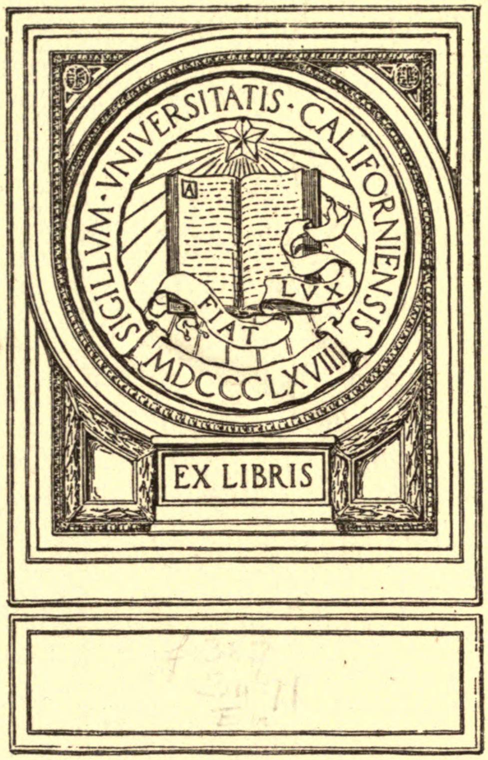
Graphical Determination of Sags and Stresses for Overhead Line Construction

by

Guido and Marco Semenza


SIGILLVM · VNIVERSITATIS · CALIFORNIENSIS
FIAT LVX
MDCCCLXVIII
EX LIBRIS

GRAPHICAL DETERMINATION OF SAGS AND STRESSES
FOR
OVERHEAD LINE CONSTRUCTION

GRAPHICAL DETERMINATION

OF

SAGS AND STRESSES

FOR

OVERHEAD LINE CONSTRUCTION

BY

GUIDO SEMENZA

KNIGHT OF THE ORDER OF THE CROWN OF ITALY, MEMBER OF A. E. I., I. E. E., A. I. E. E., ETC.; SECRETARY OF TURIN INTERNATIONAL ELECTRICAL CONGRESS (1911); LOCAL HONORARY SECRETARY OF A. I. E. E., FOR ITALY; CONSULTING ENGINEER AND TECHNICAL ADVISER OF SOCIETA EDISON OF MILAN, ETC.;

AND

MARCO SEMENZA

MEMBER A. E. I., ETC.; ELECTRICAL ENGINEER, MILAN, ITALY

TRANSLATED FROM THE ITALIAN

BY

C. O. MAILLOUX, E. E., M. S., D. Sc.

KNIGHT OF THE LEGION OF HONOR OF FRANCE; PAST-PRESIDENT, A. I. E. E., MEMBER A. E. I., I. E. E., S. I. E., A. E. S., ETC.; CONSULTING ELECTRICAL ENGINEER

McGRAW-HILL BOOK COMPANY, Inc.

239 WEST 39TH STREET, NEW YORK

6 BOUVERIE STREET, LONDON, E. C.

1915

TRANSLATOR'S PREFACE

Notwithstanding the great number of scientific papers and articles which have been published at various periods, on the subject of sags and stresses in overhead electric line conductors, and also on scientific methods of installing such conductors, it is a fact that the great majority of overhead electric lines are laid, one might say, with utter disregard of all scientific principle or method. Every storm of any consequence breaks down thousands of overhead electric lines, and after the storm the lineman has more "trouble" on his hands. He is kept busy for days and perhaps weeks in repairing and rebuilding the broken lines. These troubles entail important outlay on the part of the companies owning the lines, which is charged to repairs, maintenance, depreciation, and other "expense" accounts. These line breakdowns, being directly due to the "action of the elements"—wind, snow, hail, frost, cold, heat—are usually classed, not only by the general public, but by the lineman, the accountant and the manager, in the category of "dispensations of Providence," which, as such, cannot be foreseen or avoided. Indeed, only the engineer who is well informed in the science and art of electric line-construction realizes that a very large proportion of these troubles could be avoided—and without difficulty—because they are due so largely to the fact that the lines are put up without reference to the physical principles upon which their stability depends, under all conditions and at all times. In the case of expensive high-tension transmission-lines, the sags and stresses have, as a rule, received proper attention, and efforts have been made to follow proper engineering methods in the installation of such lines. In the case of the smaller and less expensive, but far more numerous, overhead lines, used for telegraph, telephone, lighting, and power circuits, there is lamentable indifference to anything approaching scientific methods of installation. In the great majority of cases, the method is empirical, and the work is done by rule-of-thumb. The "instrument of precision" most relied upon is the "eye" of the lineman or the foreman, and the criterion of quality of the work is the "looks" of the line at the time it is installed. Unfortunately, as scientific engineers know, the real criterion of the quality is not its appearance when it has just been constructed—usually at a time when the weather conditions are favorable—but it is the appearance of the line under the worst possible conditions, namely, when it is subjected to the greatest strains from the action of the elements, such as during the months when the temperature is lowest, and when high winds and storms are more prevalent. The great fault with the lineman's rule-of-thumb method of stringing the lines and adjusting the sag "by the eye" is precisely the fact that it fails to provide for the conditions and requirements at the time when the line is subjected to the worst strains. It provides for the best instead of the worst conditions.

Unfortunately, as the lineman is usually not an engineer or a technical man, the efforts made to have him recognize and follow scientific principles and adopt scientific methods in giving the lines the proper sags and stresses have proved unsatisfactory, because the methods tried expected too much from non-technical men, and could not be applied successfully by them.

The purpose of this work is to provide a method for determining sags and stresses which, while based rigorously and solidly upon sound scientific principles, is so simple in its application and in its practical use that an intelligent lineman or foreman of construction can use it without difficulty to obtain the most accurate and satisfactory results.

The subject of line sags and stresses has been handled by the authors in a very novel, ingenious and simple manner. It could not have been allotted to persons more competent and better qualified for the purpose. The senior author, Cavvaliere Guido Semenza, one of the most distinguished electrical engineers in Italy, was one of the pioneers, and he has continued to be a leader, in long-distance, high-tension, electrical transmission engineering. The transmission-lines in Italy and in other parts of Europe, designed by him, have all been models of their kind, of acknowledged originality and merit. Cavv. Semenza is recognized as one of the leading electrical transmission engineers in the world. The junior author, Signor Marco Semenza, whose experience covers a period of many years in electric line engineering and construction, under the direction of the senior author, is also a highly competent and experienced specialist.

The method of treating the subject of sags and stresses, adopted by the authors, is graphical. The work comprises some twenty-four pages of text and thirteen charts.

The text is divided into five parts:

PART I sets forth briefly the general character of the work. Its specific purpose, as clearly pointed out on Page 1, is **"to provide curves for the installation of copper conductors which require no calculation on the part of those using them."**

PART II deals with the scientific principles underlying the construction of these curves and their arrangement into groups or sets on "charts" suitable for convenient practical use.

PART III contains explanations and examples of the practical use of the charts.

PART IV contains information in regard to the allowances considered adequate in different countries for the extra loading effects due to wind and ice on line-conductors; and it gives the rules and regulations which are in force or in vogue in certain countries for the calculations of stresses and sags in electric line-conductors.

PART V contains the derivation and theoretical discussion of various formulæ used in Part III.

The presentation and discussion of the subjects treated are very simple. The mathematics employed in Part II and Part V are quite simple, involving merely elementary knowledge of algebra. Moreover, inasmuch as the parts (II and V) containing the mathematics are of special interest only to those who may desire to obtain a complete idea of the scientific principles underlying the method of the authors and the charts used by them for the practical application of this method, these portions may be passed over by those who have neither taste nor aptitude for such inquiries and investigations into fundamental principles and theories. In reality, Parts I and III will be found to contain all the instruction and explanation necessary to enable the charts to be used practically in the field by the men in charge of line-construction, to guide them in pre-determining the stresses and sags which must be given to lines at the time of their construction, in order that the sags and stresses may not exceed the prescribed limits at the time when the weather conditions are most unfavorable and severe. The value of the loading ratio, m, which is determined by the aid of chart No. I may be determined beforehand by the consulting engineer, instead of its determination being left to the foreman of line-construction. Under these circumstances, the person in charge of line-construction can use the charts with ease and precision for determining the amount of tension which must be given to the line, and the sag that will result therefrom, when the line is being strung, and for pre-determining, at the same time, what the tension and also what the corresponding sag will be, either at the time of maximum summer temperature, or of lowest winter temperature, and, in general, under the most severe weather and storm conditions.

The charts used for the English edition have been specially prepared for the purpose, to take into account the change from metric to English units. The text has been correspondingly modified wherever necessary.

The translator desires to acknowledge his indebtedness to Mr. H. W. Buck for certain changes and additions made by him to the text in Part IV, so as to make the reference to American conditions and practice complete.

THE TRANSLATOR.

CONTENTS

GRAPHICAL DETERMINATION OF SAGS AND STRESSES FOR OVERHEAD LINE CONSTRUCTION

PART I

GENERAL CONSIDERATIONS

In the case of wires suspended between two supports, the value of the mechanical tension or stress, and, consequently, the amplitude of the sag of the wires, undergo continual variations due to changes in temperature, and to the action of the wind blowing against them and of the snow and ice which may accumulate on them.

Engineering principles and methods enter into the predeterminations of the conditions and requirements for the proper installation of electric lines and their stability in a manner which it is the purpose of this work to set forth.

It is admitted generally that there is, for each metal wire, a limiting specific tension or stress which must not be exceeded. Moreover, the strain on the wire increases with a fall of temperature, and with an increase in extra load. Having assumed the lowest local temperature, and the maximum extra load which are to be expected, then, inasmuch as the wires are usually put up under temperature-conditions which are far from the lowest temperature, and with the wire unloaded, it is desirable to give to the wire, at the time of stringing it, a sag such that, under the most unfavorable conditions to be provided for, the stress attained may be neither greater nor less than the limiting tension which is allowable. It should not be greater for obvious reasons of safety, and it should not be less because this would necessitate building the supports unnecessarily high.

It is evident that the basic data which enter into the calculations vary within wide limits with the locality. Thus, while, in Southern climes, such as along the shores of Florida, the minimum temperature will rarely fall down to zero, and only the wind will have any effect on the mechanical strength of the wires, in Northern climes, especially in high altitudes, the temperatures will be very much lower, and it will be necessary to anticipate the formation of large coatings of ice.

The purpose of the set of charts or diagrams contained in this volume is to provide *curves for the installation of copper conductors* which require no calculations on the part of those using them. All that is necessary is to know the diameter of the conductor, the maximum limit of stress allowed, the distance between supports (length of span), the minimum temperature, and the additional load (wind-velocity and diameter of the ice-coating expected), in order to be able to determine, at once, by the diagrams, the curve representing the amplitude of the sag which should be given to the wire as a function of the temperature at the time the wire is strung.

An additional load applied to a line-wire always has the effect of increasing the stress on the wire; whether this additional load be due to the wind or to ice, it may always be considered as a virtual increase in the weight of the conductor itself.

To take these additional loads into account, it will be sufficient, therefore, to know what is the relation between the virtual weight of the loaded wire, and the actual weight of the unloaded wire. This ratio will be designated by the letter m. Chart No. 1 serves precisely for calculating this ratio m, for all conditions of additional load which may be reasonably anticipated in the great majority of cases.

Chart No. 2, and those which follow, give the curves for installing the wires when the value of m is known, taking into account the variations of temperature and of additional load, due to ice and wind.

PART II

CONSTRUCTION OF CHARTS

I. Chart No. 1. Calculation of m

(a) Symbols Used

d = diameter of the wire (when solid), in inches.

s = sectional area of the line-conductor (solid or stranded), in inches.

w = weight per foot of naked (unloaded) wire, in pounds ($= 0.321 \times 12 =$ 3.852 lb. for hard-drawn copper).

W = weight per foot of loaded wire, in pounds.

p = unit-weight of wire unloaded (= 3.852 lb.)

P = unit-weight of wire, loaded.

D = diameter of ice-covering, in inches.

Δ = weight, in pounds, of a cylinder or cylindroid of ice, 1 ft. in length and 1 sq. in. in sectional area.

v = wind-velocity, in miles per hour.

$V = xv^2d$ = total action of the wind on 1 ft. of the line-wire, in pounds.

$m = \frac{W}{w}$ ratio of weights of unit-lengths of loaded and unloaded line-conductor.

m_1 = value of the ratio m when the wind is not horizontal.

γ = angle between the direction of the wind and the direction of gravity.

$a = \frac{\Delta}{p}$ = ratio of weights of the unit-lengths of ice-cylinder and line-conductor defined hereinabove.

= ratio between the density of ice and the density of the metal used for the wire.

$= \frac{0.3852}{3.852} = 0.1$ in the case of copper.

$k = \frac{D}{d}$ = ratio between the external diameter of the ice-covering and the diameter of the line-conductor.

$b = 1 - \frac{\Delta}{p} = 1 - 0.1 = 0.9$

x = a constant determined by experiment (= effect, in pounds, produced on a wire 1 ft. in length having a diameter 1 in., by a wind having a velocity of 1 mile per hour).

(b) Assumptions Made

It is assumed that the wind acts on the side of the wire, and that its action is normal to that produced by the actual weight of the wire and of the covering of ice upon it. Consequently, the sag-curves determined for a loaded wire will not give sags in a vertical plane passing through the supporting points (*i.e.*, through the insulators), but in a plane determined by the points of support and the resultants of the actions just mentioned. Inasmuch as the amount of sag for a loaded wire is of no importance for our determinations, at least in ordinary cases—the maximum strain due to extra loading being alone of importance (see Appendix, page 17), the assumption made does not in any way affect the results. Nevertheless, since the supposition of a wind having any direction whatever and making an angle γ with the vertical direction, might be of interest, in certain cases, especially for lines located in mountainous regions, we will give, later, the formula which may be used in such cases for determining the value of the ratio P/p, which ratio will then be called m_1, this ratio being calculated after the value of m has been obtained, by means of the Chart (No. 1), for the same velocity of the wind on the supposition that it acts horizontally.

(c) Construction of Chart No. 1

From the demonstrations given in the Appendix (see page 17) it is seen that the value of m is expressed in a general form by the equation

$$m = \sqrt{(ak^2 + b)^2 + \cancel{X}^2k^2y^2}, \text{ where } y = \frac{v^2}{d}.$$

When it is ~~not~~ desired to take the action of the wind into account, we make $k = 1$, and the equation becomes

$$m = \sqrt{1 + X^2y^2};$$

and, when, on the other hand, it is desired to take only the ice-covering into account, we then take $v = 0$, and we have

$$m = ak^2 + b$$

From the general equation, solving for y, we have

$$y = \sqrt{\frac{m^2 - (ak^2 + b)^2}{X^2k^2}} \qquad (1)$$

Let us note that we have already assumed

$$y = \frac{v^2}{d} \qquad (2)$$

These two are the equations of the curves given in Chart No. 1.

Equation (1), when k is made constant, is the equation of a hyperbola whose axis is the axis of abscissae, and in which the abscissae represent successive values of m and the ordinates the corresponding values of y; Equation (2), when v is made constant, is the equation of an equilateral hyperbola, whose abscissae represent successive values of d and whose ordinates represent the corresponding values of y.

It is evident that, by means of these two equations, two sets of curves can be drawn, taking as abscissae, for the first set, successive values of k, and, for the second set, successive values of v. The ordinate-values are the same in both sets. These values of y, which serve merely to correlate the two sets of curves, have no importance by themselves, and for that reason no scale is given for them on the diagrams. The useful values are represented by the abscissae, namely, the values of the wire-diameter, d, for one set of curves, and the values of m for the other set.

Note.—Parallel to the scale of diameters d, a scale of sectional areas (s) of the conductors has also been indicated. For these sectional areas the resistance offered to the wind is assumed to be 25 per cent. higher than that offered by a smooth wire of the same diameter; hence the scale of sectional areas of the conductors is displaced with respect to that of their diameters in such a way as to take into account that increase in wind-resistance.

II. Calculation of Sags and Stresses

(a) Symbols Used

1. Case of spans with supports at same level

l = length of span (distance between line-supports), in feet.

$L = ml$ = length of hypothetical span, in feet.

f = true sag, in feet.

$F = mf$ = hypothetical sag, in feet.

T = stress in the wire, at its lowest point, in pounds per square inch.

α = coefficient of expansion of the metal of the line-wire, per degree Fahrenheit.

E = modulus of elasticity of the metal of the line-wire, in pounds per square inch.

p = weight per unit-length of the metal used for the line-wire (= weight, in pounds, of a wire 1 ft. in length, and having a sectional area of 1 sq. in.).

m = ratio between the weights of a conductor of unit-length when loaded and when unloaded.

$C = \theta - t$ = a constant.

$\Delta\theta = \theta_1 - \theta_2$ = actual range of temperature to be anticipated, in degrees Fahrenheit.

$t = \theta - C$ = hypothetical temperature, in degrees Fahrenheit.

2. Case of spans with supports at different levels

Symbols, same as in Case 1, with the following additions:

c = hypothetical span, in feet.

h = difference of level between line-supports, in feet.

$\Delta t' = t'_1 - t'_0$ = range of temperature, expressed in degrees Fahrenheit on the scales used in the diagrams.

l = length of span, in feet, when projected on a horizontal plane.

f = actual vertical distance, in feet, between the higher point of support and the lowest point of the curve made by the line-conductor.

$F = mf$ = hypothetical vertical distance, in feet, between the higher point of support and the lowest point of the curve made by the line-conductor.

(b) Assumptions Made in the Calculations

1. A parabola has been substituted for the catenary curve, Equation (1). The error thereby introduced in the measurement of the sag is less than 2 per cent. when the value of L/T is not greater or is less than 0.125 (see Appendix, page 18).

2. The length, l, of the *span*, *i.e.*, the distance between the two points of support of the wire, has been taken instead of the actual length of the *wire*, λ, Equation (2), (see Appendix, page 19). The error thereby introduced is negligible.

3. It has been assumed that the stress at the center of the span is equal to that at the point of support. The errors introduced by that assumption are likewise negligible (see Appendix, page 19).

(c) Construction of Charts

1. Case of spans with supports at same level

The fundamental equations for spans of line wires strung between two supports (see Appendix, pages 20 and 18) are:

$$f = \frac{Pl^2}{8T} \tag{3}$$

$$\Delta\theta = \theta_1 - \theta_0 = \left(\frac{P_1^2 l^2}{24\alpha T_1^2} - \frac{T_1}{\alpha E}\right) - \left(\frac{P_0^2 l^2}{24\alpha T_0^2} - \frac{T_0}{\alpha E}\right) \tag{4}$$

Equation (3) gives the amount of sag which is obtained with a wire, in a span of length l, when the stress is equal to T and the unit-weight of the wire is equal to P. Equation (4) gives the relation between the variations in the value of the stress T (and, consequently, also, in the value of the sag, f), and the variations of either the atmospheric temperature or the unit-weight of the wire, due to wind or ice.

From the Appendix (page 20) it will be seen that if the wire in a span of length l is subjected to a strain (due to wind, ice, or any mechanical force) that increases its virtual weight from p to mp, the sag produced will be equal to $1/m$ of that which would be produced in a span of length $ml = L$, when the specific stress in the wire remains the same in both cases.

Consequently, the process of passing from an extra load corresponding to a given value of the ratio $m = m_1$ to an extra load corresponding to another value $m = m_0$, is equivalent to passing from the consideration of a span of length $L_1 = m_1 l$ to that of a span of length $L_0 = m_0 l$, provided, however, that the sags thus found be divided by the corresponding value of m. Hence, inasmuch as this observation applies to any value of m, we will have, in general,

$$F = \frac{pL^2}{8T} \tag{5}$$

$$\Delta\theta = \theta_1 - \theta_0 = \left(\frac{p^2 L_1^2}{24\alpha T_1^2} - \frac{T_1}{\alpha E}\right) - \left(\frac{p^2 L_0^2}{24\alpha T^2} - \frac{T_0}{\alpha E}\right)$$

and also (see Appendix, page 20)

$$t = \frac{p^2L^2}{24\alpha T^2} - \frac{T}{\alpha E} \qquad (6)$$

wherein

$$t = \theta - C \qquad (7)$$

C being a constant.

It is important to note that, in (5) and in (6), we have $L = ml$, and that we may have

$$m \geq 1$$

In (5), since $F = mf$, all that is necessary, in order to obtain the value of f, the actual sag, is to take the value of F, the hypothetical sag, given in the diagrams, and divide it by the corresponding value of m.

In (6), t is not a temperature, but a quantity differing from the actual temperature, θ, by a constant, C (see Equation (7)).

In (5), solving for L^2, and substituting in (6), we have

$$t = \frac{Fp}{3\alpha T} - \frac{T}{E\alpha} \qquad (8)$$

In this equation p, α, and E are physical constants; hence, when T is also constant, (5) is the equation of a straight line containing T, but not L, and whose co-ordinates are F and t.

In (5), solving now for T, and substituting in (6), we have

$$t = \frac{8F^2}{3\alpha L^2} - \frac{pL^2}{8FE\alpha} \qquad (9)$$

This equation, when L is constant, is that of a curve of the third degree in F, which contains L but not T, and whose co-ordinates are again F and t.

Equations (8) and (9) are the actual equations of the two sets of curves contained in Diagrams Nos. 2 to 13, inclusive.

The first set is obtained from Equation (8), and consists of straight lines, each of which represents the variations of F with respect to t, for successive values of T.

The second set is obtained from Equation (9), and consists of curves, each of which represents the variations of F with respect to t, for successive values of L.

In the diagrams, the abscissae represent successive values of $t = \theta - C$ and the ordinates represent successive values of $F = mf$.

The values of t, as already seen, are not true temperatures, but differ therefrom by a constant, C. However, as we always have to deal with *differences* (representing *ranges* of temperature), $\Delta\theta = \theta_1 - \theta_2$, the value of the constant (C), is of no importance in the practical use of the diagrams.

Therefore, if, in the diagrams, we find the intersection of the curve for a given value of L, with the straight line corresponding to a given value of T, the abscissa of the point of intersection will represent a certain value of t which will be a starting point for the measurement of differences of temperature; that is to say, it will constitute a point of origin for a scale of temperatures.

If the value of the stress, T, which was employed in determining this intersection, is the maximum allowable for the wire for the span of length l (neglecting, at first, the additional loads due to ice and wind on the wire, *i.e.*, making $m = 1$, which gives $L = l$), then the value obtained for t, which we will call t_0, may, evidently, be regarded as corresponding to the lowest temperature which, it is assumed, is likely to be attained in the particular locality under consideration, since it is precisely at that minimum temperature that, as a rule, the maximum stress occurs. The ordinate of the point of intersection will represent the sag corresponding to that minimum temperature.

Still neglecting additional loads on the wire, the portion of the curve of the values of $L = l$, comprised between t_0 and the value of t which is obtained on adding to t_0 the maximum range (rise) of temperature assumed for the locality, represents the variation of the sag with respect to the temperature; and this portion of the curve is precisely that which is of practical use in stringing the line-conductors.

In the more general case, in which the additional loads due to ice and wind have to be taken into account, we will have $m > 1$, and it will be necessary to determine $L = ml$.

Even in such a case, if we find, on the diagram, the curve for that value of L, then the intersection of that curve with the straight line of the maximum value allowable for T will be a point whose abscissa determines the value of t_0.

The sag F, which is given by the ordinate of the point of intersection found, is equal to m times the actual sag for the loaded wire (see Equation (5)). This hypothetical sag is not, as a rule, of practical interest. On the contrary, it is the sag for unloaded wires that is of practical interest, when the wires are being strung.

We now proceed to determine the sag for an unloaded wire at the lowest temperature assumed to be reached, *i.e.*, at the very temperature at which the maximum loading due to ice and wind is supposed likely to occur.

We have stated that in a span of length l, having an extra load which increases the actual weight of the wire by the ratio m, the sag is $1/m$ of that of a span of unloaded wire of length ml, when T is the same in both cases.

Starting from this consideration, we have sought to determine the sag corresponding to the extra-load ratio m, for a span of length l, at the minimum temperature t_0, and at the maximum stress T_m, on the curve of values of $L = ml$. If, for that same span of length l, we now assume the mechanical load to be reduced so as to give $m_1 < m$, the temperature t_0 remaining the same, the result will be a decrease of the stress to a value T_1, and a decrease of the sag to a value f_1. In that new condition, we can always resort to the consideration of a span of unloaded wire whose length is $m_1 l = L$, in which the stress will be T_1 and the sag mf_1. The curve corresponding to that span is to be found on the diagram and in the very same manner that the diagrams were constructed; and the point which is to be considered is always on the vertical line corresponding to t_0, since it has been assumed that the temperature is maintained constant.

On reducing gradually the extra load, and, therefore, the ratio m, the stress and the sag will continue to decrease; and the length of the hypothetical span will approach more and more to that of the actual span, until the two coincide, when m becomes equal to 1. In doing this we will have arrived at the curve of values for $L = l$, the actual length of the span.

Therefore, to pass from the consideration of a line wire which is loaded to that of one which is unloaded, when the temperature remains unchanged, all that is necessary is to follow downward, on the diagram, the vertical line corresponding to that temperature, until it meets the curve of values of L corresponding to the actual length, l, of the span. The ordinate of that point gives the true sag for the wire, when unloaded, at the same temperature, t_0.

This point being determined, the portion of the curve of values of $L = l$, comprised between the ordinates of t_0 and of $t_1 = t_0 + \Delta t$, where $\Delta t = \Delta\theta$ represents the maximum range of temperature that is assumed and is likely to occur in the locality; and it will give the range of variation of the sag for the unloaded wire, as a function of the temperature, that is to say, *the amounts of sag which should be given to the wire as a function of its actual temperature at the time when it is being strung.*

2. Case of spans with supports not placed at the same level

It will be seen, from page 20 of the Appendix, that in calculating the amount of sag, the actual span, of length l, can be replaced by a hypothetical span of length c, the second (lower) support being then supposed to be at the level of the higher support. The length of that hypothetical span is given by the following relation:

$$c = l + \frac{2}{mp} \frac{h}{l} T \tag{10}$$

If, for a given case, we have

$$l = \frac{2}{mp} \frac{h}{l} T,$$

which gives $c = 2l$,
then the lowest point of the curve of sag described by the wire falls on the lower support of the actual span; and if we have

$$l > \frac{2}{mp} \frac{h}{l} T,$$

then the lowest point of the wire falls in the space inside the actual span; and if, on the other hand, we have

$$l < \frac{2}{mp} \frac{h}{l} T, \tag{11}$$

then the lowest point of the wire falls outside (beyond) the space of the actual span.

The last condition (11) is that which obtains in the greater number of practical cases, for reasons which can be readily made apparent. The quantity $2/p$ represents a physical constant (which, in the case of copper, has a value $2/p = 0.519$). The quantity h/l, representing the percentage of the "grade" to which the difference in levels (h) is due, will have a still smaller decimal value. Their product will be a small decimal quantity which will be further decreased as m increases. However, while the combined product of the three factors ($1/m$, $2/p$, and h/l) will usually be a quantity ranging between *hundredths* and *thousandths*, the fourth factor T, representing the stress in the wire, will range in value between *thousands* and *tens of thousands;* hence, the total product may still have a value in the hundreds or even in the thousands.

It is necessary, therefore, to have high values of m and l and low values of h and T to avoid the inequality (11).

When this condition occurs it is evident that there is no interest in knowing the successive values of the sag, since the lowest part of the curve then falls outside the actual span, and the sag-values refer to the hypothetical part of the curve. *It will be necessary, therefore, in the greater number of cases, to string the wires by reference to the values of the stress, T.*

The values of T corresponding to various temperatures and extra loads can be readily determined by means of the diagrams. If desired, at any time, a special curve representing the sag-variations can be constructed by the aid of the diagrams and on the diagrams themselves. Such curves cannot be put beforehand on the diagrams, because the value of c varies continually for the same span.

We know (see Appendix, pages 22–23) that, for calculating the stresses by the aid of the diagrams, a span with supports at different levels can be replaced by the same span with supports at the same level. Hence, to determine the successive values of the stress, when the values of l, T_m, $\Delta\theta$ and θ are given, we may proceed as in the case of a span with supports at the same level.

If the minimum value of T thus found leads to the inequality (11), all higher values of T will, obviously, do it also, and it will be useless to consider the corresponding sag-values. In other words, the only sag-values which it is necessary to know are those corresponding to values of T which do *not* lead to the inequality (11) When these values of T have been determined, then, by means of Equation (10), we will find the hypothetical spans, c, with supports at the same level, which correspond to those values, and, therefore, we can obtain the successive sag-values by means of Equation (3) and the diagrams.

