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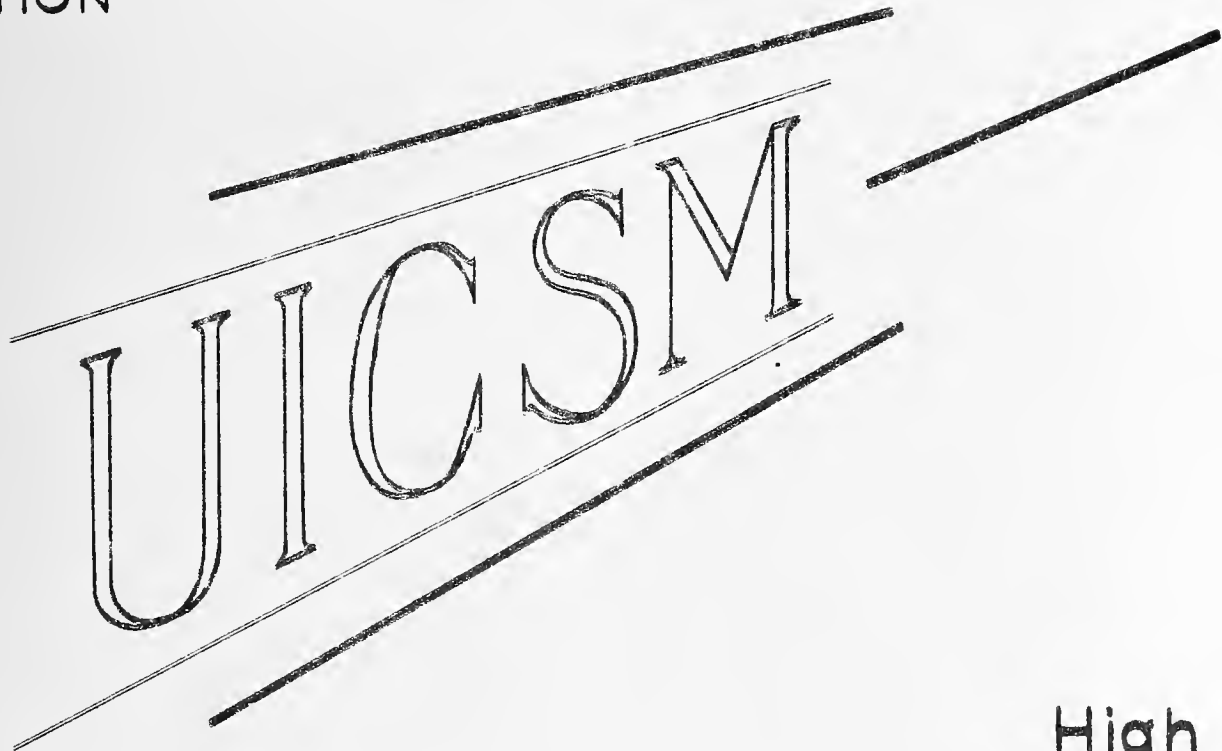
<http://www.archive.org/details/highschoolmathem03univ>







EXPERIMENTAL  
PROGRAMMED  
EDITION

The title 'UICSM' is rendered in a large, outlined, serif font. It is centered between two sets of parallel lines that create a perspective effect, sloping downwards from left to right. The top set of lines is above the letters, and the bottom set is below them, with the letters themselves appearing to sit on the bottom line.

# UICSM

High  
School  
Mathematics

PART 43

Comparative Studies of Principles  
for  
Programing Mathematics  
in  
Automated Instruction

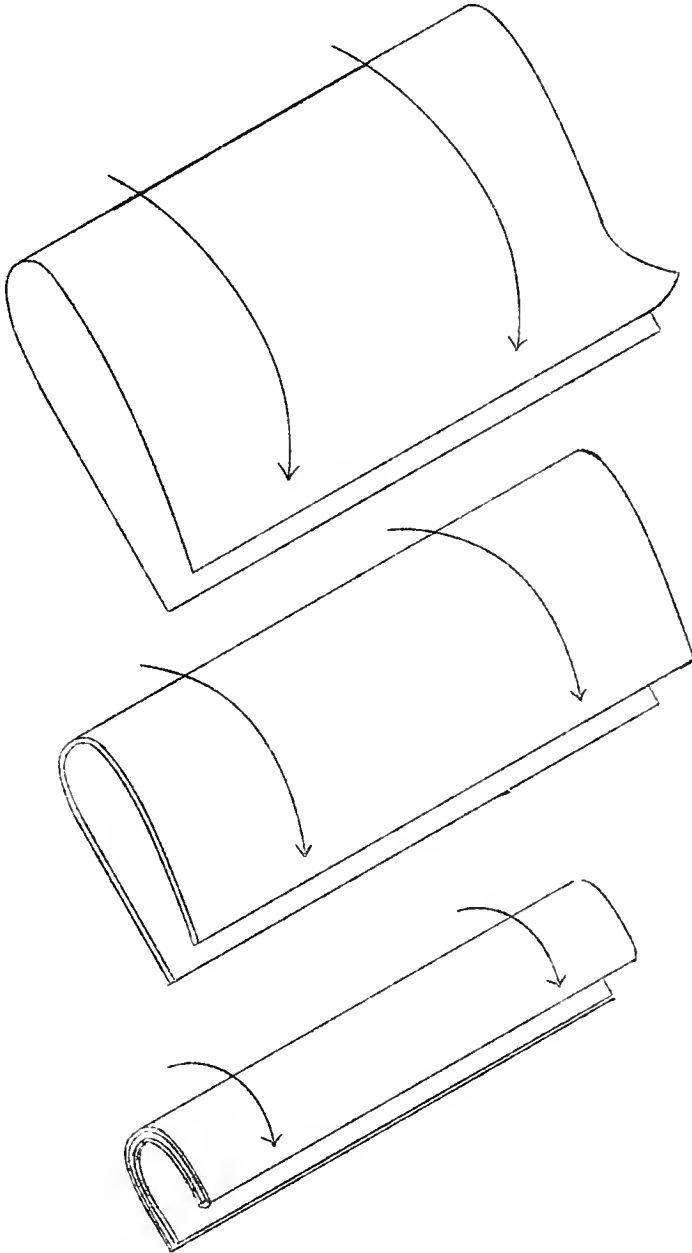
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National Science Foundation

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Fill in the heading on your work sheet.

Some of the exercises in this book will ask you to draw a line segment between two dots. You will be able to do a neater job on these exercises if you have a ruler or some other kind of "straight-edge". If you don't have a wooden or plastic straight-edge with you, you can make a perfectly good straight-edge by folding a sheet of paper several times.



Turn to PAGE 2.

The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that every entry should be supported by a valid receipt or invoice. This ensures transparency and allows for easy verification of the data.

In addition, the document outlines the procedures for handling discrepancies. If there is a difference between the recorded amount and the actual amount received or paid, it is crucial to investigate the cause immediately. This could be due to a clerical error, a missing receipt, or a change in the terms of the agreement.

