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AND FORMING A COMPLETE TREATISE ON ARITHMETICAL SCIENCE, AND ITS COMMERCIAL AND BUSINESS APPLICATIONS.

BY

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PREFACE.

THIS work is intended to complete a well graded and progressive series of Arithmetics, and to furnish to advanced students a more full and comprehensive text-book on the Science of Numbers than has before been published; a work that shall embrace those subjects necessary to give the pupil a thoroughly practical and scientific arithmetical education, either for the farm, the workshop, or a profession, or for the more difficult operations of the counting-room and of mercantile and commercial life.

There are two general methods of presenting the elements of arithmetical science, the *Synthetic* and the *Analytic.* Comparison enters into every operation, from the simplest combination of numbers to the most complicated problems in the Higher Mathematics. Analysis first generalizes a subject and then develops the particulars of which it consists; Synthesis first presents particulars, from which, by easy and progressive steps, the pupil is led to a general and comprehensive view of the subject. Analysis separates truths and properties into their elements or first principles; Synthesis constructs general principles from particular cases. Analysis appeals more to the reason, and cultivates the desire to search for first principles, and to *understand the reason* for every process rather than to *know the rule.* Hence, the leading method in an elementary course of instruction should be the Synthetic, while in an advanced course it should be the Analytic.

The following characteristics of a first class text-book will be obvious to all who examine this work: *the typogra-*

phy and mechanical execution; the philosophical and scientific arrangement of the subjects; clear and concise definitions; full and rigid analyses; exact and comprehensive rules; brief and accurate methods of operation: the wide range of subjects and the large number and practical character of the examples—in a word, SCIENTIFIC ACCURACY *combined with* PRACTICAL UTILITY, *throughout the entire work.*

Much labor and attention have been devoted to obtaining correct and adequate information pertaining to mercantile and commercial transactions, and the Government Standard units of measures, weights, and money. The counting-room, the bank, the insurance and broker's office, the navy and ship-yard, the manufactory, the wharves, the custom-house, and the mint, have all been visited, and the most reliable statistics and the latest statutes have been consulted, for the purpose of securing *entire accuracy* in those parts of this work which relate to these subjects and departments. As the result of this thorough investigation, many statements found in most other arithmetics of a similar grade will not agree with the facts presented in this work, and simply because the statements in these other books have been copied from older works, while laws and customs have undergone great changes since the older works were written.

New material and new methods will be found in the several subjects throughout the entire work. Considerable prominence has been given to Percentage and its numerous applications, especially to Stocks, Insurance, Interest, Averaging Accounts, Domestic and Foreign Exchange, and several other subjects necessary to qualify students to become good accountants or commercial business men. And while this work may embrace many subjects not necessary to the course usually prescribed in Mercantile and Commercial Colleges, yet those subjects requisite to make good accountants, and which have been taught orally in that class of institutions from want of a suitable text-book, are fully discussed and practically applied in this work; and it is therefore believed to be better adapted to the wants of Mercantile Colleges than any similar work yet published. And while it is due, it is also proper here to state that J. C. Porter, A. M., an experienced and successful teacher of Mathematics in this State, and formerly professor of Commercial Arithmetic, in Iron City Commercial College, Pittsburgh, Penn., has rendered valuable aid in the preparation of the above-named subjects, and of other portions of the work. He is likewise the author of the Factor Table on pages 72 and 73, and of the new and valuable improvement in the method of Cube Root.

Teachers entertain various views relative to having the answers to problems and examples inserted in a text-book. Some desire the answers placed immediately after the examples; others wish them placed together in the back part of the book; and still others desire them omitted altogether. All these methods have their advantages and their disadvantages.

If all the answers are given, there is danger that the pupil will become careless, and not depend enough upon the accuracy of his own computations. Hence he is liable to neglect the cultivation of those habits of patient investigation and self-reliance which would result from his being obliged to test the truth and accuracy of his own processes by proof,—the only test he will have to depend upon in all the computations in real business transactions in after life. Besides, the work of proving the correctness of a result is often of quite as much value to the pupil as the work of performing the operation; as the proof may render simple and clear some part or the whole of an operation that was before complicated and obscure.

If answers are placed in the back part of the book, the pupil will at once refer to them whenever he is in any doubt or difficulty in performing an operation. Hence the object aimed at is not accomplished by placing the answers together in this manner.

Again, if all the answers are omitted, the pupil may become involved in doubt and uncertainty, and acquire a distaste for the study; and from this discouragement, subsequently make but limited advancement in Mathematical Science.

In order, therefore, that pupils may receive the advantages of both methods, the answers to nearly one half of the examples in this book are omitted. They will be found, together with full and clear solutions of all the examples, in a Key to this work, which has been prepared for the use of teachers and private learners.

Many valuable hints and suggestions which have been received from teachers and friends of education, have been incorporated into this work. The author desires to make especial acknowledgment of the valuable services rendered in the preparation of this work by D. W. Fish, A.M., of Rochester, N. Y., a gentleman who has had long and successful experience as a teacher, and an intimate acquaintance with the plans and operations of some of the best schools in the country.

August 1, 1860.

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DUODECIMALS.

SHORT METHODS.

PERCENTAGE.

HIGHER ARITHMETIC.

DEFINITIONS.

1. **Quantity** is any thing that can be increased, diminished, or measured; as distance, space, weight, motion, time.

2. A **Unit** is one, a single thing, or a definite quantity.

3. A **Number** is a unit, or a collection of units.

4. The **Unit of a Number** is *one* of the collection constituting the number. Thus, the unit of 34 is 1; of 34 days is 1 day.

5. An **Abstract Number** is a number used without reference to any particular thing or quantity; as 3, 24, 756.

6. A **Concrete Number** is a number used with reference to some particular thing or quantity; as 21 hours, 4 cents, 230 miles.

7. **Unity** is the unit of an abstract number.

8. The **Denomination** is the name of the unit of a concrete number.

9. A **Simple Number** is either an abstract number, or a concrete number of but one denomination; as 48, 52 pounds, 36 days.

10. A **Compound Number** is a concrete number expressed in two or more denominations; as, 4 bushels 3 pecks, 8 rods 4 yards 2 feet 3 inches.

11. An **Integral Number,** or **Integer,** is a number which expresses whole things; as 5, 12 dollars, 17 men.

12. A **Fractional Number,** or **Fraction,** is a number which expresses equal parts of a whole thing or quantity; as $\frac{1}{2}$, $\frac{3}{4}$ of a pound, $\frac{5}{12}$ of a bushel.

13. **Like Numbers** have the same kind of unit, or express the same kind of quantity. Thus, 74 and 16 are like numbers; so are 74 pounds, 16 pounds, and 12 pounds; also, 4 weeks 3 days, and 16 minutes 20 seconds, both being used to express units of time.

14. **Unlike Numbers** have different kinds of units, or are used to express different kinds of quantity. Thus, 36 miles, and 15 days; 5 hours 36 minutes, and 7 bushels 3 pecks.

15. A **Power** is the product arising from multiplying a number by itself, or repeating it any number of times as a factor.

16. A **Root** is a factor repeated to produce a power.

17. A **Scale** is the order of progression on which any system of notation is founded. Scales are uniform and varying.

18. A **Uniform Scale** is one in which the order of progression is the same throughout the entire succession of units.

19. A **Varying Scale** is one in which the order of progression is not the same throughout the entire succession of units.

20. A **Decimal Scale** is one in which the order of progression is uniformly ten.

21. **Mathematics** is the science of quantity.

The two fundamental branches of Mathematics are Geometry and Arithmetic. Geometry considers quantity with reference to positions, form, and extension. Arithmetic considers quantity as an assemblage of definite *portions*, and treats only of those conditions and attributes which may be investigated and expressed by numbers. Hence,

22. **Arithmetic** is the *Science* of numbers, and the *Art* of computation.

NOTE.—When Arithmetic treats of operations on abstract numbers it is a science, and is then called *Pure Arithmetic.* When it treats of operations on concrete numbers it is an art, and is then called *Applied Arithmetic.* Pure and Applied Arithmetic are also called *Theoretical* and *Practical* Arithmetic.

23. A **Demonstration** is a process of reasoning by which a truth or principle is established.

24. An **Operation** is a process in which figures are employed to make a computation, or obtain some arithmetical result.

25. A **Problem** is a question requiring an operation.

26. A **Rule** is a prescribed method of performing an operation.

27. **Analysis**, in arithmetic, is the process of investigating principles, and solving problems, independently of set rules.

28. The **Five Fundamental Operations** of Arithmetic are, Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

SIGNS.

29. A **Sign** is a character indicating the relation of numbers, or an operation to be performed.

30. The **Sign of Numeration** is the comma (,). It indicates that the figures set off by it express units of the same general name, and are to be read together, as *thousands, millions, billions,* etc.

31. The **Decimal Sign** is the period (.). It indicates that the number after it is a decimal.

32. The **Sign of Addition** is the perpendicular cross, $+$, called *plus.* It indicates that the numbers connected by it are to be added; as $3 + 5 + 7$, read 3 plus 5 plus 7.

33. The **Sign of Subtraction** is a short horizontal line, $-$, called *minus.* It indicates that the number after it is to be subtracted from the number before it; as $12 - 7$, read 12 minus 7.

34. The **Sign of Multiplication** is the oblique cross, $\times$. It indicates that the numbers connected by it are to be multiplied together; as $5 \times 3 \times 9$, read 5 multiplied by 3 multiplied by 9.

35. The **Sign of Division** is a short horizontal line, with a point above and one below, $\div$. It indicates that the number before it is to be divided by the number after it; as $18 \div 6$, read 18 divided by 6.

Division is also expressed by writing the dividend *above,* and the divisor *below,* a short horizontal line. Thus, $\frac{18}{6}$, read 18 divided by 6.

36. The **Sign of Equality** is two short, parallel, horizontal lines, $=$. It indicates that the numbers, or combinations of numbers, connected by it are equal; as $4 + 8 = 15 - 3$, read 4 plus 8 is equal to 15 minus 3. Expressions connected by the sign of equality are called *equations.*

37. The **Sign of Aggregation** is a parenthesis, ( ). It indicates that the numbers included within it are to be considered together, and subjected to the same operation. Thus, $(8 + 4) \times 5$ indicates that both 8 and 4, or their sum, is to be multiplied by 5. A vinculum or bar, $\overline{\quad\quad}$, has the same signification. Thus, $\overline{7 \times 9} \div 3 = 21$.

38. The **Sign of Ratio** is two points, : . Thus, 7 : 4 is read, the ratio of 7 to 4.

39. The **Sign of Proportion** is four points, : : . Thus, 3 : 6 : : 4 : 8, is read, 3 is to 6 as 4 is to 8.

40. The **Sign of Involution** is a number written above, and a little to the right, of another number. It indicates the power to which the latter is to be raised. Thus, 12^3 indicates that 12 is to be taken 3 times as a factor; the expression is equivalent to $12 \times 12 \times 12$. The number expressing the sign of involution is called the *Index* or *Exponent*.

41. The **Sign of Evolution,** $\surd$, is a modification of the letter *r*. It indicates that some root of the number after it is to be extracted. Thus, $\sqrt{25}$ indicates that the square root of 25 is to be extracted; $\sqrt[3]{64}$ indicates that the cube root of 64 is to be extracted.

AXIOMS.

42. An **Axiom** is a self-evident truth. The principal axioms required in arithmetical investigations are the following:

1. If the same quantity or equal quantities be added to equal quantities, the sums will be equal.
2. If the same quantity or equal quantities be subtracted from equal quantities, the remainders will be equal.
3. If equal quantities be multiplied by the same number, the products will be equal.
4. If equal quantities be divided by the same number, the quotients will be equal.
5. If the same number be added to a quantity and subtracted from the sum, the remainder will be that quantity.
6. If a quantity be multiplied by a number and the product divided by the same number, the quotient will be that quantity.
7. Quantities which are respectively equal to any other quantity are equal to each other.
8. Like powers or like roots of equal quantities are equal.
9. The whole of any quantity is greater than any of its parts.
10. The whole of any quantity is equal to the sum of all its parts.

NOTATION AND NUMERATION.

43. Notation is a system of *writing* or expressing numbers by characters; and,

44. Numeration is a method of *reading* numbers expressed by characters.

45. Two systems of notation are in general use — the *Roman* and the *Arabic.*

NOTE. — The Roman Notation is supposed to have been first used by the Romans: hence its name. The Arabic Notation was first introduced into Europe by the Moors or Arabs, who conquered and held possession of Spain during the 11th century. It received the attention of scientific men in Italy at the beginning of the 13th century, and was soon afterward adopted in most European countries. Formerly it was supposed to be an invention of the Arabs; but investigations have shown that the Arabs adopted it from the Hindoos, among whom it has been in use more than 2000 years. From this undoubted origin it is sometimes called the *Indian Notation.*

THE ROMAN NOTATION.

46. Employs seven capital letters to express numbers. Thus,

Letters,	I	V	X	L	C	D	M
Values,	one,	five,	ten,	fifty,	one hundred,	five hundred,	one thousand.

47. The Roman notation is founded upon five principles, as follows:

1st. Repeating a letter repeats its value. Thus, II represents two, XX twenty, CCC three hundred.

2d. If a letter of any value be placed *after* one of greater value, its value is to be *united to* that of the greater. Thus, XI represents eleven, LX sixty, DC six hundred.

3d. If a letter of any value be placed *before* one of greater value, its value is to be *taken from* that of the greater. Thus, IX represents nine, XL forty, CD four hundred.

4th. If a letter of any value be placed *between* two letters, each of greater value, its value is to be *taken from* the *united value* of the other two. Thus, XIV represents fourteen, XXIX twenty-nine, XCIV ninety-four.

5th. A bar or dash placed over a letter increases its value one thousand fold. Thus, V signifies five, and $\overline{\text{V}}$ five thousand; L fifty, and $\overline{\text{L}}$ fifty thousand.

TABLE OF ROMAN NOTATION.

I	is	One.	XX	is	Twenty.
II	"	Two.	XXI	"	Twenty-one.
III	"	Three.	XXX	"	Thirty.
IV	"	Four.	XL	"	Forty.
V	"	Five.	L	"	Fifty.
VI	"	Six.	LX	"	Sixty.
VII	"	Seven.	LXX	"	Seventy.
VIII	"	Eight.	LXXX	"	Eighty.
IX	"	Nine.	XC	"	Ninety.
X	"	Ten.	C	"	One hundred.
XI	"	Eleven.	CC	"	Two hundred.
XII	"	Twelve.	D	"	Five hundred.
XIII	"	Thirteen.	DC	"	Six hundred.
XIV	"	Fourteen.	M	"	One thousand.
XV	"	Fifteen.	MC	"	One thousand one hundred.
XVI	"	Sixteen.	MM	"	Two thousand.
XVII	"	Seventeen.	$\overline{X}$	"	Ten thousand.
XVIII	"	Eighteen.	$\overline{C}$	"	One hundred thousand.
XIX	"	Nineteen.	$\overline{M}$	"	One million.

NOTES.—1. Though the letters used in the above table have been employed as the Roman numerals for many centuries, the marks or characters used originally in this notation are as follows:

Modern numerals,	I	V	X	L	C	D	M
Primitive characters,	I	V	X	L	⊏	N	M

2. The system of Roman Notation is not well adapted to the purposes of numerical calculation; it is principally confined to the numbering of chapters and sections of books, public documents, etc.

EXAMPLES FOR PRACTICE.

Express the following numbers by the Roman notation:

1. Fourteen.
2. Nineteen.
3. Twenty-four.
4. Thirty-nine.
5. Forty-six.
6. Fifty-one.
7. Eighty-eight.
8. Seventy-three.
9. Ninety-five.
10. One hundred one.
11. Five hundred fifty-five.
12. Seven hundred ninety-eight.
13. One thousand three.
14. Twenty thousand eight hundred forty-five.

THE ARABIC NOTATION

48. Employs ten characters or figures to express numbers. Thus,

Figures,	0	1	2	3	4	5	6	7	8	9
Names and values.	naught or cipher.	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.

49. The cipher, or first character, is called *naught*, because it has no value of its own. It is otherwise termed *nothing*, and *zero*. The other nine characters are called *significant figures*, because each has a value of its own. They are also called *digits*, a word derived from the Latin term *digitus*, which signifies *finger*.

50. The ten Arabic characters are the Alphabet of Arithmetic. Used independently, they can express only the nine numbers that correspond to the names of the nine digits. But when combined according to certain principles, they serve to express all numbers.

51. The notation of all numbers by the ten figures is accomplished by the formation of a series of units of different values, to which the digits may be successively applied. First, ten simple units are considered together, and treated as a single superior unit; then, a collection of ten of these new units is taken as a still higher unit; and so on, indefinitely. A regular series of units, in ascending orders, is thus formed, as shown in the following

TABLE OF UNITS.

Primary units are called					units of the		first	order.	
Ten units of the first	order make	1	unit	"	"		second	"	
Ten " " "	second "	"	1	"	"	"	third	"	
Ten " " "	third "	"	1	"	"	"	fourth	"	
etc., etc.					etc.,		etc.		

52. The various orders of units, when expressed by figures, are distinguished from each other by their *location*, or the place they occupy in a horizontal row of figures. Units of the first order are written at the right hand; units of the second order occupy the second place; units of the third order the third place; and so on, counting from right to left, as shown on the following page:

9th order.	8th order.	7th order.	6th order.	5th order.	4th order.	3d order.	2d order.	1st order.
0	0	0	0	0	0	0	0	0

53. In this notation we observe—

1st. That a figure written in the place of any order, expresses as many units of that order as is denoted by the name of the figure used. Thus, 436 expresses 4 units of the 3d order, 3 units of the 2d order, and 6 units of the 1st order.

2d. The cipher, having no value of its own, is used to fill the places of vacant orders, and thus preserve the relative positions of the significant figures. Thus, in 50, the cipher shows the absence of simple units, and at the same time gives to the figure 5 the local value of the second order of units.

54. Since the number expressed by any figure depends upon the place it occupies, it follows that figures have two values, Simple and Local.

55. The **Simple Value** of a figure is its value when taken alone; thus, 4, 7, 2.

56. The **Local Value** of a figure is its value when used with another figure or figures in the same number. Thus, in 325, the local value of the 3 is 300, of the 2 is 20, and of the 5 is 5 units.

Note.—When a figure occupies units' place, its simple and local values are the same.

57. The leading principles upon which the Arabic notation is founded are embraced in the following

GENERAL LAWS.

I. *All numbers are expressed by applying the ten figures to different orders of units.*

II. *The different orders of units increase from right to left, and decrease from left to right, in a tenfold ratio.*

III. *Every removal of a figure one place to the left, increases its local value tenfold; and every removal of a figure one place to the right, diminishes its local value tenfold.*

58. In *numerating*, or expressing numbers verbally, the various orders of units have the following names:

ORDERS.	NAMES.
1st order is called	Units.
2d order " "	Tens.
3d order " "	Hundreds.
4th order " "	Thousands.
5th order " "	Tens of thousands.
6th order " "	Hundreds of thousands.
7th order " "	Millions.
8th order " "	Tens of millions.
9th order " "	Hundreds of millions.
etc., etc.	etc., etc.

59. This method of numerating, or naming, groups the successive orders into *periods* of three figures each, there being three orders of thousands, three orders of millions, and so on in all higher orders. These periods are commonly separated by commas, as in the following table, which gives the names of the orders and periods to the twenty-seventh place.

of septillions.	of sextillions.	of quintillions.	of quadrillions.	of trillions.	of billions.	of millions.	of thousands.	of units.
tens units	hundreds tens units	hundreds tens units	hundreds tens units	hundreds tens units	hundreds tens units	hundreds tens units	hundreds tens units	hundreds tens units
9 8,	7 6 5,	4 3 2,	1 0 9,	8 7 6,	5 5 6,	7 8 9,	0 1 2,	3 4 5
ninth period.	eighth period.	seventh period.	sixth period.	fifth period.	fourth period.	third period.	second period.	first period.

NOTE.—This is the French method of numerating, and is the one in general use in this country. The English numerate by periods of six figures each.

60. The names of the periods are derived from the Latin numerals. The twenty-two given on the following page extend the numeration table to the sixty-sixth place or order, inclusive.

PERIODS.	NAMES.	PERIODS.	NAMES.
1st	Units.	12th	Decillions.
2d	Thousands.	13th	Undecillions.
3d	Millions.	14th	Duodecillions.
4th	Billions.	15th	Tredecillions.
5th	Trillions.	16th	Quatuordecillions.
6th	Quadrillions.	17th	Quindecillions.
7th	Quintillions.	18th	Sexdecillions.
8th	Sextillions.	19th	Septendecillions.
9th	Septillions.	20th	Octodecillions.
10th	Octillions.	21st	Novendecillions.
11th	Nonillions.	22d	Vigintillions.

61. From this analysis of the principles of Notation and Numeration, we derive the following rules:

RULE FOR NOTATION.

I. *Beginning at the left hand, write the figures belonging to the highest period.*

II. *Write the hundreds, tens, and units of each successive period in their order, placing a cipher wherever an order of units is omitted.*

RULE FOR NUMERATION.

I. *Separate the number into periods of three figures each, commencing at the right hand.*

II. *Beginning at the left hand, read each period separately, and give the name to each period, except the last, or period of units.*

NOTE.—Omit *and* in reading the orders of units and periods of a number.

EXAMPLES FOR PRACTICE.

Write and read the following numbers:—

1 One unit of the 3d order, two of the 2d, five of the 1st.
Ans. 125; read, *one hundred twenty-five.*

2. Two units of the 5th order, four of the 4th, five of the 2d, six of the 1st. *Ans.* 24056; read, *twenty-four thousand fifty-six.*

3. Seven units of the 4th order, five of the 3d, three of the 2d, eight of the 1st.

4. Nine units of the 4th order, two of the 3d, four of the 1st.

5. Five units of the 4th order, eight of the 2d.

6. Five units of the 5th order, one of the 3d, eight of the 1st.

7. Three units of the 5th order, six of the 4th, four of the 3d, seven of the 1st.

8. Two units of the 6th order, four of the 5th, nine of the 4th, three of the 3d, five of the 1st.

9. Three units of the 8th order, five of the 7th, four of the 6th, three of the 5th, eight of the 4th, five of the 3d, eight of the 2d, seven of the 1st.

10. Three units of the 9th order, eight of the 7th, four of the 6th, six of the 5th, nine of the 1st.

11. Five units of the 12th order, three of the 11th, six of the 10th.

12. Four units of the 12th order, five of the 10th, eight of the 5th, nine of the 4th, four of the 3d.

13. Three units of the 15th order, six of the 14th, five of the 13th, three of the 9th, six of the 8th, five of the 7th, three of the 3d, six of the 2d, five of the 1st.

14. Five units of the 18th order, three of the 17th, six of the 16th, four of the 15th, seven of the 14th, eight of the 13th, four of the 12th, five of the 11th, six of the 10th, seven of the 9th, eight of the 8th, nine of the 7th, five of the 6th, six of the 5th, three of the 4th, two of the 3d, four of the 2d, eight of the 1st.

15. Two units of the 20th order, seven of the 19th, four of the 18th, eight of the 13th, five of the 6th, five of the 5th, five of the 4th, nine of the 1st.

Write the following numbers in figures:

16. Forty-eight.

17. One hundred sixty-four.

18. Forty-eight thousand seven hundred eighty-nine.

19. Five hundred thirty-six million three hundred forty-seven thousand nine hundred seventy-two.

20. Ninety-nine billion thirty-seven thousand four.

21. Eight hundred sixty-four billion five hundred thirty-eight million two hundred seventeen thousand nine hundred fifty-three.

22. One hundred seventeen quadrillion two hundred thirty-five trillion one hundred four billion seven hundred fifty million sixty-six thousand ten.

23. Ninety-nine quintillion seven hundred forty-one trillion fifty-four billion one hundred eleven million one hundred one.

24. One hundred octillion one hundred septillion one hundred quintillion one hundred quadrillion one hundred trillion one hundred billion one hundred million one hundred thousand one hundred.

25. Four decillion seventy-five nonillion three octillion fifty-two septillion one sextillion four hundred seventeen quintillion ten quadrillion twelve trillion fourteen billion three hundred sixty million twenty-two thousand five hundred nineteen.

Write the following numbers in figures, and read them:

26. Twenty-five units in the 2d period, four hundred ninety-six in the 1st. *Ans.* 25,496.

27. Three hundred sixty-four units in the 3d period, seven hundred fifteen in the 2d, eight hundred thirty-two in the 1st.

28. Four hundred thirty-six units in the 4th period, twelve in the 3d, one hundred in the 2d, three hundred one in the 1st.

29. Eighty-one units in the 5th period, two hundred nineteen in the 4th, fifty-six in the 2d.

30. Nine hundred forty-five units in the 7th period, eighteen in the 5th, one hundred three in the 3d.

31. One unit in the 10th period, five hundred thirty-six in the 9th, two hundred forty-seven in the 8th, nine hundred twenty-four in the 7th.

Point off and read the following numbers:

32.	564.	37.	2005.
33.	24835.	38.	100103.
34.	2474783.	39.	53000008.
35.	247843112.	40.	1001005003.
36.	23678542789.	41.	750000040003.

42. 247364582327896438542721.

43. 379403270506038009503070.

44. 20005700032004673000430512500000567304705030040.

ADDITION.

62. **Addition** is the process of uniting *several* numbers of the same kind into *one* equivalent number.

63. The **Sum** or **Amount** is the result obtained by the process of addition.

64. When the given numbers contain several orders of units, the method of addition is based upon the following principles:

I. If the like orders of units be added separately, the sum of all the results must be equal to the entire sum of the given numbers. (Ax. 10).

II. If the sum of the units of any order contain units of a higher order, these higher units must be combined with units of like order. Hence,

III. The work must commence with the lowest unit, in order to combine the partial sums in a single expression, at one operation.

1. Find the sum of 397, 476, and 873.

OPERATION

$$\begin{array}{r} 397 \\ 476 \\ \underline{873} \\ 1746 \end{array}$$

ANALYSIS. We arrange the numbers so that units of like order shall stand in the same column. We then add the first, or right hand column, and find the sum to be 16 units, or 1 ten and 6 units; writing the 6 units under the column of units, we add the 1 ten to the column of tens, and find the sum to be 24 tens, or 2 hundreds and 4 tens; writing the 4 tens under the column of tens, we add the 2 hundreds to the column of hundreds, and find the sum to be 17 hundreds, or 1 thousand and 7 hundreds; writing the 7 hundreds under the column of hundreds, and the 1 in thousands' place, we have the entire sum, 1746.

65. From these principles we deduce the following

RULE. I. *Write the numbers to be added so that all the units of the same order shall stand in the same column; that is, units under units, tens under tens, etc.*

II. *Commencing at units, add each column separately, and write the sum underneath, if it be less than ten.*

III. *If the sum of any column be ten or more than ten, write the unit figure only, and add the ten or tens to the next column.*

IV. *Write the entire sum of the last column.*

NOTES.—1. In adding, learn to pronounce the partial results without naming the *figures* separately. Thus, in the operation given for illustration, say 3, 9, 16; 8, 15, 24; 10, 14, 17.

2. When the sum of any column is greater than 9, the process of adding the tens to the next column is called *carrying*.

66. PROOF. There are two principal methods of proving Addition.

1st. By varying the combinations.

Begin with the right hand or unit column, and add the figures in each column in an opposite direction from that in which they were first added; if the two results agree, the work is supposed to be right.

2d. By excess of 9's.

67. This method depends upon the following properties of the number 9:*

I. If a number be divided by 9, the remainder will be the same as when the sum of its digits is divided by 9. Therefore,

II. If several numbers be added, the excess of 9's in the sum must be equal to the excess of 9's in the sum of all the digits in the numbers.

1. Add 34852, 24784, and 72456, and prove the work by the excess of 9's.

OPERATION.

```
 34852
 24784
 72456 ... 8, excess of 9's in all the digits of the numbers.
132092 ... 8,   "    "   "   "  sum     "     "
```

ANALYSIS. Commencing with the first number, at the left hand, we say 3 and 4 are 7, and 8 are 15; dropping 9, the excess is 6, which added to 5, the next digit, makes 11; dropping 9, the excess is 2; then 2 and 2 are 4, and 2 (the left hand digit of the second number)

* For a demonstration of these properties, see **136, IX.**

are 6, and 4 are 10; dropping 9, the excess is 1. Proceeding in like manner through all the digits, the final excess is 8; and as 8 is also the excess of 9's in the sum, the work of addition is correct. It is evident that the same result will be obtained by adding the digits in *columns* as in *rows*. Hence, to prove Addition by excess of 9's:—

Commencing at any figure, add the digits of the given numbers in any order, dropping 9 as often as the amount exceeds 9. If the final excess be equal to the excess of 9's in the sum, the work is right.

NOTE.—This method of proving addition by the excess of 9's, fails in the following cases: 1st, when the figures of the answer are misplaced; 2d, when the value of one figure is as much too great as that of another is too small.

EXAMPLES FOR PRACTICE

(1.)	(2.)	(3.)	(4.)
8635	1234567	67	24603
2194	723456	123	298765
7421	34565	4567	47321
5063	45666	89093	58653
2196	333	654321	5376
1245	90	1234567	340
26754	2038677		

5. 123+456+785+12+345+901+567=how many?

6. 12345+67890+8763+347+1037+198760=how many?

7. 172+4005+3761+20472+367012+19762=how many?

8. What is the sum of thirty-seven thousand six, four hundred twenty-nine thousand nine, and two millions thirty-six?

Ans. 2,466,051.

9. Add eight hundred fifty-six thousand nine hundred thirty-three, one million nine hundred seventy-six thousand eight hundred fifty-nine, two hundred three millions eight hundred ninety-five thousand seven hundred fifty-two. *Ans.* 206,729,544.

10. What is the sum of one hundred sixty-seven thousand, three hundred sixty-seven thoasand, nine hundred six thousand, two hundred forty-seven thousand, seventeen thousand, one hundred six thousand three hundred, forty thousand forty-nine, ten thousand four hundred one? *Ans.* 1,860,750.

11. What number of square miles in New England, there

being in Maine 31766, in New Hampshire 9280, in Vermont 10212, in Massachusetts 7800, in Rhode Island 1306, and in Connecticut 4674? *Ans.* 65,038.

12. The estimated population of the above States, in 1855, was as follows; Maine 653000, New Hampshire 338000, Vermont 327000, Massachusetts 1133123, Rhode Island 166500, and Connecticut 384000. What was the entire population?

13 At the commencement of the year 1858 there were in operation in the New England States, 3751 miles of railroad; in New York, 2590 miles; in Pennsylvania, 2546; in Ohio, 2946; in Virginia, 1233; in Illinois, 2678; and in Georgia, 1233. What was the aggregate number of miles in operation in all these States?

14. The Grand Trunk Railway is 962 miles long, and cost \$60000000; the Great Western Railway is 229 miles long, and cost \$14000000; the Ontario, Simcoe and Huron, is 95 miles long, and cost \$3300000; the Toronto and Hamilton is 38 miles long, and cost \$2000000. What is the aggregate length, and what the cost, of these four roads?

Ans. Length, 1,324 miles; cost, \$79,300,000.

15. A man bequeathed his estate as follows; to each of his two sons, \$12450; to each of his three daughters, \$6500; to his wife, \$650 more than to both the sons, and the remainder, which was \$1000 more than he had left to all his family, he gave to benevolent institutions. What was the whole amount of his property? *Ans.* \$140,900.

16. How many miles from the southern extremity of Lake Michigan to the Gulf of St. Lawrence, passing through Lake Michigan, 330 miles; Lake Huron, 260 miles; River St. Clair, 24 miles; Lake St. Clair, 20 miles; Detroit River, 23 miles; Lake Erie, 260 miles; Niagara River, 34 miles; Lake Ontario, 180 miles; and the River St. Lawrence, 750 miles?

17. The United States exported molasses, in the year 1856, to the value of \$154630; in 1857, \$108003; in 1858, \$115893; and tobacco, during the same years respectively, to the value of \$1829207, \$1458553, and \$2410224. What was the entire value of the molasses and tobacco exported in these three years?

18. The population of Boston, in 1855, was 162629; Providence, 50000; New York, 629810; Philadelphia, 408815; Brooklyn, 127618; Cleveland, 43740; and New Haven, 25000. What was the entire population of these cities? *Ans.* 1,447,612.

19. Iron was discovered in Greece by the burning of Mount Ida, B. C. 1406; and the electro-magnetic telegraph was invented by Morse, A. D. 1832. What period of time elapsed between the two events? *Ans.* 3,238 years.

20. The number of pieces of silver coin made at the United States Mint at Philadelphia, in the year 1858, were as follows: 4628000 half dollars, 10600000 quarter dollars, 690000 dimes, 4000000 half dimes, and 1266000 three-cent pieces. What was the total number of pieces coined?

21. The cigars imported by the United States, in the year 1856, were valued at $3741460; in 1857, at $4221096; and in 1858, at $4123208. What was the total value of the importations for the three years? *Ans.* $12,085,764.

22. In the appropriations made by Congress for the year ending June 30, 1860, were the following; for salary and mileage of members of Congress, $1557861; to officers and clerks of both Houses, $157639; for paper and printing of both Houses, $170000; to the President of the United States, $31450; and to the Vice President, $8000. What is the total of these items?

ADDING TWO OR MORE COLUMNS AT ONE OPERATION.

68. 1. What is the sum of 4632, 2553, 4735, and 2863?

OPERATION.

```
 4632
 2553
 4735
 2863
-----
14783
```

ANALYSIS. Beginning with the units and tens of the number last written, we add first the tens above, then the units, thus; 63 and 30 are 93, and 5 are 98, and 50 are 148, and 3 are 151, and 30 are 181, and 2 are 183. Of this sum, we write the 83 under the columns added, and carry the 1 to the next columns, thus; 28 and 1 are 29, and 40 are 69, and 7 are 76, and 20 are 96, and 5 are 101, and 40 are 141, and 6 are 147, which we write in its place, and we have the whole amount, 14783.

EXAMPLES FOR PRACTICE.

(1.)	(2.)	(3.)	(4.)
8450	75634	123456	7349042
5425	86213	47021	2821986
8595	92045	82176	1621873
6731	73461	570914	236719
7963	34719	379623	401963
5143	26054	7542	67254
4561	19732	25320	45067
6783	84160	57644	910732
4746	97013	908176	6328419
2373	34567	73409	1437651
3021	43651	3147	9716420
7273	52170	67039	3191232
71064	719419	2345467	34128358

5. What is the total number of churches, the number of persons accommodated, and the value of church property in the United States, as shown by the following statistics?

	No. of churches.	No. of persons accommodated.	Value of church property.
Methodist	12484	4220293	$14636671
Baptist	8798	3134438	10931382
Presbyterian	4591	2045516	14469889
Congregational	1675	795677	7973962
Episcopal	1430	631613	11261970
Roman Catholic	1269	705983	8973838
Lutheran	1205	532100	2867886
Christians	812	296050	845810
Friends	715	283023	1709867
Union	619	213552	690065
Universalist	494	205462	1767015
Free Church	361	108605	252255
Moravian	331	112185	443347
German Reformed	327	156932	965880
Dutch Reformed	324	181986	4096730
Unitarian	244	137867	3268122
Mennonite	110	29900	94245
Tunkers	52	35075	46025
Jewish	31	16575	371600
Swedenborgian	15	5070	108100

6. Give the amounts of the productions of the United States and Territories for the year 1850, as expressed in the following columns:

	Pounds of Butter.	Pounds of Cheese.	Pounds of Wool.	Bushels of Wheat.
Alabama......	4,008,811	31,412	657,118	294,044
Arkansas......	1,854,239	30,088	182,595	199,639
California.....	705	150	5,520	17,228
Columb.,Dist.	14,872	1,500	525	17,370
Connecticut...	6,498,119	5,363,277	497,454	41,762
Delaware......	1,055,308	3,187	57,768	482,511
Florida........	371,498	18,015	23,247	1,027
Georgia.......	4,640,559	46,976	990,019	1,088,534
Illinois........	12,526,543	1,278,225	2,150,113	9,414,575
Indiana.......	12,881,535	624,564	2,610,287	6,214,458
Iowa..........	2,171,188	209,840	373,898	1,530,581
Kentucky....	9,947,523	213,954	2,297,433	2,142,822
Louisiana.....	683,069	1,957	109,897	417
Maine.........	9,243,811	2,434,454	1,364,034	296,259
Maryland.....	3,086,160	3,975	477,438	4,494,680
Massachusetts	8,071,370	7,088,142	585,138	31,211
Michigan.....	7,065,878	1,011,492	2,043,283	4,925,889
Mississippi...	4,346,234	21,191	559,619	137,990
Missouri......	7,834,359	203,572	1,627,164	2,981,652
N. Hampshire	6,977,056	3,196,563	1,108,476	185,658
New Jersey...	9,487,210	365,756	375,396	1,601,190
New York....	79,766,094	49,741,413	10,071,301	13,121,498
N. Carolina...	4,146,290	95,921	970,738	2,130,102
Ohio..........	34,449,379	20,819,542	10,196,371	14,487,351
Pennsylvania	39,878,418	2,505,034	4,481,570	15,367,691
Rhode Island	995,670	316,508	129,692	49
S. Carolina...	2,981,850	4,970	487,233	1,066,277
Tennessee....	8,139,585	177,681	1,364,378	1,619,386
Texas........	2,344,900	95,299	131,917	41,729
Vermont......	12,137,980	8,720,834	3,400,717	535,955
Virginia.....	11,089,359	436,292	2,860,765	11,212,616
Wisconsin....	3,633,750	400,283	253,963	4,286,131
Territories...	295,984	73,826	71,894	517,562

SUBTRACTION.

69. Subtraction is the process of determining the difference, between two numbers of the same unit value.

70. The **Minuend** is the number to be subtracted from.

71. The **Subtrahend** is the number to be subtracted.

72. The **Difference** or **Remainder** is the result obtained by the process of subtraction.

73. When the given numbers contain more than one figure each, the method of subtraction depends upon the following principles:

I. If the units of each order in the subtrahend be taken separately from the units of like order in the minuend, the sum of the differences must be equal to the entire difference of the given numbers. (Ax. 10.)

II. If both minuend and subtrahend be equally increased, the remainder will not be changed.

1. From 928 take 275.

OPERATION.

Minuend,	928
Subtrahend,	275
Remainder,	653

ANALYSIS. We first subtract 5 units from 8 units, and obtain 3 units for a partial remainder. As we cannot take 7 tens from 2 tens, we add 10 tens to the 2 tens, making 12 tens; then 7 tens from 12 tens leave 5 tens, the second partial remainder. Now, since we added 10 tens, or 1 hundred, to the minuend, we will add 1 hundred to the subtrahend, and the true remainder will not be changed (II); thus, 1 hundred added to 2 hundreds makes 3 hundreds, and this sum subtracted from 9 hundreds leaves 6 hundreds; and we have for the total remainder, 653.

NOTE.—The process of adding 10 to the minuend is sometimes called *borrowing* 10, and that of adding 1 to the next figure of the subtrahend, *carrying* 1.

74. From these principles and illustrations we deduce the following

RULE. I. *Write the less number under the greater, placing units of the same order under each other.*

II. *Begin at the right hand, and take each figure of the subtrahend from the figure above it, and write the result underneath.*

III. *If any figure in the subtrahend be greater than the corresponding figure above it, add* 10 *to that upper figure before subtracting, and then add* 1 *to the next left hand figure of the subtrahend.*

75. Proof. It is evident that the subtrahend and remainder must together contain as many units as the minuend; hence, to prove subtraction, we have three methods:

1st. Add the remainder to the subtrahend; the sum will be equal to the minuend. Or,

2d. Subtract the remainder from the minuend; the difference will be equal to the subtrahend. Or,

3d. Find the excess of 9's in the remainder and subtrahend together, and it will be equal to the excess of 9's in the minuend.

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)	(4.)
From	47965	103767	57610218	89764321
Take	26714	98731	8306429	83720595
Rem.	21251	5036	49303789	6043726

5. From 180037561 take 5703746.

6. From 2460371219 take 98720342.

7. 89037426175 — 2435036749 = how many?

8. 10000033421 — 999044110 = how many?

9. A certain city contains 146758 inhabitants, which is 3976 more than it contained last year; how many did it contain last year? *Ans.* 142,782.

10. The first newspaper published in America was issued at Boston in 1704; how long was that before the death of Benjamin Franklin, which occurred in 1790?

11. A merchant sold a quantity of goods for $42017, which was $1675 more than they cost him; how much did they cost him? *Ans.* $40,342.

12. In 1858 the exports of the United States amounted to

$324644421, and the imports to $282613150; how much did the exports exceed the imports? *Ans.* $42,031,271.

13. In 1858 the gold coinage of the United States amounted to $52889800, and the silver to $8233287; how much did the gold exceed the silver coinage?

14. The South in 1850 produced 978311690 pounds of cotton, valued at $101834616, and 237133000 pounds of sugar valued at $16599310; how much did the cotton exceed the sugar in quantity and in value? *Ans.* 741,178,690 pounds; $85,235,306.

15. The area of the Chinese Empire is 5110000 square miles, and that of the United States 2988892 square miles; the estimated population of the former is 340000000, and that of the latter in 1850 was 23363714. What is the difference in area and in population?

16. The population of London in 1850 was 2362000, and that of New York city 515547; how many more inhabitants had London than New York? *Ans.* 1,846,453.

17. The total length of railroads in operation in the United States, January 1, 1859, was 27857 miles, and the total length of the canals 5131 miles; how many miles more of railroad than of canal? *Ans.* 22,726.

18. The entire deposit of domestic gold at the United States Mint and its branches, to June, 1859, was $470341478, of which $451310840 was from California; how much was received from other sources? *Ans.* $19,030,638.

19. During the year ending September 30, 1858, the number of letters exchanged between the United States and Great Britain were 1765015 received, and 1603609 sent; between the United States and France, 624795 received, and 639906 sent. How many letters did the exchange with Great Britain exceed those with France? *Ans.* 2,103,923.

20. The Southern States in 1850 had a population of 6696061, the Middle States 6624988, and the Eastern States 2728116; how many more inhabitants had the Middle and Eastern States than the Southern States?

21. Having $20000, I wish to know how much more I must

accumulate to be able to purchase a piece of property worth $23470, and have $5400 left? *Ans.* $8,870.

22. A has $3540 more than B, and $1200 less than C, who has $20600; D has as much as A and B together. How much has D? *Ans.* $35,260.

TWO OR MORE SUBTRAHENDS.

76. Two or more numbers may be taken from another at a single operation, as shown by the following example:

1. A man having 1278 barrels of flour, sold 236 barrels to A, 362 to B, and 387 to C; how many had he left?

OPERATION.

Minuend,	1278
Subtrahends,	236
	362
	387
Remainder,	293

ANALYSIS. Since the remainder sought, added to the subtrahends, must be equal to the minuend, we add the columns of the subtrahends, and supply such figures in the remainder as, combined with these sums, will produce the minuend. Thus, 7 and 2 are 9, and 6 are 15, and 3 (supplied in the remainder sought) are 18; then, carrying the tens' figure of the 18, 1 and 8 are 9, and 6 are 15, and 3 are 18, and 9 (supplied in the remainder) are 27; lastly, 2 to carry to 3 are 5, and 3 are 8, and 2 are 10, and 2 (supplied in the remainder) are 12; and the whole remainder is 293. Hence, the following

RULE. I. *Having written the several subtrahends under the minuend, add the first column of the subtrahends, and supply such a figure in the remainder sought, as, added to this partial sum, will give an amount having for its unit figure the figure above in the minuend.*

II. *Carry the tens' figure of this amount to the next column of the subtrahends, and proceed as before till the entire remainder is obtained.*

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)	(4.)
From	47962	127368	903486	2503734
Take	21435	56304	430164	89763
	15672	4782	132875	94207
	456	9156	67321	237564
Rem.	10399	57126	273126	2082200

5. From 4568 take 1320 + 275 + 320.

6. Subtract 1200 + 750 + 96 from 4756 + 575 + 140 + 84.

7. A man bought four city lots, for which he paid \$15760. For the first he paid \$2175, for the second \$3794, and for the third \$4587; how much did he pay for the fourth? *Ans.* \$5,204.

8. John Wise owns property to the amount of \$75860, of which he has \$45640 invested in real estate, \$25175 in personal property, and the remainder he has in bank; how much has he in bank?

9. Lake Huron contains 20000 square miles; by how much does it exceed the area of Lake Erie and Lake Ontario, the former containing 11000 square miles, and the latter 7000?

Ans. 2000 square miles.

10. In the year 1852, there arrived in the United States 398470 immigrants, of whom 157548 were born in Ireland, and 143429 were born in Germany; how many were born in other countries?

Ans. 97,493.

11. The entire amount of coinage in the United States for the year ending June, 1858, was \$61357088, of which \$52889800 was of gold, \$234000 of copper, and the remainder of silver; how much was of silver?

12 A speculator gained \$5760, and afterward lost \$2746; at another time he gained \$3575, and then lost \$4632. How much did his gains exceed his losses? *Ans.* \$1,957.

13. The Eastern States have an area of 65038 square miles, the Middle States 114624 square miles, and the Southern States 643166 square miles; how many more square miles have the Southern than the Middle and Eastern States?

14. The entire revenue of the United States Post Office Department for the year ending Sept. 30, 1858, was \$8186793, of which sum \$5700314 was received for stamps and stamped letters, and \$904299 for letter-postage in money; how much was received from all other sources? *Ans.* \$1,582,180.

15. The total expenditures of the Department for the same year were \$12722470, of which sum \$7821556 was paid for the transportation of inland mails, \$424497 for the transportation of foreign mails and \$2355016 as compensation to postmasters; how much was expended for all other purposes? **\$2,121,401.**

MULTIPLICATION.

77. **Multiplication** is the process of taking one of two given numbers as many times as there are units in the other.

78. The **Multiplicand** is the number to be taken.

79. The **Multiplier** is the number which shows how many times the multiplicand is to be taken.

80. The **Product** is the result obtained by the process of multiplication.

81. The **Factors** are the multiplicand and multiplier.

NOTES. — 1. Factors are producers, and the multiplicand and multiplier are called factors because they produce the product.

2. Multiplication is a short method of performing addition when the numbers to be added are equal.

82. The method of multiplying when either factor contains more than one figure, depends upon the following principles:

It is evident that 5 units taken 3 times is the same as 3 units taken 5 times; and the same is true of any two factors. Hence,

I. The product of any two factors is the same, whichever is used as the multiplier. If units be multiplied by units, the product will be units; if tens be multiplied by units, or units by tens, the product will be tens; and so on. That is,

II. If either factor be units of the first order, the product will be units of the same order as the other factor.

III. If the units of each order in the multiplicand be taken separately as many times as there are units in the multiplier, the sum of the products must be equal to the entire product of the given numbers, (Ax. 10).

1. Multiply 346 by 8.

OPERATION.

Multiplicand,	346
Multiplier,	8
Product,	2768

ANALYSIS. In this example it is required to take 346 eight times. If we take the units of each order 8 times, we shall take the entire number 8 times, (III). Therefore, commencing at the right hand, we say; 8 times 6 units are 48 units, or 4 tens and 8 units; writing the 8 units in the product in units' place, we reserve the 4 tens to add to the next product; 8 times 4 tens are 32 tens, and the 4 tens reserved in the last product added, are 36 tens, or 3 hundreds and 6 tens; we write the 6 tens in the product in tens' place, and reserve the 3 hundreds to add to the next product; 8 times 3 hundreds are 24 hundreds, and the 3 hundreds reserved in the last product added, are 27 hundreds, which being written in the product, each figure in the place of its order, gives, for the entire product, 2768.

2. Multiply 758 by 346.

OPERATION.

```
   758
   346
  ----
  4548
 3032
2274
------
262268
```

ANALYSIS. In this example the multiplicand is to be taken 346 times, which may be done by taking the multiplicand separately as many times as there are units expressed by each figure of the multiplier. 758 multiplied by 6 units is 4548 *units*, (II); 758 multiplied by 4 tens is 3032 *tens*, (II), which we write with its lowest order in tens' place, or under the figure used as a multiplier; 758 multiplied by 3 hundreds is 2274 *hundreds*, (II), which we write with its lowest order in hundreds' place. Since the sum of these products must be the entire product of the given numbers, (III), we add the results, and obtain 262268, the answer.

NOTES.—1. When the multiplier contains two or more figures, the several results obtained by multiplying by each figure are called *partial products*.

2. When there are ciphers between the significant figures of the multiplier, pass over them, and multiply by the significant figures only.

83. From these principles and illustrations we deduce the following general

RULE. I. *Write the multiplier under the multiplicand, placing units of the same order under each other.*

II. *Multiply the multiplicand by each figure of the multiplier successively, beginning with the unit figure, and write the first figure of each partial product under the figure of the multiplier used, writing down and carrying as in addition.*

III. *If there are partial products, add them, and their sum will be the product required.*

NOTE. — The multiplier denotes simply the number of times the multiplicand is to be taken; hence, in the analysis of a problem, the multiplier must be considered as abstract, though the multiplicand may be either abstract or concrete.

84. PROOF. There are two principal methods of proving multiplication

1st. By varying the partial products.

Invert the order of the factors; that is, multiply the multiplier by the multiplicand; if the product is the same as the first result, the work is correct.

2d. By excess of 9's.

85. The illustration of this method depends upon the following principles:

I. If the excess of 9's be subtracted from a number, the remainder will be a number having no excess of 9's.

II. If a number having no excess of 9's be multiplied by any number, the product will have no excess of 9's.

1. Let it be required to multiply 473 by 138.

OPERATION.

$$473 = 468 + 5$$
$$138 = 135 + 3$$

$$\text{Partial products.}\left\{\begin{array}{rcr} 468 \times 135 & = & 63180 \\ 5 \times 135 & = & 675 \\ 468 \times 3 & = & 1404 \\ 5 \times 3 & = & 15 \end{array}\right.$$

$$\text{Entire product,} \qquad 65274$$

ANALYSIS. The excess of 9's in 473 is 5, and $473 = 468 + 5$, of which the first part, 468, contains no excess of 9's, (I). The excess of 9's in 138 is 3, and $138 = 135 + 3$, of which the first part, 135, contains no excess of 9's, (I). Multiplying both parts of the multiplicand by each part of the multiplier, we have four partial products, of which the first three have no excess of 9's, because each contains a factor having no excess of 9's, (II). Therefore, the excess of 9's in the entire product must be the same as the excess of 9's in the last partial product, 15, which we find to be $1 + 5 = 6$. The same may be shown of any two numbers. Hence, to prove multiplication by excess of 9's,

Find the excess of 9's in each of the two factors, and multiply them together; if the excess of 9's in this product is equal to the excess of 9's in the product of the factors, the work is supposed to be right.

NOTE.—If the excess of 9's in either factor is 0, the excess of 9's in the product will be 0, (II).

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)	(4.)
Multiply	475	3172	9827	7198
By	9	14	84	216
Prod.	4275	44408	825468	1554768

5. Multiply 31416 by 175. *Ans.* 5497800.

6. Multiply 40930 by 779. *Ans.* 31884470.

7. Multiply 46481 by 936.

8. Multiply 15607 by 3094.

9. Multiply 281216 by 978. *Ans.* 275029248.

10. Multiply 30204 by 4267. *Ans.* 128,880,468.

11. What is the product of 4444 × 2341? *Ans.* 10,403,404.

12. What is the product of 4567 × 9009? *Ans.* 41,144,103.

13. What is the product of 2778588 × 9867? *Ans.* 27,416,327,796.

14. What is the product of 7060504 × 30204? *Ans.* 213,255,462,816.

15. What will be the cost of building 276 miles of railroad at $61320 per mile? *Ans.* $16,924,320.

16. If it require 125 tons of iron rail for one mile of railroad, how many tons will be required for 196 miles?

17. A merchant tailor bought 36 pieces of broadcloth, each piece containing 47 yards, at 7 dollars a yard; how much did he pay for the whole? *Ans.* $11,844.

18. The railroads in the State of New York, in operation in 1858, amounted to 2590 miles in length, and their average cost was about $52916 per mile; what was the total cost of the railroads in New York? *Ans.* $137,052,440.

19. The Illinois Central Railroad is 700 miles long, and cost $45210 per mile; what was its total cost?

20. The salary of a member of Congress is $3000, and in 1860 there were 303 members; how much did they all receive?

21. The United States contain an area of 2988892 square miles, and in 1850 they contained 8 inhabitants to each square mile; what was their entire population? *Ans.* 23,911,136.

22. Great Britain and Ireland have an area of 118949 square miles, and in 1850 they contained a population of 232 to the square mile; what was their entire population? *Ans.* 27,596,168.

23. The national debt of France amounts to $32 for each indi-

vidual, and the population in 1850 was 35781628; what was the entire debt of France? *Ans.* 1,145,012,096.

POWERS OF NUMBERS.

86. We have learned (**15**) that a power is the product arising from multiplying a number by itself, or repeating it any number of times as a factor; (**16**), that a root is a factor repeated to produce a power; and (**40**) an index or exponent is the number indicating the power to which a number is to be raised.

87. The **First Power** of any number is the number itself, or the root; thus, 2, 3, 5, are first powers or roots.

88. The **Second Power,** or **Square,** of a number is the product arising from using the number two times as a factor; thus, $2^2 = 2 \times 2 = 4$; $5^2 = 5 \times 5 = 25$.

89. The **Third Power,** or **Cube,** of a number is the product arising from using the number three times as a factor; thus, $4^3 = 4 \times 4 \times 4 = 64$.

90. The higher powers are named in the order of their numbers, as *Fourth Power, Fifth Power, Sixth Power,* etc.

91. 1. What is the third power or cube of 23?

OPERATION.

$23 \times 23 \times 23 = 12167$

ANALYSIS. We multiply 23 by 23, and the product by 23; and, since 23 has been taken 3 times as a factor, the last product, 12167, must be the third power or cube of 23. Hence,

RULE. *Multiply the number by itself as many times, less* 1, *as there are units in the exponent of the required power.*

NOTE.—The process of producing any required power of a number by multiplication is called *Involution.*

EXAMPLES FOR PRACTICE.

1. What is the square of 72? *Ans.* 5184.
2. What is the fifth power of 12? *Ans.* 248832.
3. What is the cube of 25?
4. What is the seventh power of 7? *Ans.* 823543.

5. What is the fourth power of 19? *Ans.* 130321.

6. Required the sixth power of 3. *Ans.* 729.

7. Find the powers indicated in the following expressions: 9^5, 11^3, 18^2, 125^4, 786^2, 94^6, 100^4, 17^3, 251^5.

8. Multiply 8^3 by 15^2. *Ans.* 115200.

9. What is the product of $25^2 \times 3^4$?

10. $7^3 \times 200 = 4^4 \times 11^2$, and how many? *Ans.* 37.624.

GENERAL PRINCIPLES OF MULTIPICATION.

92. There are certain general principles of multiplication, of use in various contractions and applications which occur in subsequent portions of this work. These relate, 1st, to changing the factors by addition or subtraction; 2d, to the use of successive factors in continued multiplication.

CHANGING THE FACTORS BY ADDITION OR SUBTRACTION.

93. The product is equal to either factor taken as many times as there are units in the other factor. (**82**, I). Hence,

I. *Adding* 1 *to either factor, adds the other factor to the product.*

II. *Subtracting* 1 *from either factor, subtracts the other factor from the product.* Hence,

III. ADDING *any number to either factor,* INCREASES *the product by as many times the other factor as there are units in the number added; and* SUBTRACTING *any number from either factor,* DIMINISHES *the product by as many times the other factor as there are units in the number subtracted.*

CONTINUED MULTIPLICATION.

94. A **Continued Multiplication** is the process of finding the product of three or more factors, by multiplying the first by the second, this result by the third, and so on.

95. To show the nature of continued multiplication, we observe:

1st. If any number, as 17, be multiplied by any other number, as 3, the result will be 3 times 17; if this result be multiplied by another number, as 5, the new product will be 5 times 3 times 17, which is evidently 15 times 17. Hence, $17 \times 3 \times 5 = 17 \times 15$; the same reasoning would extend to three or more multipliers.

2d. Since 5 times 3 is equal to 3 times 5, (**82**, I), it follows that 17 multiplied by 5 times 3 is the same as 17 multiplied by 3 times 5; or $17 \times 3 \times 5 = 17 \times 5 \times 3$. Hence, the product is not changed by changing the orders of the factors.

These principles may be stated as follows:

I. If a given number be multiplied by several factors in continued multiplication, the result will be the same as if the given number were multiplied by the product of the several multipliers.

II. The product of several factors in continued multiplication will be the same, in whatever order the factors are taken.

CONTRACTIONS IN MULTIPLICATION.

CASE I.

96. When the multiplier is a composite number.

A **Composite Number** is one that may be produced by multiplying together two or more numbers. Thus, 18 is a composite number, since $6 \times 3 = 18$; or, $9 \times 2 = 18$; or, $3 \times 3 \times 2 = 18$.

97. The **Component Factors** of a number are the several numbers which, multiplied together, produce the given number; thus, the component factors of 20 are 10 and 2 ($10 \times 2 = 20$); or, 4 and 5 ($4 \times 5 = 20$); or, 2 and 2 and 5 ($2 \times 2 \times 5 = 20$).

NOTE.—The pupil must not confound the *factors* with the *parts* of a number. Thus, the *factors* of which 12 is composed, are 4 and 3 ($4 \times 3 = 12$); while the *parts* of which 12 is composed are 8 and 4 ($8 + 4 = 12$); or 10 and 2 ($10 + 2 = 12$). The *factors* are *multiplied*, while the *parts* are *added*, to produce the number.

98. 1. Multiply 327 by 35.

OPERATION.

$$\begin{array}{r} 327 \\ 7 \\ \hline 2289 \\ 5 \\ \hline 11445 \end{array}$$

ANALYSIS. The factors of 35 are 7 and 5. We multiply 327 by 7, and this result by 5, and obtain 11445, which must be the same as the product of 327 by 5 times 7, or 35. (**95**, I). Hence we have the following

RULE. I. *Separate the composite number into two or more factors.*

II. *Multiply the multiplicand by one of these factors, and that product by another, and so on until all the factors have been used successively; the last product will be the product required.*

NOTE.—The factors may be used in any order that is most convenient, (**95**, II).

EXAMPLES FOR PRACTICE.

1. Multiply 736 by 24. *Ans.* 17664.
2. Multiply 538 by 56. *Ans.* 30128.
3. Multiply 27865 by 84.
4. Multiply 7856 by 144. *Ans.* 1131264.
5. What will 56 horses cost at 185 each?
6. If a river discharge 17740872 cubic feet of water in one hour, how much will it discharge in 96 hours?

Ans. 1703123712 cubic feet.

CASE II.

99. When the multiplier is a unit of any order.

If we annex a cipher to the multiplicand, each figure is removed *one* place toward the left, and consequently the value of the whole number is increased tenfold, (**57**, III). If two ciphers are annexed, each figure is removed *two* places toward the left, and the value of the number is increased one hundred fold; and every additional cipher increases the value tenfold. Hence, the

RULE *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier.*

EXAMPLES FOR PRACTICE.

1. Multiply 364 by 100. *Ans.* 36400.
2. Multiply 248 by 1000. *Ans.* 248000.
3. What cost 1000 head of cattle at 50 dollars each?
4. Multiply one million by one hundred thousand?
5. How many letters will there be on 100 sheets, if each sheet have 100 lines, and each line 100 letters? *Ans.* 1000000.

CASE III.

100. When there are ciphers at the right hand of one or both of the factors.

1. Multiply 7200 by 40.

OPERATION.
7200
40
288000

ANALYSIS. The multiplicand, factored, is equal to 72×100; the multiplier, factored, is equal to 4×10; and as these factors taken in any order will give the same product, (**95**, II), we first multiply 72 by 4, then this product by 100 by annexing two ciphers, and this product by 10 by annexing one cipher. Hence, the following

RULE. *Multiply the significant figures of the multiplicand by those of the multiplier, and to the product annex as many ciphers as there are ciphers on the right of both factors.*

EXAMPLES FOR PRACTICE.

1. Multiply 740 by 300. *Ans.* 222000.
2. Multiply 36000 by 240. *Ans.* 8640000.
3. Multiply 20700 by 500.
4. Multiply 4007000 by 3002. *Ans.* 12029014000.
5. Multiply 300200 by 640.

CASE IV.

101. When one part of the multiplier is a factor of another part.

1. Multiply 4739 by 357.

OPERATION.	
4739	
357	
33173	Prod. by 7 units.
165865	Prod. by 35 tens.
1691823	*Ans.*

ANALYSIS. In this example, 7, one part of the multiplier, is a factor of 35, the other part. We first find, in the usual manner, the product of the multiplicand by the 7 units; multiplying this product by 5, and writing the first figure of the result in tens' place, we obtain the product of the multiplicand by $7 \times 5 \times 10 = 35$ tens; and the sum of these two partial products must be the whole product required.

2. Multiply 58327 by 21318.

OPERATION.

58327	
21318	
174981	Prod. by 3 hundreds.
1049886	Prod. by 18 units.
1224867	Prod. by 21 thousands.
1243414986	*Ans.*

ANALYSIS. In this example, the 3 hundreds is a factor of 18, the part on the right of it, and also of 21, the part on the left of it. We first multiply by 3, writing the first figure in hundreds' place; multiplying this product by 6, and writing the first figure in units' place, we obtain the product of the multiplicand by $3 \times 6 =$ 18 units; multiplying the first partial product by 7, and writing the first figure in thousands' place, we obtain the product of the multiplicand by $7 \times 3 \times 1000 = 21$ thousands, and the sum of these three partial products must be the entire product required.

NOTE.—The product obtained by multiplying any partial product is called a *derived product.*

102. From these illustrations we have the following

RULE. I. *Find the product of the multiplicand by some figure of the multiplier which is a factor of one or more parts of the multiplier.*

II. *Multiply this product by that factor which, taken with the figure of the multiplier first used, will produce other parts of the multiplier, and write the first figure of each result under the first figure of the part of the multiplier thus used.*

III. *In like manner, find the product, either direct or derived, for every figure or part of the multiplier; the sum of all the products will be the whole product required.*

EXAMPLES FOR PRACTICE.

1. Multiply 5784 by 246. *Ans.* 1422864.
2. Multiply 3785 by 721. *Ans.* 2728985.
3. Multiply 472856 by 54918. *Ans.* 25968305808.
4. Multiply 43785 by 7153. *Ans.* 313194105.
5. Multiply 573042 by 24816. *Ans.* 14220610272.
6. Multiply 78563721 by 127369.
7. Multiply 43725652 by 5187914.

8. Multiply 3578426785 by 64532164.

9. Multiply 2703605 by 4249784.

10. What is the product of 9462108 multiplied by 16824?
Ans. 159,190,504,992.

EXAMPLES COMBINING THE PRECEDING RULES.

1. A man bought two farms, one containing 175 acres at $28 per acre, and the other containing 320 acres at $37 per acre; what was the cost of both? *Ans.* $16,740.

2. If a man receive $1200 salary, and pay $364 for board, $275 for clothing, $150 for books, and $187 for other expenses, how much can he save in 5 years? *Ans.* $1,120.

3. Two persons start from the same point, and travel in opposite directions; one travels 29 miles a day, and the other 32 miles. How far apart will they be in 17 days? *Ans.* 1,037 miles.

4. A drover bought 127 head of cattle at $34 a head, and 97 head at $47 a head, and sold the whole lot at $40 a head; what was his entire profit or loss? *Ans.* $83 profit.

5. Multiply $675 - (77 + 56)$ by $(3 \times 156) - (214 - 28)$.
Ans. 152844.

6. Multiply $98 + \overline{6 \times (37 + 50)}$ by $\overline{(64 - 50) \times 5} - 10$.
Ans. 37200.

7. What is the product of $\overline{(14 \times 25) - (9 \times 36) + 4324} \times \overline{(280 - 112) + (376 + 42) \times 4}$? *Ans.* 8,004,000.

8. In 1850 South Carolina cultivated 29967 farms and plantations, containing an average of 541 acres each, at an average value of $2751 for each farm; New Jersey cultivated 23905 farms, containing an average of 115 acres each, at an average value of $5030 per farm. How much more were the farming lands of the latter valued at, than those of the former?

9. There are in the United States 1922890880 acres of land; of this there were reported under cultivation, in 1850, 1449075 farms, each embracing an average of 203 acres. How many acres were still uncultivated?

10. Each of the above farms in the United States was valued at an average of $2258, and upon each farm there was an average

of \$105 in implements and machinery. What was the aggregate value of the farms and implements? *Ans.* \$3,424,164,225.

Find the values of the following expressions:

11. $2^4 \times 5^5 - 7^3$? *Ans.* 49,657.

12. $15^3 - (3^2 \times 2^5) + 208^2 - 9 \times 2^4$? *Ans.* 46,207.

13. $2^2 + 3^3 + 4^4 + 5^5 + 6^6$?

14. In 1852 Great Britain consumed 1200000 bales of American cotton; allowing each bale to contain 400 pounds, what was its total weight?

15. If a house is worth \$2450, and the farm on which it stands 6 times as much, lacking \$500, and the stock on the farm twice as much as the house, what is the value of the whole?

Ans. \$21550.

16. A flour merchant bought 1500 barrels of flour at 7 dollars a barrel; he sold 800 barrels at 10 dollars a barrel, and the remainder at 6 dollars a barrel. How much was his gain?

17. A man invests in trade at one time \$450, at another \$780, at another \$1250, and at another \$2275; how much must he add to these sums, that the amount invested by him shall be increased fourfold? *Ans.* \$14,265.

18. At the commencement of the year 1858 there were in operation in the United States 35000 miles of telegraph; allowing the average cost to be \$115 per mile, what was the total cost?

19. The cost of the Atlantic Telegraph Cable, as originally made, was as follows; 2500 miles at \$485 per mile, 10 miles deep-sea cable at \$1450 per mile, and 25 miles shore ends at \$1250 per mile. What was its total cost? *Ans.* \$1,258,250.

20. For the year ending June 30, 1859, there were coined in the United States 1401944 double eagles valued at twenty dollars each, 62990 eagles, 154555 half eagles, and 22059 three dollar pieces; what was the total value of this gold coin?

Ans. \$29,507,732.

DIVISION.

103. **Division** is the process of finding how many times one number is contained in another.

104. The **Dividend** is the number to be divided.

105. The **Divisor** is the number to divide by.

106. The **Quotient** is the result obtained by the process of division.

107. The **Reciprocal** of a number is 1 divided by the number. Thus, the reciprocal of 15 is $1 \div 15$, or $\frac{1}{15}$.

NOTES.—1. When the dividend does not contain the divisor an exact number of times, the part of the dividend left is called the *Remainder*, which must be less than the divisor.

2. As the remainder is always a part of the dividend, it is always of the same name or kind.

3. When there is no remainder the division is said to be *exact*.

108. The method of dividing any number by another depends upon the following principles:

I. Division is the reverse of multiplication, the dividend corresponding to the product, and the divisor and quotient to the factors.

II. If all the parts of a number be divided, the entire number will be divided.

Since the remainder in dividing any part of the dividend must be less than the divisor, it can be divided only by being expressed in units of a lower order. Hence,

III. The operation must commence with the units of the highest order.

1. Divide 2742 by 6.

OPERATION.

$$\begin{array}{r} 6)\underline{2742} \\ 457 \end{array}$$ *Ans.*

ANALYSIS. We write the divisor at the left of the dividend, separated from it by a line. As 6 is not contained in 2 thousands, we take the 2 thousands and 7 hundreds together, and proceed thus; 6 is contained in 27 hundreds 4 hundred times, and the remainder is 3 hundreds; we write 4 in hundreds' place in the quotient, and unite the remainder, 3 hundreds, to the next figure of the dividend, making 34 tens; then, 6 is contained in 34 tens 5 tens times, and the remainder is 4 tens; writing 5 tens in its place in the quotient, we unite the remainder to the next figure in the dividend, making 42; 6 is contained in 42 units 7 times, and there is no remainder; writing 7 in its place in the quotient, we have the entire quotient, 457.

NOTE 1.—The different numbers which we divide in obtaining the successive figures of the quotient, are called *partial dividends*.

2. Divide 18149 by 56.

OPERATION.

```
56)18149(324 5/56 Ans.
   168
   ---
    134
    112
    ---
     229
     224
     ---
       5
```

ANALYSIS. As neither 1 nor 18 will contain the divisor, we take three figures, 181, for the first partial dividend. 56 is contained in 181 3 times, and a remainder; we write the 3 as the first figure in the quotient, and then multiply the divisor by this quotient figure; 3 times 56 is 168, which subtracted from 181, leaves 13; to this remainder we annex or bring down 4, the next figure of the dividend, and thus form 134, the next partial dividend; 56 is contained in 134 2 times, and a remainder; 2 times 56 is 112, which subtracted from 134, leaves 22; to this remainder we bring down 9, the last figure of the dividend, and we have 229, the last partial dividend; 56 is contained in 229 4 times, and a remainder; 4 times 56 is 224, which subtracted from 229, gives 5, the final remainder, which we write in the quotient with the divisor below it, thus completing the division, (**35**).

NOTE 2.—When the multiplication and subtraction are performed mentally, as in the first example, the operation is called *Short Division*; but when the work is written out in full, as in the second example, the operation is called *Long Division*. The principles governing the two methods are the same.

109. From these principles and illustrations we derive the following general

RULE. I. *Beginning at the left hand, take for the first partial dividend the fewest figures of the given dividend that will contain the divisor one or more times; find how many times the divisor is contained in this partial dividend, and write the result in the quotient; multiply the divisor by this quotient figure, and subtract the product from the partial dividend used.*

II. *To the remainder bring down the next figure of the dividend, with which proceed as before; and thus continue till all the figures of the dividend have been divided.*

III. *If the division is not exact, place the final remainder in the quotient, and write the divisor underneath.*

110. Proof. There are two principal methods of proving division.

1st. By multiplication.

Multiply the divisor and quotient together, and to the product add the remainder, if any; if the result be equal to the dividend, the work is correct. (**108**, I.)

Note.—In multiplication, the two factors are given to find the product; in division, the product and one of the factors are given to find the other factor.

2d. By excess of 9's.

111. Subtract the remainder, if any, from the dividend, and find the excess of 9's in the result. Multiply the excess of 9's in the divisor by the excess of 9's in the quotient, and find the excess of 9's in the product; if the latter excess is the same as the former, the work is supposed to be correct. (**85.**)

EXAMPLES FOR PRACTICE.

(1.)	(2.)	(3.)	(4.)
6)473832	8)972496	9)1370961	12)73042164

	Quotients.
5. Divide 170352 by 36.	4732.
6. Divide 409887 by 47.	8721.
7. Divide 443520 by 84.	5280.
8. Divide 36380250 by 125.	291042.
9. Divide 1554768 by 216.	
10. Divide 3931476 by 556.	

	Rem.
11. Divide 48288058 by 3094.	
12. Divide 11214887 by 232.	7.
13. Divide 27085946 by 216.	194.
14. Divide 29137062 by 5317.	5219.
15. Divide 4917968967 by 2359.	1255.

16. What is the value of $721198 \div 291$? Rem. 100.
17. What is the value of $3844449 \div 657$? 342.
18. What is the value of $536819237 \div 907$? 403.
19. What is the value of $571943007145 \div 37149$? 12214.
20. What is the value of $48659910 \div 54001$? 5009.

21. The annual receipts of a manufacturing company are \$147675; how much is that per day, there being 365 days in the year? *Ans.* $\$404\frac{215}{365}$.

22. The New York Central Railroad Company, in 1859, owned 556 miles in length of railroad, which cost, for construction and equipment, \$30732518; what was the average cost per mile? *Ans.* $\$55,274\frac{174}{556}$.

23. The Memphis and Charleston Railroad is 287 miles in length, and cost \$5572470; what was the average cost per mile? *Ans.* $\$19,416\frac{78}{287}$.

24. The whole number of Post offices in the United States, in 1858, was 27977, and the revenue was \$8186793; what was the average income to an office?

ABBREVIATED LONG DIVISION.

112. We may avoid writing the products in long division, and obtain the successive remainders by the method of subtraction employed in the case of several subtrahends. (**76.**)

1. Divide 261249 by 487.

OPERATION.

```
487)261249(536
      177
       313
        217 Rem.
```

ANALYSIS. Dividing the first partial dividend, 2612, we obtain 5 for the first figure of the quotient. We now multiply 487 by 5; but instead of writing the product, and subtracting it from the partial dividend, we simply observe what figures must be added to the figures of the product, as we proceed, to give the figures of the partial dividend, and write them for the remainder sought. Thus, 5 times 7 are 35, and 7 (written in the remainder,) are 42, a number whose unit figure is the same as the right hand figure of the partial dividend; 5 times 8 are 40, and 4, the tens of the 42, are 44, and 7 (written in the remainder,) are 51; 5

times 4 are 20, and 5, the tens of the 51, are 25, and 1 (written in the remainder,) are 26. We next consider the whole remainder, 177, as joined with 4, the next figure of the dividend, making 1774 for the next partial dividend. Proceeding as before, we obtain 313 for the second remainder, 217 for the final remainder, and 536 for the entire quotient. Hence, the following

RULE. I. *Obtain the first figure in the quotient in the usual manner.*

II. *Multiply the first figure of the divisor by this quotient figure, and write such a figure in the remainder as, added to this partial product, will give an amount having for its unit figure the first or right hand figure of the partial dividend used.*

III. *Carry the tens' figure of the amount to the product of the next figure of the divisor, and proceed as before, till the entire remainder is obtained.*

IV. *Conceive this remainder to be joined to the next figure of the dividend, for a new partial dividend, and proceed as with the former, till the work is finished.*

EXAMPLES FOR PRACTICE.

1. Divide 77112 by 204. *Ans.* 378.
2. Divide 65664 by 72. *Ans.* 912.
3. Divide 7913576 by 209. *Ans.* 37864.
4. Divide 6636584 by 698.
5. Divide 4024156 by 8903. *Ans.* 452.
6. Divide 760592 by 6791.
7. Divide 101443929 by 25203. *Ans.* $4025\frac{1854}{25203}$.
8. Divide 1246038849 by 269181. *Ans.* 4629.
9. Divide 2318922 by 56240.
10. Divide 1454900 by 17300. *Ans.* $84\frac{1700}{17300}$.

GENERAL PRINCIPLES OF DIVISION.

113. The general principles of division most important in their application, relate; 1st, to changing the terms of division by addition or subtraction; 2d, to changing the terms of division by multiplication or division; 3d, to successive division.

114. The quotient in division depends upon the relative values of the dividend and divisor. Hence, any change in the value of either dividend or divisor must produce a change in the value of the quotient; though certain changes may be made in both dividend and divisor, at the same time, that will not affect the quotient.

CHANGING THE TERMS BY ADDITION OR SUBTRACTION.

115. Since the dividend corresponds to a product, of which the divisor and quotient are factors, we observe,

1st. If the divisor be increased by 1, the dividend must be increased by as many units as there are in the quotient, in order that the quotient may remain the same, (**93**, I); and if the dividend be *not* thus increased, the quotient will be *diminished* by as many units as the number of times the new divisor is contained in the quotient. Thus,

$$84 \div 6 = 14$$
$$84 \div 7 = 14 - \tfrac{14}{7} = 12$$

2d. If the divisor be diminished by 1, the dividend must be diminished by as many units as there are in the quotient, in order that the quotient may remain the same, (**93**, II); and if the dividend be *not* thus diminished, the quotient will be *increased* by as many units as the number of times the new divisor is contained in the quotient. Thus,

$$144 \div 9 = 16$$
$$144 \div 8 = 16 + \tfrac{16}{8} = 18$$

These principles may be stated as follows:

I. *Adding* 1 *to the divisor takes as many units from the quotient as the new divisor is contained times in the quotient.*

II. *Subtracting* 1 *from the divisor adds as many units to the quotient as the new divisor is contained times in the quotient.* Hence,

III. ADDING *any number to the divisor* SUBTRACTS *as many units from the quotient as the new divisor is contained times in the product of the quotient by the number added; and* SUBTRACTING *any number from the divisor* ADDS *as many units to the quotient as the new divisor is contained times in the product of the quotient by the number subtracted.*

CHANGING THE TERMS BY MULTIPLICATION OR DIVISION.

116. There are six cases:

1st. If any divisor is contained in a given dividend a certain number of times, the same divisor will be contained in twice the dividend twice as many times; in three times the dividend, three times as many times; and so on. Hence,

Multiplying the dividend by any number, multiplies the quotient by the same number.

2d. If any divisor is contained in a given dividend a certain number of times, the same divisor will be contained in one half the dividend one half as many times; in one third the dividend, one third as many times; and so on. Hence,

Dividing the dividend by any number, divides the quotient by the same number.

3d. If a given divisor is contained in any dividend a certain number of times, twice the divisor will be contained in the same dividend one half as many times; three times the divisor, one third as many times; and so on. Hence,

Multiplying the divisor by any number, divides the quotient by the same number.

4th. If a given divisor is contained in any dividend a certain number of times, one half the divisor will be contained in the same dividend twice as many times; one third of the divisor, three times as many times; and so on. Hence,

Dividing the divisor by any number, multiplies the quotient by the same number.

5th. It a given divisor is contained in a given dividend a certain number of times, twice the divisor will be contained the same number of times in twice the dividend; three times the divisor will be contained the same number of times in three times the dividend; and so on. Hence, *Multiplying both dividend and divisor by the same number does not alter the quotient.*

6th. If a given divisor is contained in a given dividend a certain number of times, one half the divisor will be contained the same number of times in one half the dividend; one third of the divisor will be contained the same number of times in one third of the dividend; and so on. Hence,

Dividing both dividend and divisor by the same number does not alter the quotient.

NOTE.—If a number be multiplied and the product divided by the same number, the quotient will be equal to the number multiplied; hence the 5th case may be regarded as a direct consequence of the 1st and 3d; and the 6th, as the direct consequence of the 2d and 4th.

To illustrate these cases, take 24 for a dividend and 6 for a divisor; then the quotient will be 4, and the several changes may be represented in their order as follows:

	Dividend.	Divisor.	Quotient.	
	24 ÷	6 =	4	
1.	48 ÷	6 =	8	Multiplying the dividend by 2 multiplies the quotient by 2.
2.	12 ÷	6 =	2	Dividing the dividend by 2 divides the quotient by 2.
3.	24 ÷	12 =	2	Multiplying the divisor by 2 divides the quotient by 2.
4.	24 ÷	3 =	8	Dividing the divisor by 2 multiplies the quotient by 2.
5.	48 ÷	12 =	4	Multiplying both dividend and divisor by 2 does not alter the quotient.
6.	12 ÷	3 =	4	Dividing both dividend and divisor by 2 does not alter the quotient.

117. These six cases constitute three general principles, which may now be stated as follows:

PRIN. I. *Multiplying the dividend multiplies the quotient; and dividing the dividend divides the quotient.*

PRIN. II. *Multiplying the divisor divides the quotient; and dividing the divisor multiplies the quotient.*

PRIN. III. *Multiplying or dividing both dividend and divisor by the same number, does not alter the quotient.*

118. These three principles may be embraced in one

GENERAL LAW.

A change in the dividend produces a LIKE *change in the quotient; but a change in the divisor produces an* OPPOSITE *change in the quotient.*

SUCCESSIVE DIVISION.

119. Successive Division is the process of dividing one number by another, and the resulting quotient by a second divisor, and so on.

Successive division is the reverse of continued multiplication. Hence,

I. If a given number be divided by several numbers in successive division, the result will be the same as if the given number were divided by the product of the several divisors, (**95**, I).

II. The result of successive division is the same, in whatever order the divisors are taken, (**95**, II).

CONTRACTIONS IN DIVISION.

CASE I.

120. When the divisor is a composite number.

1. Divide 1242 by 54.

OPERATION.

```
6) 1242
   ----
9)  207
   ----
     23 Ans.
```

ANALYSIS. The component factors of 54 are 6 and 9. We divide 1242 by 6, and the resulting quotient by 9, and obtain for the final result, 23, which must be the same as the quotient of 1242 divided by 6 times 9, or 54, (**119**, I). We might have obtained the same result by dividing first by 9, and then by 6, (**119**, II). Hence the following

RULE. *Divide the dividend by one of the factors, and the quo-*

tient thus obtained by another, and so on if there be more than two factors, until every factor has been made a divisor. The last quotient will be the quotient required.

TO FIND THE TRUE REMAINDER.

121. If remainders occur in successive division, it is evident that the true remainder must be the least number, which, subtracted from the given dividend, will render all the divisions exact

1. Divide 5855 by 168, using the factors 3, 7, and 8, and find the true remainder.

OPERATION.

3) 5855		
7) 1951	....	2
8) 278	.... $5 \times 3 =$	15
34	... $6 \times 7 \times 3 =$	126
True remainder	....	143

ANALYSIS. Dividing the given dividend by 3, we have 1951 for a quotient, and a remainder of 2. Hence, 2 subtracted from 5855 would render the first division exact, and we therefore write 2 for a part of the true remainder. Dividing 1951 by 7, we have 278 for a quotient, and a remainder of 5. Hence, 5 subtracted from 1951 would render the second division exact. But to diminish 1951 by 5 would require us to diminish 1951×3, the dividend of the first exact division, by $5 \times 3 = 15$, (**93**, III); and we therefore write 15 for the second part of the true remainder. Dividing 278 by 8, we have 34 for a quotient, and a remainder of 6. Hence, 6 subtracted from 278 would render the third division exact. But to diminish 278 by 6 would require us to diminish 278×7, the dividend of the second exact division, by 6×7; or $278 \times 7 \times 3$, the dividend of the first exact division, by $6 \times 7 \times 3 = 126$; and we therefore write 126 for the third part of the true remainder. Adding the three parts, we have 143 for the entire remainder.

Hence the following

RULE. I. *Multiply each partial remainder by all the preceding divisors.*

II. *Add the several products; the sum will be the true remainder.*

EXAMPLES FOR PRACTICE.

1. Divide 435 by $15 = 3 \times 5$.	*Ans.* 29.
2. Divide 4256 by $56 = 7 \times 8$.	
3. Divide 17856 by $72 = 9 \times 8$.	
4. Divide 15288 by $42 = 2 \times 3 \times 7$.	*Ans.* 364.
5. Divide 972552 by $168 = 8 \times 7 \times 3$.	*Ans.* 5789.
6. Divide 526050 by $126 = 9 \times 7 \times 2$.	
7. Divide 612360 by $105 = 7 \times 5 \times 3$.	*Ans.* 5832.
8. Divide 553 by $15 = 3 \times 5$.	Rem. 13.
9. Divide 10183 by $105 = 3 \times 5 \times 7$.	103.
10. Divide 10197 by $120 = 2 \times 3 \times 4 \times 5$.	117.
11. Divide 29792 by $144 = 3 \times 8 \times 6$.	128.
12. Divide 73522 by $168 = 4 \times 6 \times 7$.	106.
13. Divide 63844 by $135 = 3 \times 5 \times 9$.	124.
14. Divide 386639 by $720 = 2 \times 3 \times 4 \times 5 \times 6$.	719.
15. Divide 734514 by $168 = 4 \times 6 \times 7$.	18.
16. Divide 636388 by $729 = 9^3$.	700.
17. Divide 4619 by $125 = 5^3$.	119.
18. Divide 116423 by $10584 = 3 \times 7^2 \times 8 \times 9$.	10583.
19. Divide 79500 by $6125 = 5^3 \times 7^2$.	6000.

CASE II.

122. When the divisor is a unit of any order.

If we cut off or remove the right hand figure of a number, each of the other figures is removed one place toward the right, and, consequently, the value of each is diminished tenfold, or divided by 10, (**57**, III). For a similar reason, by cutting off *two* figures we divide by 100; by cutting off *three*, we divide by 1000, and so on; and the figures cut off will constitute the remainder. Hence the

RULE. *From the right hand of the dividend cut off as many figures as there are ciphers in the divisor. Under the figures so cut off, place the divisor, and the whole will form the quotient.*

EXAMPLES FOR PRACTICE.

1. Divide 79 by 10. *Ans.* $7\frac{9}{10}$.
2. Divide 7982 by 100.
3. Divide 4003 by 1000. *Ans.* $4\frac{3}{1000}$.
4. Divide 2301050 by 10000.
5. Divide 3600036 by 1000. *Ans.* $3600\frac{36}{1000}$.

CASE III.

123. When there are ciphers on the right hand of the divisor.

1. Divide 25548 by 700.

OPERATION.

$$7|00)\underline{255|48}$$

$$36 \text{ Quotient.} \quad 3 \text{ 2d rem.}$$

$$3 \times 100 + 48 = 348 \text{ true rem.}$$

Analysis. We resolve 700 into the factors 100 and 7. Dividing first by 100, the quotient is 255, and the remainder 48. Dividing 255 by 7, the final quotient is 36, and the second remainder 3. Multiplying the last remainder, 3, by the preceding divisor, 100, and adding the preceding remainder, we have $300 + 48 = 348$, the true remainder, (**121**). In practice, the true remainder may be obtained by prefixing the second remainder to the first. Hence the

Rule. I. *Cut off the ciphers from the right of the divisor, and as many figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisor, for the final quotient.*

III. *Prefix the remainder to the figures cut off, and the result will be the true remainder.*

EXAMPLES FOR PRACTICE.

1. Divide 7856 by 900. *Ans.* $8\frac{656}{900}$.
2. Divide 13872 by 500.
3. Divide 83248 by 2600. *Ans.* $32\frac{48}{2600}$.
4. Divide 1548036 by 4300. *Ans.* $360\frac{36}{4300}$.
5. Divide 436000 by 300. *Ans.* $1453\frac{100}{300}$.
6. Divide 66472000 by 8100.
7. Divide 10818000 by 3600.

EXAMPLES COMBINING THE PRECEDING RULES.

1. How many barrels of flour at \$8 a barrel, will pay for 25 tons of coal at \$4 a ton, and 36 cords of wood at \$3 a cord?

Ans. 26.

2. A grocer bought 12 barrels of sugar at \$16 per barrel, and 17 barrels at \$13 per barrel; how much would he gain by selling the whole at \$18 per barrel?

3. A farmer sold 300 bushels of wheat at \$2 a bushel, corn and oats to the amount of \$750; with the proceeds he bought 120 head of sheep at \$3 a head, one pair of oxen for \$90, and 25 acres of land for the remainder How much did the land cost him per acre?

Ans. \$36.

4. Divide $450 + \overline{(24 - 12) \times 5}$ by $(90 \div 6) + \overline{(3 \times 11) - 18}$.

Ans. 17.

5. Divide $648 \times \overline{(3^2 \times 2^3) \div 9} - (2910 \div 15)$ by $2863 \div \overline{(4375 \div 175) \times 4^2 + 3^2}$.

Ans. $712\frac{6}{7}$.

6. The product of three numbers is 107100; one of the numbers is 42, and another 34. What is the third number?

Ans. 75.

7. What number is that which being divided by 45, the quotient increased by $7^2 + 1$, the sum diminished by the difference between 28 and 16, the remainder multiplied by 6, and the product divided by 24, the quotient will be 12?

Ans. 450.

8. A mechanic earns \$60 a month, but his necessary expenses are \$42 a month. How long will it take him to pay for a farm of 50 acres worth \$36 an acre?

9. What number besides 472 will divide 251104 without a remainder?

Ans. 532.

10. Of what number is 3042 both divisor and quotient?

Ans. 9253764.

11. What must the number be which, divided by 453, will give the quotient 307, and the remainder 109?

Ans. 139180.

12. A farmer bought a lot of sheep and hogs, of each an equal number, for \$1276. He gave \$4 a head for the sheep, and \$7 a

head for the hogs; what was the whole number purchased, and how much was the difference in the total cost of each?

Ans. 232 purchased; $348 difference in cost.

13. According to the census of 1850 the total value of the tobacco raised in the United States was $13,982,686. How many school-houses at a cost of $950, and churches at a cost of $7500, of each an equal number, could be built with the proceeds of the tobacco crop of 1850? *Ans.* 1654, and a remainder of $6386.

14. The entire cotton crop in the United States in 1859 was 4,300,000 bales, valued at $54 per bale. If the entire proceeds were exchanged for English iron, at $60 per ton, how many tons would be received?

15. The population of the United States in 1850 was 23,191,876. It was estimated that 1 person in every 400 died of intemperance. How many deaths may be attributed to this cause in the United States, during that year?

16. In 1850, there were in the State of New York, 10,593 public schools, which were attended during the winter by 508464 pupils; what was the average number to each school?

Ans. 48.

17. A drover bought a certain number of cattle for $9800, and sold a certain number of them for $7680, at $64 a head, and gained on those he sold $960. How much did he gain a head, and how many did he buy at first?

Ans. Gained $8 per head; bought 175.

18. A house and lot valued at $1200, and 6 horses at $95 each, were exchanged for 30 acres of land. At how much was the land valued per acre?

19. If 16 men can perform a job of work in 36 days, in how many days can they perform the same job with the assistance of 8 more men? *Ans.* 24.

20. Bought 275 barrels of flour for $1650, and sold 186 barrels of it at $9 a barrel, and the remainder for what it cost. How much was gained by the bargain? *Ans.* $558.

21. A grocer wishes to put 840 pounds of tea into three kinds of boxes, containing respectively 5, 10, and 15 pounds, using the

same number of boxes of each kind. How many boxes can he fill? *Ans.* 84.

22. A coal dealer paid $965 for some coal. He sold 160 tons for $5 a ton, when the remainder stood him in but $3 a ton. How many tons did he buy? *Ans.* 215.

23. A dealer in horses gave $7560 for a certain number, and sold a part of them for $3825, at $85 each, and by so doing, lost $5 a head; for how much a head must he sell the remainder, to gain $945 on the whole? *Ans.* $120.

24. Bought a Western farm for $22,360, and after expending $1742 in improvements upon it, I sold one half of it for $15480, at $18 per acre. How many acres of land did I purchase, and at what price per acre?

PROBLEMS IN SIMPLE INTEGRAL NUMBERS.

124. The four operations that have now been considered, viz., Addition, Subtraction, Multiplication, and Division, are all the operations that can be performed upon numbers, and hence they are called the *Fundamental Rules.*

125. In all cases, the numbers operated upon and the results obtained, sustain to each other the relation of a whole to its parts. Thus,

I. *In Addition,* the numbers added are the parts, and the sum or amount is the whole.

II. *In Subtraction,* the subtrahend and remainder are the parts, and the minuend is the whole.

III. *In Multiplication,* the multiplicand denotes the value of one part, the multiplier the number of parts, and the product the total value of the whole number of parts.

IV. *In Division,* the dividend denotes the total value of the whole number of parts, the divisor the value of one part, and the quotient the number of parts; or the divisor the number of parts, and the quotient the value of one part.

126. Every example that can possibly occur in Arithmetic, and every business computation requiring an arithmetical operation, can be classed under one or more of the four Fundamental Rules, as follows:

I. Cases requiring Addition.

There may be given	*To find*
1. The parts,	the whole, or the sum total.
2 The less of two numbers and their difference, or the subtrahend and remainder,	the greater number or the minuend.

II. Cases requiring Subtraction.

There may be given	*To find*
1. The sum of two numbers and one of them,	the other.
2. The greater and the less of two numbers, or the minuend and subtrahend,	the difference or remainder

III. Cases requiring Multiplication.

There may be given	*To find*
1. Two numbers,	their product.
2. Any number of factors,	their continued product.
3. The divisor and quotient,	the dividend.

IV. Cases requiring Division.

There may be given	*To find*
1. The dividend and divisor,	the quotient.
2. The dividend and quotient,	the divisor.
3. The product and one of two factors,	the other factor.
4. The continued product of several factors, and the product of all but one factor,	that one factor.

127. Let the pupil be required to illustrate the following problems by original examples.

Problem 1. Given, several numbers, to find their sum.

Prob. 2. Given, the sum of several numbers and all of them but one, to find that one.

Prob. 3. Given, the parts, to find the whole.

Prob. 4. Given, the whole and all the parts but one, to find that one.

Prob. 5. Given, two numbers, to find their difference.

Prob. 6. Given, the greater of two numbers and their difference, to find the less number.

Prob. 7. Given, the less of two numbers and their difference, to find the greater number.

Prob. 8. Given, the minuend and subtrahend, to find the remainder.

Prob. 9. Given, the minuend and remainder, to find the subtrahend.

Prob. 10. Given, the subtrahend and remainder, to find the minuend.

Prob. 11. Given, two or more numbers, to find their product.

Prob. 12. Given, the product and one of two factors, to find the other factor.

Prob. 13. Given, the continued product of several factors and all the factors but one, to find that factor.

Prob. 14. Given, the factors, to find their product.

Prob. 15 Given, the multiplicand and multiplier, to find the product.

Prob. 16. Given, the product and multiplicand, to find the multiplier.

Prob. 17. Given, the product and multiplier, to find the multiplicand.

Prob. 18. Given, two numbers, to find their quotients.

Prob. 19. Given, the divisor and dividend, to find the quotient.

Prob. 20. Given, the divisor and quotient, to find the dividend.

Prob. 21. Given, the dividend and quotient, to find the divisor.

Prob. 22. Given, the divisor, quotient, and remainder, to find the dividend.

Prob. 23. Given, the dividend, quotient, and remainder, to find the divisor.

Prob. 24. Given, the final quotient of a continued division and the several divisors, to find the dividend.

Prob 25. Given, the final quotient of a continued division, the first dividend, and all the divisors but one, to find that divisor.

Prob. 26. Given, the dividend and several divisors of a continued division, to find the quotient.

Prob. 27. Given, two or more sets of numbers, to find the difference of their sums.

Prob. 28. Given, two or more sets of factors, to find the sum of their products.

Prob. 29. Given, one or more sets of factors and one or more numbers, to find the sum of the products and the given numbers.

Prob. 30. Given, two or more sets of factors, to find the difference of their products.

Prob. 31. Given, one or more sets of factors and one or more numbers, to find the sum of the products and the given number or numbers.

Prob. 32. Given, two or more sets of factors and two or more other sets of factors, to find the difference of the sums of the products of the former and latter.

Prob. 33. Given, the sum and the difference of two numbers, to find the numbers.

Analysis. If the difference of two unequal numbers be added to the less number, the sum will be equal to the greater; and if this sum be added to the greater number, the result will be twice the greater number. But this result is the sum of the two numbers *plus* their difference.

Again, if the difference of two numbers be subtracted from the greater number, the remainder will be equal to the less number; and if this remainder be added to the less number, the result will be twice the less number. But this result is the sum of the two numbers *minus* their difference. Hence,

I. The sum of two numbers *plus* their difference is equal to twice the greater number.

II. The sum of two numbers *minus* their difference is equal to twice the less number.

PROPERTIES OF NUMBERS.

EXACT DIVISORS.

128. An **Exact Divisor** of a number is one that gives an integral number for a quotient. And since division is the reverse of multiplication, it follows that all the exact divisors of a number are factors of that number, and that all its factors are exact divisors.

NOTES.—1. Every number is divisible by itself and unity; but the number itself and unity are not generally considered as factors, or exact divisors of the number.

2. An exact divisor of a number is sometimes called the *measure* of the number.

129. An **Even Number** is a number of which 2 is an exact divisor; as 2, 4, 6, or 8.

130. An **Odd Number** is a number of which 2 is not an exact divisor; as 1, 3, 5, 7, or 9.

131. A **Perfect Number** is one that is equal to the sum of all its factors plus 1; as $6 = 3 + 2 + 1$, or $28 = 14 + 7 + 4 + 2 + 1$.

NOTE.—The only perfect numbers known are 6, 28, 496, 8128, 33550336, 8589869056, 137438691328, 2305843008139952128, 2417851639228158837784576, 9903520314282971830448816128.

132. An **Imperfect Number** is one that is not equal to the sum of all its factors plus 1, as 12, which is not equal to $6 + 4 + 3 + 2 + 1$.

133. An **Abundant Number** is one which is less than the sum of all its factors plus 1; as 18, which is less than $9 + 6 + 3 + 2 + 1$.

134. A **Defective Number** is one which is greater than the sum of all its factors plus 1; as 27, which is greater than $9+3+1$.

135. To show the nature of exact division, and furnish tests of divisibility, observe that if we begin with any number, as 4, and take once 4, two times 4, three times 4, four times 4, and so on indefinitely, forming the series 4, 8, 12, 16, etc., we shall have all the numbers that are divisible by 4; and from the manner of forming this series, it is evident,

1st. That the *product* of any one number of the series by any integral number whatever, will contain 4 an exact number of times;

2d. The *sum* of any two numbers of the series will contain 4 an exact number of times; and

3d. The *difference* of any two will contain 4 an exact number of times. Hence,

I Any number which will exactly divide one of two numbers will divide their product.

II. Any number which will exactly divide each of two numbers will divide their sum.

III. Any number which will exactly divide each of two numbers will divide their difference.

136. From these principles we derive the following properties:

I. Any number terminating with 0, 00, 000, etc., is divisible by 10, 100, 1000, etc., or by any factor of 10, 100, or 1000.

For by cutting off the cipher or ciphers, the number will be divided by 10, 100, or 1000, etc., without a remainder, (**122**); and a number of which 10, 100, or 1000, etc., is a factor, will contain any factor of 10, 100, or 1000, etc., (I).

II. A number is divisible by 2 if its right hand figure is even or divisible by 2.

For, the part at the left of the units' place, taken alone, with its local value, is a number which terminates with a cipher, and is divisible by 2, because 2 is a factor of 10, (I); and if both parts, taken separately, with their local values, are divisible by 2, their sum, which is the entire number, is divisible by 2, (**135**, II).

NOTE.—Hence, all numbers terminating with 0, 2, 4, 6, or 8, are *even*, and all numbers terminating with 1, 3, 5, 7, or 9, are *odd*.

III. A number is divisible by 4 if the number expressed by its two right hand figures is divisible by 4.

For, the part at the left of the tens' place, taken alone, with its local value, is a number which terminates with two ciphers, and is divisible by 4, because 4 is a factor of 100, (I); and if both parts, taken separately, with their local values, are divisible by 4, their sum which is the entire number, is divisible by 4, (**135**, II)

IV. A number is divisible by 8 if the number expressed by its three right hand figures is divisible by 8.

For, the part at the left of the hundreds' place, taken alone, with its local value, is a number which terminates with three ciphers, and is divisible by 8, because 8 is a factor of 1000, (I); and if both parts, taken separately, with their local values, are divisible by 8, their sum, or the entire number, is divisible by 8, (**135**, II).

V. A number is divisible by any power of 2, if as many right hand figures of the number as are equal to the index of the given power, are divisible by the given power.

For, as 2 is a factor of 10, any power of 2 is a factor of the corresponding power of 10, or of a unit of an order one higher than is indicated by the index of the given power of 2; and if both parts of a number, taken separately, with their local values, are divisible by a power of 2, their sum, or the entire number, is divisible by the same power of 2, (**135**, II).

VI. A number is divisible by 5 if its right hand figure is 0, or 5.

For, if a number terminates with a cipher, it is divisible by 5, because 5 is a factor of 10, (I); and if it terminates with 5, both parts, the units and the figures at the left of units, taken separately, with their local values, are divisible by 5, and consequently their sum, or the entire number, is divisible by 5, (**135**, II).

VII. A number is divisible by 25 if the number expressed by its two right hand figures is divisible by 25.

For, the part at the left of the tens' figure, taken with its local value, is a number terminating with two ciphers, and is divisible by 25, because 25 is a factor of 100, (I); and if both parts, taken separately, with their local values, are divisible by 25, their sum, or the entire number, is divisible by 25, (**135**, II).

VIII. A number is divisible by any power of 5, if as many right hand figures of the number as are equal to the index of the given power are divisible by the given power.

For, as 5 is a factor of 10, any power of 5 is a factor of the corresponding power of 10, or of a unit of an order one higher than is indicated by the index of the given power of 5; and if both parts of a number, taken separately, with their local values, are divisible by a power of 5, their sum, or the entire number, is divisible by the same power of 5, (**135**, II).

IX. A number is divisible by 9 if the sum of its digits is divisible by 9.

For, if any number, as 7245, be separated into its parts, 7000 + 200 + 40 + 5, and each part be divided by 9, the several remainders will be the digits 7, 2, 4, and 5, respectively; hence, if the sum of these digits, or remainders, be 9 or an exact number of 9's, the entire number must contain an exact number of 9's, and will therefore be divisible by 9.

NOTE. — Whence it follows that if a number be divided by 9, the remainder will be the same as the excess of 9's in the sum of the digits of the number. Upon this property depends one of the methods of proving the operations in the four Fundamental Rules.

X. A number is divisible by a composite number, when it is divisible, successively, by all the component factors of the composite number.

For, dividing any number successively by several factors, is the same as dividing by the product of these factors, (**119**, I).

XI. An odd number is not divisible by an even number.

For, the product of any even number by any odd number is even; and, consequently, any composite odd number can contain only odd factors.

XII. An even number that is divisible by an odd number, is also divisible by twice that odd number.

For, if any even number be divided by an odd number, the quotient must be even, and divisible by 2; hence, the given even number, being divisible successively by the odd number and 2, will be divisible by their product, or twice the odd number, (**119**, I).

PRIME NUMBERS.

137. A **Prime Number** is one that can not be resolved or separated into two or more integral factors.

NOTE. — Every number must be either prime or composite.

138. To find all the prime numbers within any given limit, we observe that all even numbers except 2 are composite; hence, the prime numbers must be sought among the odd numbers.

139. If the odd numbers be written in their order, thus; 1, 3, 5, 7, 9, 11, 13, 15 17, etc., we observe,

1st. Taking every third number after 3, we have 3 times 3, 5 times 3, 7 times 3, and so on; which are the only odd numbers divisible by 3.

2d. Taking every fifth number after 5, we have 3 times 5, 5 times 5, 7 times 5, and so on; which are the only odd numbers divisible by 5. And the same will be true of every other number in the series. Hence,

3d. If we cancel every third number, counting from 3, no number divisible by 3 will be left; and since 3 times 5 will be canceled, 5 times 5, or 25, will be the least composite number left in the series. Hence,

4th. If we cancel every fifth number, counting from 25, no number divisible by 5 will be left; and since 3 times 7, and 5 times 7, will be canceled, 7 times 7, or 49, will be the least composite number left in the series. And thus with all the prime numbers. Hence,

140. To find all the prime numbers within any given limit, we have the following

Rule. I. *Write all the odd numbers in their natural order.*

II. *Cancel, or cross out,* 3 *times* 3, *or* 9, *and every third number after it;* 5 *times* 5, *or* 25, *and every fifth number after it;* 7 *times* 7, *or* 49, *and every seventh number after it; and so on, beginning with the second power of each prime number in succession, till the given limit is reached. The numbers remaining, together with the number* 2, *will be the prime numbers required.*

Notes.—1. It is unnecessary to count for every ninth number after 9 times 9, for being divisible by 3, they will be found already canceled; the same may be said of any other canceled, or composite number.

2. This method of obtaining a list of the prime numbers was employed by Eratosthenes (born B. C., 275), and is called *Eratosthenes' Sieve.*

TABLE OF PRIME NUMBERS LESS THAN 1000.

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	

FACTORING.

CASE I.

141. To resolve any composite number into its prime factors.

The **Prime Factors** of a number are those prime numbers which multiplied together will produce the given number.

142. The process of factoring numbers depends upon the following principles:

I. Every prime factor of a number is an exact divisor of that number.

II. The only exact divisors of a number are its prime factors, or some combination of its prime factors.

1. What are the prime factors of 798?

OPERATION.

$$\begin{array}{r|l} 2 & 798 \\ \hline 3 & 399 \\ \hline 7 & 133 \\ \hline 19 & 19 \\ \hline & \end{array}$$

Analysis. Since the given number is even, we divide by 2, and obtain an odd number, 399, for a quotient. We then divide by the prime numbers 3, 7, and 19, successively, and the last quotient is 1. The divisors, 2, 3, 7, and 19, are the prime factors required, (II). Hence, the

RULE. *Divide the given number by any prime factor; divide the quotient in the same manner, and so continue the division until the quotient is a prime number. The several divisors and the last quotient will be the prime factors required.*

PROOF. The product of all the prime factors will be the given number.

EXAMPLES FOR PRACTICE.

1. What are the prime factors of 2150?
2. What are the prime factors of 2445?
3. What are the prime factors of 6300?
4. What are the prime factors of 21504?
5. What are the prime factors of 2366?
6. What are the prime factors of 1000?
7. What are the prime factors of 390625?
8. What are the prime factors of 999999?

143. If the prime factors of a number are small, as 2, 3, 5, 7, or 11, they may be easily found by the tests of divisibility, (**136**), or by trial. But numbers may be proposed requiring many trials to find their prime factors. This difficulty is obviated, within a certain limit, by the Factor Table given on pages 72, 73.

By prefixing each number in bold-face type in the column of Numbers, to the several numbers following it in the same division of the column, we shall form all the *composite numbers* less than 10,000, and *not divisible* by 2, 3, 5, 7, or 11; the numbers in the columns of Factors are the *least prime factors* of the numbers thus formed respectively. Thus, in one of the columns of Numbers we find 39, in bold-face type, and below 39, in the same column, is 77, which annexed to 39, forms 3977, a composite number. The least prime factor of this number is 41, which we find at the right of 77, in the column of Factors.

144. Hence, for the use of this table, we have the following

RULE. I. *Cancel from the given number all factors less than* 13, *and then find the remaining factors by the table.*

II. *If any number less than* 10,000 *is not found in the table, and is not divisible by* 2, 3, 5, 7, *or* 11, *it is prime.*

FACTOR TABLE.

Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.
1		99	29	11	17	43	29	79	37	41	17	83	17	41	23	09	31	17	53	77	31
69	13	**9**		17	13	57	19	83	13	51	13	97	43	59	17	13	19	27	29	83	71
2		01	17	57	31	61	37	89	19	77	13	**34**		69	53	21	29	47	47	91	29
21	13	23	13	69	13	63	13	91	47	83	19	01	19	87	13	31	61	57	67	**52**	
47	13	43	23	**15**		**20**		**25**		87	29	03	41	93	17	43	43	69	19	07	41
89	17	49	13	01	19	21	43	01	41	93	41	19	13	**39**		51	19	71	13	13	13
99	13	61	31	13	17	33	19	07	23	**30**		27	23	01	47	69	17	77	17	19	17
3		89	23	17	37	41	13	09	13	07	31	31	47	37	31	79	29	**48**		21	23
23	17	**10**		37	29	47	23	33	17	13	23	39	19	53	59	81	13	11	17	39	13
61	19	03	17	41	23	59	29	37	43	29	13	73	23	59	37	87	41	19	61	49	29
77	13	07	19	77	19	71	19	61	13	43	17	81	59	61	17	93	23	41	47	51	59
91	17	27	13	91	37	77	31	67	17	53	43	97	13	73	29	99	53	43	29	63	19
4		37	17	**16**		**21**		73	31	71	37	**35**		77	41	**44**		47	37	67	23
03	13	73	29	33	23	17	29	81	29	77	17	03	31	79	23	27	19	49	13	87	17
37	19	79	13	43	31	19	13	87	13	97	19	23	13	91	13	29	43	53	23	93	67
81	13	81	23	49	17	47	19	99	23	**31**		51	53	**40**		39	23	59	43	**53**	
93	17	**11**		51	13	59	17	**26**		03	29	69	43	09	19	53	61	67	31	11	47
5		21	19	79	23	71	13	03	19	07	13	87	17	31	29	69	41	83	19	17	13
27	17	39	17	81	41	73	41	23	43	27	53	89	37	33	37	71	17	91	67	21	17
29	23	47	31	91	19	83	37	27	37	31	31	99	59	43	13	89	67	97	59	29	73
33	13	57	13	**17**		97	13	41	19	33	13	**36**		61	31	**45**		**49**		39	19
51	19	59	19	03	13	**22**		69	17	39	43	01	13	63	17	11	13	01	13	53	53
59	13	89	29	11	29	01	31	**27**		49	47	11	23	69	13	31	23	13	17	59	23
89	19	**12**		17	17	09	47	01	37	51	23	29	19	87	61	37	13	27	13	63	31
6		07	17	39	37	27	17	43	13	61	29	49	41	97	17	41	19	79	13	71	41
11	13	19	23	51	17	31	23	47	41	73	19	53	13	**41**		53	29	81	17	77	19
29	17	41	17	63	41	49	13	59	31	93	31	67	19	17	23	59	47	97	19	89	17
67	23	47	29	69	29	57	37	71	17	97	23	79	13	21	13	73	17	**50**		**54**	
89	13	61	13	81	13	63	31	73	47	**32**		83	29	41	41	77	23	17	29	29	61
97	17	71	31	**18**		79	43	**28**		11	13	**37**		63	23	79	19	29	47	47	13
7		73	19	07	13	91	29	09	53	33	53	13	47	71	43	89	13	41	71	59	53
03	19	**13**		17	23	**23**		13	29	39	41	21	61	81	37	**46**		53	31	61	43
13	23	13	13	19	17	23	23	31	19	47	17	37	37	83	47	01	43	57	13	73	13
31	17	33	31	29	31	27	13	39	17	63	13	43	19	87	53	07	17	63	61	91	17
67	13	39	13	43	19	29	17	67	47	77	29	49	23	89	59	19	31	69	37	97	23
79	19	43	17	49	43	53	13	69	19	81	17	57	13	99	13	33	41	83	13	**55**	
93	13	49	19	53	17	63	17	73	13	87	19	63	53	**42**		61	59	**51**		13	37
99	17	57	23	91	31	69	23	81	43	93	37	81	19	23	41	67	13	11	19	39	29
8		63	29	**19**		**24**		99	13	**33**		91	17	37	19	81	31	23	47	43	23
17	19	69	37	09	23	07	29	**29**		17	31	99	29	47	31	87	43	29	23	49	31
41	29	87	19	19	19	13	19	11	41	37	47	**38**		67	17	93	13	41	53	61	67
51	23	91	13	21	17	19	41	21	23	41	13	09	13	**43**		99	37	43	37	67	19
71	13	**14**		27	41	49	31	23	37	49	17	11	37	03	13	**47**		49	19	87	37
93	19	03	23	37	13	61	23	29	29	79	31	27	43	07	59	09	17	61	13	97	29

FACTOR TABLE — Continued.

Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers.	Factors.	Numbers	Factors.	Numbers.	Factors.
56		**60**		39	47	51	13	61	53	57	13	23	71	13	47	51	53	53	19	59	13
03	13	01	17	43	17	59	19	67	13	61	47	27	23	17	19	57	17	59	47	71	19
09	71	19	13	63	23	77	13	77	19	63	79	33	29	41	23	73	19	63	59	73	17
11	31	23	19	67	29	87	71	79	29	97	43	47	13	53	79	79	13	69	13	83	23
17	41	31	37	87	13	89	83	89	37	**77**		51	83	71	43	81	83	71	73	**97**	
27	17	49	23	93	43	93	61	91	23	09	13	77	41	73	37	91	17	87	37	01	89
29	13	59	73	97	73	**69**		**73**		29	59	83	59	79	61	**89**		99	17	03	31
33	43	71	13	99	67	01	67	03	67	39	71	**81**		83	17	03	29	**93**		07	17
71	53	77	59	**65**		13	31	13	71	47	61	19	23	89	13	09	59	01	71	27	71
81	13	**61**		09	23	29	13	19	13	51	23	31	47	97	29	17	37	07	41	31	37
99	41	03	17	11	17	31	29	27	17	69	17	37	79	**85**		27	79	13	67	61	43
57		07	31	27	61	43	53	39	41	71	19	43	17	07	47	47	23	22	19	63	13
07	13	09	41	33	47	53	17	61	17	81	31	49	29	09	67	57	13	47	13	73	29
13	29	19	29	39	13	73	19	63	37	83	43	53	31	31	19	59	17	53	47	97	97
23	59	37	17	41	31	89	29	67	53	87	13	59	41	49	83	77	47	67	17	99	41
29	17	57	47	57	79	**70**		73	73	**78**		77	13	51	17	83	13	79	83	**98**	
59	13	61	61	83	29	03	47	79	47	01	29	89	19	57	43	89	89	89	41	09	17
67	73	69	31	93	19	09	43	87	83	07	37	**82**		67	13	93	17	**94**		27	31
71	29	79	37	**66**		31	79	91	19	11	73	01	59	79	23	**90**		07	23	41	13
73	23	87	23	13	17	33	13	97	13	13	13	03	13	87	31	17	71	09	97	47	43
77	53	91	41	17	13	37	31	**74**		31	41	07	29	93	13	19	29	51	13	53	59
58		**62**		23	37	61	23	09	31	37	17	13	43	**86**		47	83	69	17	69	71
09	37	27	13	31	19	67	37	21	41	49	47	27	19	11	79	61	13	81	19	81	41
33	19	33	23	41	29	81	73	23	13	59	29	49	73	21	37	71	47	87	53	93	13
37	13	39	17	47	17	87	19	29	17	71	17	51	37	33	89	73	43	**95**		99	19
91	43	41	79	49	61	93	41	39	43	91	13	57	23	39	53	77	29	03	13	**99**	
93	71	53	13	67	59	97	47	53	29	97	53	79	17	51	41	83	31	09	37	13	23
99	17	83	61	83	41	99	31	63	17	**79**		99	43	53	17	89	61	17	31	17	47
59		89	19	97	37	**71**		71	31	13	41	**83**		71	13	**91**		23	89	37	19
09	19	**63**		**67**		11	13	93	59	21	89	03	19	83	19	01	19	29	13	43	61
11	23	13	59	07	19	23	17	**75**		39	17	21	53	**87**		13	13	53	41	53	37
17	61	19	71	31	53	41	37	01	13	43	13	33	13	11	31	31	23	57	19	59	23
21	31	31	13	39	23	53	23	19	73	57	73	39	31	17	23	39	13	63	73	71	13
33	17	41	17	49	17	57	17	31	17	61	19	41	19	49	13	43	41	71	17	79	17
41	13	71	23	51	43	63	13	43	19	67	31	47	17	59	19	67	89	77	61	83	67
47	19	83	13	57	29	69	67	71	67	69	13	57	61	73	31	69	53	89	43	91	97
59	59	**64**		67	67	71	71	97	71	79	79	59	13	77	67	79	67	93	53	97	13
63	67	01	37	73	13	81	43	**76**		81	23	81	17	91	59	93	29	99	29		
69	47	03	19	99	13	99	23	13	23	91	61	83	83	97	19	97	17	**96**			
77	43	07	43	**68**		**72**		19	19	99	19	99	37	**88**		**92**		07	13		
83	31	09	13	17	17	01	19	27	29	**80**		**84**		01	13	11	61	17	59		
89	53	31	59	21	19	23	31	31	13	03	53	01	31	09	23	17	13	37	23		
93	13	37	41	47	41	41	13	33	17	21	13	11	41	43	37	23	23	41	31		

1. Resolve 1961 into its prime factors.

OPERATION.

$$1961 \div 37 = 53$$
$$1961 = 37 \times 53, \textit{ Ans.}$$

ANALYSIS. Cutting off the two right hand figures of the given number, and referring to the table, column No., we find the other part, 19, in bold-face type; and under it, in the same division of the column, we find 61, the figures cut off; at the right of 61, in column Fac., we find 37, the least prime factor of the given number. Dividing by 37, we obtain 53, the other factor.

2. Resolve 188139 into its prime factors.

OPERATION.

$$\begin{array}{r|r} 3 & 188139 \\ \hline 7 & 62713 \\ \hline 17 & 8959 \\ \hline 17 & 527 \\ \hline & 31 \end{array}$$
$$3 \times 7 \times 17 \times 17 \times 31, \textit{ Ans.}$$

ANALYSIS. We find by trial that the given number is divisible by 3 and 7; dividing by these factors, we have for a quotient 8959. By referring to the factor table, we find the least prime factor of this number to be 17; dividing by 17, we have 527 for a quotient. Referring again to the table, we find 17 to be the least factor of 527, and the other factor, 31, is prime.

EXAMPLES FOR PRACTICE.

1. Resolve 18902 into its prime factors. *Ans.* 2, 13, 727.
2. Resolve 352002 into its prime factors.
3. Resolve 6851 into its prime factors.
4. Resolve 9367 into its prime factors.
5. Resolve 203566 into its prime factors.
6. Resolve 59843 into its prime factors.
7. Resolve 9991 into its prime factors.
8. Resolve 123015 into its prime factors.
9. Resolve 893235 into its prime factors.
10. Resolve 390976 into its prime factors.
11. Resolve 225071 into its prime factors.
12. Resolve 81770 into its prime factors.
13. Resolve 6409 into its prime factors.
14. Resolve 178296 into its prime factors.
15. Resolve 714210 into its prime factors.

CASE II.

145. To find all the exact divisors of a number.

It is evident that all the prime factors of a number, together with all the possible combinations of those prime factors, will constitute all the exact divisors of that number, (**142,** II).

1. What are all the exact divisors of 360?

OPERATION.

$$360 = 1 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5.$$

Ans.	1,	2,	4,	8	Combinations of 1 and 2.
	3,	6,	12,	24	" " 1 and 2 and 3.
	9,	18,	36,	72	
	5,	10,	20,	40	" " 1 and 2 and 5.
	15,	30,	60,	120	" " 1 and 2 and 3 and 5.
	45,	90,	180,	360	

ANALYSIS. By Case I we find the prime factors of 360 to be 1, 2, 2, 2, 3, 3, and 5. As 2 occurs three times as a factor, the different combinations of 1 and 2 by which 360 is divisible will be 1, $1 \times 2 = 2$, $1 \times 2 \times 2 = 4$, and $1 \times 2 \times 2 \times 2 = 8$; these we write in the first line. Multiplying the first line by 3 and writing the products in the second line, and the second line by 3, writing the products in the third line, we have in the first, second and third lines all the different combinations of 1, 2, and 3, by which 360 is divisible. Multiplying the first, second and third lines by 5, and writing the products in the fourth, fifth and sixth lines, respectively, we have in the six lines together, every combination of the prime factors by which the given number, 360, is divisible.

Hence the following

RULE. I. *Resolve the given number into its prime factors.*

II *Form a series having* 1 *for the first term, that prime factor which occurs the greatest number of times in the given number for the second term, the square of this factor for the third term, and so on, till a term is reached containing this factor as many times as it occurs in the given number.*

III. *Multiply the numbers in this line by another factor, and these results by the same factor, and so on, as many times as this factor occurs in the given number.*

IV. *Multiply all the combinations now obtained by another factor in continued multiplication, and thus proceed till all the different factors have been used.* ALL *the combinations obtained will be the exact divisors sought.*

EXAMPLES FOR PRACTICE.

1. What are all the exact divisors of 120?
Ans. 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120.
2. Find all the exact divisors of 84.
Ans. 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84.
3. Find all the exact divisors of 100.
Ans. 1, 2, 4, 5, 10, 20, 25, 50, 100.
4. Find all the exact divisors of 420.
Ans. $\begin{cases} 1, 2, 3, 4, 5, 6, 7, 10, 12, 14, 15, 20, 21, 28, \\ 30, 35, 42, 60, 70, 84, 105, 140, 210, 420. \end{cases}$
5. Find all the exact divisors of 1050.
Ans. $\begin{cases} 1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 25, 30, 35, 42, 50, 70, \\ 75, 105, 150, 175, 210, 350, 525, 1050. \end{cases}$

GREATEST COMMON DIVISOR.

146. A **Common Divisor** of two or more numbers is a number that will exactly divide each of them.

147. The **Greatest Common Divisor** of two or more numbers is the greatest number that will exactly divide each of them.

148. Numbers Prime to each other are such as have no common divisor.

NOTE.—A common divisor is sometimes called a *Common Measure;* and the greatest common divisor, the *Greatest Common Measure.*

CASE I.

149. When the numbers can be readily factored.

It is evident that if several numbers have a common divisor, they may all be divided by any component factor of this divisor, and the resulting quotients by another component factor, and so on, till all the component factors have been used.

1. What is the greatest common divisor of 28, 140, and 420?

OPERATION.

$$\begin{array}{r|ccc} 7 & 28 & 140 & 420 \\ \hline 4 & 4 & 20 & 60 \\ \hline & 1 & 5 & 15 \end{array}$$

$4 \times 7 = 28$, *Ans.*

ANALYSIS. We readily see that 7 will exactly divide each of the given numbers; and then, 4 will exactly divide each of the resulting quotients. Hence, each of the given numbers can be exactly divided by 7 times 4; and these numbers must be component factors of the greatest common divisor. Now, if there were any other component factor of the greatest common divisor, the quotients, 1, 5 and 15, would be divisible by it. But these quotients are prime to each other; therefore, 7 and 4 are all the component factors of the greatest common divisor sought.

From this analysis we derive the following

RULE. I. *Write the numbers in a line, with a vertical line at the left, and divide by any factor common to all the numbers.*

II. *Divide the quotients in like manner, and continue the division till a set of quotients is obtained that are prime to each other.*

III. *Multiply all the divisors together, and the product will be the greatest common divisor sought.*

EXAMPLES FOR PRACTICE.

1. What is the greatest common divisor of 40, 75, and 100? *Ans.* 5.

2. What is the greatest common divisor of 18, 30, 36, 42, and 54?

3. What is the greatest common divisor of 42, 63, 126, and 189? *Ans.* 21.

4. What is the greatest common divisor of 135, 225, 270, and 315? *Ans.* 45.

5. What is the greatest common divisor of 84, 126, 210, 252, 294, and 462?

6. What is the greatest common divisor of 216, 360, 432, 648, and 936? *Ans.* 72.

7. What is the greatest common divisor of 102, 153, and 255? *Ans.* 51.

7*

8. What is the greatest common divisor of 756, and 1575?

9. What is the greatest common divisor of 182, 364, and 455?

10. What is the greatest common divisor of 2520, and 3240?
Ans. 360.

11. What is the greatest common divisor of 1428, and 1092?

12. What is the greatest common divisor of 1008, and 1036?
Ans. 28.

CASE II.

150. When the numbers cannot be readily factored.

The analysis of the method in this case depends upon the following properties of divisors.

I. An exact divisor divides any number of times its dividend.

II. A common divisor of two numbers is an exact divisor of their *sum.*

III. A common divisor of two numbers is an exact divisor of their *difference.*

NOTE.—The last two properties are essentially the same as **102**, II, III.

1. What is the greatest common divisor of 527, and 1207?

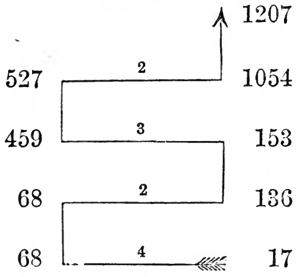
OPERATION.

		1207
527	2	1054
459	3	153
68	2	136
68	4	17

ANALYSIS. We will first describe the process, and then examine the reasons for the several steps in the operation. Drawing two vertical lines, we place the greater number on the right, and the less number on the left, one line lower down. We then divide 1207, the greater number, by 527, the less, and write the quotient, 2, between the verticals, the product, 1054, opposite the less number and under the greater, and the remainder, 153, below. We next divide 527 by this remainder, writing the quotient, 3, between the verticals, the product, 459, on the left, and the remainder, 68, below. We again divide the last divisor, 153, by 68, and obtain 2 for a quotient, 136 for a product, and 17 for a remainder, all of which we write in the same order as in the former steps. Finally, dividing the last divisor, 68, by the last remainder, 17, we have no remainder, and 17, the last divisor, is the greatest common divisor of the given numbers.

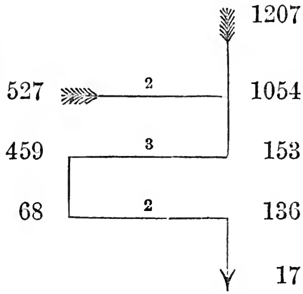
Now, observing that the dividend is always the *sum* of the product and remainder, and that the remainder is always the *difference* of the dividend and product, we first trace the work in the reverse order, as indicated by the arrow line in the diagram below.

OPERATION.



17 divides 68, as proved by the last division; it will also divide 2 times 68, or 136, (I). Now, as 17 divides both itself and 136, it will divide 153, their sum, (II). It will also divide 3 times 153, or 459, (I); and since it is a common divisor of 459 and 68, it must divide their sum, 527, which is one of the given numbers. It will also divide 2 times 527, or 1054, (I); and since it is a common divisor of 1054 and 153, it must divide their sum, 1207, the greater number, (II). Hence, 17 is a *common divisor* of the given numbers.

Again, tracing the work in the direct order, as indicated in the following diagram, we know that



the *greatest* common divisor, *whatever it be*, must divide 2 times 527, or 1054, (I). And since it will divide both 1054 and 1207, it must divide their difference, 153, (III). It will also divide 3 times 153, or 459, (I); and as it will divide both 459 and 527, it must divide their difference, 68, (III). It will also divide 2 times 68, or 136, (I); and as it will divide both 136 and 153, it must divide their difference, 17, (III); *hence, it cannot be greater than* 17.

Thus we have shown,

1st. That 17 is a *common divisor* of the given numbers.

2d. That their greatest common divisor, whatever it be, cannot be *greater than* 17. Hence it must be 17.

From this example and analysis, we derive the following

RULE. I. *Draw two verticals, and write the two numbers, one on each side, the greater number one line above the less.*

II. *Divide the greater number by the less, writing the quotient between the verticals, the product under the dividend, and the remainder below.*

III. *Divide the less number by the remainder, the last divisor by the last remainder, and so on, till nothing remains. The last divisor will be the greatest common divisor sought.*

IV. *If more than two numbers be given, first find the greatest common divisor of two of them, and then of this divisor and one of the remaining numbers, and so on to the last; the last common divisor found will be the greatest common divisor required.*

NOTES.—1. When more than two numbers are given, it is better to begin with the least two.

2. If at any point in the operation a *prime* number occur as a remainder, it must be a common divisor, or the given numbers have no common divisor.

EXAMPLES FOR PRACTICE.

1. What is the greatest common divisor of 18607 and 417979?

OPERATION.

			417979
	18607	2	37214
			45839
		2	37214
	17250	2	8625
	1357	6	8142
	966	2	483
	391	1	391
	368	4	92
Ans.	23	4	92

2. What is the greatest common divisor of 10661 and 12303?

OPERATION.

			12303
	10661	1	10661
	9852	6	1642
Prime	809		

Ans. 1.

3. What is the greatest common divisor of 336 and 812?

Ans. 28.

4. What is the greatest common divisor of 407 and 1067?
5. What is the greatest common divisor of 825 and 1372?
6. What is the greatest common divisor of 2041 and 8476?

Ans. 13.

7. What is the greatest common divisor of 3281 and 10778?
8. Find the greatest common divisor of 22579, and 116939.
9. What is the greatest common divisor of 49373 and 147731?

Ans. 97.

10. What is the greatest common divisor of 1005973 and 4616175?

11. Find the greatest common divisor of 292, 1022, and 1095.

Ans. 73.

12. What is the greatest common divisor of 4718, 6951, and 8876? *Ans.* 7.

13. Find the greatest common divisor of 141, 799, and 940.

14. What is the greatest common divisor of 484391 and 684877?

Ans. 701.

15. A farmer wishes to put 364 bushels of corn and 455 bushels of oats into the least number of bins possible, that shall contain the same number of bushels without mixing the two kinds of grain; what number of bushels must each bin hold?

Ans. 91.

16. A gentleman having a triangular piece of land, the sides of which are 165 feet, 231 feet, and 385 feet, wishes to inclose it with a fence having pannels of the greatest possible uniform length; what will be the length of each pannel?

17. B has $620, C $1116, and D $1488, with which they agree to purchase horses, at the highest price per head that will allow each man to invest all his money; how many horses can each man purchase? *Ans.* B 5, C 9, and D 12.

18. How many rails will inclose a field 14599 feet long by 10361 feet wide, provided the fence is straight, and 7 rails high, and the rails of equal length, and the longest that can be used?

Ans. 26880.

LEAST COMMON MULTIPLE.

151. A **Multiple** is a number exactly divisible by a given number; thus, 20 is a multiple of 4.

NOTES. — 1. A multiple is necessarily composite; a divisor may be either prime or composite.

2. A number is a divisor of all its multiples and a multiple of all its divisors.

152. A **Common Multiple** is a number exactly divisible by two or more given numbers; thus, 20 is a common multiple of 2, 4, 5, and 10.

153. The **Least Common Multiple** of two or more numbers is the least number exactly divisible by those numbers; thus, 24 is the least common multiple of 3, 4, 6, and 8.

154. From the definition it is evident that the product of two or more numbers, or any number of times their product, must be a common multiple of the numbers. Hence, *A common multiple of two or more numbers may be found by multiplying the given numbers together.*

155. To find the least common multiple.

FIRST METHOD.

From the relations of multiple and divisor we have the following properties:

I. A multiple of a number must contain all the prime factors of that number.

II. A common multiple of two or more numbers must contain all the prime factors of each of those numbers.

III. The least common multiple of two or more numbers must contain all the prime factors of each of those numbers, and no other factors.

1. Find the least common multiple of 63, 66, and 78.

OPERATION.

$$63 = 3 \times 3 \times 7$$
$$66 = 2 \times 3 \times 11$$
$$78 = 2 \times 3 \times 13$$
$$2 \times 3 \times 13 \times 11 \times 3 \times 7 = 18018\ \textit{Ans.}$$

ANALYSIS. The number cannot be less than 78, because it must contain 78; and if it contains 78, it must contain all its prime factors, viz.; $2 \times 3 \times 13$.

We here have all the prime factors, and also all the factors of 66 except 11. Annexing 11 to the series of factors,

$$2 \times 3 \times 13 \times 11,$$

and we have all the prime factors of 78 and 66, and also all the factors of 63 except one 3, and 7. Annexing 3 and 7 to the series of factors,

$$2 \times 3 \times 13 \times 11 \times 3 \times 7,$$

and we have all the prime factors of each of the given numbers, and *no others;* hence the product of this series of factors is the least common multiple of the given numbers, (III).

From this example and analysis we deduce the following

RULE. I. *Resolve the given numbers into their prime factors.*

II. *Multiply together all the prime factors of the largest number, and such prime factors of the other numbers as are not found in the largest number, and their product will be the least common multiple.*

NOTE.—When a prime factor is repeated in any of the given numbers, it must be taken as many times in the multiple, as the greatest number of times it appears in any of the given numbers.

EXAMPLES FOR PRACTICE.

1. Find the least common multiple of 60, 84, and 132.
 Ans. 4620.
2. Find the least common multiple of 21, 30, 44, and 126.
 Ans. 13,860.
3. Find the least common multiple of 8, 12, 20, and 30.
4. Find the least common multiple of 16, 60, 140, and 210.
 Ans. 1,680.
5. Find the least common multiple of 7, 15, 21, 25, and 35.
6. Find the least common multiple of 14, 19, 38, 42, and 57.
 Ans. 798.
7. Find the least common multiple of 144, 240, 480, 960.

SECOND METHOD.

156. 1. What is the least common multiple of 4, 9, 12, 18, and 36?

OPERATION.

2	4	9	12	18	36
2	2	9	6	9	18
3		9	3	9	9
3		3	3		3

$2 \times 2 \times 3 \times 3 = 36$ *Ans.*

ANALYSIS. We first write the given numbers in a series with a vertical line at the left. Since 2 is a factor of *some* of the given numbers, it must be a factor of the least common multiple sought, (**155**, III). Dividing as many of the numbers as are divisible by 2, we write the quotients, and the undivided number, 9, in a line underneath. Now, since some of the numbers in the second line contain the factor 2, the least common multiple must contain another 2, and we again divide by 2, omitting to write any quotient when it is 1. We next divide by 3 for a like reason, and still again by 3. By this process we have transferred all the factors of each of the numbers to the left of the vertical; and their product, 36, must be the least common multiple sought, (**155**, III).

2. What is the least common multiple of 20, 12, 15, and 75?

OPERATION.

2 , 5	20	12	15	75
2 , 3	2	6	3	15
5				5

$2 \times 5 \times 2 \times 3 \times 5 = 300$, *Ans.*

ANALYSIS. We readily see that 2 and 5 are among the factors of the given numbers, and must be factors of the least common multiple; hence, writing 2 and 5 at the left, we divide every number that is divisible by *either* of these factors or by their *product;* thus, we divide 20 by both 2 and 5; 12 by 2; 15 by 5; and 75 by 5. We next divide the second line in like manner by 2 and 3; and afterward the third line by 5. By this process we collect the factors of the given numbers into *groups;* and the product of the factors at the left of the vertical is the least common multiple sought.

3. What is the least common multiple of 7, 10, 15, 42, and 70?

OPERATION.

3 , 7	15	42	70
2 , 5	5	2	10

$3 \times 7 \times 2 \times 5 = 210$, *Ans.*

ANALYSIS. In this operation we omit the 7 and 10, because they are exactly contained in some of the other given numbers; thus, 7 is contained in 42, and 10 in 70; and whatever will contain 42 and 70 must contain 7 and 10. Hence we have only to find the least common multiple of the remaining numbers, 15, 42, and 70.

From these examples we derive the following

RULE. I. *Write the numbers in a line, omitting such of the smaller numbers as are factors of the larger, and draw a vertical line at the left.*

II. *Divide by any prime factor or factors that may be contained in one or more of the given numbers, and write the quotients and undivided numbers in a line underneath, omitting the 1's.*

III. *In like manner divide the quotients and undivided numbers, and continue the process till all the factors of the given numbers have been transferred to the left of the vertical. Then multiply these factors together, and their product will be the least common multiple required.*

NOTE. — We may use a composite number for a divisor, when it is contained in all the given numbers.

EXAMPLES FOR PRACTICE.

1. What is the least common multiple of 15, 18, 21, 24, 35, 36, 42, 50, and 60? *Ans.* 12600.

2. What is the least common multiple of 6, 8, 10, 15, 18, 20, and 24? *Ans.* 360.

3. What is the least common multiple of 9, 15, 25, 35, 45, and 100? *Ans.* 6300.

4. What is the least common multiple of 18, 27, 36, and 40?

5. What is the least common multiple of 12, 26, and 52?

6. What is the least common multiple of 32, 34, and 36? *Ans.* 4896.

7. What is the least common multiple of 8, 12, 18, 24, 27, and 36?

8. What is the least common multiple of 22, 33, 44, 55, and 66?

9. What is the least common multiple of 64, 84, 96, and 216?

10. If A can build 14 rods of fence in a day, B 25 rods, C 8 rods, and D 20 rods, what is the least number of rods that will furnish a number of whole days' work to either one of the four men? *Ans.* 1400.

11. What is the smallest sum of money for which I can purchase either sheep at \$4 per head, or cows at \$21, or oxen at \$49, or horses at \$72? *Ans.* \$3528.

12. A can dig 4 rods of ditch in a day, B can dig 8 rods, and C can dig 6 rods; what must be the length of the shortest ditch, that will furnish exact days' labor either for each working alone or for all working together? *Ans.* 72 rods.

13. The forward wheel of a carriage was 11 feet in circumference, and the hind wheel 15 feet; a rivet in the tire of each was up when the carriage started, and when it stopped the same rivets were up together for the 575th time; how many miles had the carriage traveled, allowing 5280 feet to the mile?

Ans. 17 miles 5115 feet.

CANCELLATION.

157. **Cancellation** is the process of rejecting equal factors from numbers sustaining to each other the relation of dividend and divisor.

158. It is evident that factors common to the dividend and divisor may be rejected without changing the quotient, (**117**, III).

1. Divide 1365 by 105.

OPERATION.

$$\frac{1365}{105} = \frac{\cancel{3} \times \cancel{5} \times \cancel{7} \times 13}{\cancel{3} \times \cancel{5} \times \cancel{7}} = 13$$

ANALYSIS. We first indicate the division by writing the dividend above a horizontal line and the divisor below. Then factoring each term, we find that 3, 5, and 7 are common factors; and crossing, or canceling these factors, we have 13, the remaining factor of the dividend, for a quotient.

159. If the product of several numbers is to be divided by the product of several other numbers, the common factors should be canceled before the multiplications are performed, for two reasons:

1st. The operations in multiplication and division will thus be abridged.

2d. The factors of small numbers are generally more readily detected than those of large numbers.

2. Divide 20 times 56 by 7 times 15.

OPERATION.

$$\frac{\overset{4}{\cancel{20}} \times \overset{8}{\cancel{56}}}{\cancel{7} \times \underset{3}{\cancel{15}}} = \frac{32}{3} = 10\tfrac{2}{3}$$

ANALYSIS. Having first indicated all the operations required by the question, we cancel 7 from 7 and 56, and 5 from 15 and 20, leaving the factors 3 in the divisor, and 8 and 4 in the dividend. Then $8 \times 4 = 32$, which divided by 3, gives $10\frac{2}{3}$, the quotient required. Hence the following

RULE I. *Write the numbers composing the dividend above a horizontal line, and the numbers composing the divisor below it.*

II. *Cancel all the factors common to both dividend and divisor.*

III. *Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

NOTES.—1. When a factor is canceled, the unit, 1, is supposed to take its place.

2. By many it is thought more convenient to write the factors of the dividend on the right of a *vertical* line, and the factors of the divisor on the left.

EXAMPLES FOR PRACTICE.

1. What is the quotient of $18 \times 6 \times 4 \times 42$ divided by $4 \times 9 \times 3 \times 7 \times 6$?

FIRST OPERATION.

$$\frac{{}^{2}\cancel{18} \times \cancel{6} \times \cancel{4} \times \cancel{42}^{2}}{\cancel{4} \times \cancel{9} \times \cancel{3} \times \cancel{7} \times \cancel{6}} = 4$$

SECOND OPERATION.

$$\begin{array}{c|c} \cancel{4} & \cancel{18}^{2} \\ \cancel{9} & \cancel{6} \\ \cancel{3} & \cancel{4} \\ \cancel{7} & \cancel{42}^{2} \\ \cancel{6} & \\ \hline & 4,\ \textit{Ans.} \end{array}$$

2. Divide the product of $21 \times 8 \times 60 \times 8 \times 6$ by $7 \times 12 \times 3 \times 8 \times 3$. *Ans.* 80.

3. The product of the numbers 16, 5, 14, 40, 16, 60, and 50, is to be divided by the product of the numbers 40, 24, 50, 20, 7, and 10; what is the quotient? *Ans.* 32.

4. Divide the continued product of 12, 5, 183, 18, and 70 by the continued product of 3, 14, 9, 5, 20, and 6.

5. If $213 \times 84 \times 190 \times 264$ be divided by $30 \times 56 \times 36$, what will be the quotient?

6. Multiply 64 by 7 times 31 and divide the product by 8 times 56, multiply this quotient by 15 times 88 and divide the product by 55, multiply this quotient by 13 and divide the product by 4 times 6. *Ans.* 403.

7. How many cords of wood at $4 a cord, must be given for 3 tons of hay at $12 a ton?

8. How many firkins of butter, each containing 56 pounds, at 15 cents a pound, must be given for 8 barrels of sugar, each containing 195 pounds, at 7 cents a pound? *Ans.* 13.

9 A grocer sold 16 boxes of soap, each containing 66 pounds at 9 cents a pound, and received as pay 99 barrels of potatoes, each containing 3 bushels; how much were the potatoes worth a bushel?

10. A farmer exchanged 480 bushels of corn worth 70 cents a bushel, for an equal number of bushels of barley worth 84 cents a bushel, and oats worth 56 cents a bushel; how many bushels of each did he receive? *Ans.* 240.

11. A merchant sold to a farmer two kinds of cloth, one kind at 75 cents a yard, and the other at 90 cents, selling him twice as many yards of the first kind as of the second. He received as pay 132 pounds of butter at 20 cents a pound; how many yards of each kind of cloth did he sell?

Ans. 22 yards of the first, and 11 yards of the second.

12. A man took six loads of potatoes to market, each load containing 20 bags, and each bag 2 bushels. He sold them at 44 cents a bushel, and received in payment 8 chests of tea, each containing 22 pounds; how much was the tea worth a pound?

Ans. 60 cents.

FRACTIONS.

DEFINITIONS, NOTATION, AND NUMERATION.

160. When it is necessary to express a quantity less than a unit, we may regard the unit as divided into some number of equal parts, and use one of these parts as a new unit of less value than the unit divided. Thus, if a yard, considered as an integral unit, be divided into 4 equal parts, then one, two, or three of these parts will constitute a number less than a unit. The *parts of a unit* thus used are called *fractional units;* and the numbers formed from them, *fractional numbers.* Hence

161. A **Fractional unit** is one of the equal parts of an integral unit.

162. A **Fraction** is a fractional unit, or a collection of fractional units.

163. Fractional units take their *name*, and their *value*, from the *number* of parts into which the integral unit is divided. Thus,

If a unit be divided into 2 equal parts, one of the parts is called *one half.* If a unit be divided into 3 equal parts, one of the parts is called *one third.* If a unit be divided into 4 equal parts, one of the parts is called *one fourth.*

And it is evident that one *third* is less in value than one *half*, one *fourth* less than one *third*, and so on.

164. To express a fraction by figures, two integers are required; one to denote the number of parts into which the integral unit is divided, the other to denote the number of parts taken, or the number of fractional units in the collection. The former is written below a horizontal line, the latter above. Thus,

One half is written		$\frac{1}{2}$	One fifth is written		$\frac{1}{5}$
One third	"	$\frac{1}{3}$	Two fifths	"	$\frac{2}{5}$
Two thirds	"	$\frac{2}{3}$	One seventh	"	$\frac{1}{7}$
One fourth	"	$\frac{1}{4}$	Three eighths	"	$\frac{3}{8}$
Two fourths	"	$\frac{2}{4}$	Five ninths	"	$\frac{5}{9}$
Three fourths	"	$\frac{3}{4}$	Eight tenths	"	$\frac{8}{10}$

165. The **Denominator** of a fraction is the number below the line.

It denominates or names the fractional unit, and it shows how many fractional units are equal to an integral unit.

166. The **Numerator** is the number above the line.

It numerates or numbers the fractional units; and it shows how many are taken.

167. The **Terms** of a fraction are the numerator and denominator, taken together.

168. Since the denominator of a fraction shows how many fractional units in the numerator are equal to 1 integral unit, it follows,

I. That the value of a fraction in integral units, is the quotient of the numerator divided by the denominator.

II. That fractions indicate division, the numerator being a dividend and the denominator a divisor.

169. To analyze a fraction is to designate and describe its numerator and denominator. Thus $\frac{5}{7}$ is analyzed as follows:—

7 is the *denominator*, and shows that the units expressed by the numerator are *sevenths*.

5 is the *numerator*, and shows that 5 sevenths are taken.

5 and 7 are the *terms* of the fraction considered as an expression of division, 5 being the dividend and 7 the divisor.

EXAMPLES FOR PRACTICE.

Express the following fractions by figures:—

1. Four *ninths*. *Ans.* $\frac{4}{9}$.
2. Seven *fifty-sixths*. *Ans.* $\frac{7}{56}$.
3. Sixteen *forty-eighths*.
4. Ninety-five *one hundred seventy-ninths*.
5. Five hundred thirty-six *four hundredths*.
6. One thousand eight hundred fifty-seven *nine thousand five hundred twenty-firsts*.
7. Twenty-five thousand *eighty-sevenths*.
8. Thirty *ten thousand eighty-seconds*.
9. One hundred one *ten millionths*.

Read and analyze the following fractions :—

10. $\frac{4}{9}$; $\frac{7}{12}$; $\frac{17}{38}$; $\frac{45}{100}$; $\frac{72}{375}$; $\frac{48}{1009}$; $\frac{84}{7863}$; $\frac{456}{537}$.

11. $\frac{20}{4}$; $\frac{87}{30}$; $\frac{95}{100}$; $\frac{48}{12}$; $\frac{75}{437}$; $\frac{175}{2}$; $\frac{436}{50}$; $\frac{766}{4879}$.

12. $\frac{467}{936}$; $\frac{536}{248}$; $\frac{10000}{75}$; $\frac{75}{10000}$; $\frac{5007}{3007}$.

13. $\frac{150}{537}$; $\frac{436}{972}$; $\frac{13785}{47956}$; $\frac{150072}{475000}$; $\frac{100001}{200002}$.

Fractions are distinguished as *Proper* and *Improper.*

170. A **Proper Fraction** is one whose numerator is less than its denominator; its value is less than the unit 1.

171. An **Improper Fraction** is one whose numerator equals or exceeds its denominator; its value is never less than the unit 1.

NOTES.—1. The value of a proper fraction, always being less than a unit, can only be expressed in a fractional form, hence, its name.

2. The value of an improper fraction, always being equal to, or greater than a unit, can always be expressed in some other form; hence its name.

172. A **Mixed Number** is a number expressed by an integer and a fraction.

173. Since fractions indicate division, (**168**, II), all changes in the *terms* of a fraction will affect the *value* of that fraction according to the laws of division; and we have only to modify the language of the General Principles of Division, by substituting the words *numerator*, *denominator*, and *fraction*, or *value of the fraction*, for the words *dividend*, *divisor*, and *quotient*, respectively, and we shall have the following

GENERAL PRINCIPLES OF FRACTIONS.

174. PRIN. I. *Multiplying the numerator multiplies the fraction, and dividing the numerator divides the fraction.*

PRIN. II. *Multiplying the denominator divides the fraction, and dividing the denominator multiplies the fraction.*

PRIN. III. *Multiplying or dividing both terms of the fraction by the same number, does not alter the value of the fraction.*

175. These three principles may be embraced in one

GENERAL LAW.

A change in the* NUMERATOR *produces a* LIKE *change in the value of the fraction; but a change in the* DENOMINATOR *produces an* OPPOSITE *change in the value of the fraction.

REDUCTION.

176. The **Reduction** of a fraction is the process of changing its terms, or its form, without altering its value.

CASE I.

177. To reduce fractions to their lowest terms.

A fraction is in its *lowest terms* when its numerator and denominator are prime to each other; that is, when both terms have no common divisor.

1. Reduce the fraction $\frac{60}{105}$ to its lowest terms.

OPERATION.

$\frac{60}{105} = \frac{12}{21} = \frac{4}{7}$

Or,

$15)\frac{60}{105} = \frac{4}{7}$

ANALYSIS. Dividing both terms of the fraction by the same number does not alter the value of the fraction, (**174**, III); hence, we divide both terms of $\frac{60}{105}$ by 5, and both terms of the result, $\frac{12}{21}$, by 3, and obtain $\frac{4}{7}$ for the final result. As 4 and 7 are *prime* to each other, the lowest terms of $\frac{60}{105}$ are $\frac{4}{7}$.

Instead of dividing by the factors 5 and 3 successively, we may divide by the greatest common divisor of the given terms, and reduce the fraction to its lowest terms at a single operation. Hence, the

RULE. *Cancel or reject all factors common to both numerator and denominator.* Or,

Divide both terms by their greatest common divisor.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{72}{120}$ to its lowest terms. *Ans.* $\frac{3}{5}$
2. Reduce $\frac{84}{96}$ to its lowest terms. *Ans.* $\frac{7}{8}$.
3. Reduce $\frac{168}{252}$ to its lowest terms. *Ans.* $\frac{2}{3}$.
4. Reduce $\frac{75}{135}$ to its lowest terms. *Ans.* $\frac{5}{9}$.
5. Reduce $\frac{156}{208}$ to its lowest terms. *Ans.* $\frac{3}{4}$.
6. Reduce $\frac{595}{665}$ to its lowest terms.
7. Reduce $\frac{258}{282}$ to its lowest terms.
8. Reduce $\frac{639}{1737}$ to its lowest terms.
9. Reduce $\frac{5222}{7042}$ to its lowest terms.
10. Reduce $\frac{247}{403}$ to its lowest terms. *Ans.* $\frac{19}{31}$.

NOTE. — Consult the factor table.

11. Reduce $\frac{851}{1219}$ to its lowest terms. *Ans.* $\frac{37}{53}$.

12. Reduce $\frac{3127}{3233}$ to its lowest terms. *Ans.* $\frac{59}{61}$.
13. Reduce $\frac{4489}{5561}$ to its lowest terms.
14. Reduce $\frac{3977}{5917}$ to its lowest terms. *Ans.* $\frac{41}{61}$.
15. Reduce $\frac{41369}{62443}$, and $\frac{47166}{43418}$ to their lowest terms.

CASE II.

178. To reduce an improper fraction to a whole or mixed number.

1. Reduce $\frac{297}{12}$ to a whole or mixed number.

OPERATION.

$$\tfrac{297}{12} = 297 \div 12 = 24\tfrac{9}{12} = 24\tfrac{3}{4}.$$

ANALYSIS. Since the value of a fraction in integral units is equal to the quotient of the numerator divided by the denominator, (**168**, 1,) we divide the given numerator, 297, by the given denominator, 12, and obtain for the value of the fraction, the mixed number $24\frac{9}{12} = 24\frac{3}{4}$. Hence the

RULE. *Divide the numerator by the denominator.*

NOTES. 1. When the denominator is an exact divisor of the numerator, the result will be a whole number.

2. In all answers containing fractions, reduce the fractions to their lowest terms.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{96}{6}$ to an equivalent integer. *Ans.* 16.
2. Reduce $\frac{56}{4}$ to an equivalent integer.
3. Reduce $\frac{154}{9}$ to a mixed number. *Ans.* $17\frac{1}{9}$.
4. Reduce $\frac{349}{13}$ to a mixed number. *Ans.* $26\frac{11}{13}$.
5. Reduce $\frac{588}{24}$ to a mixed number. *Ans.* $24\frac{1}{2}$.
6. Reduce $\frac{978}{56}$ to a mixed number. *Ans.* $17\frac{13}{28}$.
7. Reduce $\frac{2431}{85}$ to a mixed number.
8. Reduce $\frac{16315}{104}$ to a mixed number. *Ans.* $156\frac{7}{8}$.
9. Reduce $\frac{3802}{15}$ to a mixed number.
10. Reduce $\frac{3828}{836}$ to a mixed number. *Ans.* $4\frac{11}{19}$.
11. Reduce $\frac{22625}{225}$ to a mixed number. *Ans.* $100\frac{5}{9}$.
12. Reduce $\frac{10302}{10201}$ to a mixed number.
13. In $\frac{7859}{8}$ of a day how many days? *Ans.* $982\frac{3}{8}$ days.
14. In $\frac{407}{13}$ of a dollar how many dollars? *Ans.* $\$31\frac{4}{13}$.
15. If 1000 dollars be distributed equally among 36 men, what part of a dollar must each man receive in change? *Ans.* $\frac{7}{9}$.

CASE III.

179. To reduce a whole number to a fraction having a given denominator.

1. Reduce 37 to an equivalent fraction whose denominator shall be 5.

OPERATION.

$37 \times 5 = 185$

$37 = \frac{185}{5}$, *Ans.*

ANALYSIS. Since in each unit there are 5 fifths, in 37 units there must be 37 times 5 fifths, or 185 fifths $= \frac{185}{5}$. The numerator, 185, is obtained in the operation by multiplying the whole number, 37, by the given denominator, 5. Hence the

RULE. *Multiply the whole number by the given denominator; take the product for a numerator, under which write the given denominator.*

NOTE.—A whole number may be reduced to a fractional form by writing 1 under it for a denominator; thus, $9 = \frac{9}{1}$.

EXAMPLES FOR PRACTICE.

1. Reduce 17 to an equivalent fraction whose denominator shall be 6. *Ans.* $\frac{102}{6}$.
2. Change 375 to a fraction whose denominator shall be 8.
3. Change 478 to a fraction whose denominator shall be 24.
4. Reduce 36 pounds to ninths of a pound.
5. Reduce 359 days to sevenths of a day. *Ans.* $\frac{2513}{7}$.
6. Reduce 763 feet to fourteenths of a foot. *Ans.* $\frac{10682}{14}$.
7. Reduce 937 to a fractional form. *Ans.* $\frac{937}{1}$.

CASE IV.

180. To reduce a mixed number to an improper fraction.

1. In $12\frac{5}{7}$ how many sevenths?

OPERATION.

$$\begin{array}{r} 12\frac{5}{7} \\ 7 \\ \hline \frac{89}{7} \end{array}$$

ANALYSIS. In the whole number 12, there are 12×7 sevenths $= 84$ sevenths, (Case III), and 84 sevenths $+$ 5 sevenths $=$ 89 sevenths, or $\frac{89}{7}$. Hence the following

RULE. *Multiply the whole number by the denominator of the fraction; to the product add the numerator, and under the sum write the denominator.*

EXAMPLES FOR PRACTICE.

1. Reduce $15\frac{4}{5}$ to fifths. *Ans.* $\frac{79}{5}$.
2. Reduce $24\frac{3}{4}$ to an improper fraction. *Ans.* $\frac{99}{4}$.
3. Reduce $57\frac{5}{7}$ to an improper fraction.
4. Reduce $356\frac{12}{17}$ to an improper fraction. *Ans.* $\frac{6064}{17}$.
5. Reduce $872\frac{5}{12}$ to an improper fraction.
6. Reduce $300\frac{1}{300}$ to an improper fraction. *Ans.* $\frac{90001}{300}$.
7. Reduce $434\frac{18}{23}$ to an improper fraction. *Ans.* $\frac{10000}{23}$.
8. In $15\frac{7}{8}$ how many eighths?
9. In $135\frac{3}{20}$ how many twentieths? *Ans.* $\frac{2703}{20}$.
10. In $43\frac{3}{4}$ bushels how many fourths of a bushel?
11. In $760\frac{9}{10}$ days how many tenths of a day?

CASE V.

181. To reduce a fraction to a given denominator.

We have seen that fractions may be reduced to lower terms by division. Conversely,

I. Fractions may be reduced to higher terms by multiplication.

II. All higher terms of a fraction must be multiples of its lowest terms.

1. Reduce $\frac{3}{8}$ to a fraction whose denominator is 40.

OPERATION.

$$40 \div 8 = 5$$

$$\frac{3 \times 5}{8 \times 5} = \frac{15}{40}, \textit{ Ans.}$$

ANALYSIS. We first divide 40, the required denominator, by 8, the denominator of the given fraction, to ascertain if it be a multiple of this term, 8. The division shows that it is a multiple, and that 5 is the factor which must be employed to produce it. We therefore multiply both terms of $\frac{3}{8}$ by 5, (**174,** III), and obtain $\frac{15}{40}$, the required result. Hence the

RULE. *Divide the required denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{2}{3}$ to a fraction having 24 for a denominator. *Ans.* $\frac{16}{24}$.
2. Reduce $\frac{7}{12}$ to a fraction whose denominator is 96. *Ans.* $\frac{56}{96}$.
3. Reduce $\frac{12}{17}$ to a fraction whose denominator is 51.
4. Reduce $\frac{9}{13}$ to a fraction whose denominator is 78.
5. Reduce $\frac{62}{375}$ to a fraction whose denominator is 3000. *Ans.* $\frac{496}{3000}$.
6. Change $7\frac{3}{4}$ to a fraction whose denominator is 8.
7. Change $16\frac{7}{28}$ to a fraction whose denominator is 176.
8. Change $5\frac{3}{11}$ to a fraction whose denominator is 363.
9. Change $36\frac{5}{7}$ to a fraction whose denominator is 42. *Ans.* $\frac{1542}{42}$.

CASE VI.

182. To reduce two or more fractions to a common denominator.

A **Common Denominator** is a denominator common to two or more fractions.

1. Reduce $\frac{3}{5}$ and $\frac{7}{9}$ to a common denominator.

OPERATION.

$$\frac{3 \times 9}{5 \times 9} = \frac{27}{45}$$

$$\frac{7 \times 5}{9 \times 5} = \frac{35}{45}$$

Analysis. We multiply the terms of the first fraction by the denominator of the second, and the terms of the second fraction by the denominator of the first, (**174**, III). This must reduce each fraction to the same denominator, for each new denominator will be the product of the given denominators. Hence the

Rule. *Multiply the terms of each fraction by the denominators of all the other fractions.*

Note.—Mixed numbers must first be reduced to improper fractions.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{2}{3}$ and $\frac{2}{5}$ to a common denominator. *Ans.* $\frac{10}{15}$, $\frac{6}{15}$.
2. Reduce $\frac{2}{7}$ and $\frac{5}{9}$ to a common denominator.

3. Reduce $\frac{3}{5}$, $\frac{5}{12}$ and $\frac{1}{2}$ to a common denominator.

Ans. $\frac{72}{120}$, $\frac{50}{120}$, $\frac{60}{120}$.

4. Reduce $\frac{1}{2}$, $5\frac{2}{3}$ and $1\frac{3}{5}$ to equivalent fractions having a common denominator. *Ans.* $\frac{15}{30}$, $\frac{170}{30}$, $\frac{48}{30}$.

5. Reduce $\frac{4}{13}$ and $\frac{3}{17}$ to a common denominator.

Ans. $\frac{68}{221}$, $\frac{39}{221}$.

6. Reduce $\frac{1}{3}$, $\frac{1}{7}$ and $\frac{1}{11}$ to a common denominator.

7. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{12}$ and $\frac{9}{16}$ to a common denominator.

Ans. $\frac{768}{1536}$, $\frac{1152}{1536}$, $\frac{896}{1536}$, $\frac{864}{1536}$.

CASE VII.

183. To reduce fractions to their least common denominator.

The **Least Common Denominator** of two or more fractions is the least denominator to which they can all be reduced.

184. We have seen that all higher terms of a fraction must be multiples of its lowest terms, (**181**, II). Hence,

I. If two or more fractions be reduced to a common denominator, this common denominator will be a common multiple of the several denominators.

II. The least common denominator must therefore be the least common multiple of the several denominators.

1. Reduce $\frac{5}{6}$, $\frac{7}{12}$ and $\frac{2}{15}$ to their least common denominator.

OPERATION.

$$\begin{array}{l|l} 3\ ,\ 5 & 12\ ..\ 15 \\ \hline 2\ ,\ 2 & 4 \end{array}$$

$$3 \times 5 \times 2 \times 2 = 60$$

$$\left.\begin{array}{l} \frac{5}{6} = \frac{50}{60} \\ \frac{7}{12} = \frac{35}{60} \\ \frac{2}{15} = \frac{8}{60} \end{array}\right\} \textit{Ans.}$$

ANALYSIS. We first find the least common multiple of the given denominators, which is 60. This must be the least common denominator to which the given fractions can be reduced, (II). Reducing each fraction to the denominator 60, by Case V, we obtain $\frac{50}{60}$, $\frac{35}{60}$ and $\frac{8}{60}$ for the answer. Hence the following

RULE. I. *Find the least common multiple of the given denominators, for the least common denominator.*

II. *Divide this common denominator by each of the given denominators, and multiply each numerator by the corresponding quotient. The products will be the new numerators.*

NOTES.—1. If the several fractions are not in their lowest terms, they should be reduced to their lowest terms before applying the rule.

2. When two or more fractions are reduced to their least common denominator, their numerators and the common denominator will be prime to each other.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{5}{8}$ and $\frac{3}{10}$ to their least common denominator.
Ans. $\frac{25}{40}$, $\frac{12}{40}$.

2. Reduce $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ to their least common denominator.

3. Reduce $\frac{3}{5}$, $\frac{7}{12}$ and $\frac{11}{15}$ to their least common denominator.

4. Reduce $\frac{2}{3}$, $\frac{8}{9}$ and $\frac{3}{8}$ to their least common denominator.

5. Reduce $\frac{6}{14}$, $\frac{10}{24}$ and $\frac{13}{42}$ to their least common denominator.
Ans. $\frac{36}{84}$, $\frac{35}{84}$, $\frac{26}{84}$.

6. Reduce $\frac{2}{3}$, $\frac{4}{13}$, $\frac{25}{26}$ and $\frac{4}{39}$ to their least common denominator.
Ans. $\frac{52}{78}$, $\frac{24}{78}$, $\frac{75}{78}$, $\frac{8}{78}$.

7. Reduce $2\frac{3}{5}$, $\frac{7}{15}$, $\frac{5}{24}$ and $\frac{37}{60}$ to their least common denominator.
Ans. $\frac{312}{120}$, $\frac{56}{120}$, $\frac{25}{120}$, $\frac{74}{120}$.

8. Reduce $\frac{20}{21}$, $\frac{9}{56}$ and $\frac{51}{84}$ to their least common denominator.

9. Reduce $\frac{25}{40}$, $\frac{25}{120}$ and $\frac{14}{64}$ to their least common denominator.
Ans. $\frac{60}{96}$, $\frac{20}{96}$, $\frac{21}{96}$.

10. Reduce $\frac{38}{247}$, $\frac{77}{119}$ and $\frac{120}{442}$ to their least common denominator.
Ans. $\frac{34}{221}$, $\frac{143}{221}$, $\frac{60}{221}$.

11. Reduce $\frac{161}{529}$, $\frac{289}{391}$ and $\frac{1147}{1961}$ to their least common denominator.
Ans. $\frac{371}{1219}$, $\frac{901}{1219}$, $\frac{713}{1219}$.

12. Reduce $2\frac{5}{7}$, $\frac{3}{14}$ and $1\frac{7}{10}$ to their least common denominator.

13. Reduce $\frac{931}{1829}$, $\frac{3127}{5723}$ and $\frac{5133}{5626}$ to their least common denominator.
Ans. $\frac{180614}{354826}$, $\frac{193874}{354826}$, $\frac{323733}{354826}$.

14. Reduce $\frac{5}{7}$, $\frac{11}{12}$, $\frac{2}{15}$, $\frac{8}{27}$, $\frac{9}{35}$ and $\frac{17}{40}$ to their least common denominator.
Ans. $\frac{5400}{7560}$, $\frac{6930}{7560}$, $\frac{1008}{7560}$, $\frac{2240}{7560}$, $\frac{1944}{7560}$, $\frac{3213}{7560}$

15. Reduce $\frac{4}{7}$, $\frac{3}{13}$, $\frac{5}{28}$, $\frac{7}{52}$ and $\frac{15}{182}$ to their least common denominator.

16. Reduce $\frac{4}{15}$, $\frac{5}{75}$, $\frac{32}{56}$ and $4\frac{1}{3}$ to their least common denominator.
Ans. $\frac{28}{105}$, $\frac{7}{105}$, $\frac{60}{105}$, $\frac{455}{105}$.

ADDITION.

185. The denominator of a fraction determines the value of the fractional unit, (**165**); hence,

I. If two or more fractions have the same denominator, their numerators express fractional units of the same value.

II. If two or more fractions have different denominators, their numerators express fractional units of different values.

And since units of the same value only can be united into one sum, it follows,

III. That fractions can be added only when they have a common denominator.

1. What is the sum of $\frac{1}{5}$, $\frac{5}{12}$ and $\frac{2}{15}$?

OPERATION.

$$\tfrac{1}{5} + \tfrac{5}{12} + \tfrac{2}{15} = \frac{12 + 25 + 8}{60} = \tfrac{45}{60} = \tfrac{3}{4}.$$

ANALYSIS. We first reduce the given fractions to a common denominator, (III). And as the resulting fractions, $\frac{12}{60}$, $\frac{25}{60}$, and $\frac{8}{60}$ have the same fractional unit, (I), we add them by uniting their numerators into one sum, making $\frac{45}{60} = \frac{3}{4}$, the answer.

2. Add $5\frac{3}{4}$, $3\frac{7}{8}$ and $4\frac{7}{12}$.

OPERATION.

$$\begin{array}{l} 5 + 3 + 4 = 12 \\ \tfrac{3}{4} + \tfrac{7}{8} + \tfrac{7}{12} = \underline{\ 2\tfrac{5}{24}} \\ \qquad\qquad\quad 14\tfrac{5}{24}, \textit{ Ans.} \end{array}$$

ANALYSIS. The sum of the integers, 5, 3, and 4, is 12; the sum of the fractions, $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{7}{12}$, is $2\frac{5}{24}$. Hence, the sum of both fractions and integers is $12 + 2\frac{5}{24} = 14\frac{5}{24}$.

186. From these principles and illustrations we derive the following general

RULE. I. To add fractions.—*When necessary, reduce the fractions to their least common denominator; then add the numerators and place the sum over the common denominator.*

II. To add mixed numbers.—*Add the integers and fractions separately, and then add their sums.*

NOTE.—All fractional results should be reduced to their lowest terms, and if improper fractions, to whole or mixed numbers.

EXAMPLES FOR PRACTICE.

1. What is the sum of $\frac{7}{12}$, $\frac{4}{12}$, $\frac{5}{12}$ and $\frac{11}{12}$? *Ans.* $2\frac{1}{4}$.
2. What is the sum of $\frac{13}{15}$, $\frac{4}{15}$, $\frac{2}{15}$ and $\frac{8}{15}$? *Ans.* $1\frac{4}{5}$.
3. What is the sum of $\frac{5}{21}$, $\frac{8}{21}$, $\frac{16}{21}$ and $\frac{19}{21}$?
4. What is the sum of $7\frac{11}{36}$, $8\frac{23}{36}$, $2\frac{17}{36}$, $5\frac{19}{36}$ and $4\frac{29}{36}$? *Ans.* $28\frac{3}{4}$.
5. What is the sum of $37\frac{9}{56}$, $12\frac{27}{56}$, $13\frac{37}{56}$ and $\frac{53}{56}$?
6. Add $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$.
7. Add $\frac{3}{5}$, $\frac{4}{7}$, $\frac{13}{21}$ and $\frac{7}{15}$. *Ans.* $2\frac{9}{35}$.
8. Add $\frac{1}{3}$, $\frac{2}{9}$ and $\frac{1}{15}$.
9. Add $\frac{5}{12}$, $\frac{13}{15}$ and $\frac{7}{20}$. *Ans.* $1\frac{19}{30}$.
10. Add $\frac{1}{3}$, $\frac{3}{4}$, $\frac{11}{12}$ and $\frac{29}{42}$. *Ans.* $2\frac{29}{42}$.
11. Add $\frac{7}{8}$, $\frac{11}{12}$, $\frac{17}{18}$, $\frac{23}{24}$ and $\frac{26}{27}$. *Ans* $4\frac{71}{108}$.
12. Add $3\frac{1}{2}$, $4\frac{2}{3}$ and $2\frac{2}{15}$.
13. Add $16\frac{1}{12}$ and $24\frac{1}{18}$. *Ans.* $40\frac{5}{36}$.
14. Add $1\frac{1}{2}$, $2\frac{2}{3}$, $3\frac{3}{4}$, $4\frac{4}{5}$ and $5\frac{5}{6}$.
15. Add $4\frac{7}{15}$, $8\frac{5}{21}$ and $2\frac{8}{35}$. *Ans.* $14\frac{14}{15}$.
16. Add $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{12}$ and $\frac{1}{17}$. *Ans.* $\frac{37}{51}$.
17. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{2}{13}$ and $\frac{5}{17}$.
18. Add $\frac{1}{2}$, $\frac{2}{9}$, $\frac{3}{11}$, $\frac{7}{18}$ and $\frac{17}{33}$. *Ans.* $1\frac{89}{99}$.
19. Add $\frac{1}{2}$, $\frac{5}{14}$, $\frac{8}{19}$ and $\frac{2}{3}$.
20. Add $41\frac{1}{2}$, $105\frac{2}{9}$, $300\frac{3}{4}$, $241\frac{3}{5}$ and $472\frac{1}{4}$. *Ans.* $1161\frac{29}{90}$.
21. Add $4\frac{1}{6}$, $2\frac{1}{4}$, $1\frac{1}{16}$, $2\frac{5}{24}$, $5\frac{7}{16}$, $7\frac{2}{3}$, $4\frac{1}{2}$ and $6\frac{5}{6}$.
22. Four cheeses weighed respectively $36\frac{5}{8}$, $42\frac{2}{3}$, $39\frac{7}{16}$ and $51\frac{1}{4}$ pounds; what was their entire weight? *Ans.* $169\frac{47}{48}$ pounds.
23. What number is that from which if $4\frac{4}{5}$ be taken, the remainder will be $3\frac{22}{35}$? *Ans.* $8\frac{3}{7}$.
24. What fraction is that which exceeds $\frac{5}{18}$ by $\frac{5}{54}$?
25. A beggar obtained $\frac{1}{8}$ of a dollar from one person, $\frac{1}{6}$ from another, $\frac{1}{5}$ from another, and $\frac{2}{15}$ from another; how much did he get from all?
26. A merchant sold $46\frac{4}{9}$ yards of cloth for $\$127\frac{7}{18}$, $64\frac{11}{27}$ yards for $\$226\frac{5}{6}$, and $76\frac{5}{6}$ yards for $\$312\frac{2}{3}$; how many yards of cloth did he sell, and how much did he receive for the whole? *Ans.* $187\frac{37}{54}$ yards, for $\$666\frac{15}{18}$.

SUBTRACTION.

187. The process of subtracting one fraction from another is based upon the following principles:

I. One number can be subtracted from another only when the two numbers have the same unit value. Hence,

II. In subtraction of fractions, the minuend and subtrahend must have a common denominator, (**185,** I).

1. From $\frac{4}{5}$ subtract $\frac{2}{3}$.

OPERATION.

$$\frac{4}{5}-\frac{2}{3}=\frac{12-10}{15}=\frac{2}{15}$$

Analysis. Reducing the given fractions to a common denominator, the resulting fractions $\frac{12}{15}$ and $\frac{10}{15}$ express fractional units of the same value, (**185,** I). Then 12 fifteenths less 10 fifteenths equals 2 fifteenths $=\frac{2}{15}$, the answer.

2. From $238\frac{1}{4}$ take $24\frac{5}{6}$.

OPERATION.

$$\begin{aligned} 238\tfrac{1}{4} &= 238\tfrac{3}{12} \\ 24\tfrac{5}{6} &= \underline{\ \ 24\tfrac{10}{12}} \\ & \quad 213\tfrac{5}{12} \text{ Ans.} \end{aligned}$$

Analysis. We first reduce the fractional parts, $\frac{1}{4}$ and $\frac{5}{6}$, to the common denominator, 12. Since we cannot take $\frac{10}{12}$ from $\frac{3}{12}$, we add $1=\frac{12}{12}$, to $\frac{3}{12}$, making $\frac{15}{12}$. Then, $\frac{10}{12}$ subtracted from $\frac{15}{12}$ leaves $\frac{5}{12}$; and carrying 1 to 24, the integral part of the subtrahend, (**73,** II), and subtracting, we have $213\frac{5}{12}$ for the entire remainder.

188. From these principles and illustrations we derive the following general

Rule. I. To subtract fractions.—*When necessary, reduce the fractions to their least common denominator. Subtract the numerator of the subtrahend from the numerator of the minuend, and place the difference of the new numerators over the common denominator.*

II. To subtract mixed numbers.—*Reduce the fractional parts to a common denominator, and then subtract the fractional and integral parts separately.*

Note.—We may reduce mixed numbers to improper fractions, and subtract by the rule for fractions. But this method generally imposes the useless labor of reducing integral numbers to fractions, and fractions to integers again.

EXAMPLES FOR PRACTICE.

1. From $\frac{7}{15}$ take $\frac{3}{15}$. *Ans.* $\frac{4}{15}$.
2. From $\frac{35}{56}$ take $\frac{21}{56}$. *Ans.* $\frac{1}{4}$.
3. From $\frac{35}{52}$ take $\frac{9}{52}$.
4. From $\frac{5}{6}$ take $\frac{3}{8}$. *Ans.* $\frac{11}{24}$.
5. From $\frac{3}{4}$ take $\frac{7}{20}$.
6. From $\frac{13}{15}$ take $\frac{7}{24}$. *Ans.* $\frac{23}{40}$.
7. From $\frac{9}{14}$ take $\frac{16}{63}$. *Ans.* $\frac{7}{18}$.
8. From $\frac{14}{39}$ take $\frac{19}{65}$. *Ans.* $\frac{1}{15}$.
9. From $\frac{5}{21}$ take $\frac{1}{15}$.
10. From $\frac{7}{12}$ take $\frac{5}{42}$. *Ans.* $\frac{13}{28}$.
11. From $\frac{7}{64}$ take $\frac{13}{120}$. *Ans.* $\frac{1}{960}$.
12. From $\frac{9}{35}$ take $\frac{7}{30}$.
13. From $16\frac{5}{6}$ take $7\frac{1}{6}$. *Ans.* $9\frac{2}{3}$.
14. From $36\frac{11}{24}$ take $8\frac{19}{24}$. *Ans.* $27\frac{2}{3}$.
15. From $25\frac{7}{10}$ subtract $14\frac{13}{15}$.
16. From 75 subtract $4\frac{3}{7}$. *Ans.* $70\frac{4}{7}$.
17. From $18\frac{2}{9}$ subtract $5\frac{5}{6}$.
18. From $26\frac{7}{24}$ subtract $25\frac{13}{15}$.
19. From $28\frac{16}{63}$ subtract $3\frac{9}{14}$. *Ans.* $24\frac{11}{18}$.
20. From $78\frac{7}{15}$ subtract $32\frac{2}{3}$.

21. The sum of two numbers is $26\frac{1}{4}$, and the less is $7\frac{1}{13}$; what is the greater? *Ans.* $19\frac{9}{52}$.

22. What number is that to which if you add $18\frac{4}{7}$, the sum will be $97\frac{8}{9}$?

23. What number must you add to the sum of $126\frac{1}{7}$ and $240\frac{3}{4}$, to make $560\frac{5}{8}$? *Ans.* $193\frac{41}{56}$.

24. What number is that which, added to the sum of $\frac{1}{8}$, $\frac{1}{12}$, and $\frac{1}{18}$, will make $\frac{25}{36}$? *Ans.* $\frac{31}{72}$.

25. To what fraction must $\frac{2}{5}$ be added, that the sum may be $\frac{8}{9}$?

26. From a barrel of vinegar containing $31\frac{1}{2}$ gallons, $14\frac{7}{8}$ gallons were drawn; how much was then left? *Ans.* $16\frac{5}{8}$ gallons.

27. Bought a quantity of coal for $\$140\frac{3}{8}$, and of lumber for $\$456\frac{2}{3}$. Sold the coal for $\$775\frac{1}{3}$, and the lumber for $\$516\frac{3}{16}$; how much was my whole gain? *Ans.* $\$694\frac{23}{48}$.

THEORY OF MULTIPLICATION AND DIVISION OF FRACTIONS.

189. In multiplication and division of fractions, the various operations may be considered in two classes:

1st. Multiplying or dividing a fraction.

2d. Multiplying or dividing *by* a fraction.

190. The methods of multiplying and dividing fractions may be derived directly from the General Principles of Fractions, (**174**); as follows:

I. To multiply a fraction.—*Multiply its numerator or divide its denominator*, (**174**, I. *and* II).

II. To divide a fraction.—*Divide its numerator or multiply its denominator*, (**174**, I. *and* II).

GENERAL LAW.

III. *Perform the required operation upon the numerator, or the opposite upon the denominator*, (**174**, III).

191. The methods of multiplying and dividing *by* a fraction may be deduced as follows:

1st. The value of a fraction is the quotient of the numerator divided by the denominator (**168**, I). Hence,

2d. The numerator alone is as many times the value of the fraction, as there are units in the denominator.

3d. If, therefore, in *multiplying* by a fraction, we multiply by the numerator, this result will be *too great*, and must be divided by the denominator.

4th. But if in *dividing* by a fraction, we divide by the numerator, the resulting quotient will be *too small*, and must be multiplied by the denominator.

Hence, the methods of multiplying and dividing *by* a fraction may be stated as follows:

I. To multiply by a fraction.—*Multiply by the numerator and divide by the denominator*, (3d).

II. To divide by a fraction.—*Divide by the numerator and multiply by the denominator*, (4th).

GENERAL LAW.

III. *Perform the required operation by the numerator and the opposite by the denominator.*

MULTIPLICATION.

192. 1. Multiply $\frac{5}{12}$ by 4.

FIRST OPERATION.

$$\frac{5}{12} \times 4 = \frac{20}{12} = 1\frac{2}{3}$$

SECOND OPERATION.

$$\frac{5}{12} \times 4 = \frac{5}{3} = 1\frac{2}{3}$$

THIRD OPERATION.

$$\frac{5}{\underset{3}{\cancel{12}}} \times \frac{\cancel{4}}{1} = \frac{5}{3} = 1\frac{2}{3}$$

ANALYSIS. In the first operation, we multiply the fraction by 4 by multiplying its numerator by 4; and in the second operation, we multiply the fraction by 4 by dividing its denominator by 4, (**190**, I or III).

In the third operation, we express the multiplier in the form of a fraction, indicate the multiplication, and obtain the result by cancellation.

2. Multiply 21 by $\frac{4}{7}$.

FIRST OPERATION.

$$21 \times \frac{4}{7} = \frac{84}{7} = 12$$

SECOND OPERATION.

$$21 \times \frac{4}{7} = 3 \times 4 = 12$$

THIRD OPERATION.

$$\frac{\overset{3}{\cancel{21}}}{1} \times \frac{4}{\cancel{7}} = 12$$

ANALYSIS. To multiply by $\frac{4}{7}$, we must multiply by 4 and divide by 7, (**191**, I or III).

In the first operation, we first multiply 21 by 4, and then divide the product, 84, by 7.

In the second operation, we first divide 21 by 7, and then multiply the quotient, 3, by 4.

In the third operation, we express the whole number, 21, in the form of a fraction, indicate the multiplication, and obtain the result by cancellation.

3. Multiply $\frac{5}{14}$ by $\frac{7}{8}$.

FIRST OPERATION.

1st step, $\frac{5}{14} \times 7 = \frac{35}{14}$

2d step, $\frac{35}{14} \div 8 = \frac{35}{112}$

$\frac{35}{112} = \frac{5}{16}$ *Ans.*

ANALYSIS. To multiply by $\frac{7}{8}$, we must multiply by 7 and divide by 8, (**191**, I or III). In the first operation, we multiply $\frac{5}{14}$ by 7 and obtain $\frac{35}{14}$;

SECOND OPERATION.

$$\frac{5}{14} \times \frac{7}{8} = \frac{35}{112} = \frac{5}{16}$$

THIRD OPERATION.

$$\frac{5}{\cancel{14}_{2}} \times \frac{\cancel{7}}{8} = \frac{5}{16}$$

we then divide $\frac{35}{14}$ by 8 and obtain $\frac{35}{112}$, which reduced gives $\frac{5}{16}$, the required product. In the second operation we obtain the same result by multiplying the numerators together for the numerator of the product, and the denominators together for the denominator of the product. In the third operation, we *indicate* the multiplication, and obtain the result by cancellation.

193. From these principles and illustrations we derive the following general

RULE. I. *Reduce all integers and mixed numbers to improper fractions.*

II. *Multiply together the numerators for a new numerator, and the denominators for a new denominator.*

NOTES.—1. Cancel all factors common to numerators and denominators.
2. If a fraction be multiplied by its denominator, the product will be the numerator.

EXAMPLES FOR PRACTICE.

1. Multiply $\frac{3}{9}$ by 8. *Ans.* $2\frac{2}{3}$
2. Multiply $\frac{4}{9}$ by 27, $\frac{3}{16}$ by 4, and $\frac{5}{36}$ by 9.
3. Multiply $\frac{2}{75}$ by 15. *Ans.* $\frac{2}{5}$.
4. Multiply 8 by $\frac{3}{4}$. *Ans.* 6.
5. Multiply 75 by $\frac{3}{15}$, 7 by $\frac{8}{21}$, 756 by $\frac{5}{6}$, and 572 by $\frac{5}{24}$.
6. Multiply $\frac{3}{7}$ by $\frac{5}{6}$.
7. Multiply $\frac{11}{15}$ by $\frac{20}{33}$, and $\frac{15}{17}$ by $\frac{51}{75}$.
8. Multiply $\frac{7}{12}$ by $\frac{17}{42}$, and $\frac{8}{35}$ by $\frac{21}{20}$.
9. Multiply $2\frac{4}{7}$ by $3\frac{8}{9}$. *Ans.* 10.
10. Multiply $1\frac{2}{7}$ by $1\frac{17}{18}$. *Ans.* $2\frac{1}{2}$.
11. Multiply $\frac{9}{17}$ by $2\frac{13}{36}$.
12. Find the value of $\frac{2}{3} \times \frac{5}{6} \times \frac{9}{15} \times \frac{7}{8}$. *Ans.* $\frac{7}{24}$.
13. Find the value of $\frac{2}{3} \times \frac{7}{8} \times \frac{15}{28} \times \frac{4}{11} \times \frac{44}{75}$. *Ans.* $\frac{1}{15}$.
14. Find the value of $\frac{34}{35} \times \frac{7}{53} \times \frac{106}{119}$.
15. Find the value of $2\frac{2}{5} \times 2\frac{4}{7} \times \frac{2}{11} \times \frac{5}{108} \times 1\frac{7}{15} \times 26\frac{1}{4}$. *Ans.* 2.
16. Find the value of $\frac{7}{11} \times \frac{5}{21} \times 4\frac{2}{5} \times 15 \times \frac{3}{16}$.
17. Find the value of $\frac{43}{126} \times \frac{7}{860} \times \frac{540}{53}$. *Ans.* $\frac{3}{106}$.

18. Find the value of $\overline{(4\frac{1}{2} \times \frac{7}{8}) + 1\frac{3}{5}} \times (3\frac{1}{3} - \frac{9}{10})$.

19. Find the value of $28 + (7\frac{3}{4} - 2\frac{2}{5}) \times \frac{5}{6} \times (\frac{4}{5} + \frac{1}{3})$.

NOTE 3. — The word *of* between fractions is equivalent to the sign of multiplication; **and such an expression is** sometimes called a *compound fraction.*

Find the values of the following indicated products: —

20. $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{5}{6}$. *Ans.* $\frac{2}{9}$.

21. $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{3}{21}$. *Ans.* $\frac{1}{12}$.

22. $\frac{5}{8}$ of $\frac{4}{15}$ of $\frac{6}{7}$.

23. $\frac{11}{12}$ of $\frac{5}{33}$ of $\frac{18}{35}$ of $\frac{21}{28}$. *Ans.* $\frac{3}{56}$.

24. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$.

In the following examples, cancellation may be employed by the aid of the Factor Table.

25. What is the value of $\frac{901}{713} \times \frac{1219}{1343} \times \frac{2449}{5353}$? *Ans.* $\frac{53}{101}$.

26. What is the value of $\frac{2501}{2537} \times \frac{2881}{3403} \times \frac{4897}{6499}$? *Ans.* $\frac{61}{97}$.

27. What is the value of $\frac{7379}{9991} \times \frac{9409}{8413} \times \frac{8549}{9797}$?
Ans. $\frac{13031}{18079}$.

28. What will 7 cords of wood cost at $\$3\frac{5}{8}$ per cord?
Ans. $\$25\frac{3}{8}$.

29. What is the value of $(\frac{3}{4})^2 \times \frac{16}{21} \times (\frac{1}{5})^5$? *Ans.* $\frac{3}{21875}$.

30. If 1 horse eat $\frac{3}{7}$ of a bushel of oats in a day, how many bushels will 10 horses eat in 6 days? *Ans.* $25\frac{5}{7}$.

31. What is the cube of $12\frac{7}{8}$?

32. At $\$9\frac{3}{5}$ per ton, what will be the cost of $\frac{1}{2}$ of $\frac{5}{6}$ of a ton of hay? *Ans.* $4.

33. At $\$\frac{9}{16}$ a bushel, what will be the cost of $1\frac{3}{5}$ bushels of corn?

34. When peaches are worth $\$\frac{7}{8}$ per basket, what is $\frac{4}{7}$ of a basket worth?

35. A man owning $\frac{4}{5}$ of $156\frac{2}{3}$ acres of land, sold $\frac{1}{2}$ of $\frac{3}{4}$ of his share; how many acres did he sell? *Ans.* 47.

36. What is the product of $(\frac{3}{8})^3 \times (\frac{1}{2})^2 \times (\frac{9}{10})^2 \times (3\frac{1}{3})^4$?
Ans. $\frac{675}{512}$.

37. If a family consume $1\frac{7}{9}$ barrels of flour a month, how many barrels will 6 families consume in $8\frac{9}{10}$ months?

38. What is the product of $\overline{150\frac{1}{2} - (\frac{6}{7} \text{ of } 121\frac{4}{5} + \frac{3}{4} \text{ of } 48\frac{2}{3}) - 75}$, multiplied by $3 \times \overline{(\frac{4}{5} \text{ of } 1\frac{1}{8} \times 4) - 2\frac{1}{4}}$? *Ans.* $342\frac{63}{100}$.

39. A man at his death left his wife \$12,500, which was $\frac{1}{2}$ of $\frac{5}{6}$ of his estate; she at her death left $\frac{5}{7}$ of her share to her daughter; what part of the father's estate did the daughter receive? *Ans.* $\frac{25}{84}$.

40. A owned $\frac{5}{6}$ of a cotton factory, and sold $\frac{3}{4}$ of his share to B, who sold $\frac{1}{2}$ of what he bought to C, who sold $\frac{2}{3}$ of what he bought to D; what part of the whole factory did each then own?

Ans. A, $\frac{5}{24}$; B, $\frac{5}{16}$; C, $\frac{5}{48}$; D, $\frac{5}{24}$.

41. What is the value of $\overline{2\frac{1}{4}\times\frac{1}{5}+\frac{5}{6}}$ of $4\frac{1}{4}\times(\frac{2}{5})^2+\overline{(3\frac{2}{3})^3-(3\frac{2}{3})^2}$? *Ans.* $36\frac{469}{540}$.

DIVISION.

194. 1. Divide $\frac{21}{25}$ by 3.

FIRST OPERATION.

$$\frac{21}{25}\div 3=\frac{7}{25}$$

SECOND OPERATION.

$$\frac{21}{25}\div 3=\frac{21}{75}=\frac{7}{25}$$

ANALYSIS. In the first operation we divide the fraction by 3 by dividing its numerator by 3, and in the second operation we divide the fraction by 3 by multiplying its denominator by 3, (**190**, II or III).

2. Divide 15 by $\frac{3}{7}$.

FIRST OPERATION.

$$15\div\frac{3}{7}=5\times 7=35$$

SECOND OPERATION.

$$15\div\frac{3}{7}=105\div 3=35$$

ANALYSIS. To divide by $\frac{3}{7}$, we must divide by 3 and multiply by 7, (**191**, II or III).

In the first operation, we first divide 15 by 3, and then multiply the quotient by 7. In the second operation we first multiply 15 by 7, and then divide the product by 3.

3. Divide $\frac{4}{15}$ by $\frac{3}{5}$.

FIRST OPERATION.

1st step, $\frac{4}{15}\div 3=\frac{4}{45}$

2d step, $\frac{4}{45}\times 5=\frac{20}{45}=\frac{4}{9}$ *Ans.*

SECOND OPERATION.

$$\frac{4}{15}\times\frac{5}{3}=\frac{20}{45}=\frac{4}{9}$$

ANALYSIS. To divide by $\frac{3}{5}$, we must divide by 3 and multiply by 5, (**191**, II or III). In the first operation we first divide $\frac{4}{15}$ by 3 by multiplying the denominator by 3. We then multi-

THIRD OPERATION.

$$\frac{4}{\cancel{15}_{\,3}} \times \frac{\cancel{5}}{3} = \frac{4}{9}$$

ply the result, $\frac{4}{45}$, by 5, by multiplying the numerator by 5, giving $\frac{20}{45} = \frac{4}{9}$ for the required quotient. By inspecting this operation, we observe that the result, $\frac{20}{45}$, is obtained by multiplying the denominator of the given dividend by the numerator of the divisor, and the numerator of the dividend by the denominator of the divisor. Hence, in the second operation, we invert the terms of the divisor, $\frac{3}{5}$, and then multiply the upper terms together for a numerator, and the lower terms together for a denominator, and obtain the same result as in the first operation. In the third operation, we shorten the process by cancellation.

We have learned (**107**) that the reciprocal of a number is 1 divided by the number. If we divide 1 by $\frac{3}{5}$, we shall have $1 \div \frac{3}{5} = 1 \times \frac{5}{3} = \frac{5}{3}$. Hence

195. The **Reciprocal of a Fraction** is the fraction inverted.

From these principles and illustrations we derive the following general

RULE. I. *Reduce integers and mixed numbers to improper fractions.*

II. *Multiply the dividend by the reciprocal of the divisor.*

NOTES.—1. If the vertical line be used, the numerators of the dividend and the denominators of the divisor must be written on the right of the vertical.

2. Since a compound fraction is an indicated product of several fractions, its reciprocal may be obtained by inverting each factor of the compound fraction.

EXAMPLES FOR PRACTICE.

1. Divide $\frac{12}{35}$ by 4. $\frac{12}{35} \times \frac{1}{4} = \frac{3}{35}$, *Ans.*
2. Divide $\frac{10}{11}$ by 5, and $\frac{128}{135}$ by 80.
3. Divide 10 by $\frac{2}{7}$. *Ans.* 35.
4. Divide 28 by $\frac{3}{4}$, and 3 by $\frac{5}{12}$.
5. Divide 56 by $1\frac{5}{9}$. *Ans.* 36.
6. Divide $\frac{15}{24}$ by $\frac{5}{6}$.
7. Divide $\frac{16}{27}$ by $\frac{8}{9}$, $\frac{12}{55}$ by $\frac{9}{77}$, and $3\frac{5}{6}$ by $5\frac{1}{9}$.
8. Divide $1\frac{7}{8}$ by $1\frac{1}{8}$. *Ans.* $1\frac{2}{3}$.
9. Divide $1\frac{14}{91}$ by $\frac{35}{52}$. *Ans.* $1\frac{5}{7}$.

10. Divide $\frac{3}{5}$ of $\frac{5}{9}$ by $\frac{7}{9}$ of $\frac{5}{14}$.

OPERATION.

$$\frac{3}{5} \times \frac{5}{9} = \frac{1}{3}$$
$$\frac{7}{9} \times \frac{5}{14} = \frac{5}{18}$$
$$\frac{1}{3} \times \frac{18}{5} = \frac{6}{5} = 1\frac{1}{5} \quad Ans.$$

Or,

$$\frac{3}{5} \times \frac{5}{9} \times \frac{9}{7} \times \frac{14}{5} = 1\frac{1}{5}$$

ANALYSIS. The dividend, reduced to a simple fraction, is $\frac{1}{3}$; the divisor, reduced in like manner, is $\frac{5}{18}$; and $\frac{1}{3}$ divided by $\frac{5}{18}$ is $1\frac{1}{5}$, the quotient required. Or, we may apply the general rule directly by inverting both factors of the divisor.

NOTE 3.—The second method of solution given above has two advantages. 1st, It gives the answer by a single operation; 2d, It affords greater facility for cancellation.

11. Divide $\frac{4}{9}$ of $\frac{5}{11}$ by $\frac{8}{11}$ of $\frac{5}{18}$. *Ans.* 1.

12. Divide $\frac{7}{12}$ of $\frac{8}{13}$ by $\frac{7}{9}$ of $\frac{5}{21}$. *Ans.* $1\frac{61}{65}$.

13. Divide $2\frac{1}{2} \times 7\frac{1}{5}$ by $3\frac{1}{3} \times 3\frac{3}{10}$.

14. Divide 11 by $\frac{2}{3} \times 5\frac{1}{2} \times 7$.

15. Divide $3\frac{1}{4} \times 19$ by $\frac{1}{5} \times 7\frac{3}{5} \times 1\frac{5}{8}$. *Ans.* 25.

16. Divide $\frac{5}{12} \times \frac{18}{25}$ by $\frac{1}{2} \times \frac{7}{8} \times \frac{5}{17} \times \frac{34}{35} \times \frac{51}{72}$. *Ans.* $3\frac{33}{85}$.

17. Divide $\frac{391}{589}$ by $\frac{667}{1178}$. *Ans.* $1\frac{5}{29}$.

18. Divide $\frac{5239}{27671}$ by $\frac{31}{59} \times \frac{39}{56} \times \frac{64}{67}$. *Ans.* $\frac{13}{24}$.

19. Divide $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$ by $\frac{5}{6} \times \frac{6}{7} \times \frac{7}{8} \times \frac{8}{9} \times \frac{9}{10}$.

20. What is the value of $\dfrac{5\frac{1}{2}}{4\frac{2}{5}}$?

OPERATION.

$$\frac{5\frac{1}{2}}{4\frac{2}{5}} = \frac{\frac{11}{2}}{\frac{22}{5}} = \frac{11}{2} \times \frac{5}{22} = \frac{5}{4} = 1\frac{1}{4}$$

ANALYSIS. The fractional form indicates division, the numerator being the dividend and the denominator the divisor, (168, II); hence, we reduce the mixed numbers to improper fractions, and then treat the denominator, $\frac{22}{5}$, as a divisor, and obtain the result, $1\frac{1}{4}$, by the general rule for division of fractions.

NOTE 4.—Expressions like $\dfrac{5\frac{1}{2}}{4\frac{2}{5}}$ and $\dfrac{\frac{11}{2}}{\frac{22}{5}}$ are sometimes called *complex fractions*.

5. In the reduction of complex fractions to simple fractions, if either the numerator or denominator consists of one or more parts connected by + or —, the operations indicated by these signs must first be performed, and afterward the division.

21. What is the value of $\dfrac{\frac{3}{4}}{\frac{7}{8}}$? *Ans.* $\frac{6}{7}$.

22. What is the value of $\dfrac{\frac{2}{3} \times \frac{11}{12}}{\frac{1}{18} \times 5\frac{1}{2}}$? *Ans.* 2.

23. What is the value of $\dfrac{7 + 3\frac{5}{8}}{1\frac{5}{12}}$? *Ans.* $7\frac{1}{2}$.

24. Reduce $\dfrac{\frac{3}{4} - \frac{3}{5}}{\frac{1}{3} + \frac{3}{8}}$ to its simplest form.

25. Reduce $\dfrac{\frac{5}{7} - \frac{2}{3}}{\frac{1}{3} \times \frac{2}{7}}$ to its simplest form.

26. Reduce $\dfrac{\frac{5}{9} \text{ of } \frac{3}{7}}{6\frac{1}{5} - 5\frac{4}{15}}$ to its simplest form. *Ans.* $\frac{25}{98}$.

27. If 7 pounds of coffee cost $\$\frac{4}{5}$, how much will 1 pound cost?

28. If a boy earn $\$\frac{3}{8}$ a day, how many days will it take him to earn $\$6\frac{1}{2}$? *Ans.* $17\frac{1}{3}$.

29. If $\frac{4}{9}$ of an acre of land sell for $30, what will an acre sell for at the same rate? *Ans.* $\$67\frac{1}{2}$.

30. At $\frac{1}{2}$ of $\frac{3}{4}$ of a dollar a pint, how much wine can be bought for $\$\frac{9}{10}$? *Ans.* $2\frac{2}{5}$ pints.

31. If $\frac{3}{10}$ of a barrel of flour be worth $\$2\frac{1}{6}$, how much is 1 barrel worth? *Ans.* $\$7\frac{2}{9}$.

32. Bought $\frac{1}{5}$ of $4\frac{1}{3}$ cords of wood, for $\frac{2}{5}$ of $\frac{1}{3}$ of $30; what was 1 cord worth at the same rate? *Ans.* $\$4\frac{8}{13}$.

33. If $235\frac{1}{2}$ acres of land cost $\$1725\frac{3}{8}$, how much will $125\frac{1}{3}$ acres cost? *Ans.* $\$918\frac{116}{471}$.

34. Of what number is $26\frac{1}{4}$ the $\frac{5}{6}$ part? *Ans.* $31\frac{1}{2}$.

35. The product of two numbers is 27, and one of them is $2\frac{5}{9}$; what is the other?

36. By what number must you multiply $16\frac{11}{12}$ to produce $148\frac{3}{4}$?

37. What number is that which, if multiplied by $\frac{3}{8}$ of $\frac{5}{6}$ of 2, will produce $\frac{7}{9}$? *Ans.* $1\frac{11}{45}$.

38. Divide $720 - (\frac{5}{8} \times \overline{28 - 7\frac{1}{2}})$ by $\overline{40\frac{1}{4} + (\frac{9}{10} \div \frac{3}{5})} \times (\frac{1}{2})^4$.

39. What is the value of $\left(3\frac{1}{2} \times (\frac{2}{5})^2 + \frac{3}{4} \text{ of } \frac{1}{3}\right)^3 \div \left(17\frac{1}{8} - \frac{3}{5} + \overline{\frac{2}{7} \div (\frac{2}{3})^3} \times 5\right)$?

40. Divide $\dfrac{\frac{1}{3} \text{ of } (\frac{2}{5})^3 \times 3\frac{1}{4}}{\frac{1}{2} \text{ of } (9\frac{1}{4})^2 \times (\frac{2}{3})^4}$ by $\dfrac{\frac{3}{4} \text{ of } 5\frac{1}{3}}{\frac{8}{11} \times 2^3}$. *Ans.* $\frac{22464}{1882375}$.

GREATEST COMMON DIVISOR OF FRACTIONS.

196. The **Greatest Common Divisor** of two or more fractions is the greatest number which will exactly divide each of them, giving a whole number for a quotient.

NOTE.—The definition of an exact divisor, (**128**), is general, and applies to fractions as well as to integers.

197. In the division of one fraction by another the quotient will be a whole number, if, when the divisor is inverted, the two lower terms may both be canceled. This will be the case when the numerator of the divisor is exactly contained in the numerator of the dividend, and the denominator of the divisor exactly contains, *or is a multiple of*, the denominator of the dividend. Hence,

I. A fraction is an exact divisor of a given fraction when its numerator is a *divisor* of the given *numerator*, and its denominator is a *multiple* of the given *denominator*. And,

II. A fraction is a common divisor of two or more given fractions when its numerator is a *common divisor* of the given *numerators*, and its denominator is a *common multiple* of the given *denominators*. Therefore,

III. The greatest common divisor of two or more given fractions is a fraction whose numerator is the *greatest common divisor* of the given *numerators*, and whose denominator is the *least common multiple* of the given *denominators*.

1. What is the greatest common divisor of $\frac{5}{6}$, $\frac{5}{12}$, and $\frac{15}{16}$?

ANALYSIS. The greatest common divisor of 5, 5, and 15, the given numerators, is 5. The least common multiple of 6, 12, and 16, the given denominators, is 48. Therefore the greatest common divisor of the given fractions is $\frac{5}{48}$, *Ans.* (III).

PROOF.

$$\left.\begin{array}{l} \frac{5}{6} \div \frac{5}{48} = 8 \\ \frac{5}{12} \div \frac{5}{48} = 4 \\ \frac{15}{16} \div \frac{5}{48} = 9 \end{array}\right\} \textit{Prime}\text{ to each other.}$$

198. From these principles and illustrations, we derive the following

RULE. *Find the greatest common divisor of the given numerators for a new numerator, and the least common multiple of the given denominators for a new denominator. This fraction will be the greatest common divisor sought.*

NOTE.—Whole and mixed numbers must first be reduced to improper fractions, and all fractions to their lowest terms.

EXAMPLES FOR PRACTICE.

1. What is the greatest common divisor of $\frac{7}{9}$, $\frac{14}{27}$, and $\frac{28}{45}$?
Ans. $\frac{7}{135}$.
2. What is the greatest common divisor of $3\frac{1}{5}$, $1\frac{5}{7}$, and $\frac{24}{35}$?
3. What is the greatest common divisor of 4, $2\frac{2}{9}$, $2\frac{2}{5}$, and $\frac{4}{90}$?
Ans. $\frac{2}{45}$.
4. What is the greatest common divisor of $109\frac{1}{5}$ and $122\frac{4}{7}$?
5. What is the length of the longest measure that can be exactly contained in each of the two distances, $18\frac{2}{5}$ feet and $57\frac{1}{2}$ feet? *Ans.* $2\frac{3}{10}$ feet.
6. A merchant has three kinds of wine, of the first $134\frac{3}{4}$ gallons, of the second $128\frac{1}{3}$ gallons, of the third $115\frac{1}{2}$ gallons; he wishes to ship the same in full casks of equal size; what is the least number he can use without mixing the different kinds of wine? How many kegs will be required? *Ans.* 59.

LEAST COMMON MULTIPLE OF FRACTIONS.

199. The **Least Common Multiple** of two or more fractions is the least number which can be exactly divided by each of them, giving a whole number for a quotient.

200. Since in performing operations in division of fractions the divisor is inverted, it is evident that one fraction will exactly contain another when the numerator of the dividend exactly contains the numerator of the divisor, and the denominator of the dividend is exactly contained in the denominator of the divisor Hence,

I. A fraction is a multiple of a given fraction when its numerator is a *multiple* of the *given numerator*, and its denominator is a *divisor* of the *given denominator*. And

II. A fraction is a common multiple of two or more given fractions when its numerator is a *common multiple* of the *given numerators,* and its denominator is a *common divisor* of the *given denominators.* Therefore,

III. The least common multiple of two or more given fractions is a fraction whose numerator is the *least common multiple* of the *given numerators,* and whose denominator is the *greatest common divisor* of the *given denominators.*

NOTE.—The least whole number that will exactly contain two or more given fractions in their lowest terms, is the least common multiple of their numerators, (**193**, Note 2).

1. What is the least common multiple of $\frac{3}{4}$, $\frac{5}{12}$, and $\frac{15}{16}$?

ANALYSIS. The least common multiple of 3, 5, and 15, the given numerators, is 15; the greatest common divisor of 4, 12, and 16, the given denominators, is 4. Hence, the least common multiple of the given fractions is $\frac{15}{4} = 3\frac{3}{4}$, *Ans.* (III).

201. From these principles and illustrations we derive the following

RULE. *Find the least common multiple of the given numerators for a new numerator, and the greatest common divisor of the given denominators for a new denominator. This fraction will be the least common multiple sought.*

NOTE.—Mixed numbers and integers should be reduced to improper fractions, and all fractions to their lowest terms, before applying the rule.

EXAMPLES FOR PRACTICE.

1. What is the least common multiple of $\frac{2}{5}$, $\frac{7}{10}$, $\frac{14}{15}$, and $\frac{8}{25}$. *Ans.* $11\frac{1}{5}$.

2. What is the least common multiple of $\frac{7}{24}$, $\frac{35}{36}$, and $\frac{49}{60}$?

3. What is the least common multiple of $2\frac{2}{25}$, $1\frac{37}{75}$, and $\frac{63}{100}$?

4. What is the least common multiple of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, and $\frac{9}{10}$? *Ans.* 2520.

5. The driving wheels of a locomotive are $15\frac{5}{16}$ feet in circumference, and the trucks $9\frac{3}{8}$ feet in circumference. What distance must the train move, in order to bring the wheel and truck in the same relative positions as at starting? *Ans.* $459\frac{3}{8}$ feet.

PROMISCUOUS EXAMPLES.

1. Change $\frac{7}{9}$ of $\frac{5}{7}$ to an equivalent fraction having 135 for its denominator. *Ans.* $\frac{75}{135}$.

2. Reduce $\frac{3}{4}$, $\frac{1}{6}$, $\frac{5}{8}$, and $\frac{11}{12}$ to equivalent fractions, whose denominators shall be 48.

3. Find the least common denominator of $1\frac{1}{3}$, $\frac{6}{7}$, 2, $\frac{7}{10}$, $\frac{2}{3}$ of $\frac{4}{5}$, $\frac{4}{9}$ of $\frac{1}{4}$.

4. The sum of $\dfrac{\frac{2}{3}\text{ of }\frac{3}{4}}{\frac{1}{2}}$ and $\dfrac{\frac{2}{5}\text{ of }\frac{5}{6}}{\frac{2}{9}\text{ of }4\frac{1}{2}}$ is equal to how many times their difference? *Ans.* 2.

5. The less of two numbers is $\dfrac{54\frac{3}{5}}{\frac{1}{5}\text{ of }8\frac{2}{3}}$, and their difference $\dfrac{1\frac{5}{9}}{\frac{9}{16}}$; what is the greater number? *Ans.* $34\frac{43}{162}$.

6. What number multiplied by $\frac{2}{9}$ of $\frac{5}{8}\times 3\frac{2}{7}$, will produce $\frac{23}{84}$? *Ans.* $\frac{3}{5}$.

7. Find the value of $\dfrac{2-\frac{1}{4}}{2}\times\dfrac{(8\frac{4}{7})^2}{12}+\overline{(2+\frac{1}{5})\div(3+\frac{1}{7})}+\dfrac{11\frac{3}{32}}{8\frac{7}{8}}$. *Ans.* $7\frac{43}{140}$.

8. What number diminished by the difference between $\frac{2}{7}$ and $\frac{7}{9}$ of itself, leaves a remainder of 144? *Ans.* $283\frac{1}{2}$.

9. A person spending $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{1}{8}$ of his money, had \$119 left; how much had he at first?

10. What will $\frac{1}{2}$ of $10\frac{7}{8}$ cords of wood cost, at $\frac{4}{29}$ of \$42 per cord? *Ans.* \$$31\frac{1}{2}$.

11. There are two numbers whose difference is $25\frac{7}{15}$, and one number is $\frac{5}{7}$ of the other; what are the numbers? *Ans.* $63\frac{2}{3}$ and $89\frac{2}{15}$.

12. Divide \$2000 between two persons, so that one shall have $\frac{7}{9}$ as much as the other. *Ans.* \$1125 and \$875.

13. If a man travel 4 miles in $\frac{3}{5}$ of an hour, how far would he travel in $1\frac{1}{2}$ hours at the same rate? *Ans.* 10 miles.

14. At \$$\frac{7}{9}$ a yard, how many yards of silk can be bought for \$$10\frac{5}{6}$?

15. How many bushels of oats worth \$$\frac{2}{5}$ a bushel, will pay for $\frac{2}{3}$ of a barrel of flour at \$$7\frac{1}{3}$ a barrel?

16. If $\frac{3}{5}$ of a bushel of barley be worth $\frac{3}{4}$ of a bushel of corn, and corn be worth $\$\frac{2}{3}$ per bushel, how many bushels of barley will \$15 buy? *Ans.* 18.

17. If 48 is $\frac{5}{7}$ of some number, what is $\frac{3}{4}$ of the same number?

18. If cloth $1\frac{1}{4}$ yards in breadth require $20\frac{1}{2}$ yards in length to make a certain number of garments, how many yards in length will cloth $\frac{3}{4}$ of a yard wide require to make the same?

19. A gentleman owning $\frac{5}{8}$ of an iron foundery, sold $\frac{4}{5}$ of his share for $\$2570\frac{2}{3}$; how much was the whole foundery worth? *Ans.* $\$5141\frac{1}{3}$.

20. Suppose the cargo of a vessel to be worth \$10,000, and $\frac{4}{9}$ of $\frac{5}{7}$ of $\frac{9}{10}$ of the vessel be worth $\frac{1}{4}$ of $\frac{4}{5}$ of $1\frac{5}{7}$ of the cargo; what is the whole value of the ship and cargo? *Ans.* \$22000.

21. A gentleman divided his estate among his three sons as follows: to the first he gave $\frac{3}{8}$ of it; to the second $\frac{2}{3}$ of the remainder. The difference between the portions of first and second was \$500. What was the whole estate, and how much was the third son's share? *Ans.* Whole estate, \$12000. Third son's share, \$2500.

22. If $7\frac{1}{2}$ tons of hay cost \$60, how many tons can be bought for \$78, at the same rate?

23. If a person agree to do a job of work in 30 days, what part of it ought he to do in $16\frac{1}{2}$ days? *Ans.* $\frac{11}{20}$.

24. A father divided a piece of land among his three sons; to the first he gave $12\frac{1}{4}$ acres, to the second $\frac{3}{8}$ of the whole, and to the third as much as to the other two; how many acres did the third have? *Ans.* 49 acres.

25. If $\frac{3}{4}$ of 6 bushels of wheat cost $\$4\frac{1}{2}$, how much will $\frac{4}{5}$ of 1 bushel cost?

26. A man engaging in trade lost $\frac{2}{5}$ of his money invested, after which he gained \$740, when he had \$3500; how much did he lose? *Ans.* \$1840.

27. A cistern being full of water sprung a leak, and before it could be stopped, $\frac{5}{8}$ of the water ran out, but $\frac{3}{5}$ as much ran in at the same time; what part of the cistern was emptied? *Ans.* $\frac{1}{4}$.

28. A can do a certain piece of work in 8 days, and B can do the same in 6 days; in what time can both together do it?

Ans. $3\frac{3}{7}$ days.

29. A merchant sold 5 barrels of flour for \$$32\frac{1}{2}$, which was $\frac{5}{8}$ as much as he received for all he had left, at \$4 a barrel; how many barrels in all did he sell? *Ans.* 18.

30. What is the least number of gallons of wine, expressed by a whole number, that will exactly fill, without waste, bottles containing either $\frac{5}{6}$, $\frac{3}{8}$, $\frac{6}{7}$, or $\frac{4}{5}$ gallons? *Ans.* 60.

31. A, B, and C start at the same point in the circumference of a circular island, and travel round it in the same direction. A makes $\frac{2}{7}$ of a revolution in a day, B $\frac{4}{17}$, and C $\frac{8}{51}$. In how many days will they all be together at the point of starting?

Ans. $178\frac{1}{2}$ days.

32. Two men are $64\frac{3}{4}$ miles apart, and travel toward each other; when they meet one has traveled $5\frac{1}{2}$ miles more than the other; how far has each traveled?

Ans. One $29\frac{5}{8}$ miles, the other $35\frac{1}{8}$ miles.

33. There are two numbers whose sum is $1\frac{1}{10}$, and whose difference is $\frac{2}{5}$; what are the numbers? *Ans.* $\frac{3}{4}$ and $\frac{7}{20}$.

34. A, B, and C own a ferry boat; A owns $\frac{13}{126}$ of the boat, and B owns $\frac{7}{18}$ of the boat more than C. What shares do B and C own respectively? *Ans.* B, $\frac{9}{14}$; C, $\frac{16}{63}$.

35. A schoolboy being asked how many dollars he had, replied, that if his money be multiplied by $\frac{14}{15}$, and $\frac{1}{6}$ of a dollar be added to the product, and $\frac{2}{5}$ of a dollar taken from the sum, this remainder divided by $\frac{6}{25}$ would be equal to the reciprocal of $\frac{4}{5}$ of a dollar. How much money had he?

36. If a certain number be increased by $1\frac{3}{4}$, this sum diminished by $\frac{3}{8}$, this remainder multiplied by $5\frac{2}{5}$, and this product divided by $1\frac{2}{7}$, the quotient will be $7\frac{1}{2}$; what is the number? *Ans.* $\frac{23}{56}$.

37. If $\frac{2}{3}$ of $\frac{4}{5}$ of $3\frac{1}{2}$ times any number be multiplied by $\frac{7}{8}$, the product divided by $\frac{2}{9}$, the quotient increased by $4\frac{1}{6}$, and the sum diminished by $\frac{3}{7}$ of itself, the remainder will be how many times the number? *Ans.* $6\frac{61}{105}$ times.

DECIMAL FRACTIONS.

NOTATION AND NUMERATION.

202. A **Decimal Fraction** is one or more of the decimal divisions of a unit.

NOTES.—1. The word *decimal* is derived from the Latin *decem*, which signifies *ten*.
2. Decimal fractions are commonly called *decimals*.

203. In the formation of decimals, a simple unit is divided into ten equal parts, forming decimal units of the first order, or tenths, each tenth is divided into ten equal parts, forming decimal units of the second order, or hundredths; and so on, according to the following

TABLE OF DECIMAL UNITS.

1 single unit	equals	10 tenths;
1 tenth	"	10 hundredths;
1 hundredth	"	10 thousandths;
1 thousandth	"	10 ten thousandths.
etc.		etc.

204. In the notation of decimals it is not necessary to employ denominators as in common fractions; for, since the different orders of units are formed upon the decimal scale, the same law of local value as governs the notation of simple integral numbers, (**57**), enables us to indicate the relations of decimals by place or position.

205. The Decimal Sign (.) is always placed before decimal figures to distinguish them from integers. It is commonly called the *decimal point*. When placed between integers and decimals in the same number, is sometimes called the *separatrix*.

206. The law of local value, extended to decimal units, assigns the first place at the right of the decimal sign to tenths; the second, to hundredths; the third, to thousandths; and so on, as shown in the following

DECIMAL NUMERATION TABLE.

Millions.	Hund. of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	*Decimal Sign.*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
5	7	3	2	7	5	4	.	5	7	3	2	5	6
7th order,	6th order,	5th order,	4th order,	3d order,	2d order,	1st order,		1st order,	2d order,	3d order,	4th order,	5th order,	6th order,
Integers.								Decimals.					

207. The denominator of a decimal fraction, when expressed, is necessarily 10, 100, 1000, or some power of 10. By examining the table it will be seen, that the number of places in a decimal is equal to the number of ciphers required to express its denominator. Thus, tenths occupy the first place at the right of units, and the denominator of $\frac{1}{10}$ has one cipher; hundredths in the table extend two places from units, and the denominator of $\frac{1}{100}$ has two ciphers; and so on.

208. A decimal is usually read as expressing a certain number of decimal units of the lowest order contained in the decimal. Thus, 5 tenths and 4 hundredths, or .54, may be read, fifty-four hundredths. For, $\frac{5}{10} + \frac{4}{100} = \frac{54}{100}$.

209. From the foregoing explanations and illustrations we derive the following

PRINCIPLES OF DECIMAL NOTATION AND NUMERATION.

I. Decimals are governed by the same law of local value that governs the notation of integers.

II. The different orders of decimal units decrease from left to right, and increase from right to left, in a tenfold ratio.

III. The value of any decimal figure depends upon the place it occupies at the right of the decimal sign.

IV. Prefixing a cipher to a decimal diminishes its value tenfold, since it removes every decimal figure one place to the right.

V. Annexing a cipher to a decimal does not alter its value, since it does not change the place of any figure in the decimal.

VI. The denominator of a decimal, when expressed, is the unit, 1, with as many ciphers annexed as there are places in the decimal.

VII. To read a decimal requires two numerations; first, *from* units, to find the name of the denominator; second, *towards* units, to find the value of the numerator.

210. Having analyzed all the principles upon which the writing and reading of decimals depend, we will now present these principles in the form of rules.

RULE FOR DECIMAL NOTATION.

I. *Write the decimal the same as a whole number, placing ciphers in the place of vacant orders, to give each significant figure its true local value.*

II. *Place the decimal point before the first figure.*

RULE FOR DECIMAL NUMERATION.

I. *Numerate from the decimal point, to determine the denominator.*

II. *Numerate towards the decimal point, to determine the numerator.*

III. *Read the decimal as a whole number, giving it the name of its lowest decimal unit, or right hand figure.*

EXAMPLES FOR PRACTICE.

Express the following decimals by figures, according to the decimal notation.

1. Five tenths. *Ans.* .5.
2. Thirty-six hundredths. *Ans.* .36.
3. Seventy-five ten-thousandths. *Ans.* .0075.

4. Four hundred ninety-six thousandths.

5. Three hundred twenty-five ten-thousandths.

6. One millionth.

7. Seventy-four ten-millionths.

8. Four hundred thirty-seven thousand five hundred forty-nine millionths.

9. Three million forty thousand ten ten-millionths.

10. Twenty-four hundred-millionths.

11. Eight thousand six hundred forty-five hundred-thousandths.

12. Four hundred ninety-five million seven hundred five thousand forty-eight billionths.

13. Ninety-nine thousand nine ten-billionths.

14. Four million seven hundred thirty-five thousand nine hundred one hundred-millionths.

15. One trillionth.

16. One trillion one billion one million one thousand one ten-trillionths.

17. Eight hundred forty-one million five hundred sixty-three thousand four hundred thirty-six trillionths.

18. Nine quintillionths.

Express the following fractions and mixed numbers decimally:

19. $\frac{3}{10}$. *Ans.* .3.

20. $\frac{105}{1000}$.

21. $\frac{11}{10000}$.

22. $\frac{85}{1000000}$.

23. $\frac{100004}{10000000}$.

24. $\frac{704}{100000000}$.

25. $46\frac{4}{10}$. *Ans.* 46.4.

26. $205\frac{65}{100}$.

27. $60\frac{36}{1000000}$.

28. $705\frac{5}{10000000000}$.

29. $300\frac{10001001}{1000000000}$.

30. $52\frac{5}{10000000000000}$.

Read the following numbers:

31. .24.

32. .075.

33. .503.

34. .00725.

35. .40000004.

36. .0000256.

37. .0010075.

38. 8.25.

39. 75.368.

40. 42.0637.

41. 8.0074.

42. 30.4075.

43. 26.00005.

44. 100.00000001.

REDUCTION.

CASE I.

211. To reduce decimals to a common denominator.

1. Reduce .5, .24, .7836 and .375 to a common denominator.

OPERATION.
.5000
.2400
.7836
.3750

ANALYSIS. A common denominator must contain as many decimal places as are equal to the greatest number of decimal figures in any of the given decimals. We find that the third number contains four decimal places, and hence 10000 must be a common denominator. As annexing ciphers to decimals does not alter their value, we give to each number four decimal places, by annexing ciphers, and thus reduce the given decimals to a common denominator. Hence,

RULE. *Give to each number the same number of decimal places, by annexing ciphers.*

NOTES.—1. If the numbers be reduced to the denominator of that one of the given numbers having the greatest number of decimal places, they will have their least common decimal denominator.

2. An integer may readily be reduced to decimals by placing the decimal point after units, and annexing ciphers; one cipher reducing it to tenths, two ciphers to hundredths, three ciphers to thousandths, and so on.

EXAMPLES FOR PRACTICE.

1. Reduce .18, .456, .0075, .000001, .05, .3789, .5943786, and .001 to their least common denominator.

2. Reduce 12 thousandths, 185 millionths, 936 billionths, and 7 trillionths to their least common denominator.

3. Reduce 57.3, 900, 4.7555, and 100.000001 to their least common denominator.

CASE II.

212. To reduce a decimal to a common fraction.

1. Reduce .375 to an equivalent common fraction.

OPERATION.

$$.375 = \frac{375}{1000} = \frac{3}{8}.$$

ANALYSIS. Writing the decimal figures, .375, over the common denominator, 1000, we have $\frac{375}{1000} = \frac{3}{8}$.

Hence,

Rule. *Omit the decimal point, and supply the proper denominator.*

EXAMPLES FOR PRACTICE.

1. Reduce .75 to a common fraction. *Ans.* $\frac{3}{4}$.
2. Reduce .625 to a common fraction. *Ans.* $\frac{5}{8}$.
3. Reduce .12 to a common fraction.
4. Reduce .68 to a common fraction.
5. Reduce .5625 to a common fraction.
6. Reduce .024 to a common fraction. *Ans.* $\frac{3}{125}$.
7. Reduce .00032 to a common fraction. *Ans.* $\frac{1}{3125}$.
8. Reduce .002624 to a common fraction. *Ans.* $\frac{41}{15625}$.
9. Reduce $.13\frac{1}{3}$ to a common fraction.

OPERATION.

$$.13\frac{1}{3} = \frac{13\frac{1}{3}}{100} = \frac{40}{300} = \frac{2}{15}.$$

Note.—The decimal $.13\frac{1}{3}$ may properly be called a *complex decimal.*

10. Reduce $.57\frac{1}{7}$ to a common fraction. *Ans.* $\frac{4}{7}$.
11. Reduce $.66\frac{2}{3}$ to a common fraction. *Ans.* $\frac{2}{3}$.
12. Reduce $.444\frac{4}{9}$ to a common fraction.
13. Reduce $.024\frac{2}{3}$ to a common fraction. *Ans.* $\frac{37}{1500}$.
14. Reduce $.984\frac{3}{8}$ to a common fraction.
15. Express 7.4 by an integer and a common fraction. *Ans.* $7\frac{2}{5}$.
16. Express 24.74 by an integer and a common fraction.
17. Reduce 2.1875 to an improper fraction. *Ans.* $\frac{35}{16}$.
18. Reduce 1.64 to an improper fraction.
19. Reduce 7.496 to an improper fraction. *Ans.* $\frac{937}{125}$.

CASE III.