PART III

MANNER OF USING THE CHARTS

Diagram No. 1. Calculation of m

Diagram No. 1 serves for hard-drawn copper-wires whose physical constants have the following values:

α = 00.00010 = coefficient of expansion per degree Fahrenheit.

E = 16,800,000 = modulus of elasticity.

In calculating the values plotted in the curves shown in the diagram, the following constants and coefficients were assumed:

Δ = 0.3852 lb. = weight of ice-covering 1 ft. long, 1 sq. in. in sectional area (12 cu. in. at 0.0321 lb. per cubic inch).

value of x in the formula, $P_v = xv^2d$ = pressure in pounds per square foot:

x = 0.0002 when v = miles per hour, and d = diameter of wire, in inches.

The curves in the diagram are applicable within the following limits:

Diameter of cable; from No. 2 to No. 000000 B. & S. G.

Diameter of wire; from No. 12 to No. 0000 B. & S. G.

Sectional area of wire; from 6530 to 212,000 circular mils.

Ratio ($k = D/d$) between the diameter of the ice-coating and that of the wire, from 1 to 10.

Wind-velocity, v; from 20 miles per hour to 120 miles per hour.

Value of ratio m; from 1 to 17.

Data necessary for using Chart No. 1

(a) The gauge number (B. & S. G.) or the diameter, d, or the sectional area, s, of the wire; (b) the wind-velocity, v (in miles per hour); (c) the ratio, k, between the external diameter of the ice-coating and the diameter of the conductor.

Directions

Find the point corresponding to the number (B. & S. G.) of the wire, i.e., to the value of d or of s, on the scale of abscissae at the bottom of the diagram; from this point rise along a vertical line until the line meets the hyperbola corresponding to the value of the assumed maximum wind-velocity (v); from this point of intersection follow a horizontal line to the curve of the assumed ice-loading ratio (k); from this point of intersection follow a vertical line to the upper scale of abscissae; the desired value of m can be read off directly by reference to that scale.

NOTE 1.—*When it is desired to take into account only the extra loading due to wind-pressure (and to neglect the ertra loading due to ice), proceed as follows: after having found the point of intersection of the ordinate from d or s with the curve of v, follow a horizontal line from this point of intersection until it meets the curve corresponding to k = 1; the abscissa of the point of intersection, read off at the upper scale, will give the desired value of m.*

NOTE 2.—*When, on the other hand, it is desired to take into account only the extra loading due to ice (and to neglect the loading due to wind-pressure), all that is necessary is to read off directly on the upper scale of abscissae the value of m corresponding to the intersection of the curve of the assumed ice-loading ratio (k) with the axis of abscissae itself, since in such a case the extra-load ratio m is independent of the diameter of the wire.*

NOTE 3.—*If it were desired to know the direction of the resultant of the loading actions exerted upon the wire, i.e., the direction of the straight line which, with the two points of support, determines the plane in which the wire lies when subjected to the action of the wind—all that would be necessary would be to construct the right-angled triangle having for its hypothenuse the value of m and for its vertical side the value of* $(ak^2 + b)$, *both of these values being obtainable from the chart, since* $ak^2 + b = m$, *when only the loading action of the ice-covering is considered.*

Examples

Suppose the following conditions:

Wire = No. 6 B. & S. G.; v = 90 miles per hour; k = 3.

EXAMPLE I.—From the point corresponding to No. 6 B. & S. G., at the bottom of the chart, rise vertically to the point of intersection with the curve for v = 90; then follow a horizontal line to the point of intersection with the curve for k = 3; thence rise vertically to the scale of m values at the top of the chart, and find the value $m = 10$.

If only the loading effect of the wind is to be considered, then $k = 1$, and, in that case, from the intersection with the line for v, follow a horizontal line to the curve for $k = 1$; thence rise vertically to the top of the chart and find the value of m, which is $= 3.4$. If only the loading effect of ice is to be considered then $v = 0$, and all that is necessary is to follow the curve for $k = 3$ to the upper end of the chart and read the value for m, which is $= 1.8$.

EXAMPLE II.—Suppose the following conditions:

Wire = No. 2 B. & S.; $v = 70$ miles per hour; $k = 5$.

When both wind and ice loading are to be considered, the value of m will be found to be $m = 7.1$.

When the wind-loading alone is considered, the value of m will be $= 1.6$.

When the ice-loading alone is considered, the value of m will be $= 3.4$.

Charts Nos. 2 to 13. Calculation of Sags and Stresses

Charts Nos. 2 to 13 serve for copper wires having the physical constants given on page 8.

Chart No. 2* is a general diagram which covers the entire range of conditions. It serves as an *index* for Charts Nos. 3 to 13, inclusive, which show portions of the same general diagram on an *enlarged scale.* The particular portion reproduced on each enlarged diagram is indicated by red lines on Chart No. 2. After locating on Chart No. 2 the portion of the general diagram which is most suitable for a given case, it will be easy, by reference to the chart outlines and numbers in red, to select the particular enlarged chart which is most suitable for a given purpose. The enlarged diagrams provided in Charts Nos. 3 to 13 should be sufficient for nearly all practical purposes. In a few exceptional cases, if the portion of the general diagram to be utilized is not comprised in any enlarged diagram, Chart No. 2 itself can be utilized for determining the data required. The charts are applicable to all cases comprised within the following limits:

Hypothetical spans, $L = ml$, from 80 ft. to 4000 ft.

Hypothetical sags, $F = mf$, from 0 ft. to 300 ft.

Mechanical stresses, T, from 1000 lb. to 27,000 lb. per square inch.

*NOTE—Subdivided, for convenience, into two parts, A and B, shown on separate sheets.

1. Case of spans with supports at the same level

Data required

l = the span.

m = the coefficient of extra load at the minimum temperature, determined by the aid of Chart No. 1.

T_m = the maximum stress allowable.

θ_0 = the lowest winter temperature.

θ_1 = the highest summer temperature.

Directions

Multiply m by l, which gives L; find on Chart No. 2 the point of intersection of the straight line for the value of T_m with the curve for the value of L; it will fall in one or more of the enlarged diagrams; select the one which contains the whole range of temperature variation allowable, starting from the point of intersection just mentioned. On that enlarged diagram the point of intersection determines a value of t which we will call t_0, and which is to be taken as the minimum temperature, i.e., as being equal to θ_0.

The steps of this operation will be simplified by using a scale made of paper or cardboard, like those furnished with the charts. These scales give divisions in degree of temperature, from $-10°$ F. to $+110°$ F. This scale is to be laid on the scale of abscissae in such a way that the lowest value selected for the local temperature (namely, $\theta_0 = 32°$, or $15°$, or $0°$, etc., according to the case) shall coincide with the point t_0, determined as already explained.

The following relations will be found to hold:

1. *The ordinate of the point of intersection between the straight line for the value of T_m and the curve for the value of L is a hypothetical sag-value equal to m times the actual sag, when the wire has its maximum extra load, and when it is at the lowest winter temperature θ_0.*

2. *The ordinate of the curve of $L = l$ (effective span) corresponding to the value of t_0, is equal to the sag f_0 which the wire assumes when unloaded and at the lowest temperature, θ_0.*

3. *For each value of θ that is higher than θ_0, and which is to be read off, either directly or by means of the auxiliary scale, the ordinate of the curve of effective span $L = l$, gives the value of the actual sag for the wire at that temperature, θ, when not loaded.*

4. *The maximum sag for the unloaded wire is therefore that which corresponds to θ_1, the maximum temperature for the locality.*

5. *The mechanical tension in the wire is measured off at some point along the curve of $L = l$ by the straight line of T-values passing through that point.*

Example.—It is required to find the curve for erecting a line which is to be subject to the following conditions:

l = span = 180 ft.; T_m = maximum allowable stress = 13,500 lb. per square inch; θ_0 = lowest winter temperature = 5° F.; θ_1 = maximum summer temperature = 105° F.; m = coefficient of extra loading (obtained from Chart No. 1) = 3.

We will have:

L = hypothetical span = $ml = 3 \times 180 = 540$ ft.

On referring to Chart No. 2, it is found that the point of intersection of the straight line for T = 13,500, with the curve for $L = 540$, lies in the space of Chart No. 3, and also of Chart No. 5. We select Chart No. 3. (The reason for this choice is that Chart No. 5 does not contain the curve for $L = l$ = 180 = actual span, which is also needed.) The abscissa of the point mentioned gives $t_0 = 179°$; the ordinate of the same point gives $F_m = 10.42$; whence, we have

$$\frac{F_m}{m} = \frac{10.42}{3} = 3.47 \text{ ft.},$$

which is the sag for the wire *when loaded, at the lowest winter temperature* (50° F.).

The intersection of t_0 with the curve $L = 180$ gives:

$f_0 = 2.57$ ft.; and $T_0 = 6000$ lb. per square inch.

These values represent the sag and the stress for the wire, *when not loaded, at the lowest winter temperature.*

We now place the movable scale of t-values on the scale of abscissae on the chart in such manner that the lowest winter temperature assumed (5° F.) corresponds to the abscissa 179°. Then the portion of the curve for $L = 180$ which is comprised between 5° and 105° on the movable scale of t-values (or between 179° and 279° on the t-scale of the chart) will be the portion of curve that is required and is useful for stringing the wire.

By reference to the ordinate scale on the chart, it will be found that the sag to be given to the spans in stringing the wire will vary between 2.57 ft. for 5° F., and 4.3 ft. for 105° F.; and by reference to the T-curves intersected by the portion of L-curve above mentioned, it will be found that the corresponding stresses will be: 6000 lb. per square inch for 5° F., and 3800 lb. per square inch for 105° F.

2. Case of spans with supports at different levels

Data required

l = length of horizontal projection of span.

h = difference of level between the two supports.

m = coefficient of extra load.

T_m = maximum allowable stress.

θ_0 = lowest winter temperature for the locality.

θ_1 = highest summer temperature for the locality.

Directions

Proceed as in the case of a span with supports at the same level, of length l, seeking to ascertain only the values of the stress, T, and paying no attention at all to the sag-values; determine, in this manner, the lowest value of the stress and call it T_1. Next, calculate the product $(2/p \times h/l \times T_1)$. If that product be equal to or greater than the value of the effective span, l, no attention need be paid to the sag; if the product be less than the value of l, then it will be necessary to determine the correspond-

ing sag-value, by finding, on Chart No. 2, the point of intersection of the straight line of T-values with the curve of the values of c_1, *bearing in mind that* $c_1 = l + 2/p \times h/l \times T$.

Note.—It may happen that the inequality (11) is not produced by the value of T_m; and this may be due to the existence of extra load, giving a high value for m (see Equation (10)). In such a case, it will be well to ascertain the value of the sag for the wire when loaded, to serve for subsequent comparison with the sag for the wire when not loaded, at the lowest stress-value, T (see page 18).

Example I.—Suppose a span of length $l = 200$ ft., with a difference of level between supports of $h = 18$ ft. It is required to find the variations of stress, and, incidentally, the variations of sag, between the lowest winter temperature of $-10°$ F., and the highest summer temperature of 100° F., *i.e.*, for a temperature range $\Delta\theta = 110°$ F. Let $m = 2.5$, and let the maximum allowable stress $T_m = 15{,}000$ lb. per square inch.

We have $L = ml = 2.5 \times 200 = 500$ ft. On referring to Chart No. 2, the point of intersection of the curve of $L = 500$ with the straight line of $T = 15{,}000$ is found to be in the portion corresponding to Chart No. 3. From the latter we obtain, directly, $t_0 = 139°$. Following along the ordinate of t_0, downward, we meet the curve of $L = 200$. From the point of intersection, we obtain the value $T = 8850$ lb. per square inch. (The point lies between the T-curves for 9000 and 7500, being nearer the former in the proportion of approximately nine-tenths the distance between the two curves; hence adding 9/10 of (9000−7500) to 7500 gives 8850). Adding t_0 to Δt, we have $t_1 = 139° + 110° = 249°$; or else, placing the movable t-scale on the chart in such manner that $-10°$ coincides with 139°, we will have $+100°$, coinciding with 249°. From the point of intersection of the ordinate of $t_1 = 249°$ (corresponding to $+100°$) with the curve of $L = 200$, it will be found that $T_1 = 4650$ lb. per square inch for that value of t, which is the minimum value of the stress (corresponding to the highest temperature). Now, calculating the value of the product $2/p \times h/l \times T_1$, we find

$$\frac{2}{3.852} \times \frac{18}{200} \times 4650 = 217 \text{ ft.}$$

which is greater than $l = 200$ ft. Hence it is not necessary to find the sag-values.

Example II.—Suppose a span of length $l = 300$ ft. with a difference of level between supports of $h = 22$ ft. Let the maximum allowable stress be $T_m = 17{,}000$ lb. per square inch, at the lowest temperature $\theta_0 = 15°$ F., with a coexistent extra loading that corresponds to $m = 3$. It is required to find the variations of sag and of stress that will occur between the minimum winter temperature of 15° F. and the maximum summer temperature, $\theta_1 = 115°$ F. From equation (10), taking $2/p = 0.5192$ (for copper) and inserting values for m, h, l, T, we have

$$c_1 = 300 + \frac{0.5192}{3} \times \frac{22}{300} \times 17{,}000 = 300 + 216 = 516$$

Since $216 < 300$, it will be necessary to ascertain the sag-value. We look on Chart No. 2 to find the point of intersection of the curve giving the values of $mc_m = 3 \times 516 = 1548$ ft., with the straight line representing $T_m = 17{,}000$. That point is found in the space which corresponds to Chart No. 9. The ordinate of the point of intersection on Chart No. 9, gives, by the scale of F-values, $F_m = mf_m = 66.6$ ft.; whence, dividing by m, we have $f_m = 66.6/3 = 22.2$ ft.

We have $L = ml = 3 \times 300 = 900$ ft. We look on Chart No. 2 for the point of intersection of the curve corresponding to $L = 900$ with the straight line corresponding to $T_m = 17{,}000$. This point is in the space corresponding to Chart No. 7. From that chart, we obtain, directly, $t_0 = 233°$. The next step is to find the point of intersection of the ordinate through t_0 with the curve of the actual span $L = 300$ ft. As Chart No. 7 does not contain the L curves for spans under 500 ft., we refer to Index Chart No. 2; we find that Chart No. 3 is the proper chart. The point of intersection between the ordinate through t_0 and the curve for $L = 300$ determines a value of the stress, T, which is that corresponding to the lowest winter temperature assumed (15° F.), when the wire has no extra load.

We will have

$$T_0 = 7000$$

We also have

$$\Delta\theta = \theta_1 - \theta_0 = 115° - 15° = 100°$$

and also

$$t_1 = t_0 + \Delta\theta = 233° + 100° = 333°.$$

We look on the Index Chart (No. 2) for the intersection of the curve for $L = 300$ with the ordinate of $t_1 = 333°$, and find it in the space corresponding to Charts No. 3 and 4. From either of these charts we obtain directly

$$T_1 = 5300 \text{ lb. per square inch.}$$

This is the minimum stress to which the wire will be subjected. By means of (10) we calculate the corresponding hypothetical span

$$c_1 = 300 + \frac{0.5192}{3} \times \frac{22}{300} \times 5300 = 300 + 67.3 = 367.3$$

Since $67.3 < 300$, it will be necessary to calculate the sag. On Chart No. 2, we find that the intersection of the curve for $L = 367.3$ with the straight line corresponding to $T = 5300$ is on Chart No. 5. From that chart we obtain

$$f_1 = 12.3 \text{ ft.}$$

Degree of Accuracy Attainable by Means of the Charts

The charts were designed on the assumption that it is possible, with the naked eye, to distinguish lines which are one-fiftieth of an inch apart; and the scales were varied in the different charts in such a way as to enable the values to be read off within 1 per cent. This degree of accuracy is sufficient, considering the fact that the systematic errors incidental to the use of simplified formulae involve errors which may amount to 2 percent. for large spans and heavy extra loading (see page 19, etc.).

The value of m can be obtained from the charts with an error not exceeding 5 per cent.

PART IV

NOTES CONCERNING THE SELECTION AND THE VALUES OF COEFFICIENTS AND CONSTANTS

1. Wind-velocity and pressure. Ice-coverings

Efforts have been made, by certain public authorities and electrotechnical societies, to prescribe definite values, more or less related to local conditions, for use in making calculations and allowances for the additional loading on line wires, due to wind and ice. This has been done, notably, by the Public Works Department in France, by the Italian State Railways, by the Italian, German and Austrian Electrotechnical societies, etc.

These attempts at standardization are intended to simplify calculations, but they are not very successful, and they are scarcely warranted, in countries where meteorological conditions vary greatly, as is the case in Italy, in Northern Europe, in America, etc. Rules for simplifying calculations become unnecessary when it is possible to prepare and use charts which, like those contained in this book, enable all conditions to be taken into account.

The engineer who is planning the construction of an electric line will be able, easily, to deduce, from the meteorological data for the locality in which the line is to be built, the maximum wind-velocity and the maximum thickness of ice-covering for which provision must be made, and to base his calculations on those data. In doing this, he should bear in mind that, as a rule, the conditions for maximum wind-velocity and pressure do not occur at the same time as the conditions for the formation of ice-coverings. Indeed, in those countries where the maximum wind-pressure is due to summer storms (tornadoes, gales, wind storms, etc.), it would be allowing too much to combine, in the calculations, the effects of maximum loading due to both wind and ice. In such cases, therefore, it is the maximum wind-velocity and pressure during the winter only that should be taken into account in the calculations.

In every case, Chart No. 1 will show clearly the influence and effect produced on the value of the coefficient of extra loading, m, under all conditions, imposed or assumed, which can possibly affect the result; and it enables the conditions under which m is really a maximum to be determined and utilized for the calculations.

In temperate regions, where there is snow but no formation of ice-coverings on wires, it will be necessary to consider only the maximum wind-velocity during the winter. In fact, for falling snow, k cannot exceed the value of 2, and it is not to be expected that any deposit of fallen snow can form on the wire when a wind of high velocity is blowing against it.

When it is a question of providing for extreme conditions, and there are no meteorological statistics and data at hand for the locality, it is necessary to have recourse to general data. A partial summary is given, herein below, of observations made at different times and places, concerning maximum wind-velocity and ice-formation.

(*a*) *Maximum Wind-velocity.*—The most recent investigations tend to demonstrate that the maximum wind-velocities assumed hitherto are, as a rule, exaggerated. (In Italy, the most violent winds in the valley of the River Po, certainly do not exceed 100 km. (62 miles) per hour, and it is only in mountain gorges and on the sea-coast that wind-velocities of more than 120 km. (74.6 miles) per hour may be expected. The maximum wind-velocities occur only in summer everywhere except in localities exposed to winds from the sea. In making calculations for extreme conditions, for Italy, it would be safe to take 80 km. (49.7 miles) per hour for inland localities, and 120 km. (74.6 miles) per hour for lines exposed directly to sea-winds or to winds in mountain gorges, as in the Alps and the Apennine Mountains).

In America, the records of the U. S. Weather Bureau contain much detailed information in regard to the maximum wind-velocities attained in different parts of the country. The most complete and authoritative investigations of the effect of wind-pressure on electric lines are those conducted by Mr. H. W. Buck and reported in the paper on "Aluminum as a Conductor" presented by him at the St. Louis International Electrical Congress in 1904. The following points are noted by Mr. Buck: (1) The *actual* wind-velocity is lower than that *indicated* by the instruments used in measuring it because of the inherent characteristics of the anemometer; thus, an *indicated* velocity of 100 miles per hour was found to correspond to an *actual* velocity of only 76.2 miles per hour; (2) maximum velocities do not occur at very low temperature; (3) the highest regular winds occur on the actual sea-coast, the exception being tornadoes, which usually occur inland and which blow at unknown velocities, probably 200 miles per hour, or more; (4) with the exception of tornadoes and of gales which blow on the tops of high peaks in places which might be considered "freak localities," the highest winds recorded do not exceed 100 miles per hour *indicated*, or about 76 miles per hour of *actual* velocity; (5) the records of the U. S. Weather Bureau are all taken at points which are high—100 ft. or more—above the ground; the wind-velocity decreases rapidly as the ground-level is approached, and, at the level of an ordinary transmission line, the velocity is about 30 per cent. less than at a point 100 ft. or more above the ground; on this basis, a maximum velocity of 100 miles per hour indicated velocity at an elevation of 100 ft. would be only about 55 miles per hour actual velocity, for an ordinary transmission-line. Mr. Buck's calculations as given in his paper were based upon a maximum actual wind-velocity of 65 miles per hour, at the minimum temperature, which corresponds to about 80 miles per hour at the maximum temperature, in the stress which it produces in the wire. In regard to tornadoes, he considers that, owing to their high velocity, it is commercially impossible to build all lines strong enough to withstand them.

(*b*) *Formation of Ice.*—In Italy, it is only in the high portions of the Alps and Apennines that coverings of sleet and ice are formed on electric wires. The coverings vary greatly in importance, because their formation depends upon the concurrence of so many conditions. Thus, while no sleet or ice-coverings have ever been noticed on some of the very high points in the Alps, they have been found repeatedly in certain high places in the Apennines (which are farther South than the Alps), the size of covering being large enough to raise the value of k to 5 or 6.

In the United States it is customary to make allowance for sleet-covering in calculating transmission lines, although sleet is rarely experienced. This is probably due, as elsewhere, to the static repulsion of the particles of water away from the conductor, caused by the high electric stress, thus preventing the accumulation of water on the conductor surface necessary for sleet-formation. Furthermore, lines carrying electric power currents, as distinguished from telephone and telegraph lines and lighting distribution circuits, which usually carry no load during the day, are appreciably heated by the current; and this heat tends to melt the sleet, if formed. A rise in temperature of even only one degree above the atmosphere might make the formation of sleet impossible. Sleet has been known to form at times on high tension lines in America, but rarely more than a thin film in thickness. In mountain regions in still damp cold weather, a light frost fabric has been observed to form on conductors at certain critical elevations, in the mountains. This, however, is a light open crystalline structure, although it has been observed to form to a thickness of as much as two inches around the conductor. It is, however, thrown off readily by any agitation due to wind. It may be regarded as probable, therefore, that the effect of ice and sleet will seldom if ever raise the value of "k" to as much as 5. There may be a few exceptional cases where "k" would rise above 5. In the great majority of cases, the value of "k" will probably remain below 3.

2. Maximum stress allowable in copper conductors

Different rules are followed in determining the values of the maximum stress allowable.

In some cases, the "factors of safety" required for iron serve as the basis for the rules, which limit the stress to one-fourth or one-fifth the tensile strength of the metal. This rule does not do justice to the difference in physical properties of iron and copper. The limit of elasticity is proportionately nearer to the ultimate tensile strength for copper than for iron; and it is the limit of elasticity rather than the ultimate strength that is of importance.

A method frequently followed in Italy, is to take, as the maximum or limiting value of stress, 80 per cent. of the stress corresponding to the elastic limit. The limit of stress, T, vaies with the quality of the copper used and with the diameter of the wire, the mechanical constants of the wire being altered somewhat by the process of drawing the wire.

In practice, the load at the elastic limit ranges between 12 and 20 kg. per square millimeter (*i.e.*, between 17,000 to 28,000 lb. per square inch); hence, the maximum value of T may range between 9.6 and 16 kg. per square millimeter (*i.e.*, between about 14,000 and 23,000 lb. per square inch). The values recommended for use in calculations for limiting conditions are, for large wires, 10 kg. per square millimeter (about 14,000 lb. per square inch); for small wires, and stranded conductors, 12 kg. per square millimeter (about 17,000 lb. per square inch).

As a general rule, it is desirable to assume for T the maximum value allowable or agreed upon, in order to reduce the height of supports as much as possible.

In the United Stages it is becoming common practice to assume certain maximum wind stresses, ice coverings, and a minimum probable temperature coincident therewith, and then to construct the line so that under these conditions the stress in the conductor will just equal its elastic limit. The justification for this is that even if some abnormal conditions of stress arose above those allowed for in the design, where the conductor-tension should exceed the elastic limit, it would merely result in a slight increase in conductor sag through stretching without risk of breakage.

3. Notes on the calculation of supports

It should be remembered that the expression for the wind-pressure exerted on the wires is derived from the formula

$P_v = 0.0002v^1ld$ (see page 8).

where P_v = total pressure, in pounds, exerted against the area ld.

v = wind-velocity, in miles per hour.

d = diamere of wire in inches.

l = length of wire in feet.

In the calculations for the line-supports, it will be necessary to use, for v, the very highest wind-velocity attained in the locality traversed by the line.

4. Measurement of temperature

While the wires are being strung, the temperature will be measured, generally, with a thermometer kept in the shade. However, a copper wire exposed to the mid-day sun in summer may attain a temperature that is sensibly higher than that of the air. It will therefore be desirable, under those conditions, to provide a thermometer whose bulb is enclosed in the hollow of a coil of copper wire that is exposed to the sun.

5. Regulations prescribed by various societies, etc.

I. Regulations of the Italian Electrotechnical Association (Associazione Elettrotecnica Italiana).

Calculations of the mechanical resistance of over-head wires must be based on a wind exerting a pressure of 72 kg. per square meter (14.75 lb. per square foot) of longitudinal projected area of the wire. This corresponds to a wind-velocity of 126.5 km. per hour (78.6 miles per hour). The lowest temperature attained in the locality is taken for calculating the total range of temperature; and, in the absence of reliable data, a temperature of $-15°$ C. (5° F.) is assumed, and the additional loading due to ice should also be taken into account, when there is a possibility of such loading. The maximum stress to which the wire is subjected must not exceed one-third the ultimate tensile strength, or two-thirds of the elastic limit.

II. Regulations of the Society of German Electricians (Verband Deutscher Elektrotechniker).

Additional loading due to wind-pressure is not taken into account. The calculations are to be made in two ways, based upon different assumptions, and the figures to be used will be those giving the most unfavorable result in respect to the stress on the wire. The maximum stress T_m, allowable is: (1) For annealed copper, 5 kg. per square millimeter = 7,112 lb. per square inch; (2) for hard-drawn copper, 12 kg. per square millimeter = 17,068 lb. per square inch.

Method I.—This is to be based upon the following assumptions:

Lowest temperature, $t_0 = -5°$ C.

An ice-covering of weight, in kilograms per meter of wire, equal to $0.115s$, where s = sectional area of wire, in square millimeters. The value of m, calculated on that basis, will be

$$m = \frac{0.0089 + 0.15}{0.0089} = 2.685$$

The value of m will therefore be constant, independently of the size of the wire. Taking that value of m, and selecting the value of T_m, the calculations can all be made by means of the charts.

Method II.—This is to be based upon the following assumptions:

Lowest temperature, $-20°$ C. ($= -4°$ F.), without extra loading.

In that case, $m = 1$; hence, $L = l$, and the charts may be used directly.

It is interesting to note that the second method of calculation gives higher stress-values for short spans—up to 40 meters (128 ft.) in length—while the first method gives higher stress-values for longer spans.

III. Regulations of the Society of Austrian Electricians (Verein Oesterreischischer Elektrotechniker).

Maximum stress allowable for hard-drawn copper wire

T_m = 8 kg. per square millimeter,
= 11,379 lb. per square inch.

No extra loading due to ice-covering is taken into account, and the calculation is to be based upon the following assumptions:

Lowest temperature, $\theta_0 = -25°$ C. (= $-13°$ F.).

The wind is supposed to produce a pressure of 150 kg. per square meter (= 30.72 lb. per square foot) of longitudinal projection of the wire; which corresponds, according to our system of calculation, to a wind-velocity of 182.6 km. (= 113.5 miles) per hour. The value of m, in this case, evidently, depends upon the diameter of the wire.

IV. Regulation of the French Government.

Two methods of calculation are prescribed, and the figures corresponding to the greater stress-value are those to be used.

Method I.—The mean temperature in the locality is to be taken, with a wind producing a pressure of 72 kg. per square meter (= 14.75 lb. per square foot) of longitudinal projection of the wire, which corresponds to a wind-velocity of 126.5 km. (= 78.6 miles) per hour.

Method II.—The lowest temperature in the locality is to be taken, with a wind producing a pressure of 18 kg. per square meter (= 3.69 lb. per square foot) of longitudinal projection of the wire; which corresponds to a wind-velocity of 63.3 km. (= 39.3 miles) per hour. The value of m can be found, for each case, by means of Chart No. 1.

