H. O. Pub. No. 605

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UNITED STATES NAVY DEPARTMENT
HYDROGRAPHIC OFFICE

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TECHNICAL REPORT NUMBER 2

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GRAPHICAL CONSTRUCTION OF WAVE REFRACTION DIAGRAMS BY THE WAVE FRONT METHOD

INTRODUCTION

The height, period, and direction of waves in deep water at an offshore point may be estimated, either by direct measurement with suitable instruments or directly from synoptic weather maps by the forecasting method of Sverdrup and Munk.[1] When waves move shoreward from deep water and approach the shore line at an angle, the wave crests are bent because the inshore portion of the wave travels at a lower velocity than the portion in deep water; consequently, the crests tend to conform to the bottom contours. Figure 1 shows the bending, or what is called "refraction," of waves near a shore line. The results of refraction are a change in wave height and in direction of travel. The amount of these changes is best estimated by use of a "refraction diagram." Such a diagram might be prepared entirely from aerial photographs, as was done in figure 2, but generally they are constructed graphically. A refraction diagram may be considered to be a map showing the wave crests at a given time, or the successive positions of a particular wave crest as it moves shoreward. Only crests several wave lengths apart are required to show the bending of the waves and thereby permit the construction of a set of lines which are everywhere perpendicular to the wave crests (fig. 2). These lines are known as "orthogonals", and the wave energy between any two orthogonals is considered to remain constant in estimating variations in wave height. The power transmitted by a train of sinusoidal waves is,

$$P=C_g\cdot\frac{w}{8}\cdot bH^2$$

Here, C_g is the velocity of transmission of the energy, w is the weight of water per unit volume, b is the length of crest (perpendicular to the local direction of travel) and H is the height from trough to crest. Ocean waves are not exactly sinusoidal, and their departure from this form increases as they approach the condition of breaking, but this formula is sufficiently precise for estimating wave heights and can be corrected by empirical results in the vicinity of the line of breakers. If no energy flows laterally along the wave crest, then, in a steady state of wave motion the same power should flow past all positions between two orthogonals. Indicating the conditions in deep water by the subscript zero,

$$P=P_o$$

$$H=H_o\sqrt{\frac{C_{g_o}}{C_g}}\cdot\sqrt{\frac{b_o}{b}}$$

The quantity $\sqrt{\frac{b_o}{b}}$ is termed the refraction coefficient. It will be designated at K_d. The quantity $\sqrt{\frac{C_{g_o}}{C_g}}$ represents the effect of a change in depth on the wave height. It will be designated as D. The wave height in any depth of water then may be written as,

$$H=H_o\cdot D\cdot K_d$$

It is the purpose of this report to present methods of determining K_d, the refraction coefficient.

It should be noted that the values of both D and K_d depend upon the depth and that they are usually opposite in effect. Refraction commonly tends to increase the length of the wave crest and thus to reduce the height while D, representing the effect of shoaling, tends to increase the height, except in a relatively unimportant range of depths where the waves first "feel the bottom." For reference, values of D are presented in table 1. (For additional details, see plate I, Breakers and Surf, H. O. No. 234, U. S. Navy, where it is

[1] Wind Waves and Swell, Principles in Forecasting, Hydrographic Office, Misc. Pub. 11275.

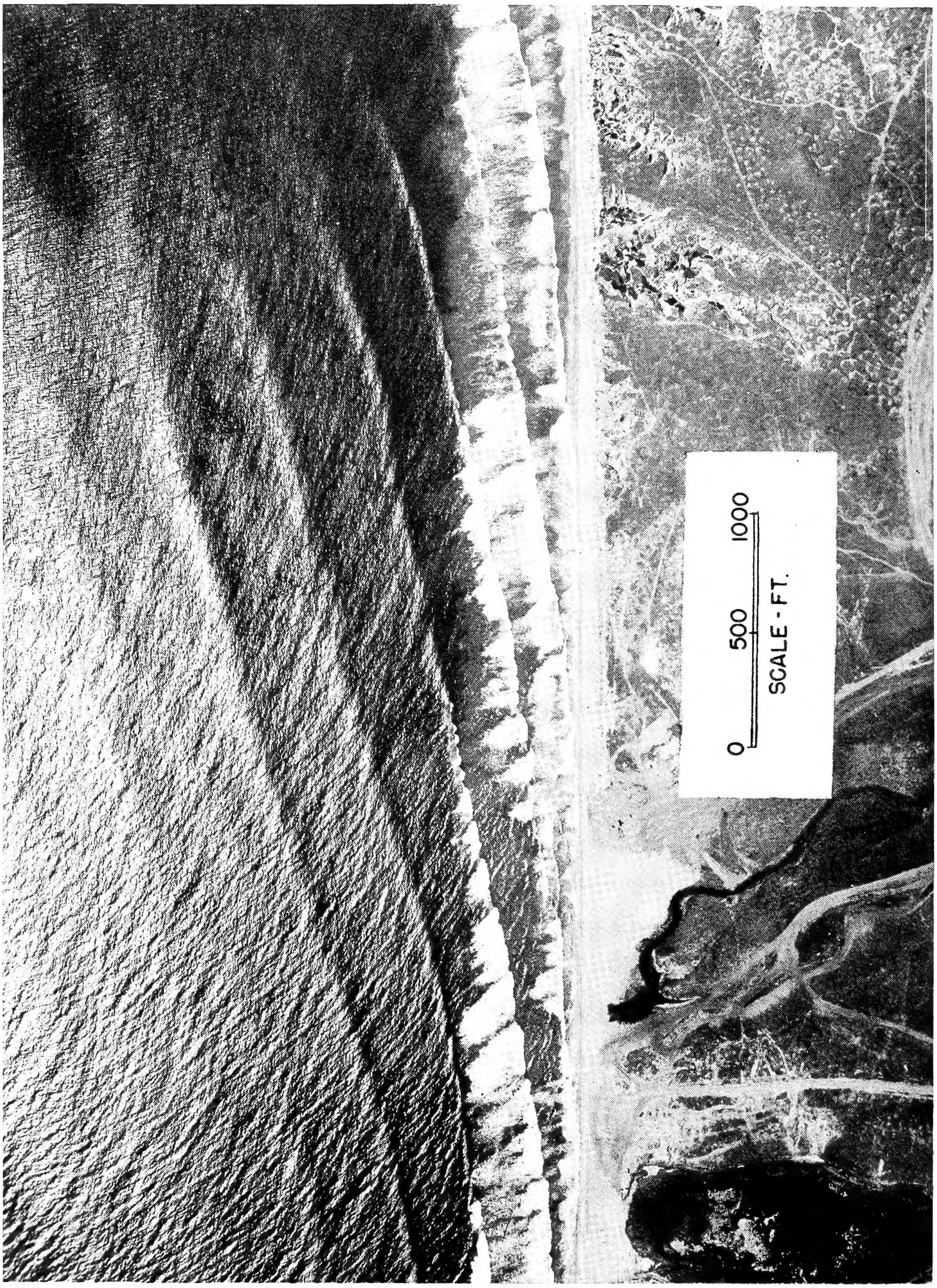


Figure 1.—Aerial photograph of swell, breakers, and surf north of Oceanside, Calif., 17 August 1945 (Utility Squadron 12).

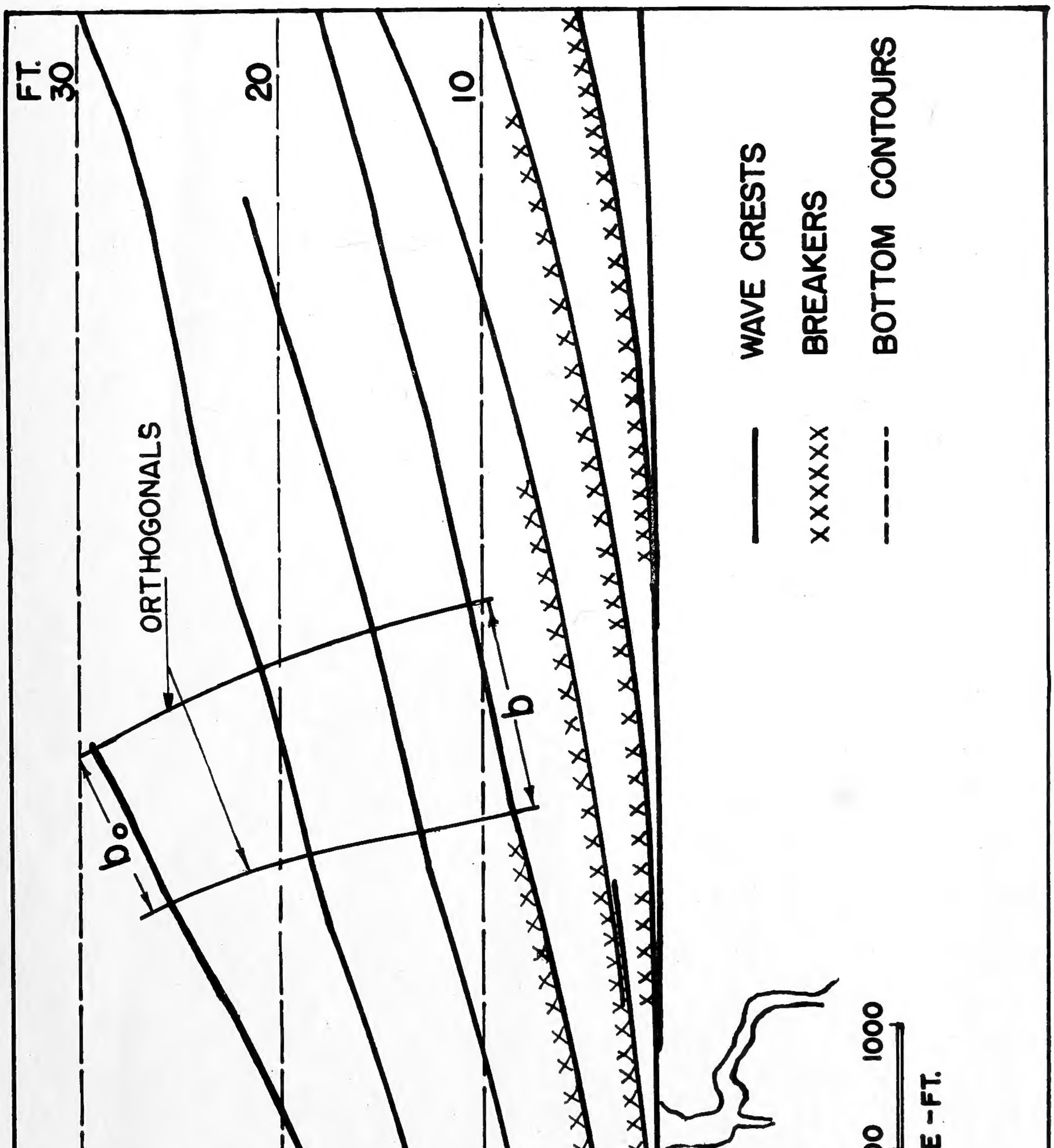


Figure 2.—Wave pattern from aerial photograph shown in figure 1.

designated as H/H'_o. or Tables of the Functions of d/L and d/L_o, HE-116-265.)

TABLE 1

Coefficient of shoaling, D

d/L_o	0.002	0.005	0.007	0.01	0.02	0.04
D	2.12	1.69	1.57	1.45	1.23	1.06

d/L_o	0.056	0.08	0.1	0.15	0.2	0.3	0.4
D	1.0	.94	.92	.91	.92	.93	.96

The graphical or analytical determination of wave refraction coefficients assumes (1) that the velocity of the wave crest depends only upon the still water depth under the crest at each point, (2) that the wave crest advances perpendicular to itself, and (3) that the wave energy is confined between orthogonals.

REFRACTION AT A STRAIGHT SHORE LINE

When the shore line and offshore contours are straight and parallel, refraction may be treated analytically by utilizing what is known as Snell's Law,

$$\frac{\text{Sin } \alpha}{\text{Sin } \alpha_o} = \frac{C}{C_o}$$

Here, α is the angle between the wave crest and the shore line in a depth such that the wave velocity is C (fig. 3). The change in angle determines the increase in crest length, and thus the value of K_d is fixed by the depth, which determines C for a particular C_o, and α_o, the angle in deep water. Referring to figure 3, the value of K_d may be computed from the relationship,

$$\frac{b_o}{\text{Cos } \alpha_o} = b_s = \frac{b}{\text{Cos } \alpha}$$

or

$$K_d = \sqrt{\frac{b_o}{b}} = \sqrt{\frac{\text{Cos } \alpha_o}{\text{Cos } \alpha}}$$

where

$$\alpha = \text{Sin}^{-1}\left(\frac{C}{C_o} \text{ Sin } \alpha_o\right)$$

For example, if $\alpha_o = 45°$ and the depth and period at the point for which K_d is to be computed, are such that $C/C_o = 0.5$,

$$\alpha = \text{Sin}^{-1}(0.5 \times 0.71) = 20.8 \text{ deg.}$$

$$\text{Cos } \alpha = 0.935 \text{ and Cos } \alpha_o = 0.707$$

$$K_d = 0.87$$

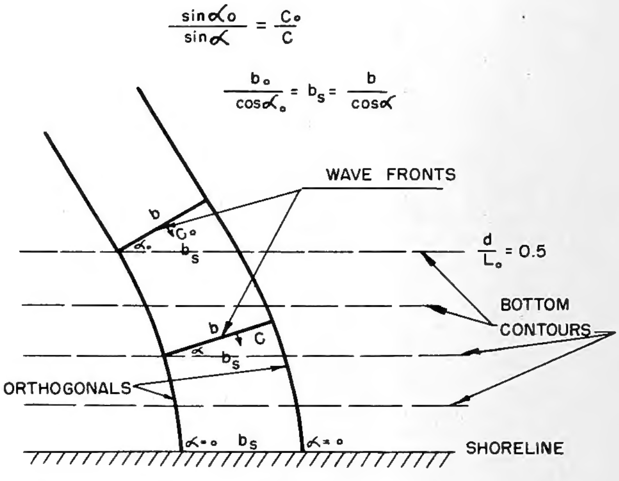


Figure 3.—Wave refraction assuming a gradual change in wave velocity.

For convenience the relationship between α, α_o, depth, period, and K_d have been summarized in graphical form in plate II, Breakers and Surf. This graph is included herein as figure 4.

A thorough understanding of the nature and magnitude of refraction effects at straight coast lines is helpful in constructing refraction diagrams for complex hydrography. The beginner should study figure 4 in order to develop judgment regarding the hydrographic conditions necessitating graphical analysis and as a basis for checking approximately the numerical values of K_d resulting from a graphical determination.

It is noteworthy that, on a straight shore line, the reduction in wave height by refraction is less than 10 percent when the initial angle in deep water is less than 36 degrees.

CONSTRUCTION OF REFRACTION DIAGRAMS

Ideal waves in deep water move forward with their crests parallel, but over a shoaling bottom the reduction in wave velocity causes the crest to swing around in the direction which will decrease the angle between the crest and the bottom contour. The preceding statement obviously requires a quantitative definition of what is meant by "deep water" and by "shallow water." The usual definition is that in

$$\left.\begin{array}{l}\text{deep water, } d > \frac{L_o}{2} \\ \text{shallow water, } d < \frac{L_o}{2}\end{array}\right| L_o = \frac{g}{2\pi} T^2 = 5.12 T^2$$

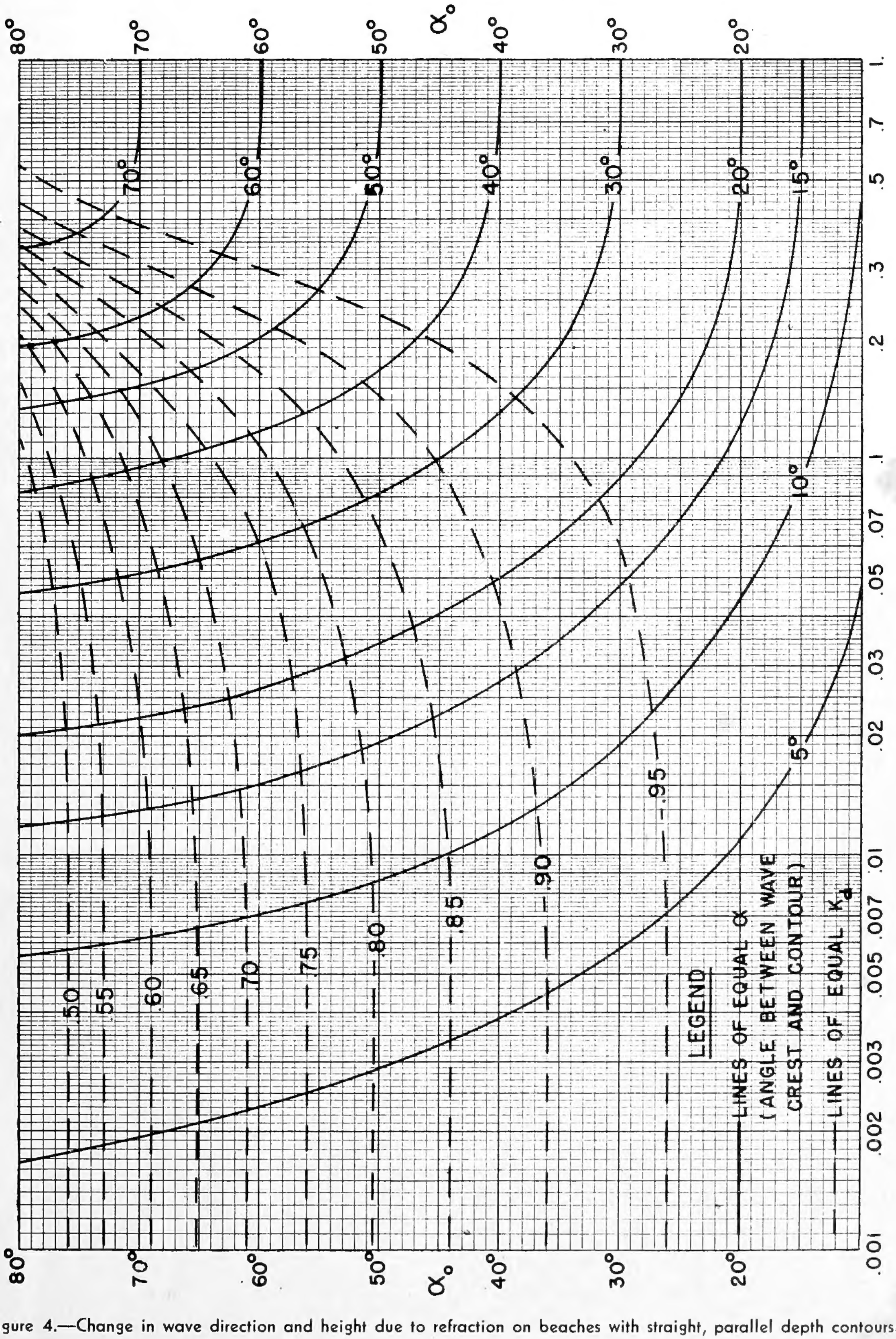


Figure 4.—Change in wave direction and height due to refraction on beaches with straight, parallel depth contours.

However, the meaning of these terms may be examined in the light of refractive effects in shallow water by considering the velocities and angles involved.

The velocity and length of a wave decreases in shallow water as shown in the following tables:

d/L_o	0. 4	0. 3	0. 2	0. 1
C/C_o	0. 98	0. 96	0. 89	0. 71

The angle through which a wave crest will turn between deep water and any value of d/L_o may be obtained from figure 4. By selecting a few values to represent the effect, table 2 shows that the limits of "shallow water" depend upon the angle α_o and the accuracy to which the diagram is to be constructed. If $\alpha_o=70$ degrees and if the construction is at an accuracy of ± 1 degree, the diagram should start in depths even greater than $d=0.5\ L_o$, but if α_o is only 10 degrees, a negligible error is introduced if the diagram is started from $d=0.3\ L_o$.

As a working rule, to be modified if circumstances so indicate, refraction diagrams should start from straight wave crests in a depth equal to half the deep water wave length or at $d=0.5\times 5.12\ T^2$.

TABLE 2

Values of α as a function of d/L_o and α_o

α_o \ d/L_o	0. 5	0. 4	0. 3	0. 2	0. 1
70	69	68	64	57	41
50	49. 5	48	46	42	32
30	30	29	28	26	21
10	10	10	9. 95	8. 5	7

The velocity of a wave in deep water is 5.12 T. As the wave moves into shallow water, its velocity decreases and, if the crest makes an angle with the bottom contours, the wave velocity will vary from point to point along the crest. Graphical construction of a refraction diagram consists simply in moving each point of the crest in a direction perpendicular to the crest by a distance equal to the wave velocity times the time interval selected. The initial form of the wave is a straight line in the deep water area, as previously defined. Figure 5 shows scales constructed in such manner as to give the advance of the wave crest at any value of d/L_o on a chart of any scale S. (These scales have been printed on thin paper and are available for distribution.) The two scales of figure 5 differ only in that scale A gives the wave advance during an interval which is twice that of scale B.