213. To reduce a common fraction to a decimal.

1. Reduce $\frac{5}{8}$ to its equivalent decimal.

FIRST OPERATION.

$$\frac{5}{8} = \frac{5000}{8000} = \frac{625}{1000} = .625$$

Analysis. We first annex the same number of ciphers to both terms of the fraction; this does not alter its value, **(174,**

SECOND OPERATION.

$$8\,)\underline{5.000}$$
$$.625$$

III); we then divide both resulting terms by 8, the significant figure of the denominator, and obtain the decimal denominator, 1000. Omitting the denominator, and prefixing the sign, we have the equivalent decimal, .625.

In the second operation, we omit the intermediate steps, and obtain the result, practically, by annexing the three ciphers to the numerator, 5, and dividing the result by the denominator, 8.

2. Reduce $\frac{3}{125}$ to a decimal.

OPERATION.

$$125\,)\underline{3.000}$$
$$.024$$

ANALYSIS. Dividing as in the former example, we obtain a quotient of 2 figures, 24. But since 3 ciphers have been annexed to the numerator, 3, there must be three places in the required decimal; hence we prefix 1 cipher to the quotient figures, 24. The reason of this is shown also in the following operation.

$$\tfrac{3}{125} = \tfrac{3000}{125000} = \tfrac{24}{1000} = .024$$

214. From these illustrations we derive the following

RULE. I. *Annex ciphers to the numerator, and divide by the denominator.*

II. *Point off as many decimal places in the result as are equal to the number of ciphers annexed.*

NOTE.—If the division is not exact when a sufficient number of decimal figures have been obtained, the sign, +, may be annexed to the decimal to indicate that there is still a remainder. When this remainder is such that the next decimal figure would be 5 or greater than 5, the last figure of the terminated decimal may be increased by 1, and the sign, —, annexed. And in general, + denotes that the written decimal is too small, and — denotes that the written decimal is too large; the error always being less than one half of a unit in the last place of the decimal.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{3}{4}$ to a decimal. *Ans.* .75.
2. Reduce $\frac{5}{16}$ to a decimal. *Ans.* .3125.
3. Reduce $\frac{7}{8}$ to a decimal.
4. Reduce $\frac{14}{25}$ to a decimal.
5. Reduce $\frac{13}{16}$ to a decimal.
6. Reduce $\frac{1}{25}$ to a decimal. *Ans.* .04.
7. Reduce $\frac{17}{250}$ to a decimal. *Ans.* .068.

8. Reduce $\frac{19}{32}$ to a decimal. *Ans.* .59375
9. Reduce $\frac{23}{12800}$ to a decimal.
10. Reduce $\frac{7}{24}$ to a decimal. *Ans.* .29167—.
11. Reduce $\frac{97}{160}$ to a decimal.
12. Reduce $\frac{43}{56}$ to a decimal. *Ans.* .767857+.
13. Reduce $7\frac{1}{8}$ to the decimal form. *Ans.* 7.125.
14. Reduce $56\frac{5}{64}$ to the decimal form. *Ans.* 56.078125.
15. Reduce $32\frac{5}{7}$ to the decimal form.
16. Reduce $.24\frac{1}{2}$ to a simple decimal.
17. Reduce $5.78\frac{10}{32}$ to a simple decimal.
18. Reduce $.3\frac{11}{1250}$ to a simple decimal. *Ans.* .30088.
19. Reduce $4.0\frac{2}{25}$ to a simple decimal. *Ans.* 4.008.
20. Reduce $.30\frac{1001}{143000}$ to a simple decimal.

ADDITION.

215. Since the same law of local value extends both to the right and left of units' place; that is, since decimals and simple integers increase and decrease uniformly by the scale of ten, it is evident that decimals may be added, subtracted, multiplied and divided in the same manner as integers.

216. 1. What is the sum of 4.75, .246, 37.56 and 12.248?

OPERATION.

$$\begin{array}{r} 4.75 \\ .246 \\ 37.56 \\ 12.248 \\ \hline 54.804 \end{array}$$

ANALYSIS. We write the numbers so that units of like orders, whether integral or decimal, shall stand in the same columns; that is, units under units, tenths under tenths, etc. This brings the decimal points directly under each other. Commencing at the right hand, we add each column separately, carrying 1 for every ten, according to the decimal scale; and in the result we place the decimal point between units and tenths, or directly under the decimal points in the numbers added. Hence the following

RULE. I. *Write the numbers so that the decimal points shall stand directly under each other.*

II. *Add as in whole numbers, and place the decimal point, in the result, directly under the points in the numbers added.*

EXAMPLES FOR PRACTICE.

1. Add .375, .24, .536, .78567, .4637, and .57439.

Ans. 2.97476.

2. Add 5.3756, 85.473, 9.2, 46.37859, and 45.248377.

Ans. 191.675567.

3. Add .5, .37, .489, .6372, .47856, and .02524.

4. Add $.46\frac{3}{4}$, $.325\frac{1}{8}$, $.16\frac{4}{25}$, and $.275\frac{7}{16}$. *Ans.* 1.2296625.

5. Add $4.6\frac{1}{2}$, $7.32\frac{7}{35}$, $5.3784\frac{1}{8}$, and $2.64878\frac{3}{4}$.

6. Add 4.3785, $2\frac{2}{3}$, $5\frac{3}{7}$, and 12.4872. *Ans.* 24.9609+.

7. What is the sum of 137 thousandths, 435 thousandths, 836 thousandths, 937 thousandths, and 496 thousandths?

Ans. 2.841.

8. What is the sum of one hundred two ten-thousandths, thirteen thousand four hundred twenty-six hundred thousandths, five hundred sixty-seven millionths, three millionths, and twenty-four thousand seven hundred-thousandths?

9. A farm has five corners; from the first to the second is 34.72 rods; from the second to the third, 48.44 rods; from the third to the fourth, 152.17 rods; from the fourth to the fifth, 95.36 rods; and from the fifth to the first, 56.18 rods. What is the whole distance around the farm?

10. Find the sum of $\frac{24}{97}$, $\frac{75}{436}$, $\frac{37}{150}$, and $\frac{1}{1728}$ in decimals, correct to the fourth place. *Ans.* .6669+.

NOTE.—In the reduction of each fraction, carry the decimal to at least the fifth place, in order to insure accuracy in the fourth place.

11. A man owns 4 city lots, containing $16\frac{6}{19}$ rods, $15\frac{2}{17}$ rods, $18\frac{15}{31}$ rods, and $14\frac{7}{45}$ rods of land, respectively; how many rods in all?

12. What is the sum of $4\frac{1}{2}$ decimal units of the first order, $2\frac{3}{4}$ of the second order, $9\frac{1}{8}$ of the third order, and $3\frac{1}{25}$ of the fourth order? *Ans.* .486929.

13. What is the approximate sum of 1 decimal unit of the first order, $\frac{1}{2}$ of a unit of the second order, $\frac{1}{3}$ of a unit of the third order, $\frac{1}{4}$ of a unit of the fourth order, $\frac{1}{5}$ of a unit of the fifth order, $\frac{1}{6}$ of a unit of the sixth order, and $\frac{1}{7}$ of a unit of the seventh order?

Ans. .1053605143—.

SUBTRACTION.

217. 1. From 4.156 take .5783.

OPERATION.

$$\begin{array}{r} 4.1560 \\ .5783 \\ \hline 3.5777 \end{array}$$

Or,

$$\begin{array}{r} 4.156 \\ .5783 \\ \hline 3.5777 \end{array}$$

ANALYSIS. We write the given numbers as in addition, reduce the decimals to a common denominator, and subtract as in integers. Or, we may, in practice, omit the ciphers necessary to reduce the decimals to a common denominator, and merely conceive them to be annexed, subtracting as otherwise. Hence the following

RULE. I. *Write the numbers so that the decimal points shall stand directly under each other.*

II. *Subtract as in whole numbers, and place the decimal point in the result directly under the points in the given numbers.*

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)
Minuend,	.9876	48.3676	36.5
Subtrahend,	.3598	23.98	35.875632
Remainder,	.6278	24.3876	.624368

4. From 37.456 take 24.367. *Ans.* 13.089.
5. From 1.0066 take .15.
6. From 1000 take .001. *Ans.* 999.999.
7. From $36\frac{3}{4}$ take $22\frac{12}{25}$. *Ans.* 14.27.
8. From $.56\frac{7}{8}$ take $.55\frac{124}{125}$.
9. From $7\frac{1}{3}$ take $5\frac{9}{16}$. *Ans.* 1.7708+.
10. From $\frac{991}{997}$ take $\frac{719}{761}$.
11. From one take one trillionth. *Ans.* .999999999999.
12. A speculator having 57436 acres of land, sold at different times 536.74 acres, 1756.19 acres, 3678.47 acres, 9572.15 acres, 7536.59 acres, and 4785.94 acres; how much land has he remaining?
13. Find the difference between $\frac{54321}{12345}$ and $\frac{12345}{54321}$, correct to the fifth decimal place. *Ans.* 4.17298+.

MULTIPLICATION.

218. In multiplication of decimals, the location of the decimal point in the product depends upon the following principles:

I. The number of ciphers in the denominator of a decimal is equal to the number of decimal places, (**209**, VI).

II. If two decimals, in the fractional form, be multiplied together, the denominator of the product must contain as many ciphers as there are decimal places in both factors. Therefore,

III. The product of two decimals, expressed in the decimal form, must contain as many decimal places as there are decimals in both factors.

1. Multiply .45 by .7.

OPERATION.

$$\begin{array}{r} .45 \\ .7 \\ \hline .315 \end{array}$$

PROOF.

$$\tfrac{45}{100} \times \tfrac{7}{10} = \tfrac{315}{1000} = .315$$

ANALYSIS. We first multiply as in whole numbers; then, since the multiplicand has 2 decimal places and the multiplier 1, we point off $2 + 1 = 3$ decimal places in the product, (III). The reason of this is further illustrated in the proof, a method applicable to all similar cases.

219. Hence the following

RULE. *Multiply as in whole numbers, and from the right hand of the product point off as many figures for decimals as there are decimal places in both factors.*

NOTES.—1. If there be not as many figures in the product as there are decimals in both factors, supply the deficiency by prefixing ciphers.

2. To multiply a decimal by 10, 100, 1000, etc., remove the point as many places to the right as there are ciphers on the right of the multiplier.

EXAMPLES FOR PRACTICE.

1. Multiply .75 by .41. *Ans.* .3075.
2. Multiply .436 by .24.
3. Multiply 5.75 by .35. *Ans.* 2.0125.
4. Multiply .756 by .025. *Ans.* .0189.
5. Multiply 3.784 by 2.475.

6. Multiply 7.23 by .0156. *Ans.* .112788.
7. Multiply .0075 by .005. *Ans.* .0000375.
8. Multiply 324 by .324.
9. Multiply 75.64 by .225.
10. Multiply 5.728 by 100. *Ans.* 572.8.
11. Multiply .36 by 1000.
12. Multiply .000001 by 1000000.
13. Multiply .576 by 100000.
14. Multiply $7\frac{3}{4}$ by $5\frac{1}{2}$. *Ans.* 42.625.
15. Multiply $.63\frac{1}{8}$ by 24.
16. Multiply $4\frac{5}{16}$ by $7\frac{9}{25}$. *Ans.* 31.74.
17. Find the value of $3.425 \times 1.265 \times 64$. *Ans.* 277.288.
18. Find the value of $32 \times .57825 \times .25$.
19. Find the value of $18.375 \times 5.7 \times 1.001$.
Ans. 104.8422375.
20. If a cubic foot of granite weigh 168.48 pounds, what will be the weight of a granite block that contains $27\frac{3}{8}$ cubic feet?
21. When a bushel of corn is worth 2.8 bushels of oats, how many bushels of oats must be given in exchange for 36 bushels of corn and 48 bushels of oats? *Ans.* 148.8.

CONTRACTED MULTIPLICATION.

220. To obtain a given number of decimal places in the product.

It is frequently the case in multiplication, that a greater number of decimal figures is obtained in the product, than is necessary for practical accuracy. This may be avoided by contracting each partial product to the required number of decimal places.

To investigate the principles of this method, let us take the two decimals .12345 and .54321, and having reversed the order of the digits in the latter, and written it under the former, multiply each figure of the direct number by the figure below in the *reversed* number, placing the products with like orders of units in the same column, thus:

$$
\begin{array}{llll}
.12345 \text{ direct} & = .12345 & & \\
.54321 \text{ reversed} = & \underline{12345.} & & \\
& .000025 = .00005 & \times & .5 \\
& .000016 = .0004 & \times & .04 \\
& .000009 = .003 & \times & .003 \\
& .000004 = .02 & \times & .0002 \\
& .000001 = .1 & \times & .00001.
\end{array}
$$

In this operation we perceive that all the products are of the same order; and this must always be, whether the numbers used be fractional, integral, or mixed. For, as we proceed from right to left in the multiplication, we pass regularly from lower to higher orders in the *direct* number, and from higher to lower in the *reversed* number. Hence

221. If one number be written under another with the order of its digits reversed, and each figure of the reversed number be multiplied by the figure above it in the direct number, the products will all be of the same order of units.

1. Multiply 4.78567 by 3.25765, retaining only 3 decimal places in the product.

OPERATION.

$$
\begin{array}{rl}
4.78567 & \\
\underline{56752.3} & \\
14357 = & 4785 \times 3 + 2 \\
957 = & 478 \times 2 + 1 \\
239 = & 47 \times 5 + 4 \\
33 = & 4 \times 7 + 5 \\
3 = & 0 \times 6 + 3 \\
\hline
15.589 \pm, & \textit{Ans.}
\end{array}
$$

ANALYSIS. Since the product of any figure by units is of the same order as the figure multiplied, (**82**, II,) we write 3, the units of the multiplier, under 5, the third decimal figure of the multiplicand, and the lowest order to be retained in the product; and the other figures of the multiplier we write in the inverted order, extending to the left. Then, since the product of 3 and 5 is of the third order, or thousandths, the products of the other corresponding figures at the left, 2 and 8, 5 and 7, 7 and 4, etc., will be thousandths; and we therefore multiply each figure of the multiplier by the figures above and to the left of it in the multiplicand, carrying from the rejected figures of the multiplicand, as follows: 3 times 6 are 18, and as this is nearer 2 units than one of the next higher order, we must carry 2 to the first contracted product; 3 times 5 are 15, and 2 to be carried are 17; writing the 7 under the 3, and multiplying the other figures at the left in the usual manner, we obtain 14357 for the first partial product. Then, beginning with the next figure of the multiplier, 2 times 5 are 10, which gives 1 to be carried to the second partial product; 2 times 8 are 16, and 1 to be carried are 17; writing the 7 under the first figure of the former product, and multiplying the remaining left-hand figures of the multiplicand, we obtain 957 for the second partial product. Then, 5 times 8 are 40, which gives 4 to be carried to the third partial product; 5 times 7 are 35 and 4 are 39; writing the 9 in the first column of the products, and proceeding as in the former steps, we obtain 239 for the third partial product. Next, multiplying by 7 in the same manner, we obtain 33 for the fourth partial product. Lastly, beginning 2 places to the right in the multiplicand, 6 times 7 are 42; 6 times 4 are 24, and 4 are 28, which gives 3 to be carried to the fifth partial product; 6 times 0 is 0, and 3 to be carried are 3, which we write for the last partial product. Adding the several partial products, and pointing off 3 decimal places, we have 15.589, the required product.

222. From these principles and illustrations we derive the following

RULE. I. *Write the multiplier with the order of its figures reversed, and with the units' place under that figure of the multiplicand which is the lowest decimal to be retained in the product.*

II. *Find the product of each figure of the multiplier by the figures above and to the left of it in the multiplicand, increasing each partial product by as many units as would have been carried from the rejected part of the multiplicand, and one more when the highest figure in the rejected part of any product is 5 or greater than 5; and write these partial products with the lowest figure of each in the same column.*

III. *Add the partial products, and from the right hand of the result point off the required number of decimal figures.*

NOTES.—1. In obtaining the number to be carried to each contracted partial product, it is generally necessary to multiply (mentally) only one figure at the right of the figure above the multiplying figure; but when the figures are large, the multiplication should commence at least *two* places to the right.

2. Observe, that when the number of units in the highest order of the rejected part of the product is between 5 and 15, carry 1; if between 15 and 25 carry 2; if between 25 and 35 carry 3; and so on.

3. There is always a liability to an error of one or two units in the last place; and as the answer may be either too great or too small by the amount of this

error, the uncertainty may be indicated by the double sign, ±, read, *plus, or minus*, and placed after the product.

4. When the number of decimal places in the multiplicand is less than the number to be retained in the product, supply the deficiency by annexing ciphers.

EXAMPLES FOR PRACTICE.

1. Multiply 236.45 by 32.46357, retaining 2 decimal places, and 2.563789 by .0347263, retaining 6 decimal places in the product.

OPERATION.	OPERATION.
236.450	2.563789
75364.23	362 7430.
709350	76914
47290	10255
9458	1795
1419	51
71	15
12	1
2	.089031 ±
7676.02 ±	

2. Multiply 36.275 by 4.3678, retaining 1 decimal place in the product. *Ans.* 158.4 ±.

3. Multiply .24367 by 36.75, retaining 2 decimal places in the product.

4. Multiply 4256.785 by .00564, rejecting all beyond the third decimal place in the product. *Ans.* 24.008 ±.

5. Multiply 357.84327 by 1.007806, retaining 4 decimal places in the product.

6. Multiply 400.756 by 1.367583, retaining 2 decimal places in the product. *Ans.* 548.07 ±.

7. Multiply 432.5672 by 1.0666666, retaining 3 decimal places in the product.

8. Multiply 48.4367 by $2\frac{5}{37}$, extending the product to three decimal places. *Ans.* 103.418 ±.

9. Multiply $7\frac{5}{113}$ by $3\frac{376}{439}$, extending the product to three decimal places.

10. The first satellite of Uranus moves in its orbit 142.8373 +

degrees in 1 day; find how many degrees it will move in 2.52035 days, carrying the answer to two decimal places.

Ans. 360.00 degrees.

11. A gallon of distilled water weighs 8.33888 pounds; how many pounds in 35.8756 gallons? *Ans.* 299.16 ± pounds.

12. One French metre is equal to 1.09356959 English yards; how many yards in 478.7862 metres. *Ans.* 523.58 ± yards.

13. The polar radius of the earth is 6356078.96 metres, and the equatorial radius, 6377397.6 metres; find the two radii, and their difference, to the nearest hundredth of a mile, 1 metre being equal to 0.000621346 of a mile.

DIVISION.

223. In division of decimals the location of the decimal point in the quotient depends upon the following principles:

I. If one decimal number in the fractional form be divided by another also in the fractional form, the denominator of the quotient must contain as many ciphers as the number of ciphers in the denominator of the dividend exceeds the number in the denominator of the divisor. Therefore,

II. The quotient of one number divided by another in the decimal form must contain as many decimal places as the number of decimal places in the dividend exceed the number in the divisor.

1. Divide 34.368 by 5.37.

OPERATION.

```
5.37 ) 34.368 ( 6.4
       32 22
       ------
        2 148
        2 148
        -----
```

PROOF.

$$\frac{34368}{1000} \times \frac{100}{537} = \frac{64}{10} = 6.4$$

ANALYSIS. We first divide as in whole numbers; then, since the dividend has 3 decimal places and the divisor 2, we point off 3 — 2 = 1 decimal place in the quotient, (II). The correctness of the work is shown in the proof, where the dividend and divisor are written as common fractions. For, when we have canceled the denominator of the divisor from the denominator of the dividend, the denominator of the quotient must contain as many ciphers as the number in the dividend exceeds those in the divisor.

224. Hence the following

RULE. *Divide as in whole numbers, and from the right hand of the quotient point off as many places for decimals as the decimal places in the dividend exceed those in the divisor.*

NOTES.—1. If the number of figures in the quotient be less than the excess of the decimal places in the dividend over those in the divisor, the deficiency must be supplied by prefixing ciphers.

2. If there be a remainder after dividing the dividend, annex ciphers, and continue the division: the ciphers annexed are decimals of the dividend.

3. The dividend should always contain at least as many decimal places as the divisor, before commencing the division; the quotient figures will then be integers till all the decimals of the dividend have been used in the partial dividends.

4. To divide a decimal by 10, 100, 1000, etc., remove the point as many places to the left as there are ciphers on the right of the divisor.

EXAMPLES FOR PRACTICE.

1. Divide 9.6188 by 3.46. *Ans.* 2.78.
2. Divide 46.1975 by 54.35. *Ans.* .85.
3. Divide .014274 by .061. *Ans.* .234.
4. Divide .952 by 4.76.
5. Divide 345.15 by .075. *Ans.* 4602.
6. Divide .8 by 476.3. *Ans.* .001679+.
7. Divide .0026 by .003.
8. Divide 3.6 by .00006. *Ans.* 60000.
9. Divide 3 by 450.
10. Divide 75 by 10000.
11. Divide 4.36 by 100000.
12. Divide .1 by .12.
13. Divide 645.5 by 1000.
14. If 25 men build 154.125 rods of fence in a day, how much does each man build?
15. How many coats can be made from 16.2 yards of cloth, allowing 2.7 yards for each coat?
16. If a man travel 36.34 miles a day, how long will it take him to travel 674 miles? *Ans.* 18.547+days.
17. How many revolutions will a wheel 14.25 feet in circumference make in going a distance of 1 mile or 5280 feet?

12

CONTRACTED DIVISION.

225. To obtain a given number of decimal places in the quotient.

In division, the products of the divisor by the several quotient figures may be contracted, as in multiplication, by rejecting at each step the unnecessary figures of the divisor, (**220**).

1. Divide 790.755197 by 32.4687, extending the quotient to two decimal places.

FIRST CONTRACTED METHOD.

```
32.4687 ) 790.755197 ( 24.35
          649 4
          -----
          141 3
          129 9
          -----
           11 4
            9 7
          -----
            1 7
            1 6
          -----
              1
```

COMMON METHOD.

```
32.4687 ) 790.7|55198 ( 24.35
          649 3|74
          -----------
          141 3|811
          129 8|748
          -----------
           11 5|0639
            9 7|4061
          -----------
            1 7|65787
            1 6|23435
          -----------
              1|42352
```

SECOND CONTRACTED METHOD.

```
32.4687 ) 790.755197
53.42       141 3
             11 4
              1 7
                1
```

Analysis. In the first method of contraction, we first compare the 3 tens of the divisor with the 79 tens of the dividend, and ascertain that there will be 2 integral places in the quotient; and as 2 decimal places are required, the quotient must contain 4 places in all. Then assuming the four left hand figures of the divisor, we say 3246 is contained in 7907, 2 times; multiplying the assumed part of the divisor by 2, and carrying 2 units from the rejected part, as in Contracted Multiplication of Decimals, we have 6494 for the product, which subtracted from the dividend, leaves 1413 for a new dividend. Now, since the next quotient figure will be of an order next below the former, we reject one more place in the divisor, and divide by 324, obtaining 4 for a quotient, 1299 for a product, and 114 for a new dividend. Continuing this process till all the figures of the divisor are rejected, we have, after pointing off 2 decimals as required, 24.35 for a quotient. Comparing the contracted with the common method, we see the extent of the abbreviation, and the agreement of the corresponding intermediate results.

In the second method of contraction, the quotient is written with its first figure under the lowest order of the assumed divisor, and the other figures at the left in the reverse order. By this arrangement, the several products are conveniently formed, by multiplying each quotient figure by the figures above and to the left of it in the divisor, by the rule for contracted multiplication, (**222**), and the *remainders* only are written as in (**112**).

226. From these illustrations we derive the following

RULE. I. *Compare the highest or left hand figure of the divisor with the units of like order in the dividend, and determine how many figures will be required in the quotient.*

II. *For the first contracted divisor, take as many significant figures from the left of the given divisor as there are places required in the quotient; and at each subsequent division reject one place from the right of the last preceding divisor.*

III. *In multiplying by the several quotient figures, carry from the rejected figures of the divisor as in contracted multiplication.*

NOTES.—1. Supply ciphers, at the right of either divisor or dividend, when necessary, before commencing the work.

2. If the first figure of the quotient is written under the lowest assumed figure of the divisor, and the other figures at the left in the inverted order, the several products will be formed with the greatest convenience, by simply multiplying each quotient figure by the figures above and to the left of it in the divisor.

EXAMPLES FOR PRACTICE.

1. Divide 27.3782 by 4.3267, extending the quotient to 3 decimal places. *Ans.* 6.328 ±.

2. Divide 487.24 by 1.003675, extending the quotient to 2 decimal places.

3. Divide 8.47326 by 75.43, extending the quotient to 5 decimal places.

4. Divide .8487564 by .075637, extending the quotient to 3 decimal places. *Ans.* 11.221 ±.

5. Divide 478.325 by $1.43\frac{2}{3}$, extending the quotient to 3 decimal places. *Ans.* 332.942 ±.

6. Divide 8972.436 by 756.3452, extending the quotient to 4 decimal places.

7. Divide 1 by 1.007633, extending the quotient to 6 decimal places. *Ans.* .992425 ±.

8. Find the quotient of .95372843 divided by 44.736546, true to 8 decimal places.

9. Reduce $\frac{4273}{5737}$ to a decimal of 4 places. *Ans.* .7448 ±.

CIRCULATING DECIMALS.

227. Common fractions can not always be exactly expressed in the decimal form; for in some instances the division will not be exact if continued indefinitely.

228. A **Finite Decimal** is a decimal which extends a limited number of places from the decimal point.

229. An **Infinite Decimal** is a decimal which extends an unlimited number of places from the decimal point.

230. A **Circulating Decimal** is an infinite decimal in which a figure or set of figures is continually repeated in the same order; as .3333+, or .437437437+.

231. A **Repetend** is the figure or set of figures continually repeated. When a repetend consists of a single figure, it is indicated by a point placed over it; when it consists of more than one figure, a point is placed over the first, and one over the last figure. Thus, the circulating decimals .55555+ and .324324324+, are written, $\dot{5}$ and $.\dot{3}2\dot{4}$.

232. A repetend is said to be *expanded* when its figures are continued in their proper order any number of places toward the right; thus, $.\dot{2}\dot{4}$, expanded is .2424+, or .242424242+.

233. **Similar Repetends** are those which begin at the same decimal place or order; as $.\dot{3}\dot{7}$ and $.5$, $.2\dot{4}$ and $.3\dot{7}\dot{5}$, $\dot{1}.5\dot{6}$ and $2\dot{4}.\dot{3}$.

234. **Conterminous Repetends** are those which end at the same decimal place or order; as $.7\dot{5}$ and $1.5\dot{3}$, $.5\dot{6}\dot{7}$, and $\dot{3}.24\dot{5}$.

NOTE.—Two or more repetends are *Similar and Conterminous* when they *begin* and *end* at the same decimal places or orders.

235. A **Pure Circulating Decimal** is one which contains no figures but the repetend; as $.\dot{7}$, or $.\dot{7}0\dot{4}$.

236. A **Mixed Circulating Decimal** is one which contains other figures, called *finite places*, before the repetend; as $.5\dot{4}$, or $.01\dot{3}24\dot{5}$, in which .5 and .01 are the finite places.

PROPERTIES OF FINITE AND CIRCULATING DECIMALS.

237. The operations in circulating decimals depend upon the following properties.

NOTE.—1. The common fractions referred to are understood to be *proper* fractions, in their *lowest terms.*

I. Every fraction whose denominator contains no other prime factor than 2 or 5 will give rise to a finite decimal; and the number of decimal places will be equal to the greatest number of equal factors, 2 or 5, in the denominator.

For, in the reduction, every cipher annexed to the numerator multiplies it by 10, or introduces the two prime factors, 2 and 5, and also gives 1 decimal place in the result. Hence the division will be exact when the number of ciphers annexed, or the number of decimal places obtained, shall be equal to the greatest number of equal factors, 2 or 5, to be canceled from the denominater.

II. Every fraction whose denominator contains any other prime factor than 2 or 5, will give rise to an infinite decimal.

For, annexing ciphers to the numerator introduces no other prime factors than 2 and 5; hence the numerator will never contain all the prime factors of the denominator.

III. Every infinite decimal derived from a common fraction is also a circulating decimal; and the number of places in the repetend must be less than the number of units in the denominator of the common fraction.

For, in every division, the number of possible remainders is limited to the number of units in the divisor, less 1; thus, in dividing by 7, the only possible remainders are 1, 2, 3, 4, 5, and 6. Hence, in the reduction of a common fraction to a decimal, some of the remainders must repeat before the number of decimal places obtained equals the number of units in the denominator; and this will cause the intermediate quotient figures to repeat.

NOTES.—2. It will be found that the number of places in the repetend is always equal to the denominator less 1, or to some factor of this number. Thus, the repetend arising from $\frac{3}{7}$ has $7-1=6$ places; the repetend arising from $\frac{29}{41}$ has $\frac{41-1}{8}=5$ places.

3. A *perfect repetend* is one which consists of as many places, less 1, as there are units in the denominator of the equivalent fraction.

4. If the denominator of a fraction contains neither of the factors 2 and 5, it will give rise to a pure repetend. But if a circulating decimal is derived from a fraction whose denominator contains either of the factors 2 or 5, it will contain as many finite places as the greatest number of equal factors 2 or 5 in the denominator.

IV. If to any number we annex as many ciphers as there are places in the number, or more, and divide the result by as many 9's as the number of ciphers annexed, both the quotient and remainder will be the same as the given number.

For, if we take any number of two places, as 74, and annex two ciphers, the result divided by 100 will be equal to 74; thus,

$$7400 \div 100 = 74.$$

Now subtracting 1 from the divisor, 100, will add as many units to the quotient, 74, as the new divisor, 99, is contained times in 74, (**115**, II); thus,

$$7400 \div 99 = 74 + \tfrac{74}{99}, \text{ or } 74\tfrac{74}{99};$$

that is, if two ciphers be annexed to 74, and the result be divided by 99, both quotient and remainder will be 74. In like manner, annexing three ciphers to 74, and dividing by 999, we have

$$74000 \div 999 = 74\tfrac{74}{999};$$

and the same is true of any number whatever.

V. Every pure circulating decimal is equal to a common fraction whose numerator is the repeating figure or figures, and whose denominator is as many 9's as there are places in the repetend.

For, if we take any fraction whose denominator is expressed by some number of 9's, as $\frac{24}{99}$, then according to the last property, annexing two ciphers to the numerator, and reducing to a decimal, we have $\frac{24}{99} = 24\frac{24}{99}$. In like manner, carrying the decimal two places farther, $.24\frac{24}{99} = .2424\frac{24}{99}$; hence, $\frac{24}{99} = \dot{2}\dot{4}$. By the same principle, we have $\frac{2}{9} = .\dot{2}$; $\frac{1}{99} = .\dot{0}\dot{1}$; $\frac{2}{99} = .\dot{0}\dot{2}$; $\frac{1}{999} = .\dot{0}0\dot{1}$; $\frac{324}{999} = .\dot{3}2\dot{4}$; and so on. And it is evident that *all possible repetends* can thus be derived from fractions whose numerators are the repeating figures, and whose denominators are as many 9's as there are repeating figures.

NOTE 5.—It follows from the last property, that any fraction from which a pure repetend can be derived is reducible to a form in which the denominator is some number of 9's; thus $\frac{3}{13} = \frac{615384}{999999}$; $\frac{5}{37} = \frac{135}{999}$. This is true of every fraction whose denominator terminates with 1, 3, 7, or 9.

VI. Any repetend may be reduced to another equivalent repetend, by expanding it, and moving either the second point, or both points, to the right; provided that in the result they be so placed as to include the same number of places as are contained in the given repetend, or some multiple of this number.

For, in every such reduction, the new repetend and the given repetend, when expanded indefinitely, will give results which are identical. *Thus*, $.\dot{5}3\dot{6} = .\dot{5}3653\dot{6}$, or $.\dot{5}3653653\dot{6}$, or $.5\dot{3}6\dot{5}$, or $.53\dot{6}5\dot{3}$, or $.5\dot{3}6536\dot{5}$, or $.53\dot{6}536536\dot{5}\dot{3}$; because each of these new repetends, when expanded, gives .5365365365365365 3+.

NOTE 6.—If in any reduction, the new repetend should not contain the same number of places, or some multiple of the same number, as the given repetend, we should not have, in the expansions, the *same figures* repeated in the *same order*.

REDUCTION.

CASE I.

238. To reduce a pure circulating decimal to a common fraction.

1. Reduce $.\dot{6}7\dot{5}$ to a common fraction.

OPERATION.

$$.\dot{6}7\dot{5} = \frac{675}{999} = \frac{25}{37}$$

ANALYSIS. Since the repetend has 3 places, we take for the denominator of the required fraction the number expressed by three 9's, (**237**, V). Hence,

RULE. *Omit the points and the decimal sign, and write the figures of the repetend for the numerator of a common fraction, and as many 9's as there are places in the repetend for the denominator.*

EXAMPLES FOR PRACTICE.

1. Reduce $.\dot{4}\dot{5}$ to a common fraction. *Ans.* $\frac{5}{11}$.
2. Reduce $.\dot{6}\dot{6}$ to a common fraction.
3. Reduce $.\dot{2}7\dot{9}$ to a common fraction. *Ans.* $\frac{31}{111}$.

4. Reduce $.\dot{4}2\dot{3}$ to a common fraction. *Ans.* $\frac{47}{111}$.
5. Reduce $.\dot{9}2307\dot{6}$ to a common fraction. *Ans.* $\frac{12}{13}$.
6. Reduce $.\dot{9}512\dot{1}$ to a common fraction.
7. Reduce $4.\dot{7}\dot{2}$ to a mixed number. *Ans.* $4\frac{8}{11}$.
8. Reduce $2.\dot{2}9\dot{7}$ to an improper fraction. *Ans.* $\frac{85}{37}$.
9. Reduce $\dot{2}.9\dot{7}$ to an improper fraction. *Ans.* $\frac{110}{37}$.

NOTE. — According to **237**, VI, $\dot{2}.9\dot{7} = 2.\dot{9}7\dot{2}$.

10. Reduce $\dot{1}5.\dot{0}$ to a mixed number. *Ans.* $15\frac{5}{333}$.

CASE II.

239. To reduce a mixed circulating decimal to a common fraction.

1. Reduce $.0\dot{7}5\dot{6}$ to a common fraction.

OPERATION.

$$.0\dot{7}5\dot{6} = \tfrac{756}{9990} = \tfrac{14}{185}$$

ANALYSIS. Since $.\dot{7}5\dot{6}$ is equal to $\frac{756}{999}$, $.0\dot{7}5\dot{6}$ will be $\frac{1}{10}$ of $\frac{756}{999}$, or $\frac{756}{9990} = \frac{14}{185}$.

2. Reduce $.64\dot{7}$ to a common fraction.

OPERATION.

$$\begin{aligned} .64\dot{7} &= \tfrac{64}{100} + \tfrac{7}{900} \\ &= \frac{640 - 64}{900} + \frac{7}{900} \\ &= \frac{640 - 64 + 7}{900} \\ &= \frac{647 - 64}{900} \\ &= \frac{583}{900}, \textit{ Ans.} \end{aligned}$$

Or,

$.64\dot{7}$ given decimal.
$\underline{64}$ finite figures.
583
$\frac{583}{900}$, *Ans.*

ANALYSIS. Reducing the finite part and the repetend separately to fractions, we have $\frac{64}{100} + \frac{7}{900}$. To reduce these fractions to a common denominator, we must multiply the terms of the first by 9; but the numerator, 64, may be multiplied by 9 by annexing 1 cipher and subtracting 64 from the result, giving $\frac{640-64}{900}$, for the first fraction reduced. The numerator of the sum of the two fractions will therefore be $640 - 64 + 7 = 583$, and supplying the common denominator, we have $\frac{583}{900}$. In the second operation, the intermediate steps are omitted. Hence the following

RULE. I. *From the given circulating decimal subtract the finite part, and the remainder will be the required numerator.*

II. *Write as many 9's as there are figures in the repetend, with as many ciphers annexed as there are finite decimal figures, for the required denominator.*

EXAMPLES FOR PRACTICE.

1. Reduce $.5\dot{7}$ to a common fraction. *Ans.* $\frac{26}{45}$.
2. Reduce $.04\dot{8}$ to a common fraction. *Ans.* $\frac{11}{225}$.
3. Reduce $.6\dot{4}7\dot{2}$ to a common fraction.
4. Reduce $.65\dot{9}\dot{0}$ to a common fraction. *Ans.* $\frac{29}{44}$.
5. Reduce $.04\dot{6}4\dot{8}$ to a common fraction. *Ans.* $\frac{43}{925}$.
6. Reduce $.100\dot{4}$ to a common fraction.
7. Reduce $.9\dot{2}8571\dot{4}$ to a common fraction. *Ans.* $\frac{13}{14}$.
8. Reduce $5.2\dot{7}$ to a common fraction. *Ans.* $\frac{95}{18}$.
9. Reduce $7.01\dot{2}\dot{6}$ to a mixed number. *Ans.* $7\frac{5}{396}$.
10. Reduce $1.582\dot{3}170\dot{7}$ to an improper fraction. *Ans.* $\frac{519}{328}$.
11. Reduce $2.0\dot{2}926\dot{8}$ to an improper fraction.

CASE III.

240. To make two or more repetends similar and conterminous.

1. Make $.\dot{4}\dot{7}$, $.53\dot{6}7\dot{5}$, and $.3\dot{7}23\dot{4}$ similar and conterminous.

OPERATION.

$$\left.\begin{array}{l} .\dot{4}\dot{7} \quad\;\;\; = .47\dot{4}74747474747\dot{7} \\ .53\dot{6}7\dot{5} = .53\dot{6}75675675675\dot{5} \\ .3\dot{7}23\dot{4} = .37\dot{2}34723472347\dot{7} \end{array}\right\} \textit{Ans.}$$

ANALYSIS. The first of the given repetends begins at the place of tenths, the second at the place of thousandths, and the third at the place of hundredths; and as the points in any repetend cannot be moved to the left over the finite places, we can make the given repetends *similar*, only by moving the points of at least two of them to the right.

Again, the first repetend has 2 places, the second 3 places, and the third 4 places; and the number of places in the new repetends must be at least 12, which is the least common multiple of 2, 3, and 4. We therefore expand the given repetends, and place the first point in each new repetend over the third place in the decimal, and the second point over the fourteenth, and thus render them similar and conterminous. Hence the following

RULE. I. *Expand the repetends, and place the first point in each over the same order in the decimal.*

II. *Place the second point so that each new repetend shall contain as many places as there are units in the least common multiple of the number of places in the several given repetends.*

NOTE.—Since none of the points can be carried to the left, some of them must be carried to the right, so that each repetend shall have at least as many finite places as the greatest number in any of the given repetends.

EXAMPLES FOR PRACTICE.

1. Make $.4\dot{3}$, $.\dot{5}\dot{7}$, $.4\dot{5}6\dot{7}$, and $.50\dot{3}\dot{7}$ similar and conterminous.
2. Make $.57\dot{8}$, $.\dot{3}\dot{7}$, $.248\dot{5}$, and $\dot{0}\dot{4}$ similar and conterminous.
3. Make $\dot{1}.34$, $\dot{4}.5\dot{6}$, and $.3\dot{4}\dot{1}$ similar and conterminous.
4. Make $.\dot{5}67\dot{4}$, $.3\dot{4}$, $.\dot{2}4\dot{7}$, and $.\dot{6}\dot{7}$ similar and conterminous.
5. Make $\dot{1}.2\dot{4}$, $.0\dot{5}7\dot{8}$, $.\dot{4}$, and $.47\dot{3}214\dot{7}$ similar and conterminous.
6. Make $.\dot{7}$, $.456\dot{7}$, $.\dot{2}\dot{4}$, and $.3\dot{4}678\dot{9}$ similar and conterminous.
7. Make $.\dot{8}$, $.\dot{3}\dot{6}$, $.\dot{4}85\dot{7}$, $.\dot{3}456\dot{7}$, and $.\dot{2}78467894\dot{3}$ similar and conterminous.

ADDITION AND SUBTRACTION.

241. The processes of adding and subtracting circulating decimals depend upon the following properties of repetends:

I. If two or more repetends are similar and conterminous, their denominators will consist of the same number of 9's, with the same number of ciphers annexed. Hence,

II. Similar and conterminous repetends have the same denominators and consequently the same fractional unit.

1. Add $.5\dot{4}$, $3.\dot{2}\dot{4}$ and, $\dot{2}.78\dot{5}$.

OPERATION.

$$
\begin{array}{rcr}
.5\dot{4} & = & .5\dot{4}44\dot{4} \\
3.\dot{2}\dot{4} & = & 3.2\dot{4}24\dot{2} \\
\dot{2}.78\dot{5} & = & 2.7\dot{8}52\dot{7} \\
\hline
 & & 6.5\dot{7}21\dot{4}
\end{array}
$$

ANALYSIS. Since fractions can be added only when they have the same fractional unit, we first make the repetends of the given decimals similar and conterminous. We then add as in finite decimals, observing, however, that the 1 which we carry from the left hand column of the repetends, must also be added to the right hand column; for this would be required if the repetends were further expanded before adding.

2. From $7.\dot{4}$ take $2.7\dot{8}5\dot{2}$.

OPERATION.

$$\begin{array}{r} 7.4\dot{4}4\dot{4} \\ 2.7\dot{8}5\dot{2} \\ \hline 4.6\dot{5}8\dot{1} \end{array}$$

ANALYSIS. Since one fraction can be subtracted from another only when they have the same fractional unit, we first make the repetends of the given decimals similar and conterminous. We then subtract as in finite decimals; observing that if both repetends were expanded, the next figure in the subtrahend would be 8, and the next in the minuend 4; and the subtraction in this form would require 1 to be carried to the 2, giving 1 for the right hand figure in the remainder.

242. From these principles and illustrations we derive the following

RULE. I. *When necessary, make the repetends similar and conterminous.*

II. To add;—*Proceed as in finite decimals, observing to increase the sum of the right hand column by as many units as are carried from the left hand column of the repetends.*

III. To subtract;—*Proceed as in finite decimals, observing to diminish the right hand figure of the remainder by* 1, *when the repetend in the subtrahend is greater than the repetend of the minuend.*

IV. *Place the points in the result directly under the points above.*

NOTE.—When the sum or difference is required in the form of a common fraction, proceed according to the rule, and reduce the result.

EXAMPLES FOR PRACTICE.

1. What is the sum of $2.\dot{4}$, $.\dot{3}\dot{2}$, $.\dot{5}6\dot{7}$, $7.0\dot{5}\dot{6}$, and $4.3\dot{7}$? *Ans.* $14.7\dot{6}9587\dot{7}$.
2. What is the sum of $.4\dot{7}\dot{8}$, $.\dot{3}2\dot{1}$, $.7\dot{8}56\dot{4}$, $.3\dot{2}$, $.\dot{5}$, and $.\dot{4}32\dot{6}$? *Ans.* $2.8\dot{9}6178807069\dot{8}$.
3. From $.7\dot{8}5\dot{4}$ subtract $.\dot{5}\dot{9}$. *Ans.* $.1\dot{8}9525\dot{8}$.
4. From $57.\dot{0}58\dot{7}$ subtract $27.\dot{3}\dot{1}$. *Ans.* $29.\dot{7}45\dot{5}$.
5. What is the sum of $.\dot{5}$, $.\dot{3}\dot{2}$, and $.\dot{1}\dot{2}$? *Ans.* 1.
6. What is the sum of $.43\dot{8}\dot{7}$, $.86\dot{3}$, $.2\dot{1}$, and $.35\dot{5}\dot{4}$?
7. What is the sum of $3.6\dot{5}3\dot{7}$, $3.\dot{1}3\dot{5}$, $2.5\dot{6}\dot{4}$, and $.\dot{5}\dot{3}$?
8. From $.\dot{4}3\dot{2}$ subtract .25. *Ans.* $.18\dot{2}4\dot{3}$.
9. From $7.24\dot{5}7\dot{4}$ subtract $2.\dot{6}3\dot{4}$. *Ans.* $4.6\dot{1}$.

10. From $.99$ subtract $.\dot{4}3\dot{3}$. *Ans.* $.556\dot{5}\dot{6}$.

11. What is the sum of $4.\dot{6}3\dot{8}$, $8.\dot{3}1\dot{8}$, $.\dot{0}1\dot{6}$, $.5\dot{4}$, and $.\dot{4}\dot{5}$? *Ans.* $13\frac{36}{37}$.

12. From $.\dot{4}$ subtract $.\dot{2}\dot{3}$. *Ans.* $\frac{7}{33}$.

MULTIPLICATION AND DIVISION.

243. 1. Multiply $2.\dot{4}2857\dot{1}$ by $.0\dot{6}\dot{3}$.

OPERATION.

$$2.\dot{4}2857\dot{1} = \frac{17}{7}$$
$$.0\dot{6}\dot{3} = \frac{7}{110}$$
$$\frac{17}{7} \times \frac{7}{110} = \frac{17}{110} = .1\dot{5}\dot{4} \textit{ Ans.}$$

ANALYSIS. We first reduce the multiplicand and multiplier to their equivalent fractions, and obtain $\frac{17}{7}$ and $\frac{7}{110}$; then $\frac{17}{7} \times \frac{7}{110} = \frac{17}{110} = .1\dot{5}\dot{4}$.

2. Divide $.\dot{4}7\dot{5}$ by $.3\dot{7}5\dot{3}$.

OPERATION.

$$.\dot{4}7\dot{5} = \frac{475}{999}$$
$$.3\dot{7}5\dot{0} = \frac{3750}{9990}$$
$$\frac{475}{999} \times \frac{9990}{3750} = 1.2\dot{6} \textit{ Ans.}$$

ANALYSIS. The dividend reduced to its equivalent common fraction is $\frac{475}{999}$, and the divisor reduced to its equivalent common fraction is $\frac{3750}{9990}$; and $\frac{475}{999} \div \frac{3750}{9990} = \frac{19}{15} = 1.2\dot{6}$.

244. From these illustrations we have the following

RULE. *Reduce the given numbers to common fractions; then multiply or divide, and reduce the result to a decimal.*

EXAMPLES FOR PRACTICE.

1. Multiply 3.4 by $.\dot{7}\dot{2}$. *Ans.* $2.4\dot{7}\dot{2}$.
2. Multiply $.0\dot{4}3\dot{2}$ by 18. *Ans.* $.7\dot{7}8\dot{3}$.
3. Divide $.\dot{1}5\dot{4}$ by $.\dot{2}$. *Ans.* $.\dot{6}9\dot{3}$.
4. Divide $4.\dot{5}72\dot{4}$ by $.\dot{7}$. *Ans.* $5.\dot{8}79\dot{3}$.
5. Multiply $4.3\dot{7}$ by $.2\dot{7}$. *Ans.* 1.182.
6. Divide $56.\dot{6}$ by 137. *Ans.* $.\dot{4}136253\dot{0}$.
7. Divide $.\dot{4}2857\dot{1}$ by $.\dot{5}\dot{4}$. *Ans.* $.\dot{7}85714\dot{2}$.
8. Multiply $.\dot{7}1428\dot{5}$ by $.\dot{2}\dot{7}$. *Ans.* $.\dot{1}9480\dot{5}$.
9. Multiply $3.45\dot{6}$ by $.42\dot{5}$. *Ans.* $1.4710\dot{0}3\dot{7}$.
10. Divide $9.\dot{1}704\dot{5}$ by $3.\dot{3}\dot{6}$. *Ans.* $2.\dot{7}263\dot{7}$.
11. Multiply $.\dot{2}\dot{4}$ by $.\dot{5}\dot{7}$. *Ans.* $.\dot{1}39577594123048668503\dot{2}$.

UNITED STATES MONEY.

245. By Act of Congress of August 8, 1786, the dollar was declared to be the unit of Federal or United States Money; and the subdivisions and multiples of this unit and their denominations, as then established, are as shown in the

TABLE.

10 mills	make	1	cent.
10 cents	"	1	dime.
10 dimes	"	1	dollar.
10 dollars	"	1	eagle.

246. By examining this table we find

1st. That the denominations increase and decrease in a tenfold ratio.

2d. That the dollar being the unit, dimes, cents and mills are respectively tenths, hundredths and thousandths of a dollar.

3d. That the denominations of United States money increase and decrease the same as simple numbers and decimals.

Hence we conclude that

I. *United States money may be expressed according to the decimal system of notation.*

II. *United States money may be added, subtracted, multiplied and divided in the same manner as decimals.*

NOTATION AND NUMERATION.

247. The character $ before any number indicates that it expresses United States money. Thus $75 expresses 75 dollars.

248. Since the dollar is the unit, and dimes, cents and mills are tenths, hundredths and thousandths of a dollar, the decimal point or separatrix must always be placed before dimes. Hence, in any number expressing United States money, the first figure at the right of the decimal point is dimes, the second figure is cents, the third figure is mills, and if there are others, they are ten-thousandths, hundred-thousandths, etc., of a dollar. Thus, $8.3125 expresses 8 dollars 3 dimes 1 cent 2 mills and 5 tenths of a mill or 5 ten-thousandths of a dollar.

249. The denominations, eagles and dimes, are not regarded in business operations, eagles being called tens of dollars and dimes tens of cents. Thus \$24.19 instead of being read 2 eagles 4 dollars 1 dime 9 cents, is read 24 dollars 19 cents. Hence, practically, the table of United States money is as follows:

10 mills make 1 cent.
100 cents " 1 dollar.

250. Since the cents in an expression of United States money may be any number from 1 to 99, the first two places at the right of the decimal point are always assigned to cents. Hence, when the number of cents to be expressed is less than 10, a cipher must be written in the place of tenths or dimes. Thus, 7 cents is expressed \$.07.

NOTES.—1. The half cent is frequently written as 5 mills and *vice versâ*. Thus, $\$.37\frac{1}{2} = \$.375$.

2. Business men frequently write cents as common fractions of a dollar. Thus, \$5.19 is also written $\$5\frac{19}{100}$, read 5 and $\frac{19}{100}$ dollars.

3. In business transactions, when the *final* result of a computation contains 5 mills or *more*, they are called one cent, and when *less* than 5 they are rejected. Thus, \$2.198 would be called \$2.20, and \$1.623 would be called \$1.62.

EXAMPLES FOR PRACTICE.

1. Write twenty-eight dollars thirty-six cents. *Ans.* \$28.36.
2. Write four dollars seven cents.
3. Write ten dollars four cents.
4. Write sixteen dollars four mills.
5. Write thirty-one and one-half cents.
6. Write 48 dollars $1\frac{3}{4}$ cents. *Ans.* $\$48.01\frac{3}{4}$.
7. Write 1000 dollars 1 cent 1 mill.
8. Write 3 eagles 2 dollars 5 dimes 8 cents 4 mills.
9. Write $6\frac{1}{4}$ cents.
10. Read the following numbers:

\$21.18	\$10.01	\$ .8125
\$164.05	\$201.201	$\$15.08\frac{1}{3}$
\$7.90	$\$5.37\frac{1}{2}$	\$96.005

REDUCTION.

251. Since $1 = 100 cents = 1000 mills, it is evident,

1st. That dollars may be changed or reduced to cents by annexing *two* ciphers; and to mills by annexing *three* ciphers.

2d. That cents may be reduced to dollars by pointing off *two* figures from the right; and mills to dollars by pointing off *three* figures from the right.

3d. That cents may be reduced to mills by annexing *one* cipher.

4th. That mills may be reduced to cents by pointing off *one* figure from the right.

OPERATIONS IN UNITED STATES MONEY.

252. Since United States Money may be added, subtracted, multiplied and divided in the same manner as decimals, (**246**, II), it is evident that no separate rules for these operations are required.

EXAMPLES FOR PRACTICE.

1. Paid $3475.50 for building a house, $310.20 for painting, $1287.37½ for furniture, and $207.12½ for carpets; how much was the cost of the house and furniture? *Ans.* $5280.20.

2. Bought a pair of boots for $4.62½, an umbrella for $1.75, a pair of gloves for $.87½, a cravat for $1, and some collars for $.62½; how much was the cost of all my purchases?

3. Gave $150 for a horse, $175.84 for a carriage, and $62½ for a harness, and sold the whole for $390.37½; how much did I gain? *Ans.* $2.035.

4. A man bought a farm for $3800, which was $190.87½ less than he sold it for; how much did he sell it for?

5. A lady bought a dress for $10¾, a bonnet for $5½, a veil for $2⅜, a pair of gloves for $.87½, and a fan for $⅝. She gave the shopkeeper a fifty dollar bill; how much money should he return to her? *Ans.* $29.875.

6. A farmer sold 150 bushels of oats at $.37½ a bushel, and 4 cords of wood at $3⅞ a cord. He received in payment 84 pounds of

sugar at 6¼ cents a pound, 25 pounds of tea at \$⅝ a pound, 2 barrels of flour at \$5.87½, and the remainder in cash; how much cash did he receive? *Ans.* \$39.125.

7. A speculator bought 264.5 acres of land for \$6726. He afterward sold 126.25 acres for \$31¼ an acre, and the remainder for \$33.75 an acre; how much did he gain by the transaction?

8. A merchant going to New York to purchase goods, had \$11000. He bought 40 pieces of silk, each piece containing 28½ yards, at \$.80 a yard; 300 pieces of calicoes, with an average length of 29 yards, at 11½ cents a yard; 20 pieces of broadcloths, each containing 36.25 yards, at \$3.875 a yard; 112 pieces of sheeting, each containing 30.5 yards, at \$.06¼ a yard. How much had he left with which to finish purchasing his stock? *Ans.* \$6064.62½.

9. If 139 barrels of beef cost \$2189.25, how much will 1 barrel cost? *Ans.* \$15.75.

10. If 396 pounds of hops cost \$44.748, how much are they worth per pound? *Ans.* \$.113.

11. Bought 10¾ cords of wood at \$4½ a cord, and received for it 7.74 barrels of flour; how much was the flour worth per barrel?

12. If a hogshead of wine cost \$287.4, how many hogsheads can be bought for \$4885.80? *Ans.* 17.

13. A butcher bought an equal number of calves and sheep for \$265; for the calves he paid \$3¾ a head, and for the sheep \$2⅞ a head; how many did he buy of each kind? *Ans.* 40.

14. If 128 tons of iron cost \$9632, how many tons can be bought for \$1730.75? *Ans.* 23.

15. If 125 bushels of potatoes cost \$41.25, how many barrels, each containing 2½ bushels, can be bought for \$112.20?

16. A grocer on balancing his books at the end of a month, found that his purchases amounted to \$2475.36, and his sales to \$1936.40; and that the money he now had was but ⅗ of what he had at the beginning of the month; how much money had he at the beginning of the month? *Ans.* \$1347.40.

17. A person has an income of \$3200 a year, and his expenses are \$138 a month; how much can he save in 8 years?

18. Sold 120 pieces of cloth at $\$45\frac{3}{4}$ a piece, and gained thereby \$1026; how much did it cost by the piece? *Ans.* \$37.20.

19. A flour merchant paid \$3088.25 for some flour. He sold 425 barrels at $\$6\frac{1}{4}$ a barrel, and the remainder stood him in \$4.50 a barrel; how many barrels did he purchase? *Ans.* 521.

20. If 36 engineers receive \$6315.12 for one month's work, how many engineers will \$21927.50 pay for one month at the same rate? *Ans.* 125.

21. A person having \$1378.56, wishes to purchase a house worth \$2538, and still have \$750 left with which to purchase furniture; how much more money must he have? *Ans.* \$1909.44.

22. A mechanic earns on an average $\$1.87\frac{1}{2}$ a day, and works 22 days per month. If his necessary expenses are $\$25\frac{3}{4}$ a month, how many years will it take him to save \$1116, there being 12 months in a year? *Ans.* 6 years

23. Bought 27.5 barrels of sugar for \$453.75, and sold it at a profit of $\$3.62\frac{1}{2}$ a barrel; at what price per barrel was it sold?

24. A man expended \$70.15 in the purchase of rye at \$.95 a bushel, wheat at \$1.37 a bushel, and corn at \$.73 a bushel, buying the same quantity of each kind; how many bushels in all did he purchase? *Ans* 69 bushels.

25. A farmer bought a piece of land containing $375\frac{1}{2}$ acres, at $\$22\frac{1}{4}$ per acre, and sold $\frac{1}{2}$ of it at a profit of $\$1032\frac{5}{8}$; at what price per acre was the land sold? *Ans.* \$27.75.

26 If $3\frac{1}{2}$ cords of wood cost $\$11.37\frac{1}{2}$, how much will $20\frac{1}{8}$ cords cost? *Ans.* $\$65.40\frac{5}{8}$.

27. If $\frac{3}{4}$ of a hundred pounds of sugar cost $\$6\frac{3}{8}$, how much can be bought for \$46.75, at the same rate?

Ans. 5.5 hundred pounds.

28. A man sold a wagon for \$62.50, and received in payment $12\frac{1}{2}$ yards of broadcloth at $\$3\frac{1}{4}$ per yard, and the balance in coffee at $12\frac{1}{2}$ cents per pound; how many pounds of coffee did he receive? *Ans.* 175 pounds.

29. Bought 320 bushels of barley at the rate of 16 bushels for \$10.04, and sold it at the rate of 20 bushels for $\$17\frac{1}{2}$; how much was my profit on the transaction? *Ans.* \$79.20.

13 *

PROBLEMS

INVOLVING THE RELATION OF PRICE, COST, AND QUANTITY.

PROBLEM I.

253. Given, the price and the quantity, to find the cost.

Analysis. The cost of 3 units must be 3 times the price of 1 unit; of 8 units, 8 times the price of 1 unit; of $\frac{3}{8}$ of a unit, $\frac{3}{8}$ times the price of 1 unit, etc. Hence,

Rule. *Multiply the price of* ONE *by the quantity.*

PROBLEM II.

254. Given, the cost and the quantity, to find the price.

Analysis. By Problem I, the cost is the product of the price multiplied by the quantity. Now, having the cost, which is a product, and the quantity, which is one of two factors, we have the product and one of two factors given, to find the other factor. Hence,

Rule. *Divide the cost by the quantity.*

PROBLEM III.

255. Given, the price and the cost, to find the quantity.

Analysis. Reasoning as in Problem II, we find that the cost is the product of two factors, and the price is one of the factors, Hence,

Rule. *Divide the cost by the price.*

PROBLEM IV.

256. Given, the quantity, and the price of 100 or 1000, to find the cost.

Analysis. If the price of 100 units be multiplied by the number of units in a given quantity, the product will be 100 times the required result, because the multiplier used is 100 times the true multiplier. For a similar reason, if the price of 1000 units be multiplied by the number of units in a given quantity, the product will be 1000 times the required result. These errors can be corrected in two ways,

1st. By dividing the product by 100 or 1000, as the case may be; or,

2d. By reducing the given quantity to hundreds and decimals of a hundred, or to thousands and decimals of a thousand. Hence, RULE. *Multiply the price by the quantity reduced to hundreds and decimals of a hundred, or to thousands and decimals of a thousand.*

NOTE.—In business transactions the Roman numerals C and M are commonly used to indicate hundreds and thousands, where the price is by the 100 or 1000.

PROBLEM V.

257. To find the cost of articles sold by the ton of 2000 pounds.

ANALYSIS. If the price of 1 ton or 2000 pounds be divided by 2, the quotient will be the price of $\frac{1}{2}$ ton or 1000 pounds. We then have the quantity and the price of 1000 to find the cost. Hence,

RULE. *Divide the price of 1 ton by 2, and multiply the quotient by the number of pounds expressed as thousandths.*

EXAMPLES IN THE PRECEDING PROBLEMS.

1. What cost 187 barrels of salt, at $1.32 a barrel? *Ans.* $246.84.

2. What cost 5 firkins of butter, each containing $70\frac{1}{2}$ pounds, at $\$\frac{3}{16}$ a pound? *Ans.* $\$66.09\frac{3}{8}$.

3. If the board of a family be $\$501.87\frac{1}{2}$ for 1 year, how much is it per day? *Ans.* $\$1.37\frac{1}{2}$.

4. At $\$.10\frac{1}{2}$ a dozen, how many dozen of eggs can be bought for $18.48? *Ans.* 176.

5. What is the value of 140 sacks of guano, each sack containing $162\frac{1}{2}$ pounds, at $\$17\frac{3}{4}$ a ton? *Ans.* $\$201.906\frac{1}{4}$.

6. What will be the cost of 3240 peach trees at $\$16\frac{1}{2}$ per hundred? *Ans.* $534.60.

7. At $66.44 a ton, what will be the cost of $842\frac{3}{4}$ tons of railroad iron? *Ans.* $55992.31.

8. A gentleman purchased a farm of 325.5 acres for $\$10660\frac{1}{8}$; how much did it cost per acre? *Ans.* $32.75.

9. What will be the cost of 840 feet of plank at $1.94 per C; and 1262 pickets at $\$12\frac{1}{2}$ per M? *Ans.* $32.071.

10. At $\$1\frac{1}{2}$ a bushel, how many bushels of wheat can be bought for $\$37.68\frac{3}{4}$? *Ans.* $25\frac{1}{8}$ bushels.

11. What will be the cost of 2172 pounds of plaster, at \$3.875 a ton? *Ans.* \$4.208$\frac{1}{4}$.

12. What cost $\frac{5}{8}$ of 456 bushels of potatoes at \$.37$\frac{1}{2}$ a bushel?

13. If 32$\frac{1}{2}$ barrels of apples cost \$81.25, what is the price per barrel? *Ans.* \$2.50.

14. What must be paid for 24240 feet of timber worth \$9.37$\frac{1}{2}$ per M.? *Ans.* \$227$\frac{1}{4}$.

15. At \$5$\frac{5}{8}$ an acre, how many acres of land can be bought for \$4234.37$\frac{1}{2}$? *Ans.* 752$\frac{7}{9}$.

16. How much must be paid for 972 feet of boards at \$20.25 per M, 1575 feet of scantling at \$2.87$\frac{1}{2}$ per C, and 8756 feet of lath at \$7$\frac{1}{2}$ per M? *Ans.* \$130.634$\frac{1}{4}$.

17. What is the value of 1046 pounds of beef at \$4$\frac{5}{8}$ per hundred pounds? *Ans.* \$48.37$\frac{3}{4}$.

18. What is the value of 5840 pounds of anthracite coal at \$4.7 a ton, and 4376 pounds of shamokin coal at \$5.25 a ton?

19. At \$2.50 a yard, how much cloth can be purchased for \$2?

20. What is the value of 3700 cedar rails at \$5$\frac{3}{4}$ per C?

21. How much is the freight on 3840 pounds from New York to Baltimore, at \$.96 per 100 pounds? *Ans.* \$36.864.

22. What is the value of 9 pieces of broadcloth, each piece containing 27$\frac{3}{4}$ yards, worth \$2$\frac{7}{8}$ a yard? *Ans.* \$715.87$\frac{1}{2}$.

23. At \$.42 a pound, how many pounds of wool may be bought for \$80.745? *Ans.* 192$\frac{1}{4}$.

24. What will be the cost of 327 feet of boards at \$15$\frac{1}{2}$ per M; 672 feet of siding at \$1.62$\frac{1}{2}$ per C, and 1108 bricks at \$4$\frac{1}{4}$ per M? *Ans.* \$20.69$\frac{3}{4}$.

25 At \$$\frac{7}{8}$ per yard, how many yards of silk may be bought for \$15$\frac{3}{4}$? *Ans.* 18.

26. How much must be paid for the transportation of 18962 pounds of pork from Cincinnati to New York, at \$10 a ton?

27. If 15$\frac{1}{2}$ yards of silk cost \$27.9, what is the price per yard?

28. What cost 27860 railroad ties at \$125.38 per thousand?

29. If .7 of a ton of hay cost \$13$\frac{1}{3}$, what is the price of 1 ton?

30. What is the value of 720 pounds of hay at \$12.75 a ton, and 912 pounds of mill feed at \$15$\frac{1}{2}$ a ton? *Ans.* \$11.658.

LEDGER ACCOUNTS.

258. A **Ledger** is the principal book of accounts kept by merchants and accountants. Into it are brought in summary form the accounts from the journal or day-book. The items often form long columns, and accountants in adding sometimes add more than one column at a single operation, (**68**).

(1.)	(2.)	(3.)	(4.)
$ 42.17	$ 506.76	$2371.67	$14763.84
36.24	194 32	4571.84	33276.90
18.42	427.90	1690.50	47061.39
10.71	173.26	2037.69	18242.76
194.30	71.32	5094.46	37364.96
347.16	39.46	876.54	8410.31
40.00	152.60	679.81	5724.27
12.94	271.78	155.48	56317.66
86.73	320.00	4930.71	81742.73
271.19	709.08	3104.13	22431.27
103.07	48.50	1987.67	40163.55
500.50	63.41	5142.84	32189.60
7.59	56.00	276.30	7063.21
11.44	410.10	522.71	3451.09
81.92	372.22	3114 60	9200.00
110.10	137.89	1776.82	1807.36
107.09	276.44	7152.91	56768.72
207.16	18.19	9328.42	63024.27
97.20	27.96	472.19	36180 45
21.77	157.16	321.42	90807.08
150.15	94.57	2423.79	28763.81
427.26	177.66	1600.81	37196.75
316.42	327.40	5976.27	4230 61
114.61	1132.16	4318.19	3719.84
81.13	876.57	682,45	1367.92
37.50	179.84	3174.96	8756.47

ACCOUNTS AND BILLS.

259. A **Debtor,** in business transactions, is a purchaser, or a person who receives money, goods, or services from another; and

260. A **Creditor** is a seller, or a person who parts with money, goods, or services to another.

261. An **Account** is a registry of debts and credits.

NOTES.—1. An account should always contain the names of both the parties to the transaction, the price or value of each item or article, and the date of the transaction.

2. Accounts may have only one side, which may be either debt or credit; or it may have two sides, debt and credit.

262. The **Balance of an Account** is the difference between the amount of the debit and credit sides. If the account have only one side, the balance is the amount of that side.

263. An **Account Current** is a full copy of an account, giving each item of both debit and credit sides to date.

264. A **Bill**, in business transactions, is an account of money paid, of goods sold or delivered, or of services rendered, with the price or value annexed to each item.

265. The **Footing of a Bill** is the total amount or cost of all the items.

NOTE.—A bill of goods bought or sold, or of services received or rendered at a single transaction, and containing only one date, is often called a *Bill of Parcels;* and an account current having only one side is sometimes called a *Bill of Items.*

266. In accounts and bills the following abbreviations are in general use:

Dr. for debit or debtor;

Cr. for credit or creditor;

a/c. or *acc't* for account;

@ for at or by; when this abbreviation is used it is always followed by the price of a unit. Thus, 3 yd. cloth @ $1.25, signifies 3 yards of cloth at $1.25 per yard; ¼ lb. tea @ $.75, signifies ¼ pound of tea at $.75 per pound.

267. When an account current or a bill is settled or paid, the fact should be entered on the same and signed by the creditor, or by the person acting for him. The a/c. or bill is then said to be *receipted.* Accounts and bills may be settled, balanced and receipted by the parties to the same, or by agents, clerks or attorneys authorized to transact business for the parties.

EXAMPLES FOR PRACTICE.

Required, the footings and balances of the following bills and accounts.

(1.)

Bill: receipted by clerk or agent.

NEW YORK, July 10, 1860.

MR. JOHN C. SMITH,

Bo't of HILL, GROVES & Co.,

10	yd.	Cassimere,	@	$2.85	
16	"	Blk. Silk,	"	1.12½	
72	"	Ticking,	"	.14	
42	"	Bld. Shirting,	"	.16½	
12	"	Pressed Flannel,	"	.40	
24½	"	Scotch Plaid Prints,	"	.56	
					$82.03

Rec'd Payment,

HILL, GROVES & Co.,
By J. W. HOPKINS.

(2.)

Bill: receipted by the selling party.

CHICAGO, Sept. 20, 1861.

CHASE & KENNARD,

Bo't of McDOUGAL, FENTON & Co.,

125	pr.	Boys' Thick Boots,	@	$1.25	
275	"	" Calf "	"	1.75	
180	"	" Kip "	"	1.12½	
210	"	" Brogans,	"	.87½	
80	"	Women's Fox'd Gaiters,	"	.84	
95	"	" Opera Boots,	"	.90	
175	"	" Enameled "	"	1.06	
8	cases	Men's Calf Boots,	"	30.50	
3	"	Congress Pump Boots,	"	35.75	
1	"	Drill, 958 yd.,	"	.10½	
40	gross	Silk Buttons,	"	.37½	
					$1828.79

Rec'd Payment,

McDOUGAL, FENTON & Co.

(3.)

Bill: settled by note.

NEW YORK, May 4, 1860.

SMITH & PERKINS,

Bo't of KENT, LOWBER & CO.,

40	chests	Green Tea,	@	$27.50	
25	"	Black "	"	19.20	
16	"	Imperial "	"	48.10	
12	sacks	Java Coffee,	"	17.75	
20	bbl.	Coffee Sugar, (A)	"	26.30	
15	"	Crushed "	"	31.85	
36	boxes	Lemons,	"	3.87½	
42	"	Oranges,	"	4.12½	
25	"	Raisins,	"	2.90	
					$2961.60

Rec'd Payment, by note at 6 mo.

KENT, LOWBER & CO.

(4.)

Bill: paid by draft, and receipted by Clerk.

NEW ORLEANS, April 28, 1861.

JAMES CARLTON & CO.

Bo't of WILLARD & HALE.

150	bbl.	Canada Flour,	@	$6.25	
275	"	Genesee "	"	7.16	
170	"	Philada. "	"	5.87½	
326	bu.	Wheat,	"	1.62½	
214	"	Corn,	"	.82	
300	"	Barley,	"	.91	
500	"	Rye,	"	1.06	
					$5413.48

Rec'd Payment, by Draft on N. Y.

R. S. CLARKE,
FOR WILLARD & HALE.

(5.)

Account Current; not balanced or settled.

PHILADELPHIA, Nov. 1, 1860.

MR. JAMES CORNWALL,

To DODGE & SON, Dr.

April 15,	To 24 tons Swedes Iron,	@	$64.30	$	
" "	" 15 cwt. Eng. Blister Steel,	"	10.25		
June 21,	" 7 doz. Hoes, (Trowel Steel)	"	7.78		
Aug. 10,	" 25 " Buckeye Plows,	"	8.45		
Oct. 3,	" 14 Cross-cut Saws,	"	16.12½		
" "	" 27 cwt. Bar Lead,	"	5.90		
" "	" 1840 lbs. Chain,	"	.09½		
					$

Cr.

May 25,	By 20 M. Boards,	@	$17.60	
July 14,	" 50 M. Shingles,	"	3.12½	
" "	" 42 M. Plank,	"	9.37½	
Sept. 5,	" Draft on New York,			$1000
" 12,	" 75 C. Timber,	@	3.10	
" "	" 36 C. Scantling,	"	.87½	
				$
	Bal. Due DODGE & SON,			$356.51

(6.)

Account Current, another form; balanced by note.

WM. RICHMOND & Co. in a|c. current with WOOD & POWELL.

Dr.										*Cr.*
1850					1860					
July	2	To 896 pounds butter, $.23			Nov.	3	By 61 barrels apples, $2.25			
Aug.	17	" 872 " cheese, .09¼			"	24	" 70 bushels turnips, .22			
"	24	" 481½ " lard, .11⅜			Dec.	1	" 56 " dried apples, .87½			
Oct.	4	" 509¾ " tallow, .13½			"	22	" 31 drums figs, .68¾			
"	18	" 81 dozen eggs, .16⅔			1861					
"	31	" 15 barrels salt, 1.40			Jan.	2	" Note at 3 mo. to Bal.			
Dec.	15	" 41 hams, 963⅜ pounds, .12½								
			565	25				565	25	

WOOD & POWELL.

BOSTON, Jan. 1, 1861.

PROMISCUOUS EXAMPLES.

1. What cost $12\frac{5}{8}$ cords of wood @ $\$4.87\frac{1}{2}$? *Ans.* \$61.54+.

2. At \$.37½ per bushel, how many barrels of potatoes, each containing $2\frac{1}{2}$ bushels, can be purchased for \$33.75? *Ans.* 36.

3. If 36 boxes of raisins, each containing 36 pounds, can be bought for \$97.20, what is the price per pound? *Ans.* \$.075.

4. If .625 of a barrel of flour be worth \$5.35, what is a barrel worth? *Ans.* \$8.56.

5. What is the difference between $\frac{5}{6}$ of a hundredth, and $\frac{1}{3}$ of a tenth? *Ans.* .025.

6. What is the product of $814\frac{99}{200} \times 26\frac{15}{32}$ correct to 2 decimal places?

7. A drover bought 5 head of cattle @ \$75, and 12 head @ \$68; at what price per head must he sell them to gain \$118 on the whole? *Ans.* \$77.

8. If 1 pound of tea be worth $\$.62\frac{1}{2}$, what is .8 of a pound worth? *Ans.* \$.5.

9. A person having \$27.96, was desirous of purchasing an equal number of pounds of tea, coffee, and sugar; the tea @ $\$.87\frac{1}{2}$, the coffee @ $\$.18\frac{3}{4}$, and the sugar @ $\$.10\frac{1}{4}$. How many pounds of each could he buy? *Ans.* 24.

10. If a man travel 13543.47 miles in $365\frac{1}{4}$ days, how far does he travel in $\frac{7}{8}$ of a day? *Ans.* 32.445 miles.

11. Bought 100 barrels of flour @ $\$5.12\frac{1}{2}$, and 250 bushels of wheat @ $\$1.06\frac{1}{4}$. Having sold 75 barrels of the flour @ $\$6\frac{1}{2}$, and all the wheat @ $\$1\frac{3}{8}$, at what price per barrel must the remainder of the flour be sold, to gain $\$221.87\frac{1}{2}$ on the whole investment? *Ans.* \$6.75.

12. If 114.45 acres of land produce 4580.289 bushels of potatoes, how many acres will be required to produce 120.06 bushels? *Ans.* 3.

13. Divide $.0172\frac{17}{64}$ by $.03\frac{1}{16}$, and obtain a quotient true to 4 decimal places. *Ans.* .5625±.

14. Divide 13.5 by $2\frac{1}{4}$ hundredths. *Ans.* 600.

15. A man agreed to build 59.5 rods of wall; having built 8.5

rods in 5 days, how many days will be required to finish the wall at the same rate? *Ans.* 30 days.

16. A farmer exchanged $28\frac{1}{2}$ bushels of oats worth \$.37$\frac{1}{2}$ per bushel, and 453 pounds of mill feed worth \$.75 per hundred, for 12520 pounds of plaster; how much was the plaster worth per ton? *Ans.* \$2.25.

17. A farmer sold to a merchant 3 loads of hay weighing respectively 1826, 1478, and 1921 pounds, at \$8.80 per ton, and 281 pounds of pork at \$5.25 per hundred. He received in exchange 31 yards of sheeting @ \$.09, $6\frac{1}{2}$ yards of cloth @ \$4.50, and the balance in money; how much money did he receive?

18. If 35 yards of cloth cost \$122.50, what will be the cost of 29 yards? *Ans.* \$101.50.

19. A speculator bought 1200 bushels of corn @ \$.56$\frac{1}{4}$. He sold $375\frac{1}{2}$ bushels @ \$.60. At what price must he sell the remainder, to gain \$168.675 on the whole?

20. If a load of plaster weighing 1680 pounds cost \$2.856, how much will a ton of 2000 pounds cost? *Ans.* \$3.40.

21. If .125 of an acre of land is worth \$15$\frac{7}{8}$, how much are 25.42 acres worth?

22. A farmer had 150 acres of land, which he could have sold at one time for \$100 an acre, and thereby have gained \$3900; but after keeping it for some time he was obliged to sell it at a loss of \$2250. How much an acre did the land cost him, and how much an acre did he sell it for?

23. A lumber dealer bought 212500 feet of lumber at \$14.375 per M, and retailed it out at \$1.75 per C; how much was his whole gain?

24. If 10 acres of land can be bought for \$545, how many acres can be bought for \$17712.50? *Ans.* 325.

25. How much is the half of the fourth part of 7 times 224.56? *Ans.* 196.49.

26. Sold 10450 feet of timber for \$169.8125, and gained thereby \$39.18$\frac{3}{4}$; how much did it cost per C? *Ans.* \$1.25.

27. If \$6.975 be paid for .93 of a hundred pounds of pork, how much will 1 hundred pounds cost?

28. Three hundred seventy-five thousandths of a lot of dry goods, valued at $4000, was destroyed by fire; how much would a firm lose who owned .12 of the entire lot? *Ans.* $180.

29. Reduce $\left(\frac{1\frac{3}{4}}{4\frac{1}{2}} \div \frac{2\frac{1}{3}}{2\frac{1}{4}}\right) \times \frac{4}{5}$ of $\frac{1}{2}$ to a decimal. *Ans.* .15.

30. If 7.5 tons of hay are worth 375 bushels of potatoes, and 1 bushel of potatoes is worth $.33$\frac{1}{3}$, how much is 1 ton of hay worth? *Ans.* 16.66\frac{2}{3}$.

31. A person invested a certain sum of money in trade; at the end of 5 years he had gained a sum equal to 84 hundredths of it, and in 5 years more he had doubled this entire amount. How many times the sum first invested had he at the end of the 10 years? *Ans.* 3.68 times.

32. A miller paid $54 for grain, $\frac{3}{10}$ of it being barley at $.62$\frac{1}{2}$ per bushel, and $\frac{3}{5}$ of it wheat at 1.87\frac{1}{2}$ per bushel; the balance of the money, he expended for oats at $.37$\frac{1}{2}$ per bushel. How many bushels of grain did he purchase? *Ans.* 40.

33. A merchant tailor bought 27 pieces of broadcloth, each piece containing 19$\frac{1}{3}$ yards, at 4.31\frac{1}{4}$ a yard; and sold it so as to gain 381.87\frac{1}{2}$, after deducting 9.62\frac{1}{2}$ for freight. How much was the cloth sold for per yard? *Ans.* 5.06\frac{1}{4}$.

34. Bought 1356 bushels of wheat @ 1.18\frac{3}{4}$, and 736 bushels of oats @ $.41; I had 870 bushels of the wheat floured, and disposed of it at a profit of 235.87\frac{1}{2}$, and I sold 528 bushels of the oats at a loss of 13.62\frac{1}{2}$. I afterward sold the remainder of the wheat at 1.12\frac{1}{2}$ per bushel, and the remainder of the oats at $.31 per bushel; did I gain or lose, and how much?

Ans. I gained 171.07\frac{1}{2}$.

35. The sum of two fractions is $\frac{125}{176}$, and their difference is $\frac{116}{704}$; what are the fractions?

36. A manufacturer carried on business for 3 years. The first year he gained a sum equal to $\frac{4}{9}$ of his original capital; the second year he lost $\frac{1}{5}$ of what he had at the end of the first year; the third year he gained $\frac{3}{8}$ of what he had at the end of the second year, and he then had $28585.70. How much had he gained in the 3 years? *Ans.* $10594.70.

CONTINUED FRACTIONS.

268. If we take any fraction in its lowest terms, as $\frac{13}{54}$, and divide both terms by the numerator, we shall obtain a complex fraction, thus:

$$\frac{13}{54} = \cfrac{1}{4 + \cfrac{2}{13}}$$

Reducing $\frac{2}{13}$, the fractional part of the denominator, in the same manner, we have,

$$\frac{13}{54} = \cfrac{1}{4 + \cfrac{1}{6 + \cfrac{1}{2}}}$$

Expressions in this form are called *continued fractions.* Hence,

269. A **Continued Fraction** is a fraction whose numerator is 1, and whose denominator is a whole number plus a fraction whose numerator is also 1, and whose denominator is a similar fraction, and so on.

270. The **Terms** of a continued fraction are the several simple fractions which form the parts of the continued fraction. Thus, the terms of the continued fraction given above are, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{2}$.

CASE I.

271. To reduce any fraction to a continued fraction.

1. Reduce $\frac{109}{339}$ to a continued fraction.

OPERATION.

$$\frac{109}{339} = \cfrac{1}{3 + \cfrac{1}{9 + \cfrac{1}{12}}}$$

ANALYSIS. We divide the denominator, 339, by the numerator, 109, and obtain 3 for the denominator of the first term of the continued fraction. Then in the same manner we divide the last divisor, 109, by the remainder, 12, and obtain 9 for the denominator of the second term of the continued fraction. In like manner we obtain 12 for the denominator of the final term. Hence the following

RULE. I. *Divide the greater term by the less, and the last divisor by the last remainder, and so on, till there is no remainder.*

II. *Write 1 for the numerator of each term of the continued fraction, and the quotients in their order for the respective denominators.*

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{288}{1793}$ to a continued fraction.

Ans. $\cfrac{1}{6+\cfrac{1}{4+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{9}}}}}$.

2. Reduce $\frac{1240}{6721}$ to a continued fraction.
3. Reduce $\frac{223874}{516901}$ to a continued fraction.
4. Reduce $\frac{29}{121}$ to a continued fraction.

CASE II.

272. To find the several approximate values of a continued fraction.

An **Approximate Value** of a continued fraction is the simple fraction obtained by reducing one, two, three, or more terms of the continued fraction.

273. 1. Reduce $\frac{38}{163}$ to a continued fraction, and find its approximate values.

OPERATION.

$$\frac{38}{163}=\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{5}}}},\text{ the continued fraction.}$$

$$\frac{1}{4} = \frac{1}{4},\text{ 1st approx. value.}$$

$$\cfrac{1}{4+\cfrac{1}{3}} = \frac{3}{4\times 3+1} = \frac{3}{13},\text{ 2d " "}$$

$$\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{2}}} = \cfrac{1}{4+\cfrac{2}{3\times 2+1}} = \frac{3\times 2+1}{(4\times 3+1)\times 2+4} = \frac{3\times 2+1}{13\times 2+4} = \frac{7}{30},\text{ 3d " "}$$

$$\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{5}}}} = \frac{7\times 5+3}{30\times 5+13} = \frac{38}{163},\text{ 4th " "}$$

ANALYSIS. We take $\frac{1}{4}$, the first term of the continued fraction, for the first approximate value. Reducing the complex fraction formed by the first two terms of the continued fraction, we have $\frac{3}{13}$ for the second approximate value. In like manner, reducing the first three terms, we have $\frac{7}{30}$ for the third approximate value. By examining this last process, we perceive that the third approximate value, $\frac{7}{30}$, is obtained by multiplying the terms of the preceding approximation, $\frac{3}{13}$, by the denominator of the third term of the continued fraction, 2, and adding the corresponding terms of the first approximate value. Taking advantage of this principle, we multiply the terms of $\frac{7}{30}$ by the 4th denominator, 5, in the continued fraction, and adding the corresponding terms of $\frac{3}{13}$, obtain $\frac{38}{163}$, the 4th approximate value, which is the same as the original fraction. Hence the following

RULE. I. *For the first approximate value, take the first term of the continued fraction.*

II. *For the second approximate value, reduce the complex fraction formed by the first two terms of the continued fraction.*

III. *For each succeeding approximate value, multiply both numerator and denominator of the last preceding approximation by the next denominator in the continued fraction, and add to the corresponding products respectively the numerator and denominator of the preceding approximation.*

NOTES.—1. When the given fraction is improper, invert it, and reduce this result to a continued fraction; then invert the approximate values obtained therefrom.

2. In a series of approximate values, the 1st, 3d, 5th, etc., are greater than the given fraction; and the 2d, 4th, 6th, etc., are less than the given fraction.

EXAMPLES FOR PRACTICE.

1. Find the approximate values of $\frac{67}{155}$.

Ans. $\frac{1}{2}$, $\frac{3}{7}$, $\frac{16}{37}$, $\frac{67}{155}$.

2. Find the approximate values of $\frac{83}{349}$.

Ans. $\frac{1}{4}$, $\frac{4}{17}$, $\frac{5}{21}$, $\frac{39}{164}$, $\frac{83}{349}$.

3. What are the first three approximate values of $\frac{2831}{20357}$?

Ans. $\frac{1}{7}$, $\frac{5}{36}$, $\frac{21}{151}$.

4. What are the first five approximate values of $\frac{227}{732}$?

Ans. $\frac{1}{3}$, $\frac{4}{13}$, $\frac{9}{29}$, $\frac{40}{129}$, $\frac{49}{158}$.

5. Reduce $\frac{29}{42}$ to the form of a continued fraction, and find the value of each approximating fraction.

COMPOUND NUMBERS.

274. A **Compound Number** is a concrete number expressed in two or more denominations, (**10**).

275. A **Denominate Fraction** is a concrete fraction whose integral unit is *one* of a denomination of some compound number. Thus, $\frac{3}{7}$ of a day is a denominate fraction, the integral unit being one day; so are $\frac{5}{8}$ of a bushel, $\frac{2}{3}$ of a mile, etc., denominate fractions.

276. In simple numbers and decimals the scale is uniform, and the law of increase and decrease is by 10. But in compound numbers the scale of increase and decrease from one denomination to another is varying, as will be seen in the Tables.

MEASURES.

277. **Measure** is that by which extent, dimension, capacity or amount is ascertained, determined according to some fixed standard.

NOTE.—The process by which the extent, dimension, capacity, or amount is ascertained, is called *Measuring;* and consists in comparing the thing to be measured with some conventional standard.

Measures are of seven kinds:

1. Length.
2. Surface or Area.
3. Solidity or Capacity.
4. Weight, or Force of Gravity.
5. Time.
6. Angles.
7. Money or Value.

The first three kinds may be properly divided into two classes—Measures of Extension, and Measures of Capacity.

MEASURES OF EXTENSION.

278. **Extension** has three dimensions—length, breadth, and thickness.

A **Line** has only one dimension—length.

A **Surface** or **Area** has two dimensions—length and breadth.

A **Solid** or **Body** has three dimensions — length, breadth, and thickness.

I. Linear Measure.

279. Linear Measure, also called Long Measure, is used in measuring lines or distances.

The unit of linear measure is the yard, and the table is made up of the divisors, (feet and inches,) and the multiples, (rods, furlongs, and miles,) of this unit.

TABLE.

12 inches (in.)	make	1 foot,	ft.
3 feet	"	1 yard,	yd.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet,	"	1 rod,	rd.
40 rods	"	1 furlong,	fur.
8 furlongs, or 320 rods,	"	1 statute mile,	mi.

UNIT EQUIVALENTS.

mi.		fur.		rd.		yd.		ft.		in.
								1	=	12
						1	=	3	=	36
				1	=	$5\frac{1}{2}$	=	$16\frac{1}{2}$	=	198
		1	=	40	=	220	=	660	=	7920
1	=	8	=	320	=	1760	=	5280	=	63360

Scale — ascending, 12, 3, $5\frac{1}{2}$, 40, 8; descending, 8, 40, $5\frac{1}{2}$, 3, 12.

The following denominations are also in use: —

3	barleycorns	make	1 inch,	used by shoemakers in measuring the length of the foot.
4	inches	"	1 hand,	used in measuring the height of horses directly over the fore feet.
9	"	"	1 span.	
21.888	"	"	1 sacred cubit.	
3	feet	"	1 pace.	
6	"	"	1 fathom, used in measuring depths at sea.	
1.15	statute miles	"	1 geographic mile,	used in measuring distances at sea.
3	geographic "	"	1 league.	
60	"	" or	1 degree	of latitude on a meridian or of longitude on the equator.
69.16	statute	" "	1 degree	of latitude on a meridian or of longitude on the equator.
360	degrees	"	the circumference of the earth.	

Notes.—1. For the purpose of measuring cloth and other goods sold by the yard, the yard is divided into halves, fourths, eighths, and sixteenths. The old table of cloth measure is practically obsolete.

2. A span is the distance that can be reached, spanned, or measured between the end of the middle finger and the end of the thumb. Among sailors 8 spans are equal to 1 fathom.

3. The geographic mile is $\frac{1}{60}$ of $\frac{1}{360}$ or $\frac{1}{21600}$ of the distance round the center of the earth. It is a small fraction more than 1.15 statute miles.

4. The length of a degree of latitude varies, being 68.72 miles at the equator, 68.9 to 69.05 miles in middle latitudes, and 69.30 to 69.34 miles in the polar regions. The mean or average length, as stated in the table, is the standard recently adopted by the U. S. Coast Survey. A degree of longitude is greatest at the equator, where it is 69.16 miles, and it gradually decreases toward the poles, where it is 0.

SURVEYORS' LINEAR MEASURE.

280. A **Gunter's Chain**, used by land surveyors, is 4 rods or 66 feet long, and consists of 100 links.

The unit is the chain, and the table is made up of divisors and multiples of this unit.

TABLE.

7.92	inches (in.)	make	1 link,......l.
25	links	"	1 rod, ...rd.
4	rods, or 66 feet,	"	1 chain, ... ch.
80	chains	"	1 mile,.... mi.

UNIT EQUIVALENTS.

mi.		ch.		rd.		l.		in.
						1	=	7.92
				1	=	25	=	198
		1	=	4	=	100	=	792
1	=	80	=	320	=	8000	=	63360

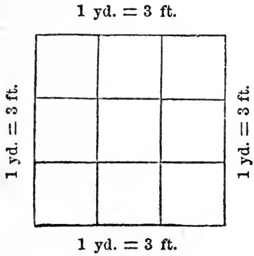
SCALE — ascending, 7.92, 25, 4, 80; descending, 80, 4, 25, 7.92.

NOTE.—The denomination, rods, is seldom used in chain measure, distances being taken in chains and links.

II. SQUARE MEASURE.

281. A **Square** is a figure having four equal sides and four equal corners or right angles.

282. **Area** or **Superficies** is the space or surface included within any given lines: as, the area of a square, of a field, of a board, etc.



1 square yard is a figure having four sides of 1 yard or 3 feet each, as shown in the diagram. Its contents are 3 × 3 = 9 square feet. Hence,

The contents or area of a square, or of any other figure having a uniform length and a uniform breadth, is found by multiplying the length by the breadth.

Thus, a square foot is 12 inches long and 12 inches wide, and the contents are $12 \times 12 = 144$ square inches. A board 20 feet long and 10 feet wide, is a rectangle, containing $20 \times 10 = 200$ square feet.

The measurements for computing area or surface are always taken in the denominations of linear measure.

283. Square Measure is used in computing areas or surfaces; as of land, boards, painting, plastering, paving, etc.

The unit is the area of a square whose side is the unit of length. Thus, the unit of square feet is 1 foot square; of square yards, 1 yard square, etc.

TABLE.

144	square inches (sq. in.)	make	1	square foot,....	sq. ft.
9	" feet	"	1	" yard,...	sq. yd.
$30\frac{1}{4}$	" yards	"	1	" rod,....	sq. rd.
40	" rods	"	1	rood,.............	R.
4	roods	"	1	acre,............	A.
640	acres	"	1	square mile,...	sq. mi.

UNIT EQUIVALENTS.

sq. mi.	A.	R.	sq. rd.	sq. yd.	sq. ft.	sq. in.
					1 =	144
				1 =	9 =	1296
			1 =	$30\frac{1}{4}$ =	$272\frac{1}{4}$ =	39204
		1 =	40 =	1210 =	10890 =	1568160
	1 =	4 =	160 =	4840 =	43560 =	6272640
1 =	640 =	2560 =	102400 =	3097600 =	27878400 =	4014489600

SCALE — ascending, 144, 9, $30\frac{1}{4}$, 40, 4, 640; descending, 640, 4, 40, $30\frac{1}{4}$, 9, 144.

Artificers estimate their work as follows:

By the square foot: glazing and stone-cutting.

By the square yard: painting, plastering, paving, ceiling, and paper-hanging.

By the square of 100 square feet: flooring, partitioning, roofing, slating, and tiling.

Bricklaying is estimated by the thousand bricks, by the square yard, and by the square of 100 square feet.

NOTES.—1. In estimating the painting of moldings, cornices, etc., the measuring-line is carried into all the moldings and cornices.

2. In estimating brick-laying by either the square yard or the square of 100 feet, the work is understood to be 12 inches or $1\frac{1}{2}$ bricks thick.

3. A thousand shingles are estimated to cover 1 square, being laid 5 inches to the weather.

SURVEYORS' SQUARE MEASURE.

284. This measure is used by surveyors in computing the area or contents of land.

TABLE.

625	square links (sq. l.)	make	1 pole, ... P.
16	poles	"	1 square chain, . sq. ch.
10	square chains	"	1 acre, ... A.
640	acres	"	1 square mile, . sq. mi.
36	square miles (6 miles square)	"	1 township, ... Tp.

UNIT EQUIVALENTS.

Tp.		sq. mi.		A.		sq. ch.		P.		sq. l.
								1	=	625
						1	=	16	=	10000
				1	=	10	=	160	=	100000
		1	=	640	=	6400	=	102400	=	64000000
1	=	36	=	23040	=	230400	=	3686400	=	2304000000

SCALE — ascending, 625, 16, 10, 640, 36; descending, 36, 640, 10, 16, 625.

NOTES.—1. A square mile of land is also called a *section*.

2. Canal and railroad engineers commonly use an engineers' chain, which consists of 100 links, each 1 foot long.

3. The contents of land are commonly estimated in square miles, acres, and hundredths; the denomination, *rood*, is rapidly going into disuse.

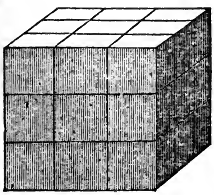
III. CUBIC MEASURE.

285. A **Cube** is a solid, or body, having six equal square sides or faces.

286. Solidity is the matter or space contained within the bounding surfaces of a solid.

The measurements for computing solidity are always taken in the denominations of linear measure.

3 ft. = 1 yd.



3 ft. = 1 yd.

If each side of a cube be 1 yard, or 3 feet, 1 foot in thickness of this cube will contain $3 \times 3 \times 1 = 9$ cubic feet; and the whole cube will contain $3 \times 3 \times 3 = 27$ cubic feet.

A solid, or body, may have the three dimensions all alike or all different. A body 4 ft. long, 3 ft. wide, and 2 ft. thick contains $4 \times 3 \times 2 = 24$ cubic or solid feet. Hence we see that

The cubic or solid contents of a body are found by multiplying the length, breadth, and thickness together.

237. Cubic Measure, also called Solid Measure, is used in computing the contents of solids, or bodies; as timber, wood stone, etc.

The unit is the solidity of a cube whose side is the unit of length. Thus, the unit of cubic feet is a cube which measures 1 foot on each side; the unit of cubic yards is 1 cubic yard, etc.

TABLE.

1728	cubic inches (cu. in.)	make	1	cubic foot	cu. ft.
27	cubic feet	"	1	cubic yard	cu. yd.
40 50	cubic feet of round timber, or " " hewn "	"	1	ton or load	T.
16	cubic feet	"	1	cord foot	cd. ft.
8 128	cord feet, or cubic feet	"	1	cord of wood	Cd.
$24\frac{3}{4}$	cubic feet	"	1	perch of stone or masonry,	Pch.

Scale — ascending, 1728, 27. The other numbers are not in a regular scale, but are merely so many times 1 foot. The unit equivalents, being fractional, are consequently omitted.

Notes. — 1. A cubic yard of earth is called a load.

2. Railroad and transportation companies estimate light freight by the space it occupies in cubic feet, and heavy freight by weight.

3. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains 1 cord; and a cord foot is 1 foot in length of such a pile.

4. A perch of stone or of masonry is $16\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet wide, and 1 foot high.

5. Joiners, bricklayers, and masons, make an allowance for windows, doors, etc., of one half the openings or vacant spaces. Bricklayers and masons, in estimating their work by cubic measure, make no allowance for the corners of the walls of houses, cellars, etc., but estimate their work by the *girt*, that is, the entire length of the wall on the *outside.*

6. Engineers, in making estimates for excavations and embankments, take the dimensions with a line or measure divided into feet and decimals of a foot. The computations are made in feet and decimals, and the results are reduced to cubic yards. In civil engineering, the cubic yard is the unit to which estimates for excavations and embankments are finally reduced.

7. In scaling or measuring timber for shipping or freighting, $\frac{1}{5}$ of the solid contents of round timber is deducted for waste in hewing or sawing. Thus, a log that will make 40 feet of hewn or sawed timber, actually contains 50 cubic feet by measurement; but its market value is only equal to 40 cubic feet of hewn or sawed timber. Hence, the cubic contents of 40 feet of round and 50 feet of hewn timber, as estimated for market, are identical.

MEASURES OF CAPACITY.

288. Capacity signifies extent of room or space.

289. Measures of capacity are all cubic measures, solidity and capacity being referred to different units, as will be seen by comparing the tables.

Measures of capacity may be properly subdivided into two classes, Measures of Liquids and Measures of Dry Substances.

I. Liquid Measure.

290. Liquid Measure, also called Wine Measure, is used in measuring liquids; as liquors, molasses, water, etc.

The unit is the gallon, and the table is made up of its divisors and multiples.

TABLE.

4	gills (gi.)	make	1 pint, ...pt.
2	pints	"	1 quart, ...qt.
4	quarts	"	1 gallon, ...gal.
$31\frac{1}{2}$	gallons	"	1 barrel, ...bbl.
2	barrels, or 63 gal.	"	1 hogshead, . .hhd.

UNIT EQUIVALENTS.

hhd.		bbl.		gal.		qt.		pt.		gi.
								1	=	4
						1	=	2	=	8
				1	=	4	=	8	=	32
		1	=	$31\frac{1}{2}$	=	126	=	252	=	1008
1	=	2	=	63	=	252	=	504	=	2016

Scale—ascending, 4, 2, 4, $31\frac{1}{2}$, 2; descending, 2, $31\frac{1}{2}$, 4, 2, 4.

The following denominations are also in use:

42	gallons	make	1 tierce.
2	hogsheads, or 126 gallons,	"	1 pipe or butt.
2	pipes or 4 hogsheads,	"	1 tun.

Notes.—1. The denominations, barrel and hogshead, are used in estimating the capacity of cisterns, reservoirs, vats, etc. In Massachusetts the barrel is estimated at 32 gallons.

2. The tierce, hogshead, pipe, butt, and tun are the names of casks, and do not express any fixed or definite measures. They are usually gauged, and have their capacities in gallons marked on them. Several of these denominations are still in use in England, (**327—330**).

BEER MEASURE.

291. Beer Measure is a species of liquid measure used in measuring beer, ale, and milk.

The unit is the gallon.

TABLE.

2	pints (pt.)	make	1 quart, ...qt.
4	quarts	"	1 gallon, ...gal.
36	gallons	"	1 barrel, ...bbl.
$1\frac{1}{2}$	barrels, or 54 gallons,	"	1 hogshead,..hhd.

UNIT EQUIVALENTS.

hhd.		bbl.		gal.		qt.		pt.
						1	=	2
				1	=	4	=	8
		1	=	36	=	144	=	288
1	=	$1\frac{1}{2}$	=	54	=	216	=	432

Scale — ascending, 2, 4, 36, $1\frac{1}{2}$; descending, $1\frac{1}{2}$, 36, 4, 2.

This measure is not a standard; it is rapidly falling into disuse.

II. Dry Measure.

292. Dry Measure is used in measuring articles not liquid; as grain, fruit, salt, roots, ashes, etc.

The unit is the bushel, of which all the other denominations in the table are divisors.

TABLE.

2	pints (pt.)	make	1 quart, ...qt.
8	quarts	"	1 peck, ...pk.
4	pecks	"	1 bushel,..bu. or bush.

UNIT EQUIVALENTS.

bu.		pk.		qt.		pt.
				1	=	2
		1	=	8	=	16
1	=	4	=	32	=	64

Scale — ascending, 2, 8, 4; descending, 4, 8, 2.

WEIGHTS.

293. Weight is the measure of the quantity of matter a body contains, determined by the force of gravity.

Note. — The process by which the quantity of matter or the force of gravity is obtained is called *Weighing;* and consists in comparing the thing to be weighed with some conventional standard.

Three scales of weight are used in the United States; namely, Troy, Avoirdupois, and Apothecaries'.

I. Troy Weight.

294. Troy Weight is used in weighing gold, silver, and jewels; in philosophical experiments, and generally where great accuracy is required.

The unit is the pound, and of this all the other denominations in the table are divisors.

TABLE.

24 grains (gr.)	make	1 pennyweight,..	pwt. or dwt.
20 pennyweights	"	1 ounce,..................	oz.
12 ounces	"	1 pound,................	lb.

UNIT EQUIVALENTS.

lb.		oz.		pwt.		gr.
				1	=	24
		1	=	20	=	480
1	=	12	=	240	=	5760

Scale — ascending, 24, 20, 12; descending, 12, 20, 24.

Note.—Troy weight is sometimes called *Goldsmiths' Weight.*

II. Avoirdupois Weight.

295. Avoirdupois Weight is used for all the ordinary purposes of weighing.

The unit is the pound, and the table is made up of its divisors and multiples.

TABLE.

16 drams (dr.)	make	1 ounce,.............	oz.
16 ounces	"	1 pound,............	lb.
100 lb.	"	1 hundred weight,..	cwt.
20 cwt., or 2000 lbs.,	"	1 ton,................	T.

UNIT EQUIVALENTS.

T.		cwt.		lb.		oz.		dr.
						1	=	16
				1	=	16	=	256
		1	=	100	=	1600	=	25600
1	=	20	=	2000	=	32000	=	512000

Scale — ascending, 16, 16, 100, 20; descending, 20, 100, 16, 16.

NOTE.—The *long* or *gross ton*, hundred weight, and quarter were formerly in common use; but they are now seldom used except in estimating English goods at the U. S. custom-houses, in freighting and wholesaling coal from the Pennsylvania mines, and in the wholesale iron and plaster trade.

LONG TON TABLE.

28 lb.	make	1 quarter,	marked qr.
4 qr. = 112 lb.	"	1 hundred weight,	" cwt.
20 cwt. = 2240 lb.	"	1 ton,	" T.

SCALE—ascending, 28, 4, 20; descending, 20, 4, 28.

296. The weight of the bushel of certain grains and roots has been fixed by statute in many of the States; and these statute weights must govern in buying and selling, unless specific agreements to the contrary be made.

TABLE OF AVOIRDUPOIS POUNDS IN A BUSHEL,

As prescribed by statute in the several States named.

COMMODITIES.	California.	Connecticut.	Delaware.	Illinois.	Indiana.	Iowa.	Kentucky.	Louisiana.	Maine.[3]	Massachusetts	Michigan.	Minnesota.	Missouri.	N. Hampshire.	New Jersey.	New York.	Ohio.	Oregon.	Pennsylvania.	Rhode Island.	Vermont.	Washington T	Wisconsin.
Barley	50			48	48	48	48	32		46	48	48	48		48	48	48	46	47		46	45	48
Beans				60	60	60	60						60			62							
Blue Grass Seed				14	14	14	14						14										
Buckwheat	40	45		40	50	52	52			46	42	42	52		50	48		42	48		46	42	42
Castor Beans				46	46	46							46										
Clover Seed				60	60	60	60				60	60	60		64	60	60	60				60	60
Dried Apples				24	25	24					28	28	24					28				28	28
Dried Peaches				33	33	33					28	28	33					28				28	28
Flax Seed				56	56	56	56						56		55	55	56						56
Hair				8					11														
Hemp Seed				44	44	44	44						44										
Indian Corn	52	56	56	52	56	56	56	56		56	56	56	52		56	58	56	56	56		56	56	56
Ind. Corn in ear				70	68	68																	
Ind. Corn Meal				48	50		50		50	50										50			
Mineral Coal[1]				80	70								80										
Oats	32	28		32	32	35	33⅓	32	30	30	32	32	35	30	30	32	32	34	32		32	36	32
Onions				57	48	57	57			52			57							50		50	
Peas																60							
Potatoes		60		60	60	60	60		60				60	60	60	60		60		60	60	60	60
Rye	54	56		54	56	56	56	32		56	56	56	56		56	56	56	56	56		56	56	56
Rye Meal									50	50										50			
Salt[2]					50	50	50						50			56							
Timothy Seed				45	45	45	45						45			44							46
Wheat	60	56	60	60	60	60	60	60		60	60	60	60		60	60	60	60	60		60	60	60
Wheat Bran				20		20	20						20										

[1] In Kentucky, 80 lbs. of bituminous coal or 70 lbs. of cannel coal make 1 bushel.
[2] In Pennsylvania, 80 lbs. coarse, 70 lbs. ground, or 62 lbs. fine salt make 1 bushel; and in Illinois, 50 lbs. common or 55 lbs. fine salt make 1 bushel.
[3] In Maine, 64 lbs. of ruta baga turnips or beets make 1 bushel.

NOTES.—1. The weight of a barrel of flour is 7 quarters of old, or long ton weight.

2. The weight of a bushel of Indian corn and rye, as adopted by most of the States, and of a bushel of salt is 2 quarters; and of a barrel of salt 10 quarters, or $\frac{1}{2}$ of a long ton.

The following denominations are also in use:

56	pounds	make	1	firkin of butter.
100	"	"	1	quintal of dried salt fish.
100	"	"	1	cask of raisins.
196	"	"	1	barrel of flour.
200	"	"	1	" " beef, pork, or fish.
280	"	"	1	" " salt at the N.Y. State salt works.

III. APOTHECARIES' WEIGHT.

297. Apothecaries' Weight is used by apothecaries and physicians in compounding medicines; but medicines are bought and sold by avoirdupois weight.

The unit is the pound, of which all the other denominations in the table are divisors.

TABLE.

20 grains (gr.)	make	1 scruple,	sc. or ℈.
3 scruples	"	1 dram,	dr. or ʒ.
8 drams	"	1 ounce,	oz. or ℥.
12 ounces	"	1 pound,	lb. or ℔.

UNIT EQUIVALENTS.

lb.		oz.		dr.		sc.		gr.
						1	=	20
				1	=	3	=	60
		1	=	8	=	24	=	480
1	=	12	=	96	=	288	=	5760

SCALE—ascending, 20, 3, 8, 12; descending, 12, 8, 3, 20.

APOTHECARIES' FLUID MEASURE.

298. The measures for fluids, as adopted by apothecaries and physicians in the United States, to be used in compounding medicines, and putting them up for market, are given in the following

TABLE.

60 minims, (♏)	make	1 fluidrachm,	fʒ.
8 fluidrachms,	"	1 fluidounce,	f℥.
16 fluidounces,	"	1 pint,	O.
8 pints,	"	1 gallon,	Cong.

UNIT EQUIVALENTS.

Cong.	O.	f℥	fʒ	♏
			1 =	60
		1 =	8 =	480
	1 =	16 =	128 =	7680
1 =	8 =	128 =	2048 =	61440

SCALE — ascending, 60, 8, 16, 8; descending, 8, 16, 8, 60.

MEASURE OF TIME.

299. **Time** is the measure of duration. The unit is the day, and the table is made up of its divisors and multiples.

TABLE.

60 seconds (sec.)	make	1 minute,	min.
60 minutes,	"	1 hour,	h.
24 hours,	"	1 day,	da.
7 days,	"	1 week,	wk.
365 days,	"	1 common year,	yr.
366 days,	"	1 leap year,	yr.
12 calendar months,	"	1 year,	yr.
100 years,	"	1 century,	C.

UNIT EQUIVALENTS.

yr.	mo.	wk.	da.	h.	min.	sec.
					1 =	60
				1 =	60 =	3600
			1 =	24 =	1440 =	86400
		1 =	7 =	168 =	10080 =	604800
1 =	12 =		365 =	8760 =	525600 =	31536000
			366 =	8784 =	527040 =	31622400

SCALE — ascending, 60, 60, 24, 7; descending, 7, 24, 60, 60.

The calendar year is divided as follows:—

No. of month.	Season.	Names of months.	Abbreviations.	No of days.
1	Winter,	January,	Jan.	31
2		February,	Feb.	28 or 29
3	Spring,	March,	Mar.	31
4		April,	Apr.	30
5		May,	——	31
6	Summer,	June,	Jun.	30
7		July,	——	31
8		August,	Aug.	31
9	Autumn,	September,	Sept.	30
10		October,	Oct.	31
11		November,	Nov.	30
12	Winter,	December,	Dec.	31

NOTES. — 1. In most business transactions 30 days are called 1 month.
2. The *civil day* begins and ends at 12 o'clock, midnight. The *astronomi-*

cal day, used by astronomers in dating events, begins and ends at 12 o'clock, noon. The civil year is composed of civil days.

BISSEXTILE OR LEAP YEAR.

300. The period of time required by the sun to pass from one vernal equinox to another, called the vernal or tropical year, is exactly 365 da. 5 h. 48 min. 49.7 sec. This is the true year, and it exceeds the common year by 5 h. 48 min. 49.7 sec.

If 365 days be reckoned as 1 year, the time lost in the calendar will be

In 1 yr.,	5 h.	48 min.	49.7 sec.
" 4 "	23 "	15 "	18.8 "

The time thus lost in 4 years will lack only 44 min. 41.2 sec. of 1 entire day. Hence,

If every fourth year be reckoned as leap year, the time *gained* in the calendar will be,

In 4 yr.,		44 min.	41.2 sec.
" 100 " (= 25 × 4 yr.)	18 h.	37 "	10 "

The time thus gained in 100 years will lack only 5 h. 22 min. 50 sec. of 1 day. Hence

If every fourth year be reckoned as leap year, the centennial years excepted, the time *lost* in the calendar will be,

In 100 yr.,	5 h.	22 min.	50 sec.
" 400 "	21 "	31 "	20 "

The time thus lost in 400 years lacks only 2 h. 28 min. 40 sec. of 1 day. Hence

If every fourth year be reckoned as leap year, 3 of every 4 centennial years excepted, the time *gained* in the calendar will be,

In 400 yr.,	2 h.	28 min.	40 sec.
" 4000 "	24 h.	46 min.	40 sec.

The following rule for leap year will therefore render the calendar correct to within 1 day, for a period of 4000 years.

I. Every year that is exactly divisible by 4 is a leap year, the centennial years excepted; the other years are common years.