V. Regulations in the United States.

Topographical and climatic conditions vary to such an extent throughout various sections of the United States that no line construction constants can be considered as standard. Assumptions which would be good engineering in one locality would be extravagant construction in another. The requirements vary over such a wide range that each instance is usually calculated as a special problem.

PART V

APPENDIX

I. Calculation of the Ratio m

Case I.—Additional loading due to ice only

Suppose a wire of diameter d, on which a covering of ice of diameter D has been formed. The weight, per unit-length, of the naked wire, is:

$$w = \frac{\pi d^2}{4} p;$$

and the weight, per unit-length, of the wire when loaded with ice, is:

$$W = \frac{\pi(D^2 - d^2)}{4} \Delta + w = \frac{\pi d^2}{4}\left(\frac{D^2}{d^2} \Delta - \Delta + p\right)$$

The ratio $m = \dfrac{W}{w}$ (a)

will have the following value:

$$m = \frac{D^2}{d^2} \frac{\Delta}{p} - \frac{\Delta}{p} + 1$$

and, taking

$$\frac{\Delta}{p} = a, \text{ also } 1 - \frac{\Delta}{p} = b, \text{ also } \frac{D}{d} = k,$$

we can write

$$m = ak^2 + b \qquad (1)$$

In the case of copper, we have $p = 3.852$, the density of the ice is assumed to be such that $\Delta = 0.3852$ (*i.e.*, less than that of solid ice), because the ice-covering which forms on wires does not have, as a rule, a compact structure. In such a case, therefore, we will have

$$a = \frac{\Delta}{p} = 0.1$$

and, consequently,

$$b = \left(1 - \frac{\Delta}{p}\right) = (1 - a) = (1 - 0.1) = 0.9$$

Introducing the values of a and b in (1), we have

$$m = (0.1k^2 + 0.9)$$

Case II.—Additional loading due to wind only

A wind-velocity v acts upon a unit-length of a wire of diameter d, with a force equal to

$$P_v = xv^2d$$

where x is a constant whose value is to be determined by experiment. If the wind is supposed to act horizontally against the wire, the resultant of the forces acting upon the wire is equal to

$$W = \sqrt{w^2 + V^2} = \sqrt{w^2 + (xv^2d)^2}$$

We can now re-write (a) as follows:

$$m = \frac{W}{w} = \sqrt{1 + \frac{V^2}{w^2}} = \sqrt{1 + \frac{x^2v^4d^2}{\frac{\pi^2d^4}{16}p^2}} = \sqrt{1 + \frac{16x^2v^4}{\pi^2d^2p^2}}$$

The constants, x, π, p can be grouped into a single constant factor; we can take

$$X = \sqrt{\frac{16x^2}{\pi^2p^2}} = \frac{4x}{\pi p} = \text{a constant.}$$

We can also take

$$y = \sqrt{\frac{v^4}{d^2}} = \frac{v^2}{d}$$

Substituting these values in the last expression for m, we have

$$m = \sqrt{1 + X^2y^2} \qquad (2)$$

The value of the constant, x, may be obtained from experiments made in Europe by Rebora, and in America by Marvin, Buck and others. Its mean value, when v is given in miles per hour and d in inches, is found to be

$$x = 0.0002$$

Inserting this value, and the value for p ($= 0.321 \times 12 = 3.852$), in the expression for X, we have

$$X = \frac{4x}{\pi p} = \frac{4 \times 0.0002}{3.1416 \times 3.852} = 0.0000661 = 661 \times 10^{-7}$$

Therefore

$$X^2 = (0.0000661)^2 = 0.437 \times 10^{-8}$$

Inserting this value in (2), we have

$$m = \sqrt{1 + (0.437 \times 10^{-8})y^2}$$

Case III.—Joint effect of both kinds of additional loading

In that case, the force, acting vertically on a unit-length of the wire is

$$= w\,(ak^2 + b);$$

and the force acting horizontally is

$$= xv^2D.$$

Hence, the resultant force will be

$$W = \sqrt{w^2(ak^2 + b)^2 + x^2v^4D^2}$$

Substituting this value in (a), we have

$$m = \frac{W}{w} = \sqrt{(ak^2 + b)^2 + \frac{x^2v^4D^2}{w^2}} \qquad (b)$$

From the relation $D/d = k$, we have $D^2 = k^2d^2$.

Substituting for D^2 and w^2 in (b), we have

$$m = \frac{W}{w} = \sqrt{(ak^2 + b)^2 + X^2k^2y^2} \qquad (3)$$

Introducing the constants for copper and ice, this becomes

$$m = \sqrt{(0.1k^2 + 0.9)^2 + (0.437 \times 10^{-8})k^2y^2}$$

Suppose, now, that the wind, instead of acting horizontally, acts in a direction which makes an angle, γ, with the vertical plane of the span. In that case the resultant force will be

$$W = \sqrt{w^2(ak^2 + b)^2 + x^2v^4D^2 - 2w(ak^2+b)x^2v^4D^2 \cos\gamma}$$

This modified value for W, when substituted in (a), leads to an expression for m which differs from (3) and which may be designated by m_1. Making the same substitutions and transformations as before, we will have

$$m_1 = \sqrt{(ak^2 + b)^2 + X^2k^2y^2 - 2X\,(ak^2+b)k\,y \cos\gamma}$$

The first two terms under the radical sign are the same as in (3); hence we can write

$$m_1 = \sqrt{m^2 - 2(ak^2 + b)Xky \cos\gamma} \qquad (4)$$

It is important to note that the *sign* of the second term under the radical will depend upon the value of the angle γ.

When the wind has an *upward* direction, *i.e.*, when $\gamma < 90°$, then $\cos\gamma$ has the positive sign and the second term retains the negative sign, which means that there will be a *reduction*, in the loading ratio, and that we will have $m_1 < m$. When the wind has a *downward* direction, *i.e.*, when $\gamma > 90°$, then $\cos \gamma$ takes the *negative* sign; hence, the sign of the second term under the radical will be *changed to positive* (+); which means that there will be an *increase* in the loading ratio and that we will have $m_1 > m$. (Incidentally, it may be noted that when $\gamma = 90°$, *i.e.*, when the wind has a perfectly horizontal direction, then $\cos \gamma = 0$, and the second term vanishes, which makes $m_1 = m$, since (4) is then identically the same as (3)).

Equation (4) enables the value of m_1 to be calculated after the value of m has been obtained on the assumption that the wind is acting horizontally.

It should be noted that the value of the sag f_m for the wire when loaded has no importance except in the case when it is greater than the value of the sag f_1, for the unloaded wire at the maximum temperature.

In the event that we found $f_m > f_1$, and also $f_m \cos \beta > f_1$, (in the case when the action of the wind is taken into account) then the height of the supports would have to be calculated not by reference to the value of the summer sag f_1, but by reference to the value of $f_m \cos \beta$, where $\beta > 0°$ and therefore $\cos \beta \geq 1$.

In practice, however, for the extra loads ordinarily assumed, such a case would rarely happen.

II. Calculation of Sags and Stresses

Case I.—Spans with supports at the same level

The general equation of the catenary of a suspended wire, expressed in cartesian coordinates and taking the lowest point of the wire as the axis of abscissae, is:

$$y = \frac{T}{P} \left\{ \cos h \frac{Px}{T} - 1 \right\} \qquad (5)$$

The length of the portion of wire extending between the origin and any point whose abscissa is x, is expressed by the relation

$$\frac{\lambda}{2} = \frac{T}{P} \sin h \frac{Px}{T} \qquad (6)$$

Developing $\cos h$ and $\sin h$ into series, we have

$$\cosh a = 1 + \frac{a^2}{2!} + \frac{a^4}{4!} + \frac{a^6}{6!} + \ldots\ldots$$

$$\sinh a = a + \frac{a^3}{3!} + \frac{a^5}{5!} + \frac{a^7}{7!} + \ldots\ldots$$

From these, we have

$$\cosh a - 1 = \frac{a^2}{2}\left(1 + \frac{a^2}{12} + \frac{a^4}{360} + \ldots\right)$$

$$\sinh a = a\left(1 + \frac{a^2}{6} + \frac{a^4}{120} + \ldots\right)$$

When a $>$ 0.48, the error made in taking

$$\cosh a - 1 = \frac{a^2}{2} \qquad (7)$$

is less than 2%; and the error made in taking

$$\sinh a = a + \frac{a^2}{6} \qquad (8)$$

is less than 1/20 of 1%.

In our case we have:

$$a = \frac{Px}{T}$$

Hence, if

$$Px < 0.48\ T \text{ (see Note*)} \qquad (9)$$

equation (5) can be written, with an error of less than 2%, under the form

$$y = \frac{T}{P}\left\{\frac{P^2x^2}{2\,T^2}\right\} = \frac{Px^2}{2\,T}$$

If $x = 1/2$, then $y = f$;

hence

$$f = \frac{Pl^2}{8T} \qquad (10)$$

which is the equation of a parabola.

*Note.—Taking $P = mp$ and $x = 1$, the inequality (9) may be written as follows:

$$mpl < 0.48\ T, \text{ or else } pL < 0.48\ T$$

where $L = ml$; but, in the case of copper, $p = 0.321 \times 12 = 3.852$, hence

$$\frac{L}{T} < 0.125$$

which condition is realized in the greater number of cases.

In like manner, if, in equation (6), we substitute for $\sinh Px/T$ its value obtained from (8), we have

$$\frac{\lambda}{2} = x + \frac{P^2x^3}{6T^2}$$

and if $x = 1/2$
then

$$\frac{\lambda}{2} = \frac{1}{2} + \frac{P^2l^2}{48T^2} \qquad (11)$$

From (10) we have

$$f^2 = \frac{P^2l^4}{64T^2}$$

or else

$$\frac{8f^2}{31} = \frac{P^2l^3}{24T^2};$$

and, substituting in (11) and multiplying by 2, we have

$$\lambda = 1 + \frac{8f^2}{3l} \qquad (12)$$

The symbol P, in (10), represents the specific weight of unit-length of the wire when loaded; *i.e.*, adopting the symbol previously used

$$P = mp, \text{ where } m \geq 1.$$

We can write

$$f = \frac{mpl^2}{8T} \quad \text{(See Note*).} \qquad (13)$$

This equation gives the sag, f, in feet, of a span of length l, in feet, when the stress at the lowest point of the wire is $= T$, in pounds per square inch, and the wire, unloaded or loaded, weighs mp pounds per foot of length and per square inch of sectional area; and this relation is independent of any considerations whatever regarding the temperature conditions.

*Note.—Equation (13) involves the assumption that the stress is constant at all points in the wire, which is not true. It can be demonstrated that the horizontal component T of the stress remains constant at all points of the wire, being equal to the stress at the lowest point. The vertical component is:

$$\frac{1}{2}\,mp = \frac{L}{2}\,p$$

The resultant stress at the point of support is

$$T_s = \sqrt{T^2 + \frac{L^2}{4}p^2}$$

which, when $L/T = 0.125$ and in the case of copper, becomes

$$T_s = 1.028T$$

Therefore, for $L/T < 0.125$, we have $T_s < 1.028T$.

The mechanical stress, T, varies, however, either with the temperature or with the additional loading; consequently, by Equation (13), the value of the sag, f, will also vary.

In passing from the temperature θ_0 to the temperature $\theta_1 > \theta_0$, the wire stretches, and the stress, T, decreases; *i.e.*, we will have $T_0 > T_1$, which produces an elastic shortening in the wire.

The stretching of the wire is, therefore, of two kinds, the first kind being due to thermal expansion, and the second kind being due to the elasticity of the wire itself; and the two oppose each other.

Therefore, neglecting infinitesimals of the second order, we can write

$$\lambda_1 - \lambda_0 = (\theta_1 - \theta_0)\alpha\lambda_0 + \frac{T_1 - T_0}{E}\lambda_0;$$

and then, substituting the length of the *span* l for the initial length of the *wire* (see Note*), we will have

$$\lambda_1 - \lambda_0 = (\theta_1 - \theta_0)\alpha\, l + \frac{T_1 - T_0}{E}\, l \qquad (14)$$

On applying Equation (11) successively to the two conditions of the wire designated by the subscripts *o* and 1, and adopting the symbols already used, we have

$$\lambda_1 = l + \frac{m^2p^2l^3}{24T_1^2}, \text{ and } \lambda_0 = l + \frac{m^2p^2l^3}{24T_0^2}$$

and, supposing the extra load and, hence, the value of m, to vary, we will have

$$\lambda_1 = l + \frac{m_1^2p^2l^3}{24T_1^2} \text{ and } \lambda_0 = l + \frac{m_0^2p^2l^3}{24T_0^2}$$

Substituting these values in Equation (14), and dividing by l, we have

$$\left\{\frac{m_1^2p^2}{T_1^2} - \frac{m_0^2p^2}{T_0^2}\right\}\frac{l^2}{24} = (\theta_1 - \theta_0)\alpha + \frac{T_1 - T_0}{E}$$

which may also be written thus

$$\theta_1 - \theta_0 = \left\{\frac{m_1^2p^2l^2}{24\alpha T_1^2} - \frac{T_1}{\alpha E}\right\} - \left\{\frac{m_0p^2l^2}{24\alpha T_0^2} - \frac{T_0}{\alpha E}\right\};$$

*Note.—The percentage of the error made by taking the length of the span instead of the actual length of the wire is given by the relation

$$\epsilon = \frac{8f_0^2}{3\,l^2}, \text{ (see Equation (12))}$$

when $f_0 = 0.06l$, $\epsilon = 0.01$; but $f_0 = 0.06l$ corresponds (see Equation (10)) to $L/T = 0.125$. Therefore, for $L/T \leq 0.125$ we have $\epsilon \leq 0.01$.

and, taking, in general, $ml = L$, and, in particular, $m_1 l = L_1$, and $m_0 l = L_0$, we have

$$\theta_1 - \theta_0 = \left\{ \frac{p^2 L_1^2}{24\alpha T_1^2} - \frac{T_1}{\alpha E} \right\} - \left\{ \frac{p^2 L_0^2}{24\alpha T_0^2} - \frac{T_0}{\alpha E} \right\} \quad (15)$$

This equation embodies the idea—due to Blondel—of the "hypothetical span," namely, that a span of length l, weighted by an additional load which multiples by m the unit-weight, p, of the naked wire, may be replaced by a span of length $L = ml$, so far as the calculation of line-stability is concerned, since the value of T remains the same for both.

In Equation (13), multiplying both sides by m, and substituting $L = ml$, we have

$$mf = \frac{m^2pl^2}{8T} = \frac{pL^2}{8T}$$

and, if we take $F = mf$, we have, in general

$$F = \frac{pL^2}{8T} \quad (16)$$

This means that replacing the actual span, l, by the hypothetical span, L, amounts to the same thing as multiplying all the sags by the value of m.

It is well to bear in mind that in all the equations thus far written, and in those which follow, in which the coefficient m appears, the latter may take any value whatever greater than 1, but that it cannot be less than 1. When we have $m = 1$, then we have $L = l$, and $F = f$.

Equation (15) can be written in the following form

$$\theta_1 - \left\{ \frac{p^2L_1^2}{24\alpha T_1^2} - \frac{T_1}{\alpha E} \right\} = \theta_0 - \left\{ \frac{p^2L_0^2}{24\alpha T_0^2} - \frac{T_0}{\alpha E} \right\},$$

or, since that equation applies for infinite values of θ, we will have, in general

$$\theta - \left\{ \frac{p^2L^2}{24\alpha T^2} - \frac{T}{\alpha E} \right\} = C$$

where c = a constant. If we take

$$t = \theta - C \quad (17)$$

we can write

$$t = \frac{p^2L^2}{24\alpha T^2} - \frac{T}{\alpha E} \quad (18)$$

in which t is not a temperature, but differs from a temperature by a constant, c (equation (17)). The difference, however, between two values of t is equal to the difference between the corresponding values of θ; in fact, from equation (17) we can write

$$\Delta t = t_1 - t_0 = (\theta_1 - C) - (\theta_0 - C) = \theta_1 - \theta_0 = \Delta\theta$$

The practical application of the principles demonstrated in the preceding section is to be found in Part I, Construction of Charts.

Case II.—Spans with supports at different levels

Suppose a span whose length, projected on a horizontal plane, is 1 ft., and whose supports have a difference of level of h feet. Such a span is equivalent, for the purpose of calculating the variations in sag, to the span with supports at the same level that is obtained by prolonging the assumed catenary of the wire until it meets the horizontal line passing through the higher support.

The length of that hypothetical span, for a given stress, T, in the wire of the span l, is given by the relation

$$c = l + \frac{2}{mp}\frac{h}{l}T \quad (19)$$

In fact, if lx = the distance between the lowest point of the curve described by the wire and the vertical line passing through the lower support, we have

$$c = 2(l - l_x) \quad (20)$$

and also

$$f_x = \frac{mpl_x^2}{2T}$$

and also, in the case of the span, c

$$h + f_x = \frac{mpc^2}{8T} = \frac{mp(l - l_x)^2}{2T}.$$

From this, substituting for f_x, and solving for l_x, we have

$$l_x = \frac{1}{2lmp}(mpl^2 - 2Th);$$

and, substituting in (20), we obtain (19), again.

Referring to Equation (19), it is seen that when

$$l = \frac{2}{mp}\frac{h}{l}T \quad (21)$$

the lowest point of the curve described by the wire coincides with the lower support; whereas, when

$$l > \frac{2}{mp}\frac{h}{l}T \tag{22}$$

the lowest point of the wire falls inside the span l; and, when

$$l < \frac{2}{mp}\frac{h}{l}T \tag{23}$$

the lowest point of the curve described by the wire falls beyond the span, l.

The hypothetical span, c, which takes the place of the actual span l, in the calculation of the sag, is different for different values of T and of m; in reality, it increases when T increases, and when m decreases. In consequence, when Equation (16) is used for calculating the sag-values (which in that case are equal to the vertical distances between the higher support and the lowest point of the curve described by the wire), if the inequality (23) obtains for the initial value of T, then, as T decreases, the sag will continue to decrease until the equality (21) obtains, after which it will go on increasing. The contrary will be the case if the value of T increases.

In general, therefore, there will be, for constant additional loading, two equal values of f corresponding to values of T which are very different from each other. For this result it is only necessary that the conditions of the following equation (see Equation (13)) should be fulfilled:

$$\frac{\left\{l + \frac{2}{mp}\frac{h}{l}T_1\right\}^2}{T_1} = \frac{\left\{l + \frac{2}{mp}\frac{h}{l}T_0\right\}^2}{T_0};$$

from which we have

$$T_1 T_0 = \frac{l^4}{4h^2}m^2p^2$$

However, one of the sags corresponding to all the stresses given by this last equation is hypothetical, because the lowest point of the curve described by the wire falls *beyond* the span l.

When $T_1 = T_0$, we have

$$T^2 = \frac{l^4}{4h^2}m^2p^2$$

whence

$$T = \frac{l^2 mp}{2h}$$

which is only another form of the inequality (21) already found.

When the values of T and of m are fixed, the value of c can be obtained by Equation (19), and, taking $L = mc$, Equation (16) is then available for calculating the sag.

Passing to the variations of temperature and of additional loading, and to the corresponding variations of stress and sag, we substitute for L, in (15), the value of mc obtained by Equation (19). Inserting values and taking, as before, $t' = \theta - c$, we arrive at the equation

$$t' = \left(\frac{p^2L^2}{24\alpha T} - \frac{T}{\alpha E}\right) + \frac{1}{6}\frac{ph}{\alpha}\frac{m}{T}$$

where $L = ml$, and $m \geq 1$, and, therefore, (see Equation (18)),

$$t' = t + \frac{1}{6}\frac{ph}{\alpha}\frac{m}{T} \tag{24}$$

This equation, together with Equation (19), shows that, unless it be done in a special way, it is not possible to use for spans with supports at different levels, the charts made for spans with supports at the same level, because the hypothetical span is a function of the stress T existing in the wire (Equation (19)), and, therefore, it varies at all times as T varies, and t' differs from t by a quantity which is a function of the value of T, and is, therefore, always variable.

We may note, at the outset, that, when we wish to make use of the charts, the scale of F-values can be used just as it is, because Equation (16) was used in making the calculations for these tables. On the other hand, the scale of t-values will evidently require to be modified in some way (see Equation (24)).

Now, as we cannot modify the scale of t-values used in the charts, it will be necessary to *interpret* them in a different way. To this end, we will call $\Delta t'$ the portion of the scale which represents, on the charts, the range of temperature having the value $\Delta\theta = \Delta t$, when, instead of a span with supports at the same level, we have to deal with a span of the same horizontal length but with supports at different levels.

In like manner, we will call t'_0 and t'_1 the extreme points of the portion of scale for values of $\Delta t'$ which correspond, respectively, to t_0 and t_1, the extreme limits of the range of temperature.

In general, therefore, we will call t' that point on the scale of the chart which corresponds, for a span with supports at different levels, to the point t of the same span when it is supposed to be with supports at the same level (see Equation (24)).

Aided by these considerations, we will now be able to use the charts even in the case of spans with supports at different levels.

We shall see that the charts may be used just as they are for the calculation of stresses in the wire. If the curve of sags is also desired (for spans with supports at different levels) it will be necessary to construct it point by point, since the hypothetical span, c, varies continuously.

The extra loading is, at first, neglected. Suppose a span of horizontal length, l, with a difference of levels, h, and suppose the maximum stress to be limited to a given value, T_m, which must not be exceeded, at the lowest temperature, θ_0. From these data, we calculate the hypothetical span, c, by means of Equation (19). On the charts, the straight line corresponding to T_m and the curve for c determine, by their point of intersection, a sag, $F = f$, which is the actual sag, measured from the highest point to the lowest point of the curve described by the wire; and the abscissa of that point fixes a value of t' which we will call t'_0, and which will be the starting point of a section of the scale of values of $\Delta t'$, for the span with supports at different levels.

That point of intersection will be also the starting point of the curve of sags.

To that value of t'_0, corresponds a value t_0, which is the initial point of the actual range of temperature $\Delta\theta = \Delta t$, for the span of length $L = l$, with supports supposed to be at the same level, and with the same stress, T_m. To find that point, we look for the point of intersection of the curve $L = l$, with the straight line for P_m; and the abscissa of that point of intersection determines t_0. The difference $(t'_0 - t_0)$, in the scale of the chart, is proportional to the term

$$\left(\frac{1}{6}\ \frac{ph}{\alpha}\ \frac{m}{T_m}\right) \text{ (see Equation (24)).}$$

The maximum range of temperature, $\Delta\theta = \Delta t$, assumed for the locality, is to be measured off, in degrees Fahrenheit, starting from the point t_0 and ending at the point t_1, on the scale of abscissae, on the chart.

The ordinate corresponding to t_1 will intersect the curve of $L = l$ at a point through which passes a straight line of stress-values which may be designated T_1. To that stress corresponds a span c_1, which can be calculated by means of Equation (19). The point of intersection of the straight line of T_1 with the curve of c_1 is the last point of the curve sought. The ordinate of that intersection determines, on the scale of t-values (*i.e.*, on the scale of abscissae) a value of t'_1 which is the last (highest) point of the rise along the scale $\Delta t'$.

The difference $(t'_1 - t_1)$ on the scale of the charts is proportional to the term

$$\left(\frac{1}{6}\ \frac{ph}{\alpha}\ \frac{m}{T_1}\right) \text{ (see Equation (24)).}$$

It is sufficient to take a few intermediate points, making the value of $\Delta\theta$ vary, to obtain the different values of T, and, consequently, the different points of the curve desired. In the curve thus constructed, the whole of the scale of $\Delta t'_1$, in accordance with what has already been stated, will be divided into as many degrees as are comprised in the range of temperature $\Delta\theta = \Delta t$, and the scale will vary from point to point in accordance with Equation (24). However, it is evident that if the sag-values do not really need to be known (see remark on page 7, Part I. Construction of Charts), then the practical use of the charts becomes quite simple in that case also. In fact, in such a case, we deal with the curve of $L = l$, as if it related to a span with supports at the same level; to each value of Δt corresponds a value of T, and the portion of the curve $L = l$, comprised between t_0 and t_1 gives the different values of the stress.

We proceed in similar manner when we wish to take into account the extra loading.

In that case, there will be, in addition to the preceding data, a coefficient of extra loading having the value m. We determine, by the aid of Equation (19), the hypothetical span c_m, corresponding to the maximum stress T_m allowed in the wire.

On the charts, the ordinate of the point of intersection of the curve of $L = mc_m$ with the straight line for T_m gives, for the wire when loaded, the true sag multiplied by m; and the abscissa of that point gives a value of t' which we will call t'_m.

We must now find the stress T_0 to which the wire is subjected when, while keeping the temperature constant, the extra load is removed. Equation (24), when applied to the condition of the wire which corresponds to an additional load corresponding to m, gives

$$t'_m = \left\{ \frac{p^2m^2l^2}{24\alpha T_m^2} - \frac{T_m}{\alpha E} \right\} + \frac{1}{6}\,\frac{ph}{\alpha}\,\frac{m}{T_m}$$

which may be also written as follows:

$$t'_m = \left\{ \frac{p^2L^2}{24\alpha T_m^2} - \frac{T_m}{\alpha E} \right\} + \frac{1}{6}\,\frac{ph}{\alpha}\,\frac{m}{T_m} = t_0 + \frac{1}{6}\,\frac{ph}{\alpha}\,\frac{m}{T_m} \qquad (25)$$

This means that, on the chart, a certain value (t_0) of t, given by Equation (25), corresponds to the value (t'_m) of t', already found. To find that value on the chart, all that is necessary, evidently, is to follow the straight line of T_m to its intersection with the curve of $L = ml$; the abscissa of that point corresponds to the value of t which satisfies the equation

$$t = \frac{p^2L^2}{24\alpha T_m^2} - \frac{T_m}{\alpha E}$$

that is to say (see Equation (25)), it represents the desired value, t_0. Under these circumstances, we can proceed as in the case of spans with supports at the same level. In fact, to unload the wire, while maintaining the temperature constant, is equivalent to changing from the span $L = ml$ to the span $L = l$ by following downward along the ordinate of t_0; the point of intersection of that ordinate with the curve of $L = l$ determines the value of the stress T_0 to be found. For that value we have

$$t_0 = \left(\frac{p^2l^2}{24\alpha T_0^2} - \frac{T_0}{\alpha E}\right) = \left(\frac{p^2L^2}{24\alpha T_m^2} - \frac{T_m}{\alpha E}\right)$$

The value of t'_0 corresponding to t_0 will be determined by the abscissa of the point of intersection of the straight line of T_0 with the curve of $L = c_0$, where c_0 has the value

$$c_0 = l + \frac{2}{p}\,\frac{h}{l}\,T_0$$

For that point, by Equation (24), we can write

$$t'_0 = \frac{p^2l^2}{24\alpha T_0^2} - \frac{T_0}{\alpha E} + \frac{1}{6}\,\frac{ph}{\alpha}\,\frac{1}{T_0} = t_0 + \frac{1}{6}\,\frac{ph}{\alpha}\,\frac{1}{T_0}$$

whence

$$t'_0 - t_0 = \frac{1}{6}\,\frac{ph}{\alpha}\,\frac{1}{T_0}$$

Therefore, neglecting the extra load, we find the same result as before.

BIBLIOGRAPHY

F. LOPPÉ: Formules et tables pour le calcul des conducteurs aériens. Paris, 1902.

A. BLONDEL: Calcul rapide des conducteurs aériens au moyen d'un abaque unique. Lumière Electrique, Feb. 19, 1910; Dec. 31, 1910; Jan. 7, 1911. Calcul rapide des lignes aériennes au point de vue mécanique. Lumière Electrique, Aug. 9, 1913; Aug. 16, 1913; Aug. 23, 1913.

R. S. Brown: A Graphical Method for Finding the Sag and Tension in Wire Spans. Electrical World, Aug. 12, 1911.

Milliken Brothers: A Handbook Appertaining to the Design and Construction of Overhead Electric Transmission Lines.

Wm. Le R. Robertson: Solution to Problems in Sags and Spans.

Percy H. Thomas: Sag-calculations for Suspended Wires.

H. Pender and H. F. Thomson: The Mechanical and Electric Characteristics of Transmission Lines. Proceedings of A. I. E. E., June, 1911.

