GRAPHS FOR BEGINNERS

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LONDON

BLACKIE & SON, LIMITED, 50 OLD BAILEY, E.C.

GLASGOW AND DUBLIN

1906

PREFACE

Graphs are the illustrations of mathematics, and as in the early stages of education great recourse is made to the picture-book, so graphs should take a prominent place in the early mathematical training of pupils. Used aright they create interest, cultivate habits of observation, stimulate the reasoning powers, and are a powerful factor in obtaining neatness and accuracy in general work.

This little book treats of graphs from a general point of view and not as a branch of pure mathematics. Discontinuous graphs are given a prominent place, especially those generally used in commercial and technical work. This has necessitated the exclusion of much matter purely algebraical and more suitable for an advanced course of study.

It will be necessary for the teacher to elaborate the text at some points, notably Exercise VI of Chapter V, where some instruction should be given in the rearrangement of formulæ. Every opportunity also should be taken to apply the graphic methods given, to the elucidation of problems in Arithmetic, Geometry, and Mensuration. The teacher should also devise simple experiments, the results of which may be expressed graphically. Suggestions for such are contained in Exercise 23 of the Miscellaneous Examples, and they will be found to intensify the interest of the pupil in his work.

The book is intended for one year's course, and the pupil's exercises throughout should be carefully preserved. Each pupil should have two exercise books, one ruled in $\frac{1}{10}''$ and another in millimetre squares, and these should be indexed. Loose sheets of squared paper should be used for the preliminary work of each chapter.

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GRAPHS FOR BEGINNERS

CHAPTER I

Have you ever noticed at the corner of a street an enamelled plate with such symbols as F.P. 8.14 on it? No doubt you have, and perhaps wondered what these figures meant. The meaning is simply this. If you walk 8 feet to the right of the plate, and then 14 feet towards the middle of the street, you come to a "fire-plug". Even

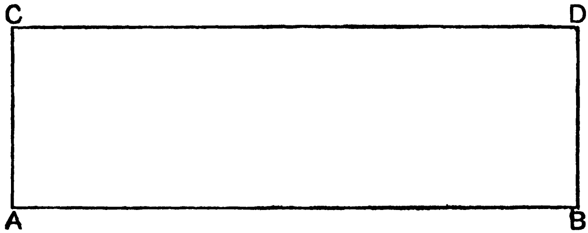


Fig 1

in the depth of winter, when the streets are covered with snow, the firemen have no difficulty in finding the nearest fire-plug with this plate to guide them. In fact, it would be quite easy to set down in this way all the fire-plugs, gas-plugs, sewer-openings, gratings, &c., in a street, and should they at any time be accidentally covered it would be an easy matter to find them.

For example, draw A B C D to represent a section of a street 60 yards long and 20 yards broad. Let A be the corner from which we are to measure, and to keep the drawing a reasonable size make

$\frac{1}{2}$ inch represent 10 yards. Here is a list of plugs, &c., given in the manner already indicated; put them in your drawing.

Gratings.—0 . $2\frac{1}{2}$, 0 . $17\frac{1}{2}$, 20 . $2\frac{1}{2}$, 60 . $17\frac{1}{2}$, 60 . $2\frac{1}{2}$ yards.

Fire-plugs.—10 . 10, 50 . 10.

Gas-plugs.—10 . 5, 20 . 5, 40 . 5, 50 . 5, 60 . 5.
10 . 15, 20 . 15, 40 . 15, 50 . 15, 60 . 15.

Water-plugs.—15 . 5, 25 . 5, 45 . 5, 55 . 5.
15 . 15, 25 . 15, 45 . 15, 55 . 15.

Mark gratings G., fire-plugs F.P., gas-plugs G.P., and water-plugs W.P.

You will notice that each plug requires two measurements to determine its position, one distance being measured at right angles to the other.

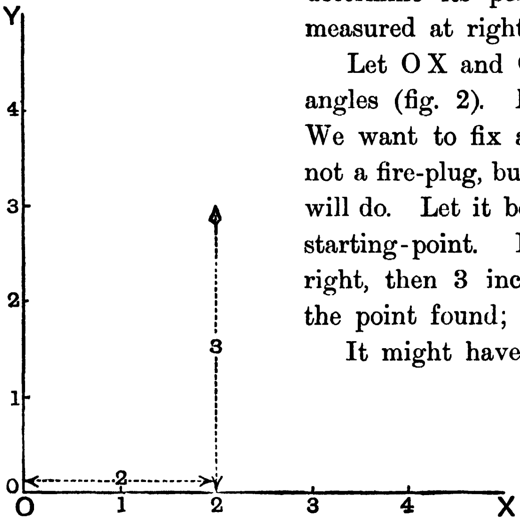


Fig. 2

Let O X and O Y be two lines at right angles (fig. 2). Mark them off in inches. We want to fix a certain *point* this time, not a fire-plug, but exactly the same method will do. Let it be the point **2 . 3**. O is our starting-point. Measure 2 inches to the right, then 3 inches up. Put a mark at the point found; it is the point **2 . 3**.

It might have been more convenient, if we had a number of points to find, to draw lines horizontally and vertically through the inch divisions, as shown in fig. 3. Where the vertical line through 2 cuts the horizontal line through 3 is, as before, the point **2 . 3**. Paper may be obtained carefully ruled in this way with inches, centimetres, millimetres, or any divisions we choose. It will be evident, however, that the divisions need not be any particular size, as we may call them inches, yards, miles, centimetres, or kilometres, provided we remember what scale we have fixed on. Such paper is called "squared paper", and will be much used hereafter.

The line O X is called an **axis** or centre line.

The line O Y is also called an **axis** or centre line.

O X is called the **x axis.**

O Y is called the **y axis.**

We may therefore say that measurements along O X are made along the x axis, and measurements up O Y are made up the y axis. Further, since we require to make two measurements to fix a certain point, one along the x axis and one along the y axis, these two measurements are called "Co-ordinates". We could have said that the co-ordinates of the point A found in figs. 2 and 3 were **2.3**.

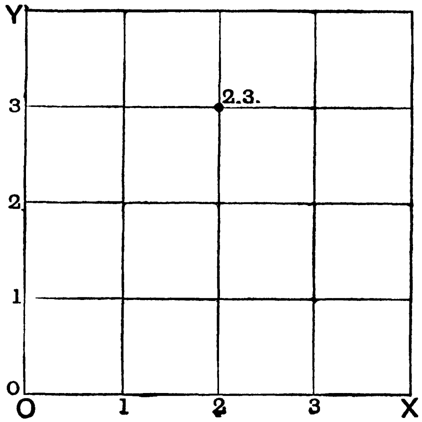


Fig. 3

Take some squared paper, ruled, say, in tenths of an inch. Draw the lines O X and O Y, and find the points whose co-ordinates are given in Exercise I.

In doing this, be most exact and neat in your methods; use a sharp pencil, and test every point after finding it. While it is not necessary to do so, you may join the points in the order they are given, and thus determine whether you are correct or not; a glance will tell you. After doing one question, take a new Y axis to the right; this will save paper.

EXERCISE I

1. 3.13, 5.15, 5.5, 3.5, 7.5,—15.7, 8.7, 13.15, 13.5, 11.5, 15.5.
2. 5.5, 10.5, 10.10, 5.10, 5.15, 10.15.
3. 5.5, 4.6, 3.8, 3.12, 4.14, 5.15, 6.16, 8.17, 12.17, 14.16, 16.14, 17.12, 17.8, 16.6, 15.5, 14.4, 12.3, 8.3, 6.4.

4. 10.10, 15.20, 20.10,—10.15, 20.15, 15.5.

5. 10.10, 8.10, 7.15, 13.15, 12.10, 10.10, 10.2, 8.2, 12.2.

6. Draw your initials with a ruler on squared paper, using straight lines only. Take an x and y axis, and mark the chief points in the letters. Find the co-ordinates of these points, note them down in their order, and give the list to your neighbour to plot out. Remember a straight line requires only 2 points in order to draw it correctly.

CHAPTER II

Suppose you find a number of points by the method of co-ordinates, a glance may show that these points when joined form a line or a curve. For example, you no doubt noticed that question 3, Exercise I, was a circle, question 4, two triangles, and so on. When a number of points are thus joined, the line so formed is termed a "*Graph*". In question 3, Exercise I, we would say that the "*Graph*" obtained by joining all the points given is a circle. The word "Graph" is generally used to indicate one line either straight or curved fulfilling certain conditions, but to simplify our work let us apply the word graph *to any line*—straight, curved, or broken, provided it is found by the method of co-ordinates. The study of graphs is both interesting and instructive, for valuable information may often be derived from them. Just as a detective ferrets out all the "points" of a criminal case, puts them together, and sees if from them he can gain such information as will lead to the detection of the criminal, so in the realms of science, in the workshop, in the office, we may often by observation or experiment obtain certain numbers. These, when set down by the method of co-ordinates, and the points so found joined, may give us a graph leading to the detection of the law or laws underlying or governing the phenomena or facts we have been considering.

Perhaps the most interesting case to start with will be the making of "Contour Graphs" of the roads in the district in which you live. In the case of the street plugs, the line O X represented distances along the street, O Y distances across the street; now O X will represent distances from our starting-point and O Y heights above the sea-level. Take, for example, the road from Glasgow to Prestwick, starting from the Broomielaw Bridge. This is 50 feet above the level of the sea, and being the starting-point is no distance along; we may therefore term it the point **0.50**. One mile farther on the height is still 50 feet; call this the point **1.50**. We may, in fact, set down our information regarding the road thus—

Route.—GLASGOW TO PRESTWICK. Distance, 30 miles.

0.50, **1**.50, **2**.75, **3**.100, **4**.150, **5**.200, **6**.300, **7**.450, **8**.550, **9**.625, **10**.650, **11**.700, **12**.725, **13**.700, **14**.650, **15**.550, **16**.500, **17**.400, **18**.350, **19**.300, **20**.200, **21**.125, **22**.100, **23**.100, **24**.200, **25**.275, **26**.300, **27**.225, **28**.150, **29**.50, **30**.50.

Places on the Route.—$1\frac{1}{2}$ miles, Strathbungo; $2\frac{1}{2}$, Shawlands; $4\frac{1}{2}$, Giffnock; 7, Newton Mearns; $7\frac{3}{4}$, Mallctsheugh; $9\frac{3}{4}$, Loganswell; 17, Fenwick; 21, Kilmarnock; 22, Riccarton; 26, Whitelea; 29, Monkton; 30, Prestwick.

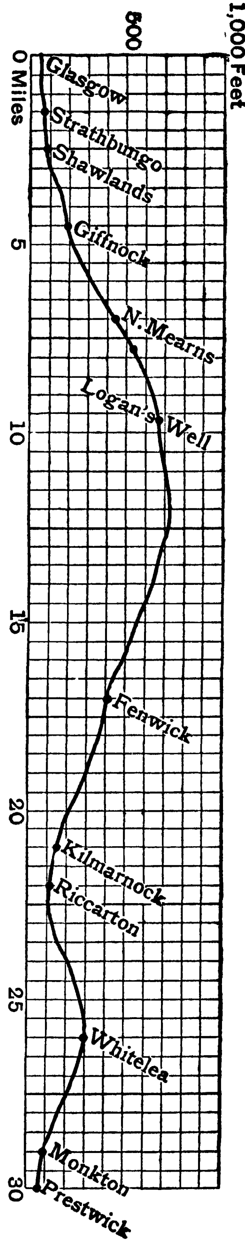


Fig 4

Now take squared paper, as previously mentioned. Let every 2 divisions on the O X line represent a mile, and every division on the O Y line 100 feet. Make an oblong as in fig. 4 to contain the "contour graph", marking the miles 0, 5, 10, &c., and the heights 0, 100, 200, to 1000 feet. Mark in the points given by the method of co-ordinates, and join them with a clear line in red ink. Put in any places of note as shown. Note that the scales of heights and distances are quite different, but as long as this is remembered no difficulty need result. Thus 75 feet will be ¾ of a division up; 325 feet, 3¼ divisions up, and so on. Further, it should be noted that this is the graph connecting two things, distance and height, and it shows how the height changes as the distance changes. All the graphs to be plotted later on will tell us how one thing changes as another changes. It will be seen also that the slope gives us an idea of the rate of the change. As the slope of a graph will be found later to be of the utmost importance, it would be well that the pupil should find the slope in degrees at different points.

In the same manner as indicated in fig. 4 plot out the following contour graphs:—

EXERCISE II

I—Glasgow to Greenock. 22 miles.

0.20, **8**.50, **11½**.50, **12**.100, **12¾**.175, **13**.150, **13½**.100, **14**.50, **17¼**.75, **19**.50, **22**.25.

Note.—**0**.20, **8**.50 signifies road level 0 to 7. (Put 4 divisions to mile.)

Places on Route.—2½, Govan; 6, Renfrew; 7½, Inchinnan; 9, Wardhouse; 12, Bishopton; 15, Langbank; 19, Port-Glasgow; 22, Greenock.

II—Glasgow to Largs. 29¼ miles.

0 to **5**.30, **6** to **9**.50, **9¼**.75, **10**.100, **10½**.125, **11**.100, **12**.100, **13**.125, **13¼**.150, **14**.125, **14⅛**.100, **14¼**.130, **15**.175, **15½**.175, **16**.125, **16¼**.100, **17**.150, **17½**.150, **18**.225, **18¼**.200, **18¾**.175, **19**.200, **19½**.250, **20**.200, **20¾**.300, **21**.350, **22**.475, **23**.600, **24**.625, **25**.700, **26**.750, **26½**.700, **27**.600, **28**.300, **28½**.100, **29**.75, **29¼**.25.

Places on Route.—6, Paisley; 9¼, Elderslie; 13½, Elliston; 16½, Lochwinnoch; 20, Kilbirnie; 22½, Howrat; 24, Whitehill; 29¼, Largs.

III—Glasgow to Edinburgh. 44 miles.

0.20, **1**.50, **2**.75, **3**.75, **4**.100, **5**.200, **6**.250, **7**.250, **8**.250, **9**.250, **10**.300, **11**.420, **12**.475, **13**.550, **14**.550, **15**.575, **16**.650, **17**.**18**.650, **19**.625, **20**.600, **21**.600, **22**.575, **23**.550, **24**.500, **25**.450, **26**.450, **27**.500, **28**.**29**.550, **30**.450, **31**.400, **32**.300, **33**.200, **34**.**35**.150, **36**.120, **37**.150, **38**.**39**.**40**.**41**.**42**.150, **43**.200, **44**.250.

Places on Route.—3, Shettleston; 9, Coatbridge; 11, Airdrie; 25½, Bathgate; 31½, Uphall; 33, Broxburn; 44, Edinburgh.

IV—London to Brighton. 53 miles.

0.50, **1**.50, **2**.50, **3**.50, **4**.75, **5**.100, **6**.175, **7**.150, **8**.100, **9**.150, **10**.175, **11**.175, **12**.175, **13**.200, **14**.225, **15**.250, **16**.300, **17**.400, **18**.425, **19**.350, **20**.300, **21**.275, **22**.350, **23**.200, **24**.200, **25**.200, **26**.200, **27**.200, **28**.200, **29**.200, **30**.225, **31**.300, **32**.350, **33**.450, **34**.450, **35**.500, **36**.250, **37**.200, **38**.300, **39**.100, **40**.75, **41**.75, **42**.100, **43**.100, **44**.100, **45**.150, **46**.250, **47**.350, **48**.250, **49**.150, **50**.150, **51**.100, **52**.50, **53**.50.

