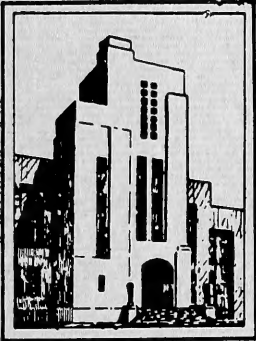


1106



NAVY DEPARTMENT  
DAVID TAYLOR MODEL BASIN



HYDROMECHANICS

○

AERODYNAMICS

○

STRUCTURAL  
MECHANICS

○

APPLIED  
MATICS

A GRAPHICAL METHOD FOR DETERMINING THE  
GENERAL-INSTABILITY STRENGTH OF  
STIFFENED CYLINDRICAL SHELLS

by

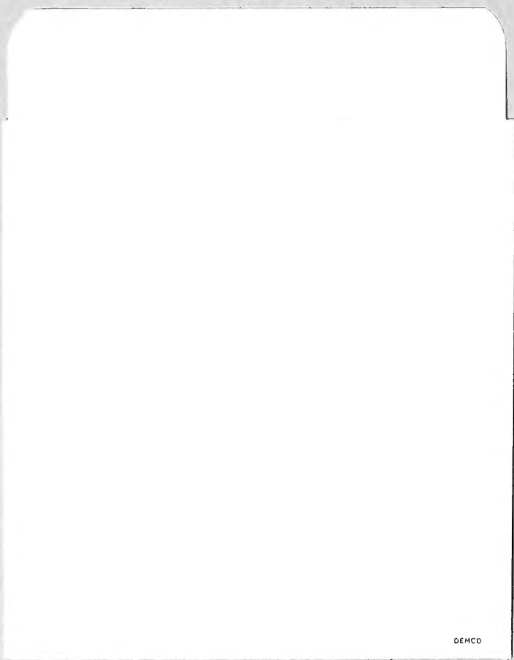
Thomas E. Reynolds

STRUCTURAL MECHANICS LABRATORY  
RESEARCH AND DEVELOPMENT REPORT

September 1957

Report 1106

GC  
1  
103  
1106



GENCO

MBL/WHOI



0 0301 0037599 4

**A GRAPHICAL METHOD FOR DETERMINING THE  
GENERAL-INSTABILITY STRENGTH OF  
STIFFENED CYLINDRICAL SHELLS**

by

**Thomas E. Reynolds**

**September 1957**

**Report 1106  
NS731-038**

## TABLE OF CONTENTS

|  | Page |
|--|------|
| ABSTRACT .....                                   | 1    |
| INTRODUCTION .....                               | 1    |
| METHOD .....                                     | 2    |
| ACCURACY OF RESULTS .....                        | 7    |
| ACKNOWLEDGMENT .....                             | 7    |
| APPENDIX A - NUMERICAL EXAMPLE.....              | 9    |
| APPENDIX B - APPROXIMATE FORMULA FOR $I_e$ ..... | 10   |
| REFERENCES .....                                 | 15   |

## LIST OF ILLUSTRATIONS

|   |    |
|---|----|
| Figure 1 - Shell Pressure Factor ( $p_s$ ) as a Function of Bulkhead Spacing ( $L_b/R$ ) .....      | 4  |
| Figure 2 - Frame Pressure Factor ( $p_f$ ) as a Function of Frame Stiffness ( $I_e/L_f R^3$ ) ..... | 5  |
| Figure 3 - Curves of Constant Pressure .....  | 6  |
| Figure 4 - Identification of Symbols Used in Calculating $I_e$ .....                                | 13 |

## LIST OF TABLES

|   |    |
|---|----|
| Table 1 - Comparison of Numerical Results with Kendrick's (Part III) Theory ..... | 8  |
| Table 2 - Example of Results Given by Various Methods .....                       | 9  |
| Table 3 - Dimensions of Typical Frame Cross Sections .....                        | 14 |
| Table 4 - Comparison of Approximate and Exact Values for $I_f$ and $I_e$ .....    | 14 |

## ABSTRACT

The results of numerical calculations of the general-instability strength of ring-stiffened circular cylinders are presented in graphical form. The calculations are based on Kendrick's "second solution" which is published in Naval Construction Research Establishment Report No. 244 (Part III). The collapse pressures from these graphs agree within 10 percent with those computed by Kendrick's theory throughout the normal range of submarine geometries.

## INTRODUCTION

One consideration in the structural design of submarine pressure hulls is the possibility of collapse by general instability, i.e., large deformations of frames and shell between holding bulkheads. It is also recognized that the elastic general-instability pressure governs the extent to which imperfections reduce the load-carrying capacity of the frames. For this reason, an accurate determination of this pressure can be useful in frame design even though it is far greater than the pressure encountered at normal operating depth.

The most reliable of several theoretical investigations of the general-instability problem appears to be the recent work of Kendrick conducted at the Naval Construction Research Establishment, Rosyth, Scotland. This was published in the form of two separate analyses.<sup>1,2</sup> Both employ the same general approach, but Reference 2 (Kendrick, Part III) is a more general treatment than Reference 1 (Kendrick, Part I) and always gives a lower collapse pressure.

While Kendrick's analysis is a significant advance in the study of stiffened cylinders, its application to submarine design is difficult because of the lengthy calculations required. Recently Bryant,<sup>3</sup> also of the Naval Construction Research Establishment, developed an approximation which agrees closely with Kendrick's Part I solution. Because of its simplicity, Bryant's formula is a valuable aid in design calculations.

These solutions have been examined at the Taylor Model Basin, and several tests<sup>4</sup> have been conducted with machined models to provide experimental evaluation. In general, agreement between experiment and theory was good, the failure pressures usually being slightly lower than the prediction of Kendrick Part I or Bryant, and slightly higher than that of Kendrick Part III.\* The difference in the pressures given by these solutions is insignificant for cylinders with light frames but becomes much larger for heavy frames such as used on submarines. It was found, in fact, that for contemporary submarine geometries, Bryant's formula

---

<sup>1</sup> References are listed on page 15.

\* It should be noted that this analysis contains two solutions. The first represents a physically impossible buckling configuration but is presented because it gives a lower collapse pressure. In this report, all mention of Kendrick Part III will be confined to the second solution of that analysis.

and Kendrick's Part I solution give pressures as much as 35 percent higher than the Part III solution. In such cases use of the more exact solution in design work is desirable, but it is impractical because of the extensive calculations required.