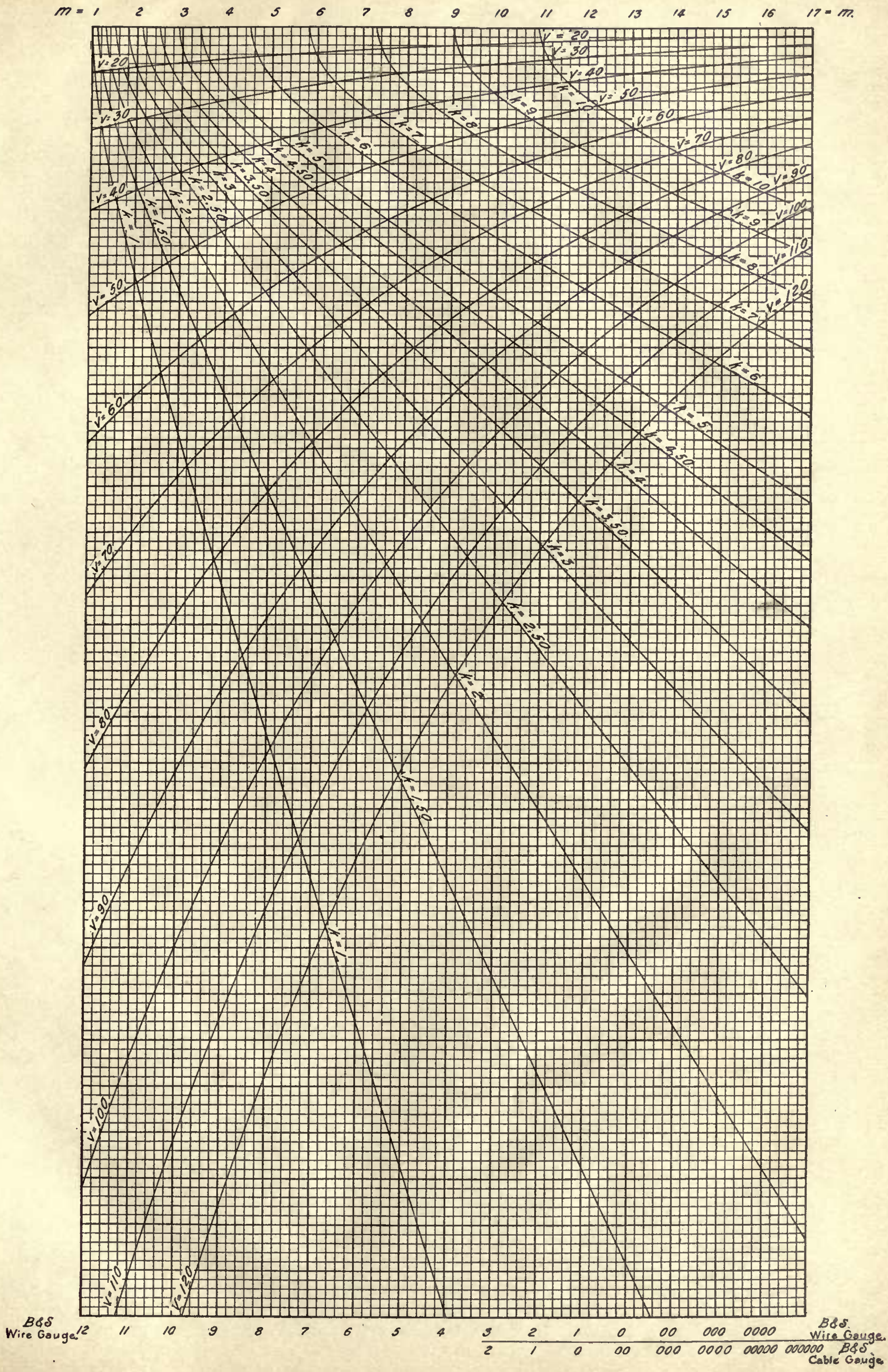
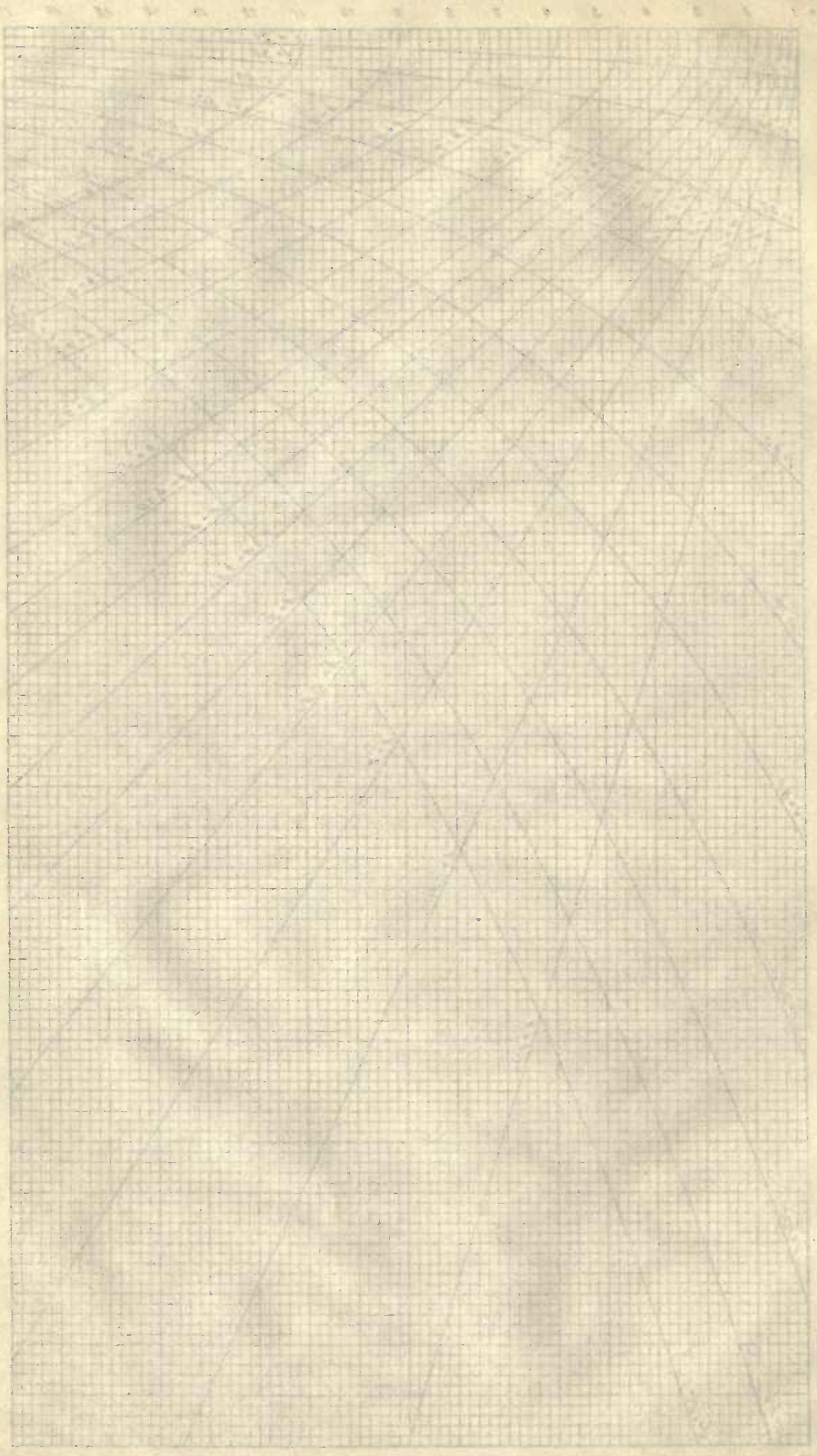
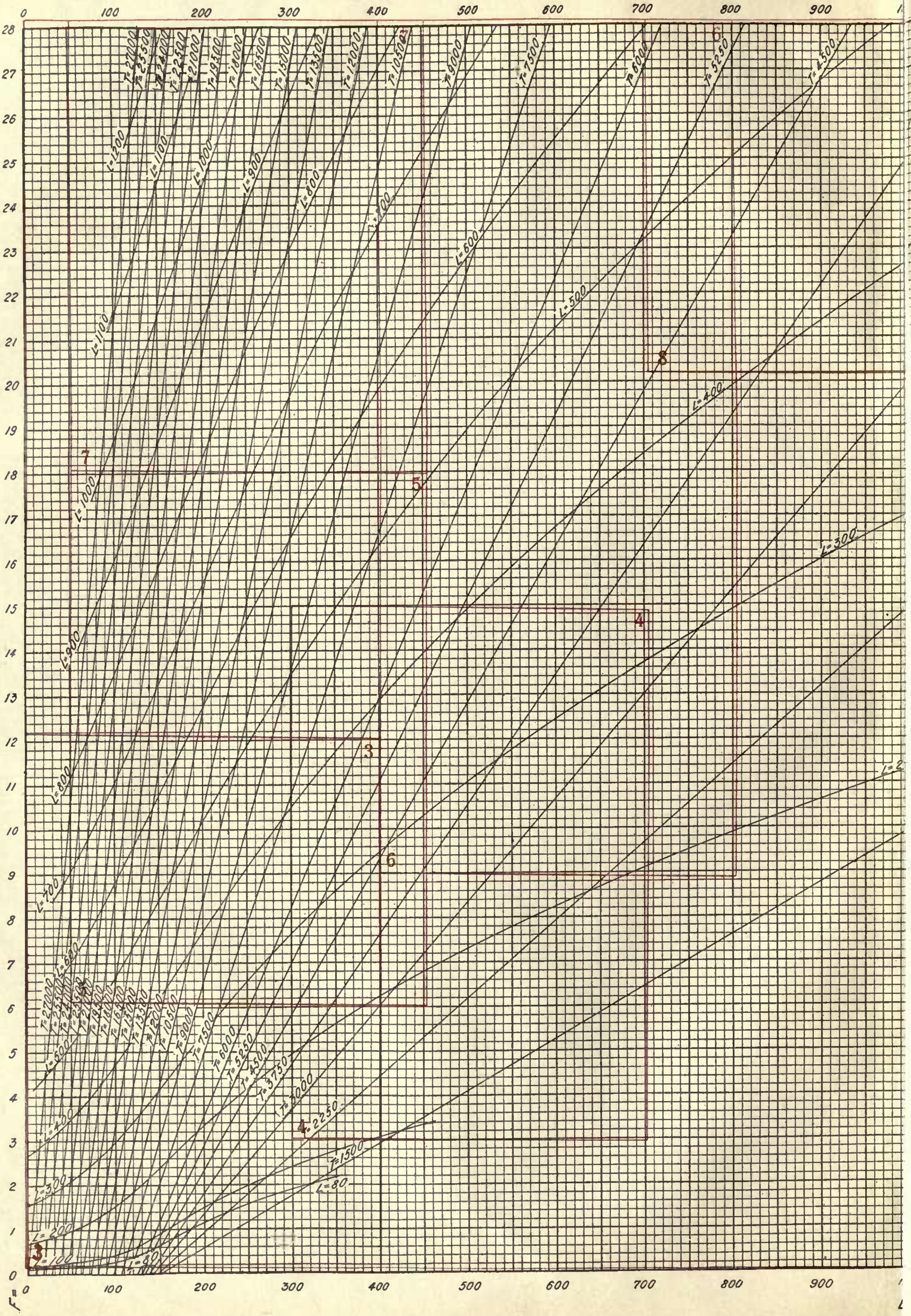
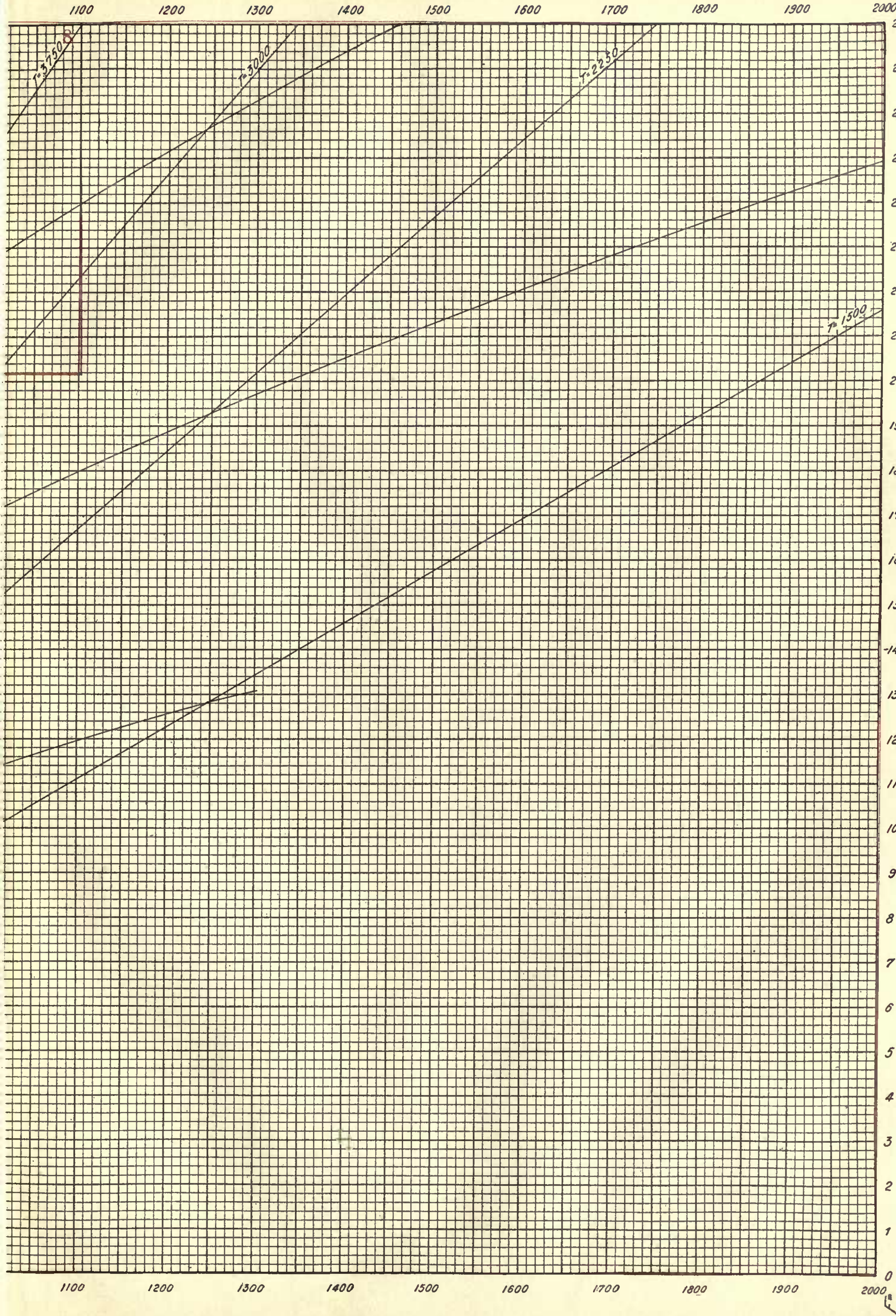

m = 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 = m.
v = 20
v = 30
v = 40
v = 50
v = 60
v = 70
v = 80
v = 90
v = 100
v = 110
v = 120
k = 1
k = 1.50
k = 2
k = 2.50
k = 3
k = 3.50
k = 4
k = 4.50
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
B&S Wire Gauge 12 11 10 9 8 7 6 5 4 3 2 1 0 00 000 0000 B&S Wire Gauge.
2 1 0 00 000 0000 00000 000000 B&S Cable Gauge

Chart No 1



Lower portion (A) of Index Chart (No. 2) For upper portion see Chart No. 2-B





1100 1200 1300 1400 1500 1600 1700 1800 1900 2000
T=3750
T=3000
T=2250
T=1500
28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0
F

Upper portion (B) of Index Chart (No. 2) For lower portion see Chart No. 2-A

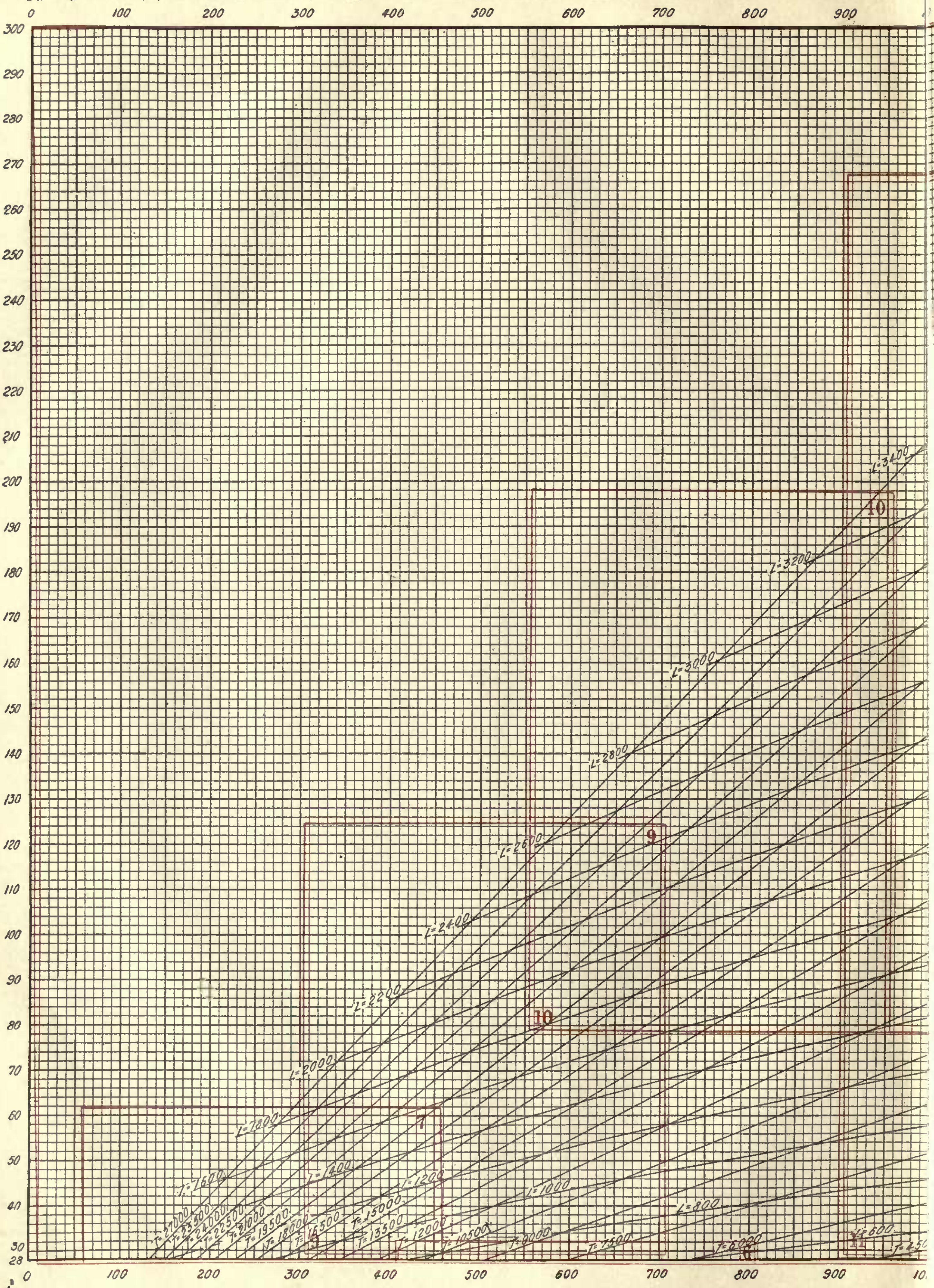
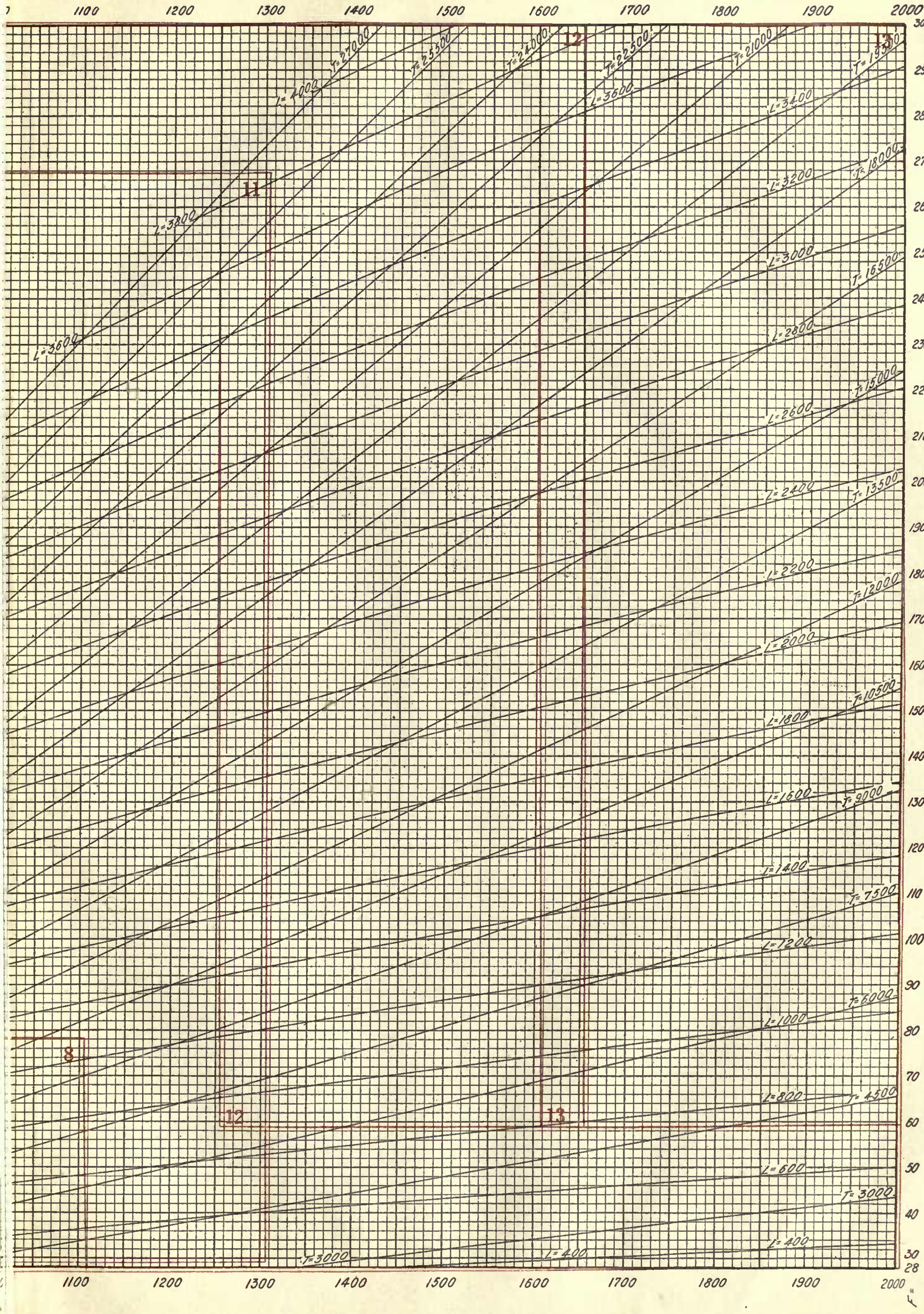
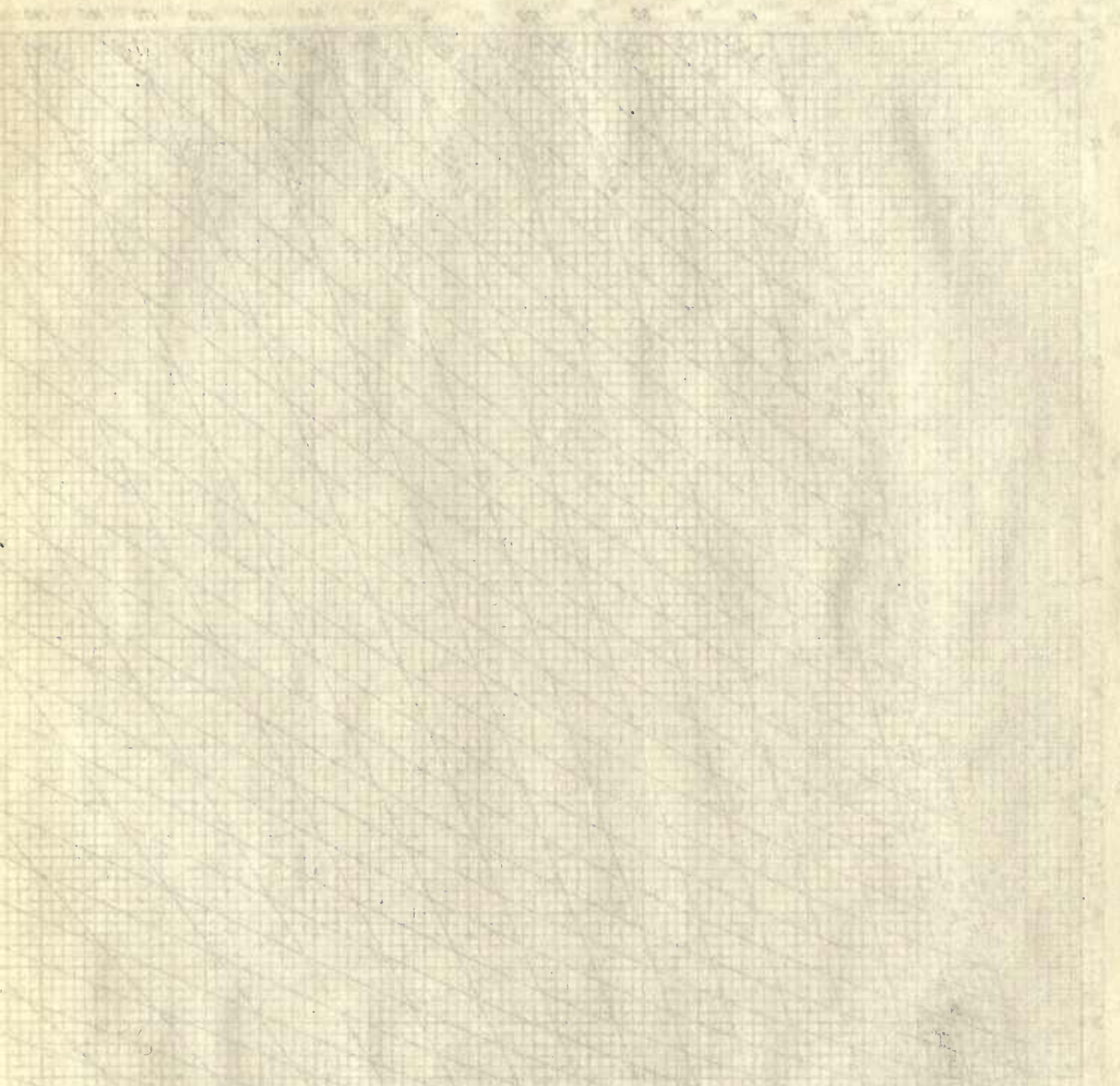
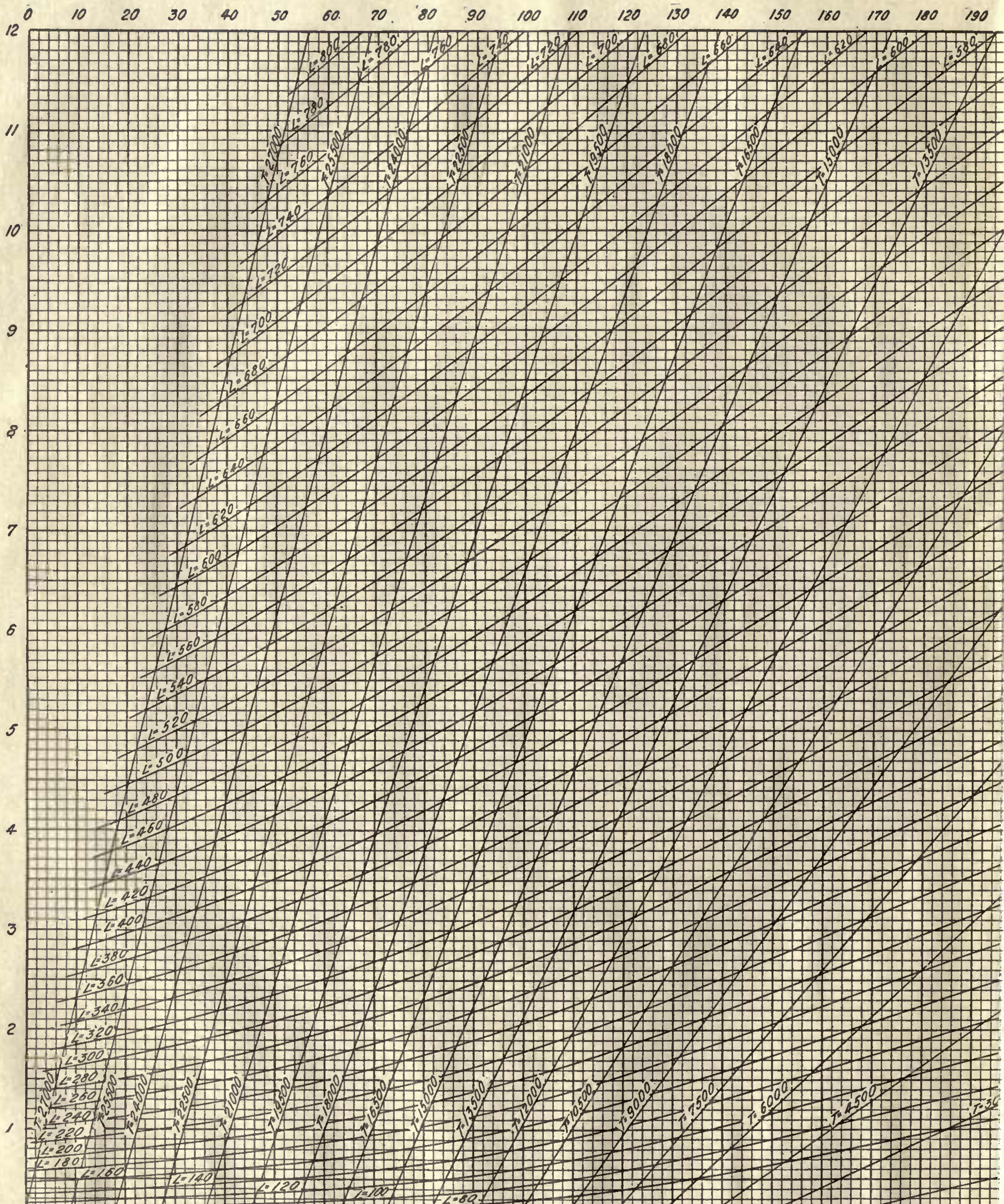


