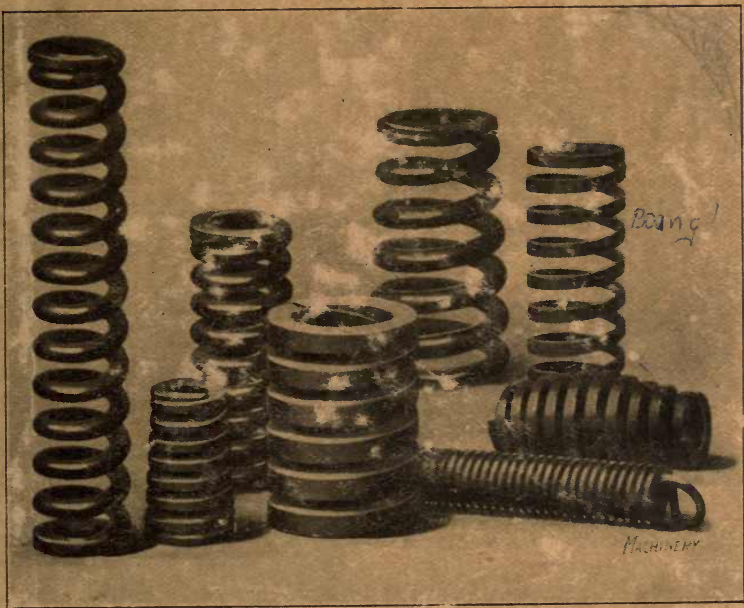
25 CENTS

HELICAL AND ELLIPTIC SPRINGS

THEORETICAL PRINCIPLES OF SPRING CALCULATIONS

SECOND EDITION



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NUMBER 58

HELICAL AND ELLIPTIC SPRINGS

SECOND EDITION

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CHAPTER I

PRINCIPLES OF SPRING CALCULATIONS*

Although made in a great variety of shapes, the working and efficiency of any spring can be readily understood and investigated if a few fundamental principles determining the resistance to bending or twisting, and the deflection of elastic bodies are understood. Springs are generally made of steel or brass, and when under tension are either bent or twisted. Let us, therefore, first consider a flat piece of tempered tool steel of even thickness and width, firmly clamped at one end, and with a weight suspended at the free end, as shown in Fig. 1. The free length is a little over 12 inches, the width 1½ inch, the thickness 1/16 inch, and the suspended weight 10 pounds. The deflection at the free end will be about 4½ inches, and the curvature will be as

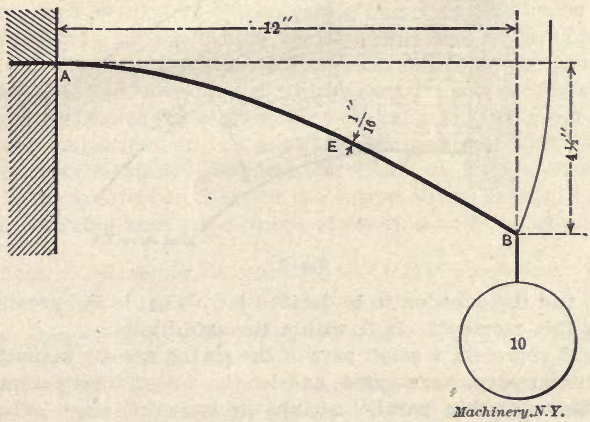

Fig. 1

shown in the figure. If made of high carbon crucible steel, properly tempered, 10 pounds is a safe load on this spring, but it may carry considerably more before the elastic limit is reached. These facts were obtained by calculation, by methods explained later.

It is obvious that the curvature of any part of the spring depends on the leverage or arm of bending, or rather on the "moment of bending," which is the weight multiplied by its arm of leverage. At *A* the arm is 12 inches, at *E* it is about 6 inches; the moment of bending at *E*, therefore, is only half the moment of bending at *A*; at *B* there is no arm, and therefore no bending. Consider a small part—an element—of the curve at *A*. This element will be bent to the arc of a circle, and the radius of this arc is called the radius of curvature at *A*; any ele-

* Machinery, May, July and August, 1898.

ment nearer *B* will have a larger radius of curvature. At *A* the radius of curvature is about 11 inches, at *E* it is about 22 inches, and at *B* it is infinite.

Carrying Capacity of a Flat Spring

Considering any spring, we must first know whether it is strong enough to carry the load or will stand the work for which it is intended. The mistake is often made of using, or attempting to use, a spring which has not sufficient strength or endurance for the work it has to do, and which, consequently, gives out after being in use a short time. The spring shown in Fig. 1 being of even thickness through its entire length, is evidently weakest where it is bent most, that is at *A*. The moment of bending at this point is $10 \times 12 = 120$, and the bending brings forth a moment of resistance or internal resisting moment in the steel equal in magnitude to the bending moment of the extrane-

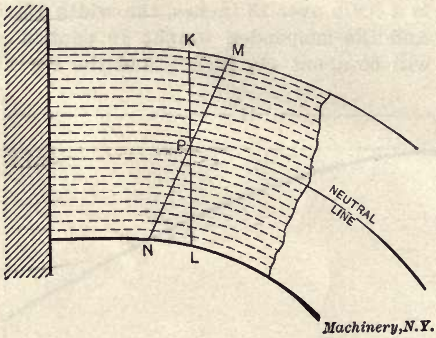

Fig. 2

ous force, and the question to be decided is: What is the greatest fiber stress for this moment? Is it within the safe limit?

Let Fig. 2 represent a small part of the spring greatly magnified and the bending greatly exaggerated, and let the dotted lines represent imaginary fibers or thin parallel strings or strips of steel. The upper half of these will be stretched, and the lower half will be compressed; but right in the center line of the thickness of the spring the fibers will neither be stretched nor compressed, and this line is therefore called the neutral line. We may consider any point in this line as a pivot for a double-armed lever to which the fibers are attached. Let *P* be the pivot and let *KL* represent the position of the lever before bending, and suppose that this lever, by the bending of the spring, is thrown in the position *MN*, and then *KM* represents the amount of stretching of the extreme upper fibers, and *NL* represents the compression of the extreme lower fibers, or rather of a small part of these. Steel is not fibrous, but we may call a string of molecules a fiber. All the fibers will be stretched or compressed in proportion to their distance from the neutral line, and they will therefore exert a certain resistance on the imaginary lever *MN*, and this collective resistance will exactly counterbalance the weight on the end of the spring acting on the lever *AB*, Fig. 1. The outside fibers will be stretched or compressed most, and if they are stretched beyond a certain limit, the spring will break or receive a permanent set. If we double the thickness of the spring, it can evidently only bend half as much before the limit of fiber stress is reached; but the average distance of the fibers from pivot *P* will, in this case, be doubled—that is, the leverage of resistance will be doubled and the number of fibers will also be doubled. The total resistance to bending at the same limit of fiber stress will therefore be twice doubled; that is, it will be $2 \times 2 = 2^2 = 4$ times as great, or, in other words, doubling the thickness of a spring quadruples its carrying capacity. If we had increased the thickness by one-half only, we should have $(1\frac{1}{2})^2 = 2\frac{1}{4}$ times greater strength. In general, let T and U represent the respective thicknesses of two similar springs of same width and length; then

$$\frac{\text{carrying capacity of spring } T}{\text{carrying capacity of spring } U} = \frac{T^2}{U^2}$$

or, the carrying capacities of otherwise similar springs are as the square of their respective thicknesses. This rule applies to bending only, and not to springs which are twisted. The strength of a flat spring is in simple proportion to its width, which is obvious without demonstration, and therefore, if thickness $= t$ and width $= b$, the moment of resistance for a given fiber stress $= cbt^2$, where c is a constant factor dependent on the allowable fiber stress. This factor can be found experimentally. Suppose, for instance, it is known that 10 pounds is the greatest load which the spring shown in Fig. 1 ought to carry, then in this case, the moment of bending $= 10 \times 12 = 120$, and the moment of resistance $= c \times 1\frac{1}{2} \times (1/16)^2 = \frac{3}{512} c$. Equating these two quantities we have $120 = \frac{3}{512} c$, or $c = 20{,}480$. Now c being a known constant factor, we can always find the moment of resistance from the formula cbt^2, and this product divided by the leverage of the load gives the carrying capacity or admissible load. For instance, let the length or leverage be 10 inches, the width 1 inch, and the thickness ⅛ inch, then,

$$\frac{20{,}480 \times 1 \times (\frac{1}{8})^2}{10} = \frac{20{,}480}{640} = 32 \text{ pounds,}$$

which is the safe load on the free end of the spring. Great exactness is not necessary in such calculations, and the factor 20,500, being easier to remember, may be used instead of 20,480. If a spring is continually working, a smaller factor must be used than would be admissible if it were only occasionally in action, and a much higher factor may be used if it has only to exert a constant pressure without any bending motion. If a spring is sufficiently strong and durable under certain conditions, we may, from the formula here given, design any number of springs equally strong under similar conditions.

Deflection of a Flat Spring

We will now consider the amount of deflection of a flat spring. Referring to Fig. 3, suppose there be one flexible element at *A*, and suppose the rest of the spring to be perfectly stiff or unelastic, which part we will call the "arm," and suppose the deflection at *A* will bring the arm in the position *AF*. If there now, instead of one flexible element, be two such elements at *A*, the inclination of the arm will be on line *AG*, and deflection $GB = 2BF$. For three flexible elements the deflection would be three times *BF*, and so on, provided the length of the arm remains the same; that is, the deflection of the arm *AB* is directly proportional to the number of flexible elements at *A*. Now suppose we double the length of the arm, as shown by the dotted lines; then we also double the moment of bending, and the deflection at the end of the arm will therefore be twice doubled. Therefore, by doubling the number of elements at *A* and by doubling the arm, we increase the linear deflection at the free end $2 \times 2 \times 2 = 2^3 = 8$ times. In reality the

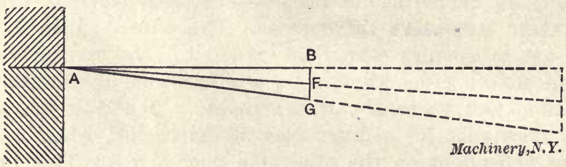

Fig. 3

arm itself is flexible, and considering it as made up of flexible elements, we may imagine the deflection at the free end as made up of a series of decreasing elementary deflections corresponding to a series of flexible elements of the spring and their respective arms.

Fig. 4 shows the curve of two similar springs of different lengths similarly loaded. *AC* represents the curve when the length is double that of *AE*. Suppose we divide *AE* into a number of small parts and call these elements, and divide *AC* into the same number of parts; each of these will contain two elements, that is, for each element of *AE* there will be two corresponding elements of *AC*, and the distance of any two such elements from the end of the spring will be twice the distance of the corresponding element of *AE* from *E*. That is, the arm and the moment of bending of any two elements of *AC* will be twice the arm and moment of bending of the corresponding element of *AE*. The deflection of spring *AC* will therefore be $2 \times 2 \times 2 = 2^3 = 8$ times the deflection of spring *AE*. If the deflection of spring *AE* is $\frac{3}{8}$ inch, the deflection of spring *AC* will be $2^3 \times \frac{3}{8} = 3$ inches. If the deflection of *AE* is $1\frac{1}{4}$ inch, the deflection of *AC* will be $2^3 \times 1\frac{1}{4} = 10$ inches, provided it is strong enough to carry the load.

If spring *AC* had been three times as long as *AE* there would, for each element of *AE*, be three corresponding elements of *AC*, and the moment of bending of any such group of elements would be three times the moment of bending of the corresponding single element of *AE*, and the distance of any group of three elements of *AC* from *C* would be three times the corresponding distance on AE; we should, therefore, in this case have a deflection at the free end of $AC = 3^3 = 27$ times the corresponding deflection of AE. If in this case the deflection of AE were ¼ inch, the deflection of AC would be $3^3 \times$ ¼ inch $= 6\frac{3}{4}$ inches. If the length of AC were 1½ times the length of AE, we should have the deflection of $AC = (1\frac{1}{2})^3 \times \frac{1}{4}$ inch $= \frac{27}{32}$ inch.