In view of these difficulties, it seemed worthwhile to look for a short method of approximating Kendrick's Part III solution. Accordingly, an extensive program was begun at the Model Basin to obtain numerical solutions over a wide range of geometries, the objective being to summarize the results in some graphical form which would be of practical use to the designer. Such a presentation would not only provide a quicker and more accurate means of determining the general-instability strength of a structure but would present a better picture of how variations in the scantlings affect this strength. Because of the very large amount of computation involved, this program would have been virtually impossible without the aid of the high-speed computer UNIVAC.\*

The results of the calculations are summarized in this report in the form of graphs which relate general-instability strength to variations in frame size, frame spacing, shell radius and thickness, and compartment length. All calculations were for externally framed steel cylinders with a Young's modulus of  $30 \times 10^6$  psi. Since the elastic general-instability pressure is directly proportional to the modulus, these results are readily applicable to other elastic materials having Poisson's ratio  $\nu = 0.3$ . Moreover, since internal frames theoretically provide slightly higher general-instability strength than external frames of the same dimensions, the results can be safely applied to internally framed cylinders. The accuracy of the graphical results is demonstrated by a comparison with numerical solutions of Kendrick's Part III theory for a wide range of geometry. The use of the graphs is illustrated in Appendix A by a numerical example. In Appendix B, approximate formulas are given whereby a frame strength parameter can be determined, and several numerical examples are provided to demonstrate the accuracy of the formulas.

## METHOD

Since Kendrick's Part III analysis cannot be reduced to a simple algebraic expression, the problem of presenting the theory in graphical form must be approached somewhat indirectly. The method adopted consisted of plotting the results of many numerical calculations against various geometrical parameters until, by a process of trial and error, a system of coordinates was found in which the points followed closely a set of single-valued curves.

In attempting to define general-instability strength in terms of the shape and size of the structure, at least five quantities must be considered, i.e., shell radius, shell thickness, compartment length, frame spacing, and some measure of frame size. This situation is further complicated by the variability of frame shape and the fact that the number of circumferential

---

\* Numerical results for more than 200 different geometries were obtained on UNIVAC through the solution of a fifth-order matrix by iteration methods.

waves into which the cylinder buckles (also a function of the relative dimensions) is a vital factor in the determination of general-instability strength. Thus it is clear that the problem would be greatly simplified if a few fundamental parameters could be obtained by combining some of the dimensions in a rational manner.

It is shown in Bryant's paper<sup>3</sup> that the general-instability pressure can be treated approximately as the sum of two terms, one involving the strength of the shell and the other the strength of frame per unit length of shell. This latter term was defined as the moment of inertia about the centroid of a section comprising one frame plus a length of shell equal to one frame spacing. Numerical results indicate that Kendrick's second solution in Reference 2 can be broken down fairly successfully in this way. However, a slightly different parameter used by Bijlaard<sup>5</sup> in treating the same problem lent itself better to a graphical presentation and gave less scatter in the results. Bijlaard expresses frame strength as the moment of inertia  $I_e$  about the centroid of the combined section of a frame plus an effective length  $L_e$  of shell.  $L_e$  is taken to be  $1.57 \sqrt{R\bar{h}}$  so long as the frame spacing  $L_f$  exceeds  $2 \sqrt{R\bar{h}}$ . For smaller values of  $L_f$ ,  $L_e$  can be determined from Table 46 of Reference 6. The quantity  $I_e$  can be written

$$I_e = \frac{A_f e^2}{1 + \frac{A_f}{L_e \bar{h}}} + I_f + \frac{L_e \bar{h}^3}{12} \quad [1]$$

Where  $R$  is the radius to the median surface of the shell,

$\bar{h}$  is the shell thickness,

$A_f$  is the frame area,

$I_f$  is the moment of inertia of the frame about its own centroid, and

$e$  is the distance from the median surface of the shell to the centroid of the frame.

With the parameters as defined above, the general-instability pressure  $p_{cr}$  of a stiffened cylinder can be expressed as the sum of two terms:

$$p_{cr} = p_s \left( \frac{L_b}{R}, \frac{\bar{h}}{R}, n \right) + p_f \left( \frac{I_e}{L_f R^3}, n \right) \quad [2]$$

where  $n$  is the number of circumferential waves,

$L_b$  is the bulkhead spacing, and

$p_s$  is a linear function of  $\bar{h}/R$ .

The quantity  $p_s$  can be determined readily from Figure 1 in which  $p_s \cdot \frac{R}{100\bar{h}}$  is plotted against  $L_b/R$  for values of  $n$  of 2, 3, 4, and 5. Similarly, Figure 2 shows the variation of  $p_f$  with  $I_e/L_f R^3$  for the different values of  $n$ . These curves were drawn to fit a large number of calculated points.\* The same information is presented more concisely in Figure 3 where the results from Figures 1 and 2 are combined in one graph having the coordinates

---

\*Since only values for  $p_{cr}$  were obtained from the calculations,  $p_{cr}$  was plotted against  $I_e/L_f R^3$ , and  $p_s$  and  $p_f$  were determined by extrapolation to  $I_e/L_f R^3 = 0$ .





