{\sin 28°} = \frac{.95 - .88295}{.46947} = .14282$

and, therefore, $A = 81° 52' 19''$. Ans.

EXAMPLE 2.—To transform the expression $\tan A + \tan B$ into the expression $\frac{\sin(A+B)}{\cos A \cos B}$.

NOTE.—Transformations of this kind are very often useful, when logarithms are employed. Thus, if $\tan A + \tan B$ were to be multiplied by 39.578, it would be necessary first to find the natural tangent of A, then that of B, add the two together, take the logarithm of the sum thus obtained, and add this logarithm to that of 39.578. If, however, the expression $\frac{\sin(A+B)}{\cos A \cos B}$ is used, the logarithms of $\sin(A+B)$, $\cos A$ $\cos B$ can be taken from the table, and the operation performed without having recourse to natural functions, which are often inconvenient.

SOLUTION.—We have (*Plane Trigonometry*, Part 1),

$$\tan A + \tan B = \frac{\sin A}{\cos A} + \frac{\sin B}{\cos B} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}$$

According to formula **1**, the numerator of this last fraction is equal to $\sin(A+B)$. Therefore,

$$\tan A + \tan B = \frac{\sin(A+B)}{\cos A \cos B}. \quad \text{Ans.}$$

17. Sine and Cosine of 2 A and of $\frac{1}{2}$ A.—From the formulas for the sine and cosine of the sum of two angles, the following are deduced:

$$\sin 2A = 2 \sin A \cos A \qquad (1)$$

$$\cos 2A = \cos^2 A - \sin^2 A \qquad (2)$$

$$\cos 2A = 1 - 2\sin^2 A \qquad (3)$$

$$\cos 2A = 2\cos^2 A - 1 \qquad (4)$$

$$\sin A = 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A \qquad (5)$$

$$\cos A = \cos^2 \tfrac{1}{2} A - \sin^2 \tfrac{1}{2} A \qquad (6)$$

$$\cos A = 1 - 2 \sin^2 \tfrac{1}{2} A \qquad (7)$$

$$\cos A = 2 \cos^2 \tfrac{1}{2} A - 1 \qquad (8)$$

As in the case of formulas **1** to **4**, Art. **16**, these formulas are used mainly for the purposes of transformation. They are very simply derived as follows:

When B is made equal to A, formula **1**, Art. **16**, becomes

$$\sin (A + A) = \sin A \cos A + \cos A \sin A$$

that is, $\sin 2 A = 2 \sin A \cos A$

Similarly, formula **2**, Art. **16**, becomes

$$\cos (A + A) = \cos A \cos A - \sin A \sin A$$

that is, $\cos 2 A = \cos^2 A - \sin^2 A$

Formula **3** follows from this, by writing $1 - \sin^2 A$ instead of $\cos^2 A$ (since $\sin^2 A + \cos^2 A = 1$); and formula **4**, by writing $1 - \cos^2 A$ instead of $\sin^2 A$.

Formulas **1** to **4** give the sine and cosine of twice any angle in terms of the sine and cosine of the angle. If the angle is denoted by $\frac{1}{2} A$, twice the angle will be A, and formulas **1** to **4** take the forms of formulas **5** to **8**.

OBLIQUE TRIANGLES

FUNDAMENTAL PRINCIPLES

NOTE.—For the general method of marking and naming the sides and angles of a triangle, see *Plane Trigonometry*, Part 1.

18. Principle of Sines.—*In any triangle, the sides are proportional to the sines of the opposite angles.* That is,

$$\frac{a}{b} = \frac{\sin A}{\sin B}, \quad \frac{a}{c} = \frac{\sin A}{\sin C}, \quad \frac{b}{c} = \frac{\sin B}{\sin C}$$

Let $A B C$, Fig. 6, be any triangle and p the perpendicular from C on the opposite side. Then, in (a), the right triangles $A C D$ and $B C D$ give, respectively,

$$p = b \sin A, p = a \sin B$$

whence, putting the two values of p equal to each other,

$$a \sin B = b \sin A$$

and, therefore, dividing by $b \sin B$,

$$\frac{a}{b} = \frac{\sin A}{\sin B}$$

In (b), the right triangles ACD and BCD give, respectively,

$$p = b \sin A, p = a \sin CBD$$

whence,

$$a \sin CBD = b \sin A$$

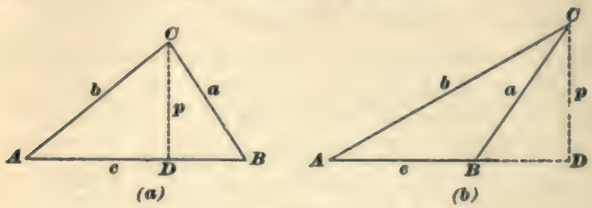

FIG. 6

But, as $CBD = 180° - B$, we may write $\sin B$ instead of $\sin CBD$ (Art. **13**), and, therefore,

$$a \sin B = b \sin A$$

whence, as before,

$$\frac{a}{b} = \frac{\sin A}{\sin B} \qquad (1)$$

By drawing a perpendicular from B on AC, and reasoning in the same manner, it may be shown that

$$\frac{a}{c} = \frac{\sin A}{\sin C} \qquad (2)$$

Similarly,

$$\frac{b}{c} = \frac{\sin B}{\sin C}$$

By transforming equation (1), we obtain

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

and by a similar transformation of equation (2),

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

We have, therefore,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The principle of sines may, then, be stated in this form: *In every triangle, the quotient obtained by dividing the length of any side by the sine of the opposite angle is the same, whatever the side taken.*

This quotient is called the **modulus** of the triangle, and will here be denoted by M. The modulus can be found when any of the sides and the opposite angle are known.

The principle of sines is one of the most important in trigonometry, and both forms in which it is stated in this article should be committed to memory.

19. The Cosine Principle.—*In any triangle, the square of one side is equal to the sum of the squares of the other two sides minus twice the product of these two sides and the cosine of their included angle.* That is (Fig. 6),

$$a^2 = b^2 + c^2 - 2\,b\,c \cos A$$
$$b^2 = a^2 + c^2 - 2\,a\,c \cos B$$
$$c^2 = a^2 + b^2 - 2\,a\,b \cos C$$

These formulas are derived in Appendix II.

20. Principle of Tangents.—*The sum of any two sides of a triangle is to their difference as the tangent of half the sum of the opposite angles is to the tangent of half their difference.* That is (Fig. 6),

$$\frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$$

The derivation of this formula is given in Appendix III. The student should have no difficulty in committing the formula to memory, as its symmetry makes it very easy to remember.

SOLUTION OF OBLIQUE TRIANGLES

21. The solution of oblique triangles is treated under four cases:

Case I: Given Two Sides and the Included Angle. Let a, b, and C, Fig. 6, be given and A, B, and c be required. Of the two methods given below, the first is preferable in most cases.

First Method.—From the formula in Art. **20,** the following is readily derived:

$$\tan \tfrac{1}{2}(A - B) = \frac{a-b}{a+b} \tan \tfrac{1}{2}(A + B) \qquad (1)$$

Now, since $A + B + C = 180°$, we have also,

$$A + B = 180° - C; \text{ and } \tfrac{1}{2}(A + B) = \tfrac{1}{2}(180° - C)$$
$$= 90° - \tfrac{1}{2}C$$

Therefore, $\frac{1}{2}C$ is the complement of $\frac{1}{2}(A + B)$, and hence, $\tan \frac{1}{2}(A + B) = \cot \frac{1}{2}C$. Substituting this value in equation (1), the following formula is derived:

$$\tan \tfrac{1}{2}(A - B) = \frac{a - b}{a + b} \cot \tfrac{1}{2}C \qquad \textbf{(1)}$$

If the student remembers the formula in Art. **20**, or the principle of tangents, he will have no difficulty in remembering this formula, which is derived from the formula in Art. **20**, by simply writing $\cot \frac{1}{2}C$ instead of $\tan \frac{1}{2}(A + B)$.

From this formula $\frac{1}{2}(A - B)$ can be found. Let this value of $\frac{1}{2}(A - B)$ be denoted by D. We have also, as explained above, $\frac{1}{2}(A + B) = \frac{1}{2}(180° - C) = 90° - \frac{1}{2}C$.

$$\tfrac{1}{2}(A + B) = 90° - \tfrac{1}{2}C \qquad (2)$$
$$\tfrac{1}{2}(A - B) = D \qquad (3)$$

Adding equations (2) and (3) gives

$$A = (90° - \tfrac{1}{2}C) + D$$

Subtracting equation (3) from (2) gives

$$B = (90° - \tfrac{1}{2}C) - D$$

Knowing A and B, the side c may be found from the relation (Art. **18**),

$$\frac{c}{\sin C} = \frac{a}{\sin A}, \text{ which gives } c = \frac{a \sin C}{\sin A}$$

It is, however, more convenient to find c from the following formula, the derivation of which is given in Appendix IV:

$$c = \frac{(a - b) \cos \tfrac{1}{2}C}{\sin \tfrac{1}{2}(A - B)} \qquad \textbf{(2)}$$

It will be noticed that, for calculating $\tan \frac{1}{2}(A - B)$, the logarithms of $(a - b)$ and $\cot \frac{1}{2}C$ have to be found. The logarithm of $\cos \frac{1}{2}C$ may be taken out of the table at the same time as that of $\cot \frac{1}{2}C$. Also, when the angle $\frac{1}{2}(A - B)$ is taken from the table, its logarithmic sine should be taken at the same time. This greatly simplifies the application of formula **2.**

Second Method.—The third side c can be found directly from the formula in Art. **19**, which gives

$$c = \sqrt{a^2 + b^2 - 2\,a\,b\cos C}$$

Then, by the principle of sines,

$$\sin A = \frac{a \sin C}{c},\ \sin B = \frac{b \sin C}{c}$$

This method is of value when the only required part is the side c, especially if a and b are convenient numbers to square.

EXAMPLE 1.—In a triangle, $a = 17$ feet, $b = 12$ feet, and the included angle $C = 59° 23'$. To find the other parts of the triangle.

SOLUTION.—Here $\frac{1}{2}\ C = 29° 41' 30''$; $a + b = 17 + 12 = 29$, and $a - b = 17 - 12 = 5$. Then, by the first method,

$$\tan \tfrac{1}{2}(A - B) = \frac{5}{29} \times \cot 29° 41' 30''$$

$$\begin{aligned} \log 5 &= .69897 \\ \log 29 &= 1.46240 \\ &\overline{\bar{1}.23657} \\ \log \cot 29° 41' 30'' &= .24397 \\ \log \tan \tfrac{1}{2}(A - B) &= \bar{1}.48054 \\ D = \tfrac{1}{2}(A - B) &= 16° 49' 25''; \end{aligned}$$

$$\begin{aligned} \log 5 &= .69897 \\ \log \cos 29° 41' 30'' &= \bar{1}.93887 \\ &\ .63784 \\ \log \sin D &= \bar{1}.46154 \\ \log c &= 1.17630 \\ c &= 15.007. \text{ Ans.} \end{aligned}$$

$$A = (90° - 29° 41' 30'') + 16° 49' 25'' = 77° 7' 55''. \text{ Ans.}$$
$$B = (90° - 29° 41' 30'') - 16° 49' 25'' = 43° 29' 5''. \text{ Ans.}$$

EXAMPLE 2.—Given $a = 10$, $b = 15$, and $C = 60°$; to find c.

SOLUTION.—By the second method,

$$c = \sqrt{10^2 + 15^2 - 2 \times 10 \times 15 \cos 60°}$$
$$= \sqrt{325 - 300 \times .5} = \sqrt{175} = 13.229 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1. Given $a = 37.46$ feet, $b = 59.17$ feet, and $C = 69° 13'$; find A, B, and c.
Ans. $\begin{cases} A = 37° 21' 30'' \\ B = 73° 25' 30'' \\ c = 57.72 \text{ ft.} \end{cases}$

2. Two sides of a triangle are, respectively, 687.64 and 319.58 feet long, and their included angle is 47° 15′ 8″; find the other two angles and the third side.
Ans. {Angles, 106° 14′ 56″ and 26° 29′ 56″; Third side = 525.97}

3. Given $c = 4$ chains, $a = 6$ chains, and $B = 45° 18'$; find b.
Ans. $b = 4.271$ ch.

4. Given $b = 43.16$ chains, $c = 51.29$ chains, and $A = 35° 8' 10''$; find B, C, and a.

Ans. $\begin{cases} B = 57° 13' 20'' \\ C = 87° 38' 30'' \\ a = 29.544 \text{ ch.} \end{cases}$

---

22. Case II: Given a Side and Two Angles.—Let c, A, and B be known, to find a, b, and C. The angle $C = 180° - A - B$. By the principle of sines,

$$\frac{a}{\sin A} = \frac{c}{\sin C}, \text{ whence } a = \frac{c}{\sin C} \sin A$$

Similarly,
$$b = \frac{c}{\sin C} \sin B$$

Since $\frac{c}{\sin C}$ is the modulus of the triangle (Art. 18), these formulas may be thus stated: *Any side of a triangle is equal to the modulus of the triangle multiplied by the sine of the angle opposite that side.*

EXAMPLE.—Given $a = 98.48$, $B = 60° 45'$, and $C = 39° 15'$; to find b, c, and A.

SOLUTION.— $A = 180° - (60° 45' + 39° 15') = 80°$. Ans.

$$M = \frac{98.48}{\sin 80°};\ b = \frac{98.48}{\sin 80°} \sin 60° 45';\ c = \frac{98.48}{\sin 80°} \sin 39° 15'$$

$\log 98.48 = 1.99335$
$\log \sin 80° = \bar{1}.99335$
$\log M = 2.00000$

$\log b = 1.94076$; $b = 87.248$. Ans.
$\log \sin 60° 45' = \bar{1}.94076$
$\log M = 2.00000$
$\log \sin 39° 15' = \bar{1}.80120$
$\log c = 1.80120$; $c = 63.27$. Ans.

NOTE.—Attention is called to the convenient way in which the work is here arranged. Having determined log M, this logarithm is copied, and then one of the logarithms to be added to it is written above it, the other under it, the addition being performed upwards in one case, and downwards in the other.

EXAMPLES FOR PRACTICE

1. Given $a = 45.39$ feet, $B = 38° 12'$, and $C = 11° 11' 34''$; find A, b, and c.

Ans. $\begin{cases} A = 130° 36' 26'' \\ b = 36.973 \text{ ft.} \\ c = 11.605 \text{ ft.} \end{cases}$

2. Given $c = 101.11$ chains, $C = 55° 55' 55''$, and $A = 10° 10' 10''$; find B, a, and b.

Ans. $\begin{cases} B = 113° 53' 55'' \\ a = 21.551 \text{ ch.} \\ b = 111.59 \text{ ch.} \end{cases}$

23. Case III: Given Three Sides.—Let a, b, and c be given, to find A, B, and C.

First Method.—The angles can be found directly from the cosine formulas (Art. **19**), which, being solved for cos A, cos B, and cos C, respectively, give

$$\left.\begin{aligned}\cos A &= \frac{b^2+c^2-a^2}{2\,bc}\\ \cos B &= \frac{a^2+c^2-b^2}{2\,ac}\\ \cos C &= \frac{a^2+b^2-c^2}{2\,ab}\end{aligned}\right\} \qquad (1)$$

These formulas are to be used when the numbers a, b, c are convenient to square; otherwise, they are too cumbersome, and those given below for the functions of half the angles should be employed. It is necessary to apply the formulas in determining only two of the angles, as the third follows from the relation $A + B + C = 180°$. As a check, however, the formulas should be applied to the third angle also.

It should be borne in mind that, if the cosine of an angle is found to be negative, this implies that the angle is obtuse (Art. **13**). In such case, the cosine is treated as positive, and the corresponding angle taken from the table is subtracted from 180° to obtain the required angle. Thus, if $\cos A = -.97030$, we look for the angle whose cosine is $+.97030$, which is 14°. Then, $A = 180° - 14° = 166°$.

EXAMPLE.—Given $a = 4$ inches, $b = 5$ inches, and $c = 7$ inches; to find A, B, and C.

SOLUTION.— $\cos A = \dfrac{b^2+c^2-a^2}{2\,bc} = \dfrac{5^2+7^2-4^2}{2\times5\times7} = \dfrac{58}{70} = .82857,$

and, therefore, $A = 34° 2' 53''$. Ans.

$$\cos B = \frac{a^2+c^2-b^2}{2\,ac} = \frac{4^2+7^2-5^2}{2\times4\times7} = \frac{40}{56} = .71429$$

and, therefore, $B = 44° 24' 54''$. Ans.

$$C = 180° - A - B = 101° 32' 13''. \text{ Ans.}$$

As a check, we have

$$\cos C = \frac{a^2+b^2-c^2}{2\,ab} = \frac{4^2+5^2-7^2}{2\times4\times5} = -\frac{8}{40} = -.20000$$

The angle whose cosine is .20000 is 78° 27′ 47″. Therefore, $C = 180° - 78° 27' 47'' = 101° 32' 13''$.

Second Method.—As said before, this method is to be applied when the operations required by formula **1** involve too much labor, which happens when the lengths of the given sides consist of three or more significant figures—the usual case. If the sum of the sides is denoted by $2s$, or half their sum by s, the angles A, B, C may be found by the following formulas, which are derived in Appendix V:

$$\left.\begin{aligned}\tan\tfrac{1}{2}A &= \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\\ \tan\tfrac{1}{2}B &= \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}\\ \tan\tfrac{1}{2}C &= \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}\end{aligned}\right\} \qquad (2)$$

$$\left.\begin{aligned}\cos\tfrac{1}{2}A &= \sqrt{\frac{s(s-a)}{bc}}\\ \cos\tfrac{1}{2}B &= \sqrt{\frac{s(s-b)}{ac}}\\ \cos\tfrac{1}{2}C &= \sqrt{\frac{s(s-c)}{ab}}\end{aligned}\right\} \qquad (3)$$

For angles differing but little from 90° (say between 85° and 90°), use the cosine formulas **3**; in all other cases, the tangent formulas **2**.

We have also,

$$\sin\tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}} \qquad (4)$$

with similar formulas for $\sin\frac{1}{2}B$ and $\sin\frac{1}{2}C$. These formulas are of value for deriving the tangent formulas **2**, as well as for deriving an expression for the area of a triangle when the sides are given. They may also be used instead of the tangent formulas **2** for the determination of the angles, but the latter are preferable.

EXAMPLE.—In the triangle ABC, $a = 567$ feet, $b = 736$ feet, and $c = 264$ feet; to find the angles A, B, and C.

SOLUTION.—The tangent formulas will be used.

To find A

$$\begin{array}{rlrl}
a = & 567 & \log(s-c) = & 2.71559 \\
b = & 736 & \log(s-b) = & 1.67669 \\
c = & 264 & & 4.39228 \\
2s = & 1{,}567 & \log s = & 2.89404 \\
s = & 783.5 & \log(s-a) = & 2.33546 \\
s-a = & 216.5 & & 5.22950 \\
s-b = & 47.5 & & 2)\bar{1}.16278 \\
s-c = & 519.5 & \log\tan\frac{1}{2}A = & \bar{1}.58139
\end{array}$$

$\frac{1}{2}A = 20° 52' 38''$, $A = 41° 45' 16''$. Ans.

To find B

$$\begin{array}{rl}
\log(s-a) = & 2.33546 \\
\log(s-c) = & 2.71559 \\
& 5.05105 \\
\log s = & 2.89404 \\
\log(s-b) = & 1.67669 \\
& 4.57073 \\
& 2)0.48032 \\
\log\tan\frac{1}{2}B = & 0.24016
\end{array}$$

$\frac{1}{2}B = 60° 5' 29''$; $B = 120° 10' 58''$ Ans.

To find C

$$\begin{array}{rl}
\log(s-a) = & 2.33546 \\
\log(s-b) = & 1.67669 \\
& 4.01215 \\
\log s = & 2.89404 \\
\log(s-c) = & 2.71559 \\
& 5.60963 \\
& 2)\bar{2}.40252 \\
\log\tan\frac{1}{2}C = & \bar{1}.20126
\end{array}$$

$\frac{1}{2}C = 9° 1' 54''$; $C = 18° 3' 48''$ Ans.

To check, add the angles:

$$\begin{array}{rrr}
41° & 45' & 16'' \\
120 & 10 & 58 \\
18 & 3 & 48 \\
\hline
180° & 00' & 2''
\end{array}$$

The triangle closes within 2 sec. This error is due to the use of five-place tables, and to the fact that the angle in each case was taken out to the nearest second.

EXAMPLES FOR PRACTICE

1. Given $a = 1$ mile, $b = 2$ miles, and $c = 1.5$ miles; find A, B, and C. (Use first method.)

Ans. $\begin{cases} A = 28° 57' 17'' \\ B = 104° 28' 39'' \\ C = 46° 34' 4'' \end{cases}$

2. Given $a = 50$ chains, $b = 30$ chains, and $c = 45$ chains; find A, B, and C. (Use first method.)

Ans. $\begin{cases} A = 80° 56' 36'' \\ B = 36° 20' 7'' \\ C = 62° 43' 17'' \end{cases}$

3. Given $a = 63.47$ feet, $b = 89.36$ feet, and $c = 109.83$ feet; find A, B, and C (Use second method.)

Ans. $\begin{cases} A = 35° 18' 10'' \\ B = 54° 27' 2'' \\ C = 90° 14' 50'' \end{cases}$

4. Given $a = 2{,}354$ feet, $b = 3{,}115$ feet, and $c = 836.6$ feet; find A, B, and C. (Use second method.)

Ans. $\begin{cases} A = 21° 7' 24'' \\ B = 151° 31' 8'' \\ C = 7° 21' 30'' \end{cases}$

---

24. Case IV: Given Two Sides and the Angle Opposite One of Them.—In the triangle ABC, let a, b, and A be given, to find B, C, and c. The angle B or C is found by means of the principle of sines; thus,

$$\frac{a}{\sin A} = \frac{b}{\sin B}, \text{ whence } \sin B = \frac{b \sin A}{a}$$

Then, $$C = 180° - A - B, \text{ and } c = \frac{a}{\sin A} \sin C$$

When the data are given as above, without any further restrictions, there may be two triangles that will answer the given conditions; and the problem is said to have two solutions. For here the angle B is determined from its sine; and as every sine corresponds to two supplementary angles, either of these angles may be taken. Thus, if $\sin B$ is found to be .64746, the corresponding angle may be either 40° 21′ or 180° − 40° 21′ = 139° 39′, since these angles both have the same sine (Art. **13**).

The same result is obtained from geometrical considerations. On any line AX, Fig. 7, construct an angle equal to the given angle A, and on its side AC take AC equal to one of the given sides b. From C as a center, with a radius equal to the side a, describe an arc. This arc will generally cut AX at two points, B_1 and B_2, and either of the triangles ACB_1 or ACB_2 will answer the conditions of the problem, for they both contain the given sides b and a, and the angle A opposite a.

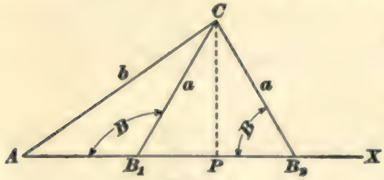

FIG. 7

The problem will have but one solution in the following cases:

1. If $a = b \sin A$. For in this case a will be equal to the perpendicular CP, Fig. 7, and the arc described from C will touch AX at P only.

2. If $a = b$. For in this case the angles A and B must be equal, and therefore both acute, since a triangle cannot have two obtuse angles. In this case B_1 coincides with A in Fig. 7, since $CB_1 = CA$.

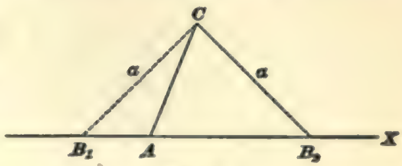

FIG. 8

3. When a is greater than b. For in this case A must be greater than B, and the latter angle must therefore be acute. This is shown by Fig. 8; the arc described from C cuts AX produced at B_1, and CB_1, although equal to a, is not opposite A $(= CAB_2)$.

When a is less than $b \sin A$, the problem is impossible. For then a is less than CP, Fig. 7, and the arc does not cut AX at all. This is also shown by the formula $\sin B = \frac{b \sin A}{a}$, which would give $\sin B$ a value greater than 1, which is an impossible value, for no sine can be greater than 1.

EXAMPLE.—Given $a = 273$ feet, $b = 392$ feet, and $A = 37° 14'$; to find B, C, and c.

SOLUTION.—Here a is less than b, and, unless $\sin B$ is found to be greater than 1 (in which case the problem is impossible), there are two solutions.

$$\sin B = \frac{b \sin A}{a} = \frac{392 \times \sin 37° 14'}{273};\ B = \begin{cases} 60° 19' 17'', \text{ or} \\ 180° - 60° 19' 17'' \\ = 119° 40' 43''. \text{ Ans.} \end{cases}$$

$$C = \begin{cases} 180° - 37° 14' - 60° 19' 17'' = 82° 26' 43'', \text{ or} \\ 180° - 37° 14' - 119° 40' 43'' = 23° 5' 17''. \text{ Ans.} \end{cases}$$

$$c = \frac{a}{\sin A} \sin C = \frac{273}{\sin 37° 14'} \sin \begin{Bmatrix} 82° 26' 43'', \text{ or} \\ 23° 5' 17'' \end{Bmatrix} = \begin{matrix} 447.27 \text{ ft., or} \\ 176.93 \text{ ft. Ans.} \end{matrix}$$

PRACTICAL EXAMPLES

EXAMPLE 1.—The distance between two points A and B, Fig. 9, is 360.38 feet, the angles from A and B to a station C are found, with a transit, to be, respectively, 62° 17′ and 39° 51′. What are the distances of C from A and B?

SOLUTION.— $C = 180° - 62° 17' - 39° 51' = 77° 52'$. Modulus ($M$) of triangle $= \frac{360.38}{\sin 77° 52'}$. Then (Art. 18),

$$a = \frac{360.38}{\sin 77° 52'} \sin 62° 17' = 326.32 \text{ ft.} \quad \text{Ans.}$$

$$b = \frac{360.38}{\sin 77° 52'} \sin 39° 51' = 236.2 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 2.—The distances of a fort C from two other forts A and B are as marked in Fig. 10; the lines of sight from C to A and B make an angle of 53° 8′ 16″. What is the distance between the two forts A and B?

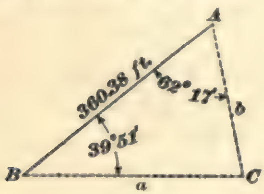

FIG. 9

FIG. 10

SOLUTION.—The two sides and the included angle are given, and formulas 1 and 2, Art. 21, will be applied. It is not necessary to determine the angles A and B, for they are not required. Formula 1, Art. 21,

$$\tan \tfrac{1}{2}(A - B) = \frac{a - b}{a + b} \cot \tfrac{1}{2} C$$

$$= \frac{3{,}439 - 2{,}378}{3{,}439 + 2{,}378} \cot \frac{53° 8' 16''}{2}$$

$$= \frac{1{,}061}{5{,}817} \cot 26° 34' 8''$$

$$\tfrac{1}{2}(A - B) = 20° 2' 20''$$

Formula 2, Art. 21,

$$c = AB = \frac{(a - b) \cos \tfrac{1}{2} C}{\sin \tfrac{1}{2}(A - B)}$$

$$= \frac{1{,}061 \cos 26° 34' 8''}{\sin 20° 2' 20''}$$

$$= 2{,}769.4 \text{ ft.} \qquad \text{Ans.}$$

FIG. 11

EXAMPLE 3.—A weight W, Fig. 11, is to be hung from a pulley sliding freely on the rope $O Q P$. The length of the rope is l, and its ends are fastened at two points O and P, whose horizontal distance is d and whose vertical distance is h, as shown. It being proved in mechanics that the pulley will rest in equilibrium when the vertical line WQ bisects the angle OQP, what are the lengths x $(= OQ)$ and y $(= PQ)$ of the two segments of the rope for which that condition obtains?

NOTE.—This problem is given here as an illustration of the many problems that occur in practice requiring the exercise of some ingenuity and the performance of some transformations, both algebraical and trigonometrical, before the required results are obtained.

SOLUTION.—Let QD be a vertical line through Q. According to the data, this line makes equal angles with OQ and QP. These angles are denoted by Z. The angles made by OQ and PQ with OP are denoted by Y and X, respectively. The line OR is horizontal, and PR vertical

As OR and RP are known, the right triangle OPR gives

$$\tan M = \frac{h}{d}$$

Also, in the triangle OED, $N = 90° - M$.

The angles M and N may, therefore, be assumed to be known.

Drawing PF parallel to RO, we have

$$d\ (= OR) = OD + DR = OD + PF$$

or, substituting the values of OD and PF from the triangles ODQ and PFQ,

$$d = x \sin Z + y \sin Z = (x + y) \sin Z = l \sin Z$$

whence,
$$\sin Z = \frac{d}{l}$$

Having found Z, we have

$$X = 180° - (N + Z) \text{ (triangle } PEQ)$$
$$Y = N - Z \text{ (triangle } OEQ)$$

The modulus of the triangle OPQ is

$$\frac{OP}{\sin OQP} = \frac{OP}{\sin 2Z} = \frac{d \div \cos M}{\sin 2Z} = \frac{d}{\cos M \sin 2Z} = \frac{d}{\sin N \sin 2Z}$$

Therefore (Art. 18), $x = \dfrac{d}{\sin N \sin 2Z} \sin X$

or, substituting the value of X, and noticing that $\sin[180° - (N + Z)] = \sin(N + Z)$,

$$x = \frac{d}{\sin N \sin 2Z} \sin(N + Z). \quad \text{Ans.}$$

Likewise,

$$y = \frac{d}{\sin N \sin 2Z} \sin(N - Z). \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the distance MN across the lake from the data shown in Fig. 12.

Ans. $MN = 669.51$ ft.

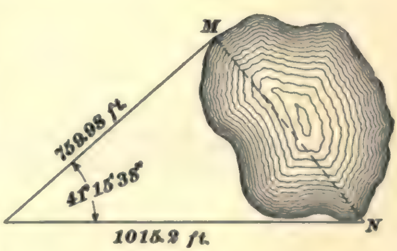

FIG. 12

2. The angles from two stations M and N, Fig. 13, to two inaccessible

points P and Q being as shown, and the distance MN being 550 feet find the distance PQ.

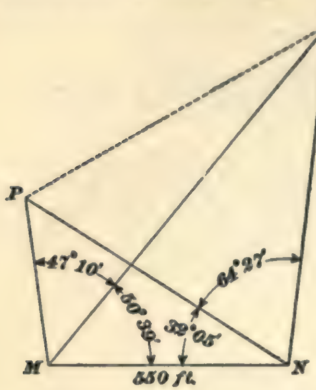

FIG. 13

HINT.—First calculate MP, then MQ, and finally PQ.

Ans. PQ = 799.7 ft.

3. In Fig. 14, the sides AB and DE were measured and the angles were turned as marked. Find the lengths of the sides BC, CA, CF, AF, CD, FD, EF.

Ans. BC = 677.92 ft.
CA = 1,065.8 ft.
CF = 905.46 ft.
AF = 703.1 ft.
CD = 696.83 ft.
FD = 1,019.7 ft.
EF = 687.97 ft.

4. Two observers on the same side of a steeple, and in the same vertical plane with it, are 100 feet apart, and find that the angles of elevation are 26° 28′ and 49° 14′. What is the height of the steeple?

Ans. 87.225 ft.

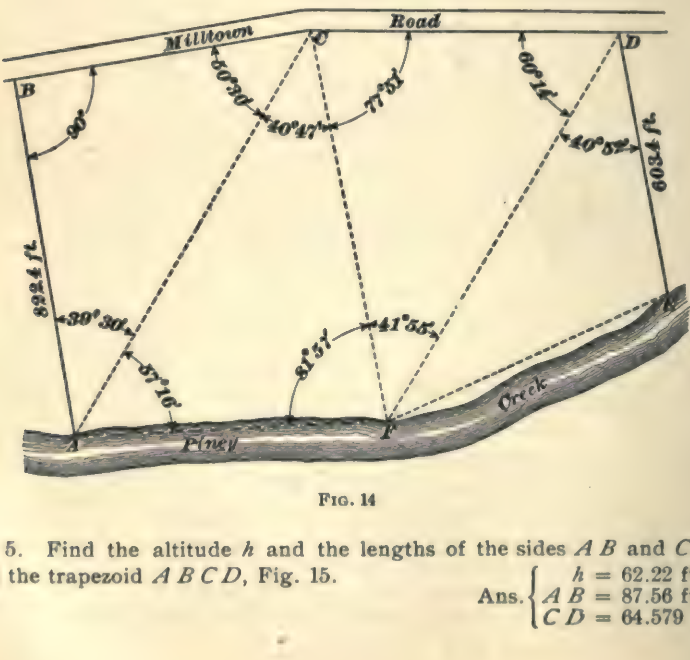

FIG. 14

5. Find the altitude h and the lengths of the sides AB and CD of the trapezoid $ABCD$, Fig. 15.

Ans. h = 62.22 ft.
AB = 87.56 ft.
CD = 64.579 ft.

6. The connecting-rod AB, Fig. 16, of an engine is 9 feet 3 inches, and the crank-arm CB is $10\frac{1}{2}$ inches; the figure shows the crank after

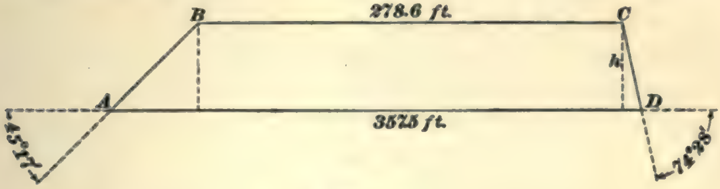

FIG. 15

it has performed one-eighth of a revolution, starting from the position CB'. Find: (*a*) the inclination M of the connecting-rod to the axis

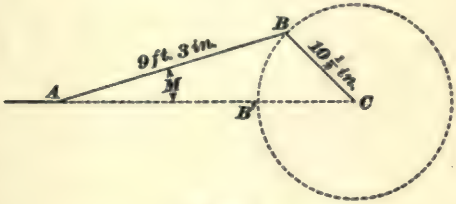

FIG. 16

of the piston rod, which is in line with CA; (*b*) the distance AC of the joint A from the center of the crank-circle.

Ans. $\begin{cases} (a) & M = 3° 50' 7'' \\ (b) & AC = 9 \text{ ft. } 10\frac{1}{2} \text{ in., nearly} \end{cases}$

AREAS

LAND MEASURE

25. In surveying the public lands of the United States and Canada, all linear measurements are made with the surveyors' chain, also known as Gunter's chain, from the name of the inventor. This chain is 66 feet in length and contains 100 links, each 7.92 inches long. In private surveys, the foot is commonly taken as the unit of linear measure, and small land areas are expressed in square feet.

Land areas of considerable extent in the countries mentioned are generally expressed in acres. Fractional parts of an acre, which formerly were expressed in roods, square rods or perches, and square links, are now expressed decimally by nearly all surveyors. Thus, 40.35 acres is written instead of 40 acres, 1 rood, and 16 square rods.

Tables of linear and square measure are given in *Arithmetic*, and to those tables the student is referred for detailed information regarding the subject. The following table gives the relative values of the units of area used in land surveying in the countries referred to above. As already stated, the square foot and acre are now the units most commonly employed.

TABLE OF LAND MEASURE

1 square yard (sq. yd.) .	=	9	square feet (sq. ft.)
1 square rod* (sq. rd) .	=	30¼	square yards = 272¼ square feet
1 square chain (sq. ch.) .	=	16	square rods = 4,356 square feet
1 acre (A.)	=	10	square chains = 43,560 square feet
1 rood (R.)	=	40	square rods = 10,890 square feet
1 acre	=	4	roods = 160 square rods
1 square mile (sq. mi.)	=	640	acres = 6,400 square chains
1 township (Tp.)	=	36	square miles = 23,040 acres (app)

*Sometimes called a perch or pole, and designated by the abbreviation P.

As will be observed, there are 10 square chains in an acre. In order, therefore, to reduce to acres any number of square chains, it is sufficient to move the decimal point one place toward the left, which is equivalent to dividing by 10. It must also be borne in mind that, since there are 100 links in 1 chain, links are usually expressed decimally as hundredths of a chain. Thus, 6.72 chains is written instead of 6 chains 72 links.

EXAMPLE 1.—A rectangular piece of land is 1,060 feet in length by 820 feet in breadth; what is its area: (*a*) in acres and decimals? (*b*) in acres, roods, and perches?

SOLUTION.— (*a*) $1,060 \times 820 = 869,200$ sq. ft.; $869,200 \div 43,560 = 19.954$ A. Ans.

(*b*) $.954$ A. $= .954 \times 4 = 3.816$ R.; .816 R. is equal to $.816 \times 40 = 32.64$ P. Hence, the area is 19 A. 3 R. 32.64 P. Ans.

EXAMPLE 2.—A rectangular piece of land is 12 chains and 6 links (12.06 chains) in length by 8 chains and 55 links (8.55 chains) in breadth; what is its area: (*a*) in acres and decimals? (*b*) in acres, roods, and perches?

SOLUTION.— (*a*) $12.06 \times 8.55 = 103.11$ sq. ch.; $103.11 \div 10 = 10.311$ A. Ans.

(*b*) $.311$ A. $= .311 \times 4 = 1.244$ R.; .244 R. is equal to $.244 \times 40 = 9.76$ P. Hence, the area is 10 A. 1 R. 9.76 P. Ans.

EXAMPLES FOR PRACTICE

1. A rectangular piece of land is 1,190 feet in length by 700 feet in breadth; what is its area: (*a*) in acres and decimals? (*b*) in acres, roods, and perches?

Ans. { (*a*) 19.123 A.
(*b*) 19 A. 0 R. 19.7 P.

2. A rectangular piece of land is 525 feet long by 250 feet wide, what is its area: (*a*) in acres and decimals? (*b*) in acres, roods, and perches?

Ans. { (*a*) 3.013 A.
(*b*) 3 A. 0 R. 2.08 P.

3. A rectangular piece of land is 15 chains and 65 links in length by 8 chains and 16 links in breadth; what is its area: (*a*) in acres and decimals? (*b*) in acres, roods, and perches?

Ans. { (*a*) 12.77 A.
(*b*) 12 A. 3 R. 3.2 P.

AREAS OF POLYGONS

THE TRIANGLE

NOTE.—In all that follows, the area of any figure under consideration will be designated by S, unless otherwise stated.

26. Given the Base and Altitude.—Any of the sides of a triangle may be taken as the base, the altitude being the length of the perpendicular drawn on the base from the vertex of the opposite angle. In Fig. 17, b is taken as the base, and the perpendicular BH, denoted by h, is the altitude.

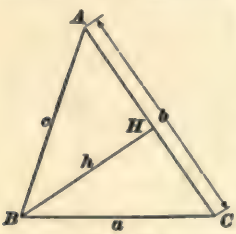

FIG. 17

It was shown in *Geometry*, Part 2, that the area of a triangle, when the base b and altitude h are known, is given by the formula

$$S = \tfrac{1}{2}\, b\, h$$

27. Given Two Sides and the Included Angle.—Let b, c, and A, Fig. 17, be given. In the right triangle ABH, we have $h = c \sin A$. The substitution of this value of h in the formula in Art. 26 gives

$$S = \tfrac{1}{2}\, b c \sin A$$

In words, *the area of a triangle is equal to one-half the product of any two sides and the sine of their included angle.*

EXAMPLE.—Two of the sides of a triangular field are 39.47 and 59.23 chains, respectively, and their included angle is 65° 10′ 40″. To find the contents of the field, in acres.

SOLUTION.—By the formula, S (square chains) $= \frac{1}{2} \times 39.47 \times 59.23$ sin 65° 10′ 40″ = 1,060.9 sq. ch.; whence, dividing by 10 (Art. 25),

S (acres) = 106.09 A. Ans.

28. Given One Side and Two Angles.—The other angle may be at once found by subtracting the sum of the two given angles from 180°. It may, therefore, be assumed that the three angles are known. Let b, Fig. 17, be the given side. From Art. 22, the value of c is equal to the modulus of the triangle multiplied by sin C, or,

$$c = \frac{b}{\sin B} \sin C$$

Substituting this value in the formula in Art. **27**, we obtain

$$S = \frac{b^2 \sin A \sin C}{2 \sin B}$$

29. The formula in Art. **28** is convenient when logarithmic functions are employed. For the use of natural functions, the following is preferable:

In the right triangles ABH and CBH, Fig. 17, we have,

$$AH = h \cot A, \; CH = h \cot C$$

whence, adding these two equations,

$$AH + CH = h \cot A + h \cot C$$

that is,

$$b = h(\cot A + \cot C)$$

and, therefore,

$$h = \frac{b}{\cot A + \cot C} \qquad (1)$$

This formula is useful and should be committed to memory. It may be stated in words thus: *The altitude of a triangle is equal to the base divided by the sum of the cotangents of the adjacent angles.*

By substituting, in the formula in Art. **26**, the value of h given in formula **1**, we obtain

$$S = \frac{b^2}{2(\cot A + \cot C)} \qquad (2)$$

In words, *the area of a triangle is equal to the square of any side divided by twice the sum of the cotangents of the angles adjacent to that side.*

EXAMPLE.—One side of a triangular field is 127.64 chains, and the adjacent angles are 46° 15′ and 60° 41′. To find the area.

SOLUTION BY LOGARITHMIC FUNCTIONS.—Here, $b = 127.64$, $A = 46° 15'$, $C = 60° 41'$, and $B = 180° - 46° 15' - 60° 41' = 73° 4'$. Formula of Art. **28**,

$$S = \frac{127.64^2 \sin 46° 15' \sin 60° 41'}{2 \sin 73° 4'}$$

$$= 5{,}363.4 \text{ sq. ch.} = 536.34 \text{ A. Ans.}$$

SOLUTION BY NATURAL FUNCTIONS.—By formula **2**,

$$S = \frac{127.64^2}{2(\cot 46° 15' + \cot 60° 41')} = \frac{127.64^2}{2(.95729 + .56156)}$$

$$= \frac{127.64^2}{3.0377} = 5{,}363.4 \text{ sq. ch.} = 536.34 \text{ A. Ans.}$$

NOTE.—Even if natural functions are used, the division is advantageously performed by means of logarithms.

EXAMPLES FOR PRACTICE

1. Two sides of a triangular field are 3,760 and 2,757 feet, respectively, and their included angle is 54° 13′ 13″. What is the area of the field, in acres? Ans. $S = 96.534$ A.

2. One side of a triangle is 96.34 chains; the opposite angle is 49° 10′, and one of the adjacent angles, 69° 45′ 30″. What is the area of the triangle, in acres? Ans. $S = 503.69$ A.

3. One side of a triangle is 8.93 inches, and the adjacent angles are 34° 16′ and 17° 37′ 18″. What is the area of the triangle?
Ans. $S = 8.638$ sq. in.

4. Two sides of a triangle are 17 and 25 feet, respectively, and the included angle is 76° 13′. What is the area of the triangle?
Ans. $S = 206.38$ sq. ft.

---

30. Given the Three Sides.—Let a, b, and c, Fig. 17, be given, and denote $\frac{1}{2}(a+b+c)$ by s. The area S of the triangle is given by the following formula, which is derived in Appendix VI:

$$S = \sqrt{s(s-a)(s-b)(s-c)}$$

EXAMPLE.—The sides of a triangular tract are 1,634.6 ($= a$, say), 978.28 ($= b$, say), and 2,176.4 ($= c$, say) feet, respectively; to find the area, in acres.

SOLUTION.—The work may be conveniently arranged as shown below. The numbers in marks of parenthesis indicate the order in which the several quantities are set down. In (6), s is placed above a, b, c in order to facilitate the subtractions. The differences $s-a$, $s-b$, $s-c$ are written, as the subtractions are performed, horizontally opposite a, b, and c, respectively.

(6) $s = 2{,}394.64$	
(1) $a = 1{,}634.60$	(7) $s - a = 760.04$
(2) $b = 978.28$	(8) $s - b = 1{,}416.36$
(3) $c = 2{,}176.40$	(9) $s - c = 218.24$
(4) $2s = 4{,}789.28$	
(5) $s = 2{,}394.64$	

(10) $\log s = 3.37924$
(11) $\log (s-a) = 2.88083$
(12) $\log (s-b) = 3.15117$
(13) $\log (s-c) = 2.33893$
2)11.75017

$\log S = 5.87509$; $S = 750{,}050$ sq. ft. $= 17.22$ A. Ans.

EXAMPLES FOR PRACTICE

1. Find the area of a triangular tract whose sides are 54.36, 73.19, and 101.76 chains, respectively. Ans. $S = 192.26$ A.

2. Find the area of a triangular plate whose sides are 17.12, 12.75, and 8.95 inches, respectively. Ans. $S = 55.646$ sq. in.

THE TRAPEZOID

31. Notation.—In Fig. 18, the bases, or parallel sides, of the trapezoid $ABCD$ are denoted by b_1 and b_2; the altitude, by h; and the sides AD and BC, by a and c, respectively. The angles will be designated by the letters A, B, C, D at the vertexes. The line DB' is drawn through D parallel to CB, thus forming a parallelogram in which $B'B = DC = b_2$, and $DB' = CB = c$. Also, angle $DB'A = B$, and $AB' = AB - B'B = b_1 - b_2$. For some purposes, it is convenient to represent this difference by a single letter d, as shown in the figure.

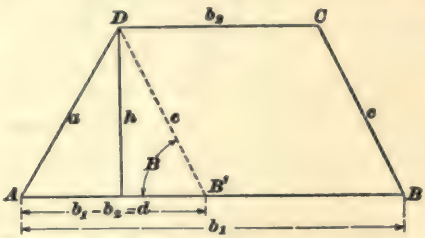

Fig. 18

32. Given the Bases and the Altitude.—As shown in *Geometry*, Part 2, the area of a trapezoid is equal to one-half the product of the altitude by the sum of the bases; that is,

$$S = \tfrac{1}{2}(b_1 + b_2)h$$

33. Given the Bases and the Angles Adjacent to One of Them.—Let b_1, b_2, A, and B, Fig. 18, be given. In the triangle ADB' we have (formula **1**, Art. **29**),

$$h = \frac{b_1 - b_2}{\cot A + \cot B}$$

If this value of h is substituted in the formula of Art. **32**, the result is,

$$S = \frac{(b_1 - b_2)(b_1 + b_2)}{2(\cot A + \cot B)} \qquad \textbf{(1)}$$

As the product of the sum of two quantities by their difference is equal to the difference between the squares of the quantities, $(b_1 - b_2)(b_1 + b_2)$ is equal to $b_1^2 - b_2^2$; and, therefore, formula **1** may also be written:

$$S = \frac{b_1^2 - b_2^2}{2(\cot A + \cot B)} \qquad \textbf{(2)}$$

For the use of logarithmic functions, formula **1** may be transformed into the following (see Appendix VII):

$$S = \frac{(b_1 - b_2)(b_1 + b_2)\sin A \sin B}{2\sin(A + B)} \qquad \textbf{(3)}$$

In the application of these formulas, the student should bear in mind that the cotangent of an angle greater than 90° is negative, and numerically equal to the cotangent of the supplement of the angle; also, that the sine of an angle greater than 90° is equal to the sine of its supplement. Thus, $\cot 105° = -\cot(180° - 105°) = -\cot 75° = -.26795$; and $\sin 105° = \sin 75° = .96593$.

EXAMPLE 1.—The two bases of a trapezoid are 350 and 137 chains, respectively; the angles adjacent to the longer base are 75° 10′ and 63° 54′. What is the area of the trapezoid?

SOLUTION BY NATURAL FUNCTIONS.—Let $350 = b_1$, $137 = b_2$, $A = 75° 10'$, $B = 63° 54'$. As b_1 and b_2 are not convenient numbers to square, formula **1**, which is better adapted to logarithmic work, will be used.

$$S = \frac{(350 - 137)(350 + 137)}{2(\cot 75° 10' + \cot 63° 54')} = \frac{213 \times 487}{2(.26483 + .48989)} = 68{,}721 \text{ sq. ch.}$$
$$= 6{,}872.1 \text{ A. Ans.}$$

SOLUTION BY LOGARITHMIC FUNCTIONS.—By formula **3**,

$$S = \frac{(350 - 137)(350 + 137)\sin 75° 10' \sin 63° 54'}{2\sin 139° 4'}$$

or, replacing sin 139° 4′ by sin (180° − 139° 4′) = sin 40° 56′,

$$S = \frac{213 \times 487 \sin 75° 10' \sin 63° 54'}{2\sin 40° 56'} = 68{,}721 \text{ sq. ch.} = 6{,}872.1 \text{ A. Ans.}$$

EXAMPLE 2.—The bases of a trapezoid are 100 and 70 feet, the angles adjacent to the shorter base being 52° 47′ and 143° 14′. What is the area of the trapezoid?

SOLUTION.—Since the bases are parallel, the two angles adjacent to each of the non-parallel sides are supplementary. Thus, in Fig. 18, $A + D = 180°$, $B + C = 180°$; and, therefore, $A = 180° - D$; $B = 180° - C$. Let $52° 47' = D$, $143° 14' = C$. Then,

$$A = 180° - 52° 47' = 127° 13'$$
$$B = 180° - 143° 14' = 36° 46'$$
$$\cot A = -\cot(180° - 127° 13') = -\cot 52° 47' = -.75950$$
$$\cot B = \cot 36° 46' = 1.33835$$

Formula 2,

$$S = \frac{100^2 - 70^2}{2(+1.33835 - .7595)} = \frac{5{,}100}{1.1577} = 4{,}405.3 \text{ sq. ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The bases of a trapezoidal tract are 78.63 and 54.71 chains, respectively; the angles adjacent to the longer base are 55° 18′ and 62° 53′. Find the area, in acres. Ans. $S = 132.4$ A.

2. Find the number of square feet in a trapezoidal cross-section of a canal 40 feet wide at the bottom, 65 feet wide at the top, and whose non-parallel sides are inclined to the horizontal at an angle of 50°. (The dimensions across the top and bottom are measured horizontally.) Ans. $S = 782.09$ sq. ft.

3. The two bases of a trapezoid are 10.25 and 18.76 inches, respectively; one of the angles adjacent to the shorter base is 76° 45′ 10″, and the angle diagonally opposite is 66° 8′ 9″; find the area of the trapezoid. (Use logarithmic functions.) Ans. $S = 596.4$ sq. in.

---

34. Given the Four Sides.—If the difference between the two bases added to the sum of the non-parallel sides is denoted by $2s$; that is, if the expression $\frac{1}{2}(a + c + d)$, Fig. 18, is denoted by s, the area of the trapezoid is given by the following formula (see Appendix VIII):

$$S = \frac{b_1 + b_2}{d}\sqrt{s(s-a)(s-c)(s-d)}$$

EXAMPLE FOR PRACTICE

The bases of a trapezoidal field are 136.43 and 210.18 chains, respectively; one of the non-parallel sides is 96.73 chains, and the other 164.37 chains. Find the area of the tract, in acres.

Ans. $S = 864.97$ A.

THE REGULAR POLYGON

35. Given the Number of Sides and the Radius. Let MN, Fig. 19, be one of the sides of a regular polygon of n sides; O, the center, and r the radius, of the circumscribed circle (called also the center and radius, respectively, of the polygon); and A, the angle at the center subtended by a side of the polygon. The length of the side MN will be denoted by l.

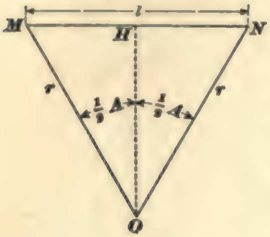

FIG. 19

Let n and r be given, to find the area S of the polygon and the length l of each of its sides. From *Geometry*, Part 2, the angle MON, or A, is found by dividing 360° by the number of sides in the polygon; that is,

$$A = \frac{360°}{n}$$

The area of the triangle MON is (Art. **27**) $\frac{1}{2} OM \times ON \sin MON$, or $\frac{1}{2} r \times r \sin A = \frac{1}{2} r^2 \sin A = \frac{1}{2} r^2 \sin \frac{360°}{n}$. Since the polygon consists of n triangles equal to MON, its area S is equal to n times the area of MON; that is,

$$S = n \times \tfrac{1}{2} r^2 \sin \frac{360°}{n}$$

or

$$S = \tfrac{1}{2} n r^2 \sin \frac{360°}{n} \qquad (1)$$

In the right triangle MOH, we have,

$$MH = r \sin \frac{A}{2}$$

or, since MH is one-half of MN, or of l,

$$\frac{l}{2} = r \sin \frac{A}{2}$$

whence, multiplying by 2,

$$l = 2 r \sin \frac{A}{2}$$

Finally, $\frac{A}{2} = \frac{1}{2} \frac{360°}{n} = \frac{180°}{n}$. By the substitution of this value in the expression for l just found, we get, finally,

$$l = 2r \sin \frac{180°}{n} \qquad (2)$$

36. When the Number of Sides and Their Common Length Are Given.—Let n and l, Fig. 19, be given, to find the radius r and the area S. The radius is found by solving formula **2**, Art. **35**, for r, which gives,

$$r = \frac{l}{2 \sin \frac{180°}{n}} \qquad (1)$$

In the triangle MOH, we have,

$$OH = MH \cot \tfrac{1}{2} A = \frac{MN}{2} \cot \tfrac{1}{2} A$$

The area of MON is $\frac{1}{2} MN \times OH$. Writing instead of OH the value just found,

$$\tfrac{1}{2} MN \times \frac{MN}{2} \cot \tfrac{1}{2} A = \frac{MN^2}{4} \cot \tfrac{1}{2} A$$

$$= \frac{l^2}{4} \cot \tfrac{1}{2} A = \frac{l^2}{4} \cot \frac{180°}{n}$$

Multiplying this by n, we obtain, for the area of the polygon,

$$S = \frac{n l^2}{4} \cot \frac{180°}{n} \qquad (2)$$

EXAMPLE 1.—Find the area, and also the length of the side, of a regular decagon inscribed in a 15-inch circle.

SOLUTION.—In practice, it is usual to refer to a circle by its diameter, and so a 15-in. circle is a circle whose diameter is 15 in. We have, therefore, $r = \frac{15}{2} = 7.5$, $n = 10$, $\frac{360°}{n} = \frac{360°}{10} = 36°$, $\frac{180°}{n} = 18°$, and formulas **1** and **2**, Art. **35**, give

$$S = \tfrac{1}{2} \times 10 \times 7.5^2 \sin 36° = 165.32 \text{ sq. in. Ans.}$$

$$l = 2 \times 7.5 \sin 18° = 4.635 \text{ in. Ans.}$$

EXAMPLE 2.—Each of the sides of an octagonal park is 150 feet; what is the area of the park, in acres?

SOLUTION.—Here $l = 150$ ft., $n = 8$, $\frac{180°}{n} = \frac{180°}{8} = 22\tfrac{1}{2}° = 22° \ 30'$, and formula **2**, Art. **36**, gives,

$S = \frac{1}{4} \times 8 \times 150^2 \cot 22° 30' = 2 \times 22{,}500 \cot 22° 30' = (45{,}000 \cot 22° 30')$ sq. ft. $= \frac{45{,}000 \cot 22° 30'}{43{,}560}$ A. $= 2.494$ A. Ans.

EXAMPLES FOR PRACTICE

1. Find the side and area of an equilateral triangle inscribed in a 20-inch circle.

Ans. $\begin{cases} l = 17.321 \text{ in.} \\ S = 129.9 \text{ sq. in.} \end{cases}$

2. What must be the length of the side and the radius of a regular pentagon, that its area may be 46.97 square feet?

Ans. $\begin{cases} l = 5.225 \text{ ft.} \\ r = 4.445 \text{ ft.} \end{cases}$

3. An eight-sided drive is to be built around a circular park 1,500 feet in diameter, the drive to be 15 feet wide, with its outer corners on the circumference of the park. Find: (*a*) the length of each of the sides of the outer boundary of the drive; (*b*) the length of each of the sides of the inner boundary; (*c*) the cost of paving the drive with asphalt, at $2.25 per square yard; (*d*) the difference between the exact area of the drive and the approximate area found by assuming the polygonal boundaries to coincide with the circumferences of their respective circumscribed circles.

Ans. (*a*) 574.02 ft.; (*b*) 561.60 ft.; (*c*) $17,025; (*d*) 844 sq. yd.

OTHER POLYGONS

37. The area of any polygon can be determined by dividing the polygon into triangles, and measuring in each triangle whatever parts are necessary for the determination of its area. The parts to be measured depend on special conditions and on the instruments used. The polygon may be divided into triangles either by diagonals or by lines drawn from a convenient interior point to the different vertexes. Illustrations of these methods of division will be given in connection with surveying. When the area is to be determined from a plat, the base and altitude of each triangle are usually the most convenient parts to measure.

AREAS BOUNDED BY IRREGULAR OUTLINES

AREA INCLUDED BETWEEN A STRAIGHT LINE AND AN IRREGULAR CURVE

38. By Selected Ordinates.—Let it be required to determine the area between the curve DC and the straight line AB, Fig. 20. A very convenient method is to draw perpendiculars on AB from the points of the curve at which its direction changes appreciably, and to consider the portion of the curve between two consecutive perpendiculars to be a straight line. The figure is then treated as if divided into a number of trapezoids, whose areas can be computed by the rules of geometry. The perpendiculars are called **ordinates.** Both the lengths of the ordinates and the distances between every two consecutive ordinates should be measured. The area of any of the (approximate) trapezoids into which the figure is thus divided is equal to one-half the sum of the two ordinates enclosing it multiplied by the distance between them. It should be understood that both this rule and those given further on relating to the same subject are only approximate. Since the bounding curve is irregular, that is, does not follow any mathematical law, no exact formula can be found for the area.

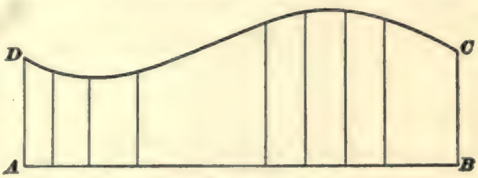

FIG. 20

EXAMPLE.—Referring to Fig. 20, suppose that, beginning at the left of the figure, the successive ordinates measure 15, 13, 12, 13.5, 20, 21.5, 22, 20, and 16 feet, respectively, and that the successive distances between the offsets, from left to right, measure 7.5, 10, 15, 41, 10.5, 11.5, 11.5, and 21 feet, respectively; what is the area of the surface?

SOLUTION.—The area of the figure is approximately equal to the sum of the areas of the trapezoids into which it is divided, and the area of each trapezoid is equal to one-half the sum of its parallel sides multiplied by the perpendicular distance between them. Therefore, the area of the figure is equal to

$$\frac{15+13}{2}\times 7.5+\frac{13+12}{2}\times 10+\frac{12+13.5}{2}\times 15+\frac{13.5+20}{2}\times 41$$
$$+\frac{20+21.5}{2}\times 10.5+\frac{21.5+22}{2}\times 11.5+\frac{22+20}{2}\times 11.5+\frac{20+16}{2}\times 21$$
$$= 2{,}195.5 \text{ sq. ft.} \quad \text{Ans.}$$

39. Trapezoidal Rule: Sigma Notation.—In order to facilitate the calculations, the ordinates are often measured at regular intervals along the straight line, as shown in Fig. 21. The area *A B C D* included between the straight line and the irregular boundary can then be more easily calculated by what is commonly known as the **trapezoidal rule.** This

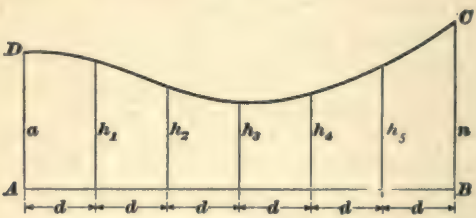

FIG. 21

is merely a rule for calculating the combined area of a series of trapezoids that have the same altitude, the areas being combined for convenience of calculation. The result given by this rule is closer the smaller the distance between the ordinates. The rule is as follows:

Rule.—*Add together one-half the two end ordinates and all the intermediate ordinates, and multiply the sum by the common distance between the ordinates.*

Let a = first ordinate;
n = last ordinate;
h_1, h_2, h_3 = intermediate ordinates;
a = common distance between ordinates;
S = area of surface.

Then, $S = [\frac{1}{2}(a+n) + h_1 + h_2 + h_3 + \ldots]d$

This expression may be put in a simpler form by using the **sigma notation,** which is as follows: As will be noticed, all the intermediate ordinates are denoted by h, different subscripts being used to indicate different values of h. We may, therefore, write the value of S thus,

$$S = [\tfrac{1}{2}(a + n) + \text{sum of all values of } h]d$$

Instead of the phrase *sum of all values of h*, the expression Σh, read *sigma h*, is used. The symbol Σ is the Greek letter sigma, corresponding to English S, and is very commonly used, as here, to indicate the addition of several quantities of the same character, denoted by a single symbol; hence, the name **sign of summation,** which also is often given to that letter.

By using the sigma notation, the value of S may be written

$$S = \left(\frac{a + n}{2} + \Sigma h\right)d$$

EXAMPLE.—If the ordinates from the straight line AB to the curved boundary DC, Fig. 21, are 19, 18, 14, 12, 13, 17, and 23 links, respectively, and are at equal distances of 50 links, what is the area included between the curved boundary and the straight line?

SOLUTION.—Area $ABCD = \left(\frac{19 + 23}{2} + 18 + 14 + 12 + 13 + 17\right) \times 50 = 4{,}750$ sq. li. Ans.

40. Simpson's Rule.—The foregoing rule assumes that all the small figures into which the area is divided are perfect trapezoids, which assumption always involves more or less error, since the irregular boundary is in nearly all cases an irregular curve. When the offsets are taken at

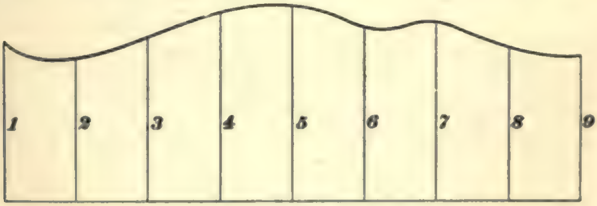

FIG. 22

regular intervals, the following rule, known as **Simpson's one-third rule,** gives a closer approximation. In applying this rule, the base line must be divided into an even number of equal parts; the ordinates measured at the points of division are numbered consecutively, as shown in Fig. 22.