II. Every centennial year that is exactly divisible by 400 is a leap year; the other centennial years are common years.

NOTES.—1. Julius Cæsar, the Roman Emperor, decreed that the year should consist of 365 days 6 hours; that the 6 hours should be disregarded for 3 successive years, and an entire day be added to every fourth year. This day was inserted in the calendar between the 24th and 25th days of February, and is called the *intercalary* day. As the Romans counted the days backward from the first day of the following month, the 24th of February was called by them *sexto* *calendas Martii*, the sixth before the calends of March. The intercalary day which followed this was called *bis-sexto calendas Martii;* hence the name *bissextile.*

2. In 1582 the error in the calendar as established by Julius Cæsar had increased to 10 days; that is, too much time had been reckoned as a year, until the civil year was 10 days behind the solar year. To correct this error, Pope Gregory decreed that 10 entire days should be stricken from the calendar, and that the day following the 3d day of October, 1582, should be the 14th. This brought the vernal equinox at March 21 — the date on which it occurred in the year 325, at the time of the Council of Nice.

3. The year as established by Julius Cæsar is sometimes called the *Julian year;* and the period of time in which it was in force, namely from 46 years B. C. to 1582, is called the *Julian Period.*

4. The year as established by Pope Gregory is called the *Gregorian year,* and the calendar now used is the *Gregorian Calendar.*

5. Most Catholic countries adopted the Gregorian Calendar soon after it was established. Great Britain, however, continued to use the Julian Calendar until 1752. At this time the civil year was 11 days behind the solar year. To correct this error, the British Government decreed that 11 days should be stricken from the calendar, and that the day following the 2d day of September, 1752, should be the 14th.

6. Time before the adoption of the Gregorian Calendar is called *Old Style* (O. S), and since, *New Style,* (N. S.) In Old Style the year commenced March 25, and in New Style it commences January 1.

7. Russia still reckons time by Old Style, or the Julian Calendar; hence their dates are now 12 days behind ours.

8. The centuries are numbered from the commencement of the Christian era; the months from the commencement of the year; the days from the commencement of the month, and the hours from the commencement of the day, (12 o'clock, midnight.) Thus, May 23, 1860, 9 o'clock A. M., is the 9th hour of the 23d day of the 5th month of the 60th year of the 19th century.

MEASURE OF ANGLES.

301. Circular Measure, or Circular Motion, is used principally in surveying, navigation, astronomy, and geography, for reckoning latitude and longitude, determining locations of places and vessels, and computing difference of time.

Every circle, great or small, is divisible into the same number of equal parts: as quarters, called quadrants; twelfths, called signs; 360ths, called degrees, etc. Consequently the parts of different circles, although having the same names, are of different lengths.

The unit is the degree, which is $\frac{1}{360}$ part of the space about a point in any plane. The table is made up of divisors and multiples of this unit.

TABLE.

60 seconds (″)	make	1 minute, ... ′.
60 minutes	"	1 degree, ... °.
30 degrees	"	1 sign, ... S.
12 signs, or 360°,	"	1 circle, ... C.

UNIT EQUIVALENTS.

C.		S.		°		′		″
						1	=	60
				1	=	60	=	3600
		1	=	30	=	1800	=	108000
1	=	12	=	360	=	21600	=	1296000

SCALE — ascending, 60, 60, 30, 12; descending, 12, 30, 60, 60.

NOTES.—1. Minutes of the earth's circumference are called geographic or nautical miles.

2. The denomination, *signs*, is confined exclusively to Astronomy.

3. A degree has no fixed linear extent. When applied to any circle it is always $\frac{1}{360}$ part of the circumference. But, strictly speaking, it is not any part of a circle.

4. 90° make a quadrant or right-angle;
60° " " sextant " $\frac{1}{6}$ of a circle.

MISCELLANEOUS TABLES.

302. COUNTING.

12 units or things make 1 dozen.
12 dozen " 1 gross.
12 gross " 1 great gross.
20 units " 1 score.

303. PAPER.

24 sheets..... make .....1 quire.
20 quires " 1 ream.
2 reams " 1 bundle.
5 bundles " 1 bale.

304. BOOKS.

The terms *folio*, *quarto*, *octavo*, *duodecimo*, etc., indicate the number of leaves into which a sheet of paper is folded.

A sheet folded in 2 leaves is called a folio.
A sheet folded in 4 leaves " a quarto, or 4to.
A sheet folded in 8 leaves " an octavo, or 8vo.
A sheet folded in 12 leaves " a 12mo.
A sheet folded in 16 leaves " a 16mo.
A sheet folded in 18 leaves " an 18mo.
A sheet folded in 24 leaves " a 24mo.
A sheet folded in 32 leaves " a 32mo.

305. COPYING.

72 words make 1 folio or sheet of common law.
90 " " 1 " " " " chancery.

GOVERNMENT STANDARDS OF MEASURES AND WEIGHTS.

306. In early times, almost every province and chief city had its own measures and weights; but these were neither definite nor uniform. This variety in the weights and measures of different countries has always proved a serious embarrassment to commerce; hence the many attempts that have been made in modern times to establish uniformity.

The English, American, and French Governments, in establishing their standards of measures and weights, founded them upon unalterable principles or laws of nature, as will be seen by examining the several standards.

UNITED STATES STANDARDS.

307. In the year 1834 the U. S. Government adopted a uniform standard of weights and measures, for the use of the custom houses, and the other branches of business connected with the General Government. Most of the States which have adopted any standards have taken those of the General Government.

308. *The invariable standard unit* from which the standard units of measure and weight are derived is *the day.*

Astronomers have proved that the diurnal revolution of the earth is *entirely uniform,* always performing equal parts of a revolution on its axis in equal periods of duration.

Having decided upon the invariable standard unit, a measure of this unit was sought that should in some manner be connected with extension as well as with this unit. A clock pendulum whose rod is of any given length, is found always to vibrate the same number of times in the same period of duration. Having now the day and the pendulum, the different standards hereafter given have been determined and adopted.

STANDARD OF EXTENSION.

309. *The U. S. standard unit of measures of extension,* whether linear, superficial, or solid, is the yard of 3 feet, or 36 inches, and is the same as the Imperial standard yard of Great Britain. It is determined as follows: The rod of a pendulum vibrating seconds of mean time, in the latitude of London, in a vacuum, at the level of the sea, is divided into 391393 equal parts, and 360000 of these parts are 36 inches, or 1 standard yard. Hence, such a pendulum rod is 39.1393 inches long, and the standard yard is $\frac{360000}{391393}$ of the length of the pendulum rod.

STANDARDS OF CAPACITY.

310. *The U. S. standard unit of liquid measure* is the old English wine gallon, of 231 cubic inches, which is equal to 8.33888 pounds avoirdupois of distilled water at its maximum density; that is, at the temperature of 39.83° Fahrenheit, the barometer at 30 inches.

311. *The U. S. standard unit of dry measure* is the British Winchester bushel, which is $18\frac{1}{2}$ inches in diameter and 8 inches deep, and contains 2150.42 cubic inches, equal to 77.6274 pounds avoirdupois of distilled water, at its maximum density. A gallon, dry measure, contains 268.8 cubic inches.

NOTES.—1. Grain and some other commodities are sold by *stricken measure*, and in such cases the "measure is to be stricken with a round stick or roller, straight, and of the same diameter from end to end."

2. Coal, ashes, marl, manure, corn in the ear, fruit and roots are sold by *heap measure*. The bushel, heap measure, is the Winchester bushel heaped in the form of a cone, which cone must be $19\frac{1}{2}$ inches in diameter (= to the outside diameter of the standard bushel measure,) and at least 6 inches high. A bushel, heap measure, contains 2747.7167 cubic inches, or 597.2967 cubic inches more than a bushel stricken measure. Since 1 peck contains $\frac{2150.4}{4}$ = 537.605 cubic inches, the bushel, heap measure, contains 59.6917 cubic inches more than 5 pecks. As this is about 1 bu. 1 pk. $1\frac{3}{4}$ pt., it is sufficiently accurate in practice, to call 5 pecks stricken measure a heap bushel.

3. A standard bushel, stricken measure, is commonly estimated at 2150.4 cubic inches. The old English standard bushel from which the United States standard bushel was derived, was kept at Winchester, England; hence the name.

4. The wine and dry measures of the same denomination are of differentcapacities. The exact and the relative size of each may be readily seen by the following

312. COMPARATIVE TABLE OF MEASURES OF CAPACITY.

	Cubic in. in one gallon.	Cubic in. in one quart.	Cubic in. in one pint.	Cubic in. in one gill.
Wine measure,........	231	$57\frac{3}{4}$	$28\frac{7}{8}$	$7\frac{7}{32}$
Dry measure ($\frac{1}{2}$ pk.,)..	$268\frac{4}{5}$	$67\frac{1}{5}$	$33\frac{3}{5}$	$8\frac{2}{5}$

NOTE. — The beer gallon of 282 inches is retained in use only by custom.

STANDARDS OF WEIGHT.

313. It has been found that a given volume or quantity of distilled rain water at a given temperature always weighs the same. Hence, a cubic inch of distilled rain water has been adopted as the standard of weight.

314. *The U. S. standard unit of weight* is the Troy pound of the Mint, which is the same as the Imperial standard pound of Great Britain, and is determined as follows: A cubic inch of distilled water in a vacuum, weighed by brass weights, also in a vacuum, at a temperature of 62° Fahrenheit's thermometer, is equal to 252.458 grains, of which the standard Troy pound contains 5760.

315. *The U. S. Avoirdupois pound* is determined from the standard Troy pound, and contains 7000 Troy grains. Hence, the Troy pound is $\frac{5760}{7000} = \frac{144}{175}$ of an avoirdupois pound. But the Troy ounce contains $\frac{5760}{12} = 480$ grains, and the avoirdupois ounce $\frac{7000}{16} = 437.5$ grains; and an ounce Troy is $480 - 437.5 = 42.5$ grains greater than an ounce avoirdupois. The pound, ounce, and grain, Apothecaries' weight, are the same as the like denominations in Troy weight, the only difference in the two tables being in the divisions of the ounce.

316. COMPARATIVE TABLE OF WEIGHTS.

		Troy.		Avoirdupois.		Apothecaries'.
1 pound	=	5760 grains,	=	7000 grains.	=	5760 grains,
1 ounce	=	480 "	=	437.5 "	=	480 "
		175 pounds,	=	144 pounds.	=	175 pounds,

STANDARD SETS OF WEIGHTS AND MEASURES.

317. A uniform set of weights and measures for all the States was approved by Congress, June 14, 1836, and furnished to the States in 1842. The set furnished consisted of

A yard.

A set of Troy weights.

A set of Avoirdupois weights.

A wine gallon, and its subdivisions.

A half bushel, and its subdivisions.

318. State Sealers of Weights and Measures furnish standard sets of weights and measures to counties and towns.

A County Standard consists of

1. A large balance, comprising a brass beam and scale dishes, with stand and lever.

2. A small balance, with a drawer stand for small weights.

3. A set of large brass weights, namely, 50, 20, 10, and 5 lb.

4. A set of small brass weights, avoirdupois, namely, 4, 2, and 1 lb., 8, 4, 2, 1, $\frac{1}{2}$, and $\frac{1}{4}$ oz.

5. A brass yard measure, graduated to feet and inches, and the first foot graduated to eighths of an inch, and also decimally; with a graduation to cloth measure on the opposite side; in a case.

6. A set of liquid measures, made of copper, namely, 1 gal., $\frac{1}{2}$ gal., 1 qt., 1 pt., $\frac{1}{2}$ pt., 1 gi.; in a case.

7. A set of dry measures, of copper, namely, $\frac{1}{2}$ bu., 1 pk., $\frac{1}{2}$ pk. (or 1 gal.), 2 qt. (or $\frac{1}{2}$ gal.), 1 qt.; in a case.

ENGLISH MEASURES AND WEIGHTS.

GOVERNMENT STANDARDS.

319. The English act establishing standard measures and weights, called "The Act of Uniformity," took effect Jan. 1, 1826, and the standards then adopted, form what is called the *Imperial System.*

320. *The Invariable Standard Unit* of this system is the same as that of the United States, and is described in the Act of Uniformity as follows: "Take a pendulum which will vibrate seconds in London, on a level of the sea, in a vacuum; divide all that part thereof which lies between the axis of suspension and the center of oscillation, into 391393 equal parts; then will 10000 of those parts be an imperial inch, 12 whereof make a foot, and 36 whereof make a yard."

STANDARD OF EXTENSION.

321. *The English Standard Unit of Measures of Extension,* whether linear, superficial, or solid, is identical with that of the United States, (**309**).

STANDARDS OF CAPACITY.

322. *The Imperial Standard Gallon,* for liquids and all dry substances, is a measure that will contain 10 pounds avoirdupois weight of distilled water, weighed in air, at 62° Fahrenheit, the barometer at 30 inches. It contains 277.274 cubic inches.

323. *The Imperial Standard Bushel* is equal to 8 gallons or 80 pounds of distilled water, weighed in the manner above described. It contains 2218.192 cubic inches.

STANDARDS OF WEIGHT.

324. *The Imperial Standard Pound* is the pound Troy, which is identical with that of the United States Standard Troy pound of the Mint, (**314.**)

325. *The Imperial Avoirdupois Pound* contains 7000 Troy grains, and the Troy pound 5760. It also is identical with the United States avoirdupois pound.

TABLES.

326. The denominations in the standard tables of measures of extension, capacity, and weights, are the same in Great Britain and the United States. But some denominations in several of the tables are in use in various parts of Great Britain that are not known in the United States.

These denominations are retained in use by common consent, and are recognized by the English common law. They are as follows:

327. MEASURES OF EXTENSION.

18 inches	make 1 cubit.
45 inches or 5 quarters of the standard yard	" 1 ell.

NOTE.—The cubit was originally the length of a man's forearm and hand; or the distance from the elbow to the end of the middle finger.

328. MEASURES OF CAPACITY.

LIQUID MEASURES.

9 old ale gallons	make	1 firkin.
4 firkins	"	1 barrel of beer.
$7\frac{1}{2}$ Imperial "	"	1 firkin.
$52\frac{1}{2}$ Imperial gallons or 63 wine "	"	1 hogshead.
70 Imperial gallons or 84 wine "	"	1 puncheon or $\frac{1}{3}$ of a tun.
2 hogsheads, that is 105 Imperial gallons or 126 wine "	"	1 pipe.
2 pipes	"	1 tun.

Pipes of wine are of different capacities, as follows:

110	wine gallons	make	1	pipe of Madeira.
120	"	"	1	" Barcelona, Vidonia, or Teneriffe.
130	"	"	1	" Sherry.
138	"	"	1	" Port.
140	"	"	1	" Bucellas, or Lisbon.

329. DRY MEASURE.

8 bushels of 70 pounds each	make	1 quarter of wheat.
36 " heaped measure,	"	1 chaldron of coal.

NOTE.—The quarter of wheat is 560 pounds, or $\frac{1}{4}$ of a ton of 2240 pounds.

330. WEIGHTS.

8 pounds of butchers' meat	make	1 stone.
14 " " other commodities	"	1 " or $\frac{1}{8}$ of a cwt.
2 stone, or 28 pounds	"	1 todd of wool.
70 pounds of salt	"	1 bushel.

NOTE.—The English quarter is 28 pounds, the hundred weight is 112 pounds, and the ton is 20 hundred weight, or 2240 pounds.

FRENCH MEASURES AND WEIGHTS.

GOVERNMENT STANDARDS.

331. The tables of standard measures and weights adopted by the French Government are all formed upon a decimal scale, and constitute what is called the *French Metrical System.*

332. *Invariable Standard Unit.* The French metrical system has, for its unit of all measures, whether of length, area, solidity, capacity, or weight, a uniform invariable standard, adopted from nature and called the *mètre.* It was determined and established as follows: a very accurate survey of that portion of the terrestrial meridian, or north and south circle, between Dunkirk and Barcelona, France, was made, under the direction of Government, and from this measurement the exact length of a quadrant of the entire meridian, or the distance from the equator to the north pole, was computed. The ten millionth part of this arc was denominated a *mètre,* and from this all the standard units of measure and weight are derived and determined.

STANDARDS OF EXTENSION.

333. *The French Standard Linear Unit* is the mètre.

334. *The French Standard Unit of Area* is the *Are,* which is a unit 10 mètres square, and contains 100 square mètres.

335. *The French Standard Unit of Solidity and Capacity* is the *Litre,* which is the cube of the tenth part of the mètre.

STANDARD OF WEIGHT.

336 *The French Standard Unit of Weight* is the *Gramme,* which is determined as follows: the weight in a vacuum of a cubic decimetre or litre of distilled water, at its maximum density, was called a *kilogramme,* and the thousandth part of this was called a *Gramme,* and was declared to be the unit of weight.

NOMENCLATURE OF THE TABLES.

337. It has already been remarked, (**331**), that the tables are all formed upon a decimal scale. The names of the multiples and divisors of the Government standard units in the tables are formed, by combining the names of the standard units with prefixes; the names of the multiples being formed by employing the prefixes *deca,* (ten), *hecto,* (hundred), *kilo,* (thousand), and *myria,* (ten thousand), taken from the Greek numerals; and the names of the divisors by employing the prefixes *deci,* (tenth), *centi,* (hundredth), *mili*, (thousandth), from the Latin numerals. Hence the name of any denomination indicates whether a unit of that denomination is greater or less than the standard unit of the table.

338. I. French Linear Measure.

TABLE.

10	millimetres	make	1 centimetre.
10	centimetres	"	1 decimetre.
10	decimetres	"	1 metre.
10	metres	"	1 decametre.
10	decametres	"	1 hectometre.
10	hectometres	"	1 kilometre.
10	kilometres	"	1 myriametre.

Notes.—1. The metre is equal to 39.3685 inches, the standard rod of brass on which the former is measured being at the temperature of 32° Fahrenheit, and the English standard brass yard or "Scale of Troughton" at 62°. Hence, a metre is equal to 3.2807 feet English measure.

2. The length of a metre being 39.3685 inches, and of a clock pendulum vibrating seconds at the level of the sea in the latitude of London 39.1393 inches, the two standards differ only .2292, or less than ¼ of an inch.

339. II. French Square Measure.

TABLE.

100 square metres, or centiares (10 metres square) make 1 are.
100 ares (10 ares square) " 1 hectoare.

Note.—A square metre or centiare is equal to 1.19589444 square yards, and an are to 119.589444 square yards.

340. III. French Liquid and Dry Measure.

TABLE.

10	decilitres	make	1 litre.
10	litres	"	1 decalitre.
10	decalitres	"	1 hectolitre.
10	hectolitres	"	1 kilolitre.

Notes.—1. A litre is equal to 61.53294 cubic inches, or 1.06552 quarts of a U. S. liquid gallon.

2. A table of Solid or Cubic Measure is also in use in some parts of France, although it is not established or regulated by government enactments or decrees. The unit of this table is a cubic metre, which is equal to 61532.94238 cubic inches, or 35.60934 cubic feet. This unit is called a *Stere*.

TABLE.

10	decisteres	make	1 stere.
10	steres	"	1 decastere.

341. IV. French Weight.

TABLE.

10 milligrammes	make	1 centigramme.
10 centigrammes	"	1 decigramme.
10 decigrammes	"	1 gramme.
10 grammes	"	1 decagramme.
10 decagrammes	"	1 hectogramme.
10 hectogrammes	"	1 kilogramme.
100 kilogrammes	"	1 quintal.
10 quintals	"	1 millier, or 1 ton of sea water.

Notes.—1. A gramme is equal to 15.433159 Troy grains.

2. A kilogramme is equal to 2 lb. 8 oz. 3 pwt. 1.159 gr. Troy, or 2 lb. 3 oz. 4.1549 dr. Avoirdupois.

342. Comparative Table of the United States, English, and French Standard Units of Measures and Weights.

	United States.	English.	French.
Extension,	Yd. of 3 ft., or 36 in.	Same as U. S.	Metre, 39.3685 in.
Capacity,	Wine gal., 231 cu. in.	Imp'l gal., 277.274 cu. in.	Litre, 61.53294 cu. in.
	Winch'r bu., 2150.42 cu. in.	Imp'l bu., 2218.192 cu. in.	
Weight,	Troy lb., 5760 gr.	Imperial lb., 5760 gr.	Gramme, 15.433159 T. gr.

Notes.—1. An Imperial gallon is equal to 1.2 wine gallons.

2. An old ale or beer gallon is very nearly 1.221 wine gallons, or 1.017 Imperial gallons.

3. In ordinary computations 2150.4 cu. in. may be taken as a Winchester bushel, and 2218.2 cu. in. as an Imperial bushel.

MONEY AND CURRENCIES.

343. Money is the commodity adopted to serve as the universal equivalent or measure of value of all other commodities, and for which individuals readily exchange their surplus products or their services.

344. Coin is metal struck, stamped, or pressed with a die, to give it a legal, fixed value, for the purpose of circulating as money.

Note.—The coins of civilized nations consist of gold, silver, copper, and nickel.

345. A **Mint** is a place in which the coin of a country or government is manufactured.

Note.—In all civilized countries mints and coinage are under the exclusive direction and control of government.

346. **An Alloy** is a metal compounded with another of greater value. In coinage, the less valuable or *baser metal* is not reckoned of any value.

NOTE.—Gold and silver, in their pure state, are too soft and flexible for coinage; hence they are hardened by compounding them with an alloy of baser metal, while their color and other valuable qualities are not materially impaired.

347. An **Assayer** is a person who determines the composition and consequent value of alloyed gold and silver.

The fineness of gold is estimated by carats, as follows:—

Any mass or quantity of gold, either pure or alloyed, is divided into 24 equal parts, and each part is called a *carat.*

Fine gold is pure, and is 24 carats fine.

Alloyed gold is as many carats fine as it contains parts in 24 of fine or pure gold. Thus, gold 20 carats fine contains 20 parts or carats of fine gold, and 4 parts or carats of alloy.

348. An **Ingot** is a small mass or bar of gold or silver, intended either for coinage or exportation. Ingots for exportation usually have the assayer's or mint value stamped upon them.

349. **Bullion** is uncoined gold or silver.

350. **Bank Bills** or **Bank Notes** are bills or notes issued by a banking company, and are payable to the bearer in gold or silver, at the bank, on demand. They are substitutes for coin, but are not legal tender in payment of debts or other obligations.

351. **Treasury Notes** are notes issued by the General Government, and are payable to the bearer in gold or silver, at the general treasury, at a specified time.

352. **Currency** is coin, bank bills, treasury notes, and other substitutes for money, employed in trade and commerce.

353. A **Circulating Medium** is the currency or money of a country or government.

354. A **Decimal Currency** is a currency whose denominations increase and decrease according to the decimal scale.

I. UNITED STATES MONEY.

355. The currency of the United States is decimal currency, and is sometimes called *Federal Money.*

The unit is the dollar, and all the other denominations are either divisors or multiples of this unit.

TABLE.

10 mills (m.)	make	1 cent,	ct.
10 cents	"	1 dime,	d.
10 dimes	"	1 dollar,	$.
10 dollars	"	1 eagle,	E.

UNIT EQUIVALENTS.

E.	$	d.	ct.	m.
			1 =	10
		1 =	10 =	100
	1 =	10 =	100 =	1000
1 =	10 =	100 =	1000 =	10000

SCALE — uniformly 10.

NOTES. — 1. Federal Money was adopted by Congress in 1786.
2. The character $ is supposed to be a contraction of U. S., (United States,) the U being placed upon the S.

COINS. *The gold coins* are the double eagle, eagle, half eagle, quarter eagle, three dollar piece and dollar.

The silver coins are the half and quarter dollar, dime and half dime, and three-cent piece.

The nickel coin is the cent.

NOTES.—1. The following pieces of gold are in circulation, but are not legal coin, viz.: the fifty dollar piece, and the half and quarter dollar pieces.
2. The silver dollar, and the copper cent and half cent, are no longer coined for general circulation.
3. The mill is a denomination used only in computations; it is not a coin.

356. *Government Standard.* By Act of Congress, January 18, 1837, all gold and silver coins must consist of 9 parts (.900) pure metal, and 1 part (.100) alloy. The alloy for gold must consist of equal parts of silver and copper, and the alloy for silver of pure copper.

The three-cent piece is 3 parts ($\frac{3}{4}$) silver, and 1 part ($\frac{1}{4}$) copper. The nickel cent is 88 parts copper and 12 parts nickel.

STATE CURRENCIES.

357. United States money is reckoned in dollars, dimes, cents, and mills, one dollar being uniformly valued in all the States at 100 cents; but in many of the States money is sometimes reckoned in dollars, shillings, and pence.

NOTE. — At the time of the adoption of our decimal currency by Congress, in 1786, the *colonial currency*, or *bills of credit*, issued by the colonies, had depreciated in value, and this depreciation, being unequal in the different colonies, gave rise to the different values of the State currencies; this variation continues wherever the denominations of shillings and pence are in use.

Georgia Currency.

Georgia, South Carolina, ... $1 = 4s. 8d. = 56d.

Canada Currency.

Canada, Nova Scotia, ... $1 = 5s. = 60d.

New England Currency.

New England States, Indiana, Illinois, Missouri, Virginia, Kentucky, Tennessee, Mississippi, Texas, ... } .... $1 = 6s. = 72d.

Pennsylvania Currency.

New Jersey, Pennsylvania, Delaware, Maryland, ... } .... $1 = 7s. 6d. = 90d.

New York Currency.

New York, Ohio, Michigan, North Carolina, ... } .... $1 = 8s. = 96d.

II. CANADA MONEY.

358. The currency of the Canadian provinces is decimal, and the table and denominations are the same as those of the United States money.

NOTE. — The decimal currency was adopted by the Canadian Parliament in 1858, and the Act took effect in 1859. Previous to the latter year the money of Canada was reckoned in pounds, shillings, and pence, the same as in England.

COINS. The new Canadian coins are of silver and copper.

The *silver coins* are the shilling or 20-cent piece, the dime, and half dime.

The *copper coin* is the cent.

NOTE. — The 20-cent piece represents the value of the shilling of the old Canada Currency.

359. *Government Standard.* The silver coins consist of 925 parts (.925) pure silver and 75 parts (.075) copper. That is, they are .925 fine.

NOTE. — The value of the 20-cent piece in United States money is $18\frac{2}{3}$ cents, of the dime $9\frac{1}{3}$ cents, and of the half dime $4\frac{2}{3}$ cents.

III. English Money.

360. English or Sterling Money is the currency of Great Britain.

The unit is the pound sterling, and all the other denominations are divisors of this unit.

TABLE.

4 farthings (far. or qr.)	make	1 penny,	d.
12 pence	"	1 shilling,	s.
20 shillings	"	1 pound or sovereign	£ or sov.

UNIT EQUIVALENTS.

	s.	d.	far.
		1 =	4
£, or sov.	1 =	12 =	48
1 =	20 =	240 =	960

Scale — ascending, 4, 12, 20; descending, 20, 12, 4.

Notes. — 1. Farthings are generally expressed as fractions of a penny; thus, 1 far., sometimes called 1 quarter, (qr.) = $\frac{1}{4}$d.; 3 far. = $\frac{3}{4}$d.

2. The old /, the original abbreviation for shillings, was formerly written between shillings and pence, and *d*, the abbreviation for pence, was omitted. Thus 2s. 6d. was written 2/6. A straight line is now used in place of the /, and shillings are written on the left of it and pence on the right. Thus, 2/6, 10/3, etc.

Coins. The *gold coins* are the sovereign (= £1) and the half sovereign, (= 10s.)

The *silver coins* are the crown (= 5s.), the half crown (= 2s. 6d.), the shilling, and the 6 penny piece.

The *copper coins* are the penny, half penny, and farthing.

Note.—The guinea (= 21s.) and the half guinea (= 10s. 6d. sterling) are old gold coins, that are still in circulation, but are no longer coined.

361. *Government Standard.* The standard fineness of English gold coin is 11 parts pure gold and 1 part alloy; that is, it is 22 carats fine. The standard fineness of silver coin is 11 oz. 2 pwt. (= 11.1 oz.) pure silver to 18 pwt. (= .9 oz.) alloy. Hence the silver coins are 11 oz. 2 pwt. fine; that is, 11 oz. 2 pwt. pure silver in 1 lb. standard silver.

This standard is 37 parts ($\frac{37}{40}$ = .925) pure silver and 3 parts ($\frac{3}{40}$ = .075) copper.

Note. — A pound of English standard gold is equal in value to 14.2878 lb. = 14 lb. 3 oz. 9 pwt. 1.727 gr. of silver.

IV. French Money.

362. The currency of France is decimal currency.

The unit is the franc, of which the other denominations are divisors.

TABLE.

10 millimes make 1 centime.
100 centimes " 1 franc.

Scale — ascending, 10, 100; descending, 100,10.

Coins. The *gold coin* is the 20-franc piece, or Louis.

The *silver coins* are the franc and the demi franc.

Note.—In France accounts are kept in francs and decimes. A franc is equal to 18.6 cents U. S. money.

363. COMPARATIVE TABLE OF MONEYS.

English.		U. S.	French.		U. S.
1qr.	=	$ $.00\frac{121}{240}$	1 millime	=	$ .000186
1d.	=	$\cdot 02\frac{1}{60}$	1 centime	=	.00186
1s.	=	.242	1 franc	=	.186
4s. 1d. $2\frac{42}{121}$qr.	=	1.00			
£1	=	4.84			

REDUCTION.

364. Reduction is the process of changing a number from one denomination to another without altering its value.

Reduction is of two kinds, Descending and Ascending.

365. Reduction Descending is changing a number of one denomination to another denomination of *less unit value;* thus, $1 = 10 dimes = 100 cents = 1000 mills.

366. Reduction Ascending is changing a number of one denomination to another denomination of *greater unit value;* thus, 1000 mills = 100 cents = 10 dimes = $1.

REDUCTION DESCENDING.

CASE I.

367. To reduce a compound number to lower denominations.

1. Reduce 3 mi. 1 fur. 17 rd. 2 yd. 1 ft. 8. in. to inches.

OPERATION.

$$
\begin{array}{rl}
 & 3 \text{ mi. } 1 \text{ fur. } 17 \text{ rd. } 2 \text{ yd. } 1 \text{ ft. } 8 \text{ in.} \\
 & 8 \\
\hline
25 & \text{fur.} \\
40 & \\
\hline
1017 & \text{rd.} \\
5\frac{1}{2} & \\
\hline
5087 & \\
508\frac{1}{2} & \\
\hline
5595\frac{1}{2} & \text{yd.} \\
3 & \\
\hline
16787\frac{1}{2} & \text{ft.} \\
12 & \\
\hline
201458 & \text{in.}
\end{array}
$$

ANALYSIS. Since in 1 mi. there are 8 fur., in 3 miles there are 3×8 fur. $= 24$ fur., and the 1 fur. in the given number, added, makes 25 fur. in 3 mi. 1 fur. Since in 1 fur. there are 40 rd., in 25 fur. there are 25×40 rd. $= 1000$ rd., and the 17 rd. in the given number added, makes 1017 rd. in 3 mi. 1 fur. 17 rd. Since in 1 rd. there are $5\frac{1}{2}$ yd., in 1017 rd. there are $1017 \times 5\frac{1}{2}$ yd. $= 5593\frac{1}{2}$ yd., which plus the 2 yd. in the given number $= 5595\frac{1}{2}$ yd. in 3 mi. 1 fur. 17 rd. 2 yd. Since in 1 yd. there are 3 ft., in $5595\frac{1}{2}$ yd. there are $5595\frac{1}{2} \times 3$ ft. $= 16786\frac{1}{2}$ ft., which plus the 1 ft. in the given number $= 16787\frac{1}{2}$ ft. in 3 mi. 1 fur. 17 rd. 2 yd. 1 ft. And since in 1 ft. there are 12 in., in $16787\frac{1}{2}$ ft. there are $16787\frac{1}{2} \times 12$ in. $= 201450$ in., which plus the 8 in. in the given number $= 201458$ in. in the given compound number. On examining the operation, we find that we have successively multiplied by the numbers in the descending scale of linear measure from miles to inches, inclusive. But, as either factor may be used as a multiplicand, (**82**, I), we may consider the numbers in the descending scale as multipliers. Hence the following

RULE. I. *Multiply the highest denomination of the given compound number by that number of the scale which will reduce it to the next lower denomination, and add to the product the given number, if any, of that lower denomination.*

II. *Proceed in the same manner with the results obtained in each lower denomination, until the reduction is brought to the denomination required.*

EXAMPLES FOR PRACTICE.

1. In 16 lb. 10 oz. 18 pwt. 5 gr., how many grains?

2. In £133 6 s. 8d., how many farthings? *Ans.* 128,000.

3. Change 100 mi. to inches. *Ans.* 6336000 in.

4. How many rods of fence will inclose a farm $1\frac{1}{2}$ miles square? *Ans.* 1920 rd.

5. The grey limestone of Central New York weighs 175 lbs. to the cubic foot; what is the weight of a block 8 ft. long and 1 yd. square? *Ans.* 6 T. 6 cwt.

6. What will be the cost of 1 hhd. of molasses at \$.28 per gal.?

7. A man wishes to ship 1548 bu. 1 pk. of potatoes in barrels containing 2 bu. 3 pk. each; how many barrels must he obtain?

8. A grocer bought 10 bu. of chestnuts at \$3.75 a bushel, and retailed them at \$.06$\frac{1}{4}$ a pint; how much was his whole gain?

9. Reduce 90° 17′ 40″ to seconds. *Ans.* 325060″.

10. In the 18th century how many days? *Ans.* 36524 da.

11. At $6\frac{1}{4}$ cts. each, what will be the cost of a great-gross of writing books? *Ans.* \$108.

12. How large an edition of an octavo book can be printed from 4 bales 4 bundles 1 ream 10 quires of paper, allowing 8 sheets to the volume? *Ans.* 2970 vol.

13. Suppose your age to be 18 yr. 24 da.; how many minutes old are you, allowing 4 leap years to have occurred in that time?

14. How many pence in 481 sovereigns? *Ans.* 115,440 d.

15. Reduce \$$7\frac{3}{8}$ to mills. *Ans.* 7375 mills.

16. In 3 P. of Sherry wine, how many qt.? *Ans.* 1560 qt.

17. Reduce 37 Eng. ells 1 qr. to yd. *Ans.* 46 yd. 2 qr.

18. In £6 10s. 10d. how many dollars U. S. currency?

19. Reduce 6 O. 14f℥ 3fʒ 45♏ to minims.

20. Reduce 1 T. 1 P. 1 hhd. to Imperial gallons.

Ans. $367\frac{1}{2}$ Imperial gal.

21. How many dollars Canada currency are equal to £126 12s. 6d.? *Ans.* \$$506\frac{1}{2}$.

22. How many pint, quart, and two-quart bottles, of each an equal number, may be filled from a hogshead of wine?

Ans. 72.

23. How many steps of 2 ft. 9 in. each, will a man take, in walking from Erie to Cleveland, the distance being 95 mi.?

24. A grocer bought 12 bbl. of cider at $\$1\frac{3}{4}$ a barrel, and after converting it into vinegar, he retailed it at 6 cents a quart; how much was his whole gain? *Ans.* \$69.72.

25. In 75 A. 4 sq. ch. 18 P. 118 sq. l. how many square links?

26. How many inches high is a horse that measures 16 hands?

27. If a vessel sail 150 leagues in a day, how many statute miles does she sail? *Ans.* 517.5.

28. If 14 A. be sold from a field containing 50 A., how many square rods will the remainder contain? *Ans.* 5,760 sq. rd.

29. A man returning from Pike's Peak has 36 lb. 8 oz. of pure gold; what is its value at $\$1.04\frac{1}{5}$ per pwt.? *Ans.* \$9169.60.

30. A person having 8 hhd. of tobacco, each weighing 9 cwt. 42 lb., wishes to put it into boxes containing 48 lb. each; how many boxes must he obtain? *Ans.* 157.

31. A merchant bought 12 bbl. of salt at $\$1\frac{1}{4}$ a barrel, and retailed it at $\frac{3}{4}$ of a cent a pound; how much was his whole gain?

32. A physician bought 1℔ 10℥ of quinine at \$2.25 an ounce, and dealt it out in doses of 10 gr. at $\$.12\frac{1}{2}$ each; how much more than cost did he receive? *Ans.* \$82.50.

CASE II.

368. To reduce a denominate fraction from a greater to a less unit.

1. Reduce $\frac{1}{44}$ of a gallon to the fraction of a gill.

OPERATION.

$$\tfrac{1}{44}\text{ gal.} \times \tfrac{4}{1} \times \tfrac{2}{1} \times \tfrac{4}{1} = \tfrac{8}{11}\text{ gi.}$$

Or,

$$\begin{array}{r|l} {}^{11}\cancel{44} & 1 \\ & 4 \\ & 2 \\ & \cancel{4} \\ \hline 11 & 8 = \tfrac{8}{11}\text{ gi., } Ans. \end{array}$$

ANALYSIS. To reduce gallons to gills, we multiply successively by 4, 2, and 4, the numbers in the descending scale. And since the given number is a fraction, we indicate the process, as in multiplication of fractions, after which we perform the indicated operations, and obtain $\frac{3}{11}$, the answer. Hence, RULE. *Multiply the fraction of the higher denomination by the numbers in the descending scale successively, between the given and the required denomination.*

NOTE. — Cancellation may be applied wherever practicable.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{1}{300}$ of a lb. Troy to the fraction of a pennyweight. *Ans.* $\frac{4}{5}$ pwt.
2. Reduce $\frac{1}{672}$ of a hhd. to the fraction of a pint.
3. Reduce $\frac{1}{2112}$ of a mile to the fraction of a yard. *Ans.* $\frac{5}{6}$ yd.
4. Reduce $\frac{9}{342}$ of a gallon to the fraction of a gill.
5. What part of a dram is $\frac{1}{5000}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{6}{11}$ of $3\frac{1}{7}$ pounds avoirdupois weight? *Ans.* $\frac{192}{4375}$ dr.
6. Reduce $\frac{1}{18000}$ of a dollar to the fraction of a cent.
7. Reduce $\frac{1}{45}$ of a rod to the fraction of a link. *Ans.* $\frac{5}{9}$ l.
8. Reduce $\frac{1}{35}$ of a scruple to the fraction of a grain.
9. What fraction of a yard is $\frac{6}{7}$ of $\frac{4}{11}$ of a rod?
10. $\frac{5}{18}$ of a week is $\frac{2}{9}$ of how many days? *Ans.* $8\frac{3}{4}$ da.
11. What fraction of a square rod is $\frac{3}{1360}$ of $4\frac{1}{4}$ times $\frac{2}{19}$ of an acre? *Ans.* $\frac{3}{19}$ sq. rd.

CASE III.

369. To reduce a denominate fraction to integers of lower denominations.

1. What is the value of $\frac{2}{5}$ of a bushel?

OPERATION.

$\frac{2}{5}$ bu. $\times 4 = \frac{8}{5}$ pk. $= 1\frac{3}{5}$ pk.
$\frac{3}{5}$ pk. $\times 8 = \frac{24}{5}$ qt. $= 4\frac{4}{5}$ qt.
$\frac{4}{5}$ qt. $\times 2 = \frac{8}{5}$ pt. $= 1\frac{3}{5}$ pt.
1 pk. 4 qt. $1\frac{3}{5}$ pt., *Ans.*

ANALYSIS. $\frac{2}{5}$ bu. $= \frac{2}{5}$ of 4 pk., or $1\frac{3}{5}$ pk.; $\frac{3}{5}$ pk. $= \frac{3}{5}$ of 8 qt. $= 4\frac{4}{5}$ qt.; and $\frac{4}{5}$ qt. $= \frac{4}{5}$ of 2 pt. $= 1\frac{3}{5}$ pt. The units, 1 pk., 4 qt., 1 pt., with the last denominate fraction, $\frac{3}{5}$ pt., form the answer. Hence,

RULE. I. *Multiply the fraction by that number in the scale which will reduce it to the next lower denomination, and if the result be an improper fraction, reduce it to a whole or mixed number.*

II. *Proceed with the fractional part, if any, as before, until reduced to the denominations required.*

III. *The units of the several denominations, arranged in their order, will be the required result.*

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{9}{10}$ of a yard to integers of lower denominations.
   Ans. 2 ft. $8\frac{2}{5}$ in.
2. Reduce $\frac{4}{5}$ of a month to lower denominations.
3. Reduce $\frac{607}{640}$ of a short ton to lower denominations.
4. What is the value of $\frac{5}{9}$ of a long ton?
   Ans. 11 cwt. 12 lb. $7\frac{1}{9}$ oz.
5. What is the value of $\frac{3}{8}$ of $2\frac{1}{2}$ pounds apothecaries' weight?
6. What is the value of $\frac{7}{13}$ of an acre?
   Ans. 2 R. 6 P. 4 sq. yd. 5 sq. ft. $127\frac{5}{13}$ sq. in.
7. Reduce $\frac{3}{7}$ of a mile to integers of lower denominations.
8. What is the value of $\frac{4}{7}$ of a great gross?
   Ans. 6 gross 10 doz. $3\frac{3}{7}$.
9. What is the value in geographic miles of $\frac{9}{16}$ of a great circle?
   Ans. 12150 mi.
10. What is the value of $\frac{4}{5}$ of $3\frac{3}{8}$ cords of wood?
    Ans. 2 Cd. 5 cd. ft. $9\frac{3}{5}$ cu. ft.
11. The distance from Buffalo to Cincinnati is 438 miles; having traveled $\frac{2}{5}$ of this distance, how far have I yet to travel?
    Ans. 262 mi. 6 fur. 16 rd.
12. What is the value of $\frac{43}{96}$ f℥? *Ans.* 3 fʒ 35 ♏.
13. What is the value of $\frac{3}{7}$ of a sign?
    Ans. $12° 51' 25\frac{5}{7}''$.
14. A man having a hogshead of wine, sold $\frac{6}{13}$ of it; how much remained? *Ans.* 33 gal. 3 qt. 1 pt. $1\frac{7}{13}$ gi.

CASE IV.

370. To reduce a denominate decimal to integers of lower denominations.

1. Reduce .125 of a barrel to integers of lower denominations.

OPERATION.

$$
\begin{array}{r}
.125 \\
31.5 \\
\hline
3.9375 \text{ gal.} \\
4 \\
\hline
3.7500 \text{ qt.} \\
2 \\
\hline
1.50 \text{ pt.} \\
4 \\
\hline
2.0 \text{ gi.}
\end{array}
$$

3 gal. 3 qt. 1 pt. 2 gi., *Ans.*

ANALYSIS. We first multiply the given decimal, .125 of a barrel, by 31.5 (= $31\frac{1}{2}$) to reduce it to gallons, and obtain 3.9375 gallons. Omitting the 3 gallons, we multiply the decimal, .9375 gal., by 4 to reduce it to quarts, and obtain 3.75 quarts. We next multiply the decimal part of this result by 2, to reduce it to pints, and obtain 1.5 pints. And the decimal part of this result we multiply by 4 to reduce it to gills, and obtain 2 gills. The integers of the several denominations, arranged in their order, form the answer. Hence,

RULE. I. *Multiply the given denominate decimal by that number in the descending scale which will reduce it to the next lower denomination, and point off the result as in multiplication of decimals.*

II. *Proceed with the decimal part of the product in the same manner until reduced to the required denominations. The integers at the left will be the answer required.*

EXAMPLES FOR PRACTICE.

1. What is the value of .645 of a day?
Ans. 15 h. 28 min. 48 sec.
2. What is the value of .765 of a pound Troy?
3. What is the value of .6625 of a mile?
4. What is the value of .8469 of a degree?
Ans. 50′ 48.84″.
5. What is the value of .875 of a hhd.?
6. What is the value of £.85251?
Ans. 17 s. 2.4 + far.
7. What is the value of .715°? *Ans.* 42′ 54″.
8. What is the value of 7.88125 acres?
Ans. 7 A. 3 R. 21 P.
9. What is the value of .625 of a fathom? *Ans.* $3\frac{3}{4}$ ft.

10. What is the value of .375625 of a barrel of pork?

11. What is the value of .1150390625 Cong.?

Ans. 14 f℥ 5 fʒ 48 ♏.

12. What is the value of .61 of a tun of wine?

Ans. 1 P. 27 gal. 2 qt. 1 pt. 3.04 gi.

REDUCTION ASCENDING.

CASE I.

371. To reduce a denominate number to a compound number of higher denominations.

1. Reduce 157540 minutes to weeks.

OPERATION.

60) 157540 min.
24) 2625 h. + 40 min.
7) 109 da. + 9 h.
15 wk. + 4 da.

15 wk. 4 da. 9 h. 40 min., *Ans.*

ANALYSIS. Dividing the given number of minutes by 60, because there are $\frac{1}{60}$ as many hours as minutes, and we obtain 2645 h. plus a remainder of 40 min. We next divide the 2645 h. by 24, because there are $\frac{1}{24}$ as many days as hours, and we find that 2645 h. = 109 da. plus a remainder of 9 h. Lastly we divide the 109 da. by 7, because there are $\frac{1}{7}$ as many weeks as days, and we find that 109 da. = 15 wk. plus a remainder of 4 da. The last quotient and the several remainders annexed in the order of the succeeding denominations, form the answer.

2. Reduce 201458 inches to miles.

OPERATION.

12) 201458 in.
3) 16788 ft. 2 in.
5½ or 5.5) 5596 yd.
40) 1017 rd. 2 yd. 1 ft. 6 in.
8) 25 fur. 17 rd.
3 mi. 1 fur.

3 mi. 1 fur. 17 rd. 2 yd. 1 ft. 8 in., *Ans.*

ANALYSIS. We divide successively by the numbers in the ascending scale of linear measure, in the same manner as in the last preceding operation. But, in dividing the 5596 yd. by 5½ or 5.5, we have a remainder of $2\frac{1}{2}$ yd., and this reduced to its equivalent compound number, (**369**) = 2 yd. 1 ft. 6 in. In forming our final result, the 6 in. of this number are added to the first remainder, 2 in., making the 8 in. as given in the answer. From these examples and analyses we deduce the following

RULE. I. *Divide the given concrete or denominate number by that number of the ascending scale which will reduce it to the next higher denomination.*

II. *Divide the quotient by the next higher number in the scale; and so proceed to the highest denomination required. The last quotient, with the several remainders annexed in a reversed order, will be the answer.*

NOTE.—The several corresponding cases in reduction descending and reduction ascending, being opposites, mutually prove each other.

EXAMPLES FOR PRACTICE.

1. Reduce 1913551 drams to tons.
2. In 97920 gr. of medicine how many lb.? *Ans.* 17 lb.
3. Reduce 1000000 in. to mi.
4. How many acres in a field 120 rd. long and 56 rd. wide?
5. In a pile of wood 60 ft. long, 15 ft. wide, and 10 ft. high, how many cords? *Ans.* 70 Cd. 2 cd. ft. 8 cu. ft.
6. How many fathoms deep is a pond that measures 28 ft. 6 in.? *Ans.* $4\frac{3}{4}$ fath.
7. In 30876 gi. how many hhd.?
8. How many bushels of corn in 27072 qt.? *Ans.* 846 bu.
9. At 2 cts. a gill, how much alcohol may be bought for $2.54?
10. In 1234567 far. how many £? *Ans.* £1286 $1\frac{3}{4}$ d.
11. Reduce 2468 pence to half crowns.
12. In $88.35 how many francs? *Ans.* 475.
13. In 622080 cu. in. how many tons of round timber?
14. In 84621 ♍ how many Cong.?
15. If 135 million Gillott steel pens are manufactured yearly, how many great-gross will they make? *Ans.* 78125.
16. Reduce 1020300″ to S. *Ans.* 9 S. 13° 25′
17. In 411405 sec. how many da.?

18. During a storm at sea, a ship changed her latitude 412 geographic miles; how many degrees and minutes did she change? *Ans.* 6° 52′.

19. If a man travel at the rate of a minute of distance in 20 minutes of time, how much time would he require to travel round the earth? *Ans.* 300 days.

20. In 120 gross how many score? *Ans.* 864.

21. How many miles in the semi-circumference of the earth?

22. How much time will a person gain in 36 yr. by rising 45 min. earlier, and retiring 25 min. later, every day, allowing for 9 leap years? *Ans.* 639 da. 4 h. 30 min.

23. A grocer bought 20 gal. of milk by beer measure, and sold it by wine measure; how many quarts did he gain? *Ans.* $17\frac{5}{7}\frac{1}{7}$.

24. How many bushels of oats in Connecticut are equivalent to 1500 bushels in Iowa? *Ans.* 1875 bu.

25. Reduce 120 leagues to statute miles. *Ans.* 414 mi.

26. In 1 bbl. 1 gal. 2 qt. wine measure, how many beer gallons? *Ans.* $27\frac{3}{94}$.

27. Reduce 150 U. S. bushels to Imperial bushels. *Ans.* 145.415 + Imp'l. bu.

28. How many squares in a floor 68 ft. 8 in. long, and 33 ft. wide? *Ans.* $22\frac{33}{50}$.

29. How many cubic inches in a solid 4 ft. long 3 ft. wide, and 1 ft. 6 in. thick?

30. How many acres in a field 120 rd. long and 56 rd. wide?

31. Change 356 dr. apothecaries weight, to Troy weight.

32. A coal dealer bought 175 tons of coal at $3.75 by the long ton, and sold it at $4.50 by the short ton; how much was his whole gain? *Ans.* $225.75.

33. How many acres of land can be purchased in the city of New York for $73750, at $1.25 a square foot? *Ans.* 1 A. 56 P. 194 sq. ft.

34. An Ohio farmer sold a load of corn weighing 2492 lb., and a load of wheat weighing 2175 lb.; for the corn he received $.60 a bushel, and for the wheat $1.20 a bushel; how much did he receive for both loads? *Ans.* $70.20.

The following examples are given to illustrate a short and practical method of reducing currencies.

35. What will be the cost of 54 bu. of corn at 5s. a bushel, New England currency?

OPERATION.

$$54 \times 5 = 270\text{s.}$$
$$270\text{s.} \div 6 = \$45$$

Or,
$$\begin{array}{r|l} & 9 \\ & \cancel{54} \\ \cancel{6} & 5 \\ \hline & \$45 \end{array}$$

ANALYSIS. Since 1 bu. costs 5s., 54 bu. cost 54×5s. = 270s.; and since 6s. make \$1 N. E. currency, 270 ÷ 6 = \$45, *Ans.*

36. What will 270 bu. of wheat cost, @ 8s. 4d. Penn. currency?

OPERATION.

$$\begin{array}{r|l} & 3 \\ & \cancel{270} \\ \cancel{90} & 100 \\ \hline & \$300 \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} & 6 \\ & \cancel{270} \\ \cancel{3} & 25 \\ \cancel{15} & 2 \\ \hline & \$300 \end{array}$$

ANALYSIS. Multiply the quantity by the price in Penn. currency, and divide the cost by the value of \$1 in the same currency; or reduce the shillings and pence to a fraction of a shilling, before multiplying and dividing.

37. Bought 5 hhd. of rum at the rate of 2s. 4d. a quart, Georgia currency; how much was the whole cost?

OPERATION.

$$\begin{array}{r|l} & 5 \\ & 63 \\ & 2 \\ & \cancel{4} \\ \cancel{56} & \cancel{28} \\ \hline & \$630 \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} & 5 \\ & 63 \\ & 2 \\ & \cancel{4} \\ \cancel{3} & \cancel{7} \\ \cancel{14} & \cancel{3} \\ \hline & \$630 \end{array}$$

ANALYSIS. In this example we first reduce 5 hhd. to quarts by multiplying by 63 and 4, and then proceed as in the preceding examples.

38. Sold 120 barrels of apples, each containing 2 bu. 2 pk., at 4s. 7d. a bushel, and received pay in cloth at 10s. 5d. a yard; how many yards of cloth did I receive?

OPERATION.

$$\begin{array}{r|l} & 12 \\ & \cancel{120} \\ \cancel{2} & \cancel{5} \\ & 11 \\ \cancel{125} & \cancel{55} \\ \hline & \$132 \end{array}$$

ANALYSIS. The operation in this example is similar to the preceding examples, except that we divide the *cost* of the apples by the *price of a unit* of the article received in payment, reduced to units of the same denomination as the price of a *unit* of the article sold. The result will be the same in whatever currency.

39. What cost 75 yards of flannel at 3s. 6d. per yard, New England currency? *Ans.* \$43.75.

40. A man in Philadelphia worked 5 weeks at 6s. 4d. a day; how much did his wages amount to? *Ans.* \$25.33$\frac{1}{3}$.

41. A farmer exchanged 2 bushels of beans worth 10s. 6d. per bushel, for two kinds of sugar, the one at 10d. and the other at 11d. per pound, taking the same quantity of each kind; how many pounds of sugar did he receive? *Ans.* 24 lb.

42. If corn be rated at 5s. 10d. per bushel in Vermont, at what price in the currency of New Jersey must it be sold, in order to gain \$7.50 on 54 bushels?

CASE II.

372. To reduce a denominate fraction from a less to a greater unit.

1. Reduce $\frac{8}{11}$ of a gill to the fraction of a gallon.

OPERATION.

$$\frac{8}{11} \text{ gi.} \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{44} \text{ gal.}$$

Or,

$$\begin{array}{r|l} 11 & \cancel{8} \\ 4 & \\ \cancel{2} & \\ \cancel{4} & \\ \hline 44 & 1 = \frac{1}{44}, \textit{ Ans.} \end{array}$$

ANALYSIS. To reduce gills to gallons, we divide successively by 4, 2, and 4, the numbers in the ascending scale. And since the given number is a fraction, we indicate the process, as in division of fractions, after which we perform the indicated operations, and obtain $\frac{1}{44}$, the answer. Hence,

RULE. *Divide the fraction of the lower denomination by the numbers in the ascending scale successively, between the given and the required denomination.*

NOTE.—The operation may frequently be shortened by cancellation.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{2}{9}$ of a shilling to the fraction of a pound. *Ans.* £$\frac{1}{90}$.

2. Reduce $\frac{5}{7}$ of a pennyweight to the fraction of a pound Troy. *Ans.* $\frac{1}{336}$ lb.

3. What part of a ton is $\frac{4}{5}$ of a pound avoirdupois weight?

4. What fraction of an hour is $\frac{4}{9}$ of 20 seconds?

5. What is the fractional difference between $\frac{1}{680}$ of a hhd. and $\frac{3}{5}$ of a pt.? *Ans.* $\frac{1}{3570}$ hhd.

6. $\frac{224}{3}$ of $\frac{1}{3}$ of $\frac{2}{7}$ of a pint is what fraction of 2 pecks? *Ans.* $\frac{2}{9}$

7. Reduce $\frac{2}{3}$ of $\frac{4}{7}$ of $\frac{9}{14}$ of a cord foot to the fraction of a cord. *Ans.* $\frac{3}{98}$ Cd.

8. What part of an acre is $\frac{3}{19}$ of $\frac{4}{17}$ of $9\frac{1}{2}$ square rods?

9. $\frac{3}{4}$ of $5\frac{1}{2}$ furlongs is $\frac{1}{2}$ of $\frac{1}{12}$ of how many miles? *Ans.* $12\frac{3}{8}$ mi.

10. A block of granite containing $\frac{3}{4}$ of $\frac{6}{7}$ of $20\frac{1}{6}$ cubic feet, is what fraction of a perch? *Ans.* $\frac{11}{21}$ Pch.

11. What part of a cord of wood is a pile $7\frac{1}{2}$ ft. long, 2 ft. high, and $3\frac{1}{4}$ feet wide? *Ans.* $\frac{195}{512}$ Cd.

12. Reduce $\frac{5}{7}$ of an inch to the fraction of an Ell English.

CASE III.

373. To reduce a compound number to a fraction of a higher denomination.

1. Reduce 2 oz. 12 pwt. 12 gr. to the fraction of a pound Troy.

OPERATION.

2 oz. 12 pwt. 12 gr. = 1260 gr.
1 lb. Troy = 5760 gr.
$\frac{1260}{5760}$ lb. = $\frac{7}{32}$ lb., *Ans.*

ANALYSIS. To find what part one compound number is of another, they must be like numbers and reduced to the same denomination. In 2 oz. 12 pwt. 12 gr. there are 1260 gr., and in 1 lb. there are 5760 gr. Therefore 1 gr. is $\frac{1}{5760}$ lb., and 1260 gr. are $\frac{1260}{5760}$ lb. = $\frac{7}{32}$ lb., the answer. Hence,

RULE. *Reduce the given number to its lowest denomination for the numerator, and a unit of the required denomination to the same denomination for the denominator of the required fraction.*

NOTE.—If the given number contain a fraction, the denominator of this fraction must be regarded as the lowest denomination.

EXAMPLES FOR PRACTICE.

1. Reduce 2 R. 20 P. to the fraction of an acre.
Ans. $\frac{5}{8}$ A.

2. What part of a mile is 6 fur. 26 rd. 3 yd. 2 ft.?

3. What part of a £ is 18s. 5d. $2\frac{2}{13}$ far.? *Ans.* £$\frac{12}{13}$.

4. What part of 21 ℔. Apothecaries' weight is 7℥ 7ʒ 2℈ 14 gr.? *Ans.* $\frac{71}{2240}$ ℔.

5. What part of 3 weeks is 4 da. 16 h. 30 min.?

6. Reduce $1\frac{3}{4}$ pecks to the fraction of a bushel.

7. From a hogshead of molasses 28 gal. 2 qt. were drawn; what part of the whole remained in the hogshead? *Ans.* $\frac{23}{42}$.

8. Reduce 4 bundles 6 quires 16 sheets of paper to the fraction of a bale. *Ans.* $\frac{5}{6}$ of a bale.

9. What part of 54 cords of wood is 4800 cubic feet?

10. What is the value of $\dfrac{7\frac{4}{11}}{1\frac{13}{77}}$ of a dollar? *Ans.* \$6.30.

11. Reduce 3O. 3f℥ 1fʒ 36♏ to the fraction of a Cong.

12. What part of a ton of hewn timber is 36 cu ft. 864 cu. in.?

CASE IV.

374. To reduce a compound number to a decimal of a higher denomination.

1. Reduce 3 cd. ft. 8 cu. ft. to the decimal of a cord.

OPERATION.

$$\begin{array}{r|ll} 16 & 8.0 & \text{cu. ft.} \\ \hline 8 & 3.5000 & \text{cd. ft.} \\ \hline & .4375 & \text{Cd., } Ans. \end{array}$$

Or,

3 cd. ft. 8 cu. ft. = 56 cu. ft.
1 Cd. = 128 cu. ft.
$\frac{56}{128}$ Cd. = $\frac{7}{16}$ Cd. = .4375 Cd., *Ans.*

ANALYSIS. We reduce the 8 cu. ft. to the decimal of a cd. ft., by annexing a cipher, and dividing by 16, the number of cu. ft. in 1 cd. ft., annexing the decimal quotient to the 3 cd. ft. We now reduce the 3. 5 cd. ft. to Cd. or a decimal of a Cd., by dividing by 8, the number of cd. ft. in 1 Cd., and we have .4375 Cd., the answer.

Or, we may reduce the 3 cd. ft. 8 cu. ft., to the fraction of a Cd., (as in **373**), and we shall have $\frac{56}{128}$ Cd. = $\frac{7}{16}$ Cd., which, reduced to its equivalent decimal, equals .4375 Cd., the same as before. Hence,

RULE. *Divide the lowest denomination given by that number in the scale which will reduce it to the next higher denomination, and annex the quotient as a decimal to that higher. Proceed in the same manner until the whole is reduced to the denomination required.* Or,

Reduce the given number to a fraction of the required denomination, and reduce this fraction to a decimal.

EXAMPLES FOR PRACTICE.

1. Reduce 5 da. 9 h. 46 min. 48 sec. to the decimal of a week. *Ans.* .7725 wk.
2. Reduce 3° 27′ 46.44″ to the decimal of a sign.
3. Reduce 1 R. 11.52 P. to the decimal of an acre.
4. What part of 4 oz. is 2 oz. 16 pwt. 19.2 gr? *Ans.* .71.
5. What part of a furlong is 28 rd. 2 yd. 1 ft. 11.04 in.?
6. Reduce $3\frac{1}{4}$℥ to the decimal of a pound.
7. Reduce 126 A. 4 sq. ch. 12 P. to the decimal of a township. *Ans.* .0054893 + Tp.
8. What part of a fathom is $3\frac{3}{4}$ ft.? *Ans.* .625 fath.
9. What part of $1\frac{1}{4}$ bushels is .45 of a peck? *Ans.* .09.
10. What part of 3 A. 2 R. is 1 R. 11.52 P.? *Ans.* .092.
11. Reduce $\frac{2}{7}$ of $\frac{1}{2}$ of $22\frac{3}{4}$ lb. to the decimal of a short ton.
12. What part of a f℥ is 5 fʒ 36 ♏? *Ans.* .7 f℥.
13. Reduce 50 gal. 3 qt. 1 pt. to the decimal of a tun. *Ans.* .20188 + T.

ADDITION.

375. Compound numbers are added, subtracted, multiplied, and divided by the same general methods as are employed in simple numbers. The corresponding processes are based upon the same principles; and the only modification of the operations and rules is that required for borrowing, carrying, and reducing by a *varying*, instead of a *uniform scale.*

376. 1. What is the sum of 50 hhd. 32 gal. 3 qt. 1 pt., 2 hhd. 19 gal. 1 pt., 15 hhd. $46\frac{1}{4}$ gal., and 9 hhd. 39 gal. $2\frac{1}{2}$ qt.?

OPERATION.			
hhd.	gal.	qt.	pt.
50	32	3	1
2	19	0	1
15	46	1	0
9	39	2	1
78	11	3	1

ANALYSIS. Writing the numbers so that units of the same denomination shall stand in the same column, we add the numbers of the right hand or lowest denomination, and find the amount to be 3 pints, which is equal to 1 qt. 1 pt. We write the 1 pt. under the column of pints, and add the 1 qt. to the column of quarts. The amount of the numbers of the next higher denomination is 7 qt., which is equal to 1 gal. 3 qt. We write the 3 qt. under the column of quarts, and add the 1 gal. to the column of gallons. Adding the gallons, we find the amount to be 137 gal., equal to 2 hhd. 11 gal. Writing the 11 gal. under the gallons in the given numbers, we add the 2 hhd. to the column of hogsheads. Adding the hogsheads, we find the amount to be 78 hhd., which we write under the left hand denomination, as in simple numbers.

2. What is the sum of $\frac{7}{10}$ wk., $\frac{3}{5}$ da., and $\frac{3}{8}$ h.

OPERATION.

$\frac{7}{10}$ wk. =	4 da.	21 h.	36 min.	
$\frac{3}{5}$ da. =		14 "	24 min.	
$\frac{3}{8}$ h. =			22 "	30 sec.
	5	12	22	30

Or,

$\frac{3}{5}$ da. $\times \frac{1}{7} = \frac{3}{35}$ wk;

$\frac{3}{8}$ h. $\times \frac{1}{24} \times \frac{1}{7} = \frac{1}{56}$ wk;

$\frac{7}{10}$ wk. $+ \frac{3}{35}$ wk. $+ \frac{1}{56}$ wk. $= \frac{11}{16}$ wk;

$\frac{11}{16}$ wk. = 5 da. 12 h. 22 min. 30 sec.

ANALYSIS. We first find the value of each fraction in integers of less denominations, (**369**), and then add the resulting or equivalent compound numbers.

Or, we may reduce the given fractions to fractions of the same denomination, (**368** or **372**), then add them, and find the value of their sum in lower denominations.

377. From these examples and illustrations we derive the following

RULE. I. *If any of the numbers are denominate fractions, or if any of the denominations are mixed numbers, reduce the fractions to integers of lower denominations.*

II. *Write the numbers so that those of the same unit value will stand in the same column.*

III. *Beginning at the right hand, add each denomination as in* *simple numbers, carrying to each succeeding denomination one for as many units as it takes of the denomination added, to make one of the next higher denomination.*

NOTE. — The pupil cannot fail to see that the principles involved in adding compound numbers are the same as those in addition of simple numbers; and that the *only difference* consists in the different carrying units.

EXAMPLES FOR PRACTICE.

(1.)

	lb.	oz.	pwt.	gr.
	14	6	12	13
	17	5	3	12
	15		9	16
	2	7	15	20
	13	2	1	19
	4	1	5	21
Sum,	66	11	9	5

(2.)

℔	℥	ʒ	℈	gr.
10	8	5	1	8
7	7	6	2	13
5	11	7		
21	10			16
12	1	2	2	3
		7	1	19
58	4	5	2	19

(3.)

fur.	rd.	ft.	in.
7	26	11	9
4	16	7	11
	36	14	3
1	9	2	8
5		10	1
6	2	5	
1	15	13	10

(4.)

A.	R.	P.	sq. yd.	sq. ft.
140	3	17	27	6
320	1	30	14	2
111		7	3	
214	2	15	22	7
100	3		6	1
25	1	36		8
104	2	9	1	4

5. Add 1 T. 17 cwt. 8 lb., 5 cwt. 29 lb. 8 oz., 1 cwt. 42 lb. 6 oz., and 17 lb. 8 oz. *Ans.* 2 T. 3 cwt. 97 lb. 6 oz.

6. Add 6 yd. 2 ft., 3 yd. 1 ft. 8 in., 1 ft. $10\frac{1}{2}$ in., 2 yd. 2 ft. $6\frac{1}{2}$ in., 2 ft. 7 in., and 2 yd. 5 in. *Ans.* 16 yd. 2 ft. 1 in.

7. Add 4 Cd. 7 cd. ft., 2 Cd. 2 cd. ft. 12 cu. ft., 6 cd. ft. 15 cu. ft., 5 Cd. 3 cd. ft. 8 cu. ft., and 2 Cd. 1 cu. ft.

8. What is the sum of $1\frac{3}{8}$ hhd. 42 gal. 3 qt. $1\frac{1}{4}$ pt., $\frac{7}{8}$ gal. 2 qt. $\frac{3}{4}$ pt., and 1.75 pt.? *Ans.* 2 hhd. 23 gal. 2 qt. 3 gi.

9. What is the sum of $145\frac{7}{8}$ A., 7 A. 2 R. $29\frac{1}{2}$ P., 1 A. 3 R. 16.5 P., and $\frac{5}{6}$ A.? *Ans.* 156 A. $39\frac{1}{3}$ P.

10. Required the sum of 31 bu. 2 pk., $10\frac{7}{8}$ bu., 5 bu. $6\frac{1}{2}$ qt., 14 bu. 2.75 pk., and $\frac{2}{3}$ pk. *Ans.* 62 bu. 1 pk. 5 qt. $1\frac{2}{3}$ pt.

11. Required the value of 42 yr. $7\frac{1}{2}$ mo. + 10 yr. 3 wk. 5 da. + $9\frac{3}{4}$ mo. + 1 wk. 16 h. 40 min. + $\frac{5}{6}$ mo. + $3\frac{4}{5}$ da.
Ans. 53 yr. 7 mo. 9 da. 23 h. 52 min.

12. Add 3 S. 22° 50′, 24° 36′ 25.7″, 17′ 18.2″, 1 S. 3° 12′ 15.5″, 12° 36′ 17.8″, and 57.3″. *Ans.* 6 S. 3° 33′ 14.5″.

13. How many units in $1\frac{1}{2}$ gross $7\frac{1}{3}$ doz., 3 gross $1\frac{3}{4}$ doz., $\frac{3}{4}$ of a great gross, $6\frac{1}{4}$ doz., and 4 doz. 7 units? *Ans.* 2183.

14. What is the sum of 240 A. 6 sq. ch., 212.1875 sq. ch., and 5 sq. ch. $10\frac{4}{5}$ P.? *Ans.* 262 A. 3 sq. ch. 13.8 P.

15. Add $3\frac{1}{3}$ Pch. 18 cu. ft., 84.6 cu. ft., $\frac{5}{6}$ Pch., and $\frac{20}{27}$ cu. ft.

16. Add $\$3\frac{3}{4}$, $\$25\frac{1}{2}$, $\$12\frac{7}{8}$, $\$2\frac{2}{5}$, and $\$2.54\frac{3}{4}$. *Ans.* \$47.0725.

17. What is the sum of 3 ℔ 5 ℥ 4 ʒ 2 ℈ 17 gr., 2 ℔ 5 ʒ 12 gr., 4 ℥ 2 ʒ 1 ℈ 16 gr.? *Ans.* 5 ℔ 10 ℥ 4 ʒ 2 ℈ 5 gr.

18. A N. Y. farmer received \$.60 a bushel for 4 loads of corn; the first contained 42.4 bu., the second 2866 lb., the third $36\frac{3}{4}$ bu., and the fourth 39 bu. 29 lb. How much did he receive for the whole? *Ans.* \$100.84 –.

19. Bought three loads of hay at \$8 per ton. The first weighed 1.125 T., the second $1\frac{2}{5}$ T., and the third 2500 pounds; how much did the whole cost? *Ans.* \$30.20.

20. A man in digging a cellar removed $140\frac{4}{5}$ cu. yd. of earth, in digging a cistern 24.875 cu. yd., and in digging a drain 46 cu. yd. $20\frac{1}{4}$ cu. ft. What was the amount of earth removed, and how much the cost at 18 cts. a cu. yd.?
Ans. 212.425 cu. yd. removed; \$38.24 — cost.

SUBTRACTION.

378. 1. From 18 lb. 5 oz. 4 pwt. 14 gr. take 10 lb. 6 oz. 10 pwt. 8 gr.

OPERATION.

lb.	oz.	pwt.	gr.
18	5	4	14
10	6	10	8
7	10	14	6

ANALYSIS. Writing the subtrahend under the minuend, placing units of the same denomination under each other, we subtract 8 gr. from 14 gr. and write the remainder, 6 gr., underneath. Since we cannot subtract 10 pwt. from 4 pwt., we add 1 oz. or 20 pwt. to the 4 pwt., subtract 10 pwt. from the sum, and write the remainder, 14 pwt., underneath. Having added 20 pwt. or 1 oz. to the 6 oz. in the subtrahend, we find that we cannot subtract the sum, 7 oz., from the 5 oz. in the minuend; we therefore add 1 lb. or 12 oz. to the 5 oz., subtract 7 oz. from the sum, and write the remainder, 10 oz., underneath. Adding 12 oz. or 1 lb. to the 10 lb. in the subtrahend, we subtract the sum, 11 lb., from the 18 lb. in the minuend, as in simple numbers, and write the remainder, 7 lb., underneath.

2. From 12 bar. 15 gal. 3 qt. take 7 bar. 18 gal. 1 qt.

OPERATION.

bar.	gal.	qt.
12	15	3
7	18	1
4	$28\frac{1}{2}$	2
4	29	

ANALYSIS. Proceeding as in the last operation, we obtain a remainder of 4 bar. $28\frac{1}{2}$ gal. 2 qt. But, $\frac{1}{2}$ gal. = 2 qt., which added to the 2 qt. in the remainder makes 1 gal., and this added to the 28 gal. makes 29 gal.; and the answer is 4 bar. 29 gal.

3. From $\frac{3}{4}$ of a rod subtract $\frac{3}{4}$ of a yard.

OPERATION.

	yd.	ft.	in.
$\frac{3}{4}$ rd. =	4 yd.	0 ft.	$4\frac{1}{2}$ in.
$\frac{3}{4}$ yd. =		2 "	3 "
	3	1	$1\frac{1}{2}$

Or,

$$\frac{3}{4}\text{ yd.} \times \frac{1}{5\frac{1}{2}} = \frac{3}{4}\text{ yd.} \times \frac{2}{11} = \frac{3}{22}\text{ rd.};$$

$$\frac{3}{4}\text{ rd.} - \frac{3}{22}\text{ rd.} = \frac{27}{44}\text{ rd.};$$

$$\frac{27}{44}\text{ rd.} = 3\text{ yd. } 1\text{ ft. } 1\tfrac{1}{2}\text{ in.}$$

ANALYSIS. We first find the value of each fraction in integers of lower denominations, (**369**), and then subtract the less value from the greater. Or, we may reduce the given fractions to fractions of the same denomination, subtract the less value from the greater, and find the value of the remainder in integers of lower denominations.

379. From these illustrations we deduce the following

RULE. I. *If any of the numbers are denominate fractions, or if any of the denominations are mixed numbers, reduce the fractions to integers of lower denominations.*

II. *Write the subtrahend under the minuend, so that units of the same denomination shall stand under each other.*

III. *Beginning at the right hand, subtract each denomination separately, as in simple numbers.*

IV. *If the number of any denomination in the subtrahend exceed that of the same denomination in the minuend, add to the number in the minuend as many units as make one of the next higher denomination, and then subtract; in this case add 1 to the next higher denomination of the subtrahend before subtracting. Proceed in the same manner with each denomination.*

EXAMPLES FOR PRACTICE.

(1.)

	mi.	fur.	rd.	ft.	in.
From	175	3	27	11	4
Take	59	6	10	12	9
Rem.	115	5	16	15	1

(2.)

A.	R.	P.
320	3	26.4
150	2	31.86
170	0	34.54

(3.)

hhd.	gal.	qt.
5	36	$3\frac{1}{4}$
2	45	$1\frac{7}{8}$

(4.)

yr.	mo.	wk.	da.	h.
45	1	3	0	$17\frac{1}{2}$
10	9	1	22	6.8

5. Subtract 15 rd. 10 ft. $3\frac{1}{4}$ in. from 26 rd. 11 ft. 3 in.
Ans. 11 rd. $11\frac{3}{4}$ in.

6. From 1 T. 11 cwt. 30 lbs. 6 oz. take 18 cwt. 45 lb.

7. Subtract .659 wk. from 2 wk. $3\frac{5}{6}$ da.
Ans. 1 wk. 6 da. 5 h. 17 min. $16\frac{4}{5}$ sec.

8. From $\frac{117}{224}$ hhd. take .90625 gal. *Ans.* 32 gal.

9. From $\frac{3}{8}$ of $3\frac{3}{5}$ A. take 3 R. 12.56 P.

10. Subtract $\frac{3}{40}$ lb. Troy, from 10 lb. 8 oz. 8 pwt.

11. From a pile of wood containing 36 Cd. 4 cd. ft., there was sold 10 Cd. 6 cd. ft. 12 cu. ft.; how much remained?

12. From $5\frac{1}{2}$ barrels take $\frac{4}{7}$ of a hogshead.
Ans. 4 bbl. 11 gal. 1 qt.

13. Subtract $\frac{607}{960}$ of a day from $\frac{2}{3}$ of a week.
Ans. 4 da. 49 min. 30 sec.

14. From $\frac{5}{8}$ of a gross subtract $\frac{2}{3}$ of a dozen. *Ans.* $6\frac{5}{6}$ doz.

15. From $\frac{7}{9}$ of a mile take $\frac{21}{22}$ of a rod.

16. Subtract 2 A. 3 R. 5.76 P. from 5 A. 1 R. 24.24 P.
Ans. 2 A. 2 R. 18.48 P.

17. Subtract .0625 bu. from $\frac{3}{4}$ pk. *Ans.* 4 qt.

18. From the sum of $\frac{5}{9}$ of $365\frac{1}{4}$ da. and $\frac{3}{4}$ of $5\frac{5}{9}$ wk. take $49\frac{1}{7}$ min. *Ans.* 33 wk. 1 da. 1 h. $10\frac{6}{7}$ min.

19. From the sum of $\frac{2}{7}$ of $3\frac{3}{4}$ mi. and $17\frac{1}{7}$ rd., take $5\frac{1}{3}$ fur.

20. From 15 bbl. 3.25 gal. take 14 bbl. 24 gal. 3.54 qt.

21. A farmer in Ohio having 200 bu. of barley, sold 3 loads, the first weighing 1457 lb., the second 1578 lb., and the third 1420 lb.; how many bushels had he left? *Ans.* 107 bu. 9 lb.

22. Of a farm containing 200 acres two lots were reserved, one containing 50 A. 136.4 P. and the other 48 A. 123.3 P.; the remainder was sold at $35 per acre. How much did it bring? *Ans.* $3513.19+.

23. An excavation 58 ft. long, 37 ft. wide, and 6 ft. deep is to be made for a cellar; after 471 cu. yd. 16 cu. ft. 972 cu. in. of earth have been removed, how much more still remains to be taken out? *Ans.* 5 cu. yd. 7 cu. ft. 756 cu. in.

24. From the sum of $\frac{4}{5}$ lb., $4\frac{5}{6}$ oz., and $31\frac{1}{3}$ pwt., take the difference between $\frac{3}{5}$ oz. and $\frac{7}{8}$ pwt. *Ans.* 1 lb. 3 oz. 8 pwt. 21 gr.

25. From the sum of $5\frac{7}{12}$ A., $\frac{2}{3}$ of $6\frac{1}{4}$ A., $\dfrac{9\frac{1}{2}}{12\frac{2}{3}}$ R., and $\frac{3}{11}$ of $2\frac{2}{11}$ P., take 4 A. 25 P. 12 sq. yd.

Ans. 5 A. 3 R. 5 P. 6 sq. yd.

380. To find the difference in dates.

1. How many years, months, days and hours from 3 o'clock P. M. of June 15, 1852, to 10 o'clock A. M. of Feb. 22, 1860?

OPERATION.

yr.	mo.	da.	h.
1860	2	22	10
1852	6	15	15
7	8	6	19

ANALYSIS. Since the later of two dates always expresses the greater period of time, we write the later date for a minuend and the earlier date for a subtrahend, placing the denominations in the order of the descending scale from left to right, (**300,** NOTE 8). We then subtract by the rule for subtraction of compound numbers.

When the *exact number of days* is required for any period not exceeding one ordinary year, it may be readily found by the following

TABLE,

Showing the number of days from any day of one month to the same day of any other month within one year.

FROM ANY DAY OF	TO THE SAME DAY OF THE NEXT											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January.........	365	31	59	90	120	151	181	212	243	273	304	334
February........	334	365	28	59	89	120	150	181	212	242	273	303
March...........	306	337	365	31	61	92	122	153	184	214	245	275
April............	275	306	335	365	30	61	91	122	153	183	214	244
May.............	245	276	304	335	365	31	61	92	123	153	184	214
June............	214	245	273	304	334	365	30	61	92	122	153	183
July............	184	215	243	274	304	335	365	31	62	92	123	153
August..........	153	184	212	243	273	304	334	365	31	61	92	122
September......	122	153	181	212	242	273	303	334	365	30	61	91
October.........	92	123	151	182	212	243	273	304	335	365	31	61
November......	61	92	120	151	181	212	242	273	304	334	365	30
December......	31	62	90	121	151	182	212	243	274	304	335	365

If the days of the different months are not the same, the number of days of difference should be *added* when the earlier day belongs to the month *from* which we reckon, and *subtracted* when it belongs to the month *to* which we find the time. If the 29th of February is to be included in the time computed, one day must be added to the result.

EXAMPLES FOR PRACTICE.

1. War between England and America was commenced April 19, 1775, and peace was restored January 20, 1783; how long did the war continue? *Ans.* 7 yr. 9 mo. 1 da.

2. The Pilgrims landed at Plymouth Dec. 22, 1620, and Gen. Washington was born Feb. 22, 1732; what was the difference in time between these events?

3. The first settlement made in the U. S. was at Jamestown, Va., May 23, 1607; how many years from that time to July 4, 1860?

4. How long has a note to run, dated Jan. 30, 1859, and made payable June 3, 1861? *Ans.* 2 yr. 4 mo. 3 da.

5. How many years, months, and days from your birthday to this date?

6. What length of time elapsed from 16 minutes past 10 o'clock, A. M., July 4, 1855, to 22 minutes before 8 o'clock, P. M., Dec. 12, 1860? *Ans.* 1988 da. 9 h. 22 min.

7. What length of time will elapse from 40 minutes 25 seconds past 12 o'clock, noon, April 21, 1860, to 4 minutes 36 seconds before 5 o'clock, A. M., Jan. 1, 1862?

8. How many days from the 4th September, to the 27th of May following? *Ans.* 265 da.

MULTIPLICATION.

381. 1. Multiply 5 mi. 4 fur. 18 rd. 15 ft. by 6.

OPERATION.

mi.	fur.	rd.	ft.
5 mi.	4 fur.	18 rd.	15 ft.
			6
33	2	33	$7\frac{1}{2}$

ANALYSIS. Writing the multiplier under the lowest denomination of the multiplicand, we multiply each denomination in the multiplicand separately in order from lowest to highest, as in simple numbers, and carry from lower denominations to higher, according to the ascending scale of the multiplicand, as in addition of compound numbers. Hence,

RULE. I. *Write the multiplier under the lowest denomination of the multiplicand.*

II. *Multiply as in simple numbers, and carry as in addition of compound numbers.*

NOTES.—1. When the multiplier is large, and is a *composite* number, we may shorten the work by multiplying by the component factors.

2. The multiplier must be an abstract number.

3. If any of the denominations are mixed numbers, they may either be reduced to integers of lower denominations before multiplying, or they may be multiplied as directed in **193**.

4. The multiplication of a denominate fraction is the most readily performed by **193**, after which the product may be reduced to integers of lower denominations by **369**.

382. As the work of multiplying by large prime numbers is somewhat tedious, the following method may often be so modified and adapted as to greatly shorten the operation.

1. How many bushels of grain in 47 bags, each containing 2 bu. 1 pk. 4 qt.?

FIRST OPERATION.

$47 = (5 \times 9) + 2$

2 bu.	1 pk.	4 qt. × 2			
		5			
11 bu.	3 pk.	4 qt.	in	5	bags.
		9			
106 bu.	3 pk.	4 qt.	in	45	bags.
4 "	3 "		"	2	"
111 bu.	2 pk.	4 qt.	"	47	"

SECOND OPERATION.

$47 = (6 \times 8) - 1$

2 bu.	1 pk.	4 qt. × 1			
		6			
14 bu.	1 pk.		in	6	bags.
	8				
114 bu.			in	48	bags.
2 "	1 pk.	4 qt.	"	1	bag.
111 bu.	2 pk.	4 qt.	"	47	bags.

ANALYSIS. Multiplying the contents of 1 bag by 5, and the resulting product by 9, we have the contents of 45 bags, which is the composite number *next less* than the given prime number, 47. Next, multiplying the contents of 1 bag by 2, we have the contents of 2 bags, which, added to the contents of 45 bags, gives us the contents of $45 + 2 = 47$ bags.

Or, we may multiply the contents of 1 bag by the factors of the composite number *next greater* than the given prime number, 47, and from the last product subtract the multiplicand.

EXAMPLES FOR PRACTICE.

(1.)

	T.	cwt.	lb.	oz.
	12	15	27	9
				8
Prod.	102	2	20	8

(2.)

mi.	fur.	rd.	ft.
14	6	36	14
			9
133	6	11	10½

(3.)

A.	R.	P.	sq. yd.	sq. ft.
7	1	33	21	7
				6

(4.)

Cd.	cd. ft.	cu ft.
10	7	13
		12

5. Multiply 34 bu. 3 pk. 6 qt. 1 pt. by 14.

6. Multiply 4 lb. 10 oz. 18.7 pwt. by 27.

Ans. 132 lb. 7 oz. 4.9. pwt.

7. Multiply 9 ℥ 3 ʒ 2 ℈ 13 gr. by 35.

8. Multiply 5 gal. 2 qt. 1 pt. 3.25 gi. by 96.

9. Multiply 78 A. 3 R. 15 P. 15 sq. yd. by 15$\frac{3}{5}$.

Ans. 1235 A. 1 R. 2 P. 23$\frac{1}{4}$ sq. yd.

10. What is 73 times 9 cu. yd. 10 cu. ft. 1424 cu. in.?

11. Multiply 2 lb. 8 oz. 13 pwt. 18 gr. by 59.

12. Multiply 4 yd. 1 ft. 4.7 in. by 125.

13. If 1 qt. 2 gi. of wine fill 1 bottle, how much will be required to fill a gross of bottles of the same capacity?

14. Multiply 7 O. 10 f℥ 4 fʒ 25 ♏ by 24.

Ans. 22 Cong. 7 O. 13 f℥ 2 fʒ.

15. Multiply 3 hhd. 43 gal. 2.6 gi. by 17.

16. Multiply 9 T. 13 cwt. 1 qr. 10.5 lb. by 1.7.

Note.—When the multiplier contains a decimal, the multiplicand may be reduced to the lowest denomination mentioned, or the lower denominations to a decimal of the higher, before multiplying. The result can be reduced to the compound number required.

Ans. 16 T. 8 cwt. 2 qr. 20.65 lb.

17. If a pipe discharge 2 hhd. 23 gal. 2 qt. 1 pt. of water in 1 hour, how much will it discharge in 4.8 hours, if the water flow with the same velocity? *Ans.* 11 hhd. 25 gal. 1 pt. 2.4 gi.

18. What will be the value of 1 dozen gold cups, each cup weighing 9 oz. 13 pwt. 8 gr., at $212.38 a pound?

19. What cost 5 casks of wine, each containing 27 gal. 3 qt. 1 pt., at 1.37\frac{1}{2}$ a gallon? *Ans.* $191.64+.

20. A farmer sold 5 loads of oats, averaging 37 bu. 3 pk. 5 qt. each, at $.65 per bushel; how much money did he receive for the grain? *Ans.* $123.20—.

DIVISION.

383. 1. Divide 37 A. 1 R. 16 P. by 8.

OPERATION.

8)	37 A.	1 R.	16 P.
	4	2	27

Analysis. Writing the divisor on the left of the dividend, we divide the highest denomination, and obtain a quotient of 4 A. and a remainder of 5 A. Writing the quotient under the denomination divided, we reduce the remainder to roods, making 20 R., which added to the 1 R. of the dividend, equals 21 R. Dividing this, we have a quotient of 2 R. and a remainder of 5 R. Writing the 2 R. under the denomination divided, we reduce the remainder to rods, making 200 P., which added to the 16 P. of the dividend, equals 216 P. Dividing this, we have a quotient of 27 P. and no remainder.

2. Divide 111 bu. 2 pk. 4 qt. by 47.

OPERATION.

```
47 ) 111 bu. 2 pk. 4 qt. ( 2 bu. 1 pk. 4 qt.
      94
      ——
      17 bu. rem.
       4
      ——
      70 pk. in 17 bu. 2 pk.
      47
      ——
      23 pk. rem.
       8
     ———
     188 qt. in 23 pk. 4 qt.
     188
     ———
```

ANALYSIS. The divisor being large, and a prime number, we divide by long division, setting down the whole work of subtracting and reducing.

From these examples and illustrations we derive the following

RULE. I. *Divide the highest denomination as in simple numbers, and each succeeding denomination in the same manner, if there be no remainder.*

II. *If there be a remainder after dividing any denomination, reduce it to the next lower denomination, adding in the given number of that denomination, if any, and divide as before.*

III. *The several partial quotients will be the quotient required.*

NOTES. — 1. When the divisor is large and is a *composite* number, we may shorten the work by dividing by the factors.

2. When the divisor and dividend are both compound numbers, they must both be reduced to the same denomination before dividing, and then the process is the same as in simple numbers.

3. The division of a denominate fraction is most readily performed by **195**, after which the quotient may be reduced to its equivalent compound number, by **369**.

EXAMPLES FOR PRACTICE.

(1.)

	£	s.	d.	far.
5)	62	7	9	3
Quotient,	12	9	6	3
	19			

(2.)

	lb.	oz.	pwt.	gr.
9)	56	6	17	6
	6 lb.	3 oz.	8 pw.	14 gr.

(3.)

	hhd.	gal.	qt.	pt.
12)	9	28	2	
		49	2	1

(4.)

	T.	cwt.	qr.	lb.
19)	373	19	2	4
	19	13	2	16

5. Divide 358 A. 1 R. 17 P. 6 sq. yd. 2 sq. ft. by 7.
Ans. 51 A. 31 P. 8 sq. ft.

6. Divide 192 bu. 3 pk. 1 qt. 1 pt. by 9.

7. Divide 336 yd. 4 ft. $3\frac{1}{3}$ in. by 21. *Ans.* 16 yd. $2\frac{4}{9}$ in.

8. Divide 77 sq. yd. 5 sq. ft. 82 sq. in. by 13.
Ans. 5 sq. yd. 8 sq. ft. 106 sq. in.

9. Divide 678 cu. yd. 1 cu. ft. 1038.05 cu. in. by 67.

10. Divide 1986 mi. 3 fur. 20 rd. 1 yd. by 108.

11. Divide 12 sq. mi. 1 R. 30 P. by $22\frac{1}{2}$.
Ans. 341 A. 1 R. $16\frac{4}{9}$ P.

NOTE 4.—Observe that $22\frac{1}{2} = \frac{45}{2}$; hence, multiply by 2, and divide the result by 45.

12. Divide 365 da. 6 h. by 240.

13. Divide 3794 cu. yd. 20 cu. ft. $709\frac{1}{3}$ cu. in. by $33\frac{1}{3}$.

14. Divide 121 lb. 3℥ 2ʒ 1℈ 4 gr. by $13\frac{3}{4}$.

15. Divide 28° 51′ 27.765″ by 2.754. *Ans.* 10° 28′ $42\frac{1}{2}$″.

16. Divide 202 yd. 1 ft. $6\frac{3}{4}$ in. by $\frac{3}{5}$.
Ans. 337 yd. 1 ft. $7\frac{1}{4}$ in.

17. Divide 1950 da. 15 h. $15\frac{5}{9}$ min. by 100.

18. If a town 4 miles square be divided equally into 124 farms, how much will each farm contain? *Ans.* 82 A. 2 R. $12\frac{28}{31}$ P.

19. A cellar 48 ft. 6 in. long, 24 ft. wide, and $6\frac{1}{2}$ ft. deep, was excavated by 6 men in 8 days; how many cubic yards did each man excavate daily? *Ans.* 5 cu. yd. 22 cu. ft. 1080 cu. in.

20. How many casks, each containing 2 bu. 3 pk. 6 qt., can be filled from 356 bu. 3 pk. 5 qt. of cherries? *Ans.* $121\frac{1}{2}$.

LONGITUDE AND TIME.

384. Since the earth performs one complete revolution on its axis in a day or 24 hours, the sun *appears to pass* from east to west round the earth, or through 360° of *longitude* once in every

24 hours of *time*. Hence the relation of time to the *real* motion of the earth or the *apparent* motion of the sun, is as follows:

Time.			Longitude.	
24 h.			$= 360°$	
1 h. or 60 min.		$= \frac{3600°}{24}$	$= 15°$	$= 900'$
1 min. or 60 sec.	$= \frac{15°}{60}$	$= \frac{900'}{60}$	$= 15'$	$= 900''$
1 sec.	$= \frac{15'}{60}$	$= \frac{900''}{60}$	$= 15''$	

Hence, 1 h. of time = 15° of longitude.
1 min. " = 15′ " "
1 sec. " = 15″ " "

CASE I.

385. To find the difference of longitude between two places or meridians, when the difference of time is known.

ANALYSIS. A difference of 1 h. of time corresponds to a difference of 15° of longitude, of 1 min. of time to a difference of 15′ min. of longitude, and of 1 sec. of time to a difference of 15″ of longitude, (**384**). Hence, the

RULE. *Multiply the difference of time, expressed in hours, minutes, and seconds, by* 15, *according to the rule for multiplication of compound numbers; the product will be the difference of longitude in degrees, minutes, and seconds.*

NOTES. — 1. If one place be in east, and the other in west longitude, the difference of longitude is found by *adding* them, and if the sum be greater than 180°, it must be subtracted from 360°.

2. Since the sun appears to move from east to west, when it is exactly 12 o'clock at one place, it will be *past* 12 o'clock at all places east, and *before* 12 at all places west. Hence, if the difference of time between two places, be *subtracted* from the time at the *easterly* place, the result will be the time at the westerly place; and if the difference be *added* to the time at the *westerly* place the result will be the time at the easterly place.

EXAMPLES FOR PRACTICE.

1. When it is 9 o'clock at Washington, it is 8 h. 7 min. 4 sec. at St. Louis; the longitude of Washington being 77° 1′, west, what is the longitude of St. Louis? *Ans.* 90° 15′ west.

2. The sun rises at Boston 1 h. 11 min. 56 sec. sooner than at New Orleans; the longitude of New Orleans being 89° 2′ west, what is the longitude of Boston? *Ans.* 71° 3′ west.

3. When it is half past 2 o'clock in the morning at Havana, it is 9 h. 13 min. 20 sec. A. M. at the Cape of Good Hope; the longitude of the latter place being 18° 28′ east, what is the longitude of Havana? *Ans.* 82° 22′ west.

4. The difference of time between Valparaiso and Rome is 6 h. 8 min. 28 sec.; what is the difference in longitude?

5. A gentleman traveling East from Fort Leavenworth, which is in 94° 44′ west longitude, found, on arriving at Philadelphia, that his watch, an accurate time keeper, was 1 h. 18 min. 16 sec. slower than the time at Philadelphia; what is the longitude of Philadelphia? *Ans.* 75° 10′ west.

6. When it is 12 o'clock M. at San Francisco it is 2 h. 58 min. $23\frac{1}{3}$ sec. P. M. at Rochester, N. Y; the longitude of the latter place being 77° 51′ W., what is the longitude of San Francisco?

7. A gentleman traveling West from Quebec, which is in 71° 12′ 15″ W. longitude, finds, on his arrival at St. Joseph, that his watch is 2 h. 33 min. $53\frac{14}{15}$ sec. faster than true time at the latter place. If his watch has kept accurate time, what is the longitude of St. Joseph? *Ans.* 109° 40′ 44″ W.

8. A ship's chronometer, set at Greenwich, points to 5 h. 40 min. 20 sec. P. M., when the sun is on the meridian; what is the ship's longitude? *Ans.* 85° 5′ west.

NOTE 3.—Greenwich, Eng., is on the meridian of 0°, and from this meridian longitude is reckoned.

9. The longitude of Stockholm being 18° 3′ 30″ E., when it is midnight there, it is 5 h. 51 min. $41\frac{3}{5}$ sec. A. M. at New York; what is the longitude of New York from Greenwich?

Ans. 74° 1′ 6″ W.

10. A vessel set sail from New York, and proceeded in a southeasterly direction for 24 days. The captain then took an observation on the sun, and found the local time at the ship's meridian to be 10 h. 4 min. 36.8 sec. A. M.; at the moment of the observation, his chronometer, which had been set for New York time, showed 8 h. 53 min. 47 sec. A. M. Allowing that the chronometer had gained 3.56 sec. per day, how much had the ship changed her longitude since she set sail? *Ans.* 18° 3′ 48.6″.

CASE II.

386. To find the difference of time between two places or meridians, when the difference of longitude is known.

ANALYSIS. A difference of 15° of longitude produces a difference of 1 h. of time, 15′ of longitude a difference of 1 min. of time, and 15′′ of longitude a difference of 1 sec. of time, (**384**). Hence the

RULE. *Divide the difference of longitude, expressed in degrees, minutes, and seconds, by* 15, *according to the rule for division of compound numbers; the quotient will be the difference of time in hours, minutes, and seconds.*

EXAMPLES FOR PRACTICE.

1. Washington is 77° 1′ and Cincinnati is 84° 24′ west longitude; what is the difference of time? *Ans.* 29 min. 32 sec.

2. Paris is 2° 20′ and Canton 113° 14′ east longitude; what is the difference in time?

3. Buffalo is 78° 55′ west, and the city of Rome 20° 30′ east longitude; what is the difference in time?

Ans. 6 h. 37 min. 40 sec.

4. A steamer arrives at Halifax, 63° 36′ west, at 4 h. 30 min. P. M.; the fact is telegraphed to New York, 74° 1′ west, without loss of time; what is the time of its receipt at New York?

5. The longitude of Cambridge, Mass., is 71° 7′ west, and of Cambridge, England, is 5′ 2″ east; what time is it at the former place when it is 12 M. at the latter?

Ans. 7 h. 15 min. $11\frac{13}{15}$ sec. A. M.

6. The longitude of Pekin is 118° east, and of Sacramento City 120° west; what is the difference in time?

7. The longitude of Jerusalem is 35° 32′ east, and that of Baltimore 76° 37′ west; when it is 40 minutes past 6 o'clock A. M. at Baltimore, what is the time at Jerusalem?

8. What time is it in Baltimore when it is 6 o'clock P. M. at Jerusalem? *Ans.* 10 h. 31 min. 24 sec. A. M.

9. The longitude of Springfield, Mass., is 72° 35′ 45″ W., and of Galveston, Texas, 94° 46′ 34″ W.; when it is 20 min. past 6 o'clock A. M. at Springfield, what time is it at Galveston?

10. The longitude of Constantinople is 28° 49′ east, and of St. Paul 93° 5′ west; when it is 3 o'clock P. M. at the latter place, what time is it at the former?

11. What time is it at St. Paul when it is midnight at Constantinople? *Ans.* 3 h. 52 min. 24 sec. P. M.

12. The longitude of Cambridge, Eng., is 5′ 2″ E., and of Mobile, Ala., 88° 1′ 29″ W.; when it is 12 o'clock M. at Mobile, what is the time at Cambridge?

PROMISCUOUS EXAMPLES IN COMPOUND NUMBERS.

1. In 9 lb. 8℥ 1ʒ 2℈ 19 gr. how many grains?
2. How much will 3 cwt. 12 lb. of hay cost, at $\$15\frac{1}{2}$ a ton?
3. In 27 yd. 2 qr. how many Eng. ells? *Ans.* 22.
4. Reduce \$18.945 to sterling money. *Ans.* £3 18s. $3\frac{51}{121}$d.
5. In 4 yr. 48 da. 10 h. 45 sec. how many seconds?
6. How many printed pages, 2 pages to each leaf, will there be in an octavo book having 24 fully printed sheets? *Ans.* 384.
7. At 1/6 sterling per yard, how many yards of cloth may be bought for £5 6s. 6d.? *Ans.* 71 yd.
8. In 4 mi. 51 ch. 73 l. how many links?
9. In 22 A. 3 R. 33 sq. rd. $2\frac{3}{4}$ sq. yd. how many square yards?
10. How many demijohns, each containing 3 gal. 1 qt. 1 pt., can be filled from 3 hhd. of currant wine? *Ans.* 56.
11. Paid \$375.75 for $2\frac{1}{2}$ tons of cheese, and retailed it at $9\frac{1}{2}$ cts. a pound; how much was my whole gain?
12. A gentleman sent a silver tray and pitcher, weighing 3 lb. 9 oz., to a jeweler, and ordered them made into tea spoons, each weighing 1 oz. 5 pwt.; how many spoons ought he to receive? *Ans.* 3 doz.
13. What part of 4 gal. 3 qt. is 2 qt. 1 pt. 2 gi.? *Ans.* $\frac{11}{76}$.
14. Reduce $\frac{3}{7}$ of $\frac{4}{11}$ of a rod to the fraction of yard.
15. How many yards of carpeting 1 yd. wide, will be required to cover a floor $26\frac{1}{2}$ ft. long, and 20 ft. wide? *Ans.* $58\frac{8}{9}$.

16. If I purchase 15 T. 3 cwt. 3 qr. 24 lb. of English iron, by long ton weight, at 6 cents a pound, and sell the same at $140 per short ton, how much will I gain by the transaction?

17. What will be the expense of plastering a room 40 ft. long, $36\frac{1}{2}$ ft. wide, and $22\frac{1}{4}$ ft. high, at 18 cents a sq. yd., allowing 1375 sq. ft. for doors, windows, and base board? *Ans.* $69.78\frac{1}{2}$.

18. How much tea in 23 chests, each weighing 78 lb. 9 oz.?

19. Valparaiso is in latitude 33° 2′ south, and Mobile 30° 41′ north; what is their difference of latitude? *Ans.* 63° 43′.

20. If a druggist sell 1 gross 4 dozen bottles of Congress water a day, how many will he sell during the month of July?

21. Eighteen buildings are erected on an acre of ground, each occupying, on an average, 4 sq. rd. 120 sq. ft. 84 sq. in.; how much ground remains unoccupied?

22. At $13 per ton, how much hay may be bought for 12.02\frac{1}{2}$?

23. If 1 pk. 4 qt. of wheat cost $.72, how much will a bushel cost? *Ans.* $1.92.

24. How many bushels, Indiana standard, in 36244 lbs. of wheat?

25. At 20 cents a cubic yard, how much will it cost to dig a cellar 32 ft. long, 24 ft. wide, and 6 ft. deep? *Ans.* $34.13+.

26. If the wall of the same cellar be laid $1\frac{1}{2}$ feet thick, what will it cost at $1.25 a perch? *Ans.* 50.90\frac{10}{11}$.

27. The forward wheels of a wagon are 10 ft. 4 in. in circumference, and the hind wheels $15\frac{1}{2}$ ft.; how many more times will the forward wheels revolve than the hind wheels in running from Boston to N. Y., the distance being 248 miles? *Ans.* 42240.

28. Bought 15 cwt. 22 lb. of rice at $3.75 a cwt., and 7 cwt. 36 lb. of pearl barley at $4.25 a cwt. How much would be gained by selling the whole at $4\frac{1}{2}$ cents a pound? *Ans.* $13.255.

29. From $\frac{5}{7}$ of 3 T. 10 cwt. subtract $\frac{4}{13}$ of 7 T. 3 cwt. 26 lb.

30. What is the value in avoirdupois weight of 16 lb. 5 oz. 10 pwt. 13 gr. Troy? *Ans.* 13 lb. 8 oz. 11.4+dr.

31. What decimal of a rod is 1 ft. 7.8 in.?

32. If a piece of timber be 9 in. wide and 6 in. thick, what length of it will be required to make 3 cu. ft.? *Ans* 8 ft.

33. If a board be 16 in. broad, what length of it will make 7 sq. ft.? *Ans.* $5\frac{1}{4}$ ft.

34. If a hogshead contain 10 cubic feet, how many more gallons of dry measure will it contain than of beer measure?

35. How many tons in a stick of hewn timber 60 ft. long, and 1 ft. 9 in. by 1 ft. 1 in.? *Ans.* 2.275 tons.

36. Subtract $\dfrac{7\frac{1}{2}}{8\frac{1}{3}}$ bu. + $\frac{5}{8}$ of $\frac{56}{75}$ of $3\frac{1}{3}$ qt. from 5 bu. $3\frac{31}{45}$ qt.

Ans. $16\frac{2}{3}$ pk.

37. What is the difference between $\frac{7}{8}$ of 5 sq. mi. 250 A. 3 R., and $3\frac{1}{4}$ times 456 A. 3 R. 14 P. 25 sq. yd.?

Ans. 2 sq. mi. 254 A. 2 R. 26 P. $24\frac{5}{8}$ sq. yd.

38. How many pounds of silver, Troy weight, are equivalent in value to 5.6 lb. of gold by the English government standard?

Ans. 80 lb. 2 pwt. 19.2768 gr.

39. If a piece of gold is $\frac{5}{8}$ pure, how many carats fine is it?

40. In gold 16 carats fine what part is pure, and what part is alloy?

41. A man having a piece of land containing $384\frac{4}{5}$ A., divided it between his two sons, giving to the elder 22 A. 1 R. 20 P. more than to the younger; how many acres did he give to each?

Ans. 203 A. 2 R. 14 P., elder; 181 A. 0 R. 34 P., younger.

42. 4000 bushels of corn in Illinois is equal to how many bushels in New York? *Ans.* $3586\frac{6}{29}$ bu.

43. The market value being the same in both States, a farmer in New Jersey exchanged 110 bu. of cloverseed, worth \$4 a bushel, with a farmer in New York for corn, worth \$$\frac{3}{4}$ a bushel, which he sold in his own State for cash. The exchange being made by weight, in whose favor was the difference, and how much in cash value?

Ans. The N. J. farmer gained $69\frac{1}{7}$ bu. corn, worth \$$46\frac{2}{21}$.

44. The great pyramid of Cheops measures 763.4 feet on each side of its base, which is square. How many acres does it cover?

45. The roof of a house is 42 ft. long, and each side 20 ft. 6 in. wide; what will the roofing cost at \$$4.62\frac{1}{2}$ a square?

46. If 17 T. 15 cwt. 62½ lb. of iron cost $1333.593, how much will 1 ton cost?

47. How many wine gallons will a tank hold, that is 4 ft. long by $3\frac{3}{4}$ ft. wide, and $1\frac{2}{3}$ ft. deep? *Ans.* $187\frac{1}{77}$ gal.

48. What will be the cost of 300 bushels of wheat at 9s. 4d. per bushel, Michigan currency? *Ans.* $350.

49. What will be the cost in Missouri currency?

50. What will be the cost in Delaware currency?

51. What will be the cost in Georgia currency? *Ans.* $600.

52. What will be the cost in Canada currency? *Ans.* $560.

53. Bought the following bill of goods in Boston:

6½	yd. Irish linen	@	5/4
12	" flannel	"	3/9
8¼	" calico	"	1/7
9	" ribbon	"	/9
4½	lb. coffee	"	1/5
6¾	gal. molasses	"	3/8

What was the amount of the bill? *Ans.* $21.76 +.

54. How many pipes of Madeira are equal to 22 pipes of sherry?

55. A cubic foot of distilled water weighs 1000 ounces avoirdupois; what is the weight of a wine gallon? *Ans.* 8 lb. $5\frac{49}{72}$ oz.

56. There is a house 45 feet long, and each of the two sides of the roof is 22 feet wide. Allowing each shingle to be 4 inches wide and 15 inches long, and to lie one third to the weather, how many half-thousand bunches will be required to cover the roof?

Ans. $28\frac{64}{125}$.

57. A cistern measures 4 ft. 6 in. square, and 6 ft. deep; how many hogsheads of water will it hold?

58. If the driving wheels of a locomotive be 18 ft. 9 in. in circumference, and make 3 revolutions in a second, how long will the locomotive be in running 150 miles?

Ans. 3 h. 54 min. 40 sec.

59 In traveling, when I arrived at Louisville my watch, which was exactly right at the beginning of my journey, and a correct

timekeeper, was 1 h. 6 min. 52 sec. fast; from what direction had I come, and how far? *Ans.* From the east, 16° 43′.

60. How many U. S. bushels will a bin contain that is 8.5 ft. long, 4.25 ft. wide, and $3\frac{3}{4}$ ft. deep?

61. Reduce 3 hhd. 9 gal. 3 qt. wine measure to Imperial gallons. *Ans.* 165.5807+ Imp'l gal.

62. A man owns a piece of land which is 105 ch. 85 l. long, and 40 ch. 15 l. wide; how many acres does it contain?

63. A and B own a farm together; A owns $\frac{7}{12}$ of it and B the remainder, and the difference between their shares is 15 A. 1 R. $28\frac{1}{2}$ P. How much is B's share? *Ans.* 38 A. 2 R. $11\frac{1}{4}$ P.

64. At $3.40 per square, what will be the cost of tinning both sides of a roof 40 ft. in length, and whose rafters are 20 ft. 6 in. long? *Ans.* $55.76.

65. What is the value of a farm 189.5 rd. long and 150 rd. wide, at $\$31\frac{3}{4}$ per acre?

66. Reduce 9.75 tons of hewn timber to feet, board measure, that is, 1 inch thick. *Ans.* 5850 ft.

67. How many wine gallons will a tank contain that is 4 ft. long, $3\frac{1}{2}$ ft. wide, and $2\frac{6}{7}$ ft. deep? *Ans.* $299\frac{17}{77}$ gal.

68. If a load of wood be 12 ft. long, and 3 ft. 6 in. wide, how high must it be to make a cord?

69. In a school room 32 ft. long, 18 ft. wide, and 12 ft. 6 in. high, are 60 pupils, each breathing 10 cu. ft. of air in a minute. In how long a time will they breathe as much air as the room contains?

70. A man has a piece of land $201\frac{2}{3}$ rods long and $41\frac{1}{4}$ rods wide, which he wishes to lay out into square lots of the greatest possible size. How many lots will there be? *Ans.* 396.

71. A man has 4 pieces of land containing 4 A. 3 R. 20 P., 6 A. 3 R. 12 P., 9 A. 3 R., and 11 A. 2 R. 32 P. respectively. It is required to divide each piece into the largest sized building lots possible, each lot containing the same area, and an exact number of square rods. How much land will each lot contain?

Ans. 156 P.

DUODECIMALS.

387. **Duodecimals** are the parts of a unit resulting from continually dividing by 12; as 1, $\frac{1}{12}$, $\frac{1}{144}$, $\frac{1}{1728}$, etc. In practice, duodecimals are applied to the measurement of extension, the foot being taken as the unit.

In the duodecimal divisions of a foot, the different orders of units are related as follows:

1′ (inch or prime).......... is $\frac{1}{12}$ of a foot, or 1 in. linear measure.
1″ (second) or $\frac{1}{12}$ of $\frac{1}{12}$,...... " $\frac{1}{144}$ of a foot, or 1 " square "
1‴ (third) or $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$,.. " $\frac{1}{1728}$ of a foot, or 1 " cubic "

TABLE.

12 fourths, (⁗),	make 1 third	1‴
12 thirds	" 1 second, ...	1″
12 seconds	" 1 prime, ...	1′
12 primes,	" 1 foot, ...	ft.

SCALE — uniformly 12.

The marks ′, ″, ‴, ⁗, are called indices.

NOTES. — 1. Duodecimals are really common fractions, and can always be treated as such; but usually their denominators are not expressed, and they are treated as compound numbers.

2. The word *duodecimal* is derived from the Latin term *duodecim*, signifying 12.

ADDITION AND SUBTRACTION.

388. Duodecimals are added and subtracted in the same manner as compound numbers.

EXAMPLES FOR PRACTICE.

1. Add 12 ft. 7′ 8″, 15 ft. 3′ 5″, 17 ft. 9′ 7″.
Ans. 45 ft. 8′ 8″.

2. Add 136 ft. 11′ 6″ 8‴, 145 ft. 10′ 8″ 5‴, 160 ft. 9′ 5″ 5‴.
Ans. 443 ft. 7′ 8″ 6‴.

3. From 36 ft. 7′ 11″ take 12 ft. 9′ 5″. *Ans.* 23 ft. 10′ 6″.

4. A certain room required 300 sq. yd. 2 sq. ft. 5′ of plastering. The walls required 50 sq. yd. 1 sq. ft. 7′ 4″, 62 sq. yd. 5′ 3″, 48 sq. yd. 2 sq. ft., and 42 sq. yd. 2 sq. ft. 3′ 4″, respectively. Required the area of the ceiling. *Ans.* 97 sq. yd. 5 sq. ft. 1′ 1″.

MULTIPLICATION.

389. In the multiplication of duodecimals, the product of two dimensions is area, and the product of three dimensions is solidity (**282, 286**).