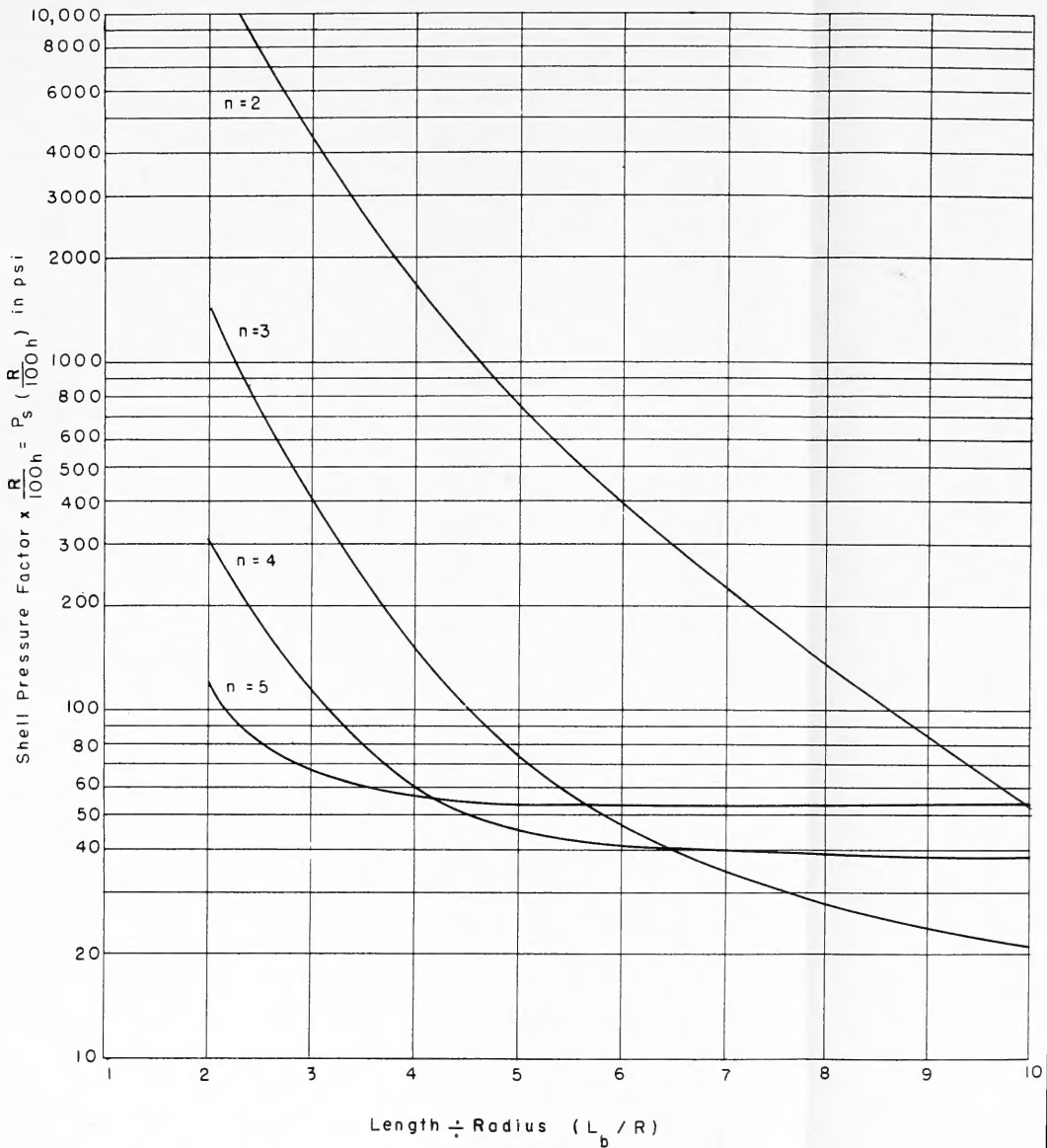


Figure 1 - Shell Pressure Factor ( $p_s$ ) as a Function of Bulkhead Spacing ( $L_b/R$ )



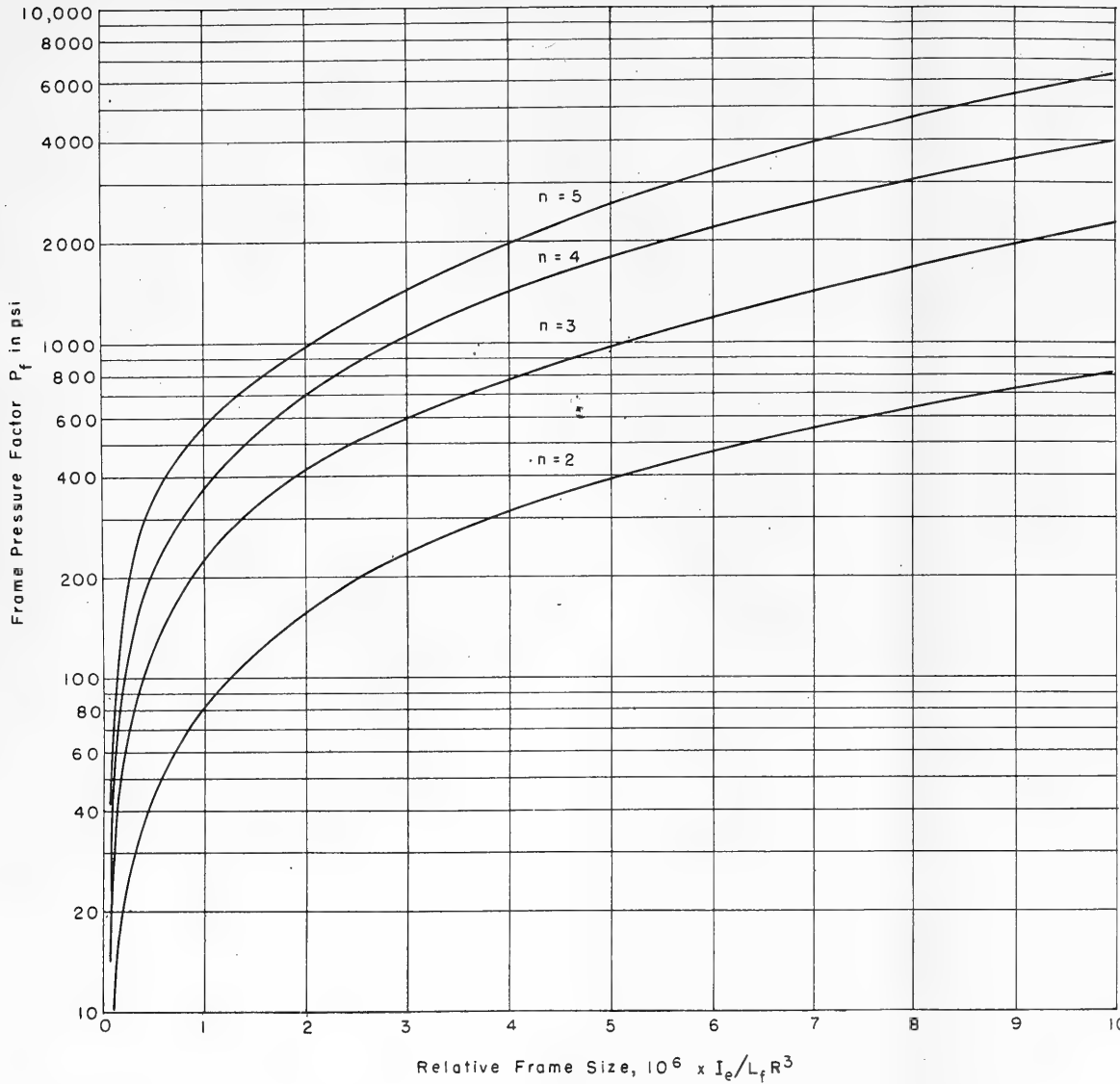


Figure 2 - Frame Pressure Factor ( $p_f$ ) as a Function of Frame Stiffness ( $I_e/L_f R^3$ )