In general, the deflection at the end is proportional to the third power of the length of the spring. The amount of deflection can be expressed by the formula al^3, in which l is the length and a is a factor dependent on the load, width, thickness and material of the spring. If we double the width, the bending moment for each element of the width will be halved, and the deflection will consequently be half of

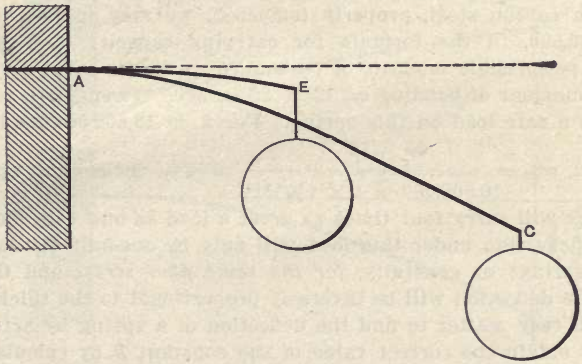

Fig. 4

that of the narrower spring. If we double the thickness of the spring, its area of cross-section is doubled, as is also the average distance of the fibers from the neutral axis, and besides for a given curvature or deflection the outer fibers will be stretched twice as much and will therefore offer twice the resistance (see Fig. 2). The total resisting moment is therefore increased $2 \times 2 \times 2 = 2^3 = 8$ times, or, in general, the moment of resistance is proportional to the third power of the thickness, and the deflection will be inversely proportional to this. If, for instance, we double the thickness and double the length of a flat spring, the deflection will remain the same for the same load. The deflection will also be directly proportional to the load.

It should be observed that we have here the moment of resistance in its general sense, that is, without any restriction in regard to the fiber stress. If we impose the condition that the fibers shall be stretched to a certain extent, as in the formula for strength, we have, in that case, the thickness squared in the moment of resistance, and it should be remembered that this is only in the formula for strength.

We are now able to calculate the deflection of any flat spring if we know the deflection of any other flat spring of the same material.

Let f = deflection of the free end, b = the width of the spring, t = the thickness, l = the length, w = the load, and k = a constant factor depending on the material, then,

$$f = \frac{wl^3}{kbt^3}$$

Suppose we have a spring 1½ inch wide, 1/16 inch thick and 12 inches long, and find that it deflects 4½ inches under a load of 10 pounds, then we have:

$$4\tfrac{1}{2} = \frac{10 \times 12^3}{k \times 1\tfrac{1}{2} \times (1/16)^3}$$

from which we deduce k = 10,500,000.

Suppose we have a spring 1 inch wide, ⅛ inch thick, 12 inches long and a load of 25 pounds at the end of it. If this spring is made of best high carbon steel, properly tempered, we may use the constant factor, 20,500, in the formula for carrying capacity, and have the greatest permissible moment of resistance = $20{,}500 \times (1/8)^2 = 320.3$, and the moment of bending = $12 \times 25 = 300$. Twenty-five pounds is therefore a safe load on this spring. For k = 10,500,000 we have the

$$\text{deflection} = \frac{25 \times 12^3}{10{,}500{,}000 \times 1 \times (\tfrac{1}{8})^3} = 2\tfrac{1}{8} \text{ inches.}$$

A spring ⅛ inch thick will carry four times as great a load as one 1/16 inch thick, and the deflection under this load will only be one-half of that of the thinner spring; or generally, *for the same fiber stress* and the same length, the deflection will be inversely proportional to the thickness.

It is an easy matter to find the deflection of a spring by actual trial and then obtain the correct value of the constant k by calculation, as here explained; but the thickness of the spring must be very carefully measured, for it will be observed that a small variation in the thickness has a great effect on the deflection, and particularly so if thin springs are used. If, for instance, the deflection of a spring 1/16 inch thick is 4 inches, then the deflection of a similar spring which is 1/100 inch thicker will only be two inches for the same load.

The Modulus of Elasticity

The deflection may also be found if the "modulus of elasticity" of the material is known. The modulus of elasticity is the ratio of a direct pulling force to the extension per unit of length of a rod of 1 square inch sectional area. The extension must be within the elastic limit of the material, and is a very small quantity which can only be found by very careful measurement in a testing machine, but as it is obtained by a straight pull, it cannot furnish so trustworthy a constant for the calculation of bending deflection as that obtained by the method just explained. If a steel bar 1 inch square and 10 inches long is stretched one-hundredth of an inch by a pull of 30,000 pounds, the modulus of elasticity is $30{,}000 \div 0.001 = 30{,}000{,}000$ which is the approximately correct figure for unhardened steel. For hardened tool steel it is about 42,600,000 according to Reuleaux. The tensile strength of different steels varies considerably. The strength of high carbon steel is greatly increased by hardening. The so-called spring steel is probably more elastic, but less strong, and it is doubtful whether hardening changes its elasticity, while it no doubt increases the elastic limit and the tensile strength; but no spring steel can compare in strength with high-grade high carbon crucible tool steel, properly tempered. The allowable fiber stress depends to a great extent on the treatment of the steel; it may, according to Reuleaux, exceed 200,000 pounds per square inch at the elastic limit. In the formula for carrying capacity, the factor c should be one-sixth of the allowable fiber stress, and in

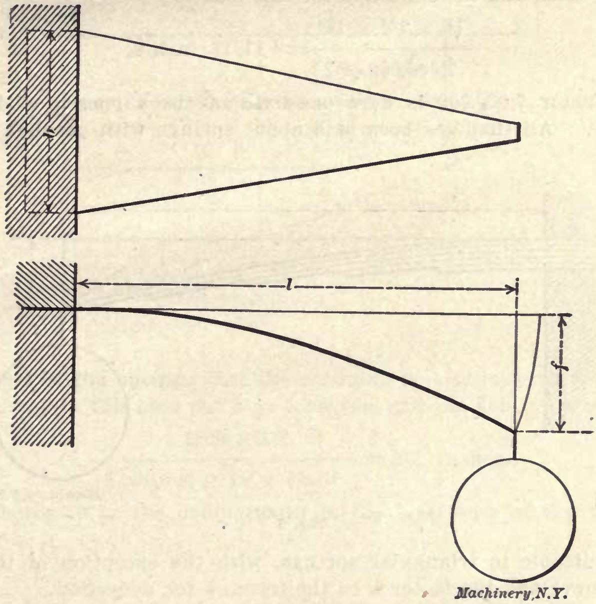

Fig. 5

the formula for deflection of a flat spring of even thickness and width the factor k should be one-fourth of the modulus of elasticity. The allowable fiber stress, is, of course, always less than the elastic limit.

Springs of Uniform Strength

A single steel band of even thickness and width does not always make a desirable spring, for if it is made just strong enough at its base, it will be stronger than necessary at other points, and the deflection at the free end will be less than if every part of the spring were equally strong—that is, if the fiber stress were uniform throughout the entire length. The spring shown in Fig. 5 is of nearly correct form; it is of even thickness and the edges converge nearly to a point.

It is obvious that, in practice, the end must be made a little blunt, but if it were continued to a sharp point, it will be seen that the width would be at any point of the length, proportional to the arm of leverage;

the radius of curvature would, therefore, be the same at any point—that is, the spring would bend to the arc of a circle. The strength is the same as that of a spring with parallel sides, but the deflection at the end will be one-half greater, and in the formula for deflection the factor k becomes two-thirds of that for parallel sides. Let the triangular spring be 2 inches wide at the base, 10 inches long, 1/16 inch thick, and made of high carbon steel, hardened, then,

$$\frac{2 \times 20{,}500}{16^2 \times 10} = 16 \text{ pounds}$$

is a safe load, and the deflection for this load is

$$\frac{16 \times 10^3 \times 16^3}{7{,}000{,}000 \times 2} = 4\ 11/16 \text{ inches.}$$

The factor 7,000,000 is here one-sixth of the supposed modulus of elasticity. All that has been said about springs with parallel sides is

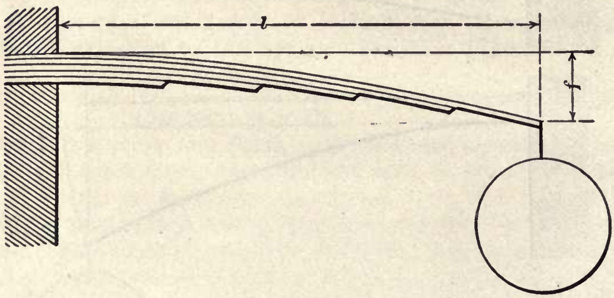

Fig. 6

also applicable to triangular springs, with the exception of the form of the curve and the factor k in the formula for deflection.

Built-up Leaf Springs

To get the most work out of a spring of given length or weight, it will often be found advantageous to use thin wide springs instead of thicker narrow ones, for it will be noticed that it is only the outside fibers which can be fully stretched or compressed, while all the others will be less useful in proportion to their proximity to the neutral axis. Instead of one broad triangular spring we may use a number of parallel springs, one on top of the other, as shown in Fig. 6. Each leaf or plate of this spring will be bent nearly to the same curve, and the deflection will be nearly equal to that of a triangular spring with a base equal to the collective width of all the leaves. Fig. 7 shows the leaves in the same plane laid side by side, and the dotted lines show the approximate size of the equivalent triangular spring. Suppose there be five leaves of tempered spring steel 2 inches wide and ⅜ inch thick, and let the working length of the main leaf be 18 inches; also suppose that the safe working fiber stress for this spring is 96,000 pounds per square inch; then we may, in the formula for strength, put

$$\text{factor } c = \frac{96{,}000}{6} = 16{,}000,$$

and the safe moment of resistance becomes $5 \times 2 \times (\frac{3}{8})^2 \times 16{,}000 = 22{,}500$, which, divided by 18 gives 1250 pounds as a safe working load

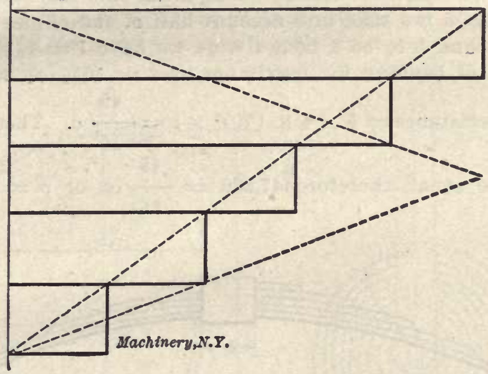

Fig. 7

on the end of the spring. Let the modulus of elasticity be 30,000,000, then we may in this case put $k = 5{,}000{,}000$, and the deflection equals

$$\frac{1250 \times 18^3}{5{,}000{,}000 \times 10 \times (\frac{3}{8})^3} = 2\frac{3}{4} \text{ inches.}$$

The factor 10 in the denominator is the total sum of the width of the leaves.

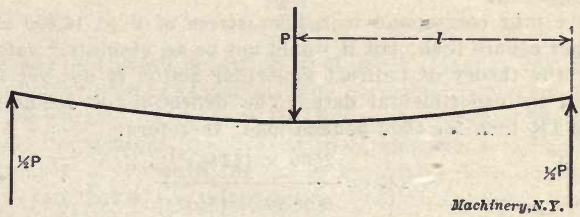

Fig. 8

It should be remarked that the deflection of such springs may vary considerably from that of the supposed equivalent triangular spring, and to get fairly correct results the factor k should be obtained by actual trial, and not from the supposed modulus of elasticity.

Fig. 8 represents a steel plate supported at both ends and a load P applied at the center. The upward pressure or reaction of each support is $\frac{1}{2}$ P, and it will readily be seen that the deflection of this spring must be exactly as if it had been supported at the center and loaded with ½ P at each end; that is, the moment of bending at the center is ½ Pl.

Fig. 9 represents a so-called elliptic spring, of a type used on carriages, automobiles and railroad cars. It is made of steel plates 4 inches wide and ⅜ inch thick. The distance between centers is 30 inches, and there are five plates in each part. The following experimental data have been ascertained for this spring: light load = 2000 pounds; maximum working load = 7000 pounds; deflection due to a load of 5000 pounds = 3 inches. Comparing this case with that represented by Fig. 8, we take into account half of the ellipse only, and assuming the band b to be 3 inches wide we have l = 13½ inches,, and the moment of bending for maximum load = 13½ × 3500 = 47,250.