We observe that

$$1' \times 1 \text{ ft.} = \tfrac{1}{12} \text{ of } 1 \text{ ft.} = 1'.$$
$$1'' \times 1 \text{ ft.} = \tfrac{1}{144} \text{ of } 1 \text{ ft.} = 1''.$$
$$1' \times 1' = \tfrac{1}{12} \times \tfrac{1}{12} \text{ of } 1 \text{ ft.} = 1''.$$
$$1'' \times 1' = \tfrac{1}{144} \times \tfrac{1}{12} \text{ of } 1 \text{ ft.} = 1'''. \quad \text{Hence,}$$

The product of any two orders is of the order denoted by the sum of their indices.

390. 1. Multiply 9 ft. 8′ by 4 ft. 7′.

OPERATION.

9 ft.	8′	
4 ft.	7′	
5 ft.	7′	8″
38 ft.	8′	
44 ft.	3′	8″, *Ans.*

ANALYSIS. Beginning at the right, $8' \times 7' = 56'' = 4'\ 8''$; writing the 8″ one place to the right, we reserve the 4′ to be added to the next product. Then, $9 \text{ ft.} \times 7' + 4' = 67' = 5 \text{ ft. } 7'$, which we write in the places of feet and primes. Next multiplying by 4 ft., we have $8' \times 4 \text{ ft.} = 32' = 2 \text{ ft. } 8'$; writing the 8′ in the place of primes, we reserve the 2 ft. to be added to the next product. Then, $9 \text{ ft.} \times 4 \text{ ft.} + 2 \text{ ft.} = 38 \text{ ft.}$, which we write in the place of feet. Adding the partial products, we have 44 ft. 3′ 8″ for the product required. Hence the

RULE. I. *Write the several terms of the multiplier under the corresponding terms of the multiplicand.*

II. *Multiply each term of the multiplicand by each term of the multiplier, beginning with the lowest term in each, and call the product of any two orders, the order denoted by the sum of their indices, carrying* 1 *for every* 12.

III. *Add the partial products; their sum will be the required answer.*

EXAMPLES FOR PRACTICE.

1. How many square feet in a floor 16 ft. 8′ wide, and 18 ft. 5′ long?

2. How much wood in a pile 4 ft. wide, 3 ft. 8′ high, and 23 ft 7′ long?

3. If a floor be 79 ft. 8′ by 38 ft. 11′, how many square yards does it contain? *Ans.* 344 yd. 4 ft. 4′ 4″.

4. If a block of marble be 7 ft. 6′ long, 3 ft. 3′ wide, and 1 ft. 10′ thick, what are the solid contents? *Ans.* 44 ft. 8′ 3″.

5. How many solid feet in 7 sticks of timber, each 56 ft. long, 11 inches wide, and 10 inches thick? *Ans.* 299 ft. 5′ 4″.

6. How many feet of boards will it require to inclose a building 60 ft. 6′ long, 40 ft. 3′ wide, 22 ft. high, and each side of the roof 24 ft. 2′, allowing 523 ft. 3′ for the gables, and making no deduction for doors and windows? *Ans.* 7880 ft. 5′.

CONTRACTED METHOD.

391. The method of contracting the multiplication of decimals may be applied to duodecimals, the only modification being in carrying according to the duodecimal, instead of the decimal, scale.

1. Multiply 7 ft. 3′ 5″ 8‴ by 2 ft. 4′ 7″ 9‴, rejecting all denominations below seconds in the product.

OPERATION.

7 ft.	3′	5″	8‴
9‴	7″	4′	2 ft
14 ft.	6′	11″	
2 ft.	5′	2″	
	4′	3″	
		5″	
17 ft.	4′	9″±, *Ans.*	

ANALYSIS. We write 2 ft., the units of the multiplier, under the lowest order to be reserved in the product, and the other terms at the left, with their order reversed. Then it is obvious that the product of each term by the one above it is seconds. Hence we multiply each term of the multiplier into the terms above and to the left of it in the multiplicand, carrying from the rejected terms, thus; in multiplying by 2 ft., we have 8‴ × 2 ft. = 16‴ = 1″ 4‴, which being nearer 1″ than 2″, gives 1″ to be carried to the first contracted product. In multiplying by 4′, we have 5″ × 4′ = 20‴ = 1″ 8‴, which being nearer 2″ than 1″, gives 2″ to be carried to the second contracted product, and so on.

EXAMPLES FOR PRACTICE.

1. Multiply 7 ft. 3′ 4″ 5‴ by 5 ft. 8′ 6″, extending the product only to primes. *Ans.* 41 ft. 7′±

2. How many yards of carpeting will cover a floor 36 ft. 9′ 4″ long, and 26 ft. 6′ 9″ wide?

3. How many cu. ft. in a block of marble measuring 6 ft. 2′ 7″ in length, 3 ft. 3′ 4″ wide, and 2 ft. 8′ 6″ thick?

4. Find the product of 7 ft. 6′ 8″, 3 ft. 2′ 11″, and 3 ft. 8′ 4″, correct to within 1′. *Ans.* 90 ft. 6′±.

DIVISION.

392. 1. Divide 41 ft. 8′ 7″ 6‴ by 7 ft. 5′.

OPERATION.

```
7 ft. 5′)41 ft. 8′ 7″ 6‴(5 ft. 7′ 6″
         37 ft. 1′
         ---------
          4 ft. 7′  7″
          4 ft. 3′ 11″
          ------------
                3′  8″ 6‴
                3′  8″ 6‴
                ---------
```

ANALYSIS. Dividing the units of the dividend by the units of the divisor, we obtain 5 ft. for the first term of the quotient, and 4 ft. 7′ for a remainder. Bringing down the next term of the dividend, we have 4 ft. 7′ 7″ for a new dividend. Reducing the first two terms to primes, we have 55′ 7″, whence by trial division we obtain 7′ for the second term of the quotient, and 3′ 8″ for a remainder. Completing the division in like manner, we have 5 ft. 7′ 6″ for the entire quotient Hence the following

RULE. I. *Write the divisor on the left hand of the dividend, as in simple numbers.*

II. *Find the first term of the quotient either by dividing the first term of the dividend by the first term of the divisor, or by dividing the first two terms of the dividend by the first two terms of the divisor; multiply the divisor by this term of the quotient, subtract the product from the corresponding terms of the dividend, and to the remainder bring down another term of the dividend.*

III. *Proceed in like manner till there is no remainder. or till a quotient has been obtained sufficiently exact.*

EXAMPLES FOR PRACTICE.

1. Divide 287 ft. 7′ by 17 ft. *Ans.* 16 ft. 11′.
2. Divide 29 ft. 5′ 4″ by 6 ft. 8′. *Ans.* 4 ft. 5′.

3. A floor whose length is 48 ft. 6′ has an area of 1176 ft. 1′ 6″; what is its width? *Ans.* 24 ft. 3′.

4. From a cellar 38 ft. 10′ long and 9 ft. 4′ deep, were excavated 275 cu. yd. 5 cu. ft. 1′ 4″ of earth; how wide was the cellar? *Ans.* 20 ft. 6′.

CONTRACTED METHOD.

393. Division of Duodecimals may be abbreviated after the manner of contracted division of decimals.

1. Divide 35 ft. 11′ 11″ by 4 ft. 3′ 7″ 3‴, and find a quotient correct to seconds.

OPERATION.

```
4 ft. 3′ 7″ 3‴ ) 35 ft. 11′ 11″ ( 8 ft. 4′ 5″
                 34 ft.  4′ 10″
                 --------------
                  1 ft.  7′  1″
                  1 ft.  5′  2″
                 --------------
                         1′ 11″
                         1′  9″
                         ------
                             2″, rem.
```

Analysis. Having obtained by trial, 8 ft. for the first term of the quotient, we multiply three terms of the divisor, 4 ft. 3′ 7″, carrying from the rejected term, 3‴ × 8 = 24‴ = 2″, making 34 ft. 4′ 10″, which subtracted from the dividend leaves 1 ft. 7′ 1″ for a new dividend. In the next division, we reject 2 terms from the right of the divisor, and at the last division, 3 terms, and obtain for the required quotient, 8 ft. 4′ 5″.

EXAMPLES FOR PRACTICE.

1. Divide 7 ft. 7′ 3″ by 2 ft. 10′ 7″, extending the quotient to seconds. *Ans.* 2 ft. 7′ 8″±.

2. Separate 64 ft. 9′ 8″ into three factors, the first and second of which shall be 7 ft. 2′ 4″ and 4 ft. 7′ 9″ 8‴ respectively, and obtain the third factor correct to within 1 second.

Ans. 1 ft. 11′ 3″±.

3. What is the width of a room whose area is 36 ft. 4′ 8″ and whose length 7 ft. 2′ 11″?

SHORT METHODS.

394. Under the heads of Contractions in Multiplication and Contractions in Division, are presented only such short methods as are of the most extensive application. The short methods which follow, although limited in their application, are of much value in computations.

FOR SUBTRACTION.

395. When the minuend consists of one or more digits of any order higher than the highest order in the subtrahend.

The difference between any number and a unit of the next higher order is called an *Arithmetical Complement.* Thus, 4 is the arithmetical complement of 6, 31 of 69, 2792 of 7208, etc.

1. Subtract 29876 from 400000.

OPERATION.

$$\begin{array}{r} 400000 \\ 29876 \\ \hline 370124 \end{array}$$

ANALYSIS. To subtract 29876 from 400000 is the same as to subtract a number one less than 29876, or 29875, from 399999 (Ax. 2). We therefore diminish the 4 of the minuend by 1, and then take each figure of the subtrahend from 9, except the last or right-hand digit, which we subtract from 10. Hence the

RULE. I. *Subtract* 1 *from the significant part of the minuend and write the remainder, if any, as a part of the result.*

II. *Proceeding to the right, subtract each figure in the subtrahend from* 9, *except the last significant figure, which subtract from* 10.

EXAMPLES FOR PRACTICE.

1. Subtract 756 from 1000. *Ans.* 244.
2. Subtract 8576 from 4000000. *Ans.* 3991424.
3. Subtract .5768 from 10.
4. Subtract 13057 from 1700000.
5. Subtract 90.59876 from 64000.
6. Subtract 599948 from 1000000.
7. What is the arithmetical complement of 271? Of 18365? Of 3401250?

FOR MULTIPLICATION.

CASE I.

396. When the multiplier is 9, 99, or any number of 9's.

Annexing 1 cipher to a number multiplies it by 10, two ciphers by 100, three ciphers by 1000, etc. Since 9 is 10 — 1, any number may be multiplied by 9 by annexing 1 cipher to it and subtracting the number from the result. For similar reasons, 100 times a number — 1 time the number = 99 times the number, etc. Hence,

RULE. *Annex to the multiplicand as many ciphers as the multiplier contains 9's, and subtract the multiplicand from the result.*

EXAMPLES FOR PRACTICE.

1. Multiply 784 by 99. *Ans.* 77616.
2. Multiply 5873 by .999.
3. Multiply 4783 by 99999. *Ans.* 478295217.
4. Multiply 75 by 999.999.

CASE II.

397. When the multiplier is a number a few units less than the next higher unit.

Were we required to multiply by 97, which is 100 — 3, we could evidently annex 2 ciphers to the multiplicand, and subtract 3 times the multiplicand from the result. Were our multiplier 991, which is 1000 — 9, we could subtract 9 times the multiplicand from 1000 times the multiplicand. Hence,

RULE. I. *Multiply by the next higher unit by annexing ciphers.*

II. *From this result subtract as many times the multiplicand as there are units in the difference between the multiplier and the next higher unit.*

EXAMPLES FOR PRACTICE.

1. Multiply 786 by 98. *Ans.* 77028.
2. Multiply 4327 by 96. *Ans.* 415392.
3. Multiply 7328 by 997.

4. Multiply 7873.586 by 9.95. *Ans.* 78342.18070.
5. Multiply 43789 by 9994.
6. Multiply 7077364 by .999993.

CASE III.

398. When the left hand figure of the multiplier is the unit, 1, the right hand figure is any digit whatever, and the intervening figures, if any, are ciphers.

1. Multiply 3684 by 17.

OPERATION.

$$\frac{3684 \times 17}{62628}$$

ANALYSIS. If we multiply by the usual method, we obtain, separately, 7 times and 10 times the multiplicand, and add them. We may therefore multiply by the 7 units, and to the product add the multiplicand regarded as tens, thus: 7 times 4 is 28, and we write the 8 as the unit figure of the product. Then, 7 times 8 is 56, and the 2 reserved being added is 58, and the 4 in the multiplicand, added, is 62, and we write 2 in the product. Next, 7 times 6, plus the 6 reserved, plus the 8 in the multiplicand, is 56, and we write 6 in the product. Next, 7 times 3, plus the 5 reserved, plus the 36 in the multiplicand, is 62, which we write in the product, and the work is done.

Had the multiplier been 107, we should have multiplied two figures of the multiplicand by 7, before we commenced adding the digits of the multiplicand to the partial products; 3 figures had the multiplier been 1007, etc. Hence the

RULE. I. *Write the multiplier at the right of the multiplicand, with the sign of multiplication between them.*

II. *Multiply the multiplicand by the unit figure of the multiplier, and to the product add the multiplicand, regarding its local value as a product by the left hand figure of the multiplier.*

EXAMPLES FOR PRACTICE.

1. Multiply 567 by 13. *Ans.* 7371.
2. Multiply 439603 by 10.5. *Ans.* 4615831.5.
3. Multiply 7859 by 107.
4. Multiply 18075 by 1008. *Ans.* 18219600.
5. Multiply 3907 by 10.002.

CASE IV.

399. When the left hand figure of the multiplier is any digit, the right hand figure is the unit, 1, and the intermediate figures, if any, are ciphers.

1. Multiply 834267 by 301.

OPERATION.

$$\underline{834267 \times 301}$$
$$251114367$$

ANALYSIS. Regarding the multiplicand as a product by the unit, 1, of the multiplier, we multiply the multiplicand by 3 hundreds, and add the digits of the multiplicand to the several products as we proceed. Since the 3 is hundreds, the two right hand figures of the multiplicand will be the two right hand figures of the product; and the product of 3×7 will be increased by 2, the hundreds of the multiplicand.

Had the multiplier been 31, the *tens* of the multiplicand would have been added to 3×7; had the multiplier been 3001 the *thousands* of the multiplicand would have been added to 3×7; and so on. Hence the

RULE. I. *Write the multiplier at the right of the multiplicand, with the sign of multiplication between them.*

II. *Multiply the multiplicand by the left hand figure of the multiplier, and to the product add the multiplicand, regarding its local value as a product by the unit figure of the multiplier.*

EXAMPLES FOR PRACTICE.

1. Multiply 56783 by 71.
2. Multiply 47.89 by 60.1. *Ans.* 2878.189.
3. Multiply 3724.5 by .901
4. Multiply 103078 by 40001. *Ans.* 4123223078.

CASE V.

400. When the digits of the multiplier are all the same figure.

1. Multiply 81362 by 333.

OPERATION.

$$\begin{array}{r} 81362000 \\ \underline{81362} \\ 3\,)\,\underline{81280638} \\ 27093546 \end{array}$$

ANALYSIS. We first multiply by 999, by (**396**). Then, since 333 is $\frac{1}{3}$ of 999, we take $\frac{1}{3}$ of the product.

Had our multiplier been 444, we would have taken $\frac{4}{9}$ of 999 times the multiplicand. Had it been 66, we would have taken $\frac{6}{9} = \frac{2}{3}$ of 99 times the multiplicand, etc. Hence

Rule I. *Multiply by as many 9's as the multiplier contains digits, by* (**396**).

II. *Take such a part of the product as* 1 *digit of the multiplier is part of* 9.

EXAMPLES FOR PRACTICE.

1. Multiply 432711 by 222. *Ans.* 96061842.
2. Multiply 578 by 1111.
3. Multiply .6732 by 88.888. *Ans.* 59.8394016.
4. Multiply 8675 by 77.7.
5. Multiply 44444 by 88888.

CASE VI.

401. To square a number consisting of only two digits.

1. What is the square of 18?

Analysis. According to (**86**), we have

$$18^2 = 18 \times 18$$

Now if one of these factors be diminished by 2, the product will be less than the square of 18 by 2 times the other factor, (**93**, I); that is,

$$18^2 = (16 \times 18) + (2 \times 18).$$

Next, if we increase the other factor, 18, in this result, by 2, the whole result will exceed the square of 18, by 2 times the other factor, 16, (**93**, III); that is,

$$18^2 = (16 \times 20) + (2 \times 18) - (2 \times 16).$$

But as 2 times 18 minus 2 times 16 is equal to 2×2, or 2^2, we have

$$18^2 = 16 \times 20 + 2^2.$$ Hence the

Rule. I. *Take two numbers, one of which is as many units less than the number to be squared as the other is units greater, and one of the numbers taken an exact number of tens.*

II. *Multiply these two numbers together, and to the product add the square of the difference between the given and one of the assumed numbers.*

Note.—A little practice will enable the pupil to readily square any number less than 100 mentally by this rule.

EXAMPLES FOR PRACTICE.

1. What is the square of 27? *Ans.* 729.
2. What is the square of 49? *Ans.* 2401.
3. Square 28, 26, 39, 38, 37, 36, and 35.
4. Square 77, 88, 8.6, 99, 98, 69, 68, 6.7, and 62.

CASE VII.

402. When the multiplier is an aliquot part of some higher unit.

An **Aliquot** or **Even Part** of a number is such a part as will exactly divide that number. Thus, 5, $8\frac{1}{3}$, and $12\frac{1}{2}$ are aliquot parts of 25 and of 100, etc.

NOTE.—An *aliquot part* may be either a whole or a mixed number, while a *component factor* must be a whole number.

403. The aliquot parts of 10 are 5, $3\frac{1}{3}$, $2\frac{1}{2}$, 2, $1\frac{2}{3}$, $1\frac{3}{7}$, $1\frac{1}{4}$, $1\frac{1}{9}$. The aliquot parts of 100, 1000, or of any other number, may be found by dividing the number by 2, 3, 4, etc., until it has been divided by all the integral numbers between 1 and itself.

1. Multiply 78 by $3\frac{1}{3}$, and by 25 separately.

OPERATION.

$$3\,)\underline{780}\qquad\text{and}\qquad 4\,)\underline{7800}$$
$$\quad 260 \qquad\qquad\qquad\quad 1950$$

ANALYSIS. Since $3\frac{1}{3}$ is $\frac{1}{3}$ of 10, the next higher unit, we multiply 78 by 10 and take $\frac{1}{3}$ of the product. Again, since 25 is $\frac{1}{4}$ of 100, we multiply 78 by 100 and take $\frac{1}{4}$ of the product. Hence the

RULE. I. *Multiply the given multiplicand by the unit next higher than the multiplier, by annexing ciphers.*

II. *Take such a part of this product as the given multiplier is part of the next higher unit.*

EXAMPLES FOR PRACTICE.

1. Multiply 437 by 25. *Ans.* 10925.
2. Multiply 6872 by $2\frac{1}{2}$. *Ans.* 17180.
3. Multiply 5734154 by $333\frac{1}{3}$. *Ans.* $1911384666\frac{2}{3}$.
4. Multiply 758642 by $12\frac{1}{2}$.
5. Multiply 78563 by 125. *Ans.* 9820375.
6. Multiply 57687 by $142\frac{6}{7}$.

CASE VIII.

404. When the right hand figure or figures of the multiplier are aliquot parts of 10, 100, 1000, etc.

1. Multiply 2183 by $1233\frac{1}{3}$.

OPERATION.

$$\begin{array}{r} 218300 \\ 12\frac{1}{3} \\ \hline 72766\frac{2}{3} \\ 26196\phantom{\frac{2}{3}} \\ \hline 2692366\frac{2}{3} \end{array}$$

ANALYSIS. $1233\frac{1}{3} = 12\frac{1}{3} \times 100$. We therefore multiply by 100, and by $12\frac{1}{3}$, in continued multiplication. Hence the

RULE. I. *Reject from the right hand of the multiplier such figure or figures as are an aliquot part of some higher unit, and to the remaining figures of the multiplier annex a fraction which expresses the aliquot part thus rejected, for a reserved multiplier.*

II. *Annex to the multiplicand as many ciphers as are equal to the number of figures rejected from the right hand of the multiplier, and multiply the result by the reserved multiplier.*

EXAMPLES FOR PRACTICE.

1. Multiply 43789 by 825. *Ans.* 36125925.
2. Multiply 58730 by 7125.
3. Multiply 7854 by $34.2\frac{1}{2}$. *Ans.* 268999.5.
4. Multiply 30724 by $7333\frac{1}{3}$.
5. Multiply 47836 by $712\frac{1}{2}$. *Ans.* 34083150.
6. Multiply 53727 by $2416\frac{2}{3}$.

CASE IX.

405. To find the cost of a quantity when the price is an aliquot part of a dollar.

1. What cost a case of muslins containing 1627 yds., at $\$.12\frac{1}{2}$ per yard?

OPERATION.

$$\begin{array}{r|l} 8 & \$1627 \\ \hline & \$203.37\frac{1}{2} \end{array}$$

ANALYSIS. At \$1 per yard the case would cost as many dollars as it contained yards; and at $\$.12\frac{1}{2} = \$\frac{1}{8}$ per yard, it would cost $\frac{1}{8}$ as many dollars as it contained yards. We therefore regard the yards as dollars, which we divide by 8. Hence, RULE. *Take such a part of the given quantity as the price is part of one dollar.*

NOTE.—Since the shilling in most of the different currencies is some aliquot part of the dollar, this rule is of much practical use in making out bills and accounts where the prices of the items are given in State Currency, and the amounts are required in United States Money.

EXAMPLES FOR PRACTICE.

1. What cost 568 pounds of butter at 25 cents a pound? *Ans.* $142.

2. A merchant sold 51 yards of prints at $16\frac{2}{3}$ cents per yard, 8 pieces of sheeting, each piece containing 33 yards, at $6\frac{1}{4}$ cents per yard, and received in payment 18 bushels of oats at $33\frac{1}{3}$ cents per bushel, and the balance in money; how much money did he receive? *Ans.* $19.

3. Required the cost of 28 dozen candles, at 1 shilling per dozen, New York currency. *Ans.* $3.50.

4. What cost 576 lbs. of beef at 10d. per pound, Pennsylvania currency? *Ans.* $64.

5. If a grocer in New York gain $7.875 on a hogshead of molasses containing 63 gallons, how much will he gain on 576 gallons at the same rate? *Ans.* $72.

CASE X.

406. To find the cost of a quantity, when the quantity is a compound number, some part or all of which is an aliquot part of the unit of price.

1. What cost 5 bu. 3 pk. 4 qt. of cloverseed, at $3.50 per bu.?

OPERATION.

2,, 4,, 8)	$3.50	price.
	5	
	$17.50	cost of 5 bu.
	1.75	" " 2 pk.
	.875	" " 1 "
	.4375	" " 4 qt.
	$20.5625, *Ans.*	

ANALYSIS. Multiplying the price by 5, we have the cost of 5 bu. Dividing the price by 2, we have the cost of $\frac{1}{2}$ bu. = 2 pk. Dividing the price by 4, or the cost of 2 pk. by 2, we have the cost of 1 pk. Dividing the price by 8, or the cost of 2 pk. by 4, or the cost of 1 pk. by 2, we have the cost of ½ pk. = 4 qt. And the sum of these several values is the entire cost required.

2. At £6 7s. 5½d. Sterling per hhd., how much will 4 hhd. 9 gal. 3 qt. of West India Molasses cost?

OPERATION.

7)	£6	7s.	5½d.	price.
			4	
	" 25	9 "	10 "	cost of 4 hhd.
12)	"	18 "	2 " 2 qr.	" " 9 gal.
		1 "	6 " $\frac{5}{6}$	" " " 3 qt.
	" 26	9 "	6 " $2\frac{5}{6}$,	" *Ans.*

ANALYSIS. Multiplying the price by 4, we have the cost of 4 hhd. Dividing the price by 7, we have the cost of $\frac{1}{7}$ hhd. = 9 gal. Dividing the cost of 9 gal. by 12, we have the cost of $\frac{1}{12}$ of 36 qt. = 3 qt. And the sum of these several results is the entire cost required.

From these illustrations we deduce the following

RULE. I. *Multiply the price by the number of units of the denomination corresponding to the price.*

II. *For the lower denominations, take aliquot parts of the price; the sum of the several results will be the entire cost.*

NOTE.—This method is applicable in certain cases of multiplication, where one compound number is taken as many times as there are units and parts of a unit of a certain kind, in another compound number. This will be seen in the first example below.

EXAMPLES FOR PRACTICE.

1. A chemist filtered 18 gal. 3 qt. 1 pt. of rain-water in 1 day; at the same rate how much could he filter in 4 da. 6 h. 30 min.?

OPERATION.

18 gal.	3 qt.	1 pt.		in 1 da.	
		4			
75 "	2 "	0 "		" 4 "	
4 "	2 "	1 "	3 gi.	" 6 h.	
	1 "	1 "	$\frac{7}{12}$ "	" 30 min.	
80 "	2 "	0 "	$3\frac{7}{12}$ "		*Ans.*

ANALYSIS. Multiplying the quantity filtered in 1 day by 4, we have the quantity filtered in 4 days. Dividing the quantity filtered in 1 day by 4, we have the quantity filtered in ¼ da. = 6 h. Dividing the quantity filtered in 6 hours by 12, we have the quantity filtered in ½ h. = 30 min. And the sum of these several results is the entire result required.

2. What will be the cost of 3 lb. 10 oz. 8 pwt. $5\frac{1}{2}$ gr. of gold at \$15.46 per oz.? *Ans.* \$717.52.

3. A man bought 5 cwt. 90 lb. of hay at \$.56 per cwt.; what was the cost? *Ans.* \$3.304.

4. What must be given for 3 bu. 1 pk. 3 qt. of cloverseed, at \$4.48 per bushel? *Ans.* \$14.98.

5. A gallon of distilled water weighs 8 lb. 5 oz. 6.74 dr.; required the weight of 5 gal. 3 qt. 1 pt. 3 gi.

Ans. 49 lb. 12 oz. 5.73— dr.

6. At \$17.50 an acre, what will 3 A. 1 R. 35.4 P. of land cost?

7. If an ounce of English standard gold be worth £3 17s. $10\frac{1}{2}$d., what will be the value of an ingot weighing 7 oz. 16 pwt. 18 gr.?

Ans. £30 10s. 4.14375d.

8. If a comet move through an arc of $4° 36' 40''$ in 1 day, how far will it move in 5 da. 15 h. 32 min. 55 sec.?

9. What will be the cost of 7 gal. 1 qt. 1 pt. 3 gi. of burning fluid, at 4s. 8d. per gallon, N. Y. currency? *Ans.* \$4.35+.

10. What must be paid for $12\frac{1}{2}$ days' labor, at 5s. 3d. per day, New England currency?

FOR DIVISION.

CASE I.

407. When the divisor is an aliquot part of some higher unit.

1. Divide 260 by $3\frac{1}{3}$, and 1950 by 25.

OPERATION.

26|0		19|50
3	and	4
78		78

ANALYSIS. Since $3\frac{1}{3}$ is $\frac{1}{3}$ of 10, the next higher unit, we divide 260 by 10; and having used 3 times our true divisor, we obtain only $\frac{1}{3}$ of our true quotient. Multiplying the result, 26, by 3, we have 78, the true quotient. Again, since 25 is $\frac{1}{4}$ of 100, the next higher unit, we divide 1950 by 100; and having used 4 times our true divisor, the result, 19.5, is only $\frac{1}{4}$ of our true quotient. Multiplying 19.5 by 4, we have 78, the true quotient. Hence the

RULE. I. *Divide the given dividend by a unit of the order next higher than the divisor, by cutting off figures from the right.*

II. *Take as many times this quotient as the divisor is contained times in the next higher unit.*

EXAMPLES FOR PRACTICE.

1. Divide 63475 by 25. *Ans.* 2539.
2. Divide 7856 by 1.25. *Ans.* 6284.8.
3. Divide 516 by $33.3\frac{1}{3}$.
4. Divide 16.7324 by $12\frac{1}{2}$.
5. Divide 1748 by $.14\frac{2}{7}$. *Ans.* 12236.
6. Divide 576.34 by $1.6\frac{2}{3}$.

CASE II.

408. When the right hand figure or figures of the divisor are an aliquot part of 10, 100, 1000, etc.

1. Divide $2692366\frac{2}{3}$ by $1233\frac{1}{3}$.

OPERATION.

$$\begin{array}{r@{\,}l}
1233\frac{1}{3}\;) & 2692366\frac{2}{3} \\
3 & \qquad\qquad 3 \\
\hline
37|00\;) & 80771|00\;(\,2183,\ \textit{Ans.} \\
 & \;\;67 \\
 & \;\;307 \\
 & \;\;\;111
\end{array}$$

ANALYSIS. Since $33\frac{1}{3}$ is $\frac{1}{3}$ of 100, we multiply both dividend and divisor by 3, (**117**, III), and we obtain a divisor the component factors of which are 100 and 37. We then divide after the manner of contracted division, (**112**).

2. Divide 601387 by 1875.

OPERATION.

$$\begin{array}{r@{\,}r}
1875\;) & 601387 \\
4 & 4 \\
\hline
7500\;) & 2405548 \\
4 & 4 \\
\hline
3|0000\;) & 962|2192 \\
\hline
 & 320\frac{1387}{1875},\ \textit{Ans.}
\end{array}$$

ANALYSIS. Multiplying both dividend and divisor by 4, we obtain a new divisor, 7500, having 2 ciphers on the right of it. Multiplying again by 4, we obtain a new divisor, 30000, having 4 ciphers on the right. Then dividing the new dividend by the new divisor, we obtain 320 for a quotient, and 22192 for a remainder. As this remainder is a part of the new dividend, it must be $4 \times 4 = 16$ times the true remainder; we therefore divide it by 16, and write the result over the given divisor, 1875, and annex the fraction thus formed to the integers of the quotient.

From these illustrations we derive the following

RULE. I. *Multiply both dividend and divisor by a number or numbers that will produce for a new divisor a number ending in a cipher or ciphers.*

II. *Divide the new dividend by the new divisor.*

NOTE.—If the divisor be a whole number, or a finite decimal, the multiplier will be 2, 4, 5, or 8, or some multiple of one of these numbers.

EXAMPLES FOR PRACTICE.

1. Divide 64375 by 2575.
2. Divide 76394 by 3625. *Ans.* $21\frac{269}{3625}$.
3. Divide 7325 by $433\frac{1}{3}$.
4. Divide 5736 by 431.25. *Ans.* $13\frac{173}{575}$.
5. Divide 42.75 by $566\frac{2}{3}$.
6. Divide 24409375 by .21875.
7. Divide 785 by $3.14\frac{2}{7}$. *Ans.* $249\frac{17}{22}$.

RATIO.

409. **Ratio** is the relation of two like numbers with respect to comparative value.

NOTE.—There are two methods of comparing numbers with respect to value; 1st, by subtracting one from the other; 2d, by dividing one by the other. The relation expressed by the difference is sometimes called *Arithmetical Ratio*, and the relation expressed by the quotient, *Geometrical Ratio*.

410. When one number is compared with another, as 4 with 12, by means of division, thus, $12 \div 4 = 3$, the quotient, 3, shows the relative value of the dividend when the divisor is considered as a *unit* or *standard*. The ratio in this case shows that 12 is 3 times 4; that is, if 4 be regarded as a unit, 12 will be 3 units, or the relation of 4 to 12 is that of 1 to 3.

411. Ratio is indicated in two ways:

1st. By placing two points between the two numbers compared, writing the divisor before and the dividend after the points. Thus, the ratio of 8 to 24 is written 8 : 24; the ratio of 7 to 5 is written 7 : 5.

2d. In the form of a fraction. Thus, the ratio of 8 to 24 is written $\frac{24}{8}$; the ratio of 7 to 5 is $\frac{5}{7}$.

412. The **Terms** of a ratio are the two numbers compared.

The **Antecedent** is the first term; and

The **Consequent** is the second term.

The two terms of a ratio taken together are called a *couplet.*

413. A **Simple Ratio** consists of a single couplet; as 5 : 15.

414. A **Compound Ratio** is the product of two or more simple ratios. Thus, from the two simple ratios, 5 : 16 and 8 : 2, we

$$\begin{array}{r} 5 : 16 \\ 8 : \ \ 2 \end{array}$$

may form the compound ratio $\overline{5 \times 8 : 16 \times 2}$, or $\frac{16}{5} \times \frac{2}{8} = \frac{32}{40} = \frac{4}{5}$.

415. The **Reciprocal** of a ratio is 1 divided by the ratio; or, which is the same thing, it is the antecedent divided by the consequent. Thus, the ratio of 7 to 9 is 7 : 9 or $\frac{9}{7}$, and its reciprocal is $\frac{7}{9}$.

NOTE.—The quotient of the second term divided by the first is sometimes called a *Direct Ratio,* and the quotient of the first term divided by the second, an *Inverse* or *Reciprocal Ratio.*

416. One quantity is said to *vary directly* as another, when the two increase or decrease together in the same ratio; and one quantity is said to *vary inversely* as another, when one increases in the same ratio as the other decreases. Thus time varies *directly* as wages; that is, the greater the time the greater the wages, and the less the time the less the wages. Again, velocity varies *inversely* as the time, the distance being fixed; that is, in traveling a given distance, the greater the velocity the less the time, and the less the velocity the greater the time.

417. Ratio can exist only between like numbers, or between two quantities of the same kind. But of two unlike numbers or quantities, one may *vary* either *directly* or *inversely* as the other. Thus, cost varies directly as quantity, in the purchase of goods; time varies inversely as velocity, in the descent of falling bodies. In all cases of this kind, the quantities, though unlike in kind, have a mutual dependence, or sustain to each other the relation of *cause* and *effect.*

418. In the comparison of like numbers we observe,

I. If the numbers are *simple*, whether abstract or concrete, their ratio may be found directly by division.

II. If the numbers are *compound*, they must first be reduced to the same unit or denomination.

III. If the numbers are *fractional*, and have a common denominator, the fractions will be to each other as their numerators; if they have not a common denominator, their ratio may be found either directly by division, or by reducing them to a common denominator and comparing their numerators.

419. Since the antecedent is a divisor and the consequent a dividend, any change in either or both terms will be governed by the general principles of division, (**117**). We have only to substitute the terms *antecedent*, *consequent*, and *ratio*, for *divisor*, *dividend*, and *quotient*, and these principles become

GENERAL PRINCIPLES OF RATIO.

PRIN. I. *Multiplying the consequent multiplies the ratio; dividing the consequent divides the ratio.*

PRIN. II. *Multiplying the antecedent divides the ratio; dividing the antecedent multiplies the ratio.*

PRIN. III. *Multiplying or dividing both antecedent and consequent by the same number does not alter the ratio.*

420. These three principles may be embraced in one

GENERAL LAW.

A change in the consequent by multiplication or division produces a LIKE *change in the ratio; but a change in the antecedent produces an* OPPOSITE *change in the ratio.*

421. Since the *ratio* of two numbers is equal to the consequent divided by the antecedent, it follows, that

I. The *antecedent* is equal to the consequent divided by the ratio; and that,

II. The *consequent* is equal to the antecedent multiplied by the ratio.

EXAMPLES FOR PRACTICE.

1. What part of 28 is 7?

$\frac{7}{28} = \frac{1}{4}$; or, 28 : 7 as 1 : $\frac{1}{4}$; that is, 28 has the same ratio to 7 that 1 has to $\frac{1}{4}$. *Ans.* $\frac{1}{4}$.

2. What part of 42 is 6?

3. What is the ratio of 120 to 80? *Ans.* $\frac{2}{3}$.

4. What is the ratio of $8\frac{4}{7}$ to 60? *Ans.* 7.

5. What is the ratio of $\frac{7}{13}$ to 26?

6. What is the ratio of $7\frac{1}{8}$ to $2\frac{1}{2}$? *Ans.* $\frac{20}{57}$.

7. What is the ratio of $\frac{1}{6}$ to $\frac{7}{10}$? *Ans.* $4\frac{1}{5}$.

8. What is the ratio of 1 mi. to 3 fur.? *Ans.* $\frac{3}{8}$.

9. What is the ratio of 1 wk. 3 da. 12 h. to 9 wk.? *Ans.* 6.

10. What is the ratio of 10 A. 1 R. 20 P. to 6 A. 2 R. 30 P.?

11. What is the ratio of 25 bu. 2 pk. 6 qt. to 40 bu. 4.5 pk.?

12. What is the ratio of $18\frac{3}{4}°$ to 45′ 30″?

13. What part of $\dfrac{12\frac{1}{2}}{\frac{4}{7}}$ is $\dfrac{\frac{2}{3} \text{ of } \frac{3}{4}}{\frac{1}{2}}$? *Ans.* $\frac{8}{175}$.

14. What is the ratio of $\dfrac{11\frac{3}{2}}{8\frac{7}{8}}$ to $\frac{5}{8}$ of $\frac{3}{10}$ of $\dfrac{9\frac{3}{4}}{13}$? *Ans.* $\frac{639}{6400}$.

15. Find the reciprocal of the ratio of 42 to 28. *Ans.* $1\frac{1}{2}$.

16. Find the reciprocal of the ratio of 3 qt. to 43 gal.

17. If the antecedent be 15 and the ratio $\frac{4}{5}$, what is the consequent? *Ans.* 12.

18. If the consequent be $3\frac{1}{4}$ and the ratio 7, what is the antecedent? *Ans.* $\frac{13}{28}$.

19. If the antecedent be $\frac{1}{2}$ of $\frac{5}{8}$ and the consequent .75, what is the ratio?

20. If the consequent be $6.12\frac{1}{2}$ and the ratio 25, what is the antecedent? *Ans.* \$.245.

21. If the ratio be $\frac{1}{6}$ and the antecedent $\frac{3}{5}$, what is the consequent?

22. If the antecedent be 13 A. 3 R. 25 P. and the ratio $\frac{42}{89}$, what is the consequent? *Ans.* 6 A. 2 R. 10 P.

PROPORTION.

422. **Proportion** is an equality of ratios. Thus, the ratios 5 : 10 and 6 : 12, each being equal to 2, form a proportion.

NOTE.—When four numbers form a proportion, they are said to be *proportional*.

423. Proportion is indicated in three ways:

1st. By a double colon placed between the two ratios; thus, 3 : 4 : : 9 : 12 expresses the proportion between the numbers 3, 4, 9, and 12, and is read, 3 is to 4 as 9 is to 12.

2d. By the sign of equality placed between two ratios; thus, 3 : 4 = 9 : 12 expresses proportion, and may be read as above, or, the ratio of 3 to 4 equals the ratio of 9 to 12.

3d. By employing the second method of indicating ratio; thus, $\frac{4}{3} = \frac{12}{9}$ indicates proportion, and may be read as either of the above forms.

424. Since each ratio consists of two terms, every proportion must consist of at least four terms. Of these

The **Extremes** are the first and fourth terms; and

The **Means** are the second and third terms.

425. Three numbers are proportional when the first is to the second as the second is to the third. Thus, the numbers 4, 6, and 9 are proportional, since 4 : 6 = 6 : 9, the ratio of each couplet being $\frac{3}{2}$, or $1\frac{1}{2}$.

426. When three numbers are proportional, the second term is called the *Mean Proportional* between the other two.

427. If we have any proportion, as

$$3 : 15 = 4 : 20,$$

Then, indicating this ratio by the second method, we have

$$\frac{15}{3} = \frac{20}{4}.$$

Reducing these fractions to a common denominator,

$$\frac{15 \times 4}{12} = \frac{20 \times 3}{12}.$$

And since these two equal fractions have the same denominator, the numerator of the first, which is the product of the means, must be equal to the numerator of the second, which is the product of the extremes; or, $15 \times 4 = 20 \times 3$. Hence,

I. In every proportion the product of the means equals the product of the extremes.

Again, take any three terms in proportion, as

$$4 : 6 = 6 : 9$$

Then, since the product of the means equals the product of the extremes,

$$6^2 = 4 \times 9. \quad \text{Hence,}$$

II. The square of a mean proportional is equal to the product of the other two terms.

428. Since in every proportion the product of the means equals the product of the extremes, (**427**, I), it follows that, any three terms of a proportion being given, the fourth may be found by the following

RULE. I. *Divide the product of the extremes by one of the means, and the quotient will be the other mean.* Or,

II. *Divide the product of the means by one of the extremes, and the quotient will be the other extreme.*

EXAMPLES FOR PRACTICE.

The required term in an operation will be denoted by (?), which may be read "how many," or "how much."

Find the term not given in each of the following proportions:

1. 4 : 26 = 10 : (?). *Ans.* 65.
2. \$8865 : \$720 = (?) : 16 A. *Ans.* 197 A.
3. $4\frac{1}{2}$ yd. : (?) : : \$9.75 : \$29.25. *Ans.* $13\frac{1}{2}$ yd.
4. (?) : 21 A. 3 R. 20 P. : : \$1260 : \$750. *Ans.* 36 A. 3 R.
5. 7.50 : 18 = (?) : $7\frac{1}{15}$ oz.
6. 7 oz : (?) : : £30 : £407 2s. $10\frac{2}{7}$d. *Ans.* 7 lb. 11 oz.
7. (?) : .15 hhd. : : \$2.39 : \$.3585. *Ans.* 1 hhd.
8. 1 T. 7 cwt. 3 qr. 20 lb. : 13 T. 5 cwt. 2 qr. = \$9.50 : (?).
9. \$175.35 : (?) = $\frac{1}{8}$: $\frac{3}{7}$. *Ans.* \$601.20.
10. (?) : $\$12\frac{1}{2}$ = $240\frac{1}{7}$: $149\frac{177}{1127}$. *Ans.* $\$20\frac{1}{8}$.
11. $\frac{3}{5}$ yd. : (?) : : $\$\frac{7}{8}$: \$59.0625. *Ans.* $40\frac{1}{2}$ yd.

CAUSE AND EFFECT.

429. Every question in proportion may be considered as a comparison of two *causes* and two *effects*. Thus, if 3 dollars as a *cause* will buy 12 pounds as an *effect*, 6 dollars as a *cause* will buy 24 pounds as an *effect*. Or, if 5 horses as a *cause* consume 10 tons as an *effect*, 15 horses as a *cause* will consume 30 tons as an *effect*.

Causes and effects in proportion are of two kinds — simple and compound.

430. A **Simple Cause** or **Effect** contains but one element; as price, quantity, cost, time, distance, or any single factor used as a term in proportion.

431. A **Compound Cause** or **Effect** is the product of two or more elements; as the number of workmen taken in connection with the time employed, length taken in connection with breadth and depth, capital considered with reference to the time employed, etc.

432. Since *like causes* will always be connected with *like effects*, every question in proportion must give one of the following statements:

1st Cause : 2d Cause = 1st Effect : 2d Effect.
1st Effect : 2d Effect = 1st Cause : 2d Cause.

in which the two causes or the two effects forming one couplet, must be like numbers and of the same denomination.

Considering all the terms of a proportion as abstract numbers, we *may* say that

1st Cause : 1st Effect = 2d Cause : 2d Effect.

But as ratio is the result of comparing two numbers or things of the *same kind*, (**417**), the first form is regarded as the more natural and philosophical.

SIMPLE PROPORTION.

433. **Simple Proportion** is an equality of two simple ratios, and consists of four terms.

Questions in simple proportion involve only simple causes and simple effects.

FIRST METHOD.

1. If \$8 will buy 36 yards of velvet, how many yards may be bought for \$12?

STATEMENT.

$$\underset{\text{1st cause.}}{\overset{\$}{8}} : \underset{\text{2d cause.}}{\overset{\$}{12}} = \underset{\text{1st effect.}}{\overset{\text{yds.}}{36}} : \underset{\text{2d effect.}}{\overset{\text{yds.}}{(?)}}$$

OPERATION.

$$8 \times (?) = 12 \times 36$$

$$(?) = \frac{\overset{6}{\cancel{12}} \times \overset{9}{\cancel{36}}}{\cancel{8}} = 54 \text{ yd.}$$

ANALYSIS. The required term in this example is an effect; and the statement is, \$8 is to \$12 as 36 yards is to (?), or how many yards. Dividing 12×36, the product of the means, by 8, the given extreme, we have (?) = 54 yards, the required term, (**428, II**).

2. If 6 horses will draw 10 tons, how many horses will draw 15 tons?

STATEMENT.

$$\underset{\text{1st cause.}}{\overset{\text{horses.}}{6}} : \underset{\text{2d cause.}}{\overset{\text{horses.}}{(?)}} = \underset{\text{1st effect.}}{\overset{\text{tons.}}{10}} : \underset{\text{2d effect.}}{\overset{\text{tons.}}{15}}$$

OPERATION.

$$\begin{array}{c|c} \cancel{10} & \cancel{15}\,3 \\ (?) & \cancel{6}^{3} \\ \hline \end{array}$$

$$(?) = 9 \text{ horses.}$$

ANALYSIS. In this example a cause is required; and the statement is, 6 horses is to (?), or how many horses, as 10 tons is to 15 tons. Dividing 15×6, the product of the extremes, by 10, the given mean, we have 9, the required term, (**428, I**).

434. Hence the following

RULE. I. *Arrange the terms in the statement so that the causes shall compose one couplet, and the effects the other, putting (?) in the place of the required term.*

II. *If the required term be an extreme, divide the product of the means by the given extreme; if the required term be a mean, divide the product of the extremes by the given mean.*

NOTES.—1. If the terms of any couplet be of different denominations, they must be reduced to the same unit value.

2. If the odd term be a compound number, it must be reduced either to its lowest unit, or to a fraction or a decimal of its highest unit.

3. If the divisor and dividend contain one or more factors common to both, they should be canceled. If any of the terms of a proportion contain mixed

numbers, they should first be changed to improper fractions, or the fractional part to a decimal.

4. When the vertical line is used, the divisor and (?) are written on the left, and the factors of the dividend on the right.

SECOND METHOD.

435. The following method of solving examples in simple proportion without making the statement in form, may be used by those who prefer it.

Every question in simple proportion gives *three* terms to find a *fourth*. Of the three given terms, two will always be *like* numbers, forming the complete ratio, and the third will be of the same name or kind as the required term, and may be regarded as the antecedent of the incomplete ratio; hence the required term may be found by multiplying this third term, or antecedent, by the ratio of the other two, (**421**, II).

From the conditions of the question we can readily determine whether the answer, or required term, will be greater or less than the third term; if greater, then the ratio will be greater than 1, and the two like numbers must be arranged in the form of an improper fraction, as a multiplier; if less, then the ratio will be less than 1, and the two like numbers must be arranged in the form of a proper fraction, as a multiplier.

1. If 4 tons of hay cost $36, what will 5 tons cost?

OPERATION.

$\$36 \times \frac{5}{4} = \$45$, *Ans.*

ANALYSIS. In this example, 4 tons and 5 tons are the like terms, and $36 is the third term, and of the same kind as the answer sought. Now if 4 tons cost $36, will 5 tons cost more, or less, than $36? Evidently more: and the required term will be greater than the third term, $36, and the ratio greater than 1. We therefore arrange the like terms in the form of an improper fraction, $\frac{5}{4}$, for a multiplier, and obtain $45, the answer.

2. If 7 men build 21 rods of wall in a day, how many rods will 4 men build in the same time?

OPERATION.

$21 \times \frac{4}{7} = 12$ rods, *Ans.*

ANALYSIS. In this example, 7 men and 4 men are the like terms, and 21 rods is the third term, and of the same kind as the answer sought. Since 4 men will perform less work than 7 men in the same time, the required term will be less than

21, and the ratio less than 1. We therefore arrange the like terms in the form of a proper fraction, $\frac{4}{7}$, and obtain by multiplication, 12 rods, the answer.

436. Hence the following

RULE. I. *With the two given numbers, which are of the same name or kind, form a ratio greater or less than* 1, *according as the answer is to be greater or less than the third given number.*

II. *Multiply the third number by this ratio; the product will be the required number or answer.*

NOTES.—1. Mixed numbers should first be reduced to improper fractions, and the ratio of the fractions found according to **418.**

2. Reductions and cancellation may be applied as in the first method.

The following examples may be solved by either of the foregoing methods.

EXAMPLES FOR PRACTICE.

1. If 12 gallons of wine cost \$30, what will 63 gallons cost? 157

2. If 9 bushels of wheat make 2 barrels of flour, how many barrels of flour will 100 bushels make? *Ans.* $22\frac{2}{9}$.

3. If 18 bushels of wheat be bought for \$22.25, and sold for \$26.75, how much will be gained on 240 bushels, at the same rate of profit? *Ans.* \$60.

4. If $6\frac{1}{2}$ bushels of oats cost \$3, what will $9\frac{1}{4}$ bushels cost?

5. What will 87.5 yards of cloth cost, if $1\frac{3}{4}$ yards cost \$.42? 21.

6. If by selling \$1500 worth of dry goods I gain \$275.40, what amount must I sell to gain \$1000? \$4446 $\frac{858}{1377}$

7. If 20 men can perform a piece of work in 15 days, how many men must be added to the number, that the work may be accomplished in $\frac{4}{5}$ of the time? *Ans.* 5.

8. If 100 yd. of broadcloth cost $\$473.07\frac{9}{13}$, how much will 3.25 yd. cost? \$15.375000 5757

9. If 1 lb. 4 oz. 10 pwt. of gold may be bought for \$260.70, how much may be bought for \$39.50? *Ans.* 2 oz. 10 pwt.

10. In what time can a man pump 54 barrels of water, if he pump 24 barrels in 1 h. 14 min.? *Ans.* 2 h. 46 min. 30 sec.

11. If $\frac{4}{5}$ of a bushel of peaches cost $\$\frac{12}{25}$, what part of a bushel can be bought for $\$\frac{7}{20}$? *Ans.* $\frac{7}{12}$ bu.

12. If the annual rent of 46 A. 3 R. 14 P. of land be \$374.70, how much will be the rent of 35 A. 2 R. 10 P.? 49

13. If a man gain \$1870.65 by his business in 1 yr. 3 mo., how much would he gain in 2 yr. 8 mo., at the same rate? 3990.

14. Two numbers are to each other as 5 to $7\frac{1}{2}$, and the less is 164.5, what is the greater? *Ans.* 246.75.

15. If 16 head of cattle require 12 A. 3 R. 36 P. of pasture during the season, how many acres will 132 head of cattle require? *Ans.* 107 A. 7 P.

16. If a speculator in grain gain \$26.32 by investing \$325, how much would he gain by investing \$2275? 184.24

17. What will be the cost of paving an open court 60.5 ft. long and 44 ft. wide, if 14.25 sq. yd. cost $\$34\frac{1}{5}$? 219.19+

18. At $6\frac{1}{4}$ cents per dozen, what will be the cost of $10\frac{3}{4}$ gross of steel pens? 7.64 1/16

19. If when wheat is 7s. 6d. per bushel, the bakers' loaf will weigh 9 oz., what ought it to weigh when wheat is 6s. per bushel? *Ans.* $11\frac{1}{4}$ oz.

COMPOUND PROPORTION.

437. Compound Proportion is an expression of equality between a compound and a simple ratio, or between two compound ratios.

It embraces the class of questions in which the causes, or the effects, or both, are compound. The required term must be either a simple cause or effect, or a single element of a compound cause or effect.

FIRST METHOD.

1. If 8 men mow 40 acres of grass in 3 days, how many acres will 9 men mow in 4 days?

STATEMENT.

1st cause.	2d cause.		1st effect.	2d effect.
$\left\{\begin{matrix}8\\3\end{matrix}\right.$:	$\left\{\begin{matrix}9\\4\end{matrix}\right.$	=	40 :	(?)

Or, $8 \times 3 : 9 \times 4 = 40 : (?)$

OPERATION.

$$(?) = \frac{9 \times 4 \times 40}{8 \times 3} = 60, \ Ans.$$

ANALYSIS. In this example the required term is the second effect; and the statement is, 8 men 3 days is to 9 men 4 days, as 40 acres is to (?), or how many acres. Dividing the continued product of all the elements of the means by the elements of the given extreme, we obtain (?) = 60 acres.

2. If 6 compositors in 14 hours can set 36 pages of 56 lines each, how many compositors, in 12 hours, can set 48 pages of 54 lines each?

STATEMENT.

$$\overset{\text{1st cause.}}{\begin{cases} 6 \\ 14 \end{cases}} : \overset{\text{2d cause.}}{\begin{cases} (?) \\ 12 \end{cases}} :: \overset{\text{1st effect.}}{\begin{cases} 36 \\ 56 \end{cases}} : \overset{\text{2d effect.}}{\begin{cases} 48 \\ 54 \end{cases}}$$

OPERATION.

$$\begin{array}{r|l} (?) & \cancel{6} \\ \cancel{12} & \cancel{14} \\ \cancel{36} & \cancel{48} \\ \cancel{56} & \cancel{54} \\ \hline \end{array}$$

$(\ ) = 9$, *Ans.*

ANALYSIS. In this example, an element of the second cause is required; and the statement is, 6 compositors 14 hours is to (?) compositors 12 hours as 36 pages of 56 lines each is to 48 pages of 54 lines each. Now, since the required term is an element of one of the means, we divide the continued product of all the elements of the extremes by the continued product of all the given elements of the means. Placing the dividend on the right of the vertical line and the divisors on the left, and canceling equal factors we obtain (?) = 9.

438. From these illustrations we deduce the following

RULE. I. *Of the given terms, select those which constitute the causes, and those which constitute the effects, and arrange them in couplets, putting* (?) *in place of the required term.*

II. *Then, if the blank term* (?) *occur in either of the extremes, divide the product of the means by the product of the extremes; but if the blank term occur in either mean, divide the product of the extremes by the product of the means.*

NOTES.—1. The causes must be exactly alike in the *number* and *kind* of their terms: the same is true of the effects.

2. The same preparation of the terms by reduction is to be observed as in simple proportion.

SECOND METHOD.

439. The second method given in Simple Proportion, is also applicable in Compound Proportion.

In every example in compound proportion all the terms appear in *couplets*, except one, called the *odd* term, which is always of the same kind as the answer sought. Hence the required term in a compound proportion may be found, by multiplying the odd term by the compound ratio composed of all the simple ratios formed by these couplets, each couplet being arranged in the form of a fraction.

The fraction formed by any couplet will be improper when the required term, considered as depending on this couplet alone, should be greater than the odd term; and proper, when the required term should be less than the odd term.

1. If it cost \$4320 to supply a garrison of 32 men with provisions for 18 days, when the rations are 15 ounces per day, what will it cost to supply a garrison of 24 men 34 days, when the rations are 12 ounces per day?

OPERATION.

$$\$4320 \times \underset{\text{men.}}{\tfrac{24}{32}} \times \underset{\text{days.}}{\tfrac{34}{18}} \times \underset{\text{ounces.}}{\tfrac{12}{15}} = \$4896$$

BY CANCELLATION.

(?)	4320
32	24
18	34
15	12

( ) = \$4896, *Ans.*

ANALYSIS. In this example there are three pairs of terms, or couplets, viz., 32 men and 24 men, 18 days and 34 days, 15 ounces and 12 ounces; and there is an odd term, \$4320, which is of the same kind as the required term. We arrange each couplet as a multiplier of this term, thus; First, if it cost \$4320 to supply 32 men, will it cost more, or less, to supply 24 men? Less; we therefore arrange the couplet in the form of a proper fraction as a multiplier, and we have \$4320 $\times \frac{24}{32}$. Next, if it cost \$4320 to supply a garrison 18 days, will it cost more, or less, to supply it 34 days? More; hence the multiplier is the improper fraction $\frac{24}{18}$, and we have \$4320 $\times \frac{24}{32} \times \frac{34}{18}$. Next, if it cost \$4320 to supply a garrison with rations of 15 ounces, will it cost more, or less, when the rations are 12 ounces? Less; consequently, the multiplier is the proper fraction $\frac{12}{15}$, and we have \$4320 $\times \frac{24}{32} \times \frac{34}{18} \times \frac{12}{17} =$ \$4896, the required term. Hence the following

RULE. I. *Of the terms composing each couplet form a ratio greater or less than 1, in the same manner as if the answer depended on those two and the third or odd term.*

II. *Multiply together the third or odd term and these ratios; the product will be the answer sought.*

EXAMPLES FOR PRACTICE.

1. If 12 horses plow 11 acres in 5 days, how many horses would plow 33 acres in 18 days? *Ans.* 10.

2. If 480 bushels of oats will last 24 horses 40 days, how long will 300 bushels last 48 horses, at the same rate?
Ans. $12\frac{1}{2}$ days.

3. If 7 reaping machines can cut 1260 acres in 12 days, in how many days can 16 machines reap 4728 acres?
Ans. 19.7 days.

4. If 144 men in 6 days of 12 hours each, build a wall 200 ft. long, 3 ft. high, and 2 ft. thick, in how many days of 7 hours each can 30 men build a wall 350 ft. long, 6 ft. high, and 3 ft. thick? *Ans.* 259.2 da.

5. In how many days will 6 persons consume 5 bu. of potatoes, if 3 bu. 3 pk. last 9 persons 22 days?

6. How many planks $10\frac{2}{3}$ ft. long and $1\frac{1}{2}$ in. thick, are equivalent to 3000 planks 12 ft. 8 in. long and $2\frac{1}{4}$ in. thick?
Ans. $6531\frac{1}{4}$.

7. If 300 bushels of wheat @ $1.25 will discharge a certain debt, how many bushels @ $.90 will discharge a debt 3 times as great? *Ans.* 1250 bu.

8. If 468 bricks, 8 inches long and 4 inches wide, are required for a walk 26 ft. long and 4 ft. wide, how many bricks will be required for a walk 120 ft. long and 6 ft. wide?

9. If a cistern $17\frac{1}{2}$ ft. long, $10\frac{1}{2}$ ft. wide, and 13 ft. deep, hold 546 barrels, how many barrels will a cistern hold that is 16 ft. long, 7 ft. wide, and 15 ft. deep? *Ans.* 384 bbl.

10. If 11 men can cut 147 cords of wood in 7 days, when they work 14 hours per day, how many days will it take 5 men to cut 150 cords, working 10 hours each day?

PROMISCUOUS EXAMPLES IN PROPORTION.

1. If a staff 4 ft. long cast a shadow 7 ft. in length, what is the hight of a tower that casts a shadow of 198 ft. at the same time? *Ans.* $113\frac{1}{7}$ ft.

2. A person failing in business owes \$972, and his entire property is worth but \$607.50; how much will a creditor receive on a debt of $\$11.33\frac{1}{3}$? *Ans.* \$7.08+.

3. If 3 cwt. can be carried 660 mi. for \$4, how many cwt. can be carried 60 mi. for \$12? *Ans.* 99.

4. A man can perform a certain piece of work in 18 days by working 8 hours a day; in how many days can he do the same work by working 10 hours a day? *Ans.* $14\frac{2}{5}$.

5. How much land worth \$16.50 an acre, should be given in exchange for 140 acres, worth \$24.75 an acre?

6. If I gain \$155.52 on \$1728 in 1 yr. 6 mo., how much will I gain on \$750 in 4 yr. 6 mo.? *Ans.* \$202.50.

7. If 1 lb. 12 oz. of wool make $2\frac{1}{2}$ yd. of cloth 6 qr. wide, how many lb. of wool will it take for 150 yd. of cloth 4 qr. wide?

8. What number of men must be employed to finish a piece of work in 5 days, which 15 men could do in 20 days? *Ans.* 60.

9. At 12s. 7d. per oz., N. Y. currency, what will be the cost of a service of silver plate weighing 15 lb. 11 oz. 13 pwt. 17 gr.?

10. If a cistern 16 ft. long, 7 ft. wide, and 15 ft. deep, cost \$36.72, how much, at the same rate per cubic foot, would another cistern cost that is $17\frac{1}{2}$ ft. long, $10\frac{1}{2}$ ft. wide, and 16 ft. deep?

11. A borrows \$1200 and keeps it 2 yr. 5 mo. 5 da.; what sum should he lend for 1 yr. 8 mo. to balance the favor?

12. A farmer has hay worth \$9 a ton, and a merchant has flour worth \$5 per barrel. If in trading the former asks \$10.50 for his hay, how much should the merchant ask for his flour?

13. If 12 men, working 9 hours a day for $15\frac{5}{9}$ days, were able to execute $\frac{2}{5}$ of a job, how many men may be withdrawn and the job be finished in 15 days more, if the laborers are employed only 7 hours a day? *Ans.* 4.

14. If the use of \$300 for 1 yr. 8 mo. is worth \$30, how much is the use of \$210.25 for 3 yr. 4 mo. 24 da. worth?

15. What quantity of lining $\frac{3}{4}$ yd. wide, will it require to line $9\frac{1}{2}$ yd. of cloth, $1\frac{1}{4}$ yd. wide? *Ans.* $15\frac{5}{6}$ yd.

16. If it cost $95.60 to carpet a room 24 ft. by 18 ft., how much will it cost to carpet a room 38 ft. by 22 ft. with the same material? *Ans.* $185.00+.

17. If $16\frac{7}{18}$ cords of wood last as long as $11\frac{9}{26}$ tons of coal, how many cords of wood will last as long as $15\frac{7}{13}$ tons of coal?

18. A miller has a bin 8 ft. long, $4\frac{1}{5}$ ft. wide, and $2\frac{1}{2}$ ft. deep, and its capacity is 75 bu.; how deep must he make another bin which is to be 18 ft. long and $3\frac{5}{6}$ feet wide, that its capacity may be 450 bu.? *Ans.* $7\frac{7}{23}$ ft.

19. If 4 men in $2\frac{1}{2}$ days, mow $6\frac{2}{3}$ acres of grass, by working $8\frac{1}{4}$ hours a day, how many acres will 15 men mow in $3\frac{3}{4}$ days, by working 9 hours a day? *Ans.* $40\frac{10}{11}$ acres.

20. If an army of 600 men have provisions for 5 weeks, allowing each man 12 oz. a day, how many men may be maintained 10 weeks with the same provisions, allowing each man 8 oz. a day?

21. A cistern holding 20 barrels has two pipes, by one of which it receives 120 gallons in an hour, and by the other discharges 80 gallons in the same time; in how many hours will it be filled?

22. A merchant in selling groceries sells $14\frac{9}{16}$ oz. for a pound; how much does he cheat a customer who buys of him to the amount of $38.40? *Ans.* $3.45.

23. If 5 lb. of sugar costs $.62$\frac{1}{2}$, and 8 lb. of sugar are worth 5 lb. of coffee, how much will 75 lb. of coffee cost?

24. B and C have each a farm; B's farm is worth $32.50 an acre, and C's $28.75; but in trading B values his at $40 an acre. What value should C put upon his?

25. If it require $859\frac{3}{8}$ reams of paper to print 12000 copies of an 8vo. book containing 550 pages, how many reams will be required to print 3000 copies of a 12mo. book containing 320 pages?

26. If 248 men, in $5\frac{1}{2}$ days of 12 hours each, dig a ditch of 7 degrees of hardness, $232\frac{1}{2}$ yd. long, $3\frac{2}{3}$ yd. wide, and $2\frac{1}{3}$ yd. deep; in how many days of 9 hours each, will 24 men dig a ditch of 4 degrees of hardness, $387\frac{1}{2}$ yd. long, $5\frac{1}{4}$ yd. wide, and $3\frac{1}{2}$ yd deep? *Ans.* 155

PERCENTAGE.

440. Per Cent. is a contraction of the Latin phrase *per centum*, and signifies *by the hundred;* that is, a certain part of every hundred, of any denomination whatever. Thus, 4 per cent means 4 of every hundred, and may signify 4 cents of every 100 cents, 4 dollars of every 100 dollars, 4 pounds of every 100 pounds, etc.

NOTATION.

441. The character, %, is generally employed in business transactions to represent the words *per cent.;* thus 6 % signifies 6 per cent.

442. Since any per cent. is some number of hundredths, it is properly expressed by a *decimal fraction;* thus 5 per cent. = 5 % = .05. Per cent. may always be expressed, however, either by a *decimal* or a *common fraction*, as shown in the following

TABLE.

Words.		Symbols.		Decimals.		Common fractions.		
1 per cent.	=	1 %	=	.01	=	$\frac{1}{100}$	=	$\frac{1}{100}$
2 per cent.	=	2 %	=	.02	=	$\frac{2}{100}$	=	$\frac{1}{50}$
4 per cent.	=	4 %	=	.04	=	$\frac{4}{100}$	=	$\frac{1}{25}$
5 per cent.	=	5 %	=	.05	=	$\frac{5}{100}$	=	$\frac{1}{20}$
6 per cent.	=	6 %	=	.06	=	$\frac{6}{100}$	=	$\frac{3}{50}$
7 per cent.	=	7 %	=	.07	=	$\frac{7}{100}$	=	$\frac{7}{100}$
8 per cent.	=	8 %	=	.08	=	$\frac{8}{100}$	=	$\frac{2}{25}$
10 per cent.	=	10 %	=	.10	=	$\frac{10}{100}$	=	$\frac{1}{10}$
20 per cent.	=	20 %	=	.20	=	$\frac{20}{100}$	=	$\frac{1}{5}$
25 per cent.	=	25 %	=	.25	=	$\frac{25}{100}$	=	$\frac{1}{4}$
50 per cent.	=	50 %	=	.50	=	$\frac{50}{100}$	=	$\frac{1}{2}$
100 per cent.	=	100 %	=	1.00	=	$\frac{100}{100}$	=	1
125 per cent.	=	125 %	=	1.25	=	$\frac{125}{100}$	=	$\frac{5}{4}$
$\frac{1}{2}$ per cent.	=	$\frac{1}{2}$ %	=	$.00\frac{1}{2}$	=	$\frac{\frac{1}{2}}{100}$	=	$\frac{1}{200}$
$\frac{3}{4}$ per cent.	=	$\frac{3}{4}$ %	=	$.00\frac{3}{4}$	=	$\frac{\frac{3}{4}}{100}$	=	$\frac{3}{400}$
$12\frac{1}{2}$ per cent.	=	$12\frac{1}{2}$ %	=	$.12\frac{1}{2}$	=	$\frac{12\frac{1}{2}}{100}$	=	$\frac{1}{8}$

EXAMPLES FOR PRACTICE.

1. Express decimally 3 per cent.; 9 per cent.; 12 per cent.; 16 per cent.; 23 per cent.; 37 per cent.; 75 per cent.; 125 per cent.; 184 per cent.; 205 per cent.

2. Express decimally 15 %; 11 %; $4\frac{1}{2}$ %; $5\frac{1}{4}$ %; $8\frac{3}{4}$ %; $20\frac{1}{2}$ %; $25\frac{5}{8}$ %; $35\frac{3}{5}$ %; $24\frac{7}{8}$ %; $130\frac{1}{2}$ %.

3. Express decimally $\frac{1}{4}$ per cent.; $\frac{3}{4}$ per cent.; $\frac{1}{2}$ per cent.; $\frac{2}{5}$ per cent.; $\frac{3}{8}$ per cent.; $\frac{7}{25}$ per cent.; $\frac{36}{125}$ per cent.; $1\frac{5}{16}$ per cent.; $10\frac{1}{5}$ per cent.

4. Express by common fractions, in their lowest terms, 4 %; $37\frac{1}{2}$ %; $16\frac{4}{5}$ %; $11\frac{1}{9}$ %; $42\frac{6}{7}$ %; $45\frac{5}{11}$ %; $43\frac{9}{37}$ %.

5. What per cent. is .0725?

ANALYSIS. $.0725 = .07\frac{1}{4} = 7\frac{1}{4}$ %, *Ans.*

6. What per cent. is .065? *Ans.* $6\frac{1}{2}$ %.
7. What per cent. is .14375? *Ans.* $14\frac{3}{8}$ %.
8. What per cent. is .0975?
9. What per cent. is .014?
10. What per cent. is .1025?
11. What per cent. is .004?
12. What per cent. is .028?
13. What % is .1324?
14. What % is $.084\frac{2}{7}$?
15. What % is $.004\frac{6}{11}$? *Ans.* $\frac{5}{11}$ %
16. What % is $.003\frac{1}{13}$?

GENERAL PROBLEMS IN PERCENTAGE.

443. In the operations of Percentage there are five parts or elements, namely: Rate per cent., Percentage, Base, Amount, and Difference.

444. Rate per Cent., or Rate, is the decimal which denotes how many hundredths of a number are to be taken.

NOTES.—1. Such expressions as 6 per cent., and 5 %, are essentially *decimals*, the words *per cent.*, or the character %, indicating the decimal denominator.

2. If the decimal be reduced to a common fraction in its *lowest terms*, this fraction will still be the equivalent *rate*, though not the rate *per cent.*

445. **Percentage** is that part of any number which is indicated by the rate.

446. The **Base** is the number on which the percentage is computed.

447. The **Amount** is the sum obtained by adding the percentage to the base.

448. The **Difference** is the remainder obtained by subtracting the percentage from the base.

PROBLEM I.

449. Given, the base and rate, to find the percentage.

1. What is 5 % of 360?

OPERATION.

$$\begin{array}{r} 360 \\ .05 \\ \hline 18.00, \textit{ Ans.} \end{array}$$

Or,

$360 \times \frac{1}{20} = 18$, *Ans.*

ANALYSIS. Since 5 % of any number is .05 of that number, (**442**), we multiply the *base*, 360, by the *rate*, .05, and obtain the *percentage*, 18. Or, since the rate is $\frac{5}{100} = \frac{1}{20}$, we have $360 \times \frac{1}{20} =$ 18, the percentage. Hence the following

RULE. *Multiply the base by the rate.*

NOTE 1.—Percentage is always a *product*, of which the *base* and *rate* are the *factors*.

EXAMPLES FOR PRACTICE.

1. What is 4 per cent. of 250? *Ans.* 10.
2. What is 7 per cent. of 3500? *Ans.* 245.
3. What is 16 per cent. of 324? *Ans.* 51.84.
4. What is $12\frac{1}{2}$ per cent. of \$5600? *Ans.* \$700.
5. What is 9 % of 785 lbs.?
6. What is 25 % of 960 mi.?
7. What is 75 % of 487 bu.? *Ans.* 365.25 bu.
8. What is $33\frac{1}{3}$ % of 2757 men?
9. What is 125 % of 756?
10. What is $\frac{1}{4}$ % of \$2364? *Ans.* \$5.91.

11. What is $3\frac{2}{3}$ % of \$856? *Ans.* \$31.39 —.

12. What is $\frac{3}{4}$ % of $\frac{5}{6}$? *Ans.* $\frac{1}{160}$.

13. What is $14\frac{2}{7}$ % of $5\frac{1}{4}$?

14. If the base is \$375, and the rate .05, what is the percentage? *Ans.* \$18.75.

15. A man owed \$536 to A, \$450 to B, and \$784 to C; how much money will be required to pay 54 % of his debts?

16. My salary is \$1500 a year; if I pay 15 % for board, 5 % for clothing, 6 % for books, and 8 % for incidentals, what are my yearly expenses? *Ans.* \$510.

NOTE 2.—15 % + 5 % + 6 % + 8 % = 34 %. In all cases where several rates refer to the same *base*, they may be added or subtracted, according to the conditions of the question.

17. A man having a yearly income of \$3500, spends 10 per cent of it the first year, 12 per cent. the second year, and 18 per cent. the third year; how much does he save in the 3 years?

18. A had \$6000 in a bank. He drew out 25 % of it, then 30 % of the remainder, and afterward deposited 10 % of what he had drawn; how much had he then in bank? *Ans.* \$3435.

19. A merchant commenced business, Jan. 1, with a capital of \$5400, and at the end of 1 year his ledger showed the condition of his business as follows: For Jan., 2 % gain; Feb., $3\frac{1}{4}$ % gain; March, $\frac{1}{2}$ % loss; Apr., 2 % gain; May, $2\frac{1}{2}$ % gain; June, $1\frac{3}{4}$ % loss; July, $1\frac{1}{2}$ % gain; Aug., 1 % loss; Sept., $2\frac{3}{4}$ % gain; Oct., 4 % gain; Nov., $\frac{3}{4}$ % loss; Dec., 3 % gain. What were the net profits of his business for the year? *Ans.* \$918.

PROBLEM II.

450. Given, the percentage and base, to find the rate.

1. What per cent of 360 is 18?

OPERATION.

$18 \div 360 = .05 = 5$ %

Or,

$\frac{18}{360} = \frac{1}{20} = .05 = 5$ %

ANALYSIS. Since the percentage is always the *product* of the base and rate, (**449**), we divide the given percentage, 18, by the given base, 360, and obtain the required rate, .05 = 5 %. Hence the

RULE. *Divide the percentage by the base.*

EXAMPLES FOR PRACTICE.

1. What per cent. of \$720 is \$21.60? *Ans.* 3.
2. What per cent. of 1560 lb. is 234 lb.?
3. What per cent. of 980 rd. is 49 rd.?
4. What per cent. of £320 10s. is £25 12.8s.? *Ans.* 8.
5. What per cent. of 46 gal. is 5 gal. 3 qt.? *Ans.* $12\frac{1}{2}$.
6. What per cent. of 7.85 mi. is 5.495 mi.? *Ans.* 70.
7. What per cent. of $\frac{8}{15}$ is $\frac{2}{5}$? *Ans.* 75.
8. What per cent. of $\frac{4}{7}$ is $\frac{3}{25}$?
9. What per cent. of 560 is 80?
10. The base is \$578, and the percentage is \$26.01; what is the rate? *Ans.* $4\frac{1}{2}$ %.
11. The base is \$972.24, and the percentage is \$145.836; what is the rate?
12. An editor having 5600 subscribers, lost 448; what was his loss per cent? *Ans.* 8.
13. A merchant owes \$7560, and his assets are \$4914; what per cent. of his debts can he pay? *Ans.* 65.
14. A man shipped 2600 bushels of grain from Chicago, and 455 bushels were thrown overboard during a gale; what was the rate per cent. of his loss?
15. A miller having 720 barrels of flour, sold 288 barrels; what per cent. of his stock remained unsold? *Ans.* 60.
16. What per cent. of a number is 30 % of $\frac{4}{5}$ of it?
17. The total expenditures of the General Government, for the year ending June 30, 1858, were \$83,751,511.57; the expenses of the War Department were \$23,243,822.38, and of the Navy Department, \$14,712,610.21. What per cent. of the whole expense of government went for armed protection?

Ans. $45\frac{1}{3}$, nearly.

18. In the examination of a class, 165 questions were submitted to each of the 5 members; A answered 130 of them, B 125, C 96, D 110, and E 160. What was the standing of the class? *Ans.* $75.2\dot{7}$ %.

PROBLEM III.

451. Given, the percentage and rate, to find the base.

1. 18 is 5 % of what number?

OPERATION.

$18 \div .05 = 360$, *Ans.*

Or,

$18 \div \frac{1}{20} = 360$, *Ans.*

ANALYSIS. Since the percentage is always the *product* of the base and rate, (**449**), we divide the given percentage, 18, by the given rate, .05, or $\frac{1}{20}$, and obtain the base, 360. Hence the

RULE. *Divide the percentage by the rate.*

EXAMPLES FOR PRACTICE.

1. 18 is 25 % of what number? *Ans.* 72.
2. 54 is 15 % of what number?
3. 17.5 is $2\frac{1}{3}$ % of what number? *Ans.* 750.
4. 2.28 is 5 % of what number?
5. 414 is 120 % of what number?
6. 6119 is $105\frac{1}{2}$ % of what number? *Ans.* 5800.
7. .43 is $71\frac{2}{3}$ % of what number? *Ans.* .6.
8. The percentage is \$18.75, and the rate is $2\frac{1}{2}$ %; what is the base? *Ans.* \$750.
9. The percentage is $31\frac{1}{4}$, and the rate $31\frac{1}{4}$ %; what is the base?
10. I sold my house for \$4578, which was 84 % of its cost; what was the cost? *Ans.* \$5450.
11. A wool grower sold 3150 head of sheep, and had 30 % of his original flock left; how many sheep had he at first?
12. A man drew 40 % of his bank deposits, and expended $13\frac{1}{3}$ % of the money thus drawn in the purchase of a carriage worth \$116; how much money had he in bank? *Ans.* \$2175.
13. If \$147.56 is $13\frac{1}{3}$ % of A's money, and $4\frac{2}{3}$ % of A's money is 8 % of B's, how much more money has A than B? *Ans.* $\$461.12\frac{1}{2}$.

14. In a battle 4 % of the army were slain upon the field; and 5 % of the remainder died of wounds, in the hospital. The difference between the killed and the mortally wounded was 168; how many men were there in the army? *Ans.* 21000.

NOTE.—100 % — 4 % = 96 %, left after the battle; and 5 % of 96 % = $4\frac{4}{5}$ %, the part of the army that died of wounds.

15. A owns $\frac{3}{4}$ of a prize and B the remainder; after A has taken 40 % of his share, and B 20 % of his share, the remainder is equitably divided between them by giving A $1950 more than B; what is the value of the prize? *Ans.* $7800.

PROBLEM IV.

452. Given, the amount and rate, to find the base.

1. What number increased by 5 % of itself is equal to 378?

OPERATION.

$$1 + .05 = 1.05$$
$$378 \div 1.05 = 360, \textit{ Ans.}$$

Or,

$$1 + \tfrac{1}{20} = \tfrac{21}{20}$$
$$378 \div \tfrac{21}{20} = 360, \textit{ Ans.}$$

ANALYSIS. If any number be increased by 5 % of itself the amount will be 1.05 times the number. We therefore divide the given *amount*, 378, by 1.05, or $\frac{21}{20}$, and obtain the *base*, 360, which is the number required. Hence the

RULE. *Divide the amount by* 1 *plus the rate.*

NOTE 1.—The amount is always a *product*, of which the base is one factor, and 1 plus the rate the other factor.

EXAMPLES FOR PRACTICE.

1. What number increased by 15 % of itself is equal to 644? *Ans.* 560.

2. A has $815.36, which is 4 % more than B has; how much money has B? *Ans.* $784.

3. Having increased my stock in trade by 12 % of itself, I find that I have $3800; how much had I at first?

4. In 1860 the population of a certain city was 39600, which was an increase of 10 % during the 10 years preceding; what was the population in 1850?

5. My crop of wheat this year is 8 % greater than my crop of last year, and I have raised during the two years 5200 bushels; what was my last year's crop? *Ans.* 2500 bu.

NOTE 2.—1.00 + 1.08 = 2.08. Hence, 5200 bu. = 2.08 % of last year's crop.

6. The net profits of a nursery in two years were $6970, and the profits the second year were 5 % greater than the profits the first year; what were the profits each year?

Ans. 1st year, $3400; 2d year, $3570.

7. If a number be increased 8 %, and the amount be increased 7 %, the result will be 86.67; required the number.

NOTE 3.—The whole amount will be 1.08 × 1.07 = 1.1556 times the original number.

8. A produce dealer bought grain by measure, and sold it by weight, thereby gaining $1\frac{1}{2}$ % in the number of bushels. He sold at a price 5 % above his buying price, and received $4910.976 for the grain; required the cost. *Ans.* $4608.

9. B has 6 %, and C 4 % more money than A, and they all have $11160; how much money has A? *Ans.* $3600.

10. In the erection of a house I paid twice as much for material as for labor. Had I paid 6 % more for material, and 9 % more for labor, my house would have cost $1284; what was its cost?

Ans. $1200.

PROBLEM V.

453. Given, the difference and rate, to find the base.

1. What number diminished by 5 % of itself, is equal to 342?

OPERATION.

$1 - .05 = .95$

$342 \div .95 = 360$, *Ans.*

Or,

$1 - \frac{1}{20} = \frac{19}{20}$

$342 \div \frac{19}{20} = 360$, *Ans.*

ANALYSIS. If any number be diminished by 5 % of itself, the difference will be .95 of the number. We therefore divide the given difference, 342, by .95, or $\frac{19}{20}$, and obtain the base, 360, which is the required number. Hence the

RULE. *Divide the difference by 1 minus the rate.*

NOTE.—The difference is always a *product*, of which the base is one factor, and 1 minus the rate the other.

EXAMPLES FOR PRACTICE.

1. What number diminished by 10 % of itself is equal to 504?
Ans. 560.

2. The rate is 8 %, and the difference \$4.37; what is the base?

3. After taking away 15 % of a heap of grain, there remained 40 bu. $3\frac{1}{5}$ pk.; how many bushels were there at first?
Ans. 48 bu.

4. Having sold 36 % of my land, I have 224 acres left; how much land had I at first?

5. After paying 65 % of my debts, I find that \$2590 will discharge the remainder; how much did I owe in all?
Ans. \$7400.

6. A young man having received a fortune, deposited 80 % of it in a bank. He afterward drew 20 % of his deposit, and then had \$5760 in bank; what was his entire fortune?
Ans. \$9000.

7. A man owning $\frac{7}{8}$ of a ship, sold 12 % of his share to A, and the remainder to B, at the same rate, for \$20020; what was the estimated value of the whole ship? *Ans.* \$26000.

8. An army which has been twice decimated in battle, now contains only 6480 men; what was the original number in the army? *Ans.* 8000.

9. Each of two men, A and B, desired to sell his horse to C. A asked a certain price, and B asked 50 % more. A then reduced his price 20 %, and B his price 30 %, at which prices C took both horses, paying for them \$148; what was each man's asking price?

$$\textit{Ans.}\ \begin{cases}\text{A, \$80.}\\ \text{B, \$120.}\end{cases}$$

10. A buyer expended equal sums of money in the purchase of wheat, corn, and oats. In the sales, he cleared 6 % on the wheat, and 3 % on the corn, but lost 17 % on the oats; the whole amount received was \$2336. What sum did he lay out in each kind of grain? *Ans.* \$800.

APPLICATIONS OF PERCENTAGE.

454. The principal applications of Percentage, where time is not considered, are Commission, Stocks, Profit and Loss, Insurance, Taxes, and Duties. And since the five problems in Percentage involve all the essential relations of the parts or elements, we have for the above applications the following

GENERAL RULE. *Note what elements of Percentage are given in the example, and what element is required; then apply the special rule for the corresponding case.*

COMMISSION.

455. An **Agent, Factor,** or **Broker,** is a person who transacts business for another.

456. A **Commission Merchant** is an agent who buys and sells goods for another.

457. **Commission** is the fee or compensation of an agent, factor, or commission merchant.

458. A **Consignment** is a quantity of goods sent to one person to be sold on commission for another person.

459. A **Consignee** is a person who receives goods to sell for another; and

460. A **Consignor** is a person who sends goods to another to be sold.

461. The **Net Proceeds** of a sale or collection is the sum left, after deducting the commission and other charges.

NOTE.—A person who is employed in establishing mercantile relations between others living at a distance from each other, is called the *Correspondent* of the party in whose behalf he acts. A correspondent is the *agent* of those whose custom or patronage he secures to the party in whose interest he is employed.

462. Commission is usually reckoned at a certain per cent. of the money involved in the transaction; hence we have the following relations:

I. Commission is *percentage*, (**445**).

II. The sum received by the agent as the price of property sold, or the sum invested by the agent in the purchase or exchange of property, is the *base* of commission, (**446**).

III. The sum remitted to an agent, and including both the purchase money and the agent's commission, is the *amount*, (**447**).

IV. The sum due the employer or consignor as the net proceeds of a sale or collection, is the *difference*, (**448**).

EXAMPLES FOR PRACTICE.

1. My agent sells goods to the amount of \$6250; what is his commission at 3 %?

OPERATION.

\$6250 × .03 = \$187.50

ANALYSIS. According to Prob. I, (**449**), we multiply the sum obtained for the goods, \$6250, which is the *base* of the commission, (II), by the rate of the commission, .03, and obtain the commission or percentage, \$187.50.

2. A flour merchant remits to his agent in Chicago \$3796, for the purchase of grain, after deducting the commission at 4 %; how much will the agent expend for his employer, and what will be his commission?

OPERATION.

1.00 + .04 = 1.04
\$3796 ÷ 1.04 = \$3650, for grain,
\$3796 — \$3650 = \$146, commission.

ANALYSIS. According to Prob. IV, (**452**), we divide the remittance, \$3796, which is *amount*, (III), by 1 plus the *rate* of commission, or 1.04, and obtain the *base* of commission, \$3650, which is the sum to be expended in the purchase. Subtracting this from the remittance, we have \$146, the commission.

NOTE 1.—It is evident that the whole remittance, \$3796, should not be taken as the base of commission; for that would be computing commission on commission. A person must charge commission only on what he *expends* or *collects*, in his capacity as agent.

3. A factor sold real estate on commission of 5 %, and returned to the owner, as the net proceeds, \$8075; for what price did he sell the property, and what was his commission?

OPERATION.

1.00 — .05 = .95
\$8075 ÷ .95 = \$8500, price.
\$8500 — \$8075 = \$425, com.

ANALYSIS. According to Prob. V, (**453**), we divide the net proceeds, \$8075, which is *difference*, (IV), by 1 minus the *rate* of commission, and obtain the *base*, \$8500, which is the price of the property sold; whence by subtraction, we obtain the commission, \$425.

4. An agent sold my house and lot for \$8600; what was his commission at $2\frac{1}{4}$ %? *Ans.* \$193.50.

5. A lawyer collects \$750.75; what is his commission at $3\frac{3}{4}$ %? *Ans.* \$28.15+.

6. My agent in New York has sold 3500 bushels of Indiana wheat @ \$1.40, and 3600 bushels of dent corn @ \$.74; what is his commission at $2\frac{1}{4}$ %?

7. A dealer in Philadelphia sells hides on commission of $8\frac{1}{3}$ %, as follows: 2000 lb. Orinoco @ \$.23½, 5650 lb. Central American @ \$.22, 450 lb. Texas @ \$.23, and 650 lb. city slaughter @ \$.21; what does he receive for his services? *Ans.* \$162.75.

8. A commission merchant sold a consignment of flour and pork for \$25372. He charged \$132 for storage, and $6\frac{1}{4}$ % commission; what were the net proceeds of the sale?

9. An agent for a Rochester nurseryman sells 4000 apple trees at \$25 per hundred, 2000 pear trees at \$50 per hundred, 1600 peach trees at \$20 per hundred, 1800 cherry trees at \$50 per hundred, and 500 plum trees at \$50 per hundred; what is his commission at 30 %, and how much should he return to his employer as the net proceeds, after deducting \$203.50 for expenses?

Ans. Commission, \$1041; Net proceeds, \$2225.50.

10. A lawyer having a debt of \$785 to collect, compromises for 82 %; what is his commission, at 5 %? *Ans.* \$32.185.

11. I purchased in Chicago 4000 bushels of wheat @ \$1.25, and shipped the same to my agent in Oswego, N. Y., who sold it @ \$1.50; how much did I make, after paying expenses amounting to \$415, and a commission of 3 %? *Ans.* \$405.

12. An agent received \$63 for collecting a debt of \$1260; what was the rate of his commission? *Ans.* 5 %.

13. My Charleston agent has charged \$74.25 for purchasing 26400 lb. of rice at \$4.50 per 100 lb.; required the rate of his commission. $6\frac{1}{4}$ %

14. A house and lot were sold for \$7850, and the owner re-

ceived \$7732.25 as the net proceeds; what was the rate of commission? .015

15. A commission merchant in Boston having received 28000 lb. of Mobile cotton, effects a sale at \$.12½ per pound. After deducting \$35.36 for freight and cartage, \$10.50 for storage, and his commission, he remits to his employer \$3252.89 as the net proceeds of the sale; at what rate did he charge commission?

Ans. 5¾ %.

16. The net proceeds of a sale were \$5635, the commission was \$115; what was the rate of commission?

17. An agent received \$22.40 for selling grain at a commission of 4 %; what was the value of the grain sold? *Ans.* 560.

18. My attorney, in collecting a note for me at a commission of 8 %, received as his fee \$6.80; what was the face of the note?

19. Sent to my agent in Boston \$255, to be invested in French prints at \$.15 per yard, after deducting his commission of 2 %; how many yards shall I receive? *Ans.* 1666⅔.

20. John Kennedy, commission merchant, sells for Ladd & Co. 860 barrels of flour @ \$7.50, on a commission of 2½ %. He invests the proceeds in dry goods, after deducting his commission of 1½ % for purchasing; how many dollars' worth of goods do Ladd & Co. receive? *Ans.* \$6195.81+.

21. A commission merchant, whose rate both for selling and investing is 5 %, receives 24000 lbs. of pork, worth 6 cents, and \$3000 in cash, with instructions to invest in a shipment of cotton to London. What will be his entire commission? *Ans.* \$280.

22. A speculator received \$3290 as the net proceeds of a sale, after allowing a commission of 6 %; what was the value of the property? *Ans.* \$3500.

23. The net proceeds of a shipment of 500 tons of pressed hay, after deducting a commission of 3 %, and \$500 for other charges, were \$6290; what was the selling price per ton?

24. I send a quantity of dry goods into the country to be sold at auction, on commission of 9 %. What amount of goods must be sold, that my agent may buy produce with the avails, to the value of \$3500, after retaining his purchase commission of 4 %?

NOTE 2. — $3500 plus the agent's commission equals the *net proceeds* of the *sale.*

25. Having sold a consignment of cotton on 3 % commission, I am instructed to invest the proceeds in city lots, after deducting my purchase commission of 2 %. My whole commission is $265; what is the price of the city lots? *Ans.* $5141.

26. What tax must be assessed to raise $50000, the collector's commission being $\frac{3}{4}$ %? *Ans.* $50377.83+.

STOCKS.

463. A **Company** is an association of individuals for the prosecution of some industrial undertaking. Companies may be incorporated or unincorporated.

464. A **Corporation** is a body formed and authorized by law to act as a single person.

465. A **Charter** is the legal act of incorporation, and defines the powers and obligations of the incorporated body.

466. A **Firm** is the name under which an unincorporated company transacts business.

NOTE.—A private banking company, or a manufacturing or commercial firm is also called a *House.*

467. The **Capital Stock** of a corporation is the money contributed and employed to carry on the business of the company.

468. **Joint Stock** is the money or capital of any company, incorporated or unincorporated.

469. **Scrip** or **Certificates of Stock** are the papers or documents issued by a corporation, giving the members their respective titles or claims to the joint capital.

470. A **Share** is one of the equal parts into which capital stock is divided. The value of a share in the original contribution of capital varies in different companies; in bank, insurance, and railroad companies of recent organization, it is usually $100.

471. **Stockholders** are the owners of stock, either by original title or by subsequent purchase. The stockholders constitute the company.

NOTES.—1. The capital stock of any corporation is limited by the charter. As a general rule, only a portion is paid at the time of subscription, the residue being reserved for future outlays or disbursements.

2. When the capital stock has been all paid in, money may be raised, if necessary, by *loans*, secured by mortgage upon the property. The *bonds* issued for these loans entitle the holders to a fixed rate of interest.

3. Stocks, as a general name, applies to the scrip and bonds of a corporation, to government bonds and public securities, and to all paper representing joint capital or claims upon corporate bodies.

4. The members of an incorporated company are individually liable for the debts and obligations of the company, to the amount of their interest or stock in the company, and to no greater amount. But the members of a firm or house are individually liable for all the debts and obligations of the company, without regard to the amount of their share or interest in the concern.

The calculations of percentage in stocks are treated in this work under the heads of

Stock-jobbing, Assessments and Dividends, and Stock Investments.

STOCK-JOBBING.

472. **Stock-jobbing** is the buying and selling of stocks with a view to realize gain from their rise and fall in the market.

473. The **Nominal** or **Par value** of stock is the sum for which the scrip or certificate is issued.

474. The **Market** or **Real value** of stock is the sum for which it will sell.

475. Stock is **At Par** when it sells for its first cost, or nominal value.

476. Stock is **Above Par**, at a premium or advance, when it sells for more than its nominal value.

477. Stock is **Below Par**, or at a discount, when it sells for less than its nominal value.

Note.—When the business of a company pays large profits to the stockholders, the stock will be worth more than its original cost; but when the business does not pay expenses, the value of the stock will be less than its original cost. The average market value of stock generally varies directly as the rate of profit which the business pays.

478. A **Stock Broker** is a person who buys and sells stocks, either for himself, or as the agent of another.

Note.—A person employed by a manufacturer, wholesale dealer, or commission merchant, to seek customers and close bargains, at or from his place of business, is called a *broker*, of the class or kind corresponding to his business.

479. **Brokerage** is the fee or compensation of a broker.

480. The calculations in stock-jobbing are based upon the following relations:

I. Premium, discount, and brokerage are each a *percentage*, computed upon the par value of the stock as the *base*.

II. The market value of stock, or the proceeds of a sale, is the *amount* or *difference*, according as the sum is greater or less than the par value.

NOTE 1.—In all examples relating to stocks, $100 will be considered a share, unless otherwise stated.

EXAMPLES FOR PRACTICE.

1. What cost 54 shares of Reading Railroad stock, at $4\frac{1}{2}$ % premium?

OPERATION.

$$\$5400 \times .045 = \$243, \text{ premium.}$$
$$\$5400 + \$243 = 5643, \textit{ Ans.}$$
Or,
$$5400 \times \$1.045 = \$5643, \textit{ Ans.}$$

ANALYSIS. We first compute the premium upon the par value of the stock, and find it to be $243; adding this to the $5400, we obtain the cost, or market value, $5643. Or, since every dollar of the stock will cost $1 plus the premium, or $1.045, $5400 will cost 5400 × $1.045 = $5643.

2. What do I receive for 32 shares of telegraph stock, which a broker sells for me at 15 % discount charging $\frac{1}{4}$ % brokerage?

OPERATION.

$$.15 + .0025 = .1525$$
$$\$1.00 - \$.1525 = \$.8475 \text{ proceeds of \$1 of stock.}$$
$$3200 \times \$.8475 = \$2712, \textit{ Ans.}$$

ANALYSIS. Adding the rate of brokerage to the rate of discount, we have .1525; hence $1 will bring $1—$.1525= $.8475, and $3200 will bring 3200 × $.8475 = $2712.

3. I put $35400 into the hands of a broker to be invested in Missouri State Bonds, when their market value is 12 % below par; how many shares shall I receive, if the broker charges $\frac{1}{2}$ % for his services?

OPERATION.

$$\$1.00 - \$.12 = \$.88, \text{ market value of \$1.}$$
$$\$\ .88 + \$.00\tfrac{1}{2} = .885, \text{ cost of \$1.}$$
$$\$35400 \div .885 = \$40000 = 400 \text{ shares, } \textit{Ans.}$$

ANALYSIS. Since the stock is 12 % below par, the market value of $1 is $.88; adding the rate of brokerage, we find that every dollar of the stock will cost me \$.885. Hence for \$35400 the broker can buy \$35400 ÷ .885 = \$40000 = 400 shares.

NOTES. — 2. The rate of brokerage in New York city has been fixed by custom at $\frac{1}{4}$ per cent.

3. Since brokerage has the same base as the premium or discount, the *rate* of brokerage may always be combined with the rate of premium or discount, by addition or subtraction, as the nature of the question may require.

4. The price of stock is usually quoted at a certain per cent. of the *face*, or nominal value. Thus stock at 4 % above par is quoted at 104 %; stock at 5 % below par is quoted at 95 %; and so on.

4. What is the market value of 15 Ohio State bonds at 112 %? *Ans.* \$1680.

5. What shall I realize on 20 shares of Panama railroad stock at 135 %, brokerage at $1\frac{3}{4}$ %? *Ans.* \$2665.

6. My agent bought for me 120 shares of N. Y. Central railroad stock, paying $80\frac{3}{4}$ %, and charging brokerage at $\frac{1}{2}$ %; what did the stock cost me? *Ans.* \$9750.

7. What cost 36 shares in the Merchants' Bank, at a premium of $7\frac{1}{2}$ %, brokerage $\frac{1}{4}$ %?

8. A speculator invested \$21910 in shares of the Harlem railroad, at a discount of $60\frac{7}{8}$ %; how many shares did he buy?

9. If 400 shares of the Bank of Commerce sell for \$40150, what is the rate of premium? *Ans.* $\frac{3}{8}$ %.

10. A broker receives \$48447 to be invested in bonds of the Michigan Central railroad, at $94\frac{1}{8}$ %; how much stock can he buy, allowing $1\frac{1}{4}$ % brokerage?

11. My agent sells 830 barrels of Genesee flour at \$6 per barrel, commission 5 %, and invests the proceeds in stock of the Pennsylvania Coal Company, at $82\frac{3}{4}$ %, charging $\frac{1}{4}$ % for making the purchase; how many shares do I receive? *Ans.* 57.

12. I purchased 18 shares of Ocean Telegraph stock, par value \$500 per share, at a premium of 2 %, and sold the same at a discount of 28 %; what was my loss? *Ans.* \$2700.

NOTE 5. — The rate of loss is .02. + .28 = .30, or .30 %.

13. A speculator exchanged \$3600 of railroad bonds, at 5 % discount, for 27 shares of stock of the Suffolk Bank, at 3 % premium, receiving the difference in cash; how much money did he receive?

14. A merchant owning 525 shares in the American Exchange

Bank, worth 104 %, exchanges them for United States bonds worth 105 %; how much of the latter stock does he receive?

15. I purchased 12 shares of stock at a premium of 5 %, and sold the same at a loss of $96; what was the selling price?

16. Having bought $64000 stock in the Cunard Line, at 2 % premium, at what price must I sell it, to gain $2560?

Ans. 106 %.

17. A speculator bought 250 shares in a Carson Valley mining company at 103 %, and 150 shares of the Western Railroad stock at 95 %; he exchanged the whole at the same rates, for shares in the N. Y. Central Railroad at 80 %, which he afterward sold at 85 %. How much did he gain? *Ans.* $2500.

18. I purchased stock at par, and sold the same at 3 % premium, thereby gaining $750; how many shares did I purchase?

19. A broker bought Illinois State bonds at 103 %, and sold at 105 %. His profits were $240; what was the amount of his purchase? *Ans.* $12000.

20. A man invested in mining stock when it was 4 % above par, and afterward sold his shares at 5½ % discount. His loss in trade was $760; how many shares did he purchase?

21. I invested $6864 in Government bonds at 106 %, paying 1¼ % brokerage, and afterward sold the stock at 112 %, paying 1½ % brokerage; what was my gain? *Ans.* $208.

22. How much money must be invested in stocks at 3 % advance, in order to gain $480 by selling at 7 % advance?

23. How many shares of stock must be sold at 4 % discount, brokerage ½ %, to realize $4775? *Ans.* 50.

INSTALLMENTS, ASSESSMENTS, AND DIVIDENDS.

481. An **Installment** is a portion of the capital stock required of the stockholders, as a payment on their subscription.

482. An **Assessment** is a sum required of stockholders, to meet the losses or the business expenses of the company.

483. A **Dividend** is a sum paid to the stockholders from the profits of the business.

484. **Gross Earnings** are all the moneys received from the regular business of the company.

485. **Net Earnings** are the moneys left after paying expenses, losses, and the interest upon the bonds, if there be any.

486. In the division of the net earnings, or the apportionment of dividends and assessments, the calculations are made by finding the rate per cent. which the sum to be distributed or assessed bears to the entire capital stock. Hence,

487. Dividends and assessments are a *percentage* computed upon the par value of the stock as the *base*.

EXAMPLES FOR PRACTICE.

1. The Long Island Insurance Company declares a dividend of 6 %; what does A receive, who owns 14 shares?

OPERATION.

$\$1400 \times .06 = \$84$

ANALYSIS. According to **449**, we multiply the base, \$1400, by the rate, .06, and obtain the dividend, \$84.

2. A canal company whose subscribed funds amount to \$84000, requires an installment of \$6300; what per cent. must the stockholders pay?

OPERATION.

$\$6300 \div 84000 = .07\frac{1}{2}$

ANALYSIS. According to **450**, we divide the installment, \$6300, which is *percentage*, by the *base*, \$84000, and obtain the rate, $.07\frac{1}{2} = 7\frac{1}{2}$ %.

3. A man owns 56 shares of railroad stock, and the company has declared a dividend of 8 %; what does he receive?

Ans. \$448.

4. I own \$15000 in a mutual insurance company; how many shares shall I possess after a dividend of 6 % has been declared, payable in stock? *Ans.* 159 shares.

5. The Pittsburgh Gas Company declares a dividend of 15 %; what will be received on 65 shares?

6. A received \$600 from a 4 % dividend; how much stock did he own? *Ans.* \$15000.

7. The paid-in capital of an insurance company is $536000. Its receipts for one year are $99280, and its losses and expenses are $56400; what rate of dividend can it declare? *Ans.* 8 %.

8. The net earnings of a western turnpike are $3616, and the amount of stock is $56000; if the company declare a dividend of 6 %, what surplus revenue will it have? *Ans.* $256.

9. The capital stock of the Boston and Lowell Railroad Co. is $1830000, and its debt is $450000. Its gross earnings for the year 1858 were $407399, and its expenses $217621. If the company paid expenses, and interest on its debt at $5\frac{5}{9}$ %, and reserved $78, what dividend would a stockholder receive who owned 30 shares? *Ans.* $270.

10. The charter of a new railroad company limits the stock to $800,000, of which 3 installments of 10 %, 25 %, and 35 %, respectively, have been already paid in. The expenditures in the construction of the road have reached the sum of $540,000, and the estimated cost of completion is $400,000. If the company call in the final installment of its stock, and assess the stockholders for the remaining outlay, what will be the rate %? *Ans.* $17\frac{1}{2}$.

11. The Bank of New York, having $156753.19 to distribute to the stockholders, declares a dividend of $5\frac{1}{4}$ %; what is the amount of its capital? *Ans.* $2,985,775 nearly.

12. The passenger earnings of a western railroad in one year were $574375.25, the freight and mail earnings were $643672.36, the whole amount of disbursements were $651113.53, and the company was able to declare a dividend of 8 %; how much scrip had the company issued? *Ans.* $7086676.

13. Having received a stock dividend of 5 %, I find that I own 504 shares; how many shares had I at first? *Ans.* 480.

14. I received a 6 % dividend on Philadelphia City railroad stock, and invested the money in the same stock at 75 %. My stock had then increased to $16200; what was the amount of my dividend? *Ans.* $900.

15. A ferry company, whose stock is $28000, pays 5 % dividends semi-annually. The annual expenses of the ferry are $2950; what are the gross earnings? *Ans.* $5750.

STOCK INVESTMENTS.*

488. The net earnings of a corporation are usually divided among the stockholders, in semi-annual dividends. The income of *capital stock* is therefore fluctuating, being dependent upon the condition of business; while the income arising from *bonds*, whether of government or corporations, is fixed, being a certain rate per cent., annually, of the par value, or face of the bonds.

489. Federal or United States Securities are of two kinds: viz., Bonds and Notes.

Bonds are of two kinds.

First, Those which are payable at a fixed date, and are known and quoted in commercial transactions by the rate of interest they bear, thus,: U. S. 6's, that is, United States Bonds bearing 6 % interest.

Second, Those which are payable at a fixed date, but which may be paid at an earlier specified time, as the Government may elect. These are known and quoted in commercial transactions by a combination of the two dates, thus: U. S. 5–20's, or a combination of the rate of interest and the two dates, thus: U. S. 6's 5–20; that is, bonds bearing 6 % interest, which are payable in twenty years, but may be paid in five years, if the Government so elect.

When it is necessary, in any transaction, to distinguish from each other different issues which bear the same rate of interest, this is done by adding the year in which they become due, thus: U. S. 5's of '71; U. S. 5's of '74; U. S. 6's 5–20 of '84; U. S. 6's 5–20 of '85.

Notes are of two kinds.

First, Those payable on demand, without interest, known as United States Legal-tender Notes, or, in common language, "Green Backs."

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* The following eight pages contain *four* pages of *new matter*, on U. S. Securities, Bonds, Treasury Notes, Gold Investments, &c., to meet a necessity which did not exist at the time this book was written.

The pupil will find the Cases, Rules, and Operations of the previous editions essentially the same in this, with *additional* examples, and other matter, which may be used or omitted; so that the present may be used with the previous editions with little or no inconvenience.

Second, Notes payable at a specified time, with interest, known as Treasury Notes. Of these, there are two kinds, — Six-per-cent. Compound-interest Notes, and Notes bearing $7\frac{3}{10}$ % interest, the latter known and quoted in commercial transactions as 7.30's.

The nomenclature here explained is the one used in commercial transactions, which involve similar securities of States or corporations.

The interest on all bonds is payable in gold.

The interest on notes is payable in Legal-tender Notes.

When Bonds or Stocks are sold, a revenue stamp must be used equal in value to one cent on each $100, or fraction of $100, of their currency value. If sold by a broker, this is charged to the person for whom they are sold.

The following are the principal United States Securities: —

BONDS.

U. S. 6's of 1867.
U. S. 6's of 1868.
U. S. 6's of 1880.
U. S. 6's of 1881.
U. S. 5's of 1871.
U. S. 5's of 1874.
U. S. 5–20's, due in 1882, interest 5 %.
U. S. 5–20's, due in 1884, interest 6 %.
U. S. 5–20's, due in 1885, interest 6 %.
U. S. 10–40's, due in 1904, interest 5 %.
Pacific Railroad 6's of 1895.
Pacific Railroad 6's of 1896.

NOTES.

Compound-interest Notes of 1867.
Compound-interest Notes of 1868.
7.30 Notes of 1867.
7.30 Notes of 1868.

CASE I.

490. To find what income any investment will produce.

1. What income will be obtained by investing \$6840 in stock bearing 6 %, and purchased at 95 % ?

OPERATION.

$\$6840 \div .95 = \$7200$, stock purchased.
$\$7200 \times .06 = \$432$, annual income.

ANALYSIS. We divide the investment, \$6840, by the cost of \$1, and obtain \$7200, the stock which the investment will purchase, (**452**). And since the stock bears 6 % interest, we have $\$7200 \times .06 = \$432$, the annual income obtained by the investment. Hence,

RULE. — *Find how much stock the investment will purchase, and then compute the income at the given rate upon the par value.*

EXAMPLES FOR PRACTICE.

1. The trustees of a school invested \$35374.80 in the U. S. 5 % bonds as a teachers' fund, purchasing the stock at $102\frac{1}{2}$ if the salary of the Principal be \$1000, what sum will be left to pay assistants? *Ans.* \$725.60.

2. A young man, receiving a legacy of \$48000, invested one half in 5 % stock at $95\frac{1}{2}$ %, and the other half in 6 % stock at 112 %, paying brokerage at $\frac{1}{2}$ % ; what annual income did he secure from his legacy? *Ans.* \$2530.

3. I have \$2300 to invest, and can buy New York Central 6's at 85 %, or New York Central 7's at 95 % ; how much more profitable will the latter be than the former, per year?

4. A owns a farm which rents for \$411.45 per annum. If he sell the same for \$8229, and invest the proceeds in U. S. 5–20's of '84, at 105 %, paying $\frac{1}{2}$ % brokerage, will his yearly income be increased or diminished, and how much! *Ans.* Increased \$56.55.

5. A sold \$8700 of U. S. 5–20's of '84 at 104 %, paying for necessary revenue stamps, and invested the proceeds in U. S. 10–40's at 94 %, brokerage $\frac{1}{2}$ % both for selling and buying. Did he gain or lose by the exchange, and how much annually?
Ans. \$45.62—.

CASE II.

491. To find what sum must be invested to obtain a given income.

1. What sum must be invested in Virginia 5 per cent. bonds, purchasable at 80 %, to obtain an income of $600?

OPERATION.

$600 ÷ .05 = $12000, stock required.
$1200 × .80 = $9600, cost or investment.

ANALYSIS. Since $1 of the stock will obtain $.05 income, to obtain $600 will require $600 ÷ .05 = $12000, (Case 1). Multiplying the par value of the stock by the market price of $1, we have $12000 × .80 = $9600, the cost of the required stock, or the sum to be invested. Hence the

RULE. I. *Divide the given income by the % which the stock pays; the quotient will be the par value of the stock required.*

II. *Multiply the par value of the stock by the market value of one dollar of the stock; the product will be the required investment.*

EXAMPLES FOR PRACTICE.

1. If Missouri State 6's are 16 % below par, what sum must be invested in this stock to obtain an income of $960?

2. What sum must I invest in U. S. 5–20's of '82 at $96\frac{3}{4}$ %, brokerage $\frac{1}{4}$ %, to secure an annual income of $1500.

Ans. $29100.

3. How much must I invest in U. S. 7–30's, at 106 %, that my annual income may be $1752? *Ans.* $25440.

4. If I sell $15600 U. S. 10–40's at 97 %, and invest a sufficient amount of the proceeds in U. S. 5–20's of '85 at 107 % to yield an annual income of $540, and buy a house with the remainder, how much will the house cost me? Ans. $5502.

5. Charles C. Thomson, through his broker, invested a certain sum of money in U. S. 6's 5–20 at 107 %, and twice as much in U. S. 10–40's at $98\frac{1}{2}$ %, brokerage in each case $\frac{1}{2}$ %. His income from both investments was $1674. How much did he invest in each kind of stock?

Ans. First kind, $10692. Second kind, $21384.

CASE III.

492. To find what per cent. the income is of the investment, when stock is purchased at a given price.

1. What per cent. of my investment shall I secure by purchasing the New York 7 per cents. at 105 %?

OPERATION.

$.07 \div 1.05 = 6\frac{2}{3}$ %.

ANALYSIS. Since \$1 of the stock will *cost* \$1.05, and *pay* \$.07, the income is $\frac{7}{105} = 6\frac{2}{3}$ % of the investment. Hence the

RULE. *Divide the annual rate of income which the stock bears by the price of the stock; the quotient will be the rate upon the investment.*

EXAMPLES FOR PRACTICE.

1. What per cent. of his money will a man obtain by investing in 6 per cent. stock at 108 %? *Ans.* $5\frac{5}{9}$ %.

2. What is the rate of income upon money invested in 6 per cent. bonds, purchased at a discount of 16 %? *Ans.* $7\frac{1}{7}$ %.

3. Panama railroad stock is at a premium of $34\frac{1}{2}$ %, and the charge for brokerage is $1\frac{1}{2}$ %; what will be the rate of income on an investment in these funds if the stock pays a dividend of $8\frac{1}{2}$ % annually? *Ans.* $6\frac{1}{4}$ %.