Rule.—*Divide the base line into an even number of equal parts, and at the points of division erect ordinates terminating in the curve. Number the ordinates 1, 2, 3, etc., from left to right, including those at the ends of the base. Add together the end ordinates, four times the sum of all intermediate even-numbered ordinates, and twice the sum of all intermediate odd-numbered ordinates; multiply the total sum by one-third the common distance between adjacent ordinates.*

This rule has been used extensively; it can be expressed by a formula as follows:

Let h_2 = any intermediate even-numbered ordinate;
h_3 = any intermediate odd-numbered ordinate;
and let all other quantities be represented by the same letters as in the preceding article. Then,

$$S = (a + n + 4\,\Sigma\,h_2 + 2\,\Sigma\,h_3)\,\frac{d}{3}$$

The notation will be readily understood by reference to Fig. 23. The expression $4\,\Sigma\,h_2$ means four times the sum of all the ordinates h_2, or, in other words, four times the sum of all the even-numbered ordinates.

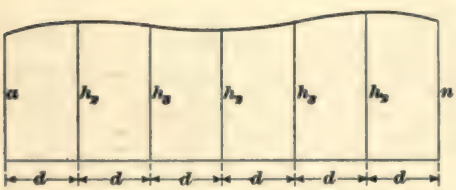

FIG. 23

EXAMPLE.—What is the area $ABCD$, Fig. 21, by Simpson's rule, using the same values as in the example in Art. 39?

SOLUTION.— $S = [19 + 23 + 4\,(18 + 12 + 17) + 2\,(14 + 13)] \times \frac{50}{3}$ = 4,733 sq. li. Ans.

EXAMPLES FOR PRACTICE

1. A figure included between a straight base line, a curve, and two perpendiculars to the base at the ends has nine ordinates, including the two end perpendiculars, whose lengths are 43, 48, 39, 50, 41, 32, 37, 31, and 22 feet, respectively; the common distance between the ordinates is 60 feet. Find the area: (*a*) by the trapezoidal rule; (*b*) by Simpson's rule.

Ans. { (*a*) 18,630 sq. ft.
{ (*b*) 18,860 sq. ft.

2. In order to determine the area included between an irregular boundary, a straight base line, and two perpendiculars to the base at the ends, eight ordinates, including the two end perpendiculars, are measured from the straight line to the boundary. The ordinates are found to measure 16, 18, 12, 13, 15, 17, 19, and 20.5 feet, and the successive distances between them are found to measure 7.8, 10, 15, 20, 12, 40, and 5 feet, respectively. What is the area of the surface?

Ans. 1,760.9 sq. ft.

3. A surface lying between a straight base line and a curve is limited by two perpendiculars to the base line at the ends; the base line is divided into eight parts 50 feet each, and at the points of division ordinates are measured. The lengths of the successive ordinates, including the two end perpendiculars, are 10, 25, 38, 49, 58, 65, 70, 73, and 74 feet, respectively. Find the area of the surface: (*a*) by the trapezoidal rule; (*b*) by Simpson's rule.

Ans. { (*a*) 21,000 sq. ft.; (*b*) 21,067 sq. ft.

AREA BOUNDED BY AN IRREGULAR CURVE

41. By Ordinates.—Suppose that it is required to find the area enclosed by the heavy irregular curve shown in Fig. 24. A broken line *AEFMGHIA* is drawn around

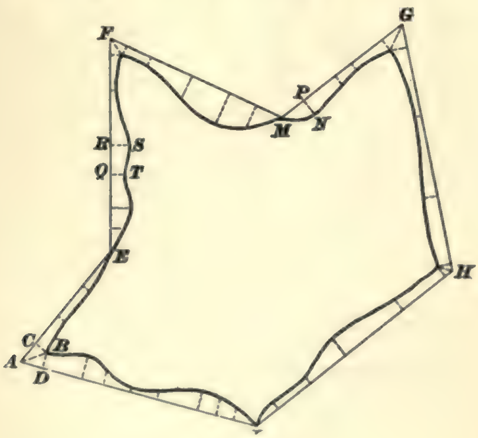

FIG. 24

the curved boundary line, and as close to it as convenient. Ordinates to the straight lines thus drawn are measured from the points where the direction of the curved boundary changes materially, as shown. The area of the polygon

AEFMGHIA is çalculated by one of the methods explained in preceding articles, and from it is subtracted the sum of the areas included between the curved boundary and the broken line, calculated as in Art. **39**.

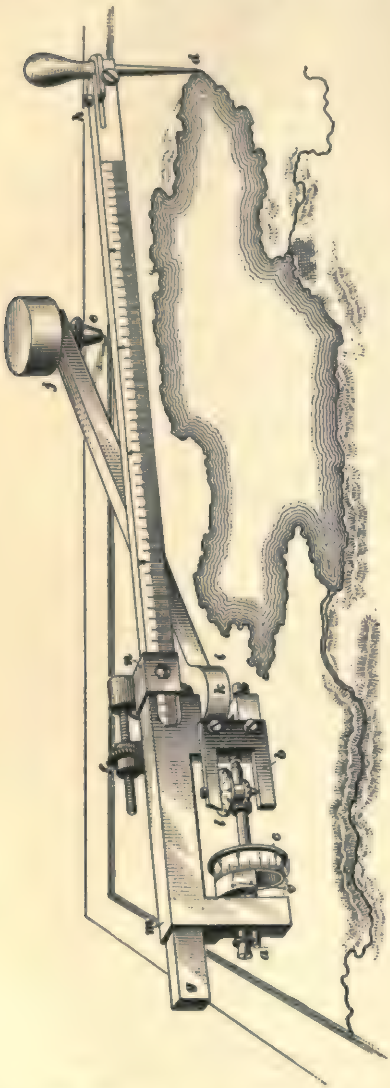

Fig. 25

At such corners as *A*, the triangles *ABC* and *ABD* are computed from the measured bases *AC* and *AD* and the altitudes *BC* and *BD*. All the quadrilaterals, as *QRST*, are treated as trapezoids; and such three-sided figures as *MPN*, as triangles. The process is so simple that it does not require any further explanation.

42. By the Planimeter.—The most convenient way to find the area of a plane surface having an irregular boundary is by the **planimeter.** There are several forms of planimeters; the one most commonly used is the **polar planimeter** (see Fig. 25). As will be seen from the illustration, this instrument has two arms *ij* and *gh* connected by a hinge joint. The point *e* at the end of the bar *ij* is called the **anchor**

point; it remains stationary while the point *d*, called the **pointer** or **tracer**, at the end of the bar *g h* is moved over the outline of the figure whose area is to be determined. The movement of the pointer *d* causes the wheel *c* on the opposite end of the bar to roll on the paper; this wheel is called the **measuring wheel** or **counter wheel**. The graduated bar *g h* can be adjusted by sliding it in or out through the socket *m* in the top of the frame. This bar is clamped by means of a clamp screw, a part of which is shown back of the small movable socket *n*, and is set at the exact length required by means of the thumbscrew *f*. The bar *i j* is of fixed length; it is pivoted at *k*, the junction of the two bars. The measuring wheel *c* is mounted on the main axis *a b*, which is parallel with the bar *g h*. The complete revolutions of the wheel *c* are read on the disk *l*, and the fractional parts of revolutions are read on the wheel *c* and the vernier *v*, the tenths and hundredths being read on the wheel itself, and the thousandths on the vernier.

To use the planimeter, the anchor point *e* is fixed on the paper or drawing board, preferably outside the figure to be measured, the pointer *d* is placed on some point in the periphery of the figure, and a reading of the wheel *c* is taken. The point *d* is then moved carefully around the periphery of the figure, in a clockwise direction, or from left to right, to the point of beginning. A second reading of the wheel *c* is then taken, and the difference between the two readings is the number of revolutions of the wheel. If the wheel is set to read zero, the number of revolutions is given directly by the second reading.

If the anchor point is outside the area to be measured, the distance traversed by the wheel, or the product of the number of revolutions by the circumference of the wheel, in inches, multiplied by the length of the bar *n h*, in inches, is the area, in square inches, bounded by the path of the pointer *d*.

If the anchor point is inside the area, the product just referred to must be added to the area of the **zero circle**, whose radius is equal to $\sqrt{p^2 + q^2 + 2\,p\,r}$, p being the length of the arm $n\,h$; r, the distance from the center of the wheel c to the center of the joint k; and q, the length of the bar $k\,j$. The bar $g\,h$ is generally set at such a length that ten times the number of revolutions of the wheel c is the area measured. This area is the actual area of the figure measured, and the area *represented* by the figure is determined from the scale of the plat. The area given by the planimeter, in square inches, must be multiplied by the square of the scale of the plat, in order to get the area sought. Thus, if the plat has been drawn to a scale of 50 feet to an inch, each square inch of the plat is equivalent to $50 \times 50 = 2{,}500$ square feet of area.

Suppose that the area bounded by the irregular line in Fig. 25, as measured by the planimeter, is 2.535 square inches, and that the scale of the plat is 100 feet to an inch; then the area represented by a square inch of the plat is $100 \times 100 = 10{,}000$ square feet, and the area represented by the closed figure is $10{,}000 \times 2.535 = 25{,}350$ square feet.

Full directions for using the planimeter are usually furnished by the maker.

APPENDIX: DERIVATION OF FORMULAS

I—FORMULAS 1 TO 4 OF ART. 16

Let $R\,O\,Q$, Fig. 26, be any angle A, and $Q\,O\,S$ any angle B. Then, $A + B = R\,O\,S$. From any point P on $O\,S$, draw $P\,N$ and $P\,M$, perpendicular, respectively, to $O\,R$ and $O\,Q$. Draw $M\,K$ parallel to $O\,R$ and therefore perpendicular to $P\,N$; also, $M\,L$ perpendicular to $O\,R$. The angles $M\,P\,K$ and $R\,O\,Q$, having their sides perpendicular each to each, are equal. Now,

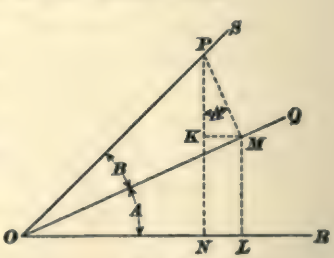

FIG. 26

$$\sin(A+B) = \frac{NP}{OP} = \frac{NK + KP}{OP} = \frac{ML}{OP} + \frac{KP}{OP} = \frac{OM \sin A}{OP} + \frac{PM \cos A}{OP}$$

(triangles MLO and PMK) $= \sin A \frac{OM}{OP} + \cos A \frac{PM}{OP} = \sin A \cos B + \cos A \sin B$ (triangle OPM)

This is formula **1.**

Also,

$$\cos(A - B) = \sin[90° - (A - B)] = \sin[(90° - A) + B]$$

or, by formula **1**,

$$\cos(A - B) = \sin(90° - A)\cos B + \cos(90° - A)\sin B$$
$$= \cos A \cos B + \sin A \sin B$$

which is formula **4.**

Formula **3** follows from this; for

$$\sin(A - B) = \cos[90° - (A - B)] = \cos[(90° + B) - A]$$
$$= \cos(90° + B)\cos A + \sin(90° + B)\sin A$$

or, because $\cos(90° + B) = -\sin B$, and $\sin(90° + B) = \cos B$ (Art. **14**), $\sin(A - B) = -\sin B \cos A + \cos B \sin A = \sin A \cos B - \cos A \sin B$.

Finally, applying this formula,

$$\cos(A + B) = \sin[90° - (A + B)] = \sin[(90° - A) - B]$$
$$= \sin(90° - A)\cos B - \cos(90° - A)\sin B$$
$$= \cos A \cos B - \sin A \sin B$$

which is formula **2.**

II—FORMULAS OF ART. 19

Referring to Fig. 6 (*a*) and (*b*), Art. **18**,

$$a^2 = p^2 + BD^2 \qquad (1)$$

In (*a*), $BD = c - AD$, whence $\overline{BD}^2 = c^2 - 2c \times AD + \overline{AD}^2$.

In (*b*), $BD = AD - c$, whence $\overline{BD}^2 = \overline{AD}^2 - 2c \times AD + c^2$.

Substituting this value of BD in equation (1),

$$a^2 = p^2 + \overline{AD}^2 + c^2 - 2c \times AD \qquad (2)$$

But $p^2 + \overline{AD}^2 = b^2$, and $AD = b \cos A$; therefore,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

When the angle opposite the side is obtuse, as B in Fig. 6 (*b*), the same reasoning leads to the relation,

$$b^2 = a^2 + c^2 + 2ac \times \cos CBD$$

the second member of which becomes $a^2 + c^2 - 2ac \cos B$, when $\cos CBD$ is replaced by its equal $-\cos B$ (Art. **13**).

III—FORMULAS OF ART. 20

Let ABC, Fig. 27, be any triangle. As usual, the angles of the triangle will be denoted by A, B, C, and the opposite sides by a, b, c, respectively; that is, angle $CAB = A$, $BC = a$, etc. Produce AC to A', making $CA' = BC = a$. Draw BA', and AP perpendicular to it, meeting BC at Q.

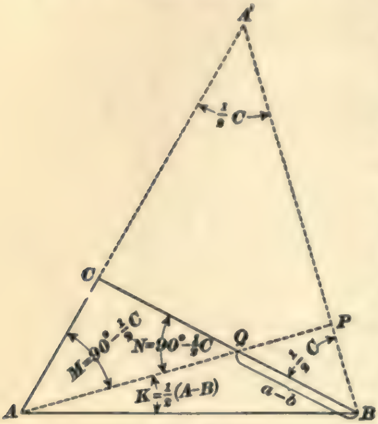

FIG. 27

Since $BC = CA'$, the triangle BCA' is isosceles, and, therefore, the angles $CA'B$ and CBA' are equal. The sum of these two angles, or twice either of them, is equal to the external angle BCA, or C, and therefore each of these two angles is equal to $\frac{1}{2}C$. In the right triangle APA', the angle M, being the complement of A', is equal to $90° - \frac{1}{2}C$. We have also,

$$K = A - M = A - (90° - \tfrac{1}{2}C),$$

or, since $C = 180° - (A + B) = 180° - A - B$,

$$K = A - [90° - \tfrac{1}{2}(180° - A - B)] = \tfrac{1}{2}(A - B)$$

The angle N being external to the triangle AQB, we have

$$N = K + B = \tfrac{1}{2}(A - B) + B = \tfrac{1}{2}(A + B)$$
$$= \tfrac{1}{2}(180° - C) = 90° - \tfrac{1}{2}C = M$$

Therefore, the triangle AQC is isosceles, and $QC = AC = b$; and, consequently, $BQ = a - b$.

The right triangle ABP gives,

$$\tan \tfrac{1}{2}(A - B) = \frac{BP}{AP}$$

or, writing the values of BP and AP from the triangles BQP and APA',

$$\tan \tfrac{1}{2}(A - B) = \frac{BQ \cos \frac{1}{2}C}{AA' \sin \frac{1}{2}C} = \frac{BQ}{AA'} \cot \tfrac{1}{2}C$$

that is,

$$\tan \tfrac{1}{2}(A - B) = \frac{a - b}{a + b} \cot \tfrac{1}{2}C \qquad (1)$$

Now, $\frac{1}{2}C = \frac{1}{2}[180° - (A + B)] = 90° - \frac{1}{2}(A + B)$, and therefore, $\cot \frac{1}{2}C = \tan \frac{1}{2}(A + B)$. By substituting this value in equation (1), and transforming, the formula in Art. **20** is obtained.

IV—FORMULA 2 OF ART. 21

This formula is derived from Fig. 27 as follows: In the triangle BPQ,

$$BP = BQ \cos \tfrac{1}{2} C = (a - b) \cos \tfrac{1}{2} C \qquad (1)$$

and, in the triangle ABP,

$$c\,(= AB) = \frac{BP}{\sin \frac{1}{2}(A - B)}$$

which becomes formula **2** when BP is replaced by its value (1).

V—FORMULAS 2 TO 4 OF ART. 23

We have (formula **8**, Art. **17**),

$$2 \cos^2 \tfrac{1}{2} A = 1 + \cos A$$

or, substituting the value of $\cos A$ from formula **1**, Art. **23**,

$$2 \cos^2 \tfrac{1}{2} A = 1 + \frac{b^2 + c^2 - a^2}{2bc} = \frac{2bc + b^2 + c^2 - a^2}{2bc} = \frac{(b + c)^2 - a^2}{2bc}$$

or, remembering that the difference between the squares of two numbers is equal to their sum multiplied by their difference,

$$2 \cos^2 \tfrac{1}{2} A = \frac{(b + c + a)(b + c - a)}{2bc} \qquad (1)$$

Now, since $a + b + c = 2s$, we have, subtracting $2a$ from both members, $b + c - a = 2s - 2a = 2(s - a)$. Likewise, $a + b - c = 2(s - c)$, and $a + c - b = 2(s - b)$. Substituting these values in equation (1),

$$2 \cos^2 \tfrac{1}{2} A = \frac{2s \times 2(s - a)}{2bc} = \frac{2s(s - a)}{bc}$$

whence,
$$\cos \tfrac{1}{2} A = \sqrt{\frac{s(s - a)}{bc}} \qquad (2)$$

which is formula **3**, Art. **23**.

Likewise (formula **7**, Art. **17**),

$$2 \sin^2 \tfrac{1}{2} A = 1 - \cos A = 1 - \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{2bc - b^2 - c^2 + a^2}{2bc} = \frac{a^2 - (b^2 - 2bc + c^2)}{2bc} = \frac{a^2 - (b - c)^2}{2bc}$$

$$= \frac{(a + b - c)(a - b + c)}{2bc} = \frac{2(s - c) \times 2(s - b)}{2bc} = \frac{2(s - b)(s - c)}{bc}$$

whence,
$$\sin \tfrac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{bc}} \qquad (3)$$

which is formula **4**, Art. **23**.

Formula **2** is obtained by dividing equation (3) by equation (2).

VI—FORMULA OF ART. 30

Formulas **3** and **4** of Art. **23** are:

$$\sin \tfrac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{bc}} \qquad (1)$$

$$\cos \tfrac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}} \qquad (2)$$

Also (formula **5**, Art. **17**),

$$\sin A = 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A \qquad (3)$$

Substituting in equation (3) the values of $\sin \frac{1}{2} A$ and $\cos \frac{1}{2} A$ from equations (1) and (2),

$$\sin A = 2\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{s(s-a)}{bc}} = 2\sqrt{\frac{s(s-a)(s-b)(s-c)}{b^2c^2}}$$

$$= 2\,\frac{\sqrt{s(s-a)(s-b)(s-c)}}{bc}$$

Substituting this value in formula of Art. **27**,

$$S = \sqrt{s(s-a)(s-b)(s-c)}$$

VII—FORMULA 3 OF ART. 33

We have, since $\cot = \frac{\cos}{\sin}$,

$$\frac{1}{\cot A + \cot B} = \frac{1}{\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}} = \frac{\sin A \sin B}{\sin B \cos A + \cos B \sin A}$$

$$= \frac{\sin A \sin B}{\sin (A+B)}$$

By substituting this value in formula **1**, we obtain

$$S = \frac{(b_1 - b_2)(b_1 + b_2) \sin A \sin B}{2 \sin (A+B)}$$

VIII—FORMULA OF ART. 34

Let the area of the triangle ADB', Fig. 18, be denoted by T, and that of the parallelogram $BCDB'$ by P. Then,

$$S = P + T \qquad (1)$$

Now, $$P = b_2 h,\ T = \tfrac{1}{2} d h$$

Dividing the first of these equations by the second,

$$\frac{P}{T} = \frac{b_2}{\frac{1}{2} d} = \frac{2 b_2}{d} = \frac{2 b_2}{b_1 - b_2}$$

whence, $$P = \frac{2 b_2}{b_1 - b_2} T$$

Substituting this value of P in equation (1),

$$S = \frac{2b_2}{b_1 - b_2}T + T = \left(\frac{2b_2}{b_1 - b_2} + 1\right)T = \frac{b_1 + b_2}{b_1 - b_2}T = \frac{b_1 + b_2}{d}T \qquad (2)$$

Let $\frac{1}{2}(a + c + d) = s$. Then (formula of Art. **30**),

$$T = \sqrt{s(s-a)(s-c)(s-d)}$$

and, substituting this value in equation (2),

$$S = \frac{b_1 + b_2}{d}\sqrt{s(s-a)(s-c)(s-d)}$$

TABLE OF TRIGONOMETRIC FORMULAS

The principal formulas occurring in the text, and others that can be readily derived from these, are tabulated in the following pages for convenient reference. As these formulas, which include those for the solution of triangles, are here systematically classified and arranged, the student will find this table useful in the solution of all kinds of problems requiring the application of trigonometry. He is advised to refer to it often, so as to become familiar with its contents and use.

FORMULAS DEFINING THE TRIGONOMETRIC FUNCTIONS

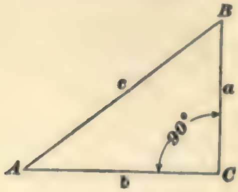

FIG. 28

1. $\sin A = \frac{a}{c}$
2. $\tan A = \frac{a}{b}$
3. $\cos A = \sin (90° - A) = \frac{b}{c}$
4. $\cot A = \tan (90° - A) = \frac{b}{a}$
5. $\sec A = \frac{c}{b}$
6. $\csc A = \sec (90° - A) = \frac{c}{a}$
7. $\text{vers } A = 1 - \cos A = 1 - \frac{b}{c}$
8. $\text{covers } A = \text{vers } (90° - A) = 1 - \sin A = 1 - \frac{a}{c}$

FUNCTIONS OF 0° AND 90°

9. $\sin 0° = 0$
10. $\tan 0° = 0$
11. $\cos 0° = 1$
12. $\cot 0° = \infty$
13. $\sec 0° = 1$
14. $\csc 0° = \infty$
15. $\sin 90° = 1$
16. $\tan 90° = \infty$
17. $\cos 90° = 0$
18. $\cot 90° = 0$
19. $\sec 90° = \infty$
20. $\csc 90° = 1$

FUNCTIONS OF NEGATIVE ANGLES

21. $\sin (-A) = -\sin A$
22. $\tan (-A) = -\tan A$
23. $\cos (-A) = \cos A$
24. $\cot (-A) = -\cot A$
25. $\sec (-A) = \sec A$
26. $\csc (-A) = -\csc A$

FUNCTIONS OF $90° + A$

27. $\sin(90° + A) = \cos A$
28. $\tan(90° + A) = -\cot A$
29. $\cos(90° + A) = -\sin A$
30. $\cot(90° + A) = -\tan A$
31. $\sec(90° + A) = -\csc A$
32. $\csc(90° + A) = \sec A$

FUNCTIONS OF $180° - A$ AND OF $180° + A$

33. $\sin(180° - A) = \sin A$
34. $\tan(180° - A) = -\tan A$
35. $\cos(180° - A) = -\cos A$
36. $\cot(180° - A) = -\cot A$
37. $\sec(180° - A) = -\sec A$
38. $\csc(180° - A) = \csc A$
39. $\sin(180° + A) = -\sin A$
40. $\tan(180° + A) = \tan A$
41. $\cos(180° + A) = -\cos A$
42. $\cot(180° + A) = \cot A$
43. $\sec(180° + A) = -\sec A$
44. $\csc(180° + A) = -\csc A$

FUNCTIONS OF $360° - A$ AND OF $360° + A$

45. $\sin(360° - A) = -\sin A$
46. $\tan(360° - A) = -\tan A$
47. $\cos(360° - A) = \cos A$
48. $\cot(360° - A) = -\cot A$
49. $\sec(360° - A) = \sec A$
50. $\csc(360° - A) = -\csc A$
51. $\sin(360° + A) = \sin A$
52. $\tan(360° + A) = \tan A$
53. $\cos(360° + A) = \cos A$
54. $\cot(360° + A) = \cot A$
55. $\sec(360° + A) = \sec A$
56. $\csc(360° + A) = \csc A$

FUNCTIONS OF $(A + B)$ AND OF $(A - B)$

57. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
58. $\sin(A - B) = \sin A \cos B - \cos A \sin B$
59. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
60. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
61. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
62. $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

FUNCTIONS OF 2 A AND OF ½ A

63. $\sin 2A = 2\sin A\cos A$
64. $\cos 2A = \cos^2 A - \sin^2 A$
65. $\cos 2A = 2\cos^2 A - 1$
66. $\cos 2A = 1 - 2\sin^2 A$
67. $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$
68. $\sin \tfrac{1}{2}A = \sqrt{\dfrac{1-\cos A}{2}}$
69. $\cos \tfrac{1}{2}A = \sqrt{\dfrac{1+\cos A}{2}}$
70. $\tan \tfrac{1}{2}A = \sqrt{\dfrac{1-\cos A}{1+\cos A}}$
71. $\tan \tfrac{1}{2}A = \dfrac{1-\cos A}{\sin A}$

SUMS AND DIFFERENCES OF FUNCTIONS

72. $\sin A + \sin B = 2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$
73. $\sin A - \sin B = 2\sin\tfrac{1}{2}(A-B)\cos\tfrac{1}{2}(A+B)$
74. $\cos A + \cos B = 2\cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$
75. $\cos A - \cos B = 2\sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(B-A)$
76. $\tan A + \tan B = \dfrac{\sin(A+B)}{\cos A\cos B}$
77. $\tan A - \tan B = \dfrac{\sin(A-B)}{\cos A\cos B}$
78. $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$
79. $\cos^2 A - \cos^2 B = \sin(A+B)\sin(B-A)$
80 $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$

RELATIONS AMONG THE FUNCTIONS OF AN ANGLE

$\sin A =$	$\tan A =$	$\cos A =$	$\cot A =$	$\sec A =$	$\csc A =$
81. $\frac{\tan A}{\sqrt{1+\tan^2 A}}$	86. $\frac{\sin A}{\sqrt{1-\sin^2 A}}$	91. $\sqrt{1-\sin^2 A}$	96. $\frac{\sqrt{1-\sin^2 A}}{\sin A}$	101. $\frac{1}{\sqrt{1-\sin^2 A}}$	106. $\frac{1}{\sin A}$
82. $\sqrt{1-\cos^2 A}$	87. $\frac{\sqrt{1-\cos^2 A}}{\cos A}$	92. $\frac{1}{\sqrt{1+\tan^2 A}}$	97. $\frac{1}{\tan A}$	102. $\sqrt{1+\tan^2 A}$	107. $\frac{\sqrt{1+\tan^2 A}}{\tan A}$
83. $\frac{1}{\sqrt{1+\cot^2 A}}$	88. $\frac{1}{\cot A}$	93. $\frac{\cot A}{\sqrt{1+\cot^2 A}}$	98. $\frac{\cos A}{\sqrt{1-\cos^2 A}}$	103. $\frac{1}{\cos A}$	108. $\frac{1}{\sqrt{1-\cos^2 A}}$
84. $\frac{\sqrt{\sec^2 A-1}}{\sec A}$	89. $\sqrt{\sec^2 A-1}$	94. $\frac{1}{\sec A}$	99. $\frac{1}{\sqrt{\sec^2 A-1}}$	104. $\frac{\sqrt{1+\cot^2 A}}{\cot A}$	109. $\sqrt{1+\cot^2 A}$
85. $\frac{1}{\csc A}$	90. $\frac{1}{\sqrt{\csc^2 A-1}}$	95. $\frac{\sqrt{\csc^2 A-1}}{\csc A}$	100. $\sqrt{\csc^2 A-1}$	105. $\frac{\csc A}{\sqrt{\csc^2 A-1}}$	110. $\frac{\sec A}{\sqrt{\sec^2 A-1}}$

FORMULAS FOR THE SOLUTION OF RIGHT TRIANGLES

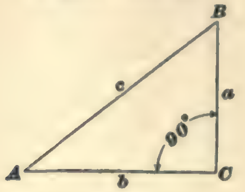

FIG. 29

Given	Required	Formula
a, A	B, b, c	111. $B = 90° - A$ 112. $b = a \cot A$ 113. $c = \dfrac{a}{\sin A} = a \csc A$
a, B	A, b, c	114. $A = 90° - B$ 115. $b = a \tan B$ 116. $c = \dfrac{a}{\cos B} = a \sec B$
c, A	B, a, b	117. $B = 90° - A$ 118. $a = c \sin A$ 119. $b = c \cos A$
a, b	A, B, c	120. $\tan A = \dfrac{a}{b}$ 121. $\tan B = \dfrac{b}{a}$, or $B = 90° - A$ 122. $c = \sqrt{a^2 + b^2}$ 123. $c = \dfrac{a}{\sin A} = a \csc A$
a, c	A, B, b	124. $\sin A = \dfrac{a}{c}$ 125. $\cos B = \dfrac{a}{c}$, or $B = 90° - A$ 126. $b = \sqrt{c^2 - a^2} = \sqrt{(c + a)(c - a)}$ 127. $b = a \cot A$

FORMULAS FOR THE SOLUTION OF OBLIQUE TRIANGLES

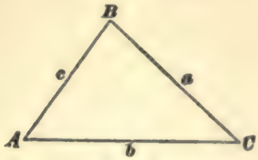

FIG. 30

Given	Required	Formulas
a, b, C	A, B, c	128. $\tan \frac{1}{2}(A - B) = \frac{a-b}{a+b} \cot \frac{1}{2} C$; $A = (90° - \frac{1}{2} C) + \frac{1}{2}(A - B)$; $B = (90° - \frac{1}{2} C) - \frac{1}{2}(A - B)$ 129. $c = \frac{(a-b) \cos \frac{1}{2} C}{\sin \frac{1}{2}(A - B)}$ 130. $c = \sqrt{a^2 + b^2 - 2ab \cos C}$
c, A, B	C, a, b	131. $C = 180° - (A + B)$ 132. $a = \frac{c}{\sin C} \sin A$ 133. $b = \frac{c}{\sin C} \sin B$
a, b, A	B, C, c	134. $\sin B = \frac{b}{a} \sin A$ 135. $C = 180° - A - B$ 136. $c = \frac{a}{\sin A} \sin C$
a, b, c $\frac{1}{2}(a + b + c) = s$	A	137. $\tan \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ 138. $\cos \frac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}}$ 139. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

NATURAL TRIGONOMETRIC FUNCTIONS

′	0°		1°		2°		3°		4°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.00000	1.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	1.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	1.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	1.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	1.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	1.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	1.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	1.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	1.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	1.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	1.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	89°		88°		87°		86°		85°		′

′	5°		6°		7°		8°		9°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	84°		83°		82°		81°		80°		

′	10° Sine	10° Cosine	11° Sine	11° Cosine	12° Sine	12° Cosine	13° Sine	13° Cosine	14° Sine	14° Cosine	′
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	79°		78°		77°		76°		75°		

′	15° Sine	15° Cosine	16° Sine	16° Cosine	17° Sine	17° Cosine	18° Sine	18° Cosine	19° Sine	19° Cosine	′
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	74°		73°		72°		71°		70°		

′	20° Sine	20° Cosine	21° Sine	21° Cosine	22° Sine	22° Cosine	23° Sine	23° Cosine	24° Sine	24° Cosine	′
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	69° Cosine	69° Sine	68° Cosine	68° Sine	67° Cosine	67° Sine	66° Cosine	66° Sine	65° Cosine	65° Sine	′

′	25°		26°		27°		28°		29°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	64°		63°		62°		61°		60°		′

′	30°		31°		32°		33°		34°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	59°		58°		57°		56°		55°		

′	35° Sine	35° Cosine	36° Sine	36° Cosine	37° Sine	37° Cosine	38° Sine	38° Cosine	39° Sine	39° Cosine	′
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	49°		48°		47°		46°		45°		′

′	0° Tang	0° Cotang	1° Tang	1° Cotang	2° Tang	2° Cotang	3° Tang	3° Cotang	4° Tang	4° Cotang	′
0	.00000	Infinite	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	.07022	14.2411	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	.07051	14.1821	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	.07080	14.1235	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	.07110	14.0655	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	.07139	14.0079	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	.07168	13.9507	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	.07197	13.8940	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	.07227	13.8378	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	.07256	13.7821	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	.07285	13.7267	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	.07314	13.6719	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	.07344	13.6174	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	.07373	13.5634	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	.07402	13.5098	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	.07431	13.4566	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	.07461	13.4039	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	.07490	13.3515	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	.07519	13.2996	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	.07548	13.2480	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	.07578	13.1969	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	.07607	13.1461	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	.07636	13.0958	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	.07665	13.0458	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	.07695	12.9962	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	.07724	12.9469	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	.07753	12.8981	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	.07782	12.8496	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	.07812	12.8014	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	.07841	12.7536	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	.07870	12.7062	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	.07899	12.6591	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	.07929	12.6124	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	.07958	12.5660	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	.07987	12.5199	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	.08017	12.4742	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	.08046	12.4288	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	.08075	12.3838	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	.08104	12.3390	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	.08134	12.2946	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	.08163	12.2505	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	.08192	12.2067	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	.08221	12.1632	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	.08251	12.1201	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	.08280	12.0772	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	.08309	12.0346	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	.08339	11.9923	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	.08368	11.9504	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	.08397	11.9087	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	.08427	11.8673	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	.08456	11.8262	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	.08485	11.7853	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	.08514	11.7448	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	.08544	11.7045	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	.08573	11.6645	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	.08602	11.6248	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	.08632	11.5853	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	.08661	11.5461	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	.08690	11.5072	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	.08720	11.4685	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	.08749	11.4301	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	60
1	.08778	11.3919	.10540	9.48781	.12308	8.12481	.14084	7.10038	.15868	6.30189	59
2	.08807	11.3540	.10569	9.46141	.12338	8.10536	.14113	7.08546	.15898	6.29007	58
3	.08837	11.3163	.10599	9.43515	.12367	8.08600	.14143	7.07059	.15928	6.27829	57
4	.08866	11.2789	.10628	9.40904	.12397	8.06674	.14173	7.05579	.15958	6.26655	56
5	.08895	11.2417	.10657	9.38307	.12426	8.04756	.14202	7.04105	.15988	6.25486	55
6	.08925	11.2048	.10687	9.35724	.12456	8.02848	.14232	7.02637	.16017	6.24321	54
7	.08954	11.1681	.10716	9.33155	.12485	8.00948	.14262	7.01174	.16047	6.23160	53
8	.08983	11.1316	.10746	9.30599	.12515	7.99058	.14291	6.99718	.16077	6.22003	52
9	.09013	11.0954	.10775	9.28058	.12544	7.97176	.14321	6.98268	.16107	6.20851	51
10	.09042	11.0594	.10805	9.25530	.12574	7.95302	.14351	6.96823	.16137	6.19703	50
11	.09071	11.0237	.10834	9.23016	.12603	7.93438	.14381	6.95385	.16167	6.18559	49
12	.09101	10.9882	.10863	9.20516	.12633	7.91582	.14410	6.93952	.16196	6.17419	48
13	.09130	10.9529	.10893	9.18028	.12662	7.89734	.14440	6.92525	.16226	6.16283	47
14	.09159	10.9178	.10922	9.15554	.12692	7.87895	.14470	6.91104	.16256	6.15151	46
15	.09189	10.8829	.10952	9.13093	.12722	7.86064	.14499	6.89688	.16286	6.14023	45
16	.09218	10.8483	.10981	9.10646	.12751	7.84242	.14529	6.88278	.16316	6.12899	44
17	.09247	10.8139	.11011	9.08211	.12781	7.82428	.14559	6.86874	.16346	6.11779	43
18	.09277	10.7797	.11040	9.05789	.12810	7.80622	.14588	6.85475	.16376	6.10664	42
19	.09306	10.7457	.11070	9.03379	.12840	7.78825	.14618	6.84082	.16405	6.09552	41
20	.09335	10.7119	.11099	9.00983	.12869	7.77035	.14648	6.82694	.16435	6.08444	40
21	.09365	10.6783	.11128	8.98598	.12899	7.75254	.14678	6.81312	.16465	6.07340	39
22	.09394	10.6450	.11158	8.96227	.12929	7.73480	.14707	6.79936	.16495	6.06240	38
23	.09423	10.6118	.11187	8.93867	.12958	7.71715	.14737	6.78564	.16525	6.05143	37
24	.09453	10.5789	.11217	8.91520	.12988	7.69957	.14767	6.77199	.16555	6.04051	36
25	.09482	10.5462	.11246	8.89185	.13017	7.68208	.14796	6.75838	.16585	6.02962	35
26	.09511	10.5136	.11276	8.86862	.13047	7.66466	.14826	6.74483	.16615	6.01878	34
27	.09541	10.4813	.11305	8.84551	.13076	7.64732	.14856	6.73133	.16645	6.00797	33
28	.09570	10.4491	.11335	8.82252	.13106	7.63005	.14886	6.71789	.16674	5.99720	32
29	.09600	10.4172	.11364	8.79964	.13136	7.61287	.14915	6.70450	.16704	5.98646	31
30	.09629	10.3854	.11394	8.77689	.13165	7.59575	.14945	6.69116	.16734	5.97576	30
31	.09658	10.3538	.11423	8.75425	.13195	7.57872	.14975	6.67787	.16764	5.96510	29
32	.09688	10.3224	.11452	8.73172	.13224	7.56176	.15005	6.66463	.16794	5.95448	28
33	.09717	10.2913	.11482	8.70931	.13254	7.54487	.15034	6.65144	.16824	5.94390	27
34	.09746	10.2602	.11511	8.68701	.13284	7.52806	.15064	6.63831	.16854	5.93335	26
35	.09776	10.2294	.11541	8.66482	.13313	7.51132	.15094	6.62523	.16884	5.92283	25
36	.09805	10.1988	.11570	8.64275	.13343	7.49465	.15124	6.61219	.16914	5.91236	24
37	.09834	10.1683	.11600	8.62078	.13372	7.47806	.15153	6.59921	.16944	5.90191	23
38	.09864	10.1381	.11629	8.59893	.13402	7.46154	.15183	6.58627	.16974	5.89151	22
39	.09893	10.1080	.11659	8.57718	.13432	7.44509	.15213	6.57339	.17004	5.88114	21
40	.09923	10.0780	.11688	8.55555	.13461	7.42871	.15243	6.56055	.17033	5.87080	20
41	.09952	10.0483	.11718	8.53402	.13491	7.41240	.15272	6.54777	.17063	5.86051	19
42	.09981	10.0187	.11747	8.51259	.13521	7.39616	.15302	6.53503	.17093	5.85024	18
43	.10011	9.98931	.11777	8.49128	.13550	7.37999	.15332	6.52234	.17123	5.84001	17
44	.10040	9.96007	.11806	8.47007	.13580	7.36389	.15362	6.50970	.17153	5.82982	16
45	.10069	9.93101	.11836	8.44896	.13609	7.34786	.15391	6.49710	.17183	5.81966	15
46	.10099	9.90211	.11865	8.42795	.13639	7.33190	.15421	6.48456	.17213	5.80953	14
47	.10128	9.87338	.11895	8.40705	.13669	7.31600	.15451	6.47206	.17243	5.79944	13
48	.10158	9.84482	.11924	8.38625	.13698	7.30018	.15481	6.45961	.17273	5.78938	12
49	.10187	9.81641	.11954	8.36555	.13728	7.28442	.15511	6.44720	.17303	5.77936	11
50	.10216	9.78817	.11983	8.34496	.13758	7.26873	.15540	6.43484	.17333	5.76937	10
51	.10246	9.76009	.12013	8.32446	.13787	7.25310	.15570	6.42253	.17363	5.75941	9
52	.10275	9.73217	.12042	8.30406	.13817	7.23754	.15600	6.41026	.17393	5.74949	8
53	.10305	9.70441	.12072	8.28376	.13846	7.22204	.15630	6.39804	.17423	5.73960	7
54	.10334	9.67680	.12101	8.26355	.13876	7.20661	.15660	6.38587	.17453	5.72974	6
55	.10363	9.64935	.12131	8.24345	.13906	7.19125	.15689	6.37374	.17483	5.71992	5
56	.10393	9.62205	.12160	8.22344	.13935	7.17594	.15719	6.36165	.17513	5.71013	4
57	.10422	9.59490	.12190	8.20352	.13965	7.16071	.15749	6.34961	.17543	5.70037	3
58	.10452	9.56791	.12219	8.18370	.13995	7.14553	.15779	6.33761	.17573	5.69064	2
59	.10481	9.54106	.12249	8.16398	.14024	7.13042	.15809	6.32566	.17603	5.68094	1
60	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	.17633	5.67128	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	17633	5.67128	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	60
1	.17663	5.66165	.19468	5.13658	.21286	4.69791	.23117	4.32573	.24964	4.00582	59
2	.17693	5.65205	.19498	5.12862	.21316	4.69121	.23148	4.32001	.24995	4.00086	58
3	.17723	5.64248	.19529	5.12069	.21347	4.68452	.23179	4.31430	.25026	3.99592	57
4	.17753	5.63295	.19559	5.11279	.21377	4.67786	.23209	4.30860	.25056	3.99099	56
5	.17783	5.62344	.19589	5.10490	.21408	4.67121	.23240	4.30291	.25087	3.98607	55
6	.17813	5.61397	.19619	5.09704	.21438	4.66458	.23271	4.29724	.25118	3.98117	54
7	.17843	5.60452	.19649	5.08921	.21469	4.65797	.23301	4.29159	.25149	3.97627	53
8	.17873	5.59511	.19680	5.08139	.21499	4.65138	.23332	4.28595	.25180	3.97139	52
9	.17903	5.58573	.19710	5.07360	.21529	4.64480	.23363	4.28032	.25211	3.96651	51
10	.17933	5.57638	.19740	5.06584	.21560	4.63825	.23393	4.27471	.25242	3.96165	50
11	.17963	5.56706	.19770	5.05809	.21590	4.63171	.23424	4.26911	.25273	3.95680	49
12	.17993	5.55777	.19801	5.05037	.21621	4.62518	.23455	4.26352	.25304	3.95196	48
13	.18023	5.54851	.19831	5.04267	.21651	4.61868	.23485	4.25795	.25335	3.94713	47
14	.18053	5.53927	.19861	5.03499	.21682	4.61219	.23516	4.25239	.25366	3.94232	46
15	.18083	5.53007	.19891	5.02734	.21712	4.60572	.23547	4.24685	.25397	3.93751	45
16	.18113	5.52090	.19921	5.01971	.21743	4.59927	.23578	4.24132	.25428	3.93271	44
17	.18143	5.51176	.19952	5.01210	.21773	4.59283	.23608	4.23580	.25459	3.92793	43
18	.18173	5.50264	.19982	5.00451	.21804	4.58641	.23639	4.23030	.25490	3.92316	42
19	.18203	5.49356	.20012	4.99695	.21834	4.58001	.23670	4.22481	.25521	3.91839	41
20	.18233	5.48451	.20042	4.98940	.21864	4.57363	.23700	4.21933	.25552	3.91364	40
21	.18263	5.47548	.20073	4.98188	.21895	4.56726	.23731	4.21387	.25583	3.90890	39
22	.18293	5.46648	.20103	4.97438	.21925	4.56091	.23762	4.20842	.25614	3.90417	38
23	.18323	5.45751	.20133	4.96690	.21956	4.55458	.23793	4.20298	.25645	3.89945	37
24	.18353	5.44857	.20164	4.95945	.21986	4.54826	.23823	4.19756	.25676	3.89474	36
25	.18384	5.43966	.20194	4.95201	.22017	4.54196	.23854	4.19215	.25707	3.89004	35
26	.18414	5.43077	.20224	4.94460	.22047	4.53568	.23885	4.18675	.25738	3.88536	34
27	.18444	5.42192	.20254	4.93721	.22078	4.52941	.23916	4.18137	.25769	3.88068	33
28	.18474	5.41309	.20285	4.92984	.22108	4.52316	.23946	4.17600	.25800	3.87601	32
29	.18504	5.40429	.20315	4.92249	.22139	4.51693	.23977	4.17064	.25831	3.87136	31
30	.18534	5.39552	.20345	4.91516	.22169	4.51071	.24008	4.16530	.25862	3.86671	30
31	.18564	5.38677	.20376	4.90785	.22200	4.50451	.24039	4.15997	.25893	3.86208	29
32	.18594	5.37805	.20406	4.90056	.22231	4.49832	.24069	4.15465	.25924	3.85745	28
33	.18624	5.36936	.20436	4.89330	.22261	4.49215	.24100	4.14934	.25955	3.85284	27
34	.18654	5.36070	.20466	4.88605	.22292	4.48600	.24131	4.14405	.25986	3.84824	26
35	.18684	5.35206	.20497	4.87882	.22322	4.47986	.24162	4.13877	.26017	3.84364	25
36	.18714	5.34345	.20527	4.87162	.22353	4.47374	.24193	4.13350	.26048	3.83906	24
37	.18745	5.33487	.20557	4.86444	.22383	4.46764	.24223	4.12825	.26079	3.83449	23
38	.18775	5.32631	.20588	4.85727	.22414	4.46155	.24254	4.12301	.26110	3.82992	22
39	.18805	5.31778	.20618	4.85013	.22444	4.45548	.24285	4.11778	.26141	3.82537	21
40	.18835	5.30928	.20648	4.84300	.22475	4.44942	.24316	4.11256	.26172	3.82083	20
41	.18865	5.30080	.20679	4.83590	.22505	4.44338	.24347	4.10736	.26203	3.81630	19
42	.18895	5.29235	.20709	4.82882	.22536	4.43735	.24377	4.10216	.26235	3.81177	18
43	.18925	5.28393	.20739	4.82175	.22567	4.43134	.24408	4.09699	.26266	3.80726	17
44	.18955	5.27553	.20770	4.81471	.22597	4.42534	.24439	4.09182	.26297	3.80276	16
45	.18986	5.26715	.20800	4.80769	.22628	4.41936	.24470	4.08666	.26328	3.79827	15
46	.19016	5.25880	.20830	4.80068	.22658	4.41340	.24501	4.08152	.26359	3.79378	14
47	.19046	5.25048	.20861	4.79370	.22689	4.40745	.24532	4.07639	.26390	3.78931	13
48	.19076	5.24218	.20891	4.78673	.22719	4.40152	.24562	4.07127	.26421	3.78485	12
49	.19106	5.23391	.20921	4.77978	.22750	4.39560	.24593	4.06616	.26452	3.78040	11
50	.19136	5.22566	.20952	4.77286	.22781	4.38969	.24624	4.06107	.26483	3.77595	10
51	.19166	5.21744	.20982	4.76595	.22811	4.38381	.24655	4.05599	.26515	3.77152	9
52	.19197	5.20925	.21013	4.75906	.22842	4.37793	.24686	4.05092	.26546	3.76709	8
53	.19227	5.20107	.21043	4.75219	.22872	4.37207	.24717	4.04586	.26577	3.76268	7
54	.19257	5.19293	.21073	4.74534	.22903	4.36623	.24747	4.04081	.26608	3.75828	6
55	.19287	5.18480	.21104	4.73851	.22934	4.36040	.24778	4.03578	.26639	3.75388	5
56	.19317	5.17671	.21134	4.73170	.22964	4.35459	.24809	4.03076	.26670	3.74950	4
57	.19347	5.16863	.21164	4.72490	.22995	4.34879	.24840	4.02574	.26701	3.74512	3
58	.19378	5.16058	.21195	4.71813	.23026	4.34300	.24871	4.02074	.26733	3.74075	2
59	.19408	5.15256	.21225	4.71137	.23056	4.33723	.24902	4.01576	.26764	3.73640	1
60	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.26795	3.73205	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.26826	3.72771	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.26857	3.72338	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.26888	3.71907	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.26920	3.71476	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.26951	3.71046	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.26982	3.70616	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.27013	3.70188	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.27044	3.69761	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.27076	3.69335	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.27107	3.68909	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.27138	3.68485	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.27169	3.68061	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.27201	3.67638	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.27232	3.67217	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.27263	3.66796	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.27294	3.66376	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.27326	3.65957	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.27357	3.65538	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.27388	3.65121	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.27419	3.64705	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.27451	3.64289	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.27482	3.63874	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.27513	3.63461	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.27545	3.63048	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.27576	3.62636	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.27607	3.62224	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.27638	3.61814	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.27670	3.61405	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.27701	3.60996	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.27732	3.60588	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.27764	3.60181	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.27795	3.59775	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.27826	3.59370	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.27858	3.58966	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.27889	3.58562	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.27921	3.58160	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.27952	3.57758	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.27983	3.57357	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.28015	3.56957	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.28046	3.56557	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.28077	3.56159	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.28109	3.55761	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.28140	3.55364	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.28172	3.54968	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.28203	3.54573	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.28234	3.54179	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.28266	3.53785	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.28297	3.53393	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.28329	3.53001	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.28360	3.52609	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.28391	3.52219	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.28423	3.51829	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.28454	3.51441	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.28486	3.51053	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.28517	3.50666	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.28549	3.50279	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.28580	3.49894	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.28612	3.49509	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.28643	3.49125	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	.44558	2.24428	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	.44593	2.24252	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	.44627	2.24077	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	.44662	2.23902	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	.44697	2.23727	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	.44732	2.23553	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	.44767	2.23378	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	.44802	2.23204	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	.44837	2.23030	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	.44872	2.22857	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	.44907	2.22683	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	.44942	2.22510	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2:33130	.44977	2.22337	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	.45012	2.22164	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	.45047	2.21992	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	.45082	2.21819	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	.45117	2.21647	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	.45152	2.21475	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	.45187	2.21304	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	.45222	2.21132	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	.45257	2.20961	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	.45292	2.20790	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43230	2.31271	.45327	2.20619	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	.45362	2.20449	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	.45397	2.20278	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	.45432	2.20108	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	.45467	2.19938	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	.45502	2.19769	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	.45538	2.19599	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	.45573	2.19430	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	.45608	2.19261	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	.45643	2.19092	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	.45678	2.18923	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	.45713	2.18755	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	.45748	2.18587	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	.45784	2.18419	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	.45819	2.18251	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	.45854	2.18084	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	.45889	2.17916	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	.45924	2.17749	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	.45960	2.17582	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	.45995	2.17416	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	.46030	2.17249	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	.46065	2.17083	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	.46101	2.16917	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	.46136	2.16751	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	.46171	2.16585	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	.46206	2.16420	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	.46242	2.16255	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	.46277	2.16090	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	.46312	2.15925	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	.46348	2.15760	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	.46383	2.15596	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	.46418	2.15432	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	.46454	2.15268	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	.46489	2.15104	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	.46525	2.14940	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	.46560	2.14777	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	.46595	2.14614	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	.46631	2.14451	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
′	69°		68°		67°		66°		65°		′

′	25°		26°		27°		28°		29°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	60
1	.46666	2.14288	.48809	2.04879	.50989	1.96120	.53208	1.87941	.55469	1.80281	59
2	.46702	2.14125	.48845	2.04728	.51026	1.95979	.53246	1.87809	.55507	1.80158	58
3	.46737	2.13963	.48881	2.04577	.51063	1.95838	.53283	1.87677	.55545	1.80034	57
4	.46772	2.13801	.48917	2.04426	.51099	1.95698	.53320	1.87546	.55583	1.79911	56
5	.46808	2.13639	.48953	2.04276	.51136	1.95557	.53358	1.87415	.55621	1.79788	55
6	.46843	2.13477	.48989	2.04125	.51173	1.95417	.53395	1.87283	.55659	1.79665	54
7	.46879	2.13316	.49026	2.03975	.51209	1.95277	.53432	1.87152	55697	1.79542	53
8	.46914	2.13154	.49062	2.03825	.51246	1.95137	.53470	1.87021	.55736	1.79419	52
9	.46950	2.12993	.49098	2.03675	.51283	1.94997	.53507	1.86891	.55774	1.79296	51
10	.46985	2.12832	.49134	2.03526	.51319	1.94858	.53545	1.86760	.55812	1.79174	50
11	.47021	2.12671	.49170	2.03376	.51356	1.94718	.53582	1.86630	.55850	1.79051	49
12	.47056	2.12511	.49206	2.03227	.51393	1.94579	.53620	1.86499	.55888	1.78929	48
13	.47092	2.12350	.49242	2.03078	.51430	1.94440	.53657	1.86369	.55926	1.78807	47
14	.47128	2.12190	.49278	2.02929	.51467	1.94301	.53694	1.86239	.55964	1.78685	46
15	.47163	2.12030	.49315	2.02780	.51503	1.94162	.53732	1.86109	.56003	1.78563	45
16	.47199	2.11871	.49351	2.02631	.51540	1.94023	.53769	1.85979	.56041	1.78441	44
17	.47234	2.11711	.49387	2.02483	.51577	1.93885	.53807	1.85850	.56079	1.78319	43
18	.47270	2.11552	.49423	2.02335	.51614	1.93746	.53844	1.85720	.56117	1.78198	42
19	.47305	2.11392	.49459	2.02187	.51651	1.93608	.53882	1.85591	.56156	1.78077	41
20	.47341	2.11233	.49495	2.02039	.51688	1.93470	.53920	1.85462	.56194	1.77955	40
21	.47377	2.11075	.49532	2.01891	.51724	1.93332	.53957	1.85333	.56232	1.77834	39
22	.47412	2.10916	.49568	2.01743	.51761	1.93195	.53995	1.85204	.56270	1.77713	38
23	.47448	2.10758	.49604	2.01596	.51798	1.93057	.54032	1.85075	.56309	1.77592	37
24	.47483	2.10600	.49640	2.01449	.51835	1.92920	.54070	1.84946	.56347	1.77471	36
25	.47519	2.10442	.49677	2.01302	.51872	1.92782	.54107	1.84818	.56385	1.77351	35
26	.47555	2.10284	.49713	2.01155	.51909	1.92645	.54145	1.84689	.56424	1.77230	34
27	.47590	2.10126	.49749	2.01008	.51946	1.92508	.54183	1.84561	.56462	1.77110	33
28	.47626	2.09969	.49786	2.00862	.51983	1.92371	.54220	1.84433	.56501	1.76990	32
29	.47662	2.09811	.49822	2.00715	.52020	1.92235	.54258	1.84305	.56539	1.76869	31
30	.47698	2.09654	.49858	2.00569	.52057	1.92098	.54296	1.84177	.56577	1.76749	30
31	.47733	2.09498	.49894	2.00423	.52094	1.91962	.54333	1.84049	.56616	1.76629	29
32	.47769	2.09341	.49931	2.00277	.52131	1.91826	.54371	1.83922	.56654	1.76510	28
33	.47805	2.09184	.49967	2.00131	.52168	1.91690	.54409	1.83794	.56693	1.76390	27
34	.47840	2.09028	.50004	1.99986	.52205	1.91554	.54446	1.83667	.56731	1.76271	26
35	.47876	2.08872	.50040	1.99841	.52242	1.91418	.54484	1.83540	.56769	1.76151	25
36	.47912	2.08716	.50076	1.99695	.52279	1.91282	.54522	1.83413	.56808	1.76032	24
37	.47948	2.08560	.50113	1.99550	.52316	1.91147	.54560	1.83286	.56846	1.75913	23
38	.47984	2.08405	.50149	1.99406	.52353	1.91012	.54597	1.83159	.56885	1.75794	22
39	.48019	2.08250	.50185	1.99261	.52390	1.90876	.54635	1.83033	.56923	1.75675	21
40	.48055	2.08094	.50222	1.99116	.52427	1.90741	.54673	1.82906	.56962	1.75556	20
41	.48091	2.07939	.50258	1.98972	.52464	1.90607	.54711	1.82780	.57000	1.75437	19
42	.48127	2.07785	.50295	1.98828	.52501	1.90472	.54748	1.82654	.57039	1.75319	18
43	.48163	2.07630	.50331	1.98684	.52538	1.90337	.54786	1.82528	.57078	1.75200	17
44	.48198	2.07476	.50368	1.98540	.52575	1.90203	.54824	1.82402	.57116	1.75082	16
45	.48234	2.07321	.50404	1.98396	.52613	1.90069	.54862	1.82276	.57155	1.74964	15
46	.48270	2.07167	.50441	1.98253	.52650	1.89935	.54900	1.82150	.57193	1.74846	14
47	.48306	2.07014	.50477	1.98110	.52687	1.89801	.54938	1.82025	.57232	1.74728	13
48	.48342	2.06860	.50514	1.97966	.52724	1.89667	.54975	1.81899	.57271	1.74610	12
49	.48378	2.06706	.50550	1.97823	.52761	1.89533	.55013	1.81774	.57309	1.74492	11
50	.48414	2.06553	.50587	1.97681	.52798	1.89400	.55051	1.81649	.57348	1.74375	10
51	.48450	2.06400	.50623	1.97538	.52836	1.89266	.55089	1.81524	.57386	1.74257	9
52	.48486	2.06247	.50660	1.97395	.52873	1.89133	.55127	1.81399	.57425	1.74140	8
53	.48521	2.06094	.50696	1.97253	.52910	1.89000	.55165	1.81274	.57464	1.74022	7
54	.48557	2.05942	.50733	1.97111	.52947	1.88867	.55203	1.81150	.57503	1.73905	6
55	.48593	2.05790	.50769	1.96969	.52985	1.88734	.55241	1.81025	.57541	1.73788	5
56	.48629	2.05637	.50806	1.96827	.53022	1.88602	.55279	1.80901	.57580	1.73671	4
57	.48665	2.05485	.50843	1.96685	.53059	1.88469	.55317	1.80777	.57619	1.73555	3
58	.48701	2.05333	.50879	1.96544	.53096	1.88337	.55355	1.80653	.57657	1.73438	2
59	.48737	2.05182	.50916	1.96402	.53134	1.88205	.55393	1.80529	.57696	1.73321	1
60	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	.57735	1.73205	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.57735	1.73205	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	60
1	.57774	1.73089	.60126	1.66318	.62527	1.59930	.64982	1.53888	.67493	1.48163	59
2	.57813	1.72973	.60165	1.66209	.62568	1.59826	.65024	1.53791	.67536	1.48070	58
3	.57851	1.72857	.60205	1.66099	.62608	1.59723	.65065	1.53693	.67578	1.47977	57
4	.57890	1.72741	.60245	1.65990	.62649	1.59620	.65106	1.53595	.67620	1.47885	56
5	.57929	1.72625	.60284	1.65881	.62689	1.59517	.65148	1.53497	.67663	1.47792	55
6	.57968	1.72509	.60324	1.65772	.62730	1.59414	.65189	1.53400	.67705	1.47699	54
7	.58007	1.72393	.60364	1.65663	.62770	1.59311	.65231	1.53302	.67748	1.47607	53
8	.58046	1.72278	.60403	1.65554	.62811	1.59208	.65272	1.53205	.67790	1.47514	52
9	.58085	1.72163	.60443	1.65445	.62852	1.59105	.65314	1.53107	.67832	1.47422	51
10	.58124	1.72047	.60483	1.65337	.62892	1.59002	.65355	1.53010	.67875	1.47330	50
11	.58162	1.71932	.60522	1.65228	.62933	1.58900	.65397	1.52913	.67917	1.47238	49
12	.58201	1.71817	.60562	1.65120	.62973	1.58797	.65438	1.52816	.67960	1.47146	48
13	.58240	1.71702	.60602	1.65011	.63014	1.58695	.65480	1.52719	.68002	1.47053	47
14	.58279	1.71588	.60642	1.64903	.63055	1.58593	.65521	1.52622	.68045	1.46962	46
15	.58318	1.71473	.60681	1.64795	.63095	1.58490	.65563	1.52525	.68088	1.46870	45
16	.58357	1.71358	.60721	1.64687	.63136	1.58388	.65604	1.52429	.68130	1.46778	44
17	.58396	1.71244	.60761	1.64579	.63177	1.58286	.65646	1.52332	.68173	1.46686	43
18	.58435	1.71129	.60801	1.64471	.63217	1.58184	.65688	1.52235	.68215	1.46595	42
19	.58474	1.71015	.60841	1.64363	.63258	1.58083	.65729	1.52139	.68258	1.46503	41
20	.58513	1.70901	.60881	1.64256	.63299	1.57981	.65771	1.52043	.68301	1.46411	40
21	.58552	1.70787	.60921	1.64148	.63340	1.57879	.65813	1.51946	.68343	1.46320	39
22	.58591	1.70673	.60960	1.64041	.63380	1.57778	.65854	1.51850	.68386	1.46229	38
23	.58631	1.70560	.61000	1.63934	.63421	1.57676	.65896	1.51754	.68429	1.46137	37
24	.58670	1.70446	.61040	1.63826	.63462	1.57575	.65938	1.51658	.68471	1.46046	36
25	.58709	1.70332	.61080	1.63719	.63503	1.57474	.65980	1.51562	.68514	1.45955	35
26	.58748	1.70219	.61120	1.63612	.63544	1.57372	.66021	1.51466	.68557	1.45864	34
27	.58787	1.70106	.61160	1.63505	.63584	1.57271	.66063	1.51370	.68600	1.45773	33
28	.58826	1.69992	.61200	1.63398	.63625	1.57170	.66105	1.51275	.68642	1.45682	32
29	.58865	1.69879	.61240	1.63292	.63666	1.57069	.66147	1.51179	.68685	1.45592	31
30	.58905	1.69766	.61280	1.63185	.63707	1.56969	.66189	1.51084	.68728	1.45501	30
31	.58944	1.69653	.61320	1.63079	.63748	1.56868	.66230	1.50988	.68771	1.45410	29
32	.58983	1.69541	.61360	1.62972	.63789	1.56767	.66272	1.50893	.68814	1.45320	28
33	.59022	1.69428	.61400	1.62866	.63830	1.56667	.66314	1.50797	.68857	1.45229	27
34	.59061	1.69316	.61440	1.62760	.63871	1.56566	.66356	1.50702	.68900	1.45139	26
35	.59101	1.69203	.61480	1.62654	.63912	1.56466	.66398	1.50607	.68942	1.45049	25
36	.59140	1.69091	.61520	1.62548	.63953	1.56366	.66440	1.50512	.68985	1.44958	24
37	.59179	1.68979	.61561	1.62442	.63994	1.56265	.66482	1.50417	.69028	1.44868	23
38	.59218	1.68866	.61601	1.62336	.64035	1.56165	.66524	1.50322	.69071	1.44778	22
39	.59258	1.68754	.61641	1.62230	.64076	1.56065	.66566	1.50228	.69114	1.44688	21
40	.59297	1.68643	.61681	1.62125	.64117	1.55966	.66608	1.50133	.69157	1.44598	20
41	.59336	1.68531	.61721	1.62019	.64158	1.55866	.66650	1.50038	.69200	1.44508	19
42	.59376	1.68419	.61761	1.61914	.64199	1.55766	.66692	1.49944	.69243	1.44418	18
43	.59415	1.68308	.61801	1.61808	.64240	1.55666	.66734	1.49849	.69286	1.44329	17
44	.59454	1.68196	.61842	1.61703	.64281	1.55567	.66776	1.49755	.69329	1.44239	16
45	.59494	1.68085	.61882	1.61598	.64322	1.55467	.66818	1.49661	.69372	1.44149	15
46	.59533	1.67974	.61922	1.61493	.64363	1.55368	.66860	1.49566	.69416	1.44060	14
47	.59573	1.67863	.61962	1.61388	.64404	1.55269	.66902	1.49472	.69459	1.43970	13
48	.59612	1.67752	.62003	1.61283	.64446	1.55170	.66944	1.49378	.69502	1.43881	12
49	.59651	1.67641	.62043	1.61179	.64487	1.55071	.66986	1.49284	.69545	1.43792	11
50	.59691	1.67530	.62083	1.61074	.64528	1.54972	.67028	1.49190	.69588	1.43703	10
51	.59730	1.67419	.62124	1.60970	.64569	1.54873	.67071	1.49097	.69631	1.43614	9
52	.59770	1.67309	.62164	1.60865	.64610	1.54774	.67113	1.49003	.69675	1.43525	8
53	.59809	1.67198	.62204	1.60761	.64652	1.54675	.67155	1.48909	.69718	1.43436	7
54	.59849	1.67088	.62245	1.60657	.64693	1.54576	.67197	1.48816	.69761	1.43347	6
55	.59888	1.66978	.62285	1.60553	.64734	1.54478	.67239	1.48722	.69804	1.43258	5
56	.59928	1.66867	.62325	1.60449	.64775	1.54379	.67282	1.48629	.69847	1.43169	4
57	.59967	1.66757	.62366	1.60345	.64817	1.54281	.67324	1.48536	.69891	1.43080	3
58	.60007	1.66647	.62406	1.60241	.64858	1.54183	.67366	1.48442	.69934	1.42992	2
59	.60046	1.66538	.62446	1.60137	.64899	1.54085	.67409	1.48349	.69977	1.42903	1
60	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.70021	1.42815	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.70064	1.42726	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.70107	1.42638	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.70151	1.42550	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.70194	1.42462	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.70238	1.42374	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.70281	1.42286	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.70325	1.42198	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.70368	1.42110	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.70412	1.42022	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.70455	1.41934	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.70499	1.41847	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.70542	1.41759	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.70586	1.41672	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.70629	1.41584	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.70673	1.41497	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.70717	1.41409	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.70760	1.41322	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.70804	1.41235	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.70848	1.41148	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.70891	1.41061	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.70935	1.40974	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.70979	1.40887	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.71023	1.40800	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.71066	1.40714	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.71110	1.40627	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.71154	1.40540	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.71198	1.40454	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.71242	1.40367	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.71285	1.40281	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.71329	1.40195	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.71373	1.40109	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.71417	1.40022	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.71461	1.39936	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.71505	1.39850	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.71549	1.39764	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.71593	1.39679	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.71637	1.39593	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.71681	1.39507	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.71725	1.39421	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.71769	1.39336	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.71813	1.39250	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.71857	1.39165	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.71901	1.39079	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.71946	1.38994	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.71990	1.38909	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.72034	1.38824	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.72078	1.38738	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.72122	1.38653	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.72167	1.38568	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.72211	1.38484	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.72255	1.38399	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.72299	1.38314	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.72344	1.38229	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.72388	1.38145	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.72432	1.38060	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.72477	1.37976	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.72521	1.37891	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.72565	1.37807	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.72610	1.37722	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	.96625	1.03493	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	.96681	1.03433	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	.96738	1.03372	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	.96794	1.03312	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	.96850	1.03252	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	.96907	1.03192	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	.96963	1.03132	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	.97020	1.03072	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	.97076	1.03012	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	.97133	1.02952	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	.97189	1.02892	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	.97246	1.02832	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	.97302	1.02772	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	.97359	1.02713	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	.97416	1.02653	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	.97472	1.02593	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	.97529	1.02533	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09399	.94235	1.06117	.97586	1.02474	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	.97643	1.02414	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	.97700	1.02355	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	.97756	1.02295	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	.97813	1.02236	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	.97870	1.02176	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	.97927	1.02117	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	.97984	1.02057	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	.98041	1.01998	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	.98098	1.01939	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	.98155	1.01879	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	.98213	1.01820	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	.98270	1.01761	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	.98327	1.01702	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	.98384	1.01642	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	.98441	1.01583	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	.98499	1.01524	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	.98556	1.01465	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	.98613	1.01406	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	.98671	1.01347	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	.98728	1.01288	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	.98786	1.01229	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	.98843	1.01170	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	.98901	1.01112	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	.98958	1.01053	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	.99016	1.00994	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	.99073	1.00935	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	.99131	1.00876	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	.99189	1.00818	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	.99247	1.00759	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	.99304	1.00701	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	.99362	1.00642	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	.99420	1.00583	10
51	.86470	1.15647	.89567	1.11648	.02763	1.07801	.96064	1.04097	.99478	1.00525	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	.99536	1.00467	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	.99594	1.00408	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	.99652	1.00350	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	.99710	1.00291	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	.99768	1.00233	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	.99826	1.00175	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	.99884	1.00116	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	.99942	1.00058	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	1.00000	1.00000	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		45°		