Chart No. 2-B



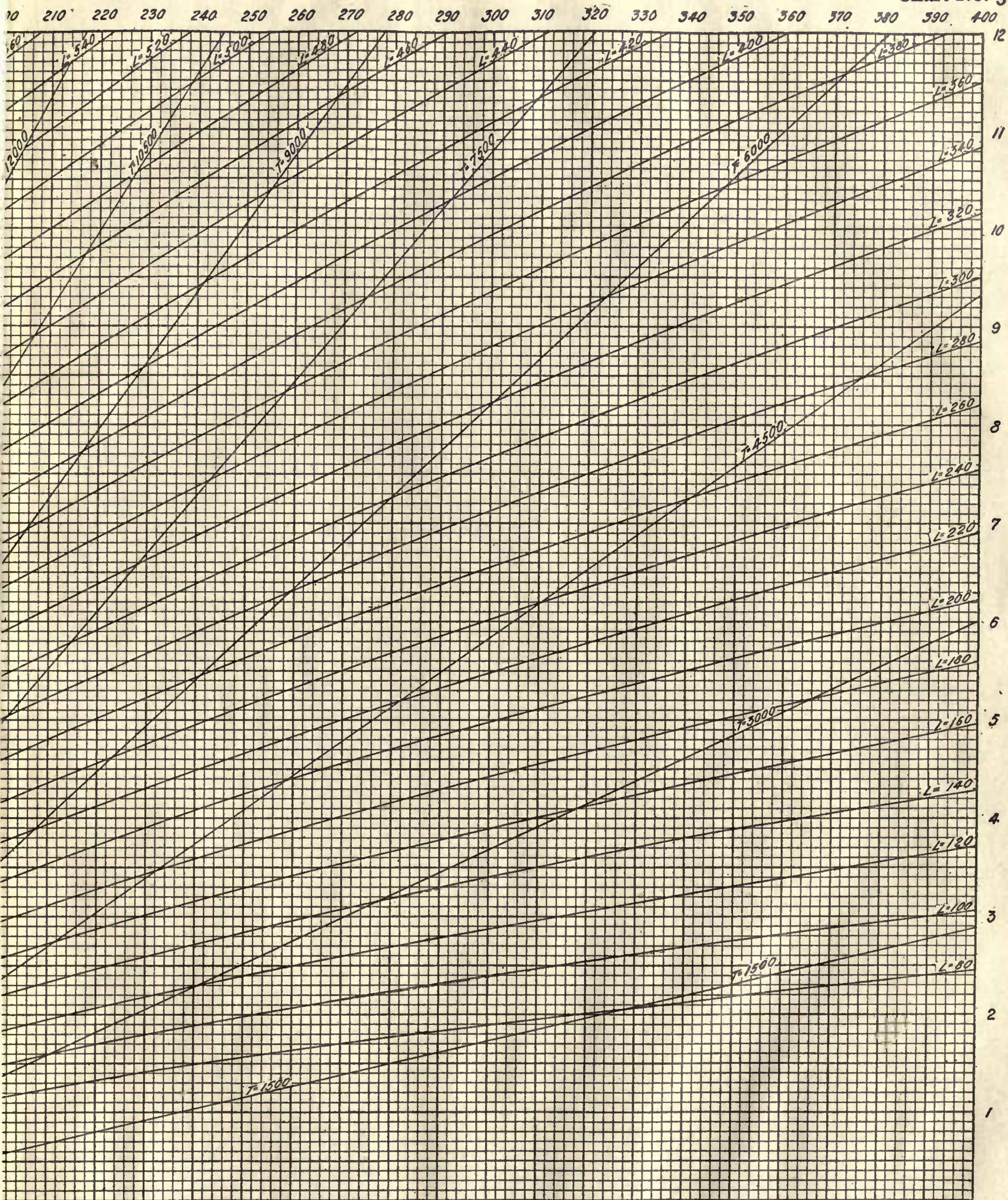
1100
1200
1300
1400
1500
1600
1700
1800
1900
2000
300
290
280
270
260
250
240
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
28
11
12
13
8
L=4000
T=27000
T=25500
T=24000
T=22500
T=21000
T=19500
L=3800
L=3600
L=3400
L=3200
T=18000
L=3000
T=16500
L=2800
T=15000
L=2600
T=13500
L=2200
T=12000
L=2000
T=10500
L=1800
L=1600
T=9000
L=1400
T=7500
L=1200
T=6000
L=1000
L=800
T=4500
L=600
T=3000
L=400




0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190
12 11 10 9 8 7 6 5 4 3 2 1 0
L=800
L=780
L=760
L=740
L=720
L=700
L=680
L=660
L=640
L=620
L=600
L=580
L=560
L=540
L=520
L=500
L=480
L=460
L=440
L=420
L=400
L=380
L=360
L=340
L=320
L=300
L=280
L=260
L=240
L=220
L=200
L=180
L=160
L=140
L=120
L=100
L=80
T=27000
T=25500
T=24000
T=22500
T=21000
T=19500
T=18000
T=16500
T=15000
T=13500
T=12000
T=10500
T=9000
T=7500
T=6000
T=4500

t

Chart No. 3



210 220 230 240 250 260 270 280 290 300 310 320 330 340 350 360 370 380 390 400
L=540
L=520
L=500
L=480
L=460
L=440
L=420
L=400
L=380
L=360
L=340
L=320
L=300
L=280
L=260
L=240
L=220
L=200
L=180
L=160
L=140
L=120
L=100
L=80
T=12000
T=10500
T=9000
T=7500
T=6000
T=4500
T=3000
T=1500
12 11 10 9 8 7 6 5 4 3 2 1 0

t.

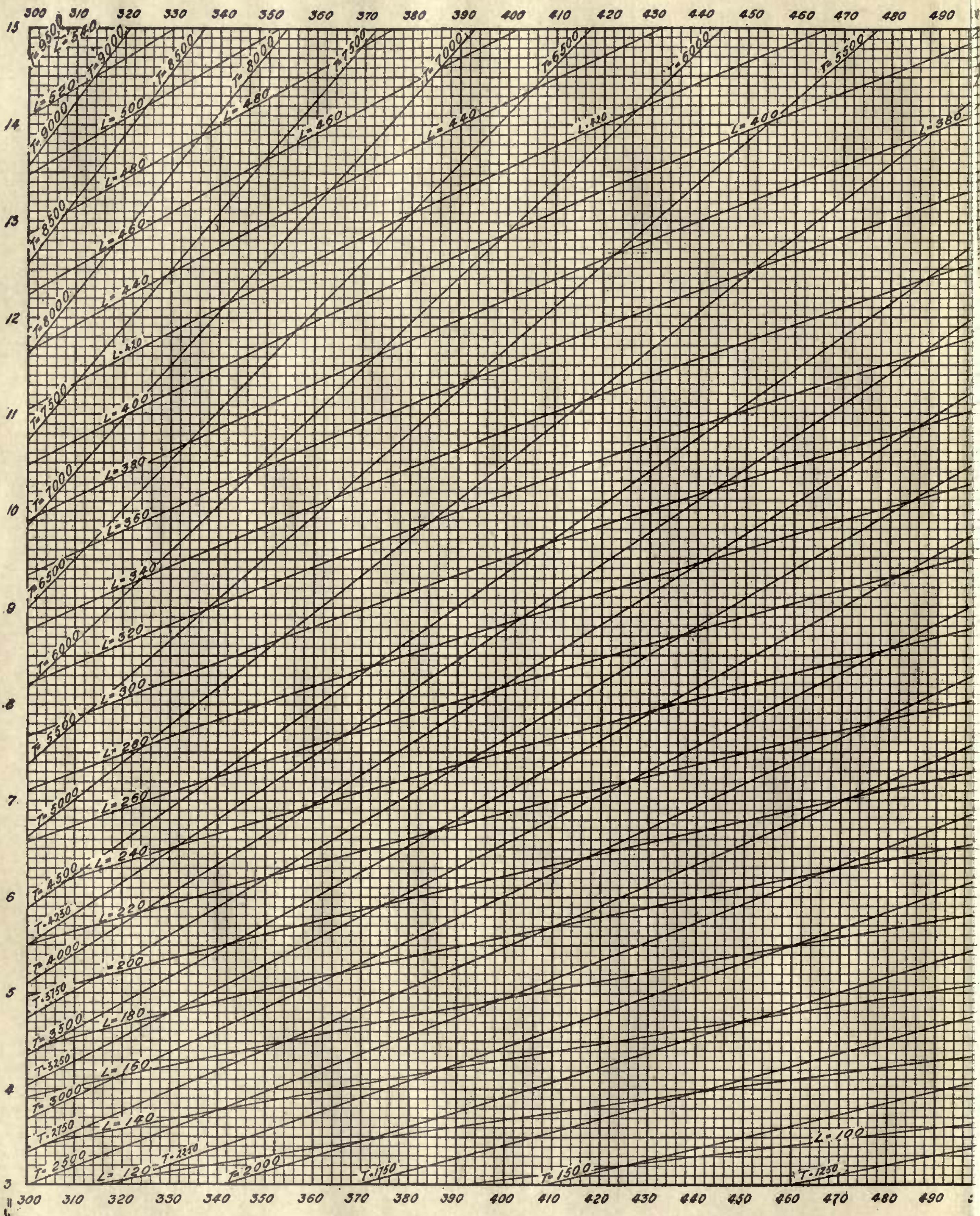
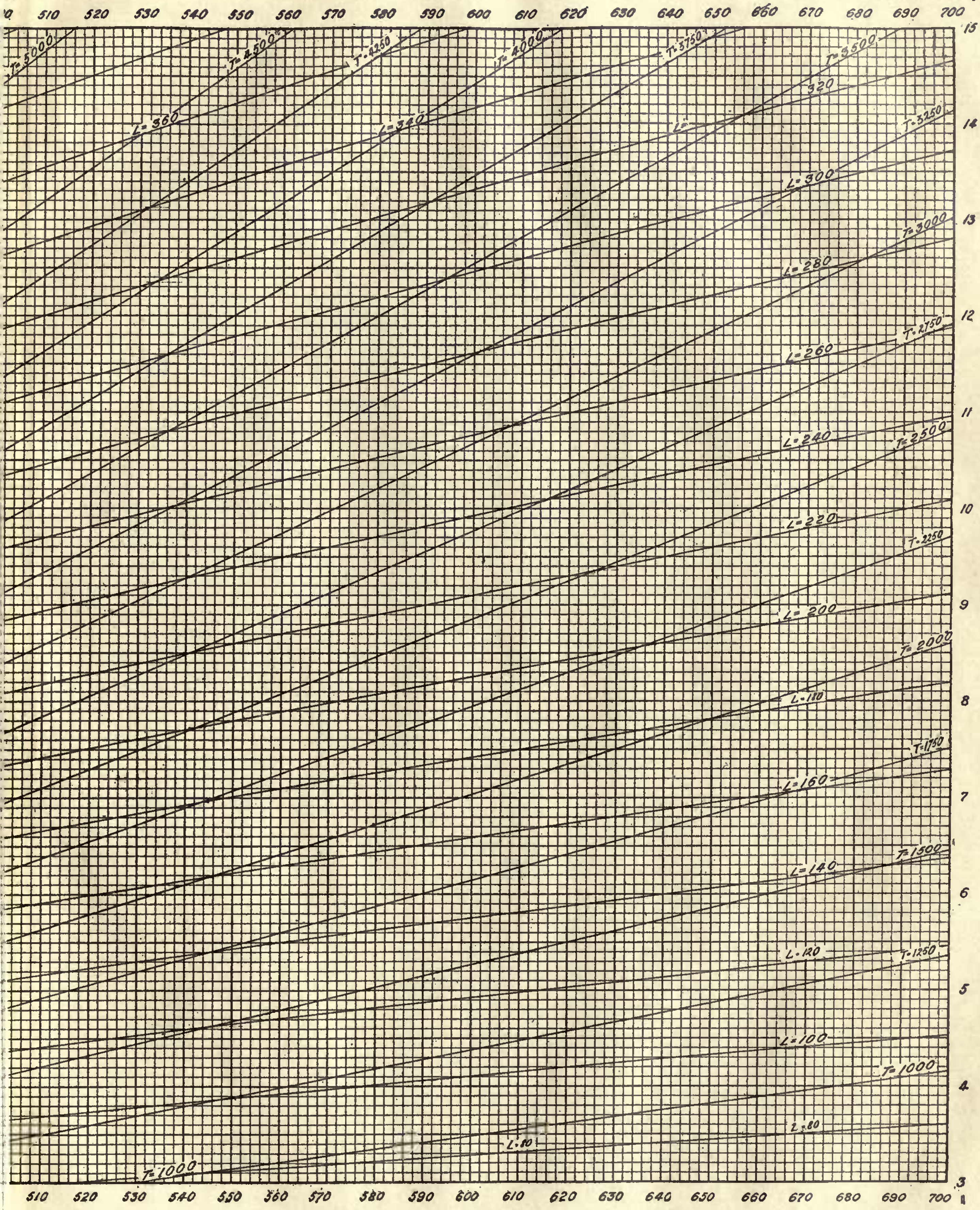

300 310 320 330 340 350 360 370 380 390 400 410 420 430 440 450 460 470 480 490
15 14 13 12 11 10 9 8 7 6 5 4 3
T=9000 T=8500 T=8000 T=7500 T=7000 T=6500 T=6000 T=5500 T=5000 T=4500 T=4250 T=4000 T=3750 T=3500 T=3250 T=3000 T=2750 T=2500 T=2250 T=2000 T=1750 T=1500 T=1250
L=520 L=500 L=480 L=460 L=440 L=420 L=400 L=380 L=360 L=340 L=320 L=300 L=280 L=260 L=240 L=220 L=200 L=180 L=160 L=140 L=120 L=100

Chart No. 4


T=5000
T=4500
T=4250
T=4000
T=3750
T=3500
T=3250
T=3000
T=2750
T=2500
T=2250
T=2000
T=1750
T=1500
T=1250
T=1000
L=360
L=340
320
L=300
L=280
L=260
L=240
L=220
L=200
L=180
L=160
L=140
L=120
L=100
L=80
510 520 530 540 550 560 570 580 590 600 610 620 630 640 650 660 670 680 690 700
15 14 13 12 11 10 9 8 7 6 5 4 3

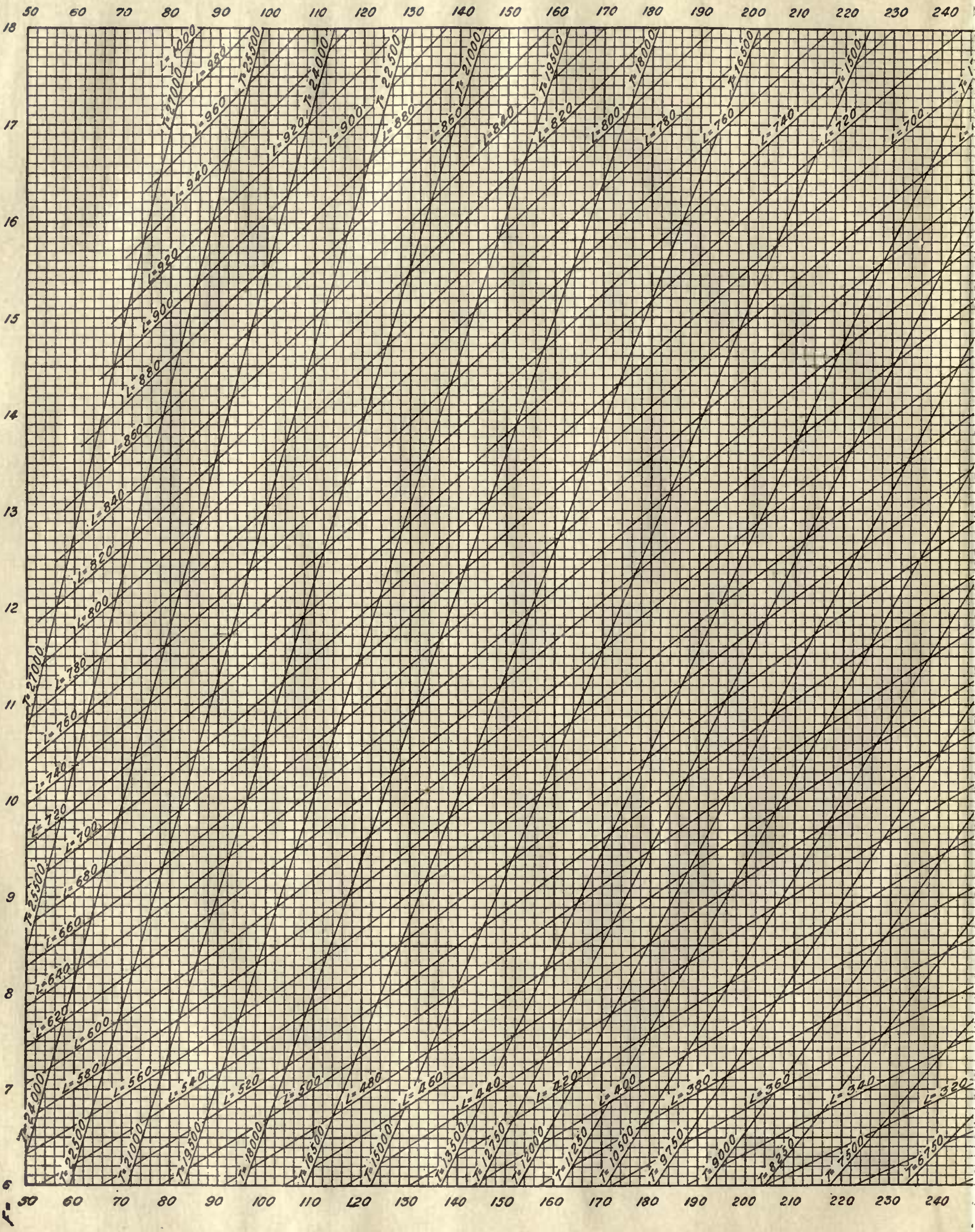
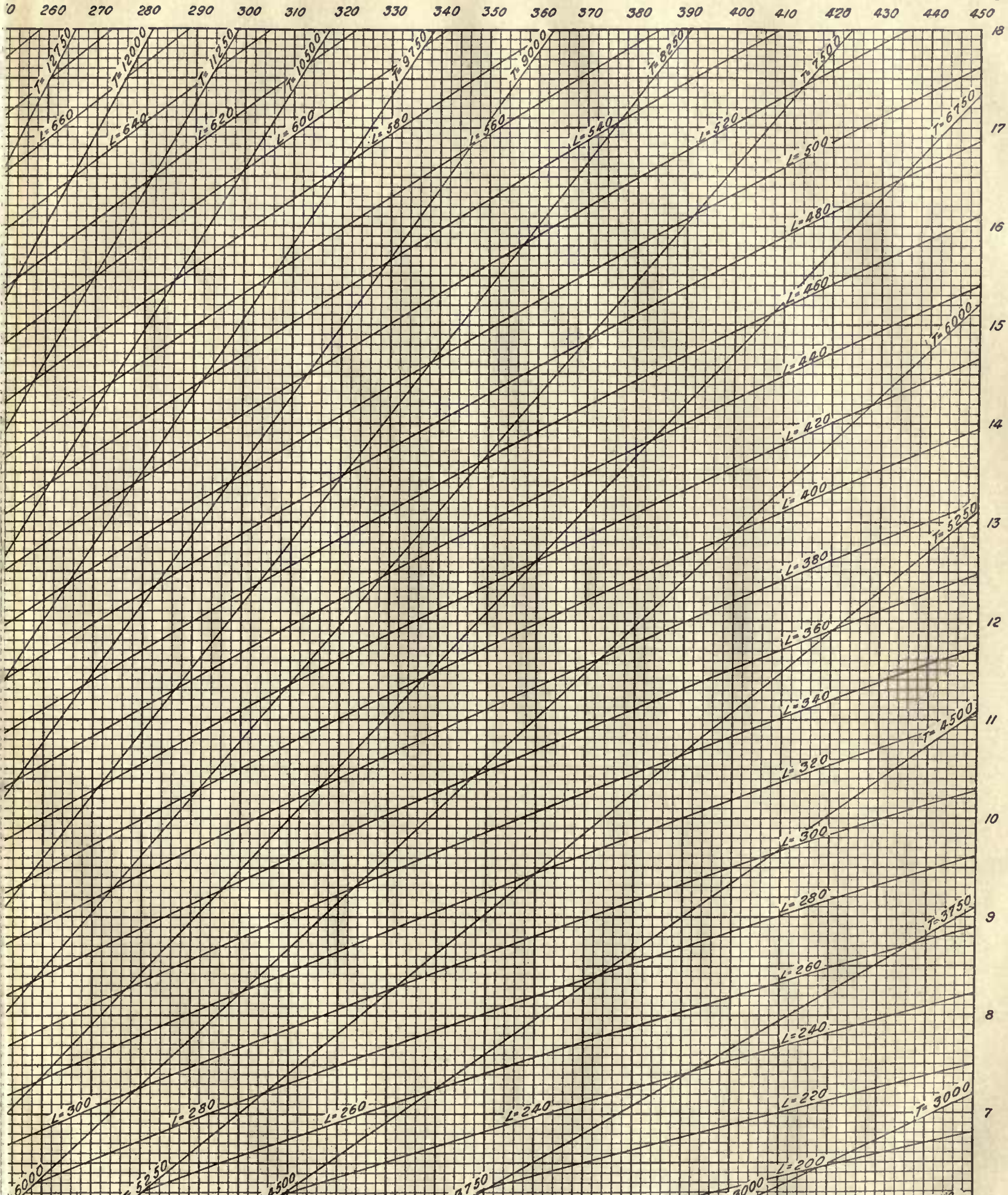
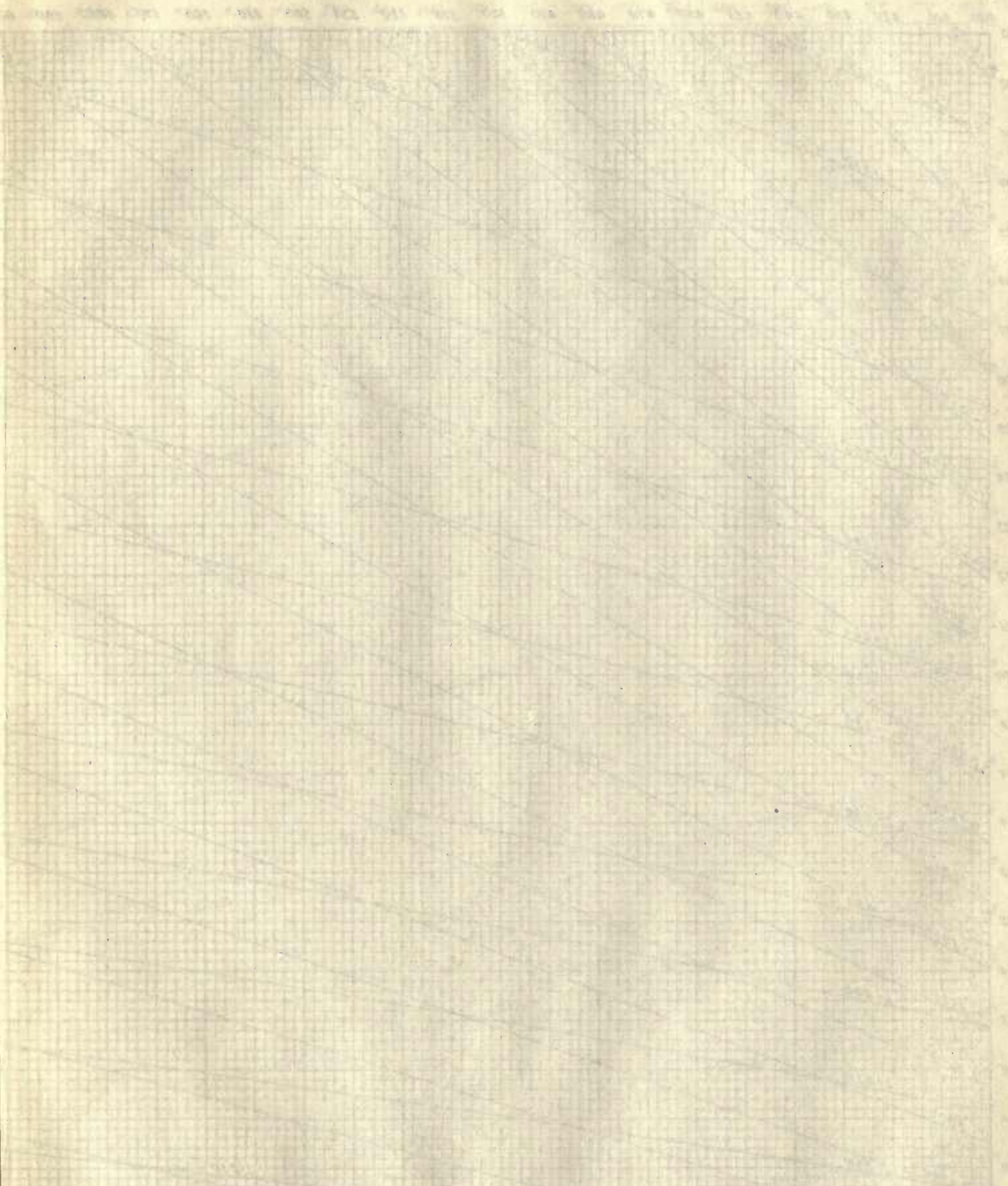

50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230 240
6 7 8 9 10 11 12 13 14 15 16 17 18
L=1000
L=980
L=960
L=940
L=920
L=900
L=880
L=860
L=840
L=820
L=800
L=780
L=760
L=740
L=720
L=700
L=680
L=660
L=640
L=620
L=600
L=580
L=560
L=540
L=520
L=500
L=480
L=460
L=440
L=420
L=400
L=380
L=360
L=340
L=320
T=27000
T=25500
T=24000
T=22500
T=21000
T=19500
T=18000
T=16500
T=15000
T=13500
T=12750
T=12000
T=11250
T=10500
T=9750
T=9000
T=8250
T=7500
T=6750