0, G.P.O; 6, Streatham; 11, Croydon; 21, Redhill; 30, Crawley; 35, Handcross Hill; 53, Brighton.

V—Manchester to Buxton. 24½ miles.

0.150, **1**.100, **2**.150, **3**.150, **4**.200, **5**.250, **6**.200, **7**.250, **8**.250, **9** 300, **10**.300, **11**.450, **12**.600, **13**.600, **14**.600, **15**.600, **16**.550, **17**.550, **18**.600, **19**.800, **20**.1000, **21**.1200, **22**.1300, **22½**.1400, **23**.1350, **24**.1100, **24½**.1050.

0, Manchester; 7, Stockport; 13, Disley; 24½, Buxton.

VI—Liverpool to Warrington. 17½ miles.

0.50, **1**.200, **2**.150, **3**.200, **4**.100, **5**.100, **6**.125, **7**.200, **8**.300, **9**.225, **10**.200, **11**.200, **12**.150, **13**.100, **14**.75, **15**.75, **16**.50, **17**.50, **17½**.50.

0, Liverpool; 7½, Prescot; 9, Rainhill; 16, Sankey Bridge; 17½, Warrington.

VII—Newcastle to Wolsingham. 23¼ miles.

0.100, **1**.300, **2**.50, **3**.200, **4**.350, **5**.500, **6**.700, **7**.600, **8**.750, **9**.800, **10**.850, **11**.750, **12**.600, **13**.500, **14**.400, **15**.650, **16**.700, **16½**.500, **17**.650, **18**.750, **19**.850, **20**.1000, **21**.900, **21¼**.750, **22**.850, **23**.500, **23¼**.500.

0, Newcastle; 1, Gateshead; 14, Lanchester; 15¾, Coldpike Hall; 23¼, Wolsingham.

VIII—Edinburgh to Kinross. 26 miles.

0.250, **1**.200, **2**.150, **3**.200, **4**.210, **5**.150, **5½**.100, **6**.200, **7**.100, **8**.200, **9**.0, **10**.0, **11**.100, **11½**.20, **12**.100, **13**.50, **14**.300, **15**.350, **16**.400, **17**.450, **18**.450, **19**.**20**.400, **21**.375, **22**.400, **23**.**24**.**25**.**26**.400.

5½, Cramond Bridge; 7, Dalmeny; 9, Queensferry; 10, North Queensferry (ferry over Firth of Forth); 12, Inverkeithing; 18, Cowdenbeath; 26, Kinross.

IX—Dundee to Aberdeen. 66½ miles.

0.50, **1**.**2**.150, **3**.120, **4**.**5**.100, **6**.150, **7**.120, **8**.**9**.**10**.**11**.**12**.150, **13**.100, **14**.75, **15**.**16**.**17**.50, **18**.100, **19**.150, **20**.200, **21**.**22**.100, **23**.50, **24**.150,

25.200, **26**.300, **27**.275, **28**.**29**.**30**.**31**.**32**.50, **33**.100, **34**.**35**.250, **36**.**37**. **38**.200, **39**.150, **40**.120, **41**.150, **42**.100, **43**.200, **44**.350, **45**.325, **46**.300, **47**.250, **48**.220, **49**.250, **50**.220, **51**.100, **52**.50, **53**.150, **54**.250, **55**.225, **55½**.150, **56**.200, **57**.250, **58**.200, **59**.**60**.250, **61**.300, **62**.325, **63**.250, **64**.100, **65**.50, **66**.100, **67**.50.

Places on Route.—17, Arbroath; 30, Montrose; 42, Bervie; 52, Stonehaven; 65, Brig of Dee; 66½, Aberdeen.

CHAPTER III

It has been already said that a graph shows the rate of change of two things, and conversely if any one thing change and thus cause another to change, the relation of the two may be set down in the form of a graph. For example, the price of pig-iron varies almost every day according to the supply and demand. Here price changes with time, and we may draw a graph to show this, plotting time horizontally and price vertically. The time may be expressed in days, weeks, or months, and the price in pounds or shillings per ton. We could also on the same paper plot out the prices of copper, zinc, tin, lead, &c., during the same time. By comparing the graphs we could easily ascertain which metals fall or rise in price at the same time, and which do not. We could also plot out on one sheet of paper the price of one metal for, say, 10 years, drawing a new graph for each year. By studying these graphs we could find out when, say, iron is dearest in any year, when cheapest. Diligent enquiry might then lead us to discover causes that arise every year, making iron dear or cheap at certain periods.

Here are the prices of pig-iron, ingot copper, steel, and tin, &c., for 1901. Prices are given on the average of every month. For your guidance the graphs of silver, lead, and tin are drawn (fig. 5). Put in these and the others on one sheet of squared paper, if possible, using the scale for pounds, shillings, or pence as required.

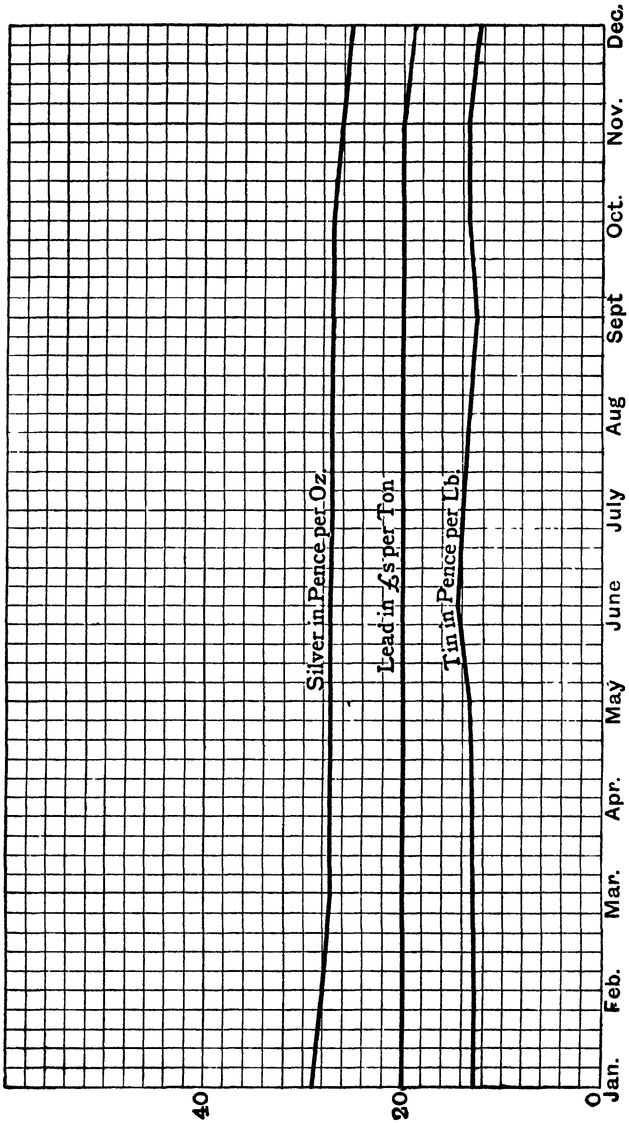

Silver in Pence per Oz.
Lead in £s per Ton
Tin in Pence per Lb.
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug
Sept
Oct.
Nov.
Dec.
0
20
40

Fig 5

EXERCISE III

Jan.	Feb.	Mar	Apr.	May.	June.	July	Aug.	Sept	Oct.	Nov	Dec
Price of Iron in shillings per ton—											
66·2	73·7	85	85	81·2	80	80	80	80	81·2	83·7	83·7
Price of Copper in £s per ton—											
71·7	71	69·5	69·6	69·6	68·8	67·6	66·3	65·9	64	64·5	52
Price of Lead in £s per ton—											
20	20	20	20	20	20	20	20	20	20	20	19·5
Price of Steel in shillings per ton—											
98·7	105	120	120	120	120	120	122·5	132·5	137·5	140	140
Price of Tin per pound in pence—											
13·25	13·3	13	13	13·5	14·3	14	13·3	12·6	13·3	13·3	12
Price of Silver per ounce in pence—											
28·9	28	27	27·3	27·4	27·4	26·9	26·9	26·9	26·9	26·1	25·4

Much time may be usefully spent in drawing graphs of the kind just given. Find some for yourselves, and suggest them to your teacher. For example, the height of the barometer daily for a month, the temperature of your room from 9 A.M. till 4 P.M., taking observations every five minutes; the attendances of the pupils in your class for a fortnight, with your own attendances on the same paper. In fact, from your daily life and surroundings you will be able to make numerous graphs. Do not, however, plot out the graphs mechanically; endeavour to extract information from them. For example, the temperature of your class-room from 9 A.M. till 4 P.M. will show the effects of the breaths of the pupils; the rapidity of the rise will show how far the ventilation is reliable. The effect of the intervals for play should be noted as to whether the temperature falls to its normal, or if not, how far it falls in the interval. Further, borrow several thermometers, place them in different parts of the room, and take simultaneous readings for each. Plot out the graphs on one sheet of squared paper, and thus determine the effects of ventilation in different parts of the room. In the graph of attendances see if your graph is over or under the average of the class; see if there is any day or days in the week uniformly good or bad in attendance, and endeavour to ascertain why. Trace any abnormally good or bad attendance. It will occur to you that the graph of attendances is often the graph of many other things besides. Thus it may be partly a graph of the weather prevailing, it may be a graph of the infectious diseases prevalent. There is almost no limit to the information about other things thus locked up, and only to be had by careful research. Let then your aim, in making a graph, be to find out how much the graph can tell you; and this may only be extracted by much patient and earnest thought.

EXERCISE IV

Suggested graphs. Find the data—

1. Height of barometer for a month taken daily.
2. Temperature of class-room from 9 A.M. till 4 P.M.
3. Heights of boys in your school according to ages, taking, say, 6 of each age to get an average.
4. The distance you walk from 7.30 A.M. till 10.30 P.M., from notes made every half-hour.
5. The rainfall in your district for a year.
6. The cases of infectious disease (ascertained weekly from the newspapers) for a year, each disease having a graph of its own, but all on one paper.
7. The passengers in the tramcars weekly for a year (from newspapers).
8. The prices of coal and iron (on one sheet of paper) for a year.

A large number of examples of the above type are given at the end of the book. Selections may be made to suit the other subjects taken by the student.

The following graph is worthy of careful study:—Take a railway time-table and a good cycling map. Select some route, for example, from Glasgow (St. Enoch) to Greenock (Princes Pier). Set down the time of starting of one train and the times at all stations *en route*, with their distances. Make a graph connecting distance and time, plotting distances in miles vertically and minutes horizontally.

Train, 5.58 P.M.—GLASGOW TO GREENOCK (Princes Pier)

Places	Miles from Terminus	Time	Time from Terminus.
Glasgow	0	5.58 P.M.	0
Shields	2	6.1½ „	3½
Bellahouston	3	6.3 „	5
Crookston	5½	6.6½ „	8½
Paisley (Canal)	6½	Arrive 6.10. Leave 6.12	12.14
Paisley (West)	7½	6.14 P.M.	16
Elderslie	9	6.16 „	18
Houston	10	6.18½ „	20
Bridge of Weir	13½	6.21 „	23½
Kilmalcolm	16½	6.26½ „	29
Lynedoch	22½	6.35 „	37
Greenock (Princes Pier)	23½	6.38 „	40

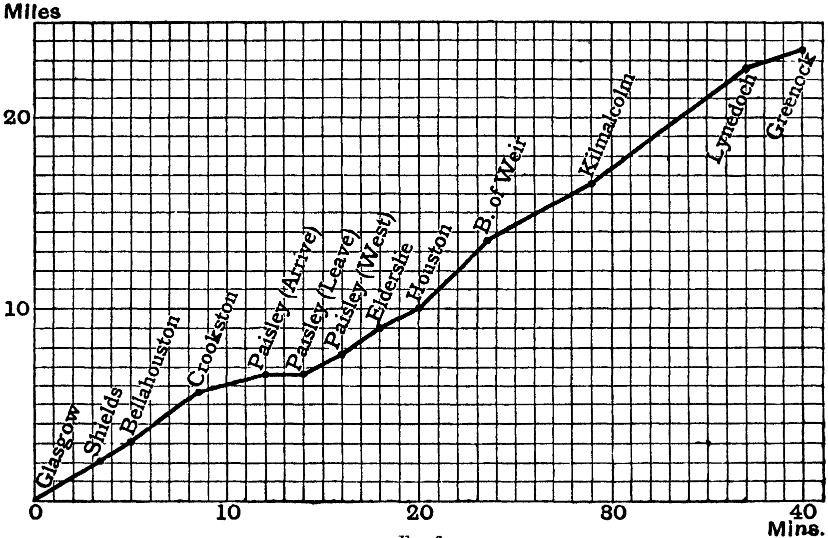


Fig. 6

Fig. 6 shows the graph. Note that the slope is entirely upward, meaning that the distance always increases as the time increases. It will be noticed that the slope is greatest between Houston and Bridge of Weir. Now from Houston to Bridge of Weir is 3½ miles, and we notice it is done in 3½ minutes—that is, the average speed is 60 miles per hour. But if you look carefully you will see that the slope tells us that 3½ miles is gone in 3½ minutes, therefore the slope is a measure of the speed of the train, and the part with steepest slope is the part where the highest average speed has been reached. *Note also most carefully* that at Paisley the graph is level *since the train stops* for two minutes.

EXERCISE V

The following are extracts from time-tables. Plot out the graphs, connecting distance and time; ascertain the points of maximum average speed in each case:—

1.—Express (GLASGOW TO EDINBURGH) leaving Central Station at 11 o'clock A.M.

Station	Distance from Glasgow	Time Leaving.
Glasgow..............	0 miles	11.0 A.M
Eglinton St.	1 mile	11.2½ ,,
Rutherglen	2½ miles	11.6 ,,
Cambuslang..........	5 ,,	11.8½ ,,
Newton	7 ,,	11.10½ ,,
Uddingston	8 ,,	11.13 ,,
Bellshill...	10½ ,,	11.16½ ,,
Holytown	13 ,,	11.19 ,,
Omoa.	15 ,,	11.23½ ,,
Hartwood	18 ,,	11.27 ,,
Shotts...	20 ,,	11.30 ,,
Fauldhouse	23 ,,	11.34 ,,
Breich	25 ,,	11.36½ ,,
Addiewell	27½ ,,	11.39½ ,,
West-Calder	29 ,,	11.42 ,,
New-Park	31 ,,	11.44½ ,,
Mid-Calder...	34½ ,,	11.48 ,,
Curriehill.............	38½ ,,	11.52 ,,
Kingsknowe	40½ ,,	11.55 ,,
Slateford..	41½ ,,	11.59 ,,
Merchiston..	42½ ,,	12.0 ,,
Edinburgh	44 ,,	12.5 ,,

2.—South Morning Express (GLASGOW TO CARLISLE)

Station.	Distance from Glasgow	Time Leaving
Glasgow (Central)	0 miles	10.0 A.M.
Eglinton St.	1 mile	10.2½ ,,
Rutherglen...........	2½ miles	10.6 ,,
Motherwell	12¾ ,,	10.30 ,,
Wishaw..........	16 ,,	10.36 ,,
Carstairs..............	29 ,,	10.52 ,,
Beattock	65 ,,	11.47 ,,
Lockerbie	79 ,,	12.8 P.M.
Carlisle	102 ,,	12.35 ,,

3.—The stopping-places on an electric tramway are $\frac{1}{4}$ mile apart. Starting from the terminus, I take the time every $\frac{1}{4}$ mile till car stops.