The final section provides a summary of the key points and reiterates the commitment to accuracy and integrity in all financial reporting. It concludes by stating that the information provided is for informational purposes only and should not be used as a substitute for professional advice.



Imagine that you own a very smart grasshopper. This grasshopper is so smart that he has learned to play a game called a 'number plane lattice game'. In playing this game, your grasshopper needs this equipment:

- (a) a large picture of part of the number plane lattice,
- (b) a pair of dice, one red and one green, and
- (c) a pack of small cards, each card containing a rule and "jumping" instructions.

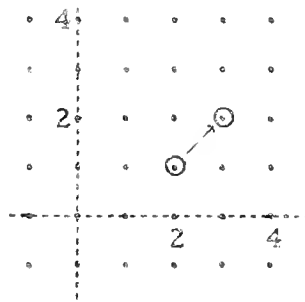
Here is how your grasshopper plays the game. First, he rolls the dice. [This is a big grasshopper, or he has very small dice!] Suppose that 2 comes up on the red die and 1 on the green die. This means that he is to start the game sitting on the dot corresponding to the point  $(2, 1)$  on the number plane lattice. Next, he turns a card face up and reads the rule and instructions. Suppose that the card says:

Rule: A jump takes you from  
 $(x, y)$  to  $(x + 1, y + 1)$ .

Instructions: Make one jump.

The grasshopper will finish this game on the dot corresponding to  $(3, 2)$ . He starts at  $(2, 1)$  and makes one jump according to the given rule. This takes him to  $(2 + 1, 1 + 1)$  or  $(3, 2)$ . Since the instructions were to make just one jump, he finishes at  $(3, 2)$ .

Here is a diagram showing his jump.



Turn to PAGE 3.



Here is how your grasshopper might play another game.

Start:  $(-4, -1)$

Rule: A jump takes you from  $(x, y)$  to  $(x + 2, y + 1)$ .

Instructions: Make 3 jumps.

Where does he finish?

Solution.

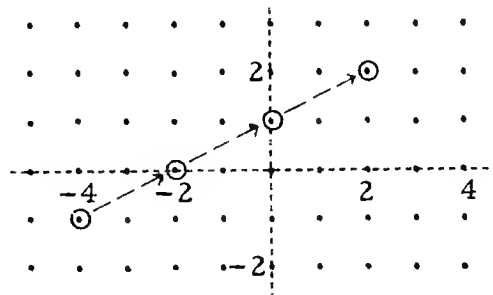
First jump: From  $(-4, -1)$  to  $(-4 + 2, -1 + 1)$  or  $(-2, 0)$

Second jump: From  $(-2, 0)$  to  $(-2 + 2, 0 + 1)$  or  $(0, 1)$

Third jump: From  $(0, 1)$  to  $(2, 2)$

So, after 3 jumps, he finishes on  $(2, 2)$ .

Here is a diagram showing his jumps.



The exercises below are about a game your grasshopper played. Answer them on your work sheet.

Start:  $(4, 1)$

Rule: A jump takes you from  $(x, y)$  to  $(x - 1, y + 1)$ .

Instructions: Make 2 jumps.

- (1) First jump: From  $(4, 1)$  to  $(3, 2)$   
 Second jump: From  $(3, 2)$  to  $(\underline{\quad ? \quad}, \underline{\quad ? \quad})$
- (2) Where did he finish?
- (3) Draw a diagram showing his jumps.

Turn to PAGE 4.



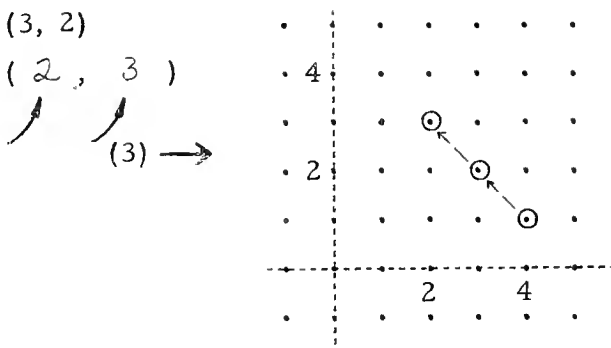
Check your answers.

Start: (4, 1)

Rule: A jump takes you from (x, y) to (x - 1, y + 1).

Instructions: Make 2 jumps.

- (1) First jump: From (4, 1) to (3, 2)  
 Second jump: From (3, 2) to (2, 3)
- (2) Finish: (2, 3)



Record your results on your work sheet.

\* \* \*

Do these exercises about another game the grasshopper played. Write your answers on your work sheet.

Start: (0, 1)

Rule: A jump takes you from (x, y) to (x + 3, y).

Instructions: Make 3 jumps.

- (1) First jump: From (0, 1) to (3, 1)  
 Second jump: From (3, 1) to (   ? ,   ? )
- (2) Third jump: From (   ? ,   ? ) to (   ? ,   ? )
- (3) Where did he finish?
- (4) Draw a diagram showing his jumps.

Turn to PAGE   5  .





Check your answers.

Start:  $(0, 1)$

Rule: A jump takes you from  $(x, y)$  to  $(x + 3, y)$ .

Instructions: Make 3 jumps.

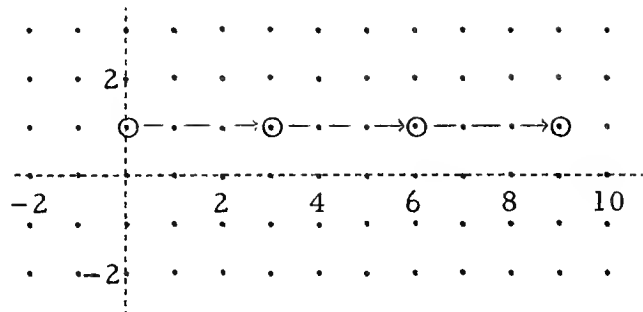
(1) First jump: From  $(0, 1)$  to  $(3, 1)$

Second jump: From  $(3, 1)$  to  $(6, 1)$

(2) Third jump: From  $(6, 1)$  to  $(9, 1)$

(3) Finish:  $(9, 1)$

(4)



Record your results on your work sheet.

Turn to PAGE 6.

The first part of the paper is devoted to the study of the asymptotic behaviour of the eigenvalues of the Dirac operator  $D_{\mu}$  on a Riemannian manifold  $(M, g)$  with a magnetic field  $\mu$ . The second part is devoted to the study of the asymptotic behaviour of the eigenvalues of the Dirac operator  $D_{\mu}$  on a Riemannian manifold  $(M, g)$  with a magnetic field  $\mu$  and a potential  $V$ . The third part is devoted to the study of the asymptotic behaviour of the eigenvalues of the Dirac operator  $D_{\mu}$  on a Riemannian manifold  $(M, g)$  with a magnetic field  $\mu$  and a potential  $V$  and a scalar curvature  $R$ . The fourth part is devoted to the study of the asymptotic behaviour of the eigenvalues of the Dirac operator  $D_{\mu}$  on a Riemannian manifold  $(M, g)$  with a magnetic field  $\mu$  and a potential  $V$  and a scalar curvature  $R$  and a volume element  $dV$ .