Chart No. 5


T=12750
T=12000
T=11250
T=10500
T=9750
T=9000
T=8250
T=7500
T=6750
T=6000
T=5250
T=4500
T=3750
T=3000
L=660
L=640
L=620
L=600
L=580
L=560
L=540
L=520
L=500
L=480
L=460
L=440
L=420
L=400
L=380
L=360
L=340
L=320
L=300
L=280
L=260
L=240
L=220
L=200
L=180
260 270 280 290 300 310 320 330 340 350 360 370 380 390 400 410 420 430 440 450
6 7 8 9 10 11 12 13 14 15 16 17 18
F=



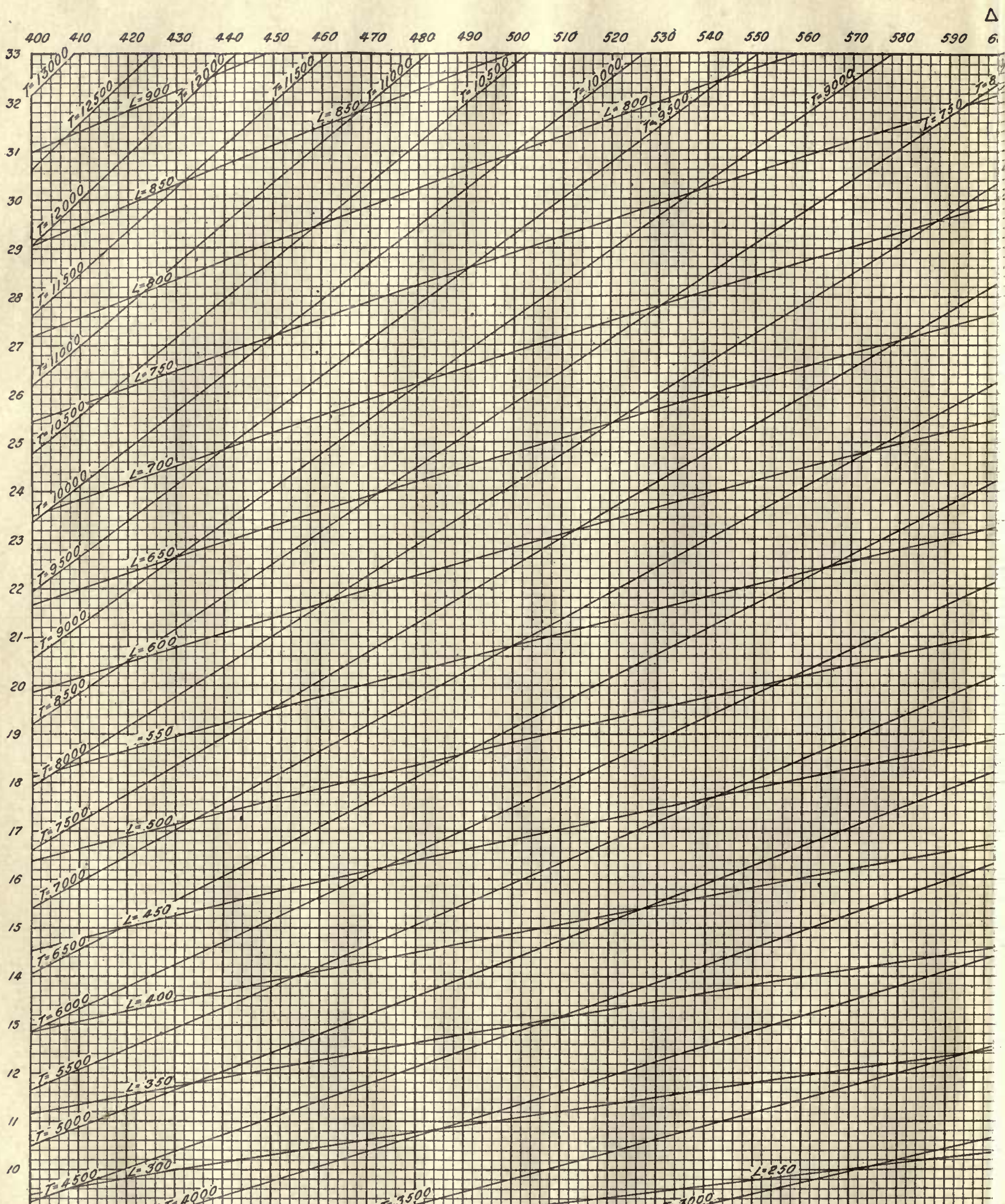
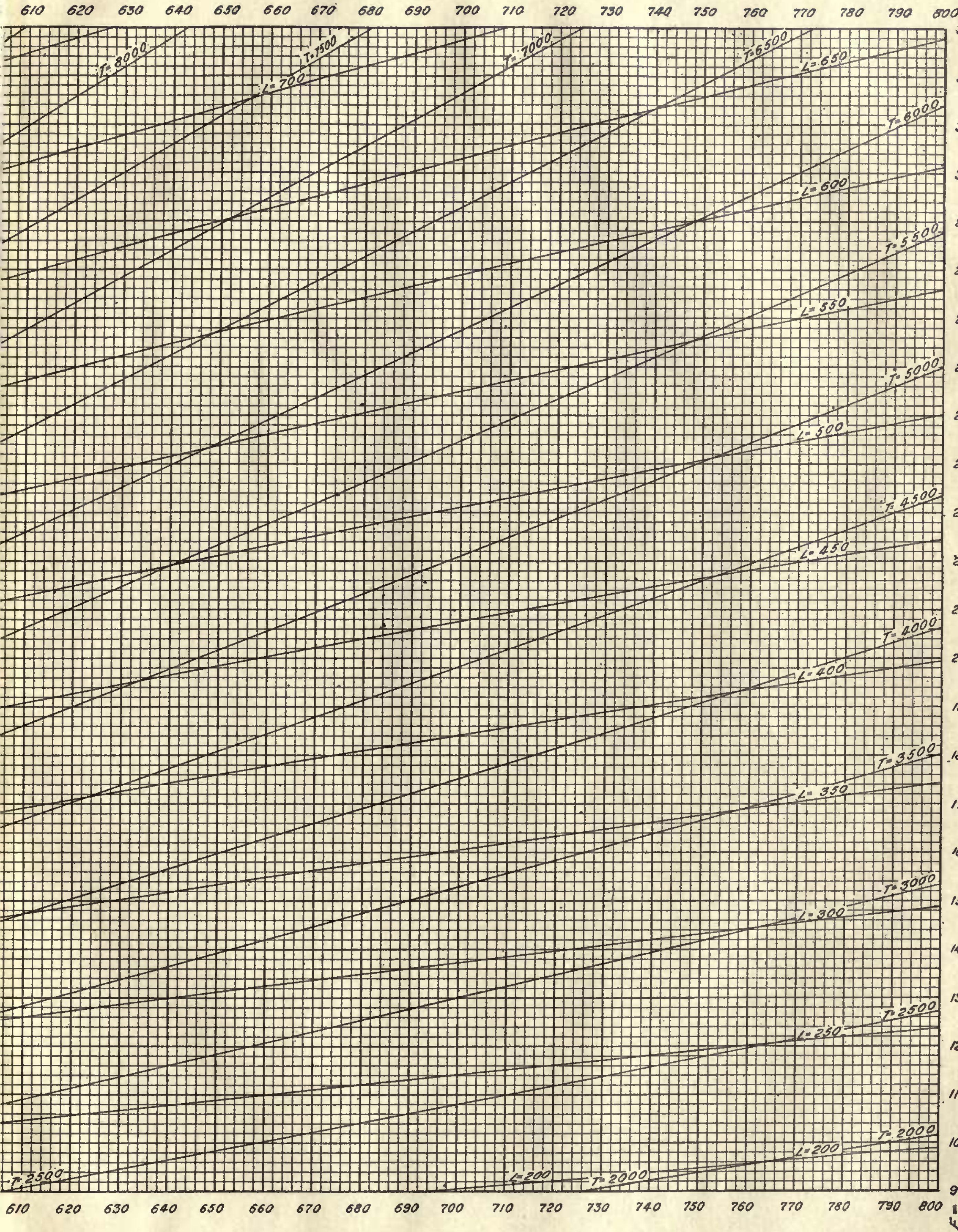
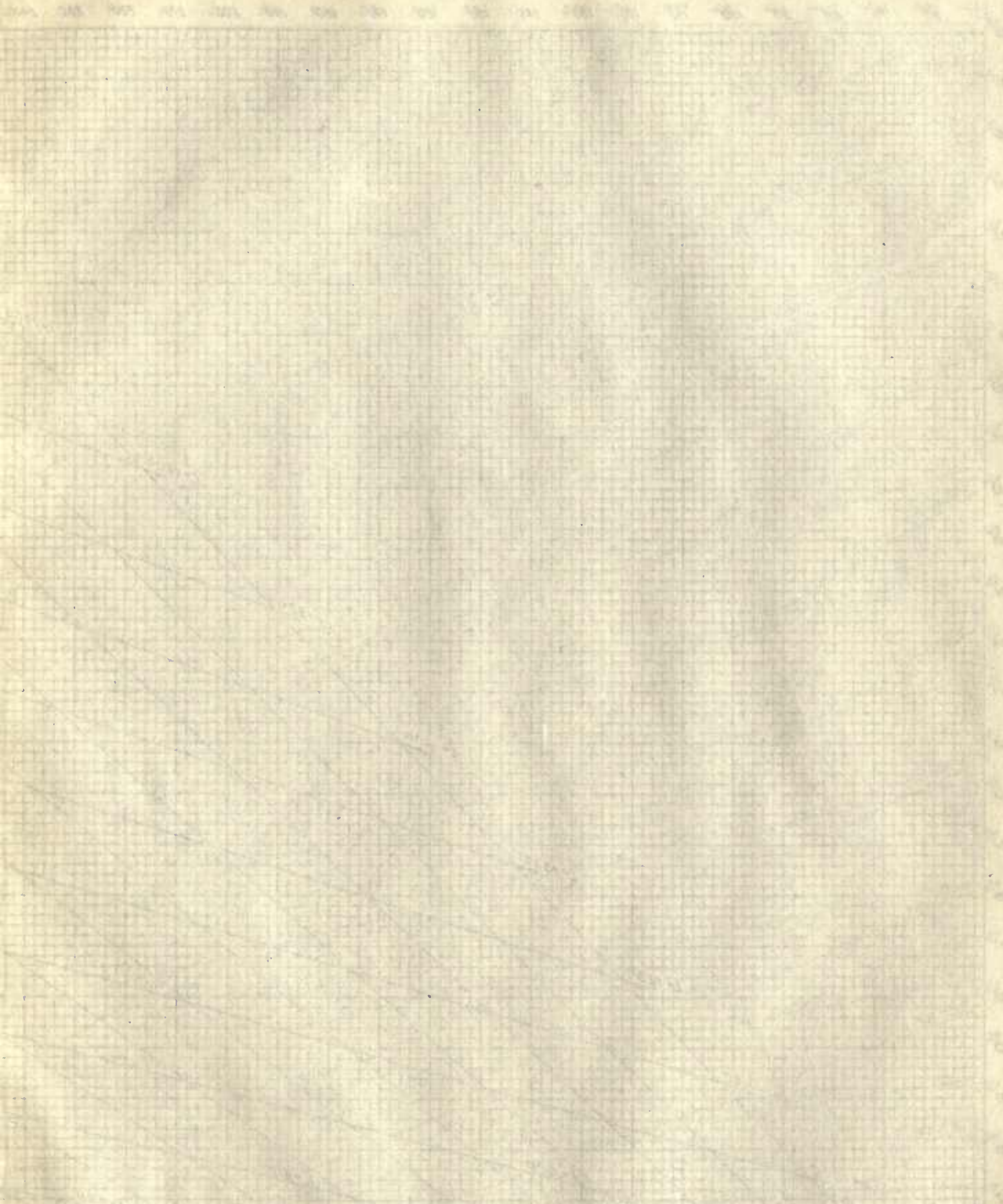

Δ
T=13000
T=12500
L=900
T=12000
T=11500
L=850
T=11000
T=10500
T=10000
L=800
T=9500
T=9000
L=750
L=850
L=800
L=750
L=700
L=650
L=600
=550
L=500
L=450
L=400
L=350
L=300
L=250
T=12000
T=11500
T=11000
T=10500
T=10000
T=9500
T=9000
T=8500
T=8000
T=7500
T=7000
T=6500
T=6000
T=5500
T=5000
T=4500
T=4000
T=3500
T=3000
400
410
420
430
440
450
460
470
480
490
500
510
520
530
540
550
560
570
580
590
Δ

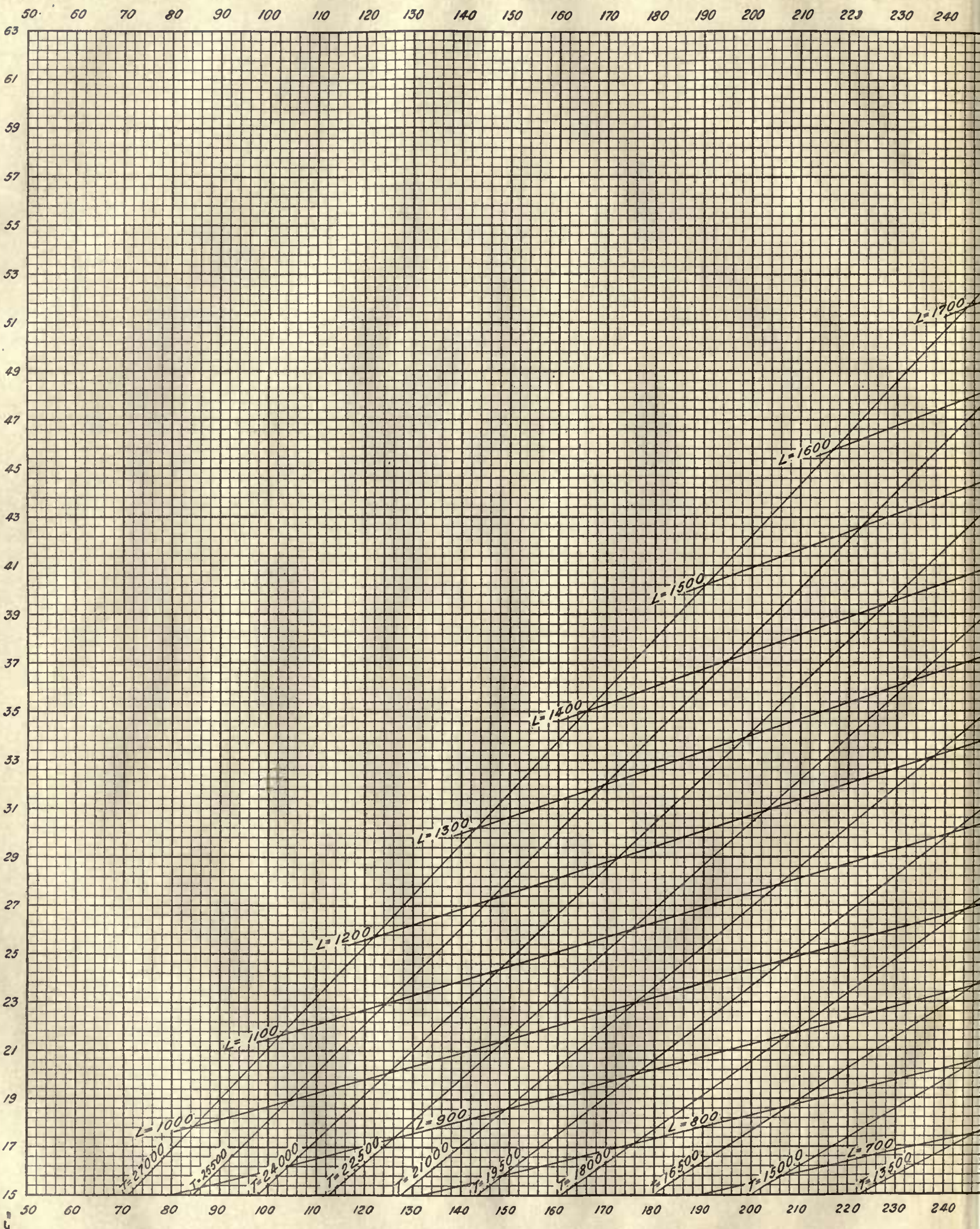
Chart No. 6



610 620 630 640 650 660 670 680 690 700 710 720 730 740 750 760 770 780 790 800
T=8000
T=7500
T=7000
T=6500
T=6000
T=5500
T=5000
T=4500
T=4000
T=3500
T=3000
T=2500
T=2000
L=700
L=650
L=600
L=550
L=500
L=450
L=400
L=350
L=300
L=250
L=200
33 32 31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9

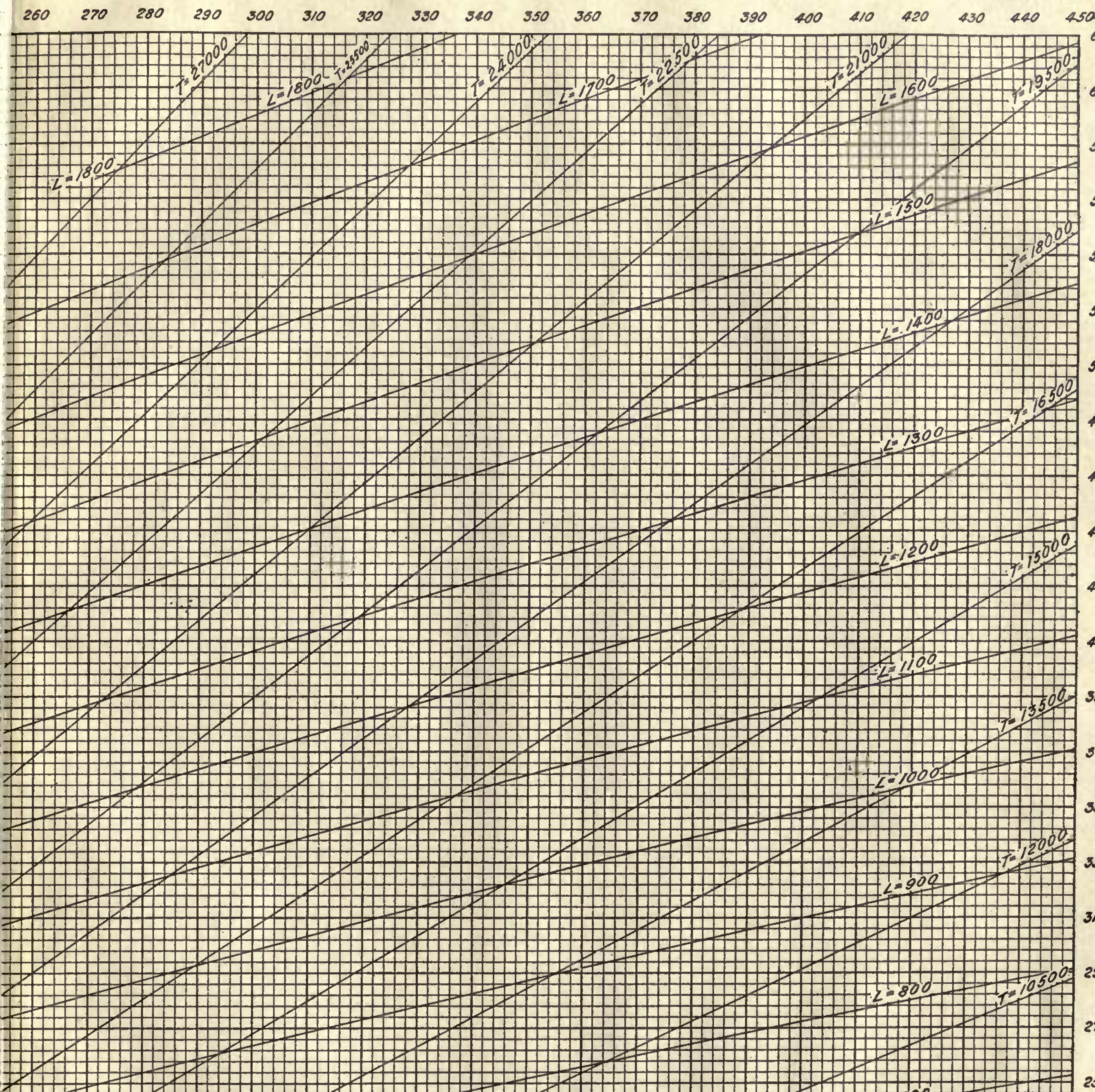
Chart No.

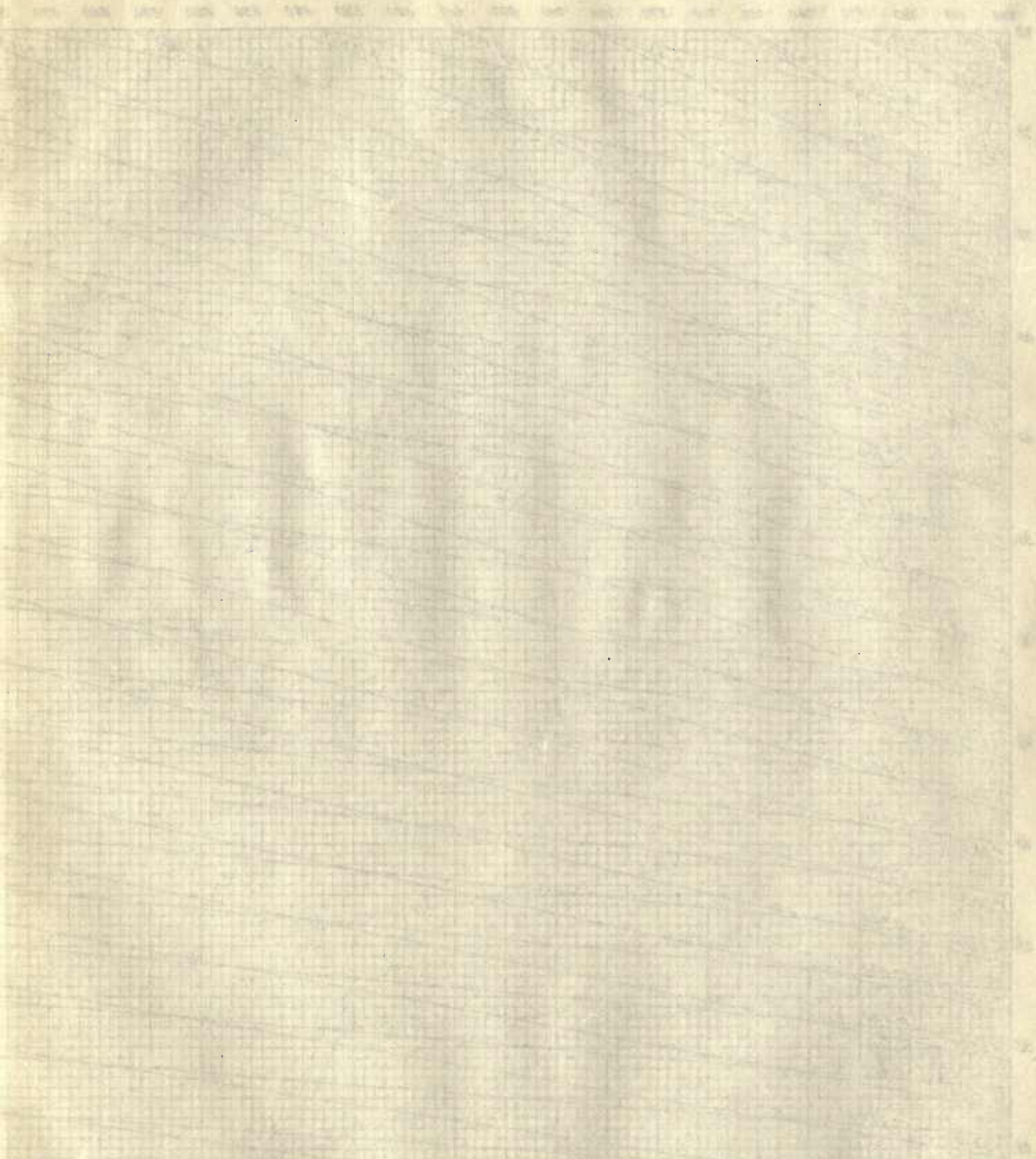


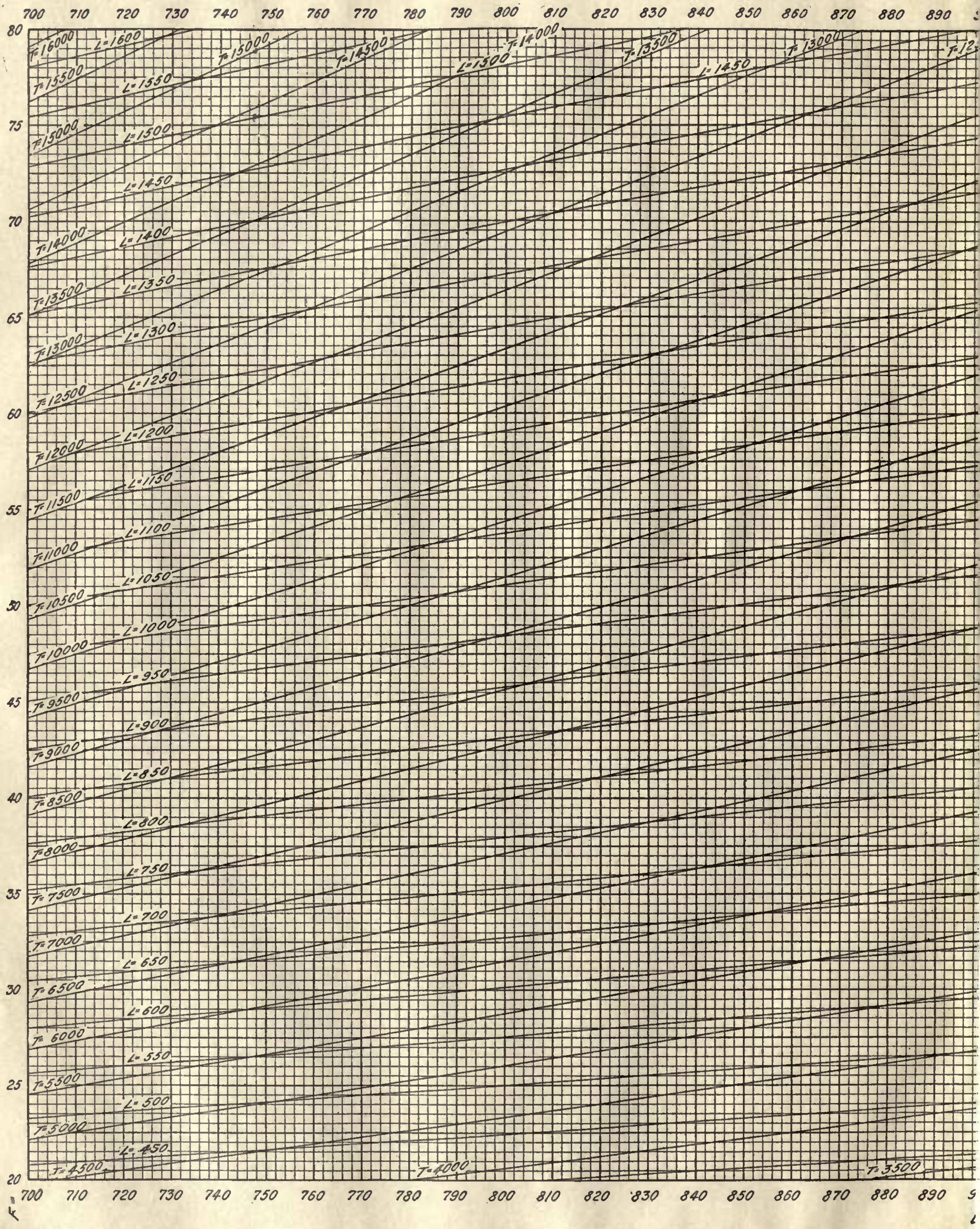


50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 223 230 240
63 61 59 57 55 53 51 49 47 45 43 41 39 37 35 33 31 29 27 25 23 21 19 17 15
L=1700
L=1600
L=1500
L=1400
L=1300
L=1200
L=1100
L=1000
L=900
L=800
L=700
T=27000
T=25500
T=24000
T=22500
T=21000
T=19500
T=18000
T=16500
T=15000
T=13500
50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230 240