The following are times:—Draw a graph showing distance in given times. Indicate approximately where the tram passes through the busy streets, where country roads. *Start*—P.M. 5.0, 5.1½, 5.3, 5.5, 5.7, 5.9, 5.11, 5.13½, 5.16½, 5.20, 5.24, 5.28, 5.31, 5.33½, 5.35, 5.37, 5.40, 5.42, 5.44, 5.45½, 5.47, 5.48½, 5.49¾, 5.51¼—*Terminus.*

4.—LONDON TO CARLISLE (Express)

Station.	Distance from London	Time Leaving.
London (St. Pancras)....	0 miles	12.0 midnight
Leicester	99 „	Arrive 1.55. Leave 2.0
Trent.........................	120 „	„ 2.24. „ 2.28
Leeds..........	198 „	4.0
Bradford	211 „	4.40
Carlisle......................	310 „	6.25

5.—LIVERPOOL TO MANCHESTER.

Station.	Distance from Liverpool	Time Leaving.
Liverpool (Central)......	0 miles	7.15 P.M.
St. Michaels...............	2½ „	7.21 „
Cressington................	5 „	7.25 „
Garston.....................	5½ „	7.28 „
Hunt's Cross....	7 „	7.33 „
Halewood..................	8½ „	7.37 „
Ditton....	10½ „	7.42 „
Farnworth.................	12 „	7.48 „
Sankey......................	16 „	7.55 „
Warrington................	18 „	8.0 „
Padgate....	20 „	8.7 „
Glazebrook................	24½ „	8.15 „
Irlam.........................	25½ „	8.19 „
Flixton.....	28 „	8.26 „
Urmston	29 „	8.34 „
Trafford Park.............	31½ „	8.39 „
Manchester....	34 „	8.48 „

6.—London to Brighton

Station	Distance from London.	Time Leaving.
London Bridge........	0 miles	5.20 P.M.
New Cross.........	$2\frac{3}{4}$ "	5.26 "
Norwood...............	$8\frac{1}{2}$ "	5.43 "
Croydon................	10 "	5.52 "
Purley....................	13 "	5.59 "
Red Hill...	21 "	6.14 "
Earlswood.............	$21\frac{1}{2}$ "	6.21 "
Horley..	$25\frac{1}{2}$ "	6.30 "
Three Bridges.	29 "	Arrive 6.39. Depart 6.43
Balcombe..	34 "	6 53 P.M.
Haywards Heath... .	38 "	Arrive 7.1. Depart 7.3
Burgess Hill.	$41\frac{1}{2}$ "	7 9 P.M.
Hassocks....	$43\frac{1}{2}$ "	7.14 "
Preston Park.........	49 "	7.25 "
Brighton.......	$50\frac{1}{2}$ "	7 30 "

CHAPTER IV

Take a sheet of squared paper, draw the axes O X and O Y; call the vertical divisions shillings and the horizontal divisions, say, "articles".

Now 4 articles at 3*d.* = 1*s.* Find the point **4 . 1**,
and 12 " = 3*s.* " **12 . 3.**

Join these points, and produce the line joining them both ways (fig. 7). We have a graph of some kind; let us test it. First of all it passes through O, in fact seems to take its origin from O; O is therefore the "Origin". Take now any number of articles, say 40. Trace the vertical from 40 till it cuts the graph; run along horizontally to O Y, and you find 10—that is, 10*s.* Hence 40 articles cost 10*s.* Try this with other numbers of articles, and you find the correct answer each time. Fig. 7 is the graph connecting articles and their value; in this particular case at 3*d.* each. Hence this graph may be used as a ready reckoner. Note how easy it is to

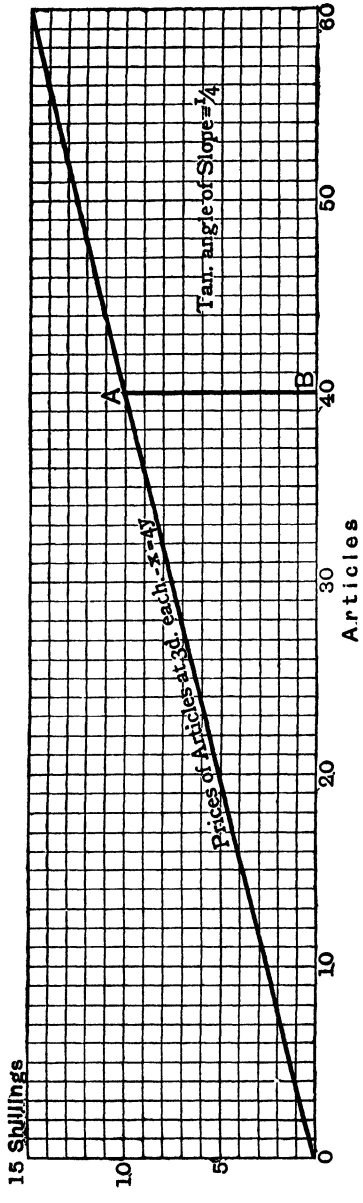


Fig 7

obtain the graph. It must pass through the origin O, for 0 articles cost 0*s.* One other point only is needed to determine the graph (since it is a straight line); select an easy number as shown at the beginning of the chapter, and ascertain this point. Now on the same sheet of squared paper plot out graphs of any number of articles at 6*d.*, 1*s.*, 1*s.* 6*d.*, 2*s.* 6*d.*, 3*s.* 4*d.*, &c. Make as many as the sheet will *conveniently* hold. Use the graphs to find the cost of any number of articles at any price.

At any point on the x axis, say 40, draw a perpendicular A B meeting the graph. A B O is a right-angled triangle.

O B is 40 divisions and A B 10 divisions, hence O B equals 4 times A B—that is, the distance measured along the x axis is 4 times the distance measured along the y axis.

For short, call "the distance along the x axis" x, and call "the distance along the y axis" y.

It will be seen that with this proviso, "O B is 4 times A B" may be stated as $x = 4y$.

Test *any* point by measuring its distances along the x axis and y axis, and the above holds good. Hence $x = 4y$ might be called a *formula* for the graph.

To make the graph, given its "formula", is now easy, but the process should be done in a systematic manner. If y is the distance of some point along the y axis, then we know that $4y$ is its distance along the x axis.

$x = 4y$	x	y	
$x = 4 \times 1 = 4$	4	1	Make $y = 1$
$x = 4 \times 2 = 8$	8	2	" $y = 2$
$x = 4 \times 3 = 12$	12	3	" $y = 3$
$x = 4 \times 4 = 16$	16	4	" $y = 4$.

Hence give y any values you choose, say, 1, 2, 3, 4, and find the corresponding values of x. Put the results in tabular form as shown. You have obtained points **4.1**, **8.2**, **12.3**, **16.4**. Test if these are on the graph $x = 4y$ (fig. 7). Clearly they are, and we have a method of obtaining the graph from its "formula", for we have but to find the points indicated, and join them.

Further, A B and B O are a measure of the slope of A O, that is, of the size of the angle A O B—

$$\frac{AB}{OB} = \frac{10}{40} = \frac{1}{4}.$$

This means that if you go along 4 units you go up one unit. Test 4 or 5 points and this will be found true for all. $\frac{AB}{OB}$, or in this case $\frac{1}{4}$, is called the "Tangent" of the angle of slope A O B. This angle of slope is very important because at a glance we may tell from the graph at what rate the quantity measured on the y axis is changing. The steeper the slope the greater the quantity on the y axis is, compared with the quantity on the x axis.

Since A B is 10 and O B 40

$$AB = \tfrac{1}{4}\ OB$$
$$\text{therefore } y = \tfrac{1}{4}x.$$

Work this out by the method shown on p. 23, and it will be found to be the same graph as $x = 4y$.

We may put y or x first as we choose. Generally y is put on the left-hand side of the formula.

On each of the graphs you have done, showing the cost of articles at various prices, put in—

1. The formula for the graph.
2. The tangent of the angle of slope.
3. A practical interpretation of the graph.

With regard to 3, while we have made $x = 4y$ represent the value of any number of articles at 3*d.* each, this is not the only interpretation we may put on this graph. It represents the interest on £1 for any number of years at $1\frac{1}{4}$ %, and this fact may be used to find the interest on any sum for a given time at $1\frac{1}{4}$ %. It may represent many other things, no doubt, as will be seen later on.

In mensuration we learn that the circumference of a circle equals $3\frac{1}{7}$ times the diameter, or in formula form

$$\mathrm{C} = 3\tfrac{1}{7}\,\mathrm{D}.$$

This is similar to $$x = 3\tfrac{1}{7}y.$$

Using the method indicated on p. 23, plot out the graph $x = 3\frac{1}{7}y$, but to obviate fractions make $y = 7, 14, 21$, &c. When you have found the graph use it to find the circumference of any circle, given the diameter. Note that measurements along the x axis may be in inches, feet, miles, centimetres, or metres, as long as we express the circumferences in the same units. Also only 2 points are required, as stated on p. 22, to find the graph, and the origin is one of them.

Many useful graphs of this kind should be drawn, for example take a large sheet of squared paper and put "feet" horizontally and lbs. vertically. Suppose we had rods of different metals all 1 sq. inch in section, then 9 feet of aluminium rod 1 sq. inch section weighs 10 lbs. Find the point **9.10**. Join to the origin and produce upwards as far as convenient. The graph so drawn gives the weights of any length of aluminium rod 1 sq. inch in section. Use this to

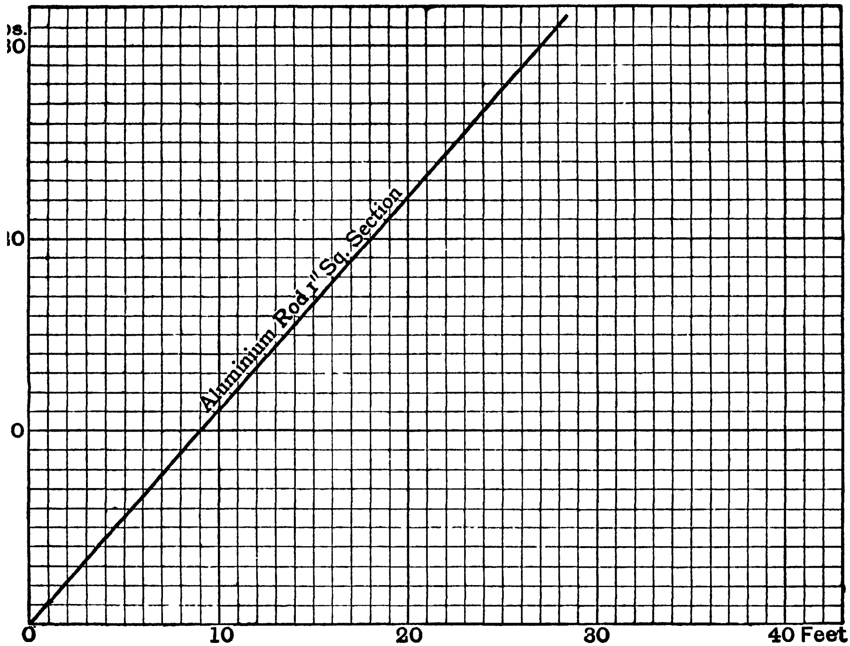


Fig. 8

find the weight of any piece of aluminium. Thus, an ingot of aluminium is 27′ long by 5″ × 3″, find its weight.

27′ aluminium 1 sq. inch section weighs 30 lbs.

∴ 27′ aluminium 5″ × 3″ = 15 sq. inches section weighs

30 × 15 lbs. = 450 lbs.

On the same sheet of paper draw graphs from the following data:—

A 4 foot	rod	copper	1 sq. inch section	weighs	15 lbs.
„ 9	„	cast-iron	„	„	28 „
„ 2	„	lead	„	„	10 „
„ 5	„	brass	„	„	18 „

Use millimetre ruled paper by preference.

Similar information with regard to woods is appended. Put all the graphs on one sheet squared paper.

A 10 foot rod ash 1 sq. inch section weighs 3 lbs. This is approximately true for beech, birch, cedar, red pine, teak, pitch pine.

A	10	foot rod	ebony	1 sq. inch section	weighs	5	lbs.
„	20	„	elm	„	„	5	„
An	11	„	mahogany	„	„	4	„
A	6	„	oak	„	„	2	„
An	11	„	white pine	„	„	2	„

When these graphs are drawn ascertain which are the heaviest and lightest metals, and which the heaviest and lightest woods.

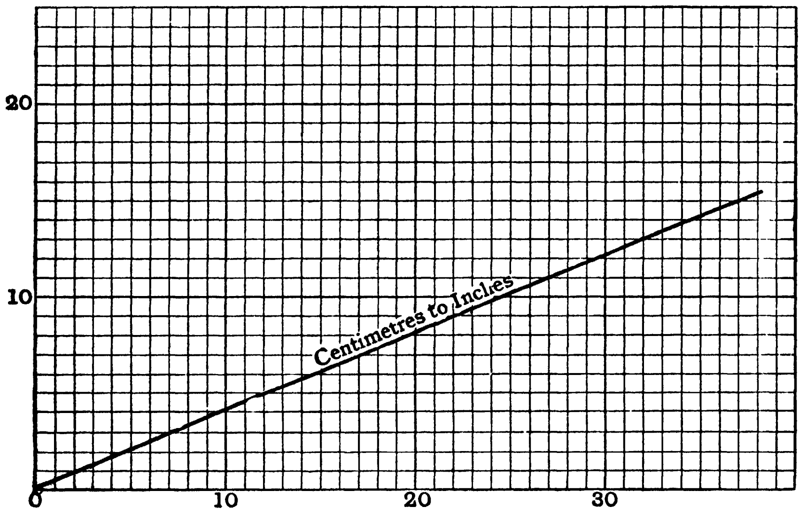


Fig 9

In the same way make graphs connecting two different measures, and use them to convert one into another. By putting numbers only, vertically and horizontally, all the subjoined may be put on one large sheet of squared paper and preserved. Thus 10 inches equal 25 centimetres. Find the point **25 . 10**. Join to origin and produce as before.

Write on this graph "Centimetres to Inches", and this indicates that centimetres are to be read on the x axis and inches on the y axis.

In the same way and on the same paper draw "Conversion Graphs" for the following:—

1. Kilometres to miles: 140 = 87.
2. Kilogrammes to pounds: 127 = 280.
3. Litres to cubic feet: 85 = 3.
4. Litres to gallons: 50 = 11.
5. Cubic feet to gallons: 17 = 106.
6. Lbs. water to cubic feet: 1060 = 17.

Geometrical conversion graphs:—

7. Side of square and diagonal of square: 70 = 99.
8. Area of circle and area of square inscribed in it: 300 = 191.
9. Circumference of circle and side of square equal in area to the circle: 39 = 11.

Having now studied "Ready Reckoner" and "Conversion" graphs, the student should note that all sums involving proportion may be readily solved by their use. Thus if 12 bushels are consumed by 19 horses, how many bushels will 47 horses consume in the same time? Evidently more. Find the point 47 . 19 and join to the origin. Now note where 12 (bushels) on the y axis cuts this graph. It is approximately at 29½ along. Then 29½ bushels is the answer. The process is: if the expected answer is more, then the larger number is marked off on the x axis; if less, the smaller number is marked off on the x axis. The answer is always on the x axis, as shown above. Compound proportion is as readily done, though the explanation is somewhat involved. Thus—

If 6 men build a wall 20 feet high in 6 days, working 12 hours per day, how many men could build one 30 feet high in 3 days, working 9 hours per day?