To construct a refraction diagram, the chart first is contoured with an interval which will represent accurately the details of the bottom topography. Each contour on the map is converted to mean sea level—or any other desired stage of the tide—adding the proper constant to the chart soundings. For the wave period selected the deep water wave length is computed from the relationship, $L_o=5.12\ T^2$. The contour values, in depth in feet below the tide stage selected, are then divided by L_o (in feet) to give contours in terms of d/L_o. Thus, in figure 6, for example, the contours in fathoms have been re-labeled in terms of values of d/L_o. Additional contours of d/L_o may be added if considered desirable. In figure 6, for example, contours of d/L_o of 0.5 and 0.4 have been added.

Generally, it is sufficient to draw every nth crest, where the value of the crest interval, n, depends upon the scale of the chart and the complexity of the bottom topography. The crest interval is determined by the scales used and may be expressed as n, a multiple value of wave length, or as a time interval, t. The crest interval does not have to be an even value, nor does it have to be the same for the entire chart, since more crests often should be drawn where the bottom topography is particularly complex. The two transparent scales (fig. 5) for plotting the wave advance are provided so that the crest interval in one scale (scale A) is just twice that for the second scale (scale B). These scales are applicable to charts of any scale and of any wave period. The only variable between refraction diagrams prepared by the use of the scales is the crest interval, this interval being a function of the scale of the hydrographic chart. Formulas are provided for computation of the crest interval, n, or time interval, t, for any particular refraction diagram (fig. 5).

It is often advantageous, as well as sometimes necessary, to draw a refraction diagram for a particular locality in several steps; First, the over-all pattern for a long stretch of coast line is drawn on a relatively small scale chart, following the waves from deep water to within a few thousand feet from shore; finally, the results are transferred to larger scale charts, and a detailed pattern is constructed of the waves close to shore in bays, harbors, and other areas of particular importance. Where the tidal range is large, it

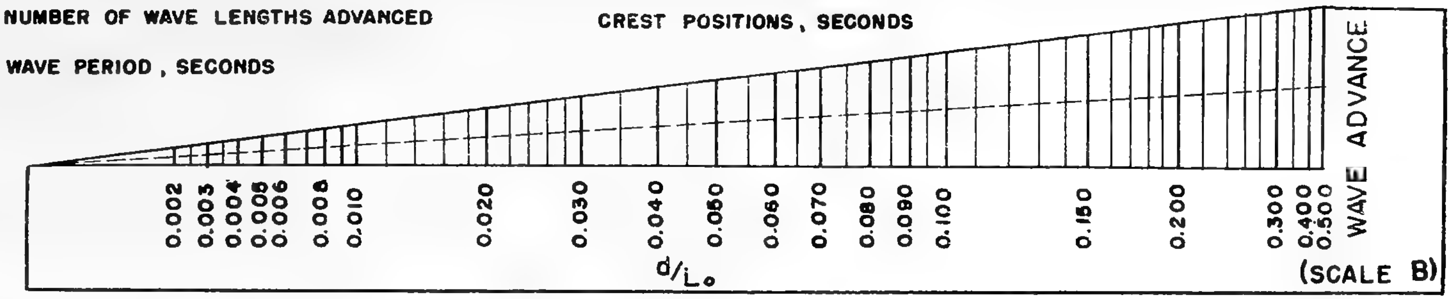
FOR SCALE A; $n = 0.0326 \frac{s}{T^2}$ AND $t = 0.0326 \frac{s}{T}$ FOR SCALE B; $n = 0.0163 \frac{s}{T^2}$ AND $t = 0.0163 \frac{s}{T}$

WHERE S = MAP SCALE IN THE FORM $1/S$

n = NUMBER OF WAVE LENGTHS ADVANCED

T = WAVE PERIOD, SECONDS

t = TIME INTERVAL BETWEEN CREST POSITIONS, SECONDS



EXAMPLE

FOR A MAP SCALE OF $\frac{1}{50,000}$ AND T = 10 SEC.; WHEN USING SCALE B, THE DISTANCE BETWEEN SUCCESSIVE CREST POSITIONS IS –

$$n = \frac{(0.0163)(50,000)}{(10)^2} = 8.14 \text{ WAVE LENGTHS}$$

AND

$$t = \frac{(0.0163)(50,000)}{(10)} = 81.4 \text{ SECONDS}$$

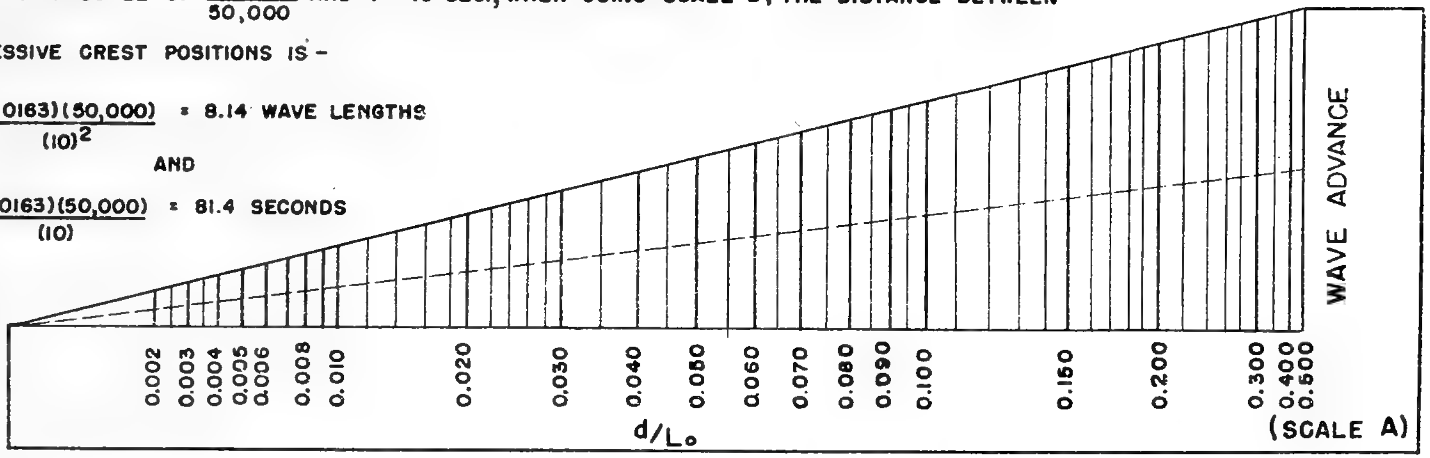


FIGURE 5

SCALES FOR PREPARATION OF WAVE REFRACTION DIAGRAMS

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may be necessary to construct several diagrams for different stages of the tide. On the California coast, where the range of tide is approximately 5 feet, diagrams prepared for an average stage of tide usually will suffice.

REFRACTION DIAGRAM, MONTEREY BAY, CALIF.

It is desired to prepare a refraction diagram for Monterey Bay and obtain values of K_d for points along the coast from Monterey to Santa Cruz. The diagram is to be prepared for a mean tide condition of 2 feet above M. L. L. W., direction of advance in deep water from W. N. W., and $T=14$ secs. Thus, $L_o=5.12\ T^2=5.12\ (14)^2=1{,}000$ feet and the depth which is the dividing contour between deep and shallow water is $L_o/2=500$ feet.

U. S. C. and G. S. charts that are available to show bottom topography are No. 5402, scale 1:214,000 and No. 5403, scale 1:50,000.

The contours appearing on these charts are in fathoms. The equivalent values in terms of d/L_o are as follows:

$$\text{100 fathoms; } \frac{d}{L_o}=\frac{(6)(100)+2}{1{,}000}=0.602$$

$$\text{50 fathoms; } \frac{d}{L_o}=\frac{(6)(50)+2}{1{,}000}=0.302$$

$$\text{40 fathoms; } \frac{d}{L_o}=\frac{(6)(40)+2}{1{,}000}=0.242$$

$$\text{30 fathoms; } \frac{d}{L_o}=\frac{(6)(30)+2}{1{,}000}=0.182$$

$$\text{20 fathoms; } \frac{d}{L_o}=\frac{(6)(20)+2}{1{,}000}=0.122$$

$$\text{15 fathoms; } \frac{d}{L_o}=\frac{(6)(15)+2}{1{,}000}=0.092$$

$$\text{10 fathoms; } \frac{d}{L_o}=\frac{(6)(10)+2}{1{,}000}=0.062$$

$$\text{5 fathoms; } \frac{d}{L_o}=\frac{(6)(5)+2}{1{,}000}=0.032$$

For accuracy in preparing the refraction diagram, it is necessary to use the chart with the largest scale (chart 5403); however, this chart does not extend to deep water (that is, beyond a depth of 500 feet); hence chart No. 5402 must be used, necessitating carrying the waves part way on this smaller scale chart and then transferring the front to the larger scale chart (No. 5403) near to the shore line. (Usually it is desirable to draw refraction diagrams on tracing paper placed over the hydrographic chart; however, for illustrative purposes in this report, the diagrams are drawn directly on the charts.)

Figure 6 shows a portion of USC and GS Chart 5402 with 14 wave crests marked 1, 2, ... 14. Crest 1 lies in deep water and is drawn as a straight line. The southerly portion of crests 1–14 remain in deep water (that is, where the contours of d/L_o have values greater than 0.5), and the distances between them are equal to a constant multiple of L_o. The northerly portions of the crests advance into shallow water with the result that the distances between the crests decrease. The position of each crest is determined from that of the crest behind it by locating a few points on the new crest and drawing a smooth curve through them. These points are shown as small circles in figure 6. The points are located by means of scale B in figure 5 which has been cut out and placed on the refraction diagram as illustrated in figure 6. For example, to locate point *c* on crest 12:

1. Lay the scale on the chart so that the dashed center line and the line of $d/L_o=0.302$ on the scale intersect with the contour $d/L_o=0.302$ on the chart (point *a*).
2. Move the scale so that condition (1) remains satisfied and the lower side of the scale is tangent to crest 11 on the diagram at the end of the $d/L_o=$ 0.302 line on the scale (point *b*).
3. Mark point *c* where the $d/L_o=0.302$ line on the scale reaches the upper side of the scale.
4. Thus, other points as *d*, *e*, and *f* are found by making use of the d/L_o contours of 0.242, 0.182, and 0.122, respectively. The process is repeated until a sufficient number of points are found to determine the position of crest 12.

In a similar manner to that outlined in items 1–4, above, other crests are located until the diagram is carried into a locality which is within the limits of a larger scale chart. On figure 6 it is noted that crest 14 is within the limits of the area shown on chart 5403. This crest is then transferred from chart 5402 by taking offsets from a convenient longitude (122°), correcting for scale ratio, and replotting on chart 5403 (fig. 7). Thus, the offsets in inches at each minute of latitude on chart 5402 are multiplied by the ratio 214,000/-50,000=4.28 and are shown plotted on chart 5403

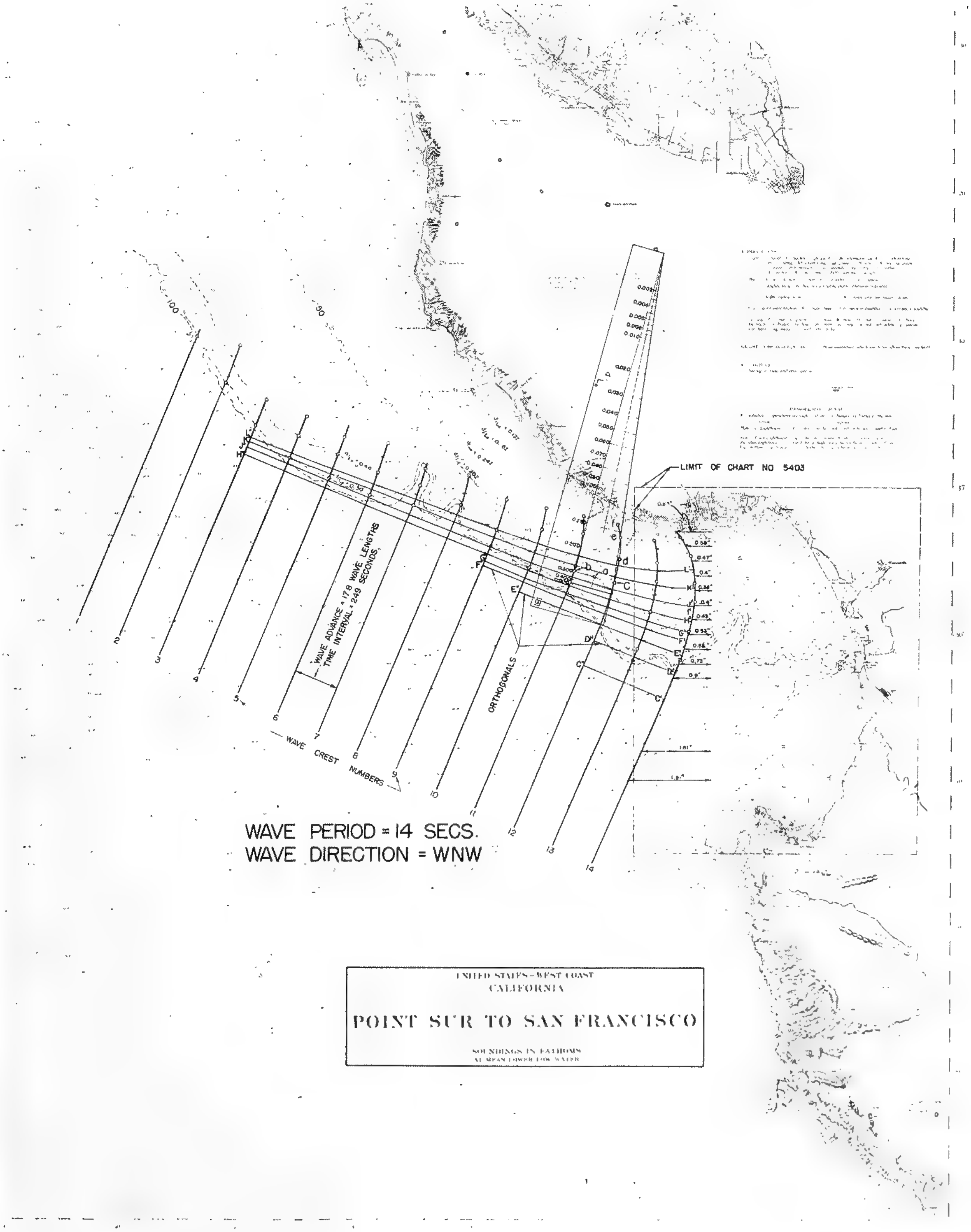


Figure 6.—Chart 5402.

with a smooth curve drawn to give the position of crest 14. (Note that charts 5402 and 5403 have been reduced by photostat for inclusion in this report.)

Starting with wave crest 14 on chart 5403, crests 15 to 23 are plotted by the method illustrated in steps 1 to 4, above. Because the bottom slope in the area covered by this chart, in general, is relatively uniform, crests can be plotted using a larger interval between crests; thus, crests 15 to 23 were plotted using scale A of figure 5.

On chart 5402 the spacing between crests was determined by means of scale B and the corresponding values of n and t are:

$$n=0.0163\,\frac{214{,}000}{(14)^2}=17.8 \text{ wave lengths}$$

$$t=0.0163\,\frac{(214{,}000)}{14}=249 \text{ seconds}$$

Similarly, on chart 5403 scale A was used, and the values of n and t are:

$$n=0.0326\,\frac{(50{,}000)}{(14)^2}=8.3 \text{ wave lengths}$$

$$t=0.0326\,\frac{(50{,}000)}{14}=117 \text{ seconds}$$

At a few localities where the bottom configuration is irregular, intermediate crests on chart 5403 (fig. 7) have been added by the use of scale B. These localities are between crests 16 and 17 near a side canyon of the Monterey Canyon and shoreward from crests 21, 22, and 23. Even this spacing is inadequate to describe the refraction which probably occurs at such locations as Santa Cruz Harbor, Monterey Harbor, and at the head of the Monterey Canyon at Moss Landing. For greater detail in these areas, charts with a larger scale should be used. Photostat enlargements, of the areas from chart No. 5403 could be made, but for more accuracy, the original USC and GS work sheets should be used. Thus, figure 8 shows contours from Hydrographic Chart 5415, which covers Monterey Harbor and vicinity with a scale of 1:5000. Inspection of chart No. 5415 shows that the bottom contours extend seaward only to the 17-fathom contour, which is about the location of crest 23 on chart 5403. Crest 23, consequently, has been transferred to chart 5415 by the offset method described above. Crests 24 to 33 have been constructed by the use of scale A. The crest interval on this diagram is as follows:

$$n=0.0326\,\frac{(5{,}000)}{(14)^2}=0.83 \text{ wave lengths}$$

$$t=0.0326\,\frac{(5{,}000)}{14}=11.7 \text{ seconds}$$

Further detail near the Monterey breakwater and within the harbor could be obtained by enlarging that area from chart 5415.

Upon completion of a plot showing the wave crests, the orthogonals are drawn on the diagrams. The orthogonals are started at the shore, or at some specified depth contour, and carried seaward as perpendiculars to the wave crests until deep water is reached. In shallow water they are curved lines but become straight lines in deep water. A small triangle and a short straight edge are convenient in constructing the orthogonals. The triangle is adjusted to be tangent to a wave crest, such as at a point a (fig. 9) where the orthogonal is to be started (crest 1). The straight-edge is held against the triangle and the triangle then slid along the straight-edge to permit a perpendicular to be drawn through point a and to point b half way to crest 2. The process is repeated for crest 2 with the perpendicular being drawn shoreward to point b and then extended seaward to a point c midway between crests 2 and 3. The procedure is repeated until the wave crest in deep water is reached. If desired, a smooth curve then could be drawn through the points where the perpendiculars cross the crests. The irregularities in the wave crests indicate how closely the orthogonals should be located. In figure 7 orthogonals have been drawn every two nautical miles starting at the Monterey Municipal Pier. With the exception of the vicinity of Moss Landing and northward this spacing appears suitable for most of the coast between the orthogonals A–F (fig. 7). As mentioned above, if the refraction coefficients are desired for the Moss Landing shore line, a refraction diagram should be prepared on a larger scale map, and the orthogonals should then be drawn at a smaller spacing. Additional orthogonals should be constructed northward from Moss Landing to better define the refraction coefficients in this area.

It is to be noted that the orthogonals shown on chart 5403 (fig. 7) ended at crest 14 and then had to be transferred to chart 5402 (fig. 6). In all probability, not all the orthogonals can be carried to deep water on the smaller scale chart. Only a few of the orthogonals actually are required to give a measure of refraction between

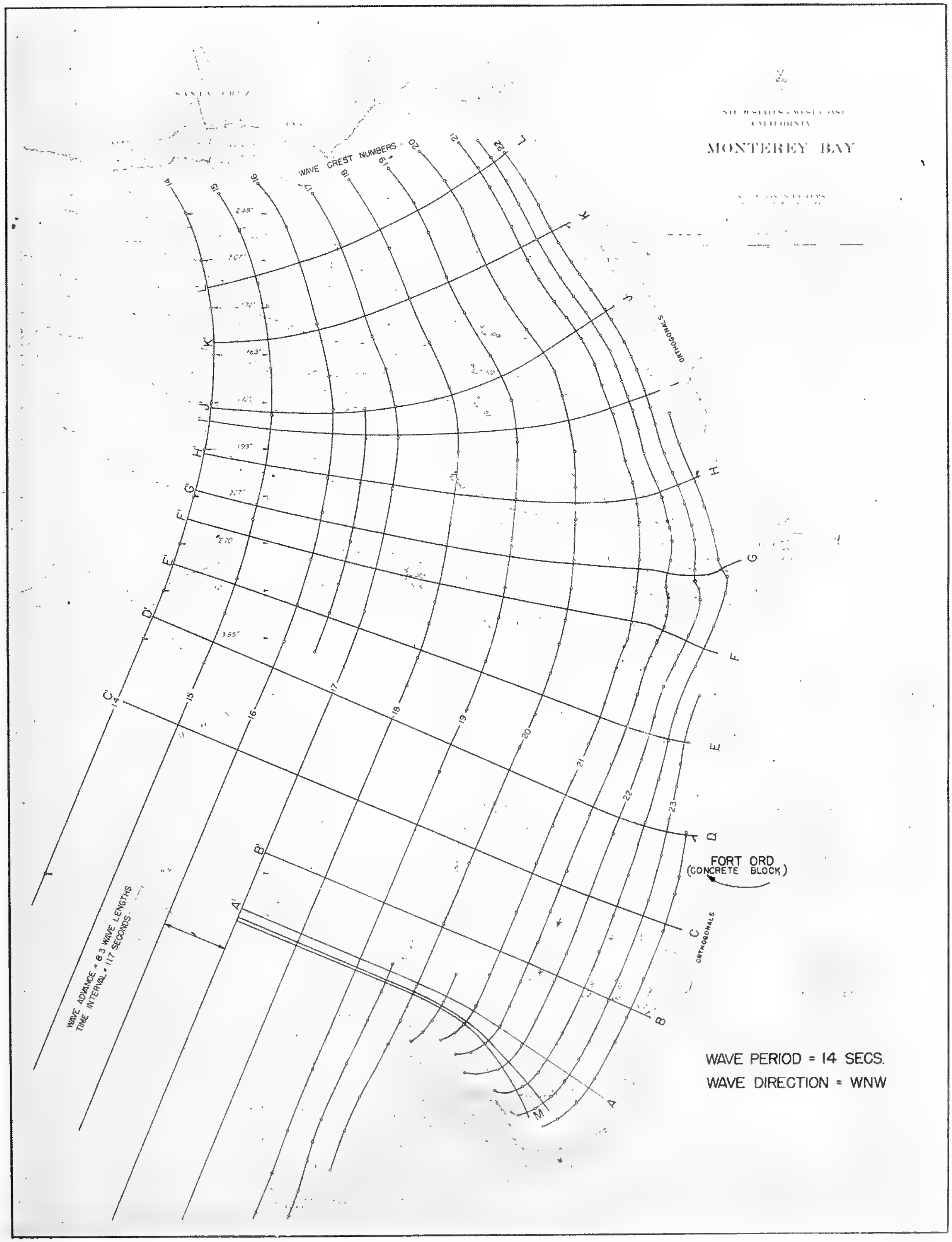


Figure 7.—Chart 5403.