Moment of resistance $= 5 \times 4 \times (\frac{3}{8})^2 \times c = \frac{45}{16} c$. These two quantities must be equal, therefore $47{,}250 = \frac{45}{16} c$, or $c = 16{,}800$. This

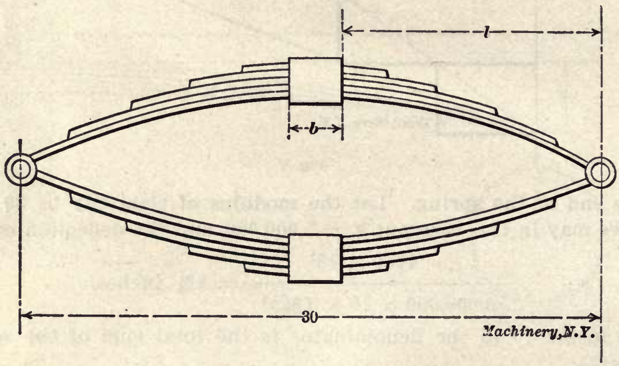

Fig. 9

value of c may correspond to a fiber stress of 6 × 16,800 = 100,800 pounds per square inch; but it would not be an absolutely safe assumption, for the theory of indirect molecular action is not yet fully substantiated by experimental data. The deflection of one-half of this spring is 1½ inch for 5000 pounds load; therefore:

$$1\tfrac{1}{2} = \frac{2500 \times (13\tfrac{1}{2})^3}{k \times 20 \times (\tfrac{3}{8})^3}$$

that is, k = 3,888,000. Assuming the curve of deflection similar to that of a single triangular spring, we should have, approximately, the modulus of elasticity = 6 × 3,888,000 = 23,328,000, but this is probably too low a figure. By using the constant factor 3,888,000, sufficiently accurate results would be obtained for similar springs of similar material.

Miscellaneous Classes of Springs

The available space for a spring may determine its shape and size. A long straight spring cannot often be used. Fig. 10 shows a spring which may be useful in a limited space. It is supposed to be made of a strip of high carbon crucible steel 1½ inch wide and 1/16 inch thick, and to be spring tempered. The moment of resistance is $1\frac{1}{2} \times (1/16)^2 \times c$, where c is supposed to be one-sixth of the allowable fiber stress per square inch, or the allowable unit-stress. For $c = 15{,}000$ we have the moment of resistance = 88. At *A* the lever arm is 4 inches, and the permissible load at *B* is therefore about $\frac{88}{4} = 22$ pounds. The moment of bending varies directly as the distance from *B*; at *E* and *F* it is $2 \times 22 = 44$ inch-pounds. If we imagine the spring divided into a number of small parts or elements, there will, for

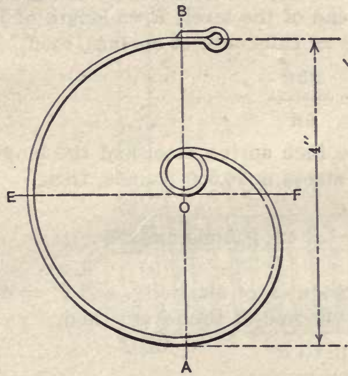

Fig.10

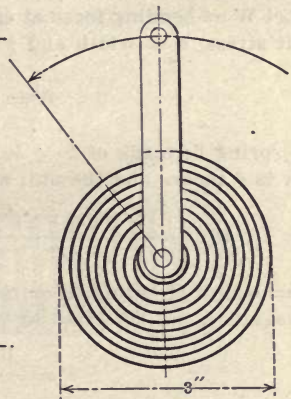

Fig.12

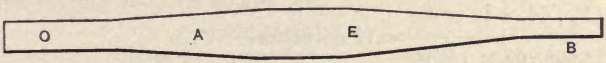

Fig.11 *Machinery,N.Y.*

Figs. 10 to 12

each of these, be a small deflection at *B* proportional to the square of its perpendicular distance from *B*. The horizontal deflection at *F* will be as if that element had been at *O*. But as the curve of the spring is longer than the straight line from *A* to *B*, and has a correspondingly greater number of elements, the entire horizontal deflection at *B* will be greater than that of a straight spring fixed at *A*. For a modulus of elasticity of 42,000,000, the deflection at *B* is about 1¼ inch, or about three times the deflection of a straight spring 4 inches long. A similar spring of the same thickness and width, but twice as large, would only carry 11 pounds, but it would deflect $2^2 \times 1\frac{1}{4} = 5$ inches under that load. Generally, for the same thickness and same unit stress the bending deflection of similar springs of this type varies as the square of their lengths.

It will be noticed that the bending moment for different parts of this spring varies considerably, while the moment of resistance is constant.

At *A* the lever arm is greatest and the unit stress is there at the safe limit, but at other points the spring will be stiffer than necessary; we may therefore improve it by varying the width in proportion to the bending moments; for then the same unit stress is obtained at any point of the length, whereby the deflection is increased without reducing the strength. Fig. 11 shows the spring when straightened out and shaped so as to give a nearly constant unit stress. This will make the deflection at *B* about one-third greater.

A great deal of potential energy may be stored in a small space by coiling a strip of steel like the main spring of a watch. If the ends are fixed and guided concentrically, the moment of bending will be constant for the whole length; and as the spring can be very long, it may be very efficient in a limited space. Fig. 12 represents a spring of this kind. Let W = bending force at end of the lever, R = length of lever, S = unit stress, b = width and t = thickness of spring; then

$$W = \frac{Sbt^2}{6R}$$

If the spring be made of $1 \times \frac{1}{8}$ inch spring steel and the length of the lever is 6 inches and the unit stress is 96,000 pounds, then,

$$W = \frac{96{,}000 \times (\frac{1}{8})^2}{6 \times 6} = 42 \text{ pounds, nearly.}$$

Let l = length of spring, E = modulus of elasticity, and F = deflection or length of arc described by the end of the lever; then,

$$F = \frac{12\,lWR^2}{Ebt^3}$$

$E = 28{,}000{,}000$ and l = 56 inches gives

$$F = \frac{12 \times 56 \times 42 \times 6^2}{28{,}000{,}000 \times (\frac{1}{8})^3} = 18\tfrac{1}{2} \text{ inches.}$$

Hence the lever turns nearly one-half of a revolution. This result may be found more directly from the formula

$$U = \frac{Sl}{\pi Et}$$

where U is the deflection expressed in revolutions and $\pi = 3.1416$. By substitution as above,

$$U = \frac{96{,}000 \times 56 \times 8}{\pi \times 28{,}000{,}000} = 0.49,$$

or nearly one-half revolution, which agrees with the former result. From this formula it appears that the deflection for a given unit stress varies directly as the length and inversely as the thickness, and is independent of the width of the spring. If we had this spring 1/16 inch thick, the lever could be turned nearly a whole revolution, but the force would be only $10\frac{1}{2}$ pounds. If we then had twice as many turns in the spiral, the lever would turn nearly two revolutions before the limit of stress would be reached. Such springs may also be useful when a nearly constant pressure through a shorter motion is desired, for this can be obtained by a considerable initial deflection. The great efficiency of watch springs is due to the high elastic limit and careful treatment of the steel.

It is sometimes preferable to coil the spring in a screw-line, as shown in Fig. 13. As in the former case, the motion is supposed to be about a fixed center, and the same formulas may be used in both cases. Let there be 72 inches of ¼ inch square spring steel, and let the lever be 3 inches long, then,

$$W = \frac{96{,}000 \times (\tfrac{1}{4})^2 \times \tfrac{1}{4}}{6 \times 3} = 83 \text{ pounds.}$$

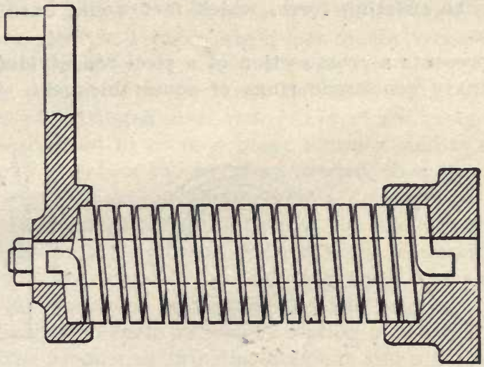

Fig. 13

We have further for this spring

$$U = \frac{96{,}000 \times 72 \times 4}{\pi \times 28{,}000{,}000} = 0.31,$$

or about 5/16 of one revolution of the lever, which is the maximum allowable motion for a unit stress of 96,000 pounds.

For round steel $W = \frac{S\pi t^3}{32\,R} = \frac{St^3}{10\,R}$, nearly. Therefore if this spring is made of ¼-inch round steel, then,

$$W = \frac{96{,}000}{10 \times 3 \times 4^3} = 50 \text{ pounds.}$$

Round steel has only 3/5 of the strength of square steel of the same diameter under bending action, but the value of U is the same in both cases.

The various springs treated of here are all of uniform thickness throughout their entire length. Good results may also be obtained by varying the thickness of a spring so as to correspond with a variable bending moment; but as such springs cannot be rolled to shape and can only receive the correct shape by skillful hand work, they are used very little. The forging down of the ends of flat springs is a simple matter and is often done. It improves the appearance of leaf springs and is preferable to blunt ends.

Torsional Springs

What really happens to the molecules of a bar when it is twisted within the elastic limit is a matter of conjecture, but all formulas for strength and deflection of torsional springs are based upon the assumption that the molecules receive a sort of lateral or sliding displacement, as if subjected to a shearing action. Whether or not this assumption is correct, it is certainly supported by experimental results. It is, for instance, known that the angle of deflection is directly proportional to the twisting force, which fact would hardly agree with other theories.

Fig. 15 represents a cross-section of a steel rod divided into a number of imaginary concentric rings of equal thickness. The torsional

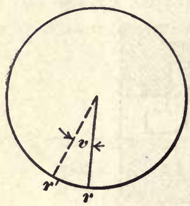

Fig. 14

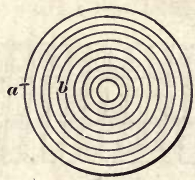

Fig. 15

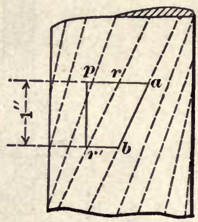

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Fig. 16

strength of this rod depends on the resistance to shearing of the rings and on their respective distances from the center, which are their leverages of resistance. Ring *a* is twice as large as ring *b* and is twice as far removed from the center, and offers, therefore, $2 \times 2 = 2^2 = 4$ times the resistance to a twisting force. Suppose we have another rod twice as large in diameter, and divided it into the same number of rings, then each ring will be twice as thick, twice as long and twice as far removed from the center as the corresponding ring of the first rod; the torsional resistance will, therefore, be $2 \times 2 \times 2 = 2^3 = 8$ times that of the first rod, provided the resistance per unit area is the same in both cases. In other words, by increasing the diameter of the rod we increase both the thickness, length and leverage of resistance of the rings in the same proportion. The torsional strength, therefore, is proportional to the third power of the diameter.

The formulas for torsional strength are,

$$\text{For round bars } RW = \frac{\pi}{16} Zd^3 = \frac{1}{5} Zd^3, \text{ nearly.} \qquad (1)$$

$$\text{For square bars } RW = \frac{1}{3\sqrt{2}} Zd^3 = \frac{1}{4} Zd^3, \text{ nearly.} \qquad (2)$$

in which W = twisting force in pounds,

R = lever arm in inches,

Z = shearing unit stress of outside ring, in pounds,

d = diameter or size of bar, in inches,

For tool steel we may put $Z = 80{,}000$ pounds, and the moment of resistance of round steel $= 1/5 \times 80{,}000\ d^3 = 16{,}000\ d^3$. This gives for a ½-inch rod, the safe moment of resistance $= (½)^3 \times 16{,}000 = 2000$. If we twist the rod with a 6-inch lever, the safe load on the end of the lever $= \frac{2000}{6} = 333$ pounds. A ⅝-inch rod would carry $\frac{16{,}000}{6} \times (⅝)^3 = 650$ pounds on the end of a 6-inch lever. It will be noticed that a small increase of diameter greatly increases the strength, and that square steel will carry about one-fourth more than round steel of the same diameter.