Method.—Take feet, days, and hours separately (fig. 10).

1. Find the point 30 . 20 (more) and join to origin.
2. 6 (men) on the y axis cuts this graph at 9 along.
3. Mark 9 on the y axis.
4. Find point 6 . 3 (more and days) and join to origin.
5. 9 cuts this graph at 18 along.
6. Mark 18 on the y axis.

7. Find the point 9 . 12 (more and hours) and join to origin.
8. 18 cuts this at 24 along.
24 men is the answer.

It is not intended to give further examples here. Any arithmeti will furnish abundance. Moreover, this method is sometimes cum

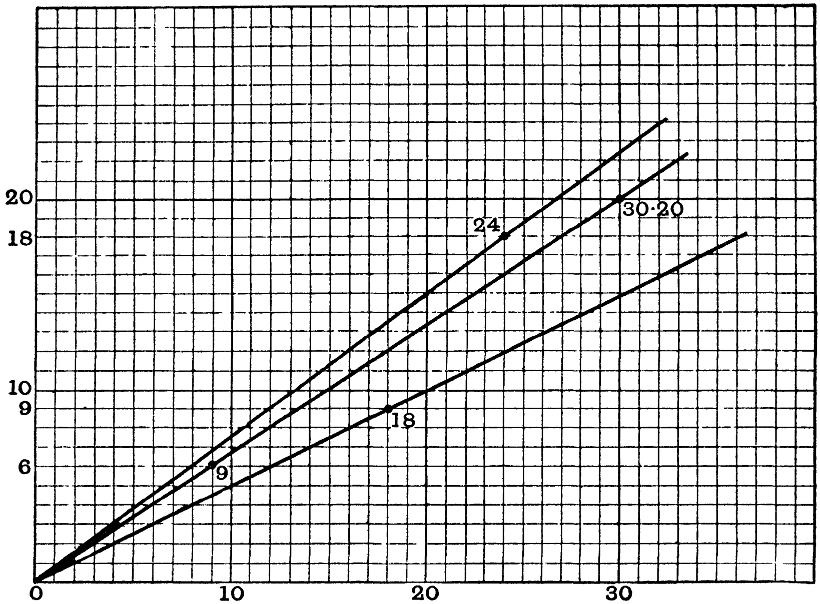


Fig. 10

brous and slow. As an approximation to the answer, however, th method may be used in Proportion, Percentages, Profit and Loss, an Stocks and Shares with much practical benefit. It gives a vivi picture of the mechanism of proportion to the student, far cleare than any verbal explanation could possibly do.

In work with fractions, decimal and vulgar, the methods alread given are useful, rapid, and accurate in the hands of a careful worke For example, to reduce $\frac{17}{43}$ to a simpler fraction with the least possibl error, find the point 43 . 17, and join to the origin. Find where th graph has approximately even co-ordinates, and choose the nearest to the origin (say $\frac{2}{5}$, which is a close approximation).

Of the fractions $\frac{4}{9}$, $\frac{13}{73}$, $\frac{116}{225}$, $\frac{18}{57}$, which is the greatest and least?

Draw graphs for each as before. The steepest graph is that of the greatest fraction, and the others are in order of magnitude.

To bring vulgar fractions to decimals, or *vice versa*, the same method may usefully be employed.

For example, bring $\frac{3}{8}$ to a decimal. Find the point 8 . 3, and join to the origin. Now we wish to bring eighths to tenths, hence we measure the perpendicular above 10 on the x axis. It is 375 ($3\frac{3}{4}$), therefore the answer is ·375. Millimetre ruled paper should be used and as large a scale as possible, at least 10 mm. to 1 unit.

To reverse the process, convert ·25 to a vulgar fraction. Find the point 10 . ·25 (that is, 10 · $2\frac{1}{2}$). Join to the origin. Select the fraction nearest the origin whose co-ordinates are even. It will be found to be $\frac{1}{4}$.

Note the decimals are reckoned as tenths, hence 10 is always the standard point on the x axis.

No further examples need be given here, but the student is strongly advised to work as many as possible for himself. It will be of the greatest possible service later on.

CHAPTER V

In all the graphs already plotted we have first found a definite meaning for them, making formulæ a secondary consideration. It must be evident, however, that cases will frequently occur in which we require to plot graphs from formulæ to which no specific meaning can be assigned, or which do not immediately require any. For example, you might write down $y = 7x + 4\frac{1}{4}$. No doubt this represents a graph, but what, you do not at present know.

Let us find out what $x = y + 5$ means (fig. 11).

	$x = y + 5$	x	y	
Then	$x = 1 + 5 = 6$	6	1	Make $y = 1$
	$x = 2 + 5 = 7$	7	2	" $y = 2$
	$x = 3 + 5 = 8$	8	3	" $y = 3$
	$x = 4 + 5 = 9$	9	4	" $y = 4$

Find the points **6.1, 7.2, 8.3, 9.4.**

Join these points, and you find a graph similar to one already plotted, viz., $x = y$, but in this case shifted *5* squares to the right of the former position. Instead of passing through the origin as $x = y$ did, it cuts the x axis 5 squares to the right of the point 0.

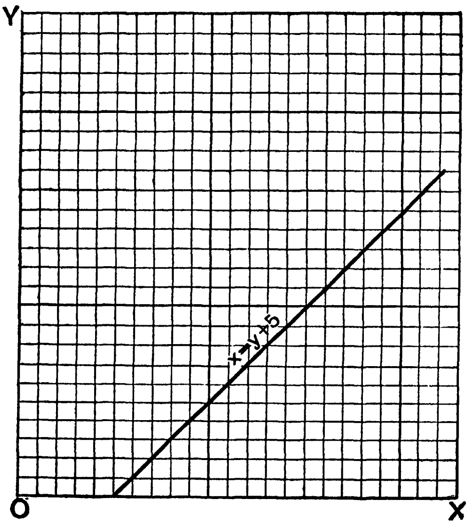


Fig. 11

Try $x = y + 7$, $x = y + 9$, $x = y + 11$, and you find these give you graphs parallel to $x = y$, but cutting the x axis 7, 9, and 11 divisions or squares to the right of 0 respectively.

In the same way, find out what $y = x + 5$ means, also $y = x + 7$, $y = x + 9$, $y = x + 11$.

Put these on the same squared paper as the previous four, giving x values now instead of y.

Plot out the following, each set on one piece of squared paper.

1. $x = 2y + 1$, $x = 2y + 5$, $x = 2y + 9$.
2. $y = 2x + 1$, $y = 2x + 5$, $y = 2x + 9$.

It must be evident to you now that $x = 2y + 1$ is a graph parallel to $x = 2y$, and cutting the x axis one square to the right of the origin; $y = 2x + 1$ is a graph parallel to $y = 2x$, and cutting the y axis one square up.

What meaning, however, can we assign to $x = y - 5$?

Plot out this graph, giving y greater values than 5. Thus—

$x = y - 5$	x	y	
$x = 6 - 5 = 1$	1	6	Let $y = 6$
$x = 7 - 5 = 2$	2	7	„ $y = 7$
$x = 8 - 5 = 3$	3	8	„ $y = 8$
$x = 9 - 5 = 4$	4	9	„ $y = 9$

Take a sheet of squared paper, but now extend OX and OY to the left and downwards, as shown in fig. 12.

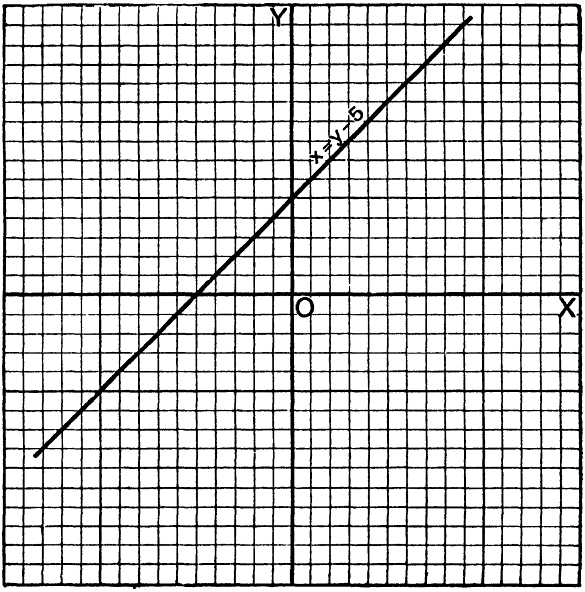


Fig. 12

Plot out the points **1.6, 2.7, 3.8, 4.9.**

Join and produce both ways.

Notice this graph cuts the x axis 5 squares to the left from **0,** and, moreover, it is parallel to the graph $x = y$, as may be seen it the slope is tested. Now, comparing this with $x = y + 5$ it would seem that -5 means 5 squares to the left from 0.

We have already seen that $y = 2x + 1$ is parallel to $y = 2x$, but cuts the y axis one square *up*. By the same reasoning could we not say $y = 2x - 1$ is parallel to $y = 2x$, but cuts the y axis one square *down* from 0? Test this by plotting $y = 2x - 1$.

We may conclude, then, that the negative sign signifies that measurements are to be made to the left in the case of the x axis and down in the case of the y axis.

Thus the point **4.−3** signifies 4 units to the right from 0 along the x axis and 3 units down from 0 along the y axis. For practice put in the points **4.3**, **4.−3**, **−4.3**, **−4.−3**.

Jot down a number of points at random, and put in the co-ordinates with proper sign attached.

Now draw the following graphs:—

1. $y = 2x + 4$, $y = 2x - 4$, $y = 2x$.
2. $y = 3x + 6$, $y = 3x - 6$, $y = 3x$.

Summarizing, we may say—

x	represents	a	measurement to the	right,	on the x axis.	
$-x$	”	”	”	left,	”	”
y	”	”	”	up	on the y axis.	
$-y$	”	”	”	down	”	”

EXERCISE VI

Plot the following graphs:—

1. $y = x + 8$.
2. $y = 3x + 2$.
3. $3y = 2x + 1$.
4. $y + 2 = x + 4$.
5. $2y + 4 = 3x + 2$.
6. $y = 6x - 4$.
7. $3y = 4x - 2$.
8. $\frac{4y}{5} = x - 1\frac{1}{5}$.
9. $y = 6$. (Note x is not mentioned here, hence give x any values you choose, and y still equals 6.)
10. $x = 6$ (see 9) is a line parallel to the y axis and 6 divisions to the right of it.
11. $y = -6$.
12. $x = -6$.

CHAPTER VI

We may now utilize graphs to solve some simple problems in arithmetic. For example—

A mother is 3 times as old as her daughter. In 10 years, however, she will be twice as old. Find the age of each.

Set down the above statements thus—

$$\text{(Age of mother)} = 3 \text{ times (age of daughter)}$$
$$\text{or } y = 3x$$

where we put y in place of the mother's age, for we do not know it, and x in place of the daughter's. Again—

$$\text{(Age of mother in 10 years)} = 2 \text{ times (age of daughter in 10 years)}$$
$$\text{or } (y + 10) = 2(x + 10)$$
$$\text{which simplified becomes } y = 2x + 10.$$

Now plot out on one piece of squared paper the two graphs—

$$y = 3x \text{ and } y = 2x + 10 \text{ (fig. 13).}$$

Notice carefully that one graph gives us all the ages on the y axis, which are 3 times those on the x axis; the other gives us the graph of all the ages on the y axis, which are twice those on the x axis in 10 years. Where the graphs intersect we get two ages, viz. 30 and 10, which not only give us the one relation but also the other, hence the mother's age is 30 years and the daughter's 10 years. It is to be noted, then, that when two graphs contain each some special information, as in the above problem, the intersection of the graphs supplies the solution. Perhaps this information will make the reason plainer. When letters go amissing in any district in the United States, the postal officials send secretly marked letters from all points to this district, generally containing money. A map is also made, and the course of each letter carefully drawn out on it. If a letter disappears, its course is marked out in red on this map. After

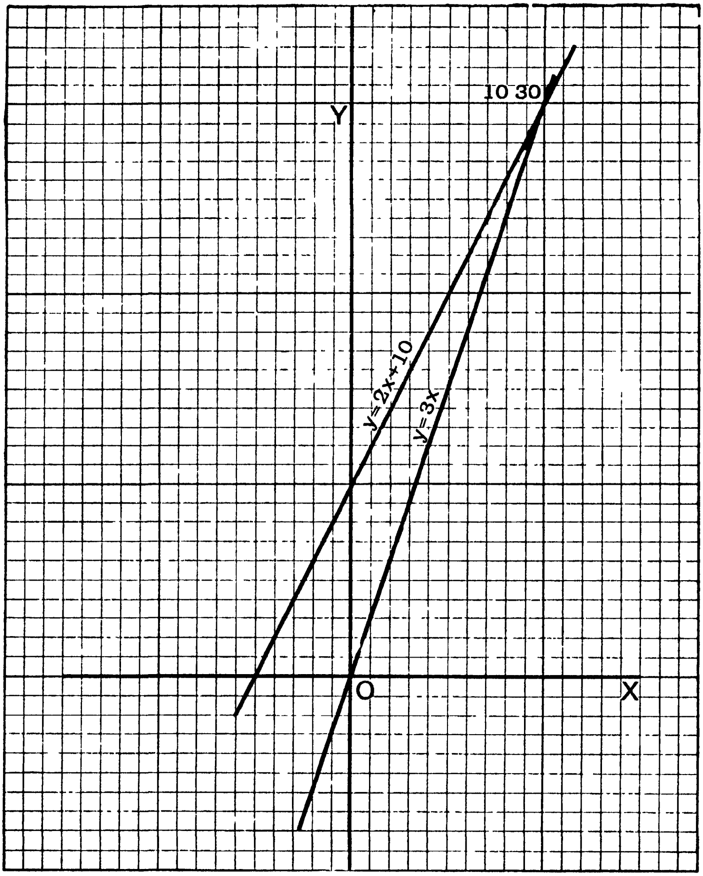


Fig 13

a few weeks it is generally found that all the red lines cross at a certain place. This is the town where the thefts are taking place. Each red line indicates the "path of theft" of one letter, but the intersection gives the actual point. So may graphs be used to solve problems.

Example 2.—There are 2 milk cans. If I take 2 gallons from A and put them in B, then B contains 3 times as much as A. But

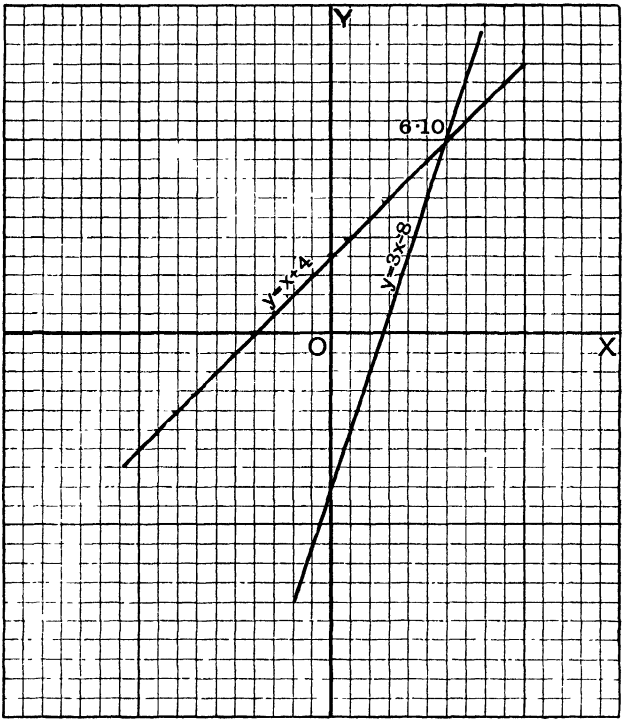


Fig. 14

if I take 2 gallons from B and put in A, both cans contain the same amount. Find how much milk is in each.