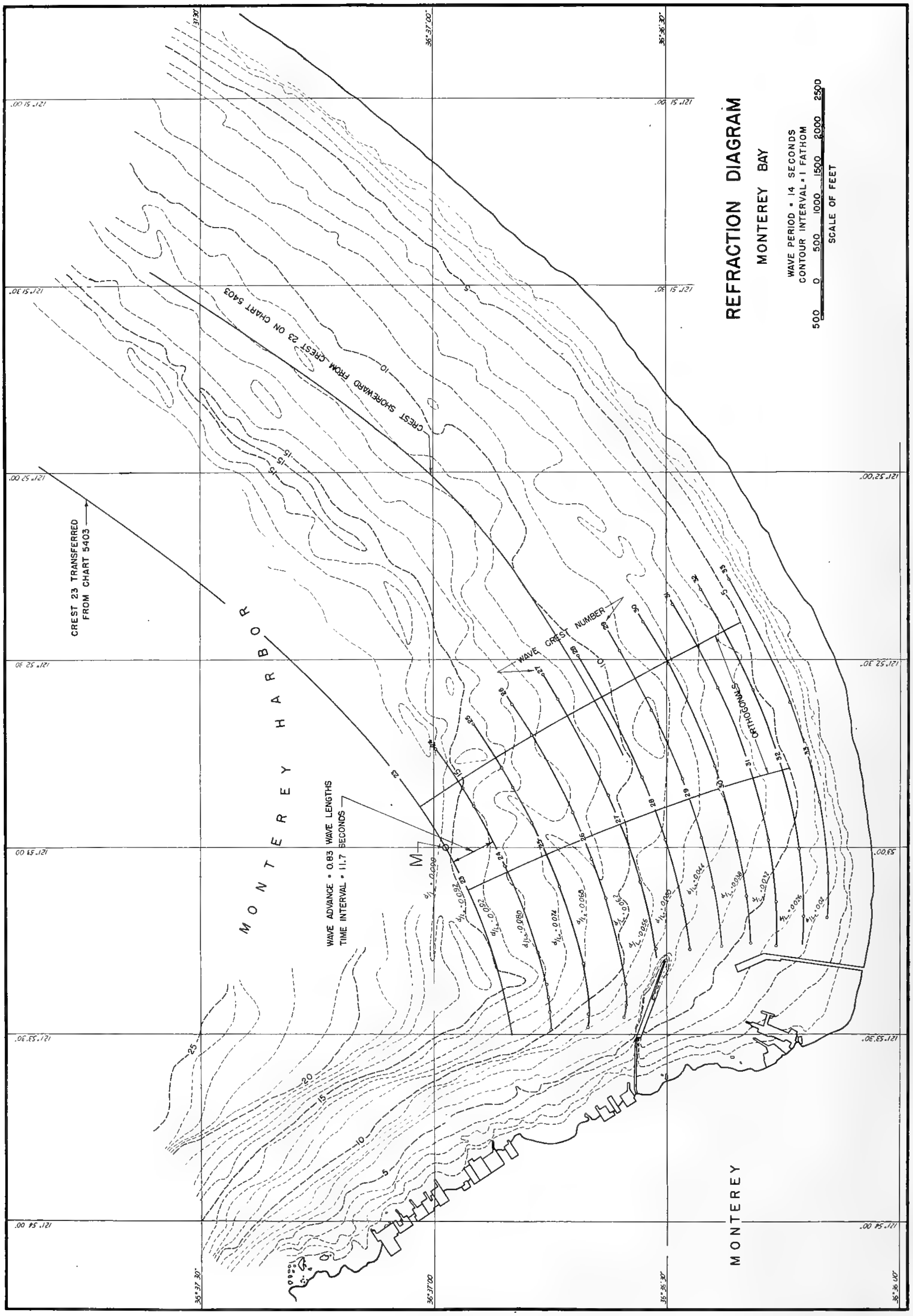

REFRACTION DIAGRAM
MONTEREY BAY
WAVE PERIOD = 14 SECONDS
CONTOUR INTERVAL = 1 FATHOM
500 0 500 1000 1500 2000 2500
SCALE OF FEET
CREST 23 TRANSFERRED FROM CHART 5403
CREST SHOREWARD FROM CREST 23 ON CHART 5403
MONTEREY HARBOR
WAVE CREST NUMBER
ORTHOGONALS
WAVE ADVANCE = 0.83 WAVE LENGTHS
TIME INTERVAL = 11.7 SECONDS
MONTEREY

Figure 8.

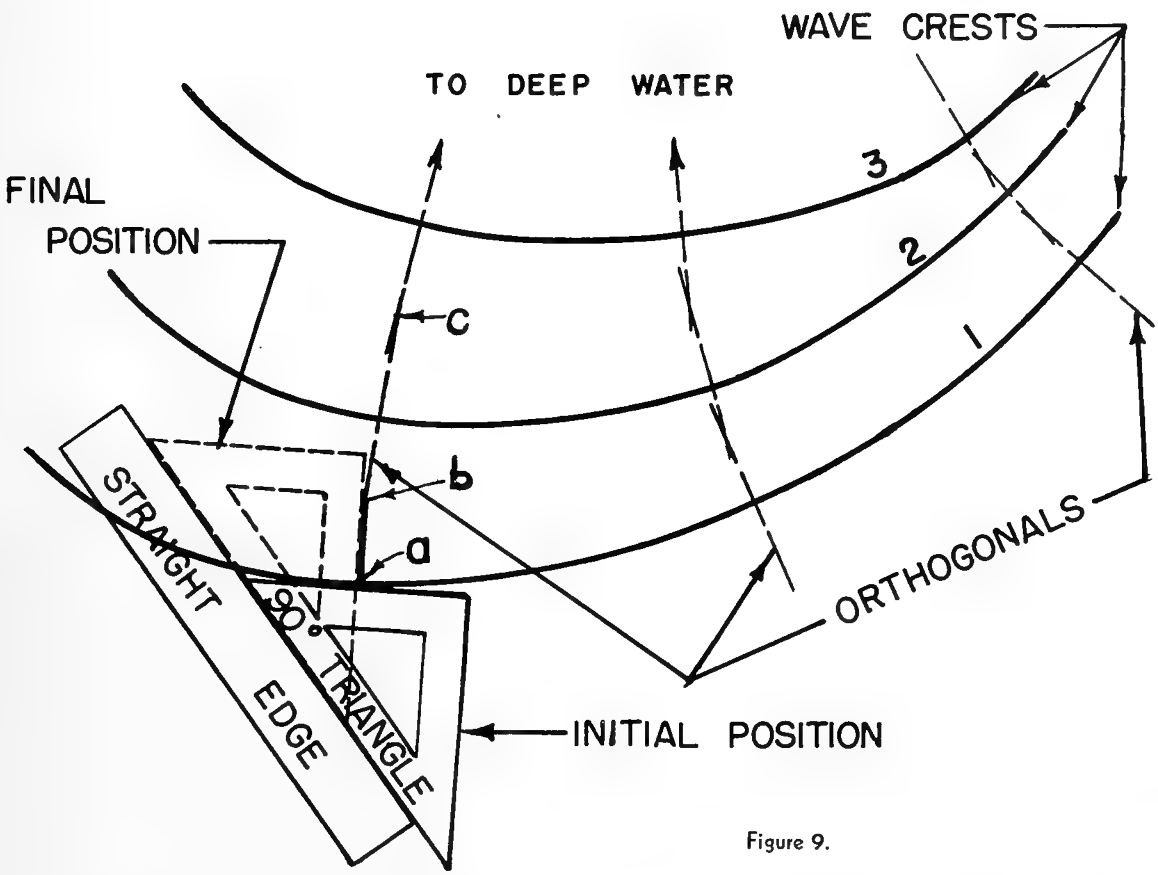


Figure 9.

deep water and crest 14, thus, the points on crest 14 which are midway between orthogonals *A–L* (fig. 7) have been transferred to figure 6. It has not been necessary to carry orthogonal *I*, seaward from crest 14 as orthogonals *H* and *J* give a measure of the refraction for this portion of the wave.

As an example of the computation of the refraction coefficient, K_d, a point on the 5-fathom contour midway between orthogonals *K* and *L* is considered. The procedure is as follows:

> On chart 5403 (fig. 7) the distance between orthogonals *K* and *L* at the 5-fathom line is 2.9 inches and at crest 14 the distance is 1.72 inches.
>
> On figure 6 (chart 5402) the distance between orthogonals *K′* and *L′* is 0.49 inch at crest 14 and 0.12 inch at crest 1. Therefore, the refraction coefficient for the point midway between orthogonals *K* and *L* at the 5-fathom line is:

$$K_d=\sqrt{\frac{1.72}{2.9}\times\frac{0.12}{0.49}}=0.38$$

Computations for refraction coefficients at other points along the 5-fathom contour are summarized in table 3. To obtain a refraction coefficient for a point nearer to the Municipal Pier than obtained by orthogonals *A* and *B*, additional orthogonals have been drawn on charts 5403 and 5415 (fig. 8). Point *M* on crest 23 has been selected as a starting point. On chart 5403 two orthogonals, 0.3 inch on either side of point *M*, have been carried seaward to crest 17 in deep water, where the distance between orthogonals is 0.13 inch. On chart 5415 the two orthogonals, 1½ inches on either side of point *M*, have been carried to the 5-fathom contour where the distance between orthogonals is 4.72 inches. The value of K_d at this location on the 5-fathom contour, which is 0.8 nautical mile from the Municipal Pier, is

$$K_d=\sqrt{\frac{0.13}{0.6}\times\frac{3}{4.72}}=0.37$$

For the benefit of the forecaster, refraction diagrams should be prepared for various periods and

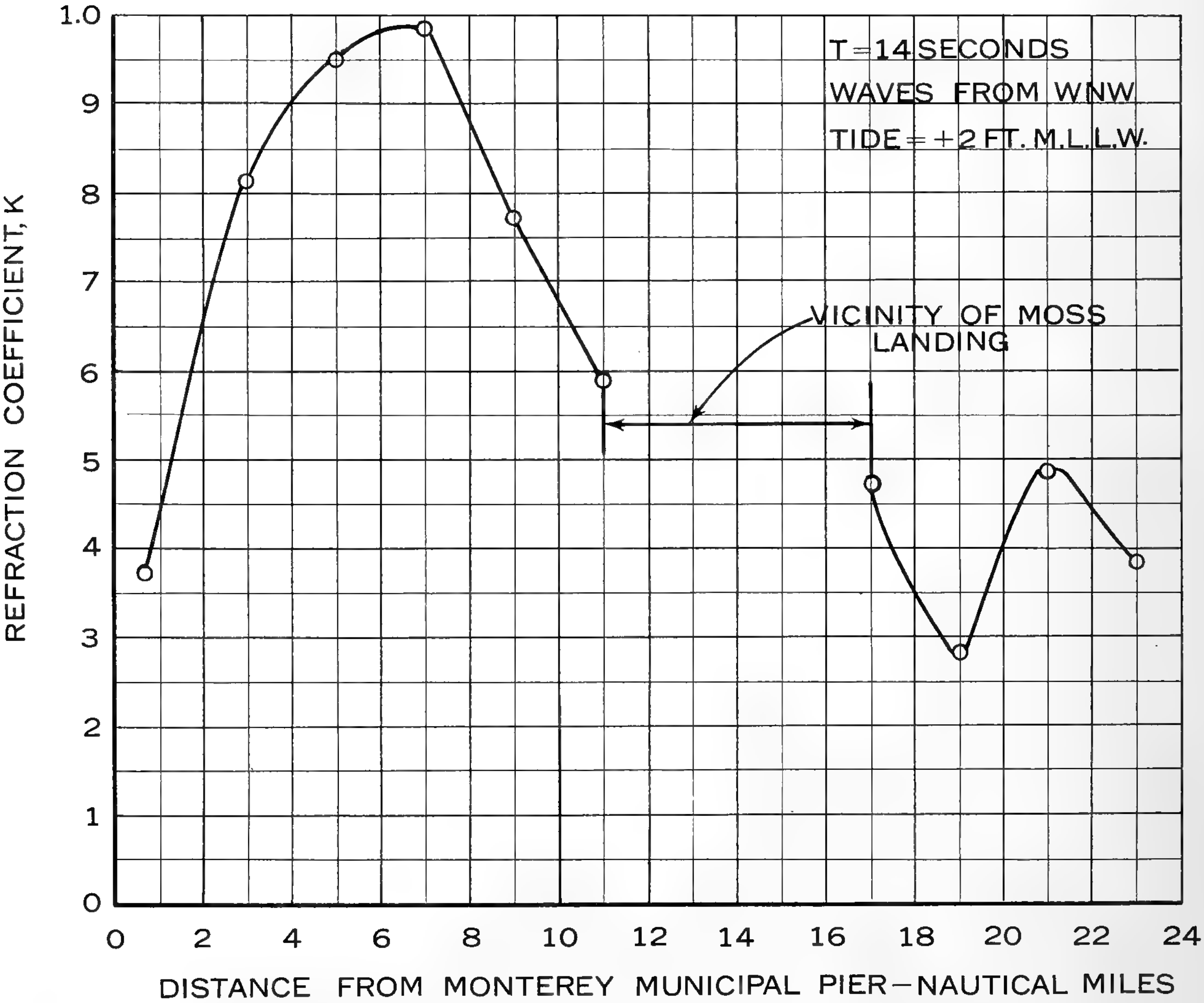


Figure 10.—Refraction coefficient at 5-fathom line—Monterey Bay.

several deep water wave directions. The coefficients should be summarized in convenient table or graph form. If forecasts are being made for only one point on shore, a plot of K_d as a function of wave direction and period is sufficient. If forecasts are being made for several points along a coast, a convenient means of summarizing the data is a graph which shows K_d factors plotted against distance along the coast for various wave periods (fig. 10). A separate graph for each wave direction would be necessary. Another possible method of summarizing data would be to show a map of an area with contours of equal K_d values indicated. A map would have to be prepared for each wave direction and period. Figure 11 shows such a map for Monterey Bay as prepared from the refraction diagrams shown in figures 6 and 7.

As previously stated, it may be necessary, where the tidal range is large, to construct separate diagrams for different stages of the tide. It is important to note that the assumption of constant wave energy between orthogonals does not apply after a wave breaks. If waves pass over a submerged reef, it may be necessary to examine this area critically to determine whether the waves break at some, or all, stages of the tide. Should breaking occur, wave heights beyond the reef would be lower than that determined by use of K_d factors from a refraction diagram (for examples, see figs. 25 and 33). As a wave passes over a reef, whether or not breaking occurs, the crest may break into several crests. Thus, the further refraction of the wave may not be simple.

The refraction coefficient, K_d, is a function of

Table 3

Computation of refraction coefficients for Monterey Bay

[K_d values apply to the 5-fathom contour for waves of 14-second period from W. N. W.]

Data from chart 5403					Data from chart 5402					K_d	Distance at midpoint between orthogonals (nautical miles from municipal pier)
Segment at 5-fathom line		Segment at crest 14		Col. (4) ÷ Col. (2)	Segment at crest 14		Segment in deep water		Col. (9) ÷ Col. (7)	$\sqrt{\text{Col. (10)}} \times \sqrt{\text{Col. (5)}}$	
No.	Length (inches)	No.	Length (inches)		No.	Length (inches)	No.	Length (inches)			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
A–B	2.9	A′–B′	1.87	0.65	A′–B′	0.44	A″–B″	0.44	1.0	0.81	3
B–C	2.9	B′–C′	2.61	.9	B′–C′	.61	B″–C″	.61	1.0	.95	5
C–D	2.9	C′–D′	2.78	.96	C′–D′	.65	C″–D″	.64	.99	.98	7
D–E	2.9	D′–E″	1.73	.60	D′–E′	.41	D″–E″	.40	.98	.77	9
E–F	2.9	E′–F′	1.46	.50	E′–F′	.34	E″–F″	.24	.71	.59	11
F–G	[1] 1.42	F′–G′	.90	.63	F′–G′	.20	F″–G″	.11	.55	[2] .59	13
G–H	[1] 1.88	G′–H′	1.15	.61	G′–H′	.26	G″–H″	.17	.65	[2] .63	15
H–I	2.9	H′–I′	1.02	.35	H′–J′	.34	H″–J″	.18	.53	[3] .43	17
I–J	2.9	I′–J′	.42	.15	H′–J′	.34	H″–J″	.18	.53	[3] .28	19
J–K	2.9	J′–K′	2.0	.69	J′–K′	.43	J″–K″	.14	.33	.48	21
K–L	2.9	K′–L′	1.72	.59	K′–L′	.49	K″–L″	.12	.25	.38	23

[1] Distance between orthogonals measured at crest 20. Refraction shoreward of this crest should be determined from larger scale chart.
[2] Coefficients apply to crest 20.
[3] Refraction seaward from crest 14 is based on distance between orthogonals *H* and *J*.

depth, of period, and of the initial angle of the wave crest. In the preceding examples, representation of the results was simplified by reporting the coefficient of refraction at a constant depth of 5 fathoms, thus eliminating one variable. The question now arises as to the magnitude of the difference in this coefficient if the wave breaks in a lesser depth, say, 10 feet. For this purpose, the 30-foot contour may be assumed as parallel to the shore line and the effect worked out approximately from figure 4. Using the example of Monterey Bay, with a period of 14 seconds and waves from W. N. W., the refraction coefficient between stations E and F is around 0.60 at $d/L_o=0.03$. On a straight shore line, from figure 4 this coefficient and depth would correspond to $\alpha_o=71°$ and $\alpha=24°$. At $d/L_o=0.01$, $\alpha=13$ degrees so that the further change in α between $d/L_o=0.03$ and $d/L_o=0.01$ would be about 11 degrees. The change in coefficient would be determined from

$$K_{30}=\sqrt{\frac{b_o}{b_{30}}} \qquad K_{10}=\sqrt{\frac{b_o}{b_{10}}}$$

$$\frac{K_{10}}{K_{30}}=\sqrt{\frac{b_{30}}{b_{10}}}=\sqrt{\frac{\cos \alpha_{30}}{\cos \alpha_{10}}}=0.97$$

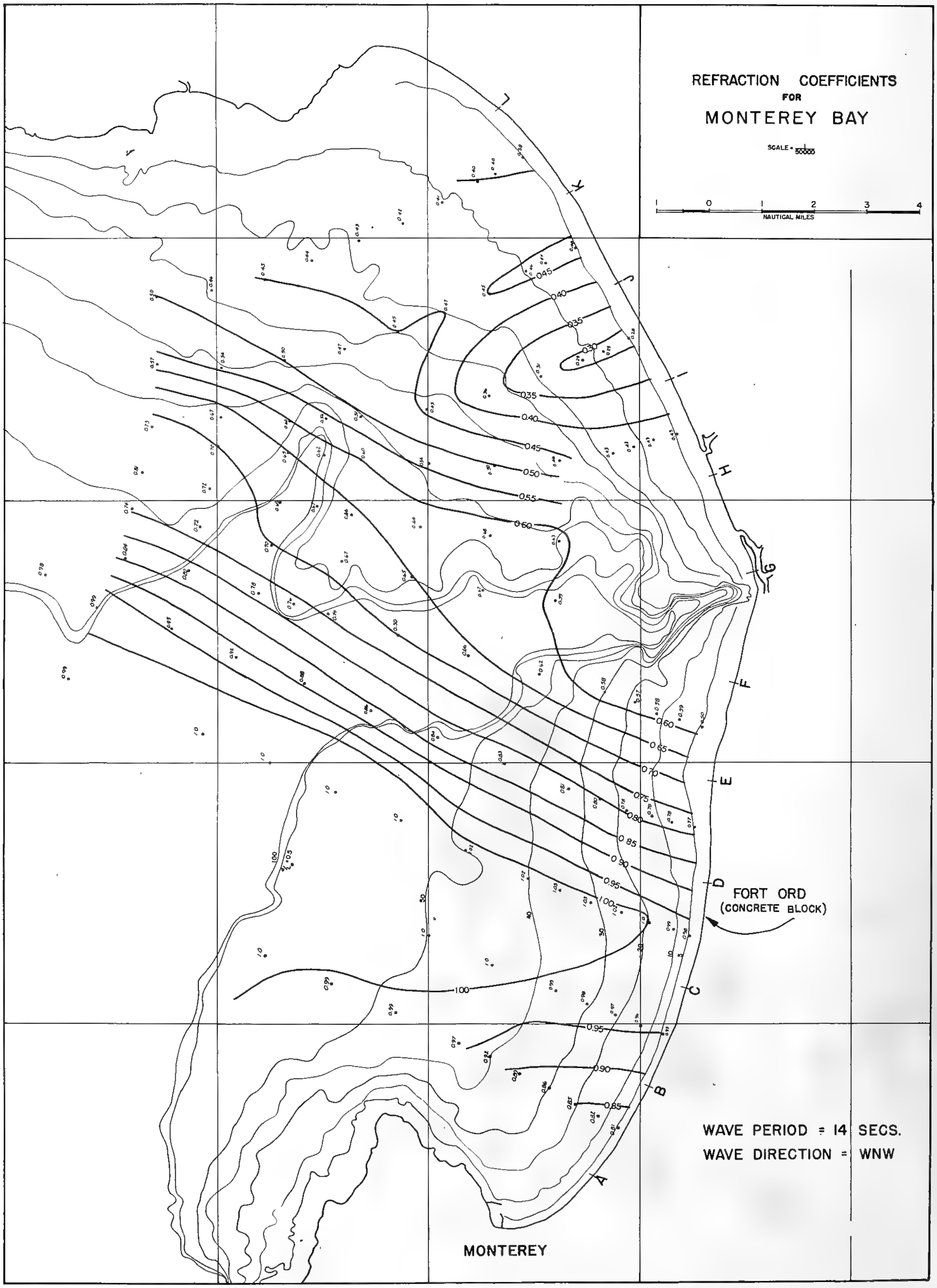
With the change in angle from 24 degrees at $d=30$ feet to 13 degrees at $d=10$ feet, the value of K_d at the 10-foot contour is only 97 percent of the value at the 30-foot contour. Evidently, the exact depth to which the refraction diagram is carried does not greatly affect the value. As a working rule, to be modified if special circumstances so indicate, carry refraction diagrams shoreward to at least $d/L_o=0.03$.