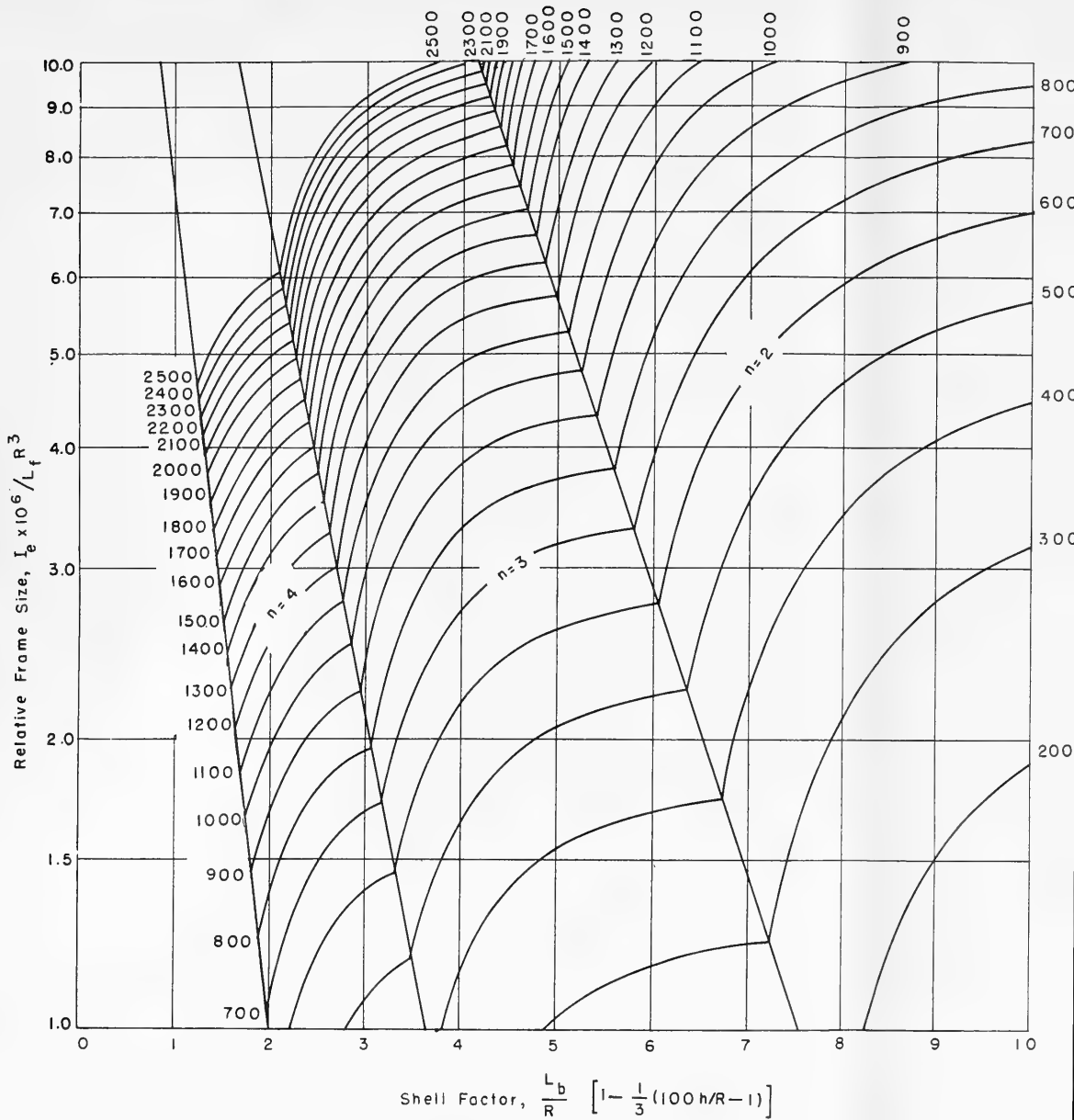


Figure 3 - Curves of Constant Pressure



$\left[1 - \frac{1}{3}(100 h/R - 1)\right] L_b/R$  and  $I_e/L_f R^3$ . Here plots of constant values for  $p_{cr}$  appear as families of curves intersecting along straight lines. These lines divide the graph into regions of  $n = 2, 3, 4,$  and  $5$ . A close estimate of the collapse pressure can thus be obtained directly from this figure once the coordinates are known. For a more precise determination of  $p_{cr}$  Figures 1 and 2 should be used after the appropriate value of  $n$  has been found from Figure 3.

### ACCURACY OF RESULTS

In the construction of Figures 1 through 3, some scatter in the calculated points was unavoidable, as might be expected. However, in only a few isolated instances was the difference between the graphical results and those from Kendrick Part III as high as 15 percent. In the great majority of cases, it was less than 10 percent. A representative set of results obtained from Figures 1 to 3 is presented for comparison with Kendrick's theory in Table 1.

### ACKNOWLEDGMENT

The author is indebted to Dr. E. Wenk, Jr., at whose suggestion this project was initiated, and to the members of the Applied Mathematics Laboratory for their valuable assistance in programming the numerous calculations on UNIVAC.