Suppose A has x gallons and B has y gallons.

$$\begin{aligned} \text{Then } y \text{ gets 2 gallons} &= (y + 2) \\ x \text{ loses 2 gallons} &= x - 2 \\ \text{But B} &= 3 \text{ times A.} \\ \therefore\ \underline{(y + 2)} &\underline{= 3\,(x - 2).} \end{aligned}$$

$$\begin{aligned} \text{Again, } y \text{ loses 2 gallons} &= y - 2 \\ x \text{ gains 2 gallons} &= x + 2, \\ \text{and B} &= \text{A} \\ \underline{(y - 2)} &\underline{= x + 2.} \end{aligned}$$

Now, simplify each statement and we find—

$$(a)\ y = 3x - 8,$$
$$(b)\ y = x + 4.$$

Plot out each of these graphs. They intersect in the points 6 . 10.

Then A contains 6 gallons
and B contains 10 gallons (fig. 14).

EXERCISE VII

1. Find 2 numbers whose sum is 42 and whose difference is 24.

2. There are 2 numbers and 3 times the first plus the second equals 62, while 3 times the second plus the first equals 42. Find the numbers.

3. If I make a bell with 16 cwts. copper and 5 cwts. tin it costs £62. If I make engine bearings with 7 cwts. copper and 10 cwts. tin they cost £74. Find the price of copper and tin per cwt.

4. A merchant mixes 3 gallons No. 1 vinegar with 2 gallons No. 2, costing in all 10*s.* (120 pence). He also sells a quality consisting of 2 gallons No. 1 and 1 gallon No. 2, costing in all 6*s.* 2*d.* (74 pence). Find prices per gallon of two vinegars.

5. An oil merchant sells wagon grease consisting of 6 parts oil, 2 parts soda liquor, at 60*s.* per barrel. If he uses 5 parts oil and 3 parts soda solution he charges 54*s.* per barrel. Compare the prices of oil and soda.

6. Two pounds tea and 5 pounds sugar cost 4*s.* Four pounds tea and 2 pounds sugar cost 6*s.* 8*d.* Find cost of tea and sugar per pound (in pence).

7. If I give A 6*s.* he has now twice what I have, but if he gives me 9*s.* I have now twice what he has. How much has each?

8. John and James have 11*s.* between them, but if John's money were five times what it is, and James's money three times what it is, they would have 37*s.* between them. How much has each?

9. A shopkeeper finds that if he burns 5 electric arc lamps and 6 incandescent electric lamps it will cost him 5*s.* 9*d.* per hour, but if he burns 10 arcs and 2 incandescents it will cost him 4*s.* per hour. Find cost of arc and incandescent lighting per hour. (Note arcs are 1000 candle power, incandescent 500 candle power.)

10. Six dollars and 3 rupees are worth 30*s.*, and 3 dollars and 6 rupees are worth 22*s.* 6*d.* Find value of rupee and dollar.

11. A confectioner mixes 3 cwts. sugar and 1 cwt. glucose, selling mixture at 61*s.* for 4 cwts. A poorer quality consists of 1 cwt. sugar and 3 cwts. glucose, and sells at 39*s.* for 4 cwts. Find price of sugar and glucose per cwt.

12. Soft solder made by mixing 1 cwt. lead and 1 cwt. tin costs 76*s.* per cwt. Harder solder made by mixing 3 cwts. of lead and 2 cwts. of tin costs 43*s.* per cwt. Compare the prices of lead and tin.

There is another class of sums which is placed at this stage (though it might well have been taken earlier) in order that the student may reverse the process just gone through in Exercise VII. That is, the graphs are plotted first, and the formulæ found later.

By referring to the chapter on "Ready Reckoner" graphs the student should be able to make a graph showing the number of miles traversed (at a given rate per hour) in a certain time. Plot miles horizontally and hours or minutes vertically. Fig. 15 shows such a graph for 20 miles per hour. To plot it, the point 20.1 is joined to the origin and produced. Call this line "Train A". Instead of taking 0 as the origin make 1 hour the origin and plot the same graph. Note it is parallel to "Train A" and may be used to calculate the distance gone by a train, at 20 miles per hour, starting one hour after "Train A". Call this graph "Train B". Again, instead of 0 make 20 miles the origin, and draw a "20 miles per hour" graph. It is parallel to "Train A" and may be used to calculate the distance gone by a train which starts 20 miles ahead of "Train A" at the same time and speed. Call this "Train C". Lastly, take 60 miles as the origin and plot the graph backwards, as "Train D". This graph may be used to calculate the distance traversed by a train going at the same rate as A, B, and C but in the opposite direction. It will be noticed that the "Train D" graph crosses the others. These points of crossing give the distance from the origin that the trains meet and the time of meeting. Thus D meets C 40 miles from the origin, one hour from starting, A 1½ hours from starting and 30 miles from the origin, and B 2 hours from starting and 20 miles from the origin. Suppose "Train D" stops for an hour after going 20 miles, then "Time" changes while "Distance" does not. The graph is therefore a perpendicular line for one hour. If "Train D" starts again at its previous speed, the graph takes its origin from this new time, but is parallel to the first graph, as shown.

The origin and 60 miles might be towns 60 miles apart. If they are connected by a single line of railway then stations or sidings

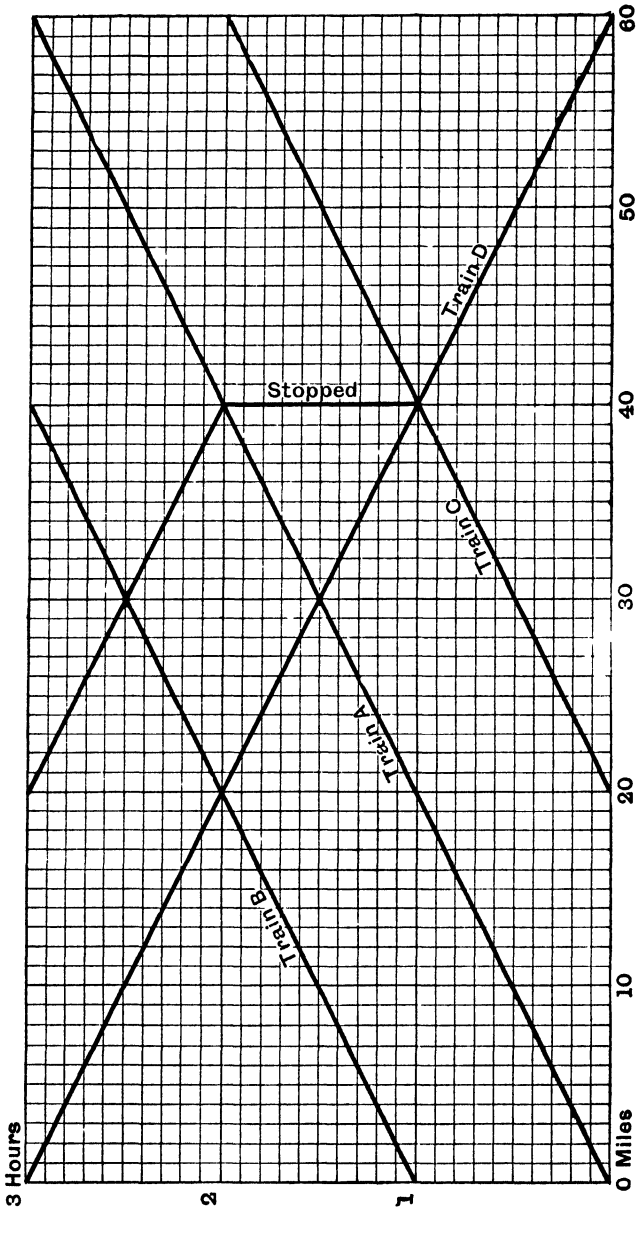

3 Hours
2
1
0 Miles
10
20
30
40
50
60
Train A
Train B
Train C
Train D
Stopped

Fig 15

would be required at 20, 30 and 40 miles from the first town to permit the trains to pass each other.

EXERCISE VIIA

1. In a cycle race between two towns 40 miles apart, A gets 10 miles start and travels at 15 miles per hour. B gets 5 miles start and travels at 20 miles per hour. C is scratch and travels at 22 miles per hour. Who was the winner and by how much?

2. Two towns are 80 miles apart. A cyclist starts at 9 A.M. from A at 16 miles per hour. After cycling an hour he is delayed half an hour by a puncture, then proceeds. At 9 A.M. also, a motor-car starts from B at 20 miles per hour. After going half an hour it breaks down and is delayed an hour. When and where do cyclist and motorist meet?

3. Two towns, A and B, are 20 miles apart and a single line of railway connects them. The morning trains from A leave at 9, 9.30, and 10. From B they leave at 9.15 and 10.15. If they are timed to meet at stations, where are the stations?

4. A pedestrian, a cyclist, and a motorist decide on a 30 mile race. The pedestrian gets 27 miles start, the cyclist 12 miles, while the motorist is scratch. The average speeds were 3 miles, 18 miles, and 30 miles per hour respectively. What was the result of the race?

Note.—Students should investigate the formulæ of the graphs in each question.

CHAPTER VII

When a number is multiplied by itself the operation is termed "squaring" the number. Thus 4×4 equals the square of $4 = 16$. This is written $4^2 = 16$.

Similarly, $4 \times 4 \times 4$ is called the "cube" of 4, and is written thus—$4^3 = 64$.

The "2" above the 4 means that two fours are to be multiplied; the "3" that three fours are to be multiplied, and so on. This perhaps you have already learnt.

$$\begin{aligned}
\text{Thus}\quad 10^2 &\text{ means } 10 \times 10 = 100 \\
10^3 &\quad ,, \quad 10 \times 10 \times 10 = 1000 \\
\text{Hence}\quad 10^2 \times 10^3 &= 100 \times 1000 = 100{,}000. \\
\text{But}\quad 100{,}000 &= 10 \times 10 \times 10 \times 10 \times 10 = 10^5, \\
\text{hence}\quad 10^2 \times 10^3 &= 10^5.
\end{aligned}$$

Try $10^4 \times 10^3$ in the same manner, and you find it equals 10^7.

When we multiply 10 any number of times by itself we are said to raise it to some "Power", and the little figure placed above the 10 is called the Index (indicator) of the power.

Thus 10^2 is the second power of 10, and 2 is the index,
10^6 is the sixth power of 10, and 6 is the index.

To multiply 10^2 by 10^6 we merely *add* the indices (indexes).

$$\text{Thus}\quad 10^2 \times 10^6 = 10^8$$
$$10^7 \times 10^3 = 10^{10}$$
$$10^8 \times 10^9 = 10^{17}$$

Note how much easier it is to say—

$$10^6 \times 10^3 = 10^9 \text{ than } 1000000 \times 1000 = 1000000000.$$

The method of working with indices turns multiplication into addition.

EXERCISE VIII

Simplify—

1. $10^8 \times 10^{19}$	3. $10^6 \times 10^{15}$	5. $10^{\frac{1}{2}} \times 10^{\frac{1}{4}}$
2. $10^4 \times 10^{22}$	4. $10^{1\frac{1}{2}} \times 10^{6\frac{1}{2}}$	6. $10^{\frac{1}{3}} \times 10^{\frac{2}{3}}$

Perhaps you wonder what $10^{\frac{1}{2}}$ means.

$$10^1 \text{ must equal } 10, \text{ since } 10^2 = 10 \times 10.$$

Therefore, $10^{\frac{1}{2}}$ must be some number less than 10, *not half of ten*, but (if the expression may be used) 10 multiplied by itself *half a time.* This is, of course, impossible in arithmetic, but getting the value of $10^{\frac{1}{2}}$ by other methods we find it equals 3·16.

$10^{\frac{1}{2}} = 3{\cdot}16$	$10^{\frac{1}{8}} = 1{\cdot}33$	$10^{\frac{1}{32}} = 1{\cdot}07$
$10^{\frac{1}{4}} = 1{\cdot}78$	$10^{\frac{1}{16}} = 1{\cdot}15$	

Note that each index is half the one before, but that the numbers given do not bear this relation at all.

To face p 40

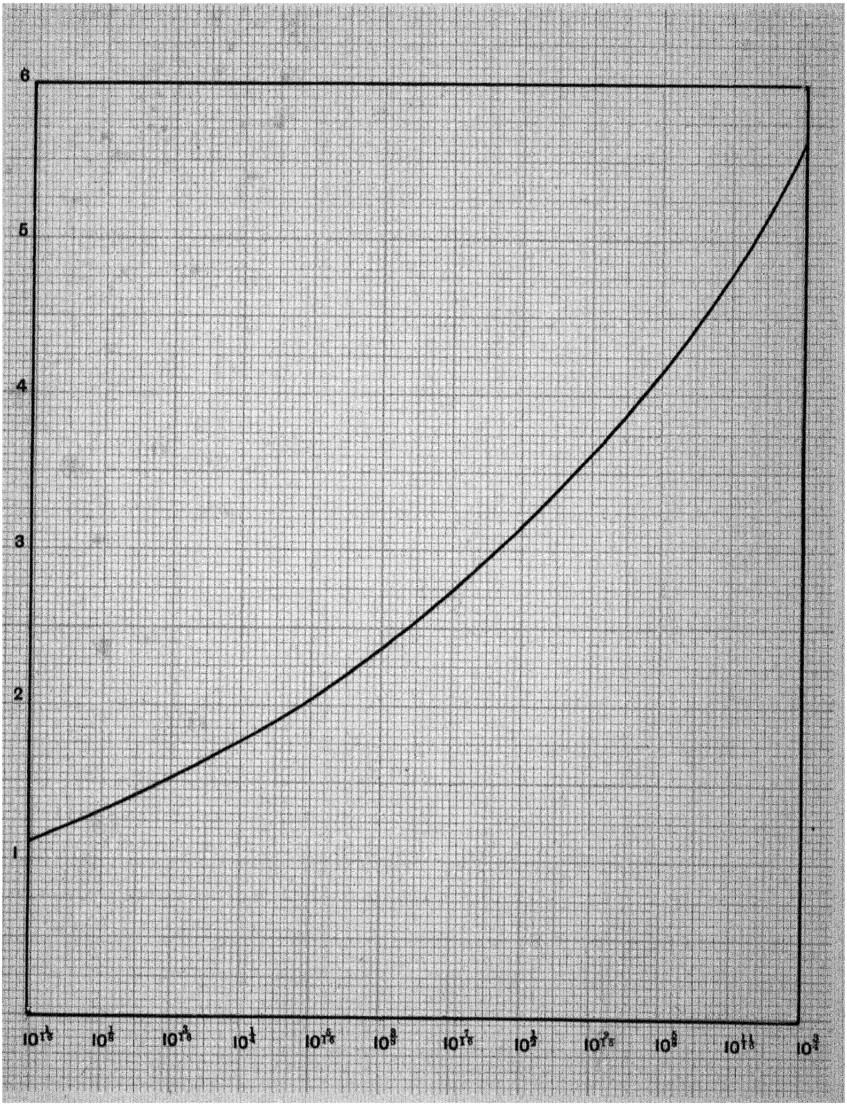


FIG. 16

To show this more clearly let us make a graph of them.

$$
\begin{array}{lll}
10^{\frac{1}{16}} = 1{\cdot}15, & 10^{\frac{1}{8}} = 1{\cdot}33, & 10^{\frac{3}{16}} = 1{\cdot}54, \\
10^{\frac{1}{4}} = 1{\cdot}78, & 10^{\frac{5}{16}} = 2{\cdot}05, & 10^{\frac{3}{8}} = 2{\cdot}37, \\
10^{\frac{7}{16}} = 2{\cdot}74, & 10^{\frac{1}{2}} = 3{\cdot}16, & 10^{\frac{9}{16}} = 3{\cdot}65, \\
10^{\frac{5}{8}} = 4{\cdot}22, & 10^{\frac{11}{16}} = 4{\cdot}87, & 10^{\frac{3}{4}} = 5{\cdot}63.
\end{array}
$$

Since each index differs by $\frac{1}{16}$, set off on squared paper to a suitable scale $10^{\frac{1}{16}}$, $10^{\frac{1}{8}}$, &c. (horizontally). Vertically with 10 divisions to every unit set off the corresponding numbers. Neglecting the origin, trace the graph (fig. 16). It is a curve gradually becoming steeper and steeper. From previous lessons we learned that the slope indicates the rate at which the two quantities involved are changing. When the slope is $\frac{1}{1}$ both are changing at the same rate; $\frac{2}{1}$, one twice as fast as the other. Since the slope here gets steeper and steeper, we learn that as we raise 10 to higher and higher powers, those powers represent numbers which latterly increase at a much more rapid rate than their indices do. Stretch your imagination a little, and carry the curve in fig. 14 far out into space. By and by it will become almost vertical, indicating that a very small addition to the index of 10 will mean an enormous addition to the corresponding number.