If *the refraction diagram is to be drawn for the purpose of determining the local angle between the shore line and the the breaking crest*, then it is obviously necessary to continue the diagram to the depth in which the wave breaks. This refinement is unnecessary in determining wave heights but is necessary in estimating the strength of the littoral current which depends upon the angle between the breaking crests and the shore line.

CONSTRUCTION OF REFRACTION DIAGRAMS FROM AERIAL PHOTOGRAPHS

The graphical method of preparing refraction diagrams may be replaced by a purely photographic method, using accurately timed aerial photographs. Various components of the method have been described in other reports and the procedure will be summarized only briefly here. Steps in the analysis are:

1. Obtain aerial photographs of the shore line and offshore area, preferably verticals, taken at an accurately timed interval of approximately 3 seconds and with about 85 percent overlap.
2. Check photographs for altitude and tilt and determine ground scale. Enlargements corrected for tilt are desirable.
3. Trace crest of major wave train from selected photographs in the set and transfer to overlay of hydrographic chart. The set of photographs will show different crest angles at the same position and averaged curves should be drawn.



REFRACTION COEFFICIENTS
FOR
MONTEREY BAY
SCALE
NAUTICAL MILES
FORT ORD
(CONCRETE BLOCK)
WAVE PERIOD = 14 SECS.
WAVE DIRECTION = WNW
MONTEREY

Figure 11.

4. Draw the orthogonals and from the spacing of the orthogonals determine K_d at a selected depth contour.

5. Determine the average wave period of the major wave train by (*a*) measuring from the photographs the time interval between breakers or between the instants at which crests pass identifiable points such as rocks or small patches of foam or (*b*) measuring the wave length at points where the depth is known and compute the period (or obtain it from available graphs).

Except for 5*b*, this procedure may be followed even when the hydrography is unknown.

This aerial method of preparing a refraction diagram has the practical advantage that it deals with real waves, which vary in period and direction, and it truly represents the effect of local irregularities in the bottom. It has a number of disadvantages among which may be mentioned:

1. Obtaining photographs representing a range of periods and directions, and possibly low and high tide, will require considerable flying time and, almost invariably, a long period of waiting for the desired wave conditions to occur in good photographic weather.

2. The swells, which dominate the breaker zone, are frequently obscured at a relatively short distance from shore by small steep waves. It is usually the long period waves which are of interest.

If the refractive effect occurs in a limited area, as in a small bay or near a harbor entrance, direct use of aerial photographs as outlined here is feasible. If the refractive effect takes place gradually over large areas, the aerial photographs become too time consuming because of the problem of ground control. A judicious combination of the graphical method with measurements of aerial photographs will yield the most reliable results at reasonable cost.

DETERMINATION OF REFRACTION COEFFICIENTS FROM AERIAL PHOTOGRAPHS

In the preceding section, the method outlined utilized the aerial photographs only for the purpose of determining crest positions. The refraction coefficients, K_d, were then obtained by drawing orthogonals and measuring the spacing between them just as in the completely graphical method. Accurately timed aerial photographs, preferably corrected verticals, may be used to go a step further and permit measurement of the refraction coefficients directly. The steps in the procedure are as follows:

1. Secure accurately timed aerial photographs of the shore line and offshore area.

2. Determine offshore direction directly from photographs if photography shows waves in deep water or indirectly from the period, depth, and angle at the offshore edge of the photographs.

3. Determine the depth by the wave velocity method ("Underwater Depth Determination," U. S. Navy Photographic Interpretation Center Report 46) if the available charts do not show hydrography in and near the line of breakers.

4. Measure the photographs to determine the depth in which the waves break and compute breaker height from $H_b=1.3\ d_b$. Repeat for as many waves at each shore-line point as photography permits.

5. Determine period (*a*) from interval between breakers or (*b*) from water depth and wave length outside breaker line.

6. Compute L_o, from T ($L_o=5.12T^2$), and d_b/L_o and obtain H_b/H_o' from plate I, Breakers and Surf, H. O. 234.

7. From the value of H_b obtained in step 5 and H_b/H_o' compute H_o', the wave height which would have been required to produce the observed breaker had the original crest been parallel to the shore and the shoreline straight.

8. If it is assumed that the breakers observed were generated by a train of waves of uniform period and height, the relative refraction coefficient is obtained by taking the ratio of H_o' at each point along the shore to its value at a single reference point, say, the point where H_o' is maximum.

9. To obtain an absolute value of K_d for each point, it is necessary to find at least one reliable value by (*a*) computing the refraction coefficient for a point along a straight stretch to which figure 4 applies or (*b*) obtaining an independent measure of H_o from an offshore recorder, from photographic measurements of breakers on an adjacent straight beach, or other means. Divide H_o' from step 8 by H_o from step 9 to obtain the absolute value of K_d.

10. The experimental values of K_d in step 9 apply to the varying depths in which the waves broke. Usually, no further analysis is necessary, but it should be remembered that K_d should be corrected back to a common depth, if the difference between the standard depth and the depth of breaking changes K_d.

In addition to the disadvantages, previously mentioned, of the aerial method of determining the refraction pattern, measurement of the coefficient from aerial photographs suffers because of the fact that waves vary in height along their crests and there is not a single value of H_o applicable to all breakers shown in the photographs. At a scale of 1:12,000 a 7-inch print, and a speed of 15,000 feet per minute, for example, each point on the shore remains in the field of the camera for about 30 seconds, or long enough for 2 or 3 waves to break. By measuring all the waves visible at many points, random variations may, in part, be eliminated. The accuracy is increased by making several photographic sorties in quick succession while the same wave train prevails.

GRAPHICAL CONSTRUCTION OF REFRACTION DIAGRAMS DIRECTLY BY ORTHOGONALS

I. INTRODUCTION

A system has been devised whereby the orthogonals to refracted wave fronts may be constructed directly without first drawing the wave fronts. This method has the advantage of eliminating an entire graphical step and its attendant inaccuracies. In trained hands a refraction diagram can be constructed by this method in about a quarter of the time required for its construction by the wave front method. The method requires a higher degree of training, however, and the operation does not become so nearly automatic as does the wave front method. The method is thus suitable for use by specialist draftsmen or engineers.

Physically the method is carried out by a special protractor which incorporates the requisite scales. The protractor is manipulated in steps from contour to contour and at each step indicates the direction of the orthogonal. One orthogonal is thus drawn from deep water to shore in each series of operations. The device has been made in the form of a protractor so that it would be entirely adequate for the construction of refraction diagrams. A drafting machine can be employed to an advantage, however, and the protractor used merely as a graph and tables for obtaining the required values for the manipulation of the drafting machine.

A detailed description of the development of the method and its application follows:

II. DEVELOPMENT OF THE METHOD

A. Assumptions

1. That contours can be drawn at every abrupt change in slope of the chart.

(*a*) The depth at a point between contours is a linear function of its distance from the contours.

2. That, between contours, wave length and velocity may be considered to vary linearly (the usual assumption).

(*a*) The wave length and velocity may be considered a linear function of its distance from the contours.

(*b*) The radius of curvature of the orthogonal between contours may be considered constant (a circular arc).

(*c*) The angle of the arc is equal to the change of angle of the orthogonal.

3. That the undulations of small magnitude extant in most contours are more likely to be a measure of observational inaccuracies magnified by rigorous drafting than an indication of the direction of level bottom. Also, that bottom features whose dimensions are small compared with the wave length do not influence the motion of the wave to any appreciable extent.

(*a*) If the contours are smoothed out and only those features preserved that are obviously characteristics of the hydrography, the result is more nearly accurate.

4. That the angle of convergence or divergence

of orthogonals at differential intervals is small compared to the angle of refraction. (Please note that convergence or divergence is not considered non existent).

5. That a line drawn through any point midway between two contours and making equal angles with the adjacent contours is closely the direction of a level line at that point (provided the requirement implied in assumption No. 1 is accomplished).

B. Derivation

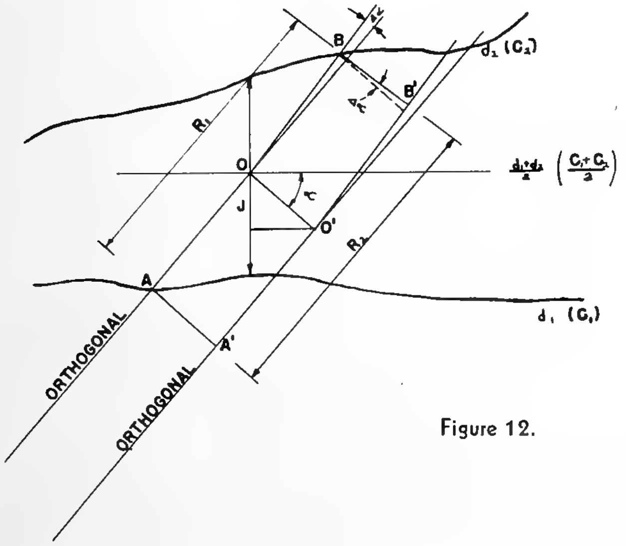


Figure 12.

considering $\Delta\alpha < 13°$ for two-place accuracy, or $\Delta\alpha < 6°$ for three-place accuracy, $\tan \Delta\alpha = \sin \Delta\alpha = \Delta\alpha = \frac{R_2 - R_1}{BB'}$

$AA' = 00' = BB' = \text{d}$ (a differential distance). Assumption No. 4.

and: $\Delta\alpha = \frac{R_2 - R_1}{\text{d}}$

let c' be the velocity from A to B (effective)

let c'' be the velocity from A' to B' (effective)

let t be the time required for wave front to move from AA' to BB'

$R_2 = c''t$; $R_1 = c't$; then $\Delta\alpha = \frac{c''t - c't}{\text{d}}$

$$c'' = c' + \frac{c_1 - c_2}{J} \text{d} \sin \alpha$$

then $\Delta\alpha = \frac{c_1 - c_2}{J} t \sin \alpha$; but $t = \frac{R_1}{c'}$

and $c' = \frac{c_1 + c_2}{2}$

$$\therefore \Delta\alpha = \frac{R_1}{J} \frac{(c_1 - c_2)}{\frac{c_1 + c_2}{2}} \sin \alpha; \text{ or } \Delta\alpha = \frac{R}{J} \frac{\Delta c}{c_{ave}} \sin \alpha$$

$$\text{or } \Delta\alpha = \frac{R}{J} \frac{\Delta L}{L_{ave}} \sin \alpha \qquad (1)$$

but for the general case $\frac{R}{J} = \sec \alpha$

$$\text{therefore, } \Delta\alpha = \frac{\Delta L}{L_{ave}} \tan \alpha \qquad (2)$$

C. Utilization

These two equations are thus seen to be independent of the scale of the chart, and they have been made independent of the wave period by the reduction of this factor to the dimensionless ratio $\frac{\Delta L}{L_{ave}}$.

Equation (1) is applicable to all cases where $\Delta\alpha$ is less than some predetermined limit depending upon the accuracy desired. In general, good results are obtained when $\Delta\alpha$ is less than 13°. In practice $\Delta\alpha$ rarely approaches this limit.

Equation (2) is more readily applied under ordinary conditions than is equation (1). The limitation of equation (2) stems from the fact that as α approaches 90°, $\tan \alpha$ becomes infinite. Physically this situation may occur but is instantaneously altered by refraction so that α becomes less than 90°. However, the application of equation (2) normally necessitates crossing an entire contour interval at each step. The value of $\Delta\alpha$ changes very rapidly in the region when α approaches 90°. Thus the instantaneous refraction over a small distance results in a great change in the rate of refraction, and the interval must be crossed in a series of shorter steps. It is therefore desirable to employ equation (1) whenever α exceeds about 80°. Equation (1) readily lends itself to crossing a contour interval in partial steps by the judicious selection of R (the distance of wave advance).

The protractor has thus been constructed with graphs for equation (2) and a special table for use when α exceeds 80°, which adapts equation (1) to the graph of equation (2).

The use of the graph suffices for the great majority of diagrams and, for all ordinary cases, there is never any necessity to refer to the table. The table therefore has been made very simple. The graph requires the measurement of one factor only (α) and thus is much faster to use than the table, which requires the measurement of three factors (α, R and J).

The uses of these two component operations are summarized in the following:

TABLE I

Equation	Measured quantities	Use
Graph $\Delta\alpha = \frac{\Delta L}{L_{ave}} \tan \alpha$	α	For values of α less than 80°.
Table $\Delta\alpha = \frac{R}{J} \frac{\Delta L}{L_{ave}} \sin \alpha$ or $\Delta\alpha = \frac{\Delta L}{L_{ave}} \tan \alpha_e$ where $\alpha_e = \tan^{-1} \frac{R^*}{J}$	α, R and J	For values of α greater than 80°.

*Where the angle made by the orthogonal and the contours is greater than 80°, the graph is still used by crossing the contour interval in a series of steps and progressing a distance R which has some definite relation to J such as 1, 0.5, etc. Equivalent α for such cases comprise the t.ble. Here α_e (equivalent α) $=\tan^{-1} \frac{R}{J} \sin \alpha$, but, as α is between 80° and 90° for these cases, $\sin \alpha$ may be given the value of 1 and the equation becomes

$$\alpha_e = \tan^{-1} \frac{R}{J} \tag{3}$$

Figure 13 is a reproduction of the type I protractor showing the component scales and their functions. The protractor is 14 inches in diameter.

Figure 22 shows the type II protractor. This protractor involves the use of a moveable arm. When this arm is aligned along the direction of level bottom, $\Delta\alpha$ is read directly along the pointer at the appropriate values of $\frac{\Delta L}{L_{ave}}$. α and $\Delta\alpha$ are entered on the graphs in their actual dimensions, and it is thus unnecessary to determine their numerical values. The factors $\frac{R}{J}$ are entered on a circular scale. In this way α_e is indicated directly on the graph for a value of R/J. The type II protractor facilitates the operation but is more difficult to construct.

III. APPLICATION OF THE METHOD

A. General Preparation

Contours are drawn upon the chart at such intervals that adequately will represent the details of the bottom topography and which are consistent with assumption 1, above. Normally (as a rule of thumb) there will be about as many contours required as the period in seconds of the longest period wave to be studied. These contours must extend to a condition of deep water for the longest period wave. That is, to $d=2.56T^2$, where d is the depth of the deepest contour required and T is the period of the longest period wave to be studied. A table is then prepared for each wave period to be studied as shown in table II following:

TABLE II

Computation for use of protractor in example

Period: 10 seconds $L_o=512$ feet

1	2	3	4	5	6	7
d Fathom	d Feet	$\frac{d}{L_o}$	$\frac{L}{L_o}$	$\frac{\Delta L}{L_o}$	$\frac{L}{L_o}$ ave	$\frac{\Delta L}{L_{ave}}$
1	6	0.0117	0.26			
				0.11	0.31	0.35
2	12	.0235	.37			
				.20	.47	.43
5	30	.0587	.57			
				.18	.66	.27
10	60	.117	.75			
				.18	.84	.21
20	120	.235	.93			
				.05	.95	.05
30	180	.352	.98			
				.02	.99	.02
50	300	.587	1.00			

EXPLANATION OF TABLE

The first column, d, fathom, is a list of the contours of the chart. Should the chart be in feet, this column naturally is eliminated. In practical problems it would be advisable to have contours at 3, 4, and 7 fathom as the values of $\frac{\Delta L}{L_{ave}}$ should not exceed about 0.20. For brevity in the following figures, however, fewer contour intervals have been used.

Column 2, d, feet, is the result of multiplying column 1 by the factor 6. If a stage of tide is used other than that of the datum of the chart, a certain constant must be added or subtracted from this column.

Column 3, $\frac{d}{L_o}$, is the ratio of the depth to the deep water wave length for the wave period. In this case (i. e., 10-second period) the wave length is 512 feet and is computed from the equation $L_o=5.12T^2$, where T is the period in seconds, and L_o is the deep water wave length in feet.

Column 4, $\frac{L}{L_o}$, is taken from a graph of this function such as that in H. O. Report No. 234, "Breakers and Surf, Principles in Forecasting," plate 1, or HE–116–265, "Tables of the Functions of d/L and d/L_o.", or they can be obtained by interpolation in Table 4 in the appendix of this publication.

Column 5, $\frac{\Delta L}{L_o}$, is the change in the $\frac{L}{L_o}$ ratio and is written between the lines of the depths between which the change occurred.

Column 6, $\frac{L}{L_o}$ ave, is the average of the $\frac{L}{L_o}$ ratio between any two depths.

This is also written between the lines. This figure can be obtained simply by adding one-half of the $\Delta \frac{L}{L_o}$ figure to the previous value for $\frac{L}{L_o}$.

Column 7, $\frac{\Delta L}{L_{ave}}$, is obtained by dividing column 5 by column 6.

