GRAPHIC STATICS

ALEXANDER AND THOMSON

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A GRADUATED SERIES OF PROBLEMS AND PRACTICAL EXAMPLES, WITH NUMEROUS DIAGRAMS ALL DRAWN TO SCALE.

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PREFACE.

IN our "Elementary Applied Mechanics" much of the work is done by Graphic Methods; but for want of space, the subject Graphic Statics is not treated systematically. The following problems and applications are intended to form an introduction to our larger work; but as they are complete in themselves, they may be studied as a separate subject.

For many years Graphic Statics has formed an important part of the regular work done by the students who have attended our classes; we find the subject exceedingly useful, readily followed, and easily understood by all.

Many problems, for the solution of which the analytical methods are laborious and difficult to follow, are easily solved by drawing; and students have a better appreciation of many of these problems after they have solved them by graphic methods. The best results are often obtained by using analytical *and* graphic methods on the same problem; part of the work being done by one method, part by the other. Frequently a large number of results are obtained by drawing; and the accuracy of the whole drawing may sometimes be tested by checking one of these results by calculation.

The authors have prepared a more advanced set of Graphical Exercises, drawn on a large scale, on sheets of quarter double elephant size: many of them in two colours. These exercises are nearly ready for publication.

T. ALEXANDER.
A. W. THOMSON.

CONTENTS.

PRACTICAL APPLICATIONS.

INTRODUCTION.

In the examples given in the following pages, a Force is represented by two lines; one, a line of indefinite length to represent the line of action of the force; the other, a line drawn parallel to it, whose length gives the magnitude of the force upon a scale which appears on the drawing. For accuracy, these lines should be drawn as thin as possible; the direction of the force should be indicated by an arrow head; and the point of the arrow should indicate the end of the line which represents the magnitude of the force.

A number of forces which act simultaneously upon a body is called a "set of forces;" and when the lines of action of these forces are all in one plane, they are called a "plane set of forces."

A plane set of forces cannot produce change of motion, or tendency to change of motion, normal to their plane. Two sets of forces which produce the same tendency to change of motion are called "equivalent sets." Of two equivalent sets of forces that which consists of the fewer number of forces is called the "simpler;" and of all equivalent sets, that which consists of the fewest forces is called the simplest equivalent set.

In some cases the simplest equivalent set consists of one force—the resultant force; in others, it consists of two forces,—the resultant couple.

In numbering and arranging the forces, we will take them in *cyclic* order; that is—the numbers will follow each other as the figures on a watch dial; this is frequently of much importance.

The diagrams given have all been drawn to a large scale and reduced; for accuracy of work, large scale drawings are essential.

PROBLEM I.

GIVEN—A plane set of four forces P_1, P_2, P_3, P_4 acting at a point.
FIND—The simplest equivalent set.

Let the forces intersect at point A, Fig. 1; and let $P_1 = 7$ lbs. inclined at 30° from the line marked 0° on the protractor; similarly $P_2 = 10$, 75°; $P_3 = 9$, 105°; and $P_4 = 15$, 135°.

In the diagram, the letters P are omitted; and the forces are numbered in cyclic order.

Construct the force polygon; begin at any convenient point B, and draw $P_1 = 7$ lbs. to scale, and parallel to P_1 of the left hand figure; then draw in succession P_2, P_3, P_4 of the proper lengths to scale, and parallel to the lines of the original figure. Join the two ends of the figure thus formed by a dotted line; this gives $P_5 = 32{\cdot}7$ lbs. at 97°. Draw the dotted line on the original figure, parallel to the dotted line of the Force Polygon; the forces P_1, P_2, P_3, P_4, and $P_{5'}$ upwards, are in equilibrium; and P_5 downwards through A is equivalent to P_1, P_2, P_3, and P_4.

If the original lines P_1, P_2, P_3, and P_4 of the force polygon form a closed figure, the set is a balanced set.

PROBLEM II.

GIVEN—A plane set of three forces P_1, P_2, P_3, acting at a point.
FIND—A simpler equivalent set of two, P_4, P_5 at the same point, whose lines of action are given.

Let the forces intersect at point A, Fig. 2; let $P_1 = 15$ lbs. at 330°; $P_2 = 20$ lbs. at 45°; $P_3 = 33$ lbs. at 90°; and let P_4 act at 0°, and P_5 at 120°.

Construct the force polygon; from any point B draw $P_1 = 15$ lbs. to scale, and parallel to P_1 of the left hand figure; similarly draw, in succession, P_2 and P_3. From the starting point, draw a dotted line P_5; and from the last point, a dotted line P_4; these two dotted lines will intersect at C; and their lengths will give $P_4 = 50$ lbs. and $P_5 = 46$ lbs. to scale.

The five lines $P_1, P_2, P_3, P_{4'}$, and $P_{5'}$ of the force polygon form a closed figure; these five forces acting at A are in equilibrium; reversing directions of $P_{4'}$ and $P_{5'}$ we get P_4 and P_5 in the required directions, and forming a set equivalent to P_1, P_2. and P_3.

PROBLEM III.

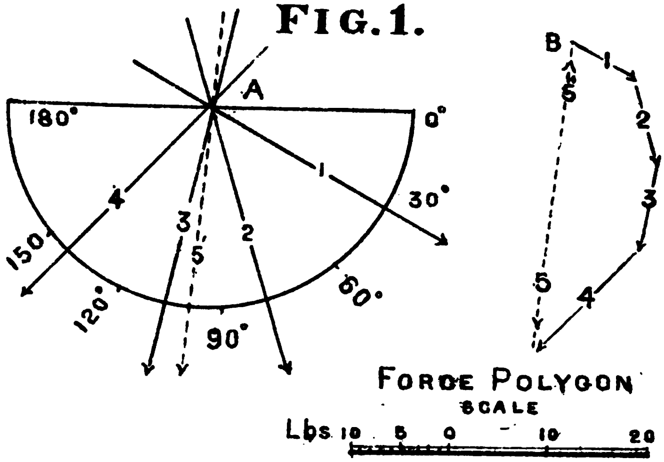
GIVEN—A plane parallel set of forces P_1, P_2; FIND—A balancing set of two P_3, P_4 whose lines of action are given.

For convenience we take the parallel forces to be vertical.

Let $P_1 = 5$ tons, $P_2 = 10$ tons; and let the positions of the lines of action of these forces and of the forces P_3 and P_4 be given by measurements along a horizontal line, thus—$x_4 = 0$, $x_1 = 10$, $x_2 = 16$, and $x_3 = 30$ feet. Fig. 3.

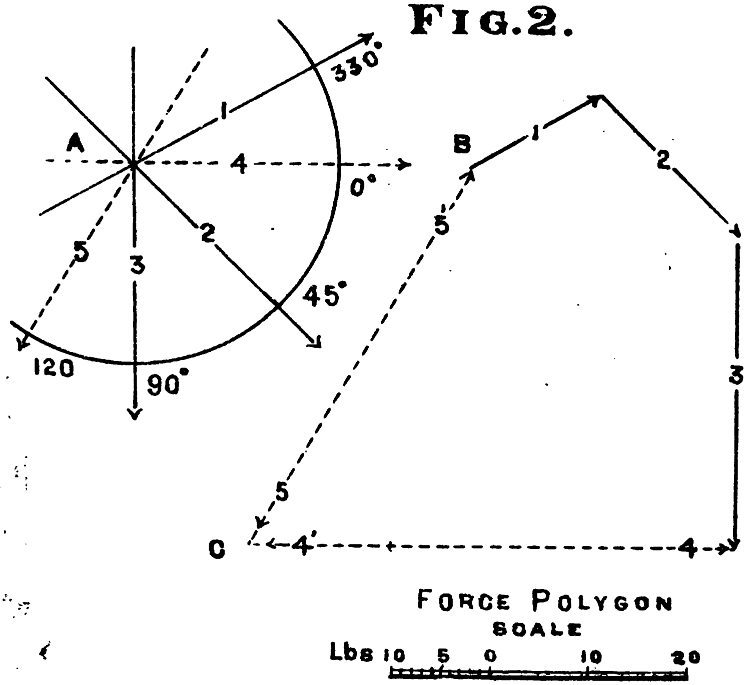
Construct the force polygon—a vertical straight line,—to scale; $P_1 = 5$, and $P_2 = 10$ tons. For equilibrium $P_3 + P_4 = P_1 + P_2$; but as yet we do not know the amounts of P_3 and P_4 separately. P_3 and P_4 in the force polygon should be in the

FIG. 1.



A
180°
0°
30°
60°
90°
120°
150
1
2
3
4
5
B
FORCE POLYGON
SCALE
Lbs 10 5 0 10 20

FIG. 2.



A
330°
0°
45°
90°
120
1
2
3
4
5
B
C
4'
FORCE POLYGON
SCALE
Lbs 10 5 0 10 20

same vertical line with P_1 and P_2; but for clearness they are drawn a little to the right.

Choose *any* point O as a *pole*, No. 1 and draw rays 5, 6, and 7.

Construct the link polygon DABC; begin at D *any* point in the line of action of P_4, and draw link DA parallel to ray 5, link AB parallel to ray 6, and link BC parallel to ray 7; and number the links 5, 6, and 7 to correspond with the rays to which they are parallel. The ray 5 joins the vertical force line between 4 and 1, the link 5 joins the lines of action 4 and 1; the ray 6 joins between 1 and 2, the link 6 joins 1 and 2; and so on. Join the ends DC of the link polygon by the dotted line 8; and from pole O draw the dotted ray 8, parallel to DC, to meet the vertical force line; this gives the amounts $P_3 = 7$ tons and $P_4 = 8$ tons measured by scale. If P_3 and P_4 act upwards, they balance P_1 and P_2; if P_3 and P_4 act downwards, they form a set equivalent to P_1 and P_2; the lines of action of P_3 and P_4 were originally fixed by the conditions of the problem.

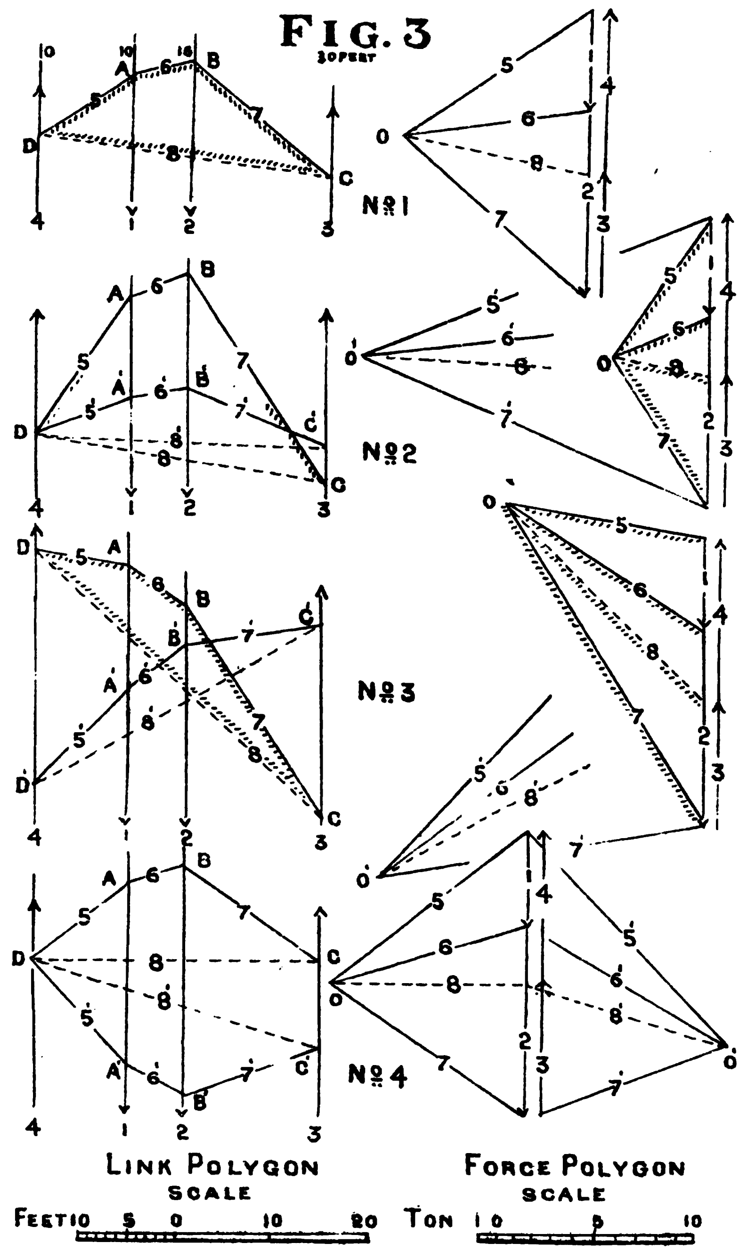
Consider the link polygon to be a framed structure, made of four pieces DC, DA, AB, BC, with free hinged joints D, A, B, and C, with loads P_1 and P_2 at A and B, and with supporting forces P_3 and P_4 at C and D; then the structure is in equilibrium. The lines 5, 6, 7, 8 of the force polygon represent, in direction and amount, the stresses in the corresponding members of the link polygon, induced by the given loads and supporting forces. Each joint of the structure is in equilibrium; thus—the three forces 5, 1, 6, acting at A, form in the force polygon a triangle, and therefore produce equilibrium; similarly 6, 2, 7 at B; 7, 3, 8 at C; and 8, 4, 5, at D; by going *round* each triangle, we get the direction in which each force acts at each joint. It will be observed that each of the forces 5, 6, 7, and 8 appears twice and in opposite directions; and as each pair 5, 5; 6, 6; 7, 7; 8, 8 act in one straight line, the length of the link, they may all be removed without impairing the equilibrium of the frame; this leaves P_1, P_2, P_3, and P_4 in equilibrium; that is to say P_3 and P_4 have been determined.

In the frame shown No. 1, DA, AB,BC are struts and DC is a tie. The amounts of P_3 and P_4 found by diagram may be checked by calculation; thus—taking moments about D we have

$$P_1 x_1 + P_2 x_2 = P_3 x_3 ; \; 5 \times 10 + 10 \times 16 = P_3 \times 30 ;$$
$$P_3 = 7 \text{ tons; and similarly } P_4 = 8 \text{ tons.}$$

The shape of the link polygon depends on the position chosen for the pole on the force polygon; if the pole is taken near the vertical load line, No. 2, the height of the link polygon is great; if taken far away from the load line, No. 2, the height is small; if taken high up, or low down on the figure, No. 3, the link polygon is much distorted; if taken on the left hand side, then DA, AB, BC are struts, DC is a tie; if taken on the right hand side of the load line, No 4, then the link polygon is inverted, and DA′, A′B′, B′C′ are ties, and DC′ is a strut. If we suppose that P_3 and P_4 have already been found, and if pole O be now taken on the same level as the top of P_3 in the force line, No. 4, then the line 8 is horizontal; the link polygon has the side DC horizontal, that is, it has a horizontal base.

FIG. 3



PROBLEM IV.

GIVEN—A plane set of three forces P_1, P_2, P_3; of which P_2 and P_3 are equal, parallel, opposite, and not in one line.

FIND—The simplest equivalent set.

The forces P_2 and P_3 form a couple; it will be balanced by any opposite couple, in the same plane, and of equal moment. Since P_1 is given, the arm of the required couple, that is the distance between P_1 and P_4, remains to be found.

Let $P_1 = 5$ lbs; $P_2 = P_3 = 10$ lbs.; and let the distance between P_2 and P_3 be 10 feet. Fig, 4.

Construct the force polygon. The lines P_1, P_2, and P_3 are drawn to scale; to close the figure it is evident that $P_1 = P_4$. The lines P_1 and P_4, and the lines P_2 and P_3 coincide; but for clearness, P_3 and P_4 are drawn slightly out of position. Any point O may be taken as the pole; but in this case it is convenient to take O the pole at the intersection of a and c, lines drawn at right angles to P_1 and P_2.

Construct the link polygon. From A any point in P_1 draw AB parallel to b; draw BC $\parallel c$; and from A and C draw dotted lines $\parallel a$ and b, intersecting at D. The force P_4 acts through D, and is $\parallel P_1$.

Produce AD and BC to meet at E. Then in the triangle AEB

EB : EA : : CB : DA : : sin α : sin β; DA·sin α = CB sin β.

In the force polygon $- P_1 : P_2 : :$ sin α : sin β; $\therefore$

$P_1 \times DA = P_2 \times CB$; and DA $= 20$ feet.

that is—the two couples have equal moments.

The force $P_{4'}$ acting downwards is equivalent to P_1, P_2, and P_3; hence compounding a force and a couple in the same plane merely shifts the force parallel to itself through a distance equal to the moment of the couple divided by the force; to the right if the couple be right handed; to the left if left handed.

PROBLEM V.

GIVEN—A plane set of four forces P_1, P_2, P_3, P_4, for which the force polygon closes.