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Now, let's take over from the grasshopper and play some number plane lattice games.

We shall make "moves" instead of "jumps", and we shall use an abbreviated form for the rule. For example, the rule:

A move takes you from  $(x, y)$  to  $(x + 2, y - 3)$   
will be written:

$$(x, y) \rightarrow (x + 2, y - 3)$$

Do these exercises on your work sheet.

$$\text{Rule: } (x, y) \rightarrow (x + 2, y - 3)$$

Start at  $(3, 3)$  and make 3 moves.

(1) First move takes you to  $(3 + 2, 3 - 3)$  or  $(5, 0)$ .

Second move takes you to  $(\underline{\quad}, \underline{\quad})$ .

(2) Third move takes you to  $(\underline{\quad}, \underline{\quad})$ .

(3) Where do you finish?

(4) Draw a diagram showing your moves.

Turn to PAGE 7.

1. The first part of the document is a list of names.

2.

3. The second part of the document is a list of names.

4. The third part of the document is a list of names.

5. The fourth part of the document is a list of names.

6.

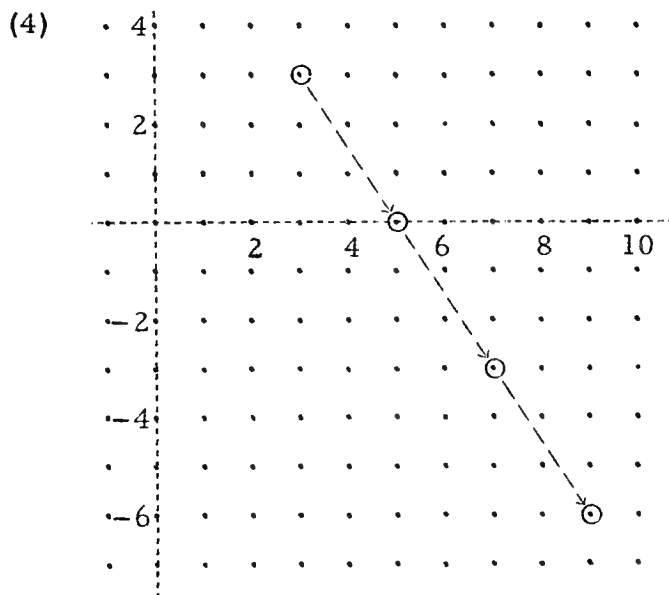
Check your answers.

Rule:  $(x, y) \rightarrow (x + 2, y - 3)$

Start at  $(3, 3)$  and make 3 moves.

(1) First move takes you to  $(5, 0)$ .      Second move takes you to  $(7, -3)$ .

(2) Third move takes you to  $(9, -6)$ .      (3) Finish:  $(9, -6)$



Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

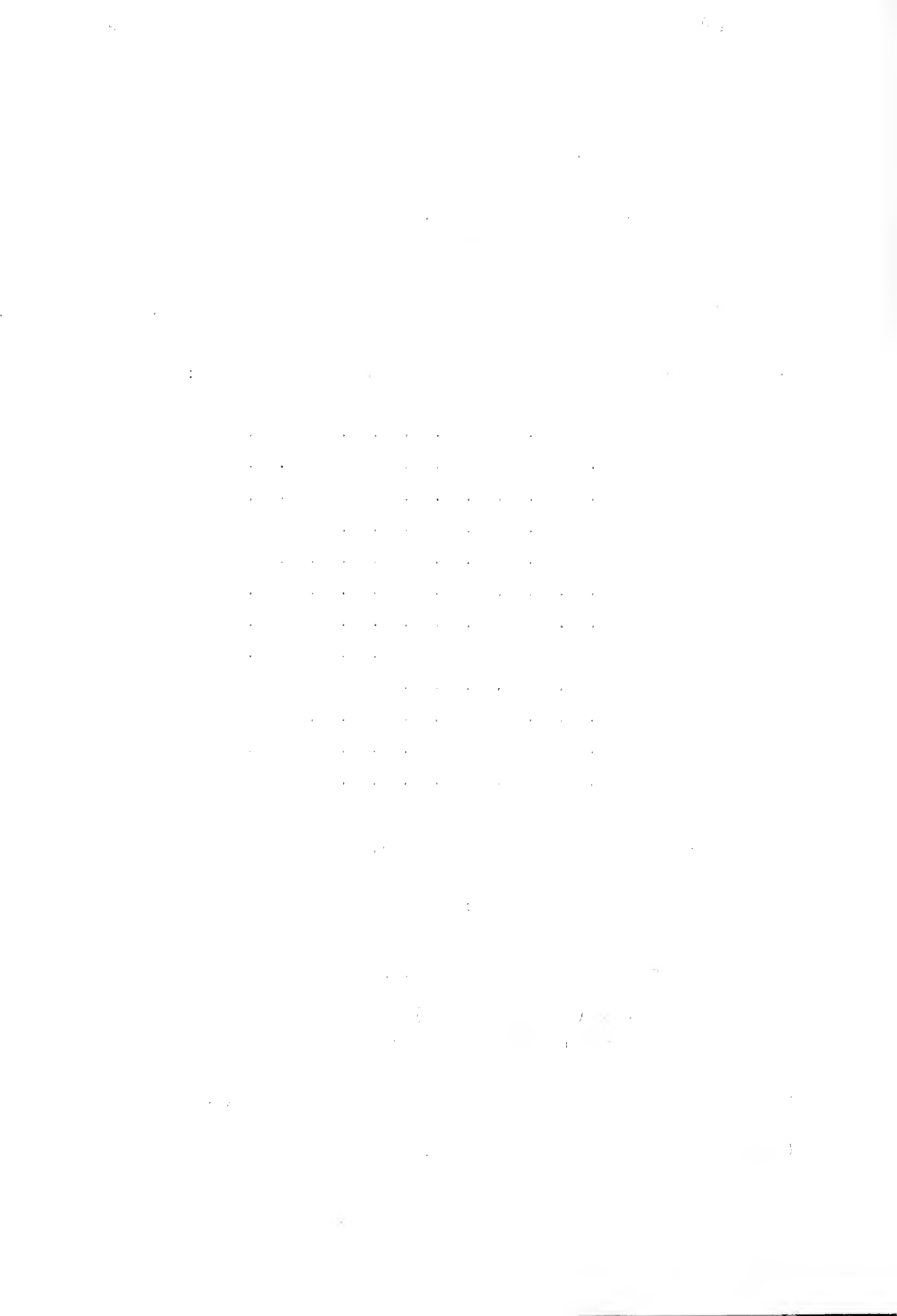
Rule:  $(x, y) \rightarrow (2x, 2y)$

Start at  $(1, 2)$  and make 2 moves.

(1) Where do you finish?      [First move takes you to  $(2, 4)$ .]

(2) Draw a diagram of your moves.

Turn to PAGE 8.