B. General Procedure

The general method of drawing a refraction dia-

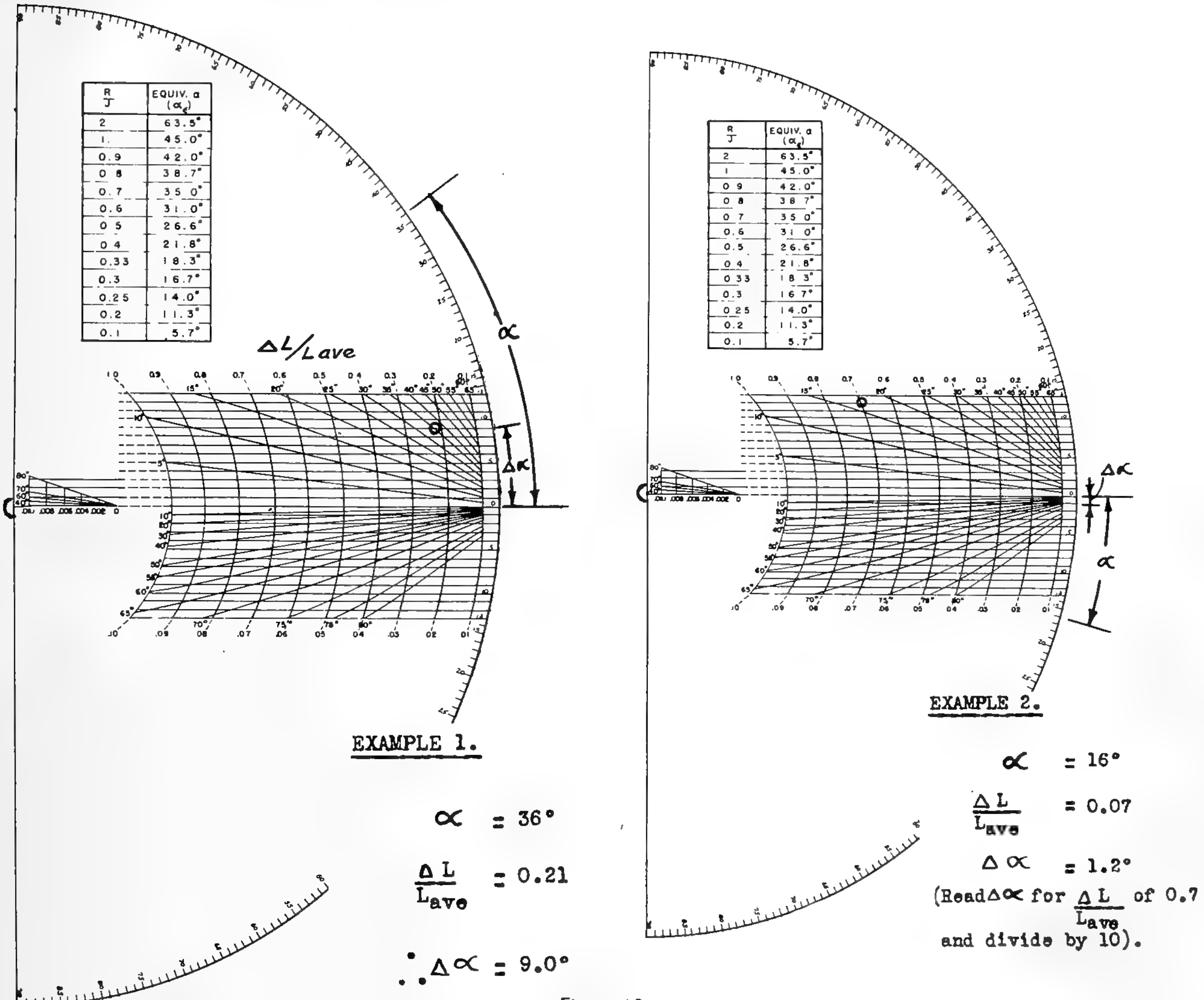


Figure 13.

gram by use of a protractor is carried out below for the case where contours are simple and α is always less than 80°. The steps are fully explained under each figure. The protractor shown is the type I.

C. Special Procedure

The special method is shown for the two special cases. First, when the angle of approach is greater than 80° to the contours and, second, where complex hydrography is encountered. The steps are again explained under each figure.

IV. DISCUSSION OF THE METHOD

The method has given results in close agreement with those obtained by the wave front method, and apparently more nearly correct. See figures 19 and 20.

Its particular advantages are:

1. Speed—orthogonals can be constructed in about a fourth of the time required by the wave front method.

2. Independence of the instrument from the chart scale and wave period.

3. The elimination of an entire step in the construction of refraction diagrams.

The disadvantages are:

1. A higher degree of training is required for the use of the protractor than for the wave front method.

2. It is not possible to draw an orthogonal from shore seaward as is the case of the final step in the wave front method. This is not an actual comparative disadvantage, however, for in the wave front method the wave fronts first must be drawn to shore before the orthogonals can be drawn seaward.

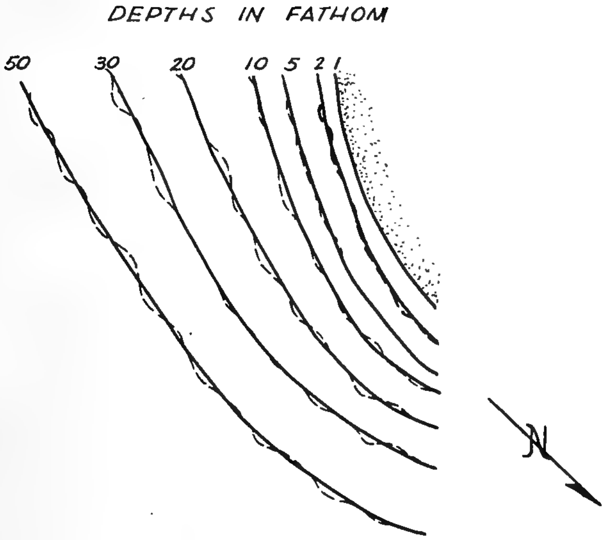
V. CONCLUSIONS

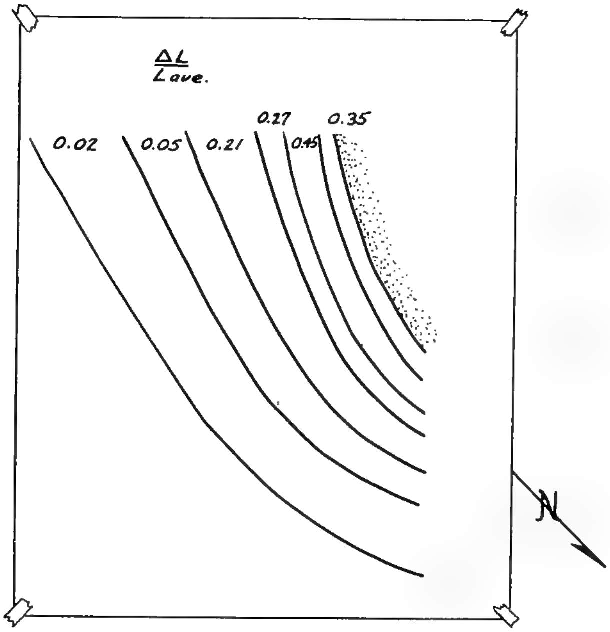
In the hands of individuals who thoroughly understand the method and its application the method gives speedy and accurate results.

VI. REFERENCES

H. O. Report No. 234, Breakers and Surf, Principles in Forecasting.

HE–116–265, Tables of Functions of d/L and d/L_o. Navy Wave Project, University of California. January 1948.



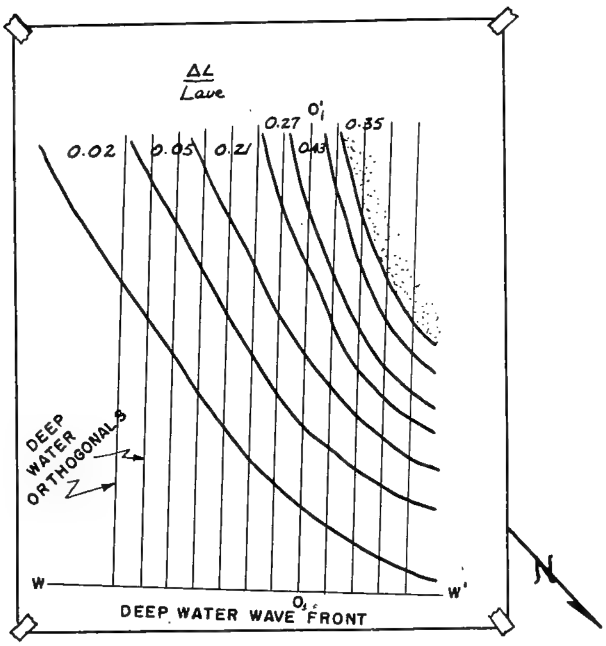


Step 1.—The chart has been checked carefully for changes in slope and the requisite contours drawn in heavily in ink smoothing them off as is consistent with assumption No. 3.

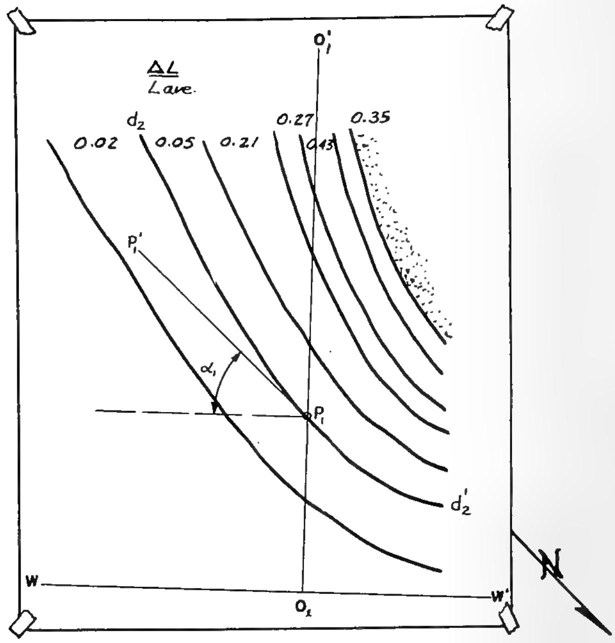
This chart is then used for all periods and directions without further work.

Step 2.—A tracing paper overlay is placed on the contour map and the $\frac{\Delta L}{L_{ave}}$ is written between each of the contours as computed in Table II. This overlay is then good for all directions of approach for a particular period, and can be prepared in a very few minutes. All further work is performed on the overlay.

Figure 14.



Step 3.—A deep water wave front (WW′) is drawn in for the direction to be studied. A suitable interval for orthogonals is stepped off and the directions of the deep water orthogonals are drawn in. These are all straight lines, of course. The direction selected was from the NW. Deep water refers to depths greater than $L_o/2$ as described in the text, in this case 50 fathoms.

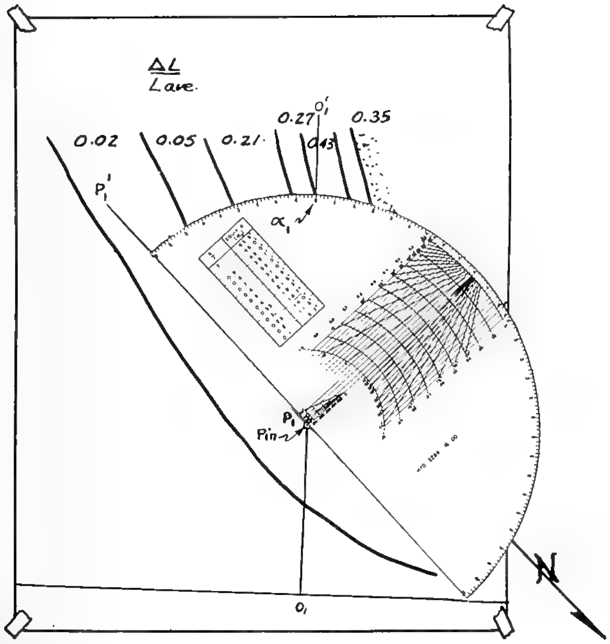


Case 1.—Regular, simple hydrography, no angles in excess of 80°.

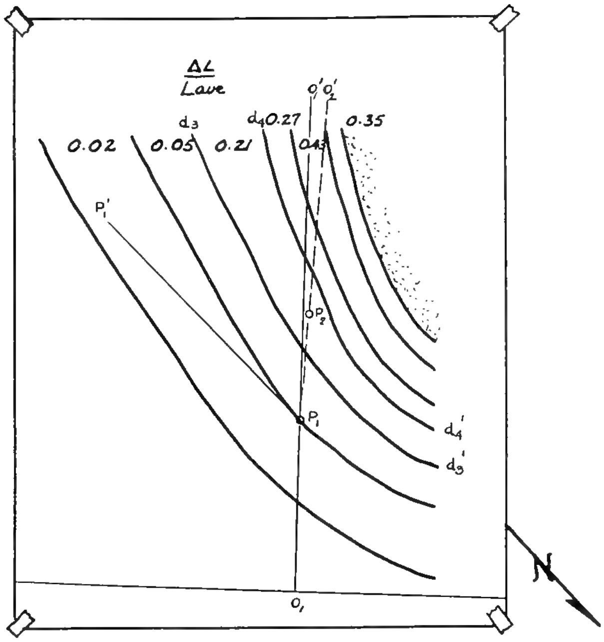
Step 4.—The diagram is started on any orthogonal. If the K value is required for a partiular point on the beach, orthogonals can be selected which will reach the point.

Where the $\frac{\Delta L}{L_{ave}}$ values are small, two contour intervals can be crossed simultaneously. Thus, the first point selected is P_1, where the orthogonal intersects the second contour. α is measured to an estimated line P_1P_1' which is the direction of level bottom at the point P_1. In this case it is a line tangent to d_2d_2'.

Figure 15.

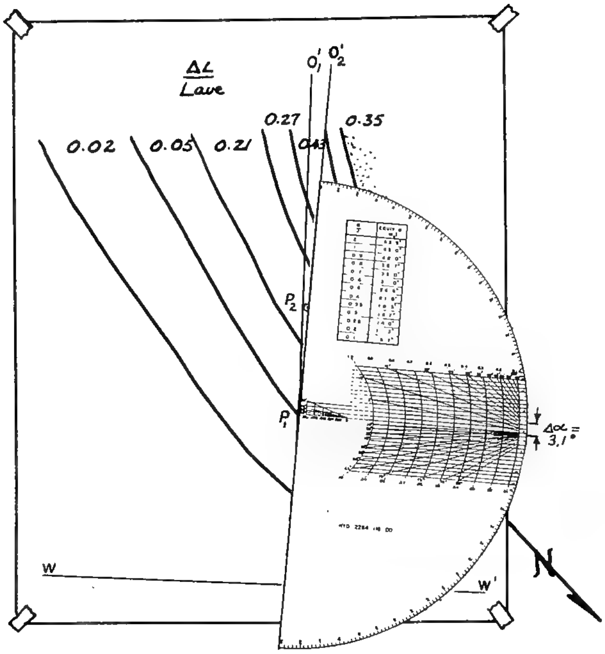


Step 4.—Measuring α_1 with protractor pinned at P_1, $\triangle\alpha$ is then turned from the line P_1O_1' by use of the degree scale. When a drafting machine is used, α_1 is measured with an ordinary protractor.



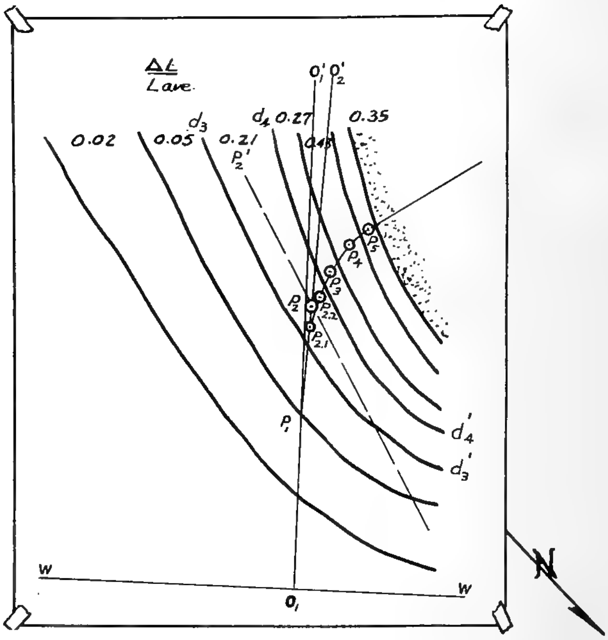
Step 5.—$\triangle\alpha_1$ at P_1 is determined from the graph on the protractor. Since two contour intervals are being crossed, the $\frac{\triangle L}{L_{ave}}$ values for the two are added (i. e., 0.07). $\triangle\alpha$ is turned at P_1 to the right or *so that α_1 is decreased.* The orthogonal is carried into P_2, midway between d_3d_3' and d_4d_4'. Two contour intervals cannot be crossed simultaneously from P_2 shoreward as the refraction is too great. If it were not so great to shoreward, P_2, could be established at the intersection of P_1O_1', and the contour d_4d_4'.

Figure 16.



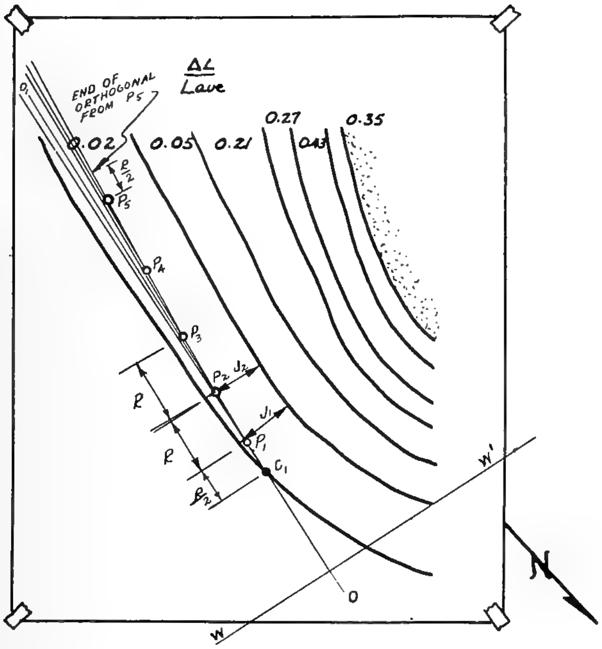
Step 5.—Measuring $\triangle\alpha_1$ with protractor pinned at P_1, the line P_2O_2' is the new direction of the orthogonal.

When a drafting machine is used, $\triangle\alpha$ is turned on the protractor head. $\triangle\alpha$ is always turned in a direction so that α is *decreased* except in the rare case when the wave is progressing from shallow to deeper water. In this case $\triangle\alpha$ is turned so that α is *increased.*



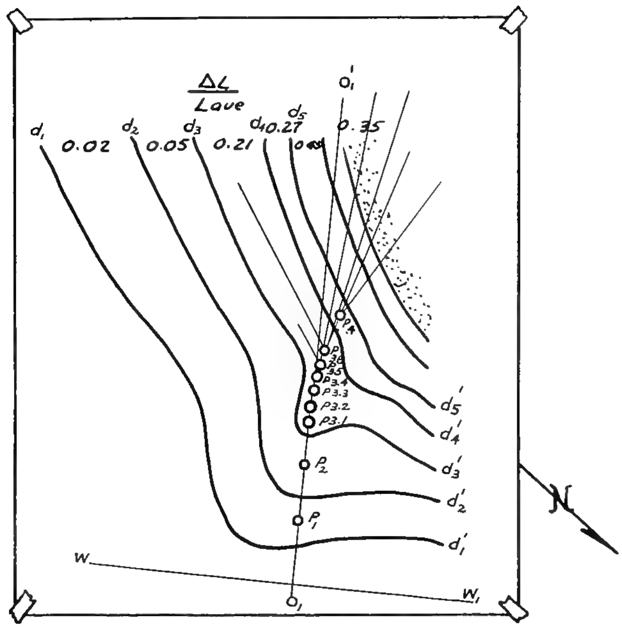
Step 6.—At P_2, α_2 is measured in respect to a line P_2P_2' which is drawn in by eye. P_2P_2' has a direction midway between the directions of the contours d_3d_3' and d_4d_4' at the points where they intersect the line P_1O_1. $\triangle\alpha_2$ is turned off at P_2 and P_3 is established in the same manner as P_2. This is continued to the beach. In the example $\triangle\alpha$ at P_2 was greater than 13°. Thus P_2' and P_2'' were established using the line P_2P_2' as an intermediate contour with $\frac{\triangle L}{L_{ave}}$ values of 10.5 on each side. This step is rarely necessary in practice. The true orthogonal corresponds with the line $P_1 \ldots P_5$ only at the intersections with contours. For all ordinary purposes, however, the line $P_1 \ldots P_5$ is an orthogonal.

Figure 17.



Case II. α greater than 80°.

Step 4.—(Steps 1 to 3 are the same as in Case I). P_1 is established along the deep water orthogonal at a distance from C_1 equal to half of one of the proportions of J shown on the protractor. In this case R was selected equal to J. The $\Delta\alpha$ for $R/J=1$ and $\frac{\Delta L}{L_{ave}}=0.02$ is 1.15°. Thus P_2P_3, etc., can be established by turning 1.15° at each point and progressing a distance R_1, R_2 . . . R_5, etc., equal to J_1, J_2 . . . J_5 until α becomes less than 80°. Any other proportion of R/J can be selected at any time. It is to be remembered that the angle turned at any P carries the orthogonal a distance of $\frac{1}{2}R$ beyond the point.



Case III.—Contours are not simple.

Step 4.—(Steps 1 to 3 are the same as for Case I.) As α_1 and α_2 are zero no refraction occurs between d_1d_1' and d_3d_3'. $P_{3.1}$ can be established on the line O_1O_1' as the limit of no refraction. $\alpha_{3.2}$ and $\alpha_{3.4}$ are approximately 90° so $P_{3.2}$ and $P_{3.4}$ are drawn in using $R/J=0.5$. At $P_{3.5}$, conditions change and $P_{3.6}$ is established as though a contour existed at $P_{3.5}$ with a $\frac{\Delta L}{L_{ave}}$ between it and d_4d_4' of $\frac{5}{8}\times 0.21=0.13$.

That is, $\frac{\Delta L}{L_{ave}}$ is reduced by a simple proportion to establish an intermediate contour interval. The diagram then progresses as usual to P_4, P_5, etc.

Figure 18.