FIND—The simplest equivalent set.

Let the forces P_1, P_2, P_3, P_4, amount to 12, 20, 22, 28 lbs. intersect a horizontal line at 0, 10, 18, 25 feet, and be inclined to horizontal at 90°, 45°, 90°, 60°. Fig. 5.

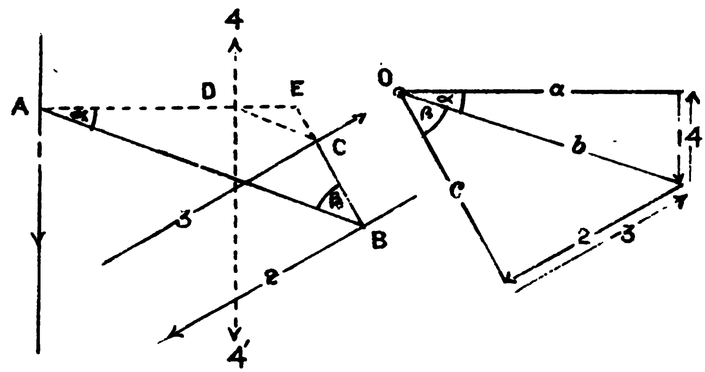
Leave out any one of the given forces,—say P_1; find as in problem IV. the position of $P_{1'}$ equivalent to P_2, P_3, and P_4.

Construct the force polygon. Take any point O as pole and draw rays 5, 6, 7, and 8.

Construct the link polygon. From B any point in P_2, draw BC $\parallel$ 6, and CD $\parallel$ 7; draw the dotted lines BA $\parallel$ 5, and DA $\parallel$ 8; they intersect at A; and $P_{1'}$ acting downwards through A is equivalent to P_2, P_3, and P_4.

$P_{1'}$ and the given force P_1 are equivalent to original set of four forces; they form a left hand couple whose moment is

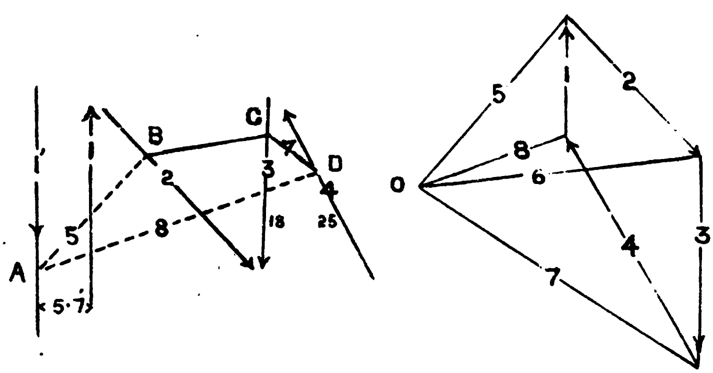
FIG. 4.



LINK POLYGON
SCALE
FEET 10 5 0 10

FORCE POLYGON
SCALE
Lbs 10 5 10

FIG. 5.



LINK POLYGON
SCALE
FEET 10 5 0 10

FORCE POLYGON
SCALE
Lbs 10 5 0 10

$P_1 \times d = 12 \times 5{\cdot}7 = 68{\cdot}4$ foot-pounds.

By leaving out any of the other forces P_2, or P_3, or P_4 a left hand couple of moment equal to the above will be obtained.

It appears then that *any* left hand couple in the given plane, whose moment is 68·4 ft. lbs. is equivalent to P_1, P_2, P_3, and P_4. If the point A happens to fall on the line of action of P_1, then the four given forces are in equilibrium.

PROBLEM VI.

GIVEN—A plane set of four forces P_1, P_2, P_3, and P_4. FIND—An equivalent set of two forces P_5 and P_6; the line of action of P_5 being given, and a point A on the line of action of P_6.

Let the forces (Fig. 6)	P_6	P_1	P_2	P_3	P_4	P_5	
amount to	—	10	8	6	12	—	lbs.
intersect a horizontal line at	0	8	15	23	30	35	feet.
and be inclined to horizon at	—	60°	75°	90°	60°	75°	

Construct the force polygon. Begin at any point G; draw P_1, P_2, P_3, and P_4 to scale and at the proper inclinations; from H draw a dotted line at slope given for P_5, and of indefinite length; choose any point O as pole, and draw rays 7, 8, 9, 10, and 11.

Construct the link polygon. Begin at the given point A; draw links AB, BC, CD, DE, and EF; join AF by a dotted line.

From O, draw a dotted line OK ‖ AF, to meet the dotted line P_5 at K; join KG; then HK represents the amount of P_5, and KG the amount and direction of P_6. Through A draw a dotted line parallel to KG; this gives the line of action of P_6.

$P_5 = 18\frac{1}{2}$ lbs.; $P_6 = 16$ lbs. inclined at $68\frac{1}{2}^\circ$ to the horizon. P_5 and P_6 acting upwards balance P_1, P_2, P_3, and P_4; P_5, and P_6, acting downwards form the equivalent set required.

PROBLEM VII.

GIVEN—A plane parallel set of six forces P_1, P_2, P_3, P_4, P_5, and P_6. FIND—The simplest equivalent set.

Let $P_1 = 3$, $P_2 = 5$, $P_3 = -6$, $P_4 = 10$, $P_5 = 8$, and $P_6 = -4$ tons.

$x_1 = 0$, $x_2 = 6$, $x_3 = 13$, $x_4 = 20$, $x_5 = 32$, and $x_6 = 40$ feet.

The negative sign denotes that the force is upwards. Fig. 7.

Construct the force polygon. Note that for distinctness there are four vertical lines on which the forces are shewn; the ends of all the forces are projected on to the left hand line, and to these points the rays are drawn.

Construct the link polygon. Begin at A, any point in the line of action of P_1; draw the links 9, 10, 11, 12, and 13. From A draw the dotted line 8, and from B the dotted line 14, parallel to the corresponding rays; they intersect at C, a point in the line of action of P_7. $P_7 = 16$ tons; $x_7 = 15\frac{1}{2}$ feet.

If P_7 acts upwards, the seven forces are balanced; if P_7 acts down, it is the simplest set equivalent to the six given forces.

Fig. 6.

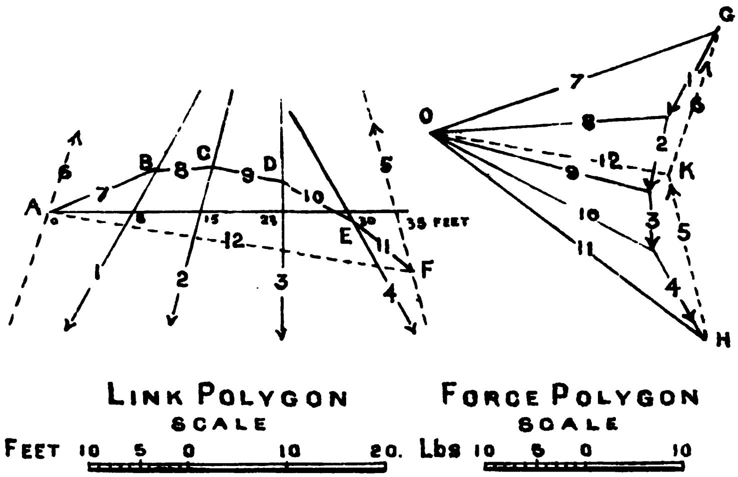
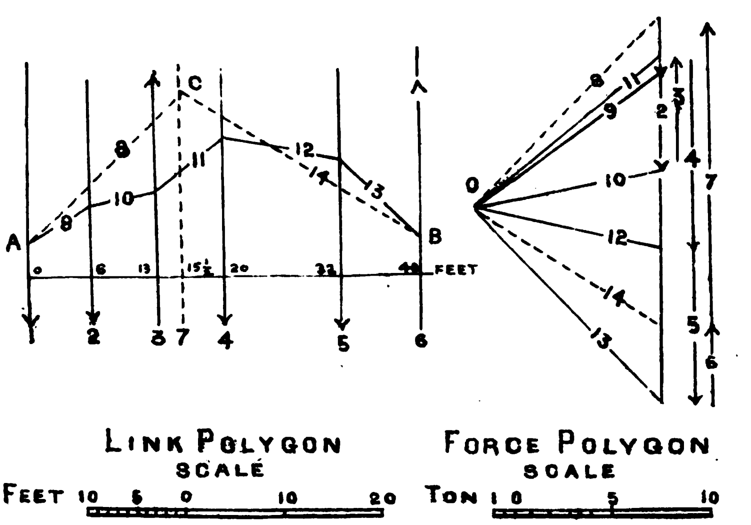

A
B
C
D
E
F
G
H
K
O
35 FEET
Link Polygon
Scale
Feet 10 5 0 10 20.
Force Polygon
Scale
Lbs 10 5 0 10

Fig. 7.


A
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C
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FEET
Link Polygon
Scale
Feet 10 5 0 10 20
Force Polygon
Scale
Ton 1 0 5 10

PROBLEM VIII.

GIVEN—A plane parallel set of five forces P_1, P_2, P_3, P_4, and P_5.
FIND—An equivalent set of two P_6 and P_7, two points E and F one on each line of action being given; and draw a link polygon which shall have the given line EF as its base.

Let $P_1 = 5$; $P_2 = 7$; $P_3 = -9$; $P_4 = 6$; $P_5 = 9$ tons. Fig. 8. $x_7 = 0$; $x_1 = 5$; $x_2 = 17$; $x_3 = 13$; $x_4 = 25$; $x_5 = 35$; $x_6 = 40$ feet.

Construct the force polygon. Take O *any* point for the pole.

Construct link polygon N° 1, beginning at A any point in the line of action of P_7; observe that P_2 is on the right side of P_3. Join AB by the dotted line 14; and from the pole O, draw 14 to determine the amount of P_6 and of P_7. By measuring on the force polygon, $P_6 = 12{\cdot}3$ lbs. and $P_7 = 5{\cdot}7$ lbs. If P_6 and P_7 act upwards, the seven forces balance; if downwards, they form the required set equivalent to the five given forces.

The two points E and F on the lines of action of P_6 and P_7 being given; the link polygon N° 2 is readily drawn with EF as base. From point C on the force line, that is from the top of P_6, draw a dotted line parallel to EF; and on it take any point O′ as pole; link polygon N° 2 satisfies the given condition.

If the forces forming the given set are arranged symmetrically, the two forces P_6 and P_7 are equal to each other, and it will not be necessary to draw link polygon No. 1.

PROBLEM IX.

GIVEN—A plane symmetrical parallel set of equal forces at equal distances apart; FIND—The form of the link polygon.

Let the number of parallel vertical downward forces be eleven, P_1, P_2,......P_{11}; and the two upward balancing forces P_{12}, and P_{13}; let each of the eleven forces be 2 tons, then $P_{12} = P_{13} = 11$ tons; let interval between each pair of forces be 4 feet. Construct the force polygon. Draw a horizontal dotted line from the top of P_{12}; fix the pole at any point O in this line.

Construct link polygon No. 1. Begin at A any point in the line of action of P_{13}. The base AC, 26, will be horizontal. It is evident that B is the highest point in the link polygon. Draw the short horizontal line BD, Taking the slope of a line as vertical ÷ horizontal; then if the slope of 19 is 1 that of 18 is 3, that of 17 is 5, and so on; the depths measured from line BD, to E, F, and G are therefore in the proportion of 1, 4, 9; these increase as the square of the horizontal distance from B measured along BD; ABC is a parabola, and B is its vertex.

For link polygon No. 2, the pole O′ is fixed, for convenience, on the level of the middle of any of the vertical force lines of the force polygon, say on the middle of P_8; draw rays 14′......25′. The point H is the highest point in the link polygon; draw HK; the depths from HK to the link polygon are in the same proportion as in the case of link polygon No. 1. The figure is therefore a parabola, and H is its vertex.

FIG. 8

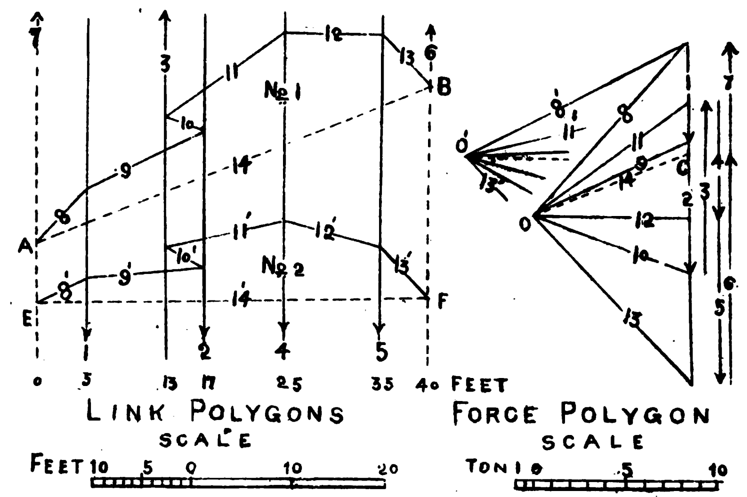
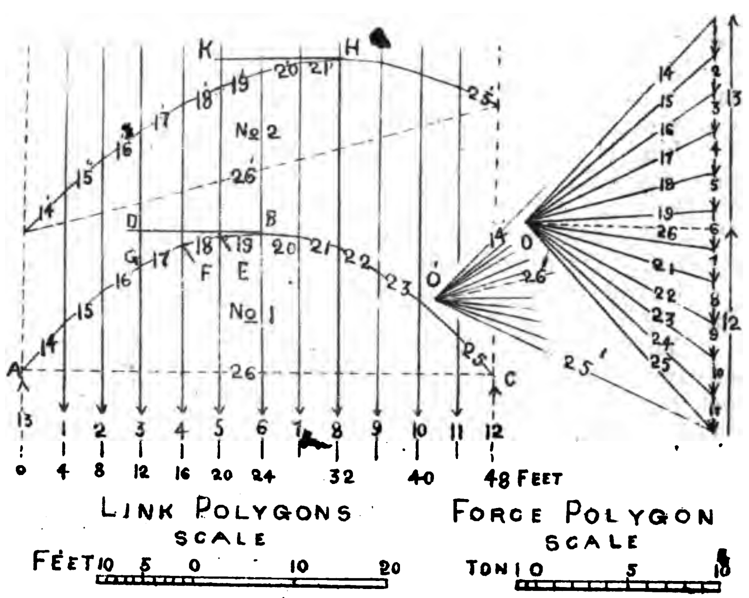

No 1
No 2
A
B
E
F
O
O′
C
0 5 13 17 25 35 40 FEET
LINK POLYGONS
SCALE
FEET 10 5 0 10 20
FORCE POLYGON
SCALE
TON 1 0 5 10

FIG. 9


K
H
D
B
G
F
E
No 2
No 1
A
C
O
O′
0 4 8 12 16 20 24 32 40 48 FEET
LINK POLYGONS
SCALE
FEET 10 5 0 10 20
FORCE POLYGON
SCALE
TON 1 0 5 10

PROBLEM X.

GIVEN—A plane set of parallel forces.
FIND—The centre of the forces.

Let $P_1 = 6$, $P_2 = 5$, $P_3 = 3$, $P_4 = 4$, and $P_5 = 2$ tons.
$x_1 = 0$, $x_2 = 9$, $x_3 = 15$, $x_4 = 25$, and $x_5 = 35$ feet.

Construct the force polygon. Fig. 10. Take any point O for the pole.

Construct the link polygon. Begin at A any point in the line of action of P_1; draw the links 8, 9, 10, and 11; from A draw 7, and from B draw 12 to intersect at C; then P_6 upwards, acting through C, balances the given forces; and P_6 downwards, acting through C is equivalent to the given forces. Any point in the line of action of P_6 is in the centre of the given forces.

$P_6 = 20$ tons; $x_6 = 13$ feet.

PROBLEM XI.

To find the centre of gravity of a plane area with straight boundaries.

Let the five-sided figure ABCDE represent the area; divide it into triangles by the lines EB and EC; find the area and the centre of gravity of each triangle. Fig. 11.

Number the triangles (1), (2), and (3) as in the figure; and draw vertical lines 1, 2, 3 through their centres of gravity.

Construct the force polygon. Draw $P_1 = 70$, $P_2 = 72$, and $P_3 = 48$ square feet; these areas will represent weights if the figure ABCDE represents a plate of uniform thickness.

Construct link polygon No. 1. The centre of the three forces is in the line 4.

Construct link polygon No. 2. The lines 1′, 2′, and 3′ are drawn horizontally; and instead of drawing a new force polygon, the links 5′, 6′, 7′, and 8′, are drawn at right angles to the rays on the force polygon. The centre of forces is in the line 4′.

The centre of gravity of the given figure is at G the point at which the dotted lines 4 and 4′ intersect.

FIG.10.