It will be useful to determine where the curve has certain slopes. Thus find *the point* where the slope is $\frac{1}{1}$ (note it is only $\frac{1}{1}$ at one point), $\frac{2}{1}$, &c.

$$\frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10} = \frac{10^5}{10^2} = \frac{100000}{100} = 1000.$$

$$\text{Hence } \frac{10^5}{10^2} = 1000.$$

$$\text{But } 1000 = 10^3,$$

$$\therefore \frac{10^5}{10^2} = 10^3.$$

Now $5 - 2 = 3$, which seems to show that to divide 10^5 by 10^3 we subtract the indices.

$$\frac{10 \times 10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10 \times 10} = \frac{10^6}{10^4} = 100 = 10^2,$$

but $6 - 4 = 2$.

Hence the rule:—To divide 10 raised to any power by 10 raised to any other power, subtract the index of the second from the first.

EXERCISE IX

Simplify—

1. $\frac{10^4}{10}$.
2. $10^{19} \div 10^6$.
3. $\frac{10^{\frac{1}{2}}}{10^{\frac{1}{4}}}$.
4. $\frac{10}{10}$.
5. $\frac{10^{1\frac{1}{2}}}{10^{\frac{1}{4}}}$.
6. $10^{17} \div 10^2$.
7. $10^{\frac{2}{3}} \div 10^{\frac{1}{3}}$.
8. $\frac{10^{5}}{10^{2}}$.
9. $\frac{10^{214}}{10^{210}}$.
10. $\frac{10^{6\ 514}}{10^{4\ 216}}$.
11. $10^{11} \times 10^{24}$.
12. $10^{411} \times 10^{6\ 24}$.
13. $10^{2\ 416} \times 10^{3\ 417}$.
14. $10^{\cdot 301} \times 10^{\cdot 477}$.
15. $10^{477} \div 10^{301}$.
16. $10^{6\ 301} \div 10^{4\ 477}$.

It has been found by calculation that—

$10^{3010} = 2$	$10^{6990} = 5$	$10^{\cdot 9031} = 8$
$10^{4771} = 3$	$10^{7781} = 6$	$10^{9452} = 9$
$10^{6020} = 4$	$10^{8451} = 7$	and $10^1 = 10$

Take this multiplication—

$$2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880.$$

We could replace this by—

$$10^{301} \times 10^{477} \times 10^{602} \times 10^{\cdot 699} \times 10^{778} \times 10^{\cdot 845} \times 10^{903} \times 10^{954}.$$

Add all the indices, and we find this equals $10^{5\cdot 559}$.

$$\text{And } 10^{5\cdot 559} = 362880.$$

$$\text{Again, } \frac{6}{3} = \frac{10^{\cdot 7781}}{10^{4771}} = 10^{\cdot 3010} = 2.$$

Those examples are very simple, but it will be seen that the most complicated multiplication or division sum may thus be transformed into addition or subtraction.

Now if $10^{\cdot 3010} = 2$, then $\cdot 3010$ is called the *logarithm* of 2, or shortly log 2. Similarly, $\cdot 4771$ is log 3.

$$10 = 10^1, \text{ therefore } 1 \text{ is } \log 10,$$
$$100 = 10^2 \quad ,, \quad 2 \text{ is } \log 100.$$
$$1000 = 10^3, \quad ,, \quad 3 \text{ is } \log 1000, \text{ and so on.}$$

$$20 = 2 \times 10$$
$$= 10^{\cdot 3010} \times 10^1$$
$$= (\text{by our rule of addition})\ 10^{1\cdot 3010}.$$
$$\text{But } 1{\cdot}3010 = 1 + {\cdot}3010 = \log 10 + \log 2,$$
$$\text{Therefore } \log 20 = \log 10 + \log 2.$$

$$200 = 100 \times 2, \therefore \log 200 = \log 100 + \log 2 = 2{\cdot}3010$$
$$2000 = 1000 \times 2, \therefore \log 2000 = \log 1000 + \log 2 = 3{\cdot}3010.$$

Note then—		Also—	
Log 2	= $\cdot 3010$	Log 3	= $\cdot 477$
Log 20	= $1{\cdot}3010$	Log 30	= $1{\cdot}477$
Log 200	= $2{\cdot}3010$	Log 300	= $2{\cdot}477$
Log 2000	= $3{\cdot}3010$	Log 3000	= $3{\cdot}477$
Log 20000	= $4{\cdot}3010$	Log 30000	= $4{\cdot}477$

In the same way, we may say—

$$864 = 8{\cdot}64 \times 100,$$
$$\therefore \log 864 = \log 8{\cdot}64 + \log 100.$$

If we know log 8·64, then we can immediately find log 86·4, log 864, log 8640, &c. &c.

Further, since we notice that log 2 is a decimal only without a whole number,

Log 20,	1 plus a decimal		
Log 200,	2	,,	,,
Log 2000,	3	,,	,,
Log 20000,	4	,,	,,

it is clear that the logarithm of any number consists generally of two parts, a whole number and a decimal. The decimal part must be got from tables, but the *whole number* is one less than the number of figures in the number (excluding decimals).

To summarize this—

Log 7	=	some decimal	
Log 23	= 1·	„	
Log 636	= 2·	„	
Log 8745	= 3·	„	
Log 61725	= 4·	„	and so on.

As the decimal part of the log must be got from tables, such tables are given on pages 63 and 64.

To explain the method of use take one line—

	Logs										Differences								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	&c.													
12																			

Columns 0 to 9 give the decimal part of the logs; you are expected to put in the whole number yourself.

Thus, from the first line—

$$\left.\begin{array}{lcl} \text{Log } 10 & = & 1{\cdot}0000 \\ \text{Log } 10{\cdot}1 & = & 1{\cdot}0043 \\ \text{Log } 10{\cdot}2 & = & 1{\cdot}0086 \\ \text{Log } 10{\cdot}3 & = & 1{\cdot}0128 \\ \text{Log } 10{\cdot}4 & = & 1{\cdot}0170 \\ \text{Log } 10{\cdot}5 & = & 1{\cdot}0212 \end{array}\right\} \text{ or } \left\{\begin{array}{lcl} \text{Log } 1 & = & {\cdot}0000 \\ \text{Log } 1{\cdot}01 & = & {\cdot}0043 \\ \text{Log } 1{\cdot}02 & = & {\cdot}0086 \\ \text{Log } 1{\cdot}03 & = & {\cdot}0128 \\ \text{Log } 1{\cdot}04 & = & {\cdot}0170 \\ \text{Log } 1{\cdot}05 & = & {\cdot}0212 \end{array}\right.$$

and so on.

$$\text{Log } 10{\cdot}9 = 1{\cdot}0374 \qquad \text{Log } 1{\cdot}09 = {\cdot}0374$$

Now going to next line—

$$\text{Log } 11 = 1{\cdot}0414 \text{ or Log } 1{\cdot}1 = {\cdot}0414.$$

Further— Log 106 = 2·0253
Log 1060 = 3·0253
Log 10600 = 4·0253, &c.

The above only gives us the logarithms of 3-figure numbers. Suppose we desire the log of 1065, proceed thus: Log 1060 = 3·0253, as above. The difference of 1060 and 1065 = 5. Under 5 in the column of differences we get 21. Add this to 3·0253, putting the last figure 1 under the last figure 3, thus—

$$\begin{array}{r} 3{\cdot}0253 \\ 21 \\ \hline \underline{3{\cdot}0274} \end{array} = \log 1065.$$

Putting it down systematically; find log 1032.

	Log 1032	=	?
	Log 1030	=	3·0128 from tables.
Difference	2 corresponds to		8
	Log 1032	=	3·0136

To find a number corresponding to a logarithm.

What number corresponds to 3·0298?

The nearest log below this in the tables is ·0294 corresponding to 1·07.

·0298 − ·0294 = difference of 4.

Running along from ·0294 to the difference column, we see 4 is under column "1". Hence the number is 1071.

Systematically.—Find number corresponding to 2·0182.

	2·0182	=	Log ?
	2·0170	=	Log 104
Difference	12 corresponds to		3
∴	2·0182	=	Log 104·3

Note.—Great care should be taken at first to distinguish numbers and logs of numbers.

CHAPTER VIII

The interest on £1 for a year at 10% is 2*s*. = £·1.

Therefore the amount of £1 for a year at 10% = £1·1.

To find the amount of any sum of money for 1 year at 10%, we could multiply it by 1·1. (Test this.)

Now put £1 in the bank at 10% interest.

At the end of one year it becomes £1·1.

Leaving the £1·1 in the bank, at the end of the 2nd year it becomes £1·1 × 1·1 = £1·21.

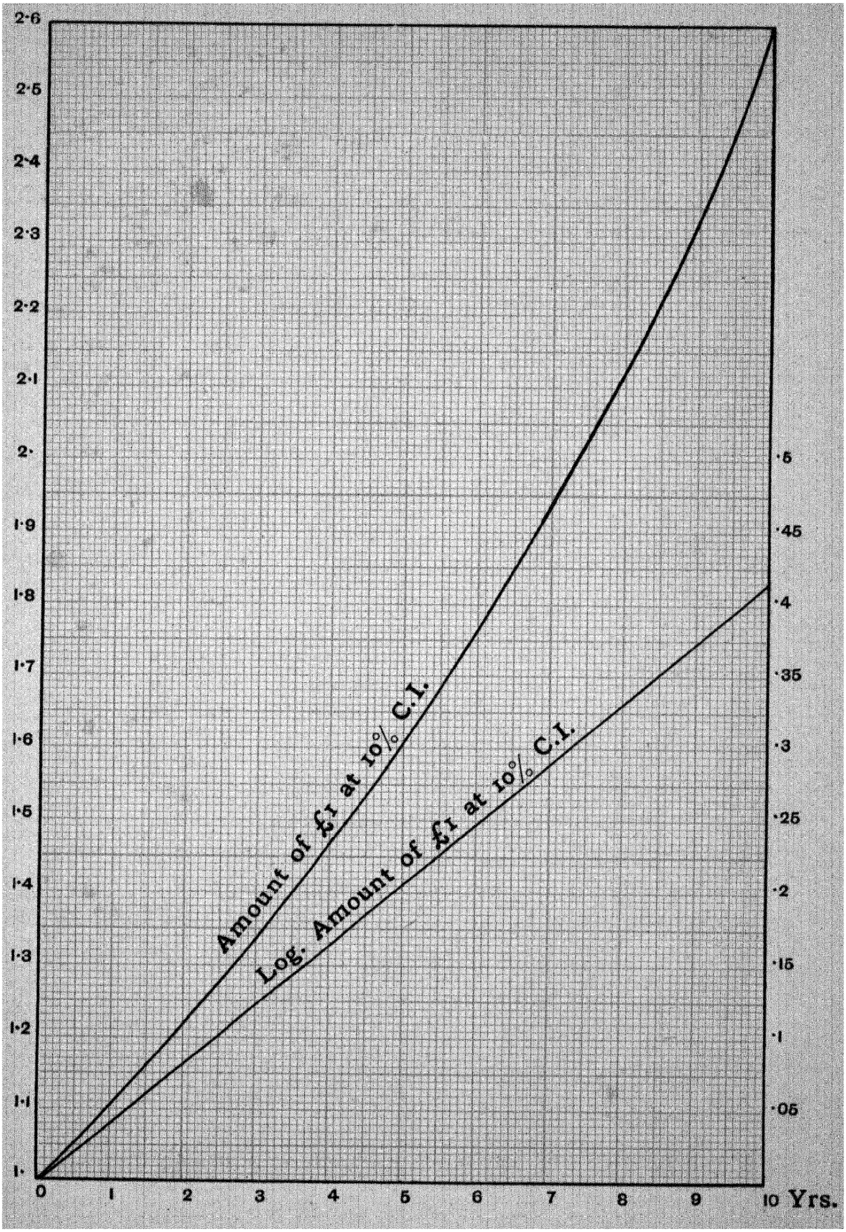
Leaving the £1·21 in the bank, at the end of the 3rd year it becomes £1·21 × 1·1 = £1·331.

Putting this in tabular form, we find, starting with £1, interest 10%—

£1 becomes at end of	1st	year	£1·1	= $(1{\cdot}1)^1$
End of	2nd	„	£1·21	= $(1{\cdot}1)^2$
„	3rd	„	£1·33	= $(1{\cdot}1)^3$
„	4th	„	£1·46	= $(1{\cdot}1)^4$
„	5th	„	£1·61	= $(1{\cdot}1)^5$
„	6th	„	£1·77	= $(1{\cdot}1)^6$
„	7th	„	£1·95	= $(1{\cdot}1)^7$
„	8th	„	£2·10	= $(1{\cdot}1)^8$
„	9th	„	£2·36	= $(1{\cdot}1)^9$
„	10th	„	£2·6	= $(1{\cdot}1)^{10}$

Take squared paper and plot vertically £s, and horizontally years. As the maximum £s are £2·6, a large scale is advisable, say £1 to £2·6 vertically (fig. 17), using mm. paper.