TABLE 1

Comparison of Numerical Results with Kendrick's (Part III) Theory

| Relative Thickness<br>$100h/R$ | Relative Length<br>$L_y/R$ | Relative Frame Stiffness<br>$I_e/L_y R^3$ | Collapse Pressure $p_{cr}$ |                        | Mode of Collapse $n$ |                        |
|--------------------------------|----------------------------|---|----------------------------|------------------------|----------------------|------------------------|
|                                |                            |   | From Figures 1 and 2       | From Kendrick's Theory | From Figure 3        | From Kendrick's Theory |
| 0.6                            | 2.194                      | $4.93 \times 10^{-6}$                     | 1632                       | 1558                   | 3                    | 3                      |
| 0.8                            |                            | $5.99 \times 10^{-6}$                     | 2096                       | 2036                   | 3                    | 3                      |
| 1.0                            |                            | $6.62 \times 10^{-6}$                     | 2475                       | 2409                   | 3                    | 3                      |
| 1.22                           |                            | $7.15 \times 10^{-6}$                     | 2850                       | 2766                   | 3                    | 3                      |
| 0.6                            | 4.539                      | $4.93 \times 10^{-6}$                     | 1015                       | 959                    | 3*                   | 2                      |
| 0.8                            |                            | $5.99 \times 10^{-6}$                     | 1260                       | 1218                   | 3*                   | 2                      |
| 1.0                            |                            | $6.62 \times 10^{-6}$                     | 1430                       | 1464                   | 3*                   | 2                      |
| 1.22                           |                            | $7.15 \times 10^{-6}$                     | 1572                       | 1671                   | 3                    | 3                      |
| 0.6                            | 6.884                      | $4.93 \times 10^{-6}$                     | 529                        | 520                    | 2                    | 2                      |
| 0.8                            |                            | $5.99 \times 10^{-6}$                     | 662                        | 623                    | 2                    | 2                      |
| 1.0                            |                            | $6.62 \times 10^{-6}$                     | 757                        | 715                    | 2                    | 2                      |
| 1.22                           |                            | $7.15 \times 10^{-6}$                     | 853                        | 804                    | 2                    | 2                      |
| 0.6                            | 9.229                      | $4.93 \times 10^{-6}$                     | 430                        | 455                    | 2                    | 2                      |
| 0.8                            |                            | $5.99 \times 10^{-6}$                     | 530                        | 534                    | 2                    | 2                      |
| 1.0                            |                            | $6.62 \times 10^{-6}$                     | 592                        | 599                    | 2                    | 2                      |
| 1.22                           |                            | $7.15 \times 10^{-6}$                     | 651                        | 659                    | 2                    | 2                      |
| 0.6                            | 2.194                      | $1.145 \times 10^{-6}$                    | 561                        | 539                    | 4                    | 4                      |
| 0.8                            |                            | $1.385 \times 10^{-6}$                    | 703                        | 660                    | 4                    | 4                      |
| 1.0                            |                            | $1.581 \times 10^{-6}$                    | 810                        | 781                    | 4                    | 4                      |
| 1.22                           |                            | $1.773 \times 10^{-6}$                    | 937                        | 923                    | 4                    | 4                      |
| 0.6                            | 4.539                      | $1.145 \times 10^{-6}$                    | 310                        | 293                    | 3                    | 3                      |
| 0.8                            |                            | $1.385 \times 10^{-6}$                    | 377                        | 347                    | 3                    | 3                      |
| 1.0                            |                            | $1.581 \times 10^{-6}$                    | 435                        | 401                    | 3                    | 3                      |
| 1.22                           |                            | $1.773 \times 10^{-6}$                    | 492                        | 464                    | 3                    | 3                      |
| 0.6                            | 6.884                      | $1.145 \times 10^{-6}$                    | 235                        | 223                    | 2                    | 2                      |
| 0.8                            |                            | $1.385 \times 10^{-6}$                    | 290                        | 281                    | 2                    | 2                      |
| 1.0                            |                            | $1.581 \times 10^{-6}$                    | 363                        | 338                    | 2                    | 2                      |
| 1.22                           |                            | $1.773 \times 10^{-6}$                    | 413                        | 391                    | 3                    | 3                      |
| 0.6                            | 9.229                      | $1.145 \times 10^{-6}$                    | 136                        | 136                    | 2                    | 2                      |
| 0.8                            |                            | $1.385 \times 10^{-6}$                    | 168                        | 164                    | 2                    | 2                      |
| 1.0                            |                            | $1.581 \times 10^{-6}$                    | 198                        | 191                    | 2                    | 2                      |
| 1.22                           |                            | $1.773 \times 10^{-6}$                    | 229                        | 223                    | 2                    | 2                      |

\*For these three geometries, collapse pressures for  $n = 2$  were nearly the same as for  $n = 3$ .



## APPENDIX A NUMERICAL EXAMPLE

A numerical example is provided to illustrate the use of the curves presented herein. A typical case has been chosen in which the dimensions could be those of a submarine pressure hull with external frames. The pertinent scantlings are:

$$\begin{array}{ll}
 R = 96 \text{ in.} & A_f = 9.63 \text{ in.}^2 \\
 h = 0.768 \text{ in.} & I_f = 80.48 \text{ in.}^4 \\
 L_f = 30 \text{ in.} & e = 4.49 \text{ in.} \\
 L_b = 384 \text{ in.} &
 \end{array}$$

The cylinder has 13 frames with one short bay at each end. First the value of the effective length  $L_e$  is found to be 13.48 in. Then

$$\frac{I_e}{L_f R^3} \text{ is } 6.85 \times 10^{-6}$$

and

$$\left[ 1 - \frac{1}{3} (100 h/R - 1) \right] \frac{L_b}{R} \text{ is } 4.26$$

From Figure 3,  $p_{cr}$  is found to be approximately 1500 psi corresponding to the mode  $n = 3$ . Using the ratio  $L_b/R = 4$  in Figure 1,  $p_s \cdot R/100h$  is 150 psi. This is multiplied by 0.80, the value of  $100h/R$ , to give  $p_s = 120$  psi. From Figure 2, the value  $p_f$  for  $n = 3$  is 1370 psi. Adding these two pressures,  $p_{cr}$  is found to be 1490 psi as compared with 1500 psi found directly from Figure 3.

Table 2 compares the values of  $p_{cr}$  for this example as determined by Kendrick Part I, Kendrick Part III, Bryant, and this graphical method. While the pressure obtained from the graphs agrees closely with Kendrick Part III, it is interesting to note that both the Kendrick Part I and Bryant values are considerably higher and predict a different number of lobes.

TABLE 2  
Example of Results Given by Various Methods

|                   | Bryant | Kendrick<br>Part I | Graphical<br>Method | Kendrick<br>Part III |
|-------------------|--------|--------------------|---------------------|----------------------|
| $p_{cr}$<br>(psi) | 2045   | 1855               | 1490                | 1409                 |
| $n$               | 2      | 2                  | 3                   | 3                    |

**APPENDIX B**  
**APPROXIMATE FORMULA FOR  $I_e$**

Considerable computation is required to determine the parameter  $I_e$ , which is a measure of frame strength. To simplify this work, an approximate formula has been derived which is applicable to frames of various cross-sectional shapes.

As previously defined,  $I_e$  is the moment of inertia of the combination of a frame plus an effective length of shell  $L_e$  about the centroid of the combined section. Figure 4 is a diagram of the most general frame shape in practical use, i.e., the builtup H-section with unequal flanges. Other shapes, such as T- or I-sections, are special cases of this shape. Referring to Figure 4, the quantity  $e$  of Equation [1] on page 3 can be written:

$$e = \frac{1}{A_f} \left[ bt \left( d - \frac{t}{2} \right) + \frac{as}{2} (d + u - t) + \frac{cu^2}{2} \right] + \frac{h}{2} \quad [3]$$

where  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $s$ ,  $t$ , and  $u$  are defined in Figure 4. Using the relationship

$$A_f = as + bt + cu \quad [4]$$

Equation [3] can be rearranged so that

$$e = \frac{1}{A_f} \left[ (d-t) \left( bt + \frac{as}{2} \right) + \frac{uA_f}{2} + \frac{bt}{2} (t-u) \right] + \frac{h}{2} \quad [5]$$

If the small quantity  $t - u$  is neglected, Equation [5] becomes

$$e = \frac{1}{2} \left\{ (d-t) \left[ 1 + \frac{bt - cu}{A_f} \right] + u + h \right\} \quad [6]$$

and  $A_f e^2$  is written

$$A_f e^2 = \frac{A_f}{4} (d-t)^2 \left[ 1 + \frac{bt - cu}{A_f} + \frac{u + h}{d-t} \right]^2 \quad [7]$$