LOGARITHMIC TRIGONOMETRIC FUNCTIONS

″	′	log sin	d	S	T	log tan	c. d.	log cot	C	log cos	
				$\bar{6}$.685					5.314		
0	0									0.00000	60
60	1	$\bar{4}$.46373	30103	57	57	$\bar{4}$.46373	30103	3.53627	43	0.00000	59
120	2	$\bar{4}$.76476	17609	57	57	$\bar{4}$.76476	17609	3.23524	43	0.00000	58
180	3	$\bar{4}$.94085	12494	57	58	$\bar{4}$.94085	12494	3.05915	43	0.00000	57
240	4	$\bar{3}$.06579	9691	57	58	$\bar{3}$.06579	9691	2.93421	42	0.00000	56
300	5	$\bar{3}$.16270	7918	57	58	$\bar{3}$.16270	7918	2.83730	42	0.00000	55
360	6	$\bar{3}$.24188	6694	57	58	$\bar{3}$.24188	6694	2.75812	42	0.00000	54
420	7	$\bar{3}$.30882	5800	57	58	$\bar{3}$.30882	5800	2.69118	42	0.00000	53
480	8	$\bar{3}$.36682	5115	57	58	$\bar{3}$.36682	5115	2.63318	42	0.00000	52
540	9	$\bar{3}$.41797	4576	57	58	$\bar{3}$.41797	4576	2.58203	42	0.00000	51
600	10	$\bar{3}$.46373	4139	57	58	$\bar{3}$.46373	4139	2.53627	42	0.00000	50
660	11	$\bar{3}$.50512	3779	57	58	$\bar{3}$.50512	3779	2.49488	42	0.00000	49
720	12	$\bar{3}$.54291	3476	57	58	$\bar{3}$.54291	3476	2.45709	42	0.00000	48
780	13	$\bar{3}$.57767	3218	57	58	$\bar{3}$.57767	3219	2.42233	42	0.00000	47
840	14	$\bar{3}$.60985	2997	57	58	$\bar{3}$.60986	2996	2.39014	42	0.00000	46
900	15	$\bar{3}$.63982	2802	57	58	$\bar{3}$.63982	2803	2.36018	42	0.00000	45
960	16	$\bar{3}$.66784	2633	57	58	$\bar{3}$.66785	2633	2.33215	42	0.00000	44
1020	17	$\bar{3}$.69417	2483	57	58	$\bar{3}$.69418	2482	2.30582	42	$\bar{1}$.99999	43
1080	18	$\bar{3}$.71900	2348	57	58	$\bar{3}$.71900	2348	2.28100	42	$\bar{1}$.99999	42
1140	19	$\bar{3}$.74248	2227	57	58	$\bar{3}$.74248	2228	2.25752	42	$\bar{1}$.99999	41
1200	20	$\bar{3}$.76475	2119	57	58	$\bar{3}$.76476	2119	2.23524	42	$\bar{1}$.99999	40
1260	21	$\bar{3}$.78594	2021	57	58	$\bar{3}$.78595	2020	2.21405	42	$\bar{1}$.99999	39
1320	22	$\bar{3}$.80615	1930	57	58	$\bar{3}$.80615	1931	2.19385	42	$\bar{1}$.99999	38
1380	23	$\bar{3}$.82545	1848	57	58	$\bar{3}$.82546	1848	2.17454	42	$\bar{1}$.99999	37
1440	24	$\bar{3}$.84393	1773	57	58	$\bar{3}$.84394	1773	2.15606	42	$\bar{1}$.99999	36
1500	25	$\bar{3}$.86166	1704	57	58	$\bar{3}$.86167	1704	2.13833	42	$\bar{1}$.99999	35
1560	26	$\bar{3}$.87870	1639	57	58	$\bar{3}$.87871	1639	2.12129	42	$\bar{1}$.99999	34
1620	27	$\bar{3}$.89509	1579	57	58	$\bar{3}$.89510	1579	2.10490	42	$\bar{1}$.99999	33
1680	28	$\bar{3}$.91088	1524	57	58	$\bar{3}$.91089	1524	2.08911	42	$\bar{1}$.99999	32
1740	29	$\bar{3}$.92612	1472	57	59	$\bar{3}$.92613	1473	2.07387	41	$\bar{1}$.99998	31
1800	30	$\bar{3}$.94084	1424	57	59	$\bar{3}$.94086	1424	2.05914	41	$\bar{1}$.99998	30
1860	31	$\bar{3}$.95508	1379	57	59	$\bar{3}$.95510	1379	2.04490	41	$\bar{1}$.99998	29
1920	32	$\bar{3}$.96887	1336	57	59	$\bar{3}$.96889	1336	2.03111	41	$\bar{1}$.99998	28
1980	33	$\bar{3}$.98223	1297	57	59	$\bar{3}$.98225	1297	2.01775	41	$\bar{1}$.99998	27
2040	34	$\bar{3}$.99520	1259	57	59	$\bar{3}$.99522	1259	2.00478	41	$\bar{1}$.99998	26
2100	35	$\bar{2}$.00779	1223	57	59	$\bar{2}$.00781	1223	1.99219	41	$\bar{1}$.99998	25
2160	36	$\bar{2}$.02002	1190	57	59	$\bar{2}$.02004	1190	1.97996	41	$\bar{1}$.99998	24
2220	37	$\bar{2}$.03192	1158	57	59	$\bar{2}$.03194	1159	1.96806	41	$\bar{1}$.99997	23
2280	38	$\bar{2}$.04350	1128	57	59	$\bar{2}$.04353	1128	1.95647	41	$\bar{1}$.99997	22
2340	39	$\bar{2}$.05478	1100	57	59	$\bar{2}$.05481	1100	1.94519	41	$\bar{1}$.99997	21
2400	40	$\bar{2}$.06578	1072	57	59	$\bar{2}$.06581	1072	1.93419	41	$\bar{1}$.99997	20
2460	41	$\bar{2}$.07650	1046	56	60	$\bar{2}$.07653	1047	1.92347	40	$\bar{1}$.99997	19
2520	42	$\bar{2}$.08696	1022	56	60	$\bar{2}$.08700	1022	1.91300	40	$\bar{1}$.99997	18
2580	43	$\bar{2}$.09718	999	56	60	$\bar{2}$.09722	998	1.90278	40	$\bar{1}$.99997	17
2640	44	$\bar{2}$.10717	976	56	60	$\bar{2}$.10720	976	1.89280	40	$\bar{1}$.99996	16
2700	45	$\bar{2}$.11693	954	56	60	$\bar{2}$.11696	955	1.88304	40	$\bar{1}$.99996	15
2760	46	$\bar{2}$.12647	934	56	60	$\bar{2}$.12651	934	1.87349	40	$\bar{1}$.99996	14
2820	47	$\bar{2}$.13581	914	56	60	$\bar{2}$.13585	915	1.86415	40	$\bar{1}$.99996	13
2880	48	$\bar{2}$.14495	896	56	60	$\bar{2}$.14500	895	1.85500	40	$\bar{1}$.99996	12
2940	49	$\bar{2}$.15391	877	56	60	$\bar{2}$.15395	878	1.84605	40	$\bar{1}$.99996	11
3000	50	$\bar{2}$.16268	860	56	61	$\bar{2}$.16273	860	1.83727	39	$\bar{1}$.99995	10
3060	51	$\bar{2}$.17128	843	56	61	$\bar{2}$.17133	843	1.82867	39	$\bar{1}$.99995	9
3120	52	$\bar{2}$.17971	827	56	61	$\bar{2}$.17976	828	1.82024	39	$\bar{1}$.99995	8
3180	53	$\bar{2}$.18798	812	56	61	$\bar{2}$.18804	812	1.81196	39	$\bar{1}$.99995	7
3240	54	$\bar{2}$.19610	797	56	61	$\bar{2}$.19616	797	1.80384	39	$\bar{1}$.99995	6
3300	55	$\bar{2}$.20407	782	56	61	$\bar{2}$.20413	782	1.79587	39	$\bar{1}$.99994	5
3360	56	$\bar{2}$.21189	769	56	61	$\bar{2}$.21195	769	1.78805	39	$\bar{1}$.99994	4
3420	57	$\bar{2}$.21958	755	55	61	$\bar{2}$.21964	756	1.78036	39	$\bar{1}$.99994	3
3480	58	$\bar{2}$.22713	743	55	62	$\bar{2}$.22720	742	1.77280	38	$\bar{1}$.99994	2
3540	59	$\bar{2}$.23456	730	55	62	$\bar{2}$.23462	730	1.76538	38	$\bar{1}$.99994	1
3600	60	$\bar{2}$.24186		55	62	$\bar{2}$.24192		1.75808	38	$\bar{1}$.99993	0
				$\bar{6}$.685					5.314		
		log cos	d	S	T	log cot	c. d.	log tan	C	log sin	′

″	′	log sin	d	S	T	log tan	c. d.	log cot	C	log cos	
				$\bar{6}.685$					5.314		
3600	0	$\bar{2}.24186$	717	55	62	$\bar{2}.24192$	718	1.75808	38	$\bar{1}.99993$	60
3660	1	$\bar{2}.24903$	706	55	62	$\bar{2}.24910$	706	1.75090	38	$\bar{1}.99993$	59
3720	2	$\bar{2}.25609$	695	55	62	$\bar{2}.25616$	696	1.74384	38	$\bar{1}.99993$	58
3780	3	$\bar{2}.26304$	684	55	62	$\bar{2}.26312$	684	1.73688	38	$\bar{1}.99993$	57
3840	4	$\bar{2}.26988$	673	55	63	$\bar{2}.26996$	673	1.73004	37	$\bar{1}.99992$	56
3900	5	$\bar{2}.27661$	663	55	63	$\bar{2}.27669$	663	1.72331	37	$\bar{1}.99992$	55
3960	6	$\bar{2}.28324$	653	55	63	$\bar{2}.28332$	654	1.71668	37	$\bar{1}.99992$	54
4020	7	$\bar{2}.28977$	644	55	63	$\bar{2}.28986$	643	1.71014	37	$\bar{1}.99992$	53
4080	8	$\bar{2}.29621$	634	55	63	$\bar{2}.29629$	634	1.70371	37	$\bar{1}.99992$	52
4140	9	$\bar{2}.30255$	624	55	63	$\bar{2}.30263$	625	1.69737	37	$\bar{1}.99991$	51
4200	10	$\bar{2}.30879$	616	54	63	$\bar{2}.30888$	617	1.69112	37	$\bar{1}.99991$	50
4260	11	$\bar{2}.31495$	608	54	64	$\bar{2}.31505$	607	1.68495	36	$\bar{1}.99991$	49
4320	12	$\bar{2}.32103$	599	54	64	$\bar{2}.32112$	599	1.67888	36	$\bar{1}.99990$	48
4380	13	$\bar{2}.32702$	590	54	64	$\bar{2}.32711$	591	1.67289	36	$\bar{1}.99990$	47
4440	14	$\bar{2}.33292$	583	54	64	$\bar{2}.33302$	584	1.66698	36	$\bar{1}.99990$	46
4500	15	$\bar{2}.33875$	575	54	64	$\bar{2}.33886$	575	1.66114	36	$\bar{1}.99990$	45
4560	16	$\bar{2}.34450$	568	54	65	$\bar{2}.34461$	568	1.65539	35	$\bar{1}.99989$	44
4620	17	$\bar{2}.35018$	560	54	65	$\bar{2}.35029$	561	1.64971	35	$\bar{1}.99989$	43
4680	18	$\bar{2}.35578$	553	54	65	$\bar{2}.35590$	553	1.64410	35	$\bar{1}.99989$	42
4740	19	$\bar{2}.36131$	547	54	65	$\bar{2}.36143$	546	1.63857	35	$\bar{1}.99989$	41
4800	20	$\bar{2}.36678$	539	54	65	$\bar{2}.36689$	540	1.63311	35	$\bar{1}.99988$	40
4860	21	$\bar{2}.37217$	533	53	66	$\bar{2}.37229$	533	1.62771	34	$\bar{1}.99988$	39
4920	22	$\bar{2}.37750$	526	53	66	$\bar{2}.37762$	527	1.62238	34	$\bar{1}.99988$	38
4980	23	$\bar{2}.38276$	520	53	66	$\bar{2}.38289$	520	1.61711	34	$\bar{1}.99987$	37
5040	24	$\bar{2}.38796$	514	53	66	$\bar{2}.38809$	514	1.61191	34	$\bar{1}.99987$	36
5100	25	$\bar{2}.39310$	508	53	66	$\bar{2}.39323$	509	1.60677	34	$\bar{1}.99987$	35
5160	26	$\bar{2}.39818$	502	53	67	$\bar{2}.39832$	502	1.60168	33	$\bar{1}.99986$	34
5220	27	$\bar{2}.40320$	496	53	67	$\bar{2}.40334$	496	1.59666	33	$\bar{1}.99986$	33
5280	28	$\bar{2}.40816$	491	53	67	$\bar{2}.40830$	491	1.59170	33	$\bar{1}.99986$	32
5340	29	$\bar{2}.41307$	485	53	67	$\bar{2}.41321$	486	1.58679	33	$\bar{1}.99985$	31
5400	30	$\bar{2}.41792$	480	53	67	$\bar{2}.41807$	480	1.58193	33	$\bar{1}.99985$	30
5460	31	$\bar{2}.42272$	474	52	68	$\bar{2}.42287$	475	1.57713	32	$\bar{1}.99985$	29
5520	32	$\bar{2}.42746$	470	52	68	$\bar{2}.42762$	470	1.57238	32	$\bar{1}.99984$	28
5580	33	$\bar{2}.43216$	464	52	68	$\bar{2}.43232$	464	1.56768	32	$\bar{1}.99984$	27
5640	34	$\bar{2}.43680$	459	52	68	$\bar{2}.43696$	460	1.56304	32	$\bar{1}.99984$	26
5700	35	$\bar{2}.44139$	455	52	69	$\bar{2}.44156$	455	1.55844	31	$\bar{1}.99983$	25
5760	36	$\bar{2}.44594$	450	52	69	$\bar{2}.44611$	450	1.55389	31	$\bar{1}.99983$	24
5820	37	$\bar{2}.45044$	445	52	69	$\bar{2}.45061$	446	1.54939	31	$\bar{1}.99983$	23
5880	38	$\bar{2}.45489$	441	52	69	$\bar{2}.45507$	441	1.54493	31	$\bar{1}.99982$	22
5940	39	$\bar{2}.45930$	436	51	69	$\bar{2}.45948$	437	1.54052	31	$\bar{1}.99982$	21
6000	40	$\bar{2}.46366$	433	51	70	$\bar{2}.46385$	432	1.53615	30	$\bar{1}.99982$	20
6060	41	$\bar{2}.46799$	427	51	70	$\bar{2}.46817$	428	1.53183	30	$\bar{1}.99981$	19
6120	42	$\bar{2}.47226$	424	51	70	$\bar{2}.47245$	424	1.52755	30	$\bar{1}.99981$	18
6180	43	$\bar{2}.47650$	419	51	70	$\bar{2}.47669$	420	1.52331	30	$\bar{1}.99981$	17
6240	44	$\bar{2}.48069$	416	51	71	$\bar{2}.48089$	416	1.51911	29	$\bar{1}.99980$	16
6300	45	$\bar{2}.48485$	411	51	71	$\bar{2}.48505$	412	1.51495	29	$\bar{1}.99980$	15
6360	46	$\bar{2}.48896$	408	51	71	$\bar{2}.48917$	408	1.51083	29	$\bar{1}.99979$	14
6420	47	$\bar{2}.49304$	404	50	72	$\bar{2}.49325$	404	1.50675	28	$\bar{1}.99979$	13
6480	48	$\bar{2}.49708$	400	50	72	$\bar{2}.49729$	401	1.50271	28	$\bar{1}.99979$	12
6540	49	$\bar{2}.50108$	396	50	72	$\bar{2}.50130$	397	1.49870	28	$\bar{1}.99978$	11
6600	50	$\bar{2}.50504$	393	50	72	$\bar{2}.50527$	393	1.49473	28	$\bar{1}.99978$	10
6660	51	$\bar{2}.50897$	390	50	73	$\bar{2}.50920$	390	1.49080	27	$\bar{1}.99977$	9
6720	52	$\bar{2}.51287$	386	50	73	$\bar{2}.51310$	386	1.48690	27	$\bar{1}.99977$	8
6780	53	$\bar{2}.51673$	382	50	73	$\bar{2}.51696$	383	1.48304	27	$\bar{1}.99977$	7
6840	54	$\bar{2}.52055$	379	50	73	$\bar{2}.52079$	380	1.47921	27	$\bar{1}.99976$	6
6900	55	$\bar{2}.52434$	376	49	74	$\bar{2}.52459$	376	1.47541	26	$\bar{1}.99976$	5
6960	56	$\bar{2}.52810$	373	49	74	$\bar{2}.52835$	373	1.47165	26	$\bar{1}.99975$	4
7020	57	$\bar{2}.53183$	369	49	74	$\bar{2}.53208$	370	1.46792	26	$\bar{1}.99975$	3
7080	58	$\bar{2}.53552$	367	49	75	$\bar{2}.53578$	367	1.46422	25	$\bar{1}.99974$	2
7140	59	$\bar{2}.53919$	363	49	75	$\bar{2}.53945$	363	1.46055	25	$\bar{1}.99974$	1
7200	60	$\bar{2}.54282$		49	75	$\bar{2}.54308$		1.45692	25	$\bar{1}.99974$	0
				$\bar{6}.685$					5.314		
		log cos	d	S	T	log cot	c. d.	log tan	C	log sin	′

88°

″	′	log sin	d	S	T	log tan	c. d.	log cot	C	log cos	
				$\bar{6}$.685					5.314		
7200	0	$\bar{2}.54282$	360	49	75	$\bar{2}.54308$	361	1.45692	25	$\bar{1}.99974$	60
7260	1	$\bar{2}.54642$	357	49	75	$\bar{2}.54669$	358	1.45331	25	$\bar{1}.99973$	59
7320	2	$\bar{2}.54999$	355	48	76	$\bar{2}.55027$	355	1.44973	24	$\bar{1}.99973$	58
7380	3	$\bar{2}.55354$	351	48	76	$\bar{2}.55382$	352	1.44618	24	$\bar{1}.99972$	57
7440	4	$\bar{2}.55705$	349	48	76	$\bar{2}.55734$	349	1.44266	24	$\bar{1}.99972$	56
7500	5	$\bar{2}.56054$	346	48	77	$\bar{2}.56083$	346	1.43917	23	$\bar{1}.99971$	55
7560	6	$\bar{2}.56400$	343	48	77	$\bar{2}.56429$	344	1.43571	23	$\bar{1}.99971$	54
7620	7	$\bar{2}.56743$	341	48	77	$\bar{2}.56773$	341	1.43227	23	$\bar{1}.99970$	53
7680	8	$\bar{2}.57084$	337	47	78	$\bar{2}.57114$	338	1.42886	22	$\bar{1}.99970$	52
7740	9	$\bar{2}.57421$	336	47	78	$\bar{2}.57452$	336	1.42548	22	$\bar{1}.99969$	51
7800	10	$\bar{2}.57757$	332	47	78	$\bar{2}.57788$	333	1.42212	22	$\bar{1}.99969$	50
7860	11	$\bar{2}.58089$	330	47	79	$\bar{2}.58121$	330	1.41879	21	$\bar{1}.99968$	49
7920	12	$\bar{2}.58419$	328	47	79	$\bar{2}.58451$	328	1.41549	21	$\bar{1}.99968$	48
7980	13	$\bar{2}.58747$	325	47	79	$\bar{2}.58779$	326	1.41221	21	$\bar{1}.99967$	47
8040	14	$\bar{2}.59072$	323	46	79	$\bar{2}.59105$	323	1.40895	21	$\bar{1}.99967$	46
8100	15	$\bar{2}.59395$	320	46	80	$\bar{2}.59428$	321	1.40572	20	$\bar{1}.99967$	45
8160	16	$\bar{2}.59715$	318	46	80	$\bar{2}.59749$	319	1.40251	20	$\bar{1}.99966$	44
8220	17	$\bar{2}.60033$	316	46	80	$\bar{2}.60068$	316	1.39932	20	$\bar{1}.99966$	43
8280	18	$\bar{2}.60349$	313	46	81	$\bar{2}.60384$	314	1.39616	19	$\bar{1}.99965$	42
8340	19	$\bar{2}.60662$	311	46	81	$\bar{2}.60698$	311	1.39302	19	$\bar{1}.99964$	41
8400	20	$\bar{2}.60973$	309	45	82	$\bar{2}.61009$	310	1.38991	18	$\bar{1}.99964$	40
8460	21	$\bar{2}.61282$	307	45	82	$\bar{2}.61319$	307	1.38681	18	$\bar{1}.99963$	39
8520	22	$\bar{2}.61589$	305	45	82	$\bar{2}.61626$	305	1.38374	18	$\bar{1}.99963$	38
8580	23	$\bar{2}.61894$	302	45	83	$\bar{2}.61931$	303	1.38069	17	$\bar{1}.99962$	37
8640	24	$\bar{2}.62196$	301	45	83	$\bar{2}.62234$	301	1.37766	17	$\bar{1}.99962$	36
8700	25	$\bar{2}.62497$	298	45	83	$\bar{2}.62535$	299	1.37465	17	$\bar{1}.99961$	35
8760	26	$\bar{2}.62795$	296	44	84	$\bar{2}.62834$	297	1.37166	16	$\bar{1}.99961$	34
8820	27	$\bar{2}.63091$	294	44	84	$\bar{2}.63131$	295	1.36869	16	$\bar{1}.99960$	33
8880	28	$\bar{2}.63385$	293	44	84	$\bar{2}.63426$	292	1.36574	16	$\bar{1}.99960$	32
8940	29	$\bar{2}.63678$	290	44	85	$\bar{2}.63718$	291	1.36282	15	$\bar{1}.99959$	31
9000	30	$\bar{2}.63968$	288	44	85	$\bar{2}.64009$	289	1.35991	15	$\bar{1}.99959$	30
9060	31	$\bar{2}.64256$	287	44	85	$\bar{2}.64298$	287	1.35702	15	$\bar{1}.99958$	29
9120	32	$\bar{2}.64543$	284	43	86	$\bar{2}.64585$	285	1.35415	14	$\bar{1}.99958$	28
9180	33	$\bar{2}.64827$	283	43	86	$\bar{2}.64870$	284	1.35130	14	$\bar{1}.99957$	27
9240	34	$\bar{2}.65110$	281	43	87	$\bar{2}.65154$	281	1.34846	13	$\bar{1}.99956$	26
9300	35	$\bar{2}.65391$	279	43	87	$\bar{2}.65435$	280	1.34565	13	$\bar{1}.99956$	25
9360	36	$\bar{2}.65670$	277	43	87	$\bar{2}.65715$	278	1.34285	13	$\bar{1}.99955$	24
9420	37	$\bar{2}.65947$	276	42	88	$\bar{2}.65993$	276	1.34007	12	$\bar{1}.99955$	23
9480	38	$\bar{2}.66223$	274	42	88	$\bar{2}.66269$	274	1.33731	12	$\bar{1}.99954$	22
9540	39	$\bar{2}.66497$	272	42	88	$\bar{2}.66543$	273	1.33457	12	$\bar{1}.99954$	21
9600	40	$\bar{2}.66769$	270	42	89	$\bar{2}.66816$	271	1.33184	11	$\bar{1}.99953$	20
9660	41	$\bar{2}.67039$	269	42	89	$\bar{2}.67087$	269	1.32913	11	$\bar{1}.99952$	19
9720	42	$\bar{2}.67308$	267	41	90	$\bar{2}.67356$	268	1.32644	10	$\bar{1}.99952$	18
9780	43	$\bar{2}.67575$	266	41	90	$\bar{2}.67624$	266	1.32376	10	$\bar{1}.99951$	17
9840	44	$\bar{2}.67841$	263	41	90	$\bar{2}.67890$	264	1.32110	10	$\bar{1}.99951$	16
9900	45	$\bar{2}.68104$	263	41	91	$\bar{2}.68154$	263	1.31846	09	$\bar{1}.99950$	15
9960	46	$\bar{2}.68367$	260	41	91	$\bar{2}.68417$	261	1.31583	09	$\bar{1}.99949$	14
10020	47	$\bar{2}.68627$	259	40	92	$\bar{2}.68678$	260	1.31322	08	$\bar{1}.99949$	13
10080	48	$\bar{2}.68886$	258	40	92	$\bar{2}.68938$	258	1.31062	08	$\bar{1}.99948$	12
10141	49	$\bar{2}.69144$	256	40	92	$\bar{2}.69196$	257	1.30804	08	$\bar{1}.99948$	11
10200	50	$\bar{2}.69400$	254	40	93	$\bar{2}.69453$	255	1.30547	07	$\bar{1}.99947$	10
10260	51	$\bar{2}.69654$	253	40	93	$\bar{2}.69708$	254	1.30292	07	$\bar{1}.99946$	9
10320	52	$\bar{2}.69907$	252	39	94	$\bar{2}.69962$	252	1.30038	06	$\bar{1}.99946$	8
10380	53	$\bar{2}.70159$	250	39	94	$\bar{2}.70214$	251	1.29786	06	$\bar{1}.99945$	7
10440	54	$\bar{2}.70409$	249	39	95	$\bar{2}.70465$	249	1.29535	05	$\bar{1}.99944$	6
10500	55	$\bar{2}.70658$	247	39	95	$\bar{2}.70714$	248	1.29286	05	$\bar{1}.99944$	5
10560	56	$\bar{2}.70905$	246	39	95	$\bar{2}.70962$	246	1.29038	05	$\bar{1}.99943$	4
10620	57	$\bar{2}.71151$	244	38	96	$\bar{2}.71208$	245	1.28792	04	$\bar{1}.99942$	3
10680	58	$\bar{2}.71395$	243	38	96	$\bar{2}.71453$	244	1.28547	04	$\bar{1}.99942$	2
10740	59	$\bar{2}.71638$	242	38	97	$\bar{2}.71697$	243	1.28303	03	$\bar{1}.99941$	1
10800	60	$\bar{2}.71880$		38	97	$\bar{2}.71940$		1.28060	03	$\bar{1}.99940$	0
				$\bar{6}$.685					5.314		
		log cos	d	S	T	log cot	c. d.	log tan	C	log sin	′

′	log sin	d	log tan	c. d.	log cot	log cos	
0	2̄.71880	240	2̄.71940	241	1.28060	1̄.99940	60
1	2̄.72120	239	2̄.72181	239	1.27819	1̄.99940	59
2	2̄.72359	238	2̄.72420	239	1.27580	1̄.99939	58
3	2̄.72597	237	2̄.72659	237	1.27341	1̄.99938	57
4	2̄.72834	235	2̄.72896	236	1.27104	1̄.99938	56
5	2̄.73069	234	2̄.73132	234	1.26868	1̄.99937	55
6	2̄.73303	232	2̄.73366	234	1.26634	1̄.99936	54
7	2̄.73535	232	2̄.73600	232	1.26400	1̄.99936	53
8	2̄.73767	230	2̄.73832	231	1.26168	1̄.99935	52
9	2̄.73997	229	2̄.74063	229	1.25937	1̄.99934	51
10	2̄.74226	228	2̄.74292	229	1.25708	1̄.99934	50
11	2̄.74454	226	2̄.74521	227	1.25479	1̄.99933	49
12	2̄.74680	226	2̄.74748	226	1.25252	1̄.99932	48
13	2̄.74906	224	2̄.74974	225	1.25026	1̄.99932	47
14	2̄.75130	223	2̄.75199	224	1.24801	1̄.99931	46
15	2̄.75353	222	2̄.75423	222	1.24577	1̄.99930	45
16	2̄.75575	220	2̄.75645	222	1.24355	1̄.99929	44
17	2̄.75795	220	2̄.75867	220	1.24133	1̄.99929	43
18	2̄.76015	219	2̄.76087	219	1.23913	1̄.99928	42
19	2̄.76234	217	2̄.76306	219	1.23694	1̄.99927	41
20	2̄.76451	216	2̄.76525	217	1.23475	1̄.99926	40
21	2̄.76667	216	2̄.76742	216	1.23258	1̄.99926	39
22	2̄.76883	214	2̄.76958	215	1.23042	1̄.99925	38
23	2̄.77097	213	2̄.77173	214	1.22827	1̄.99924	37
24	2̄.77310	212	2̄.77387	213	1.22613	1̄.99923	36
25	2̄.77522	211	2̄.77600	211	1.22400	1̄.99923	35
26	2̄.77733	210	2̄.77811	211	1.22189	1̄.99922	34
27	2̄.77943	209	2̄.78022	210	1.21978	1̄.99921	33
28	2̄.78152	208	2̄.78232	209	1.21768	1̄.99920	32
29	2̄.78360	208	2̄.78441	208	1.21559	1̄.99920	31
30	2̄.78568	206	2̄.78649	206	1.21351	1̄.99919	30
31	2̄.78774	205	2̄.78855	206	1.21145	1̄.99918	29
32	2̄.78979	204	2̄.79061	205	1.20939	1̄.99917	28
33	2̄.79183	203	2̄.79266	204	1.20734	1̄.99917	27
34	2̄.79386	202	2̄.79470	203	1.20530	1̄.99916	26
35	2̄.79588	201	2̄.79673	202	1.20327	1̄.99915	25
36	2̄.79789	201	2̄.79875	201	1.20125	1̄.99914	24
37	2̄.79990	199	2̄.80076	201	1.19924	1̄.99913	23
38	2̄.80189	199	2̄.80277	199	1.19723	1̄.99913	22
39	2̄.80388	197	2̄.80476	198	1.19524	1̄.99912	21
40	2̄.80585	197	2̄.80674	198	1.19326	1̄.99911	20
41	2̄.80782	196	2̄.80872	196	1.19128	1̄.99910	19
42	2̄.80978	195	2̄.81068	196	1.18932	1̄.99909	18
43	2̄.81173	194	2̄.81264	195	1.18736	1̄.99909	17
44	2̄.81367	193	2̄.81459	194	1.18541	1̄.99908	16
45	2̄.81560	192	2̄.81653	193	1.18347	1̄.99907	15
46	2̄.81752	192	2̄.81846	192	1.18154	1̄.99906	14
47	2̄.81944	190	2̄.82038	192	1.17962	1̄.99905	13
48	2̄.82134	190	2̄.82230	190	1.17770	1̄.99904	12
49	2̄.82324	189	2̄.82420	190	1.17580	1̄.99904	11
50	2̄.82513	188	2̄.82610	189	1.17390	1̄.99903	10
51	2̄.82701	187	2̄.82799	188	1.17201	1̄.99902	9
52	2̄.82888	187	2̄.82987	188	1.17013	1̄.99901	8
53	2̄.83075	186	2̄.83175	186	1.16825	1̄.99900	7
54	2̄.83261	185	2̄.83361	186	1.16639	1̄.99899	6
55	2̄.83446	184	2̄.83547	185	1.16453	1̄.99898	5
56	2̄.83630	183	2̄.83732	184	1.16268	1̄.99898	4
57	2̄.83813	183	2̄.83916	184	1.16084	1̄.99897	3
58	2̄.83996	181	2̄.84100	182	1.15900	1̄.99896	2
59	2̄.84177	181	2̄.84282	182	1.15718	1̄.99895	1
60	2̄.84358		2̄.84464		1.15536	1̄.99894	0
	log cos	d	log cot	c. d.	log tan	log sin	′

p. p.

	238	234	229	225	220	216	212	208	204
6	23.8	23.4	22.9	22.5	22.0	21.6	21.2	20.8	20.4
7	27.8	27.3	26.7	26.3	25.7	25.2	24.7	24.3	23.8
8	31.7	31.2	30.5	30.0	29.3	28.8	28.3	27.7	27.2
9	35.7	35.1	34.4	33.8	33.0	32.4	31.8	31.2	30.6
10	39.7	39.0	38.2	37.5	36.7	36.0	35.3	34.7	34.0
20	79.3	78.0	76.3	75.0	73.3	72.0	70.7	69.3	68.0
30	119.0	117.0	114.5	112.5	110.0	108.0	106.0	104.0	102.0
40	158.7	156.0	152.7	150.0	146.7	144.0	141.3	138.7	136.0
50	198.3	195.0	190.8	187.5	183.3	180.0	176.7	173.3	170.0

	201	197	193	189	185	181
6	20.1	19.7	19.3	18.9	18.5	18.1
7	23.5	23.0	22.5	22.1	21.6	21.1
8	26.8	26.3	25.7	25.2	24.7	24.1
9	30.2	29.6	29.0	28.4	27.8	27.2
10	33.5	32.8	32.2	31.5	30.8	30.2
20	67.0	65.7	64.3	63.0	61.7	60.3
30	100.5	98.5	96.5	94.5	92.5	90.5
40	134.0	131.3	128.7	126.0	123.3	120.7
50	167.5	164.2	160.8	157.5	154.2	150.8

	4	3	2	1
6	0.4	0.3	0.2	0.1
7	0.5	0.4	0.2	0.1
8	0.5	0.4	0.3	0.1
9	0.6	0.5	0.3	0.2
10	0.7	0.5	0.3	0.2
20	1.3	1.0	0.7	0.3
30	2.0	1.5	1.0	0.5
40	2.7	2.0	1.3	0.7
50	3.3	2.5	1.7	0.8

′	log sin	d	log tan	c. d.	log cot	log cos	
0	$\bar{2}.84358$	181	$\bar{2}.84464$	182	1.15536	$\bar{1}.99894$	60
1	$\bar{2}.84539$	179	$\bar{2}.84646$	180	1.15354	$\bar{1}.99893$	59
2	$\bar{2}.84718$	179	$\bar{2}.84826$	180	1.15174	$\bar{1}.99892$	58
3	$\bar{2}.84897$	178	$\bar{2}.85006$	179	1.14994	$\bar{1}.99891$	57
4	$\bar{2}.85075$	177	$\bar{2}.85185$	178	1.14815	$\bar{1}.99891$	56
5	$\bar{2}.85252$	177	$\bar{2}.85363$	177	1.14637	$\bar{1}.99890$	55
6	$\bar{2}.85429$	176	$\bar{2}.85540$	177	1.14460	$\bar{1}.99889$	54
7	$\bar{2}.85605$	175	$\bar{2}.85717$	176	1.14283	$\bar{1}.99888$	53
8	$\bar{2}.85780$	175	$\bar{2}.85893$	176	1.14107	$\bar{1}.99887$	52
9	$\bar{2}.85955$	173	$\bar{2}.86069$	174	1.13931	$\bar{1}.99886$	51
10	$\bar{2}.86128$	173	$\bar{2}.86243$	174	1.13757	$\bar{1}.99885$	50
11	$\bar{2}.86301$	173	$\bar{2}.86417$	174	1.13583	$\bar{1}.99884$	49
12	$\bar{2}.86474$	171	$\bar{2}.86591$	172	1.13409	$\bar{1}.99883$	48
13	$\bar{2}.86645$	171	$\bar{2}.86763$	172	1.13237	$\bar{1}.99882$	47
14	$\bar{2}.86816$	171	$\bar{2}.86935$	171	1.13065	$\bar{1}.99881$	46
15	$\bar{2}.86987$	169	$\bar{2}.87106$	171	1.12894	$\bar{1}.99880$	45
16	$\bar{2}.87156$	169	$\bar{2}.87277$	170	1.12723	$\bar{1}.99879$	44
17	$\bar{2}.87325$	169	$\bar{2}.87447$	169	1.12553	$\bar{1}.99879$	43
18	$\bar{2}.87494$	167	$\bar{2}.87616$	169	1.12384	$\bar{1}.99878$	42
19	$\bar{2}.87661$	168	$\bar{2}.87785$	168	1.12215	$\bar{1}.99877$	41
20	$\bar{2}.87829$	166	$\bar{2}.87953$	167	1.12047	$\bar{1}.99876$	40
21	$\bar{2}.87995$	166	$\bar{2}.88120$	167	1.11880	$\bar{1}.99875$	39
22	$\bar{2}.88161$	165	$\bar{2}.88287$	166	1.11713	$\bar{1}.99874$	38
23	$\bar{2}.88326$	164	$\bar{2}.88453$	165	1.11547	$\bar{1}.99873$	37
24	$\bar{2}.88490$	164	$\bar{2}.88618$	165	1.11382	$\bar{1}.99872$	36
25	$\bar{2}.88654$	163	$\bar{2}.88783$	165	1.11217	$\bar{1}.99871$	35
26	$\bar{2}.88817$	163	$\bar{2}.88948$	163	1.11052	$\bar{1}.99870$	34
27	$\bar{2}.88980$	162	$\bar{2}.89111$	163	1.10889	$\bar{1}.99869$	33
28	$\bar{2}.89142$	162	$\bar{2}.89274$	163	1.10726	$\bar{1}.99868$	32
29	$\bar{2}.89304$	160	$\bar{2}.89437$	161	1.10563	$\bar{1}.99867$	31
30	$\bar{2}.89464$	161	$\bar{2}.89598$	162	1.10402	$\bar{1}.99866$	30
31	$\bar{2}.89625$	159	$\bar{2}.89760$	160	1.10240	$\bar{1}.99865$	29
32	$\bar{2}.89784$	159	$\bar{2}.89920$	160	1.10080	$\bar{1}.99864$	28
33	$\bar{2}.89943$	159	$\bar{2}.90080$	160	1.09920	$\bar{1}.99863$	27
34	$\bar{2}.90102$	158	$\bar{2}.90240$	159	1.09760	$\bar{1}.99862$	26
35	$\bar{2}.90260$	157	$\bar{2}.90399$	158	1.09601	$\bar{1}.99861$	25
36	$\bar{2}.90417$	157	$\bar{2}.90557$	158	1.09443	$\bar{1}.99860$	24
37	$\bar{2}.90574$	156	$\bar{2}.90715$	157	1.09285	$\bar{1}.99859$	23
38	$\bar{2}.90730$	155	$\bar{2}.90872$	157	1.09128	$\bar{1}.99858$	22
39	$\bar{2}.90885$	155	$\bar{2}.91029$	156	1.08971	$\bar{1}.99857$	21
40	$\bar{2}.91040$	155	$\bar{2}.91185$	155	1.08815	$\bar{1}.99856$	20
41	$\bar{2}.91195$	154	$\bar{2}.91340$	155	1.08660	$\bar{1}.99855$	19
42	$\bar{2}.91349$	153	$\bar{2}.91495$	155	1.08505	$\bar{1}.99854$	18
43	$\bar{2}.91502$	153	$\bar{2}.91650$	153	1.08350	$\bar{1}.99853$	17
44	$\bar{2}.91655$	152	$\bar{2}.91803$	154	1.08197	$\bar{1}.99852$	16
45	$\bar{2}.91807$	152	$\bar{2}.91957$	153	1.08043	$\bar{1}.99851$	15
46	$\bar{2}.91959$	151	$\bar{2}.92110$	152	1.07890	$\bar{1}.99850$	14
47	$\bar{2}.92110$	151	$\bar{2}.92262$	152	1.07738	$\bar{1}.99848$	13
48	$\bar{2}.92261$	150	$\bar{2}.92414$	151	1.07586	$\bar{1}.99847$	12
49	$\bar{2}.92411$	150	$\bar{2}.92565$	151	1.07435	$\bar{1}.99846$	11
50	$\bar{2}.92561$	149	$\bar{2}.92716$	150	1.07284	$\bar{1}.99845$	10
51	$\bar{2}.92710$	149	$\bar{2}.92866$	150	1.07134	$\bar{1}.99844$	9
52	$\bar{2}.92859$	148	$\bar{2}.93016$	149	1.06984	$\bar{1}.99843$	8
53	$\bar{2}.93007$	147	$\bar{2}.93165$	148	1.06835	$\bar{1}.99842$	7
54	$\bar{2}.93154$	147	$\bar{2}.93313$	149	1.06687	$\bar{1}.99841$	6
55	$\bar{2}.93301$	147	$\bar{2}.93462$	147	1.06538	$\bar{1}.99840$	5
56	$\bar{2}.93448$	146	$\bar{2}.93609$	147	1.06391	$\bar{1}.99839$	4
57	$\bar{2}.93594$	146	$\bar{2}.93756$	147	1.06244	$\bar{1}.99838$	3
58	$\bar{2}.93740$	145	$\bar{2}.93903$	146	1.06097	$\bar{1}.99837$	2
59	$\bar{2}.93885$	145	$\bar{2}.94049$	146	1.05951	$\bar{1}.99836$	1
60	$\bar{2}.94030$		$\bar{2}.94195$		1.05805	$\bar{1}.99834$	0
	log cos	d	log cot	c. d.	log tan	log sin	′

p. p.

	181	179	177
6	18.1	17.9	17.7
7	21.1	20.9	20.7
8	24.1	23.9	23.6
9	27.2	26.9	26.6
10	30.2	29.8	29.5
20	60.3	59.7	59.0
30	90.5	89.5	88.5
40	120.7	119.3	118.0
50	150.8	149.2	147.5

	175	173	171
6	17.5	17.3	17.1
7	20.4	20.2	20.0
8	23.3	23.1	22.8
9	26.3	26.0	25.7
10	29.2	28.8	28.5
20	58.3	57.7	57.0
30	87.5	86.5	85.5
40	116.7	115.3	114.0
50	145.8	144.2	142.5

	168	166	164
6	16.8	16.6	16.4
7	19.6	19.4	19.1
8	22.4	22.1	21.9
9	25.2	24.9	24.6
10	28.0	27.7	27.3
20	56.0	55.3	54.7
30	84.0	83.0	82.0
40	112.0	110.7	109.3
50	140.0	138.3	136.7

	162	159	157
6	16.2	15.9	15.7
7	18.9	18.6	18.3
8	21.6	21.2	20.9
9	24.3	23.9	23.6
10	27.0	26.5	26.2
20	54.0	53.0	52.3
30	81.0	79.5	78.5
40	108.0	106.0	104.7
50	135.0	132.5	130.8

	155	153	151
6	15.5	15.3	15.1
7	18.1	17.9	17.6
8	20.7	20.4	20.1
9	23.3	23.0	22.7
10	25.8	25.5	25.2
20	51.7	51.0	50.3
30	77.5	76.5	75.5
40	103.3	102.0	100.7
50	129.2	127.5	125.8

	149	147	1
6	14.9	14.7	0.1
7	17.4	17.2	0.1
8	19.9	19.6	0.1
9	22.4	22.1	0.2
10	24.8	24.5	0.2
20	49.7	49.0	0.3
30	74.5	73.5	0.5
40	99.3	98.0	0.7
50	124.2	122.5	0.8

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	
0	$\bar{2}$.94030	144	$\bar{2}$.94195	145	1.05805	$\bar{1}$.99834	60
1	$\bar{2}$.94174	143	$\bar{2}$.94340	145	1.05660	$\bar{1}$.99833	59
2	$\bar{2}$.94317	144	$\bar{2}$.94485	145	1.05515	$\bar{1}$.99832	58
3	$\bar{2}$.94461	142	$\bar{2}$.94630	143	1.05370	$\bar{1}$.99831	57
4	$\bar{2}$.94603	143	$\bar{2}$.94773	144	1.05227	$\bar{1}$.99830	56
5	$\bar{2}$.94746	141	$\bar{2}$.94917	143	1.05083	$\bar{1}$.99829	55
6	$\bar{2}$.94887	142	$\bar{2}$.95060	142	1.04940	$\bar{1}$.99828	54
7	$\bar{2}$.95029	141	$\bar{2}$.95202	142	1.04798	$\bar{1}$.99827	53
8	$\bar{2}$.95170	140	$\bar{2}$.95344	142	1.04656	$\bar{1}$.99825	52
9	$\bar{2}$.95310	140	$\bar{2}$.95486	141	1.04514	$\bar{1}$.99824	51
10	$\bar{2}$.95450		$\bar{2}$.95627		1.04373	$\bar{1}$.99823	50
		139		140			
11	$\bar{2}$.95589	139	$\bar{2}$.95767	141	1.04233	$\bar{1}$.99822	49
12	$\bar{2}$.95728	139	$\bar{2}$.95908	139	1.04092	$\bar{1}$.99821	48
13	$\bar{2}$.95867	138	$\bar{2}$.96047	140	1.03953	$\bar{1}$.99820	47
14	$\bar{2}$.96005	138	$\bar{2}$.96187	138	1.03813	$\bar{1}$.99819	46
15	$\bar{2}$.96143	137	$\bar{2}$.96325	139	1.03675	$\bar{1}$.99817	45
16	$\bar{2}$.96280	137	$\bar{2}$.96464	138	1.03536	$\bar{1}$.99816	44
17	$\bar{2}$.96417	136	$\bar{2}$.96602	137	1.03398	$\bar{1}$.99815	43
18	$\bar{2}$.96553	136	$\bar{2}$.96739	138	1.03261	$\bar{1}$.99814	42
19	$\bar{2}$.96689	136	$\bar{2}$.96877	136	1.03123	$\bar{1}$.99813	41
20	$\bar{2}$.96825		$\bar{2}$.97013		1.02987	$\bar{1}$.99812	40
		135		137			
21	$\bar{2}$.96960	135	$\bar{2}$.97150	135	1.02850	$\bar{1}$.99810	39
22	$\bar{2}$.97095	134	$\bar{2}$.97285	136	1.02715	$\bar{1}$.99809	38
23	$\bar{2}$.97229	134	$\bar{2}$.97421	135	1.02579	$\bar{1}$.99808	37
24	$\bar{2}$.97363	133	$\bar{2}$.97556	135	1.02444	$\bar{1}$.99807	36
25	$\bar{2}$.97496	133	$\bar{2}$.97691	134	1.02309	$\bar{1}$.99806	35
26	$\bar{2}$.97629	133	$\bar{2}$.97825	134	1.02175	$\bar{1}$.99804	34
27	$\bar{2}$.97762	132	$\bar{2}$.97959	133	1.02041	$\bar{1}$.99803	33
28	$\bar{2}$.97894	132	$\bar{2}$.98092	133	1.01908	$\bar{1}$.99802	32
29	$\bar{2}$.98026	131	$\bar{2}$.98225	133	1.01775	$\bar{1}$.99801	31
30	$\bar{2}$.98157		$\bar{2}$.98358		1.01642	$\bar{1}$.99800	30
		131		132			
31	$\bar{2}$.98288	131	$\bar{2}$.98490	132	1.01510	$\bar{1}$.99798	29
32	$\bar{2}$.98419	130	$\bar{2}$.98622	131	1.01378	$\bar{1}$.99797	28
33	$\bar{2}$.98549	130	$\bar{2}$.98753	131	1.01247	$\bar{1}$.99796	27
34	$\bar{2}$.98679	129	$\bar{2}$.98884	131	1.01116	$\bar{1}$.99795	26
35	$\bar{2}$.98808	129	$\bar{2}$.99015	130	1.00985	$\bar{1}$.99793	25
36	$\bar{2}$.98937	129	$\bar{2}$.99145	130	1.00855	$\bar{1}$.99792	24
37	$\bar{2}$.99066	128	$\bar{2}$.99275	130	1.00725	$\bar{1}$.99791	23
38	$\bar{2}$.99194	128	$\bar{2}$.99405	129	1.00595	$\bar{1}$.99790	22
39	$\bar{2}$.99322	128	$\bar{2}$.99534	128	1.00466	$\bar{1}$.99788	21
40	$\bar{2}$.99450		$\bar{2}$.99662		1.00338	$\bar{1}$.99787	20
		127		129			
41	$\bar{2}$.99577	127	$\bar{2}$.99791	128	1.00209	$\bar{1}$.99786	19
42	$\bar{2}$.99704	126	$\bar{2}$.99919	127	1.00081	$\bar{1}$.99785	18
43	$\bar{2}$.99830	126	$\bar{1}$.00046	128	0.99954	$\bar{1}$.99783	17
44	$\bar{2}$.99956	126	$\bar{1}$.00174	127	0.99826	$\bar{1}$.99782	16
45	$\bar{1}$.00082	125	$\bar{1}$.00301	126	0.99699	$\bar{1}$.99781	15
46	$\bar{1}$.00207	125	$\bar{1}$.00427	126	0.99573	$\bar{1}$.99780	14
47	$\bar{1}$.00332	124	$\bar{1}$.00553	126	0.99447	$\bar{1}$.99778	13
48	$\bar{1}$.00456	125	$\bar{1}$.00679	126	0.99321	$\bar{1}$.99777	12
49	$\bar{1}$.00581	123	$\bar{1}$.00805	125	0.99195	$\bar{1}$.99776	11
50	$\bar{1}$.00704		$\bar{1}$.00930		0.99070	$\bar{1}$.99775	10
		124		125			
51	$\bar{1}$.00828	123	$\bar{1}$.01055	124	0.98945	$\bar{1}$.99773	9
52	$\bar{1}$.00951	123	$\bar{1}$.01179	124	0.98821	$\bar{1}$.99772	8
53	$\bar{1}$.01074	122	$\bar{1}$.01303	124	0.98697	$\bar{1}$.99771	7
54	$\bar{1}$.01196	122	$\bar{1}$.01427	123	0.98573	$\bar{1}$.99769	6
55	$\bar{1}$.01318	122	$\bar{1}$.01550	123	0.98450	$\bar{1}$.99768	5
56	$\bar{1}$.01440	121	$\bar{1}$.01673	123	0.98327	$\bar{1}$.99767	4
57	$\bar{1}$.01561	121	$\bar{1}$.01796	122	0.98204	$\bar{1}$.99765	3
58	$\bar{1}$.01682	121	$\bar{1}$.01918	122	0.98082	$\bar{1}$.99764	2
59	$\bar{1}$.01803	120	$\bar{1}$.02040	122	0.97960	$\bar{1}$.99763	1
60	$\bar{1}$.01923		$\bar{1}$.02162		0.97838	$\bar{1}$.99761	0

	log cos	d	log cot	c. d.	log tan	log sin	′

p. p.

	145	143	141
6	14.5	14.3	14.1
7	16.9	16.7	16.5
8	19.3	19.1	18.8
9	21.8	21.5	21.2
10	24.2	23.8	23.5
20	48.3	47.7	47.0
30	72.5	71.5	70.5
40	96.7	95.3	94.0
50	120.8	119.2	117.5
	139	**138**	**136**
6	13.9	13.8	13.6
7	16.2	16.1	15.9
8	18.5	18.4	18.1
9	20.9	20.7	20.4
10	23.2	23.0	22.7
20	46.3	46.0	45.3
30	69.5	69.0	68.0
40	92.7	92.0	90.7
50	115.8	115.0	113.3
	135	**133**	**131**
6	13.5	13.3	13.1
7	15.8	15.5	15.3
8	18.0	17.7	17.5
9	20.3	20.0	19.7
10	22.5	22.2	21.8
20	45.0	44.3	43.7
30	67.5	66.5	65.5
40	90.0	88.7	87.3
50	112.5	110.8	109.2
	129	**128**	**126**
6	12.9	12.8	12.6
7	15.1	14.9	14.7
8	17.2	17.1	16.8
9	19.4	19.2	18.9
10	21.5	21.3	21.0
20	43.0	42.7	42.0
30	64.5	64.0	63.0
40	86.0	85.3	84.0
50	107.5	106.7	105.0
	125	**123**	**122**
6	12.5	12.3	12.2
7	14.6	14.4	14.2
8	16.7	16.4	16.3
9	18.8	18.5	18.3
10	20.8	20.5	20.3
20	41.7	41.0	40.7
30	62.5	61.5	61.0
40	83.3	82.0	81.3
50	104.2	102.5	101.7
	121	**120**	**1**
6	12.1	12.0	0.1
7	14.1	14.0	0.1
8	16.1	16.0	0.1
9	18.2	18.0	0.2
10	20.2	20.0	0.2
20	40.3	40.0	0.3
30	60.5	60.0	0.5
40	80.7	80.0	0.7
50	100.8	100.0	0.8

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	
0	$\bar{1}.01923$	120	$\bar{1}.02162$	121	0.97838	$\bar{1}.99761$	60
1	$\bar{1}.02043$	120	$\bar{1}.02283$	121	0.97717	$\bar{1}.99760$	59
2	$\bar{1}.02163$	120	$\bar{1}.02404$	121	0.97596	$\bar{1}.99759$	58
3	$\bar{1}.02283$	119	$\bar{1}.02525$	120	0.97475	$\bar{1}.99757$	57
4	$\bar{1}.02402$	118	$\bar{1}.02645$	121	0.97355	$\bar{1}.99756$	56
5	$\bar{1}.02520$	119	$\bar{1}.02766$	119	0.97234	$\bar{1}.99755$	55
6	$\bar{1}.02639$	118	$\bar{1}.02885$	120	0.97115	$\bar{1}.99753$	54
7	$\bar{1}.02757$	117	$\bar{1}.03005$	119	0.96995	$\bar{1}.99752$	53
8	$\bar{1}.02874$	118	$\bar{1}.03124$	118	0.96876	$\bar{1}.99751$	52
9	$\bar{1}.02992$	117	$\bar{1}.03242$	119	0.96758	$\bar{1}.99749$	51
10	$\bar{1}.03109$	117	$\bar{1}.03361$	118	0.96639	$\bar{1}.99748$	50
11	$\bar{1}.03226$	116	$\bar{1}.03479$	118	0.96521	$\bar{1}.99747$	49
12	$\bar{1}.03342$	116	$\bar{1}.03597$	117	0.96403	$\bar{1}.99745$	48
13	$\bar{1}.03458$	116	$\bar{1}.03714$	118	0.96286	$\bar{1}.99744$	47
14	$\bar{1}.03574$	116	$\bar{1}.03832$	116	0.96168	$\bar{1}.99742$	46
15	$\bar{1}.03690$	115	$\bar{1}.03948$	117	0.96052	$\bar{1}.99741$	45
16	$\bar{1}.03805$	115	$\bar{1}.04065$	116	0.95935	$\bar{1}.99740$	44
17	$\bar{1}.03920$	114	$\bar{1}.04181$	116	0.95819	$\bar{1}.99738$	43
18	$\bar{1}.04034$	115	$\bar{1}.04297$	116	0.95703	$\bar{1}.99737$	42
19	$\bar{1}.04149$	113	$\bar{1}.04413$	115	0.95587	$\bar{1}.99736$	41
20	$\bar{1}.04262$	114	$\bar{1}.04528$	115	0.95472	$\bar{1}.99734$	40
21	$\bar{1}.04376$	114	$\bar{1}.04643$	115	0.95357	$\bar{1}.99733$	39
22	$\bar{1}.04490$	113	$\bar{1}.04758$	115	0.95242	$\bar{1}.99731$	38
23	$\bar{1}.04603$	112	$\bar{1}.04873$	114	0.95127	$\bar{1}.99730$	37
24	$\bar{1}.04715$	113	$\bar{1}.04987$	114	0.95013	$\bar{1}.99728$	36
25	$\bar{1}.04828$	112	$\bar{1}.05101$	113	0.94899	$\bar{1}.99727$	35
26	$\bar{1}.04940$	112	$\bar{1}.05214$	114	0.94786	$\bar{1}.99726$	34
27	$\bar{1}.05052$	112	$\bar{1}.05328$	113	0.94672	$\bar{1}.99724$	33
28	$\bar{1}.05164$	111	$\bar{1}.05441$	112	0.94559	$\bar{1}.99723$	32
29	$\bar{1}.05275$	111	$\bar{1}.05553$	113	0.94447	$\bar{1}.99721$	31
30	$\bar{1}.05386$	111	$\bar{1}.05666$	112	0.94334	$\bar{1}.99720$	30
31	$\bar{1}.05497$	110	$\bar{1}.05778$	112	0.94222	$\bar{1}.99718$	29
32	$\bar{1}.05607$	110	$\bar{1}.05890$	112	0.94110	$\bar{1}.99717$	28
33	$\bar{1}.05717$	110	$\bar{1}.06002$	111	0.93998	$\bar{1}.99716$	27
34	$\bar{1}.05827$	110	$\bar{1}.06113$	111	0.93887	$\bar{1}.99714$	26
35	$\bar{1}.05937$	109	$\bar{1}.06224$	111	0.93776	$\bar{1}.99713$	25
36	$\bar{1}.06046$	109	$\bar{1}.06335$	110	0.93665	$\bar{1}.99711$	24
37	$\bar{1}.06155$	109	$\bar{1}.06445$	111	0.93555	$\bar{1}.99710$	23
38	$\bar{1}.06264$	108	$\bar{1}.06556$	110	0.93444	$\bar{1}.99708$	22
39	$\bar{1}.06372$	109	$\bar{1}.06666$	109	0.93334	$\bar{1}.99707$	21
40	$\bar{1}.06481$	108	$\bar{1}.06775$	110	0.93225	$\bar{1}.99705$	20
41	$\bar{1}.06589$	107	$\bar{1}.06885$	109	0.93115	$\bar{1}.99704$	19
42	$\bar{1}.06696$	108	$\bar{1}.06994$	109	0.93006	$\bar{1}.99702$	18
43	$\bar{1}.06804$	107	$\bar{1}.07103$	108	0.92897	$\bar{1}.99701$	17
44	$\bar{1}.06911$	107	$\bar{1}.07211$	109	0.92789	$\bar{1}.99699$	16
45	$\bar{1}.07018$	106	$\bar{1}.07320$	108	0.92680	$\bar{1}.99698$	15
46	$\bar{1}.07124$	107	$\bar{1}.07428$	108	0.92572	$\bar{1}.99696$	14
47	$\bar{1}.07231$	106	$\bar{1}.07536$	107	0.92464	$\bar{1}.99695$	13
48	$\bar{1}.07337$	105	$\bar{1}.07643$	108	0.92357	$\bar{1}.99693$	12
49	$\bar{1}.07442$	106	$\bar{1}.07751$	107	0.92249	$\bar{1}.99692$	11
50	$\bar{1}.07548$	105	$\bar{1}.07858$	106	0.92142	$\bar{1}.99690$	10
51	$\bar{1}.07653$	105	$\bar{1}.07964$	107	0.92036	$\bar{1}.99689$	9
52	$\bar{1}.07758$	105	$\bar{1}.08071$	106	0.91929	$\bar{1}.99687$	8
53	$\bar{1}.07863$	105	$\bar{1}.08177$	106	0.91823	$\bar{1}.99686$	7
54	$\bar{1}.07968$	104	$\bar{1}.08283$	106	0.91717	$\bar{1}.99684$	6
55	$\bar{1}.08072$	104	$\bar{1}.08389$	106	0.91611	$\bar{1}.99683$	5
56	$\bar{1}.08176$	104	$\bar{1}.08495$	105	0.91505	$\bar{1}.99681$	4
57	$\bar{1}.08280$	103	$\bar{1}.08600$	105	0.91400	$\bar{1}.99680$	3
58	$\bar{1}.08383$	103	$\bar{1}.08705$	105	0.91295	$\bar{1}.99678$	2
59	$\bar{1}.08486$	103	$\bar{1}.08810$	104	0.91190	$\bar{1}.99677$	1
60	$\bar{1}.08589$		$\bar{1}.08914$		0.91086	$\bar{1}.99675$	0
	log cos	d	log cot	c. d.	log tan	log sin	′

p. p.			
	121	120	119
6	12.1	12.0	11.9
7	14.1	14.0	13.9
8	16.1	16.0	15.9
9	18.2	18.0	17.9
10	20.2	20.0	19.8
20	40.3	40.0	39.7
30	60.5	60.0	59.5
40	80.7	80.0	79.3
50	100.8	100.0	99.2
	118	117	116
6	11.8	11.7	11.6
7	13.8	13.7	13.5
8	15.7	15.6	15.5
9	17.7	17.6	17.4
10	19.7	19.5	19.3
20	39.3	39.0	38.7
30	59.0	58.5	58.0
40	78.7	78.0	77.3
50	98.3	97.5	96.7
	115	114	113
6	11.5	11.4	11.3
7	13.4	13.3	13.2
8	15.3	15.2	15.1
9	17.3	17.1	17.0
10	19.2	19.0	18.8
20	38.3	38.0	37.7
30	57.5	57.0	56.5
40	76.7	76.0	75.3
50	95.8	95.0	94.2
	112	111	110
6	11.2	11.1	11.0
7	13.1	13.0	12.8
8	14.9	14.8	14.7
9	16.8	16.7	16.5
10	18.7	18.5	18.3
20	37.3	37.0	36.7
30	56.0	55.5	55.0
40	74.7	74.0	73.3
50	93.3	92.5	91.7
	109	108	107
6	10.9	10.8	10.7
7	12.7	12.6	12.5
8	14.5	14.4	14.3
9	16.4	16.2	16.1
10	18.2	18.0	17.8
20	36.3	36.0	35.7
30	54.5	54.0	53.5
40	72.7	72.0	71.3
50	90.8	90.0	89.2
	106	105	104
6	10.6	10.5	10.4
7	12.4	12.3	12.1
8	14.1	14.0	13.9
9	15.9	15.8	15.6
10	17.7	17.5	17.3
20	35.3	35.0	34.7
30	53.0	52.5	52.0
40	70.7	70.0	69.3
50	88.3	87.5	86.7

′	log sin	d	log tan	c. d.	log cot	log cos	
0	1̄.08589	103	1̄.08914	105	0.91086	1̄.99675	60
1	1̄.08692	103	1̄.09019	104	0.90981	1̄.99674	59
2	1̄.08795	102	1̄.09123	104	0.90877	1̄.99672	58
3	1̄.08897	102	1̄.09227	103	0.90773	1̄.99670	57
4	1̄.08999	102	1̄.09330	104	0.90670	1̄.99669	56
5	1̄.09101	101	1̄.09434	103	0.90566	1̄.99667	55
6	1̄.09202	102	1̄.09537	103	0.90463	1̄.99666	54
7	1̄.09304	101	1̄.09640	102	0.90360	1̄.99664	53
8	1̄.09405	101	1̄.09742	103	0.90258	1̄.99663	52
9	1̄.09506	100	1̄.09845	102	0.90155	1̄.99661	51
10	1̄.09606	101	1̄.09947	102	0.90053	1̄.99659	50
11	1̄.09707	100	1̄.10049	101	0.89951	1̄.99658	49
12	1̄.09807	100	1̄.10150	102	0.89850	1̄.99656	48
13	1̄.09907	99	1̄.10252	101	0.89748	1̄.99655	47
14	1̄.10006	100	1̄.10353	101	0.89647	1̄.99653	46
15	1̄.10106	99	1̄.10454	101	0.89546	1̄.99651	45
16	1̄.10205	99	1̄.10555	101	0.89445	1̄.99650	44
17	1̄.10304	98	1̄.10656	100	0.89344	1̄.99648	43
18	1̄.10402	99	1̄.10756	100	0.89244	1̄.99647	42
19	1̄.10501	98	1̄.10856	100	0.89144	1̄.99645	41
20	1̄.10599	98	1̄.10956	100	0.89044	1̄.99643	40
21	1̄.10697	98	1̄.11056	99	0.88944	1̄.99642	39
22	1̄.10795	98	1̄.11155	99	0.88845	1̄.99640	38
23	1̄.10893	97	1̄.11254	99	0.88746	1̄.99638	37
24	1̄.10990	97	1̄.11353	99	0.88647	1̄.99637	36
25	1̄.11087	97	1̄.11452	99	0.88548	1̄.99635	35
26	1̄.11184	97	1̄.11551	98	0.88449	1̄.99633	34
27	1̄.11281	96	1̄.11649	98	0.88351	1̄.99632	33
28	1̄.11377	97	1̄.11747	98	0.88253	1̄.99630	32
29	1̄.11474	96	1̄.11845	98	0.88155	1̄.99629	31
30	1̄.11570	96	1̄.11943	97	0.88057	1̄.99627	30
31	1̄.11666	95	1̄.12040	98	0.87960	1̄.99625	29
32	1̄.11761	96	1̄.12138	97	0.87862	1̄.99624	28
33	1̄.11857	95	1̄.12235	97	0.87765	1̄.99622	27
34	1̄.11952	95	1̄.12332	96	0.87668	1̄.99620	26
35	1̄.12047	95	1̄.12428	97	0.87572	1̄.99618	25
36	1̄.12142	94	1̄.12525	96	0.87475	1̄.99617	24
37	1̄.12236	95	1̄.12621	96	0.87379	1̄.99615	23
38	1̄.12331	94	1̄.12717	96	0.87283	1̄.99613	22
39	1̄.12425	94	1̄.12813	96	0.87187	1̄.99612	21
40	1̄.12519	93	1̄.12909	95	0.87091	1̄.99610	20
41	1̄.12612	94	1̄.13004	95	0.86996	1̄.99608	19
42	1̄.12706	93	1̄.13099	95	0.86901	1̄.99607	18
43	1̄.12799	93	1̄.13194	95	0.86806	1̄.99605	17
44	1̄.12892	93	1̄.13289	95	0.86711	1̄.99603	16
45	1̄.12985	93	1̄.13384	94	0.86616	1̄.99601	15
46	1̄.13078	93	1̄.13478	95	0.86522	1̄.99600	14
47	1̄.13171	92	1̄.13573	94	0.86427	1̄.99598	13
48	1̄.13263	92	1̄.13667	94	0.86333	1̄.99596	12
49	1̄.13355	92	1̄.13761	93	0.86239	1̄.99595	11
50	1̄.13447	92	1̄.13854	94	0.86146	1̄.99593	10
51	1̄.13539	91	1̄.13948	93	0.86052	1̄.99591	9
52	1̄.13630	92	1̄.14041	93	0.85959	1̄.99589	8
53	1̄.13722	91	1̄.14134	93	0.85866	1̄.99588	7
54	1̄.13813	91	1̄.14227	93	0.85773	1̄.99586	6
55	1̄.13904	90	1̄.14320	92	0.85680	1̄.99584	5
56	1̄.13994	91	1̄.14412	92	0.85588	1̄.99582	4
57	1̄.14085	90	1̄.14504	93	0.85496	1̄.99581	3
58	1̄.14175	91	1̄.14597	91	0.85403	1̄.99579	2
59	1̄.14266	90	1̄.14688	92	0.85312	1̄.99577	1
60	1̄.14356		1̄.14780		0.85220	1̄.99575	0
	log cos	d	log cot	c. d.	log tan	log sin	′

p. p.