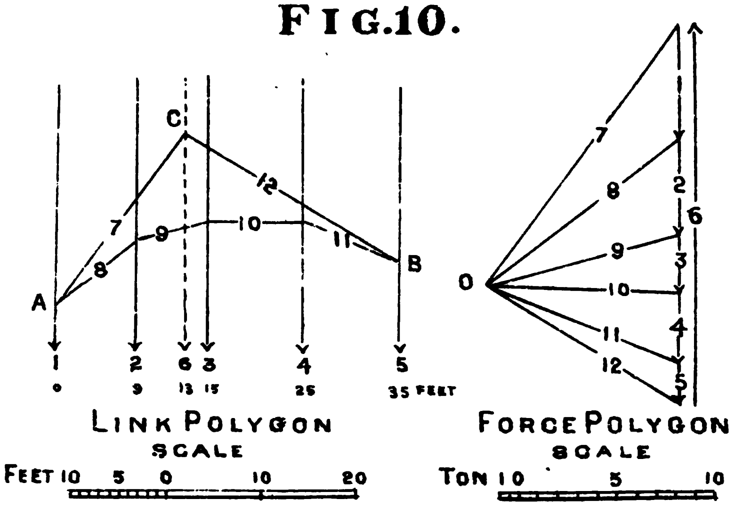
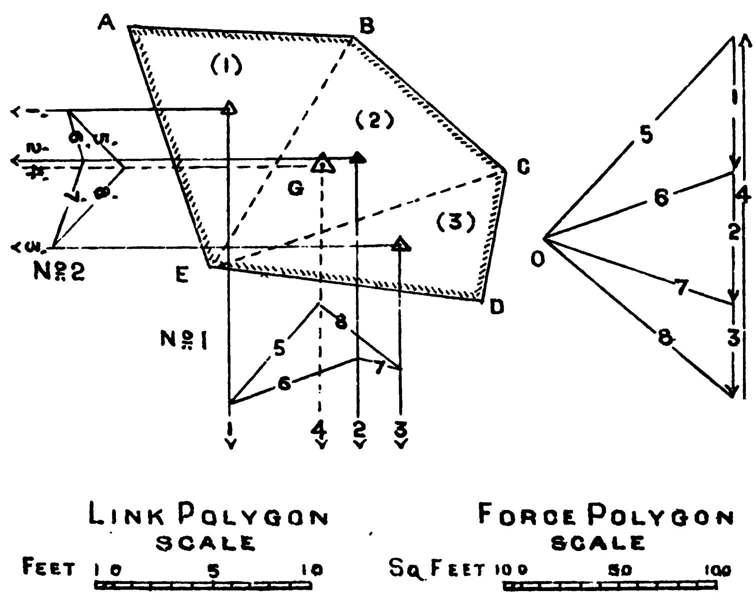

C
A
B
O
7
8
9
10
11
12
1
2
6
3
4
5
0
9
13
15
25
35 FEET
LINK POLYGON
SCALE
FEET 10 5 0 10 20
FORCE POLYGON
SCALE
TON 10 5 10

FIG.11.


A
B
C
D
E
G
(1)
(2)
(3)
O
No 2
No 1
5
6
7
8
1
4
2
3
LINK POLYGON
SCALE
FEET 10 5 10
FORCE POLYGON
SCALE
SQ. FEET 100 50 100

PROBLEM XII.

GIVEN—A plane set of parallel forces P_1, P_2, P_3, and P_4.
FIND—The moments of the forces about any given point A.

Let the four forces

$P_1 = 10$, $P_2 = 12$, $P_3 = 8$, and $P_4 = 4$ lbs.

and $x_1 = 1$, $x_2 = 1\frac{1}{2}$, $x_3 = 1\frac{3}{4}$, and $x_4 = 2\frac{1}{4}$ feet. Fig. 12.

Construct the force polygon. The pole O is taken at unit distance from the force line, that is $d = 1$ foot.

Construct the link polygon. Begin at A; draw the links 5, 6, 7, and 8, and thus find B; from B draw 9, and produce 8, 7, 6, to meet the vertical line through A. The moments of the forces are given by the intercepts on the vertical line AC.

The four triangles above the link polygon, given by the dotted lines and intercepts on the vertical line AC, are similar to the four triangles on the force polygon given by the rays and the forces P. For example—the triangle $M_1 56$ on the link polygon is similar to $P_1 56$ on the force polygon.

$M_1 : P_1 :: l_1 : d$; M_1 and P_1 may be measured on the scale for forces, and l_1 and d on the scale for dimensions.

$M_1 . d = P_1 . l_1$; but d was taken as unity, therefore

$M_1 = P_1 \, l_1$ the moment required.

Similarly $M_2 = P_2 \, l_2$; $M_3 = P_3 \, l_3$; and $M_4 = P_4 \, l_4$.

The scale for moments is the *same* as the scale for forces since the polar distance is *unity*, that is since $d = 1$ foot.

$M_1 = 10$, $M_2 = 18$, $M_3 = 14$, and $M_4 = 9$ foot pounds.

PROBLEM XIII.

GIVEN—A plane set of parallel forces P_1, P_2, P_3, and P_4.
FIND—The moments of the forces about any given point A.
Let the forces $P_1 = 10$, $P_2 = 8$, $P_3 = 7$, and $P_4 = 5$ lbs.

$x_1 = 8$, $x_2 = 12$, $x_3 = 15$, and $x_4 = 20$ feet.

If the polar distance is *increased*, the intercepts on the vertical line A C, figs. 12 and 13, *diminish* in the same proportion. It is seldom convenient to take the polar distance equal to *unity* on the dimension scale; in this example, fig. 13, it is taken equal to *ten* feet. The intercepts on AC, measured on the force scale, will be *one tenth* of the moments; but by preparing a new scale for moments with the divisions *one tenth* of those on the force scale, the moments may be read off at once on the new scale.

From problem XII we have $M_1 \times d = P_1 \times l_1$;

in fig. 13, $d = 10$; thus $M_1 \times 10 = P_1 \times l_1$.

in which M_1 and P_1 are measured on the scale for forces, and $d = 10$ feet and l_1 on the scale for dimensions; if we now measure M_1 on the scale for moments we get

$M_1 = P_1 \times l_1 = 10 \times 8 = 80$ ft. lbs.

Similarly $M_2 = 96$, $M_3 = 105$, and $M_4 = 100$ ft. lbs.

FIG. 12

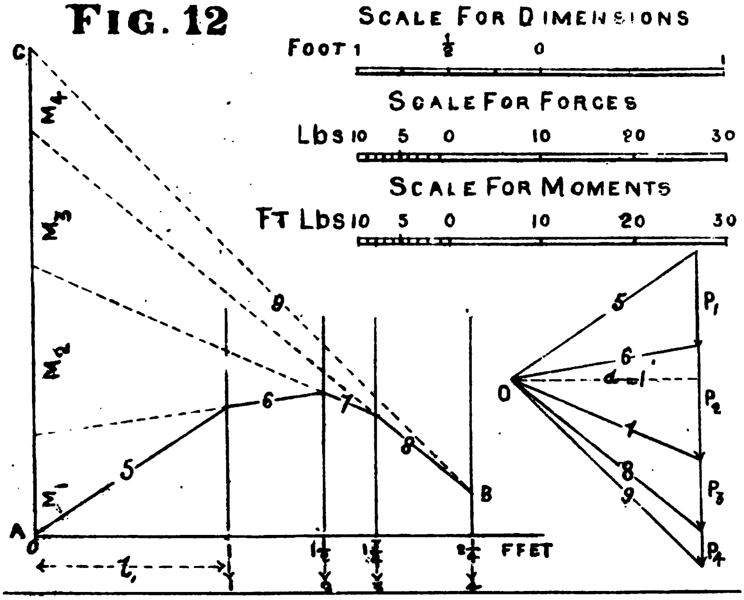
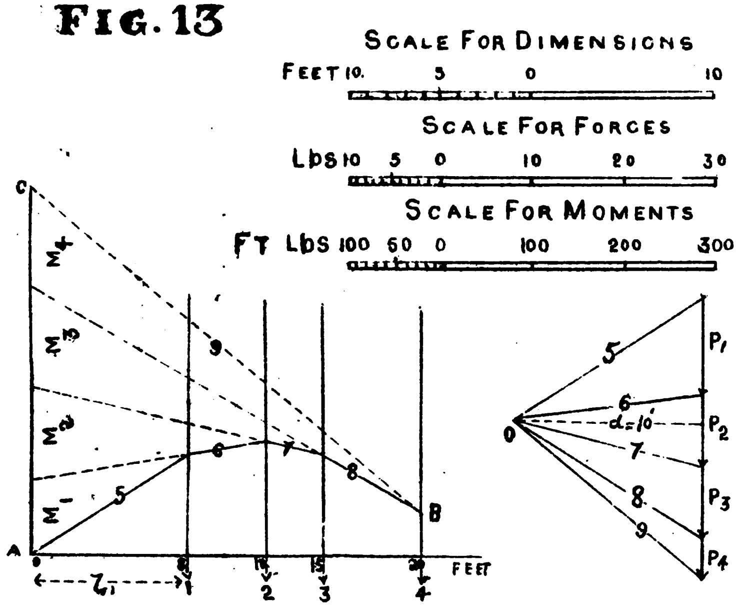

SCALE FOR DIMENSIONS
FOOT 1 ½ 0 1
SCALE FOR FORCES
Lbs 10 5 0 10 20 30
SCALE FOR MOMENTS
FT Lbs 10 5 0 10 20 30
FEET
B
O
d=1
P1
P2
P3
P4

FIG. 13


SCALE FOR DIMENSIONS
FEET 10 5 0 10
SCALE FOR FORCES
Lbs 10 5 0 10 20 30
SCALE FOR MOMENTS
FT Lbs 100 50 0 100 200 300
FEET
B
O
d=10'
P1
P2
P3
P4

PROBLEM XIV.

GIVEN—A plane set of parallel forces P_1, P_2, P_3, P_4, and P_5.

FIND—(*a*) The moments of the forces about any point A among the forces; some moments will be positive and some negative.

(*b*) The centre of the forces; and the moments of the forces about this point.

Let the five forces $P_1=3$, $P_2=4$, $P_3=2$, $P_4=6$, and $P_5=5$ lbs.
$x_1=0$, $x_2=7$, $x_3=20$, $x_4=32$, and $x_5=40$ feet, and for the point A, let $x=15$ feet. Fig. 14.

Construct the force polygon. Choose any point O for pole such that $d=20$ feet; this determines the ratio between the scale for forces and the scale for moments.

Construct link polygon No. 1. Begin at B, any point in the line of action of P_1; draw links 7, 8, 9, and 10 and fix on C; draw BE and CD parallel to rays 6 and 11, and produce links 7, 10, and 9 to meet the vertical through A.

The intercepts give the moments. M_1 and M_2 are negative; M_3, M_4, and M_5 are positive. A length 10 lbs. on the scale for forces is marked $10 \times 20 = 200$ ft. lbs. on the scale for moments, since *d* the polar distance is 20 feet.

$M_1 = -45$, $M_2 = -32$, $M_3 = 10$, $M_4 = 102$, and $M_5 = 125$ ft. lbs.

Construct link polygon No. 2. Begin at B as before; draw links 7, 8, 9, and 10, and fix on C. From B and C draw links parallel to rays 6 and 11 to intersect at G; draw a vertical line through G, and produce links 7, 8, and 10 to meet it. The centre of the forces is at G for which $\bar{x} = 23$ feet; and $M_1 = -69$, $M_2 = -64$, $M_3 = -6$, $M_4 = 54$, $M_5 = 85$ ft. lbs.

PROBLEM XV.

GIVEN—A beam, 42 feet span, supporting five loads, and supported at its ends.

$W_1 = 5$, $W_2 = 5$, $W_3 = 11$, $W_4 = 12$, and $W_5 = 9$ tons.

$x_1 = 1$, $x_2 = 6$, $x_3 = 14$, $x_4 = 24$, and $x_5 = 31$ feet.

FIND—The supporting forces P and Q at the ends of the span; the centre of gravity of the loads; and the bending moment at any point $x = 14$ feet. Fig. 15.

Construct the force polygon. Choose any point O′ as pole, and draw rays 8′, 9′.........13′.

Construct the link polygon. Begin at H′, any point in the line of action of P; draw links 8′, 9′, 10′......13′, and determine the point K′; join H′K′, and draw dotted ray 14′ on force polygon; this gives P = 24 tons, and Q = 18 tons.

Construct the bending moment diagram. Choose O as pole on the horizontal line drawn from top of Q; and make the polar distance $d = 20$ feet; draw rays 8, 9......13.

Begin at H, any point in the line of action of P; draw links 8, 9, 10......13, and determine K; the figure HABCDEK is the bending moment diagram.

Produce links 8 and 13 to meet at *h*; the point G for which $\bar{x} = 18$ feet is the centre of gravity of the loads.

The bending moment at any point of span is usually defined

FIG. 14.

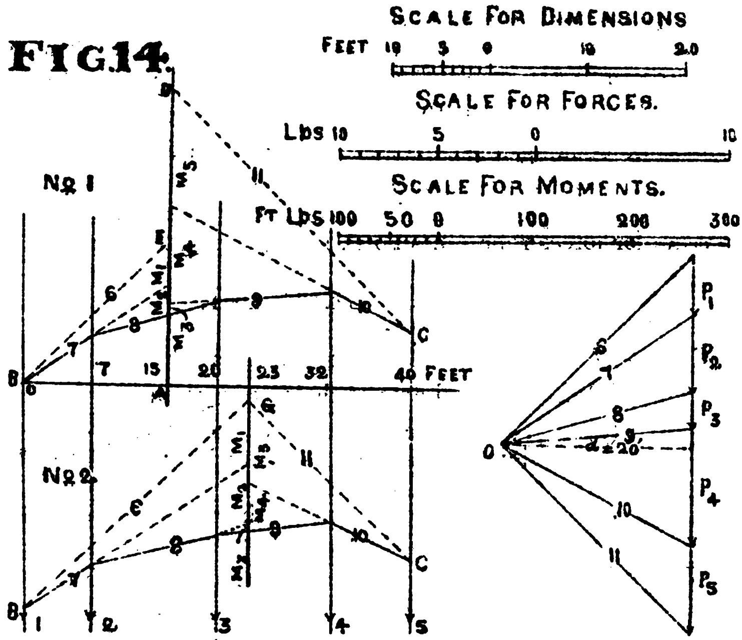
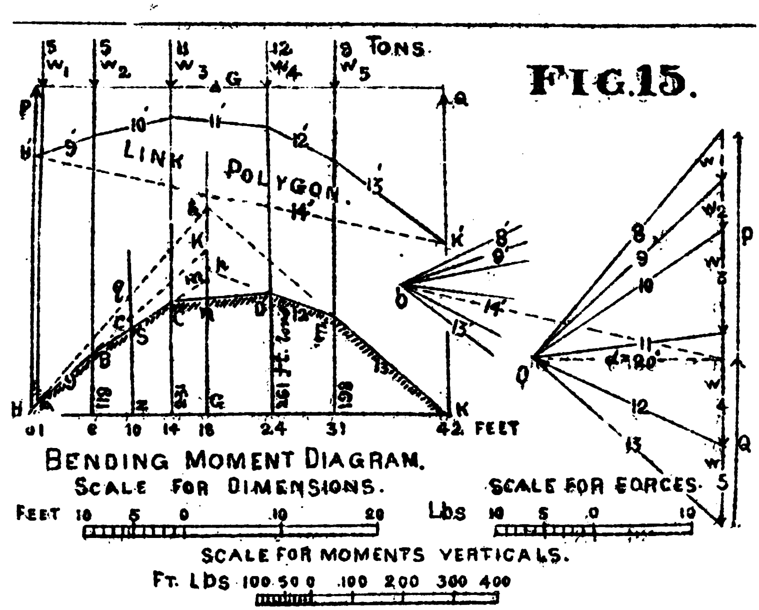

SCALE FOR DIMENSIONS
FEET 10 5 0 10 20
SCALE FOR FORCES.
Lbs 10 5 0 10
SCALE FOR MOMENTS.
FT LBS 100 50 0 100 200 300
No 1
No 2
FEET

FIG. 15.


TONS.
LINK POLYGON.
BENDING MOMENT DIAGRAM.
SCALE FOR DIMENSIONS.
FEET 10 5 0 10 20
SCALE FOR FORCES.
Lbs 10 5 0 10
SCALE FOR MOMENTS VERTICALS.
FT. LBS 100 50 0 100 200 300 400

as — The sum of the moments about that point of all the external forces, acting on the portion of the span on either side of the point. Let z be the given point for which $x = 10$ feet; the height $\overline{zq}$ represents the moment of P; $\overline{qr}$ the moment of W_1; and $\overline{rs}$ the moment of W_2 all about the given point; and $\overline{zs} = 175$ foot tons is the bending moment at the point.