Note that the graph is a curve, and the points we have found on it though numerous are not sufficient to ensure very great accuracy, yet they involved a great amount of calculation. As a ready reckoner this graph is by no means as accurate as the ready reckoner graphs already made with straight lines. Also in the straight-line graphs


Amount of £1 at 10% C.I.
Log. Amount of £1 at 10% C.I.
2·6
2·5
2·4
2·3
2·2
2·1
2·
1·9
1·8
1·7
1·6
1·5
1·4
1·3
1·2
1·1
1·
·5
·45
·4
·35
·3
·25
·2
·15
·1
·05
0
1
2
3
4
5
6
7
8
9
10 Yrs.

already used as reckoners only one point was found, the origin being the other point necessary. We could then extend the graph to the limits of the paper, and thus have a wide range of usefulness. Here we have no means of extending the graph mechanically with any hope of accuracy. Now looking at the column of amounts (p. 46), we see that the amount at the end of each year is £1·1 raised to the power indicated by the year. Thus in the 6th year the amount is £1·77 = £$(1{\cdot}1)^6$.

$$(1{\cdot}1)^6 = 1{\cdot}1 \times 1{\cdot}1 \times 1{\cdot}1 \times 1{\cdot}1 \times 1{\cdot}1 \times 1{\cdot}1,$$

and $\log (1{\cdot}1)^6 = \log 1{\cdot}1 + \log 1{\cdot}1 + \log 1{\cdot}1 + \log 1{\cdot}1 + \log 1{\cdot}1 + \log 1{\cdot}1$

$$= 6 \text{ times } \log 1{\cdot}1$$
$$= 6 \log 1{\cdot}1.$$

Therefore $\log (1{\cdot}1)^6 = 6 \log 1{\cdot}1$.

To make this quite clear, note—

$$\begin{aligned} \text{Log } (31)^4 &= 4 \log 31 \\ \text{Log } (1116)^{18} &= 18 \log 1116 \\ \text{Log } (42)^{\frac{1}{3}} &= \tfrac{1}{3} \log 42 \\ \text{Log } (675)^{\frac{4}{5}} &= \tfrac{4}{5} \log 675 \end{aligned}$$

Now from the table of logs we find—

$$\begin{aligned} \text{Log } 1{\cdot}1 &= {\cdot}0414 \\ \text{Therefore } \log (1{\cdot}1)^2 &= 2 \log 1{\cdot}1 = {\cdot}0828 \\ \therefore \log (1{\cdot}1)^3 &= {\cdot}1242 \\ \log (1{\cdot}1)^4 &= {\cdot}1656 \\ \log (1{\cdot}1)^5 &= {\cdot}2070 \\ \log (1{\cdot}1)^6 &= {\cdot}2484 \\ \log (1{\cdot}1)^7 &= {\cdot}29 \\ \log (1{\cdot}1)^8 &= {\cdot}3212 \\ \log (1{\cdot}1)^9 &= {\cdot}3726 \\ \log (1{\cdot}1)^{10} &= {\cdot}4140 \end{aligned}$$

On the right-hand side of fig. 15 mark vertically a new scale from

0 to ·5, making 20 divisions = ·1. Now leaving the horizontal scale as before, plot the graph of logs of amount of £1. Thus—

0 years, log £1 = 0. Find point (0·0)
1 year, log £1·1 = ·0828. „ (1.·0828), &c.

It will immediately be noticed that the graph is a straight line, hence only one point was necessary. The 7th year would have been most suitable since

$$\text{Log } (1{\cdot}1)^7 = {\cdot}29. \quad \text{(An easy quantity to plot.)}$$

We may therefore use this graph as a ready reckoner, and extend it to any limits we please without impairing its accuracy. Use it to find the amount and Compound Interest of any sum for any number of years at 10%. Thus log amount of £1 for 4 years $10\frac{1}{2}$ months at 10% Compound Interest = ·2. From the tables we find ·2 = log 1·59. Hence the amount of £1 for 4 years $10\frac{1}{2}$ months at 10% Compound Interest is £1·59, and the amount of £600 for same time and rate

$$= £1{\cdot}59 \times 600 = £954.$$

The Compound Interest

$$= £954 - £600 = £354.$$

It must be carefully observed that the graph gives us not the amount of £1, but the log of the amount. To get the amount we must consult the tables.

On one sheet of squared paper plot out the graphs of logs of amount of £1 for any number of years at 3, 4, 5, and 6%. Use them to find Compound Interest on any sum thus. Find the log of amount of £1 for the given time at the given rate. Find the sum corresponding from the tables. Multiply the Principal by this sum (keeping both in decimals). This gives the amount.

Subtract the Principal from the amount, and the Compound Interest is obtained.

Let R stand for the amount of £1 for a year.

Then evidently $(R)^x$ is the amount in x years. We might say that the graphs we have just plotted are (fig. 17)—

$$y = (R)^x,$$
$$\text{and } \log y = \log (R)^x \text{ or } x \log R.$$

EXERCISE X

Find, by using graphs, amount and compound interest on—

1.	£650	for	6	years at	5	per cent.
2.	£1145	,,	4	,,	3	,,
3.	£3600	,,	7	,,	6	,,
4.	£198	,,	3	,,	10	,,
5.	£2194	,,	9	,,	4	,,
6.	£371, 15*s.*	,,	4	,,	5	,,
7.	£1916, 10*s.*	,,	9	,,	4	,,
8.	£18,146, 10*s.*	,,	6	,,	3	,,
9.	£236, 5*s.*	,,	3	,,	10	,,
10.	£155, 7*s.* 6*d.*	,,	7	,,	4	,,
11.	£27, 10*s.*	,,	4	,,	4	,,
12.	£1750	,,	25	,,	3	,,
13.	£99, 10*s.*	,,	16	,,	4	,,
14.	£1165, 15*s.*	,,	15	,,	6	,,
15.	13*s.* 4*d.*	,,	7	,,	6	,,
16.	£10,000	,,	20	,,	3	,,

The "Compound Interest Graph", as we might term fig. 17, is of very great importance. It is the graph of a sum increasing at a rate proportional to itself, that is to say, the more it increases the greater becomes the rate of increase, and looking at the curve in fig. 15 we clearly see this, for the graph becomes steeper and steeper. Carrying it mentally far out into space we see that in time this graph will approach the vertical more and more, till a very small addition of time will mean an enormous increase in the amount. This is illustrated in the fact that 1*d.* put in the bank at the birth of Christ would now (at ordinary rate of interest) amount to more than all the gold in the world in value.

Very many things increase (or decrease) by the "Compound Interest Law" (as Lord Kelvin calls it). Thus, if no other influences are at work, the population of a country increases at a rate proportional to itself. Now the larger the population the more food it requires, the more ships to carry produce, the more coal, iron and steel. Thus it will be found that to a certain extent (although affected by other causes) the food supply, the tonnage of shipping, the coal, iron and steel output all tend to increase in accordance with the Compound Interest Law.

While the graph of two things which change according to the Compound Interest Law is a curve, great attention should be paid to the fact that the log graph is a straight line. Hence if we know two points on the log graph we are entitled to join them and produce the graph to such limits as we please. The intermediate points will be found correct.

Plot out the following examples both in arithmetical and log forms, using suitable scales.

EXERCISE XI

1. The temperature of hot water cooling falls at a rate proportional to the excess of temperature above surrounding objects (say the room it is in). Take a glass of water nearly at boiling point, put a thermometer in it and take the temperature (every two minutes) ABOVE THAT OF THE ROOM. Plot a graph of times and temperatures—temperatures in degrees vertically, time in minutes horizontally. Do this between the limits of the temperature of the room and the highest temperature of the water. On the same paper plot the log graph and it will be found to be a straight line.

Take another glass of water at boiling point; put a thermometer in it and get the temperature. Have squared paper marked off as before, and mark the log of excess of temperature on the y axis. In five minutes more, again read the temperature, get the excess above room and mark the point (5.-log excess temperature). Join these two points and produce downwards. The graph is now completed without actual observations. Now leave someone to watch the thermometer and calculate from the log tables the temperature excess for every two minutes. Do this rapidly. See if the temperatures observed correspond with those calculated. Also predict at what time the water will reach approximately the temperature of the room.

(NOTE.—In marking temperature, vertically start with that of the room, *not* 0°, since the water cannot fall below the temperature of the room it is in.)

2. The population of a country is 28 millions (1903). Five years ago it was 25 millions. Draw a log graph showing theoretical rate of increase, and estimate probable population in other 5 years.

(Plot years horizontally up to 10, five divisions to 1 year. Plot logs vertically, five divisions to ·1, or 50 divisions to 1. Make 5th year 1903 and set off log 28.

At 1st year set off log 25. Join points and produce to 10th year, when log of population in millions will be found.)

3. Scotland has a population of approximately 4·6 millions (1903). Ten years ago it had a population of 4 millions. At another period its population was 3 millions. When was this?

4. A cistern containing 4000 gallons of water springs a leak. When full it leaks at the rate of 8 gallons per minute, and when three-quarters full at 6 gallons per minute. At what rate will it leak when it is quarter full if it leaks in accordance with the Compound Interest Law? Find also average rate of leakage and when cistern will be empty.

(Plot graphs of leakage and fullness, taking logs of leakage.)

5. The tyre of a motor car has an internal air pressure of 60 lbs. per sq. inch above external pressure, at 10 P.M. At 8 A.M. next morning it is found to have fallen to 10 lbs. per sq. inch, through a puncture. If a tyre leaks at a rate in accordance with the Compound Interest Law, find the pressure every hour from 10 P.M.

6. A cup of tea is found to be at a temperature of 180°, the room being at 65°. In five minutes it has fallen to 160°. What will it be in ten minutes, and when will it be 80°?

The following graphs should be plotted both in arithmetical and log forms. The difference between them and the Compound Interest Graph will then be evident. The formula for the Compound Interest Graph might be written $y = (c)^x$ where c is some constant quantity, generally easily obtained.

7. Area of circle $= \pi r^2$ where $\pi = 3\frac{1}{7}$, $r =$ radius. *Formula*, $y = \pi x^2$.

8. The distance (s) a stone falls in a given time (t seconds) is found by $s = 16t^2$. *Formula*, $y = 16x^2$. (Use contracted scale for feet vertically, or plot yards.

9. Draw a graph showing squares of numbers. *Formula* ($y = x^2$).

CHAPTER IX

In the graph of the road from Glasgow to Prestwick (fig. 4) it will be noticed that the road first slopes upward, at 12 miles becomes level, and then slopes downward. Now at this level point, *where there is no slope whatever*, the road has reached its highest point. Suppose any graph slopes upwards, then becomes level, then slopes downwards, we may say, as in the case of the road, that the quantity corresponding to height first increases, becomes a maximum, then decreases. We may thus discover when the quantity in question is a maximum, and what that maximum is.

For example, a rectangle is made out of a piece of wire 20 inches long. What should the length and breadth be to form the greatest possible rectangle?

Now perimeter = 20 inches,

∴ Length + breadth = 10 „

Make a number of possible combinations with this. Thus—

When the	length is	1″	the breadth is	9″,	∴	area	=	9	sq. ins.
„	„	2″	„	8″,	∴	„	=	16	„
„	„	3″	„	7″,	∴	„	=	21	„
„	„	4″	„	6″,	∴	„	=	24	„
„	„	5″	„	5″,	∴	„	=	25	„
„	„	6″	„	4″,	∴	„	=	24	„
„	„	7″	„	3″,	∴	„	=	21	„
„	„	8″	„	2″,	∴	„	=	16	„
„	„	9″	„	1″,	∴	„	=	9	„
„	„	10″	„	0″,	∴	„	=	0	„
„	„	0″	„	10″,	∴	„	=	0	„

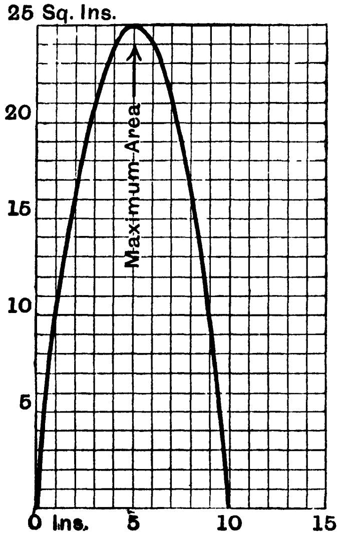


Fig. 18

Take squared paper, and set off length horizontally and area vertically. Draw the graph connecting area with length (fig. 18). It will be seen that the area gradually increases, then decreases. At 5 inches the area is a maximum, namely, 25 sq. inches. Therefore the length required is 5 inches, and since length and breadth together equal 10 inches, the breadth should be 5 inches. Therefore the greatest rectangle which can be made out of a given length of wire is a square. In the same way, plot graphs for the following questions, and ascertain the point of least slope, that is, maximum height.

EXERCISE XII

Examples

1. A tree is 5 feet in diameter. Find the largest beam that can be cut out of it. (Express length and breadth in terms of diameter.)

2. Find greatest rectangle contained by a rope 240 feet long.

3. I have a parcel to tie with one piece of string 22 inches long. What length and breadth should I make the parcel just to use up all the string, allowing 2 inches for tying, the area so enclosed being a maximum?

4. A tree is 8″ diameter, and I wish to cut the strongest possible beam out of it. If the beam is strongest when bd^2 is greatest, what breadth (b) and depth (d) should I make the beam? (Express b and d in terms of diameter.)

5. A cannon is fired at a target 8 miles off. The height of the projectile above the firing point is: At 1 mile, 85 yards; 2 miles, 145 yards; 3 miles, 185 yards; 4 miles, 200 yards; 5 miles, 185 yards; 6 miles, 145 yards; 7 miles, 85 yards; at 8 miles it strikes target at the same level as firing-point. Find where projectile was highest, and what was the height at that point. At what angle did it strike the target?

6. An engine crosses a single-span bridge 80 feet long. The stress to which the bridge is subjected as it crosses is proportional to the following numbers:—

At 5 feet from beginning,	10 units.				At 45 feet from beginning,	39 units.			
10	„	„	17	„	50	„	„	37	„
15	„	„	25	„	55	„	„	33	„
20	„	„	29	„	60	„	„	29	„
25	„	„	33	„	65	„	„	25	„
30	„	„	37	„	70	„	„	17	„
35	„	„	39	„	75	„	„	10	„
40	„	„	40	„	80	„	„	0	„

Draw a graph connecting stress and distance. When is the stress greatest?

7. A boy throws up a cricket ball straight in the air. The following table gives height of the ball at different times from its start:—

$\frac{1}{4}$ second,	9 feet.			$1\frac{1}{2}$ seconds,	24 feet.		
$\frac{1}{2}$	„	16	„	$1\frac{3}{4}$	„	21	„
$\frac{3}{4}$	„	21	„	2	„	16	„
1	„	24	„	$2\frac{1}{4}$	„	9	„

Draw a graph connecting height and time. When was ball highest; how high was it; and when did it reach the ground?

8. A pendulum which beats seconds is pulled aside and let go. Its velocity

is observed every twelfth of a second as it travels from the one side to the other. Thus—

Start,		0	millimetres	per second.
$\frac{1}{12}$	second later,	10	,,	,,
$\frac{2}{12}$	,,	20	,,	,,
$\frac{3}{12}$	,,	28	,,	,,
$\frac{4}{12}$	,,	34	,,	,,
$\frac{5}{12}$	,,	39	,,	,,
$\frac{6}{12}$	,,	40	,,	,,
$\frac{7}{12}$	,,	39	,,	,,
$\frac{8}{12}$	,,	34	,,	,,
$\frac{9}{12}$	,,	28	,,	,,
$\frac{10}{12}$	,,	20	,,	,,
$\frac{11}{12}$	,,	10	,,	,,
$\frac{12}{12}$	,,	0	,,	,,

Draw a graph connecting velocity and time. (This graph is the graph of Simple Harmonic Motion, and is of very great importance in Physics and Electrical Engineering.) When is velocity greatest?