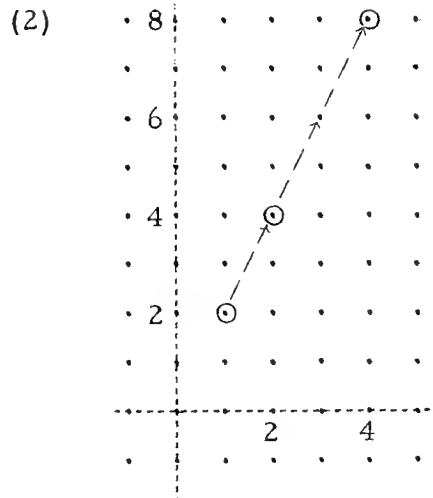


Check your answers.

Rule:  $(x, y) \rightarrow (2x, 2y)$

Start at  $(1, 2)$  and make 2 moves.

(1) Finish:  $(4, 8)$



Record your results on your work sheet.

\* \* \*

Answer this question on your work sheet.

Rule:  $(x, y) \rightarrow (3x, 2y)$

Start at  $(0, 0)$  and make 10 moves.

What is the final point?

Turn to PAGE 9.





Check your answer.

$$\text{Rule: } (x, y) \rightarrow (3x, 2y)$$

Start at  $(0, 0)$  and make 10 moves.

Finish:  $(0, 0)$

Record your results on your work sheet.

\* \* \*

Here is a sample number plane lattice game where the rule is a bit more complicated.

$$\text{Rule: } (x, y) \rightarrow (2x - 3, 3y + 1)$$

Start at  $(2, 0)$  and make 2 moves. What is the final point?

Solution.

First move: From  $(2, 0)$  to  $(2 \cdot 2 - 3, 3 \cdot 0 + 1)$ , or  $(1, 1)$

Second move: From  $(1, 1)$  to  $(2 \cdot 1 - 3, 3 \cdot 1 + 1)$ , or  $(-1, 4)$

So, the final point is  $(-1, 4)$ .

Do these exercises on your work sheet.

$$\text{Rule: } (x, y) \rightarrow (3x - 5, y + 2)$$

Start at  $(2, -2)$  and make 3 moves.

(1) First move takes you to  $(1, 0)$ .

Second move takes you to  $(\underline{\quad}, \underline{\quad})$ .

(2) Third move takes you to  $(\underline{\quad}, \underline{\quad})$ .

(3) What is the final point?

(4) Make a diagram showing your moves.

Turn to PAGE 10.

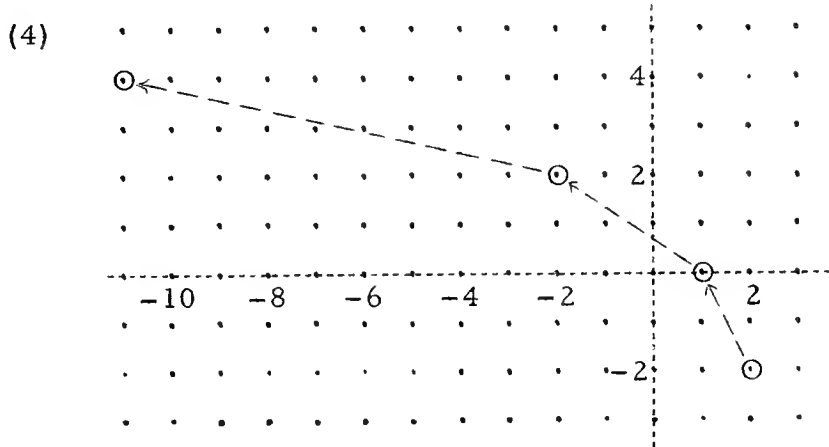


Check your answers.

Rule:  $(x, y) \rightarrow (3x - 5, y + 2)$

Start at  $(2, -2)$  and make 3 moves.

- (1) First move takes you to  $(1, 0)$ .  
Second move takes you to  $(-2, 2)$ .
- (2) Third move takes you to  $(-11, 4)$ .
- (3) Final point:  $(-11, 4)$



Record your results on your work sheet.

\* \* \*

Answer this question on your work sheet.

Rule:  $(j, k) \rightarrow (2j - 5, 2k + 3)$

Start at  $(5, -3)$  and make 7 moves.

What is the final point?

Turn to PAGE 11.



Check your answer.

$$\text{Rule: } (j, k) \rightarrow (2j - 5, 2k + 3)$$

Start at  $(5, -3)$  and make 7 moves.

Final point:  $(5, -3)$

[The final point would be  $(5, -3)$  if you made 101 moves!]

Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$\text{Rule: } (x, y) \rightarrow (x^2, 3y - 1)$$

Start at  $(2, 2)$  and make 3 moves.

- (1) First move takes you to  $(4, 5)$ .  
Second move takes you to  $(16, \underline{\quad ? \quad})$ .
- (2) Third move takes you to  $(\underline{\quad ? \quad}, \underline{\quad ? \quad})$ .
- (3) So, the final point is  $(\underline{\quad ? \quad}, \underline{\quad ? \quad})$ .

Turn to PAGE 12.



Check your answers.

$$\text{Rule: } (x, y) \rightarrow (x^2, 3y - 1)$$

Start at (2, 2) and make 3 moves.

- (1) First move takes you to (4, 5).  
 Second move takes you to (16, 14).
- (2) Third move takes you to (256, 41).
- (3) So, the final point is (256, 41).

Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$\text{Rule: } (x, y) \rightarrow (x^2, 3 - y).$$

Start at (-1, -1) and make 3 moves.

- (1) First move takes you to (1, 4). Do you agree? [Yes or No?]
- (2) Second move takes you to \_\_\_\_\_?
- (3) Third move takes you to \_\_\_\_\_?
- (4) What would be the final point if you made 4 moves?
- (5) What would be the final point if you made 20 moves?
- (6) Draw a diagram showing your first 3 moves.

Turn to PAGE 13.



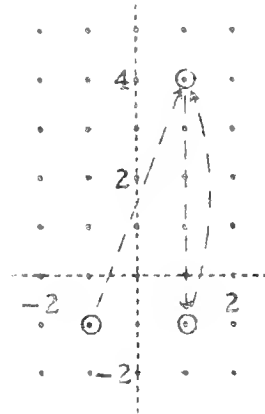


Check your answers.

Rule:  $(x, y) \rightarrow (x^2, 3 - y)$

Start at  $(-1, -1)$  and make 3 moves.

- (1) First move takes you to  $(1, 4)$ . Yes (6)
- (2) Second move takes you to  $(1, -1)$ .
- (3) Third move takes you to  $(1, 4)$ .
- (4) The final point after 4 moves would be  $(1, -1)$ .
- (5) The final point after 20 moves would be  $(1, -1)$ .



Record your results on your work sheet.

\* \* \*

The exercise below introduces a type of number plane lattice game which might be a bit more challenging than the kind you have tried up to now. Be on the lookout for a system to use in solving this new type. Do this exercise on your work sheet.

Rule:  $(x, y) \rightarrow (x + 1, y + 2)$

After making 1 move, the final point is  $(5, 8)$ .

What was the starting point?

Turn to PAGE 14.