We will now consider the torsional deflection. Fig. 14 is an end view or section of a twisted steel rod, r and r' are imaginary radial lines, and r is supposed to be in a plane above r' and is supposed to have just covered r' before the rod was twisted, that is, a small particle directly over r' is moved horizontally a distance $r'r$ through an angle v. Fig. 16 is an elevation of part of the rod where the dotted lines indicate the twisting of the surface much exaggerated. The planes of r and r' are supposed to be 1 inch apart and rp represents the transverse displacement of a small particle originally at p. The maximum unit stress in each transverse section of the rod is supposed to be equal to the product of this displacement and a certain constant multiplier. If the material be tool steel and $Z = 80{,}000$, the distance pr is about 1/150 inch. It varies directly as Z, and is independent of the diameter of the rod. The multiplier is in this case 12,000,000, which, according to our hypothesis, is a constant for tool steel. It is a purely hypothetical quantity, which bears no rational relation to the modulus of elasticity of the material, but we may call it the torsional modulus of elasticity, because it takes the same place in the calculation of torsional deflection as the modulus of elasticity takes in the calculation of bending deflection. It will be seen that a rectangular area in the surface of the rod becomes a rhomboid when the rod is twisted. Area $rr'ba$ is a rhomboid, or deformed rectangle; suppose that pr' represents the unit of length, and let distance pr be the displacement caused for a torsional unit stress of one pound at the surface of the rod, then this displacement becomes the modulus or measure of deformation, which is the reciprocal of the torsional modulus of elasticity; but it will be readily inferred that such deformation does not produce a lateral or shearing stress, as if the surface had been stretched lengthwise of the rod a distance equal to pr, and that the torsional modulus of elasticity must be considerably less than the modulus of elasticity for bending. We have seen that for a given maximum unit stress Z, the moment of torsional resistance varies as the third power of the diameter; but without this limitation of stress the mean unit stress for any given angular deflection varies directly as the diameter of the rod, and under this condition the moment of resistance, therefore, becomes proportional to the fourth power of the diameter; and the deflection will be inversely proportional to this. That the entire angle of deflection must be proportional to the length of the rod requires no demonstration. It is also directly proportional to the load.

The following are convenient formulas for torsional deflection:

For round steel

$$F = \frac{32\,WR^2l}{\pi G d^4} = \frac{10\,WR^2l}{Gd^4}, \text{ nearly.} \qquad (3)$$

$$F = \frac{2ZlR}{Gd} \qquad (4)$$

For square steel

$$F = \frac{6\,WR^2l}{Gd^4} \qquad (5)$$

$$F = \frac{\sqrt{2}ZlR}{Gd} \qquad (6)$$

in which F = linear deflection at end of lever,
W = twisting force at end of lever,
R = length of lever,
l = length of rod,
G = torsional modulus of elasticity,
Z = unit shearing stress in periphery of cross-section,
d = diameter of rod.

For spring steel $G = 12{,}000{,}000$ is a nearly correct mean value. The proper value of Z depends on the working conditions. A spring that is continually working should be strained less than one whose action is intermittent or irregular; and it should be observed that shearing resistance at the elastic limit is somewhat less than tensile strength at the same limit. $Z = 80{,}000$ is probably not too much, unless the spring is continually working to its full capacity. But when the construction and circumstances are such as to admit of a lower stress, it is always preferable.

As a simple example of torsional springs, take a rod of ½-inch round steel 3 feet long, fixed solidly at one end, and the other end so guided as to prevent lateral motion, and let there be a 6-inch lever keyed to this end. How much will it be safe to load the end of the lever if the rod is twisted 100 times a minute? The rod is not supposed to be hardened, and though its ultimate strength is considerable, the elastic limit is comparatively low. Let $Z = 30{,}000$ and $E = 12{,}000{,}000$. Then, substituting in Formula (1) we have:

$$6\,W = 1/5 \times 30{,}000 \times (\tfrac{1}{2})^3 = \frac{6000}{8}, \text{ and } W = \frac{1000}{8} = 125 \text{ pounds}$$

which is the admissible force on the end of the lever. The deflection for this force can easily be found from Formula (4), because the value of

Z is known. We have

$$F = \frac{2 \times 30{,}000 \times 36 \times 6 \times 2}{12{,}000{,}000} = 2.16 \text{ inches.}$$

If this rod were of hardened steel we might put $Z = 70{,}000$, and would then have $W = 125 \times 7/3 = 292$ pounds, and $F = 2.16 \times 7/3 = 5.04$ inches.

Steel used for springs should have a high elastic limit and preferably a low modulus of elasticity, for the deflection is proportional to the quotient $\frac{Z}{G}$ and the greater efficiency of torsional springs is due to the smaller modulus of elasticity, as compared with that of bending. For the same unit stress at the surface of the rod the angular deflection will vary inversely as the diameter, which is an important rule easy to remember. But for the same load and varying diameters the deflection varies inversely as the fourth power of the diameter. The torsional deflection of a 5/8-inch rod, for instance, would only be about 2/5 of that of a 1/2-inch rod under the same load.

Helical Springs

The rod would in many cases have to be very long to give the desired deflection, and a straight rod would therefore often be impracticable; but fortunately it can be bent so as to make a comparatively short spring, easy to make and easy to harden. This is obtained by bending it in the form of a cylindrical helix, or screw-line, as shown in Figs. 17 and 18. One of these springs will be compressed and the other will be stretched, but the former may, by a slight change in the connections, be used both ways. These are true torsional springs, though it may not appear so at first sight. The following analogous case will explain it. Fig. 19 shows an open ring of steel wire firmly fixed and supported at A, and a radial lever firmly attached to the free end at B. A pressure exerted on this lever at the center of the ring perpendicular to its plane will twist the wire while it pushes point B back. This will be better understood by reference to the bent wire, shown in the dotted line. At a point N is drawn a tangent and from C a perpendicular CM. There will be a bending moment at N represented by line MN and a twisting moment represented by line CM; but when the curve becomes a circle with center at C the bending moment disappears and there is nothing but a twisting moment left, and this twisting moment is constant for any part of the concentric ring. We see that when the rod is coiled, the twisting lever is equal to the mean radius, and the deflection will be in line with the axis of the helix. The helical form is compact, and the weight of a helical spring of round steel is only about 5/12 of that of a leaf spring of the same capacity.

In the following are given a number of formulas for helical springs. Calculated values based on these formulas are given in MACHINERY'S Data Sheet Book No. 9, pages 8 to 11, inclusive.

The following formulas apply to helical springs:

For round steel

$$W = \frac{40\,Zd^3}{100\,(D-d)} \quad (7)$$

$$F = \frac{8\,W\,(D-d)^3}{Gd^4} \quad (8)$$

$$F = \frac{314\,Z\,(D-d)^2}{100\,Gd} \quad (9)$$

For square steel

$$W = \frac{47\,Zd^3}{100\,(D-d)} \quad (10)$$

$$F = \frac{47\,W\,(D-d)^3}{10\,Gd^4} \quad (11)$$

$$F = \frac{222\,Z\,(D-d)^2}{100\,Gd} \quad (12)$$

In these formulas F is the deflection of one coil, and D is the outside diameter of the coil, and the meaning of the other letters is the same as in Formulas (1) to (6). It appears from these formulas that square steel is about 17 per cent stronger than round steel, but for the same unit stress the deflection of square steel is about 30 per cent less. Round steel is, therefore, better adapted to helical springs. This may easily be perceived without any calculation, considering that when square steel is twisted, the corners cannot add very much to the strength on account of the smallness of their areas, which terminate in four points; but these points, being furthest removed from the center, will take the greatest strain, and will limit the angle of deflection as much as a full circle including the points, would do.

Fig. 18 shows a car spring of, say, 1-inch round steel, 5 inches outside diameter. How much will it carry? It must not close under the maximum static load, but it may close entirely by the jolting of the car, and we will therefore put $Z = 50{,}000$ pounds for the maximum static load, assuming the elastic limit to be above 100,000 pounds unit stress. Substituting these values in Formula (7) we have:

$$W = \frac{40 \times 50{,}000}{100 \times 4} = 5000 \text{ pounds,}$$

and assuming $Z = 100{,}000$ pounds when the spring is entirely closed, we have from Formula (9):

$$F = \frac{314 \times 100{,}000 \times 16}{100 \times 12{,}000{,}000} = 7/16 \text{ inch, nearly.}$$

That is, the coils should be 7/16 inch apart without load, and they will be 7/32 inch apart under maximum load.

The spring in Fig. 17 is, say, 3 inches in diameter and is made of ½-inch round steel, and there are 24 coils. How much may this spring be extended if used on a shaft governor? As its work is intermittent, and as it very seldom is fully extended, we may put $Z = 70,000$, and we have from Formula (9):

$$F = \frac{314 \times 70,000 \times (2\frac{1}{2})^2}{100 \times 12,000,000 \times \frac{1}{2}} = 0.23 \text{ inch.}$$

which is the allowable deflection of one coil, and $0.23 \times 24 = 5\frac{1}{2}$ inches is, therefore, the safe extension of this spring. From Formula (7) we find the maximum load to be 1400 pounds. Closed coil springs, as represented by Fig. 17, are sometimes distinguished by a considerable initial tension; that is, it takes some initial force to separate the coils, and the elongation cannot be calculated from the above formulas. The probabilities are that they are made from cold rolled wire, un-

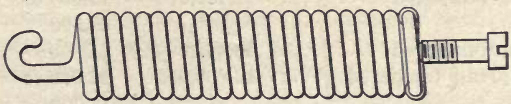

Fig.17

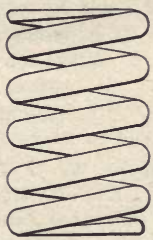

Fig.18

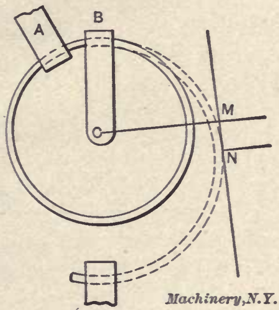

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Fig.19

Figs. 17 to 19

tempered, for the initial tension would be removed by the process of tempering. Such springs are easily distinguishable by their resistance to bending before they are stretched.

It will be noticed that in the calculations of springs the supposed elastic limit is approached closer than would be judicious in the calculation of other machine parts; but the results agree with the average common practice, and there are several reasons why this is so. In the first place, springs are made of tool steel of moderate dimensions, which is a most reliable material. In the second place, the form is such that no part can be subjected to unexpected or unaccountable strains, and on account of their great elasticity springs do not suffer materially by shocks or blows.

There seems to be considerable uncertainty or lack of knowledge as to the proper modulus of elasticity of hardened steel. The comparatively small demand for such knowledge except for the calculation of springs is a probable reason for its scarcity. According to various tests the modulus of elasticity of untempered steel is from 28,000,000 to 32,000,000, and it appears from calculations of bending and twisting deflection of ordinary springs that the modulus of elasticity is not increased by tempering. Still it will hardly do to overlook the figures given by Reuleaux, which appear in his "Constructor." His figures for the elastic limit and ultimate tensile strength are also interesting. In the heading, he states that the figures are mean values of numerous experiments by various experimenters on materials of different make, and in actual use.

	Modulus of Elasticity	Elastic Limit	Ultimate Tensile Strength, Pounds per Square Inch
Spring steel, tempered	28,440,000	71,000 to 99,500	113,700
Tool steel, untempered	28,440,000	35,500	113,700
Tool steel, spring tempered	42,600,000	92,000 to 213,000	142,000

CHAPTER II

THE DESIGN OF HEAVY HELICAL SPRINGS*

A spring is usually specified by three dimensions, although some specifications complete the design by a fourth. The dimensions usually given are the outside diameter, free height, and diameter of bar. The fourth dimension, the solid height, is not generally given, so that the actual design of the spring is really left to the manufacturer. In some cases the number of coils or "rings" is specified, but this should never be done, as a tapered coil may be considered by one as a full coil and by another as a partial coil, thus causing confusion.

Investigation of such formulas as are found in the general text-books, hand-books, and books of reference, indicates the need of more direct formulas to facilitate the design of springs. It is the writer's intention to present the derivation of such formulas with parallel examples, showing the ease of application. For this purpose we adopt the following notation:

d = diameter of bar,
D = mean diameter of coil,
f = total deflection,
h = solid height,
H = free height,
L = blunt length of bar,
W = weight of bar, or spring,
P = capacity of coil,
P_1 = any load less than capacity,
h_1 = height of coil under load P_1,
S = maximum fiber stress,
G = torsional modulus,
w = weight of steel per cubic inch.

Only round bar coils will be considered.

I. Length of Bar when Solid Height is Given

$$\text{Total number of coils} = \frac{L}{\pi D}.$$

$$\text{Total number of coils} = \frac{h}{d}.$$

Hence,

$$\frac{L}{\pi D} = \frac{h}{d}.$$

$$L = \pi \left(\frac{D}{d}\right) h = 3.1416 \left(\frac{D}{d}\right) h$$

* MACHINERY, January, 1910, Railway Edition.