The bending moments at the loads are—$M_1 = 20$, $M_6 = 119$, $M_{14} = 231$; $M_{24} = 261$; and $M_{31} = 198$ foot tons.

Since the polar distance is 20 feet, unit length or 1 lb. on the scale for forces represents 20 ft. lbs. on the scale for moments.

PROBLEM XVI.

GIVEN—A cross section consisting of rectangles.

FIND—(*a*) The neutral axis of the cross section;

(*b*) The geometrical moment of that portion of the section lying above the neutral axis with respect to the neutral axis; G'_o

(*c*) The moment of inertia of the section also with respect to the the neutral axis; I_o.

NOTE—The neutral axis of a cross section passes through the geometrical centre (or centre of gravity) of the cross section.

The geometrical moment of any surface which can be divided into simpler figures whose geometrical centres are known, about any axis in its plane, is—The algebraic sum of the products of each of such areas multiplied by the distance of its geometrical centre from the axis; leverages on one side of the axis are positive—on the other side negative.

The moment of inertia of a surface, about a line in its plane as axis, is the sum of the products of each small area into which the surface can be divided into the square of its distance from the axis. For a rectangle of breadth b, height h, and area $S = bh$, the moment of inertia about an axis parallel to the side b and passing through the centre of gravity, is

$$I_o = \tfrac{1}{12} bh^3 = \tfrac{1}{12} Sh^2;$$

and about any other parallel axis it becomes

$$I = I_o + Sd^2 = \tfrac{1}{12} Sh^2 + Sd^2,$$

where d = distance between the given axis and the centre of gravity of the area.

Let the given cross section consist of five rectangles, Fig. 16. (1) 12″ × 3″; (2) 6″ × 3″; (3) 1½″ × 10″; (4) 4″ × 2″; and (5) 8″ × 2″. areas 36, 18, 15, 8, 16 square inches; total height 20 inches;

Construct the line of areas to any scale; with polar distance ten inches on scale for dimensions, fix any point O as pole; and draw rays.

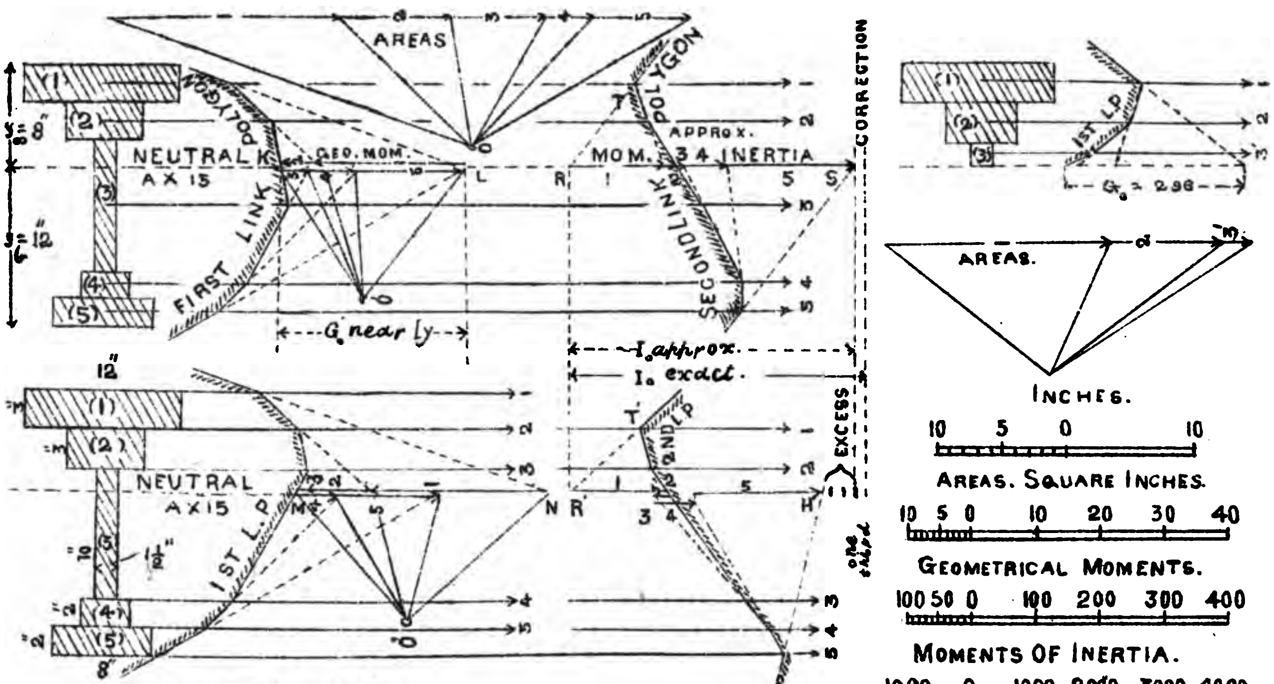

AREAS
NEUTRAL K
AX 15
FIRST LINK POLYGON
GEO. MOM.
G₀ near Ly
MOM. 3 4 INERTIA
APPROX.
SECOND LINK POLYGON
CORRECTION
I₀ approx.
I₀ exact.
EXCESS
NEUTRAL
AXIS
1ST L.P
2ND L.P
one third
POLAR DISTANCES
TEN INCHES.
1ST L.P
G₀ = 2.98
AREAS.
INCHES.
AREAS. SQUARE INCHES.
GEOMETRICAL MOMENTS.
MOMENTS OF INERTIA.

FIG.16.

Construct the first link polygon. The lines of action 1,2,3,4,5 are drawn from the centre of each area; the two end links produced intersect at L, a point on the neutral axis which may now be drawn. The intercepts 1 and 2 give the geometrical moments of the areas whose centres are above the neutral axis; 3,4,5 give those below; and it is observed that KL, the sum of intercepts 1 and 2 equals the sum of intercepts 3, 4, and 5.

Construct the second link polygon. For O′ the pole, choose any point whose distance from KL the neutral axis is ten inches and draw rays. Begin at T, any point in the line of action of 1, and draw links ending at V on line from centre of (5); produce the end links to cut the neutral axis line in R and S; then RS, the sum of intercepts 1, 2, 3, 4, 5 is approximately equal to I_0 on the moment of inertia scale.

To determine the exact value of I_0; draw the lower diagrams in Fig. 16; the first link polygon has the lines of action 1, 2, 3, 4. 5 drawn from the upper side of each area; the second link polygon has the lines of action drawn from the lower side of each area. The intercepts on MN the neutral axis are 1, 2, 3 on one side, and 4, 5 on the other side. The third pole O″ is any point with ten inches as its polar distance, as before. The second derived link polygon is constructed with its first link drawn from R′ a point exactly *under* R of upper figure. The sum of the intercepts R′H will be less than RS. One third part of the excess of RS over R′H added to RS gives the exact value of I_0 the moment of inertia of the cross section relatively to the neutral axis.

The proof is as follows:—Let S be the area of any one of the rectangles and d the distance of its centre from the neutral axis; the intercept on KL, measured on the scale for moments, is Sd the geometrical moment of the area with respect to the axis. Similarly the intercept on RS is—$Sd \times d = Sd^2$, a quantity corresponding to the second term for I as given above. Since h is the height of the given rectangle, the intercept on MN of the first link polygon, lower diagram, is $S\left(d + \frac{h}{2}\right)$; and the intercept on R′H is $S\left(d+\frac{h}{2}\right)\left(d-\frac{h}{2}\right) = S\left(d^2-\frac{h^2}{4}\right) = Sd^2 - \frac{Sh^2}{4}$ a quantity less than Sd^2. In the diagram, the length marked *excess* corresponds to $\frac{1}{4}Sh^2$; and one third of this quantity $\frac{1}{12}Sh^2$ gives the first term of the expression for I given above. The small link polygon, on the right of the figure, and above the scales, gives G'_0 the geometrical moment about the neutral axis of the part of the section lying above the neutral axis.

In the above example the total area is 93 sq. inches; $G'_0 = 296$; and $I_0 = 4634$.

PRACTICAL APPLICATIONS.

ROOF TRUSSES.

A roof truss is a frame, supported at its two ends, of some arbitrary shape usually symmetrical, and arranged to carry off rain water. It is in equilibrium as a whole under the given loading; equilibrium for the various parts of the frame is obtained by *trussing*, that is by introducing additional pieces to make the truss indeformable. For this purpose, the truss is divided into a series of triangles; since in a triangle, two sides cannot change their relative positions without lengthening or shortening, that is straining, the third side.

In the examples which follow, the joints at all the apexes are considered as flexible or free; in practice the joints are stiffened by making two pieces continuous over a joint, by plates and straps, and by properly shaping the pieces that form the joint.

In roof trusses, the load may be spread over the rafters, or concentrated at points; in either case it is to be *reduced* to the joints. That part of the load which is *reduced* to the points of support, and which is *directly* resisted there, may be left out of account so far as concerns the truss. For vertical loading, the points of support always give the necessary direct vertical resistance; for horizontal or inclined loading, due perhaps to wind, the friction between the ends of the truss and the walls will probably give the necessary direct resistance to prevent the truss from moving; holding down bolts or some such method may also be employed.

In a few of the diagrams of trusses, the pieces which are in compression, struts, are shewn by thick lines; those which are in tension, ties, by thin lines.

EXAMPLE I.

King Post Truss with symmetrical vertical loading.

In a king post truss, Fig. 17, BF and BD are called the rafters; FD, the tie beam; AE and CE, the struts; and BE the king post. Suppose a load is placed on B;—the point B tends to move downwards; F and D, the feet of the rafters, tend to move outwards; this is prevented by the tie beam FD. If loads are placed at A and C, the rafters will tend to bend; this is prevented by the struts EA and EC pushing upwards; the feet of the struts are held up by the king post EB, and the top of the king post is held up by the rafters; the tie beam is prevented from bending by its own weight, by being fixed to the foot of the king post.

Since the length of each section of the rafters, FA, AB, BC, and CD is usually from about 6 to about 10 feet, the span for a king post truss may be from 20 to 35 feet.

The truss shewn in Fig. 17 has a span of 25 feet; the rafters are inclined at 30° to the horizon; and three loads $W_1 = W_2 = W_3 = 6000$ lbs. are supported at the joints A, B, and C.

Construct the stress diagram. Draw the force line 1, 2, 3 each equal to 6000 lbs. to scale; since the loading is symmetrical the supporting forces 4 and 5 are 9000 lbs. each.

Construct a table of "apex, known, found" as given below. At the point F force 5, and the directions of the stresses 6 and 7 are known; by drawing the triangle 5, 6, 7 on the stress diagram, the amounts of 6 and 7 are found by scale; the directions of 6 and 7 are found by *going round* the triangle of forces; these directions should now be shewn by arrow heads near F. Reverse these arrows at the other ends of FA and FE; that is FA is pushing upwards at A, and FE is pulling to the left at E. Pass on to the next apex A; we may proceed to any apex at which not more than two stresses are unknown. At apex A, taking the forces in cyclic order, 6 and 1 are known, 9 and 8 are to be found; on the stress diagram 6, 1 appear already, and the figure is to be closed by 9, 8 drawn parallel to 9, 8 on the truss. Go *round* the figure 6, 1, 9, 8 on stress diagram; this gives directions of 9, 8 acting at A, and shewn by arrow heads. Reverse arrows near other ends of AB and AE.

Pass on to next apex B; taking the forces in cyclic order 9 and 2 are known, 11 and 10 are to be found; on the stress diagram 9 and 2 appear already, and figure is closed by 11, 10 drawn parallel to 11, 10 on the truss. Mark the arrows on 11 and 10 near B; reverse the arrows. Pass on to next apex C; 11 and 3 are known, 14 and 12 are to be found; on the stress diagram 11 and 3 already appear, close the figure by 14 and 12; mark the arrows near C, and reverse them near D and E.

Pass on to next apex D; 14 and 4 are known, 13 is to be found; and since *only one* force is to be found, we have a check on the accuracy of the drawing; the line 13 on the stress diagram should be parallel with 13 on the truss. Mark the arrow at D, and reverse it near E.

Pass on to the last apex E; all the forces 7, 8, 10, 12, and 13 are known, and their directions are shewn by arrow heads; go *round* the lines on the stress diagram; we begin at the right hand end of 7, and end at the right hand end of 13; this gives a closed figure, and the point E is in equilibrium under the five forces acting at it.

The following is the proper order of operations;

(a) Write the first line on Table No. 1.

(b) on the stress diagram draw the lines which are to be found;

(c) mark the arrows; and *(d)* reverse the arrows.

And so on with each succeeding line in the table.

Construct another Table No. 2 shewing the number of each piece forming the truss; its length; and the kind and amount of stress acting along it:—

FIG. 17.

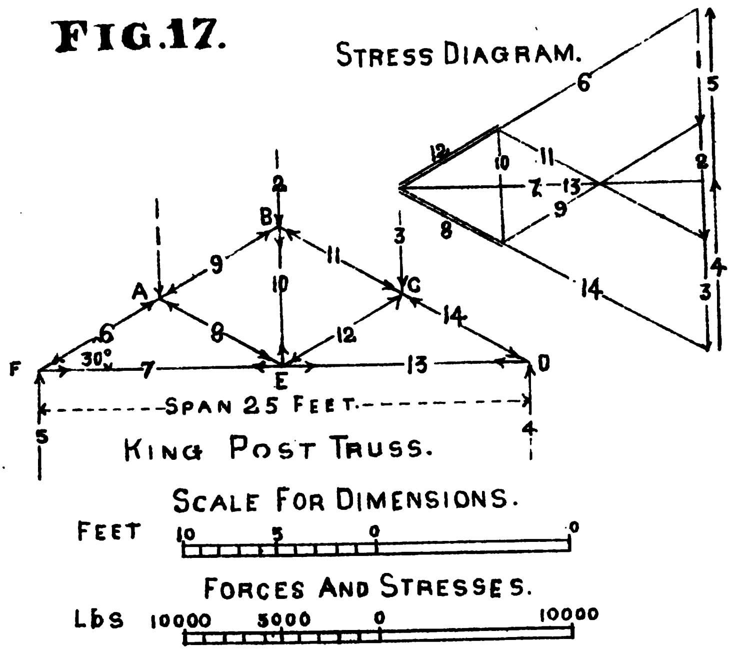


TABLE NO. 1.

APEX.	KNOWN.	FOUND.
F	5	6.7
A	6.1	9.8
B	9.2	11.10
C	11.3	14.12
D	14.4	13 check
E	7.8.10. 12.13.	

TABLE NO. 2.

No.	Length Feet.	Kind.	Amount of stress ; lbs.
6	7·2	strut.	18000
7	12·5	tie.	15588
8	7·2	strut.	6000
9	7·2	strut.	12000
10	7·2	tie.	6000
11	7·2	strut.	12000
12	7·2	strut.	6000
13	12·5	tie.	15588
14	7·2	strut.	18000

The length of each piece is obtained by measuring the drawing shewing the truss. The *kind* is found by inspecting the arrow heads on the piece; if the arrow heads point away from each other, the piece is a strut; if towards each other, a tie. The amount of stress is found from the stress diagram by scale.

EXAMPLE II.

King Post Truss with unsymmetrical vertical loads.

Span 25 feet; rafters at 30°; loads at apexes $W_1 = 1000$, $W_2 = 4000$, $W_3 = 5000$, $W_4 = 6000$, and $W_5 = 2000$ lbs.

Draw the loads 1, 2, 3, 4, 5 to scale; choose any point O as pole, and draw rays. Construct the link polygon; and find the supporting forces 6 = 10,000 lbs. and 7 = 8000 lbs. Put letters AB......F at the apexes, and number the pieces 8, 9......16 on the truss. Fig. 18.

Construct the table—apex, known, found as before; and put arrows to shew direction of stresses on the pieces at the apexes.

Apex.	Known.	Found.
F	7.1	8.9
A	8.2	11.10
B	11.3	13.12
C	13.4	16.14
D	16.5.6	15 check
E	9.10.12 14.15	

Construct the table giving number, length, kind, and amount. Observe that the links of the link polygon parallel to the rays drawn from O to the top of 7 and 1, and to the bottom of 5 and 6, are of zero length, and so do not appear; that the loads W_1 and W_5 do not affect the lengths of the lines on the stress diagram; and that the lines 9 and 15 on the stress diagram coincide; 15 is drawn slightly out of position for clearness.

EXAMPLE III.

King post roof truss, distorted, with unsymmetrical vertical loads at apexes, and a load at foot of king post.

A truss of this shape is sometimes used in mills for the purpose of getting *a north* light; the load at the middle of the tie beam may be a line of shafting. Fig. 19.