Chart No. 7

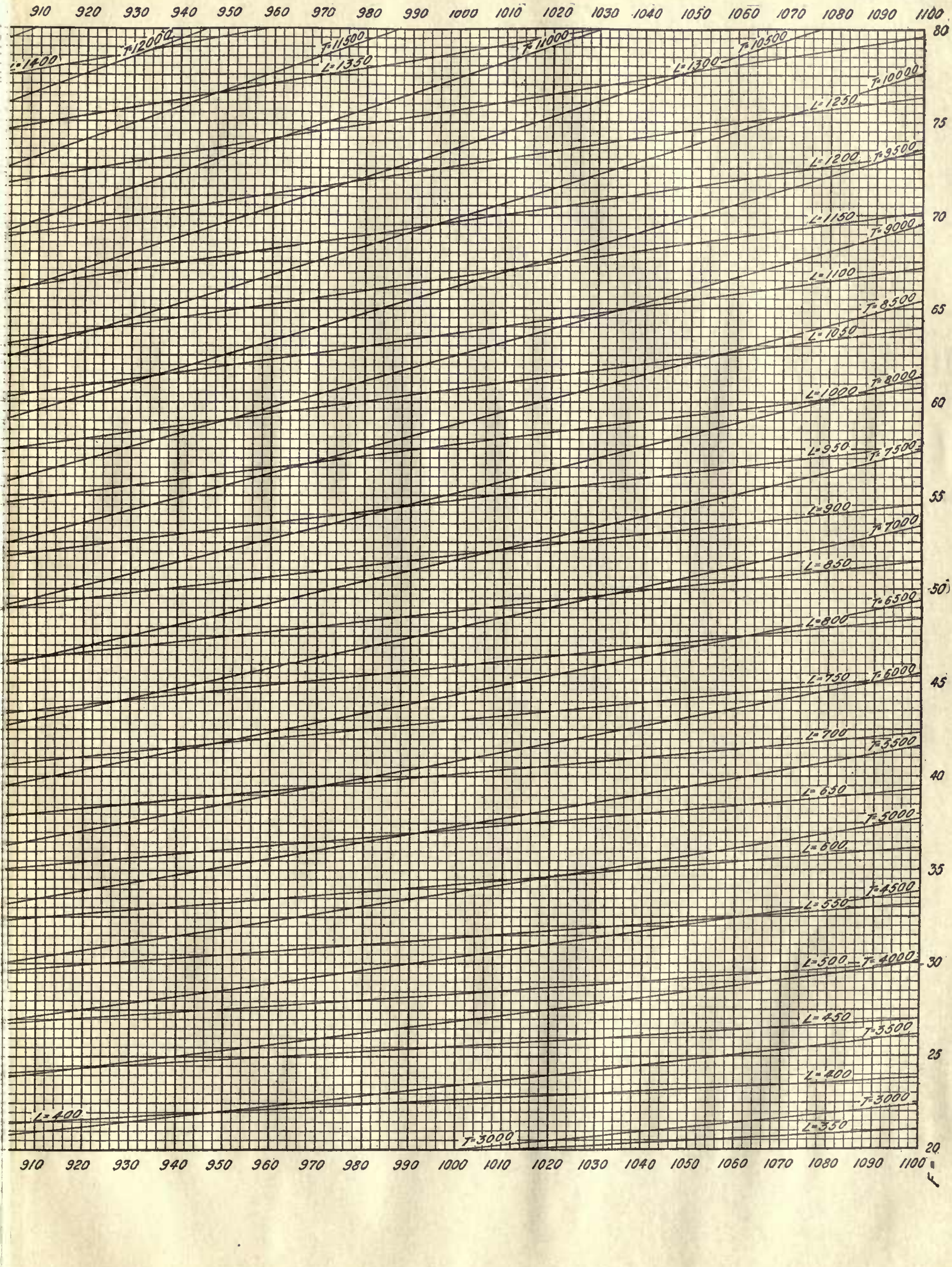

260 270 280 290 300 310 320 330 340 350 360 370 380 390 400 410 420 430 440 450
63 61 59 57 55 53 51 49 47 45 43 41 39 37 35 33 31 29 27 25 23 21 19 17 15
T=27000
L=1800
T=25500
T=24000
L=1700
T=22500
T=21000
L=1600
T=19500
L=1500
T=18000
L=1400
T=16500
L=1300
L=1200
T=15000
L=1100
T=13500
L=1000
T=12000
L=900
L=800
T=10500
L=700
T=9000
L=600
T=7500
L=500
2000
T=10500
L=600
T=9000
L=500
T=7500



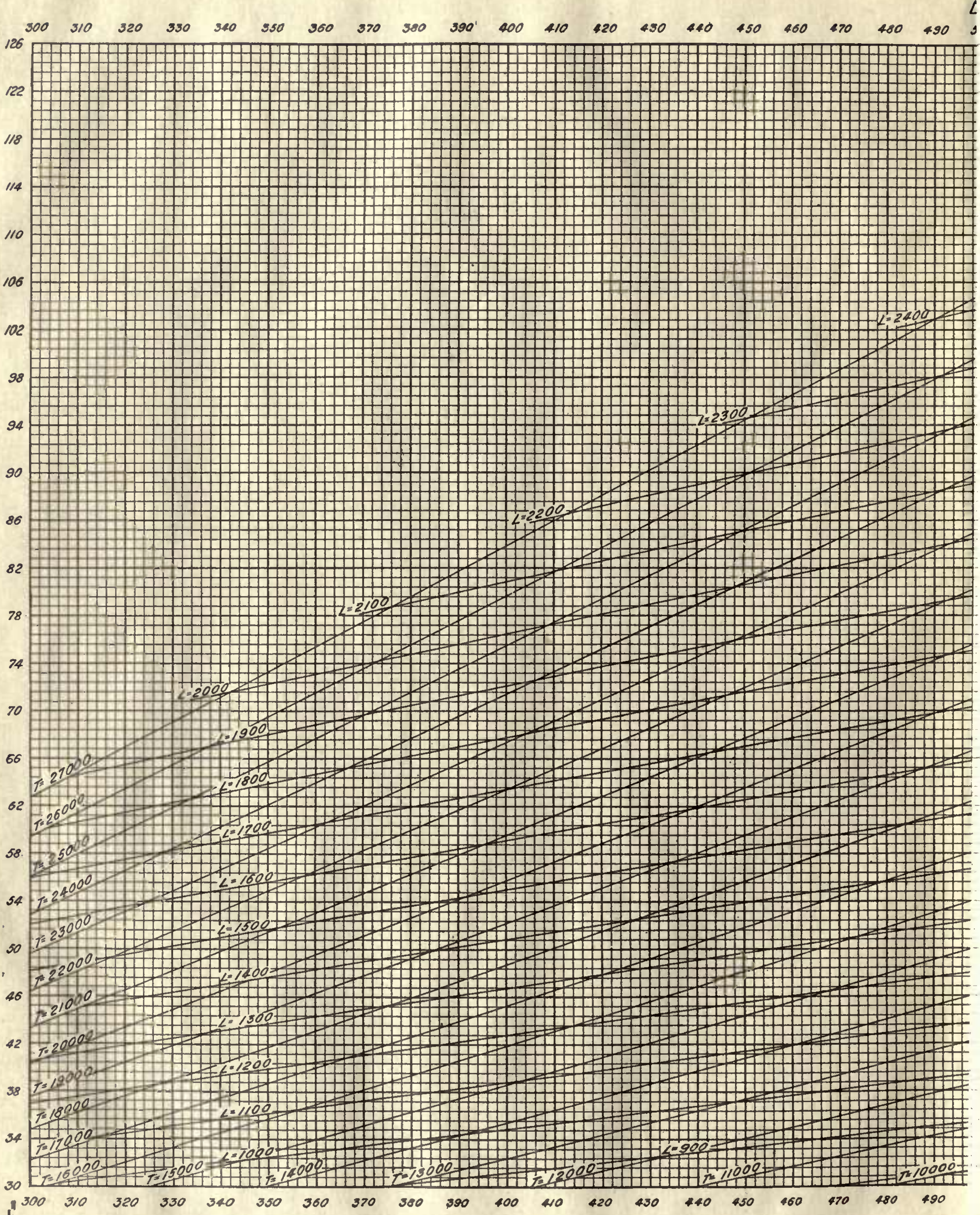


700 710 720 730 740 750 760 770 780 790 800 810 820 830 840 850 860 870 880 890
80 75 70 65 60 55 50 45 40 35 30 25 20
T=16000
T=15500
T=15000
T=14500
T=14000
T=13500
T=13000
T=12500
T=12000
T=11500
T=11000
T=10500
T=10000
T=9500
T=9000
T=8500
T=8000
T=7500
T=7000
T=6500
T=6000
T=5500
T=5000
T=4500
T=4000
T=3500
L=1600
L=1550
L=1500
L=1450
L=1400
L=1350
L=1300
L=1250
L=1200
L=1150
L=1100
L=1050
L=1000
L=950
L=900
L=850
L=800
L=750
L=700
L=650
L=600
L=550
L=500
L=450

Chart No. 8

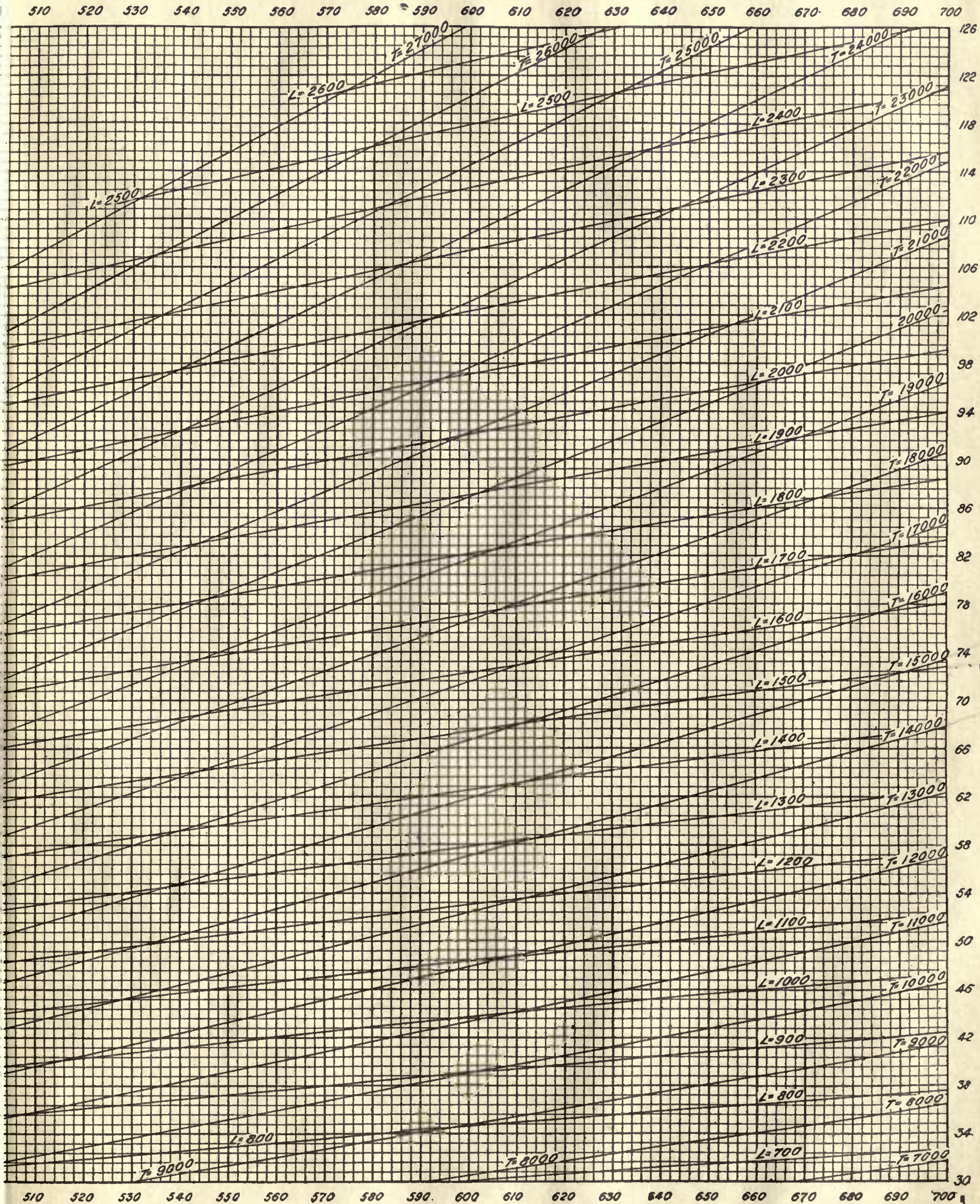


910 920 930 940 950 960 970 980 990 1000 1010 1020 1030 1040 1050 1060 1070 1080 1090 1100
80 75 70 65 60 55 50 45 40 35 30 25 20
T=12000
T=11500
T=11000
T=10500
T=10000
T=9500
T=9000
T=8500
T=8000
T=7500
T=7000
T=6500
T=6000
T=5500
T=5000
T=4500
T=4000
T=3500
T=3000
L=1400
L=1350
L=1300
L=1250
L=1200
L=1150
L=1100
L=1050
L=1000
L=950
L=900
L=850
L=800
L=750
L=700
L=650
L=600
L=550
L=500
L=450
L=400
L=350

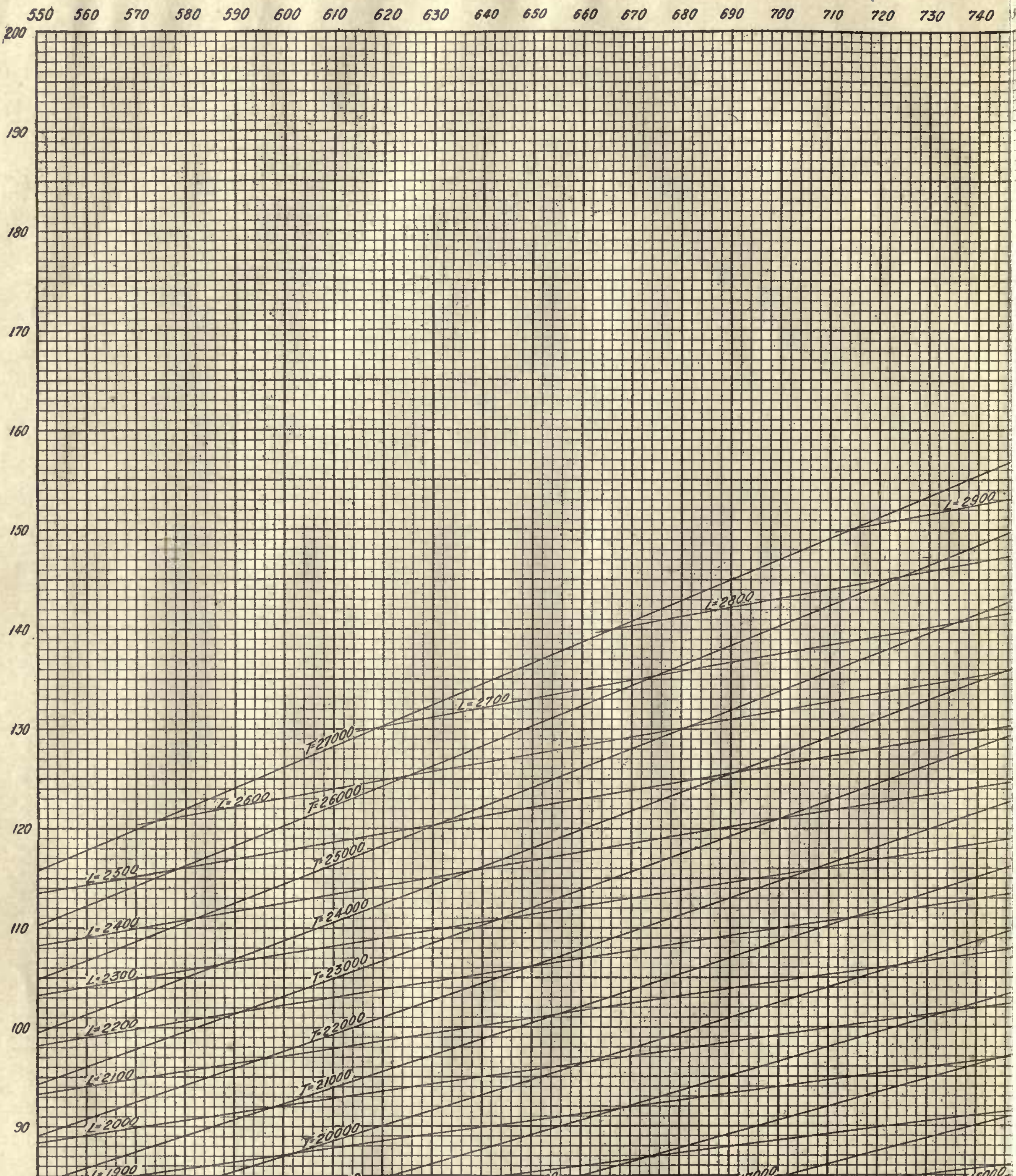


L=2400
L=2300
L=2200
L=2100
L=2000
L=1900
L=1800
L=1700
L=1600
L=1500
L=1400
L=1300
L=1200
L=1100
L=1000
L=900
T=27000
T=26000
T=25000
T=24000
T=23000
T=22000
T=21000
T=20000
T=19000
T=18000
T=17000
T=16000
T=15000
T=14000
T=13000
T=12000
T=11000
T=10000

Chart No. 9



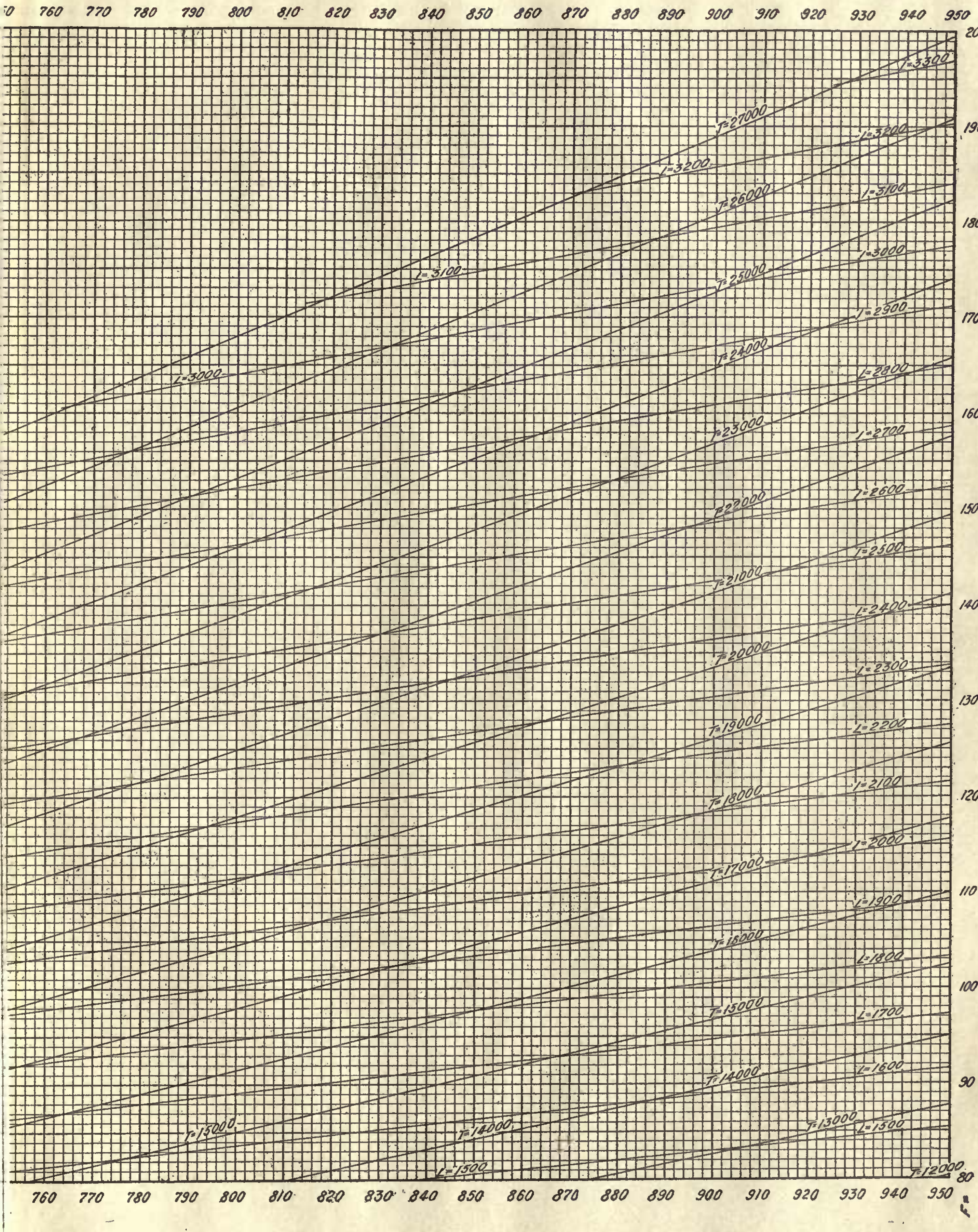
510 520 530 540 550 560 570 580 590 600 610 620 630 640 650 660 670 680 690 700
126 122 118 114 110 106 102 98 94 90 86 82 78 74 70 66 62 58 54 50 46 42 38 34 30
T=27000
T=26000
T=25000
T=24000
T=23000
T=22000
T=21000
20000
T=19000
T=18000
T=17000
T=16000
T=15000
T=14000
T=13000
T=12000
T=11000
T=10000
T=9000
T=8000
T=7000
L=2600
L=2500
L=2400
L=2300
L=2200
L=2100
L=2000
L=1900
L=1800
L=1700
L=1600
L=1500
L=1400
L=1300
L=1200
L=1100
L=1000
L=900
L=800
L=700

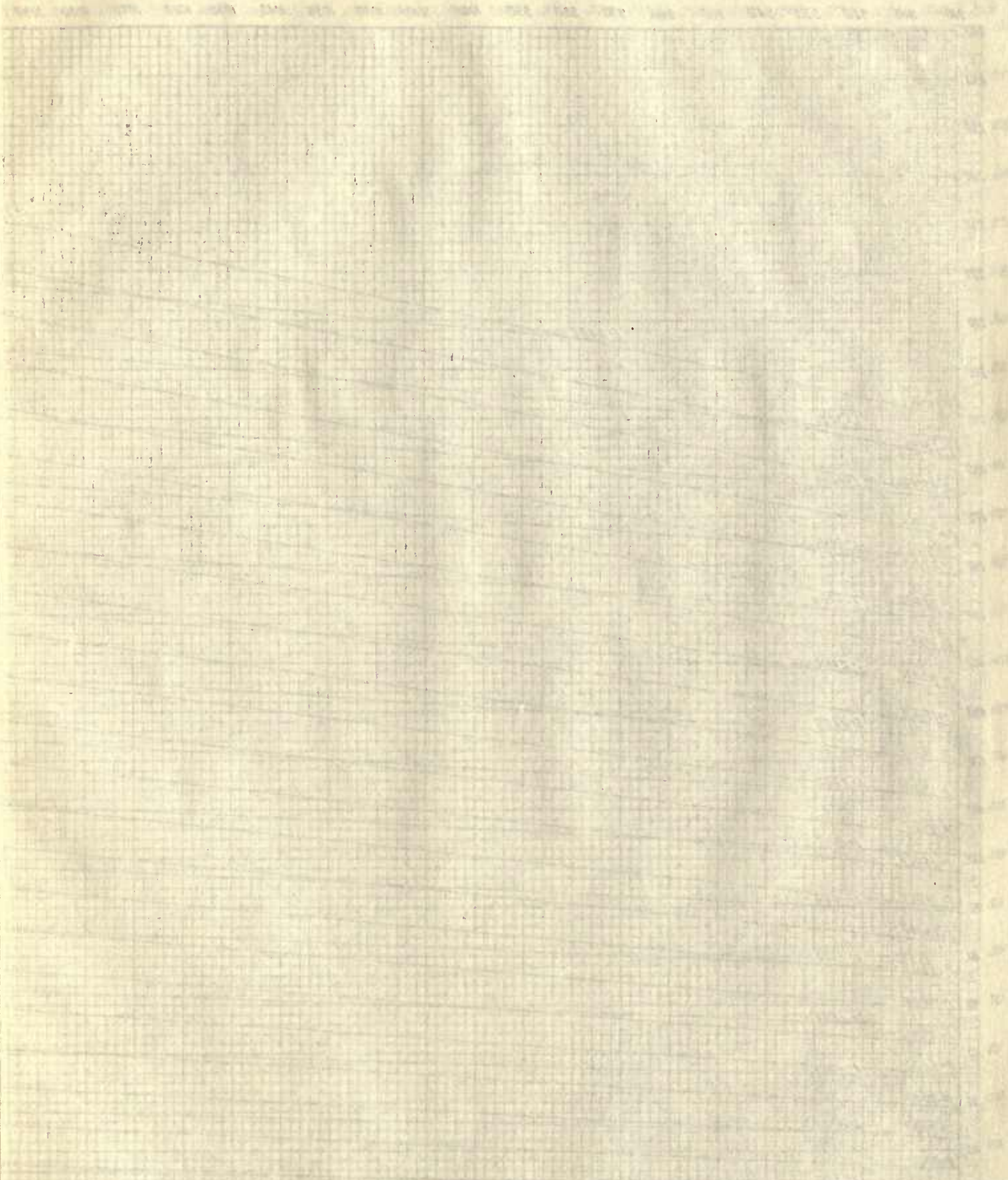


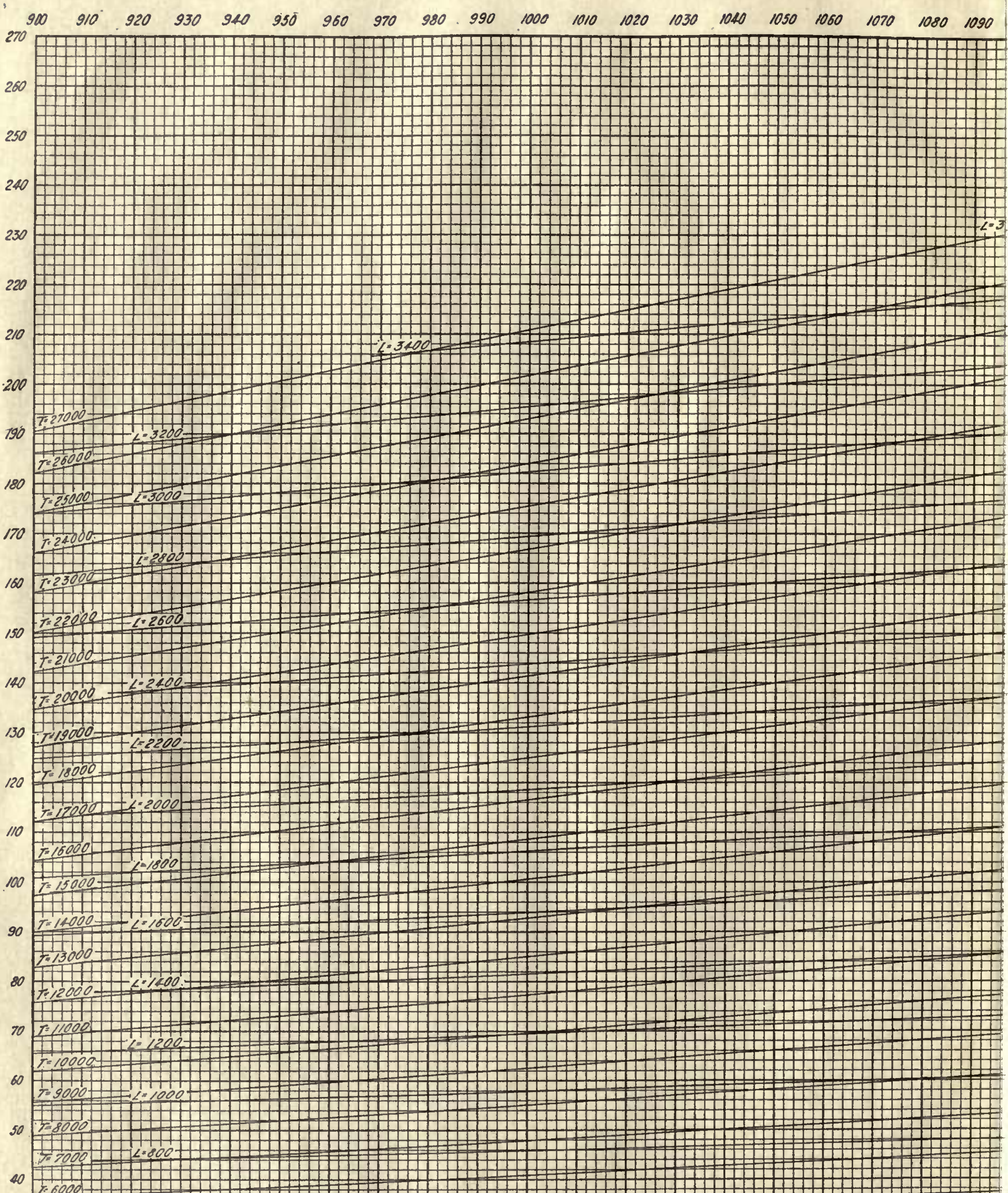
550 560 570 580 590 600 610 620 630 640 650 660 670 680 690 700 710 720 730 740
200
190
180
170
160
150
140
130
120
110
100
90
80
L=2900
L=2800
L=2700
T=27000
L=2600
T=26000
T=25000
L=2500
L=2400
T=24000
L=2300
T=23000
L=2200
T=22000
L=2100
T=21000
L=2000
T=20000
L=1900
T=19000
T=18000
T=17000
T=16000
L=1800
L=1700

t.