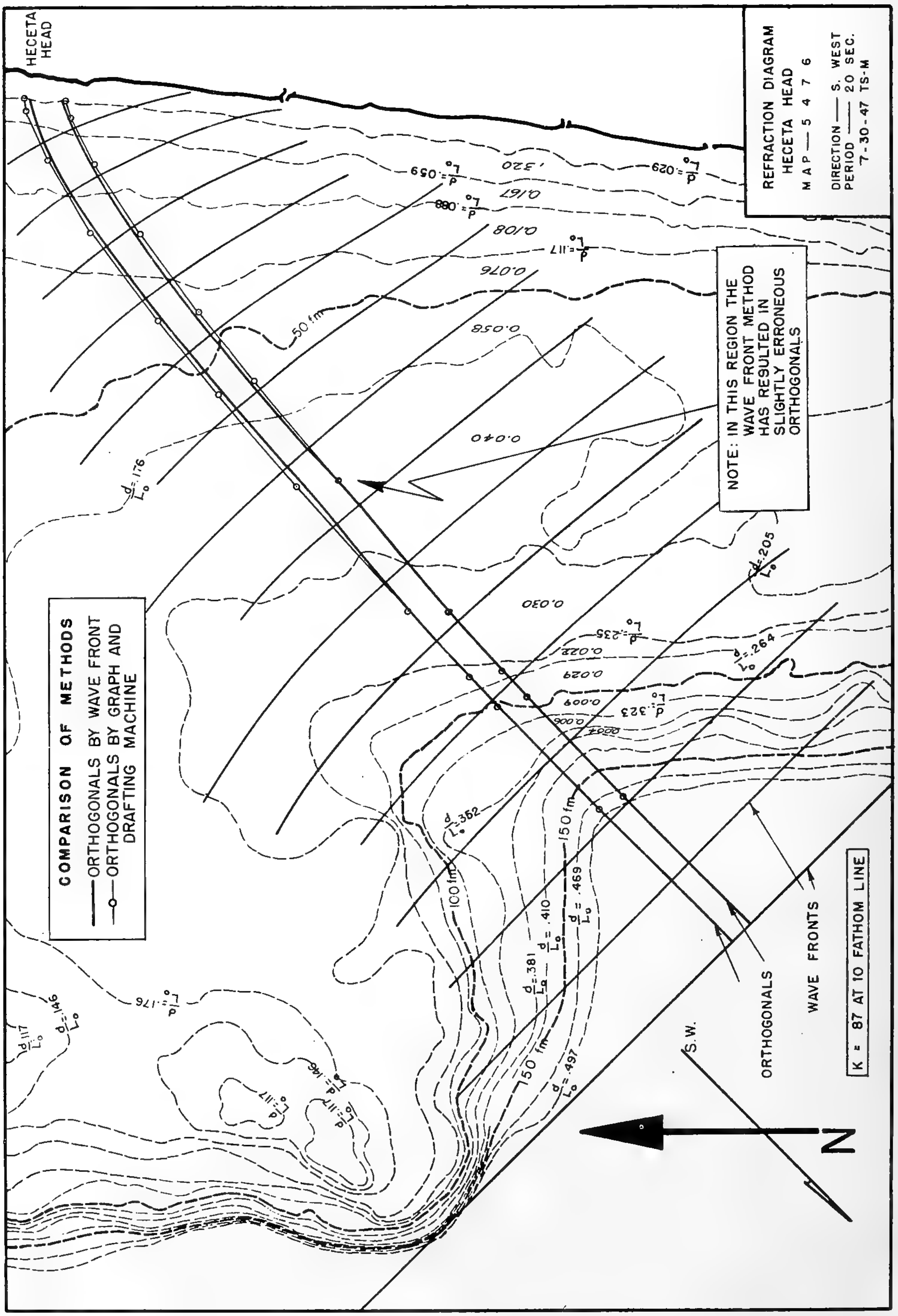

HECETA HEAD
REFRACTION DIAGRAM
HECETA HEAD
MAP — 5 4 7 6
DIRECTION —— S. WEST
PERIOD —— 20 SEC.
7-30-47 TS-M
NOTE: IN THIS REGION THE WAVE FRONT METHOD HAS RESULTED IN SLIGHTLY ERRONEOUS ORTHOGONALS
COMPARISON OF METHODS
ORTHOGONALS BY WAVE FRONT
ORTHOGONALS BY GRAPH AND DRAFTING MACHINE
50 fm
100 fm
150 fm
ORTHOGONALS
WAVE FRONTS
K = 87 AT 10 FATHOM LINE
S.W.
N

Figure 19.

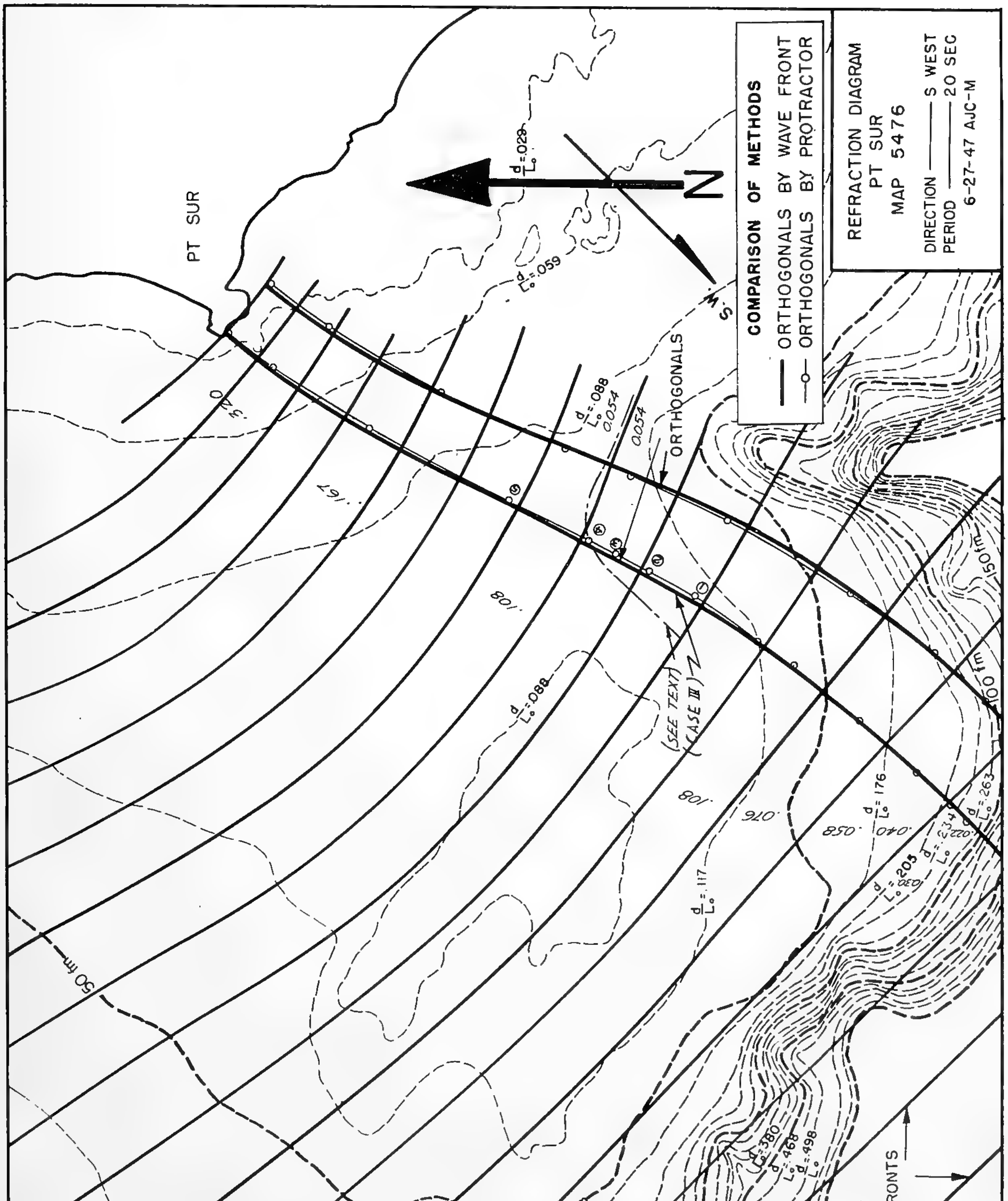
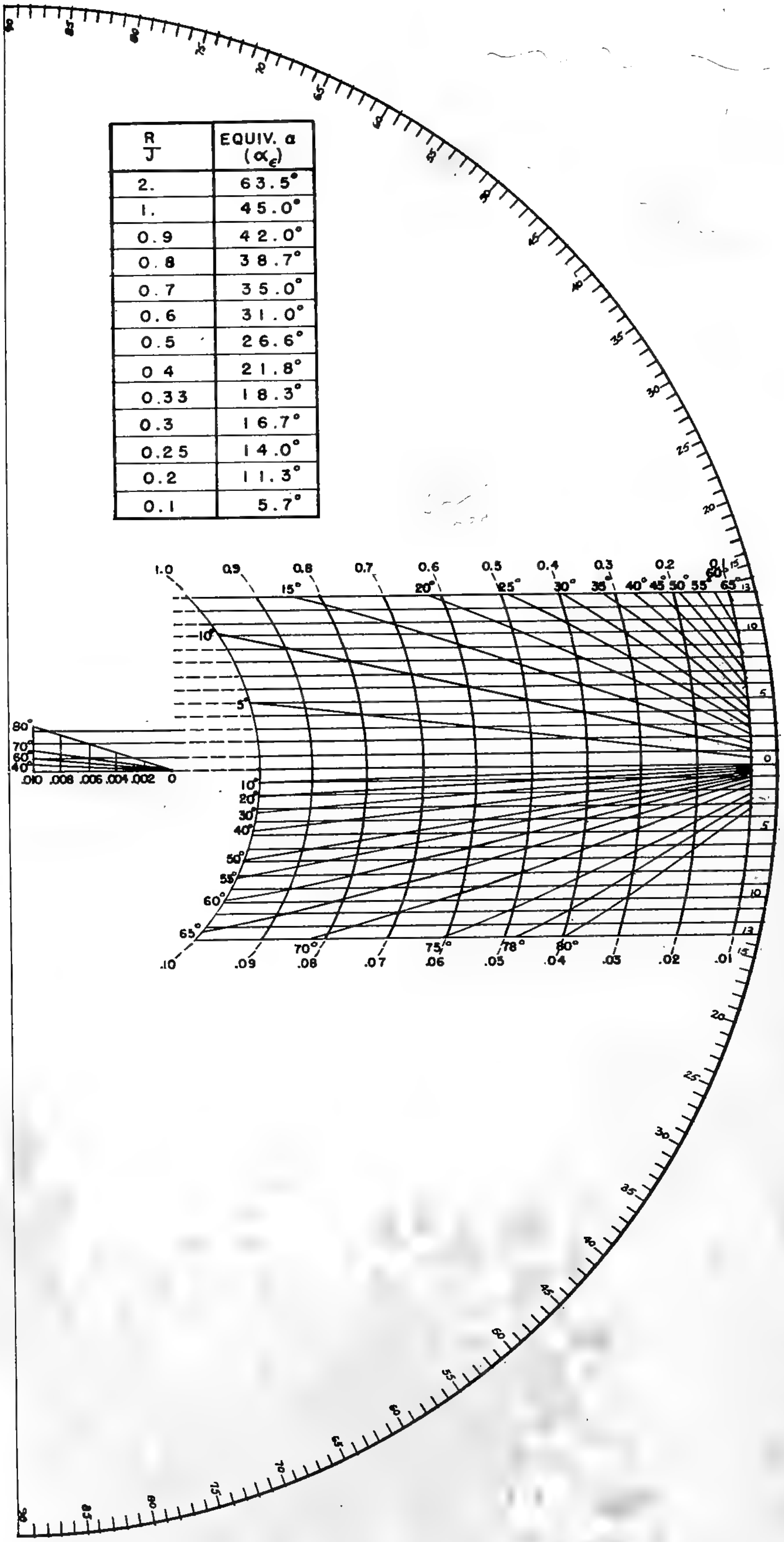
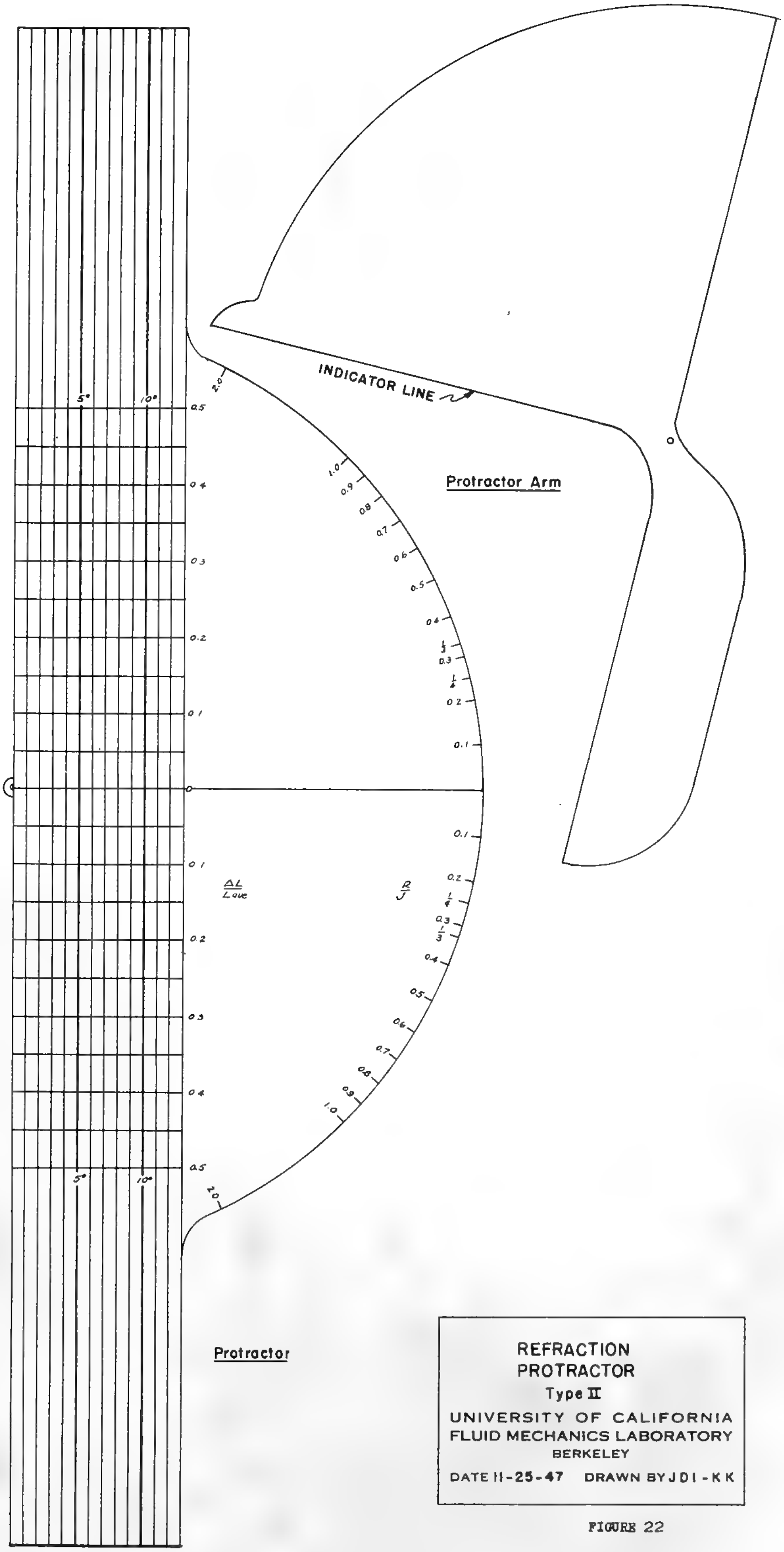

REFRACTION DIAGRAM
PT SUR
MAP 5476
DIRECTION ——— S WEST
PERIOD ——— 20 SEC
6-27-47 AJC-M
COMPARISON OF METHODS
ORTHOGONALS BY WAVE FRONT
ORTHOGONALS BY PROTRACTOR
PT SUR
N
S.W.
ORTHOGONALS
(SEE TEXT)
(CASE III)
WAVE FRONTS
50 fm
100 fm
150 fm
200 fm

Figure 20.



$\frac{R}{J}$	EQUIV. α (α_ϵ)
2.	63.5°
1.	45.0°
0.9	42.0°
0.8	38.7°
0.7	35.0°
0.6	31.0°
0.5	26.6°
0.4	21.8°
0.33	18.3°
0.3	16.7°
0.25	14.0°
0.2	11.3°
0.1	5.7°

Figure 21.



REFRACTION
PROTRACTOR
Type II
UNIVERSITY OF CALIFORNIA
FLUID MECHANICS LABORATORY
BERKELEY
DATE 11-25-47 DRAWN BY J D I - K K

FIGURE 22

Figure 22.

EXAMPLES OF REFRACTION

Various aerial photographs and refraction diagrams are of interest in providing typical examples. Figures 23 and 24 show refraction diagrams for Little Placentia Harbor, New Foundland. Figure 23 shows waves of 10 second period from NW carried from deep water, on Hydrographic Chart 2376, to the mouth of the harbor. Wave fronts then were transferred to a larger scale chart (chart 5621) and carried into the harbor as shown in figure 24.

Aerial photographs which show examples of wave refraction often are of value in the preparation of refraction diagrams. With the exception of figure 27 such examples are shown in figures 25–33.

Figure 25 shows a mosaic prepared from aerial photographs of refraction effects at Half Moon Bay, Calif. Note that the waves are breaking over a submerged reef offshore.

Figure 26 shows waves which have passed through the entrance of Humboldt Bay and into the bay. Note that waves are breaking inside the inlet. This breaking is probably the result of a combination of shoal water and a tidal current running opposite to the direction of wave travel. Very often in the preparation of a refraction diagram, when waves are carried over a shoal of limited area, the wave crests appear to cross each other. That such a condition can occur is illustrated in the upper right-hand corner of figure 26 where the waves do cross. That a shoal area exists at this locality is shown by the hydrographic chart in figure 27 which covers the section of the bay appearing in figure 26. Note that where the waves cross, they augment each other and breaking results.

The important features of refraction illustrated by the photos in figures 28–33, inclusive, are indicated in the caption under each photo.