The quantity  $I_f$  can also be simplified if a few small terms are neglected. The exact expression for  $I_f$  is

$$I_f = \frac{1}{12} (bt^3 + a^3s + cu^3) + bt \left[ d - \frac{t}{2} - e + \frac{h}{2} \right]^2 \quad [8]$$

$$+ as \left[ \frac{1}{2} (d-t + u + h) - e \right]^2 + cu \left[ \frac{1}{2} (u + h) - e \right]^2$$

Using the expression for  $e$  of Equation [6] the above becomes

$$I_f = \frac{1}{12} (bt^3 + a^3s + cu^3) + \frac{bt}{4} \left[ d - u - (d-t) \frac{(bt-cu)}{A_f} \right]^2 \quad [9]$$

$$+ \frac{as}{4} \left[ (d-t) \frac{(bt-cu)}{A_f} \right]^2 + \frac{cu}{4} (d-t)^2 \left[ 1 + \frac{bt-cu}{A_f} \right]^2$$

With the approximation  $u = t$ , Equation [9] becomes

$$I_f = \frac{1}{12} (bt^3 + a^3s + cu^3) + \frac{bt}{4} (d-t)^2 \left[ 1 - \frac{bt-cu}{A_f} \right]^2 \quad [10]$$

$$+ \frac{as}{4} \left[ (d-t) \frac{(bt-cu)}{A_f} \right]^2 + \frac{cu}{4} (d-t)^2 \left[ 1 + \frac{bt-cu}{A_f} \right]^2$$

or,

$$I_f = \frac{1}{12} (bt^3 + a^3s + cu^3) + \frac{(d-t)^2}{4} \left[ bt + cu - \frac{(bt-cu)^2}{A_f} \right] \quad [11]$$

With a further simplification that the first term of Equation [11] is approximately  $\frac{as}{12} (d-t)^2$ , the resulting expression for  $I_f$  is

$$I_f = \frac{(d-t)^2}{12} A_f \left[ 1 + 2 \frac{(bt+cu)}{A_f} - \frac{3}{A_f^2} (bt-cu)^2 \right] \quad [12]$$

In calculating  $I_e$  from Equation [1], the term  $L_e h^3/12$  is usually so small that it can be neglected. With the substitution of Equations [7] and [12] into Equation [1], the terms can be rearranged so that

$$I_e = \frac{A_f(d-t)^2}{4} \left\{ \frac{1}{1 + \frac{L_e}{L_e h}} \left[ 1 + \frac{bt-cu}{A_f} + \frac{h+u}{d-t} \right]^2 \right. \quad [13]$$

$$\left. + \frac{1}{3} \left[ 1 + 2 \frac{(bt+cu)}{A_f} - 3 \left( \frac{bt-cu}{A_f} \right)^2 \right] \right\}$$

For an I-beam a simpler expression is obtained. Since  $b = c$  and  $u = t$  Equation [3] becomes

$$e = \frac{d + h}{2} \quad [14]$$

while the Equation [12] for  $I_f$  reduces to

$$I_f = \frac{(d - t)^2}{12} A_f \left( 1 + \frac{4bt}{A_f} \right) \quad [15]$$

The expression for  $I_e$  is

$$I_e = \frac{A_f}{4} \left[ \frac{(d + h)^2}{1 + \frac{L_e h}{A_f}} + \frac{(d - t)^2}{3} \left( 1 + \frac{4bt}{A_f} \right) \right] \quad [16]$$

For a T-section, the quantities  $c$  and  $u$  are zero and Equation [5] can be written

$$e = \frac{1}{A_f} \left[ (d - t) \left( \frac{bt + A_f}{2} + \frac{bt^2}{2} \right) + \frac{h}{2} \right] \quad [17]$$

which reduces to

$$e = \frac{d}{2} \left( 1 + \frac{bt}{A_f} - \frac{t}{d} \right) + \frac{h}{2} \quad [18]$$

Substituting this expression in Equation [8]

$$I_f = \frac{1}{12} (bt^3 + a^3s) + \frac{d^2}{4} bt \left( 1 - \frac{bt}{A_f} \right) \quad [19]$$

or

$$I_f = \frac{d^2 A_f}{12} \left[ \left( 1 - \frac{2t}{d} \right) \left( 1 - \frac{bt}{A_f} \right) + \frac{t^2}{d^2} \right] \quad [20]$$

$$+ \frac{d^2}{4} bt \left( 1 - \frac{bt}{A_f} \right)$$

If the small quantity  $t^2/d^2$  is neglected, this can be reduced to

$$I_f = \frac{d^2 A_f}{12} \left( 1 - \frac{bt}{A_f} \right) \left( 1 - \frac{2t}{d} + \frac{3bt}{A_f} \right) \quad [21]$$

and

$$I_e = \frac{A_f d^2}{12} \left[ \frac{3}{1 + \frac{A_f}{L_e h}} \left( 1 + \frac{bt}{A_f} + \frac{h-t}{d} \right)^2 + \left( 1 - \frac{bt}{A_f} \right) \left( 1 - \frac{2t}{d} + \frac{3bt}{A_f} \right) \right] \quad [22]$$

To demonstrate the accuracy of these frame formulas some numerical examples are provided. Table 3 lists the dimensions of several frames representing shapes and sizes which might be used in submarine construction. Type 1 is an H-section with unequal flanges which was used as the example in Appendix A. Types 2 - 5 are I- and T-sections taken from the steel construction manual of the American Institute of Steel Construction.<sup>7</sup> Table 4 lists shell dimensions for each of these frames and compares the values of  $I_f$  and  $I_e$  calculated from the simplified formulas with the exact values from Reference 7. The error in each case is found to be less than 2 percent.