MISCELLANEOUS EXAMPLES

The following examples, though as a whole fairly easy, are typical of what is daily required in commercial and technical life. All are taken from the most accurate sources obtainable, and in most cases from actual graphs:—

I—COMMERCIAL, BOARD OF TRADE, &c.

1. Tonnage of ships launched on the Clyde since 1878 (the introduction of steel).

Year	Tons	Year	Tons	Year	Tons
1878	222,353	1887	185,362	1896	420,841
1879	174,750	1888	280,037	1897	340,037
1880	241,114	1889	335,201	1898	466,832
1881	341,022	1890	349,995	1899	491,074
1882	391,934	1891	326,475	1900	486,337
1883	419,664	1892	336,414	1901	511,990
1884	296,854	1893	280,160	1902	516,977
1885	193,453	1894	340,885	1903	446,869
1886	172,440	1895	360,152		

Draw a graph showing progress of ship-building from 1878. Note, 1886 bad trade, 1893 dull trade and strike, 1896–97 engineers' strike.

2. The following are prices of iron, copper, tin, and silver since 1888, the highest price in each year being given. Put graphs on one sheet, noting price of iron is in shillings per ton, copper and tin £s per ton, silver pence per ounce:—

Iron	Per Ton. s.	d	Copper	Per Ton. £	s	d
1903	61	7	1903 (to date)	66	12	6
1902	61	$10\frac{1}{2}$	1902	56	15	0
1901	63	0	1901	72	17	6
1900	86	10	1900	79	2	6
1899	80	$1\frac{1}{2}$	1899	79	5	0
1898	59	$4\frac{1}{2}$	1898	57	10	0
1897	51	9	1897	51	3	9
1896	51	8	1896	50	3	9
1895	51	6	1895	47	7	6
1894	46	1	1894	42	8	9
1893	46	6	1893	46	17	6
1892	53	0	1892	47	15	0
1891	54	3	1891	56	2	6
1890	82	0	1890	59	0	0
1889	78	4	1889	77	10	0
1888	46	0	1888	100	10	0

Tin	Per Ton. £	s	d	Silver	Per Oz d
1903 (to date)	140	10	0	1903	$28\frac{1}{2}$
1902	137	5	0	1902	$26\frac{1}{8}$
1901	140	0	0	1901	$29\frac{9}{16}$
1900	153	0	0	1900	$30\frac{3}{16}$
1899	150	10	0	1899	$28\frac{7}{8}$
1898	86	10	0	1898	$28\frac{5}{16}$
1897	63	3	9	1897	$29\frac{13}{16}$
1896	61	12	8	1896	$31\frac{7}{16}$
1895	68	0	0	1895	$31\frac{3}{8}$
1894	77	10	0	1894	$31\frac{3}{4}$
1893	95	5	0	1893	$38\frac{3}{4}$
1892	103	0	0	1892	$43\frac{3}{4}$
1891	93	15	0	1891	$48\frac{3}{4}$
1890	105	0	0	1890	$54\frac{5}{8}$
1889	98	5	0	1889	$44\frac{3}{8}$
1888	168	10	0	1888	$44\frac{5}{8}$

3. Steerage passengers landed at New York 1893–1903. Draw graph showing fluctuations.

Year.	Passengers	Year	Passengers
1893	364,700	1899	305,760
1894	188,164	1900	403,190
1895	288,500	1901	438,868
1896	252,350	1902	574,276
1897	192,000	1903	643,358
1898	249,650		

Set up scale of thousands, commencing with 188, the lowest (1894).

4. Production of gold in South Africa, 1887 to 1902.

Year.	Millions of £s.	Year	Millions of £s.	Year	Millions of £s
1887	1	1893	$5\frac{1}{3}$	1899	$14\frac{3}{4}$
1888	1	1894	$7\frac{1}{3}$	1900	$1\frac{1}{2}$
1889	$1\frac{1}{2}$	1895	$8\frac{1}{4}$	1901	1
1890	2	1896	$8\frac{1}{4}$	1902	2
1891	3	1897	$10\frac{3}{4}$	1903	3
1892	$4\frac{3}{4}$	1898	$15\frac{1}{2}$		

5. Export of home products per head of population. Put graphs on one sheet of squared paper.

Year	Great Britain	France	Germany	United States
1870	£7·36	£3·75	£2·83	£2·5
1875	6	3·75	3·15	2·81
1880	6·66	3·67	3·43	3·29
1885	6·18	3·46	3·27	2·59
1890	6·14	3·56	3·14	2·95
1895	5·97	3·73	3·36	2·91
1900	6·85	4·23	4·05	3·81

6. Growth of our trade.

Year	Britain's Exports. Millions of £s	Britain's Imports. Millions of £s
1897	227	371
1898	233	410
1899	255	420
1900	291	461
1901	280	454
1902	284	463

7. British ships lost, with numbers of crew and passengers, 1894 to 1900.

Year	No of Ships.	No. of Crew Lost.	No of Passengers Lost
1894	539	1481	1254
1895	478	1340	104
1896	433	833	410
1897	475	828	48
1898	413	872	100
1899	397	1183	125
1900	387	1128	50

Put above on one sheet squared paper. Are passenger ships becoming safer? Are trading ships?

8. Coal output of United Kingdom, 1892 to 1901.

Year	Millions of Tons	Year	Millions of Tons
1892	$181\frac{3}{4}$	1897	$202\frac{1}{4}$
1893	$164\frac{1}{3}$	1898	203
1894	$188\frac{1}{4}$	1899	220
1895	$189\frac{3}{4}$	1900	$225\frac{1}{4}$
1896	$195\frac{1}{4}$	1901	219

9. World's production of gold, 1890-1902.

Year	Millions of £s	Year.	Millions of £s.
1890	$24\frac{1}{4}$	1898	$59\frac{1}{2}$
1892	$29\frac{9}{10}$	1900	$53\frac{2}{3}$
1894	$36\frac{3}{4}$	1902	$62\frac{1}{2}$
1896	$41\frac{3}{4}$		

II—ENGINEERING

10. Pressure of water in a river against the side of a bridge at different rates of flow.

Speed of River – Miles per Hour	Pressure in Pounds per Sq Foot.	Speed of River—Miles per Hour	Pressure in Pounds per Sq Foot.	Speed of River—Miles per Hour	Pressure in Pounds per Sq Foot
1	3·8	4	62	8	248
2	15·5	5	97	9	314
3	35	6	139	10	387
		7	190		

Show the above relations graphically.

11. The horse power required to drive a certain vessel at certain speeds is given.

Knots per Hour	Horse Power	Knots per Hour	Horse Power.	Knots per Hour.	Horse Power
4	22	7	60	10	140
5	30	8	80	11	170
6	40	9	110	12	230

Show the above relations graphically. Produce the graph in order to tell horse power required at higher speeds. What horse power would be required for 14, 15, 16 and 17 knots per hour respectively?

12. *Force of the Wind.*—Pressure on every square foot at different velocities.

Velocity—Miles per Hour.	Pressure—lbs.	Velocity—Miles per Hour	Pressure—lbs
10	$\frac{1}{2}$	40	$7\frac{3}{4}$
15	1	45	10
20	2	50	$12\frac{1}{4}$
25	3	55	$14\frac{3}{4}$
30	$4\frac{1}{2}$	60	$17\frac{3}{4}$
35	6		

Show this graphically.

13. Velocity of water out of hole in bottom of tank. As the tank empties the velocity decreases. Correct any experimental errors.

Depth—Feet	Velocity—Feet per Second	Depth—Feet	Velocity—Feet per Second	Depth—Feet.	Velocity—Feet per Second
12	27·8	8	22·75	4	16
11	26·6	7	21 2	3	13·9
10	25·4	6	19·66	2	11·3
9	24	5	17·9	1	8

What should the velocity be when the depth is 16 and 35 feet?

14. A brassfounder wishes to know at what temperature alloys of copper and zinc melt, and his chemist experiments with different mixtures. The results are given below.

Percentage of Copper	Melting Point
0 (zinc only)	730° F.
10	1000° ,,
20	1300° ,,
30	1450° ,,
40	1500° ,,
50	1600° ,,
60	1650° ,,
70	1700° ,,
80	1800° ,,
90	1900° ,,
100 (copper only)	1950° ,,

Make a graph so that the brassfounder may determine the melting point of any mixture.

15. An iron-merchant wishes to test the strength of some steel he has bought. He takes a piece one inch square and a foot long, fixes it firmly at one end (vertically) and applies heavy weights to the other. As the weights increase the steel stretches slightly, then suddenly snaps. The following figures show the weights applied and the amount the steel stretches in thousandths of an inch. Draw a graph showing the behaviour of the steel under test.

Weight Applied —Tons	Stretch	Weight Applied —Tons.	Stretch	Weight Applied —Tons	Stretch
10	$\frac{1}{4}$	46	3	56	8
20	$\frac{1}{2}$	47	4	57	9
30	$\frac{3}{4}$	50	5	58	10
40	1	52	6	59	Suddenly snapped.
45	2	54	7		

Note any points of sudden change of slope.

16. The following table shows the safe weight hemp and steel ropes should be allowed to carry. Make a graph for office use.

Diameter	Safe Load—Hemp	Safe Load—Steel
1 inch	$\frac{3}{4}$ cwt.	1 ton
$1\frac{1}{4}$,,	$1\frac{1}{4}$,,	$1\frac{3}{4}$,,
$1\frac{1}{2}$,,	2 ,,	2 ,,
$1\frac{3}{4}$,,	$2\frac{1}{3}$,,	$2\frac{1}{3}$,,
2 ,,	$3\frac{1}{2}$,,	$3\frac{1}{3}$,,
$2\frac{1}{4}$,,	$4\frac{1}{3}$,,	$4\frac{1}{2}$,,
$2\frac{1}{2}$,,	$5\frac{1}{3}$,,	$5\frac{1}{2}$,,
$2\frac{3}{4}$,,	$6\frac{3}{4}$,,	$6\frac{3}{4}$,,
3 ,,	8 ,,	8 ,,

17. A steel bar 5 feet long is fixed at one end horizontally, and weights h on the other. As the weights increase it bends down, then snaps. Draw graph and tell the story of its behaviour, given the weights in pounds and dip of the loaded end in millimetres.

Weight Applied	Dip in Millimetres
200 lbs.	6
400 „	11
600 „	16
800 „	22
1000 „	28
1200 „	33
1400 „	38
1600 „	45
1700 „	52
1800 „	62
1900 „	80
2000 „	Suddenly snapped.

18. Aluminium wire was tried for telegraph wire, but was a failure, as it continually breaking, seeming to be unable to bear its own weight. To test t two wires 36 feet long, one aluminium, one copper, were hung from the roo a factory, with 110 pound weights at the end. The stretch of the wires observed each day for 50 days. A graph of the results showed the reason of failure. What was it?

Days	Stretch of Aluminium	Stretch of Copper in 64ths of one inch.
0	18	14
5	22	16
10	23	17
15	24	
20	26	gradual stretch
25	$26\frac{1}{2}$	on to
30	27	
35	28	18
40	28	18
45	$28\frac{1}{4}$	18

III—ELECTRICAL

19. An electric motor car is designed to average 12 miles per hour, but the maker gives the following figures, showing total distance it will go at various speeds.

Speed	Distance Car will Travel without Recharge
5 miles per hour.	107 miles.
10 " "	100 "
15 " "	92 "
20 " "	80 "
25 " "	67 "
30 " "	50 "

Draw a graph which may be used to give distance for any speed. The faster the car goes the less total distance it covers.

20. An electrician keeps a 100 candle-power glow lamp burning night and day to test how long it will last, and if the candle power keeps constant. He gets the following results. Plot them in graphic form.

Hours.	Candle Power	Hours	Candle Power
0	100	800	61
200	86	1000	55
400	76	1200	50
600	68	1201	Lamp burst

21. He also tests at the same time a new Nernst lamp, and gets the results below. Compare the falling off of candle power in the two lamps.

Hours	Candle Power.	Hours	Candle Power
0	140	400	110
100	110	500	100
200	110	600	95
300	110	700	80 (burst)

22. The chief electrician to the corporation of a large town measures carefully the electricity supplied to the public throughout the 24 hours. He does this on the 14th July, 14th September, and 14th December. Below are given the equivalent number of lamps every hour. On one sheet put all the graphs, one in pencil, one in black, and one in red ink. Study these graphs very carefully, as

they are of great importance. Where the graphs coincide use black ink for part common to the three.

Hour	Lamps—July.	Lamps—September.	Lamps—December
6 A.M.	50 (hundreds)	75 (hundreds)	250 (hundreds)
7 „	50 „	75 „	300† „
8 „	50 „	75 „	200 „
9 „	50 „	75 „	150 „
10 „	50 „	75 „	175 „
11 „	50 „	75 „	100 „
12 Noon	50 „	75 „	100 „
1 P.M.	50 „	75 „	100 „
2 „	50 „	75 „	150 „
3 „	50 „	75 „	300 „
4 „	50 „	75 „	850* „
5 „	50 „	150 „	1250 „
6 „	50 „	500 „	1200 „
7 „	75 „	1000* „	1200 „
8 „	150 „	1200 „	1200 „
9 „	650 „	1000 „	1000 „
10 „	650* „	700 „	700 „
11 „	450 „	450 „	450 „
12 Midnight	300 „	300 „	300 „
1 A.M.	200 „	200 „	250 „
2 „	150 „	200 „	250 „
3 „	100† „	175 „	250 „
4 „	50 „	175 „	250 „
5 „	50 „	150† „	250 „
6 „	50 „	75 „	250 „

23. The student is advised to do the following simple experiments, express the results in graph form:—

(*a*) Raise some water to boiling point, and take the temperature. ⌟ common salt 1 oz. at a time, and find new boiling points. Carry out experin quickly. Show how addition of salt alters boiling point.

(*b*) Attach bullet to a thread 1 foot long. Fix thread to a nail, and vibı as pendulum, gently. Count vibrations in minute. Repeat with threads 2′, 2′ 6″, 3′, &c., long. Express results graphically.

(*c*) Fix up a pencil in front of a gas jet. Measure shadow it casts on ne paper held 1 foot away, 2 feet, 3 feet, &c. Plot results as a graph.

(*d*) Bore very small hole in bottom of cocoa-tin. Fill with water and h up. Measure depth every minute (or five minutes if more convenient) till em Draw the graph of depths and times.

(*e*) Get some pieces of wood ½ inch square and a foot long. Fix one horizontally, and hang weights on the other end (gradually) till the wood sn

* Hour when street lamps are turned on
† Hour when street lamps are turned off Note effect of this.

Try this with as many different woods as possible. Express results as a graph of weights, and distance end dips down.

LOGARITHMS

From *Mathematical Tables for the Use of Students*, by permission of the Board of Education.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969					3	7	11	14	18	21	25	28	32
							1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553							3	6	9	12	15	18	21	24	28
					1584	1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903					3	6	9	11	14	17	20	23	26
							1931	1959	1987	2014	3	5	8	11	14	16	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	15	18	21	23
17	2304	2330	2355	2380	2405	2430					3	5	8	10	13	15	18	20	23
							2455	2480	2504	2529	2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Note —The numbers from 10 to 19 have two rows of differences Use the first row for the upper set of logs (columns 0 to 4), and the second row for the lower set of logs (columns 5 to 9).