Example: Outside diameter = 4⅜ inches,
Bar = 7/16 inch,
Solid height = 10 inches.

$$L = 3.1416 \times \left(\frac{3\frac{15}{16}}{\frac{7}{16}}\right) \times 10 = 282.74 \text{ inches.}$$

II. Deflection when Solid Height is Given

Fundamentally, as given in most text-books,

$$f = \frac{LDS}{Gd}.$$

But

$$L = \pi\left(\frac{D}{d}\right)h$$

Hence,

$$f = \frac{\pi S}{G}\left(\frac{D}{d}\right)^2 h.$$

Or, for steel springs,

$$f = 0.019946\left(\frac{D}{d}\right)^2 h$$

Example: Outside diameter = 4¼ inches,
Diameter of bar = ¾ inch,
Solid height = 10 inches.

$$f = 0.019946\left(\frac{3\frac{1}{2}}{\frac{3}{4}}\right)^2 \times 10 = 4.34 \text{ inches.}$$

III. Ratio between Free and Solid Heights

$$H = h + f$$

$$f = \frac{\pi S}{G}\left(\frac{D}{d}\right)^2 h$$

Hence,

$$H = h + \frac{\pi S}{G}\left(\frac{D}{d}\right)^2 h$$

$$H = \left[1 + \frac{\pi S}{G}\left(\frac{D}{d}\right)^2\right] h$$

Or, for steel springs,

$$H = \left[1 + 0.019946\left(\frac{D}{d}\right)^2\right] h$$

and

$$h = \frac{H}{1 + 0.019946\left(\frac{D}{d}\right)^2}$$

Example 1: Outside diameter = 6 inches,
Diameter of bar = $1\frac{1}{8}$ inch,
Free height = $13\frac{3}{4}$ inches.
Find solid height h.

$$h = \frac{13.75}{1 + 0.019946 \left(\frac{4\frac{7}{8}}{1\frac{1}{8}}\right)^2} = 10 \text{ inches.}$$

Example 2: Outside diameter = $7\frac{1}{8}$ inches,
Diameter of bar = $1\frac{1}{8}$ inch,
Solid height = 10 inches.
Find free height H.

$$H = \left[1 + 0.019946 \left(\frac{6}{1\frac{1}{8}}\right)^2\right] \times 10 = 15.67 \text{ inches.}$$

IV. Deflection when only Free Height is Given

$$f = \frac{\pi S}{G}\left(\frac{D}{d}\right)^2 h$$

But

$$h = \frac{H}{1 + \frac{\pi S}{G}\left(\frac{D}{d}\right)^2}$$

Hence,

$$f = \frac{\frac{G}{\pi S}\left(\frac{D}{d}\right)^2 H}{1 + \frac{\pi S}{G}\left(\frac{D}{d}\right)^2}$$

$$f = \frac{H}{1 + \frac{G}{\pi S}\left(\frac{d}{D}\right)^2}$$

Or, for steel springs,

$$f = \frac{H}{1 + 50.1337\left(\frac{d}{D}\right)^2}$$

Example: Outside diameter = $5\frac{1}{2}$ inches,
Diameter of bar = $1\frac{3}{8}$ inch,
Free height = $11\frac{3}{4}$ inches.

$$f = \frac{11\frac{3}{4}}{1 + 50.1337\left(\frac{1\frac{3}{8}}{4\frac{1}{8}}\right)^2} = 1\frac{3}{4} \text{ very nearly.}$$

V. Weight when Solid Height is Given

$$\text{Area of cross section} = \frac{\pi d^2}{4}.$$

$$\text{Cubical contents of bar} = \frac{L \pi d^2}{4}.$$

$$\text{Then } W = \frac{L \pi d^2 w}{4}$$

$$\text{But } L = \pi \left(\frac{D}{d}\right) h$$

$$\text{Hence, } W = \frac{\pi^2 w}{4} d D h$$

For steel springs, where one cubic foot of steel weighs 486.6 pounds,

$$W = 0.694\ dDh.$$

Example: Outside diameter = 3¾ inches,
Diameter of bar = 15/16 inch,
Solid height = 10 inches.

$$W = 0.694 \times \frac{15}{16} \times 2\frac{13}{16} \times 10 = 18.3 \text{ pounds.}$$

VI. When Free and Solid Heights are Given to Determine Stress

$$h = \frac{H}{1 + \frac{\pi S}{G}\left(\frac{D}{d}\right)^2}$$

$$S = \frac{(H - h)\, G}{\pi h} \times \left(\frac{d}{D}\right)^2$$

$$S = \frac{Gf}{\pi h} \times \left(\frac{d}{D}\right)^2$$

For steel springs,

$$S = 4{,}010{,}700 \frac{f}{h}\left(\frac{d}{D}\right)^2$$

Example: Outside diameter = 4½ inches,
Diameter of bar = ½ inch,
Free height = 22¾ inches,
Solid height = 10 inches.

$$S = 4{,}010{,}700 \times \frac{12.75}{10}\left(\frac{0.5}{4}\right)^2 = 80{,}000 \text{ pounds.}$$

VII. When Free and Solid Heights are Given to Determine Capacity

$$P = \frac{S \pi d^3}{8 D}$$

and

$$S = \frac{G f}{\pi h} \left(\frac{d}{D}\right)^2$$

Hence,

$$P = \frac{G f d^5}{8 h D^3}$$

For steel springs,

$$P = 1{,}575{,}000 \frac{f d^5}{h D^3}$$

Example: Outside diameter = 2⅞ inches,
Diameter of bar = ½ inch,
Free height = 14½ inches,
Solid height = 10 inches.

$$P = 1{,}575{,}000 \times \frac{4.5 \times 0.5^5}{10 \times 2.375^3} = 1653 \text{ pounds.}$$

These last two formulas are very useful in ascertaining the stresses and loads of the separate coils of double and triple coil springs.

VIII. Given Free Height, Diameter of Spring and Bar, and Load Carried at Given Height. To Find Proper Solid Height

$$\frac{P_1}{P} = \frac{f_1}{f}$$

$$H = f + h$$

$$H = f_1 + h_1$$

Hence, $f_1 = f + h - h_1$

Then $P\,(f + h - h_1) = P_1 f$

Hence $h = \dfrac{P_1 f - P f + P h_1}{P}$

$$h = \frac{P_1 - P}{P} \times f + h_1$$

But $f = \dfrac{\pi S}{G} \left(\dfrac{D}{d}\right)^2 h$

Hence,

$$h = \frac{\pi S}{G} \left(\frac{D}{d}\right)^2 \left(\frac{P_1 - P}{P}\right) h + h_1$$

$$h = \frac{h_1}{1 + \dfrac{\pi S}{G}\left(\dfrac{P-P_1}{P}\right)\left(\dfrac{D}{d}\right)^2}$$

For steel springs,

$$h = \frac{h_1}{1 + 0.019946\left(\dfrac{P-P_1}{P}\right)\left(\dfrac{D}{d}\right)^2}$$

Example: Outside diameter = 5½ inches,
Diameter of bar = ¾ inch,
Free height = 18 inches.

What solid height is required for carrying 1395 pounds at 14 inches?

$P = 2790$ pounds by formula $P = \dfrac{S \pi d^3}{8 D}$.

Then,

$$h = \frac{14}{1 + 0.019946\left(\dfrac{2790-1395}{2790}\right)\left(\dfrac{4\frac{3}{4}}{\frac{3}{4}}\right)^2} = 10 \text{ inches.}$$

IX. To Determine the Quality of the Steel

The value of G is the index to the quality of the steel, and upon this value depend all properties of the spring. By transposing either the formula given in (VII) for capacity, or that for load, we find a method for ascertaining this value, *i. e.:*

$$G = \pi S \frac{h}{f}\left(\frac{D}{d}\right)^2$$

or

$$G = 8 P \frac{h D^3}{f d^5}$$

Example: Outside diameter = 4⅞ inches,
Diameter of bar = 11/16 inch,
Load = 1219 pounds,
Deflection = 3.7 inches,
Solid height = 10 inches.

$$G = 8 \times 1219 \times \frac{10 \times (4\frac{3}{16})^3}{3.7 \times (\frac{11}{16})^5} = 12{,}600{,}000.$$

General Remarks

Concentric coils, as shown in Fig. 21, are made generally of the same free and solid heights. Presuming that such coils are all made of the same quality of steel, the ratio of $\dfrac{D}{d}$ should be the same throughout, for the formula in (II) clearly shows that this is necessary to obtain equal stresses in all coils.

The formula in (I) shows that after all values of $\frac{D}{d}$ are made the same, the lengths of all bars will be the same before tapering. A study of all the formulas reveals the fact that the ratio of $\frac{D}{d}$ determines everything; this ratio might well be called the *spring index.*

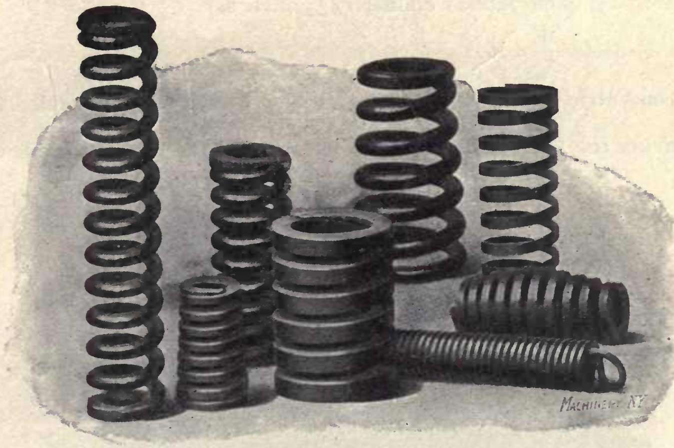

Fig. 20. Types of Coil Springs for Railroad Cars

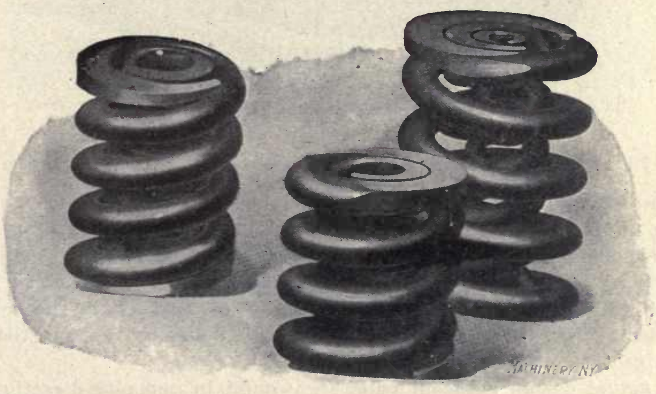

Fig. 21. Concentric Coil Springs for Railroad Cars

The absolutely perfect design of concentric springs is seldom possible where a scale of sixteenths inch for dimensions is used, with the customary one-eighth inch between inside diameter of one spring and outside diameter of the next. As cases of perfect design, however, the following springs are given as examples:

Spring No. 1

Outer: 5 inches outside diameter, 15/16 inch bar.
Inner: 3 inches outside diameter, 9/16 inch bar.

In this design $\frac{D}{d} = 4\ 1/3$.

Spring No. 2

Outer: 2⅝ inches outside diameter, ⅜ inch bar.
Inner: 1¾ inch outside diameter, ¼ inch bar.

In this design $\frac{D}{d} = 6$.

In concentric coil springs where perfect design is impossible, the coil having the least value of $\frac{D}{d}$ will be stressed the highest, as shown

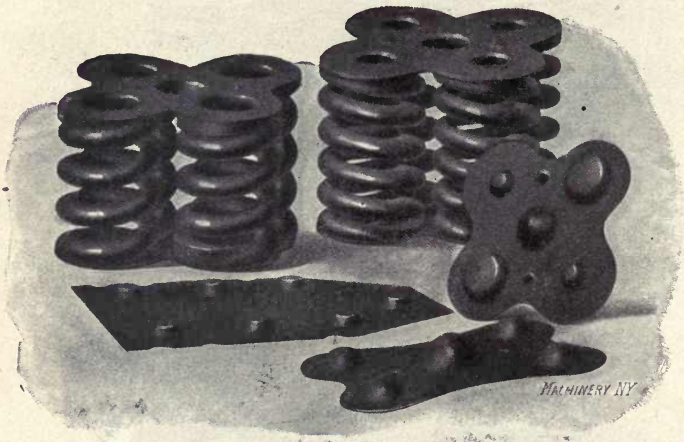

Fig. 22. Groups of Coil Springs held together by Plates at Top and Bottom

by the formula in (VI); this coil may therefore be called the governing coil, inasmuch as the motion, or deflection, of the spring as a whole depends upon this coil. To estimate the capacity of such concentric coils we have recourse to the formula in (VII), while the formula in (VI) shows the separate stresses. The load which the concentric spring will carry at any height is then found by the fact that all loads are proportional to deflection.