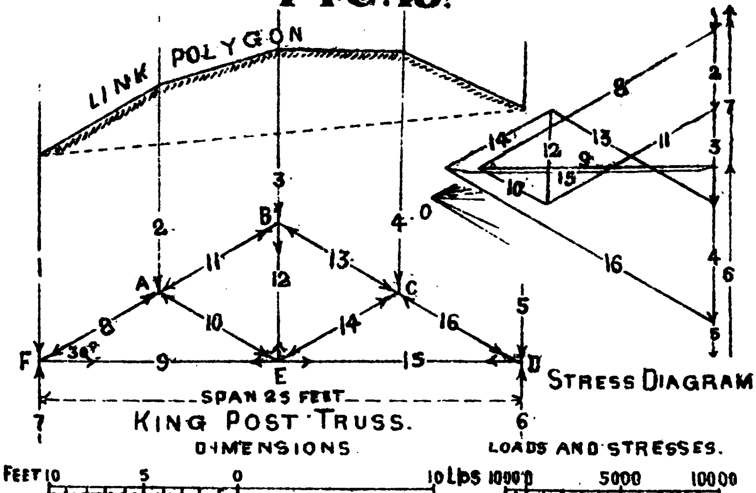
Span 25 feet; rafters at 60° and 30° to horizon; vertical loads 1 = 3000, 2 = 4000, 3 = 6000, and 4 = 2000 lbs.

From any point K draw the load line, 1, 2, 3, 4; take O any point as pole and draw rays; draw the link polygon from M any point in the line of action of 6; from O draw the dotted line 12, and determine the amounts of the supporting forces—5 = 6125, and 6 = 8875 lbs.

From any point L draw another force line, and number the forces in cyclic order; 5, 4, and 6 are in the same line as 1, 2, and 3, but are drawn to one side for clearness. Construct the stress diagram.

Apex.	Known.	Found.
F	6	13.14
A	13.1	16.15
B	16.2	18.17
C	18.3	21.19
D	21.5	20 check
E	4.14.15 17.19.20	——

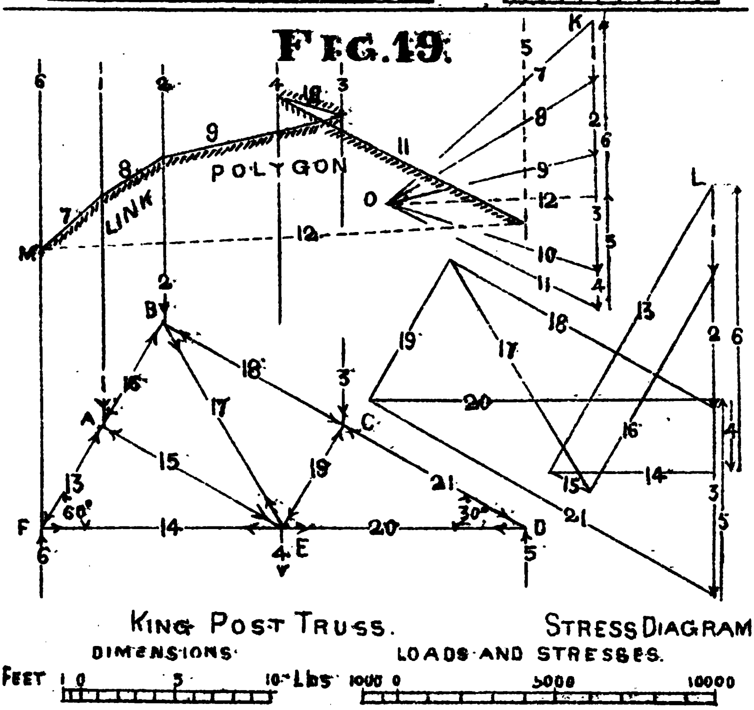
FIG. 18.



KING POST TRUSS.
DIMENSIONS

STRESS DIAGRAM
LOADS AND STRESSES.

FIG. 19.



KING POST TRUSS.
DIMENSIONS

STRESS DIAGRAM
LOADS AND STRESSES.

EXAMPLE IV.

King Post Truss with vertical load and wind load.

Span 25 feet; rafters at 30°; length of each rafter 15 feet nearly; half of this length 7½ feet is supported at each joint A, B, C; and if the distance between the trusses be taken as 10 feet, the area of roof allotted to each of the three joints is 75 sq. feet. Fig. 20.

The weight of roof per square foot varies with the roofing material from 10 to 40 or 50 lbs. per sq. foot; for double tiles, cleading, purlins, and rafters it may be taken at 40 lbs. The wind load is taken at 40 lbs. per sq. foot normal to rafter.

The vertical loads are 1 = 2 = 3 = 3000 lbs. at A, B, and C; the wind loads are 2′ = 4′ = 1500 lbs. and 3′ = 3000 lbs. at B, D, and C. The resultant loads 2_a and 3_a at B and C are obtained by drawing the parallelogram of forces for each point. The horizontal resistance to the wind is obtained at the supports F and D; and in the present example it is *assumed* to be *equally* divided.

Construct the line of loads MN; the horizontal distance between N and M is the horizontal component of the wind load; NR = SM = 5′ = 6′ are the horizontal resistances at D and F; the vertical line RS represents the vertical supporting forces 5 and 6 which are now to be determined. Choose any point O as pole and draw rays 7, 8..... 13. Construct the link polygon; begin at K any point in the line of action of 6 and draw link 7; draw links 8, 9......13 and find L; draw the dotted line KL, 14, and draw the line parallel to 14 from pole O; this gives T; and the forces 5 = 7965, 5′ = 1500, 6 = 6230, and 6′ = 1500 lbs. are obtained from the diagram.

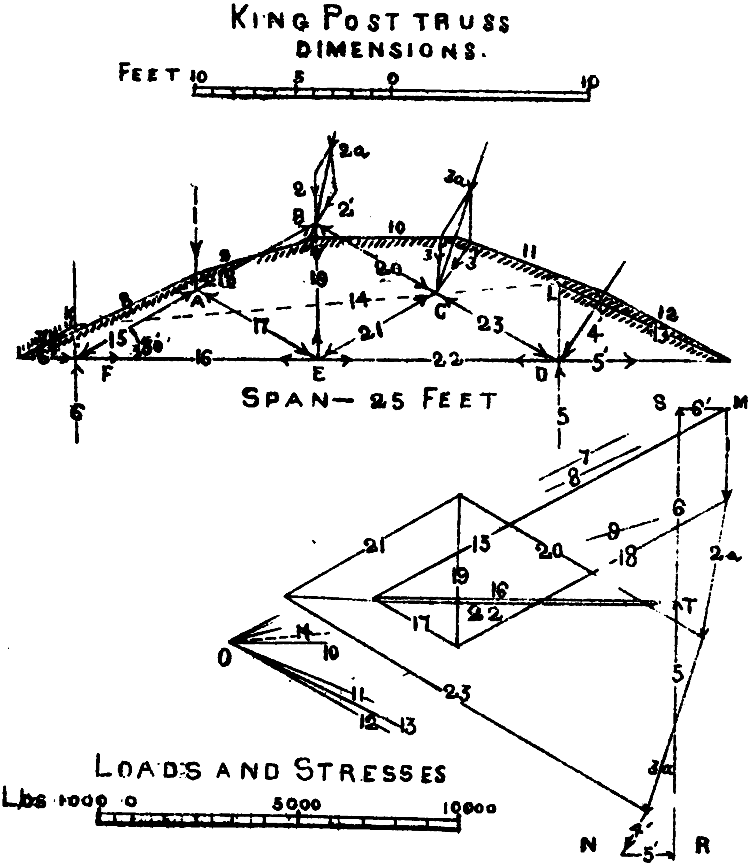
Construct the table—apex, known, found; and draw the stress, diagram. Construct the table-number, length, kind, amount of stress; as described in Example I.

Apex.	Known.	Found.
F	6.6′	15.16
A	15.1	18.17
B	18.2a	20.19
C	20.3a	23.21
D	23, 4′	
	5′.5	22 check
E	16.17.19	
	21.22	

No.	Length feet.	Kind.	Amount of stress. Lbs.	Area required Square inches.	Sizes required.	Sizes adopted allowing for joints.
15	7·2	Sruts.	12500	13·4	6″ × 2.5″	7″ × 3½″
18	7·2		9500			
20	7·2		8600			
23	7·2		13350			
16	12·5	tie	9350	10·3	4.2″ × 2.5″	5″ × 3½″
22	12·5	,,	12300			
17	7·2	strut	3000	6·5	2.6″ × 2.5″	4″ × 3½″
21	7·2	,,	6500			
19	7·2	tie	4800	4	1.6″ × 2.5″	3″ × 3½″*

* This is the size at middle; at top and bottom the king post is enlarged.

FIG. 20



The cross sectional area required for the pieces forming the truss are readily obtained when we know the *working* strength of the timber. We here assume $f_{t.}$ = 1200 lbs. per square inch for pieces in tension, and f_c = 1000 lbs. for compression. In the "sizes required" column, a convenient thickness $2\frac{1}{2}''$ is taken, the breadth of the piece is then fixed so as to give the required area. Allowance is now made for cutting at joints, and sizes to be used in constructing the pieces are found in the last column.

The rafters 15, 18, 20, and 23 are all made *one* size; similarly 16 and 22, and 17 and 21.

When the wind blows from left to right, the stress on 15 will correspond with that given above for 23; and so on.

EXAMPLE V.

Queen Post Truss with equal symmetrical vertical loads.

A queen post truss is suitable for spans varying from about 30 to 45 feet and is represented in fig. 21. The feet of the rafters CK and CF are prevented from spreading outwards by the tie beam KF; the rafters are prevented from bending downwards by the struts HA and GE, and by the straining beam BD; the feet of the struts are pulled upwards by the queen posts HB and GD; the feet of the struts are pushed outwards by fastenings at H and G, or by a straining sill HG placed on the upper side of the tie beam; the tie beam KF is prevented from sagging by its own weight by the queen posts HB and GD pulling upwards. Sometimes the members BC and CD are omitted in the main truss, and are formed by secondary rafters in a slightly higher position.

When loading is symmetrical there is no tendency to distort the rectangle HBDG; and a diagonal HD *or* BG is not required. When the loading is not symmetrical one of these diagonals is necessary. In practice the truss is frequently constructed without a diagonal; and the stiffness of the joints H, B, D, and G is relied upon to prevent distortion by the variations of loading met with in practice.

In fig. 21 the span is 40 feet; five loads 1, 2, 3, 4, 5 each equal to 3000 lbs. at apexes; the supporting forces 6 and 7 are each equal to 7500 lbs.

Construct the table—apex, known, found; and the stress diagram.

Apex.	Known	Found.
K	7	8.11
A	8.1	9.12
H	11.12	13.15
B	13.9.2	10.14
C	10.3	10′
D	14.10′.4	9′.13′
E	9′.5	8′.12
F	8′.6	11′ check.
G	15 13′ 12′ 11′	

EXAMPLE VI.

Queen Post Truss with irregular vertical loads at apexes; and with two loads from tie beam, one at foot of each queen post.

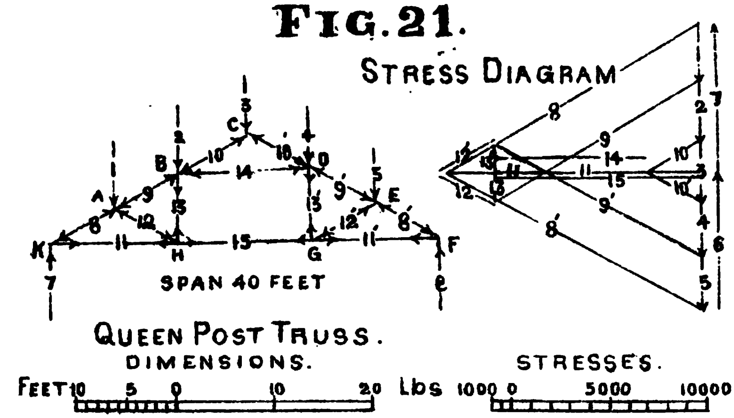
Span 40 feet; rafters at 30°; loads 1 = 2000, 2 = 2500, 3 = 4000, 4 = 3000, 5 = 3000, 6 = 1500, and 7 = 1000 lbs.; total load 17000 lbs. Fig. 22.

Draw the load line LM to scale; chose any point O as pole, and draw rays 10, 11,......17.

Construct link polygon. Begin at N any point in the line of action of 9; draw links 10, 11,......17 and thus fix position of R; join NR by dotted line 18. From pole O draw ray 18, and fix the point S; this gives 8 = 9000, and 9 = 8000 lbs.

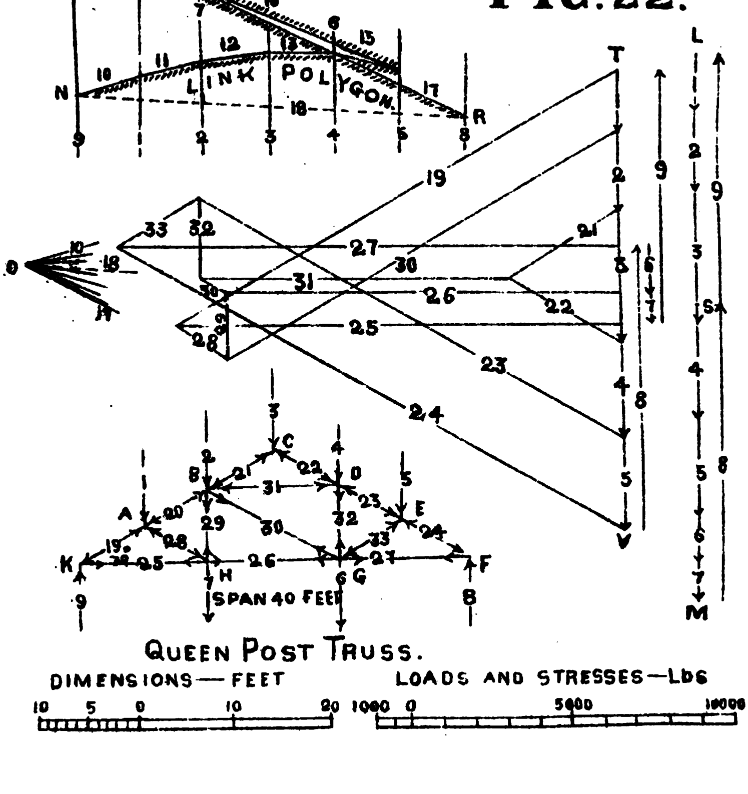
Draw another load line from any point T. Shew loads and supporting forces in cyclic order; they are all on the line TV, but for clearness 8, 6, 7, 9 are drawn to the side.

FIG. 21.



QUEEN POST TRUSS.

FIG. 22.



QUEEN POST TRUSS.

Construct the table apex, known, found; and the stress diagram.

APEX.	KNOWN.	FOUND.
K	9	19.25
A	19.1	20.28
H	7.25.28	29.26
C	3	22.21
B	29, 20, 2, 21	31.30
D	31.22, 4	23.32
E	23.5	24.33
F	24.8	27 check.
G	6.26.30. 32.33.27	

The student should construct a table like Table No. 2 shewn on page 23; and find among others 19 = 16000, 24 = 18000, 25 = 13860, and 27 = 15590 lbs.

The problem may be varied by substituting diagonal HD for No. 30; and taking only one load suspended from the tie beam.

EXAMPLE VII.

Queen Post Truss with vertical load and wind load. Span 40 feet, rafters at 30° ; length of each rafter 23½ feet giving about 8 feet of rafter to each joint; taking distance between trusses as 9½ feet, 75 square feet of roof are supported at each joint as in example IV. The weight of roof is again taken at 40 lbs. per square foot, and wind load 40 lbs. per square foot normal to rafter. Fig 23.

Vertical loads 1, 2, 3, 4, 5, each 3000 lbs.

Wind loads 3′, 6′ each 1500 lbs., 4′, 5′ each 3000; loads 3 and 3′ give 3_a = 4365 lbs.; and 4_a = 5_a = 5795 lbs.

In example IV the horizontal resistance to the wind is equally shared at the two supports; in this example we assume that it is given at the right end only. The right end may be taken as fixed, the left end supported only; this arrangement gives the greatest stresses on the members of the truss.

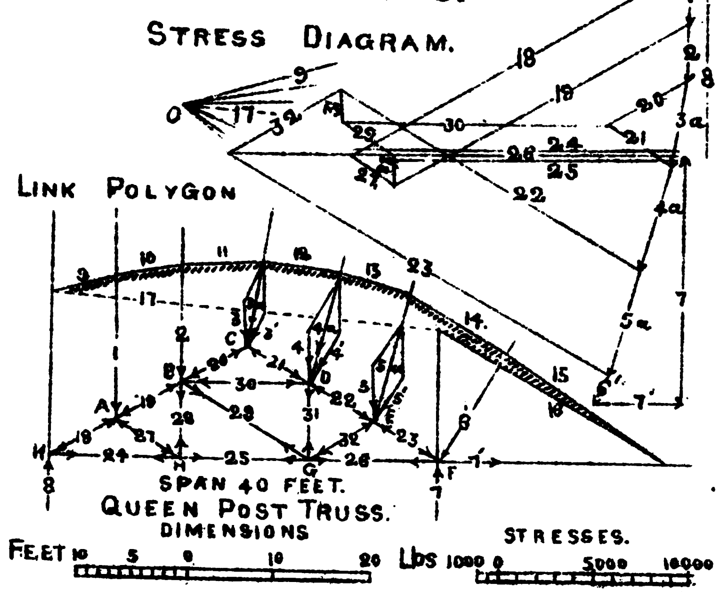
Construct the load diagram; and find 7′ = 4500 lbs. and vertical supporting forces 7 + 8 = 22800 lbs.