4. Which is the better investment, to buy 5's at 70 %, or 6's at 80 %?

5. Which is the more profitable, to buy 8's at 120 %, or 5's at 75 %?

6. What is the rate of income upon money invested in U. S. 7–30's at 106 %? *Ans.* $6\frac{47}{53}$ %.

7. Which is the better investment, U. S. 5–20's of '84 at $108\frac{1}{2}$ %, or U. S. 10–40's at 98 %, and how much per cent. per annum? *Ans.* U. S. 5–20's, $\frac{650}{1519}$ %.

8. If a man invest \$10000 in U. S. 10–40's at 98 %, and exchanges them at par for U. S. 7–30's at 102 %, what is his rate of income?

9. What per cent of his money will a man gain by investing in Pacific Railroad 6's at 105 %?

CASE IV.

493. To find the price at which stock must be purchased to obtain a given rate upon the investment.

1. At what price must 6 per cent. stocks be purchased in order to obtain 8 % income on the investment?

OPERATION.

$.06 ÷ .08 = $75.

ANALYSIS. Since $.06, the income of $1 of the stock, is 8 % of the sum paid for it, we have, (**449**), $.06 ÷ .08 = $75, the purchase price. Hence,

RULE. *Divide the annual rate of income which the stock bears by the rate required on the investment; the quotient will be the price of the stock.*

EXAMPLES FOR PRACTICE.

1. What must I pay for Government 5 per cents., that my investment may yield 8 %? *Ans.* $62\frac{1}{2}$ %.
2. At what rate of discount must the Vermont 6 per cent. bonds be purchased that the person investing may secure $6\frac{1}{4}$ % upon his money? *Ans.* 4 %.
3. What rate of premium does 7 per cent. stock bear in the market when an investment pays 6 %?
4. A speculator invested in a Life Insurance Company, and received a dividend of 6 %, which was $8\frac{1}{3}$ % on his investment; at what price did he purchase? *Ans.* 72 %.
5. What must I pay for U. S. 10–40's, that my investment may yield 6 %? *Ans.* $83\frac{1}{3}$ %.
6. What rate of premium does U. S. 6's 5-20 bear in market when an investment pays 5 %?
7. At what rate of discount must U. S. 7–30's be purchased, that the investment shall yield 10 %?
8. What must I pay for government 6's of '81, that my investment may yield 7 %?

GOLD INVESTMENTS.

493 a. **Currency** is a term used in commercial language, *First,* To denote the aggregate of Specie and Bills of Exchange, Bank Bills, Treasury Notes, and other substitutes for money employed in buying, selling, and carrying on exchange of commodities between various nations. *Second,* To denote whatever circulating medium is used in any country as a substitute for the government standard. In this latter sense, the paper circulating medium, when below par, is called Currency, to distinguish it from gold and silver. If, from any cause, the paper medium depreciates in value, as it has done in the United States, gold becomes an object of investment, the same as stocks. In commercial language, gold is represented as rising and falling; but gold being the standard of value, it cannot vary. The variation is in the medium of circulation substituted for gold; hence, when gold is said to be at a premium, the currency, or circulating medium, is made the standard, while it is virtually below par.

CASE I.

To change gold into currency.

1. How much currency can be bought for \$150 in gold when gold is at 170 %?

OPERATION.

$\$1.70 \times 150 = \$255$

ANALYSIS. Since a dollar of gold is worth \$1.70 in currency, there can be as many times \$170 of currency bought as there are dollars of gold. Therefore, $\$1.70 \times 150 = \$255$ is the amount of currency which can be purchased for \$150 in gold.

RULE. *Multiply the value of one dollar of gold in currency by the number of dollars of gold.*

2. What is the value, in current funds, of \$250.47 gold, when gold is at $142\frac{3}{4}$ %? *Ans.* \$357.546 —.

3. If a person holds \$6000 U. S. 10–40's, what would be his annual income in current funds if gold is at 157 %? *Ans.* \$471.

4. A merchant purchased a bill of goods for which he was to pay \$7000 in currency, or \$5500 in gold, at his option. Will he gain or lose by accepting the latter proposition, gold being at $138\frac{1}{2}$ %, and how much in currency? *Ans.* Lose \$617.50.

5. Bought broadcloth @ \$3 in gold, and sold the same @ \$4 in currency. Did I gain or lose by the transaction, and how much per cent. in currency, gold being at 140 % ? *Ans.* Lost $4\frac{16}{21}$ %.

6. A broker invested \$3000 of gold in U. S. 6's, which were worth 102 % in currency. What was his annual income from the investment, gold being at 134 % ? and what the rate per cent. ?

Ans. to first, \$236.47. *Ans. to last*, $5\frac{45}{51}$ %.

7. A gentleman invested \$10,000, current funds, in U. S. 5–20's of '85, at 104 %. What will be his annual income in currency when gold is at 137 % ? *Ans.* $\$790.38\frac{6}{13}$.

CASE II.

To change currency into gold.

1. How much gold can be purchased for \$75 current funds, gold being at 150 % ?

OPERATION.

$\$75 \div \$1.50 = 50$

ANALYSIS. A dollar of gold cost \$1.50 in currency, therefore there can be as many dollars of gold purchased for \$75 in currency as \$1.50 is contained times in \$75.

RULE. *Divide the amount in currency by the price of gold.*

2. What is the value in gold of a dollar in currency when gold is at 203 % ? *Ans.* $\$.49\frac{53}{203}$.

3. Gold being the standard, what is the rate of discount upon current funds when gold is at 134 %, 150 %, 175 %, 180 %, 215 %, and 398 % ? *Ans. to first*, $25\frac{25}{67}$ %. *Ans. to last*, $74\frac{174}{199}$ %.

4. What is gold quoted at when a dollar in currency is worth 20 cents in gold, 30 cts., 50 cts., 15 cts., 25 cts., and 72 cts. ?

5. How many yards of cotton, at 25 cts. in gold, can be purchased for \$250 current funds, when gold is at 175 % ?

Ans. $571\frac{3}{7}$ yds.

6. Sold \$7800 7.30 Treasury Notes, at 105 %, and invested the proceeds in gold at 145 %, with which I bought U. S. 10–40's at 60 % in gold. Will my yearly income be increased or diminished by the transaction, and how much in gold? *Ans.* Increased \$78.

7. Which is the better investment, a bond and mortgage at 7 %, or U. S. 10–40's, gold being 134 ; and what per cent. in gold?

PROFIT AND LOSS.

494. **Profit** and **Loss** are commercial terms, used to express the gain or loss in business transactions.

495. Gains and losses are usually estimated at some rate per cent. on the money first expended or invested. Hence

I. Profit and loss are reckoned as *percentage* upon the prime or first cost of the goods as the *base*.

II. The selling price of the goods is *amount* or *difference*, according as it is greater or less than the prime cost.

EXAMPLES FOR PRACTICE.

1. A merchant bought cloth for \$3.25 per yard, and gained 8 % in selling; what was the selling price?

OPERATION.

$\$3.25 \times .08 = \$.26$, advance in price.
$\$3.25 + .26 = \$3.51$, selling price.
Or,
$\$3.25 \times 1.08 = \$3.51$, selling price.

ANALYSIS. Multiplying the prime cost, \$3.25, which is the *base* of gain, (I), by the *rate*, .08, we have \$.26, the gain, which added to the cost gives \$3.51, the selling price. Or, since the rate of gain is 8 %, that which cost \$1 will bring \$1.08, and the selling price will be 1.08 times the buying price. Hence $\$3.25 \times 1.08 = \$3.51$, the selling price.

2. A jobber invested \$2560 in dry goods, and realized \$384 net profit; what was the rate per cent. of his gain?

OPERATION.

$\$384 \div \$2560 = 15$ %

ANALYSIS. According to (Prob. II, **450**), we divide the gain, \$384, which is *percentage*, by the cost, \$2560, which is the *base*, and obtain 15 = 15 %, the *rate* of gain.

3. A produce dealer sold a shipment of wheat at a loss of 5 %, realizing as the net proceeds, \$8170; what was the cost?

OPERATION.

$\$1.00 - .05 = .95$
$8170 \div .95 = \$8600$, *Ans.*

ANALYSIS. According to (Prob. V, **453**), we divide the net proceeds, \$8170, which is *difference*, (**448**), by 1 minus the *rate* of loss, or .95, and obtain the *base*, or prime cost, \$8600.

4. A merchant pays \$7650 for a stock of spring goods; if he sell at an advance of 20 % upon the purchase price, what will be his profits, after deducting \$480 for expenses? *Ans.* \$1050.

5. Bought 320 yards of calico @ 15 cents, and sold it at a reduction of $2\frac{1}{2}$ %; what was the entire loss?

6. A dealer having bought 30 barrels of apples at \$3.50 per barrel, and shipped them at an expense of \$5.38, to be sold on commission of 5 %, what will be his whole loss if the selling price is 10 % below the purchase price? *Ans.* \$20.60$\frac{1}{2}$.

7. Bought corn at \$.50 a bushel; at what price must it be sold to gain $33\frac{1}{3}$ per cent.?

8. Bought fish at \$4.25 per quintal, and sold the same at \$4.93; what was my gain per cent.? *Ans.* 16 %.

9. Bought a hogshead of sugar containing 9 cwt. 44 lb., for \$59; paid \$4.72 for freight and cartage; at what price per pound must it be sold to gain 20 per cent. on the buying price?

10. A wine merchant bought a hogshead of wine for \$157.50; a part having leaked out he sold the remainder for \$3.32$\frac{1}{2}$ a gallon, and found his loss to be 5 per cent. on the cost; how many gallons leaked out? *Ans.* 18.

11. Sold a farm of 106 A. 3 R. 30 P. for \$96 an acre, and gained 18 per cent. on the cost; how much did the whole farm cost? *Ans.* \$8700.

12. A lumberman sold 36840 feet of lumber at \$21.12 per M, and gained 28 per cent.; how much would he have gained or lost, had he sold it at \$17 per M? *Ans.* \$18.42, gained.

13. A speculator bought shares in a mining company when the stock was 4 % below par, and sold the same when it was 28 % below par; what per cent. did he lose on his investment?

14. A machinist sold a fire engine for \$7050, and lost 6 per cent. on its cost; for how much ought he to have sold it to gain $12\frac{1}{2}$ per cent.? *Ans.* \$8437.50.

15. Sold my carriage at 30 per cent. gain, and with the money bought another, which I sold for \$182, and lost $12\frac{1}{2}$ per cent.; how much did each carriage cost me? *Ans.* { First, \$160; Second, \$208.

16. Gaffney, Burke & Co. bought a quantity of dry goods for \$6840; they sold $\frac{1}{4}$ of them at 15 per cent. profit, $\frac{1}{3}$ at $18\frac{3}{4}$ per cent., $\frac{1}{6}$ at 20 per cent., and the remainder at $33\frac{1}{3}$ per cent. profit; how much was the average gain per cent., and how much the whole gain? *Ans.* $21\frac{2}{3}$ % gain; \$1482, entire gain.

17. If I buy a piece of land, and it increases in value each year at the rate of 50 per cent. on the value of the previous year, for 4 years, and then is worth \$12000, how much did it cost?

18. A Western merchant bought wheat as follows: 600 bushels of red Southern @ \$1.80, 1200 bushels of white Michigan @ \$$1.62\frac{1}{2}$, and 200 bushels of Chicago spring, @ \$1.25. He shipped the whole to his correspondent in Buffalo, who sold the first two kinds at an advance of 20 % in the price, and the balance at \$1.20 per bushel, and deducting from the gross avails his commission at 5 %, and \$254.60 for expenses, returned to the consignor the net proceeds. What was the rate of the merchant's gain? *Ans.* $4\frac{1}{2}$ %.

19. A broker buys stock when it is 20 % below par, and sells it when it is 16 % below par; what is his rate of gain?

20. A man has 5 per cent. stock the market value of which is 78 %; if he sells it, and takes in exchange 6 % stock at 4 % premium, what per cent. of his annual income does he lose?

21. A machinist sold 24 grain-drills for \$125 each. On one half of them he gained 25 per cent., and on the remainder he lost 25 per cent.; did he gain or lose on the whole, and how much? *Ans.* Lost \$200.

22. Bought land at \$30 an acre; how much must I ask an acre, that I may abate 25 per cent. from my asking price, and still make 20 per cent. on the purchase money? *Ans.* \$48.

23. A salesman asked an advance of 20 per cent. on the cost of some goods, but was obliged to sell at 20 per cent. less than his asking price; did he gain or lose, and how much per cent.?

24. A Southern merchant ships to his agent in Boston, a quantity of sugar consisting of 200 bbl. of New Orleans, each containing 216 lb., purchased at 5 cents per pound, and 560 bbl. of West India, each containing 200 lb., purchased at $5\frac{3}{4}$ cents per pound.

The agent's account of sales shows a loss of 1 % on the New Orleans, and a profit of $\frac{20}{23}$ % on the West India sugar; does the merchant gain or lose on the whole consignment, and what per cent.? *Ans.* Gains $\frac{2}{5}$ %.

25. A grocer sold a hogshead of molasses for $31.50, which was a reduction of 30 % from the prime cost; what was the purchase price paid per gallon?

26. A speculator sold stock at a discount of $7\frac{3}{5}$ %, and made a profit of 5 %; at what rate of discount had he purchased the stock? *Ans.* 12 %.

27. A dry-goods merchant sells delaines for $2\frac{1}{2}$ cents per yard more than they cost, and realizes a profit of 8 %; what was the cost per yard? *Ans.* $.31$\frac{1}{4}$.

28. If I make a profit of $18\frac{3}{4}$ % by selling broadcloth for $.75 per yard above cost, how much must I advance on this price to realize a profit of $31\frac{1}{4}$ %?

29. A speculator gained 30 % on $\frac{3}{5}$ of his investment, and lost 5 % on the remainder, and his net profits were $720. What would have been his profits, had he gained 30 % on $\frac{2}{5}$ and lost 5 % on the remainder? *Ans.* $405.

30. A man wishing to sell his real estate asked 36 per cent. more than it cost him, but he finally sold it for 16 per cent. less than his asking price. He gained by the transaction $740.48. How much did the estate cost him, what was his asking price, and for how much did he sell it?

Ans. Cost, $5200; asking price, $7072; sold for $5940.48.

31. Sold $\frac{5}{8}$ of a barrel of beef for what the whole barrel cost; what per cent. did I gain on the part sold?

32. Bought 4 hogsheads of molasses, each containing 84 gallons, at $.37$\frac{1}{2}$ a gallon, and paid $7.50 for freight and cartage. Allowing 5 per cent. for leakage and waste, 4 per cent. of the sales for bad debts, and 1 per cent. of the remainder for collecting, for how much per gallon must I sell it to make a net gain of 25 per cent. on the whole cost? *Ans.* $.55+.

INSURANCE.

496. **Insurance** is security guaranteed by one party to another, against loss, damage, or risk. It is of two kinds; insurance on property, and insurance on life.

497. The **Insurer** or **Underwriter** is the party taking the risk.

498. The **Insured** or **Assured** is the party protected.

499. The **Policy** is the written contract between the parties.

500. **Premium** is the sum paid for insurance. It is always a certain per cent. of the sum insured, varying according to the degree or nature of risk assumed, and payable annually or at stated intervals.

NOTES.—1. Insurance business is generally conducted by joint stock companies, though sometimes by individuals.

2. A *Mutual Insurance* company is one in which each person insured is entitled to a share in the profits of the concern.

3. The act of insuring is sometimes called *taking a risk.*

FIRE AND MARINE INSURANCE.

501. Insurance on property is of two kinds; *Fire Insurance* and *Marine Insurance.*

Fire Insurance is security against loss of property by fire.

Marine Insurance is security against the loss of vessel or cargo by the casualties of navigation.

502. The **Sum Covered** by insurance is the difference between the sum insured and the premium paid.

NOTES.—1. As security against fraud, most insurance companies take risks at not more than two-thirds of the full value of the property insured.

2. When insured property suffers damage less than the amount of the policy, the insurers are required to pay only the estimated loss.

503. The calculations in insurance are based upon the following relations:

I. Premium is *percentage* (**445**).

II. The sum insured is the *base* of premium.

III. The sum covered by insurance is *difference.*

EXAMPLES FOR PRACTICE.

1. What premium must be paid for insuring my stock of goods to the amount of $5760 at $1\frac{1}{4}$ %?

OPERATION.

$\$5760 \times .0125 = \$72$, *Ans.*

ANALYSIS. According to Prob. I, (**449**), we multiply $5760, the base of premium, by .0125, the rate, and obtain $72, the premium.

2. For what sum must a granary be insured at 2 % in order to cover the loss of the wheat, valued at $1617?

OPERATION.

$1.00 - .02 = .98$

$\$1617 \div .98 = \$1650$, *Ans.*

ANALYSIS. According to Prob. V, (**453**), we divide the sum to be covered, $1617, which is difference, by 1 minus the *rate* of premium, and obtain $1650, the *base* of premium, or the sum to be insured.

PROOF. $\$1650 \times .02 = \$33$, premium; $\$1650 - \$33 = \$1617$, the sum covered.

3. What must be paid for an insurance of $5860 at $1\frac{1}{2}$ %?

4. What is the premium of $860 at $\frac{1}{2}$ %? *Ans.* $4.30.

5. What is the premium for an insurance of $3500 on my house and barn, at $1\frac{1}{4}$ %? *Ans.* $43.75.

6. A fishing craft, insured for $10000 at $2\frac{1}{4}$ %, was totally wrecked; how much of the loss was covered? *Ans.* $9775.

7. A hotel valued at $10000 has been insured for $6000 at $1\frac{1}{4}$ %, $5.50 being charged for the policy and the survey of the premises; if it should be destroyed by fire, what loss would the owner suffer? *Ans.* $4080.50.

8. A merchant whose stock in trade is worth $12000, gets the goods insured for $\frac{4}{5}$ of their value, at $\frac{3}{4}$ %; if in a conflagration he saves only $2000 of the stock, what actual loss will he sustain?

9. If I take a risk of $36000 at $2\frac{1}{2}$ %, and re-insure $\frac{1}{2}$ of it at 3 %, what is my balance of the premium? *Ans.* $360.

10. I pay $12 for an insurance of $800; what is the rate of premium? *Ans.* $1\frac{1}{2}$ %.

11. A trader got a shipment of 500 barrels of flour insured for 80 % of its cost, at $3\frac{1}{4}$ %, paying $107.25 premium; at what price per barrel did he purchase the flour? *Ans.* $8.25.

12. The Astor Insurance Company took a risk of $16000, for a premium of $280; what was the rate of insurance?

13. A whaling merchant gets his vessel insured for $20000 in

the Gallatin Company, at $\frac{3}{4}$ %, and for \$30000 in the Howard Company, at $\frac{1}{2}$ %; what rate of premium does he pay on the whole insurance? *Ans.* $\frac{3}{5}$ %.

14. If it cost \$46.75 to insure a store for $\frac{1}{2}$ of its value, at $1\frac{3}{8}$ %, what is the store worth? *Ans.* \$6800.

15. For what sum must I get my library insured at $1\frac{1}{8}$ %, to cover a loss of \$7910? *Ans.* \$8000.

16. What will be the premium for insuring at $2\frac{3}{7}$ %, to cover \$27320? *Ans.* \$680.

17. A shipment of pork was insured at $4\frac{3}{8}$ %, to cover $\frac{5}{8}$ of its value. The premium paid was \$122.50; what was the pork worth? *Ans.* \$4480.

18. A gentleman obtained an insurance on his house for $\frac{3}{4}$ of its value, at $1\frac{1}{2}$ % annually. After paying 5 instalments of premium, the house was destroyed by fire, in consequence of which he suffered a loss of \$2940; what was the value of the house? *Ans.* \$9600.

19. A man's property is insured at $2\frac{1}{2}$ % payable annually; in how many years will the premium equal the policy?

20. A company took a risk at $2\frac{1}{4}$ %, and re-insured $\frac{3}{5}$ of it in another company at $2\frac{1}{2}$ %. The premium received exceeded the premium paid by \$72. What was the amount of the risk? 9600

21. The Commercial Insurance Company issued a policy of insurance on an East India merchantman for $\frac{3}{4}$ of the estimated value of ship and cargo, at $4\frac{1}{4}$ %, and immediately re-insured $\frac{1}{2}$ of the risk in the Manhattan Company, at 3 %. During the outward voyage the ship was wrecked, and the Manhattan Company lost \$1350 more than the Commercial Company; what did the owners lose? *Ans.* \$40590.

LIFE INSURANCE.

504. **Life Insurance** is a contract in which a company agrees to pay a certain sum of money on the death of an individual, in consideration of an immediate payment, or of an annual premium paid for a term of years, or during the life of the insured. The

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policy may be made payable to the heirs of the insured, or assured, person, or to any one whom he may designate.

505. The policies issued by life insurance companies are of various kinds, the principal of which are as follows:

1st. Term policies, payable on the death of the insured, if the death occurs during a specified term of years; these require the payment of an annual premium till the policy matures or expires.

2d. Life policies, payable on the death of the insured, the annual premium to continue during life.

3d. Life policies, payable on the death of the insured, the annual premium to cease at a given age.

4th. Endowment assurance policies, payable to the assured person on his attaining a given age, or to his heirs if his death occurs before that age, annual premium being required till the policy matures.

NOTE.—The premium on the first and second classes of policies may be discharged by a single payment, instead of annual installments.

506. The **Expectation of Life** is the average number of years of life that remains to persons of a given age, as determined by tables of mortality.

507. The rates of life insurance, as fixed by different companies, are based upon the expectation of life and the probable rates of interest which money will bear in future time.

508. The rates of annual premium for the assurance of $100 on a single life, according to the two kinds of *life policies* (2 and 3), as issued by the Mutual Life Insurance Company of New York, are given in the Life Table on page 291.

509. The rates of annual premium of an assurance of $100 in the same company, payable to the party assured on his attaining the age of 40, 45, 50, 55, 60, or 65, or to his representatives, in case of death before attaining these ages respectively, are shown in the Endowment Assurance Table on page 292.

NOTES.—1. The tables of the Mutual Life Insurance Company of New York have been selected, as furnishing good examples of a variety of policies; the computations by any other tables would not differ in any material respect from those introduced under these tables.

2. Since a payment is made at the issue of the policy, and another at the expiration of the first year, the number of payments on a policy will always be 1 more than the number of years.

LIFE TABLE.

ANNUAL PREMIUM ON A POLICY OF $100.					
Age at issue.	Payments during life.	Payments To cease at 65.	Payments To cease at 60.	Payments To cease at 50.	Age at issue.
14	$1.4707	$1.4999	$1.5238	$1.6150	14
15	1.5105	1.5422	1.5683	1.6681	15
16	1.5516	1.5861	1.6145	1.7240	16
17	1.5940	1.6316	1.6625	1.7826	17
18	1.6377	1.6786	1.7124	1.8444	18
19	1.6829	1.7275	1.7644	1.9096	19
20	1.7296	1.7782	1.8186	1.9785	20
21	1.7780	1.8310	1.8753	2.0516	21
22	1.8280	1.8859	1 9344	2.1292	22
23	1.8798	1.9431	1.9963	2.2118	23
24	1.9335	2 0027	2.0612	2.3000	24
25	1.9891	2.0648	2.1291	2.3944	25
26	2.0470	2.1300	2.2007	2.4959	26
27	2.1071	2.1981	2.2761	2.6054	27
28	2.1696	2.2695	2.3555	2.7238	28
29	2.2346	2.3442	2.4395	2 8525	29
30	2.3023	2.4230	2.5284	2.9928	30
31	2.3728	2.5058	2.6226	3.1466	31
32	2.4464	2.5930	2.7228	3.3163	32
33	2.5232	2.6851	2.8296	3.5044	33
34	2.6034	2.7824	2.9436	3.7142	34
35	2.6873	2.8856	3.0657	3.9503	35
36	2.7752	2.9951	3.1971	4.2182	36
37	2.8674	3.1117	3.3387	4.5251	37
38	2 9641	3.2361	3.4919	4.8807	38
39	3.0658	3.3692	3.6584	5.2981	39
40	3.1729	3.5120	3.8402	5.7959	40
41	3.2856	3.6654	4.0393		41
42	3.4046	3.8311	4.2588		42
43	3.5303	4.0106	4.5021		43
44	3.6632	4.2055	4.7735		44
45	3.8038	4.4181	5.0782		45
46	3.9530	4.6512	5.4235		46
47	4.1111	4.9075	5.8180		47
48	4.2782	5.1902	6.2726		48
49	4.4549	5.5038	6.8632		49
50	4 6417	5.8536	7.4317		50
51	4.8393	6.2470			51
52	5.0486	6.6935			52
53	5.2708	7.2061			53
54	5.5067	7.8017			54
55	5.7577	8.5048			55

ENDOWMENT ASSURANCE TABLE.

ANNUAL PREMIUM ON A POLICY OF $100.							
Age at issue.	Policy due at 40.	Policy due at 45.	Policy due at 50.	Policy due at 55.	Policy due at 60.	Policy due at 65.	Age at issue.
14		$2.475	$2.113	$1.868	$1.704		14
15		2.587	2.197	1.935	1.759		15
16	$3.356	2.707	2.285	2.004	1.816	$1.694	16
17	3.545	2.835	2.379	2.077	1.876	1.746	17
18	3.752	2.937	2.478	2.153	1.939	1.799	18
19	3.978	3.122	2.585	2.234	2.004	1.855	19
20	4.228	3.283	2.698	2.320	2.073	1.914	20
21	4.504	3.458	2.819	2.410	2.145	1.974	21
22	4.812	3.648	2.949	2.506	2.220	2.038	22
23	5 156	3 855	3.089	2.608	2.300	2.104	23
24	5.544	4.082	3.239	2.717	2.384	2.174	24
25	5.985	4.333	3.402	2.832	2.473	2.247	25
26	6.489	4.611	3.578	2.956	2.567	2.323	26
27	7.082	4.920	3.770	3.088	2.666	2.404	27
28	7.752	5.265	3.979	3.231	2.772	2.489	28
29	8.558	5.654	4.208	3.384	2.884	2.578	29
30	9.526	6.096	4.461	3.549	3.004	2.672	30
31		6.601	4.740	3.728	3.132	2.772	31
32		7.185	5.051	3.923	3.268	2.877	32
33		7.866	5.398	4.135	3.415	2.989	33
34		8.673	5.789	4.368	3.573	3.108	34
35		9.641	6.232	4.624	3.743	3.235	35
36			6.739	4.906	3.928	3.370	36
37			7.325	5.221	4.128	3.515	37
38			8.008	5.572	4.347	3.670	38
39			8.816	5.967	4.586	3.837	39
40			9.787	6.415	4.849	4.017	40
41				6.927	5.139	4.212	41
42				7.518	5.462	4.424	42
43				8 207	5.822	4.655	43
44				9.022	6.227	4.908	44
45				10.000	6.686	5.185	45
46					7.210	5.491	46
47					7.813	5.830	47
48					8.515	6.208	48
49					9.343	6.630	49
50					10.332	7.105	50
51					11.536	7.645	51
52						8.265	52
53						8.983	53
54						9.826	54
55						10.831	55

EXAMPLES FOR PRACTICE.

1. What sum must a man pay annually to the Mutual Insurance of New York, for a life policy of \$2500, his age being 33 years at the issue of the policy?

OPERATION.

$\$2500 \times .025232 = \$63.08$, *Ans.*

ANALYSIS. We multiply the face of the policy, \$2500, by the rate *per cent.* found opposite 33 years in the Life Table, expressed decimally, and obtain \$63.08, the annual premium required.

NOTE. — The examples which follow all refer to the rates given in the preceding tables.

2. A man at 30 years of age takes a policy for \$2000, the payments of premium to cease at 50; if he survives that age, how much more money will he receive from the company than he pays to the company? *Ans.* \$743.024.

3. What annual premium must a man pay during life, commencing at the age of 50, to secure \$3000 at his death?

4. A gentleman at the age of 36 gets his life insured for \$1500, premium to cease at the age of 60; if he dies at 52, how much more will his family receive than has been paid out in premiums?

5. A clergyman wishing to secure an income to his family after his death, had his life insured at the age of 54, in the sum of \$3500, premium payable during life; his decease took place at the age of 72. How much more would have been saved to his family if he had taken, instead, a policy for the same amount, with payments of premium to cease at 65? *Ans.* \$385.24.

6. How much more premium will be required to secure an endowment of \$1200 at 40, by taking out a policy at the age of 30, than if the policy be taken at 24? *Ans.* \$126.456.

7. A man 37 years old took an endowment assurance policy for \$750, due at the age of 50, and died when 49 years old; how much more would his heirs have realized if he had taken a life policy for the same amount, with payment to cease at 50?

8. A has his life insured at the age of 20, and B has his insured at the age of 30, each taking a life policy requiring annual payments of premium during life; what will be the age of each

when the amount of premium paid shall exceed the face of his policy? *Ans.* A, 77 years; B, 73 years.

9. What is the whole amount of premiums that must be paid to secure an endowment of $1000 at the age of 60, the policy being issued at the age of 45? *Ans.* $1069.76.

10. A person at the age of 34 had his life insured in the sum of $600, the premium to cease at 50. When he died, there was a net gain to his family of $421.72; how many payments of premium had he made? *Ans.* 8.

11. A gentleman obtained an insurance on his life at the age of 29, and died at the age of 40; the policy taken required annual payments of premium during life, and secured to his heirs $1829.62 more than the whole premium paid. Required the face of the policy. *Ans.* $2500.

TAXES.

510. A **Tax** is a sum of money assessed on the person or property of an individual, for public purposes.

511. A **Poll Tax** is a certain sum required of each male citizen liable to taxation, without regard to his property. Each person so taxed is called a *poll.*

512. A **Property Tax** is a sum required of each person owning property, and is always a certain *per cent.* of the estimated value of his property.

513. An **Assessment Roll** is a list or schedule containing the names of all the persons liable to taxation in the district or company to be assessed, and the valuation of each person's taxable property.

514. **Assessors** are the persons appointed to prepare the assessment roll, and apportion the taxes.

1. In a certain town a tax of $4000 is to be assessed. There are 400 polls to be assessed $.50 each, and the valuation of the taxable property, as shown by the assessment roll, is $950000; what will be the property tax on $1, and how much will be A's tax, whose property is valued at $3500, and who pays for 3 polls?

OPERATION.

$ .50 × 400 = $200, amount assessed on the polls.
$4000 — $200 = $3800, amount to be assessed on property.
$3800 ÷ $950000 = .004, rate of taxation;
$3500 × .004 = $14, A's property tax;
$ .50 × 3 = 1.50, A's poll tax;

$15.50, amount of A's tax. Hence the

RULE. I. *Find the amount of poll tax, if any, and subtract it from the whole tax to be assessed; the remainder will be the property tax.*

II. *Divide the property tax by the whole amount of taxable property; the quotient will be the rate of taxation.*

III. *Multiply each man's taxable property by the rate of taxation, and to the product add his poll tax, if any; the result will be the whole amount of his tax.*

NOTE.—When a tax is to be apportioned among a large number of individuals, the operation is greatly facilitated by first finding the tax on $1, $2, $3, etc., to $9; then on $10, $20, $30, etc., to $90, and so on, and arranging the results as in the following

TABLE.

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
$1	$.004	$10	$.04	$100	$ .40	$1000	$4.00
2	.008	20	.08	200	.80	2000	8.
3	.012	30	.12	300	1.20	3000	12.
4	.016	40	.16	400	1.60	4000	16.
5	.020	50	.20	500	2.00	5000	29.
6	.024	60	.24	600	2.40	6000	24.
7	.028	70	.28	700	2.80	7000	28.
8	.032	80	.32	800	3.20	8000	32.
9	.036	90	.36	900	3.60	9000	36.

EXAMPLES FOR PRACTICE.

1. According to the conditions of the last example, what would be the tax of a person whose property was valued at 2465, and who pays for 2 polls?

OPERATION.

From the table we find that

	The tax on	$2000		is	$8.00	
	" " "	400		"	1.60	
	" " "	60		"	.24	
	" " "	5		"	.02	
And	" " "	2	polls	"	1.00	
			Whole tax	"	$10.86,	*Ans.*

2. What would A's tax be, who is assessed for $8530, and 3 polls? *Ans.* $35.62.

3. How much will C's tax be, who is assessed for $987, and 1 poll? *Ans.* $4.448.

4. The estimated expenses of a certain town for one year are $6319, and the balance on hand in the public treasury is $354. There are 2156 polls to be assessed at $.25 each, and taxable property to the amount of $1864000. Besides the town tax, there is a county tax of $1\frac{1}{2}$ mills on a dollar, and a State tax of $\frac{1}{2}$ of a mill on a dollar. Required the whole amount of A's tax, whose property is valued at $32560, and who pays for 3 polls.

5. What does a non-resident pay, who owns property in the same town to the amount of $16840? *Ans.* $79.99.

6. What sum must be assessed in order to raise a net amount of $5561.50, and pay the commission for collecting at 2 %.

NOTE. — Since the base of the collector's commission is the sum collected, (446), the question is an example under Problem V of Percentage.

7. In a certain district a school house is to be built at an expense of $9120, to be defrayed by a tax upon property valued at $1536000. What shall be the rate of taxation to cover both the cost of the school house, and the collector's commission at 5 %?

8. The expenses of a school for one term were $1200 for salary of teachers, $57.65 for fuel, and $38.25 for incidentals; the money received from the school fund was $257.75, and the remaining part of the expense was paid by a rate-bill. If the aggregate attendance was 9568 days, what was A's tax, who sent 4 pupils 46 days each? *Ans.* $19.96 +.

9. The expense of building a public bridge was $1260.52,

which was defrayed by a tax upon the property of the town. The rate of taxation was $3\frac{1}{4}$ mills on one dollar, and the collector's commission was $3\frac{1}{2}$ %; what was the valuation of the property?

Ans. \$401920.

GENERAL AVERAGE.

515. General Average is a method of computing the loss to be sustained by the proprietors of the ship, freight, and cargo, respectively, when, in a case of common peril at sea, any portion of the property has been sacrificed or damaged for the common safety.

516. The **Contributory Interests** are the three kinds of property which are taxed to cover the loss. These are,

1st The vessel, at its value before the loss.

2d. The freight, less $\frac{1}{3}$ as an allowance for seamen's wages.

3d. The cargo, including the part sacrificed, at its market value in the port of destination.

Note.—In New York only $\frac{1}{2}$ of the freight is made contributory to the loss.

517. Jettson is the portion of goods thrown overboard.

518. The loss which is subject to general average includes,

1st. Jettson, or property thrown overboard.

2d. Repairs to the vessel, less $\frac{1}{3}$ on account of the superior worth of the new articles furnished.

3d. Expense of detention to which the vessel is subject in port.

1. The ship Nelson, valued at \$52000, and having on board a cargo worth \$18000, on which the freight was \$3600, threw overboard a portion of the goods valued at \$5000, to escape wreck in a storm; she then put into port, and underwent repairs amounting to \$1200, the expenses of detention being \$350. What portion of the loss will be sustained by each of the three contributing interests? What will be paid or received by the owners of the ship and freight? What by A, who owned \$8000 of the cargo, including \$3500 of the portion sacrificed, and by B, who owned \$6000 of the cargo, including \$1500 of the portion sacrificed, and by C, who owned \$4000, or the residue of the cargo?

OPERATION.

LOSSES.		CONTRIBUTORY INTERESTS.	
Jettson,	$5000	Vessel,	$52000
Repairs, less $\frac{1}{3}$,	800	Freight, less $\frac{1}{3}$,	2400
Cost of detention,	350	Cargo,	18000
Total,	$6150	Total,	$72400

$6150 ÷ $72400 = .0849447+, rate per cent. of loss.
$52000 × .0849447 = $4417.13, payable by vessel.
2400 × .0849447 = 203.87, " " freight.
18000 × .0849447 = 1529.00, " " cargo.
$6150.00, Total contribution.

$8000 × .0849447 = $679.56, payable by A.
6000 × .0849447 = 509.67, " " B.
4000 × .0849447 = 339.78, " " C.

$4417.13 + $203.87 = $4621.00, payable by owners of vessel and freight.
800.00 + 350.00 = 1150.00, " to " " " "
4621.00 — 1150.00 = 3471.00, balance payable by ship owners.
3500.00 — 679.56 = 2820.44, " receivable by A.
1500.00 — 509.67 = 990.33, " " " B.

Hence the following

RULE. I. *Divide the sum of the losses by the sum of the contributory interests; the quotient will be the rate of contribution.*

II. *Multiply each contributory interest by the rate; the products will be the respective contributions to the loss.*

EXAMPLES FOR PRACTICE.

1. The ship Nevada, in distress at sea, cut away her mainmast, and cast overboard $\frac{1}{4}$ of her cargo, and then put into Havana to refit; the repairs cost $1500, and the necessary expenses of detention were $420. The ship was owned and sent to sea by George Law, and was valued at $25000; the cargo was owned by Hayden & Co., and consisted of 2800 barrels of flour, valued at $9 per barrel, upon which the freight was $4200. In the adjustment of the loss by general average, how much was due from Law to Hayden & Co.? *Ans.* $2629.36.

2. A coasting vessel valued at $28000, having been disabled in a storm, entered port, and was refitted at an expense of $270 for repairs, and $120 for board of seamen, pilotage, and dockage.

Of the cargo, valued at \$5000, \$2400 belonged to A, \$1850 to B, and \$750 to C; and the amount sacrificed for the ship's safety was \$1400 of A's property, and \$170 of B's; the gross charges for freight were \$1500. Required the balance, payable or receivable, by each of the parties, the loss being apportioned by general average.

Ans. { \$1295 payable by ship owners; \$1268 receivable by A;
41.25 " " C; 68.25 " " B.

CUSTOM HOUSE BUSINESS.

519. **Duties,** or **Customs,** are taxes levied on imported goods, for the support of government and the protection of home industry.

520. A **Custom House** is an office established by government for the transaction of business relating to duties.

It is lawful to introduce merchandise into a country only at points where custom houses are established. A seaport town having a custom house, is called a *port of entry.* To carry on foreign commerce secretly, without paying the duties imposed by law, is *smuggling.*

NOTE.—Customs or duties form the principal source of revenue to the General Government of the United States; by increasing the price of imported goods they operate as an indirect tax upon consumers, instead of a general direct tax.

521. Duties are of two kinds — Ad Valorem and Specific.

Ad Valorem Duty is a sum computed on the cost of the goods in the country from which they were imported.

Specific Duty is a sum computed on the weight or measure of the goods, without regard to their cost.

522. An **Invoice** is a bill of goods imported, showing the quantity and price of each kind.

523. By the New Tariff Act, approved March 2, 1857, all duties taken at the U. S. custom houses are *ad valorem.* The principal articles of import are classified, and a fixed rate is imposed upon each list or schedule, certain articles being excepted and entered free.

In collecting customs it is the design of government to tax only so much of the merchandise as will be available to the importer in the market. The goods are weighed, measured, gauged, or inspected, in order to ascertain the actual quantity received in port; and an allowance is made in every case of waste, loss, or damage.

524. **Tare** is an allowance for the weight of the box or the covering that contains the goods. It is ascertained, if necessary, by actually weighing one or more of the empty boxes, casks, or coverings. In common articles of importation, it is sometimes computed at a certain per cent. previously ascertained by frequent trials by weighing.

525. **Leakage** is an allowance on liquors imported in casks or barrels, and is ascertained by gauging the cask or barrel in which the liquor is imported.

526. **Breakage** is an allowance on liquors imported in bottles.

527. **Gross Weight** or **Value** is the weight or value of the goods before any allowance has been made.

528. **Net Weight** or **Value** is the weight or value of the goods after all allowances have been deducted.

NOTES.—1. *Draft* is an allowance for the waste of certain articles, and is made only for *statistical purposes;* it does not affect the amount of duty.

2. Long ton measure is employed in the custom houses of the United States, in estimating goods by the ton or hundred weight.

The rates of this allowance are as follows:

On	112 lb.					1 lb.
Above	112 lb. and not exceeding				224 lb.,	2 lb.
"	224 lb.	"	"	"	336 lb.,	3 lb.
"	336 lb.	"	"	"	1120 lb.,	4 lb.
"	1120 lb.	"	"	"	2016 lb.,	7 lb.
"	2016 lb.					9 lb.

529. In all calculations where ad valorem duties are considered,

I. The net value of the merchandise is the worth of the net weight or quantity at the invoice price, allowance being made in cases of damage.

II. The duty is computed at a certain legal per cent. on the net value of the merchandise.

NOTE.—In the following examples the legal rates of duty, according to the New Tariff Act, are given.

EXAMPLES FOR PRACTICE.

1. What is the duty, at 24 %, on an invoice of cassimere goods which cost $750?

OPERATION.

$\$750 \times .24 = \$180$

ANALYSIS. According to Prob. I, (**449**), we multiply the invoice, $750, which is the *base* of the duty, by the given *rate*, and obtain the duty, $180.

2. The gross weight of 3 hogsheads of sugar is 1024 lb., 1016 lb., and 1020 lb. respectively; the invoice price of the sugar $7\frac{1}{2}$ cents, and the allowance for tare 80 lb. per hogshead; what is the duty, at 24 %?

OPERATION.

	1024	
	1016	
	1020	
	3060,	gross weight.
$80 \times 3 =$	240,	tare.
	2820,	net weight.
	$\$.07\frac{1}{2}$	
	$211.50,	net value.
	.24	
	$50.7600,	duty.

ANALYSIS. We first find the gross weight of the three hhd. from which we subtract the tare, and obtain 2820 lb., the net weight. We next find the value of the net weight, at $7\frac{1}{2}$ cents, the invoice price, and then compute the duty at 24 % on this value, and obtain $50.76, the duty required.

3. Having paid the duty at 8 % on a quantity of Malaga raisins, I find that the whole cost in store, besides freight, is $378; what were the raisins invoiced at?

OPERATION.

$\$378 \div 1.08 = \$350$

ANALYSIS. According to Prob. IV, (**452**), we divide the *amount*, $378, by 1 plus the rate, 1.08, and obtain the *base*, or invoice, $350.

4. A Boston jeweler orders from Lubec a quantity of watch movements, amounting to $2780; what will be the duty, at 4 %?

5. What will be the duty at 15 % on 1200 lb. of tapioca, invoiced at $5\frac{1}{2}$ cents per pound? *Ans.* $9.90.

6. What is the duty at 15 % on 54 boxes of candles, each weighing 1 cwt., invoiced at $8\frac{3}{4}$ cents per pound, allowing tare at $3\frac{1}{2}$ per cent.?

7. A merchant imported 50 casks of port wine, each containing originally 36 gallons, invoiced at \$2.50 per gallon. He paid freight at \$1.30 per cask, and duty at 30 %, $1\frac{1}{2}$ % leakage being allowed at the custom house, and \$8.50 for cartage; what did the wine cost him in store? *Ans.* \$5903.25.

8. A liquor dealer receives an invoice of 120 dozen bottles of porter, rated at \$1.25 per dozen; if 2 % of the bottles are found broken, what will be the duty at 24 %? *Ans.* \$35.28.

9. The duty at 19 % on an importation of Denmark satin was \$619.40; what was the invoice of the goods? *Ans.* \$3260.

10. The duty on 600 drums of figs, each containing 14 lb., invoiced at $5\frac{1}{4}$ cents per pound, was \$35.28; required, the rate of duty. *Ans.* 8 %.

11. A merchant in New York imports from Havana 200 hhd. of W. I. molasses, each containing 63 gallons, invoiced at \$.30 per gallon; 150 hhd. of B. coffee sugar, each containing 500 pounds, invoiced at \$.05 per pound; 80 boxes of lemons, invoiced at \$2.50 per box; and 75 boxes of sweet oranges, invoiced at \$3.00 per box. What was the whole amount of duty, estimated at 24 % on molasses and sugar, and at 8 % on lemons and oranges? *Ans.* \$1841.20.

12. A merchant imported 56 casks of wine, each containing 36 gallons net, the duty at 30 % amounting to \$907.20; at what price per gallon was the wine invoiced?

13. The duty on an invoice of French lace goods at 24 %, was \$132, an allowance of 12 % having been made at the custom house for damage received since the goods were shipped; what was the cost or invoice of the goods. *Ans.* \$625.

14. A quantity of Valencias, invoiced at \$1654, cost me \$1980.50 in store, after paying the duties and \$12.24 for freight; what was the rate of duty?

15. The duty on an importation of Bay rum, after allowing 2 % for breakage, was \$823.20, and the invoice price of the rum was \$.25 per bottle; how many dozen bottles did the importer receive, duty at 24 %? *Ans.* $1143\frac{1}{3}$ doz.

SIMPLE INTEREST.

530. Interest is a sum paid for the use of money.

531. **Principal** is the sum for the use of which interest is paid.

532. **Rate per cent. per annum** is the sum per cent. paid for the use of any principal for one year.

NOTE.—The rate per cent. is commonly expressed decimally as hundredths (442).

533. **Amount** is the sum of the principal and interest.

534. **Simple Interest** is the sum paid for the use of the principal only, during the whole time of the loan or credit.

535. **Legal Interest** is the rate per cent. established by law. It varies in different States, as follows:

Alabama,	8 per cent.	Minnesota,	7 per cent.
Arkansas,	6 " "	Mississippi,	8 " "
California,	10 " "	Missouri,	6 " "
Connecticut,	6 " "	New Hampshire,	6 " "
Delaware,	6 " "	New Jersey,	6 " "
Dist. of Columbia,	6 " "	New York,	7 " "
Florida,	8 " "	North Carolina,	6 " "
Georgia,	7 " "	Ohio,	6 " "
Illinois,	6 " "	Pennsylvania,	6 " "
Indiana,	6 " "	Rhode Island,	6 " "
Iowa,	6 " "	South Carolina,	7 " "
Kentucky	6 " "	Tennessee,	6 " "
Louisiana,	5 " "	Texas,	8 " "
Maine,	6 " "	United States (debts),	6 " "
Maryland,	6 " "	Vermont,	6 " "
Massachusetts,	6 " "	Virginia,	6 " "
Michigan,	7 " "	Wisconsin,	7 " "

NOTES.—1. The legal rate in Canada, Nova Scotia, and Ireland is 6 per cent., and in England and France 5 per cent.

2. When the rate per cent. is not specified in accounts, notes, mortgages, contracts, etc., the legal rate is always understood.

3. In some States the laws allow parties to give and take higher rates, by special agreement.

4. Book accounts bear interest after the expiration of the term of credit, and notes are on interest after they become due, though no mention of interest be made in them.

5. If notes are to draw interest from their date, or from a given time after date, the fact must be so stated in the body of the notes.

536. **Usury** is illegal interest, or a greater per cent. than the legal rate.

NOTE.—The taking of usury is prohibited, under various penalties, in different States.

537. In the operations of interest there are five parts or elements, namely:

I. Rate per cent. per annum; which is the fraction or decimal denoting how many hundredths of a number or sum of money are to be taken for a period of 1 year.

II. Interest; which is the whole sum taken for the whole period of time, whatever it may be.

III. Principal; which is the base or sum on which interest is computed.

IV. Amount; which is the sum of principal and interest; and

V. Time.

TO COMPUTE INTEREST.

CASE I.

538. To find the interest on any sum, at any rate per cent. per annum, for years and months.

Analysis. In interest, any rate per cent. is confined to 1 year. Therefore, if the time be more than 1 year, the per cent. will be greater than the rate per cent. per annum, and if the time be less than 1 year, the per cent. will be less than the rate per cent. per annum. From these facts, we deduce the following principles:

I. If the rate per cent. per annum be multiplied by the time, expressed in years and fractions or decimals of a year, the product will be the rate for the required time. And

II. If the principal be multiplied by the rate for the required time, the product will be the required interest. Hence

III. Interest is always the product of three factors, namely, rate per cent. per annum, time, and principal.

In computing interest the three factors may be taken in any order; thus, if the principal be multiplied by the rate per cent. per annum, the product will be the interest for 1 year; and if the interest for 1 year be multiplied by the time expressed in years, the result will be the required interest. Hence the following

Rule. I. *Multiply the principal by the rate per cent., and the product will be the interest for* 1 *year.*

II. *Multiply this product by the time in years and fractions of a year; the result will be the required interest.*

Or, *Multiply together the rate per cent. per annum, time, and principal, in such order as is most convenient; the continued product will be the required interest.*

CASE II.

539. To find the interest on any sum, for any time, at any rate per cent.

The analysis of our rule is based upon the following

Obvious Relations between Time and Interest.

I The interest on any sum for 1 year at 1 per cent., is .01 of that sum, and is equal to the principal with the separatrix removed two places to the left.

II. A month being $\frac{1}{12}$ of a year, $\frac{1}{12}$ of the interest on any sum for 1 year is the interest for 1 month.

III. The interest on any sum for 3 days is $\frac{3}{30} = \frac{1}{10} = .1$ of the interest for 1 month, and any number of days may readily be reduced to *tenths* of a month by dividing by 3.

IV. The interest on any sum for 1 month, multiplied by any given time expressed in months and tenths of a month, will produce the required interest.

These principles are sufficient to establish the following

RULE. I. To find the interest for 1 yr. at 1 %:—*Remove the separatrix in the given principal two places to the left,*

II. To find the interest for 1 mo. at 1 %:—*Divide the interest for* 1 *year by* 12.

III To find the interest for any time at 1 %: — *Multiply the interest for* 1 *month by the given time expressed in months and tenths of a month.*

IV. To find the interest at any rate %: — *Multiply the interest at* 1 % *for the given time by the given rate.*

CONTRACTIONS. After removing the separatrix in the principal two places to the left, the result may be regarded either as the interest on the given principal for 12 months at 1 per cent., or for 1 month at 12 per cent. If we regard it as for 1 month at 12 per cent., and if the given rate be an aliquot part of 12 per cent., the interest on the given principal for 1 month may readily be found, by taking such an aliquot part of the interest for 1 month as the given rate is part of 12 per cent. Thus,

To find the interest for 1 month at 6 per cent., remove the separatrix two places to the left, and divide by 2.

To find it at 3 per cent., proceed as before, and divide by 4; at 4 per cent., divide by 3; at 2 per cent., divide by 6, etc.

SIX PER CENT. METHOD.*

540. By referring to **535** it will be seen that the legal rate of interest in 22 States is 6 per cent. This is a sufficient reason for introducing the following brief method into this work:

ANALYSIS. At 6 % per annum the interest on $1

For 12 months	....	is	$.06.
" 2 months ($\frac{2}{12}=\frac{1}{6}$ of 12 mo.)	....	"	.01.
" 1 month, or 30 days ($\frac{1}{12}$ of 12 mo.)		"	.00½ = $.005 ($\frac{1}{12}$ of $.06).
" 6 days ($\frac{1}{5}$ of 30 da.)	....	"	.001.
" 1 " ($\frac{1}{6}$ of 6 da. = $\frac{1}{30}$ of 30 da.)		"	.000$\frac{1}{6}$.

Hence we conclude that,

1st. The interest on $1 is $.005 per month, or $.01 for every 2 months;

2d. The interest on $1 is $.000$\frac{1}{6}$ per day, or $.001 for every 6 days.

From these principles we deduce the

RULE. I. To find the rate:— *Call every year* $.06, *every* 2 *months* $.01, *every* 6 *days* $.001, *and any less number of days sixths of* 1 *mill.*

II. To find the interest:— *Multiply the principal by the rate.*

NOTES.—1. To find the interest at any other rate % by this method, first find it at 6 %, and then increase or diminish the result by as many sixths of itself as the given rate is units greater or less than 6 %. Thus, for 7 % add $\frac{1}{6}$, for 4 % subtract $\frac{1}{3}$, etc.

2. The interest of $10 for 6 days, or of $1 for 60 days, is $.01. Therefore, if the principal be less than $10 and the time less than 6 days, or the principal less than $1 and the time less than 60 days, the interest will be less than $.01, and may be disregarded.

3. Since the interest of $1 for 60 days is $.01, the interest of $1 for any num-

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* This method of finding the interest on $1 by inspection was first published in The Scholar's Arithmetic, by Daniel Adams, M. D., in 1801, and from its simplicity it has come into very general use.

ber of days is as many cents as 60 is contained times in the number of days. Therefore, if any principal be multiplied by the number of days in any given number of months and days, and the product divided by 60, the result will be the interest in cents. That is, *Multiply the principal by the number of days, divide the product by 60, and point off two decimal places in the quotient. The result will be the interest in the same denomination as the principal.*

EXAMPLES FOR PRACTICE.

What is the interest on the following sums for the times given, at 6 per cent.?

1. $325 for 3 years. *Ans.* $58.50.
2. $1600 for 1 yr. 3 mo. *Ans.* $120.
3. $36.84 for 5 mo.
4. $35.14 for 2 yr. 9 mo. 15 da.
5. $217.15 for 3 yr. 10 mo. 1 da. *Ans.* $49.98+.
6. $721.53 for 4 yr. 1 mo. 18 da.
7. $15.125 for 15 mo. 17 da. *Ans.* $1.17+.

On the following at 7 per cent.?

8. $2000 for 5 yr. 6 mo.
9. $1436.59 for 2 yr. 5 mo. 18 da. *Ans.* $248.051+.
10. $224.14 for 8 mo. 13 da. *Ans.* $11.026.
11. $100.25 for 63 da. *Ans.* $1.228+.
12. $600 for 24 da.
13. $520 for 5 yr. 11 mo. 29 da. *Ans.* $218.298.
14. $710.01 for 3 yr. 11 mo. 8 da.

On the following at 5 per cent.?

15. $48.255 for 5 yr.
16. $750 for 1 yr. 3 mo.
17. $647.654 for 4 yr. 10 mo. 20 da. *Ans.* $158.315+.
18. $12850 for 90 da.
19. $2500 for 7 mo. 20 da. *Ans.* $79.86.
20. $850.25 for 8 mo.
21. $48.25 for 1 yr. 2 mo. 17 da. *Ans.* $2.928+.

On the following at 8 per cent.?

22. $2964.12 for 11 mo. *Ans.* $217.368+
23. $725.50 for 150 da.
24. $360 for 2 yr 6 mo. 12 da.
25. $600 for 3 yr. 2 mo. 17 da. *Ans.* $154.266⅔.

26. $1700 for 28 da. *Ans.* $10.58—.

On the following at 10 per cent.?

27. $3045.20 for 7 mo. 15 da. *Ans.* $190.32+.

28. $1247.375 for 2 yr. 26 da. *Ans.* $258.48+.

29. $2450 for 60 da.

30. $375.875 for 3 mo. 22 da.

31. $5000 for 10 da.

32. $127.65 for 1 yr. 11 mo. 3 da. *Ans.* $24.572.

33. What is the interest of $155.49 for 3 mo., at 6¼ per cent.?

34. What is the interest of $970.99 for 6 mo., at 5½ per cent.?

35. What is the amount of $350.50 for 2 yr. 10 mo., at 7 per cent.? *Ans.* $420.01+.

36. What is the interest of $95.008 for 3 mo. 24 da., at 4½ per cent.? *Ans.* $1.353+.

37. What is the amount of $145.20 for 1 yr. 9 mo. 27 da., at 12½ per cent.? *Ans.* $178.32375.

38. What is the amount of $215.34 for 4 yr. 6 mo., at 3½ per cent.? *Ans.* $249.256+.

39. What is the amount of $5000 for 20 da., at 7 per cent.?

40. What is the amount of $16941.20 for 1 yr. 7 mo. 28 da., at 4¾ per cent.? *Ans.* $18277.91—.

41. If $1756.75 be placed at interest June 29, 1860, what amount will be due Feb. 12, 1863, at 7 %?

42. If a loan of $3155.49 be made Aug. 15, 1858, at 6 per cent., what amount will be due May 1, 1866, no interest having been paid?

43. How much is the interest on a note for $257.81, dated March 1, 1859, and payable July 16, 1861, at 7 %?

44. A person borrows $3754.45, being the property of a minor who is 15 yr. 3 mo. 20 da. old. He retains it until the owner is 21 years old. How much money will then be due at 6 % simple interest? *Ans.* $5037.22+.

45. If a person borrow $7500 in Boston and lend it in Wisconsin, how much does he gain in a year?

46. A man sold a piece of property for $11320; the terms were $3200 in cash on delivery, $3500 in 6 mo., $2500 in 10 mo., and

the remainder in 1 yr. 3 mo., with 7 % interest; what was the whole amount paid? *Ans.* \$11773.83⅓.

47. May 10, 1859, I borrowed \$6840, with which I purchased flour at \$5.70 a barrel. June 21, 1860, I sold the flour for \$6.62½ a barrel, cash. How much did I gain by the transaction, interest being reckoned at 6 %?

48. If a man borrow \$15000 in New York, and lend it in Ohio, how much will he lose in 146 days, reckoning 360 days to the year in the former transaction, and 365 days in the latter?

49. Hubbard & Northrop bought bills of dry goods of Bowen, McNamee & Co., New York, as follows, viz.: July 15, 1860, \$1250; Oct. 4, 1860, \$3540.84; Dec. 1, 1860, \$575; and Jan. 24, 1861, \$816.90. They bought on time, paying legal interest; how much was the whole amount of their indebtedness, March 1, 1861?

50. A broker allows 6 per cent. per annum on all moneys deposited with him. If on an average he lend out every \$100 received on deposit 11 times during the year, for 33 days each time at 2 % a month, how much does he gain by interest on \$1000? *Ans.* \$182.

51. A man, engaged in business with a capital of \$21840, is making 12½ per cent. per annum on his capital; but on account of ill health he quits his business, and loans his money at 7¾ %. How much does he lose in 2 yr. 5 mo. 10 da. by the change?

Ans. \$2535.86⅔.

52. A speculator wishing to purchase a tract of land containing 450 acres at \$27.50 an acre, borrows the money at 5½ per cent. At the end of 4 yr. 11 mo. 20 da. he sells ⅖ of the land at \$34 an acre, and the remainder at \$32.55 an acre. How much does he lose by the transaction?

53. Bought 4500 bushels of wheat at \$1.12½ a bushel, payable in 6 months; I immediately realized for it \$1.06 a bushel, cash, and put the money at interest at 10 per cent. At the end of the 6 months I paid for the wheat; did I gain or lose by the transaction, and how much?

PARTIAL PAYMENTS OR INDORSEMENTS.

541. A **Partial Payment** is payment in part of a note, bond, or other obligation.

542. An **Indorsement** is an acknowledgment written on the back of an obligation, stating the time and amount of a partial payment made on the obligation.

543. To secure uniformity in the method of computing interest where partial payments have been made, the Supreme Court of the United States has decided that,

I. "The rule for casting interest when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due.

II. "If the payment exceeds the interest the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of the principal remaining due.

III. "If the payment be less than the interest the surplus of interest must not be taken to augment the principal, but the interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied towards discharging the principal, and the interest is to be computed on the balance as aforesaid."—*Decision of Chancellor Kent.*

This decision has been adopted by nearly all the States of the Union, the only prominent exceptions being Connecticut, Vermont, and New Hampshire. We therefore present the method prescribed by this decision as the

UNITED STATES RULE.

I. *Find the amount of the given principal to the time of the first payment, and if this payment exceed the interest then due, subtract it from the amount obtained, and treat the remainder as a new principal.*

II. *But if the interest be greater than any payment, compute the interest on the same principal to a time when the sum of the payments shall equal or exceed the interest due, and subtract the sum* *of the payments from the amount of the principal; the remainder will form a new principal, with which proceed as before.*

EXAMPLES FOR PRACTICE.

$1000. BUFFALO, N Y., May 15, 1856.

1. Two years after date I promise to pay to David Hudson, or order, one thousand dollars, with interest, for value received.

HENRY BURR.

On this note were indorsed the following payments:

Sept. 20, 1857, received,	....	$150.60
Oct. 25, 1859, "	....	200.90
July 11, 1861, "	....	75.20
Sept. 20, 1862, "	....	112.10
Dec. 5, 1863, "	....	105.

What remained due May 20, 1864?

OPERATION.

Principal on interest from May 15, 1856,..............		$1000
Interest to Sept. 20, 1857, 1 yr. 4 mo 5 da.,.............		94.31
Amount,..........		$1094.31
1st Payment, Sept 20, 1857,..........................		150.60
Remainder for a new principal,....................		$943.71
Interest from 1st paym't to Oct. 25, 1859, 2 yr. 1 mo. 5 da.,		138.54
Amount,..........		$1082.25
2d Payment, Oct. 25, 1859,...........................		200.90
Remainder for a new principal,....................		$881.35
Interest from 2d paym't to Dec. 5, 1863, 4 yr. 1 mo. 10 da.,		253.63
Amount,..........		$1134.98
3d Payment, less than interest due,...........	$75.20	
4th "	112.11	
Sum of 3d and 4th payments, less than interest due,	$187.31	
5th payment, ...	105.00	
Sums of 3d, 4th, and 5th payments,...................		292.31
Remainder for new principal,......................		$842.67
Interest to May 20, 1864, 5 mo. 15 da.,................		27.04
Balance due May 20, 1864,...........		$869.71

$1200. RICHMOND, VA., Oct. 15, 1859.

2. One year after date we promise to pay James Peterson, or order, twelve hundred dollars, for value received, with interest.

WILDER & SON.

Indorsed as follows: Oct. 15, 1860, $1000; April 15, 1861, $200. How much remained due Oct. 15, 1861? *Ans.* $82.56.

850\frac{76}{100}$. BOSTON, June 10, 1855.

3. Eighteen months after date I promise to pay Crosby, Nichols & Co., or order, eight hundred fifty and $\frac{76}{100}$ dollars, with interest, for value received. O. L. SANBORN.

Indorsed as follows: March 4, 1856, $210.93; July 9, 1857, $140; Feb. 20, 1858, $178; May 5, 1859, $154.30; Jan. 17, 1860, $259.45. How much was due Oct. 24, 1861?

384\frac{95}{100}$. SAVANNAH, GA., Sept. 4, 1860.

4. Six months after date I promise to pay John Rogers, or order, three hundred eighty-four and $\frac{95}{100}$ dollars, for value received, with interest. WM. JENKINS.

This note was settled Jan. 1, 1862, one payment of $126.50 having been made Oct. 20, 1861; how much was due at the time of settlement?

$3475. NEW ORLEANS, March 6, 1857.

5 On demand we promise to pay Evans & Hart, or order, three thousand four hundred seventy-five dollars, for value received, with interest. DAVIS & BROTHER.

Indorsed as follows: June 1, 1857, $1247.60; Sept 10, 1857, $1400. How much was due Jan. 31, 1858?

6. A gentleman gave a mortgage on his estate for $9750, dated April 1, 1860, to be paid in 5 years, with annual interest after 9 months on all unpaid balances, at 10 per cent. Six months from date he paid $846.50; Oct. 20, 1862, $2500; July 3, 1863, $1500; Jan. 1, 1864, $500; how much was due at the expiration of the given time?

$500. PHILADELPHIA, Feb. 1, 1861.

7. For value received, I promise to pay J. B. Lippincott & Co., or order, five hundred dollars three months after date, with interest.

JAMES MONROE.

Indorsed as follows: May 1, 1861, $40; Nov. 14, 1861, $8; April 1, 1862, $12; May 1, 1862, $30. How much was due Sept. 16, 1862? *Ans.* $155.57+.

544. CONNECTICUT RULE.

I. *Payments made one year or more from the time the interest commenced, or from another payment, and payments less than the interest due, are treated according to the United States rule.*

II. *Payments exceeding the interest due and made within one year from the time interest commenced, or from a former payment, shall draw interest for the balance of the year, provided the interval does not extend beyond the settlement, and the amount must be subtracted from the amount of the principal for one year; the remainder will be the new principal.*

III. *If the year extend beyond the settlement, then find the amount of the payment to the day of settlement, and subtract it from the amount of the principal to that day; the remainder will be the sum due.*

545. A note containing a promise to pay interest *annually* is not considered in law a contract for any thing more than simple interest on the principal. For partial payments on such notes the following is the

VERMONT RULE.

I. *Find the amount of the principal from the time interest commenced to the time of settlement.*

II. *Find the amount of each payment from the time it was made to the time of settlement.*

III. *Subtract the sum of the amounts of the payments from the amount of the principal; the remainder will be the sum due.*

NOTE.—This rule is in quite extensive use among merchants and others.

546. In New Hampshire interest is allowed on the annual interest if not paid when due, in the nature of damages for its detention; and if payments are made *before* one year's interest has accrued, interest must be allowed on such payments for the balance of the year. Hence the following

New Hampshire Rule.

I. *Find the amount of the principal for one year, and deduct from it the amount of each payment of that year, from the time it was made up to the end of the year; the remainder will be a new principal, with which proceed as before.*

II. *If the settlement occur less than a year from the last annual term of interest, make the last term of interest a part of a year, accordingly.*

EXAMPLES FOR PRACTICE.

$1000. New Haven, Conn., Feb. 1, 1856.

1. Two years after date, for value received, I promise to pay to Peck & Bliss, or order, one thousand dollars with interest.

John Cornwall.

Indorsed as follows: April 1, 1857, $80; Aug. 1, 1857, $30; Oct. 1, 1858, $10; Dec. 1, 1858, $600; May 1, 1859, $200. How much was due Oct. 1, 1859? *Ans.* $266.38.

$2000. Burlington, Vt., May 10, 1858.

2. For value received, I promise to pay David Camp, or order, two thousand dollars, on demand, with interest annually.

Richard Thomas.

On this note were indorsed the following payments: March 10, 1859, $800; May 10, 1860, $400; Sept. 10, 1861, $300. How much was due Jan. 10, 1863?

3. How much would be due on the above note, computing by the Connecticut rule? *Ans.* $831.58.

4. How much, computing by the New Hampshire rule? By the United States rule? *Ans.* { N. H. rule, $833.21; U. S. " $831.90.

SAVINGS BANK ACCOUNTS.

547. **Savings Banks** are institutions intended to receive in trust or on deposit, small sums of money, generally the surplus earnings of laborers, and to return the same with a moderate interest at a future time.

548. It is the custom of all savings banks to add to each depositor's account, at the end of a certain fixed term, the interest due on his deposits according to some general regulation for allowing interest. The interest term with some savings banks is 6 months, with some 3 months, and with some 1 month.

549. A savings bank furnishes each depositor with a book, in which is recorded from time to time the sums deposited and the sums drawn out. The Dr. side of such an account shews the deposits, and the Cr. side the depositor's checks or drafts. In the settlement, interest is never allowed on any sum which has not been on deposit for a full interest term. Hence, to find the amount due on any depositor's account, we have the following

RULE. *At the end of each term, add to the balance of the account one term's interest on the smallest balance on deposit at any one time during that term; the final balance thus obtained will be the sum due.*

NOTES. — 1. It will be seen that by this rule no interest is allowed for money on deposit during a partial term, whether the period be the first or the last part of the term.

2. An exception to this general rule occurs in the practice of some of the savings banks of New York city. In these, the interest term is 6 months, and the depositor is allowed not only the full term's interest on the smallest balance, but a half term's interest on any deposit, or portion of a deposit made during the first 3 months of the term, and *not drawn out during any subsequent part of the term.*

EXAMPLES FOR PRACTICE.

1. What will be due April 20, 1860, on the following account, interest being allowed quarterly at 6 per cent. per annum, the terms commencing Jan. 1, April 1, July 1, and Oct. 1?

Dr. *Savings Bank in account with James Taylor.* *Cr.*

1858,	Jan. 12,.........	$75	1858,	March 5,.........	$30
"	May 10,.........	150	"	Aug. 16,.........	50
"	Sept. 1,.........	20	"	Dec. 1,.........	48
1859,	Feb. 16,.........	130			

OPERATION.

Deposit, Jan. 12, 1858,.............................	$75
Draft, March 5, "	30
Balance, Apr. 1, 1860,..................	$45
Deposit, May 10, 1858,.............................	150
Int. on $45, for 3 mo..............................	.68
Balance, July 1, 1860, ...	$195.68
Draft, Aug. 16, 1858,.............................	50
Least balance during the current term,...........	$145.68
Deposit, Sept. 1, 1858,.............................	20.00
Int. on $145.68, for 3 mo.	2.19
Balance, Oct. 1, 1858,..................	$167.87
Draft, Dec. 1, 1858,.............................	48
Least balance during the current term,...........	119 87
Int. on $119.87, for 3 mo.	1.80
Balance, Jan. 1, 1860,..................	$121.67
Deposit, Feb. 16, 1860,.............................	130.00
Int. on $121.67, for 3 mo.	1.83
Bal. due after Apr. 1, 1860,...........	$253.50 *Ans.*

Note.—In the following examples the terms commence with the year, or on Jan. 1.

2. Allowing interest monthly at 6 % per annum, what sum will be due Sept. 1, 1860, on the book of a savings bank having the following entries?

Bay State Savings Institution, in account with Jane Ladd.

Dr. Cr.

1860.					1860.				
Jan.	3	To cash,	5	75	Jan.	28	By check,	5	00
"	8	" "	13	45	Feb.	7	" "	8	48
"	20	" "	7	60	March	20	" "	10	00
Feb.	20	" check,	16	45	April	11	" "	12	76
"	27	" cash,	8	40	June	3	" "	3	96
March	6	" check,	14	65	"	12	" "	10	48
"	29	" cash,	7	98	"	20	" draft,	17	48
April	25	" "	3	49	Aug.	17	" check,	5	64
May	7	" draft,	26	50					
"	30	" "	45	79					
July	28	" cash,	15	68					
Aug.	3	" check,	18	45					
"	26	" cash,	4	50					

Ans. $116.87.

3. Interest at 7 %, allowed quarterly, how much was due April 4, 1860, on the following savings bank account?

Detroit Savings Institution, in account with R. L. Selden.

Dr.									Cr.
1859.					1859.				
Jan.	1	To cash,	47	50	May	12	By check,	50	36
March	12	" "	124	36	Oct.	3	" "	25	78
June	20	" "	130	56	Nov.	16	" "	36	48
Aug.	3	" "	68	75	Dec.	28	" "	12	50
1860.									
Jan.	25	" "	160	80					

Ans. $423.22.

4. How much was due Jan. 1, 1860, on the following account, allowing interest semi-annually, at 6 % per annum?

Irvings Savings Institution, in account with James Taylor.

Dr.							Cr.
1858.				1858.			
June	4	To cash,	175	Sept.	14	By check,	65
Nov.	1	" "	150	1859.			
1859.				July	25	" "	120
Feb.	24	" draft.	200	Dec.	3	" "	80
Sept.	10	" check,	56				

Ans. $337.02.

5. Interest at 5 %, allowed according to Note 2, how much was due, Jan. 1, 1860, on the book of a savings bank in the city of New York, having the following entries?

Sixpenny Savings Bank, in account with William Gallup.

Dr.									Cr.
1858.					1858.				
Jan.	1	To check,	36	50	Sept.	16	By check,	36	16
March	17	" "	25	38	1859.				
Aug.	1	" cash,	84	72	Jan.	27	" "	13	48
1859.					March	1	" "	17	50
June	11	" draft,	50	00					
Nov.	16	" cash,	40	78					

Ans. $179.10.

COMPOUND INTEREST.

550. Compound Interest is interest on both principal and interest, when the interest is not paid when due.

NOTE.—The simple interest may be added to the principal annually, semi-annually, or quarterly, as the parties may agree; but the taking of compound interest is not legal.

1. What is the compound interest of $640 for 4 years, at 5 per cent.?

OPERATION.

	$640	Principal for 1st year,
$640 × 1.05 =	$672	" " 2d "
$672 × 1.05 =	$705.60	" " 3d "
$705.60 × 1.05 =	$740.88	" " 4th "
$740.88 × 1.05 =	$777.924	Amount " 4 years,
	640.	Given principal,
	$137.924	Compound interest.

This illustration is sufficient to establish the following

RULE. I. *Find the amount of the given principal at the given rate for one year, and make it the principal for the second year.*

II. *Find the amount of this new principal, and make it the principal for the third year, and so continue to do for the given number of years.*

III. *Subtract the given principal from the last amount; the remainder will be the compound interest.*

NOTES.—1. When the interest is payable semi-annually or quarterly, find the amount of the given principal for the first interval, and make it the principal for the second interval, proceeding in all respects as when the interest is payable yearly.

2. When the time contains years, months, and days, find the amount for the years, upon which compute the interest for the months and days, and add it to the last amount, before subtracting.

EXAMPLES FOR PRACTICE.

1. What is the compound interest of $750 for 4 years at 6 per cent.? *Ans.* $196.86—

2. What will $250 amount to in 3 years at 7 per cent. compound interest? *Ans.* $306.26.

3. At 7 per cent. interest, compounded semi-annually, what debt will $1475.50 discharge in 2½ years? *Ans.* $1752.43.

4. Find the compound interest of $376 for 3 yr. 8 mo. 15 da., at 6 per cent. per annum. *Ans.* $90.84.

551. A more expeditious method of computing compound interest than the preceding is by the use of the compound interest table on the following page.

TABLE,

Showing the amount of $1, or £1, at 2½, 3, 3½, 4, 5, 6, 7, and 8 per cent., compound interest, for any number of years from 1 to 40.

Years	2½ per ct.	3 per cent.	3½ per ct.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000	1.070000	1.080000
2	1.050625	1.060900	1 071225	1.081600	1.102500	1.123600	1.144900	1.166400
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016	1.225043	1.259712
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477	1.310796	1.360489
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226	1.402552	1.469328
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519	1.500730	1.586874
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630	1.605782	1.713824
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848	1.718186	1.850930
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479	1.838459	1.999005
10	1.280085	1.343916	1.410599	1.480244	1.628885	1.790848	1.967151	2.158925
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299	2.104852	2.331639
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197	2.252192	2.518170
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928	2.409845	2.719624
14	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904	2.578534	2.937194
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558	2.759032	3.172169
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352	2.952164	3.425943
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773	3.158815	3.700018
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339	3.379932	3.996020
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600	3.616528	4.315701
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136	3.869685	4.660957
21	1.679582	1.860295	2.059431	2.278768	2.785963	3.399564	4.140562	5.033834
22	1.721571	1.916103	2.131512	2.369919	2.925261	3.603537	4.430402	5.436540
23	1.764611	1.973587	2.206114	2.464716	3.071524	3.819750	4.740530	5.871464
24	1.808726	2.032794	2.283328	2.563304	3.225100	4.048935	5.072367	6.341181
25	1.853944	2.093778	2.363245	2.665836	3.386355	4.291871	5.427433	6.848475
26	1.900293	2.156591	2.445959	2.772470	3.555673	4.549383	5.807353	7.396353
27	1.947800	2.221289	2.531567	2.883369	3.733456	4.822346	6.213868	7.988062
28	1.996495	2.287928	2.620172	2.998703	3.920129	5.111687	6.648838	8.627106
29	2.046407	2.356566	2.711878	3.118651	4.116136	5.418388	7.114257	9.317275
30	2.097568	2.427262	2.806794	3.243398	4.321942	5.743491	7.612255	10.062657
31	2.150007	2.500080	2.905031	3.373133	4.538040	6.088101	8.145113	10.867669
32	2.203757	2.575083	3.006708	3.508059	4.764942	6.453387	8.715271	11.737083
33	2.258851	2.652335	3.111942	3.648381	5.003189	6.840590	9.325340	12.676050
34	2.315322	2.731905	3 220860	3.794316	5.253348	7.251025	9.978114	13.690134
35	2.373205	2.813862	3.333590	3.946089	5.516015	7.686087	10.676582	14.785344
36	2.432535	2.898278	3.450266	4.103933	5.791816	8.147252	11.423942	15.968172
37	2.493349	2.985227	3.571025	4.268090	6.081407	8.636087	12.223618	17.245626
38	2.555682	3.074783	3.696011	4.438813	6.385477	9.154252	13.079271	18.625276
39	2.619574	3.167027	3.825372	4.616366	6.704751	9.703508	13.994820	20.115298
40	2 685064	3.262038	3.959260	4.801021	7.039989	10.285718	14.974458	21.724522

EXAMPLES FOR PRACTICE.

1. What is the amount of $300 for 4 years at 6 per cent. compound interest, payable semi-annually?

OPERATION.
$1.26677
300
$380.03100

ANALYSIS. The amount of $1 at 6 per cent., compound interest payable semi-annually, is the same as the amount of $1 at 3 per cent., compound interest payable annually. We therefore take, from the table, the amount of $1 for 8 years at 3 per cent., and multiply this amount by the given principal.

2. What is the amount of $536.75 for 12 yr. at 8 per cent. compound interest? *Ans.* $1351.63.

3. What sum placed at simple interest for 2 yr. 9 mo. 12 da., at 7 per cent., will amount to the same as $1275, placed at compound interest for the same time and at the same rate, payable semi-annually? *Ans.* $1292.51—.

4. At 8 per cent. interest compounded quarterly, how much will $1840 amount to in 1 yr. 10 mo. 20 da.? *Ans.* $2137.06.

5. A father at his death left $15000 for the benefit of his only son, who was 12 yr. 7 mo. 12 da. old when the money was deposited; the same was to be paid to him when he should be 21 years of age, together with 7 per cent. interest compounded semi-annually. How much was the amount paid him?

6. What sum of money will amount to $2902.263 in 20 years, at 7 % compound interest? *Ans.* $750.

PROBLEMS IN INTEREST.

PROBLEM I.

552. Given, the time, rate per cent., and interest, to find the principal.

1. What sum of money will gain $87.42 in 4 years, at 6 per cent.?

OPERATION.

$.24, interest of $1 for 4 years.
$87.42 ÷ .24 = $364.25, *Ans.*

ANALYSIS. Since $.24 is the interest of $1 for 4 years at 6 per cent., $87.42 must be the interest of as many dollars, for the same time and at the same rate, as \$.24 is contained times in \$87.42. Dividing, we obtain \$364.25, the required principal. Hence the

RULE. *Divide the given interest by the interest of* \$1 *for the given time at the given rate.*

EXAMPLES FOR PRACTICE.

1. What sum of money, invested at 6½ per cent., will produce \$279.825 in 1 yr. 6 mo.? *Ans.* \$2870.
2. What sum will produce \$63.75 interest in 6 mo. 24 da. at 7½ per cent.?
3. What sum will produce \$12½ interest in 10 days at 10 per cent.? *Ans.* \$4500.
4. What sum must be invested in real estate paying 12½ per cent. profit in rents, to give an income of \$3125?
5. What is the value of a house and lot that pays a profit of 9½ per cent. by renting it at \$30 per month?
6. What sum of money, put at interest 6 yr. 5 mo. 11 da., at 7 per cent., will gain \$3159.14? *Ans.* \$7000.
7. What sum of money will produce \$69.67 in 2 yr. 9 mo. at 6 % compound interest? *Ans.* \$400.
8. What principal at 6 % compound interest will produce \$124.1624 in 1 yr. 6 mo. 15 da.? *Ans.* \$1314.583.

PROBLEM II.

553. Given, the time, rate per cent., and amount, to find the principal.

1. What sum of money in 2 years 6 months, at 7 per cent., will amount to \$136.535?

OPERATION.

\$1.175, amount of \$1 for 2 yr. 6 mo.
\$136.535 ÷ 1.175 = \$116.20, *Ans.*

ANALYSIS. Since \$1.175 is the amount of \$1 for 2 years 6 months, at 7 per cent., \$136.535 must be the amount of as many dollars, for the same time and at the same rate, as \$1.175 is contained times in \$136.535. Dividing, we obtain \$116.20, the required principal. Hence the

RULE. *Divide the given amount by the amount of* \$1 *for the given time at the given rate.*

EXAMPLES FOR PRACTICE.

1. What principal in 2 yr. 3 mo. 10 da., at 5 per cent., will amount to \$1893 61$\frac{1}{9}$? *Ans.* \$1700.

2. A note which had run 3 yr. 5 mo. 12 da. amounted to \$681.448, at 6 per cent.; how much was the face of the note?

3. What sum put at interest at 3$\frac{1}{2}$ per cent., for 10 yr. 2 mo., will amount to \$15660?

4. What is the interest of that sum for 2 yr. 8 mo. 29 da., at 7 per cent., which at the same time and rate, will amount to \$1568.97? *Ans.* \$253.057 +.

5. What is the interest of that sum for 243 days at 8 per cent., which at the same time and rate, will amount to \$11119.70?

6. What principal in 4 years at 6 per cent. compound interest, will amount to \$8644.62? *Ans.* \$6847.34.

7. What sum put at compound interest will amount to \$26772.96, in 10 yr. 5 mo., at 6 per cent.? *Ans.* \$14585.24.

PROBLEM III.

554. Given, the principal, time, and interest, to find the rate per cent.

1. I received \$315 for 3 years' interest on a mortgage of \$1500; what was the rate per cent.?

OPERATION.

$$\begin{array}{r} \$15.00 \\ 3 \\ \hline \$45.00 \end{array}$$, int. for 3 yr. at 1 %.

\$315 ÷ \$45 = 7 %, *Ans.*

ANALYSIS. Since \$45 is the interest on the mortgage for 3 years at 1 per cent., \$315 must be the interest on the mortgage for the same time, at as many times 1 per cent. as \$45 is contained times in \$315. Dividing, and we obtain 7, the required rate per cent. Hence the

RULE. *Divide the given interest by the interest on the principal for the given time at* 1 *per cent.*

EXAMPLES FOR PRACTICE.

1. If I loan $750 at simple interest, and at the end of 1 yr. 3 mo. receive $796.87½, what is the rate per cent.? *Ans.* 5.

2. If I pay $10.58 for the use of $1700, 28 days, what is the rate of interest? *Ans.* 8+per cent.

3. Borrowed $600, and at the end of 9 yr. 6 mo. returned $356.50; what was the rate per cent.?

4. A man invests $7266.28, which gives him an annual income of $744.7937; what rate of interest does he receive?

5. If C buys stock at 30 per cent. discount, and every 6 months receives a dividend of 4 per cent., what annual rate of interest does he receive? *Ans.* $11\frac{3}{7}$ per cent.

6. At what rate per annum of simple interest will any sum of money double itself in 4, 6, 8, and 10 years, respectively?

7. At what rate per annum of simple interest will any sum triple itself in 2, 5, 7, 12, and 20 years, respectively?

8. A house that rents for $760.50 per annum, cost $7800: what % does it pay on the investment? *Ans.* $9\frac{3}{4}$ per cent.

9. I invest $35680 in a business that pays me a profit of $223 a month; what annual rate of interest do I receive? *Ans.* 7½ %.

PROBLEM IV.

555. Given, the principal, interest, and rate, to find the time.

1. In what time will $924 gain $151.536, at 6 per cent.?

OPERATION.

$$\begin{array}{r} \$924 \\ .06 \\ \hline \end{array}$$

$55.44, int. of $924 for 1 yr. at 6 %.

$\$151.536 \div \$55.44 = 2.7\dot{3}$

$2.7\dot{3}$ yr. = 2 yr. 8 mo. 24 da., *Ans.*

ANALYSIS. Since $55.44 is the interest of $924 for 1 year at 6 per cent., $151,536 must be the interest of the same sum, at the same rate per cent., for as many years as $55.44 is contained times in $151.536, which is $2.7\dot{3}$ times. Reducing the mixed decimal to its equivalent compound number, we have 2 years 8 months 24 days, the required time. Hence the

RULE. *Divide the given interest by the interest on the principal for 1 year; the quotient will be the required time in years and decimals.*

EXAMPLES FOR PRACTICE.

1. In what time will $273.51 amount to $312.864, at 7 per cent.? *Ans.* 2 yr. 20 da.

2. How long must $650.82 be on interest to amount to $761.44, at 5 per cent.? *Ans.* 3 yr. 4 mo. 24 da.

3. How long will it take any sum of money to double itself by simple interest at 3, $4\frac{1}{2}$, 6, 7, and 10 per cent.? How long to quadruple itself? *Ans.* { To double itself at 3 %, $33\frac{1}{3}$ yr
To quadruple itself at 3 %. 100 yr.

4. In what time will $9750 produce $780 interest, at 2 per cent. a month?

5. In what time will $1000 draw $1171.353 at 6 per cent. compound interest?

ANALYSIS. $1171.353÷1000=$1.171353, the amount of $1 for the required time. From the table, $1, in 2 years, will amount to $1.1236; hence $1.171353—$1.1236=$.047753, the interest which must accrue on $1.1236 for the fraction of a year; and $1.1236 × .06 = $.067416; $.047753 ÷ $.067416 = .7083 yr. = 8 mo. 15 da.

Ans. 2 yr. 8 mo. 15 da.

6. In what time will $333 amount to $376.76 at 5 per cent compound interest, payable semi-annually?

7. In what time will any sum double itself at 6 % compound interest? At 7 %? *Ans.* to last, 10 yr. 2 mo. 26 da.

DISCOUNT.

556. **Discount** is an abatement or allowance made for the payment of a debt before it is due

557. The **Present Worth** of a debt, payable at a future time without interest, is such a sum as, being put at legal interest, will amount to the given debt when it becomes due.

1. What is the present worth and what the discount of $642.12 to be paid 4 yr. 9 mo. 27 da. hence, money being worth 7 per cent.?

OPERATION.

$1.33775, Amount of $1.
$642.12 ÷ 1.33775 = $480
$642.12, given sum.
480. present worth.
$162.12, discount.

ANALYSIS. Since $1 is the present worth of $1.33775 for the given time at the given rate of interest, the present worth of $642.12 must be as many dollars as $1.33775 is contained times in $642.12. Dividing, and we obtain $480 for the present worth, and subtracting this sum from the given sum, we have $162.12, the discount. Hence the following

RULE. I *Divide the given sum or debt by the amount of* $1 *for the given rate and time; the quotient will be the present worth of the debt.*

II. *Subtract the present worth from the given sum or debt; the remainder will be the discount.*

NOTES.—1. The terms *present worth, discount,* and *debt,* are equivalent to *principal, interest,* and *amount.* Hence, when the time, rate per cent., and amount are given, the principal may be found by Prob. II, (**553**); and the interest by subtracting the principal from the amount.

2. When payments are to be made at different times without interest, find the present worth of each payment separately. Their sum will be the present worth of the several payments, and this sum subtracted from the sum of the several payments will leave the total discount.

EXAMPLES FOR PRACTICE.

1. What is the present worth of a debt of $385.31¼, to be paid in 5 mo. 15 da., at 6 %? *Ans.* $375.

2. How much should be discounted for the present payment of a note for $429.986, due in 1 yr. 6 mo. 1 da., money being worth 5½ %? *Ans.* $32.826.

3. Bought a farm for $2964.12 ready money, and sold it again for $3665.20, payable in 1 yr. 6 mo. How much would be gained in ready money, discounting at the rate of 8 %?

4. A man bought a flouring mill for $25000 cash, or for $12000 payable in 6 mo. and $15000 payable in 1 yr. 3 mo. He accepted the latter offer; did he gain or lose, and how much, money being worth to him 10 per cent.? *Ans.* Gained $238.10.

5. B bought a house and lot April 1, 1860, for which he was to pay $1470 on the fourth day of the following September, and

$2816.80 Jan 1, 1861. If he could get a discount of 10 per cent. for present payment, how much would he gain by borrowing the sum at 7 per cent., and how much must he borrow?

6. What is the difference between the interest and the discount of $576, due 1 yr. 4 mo. hence, at 6 per cent.?

7. A merchant holds two notes against a customer, one for $243.16, due May 6, 1861, and the other for $178.64, due Sept. 25, 1861; how much ready money would cancel both the notes Oct. 11, 1860, discounting at the rate of 7 %? *Ans.* $401.29—.

8. A speculator bought 120 bales of cotton, each bale containing 488 pounds, at 9 cents a pound, on a credit of 9 months for the amount. He immediately sold the cotton for $6441.60 cash, and paid the debt at 8 % discount; how much did he gain?

9. Which is the more advantageous, to buy flour at $6.25 a barrel on 6 months, or at $6.50 a barrel on 9 months, money being worth 8 %?

10. How much may be gained by hiring money at 5 % to pay a debt of $6400, due 8 months hence, allowing the present worth of this debt to be reckoned by deducting 5 % per annum discount? *Ans.* 7.11\frac{1}{9}$.

BANKING.

558. A **Bank** is a corporation chartered by law for the purpose of receiving and loaning money, and furnishing a paper circulation.

559. A **Promissory Note** is a written or printed engagement to pay a certain sum either on demand or at a specified time.

560. **Bank Notes,** or **Bank Bills,** are the notes made and issued by banks to circulate as money. They are payable in specie at the banks.

NOTE.—A bank which issues notes to circulate as money is called a *bank of issue;* one which lends money, a *bank of discount;* and one which takes charge of money belonging to other parties, a *bank of deposit.* Some banks perform two and some all of these duties.

561. The **Maker** or **Drawer** of a note is the person by whom the note is signed;

562. The **Payee** is the person to whose order the note is made payable; and

563. The **Holder** is the owner.

564 A **Negotiable Note** is one which may be bought and sold, or negotiated. It is made payable to *the bearer* or to *the order* of the payee.

565. **Indorsing** a note by a payee or holder is the act of writing his name on its back.

NOTES.—1. If a note is payable to the bearer, it may be negotiated without indorsement.

2. An indorsement makes the indorser liable for the payment of a note, if the maker fails to pay it when it is due.

3. A note should contain the words "value received," and the sum for which it is given should be written out in words.

566. The **Face** of a note is the sum made payable by the note.

567. **Days of Grace** are the three days usually allowed by law for the payment of a note after the expiration of the time specified in the note.

568. The **Maturity** of a note is the expiration of the days of grace; a note is *due* at maturity.

NOTE.—No grace is allowed on notes payable "on demand," without grace. In some States no grace is allowed on notes, and their maturity is the expiration of the time mentioned in them.

569. Notes may contain a promise of interest, which will be reckoned from the date of the note, unless some other time be specified.

NOTE.—A note is on interest from the day it is due, even though no mention be made of interest in the note.

570. A **Notary**, or **Notary-Public**, is an officer authorized by law to attest documents or writings of any kind to make them authentic.

571. A **Protest** is a formal declaration in writing, made by a Notary-Public, at the request of the holder of a note, notifying the maker and the indorsers of its non-payment.

NOTES.—1. The failure to protest a note on the third day of grace releases the indorsers from all obligation to pay it.

2. If the third day of grace or the maturity of a note occurs on Sunday or a legal holiday, it must be paid on the day previous.

572. **Bank Discount** is an allowance made to a bank for the payment of a note before it becomes due.

573. The **Proceeds** of a note is the sum received for it when discounted, and is equal to the face of the note less the discount.

574. The transaction of borrowing money at banks is conducted in accordance with the following custom: The borrower presents a note, either made or indorsed by himself, payable at a specified time, and receives for it a sum equal to the face; *less* the interest for the time the note has to run. The amount thus withheld by the bank is in consideration of advancing money on the note prior to its maturity.

NOTES.—1. A note for discount at bank must be made payable to the order of some person, by whom it must be indorsed.

2. The business of buying or discounting notes is chiefly carried on by banks and brokers.

575. The law of custom at banks makes the bank discount of a note equal to the simple interest at the legal rate, for the time specified in the note. As the bank always takes the interest at the time of discounting a note, bank discount is equal to simple interest paid in advance. Thus, the true discount of a note for \$153, which matures in 4 months at 6 %, is $\$153 - \frac{15300}{102} =$ \$3.00, and the bank discount is $\$153 \times .02 = \$3.06$. Since the interest of \$3, the true discount, for 4 months is $\$3 \times .02 = \$.06$, we observe that the bank discount of any sum for a given time is greater than true discount, by the interest on the true discount for the same time.

NOTE. — Many banks take only true discount.

CASE I.

576. Given, the face of a note, to find the discount and the proceeds.

RULE. I. *Compute the interest on the face of the note for three days more than the specified time; the result will be the discount.*

II. *Subtract the discount from the face of the note; the remainder will be the proceeds.*

NOTES.—1. When a note is *on interest*, payable at a future specified time, the *amount* is the face of the note, or the sum made payable, and must be made the basis of discount.

2. To indicate the maturity of a note or draft, a vertical line (|) is used, with the day at which the note is nominally due on the left, and the date of maturity on the right; thus, Jan. $^{7}|_{10}$.

EXAMPLES FOR PRACTICE.

1. What is the bank discount, and what are the proceeds of a note for $1487 due in 30 days at 6 per cent.?

Ans. Discount, $8.18; Proceeds, $1478.82.

2. What are the proceeds of a note for $384.50 at 90 days, if discounted at the New York Bank?

3. Wishing to borrow $1000 of a Southern bank that is discounting paper at 8 per cent., I give my note for $975, payable in 60 days; how much more will make up the required amount?

4. A man sold his farm containing 195 A. 2 R. 25 P. for $27.50 an acre, and took a note payable in 4 mo. 15 da. at 7 % interest. Wishing the money for immediate use, he got the note discounted at a bank; how much did he receive? *Ans.* $5236.169.

5. Find the day of maturity, the term of discount, and the proceeds of the following notes:

$\$1962\frac{45}{100}$. DETROIT, July 26, 1860.

Four months after date I promise to pay to the order of James Gillis one thousand nine hundred sixty-two and $\frac{45}{100}$ dollars at the Exchange Bank, for value received. JOHN DEMAREST.

Discounted Aug. 26, at 7%.

Ans. Due Nov. $^{26}|_{29}$; term of discount 95 days; proceeds, $1926.20.

$\$1066\frac{75}{100}$. BALTIMORE, April 19, 1859.

6. Ninety days after date we promise to pay to the order of King & Dodge one thousand sixty-six and $\frac{75}{100}$ dollars at the Citizens' Bank, for value received. CASE & SONS.

Discounted May 8, at 6 %.

Ans. Due July $^{18}|_{21}$; term of discount, 74 da.; proceeds, $1053.59.

$\$784\frac{72}{100}$. MOBILE, June 20, 1861.

7. Two months after date for value received I promise to pay George Thatcher or order seven hundred eighty-four and $\frac{72}{100}$ dollars at the Traders' Bank. WM. HAMILTON.

Discounted July 5, at 8 %.

1845\frac{50}{100}$. CHICAGO, Jan. 31, 1862.

8. One month after date we jointly and severally agree to pay to W. H. Willis, or order, one thousand eight hundred forty-five and $\frac{50}{100}$ dollars at the Marine Bank.

PAYSON & WILLIAMS.

Discounted Jan. 31, at 2 % a month.

Ans. Due Feb. 28 | March 3; term of discount, 31 da.; proceeds, $1807.36.

9. What is the difference between the true and the bank discount of $950, for 3 months at 7 per cent.? *Ans.* $.29.

10. What is the difference between the true and the bank discount of $1375.50, for 60 days at 6 per cent.?

CASE II.

577. Given, the proceeds of a note, to find the face.

1. For what sum must I draw my note at 4 months, interest 6 %, that the proceeds when discounted in bank shall be $750?

OPERATION.

$1.0000
.0205, disc't on $1 for 4 mo. 3 da.
$ .9795, proceeds of $1.
$750 ÷ .9795 = $765.696, *Ans.*

ANALYSIS. We first obtain the proceeds of $1 by the last case; then, since $.9795 is the proceeds of $1, $750 is the proceeds of as many dollars as $.9795 is contained times in $750. Dividing, we obtain the required result. Hence the

RULE. *Divide the proceeds by the proceeds of* $1 *for the time and rate mentioned; the quotient will be the face of the note.*

EXAMPLES FOR PRACTICE.

1. What is the face of a note at 60 days, the proceeds of which, when discounted at bank at 6 %, are $1275? *Ans.* $1288.53.

2. If a merchant wishes to draw $5000 at bank, for what sum must he give his note at 90 days, discounting at 6 per cent.?

Ans. $5078.72.

3. The avails of a note having 3 months to run, discounted at a bank at 7 %, were $276.84; what was the face of the note?

4. James T. Fisher buys a bill of merchandise in New York at cash price, to the amount of \$1486.90, and gives in payment his note at 4 months at $7\frac{1}{2}$ %; what must be the face of the note?

5. Find the face of a 6 mo. note, the proceeds of which, discounted at 2 % a month, are \$496. *Ans.* \$564.92.

6. For what sum must a note be drawn at 30 days, to net \$1200 when discounted at 5 %?

7. Owing a man \$575, I give him a 60 day note; what should be the face of the note, to pay him the exact debt, if discounted at $1\frac{1}{2}$ % a month? *Ans.* \$593.70.

8. What must be the face of a note which, when discounted at a broker's for 110 days at 1 % a month, shall give as its proceeds \$187.50?

CASE III.

578. Given, the rate of bank discount, to find the corresponding rate of interest.

1. A broker discounts 30 day notes at $1\frac{1}{2}$ % a month; what rate of interest does his money earn him?

OPERATION.

30 day notes = 33 days' time.
\$100, base.
1.65, discount for 33 days.
\$98.35, proceeds.
$\$1.65 \div .090154\frac{1}{6} = 18\frac{594}{1967}$ %, *Ans.*

ANALYSIS. If we assume \$100 as the face of the note, the discount for 33 days at $1\frac{1}{2}$ % a month will be \$1.65 and the proceeds \$98.35. We then have \$98.35 principal, \$1.65 interest, and 33 days time, to find the rate per cent. per annum, which we do by (**554**). Hence the

RULE. I. *Find the discount and the proceeds of* \$1 *or* \$100 *for the time the note has to run.*

II. *Divide the discount by the interest of the proceeds at* 1 *per cent. for the same time.*

EXAMPLES FOR PRACTICE.

1. What rate of interest is paid, when a note payable in 30 days is discounted at 6 per cent.? *Ans.* $6\frac{22}{663}$ %.

2. A note payable in 2 months is discounted at 2 % a month; what rate of interest is paid? *Ans.* $25\frac{25}{479}$ %.

3. When a note payable in 90 days is discounted at $1\frac{1}{2}$ % a month, what rate of interest is paid? *Ans.* $18\frac{1674}{1907}$ %.

4. What rate of interest corresponds to 5, 6, 7, 10, 12 % discount on a note running 10 months without grace?

5. What rate of interest does a man pay who has a 60 day note discounted at $\frac{3}{4}$, 1, 2, $2\frac{1}{2}$, 3 % a month?

CASE IV.

579. Given, the rate of interest, to find the corresponding rate of bank discount.

1. A broker buys 60 day notes at such a discount that his money earns him 2 % a month; what is his rate % of discount?

OPERATION.

	60 da. + 3 da. = 63 da.	
$100		base.
	4.20,	interest for 63 da.
$104.20,		amount " "

$4.20 ÷ .18235 = $23\frac{17}{521}$ %, *Ans.*

ANALYSIS. If we assume $100 as the proceeds of a note, the interest for 63 days at 24 per cent. will be $4.20, and the amount or face of the note will be $104.20. We then have $104.20 the principal, $4.20 the interest, and 63 days the time, to find the rate per cent., which we do by (**549**) as in the last case. Hence the

RULE. I. *Find the interest and the amount of* $1 *or* $100 *for the time the note has to run.*

II. *Divide the interest by the interest on the amount at* 1 *per cent. for the same time.*

EXAMPLES FOR PRACTICE.

1. What rates of bank discount on 30 day notes correspond to 5, 6, 7, 10 per cent. interest?

2. At what rate should a 3 months' note be discounted to produce 8 % interest? *Ans.* $7\frac{1283}{1531}$ %.

3. At what rates should 60 day notes be discounted to pay to a broker 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$ % a month?

4. At what rate must a note payable 18 months hence, without grace, be discounted to produce 7 % interest? *Ans.* $6\frac{74}{221}$ %.

EXCHANGE.

580. **Exchange** is a method of remitting money from one place to another, or of making payments by written orders.

581. A **Bill of Exchange** is a written request or order upon one person to pay a certain sum to another person, or to his order, at a specified time.

582. A **Sight Draft** or **Bill** is one requiring payment to be made "at sight," which means, at the time of its presentation to the person ordered to pay. In other bills, the time specified is usually a certain number of days "after sight."

There are always three parties to a transaction in exchange, and usually four:

583. The **Drawer** or **Maker** is the person who signs the order or bill;

584. The **Drawee** is the person to whom the order is addressed;

585. The **Payee** is the person to whom the money is ordered to be paid; and

586. The **Buyer** or **Remitter** is the person who purchases the bill. He may be himself the *payee*, or the bill may be drawn in favor of any other person.

587. The **Indorsement** of a bill is the *writing* upon its back, by which the *payee* relinquishes his title, and transfers the payment to another. The payee may indorse *in blank* by writing his name only, which makes the bill payable to the *bearer*, and consequently transferable like a bank note; or he may accompany his signature by a special order to pay to another person, who in his turn may transfer the title in like manner. *Indorsers* become separately responsible for the amount of the bill, in case the *drawee* fails to make payment. A bill made payable to the *bearer* is transferable without indorsement.

588. The **Acceptance** of a bill is the promise which the *drawee* makes when the bill is presented to him to pay it at maturity; this obligation is usually acknowledged by writing the word "Accepted," with his signature, across the face of the bill.

NOTES.—1. In this country, and in Great Britain, three *days of grace* are allowed for the payment of a bill of exchange, after the time specified has expired. In regard to grace on *sight bills*, however, custom is variable; in New York. Pennsylvania, Virginia, and some other States, no grace is allowed on sight bills,

2. When a bill is protested for non-acceptance, the drawer is obligated to pay it immediately, even though the specified time has not expired.

Exchange is of two kinds—Domestic and Foreign.

589. **Domestic or Inland Exchange** relates to remittances made between different places of the same country.

NOTE.—An Inland Bill of Exchange is commonly called a *Draft.*

590. **Foreign Exchange** relates to remittances made between different countries.

591. A **Set of Exchange** consists of three copies of the same bill, made in foreign exchanges, and sent by different conveyances to provide against miscarriage; when one has been paid, the others are void.

592. The **Face** of a bill of exchange is the sum ordered to be paid; it is usually expressed in the currency of the place on which the draft is made.

593. The **Par of Exchange** is the estimated value of the coins of one country as compared with those of another, and is either *intrinsic* or *commercial.*

594. The **Intrinsic Par of Exchange** is the comparative value of the coins of different countries, as determined by their weight and purity.

595. The **Commercial Par of Exchange** is the comparative value of the coins of different countries, as determined by their nominal or market price.

NOTE.—The intrinsic par is always the same while the coins remain unchanged; but the commercial par, being determined by commercial usage, is fluctuating.

596. The **Course of Exchange** is the current price paid in one place for bills of exchange on another place. This price varies, according to the relative conditions of trade and commercial credit at the two places between which exchange is made. Thus, if Boston is largely indebted to Paris, bills of exchange on Paris will bear a high price in Boston.

When the course of exchange between two places is unfavorable to drawing or remitting, the disadvantage is sometimes avoided, by means of a circuitous exchange on intermediate places between which the course is favorable.

DIRECT EXCHANGE.

597. **Direct Exchange** is confined to the two places between which the money is to be remitted.

598. There are always two methods of transmitting money between two places. Thus, if A is to receive money from B,

1st. A may draw on B, and sell the draft;

2d. B may remit a draft, made in favor of A.

NOTE. — One person is said to *draw on* another person, when he is the *maker* of a draft addressed to that person.

CASE I.

599. To compute domestic exchange.

The course of exchange for inland bills, or drafts, is always expressed by the rate of premium or discount. Drafts on time, however, are subject to *bank discount*, like notes of hand, for the term of credit given. Hence, their cost is affected by both the *course of exchange* and the *discount for time.*

1. What will be the cost of the following draft, exchange on Boston being in Pittsburgh at $2\frac{1}{4}$ % premium?

$600. PITTSBURGH, June 12, 1860.

Sixty days after sight, pay to William Barnard, or order, six hundred dollars, value received, and charge the same to our account.

To the Suffolk Bank, Boston. THOMAS BAUER & Co.

OPERATION.

$$\begin{aligned} \$1 + \$.0225 = \$&1.0225, \text{ course of exchange.} \\ &\underline{\ \ .0105}, \text{ bank discount of } \$1, \text{ (63 da.)} \\ \$&1.012, \text{ cost of exchange for } \$1. \end{aligned}$$

$$\$600 \times 1.012 = \$607.20, \textit{ Ans.}$$

ANALYSIS. From $1.0225, the course of exchange, we subtract $.0105, the bank discount of $1 for the specified time, and obtain $1.012, the cost of exchange for $1; then $600 × 1.012 = $607.20, the cost of exchange for $600.

2. A commission merchant in Detroit wishes to remit to his employer in St. Louis, \$512.36 by draft at 60 days; what is the face of the draft which he can purchase with this sum, exchange being at $2\frac{1}{2}$ % discount?

OPERATION.

$$\begin{array}{rl} \$1 - \$.025 = \$.975, & \text{course of exchange.} \\ .01225, & \text{discount of } \$1. \\ \hline \$.96275, & \text{cost of exchange for } \$1. \end{array}$$

$$\$512.36 \div .96275 = \$532.18+, \textit{ Ans.}$$

Analysis. From \$.975, the course of exchange, we subtract \$.01225, the bank discount of \$1 for the specified time, at the legal rate in Detroit, and obtain \$.96275, the cost of exchange for \$1; and the face of the draft that will cost \$512.36, will be as many dollars as \$.96275 is contained times in 512.36, which is 532.18 +, times. Hence we have the following

Rule. I. To find the cost of a draft, the face being given.— *Multiply the face of the draft by the cost of exchange for* \$1.

II. To find the face of a draft, the cost being given.— *Divide the given cost by the cost of exchange for* \$1.

Note.—The cost of exchange for \$1 may always be found, by subtracting from the course of exchange the bank discount (at the legal rate where the draft is made), for the specified time. For *sight* drafts, the course of exchange is the cost of \$1.

EXAMPLES FOR PRACTICE.

1. What must be paid in New York for a draft on Boston, at 30 days, for \$5400, exchange being at $\frac{1}{2}$ % premium?

Ans. \$5392.35.

2. What is the cost of sight exchange on New Orleans, for \$3000, at $3\frac{1}{4}$ % discount?

3. What must be paid in Philadelphia for a draft on St. Paul drawn at 90 days, for \$4800, the course of exchange being $101\frac{3}{8}$ %? *Ans.* \$4791.60.

4. A sight draft was purchased for \$550.62, exchange being at a premium of $3\frac{1}{2}$ %; what was the face?

5. An agent in Syracuse, N. Y., having \$1324.74 due his employer, is instructed to remit the same by a draft drawn at 30 days; what will be the face of the draft, exchange being at $1\frac{3}{4}$ % premium? *Ans.* \$1310.22—.

6. My agent in Charleston, S. C., sells a house and lot for \$7500, on commission of $1\frac{1}{2}$ %, and remits to me the proceeds in a draft purchased at $\frac{1}{2}$ % premium; what sum do I receive from the sale of my property?

7. A man in Hartford, Conn., has \$4800 due him in Baltimore; how much more will he realize by making a draft for this sum on Baltimore and selling it at $\frac{1}{2}$ % discount, than by having a draft on Hartford remitted to him, purchased in Baltimore for this sum at $\frac{3}{4}$ % premium? *Ans.* \$11.73+.

8. The Merchants' Bank of New York having declared a dividend of $6\frac{1}{4}$ %, a stockholder in Cincinnati drew on the bank for the sum due him, and sold the draft at a premium of $1\frac{3}{4}$ %, thus realizing \$508.75 from his dividend; how many shares did he own?

9. Sight exchange on New Orleans for \$5000 cost \$5075; what was the course of exchange? *Ans.* $1\frac{1}{2}$ % premium.

10. A man in Buffalo purchased a draft on St. Paul, Minn., for \$5320, drawn at 60 days, paying \$5141.78; what was the course of exchange? *Ans.* $2\frac{1}{8}$ % discount.

CASE II.

600. To compute foreign exchange.

601. The following standards of the decimal currency of the United States were established April 2, 1792.

Coins.	Weight.	Fineness.
Gold eagle,...........	270 grains,	$916\frac{2}{3}$ thousandths.
Silver dollar,.........	416 "	" "
Copper cent,..........	264 "	" "

In 1834, the eagle was reduced in weight to 258 grains, and in 1837 its fineness was fixed at 900 thousandths pure, which is likewise the present standard of purity for all the U. S. gold and silver coins. In 1837, also, the silver dollar was reduced in weight to 412.5 grains. In 1853, the silver half dollar was reduced in weight to 192 grains, and the smaller silver coins proportionally.

NOTE. — The object of the change in the silver coinage of the United States, made in 1853, was to prevent its exportation by raising the nominal value of silver above its foreign market value.

602. The intrinsic par of exchange between the United States and different countries, is given in the following

TABLE OF FOREIGN COINS AND MONEY.

COINS.	Metal.	Lower Denominations.	Intrinsic par of U. S. gold coinage of 1837.	Intrinsic par of U. S. silver coinage of 1837.	Intrinsic par of U. S. silver coinage of 1853.
Crown, Baden	Silver.			1.077	1.157
" Bavaria	"			1.072	1.151
" England	"			1.100	1.181
" France	"			1.100	1.181
" Geneva	"			.960	1.031
" Portugal	Gold.		5.813		
" Tuscany	Silver.			1.050	1.128
" Wurtemberg	"			1.070	1.148
" Zurich	"			.960	1.031
Dollar, Argentine Republic	"	8 reals.		1.016	1.091
" Bolivia	"	8 "		1.011	1.086
" Chili	"	100 cents.		1.011	1.086
" Columbia	"	8 reals.		1.022	1.098
" Mexico	"	8 "		1.005	1.079
" Norway	"	6 marks.		1.051	1.129
" Peru	"	8 reals.		1.005	1.079
" Spain	"	10 " (old).		1.003	1.077
" Sweden	"	6 marks.		1.059	1.136
Doubloon, Bolivia	Gold.		15.580		
" Columbia (Bogota)	"		15.617		
" " (Popayan).	"		15.390		
" Chili (since 1835)	"		15.660		
" " (before 1835)	"		15.570		
" La Plata	"		14.660		
" Mexico (average)	"		15.534		
" Peru (Cuzco)	"		15.534		
" " (Lima)	"		15.551		
" Spain	"		15 570		
Drachma, Greece	Silver.			165	.177
Ducat, Austria	Gold.	60 batzen.	2.278		
" Bavaria	"		2.274		
" Cologne	"		2.250		
" Hamburg	"		2.257		
" Hungary	"		2.231		
" Netherlands	"		2.269		
" Saxony	"	4 gilders	2.264		
" Sweden	"	12 marks.	2.267		
" Wurtemberg	"		2.236		
Florin, Austria	Silver.	60 kreutzers.		.485	.521
" Bavaria	"	60 "		.395	.425
" Hanover	"	60 groshen.		.547	.587
" Italy	"	12 soldi.		.181	.194

TABLE OF FOREIGN COINS AND MONEY — CONTINUED.