In actual design adjacent coils are wound in opposite directions to prevent binding, as shown in Fig. 21. Instead of using concentric coils, groups of similar coils are sometimes used which are held together by pressed steel or cast spring-plates, as shown in Fig. 22. It is customary to suspend the static load at one-half the deflection.

A helical spring for railroad service is almost invariably made of round bar spring steel. The analysis of spring steel most frequently used is known as P. R. R. analysis, and its composition is as follows:

Carbon, 1.0 per cent (not under 0.90 per cent); phosphorous, 0.05 per cent (not over 0.07 per cent); manganese, 0.25 per cent (not over 0.50 per cent); silicon, not over 0.10 per cent; sulphur, not over 0.03 per cent.

For spring steel of this character the maximum fiber stress should not be over 80,000 pounds per square inch, and the torsional modulus should be taken as 12,600,000 pounds.

CHAPTER III

THE DESIGN OF ELLIPTIC SPRINGS*

It is doubtful if scientific calculations ever entered into the design of the original forms of such springs as are used under ordinary road carriages. Satisfactory as they are, they are not engineering results, but accepted standards born long ago of the cut-and-try methods of the blacksmith shop. Their manufacture belongs to such arts as are taught by father to son, or acquired through years of experience, during which have been gathered the "tricks of the trade." The manufacturer of this class of springs does not attempt to arrive at results by mathematics. He has learned as a part of his trade that certain styles of carriages should have certain springs.

Sufficient time did not exist during the development of railroad cars for a gradual development of definite types of springs for various types of cars. It devolved, therefore, upon the engineer to design these springs; but as soon as the spring maker found that the 70,000-, 80,000-, and 100,000-pound capacity car each had its own peculiar set of springs, and that any car could be fitted with springs according to its capacity, he adopted the engineer's designs as another class of standards. Railroad cars, while resting on springs whose dimensions were originally scientifically estimated, are now, therefore, suspended largely upon springs belonging to a few fixed classes.

With the advent of the automobile came a carriage traveling fast over uneven country roads, meeting severe usage in inexperienced hands, and demanding the extreme of comfort and safety. The question of springs and spring suspension thus becomes of primary importance, so that in these carriages each particular design requires a specially designed suspension. Automobile springs are fundamentally cantilevers, the same as all leaf springs. This class of springs more readily lends itself to an easy vibration, as well as to a better general design of the machine. It is possible to carry a load on a narrow-leafed elliptic leaf spring where there would not be room for a helical spring. Also, the addition of a leaf to an elliptic leaf spring adds to its capacity without changing its deflection, while the addition of a coil to a helical spring does not change its capacity but adds to its deflection.

Any leaf spring, tightly banded around the middle, should be considered as composed of two cantilevers of length l, where l is one-half the distance from center to center of the end bearings less one-half the width of the band. The length of each cantilever is then expressed (see Fig. 24):

$$l = \frac{c - w}{2}$$

* MACHINERY, January, 1910, Engineering Edition.

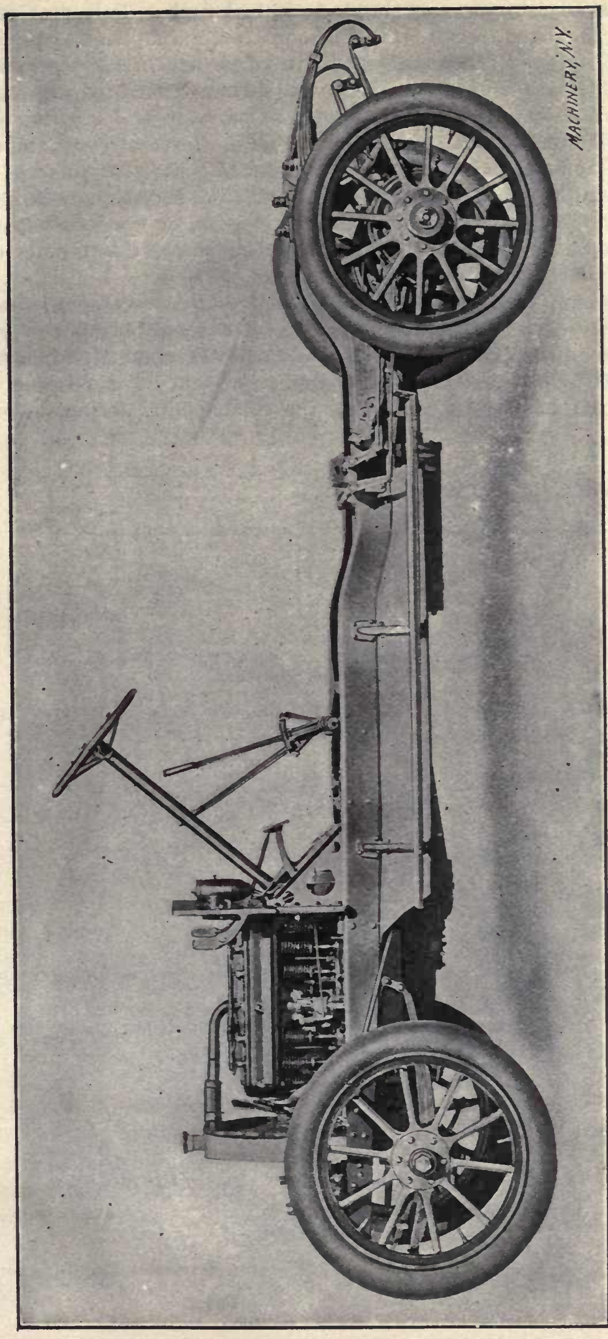

Fig. 23. Chassis of an F. B. Stearns Motor Car, showing Three-quarter Elliptic Spring Suspension in Rear and Semi-elliptic Springs in Front

To consider a spring as a simple beam of length c, is to overlook the effect of the band. It is easily demonstrated that variations in the width of the band cause corresponding variations in the strength and deflection of the spring. The elliptic spring, graduated throughout, with but *one* leaf in each section extending from end bearing to end bearing, is fundamentally a cantilever of *uniform strength;* and the formulas applicable are based on the fundamental formulas of that type of cantilever. An elliptic spring with *all* leaves in each section extending from end bearing to end bearing is, on the other hand, a cantilever of *uniform* section, and the formulas for this type of cantilever are then applicable.

The springs used in automobile practice are frequently combinations of these two forms, inasmuch as a considerable portion of the leaves extend the full length from bearing to bearing. It follows that neither of the above formulas will apply, but that the applicable formulas may be derived by combining the fundamental formulas for the

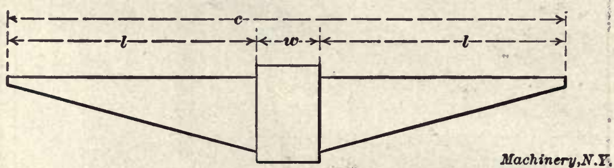

Fig. 24. Diagrammatical Sketch of Graduated Spring, giving Length Notation used in Formulas

two types of cantilevers. The load capacity of a cantilever is not affected by its form, for in either case:

$$P = \frac{S\,b\,h^2}{6\,l}$$

in which P = load,
S = allowable stress,
b = width of beam,
h = thickness of beam,
l = length of cantilever.

In other words, the load capacity is equal for like conditions, such as stress, size of beam, and length of span.

A great difference exists, however, in the deflections under the same load, one being fifty per cent more than the other:

$$f = \frac{4\,P\,l^3}{E\,b\,h^3}, \text{ for uniform section cantilevers,}$$

$$f = \frac{6\,P\,l^3}{E\,b\,h^3}, \text{ for uniform strength cantilevers,*}$$

in which f = deflection, and E = modulus of elasticity.

* The formula given is that for a cantilever of uniform strength, where the height h is uniform, but the width of the section of the cantilever decreases towards the outer end; b is the width at the support.

When such a difference as this exists, it is rather remarkable that many engineers calculate the properties of an elliptic spring no matter what the cantilever conditions, as though all elliptic springs were subject to the same rules and formulas; but, as a matter of fact, the proportion of back leaves, or the leaves on the longer side of the spring which commonly extends the full length, ranges from 5 to 50 per cent of the total number of leaves. It is not unusual to see attempts made through actual tests of the springs themselves to find the proper constant with which to modify the uniform strength equations so as to render them applicable to springs composed of uniform section cantilevers in combination with uniform strength cantilevers. The desired modifier, however, is a variable quantity, depending upon the relative size of the fundamental spring elements.

Lack of due consideration of this combination of different cantilevers accounts also for the different and conflicting formulas which various

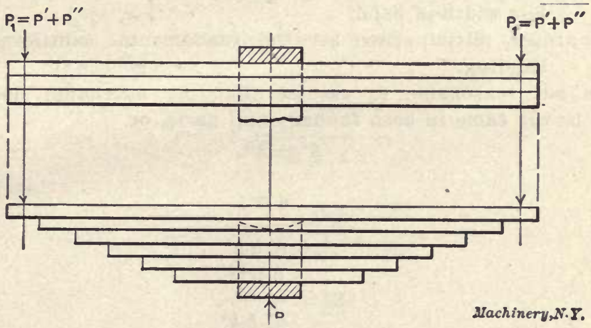

Fig. 25. Showing Division of Spring into Cantilevers of Uniform Section (Upper Portion) and Cantilevers of Uniform Strength (Lower Portion). One of the Full Length Leaves should always be considered as a Part of the Graduated Leaves

authorities advance. Thus Goodman, in "Mechanics Applied to Engineering"; Reuleaux, in his "Constructor"; and "Des Ingenieurs Taschenbuch" (Hütte), give formulas all of which reduce to uniform strength cantilevers. Molesworth and the Automotor Pocket Book base their formulas on uniform section cantilevers. Henderson, who assumed all semi-elliptic springs to contain one-fourth full length leaves, and made an approximation of the result, was the first to recognize the influence of the combination of cantilevers.

Deduction of General Formulas

For further consideration we will adopt the following notation, discussing only the semi-elliptic spring:

P = total load on spring,

P_1 = portion of load on one end of spring,

P' = portion of load on one end of full-length leaves, or on uniform section cantilever,

P'' = portion of load on one end of graduated leaves, or on uniform strength cantilever,

n = total number of leaves,
n' = number of full-length leaves,
n'' = number of graduated leaves,

$$r = \frac{n'}{n},$$

S = maximum fiber stress in spring,
S' = maximum fiber stress in full-length leaves,
S'' = maximum fiber stress in graduated leaves,
f = total deflection of banded leaves,
f' = total deflection of full-length leaves if unbanded,
f'' = total deflection of graduated leaves if unbanded,
b = width of leaves,
h = thickness of leaves,
l = length of cantilever,
L = net length of spring, *i. e.*, actual distance between end bearings, less width of band,
x = proper initial space between fundamental cantilevers before banding.

It is but reasonable to assume that the maximum fiber strain should be the same in both fundamental parts, or

$$S' = S''.$$

But

$$S' = \frac{6 P' l}{n' b h^2},$$

$$S'' = \frac{6 P'' l}{n'' b h^2}.$$

Hence,

$$\frac{P'}{P''} = \frac{n'}{n''}.$$

In a well-designed spring there should be, at full load, a division of the work proportional to the respective number of leaves in the two fundamental parts. The fundamental formulas of the two cantilevers have shown, however, that such proportional loads would produce different deflections in their respective carriers. This difference in deflection would cause a separation of the two portions of the spring were they initially together and unbanded. Were they initially together and banded the result would be internal stress under load which would mean that a division of the load proportional to the respective number of leaves in the two fundamental parts could not exist.

It is evident that by placing a space between the two fundamental parts when unloaded and unbanded, equal to the difference between the two deflections, there will result no space between the two fundamental parts at full load ; and hence if banded in this position there will be no internal stress, so that the load on each part will be proportional to the number of leaves in that part. If then the load be removed, it follows that the band alone holds the two portions together and that there must exist a resulting stress upon the band and leaves. Now

$$f' = \frac{4\,P'\,l^3}{E\,n'\,b\,h^3} \qquad (1)$$

and

$$f'' = \frac{6\,P''\,l^3}{E\,n''\,b\,h^3} \qquad (2)$$

But, as shown,

$$\frac{P'}{P''} = \frac{n'}{n''}.$$

or

$$P' = \frac{n'\,P''}{n''}$$

Hence $f' = \dfrac{4\,P''\,l^3}{E\,n''\,b\,h^3}$, as derived by substituting in (1).