Check your answer.

$$\text{Rule: } (x, y) \rightarrow (x + 1, y + 2)$$

After making 1 move, the final point is (5, 8).

The starting point was (4, 6).

If you had trouble with this exercise, here is a solution.

The first component of the final point is 5, and according to the rule, 1 was added to some number to obtain 5. That number must be 4 since  $4 + 1 = 5$ . So, the first component of the point immediately before the final point must be 4. Since only 1 move was made, the first component of the starting point must be 4.

The second component of the starting point must be 6, since according to the rule, 2 was added to some number to obtain 8, and  $6 + 2 = 8$ . So the starting point was (4, 6).

Record your result on your work sheet.

\* \* \*

Do this exercise on your work sheet.

$$\text{Rule: } (x, y) \rightarrow (x + 3, y - 2)$$

After making 2 moves the final point is (-1, 4).

Give the starting point.


Turn to PAGE 15.



Check your answer.

$$\text{Rule: } (x, y) \rightarrow (x + 3, y - 2)$$

After making 2 moves the final point is  $(-1, 4)$ .

Starting point:  $(-7, 8)$  

Solution.

After 2 moves, the first component of the final point is  $-1$ . The rule tells us that 3 was added to some number to obtain  $-1$ . That number, of course, was  $-4$ . So, the first component of the point reached after 1 move was  $-4$ . Again, the rule tells us that 3 was added to some number to obtain  $-4$ . In this case, the number was  $-7$ . So, the first component of the starting point was  $-7$ . By the same kind of reasoning, the second component of the starting point was 8.

Thus, the starting point was  $(-7, 8)$ .

In brief outline,

$$\text{After 2 moves: } (-1, 4)$$

$$\text{After 1 move: } (-4, 6)$$

$$\text{Start: } (-7, 8)$$

[We can check our solution by reading the brief outline from the bottom up to see if each move agrees with the rule.]

Record your result on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$\text{Rule: } (x, y) \rightarrow (x - 2, y + 3)$$

After 2 moves, the final point is  $(1, 8)$ .

(1) After 2 moves:  $(1, 8)$

After 1 move:  $(3, \underline{\quad ? \quad})$

(2) Start:  $(\underline{\quad ? \quad}, \underline{\quad ? \quad})$

Turn to PAGE 16.



Check your answers.

$$\text{Rule: } (x, y) \rightarrow (x - 2, y + 3)$$

After 2 moves, the final point is (1, 8).

(1) After 2 moves: (1, 8)

After 1 move: (3, 5)

(2) Start: (5, 2)

Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet. [Remember to be on the lookout for a systematic way to solve this kind of problem.]

$$\text{Rule: } (x, y) \rightarrow (x - 3, y + 1)$$

After 4 moves, the final point is (-6, 3).

(1) After 4 moves: (-6, 3)

After 3 moves: (-3,   ?)

(2) After 2 moves: (  ?,   ?)

(3) After 1 move: (  ?,   ?)

(4) Start: (  ?,   ?)

(5) Make a diagram showing the moves which must have been made to reach the final point.

Turn to PAGE 17.



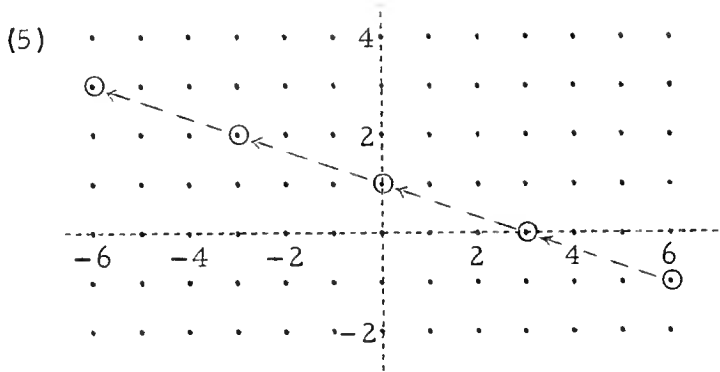


Check your answers.

Rule:  $(x, y) \rightarrow (x - 3, y + 1)$

After 4 moves, the final point is  $(-6, 3)$ .

- (1) After 4 moves:  $(-6, 3)$
- After 3 moves:  $(-3, 2)$
- (2) After 2 moves:  $(0, 1)$
- (3) After 1 move:  $(3, 0)$
- (4)        Start:  $(6, -1)$



Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

Rule:  $(x, y) \rightarrow (2x + 1, y - 1)$

After 3 moves the final point is  $(15, -3)$ .

- (1) After 3 moves:  $(15, -3)$
- After 2 moves:  $(7, \underline{\quad ? \quad})$
- (2) After 1 move:  $(\underline{\quad ? \quad}, \underline{\quad ? \quad})$
- (3)        Start:  $(\underline{\quad ? \quad}, \underline{\quad ? \quad})$

Turn to PAGE 18.



Check your answers.

$$\text{Rule: } (x, y) \rightarrow (2x + 1, y - 1)$$

After 3 moves the final point is (15, -3).

(1) After 3 moves: (15, -3)

After 2 moves: ( 7, -2)

(2) After 1 move: ( 3, -1 )

(3) Start: ( 1, 0 )

Here is how part of the solution might be done.

The first component of the final point is 15. The rule tells us that some number was multiplied by 2 and then 1 was added to obtain 15. That number must have been 7 since  $2 \cdot 7 + 1 = 15$ .

So, after 2 moves, the first component of the point reached was 7.

[You can check the completed solution by reading the answers from the starting point to the final point to see if each move followed the rule.]

Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$\text{Rule: } (x, y) \rightarrow (2x + 1, y - 1)$$

After 2 moves, the final point is (11, 3).

(1) After 2 moves: (11, 3)

After 1 move: ( 5, ? )

(2) Start: ( ?, ? )

Turn to PAGE 19.



Check your answers.

$$\text{Rule: } (x, y) \rightarrow (2x + 1, y - 1)$$

After 2 moves, the final point is (11, 3).

(1) After 2 moves: (11, 3)

After 1 move: (5, 4)

(2) Start: ( 2, 5 ) [ 2 · 2 + 1 = 5 ]

Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$\text{Rule: } (x, y) \rightarrow (2x - 1, 2y + 1)$$

After 3 moves, the final point is (9, 7).

(1) After 3 moves: (9, 7)

After 2 moves: (5,   ? ) [ 2 ·   ? + 1 = 7 ]

(2) After 1 move: (   ? ,   ? )

(3) Start:       ?

Turn to PAGE 20.



Check your answers.

$$\text{Rule: } (x, y) \rightarrow (2x - 1, 2y + 1)$$

After 3 moves, the final point is (9, 7).

(1) After 3 moves: (9, 7)

$$\text{After 2 moves: } (5, 3) \quad [2 \cdot \underline{3} + 1 = 7]$$

(2) After 1 move: (3, 1)

$$[2 \cdot \underline{3} - 1 = 5, \quad 2 \cdot \underline{1} + 1 = 3]$$

(3) Start

$$: (2, 0)$$

$$[2 \cdot \underline{2} - 1 = 3, \quad 2 \cdot \underline{0} + 1 = 1]$$

Record your results on your work sheet.