	105	104	103
6	10.5	10.4	10.3
7	12.3	12.1	12.0
8	14.0	13.9	13.7
9	15.8	15.6	15.5
10	17.5	17.3	17.2
20	35.0	34.7	34.3
30	52.5	52.0	51.5
40	70.0	69.3	68.7
50	87.5	86.7	85.8

	102	101	100
6	10.2	10.1	10.0
7	11.9	11.8	11.7
8	13.6	13.5	13.3
9	15.3	15.2	15.0
10	17.0	16.8	16.7
20	34.0	33.7	33.3
30	51.0	50.5	50.0
40	68.0	67.3	66.7
50	85.0	84.2	83.3

	99	98
6	9.9	9.8
7	11.6	11.4
8	13.2	13.1
9	14.9	14.7
10	16.5	16.3
20	33.0	32.7
30	49.5	49.0
40	66.0	65.3
50	82.5	81.7

	97	96	95
6	9.7	9.6	9.5
7	11.3	11.2	11.1
8	12.9	12.8	12.7
9	14.6	14.4	14.3
10	16.2	16.0	15.8
20	32.3	32.0	31.7
30	48.5	48.0	47.5
40	64.7	64.0	63.3
50	80.8	80.0	79.2

	94	93	92
6	9.4	9.3	9.2
7	11.0	10.9	10.7
8	12.5	12.4	12.3
9	14.1	14.0	13.8
10	15.7	15.5	15.3
20	31.3	31.0	30.7
30	47.0	46.5	46.0
40	62.7	62.0	61.3
50	78.3	77.5	76.7

	91	90	2
6	9.1	9.0	0.2
7	10.6	10.5	0.2
8	12.1	12.0	0.3
9	13.7	13.5	0.3
10	15.2	15.0	0.3
20	30.3	30.0	0.7
30	45.5	45.0	1.0
40	60.7	60.0	1.3
50	75.8	75.0	1.7

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	
0	Ī.14356	89	Ī.14780	92	0.85220	Ī.99575	60
1	Ī.14445	90	Ī.14872	91	0.85128	Ī.99574	59
2	Ī.14535	89	Ī.14963	91	0.85037	Ī.99572	58
3	Ī.14624	90	Ī.15054	91	0.84946	Ī.99570	57
4	Ī.14714	89	Ī.15145	91	0.84855	Ī.99568	56
5	Ī.14803	88	Ī.15236	91	0.84764	Ī.99566	55
6	Ī.14891	89	Ī.15327	90	0.84673	Ī.99565	54
7	Ī.14980	89	Ī.15417	91	0.84583	Ī.99563	53
8	Ī.15069	88	Ī.15508	90	0.84492	Ī.99561	52
9	Ī.15157	88	Ī.15598	90	0.84402	Ī.99559	51
10	Ī.15245	88	Ī.15688	89	0.84312	Ī.99557	50
11	Ī.15333	88	Ī.15777	90	0.84223	Ī.99556	49
12	Ī.15421	87	Ī.15867	89	0.84133	Ī.99554	48
13	Ī.15508	88	Ī.15956	90	0.84044	Ī.99552	47
14	Ī.15596	87	Ī.16046	89	0.83954	Ī.99550	46
15	Ī.15683	87	Ī.16135	89	0.83865	Ī.99548	45
16	Ī.15770	87	Ī.16224	88	0.83776	Ī.99546	44
17	Ī.15857	87	Ī.16312	89	0.83688	Ī.99545	43
18	Ī.15944	86	Ī.16401	88	0.83599	Ī.99543	42
19	Ī.16030	86	Ī.16489	88	0.83511	Ī.99541	41
20	Ī.16116	87	Ī.16577	88	0.83423	Ī.99539	40
21	Ī.16203	86	Ī.16665	88	0.83335	Ī.99537	39
22	Ī.16289	85	Ī.16753	88	0.83247	Ī.99535	38
23	Ī.16374	86	Ī.16841	87	0.83159	Ī.99533	37
24	Ī.16460	85	Ī.16928	88	0.83072	Ī.99532	36
25	Ī.16545	86	Ī.17016	87	0.82984	Ī.99530	35
26	Ī.16631	85	Ī.17103	87	0.82897	Ī.99528	34
27	Ī.16716	85	Ī.17190	87	0.82810	Ī.99526	33
28	Ī.16801	85	Ī.17277	86	0.82723	Ī.99524	32
29	Ī.16886	84	Ī.17363	87	0.82637	Ī.99522	31
30	Ī.16970	85	Ī.17450	86	0.82550	Ī.99520	30
31	Ī.17055	84	Ī.17536	86	0.82464	Ī.99518	29
32	Ī.17139	84	Ī.17622	86	0.82378	Ī.99517	28
33	Ī.17223	84	Ī.17708	86	0.82292	Ī.99515	27
34	Ī.17307	84	Ī.17794	86	0.82206	Ī.99513	26
35	Ī.17391	83	Ī.17880	85	0.82120	Ī.99511	25
36	Ī.17474	84	Ī.17965	86	0.82035	Ī.99509	24
37	Ī.17558	83	Ī.18051	85	0.81949	Ī.99507	23
38	Ī.17641	83	Ī.18136	85	0.81864	Ī.99505	22
39	Ī.17724	83	Ī.18221	85	0.81779	Ī.99503	21
40	Ī.17807	83	Ī.18306	85	0.81694	Ī.99501	20
41	Ī.17890	83	Ī.18391	84	0.81609	Ī.99499	19
42	Ī.17973	82	Ī.18475	85	0.81525	Ī.99497	18
43	Ī.18055	82	Ī.18560	84	0.81440	Ī.99495	17
44	Ī.18137	83	Ī.18644	84	0.81356	Ī.99494	16
45	Ī.18220	82	Ī.18728	84	0.81272	Ī.99492	15
46	Ī.18302	81	Ī.18812	84	0.81188	Ī.99490	14
47	Ī.18383	82	Ī.18896	83	0.81104	Ī.99488	13
48	Ī.18465	82	Ī.18979	84	0.81021	Ī.99486	12
49	Ī.18547	81	Ī.19063	83	0.80937	Ī.99484	11
50	Ī.18628	81	Ī.19146	83	0.80854	Ī.99482	10
51	Ī.18709	81	Ī.19229	83	0.80771	Ī.99480	9
52	Ī.18790	81	Ī.19312	83	0.80688	Ī.99478	8
53	Ī.18871	81	Ī.19395	83	0.80605	Ī.99476	7
54	Ī.18952	81	Ī.19478	83	0.80522	Ī.99474	6
55	Ī.19033	80	Ī.19561	82	0.80439	Ī.99472	5
56	Ī.19113	80	Ī.19643	82	0.80357	Ī.99470	4
57	Ī.19193	80	Ī.19725	82	0.80275	Ī.99468	3
58	Ī.19273	80	Ī.19807	82	0.80193	Ī.99466	2
59	Ī.19353	80	Ī.19889	82	0.80111	Ī.99464	1
60	Ī.19433		Ī.19971		0.80029	Ī.99462	0
	log cos	d	log cot	c. d.	log tan	log sin	′

p. p.

	92	91	90	89	88	87	86	85	84	83	82	81	80	2
6	9.2	9.1	9.0	8.9	8.8	8.7	8.6	8.5	8.4	8.3	8.2	8.1	8.0	0.2
7	10.7	10.6	10.5	10.4	10.3	10.2	10.0	9.9	9.8	9.7	9.6	9.5	9.3	0.2
8	12.3	12.1	12.0	11.9	11.7	11.6	11.5	11.3	11.2	11.1	10.9	10.8	10.7	0.3
9	13.8	13.7	13.5	13.4	13.2	13.1	12.9	12.8	12.6	12.5	12.3	12.2	12.0	0.3
10	15.3	15.2	15.0	14.8	14.7	14.5	14.3	14.2	14.0	13.8	13.7	13.5	13.3	0.3
20	30.7	30.3	30.0	29.7	29.3	29.0	28.7	28.3	28.0	27.7	27.3	27.0	26.7	0.7
30	46.0	45.5	45.0	44.5	44.0	43.5	43.0	42.5	42.0	41.5	41.0	40.5	40.0	1.0
40	61.3	60.7	60.0	59.3	58.7	58.0	57.3	56.7	56.0	55.3	54.7	54.0	53.3	1.3
50	76.7	75.8	75.0	74.2	73.3	72.5	71.7	70.8	70.0	69.2	68.3	67.5	66.7	1.7

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	
0	1̄.19433	80	1̄.19971	82	0.80029	1̄.99462	60
1	1̄.19513	79	1̄.20053	81	0.79947	1̄.99460	59
2	1̄.19592	80	1̄.20134	82	0.79866	1̄.99458	58
3	1̄.19672	79	1̄.20216	81	0.79784	1̄.99456	57
4	1̄.19751	79	1̄.20297	81	0.79703	1̄.99454	56
5	1̄.19830	79	1̄.20378	81	0.79622	1̄.99452	55
6	1̄.19909	79	1̄.20459	81	0.79541	1̄.99450	54
7	1̄.19988	79	1̄.20540	81	0.79460	1̄.99448	53
8	1̄.20067	78	1̄.20621	80	0.79379	1̄.99446	52
9	1̄.20145	78	1̄.20701	81	0.79299	1̄.99444	51
10	1̄.20223	79	1̄.20782	80	0.79218	1̄.99442	50
11	1̄.20302	78	1̄.20862	80	0.79138	1̄.99440	49
12	1̄.20380	78	1̄.20942	80	0.79058	1̄.99438	48
13	1̄.20458	77	1̄.21022	80	0.78978	1̄.99436	47
14	1̄.20535	78	1̄.21102	80	0.78898	1̄.99434	46
15	1̄.20613	78	1̄.21182	79	0.78818	1̄.99432	45
16	1̄.20691	77	1̄.21261	80	0.78739	1̄.99429	44
17	1̄.20768	77	1̄.21341	79	0.78659	1̄.99427	43
18	1̄.20845	77	1̄.21420	79	0.78580	1̄.99425	42
19	1̄.20922	77	1̄.21499	79	0.78501	1̄.99423	41
20	1̄.20999	77	1̄.21578	79	0.78422	1̄.99421	40
21	1̄.21076	77	1̄.21657	79	0.78343	1̄.99419	39
22	1̄.21153	76	1̄.21736	78	0.78264	1̄.99417	38
23	1̄.21229	77	1̄.21814	79	0.78186	1̄.99415	37
24	1̄.21306	76	1̄.21893	78	0.78107	1̄.99413	36
25	1̄.21382	76	1̄.21971	78	0.78029	1̄.99411	35
26	1̄.21458	76	1̄.22049	78	0.77951	1̄.99409	34
27	1̄.21534	76	1̄.22127	78	0.77873	1̄.99407	33
28	1̄.21610	75	1̄.22205	78	0.77795	1̄.99404	32
29	1̄.21685	76	1̄.22283	78	0.77717	1̄.99402	31
30	1̄.21761	75	1̄.22361	77	0.77639	1̄.99400	30
31	1̄.21836	76	1̄.22438	78	0.77562	1̄.99398	29
32	1̄.21912	75	1̄.22516	77	0.77484	1̄.99396	28
33	1̄.21987	75	1̄.22593	77	0.77407	1̄.99394	27
34	1̄.22062	75	1̄.22670	77	0.77330	1̄.99392	26
35	1̄.22137	74	1̄.22747	77	0.77253	1̄.99390	25
36	1̄.22211	75	1̄.22824	77	0.77176	1̄.99388	24
37	1̄.22286	75	1̄.22901	76	0.77099	1̄.99385	23
38	1̄.22361	74	1̄.22977	77	0.77023	1̄.99383	22
39	1̄.22435	74	1̄.23054	76	0.76946	1̄.99381	21
40	1̄.22509	74	1̄.23130	76	0.76870	1̄.99379	20
41	1̄.22583	74	1̄.23206	77	0.76794	1̄.99377	19
42	1̄.22657	74	1̄.23283	76	0.76717	1̄.99375	18
43	1̄.22731	74	1̄.23359	76	0.76641	1̄.99372	17
44	1̄.22805	73	1̄.23435	75	0.76565	1̄.99370	16
45	1̄.22878	74	1̄.23510	76	0.76490	1̄.99368	15
46	1̄.22952	73	1̄.23586	75	0.76414	1̄.99366	14
47	1̄.23025	73	1̄.23661	76	0.76339	1̄.99364	13
48	1̄.23098	73	1̄.23737	75	0.76263	1̄.99362	12
49	1̄.23171	73	1̄.23812	75	0.76188	1̄.99359	11
50	1̄.23244	73	1̄.23887	75	0.76113	1̄.99357	10
51	1̄.23317	73	1̄.23962	75	0.76038	1̄.99355	9
52	1̄.23390	72	1̄.24037	75	0.75963	1̄.99353	8
53	1̄.23462	73	.24112	74	0.75888	1̄.99351	7
54	1̄.23535	72	1̄.24186	75	0.75814	1̄.99348	6
55	1̄.23607	72	1̄.24261	74	0.75739	1̄.99346	5
56	1̄.23679	73	1̄.24335	75	0.75665	1̄.99344	4
57	1̄.23752	71	1̄.24410	74	0.75590	1̄.99342	3
58	1̄.23823	72	1̄.24484	74	0.75516	1̄.99340	2
59	1̄.23895	72	1̄.24558	74	0.75442	1̄.99337	1
60	1̄.23967		1̄.24632		0.75368	1̄.99335	0
	log cos	d	log cot	c. d.	log tan	log sin	′

p. p.

	82	81	80	79	78	77	76	75	74	73	72	71	3	2
6	8.2	8.1	8.0	7.9	7.8	7.7	7.6	7.5	7.4	7.3	7.2	7.1	0.3	0.2
7	9.6	9.5	9.3	9.2	9.1	9.0	8.9	8.8	8.6	8.5	8.4	8.3	0.4	0.2
8	10.9	10.8	10.7	10.5	10.4	10.3	10.1	10.0	9.9	9.7	9.6	9.5	0.4	0.3
9	12.3	12.2	12.0	11.9	11.7	11.6	11.4	11.3	11.1	11.0	10.8	10.7	0.5	0.3
10	13.7	13.5	13.3	13.2	13.0	12.8	12.7	12.5	12.3	12.2	12.0	11.8	0.5	0.3
20	27.3	27.0	26.7	26.3	26.0	25.7	25.3	25.0	24.7	24.3	24.0	23.7	1.0	0.7
30	41.0	40.5	40.0	39.5	39.0	38.5	38.0	37.5	37.0	36.5	36.0	35.5	1.5	1.0
40	54.7	54.0	53.3	52.7	52.0	51.3	50.7	50.0	49.3	48.7	48.0	47.3	2.0	1.3
50	68.3	67.5	66.7	65.8	65.0	64.2	63.3	62.5	61.7	60.8	60.0	59.2	2.5	1.7

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	′
0	Ī.23967	72	Ī.24632	74	0.75368	Ī.99335	60
1	Ī.24039	71	Ī.24706	73	0.75294	Ī.99333	59
2	Ī.24110	71	Ī.24779	74	0.75221	Ī.99331	58
3	Ī.24181	72	Ī.24853	73	0.75147	Ī.99328	57
4	Ī.24253	71	Ī.24926	74	0.75074	Ī.99326	56
5	Ī.24324	71	Ī.25000	73	0.75000	Ī.99324	55
6	Ī.24395	71	Ī.25073	73	0.74927	Ī.99322	54
7	Ī.24466	70	Ī.25146	73	0.74854	Ī.99319	53
8	Ī.24536	71	Ī.25219	73	0.74781	Ī.99317	52
9	Ī.24607	70	Ī.25292	73	0.74708	Ī.99315	51
10	Ī.24677	71	Ī.25365	72	0.74635	Ī.99313	50
11	Ī.24748	70	Ī.25437	73	0.74563	Ī.99310	49
12	Ī.24818	70	Ī.25510	72	0.74490	Ī.99308	48
13	Ī.24888	70	Ī.25582	73	0.74418	Ī.99306	47
14	Ī.24958	70	Ī.25655	72	0.74345	Ī.99304	46
15	Ī.25028	70	Ī.25727	72	0.74273	Ī.99301	45
16	Ī.25098	70	Ī.25799	72	0.74201	Ī.99299	44
17	Ī.25168	69	Ī.25871	72	0.74129	Ī.99297	43
18	Ī.25237	70	Ī.25943	72	0.74057	Ī.99294	42
19	Ī.25307	69	Ī.26015	71	0.73985	Ī.99292	41
20	Ī.25376	69	Ī.26086	72	0.73914	Ī.99290	40
21	Ī.25445	69	Ī.26158	71	0.73842	Ī.99288	39
22	Ī.25514	69	Ī.26229	72	0.73771	Ī.99285	38
23	Ī.25583	69	Ī.26301	71	0.73699	Ī.99283	37
24	Ī.25652	69	Ī.26372	71	0.73628	Ī.99281	36
25	Ī.25721	69	Ī.26443	71	0.73557	Ī.99278	35
26	Ī.25790	68	Ī.26514	71	0.73486	Ī.99276	34
27	Ī.25858	69	Ī.26585	70	0.73415	Ī.99274	33
28	Ī.25927	68	Ī.26655	71	0.73345	Ī.99271	32
29	Ī.25995	68	Ī.26726	71	0.73274	Ī.99269	31
30	Ī.26063	68	Ī.26797	70	0.73203	Ī.99267	30
31	Ī.26131	68	Ī.26867	70	0.73133	Ī.99264	29
32	Ī.26199	68	Ī.26937	71	0.73063	Ī.99262	28
33	Ī.26267	68	Ī.27008	70	0.72992	Ī.99260	27
34	Ī.26335	68	Ī.27078	70	0.72922	Ī.99257	26
35	Ī.26403	67	Ī.27148	70	0.72852	Ī.99255	25
36	Ī.26470	68	Ī.27218	70	0.72782	Ī.99252	24
37	Ī.26538	67	Ī.27288	69	0.72712	Ī.99250	23
38	Ī.26605	67	Ī.27357	70	0.72643	Ī.99248	22
39	Ī.26672	67	Ī.27427	69	0.72573	Ī.99245	21
40	Ī.26739	67	Ī.27496	70	0.72504	Ī.99243	20
41	Ī.26806	67	Ī.27566	69	0.72434	Ī.99241	19
42	Ī.26873	67	Ī.27635	69	0.72365	Ī.99238	18
43	Ī.26940	67	Ī.27704	69	0.72296	Ī.99236	17
44	Ī.27007	66	Ī.27773	69	0.72227	Ī.99233	16
45	Ī.27073	67	Ī.27842	69	0.72158	Ī.99231	15
46	Ī.27140	66	Ī.27911	69	0.72089	Ī.99229	14
47	Ī.27206	67	Ī.27980	69	0.72020	Ī.99226	13
48	Ī.27273	66	Ī.28049	68	0.71951	Ī.99224	12
49	Ī.27339	66	Ī.28117	69	0.71883	Ī.99221	11
50	Ī.27405	66	Ī.28186	68	0.71814	Ī.99219	10
51	Ī.27471	66	Ī.28254	69	0.71746	Ī.99217	9
52	Ī.27537	65	Ī.28323	68	0.71677	Ī.99214	8
53	Ī.27602	66	Ī.28391	68	0.71609	Ī.99212	7
54	Ī.27668	66	Ī.28459	68	0.71541	Ī.99209	6
55	Ī.27734	65	Ī.28527	68	0.71473	Ī.99207	5
56	Ī.27799	65	Ī.28595	67	0.71405	Ī.99204	4
57	Ī.27864	66	Ī.28662	68	0.71338	Ī.99202	3
58	Ī.27930	65	Ī.28730	68	0.71270	Ī.99200	2
59	Ī.27995	65	Ī.28798	67	0.71202	Ī.99197	1
60	Ī.28060		Ī.28865		0.71135	Ī.99195	0
	log cos	d	log cot	c. d.	log tan	log sin	′

p. p.

	74	73	72	71	70	69	68	67	66	65	3	2
6	7.4	7.3	7.2	7.1	7.0	6.9	6.8	6.7	6.6	6.5	0.3	0.2
7	8.6	8.5	8.4	8.3	8.2	8.1	7.9	7.8	7.7	7.6	0.4	0.2
8	9.9	9.7	9.6	9.5	9.3	9.2	9.1	8.9	8.8	8.7	0.4	0.3
9	11.1	11.0	10.8	10.7	10.5	10.4	10.2	10.1	9.9	9.8	0.5	0.3
10	12.3	12.2	12.0	11.8	11.7	11.5	11.3	11.2	11.0	10.8	0.5	0.3
20	24.7	24.3	24.0	23.7	23.3	23.0	22.7	22.3	22.0	21.7	1.0	0.7
30	37.0	36.5	36.0	35.5	35.0	34.5	34.0	33.5	33.0	32.5	1.5	1.0
40	49.3	48.7	48.0	47.3	46.7	46.0	45.3	44.7	44.0	43.3	2.0	1.3
50	61.7	60.8	60.0	59.2	58.3	57.5	56.7	55.8	55.0	54.2	2.5	1.7

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	′
0	$\bar{1}$.28060	65	$\bar{1}$.28865	68	0.71135	$\bar{1}$.99195	60
1	$\bar{1}$.28125	65	$\bar{1}$.28933	67	0.71067	$\bar{1}$.99192	59
2	$\bar{1}$.28190	64	$\bar{1}$.29000	67	0.71000	$\bar{1}$.99190	58
3	$\bar{1}$.28254	65	$\bar{1}$.29067	67	0.70933	$\bar{1}$.99187	57
4	$\bar{1}$.28319	65	$\bar{1}$.29134	67	0.70866	$\bar{1}$.99185	56
5	$\bar{1}$.28384	64	$\bar{1}$.29201	67	0.70799	$\bar{1}$.99182	55
6	$\bar{1}$.28448	64	$\bar{1}$.29268	67	0.70732	$\bar{1}$.99180	54
7	$\bar{1}$.28512	65	$\bar{1}$.29335	67	0.70665	$\bar{1}$.99177	53
8	$\bar{1}$.28577	64	$\bar{1}$.29402	66	0.70598	$\bar{1}$.99175	52
9	$\bar{1}$.28641	64	$\bar{1}$.29468	67	0.70532	$\bar{1}$.99172	51
10	$\bar{1}$.28705	64	$\bar{1}$.29535	66	0.70465	$\bar{1}$.99170	50
11	$\bar{1}$.28769	64	$\bar{1}$.29601	67	0.70399	$\bar{1}$.99167	49
12	$\bar{1}$.28833	63	$\bar{1}$.29668	66	0.70332	$\bar{1}$.99165	48
13	$\bar{1}$.28896	64	$\bar{1}$.29734	66	0.70266	$\bar{1}$.99162	47
14	$\bar{1}$.28960	64	$\bar{1}$.29800	66	0.70200	$\bar{1}$.99160	46
15	$\bar{1}$.29024	63	$\bar{1}$.29866	66	0.70134	$\bar{1}$.99157	45
16	$\bar{1}$.29087	63	$\bar{1}$.29932	66	0.70068	$\bar{1}$.99155	44
17	$\bar{1}$.29150	64	$\bar{1}$.29998	66	0.70002	$\bar{1}$.99152	43
18	$\bar{1}$.29214	63	$\bar{1}$.30064	66	0.69936	$\bar{1}$.99150	42
19	$\bar{1}$.29277	63	$\bar{1}$.30130	65	0.69870	$\bar{1}$.99147	41
20	$\bar{1}$.29340	63	$\bar{1}$.30195	66	0.69805	$\bar{1}$.99145	40
21	$\bar{1}$.29403	63	$\bar{1}$.30261	65	0.69739	$\bar{1}$.99142	39
22	$\bar{1}$.29466	63	$\bar{1}$.30326	65	0.69674	$\bar{1}$.99140	38
23	$\bar{1}$.29529	62	$\bar{1}$.30391	66	0.69609	$\bar{1}$.99137	37
24	$\bar{1}$.29591	63	$\bar{1}$.30457	65	0.69543	$\bar{1}$.99135	36
25	$\bar{1}$.29654	62	$\bar{1}$.30522	65	0.69478	$\bar{1}$.99132	35
26	$\bar{1}$.29716	63	$\bar{1}$.30587	65	0.69413	$\bar{1}$.99130	34
27	$\bar{1}$.29779	62	$\bar{1}$.30652	65	0.69348	$\bar{1}$.99127	33
28	$\bar{1}$.29841	62	$\bar{1}$.30717	65	0.69283	$\bar{1}$.99124	32
29	$\bar{1}$.29903	63	$\bar{1}$.30782	64	0.69218	$\bar{1}$.99122	31
30	$\bar{1}$.29966	62	$\bar{1}$.30846	65	0.69154	$\bar{1}$.99119	30
31	$\bar{1}$.30028	62	$\bar{1}$.30911	64	0.69089	$\bar{1}$.99117	29
32	$\bar{1}$.30090	61	$\bar{1}$.30975	65	0.69025	$\bar{1}$.99114	28
33	$\bar{1}$.30151	62	$\bar{1}$.31040	64	0.68960	$\bar{1}$.99112	27
34	$\bar{1}$.30213	62	$\bar{1}$.31104	64	0.68896	$\bar{1}$.99109	26
35	$\bar{1}$.30275	61	$\bar{1}$.31168	65	0.68832	$\bar{1}$.99106	25
36	$\bar{1}$.30336	62	$\bar{1}$.31233	64	0.68767	$\bar{1}$.99104	24
37	$\bar{1}$.30398	61	$\bar{1}$.31297	64	0.68703	$\bar{1}$.99101	23
38	$\bar{1}$.30459	62	$\bar{1}$.31361	64	0.68639	$\bar{1}$.99099	22
39	$\bar{1}$.30521	61	$\bar{1}$.31425	64	0.68575	$\bar{1}$.99096	21
40	$\bar{1}$.30582	61	$\bar{1}$.31489	63	0.68511	$\bar{1}$.99093	20
41	$\bar{1}$.30643	61	$\bar{1}$.31552	64	0.68448	$\bar{1}$.99091	19
42	$\bar{1}$.30704	61	$\bar{1}$.31616	63	0.68384	$\bar{1}$.99088	18
43	$\bar{1}$.30765	61	$\bar{1}$.31679	64	0.68321	$\bar{1}$.99086	17
44	$\bar{1}$.30826	61	$\bar{1}$.31743	63	0.68257	$\bar{1}$.99083	16
45	$\bar{1}$.30887	60	$\bar{1}$.31806	64	0.68194	$\bar{1}$.99080	15
46	$\bar{1}$.30947	61	$\bar{1}$.31870	63	0.68130	$\bar{1}$.99078	14
47	$\bar{1}$.31008	60	$\bar{1}$.31933	63	0.68067	$\bar{1}$.99075	13
48	$\bar{1}$.31068	61	$\bar{1}$.31996	63	0.68004	$\bar{1}$.99072	12
49	$\bar{1}$.31129	60	$\bar{1}$.32059	63	0.67941	$\bar{1}$.99070	11
50	$\bar{1}$.31189	61	$\bar{1}$.32122	63	0.67878	$\bar{1}$.99067	10
51	$\bar{1}$.31250	60	$\bar{1}$.32185	63	0.67815	$\bar{1}$.99064	9
52	$\bar{1}$.31310	60	$\bar{1}$.32248	63	0.67752	$\bar{1}$.99062	8
53	$\bar{1}$.31370	60	$\bar{1}$.32311	62	0.67689	$\bar{1}$.99059	7
54	$\bar{1}$.31430	60	$\bar{1}$.32373	63	0.67627	$\bar{1}$.99056	6
55	$\bar{1}$.31490	59	$\bar{1}$.32436	62	0.67564	$\bar{1}$.99054	5
56	$\bar{1}$.31549	60	$\bar{1}$.32498	63	0.67502	$\bar{1}$.99051	4
57	$\bar{1}$.31609	60	$\bar{1}$.32561	62	0.67439	$\bar{1}$.99048	3
58	$\bar{1}$.31669	59	$\bar{1}$.32623	62	0.67377	$\bar{1}$.99046	2
59	$\bar{1}$.31728	60	$\bar{1}$.32685	62	0.67315	$\bar{1}$.99043	1
60	$\bar{1}$.31788		$\bar{1}$.32747		0.67253	$\bar{1}$.99040	0
	log cos	d	log cot	c. d.	log tan	log sin	′

p. p.

	68	67	66	65	64	63	62	61	60	59	3	2
6	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	0.3	0.2
7	7.9	7.8	7.7	7.6	7.5	7.4	7.2	7.1	7.0	6.9	0.4	0.2
8	9.1	8.9	8.8	8.7	8.5	8.4	8.3	8.1	8.0	7.9	0.4	0.3
9	10.2	10.1	9.9	9.8	9.6	9.5	9.3	9.2	9.0	8.9	0.5	0.3
10	11.3	11.2	11.0	10.8	10.7	10.5	10.3	10.2	10.0	9.8	0.5	0.3
20	22.7	22.3	22.0	21.7	21.3	21.0	20.7	20.3	20.0	19.7	1.0	0.7
30	34.0	33.5	33.0	32.5	32.0	31.5	31.0	30.5	30.0	29.5	1.5	1.0
40	45.3	44.7	44.0	43.3	42.7	42.0	41.3	40.7	40.0	39.3	2.0	1.3
50	56.7	55.8	55.0	54.2	53.3	52.5	51.7	50.8	50.0	49.2	2.5	1.7

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	′
0	$\bar{1}$.31788	59	$\bar{1}$.32747	63	0.67253	$\bar{1}$.99040	60
1	$\bar{1}$.31847	60	$\bar{1}$.32810	62	0.67190	$\bar{1}$.99038	59
2	$\bar{1}$.31907	59	$\bar{1}$.32872	61	0.67128	$\bar{1}$.99035	58
3	$\bar{1}$.31966	59	$\bar{1}$.32933	62	0.67067	$\bar{1}$.99032	57
4	$\bar{1}$.32025	59	$\bar{1}$.32995	62	0.67005	$\bar{1}$.99030	56
5	$\bar{1}$.32084	59	$\bar{1}$.33057	62	0.66943	$\bar{1}$.99027	55
6	$\bar{1}$.32143	59	$\bar{1}$.33119	61	0.66881	$\bar{1}$.99024	54
7	$\bar{1}$.32202	59	$\bar{1}$.33180	62	0.66820	$\bar{1}$.99022	53
8	$\bar{1}$.32261	58	$\bar{1}$.33242	61	0.66758	$\bar{1}$.99019	52
9	$\bar{1}$.32319	59	$\bar{1}$.33303	62	0.66697	$\bar{1}$.99016	51
10	$\bar{1}$.32378	59	$\bar{1}$.33365	61	0.66635	$\bar{1}$.99013	50
11	$\bar{1}$.32437	58	$\bar{1}$.33426	61	0.66574	$\bar{1}$.99011	49
12	$\bar{1}$.32495	58	$\bar{1}$.33487	61	0.66513	$\bar{1}$.99008	48
13	$\bar{1}$.32553	59	$\bar{1}$.33548	61	0.66452	$\bar{1}$.99005	47
14	$\bar{1}$.32612	58	$\bar{1}$.33609	61	0.66391	$\bar{1}$.99002	46
15	$\bar{1}$.32670	58	$\bar{1}$.33670	61	0.66330	$\bar{1}$.99000	45
16	$\bar{1}$.32728	58	$\bar{1}$.33731	61	0.66269	$\bar{1}$.98997	44
17	$\bar{1}$.32786	58	$\bar{1}$.33792	61	0.66208	$\bar{1}$.98994	43
18	$\bar{1}$.32844	58	$\bar{1}$.33853	60	0.66147	$\bar{1}$.98991	42
19	$\bar{1}$.32902	58	$\bar{1}$.33913	61	0.66087	$\bar{1}$.98989	41
20	$\bar{1}$.32960	58	$\bar{1}$.33974	60	0.66026	$\bar{1}$.98986	40
21	$\bar{1}$.33018	57	$\bar{1}$.34034	61	0.65966	$\bar{1}$.98983	39
22	$\bar{1}$.33075	58	$\bar{1}$.34095	60	0.65905	$\bar{1}$.98980	38
23	$\bar{1}$.33133	57	$\bar{1}$.34155	60	0.65845	$\bar{1}$.98978	37
24	$\bar{1}$.33190	58	$\bar{1}$.34215	61	0.65785	$\bar{1}$.98975	36
25	$\bar{1}$.33248	57	$\bar{1}$.34276	60	0.65724	$\bar{1}$.98972	35
26	$\bar{1}$.33305	57	$\bar{1}$.34336	60	0.65664	$\bar{1}$.98969	34
27	$\bar{1}$.33362	58	$\bar{1}$.34396	60	0.65604	$\bar{1}$.98967	33
28	$\bar{1}$.33420	57	$\bar{1}$.34456	60	0.65544	$\bar{1}$.98964	32
29	$\bar{1}$.33477	57	$\bar{1}$.34516	60	0.65484	$\bar{1}$.98961	31
30	$\bar{1}$.33534	57	$\bar{1}$.34576	59	0.65424	$\bar{1}$.98958	30
31	$\bar{1}$.33591	56	$\bar{1}$.34635	60	0.65365	$\bar{1}$.98955	29
32	$\bar{1}$.33647	57	$\bar{1}$.34695	60	0.65305	$\bar{1}$.98953	28
33	$\bar{1}$.33704	57	$\bar{1}$.34755	59	0.65245	$\bar{1}$.98950	27
34	$\bar{1}$.33761	57	$\bar{1}$.34814	60	0.65186	$\bar{1}$.98947	26
35	$\bar{1}$.33818	56	$\bar{1}$.34874	59	0.65126	$\bar{1}$.98944	25
36	$\bar{1}$.33874	57	$\bar{1}$.34933	59	0.65067	$\bar{1}$.98941	24
37	$\bar{1}$.33931	56	$\bar{1}$.34992	59	0.65008	$\bar{1}$.98938	23
38	$\bar{1}$.33987	56	$\bar{1}$.35051	60	0.64949	$\bar{1}$.98936	22
39	$\bar{1}$.34043	57	$\bar{1}$.35111	59	0.64889	$\bar{1}$.98933	21
40	$\bar{1}$.34100	56	$\bar{1}$.35170	59	0.64830	$\bar{1}$.98930	20
41	$\bar{1}$.34156	56	$\bar{1}$.35229	59	0.64771	$\bar{1}$.98927	19
42	$\bar{1}$.34212	56	$\bar{1}$.35288	59	0.64712	$\bar{1}$.98924	18
43	$\bar{1}$.34268	56	$\bar{1}$.35347	58	0.64653	$\bar{1}$.98921	17
44	$\bar{1}$.34324	56	$\bar{1}$.35405	59	0.64595	$\bar{1}$.98919	16
45	$\bar{1}$.34380	56	$\bar{1}$.35464	59	0.64536	$\bar{1}$.98916	15
46	$\bar{1}$.34436	55	$\bar{1}$.35523	58	0.64477	$\bar{1}$.98913	14
47	$\bar{1}$.34491	56	$\bar{1}$.35581	59	0.64419	$\bar{1}$.98910	13
48	$\bar{1}$.34547	55	$\bar{1}$.35640	58	0.64360	$\bar{1}$.98907	12
49	$\bar{1}$.34602	56	$\bar{1}$.35698	59	0.64302	$\bar{1}$.98904	11
50	$\bar{1}$.34658	55	$\bar{1}$.35757	58	0.64243	$\bar{1}$.98901	10
51	$\bar{1}$.34713	56	$\bar{1}$.35815	58	0.64185	$\bar{1}$.98898	9
52	$\bar{1}$.34769	55	$\bar{1}$.35873	58	0.64127	$\bar{1}$.98896	8
53	$\bar{1}$.34824	55	$\bar{1}$.35931	58	0.64069	$\bar{1}$.98893	7
54	$\bar{1}$.34879	55	$\bar{1}$.35989	58	0.64011	$\bar{1}$.98890	6
55	$\bar{1}$.34934	55	$\bar{1}$.36047	58	0.63953	$\bar{1}$.98887	5
56	$\bar{1}$.34989	55	$\bar{1}$.36105	58	0.63895	$\bar{1}$.98884	4
57	$\bar{1}$.35044	55	$\bar{1}$.36163	58	0.63837	$\bar{1}$.98881	3
58	$\bar{1}$.35099	55	$\bar{1}$.36221	58	0.63779	$\bar{1}$.98878	2
59	$\bar{1}$.35154	55	$\bar{1}$.36279	57	0.63721	$\bar{1}$.98875	1
60	$\bar{1}$.35209		$\bar{1}$.36336		0.63664	$\bar{1}$.98872	0
	log cos	d	log cot	c. d.	log tan	log sin	′

p. p.

	63	62	61	60	59	58	57	56	55	3	2
6	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	0.3	0.2
7	7.4	7.2	7.1	7.0	6.9	6.8	6.7	6.5	6.4	0.4	0.2
8	8.4	8.3	8.1	8.0	7.9	7.7	7.6	7.5	7.3	0.4	0.3
9	9.5	9.3	9.2	9.0	8.9	8.7	8.6	8.4	8.3	0.5	0.3
10	10.5	10.3	10.2	10.0	9.8	9.7	9.5	9.3	9.2	0.5	0.3
20	21.0	20.7	20.3	20.0	19.7	19.3	19.0	18.7	18.3	1.0	0.7
30	31.5	31.0	30.5	30.0	29.5	29.0	28.5	28.0	27.5	1.5	1.0
40	42.0	41.3	40.7	40.0	39.3	38.7	38.0	37.3	36.7	2.0	1.3
50	52.5	51.7	50.8	50.0	49.2	48.3	47.5	46.7	45.8	2.5	1.7

′	log sin	d	log tan	c. d.	log cot	log cos	
0	1̄.35209	54	1̄.36336	58	0.63664	1̄.98872	60
1	1̄.35263	55	1̄.36394	58	0.63606	1̄.98869	59
2	1̄.35318	55	1̄.36452	57	0.63548	1̄.98867	58
3	1̄.35373	54	1̄.36509	57	0.63491	1̄.98864	57
4	1̄.35427	54	1̄.36566	58	0.63434	1̄.98861	56
5	1̄.35481	55	1̄.36624	57	0.63376	1̄.98858	55
6	1̄.35536	54	1̄.36681	57	0.63319	1̄.98855	54
7	1̄.35590	54	1̄.36738	57	0.63262	1̄.98852	53
8	1̄.35644	54	1̄.36795	57	0.63205	1̄.98849	52
9	1̄.35698	54	1̄.36852	57	0.63148	1̄.98846	51
10	1̄.35752	54	1̄.36909	57	0.63091	1̄.98843	50
11	1̄.35806	54	1̄.36966	57	0.63034	1̄.98840	49
12	1̄.35860	54	1̄.37023	57	0.62977	1̄.98837	48
13	1̄.35914	54	1̄.37080	57	0.62920	1̄.98834	47
14	1̄.35968	54	1̄.37137	56	0.62863	1̄.98831	46
15	1̄.36022	53	1̄.37193	57	0.62807	1̄.98828	45
16	1̄.36075	54	1̄.37250	56	0.62750	1̄.98825	44
17	1̄.36129	53	1̄.37306	57	0.62694	1̄.98822	43
18	1̄.36182	54	1̄.37363	56	0.62637	1̄.98819	42
19	1̄.36236	53	1̄.37419	57	0.62581	1̄.98816	41
20	1̄.36289	53	1̄.37476	56	0.62524	1̄.98813	40
21	1̄.36342	53	1̄.37532	56	0.62468	1̄.98810	39
22	1̄.36395	54	1̄.37588	56	0.62412	1̄.98807	38
23	1̄.36449	53	1̄.37644	56	0.62356	1̄.98804	37
24	1̄.36502	53	1̄.37700	56	0.62300	1̄.98801	36
25	1̄.36555	53	1̄.37756	56	0.62244	1̄.98798	35
26	1̄.36608	52	1̄.37812	56	0.62188	1̄.98795	34
27	1̄.36660	53	1̄.37868	56	0.62132	1̄.98792	33
28	1̄.36713	53	1̄.37924	56	0.62076	1̄.98789	32
29	1̄.36766	53	1̄.37980	55	0.62020	1̄.98786	31
30	1̄.36819	52	1̄.38035	56	0.61965	1̄.98783	30
31	1̄.36871	53	1̄.38091	56	0.61909	1̄.98780	29
32	1̄.36924	52	1̄.38147	55	0.61853	1̄.98777	28
33	1̄.36976	52	1̄.38202	55	0.61798	1̄.98774	27
34	1̄.37028	53	1̄.38257	56	0.61743	1̄.98771	26
35	1̄.37081	52	1̄.38313	55	0.61687	1̄.98768	25
36	1̄.37133	52	1̄.38368	55	0.61632	1̄.98765	24
37	1̄.37185	52	1̄.38423	56	0.61577	1̄.98762	23
38	1̄.37237	52	1̄.38479	55	0.61521	1̄.98759	22
39	1̄.37289	52	1̄.38534	55	0.61466	1̄.98756	21
40	1̄.37341	52	1̄.38589	55	0.61411	1̄.98753	20
41	1̄.37393	52	1̄.38644	55	0.61356	1̄.98750	19
42	1̄.37445	52	1̄.38699	55	0.61301	1̄.98746	18
43	1̄.37497	52	1̄.38754	54	0.61246	1̄.98743	17
44	1̄.37549	51	1̄.38808	55	0.61192	1̄.98740	16
45	1̄.37600	52	1̄.38863	55	0.61137	1̄.98737	15
46	1̄.37652	51	1̄.38918	54	0.61082	1̄.98734	14
47	1̄.37703	52	1̄.38972	55	0.61028	1̄.98731	13
48	1̄.37755	51	1̄.39027	55	0.60973	1̄.98728	12
49	1̄.37806	52	1̄.39082	54	0.60918	1̄.98725	11
50	1̄.37858	51	1̄.39136	54	0.60864	1̄.98722	10
51	1̄.37909	51	1̄.39190	55	0.60810	1̄.98719	9
52	1̄.37960	51	1̄.39245	54	0.60755	1̄.98715	8
53	1̄.38011	51	1̄.39299	54	0.60701	1̄.98712	7
54	1̄.38062	51	1̄.39353	54	0.60647	1̄.98709	6
55	1̄.38113	51	1̄.39407	54	0.60593	1̄.98706	5
56	1̄.38164	51	1̄.39461	54	0.60539	1̄.98703	4
57	1̄.38215	51	1̄.39515	54	0.60485	1̄.98700	3
58	1̄.38266	51	1̄.39569	54	0.60431	1̄.98697	2
59	1̄.38317	51	1̄.39623	54	0.60377	1̄.98694	1
60	1̄.38368		1̄.39677		0.60323	1̄.98690	0
	log cos	d	log cot	c. d.	log tan	log sin	′

p. p.

	58	57
6	5.8	5.7
7	6.8	6.7
8	7.7	7.6
9	8.7	8.6
10	9.7	9.5
20	19.3	19.0
30	29.0	28.5
40	38.7	38.0
50	48.3	47.5

	56	55
6	5.6	5.5
7	6.5	6.4
8	7.5	7.3
9	8.4	8.3
10	9.3	9.2
20	18.7	18.3
30	28.0	27.5
40	37.3	36.7
50	46.7	45.8

	54
6	5.4
7	6.3
8	7.2
9	8.1
10	9.0
20	18.0
30	27.0
40	36.0
50	45.0

	53	52
6	5.3	5.2
7	6.2	6.1
8	7.1	6.9
9	8.0	7.8
10	8.8	8.7
20	17.7	17.3
30	26.5	26.0
40	35.3	34.7
50	44.2	43.3

	51	4
6	5.1	0.4
7	6.0	0.5
8	6.8	0.5
9	7.7	0.6
10	8.5	0.7
20	17.0	1.3
30	25.5	2.0
40	34.0	2.7
50	42.5	3.3

	3	2
6	0.3	0.2
7	0.4	0.2
8	0.4	0.3
9	0.5	0.3
10	0.5	0.3
20	1.0	0.7
30	1.5	1.0
40	2.0	1.3
50	2.5	1.7

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	$\bar{1}$.38368		$\bar{1}$.39677		0.60323	$\bar{1}$.98690		60
1	$\bar{1}$.38418	50	$\bar{1}$.39731	54	0.60269	$\bar{1}$.98687	3	59
2	$\bar{1}$.38469	51	$\bar{1}$.39785	54	0.60215	$\bar{1}$.98684	3	58
3	$\bar{1}$.38519	50	$\bar{1}$.39838	53	0.60162	$\bar{1}$.98681	3	57
4	$\bar{1}$.38570	51	$\bar{1}$.39892	54	0.60108	$\bar{1}$.98678	3	56
5	$\bar{1}$.38620	50	$\bar{1}$.39945	53	0.60055	$\bar{1}$.98675	3	55
6	$\bar{1}$.38670	50	$\bar{1}$.39999	54	0.60001	$\bar{1}$.98671	4	54
7	$\bar{1}$.38721	51	$\bar{1}$.40052	53	0.59948	$\bar{1}$.98668	3	53
8	$\bar{1}$.38771	50	$\bar{1}$.40106	54	0.59894	$\bar{1}$.98665	3	52
9	$\bar{1}$.38821	50	$\bar{1}$.40159	53	0.59841	$\bar{1}$.98662	3	51
10	$\bar{1}$.38871	50	$\bar{1}$.40212	53	0.59788	$\bar{1}$.98659	3	50
11	$\bar{1}$.38921	50	$\bar{1}$.40266	54	0.59734	$\bar{1}$.98656	3	49
12	$\bar{1}$.38971	50	$\bar{1}$.40319	53	0.59681	$\bar{1}$.98652	4	48
13	$\bar{1}$.39021	50	$\bar{1}$.40372	53	0.59628	$\bar{1}$.98649	3	47
14	$\bar{1}$.39071	50	$\bar{1}$.40425	53	0.59575	$\bar{1}$.98646	3	46
15	$\bar{1}$.39121	50	$\bar{1}$.40478	53	0.59522	$\bar{1}$.98643	3	45
16	$\bar{1}$.39170	49	$\bar{1}$.40531	53	0.59469	$\bar{1}$.98640	3	44
17	$\bar{1}$.39220	50	$\bar{1}$.40584	53	0.59416	$\bar{1}$.98636	4	43
18	$\bar{1}$.39270	50	$\bar{1}$.40636	52	0.59364	$\bar{1}$.98633	3	42
19	$\bar{1}$.39319	49	$\bar{1}$.40689	53	0.59311	$\bar{1}$.98630	3	41
20	$\bar{1}$.39369	50	$\bar{1}$.40742	53	0.59258	$\bar{1}$.98627	3	40
21	$\bar{1}$.39418	49	$\bar{1}$.40795	53	0.59205	$\bar{1}$.98623	4	39
22	$\bar{1}$.39467	49	$\bar{1}$.40847	52	0.59153	$\bar{1}$.98620	3	38
23	$\bar{1}$.39517	50	$\bar{1}$.40900	53	0.59100	$\bar{1}$.98617	3	37
24	$\bar{1}$.39566	49	$\bar{1}$.40952	52	0.59048	$\bar{1}$.98614	3	36
25	$\bar{1}$.39615	49	$\bar{1}$.41005	53	0.58995	$\bar{1}$.98610	4	35
26	$\bar{1}$.39664	49	$\bar{1}$.41057	52	0.58943	$\bar{1}$.98607	3	34
27	$\bar{1}$.39713	49	$\bar{1}$.41109	52	0.58891	$\bar{1}$.98604	3	33
28	$\bar{1}$.39762	49	$\bar{1}$.41161	52	0.58839	$\bar{1}$.98601	3	32
29	$\bar{1}$.39811	49	$\bar{1}$.41214	53	0.58786	$\bar{1}$.98597	4	31
30	$\bar{1}$.39860	49	$\bar{1}$.41266	52	0.58734	$\bar{1}$.98594	3	30
31	$\bar{1}$.39909	49	$\bar{1}$.41318	52	0.58682	$\bar{1}$.98591	3	29
32	$\bar{1}$.39958	49	$\bar{1}$.41370	52	0.58630	$\bar{1}$.98588	3	28
33	$\bar{1}$.40006	48	$\bar{1}$.41422	52	0.58578	$\bar{1}$.98584	4	27
34	$\bar{1}$.40055	49	$\bar{1}$.41474	52	0.58526	$\bar{1}$.98581	3	26
35	$\bar{1}$.40103	48	$\bar{1}$.41526	52	0.58474	$\bar{1}$.98578	3	25
36	$\bar{1}$.40152	49	$\bar{1}$.41578	52	0.58422	$\bar{1}$.98574	4	24
37	$\bar{1}$.40200	48	$\bar{1}$.41629	51	0.58371	$\bar{1}$.98571	3	23
38	$\bar{1}$.40249	49	$\bar{1}$.41681	52	0.58319	$\bar{1}$.98568	3	22
39	$\bar{1}$.40297	48	$\bar{1}$.41733	52	0.58267	$\bar{1}$.98565	3	21
40	$\bar{1}$.40346	49	$\bar{1}$.41784	51	0.58216	$\bar{1}$.98561	4	20
41	$\bar{1}$.40394	48	$\bar{1}$.41836	52	0.58164	$\bar{1}$.98558	3	19
42	$\bar{1}$.40442	48	$\bar{1}$.41887	51	0.58113	$\bar{1}$.98555	3	18
43	$\bar{1}$.40490	48	$\bar{1}$.41939	52	0.58061	$\bar{1}$.98551	4	17
44	$\bar{1}$.40538	48	$\bar{1}$.41990	51	0.58010	$\bar{1}$.98548	3	16
45	$\bar{1}$.40586	48	$\bar{1}$.42041	51	0.57959	$\bar{1}$.98545	3	15
46	$\bar{1}$.40634	48	$\bar{1}$.42093	52	0.57907	$\bar{1}$.98541	4	14
47	$\bar{1}$.40682	48	$\bar{1}$.42144	51	0.57856	$\bar{1}$.98538	3	13
48	$\bar{1}$.40730	48	$\bar{1}$.42195	51	0.57805	$\bar{1}$.98535	3	12
49	$\bar{1}$.40778	48	$\bar{1}$.42246	51	0.57754	$\bar{1}$.98531	4	11
50	$\bar{1}$.40825	47	$\bar{1}$.42297	51	0.57703	$\bar{1}$.98528	3	10
51	$\bar{1}$.40873	48	$\bar{1}$.42348	51	0.57652	$\bar{1}$.98525	3	9
52	$\bar{1}$.40921	48	$\bar{1}$.42399	51	0.57601	$\bar{1}$.98521	4	8
53	$\bar{1}$.40968	47	$\bar{1}$.42450	51	0.57550	$\bar{1}$.98518	3	7
54	$\bar{1}$.41016	48	$\bar{1}$.42501	51	0.57499	$\bar{1}$.98515	3	6
55	$\bar{1}$.41063	47	$\bar{1}$.42552	51	0.57448	$\bar{1}$.98511	4	5
56	$\bar{1}$.41111	48	$\bar{1}$.42603	51	0.57397	$\bar{1}$.98508	3	4
57	$\bar{1}$.41158	47	$\bar{1}$.42653	50	0.57347	$\bar{1}$.98505	3	3
58	$\bar{1}$.41205	47	$\bar{1}$.42704	51	0.57296	$\bar{1}$.98501	4	2
59	$\bar{1}$.41252	47	$\bar{1}$.42755	51	0.57245	$\bar{1}$.98498	3	1
60	$\bar{1}$.41300	48	$\bar{1}$.42805	50	0.57195	$\bar{1}$.98494	4	0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	54	53
6	5.4	5.3
7	6.3	6.2
8	7.2	7.1
9	8.1	8.0
10	9.0	8.8
20	18.0	17.7
30	27.0	26.5
40	36.0	35.3
50	45.0	44.2

	52	51
6	5.2	5.1
7	6.1	6.0
8	6.9	6.8
9	7.8	7.7
10	8.7	8.5
20	17.3	17.0
30	26.0	25.5
40	34.7	34.0
50	43.3	42.5

	50	49
6	5.0	4.9
7	5.8	5.7
8	6.7	6.5
9	7.5	7.4
10	8.3	8.2
20	16.7	16.3
30	25.0	24.5
40	33.3	32.7
50	41.7	40.8

	48	47
6	4.8	4.7
7	5.6	5.5
8	6.4	6.3
9	7.2	7.1
10	8.0	7.8
20	16.0	15.7
30	24.0	23.5
40	32.0	31.3
50	40.0	39.2

	4	3
6	0.4	0.3
7	0.5	0.4
8	0.5	0.4
9	0.6	0.5
10	0.7	0.5
20	1.3	1.0
30	2.0	1.5
40	2.7	2.0
50	3.3	2.5