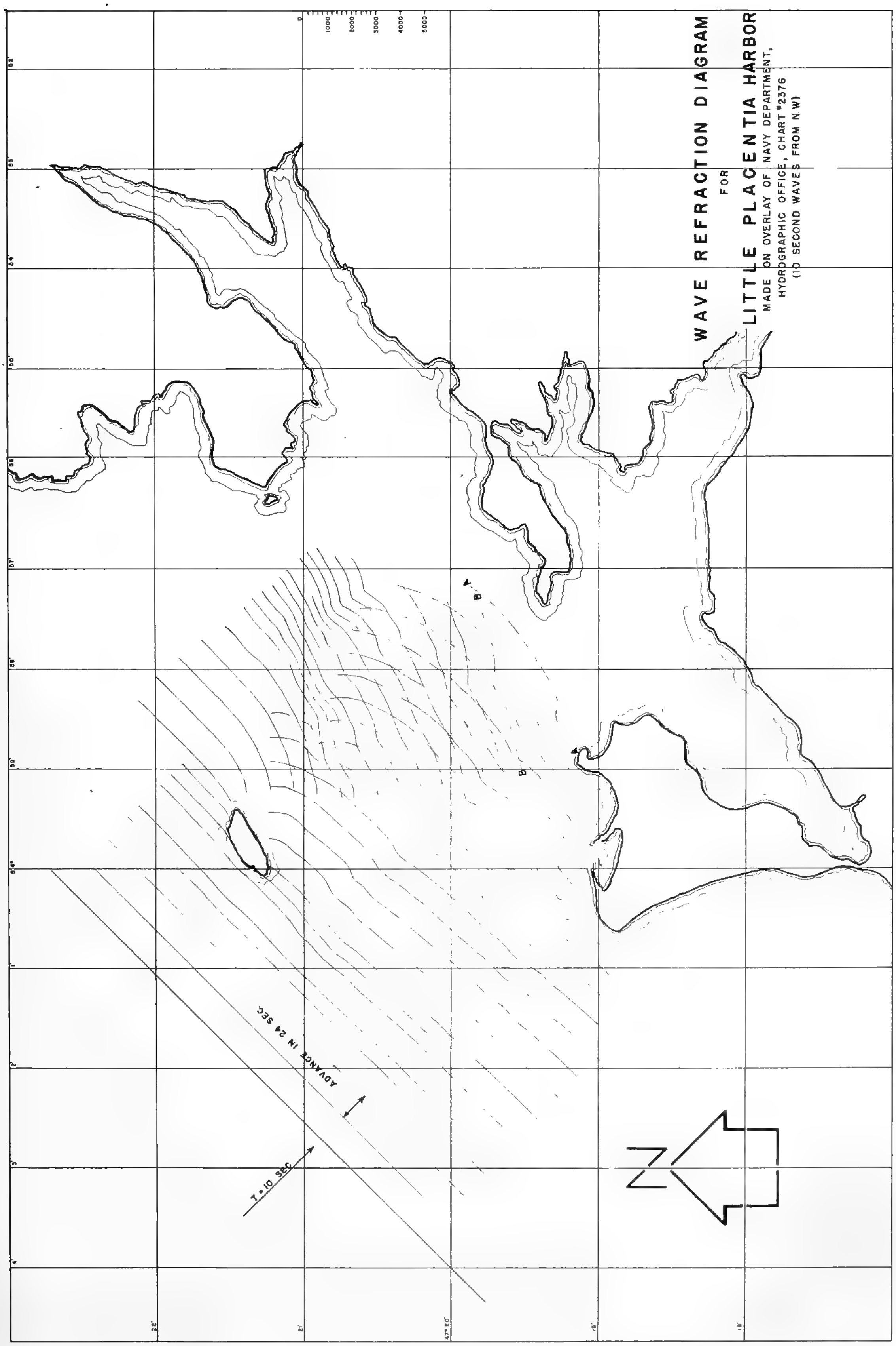
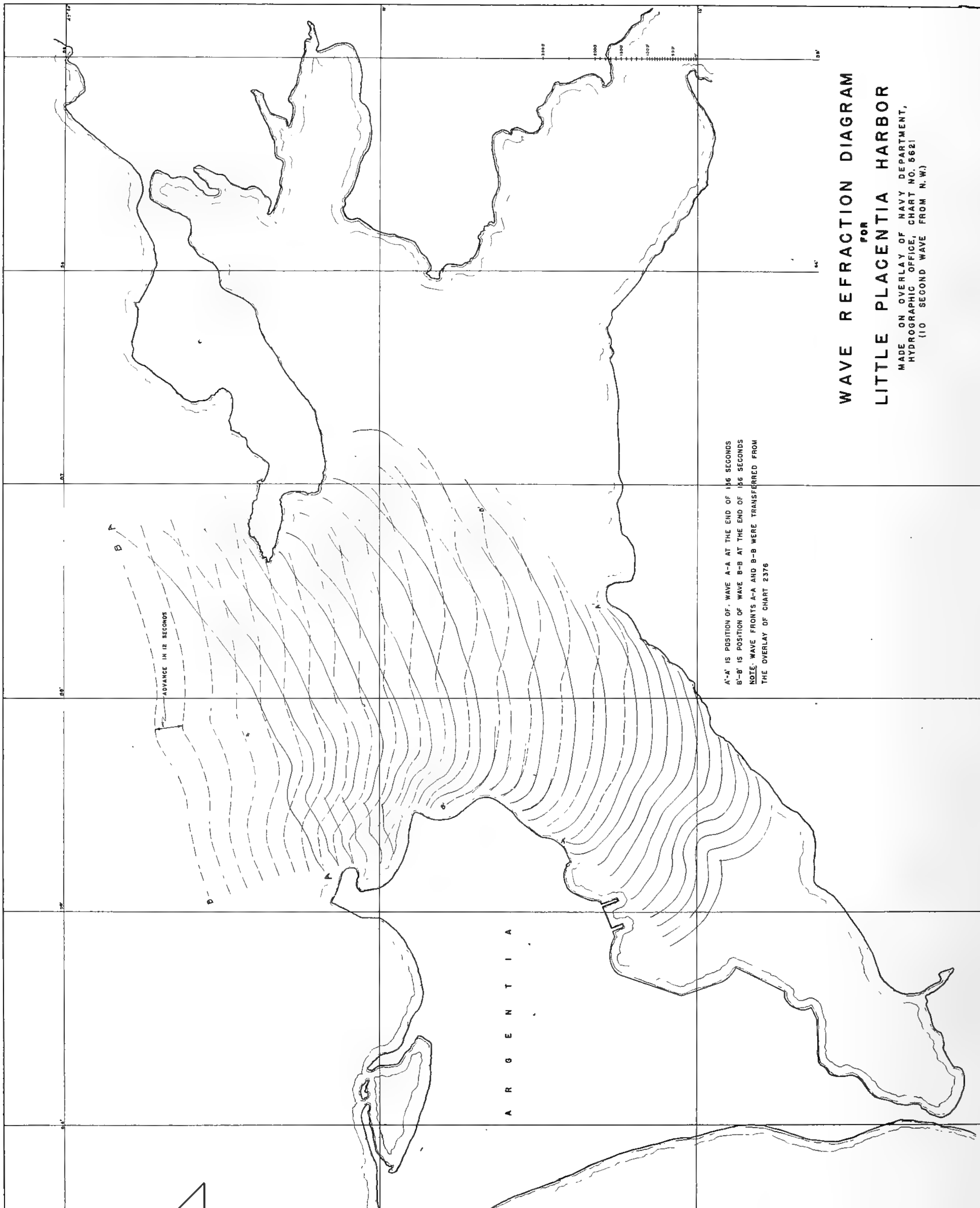

WAVE REFRACTION DIAGRAM
FOR
LITTLE PLACENTIA HARBOR
MADE ON OVERLAY OF NAVY DEPARTMENT,
HYDROGRAPHIC OFFICE, CHART #2376
(10 SECOND WAVES FROM N.W)
0
1000
2000
3000
4000
5000
T = 10 SEC
ADVANCE IN 24 SEC.
47°20'

Figure 23.



WAVE REFRACTION DIAGRAM
FOR
LITTLE PLACENTIA HARBOR
MADE ON OVERLAY OF NAVY DEPARTMENT,
HYDROGRAPHIC OFFICE, CHART NO. 5621
(10 SECOND WAVE FROM N. W.)
A'-A' IS POSITION OF WAVE A-A AT THE END OF 156 SECONDS
B'-B' IS POSITION OF WAVE B-B AT THE END OF 156 SECONDS
NOTE: WAVE FRONTS A-A AND B-B WERE TRANSFERRED FROM
THE OVERLAY OF CHART 2376
ADVANCE IN 12 SECONDS
A R G E N T I A
N

Figure 24.

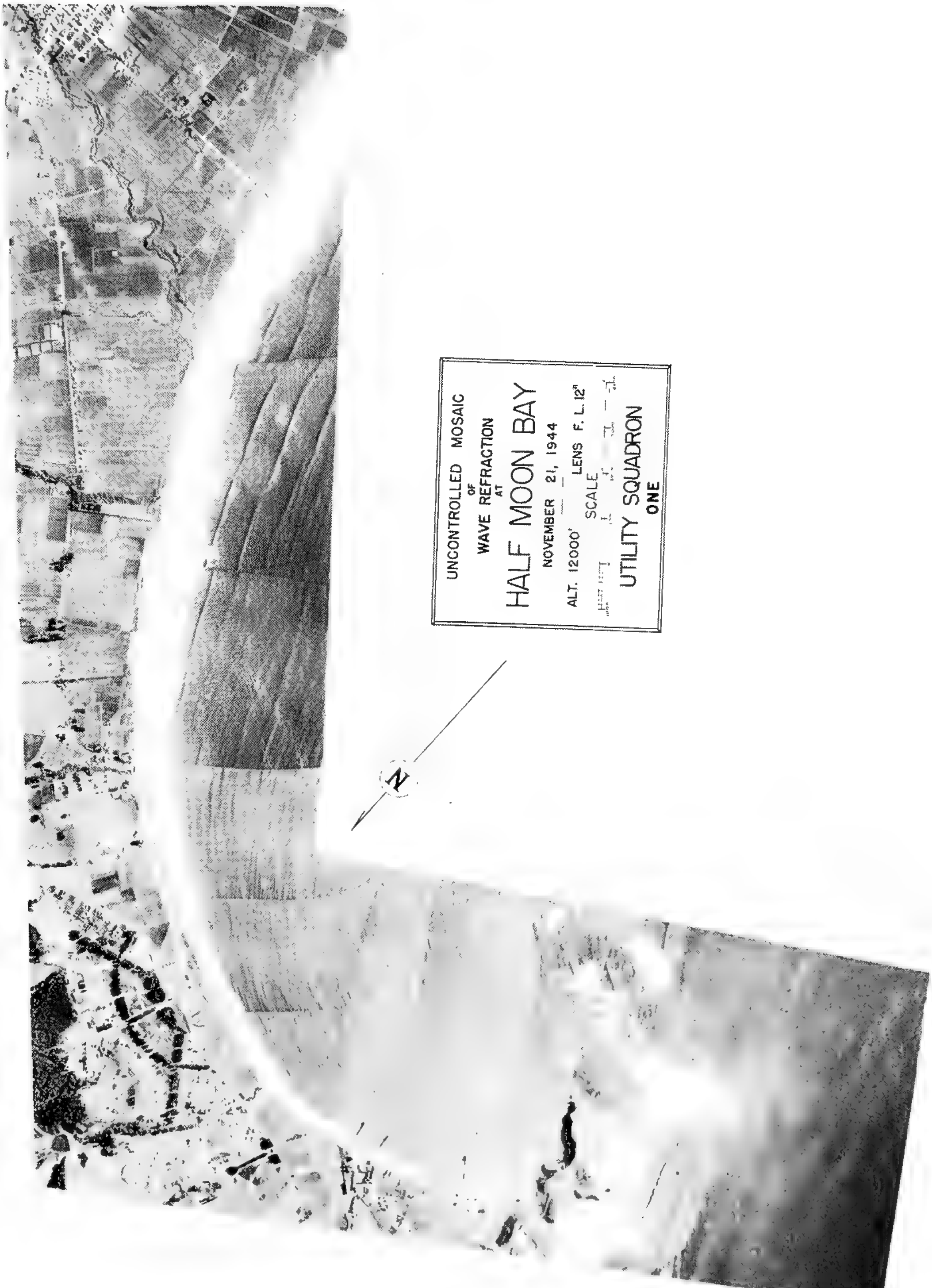

UNCONTROLLED MOSAIC
OF
WAVE REFRACTION
AT
HALF MOON BAY
NOVEMBER 21, 1944
ALT. 12000' LENS F. L. 12"
SCALE
UTILITY SQUADRON
ONE
N

Figure 25.

Figure 26.—Wave refraction inside Humboldt Bay. Note wave fronts crossing at shoal in upper right-hand corner. At this point the waves augment each other and break.

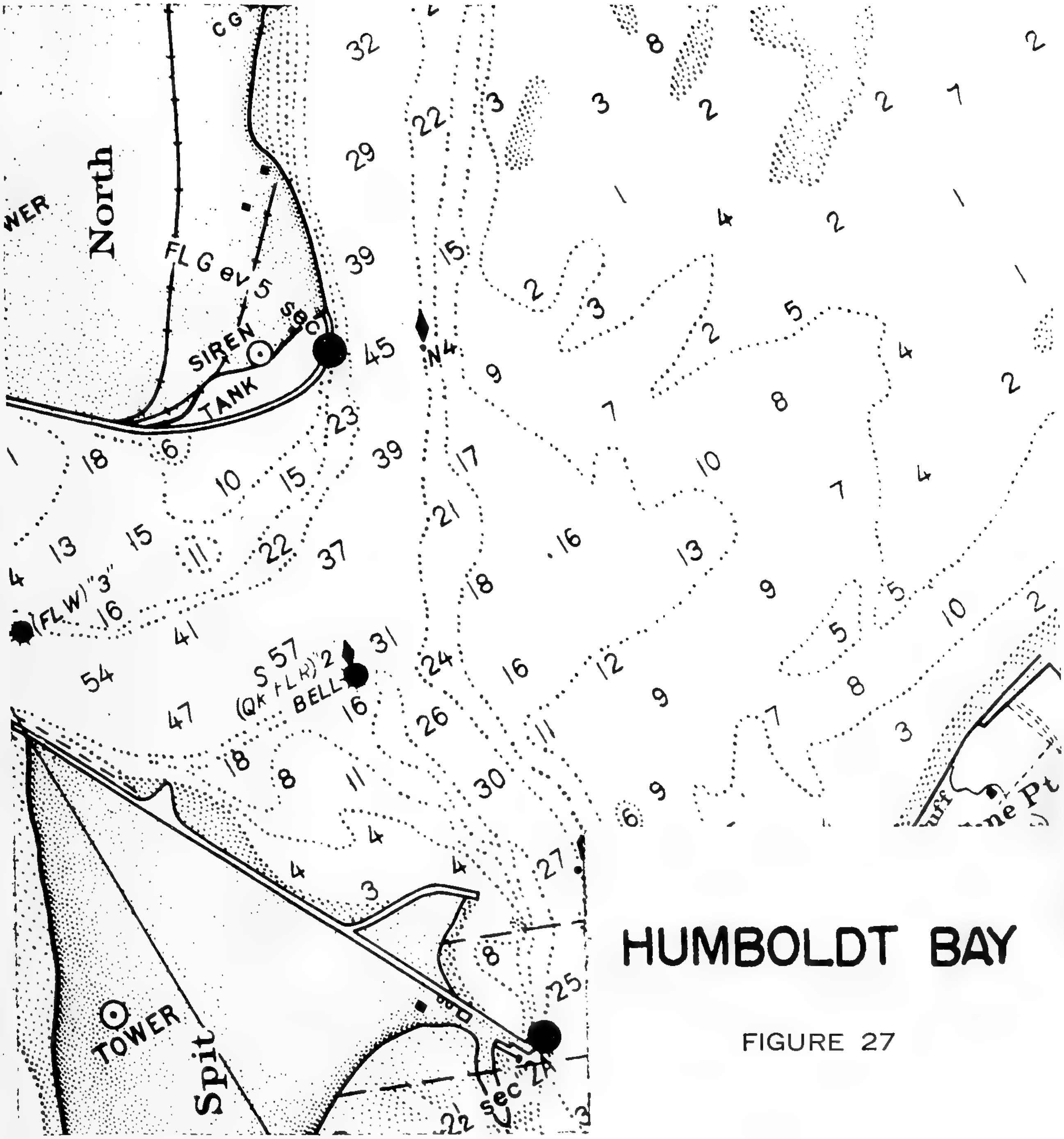

CG
North
FL G ev 5
SIREN
TANK
N4
(FL W) "3"
S 57
(Qk FL R) "2"
BELL
TOWER
Spit
HUMBOLDT BAY
FIGURE 27

Figure 27.

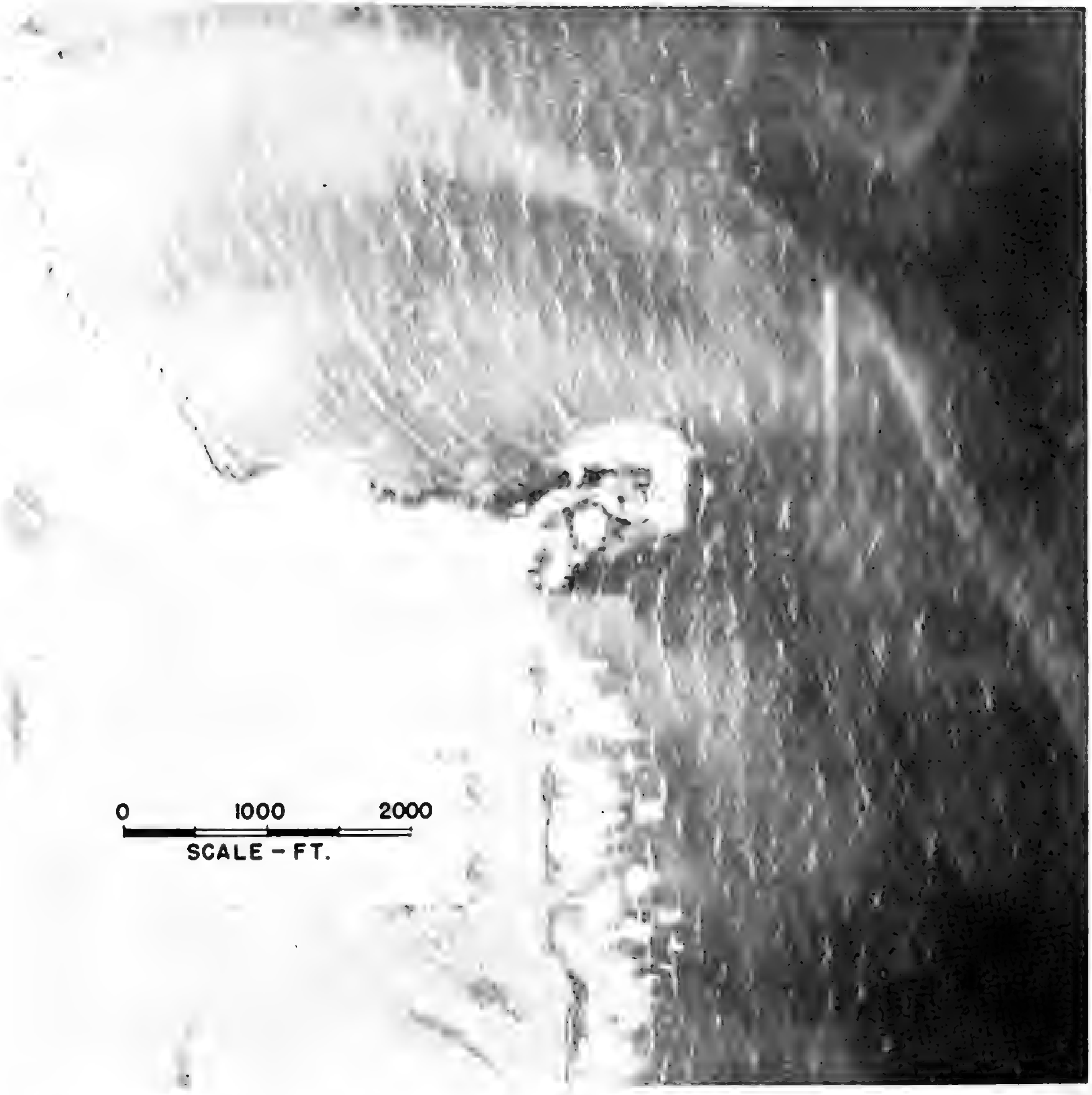


Figure 28 —Aerial photograph showing refraction of waves at Purisima Point, Calif The waves shown here are from winds blowing directly on to the coast. Note that the waves in the upper portion of the picture are almost parallel to the shore line as a result of refraction. The deep water wave length is about 600 feet; the wave height 6 feet; and the breaker height 9 feet.

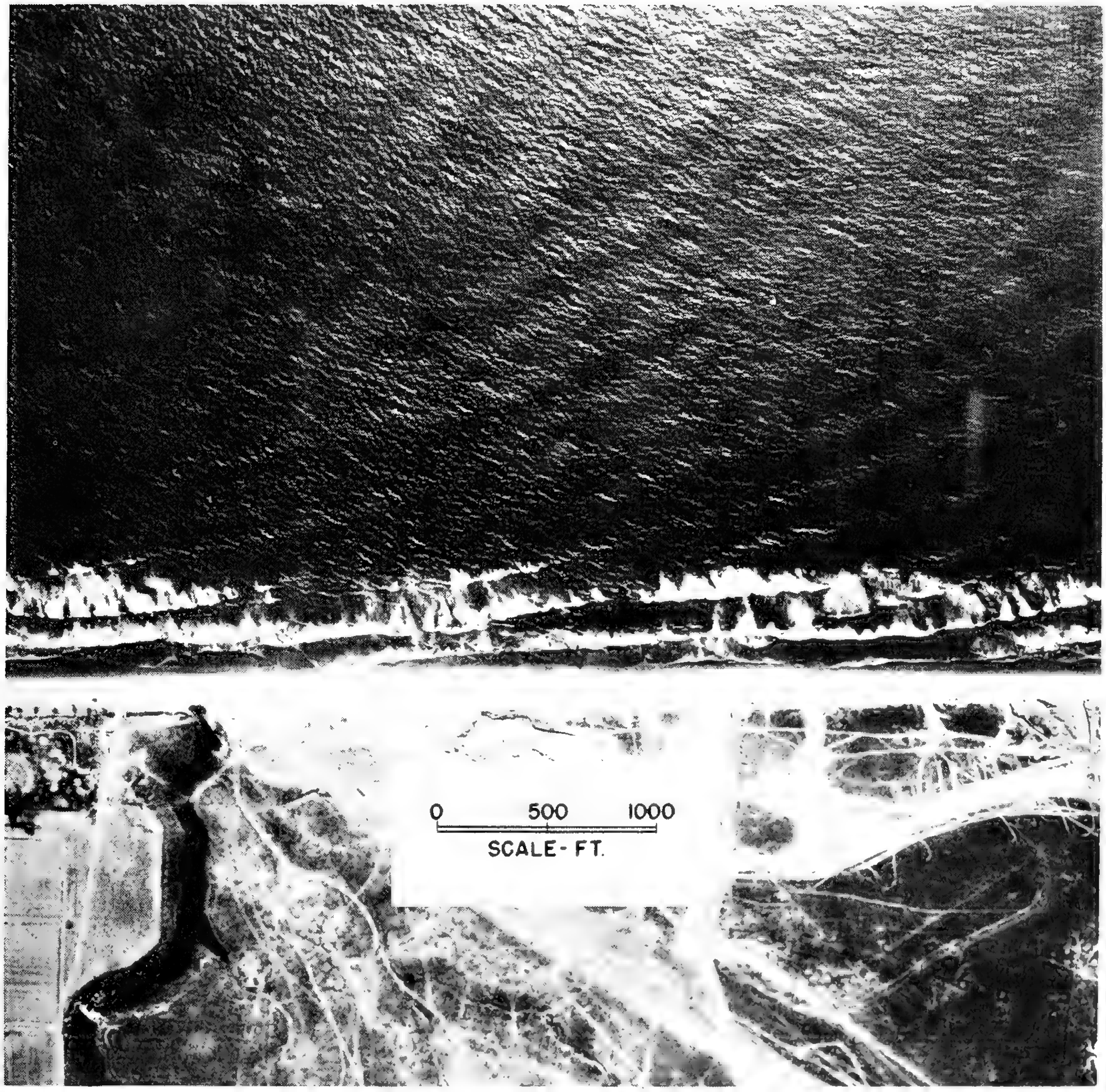


Figure 29.—Aerial photograph of surf at Oceanside, Calif. Two wave systems are present in this picture. There are small wind waves coming from the upper right, and a long low swell from the upper left. The swell is almost invisible in deep water, but peaks up near the shore to form the predominate breakers. Note how the waves break in short segments, where crests of the wind waves are superimposed on the crest of the swell. The wind waves have a deep water wave length of about 50 feet and a height of 1 to 2 feet. The swell has a deep water wave length of about 1,000 feet, with a height of 2 to 3 feet. The breaker height is about 5 feet.