Hence,

$$f'' - f' = \frac{2\,P''\,l^3}{E\,n''\,b\,h^3}.$$

Also, since

$$\frac{P'}{n'} = \frac{P''}{n''} = \frac{P_1}{n} = \frac{P}{2\,n},$$

we have

$$f'' - f' = \frac{P\,l^3}{E\,n\,b\,h^3}.$$

Also since

$$l = \frac{L}{2},$$

$$f'' - f' = \frac{P\,L^3}{8\,E\,n\,b\,h^3}$$

or

$$x = \frac{P\,L^3}{8\,E\,n\,b\,h^3}$$

This last expression is then a general expression of the proper initial distance between the two fundamental portions before banding, expressed in terms of total load on spring, total number of leaves in spring, and net span of spring. To find the actual working deflection of the entire spring it is only necessary now to ascertain how much either portion is deflected by the process of bending. For this purpose let us adopt the following notation:

P_x = force exerted by band,
f_x' = deflection of full-length leaves caused by band,
f_x'' = deflection of graduated leaves caused by band.

Then,

$$f_x' = \frac{2 P_x l^3}{E n' b h^3} \text{ and } f_x'' = \frac{3 P_x l^3}{E n'' b h^3}$$

Hence

$$\frac{P_x l^3}{E b h^3} = \frac{f_x' n'}{2} = \frac{f_x'' n''}{3}$$

or

$$f_x' = \frac{2}{3}\left(\frac{1-r}{r}\right) f_x''$$

But

$$f_x' + f_x'' = \frac{P l^3}{E n b h^3}$$

Hence

$$f_x'' + \frac{2}{3}\left(\frac{1-r}{r}\right) f_x'' = \frac{P l^3}{E n b h^3}$$

$$f_x'' = \left(\frac{3r}{2+r}\right) \frac{P l^3}{E n b h^3}$$

But

$$f_x'' = \frac{3 P_x l^3}{E n'' b h^3}$$

Hence

$$\frac{3 P_x l^3}{E n'' b h^3} = \left(\frac{3 r}{2+r}\right) \frac{P l^3}{E n b h^3}$$

or

$$\frac{3 P_x l^3}{E (1-r) n b h^3} = \left(\frac{3 r}{2+r}\right) \frac{P l^3}{E n b h^3}$$

or

$$P_x = \left(\frac{r(1-r)}{2+r}\right) P$$

The expression inside the bracket in the above equation becomes zero for either extreme value of r, as would be expected, the extreme values of r being unity and zero. The formula gives the force exerted by the band, *i. e.*, the load upon the band.

The total deflection of the graduated leaves, as already developed, is,

$$f'' = \frac{3 P l^3}{E n b h^3}$$

The deflection of the graduated leaves, caused by the band, is

$$f_x'' = \left(\frac{3 r}{2+r}\right) \frac{P l^3}{E n b h^3}$$

The difference is, therefore, the deflection left in the graduated leaves after banding, or the general formula sought for the deflection of such a spring:

$$f'' - f_x'' = \left\{ 3 - \left(\frac{3r}{2+r} \right) \right\} \frac{Pl^3}{Enbh^3}$$

or,

$$f = \left(\frac{6}{2+r} \right) \frac{Pl^3}{Enbh^3}$$

or, since $l = \frac{L}{2}$ and

$$P = 2P_1 = 2\left(\frac{Snbh^2}{6l} \right)$$

$$f = \left(\frac{6}{2+r} \right) \left(\frac{2Snbh^2}{3L} \right) \frac{L^3}{8Enbh^3}$$

Hence

$$f = \frac{1}{2(2+r)} \times \frac{SL^2}{Eh}$$

This last expression is then a general formula for the deflection of all semi-elliptic springs. If all the leaves are graduated, $r = 0$, and

$$f = 1/4 \times \frac{SL^2}{Eh}.$$

If all the leaves are full length, $r = 1$, and

$$f = 1/6 \times \frac{SL^2}{Eh}.$$

As was to be expected, the spring composed of all graduated leaves has a deflection, according to the above general formula, 50 per cent above that of a spring composed of all full-length leaves. For values of r above zero, the deflection will be found to decrease until r equals unity.

General Remarks

The general formulas given above were first deduced by the writer in the early part of 1905, at which time they were placed before Prof. C. H. Benjamin,then of the Case School of Applied Science, with a view of making extended experiments for the preparation of a thesis. It was the intention to have springs built with initial space as deduced, and compare the actual deflections of such springs with the estimated deflections. Although these experiments were not carried out, they are mentioned because it is believed that when such experiments are made, they will prove valuable. The deduction of the formulas was published for the first time in MACHINERY, in the January, 1910, issue, engineering edition. This deduction was made in connection with certain springs which were giving very poor service, although designed by the same formulas as other elliptic springs. It was the writer's conclusion that had the springs been built with the proper initial space between the fundamental parts, these springs would not have broken, and that the omission of this space caused an over-stress in the full-length leaves, and an under-stress in the graduated leaves, which caused the over-strained leaves to break, throwing an overload upon the previously under-stressed leaves which also broke when the stress became excessive. This conclusion seems to explain why springs of this type are frequently found with only the long leaves broken; the remaining leaves, all being of one type, divide the resultant overload evenly so that the over-stress is not so excessive. Perhaps the strongest indication of the correctness of the deduction lies in the well-known fact that the percentage of breakage is always much greater with semi-elliptic springs (of the combination type, usually) than with full elliptic springs. Also, it is generally found upon unbanding these springs that no initial space exists.

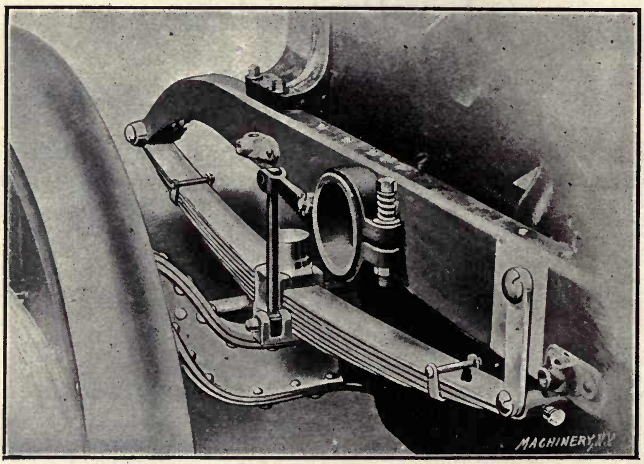

Fig. 26 Front Spring Arrangement of the 1910 Model Winton Six-cylinder Car

Comparison of deflections estimated from the above formulas, with actual deflections, has in some cases been quite satisfactory, while in other cases the actual deflections have appeared closer to those estimated by uniform strength formulas. In such cases where the writer has been able to make comparisons, however, the springs had been *made to specified deflections which evidently were estimated by the uniform strength formulas.* Experienced spring makers know that it is quite possible by putting a "pull" in the springs to vary the deflection and load. This trade term, "pull," is itself nothing more nor less than the introduction of an initial space between the leaves before banding.

Suspension of Automobiles

In road carriages, except in the heavier wagons, it is usual to find but two springs, one over each axle placed across the width of the carriage. In automobiles, one finds almost invariably at least the rear suspended upon two springs running lengthwise of the car, while, as is shown in the accompanying illustrations, it is the tendency to use the same suspension in the front. Such an arrangement takes up the forward and side lunges in a manner impossible with simple transverse springs. The further use of links and shackles, and of scroll ends, adds to the comfort, allowing the car to swing upon the springs rather than to be thrown upon them. In quite a few models, the two rear springs are attached in front to the frame and in the rear to a platform spring, which is itself attached to the center of the rear cross member of the frame. The three-quarter elliptic spring lends itself to both comfort and convenience of arrangement, and is

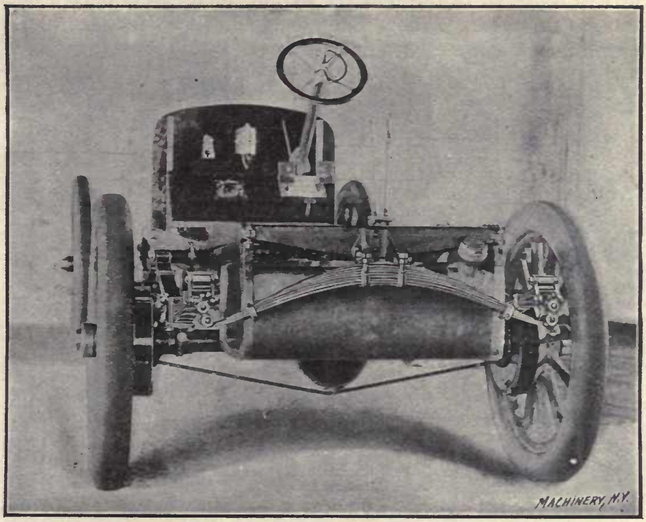

Fig. 27. Spring Support of the Lozier Motor Co.'s "Light Six" Car

rapidly coming into general use in this country, our manufacturers having apparently adopted it from foreign cars.

Steel Used in Automobile Springs

Automobile springs call for a high grade of steel, the ordinary spring steel lacking in strength and elasticity. Various grades of high carbon, silicon, manganese, nickel, chromium, and vanadium steels are used. Often such alloys are used as silico-manganese, chrome-nickel, and chrome-vanadium, the stiffening elements seeming to rank in the order given. Data as to the physical properties of such steels cannot well be given, as such properties must depend upon the proportions in the particular alloy used. Certain alloys of the vanadium group having an elastic limit of from 180,000 to 225,000 pounds per square inch, and tensile strength from 190,000 to 250,000 pounds, appear to be the most ideal steels yet produced.

Calculations of Springs

The calculation of spring properties by formulas is long and tedious. The writer appends, therefore, a table based on a modulus of elasticity of 25,400,000 and a fiber stress under maximum safe load of 80,000 pounds per square inch. Calculations of springs made of materials having other physical properties are made by simple proportion. This table is to be used only when all leaves are fully graduated.

The safe load on one leaf one inch wide is found by dividing the constant given under P_u by the net length. The corresponding deflection is found by multiplying the constant given under f_u by the square of the net length.

Example: What is the safe load on a semi-elliptic full graduated spring of five leaves if of one-quarter by two inch steel; length between end bearings, thirty-six inches; band or seat, three inches?

Net length = 36 — 3 = 33 inches.

$$\text{Load on one leaf one inch wide} = \frac{3333.33}{33} = 101.01 \text{ pounds.}$$

SEMI-ELLIPTIC SPRING TABLE

Giving safe load and deflection for 1 inch wide leaves, 1 inch net length.
Used only when all leaves are fully graduated

Thickness of Leaf	P_u	f_u	Steel	P_u	f_u
$\frac{1}{32}$	52.08	0.02519	$\frac{9}{32}$	4218.75	0.00280
$\frac{1}{16}$	208.33	0.01260	$\frac{5}{16}$	5208.33	0.00252
$\frac{3}{32}$	468.75	0.00840	$\frac{11}{32}$	6302.08	0.00229
$\frac{1}{8}$	833.33	0.00630	$\frac{3}{8}$	7500.00	0.00210
$\frac{5}{32}$	1302.08	0.00504	$\frac{13}{32}$	8802.08	0.00194
$\frac{3}{16}$	1875.00	0.00420	$\frac{7}{16}$	10208.33	0.00180
$\frac{7}{32}$	2552.08	0.00360	$\frac{15}{32}$	11718.75	0.00168
$\frac{1}{4}$	3333.33	0.00315	$\frac{1}{2}$	13333.33	0.00157

Load on one leaf two inches wide = 2 × 101.01 = 202.02 pounds.

Load on five two-inch leaves = 5 × 202.02 = 1010.10 pounds.

Corresponding deflection is:

$$0.00315 \times (33)^2 = 3.43 \text{ inches.}$$

Formulas can easily be deduced making it possible to use the accompanying table for other classes of elliptic springs than those of the semi-elliptic type with all leaves fully graduated.

The formulas for the semi-elliptic spring with all leaves graduated are:

$$P = \frac{2\,S\,n\,b\,h^2}{3\,L} \text{ and } f = \frac{S\,L^2}{4\,E\,h}.$$

To find the values of P_u given in the table, insert S = 80,000, n = 1, b = 1, h = the value given in the first column in the table, and L = 1. To find the values of f_u, insert in the second formula S = 80,000, L = 1, E = 25,400,000, and h = the value given in the first column in the table.

Now if the values in the table are to be used for other springs, constants can be deduced by which the table values may be multiplied.