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	′
0	1̄.41300	47	1̄.42805	51	0.57195	1̄.98494	3	60
1	1̄.41347	47	1̄.42856	50	0.57144	1̄.98491	3	59
2	1̄.41394	47	1̄.42906	51	0.57094	1̄.98488	4	58
3	1̄.41441	47	1̄.42957	50	0.57043	1̄.98484	3	57
4	1̄.41488	47	1̄.43007	50	0.56993	1̄.98481	4	56
5	1̄.41535	47	1̄.43057	51	0.56943	1̄.98477	3	55
6	1̄.41582	46	1̄.43108	50	0.56892	1̄.98474	3	54
7	1̄.41628	47	1̄.43158	50	0.56842	1̄.98471	4	53
8	1̄.41675	47	1̄.43208	50	0.56792	1̄.98467	3	52
9	1̄.41722	46	1̄.43258	50	0.56742	1̄.98464	4	51
10	1̄.41768	47	1̄.43308	50	0.56692	1̄.98460	3	50
11	1̄.41815	46	1̄.43358	50	0.56642	1̄.98457	4	49
12	1̄.41861	47	1̄.43408	50	0.56592	1̄.98453	3	48
13	1̄.41908	46	1̄.43458	50	0.56542	1̄.98450	3	47
14	1̄.41954	47	1̄.43508	50	0.56492	1̄.98447	4	46
15	1̄.42001	46	1̄.43558	49	0.56442	1̄.98443	3	45
16	1̄.42047	46	1̄.43607	50	0.56393	1̄.98440	4	44
17	1̄.42093	47	1̄.43657	50	0.56343	1̄.98436	3	43
18	1̄.42140	46	1̄.43707	49	0.56293	1̄.98433	4	42
19	1̄.42186	46	1̄.43756	50	0.56244	1̄.98429	3	41
20	1̄.42232	46	1̄.43806	49	0.56194	1̄.98426	4	40
21	1̄.42278	46	1̄.43855	50	0.56145	1̄.98422	3	39
22	1̄.42324	46	1̄.43905	49	0.56095	1̄.98419	4	38
23	1̄.42370	46	1̄.43954	50	0.56046	1̄.98415	3	37
24	1̄.42416	45	1̄.44004	49	0.55996	1̄.98412	3	36
25	1̄.42461	46	1̄.44053	49	0.55947	1̄.98409	4	35
26	1̄.42507	46	1̄.44102	49	0.55898	1̄.98405	3	34
27	1̄.42553	46	1̄.44151	50	0.55849	1̄.98402	4	33
28	1̄.42599	45	1̄.44201	49	0.55799	1̄.98398	3	32
29	1̄.42644	46	1̄.44250	49	0.55750	1̄.98395	4	31
30	1̄.42690	45	1̄.44299	49	0.55701	1̄.98391	3	30
31	1̄.42735	46	1̄.44348	49	0.55652	1̄.98388	4	29
32	1̄.42781	45	1̄.44397	49	0.55603	1̄.98384	3	28
33	1̄.42826	46	1̄.44446	49	0.55554	1̄.98381	4	27
34	1̄.42872	45	1̄.44495	49	0.55505	1̄.98377	4	26
35	1̄.42917	45	1̄.44544	48	0.55456	1̄.98373	3	25
36	1̄.42962	46	1̄.44592	49	0.55408	1̄.98370	4	24
37	1̄.43008	45	1̄.44641	49	0.55359	1̄.98366	3	23
38	1̄.43053	45	1̄.44690	48	0.55310	1̄.98363	4	22
39	1̄.43098	45	1̄.44738	49	0.55262	1̄.98359	3	21
40	1̄.43143	45	1̄.44787	49	0.55213	1̄.98356	4	20
41	1̄.43188	45	1̄.44836	48	0.55164	1̄.98352	3	19
42	1̄.43233	45	1̄.44884	49	0.55116	1̄.98349	4	18
43	1̄.43278	45	1̄.44933	48	0.55067	1̄.98345	3	17
44	1̄.43323	44	1̄.44981	48	0.55019	1̄.98342	4	16
45	1̄.43367	45	1̄.45029	49	0.54971	1̄.98338	4	15
46	1̄.43412	45	1̄.45078	48	0.54922	1̄.98334	3	14
47	1̄.43457	45	1̄.45126	48	0.54874	1̄.98331	4	13
48	1̄.43502	44	1̄.45174	48	0.54826	1̄.98327	3	12
49	1̄.43546	45	1̄.45222	49	0.54778	1̄.98324	4	11
50	1̄.43591	44	1̄.45271	48	0.54729	1̄.98320	3	10
51	1̄.43635	45	1̄.45319	48	0.54681	1̄.98317	4	9
52	1̄.43680	44	1̄.45367	48	0.54633	1̄.98313	4	8
53	1̄.43724	45	1̄.45415	48	0.54585	1̄.98309	3	7
54	1̄.43769	44	1̄.45463	48	0.54537	1̄.98306	4	6
55	1̄.43813	44	1̄.45511	48	0.54489	1̄.98302	3	5
56	1̄.43857	44	1̄.45559	47	0.54441	1̄.98299	4	4
57	1̄.43901	45	1̄.45606	48	0.54394	1̄.98295	4	3
58	1̄.43946	44	1̄.45654	48	0.54346	1̄.98291	3	2
59	1̄.43990	44	1̄.45702	48	0.54298	1̄.98288	4	1
60	1̄.44034		1̄.45750		0.54250	1̄.98284		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

P. P.

	51	50	49	48	47	46	45	44	4	3
6	5.1	5.0	4.9	4.8	4.7	4.6	4.5	4.4	0.4	0.3
7	6.0	5.8	5.7	5.6	5.5	5.4	5.3	5.1	0.5	0.4
8	6.8	6.7	6.5	6.4	6.3	6.1	6.0	5.9	0.5	0.4
9	7.7	7.5	7.4	7.2	7.1	6.9	6.8	6.6	0.6	0.5
10	8.5	8.3	8.2	8.0	7.8	7.7	7.5	7.3	0.7	0.5
20	17.0	16.7	16.3	16.0	15.7	15.3	15.0	14.7	1.3	1.0
30	25.5	25.0	24.5	24.0	23.5	23.0	22.5	22.0	2.0	1.5
40	34.0	33.3	32.7	32.0	31.3	30.7	30.0	29.3	2.7	2.0
50	42.5	41.7	40.8	40.0	39.2	38.3	37.5	36.7	3.3	2.5

′	log sin	d	log tan	c. d.	log cot	log cos	d	′
0	$\bar{1}.44034$	44	$\bar{1}.45750$	47	0.54250	$\bar{1}.98284$	3	60
1	$\bar{1}.44078$	44	$\bar{1}.45797$	48	0.54203	$\bar{1}.98281$	4	59
2	$\bar{1}.44122$	44	$\bar{1}.45845$	47	0.54155	$\bar{1}.98277$	4	58
3	$\bar{1}.44166$	44	$\bar{1}.45892$	48	0.54108	$\bar{1}.98273$	3	57
4	$\bar{1}.44210$	43	$\bar{1}.45940$	47	0.54060	$\bar{1}.98270$	4	56
5	$\bar{1}.44253$	44	$\bar{1}.45987$	48	0.54013	$\bar{1}.98266$	4	55
6	$\bar{1}.44297$	44	$\bar{1}.46035$	47	0.53965	$\bar{1}.98262$	3	54
7	$\bar{1}.44341$	44	$\bar{1}.46082$	48	0.53918	$\bar{1}.98259$	4	53
8	$\bar{1}.44385$	43	$\bar{1}.46130$	47	0.53870	$\bar{1}.98255$	4	52
9	$\bar{1}.44428$	44	$\bar{1}.46177$	47	0.53823	$\bar{1}.98251$	3	51
10	$\bar{1}.44472$	44	$\bar{1}.46224$	47	0.53776	$\bar{1}.98248$	4	50
11	$\bar{1}.44516$	43	$\bar{1}.46271$	48	0.53729	$\bar{1}.98244$	4	49
12	$\bar{1}.44559$	43	$\bar{1}.46319$	47	0.53681	$\bar{1}.98240$	3	48
13	$\bar{1}.44602$	44	$\bar{1}.46366$	47	0.53634	$\bar{1}.98237$	4	47
14	$\bar{1}.44646$	43	$\bar{1}.46413$	47	0.53587	$\bar{1}.98233$	4	46
15	$\bar{1}.44689$	44	$\bar{1}.46460$	47	0.53540	$\bar{1}.98229$	3	45
16	$\bar{1}.44733$	43	$\bar{1}.46507$	47	0.53493	$\bar{1}.98226$	4	44
17	$\bar{1}.44776$	43	$\bar{1}.46554$	47	0.53446	$\bar{1}.98222$	4	43
18	$\bar{1}.44819$	43	$\bar{1}.46601$	47	0.53399	$\bar{1}.98218$	3	42
19	$\bar{1}.44862$	43	$\bar{1}.46648$	46	0.53352	$\bar{1}.98215$	4	41
20	$\bar{1}.44905$	43	$\bar{1}.46694$	47	0.53306	$\bar{1}.98211$	4	40
21	$\bar{1}.44948$	44	$\bar{1}.46741$	47	0.53259	$\bar{1}.98207$	3	39
22	$\bar{1}.44992$	43	$\bar{1}.46788$	47	0.53212	$\bar{1}.98204$	4	38
23	$\bar{1}.45035$	42	$\bar{1}.46835$	46	0.53165	$\bar{1}.98200$	4	37
24	$\bar{1}.45077$	43	$\bar{1}.46881$	47	0.53119	$\bar{1}.98196$	4	36
25	$\bar{1}.45120$	43	$\bar{1}.46928$	47	0.53072	$\bar{1}.98192$	3	35
26	$\bar{1}.45163$	43	$\bar{1}.46975$	46	0.53025	$\bar{1}.98189$	4	34
27	$\bar{1}.45206$	43	$\bar{1}.47021$	47	0.52979	$\bar{1}.98185$	4	33
28	$\bar{1}.45249$	43	$\bar{1}.47068$	46	0.52932	$\bar{1}.98181$	4	32
29	$\bar{1}.45292$	42	$\bar{1}.47114$	46	0.52886	$\bar{1}.98177$	3	31
30	$\bar{1}.45334$	43	$\bar{1}.47160$	47	0.52840	$\bar{1}.98174$	4	30
31	$\bar{1}.45377$	42	$\bar{1}.47207$	46	0.52793	$\bar{1}.98170$	4	29
32	$\bar{1}.45419$	43	$\bar{1}.47253$	46	0.52747	$\bar{1}.98166$	4	28
33	$\bar{1}.45462$	42	$\bar{1}.47299$	47	0.52701	$\bar{1}.98162$	3	27
34	$\bar{1}.45504$	43	$\bar{1}.47346$	46	0.52654	$\bar{1}.98159$	4	26
35	$\bar{1}.45547$	42	$\bar{1}.47392$	46	0.52608	$\bar{1}.98155$	4	25
36	$\bar{1}.45589$	43	$\bar{1}.47438$	46	0.52562	$\bar{1}.98151$	4	24
37	$\bar{1}.45632$	42	$\bar{1}.47484$	46	0.52516	$\bar{1}.98147$	3	23
38	$\bar{1}.45674$	42	$\bar{1}.47530$	46	0.52470	$\bar{1}.98144$	4	22
39	$\bar{1}.45716$	42	$\bar{1}.47576$	46	0.52424	$\bar{1}.98140$	4	21
40	$\bar{1}.45758$	43	$\bar{1}.47622$	46	0.52378	$\bar{1}.98136$	4	20
41	$\bar{1}.45801$	42	$\bar{1}.47668$	46	0.52332	$\bar{1}.98132$	3	19
42	$\bar{1}.45843$	42	$\bar{1}.47714$	46	0.52286	$\bar{1}.98129$	4	18
43	$\bar{1}.45885$	42	$\bar{1}.47760$	46	0.52240	$\bar{1}.98125$	4	17
44	$\bar{1}.45927$	42	$\bar{1}.47806$	46	0.52194	$\bar{1}.98121$	4	16
45	$\bar{1}.45969$	42	$\bar{1}.47852$	45	0.52148	$\bar{1}.98117$	4	15
46	$\bar{1}.46011$	42	$\bar{1}.47897$	46	0.52103	$\bar{1}.98113$	3	14
47	$\bar{1}.46053$	42	$\bar{1}.47943$	46	0.52057	$\bar{1}.98110$	4	13
48	$\bar{1}.46095$	41	$\bar{1}.47989$	46	0.52011	$\bar{1}.98106$	4	12
49	$\bar{1}.46136$	42	$\bar{1}.48035$	45	0.51965	$\bar{1}.98102$	4	11
50	$\bar{1}.46178$	42	$\bar{1}.48080$	46	0.51920	$\bar{1}.98098$	4	10
51	$\bar{1}.46220$	42	$\bar{1}.48126$	45	0.51874	$\bar{1}.98094$	4	9
52	$\bar{1}.46262$	41	$\bar{1}.48171$	46	0.51829	$\bar{1}.98090$	3	8
53	$\bar{1}.46303$	42	$\bar{1}.48217$	45	0.51783	$\bar{1}.98087$	4	7
54	$\bar{1}.46345$	41	$\bar{1}.48262$	45	0.51738	$\bar{1}.98083$	4	6
55	$\bar{1}.46386$	42	$\bar{1}.48307$	46	0.51693	$\bar{1}.98079$	4	5
56	$\bar{1}.46428$	41	$\bar{1}.48353$	45	0.51647	$\bar{1}.98075$	4	4
57	$\bar{1}.46469$	42	$\bar{1}.48398$	45	0.51602	$\bar{1}.98071$	4	3
58	$\bar{1}.46511$	41	$\bar{1}.48443$	46	0.51557	$\bar{1}.98067$	4	2
59	$\bar{1}.46552$	42	$\bar{1}.48489$	45	0.51511	$\bar{1}.98063$	3	1
60	$\bar{1}.46594$		$\bar{1}.48534$		0.51466	$\bar{1}.98060$		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	48	47	46	45	44	43	42	41	4	3
6	4.8	4.7	4.6	4.5	4.4	4.3	4.2	4.1	0.4	0.3
7	5.6	5.5	5.4	5.3	5.1	5.0	4.9	4.8	0.5	0.4
8	6.4	6.3	6.1	6.0	5.9	5.7	5.6	5.5	0.5	0.4
9	7.2	7.1	6.9	6.8	6.6	6.5	6.3	6.2	0.6	0.5
10	8.0	7.8	7.7	7.5	7.3	7.2	7.0	6.8	0.7	0.5
20	16.0	15.7	15.3	15.0	14.7	14.3	14.0	13.7	1.3	1.0
30	24.0	23.5	23.0	22.5	22.0	21.5	21.0	20.5	2.0	1.5
40	32.0	31.3	30.7	30.0	29.3	28.7	28.0	27.3	2.7	2.0
50	40.0	39.2	38.3	37.5	36.7	35.8	35.0	34.2	3.3	2.5

′	log sin	d	log tan	c. d.	log cot	log cos	d	′
0	1̄.46594	41	1̄.48534	45	0.51466	1̄.98060	4	60
1	1̄.46635	41	1̄.48579	45	0.51421	1̄.98056	4	59
2	1̄.46676	41	1̄.48624	45	0.51376	1̄.98052	4	58
3	1̄.46717	41	1̄.48669	45	0.51331	1̄.98048	4	57
4	1̄.46758	42	1̄.48714	45	0.51286	1̄.98044	4	56
5	1̄.46800	41	1̄.48759	45	0.51241	1̄.98040	4	55
6	1̄.46841	41	1̄.48804	45	0.51196	1̄.98036	4	54
7	1̄.46882	41	1̄.48849	45	0.51151	1̄.98032	3	53
8	1̄.46923	41	1̄.48894	45	0.51106	1̄.98029	4	52
9	1̄.46964	41	1̄.48939	45	0.51061	1̄.98025	4	51
10	1̄.47005	40	1̄.48984	45	0.51016	1̄.98021	4	50
11	1̄.47045	41	1̄.49029	44	0.50971	1̄.98017	4	49
12	1̄.47086	41	1̄.49073	45	0.50927	1̄.98013	4	48
13	1̄.47127	41	1̄.49118	45	0.50882	1̄.98009	4	47
14	1̄.47168	41	1̄.49163	44	0.50837	1̄.98005	4	46
15	1̄.47209	40	1̄.49207	45	0.50793	1̄.98001	4	45
16	1̄.47249	41	1̄.49252	44	0.50748	1̄.97997	4	44
17	1̄.47290	40	1̄.49296	45	0.50704	1̄.97993	4	43
18	1̄.47330	41	1̄.49341	44	0.50659	1̄.97989	3	42
19	1̄.47371	40	1̄.49385	45	0.50615	1̄.97986	4	41
20	1̄.47411	41	1̄.49430	44	0.50570	1̄.97982	4	40
21	1̄.47452	40	1̄.49474	45	0.50526	1̄.97978	4	39
22	1̄.47492	41	1̄.49519	44	0.50481	1̄.97974	4	38
23	1̄.47533	40	1̄.49563	44	0.50437	1̄.97970	4	37
24	1̄.47573	40	1̄.49607	45	0.50393	1̄.97966	4	36
25	1̄.47613	41	1̄.49652	44	0.50348	1̄.97962	4	35
26	1̄.47654	40	1̄.49696	44	0.50304	1̄.97958	4	34
27	1̄.47694	40	1̄.49740	44	0.50260	1̄.97954	4	33
28	1̄.47734	40	1̄.49784	44	0.50216	1̄.97950	4	32
29	1̄.47774	40	1̄.49828	44	0.50172	1̄.97946	4	31
30	1̄.47814	40	1̄.49872	44	0.50128	1̄.97942	4	30
31	1̄.47854	40	1̄.49916	44	0.50084	1̄.97938	4	29
32	1̄.47894	40	1̄.49960	44	0.50040	1̄.97934	4	28
33	1̄.47934	40	1̄.50004	44	0.49996	1̄.97930	4	27
34	1̄.47974	40	1̄.50048	44	0.49952	1̄.97926	4	26
35	1̄.48014	40	1̄.50092	44	0.49908	1̄.97922	4	25
36	1̄.48054	40	1̄.50136	44	0.49864	1̄.97918	4	24
37	1̄.48094	39	1̄.50180	43	0.49820	1̄.97914	4	23
38	1̄.48133	40	1̄.50223	44	0.49777	1̄.97910	4	22
39	1̄.48173	40	1̄.50267	44	0.49733	1̄.97906	4	21
40	1̄.48213	39	1̄.50311	44	0.49689	1̄.97902	4	20
41	1̄.48252	40	1̄.50355	43	0.49645	1̄.97898	4	19
42	1̄.48292	40	1̄.50398	44	0.49602	1̄.97894	4	18
43	1̄.48332	39	1̄.50442	43	0.49558	1̄.97890	4	17
44	1̄.48371	40	1̄.50485	44	0.49515	1̄.97886	4	16
45	1̄.48411	39	1̄.50529	43	0.49471	1̄.97882	4	15
46	1̄.48450	40	1̄.50572	44	0.49428	1̄.97878	4	14
47	1̄.48490	39	1̄.50616	43	0.49384	1̄.97874	4	13
48	1̄.48529	39	1̄.50659	44	0.49341	1̄.97870	4	12
49	1̄.48568	39	1̄.50703	43	0.49297	1̄.97866	5	11
50	1̄.48607	40	1̄.50746	43	0.49254	1̄.97861	4	10
51	1̄.48647	39	1̄.50789	44	0.49211	1̄.97857	4	9
52	1̄.48686	39	1̄.50833	43	0.49167	1̄.97853	4	8
53	1̄.48725	39	1̄.50876	43	0.49124	1̄.97849	4	7
54	1̄.48764	39	1̄.50919	43	0.49081	1̄.97845	4	6
55	1̄.48803	39	1̄.50962	43	0.49038	1̄.97841	4	5
56	1̄.48842	39	1̄.51005	43	0.48995	1̄.97837	4	4
57	1̄.48881	39	1̄.51048	44	0.48952	1̄.97833	4	3
58	1̄.48920	39	1̄.51092	43	0.48908	1̄.97829	4	2
59	1̄.48959	39	1̄.51135	43	0.48865	1̄.97825	4	1
60	1̄.48998		1̄.51178		0.48822	1̄.97821		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	45	44
6	4.5	4.4
7	5.3	5.1
8	6.0	5.9
9	6.8	6.6
10	7.5	7.3
20	15.0	14.7
30	22.5	22.0
40	30.0	29.3
50	37.5	36.7

	43
6	4.3
7	5.0
8	5.7
9	6.5
10	7.2
20	14.3
30	21.5
40	28.7
50	35.8

	42	41
6	4.2	4.1
7	4.9	4.8
8	5.6	5.5
9	6.3	6.2
10	7.0	6.8
20	14.0	13.7
30	21.0	20.5
40	28.0	27.3
50	35.0	34.2

	40	39
6	4.0	3.9
7	4.7	4.6
8	5.3	5.2
9	6.0	5.9
10	6.7	6.5
20	13.3	13.0
30	20.0	19.5
40	26.7	26.0
50	33.3	32.5

	5	4	3
6	0.5	0.4	0.3
7	0.6	0.5	0.4
8	0.7	0.5	0.4
9	0.8	0.6	0.5
10	0.8	0.7	0.5
20	1.7	1.3	1.0
30	2.5	2.0	1.5
40	3.3	2.7	2.0
50	4.2	3.3	2.5

18°

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	$\bar{1}$.48998	39	$\bar{1}$.51178	43	0.48822	$\bar{1}$.97821	4	60
1	$\bar{1}$.49037	39	$\bar{1}$.51221	43	0.48779	$\bar{1}$.97817	5	59
2	$\bar{1}$.49076	39	$\bar{1}$.51264	42	0.48736	$\bar{1}$.97812	4	58
3	$\bar{1}$.49115	38	$\bar{1}$.51306	43	0.48694	$\bar{1}$.97808	4	57
4	$\bar{1}$.49153	39	$\bar{1}$.51349	43	0.48651	$\bar{1}$.97804	4	56
5	$\bar{1}$.49192	39	$\bar{1}$.51392	43	0.48608	$\bar{1}$.97800	4	55
6	$\bar{1}$.49231	38	$\bar{1}$.51435	43	0.48565	$\bar{1}$.97796	4	54
7	$\bar{1}$.49269	39	$\bar{1}$.51478	42	0.48522	$\bar{1}$.97792	4	53
8	$\bar{1}$.49308	39	$\bar{1}$.51520	43	0.48480	$\bar{1}$.97788	4	52
9	$\bar{1}$.49347	38	$\bar{1}$.51563	43	0.48437	$\bar{1}$.97784	5	51
10	$\bar{1}$.49385	39	$\bar{1}$.51606	42	0.48394	$\bar{1}$.97779	4	50
11	$\bar{1}$.49424	38	$\bar{1}$.51648	43	0.48352	$\bar{1}$.97775	4	49
12	$\bar{1}$.49462	38	$\bar{1}$.51691	43	0.48309	$\bar{1}$.97771	4	48
13	$\bar{1}$.49500	39	$\bar{1}$.51734	42	0.48266	$\bar{1}$.97767	4	47
14	$\bar{1}$.49539	38	$\bar{1}$.51776	43	0.48224	$\bar{1}$.97763	4	46
15	$\bar{1}$.49577	38	$\bar{1}$.51819	42	0.48181	$\bar{1}$.97759	5	45
16	$\bar{1}$.49615	39	$\bar{1}$.51861	42	0.48139	$\bar{1}$.97754	4	44
17	$\bar{1}$.49654	38	$\bar{1}$.51903	43	0.48097	$\bar{1}$.97750	4	43
18	$\bar{1}$.49692	38	$\bar{1}$.51946	42	0.48054	$\bar{1}$.97746	4	42
19	$\bar{1}$.49730	38	$\bar{1}$.51988	43	0.48012	$\bar{1}$.97742	4	41
20	$\bar{1}$.49768	38	$\bar{1}$.52031	42	0.47969	$\bar{1}$.97738	4	40
21	$\bar{1}$.49806	38	$\bar{1}$.52073	42	0.47927	$\bar{1}$.97734	5	39
22	$\bar{1}$.49844	38	$\bar{1}$.52115	42	0.47885	$\bar{1}$.97729	4	38
23	$\bar{1}$.49882	38	$\bar{1}$.52157	43	0.47843	$\bar{1}$.97725	4	37
24	$\bar{1}$.49920	38	$\bar{1}$.52200	42	0.47800	$\bar{1}$.97721	4	36
25	$\bar{1}$.49958	38	$\bar{1}$.52242	42	0.47758	$\bar{1}$.97717	4	35
26	$\bar{1}$.49996	38	$\bar{1}$.52284	42	0.47716	$\bar{1}$.97713	5	34
27	$\bar{1}$.50034	38	$\bar{1}$.52326	42	0.47674	$\bar{1}$.97708	4	33
28	$\bar{1}$.50072	38	$\bar{1}$.52368	42	0.47632	$\bar{1}$.97704	4	32
29	$\bar{1}$.50110	38	$\bar{1}$.52410	42	0.47590	$\bar{1}$.97700	4	31
30	$\bar{1}$.50148	37	$\bar{1}$.52452	42	0.47548	$\bar{1}$.97696	5	30
31	$\bar{1}$.50185	38	$\bar{1}$.52494	42	0.47506	$\bar{1}$.97691	4	29
32	$\bar{1}$.50223	38	$\bar{1}$.52536	42	0.47464	$\bar{1}$.97687	4	28
33	$\bar{1}$.50261	37	$\bar{1}$.52578	42	0.47422	$\bar{1}$.97683	4	27
34	$\bar{1}$.50298	38	$\bar{1}$.52620	41	0.47380	$\bar{1}$.97679	5	26
35	$\bar{1}$.50336	38	$\bar{1}$.52661	42	0.47339	$\bar{1}$.97674	4	25
36	$\bar{1}$.50374	37	$\bar{1}$.52703	42	0.47297	$\bar{1}$.97670	4	24
37	$\bar{1}$.50411	38	$\bar{1}$.52745	42	0.47255	$\bar{1}$.97666	4	23
38	$\bar{1}$.50449	37	$\bar{1}$.52787	42	0.47213	$\bar{1}$.97662	5	22
39	$\bar{1}$.50486	37	$\bar{1}$.52829	41	0.47171	$\bar{1}$.97657	4	21
40	$\bar{1}$.50523	38	$\bar{1}$.52870	42	0.47130	$\bar{1}$.97653	4	20
41	$\bar{1}$.50561	37	$\bar{1}$.52912	41	0.47088	$\bar{1}$.97649	4	19
42	$\bar{1}$.50598	37	$\bar{1}$.52953	42	0.47047	$\bar{1}$.97645	5	18
43	$\bar{1}$.50635	38	$\bar{1}$.52995	42	0.47005	$\bar{1}$.97640	4	17
44	$\bar{1}$.50673	37	$\bar{1}$.53037	41	0.46963	$\bar{1}$.97636	4	16
45	$\bar{1}$.50710	37	$\bar{1}$.53078	42	0.46922	$\bar{1}$.97632	4	15
46	$\bar{1}$.50747	37	$\bar{1}$.53120	41	0.46880	$\bar{1}$.97628	5	14
47	$\bar{1}$.50784	37	$\bar{1}$.53161	41	0.46839	$\bar{1}$.97623	4	13
48	$\bar{1}$.50821	37	$\bar{1}$.53202	42	0.46798	$\bar{1}$.97619	4	12
49	$\bar{1}$.50858	38	$\bar{1}$.53244	41	0.46756	$\bar{1}$.97615	5	11
50	$\bar{1}$.50896	37	$\bar{1}$.53285	42	0.46715	$\bar{1}$.97610	4	10
51	$\bar{1}$.50933	37	$\bar{1}$.53327	41	0.46673	$\bar{1}$.97606	4	9
52	$\bar{1}$.50970	37	$\bar{1}$.53368	41	0.46632	$\bar{1}$.97602	5	8
53	$\bar{1}$.51007	36	$\bar{1}$.53409	41	0.46591	$\bar{1}$.97597	4	7
54	$\bar{1}$.51043	37	$\bar{1}$.53450	42	0.46550	$\bar{1}$.97593	4	6
55	$\bar{1}$.51080	37	$\bar{1}$.53492	41	0.46508	$\bar{1}$.97589	5	5
56	$\bar{1}$.51117	37	$\bar{1}$.53533	41	0.46467	$\bar{1}$.97584	4	4
57	$\bar{1}$.51154	37	$\bar{1}$.53574	41	0.46426	$\bar{1}$.97580	4	3
58	$\bar{1}$.51191	36	$\bar{1}$.53615	41	0.46385	$\bar{1}$.97576	5	2
59	$\bar{1}$.51227	37	$\bar{1}$.53656	41	0.46344	$\bar{1}$.97571	4	1
60	$\bar{1}$.51264		$\bar{1}$.53697		0.46303	$\bar{1}$.97567		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	43	42
6	4.3	4.2
7	5.0	4.9
8	5.7	5.6
9	6.5	6.3
10	7.2	7.0
20	14.3	14.0
30	21.5	21.0
40	28.7	28.0
50	35.8	35.0

	41
6	4.1
7	4.8
8	5.5
9	6.2
10	6.8
20	13.7
30	20.5
40	27.3
50	34.2

	39	38
6	3.9	3.8
7	4.6	4.4
8	5.2	5.1
9	5.9	5.7
10	6.5	6.3
20	13.0	12.7
30	19.5	19.0
40	26.0	25.3
50	32.5	31.7

	37	36
6	3.7	3.6
7	4.3	4.2
8	4.9	4.8
9	5.6	5.4
10	6.2	6.0
20	12.3	12.0
30	18.5	18.0
40	24.7	24.0
50	30.8	30.0

	5	4
6	0.5	0.4
7	0.6	0.5
8	0.7	0.5
9	0.8	0.6
10	0.8	0.7
20	1.7	1.3
30	2.5	2.0
40	3.3	2.7
50	4.2	3.3

p. p.

71°

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.51264	37	1̄.53697	41	0.46303	1̄.97567	4	60
1	1̄.51301	37	1̄.53738	41	0.46262	1̄.97563	5	59
2	1̄.51338	36	1̄.53779	41	0.46221	1̄.97558	4	58
3	1̄.51374	37	1̄.53820	41	0.46180	1̄.97554	4	57
4	1̄.51411	36	1̄.53861	41	0.46139	1̄.97550	5	56
5	1̄.51447	37	1̄.53902	41	0.46098	1̄.97545	4	55
6	1̄.51484	36	1̄.53943	41	0.46057	1̄.97541	5	54
7	1̄.51520	37	1̄.53984	41	0.46016	1̄.97536	4	53
8	1̄.51557	36	1̄.54025	40	0.45975	1̄.97532	4	52
9	1̄.51593	36	1̄.54065	41	0.45935	1̄.97528	5	51
10	1̄.51629	37	1̄.54106	41	0.45894	1̄.97523	4	50
11	1̄.51666	36	1̄.54147	40	0.45853	1̄.97519	4	49
12	1̄.51702	36	1̄.54187	41	0.45813	1̄.97515	5	48
13	1̄.51738	36	1̄.54228	41	0.45772	1̄.97510	4	47
14	1̄.51774	37	1̄.54269	40	0.45731	1̄.97506	5	46
15	1̄.51811	36	1̄.54309	41	0.45691	1̄.97501	4	45
16	1̄.51847	36	1̄.54350	40	0.45650	1̄.97497	5	44
17	1̄.51883	36	1̄.54390	41	0.45610	1̄.97492	4	43
18	1̄.51919	36	1̄.54431	40	0.45569	1̄.97488	4	42
19	1̄.51955	36	1̄.54471	41	0.45529	1̄.97484	5	41
20	1̄.51991	36	1̄.54512	40	0.45488	1̄.97479	4	40
21	1̄.52027	36	1̄.54552	41	0.45448	1̄.97475	5	39
22	1̄.52063	36	1̄.54593	40	0.45407	1̄.97470	4	38
23	1̄.52099	36	1̄.54633	40	0.45367	1̄.97466	5	37
24	1̄.52135	36	1̄.54673	41	0.45327	1̄.97461	4	36
25	1̄.52171	36	1̄.54714	40	0.45286	1̄.97457	4	35
26	1̄.52207	35	1̄.54754	40	0.45246	1̄.97453	5	34
27	1̄.52242	36	1̄.54794	41	0.45206	1̄.97448	4	33
28	1̄.52278	36	1̄.54835	40	0.45165	1̄.97444	5	32
29	1̄.52314	36	1̄.54875	40	0.45125	1̄.97439	4	31
30	1̄.52350	35	1̄.54915	40	0.45085	1̄.97435	5	30
31	1̄.52385	36	1̄.54955	40	0.45045	1̄.97430	4	29
32	1̄.52421	35	1̄.54995	40	0.45005	1̄.97426	5	28
33	1̄.52456	36	1̄.55035	40	0.44965	1̄.97421	4	27
34	1̄.52492	35	1̄.55075	40	0.44925	1̄.97417	5	26
35	1̄.52527	36	1̄.55115	40	0.44885	1̄.97412	4	25
36	1̄.52563	35	1̄.55155	40	0.44845	1̄.97408	5	24
37	1̄.52598	36	1̄.55195	40	0.44805	1̄.97403	4	23
38	1̄.52634	35	1̄.55235	40	0.44765	1̄.97399	5	22
39	1̄.52669	36	1̄.55275	40	0.44725	1̄.97394	4	21
40	1̄.52705	35	1̄.55315	40	0.44685	1̄.97390	5	20
41	1̄.52740	35	1̄.55355	40	0.44645	1̄.97385	4	19
42	1̄.52775	36	1̄.55395	39	0.44605	1̄.97381	5	18
43	1̄.52811	35	1̄.55434	40	0.44566	1̄.97376	4	17
44	1̄.52846	35	1̄.55474	40	0.44526	1̄.97372	5	16
45	1̄.52881	35	1̄.55514	40	0.44486	1̄.97367	4	15
46	1̄.52916	35	1̄.55554	39	0.44446	1̄.97363	5	14
47	1̄.52951	35	1̄.55593	40	0.44407	1̄.97358	5	13
48	1̄.52986	35	1̄.55633	40	0.44367	1̄.97353	4	12
49	1̄.53021	35	1̄.55673	39	0.44327	1̄.97349	5	11
50	1̄.53056	36	1̄.55712	40	0.44288	1̄.97344	4	10
51	1̄.53092	34	1̄.55752	39	0.44248	1̄.97340	5	9
52	1̄.53126	35	1̄.55791	40	0.44209	1̄.97335	4	8
53	1̄.53161	35	1̄.55831	39	0.44169	1̄.97331	5	7
54	1̄.53196	35	1̄.55870	40	0.44130	1̄.97326	4	6
55	1̄.53231	35	1̄.55910	39	0.44090	1̄.97322	5	5
56	1̄.53266	35	1̄.55949	40	0.44051	1̄.97317	5	4
57	1̄.53301	35	1̄.55989	39	0.44011	1̄.97312	4	3
58	1̄.53336	34	1̄.56028	39	0.43972	1̄.97308	5	2
59	1̄.53370	35	1̄.56067	40	0.43933	1̄.97303	4	1
60	1̄.53405		1̄.56107		0.43893	1̄.97299		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	41	40
6	4.1	4.0
7	4.8	4.7
8	5.5	5.3
9	6.2	6.0
10	6.8	6.7
20	13.7	13.3
30	20.5	20.0
40	27.3	26.7
50	34.2	33.3

	39
6	3.9
7	4.6
8	5.2
9	5.9
10	6.5
20	13.0
30	19.5
40	26.0
50	32.5

	37	36
6	3.7	3.6
7	4.3	4.2
8	4.9	4.8
9	5.6	5.4
10	6.2	6.0
20	12.3	12.0
30	18.5	18.0
40	24.7	24.0
50	30.8	30.0

	35	34
6	3.5	3.4
7	4.1	4.0
8	4.7	4.5
9	5.3	5.1
10	5.8	5.7
20	11.7	11.3
30	17.5	17.0
40	23.3	22.7
50	29.2	28.3

	5	4
6	0.5	0.4
7	0.6	0.5
8	0.7	0.5
9	0.8	0.6
10	0.8	0.7
20	1.7	1.3
30	2.5	2.0
40	3.3	2.7
50	4.2	3.3

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.53405	35	1̄.56107	39	0.43893	1̄.97299	5	60
1	1̄.53440	35	1̄.56146	39	0.43854	1̄.97294	5	59
2	1̄.53475	34	1̄.56185	39	0.43815	1̄.97289	4	58
3	1̄.53509	35	1̄.56224	40	0.43776	1̄.97285	5	57
4	1̄.53544	34	1̄.56264	39	0.43736	1̄.97280	4	56
5	1̄.53578	35	1̄.56303	39	0.43697	1̄.97276	5	55
6	1̄.53613	34	1̄.56342	39	0.43658	1̄.97271	5	54
7	1̄.53647	35	1̄.56381	39	0.43619	1̄.97266	4	53
8	1̄.53682	34	1̄.56420	39	0.43580	1̄.97262	5	52
9	1̄.53716	35	1̄.56459	39	0.43541	1̄.97257	5	51
10	1̄.53751	34	1̄.56498	39	0.43502	1̄.97252	4	50
11	1̄.53785	34	1̄.56537	39	0.43463	1̄.97248	5	49
12	1̄.53819	35	1̄.56576	39	0.43424	1̄.97243	5	48
13	1̄.53854	34	1̄.56615	39	0.43385	1̄.97238	4	47
14	1̄.53888	34	1̄.56654	39	0.43346	1̄.97234	5	46
15	1̄.53922	35	1̄.56693	39	0.43307	1̄.97229	5	45
16	1̄.53957	34	1̄.56732	39	0.43268	1̄.97224	4	44
17	1̄.53991	34	1̄.56771	39	0.43229	1̄.97220	5	43
18	1̄.54025	34	1̄.56810	39	0.43190	1̄.97215	5	42
19	1̄.54059	34	1̄.56849	38	0.43151	1̄.97210	4	41
20	1̄.54093	34	1̄.56887	39	0.43113	1̄.97206	5	40
21	1̄.54127	34	1̄.56926	39	0.43074	1̄.97201	5	39
22	1̄.54161	34	1̄.56965	39	0.43035	1̄.97196	4	38
23	1̄.54195	34	1̄.57004	38	0.42996	1̄.97192	5	37
24	1̄.54229	34	1̄.57042	39	0.42958	1̄.97187	5	36
25	1̄.54263	34	1̄.57081	39	0.42919	1̄.97182	4	35
26	1̄.54297	34	1̄.57120	38	0.42880	1̄.97178	5	34
27	1̄.54331	34	1̄.57158	39	0.42842	1̄.97173	5	33
28	1̄.54365	34	1̄.57197	38	0.42803	1̄.97168	5	32
29	1̄.54399	34	1̄.57235	39	0.42765	1̄.97163	4	31
30	1̄.54433	33	1̄.57274	38	0.42726	1̄.97159	5	30
31	1̄.54466	34	1̄.57312	39	0.42688	1̄.97154	5	29
32	1̄.54500	34	1̄.57351	38	0.42649	1̄.97149	4	28
33	1̄.54534	33	1̄.57389	39	0.42611	1̄.97145	5	27
34	1̄.54567	34	1̄.57428	38	0.42572	1̄.97140	5	26
35	1̄.54601	34	1̄.57466	38	0.42534	1̄.97135	5	25
36	1̄.54635	33	1̄.57504	39	0.42496	1̄.97130	4	24
37	1̄.54668	34	1̄.57543	38	0.42457	1̄.97126	5	23
38	1̄.54702	33	1̄.57581	38	0.42419	1̄.97121	5	22
39	1̄.54735	34	1̄.57619	39	0.42381	1̄.97116	5	21
40	1̄.54769	33	1̄.57658	38	0.42342	1̄.97111	4	20
41	1̄.54802	34	1̄.57696	38	0.42304	1̄.97107	5	19
42	1̄.54836	33	1̄.57734	38	0.42266	1̄.97102	5	18
43	1̄.54869	34	1̄.57772	38	0.42228	1̄.97097	5	17
44	1̄.54903	33	1̄.57810	39	0.42190	1̄.97092	5	16
45	1̄.54936	33	1̄.57849	38	0.42151	1̄.97087	4	15
46	1̄.54969	34	1̄.57887	38	0.42113	1̄.97083	5	14
47	1̄.55003	33	1̄.57925	38	0.42075	1̄.97078	5	13
48	1̄.55036	33	1̄.57963	38	0.42037	1̄.97073	5	12
49	1̄.55069	33	1̄.58001	38	0.41999	1̄.97068	5	11
50	1̄.55102	34	1̄.58039	38	0.41961	1̄.97063	4	10
51	1̄.55136	33	1̄.58077	38	0.41923	1̄.97059	5	9
52	1̄.55169	33	1̄.58115	38	0.41885	1̄.97054	5	8
53	1̄.55202	33	1̄.58153	38	0.41847	1̄.97049	5	7
54	1̄.55235	33	1̄.58191	38	0.41809	1̄.97044	5	6
55	1̄.55268	33	1̄.58229	38	0.41771	1̄.97039	4	5
56	1̄.55301	33	1̄.58267	37	0.41733	1̄.97035	5	4
57	1̄.55334	33	1̄.58304	38	0.41696	1̄.97030	5	3
58	1̄.55367	33	1̄.58342	38	0.41658	1̄.97025	5	2
59	1̄.55400	33	1̄.58380	38	0.41620	1̄.97020	5	1
60	1̄.55433		1̄.58418		0.41582	1̄.97015		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	40	39
6	4.0	3.9
7	4.7	4.6
8	5.3	5.2
9	6.0	5.9
10	6.7	6.5
20	13.3	13.0
30	20.0	19.5
40	26.7	26.0
50	33.3	32.5

	38	37
6	3.8	3.7
7	4.4	4.3
8	5.1	4.9
9	5.7	5.6
10	6.3	6.2
20	12.7	12.3
30	19.0	18.5
40	25.3	24.7
50	31.7	30.8

	35
6	3.5
7	4.1
8	4.7
9	5.3
10	5.8
20	11.7
30	17.5
40	23.3
50	29.2

	34	33
6	3.4	3.3
7	4.0	3.9
8	4.5	4.4
9	5.1	5.0
10	5.7	5.5
20	11.3	11.0
30	17.0	16.5
40	22.7	22.0
50	28.3	27.5

	5	4
6	0.5	0.4
7	0.6	0.5
8	0.7	0.5
9	0.8	0.6
10	0.8	0.7
20	1.7	1.3
30	2.5	2.0
40	3.3	2.7
50	4.2	3.3

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.55433		1̄.58418		0.41582	1̄.97015		60
1	1̄.55466	33	1̄.58455	37	0.41545	1̄.97010	5	59
2	1̄.55499	33	1̄.58493	38	0.41507	1̄.97005	5	58
3	1̄.55532	33	1̄.58531	38	0.41469	1̄.97001	4	57
4	1̄.55564	32	1̄.58569	38	0.41431	1̄.96996	5	56
5	1̄.55597	33	1̄.58606	37	0.41394	1̄.96991	5	55
6	1̄.55630	33	1̄.58644	38	0.41356	1̄.96986	5	54
7	1̄.55663	33	1̄.58681	37	0.41319	1̄.96981	5	53
8	1̄.55695	32	1̄.58719	38	0.41281	1̄.96976	5	52
9	1̄.55728	33	1̄.58757	38	0.41243	1̄.96971	5	51
10	1̄.55761	33	1̄.58794	37	0.41206	1̄.96966	5	50
11	1̄.55793	32	1̄.58832	38	0.41168	1̄.96962	4	49
12	1̄.55826	33	1̄.58869	37	0.41131	1̄.96957	5	48
13	1̄.55858	32	1̄.58907	38	0.41093	1̄.96952	5	47
14	1̄.55891	33	1̄.58944	37	0.41056	1̄.96947	5	46
15	1̄.55923	32	1̄.58981	37	0.41019	1̄.96942	5	45
16	1̄.55956	33	1̄.59019	38	0.40981	1̄.96937	5	44
17	1̄.55988	32	1̄.59056	37	0.40944	1̄.96932	5	43
18	1̄.56021	33	1̄.59094	38	0.40906	1̄.96927	5	42
19	1̄.56053	32	1̄.59131	37	0.40869	1̄.96922	5	41
20	1̄.56085	32	1̄.59168	37	0.40832	1̄.96917	5	40
21	1̄.56118	33	1̄.59205	37	0.40795	1̄.96912	5	39
22	1̄.56150	32	1̄.59243	38	0.40757	1̄.96907	5	38
23	1̄.56182	32	1̄.59280	37	0.40720	1̄.96903	4	37
24	1̄.56215	33	1̄.59317	37	0.40683	1̄.96898	5	36
25	1̄.56247	32	1̄.59354	37	0.40646	1̄.96893	5	35
26	1̄.56279	32	1̄.59391	37	0.40609	1̄.96888	5	34
27	1̄.56311	32	1̄.59429	38	0.40571	1̄.96883	5	33
28	1̄.56343	32	1̄.59466	37	0.40534	1̄.96878	5	32
29	1̄.56375	32	1̄.59503	37	0.40497	1̄.96873	5	31
30	1̄.56408	33	1̄.59540	37	0.40460	1̄.96868	5	30
31	1̄.56440	32	1̄.59577	37	0.40423	1̄.96863	5	29
32	1̄.56472	32	1̄.59614	37	0.40386	1̄.96858	5	28
33	1̄.56504	32	1̄.59651	37	0.40349	1̄.96853	5	27
34	1̄.56536	32	1̄.59688	37	0.40312	1̄.96848	5	26
35	1̄.56568	32	1̄.59725	37	0.40275	1̄.96843	5	25
36	1̄.56599	31	1̄.59762	37	0.40238	1̄.96838	5	24
37	1̄.56631	32	1̄.59799	37	0.40201	1̄.96833	5	23
38	1̄.56663	32	1̄.59835	36	0.40165	1̄.96828	5	22
39	1̄.56695	32	1̄.59872	37	0.40128	1̄.96823	5	21
40	1̄.56727	32	1̄.59909	37	0.40091	1̄.96818	5	20
41	1̄.56759	32	1̄.59946	37	0.40054	1̄.96813	5	19
42	1̄.56790	31	1̄.59983	37	0.40017	1̄.96808	5	18
43	1̄.56822	32	1̄.60019	36	0.39981	1̄.96803	5	17
44	1̄.56854	32	1̄.60056	37	0.39944	1̄.96798	5	16
45	1̄.56886	32	1̄.60093	37	0.39907	1̄.96793	5	15
46	1̄.56917	31	1̄.60130	37	0.39870	1̄.96788	5	14
47	1̄.56949	32	1̄.60166	36	0.39834	1̄.96783	5	13
48	1̄.56980	31	1̄.60203	37	0.39797	1̄.96778	5	12
49	1̄.57012	32	1̄.60240	37	0.39760	1̄.96772	6	11
50	1̄.57044	32	1̄.60276	36	0.39724	1̄.96767	5	10
51	1̄.57075	31	1̄.60313	37	0.39687	1̄.96762	5	9
52	1̄.57107	32	1̄.60349	36	0.39651	1̄.96757	5	8
53	1̄.57138	31	1̄.60386	37	0.39614	1̄.96752	5	7
54	1̄.57169	31	1̄.60422	36	0.39578	1̄.96747	5	6
55	1̄.57201	32	1̄.60459	37	0.39541	1̄.96742	5	5
56	1̄.57232	31	1̄.60495	36	0.39505	1̄.96737	5	4
57	1̄.57264	32	1̄.60532	37	0.39468	1̄.96732	5	3
58	1̄.57295	31	1̄.60568	36	0.39432	1̄.96727	5	2
59	1̄.57326	31	1̄.60605	37	0.39395	1̄.96722	5	1
60	1̄.57358	32	1̄.60641	36	0.39359	1̄.96717	5	0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	38	37
6	3.8	3.7
7	4.4	4.3
8	5.1	4.9
9	5.7	5.6
10	6.3	6.2
20	12.7	12.3
30	19.0	18.5
40	25.3	24.7
50	31.7	30.8

	36	33
6	3.6	3.3
7	4.2	3.9
8	4.8	4.4
9	5.4	5.0
10	6.0	5.5
20	12.0	11.0
30	18.0	16.5
40	24.0	22.0
50	30.0	27.5

	32
6	3.2
7	3.7
8	4.3
9	4.8
10	5.3
20	10.7
30	16.0
40	21.3
50	26.7

	31	6
6	3.1	0.6
7	3.6	0.7
8	4.1	0.8
9	4.7	0.9
10	5.2	1.0
20	10.3	2.0
30	15.5	3.0
40	20.7	4.0
50	25.8	5.0

	5	4
6	0.5	0.4
7	0.6	0.5
8	0.7	0.5
9	0.8	0.6
10	0.8	0.7
20	1.7	1.3
30	2.5	2.0
40	3.3	2.7
50	4.2	3.3

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	$\bar{1}$.57358	31	$\bar{1}$.60641	36	0.39359	$\bar{1}$.96717	6	60
1	$\bar{1}$.57389	31	$\bar{1}$.60677	37	0.39323	$\bar{1}$.96711	5	59
2	$\bar{1}$.57420	31	$\bar{1}$.60714	36	0.39286	$\bar{1}$.96706	5	58
3	$\bar{1}$.57451	31	$\bar{1}$.60750	36	0.39250	$\bar{1}$.96701	5	57
4	$\bar{1}$.57482	32	$\bar{1}$.60786	37	0.39214	$\bar{1}$.96696	5	56
5	$\bar{1}$.57514	31	$\bar{1}$.60823	36	0.39177	$\bar{1}$.96691	5	55
6	$\bar{1}$.57545	31	$\bar{1}$.60859	36	0.39141	$\bar{1}$.96686	5	54
7	$\bar{1}$.57576	31	$\bar{1}$.60895	36	0.39105	$\bar{1}$.96681	5	53
8	$\bar{1}$.57607	31	$\bar{1}$.60931	36	0.39069	$\bar{1}$.96676	6	52
9	$\bar{1}$.57638	31	$\bar{1}$.60967	37	0.39033	$\bar{1}$.96670	5	51
10	$\bar{1}$.57669	31	$\bar{1}$.61004	36	0.38996	$\bar{1}$.96665	5	50
11	$\bar{1}$.57700	31	$\bar{1}$.61040	36	0.38960	$\bar{1}$.96660	5	49
12	$\bar{1}$.57731	31	$\bar{1}$.61076	36	0.38924	$\bar{1}$.96655	5	48
13	$\bar{1}$.57762	31	$\bar{1}$.61112	36	0.38888	$\bar{1}$.96650	5	47
14	$\bar{1}$.57793	31	$\bar{1}$.61148	36	0.38852	$\bar{1}$.96645	5	46
15	$\bar{1}$.57824	31	$\bar{1}$.61184	36	0.38816	$\bar{1}$.96640	6	45
16	$\bar{1}$.57855	30	$\bar{1}$.61220	36	0.38780	$\bar{1}$.96634	5	44
17	$\bar{1}$.57885	31	$\bar{1}$.61256	36	0.38744	$\bar{1}$.96629	5	43
18	$\bar{1}$.57916	31	$\bar{1}$.61292	36	0.38708	$\bar{1}$.96624	5	42
19	$\bar{1}$.57947	31	$\bar{1}$.61328	36	0.38672	$\bar{1}$.96619	5	41
20	$\bar{1}$.57978	30	$\bar{1}$.61364	36	0.38636	$\bar{1}$.96614	6	40
21	$\bar{1}$.58008	31	$\bar{1}$.61400	36	0.38600	$\bar{1}$.96608	5	39
22	$\bar{1}$.58039	31	$\bar{1}$.61436	36	0.38564	$\bar{1}$.96603	5	38
23	$\bar{1}$.58070	31	$\bar{1}$.61472	36	0.38528	$\bar{1}$.96598	5	37
24	$\bar{1}$.58101	30	$\bar{1}$.61508	36	0.38492	$\bar{1}$.96593	5	36
25	$\bar{1}$.58131	31	$\bar{1}$.61544	35	0.38456	$\bar{1}$.96588	6	35
26	$\bar{1}$.58162	30	$\bar{1}$.61579	36	0.38421	$\bar{1}$.96582	5	34
27	$\bar{1}$.58192	31	$\bar{1}$.61615	36	0.38385	$\bar{1}$.96577	5	33
28	$\bar{1}$.58223	30	$\bar{1}$.61651	36	0.38349	$\bar{1}$.96572	5	32
29	$\bar{1}$.58253	31	$\bar{1}$.61687	35	0.38313	$\bar{1}$.96567	5	31
30	$\bar{1}$.58284	30	$\bar{1}$.61722	36	0.38278	$\bar{1}$.96562	6	30
31	$\bar{1}$.58314	31	$\bar{1}$.61758	36	0.38242	$\bar{1}$.96556	5	29
32	$\bar{1}$.58345	30	$\bar{1}$.61794	36	0.38206	$\bar{1}$.96551	5	28
33	$\bar{1}$.58375	31	$\bar{1}$.61830	35	0.38170	$\bar{1}$.96546	5	27
34	$\bar{1}$.58406	30	$\bar{1}$.61865	36	0.38135	$\bar{1}$.96541	6	26
35	$\bar{1}$.58436	31	$\bar{1}$.61901	35	0.38099	$\bar{1}$.96535	5	25
36	$\bar{1}$.58467	30	$\bar{1}$.61936	36	0.38064	$\bar{1}$.96530	5	24
37	$\bar{1}$.58497	30	$\bar{1}$.61972	36	0.38028	$\bar{1}$.96525	5	23
38	$\bar{1}$.58527	30	$\bar{1}$.62008	35	0.37992	$\bar{1}$.96520	6	22
39	$\bar{1}$.58557	31	$\bar{1}$.62043	36	0.37957	$\bar{1}$.96514	5	21
40	$\bar{1}$.58588	30	$\bar{1}$.62079	35	0.37921	$\bar{1}$.96509	5	20
41	$\bar{1}$.58618	30	$\bar{1}$.62114	36	0.37886	$\bar{1}$.96504	6	19
42	$\bar{1}$.58648	30	$\bar{1}$.62150	35	0.37850	$\bar{1}$.96498	5	18
43	$\bar{1}$.58678	31	$\bar{1}$.62185	36	0.37815	$\bar{1}$.96493	5	17
44	$\bar{1}$.58709	30	$\bar{1}$.62221	35	0.37779	$\bar{1}$.96488	5	16
45	$\bar{1}$.58739	30	$\bar{1}$.62256	36	0.37744	$\bar{1}$.96483	6	15
46	$\bar{1}$.58769	30	$\bar{1}$.62292	35	0.37708	$\bar{1}$.96477	5	14
47	$\bar{1}$.58799	30	$\bar{1}$.62327	35	0.37673	$\bar{1}$.96472	5	13
48	$\bar{1}$.58829	30	$\bar{1}$.62362	36	0.37638	$\bar{1}$.96467	6	12
49	$\bar{1}$.58859	30	$\bar{1}$.62398	35	0.37602	$\bar{1}$.96461	5	11
50	$\bar{1}$.58889	30	$\bar{1}$.62433	35	0.37567	$\bar{1}$.96456	5	10
51	$\bar{1}$.58919	30	$\bar{1}$.62468	36	0.37532	$\bar{1}$.96451	6	9
52	$\bar{1}$.58949	30	$\bar{1}$.62504	35	0.37496	$\bar{1}$.96445	5	8
53	$\bar{1}$.58979	30	$\bar{1}$.62539	35	0.37461	$\bar{1}$.96440	5	7
54	$\bar{1}$.59009	30	$\bar{1}$.62574	35	0.37426	$\bar{1}$.96435	6	6
55	$\bar{1}$.59039	30	$\bar{1}$.62609	36	0.37391	$\bar{1}$.96429	5	5
56	$\bar{1}$.59069	29	$\bar{1}$.62645	35	0.37355	$\bar{1}$.96424	5	4
57	$\bar{1}$.59098	30	$\bar{1}$.62680	35	0.37320	$\bar{1}$.96419	6	3
58	$\bar{1}$.59128	30	$\bar{1}$.62715	35	0.37285	$\bar{1}$.96413	5	2
59	$\bar{1}$.59158	30	$\bar{1}$.62750	35	0.37250	$\bar{1}$.96408	5	1
60	$\bar{1}$.59188		$\bar{1}$.62785		0.37215	$\bar{1}$.96403		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	37	36
6	3.7	3.6
7	4.3	4.2
8	4.9	4.8
9	5.6	5.4
10	6.2	6.0
20	12.3	12.0
30	18.5	18.0
40	24.7	24.0
50	30.8	30.0

	35
6	3.5
7	4.1
8	4.7
9	5.3
10	5.8
20	11.7
30	17.5
40	23.3
50	29.2

	32	31
6	3.2	3.1
7	3.7	3.6
8	4.3	4.1
9	4.8	4.7
10	5.3	5.2
20	10.7	10.3
30	16.0	15.5
40	21.3	20.7
50	26.7	25.8

	30	29
6	3.0	2.9
7	3.5	3.4
8	4.0	3.9
9	4.5	4.4
10	5.0	4.8
20	10.0	9.7
30	15.0	14.5
40	20.0	19.3
50	25.0	24.2

	6	5
6	0.6	0.5
7	0.7	0.6
8	0.8	0.7
9	0.9	0.8
10	1.0	0.8
20	2.0	1.7
30	3.0	2.5
40	4.0	3.3
50	5.0	4.2

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	Ī.59188	30	Ī.62785	35	0.37215	Ī.96403	6	60
1	Ī.59218	29	Ī.62820	35	0.37180	Ī.96397	5	59
2	Ī.59247	30	Ī.62855	35	0.37145	Ī.96392	5	58
3	Ī.59277	30	Ī.62890	36	0.37110	Ī.96387	6	57
4	Ī.59307	29	Ī.62926	35	0.37074	Ī.96381	5	56
5	Ī.59336	30	Ī.62961	35	0.37039	Ī.96376	6	55
6	Ī.59366	30	Ī.62996	35	0.37004	Ī.96370	5	54
7	Ī.59396	29	Ī.63031	35	0.36969	Ī.96365	5	53
8	Ī.59425	30	Ī.63066	35	0.36934	Ī.96360	6	52
9	Ī.59455	29	Ī.63101	34	0.36899	Ī.96354	5	51
10	Ī.59484	30	Ī.63135	35	0.36865	Ī.96349	6	50
11	Ī.59514	29	Ī.63170	35	0.36830	Ī.96343	5	49
12	Ī.59543	30	Ī.63205	35	0.36795	Ī.96338	5	48
13	Ī.59573	29	Ī.63240	35	0.36760	Ī.96333	6	47
14	Ī.59602	30	Ī.63275	35	0.36725	Ī.96327	5	46
15	Ī.59632	29	Ī.63310	35	0.36690	Ī.96322	6	45
16	Ī.59661	29	Ī.63345	34	0.36655	Ī.96316	5	44
17	Ī.59690	30	Ī.63379	35	0.36621	Ī.96311	6	43
18	Ī.59720	29	Ī.63414	35	0.36586	Ī.96305	5	42
19	Ī.59749	29	Ī.63449	35	0.36551	Ī.96300	6	41
20	Ī.59778	30	Ī.63484	35	0.36516	Ī.96294	5	40
21	Ī.59808	29	Ī.63519	34	0.36481	Ī.96289	5	39
22	Ī.59837	29	Ī.63553	35	0.36447	Ī.96284	6	38
23	Ī.59866	29	Ī.63588	35	0.36412	Ī.96278	5	37
24	Ī.59895	29	Ī.63623	34	0.36377	Ī.96273	6	36
25	Ī.59924	30	Ī.63657	35	0.36343	Ī.96267	5	35
26	Ī.59954	29	Ī.63692	34	0.36308	Ī.96262	6	34
27	Ī.59983	29	Ī.63726	35	0.36274	Ī.96256	5	33
28	Ī.60012	29	Ī.63761	35	0.36239	Ī.96251	6	32
29	Ī.60041	29	Ī.63796	34	0.36204	Ī.96245	5	31
30	Ī.60070	29	Ī.63830	35	0.36170	Ī.96240	6	30
31	Ī.60099	29	Ī.63865	34	0.36135	Ī.96234	5	29
32	Ī.60128	29	Ī.63899	35	0.36101	Ī.96229	6	28
33	Ī.60157	29	Ī.63934	34	0.36066	Ī.96223	5	27
34	Ī.60186	29	Ī.63968	35	0.36032	Ī.96218	6	26
35	Ī.60215	29	Ī.64003	34	0.35997	Ī.96212	5	25
36	Ī.60244	29	Ī.64037	35	0.35963	Ī.96207	6	24
37	Ī.60273	29	Ī.64072	34	0.35928	Ī.96201	5	23
38	Ī.60302	29	Ī.64106	34	0.35894	Ī.96196	6	22
39	Ī.60331	28	Ī.64140	35	0.35860	Ī.96190	5	21
40	Ī.60359	29	Ī.64175	34	0.35825	Ī.96185	6	20
41	Ī.60388	29	Ī.64209	34	0.35791	Ī.96179	5	19
42	Ī.60417	29	Ī.64243	35	0.35757	Ī.96174	6	18
43	Ī.60446	28	Ī.64278	34	0.35722	Ī.96168	6	17
44	Ī.60474	29	Ī.64312	34	0.35688	Ī.96162	5	16
45	Ī.60503	29	Ī.64346	35	0.35654	Ī.96157	6	15
46	Ī.60532	29	Ī.64381	34	0.35619	Ī.96151	5	14
47	Ī.60561	28	Ī.64415	34	0.35585	Ī.96146	6	13
48	Ī.60589	29	Ī.64449	34	0.35551	Ī.96140	5	12
49	Ī.60618	28	Ī.64483	34	0.35517	Ī.96135	6	11
50	Ī.60646	29	Ī.64517	35	0.35483	Ī.96129	6	10
51	Ī.60675	29	Ī.64552	34	0.35448	Ī.96123	5	9
52	Ī.60704	28	Ī.64586	34	0.35414	Ī.96118	6	8
53	Ī.60732	29	Ī.64620	34	0.35380	Ī.96112	5	7
54	Ī.60761	28	Ī.64654	34	0.35346	Ī.96107	6	6
55	Ī.60789	29	Ī.64688	34	0.35312	Ī.96101	6	5
56	Ī.60818	28	Ī.64722	34	0.35278	Ī.96095	5	4
57	Ī.60846	29	Ī.64756	34	0.35244	Ī.96090	6	3
58	Ī.60875	28	Ī.64790	34	0.35210	Ī.96084	5	2
59	Ī.60903	28	Ī.64824	34	0.35176	Ī.96079	6	1
60	Ī.60931		Ī.64858		0.35142	Ī.96073		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	36	35	34	30	29	28	6	5
6	3.6	3.5	3.4	3.0	2.9	2.8	0.6	0.5
7	4.2	4.1	4.0	3.5	3.4	3.3	0.7	0.6
8	4.8	4.7	4.5	4.0	3.9	3.7	0.8	0.7
9	5.4	5.3	5.1	4.5	4.4	4.2	0.9	0.8
10	6.0	5.8	5.7	5.0	4.8	4.7	1.0	0.8
20	12.0	11.7	11.3	10.0	9.7	9.3	2.0	1.7
30	18.0	17.5	17.0	15.0	14.5	14.0	3.0	2.5
40	24.0	23.3	22.7	20.0	19.3	18.7	4.0	3.3
50	30.0	29.2	28.3	25.0	24.2	23.3	5.0	4.2