\* \* \*

On your work sheet, draw a diagram showing the moves which must have been made to reach the final point in the game whose solution is given near the top of this page.

Turn to PAGE 21.

1. The first part of the document discusses the importance of maintaining accurate records of all transactions.

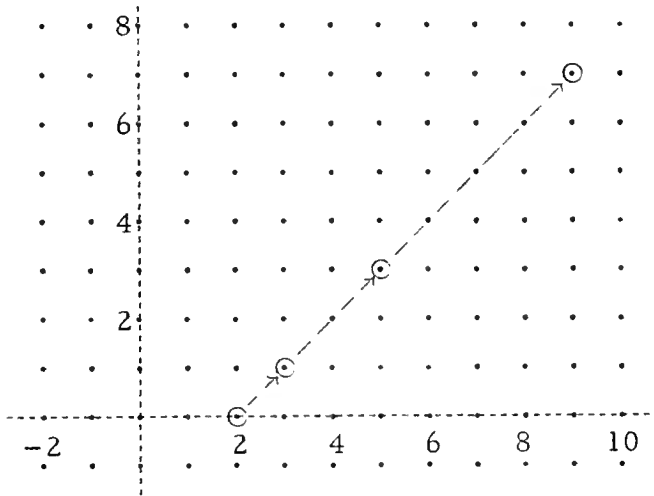
2. This section outlines the various methods used to collect and analyze data from different sources.



Check your answer.

Rule:  $(x, y) \rightarrow (2x - 1, 2y + 1)$

After 3 moves, the final point is (9, 7).



Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

Rule:  $(x, y) \rightarrow (3x + 2, 3 - 2y)$

After 3 moves, the final point is (53, -7).

(1) After 3 moves: (53, -7)

After 2 moves: (17, 5)       $[3 \cdot \underline{17} + 2 = 53. \quad 3 - 2 \cdot \underline{5} = -7.]$

After 1 move :       ?       $[3 \cdot \underline{?} + 2 = 17. \quad 3 - 2 \cdot \underline{?} = 5.]$

(2) Start :       ?

Turn to PAGE 22.



Check your answers.

$$\text{Rule: } (x, y) \rightarrow (3x + 2, 3 - 2y)$$

After 3 moves, the final point is (53, -7).

(1) After 3 moves: (53, -7)

After 2 moves: (17, 5)

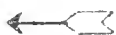
$$[3 \cdot \underline{17} + 2 = 53. \quad 3 - 2 \cdot \underline{5} = -7.]$$

After 1 move : (5, -1)



$$[3 \cdot \underline{5} + 2 = 17. \quad 3 - 2 \cdot \underline{-1} = 5.]$$

(2) Start: (1, 2)



$$[3 \cdot \underline{1} + 2 = 5. \quad 3 - 2 \cdot \underline{2} = -1.]$$

Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$\text{Rule: } (x, y) \rightarrow (3x - 2, 4 - 2y)$$

After 3 moves, the final point is (28, 12).

(1) After 3 moves: (28, 12)

After 2 moves:     ? [Remember, you are "backing up".]

(2) After 1 move :     ?

(3) Start :     ?

Turn to PAGE 23.



Check your answers.

$$\text{Rule: } (x, y) \rightarrow (3x - 2, 4 - 2y)$$

After 3 moves, the final point is (28, 12).

(1) After 3 moves: (28, 12)

After 2 moves:  $(10, -4)$

(2) After 1 move :  $(4, 4)$

(3) Start :  $(2, 0)$

Record your results on your work sheet.

Turn to PAGE 24.



You have played several number plane lattice games where you were given the final point and asked to find the starting point. Perhaps you have discovered that you can use equations to help you “back up” in a lattice game.

Here is a sample showing how equations can help. [Perhaps you discovered a different method.]

Sample.

$$\text{Rule: } (x, y) \rightarrow (3x + 4, 2 - 3y)$$

After 2 moves, the final point is (79, 68).

Give the starting point.

Solution.

After 2 moves: (79, 68)

<u>First Component</u>		<u>Second Component</u>
$3x + 4 = 79$		$2 - 3y = 68$
$3x = 75$		$-3y = 66$
$x = 25$		$y = -22$

So, the point just before (79, 68) was (25, -22).

After 1 move: (25, -22)

$3x + 4 = 25$		$2 - 3y = -22$
$x = 7$		$y = 8$

So, the starting point was (7, 8).

Do this exercise on your work sheet.

$$\text{Rule: } (x, y) \rightarrow (2x + 5, 3 - 2y)$$

After 3 moves, the final point is (19, -15).

Give the starting point.

Turn to PAGE 25.





Check your answer.

Rule:  $(x, y) \rightarrow (2x + 5, 3 - 2y)$

After 3 moves, the final point is  $(19, -15)$ .

<u>First Component</u>		<u>Second Component</u>
$2x + 5 = 19$		$3 - 2y = -15$
$x = 7$		$y = 9$
After 2 moves: $(7, 9)$		
$2x + 5 = 7$		$3 - 2y = 9$
$x = 1$		$y = -3$
After 1 move: $(1, -3)$		
$2x + 5 = 1$		$3 - 2y = -3$
$x = -2$		$y = 3$
Starting point: $(-2, 3)$		



Record your results on your work sheet.

\* \* \*

Do this exercise on your work sheet.

Rule:  $(x, y) \rightarrow (2x - 5, 3 + 4y)$

After 4 moves, the final point is  $(-27, -1)$ .

Give the starting point.

Turn to PAGE 26.



Check your answer.

$$\text{Rule: } (x, y) \rightarrow (2x - 5, 3 + 4y)$$


After 4 moves, the final point is  $(-27, -1)$ .

After 4 moves:  $(-27, -1)$

After 3 moves:  $(-11, -1)$

After 2 moves:  $(-3, -1)$

After 1 move :  $(1, -1)$

Start :  $(3, -1)$  

Record your result on your work sheet.

\* \* \*

The exercises below introduce still another type of number plane lattice game. Do these exercises on your work sheet.

$$A = \{(0, 0), (1, 1), (2, 2)\}$$

$$\text{Rule: } (x, y) \rightarrow (x + y, x - y)$$

Make one move from each point in set A. Call the new set 'X'.

(1) From  $(0, 0)$ , you move to  $(0 + 0, 0 - 0)$ , or  $(0, 0)$ .

From  $(1, 1)$ , you move to  $(1 + 1, 1 - 1)$ , or  $(2, 0)$ .

From  $(2, 2)$ , you move to  $(\underline{\quad} + \underline{\quad}, \underline{\quad} - \underline{\quad})$ , or  $(\underline{\quad}, \underline{\quad})$ .

(2) If the new set is called 'X' then  $X = \{ \underline{\quad}?, \underline{\quad}?, \underline{\quad}? \}$ .

Turn to PAGE 27.