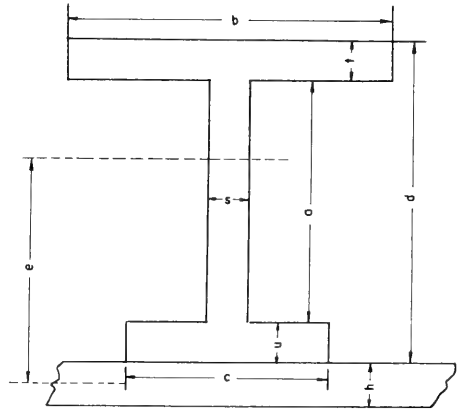


Figure 4 - Identification of Symbols Used in Calculating  $I_e$

**TABLE 3**  
Dimensions of Typical Frame Cross Sections

| Frame Type | Shape          | Frame Dimensions in inches |       |       |       |       |       |       |
|------------|----------------|----------------------------|-------|-------|-------|-------|-------|-------|
|            |                | $A_f^*$                    | $d$   | $b$   | $c$   | $s$   | $t$   | $u$   |
| 1          | H              | 9.63                       | 7.28  | 6.06  | 4.34  | 0.555 | 0.633 | 0.555 |
| 2          | I<br>(6×3 3/8) | 5.02                       | 6.00  | 3.565 | 3.565 | 0.465 | 0.359 | 0.359 |
| 3          | I<br>(6×6)     | 5.90                       | 6.20  | 6.018 | 6.018 | 0.258 | 0.367 | 0.367 |
| 4          | ST 13 WF       | 13.83                      | 13.45 | 9.99  | —     | 0.490 | 0.747 | —     |
| 5          | ST 61          | 4.63                       | 6.00  | 5.00  | —     | 0.350 | 0.544 | —     |

\*Values of  $A_f$  in inches<sup>2</sup> for Frames 2-5 are those given in the A.L.S.C. Handbook.<sup>7</sup> In some cases, these include fillets not listed in this table.

**TABLE 4**  
Comparison of Approximate and Exact Values for  $I_f$  and  $I_e$

| Frame Type | $I_f$ in in. <sup>4</sup> |             | $R$ ,<br>Shell<br>Radius<br>in<br>inches | $h$ ,<br>Shell<br>Thickness<br>in<br>inches | $I_e$ in in. <sup>4</sup> |             |
|------------|---------------------------|-------------|--|---|---------------------------|-------------|
|            | Exact*                    | Approximate |  |   | Exact                     | Approximate |
| 1          | 80.5                      | 78.9        | 96                                       | 0.768                                       | 181.5                     | 178.5       |
| 2          | 26.0                      | 26.9        | 96                                       | 1.125                                       | 77.94                     | 76.9        |
| 3          | 41.7                      | 41.8        | 96                                       | 1.125                                       | 103.5                     | 101.7       |
| 4          | 238.5                     | 241.0       | 168                                      | 2.00  | 1618                      | 1589        |
| 5          | 14.90                     | 14.80       | 96                                       | 1.125                                       | 111.3                     | 109.3       |

\*Exact values of  $I_f$  for Frames 2-5 are those given in Reference 7.

## REFERENCES

1. Kendrick, S., "The Buckling under External Pressure of Circular Cylindrical Shells with Evenly Spaced Equal Strength Circular Ring Frames - Part I," Naval Construction Research Establishment, Great Britain, Report NCRE/R No. 211 (1953).
2. Kendrick, S., "The Buckling under External Pressure of Circular Cylindrical Shells with Evenly Spaced Equal Strength Circular Ring Frames - Part III," Naval Construction Research Establishment Report NCRE/R No. 244 (1953).
3. Bryant, A.R., "Hydrostatic Pressure Buckling of a Ring-Stiffened Tube," Naval Construction Research Establishment Report NCRE/R No. 306 (1954) CONFIDENTIAL.
4. Slankard, R.C., and Galletly, G.D., "The Effect of Reinforcing Rings on the General-Instability Strength of Machined Cylindrical Shells under External Hydrostatic Pressure," David Taylor Model Basin Report C-822 (June 1957) CONFIDENTIAL.
5. Bijlaard, P.P., "Buckling under External Pressure of Cylindrical Shells Evenly Stiffened by Rings Only," Bell Aircraft Corporation Report 02-984-010 (1953).
6. Timoshenko, S., "Theory of Plates and Shells," Engineering Society Monographs, McGraw-Hill (1940).
7. "Steel Construction Manual of the American Institute of Steel Construction" (1948).





# INITIAL DISTRIBUTION

Copies

- 11 CHBUSHIPS, Library (Code 312)
  - 5 Tech Library
  - 2 Prelim Design (Code 420)
  - 1 Ship Protection (Code 423)
  - 1 Hull Design (Code 440)
  - 1 Scientific and Research (Code 442)
  - 1 Submannes (Code 525)
- 1 CHBUAER
- 1 CNO, New Dev and Op Eval, OP 37
- 1 CHBUORD
- 1 CHBUDOCKS
- 1 CHONR, Mech Br (Code 438)
- 1 CDR, USNOL
  - 1 Supt, USNGF
  - 1 DIR, USNRL, Sci Info Div
  - 1 DIR, USNEES, Annapolis, Md.
  - 1 DIR, USNAES, NAMC, Philadelphia, Pa.  
Attn: Head, Aero Matls Lab
  - 1 CO, USNAMTC, Port Mugu, Calif.
  - 1 Manager, USN Aircraft Factory, NAMC,  
Philadelphia, Pa.
  - 1 OinC, USN Civ Engin Res & Eval Lab,  
Port Hueneme, Calif.
  - 1 DIR, USNEL, San Diego, Calif.
  - 1 CDR, USNOTS, China Lake, Calif.
  - 1 Supt, USN Post Grad School, Monterey, Calif.
  - 1 CDR, USNPG, Dahlgren, Va.
  - 1 NAVSHIPYD MARE
  - 1 NAVSHIPYD NYK
  - 1 NAVSHIPYD NORVA, Attn: UERD (Code 270)
  - 1 NAVSHIPYD PTSMH
  - 2 SUPSHIPINSORD, Groton, Conn.  
1 General Dynamics Corp
  - 1 Asst Sec of Air Force (Res & Dev)
  - 1 CDR, USAF Inst of Tech, Wright-Patterson AFB,  
Dayton, O. Attn: Chief, Appl Mech Gr
  - 1 CDR, Hdq, Air Materiel Command, Wright-Patterson  
AFB, Dayton, O.
  - 1 CDR, Hdq, Air Res & Dev Command, Baltimore, Md.  
Attn: RDMPE
  - 1 Chief, Res & Dev, Dept of Army
  - 1 CO, US Engr Res & Dev Lab, Fort Belvoir, Va.
  - 1 DIR, Tech Info Br, USA Provg Gr, Aberdeen, Md.
  - 1 Chief, AFSWP
  - 2 ASTIA, Dayton, O.
  - 1 Sec of Defense, Res & Dev, Tech Library
  - 1 Dir of Research, USAEC
  - 1 Office of Tech Services, Dept of Commerce