′	log sin	d	log tan	c. d.	log cot	log cos	d	′
0	1̄.60931	29	1̄.64858	34	0.35142	1̄.96073	6	60
1	1̄.60960	28	1̄.64892	34	0.35108	1̄.96067	5	59
2	1̄.60988	28	1̄.64926	34	0.35074	1̄.96062	6	58
3	1̄.61016	29	1̄.64960	34	0.35040	1̄.96056	6	57
4	1̄.61045	28	1̄.64994	34	0.35006	1̄.96050	5	56
5	1̄.61073	28	1̄.65028	34	0.34972	1̄.96045	6	55
6	1̄.61101	28	1̄.65062	34	0.34938	1̄.96039	5	54
7	1̄.61129	29	1̄.65096	34	0.34904	1̄.96034	6	53
8	1̄.61158	28	1̄.65130	34	0.34870	1̄.96028	6	52
9	1̄.61186	28	1̄.65164	33	0.34836	1̄.96022	5	51
10	1̄.61214	28	1̄.65197	34	0.34803	1̄.96017	6	50
11	1̄.61242	28	1̄.65231	34	0.34769	1̄.96011	6	49
12	1̄.61270	28	1̄.65265	34	0.34735	1̄.96005	5	48
13	1̄.61298	28	1̄.65299	34	0.34701	1̄.96000	6	47
14	1̄.61326	28	1̄.65333	33	0.34667	1̄.95994	6	46
15	1̄.61354	28	1̄.65366	34	0.34634	1̄.95988	6	45
16	1̄.61382	29	1̄.65400	34	0.34600	1̄.95982	5	44
17	1̄.61411	27	1̄.65434	33	0.34566	1̄.95977	6	43
18	1̄.61438	28	1̄.65467	34	0.34533	1̄.95971	6	42
19	1̄.61466	28	1̄.65501	34	0.34499	1̄.95965	5	41
20	1̄.61494	28	1̄.65535	33	0.34465	1̄.95960	6	40
21	1̄.61522	28	1̄.65568	34	0.34432	1̄.95954	6	39
22	1̄.61550	28	1̄.65602	34	0.34398	1̄.95948	6	38
23	1̄.61578	28	1̄.65636	33	0.34364	1̄.95942	5	37
24	1̄.61606	28	1̄.65669	34	0.34331	1̄.95937	6	36
25	1̄.61634	28	1̄.65703	33	0.34297	1̄.95931	6	35
26	1̄.61662	27	1̄.65736	34	0.34264	1̄.95925	5	34
27	1̄.61689	28	1̄.65770	33	0.34230	1̄.95920	6	33
28	1̄.61717	28	1̄.65803	34	0.34197	1̄.95914	6	32
29	1̄.61745	28	1̄.65837	33	0.34163	1̄.95908	6	31
30	1̄.61773	27	1̄.65870	34	0.34130	1̄.95902	5	30
31	1̄.61800	28	1̄.65904	33	0.34096	1̄.95897	6	29
32	1̄.61828	28	1̄.65937	34	0.34063	1̄.95891	6	28
33	1̄.61856	27	1̄.65971	33	0.34029	1̄.95885	6	27
34	1̄.61883	28	1̄.66004	34	0.33996	1̄.95879	6	26
35	1̄.61911	28	1̄.66038	33	0.33962	1̄.95873	5	25
36	1̄.61939	27	1̄.66071	33	0.33929	1̄.95868	6	24
37	1̄.61966	28	1̄.66104	34	0.33896	1̄.95862	6	23
38	1̄.61994	27	1̄.66138	33	0.33862	1̄.95856	6	22
39	1̄.62021	28	1̄.66171	33	0.33829	1̄.95850	6	21
40	1̄.62049	27	1̄.66204	34	0.33796	1̄.95844	5	20
41	1̄.62076	28	1̄.66238	33	0.33762	1̄.95839	6	19
42	1̄.62104	27	1̄.66271	33	0.33729	1̄.95833	6	18
43	1̄.62131	28	1̄.66304	33	0.33696	1̄.95827	6	17
44	1̄.62159	27	1̄.66337	34	0.33663	1̄.95821	6	16
45	1̄.62186	28	1̄.66371	33	0.33629	1̄.95815	5	15
46	1̄.62214	27	1̄.66404	33	0.33596	1̄.95810	6	14
47	1̄.62241	27	1̄.66437	33	0.33563	1̄.95804	6	13
48	1̄.62268	28	1̄.66470	33	0.33530	1̄.95798	6	12
49	1̄.62296	27	1̄.66503	34	0.33497	1̄.95792	6	11
50	1̄.62323	27	1̄.66537	33	0.33463	1̄.95786	6	10
51	1̄.62350	27	1̄.66570	33	0.33430	1̄.95780	5	9
52	1̄.62377	28	1̄.66603	33	0.33397	1̄.95775	6	8
53	1̄.62405	27	1̄.66636	33	0.33364	1̄.95769	6	7
54	1̄.62432	27	1̄.66669	33	0.33331	1̄.95763	6	6
55	1̄.62459	27	1̄.66702	33	0.33298	1̄.95757	6	5
56	1̄.62486	27	1̄.66735	33	0.33265	1̄.95751	6	4
57	1̄.62513	28	1̄.66768	33	0.33232	1̄.95745	6	3
58	1̄.62541	27	1̄.66801	33	0.33199	1̄.95739	6	2
59	1̄.62568	27	1̄.66834	33	0.33166	1̄.95733	5	1
60	1̄.62595		1̄.66867		0.33133	1̄.95728		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	34	33
6	3.4	3.3
7	4.0	3.9
8	4.5	4.4
9	5.1	5.0
10	5.7	5.5
20	11.3	11.0
30	17.0	16.5
40	22.7	22.0
50	28.3	27.5

	29
6	2.9
7	3.4
8	3.9
9	4.4
10	4.8
20	9.7
30	14.5
40	19.3
50	24.2

	28
6	2.8
7	3.3
8	3.7
9	4.2
10	4.7
20	9.3
30	14.0
40	18.7
50	23.3

	27
6	2.7
7	3.2
8	3.6
9	4.1
10	4.5
20	9.0
30	13.5
40	18.0
50	22.5

	6	5
6	0.6	0.5
7	0.7	0.6
8	0.8	0.7
9	0.9	0.8
10	1.0	0.8
20	2.0	1.7
30	3.0	2.5
40	4.0	3.3
50	5.0	4.2

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.62595	27	1̄.66867	33	0.33133	1̄.95728	6	60
1	1̄.62622	27	1̄.66900	33	0.33100	1̄.95722	6	59
2	1̄.62649	27	1̄.66933	33	0.33067	1̄.95716	6	58
3	1̄.62676	27	1̄.66966	33	0.33034	1̄.95710	6	57
4	1̄.62703	27	1̄.66999	33	0.33001	1̄.95704	6	56
5	1̄.62730	27	1̄.67032	33	0.32968	1̄.95698	6	55
6	1̄.62757	27	1̄.67065	33	0.32935	1̄.95692	6	54
7	1̄.62784	27	1̄.67098	33	0.32902	1̄.95686	6	53
8	1̄.62811	27	1̄.67131	32	0.32869	1̄.95680	6	52
9	1̄.62838	27	1̄.67163	33	0.32837	1̄.95674	6	51
10	1̄.62865	27	1̄.67196	33	0.32804	1̄.95668	5	50
11	1̄.62892	26	1̄.67229	33	0.32771	1̄.95663	6	49
12	1̄.62918	27	1̄.67262	33	0.32738	1̄.95657	6	48
13	1̄.62945	27	1̄.67295	32	0.32705	1̄.95651	6	47
14	1̄.62972	27	1̄.67327	33	0.32673	1̄.95645	6	46
15	1̄.62999	27	1̄.67360	33	0.32640	1̄.95639	6	45
16	1̄.63026	26	1̄.67393	33	0.32607	1̄.95633	6	44
17	1̄.63052	27	1̄.67426	32	0.32574	1̄.95627	6	43
18	1̄.63079	27	1̄.67458	33	0.32542	1̄.95621	6	42
19	1̄.63106	27	1̄.67491	33	0.32509	1̄.95615	6	41
20	1̄.63133	26	1̄.67524	32	0.32476	1̄.95609	6	40
21	1̄.63159	27	1̄.67556	33	0.32444	1̄.95603	6	39
22	1̄.63186	27	1̄.67589	33	0.32411	1̄.95597	6	38
23	1̄.63213	26	1̄.67622	32	0.32378	1̄.95591	6	37
24	1̄.63239	27	1̄.67654	33	0.32346	1̄.95585	6	36
25	1̄.63266	26	1̄.67687	32	0.32313	1̄.95579	6	35
26	1̄.63292	27	1̄.67719	33	0.32281	1̄.95573	6	34
27	1̄.63319	26	1̄.67752	33	0.32248	1̄.95567	6	33
28	1̄.63345	27	1̄.67785	32	0.32215	1̄.95561	6	32
29	1̄.63372	26	1̄.67817	33	0.32183	1̄.95555	6	31
30	1̄.63398	27	1̄.67850	32	0.32150	1̄.95549	6	30
31	1̄.63425	26	1̄.67882	33	0.32118	1̄.95543	6	29
32	1̄.63451	27	1̄.67915	32	0.32085	1̄.95537	6	28
33	1̄.63478	26	1̄.67947	33	0.32053	1̄.95531	6	27
34	1̄.63504	27	1̄.67980	32	0.32020	1̄.95525	6	26
35	1̄.63531	26	1̄.68012	32	0.31988	1̄.95519	6	25
36	1̄.63557	26	1̄.68044	33	0.31956	1̄.95513	6	24
37	1̄.63583	27	1̄.68077	32	0.31923	1̄.95507	7	23
38	1̄.63610	26	1̄.68109	33	0.31891	1̄.95500	6	22
39	1̄.63636	26	1̄.68142	32	0.31858	1̄.95494	6	21
40	1̄.63662	27	1̄.68174	32	0.31826	1̄.95488	6	20
41	1̄.63689	26	1̄.68206	33	0.31794	1̄.95482	6	19
42	1̄.63715	26	1̄.68239	32	0.31761	1̄.95476	6	18
43	1̄.63741	26	1̄.68271	32	0.31729	1̄.95470	6	17
44	1̄.63767	27	1̄.68303	33	0.31697	1̄.95464	6	16
45	1̄.63794	26	1̄.68336	32	0.31664	1̄.95458	6	15
46	1̄.63820	26	1̄.68368	32	0.31632	1̄.95452	6	14
47	1̄.63846	26	1̄.68400	32	0.31600	1̄.95446	6	13
48	1̄.63872	26	1̄.68432	33	0.31568	1̄.95440	6	12
49	1̄.63898	26	1̄.68465	32	0.31535	1̄.95434	7	11
50	1̄.63924	26	1̄.68497	32	0.31503	1̄.95427	6	10
51	1̄.63950	26	1̄.68529	32	0.31471	1̄.95421	6	9
52	1̄.63976	26	1̄.68561	32	0.31439	1̄.95415	6	8
53	1̄.64002	26	1̄.68593	33	0.31407	1̄.95409	6	7
54	1̄.64028	26	1̄.68626	32	0.31374	1̄.95403	6	6
55	1̄.64054	26	1̄.68658	32	0.31342	1̄.95397	6	5
56	1̄.64080	26	1̄.68690	32	0.31310	1̄.95391	7	4
57	1̄.64106	26	1̄.68722	32	0.31278	1̄.95384	6	3
58	1̄.64132	26	1̄.68754	32	0.31246	1̄.95378	6	2
59	1̄.64158	26	1̄.68786	32	0.31214	1̄.95372	6	1
60	1̄.64184		1̄.68818		0.31182	1̄.95366		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	33	32
6	3.3	3.2
7	3.9	3.7
8	4.4	4.3
9	5.0	4.8
10	5.5	5.3
20	11.0	10.7
30	16.5	16.0
40	22.0	21.3
50	27.5	26.7

	27
6	2.7
7	3.2
8	3.6
9	4.1
10	4.5
20	9.0
30	13.5
40	18.0
50	22.5

	26
6	2.6
7	3.0
8	3.5
9	3.9
10	4.3
20	8.7
30	13.0
40	17.3
50	21.7

	7
6	0.7
7	0.8
8	0.9
9	1.1
10	1.2
20	2.3
30	3.5
40	4.7
50	5.8

	6	5
6	0.6	0.5
7	0.7	0.6
8	0.8	0.7
9	0.9	0.8
10	1.0	0.8
20	2.0	1.7
30	3.0	2.5
40	4.0	3.3
50	5.0	4.2

p. p.

26°

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	$\bar{1}$.64184	26	$\bar{1}$.68818	32	0.31182	$\bar{1}$.95366	6	60
1	$\bar{1}$.64210	26	$\bar{1}$.68850	32	0.31150	$\bar{1}$.95360	6	59
2	$\bar{1}$.64236	26	$\bar{1}$.68882	32	0.31118	$\bar{1}$.95354	6	58
3	$\bar{1}$.64262	26	$\bar{1}$.68914	32	0.31086	$\bar{1}$.95348	7	57
4	$\bar{1}$.64288	25	$\bar{1}$.68946	32	0.31054	$\bar{1}$.95341	6	56
5	$\bar{1}$.64313	26	$\bar{1}$.68978	32	0.31022	$\bar{1}$.95335	6	55
6	$\bar{1}$.64339	26	$\bar{1}$.69010	32	0.30990	$\bar{1}$.95329	6	54
7	$\bar{1}$.64365	26	$\bar{1}$.69042	32	0.30958	$\bar{1}$.95323	6	53
8	$\bar{1}$.64391	26	$\bar{1}$.69074	32	0.30926	$\bar{1}$.95317	7	52
9	$\bar{1}$.64417	25	$\bar{1}$.69106	32	0.30894	$\bar{1}$.95310	6	51
10	$\bar{1}$.64442	26	$\bar{1}$.69138	32	0.30862	$\bar{1}$.95304	6	50
11	$\bar{1}$.64468	26	$\bar{1}$.69170	32	0.30830	$\bar{1}$.95298	6	49
12	$\bar{1}$.64494	25	$\bar{1}$.69202	32	0.30798	$\bar{1}$.95292	6	48
13	$\bar{1}$.64519	26	$\bar{1}$.69234	32	0.30766	$\bar{1}$.95286	7	47
14	$\bar{1}$.64545	26	$\bar{1}$.69266	32	0.30734	$\bar{1}$.95279	6	46
15	$\bar{1}$.64571	25	$\bar{1}$.69298	31	0.30702	$\bar{1}$.95273	6	45
16	$\bar{1}$.64596	26	$\bar{1}$.69329	32	0.30671	$\bar{1}$.95267	6	44
17	$\bar{1}$.64622	25	$\bar{1}$.69361	32	0.30639	$\bar{1}$.95261	7	43
18	$\bar{1}$.64647	26	$\bar{1}$.69393	32	0.30607	$\bar{1}$.95254	6	42
19	$\bar{1}$.64673	25	$\bar{1}$.69425	32	0.30575	$\bar{1}$.95248	6	41
20	$\bar{1}$.64698	26	$\bar{1}$.69457	31	0.30543	$\bar{1}$.95242	6	40
21	$\bar{1}$.64724	25	$\bar{1}$.69488	32	0.30512	$\bar{1}$.95236	7	39
22	$\bar{1}$.64749	26	$\bar{1}$.69520	32	0.30480	$\bar{1}$.95229	6	38
23	$\bar{1}$.64775	25	$\bar{1}$.69552	32	0.30448	$\bar{1}$.95223	6	37
24	$\bar{1}$.64800	26	$\bar{1}$.69584	31	0.30416	$\bar{1}$.95217	6	36
25	$\bar{1}$.64826	25	$\bar{1}$.69615	32	0.30385	$\bar{1}$.95211	7	35
26	$\bar{1}$.64851	26	$\bar{1}$.69647	32	0.30353	$\bar{1}$.95204	6	34
27	$\bar{1}$.64877	25	$\bar{1}$.69679	31	0.30321	$\bar{1}$.95198	6	33
28	$\bar{1}$.64902	25	$\bar{1}$.69710	32	0.30290	$\bar{1}$.95192	7	32
29	$\bar{1}$.64927	26	$\bar{1}$.69742	32	0.30258	$\bar{1}$.95185	6	31
30	$\bar{1}$.64953	25	$\bar{1}$.69774	31	0.30226	$\bar{1}$.95179	6	30
31	$\bar{1}$.64978	25	$\bar{1}$.69805	32	0.30195	$\bar{1}$.95173	6	29
32	$\bar{1}$.65003	26	$\bar{1}$.69837	31	0.30163	$\bar{1}$.95167	7	28
33	$\bar{1}$.65029	25	$\bar{1}$.69868	32	0.30132	$\bar{1}$.95160	6	27
34	$\bar{1}$.65054	25	$\bar{1}$.69900	32	0.30100	$\bar{1}$.95154	6	26
35	$\bar{1}$.65079	25	$\bar{1}$.69932	31	0.30068	$\bar{1}$.95148	7	25
36	$\bar{1}$.65104	26	$\bar{1}$.69963	32	0.30037	$\bar{1}$.95141	6	24
37	$\bar{1}$.65130	25	$\bar{1}$.69995	31	0.30005	$\bar{1}$.95135	6	23
38	$\bar{1}$.65155	25	$\bar{1}$.70026	32	0.29974	$\bar{1}$.95129	7	22
39	$\bar{1}$.65180	25	$\bar{1}$.70058	31	0.29942	$\bar{1}$.95122	6	21
40	$\bar{1}$.65205	25	$\bar{1}$.70089	32	0.29911	$\bar{1}$.95116	6	20
41	$\bar{1}$.65230	25	$\bar{1}$.70121	31	0.29879	$\bar{1}$.95110	7	19
42	$\bar{1}$.65255	26	$\bar{1}$.70152	32	0.29848	$\bar{1}$.95103	6	18
43	$\bar{1}$.65281	25	$\bar{1}$.70184	31	0.29816	$\bar{1}$.95097	7	17
44	$\bar{1}$.65306	25	$\bar{1}$.70215	32	0.29785	$\bar{1}$.95090	6	16
45	$\bar{1}$.65331	25	$\bar{1}$.70247	31	0.29753	$\bar{1}$.95084	6	15
46	$\bar{1}$.65356	25	$\bar{1}$.70278	31	0.29722	$\bar{1}$.95078	7	14
47	$\bar{1}$.65381	25	$\bar{1}$.70309	32	0.29691	$\bar{1}$.95071	6	13
48	$\bar{1}$.65406	25	$\bar{1}$.70341	31	0.29659	$\bar{1}$.95065	6	12
49	$\bar{1}$.65431	25	$\bar{1}$.70372	32	0.29628	$\bar{1}$.95059	7	11
50	$\bar{1}$.65456	25	$\bar{1}$.70404	31	0.29596	$\bar{1}$.95052	6	10
51	$\bar{1}$.65481	25	$\bar{1}$.70435	31	0.29565	$\bar{1}$.95046	7	9
52	$\bar{1}$.65506	25	$\bar{1}$.70466	32	0.29534	$\bar{1}$.95039	6	8
53	$\bar{1}$.65531	25	$\bar{1}$.70498	31	0.29502	$\bar{1}$.95033	6	7
54	$\bar{1}$.65556	24	$\bar{1}$.70529	31	0.29471	$\bar{1}$.95027	7	6
55	$\bar{1}$.65580	25	$\bar{1}$.70560	32	0.29440	$\bar{1}$.95020	6	5
56	$\bar{1}$.65605	25	$\bar{1}$.70592	31	0.29408	$\bar{1}$.95014	7	4
57	$\bar{1}$.65630	25	$\bar{1}$.70623	31	0.29377	$\bar{1}$.95007	6	3
58	$\bar{1}$.65655	25	$\bar{1}$.70654	31	0.29346	$\bar{1}$.95001	6	2
59	$\bar{1}$.65680	25	$\bar{1}$.70685	32	0.29315	$\bar{1}$.94995	7	1
60	$\bar{1}$.65705		$\bar{1}$.70717		0.29283	$\bar{1}$.94988		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	32	31
6	3.2	3.1
7	3.7	3.6
8	4.3	4.1
9	4.8	4.7
10	5.3	5.2
20	10.7	10.3
30	16.0	15.5
40	21.3	20.7
50	26.7	25.8

	26
6	2.6
7	3.0
8	3.5
9	3.9
10	4.3
20	8.7
30	13.0
40	17.3
50	21.7

	25
6	2.5
7	2.9
8	3.3
9	3.8
10	4.2
20	8.3
30	12.5
40	16.7
50	20.8

	24
6	2.4
7	2.8
8	3.2
9	3.6
10	4.0
20	8.0
30	12.0
40	16.0
50	20.0

	7	6
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.1	0.9
10	1.2	1.0
20	2.3	2.0
30	3.5	3.0
40	4.7	4.0
50	5.8	5.0

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.65705	24	1̄.70717	31	0.29283	1̄.94988	6	60
1	1̄.65729	25	1̄.70748	31	0.29252	1̄.94982	7	59
2	1̄.65754	25	1̄.70779	31	0.29221	1̄.94975	6	58
3	1̄.65779	25	1̄.70810	31	0.29190	1̄.94969	7	57
4	1̄.65804	24	1̄.70841	32	0.29159	1̄.94962	6	56
5	1̄.65828	25	1̄.70873	31	0.29127	1̄.94956	7	55
6	1̄.65853	25	1̄.70904	31	0.29096	1̄.94949	6	54
7	1̄.65878	24	1̄.70935	31	0.29065	1̄.94943	7	53
8	1̄.65902	25	1̄.70966	31	0.29034	1̄.94936	6	52
9	1̄.65927	25	1̄.70997	31	0.29003	1̄.94930	7	51
10	1̄.65952	24	1̄.71028	31	0.28972	1̄.94923	6	50
11	1̄.65976	25	1̄.71059	31	0.28941	1̄.94917	6	49
12	1̄.66001	24	1̄.71090	31	0.28910	1̄.94911	7	48
13	1̄.66025	25	1̄.71121	32	0.28879	1̄.94904	6	47
14	1̄.66050	25	1̄.71153	31	0.28847	1̄.94898	7	46
15	1̄.66075	24	1̄.71184	31	0.28816	1̄.94891	6	45
16	1̄.66099	25	1̄.71215	31	0.28785	1̄.94885	7	44
17	1̄.66124	24	1̄.71246	31	0.28754	1̄.94878	7	43
18	1̄.66148	25	1̄.71277	31	0.28723	1̄.94871	6	42
19	1̄.66173	24	1̄.71308	31	0.28692	1̄.94865	7	41
20	1̄.66197	24	1̄.71339	31	0.28661	1̄.94858	6	40
21	1̄.66221	25	1̄.71370	31	0.28630	1̄.94852	7	39
22	1̄.66246	24	1̄.71401	30	0.28599	1̄.94845	6	38
23	1̄.66270	25	1̄.71431	31	0.28569	1̄.94839	7	37
24	1̄.66295	24	1̄.71462	31	0.28538	1̄.94832	6	36
25	1̄.66319	24	1̄.71493	31	0.28507	1̄.94826	7	35
26	1̄.66343	25	1̄.71524	31	0.28476	1̄.94819	6	34
27	1̄.66368	24	1̄.71555	31	0.28445	1̄.94813	7	33
28	1̄.66392	24	1̄.71586	31	0.28414	1̄.94806	7	32
29	1̄.66416	25	1̄.71617	31	0.28383	1̄.94799	6	31
30	1̄.66441	24	1̄.71648	31	0.28352	1̄.94793	7	30
31	1̄.66465	24	1̄.71679	30	0.28321	1̄.94786	6	29
32	1̄.66489	24	1̄.71709	31	0.28291	1̄.94780	7	28
33	1̄.66513	24	1̄.71740	31	0.28260	1̄.94773	6	27
34	1̄.66537	25	1̄.71771	31	0.28229	1̄.94767	7	26
35	1̄.66562	24	1̄.71802	31	0.28198	1̄.94760	7	25
36	1̄.66586	24	1̄.71833	30	0.28167	1̄.94753	6	24
37	1̄.66610	24	1̄.71863	31	0.28137	1̄.94747	7	23
38	1̄.66634	24	1̄.71894	31	0.28106	1̄.94740	6	22
39	1̄.66658	24	1̄.71925	30	0.28075	1̄.94734	7	21
40	1̄.66682	24	1̄.71955	31	0.28045	1̄.94727	7	20
41	1̄.66706	25	1̄.71986	31	0.28014	1̄.94720	6	19
42	1̄.66731	24	1̄.72017	31	0.27983	1̄.94714	7	18
43	1̄.66755	24	1̄.72048	30	0.27952	1̄.94707	7	17
44	1̄.66779	24	1̄.72078	31	0.27922	1̄.94700	6	16
45	1̄.66803	24	1̄.72109	31	0.27891	1̄.94694	7	15
46	1̄.66827	24	1̄.72140	30	0.27860	1̄.94687	7	14
47	1̄.66851	24	1̄.72170	31	0.27830	1̄.94680	6	13
48	1̄.66875	24	1̄.72201	30	0.27799	1̄.94674	7	12
49	1̄.66899	23	1̄.72231	31	0.27769	1̄.94667	7	11
50	1̄.66922	24	1̄.72262	31	0.27738	1̄.94660	6	10
51	1̄.66946	24	1̄.72293	30	0.27707	1̄.94654	7	9
52	1̄.66970	24	1̄.72323	31	0.27677	1̄.94647	7	8
53	1̄.66994	24	1̄.72354	30	0.27646	1̄.94640	6	7
54	1̄.67018	24	1̄.72384	31	0.27616	1̄.94634	7	6
55	1̄.67042	24	1̄.72415	30	0.27585	1̄.94627	7	5
56	1̄.67066	24	1̄.72445	31	0.27555	1̄.94620	6	4
57	1̄.67090	23	1̄.72476	30	0.27524	1̄.94614	7	3
58	1̄.67113	24	1̄.72506	31	0.27494	1̄.94607	7	2
59	1̄.67137	24	1̄.72537	30	0.27463	1̄.94600	7	1
60	1̄.67161		1̄.72567		0.27433	1̄.94593		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

P. P.

	32	31	30	25	24	23	7	6
6	3.2	3.1	3.0	2.5	2.4	2.3	0.7	0.6
7	3.7	3.6	3.5	2.9	2.8	2.7	0.8	0.7
8	4.3	4.1	4.0	3.3	3.2	3.1	0.9	0.8
9	4.8	4.7	4.5	3.8	3.6	3.5	1.1	0.9
10	5.3	5.2	5.0	4.2	4.0	3.8	1.2	1.0
20	10.7	10.3	10.0	8.3	8.0	7.7	2.3	2.0
30	16.0	15.5	15.0	12.5	12.0	11.5	3.5	3.0
40	21.3	20.7	20.0	16.7	16.0	15.3	4.7	4.0
50	26.7	25.8	25.0	20.8	20.0	19.2	5.8	5.0

28°

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.67161	24	1̄.72567	31	0.27433	1̄.94593	6	60
1	1̄.67185	23	1̄.72598	30	0.27402	1̄.94587	7	59
2	1̄.67208	24	1̄.72628	31	0.27372	1̄.94580	7	58
3	1̄.67232	24	1̄.72659	30	0.27341	1̄.94573	6	57
4	1̄.67256	24	1̄.72689	31	0.27311	1̄.94567	7	56
5	1̄.67280	23	1̄.72720	30	0.27280	1̄.94560	7	55
6	1̄.67303	24	1̄.72750	30	0.27250	1̄.94553	7	54
7	1̄.67327	23	1̄.72780	31	0.27220	1̄.94546	6	53
8	1̄.67350	24	1̄.72811	30	0.27189	1̄.94540	7	52
9	1̄.67374	24	1̄.72841	31	0.27159	1̄.94533	7	51
10	1̄.67398	23	1̄.72872	30	0.27128	1̄.94526	7	50
11	1̄.67421	24	1̄.72902	30	0.27098	1̄.94519	6	49
12	1̄.67445	23	1̄.72932	31	0.27068	1̄.94513	7	48
13	1̄.67468	24	1̄.72963	30	0.27037	1̄.94506	7	47
14	1̄.67492	23	1̄.72993	30	0.27007	1̄.94499	7	46
15	1̄.67515	24	1̄.73023	31	0.26977	1̄.94492	7	45
16	1̄.67539	23	1̄.73054	30	0.26946	1̄.94485	6	44
17	1̄.67562	24	1̄.73084	30	0.26916	1̄.94479	7	43
18	1̄.67586	23	1̄.73114	30	0.26886	1̄.94472	7	42
19	1̄.67609	24	1̄.73144	31	0.26856	1̄.94465	7	41
20	1̄.67633	23	1̄.73175	30	0.26825	1̄.94458	7	40
21	1̄.67656	24	1̄.73205	30	0.26795	1̄.94451	6	39
22	1̄.67680	23	1̄.73235	30	0.26765	1̄.94445	7	38
23	1̄.67703	23	1̄.73265	30	0.26735	1̄.94438	7	37
24	1̄.67726	24	1̄.73295	31	0.26705	1̄.94431	7	36
25	1̄.67750	23	1̄.73326	30	0.26674	1̄.94424	7	35
26	1̄.67773	23	1̄.73356	30	0.26644	1̄.94417	7	34
27	1̄.67796	24	1̄.73386	30	0.26614	1̄.94410	6	33
28	1̄.67820	23	1̄.73416	30	0.26584	1̄.94404	7	32
29	1̄.67843	23	1̄.73446	30	0.26554	1̄.94397	7	31
30	1̄.67866	24	1̄.73476	31	0.26524	1̄.94390	7	30
31	1̄.67890	23	1̄.73507	30	0.26493	1̄.94383	7	29
32	1̄.67913	23	1̄.73537	30	0.26463	1̄.94376	7	28
33	1̄.67936	23	1̄.73567	30	0.26433	1̄.94369	7	27
34	1̄.67959	23	1̄.73597	30	0.26403	1̄.94362	7	26
35	1̄.67982	24	1̄.73627	30	0.26373	1̄.94355	6	25
36	1̄.68006	23	1̄.73657	30	0.36343	1̄.94349	7	24
37	1̄.68029	23	1̄.73687	30	0.26313	1̄.94342	7	23
38	1̄.68052	23	1̄.73717	30	0.26283	1̄.94335	7	22
39	1̄.68075	23	1̄.73747	30	0.26253	1̄.94328	7	21
40	1̄.68098	23	1̄.73777	30	0.26223	1̄.94321	7	20
41	1̄.68121	23	1̄.73807	30	0.26193	1̄.94314	7	19
42	1̄.68144	23	1̄.73837	30	0.26163	1̄ 94307	7	18
43	1̄.68167	23	1̄.73867	30	0.26133	1̄.94300	7	17
44	1̄.68190	23	1̄.73897	30	0.26103	1̄.94293	7	16
45	1̄.68213	24	1̄.73927	30	0.26073	1̄.94286	7	15
46	1̄.68237	23	1̄.73957	30	0.26043	1̄.94279	6	14
47	1̄.68260	23	1̄.73987	30	0.26013	1̄.94273	7	13
48	1̄.68283	22	1̄.74017	30	0.25983	1̄.94266	7	12
49	1̄.68305	23	1̄.74047	30	0.25953	1̄.94259	7	11
50	1̄.68328	23	1̄.74077	30	0.25923	1̄.94252	7	10
51	1̄.68251	23	1̄.74107	30	0.25893	1̄.94245	7	9
52	1̄.68374	23	1̄.74137	29	0.25863	1̄.94238	7	8
53	1̄.68397	23	1̄.74166	30	0.25834	1̄.94231	7	7
54	1̄.68420	23	1̄.74196	30	0.25804	1̄.94224	7	6
55	1̄.68443	23	1̄.74226	30	0.25774	1̄.94217	7	5
56	1̄.68466	23	1̄.74256	30	0.25744	1̄.94210	7	4
57	1̄.68489	23	1̄.74286	30	0.25714	1̄.94203	7	3
58	1̄.68512	22	1̄.74316	29	0.25684	1̄.94196	7	2
59	1̄.68534	23	1̄.74345	30	0.25655	1̄.94189	7	1
60	1̄.68557		1̄.74375		0.25625	1̄.94182		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	31	30
6	3.1	3.0
7	3.6	3.5
8	4.1	4.0
9	4.7	4.5
10	5.2	5.0
20	10.3	10.0
30	15.5	15.0
40	20.7	20.0
50	25.8	25.0

	29
6	2.9
7	3.4
8	3.9
9	4.4
10	4.8
20	9.7
30	14.5
40	19.3
50	24.2

	24	23
6	2.4	2.3
7	2.8	2.7
8	3.2	3.1
9	3.6	3.5
10	4.0	3.8
20	8.0	7.7
30	12.0	11.5
40	16.0	15.3
50	20.0	19.2

	22
6	2.2
7	2.6
8	2.9
9	3.3
10	3.7
20	7.3
30	11.0
40	14.7
50	18.3

	7	6
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.1	0.9
10	1.2	1.0
20	2.3	2.0
30	3.5	3.0
40	4.7	4.0
50	5.8	5.0

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	$\bar{1}$.68557	23	$\bar{1}$.74375	30	0.25625	$\bar{1}$.94182	7	60
1	$\bar{1}$.68580	23	$\bar{1}$.74405	30	0.25595	$\bar{1}$.94175	7	59
2	$\bar{1}$.68603	22	$\bar{1}$.74435	30	0.25565	$\bar{1}$.94168	7	58
3	$\bar{1}$.68625	23	$\bar{1}$.74465	29	0.25535	$\bar{1}$.94161	7	57
4	$\bar{1}$.68648	23	$\bar{1}$.74494	30	0.25506	$\bar{1}$.94154	7	56
5	$\bar{1}$.68671	23	$\bar{1}$.74524	30	0.25476	$\bar{1}$.94147	7	55
6	$\bar{1}$.68694	22	$\bar{1}$.74554	29	0.25446	$\bar{1}$.94140	7	54
7	$\bar{1}$.68716	23	$\bar{1}$.74583	30	0.25417	$\bar{1}$.94133	7	53
8	$\bar{1}$.68739	23	$\bar{1}$.74613	30	0.25387	$\bar{1}$.94126	7	52
9	$\bar{1}$.68762	22	$\bar{1}$.74643	30	0.25357	$\bar{1}$.94119	7	51
10	$\bar{1}$.68784	23	$\bar{1}$.74673	29	0.25327	$\bar{1}$.94112	7	50
11	$\bar{1}$.68807	22	$\bar{1}$.74702	30	0.25298	$\bar{1}$.94105	7	49
12	$\bar{1}$.68829	23	$\bar{1}$.74732	30	0.25268	$\bar{1}$.94098	8	48
13	$\bar{1}$.68852	23	$\bar{1}$.74762	29	0.25238	$\bar{1}$.94090	7	47
14	$\bar{1}$.68875	22	$\bar{1}$.74791	30	0.25209	$\bar{1}$.94083	7	46
15	$\bar{1}$.68897	23	$\bar{1}$.74821	30	0.25179	$\bar{1}$.94076	7	45
16	$\bar{1}$.68920	22	$\bar{1}$.74851	29	0.25149	$\bar{1}$.94069	7	44
17	$\bar{1}$.68942	23	$\bar{1}$.74880	30	0.25120	$\bar{1}$.94062	7	43
18	$\bar{1}$.68965	22	$\bar{1}$.74910	29	0.25090	$\bar{1}$.94055	7	42
19	$\bar{1}$.68987	23	$\bar{1}$.74939	30	0.25061	$\bar{1}$.94048	7	41
20	$\bar{1}$.69010	22	$\bar{1}$.74969	29	0.25031	$\bar{1}$.94041	7	40
21	$\bar{1}$.69032	23	$\bar{1}$.74998	30	0.25002	$\bar{1}$.94034	7	39
22	$\bar{1}$.69055	22	$\bar{1}$.75028	30	0.24972	$\bar{1}$.94027	7	38
23	$\bar{1}$.69077	23	$\bar{1}$.75058	29	0.24942	$\bar{1}$.94020	8	37
24	$\bar{1}$.69100	22	$\bar{1}$.75087	30	0.24913	$\bar{1}$.94012	7	36
25	$\bar{1}$.69122	22	$\bar{1}$.75117	29	0.24883	$\bar{1}$.94005	7	35
26	$\bar{1}$.69144	23	$\bar{1}$.75146	30	0.24854	$\bar{1}$.93998	7	34
27	$\bar{1}$.69167	22	$\bar{1}$.75176	29	0.24824	$\bar{1}$.93991	7	33
28	$\bar{1}$.69189	23	$\bar{1}$.75205	30	0.24795	$\bar{1}$.93984	7	32
29	$\bar{1}$.69212	22	$\bar{1}$.75235	29	0.24765	$\bar{1}$.93977	7	31
30	$\bar{1}$.69234	22	$\bar{1}$.75264	30	0.24736	$\bar{1}$.93970	7	30
31	$\bar{1}$.69256	23	$\bar{1}$.75294	29	0.24706	$\bar{1}$.93963	8	29
32	$\bar{1}$.69279	22	$\bar{1}$.75323	30	0.24677	$\bar{1}$.93955	7	28
33	$\bar{1}$.69301	22	$\bar{1}$.75353	29	0.24647	$\bar{1}$.93948	7	27
34	$\bar{1}$.69323	22	$\bar{1}$.75382	29	0.24618	$\bar{1}$.93941	7	26
35	$\bar{1}$.69345	23	$\bar{1}$.75411	30	0.24589	$\bar{1}$.93934	7	25
36	$\bar{1}$.69368	22	$\bar{1}$.75441	29	0.24559	$\bar{1}$.93927	7	24
37	$\bar{1}$.69390	22	$\bar{1}$.75470	30	0.24530	$\bar{1}$.93920	8	23
38	$\bar{1}$.69412	22	$\bar{1}$.75500	29	0.24500	$\bar{1}$.93912	7	22
39	$\bar{1}$.69434	22	$\bar{1}$.75529	29	0.24471	$\bar{1}$.93905	7	21
40	$\bar{1}$.69456	23	$\bar{1}$.75558	30	0.24442	$\bar{1}$.93898	7	20
41	$\bar{1}$.69479	22	$\bar{1}$.75588	29	0.24412	$\bar{1}$.93891	7	19
42	$\bar{1}$.69501	22	$\bar{1}$.75617	30	0.24383	$\bar{1}$.93884	8	18
43	$\bar{1}$.69523	22	$\bar{1}$.75647	29	0.24353	$\bar{1}$.93876	7	17
44	$\bar{1}$.69545	22	$\bar{1}$.75676	29	0.24324	$\bar{1}$.93869	7	16
45	$\bar{1}$.69567	22	$\bar{1}$.75705	30	0.24295	$\bar{1}$.93862	7	15
46	$\bar{1}$.69589	22	$\bar{1}$.75735	29	0.24265	$\bar{1}$.93855	8	14
47	$\bar{1}$.69611	22	$\bar{1}$.75764	29	0.24236	$\bar{1}$.93847	7	13
48	$\bar{1}$.69633	22	$\bar{1}$.75793	29	0.24207	$\bar{1}$.93840	7	12
49	$\bar{1}$.69655	22	$\bar{1}$.75822	30	0.24178	$\bar{1}$.93833	7	11
50	$\bar{1}$.69677	22	$\bar{1}$.75852	29	0.24148	$\bar{1}$.93826	7	10
51	$\bar{1}$.69699	22	$\bar{1}$.75881	29	0.24119	$\bar{1}$.93819	8	9
52	$\bar{1}$.69721	22	$\bar{1}$.75910	29	0.24090	$\bar{1}$.93811	7	8
53	$\bar{1}$.69743	22	$\bar{1}$.75939	30	0.24061	$\bar{1}$.93804	7	7
54	$\bar{1}$.69765	22	$\bar{1}$.75969	29	0.24031	$\bar{1}$.93797	8	6
55	$\bar{1}$.69787	22	$\bar{1}$.75998	29	0.24002	$\bar{1}$.93789	7	5
56	$\bar{1}$.69809	22	$\bar{1}$.76027	29	0.23973	$\bar{1}$.93782	7	4
57	$\bar{1}$.69831	22	$\bar{1}$.76056	30	0.23944	$\bar{1}$.93775	7	3
58	$\bar{1}$.69853	22	$\bar{1}$.76086	29	0.23914	$\bar{1}$.93768	8	2
59	$\bar{1}$.69875	22	$\bar{1}$.76115	29	0.23885	$\bar{1}$.93760	7	1
60	$\bar{1}$.69897		$\bar{1}$.76144		0.23856	$\bar{1}$.93753		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	30
6	3.0
7	3.5
8	4.0
9	4.5
10	5.0
20	10.0
30	15.0
40	20.0
50	25.0

	29
6	2.9
7	3.4
8	3.9
9	4.4
10	4.8
20	9.7
30	14.5
40	19.3
50	24.2

	23
6	2.3
7	2.7
8	3.1
9	3.5
10	3.8
20	7.7
30	11.5
40	15.3
50	19.2

	22
6	2.2
7	2.6
8	2.9
9	3.3
10	3.7
20	7.3
30	11.0
40	14.7
50	18.3

	8	7
6	0.8	0.7
7	0.9	0.8
8	1.1	0.9
9	1.2	1.1
10	1.3	1.2
20	2.7	2.3
30	4.0	3.5
40	5.3	4.7
50	6.7	5.8

30°

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.69897		1̄.76144		0.23856	1̄.93753		60
1	1̄.69919	22	1̄.76173	29	0.23827	1̄.93746	7	59
2	1̄.69941	22	1̄.76202	29	0.23798	1̄.93738	8	58
3	1̄.69963	22	1̄.76231	29	0.23769	1̄.93731	7	57
4	1̄.69984	21	1̄.76261	30	0.23739	1̄.93724	7	56
5	1̄.70006	22	1̄.76290	29	0.23710	1̄.93717	7	55
6	1̄.70028	22	1̄.76319	29	0.23681	1̄.93709	8	54
7	1̄.70050	22	1̄.76348	29	0.23652	1̄.93702	7	53
8	1̄.70072	22	1̄.76377	29	0.23623	1̄.93695	7	52
9	1̄.70093	21	1̄.76406	29	0.23594	1̄.93687	8	51
10	1̄.70115	22	1̄.76435	29	0.23565	1̄.93680	7	50
11	1̄.70137	22	1̄.76464	29	0.23536	1̄.93673	7	49
12	1̄.70159	22	1̄.76493	29	0.23507	1̄.93665	8	48
13	1̄.70180	21	1̄.76522	29	0.23478	1̄.93658	7	47
14	1̄.70202	22	1̄.76551	29	0.23449	1̄.93650	8	46
15	1̄.70224	22	1̄.76580	29	0.23420	1̄.93643	7	45
16	1̄.70245	21	1̄.76609	29	0.23391	1̄.93636	7	44
17	1̄.70267	22	1̄.76639	30	0.23361	1̄.93628	8	43
18	1̄.70288	21	1̄.76668	29	0.23332	1̄.93621	7	42
19	1̄.70310	22	1̄.76697	29	0.23303	1̄.93614	7	41
20	1̄.70332	22	1̄.76725	28	0.23275	1̄.93606	8	40
21	1̄.70353	21	1̄.76754	29	0.23246	1̄.93599	7	39
22	1̄.70375	22	1̄.76783	29	0.23217	1̄.93591	8	38
23	1̄.70396	21	1̄.76812	29	0.23188	1̄.93584	7	37
24	1̄.70418	22	1̄.76841	29	0.23159	1̄.93577	7	36
25	1̄.70439	21	1̄.76870	29	0.23130	1̄.93569	8	35
26	1̄.70461	22	1̄.76899	29	0.23101	1̄.93562	7	34
27	1̄.70482	21	1̄.76928	29	0.23072	1̄.93554	8	33
28	1̄.70504	22	1̄.76957	29	0.23043	1̄.93547	7	32
29	1̄.70525	21	1̄.76986	29	0.23014	1̄.93539	8	31
30	1̄.70547	22	1̄.77015	29	0.22985	1̄.93532	7	30
31	1̄.70568	21	1̄.77044	29	0.22956	1̄.93525	7	29
32	1̄.70590	22	1̄.77073	29	0.22927	1̄.93517	8	28
33	1̄.70611	21	1̄.77101	28	0.22899	1̄.93510	7	27
34	1̄.70633	22	1̄.77130	29	0.22870	1̄.93502	8	26
35	1̄.70654	21	1̄.77159	29	0.22841	1̄.93495	7	25
36	1̄.70675	21	1̄.77188	29	0.22812	1̄.93487	8	24
37	1̄.70697	22	1̄.77217	29	0.22783	1̄.93480	7	23
38	1̄.70718	21	1̄.77246	29	0.22754	1̄.93472	8	22
39	1̄.70739	21	1̄.77274	28	0.22726	1̄.93465	7	21
40	1̄.70761	22	1̄.77303	29	0.22697	1̄.93457	8	20
41	1̄.70782	21	1̄.77332	29	0.22668	1̄.93450	7	19
42	1̄.70803	21	1̄.77361	29	0.22639	1̄.93442	8	18
43	1̄.70824	21	1̄.77390	29	0.22610	1̄.93435	7	17
44	1̄.70846	22	1̄.77418	28	0.22582	1̄.93427	8	16
45	1̄.70867	21	1̄.77447	29	0.22553	1̄.93420	7	15
46	1̄.70888	21	1̄.77476	29	0.22524	1̄.93412	8	14
47	1̄.70909	21	1̄.77505	29	0.22495	1̄.93405	7	13
48	1̄.70931	22	1̄.77533	28	0.22467	1̄.93397	8	12
49	1̄.70952	21	1̄.77562	29	0.22438	1̄.93390	7	11
50	1̄.70973	21	1̄.77591	29	0.22409	1̄.93382	8	10
51	1̄.70994	21	1̄.77619	28	0.22381	1̄.93375	7	9
52	1̄.71015	21	1̄.77648	29	0.22352	1̄.93367	8	8
53	1̄.71036	21	1̄.77677	29	0.22323	1̄.93360	7	7
54	1̄.71058	22	1̄.77706	29	0.22294	1̄.93352	8	6
55	1̄.71079	21	1̄.77734	28	0.22266	1̄.93344	8	5
56	1̄.71100	21	1̄.77763	29	0.22237	1̄.93337	7	4
57	1̄.71121	21	1̄.77791	28	0.22209	1̄.93329	8	3
58	1̄.71142	21	1̄.77820	29	0.22180	1̄.93322	7	2
59	1̄.71163	21	1̄.77849	29	0.22151	1̄.93314	8	1
60	1̄.71184	21	1̄.77877	28	0.22123	1̄.93307	7	0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	30	29
6	3.0	2.9
7	3.5	3.4
8	4.0	3.9
9	4.5	4.4
10	5.0	4.8
20	10.0	9.7
30	15.0	14.5
40	20.0	19.3
50	25.0	24.2

	28
6	2.8
7	3.3
8	3.7
9	4.2
10	4.7
20	9.3
30	14.0
40	18.7
50	23.3

	22
6	2.2
7	2.6
8	2.9
9	3.3
10	3.7
20	7.3
30	11.0
40	14.7
50	18.3

	21
6	2.1
7	2.5
8	2.8
9	3.2
10	3.5
20	7.0
30	10.5
40	14.0
50	17.5

	8	7
6	0.8	0.7
7	0.9	0.8
8	1.1	0.9
9	1.2	1.1
10	1.3	1.2
20	2.7	2.3
30	4.0	3.5
40	5.3	4.7
50	6.7	5.8

p. p.

31°

'	log sin	d	log tan	c. d.	log cot	log cos	d	'
0	1̄.71184	21	1̄.77877	29	0.22123	1̄.93307	8	60
1	1̄.71205	21	1̄.77906	29	0.22094	1̄.93299	8	59
2	1̄.71226	21	1̄.77935	28	0.22065	1̄.93291	7	58
3	1̄.71247	21	1̄.77963	29	0.22037	1̄.93284	8	57
4	1̄.71268	21	1̄.77992	28	0.22008	1̄.93276	7	56
5	1̄.71289	21	1̄.78020	29	0.21980	1̄.93269	8	55
6	1̄.71310	21	1̄.78049	28	0.21951	1̄.93261	8	54
7	1̄.71331	21	1̄.78077	29	0.21923	1̄.93253	7	53
8	1̄.71352	21	1̄.78106	29	0.21894	1̄.93246	8	52
9	1̄.71373	20	1̄.78135	28	0.21865	1̄.93238	8	51
10	1̄.71393	21	1̄.78163	29	0.21837	1̄.93230	7	50
11	1̄.71414	21	1̄.78192	28	0.21808	1̄.93223	8	49
12	1̄.71435	21	1̄.78220	29	0.21780	1̄.93215	8	48
13	1̄.71456	21	1̄.78249	28	0.21751	1̄.93207	7	47
14	1̄.71477	21	1̄.78277	29	0.21723	1̄.93200	8	46
15	1̄.71498	21	1̄.78306	28	0.21694	1̄.93192	8	45
16	1̄.71519	20	1̄.78334	29	0.21666	1̄.93184	7	44
17	1̄.71539	21	1̄.78363	28	0.21637	1̄.93177	8	43
18	1̄.71560	21	1̄.78391	28	0.21609	1̄.93169	8	42
19	1̄.71581	21	1̄.78419	29	0.21581	1̄.93161	7	41
20	1̄.71602	20	1̄.78448	28	0.21552	1̄.93154	8	40
21	1̄.71622	21	1̄.78476	29	0.21524	1̄.93146	8	39
22	1̄.71643	21	1̄.78505	28	0.21495	1̄.93138	7	38
23	1̄.71664	21	1̄.78533	29	0.21467	1̄.93131	8	37
24	1̄.71685	20	1̄.78562	28	0.21438	1̄.93123	8	36
25	1̄.71705	21	1̄.78590	28	0.21410	1̄.93115	7	35
26	1̄.71726	21	1̄.78618	29	0.21382	1̄.93108	8	34
27	1̄.71747	20	1̄.78647	28	0.21353	1̄.93100	8	33
28	1̄.71767	21	1̄.78675	29	0.21325	1̄.93092	8	32
29	1̄.71788	21	1̄.78704	28	0.21296	1̄.93084	7	31
30	1̄.71809	20	1̄.78732	28	0.21268	1̄.93077	8	30
31	1̄.71829	21	1̄.78760	29	0.21240	1̄.93069	8	29
32	1̄.71850	20	1̄.78789	28	0.21211	1̄.93061	8	28
33	1̄.71870	21	1̄.78817	28	0.21183	1̄.93053	7	27
34	1̄.71891	20	1̄.78845	29	0.21155	1̄.93046	8	26
35	1̄.71911	21	1̄.78874	28	0.21126	1̄.93038	8	25
36	1̄.71932	20	1̄.78902	28	0.21098	1̄.93030	8	24
37	1̄.71952	21	1̄.78930	29	0.21070	1̄.93022	8	23
38	1̄.71973	21	1̄.78959	28	0.21041	1̄.93014	7	22
39	1̄.71994	20	1̄.78987	28	0.21013	1̄.93007	8	21
40	1̄.72014	20	1̄.79015	28	0.20985	1̄.92999	8	20
41	1̄.72034	21	1̄.79043	29	0.20957	1̄.92991	8	19
42	1̄.72055	20	1̄.79072	28	0.20928	1̄.92983	7	18
43	1̄.72075	21	1̄.79100	28	0.20900	1̄.92976	8	17
44	1̄.72096	20	1̄.79128	28	0.20872	1̄.92968	8	16
45	1̄.72116	21	1̄.79156	29	0.20844	1̄.92960	8	15
46	1̄.72137	20	1̄.79185	28	0.20815	1̄.92952	8	14
47	1̄.72157	20	1̄.79213	28	0.20787	1̄.92944	8	13
48	1̄.72177	21	1̄.79241	28	0.20759	1̄.92936	7	12
49	1̄.72198	20	1̄.79269	28	0.20731	1̄.92929	8	11
50	1̄.72218	20	1̄.79297	29	0.20703	1̄.92921	8	10
51	1̄.72238	21	1̄.79326	28	0.20674	1̄.92913	8	9
52	1̄.72259	20	1̄.79354	28	0.20646	1̄.92905	8	8
53	1̄.72279	20	1̄.79382	28	0.20618	1̄.92897	8	7
54	1̄.72299	21	1̄.79410	28	0.20590	1̄.92889	8	6
55	1̄.72320	20	1̄.79438	28	0.20562	1̄.92881	7	5
56	1̄.72340	20	1̄.79466	29	0.20534	1̄.92874	8	4
57	1̄.72360	21	1̄.79495	28	0.20505	1̄.92866	8	3
58	1̄.72381	20	1̄.79523	28	0.20477	1̄.92858	8	2
59	1̄.72401	20	1̄.79551	28	0.20449	1̄.92850	8	1
60	1̄.72421		1̄.79579		0.20421	1̄.92842		0
	log cos	d	log cot	c. d.	log tan	log sin	d	'

p. p.

	29
6	2.9
7	3.4
8	3.9
9	4.4
10	4.8
20	9.7
30	14.5
40	19.3
50	24.2

	28
6	2.8
7	3.3
8	3.7
9	4.2
10	4.7
20	9.3
30	14.0
40	18.7
50	23.3

	21
6	2.1
7	2.5
8	2.8
9	3.2
10	3.5
20	7.0
30	10.5
40	14.0
50	17.5

	20
6	2.0
7	2.3
8	2.7
9	3.0
10	3.3
20	6.7
30	10.0
40	13.3
50	16.7

	8	7
6	0.8	0.7
7	0.9	0.8
8	1.1	0.9
9	1.2	1.1
10	1.3	1.2
20	2.7	2.3
30	4.0	3.5
40	5.3	4.7
50	6.7	5.8

p. p.

58°

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.72421	20	1̄.79579	28	0.20421	1̄.92842	8	60
1	1̄.72441	20	1̄.79607	28	0.20393	1̄.92834	8	59
2	1̄.72461	21	1̄.79635	28	0.20365	1̄.92826	8	58
3	1̄.72482	20	1̄.79663	28	0.20337	1̄.92818	8	57
4	1̄.72502	20	1̄.79691	28	0.20309	1̄.92810	7	56
5	1̄.72522	20	1̄.79719	28	0.20281	1̄.92803	8	55
6	1̄.72542	20	1̄.79747	29	0.20253	1̄.92795	8	54
7	1̄.72562	20	1̄.79776	28	0.20224	1̄.92787	8	53
8	1̄.72582	20	1̄.79804	28	0.20196	1̄.92779	8	52
9	1̄.72602	20	1̄.79832	28	0.20168	1̄.92771	8	51
10	1̄.72622	21	1̄.79860	28	0.20140	1̄.92763	8	50
11	1̄.72643	20	1̄.79888	28	0.20112	1̄.92755	8	49
12	1̄.72663	20	1̄.79916	28	0.20084	1̄.92747	8	48
13	1̄.72683	20	1̄.79944	28	0.20056	1̄.92739	8	47
14	1̄.72703	20	1̄.79972	28	0.20028	1̄.92731	8	46
15	1̄.72723	20	1̄.80000	28	0.20000	1̄.92723	8	45
16	1̄.72743	20	1̄.80028	28	0.19972	1̄.92715	8	44
17	1̄.72763	20	1̄.80056	28	0.19944	1̄.92707	8	43
18	1̄.72783	20	1̄.80084	28	0.19916	1̄.92699	8	42
19	1̄.72803	20	1̄.80112	28	0.19888	1̄.92691	8	41
20	1̄.72823	20	1̄.80140	28	0.19860	1̄.92683	8	40
21	1̄.72843	20	1̄.80168	27	0.19832	1̄.92675	8	39
22	1̄.72863	20	1̄.80195	28	0.19805	1̄.92667	8	38
23	1̄.72883	19	1̄.80223	28	0.19777	1̄.92659	8	37
24	1̄.72902	20	1̄.80251	28	0.19749	1̄.92651	8	36
25	1̄.72922	20	1̄.80279	28	0.19721	1̄.92643	8	35
26	1̄.72942	20	1̄.80307	28	0.19693	1̄.92635	8	34
27	1̄.72962	20	1̄.80335	28	0.19665	1̄.92627	8	33
28	1̄.72982	20	1̄.80363	28	0.19637	1̄.92619	8	32
29	1̄.73002	20	1̄.80391	28	0.19609	1̄.92611	8	31
30	1̄.73022	19	1̄.80419	28	0.19581	1̄.92603	8	30
31	1̄.73041	20	1̄.80447	27	0.19553	1̄.92595	8	29
32	1̄.73061	20	1̄.80474	28	0.19526	1̄.92587	8	28
33	1̄.73081	20	1̄.80502	28	0.19498	1̄.92579	8	27
34	1̄.73101	20	1̄.80530	28	0.19470	1̄.92571	8	26
35	1̄.73121	19	1̄.80558	28	0.19442	1̄.92563	8	25
36	1̄.73140	20	1̄.80586	28	0.19414	1̄.92555	9	24
37	1̄.73160	20	1̄.80614	28	0.19386	1̄.92546	8	23
38	1̄.73180	20	1̄.80642	27	0.19358	1̄.92538	8	22
39	1̄.73200	19	1̄.80669	28	0.19331	1̄.92530	8	21
40	1̄.73219	20	1̄.80697	28	0.19303	1̄.92522	8	20
41	1̄.73239	20	1̄.80725	28	0.19275	1̄.92514	8	19
42	1̄.73259	19	1̄.80753	28	0.19247	1̄.92506	8	18
43	1̄.73278	20	1̄.80781	27	0.19219	1̄.92498	8	17
44	1̄.73298	20	1̄.80808	28	0.19192	1̄.92490	8	16
45	1̄.73318	19	1̄.80836	28	0.19164	1̄.92482	9	15
46	1̄.73337	20	1̄.80864	28	0.19136	1̄.92473	8	14
47	1̄.73357	20	1̄.80892	27	0.19108	1̄.92465	8	13
48	1̄.73377	19	1̄.80919	28	0.19081	1̄.92457	8	12
49	1̄.73396	20	1̄.80947	28	0.19053	1̄.92449	8	11
50	1̄.73416	19	1̄.80975	28	0.19025	1̄.92441	8	10
51	1̄.73435	20	1̄.81003	27	0.18997	1̄.92433	8	9
52	1̄.73455	19	1̄.81030	28	0.18970	1̄.92425	9	8
53	1̄.73474	20	1̄.81058	28	0.18942	1̄.92416	8	7
54	1̄.73494	19	1̄.81086	27	0.18914	1̄.92408	8	6
55	1̄.73513	20	1̄.81113	28	0.18887	1̄.92400	8	5
56	1̄.73533	19	1̄.81141	28	0.18859	1̄.92392	8	4
57	1̄.73552	20	1̄.81169	27	0.18831	1̄.92384	8	3
58	1̄.73572	19	1̄.81196	28	0.18804	1̄.92376	9	2
59	1̄.73591	20	1̄.81224	28	0.18776	1̄.92367	8	1
60	1̄.73611		1̄.81252		0.18748	1̄.92359		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	29	28
6	2.9	2.8
7	3.4	3.3
8	3.9	3.7
9	4.4	4.2
10	4.8	4.7
20	9.7	9.3
30	14.5	14.0
40	19.3	18.7
50	24.2	23.3

	27
6	2.7
7	3.2
8	3.6
9	4.1
10	4.5
20	9.0
30	13.5
40	18.0
50	22.5

	21	20
6	2.1	2.0
7	2.5	2.3
8	2.8	2.7
9	3.2	3.0
10	3.5	3.3
20	7.0	6.7
30	10.5	10.0
40	14.0	13.3
50	17.5	16.7

	19
6	1.9
7	2.2
8	2.5
9	2.9
10	3.2
20	6.3
30	9.5
40	12.7
50	15.8

	9	8	7
6	0.9	0.8	0.7
7	1.1	0.9	0.8
8	1.2	1.1	0.9
9	1.4	1.2	1.1
10	1.5	1.3	1.2
20	3.0	2.7	2.3
30	4.5	4.0	3.5
40	6.0	5.3	4.7
50	7.5	6.7	5.8

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	′
0	1̄.73611	19	1̄.81252	27	0.18748	1̄.92359	8	60
1	1̄.73630	20	1̄.81279	28	0.18721	1̄.92351	8	59
2	1̄.73650	19	1̄.81307	28	0.18693	1̄.92343	8	58
3	1̄.73669	20	1̄.81335	27	0.18665	1̄.92335	9	57
4	1̄.73689	19	1̄.81362	28	0.18638	1̄.92326	8	56
5	1̄.73708	19	1̄.81390	28	0.18610	1̄.92318	8	55
6	1̄.73727	20	1̄.81418	27	0.18582	1̄.92310	8	54
7	1̄.73747	19	1̄.81445	28	0.18555	1̄.92302	9	53
8	1̄.73766	19	1̄.81473	27	0.18527	1̄.92293	8	52
9	1̄.73785	20	1̄.81500	28	0.18500	1̄.92285	8	51
10	1̄.73805	19	1̄.81528	28	0.18472	1̄.92277	8	50
11	1̄.73824	19	1̄.81556	27	0.18444	1̄.92269	9	49
12	1̄.73843	20	1̄.81583	28	0.18417	1̄.92260	8	48
13	1̄.73863	19	1̄.81611	27	0.18389	1̄.92252	8	47
14	1̄.73882	19	1̄.81638	28	0.18362	1̄.92244	9	46
15	1̄.73901	20	1̄.81666	27	0.18334	1̄.92235	8	45
16	1̄.73921	19	1̄.81693	28	0.18307	1̄.92227	8	44
17	1̄.73940	19	1̄.81721	27	0.18279	1̄.92219	8	43
18	1̄.73959	19	1̄.81748	28	0.18252	1̄.92211	9	42
19	1̄.73978	19	1̄.81776	27	0.18224	1̄.92202	8	41
20	1̄.73997	20	1̄.81803	28	0.18197	1̄.92194	8	40
21	1̄.74017	19	1̄.81831	27	0.18169	1̄.92186	9	39
22	1̄.74036	19	1̄.81858	28	0.18142	1̄.92177	8	38
23	1̄.74055	19	1̄.81886	27	0.18114	1̄.92169	8	37
24	1̄.74074	19	1̄.81913	28	0.18087	1̄.92161	9	36
25	1̄.74093	20	1̄.81941	27	0.18059	1̄.92152	8	35
26	1̄.74113	19	1̄.81968	28	0.18032	1̄.92144	8	34
27	1̄.74132	19	1̄.81996	27	0.18004	1̄.92136	9	33
28	1̄.74151	19	1̄.82023	28	0.17977	1̄.92127	8	32
29	1̄.74170	19	1̄.82051	27	0.17949	1̄.92119	8	31
30	1̄.74189	19	1̄.82078	28	0.17922	1̄.92111	9	30
31	1̄.74208	19	1̄.82106	27	0.17894	1̄.92102	8	29
32	1̄.74227	19	1̄.82133	28	0.17867	1̄.92094	8	28
33	1̄.74246	19	1̄.82161	27	0.17839	1̄.92086	9	27
34	1̄.74265	19	1̄.82188	27	0.17812	1̄.92077	8	26
35	1̄.74284	19	1̄.82215	28	0.17785	1̄.92069	9	25
36	1̄.74303	19	1̄.82243	27	0.17757	1̄.92060	8	24
37	1̄.74322	19	1̄.82270	28	0.17730	1̄.92052	8	23
38	1̄.74341	19	1̄.82298	27	0.17702	1̄.92044	9	22
39	1̄.74360	19	1̄.82325	27	0.17675	1̄.92035	8	21
40	1̄.74379	19	1̄.82352	28	0.17648	1̄.92027	9	20
41	1̄.74398	19	1̄.82380	27	0.17620	1̄.92018	8	19
42	1̄.74417	19	1̄.82407	28	0.17593	1̄.92010	8	18
43	1̄.74436	19	1̄.82435	27	0.17565	1̄.92002	9	17
44	1̄.74455	19	1̄.82462	27	0.17538	1̄.91993	8	16
45	1̄.74474	19	1̄.82489	28	0.17511	1̄.91985	9	15
46	1̄.74493	19	1̄.82517	27	0.17483	1̄.91976	8	14
47	1̄.74512	19	1̄.82544	27	0.17456	1̄.91968	9	13
48	1̄.74531	18	1̄.82571	28	0.17429	1̄.91959	8	12
49	1̄.74549	19	1̄.82599	27	0.17401	1̄.91951	9	11
50	1̄.74568	19	1̄.82626	27	0.17374	1̄.91942	8	10
51	1̄.74587	19	1̄.82653	28	0.17347	1̄.91934	9	9
52	1̄.74606	19	1̄.82681	27	0.17319	1̄.91925	8	8
53	1̄.74625	19	1̄.82708	27	0.17292	1̄.91917	9	7
54	1̄.74644	18	1̄.82735	27	0.17265	1̄.91908	8	6
55	1̄.74662	19	1̄.82762	28	0.17238	1̄.91900	9	5
56	1̄.74681	19	1̄.82790	27	0.17210	1̄.91891	8	4
57	1̄.74700	19	1̄.82817	27	0.17183	1̄.91883	9	3
58	1̄.74719	18	1̄.82844	27	0.17156	1̄.91874	8	2
59	1̄.74737	19	1̄.82871	28	0.17129	1̄.91866	9	1
60	1̄.74756		1̄.82899		0.17101	1̄.91857		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	28	27
6	2.8	2.7
7	3.3	3.2
8	3.7	3.6
9	4.2	4.1
10	4.7	4.5
20	9.3	9.0
30	14.0	13.5
40	18.7	18.0
50	23.3	22.5

	20
6	2.0
7	2.3
8	2.7
9	3.0
10	3.3
20	6.7
30	10.0
40	13.3
50	16.7

	19
6	1.9
7	2.2
8	2.5
9	2.9
10	3.2
20	6.3
30	9.5
40	12.7
50	15.8

	18
6	1.8
7	2.1
8	2.4
9	2.7
10	3.0
20	6.0
30	9.0
40	12.0
50	15.0

	9	8
6	0.9	0.8
7	1.1	0.9
8	1.2	1.1
9	1.4	1.2
10	1.5	1.3
20	3.0	2.7
30	4.5	4.0
40	6.0	5.3
50	7.5	6.7

p. p.