Choose any point O as pole, and draw rays 9, 10...........16. Construct link polygon and find dotted link 17; from the pole O draw dotted ray 17, and find vertical supporting forces 7 = 12700 and 8 = 10100 lbs.; 7 and 8 are in one vertical line, 8 is shown in the force diagram slightly out of position.

Form the table—apex, known, found; and draw the stress diagram as in example VI. The stresses 24, 25 really fall on line 26; they are shown slightly out of position for clearness.

Construct the following table, taking working strength of timber f_t = 1200 lbs. per square inch for tension: and f_c = 1000 for compression.

FIG. 23.



No.	Length feet.	Kind.	Amount of stress lbs.	Area required square inches.	Sizes required.	Sizes adopted allowing for joints.
18	7·8	Struts	20200	22·8	6″ × 3·8″	7″ × 5″
19	7·8		17200			
20	7·8		4700			
21	7·8		4000			
22	7·8		18100			
23	7·8		22800			
24	13·3	tie	17600	19·6	5″ × 3·9″	6″ × 5″
25	13·4	,,	15000			
26	13·3	,,	23500			
27	7·8	strut	3000	6·7	2″ × 3·4″	4″ × 5″
32	7·8	,,	6700			
28	7·8	tie	1600	1·5	1″ × 1·5″	3″ × 5″*
31	7·8	,,	1700			
29	16·0	,,	3500	3·0	2″ × 1·5″	3″ × 5″
30	13·4	strut	13700	13·7	4″ × 3·4″	5″ × ″5

*This is the reduced size : the ends of the queen posts are larger.

EXAMPLE VIII.

Iron Roof Truss, fixed on one side, supported only on the other; vertical loads at joints; wind load normal to that rafter whose lower end is fixed; this gives maximum stresses.

Span 25 feet; rafters at 30°; ties at 15° with rafters; trusses 10 feet apart; vertical load 20 lbs. per sq. foot; wind load 40 lbs. per sq. foot normal to right hand rafter.

Vertical loads 1 = 2 = 3 = 1500 lbs.; wind loads 2′ = 4′ = 1500 lbs., and 3′ = 3000 lbs. Compounding the loads at B, 2_a = 2900 lbs.; and 3_a = 4364 lbs. Fig. 24.

Construct the left hand force polygon. Begin at any point R, and draw 1, 2_a, 3_a, 4′; 5′ drawn to meet the vertical from R gives the horizontal resistance required at D. Choose any point O as pole and draw rays 7, 8,......12.

Construct the link polygon so as to pass through B the apex of the truss. Draw links 8, 7; and 9, 10, 11, 12; find points P and Q and join by dotted link 13. From pole O, draw ray 13; and find 5 = 5714, and 6 = 3982 lbs. Force 6 is shewn slightly out of position.

Construct another force polygon from any point S.

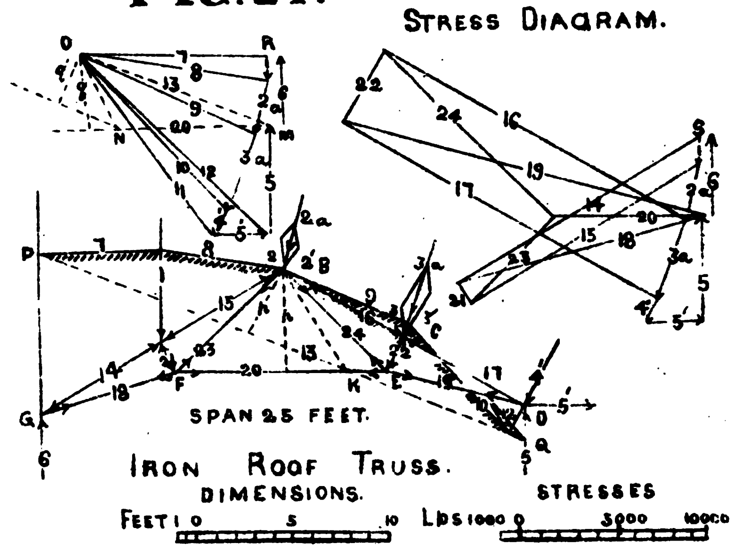
Construct the usual tables and the stress diagram. For wrought iron, the working strength is here taken at 12000 lbs. per sq. inch for tension; 10000 for compression.

Apex.	Known	Found.
G	6	14 18
A	14.1	15.21
F	18.21	23.20
B	23.15.2*a*	16·24
C	16.3*a*	17.22
D	17,4′,5′,5	19 check
E	20, 24	
	22, 19	

No.	Length Feet.	Kind.	Amount of stress —lbs.	Area required square inches.	Sizes required.	Sizes adopted allowing for joints, rusting &c.
14	7.3	Strut	14860	1·88	T Iron 5" × 3" × ¼'	T iron 5"×3½"×⅜"
15	7.3	,,	14110			
16	7.3	,,	17980			
17	7.3	,,	18730			
18	7.5	tie.	13325	1·60	1.45" diam.	1½"diam*
19	7.5	,,	19120			
20	10.6	tie.	7650	0.64	0.9" diam.	1"diam*
21	2.0	Strut	1300	0.43	L iron 1" × 1" × ¼"	L iron 2"×2"×⅜"
22	2.0	,,	4300			
23	7.5	tie.	6470	1.02	1·2" diam.	1¼" diam*
24	7.5	,,	12260			

* The ends of the tie bars are larger.

FIG. 24.



The stress on FE, bar No. 20, may be found by the special construction shewn; in the present example, this does not serve any useful purpose; but if the rafter GB is loaded and supported by struts at three intermediate points, as in Fig. 25, the ordinary method fails since more than two unknown quantities are met with at some intermediate stage in the process of constructing the link polygon. The link polygon PBQ is in equilibrium under the given loads and supporting forces: so also is the actual truss GBD. Dividing the link polygon and the truss into two parts by a vertical line through B, and taking moments round B, we get

$$\text{stress on PQ} \times p' = \text{stress on FE} \times p.$$

On the upper diagram OM, (13) represents the stress on PQ to scale and is therefore known; on the lower diagram PQ and FE (or FE produced) intersect at K; join BK. Draw ON ∥ BK; and from N draw the dotted line parallel to OM; then q' and q are proportional to p' and p. The double area of triangle OMN is

$$\text{OM} \times q' = \text{MN} \times q.$$

MN therefore represents the stress (tension) on the tie rod FK.

The student should now assume that the truss is fixed at G and supported only at D; $6' = 3000$ lbs.; $5' = 0$, the forces 5 and 6 remain unaltered. He should next assume $5' = 6' = 1500$ lbs.: again 5 and 6 remain as before. The three stresses obtained from the three stress diagrams should be written on each member of the truss and carefully compared.

EXAMPLE IX.

IRON ROOF TRUSS—Span 50 feet; rafters at 30° to horison; ties 15° to rafters; centre tie PL horizontal; each rafter is supported by three intermediate struts; interval between two trusses 20 feet; vertical load 20 lbs. per sq. ft. on both rafters; wind load 40 lbs. per sq. foot normal to right rafter; at the apexes the vertical loads are— 1 = 2 = 3 = 4 = 5 = 6 = 7 = 3000 lbs; the wind loads are 4′ = 8′ = 3000, and 5′ = 6′ = 7′ = 6000 lbs; compounding vertical and wind loads by the parallelogram of forces gives 4_a = 5795, and 5_a = 6_a = 7_a = 8728 lbs. The horizontal resistance to the wind is equally shared by the supports; 9′ = 10′ = 6000 lbs. Fig. 25.

Construct the force polygon—part of No. 4. Choose O any point as pole and draw rays.

Construct the link polygon No. 2. Begin at D the apex of the truss; draw the links left and right, and find points T and V; joint TV by a dotted line, and from pole O, No. 4, draw the parallel dotted line, find point W, and supporting forces 9 = 24360, and 10 = 17430 lbs. The line TV intersects the centre tie PL (or PL produced) at Y; join DY by a dotted line; from O draw OZ ‖ DY and meeting at Z the horizontal line from W the top of supporting force 9; this gives WZ or 21 = 22390 lbs. See Example VIII.

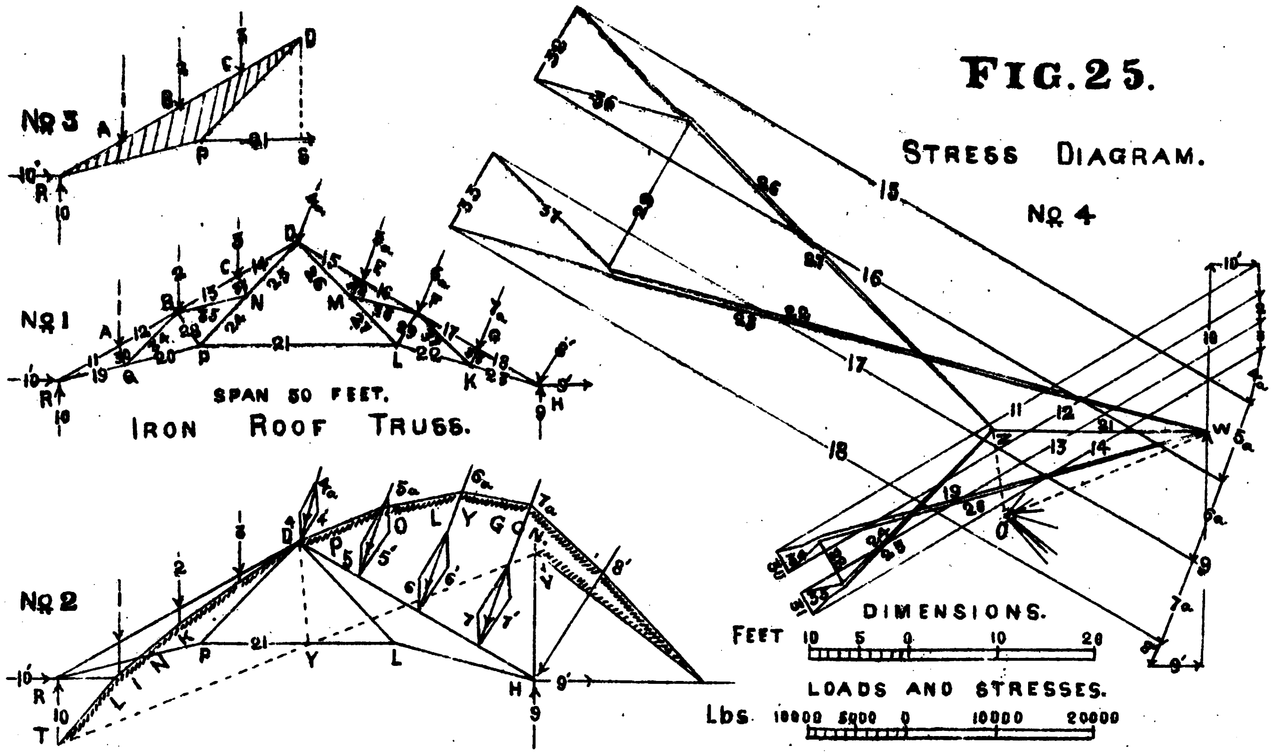
Another method of finding the stress on PL, (21) is shewn in No. 3; take moments round H, the right support, and find the vertical supporting force 10 = 17428 lbs.; take moments round D of all the forces shewn in No. 3, and find the moment of PS required for equilibrium 236640 foot pounds; divide this by the perpendicular DS = 10·57 feet, and find the pull on PS (21) = 22390 lbs. tension.

Construct the table—apex, known, found; and draw the stress diagram. The order in which the apexes come is—R, A, Q, P (21 having been found as above), B, N, C, (check) D, E, M, L, F, G, K (check).

The stresses on some of the members, found from the stress diagram, are – 11 = 59040, 14 = 54540, 15 = 81200, 18 = 85700, 19 = 46730, 23 = 81500, 25 = 26650, 26 = 61420 lbs.

In diagram No. 1, members in compression might with advantage be shewn by thick lines, members in tension by thin lines. The dimensions of the pieces may be found as in the previous example.

FIG. 25.



STRESS DIAGRAM.
No. 4
No. 3
No. 1
SPAN 50 FEET.
IRON ROOF TRUSS.
No. 2
LINK POLYGON
DIMENSIONS.
FEET 10 5 0 10 20
LOADS AND STRESSES.
Lbs. 10000 5000 0 10000 20000

EXAMPLE X.

WARREN GIRDER. A Warren girder has a top flange AF, a bottom flange PG; and a number of diagonals connecting the flanges, inclined about 60° to the horizontal, and making equilateral triangles. The top flange is in compression; the bottom flange in tension, the stresses are greatest near the middle of the span and decrease on both sides towards the supports. The diagonals are alternately struts and ties, and the stresses increase from the centre of span outwards.

Span 36 feet; live load 1 ton per foot run, dead load ½ ton per foot run; combined load equivalent to 2½ tons dead load per foot run, or 15 tons at each apex. Fig. 26.

The upper diagrams are for the load on upper flange; stresses 12, 18, 19 each equal to 78 tons; stresses 26 and 27 are zero. The lower diagrams are for the load on lower flange; stress 12 is 78 tons. Observe the manner of numbering the loads on the force polygon; the forces are really in cyclic order.

EXAMPLE XI.

FLANGED GIRDER with right angled bracing. Span 36 feet; depth ⅛th of span, 4½ feet; load as in Example X, 2½ tons per foot run, 15 tons dead load at each apex on top flange. Fig. 27.

The diagonals and verticals may be made ties and struts as in the upper figure, or struts and ties as in the lower; the first is probably preferable on account of the struts being shorter and therefore of lighter cross section.

There is no stress on 16 and 21 in the upper figure; and these members may be left out by raising supports to A and G.

There is no stress on 8, 9, 14, 15 in the lower figure; these members may not be left out if the load is rolling on the top flange. There is no stress in DM, 27; it only prevents the lower flange NL from sagging by its own weight.

The student should prepare the usual tables; the stress on 12 upper diagram, and on 18 lower diagram is 90 tons.

Only the upper half of stress diagram for each case is shown.

EXAMPLE XII.

BOWSTRING GIRDER. This form of girder has a straight flange supporting the roadway, and curved flange. The curved flange may be below, as in the upper figure; or above as in the lower figure, and forming a kind of parapet.

Span 36 feet; equivalent dead load 2½ tons per foot run, 15 tons at each apex: depth of girder at centre 6 feet. Fig. 28.

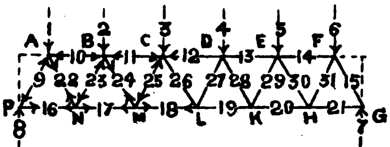
The stress on LK, 17, upper figure is 67½ tons; on DE, 11 lower figure 63 tons by scale. The upper flange is in compression; the lower flange is in tension.

The girder corresponds in a general way to a link polygon. If a link polygon be drawn for the given loads, and be of the same centre height as the girder, the curves of the link poly-

FIG. 26

WARREN GIRDERS

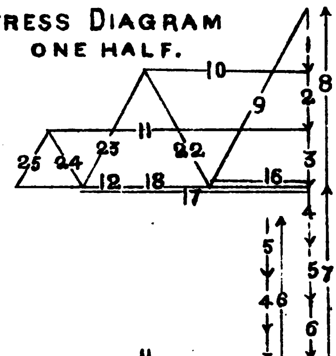
LOAD ABOVE.

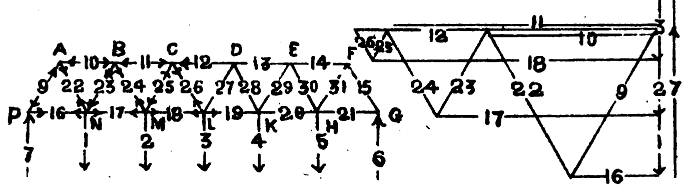


SPAN 36 FEET.

STRESS DIAGRAM

ONE HALF.

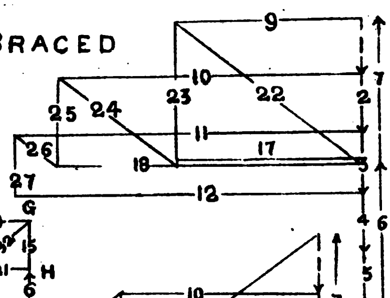




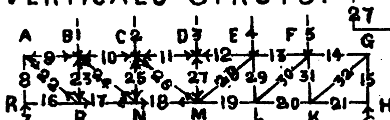
LOAD BELOW.