Check your answers.

$$A = \{(0, 0), (1, 1), (2, 2)\}$$

$$\text{Rule: } (x, y) \rightarrow (x + y, x - y)$$

Make one move from each point in set A. Call the new set 'X'.

(1) From (0, 0), you move to (0, 0).

From (1, 1), you move to (2, 0).

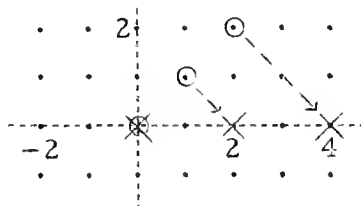
From (2, 2), you move to (2 + 2, 2 - 2), or (4, 0).

(2)  $X = \{(0,0), (2,0), (4,0)\}$

Record your results on your work sheet.

\* \* \*

Now, let's plot the points in each of the sets A and X listed above, and show the moves from each point in A to the corresponding point in X.



Do these exercises on your work sheet.

$$A = \{(0, 0), (1, -1), (2, -2), (3, -3)\}$$

$$\text{Rule: } (x, y) \rightarrow (x, |y|)$$

Make one move from each point in set A. Call the new set 'X'.

(1)  $X = \{ \underline{\quad ? \quad}, \underline{\quad ? \quad}, \underline{\quad ? \quad}, \underline{\quad ? \quad} \}$

(2) Plot the points in each set on the same diagram and indicate each move by drawing a dashed line and an arrow. [See the diagram above.]

Turn to PAGE 28.

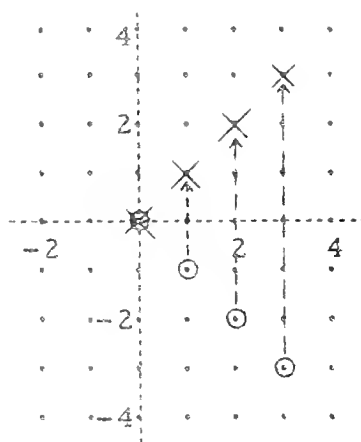


Check your answers.

$$A = \{(0, 0), (1, -1), (2, -2), (3, -3)\}$$

$$\text{Rule: } (x, y) \rightarrow (x, |y|)$$

(1)  $X = \{(0, 0), (1, 1), (2, 2), (3, 3)\}$       (2)



Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$A = \{(2, 1), (3, 2), (4, 3), (5, 3)\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

Make one move from each point in set A. Call the new set 'X'.

(1)  $X = \{(1, 2), (2, \underline{\quad}), \underline{\quad}, \underline{\quad}\}$ .

(2) Plot the points in each set on the same diagram and indicate each move. [Remember, loops for A, cross-marks for X.]

Turn to PAGE 29.



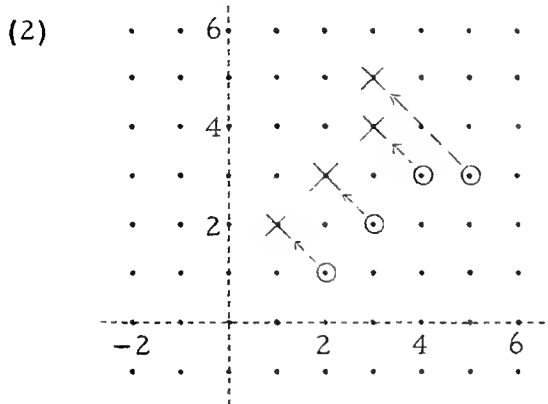


Check your answers.

$$A = \{(2, 1), (3, 2), (4, 3), (5, 3)\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

(1)  $X = \{(1, 2), (2, 3), (3, 4), (3, 5)\}$



Record your results on your work sheet.

\* \* \*

When one move is made from each point in a set according to the rule ' $(x, y) \rightarrow (y, x)$ ' and the points involved are plotted, we get an interesting picture. Study such pictures carefully and look for a "pattern".

Do these exercises on your work sheet.

$$A = \{(x, y), x \text{ and } y \text{ integers: } 0 < x < 6 \text{ and } y = 1\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

Make one move from each point in set A. Call the new set 'X'.

(1)  $A = \{ \underline{\hspace{2cm}} ? \underline{\hspace{2cm}} \}$  [List the members of set A.]

(2)  $X = \{ \underline{\hspace{2cm}} ? \underline{\hspace{2cm}} \}$  [List the members of set X.]

(3) Plot the points in each set on the same diagram and indicate the moves.

(4)  $X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2cm}} ? \underline{\hspace{2cm}}, \text{ and } x = 1\}$

Turn to PAGE 30.



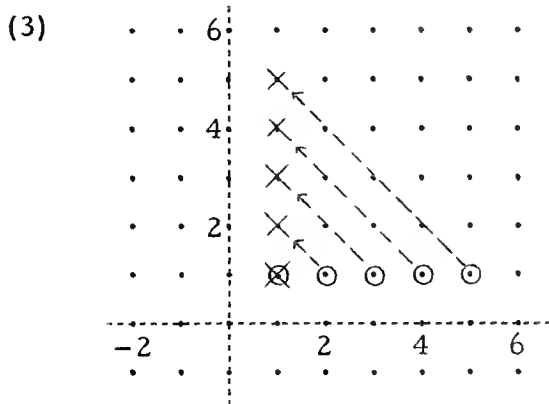
Check your answers.

$$A = \{(x, y), x \text{ and } y \text{ integers: } 0 < x < 6 \text{ and } y = 1\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

(1)  $A = \{(1,1), (2,1), (3,1), (4,1), (5,1)\}$

(2)  $X = \{(1,1), (1,2), (1,3), (1,4), (1,5)\}$



Remember, look for a "pattern".

(4)  $X = \{(x, y), x \text{ and } y \text{ integers: } 0 < y < 6 \text{ and } x = 1\}$

[Compare this description of set X with the description of set A at the top of this page.]

Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$A = \{(x, y), x \text{ and } y \text{ integers: } -2 < x < 2 \text{ and } -4 < y < -2\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

Make one move from each point in set A. Call the new set 'X'.