56°

34°

′	log sin	d	log tan	c. d.	log cot	log cos	d	′
0	$\bar{1}.74756$	19	$\bar{1}.82899$	27	0.17101	$\bar{1}.91857$	8	60
1	$\bar{1}.74775$	19	$\bar{1}.82926$	27	0.17074	$\bar{1}.91849$	9	59
2	$\bar{1}.74794$	18	$\bar{1}.82953$	27	0.17047	$\bar{1}.91840$	8	58
3	$\bar{1}.74812$	19	$\bar{1}.82980$	28	0.17020	$\bar{1}.91832$	9	57
4	$\bar{1}.74831$	19	$\bar{1}.83008$	27	0.16992	$\bar{1}.91823$	8	56
5	$\bar{1}.74850$	18	$\bar{1}.83035$	27	0.16965	$\bar{1}.91815$	9	55
6	$\bar{1}.74868$	19	$\bar{1}.83062$	27	0.16938	$\bar{1}.91806$	8	54
7	$\bar{1}.74887$	19	$\bar{1}.83089$	28	0.16911	$\bar{1}.91798$	9	53
8	$\bar{1}.74906$	18	$\bar{1}.83117$	27	0.16883	$\bar{1}.91789$	8	52
9	$\bar{1}.74924$	19	$\bar{1}.83144$	27	0.16856	$\bar{1}.91781$	9	51
10	$\bar{1}.74943$	18	$\bar{1}.83171$	27	0.16829	$\bar{1}.91772$	9	50
11	$\bar{1}.74961$	19	$\bar{1}.83198$	27	0.16802	$\bar{1}.91763$	8	49
12	$\bar{1}.74980$	19	$\bar{1}.83225$	27	0.16775	$\bar{1}.91755$	9	48
13	$\bar{1}.74999$	18	$\bar{1}.83252$	28	0.16748	$\bar{1}.91746$	8	47
14	$\bar{1}.75017$	19	$\bar{1}.83280$	27	0.16720	$\bar{1}.91738$	9	46
15	$\bar{1}.75036$	18	$\bar{1}.83307$	27	0.16693	$\bar{1}.91729$	9	45
16	$\bar{1}.75054$	19	$\bar{1}.83334$	27	0.16666	$\bar{1}.91720$	8	44
17	$\bar{1}.75073$	18	$\bar{1}.83361$	27	0.16639	$\bar{1}.91712$	9	43
18	$\bar{1}.75091$	19	$\bar{1}.83388$	27	0.16612	$\bar{1}.91703$	8	42
19	$\bar{1}.75110$	18	$\bar{1}.83415$	27	0.16585	$\bar{1}.91695$	9	41
20	$\bar{1}.75128$	19	$\bar{1}.83442$	28	0.16558	$\bar{1}.91686$	9	40
21	$\bar{1}.75147$	18	$\bar{1}.83470$	27	0.16530	$\bar{1}.91677$	8	39
22	$\bar{1}.75165$	19	$\bar{1}.83497$	27	0.16503	$\bar{1}.91669$	9	38
23	$\bar{1}.75184$	18	$\bar{1}.83524$	27	0.16476	$\bar{1}.91660$	9	37
24	$\bar{1}.75202$	19	$\bar{1}.83551$	27	0.16449	$\bar{1}.91651$	8	36
25	$\bar{1}.75221$	18	$\bar{1}.83578$	27	0.16422	$\bar{1}.91643$	9	35
26	$\bar{1}.75239$	19	$\bar{1}.83605$	27	0.16395	$\bar{1}.91634$	9	34
27	$\bar{1}.75258$	18	$\bar{1}.83632$	27	0.16368	$\bar{1}.91625$	8	33
28	$\bar{1}.75276$	18	$\bar{1}.83659$	27	0.16341	$\bar{1}.91617$	9	32
29	$\bar{1}.75294$	19	$\bar{1}.83686$	27	0.16314	$\bar{1}.91608$	9	31
30	$\bar{1}.75313$	18	$\bar{1}.83713$	27	0.16287	$\bar{1}.91599$	8	30
31	$\bar{1}.75331$	19	$\bar{1}.83740$	28	0.16260	$\bar{1}.91591$	9	29
32	$\bar{1}.75350$	18	$\bar{1}.83768$	27	0.16232	$\bar{1}.91582$	9	28
33	$\bar{1}.75368$	18	$\bar{1}.83795$	27	0.16205	$\bar{1}.91573$	8	27
34	$\bar{1}.75386$	19	$\bar{1}.83822$	27	0.16178	$\bar{1}.91565$	9	26
35	$\bar{1}.75405$	18	$\bar{1}.83849$	27	0.16151	$\bar{1}.91556$	9	25
36	$\bar{1}.75423$	18	$\bar{1}.83876$	27	0.16124	$\bar{1}.91547$	9	24
37	$\bar{1}.75441$	18	$\bar{1}.83903$	27	0.16097	$\bar{1}.91538$	8	23
38	$\bar{1}.75459$	19	$\bar{1}.83930$	27	0.16070	$\bar{1}.91530$	9	22
39	$\bar{1}.75478$	18	$\bar{1}.83957$	27	0.16043	$\bar{1}.91521$	9	21
40	$\bar{1}.75496$	18	$\bar{1}.83984$	27	0.16016	$\bar{1}.91512$	8	20
41	$\bar{1}.75514$	19	$\bar{1}.84011$	27	0.15989	$\bar{1}.91504$	9	19
42	$\bar{1}.75533$	18	$\bar{1}.84038$	27	0.15962	$\bar{1}.91495$	9	18
43	$\bar{1}.75551$	18	$\bar{1}.84065$	27	0.15935	$\bar{1}.91486$	9	17
44	$\bar{1}.75569$	18	$\bar{1}.84092$	27	0.15908	$\bar{1}.91477$	8	16
45	$\bar{1}.75587$	18	$\bar{1}.84119$	27	0.15881	$\bar{1}.91469$	9	15
46	$\bar{1}.75605$	19	$\bar{1}.84146$	27	0.15854	$\bar{1}.91460$	9	14
47	$\bar{1}.75624$	18	$\bar{1}.84173$	27	0.15827	$\bar{1}.91451$	9	13
48	$\bar{1}.75642$	18	$\bar{1}.84200$	27	0.15800	$\bar{1}.91442$	9	12
49	$\bar{1}.75660$	18	$\bar{1}.84227$	27	0.15773	$\bar{1}.91433$	8	11
50	$\bar{1}.75678$	18	$\bar{1}.84254$	26	0.15746	$\bar{1}.91425$	9	10
51	$\bar{1}.75696$	18	$\bar{1}.84280$	27	0.15720	$\bar{1}.91416$	9	9
52	$\bar{1}.75714$	19	$\bar{1}.84307$	27	0.15693	$\bar{1}.91407$	9	8
53	$\bar{1}.75733$	18	$\bar{1}.84334$	27	0.15666	$\bar{1}.91398$	9	7
54	$\bar{1}.75751$	18	$\bar{1}.84361$	27	0.15639	$\bar{1}.91389$	8	6
55	$\bar{1}.75769$	18	$\bar{1}.84388$	27	0.15612	$\bar{1}.91381$	9	5
56	$\bar{1}.75787$	18	$\bar{1}.84415$	27	0.15585	$\bar{1}.91372$	9	4
57	$\bar{1}.75805$	18	$\bar{1}.84442$	27	0.15558	$\bar{1}.91363$	9	3
58	$\bar{1}.75823$	18	$\bar{1}.84469$	27	0.15531	$\bar{1}.91354$	9	2
59	$\bar{1}.75841$	18	$\bar{1}.84496$	27	0.15504	$\bar{1}.91345$	9	1
60	$\bar{1}.75859$		$\bar{1}.84523$		0.15477	$\bar{1}.91336$		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	28	27
6	2.8	2.7
7	3.3	3.2
8	3.7	3.6
9	4.2	4.1
10	4.7	4.5
20	9.3	9.0
30	14.0	13.5
40	18.7	18.0
50	23.3	22.5

	26
6	2.6
7	3.0
8	3.5
9	3.9
10	4.3
20	8.7
30	13.0
40	17.3
50	21.7

	19
6	1.9
7	2.2
8	2.5
9	2.9
10	3.2
20	6.3
30	9.5
40	12.7
50	15.8

	18
6	1.8
7	2.1
8	2.4
9	2.7
10	3.0
20	6.0
30	9.0
40	12.0
50	15.0

	9	8
6	0.9	0.8
7	1.1	0.9
8	1.2	1.1
9	1.4	1.2
10	1.5	1.3
20	3.0	2.7
30	4.5	4.0
40	6.0	5.3
50	7.5	6.7

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	$\bar{1}$.75859	18	$\bar{1}$.84523	27	0.15477	$\bar{1}$.91336	8	60
1	$\bar{1}$.75877	18	$\bar{1}$.84550	26	0.15450	$\bar{1}$.91328	9	59
2	$\bar{1}$.75895	18	$\bar{1}$.84576	27	0.15424	$\bar{1}$.91319	9	58
3	$\bar{1}$.75913	18	$\bar{1}$.84603	27	0.15397	$\bar{1}$.91310	9	57
4	$\bar{1}$.75931	18	$\bar{1}$.84630	27	0.15370	$\bar{1}$.91301	9	56
5	$\bar{1}$.75949	18	$\bar{1}$.84657	27	0.15343	$\bar{1}$.91292	9	55
6	$\bar{1}$.75967	18	$\bar{1}$.84684	27	0.15316	$\bar{1}$.91283	9	54
7	$\bar{1}$.75985	18	$\bar{1}$.84711	27	0.15289	$\bar{1}$.91274	8	53
8	$\bar{1}$.76003	18	$\bar{1}$.84738	26	0.15262	$\bar{1}$.91266	9	52
9	$\bar{1}$.76021	18	$\bar{1}$.84764	27	0.15236	$\bar{1}$.91257	9	51
10	$\bar{1}$.76039	18	$\bar{1}$.84791	27	0.15209	$\bar{1}$.91248	9	50
11	$\bar{1}$.76057	18	$\bar{1}$.84818	27	0.15182	$\bar{1}$.91239	9	49
12	$\bar{1}$.76075	18	$\bar{1}$.84845	27	0.15155	$\bar{1}$.91230	9	48
13	$\bar{1}$.76093	18	$\bar{1}$.84872	27	0.15128	$\bar{1}$.91221	9	47
14	$\bar{1}$.76111	18	$\bar{1}$.84899	26	0.15101	$\bar{1}$.91212	9	46
15	$\bar{1}$.76129	17	$\bar{1}$.84925	27	0.15075	$\bar{1}$.91203	9	45
16	$\bar{1}$.76146	18	$\bar{1}$.84952	27	0.15048	$\bar{1}$.91194	9	44
17	$\bar{1}$.76164	18	$\bar{1}$.84979	27	0.15021	$\bar{1}$.91185	9	43
18	$\bar{1}$.76182	18	$\bar{1}$.85006	27	0.14994	$\bar{1}$.91176	9	42
19	$\bar{1}$.76200	18	$\bar{1}$.85033	26	0.14967	$\bar{1}$.91167	9	41
20	$\bar{1}$.76218	18	$\bar{1}$.85059	27	0.14941	$\bar{1}$.91158	9	40
21	$\bar{1}$.76236	17	$\bar{1}$.85086	27	0.14914	$\bar{1}$.91149	8	39
22	$\bar{1}$.76253	18	$\bar{1}$.85113	27	0.14887	$\bar{1}$.91141	9	38
23	$\bar{1}$.76271	18	$\bar{1}$.85140	26	0.14860	$\bar{1}$.91132	9	37
24	$\bar{1}$.76289	18	$\bar{1}$.85166	27	0.14834	$\bar{1}$.91123	9	36
25	$\bar{1}$.76307	17	$\bar{1}$.85193	27	0.14807	$\bar{1}$.91114	9	35
26	$\bar{1}$.76324	18	$\bar{1}$.85220	27	0.14780	$\bar{1}$.91105	9	34
27	$\bar{1}$.76342	18	$\bar{1}$.85247	26	0.14753	$\bar{1}$.91096	9	33
28	$\bar{1}$.76360	18	$\bar{1}$.85273	27	0.14727	$\bar{1}$.91087	9	32
29	$\bar{1}$.76378	17	$\bar{1}$.85300	27	0.14700	$\bar{1}$.91078	9	31
30	$\bar{1}$.76395	18	$\bar{1}$.85327	27	0.14673	$\bar{1}$.91069	9	30
31	$\bar{1}$.76413	18	$\bar{1}$.85354	26	0.14646	$\bar{1}$.91060	9	29
32	$\bar{1}$.76431	17	$\bar{1}$.85380	27	0.14620	$\bar{1}$.91051	9	28
33	$\bar{1}$.76448	18	$\bar{1}$.85407	27	0.14593	$\bar{1}$.91042	9	27
34	$\bar{1}$.76466	18	$\bar{1}$.85434	26	0.14566	$\bar{1}$.91033	10	26
35	$\bar{1}$.76484	17	$\bar{1}$.85460	27	0.14540	$\bar{1}$.91023	9	25
36	$\bar{1}$.76501	18	$\bar{1}$.85487	27	0.14513	$\bar{1}$.91014	9	24
37	$\bar{1}$.76519	18	$\bar{1}$.85514	26	0.14486	$\bar{1}$.91005	9	23
38	$\bar{1}$.76537	17	$\bar{1}$.85540	27	0.14460	$\bar{1}$.90996	9	22
39	$\bar{1}$.76554	18	$\bar{1}$.85567	27	0.14433	$\bar{1}$.90987	9	21
40	$\bar{1}$.76572	18	$\bar{1}$.85594	26	0.14406	$\bar{1}$.90978	9	20
41	$\bar{1}$.76590	17	$\bar{1}$.85620	27	0.14380	$\bar{1}$.90969	9	19
42	$\bar{1}$.76607	18	$\bar{1}$.85647	27	0.14353	$\bar{1}$.90960	9	18
43	$\bar{1}$.76625	17	$\bar{1}$.85674	26	0.14326	$\bar{1}$.90951	9	17
44	$\bar{1}$.76642	18	$\bar{1}$.85700	27	0.14300	$\bar{1}$.90942	9	16
45	$\bar{1}$.76660	17	$\bar{1}$.85727	27	0.14273	$\bar{1}$.90933	9	15
46	$\bar{1}$.76677	18	$\bar{1}$.85754	26	0.14246	$\bar{1}$.90924	9	14
47	$\bar{1}$.76695	17	$\bar{1}$.85780	27	0.14220	$\bar{1}$.90915	9	13
48	$\bar{1}$.76712	18	$\bar{1}$.85807	27	0.14193	$\bar{1}$.90906	10	12
49	$\bar{1}$.76730	17	$\bar{1}$.85834	26	0.14166	$\bar{1}$.90896	9	11
50	$\bar{1}$.76747	18	$\bar{1}$.85860	27	0.14140	$\bar{1}$.90887	9	10
51	$\bar{1}$.76765	17	$\bar{1}$.85887	26	0.14113	$\bar{1}$.90878	9	9
52	$\bar{1}$.76782	18	$\bar{1}$.85913	27	0.14087	$\bar{1}$.90869	9	8
53	$\bar{1}$.76800	17	$\bar{1}$.85940	27	0.14060	$\bar{1}$.90860	9	7
54	$\bar{1}$.76817	18	$\bar{1}$.85967	26	0.14033	$\bar{1}$.90851	9	6
55	$\bar{1}$.76835	17	$\bar{1}$.85993	27	0.14007	$\bar{1}$.90842	10	5
56	$\bar{1}$.76852	18	$\bar{1}$.86020	26	0.13980	$\bar{1}$.90832	9	4
57	$\bar{1}$.76870	17	$\bar{1}$.86046	27	0.13954	$\bar{1}$.90823	9	3
58	$\bar{1}$.76887	17	$\bar{1}$.86073	27	0.13927	$\bar{1}$.90814	9	2
59	$\bar{1}$.76904	18	$\bar{1}$.86100	26	0.13900	$\bar{1}$.90805	9	1
60	$\bar{1}$.76922		$\bar{1}$.86126		0.13874	$\bar{1}$.90796		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	27	26
6	2.7	2.6
7	3.2	3.0
8	3.6	3.5
9	4.1	3.9
10	4.5	4.3
20	9.0	8.7
30	13.5	13.0
40	18.0	17.3
50	22.5	21.7

	18
6	1.8
7	2.1
8	2.4
9	2.7
10	3.0
20	6.0
30	9.0
40	12.0
50	15.0

	17
6	1.7
7	2.0
8	2.3
9	2.6
10	2.8
20	5.7
30	8.5
40	11.3
50	14.2

	10
6	1.0
7	1.2
8	1.3
9	1.5
10	1.7
20	3.3
30	5.0
40	6.7
50	8.3

	9	8
6	0.9	0.8
7	1.1	0.9
8	1.2	1.1
9	1.4	1.2
10	1.5	1.3
20	3.0	2.7
30	4.5	4.0
40	6.0	5.3
50	7.5	6.7

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.76922	17	1̄.86126	27	0.13874	1̄.90796	9	60
1	1̄.76939	18	1̄.86153	26	0.13847	1̄.90787	10	59
2	1̄.76957	17	1̄.86179	27	0.13821	1̄.90777	9	58
3	1̄.76974	17	1̄.86206	26	0.13794	1̄.90768	9	57
4	1̄.76991	18	1̄.86232	27	0.13768	1̄.90759	9	56
5	1̄.77009	17	1̄.86259	26	0.13741	1̄.90750	9	55
6	1̄.77026	17	1̄.86285	27	0.13715	1̄.90741	10	54
7	1̄.77043	18	1̄.86312	26	0.13688	1̄.90731	9	53
8	1̄.77061	17	1̄.86338	27	0.13662	1̄.90722	9	52
9	1̄.77078	17	1̄.86365	27	0.13635	1̄.90713	9	51
10	1̄.77095	17	1̄.86392	26	0.13608	1̄.90704	10	50
11	1̄.77112	18	1̄.86418	27	0.13582	1̄.90694	9	49
12	1̄.77130	17	1̄.86445	26	0.13555	1̄.90685	9	48
13	1̄.77147	17	1̄.86471	27	0.13529	1̄.90676	9	47
14	1̄.77164	17	1̄.86498	26	0.13502	1̄.90667	10	46
15	1̄.77181	18	1̄.86524	27	0.13476	1̄.90657	9	45
16	1̄.77199	17	1̄.86551	26	0.13449	1̄.90648	9	44
17	1̄.77216	17	1̄.86577	26	0.13423	1̄.90639	9	43
18	1̄.77233	17	1̄.86603	27	0.13397	1̄.90630	10	42
19	1̄.77250	18	1̄.86630	26	0.13370	1̄.90620	9	41
20	1̄.77268	17	1̄.86656	27	0.13344	1̄.90611	9	40
21	1̄.77285	17	1̄.86683	26	0.13317	1̄.90602	10	39
22	1̄.77302	17	1̄.86709	27	0.13291	1̄.90592	9	38
23	1̄.77319	17	1̄.86736	26	0.13264	1̄.90583	9	37
24	1̄.77336	17	1̄.86762	27	0.13238	1̄.90574	9	36
25	1̄.77353	17	1̄.86789	26	0.13211	1̄.90565	10	35
26	1̄.77370	17	1̄.86815	27	0.13185	1̄.90555	9	34
27	1̄.77387	18	1̄.86842	26	0.13158	1̄.90546	9	33
28	1̄.77405	17	1̄.86868	26	0.13132	1̄.90537	10	32
29	1̄.77422	17	1̄.86894	27	0.13106	1̄.90527	9	31
30	1̄.77439	17	1̄.86921	26	0.13079	1̄.90518	9	30
31	1̄.77456	17	1̄.86947	27	0.13053	1̄.90509	10	29
32	1̄.77473	17	1̄.86974	26	0.13026	1̄.90499	9	28
33	1̄.77490	17	1̄.87000	27	0.13000	1̄.90490	10	27
34	1̄.77507	17	1̄.87027	26	0.12973	1̄.90480	9	26
35	1̄.77524	17	1̄.87053	26	0.12947	1̄.90471	9	25
36	1̄.77541	17	1̄.87079	27	0.12921	1̄.90462	10	24
37	1̄.77558	17	1̄.87106	26	0.12894	1̄.90452	9	23
38	1̄.77575	17	1̄.87132	26	0.12868	1̄.90443	9	22
39	1̄.77592	17	1̄.87158	27	0.12842	1̄.90434	10	21
40	1̄.77609	17	1̄.87185	26	0.12815	1̄.90424	9	20
41	1̄.77626	17	1̄.87211	27	0.12789	1̄.90415	10	19
42	1̄.77643	17	1̄.87238	26	0.12762	1̄.90405	9	18
43	1̄.77660	17	1̄.87264	26	0.12736	1̄.90396	10	17
44	1̄.77677	17	1̄.87290	27	0.12710	1̄.90386	9	16
45	1̄.77694	17	1̄.87317	26	0.12683	1̄.90377	9	15
46	1̄.77711	17	1̄.87343	26	0.12657	1̄.90368	10	14
47	1̄.77728	16	1̄.87369	27	0.12631	1̄.90358	9	13
48	1̄.77744	17	1̄.87396	26	0.12604	1̄.90349	10	12
49	1̄.77761	17	1̄.87422	26	0.12578	1̄.90339	9	11
50	1̄.77778	17	1̄.87448	27	0.12552	1̄.90330	10	10
51	1̄.77795	17	1̄.87475	26	0.12525	1̄.90320	9	9
52	1̄.77812	17	1̄.87501	26	0.12499	1̄.90311	10	8
53	1̄.77829	17	1̄.87527	27	0.12473	1̄.90301	9	7
54	1̄.77846	16	1̄.87554	26	0.12446	1̄.90292	10	6
55	1̄.77862	17	1̄.87580	26	0.12420	1̄.90282	9	5
56	1̄.77879	17	1̄.87606	27	0.12394	1̄.90273	10	4
57	1̄.77896	17	1̄.87633	26	0.12367	1̄.90263	9	3
58	1̄.77913	17	1̄.87659	26	0.12341	1̄.90254	10	2
59	1̄.77930	16	1̄.87685	26	0.12315	1̄.90244	9	1
60	1̄.77946		1̄.87711		0.12289	1̄.90235		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	27	26
6	2.7	2.6
7	3.2	3.0
8	3.6	3.5
9	4.1	3.9
10	4.5	4.3
20	9.0	8.7
30	13.5	13.0
40	18.0	17.3
50	22.5	21.7

	18
6	1.8
7	2.1
8	2.4
9	2.7
10	3.0
20	6.0
30	9.0
40	12.0
50	15.0

	17
6	1.7
7	2.0
8	2.3
9	2.6
10	2.8
20	5.7
30	8.5
40	11.3
50	14.2

	16
6	1.6
7	1.9
8	2.1
9	2.4
10	2.7
20	5.3
30	8.0
40	10.7
50	13.3

	10	9
6	1.0	0.9
7	1.2	1.1
8	1.3	1.2
9	1.5	1.4
10	1.7	1.5
20	3.3	3.0
30	5.0	4.5
40	6.7	6.0
50	8.3	7.5

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.77946	17	1̄.87711	27	0.12289	1̄.90235	10	60
1	1̄.77963	17	1̄.87738	26	0.12262	1̄.90225	9	59
2	1̄.77980	17	1̄.87764	26	0.12236	1̄.90216	10	58
3	1̄.77997	16	1̄.87790	27	0.12210	1̄.90206	9	57
4	1̄.78013	17	1̄.87817	26	0.12183	1̄.90197	10	56
5	1̄.78030	17	1̄.87843	26	0.12157	1̄.90187	9	55
6	1̄.78047	16	1̄.87869	26	0.12131	1̄.90178	10	54
7	1̄.78063	17	1̄.87895	27	0.12105	1̄.90168	9	53
8	1̄.78080	17	1̄.87922	26	0.12078	1̄.90159	10	52
9	1̄.78097	16	1̄.87948	26	0.12052	1̄.90149	10	51
10	1̄.78113	17	1̄.87974	26	0.12026	1̄.90139	9	50
11	1̄.78130	17	1̄.88000	27	0.12000	1̄.90130	10	49
12	1̄.78147	16	1̄.88027	26	0.11973	1̄.90120	9	48
13	1̄.78163	17	1̄.88053	26	0.11947	1̄.90111	10	47
14	1̄.78180	17	1̄.88079	26	0.11921	1̄.90101	10	46
15	1̄.78197	16	1̄.88105	26	0.11895	1̄.90091	9	45
16	1̄.78213	17	1̄.88131	27	0.11869	1̄.90082	10	44
17	1̄.78230	16	1̄.88158	26	0.11842	1̄.90072	9	43
18	1̄.78246	17	1̄.88184	26	0.11816	1̄.90063	10	42
19	1̄.78263	17	1̄.88210	26	0.11790	1̄.90053	10	41
20	1̄.78280	16	1̄.88236	26	0.11764	1̄.90043	9	40
21	1̄.78296	17	1̄.88262	27	0.11738	1̄.90034	10	39
22	1̄.78313	16	1̄.88289	26	0.11711	1̄.90024	10	38
23	1̄.78329	17	1̄.88315	26	0.11685	1̄.90014	9	37
24	1̄.78346	16	1̄.88341	26	0.11659	1̄.90005	10	36
25	1̄.78362	17	1̄.88367	26	0.11633	1̄.89995	10	35
26	1̄.78379	16	1̄.88393	27	0.11607	1̄.89985	9	34
27	1̄.78395	17	1̄.88420	26	0.11580	1̄.89976	10	33
28	1̄.78412	16	1̄.88446	26	0.11554	1̄.89966	10	32
29	1̄.78428	17	1̄.88472	26	0.11528	1̄.89956	9	31
30	1̄.78445	16	1̄.88498	26	0.11502	1̄.89947	10	30
31	1̄.78461	17	1̄.88524	26	0.11476	1̄.89937	10	29
32	1̄.78478	16	1̄.88550	27	0.11450	1̄.89927	9	28
33	1̄.78494	16	1̄.88577	26	0.11423	1̄.89918	10	27
34	1̄.78510	17	1̄.88603	26	0.11397	1̄.89908	10	26
35	1̄.78527	16	1̄.88629	26	0.11371	1̄.89898	10	25
36	1̄.78543	17	1̄.88655	26	0.11345	1̄.89888	9	24
37	1̄.78560	16	1̄.88681	26	0.11319	1̄.89879	10	23
38	1̄.78576	16	1̄.88707	26	0.11293	1̄.89869	10	22
39	1̄.78592	17	1̄.88733	26	0.11267	1̄.89859	10	21
40	1̄.78609	16	1̄.88759	27	0.11241	1̄.89849	9	20
41	1̄.78625	17	1̄.88786	26	0.11214	1̄.89840	10	19
42	1̄.78642	16	1̄.88812	26	0.11188	1̄.89830	10	18
43	1̄.78658	16	1̄.88838	26	0.11162	1̄.89820	10	17
44	1̄.78674	17	1̄.88864	26	0.11136	1̄.89810	9	16
45	1̄.78691	16	1̄.88890	26	0.11110	1̄.89801	10	15
46	1̄.78707	16	1̄.88916	26	0.11084	1̄.89791	10	14
47	1̄.78723	16	1̄.88942	26	0.11058	1̄.89781	10	13
48	1̄.78739	17	1̄.88968	26	0.11032	1̄.89771	10	12
49	1̄.78756	16	1̄.88994	26	0.11006	1̄.89761	9	11
50	1̄.78772	16	1̄.89020	26	0.10980	1̄.89752	10	10
51	1̄.78788	17	1̄.89046	27	0.10954	1̄.89742	10	9
52	1̄.78805	16	1̄.89073	26	0.10927	1̄.89732	10	8
53	1̄.78821	16	1̄.89099	26	0.10901	1̄.89722	10	7
54	1̄.78837	16	1̄.89125	26	0.10875	1̄.89712	10	6
55	1̄.78853	16	1̄.89151	26	0.10849	1̄.89702	9	5
56	1̄.78869	17	1̄.89177	26	0.10823	1̄.89693	10	4
57	1̄.78886	16	1̄.89203	26	0.10797	1̄.89683	10	3
58	1̄.78902	16	1̄.89229	26	0.10771	1̄.89673	10	2
59	1̄.78918	16	1̄.89255	26	0.10745	1̄.89663	10	1
60	1̄.78934		1̄.89281		0.10719	1̄.89653		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	27	26	17	16	10	9
6	2.7	2.6	1.7	1.6	1.0	0.9
7	3.2	3.0	2.0	1.9	1.2	1.1
8	3.6	3.5	2.3	2.1	1.3	1.2
9	4.1	3.9	2.6	2.4	1.5	1.4
10	4.5	4.3	2.8	2.7	1.7	1.5
20	9.0	8.7	5.7	5.3	3.3	3.0
30	13.5	13.0	8.5	8.0	5.0	4.5
40	18.0	17.3	11.3	10.7	6.7	6.0
50	22.5	21.7	14.2	13.3	8.3	7.5

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.78934	16	1̄.89281	26	0.10719	1̄.89653	10	60
1	1̄.78950	17	1̄.89307	26	0.10693	1̄.89643	10	59
2	1̄.78967	16	1̄.89333	26	0.10667	1̄.89633	9	58
3	1̄.78983	16	1̄.89359	26	0.10641	1̄.89624	10	57
4	1̄.78999	16	1̄.89385	26	0.10615	1̄.89614	10	56
5	1̄.79015	16	1̄.89411	26	0.10589	1̄.89604	10	55
6	1̄.79031	16	1̄.89437	26	0.10563	1̄.89594	10	54
7	1̄.79047	16	1̄.89463	26	0.10537	1̄.89584	10	53
8	1̄.79063	16	1̄.89489	26	0.10511	1̄.89574	10	52
9	1̄.79079	16	1̄.89515	26	0.10485	1̄.89564	10	51
10	1̄.79095	16	1̄.89541	26	0.10459	1̄.89554	10	50
11	1̄.79111	17	1̄.89567	26	0.10433	1̄.89544	10	49
12	1̄.79128	16	1̄.89593	26	0.10407	1̄.89534	10	48
13	1̄.79144	16	1̄.89619	26	0.10381	1̄.89524	10	47
14	1̄.79160	16	1̄.89645	26	0.10355	1̄.89514	10	46
15	1̄.79176	16	1̄.89671	26	0.10329	1̄.89504	9	45
16	1̄.79192	16	1̄.89697	26	0.10303	1̄.89495	10	44
17	1̄.79208	16	1̄.89723	26	0.10277	1̄.89485	10	43
18	1̄.79224	16	1̄.89749	26	0.10251	1̄.89475	10	42
19	1̄.79240	16	1̄.89775	26	0.10225	1̄.89465	10	41
20	1̄.79256	16	1̄.89801	26	0.10199	1̄.89455	10	40
21	1̄.79272	16	1̄.89827	26	0.10173	1̄.89445	10	39
22	1̄.79288	16	1̄.89853	26	0.10147	1̄.89435	10	38
23	1̄.79304	15	1̄.89879	26	0.10121	1̄.89425	10	37
24	1̄.79319	16	1̄.89905	26	0.10095	1̄.89415	10	36
25	1̄.79335	16	1̄.89931	26	0.10069	1̄.89405	10	35
26	1̄.79351	16	1̄.89957	26	0.10043	1̄.89395	10	34
27	1̄.79367	16	1̄.89983	26	0.10017	1̄.89385	10	33
28	1̄.79383	16	1̄.90009	26	0.09991	1̄.89375	11	32
29	1̄.79399	16	1̄.90035	26	0.09965	1̄.89364	10	31
30	1̄.79415	16	1̄.90061	25	0.09939	1̄.89354	10	30
31	1̄.79431	16	1̄.90086	26	0.09914	1̄.89344	10	29
32	1̄.79447	16	1̄.90112	26	0.09888	1̄.89334	10	28
33	1̄.79463	15	1̄.90138	26	0.09862	1̄.89324	10	27
34	1̄.79478	16	1̄.90164	26	0.09836	1̄.89314	10	26
35	1̄.79494	16	1̄.90190	26	0.09810	1̄.89304	10	25
36	1̄.79510	16	1̄.90216	26	0.09784	1̄.89294	10	24
37	1̄.79526	16	1̄.90242	26	0.09758	1̄.89284	10	23
38	1̄.79542	16	1̄.90268	26	0.09732	1̄.89274	10	22
39	1̄.79558	15	1̄.90294	26	0.09706	1̄.89264	10	21
40	1̄.79573	16	1̄.90320	26	0.09680	1̄.89254	10	20
41	1̄.79589	16	1̄.90346	25	0.09654	1̄.89244	11	19
42	1̄.79605	16	1̄.90371	26	0.09629	1̄.89233	10	18
43	1̄.79621	15	1̄.90397	26	0.09603	1̄.89223	10	17
44	1̄.79636	16	1̄.90423	26	0.09577	1̄.89213	10	16
45	1̄.79652	16	1̄.90449	26	0.09551	1̄.89203	10	15
46	1̄.79668	16	1̄.90475	26	0.09525	1̄.89193	10	14
47	1̄.79684	15	1̄.90501	26	0.09499	1̄.89183	10	13
48	1̄.79699	16	1̄.90527	26	0.09473	1̄.89173	11	12
49	1̄.79715	16	1̄.90553	25	0.09447	1̄.89162	10	11
50	1̄.79731	15	1̄.90578	26	0.09422	1̄.89152	10	10
51	1̄.79746	16	1̄.90604	26	0.09396	1̄.89142	10	9
52	1̄.79762	16	1̄.90630	26	0.09370	1̄.89132	10	8
53	1̄.79778	15	1̄.90656	26	0.09344	1̄.89122	10	7
54	1̄.79793	16	1̄.90682	26	0.09318	1̄.89112	11	6
55	1̄.79809	16	1̄.90708	26	0.09292	1̄.89101	10	5
56	1̄.79825	15	1̄.90734	25	0.09266	1̄.89091	10	4
57	1̄.79840	16	1̄.90759	26	0.09241	1̄.89081	10	3
58	1̄.79856	16	1̄.90785	26	0.09215	1̄.89071	11	2
59	1̄.79872	15	1̄.90811	26	0.09189	1̄.89060	10	1
60	1̄.79887		1̄.90837		0.09163	1̄.89050		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	26	25
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

	17
6	1.7
7	2.0
8	2.3
9	2.6
10	2.8
20	5.7
30	8.5
40	11.3
50	14.2

	16	15
6	1.6	1.5
7	1.9	1.8
8	2.1	2.0
9	2.4	2.3
10	2.7	2.5
20	5.3	5.0
30	8.0	7.5
40	10.7	10.0
50	13.3	12.5

	11
6	1.1
7	1.3
8	1.5
9	1.7
10	1.8
20	3.7
30	5.5
40	7.3
50	9.2

	10	9
6	1.0	0.9
7	1.2	1.1
8	1.3	1.2
9	1.5	1.4
10	1.7	1.5
20	3.3	3.0
30	5.0	4.5
40	6.7	6.0
50	8.3	7.5

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.79887	16	1̄.90837	26	0.09163	1̄.89050	10	60
1	1̄.79903	15	1̄.90863	26	0.09137	1̄.89040	10	59
2	1̄.79918	16	1̄.90889	25	0.09111	1̄.89030	10	58
3	1̄.79934	16	1̄.90914	26	0.09086	1̄.89020	11	57
4	1̄.79950	15	1̄.90940	26	0.09060	1̄.89009	10	56
5	1̄.79965	16	1̄.90966	26	0.09034	1̄.88999	10	55
6	1̄.79981	15	1̄.90992	26	0.09008	1̄.88989	11	54
7	1̄.79996	16	1̄.91018	25	0.08982	1̄.88978	10	53
8	1̄.80012	15	1̄.91043	26	0.08957	1̄.88968	10	52
9	1̄.80027	16	1̄.91069	26	0.08931	1̄.88958	10	51
10	1̄.80043	15	1̄.91095	26	0.08905	1̄.88948	11	50
11	1̄.80058	16	1̄.91121	26	0.08879	1̄.88937	10	49
12	1̄.80074	15	1̄.91147	25	0.08853	1̄.88927	10	48
13	1̄.80089	16	1̄.91172	26	0.08828	1̄.88917	11	47
14	1̄.80105	15	1̄.91198	26	0.08802	1̄.88906	10	46
15	1̄.80120	16	1̄.91224	26	0.08776	1̄.88896	10	45
16	1̄.80136	15	1̄.91250	26	0.08750	1̄.88886	11	44
17	1̄.80151	15	1̄.91276	25	0.08724	1̄.88875	10	43
18	1̄.80166	16	1̄.91301	26	0.08699	1̄.88865	10	42
19	1̄.80182	15	1̄.91327	26	0.08673	1̄.88855	11	41
20	1̄.80197	16	1̄.91353	26	0.08647	1̄.88844	10	40
21	1̄.80213	15	1̄.91379	25	0.08621	1̄.88834	10	39
22	1̄.80228	16	1̄.91404	26	0.08596	1̄.88824	11	38
23	1̄.80244	15	1̄.91430	26	0.08570	1̄.88813	10	37
24	1̄.80259	15	1̄.91456	26	0.08544	1̄.88803	10	36
25	1̄.80274	16	1̄.91482	25	0.08518	1̄.88793	11	35
26	1̄.80290	15	1̄.91507	26	0.08493	1̄.88782	10	34
27	1̄.80305	15	1̄.91533	26	0.08467	1̄.88772	11	33
28	1̄.80320	16	1̄.91559	26	0.08441	1̄.88761	10	32
29	1̄.80336	15	1̄.91585	25	0.08415	1̄.88751	10	31
30	1̄.80351	15	1̄.91610	26	0.08390	1̄.88741	11	30
31	1̄.80366	16	1̄.91636	26	0.08364	1̄.88730	10	29
32	1̄.80382	15	1̄.91662	26	0.08338	1̄.88720	11	28
33	1̄.80397	15	1̄.91688	25	0.08312	1̄.88709	10	27
34	1̄.80412	16	1̄.91713	26	0.08287	1̄.88699	11	26
35	1̄.80428	15	1̄.91739	26	0.08261	1̄.88688	10	25
36	1̄.80443	15	1̄.91765	26	0.08235	1̄.88678	10	24
37	1̄.80458	15	1̄.91791	25	0.08209	1̄.88668	11	23
38	1̄.80473	16	1̄.91816	26	0.08184	1̄.88657	10	22
39	1̄.80489	15	1̄.91842	26	0.08158	1̄.88647	11	21
40	1̄.80504	15	1̄.91868	25	0.08132	1̄.88636	10	20
41	1̄.80519	15	1̄.91893	26	0.08107	1̄.88626	11	19
42	1̄.80534	16	1̄.91919	26	0.08081	1̄.88615	10	18
43	1̄.80550	15	1̄.91945	26	0.08055	1̄.88605	11	17
44	1̄.80565	15	1̄.91971	25	0.08029	1̄.88594	10	16
45	1̄.80580	15	1̄.91996	26	0.08004	1̄.88584	11	15
46	1̄.80595	15	1̄.92022	26	0.07978	1̄.88573	10	14
47	1̄.80610	15	1̄.92048	25	0.07952	1̄.88563	11	13
48	1̄.80625	16	1̄.92073	26	0.07927	1̄.88552	10	12
49	1̄.80641	15	1̄.92099	26	0.07901	1̄.88542	11	11
50	1̄.80656	15	1̄.92125	25	0.07875	1̄.88531	10	10
51	1̄.80671	15	1̄.92150	26	0.07850	1̄.88521	11	9
52	1̄.80686	15	1̄.92176	26	0.07824	1̄.88510	11	8
53	1̄.80701	15	1̄.92202	25	0.07798	1̄.88499	10	7
54	1̄.80716	15	1̄.92227	26	0.07773	1̄.88489	11	6
55	1̄.80731	15	1̄.92253	26	0.07747	1̄.88478	10	5
56	1̄.80746	16	1̄.92279	25	0.07721	1̄.88468	11	4
57	1̄.80762	15	1̄.92304	26	0.07696	1̄.88457	10	3
58	1̄.80777	15	1̄.92330	26	0.07670	1̄.88447	11	2
59	1̄.80792	15	1̄.92356	25	0.07644	1̄.88436	11	1
60	1̄.80807		1̄.92381		0.07619	1̄.88425		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	26
6	2.6
7	3.0
8	3.5
9	3.9
10	4.3
20	8.7
30	13.0
40	17.3
50	21.7

	25
6	2.5
7	2.9
8	3.3
9	3.8
10	4.2
20	8.3
30	12.5
40	16.7
50	20.8

	16
6	1.6
7	1.9
8	2.1
9	2.4
10	2.7
20	5.3
30	8.0
40	10.7
50	13.3

	15
6	1.5
7	1.8
8	2.0
9	2.3
10	2.5
20	5.0
30	7.5
40	10.0
50	12.5

	11	10
6	1.1	1.0
7	1.3	1.2
8	1.5	1.3
9	1.7	1.5
10	1.8	1.7
20	3.7	3.3
30	5.5	5.0
40	7.3	6.7
50	9.2	8.3

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	$\bar{1}$.80807	15	$\bar{1}$.92381	26	0.07619	$\bar{1}$.88425	10	60
1	$\bar{1}$.80822	15	$\bar{1}$.92407	26	0.07593	$\bar{1}$.88415	11	59
2	$\bar{1}$.80837	15	$\bar{1}$.92433	25	0.07567	$\bar{1}$.88404	10	58
3	$\bar{1}$.80852	15	$\bar{1}$.92458	26	0.07542	$\bar{1}$.88394	11	57
4	$\bar{1}$.80867	15	$\bar{1}$.92484	26	0.07516	$\bar{1}$.88383	11	56
5	$\bar{1}$.80882	15	$\bar{1}$.92510	25	0.07490	$\bar{1}$.88372	10	55
6	$\bar{1}$.80897	15	$\bar{1}$.92535	26	0.07465	$\bar{1}$.88362	11	54
7	$\bar{1}$.80912	15	$\bar{1}$.92561	26	0.07439	$\bar{1}$.88351	11	53
8	$\bar{1}$.80927	15	$\bar{1}$.92587	25	0.07413	$\bar{1}$.88340	10	52
9	$\bar{1}$.80942	15	$\bar{1}$.92612	26	0.07388	$\bar{1}$.88330	11	51
10	$\bar{1}$.80957	15	$\bar{1}$.92638	25	0.07362	$\bar{1}$.88319	11	50
11	$\bar{1}$.80972	15	$\bar{1}$.92663	26	0.07337	$\bar{1}$.88308	10	49
12	$\bar{1}$.80987	15	$\bar{1}$.92689	26	0.07311	$\bar{1}$.88298	11	48
13	$\bar{1}$.81002	15	$\bar{1}$.92715	25	0.07285	$\bar{1}$.88287	11	47
14	$\bar{1}$.81017	15	$\bar{1}$.92740	26	0.07260	$\bar{1}$.88276	10	46
15	$\bar{1}$.81032	15	$\bar{1}$.92766	26	0.07234	$\bar{1}$.88266	11	45
16	$\bar{1}$.81047	14	$\bar{1}$.92792	25	0.07208	$\bar{1}$.88255	11	44
17	$\bar{1}$.81061	15	$\bar{1}$.92817	26	0.07183	$\bar{1}$.88244	10	43
18	$\bar{1}$.81076	15	$\bar{1}$.92843	25	0.07157	$\bar{1}$.88234	11	42
19	$\bar{1}$.81091	15	$\bar{1}$.92868	26	0.07132	$\bar{1}$.88223	11	41
20	$\bar{1}$.81106	15	$\bar{1}$.92894	26	0.07106	$\bar{1}$.88212	11	40
21	$\bar{1}$.81121	15	$\bar{1}$.92920	25	0.07080	$\bar{1}$.88201	10	39
22	$\bar{1}$.81136	15	$\bar{1}$.92945	26	0.07055	$\bar{1}$.88191	11	38
23	$\bar{1}$.81151	15	$\bar{1}$.92971	25	0.07029	$\bar{1}$.88180	11	37
24	$\bar{1}$.81166	14	$\bar{1}$.92996	26	0.07004	$\bar{1}$.88169	11	36
25	$\bar{1}$.81180	15	$\bar{1}$.93022	26	0.06978	$\bar{1}$.88158	10	35
26	$\bar{1}$.81195	15	$\bar{1}$.93048	25	0.06952	$\bar{1}$.88148	11	34
27	$\bar{1}$.81210	15	$\bar{1}$.93073	26	0.06927	$\bar{1}$.88137	11	33
28	$\bar{1}$.81225	15	$\bar{1}$.93099	25	0.06901	$\bar{1}$.88126	11	32
29	$\bar{1}$.81240	14	$\bar{1}$.93124	26	0.06876	$\bar{1}$.88115	10	31
30	$\bar{1}$.81254	15	$\bar{1}$.93150	25	0.06850	$\bar{1}$.88105	11	30
31	$\bar{1}$.81269	15	$\bar{1}$.93175	26	0.06825	$\bar{1}$.88094	11	29
32	$\bar{1}$.81284	15	$\bar{1}$.93201	26	0.06799	$\bar{1}$.88083	11	28
33	$\bar{1}$.81299	15	$\bar{1}$.93227	25	0.06773	$\bar{1}$.88072	11	27
34	$\bar{1}$.81314	14	$\bar{1}$.93252	26	0.06748	$\bar{1}$.88061	10	26
35	$\bar{1}$.81328	15	$\bar{1}$.93278	25	0.06722	$\bar{1}$.88051	11	25
36	$\bar{1}$.81343	15	$\bar{1}$.93303	26	0.06697	$\bar{1}$.88040	11	24
37	$\bar{1}$.81358	14	$\bar{1}$.93329	25	0.06671	$\bar{1}$.88029	11	23
38	$\bar{1}$.81372	15	$\bar{1}$.93354	26	0.06646	$\bar{1}$.88018	11	22
39	$\bar{1}$.81387	15	$\bar{1}$.93380	26	0.06620	$\bar{1}$.88007	11	21
40	$\bar{1}$.81402	15	$\bar{1}$.93406	25	0.06594	$\bar{1}$.87996	11	20
41	$\bar{1}$.81417	14	$\bar{1}$.93431	26	0.06569	$\bar{1}$.87985	10	19
42	$\bar{1}$.81431	15	$\bar{1}$.93457	25	0.06543	$\bar{1}$.87975	11	18
43	$\bar{1}$.81446	15	$\bar{1}$.93482	26	0.06518	$\bar{1}$.87964	11	17
44	$\bar{1}$.81461	14	$\bar{1}$.93508	25	0.06492	$\bar{1}$.87953	11	16
45	$\bar{1}$.81475	15	$\bar{1}$.93533	26	0.06467	$\bar{1}$.87942	11	15
46	$\bar{1}$.81490	15	$\bar{1}$.93559	25	0.06441	$\bar{1}$.87931	11	14
47	$\bar{1}$.81505	14	$\bar{1}$.93584	26	0.06416	$\bar{1}$.87920	11	13
48	$\bar{1}$.81519	15	$\bar{1}$.93610	26	0.06390	$\bar{1}$.87909	11	12
49	$\bar{1}$.81534	15	$\bar{1}$.93636	25	0.06364	$\bar{1}$.87898	11	11
50	$\bar{1}$.81549	14	$\bar{1}$.93661	26	0.06339	$\bar{1}$.87887	10	10
51	$\bar{1}$.81563	15	$\bar{1}$.93687	25	0.06313	$\bar{1}$.87877	11	9
52	$\bar{1}$.81578	14	$\bar{1}$.93712	26	0.06288	$\bar{1}$.87866	11	8
53	$\bar{1}$.81592	15	$\bar{1}$.93738	25	0.06262	$\bar{1}$.87855	11	7
54	$\bar{1}$.81607	15	$\bar{1}$.93763	26	0.06237	$\bar{1}$.87844	11	6
55	$\bar{1}$.81622	14	$\bar{1}$.93789	25	0.06211	$\bar{1}$.87833	11	5
56	$\bar{1}$.81636	15	$\bar{1}$.93814	26	0.06186	$\bar{1}$.87822	11	4
57	$\bar{1}$.81651	14	$\bar{1}$.93840	25	0.06160	$\bar{1}$.87811	11	3
58	$\bar{1}$.81665	15	$\bar{1}$.93865	26	0.06135	$\bar{1}$.87800	11	2
59	$\bar{1}$.81680	14	$\bar{1}$.93891	25	0.06109	$\bar{1}$.87789	11	1
60	$\bar{1}$.81694		$\bar{1}$.93916		0.06084	$\bar{1}$.87778		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	26
6	2.6
7	3.0
8	3.5
9	3.9
10	4.3
20	8.7
30	13.0
40	17.3
50	21.7

	25
6	2.5
7	2.9
8	3.3
9	3.8
10	4.2
20	8.3
30	12.5
40	16.7
50	20.8

	15
6	1.5
7	1.8
8	2.0
9	2.3
10	2.5
20	5.0
30	7.5
40	10.0
50	12.5

	14
6	1.4
7	1.6
8	1.9
9	2.1
10	2.3
20	4.7
30	7.0
40	9.3
50	11.7

	11	10
6	1.1	1.0
7	1.3	1.2
8	1.5	1.3
9	1.7	1.5
10	1.8	1.7
20	3.7	3.3
30	5.5	5.0
40	7.3	6.7
50	9.2	8.3

p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	1̄.81694	15	1̄.93916	26	0.06084	1̄.87778	11	60
1	1̄.81709	14	1̄.93942	25	0.06058	1̄.87767	11	59
2	1̄.81723	15	1̄.93967	26	0.06033	1̄.87756	11	58
3	1̄.81738	14	1̄.93993	25	0.06007	1̄.87745	11	57
4	1̄.81752	15	1̄.94018	26	0.05982	1̄.87734	11	56
5	1̄.81767	14	1̄.94044	25	0.05956	1̄.87723	11	55
6	1̄.81781	15	1̄.94069	26	0.05931	1̄.87712	11	54
7	1̄.81796	14	1̄.94095	25	0.05905	1̄.87701	11	53
8	1̄.81810	15	1̄.94120	26	0.05880	1̄.87690	11	52
9	1̄.81825	14	1̄.94146	25	0.05854	1̄.87679	11	51
10	1̄.81839	15	1̄.94171	26	0.05829	1̄.87668	11	50
11	1̄.81854	14	1̄.94197	25	0.05803	1̄.87657	11	49
12	1̄.81868	14	1̄.94222	26	0.05778	1̄.87646	11	48
13	1̄.81882	15	1̄.94248	25	0.05752	1̄.87635	11	47
14	1̄.81897	14	1̄.94273	26	0.05727	1̄.87624	11	46
15	1̄.81911	15	1̄.94299	25	0.05701	1̄.87613	12	45
16	1̄.81926	14	1̄.94324	26	0.05676	1̄.87601	11	44
17	1̄.81940	15	1̄.94350	25	0.05650	1̄.87590	11	43
18	1̄.81955	14	1̄.94375	26	0.05625	1̄.87579	11	42
19	1̄.81969	14	1̄.94401	25	0.05599	1̄.87568	11	41
20	1̄.81983	15	1̄.94426	26	0.05574	1̄.87557	11	40
21	1̄.81998	14	1̄.94452	25	0.05548	1̄.87546	11	39
22	1̄.82012	14	1̄.94477	26	0.05523	1̄.87535	11	38
23	1̄.82026	15	1̄.94503	25	0.05497	1̄.87524	11	37
24	1̄.82041	14	1̄.94528	26	0.05472	1̄.87513	12	36
25	1̄.82055	14	1̄.94554	25	0.05446	1̄.87501	11	35
26	1̄.82069	15	1̄.94579	25	0.05421	1̄.87490	11	34
27	1̄.82084	14	1̄.94604	26	0.05396	1̄.87479	11	33
28	1̄.82098	14	1̄.94630	25	0.05370	1̄.87468	11	32
29	1̄.82112	14	1̄.94655	26	0.05345	1̄.87457	11	31
30	1̄.82126	15	1̄.94681	25	0.05319	1̄.87446	12	30
31	1̄.82141	14	1̄.94706	26	0.05294	1̄.87434	11	29
32	1̄.82155	14	1̄.94732	25	0.05268	1̄.87423	11	28
33	1̄.82169	15	1̄.94757	26	0.05243	1̄.87412	11	27
34	1̄.82184	14	1̄.94783	25	0.05217	1̄.87401	11	26
35	1̄.82198	14	1̄.94808	26	0.05192	1̄.87390	12	25
36	1̄.82212	14	1̄.94834	25	0.05166	1̄.87378	11	24
37	1̄.82226	14	1̄.94859	25	0.05141	1̄.87367	11	23
38	1̄.82240	15	1̄.94884	26	0.05116	1̄.87356	11	22
39	1̄.82255	14	1̄.94910	25	0.05090	1̄.87345	11	21
40	1̄.82269	14	1̄.94935	26	0.05065	1̄.87334	12	20
41	1̄.82283	14	1̄.94961	25	0.05039	1̄.87322	11	19
42	1̄.82297	14	1̄.94986	26	0.05014	1̄.87311	11	18
43	1̄.82311	15	1̄.95012	25	0.04988	1̄.87300	12	17
44	1̄.82326	14	1̄.95037	25	0.04963	1̄.87288	11	16
45	1̄.82340	14	1̄.95062	26	0.04938	1̄.87277	11	15
46	1̄.82354	14	1̄.95088	25	0.04912	1̄.87266	11	14
47	1̄.82368	14	1̄.95113	26	0.04887	1̄.87255	12	13
48	1̄.82382	14	1̄.95139	25	0.04861	1̄.87243	11	12
49	1̄.82396	14	1̄.95164	26	0.04836	1̄.87232	11	11
50	1̄.82410	14	1̄.95190	25	0.04810	1̄.87221	12	10
51	1̄.82424	15	1̄.95215	25	0.04785	1̄.87209	11	9
52	1̄.82439	14	1̄.95240	26	0.04760	1̄.87198	11	8
53	1̄.82453	14	1̄.95266	25	0.04734	1̄.87187	12	7
54	1̄.82467	14	1̄.95291	26	0.04709	1̄.87175	11	6
55	1̄.82481	14	1̄.95317	25	0.04683	1̄.87164	11	5
56	1̄.82495	14	1̄.95342	26	0.04658	1̄.87153	12	4
57	1̄.82509	14	1̄.95368	25	0.04632	1̄.87141	11	3
58	1̄.82523	14	1̄.95393	25	0.04607	1̄.87130	11	2
59	1̄.82537	14	1̄.95418	26	0.04582	1̄.87119	12	1
60	1̄.82551		1̄.95444		0.04556	1̄.87107		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	26	25	15	14	12	11
6	2.6	2.5	1.5	1.4	1.2	1.1
7	3.0	2.9	1.8	1.6	1.4	1.3
8	3.5	3.3	2.0	1.9	1.6	1.5
9	3.9	3.8	2.3	2.1	1.8	1.7
10	4.3	4.2	2.5	2.3	2.0	1.8
20	8.7	8.3	5.0	4.7	4.0	3.7
30	13.0	12.5	7.5	7.0	6.0	5.5
40	17.3	16.7	10.0	9.3	8.0	7.3
50	21.7	20.8	12.5	11.7	10.0	9.2

42°

′	log sin	d	log tan	c. d.	log cot	log cos	d	′
0	Ī.82551	14	Ī.95444	25	0.04556	Ī.87107	11	60
1	Ī.82565	14	Ī.95469	26	0.04531	Ī.87096	11	59
2	Ī.82579	14	Ī.95495	25	0.04505	Ī.87085	12	58
3	Ī.82593	14	Ī.95520	25	0.04480	Ī.87073	11	57
4	Ī.82607	14	Ī.95545	26	0.04455	Ī.87062	12	56
5	Ī.82621	14	Ī.95571	25	0.04429	Ī.87050	11	55
6	Ī.82635	14	Ī.95596	26	0.04404	Ī.87039	11	54
7	Ī.82649	14	Ī.95622	25	0.04378	Ī.87028	12	53
8	Ī.82663	14	Ī.95647	25	0.04353	Ī.87016	11	52
9	Ī.82677	14	Ī.95672	26	0.04328	Ī.87005	12	51
10	Ī.82691	14	Ī.95698	25	0.04302	Ī.86993	11	50
11	Ī.82705	14	Ī.95723	25	0.04277	Ī.86982	12	49
12	Ī.82719	14	Ī.95748	26	0.04252	Ī.86970	11	48
13	Ī.82733	14	Ī.95774	25	0.04226	Ī.86959	12	47
14	Ī.82747	14	Ī.95799	26	0.04201	Ī.86947	11	46
15	Ī.82761	14	Ī.95825	25	0.04175	Ī.86936	12	45
16	Ī.82775	13	Ī.95850	25	0.04150	Ī.86924	11	44
17	Ī.82788	14	Ī.95875	26	0.04125	Ī.86913	11	43
18	Ī.82802	14	Ī.95901	25	0.04099	Ī.86902	12	42
19	Ī.82816	14	Ī.95926	26	0.04074	Ī.86890	11	41
20	Ī.82830	14	Ī.95952	25	0.04048	Ī.86879	12	40
21	Ī.82844	14	Ī.95977	25	0.04023	Ī.86867	12	39
22	Ī.82858	14	Ī.96002	26	0.03998	Ī.86855	11	38
23	Ī.82872	13	Ī.96028	25	0.03972	Ī.86844	12	37
24	Ī.82885	14	Ī.96053	25	0.03947	Ī.86832	11	36
25	Ī.82899	14	Ī.96078	26	0.03922	Ī.86821	12	35
26	Ī.82913	14	Ī.96104	25	0.03896	Ī.86809	11	34
27	Ī.82927	14	Ī.96129	26	0.03871	Ī.86798	12	33
28	Ī.82941	14	Ī.96155	25	0.03845	Ī.86786	11	32
29	Ī.82955	13	Ī.96180	25	0.03820	Ī.86775	12	31
30	Ī.82968	14	Ī.96205	26	0.03795	Ī.86763	11	30
31	Ī.82982	14	Ī.96231	25	0.03769	Ī.86752	12	29
32	Ī.82996	14	Ī.96256	25	0.03744	Ī.86740	12	28
33	Ī.83010	13	Ī.96281	26	0.03719	Ī.86728	11	27
34	Ī.83023	14	Ī.96307	25	0.03693	Ī.86717	12	26
35	Ī.83037	14	Ī.96332	25	0.03668	Ī.86705	11	25
36	Ī.83051	14	Ī.96357	26	0.03643	Ī.86694	12	24
37	Ī.83065	13	Ī.96383	25	0.03617	Ī.86682	12	23
38	Ī.83078	14	Ī.96408	25	0.03592	Ī.86670	11	22
39	Ī.83092	14	Ī.96433	26	0.03567	Ī.86659	12	21
40	Ī.83106	14	Ī.96459	25	0.03541	Ī.86647	12	20
41	Ī.83120	13	Ī.96484	26	0.03516	Ī.86635	11	19
42	Ī.83133	14	Ī.96510	25	0.03490	Ī.86624	12	18
43	Ī.83147	14	Ī.96535	25	0.03465	Ī.86612	12	17
44	Ī.83161	13	Ī.96560	26	0.03440	Ī.86600	11	16
45	Ī.83174	14	Ī.96586	25	0.03414	Ī.86589	12	15
46	Ī.83188	14	Ī.96611	25	0.03389	Ī.86577	12	14
47	Ī.83202	13	Ī.96636	26	0.03364	Ī.86565	11	13
48	Ī.83215	14	Ī.96662	25	0.03338	Ī.86554	12	12
49	Ī.83229	13	Ī.96687	25	0.03313	Ī.86542	12	11
50	Ī.83242	14	Ī.96712	26	0.03288	Ī.86530	12	10
51	Ī.83256	14	Ī.96738	25	0.03262	Ī.86518	11	9
52	Ī.83270	13	Ī.96763	25	0.03237	Ī.86507	12	8
53	Ī.83283	14	Ī.96788	26	0.03212	Ī.86495	12	7
54	Ī.83297	13	Ī.96814	25	0.03186	Ī.86483	11	6
55	Ī.83310	14	Ī.96839	25	0.03161	Ī.86472	12	5
56	Ī.83324	14	Ī.96864	26	0.03136	Ī.86460	12	4
57	Ī.83338	13	Ī.96890	25	0.03110	Ī.86448	12	3
58	Ī.83351	14	Ī.96915	25	0.03085	Ī.86436	11	2
59	Ī.83365	13	Ī.96940	26	0.03060	Ī.86425	12	1
60	Ī.83378		Ī.96966		0.03034	Ī.86413		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	26	25	14	13	12	11
6	2.6	2.5	1.4	1.3	1.2	1.1
7	3.0	2.9	1.6	1.5	1.4	1.3
8	3.5	3.3	1.9	1.7	1.6	1.5
9	3.9	3.8	2.1	2.0	1.8	1.7
10	4.3	4.2	2.3	2.2	2.0	1.8
20	8.7	8.3	4.7	4.3	4.0	3.7
30	13.0	12.5	7.0	6.5	6.0	5.5
40	17.3	16.7	9.3	8.7	8.0	7.3
50	21.7	20.8	11.7	10.8	10.0	9.2

p. p.