FIG. 27.

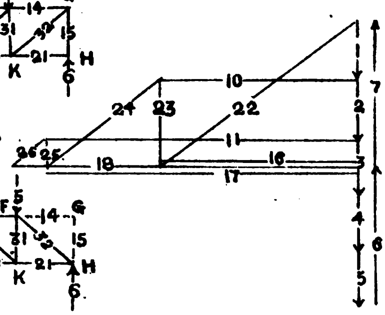
RIGHT-ANGLED BRACED GIRDERS.



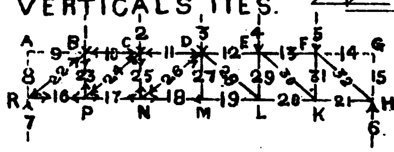
DIAGONALS TIES.
VERTICALS STRUTS.



SPAN 36 FEET.



DIAGONALS STRUTS
VERTICALS TIES.



DIMENSIONS FEET

STRESSES TONS.

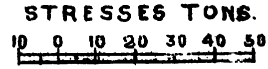
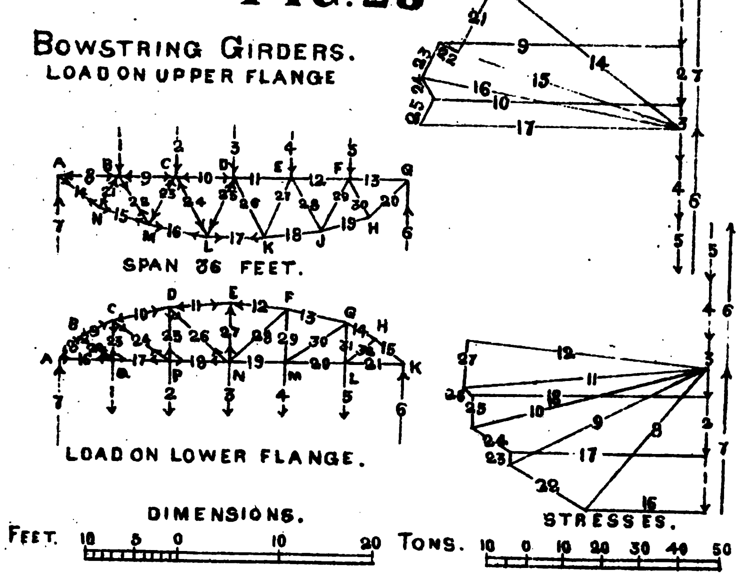


FIG. 28



gon and of the girder will nearly coincide; the link polygon will be in equilibrium without any bracing whatever; the real girder will require braces to transfer the loads to the curved flange, and to take the stresses induced by the fact that the two curves do not quite coincide. The stresses on the braces are small; and the stresses along the flanges are nearly constant.

EXAMPLE XIII.

MASONRY RETAINING WALL for supporting water. Fig. 29.

Let the wall be rectangular in cross section; depth $d = 20$ feet; weight of masonry 140, of water 62·5 lbs. per cub.foot.

The pressure of water on BC the vertical back of the wall is horizontal; its intensity at B is zero, at C it is $q = VC = 1250$ lbs. per square foot, and at any intermediate point it is represented by the horizontal breadth of the triangle BCV. The total pressure of water per foot length of wall is $Q = 1250 \times 20 \div 2 = 12500$ lbs., and the centre of this pressure is at T; $CT = 20 \div 3 = 6\frac{2}{3}$ feet.

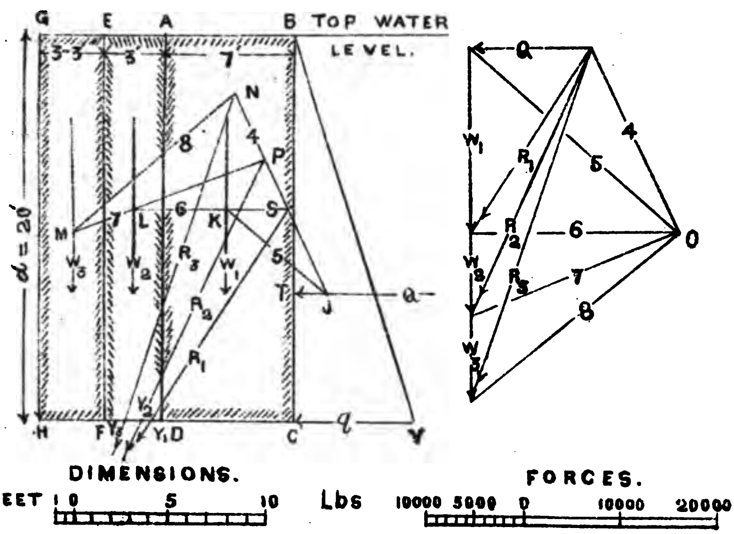
Assume a thickness of wall, say $AB = 7$ feet; weight of one foot length of wall, $W_1 = 7 \times 20 \times 140 = 19600$ lbs.

Draw the diagram of forces Q and W_1; R_1 is the resultant; take any point O as pole, and draw rays 4, 5, 6.

Construct the link polygon. Begin at J any point in Q, and draw links 5, 6, 4. The resultant R_1 acts through S; it intersects the base CD *produced* at Y_1, that is the resultant falls *outside* the base, and the wall is too thin.

FIG. 29.

RETAINING WALL.



Assume an additional thickness in front of wall, EA = 3 feet; W_2 = 3 × 20 × 140 = 8400 lbs. Proceed as before. The new resultant R_2 intersects the base of wall FC at Y_2; FY_2 = 2 feet, one-fifth of thickness of wall. The centre of pressure at base now deviates from the centre of base, three tenths of the thickness of the wall; this proportion is sufficient for works of minor importance.

For large walls retaining water, the centre of pressure at base is usually made to fall within the middle third of the base; applying this rule to this example, we require an additional thickness of wall GE = $3\frac{1}{3}$ feet. R_3 now intersects base of wall at Y_3, and HY_3 = 4·4 feet. Total thickness of wall 13·3 feet.

EXAMPLE XIV.

MASONRY RETAINING WALL for supporting earthwork.

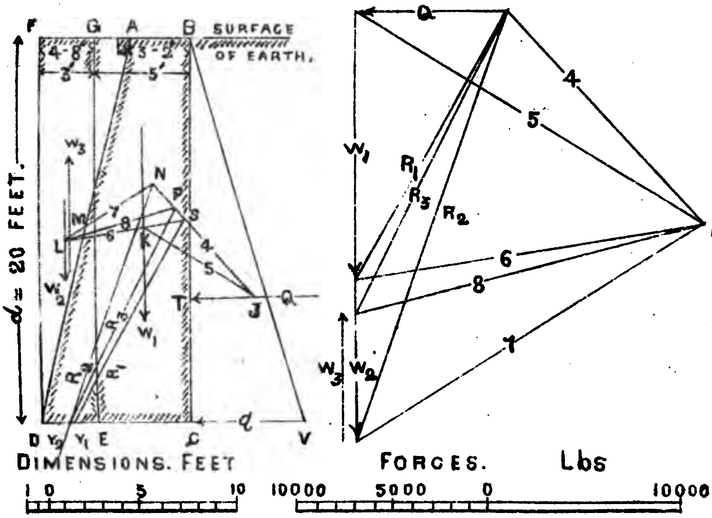
Let the wall FBCD be, at first, rectangular in cross section; depth $d = 20$ feet; weight of masonry 140 lbs. per cub. foot; earth—120 lbs. per cub. foot, spread in horizontal layers, and the ratio of the horizontal pressure to the vertical pressure one third. Fig. 30.

The pressure on BC the vertical back of the wall is horizontal; its intensity at B is zero, and at C it is $q = VC = 20 \times 120 \times \frac{1}{3} = 800$ lbs. per sq. foot; at any intermediate point, the intensity is represented by the horizontal breadth of the triangle BCV. The total pressure of earth on back of wall per foot length is $Q = 800 \times \frac{20}{2} = 8000$ lbs; and the centre of this pressure is T, CT = $6\frac{2}{3}$ feet.

Assume a thickness of wall GB = 5 feet; then W_1 = 14000

FIG. 30.

RETAINING WALL.



lbs; draw the diagram of forces Q W_1, and the link polygon JKS; R_1 drawn from S intersects CE *produced* at Y_1, and the wall is therefore too thin.

Assume an additional thickness in front of wall FG = 3 feet; $W_2 = 8400$ lbs; the resultant R_2 drawn from N intersects the base DC at Y_2, and $DY_2 =$ 1·6 feet; this is sufficient. The thickness of the rectangular wall necessary is FB = DC = 8 feet.

The triangle FAD may be taken from the front of the wall without impairing its stability. The centre of gravity of this triangle is vertically above Y_2; its base FA is $3 \times 1{\cdot}6 = 4{\cdot}8$ feet; its weighc $W_3 = 6720$ lbs. acts upwards as shewn; the resultant R_3 drawn from P passes through the point Y_2 previously obtained for the rectangular wall FBCD. The angle which R_3 makes with the vertical is about 27°; it is and should be less than 33° the angle of repose for masonry; for additional frictional resistance the courses of masonry may be sloped slightly downwards from front to back of wall.

EXAMPLE XV.

WROUGHT IRON PLATE GIRDER. Fig. 31.

Plate girders, with web plate and flanges held together by angle irons, as shewn in the enlarged cross section, are made of wrought iron or steel, for spans up to about 60 feet.

For our example—Span 36 feet; depth 36″ near supports and 37″ near centre of span; working strength of wrought iron f_a = 6 tons per sq. inch, compression; f_b = 7 tons tension; and f_c = 6 tons for shearing. The dead load on the bridge is 1 ton and the live load 2 tons per foot of span; this is equivalent to a dead load of 5 tons per foot of span; and taking two main girders, one for each side of the bridge, the dead load on each girder is $2\frac{1}{2}$ tons per foot run all over span, or 90 tons in all.

The bending moment diagram is a parabola; the maximum bending moment is at the centre of span, and its amount is

$M_o = \frac{1}{8} Wl = \frac{1}{8} \times 90 \times 36 = 405$ foot tons $= 4860$ inch tons as shewn in the diagram. The shearing force is 45 tons at each point of support, and at centre it is 18 tons.

In designing a cross section we take first an approximate and easy method; and check the results by an exact method.

Approximate Method. The bending moment is supposed to be wholly resisted by the flanges, and the shearing forces by the web; this is approximately true, and is on the safe side. The lower flange, in tension, is designed first, rivet holes being allowed for. The upper flange is made exactly the same as the lower; this is a good approximation; since little or no allowance for rivets in the compression flange needs to be made if the holes are properly filled up, and the working strength for thrust is less than that for tension.

The full working strength of the metal, 7 tons per sq. inch is attained only by the outer skin of the lower flange; it diminishes regularly to zero near the middle of the depth of the girder; we therefore take for the lower plate 7 tons, for the second plate $6\frac{3}{4}$ tons, and for angle irons $6\frac{1}{2}$ tons per sq. inch.

Taking the angle irons—the size is $4'' \times 4'' \times \frac{1}{2}''$; deducting for rivets we take $3'' \times 3'' \times \frac{1}{2}''$ which gives a cross sectional area of 6 sq. ins.; the effective distance between the angle irons of the top and of the bottom flanges is say 34 inches; and their moment of resistance to bending is $12 \times \frac{1}{2} \times 6\frac{1}{2} \times 34 = 1326$ inch tons; set up this amount in bending moment diagram.

Taking the inner plate—its breadth is 18 inches; deducting $3''$ for rivet holes gives 15 inches; its thickness is $\frac{1}{2}''$; and the effective leverage between the inner plate of the top and of the bottom flanges is $35\frac{1}{2}''$; the moment of resistance to bending for these two plates is therefore $15 \times \frac{1}{2} \times 6\frac{3}{4} \times 35\frac{1}{2} = 1797$ inch tons. Set this up in the bending moment diagram.

The sum got for the angle irons and inner plate is 3123 inch tons; and in order to give a resistance equal to 4860 inch tons, another half inch plate is evidently required.

Taking the outer plate; its moment of resistance to bending is $15 \times \frac{1}{2} \times 7 \times 36\frac{1}{2} = 1916$ inch tons. Set this up as shewn in bending moment diagram.

The flanges as now designed give 5039 inch tons; a little in excess of the required bending moment.

Taking the web—the shearing force near the supports is

45 tons; the greatest intensity of shearing resistance comes near the middle of the depth of the girder, and it is $\frac{3}{2}$ times the average intensity; this gives $6 \times \frac{2}{3} = 4$ tons per sq. inch as the average intensity of the shearing resistance over the whole cross sectional area of the web; the area required is $45 \div 4 = 11$ sq. ins. nearly and the thickness required is $\frac{5}{16}$ths inch.

ACCURATE METHOD. After having designed the cross section of the girder to suit the given conditions, by a process like the above, the moment of resistance to bending and the resistance to shearing may be checked by the following more accurate method. The cross section is divided into seven rectangles;

18″ × 1″; 8·31 × ·5; 1·31 × 3·5; ·31 × 28; 1·31 × 2·5;

6.31 × ·5; and 15 × 1; the areas are18; 4·2; 4·6; 8·7; 3·3; 3·2; 15 sq. ins; total area S = 57 sq. inches.

Following the method of Problem XVI, fig. 16;—

Draw line for areas; and choose any pole O; the polar distance in diagram is 20 inches; draw horizontal lines from *centre* of each area; construct first link polygon; find for neutral axis $y_a = 17$, and $y_b = 20$ inches; and length of intercept KL = 408 on scale prepared for geometrical moments. Choose any point O′ for the second pole; the polar distance in diagram is again 20 inches. Construct second link polygon with the lines drawn from the *centre* of each area as before; the intercept RS = 14908 and is approximately I_o the moment of inertia of cross section round the neutral axis.

Construct another first link polygon with the lines drawn from the *upper side* of each rectangle, and find MN = 549 on the scale for geometrical moments; take any point O″ for the second pole; the polar distance in the diagram is again 20 inches; construct second link polygon with lines drawn from *lower side* of areas, and find R′H = 14038 on inertia scale.

To RS add one third of difference 14908 − 14038 = 870

$$14908 + 290 = 15200 = I_o \text{ exact.}$$

The moment of resistance to bending is given by the formula $M = \frac{f_a}{y_a} I_o$ or $\frac{f_b}{y_b} I_o$ whichever is less; $\frac{f_a}{y_a} = \frac{6}{17} = \cdot353$; $\frac{f_b}{y_b} = \frac{7}{20} = \cdot350$; and $M = \cdot35 \times 15200 = 5320$ inch tons; a quantity about 5 % greater than 5039 obtained by the easy method.

The maximum intensity of the shearing resistance on the cross section is given by the formula.

$$q = \frac{F}{I_o}\frac{G'_o}{z} = \frac{45}{15200}\frac{448}{\cdot31} = 4\cdot3 \text{ tons per square inch}$$

a quantity less than 6 tons the working resistance to shearing of the material. F = shearing force at the cross section; I_o = moment of inertia of section about neutral axis; G'_o = geometrical moment of part of section above or below neutral axis, and shown in diagram; z = thickness of metal at neutral axis. The thickness of web should not be less than 5/16″; and the web is stiffened by T irons at intervals; these intervals are shorter near points of support where the shearing force is great.

The outer plate of each flange extends over the middle 24 feet of span; its length is shewn on bending moment diagram.