COINS.	Metal.	Lower Denominations.	Intrinsic par of U.S. gold coinage of 1837.	Intrinsic par of U.S. silver coinage of 1837.	Intrinsic par of U.S. silver coinage of 1853.
Florin, Mecklenburg	Silver.			.541	.571
" Prussia and Poland	"	30 groshen.		.227	.244
" Tuscany	"	12 soldi.		.262	.281
Franc, Belgium	"	100 centimes.		.186	.200
" France	"	100 "		.186	.200
Frederick d'or, Denmark	Gold.		3.932		
Gilder, Baden	Silver.	60 kreutzers.		.397	.426
" Darmstadt	"	60 "		.397	.426
" Demerara	"	20 stivers.		.263	.282
" Frankfort	"	60 kreutzers.		.397	.426
" Netherlands	"	20 stivers.		.406	.436
" Wurtemberg	"	60 kreutzers.		.395	.423
Ghersh, Tripoli	"	100 paras.		.105	.112
Guinea, England	Gold.	21 shillings.	5.059		
Lira, Lombardy	Silver.	20 soldi.		.162	.173
" Leghorn	"	20 "		.162	.173
" Milan	"	20 "		.162	.173
" Venice	"	100 centimes.		.162	.173
Livre, Genoa	"	20 soldi.		.186	.198
" Leghorn	"	20 "		.162	.173
" Switzerland	"	100 centessini.		.273	.292
Mark banco, Hamburg	"	16 skillings.		.350	.375
" current, "	"	16 "		.285	.305
Milree, Azores	"	1000 reis.		.830	.890
" Brazil	"	1000 "		.830	.890
" Madeira	"	1000 "		1.000	1.074
" Portugal	"	1000 "		1.120	1.203
Mohur, Hindostan	Gold.	16 rupees.	7.109		
Ounce, Naples	"	3 ducats.	2.485		
Pagoda, Madras	"	42 fanams.	1.840		
Piaster, Tunis	Silver.	16 carobas.		.124	.133
" Turkey	"	100 aspers.		.026	.028
Pistareen, Spain	"	4 reals vellon.		.197	.211
Pistole, Spain	Gold.		3.904		
Pound, British Provinces	"	20 shillings.	4.016		
Real, plate, Spain	Silver.	34 marvedis.		.097	.104
" vellon, "	"	34 "		.048	.051
" Egypt	"	20 piasters.		.968	1.040
Rix dollar, Austria	"	120 kreutzers.		.971	1.043
" " Batavia	"	48 stivers.		.782	.840
" " Denmark	"	96 skillings.		1.051	1.129

TABLE OF FOREIGN COINS AND MONEY — CONTINUED.

COINS.	Metal.	Lower Denominations.	Intrinsic par of U.S. gold coinage of 1837.	Intrinsic par of U.S. silver coinage of 1837.	Intrinsic par of U.S. silver coinage of 1853.
Rigsbank dollar	Silver.	48 skillings.		.526	.565
Rix dollar, Norway	"	96 "		1.051	1.129
Rouble, Russia	"	100 copecks.		.754	.806
Rupee, India	"	16 annas.		.445	.477
Ruspone, Tuscany	Gold.		6.925		
Sequin, Tuscany	"		2.301		
Scudo, Milan	Silver.	117 soldi.		.973	1.045
" Naples	"	12 carlini.		.950	1.021
" Rome	"			1.006	1.080
" Sicily	"	12 tari.		.985	1.058
Sovereign, Great Britain	Gold.	20 shillings.	4.861		
Thaler, Brunswick	Silver.	30 groschen.		.692	.743
" Hanover	"	30 "		.694	.735
" Hesse-Cassel	"	30 "		.687	.738
" Prussia	"	30 "		.692	.743
" Saxony	"	30 "		.694	.735
" Bremen	"	72 grotes.		.788	.846
Tale, China	"	10 mace, 100 candarines.		1.480	1.590
" Japan	"	10 mace, 100 candarines.		.750	.800
Tomaun, Persia	Gold.	100 mamvodis.	2.233		
Utchlik, Tripoli	Silver.	120 paras.		.149	.160
Yirmilik, Turkey	Gold.	20 piasters.	.877		

NOTES.—1. The standard value of gold as compared with silver in the United States, is as 15.407 to 1 in the coinage of 1792, as 15.988 to 1 in the coinage of 1837, and as 14.922 to 1 in the coinage of 1853.

2. The relative values of gold and silver differ in the coinage of different countries. In England the ratio is 14.288 to 1; in France it is 15.5 to 1; in Hamburg it is 15 to 1.

3. In the present gold coinage of the United States, a Troy ounce of pure gold is equal to \$20.672, and of standard gold to \$18.605. In the present silver coinage of the United States, a Troy ounce of pure silver is equal to \$1.388, and of standard silver to \$1.25.

603. It will be seen by the table that the par of exchange between the United States and Great Britain is £1 = \$4.861. Previous to the changes in the U. S. coinage, made in 1834 and in 1837, the par of exchange was £1 = $\$4.44\frac{4}{9}$, or £9 = \$40, which is called the *old par of exchange*. By the new par of exchange, sterling money is worth about $9\frac{3}{5}$ % more than by the old par.

604. The course of exchange on England is usually given with reference to the *old* par of exchange. Hence, when sterling money is really *at par*, according to present standards, it is quoted in the market at $9\frac{3}{5}$ % premium.

605. The course of exchange between different countries may be expressed either by the rate per cent. above or below par, or by giving the sum of money in one country which is equal to a certain sum in another country. In the latter case, the exchange requires simply a reduction of currencies; in the former, it requires both a reduction of currencies and a computation of percentage.

1. What will be the cost in Boston of the following bill of exchange on Liverpool, at $9\frac{1}{2}$ % premium?

£432. BOSTON, June 16, 1860.

At sight of this First of Exchange (Second and Third of same tenor and date unpaid) pay to the order of J. Simmons, Boston, Four Hundred Thirty-two Pounds, value received, and charge the same to account of

JAMES LOWELL & CO.

TO RICHARD EVANS & SON,
Liverpool, England.

OPERATION.

$$£9 = \$40 \times 1.095, \text{ course of exchange,}$$
$$£1 = \frac{\$40 \times 1.095}{9}, \text{ cost of } £1,$$
$$432 \times \frac{\$40 \times 1.095}{9} = \$2102.40, \textit{ Ans.}$$

ANALYSIS. Since exchange on Liverpool is at $9\frac{1}{2}$ % premium, £9 will cost $\$40 \times 1.095$, (603): and £1 will therefore cost $\frac{\$40 \times 1.095}{9}$. Multiplying the face of the bill, £432, by the cost of exchange of £1, we obtain \$2102.40, the required cost of the bill.

2. What is the face of a bill on London, that may be purchased in New York for \$2768.70, exchange being at 10 % premium in favor of London?

OPERATION.

$$£9 = \$40 \times 1.10, \text{ course of exchange,}$$

$$£1 = \frac{\$40 \times 1.10}{9}, \text{ cost of } £1,$$

$$\$2768.70 \div \frac{40 \times 1.10}{9} = £566 \text{ 6s. 6d., } Ans.$$

ANALYSIS. We divide \$2768.70, the given cost, by $\frac{40 \times 1.10}{9}$, the cost of exchange for £1, and obtain £566 6s. 6d., the face.

3. What cost, in Hamburg, a bill on New Orleans for \$4500, the course of exchange being 1 mark = \$.365?

OPERATION.

$\$1 = \frac{1000}{365}$ of a mark, cost of a unit.

$\$4500 \times \frac{1000}{365} = 12328$ marks 12 skillings.

ANALYSIS. Since exchange for \$1 will cost in Hamburg $\frac{1000}{365}$ of a mark, a bill for \$4500 will cost $4500 \times \frac{1000}{365} = 12328$ marks 12 skillings.

606. From these illustrations we derive the following

RULE. I. To find the cost of a bill, the face being given.—*Multiply the face by the cost of a unit of the currency in which the bill is expressed.*

II. To find the face of a bill, the cost being given.—*Divide the given cost by the cost of a unit of the currency in which the bill is to be expressed.*

EXAMPLES FOR PRACTICE.

1. What is the cost in Portland of a bill on Manchester, Eng., for £325 3s. 9d., at $9\frac{3}{4}$ % premium? *Ans.* \$1586.19+.

2. What must be paid in Charleston for a bill of exchange on Paris for 6000 francs, at $18\frac{3}{4}$ cents per franc?

3. What cost in Boston a bill on St. Petersburg for 3000 roubles at $1\frac{1}{4}$ % premium, the par of exchange being \$.754 for 1 rouble?

4. What will be the cost in Naples of a bill of exchange on New York for \$831.12, at the rate of \$.96 for 1 scudo?

Ans. 865 scudi 9 carlini.

5. A draft on Philadelphia cost £125 in Birmingham, Eng., exchange being at 8 % premium for sterling; required the face of the draft.

6. An agent in Boston, having \$7536.30 due his employer in England, is directed to remit by a bill on Liverpool; what is the face of the bill which he can purchase for this money, exchange being at 11 % premium? *Ans.* £1527 12s. $6\frac{36}{37}$d.

7. A merchant in Cincinnati has 9087 gilders 10 stivers due him in Amsterdam, and requests the remittance by draft; what sum will he receive, exchange on U. S. being in Amsterdam at $2\frac{1}{2}$ gilders for \$1?

8. A trader in London wishes to invest £2500 in merchandise in Lisbon; if he remits to his correspondent at Lisbon a bill purchased for this sum, at the rate of 64.5d. sterling per milree, what sum in the currency of Portugal will the agent receive?

Ans. 9302 milrees $325\frac{25}{43}$ reis.

9. A draft on Dublin for £360 cost \$1736; what was the course of exchange? *Ans.* $8\frac{1}{2}$ % premium.

10. A merchant in Baltimore, having received an importation of Madeira wine invoiced at 1500 milrees, allows his correspondent in Madeira to draw on him for the sum necessary to cover the cost, exchange on the United States being in Madeira 930 reis = \$1; how much would the merchant have saved, by remitting a draft on Madeira, purchased at \$1.065 per 1 milree?

Ans. \$15.40.

11. An importer received a quantity of Leghorn hats, invoiced at 25256 lire 16 soldi which was paid in U. S. gold coin, exported at a cost of 3 % for transportation and insurance, the price of fine gold in Leghorn being 131 lire per ounce Troy. How much less would the goods have cost in store, had payment been made by draft on Leghorn, purchased at the rate of 16 cents per lira? *Ans.* \$64.01.

NOTE. — In U. S. gold coinage, \$10 contains $258 \times .9 = 232.2$ grains of *fine gold*, (**601**).

12. When silver is worth in England 67d. per oz. fine, what sum of money in the U. S. silver coinage of 1853 is equal to 20 shillings, or £1 sterling? *Ans.* \$4.975.

13. At what rate of premium is Prussian coin, when \$88.23 in U. S. silver coinage of 1837 is paid for 125 thalers? *Ans.* 2 %.

ARBITRATED EXCHANGE.

607. **Arbitration of Exchange** is the process of computing exchange between two places by means of one or more intermediate exchanges.

NOTES.—1. When there is only one intermediate exchange, the process is called *Simple Arbitration;* when there are two or more intermediate exchanges, the process is called *Compound Arbitration.*

2. The arbitrated price is generally either greater or less than the price of direct exchanges; and the object of arbitration is to ascertain the best route for making drafts or remittances.

608. There are always three methods of receiving money from a place, or of transmitting money to a place, by means of indirect exchange through one intervening place. Thus,

If A is to receive money from C through B,

1st. A may draw on B, and B draw on C;

2d. A may draw on B, and C remit to B;

3d. B may draw on C, and remit to A.

If A is to transmit money to C through B,

1st. A may remit to B, and B remit to C;

2d. A may remit to B, and C draw on B;

3d. B may draw on A, and remit to C.

1. A man in Albany, N. Y., paid a demand in Paris of 5400 francs, by remitting to Amsterdam at the rate of 21 cents for 10 stivers, and thence to Paris at the rate of 28 stivers for 3 francs; how much Federal money was required?

OPERATION.

$ (?) = 5400 francs.
3 francs = 28 stivers.
10 stivers = $21.

(?) = $1058.40, *Ans.*

Or,

(?)	5400
3	28
10	.21

( ) = $1058.40, *Ans.*

ANALYSIS. We are to determine how much Federal money is equal to 5400 francs, and the question may be represented thus: $ (?) = 5400 francs. Now since 3 francs = 28 stivers, and 10 stivers = $.21, we know that if the required sum be multiplied successively by 3 francs and 10 stivers, the result will be equal to the product of 5400 francs by 28 stivers and $.21 successively, (Ax. 3). Canceling the units of currency, 1 franc, 1 stiver, and $1, and also the equal numerical factors, we have (?) = $1058.40, the sum required.

Or, since the course of exchange between Amsterdam and Paris gives 1 franc $= \frac{28}{3}$ stivers, and the course between Albany and Amsterdam gives 1 stiver $= \frac{21}{10}$ cents, we multiply the 5400 francs by $\frac{28}{3}$ and $\frac{21}{10}$ successively, using the vertical line and cancellation, and obtain $1058.40, as before.

NOTE.— In the first statement the rates of exchange are so arranged that the same unit of currency shall stand on opposite sides in each two consecutive equations, in order that these factors may all be canceled.

2. A resident of Naples having a bequest of $8720 made him in Boston, orders the remittance to be made to his agent in London, who remits the proceeds to Naples, reserving his commission of $\frac{1}{2}$ % on the draft sent. If exchange on London is 9 % in Boston, and the rate between London and Naples is £1 for 5 scudi, how much does the man realize from his bequest?

OPERATION.

$$\begin{array}{r|l} (?)\ \text{scudi} = & \$8720 \\ \$40 \times 1.09 = & £9 \\ £1 = & 5\ \text{scudi.} \\ 1.005 & \\ \hline (?) = & 8955\ \text{scudi}\ 3\ \text{carlini.} \end{array}$$

ANALYSIS. We make the statement as in the first example, according to the given rates of exchange. Then, since the agent is to deduct $\frac{1}{2}$ % commission on the face of the draft before the purchase, we place 1.005 on the left as a divisor, (**159**), and obtain by cancellation 8955 scudi 3 carlini as the proceeds of the exchange.

NOTE.—Since the par of exchange on England is £9 = $40, the course of exchange will always be £9 = $40 × 1 plus the rate of exchange.

3. A merchant in Chicago directs his agent in Albany to draw upon Baltimore at 1 % discount, for $1200 due from the sales of produce; he then draws upon the Albany agent at 2 % premium, for the proceeds, after allowing the agent to reserve $\frac{1}{2}$ % for his commission. What sum does the merchant realize from his produce?

OPERATION.

$$\begin{array}{r|l} (?)\ \text{C.} = & 1200\ \text{B.} \\ 100\ \text{B.} = & 99\ \text{A.} \\ 100\ \text{A.} = & 102\ \text{C.} \\ & .995 \\ \hline (?) = & \$1205.70,\ \textit{Ans.} \end{array}$$

ANALYSIS. According to the given rates of exchange, 100 dollars in Baltimore is equal to 99 dollars in Albany; and 100 dollars in Albany is equal to 102 dollars in Chicago; and since the unit of currency is the same in each place, being $1, we represent its exchange value in each town by the initial letter, and make the statement as in the other examples. Then, since the agent is to reserve $\frac{1}{2}$ % commission from the avails of his draft, we place 1 — .005 = .995 on the right as a multiplier, and obtain by cancellation (?) = $1205.70, the answer.

From these principles and illustrations we have the following

RULE. I. *Represent the required sum by* (?), *with the proper unit of currency affixed, and place it equal to the given sum on the right.*

II. *Arrange the given rates of exchange so that in any two consecutive equations the same unit of currency shall stand on opposite sides.*

III. *When there is commission for drawing, place* 1 *minus the rate on the left if the cost of exchange is required, and on the right if proceeds are required; and when there is commission for remitting, place* 1 *plus the rate on the right if cost is required, and on the left if proceeds are required.*

IV. *Divide the product of the numbers on the right by the product of the numbers on the left, cancelling equal factors; the result will be the answer.*

NOTES.—1. Commission for drawing is commission on the *sale* of a draft; commission for remitting is commission on the *purchase* price of a draft.

2. The above method is sometimes called the *Chain Rule*, or *Conjoined Proportion.*

EXAMPLES FOR PRACTICE.

1. A gentleman in Philadelphia wishes to deposit $5000 in a bank at Stockholm, by remitting to Liverpool and thence to Stockholm; if exchange on Liverpool is at 10 % premium in Philadelphia, and the course between Liverpool and Stockholm is 6 roubles 48 copecks per £1, how much money will the man have in bank at Stockholm, allowing the agent at Liverpool $\frac{1}{4}$ % for remitting? *Ans.* 6610 roubles 74 copecks.

2. When exchange at New York on Paris is 5 francs 16 centimes per $1, and at Paris on Hamburg $2\frac{1}{8}$ francs per marc banco, what will be the arbitrated price in New York of 7680 marc bancos of Hamburg? *Ans.* $3162.79.

3. A man in Cleveland wishes to draw on New Orleans for a bank stock dividend of $750, and exchange direct on New Orleans is $1\frac{1}{4}$ % discount; how much will he save by drawing on

his agent in New York at $1\frac{1}{2}$ % premium, allowing his agent to draw on New Orleans at 1 % discount, brokerage at $\frac{1}{2}$ %?

4. A gentleman in Boston drew on Wurtemberg for 6000 gilders at \$.415 per gilder; how much more would he have received if he had ordered remittance to London, and thence to New York, exchange at Wurtemberg on London being $11\frac{1}{4}$ gilders per £1, and at London on New York $9\frac{1}{4}$ %, in favor of sterling, brokerage at $1\frac{1}{4}$ % in London for remitting? *Ans.* \$67.66.

5. If at Philadelphia exchange on Liverpool is at $9\frac{3}{8}$ % premium, and at Liverpool on Paris 26 francs 86 centimes per £1; what is the arbitrated course of exchange between Philadelphia and Paris, through Liverpool? *Ans.* 1 franc = \$.181.

6. An American resident of Amsterdam wishing to obtain funds from the U. S. to the amount of \$6400, directs his agent in London to draw on the U. S. and remit the proceeds to him in a draft on Amsterdam, exchange on the U. S. being at 8 % in favor of London, and the course between London and Amsterdam being 18d. per gilder. If the agent charges commission at $\frac{1}{2}$ % both for drawing and remitting, how much better is this arbitration than to draw directly on the U. S. at 40 cents per gilder?

7. A speculator in Pittsburgh, having purchased 58 shares of railroad stock in New Orleans, at 95 %, remits to his agent in New York a draft purchased at 2 % premium, with orders for the agent to remit the sum due in N. O. Now, if exchange on N. O. is at $\frac{1}{2}$ % discount in N. Y., and the agent's commission for remitting is $\frac{1}{4}$ %, how much does the stock cost in Pittsburgh?

Ans. \$5606.08.

8. A banker in New York remits \$3000 to Liverpool, by arbitration, as follows: first to Paris at 5 francs 40 centimes per \$1; thence to Hamburg at 185 francs per 100 marcs; thence to Amsterdam at 35 stivers per 2 marcs; thence to Liverpool at 220 stivers per £1 sterling. How much sterling money will he have in bank at Liverpool, and what will be his gain over direct exchange at 10 % premium?

Ans. { Proceeds in Liverpool, £696 11s. 2d.
Gain by arbitration, £82 18s. 5d. }

EQUATION OF PAYMENTS.

609. Equation of Payments is the process of finding the mean or equitable time of payment of several sums, due at different times without interest.

610. The **Term of Credit** is the time to elapse before a debt becomes due.

611. The **Average Term of Credit** is the time to elapse before several debts, due at different times, may all be paid at once, without loss to debtor or creditor.

612. The **Equated Time** is the date at which the several debts may be canceled by one payment.

613. To **Average an Account** is to find the mean or equitable time of payment of the balance.

614. A **Focal Date** is a date with which all the others are compared in averaging an account.

NOTE. — Each item of a book account draws interest from the time it is due, which may be either at the date of the transaction, or after a specified term of credit.

In averaging, there are two kinds of equations, Simple and Compound.

615. A **Simple Equation** is the process of finding the average time when the payments or account contains only one side, which may be either a debit or credit.

616. A **Compound Equation** is the process of averaging when both debts and credits are to be considered.

SIMPLE EQUATIONS.

CASE I.

617. When all the terms of credit begin at the same date.

1. In settling with a creditor on the first day of April, I find that I owe him $12 due in 5 months, $15 due in 2 months, and $18 due in 10 months; at what time may I pay the whole amount?

OPERATION.

$$\begin{array}{lrl} \$12 \times & 5 = & 60 \\ 15 \times & 2 = & 30 \\ 18 \times & 10 = & 180 \\ \hline \$45 & & 270 \end{array}$$

$270 \div 45 = 6$ mo., average credit,
Apr. 1, + 6 mo. = Oct. 1, *Ans.*

ANALYSIS. The whole amount to be paid, as seen in the operation, is \$45; and we are to find how long it shall be withheld, or what term of credit it shall have, as an equivalent for the various terms of credit on the different items. Now the value of credit on any sum is measured by the product of the money and time. Therefore, the credit on \$12 for 5 mo. = the credit on \$60 for 1 mo., because $12 \times 5 = 60 \times 1$. In like manner, we have the credit on \$15 for 2 mo. = the credit on \$30 for 1 mo.; and the credit on \$18 for 10 mo. = the credit on \$180 for 1 mo. Hence, by addition, the value of the several terms of credit on their respective sums equals a credit of 1 month on \$270; and this equals a credit of 6 months on \$45, because $45 \times 6 = 270 \times 1$. Hence the following

RULE. I. *Multiply each payment by its term of credit, and divide the sum of the products by the sum of the payments; the quotient will be the average term of credit.*

II. *Add the average term of credit to the date at which all the credits begin; the result will be the equated time of payment.*

NOTES.—1. The periods of time used as multipliers must all be of the same denomination, and the quotient will be of the same denomination as the terms of credit; if these be months, and there be a remainder after the division, continue the division to days by reduction, always taking the nearest unit in the last result.

2. The several rules in equation of payments are based upon the principle of bank discount; for they imply that the *discount* of a sum paid before it is due equals the *interest* of the same amount paid after it is due.

EXAMPLES FOR PRACTICE.

1. On the first day of January, 1860, a man gave 3 notes, the first for \$500 payable in 30 days; the second for \$400 payable in 60 days; the third for \$600 payable in 90 days. What was the average term of credit, and what the equated time of payment?

Ans Term of credit, 62 da.; time of payment, Mar. 3, 1860.

2. A man purchased real estate, and agreed to pay $\frac{1}{3}$ of the price in 3 mo., $\frac{1}{4}$ in 8 mo., and the remainder in 1 year. Wishing to cancel the whole obligation at a single payment, how long shall this payment be deferred?

3. I owe $480 payable in 90 days, and $320 payable in 60 days. My creditor consents to an extension of time to 1 year, and offers to take my note for the whole amount on interest at 6 per cent. from the equated time, or a note for the true present worth of both debts, on interest from date. How much will I gain if I choose the latter condition? *Ans.* $1.14.

4. Bought merchandise April 1, as follows: $280 on 3 mo., $300 on 4 mo., $200 on 5 mo., $560 on 6 mo.; what is the equated time of payment? *Ans.* Aug. 24.

CASE II.

618. When the terms of credit begin at different dates.

1. When does the amount of the following bill become due, per average?

CHARLES CROSBY,

1860.	To BRONSON & Co., Dr.	
Jan. 12.	To Mdse.,	$400
" 16.	" Mdse. on 2 mo.,	600
Apr. 20.	" Cash,	375

FIRST OPERATION.

Due	Da.	Items.	Prod.
Jan. 12		400	
Mar. 16	64	600	38400
Apr. 20	99	375	37125
		1375	75525

$75525 \div 1375 = 55$ da.

Ans. { 55 da. after Jan. 12, or Mar. 7.

SECOND OPERATION.

Due.	Da.	Items.	Prod.
Jan. 12	99	400	39600
Mar. 16	35	600	21000
Apr. 20	0	375	
		1375	60600

$60600 \div 1375 = 44$ da.

Ans. { 44 da, before Apr. 20, or Mar. 7.

ANALYSIS. The three items of the bill are due Jan. 12, Mar. 16, and Apr. 20, respectively. In the first operation we use the *earliest* maturity, Jan. 12, for a focal date, and find the difference in days between this date and each of the others; thus, from Jan. 12 to Mar.

16 is 64 da.; from Jan. 12 to Apr. 20 is 99 da. Hence, from Jan. 12 the first item has no credit, the second has 64 days' credit, and the third 99 days' credit, as appears in the column marked *da*. We now proceed to find the products as in Case I, whence we obtain the average credit, 55 da., and the equated time, Mar. 7.

In the second operation, the *latest* maturity, Apr. 20, is taken for a focal date, and the work may be explained thus: Suppose the account to be settled Apr. 20. At that time the first item has been due 99 days, and must therefore draw interest for this time. But interest on \$400 for 99 days = the interest on \$39600 for 1 day. The second item must draw interest 35 days; but interest on \$600 for 35 days = interest on \$21000 for 1 day. Taking the sum of the products, we find that the whole amount of interest due Apr. 20 equals the interest on \$60600 for 1 day; and this is found, by division, equal to the interest on \$1375 for 44 da., which is the average term of interest. Hence the account would be settled Apr. 20, by paying \$1375, with interest on the same for 44 days. This shows that the \$1375 has been used 44 days, that is, it falls due Mar 7, *without interest*. Hence we have the following

RULE. I. *Find the time at which each item becomes due, by adding to the date of each transaction the term of credit, if any be specified, and write these dates in a column.*

II. *Assume either the earliest or the latest date for a focal date, and find the difference in days between the focal date and each of the other dates, and write the results in a second column.*

III. *Write the items of the account in a third column, and multiply each by the corresponding number of days in the preceding column, writing the products in a fourth column.*

IV. *Divide the sum of the products by the sum of the items. The quotient will be the average term of credit or interest, and must be reckoned from the focal date* TOWARD *the other dates, to find the equated time of payment.*

NOTES.—1. When dollars and cents are given, it is generally sufficient to take only dollars in the multiplicand, rejecting the cents when less than 50, and carrying 1 to the dollars, if the cents are more than 50.

2. Months in any terms of credit are understood to be calendar months; the time must therefore be carried forward to the same day of the month in which the term of credit expires.

EXAMPLES FOR PRACTICE.

1. JAMES GORDON,

1860.				To HENRY LANCEY, Dr.			
Mar. 4.	To	100 yd.	Cassimere,	@	$2 50,......	$250
" 25.	"	3000 "	French Prints,	"	.12,......	360
Apr. 16.	"	1200 "	Sheeting,	"	.08,......	96
" 30.	"	400 "	Oil Cloth,	"	.50,......	200
May 17.	"		Sundries,................................			350

When is the above bill due, per average?

Ans. Apr. 12, 1860.

2. I sell goods to A at different times, and for different terms of credit, as follows:

Sept. 12, 1859,	a bill on	30	days' credit, for	$180
Oct. 7, "	"	30	" "	300
Nov. 16, "	"	60	" "	150
Dec. 20, "	"	90	" "	350
Jan. 25, 1860,	"	30	" "	130
Feb. 24, "	"	30	" "	140

If I take his note in settlement, at what time shall interest commence?

3. What is the average of the following account?

1860,	Oct. 1.	Mdse., on 60 da.,..................	$240
"	Nov. 12.	" " "	500
"	Dec. 25.	" " "	436
1861,	Jan. 16.	" " "	325
"	Feb. 24.	" " "	436
"	Mar. 17.	" " "	537

Ans. Mar. 10, 1861.

4. I have 4 notes, as follows: the first for $350, due Aug. 16, 1859: the second for $250, due Oct. 15, 1859; the third for $300, due Dec. 14, 1859; the fourth for $248, due Feb. 12, 1860. When shall a note for which I may exchange the four, be made payable?

COMPOUND EQUATIONS.

619. 1. Average the following account.

John Lyman.

Dr.				Cr.			
1860.				1860.			
June 12	To Mdse.	530	00	June 24	By draft at 30 da.	480	00
Sept. 12	" "	428	00	Aug. 20	" cash,	280	00
Oct. 28	" Sundries,	440	00	Oct. 8	" "	140	00

OPERATION.

	Dr. Due.	Da.	Items.	Products.	Cr. Due.	Da.	Items.	Products.
	June 12	138	530	73140	July 27	93	480	44640
	Sept. 12	46	428	19688	Aug. 20	69	230	15870
Focal date,	Oct. 28	0	440		Oct. 8	20	140	2800
			1398	92828			850	63310
			850	63310				
Balances,			548	29518				

$$29518 \div 548 = 54 \text{ da., average term of interest.}$$
$$\text{Oct. } 28 - 54 \text{ da.} = \text{Sept. 4, balance due.}$$

ANALYSIS.—In this operation we have written the dates of maturity on either side, allowing 3 days' grace to the draft. The latest date, Oct. 28, is assumed as the focal date for *both sides*, and the two columns marked da. show the difference in days between the focal date and each of the other dates. The products are obtained as in simple equations, and the balance found between the items on the two sides, and also between the products. These balances, being both on the Dr. side, show that there is due on the day of the focal date, $548, with interest on $29518 for 1 day. By division, this interest is found to be equal to the interest on $548 for 54 days. Hence this balance, $548, has been due 54 days; and reckoning back from the focal date, we obtain the equated time of payment, Sept. 4.

Had we taken the *earliest* maturity, June 12, for the focal date, we should have obtained 84 days for the interval of time; and since in this case the products would represent the *credit* to which the several items are entitled *after* June 12, we should *add* 84 days to the focal date, which would give Sept. 4, as before.

2. When is the balance of the following account due, per average?

Charles Derby.

Dr. Cr.

1859.				1859.			
Jan. 21	To Mdse.	32	00	Jan. 1	By cash,	84	00
Mar. 5	" "	145	00	Feb. 4	" "	40	00
" 22	" "	194	00	Mar. 30	" "	12	00

OPERATION.

Dr. Cr.

Due.	Da.	Items.	Products.	Due.	Da.	Items.	Products.
Jan. 21	68	32	2176	Jan. 1	88	84	7392
Mar. 5	25	145	3625	Feb. 4	54	40	2160
" 22	8	194	1552	Mar. 30	0	12	
		371	7353			136	9552
		136					7353
Balance of account,		235		Balance of products,			2199

$$2199 \div 235 = 9 \text{ da.}; \text{ Mar. } 30 + 9 \text{ da.} = \text{Apr. } 8, \textit{Ans.}$$

ANALYSIS. We take the latest maturity, Mar. 30, for the focal date, and consequently the products represent the *interest* due upon the several items, at that date. We find the balance of the items upon the Dr. side, and the balance of the products upon the Cr. side. The debtor therefore owes, on Mar. 30, $235, but is entitled to such a term of interest on the same as will be equivalent to the interest on $2199 for 1 day, which by division, is found to be 9 da. Hence the balance is due Mar. 30 + 9 da. = Apr. 8. Thus we see that when the balances are on opposite sides, the interval of time is counted *from* the other dates. If we take, in this example, the *earliest* date for the focal date, the balances will both be upon the Dr. side, and the interval of time will be 97 da., which reckoned forward from the focal date, will give the equated time as before.

620. From these examples we derive the following

RULE. I. *Find the time when each item of the account is due, and write the dates, in two columns, on the sides of the account to which they respectively belong.*

II. *Use either the earliest or the latest of these dates as the focal date for both sides, and find the products as in the last case.*

III. *Divide the balance of the products by the balance of the account; the quotient will be the interval of time, which must be reckoned from the focal date* TOWARD *the other dates when both* *balances are on the same side of the account, but* FROM *the other dates when the balances are on opposite sides of the account.*

NOTES.—1. Instead of the products, we may obtain the interest, at any per cent., on the several items for the corresponding intervals of time, and divide the balance of interest by the interest on the balance of the account for 1 day; the quotient will be the interval of time to be added to, or subtracted from the focal date, according to the rule. The time obtained will be the same, at whatever rate the interest be computed.

2. There may be such a combination of debits and credits, that the equated time will be earlier or later than any date of the account.

EXAMPLES FOR PRACTICE.

1. Required, the average maturity of the following account.

A. Z. Armour.

Dr. Cr.

1859.					1859.				
Feb.	12	To Mdse.	85	75	March 15	By bal. old acc't.	97	36
"	25	" "	36	24	April 17	" cash,	56	00
April	16	" "	174	96	May 25	" "	25	00
May	20	" "	94	78	June 8	" sundries,	94	75

OPERATION.

Dr. Cr.

Due.	Da.	Items.	Int.	Due.	Da.	Items.	Int.
Feb. 12	116	85.75	1.66	March 15	85	97.36	1.38
" 25	103	36.24	.62	April 17	52	56.00	.49
April 16	53	174.96	1.55	May 25	14	25.00	.06
May 20	19	94.78	.30	June 8		94.75	
		391.73	4.13			273.11	1.93
		273.11	1.93				
	Balances,	118.62	2.20				

Int on \$118.62 for 1 da. = \$.0198.

2.20÷.0198=111 da.; June 8—111 da.=Feb. 17, 1859, *Ans.*

ANALYSIS. Taking the latest maturity, June 8, for the focal date, we find the *interest* of each item, at 6 %, from its maturity to the focal date; then, taking the balance, we find the interest due on the account to be \$2.20. Dividing this interest by the interest on the balance of the items for 1 day, we obtain 111 da., the time required for the interest, \$2.20, to accrue. The average maturity, therefore, is June 8 — 111 da. = Feb. 17, 1859.

It is evident that when the balances occur on opposite sides, the interval of time will be reckoned as in the method by products.

2. What is the balance of the following account, and when is it due?

Thomas Lardner.

Dr. Cr.

1860.				1860.			
March 1	To Sundries,	436	00	March 25	By draft, at 60 da.	400	00
April 12	" Mdse.	548	00	April 6	" " 30 "	650	00
July 16	" "	312	00	June 20	" cash,	200	00
Sept. 14	" "	536	00	Aug. 3	" "	84	00

Ans. Balance, $498; due June 22, 1860.

3. When shall a draft for the settlement of the following account be made payable?

David Sanford.

Dr. Cr.

1859.				1859.			
Jan. 1	To Mdse. on 3 mo.	54	36	April 1	By cash,	50	00
Feb. 12	" " " 2 "	23	45	May 16	" draft, at 30 da.	30	00
March 16	" Sundries,	95	75	June 12	" "	125	00
June 25	" Mdse.	26	32	" 20	" cash,	150	00

Ans. Aug. 28, 1859.

4. *Oliver Wainwright.*

Dr. Cr.

1858.				1858.			
Jan. 1	To Mdse.	36	72	Jan. 10	By cash,	98	72
Feb. 1	" "	48	25	" 21	" "	25	84
March 17	" "	72	36	March 23	" sundries,	15	17
April 1	" "	98	48	April 6	" "	8	96

If the above account were settled April 6, 1858, by draft on time, how many days' credit should be given? *Ans.* 20 da.

5. I owe $1000 due Apr. 25. If I pay $560 Apr. 1, and $324 Apr. 21, when, in equity, should I pay the balance?

Ans. Aug. 30.

NOTE.—Make the $1000 the Dr. side of an account, and the payments the Cr. side, and then average.

6. A man owes $684, payable Aug. 12, and $468, payable Oct. 15. If he pay $839, Aug. 1, what will be the equated time for the payment of the balance? *Ans.* Dec. 15.

7. A man holds 3 notes, the first for $500, due March 1, the second for $800, due June 1, and the third for $600, due Aug. 1. He wishes to exchange them for two others, one of which shall be for $1000, payable Apr. 1; what shall be the face and when the maturity of the other?

Ans. Face, $900; maturity, July 28.

8. A owes \$500, due Apr. 12, and \$1000, due Sept. 20, and wishes to discharge the obligation by two equal payments, made at an interval of 60 days; when must the two payments be made?

Ans. 1st, June 28; 2d, Aug. 27.

9. When shall a note be made payable, to balance the following account?

James Tyler.

Dr. **Cr.**

1859.				1859.			
June 12	To Mdse. on 3 mo.	530	84	Sept. 14	By cash,	436	00
" 20	" " " "	236	48	" 25	" "	320	00
" 30	" " " "	739	56	Oct. 3	" "	560	00
July 5	" " " "	273	44	" 17	" "	370	00
" 16	" " " "	194	78	Nov. 16	" "	840	00
" 29	" " " "	536	42	" 24	" "	560	00

10. I received goods from a wholesale firm in New York, in parcels, as per bills received, namely: Apr. 1, a bill for \$536.78; May 16, \$2156.94; June 12, \$843.75; July 12, \$594.37; Sept. 18, \$856.48. In part payment, I remitted cash as follows: June 3, \$500; July 1, \$1000; Nov. 1, \$1500. When is the balance payable, allowing credit of 2 months for the merchandise?

Ans. July 23.

ACCOUNT SALES.

621. An **Account Sales** is an account rendered by a commission merchant of goods sold on account of a consignor, and contains a statement of the sales, the attendant charges, and the net proceeds due the owner.

622. Guaranty is a charge made in addition to commission, for securing the owner against the risk of non-payment, in case of goods sold on credit.

623. Storage is a charge made for keeping the goods, and may be reckoned by the week or month, on each article or piece.

624. Primage is an allowance paid by a shipper or consignor of goods to the master and sailors of a vessel, for loading it.

625. A commission merchant having sold a shipment of goods by parts at different times, and on various terms, makes a final settlement by deducting all charges, and accrediting the owner with the net proceeds. It is evident, therefore,

I. That commission and guaranty should be accredited to the agent at the average maturity of the *sales*.

II. That the net proceeds should be accredited to the consignor at the average maturity of the entire account.

Hence the following

RULE. I. To compute the storage.—*Multiply each article or parcel by the time it is in store, and multiply the sum of the products by the rate per unit; the result will be the storage.*

II. To find when the net proceeds are due.—*Average the sales alone, and the result will be the date to be given to the commission and guaranty; then make the sales the Cr. side, and the charges the Dr. side, and average the entire account by a compound equation.*

NOTE.—In averaging, either the *product* method or the *interest* method may be used.

EXAMPLES FOR PRACTICE.

1. Account sales of 100 pipes of gin, received per ship Hispaniola, from Havana, on a|c. of Tyler, Jones & Co.

1860.					
April 15	Sold 32 Pipes, 4160 gal. @ $1.05, on 30 days,			4368	00
May 5	" 40 " 5240 " @ 1.02, cash,			5344	80
June 28	" 28 " 3650 " @ 1.00, "			3650	00
	100 "			13362	80
	CHARGES.				
April 1	To Freight and Primage,		$136.76		
" 1	" Wharfage and Cartage		48.54		
" 1	" Duty Bonds, at 60 days		3207.07		
June 28	" Storage from April 1, viz.:				
	On 32 Pipes, 2 wks.	64 wks.			
	" 40 " 5 "	200 "			
	" 28 " 13 "	364 "			
	100 " equal to	628 " @ 6 c.	37.68		
	" Commission on $13362.80. at $2\frac{1}{2}$ %		334.07		
	" Guaranty on $4368, at $2\frac{1}{2}$ %		109.20	3873	32

What are the net proceeds of the above account, and when due?

Ans. Net proceeds, $9489.48; due, May 20, 1860.

NOTE.—The time for which storage is charged on each part of the shipment is the interval, reduced to weeks, between Apr. 1, when the pipes were received into store, and the date of sale. Every fraction of a week is reckoned a full week.

2. A commission merchant in Boston received into his store on May 1, 1859, 1000 bbl. of flour, paying as charges on the same

day, freight \$175.48, cartage \$56.25, and cooperage \$8.37. He sold out the shipment as follows: June 3, 200 bbl. @ \$6.25; June 30, 350 bbl. @ \$6.50; July 29, 400 bbl. @ \$6.12½; Aug. 6, 50 bbl. @ \$6.00. Required the net proceeds, and the date when they shall be accredited to the owner, allowing commission at 3½ %, and storage at 2 cents per week per bbl.

Ans. Net proceeds, \$5614.28; due, July 10.

SETTLEMENT OF ACCOUNTS CURRENT.

626. To find the cash balance of an account current, at any given date.

J. Burns in account current with Tyler & Co.

Dr. Cr.

1860.				1860.			
Feb. 25	To Mdse. on 3 mo.	360	75	March 1	By cash on acct.	250	00
March 20	" " " 3 "	240	56	April 20	" accept. at 30 da.	300	00
April 26	" " " 3 "	875	24	June 12	" Sundries,	375	00
June 24	" " " 2 "	235	25	" 27	" cash on acct.	400	00

Required the cash value of the above account, July 1, 1860, interest at 6 %.

OPERATION.

Dr. Cr.

Due.	Da.	Items. Int.	Cash val	Due.	Da.	Items. Int.	Cash val.
May 25	37	360.75 + 2.22	362.97	March 1	122	250.00 + 5.08	255.08
June 20	11	240.56 + .44	241.00	May 20	42	300.00 + 2.10	302.10
July 26	25	875.24 — 3.65	871.59	June 12	19	375.00 + 1.19	376.19
Aug. 24	54	235.25 — 2.12	233.13	" 27	4	400.00 + .27	400.27
			1708.69				1333.64

\$1708.69 — \$1333.64 = \$375.05. *Ans.*

ANALYSIS. For either side of the account we write the dates at which the several items are due, and the days intervening between these dates and the day of settlement, July 1. We then compute the interest on each item for the corresponding interval of time, and add it to the item if the maturity is *before* July 1, and subtract it from the item if the maturity is after July 1; the results must be the cash values of the several items on July 1. Adding the two columns of cash values, and subtracting the less sum from the greater, we have \$375.05. the *cash balance* required. Hence the

RULE. I. *Find the number of days intervening between each maturity and the day of settlement.*

II. *Compute the interest on each item for the corresponding interval of time; add the interest to the item if the maturity is before the day of settlement, and subtract it from the item if the maturity is after the day of settlement; the results will be the cash values of the several items.*

III. *Add each column of cash values, and the difference of the amounts will be the cash balance required.*

EXAMPLES FOR PRACTICE.

1. Find the cash balance of the following account for June 1, 1858, interest at 6 per cent.?

Alvan Parke.

Dr.				Cr.			
1858.				1858.			
Jan. 12	To check,	500	36	Jan. 1	By bal. from old acct.	536	72
" 26	" "	250	48	Feb. 3	" cash,	486	57
Feb. 13	" "	400	00	March 26	" "	1260	78
March 16	" "	750	00	April 20	" "	756	36
April 25	" "	200	00	May 12	" "	248	79

Ans. $1196.67.

2. What is the cash balance of the following account on Dec. 31, at 7 per cent.?

James Hanson.

Dr.				Cr.			
1859.				1859.			
Sept. 3	To Sundries,	478	36	Sept. 17	By Sundries,	96	54
Oct. 2	" Mdse. on 3 mo.	256	37	" 20	" cash on acct.	200	00
" 21	" " " 3 "	375	26	Oct. 3	" " "	325	00
Nov. 12	" " " 3 "	80	00	Nov. 17	" " "	50	00
Dec. 15	" Sundries,	148	76	Dec. 27	" " "	84	00

PARTNERSHIP.

627. Partnership is a relation established between two or more persons in trade, by which they agree to share the profits and losses of business according to the amount of capital furnished by each, and the time it is employed.

628. The **Partners** are the individuals thus associated.

NOTE.—The terms *Capital* or *Stock*, *Dividend*, and *Assessment*, have the same signification in Partnership as in Stocks.

CASE I.

629. To find each partner's share of the profit or loss, when their capital is employed for *equal* periods of time.

1. A and B engage in trade; A furnishes \$500, and B \$700 as capital; they gain \$96; what is each man's share?

OPERATION.

\$ 500
\$ 700
\$1200, whole stock.

$\frac{500}{1200} = \frac{5}{12}$, A's part of the stock.
$\frac{700}{1200} = \frac{7}{12}$, B's " " " "
$\$96 \times \frac{5}{12} = \$40$ A's share of the gain.
$\$96 \times \frac{7}{12} = \$56$, B's " " " "

ANALYSIS. The whole amount of capital employed is \$500 + \$700 = \$1200; hence, A furnishes $\frac{500}{1200} = \frac{5}{12}$ of the capital, and B furnishes $\frac{700}{1200} = \frac{7}{12}$ of the capital. And since each man's share of the profit or loss will have the same ratio to the whole profit or loss as his part of the capital has to the whole capital, A will have $\frac{5}{12}$ of the \$96, and B $\frac{7}{12}$ of the \$96, for their respective shares of the profits.

We may also regard the whole capital as the *first cause*, and each man's share of the capital as the *second cause*, the whole profit or loss as the *first effect*, and each man's share of the profit or loss as the *second effect*, and solve by proportion thus:

1st cause.		2d cause.		1st effect.		2d effect.
\$1200	:	\$500	=	\$96	:	(?) = \$40, A's gain,
\$1200	:	\$700	=	\$96	:	(?) = \$56, B's "

Hence we have the following

RULE. *Multiply the whole profit or loss by the ratio of the whole capital to each man's share of the capital.* Or,

The whole capital is to each man's share of the capital as the whole profit or loss is to each man's share of the profit or loss.

EXAMPLES FOR PRACTICE.

1. Three men engage in trade; A puts in \$6470, B \$3780, and C \$9860, and they gain \$7890. What is each partner's share of the profit? *Ans.* A's, \$2538.453; B's, \$1483.053; C's, \$3868.493.

2. B and C buy pork to the amount of \$1847.50, of which B pays \$739, and C the remainder. They gain \$375; what is each one's share of the gain?

31 *

3. A, B, and C form a company for the manufacture of woolen cloths. A puts in \$10000, B \$12800, and C \$3200. C is allowed \$1500 a year for personal attention to the business; their expenses for labor, clerk hire, and other incidentals for 1 year are \$3400, and their receipts during the same time are \$9400. What is A's, B's, and C's income respectively from the business?

4. Four persons rent a farm of 115 A. 32 P. at \$3.75 an acre. A puts on 144, B 160, C 192, and D 324 sheep; how much rent ought each to pay?

5. Three persons gain \$2640, of which B is to have \$6 as often as C \$4, and as often as D \$2; how much is each one's share?

6. Six persons are to share among them \$6300; A is to have $\frac{1}{7}$ of it, B $\frac{1}{5}$, C $\frac{2}{9}$, D is to have as much as A and C together, and the remainder is to be divided between E and F in the ratio of 3 to 5. How much does each receive?

Ans. A, \$900; B, \$1260; C, \$1400;
D, \$2300; E, \$165; F, \$275.

7. Two persons find a watch worth \$90, and agree to divide the value of it in the ratio of $\frac{2}{3}$ to $\frac{5}{6}$; how much is each one's share?

NOTE.—If the fractions be reduced to a common denominator, they will be to each other as their numerators, (**418**, III).

8. A father divides his estate worth \$5463.80 between his two sons giving the elder $\frac{1}{8}$ more than the younger; how much is each son's share? *Ans.* Elder, \$2892.60; younger, \$2571.20.

9. Three men trade in company. A furnishes \$8000, and B \$12000 Their gain is \$1680, of which C's share is \$840; required, C's stock, and A's and B's gain. *Ans.* C's stock, \$20,000.

10. Four persons engage in the lumber trade, and invest jointly \$22500; at the expiration of a certain time, A's share of the gain is \$2000, B's \$2800.75, C's \$1685.25, and D's \$1014; how much capital did each put in? *Ans.* D put in \$3042.

11. A legacy of \$30,000 was left to four heirs in the proportion of $\frac{1}{6}$, $\frac{2}{5}$, $\frac{4}{9}$, and $\frac{1}{3}$, respectively; how much was the share of each?

12. Three men purchase a piece of land for \$1200, of which sum C pays \$500. They sell it so as to gain a certain sum of

which A takes \$71.27, and B \$142.54; how much do A and B pay, and what is C's share of the gain? *Ans.* C's gain, \$152.72$\frac{1}{7}$.

13. Three persons enter into partnership for the manufacture of coal oil, with a joint capital of \$18840. A puts in \$3 as often as B puts in \$5, and as often as C puts in \$7. Their annual gain is equal to C's stock; how much is each partner's gain?

14. A, B, and C are employed to do a piece of work for \$26.45. A and B together are supposed to do $\frac{3}{4}$ of the work, A and C $\frac{9}{10}$, and B and C $\frac{13}{20}$, and are paid proportionally; how much must each receive? *Ans.* A, \$11.50; B, \$5.75; C, \$9.20.

CASE II.

630. To find each partner's share of the profit or loss when their capital is employed for *unequal* periods of time.

It is evident that the respective shares of profit and loss will depend equally upon two conditions, viz.: *the amount of capital* invested by each, and the *time* it is employed. Hence they will be proportional to the *products* of these two elements.

1. Two men form a partnership; A puts in \$320 for 5 months, and B \$400 for 6 months. They lose \$140; what is each man's share of the loss?

OPERATION.

$\$320 \times 5 = \$1600$, A's capital for 1 mo.
$\$400 \times 6 = \$2400$, B's " " "

\$4000, entire " " "

$\$\frac{1600}{4000} = \frac{2}{5}$, A's share in the partnership
$\$\frac{2400}{4000} = \frac{3}{5}$, B's " " "
$\$140 \times \frac{2}{5} = \$56$, A's loss.
$\$140 \times \frac{3}{5} = \$84$, B's loss.

ANALYSIS The use of \$320 for 5 months is the same as the use of 5 times \$320, or \$1600, for 1 month; and the use of \$400 for 6 months is the same as the use of 6 times \$400, or \$2400, for 1 month; hence the use of the entire capital is the same as the use of $\$1600 + \$2400 = \$4000$ for 1 month. A's interest in the partnership is therefore $\frac{1600}{4000} = \frac{2}{5}$, and he will suffer $\frac{2}{5}$ of the loss, or $\$140 \times \frac{2}{5} = \$56$: and

B's interest in the partnership is $\frac{2400}{4000} = \frac{3}{5}$, and he will suffer $\frac{3}{5}$ of the loss, or $\$140 \times \frac{3}{5} = \$84$.

We may also solve by proportion, the *causes* being compounded of the two elements, *capital* and *time* · thus:

$$\$4000 : \$1600 = \$140 : (?) = \$56, \text{ A's loss.}$$
$$\$4000 : \$2400 = \$140 : (?) = \$84, \text{ B's loss.}$$

Hence the following

RULE. *Multiply each man's capital by the time it is employed in trade, and add the products. Then multiply the entire profit or loss by the ratio of each product to the sum of the products; the results will be the respective shares of profit or loss of each partner.* Or,

Multiply each man's capital by the time it is employed in trade, and regard each product as his capital, and the sum of the products as the entire capital, and solve by proportion, as in Case I.

EXAMPLES FOR PRACTICE.

1. A, B, and C enter into partnership. A puts in \$357 for 5 months, B \$371 for 7 months, and C \$154 for 11 months, and they gain \$347.20; how much is each one's share?

Ans. A's \$102; B's \$148.40; C's \$96.80.

2. Three men hire a pasture for \$55.50. A put in 5 cows, 12 weeks; B, 4 cows, 10 weeks; and C, 6 cows, 8 weeks; how much ought each to pay? *Ans.* A \$22.50; B \$15; C \$18.

3. B commenced business with a capital of \$15000. Three months afterward C entered into partnership with him, and put in 125 acres of land. At the close of the year their profits were \$4500, of which C was entitled to \$1800; what was the value of the land per acre?

4. A and B engaged in trade. A put in \$4200 at first, and 9 months afterward \$200 more. B put in at first \$1500, and at the end of 6 months took out \$500. At the end of 16 months their gain was \$772.20; how much is the share of each?

5. Four companies of men worked on a railroad. In the first company there were 30 men who worked 12 days, 9 hours a day; in the second, there were 32 men who worked 15 days, 10 hours

a day; in the third, there were 28 men who worked 18 days, 11 hours a day; and in the fourth, there were 20 men who worked 15 days, 12 hours a day. The entire amount paid to all the companies was $1500; how much were the wages of each company?

6. A and B are partners. A's capital is to B's as 5 to 8; at the end of 4 months A withdraws $\frac{1}{2}$ of his capital, and B $\frac{2}{3}$ of his; at the end of the year their whole gain is $4000; how much belongs to each? *Ans.* A, 1714\frac{2}{7}$; B, 2285\frac{5}{7}$.

7. B, C, and D form a manufacturing company, with capitals of $15800, $25000, and $30000 respectively. After 4 months B draws out $1200, and in 2 months more he draws out $1500 more, and 4 months afterward puts in $1000. C draws out $2000 at the end of 6 months, and $1500 more 4 months afterward, and a month later puts in $800. D puts in $1800 at the end of 7 months, and 3 months after draws out $5000. If their gain at the end of 18 months be $15000, how much should each receive? *Ans.* B, $3228.07; C, $5258.15; D, $6513.78.

8. The joint stock of a company was $5400, which was doubled at the end of the year. A put $\frac{1}{2}$ for $\frac{3}{4}$ of a year, B $\frac{2}{5}$ for $\frac{1}{2}$ a year, and C the remainder for one year. How much is each one's share of the entire stock at the end of the year?

9. Three men engage in merchandising. A's money was in 10 months, for which he received $456 of the profits; B's was in 8 months, for which he received $343.20 of the profits; and C's was in 12 months, for which he received $750 of the profits. Their whole capital invested was $14345; how much was the capital of each? *Ans.* A's, $4332; B's, $4075.50; C's, $5937.50.

10. Three men take an interest in a coal mine. B invests his capital for 4 months, and claims $\frac{1}{10}$ of the profits; C's capital is in 8 months; and D invests $6000 for 6 months, and claims $\frac{2}{5}$ of the profits; how much did B and C put in?

11. A, B, and C engage in manufacturing shoes. A puts in $1920 for 6 months; B, a sum not specified for 12 months; and C, $1280 for a time not specified. A received $2400 for his stock and profits, B $4800 for his, and C $2080 for his. Required, B's stock, and C's time?

ALLIGATION.

631. **Alligation** treats of mixing or compounding two or more ingredients of different values or qualities.

632. The **Mean Price** or **Quality** is the average price or quality of the ingredients, or the price or quality of a unit of the mixture.

CASE I.

633. To find the mean price or quality of a mixture, when the quantity and price of the several ingredients are given.

NOTE.—The process of finding the mean or average price of several ingredients is called *Alligation Medial.*

1. A produce dealer mixed together 84 bushels of oats worth \$.30 a bushel, 60 bushels of oats worth \$.38 a bushel, and 56 bushels of oats worth \$.40 a bushel; required, the mean price.

OPERATION.

$$\begin{array}{rrr} \$.30 \times & 84 = & \$25.20 \\ .38 \times & 60 = & 22.80 \\ .40 \times & 56 = & 22.40 \\ \hline & 200 &)\ \$70.40 \\ \hline & & \$.3520, \ \textit{Ans.} \end{array}$$

ANALYSIS. The worth of 84 bushels @ \$.30 is \$25.20, of 60 bushels @ \$.38 is \$22.80, and of 56 bushels @ \$.40 is \$22.40; and we have in the whole compound $84 + 60 + 56 = 200$ bushels, worth $\$25.20 + \$22.80 + \$22.40 = \$70.40$. One bushel of the mixture is therefore worth $\$70.40 \div 200 = \$.352$. Hence the following

RULE. *Find the entire cost or value of the ingredients, and divide it by the sum of the simples.*

EXAMPLES FOR PRACTICE.

1. A grocer mixed 4 lb. of tea at \$.60 with 3 lb. at \$.70, 1 lb. at \$1.10, and 2 lb. at \$1.20; how much is 1 lb. of the mixture worth? *Ans.* \$.80.

2. A dealer in liquors would mix 14 gal. of water with 12 gal. of wine at \$.75, 24 gal. at \$.90, and 16 gal. at \$1.10; how much is a gallon of the mixture worth? *Ans.* $\$.73\frac{1}{33}$.

3. If 3 lb. 6 oz. of gold 23 carats fine be compounded with 4 lb. 8 oz. 21 carats, 3 lb. 9 oz. 20 carats, and 2 lb. 2 oz. of alloy, what is the fineness of the composition? *Ans.* 18 carats.

4. A grain dealer mixes 15 bu. of wheat at \$1.20 with 5 bu. at \$1.10, 5 bu. at \$.90, and 10 bu. at \$.70; what will be his gain per bushel if he sell the compound at \$1.25.

5. A merchant sold 17 lb. of sugar at 5 cts. a pound, 51 lb. at 8 cts., 68 lb. at 10 cts., 17 lb. at 12 cts., and thereby gained on the whole $33\frac{1}{3}$ per cent.; how much was the average cost per pound?

6. A drover bought 42 sheep at \$2.70 per head, 48 at \$2.85, and 65 at \$3.24; at what average price per head must he sell them to gain 20 per cent.? *Ans.* \$3.567$\frac{15}{31}$.

7. A surveyor took 10 sets of observations with an instrument, for the measurement of an angle, with the following results: 1st, 36° 17′ 25.4″; 2d, 36° 17′ 24.5″; 3d, 36° 17′ 27.8″; 4th, 36° 17′ 26.9″; 5th, 36° 17′ 25.4″; 6th, 36° 17′ 24.7″; 7th, 36° 17′ 24.2″; 8th, 36° 17′ 26.3″; 9th, 36° 17′ 25.8″; 10th, 36° 17′ 26.7″. What is the average of these measurements? *Ans.* 36° 17′ 25.77″.

8. Three trials were made with chronometers to determine the difference of time between two places; the first trial gave 37 min. 54.16 sec., the second 37 min. 55.56 sec., and the third 37 min. 54.82 sec. Owing to the favorable conditions of the third trial, it is entitled to twice the degree of reliance to be placed upon either of the others; what should be taken as the difference of longitude between the two places, according to these observations? *Ans.* 9° 28′ 42.6″.

CASE II.

634. To find the proportional quantity to be used of each ingredient, when the mean price and the prices of the several simples are given.

NOTE.—The process of finding the quantities to be used in any required mixture is commonly called *Alligation Alternate.*

1. A farmer would mix oats worth 3 shillings a bushel with peas worth 8 shillings a bushel, to make a compound worth 5 shillings a bushel; what quantities of each may he take?

OPERATION.

$$5\left\{\begin{array}{c|c|c} 3 & \frac{1}{2} & 3 \\ 8 & \frac{1}{3} & 2 \end{array}\right\} Ans.$$

ANALYSIS. If a mixture, in any proportions, of oats worth 3 shillings a bushel and peas worth 8 shillings, be priced at 5 shillings, there will be a gain on the oats, the ingredient worth *less* than the mean price, and a loss on the peas, the ingredient worth *more* than the mean price; and if we take such quantities of each that the gain and loss shall each be 1 shilling, the *unit of value*, the result will be the required mixture. By selling 1 bushel of oats worth 3 shillings for 5 shillings, there will be a gain of $5 - 3 = 2$ shillings, and to gain 1 shilling would require $\frac{1}{2}$ of a bushel; hence we place $\frac{1}{2}$ opposite the 3. By selling 1 bushel of peas worth 8 shillings for 5 shillings, there will be a loss of $8 - 5 = 3$ shillings, and to lose 1 shilling will require $\frac{1}{3}$ of a bushel; hence we write $\frac{1}{3}$ opposite the 8. Therefore, $\frac{1}{2}$ bushel of oats to $\frac{1}{3}$ of a bushel of peas are the *proportional quantities* for the required mixture. It is evident that the gain and loss will be equal, if we take any number of times these proportional terms for the mixture. We may therefore multiply the fractions $\frac{1}{2}$ and $\frac{1}{3}$ by 6, the least common multiple of their denominators, and obtain the integers 3 and 2 for the proportional terms (**418**, III); that is, we may take, for the mixture, 3 bushels of oats to 2 bushels of peas.

2. What relative quantities of sugar at 7 cents, 8 cents, 11 cents, and 14 cents per pound, will produce a mixture worth 10 cents per pound?

OPERATION.

		1	2	3	4	5
10	7	$\frac{1}{3}$		4		4
	8		$\frac{1}{2}$		1	1
	11		1		2	2
	14	$\frac{1}{4}$		3		3

Or,

		1	2	3
10	7	$\frac{1}{3}$		1
	8		$\frac{1}{2}$	4
	11	1		3
	14		$\frac{1}{4}$	2

ANALYSIS. To preserve the equality of gains and losses, we must compare two prices or simples, one *greater* and one *less* than the mean rate, and treat each pair or couplet as a separate example. Thus, comparing the simples whose prices are 7 cents and 14 cents, we find that, to gain 1 cent, $\frac{1}{3}$ of a pound at 7 cents must be taken, and, to lose 1 cent, $\frac{1}{4}$ of a pound at 14 cents must be taken; and comparing the simples the prices of which are 8 cents and 11 cents, we find that $\frac{1}{2}$ pound at 8 cents must be taken to gain 1 cent, and 1 pound at 11 cents must be taken to lose 1 cent. These proportional terms are written in columns 1 and 2 We now reduce these couplets separately to integers, as in the last example, writing the results in columns 3 and 4; and arranging all the terms in column 5, we have 4, 1, 2, and 3 for the proportional quantities required. If we compare the prices 7 and 11 for the first couplet, and the prices 8 and 14 for the second couplet, as in the second operation, we shall obtain 1, 4, 3, and 2 for the proportional terms.

It will be seen that in comparing the simples of any couplet, one of which is greater and the other less than the mean rate, the proportional number finally obtained for either term is the difference between the mean rate and the other term. Thus, in comparing 7 and 14, the proportional number corresponding to the former simple is 4, which is the difference between 14 and the mean rate 10; and the proportional number corresponding to the latter simple is 3, which is the difference between 7 and the mean rate. The same is true of every other couplet. Hence, when the simples and the mean rate are integers, the intermediate steps taken to obtain the final proportional numbers as in columns 1, 2, 3, and 4, may be omitted, and the same results readily found by taking the difference between each simple and the mean rate, and placing it opposite the one with which it is compared.

From these examples and analyses we derive the following

RULE. I. *Write the prices or qualities of the several ingredients in a column, and the mean price or quality at the left.*

II. *Consider any two prices, one of which is less and the other greater than the mean rate, as forming a couplet; find the difference between each of these prices and the mean rate, and write the reciprocal of each difference opposite the given price in the couplet, as one of the proportional terms. In like manner form the couplets, till all the prices have been employed, writing each pair of proportional terms in a separate column.*

III. *If the proportional terms thus obtained are fractional, multiply each pair by the least common multiple of their denominators, and carry these integral products to a single column, observing to add any two or more that stand in the same horizontal line; the final results will be the proportional quantities required.*

NOTES.—1. If the numbers in any couplet or column have a common factor, it may be rejected.

2. We may also multiply the numbers in any couplet or column by any multiplier we choose, without affecting the equality of the gains and losses, and thus obtain an indefinite number of results, any one of which being taken will give a correct final result.

EXAMPLES FOR PRACTICE.

1. What quantities of flour worth $5\frac{1}{2}$, $6, and $7\frac{3}{4}$ per barrel, must be sold. to realize an average price of $6\frac{1}{4}$ per barrel?

OPERATION.

$6\frac{1}{4}$	$5\frac{1}{2}$	$\frac{4}{3}$		4		4
	6		4		12	12
	$7\frac{3}{4}$	$\frac{2}{3}$	$\frac{2}{3}$	2	2	4

ANALYSIS. Comparing the first price with the third, we obtain the couplet $\frac{4}{3}$ to $\frac{2}{3}$; and comparing the second price with the third, we obtain the couplet 4 to $\frac{2}{3}$. Reducing these proportional terms to integers, we find that we may take 4 barrels of the first kind with 2 of the third, and 12 of the second kind with 2 of the third; and these two combinations taken together give 4 of the first kind, 12 of the second, and 4 of the third.

2. How much sugar worth 5 cts., 7 cts., 12 cts., and 13 cts. per pound, will form a mixture worth 10 cts. per pound?

Ans. 3 lb. of each of the first and third kinds, 2 lb. of the second, and 5 lb. of the fourth.

3. How can wine worth $.60 $.90 and $1.15 per gallon be mixed with water so as to form a mixture worth $.75 a gallon?

Ans. By taking 3 gal. of each of the first two kinds of wine, 15 gal. of the third, and 8 gal. of water.

4. A farmer has 3 pieces of land worth $40, $60, and $80 an acre respectively. How many acres must he sell from the different tracts, to realize an average price of $62.50 an acre?

5. How much wine worth $.60, $.50, $.42, $.38, and $.30 per pint, will make a mixture worth $.45 a pint?

6. What relative quantities of silver $\frac{3}{4}$ pure, $\frac{5}{6}$ pure, and $\frac{9}{10}$ pure, will make a mixture $\frac{7}{8}$ pure?

Ans. 3 lb. $\frac{3}{4}$ pure, 3 lb. $\frac{5}{6}$ pure, and 20 lb. $\frac{9}{10}$ pure.

CASE III.

635. When two or more of the quantities are required to be in a certain proportion.

1. A farmer having oats worth $.30, corn worth $.60, and wheat

worth \$1.10 per bushel, desires to form a mixture worth \$.50 per bushel, which shall contain equal parts of corn and wheat; in what proportion shall the ingredients be taken?

OPERATION.

		1	2	3	4	5	6
50	30	$\frac{1}{20}$	$\frac{1}{20}$	1	3	6	7
	60	$\frac{1}{10}$		2			2
	110		$\frac{1}{60}$		1	2	2

ANALYSIS. We first obtain the proportional terms in columns 3 and 4, by Case II. Now, it is evident that the loss and gain will be equal if we take each couplet, or any multiple of each, alone; or both couplets, or any multiples of both, together. Multiplying the terms in column 4 by 2, we obtain the terms in column 5; and adding the terms in columns 3 and 5, we obtain the terms in column 6; that is, the farmer takes 7 bushels of oats to 2 of corn and 2 of wheat, which is the required proportion. Hence the following

RULE. I. *Compare the given prices, and obtain the proportional terms by couplets, as in Case II.*

II. *Reduce the couplets to higher or lower terms, as may be required; then select the columns at pleasure, and combine them by adding the terms in the same horizontal line, till a set of proportional terms is obtained, answering the required conditions.*

EXAMPLES FOR PRACTICE.

1. A grocer has four kinds of molasses, worth \$.25, \$.50, \$.62, and \$.70 per gallon, respectively; in what proportions may he mix the four kinds, to obtain a compound worth \$.58 per gallon, using equal parts of the first two kinds? *Ans.* 4, 4, 8 and 11.

2. In what proportions may we take sugars at 7 cts., 8 cts., 13 cts., and 15 cts., to form a compound worth 10 cts. per pound, using equal parts of the first three kinds? *Ans.* 5, 5, 5 and 2.

3. A miller has oats at 30 cts., corn at 50 cts., and wheat at 100 cts. per bushel. He desires to form two mixtures, each worth 70 cts. per bushel. In the first he would have equal parts of oats and corn, and in the second, equal parts of corn and wheat; what must be the proportional terms for each mixture?

Ans. For the first mixture, 1, 1 and 2.
For the second mixture, 1, 4 and 4.

CASE IV.

636. When the quantity of one of the simples is limited.

1. A miller has oats worth \$.28, corn worth \$.44, and barley worth \$.90 per bushel. He wishes to form a mixture worth \$.58 per bushel, and containing 100 bushels of corn. How many bushels of oats and barley may he take?

OPERATION.

58	28	$\frac{1}{30}$		7		7	140
	44		$\frac{1}{14}$		5	5	100
	93	$\frac{1}{35}$	$\frac{1}{35}$	6	2	8	160

ANALYSIS. By Case II, we find the proportional quantities to be 7 bushels of oats to 5 of corn and 8 of barley. But as 100 bushels of corn, instead of 5, are required, we must take $\frac{100}{5} = 20$ times each of the other ingredients, in order that the gain and loss may be equal; and we shall therefore have $7 \times 20 = 140$ bushels of oats, and $8 \times 20 = 160$ bushels of barley. Hence the following

RULE. *Find the proportional quantities by Case II or Case III. Divide the given quantity by the proportional quantity of this ingredient, and multiply each of the other proportional quantities by the quotient thus obtained.*

EXAMPLES FOR PRACTICE.

1. A dairyman bought 10 cows at \$20 a head; how many must he buy at \$16, \$18, and \$24 a head, so that the whole may cost him an average price of \$22 a head?

Ans. 10 at \$16, 10 at \$18, and 60 at \$24.

2. Bought 12 yards of cloth for \$15; how many yards must I buy at \$$1\frac{3}{4}$, and \$$\frac{3}{4}$ a yard, that the average price of the whole may be \$$1\frac{1}{5}$? *Ans.* 12 yards at \$$1\frac{3}{4}$ and 16 yards at \$$\frac{3}{4}$.

3. How much water will dilute 9 gal. 2 qt. 1 pt. of alcohol 96 per cent. strong to 84 per cent.? *Ans.* 1 gal. 1 qt. 1 pt.

4. A grocer mixed teas worth \$.30, \$.55, and \$.70 per pound respectively, forming a mixture worth \$.45 per pound, having equal parts of the first two kinds, and 12 lbs. of the third kind; how many pounds of each of the first two kinds did he take?

CASE V.

637. When the quantities of two or more of the ingredients are limited.

1. How many bushels of rye at $1.08, and of wheat at $1.44, must be mixed with 18 bushels of oats at $.48, 8 bushels of corn at $52, and 4 bushels of barley at $.85, that the mixture may be worth $.84 per bushel?

OPERATION.

$$\begin{array}{rcrcr} \$.48 & \times & 18 & = & \$\ 8.64 \\ .52 & \times & 8 & = & 4.16 \\ .85 & \times & \underline{4} & = & \underline{3.40} \\ & & 30 &) & \underline{\$16.20} \end{array}$$

$$\left.\begin{array}{r}\text{Mean price of the} \\ \text{given simples}\end{array}\right\} \$\ \ .54$$

$$84\left\{\begin{array}{r|c|c|c|c|c|r|} 54 & \frac{1}{30} & \frac{1}{30} & 4 & 2 & 6 & 30 \\ 108 & \frac{1}{24} & & 5 & & 5 & 25 \\ 144 & & \frac{1}{60} & & 1 & 1 & 5 \end{array}\right.$$

ANALYSIS. Of the given quantities there are 18 + 8 + 4 = 30 bushels, whose mean or average price we find by Case I to be $.54. We are therefore required to mix 30 bushels of grain worth $.54 per bushel, with rye at $1.08, and wheat at $1.44, to make a compound worth $.84 per bushel. Proceeding as in Case IV, we find there will be required 25 bushels of rye, and 5 bushels of wheat. Hence the following

RULE. *Consider those ingredients whose quantities and prices are given as forming a mixture, and find their mean price by Case I; then consider this mixture as a single ingredient whose quantity and price are known, and find the quantities of the other ingredients by Case IV.*

EXAMPLES FOR PRACTICE.

1. A gentleman bought 7 yards of cloth @ $2.20, and 7 yards @ $2; how much must he buy @ $1.60, and @ $1.75 that the average price of the whole may be $1.80?

2. How much wine, at $1.75 a gallon, must be added to 60 gallons at $1.14, and 30 gallons at $1.26 a gallon, so that the mixture may be worth $1.57 a gallon? *Ans.* 195 gallons.

3. A farmer has 40 bushels of wheat worth $2 a bushel, and 70 bushels of corn worth $½ a bushel. How many oats worth $¼ a bushel must he mix with the wheat and corn, to make the mixture worth $1 a bushel? *Ans.* 6⅔ bushels.

CASE VI.

638. When the quantity of the whole compound is limited.

1. A tradesman has three kinds of tea rated at \$.30, \$.45, and \$.60 per pound, respectively; what quantities of each should he take to form a mixture of 72 pounds, worth \$.40 per pound?

OPERATION.

		1	2	3	4	5	6
40	30	$\frac{1}{10}$	$\frac{1}{10}$	2	1	3	36
	45		$\frac{1}{5}$		2	2	24
	60	$\frac{1}{20}$		1		1	12
						6	72

ANALYSIS. By Case II, we find the proportional quantities to form the mixture to be 3 lb. at \$.30, 2 lb. at \$.45, and 1 lb. at \$.60. Adding these proportional quantities, we find that they would form a mixture of 6 pounds. And since the required mixture is $\frac{72}{6} = 12$ times 6 pounds, we multiply each of the proportional terms by 12, and obtain for the required quantities, 36 lb. at \$.30, 24 lb. at \$.45, and 12 lb. at \$.60. Hence the following

RULE. *Find the proportional numbers as in Case II or Case III. Divide the given quantity by the sum of the proportional quantities, and multiply each of the proportional quantities by the quotient thus obtained.*

EXAMPLES FOR PRACTICE.

1. A grocer has coffee worth 8 cts., 16 cts., and 24 cts. per pound respectively; how much of each kind must he use, to fill a cask holding 240 lb, that shall be worth 20 cts. a pound?

Ans. 40 lb. at 8 cts., 40 lb. at 16 cts., and 160 lb. at 24 cts.

2. A man bought calves, sheep, and lambs, 154 in all, for \$154. He paid \$3½ for each calf, \$1⅓ for each sheep, and \$½ for each lamb; how many did he buy of each kind?

Ans. 14 calves, 42 sheep, and 98 lambs.

3. A man paid \$165 to 55 laborers, consisting of men, women, and boys; to the men he paid \$5 a week, to the women \$1 a week, and to the boys \$½ a week; how many were there of each?

Ans. 30 men, 5 women, and 20 boys.

INVOLUTION.

639. A **Power** is the product arising from multiplying a number by itself, or repeating it any number of times as a factor.

640. **Involution** is the process of raising a number to a given power.

641. The **Square** of a number is its second power.

642. The **Cube** of a number is its third power.

643. In the process of involution, we observe,

I. That the exponent of any power is equal to the number of times the root has been taken as a factor in continued multiplication. Hence,

II. The product of any two or more powers of the same number is the power denoted by the sum of their exponents, and

III. If any power of a number be raised to any given power, the result will be that power of the number denoted by the product of the exponents.

1. What is the 5th power of 6?

OPERATION.

$$6 \times 6 \times 6 \times 6 \times 6 = 7776, \textit{ Ans.}$$

Or,

$$6 \times 6 = 6^2 = 36$$
$$36 \times 6 = 6^3 = 216$$
$$6^3 \times 6^2 = 6^5 = 216 \times 36 = 7776, \textit{ Ans.}$$

ANALYSIS. We multiply 6 by itself, and this product by 6, and so on, until 6 has been taken 5 times in continued multiplication; the final product, 7776, is the power required, (I). Or, we may first form the 2d and 3d powers; then the product of these two powers will be the 5th power required, (II).

2. What is the 6th power of 12?

OPERATION.

$$12^2 = 144$$
$$144^3 = 2985984, \textit{ Ans.}$$

ANALYSIS. We find the cube of the second power, which must be the 6th power, (III).

644. Hence for the involution of numbers we have the following

RULE. I. *Multiply the given number by itself in continued multiplication, till it has been taken as many times as a factor as there are units in the exponent of the required power.* Or,

II. *Multiply together two or more powers of the given number, the sum of whose exponents is equal to the exponent of the required power.* Or,

III *Raise some power of the given number to such a power that the product of the two exponents shall be equal to the exponent of the required power.*

NOTES. — 1. A fraction is involved to any power by involving each of its terms separately to the required power.

2. Mixed numbers should be reduced to improper fractions before involution.

3. When the number to be involved is a decimal, contracted multiplication may be applied with great advantage.

EXAMPLES FOR PRACTICE.

1. What is the square of 79? *Ans.* 6241.
2. What is the cube of 25.4? *Ans.* 16387.064.
3. What is the square of 1450?
4. Raise $16\frac{4}{5}$ to the 4th power. *Ans.* $79659\frac{261}{625}$.
5. Raise 2 to the 20th power. *Ans.* 1048576.
6. Raise .4378565 to the 8th power, reserving 5 decimals. *Ans.* .00135 +
7. Raise 1.052578 to the 6th power, reserving 4 decimals. *Ans.* 1.3600 ±.
8. Involve .029 to the 5th power? *Ans.* .000000020511149.

Find the value of each of the following expressions:

9. 4.367^4. *Ans.* 363.691178934721.
10 $(\frac{7}{8})^3$. *Ans.* $\frac{343}{512}$.
11. $(2\frac{3}{4})^5$. *Ans.* $157\frac{283}{1024}$.
12 $4.6^3 \times 25^3$ *Ans.* 1520875.
13. $(6\frac{3}{4})^4 - 7.25^2$. 14. $(8\frac{1}{2})^3 \times 2.5^2$
15. $\frac{7}{8}$ of $(\frac{4}{5})^3$ of $(3\frac{4}{7})^2$. *Ans.* $5\frac{5}{7}$.

NOTE.— Cancel like powers of the same factor.

16. $7^8 \div 3.08$.
17 $(4^3 \times 5^6 \times 12^3) \div (4^2 \times 10^4 \times 3^2)$. *Ans.* 1200

EVOLUTION.

645. A **Root** is a factor repeated to produce a power; thus, in the expression $7 \times 7 \times 7 = 343$, 7 is the root from which the power, 343, is produced.

646. **Evolution** is the process of extracting the root of a number considered as a power; it is the reverse of Involution.

Any number whatever may be considered a power whose root is to be extracted.

647. A **Rational Root** is a root that can be exactly obtained.

648. A **Surd** is an indicated root that can not be exactly obtained.

649. The **Radical Sign** is the character, $\sqrt{}$, which, placed before a number, indicates that its root is to be extracted.

650. The **Index** of the root is the figure placed above the radical sign, to denote what root is to be taken. When no index is written, the index, 2, is always understood.

651. The names of roots are derived from the corresponding powers, and are denoted by the indices of the radical sign. Thus, $\sqrt{100}$ denotes the *square root* of 100; $\sqrt[3]{100}$ denotes the *cube root* of 100; $\sqrt[4]{100}$ denotes the *fourth root* of 100; etc.

652. Evolution is sometimes denoted by a fractional exponent, the name of the root to be extracted being indicated by the denominator. Thus, the square root of 10 may be written $10^{\frac{1}{2}}$; the cube root of 10, $10^{\frac{1}{3}}$, etc.

653. Fractional exponents are also used to denote both involution and evolution in the same expression, the numerator indicating the power to which the given number is to be raised, and the denominator the root of the power which is to be taken; thus, $7^{\frac{2}{3}}$ denotes the cube root of the second power of 7, and is the same as $\sqrt[3]{7^2}$; so also $7^{\frac{5}{2}} = \sqrt{7^5}$.

654. In extracting any root of a number, any figure or figures may be regarded as tens of the next inferior order. Thus, in 2546, the 2 may be considered as tens of the 3d order, the 25 as tens of the second order, or the 254 as tens of the first order.

SQUARE ROOT.

655. The **Square Root** of a number is one of the two equal factors that produce the number. Thus, the square root of 64 is 8, for $8 \times 8 = 64$.

To derive the method of extracting the square root of a number, it is necessary to determine

1st. The relative number of places in a number and its square root.

2d. The relations of the figures of the root to the periods of the number.

3d. The law by which the parts of a number are combined in the formation of its square; and

4th. The factors of the combinations.

656. The relative number of places in a given number and its square root is shown in the following illustrations.

Roots.	Squares.	Roots.	Squares.
1	1	1	1
9	81	10	1,00
99	98,01	100	1,00.00
999	99,80,01	1000	1,00,00,00

From these examples we perceive

1st. That a root consisting of 1 place may have 1 or 2 places in the square.

2d. That in all cases the addition of 1 place to the root adds 2 places to the square. Hence,

I. *If we point off a number into two-figure periods, commencing at the right hand, the number of full periods and the left hand full or partial period will indicate the number of places in the square root.*

To ascertain the relations of the several figures of the root to the periods of the number, observe that if any number, as 2345, be decomposed at pleasure, the squares of the left hand parts will be related in local value as follows:

$$2000^2 = 4\ 00\ 00\ 00$$
$$2300^2 = 5\ 29\ 00\ 00$$
$$2340^2 = 5\ 47\ 56\ 00$$
$$2345^2 = 5\ 49\ 90\ 25: \text{ Hence,}$$

II. *The square of the first figure of the root is contained wholly in the first period of the power; the square of the first two figures* *of the root is contained wholly in the first two periods of the power; and so on.*

NOTE.—The periods and figures of the root are counted from the left hand.

The combinations in the formation of a square may be shown as follows:

If we take any number consisting of two figures, as 43, and decompose it into two parts, 40 + 3, then the square of the number may be formed by multiplying both parts by each part separately: thus,

$$\begin{array}{rrrrr}
 & & 40 & + & 3 \\
 & & 40 & + & 3 \\
\hline
 & & 120 & + & 9 \\
1600 & + & 120 & & \\
\hline
\end{array}$$

$$43^2 = 1600 + 240 + 9 = 1849.$$

Of these combinations, we observe that the first, 1600, is the square of 40; the second, 240, is twice 40 multiplied by 3; and the third, 9, is the square of 3. Hence,

III. *The square of a number composed of tens and units is equal to the square of the tens, plus twice the tens multiplied by the units, plus the square of the units.*

By observing the manner in which the square is formed, we perceive that the unit figure must always be contained as a factor in both the second and third parts; these parts taken together, may therefore be factored, thus, $240 + 9 = (80 + 3) \times 3$. Hence,

IV. *If the square of the tens be subtracted from the entire square, the remainder will be equal to twice the tens plus the units multiplied by the units.*

1. What is the square root of 5405778576?

OPERATION.

	5405778576 (73524
	49
143	505
	429
1465	7677
	7325
14702	35285
	29404
147044	588176
	588176

ANALYSIS. Pointing off the given number into periods of two figures each, the 5 periods show that there will be 5 figures in the root, (I). Since the square of the first figure of the root is always contained wholly in the first period of the power, (II), we seek for the greatest square in the first period, 54, which we find by trial to be 49, and we place its root, 7, as the first figure of the required root, and regard it as *tens* of the next inferior order, (II). We now subtract 49, the square of the first figure of the root, from the first period, 54, and bringing down the next period, obtain 505 for a remainder. And since the square of the first two figures of the root is contained wholly in the first two periods of the power, (II), the remainder, 505, must contain at least *twice the first figure* (*tens*) *plus the second figure* (*units*), *multiplied by the second figure*, (IV). Now if we could divide this remainder by *twice the first figure plus the second*, which is one of the factors, the quotient would be the *second figure*, or the other factor. But since we have not yet obtained the second figure, the complete divisor can not now be employed; and we therefore write twice the first figure, or 14, at the left of 505 for a *trial* divisor, regarding it as *tens*. Dividing the dividend, exclusive of the right hand figure, by 14, we obtain 3 for the second, or trial figure of the root, which we annex to the trial divisor, 14, making 143, the complete divisor. Multiplying the complete divisor by the trial figure 3, and subtracting the product from the dividend, we have 76 for a remainder. We have now taken the square of the first two figures of the root from the first two periods; and since the square of the first three figures of the root is contained wholly in the first three periods, (II) we bring down the third period, 77, to the remainder, 76, and obtain for a new dividend 7677, which must contain at least *twice the two figures already found plus the third, multiplied by the third*, (IV). Therefore to obtain the third figure, we must take for a new trial divisor twice the two figures, 73, considered as tens of the next inferior order, which we obtain in the operation by doubling the last figure of the last complete divisor, 143, making 146. Dividing, we obtain 5 for the next figure of the root; then regarding 735 as tens of the next inferior order, we proceed as in the former steps, and thus continue till the entire root, 73524, is obtained.

657. From these principles and illustrations we derive the following

RULE. I. *Point off the given number into periods of two figures each, counting from units' place toward the left and right.*

II. *Find the greatest square number in the left hand period, and write its root for the first figure in the root; subtract the square number from the left hand period, and to the remainder bring down the next period for a dividend.*

III. *At the left of the dividend write twice the first figure of the root, for a trial divisor; divide the dividend, exclusive of its right hand figure, by the trial divisor, and write the quotient for a trial figure in the root.*

IV. *Annex the trial figure of the root to the trial divisor for a complete divisor; multiply the complete divisor by the trial figure in the root, subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

V. *Multiply the last figure of the last complete divisor by* 2 *and add the product to* 10 *times the previous divisor, for a new trial divisor, with which proceed as before.*

NOTES.—1. If at any time the product be greater than the dividend, diminish the trial figure of the root, and correct the erroneous work.

2. If a cipher occur in the root, annex a cipher to the trial divisor, and another period to the dividend, and proceed as before.

3. If there is a remainder after all the periods have been brought down, annex periods of ciphers, and continue the root to as many decimal places as are required.

4. The decimal points in the work may be omitted, care being taken to point off in the root according to the number of decimal periods used.

5. The square root of a common fraction may be obtained by extracting the square roots of the numerator and denominator separately, provided the terms are perfect squares; otherwise, the fraction may first be reduced to a decimal.

6. Mixed numbers may be reduced to the decimal form before extracting the root; or, if the denominator of the fraction is a perfect square, to an improper fraction.

7. The pupil will acquire greater facility, and secure greater accuracy, by keeping units of like order under each other, and each divisor opposite the corresponding dividend, as shown in the operation.

EXAMPLES FOR PRACTICE.

1. What is the square root of 315844? *Ans.* 562.
2. What is the square root of 152399025? *Ans.* 12345.
3. What is the square root of 56280004? Of 597?
4. What is the square root of 10795.21? *Ans.* 103.9.
5. What is the square root of $58.1406\frac{1}{4}$? *Ans.* $7.62\frac{1}{2}$.

Find the values of the following expressions:

6. $\sqrt{.0000316969}$. *Ans.* .00563.
7. $\sqrt{3858.0769440964}$. *Ans.* 62.11342.
8. $\sqrt{\frac{5}{9}}$. *Ans.* .745355+.
9. $\sqrt{99225 - 63504}$. 10. $\sqrt{.126736} - \sqrt{.045369}$.
11. $\sqrt{\frac{169}{196}} \times \sqrt{\frac{7056}{9216}}$. *Ans.* $\frac{13}{16}$.
12. $\sqrt{81^2 \times 625^2 \times 2^4}$. *Ans.* 202500.

CONTRACTED METHOD.

658. 1. Find the square root of 8, correct to 6 decimal places.

OPERATION.

	2.828427+, *Ans.*
	8.000000
	4
48	400
	384
562	1600
	1124
5648	47600
	45184
5656	2416*
	2262
566	154
	113
57	41
	40

ANALYSIS. Extracting the square root in the usual way until we have obtained the 4 places, 2.828, the corresponding remainder is 2416, and the next trial divisor, with the cipher omitted, is 5656. We now omit to bring down a period of ciphers to the remainder, thus contracting the dividend 2 places; and we contract the divisor an equal number of places by omitting to annex the trial figure of the root, and regarding the right hand figure, 6, as a rejected or redundant figure. We now divide as in contracted division of decimals, (**226**), bringing down each divisor in its place, with one *redundant* figure increased by 1 when the *rejected* figure is 5 or more, and carrying the tens from the redundant figure in multiplication. We observe that the entire root, 2.828427+, *contains as many places as there are places in the periods used.* Hence the following

RULE. I. *If necessary, annex periods of ciphers to the given number, and assume as many figures as there are places required in the root; then proceed in the usual manner until all the assumed figures have been employed, omitting the remaining figures, if any.*

II. *Form the next trial divisor as usual, but omit to annex to it the trial figure of the root, reject one figure from the right to form each subsequent divisor, and in multiplying regard the right hand figure of each contracted divisor as redundant.*

NOTES.—1. If the rejected figure is 5 or more, increase the next left hand figure by 1.

2. Always take full periods, both of decimals and integers.

EXAMPLES FOR PRACTICE.

1. Find the square root of 32 correct to the seventh decimal place. *Ans.* 5.6568542+.

2. Find the square root of 12 correct to the seventh decimal place. *Ans.* 3.4641016+.

3. Find the square root of 3286.9835 correct to the fourth decimal place. *Ans.* 57.3322+.

4. Find the square root of $.\dot{5}$ correct to the sixth decimal place. *Ans.* .745355+.

5. Find the square root of $6\frac{4}{7}$ correct to the sixth decimal place. *Ans.* 2.563479+.

6. Find the square root of 1.06^5 correct to the sixth decimal place. *Ans.* 1.156817+.

7. Find the value of $1.0125^{\frac{3}{2}}$ correct to the fourth decimal place. *Ans.* 1.0188+.

8. Find the value of $1.023375^{\frac{1}{2}}$ correct to the sixth decimal place. *Ans.* 1.011620±.

CUBE ROOT.

659. The Cube Root of a number is one of the three equal factors that produce the number. Thus, the cube root of 343 is 7, since $7 \times 7 \times 7 = 343$.

To derive the method of extracting the cube root of a number, it is necessary to determine

1st. The relative number of places in a given number and its cube root.

2d. The relations of the figures of the root to the periods of the number.

3d. The law by which the parts of a number are combined in the formation of a cube; and

4th. The factors of these combinations.

660. The relative number of places in a given number and its cube, is shown in the following illustrations:

Roots.	Cubes.	Roots.	Cubes.
1	1	1	1
9	729	10	1,000
99	907,299	100	1,000,000
999	997,002,999	1000	1,000,000,000

From these examples, we perceive,

1st. That a root consisting of 1 place may have from 1 to 3 places in the cube.

2d. That in all cases the addition of 1 place to the root adds 3 places to the cube. Hence,

I. *If we point off a number into three-figure periods, commencing at the right hand, the number of full periods and the left hand full or partial period will indicate the number of places in the cube root.*

To ascertain the relations of the several figures of the root to the periods of the number, observe that if any number, as 5423, be decomposed, the cubes of the parts will be related in local value, as follows:

$$\begin{aligned} 5000^3 &= 125\ 000\ 000\ 000 \\ 5400^3 &= 157\ 464\ 000\ 000 \\ 5420^3 &= 159\ 220\ 088\ 000 \\ 5423^3 &= 159\ 484\ 621\ 967. \end{aligned}$$

Hence,

II. *The cube of the first figure of the root is contained wholly in the first period of the power; the cube of the first two figures of the root is contained wholly in the first two periods of the power; and so on*

To learn the combinations of tens and units in the formation of a cube, take any number consisting of two figures, as 54, and decompose it into two parts, 50+4; then having formed the square by **656**, III, multiply each part of this square by the units and tens of 54 separately, thus,

$$\begin{array}{rl} 54^2 = & 50^2 + 2 \times 50 \times 4 + 4^2 \\ & \qquad\qquad\qquad 50 + 4 \\ \hline & 50^2 \times 4 + 2 \times 50 \times 4^2 + 4^3 \\ & 50^3 + 2 \times 50^2 \times 4 + \quad 50 \times 4^2 \\ \hline 54^3 = & 50^3 + 3 \times 50^2 \times 4 + 3 \times 50 \times 4^2 + 4^3 = 156924 \end{array}$$

Of these combinations, the first is the cube of 50, the second is 3 times the square of 50 multiplied by 4, the third is 3 times 50 multiplied by the square of 4, and the fourth is the cube of 4. Hence,

III. *The cube of a number composed of tens and units is equal to the cube of the tens, plus three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.*

By observing the manner in which the cube is formed, we perceive that each of the last three parts contains the units as a factor; these parts, considered as one number, may therefore be separated into two factors, thus,

$$(3 \times 50^2 + 3 \times 50 \times 4 + 4^2) \times 4. \quad \text{Hence,}$$

IV. *If the cube of the tens be subtracted from the entire cube, the remainder will be composed of two factors, one of which will be three times the square of the tens plus three times the tens multiplied by the units plus the square of the units; and the other, the units.*

1. What is the cube root of 145780726447?

OPERATION.

145780726447 (5263, *Ans.*
125

I	II		
		7500	20780
152	304	7804	15608
		811200	5172726
1566	9396	820596	4923576
		83002800	249150447
15783	47349	83050149	249150447

ANALYSIS. Pointing off the given number into periods of 3 figures each, the four periods show that there will be four figures in the root, (I). Since the cube of the first figure of the root is contained wholly in the first period of the power, (II), we seek the greatest cube in the first period, 145, which we find by trial to be 125, and we place its root, 5, for the first figure of the required root, and regard it as *tens* of the next inferior order, (**654**). We now subtract 125, the cube of this figure, from the first period, 145, and bringing down the next period, obtain 20780 for a dividend. And since the cube of the first two figures of the root is contained wholly in the first two periods of the power, (II), the dividend, 20780, must contain at least the product of the two factors, one of which is *three times the square of the first figure* (tens), *plus three times the first figure multiplied by the second* (units), *plus the square of the second*; and the other, the *second figure* (IV). Now if we could divide this dividend by the first of these factors, the quotient would be the other factor, or the second figure of the root. But as the first factor is composed in part of the second figure, which we have not yet found, we can not now obtain the complete divisor; and we therefore write three times the square of the first figure, regarded as tens, or $50^2 \times 3 = 7500$, at the left of the dividend, for a trial divisor. Dividing the dividend by the trial divisor, we obtain 2 for the second, or trial figure of the root. To complete the divisor, we must add to the trial divisor, as a correction, three times the tens of the root already found multiplied by the units, plus the square of the units, (IV). But as $50 \times 3 \times 2 + 2^2 = (50 \times 3 + 2) \times 2$, we annex the second figure, 2, to three times the first figure, 5, and thus obtain $50 \times 3 + 2 = 152$, the first factor of the correction, which we write in the column marked I. Multiplying this result by the 2, we have 304, the correction, which we write in the column marked II. Adding the correction to the trial divisor, we obtain 7804, the complete divisor. Multiplying the complete divisor by the trial figure of the root, subtracting the product from the dividend, and bringing down the next period, we have 5172726 for a dividend.

We have now taken the cube of the first two figures of the root considered as tens of the next inferior order, from the first three periods of the number; and since the cube of the first three figures of the root is contained wholly in the first three periods of the power, (II), the dividend, 5172726 must contain at least the product of the two factors, one of which is *three times the square of the first two figures of the root* (regarded as tens of the next order) *plus three times the first two figures multiplied by the third, plus the square of the third;* and the other, *the third figure,* (IV). Therefore, to obtain the third figure, we must use for a trial divisor three times the square of the first two figures, 52, considered as tens. And we observe that the significant part of this new trial divisor may be obtained by adding the last complete divisor, the last correction, and the square of the last figure of the root, thus:

$$\begin{array}{rl}
7804 = & (50^2 \times 3) + (50 \times 3 \times 2) + 2^2 \\
304 = & \qquad\qquad\quad 50 \times 3 \times 2 \;\; + 2^2 \\
4 = & \qquad\qquad\qquad\qquad\qquad\quad 2^2 \\
\hline
8112 = & (50^2 \quad + \quad 100 \times 2 \quad + 2^2) \times 3 = 52^2 \times 3
\end{array}$$

This number is obtained in the operation without re-writing the parts, by adding the square of the second root figure mentally, and combining units of like order, thus: 4, 4, and 4 are 12, and we write the unit figure, 2, in the new trial divisor; then 1 to carry and 0 is 1; then 3 and 8 are 11, etc. Annexing two ciphers to the 8112, because 52 is regarded as tens of the next order, and dividing by this new trial divisor, 811200, we obtain 6, the third figure in the root. To complete the second trial divisor, after the manner of completing the first, we should annex the third figure of the root, 6, to three times the former figures, 52, for the first factor of the correction.

But as we have in column I three times 5 with the 2 annexed, or 152, we need only multiply the last figure, 2, by 3, and annex the third figure of the root, 6, which gives 1566, the first factor of the correction sought, or the second term in column I. Multiplying this number by the 6, we obtain 9396, the correction sought; adding the correction to the trial divisor, we have 820596, the complete divisor; multiplying the complete divisor by the 6, subtracting the product from the dividend, and bringing down the next period, we have 249150447 for a new dividend We may now regard the first three figures of the root, 526, as tens of the next inferior order, and proceed as before till the entire root, 5263, is extracted.

661. From these principles and illustrations we deduce the following

RULE. I. *Point off the given number into periods of three figures each, counting from units' place toward the left and right.*

II. *Find the greatest cube that does not exceed the left hand period, and write its root for the first figure in the required root; subtract the cube from the left hand period, and to the remainder bring down the next period for a dividend.*

III. *At the left of the dividend write three times the square of the first figure of the root, and annex two ciphers, for a trial divisor; divide the dividend by the trial divisor, and write the quotient for a trial figure in the root.*

IV. *Annex the trial figure to three times the former figure, and write the result in a column marked* I, *one line below the trial divisor, multiply this term by the trial figure, and write the product on the same line in a column marked* II; *add this term as a correction to the trial divisor, and the result will be the complete divisor.*

V. *Multiply the complete divisor by the trial figure; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

VI. *Add the square of the last figure of the root, the last term in column* II, *and the complete divisor together, and annex two ciphers, for a new trial divisor; with which obtain another trial figure in the root.*

VII. *Multiply the unit figure of the last term in column* I *by* 3, *and annex the trial figure of the root for the next term of column* I; *multiply this result by the trial figure of the root for the next term of column* II; *add this term to the trial divisor for a complete divisor, with which proceed as before.*

NOTES.—1. If at any time the product be greater than the dividend, diminish the trial figure of the root, and correct the erroneous work.

2. If a cipher occur in the root, annex two more ciphers to the trial divisor, and another period to the dividend; then proceed as before with column I, annexing both cipher and trial figure.

EXAMPLES FOR PRACTICE.

1. What is the cube root of 389017? *Ans.* 73.
2. What is the cube root of 44361864? *Ans.* 354.
3. What is the cube root of 10460353203? *Ans.* 2187.
4. What is the cube root of 98867482624? *Ans.* 4624.
5. What is the cube root of 30.625? *Ans.* 3.12866 +.
6. What is the cube root of $111\frac{1}{8}$? *Ans* 4.8076 +.
7. What is the cube root of .000148877? *Ans.* .053.

Find the values of the following expressions.

8. $\sqrt[3]{122615327232}$? *Ans.* 4968.
9. $\sqrt[3]{\sqrt[3]{134217728}}$? *Ans.* 8.
10. $\sqrt[3]{39304^2}$? *Ans.* 1156.
11. $\sqrt[3]{\frac{648}{3000}} \times \sqrt{\frac{1331}{3179}}$? *Ans.* $\frac{33}{85}$.
12. How much does the sum of the cube roots of 50 and 31 exceed the cube root of their sum? *Ans.* 2.4986 +.

CONTRACTED METHOD.

662. In applying contracted decimal division to the extraction of the cube root of numbers, we observe,

1st. For each new figure in the root, the terms in the operation extend to the right 3 places in the column of dividends, 2 places in the column of divisors, and 1 place in column I. Hence,

2d. If at any point in the operation we omit to bring down new periods in the dividend, we must shorten each succeeding divisor 1 place, and each succeeding term in column I, 2 places.

1. What is the cube root of 189, correct to 8 decimal places?

OPERATION.

			5.73879355±, *Ans.*
			189.000000
I	II		125
		7500	64 000
157	1099	8599	60 193
		974700	3 807000
1713	5139	979839	2 939517
		984987	867483*
1719	1375	986362	789090
		98774	78393
17	12	98786	69150
		9880	9243
			8892
		988	351
			296
		99	55
			50
		10	5
			5

ANALYSIS. We proceed by the usual method to extract the cube root of the given number until we have obtained the three figures, 5.73; the corresponding remainder is 867483, and the next trial divisor with the ciphers omitted is 984987. We now omit to bring down a period of ciphers, thus contracting the dividend 3 places; and we contract the divisor an equal number of places by omitting to annex the two ciphers, and regarding the right hand figure, 7, as a redundant figure. Then dividing, we obtain 8 for the next figure of the root. To complete the divisor, we obtain a correction, 1375, contracted 2 places by omitting to annex the trial figure of the root, 8, to the first factor, 1719, and regarding the right hand figure, 9, as redundant in multiplying. Adding the contraction to the contracted divisor, we have the complete divisor, 986362, the right hand figure being redundant. Multiplying by 8 and subtracting the product from the dividend, we have 78393 for a new dividend. Then to form the new trial divisor, we disregard the square of the root figure, 8, because this square consists of the same orders of units as the two rejected places in the divisor; and we simply add the correction, 1375, and the complete divisor, 986362, and rejecting 1 figure, thus obtain 98774, of which the right hand figure, 4, is redundant. Dividing, we obtain 7 for the next root figure. Rejecting 2 places from the last term in column I, we have 17 for the next contracted term in this column. We then obtain, by the manner shown in the former step, the correction 12, the complete divisor, 98786, the product, 69150, and the new dividend, 9243. We then obtain the new trial divisor, 9880; and as column I is *terminated* by rejecting the two places, 17, we continue the contracted division as in square root, and thus obtain the entire root, 5.73879355 ±, which is correct to the last decimal place, and *contains as many places as there are places in the periods used.* Hence the following

RULE. I. *If necessary, annex ciphers to the given number, and assume as many figures as there are places required in the root; then proceed by the usual method until all the assumed figures have been employed.*

II. *Form the next trial divisor as usual, but omit to annex the two ciphers, and reject one place in forming each subsequent trial divisor.*

III. *In completing the contracted divisors, omit at first to annex the trial figure of the root to the term in column* I, *and reject* 2 *places in forming each succeeding term in this column.*

IV. *In multiplying, regard the right hand figure of each contracted term, in column I and in the column of divisors, as redundant.*

NOTES.—1. After the contraction commences, the square of the last root figure is disregarded in forming the new trial divisors.

2. Employ only *full periods* in the number.

EXAMPLES FOR PRACTICE.

1. Find the cube root of 24, correct to 7 decimal places.
Ans. 2.8844992 ±.

2. Find the cube root of 12000.812161, correct to 9 decimal places. *Ans.* 22.894801334 ±.

3. Find the cube root of .171467, correct to 9 decimal places.
Ans. .555554730 ±.

4. Find the cube root of 2.42999 correct to 5 decimal places.
Ans. 1.34442±.

5. Find the cube root of 19.44, correct to 4 decimal places.
Ans. 2.6888 ±.

6. Find the value of $\sqrt[3]{\frac{5}{6}}$ to 6 places. *Ans.* .941035 ±.

7. Find the value of $\sqrt[3]{.\dot{5}7142\dot{8}}$ to 9 places.
Ans. .829826686 ±.

8. Find the value of $\sqrt[3]{1.08674325^2}$ to 7 places.

Ans. 1.057023 ±.

9. Find the value of $1.05^{\frac{5}{3}}$ to 7 places.

Ans. 1.084715 ±.

ROOTS OF ANY DEGREE.

663. Any root whatever may be extracted by means of the square and cube roots, as will be seen in the two cases which follow.

CASE I.

664. When the index of the required root contains no other factor than 2 or 3.

We have seen that if we raise any power of a given number to any required power, the result will be that power of the given number denoted by the product of the two indices, (**643**, III). Conversely, if we extract successively two or more roots of a given number, the result must be that root of the given number denoted by the product of the indices.

1. What is the 6th root of 2176782336?

OPERATION.

$$6 = 2 \times 3$$

$$\sqrt[2]{2176782336} = 46657$$

$$\sqrt[3]{46656} = 36, \textit{ Ans.}$$

Or,

$$\sqrt[3]{2176782336} = 1296$$

$$\sqrt[2]{1296} = 36, \textit{ Ans.}$$

ANALYSIS. The index of the required root is $6 = 2 \times 3$; we therefore extract the square root of the given number, and the cube root of this result, and obtain 36, which must be the 6th root required. Or, we first find the cube root of the given number, and then the square root of the result, as in the operation. Hence the following

RULE. *Separate the index of the required root into its prime factors, and extract successively the roots indicated by the several factors obtained; the final result will be the required root.*

EXAMPLES FOR PRACTICE.

1. What is the 6th root of 6321363049? *Ans.* 43.
2. What is the 4th root of 5636405776? *Ans.* 274.

3. What is the 8th root of 1099511627776? *Ans.* 32.
4. What is the 6th root of 25632972850442049? *Ans.* 543.
5. What is the 9th root of 1.577635? *Ans.* 1.051963+.

NOTE. — Extract the cube root of the cube root by the contracted method, carrying the root in each operation to 6 decimal places only.

6. What is the 12th root of 16.3939? *Ans.* 1.2624+.
7. What is the 18th root of 104.9617? *Ans.* 1.2950+.

CASE II.

665. When the index of the required root is prime, or contains any other factor than 2 or 3.

To extract any root of a number is to separate the number into as many equal factors as there are units in the index of the required root; and it will be found that if by any means we can separate a number into factors nearly equal to each other, the *average* of these factors, or their sum divided the number of factors, will be nearly equal to the root indicated by the number of factors.

1. What is the 7th root of 308?

OPERATION.

$\sqrt[6]{308} = 2.59+$
$\sqrt[8]{308} = 2.04+$
$2.59 + 2.04 = 4.63$
$4.63 \div 2 = 2.31$, assumed root.

$2.31^6 = 151.93$
$308 \div 151.93 = 2.0272+$
$2.31 \times 6 + 2.0272 = 15.8872$
$15.8872 \div 7 = 2.2696$, 1st approximation.

$2.2696^6 = 136.6748$
$308 \div 136.6748 = 2.253452+$
$2.2696 \times 6 + 2.253452 = 15.871052$
$15.871052 \div 7 = 2.267293$, 2d approx.

ANALYSIS. We first find by Case I, the 6th root, and also the 8th root of 308; and since the 7th root must be less than the former and greater than the latter, we take the average of the two, or one half of their sums, 2.31, and call it the *assumed root*. We next raise the assumed root, 2.31, to the 6th power, and divide the given number, 308, by the result, and obtain 2.0272+ for a quotient; we thus separate 308 into 7 factors, 6 of which are equal to 2.31, and the other is 2.0272. As these 7 factors are nearly equal to each other, the average of them all must be a near approximation to the 7th root. Multiplying the 2.31 by 6, adding the 2.0272 to the product, and dividing this result by 7, we find the average to be 2.2696, which is the first approximation to the required root. We next divide 308 by the 6th power of 2.2696, and obtain 2.253452+ for a quotient; and we thus separate the given number into 7 factors, 6 of which are each equal to 2.2696, and the other is 2.253452. Finding the average of these factors, as in the former steps, we have 2.267293, which is the 7th root of the given number, correct to 5 decimal places. Hence the following

RULE. I. *Find by trial some number nearly equal to the required root, and call this the assumed root.*

II. *Divide the given number by that power of the assumed root denoted by the index of the required root less* 1; *to this quotient add as many times the assumed root as there are units in the index of the required root less* 1, *and divide the amount by the index of the required root. The result will be the first approximate root required.*

III. *Take the last approximation for the assumed root, with which proceed as with the former, and thus continue till the required root is obtained to a sufficient degree of exactness.*

NOTES.—1. The involution and division in all cases will be much abridged by decimal contraction.

2. If the index of the required root contains the factors, 2 or 3, we may first extract the square or cube root as many times, successively, as these factors are found in the index, after which we must extract that root of the result which is denoted by the remaining factor of the index. Thus, if the 15th root were required, we should first find the cube root, then the 5th root of this result.

EXAMPLES FOR PRACTICE.

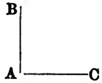
1. What is the 20th root of 617?

OPERATION.

$$20 = 2 \times 2 \times 5.$$
$$\sqrt[2]{617} = 24.839485+.$$
$$\sqrt[2]{24.839485} = 4.983923+.$$
$$\sqrt[5]{4.983923} = 1.378206+. \quad \textit{Ans.}$$

2. What is the 5th root of 120?
3. What is the 7th root of 1.95678?
4. What is the 10th root of 743044?
5. What is the 15th root of 15?
6 What is the 25th root of 100?
7. What is the 5th root of 5?

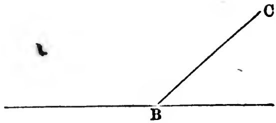
APPLICATIONS OF THE SQUARE AND CUBE ROOTS.



666. An **Angle** is the opening between two lines that meet each other.

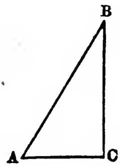
667. A **Right Angle** is an angle formed by two lines perpendicular to each other. Thus, B A C is a right angle.

668. If an angle is less than a right angle, it is *acute*; if greater than a right angle, it is *obtuse*. Thus, the angle on the right of the line C B is acute, and the angle on the left of C B is obtuse.



669. **Parallel Lines** are lines having the same direction, as A and B.

670. A **Triangle** is a figure having three sides and three angles, as A B C.



671. A **Right-Angled Triangle** is a triangle having one right angle, as at C.

672. The **Hypotenuse** is the side opposite the right angle, as A B.

673. The **Base** of a triangle is the side on which it is supposed to stand, as A C.

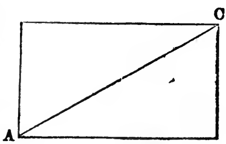
674. The **Altitude** of a triangle is the perpendicular distance from the base, or the base produced, to the angle opposite, as C B.

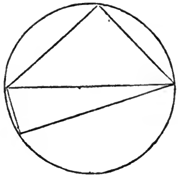
NOTE.—The altitude of a right-angled triangle is the side called the perpendicular.

675. A **Square** is a figure having four equal sides and four right angles.

676. A **Rectangle** or **Parallelogram** is a figure having four right angles, and its opposite sides equal.

677. A **Diagonal** is a line drawn through a figure, joining two opposite angles, as A C.



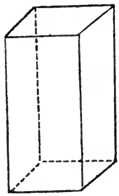


678. A **Circle** is a figure bounded by one uniform curved line.

679. The **Circumference** of a circle is the curved line bounding it.

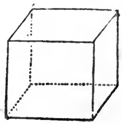
680. The **Diameter** of a circle is a straight line passing through the center, and terminating in the circumference.