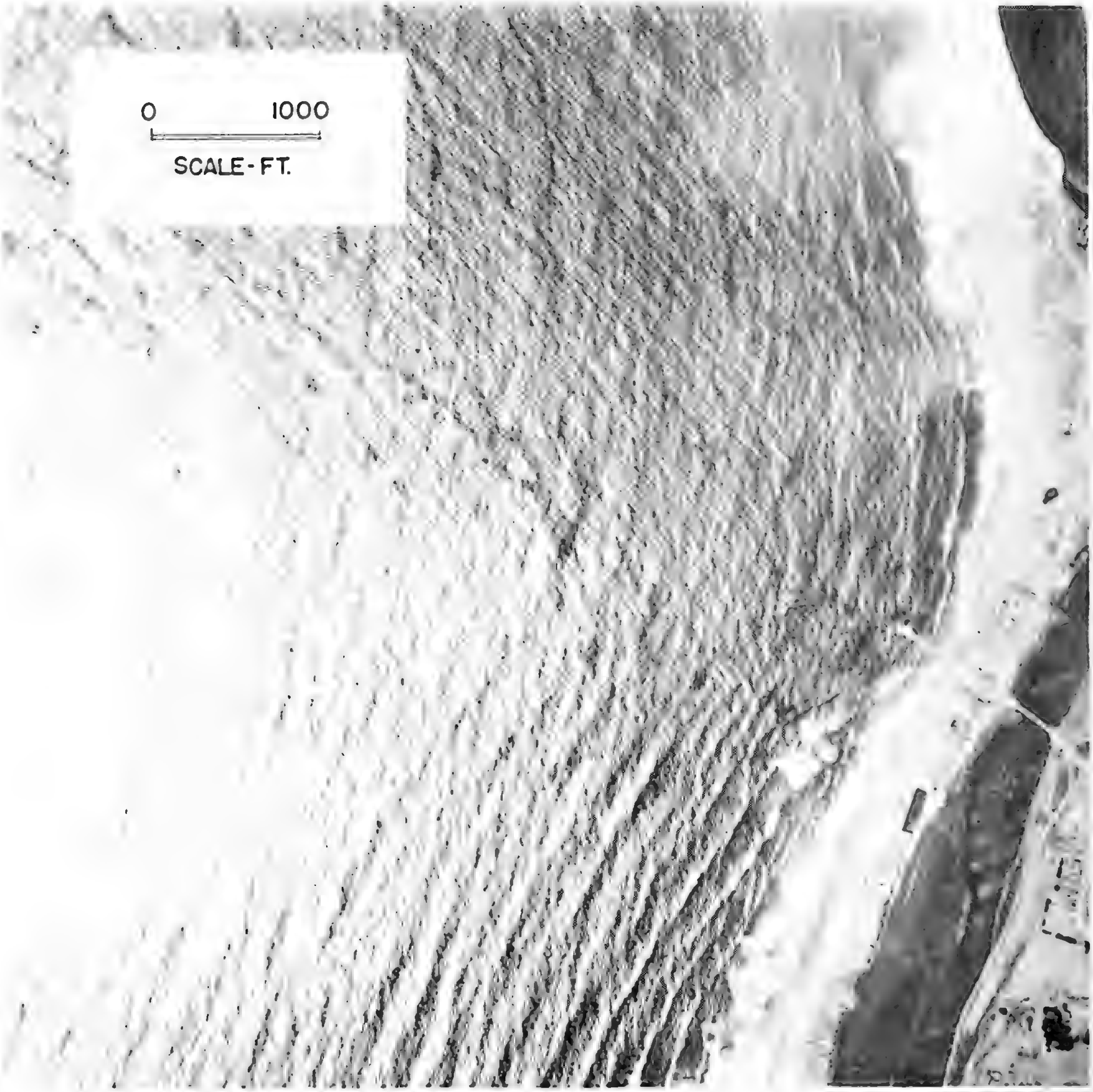


Figure 30.—Wave refraction at Moss Landing. Note the "flat" water at the end of the pier where the Monterey Canyon approaches the shore. Compare complicated refraction pattern with hydrography in figure 7.

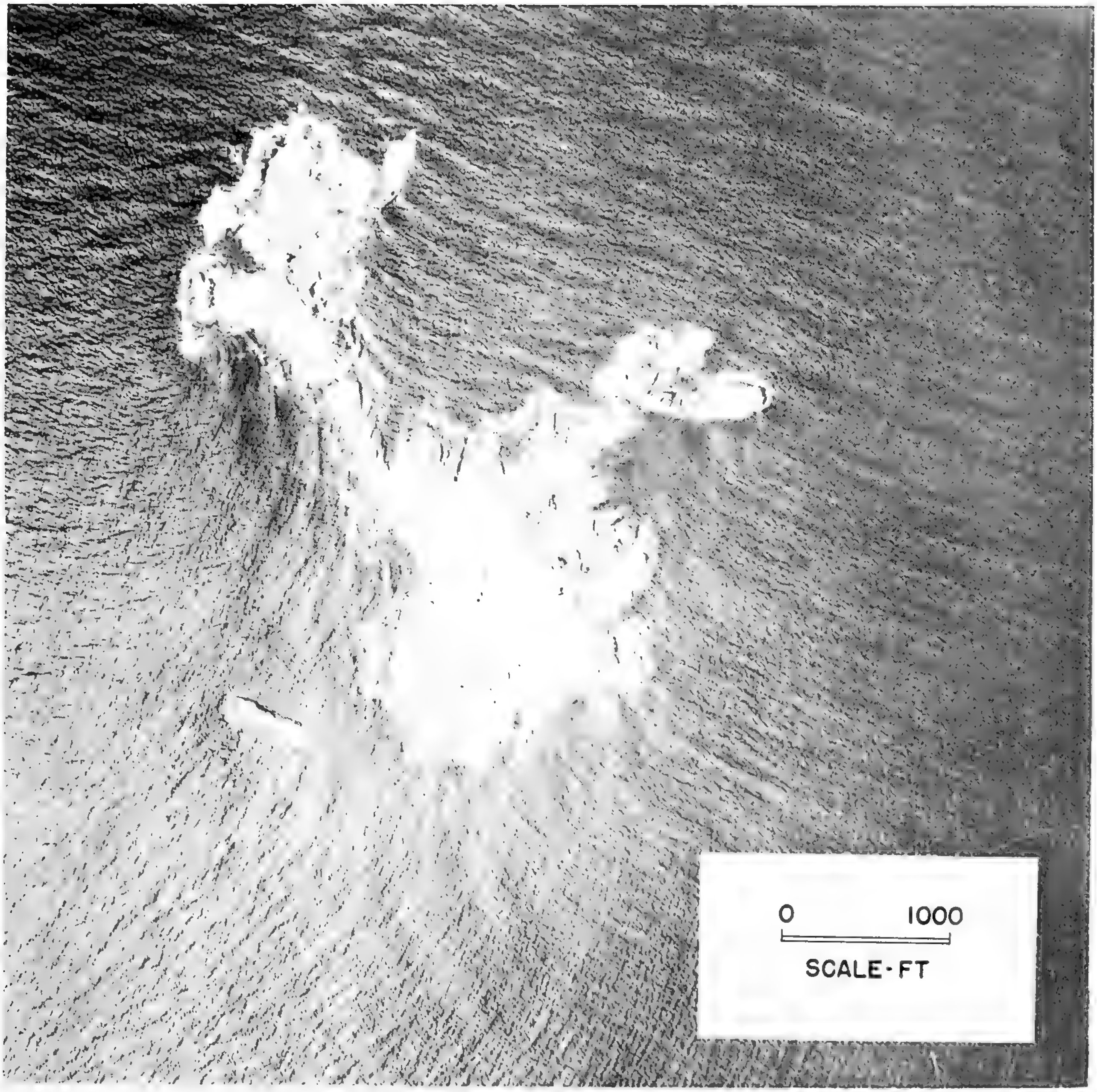


Figure 31.—Refraction and diffraction of both swell and wind waves at Farallon Island, Calif. Note reflection from small island at lower left.

Figure 32.—Wave refraction in inlet to Morro Bay. Note reflections from Morro Rock.

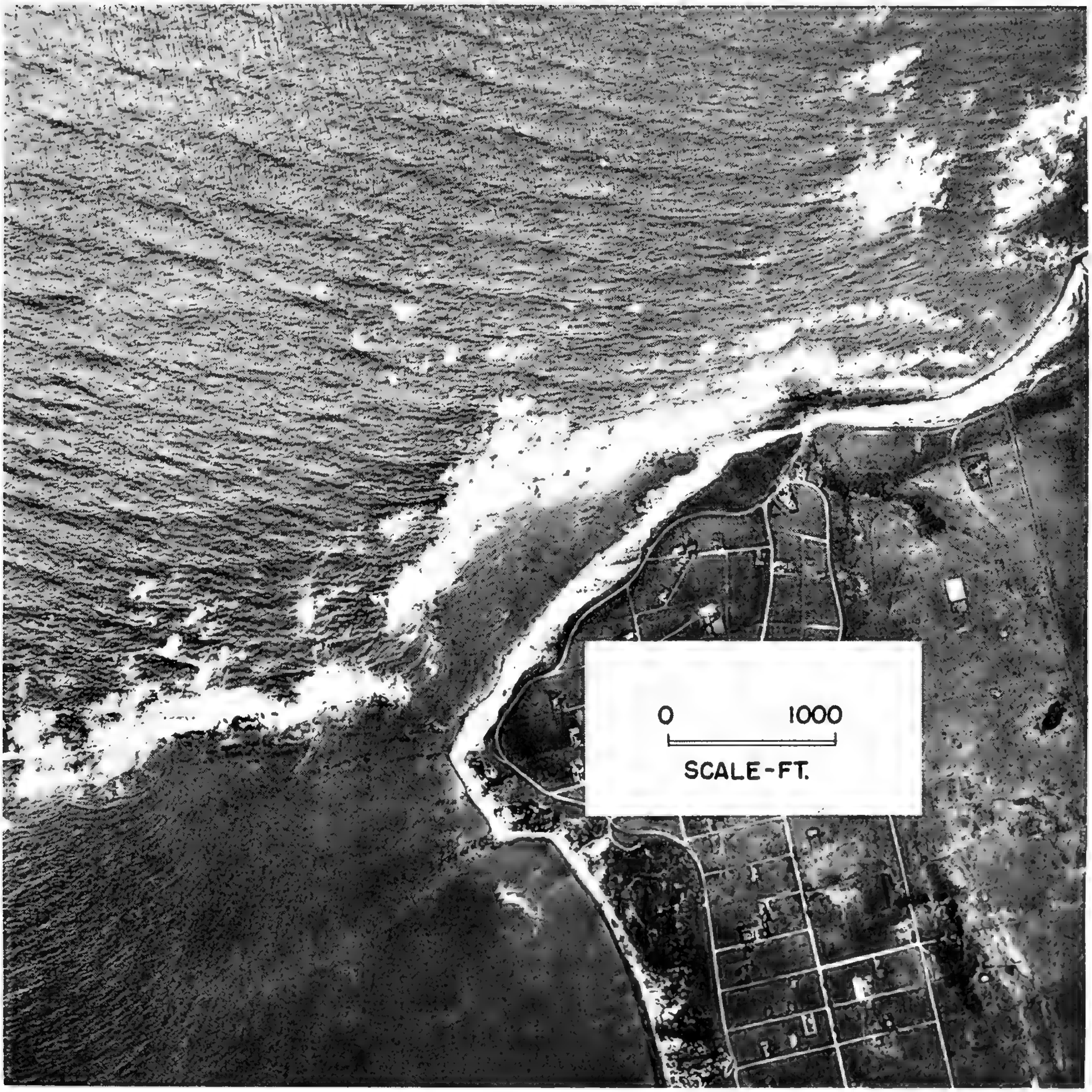


Figure 33.—Wave refraction at Duxbury reef near Bolinas, Calif. Note waves breaking on reef.

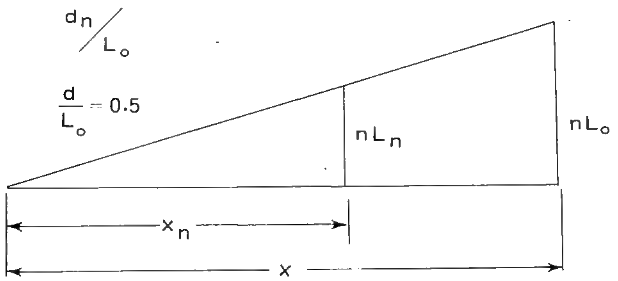
APPENDIX

THEORY AND PLOTTING DATA FOR REFRACTION SCALES

The theory involved in the construction of the scales shown in figure 5 for plotting refraction diagrams by the wave-advance method is briefly as follows:

It is desired to plot values of wave advance as a function of the ratio, d/L_o. By proper spacing of values of d/L_o, the upper plotting edge of the scale can be made a straight line, hence it can be constructed with considerable accuracy.

Referring to the following sketch, x represents the base length of the scale and the ordinate at the right-hand side represents the wave advance in deep water; that is, the advance is some multiple n of the deep water wave length, L_o.



For any particular value of the depth-length ratio, such as dn/L_o, the distance from the left-hand end of the scale to the point where the wave advance is nL_n is, by similar triangles, given by the relationship,

$$\frac{X_n}{X}=\frac{nL_n}{nL_o}$$

or

$$X_n=X\frac{L_n}{L_o} \qquad (1)$$

For any chosen length of scale (8 inches for scales A and B, fig. 5) values of X_n for various assumed values of d/L_o are calculated by the following procedure:

For shallow water, the wave velocity is

$$C^2=\frac{gL}{2\pi}\tanh\frac{2\pi d}{L} \qquad (2)$$

and

$$C=\frac{L}{T} \qquad (3)$$

or equation (3) may be written

$$C^2=\frac{L^2}{T^2}\frac{L^2}{L_o/5.12} \qquad (4)$$

hence, a combination of equations (2) and (4) gives

$$\frac{L}{L_o}=\tan h\left(\frac{2\pi d}{L}\right) \qquad (5)$$

Table 4 gives the steps in computing values of X_n by use of equation (1) and (5). Column (1) shows various values of d/L_o. From Breakers and Surf, H. O. No. 234, values of L/L_o have been obtained for the corresponding values of d/L_o and tabulated in column (2). Column (3) shows values of d/L which were obtained by dividing column (1) by column (2). Column (4) is the

TABLE 4

Computations and plotting data for wave refraction scales

$\frac{d}{L_o}$ 1	$\frac{L}{L_o}$ 2	$\frac{d}{L}$ 3	$2\pi\frac{d}{L}$ 4	$\tanh\frac{2\pi d}{L}$ 5	X_n (inches) 6
0. 0020	0. 1120	0. 1079	0. 1125	0. 1120	0. 896
. 0025	. 1250	. 0200	. 1257	. 1250	1. 000
. 0030	. 1368	. 0219	. 1376	. 1367	1. 094
. 0035	. 1478	. 0237	. 1489	. 1478	1. 182
. 0040	. 1578	. 0253	. 1590	. 1577	1. 262
. 0050	. 1764	. 0284	. 1784	. 1765	1. 412
. 0060	. 1930	. 0311	. 1954	. 1930	1. 544
. 0070	. 2082	. 0336	. 2111	. 2082	1. 666
. 0080	. 2224	. 0360	. 2262	. 2224	1. 779
. 0090	. 2355	. 0382	. 2400	. 2355	1. 884
. 010	. 2479	. 0403	. 2532	. 2479	1. 983
. 012	. 2712	. 0443	. 2790	. 2714	2. 171
. 014	. 2922	. 0479	. 3010	. 2922	2. 338
. 016	. 3116	. 0513	. 3223	. 3116	2. 493
. 018	. 3300	. 0546	. 3431	. 3302	2. 642
. 020	. 3469	. 0576	. 3619	. 3469	2. 775
. 022	. 3633	. 0606	. 3808	. 3634	2. 907
. 024	. 3786	. 0634	. 3984	. 3786	3. 029
. 026	. 3931	. 0661	. 4153	. 3930	3. 144
. 028	. 4072	. 0688	. 4323	. 4072	3. 258
. 030	. 4199	. 0714	. 4486	. 4205	3. 364
. 035	. 4518	. 0775	. 4870	. 4518	3. 614
. 040	. 4803	. 0833	. 5234	. 4803	3. 842
. 045	. 5065	. 0888	. 5580	. 5065	4. 052
. 050	. 5310	. 0942	. 5919	. 5313	4. 250
. 055	. 5538	. 0993	. 6239	. 5538	4. 430
. 060	. 5752	. 1043	. 6553	. 5752	4. 602
. 065	. 5954	. 1092	. 6861	. 5954	4. 763
. 070	. 6143	. 1139	. 7157	. 6142	4. 914
. 075	. 6323	. 1186	. 7452	. 6323	5. 058
. 080	. 6493	. 1232	. 7741	. 6493	5. 194
. 085	. 6654	. 1277	. 8024	. 6654	5. 323
. 090	. 6808	. 1322	. 8306	. 6808	5. 446
. 095	. 6954	. 1366	. 8583	. 6954	5. 563
. 100	. 7094	. 1410	. 8859	. 7094	5. 675
. 110	. 7352	. 1496	. 9400	. 7352	5. 882
. 120	. 7588	. 1581	. 9934	. 7588	6. 070
. 130	. 7805	. 1666	1. 0468	. 7806	6. 245
. 140	. 8002	. 1750	1. 0996	. 8004	6. 403
. 150	. 8183	. 1833	1. 1517	. 8183	6. 546
. 160	. 8349	. 1917	1. 2045	. 8350	6. 680
. 170	. 8501	. 2000	1. 2566	. 8501	6. 801
. 180	. 8639	. 2084	1. 3094	. 8639	6. 911
. 190	. 8768	. 2167	1. 3616	. 8768	7. 014
. 200	. 8884	. 2251	1. 4143	. 8884	7. 107
. 220	. 9089	. 2421	1. 5212	. 9089	7. 271
. 240	. 9259	. 2592	1. 6286	. 9259	7. 407
. 260	. 9400	. 2766	1. 7379	. 9400	7. 520
. 280	. 9516	. 2942	1. 8485	. 9516	7. 613
. 300	. 9612	. 3121	1. 9610	. 9612	7. 690
. 350	. 9780	. 3579	2. 2488	. 9780	7. 824
. 400	. 9878	. 4049	2. 5441	. 9878	7. 902

product of 2π and the values in column (3). Column (5) is the hyperbolic tangent of the values in column (4). Column (6) gives values of X_n in inches and is obtained by multiplying values in column (5) by 8 inches, the base length of the scale.

In plotting the scales, the value of ordinate at the right-hand side (where $d/L_o=0.5$) can be made any convenient values. Scale A was made with a height of 2 inches; whereas, scale B was made only 1 inch. This selection of heights seems suitable for the usual hydrographic charts.

The equations for determining the time interval, t, between wave crests or the number of wave lengths, n, between crests are determined as follows:

If the chart scale is in the form $\frac{1}{S}$, and y represents the ordinate in inches where $d/L_o=0.5$, then

$$\frac{y}{12}=\frac{nL_o}{S}$$

$$n=\frac{yS}{12L_o}=\frac{yS}{12(5.12T^2)}=\frac{yS}{(61.44)T^2} \quad (6)$$

The time interval, t, between crests is the distance advanced by the wave divided by the wave velocity; that is, at $d/L_o=0.5$

$$t=\frac{nL_o}{C_o}=\frac{nL_o}{L_o/T}=nT \quad (7)$$

or

$$t=\frac{yS}{(61.44)T} \quad (8)$$

Thus, for scale A (fig. 5), where $y=2$ inches

$$n=\frac{(0.0326)S}{T^2} \quad (9)$$

and

$$t=\frac{(0.0326)S}{T} \quad (10)$$

For scale B (fig. 5), where $y=1$ inch

$$n=\frac{(0.0163)S}{T^2} \quad (11)$$

and

$$t=\frac{(0.0163)S}{T} \quad (12)$$

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