47°

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	$\bar{1}.83378$	14	$\bar{1}.96966$	25	0.03034	$\bar{1}.86413$	12	60
1	$\bar{1}.83392$	13	$\bar{1}.96991$	25	0.03009	$\bar{1}.86401$	12	59
2	$\bar{1}.83405$	14	$\bar{1}.97016$	26	0.02984	$\bar{1}.86389$	12	58
3	$\bar{1}.83419$	13	$\bar{1}.97042$	25	0.02958	$\bar{1}.86377$	11	57
4	$\bar{1}.83432$	14	$\bar{1}.97067$	25	0.02933	$\bar{1}.86366$	12	56
5	$\bar{1}.83446$	13	$\bar{1}.97092$	26	0.02908	$\bar{1}.86354$	12	55
6	$\bar{1}.83459$	14	$\bar{1}.97118$	25	0.02882	$\bar{1}.86342$	12	54
7	$\bar{1}.83473$	13	$\bar{1}.97143$	25	0.02857	$\bar{1}.86330$	12	53
8	$\bar{1}.83486$	14	$\bar{1}.97168$	25	0.02832	$\bar{1}.86318$	12	52
9	$\bar{1}.83500$	13	$\bar{1}.97193$	26	0.02807	$\bar{1}.86306$	11	51
10	$\bar{1}.83513$	14	$\bar{1}.97219$	25	0.02781	$\bar{1}.86295$	12	50
11	$\bar{1}.83527$	13	$\bar{1}.97244$	25	0.02756	$\bar{1}.86283$	12	49
12	$\bar{1}.83540$	14	$\bar{1}.97269$	26	0.02731	$\bar{1}.86271$	12	48
13	$\bar{1}.83554$	13	$\bar{1}.97295$	25	0.02705	$\bar{1}.86259$	12	47
14	$\bar{1}.83567$	14	$\bar{1}.97320$	25	0.02680	$\bar{1}.86247$	12	46
15	$\bar{1}.83581$	13	$\bar{1}.97345$	26	0.02655	$\bar{1}.86235$	12	45
16	$\bar{1}.83594$	14	$\bar{1}.97371$	25	0.02629	$\bar{1}.86223$	12	44
17	$\bar{1}.83608$	13	$\bar{1}.97396$	25	0.02604	$\bar{1}.86211$	11	43
18	$\bar{1}.83621$	13	$\bar{1}.97421$	26	0.02579	$\bar{1}.86200$	12	42
19	$\bar{1}.83634$	14	$\bar{1}.97447$	25	0.02553	$\bar{1}.86188$	12	41
20	$\bar{1}.83648$	13	$\bar{1}.97472$	25	0.02528	$\bar{1}.86176$	12	40
21	$\bar{1}.83661$	13	$\bar{1}.97497$	26	0.02503	$\bar{1}.86164$	12	39
22	$\bar{1}.83674$	14	$\bar{1}.97523$	25	0.02477	$\bar{1}.86152$	12	38
23	$\bar{1}.83688$	13	$\bar{1}.97548$	25	0.02452	$\bar{1}.86140$	12	37
24	$\bar{1}.83701$	14	$\bar{1}.97573$	25	0.02427	$\bar{1}.86128$	12	36
25	$\bar{1}.83715$	13	$\bar{1}.97598$	26	0.02402	$\bar{1}.86116$	12	35
26	$\bar{1}.83728$	13	$\bar{1}.97624$	25	0.02376	$\bar{1}.86104$	12	34
27	$\bar{1}.83741$	14	$\bar{1}.97649$	25	0.02351	$\bar{1}.86092$	12	33
28	$\bar{1}.83755$	13	$\bar{1}.97674$	26	0.02326	$\bar{1}.86080$	12	32
29	$\bar{1}.83768$	13	$\bar{1}.97700$	25	0.02300	$\bar{1}.86068$	12	31
30	$\bar{1}.83781$	14	$\bar{1}.97725$	25	0.02275	$\bar{1}.86056$	12	30
31	$\bar{1}.83795$	13	$\bar{1}.97750$	26	0.02250	$\bar{1}.86044$	12	29
32	$\bar{1}.83808$	13	$\bar{1}.97776$	25	0.02224	$\bar{1}.86032$	12	28
33	$\bar{1}.83821$	13	$\bar{1}.97801$	25	0.02199	$\bar{1}.86020$	12	27
34	$\bar{1}.83834$	14	$\bar{1}.97826$	25	0.02174	$\bar{1}.86008$	12	26
35	$\bar{1}.83848$	13	$\bar{1}.97851$	26	0.02149	$\bar{1}.85996$	12	25
36	$\bar{1}.83861$	13	$\bar{1}.97877$	25	0.02123	$\bar{1}.85984$	12	24
37	$\bar{1}.83874$	13	$\bar{1}.97902$	25	0.02098	$\bar{1}.85972$	12	23
38	$\bar{1}.83887$	14	$\bar{1}.97927$	26	0.02073	$\bar{1}.85960$	12	22
39	$\bar{1}.83901$	13	$\bar{1}.97953$	25	0.02047	$\bar{1}.85948$	12	21
40	$\bar{1}.83914$	13	$\bar{1}.97978$	25	0.02022	$\bar{1}.85936$	12	20
41	$\bar{1}.83927$	13	$\bar{1}.98003$	26	0.01997	$\bar{1}.85924$	12	19
42	$\bar{1}.83940$	14	$\bar{1}.98029$	25	0.01971	$\bar{1}.85912$	12	18
43	$\bar{1}.83954$	13	$\bar{1}.98054$	25	0.01946	$\bar{1}.85900$	12	17
44	$\bar{1}.83967$	13	$\bar{1}.98079$	25	0.01921	$\bar{1}.85888$	12	16
45	$\bar{1}.83980$	13	$\bar{1}.98104$	26	0.01896	$\bar{1}.85876$	12	15
46	$\bar{1}.83993$	13	$\bar{1}.98130$	25	0.01870	$\bar{1}.85864$	13	14
47	$\bar{1}.84006$	14	$\bar{1}.98155$	25	0.01845	$\bar{1}.85851$	12	13
48	$\bar{1}.84020$	13	$\bar{1}.98180$	26	0.01820	$\bar{1}.85839$	12	12
49	$\bar{1}.84033$	13	$\bar{1}.98206$	25	0.01794	$\bar{1}.85827$	12	11
50	$\bar{1}.84046$	13	$\bar{1}.98231$	25	0.01769	$\bar{1}.85815$	12	10
51	$\bar{1}.84059$	13	$\bar{1}.98256$	25	0.01744	$\bar{1}.85803$	12	9
52	$\bar{1}.84072$	13	$\bar{1}.98281$	26	0.01719	$\bar{1}.85791$	12	8
53	$\bar{1}.84085$	13	$\bar{1}.98307$	25	0.01693	$\bar{1}.85779$	13	7
54	$\bar{1}.84098$	14	$\bar{1}.98332$	25	0.01668	$\bar{1}.85766$	12	6
55	$\bar{1}.84112$	13	$\bar{1}.98357$	26	0.01643	$\bar{1}.85754$	12	5
56	$\bar{1}.84125$	13	$\bar{1}.98383$	25	0.01617	$\bar{1}.85742$	12	4
57	$\bar{1}.84138$	13	$\bar{1}.98408$	25	0.01592	$\bar{1}.85730$	12	3
58	$\bar{1}.84151$	13	$\bar{1}.98433$	25	0.01567	$\bar{1}.85718$	12	2
59	$\bar{1}.84164$	13	$\bar{1}.98458$	26	0.01542	$\bar{1}.85706$	13	1
60	$\bar{1}.84177$		$\bar{1}.98484$		0.01516	$\bar{1}.85693$		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	26	25	14	13	12	11
6	2.6	2.5	1.4	1.3	1.2	1.1
7	3.0	2.9	1.6	1.5	1.4	1.3
8	3.5	3.3	1.9	1.7	1.6	1.5
9	3.9	3.8	2.1	2.0	1.8	1.7
10	4.3	4.2	2.3	2.2	2.0	1.8
20	8.7	8.3	4.7	4.3	4.0	3.7
30	13.0	12.5	7.0	6.5	6.0	5.5
40	17.3	16.7	9.3	8.7	8.0	7.3
50	21.7	20.8	11.7	10.8	10.0	9.2

′	log sin	d	log tan	c. d.	log cot	log cos	d	
0	$\bar{1}$.84177	13	$\bar{1}$.98484	25	0.01516	$\bar{1}$.85693	12	60
1	$\bar{1}$.84190	13	$\bar{1}$.98509	25	0.01491	$\bar{1}$.85681	12	59
2	$\bar{1}$.84203	13	$\bar{1}$.98534	26	0.01466	$\bar{1}$.85669	12	58
3	$\bar{1}$.84216	13	$\bar{1}$.98560	25	0.01440	$\bar{1}$.85657	12	57
4	$\bar{1}$.84229	13	$\bar{1}$.98585	25	0.01415	$\bar{1}$.85645	13	56
5	$\bar{1}$.84242	13	$\bar{1}$.98610	25	0.01390	$\bar{1}$.85632	12	55
6	$\bar{1}$.84255	14	$\bar{1}$.98635	26	0.01365	$\bar{1}$.85620	12	54
7	$\bar{1}$.84269	13	$\bar{1}$.98661	25	0.01339	$\bar{1}$.85608	12	53
8	$\bar{1}$.84282	13	$\bar{1}$.98686	25	0.01314	$\bar{1}$.85596	13	52
9	$\bar{1}$.84295	13	$\bar{1}$.98711	26	0.01289	$\bar{1}$.85583	12	51
10	$\bar{1}$.84308	13	$\bar{1}$.98737	25	0.01263	$\bar{1}$.85571	12	50
11	$\bar{1}$.84321	13	$\bar{1}$.98762	25	0.01238	$\bar{1}$.85559	12	49
12	$\bar{1}$.84334	13	$\bar{1}$.98787	25	0.01213	$\bar{1}$.85547	13	48
13	$\bar{1}$.84347	13	$\bar{1}$.98812	26	0.01188	$\bar{1}$.85534	12	47
14	$\bar{1}$.84360	13	$\bar{1}$.98838	25	0.01162	$\bar{1}$.85522	12	46
15	$\bar{1}$.84373	12	$\bar{1}$.98863	25	0.01137	$\bar{1}$.85510	13	45
16	$\bar{1}$.84385	13	$\bar{1}$.98888	25	0.01112	$\bar{1}$.85497	12	44
17	$\bar{1}$.84398	13	$\bar{1}$.98913	26	0.01087	$\bar{1}$.85485	12	43
18	$\bar{1}$.84411	13	$\bar{1}$.98939	25	0.01061	$\bar{1}$.85473	13	42
19	$\bar{1}$.84424	13	$\bar{1}$.98964	25	0.01036	$\bar{1}$.85460	12	41
20	$\bar{1}$.84437	13	$\bar{1}$.98989	26	0.01011	$\bar{1}$.85448	12	40
21	$\bar{1}$.84450	13	$\bar{1}$.99015	25	0.00985	$\bar{1}$.85436	13	39
22	$\bar{1}$.84463	13	$\bar{1}$.99040	25	0.00960	$\bar{1}$.85423	12	38
23	$\bar{1}$.84476	13	$\bar{1}$.99065	25	0.00935	$\bar{1}$.85411	12	37
24	$\bar{1}$.84489	13	$\bar{1}$.99090	26	0.00910	$\bar{1}$.85399	13	36
25	$\bar{1}$.84502	13	$\bar{1}$.99116	25	0.00884	$\bar{1}$.85386	12	35
26	$\bar{1}$.84515	13	$\bar{1}$.99141	25	0.00859	$\bar{1}$.85374	13	34
27	$\bar{1}$.84528	12	$\bar{1}$.99166	25	0.00834	$\bar{1}$.85361	12	33
28	$\bar{1}$.84540	13	$\bar{1}$.99191	26	0.00809	$\bar{1}$.85349	12	32
29	$\bar{1}$.84553	13	$\bar{1}$.99217	25	0.00783	$\bar{1}$.85337	13	31
30	$\bar{1}$.84566	13	$\bar{1}$.99242	25	0.00758	$\bar{1}$.85324	12	30
31	$\bar{1}$.84579	13	$\bar{1}$.99267	26	0.00733	$\bar{1}$.85312	13	29
32	$\bar{1}$.84592	13	$\bar{1}$.99293	25	0.00707	$\bar{1}$.85299	12	28
33	$\bar{1}$.84605	13	$\bar{1}$.99318	25	0.00682	$\bar{1}$.85287	13	27
34	$\bar{1}$.84618	12	$\bar{1}$.99343	25	0.00657	$\bar{1}$.85274	12	26
35	$\bar{1}$.84630	13	$\bar{1}$.99368	26	0.00632	$\bar{1}$.85262	12	25
36	$\bar{1}$.84643	13	$\bar{1}$.99394	25	0.00606	$\bar{1}$.85250	13	24
37	$\bar{1}$.84656	13	$\bar{1}$.99419	25	0.00581	$\bar{1}$.85237	12	23
38	$\bar{1}$.84669	13	$\bar{1}$.99444	25	0.00556	$\bar{1}$.85225	13	22
39	$\bar{1}$.84682	12	$\bar{1}$.99469	26	0.00531	$\bar{1}$.85212	12	21
40	$\bar{1}$.84694	13	$\bar{1}$.99495	25	0.00505	$\bar{1}$.85200	13	20
41	$\bar{1}$.84707	13	$\bar{1}$.99520	25	0.00480	$\bar{1}$.85187	12	19
42	$\bar{1}$.84720	13	$\bar{1}$.99545	25	0.00455	$\bar{1}$.85175	13	18
43	$\bar{1}$.84733	12	$\bar{1}$.99570	26	0.00430	$\bar{1}$.85162	12	17
44	$\bar{1}$.84745	13	$\bar{1}$.99596	25	0.00404	$\bar{1}$.85150	13	16
45	$\bar{1}$.84758	13	$\bar{1}$.99621	25	0.00379	$\bar{1}$.85137	12	15
46	$\bar{1}$.84771	13	$\bar{1}$.99646	26	0.00354	$\bar{1}$.85125	13	14
47	$\bar{1}$.84784	12	$\bar{1}$.99672	25	0.00328	$\bar{1}$.85112	12	13
48	$\bar{1}$.84796	13	$\bar{1}$.99697	25	0.00303	$\bar{1}$.85100	13	12
49	$\bar{1}$.84809	13	$\bar{1}$.99722	25	0.00278	$\bar{1}$.85087	13	11
50	$\bar{1}$.84822	13	$\bar{1}$.99747	26	0.00253	$\bar{1}$.85074	12	10
51	$\bar{1}$.84835	12	$\bar{1}$.99773	25	0.00227	$\bar{1}$.85062	13	9
52	$\bar{1}$.84847	13	$\bar{1}$.99798	25	0.00202	$\bar{1}$.85049	12	8
53	$\bar{1}$.84860	13	$\bar{1}$.99823	25	0.00177	$\bar{1}$.85037	13	7
54	$\bar{1}$.84873	12	$\bar{1}$.99848	26	0.00152	$\bar{1}$.85024	12	6
55	$\bar{1}$.84885	13	$\bar{1}$.99874	25	0.00126	$\bar{1}$.85012	13	5
56	$\bar{1}$.84898	13	$\bar{1}$.99899	25	0.00101	$\bar{1}$.84999	13	4
57	$\bar{1}$.84911	12	$\bar{1}$.99924	25	0.00076	$\bar{1}$.84986	12	3
58	$\bar{1}$.84923	13	$\bar{1}$.99949	26	0.00051	$\bar{1}$.84974	13	2
59	$\bar{1}$.84936	13	$\bar{1}$.99975	25	0.00025	$\bar{1}$.84961	12	1
60	$\bar{1}$.84949		0.00000		0.00000	$\bar{1}$.84949		0
	log cos	d	log cot	c. d.	log tan	log sin	d	′

p. p.

	26
6	2.6
7	3.0
8	3.5
9	3.9
10	4.3
20	8.7
30	13.0
40	17.3
50	21.7

	25
6	2.5
7	2.9
8	3.3
9	3.8
10	4.2
20	8.3
30	12.5
40	16.7
50	20.8

	14
6	1.4
7	1.6
8	1.9
9	2.1
10	2.3
20	4.7
30	7.0
40	9.3
50	11.7

	13
6	1.3
7	1.5
8	1.7
9	2.0
10	2.2
20	4.3
30	6.5
40	8.7
50	10.8

	12
6	1.2
7	1.4
8	1.6
9	1.8
10	2.0
20	4.0
30	6.0
40	8.0
50	10.0

p. p.

TABLE

OF

COMMON LOGARITHMS

OF NUMBERS

From 1 to 10,000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
0	− ∞	**20**	30 103	**40**	60 206	**60**	77 815	**80**	90 309
1	00 000	21	32 222	41	61 278	61	78 533	81	90 849
2	30 103	22	34 242	42	62 325	62	79 239	82	91 381
3	47 712	23	36 173	43	63 347	63	79 934	83	91 908
4	60 206	24	38 021	44	64 345	64	80 618	84	92 428
5	69 897	25	39 794	45	65 321	65	81 291	85	92 942
6	77 815	26	41 497	46	66 276	66	81 954	86	93 450
7	84 510	27	43 136	47	67 210	67	82 607	87	93 952
8	90 309	28	44 716	48	68 124	68	83 251	88	94 448
9	95 424	29	46 240	49	69 020	69	83 885	89	94 939
10	00 000	**30**	47 712	**50**	69 897	**70**	84 510	**90**	95 424
11	04 139	31	49 136	51	70 757	71	85 126	91	95 904
12	07 918	32	50 515	52	71 600	72	85 733	92	96 379
13	11 394	33	51 851	53	72 428	73	86 332	93	96 848
14	14 613	34	53 148	54	73 239	74	86 923	94	97 313
15	17 609	35	54 407	55	74 036	75	87 506	95	97 772
16	20 412	36	55 630	56	74 819	76	88 081	96	98 227
17	23 045	37	56 820	57	75 587	77	88 649	97	98 677
18	25 527	38	57 978	58	76 343	78	89 209	98	99 123
19	27 875	39	59 106	59	77 085	79	89 763	99	99 564
20	30 103	**40**	60 206	**60**	77 815	**80**	90 309	**100**	00 000

N.	L. 0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	*030	*072	*115	*157	*199	*242
103	01 284	326	368	410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	*036	*078
105	02 119	160	202	243	284	325	366	407	449	490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03 342	383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05 308	346	385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	*032
115	06 070	108	145	183	221	258	296	333	371	408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	*004	*041	*078	*115	*151
118	07 188	225	262	298	335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08 279	314	350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	587	621	656
125	691	726	760	795	830	864	899	934	968	*003
126	10 037	072	106	140	175	209	243	278	312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	*025
129	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
131	727	760	793	826	860	893	926	959	992	*024
132	12 057	090	123	156	189	222	254	287	320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905	937	969	*001
135	13 033	066	098	130	162	194	226	258	290	322
136	354	386	418	450	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925	956
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
141	922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	534	564	594	625	655	685	715	746	776	806
144	836	866	897	927	957	987	*017	*047	*077	*107
145	16 137	167	197	227	256	286	316	346	376	406
146	435	465	495	524	554	584	613	643	673	702
147	732	761	791	820	850	879	909	938	967	997
148	17 026	056	085	114	143	173	202	231	260	289
149	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	34	33
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2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
151	898	926	955	984	*013	*041	*070	*099	*127	*156
152	18 184	213	241	270	298	327	355	384	412	441
153	469	498	526	554	583	611	639	667	696	724
154	752	780	808	837	865	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	257	285
156	312	340	368	396	424	451	479	507	535	562
157	590	618	645	673	700	728	756	783	811	838
158	866	893	921	948	976	*003	*030	*058	*085	*112
159	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
161	683	710	737	703	790	817	844	871	898	925
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21 219	245	272	299	325	352	378	405	431	458
164	484	511	537	564	590	617	643	669	696	722
165	748	775	801	827	854	880	906	932	958	985
166	22 011	037	063	089	115	141	167	194	220	246
167	272	298	324	350	376	401	427	453	479	505
168	531	557	583	608	634	660	686	712	737	763
169	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	300	325	350	376	401	426	452	477	502	528
172	553	578	603	629	654	679	704	729	754	779
173	805	830	855	880	905	930	955	980	*005	*030
174	24 055	080	105	130	155	180	204	229	254	279
175	304	329	353	378	403	428	452	477	502	527
176	551	576	601	625	650	674	699	724	748	773
177	797	822	846	871	895	920	944	969	993	*018
178	25 042	066	091	115	139	164	188	212	237	261
179	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
181	768	792	816	840	864	888	912	935	959	983
182	26 007	031	055	079	102	126	150	174	198	221
183	245	269	293	316	340	364	387	411	435	458
184	482	505	529	553	576	600	623	647	670	694
185	717	741	764	788	811	834	858	881	905	928
186	951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27 184	207	231	254	277	300	323	346	370	393
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
191	28 103	126	149	171	194	217	240	262	285	307
192	330	353	375	398	421	443	466	488	511	533
193	556	578	601	623	646	668	691	713	735	758
194	780	803	825	847	870	892	914	937	959	981
195	29 003	026	048	070	092	115	137	159	181	203
196	226	248	270	292	314	336	358	380	403	425
197	447	469	491	513	535	557	579	601	623	645
198	667	688	710	732	754	776	798	820	842	863
199	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
1	2.5
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	23
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.0	20.7

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345	366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	244	264	284	304	325	345	365	385	405	425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	*005	*025
219	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	*005
224	35 025	044	064	083	102	122	141	160	180	199
225	218	238	257	276	295	315	334	353	372	392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
250	39 794	811	829	846	863	881	898	915	933	950
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123
252	40 140	157	175	192	209	226	243	261	278	295
253	312	329	346	364	381	398	415	432	449	466
254	483	500	518	535	552	569	586	603	620	637
255	654	671	688	705	722	739	756	773	790	807
256	824	841	858	875	892	909	926	943	960	976
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145
258	41 162	179	196	212	229	246	263	280	296	313
259	330	347	363	380	397	414	430	447	464	481
260	497	514	531	547	564	581	597	614	631	647
261	664	681	697	714	731	747	764	780	797	814
262	830	847	863	880	896	913	929	946	963	979
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144
264	42 160	177	193	210	226	243	259	275	292	308
265	325	341	357	374	390	406	423	439	455	472
266	488	504	521	537	553	570	586	602	619	635
267	651	667	684	700	716	732	749	765	781	797
268	813	830	846	862	878	894	911	927	943	959
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	185	201	217	233	249	265	281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	521	537	553	569	584	600
273	616	632	648	664	680	696	712	727	743	759
274	775	791	807	823	838	854	870	886	902	917
275	933	949	965	981	996	*012	*028	*044	*059	*075
276	44 091	107	122	138	154	170	185	201	217	232
277	248	264	279	295	311	326	342	358	373	389
278	404	420	436	451	467	483	498	514	529	545
279	560	576	592	607	623	638	654	669	685	700
280	716	731	747	762	778	793	809	824	840	855
281	871	886	902	917	932	948	963	979	994	*010
282	45 025	040	056	071	086	102	117	133	148	163
283	179	194	209	225	240	255	271	286	301	317
284	332	347	362	378	393	408	423	439	454	469
285	484	500	515	530	545	561	576	591	606	621
286	637	652	667	682	697	712	728	743	758	773
287	788	803	818	834	849	864	879	894	909	924
288	939	954	969	984	*000	*015	*030	*045	*060	*075
289	46 090	105	120	135	150	165	180	195	210	225
290	240	255	270	285	300	315	330	345	359	374
291	389	404	419	434	449	464	479	494	509	523
292	538	553	568	583	598	613	627	642	657	672
293	687	702	716	731	746	761	776	790	805	820
294	835	850	864	879	894	909	923	938	953	967
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47 129	144	159	173	188	202	217	232	246	261
297	276	290	305	319	334	349	363	378	392	407
298	422	436	451	465	480	494	509	524	538	553
299	567	582	596	611	625	640	654	669	683	698
300	712	727	741	756	770	784	799	813	828	842
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

	16
1	1.6
2	3.2
3	4.8
4	6.4
5	8.0
6	9.6
7	11.2
8	12.8
9	14.4

	15
1	1.5
2	3.0
3	4.5
4	6.0
5	7.5
6	9.0
7	10.5
8	12.0
9	13.5

	14
1	1.4
2	2.8
3	4.2
4	5.6
5	7.0
6	8.4
7	9.8
8	11.2
9	12.6

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
300	47 712	727	741	756	770	784	799	813	828	842	
301	857	871	885	900	914	929	943	958	972	986	
302	48 001	015	029	044	058	073	087	101	116	130	
303	144	159	173	187	202	216	230	244	259	273	
304	287	302	316	330	344	359	373	387	401	416	
305	430	444	458	473	487	501	515	530	544	558	
306	572	586	601	615	629	643	657	671	686	700	
307	714	728	742	756	770	785	799	813	827	841	
308	855	869	883	897	911	926	940	954	968	982	
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	
310	49 136	150	164	178	192	206	220	234	248	262	
311	276	290	304	318	332	346	360	374	388	402	
312	415	429	443	457	471	485	499	513	527	541	
313	554	568	582	596	610	624	638	651	665	679	
314	693	707	721	734	748	762	776	790	803	817	
315	831	845	859	872	886	900	914	927	941	955	
316	969	982	996	*010	*024	*037	*051	*065	*079	*092	
317	50 106	120	133	147	161	174	188	202	215	229	
318	243	256	270	284	297	311	325	338	352	365	
319	379	393	406	420	433	447	461	474	488	501	
320	515	529	542	556	569	583	596	610	623	637	
321	651	664	678	691	705	718	732	745	759	772	
322	786	799	813	826	840	853	866	880	893	907	
323	920	934	947	961	974	987	*001	*014	*028	*041	
324	51 055	068	081	095	108	121	135	148	162	175	
325	188	202	215	228	242	255	268	282	295	308	
326	322	335	348	362	375	388	402	415	428	441	
327	455	468	481	495	508	521	534	548	561	574	
328	587	601	614	627	640	654	667	680	693	706	
329	720	733	746	759	772	786	799	812	825	838	
330	851	865	878	891	904	917	930	943	957	970	
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101	
332	52 114	127	140	153	166	179	192	205	218	231	
333	244	257	270	284	297	310	323	336	349	362	
334	375	388	401	414	427	440	453	466	479	492	
335	504	517	530	543	556	569	582	595	608	621	
336	634	647	660	673	686	699	711	724	737	750	
337	763	776	789	802	815	827	840	853	866	879	
338	892	905	917	930	943	956	969	982	994	*007	
339	53 020	033	046	058	071	084	097	110	122	135	
340	148	161	173	186	199	212	224	237	250	263	
341	275	288	301	314	326	339	352	364	377	390	
342	403	415	428	441	453	466	479	491	504	517	
343	529	542	555	567	580	593	605	618	631	643	
344	656	668	681	694	706	719	732	744	757	769	
345	782	794	807	820	832	845	857	870	882	895	
346	908	920	933	945	958	970	983	995	*008	*020	
347	54 033	045	058	070	083	095	108	120	133	145	
348	158	170	183	195	208	220	233	245	258	270	
349	283	295	307	320	332	345	357	370	382	394	
350	407	419	432	444	456	469	481	494	506	518	
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351	531	543	555	568	580	593	605	617	630	642	
352	654	667	679	691	704	716	728	741	753	765	
353	777	790	802	814	827	839	851	864	876	888	
354	900	913	925	937	949	962	974	986	998	*011	**13**
355	55 023	035	047	060	072	084	096	108	121	133	1 1.3
356	145	157	169	182	194	206	218	230	242	255	2 2.6
357	267	279	291	303	315	328	340	352	364	376	3 3.9
358	388	400	413	425	437	449	461	473	485	497	4 5.2
359	509	522	534	546	558	570	582	594	606	618	5 6.5
360	630	642	654	666	678	691	703	715	727	739	6 7.8
361	751	763	775	787	799	811	823	835	847	859	7 9.1
362	871	883	895	907	919	931	943	955	967	979	8 10.4
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	9 11.7
364	56 110	122	134	146	158	170	182	194	205	217	
365	229	241	253	265	277	289	301	312	324	336	
366	348	360	372	384	396	407	419	431	443	455	**12**
367	467	478	490	502	514	526	538	549	561	573	1 1.2
368	585	597	608	620	632	644	656	667	679	691	2 2.4
369	703	714	726	738	750	761	773	785	797	808	3 3.6
370	820	832	844	855	867	879	891	902	914	926	4 4.8
371	937	949	961	972	984	996	*008	*019	*031	*043	5 6.0
372	57 054	066	078	089	101	113	124	136	148	159	6 7.2
373	171	183	194	206	217	229	241	252	264	276	7 8.4
374	287	299	310	322	334	345	357	368	380	392	8 9.6
375	403	415	426	438	449	461	473	484	496	507	9 10.8
376	519	530	542	553	565	576	588	600	611	623	
377	634	646	657	669	680	692	703	715	726	738	
378	749	761	772	784	795	807	818	830	841	852	**11**
379	864	875	887	898	910	921	933	944	955	967	1 1.1
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081	2 2.2
381	58 092	104	115	127	138	149	161	172	184	195	3 3.3
382	206	218	229	240	252	263	274	286	297	309	4 4.4
383	320	331	343	354	365	377	388	399	410	422	5 5.5
384	433	444	456	467	478	490	501	512	524	535	6 6.6
385	546	557	569	580	591	602	614	625	636	647	7 7.7
386	659	670	681	692	704	715	726	737	749	760	8 8.8
387	771	782	794	805	816	827	838	850	861	872	9 9.9
388	883	894	906	917	928	939	950	961	973	984	
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390	59 106	118	129	140	151	162	173	184	195	207	**10**
391	218	229	240	251	262	273	284	295	306	318	1 1.0
392	329	340	351	362	373	384	395	406	417	428	2 2.0
393	439	450	461	472	483	494	506	517	528	539	3 3.0
394	550	561	572	583	594	605	616	627	638	649	4 4.0
395	660	671	682	693	704	715	726	737	748	759	5 5.0
396	770	780	791	802	813	824	835	846	857	868	6 6.0
397	879	890	901	912	923	934	945	956	966	977	7 7.0
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086	8 8.0
399	60 097	108	119	130	141	152	163	173	184	195	9 9.0
400	206	217	228	239	249	260	271	282	293	304	
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400	60 206	217	228	239	249	260	271	282	293	304
401	314	325	336	347	358	369	379	390	401	412
402	423	433	444	455	466	477	487	498	509	520
403	531	541	552	563	574	584	595	606	617	627
404	638	649	660	670	681	692	703	713	724	735
405	746	756	767	778	788	799	810	821	831	842
406	853	863	874	885	895	906	917	927	938	949
407	959	970	981	991	*002	*013	*023	*034	*045	*055
408	61 066	077	087	098	109	119	130	140	151	162
409	172	183	194	204	215	225	236	247	257	268
410	278	289	300	310	321	331	342	352	363	374
411	384	395	405	416	426	437	448	458	469	479
412	490	500	511	521	532	542	553	563	574	584
413	595	606	616	627	637	648	658	669	679	690
414	700	711	721	731	742	752	763	773	784	794
415	805	815	826	836	847	857	868	878	888	899
416	909	920	930	941	951	962	972	982	993	*003
417	62 014	024	034	045	055	066	076	086	097	107
418	118	128	138	149	159	170	180	190	201	211
419	221	232	242	252	263	273	284	294	304	315
420	325	335	346	356	366	377	387	397	408	418
421	428	439	449	459	469	480	490	500	511	521
422	531	542	552	562	572	583	593	603	613	624
423	634	644	655	665	675	685	696	706	716	726
424	737	747	757	767	778	788	798	808	818	829
425	839	849	859	870	880	890	900	910	921	931
426	941	951	961	972	982	992	*002	*012	*022	*033
427	63 043	053	063	073	083	094	104	114	124	134
428	144	155	165	175	185	195	205	215	225	236
429	246	256	266	276	286	296	306	317	327	337
430	347	357	367	377	387	397	407	417	428	438
431	448	458	468	478	488	498	508	518	528	538
432	548	558	568	579	589	599	609	619	629	639
433	649	659	669	679	689	699	709	719	729	739
434	749	759	769	779	789	799	809	819	829	839
435	849	859	869	879	889	899	909	919	929	939
436	949	959	969	979	988	998	*008	*018	*028	*038
437	64 048	058	068	078	088	098	108	118	128	137
438	147	157	167	177	187	197	207	217	227	237
439	246	256	266	276	286	296	306	316	326	335
440	345	355	365	375	385	395	404	414	424	434
441	444	454	464	473	483	493	503	513	523	532
442	542	552	562	572	582	591	601	611	621	631
443	640	650	660	670	680	689	699	709	719	729
444	738	748	758	768	777	787	797	807	816	826
445	836	846	856	865	875	885	895	904	914	924
446	933	943	953	963	972	982	992	*002	*011	*021
447	65 031	040	050	060	070	079	089	099	108	118
448	128	137	147	157	167	176	186	196	205	215
449	225	234	244	254	263	273	283	292	302	312
450	321	331	341	350	360	369	379	389	398	408

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450	65 321	331	341	350	360	369	379	389	398	408	
451	418	427	437	447	456	466	475	485	495	504	
452	514	523	533	543	552	562	571	581	591	600	
453	610	619	629	639	648	658	667	677	686	696	
454	706	715	725	734	744	753	763	772	782	792	
455	801	811	820	830	839	849	858	868	877	887	
456	896	906	916	925	935	944	954	963	973	982	**10**
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	1 1.0
458	66 087	096	106	115	124	134	143	153	162	172	2 2.0
459	181	191	200	210	219	229	238	247	257	266	3 3.0 4 4.0
460	276	285	295	304	314	323	332	342	351	361	5 5.0 6 6.0
461	370	380	389	398	408	417	427	436	445	455	7 7.0
462	464	474	483	492	502	511	521	530	539	549	8 8.0
463	558	567	577	586	596	605	614	624	633	642	9 9.0
464	652	661	671	680	689	699	708	717	727	736	
465	745	755	764	773	783	792	801	811	820	829	
466	839	848	857	867	876	885	894	904	913	922	
467	932	941	950	960	969	978	987	997	*006	*015	
468	67 025	034	043	052	062	071	080	089	099	108	
469	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
471	302	311	321	330	339	348	357	367	376	385	**9**
472	394	403	413	422	431	440	449	459	468	477	1 0.9
473	486	495	504	514	523	532	541	550	560	569	2 1.8
474	578	587	596	605	614	624	633	642	651	660	3 2.7
475	669	679	688	697	706	715	724	733	742	752	4 3.6 5 4.5
476	761	770	779	788	797	806	815	825	834	843	6 5.4
477	852	861	870	879	888	897	906	916	925	934	7 6.3
478	943	952	961	970	979	988	997	*006	*015	*024	8 7.2
479	68 034	043	052	061	070	079	088	097	106	115	9 8.1
480	124	133	142	151	160	169	178	187	196	205	
481	215	224	233	242	251	260	269	278	287	296	
482	305	314	323	332	341	350	359	368	377	386	
483	395	404	413	422	431	440	449	458	467	476	
484	485	494	502	511	520	529	538	547	556	565	
485	574	583	592	601	610	619	628	637	646	655	
486	664	673	681	690	699	708	717	726	735	744	
487	753	762	771	780	789	797	806	815	824	833	**8**
488	842	851	860	869	878	886	895	904	913	922	1 0.8 2 1.6
489	931	940	949	958	966	975	984	993	*002	*011	3 2.4
490	69 020	028	037	046	055	064	073	082	090	099	4 3.2 5 4.0
491	108	117	126	135	144	152	161	170	179	188	6 4.8
492	197	205	214	223	232	241	249	258	267	276	7 5.6 8 6.4
493	285	294	302	311	320	329	338	346	355	364	9 7.2
494	373	381	390	399	408	417	425	434	443	452	
495	461	469	478	487	496	504	513	522	531	539	
496	548	557	566	574	583	592	601	609	618	627	
497	636	644	653	662	671	679	688	697	705	714	
498	723	732	740	749	758	767	775	784	793	801	
499	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
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501	984	992	*001	*010	*018	*027	*036	*044	*053	*062
502	70 070	079	088	096	105	114	122	131	140	148
503	157	165	174	183	191	200	209	217	226	234
504	243	252	260	269	278	286	295	303	312	321
505	329	338	346	355	364	372	381	389	398	406
506	415	424	432	441	449	458	467	475	484	492
507	501	509	518	526	535	544	552	561	569	578
508	586	595	603	612	621	629	638	646	655	663
509	672	680	689	697	706	714	723	731	740	749
510	757	766	774	783	791	800	808	817	825	834
511	842	851	859	868	876	885	893	902	910	919
512	927	935	944	952	961	969	978	986	995	*003
513	71 012	020	029	037	046	054	063	071	079	088
514	096	105	113	122	130	139	147	155	164	172
515	181	189	198	206	214	223	231	240	248	257
516	265	273	282	290	299	307	315	324	332	341
517	349	357	366	374	383	391	399	408	416	425
518	433	441	450	458	466	475	483	492	500	508
519	517	525	533	542	550	559	567	575	584	592
520	600	609	617	625	634	642	650	659	667	675
521	684	692	700	709	717	725	734	742	750	759
522	767	775	784	792	800	809	817	825	834	842
523	850	858	867	875	883	892	900	908	917	925
524	933	941	950	958	966	975	983	991	999	*008
525	72 016	024	032	041	049	057	066	074	082	090
526	099	107	115	123	132	140	148	156	165	173
527	181	189	198	206	214	222	230	239	247	255
528	263	272	280	288	296	304	313	321	329	337
529	346	354	362	370	378	387	395	403	411	419
530	428	436	444	452	460	469	477	485	493	501
531	509	518	526	534	542	550	558	567	575	583
532	591	599	607	616	624	632	640	648	656	665
533	673	681	689	697	705	713	722	730	738	746
534	754	762	770	779	787	795	803	811	819	827
535	835	843	852	860	868	876	884	892	900	908
536	916	925	933	941	949	957	965	973	981	989
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070
538	73 078	086	094	102	111	119	127	135	143	151
539	159	167	175	183	191	199	207	215	223	231
540	239	247	255	263	272	280	288	296	304	312
541	320	328	336	344	352	360	368	376	384	392
542	400	408	416	424	432	440	448	456	464	472
543	480	488	496	504	512	520	528	536	544	552
544	560	568	576	584	592	600	608	616	624	632
545	640	648	656	664	672	679	687	695	703	711
546	719	727	735	743	751	759	767	775	783	791
547	799	807	815	823	830	838	846	854	862	870
548	878	886	894	902	910	918	926	933	941	949
549	957	965	973	981	989	997	*005	*013	*020	*028
550	74 036	044	052	060	068	076	084	092	099	107
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550	74 036	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	**8**
561	896	904	912	920	927	935	943	950	958	966	1 0.8
562	974	981	989	997	*005	*012	*020	*028	*035	*043	2 1.6
563	75 051	059	066	074	082	089	097	105	113	120	3 2.4
564	128	136	143	151	159	166	174	182	189	197	4 3.2
565	205	213	220	228	236	243	251	259	266	274	5 4.0
566	282	289	297	305	312	320	328	335	343	351	6 4.8
567	358	366	374	381	389	397	404	412	420	427	7 5.6
568	435	442	450	458	465	473	481	488	496	504	8 6.4
569	511	519	526	534	542	549	557	565	572	580	9 7.2
570	587	595	603	610	618	626	633	641	648	656	
571	664	671	679	686	694	702	709	717	724	732	
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
581	418	425	433	440	448	455	462	470	477	485	
582	492	500	507	515	522	530	537	545	552	559	
583	567	574	582	589	597	604	612	619	626	634	
584	641	649	656	664	671	678	686	693	701	708	**7**
585	716	723	730	738	745	753	760	768	775	782	1 0.7
586	790	797	805	812	819	827	834	842	849	856	2 1.4
587	864	871	879	886	893	901	908	916	923	930	3 2.1
588	938	945	953	960	967	975	982	989	997	*004	4 2.8
589	77 012	019	026	034	041	048	056	063	070	078	5 3.5
590	085	093	100	107	115	122	129	137	144	151	6 4.2
591	159	166	173	181	188	195	203	210	217	225	7 4.9
592	232	240	247	254	262	269	276	283	291	298	8 5.6
593	305	313	320	327	335	342	349	357	364	371	9 6.3
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
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N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
600	77 815	822	830	837	844	851	859	866	873	880	
601	887	895	902	909	916	924	931	938	945	952	
602	960	967	974	981	988	996	*003	*010	*017	*025	
603	78 032	039	046	053	061	068	075	082	089	097	
604	104	111	118	125	132	140	147	154	161	168	
605	176	183	190	197	204	211	219	226	233	240	
606	247	254	262	269	276	283	290	297	305	312	**8**
607	319	326	333	340	347	355	362	369	376	383	1 | 0.8
608	390	398	405	412	419	426	433	440	447	455	2 | 1.6
609	462	469	476	483	490	497	504	512	519	526	3 | 2.4
610	533	540	547	554	561	569	576	583	590	597	4 | 3.2
611	604	611	618	625	633	640	647	654	661	668	5 | 4.0
612	675	682	689	696	704	711	718	725	732	739	6 | 4.8
613	746	753	760	767	774	781	789	796	803	810	7 | 5.6
614	817	824	831	838	845	852	859	866	873	880	8 | 6.4
615	888	895	902	909	916	923	930	937	944	951	9 | 7.2
616	958	965	972	979	986	993	*000	*007	*014	*021	
617	79 029	036	043	050	057	064	071	078	085	092	
618	099	106	113	120	127	134	141	148	155	162	
619	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
621	309	316	323	330	337	344	351	358	365	372	**7**
622	379	386	393	400	407	414	421	428	435	442	1 | 0.7
623	449	456	463	470	477	484	491	498	505	511	2 | 1.4
624	518	525	532	539	546	553	560	567	574	581	3 | 2.1
625	588	595	602	609	616	623	630	637	644	650	4 | 2.8
626	657	664	671	678	685	692	699	706	713	720	5 | 3.5
627	727	734	741	748	754	761	768	775	782	789	6 | 4.2
628	796	803	810	817	824	831	837	844	851	858	7 | 4.9
629	865	872	879	886	893	900	906	913	920	927	8 | 5.6
630	934	941	948	955	962	969	975	982	989	996	9 | 6.3
631	80 003	010	017	024	030	037	044	051	058	065	
632	072	079	085	092	099	106	113	120	127	134	
633	140	147	154	161	168	175	182	188	195	202	
634	209	216	223	229	236	243	250	257	264	271	
635	277	284	291	298	305	312	318	325	332	339	
636	346	353	359	366	373	380	387	393	400	407	
637	414	421	428	434	441	448	455	462	468	475	**6**
638	482	489	496	502	509	516	523	530	536	543	1 | 0.6
639	550	557	564	570	577	584	591	598	604	611	2 | 1.2
640	618	625	632	638	645	652	659	665	672	679	3 | 1.8
641	686	693	699	706	713	720	726	733	740	747	4 | 2.4
642	754	760	767	774	781	787	794	801	808	814	5 | 3.0
643	821	828	835	841	848	855	862	868	875	882	6 | 3.6
644	889	895	902	909	916	922	929	936	943	949	7 | 4.2
645	956	963	969	976	983	990	996	*003	*010	*017	8 | 4.8
646	81 023	030	037	043	050	057	064	070	077	084	9 | 5.4
647	090	097	104	111	117	124	131	137	144	151	
648	158	164	171	178	184	191	198	204	211	218	
649	224	231	238	245	251	258	265	271	278	285	
650	291	298	305	311	318	325	331	338	345	351	
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N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
650	81 291	298	305	311	318	325	331	338	345	351	
651	358	365	371	378	385	391	398	405	411	418	
652	425	431	438	445	451	458	465	471	478	485	
653	491	498	505	511	518	525	531	538	544	551	
654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
658	823	829	836	842	849	856	862	869	875	882	
659	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	**7**
661	82 020	027	033	040	046	053	060	066	073	079	1 0.7
662	086	092	099	105	112	119	125	132	138	145	2 1.4
663	151	158	164	171	178	184	191	197	204	210	3 2.1
664	217	223	230	236	243	249	256	263	269	276	4 2.8
665	282	289	295	302	308	315	321	328	334	341	5 3.5
666	347	354	360	367	373	380	387	393	400	406	6 4.2
667	413	419	426	432	439	445	452	458	465	471	7 4.9
668	478	484	491	497	504	510	517	523	530	536	8 5.6
669	543	549	556	562	569	575	582	588	595	601	9 6.3
670	607	614	620	627	633	640	646	653	659	666	
671	672	679	685	692	698	705	711	718	724	730	
672	737	743	750	756	763	769	776	782	789	795	
673	802	808	814	821	827	834	840	847	853	860	
674	866	872	879	885	892	898	905	911	918	924	
675	930	937	943	950	956	963	969	975	982	988	
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	
681	315	321	327	334	340	347	353	359	366	372	
682	378	385	391	398	404	410	417	423	429	436	
683	442	448	455	461	467	474	480	487	493	499	
684	506	512	518	525	531	537	544	550	556	563	**6**
685	569	575	582	588	594	601	607	613	620	626	1 0.6
686	632	639	645	651	658	664	670	677	683	689	2 1.2
687	696	702	708	715	721	727	734	740	746	753	3 1.8
688	759	765	771	778	784	790	797	803	809	816	4 2.4
689	822	828	835	841	847	853	860	866	872	879	5 3.0
690	885	891	897	904	910	916	923	929	935	942	6 3.6
691	948	954	960	967	973	979	985	992	998	*004	7 4.2
692	84 011	017	023	029	036	042	048	055	061	067	8 4.8
693	073	080	086	092	098	105	111	117	123	130	9 5.4
694	136	142	148	155	161	167	173	180	186	192	
695	198	205	211	217	223	230	236	242	248	255	
696	261	267	273	280	286	292	298	305	311	317	
697	323	330	336	342	348	354	361	367	373	379	
698	386	392	398	404	410	417	423	429	435	442	
699	448	454	460	466	473	479	485	491	497	504	
700	510	516	522	528	535	541	547	553	559	566	
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700	84 510	516	522	528	535	541	547	553	559	566
701	572	578	584	590	597	603	609	615	621	628
702	634	640	646	652	658	665	671	677	683	689
703	696	702	708	714	720	726	733	739	745	751
704	757	763	770	776	782	788	794	800	807	813
705	819	825	831	837	844	850	856	862	868	874
706	880	887	893	899	905	911	917	924	930	936
707	942	948	954	960	967	973	979	985	991	997
708	85 003	009	016	022	028	034	040	046	052	058
709	065	071	077	083	089	095	101	107	114	120
710	126	132	138	144	150	156	163	169	175	181
711	187	193	199	205	211	217	224	230	236	242
712	248	254	260	266	272	278	285	291	297	303
713	309	315	321	327	333	339	345	352	358	364
714	370	376	382	388	394	400	406	412	418	425
715	431	437	443	449	455	461	467	473	479	485
716	491	497	503	509	516	522	528	534	540	546
717	552	558	564	570	576	582	588	594	600	606
718	612	618	625	631	637	643	649	655	661	667
719	673	679	685	691	697	703	709	715	721	727
720	733	739	745	751	757	763	769	775	781	788
721	794	800	806	812	818	824	830	836	842	848
722	854	860	866	872	878	884	890	896	902	908
723	914	920	926	932	938	944	950	956	962	968
724	974	980	986	992	998	*004	*010	*016	*022	*028
725	86 034	040	046	052	058	064	070	076	082	088
726	094	100	106	112	118	124	130	136	141	147
727	153	159	165	171	177	183	189	195	201	207
728	213	219	225	231	237	243	249	255	261	267
729	273	279	285	291	297	303	308	314	320	326
730	332	338	344	350	356	362	368	374	380	386
731	392	398	404	410	415	421	427	433	439	445
732	451	457	463	469	475	481	487	493	499	504
733	510	516	522	528	534	540	546	552	558	564
734	570	576	581	587	593	599	605	611	617	623
735	629	635	641	646	652	658	664	670	676	682
736	688	694	700	705	711	717	723	729	735	741
737	747	753	759	764	770	776	782	788	794	800
738	806	812	817	823	829	835	841	847	853	859
739	864	870	876	882	888	894	900	906	911	917
740	923	929	935	941	947	953	958	964	970	976
741	982	988	994	999	*005	*011	*017	*023	*029	*035
742	87 040	046	052	058	064	070	075	081	087	093
743	099	105	111	116	122	128	134	140	146	151
744	157	163	169	175	181	186	192	198	204	210
745	216	221	227	233	239	245	251	256	262	268
746	274	280	286	291	297	303	309	315	320	326
747	332	338	344	349	355	361	367	373	379	384
748	390	396	402	408	413	419	425	431	437	442
749	448	454	460	466	471	477	483	489	495	500
750	506	512	518	523	529	535	541	547	552	558
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

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9	4.5

N.	L. 0	1	2	3	4	5	6	7	8	9
750	87 506	512	518	523	529	535	541	547	552	558
751	564	570	576	581	587	593	599	604	610	616
752	622	628	633	639	645	651	656	662	668	674
753	679	685	691	697	703	708	714	720	726	731
754	737	743	749	754	760	766	772	777	783	789
755	795	800	806	812	818	823	829	835	841	846
756	852	858	864	869	875	881	887	892	898	904
757	910	915	921	927	933	938	944	950	955	961
758	967	973	978	984	990	996	*001	*007	*013	*018
759	88 024	030	036	041	047	053	058	064	070	076
760	081	087	093	098	104	110	116	121	127	133
761	138	144	150	156	161	167	173	178	184	190
762	195	201	207	213	218	224	230	235	241	247
763	252	258	264	270	275	281	287	292	298	304
764	309	315	321	326	332	338	343	349	355	360
765	366	372	377	383	389	395	400	406	412	417
766	423	429	434	440	446	451	457	463	468	474
767	480	485	491	497	502	508	513	519	525	530
768	536	542	547	553	559	564	570	576	581	587
769	593	598	604	610	615	621	627	632	638	643
770	649	655	660	666	672	677	683	689	694	700
771	705	711	717	722	728	734	739	745	750	756
772	762	767	773	779	784	790	795	801	807	812
773	818	824	829	835	840	846	852	857	863	868
774	874	880	885	891	897	902	908	913	919	925
775	930	936	941	947	953	958	964	969	975	981
776	986	992	997	*003	*009	*014	*020	*025	*031	*037
777	89 042	048	053	059	064	070	076	081	087	092
778	098	104	109	115	120	126	131	137	143	148
779	154	159	165	170	176	182	187	193	198	204
780	209	215	221	226	232	237	243	248	254	260
781	265	271	276	282	287	293	298	304	310	315
782	321	326	332	337	343	348	354	360	365	371
783	376	382	387	393	398	404	409	415	421	426
784	432	437	443	448	454	459	465	470	476	481
785	487	492	498	504	509	515	520	526	531	537
786	542	548	553	559	564	570	575	581	586	592
787	597	603	609	614	620	625	631	636	642	647
788	653	658	664	669	675	680	686	691	697	702
789	708	713	719	724	730	735	741	746	752	757
790	763	768	774	779	785	790	796	801	807	812
791	818	823	829	834	840	845	851	856	862	867
792	873	878	883	889	894	900	905	911	916	922
793	927	933	938	944	949	955	960	966	971	977
794	982	988	993	998	*004	*009	*015	*020	*026	*031
795	90 037	042	048	053	059	064	069	075	080	086
796	091	097	102	108	113	119	124	129	135	140
797	146	151	157	162	168	173	179	184	189	195
798	200	206	211	217	222	227	233	238	244	249
799	255	260	266	271	276	282	287	293	298	304
800	309	314	320	325	331	336	342	347	352	358
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

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8	4.8
9	5.4

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4	2.0
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9	4.5

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
800	90 309	314	320	325	331	336	342	347	352	358
801	363	369	374	380	385	390	396	401	407	412
802	417	423	428	434	439	445	450	455	461	466
803	472	477	482	488	493	499	504	509	515	520
804	526	531	536	542	547	553	558	563	569	574
805	580	585	590	596	601	607	612	617	623	628
806	634	639	644	650	655	660	666	671	677	682
807	687	693	698	703	709	714	720	725	730	736
808	741	747	752	757	763	768	773	779	784	789
809	795	800	806	811	816	822	827	832	838	843
810	849	854	859	865	870	875	881	886	891	897
811	902	907	913	918	924	929	934	940	945	950
812	956	961	966	972	977	982	988	993	998	*004
813	91 009	014	020	025	030	036	041	046	052	057
814	062	068	073	078	084	089	094	100	105	110
815	116	121	126	132	137	142	148	153	158	164
816	169	174	180	185	190	196	201	206	212	217
817	222	228	233	238	243	249	254	259	265	270
818	275	281	286	291	297	302	307	312	318	323
819	328	334	339	344	350	355	360	365	371	376
820	381	387	392	397	403	408	413	418	424	429
821	434	440	445	450	455	461	466	471	477	482
822	487	492	498	503	508	514	519	524	529	535
823	540	545	551	556	561	566	572	577	582	587
824	593	598	603	609	614	619	624	630	635	640
825	645	651	656	661	666	672	677	682	687	693
826	698	703	709	714	719	724	730	735	740	745
827	751	756	761	766	772	777	782	787	793	798
828	803	808	814	819	824	829	834	840	845	850
829	855	861	866	871	876	882	887	892	897	903
830	908	913	918	924	929	934	939	944	950	955
831	960	965	971	976	981	986	991	997	*002	*007
832	92 012	018	023	028	033	038	044	049	054	059
833	065	070	075	080	085	091	096	101	106	111
834	117	122	127	132	137	143	148	153	158	163
835	169	174	179	184	189	195	200	205	210	215
836	221	226	231	236	241	247	252	257	262	267
837	273	278	283	288	293	298	304	309	314	319
838	324	330	335	340	345	350	355	361	366	371
839	376	381	387	392	397	402	407	412	418	423
840	428	433	438	443	449	454	459	464	469	474
841	480	485	490	495	500	505	511	516	521	526
842	531	536	542	547	552	557	562	567	572	578
843	583	588	593	598	603	609	614	619	624	629
844	634	639	645	650	655	660	665	670	675	681
845	686	691	696	701	706	711	716	722	727	732
846	737	742	747	752	758	763	768	773	778	783
847	788	793	799	804	809	814	819	824	829	834
848	840	845	850	855	860	865	870	875	881	886
849	891	896	901	906	911	916	921	927	932	937
850	942	947	952	957	962	967	973	978	983	988
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	6
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2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
850	92 942	947	952	957	962	967	973	978	983	988	
851	993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93 044	049	054	059	064	069	075	080	085	090	
853	095	100	105	110	115	120	125	131	136	141	
854	146	151	156	161	166	171	176	181	186	192	
855	197	202	207	212	217	222	227	232	237	242	
856	247	252	258	263	268	273	278	283	288	293	
857	298	303	308	313	318	323	328	334	339	344	6
858	349	354	359	364	369	374	379	384	389	394	1 0.6
859	399	404	409	414	420	425	430	435	440	445	2 1.2
860	450	455	460	465	470	475	480	485	490	495	3 1.8 4 2.4
861	500	505	510	515	520	526	531	536	541	546	5 3.0 6 3.6
862	551	556	561	566	571	576	581	586	591	596	7 4.2 8 4.8
863	601	606	611	616	621	626	631	636	641	646	9 5.4
864	651	656	661	666	671	676	682	687	692	697	
865	702	707	712	717	722	727	732	737	742	747	
866	752	757	762	767	772	777	782	787	792	797	
867	802	807	812	817	822	827	832	837	842	847	
868	852	857	862	867	872	877	882	887	892	897	
869	902	907	912	917	922	927	932	937	942	947	
870	952	957	962	967	972	977	982	987	992	997	
871	94 002	007	012	017	022	027	032	037	042	047	5
872	052	057	062	067	072	077	082	086	091	096	1 0.5
873	101	106	111	116	121	126	131	136	141	146	2 1.0
874	151	156	161	166	171	176	181	186	191	196	3 1.5
875	201	206	211	216	221	226	231	236	240	245	4 2.0
876	250	255	260	265	270	275	280	285	290	295	5 2.5 6 3.0
877	300	305	310	315	320	325	330	335	340	345	7 3.5 8 4.0
878	349	354	359	364	369	374	379	384	389	394	9 4.5
879	399	404	409	414	419	424	429	433	438	443	
880	448	453	458	463	468	473	478	483	488	493	
881	498	503	507	512	517	522	527	532	537	542	
882	547	552	557	562	567	571	576	581	586	591	
883	596	601	606	611	616	621	626	630	635	640	
884	645	650	655	660	665	670	675	680	685	689	
885	694	699	704	709	714	719	724	729	734	738	
886	743	748	753	758	763	768	773	778	783	787	
887	792	797	802	807	812	817	822	827	832	836	4
888	841	846	851	856	861	866	871	876	880	885	1 0.4 2 0.8
889	890	895	900	905	910	915	919	924	929	934	3 1.2
890	939	944	949	954	959	963	968	973	978	983	4 1.6 5 2.0
891	988	993	998	*002	*007	*012	*017	*022	*027	*032	6 2.4
892	95 036	041	046	051	056	061	066	071	075	080	7 2.8 8 3.2
893	085	090	095	100	105	109	114	119	124	129	9 3.6
894	134	139	143	148	153	158	163	168	173	177	
895	182	187	192	197	202	207	211	216	221	226	
896	231	236	240	245	250	255	260	265	270	274	
897	279	284	289	294	299	303	308	313	318	323	
898	328	332	337	342	347	352	357	361	366	371	
899	376	381	386	390	395	400	405	410	415	419	
900	424	429	434	439	444	448	453	458	463	468	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
900	95 424	429	434	439	444	448	453	458	463	468	
901	472	477	482	487	492	497	501	506	511	516	
902	521	525	530	535	540	545	550	554	559	564	
903	569	574	578	583	588	593	598	602	607	612	
904	617	622	626	631	636	641	646	650	655	660	
905	665	670	674	679	684	689	694	698	703	708	
906	713	718	722	727	732	737	742	746	751	756	
907	761	766	770	775	780	785	789	794	799	804	
908	809	813	818	823	828	832	837	842	847	852	
909	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	**5**
911	952	957	961	966	971	976	980	985	990	995	1 0.5
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	2 1.0
913	96 047	052	057	061	066	071	076	080	085	090	3 1.5
914	095	099	104	109	114	118	123	128	133	137	4 2.0
915	142	147	152	156	161	166	171	175	180	185	5 2.5
916	190	194	199	204	209	213	218	223	227	232	6 3.0
917	237	242	246	251	256	261	265	270	275	280	7 3.5
918	284	289	294	298	303	308	313	317	322	327	8 4.0
919	332	336	341	346	350	355	360	365	369	374	9 4.5
920	379	384	388	393	398	402	407	412	417	421	
921	426	431	435	440	445	450	454	459	464	468	
922	473	478	483	487	492	497	501	506	511	515	
923	520	525	530	534	539	544	548	553	558	562	
924	567	572	577	581	586	591	595	600	605	609	
925	614	619	624	628	633	638	642	647	652	656	
926	661	666	670	675	680	685	689	694	699	703	
927	708	713	717	722	727	731	736	741	745	750	
928	755	759	764	769	774	778	783	788	792	797	
929	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	
931	895	900	904	909	914	918	923	928	932	937	
932	942	946	951	956	960	965	970	974	979	984	
933	988	993	997	*002	*007	*011	*016	*021	*025	*030	**4**
934	97 035	039	044	049	053	058	063	067	072	077	1 0.4
935	081	086	090	095	100	104	109	114	118	123	2 0.8
936	128	132	137	142	146	151	155	160	165	169	3 1.2
937	174	179	183	188	192	197	202	206	211	216	4 1.6
938	220	225	230	234	239	243	248	253	257	262	5 2.0
939	267	271	276	280	285	290	294	299	304	308	6 2.4
940	313	317	322	327	331	336	340	345	350	354	7 2.8
941	359	364	368	373	377	382	387	391	396	400	8 3.2
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943	451	456	460	465	470	474	479	483	488	493	
944	497	502	506	511	516	520	525	529	534	539	
945	543	548	552	557	562	566	571	575	580	585	
946	589	594	598	603	607	612	617	621	626	630	
947	635	640	644	649	653	658	663	667	672	676	
948	681	685	690	695	699	704	708	713	717	722	
949	727	731	736	740	745	749	754	759	763	768	
950	772	777	782	786	791	795	800	804	809	813	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
950	97 772	777	782	786	791	795	800	804	809	813	
951	818	823	827	832	836	841	845	850	855	859	
952	864	868	873	877	882	886	891	896	900	905	
953	909	914	918	923	928	932	937	941	946	950	
954	955	959	964	968	973	978	982	987	991	996	
955	98 000	005	009	014	019	023	028	032	037	041	
956	046	050	055	059	064	068	073	078	082	087	
957	091	096	100	105	109	114	118	123	127	132	
958	137	141	146	150	155	159	164	168	173	177	
959	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	
961	272	277	281	286	290	295	299	304	308	313	5
962	318	322	327	331	336	340	345	349	354	358	1 | 0.5
963	363	367	372	376	381	385	390	394	399	403	2 | 1.0
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965	453	457	462	466	471	475	480	484	489	493	4 | 2.0
966	498	502	507	511	516	520	525	529	534	538	5 | 2.5
967	543	547	552	556	561	565	570	574	579	583	6 | 3.0
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969	632	637	641	646	650	655	659	664	668	673	8 | 4.0
970	677	682	686	691	695	700	704	709	713	717	9 | 4.5
971	722	726	731	735	740	744	749	753	758	762	
972	767	771	776	780	784	789	793	798	802	807	
973	811	816	820	825	829	834	838	843	847	851	
974	856	860	865	869	874	878	883	887	892	896	
975	900	905	909	914	918	923	927	932	936	941	
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977	989	994	998	*003	*007	*012	*016	*021	*025	*029	
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979	078	083	087	092	096	100	105	109	114	118	
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981	167	171	176	180	185	189	193	198	202	207	4
982	211	216	220	224	229	233	238	242	247	251	1 | 0.4
983	255	260	264	269	273	277	282	286	291	295	2 | 0.8
984	300	304	308	313	317	322	326	330	335	339	3 | 1.2
985	344	348	352	357	361	366	370	374	379	383	4 | 1.6
986	388	392	396	401	405	410	414	419	423	427	5 | 2.0
987	432	436	441	445	449	454	458	463	467	471	6 | 2.4
988	476	480	484	489	493	498	502	506	511	515	7 | 2.8
989	520	524	528	533	537	542	546	550	555	559	8 | 3.2
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991	607	612	616	621	625	629	634	638	642	647	
992	651	656	660	664	669	673	677	682	686	691	
993	695	699	704	708	712	717	721	726	730	734	
994	739	743	747	752	756	760	765	769	774	778	
995	782	787	791	795	800	804	808	813	817	822	
996	826	830	835	839	843	848	852	856	861	865	
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998	913	917	922	926	930	935	939	944	948	952	
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