681. A **Semi-Circle** is one half of a circle.

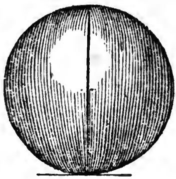


682. A **Prism** is a solid whose bases or ends are any similar, equal, and parallel plane figures, and whose sides are parallelograms.

683. A **Parallelopiped** is a solid bounded by six parallelograms, the opposite ones of which are parallel and equal to each other. Or, it is a prism whose base is a parallelogram.



684. A **Cube** is a solid bounded by six equal squares. The cube is sometimes called **a** *Right Prism.*



685. A **Sphere** or **Globe** is a solid bounded by a single curved surface, which in every part is equally distant from a point within called its center.

686. The **Diameter** of a sphere is a straight line passing through its center, and terminating at its surface.

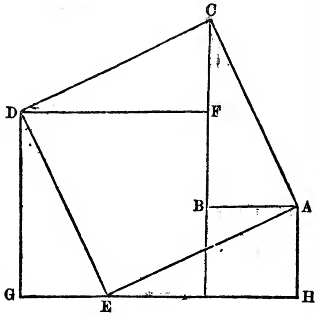
687. A **Hemisphere** is one half of a globe or sphere.

688. **Similar Figures** and **Similar Solids** are such as have their like dimensions proportional.

PROBLEM I.

689. To find either side of a right-angled triangle, the other two sides being given.

Let us take any right-angled triangle, as A B C, and form the square, A E D C, on the hypotenuse. Now take a portion, A B C, of this square, and move it as on a hinge at A, until the points B and C



are brought to the positions of H and E, respectively. Take also another portion, D F C, and move it as on a hinge at D, until the points F and C are brought to the positions of G and E, respectively. Then the figure formed by the parts thus moved and the remaining part will be composed of two new squares, one on A B, the base of the triangle, and one on D F, which is equal to the perpendicular of the triangle. Hence,

The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.

From this property we derive the following

RULE. I. To find the hypotenuse;—*Add the squares of the two sides, and extract the square root of the sum.*

II. To find either of the shorter sides;—*Subtract the square of the given side from the square of the hypotenuse, and extract the square root of the remainder.*

EXAMPLES FOR PRACTICE.

1. The top of a tower standing 22 feet from the shore of a river, is 75 feet above the water, and 256 feet in a straight line from the opposite shore; required the width of the river.

Ans. 222.76 ft.

2. Two ships set sail from the same port, and one sails due east 50 leagues, the other due north 84 leagues; how far are they apart?

3. A ladder 50 ft. long will reach a window 30 ft. from the ground on one side of the street, and without moving the foot, will reach a window 40 ft. high on the other side; what is the breadth of the street?

4. What is the distance through a cubical block, measured from one corner to the opposite diagonal corner, the side of the cube being 6 feet? *Ans.* 10.39 ft

PROBLEM II.

690. To find the side of a square equal in area to a given rectangle.

Note. — This case, arithmetically considered, requires us to find a *mean proportional* between two given numbers.

The product of the sides of the rectangle will be the area which the square is to contain; hence

Rule. *Multiply the sides of the rectangle together, and extract the square root of the product.*

EXAMPLES FOR PRACTICE.

1. There is a field whose length is 208 rods, and whose breadth is 13 rods; what is the length of the side of a square lot containing an equal area? *Ans.* 52 rods.

2. If it cost $312 to inclose a farm 216 rods long and 24 rods wide, how much less will it cost to inclose a square farm of equal area with the same kind of fence?

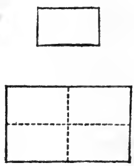
3. What is the mean proportional between 12 and 588? *Ans.* 84.

4. A and B traded together. A put in $540 for 480 days, and received $\frac{1}{3}$ of the gain; and the number of dollars which B put in was equal to the number of days it was employed in trade. What was B's capital? *Ans.* $720.

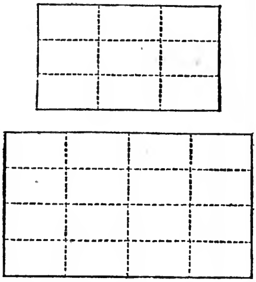
PROBLEM III.

691. To find the two sides of a rectangle, the area and the ratio of the sides being given.

Note.—This case, arithmetically considered, requires us to find two numbers whose *product* and *ratio* are given



If we multiply together the terms of the given ratio, the product will be the area of a rectangle similar in form to the rectangle whose sides are required. Now we perceive, by the accompanying figures, that multiplying both sides of any rectangle by 2, 3, 4, etc., multiplies the area by the



squares of these numbers, or 4, 9, 16, etc. If, therefore, we divide the given area by the rectangle of the terms proportional to the required sides, the quotient will be the *square* of that number which must be multiplied into these proportional terms to produce the required sides.

Hence the following

RULE. I. *Divide the given area by the product of the terms proportional to the sides, and extract the square root of the quotient.*

II. *Multiply the root thus obtained by each proportional term; the products will be the corresponding sides.*

EXAMPLES FOR PRACTICE.

1. The sides of a rectangle containing 432 square feet are as 4 to 3; required the length and breadth.

Ans. Length, 24 feet; breadth, 18 feet.

2. Separate 23 into two factors which shall be to each other as 2 to 3. *Ans.* 3.91578 +; 5.87367 +.

3. It is required to lay out 283 A. 2 R. 27 P. of land in the form of a rectangle whose length shall be 3 times the width; what will be the dimensions?

NOTE.— The proportional terms are 3 : 1.

Ans. 369 rods; 123 rods.

PROBLEM IV.

692. To find the radius, diameter, or circumference of a circle, the ratio of its area to a known circle being given.

All examples of this class relating to circles, may be solved by means of the following property:—

The areas of two circles are to each other as the squares of their radii, diameters, or circumferences.

NOTE.—This property of the circle is only a particular case of a more general principle, viz.: *That the areas of similar figures are to each other as the squares of their like dimensions.* This principle is rigidly demonstrated in Geometry, but cannot be easily proved here.

EXAMPLES FOR PRACTICE.

1. The radius of a circle containing 28.2744 sq. ft., is 6 ft.; what is the radius of a circle containing 175.7150 sq. ft.?

28.2744 : 175.7150 = 6^2 : ( ) = 225, square of radius required. Hence, $\sqrt{225} = 15$, *Ans.*

2. If it cost \$75 to inclose a circular pond containing a certain area, how much will it cost at the same rate to inclose another, containing 5 times the area of the first? *Ans.* \$167.70.

3. If a cistern 6 feet in diameter hold 80 barrels of water, what must be the diameter of a cistern of the same depth to hold 1200 barrels?

4. If a pipe 1.5 in. in diameter will fill a cistern in 5 h., what must be the diameter of a pipe that will fill the same cistern in 55 min. 6 sec.? *Ans.* 3.5 in.

PROBLEM V.

693. To find the side of a cube, the solid contents being given.

NOTE.—This case, arithmetically considered, requires us to separate a number into three equal factors.

The solid contents of a cube are found by cubing the length of one side; hence,

RULE. *Extract the cube root of the given contents.*

EXAMPLES FOR PRACTICE.

1. What must be the length of the side of a cubical bin that shall contain the same quantity as one that is 24 ft. long, 18 ft. wide, and 4 ft. deep? *Ans.* 12 ft.

2. What must be the length of the side of a cubical bin that will contain 150 bushels?

3. What must be the depth of a cubical cistern that will hold 200 bbl. of water?

4. How many sq. ft. in the surface of a cube whose solidity is 79507 cu. ft.? *Ans.* 11094.

PROBLEM VI.

694. To find the three dimensions of a parallelopiped, the solid contents and the ratio of the dimensions being given.

NOTE 1. — This case, arithmetically considered, requires us to separate a number into three factors, proportional to three given numbers.

The three dimensions will be like multiples of the proportional terms, (**691**); the product of the three dimensions, or the solid contents, will therefore contain the product of the three proportional terms, and the cube of the common ratio which the proportional terms respectively bear to the corresponding dimensions, and no other factor. Hence the

RULE. I. *Divide the given contents by the product of the terms proportional to the three dimensions, and extract the cube root of the quotient.*

II. *Multiply the root thus obtained by each proportional term; the products will be the corresponding sides.*

NOTE 2. — The dimensions are supposed to be taken in a direction perpendicular to the faces of a solid, and to each other.

EXAMPLES FOR PRACTICE.

1. A pile of bricks in the form of a parallelopiped contains 3000 cu. ft., and the length, breadth, and thickness, are to each other as 4, 3, and 2, respectively; what are the dimensions of the pile? *Ans.* 10, 15, and 20 ft.

2. Three numbers are to each other as 2, 5, and 7, and their continued product is 4480; required the numbers.
Ans. 8, 20, and 28.

3. Separate 100 into three factors which shall be to each other as 2, $2\frac{1}{2}$, and 3. *Ans.* 3.76414 +; 4.70518 +; 5.64622 —.

4. A person wishes to construct a bin that shall be of equal width and depth, and the length three times the width, and that shall contain 450 bushels of grain? what must be its dimensions?

PROMISCUOUS EXAMPLES.

1. There is a park containing an area of 10 A. 2 R. 20 P., and the breadth is equal to $\frac{3}{4}$ of the length. If two men start from one corner and travel at the rate of 3 miles per hour, one going by the walk around the park, and the other taking the diagonal path through the park, how much sooner will the latter reach the opposite corner than the former? *Ans.* 1 min. 29.3 sec.

2. What is the length of one side of a square piece of land containing 40 acres? *Ans.* 80 rd.

3. The ground situated between two parallel streets is laid out into equal rectangular lots whose front measure is 44 per cent. greater than the depth. Now, if the streets were 20 feet further apart, the ground could be laid out into square lots of the same area as the rectangular. What is the distance between the streets? *Ans.* 100 feet.

4. How much less will it cost to fence 40 acres of land in the form of a square, than in the form of a rectangle of which the breadth is $\frac{1}{4}$ the length, the price per rod being \$1.40? *Ans.* \$112.

5. If a cistern 6 feet in diameter holds 80 barrels of water, how much water will be contained in a cistern of the same depth and 18 feet in diameter?

6. What is the length of the side of a square which contains the same area as a rectangle $5\frac{1}{2}$ by 7 feet? *Ans.* 6 ft. 2.4 + in.

7. What is the length of the side of a square which can just be inclosed within a circle 42 inches in diameter? *Ans.* 29.7 — in.

8. If it costs \$75 to inclose a circular fish pond containing 3 A. 86 P., how much will it cost to inclose another containing 17 A. 110 P.? *Ans.* \$167.70.

NOTE.—It is proved in Geometry that all *similar* solids are to each other as the cubes of their like dimensions. Hence, any dimension may be found by proportion, when its ratio to the corresponding dimension of a known similar solid is given.

9. What is the length of the side of a cubical vessel that contains $\frac{1}{8}$ as much as one whose side is 6 ft.? *Ans.* 3 ft.

10. How many globes 4 in. in diameter are equal in volume to one 12 in. in diameter?

11. If an ox that weighs 900 lb. girt 6.5 ft., what is the weight of an ox that girts 8 ft.? *Ans.* 1677 lb. 14+ oz.

12. If a cable 3 in. in circumference supports a weight of 2500 lb., what must be the circumference of a cable that will support 4960 lb.?

13. If a stack of hay 4 feet high contain 4 tons, how high must a similar stack be to contain 20 tons?

SERIES.

695. A **Series** is a succession of numbers so related to each other, that each number in the succession may be formed in the same manner, from one or more preceding numbers. Thus, any number in the succession, 2, 5, 8, 11, 14, is formed by adding 3 to the preceding number. Hence, 2, 5, 8, 11, 14, is a series.

696. The **Law of a Series** is the constant relation existing between two or more terms of the series. Thus, in the series, 3, 7, 11, 15, we observe that each term after the first is greater than the preceding term by 4; this constant relation between the terms is the law of this series.

The law of a series, and the term or terms on which it depends being given, any number of terms of the series can be formed. Thus, let 64 be a term of a series whose law is, that each term is four times the preceding term. The term following 64 is 64×4, the next term 64×4^2, etc.; the term preceding 64 is $64 \div 4$. Hence the series, as far as formed, is 16, 64, 256, 1024.

697. A series is either *Ascending*, or *Descending*, according as each term is greater or less than the preceding term. Thus, 2, 6, 10, 14, is an ascending series; 32, 16, 8, 4, is a descending series.

698. An **Extreme** is either the first or last term of a series. Thus, in the series, 4, 7, 10, 13, the first extreme is 4, the last, 13.

699. A **Mean** is any term between the two extremes. Thus, in the series, 5, 10, 20, 40, 80, the means are 10, 20, and 40.

700. An **Arithmetical** or **Equidifferent Progression** is a series whose law of formation is a common difference. Thus, in the arithmetical progression, 3, 7, 11, 15, 19, each term is formed from the preceding by adding the common difference, 4.

701. An arithmetical progression is an ascending or descending series, according as each term is formed from the preceding term by adding or subtracting the common difference. Thus, the ascending series, 7, 10, 13, 16, etc., is an arithmetical progression in which the common difference, 3, is constantly added to form each succeeding term; and the descending series, 20, 17, 14, 11, 8, 5, 2, is an arithmetical progression in which the common difference is constantly subtracted, to form each succeeding term.

702. A **Geometrical Progression** is a series whose law of formation is a common multiplier. Thus, in the geometrical progression, 3, 6, 12, 24, 48, each term is formed by multiplying the preceding term by the common multiplier, 2.

703. A geometrical progression is an ascending or descending series, according as the common multiplier is a whole number or a fraction. Thus, the ascending series, 1, 2, 4, 8, 16, etc., is a geometrical progression in which the common multiplier is 2; and the descending series, 32, 16, 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, etc., is a geometrical progression in which the common multiplier is $\frac{1}{2}$.

704. The **Ratio** in a geometrical progression is the common multiplier.

705. In the solution of problems in Arithmetical or Geometrical progression, five parts or elements are concerned, viz:

In Arithmetical Progression —
1. The first term;
2. " last term;
3. " number of terms;
4. " common difference;
5. " sum of the series.

In Geometrical Progression —
1. The first term;
2. " last term;
3. " number of terms;
4. " ratio;
5. " sum of the series.

The conditions of a problem in progression may be such as to require any one of the five parts from any three of the four remaining parts; hence, in either Arithmetical or Geometrical Progression, there are $5 \times 4 = 20$ cases, or classes of problems, and no more, requiring each a different solution.

GENERAL PROBLEMS IN ARITHMETICAL PROGRESSION.

PROBLEM I.

706. Given, one of the extremes, the common difference, and the number of terms, to find the other extreme.

Let 2 be the first term of an arithmetical progression, and 3 the common difference; then,

$$\begin{array}{lll} 2 & = 2 & = \ \ 2, \text{ 1st term.} \\ 2+3 & = 2+(3\times 1) = & \ \ 5, \text{ 2d } \quad `` \\ 2+3+3 & = 2+(3\times 2) = & \ \ 8, \text{ 3d } \quad `` \\ 2+3+3+3 & = 2+(3\times 3) = & 11, \text{ 4th } \quad `` \end{array}$$

From this illustration we perceive that, in an arithmetical progression, when the series is *ascending*, the *second* term is equal to the first term plus the common difference; the *third* term is equal to the first term plus 2 times the common difference; the *fourth* term is equal to the first term plus 3 times the common difference; and so on. In a descending series, the *second* term is equal to the first term minus the common difference; the *third* term is equal to the first minus 2 times the common difference; and so on. In all cases the difference between the two extremes is equal to the product of the common difference by the number of terms less 1. Hence the

RULE. *Multiply the common difference by the number of terms less 1; add the product to the given term if it be the less extreme, and subtract the product from the given term if it be the greater extreme.*

EXAMPLES FOR PRACTICE.

1. The first term of an arithmetical progression is 5, the common difference 4, and the number of terms 8; what is the last term? *Ans.* 33.

2. If the first term of an ascending series be 2, and the common difference 3, what is the 50th term?

3. The first term of a descending series is 100, the common difference 7, and the number of terms 13; what is the last term?

4. If the first term of an ascending series be $\frac{3}{4}$, the common difference $\frac{3}{8}$, and the number of terms 20, what is the last term? *Ans.* $7\frac{19}{24}$.

PROBLEM II.

707. Given, the extremes and number of terms, to find the common difference.

Since the difference of the extremes is always equal to the common difference multiplied by the number of terms less 1, (**706**), we have the following

RULE. *Divide the difference of the extremes by the number of terms less* 1.

EXAMPLES FOR PRACTICE.

1. If the extremes of an arithmetical series are 3 and 15, and the number of terms 7, what is the common difference?

Ans. 2.

2. The extremes are 1 and 51, and the number of terms is 76; what is the common difference?

3. The extremes are .05 and .1, and the number of terms is 8; what is the common difference? *Ans.* $.00\dot{7}1428\dot{5}$.

4. If the extremes are 0 and $2\frac{1}{2}$, and the number of terms is 18, what is the common difference?

PROBLEM III.

708. Given, the extremes and common difference, to find the number of terms.

Since the difference of the extremes is equal to the common difference multiplied by the number of terms less 1, (**706**), we have the following

RULE. *Divide the difference of the extremes by the common difference, and add* 1 *to the quotient.*

EXAMPLES FOR PRACTICE.

1. The extremes of an arithmetical series are 5 and 75, and the common difference is 5; what is the number of terms?

Ans. 15.

2. The extremes are $\frac{1}{2}$ and 20, and the common difference is $6\frac{1}{2}$; find the number of terms.

3. The extremes are 2.5 and .25, and the common difference is .125; what is the number of terms?

4. Insert 5 arithmetical means between 2 and 37.

PROBLEM IV.

" one extreme No of terms & sum of series to find other extreme

709. Given, the extremes and number of terms, to find the sum of the series.

Let us take any series, as 2, 5, 8, 11, 14, and writing under it the same series in an inverse order, add each term of the inverted series to the term above it in the direct series, thus:

$$\begin{array}{rl} 2+\ 5+\ 8+11+14=40, & \text{once the sum,} \\ 14+11+\ 8+\ 5+\ 2=40, & \text{" \quad " \quad "} \\ \hline 16+16+16+16+16=80, & \text{twice the sum.} \end{array}$$

From this we perceive that 16, the sum of the extremes of the given series, multiplied by 5, the number of terms, equals 80, which is *twice* the sum of the series; and $80 \div 2 = 40$, the sum of the series. Hence

RULE. *Multiply the sum of the extremes by the number of terms, and divide the product by 2.*

X sum of series by 2 & ÷ Prod. by number of terms and − given extreme, Rem = other extreme

EXAMPLES FOR PRACTICE.

1. Find the sum of the series the first term of which is 4, the common difference 6, and the last term 40. *Ans.* 154.

2. The extremes are 0 and 250, and the number of terms is 1000; what is the sum of the series?

3. A person wishes to discharge a debt in 11 annual payments such that the last payment shall be $220, and each payment greater than the preceding by $17; find the amount of the debt, and the first payment. *Ans.* First payment, $50.

710. By reversing some one of the four problems now given, or by combining two or more of them, all of the sixteen remaining problems of Arithmetical Progression may be solved or analyzed.

GENERAL PROBLEMS IN GEOMETRICAL PROGRESSION.

PROBLEM I.

711. Given, one of the extremes, the ratio, and the number of terms, to find the other extreme.

Let 3 be the first term of a geometrical progression, and 2 the ratio: then,

3	$= 3$	$= 3$,	the 1st term,
3×2	$= 3 \times 2^1 =$	6,	" 2d "
$3 \times 2 \times 2$	$= 3 \times 2^2 =$	12,	" 3d "
$3 \times 2 \times 2 \times 2$	$= 3 \times 2^3 =$	24,	" 4th "

From this illustration we perceive that, in a geometrical progression, the *second* term is equal to the first term multiplied by the ratio; the *third* term is equal to the first term multiplied by the second power of the ratio; the *fourth* term is equal to the first term multiplied by the third power of the ratio; and so on. The same is true whether the ratio be an integer or fraction. Hence the following

RULE. I. *If the given extreme be the first term, multiply it by that power of the ratio indicated by the number of terms less* 1*; the result will be the last term.*

II. *If the given extreme be the last term, divide it by that power of the ratio indicated by the number of terms less* 1*; the result will be the first term.*

EXAMPLES FOR PRACTICE.

1. The first term of a geometrical series is 6, the ratio 4, and the number of terms 6; find the last term. *Ans.* 6144.

2. The last term of a geometrical series is 192, the ratio 2, and the number of terms 7; what is the first term?

3. If the first term be 6, the ratio $\frac{1}{3}$, and the number of terms 8, what is the last term?

4. The first term is 25, the ratio $\frac{1}{5}$, and the number of terms 5; what is the last term? *Ans.* $\frac{1}{25}$.

PROBLEM II.

712. Given, the extremes and number of terms, to find the ratio.

Since the last term is always equal to the first term multiplied by that power of the ratio indicated by the number of terms less 1, (**711**), we have the following

RULE. *Divide the last term by the first, and extract that root of the quotient indicated by the number of terms less* 1*; the result will be the ratio.*

EXAMPLES FOR PRACTICE.

1. The extremes are 2 and 512, and the number of terms is 5; what is the ratio? *Ans.* 4.

2. The extremes are $\frac{1}{48}$ and $45\frac{9}{16}$, and the number of terms is 8; what is the ratio?

3. The extremes are 7 and .0112, and the number of terms is 5; what is the ratio? *Ans.* 5.

4. Insert 3 geometrical means between 8 and 5000.

PROBLEM III.

713. Given, the extremes and ratio, to find the number of terms.

Since the quotient of the last term divided by the first term is equal to that power of the ratio indicated by the number of terms less 1, (**712**), we have the following

RULE. *Divide the last term by the first, divide this quotient by the ratio, and the quotient thus obtained by the ratio again, and so on in successive division, till the final quotient is* 1. *The number of times the ratio is used as a divisor, plus* 1, *is the number of terms.*

EXAMPLES FOR PRACTICE.

1. The extremes are 2 and 1458, and the ratio is 3; what is the number of terms? *Ans.* 7.

2. The first term is .1, the last term 100, and the ratio 10; find the number of terms.

3. The first term is $\frac{1}{640}$, the last term $\frac{1}{5}$, and the ratio 2; what is the number of terms?

4. The extremes are 196608 and 6, and the ratio is $\frac{1}{8}$; what is the number of terms? *Ans.* 6.

PROBLEM IV.

714. Given, the extremes and ratio, to find the sum of the series.

Let us take the series $5+20+80+320=425$, multiply each term by the ratio 4, and from this result subtract the given series term from term, thus:

$$
\begin{array}{lrcll}
 & 20 + 80 + 320 + 1280 & = & 1700, & \text{four times the series,} \\
 & \underline{5 + 20 + 80 + 320} & = & \underline{425,} & \text{once the series,} \\
 & 1280 - 5 & = & 1275, & \text{three times the series,} \\
\text{Then} & 1275 \div 3 & = & 425, & \text{once the series.}
\end{array}
$$

Hence the

RULE. *Multiply the greater extreme by the ratio, subtract the less extreme from the product, and divide the remainder by the ratio less* 1.

NOTE.—Let every descending series be inverted, and the first term called the last; then the ratio will be *greater* than a unit. If the series be *infinite*, the least term is a cipher.

EXAMPLES FOR PRACTICE.

1. The extremes are 3 and 384, and the ratio is 2; what is the sum of the series? *Ans.* 765.

2. If the extremes are 5 and 1080, and the ratio is 6, what is the sum of the series?

3. If the first term is $4\frac{4}{5}$, the last term $\frac{8}{405}$, and the ratio $\frac{1}{3}$, what is the sum of the series? *Ans.* $7\frac{77}{405}$.

4. What is the sum of the infinite series, 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, etc.?

PROBLEM V.

715. Given, the first term, the ratio, and the number of terms, to find the sum of the series.

If, for example, the first term be 4, the ratio 3, and the number of terms 6, then by Problem I, we have

$$4 \times 3^5 = \text{the last term.}$$

Whence by Problem IV, we have

$$\frac{4 \times 3^6 - 4}{3 - 1} = \frac{(3^6 - 1) \times 4}{3 - 1} = 1456, \text{ the sum of the series,}$$

Hence the following

RULE. *Raise the ratio to a power indicated by the number of terms, and subtract* 1 *from the result; then multiply this remainder by the first term, and divide the product by the ratio less* 1.

EXAMPLES FOR PRACTICE.

1. The first term is 7, the ratio 3, and the number of terms 4; what is the sum of the series? *Ans.* 280.

2. The first term is 375, the ratio $\frac{1}{5}$, and the number of terms 4; what is the sum of the series?

3. The first term is 175, the ratio 1.06, and the number of terms 5; what is the sum of the series? *Ans.* 986.49+.

PROBLEM VI.

716. Given, the extremes and the sum of the series, to find the ratio.

If we take the geometrical progression, 2, 6, 18, 54, 162, in which the ratio is 3, and remove the first term and the last term, successively, and then compare the results, we have

$6 + 18 + 54 + 162 =$ sum of the series minus the first term.
$2 + 6 + 18 + 54 =$ sum of the series minus the last term.

Now, since every term in the first line is 3 times the corresponding term in the second line, the sum of the terms in the first line must be 3 times the sum of the terms in the second line. Hence the

RULE. *Divide the sum of the series minus the first term, by the sum of the series minus the last term.*

EXAMPLES FOR PRACTICE.

1. The extremes are 2 and 686, and the sum of the series is 800; what is the ratio? *Ans.* 7.

2. The extremes are $\frac{1}{4}$ and 64, and the sum of the series is $127\frac{3}{4}$; what is the ratio?

3. If the sum of an infinite series be $4\frac{1}{2}$, and the greater extreme 3, what is the ratio? *Ans.* $\frac{1}{3}$.

717. Every other problem in Geometrical Progression, that admits of an arithmetical solution, may be solved either by reversing or combining some of the problems already given.

COMPOUND INTEREST BY GEOMETRICAL PROGRESSION.

718. We have seen (**550**) that if any sum at compound interest be multiplied by the amount of $1 for the given interval, the product will be the amount of the given sum or principal at the end of the first interval; and that this amount constitutes a new principal for the second interval, and so on for a third, fourth, or any other interval. Hence,

A question in compound interest constitutes a geometrical progression, whose first term is the principal; the common multiplier or ratio is one plus the rate per cent. for one interval; the number of terms is equal to the number of intervals $+1$; and the last term is the amount of the given principal for the given time. All the usual cases of compound interest and discount computed at compound interest, can therefore be solved by the rules for geometrical progression. For example,

Find the amount of $250 for 4 years, at 6 % compound interest.

OPERATION.

$$\$250 \times 1.06^4 = \$250 \times 1.262477 = \$316.21925.$$

Analysis. Here we have $250 the first term, 1.06 the ratio, and 5 the number of terms, to find the last term. Then by **711** we find the last term, which is the amount required.

EXAMPLES FOR PRACTICE.

1. What is the amount of $350 in 4 years, at 6 % per annum compound interest? *Ans.* $441.86.

2. Of what principal is $150 the compound interest for 2 years, at 7 %?

3. What sum at 6 % compound interest, will amount to $1000 in 3 years? *Ans.* $839.62.

4. In how many years will $40 amount to $53.24, at 10 % compound interest? *Ans.* 3 years.

5. At what rate per cent. compound interest will any sum double itself in 8 years? *Ans.* 9.05 + %.

6. What is the present worth of $322.51, at 5 % compound interest, due 24 years hence? *Ans.* $100.

$P = 1,\ 2 = 1,\ 9 = n \quad 2^{1/8} = 2;\ \sqrt[8]{2} = 1.0905+$

Par. 9th. $1.0905 - 1 = .0905+$

ANNUITIES.

719. An **Annuity** is literally a sum of money which is payable annually. The term is, however, applied to a sum which is payable at any equal intervals, as monthly, quarterly, semi-annually, etc.

NOTE.—The term, interval, will be used to denote the time between payments.

Annuities are of three kinds: Certain, Contingent, and Perpetual.

720. A **Certain Annuity** is one whose period of continuance is definite or fixed.

721. A **Contingent Annuity** is one whose time of commencement, or ending, or both, is uncertain; and hence the period of its continuance is uncertain.

722. A **Perpetual Annuity** or **Perpetuity** is one which continues forever.

723. Each of these kinds is subject, in reference to its commencement, to the three following conditions:

1st. *It may be deferred*, i. e., it is not to be entered upon until after a certain period of time.

2d. *It may be reversionary*, i. e., it is not to be entered upon until after the death of a certain person, or the occurrence of some certain event.

3d. *It may be in possession*, i. e., it is to be entered upon at once.

724. An **Annuity in Arrears** or **Forborne** is one on which the payments were not made when due. Interest is to be reckoned on each payment of an annuity in arrears, from its maturity, the same as on any other debt.

ANNUITIES AT SIMPLE INTEREST.

725. In reference to an annuity at simple interest, we observe:

I. The first payment becomes due at the end of the first interval, and hence will bear interest until the annuity is settled.

II. The second payment becomes due at the end of the second interval, and hence will bear interest for one interval less than the first payment.

III. The third payment will bear interest for one interval less than the second; and so on to any number of terms. Hence,

IV. All the payments being settled at one time, each will be less than the preceding, by the interest on the annuity for one interval. Therefore, they will constitute a descending arithmetical progression, whose first term is the annuity plus its interest for as many intervals less one as intervene between the commencement and settlement of the annuity; the common difference is the interest on the annuity for one interval; the number of terms is the number of intervals between the commencement and settlement of the annuity; and the last term is the annuity itself.

726. The rules in Arithmetical Progression will solve all problems in annuities at simple interest.

EXAMPLES FOR PRACTICE.

1. A man works for a farmer one year and six months, at $20 per month, payable monthly; and these wages remain unpaid until the expiration of the whole term of service. How much is due to the workman, allowing simple interest at 6 per cent. per annum?

OPERATION.

$\$20 + \$.10 \times 17 = \$21.70$, first term.

$\dfrac{\$20 + \$21.70}{2} \times 18 = 375.30$, sum.

ANALYSIS. Here the last month's wages, $20, is the last term; the number of months, 18, is the number of terms; and the interest on 1 month's wages, \$.10, is the common difference; and since the first month's wages has been on interest 17 months, the progression is a descending series. Then, by **706** we find the first term, which is the amount of the first month's wages for 17 months; and by **709** we find the sum of the series, which is the sum of all the wages and interest.

2. A father deposits annually for the benefit of his son, commencing with his tenth birthday, such a sum that on his 21st birthday the first deposit at simple interest amounts to \$210, and the sum due his son to \$1860. How much is the deposit, and at what rate per cent. is it deposited?

OPERATION.

$$\frac{\$1860 \times 2 - \$210 \times 12}{12} = \$100, \text{ deposit.}$$

$$\frac{210 - 100}{11} = 10\ \%, \text{ rate.}$$

ANALYSIS. Here the \$210, the amount of the first deposit, is the first term; 12, the number of deposits, is the number of terms; and \$1860, the amount of all the deposits and interests, is the sum of the series. By **709** we find the last term to be \$100, which is the annual deposit; and by **707** we find the common difference to be \$10, which is the annual rate %.

3. What is the amount of an annuity of \$150 for 5½ years, payable quarterly, at 1½ per cent. per quarter? *Ans.* \$3819.75.

4. In what time will an annual pension of \$500 amount to \$3450, at 6 per cent. simple interest? *Ans.* 6 years.

5. Find the rate per cent at which an annuity of \$6000 will amount to \$59760 in 8 years, at simple interest.

Ans. 7 per cent.

ANNUITIES AT COMPOUND INTEREST.

727. An Annuity at compound interest constitutes a geometrical progression whose first term is the annuity itself; the common multiplier is one plus the rate per cent. for one interval expressed decimally; the number of terms is the number of intervals for which the annuity is taken; and the last term is the first term multiplied by one plus the rate per cent. for one interval raised to a power one less than the number of terms.

728. **The Present Value of an Annuity** is such a sum as would produce, at compound interest, at a given rate, the same amount as the sum of all the payments of the annuity at compound interest. Hence, to find the present value;—*First find the amount of the annuity at the given rate and for the given time by* **715**; *then find the present value of this amount by* **553**, *taking out the amount of* $1, *or divisor*, *from* **551**.

NOTES.—1. The present value of a reversionary annuity is that principal which will amount, at the time the reversion expires, to what will then be the present value of the annuity.

2. The present value of a perpetuity is a sum whose interest equals the annuity.

729. Questions in Annuities at compound interest can be solved by the rules of Geometrical Progression.

PROMISCUOUS EXAMPLES IN SERIES.

1. Allowing 6 per cent. compound interest on an annuity of $200 which is in arrears 20 years, what is its present amount?

Ans. $7357.11.

2. Find the annuity whose amount for 25 years is $16459.35, allowing compound interest at 6 per cent. *Ans.* $300.

3. What is the present worth of an annuity of $500 for 7 years, at 6 per cent. compound interest? *Ans.* $2791.18.

4. What is the present value of a reversionary lease of $100, commencing 14 years hence, and to continue 20 years, compound interest at 5 per cent.? *Ans.* $629.426.

5. Find the sum of 21 terms of the series, 5, $4\frac{3}{4}$, $4\frac{1}{2}$, etc.

6. A man traveled 13 days; his last day's journey was 80 miles, and each day he traveled 5 miles more than on the preceding day. How far did he travel, and what was his first day's journey?

Ans. He traveled 650 miles.

7. Find the 12th term of the series, 30, 15, $7\frac{1}{2}$, etc.

Ans. $\frac{15}{1024}$.

8. The first term of a geometrical progression is 2, the last term 512, and common multiplier 4; find the sum of the series.

Ans. 682.

9. The distance between two places is 360 miles. In how many days can it be traveled, by a man who travels the first day 27 miles, and the last day 45, each day's journey being greater than the preceding by the same number of miles? *Ans.* 10.

10. The first term of a geometrical progression is 1, the last term 15625, and the number of terms 7; find the common ratio. *Ans.* 5.

11. An annual pension of $500 is in arrears 10 years. What is the amount now due, allowing 6 per cent. compound interest? *Ans.* $6590.40.

12. Find the first and last terms of an arithmetical progression whose sum is 408, common difference 6, and number of terms 8. *Ans.* First term, 30; last term, 72.

13. A farmer pays $1196, in 13 quarterly payments, in such a way that each payment is greater than the preceding by $12. What are his first and last payments? *Ans.* $20, and $164.

14. A man wishes to discharge a debt in yearly payments, making the first payment $2, the last $512, and each payment four times the preceding payment. How long will it take him to discharge the debt, and what is the amount of his indebtedness?

15. A man dying, left 5 sons, to whom he gave his property as follows: to the youngest he gave $4800, and to each of the others 1½ times the next younger son's share. What was the eldest son's fortune, and what the amount of property left? *Ans.* Eldest son's share, $24300; property, $63300.

16. Find the annuity whose amount for 5 years, at 6 per cent. compound interest, is $2818.546. *Ans.* $500.

17. A merchant pays a debt in yearly payments in such a way that each payment is 3 times the preceding; his first payment is $10, and his last $7290. What is the amount of the debt, and in how many payments is it discharged? *Ans.* Debt, $10930; 7 payments.

18. A man traveling along a road, stopped at a number of stations, but at each station he found it necessary, before proceeding to the next, to return to the place from which he first started;

the distance from the starting place to the first station was 5 miles, and to the last 25 miles; he traveled in all 180 miles. How many stations were there on the road, and what was the distance from station to station? *Ans.* 6 stations; 4 miles apart.

19. An annuity of \$200 for 12 years is in reversion 6 years What is its present worth, compound interest at 6 %?

Ans. \$1182.05+.

20. A man pays \$6 yearly for tobacco, from the age of 16 until he is 60, when he dies, leaving to his heirs \$500. What might he have left them, if he had dispensed with this useless habit and loaned the money at the end of each year at 6 % compound interest? *Ans.* \$1698.548+.

21. What is the present worth of a reversionary perpetuity of \$100, commencing 30 years hence, allowing 5 per cent. compound interest? *Ans.* \$462.75+.

22. Two boys, each 12 years old, have certain sums of money left to them; the sum left to one is put out at 7 % simple interest, and the sum left the other at 6 % compound interest, payable semi-annually, and the amount of each boy's money will be \$2000 when he is 21 years old. What is the sum left to each boy?

23. A merchant purchased 8 pieces of cloth, for which he paid \$136; the difference in the length of any two pieces was 2 yds., and the difference in the price \$4. He paid \$31 for the longest piece, and \$1 a yard for the shortest. Find the whole number of yards, and the price per yard of each piece.

24. A farmer has 600 bushels of different kinds of grain, mixed in such a way that the number of bushels of the several kinds constitute a geometrical progression, whose common multiplier is 2; the greatest number of bushels of one kind is 320. Find the number of kinds of grain in the mixture, and the number of bushels of each kind. *Ans.* 4 kinds.

MISCELLANEOUS EXAMPLES.

1. How many thousand shingles will cover both sides of a roof 36 ft. long, and whose rafters are 18 ft. in length?

2. From $\frac{2}{7}$ of $\frac{4}{7}$ of $\frac{1}{3}$ of 70 miles, subtract .73 of 1 mi. 3 fur.

3. What number is that from which if $7\frac{1}{2}$ be subtracted, $\frac{2}{3}$ of the remainder is $91\frac{1}{2}$? *Ans.* $144\frac{3}{4}$.

4. What part of 4 is $\frac{4}{9}$ of 6? *Ans.* $\frac{2}{3}$.

5 It is required to mix together brandy at $.80 a gallon, wine at $.70, cider at $.10, and water, in such proportions that the mixture may be worth $.50 a gallon; what quantity of each must be used?
Ans. 3 gal. of water, 2 of cider, 4 of wine, and 5 of brandy.

6. What number increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itself equals 125?

7. What is the hour, when the time past noon is equal to $\frac{2}{3}$ of the time to midnight? *Ans.* 4 h. 48 min. P. M.

8. A grocer mixed 12 cwt. of sugar @ $10, with 3 cwt. @ $$8\frac{2}{3}$, and 8 cwt. @ $$7\frac{1}{2}$; how much was 1 cwt. of the mixture worth?

9. If $240 gain $5.84 in 4 mo. 26 da., what is the rate %? *Ans.* 6.

10. If 24 men, in 189 da., working 10 h. a day, dig a trench $33\frac{3}{4}$ yd. long, $2\frac{2}{3}$ yd. deep, and $5\frac{1}{4}$ yd. wide; how many hours a day must 217 men work, to dig a trench $23\frac{1}{4}$ yd. long, $2\frac{1}{3}$ yd. deep, and $3\frac{2}{3}$ yd. wide, in $5\frac{1}{2}$ days? *Ans.* 16 h.

11. What is the difference between the interest and the discount of $450 at 5 per cent., for 6 yr. 10 mo.?

12. A younger brother received $6300, which was $\frac{7}{8}$ as much as his elder brother received; how much did both receive?

13. Reduce $.\dot{7}$, $.\dot{8}\dot{8}$, $.\dot{7}2\dot{7}$, $.9\dot{1}32\dot{5}$ to their equivalent common fractions.

14. A person by selling a lot of goods for $438, loses 10 %; how much should the goods have been sold for, to gain $12\frac{1}{2}$ %?

X 15. For what sum must a note be drawn at 4 mo., that the proceeds of it, when discounted at bank at 7 per cent., shall be $875.50? 896.9

16. Three persons engaged in trade with a joint capital of $2128; A's capital was in trade 5 mo., B's 8 mo., and C's 12 mo.; A's share of the gain was $228, B's $266.40, and C's $330. What was the capital of each? *Ans.* A's, $912; B's, $666; C's, $555.

17. Henry Truman purchased corn of John Bates, on 2 months' credit, as follows: Aug. 27, 300 bu. @ $.35; Aug. 31, 150 bu. @ $.40; Sept. 7, 500 bu. @ $.38; Sept. 12, 200 bu. @ $.42; Sept. 25, 250 bu. @ $.40. When was the a|c due per average? *Ans.* Nov. 8.

18. A B and C can do a job of work in 12 da., C can do it in 24 da., and A in 34 da.; in what time can B do it alone? *Ans.* $81\frac{3}{5}$ da.

19. If a man travel 7 mi. the first day, and 51 mi. the last, increasing his journey 4 mi. each day, how many days will he travel, and how far? *Ans.* 12 da., and 348 mi.

20. What is the difference between the true and bank discount of \$2500, payable in 90 days at 7 per cent.? *Ans.* \$2.21.

21. Which is the more advantageous, to buy flour at \$5 a bbl. on 6 mo., or \$4.87½ cash, money being worth 7 %? *Ans.* At \$5 on 6 mo.

22. Sold ½ of a lot of lumber for what ⅝ of it cost; what % was gained on the part sold? *Ans.* 25 %.

23. If \$500 gain \$50 in 1 yr., in what time will \$960 gain \$60?

24. Received an invoice of crockery, 12 per cent. of which was broken; at what per cent. above cost must the remainder be sold, to clear 25 per cent. on the invoice? *Ans.* $42\frac{1}{22}$.

25. The sum of two numbers is 365, and their difference is .0675; what are the numbers?

26. If the interest of \$445.62½ be \$128.99 for 7 yr., what will be the interest of \$650 for 3 yr. 10 mo. 15 da.?

27. Received from Savannah 150 bales of cotton, each weighing 540 lb., and invoiced at 7d. a pound Georgia currency. Sold it at an advance of 26 %, commission 1½ %, and remitted the proceeds by draft. What was the face of the draft, exchange being ½ % discount? *Ans.* \$12629.28+.

28. A man in Chicago has 5000 francs due him on account in Paris. He can draw on Paris for this amount, and negotiate the bill at 19⅗ cents per franc; or he can advise his correspondent in Paris to remit a draft on the United States, purchased with the sum due him, exchange on U. S. being at the rate of 5 francs 20 centimes per \$1. What sum will the man receive by each method?
Ans. By draft on Paris, \$970; by remittance from Paris, \$961.53.

29. What sum must be invested in stocks bearing 6½ %, at 105%, to produce an income of \$1000? 1000 ÷ .06½ × 105 = *Ans.* \$16153.84.

30. A person exchanges 250 shares of 6 per cent. stock, at 70 %, for stock bearing 8 per cent., at 120 %; what is the difference in his income? *Ans.* \$333.33⅓.

31. If ⅔ of A's money equals ¾ of B's, and ⅘ of B's equals ⅚ of C's, and the interest of all their money for 4 yr. 8 mo. at 6 % is \$15190, how much money has each?
Ans. A has \$18859.44+; B, \$16763.95+; C, \$18626.61.

32. A boy 14 years old is left an annuity of \$250, which is deposited in a savings bank at 6 %, interest payable semi-annually; how much will he be worth when of age? *Ans.* \$2104.227.

33. If a boy buys peaches at the rate of 5 for 2 cents, and sells them at the rate of 4 for 3 cents, how many must he buy and sell to make a profit of \$4.20? 1200

34. What % in advance of the cost must a merchant mark his goods, so that, after allowing 5 % of his sales for bad debts, an average credit of 6 months, and 7 % of the cost of the goods for his expenses, he may make a clear gain of 12½ % on the first cost of the goods, money being worth 6 %? *Ans.* 29.56+ %.

$(1.03^2)^7 = 1.03^{14}$

35. Four men contracted to do a certain job of work for $8600; the first employed 28 laborers 20 da., 10 h. a day; the second, 25 laborers 15 da., 12 h. a day; the third, 18 laborers 25 da., 11 h. a day; and the fourth, 15 laborers 24 da., 8 h. a day. How much should each contractor receive?

Ans. 1st, $2686; 2d, $2158.39; 3d, $2374.24; 4th, $1381.37.

36. If I exchange 75 railroad bonds of $500 each, at 36 % below par, for bank stock at 5 % premium, how many shares of $100 each will I receive? *Ans.* $228\frac{4}{7}$.

37. A trader has bought merchandise as follows: July 3, $35.26; July 4, $48.65, on 30 da.; Aug. 17, $6.48; Sept. 12, $50. What is due on the account Oct. 12, interest at 9 %? *Ans.* 142.60.

38. A farmer sold 34 bu. of corn, and 56 bu. of barley for $63.10, receiving 35 cents a bushel more for the barley than for the corn; what was the price of each per bushel?

39. A speculator purchased a quantity of flour, Sept. 1; Oct. 1 its value had increased 25 %; Nov. 1 its value was 30 % more than Oct. 1; Dec. 1 he sold it for 15 % less than its value Nov. 1, receiving in payment a 6 months' note, which he got discounted at a bank, at 7 %, receiving $12950 on it. How much was his profit on the flour? *Ans.* $3228.51.

40. A flour merchant bought 120 bbl. of flour for $660, paying $5.75 for first quality and $5 for second quality; how many barrels were first quality? *Ans.* 80.

41. Two mechanics work together; for 15 days' work of the first and 8 days' work of the second they receive $61, and for 6 days' work of the first and 10 days' work of the second they receive $38; how much does each man earn? *Ans.* 1st, $63; 2d, $36.

42. The duty, at 15 %, on Rio coffee, in bags weighing 180 lbs. gross, and invoiced at $.12½ per pound, was $961.87½, tare having been allowed at 5 %; how many bags were imported? *Ans.* 300.

43. A dairyman took some butter to market, for which he received $49, receiving as many cents a pound as there were pounds; how many pounds were there? *Ans.* 70 lb.

44. A mechanic received $2 a day for his labor, and paid $4 a week for his board; at the expiration of 10 weeks he had saved $72; how many days did he work, and how many was he idle?

45. To what would $250, deposited in a savings bank, amount in 10 yr., interest being allowed semi-annually at 6 % per annum?

46. How much water is there in a mixture of 100 gal. of wine and water, worth $1 per gal., if 100 gal. of the wine cost $120?

47. If a pipe 3 in. in diameter will discharge a certain quantity of water in 2 h., in what time will 3 two-inch pipes discharge 3 times the quantity? *Ans.* 4 h. 30 min.

48. Wm. Jones & Co. become insolvent and owe $8100. Their assets amount to $4981.50. What per cent. of their indebtedness can

they pay, allowing the assignees $2\frac{1}{2}$ % on the amount distributed for their services? *Ans.* 60 per cent.

49. Shipped a car load of fat cattle to Boston, and offered them for sale at 25 per cent. advance on the cost; but the market being dull I sold for 14 per cent. less than my asking price, and gained thereby \$170. How much did the cattle cost; for how much did they sell; and what was my asking price?
Ans. Cost \$2266.66$\frac{2}{3}$; sold for \$2436.66$\frac{2}{3}$; asking price, \$2833.33$\frac{1}{3}$.

50. What must be the dimensions of a cubical cistern to hold 2000 gallons? Ans. 77,3 inches +

51. A man died leaving \$5000 to be divided between his three sons, aged 13, 15, and 16 yr. 6 mo., respectively, in such a proportion that the share of each being put at simple interest at 6 %, should amount to the same sum when they should arrive at the age of 21. How much was each one's share?
Ans. Youngest, \$1536.76+; second, \$1672.36+; oldest, \$1790.88+.

52. A vessel having sailed due south and due east on alternate days, was found, after a certain time, to be 118.794 miles south-east of the place of starting; what distance had she sailed? *Ans.* 168 miles.

53. Imported 4 pipes of Madeira wine, at \$2.15 a gallon, and paid \$57.60 freight, and a duty of 24 per cent. I sold the whole for \$1980; what was my gain %?

54. If $34\frac{1}{2}$ bu. of corn are equal in value to 17 bu. wheat, 9 bu. of wheat to $59\frac{1}{2}$ bu. of oats, and 6 bu. of oats to 42 lb. of flour, how many bushels of corn will purchase 5 bbl. of flour? *Ans.* $42\frac{282}{289}$.

55. If stock bought at 8 % discount will pay 7 % on the investment, at what rate should it be bought to pay 10 %? 35,6 cts

56. A merchant in New York gave \$2000 for a bill of exchange of £400 to remit to Liverpool; what was the rate in favor of England?

57. A, B, and C start from the same point, to travel around a lake 84 miles in circumference. A travels 7 miles, and B 21 miles a day in the same direction, and C 14 miles in an opposite direction. In how many days will they all meet? *Ans.* 12.

58. The exact solar year is greater than 365 days by $\frac{10463}{43200}$ of a day; find approximately how often leap year should come, or one day be added to the common year, in order to keep the calendar right?
Ans. Once in every 4 yr.; 7 times in every 28 yr.; 8 times in every 33 yr.; 31 times in every 128 yr.; or 163 times in every 673 yr.

59. A gentleman purchases a farm for \$10000, which he sells after a certain number of years for \$14071, making on the investment 5 % compound interest. He now invests his money in a perpetuity, which is in reversion 11 years from the date of purchasing the farm. Allowing 6 % compound interest for the use of money, find the annuity and the length of time he owns the farm.
Ans. Annuity, \$1065.85; owned the farm 7 yr.

60. What will I gain % by purchasing goods on 6 mo., and selling them immediately for cash at cost, money being worth 7 %? $3\frac{1}{2}$

61. What sum must a man save annually, commencing at 21 years of age, to be worth \$30000 when he is 50 years old, his savings being invested at 5 % compound interest? *Ans.* \$481.37.

62. Three persons are to share \$10000 in the ratio of 3, 4, and 5, but the first dying it is required to divide the whole sum equitably between the other two. What are the shares of the other two?

Ans. $\$4444\frac{4}{9}$, and $\$5555\frac{5}{9}$.

63. If 50 bbl. of flour in Chicago are worth 125 yd. of cloth in New York, and 80 yd. of cloth in New York are worth 6 bales of cotton in Charleston, and 13 bales of cotton in Charleston are worth $3\frac{1}{2}$ hhd. of sugar in New Orleans, how many hhd. of sugar in New Orleans are worth 1500 bbl. of flour in Chicago? *Ans.* $75\frac{75}{104}$.

64. Seven men all start together to travel the same way round an island 120 miles in circumference, and continue to travel until they all come together again. They travel 5, $6\frac{1}{4}$, $7\frac{1}{3}$, $8\frac{1}{4}$, $9\frac{1}{2}$, $10\frac{1}{4}$ and $11\frac{1}{4}$ miles a day respectively. In how many days will they all be together again? *Ans.* 1440 da.

65. There are two clocks which keep perfect time when their pendulums beat seconds. The first loses 20 seconds a day, and the second gains 15 seconds a day. If the two pendulums beat together when both dials indicate precisely 12 o'clock, what time does each clock show when the pendulums next beat in concert?

Ans. The first shows 41 min. 8 sec. past 12; and the second 41 min. 9 sec. past 12.

66. If a body put in motion move $\frac{1}{3}$ of an inch the first second of time, 1 in. the second sec., 3 in. the third, and so continue to increase in geometrical ratio, how far would it move in 30 seconds?

Ans. $5415900730\frac{2021}{4320}$. mi.

67. If stock bought at 5 % premium will pay 6 % on the investment, what % will it pay if bought at 15 % discount? *Ans.* $7\frac{7}{17}$ %.

68. If 6 apples and 7 peaches cost 33 cts., and 10 apples and 8 peaches cost 44 cts., what is the price of one of each?

Ans. Apples, 2 cts.; peaches, 3 cts.

69. A gentleman in dividing his estate among his sons gave A \$9 as often as B \$5, and C \$3 as often as B \$7. C's share was \$3862.50; what was the value of the whole estate? *Ans.* \$29097.50.

70. A farmer sold 16 bu. of corn and 20 bu. of rye for \$30, and 24 bu. of corn and 10 bu. of rye for \$27. How much per bushel did he receive for each? *Ans.* Corn, \$.75; rye, \$.90.

71. A drover sold some oxen at \$28, cows at \$17, and sheep at \$7.50 per head, and received \$749 for the lot. There were twice as many cows as oxen, and three times as many sheep as cows. How many were there of each kind?

72. For what sum must a vessel, valued at \$25000, be insured, so

that in case of its loss, the owners may recover both the value of the vessel and the premium of 24 %? $289474.83 Ans P=7894.73

73. A boy hired to a mechanic for 20 weeks, on condition that he should receive \$20 and a coat. At the end of 12 weeks the boy quit work, when it was found that he was entitled to \$9 and the coat; what was the value of the coat? *Ans.* \$7.50.

74. An irregular piece of land, containing 540 A. 36 P., is exchanged for a square piece containing the same area; what is the length of one of its sides? If divided into 42 equal squares, what will be the length of the side of each?

75. What will be the difference in the expense of fencing two fields of 25 acres each, one square, and the other in the form of a rectangle, whose length is twice its breadth, the fence costing \$.62$\frac{1}{2}$ a rod? *Ans.* \$9.59+.

76. At what time between 5 and 6 o'clock are the hour and minute hands of a watch exactly together?

77. A general, forming his army into a square, had 284 men remaining; but increasing each side by one man, he wanted 25 men to complete the square. How many men had he? *Ans.* 24000.

78. Divide \$3648 among 3 persons, so that the share of the first to that of the second shall be as 7 to 9, and of the first to the third as 3 to 4. *Ans.* \$1008, \$1296, \$1344.

79. If a lot of land, in the form of an oblong or rectangle, contains 6 A. 3 R. 12 P., and its length is to its width as 21 to 13, what are its dimensions; and how many rods of fence will be required to inclose it? *Ans.* to last, 136 rd. of fence.

80. Five persons are employed to build a house. A, B, C, and D can build it in 13 days; A, B, C, and E in 15 days; A, B, D, and E in 12 days; A, C, D, and E in 19 days; and B, C, D, and E in 14 days. In how many days can all together build it; and which one could do the work alone in the shortest time?
Ans. 11$\frac{4813}{12137}$ da.; B in shortest time.

81. Divide \$500 among 3 persons, in such a manner that the share of the second may be $\frac{1}{2}$ greater than that of the first, and the share of the third $\frac{1}{2}$ greater than that of the second.
Ans. 1st, \$105$\frac{5}{19}$; 2d, \$157$\frac{17}{19}$; 3d, \$236$\frac{16}{19}$.

82. A and B engage in trade; A puts in \$5000, and at the end of 4 mo. takes out a certain sum. B puts in \$2500, and at the end of 5 mo. puts in \$3000 more. At the end of the year A's gain is \$1066$\frac{2}{3}$, and B's is \$1333$\frac{1}{3}$. What sum did A take out at the end of 4 mo.? *Ans.* \$2400.

83. What sum of money, with its semi-annual dividends of 5 % invested with it, will amount to \$12750 in 2 yr.? *Ans.* \$10489.459–.

84. If a speculator invests \$1500 in flour, and pays 5 % for freights, 2 % for commission, and the flour sells at 20 % advance on cost price, on a credit of 90 days, and he gets this paper discounted at bank at 7 %, and repeats the operation every 15 days, investing all the proceeds each time, how much will be his whole gain in two months?

85. If a piece of silk cost $.80 per yard, at what price shall it be marked, that the merchant may sell it at 10 % less than the marked price, and still make 20 % profit? *Ans.* 1.06\frac{2}{3}$.

86. A merchant bought 20 pieces of cloth, each piece containing 25 yd. at 4\frac{3}{8}$ per yard on a credit of 9 mo.; he sold the goods at 4\frac{5}{8}$ per yard on a credit of 4 mo. What was his net cash gain, money being worth 6 %? *Ans.* $173.85.

87. A owes B $1200, to be paid in equal annual payments of $200 each; but not being able to meet these payments at their maturities, and having an estate 10 years in reversion, he arranges with B to wait until he enters upon his estate, when he is to pay B the whole amount, with 8 % compound interest. What sum will B then receive? *Ans.* $1996.074+.

88. A gentleman who was entitled to a perpetuity of $3000 a year, provided in his will that, after his decease, his oldest son should receive it for 10 yr., then his second son for the next 10 yr., and a literary institution for ever afterward. What was the value of each bequest at the time of his decease, allowing compound interest at 6 %?

Ans. To oldest son, $22080.28; to second son, $12329.51; to institution, $15590.23.

89. B has 3 teams engaged in transportation; his horse team can perform the trip in 5 days, the mule team in 7 days, and the ox team in 11 days. Provided they start together, and each team rests a day after each trip, how many days will elapse before they all rest the same day? *Ans.* 23 days.

90. A man bought a farm for $4500, and agreed to pay principal and interest in 4 equal annual installments; how much was the annual payment, interest being 6 %? *Ans.* $1298.67+.

91. A bought a piece of property of B, and gave him his bond for $6300, dated Jan. 1, 1860, payable in 6 equal annual instalments of $1050, the first to be paid Jan. 1, 1861. A took up his bond Jan. 1, 1864, semi-annual discount at the rate of 6 % per annum on the two payments which fell due after Jan. 1, 1864, being deducted; what sum canceled the bond? *Ans.* $2972.54+.

92. A gentleman desires to set out a rectangular orchard of 864 trees, so placed that the number of rows shall be to the number of trees in a row, as 3 to 2. If the trees are 7 yards apart, how much ground will the orchard occupy? *Ans.* 39445 sq. yd.

93. S. C. Wilder bought 25 shares of bank stock at an advance of 6 % on the par value of $100. From the time of purchase until the end of 3 yr. 3 mo. he received a semi-annual dividend of 4 %, when he sold the stock at a premium of 11 %. Money being worth 7 % compound interest, how much did he gain? *Ans.* $137.31.

THE METRIC SYSTEM

OF

WEIGHTS AND MEASURES.*

INTRODUCTION.

The metric system of weights and measures — so called, because the metre is the unit from which the other units of the system are derived — had its origin in France during the Revolution, a time when all regard for institutions of the past was repudiated. In the year 1790, the French government resolved to introduce a new system; and, in order that it might be received with general favor, other countries were invited to join with it in the choice of new units. In response to this invitation, a large number of scientific men, commissioned by various countries, met in Paris, in consultation with the principal men of France. In the year 1791, a commission, nominated by the Academy of Sciences, was appointed by the Government to prepare the new system. The first work of the commission was to select a standard of lengths from which the system of units adopted might at any time be restored if from any cause the original unit should be lost. A quadrant of the earth's meridian was chosen as the standard, and the ten-millionth part of it taken as the unit of lengths, which was called a metre. In 1795, this standard and a provisional metre whose length was determined from measurements

* M. McVicar, A.M., Principal of the State Normal and Training School at Brockport, N.Y., a most thorough and critical scholar as well as teacher, prepared this article, which contains many practical improvements in Notation, Nomenclature, and Applications, not before presented to the public.

of the earth's meridian, which had already been made, was adopted by the government.

In the meantime, two eminent astronomers, Mechain and Delambre, were engaged in determining the exact length of the arc of the meridian between Dunkirk in the north of France, and Barcelona in Spain. At a later period, Biot and Arago measured the prolongation of the same meridian as far as the island of Formentara. From these measurements, together with one formerly made in Peru, they deduced, as they supposed, the exact distance from the equator to the pole, which differed slightly from the standard assumed in 1795. In 1799, a law was passed changing the length of the metre adopted in 1795 so as to conform with this difference. The metre thus determined was marked by two very fine parallel lines drawn on a platinum bar, and deposited for preservation in the national archives.

While a part of the commission were engaged in establishing the exact length of the metre, other members pursued a course of investigation for the purpose of determining a unit of weights, which would sustain an invariable relation to the unit of lengths. As the result of their investigations, the weight of a cube of pure water whose edge was one-hundredth part of a metre was the unit chosen. The water was weighed in a vacuum, at a temperature of 4° C., or 39.2° F., which was supposed to be the temperature of greatest density. This weight was called a *gramme;* and a piece of platinum weighing one thousand grammes was deposited as the standard of weights in the national archives.

Had the work of the commission ended in determining these standards of lengths and weights, their labor would have been futile. For, while the conception of basing their system upon an absolute standard in nature was good, the execution proved a failure. Later investigations have shown that the metre is less than the ten-millionth part of the earth's meridian; consequently the metric system of weights and measures is referable not to an invariable standard in nature, but to the platinum metre deposited in the national archives of France. The great benefits which result from the labors of the commission arise from the adoption of the decimal scale of units, and a simple yet general and expressive nomenclature. The amount of time and money used in carrying on exchanges between different countries, which would be saved by the universal adoption of this system, is incalculable. The system was declared obligatory throughout the whole of France after Nov. 2, 1801; but, owing to the prejudices of the people in favor of established customs, and the confusion consequent upon the use of the new measures, the Government, in 1812, adopted a compromise, in the *système usuile*, whose principal units were the new ones, while the divisions and names were nearly those formerly in use, ascending commonly in the ratios of two, three, four, eight, or twelve. In 1837, the government abolished this system, and enacted a law attaching a penalty to the use of any other than the metric system after Jan. 1, 1841. Since that time, the system has been adopted by Spain, Belgium, and Portugal, to the exclusion of other weights and measures. In Holland, other weights are used only in compounding medicines. In 1864, the system was legalized in Great Britain; and its use, either as a whole or in some of its parts, has been authorized in Greece, Italy, Norway, Sweden, Mexico, Guatemala, Venezuela, Ecuador, United States of Columbia, Brazil, Chili, San Salvador, and Argentine Republic. In 1866, Congress authorized the metric system in the United States by passing the following bills and resolution: —

An Act to authorize the use of the Metric System of Weights and Measures.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That, from and after the passage of this Act, it shall be lawful throughout the United States of America to employ the Weights and Measures of the Metric System; and no contract or dealing, or pleading in any court, shall be deemed invalid, or liable to objection, because the weights or measures expressed or referred to therein are weights or measures of the Metric System.

Section 2. *And be it further enacted,* That the tables in the schedule hereto annexed shall be recognized in the construction of contracts, and in all legal proceedings, as establishing, in terms of the weights and measures now in use in the United States, the equivalents of the weights and measures expressed therein in terms of the Metric System; and said tables may be lawfully used for computing, determining, and expressing in customary weights and measures, the weights and measures of the Metric System.

A Bill to authorize the Use in Post Offices of the Weights of the Denomination of Grammes.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That the Postmaster General be, and he is hereby, authorized and directed to furnish to the post-offices exchanging mails with foreign countries, and to such other offices as he shall think expedient, postal balances denominated in grammes of the metric system; and, until otherwise provided by law, one-half ounce avoirdupois shall be deemed and taken for postal purposes as the equivalent of fifteen grammes of the metric weights, and so adopted in progression; and the rates of postage shall be applied accordingly.

Joint Resolution to enable the Secretary of the Treasury to furnish to each State one set of the Standard Weights and Measures of the Metric System.

Be it resolved by the Senate and House of Representatives of the United States of America in Congress assembled, That the Secretary of the Treasury be, and he is hereby, authorized and directed to furnish to each State, to be delivered to the governor thereof, one set of the standard weights and measures of the metric system, for the use of the States respectively.

TABLES AUTHORIZED BY CONGRESS.

MEASURES OF LENGTHS.

Metric Denominations and Values.		Equivalents in Denominations in use.
Myriametre,...	10,000 metres,......	6.2137 miles.
Kilometre, ...	1,000 metres,......	0.62137 miles, or 3280 feet, 10 inches.
Hectometre,...	100 metres,......	328 feet and 1 inch.
Decametre, ...	10 metres,......	393.7 inches,
Metre,........	1 metre, ...	39.37 inches.
Decimetre,....	$\frac{1}{10}$ of a metre, ..	3.937 inches.
Centimetre, ...	$\frac{1}{100}$ of a metre, ..	0.3937 inch.
Millimetre,....	$\frac{1}{1000}$ of a metre, ..	0.0394 inch.

MEASURES OF SURFACES.

Metric Denominations and Values.		Equivalents in Denominations in use.
Hectare,.......	10,000 square metres,	2.471 acres.
Are, ...	100 square metres,	119.6 square yards.
Centiare, ...	1 square metre,	1550 square inches.

MEASURES OF CAPACITY.

Metric Denominations and Values.			Equivalents in Denominations in use.	
Names.	No. of litres.	Cubic Measure.	Dry Measure.	Liquid or wine measure.
Kilolitre, or stere,	1000	1 cubic metre, ...	1.308 cubic yd.	264.17 gallon.
Hectolitre, ...	100	$\frac{1}{10}$ of a cubic metre, ...	2 bu. 3.35 pk...	26.417 gallon.
Decalitre, ...	10	10 cubic decimetres, ...	9.08 quarts, ...	2.6417 gallon.
Litre, ...	1	1 cubic decimetre, ...	0.908 quart, ...	1.0567 quart.
Decilitre, ...	$\frac{1}{10}$	$\frac{1}{10}$ of a cubic decimetre,	6.1022 cubic in.	0.845 gill.
Centilitre, ...	$\frac{1}{100}$	10 cubic centimetres, ..	0.6102 cubic in.	0.338 fluid oz.
Millilitre, ...	$\frac{1}{1000}$	1 cubic centimetre, ...	0.061 cubic in. .	0.27 fluid dr.

WEIGHTS.

Metric Denominations and Values.			Equivalents in Denominations in use.
Names.	Number of grammes.	Weight of what quantity of water at maximum density.	Avoirdupois weight.
Millier, or tonneau, .	1,000,000	1 cubic metre, ...	2204.6 pounds.
Quintal, ...	100,000	1 hectolitre, ...	220.46 pounds.
Myriagramme, ...	10,000	10 litres, ...	22.046 pounds.
Kilogramme, or kilo,	1,000	1 litre, ...	2.2046 pounds.
Hectogramme, ...	100	1 decilitre, ...	3.5274 ounces.
Decagramme, ...	10	10 cubic centimetres, ...	0.3527 ounce.
Gramme, ...	1	1 cubic centimetre, ...	15.432 grains.
Decigramme, ...	$\frac{1}{10}$	1-10 of a cubic centimetre,	0.5432 grain.
Centigramme, ...	$\frac{1}{100}$	10 cubic millimetres, ...	0.1543 grain.
Milligramme, ...	$\frac{1}{1000}$	1 cubic millimetre, ...	0.0154 grain.

NOTE. — The spelling in the above tables is not the same as in the tables in the schedule annexed to the report of the committee of the House of Representatives on weights and measures. The change is not made to indicate any preference for any standard upon this subject; but to carry out what the author believes to be an essential condition to the utility and success of the system.

As remarked by a distinguished senator when the tables were adopted by Congress, "*The names are cosmopolitan;" and to retain this character fully, the spelling must also be cosmopolitan.*

The French introduced the nomenclature and spelling; and, so long as the names remain unchanged, the spelling should be retained.

NOMENCLATURE AND TABLES.

There are eight kinds of quantities for which tables are usually constructed; viz., Lengths, Surfaces, Volumes or Solids, Capacities, Weights, Values, Times, and Angles or Arcs. The table for Times is the same in the metric as in the ordinary system. The table for Angles is constructed upon a centesimal scale. The tables for the other six kinds of quantities are constructed upon a decimal scale. In each of the tables for Lengths, Surfaces, Volumes, Capacities, and Weights, there are eight denominations of units, — one principal and seven derivative. The principal units are the *metre*, which is the base of the system, and those derived directly from it. The two following tabular views present the facts regarding the principal and derivative units, which should be fixed in the memory.

PRINCIPAL UNITS.	I. METRE, ..	1. Principal unit of Lengths. 2. The base of the metric system, and nearly one ten-millionth part of a quadrant of the earth's meridian. 3. Equivalent, 39.3708 inches.
	II. ARE, . . .	1. Principal unit of surfaces. 2. A square whose side is ten metres. 3. Equivalent, 119.6 square yards.
	III. STERE, . .	1. Principal unit of volumes or solids. 2. A cube whose edge is one metre. 3. Equivalent, 1.308 cubic yards.
	IV. LITRE, . .	1. Principal unit of capacities. 2. A vessel whose volume is equal to a cube whose edge is one-tenth of a metre. 3. Equivalent, .908 quart dry measure, or 1.0567 quarts wine measure.
	V. GRAMME,	1. Principal unit of weights. 2. The weight of a cube of pure water whose edge is .01 of a metre. 3. The water must be weighed in a vacuum 4° C., or 39.2° F. 4. Equivalent, 15.432 grains.

DERIVATIVE UNITS.

I. How Derived.

1. Three orders of smaller units, or submultiples of each kind, are formed by dividing each of the principal units into tenths, hundredths, and thousandths.
2. Four orders of larger units, or multiples of each kind, are formed by considering as a unit ten times, one hundred times, one thousand times, and ten thousand times, each of the principal units.

II. Names how Formed.

1. General Principle.

The names of derivative units are formed by attaching a prefix to the name of the principal unit from which they are derived, which indicates their relation to the principal unit.

2. For Submultiples, Latin Ordinals are used as Prefixes.

1. Millesimus, one thousandth, contracted Milli. *Example*, Millilitre $= \frac{1}{1000}$ of a litre; 8 millilitres $= \frac{8}{1000}$ of a litre.
2. Centesimus, one hundredth, contracted centi. *Ex.*, Centiare $= \frac{1}{100}$ of an are; 4 centiares $= \frac{4}{100}$ of an are.
3. Decimus, tenth, contracted deci. *Ex.*, Decimetre $= \frac{1}{10}$ metre; 3 decimetres $= \frac{3}{10}$ metre.

3. For Multiples, Greek Cardinals are used as Prefixes.

1. Deca, ten. *Example*, Decametre = 10 metres; 5 decametres = 50 metres.
2. Hecaton, one hundred, contracted hecto. *Ex.*, Hectolitre = 100 litres; 7 hectolitres = 700 litres.
3. Kilioi, one thousand, contracted kilo. *Ex.* Kilogramme = 1000 grammes.
4. Myria, ten thousand. *Ex.*, Myriastere = 10,000 steres; 3 myriasteres = 30,000 steres.
5. The *a* in deca and myra, and the *o* in hecto and kilo, are dropped when prefixed to *are*.

III. Order of Progression in Tables.

The tables being constructed upon a decimal scale, ten units of a lower order make one of the next higher, thus: 10 millimetres = 1 centimetre; 10 centimetres = 1 decimetre; 10 decimetres = 1 metre; 10 metres = 1 decametre, &c.

The facts in the preceding views being mastered, the tables can be constructed by the pupil at sight. For example: The names of the derivative units are formed by attaching the seven prefixes, in their order, to the principal units of the tables. The order of progression being ten, the table of capacities will be written thus:—

10 Millilitres	= 1 Centilitre.	10 Litres	= 1 Decalitre.
10 Centilitres	= 1 Decilitre.	10 Decalitres	= 1 Hectolitre.
10 Decilitres	= 1 Litre.	10 Hectolitres	= 1 Kilolitre.
10 Kilolitres	= 1 Myrialitre.		

All the tables peculiar to the Metric System are presented together in a convenient form in the two following tables:—

TABLE OF SUBMULTIPLES AND PRINCIPAL UNITS.

Names of Units.		Pronunciation.	Symbols.
Prefix.	Base.		
	Metre	Mill′-e-mee′-ter	${}_3M$
10 Milli-	Are	Mill′-e-âre	${}_3A$
Equal	Stere	Mill′-e-stêr	${}_3S$
1 Centi-	Litre	Mill′-e-li′-ter	${}_3L$
	Gramme	Mill′-e-gram	${}_3G$
	Metre	Sent′-e-mee′-ter	${}_2M$
10 Centi-	Are	Sent′-e-âre	${}_2A$
Equal	Stere	Sent′-e-stêr	${}_2S$
1 Deci-	Litre	Sent′-e-li′-ter	${}_2L$
	Gramme	Sent′-e-gram	${}_2G$
	Metre	Des′-e-mee′-ter	${}_1M$
10 Deci-	Are	Des′-e-âre	${}_1A$
Equal	Stere	Des′-e-stêr	${}_1S$
1 Principal Unit.	Litre	Des′-e-li′-ter	${}_1L$
	Gramme	Des′-e-gram	${}_1G$
	Metre	Mee′ter	M
10 Principal Units	Are	Are	A
Equal	Stere	Stêr	S
1 Deca-	Litre	Li′-ter	L
	Gramme	Gram	G

TABLE OF MULTIPLES.

Names of Units. Prefix.	Base.	Pronunciation.	
10 Deca- Equal 1 Hecto-	Metre	Dek′-a-mee-ter	^1M
	Are	Dek′-âre	^1A
	Stere	Dek′-a-stêr	^1S
	Litre	Dek′-a-li′-ter	^1L
	Gramme	Dek′-a-gram	^1G
10 Hecto- Equal 1 Kilo-	Metre	Hec′-to-mee-ter	^2M
	Are	Hec′-târe	^2A
	Stere	Hec′-to-stêr	^2S
	Litre	Hec′-to-li′-ter	^2L
	Gramme	Hec′-to-gram	^2G
10 Kilo- Equal 1 Myria-	Metre	Kill′-o-mee-ter	^3M
	Are	Kill′-âre	^3A
	Stere	Kill′-o-stêr	^3S
	Litre	Kill′-o-li′-ter	^3L
	Gramme	Kill′-o-gram	^3G
Myria-	Metre	Mir′-e-a-mee-ter	^4M
	Are	Mir′-e-âre	^4A
	Stere	Mir′-e-a-stêr	^4S
	Litre	Mir′-e-a-li′-ter	^4L
	Gramme	Mir′-e-a-gram	^4G

ABBREVIATED NOMENCLATURE.

To secure the fullest advantage to business men by the universal adoption of the new system of weights and measures, it is necessary that the names used should be short and easy to write and pronounce, that they should express clearly the relation of the different denominations of the same table to each other, and that they should be identical in all languages.

The last two of these requirements would be secured by the universal use of the nomenclature adopted by the French. It is cosmopolitan in its character: it belongs to their language no more than to any other.* The former, however, is not secured. It is evident to all, that, for business purposes, the long names of the metric system are inconvenient, and that to shorten them would prove a great advantage. Efforts have been made to introduce short names; but these efforts have invariably sacrificed their universal and expressive character, which is of more importance to the business world than their shortness.

The only true course which seems to be open, is to abbreviate the names already introduced, in such a way as to retain their peculiar characteristics.

To secure this, the following plan of abbreviation is suggested: —

First. Let the prefixes be abbreviated thus: Myr, kil, hect, dec, des, cent, mil.

Second. Let the initial letter of the names of the five principal units be used, instead of the names themselves, thus: For metre, use a capital M; for are, use a capital A; for stere, a capital S; for litre, a capital L; and, for gramme, a capital G.

Third. For the names of multiples and sub-multiples, attach to these initial capital letters the abbreviated prefixes, thus: Kil M, pronounced kill-em′; Kil S, pronounced kill-ess′, &c.

By this method of abbreviation, the elements of the original terms are retained in such a form that each part is clearly indicated. The capital letter used after the prefix will always point to the base-word of which it is the initial, although the pronunciation is changed.

TABLES WITH ABBREVIATED NOMENCLATURE.

MEASURES OF LENGTHS.

Written.	Pronounced.		
10 Mil M,	Mill-em′,	make	1 Cent M.
10 Cent M,	Cent-em′,	"	1 Des M.
10 Des M,	Des-em′	"	1 M.
10 M,	Em	"	1 Dec M.
10 Dec M,	Dek-em′,	"	1 Hect M.
10 Hect M,	Hect-em′,	"	1 Kil M.
10 Kil M,	Kill-em′,	"	1 Myr M.*
Myr M,	Mir-em′.		

MEASURES OF SURFACES.

Written.	Pronounced.		
10 Mil A,	Mill-ā′,	make	1 Cent A.
10 Cent A,	Cent-ā′,	"	1 Des A.
10 Des A,	Des-ā′,	"	1 A.
10 A,	Ā,	"	1 Dec A.
10 Dec A,	Dek-ā′,	"	1 Hect A.
10 Hect A,	Hect-ā′,	"	1 Kil A.
10 Kil A,	Kill-ā′,	"	1 Myr A.
Myr A,	Mir-ā′.		

MEASURES OF VOLUMES, OR SOLIDS.

Written.	Pronounced.		
10 Mil S,	Mill-ess′,	make	1 Cent S.
10 Cent S,	Cent-ess′,	"	1 Des S.
10 Des S,	Des-ess′,	"	1 S.
10 S,	Ess,	"	1 Dec S.
10 Dec S,	Dek-ess′,	"	1 Hect S.
10 Hect S,	Hect-ess′,	"	1 Kil S.
10 Kil S,	Kill-ess′,	"	1 Myr S.
Myr S,	Mir-ess′.		

MEASURES OF CAPACITY.

Written.	Pronounced.		
10 Mil L,	Mill-ell′,	make	1 Cent L.
10 Cent L,	Cent-ell′,	"	1 Des L.
10 Des L,	Dess-ell′	"	1 L.
10 L,	Ell,	"	1 Dec L.
10 Dec L,	Dek-ell′,	"	1 Hect L.
10 Hect L,	Hect-ell′,	"	1 Kil L.
10 Kil L,	Kill-ell′,	"	1 Myr L.
Myr L,	Mir-ell′.		

MEASURES OF WEIGHTS.

Written.	Pronounced.		
10 Mil G,	Mill-gee′,	make	1 Cent G.
10 Cent G,	Cent-gee′,	"	1 Des G.
10 Des G,	Des-gee′,	"	1 G.
10 G,	Gee,	"	1 Dec G.
10 Dec G,	Dek-gee′,	"	1 Hect G.
10 Hect G,	Hect-gee′,	"	1 Kil G.
10 Kil G,	Kill-gee′,	"	1 Myr G.
Myr G,	Mir-gee′.		

NOTATION AND NUMERATION.

In the practical application of the metric system, it is not always convenient to use the principal units as the unit of number. For example: Should the gramme, the principal unit of weight, be used as the unit of number, in the grocery or any similar business, small quantities would be expressed by inconveniently large numbers. Example: 386 lbs. are expressed by 175,000 grammes. To avoid this inconvenience, the higher denominations are used as the unit of number when large quantities are measured.

No general system of notation is yet agreed upon. The same quantity is written in various ways by different authors. Example: 42 metres, 8 decimetres, and 5 centimetres, are written

$$42.85\text{ M.} \qquad 42^{\text{m}}85. \qquad 4\overset{\text{m}}{2}.8\overset{\text{cm}}{5}. \qquad \text{M } 42.85. \qquad \text{\&c.}$$

Inasmuch as the principal units of measure are not always used as the unit of number, it is important that a system of notation be adopted, which will apply equally well to both principal and derivative units.

It is believed that the system given below, while simple and convenient, expresses clearly the relation of the unit of number to the principal unit of measure; and, hence, has an advantage over any contractions of the names of the derivative units or arbitrary signs which might be adopted.

GENERAL PRINCIPLES OF NOTATION.

I. The scale in the metric system being decimal, the consecutive denominations are expressed by the consecutive orders of units in a number. Thus, 78642.358 metres is an expression for 7 myriametres, 8 kilometres, 6 hectometres, 4 decametres, 2 metres, 3 decimetres, 5 centimetres, 8 millimetres.

II. Whichever one of the eight denominations of units of measure is used as the unit of a number, the higher denominations are expressed as tens, hundreds, and so on; and the lower as tenths, hundredths, and so on. Example: 784.56 decametres. Here the unit of the number is a decametre; consequently the tens and hundreds are, respectively, hectometres and kilometres, and the tenths and hundredths are metres and decimetres.

From these principles and illustrations, we derive the following rule for notation: —

Rule. *Write the consecutive denominations in their order, commencing with the higher, and placing a cipher wherever a denomination is omitted, and the decimal point after the denomination which is the unit of the number.*

RULES FOR INDICATING THE DENOMINATION.

Rule I. *When a principal unit of measure is the unit of number, place the initial letter of the unit used before the number, thus:* M 342.5. *Read, three hundred and forty-two and five-tenths metres; or,* 3 *hectometres,* 4 *decametres,* 2 *metres,* 5 *decimetres.*

EXAMPLES FOR PRACTICE.

Write the numbers which represent the following quantities, considering the principal unit of measure the unit of number.

1. Seven myriametres, 4 hectometres, three decametres, and eight centimetres. *Ans.* M 70430.08.

2. Thirty-four kilometres and forty-three millimetres. *Ans.* M 34000.043.

3. Eighty-seven hectogrammes and fifty-nine centigrammes. *Ans.* G 8700.59.

4. Thirty-two myriagrammes, forty-eight decagrammes, five milligrammes. *Ans.* G 320480.005.

5. Three hundred and two kilares, eight hundred and seven centiares. *Ans.* G 302008.07.

6. Four myrialitres, sixty-two decalitres, five millilitres. *Ans.* L 40620,005.

7. Four hundred and thirty-three kilosteres, nine hundred and eighty four hectosteres, seven thousand two hundred and three centisteres. *Ans.* S 53147203.

Rule II. When a multiple of a principal unit of measure is the unit of number; — First, *Place before the number the initial letter of the principal unit from which the multiple is derived.* Second, *Indicate the order of multiple used by a small figure placed to the left and above the letter prefixed to the number.* (See symbols in table of multiples.)

Example. 42.5 kilometres, is written ^{3}M 42.5.

The M before the number indicates that the metre is the unit of measure from which the unit of the number is derived. The small 3 indicates that the third order of multiple, or kilometre, is the unit of number.

EXAMPLES FOR PRACTICE.

Write the numbers which represent the following quantities, considering the denomination named as the unit of number: —

Unit of Number, Kilogramme.

1. 43 myriagrammes, 7 decagrammes, 5 grammes. *Ans.* ^{3}G 430.075.
2. 8 kilogrammes and 3 centigrammes. *Ans.* ^{3}G 8.00003.
3. 736 hectogrammes, 243 centigrammes, and 4 milligrammes. *Ans.* ^{3}G 73.602434.
4. 2009 hectogrammes and 3 centigrammes. *Ans.* ^{3}G 200.90003.

Unit of Number, Decalitre.

5. 254 litres and 43 millilitres. *Ans.* ^{1}L 25.4043.

6. 364 myrialitres, 47 litres, 384 millilitres.
Ans. ^{1}L 364004.7384.

7. 243 decalitres, 47 centilitres. *Ans.* ^{1}L 243.047.

Unit of Number, Second Order of Multiples.

8. 23 myriametres, 72 millimetres. *Ans.* ^{2}M 2300.00072.

9. 4000007 steres and 2 millisteres. *Ans.* ^{2}S 40000.07002.

10. 3 kilares and 43 centiares. *Ans.* ^{2}A 30.0042.

Unit of Number, Myriametre.

11. 3 hectometres and 2 centimetres. *Ans.* ^{4}M .030002.

12. 5 millimetres. *Ans.* ^{4}M .0000005.

13. 3 decametres and 2 centimetres. *Ans.* ^{4}M .003002.

Rule III. When a submultiple of a principal unit of measure is the unit of number; — First, *Place before the number the initial letter of the principal unit from which the submultiple is derived.* Second, *Indicate the order of submultiple used by a small figure placed to the left and below the letter prefixed to the number.* (See symbols in table of submultiples.)

EXAMPLES FOR PRACTICE.

Write the numbers which represent the following quantities, considering the denomination named as the unit of number.

Unit of Number, Millimetre.

1. 32 decametres and 2 decimetres. *Ans.* $_{3}M$ 320200.

2. 7002 hectometres. *Ans.* $_{3}M$ 700200000.

3. 7 myriametres and 5 metres. *Ans.* $_{3}M$ 70005000.

4. 3 kilometres and 2 decametres. *Ans.* $_{3}M$ 3020000.

Unit of Number, Second Order of Submultiples.

5. 5 kilogrammes and 9 grammes. *Ans.* $_{2}G$ 500900.

6. 302 myriasteres, 5 decasteres, and 3 centisteres.
Ans. $_{2}S$ 302005003.

7. 4009 kilolitres and 5 litres. *Ans.* $_{2}L$ 400900500.

8. 2 hectares and 2 centiares. *Ans.* $_{2}A$ 20002.

Unit of Number, Decilitre.

9. 3002 hectolitres and 4 millilitres. *Ans.* $_1$L 3002000.04.
10. 6 myrialitres and 1 decalitre. *Ans.* $_1$L 600.100.
11. .404 millilitres. *Ans.* $_1$L .00004.

REDUCTION.

RULE FOR REDUCTION DESCENDING. *Multiply the given quantity by the number of the required denomination which makes a unit of the given denomination.*

Since the multiplier is always 10, 100, 1000, &c., the operation is performed by removing the decimal point as many places to the *right* as there are ciphers in the multiplier, annexing ciphers when necessary.

EXAMPLES FOR PRACTICE.

1. Reduce ^{3}M 32.58 to millimetres.
2. Reduce ^{4}M 5 to decimetres.
3. Reduce G 402 to milligrammes.
4. Reduce ^{2}A 42.3 to centiares.
5. Reduce ^{3}L 93.2 to decilitres.
6. Reduce ^{4}S 895 to decasteres.
7. Reduce ^{2}A 903.2 to milliares.
8. Reduce ^{1}G 539 to centigrammes.

RULE FOR REDUCTION ASCENDING. *Divide the given quantity by the number of its own denomination which makes a unit of the required denomination.*

Since the divisor is always 10, 100, 1000, &c., the operation is performed by removing the decimal point as many places to the *left* as there are ciphers in the divisor, prefixing ciphers when necessary.

EXAMPLES FOR PRACTICE.

1. Reduce $_2$A 5 to myriares.
2. Reduce $_3$M 403 to kilometres.
3. Reduce $_1$S 42.3 to hectosteres.
4. Reduce $_3$A 7.2 to decares.
5. Reduce $_3$G 3 to kilogrammes.
6. Reduce $_2$L 5.7 to hectolitres.
7. Reduce $_3$M 9 to myriametres.
8. Reduce $_2$S 47.3 to decasteres.

MEASURES OF SURFACES.

RELATIONS OF UNITS OF SURFACE TO UNITS OF LENGTH.

Decimilliare = One square decimetre = 100 square centimetres.

Milliare = $\left\{\begin{array}{l}\text{10 square decimetres, or a plane figure whose}\\ \text{length is one metre and breadth one decimetre.}\end{array}\right.$

Centiare = One square metre = 100 square decimetres.

Deciare	=	10 square metres, or a plane figure whose length is one decametre and breadth one metre.
Are	=	One square decametre = 100 square metres.
Decare	=	10 square decametres, or a plane figure whose length is one hectometre and breadth one decametre.
Hectare	=	One square hectometre = 100 square decametres.
Kilare	=	10 square hectometres, or a plane figure whose length is one kilometre and breadth one hectometre.
Myriare	=	One square kilometre = 100 square hectometres.

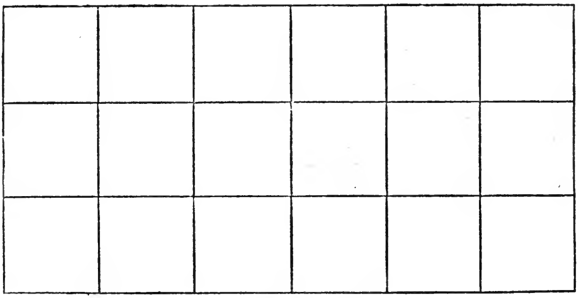
NUMERAL EXPRESSION FOR SURFACE.

The contents of a plane figure is expressed numerically by giving the number of times it contains some given area, which is assumed as the unit of surface.

The following illustrations will show how the various denominations of the table are used in numerical expressions of surface : —

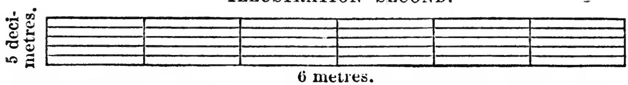
ILLUSTRATION FIRST.

Breadth 3 metres.



Length 6 metres.

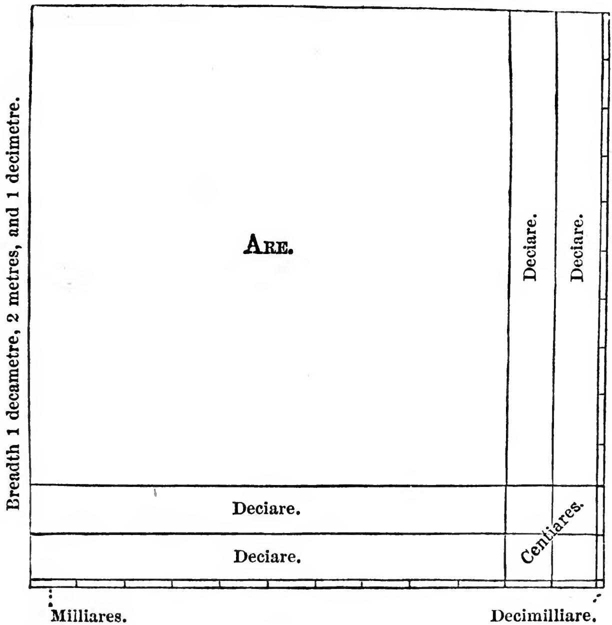
It will be seen, by examining this figure, that the lines drawn parallel to the sides, at the supposed distance of a metre from each other, divide the surface into square metres, and that there are as many rows of square metres as there are metres in the breadth, each row containing as many square metres as there are metres in the length. Hence the number of square metres in the area of the figure is equal to the product of the two numbers which indicate the length and breadth ; and A 0.18 is a numerical expression for its contents.

ILLUSTRATION SECOND.

In this figure, the lines drawn parallel to the sides divide the figure into 36 milliares, or oblongs, whose length is one metre and breadth one decimetre. It is evident that ten of these oblongs put together will constitute a centiare, or square metre. Hence the expression, 36 milliares, may be written 3.6 centiares; and read, three and six tenths centiares, or three centiares and six milliares.

By reducing the length to decimetres, the numerical expression of the contents will be, by Illustration First, 60×6, or 360 decimilliares or square decimetres.

ILLUSTRATION THIRD.

Length 1 decametre, 2 metres, and 1 decimetre.



In this figure, we have illustrated the relations of different denominations of units in expressing the contents of a given surface.

In the following analysis, each part of the contents is presented separately, as it would be obtained by multiplying the length by the breadth. The learner should carefully note each part, and analyze a sufficient number of examples to fix the principles in the mind.

ANALYSIS.

^{1}M 1.21 × ^{1}M 1.21 =

One decimetre ×	One decimetre = 1 decimilliare		= A 0.0001
	Two metres = 2 milliares		= A 0.002
	One decametre = 10 milliares = 1 centiare		= A 0.01
Two metres ×	One decimetre = 2 milliares		= A 0.002
	Two metres = 4 centiares		= A 0.04
	One decametre = 2 deciares		= A 0.2
One decametre ×	One decimetre = 10 milliare = 1 centiare		= A 0.01
	Two metres = 2 deciares		= A 0.2
	One decametre = 1 are or square metre		= A 1.

^{1}M 1.21 × ^{1}M 1.21 = A 1.4641

From these illustrations, we derive the following rule for finding a numerical expression for a given surface of uniform length and breadth: —

RULE. *Reduce the length and breadth to the same denomination; find the product of the two dimensions after reduction, and point off as many decimal places in this product as there are decimal places in the two dimensions.*

The unit of the numerical expression thus found will be a decimilliare when the unit of length is a decimetre, a centiare when the unit of length is a metre, an are when the unit of length is a decametre, a hectare when the unit of length is a hectometre, and a myriare when the unit of length is a kilometre.

EXAMPLES FOR PRACTICE.

1. How many ares in a floor M 1.25 long, and M 8.7 wide?
Ans. A .10875.

2. How many centiares, how many kilares, and how many hectares in the same floor? *Ans.* $_2$A 10.875.

3. How many ares in a board M 5.32 by $_2$M 47.?
Ans. A .025004.

4. How many milliares, how many myriares, and hectares in the same board?

5. How many metres of a carpet nine decimetres wide will cover

a floor six metres long and five and four-tenths metres wide? and what would be the cost of the carpet, at \$2.50 a centiare?

Ans. M 36. \$90.

6. In a farm consisting of four fields of the following dimensions, how many hectares? First field, length M 342, breadth M 273; second field, length M 634, breadth M 350; third field, length M 450, breadth M 329; fourth field, length M 730, breadth M 632.7.

Ans. ^{2}A 92.5187.

7. A pile of lumber was found to contain 150 boards M 4 long and ${}_1$M 4. wide, 225 boards M 6.2 long and ${}_2$M 52. wide, and 642 boards M 5.2 long and ${}_2$M 43 wide. How much was it worth, at \$42. per are, face measure.

Ans. \$1008.38 +.

8. How many bricks ${}_1$M 2.2 × ${}_1$M 1.1 would pave a side-walk M 842.6 long and M 2.2 wide? and what would be the whole cost at 82 cents per centiare.

Ans. 76600 bricks. \$1520.05 +.

MEASURES OF VOLUMES, OR SOLIDS.

RELATIONS OF UNITS OF VOLUMES TO UNITS OF LENGTHS.

Millistere = A cubic decimetre = 1000 cubic centimetres.

Centistere = 10 cubic decimetres, or a volume, or solid, whose length is one metre, and breadth and thickness one decimetre.

Decistere = 10 centisteres = 100 cubic decimetres, or a volume whose length and breadth is one metre, and thickness one decimetre.

Stere = A cube metre = 10 decisteres = 100 centisteres = 1000 millisteres or cubic decimetres.

Decastere = 10 cubic metres, or a volume whose length is one decametre, and breadth and thickness one metre.

Hectostere = 10 decasteres = 100 cubic metres, or a volume whose length and breath is one decametre, and thickness one metre.

Kilostere = A cubic decametre = 1000 cubic metres.

Myriastere = 10 kilosteres, or a volume whose length is one hectometre, and breadth and thickness each one decametre.

NUMERICAL EXPRESSION FOR VOLUME, OR SOLIDITY.

The solidity, or contents, of a volume is expressed numerically by giving the number of times it contains some given solid as the unit of volume.

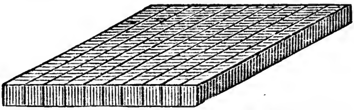
The following illustrations will show how the various denominations of the table are used in numerical expressions of volume.

Millistere, or Cubic Decimetre.

10 millisteres, placed side by side, make a volume whose length is one metre, and breadth and thickness each one decimetre, thus, —

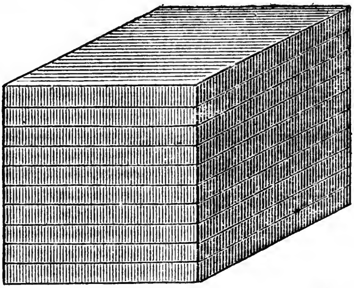
Centistere = 10 Millisteres.

10 centistere, placed side by side, make a volume whose length and breadth is each one metre, and thickness one decimetre, thus, —



Decistre = 10 Centisteres = 100 Millisteres.

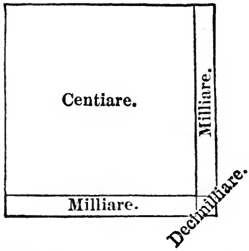
10 decisteres, placed face to face, make a cube whose edge is one metre, thus, —



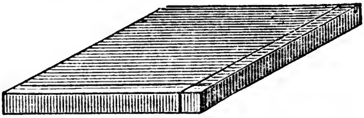
Stere = 10 Decisteres = 100 Centisteres = 1000 Millisteres.

From these illustrations, it is evident that the contents of a cubic metre may be expressed numerically, as S 1, $_1$S 10, $_2$S 100, $_3$S 1000.

The following figures illustrate the use of the same four denominations in expressing the contents of a cubic volume whose edge is one metre and one decimetre. The surface of one face of the volume contains one centiare, two milliares, and one decimilliare, thus, —



Taking a slab of the face one decimetre thick, thus, —



and we have one decistere, two centisteres, and one millistere. But the volume is eleven decimetres thick; therefore we have eleven such slabs, or eleven times one decistere, two centisteres, and one millistere.

$$= \begin{cases} 11 \text{ millisteres} = 1 \text{ centistere and } 1 \text{ millistere} = \text{S } 0.011 \\ 22 \text{ centisteres} = 2 \text{ decisteres and } 2 \text{ centisteres} = \text{S } 0.22 \\ 11 \text{ decisteres} = 1 \text{ stere and } 1 \text{ decistere} = \text{S } 1.1 \end{cases}$$

$$\text{M } 1.1 \times \text{M } 1.1 \times \text{M } 1.1 = \text{S} 1.331$$

From these illustrations, we derive the following rule for finding a numerical expression for a given volume of uniform length, breadth, and thickness: —

Rule. *Reduce the length, breadth, and thickness to the same denomination; find the product of the three dimensions, after reduction, and point off as many decimal places in this product as there are decimal places in the three dimensions.*

The unit of the numerical expression thus found will be a millistere when the unit of length is a decimetre, a stere when the unit of length is a metre, a kilostere when the unit of length is a decametre.

EXAMPLES FOR PRACTICE.

1. How many steres in a wall twenty-four metres long, eight and five-tenth metres high, and fifty-two centimetres thick? And what would be the cost of building it, at $4.25 a stere?

Ans. S 106.08. Cost, $450.84.

2. What would be the cost of a pile of wood fifteen and seven-tenths metres long, three metres high, and seven and fifty-two hundredths metres wide, at $1.50 a stere? *Ans.* $531.29.

3. What would be the cost of excavating a cellar eighteen and three-tenths metres long, ten and seventy-three hundredths metres wide, and three and four-tenths metres deep, at 15 cents per stere? *Ans.* $100.14 +.

4. How deep must a box be, whose surface is thirty-two milliares, to contain seven and thirty-six hundredths steres? *Ans.* $_1$M 23.

5. How many steres in five sticks of timber of the following dimensions: First, $_1$M 5.2 by $_1$M 7.3, and M 13 long; second, $_2$M 43. by $_2$M 65, and M 17.5 long; third, $_1$M 5.3 by $_1$M 3.7, and M 15.42 long; fourth, $_2$M 39 by $_2$M 56, and M 14 long; fifth, $_1$M 4.52 by $_1$M 3.78, and M 15 long. *Ans.* S 18.470352.

6. What must be the height of a load of wood, M 3.2 long and M 1.1 wide, to contain S 4.0128. *Ans.* M 1.14.

MEASUREMENT OF ANGLES.

In the ordinary or sexagesimal system, a right-angle, which is used as the measure of all plane angles, is divided into 90 equal parts, called degrees; a degree is divided into 60 equal parts, called minutes; and a minute into 60 equal parts, called seconds.

In the centesimal or French system, a right-angle is divided into 100 equal parts, called grades; a grade into 100 equal parts, called minutes; and a minute into 100 equal parts, called seconds.

The former is called the *sexagesimal* system, on account of the occurrence of the number *sixty* in forming the subdivisions of a degree; and the latter *centesimal*, on account of the occurrence of the number *one hundred.*

Grades, minutes, and seconds are usually written thus: $35^{g}\ 42^{\backprime}\ 24^{\backprime\backprime}$; read, thirty-five grades, forty-two minutes, twenty-four seconds.

Since the scale is centesimal, minutes may be expressed as hundredths, and seconds as ten-thousandths; hence any number of grades, minutes, and seconds may be expressed decimally thus: $73^{g}\,4569$; read, seventy-three grades, forty-five minutes, sixty-nine seconds.

In a right-angle, there are 100 grades, or 90 degrees; hence, for every 10 grades there are 9 degrees. Dividing the 10 grades into 9 equal parts or degrees, each part will contain $1\frac{1}{9}$ grades; therefore a degree is equal to $1\frac{1}{9}$ grades. Hence, in any number of grades there are as many degrees as $1\frac{1}{9}$ is contained times in the given number of grades; and, conversely, in any number of degrees there are $1\frac{1}{9}$ times as many grades as there are degrees. Hence the following rules:—

TO CHANGE THE CENTESIMAL MEASURE TO THE SEXAGESIMAL.

RULE. *Express the minutes and seconds as a decimal of a grade; divide by* $1\frac{1}{9}$: *the quotient will express the number of degrees and decimals of a degree in the given number of grades, minutes, and seconds.*

EXAMPLES.

Change the following quantities from the centesimal measure to the sexagesimal.

1.	$25^{g}\ 34^{\backprime}\ 42^{\backprime\backprime}$.	*Ans.* $22°\ 48'\ 35.208''$.
2.	$57^{\backprime}\ 93^{\backprime\backprime}$.	*Ans.* $31'\ 16.932''$.
3.	$83^{g}\ 13^{\backprime}\ 87^{\backprime\backprime}$.	*Ans.* $74°\ 49'\ 29.388''$.
4.	$36^{g}\ 98^{\backprime}\ 15^{\backprime\backprime}$.	*Ans.* $33°\ 17'\ .06''$.
5.	$14^{g}\ 15^{\backprime}\ 60^{\backprime\backprime}$.	*Ans.* $12°\ 44'\ 25.44''$.
6.	$90^{g}\ 90^{\backprime}\ 90^{\backprime\backprime}$.	*Ans.* $81°\ 49'\ 5.16''$.
7.	$18^{g}\ 50^{\backprime}\ 25^{\backprime\backprime}$.	*Ans.* $16°\ 39'\ 8.1''$.

TO CHANGE THE SEXAGESIMAL MEASURE TO THE CENTESIMAL.

RULE. *Reduce the minutes and seconds to a decimal of a degree; multiply the degrees and decimal of a degree by* $1\frac{1}{9}$: *the product is the number of grades, minutes, and seconds in the given number of degrees, minutes, and seconds.*

EXAMPLES.

Change the following quantities from the sexagesimal measure to the centesimal.

1.	$36°\ 18'\ 27''$.	*Ans.* $40^{g}\ 34^{\backprime}\ 16\frac{2}{3}^{\backprime\backprime}$.
2.	$56'\ 54''$.	*Ans.* $1^{g}\ 5^{\backprime}\ 37\frac{1}{27}^{\backprime\backprime}$.

3. $27° 36' 45''$. *Ans.* $30^{g}\ 68^{\backslash}\ 5\frac{5}{9}^{\backslash\backslash}$.
4. $189° 15' 20''$. *Ans.* $210^{g}\ 28^{\backslash}\ 39\frac{41}{81}^{\backslash\backslash}$.
5. $63° 14' 58''$. *Ans.* $70^{g}\ 27^{\backslash}\ 71\frac{49}{81}^{\backslash\backslash}$.
6. $147° 24' 48''$. *Ans.* $163^{g}\ 79^{\backslash}\ 25\frac{25}{27}^{\backslash\backslash}$.
7. $117° 36' 54'$. *Ans.* $130^{g}\ 68^{\backslash}\ 33\frac{1}{3}^{\backslash\backslash}$.

TO CHANGE THE METRIC TO THE COMMON SYSTEM.

RULE. *Reduce the given quantity to the denomination of the principal unit of the table; multiply by the equivalent, and reduce the product to the required denomination.*

1. ^{3}M 3.6, how many feet?

OPERATION.

^{3}M 3.6 × 1000 = M 3600
39.37 in. × 3600 = 141732 in.
141732 in. ÷ 12 in. = 11811 ft.

ANALYSIS. — The metre is the principal unit of the table; hence we reduce the kilometres to metres. Since there are 39.37 inches in a metre, in 3600 metres there are 3600 times 39.37 inches; and since there are 12 inches in a foot, there are as many feet as 12 inches is contained times in 141732 inches. Therefore ^{3}M 3.6 is equal to 11811 feet.

EXAMPLES FOR PRACTICE.

2. How many feet in 472 centimetres? *Ans.* $15.4855\frac{1}{3}$ ft.
3. How many cubic feet in 2 kilosteres? *Ans.* 70632 cu. ft.
4. How many gallons, wine measure, in 325 decilitres? *Ans.* 8 gals. 2.343 — qts.
5. How many gallons in 108.24 litres? *Ans.* 28.594 + gals.
6. How many bushels in 3262 kilolitres? *Ans.* 92559.25 bush.
7. How many square yards in 436 ares? *Ans.* 52145.6 sq. yds.
8. In 942325 centilitres, how many bushels? *Ans.* 267.3847 + bush.
9. In 436 myriagrammes, how many pounds? *Ans.* 9611.9314 lbs.

TO CHANGE FROM THE COMMON TO THE METRIC SYSTEM.

RULE. *Reduce the given quantity to the denomination in which the equivalent of the principal unit of the metric table is expressed; divide by this equivalent, and reduce the quotient to the required denomination.*

1. In 10 lbs. 4 oz. how many myriagrammes?

OPERATION.

10 lbs. 4 oz. = 10.25 lbs.
10.25 lbs. × 7000 = 71750 gr.
71750 gr. ÷ 15.432 gr. = G 4649.43 —
G 4649.43 — ÷ 10000 = ^{4}G .464943 — *Ans.*

ANALYSIS. — The gramme, the principal unit of the table, is expressed in grains; hence we reduce the pounds and ounces to grains. 15.432 grains make one gramme; hence there are as many grammes in 71750 grains as 15.432 grains is contained times in 71750 grains. And since there are 10000 grammes in a myriagramme, dividing G 4649.43 — by 10000 will give the myriagrammes in 10 pounds 4 ounces. Therefore, 10 lbs. 4 oz. is equal to ^{4}G 464943 —

EXAMPLES FOR PRACTICE.

2. In 6172.8 pounds, how many decagrammes?
Ans. ^{1}G 280000.
3. How many hectares in 2392 square yards? *Ans.* ^{2}A .2.
4. How many ares in a square mile?
Ans. A 25899.665552 —.
5. How many millisteres in 18924 cubic yards?
Ans. $_3$S 14467889.9082 +.
6. In 892 grains, how many hectogrammes?
Ans. ^{2}G .578019.
7. In 2 miles, 6 furlongs, 39 rods, and 5 yards, how many kilometres? *Ans.* ^{3}M 4.626416 +.
8. Bought 454 bush. wheat at $3, and sold the same at $8.75 per hectolitre; how many hectolitres did I sell? Did I gain or lose, and how much?
Ans. ^{2}L 160. Gain, $38.

MISCELLANEOUS EXAMPLES.

Required the footings of the following bills: —

(1.)

NEW YORK, April 23, 1867.

W. J. MILNE,

Bo't of L. COOLEY & SON.

M 122 Broadcloth,	@	$6.00	
" 320 Bld. Shirting,	"	.35	
" 230 White Flannel,	"	.30	
" 206.5 Ticking,	"	.31	
" 107.9 Blk. Silk,	"	2.40	
	Ans.		$1235.975

Rec'd Payment,

L. COOLEY & SON.

(2.)

BUFFALO, May 1, 1867.

CHAS. D. MCLEAN,

Bo't of WM. BENEDICT.

40 chests Tea,	each	³G 30.5	@	$ 2.50	
12 sacks Java Coffee,			"	40.00	
25 bbls. Coffee Sugar,	each	³G 110	"	.32	
10 " Crushed "	"	³G 95	"	.38	
30 boxes Raisins,	"	³G 12	"	.50	
			Ans.		$4951.00

Rec'd Payment,

WM. BENEDICT.

3. A man bought a lot of land ²M 40 long and ²M 20 wide, and sold one-third of it. How many ares had he left, and what was the cost of the lot, at $100 per acre?

Ans. to first, A 53333.33$\frac{1}{3}$. *Ans. to second,* $197685.95.

4. A farmer sold ²L 540 of wheat at $6, and invested the proceeds in coal at $8 per ton. How many myriagrammes of coal did he purchase? *Ans.* ⁴G 36741.835147 +.

5. What will be the cost of a pile of wood M 42.5 long, M 2. high, M 1.9 wide, at $2 per stere? *Ans.* $323.

6. How many metres of shirting, at \$.25 per metre, must be given in exchange for ^{2}L 300 oats, at \$1.20 per hectolitre?

Ans. M 1440.

7. A grocer buys butter at \$.28 per lb., and sells it at \$.60 per kilogramme. Does he gain or lose, and what per cent.?

Ans. Lost $2\frac{514}{643}$ %.

8. A bin of wheat measures M 5 square, and M 2.5 deep. How many hectolitres will it contain, and what will be the cost of the wheat, at \$2 per bushel? *Ans.* ^{2}L 625. \$3546.875.

9. What price per pound is equivalent to \$2.50 per ^{2}G?

Ans. \$11.34.

10. A merchant bought M 240 of silk at \$2, and sold it at \$1.95 per yard. Did he gain or lose, and how much?

Ans. Gain \$31.81.

11. Find the measure of $1^{\backprime}\ 5^{\backprime\backprime}$ in decimals of a degree.

Ans. .00945.

12. A merchant shipped to France 50 bbls. of coffee sugar, each containing 250 lbs., paying \$2 per hundred for transportation. He sold the sugar at \$.34 per kilogramme, and invested the proceeds in broadcloth at \$4 per metre How many yards of broadcloth did he purchase? *Ans.* 458.71 + yds.

13. The difference between two angles is 10 grades, and their sum is 45°. Find each angle. *Ans.* 18° and 27°.

14. Determine the number of degrees in the unit of angular measure when an angle of $66\frac{2}{3}$ grades is represented by 20.

Ans. 3°.

15. How many centiares of plastering in a house containing six rooms of the following dimensions, deducting one-twelfth for doors, windows, and base? and what would be the cost of the work at 38 cents per centiare? First room, M 6.2 × M 4.7; second room, M 4.52 × M 4; third room, M 6 × M 5.2; fourth room, M 3.82 × M 3.82; fifth room, M 7 × M 6.2; sixth room, M 4.5 × M 4.25. Height of each room, M 3.8. *Ans.* $_2$A 562.039 +. \$213.57 +.

9 Ex

100 sp = A
150 " " B
Then 90 " " A
185 " " B

∴ 185) 14800 (80 and 100 × 80 = $80 A
" 150 × 80 = $120 B

Given Sum of series and extremes to find number of terms (Arithmetical series)

Multiply

---

14 [illegible] 6% = $\frac{3}{50}$

[illegible]

[illegible] = $\frac{1}{50}$ + $\frac{3}{50}$

= $\frac{4}{50}$ + $\frac{60}{50}$ [illegible]

$\frac{1}{50}$ = $162.00 = [illegible]

3 × 3.00 = $ [illegible]

E. E. Goall

$a - b$

$x - b$

$ax - bx - ab + b^2$

$b^2 - bx - ab + ax$

$-b^2 + \ldots + a^2 b \pm abx$

$\mp a^2 b$

$a - b$

$b - bx - a^2 + ax$

G

17.57

18.60

126½

$a - b$

$x - b$

$-6x$ [illegible]

[illegible]

3 [illegible]

[illegible]

1000

18