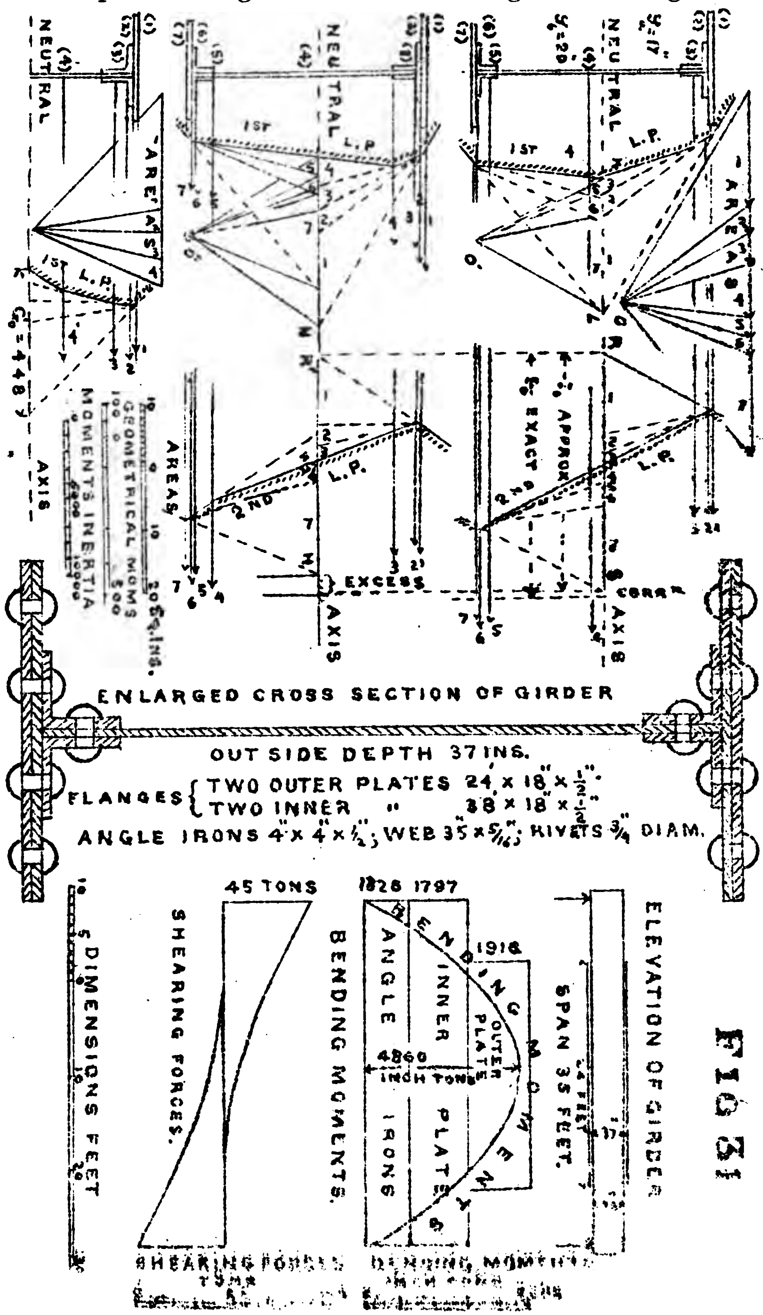


FIG. 31

EXAMPLE XVI.

MASONRY BRIDGE.—ARCH RING. Fig. 32.

Segmental Arch with vertical loading; span 50 feet: rise 10 feet. The radius r for the soffit of the arch is obtained thus:—

$$(2\,r - 10)\,10 = 25^2;\ r = 36{\cdot}25 \text{ feet.}$$

For finding thickness of arch ring, use the empirical formula given by Rankine;—$t = \sqrt{{\cdot}12\,r} = 2{\cdot}1$ feet for a single arch, and $t = \sqrt{{\cdot}17\,r} = 2{\cdot}5$ feet for an arch of a series. The thickness of the arch ring is taken as 2′ 3″.

The horizontal extrados AB, left side of No. 1, the upper line of the dead load, is 2′ 6″ above the upper surface of crown of arch; the dead load, of masonry, etc., weighs 160 lbs. per cub. ft.

The live load is a locomotive weighing one ton per foot run covering the span; this load is distributed over a breadth of say nine or ten feet of arch ring by means of sleepers and ballast; on each foot breadth of arch ring the live load is 2240 ÷ 9½ say 240 lbs. per foot run; and since this is a live load, the equivalent dead load is taken as 2 × 240 = 480 lbs. per foot run per foot breadth of arch ring; this is represented by the live load area three feet high on the right side of No. 1.

The half span of arch is divided into ten equal intervals each 2½ feet wide by the ordinates 0, 1, 2, ... 10; the ordinate 0 is at the springing, the ordinate 10 is at the crown. The heights of these ordinates from soffit to upper side of load area may be measured from the drawing; and the amount of load area extending 1¼ feet on both sides of an ordinate and taken at 160 lbs per sq. foot is easily found; these are given in the following table:—

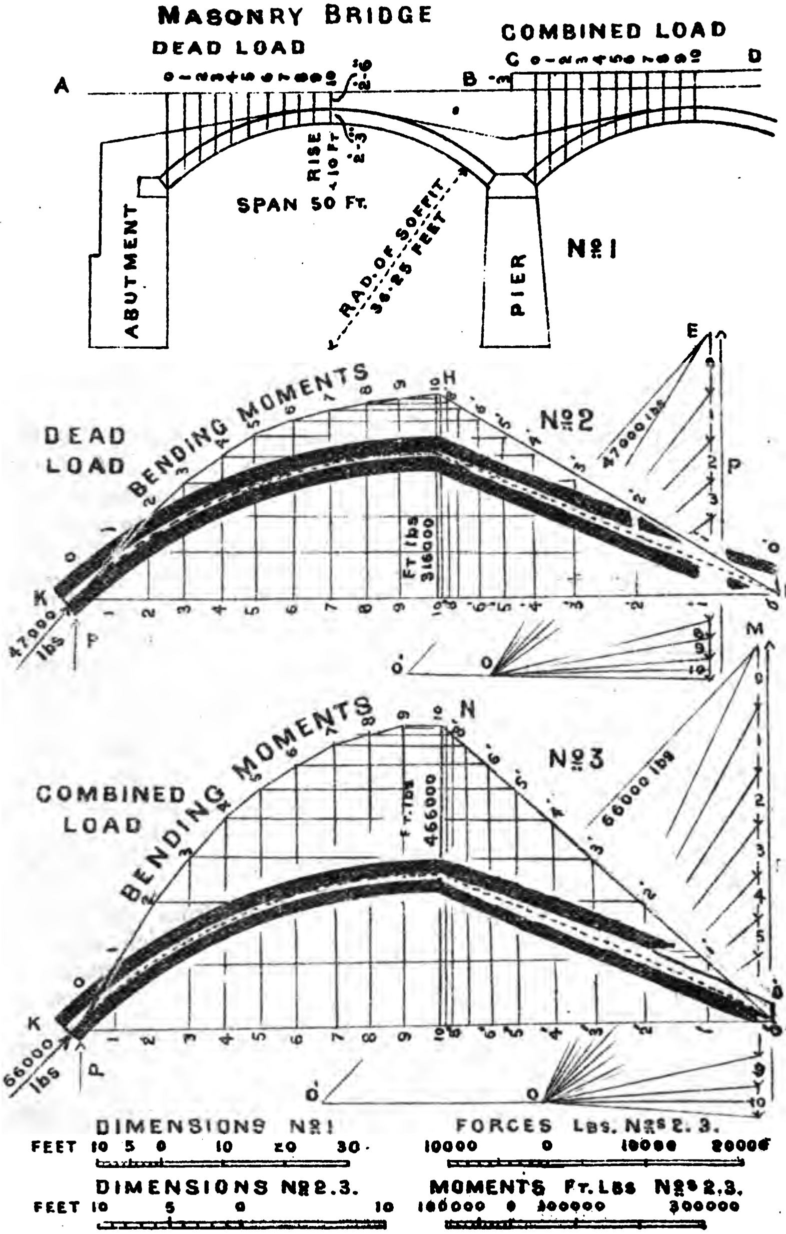
Ordinate Number	Dead Load. Height Feet.	Dead Load. Load Lbs.	Dead Load and Live Load. Height Feet.	Dead Load and Live Load. Load Lbs.
0	14·7	5880	17·7	7080
1	12·8	5120	15·8	6320
2	10·8	4320	13·8	5520
3	9·4	3760	12·4	4960
4	8·1	3240	11·1	4440
5	7·0	2800	10·0	4000
6	6·2	2480	9·2	3680
7	5·6	2240	8·6	3440
8	5·2	2080	8·2	3280
9	4·9	1960	7·9	3160
10	4·7	940*	7·7	1540*
Totals, lbs....		34820		47420

* One half of the amount of No. 10.

The totals give the dead load and combined load, length 26¼ feet from a point 1¼ feet to left of springing to the crown.

Bending Moment Diagram, for Dead Load; No. 2. From E any point, draw the vertical loads 0, 1, 2, ... 10 to scale; from middle of 10 draw P = 34820 lbs. upwards, the vertical component of the support given by the abutment or pier, when the dead load only is on the span. From foot of P draw a horizontal line; choose any point O as pole; the polar distance

FIG 32


MASONRY BRIDGE
DEAD LOAD
COMBINED LOAD
A
B
C
D
RISE 10 FT.
SPAN 50 FT.
RAD. OF SOFFIT 36.25 FEET
ABUTMENT
PIER
No. 1
DEAD LOAD
BENDING MOMENTS
No. 2
E
H
K
L
P
47000 lbs
Ft lbs 316000
COMBINED LOAD
BENDING MOMENTS
No. 3
M
N
66000 lbs
Ft. lbs 466000
DIMENSIONS No. 1
FEET 10 5 0 10 20 30
FORCES Lbs. Nos 2. 3.
10000 0 10000 20000
DIMENSIONS No. 2. 3.
FEET 10 5 0 10
MOMENTS Ft. Lbs Nos 2. 3.
100000 0 100000 300000

in the diagram is 15 feet; and draw rays. The left side of the diagram represents the arch ring to a scale somewhat larger than No. 1; the middle third is left white, the remaining two thirds are shewn in black. The half span is divided into ten equal parts as before, and the vertical ordinates 0, 1, 2, ... 10 are again drawn. These ordinates are taken as the lines of action of the loads; and a link polygon is drawn. This link polygon KH gives the bending moment at each point; and the scale for bending moments is fifteen times as fine as the scale for loads. The bending moment in the middle 316,000 ft. lbs. is marked on the diagram.

The link polygon KH thus found is in equilibrium under the given loads; and the thrust at springing is given in direction and amount by the line OE of the force polygon. We now reduce the vertical ordinates of the bending moment diagram so as to get another link polygon lying, if possible, altogether within the middle third of the arch ring; such a curve is shewn by the dotted line. This method was first given by Professor Fuller.

The horizontal base KL of the diagram is drawn from the point where the vertical 0 intersects the under side of the middle third of the arch ring. A sloping line HL is drawn from the highest point of the bending moment diagram to meet the horizontal base at *any* point L; horizontal lines 9,9′; 8,8′; ....1,1′ are drawn to meet the sloping line HL from points previously found on the bending moment diagram; vertical lines 9′9,′ 8′,8′.........1′,1′ are now drawn extending from the points already found on HL, to the horizontal base KL. The sloping straight line HL is a kind of development of the curve HK; in the same way the white area on the right lying between the two black areas is a development of the middle third of the arch ring. The heights of the upper and under sides of the middle third of arch ring on each ordinate are projected on to the corresponding ordinate of the development; *e. g.* the heights on 44 are transferred to 4′ 4′. The sides of the development are not straight lines.

If possible, a straight line is to be drawn from end to end, in this developed area; and that straight line which is lowest near L and highest near the crown is preferred as it gives maximum rise and minimum thrust; this line is shewn dotted in the diagram, and is the development of the highest link polygon corresponding to the loading, which can be drawn in the middle third of the arch ring. The link polygon itself is now to be projected from its development; it is shewn by the dotted line.

The height of crown of the original link polygon is 31·6; and the height of the new dotted curve is 22·5; the new polar distance should therefore be $31{\cdot}6 \div 22{\cdot}5 = 1{\cdot}4$ times the original polar distance 15 feet; that is 21 feet is the new polar distance, and O′ is the pole. O′E gives the direction and the amount 47000 lbs. thrust at springing of arch for dead load alone.

Bending Moment Diagram for Combined Load. No. 3.

The same procedure is to be followed for the Combined Load. The force line is drawn from M; P = 47420 lbs.; the pole O is chosen with any polar distance, 15 feet in the diagram. The link polygon KN is drawn; the bending moment at the middle of span is 466,000 foot pounds. Taking the height of N as 46·6, the height of the dotted curve in the middle third is 23·6; the ratio 46·6 ÷ 23·6 is 2·0; the polar distance of O′ is 15 × 2·0 = 30 feet. The thrust on the arch at springing is given in direction and amount by O′M = 66000 lbs.

Since the thickness of the arch ring is 2′·3″, the intensity of the thrust at the springing, if it were uniformly distributed, would be 29,333 lbs. per sq. foot; taking the thrust as of uniformly varying intensity 58666 at soffit, and zero at the outer surface of the arch ring, gives the same amount of thrust, and the centre of thrust is just within the middle third of arch ring. The working compression strength of the material of the arch ring should therefore be 58666, or say 60,000 lbs. per sq. foot.

EXAMPLE XVII.

MASONRY BRIDGE—Abutments, Piers, and Abutment Piers.

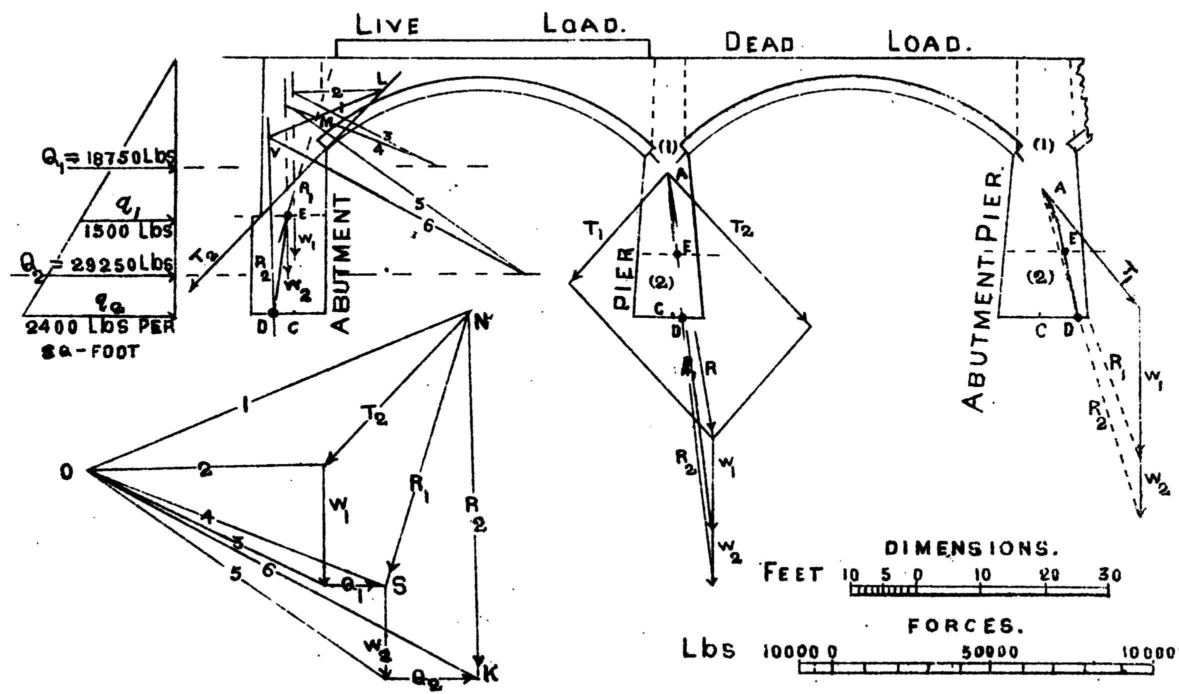
For superstructure and loads, the data of the previous example, No. 16, are taken; the foundations of the abutments and piers are 25 feet below level of springing of arches. Fig. 33.

ABUTMENT PIER. In the case of a long viaduct, consisting of many spans, every fifth or sixth pier is usually built of extra thickness to enable it to act as an abutment pier. It may not be practicable to build all the piers of a viaduct at one time; in such a case an abutment pier may form an abutment during construction; and afterwards, in case of one or more of the arches being washed away, these piers prevent the collapse of more than a few spans. The thickness of an abutment pier, at springing, is usually one fourth or one fifth of span; in this example, it is ten feet, one fifth of the span; and the sides have a batter of 1 in 12.

An abutment pier is shewn on the right hand side of the diagram; its area is divided into two parts marked (1) and (2); (1) extends from the dead load line to 10 feet above the base; its area is $7\frac{1}{2} \times 15 + 11\frac{1}{4} \times 15 = 281$ sq. feet; and its weight $W_1 = 45{,}000$ lbs.; the area (2) is $13{\cdot}3 \times 10 = 133$ sq. feet, and $W_2 = 21{,}300$ lbs.

The thrust of the arch at springing, T_1 due to the dead load only, is 47,000 lbs.; R_1 the resultant of T_1 and W_1 intersects the line 10 feet above the foundation at the point E; R_2 the resultant of T_1 and $W_1 + W_2$ cuts the base line at D; the curve AED is the *Line of centres of pressure*, and should everywhere fall a little inside the abutment pier. In this example the point E is about $2\frac{1}{2}$ feet, and D is 1 foot from the face of the

FIG. 33.



LIVE LOAD.
DEAD LOAD.
Q1 = 18750 Lbs
q1
1500 Lbs
Q2 = 29250 Lbs
q2
2400 Lbs PER SQ-FOOT
ABUTMENT
PIER
ABUTMENT PIER.
DIMENSIONS.
FEET 10 5 0 10 20 30
FORCES.
Lbs 10000 0 50000 100000