Copies

- 1 DIR, Langley Aero Lab, Structures Div,  
Langley Field, Va.
- 1 DIR, NACA, Loads and Structures Div
- 1 DIR, Natl BuStand, Attn: Dr. W.H. Ramberg
- 1 Committee on Undersea Warfare, NRC
- 1 Engineering Societies Library, New York, N.Y.
- 1 CO, USN Admin Unit, MIT, Cambridge, Mass.
- 1 Pressure Vessel Res Committee, New York, N.Y.
- 1 SNAME, New York, N.Y.
- 1 Applied Physics Lab, Johns Hopkins Univ,  
Silver Spring, Md.
- 1 DIR, ETT, SIT, Hoboken, N.J.
- 1 Admin, Webb Inst of Nav Arch, Long Island, N.Y.
- 1 American Bureau of Shipping, New York, N.Y.
- 1 Armour Research Fdn, Chicago, Ill.
- 1 Babcock & Wilcox Co., Res & Dev Dept,  
Alliance, O.
- 1 DIR, Forest Products Lab, Madison, Wis.
- 1 Gibbs & Cox, New York, N.Y.
- 1 Prof. Lynn S. Beedle, Fritz Engin Lab,  
Lehigh Univ, Bethlehem, Pa.
- 1 Prof. P.P. Bijlaard, Cornell Univ, Ithaca, N.Y.
- 1 Prof. R.L. Bisplinghoff, Dept of Aero Engin,  
MIT, Cambridge, Mass.
- 1 Prof. H.H. Bleich, Dept of Civ Engin, Columbia  
Univ, New York, N.Y.
- 1 Mr. H.C. Boardman, Chicago Bridge & Iron Co,  
Chicago, Ill.
- 1 Mr. W.R. Burrows, Standard Oil Co of Indiana,  
Whiting, Ind.
- 1 Dr. F.H. Clauser, Dept of Aero, Johns Hopkins  
Univ, Baltimore, Md.
- 1 Prof. Herbert Deresiewicz, Dept of Civ Engin,  
Columbia Univ, New York, N.Y.
- 1 Prof. T.J. Dolan, Dept of Theo & Appl Mech,  
Univ of Illinois, Urbana, Ill.
- 1 Prof. Lloyd Donnell, Dept of Mech, Illinois Inst  
of Tech, Chicago, Ill.
- 1 Prof. A.C. Eriegen, Div of Engin Sci, Purdue  
Univ, Lafayette, Ind.
- 1 Prof. J.H. Evans, Dept of NAME, MIT,  
Cambridge, Mass.
- 1 Prof. W. Flugge, Dept of Mech Engin, Stanford  
Univ, Stanford, Calif.
- 1 Mr. Martin Goland, Southwest Res Inst,  
San Antonio, Tex.
- 1 Prof. J.N. Goodier, Dept of Mech Engin,  
Stanford Univ, Stanford, Calif.
- 1 Prof. L.E. Goodman, EES, Univ of Minnesota,  
Minneapolis, Minn.

## Copies

- 1 Mr. F.V. Hartman, Aluminum Co. of America,  
Pittsburgh, Pa.
- 1 Prof. R.M. Hermes, Univ of Santa Clara,  
Santa Clara, Calif.
- 1 Mr. Harold O. Hill, Fab Steel Const, Bethlehem  
Steel Co, Bethlehem, Pa.
- 1 Prof. P.G. Hodge, Dept of Aero Engin & Appl Mech,  
PIB, Brooklyn, N.Y.
- 1 Prof. N.J. Hoff, Head, Dept of Aero Engin & Appl  
Mech, PIB, Brooklyn, N.Y.
- 1 Dr. Marshall Holt, Aluminum Res Lab,  
New Kensington, Pa.
- 1 Prof. W.H. Hoppmann, II, Dept of Mech Engin,  
Johns Hopkins Univ, Baltimore, Md.
- 1 Prof. Bruce G. Johnston, Univ of Michigan,  
Ann Arbor, Mich.
- 1 Prof. J. Kempner, Dept of Aero Engin & Appl Mech,  
PIB, Brooklyn, N.Y.
- 1 Prof. H.L. Langhaar, Dept of Theo & Appl Mech,  
Univ of Illinois, Urbana, Ill.
- 1 Prof. George H. Lee, Dir of Res, Rensselaer Polytech  
Inst, Troy, N.Y.
- 1 Prof. Aris Phillips, Dept of Civ Engin, Yale Univ,  
New Haven, Conn.
- 1 Mr. Henry Liessenberg, Stress Res Engr, Combustion  
Engin, Inc., New York, N.Y.
- 1 Mr. Frank L. Maker, Engin Specialist, Concord, Calif.
- 1 Prof. R.D. Mindin, Dept of Civ Engin, Columbia Univ,  
New York, N.Y.
- 1 Mr. James J. Murphy, Sec Engr, Dev Div, M.W. Kellogg  
Co, New York, N.Y.
- 1 Prof. Paul M. Naghdi, Dept of Engin Mech, Univ of  
Mich, Ann Arbor, Mich.
- 1 Dr. William A. Nash, Dept of Engin Mech, Univ of  
Florida, Gainesville, Fla.
- 1 Prof. N.M. Newmark, Dept of Civ Engin, Univ of  
Illinois, Urbana, Ill.
- 1 Prof. W. Prager, Chairman, Phys Sci Council,  
Brown Univ, Providence, R.I.
- 1 Prof. E. Reissner, Dept of Math, MIT, Cambridge, Mass.
- 1 Dr. R.P. Petersen, Dir, Appl Phy Div, Sandia Lab,  
Albuquerque, N.Mex.
- 1 Mr. Cyril O. Rhys, Sr., Independent Consultant, Hampton  
Hall, Cranford, N.J.
- 1 Mr. G.J. Schoessow, Babcock & Wilcox Co, Atomic  
Energy Div, Akron, O.
- 1 Mr. W. Spraragen, Dir, Welding Res Council, New York, N.Y.
- 1 Prof. R.J. Sturm Alabama Polytech Inst, Auburn Res  
Fdn & EES, Auburn, Ala.
- 1 Dr. F.K. Teichmann, Dept of Aero Engin, New York, N.Y.
- 1 Prof. S.P. Timoshenko, School of Engin, Stanford  
Univ, Stanford, Calif.

## Copies

- 1 Prof. Enrico Volterra, Dept of Mech, Rensselaer  
Polytech Inst, Troy, N.Y.
- 1 Prof. E.O. Waters, Yale Univ, New Haven, Conn.
- 1 Mr. D.B. Wesstrom, Des Div, Engin Dept,  
E.I. duPont de Nemours & Co, Wilmington, Del.
- 1 Prof. Dana Young, Yale Univ, New Haven, Conn.
- 1 Dr. E. Wenk, Jr., Dept Engin Mech, Southwest Res Inst,  
San Antonio, Tex.
- 1 ALUSNA, London, England
- 1 BSRA, London, England