Chart No. 10

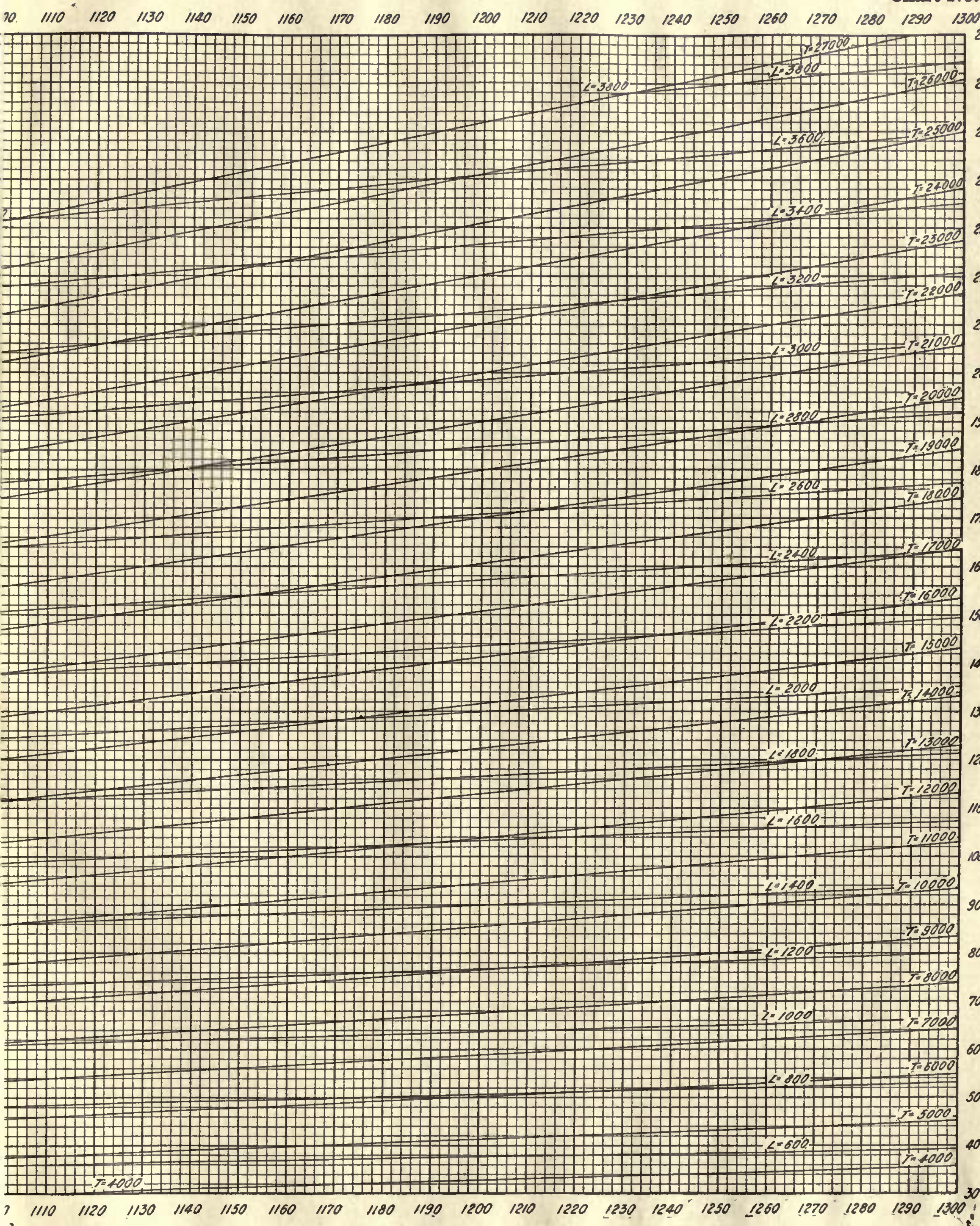

760 770 780 790 800 810 820 830 840 850 860 870 880 890 900 910 920 930 940 950
200 190 180 170 160 150 140 130 120 110 100 90 80
T=27000
T=26000
T=25000
T=24000
T=23000
T=22000
T=21000
T=20000
T=19000
T=18000
T=17000
T=16000
T=15000
T=14000
T=13000
T=12000
T=15000
T=14000
L=3300
L=3200
L=3200
L=3100
L=3100
L=3000
L=3000
L=2900
L=2800
L=2700
L=2600
L=2500
L=2400
L=2300
L=2200
L=2100
L=2000
L=1900
L=1800
L=1700
L=1600
L=1500
L=1500



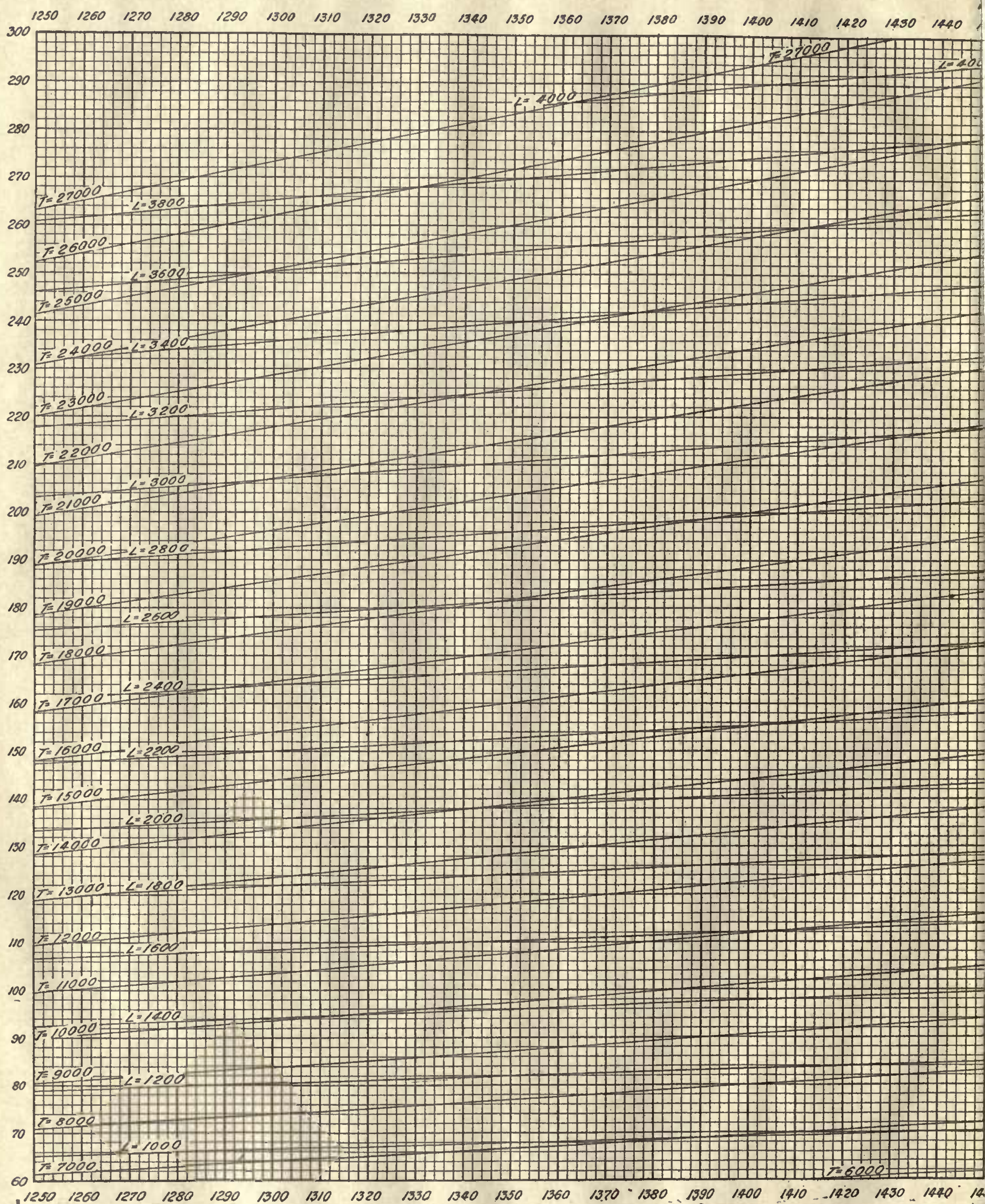


900 910 920 930 940 950 960 970 980 990 1000 1010 1020 1030 1040 1050 1060 1070 1080 1090
270 260 250 240 230 220 210 200 190 180 170 160 150 140 130 120 110 100 90 80 70 60 50 40 30
T=27000
T=26000
T=25000
T=24000
T=23000
T=22000
T=21000
T=20000
T=19000
T=18000
T=17000
T=16000
T=15000
T=14000
T=13000
T=12000
T=11000
T=10000
T=9000
T=8000
T=7000
T=6000
T=5000
L=3400
L=3200
L=3000
L=2800
L=2600
L=2400
L=2200
L=2000
L=1800
L=1600
L=1400
L=1200
L=1000
L=800
L=600

Chart No. 11

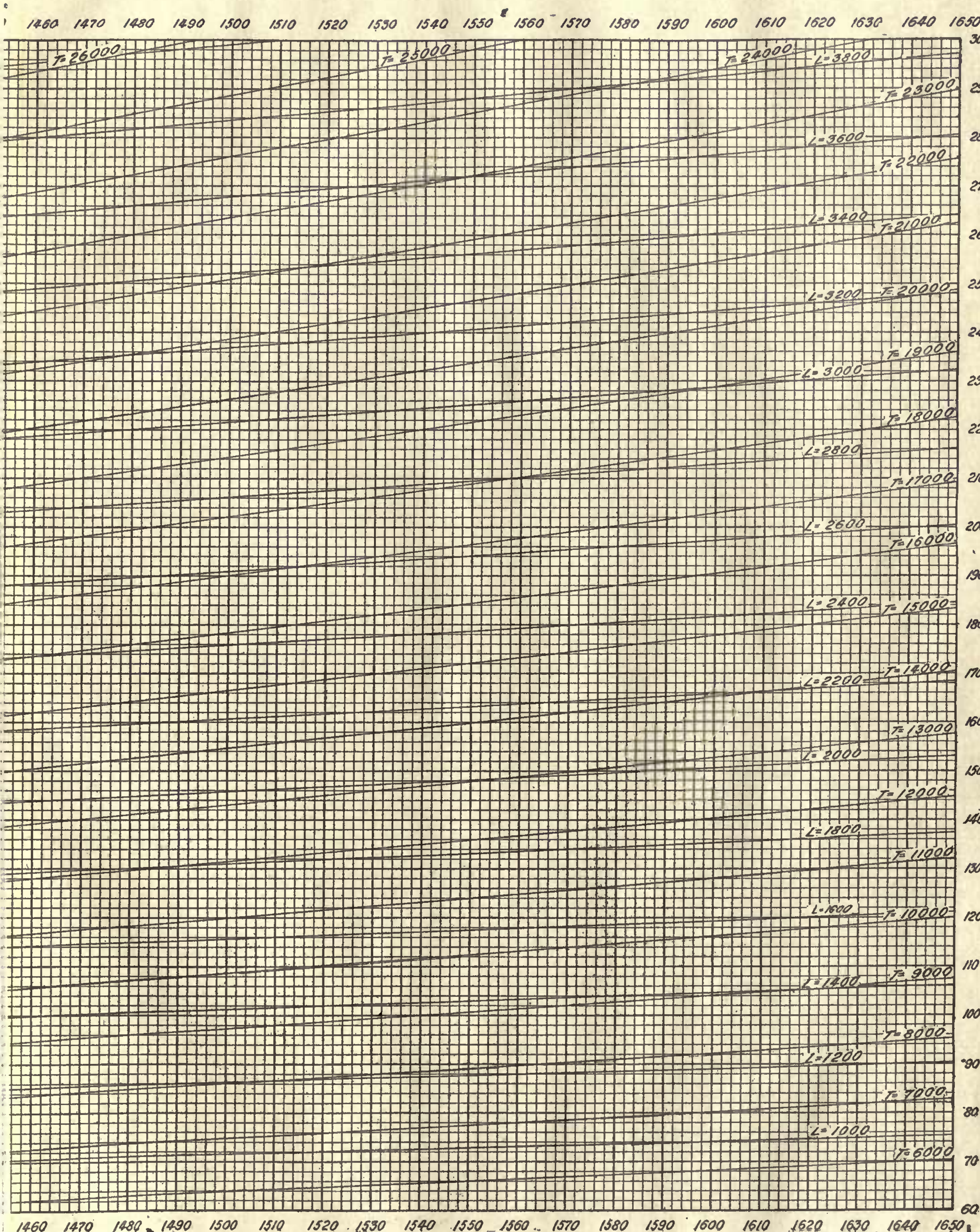

1110
1120
1130
1140
1150
1160
1170
1180
1190
1200
1210
1220
1230
1240
1250
1260
1270
1280
1290
1300
270
260
250
240
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
T=27000
L=3800
T=26000
L=3600
T=25000
T=24000
L=3400
T=23000
L=3200
T=22000
L=3000
T=21000
T=20000
L=2800
T=19000
L=2600
T=18000
L=2400
T=17000
T=16000
L=2200
T=15000
L=2000
T=14000
T=13000
L=1800
T=12000
L=1600
T=11000
L=1400
T=10000
T=9000
L=1200
T=8000
L=1000
T=7000
T=6000
L=800
T=5000
L=600
T=4000

t.



1250 1260 1270 1280 1290 1300 1310 1320 1330 1340 1350 1360 1370 1380 1390 1400 1410 1420 1430 1440
300 290 280 270 260 250 240 230 220 210 200 190 180 170 160 150 140 130 120 110 100 90 80 70 60
T=27000
T=26000
T=25000
T=24000
T=23000
T=22000
T=21000
T=20000
T=19000
T=18000
T=17000
T=16000
T=15000
T=14000
T=13000
T=12000
T=11000
T=10000
T=9000
T=8000
T=7000
T=6000
L=4000
L=3800
L=3600
L=3400
L=3200
L=3000
L=2800
L=2600
L=2400
L=2200
L=2000
L=1800
L=1600
L=1400
L=1200
L=1000

Chart No. 12



T=26000
T=25000
T=24000
L=3800
T=23000
L=3600
T=22000
L=3400
T=21000
L=3200
T=20000
T=19000
L=3000
T=18000
L=2800
T=17000
L=2600
T=16000
L=2400
T=15000
L=2200
T=14000
T=13000
L=2000
T=12000
L=1800
T=11000
L=1600
T=10000
L=1400
T=9000
T=8000
L=1200
T=7000
L=1000
T=6000

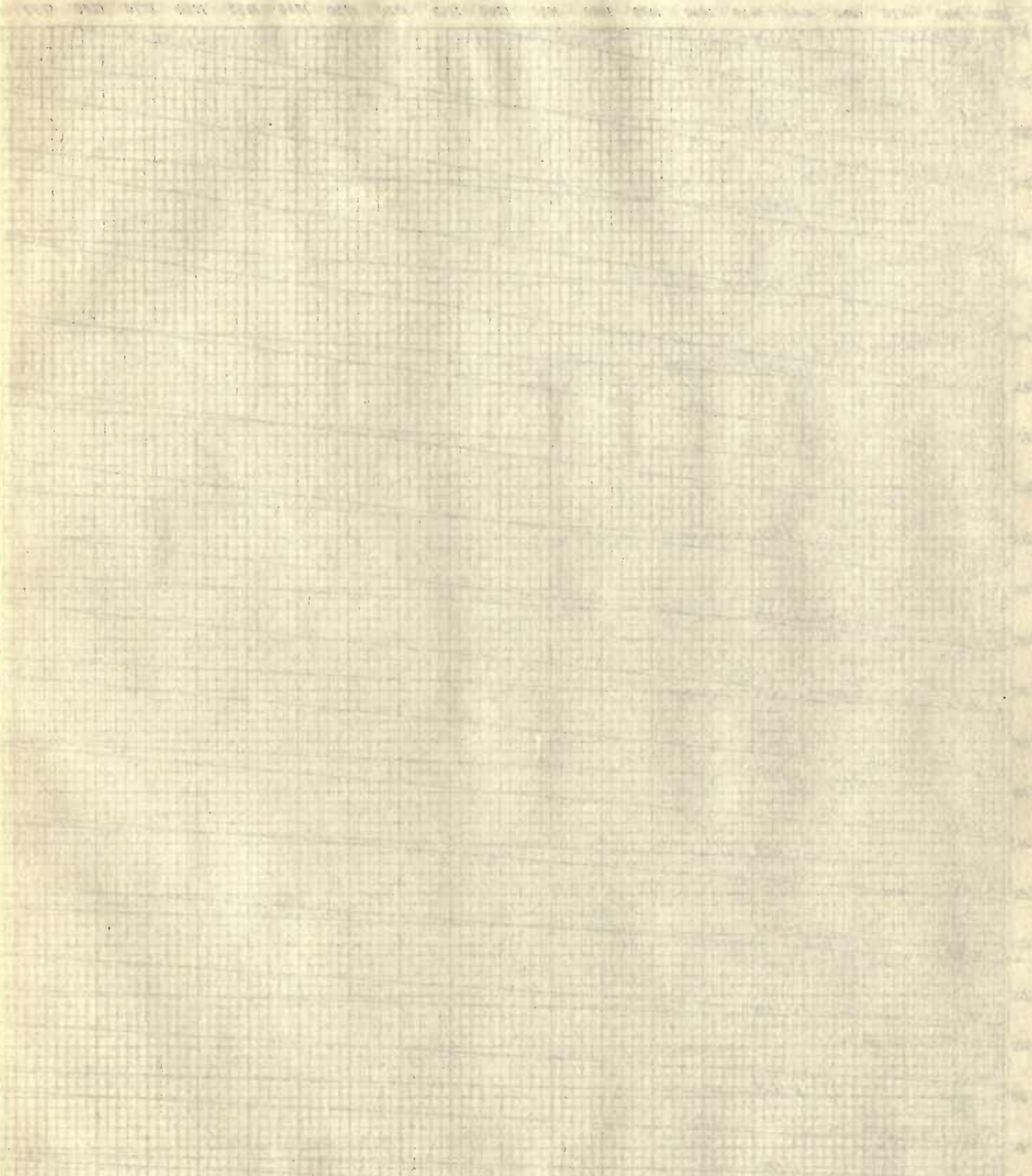


Chart No.

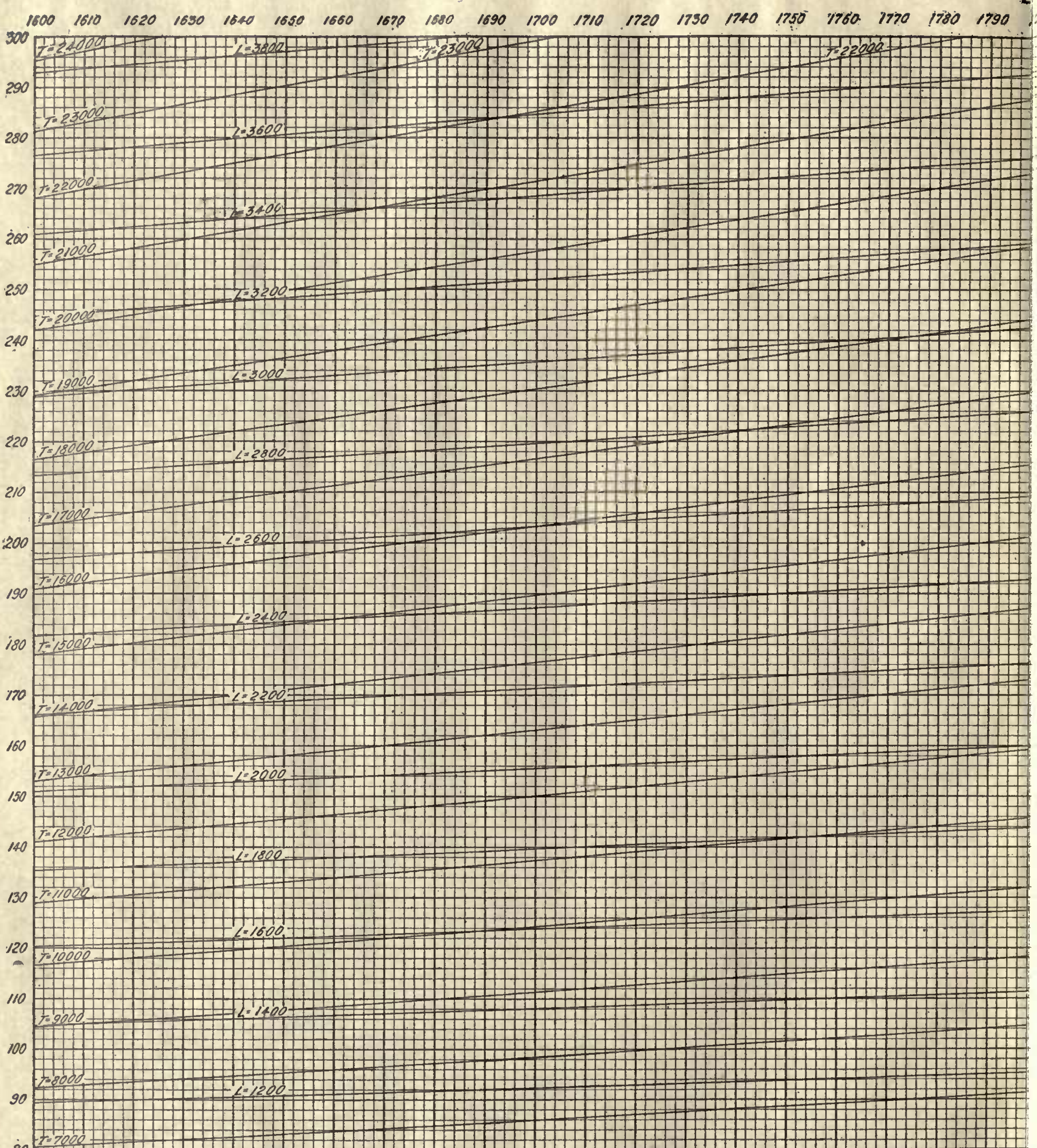
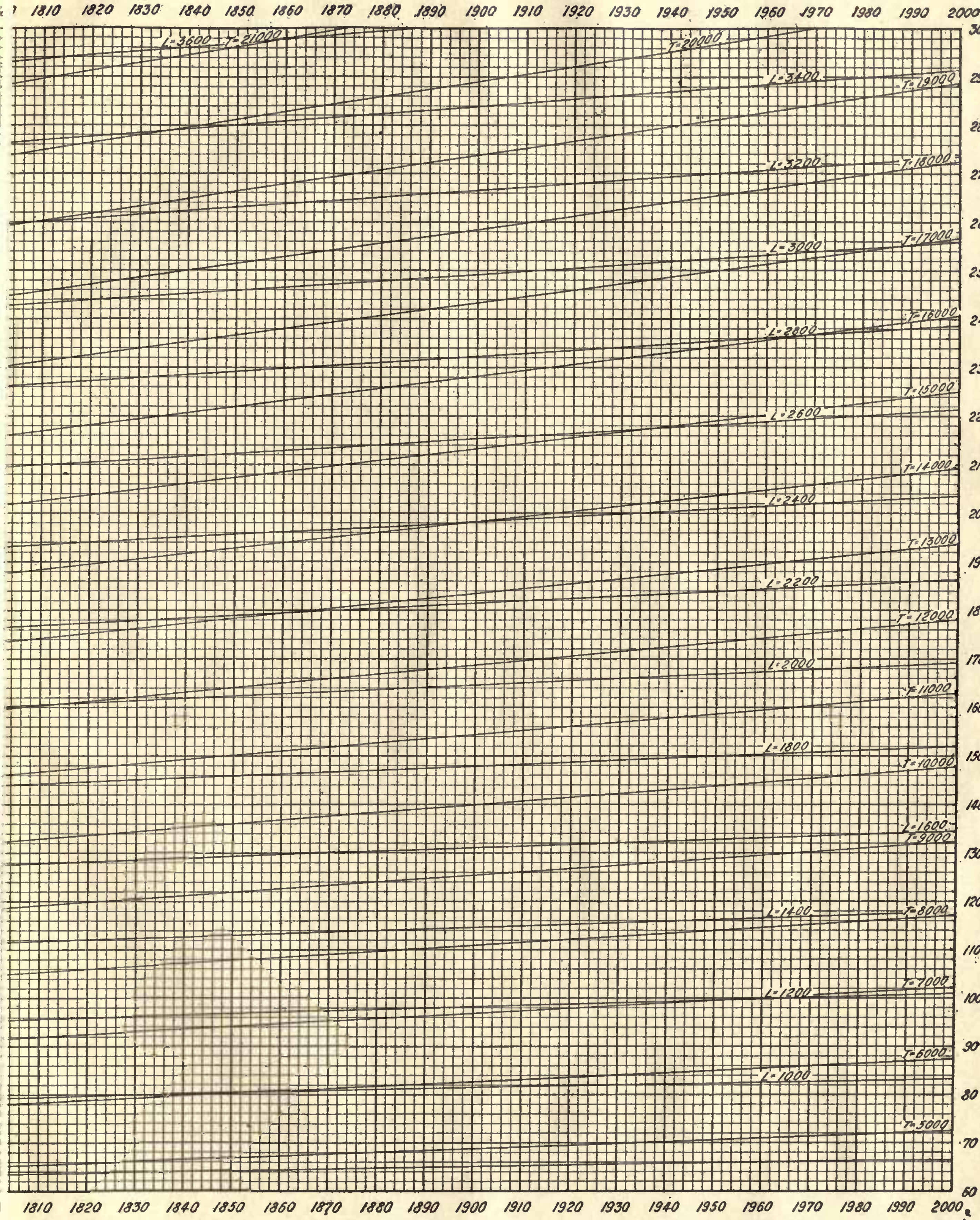

1600 1610 1620 1630 1640 1650 1660 1670 1680 1690 1700 1710 1720 1730 1740 1750 1760 1770 1780 1790
300 290 280 270 260 250 240 230 220 210 200 190 180 170 160 150 140 130 120 110 100 90 80 70 60
T=24000
T=23000
T=22000
T=21000
T=20000
T=19000
T=18000
T=17000
T=16000
T=15000
T=14000
T=13000
T=12000
T=11000
T=10000
T=9000
T=8000
T=7000
T=6000
T=5000
L=3800
L=3600
L=3400
L=3200
L=3000
L=2800
L=2600
L=2400
L=2200
L=2000
L=1800
L=1600
L=1400
L=1200
L=1000
L=800

Chart No. 13



1810 1820 1830 1840 1850 1860 1870 1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000
L=3600
T=21000
T=20000
L=3400
T=19000
L=3200
T=18000
L=3000
T=17000
T=16000
L=2800
T=15000
L=2600
T=14000
L=2400
T=13000
L=2200
T=12000
L=2000
T=11000
L=1800
T=10000
L=1600
T=9000
L=1400
T=8000
T=7000
L=1200
T=6000
L=1000
T=5000
300 290 280 270 260 250 240 230 220 210 200 190 180 170 160 150 140 130 120 110 100 90 80 70 60