For a semi-elliptic spring with a portion of the leaves graduated the load P remains the same as for a spring with all leaves graduated. The formula for the deflection, however, is:

$$f = \frac{1}{2\,(2+r)} \times \frac{S\,L^2}{E\,h}.$$

The values in the table, therefore, must be multiplied by the quantity $\frac{2}{(2+r)} \times L^2$ to find the deflection for any given combination full leaf and graduated spring of effective length L.

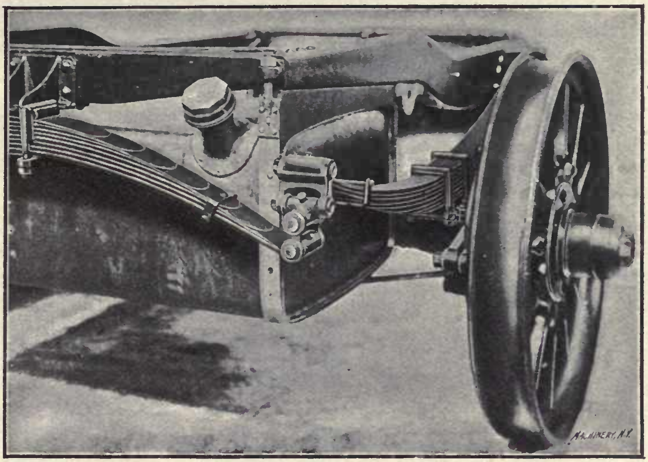

Fig. 28. Arrangement of Semi-elliptic Springs on the Lozier Motor Co.'s Four-cylinder Model

For a full elliptic spring with all leaves graduated, P still remains the same as for a semi-elliptic spring, but f doubles its value, or:

$$f = \frac{S\,L^2}{2\,E\,h}.$$

The values in the table, therefore, in this case must be multiplied by $2\,L^2$.

For the full elliptic spring with only part of the leaves graduated, the load P remains the same as before, but the deflection is twice that of a semi-elliptic spring:

$$f = \frac{1}{2\,(2+r)} \times \frac{2\,S\,L^2}{E\,h} = \frac{S\,L^2}{(2+r)\,E\,h}.$$

In this case, then, the values for the deflection in the table are to be multiplied by $\frac{4}{2+r} \times L^2$.

The flexibility of a spring is the amount of deflection as compared to the load. This may be expressed as so many inches deflection per hundred pounds, or y.

Example: Assume a full-elliptic, fully graduated spring, where

$S = 80{,}000$,
$E = 25{,}400{,}000$,
$b = 1\frac{3}{4}$ inch,
$n = 4$,
$h = \frac{1}{4}$ inch,
$L = 30$ inches.

Then the safe load equals:

$$P = 4 \times 1\frac{3}{4} \times \frac{3333.33}{30} = 778 \text{ pounds.}$$

And the deflection equals:

$$f = 30^2 \times 2 \times 0.00315 = 5.67 \text{ inches.}$$

Then,

$$y = \frac{5.67}{778} \times 100 = 0.73 \text{ inch.}$$

On the other hand, assume that the thickness and number of leaves are unknown. Then we have:

$P = 778$ pounds,
$S = 80{,}000$,
$E = 25{,}400{,}000$,
$b = 1\frac{3}{4}$ inch,
$L = 30$ inches,
$y = 0.73$ inch.

Then

$$f = \frac{778}{100} \times 0.73 = 5.67 \text{ inches.}$$

But $f = 2 f_u L^2$, where f_u is the constant for deflection in the accompanying table.

Hence,

$$f_u = \frac{f}{2 L^2} = \frac{5.67}{1800} = 0.00315.$$

The thickness of steel in the table which corresponds to this value of f_u is one-fourth inch.

The number of leaves is found by using P_u.

Load on one leaf, one inch wide is:

$$\frac{3333.333}{30} = 111.11 \text{ pounds.}$$

Load on one leaf $1\frac{3}{4}$ inch wide is:

$$111.11 \times 1\frac{3}{4} = 194.25.$$

Number of leaves is then,

$$\frac{778}{194.25} = 4.$$

The present calculation makes no allowance for the leaves of a spring varying in thickness. Where such springs are used, the deflection of the different leaves will not be uniform. Hence, in such springs also a suitable initial "pull" should exist, and such springs should be estimated by a general formula based upon a combination of different cantilevers, thus making allowance for different depths of cantilevers.

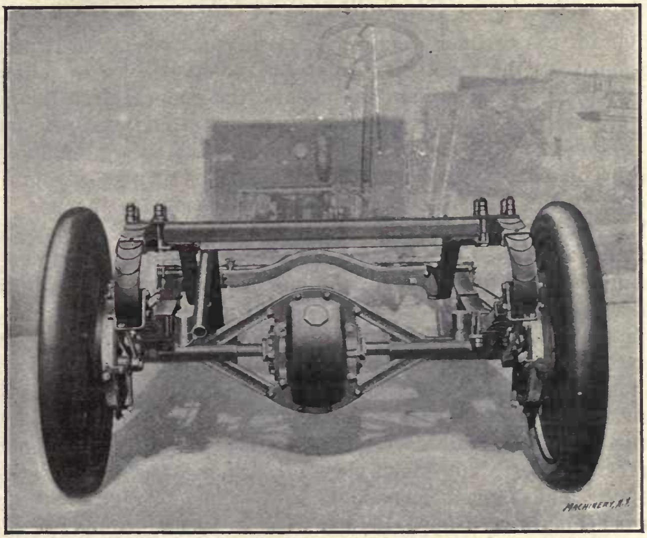

Fig. 29. Three-quarter Elliptic Spring Suspension on the F. B. Stearns Co.'s 15-30 H. P. Car

It is much better to use springs composed of but one thickness of leaves, as the combination of different thicknesses adds a complexity scarcely necessary.

Results obtained from fully graduated full elliptic springs would seem to show that the action of the friction between the leaves is not great enough to seriously affect the bending action, in that the formulas give results agreeing very closely with actual conditions.

CHAPTER IV

THE DESIGN OF SPRINGS FOR GAS ENGINE VALVES*

Springs for gas engines should be carefully designed, and if properly proportioned for the work they must do, should be just as reliable as any other part of the mechanism. While the general data for spring design are well known to engineers, yet attention may properly be given to some considerations specially applicable to gas engine valve springs. This chapter will consider compression springs of round steel wire only, as the writer knows of no valid reason for the use of any other material or section for this class of springs. It is well known that square steel is less desirable than round steel for springs, both on account of the higher cost of the springs per pound, and from the standpoint of efficiency.

The first consideration is the selection of the proper values for the fiber stress S and the torsional modulus of elasticity G. Experiments have shown that a fair value for G is 12,500,000, which value is fairly constant for the various grades and tempers of steel within their elastic limits. The safe value for S is not so easily determined, because the correct value for any given class of springs is largely a matter of experience. The highest normal value of S varies from about 120,000 pounds per square inch for 1/16-inch wire, to 90,000 pounds for ⅝-inch wire, which includes the range of sizes generally used on valves. The term "normal value" is used to distinguish these figures from the higher values which can be reached by spring makers, and which are sometimes necessary, but should never be used for rapidly vibrating springs, or for springs where safety and long life are primary considerations, as in this class of springs. In fact, even the above normal values are far too high for gas engine springs. These values are used very generally on machinery springs, etc., but should be reduced very materially to obtain springs which will give the maximum of service in gas engine work. A value of S of from 25,000 to 30,000 pounds per square inch has been found to give best results for gas engine valves.

The third variable is the length of the spring, which should be as long as practicable in order to keep the pressure on the lever or cam which operates the valve from being higher than necessary at the extreme lift of the valve. To illustrate this point we will take a valve on which a pressure of 40 pounds when closed is desired, and which opens ½ inch. If the spring is under ½ inch compression when the valve is closed, and holding 40 pounds, the pressure when the valve is open will be 80 pounds. But if we use a spring under 1¼ inch compression to hold 40 pounds when the valve is closed, when the valve is opened the ½-inch travel, the pressure will be

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increased to only 56 pounds. The diameter and assembled length of the spring will usually be determined by the general design of the engine. The diameter should be as large as convenient, which will lessen the tendency to buckle.

We will now design a spring for an exhaust valve, the lift of the valve being ½ inch, the assembled length of the spring 6 inches, the pitch diameter of the spring 2 inches, and the value of S at extreme compression 25,000 pounds per square inch. We will make the spring 7¼ inches long, thus giving a total compression of 1¾ inch, and a final pressure of 56 pounds. The following formulas will be used:

$$P = \frac{11d^3S}{28\,D} \qquad (1)$$

$$f_1 = \frac{22D^2S}{7Gd} \qquad (2)$$

in which

P = pressure at given compression,
d = diameter of wire in inches,
D = pitch diameter of spring in inches,
f_1 = deflection of one coil in inches,
S = fiber stress in pounds per square inch,
G = torsional modulus of elasticity.

The common forms of the Formulas (1) and (2) are:

$$P = \frac{S\pi d^3}{16R} \qquad (3)$$

$$f = \frac{32PR^2l}{G\pi d^4} \qquad (4)$$

In these formulas P, d, S, and G denote the same quantities as in Formulas (1) and (2), and

R = pitch radius of spring in inches,
f = deflection of the whole spring under load,
l = full length of wire in spring.

The Formulas (3) and (4) can easily be transformed to the form in (1) and (2) by writing $\pi = 22/7$, $R = D/2$, and $l = \pi Dn$ (n being the number of coils in the spring).

We use Formula (1) to determine the size of the wire. Substituting the known values, we have:

$$56 = \frac{11d^3 \times 25{,}000}{28 \times 2}, \text{ or } d = 0.225.$$

We therefore will use No. 4 Washburn & Moen gage wire, which is 0.225. To determine the deflection per coil, we will substitute the known values in Formula (2), as follows:

$$f_1 = \frac{22 \times 4 \times 25{,}000}{7 \times 12{,}500{,}000 \times 0.225} = 0.112 \text{ inch.}$$

The free length of the spring is 7¼ inches, and the length with the valve open is 5½ inches; the compression therefore is 1¾ inch. Then 1¾ ÷ 0.112 (the compression per coil) gives 15¾ acting coils approximately, and adding one coil on each end, for a flat bearing to be ground at right angles to the axis of the spring, gives 17¾ total coils. Therefore the spring will be 2 inches pitch diameter, 7¼ inches free length, No. 4 W. & M. gage wire, 17¾ total coils, squared and ground ends, holding 40 pounds at 6 inches long, and 56 pounds at 5½ inches long, with a fiber stress at 5½ inches long of 25,000 pounds per square inch.

If it is desirable that the pressure, when the valve is open, rise *as little as possible* above 40 pounds, we must make the spring as long as possible and still compress to the closed length given. We will assume a spring 2 inches pitch diameter, to hold 40 pounds when 6 inches long, and as little over 40 pounds as possible at 5½ inches long. As we do not know the pressure at 5½ inches long, we will take the fiber stress 25,000 pounds at 6 inches long, instead of at total compression. Using Formula (1): $40 = \frac{11d^3 \times 25,000}{28 \times 2}$, or $d^3 = \frac{224}{27,500}$, and $d = 0.200$. We will therefore use No. 5 W. & M. gage wire, which is 0.207. Using Formula (2): $f_1 = \frac{22 \times 4 \times 25,000}{7 \times 12,500,000 \times 0.207} = 0.1215$ inch compression per coil when holding 40 pounds. Then 5½ inches solid length less twice 0.207 gives the length occupied by the acting coils when solid, or 5.086 inches, and 5.086 ÷ 0.207 = 24.5 acting coils. Further, 24.5 × 0.1215 = 2.975 inches compression, which added to 6 inches gives 8.975 inches free length of the spring, say 9 inches. The spring therefore compresses 3 inches when holding 40 pounds, with a value of S of 25,000 pounds and at 5½ inches long, being compressed 3½ inches, holds 46 2/3 pounds, with a value of S of $\frac{46\ 2/3}{40} \times 25,000$ or 29,166 2/3 pounds.

In these examples we have not corrected the values of S to allow for the variation in sizes of wire used, from the theoretical sizes obtained, as it is not necessary to do so in practice. It is interesting to note, however, the difference in this value at final compression, obtained by the above method of proportion based on 25,000 pounds at 40 pounds pressure, from that obtained by using the original formula with the final pressure at 46 2/3 pounds, and wire of 0.207 inch diameter. The first method gives 29,166 pounds, while the second method gives 26,782 pounds, this difference being caused by the difference of 0.007 in the size of wire.

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