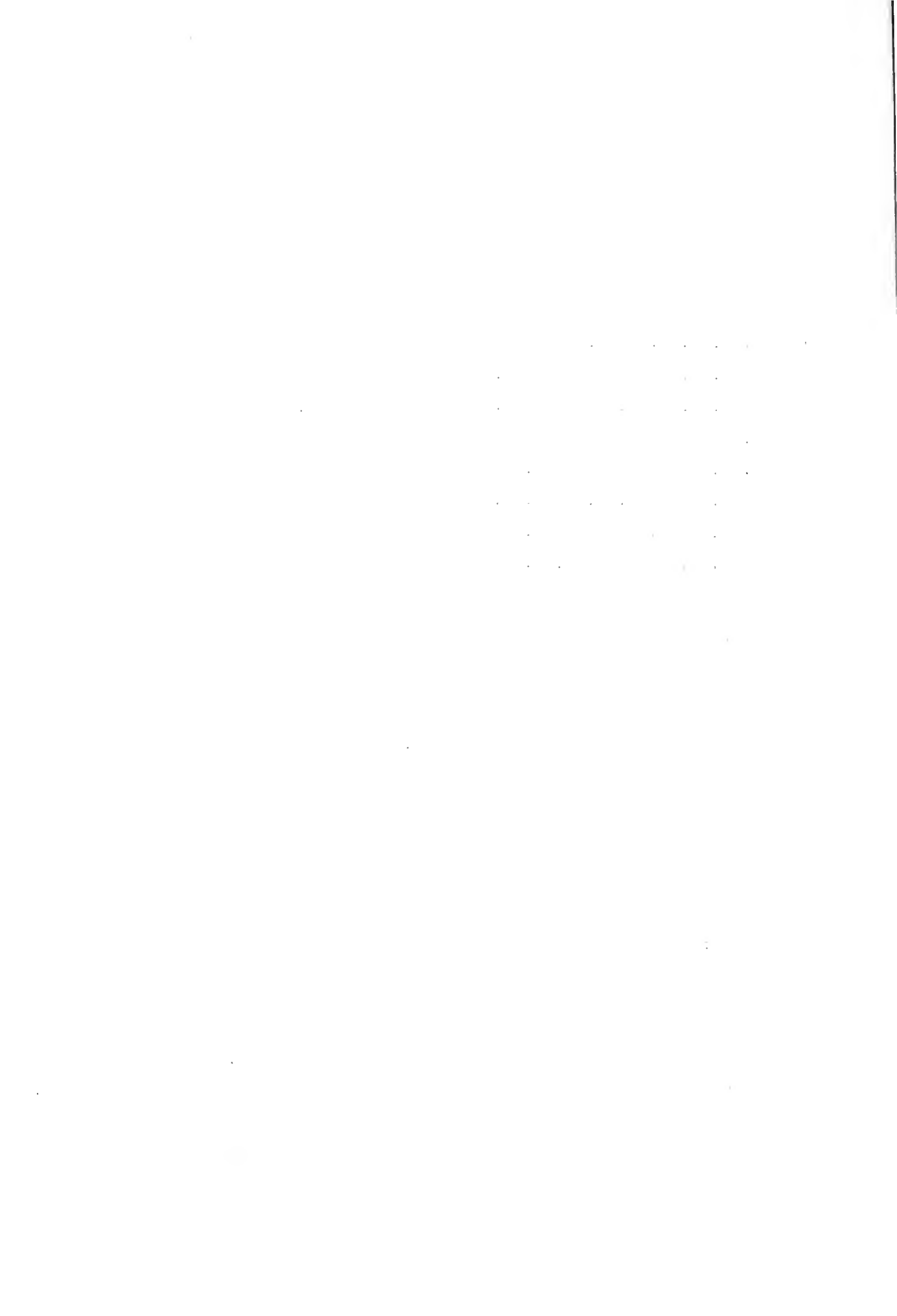
(1)  $A = \{ \text{_____?} \}$  [List the members of set A.]

(2)  $X = \{ \text{_____?} \}$  [List the members of set X.]

(3) Plot the points in each set on the same diagram and indicate the moves. [Remember, loops for A, cross-marks for X.]

(4)  $X = \{(x, y), x \text{ and } y \text{ integers: } -2 < y < 2 \text{ and } \text{_____?} \}$

Turn to PAGE 31.



Check your answers.

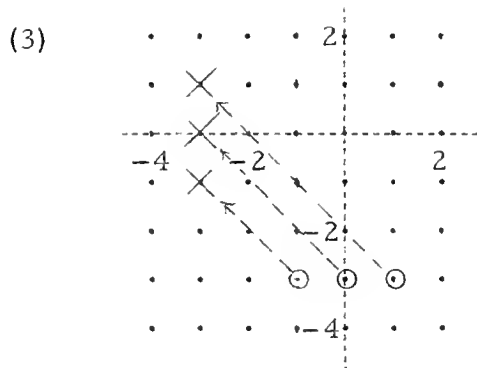
$$A = \{(x, y), x \text{ and } y \text{ integers: } -2 < x < 2 \text{ and } -4 < y < -2\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

Make one move from each point in set A. Call the new set 'X'.

(1)  $A = \{ (-1, -3), (0, -3), (1, -3) \}$

(2)  $X = \{ (-3, -1), (-3, 0), (-3, 1) \}$



(4)  $X = \{(x, y), x \text{ and } y \text{ integers: } -2 < y < 2 \text{ and } -4 < x < -2\}$

[Compare this description of set X with the description of Set A at the top of this page.]

Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

After making one move from each point in set A, the result in set X where

$$X = \{(2, 7), (3, 7), (4, 7)\}.$$

(1)  $A = \{ \underline{\quad ? \quad}, \underline{\quad ? \quad}, \underline{\quad ? \quad} \}$

(2)  $X = \{(x, y), x \text{ and } y \text{ integers: } 1 < x < 5 \text{ and } \underline{\quad ? \quad}\}$

(3)  $A = \{(x, y), x \text{ and } y \text{ integers: } \underline{\quad ? \quad} \text{ and } x = 7\}$

Turn to PAGE 32.



Check your answers.

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

After making one move from each point in set A, the result is set X where

$$X = \{(2, 7), (3, 7), (4, 7)\}.$$

$$(1) A = \{(7, 2), (7, 3), (7, 4)\}$$

$$(2) X = \{(x, y), x \text{ and } y \text{ integers: } 1 < x < 5 \text{ and } y = 7\}$$

$$(3) A = \{(x, y), x \text{ and } y \text{ integers: } 1 < y < 5 \text{ and } x = 7\}$$

Record your results on your work sheet.

\* \* \*

Do this exercise on your work sheet.

$$A = \{(x, y), x \text{ and } y \text{ integers: } 2 < x < 5 \text{ and } -3 < y < 0\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

If you make one move from each point in set A, and call the new set 'X', then  $X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2cm}} ? \underline{\hspace{2cm}} \text{ and } \underline{\hspace{2cm}} ? \underline{\hspace{2cm}}\}$ .

[Try to complete the description of set X without listing the members of either set A or set X.]

Turn to PAGE 33.





Check your answers.

$$A = \{(x, y), x \text{ and } y \text{ integers: } 2 < x < 5 \text{ and } -3 < y < 0\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

If you make one move from each point in set A, and call the new set 'X', then

$$X = \{(x, y), x \text{ and } y \text{ integers: } 2 < y < 5 \text{ and } -3 < x < 0\}.$$

[Of course, 'X = {x, y}, x and y integers:  $-3 < x < 0$  and  $2 < y < 5$ ' is also correct.]

Record your results on your work sheet.

\* \* \*

If you were able to complete the description of set X above without listing the members of set A or set X, you have probably made an interesting discovery. We hope you have discovered that when you are given a brace-notation description of a set of points and you make one move from each point in the set according to the rule  $(x, y) \rightarrow (y, x)$ , you can easily get a brace-notation description of the new set. You simply copy the description of the given set except that after the ' $\}$ ' you substitute the pronomeral which indicates second components for the pronomeral which indicates first components, and you substitute the first component pronomeral for the second component pronomeral. The exercise answered at the top of this page is a good example.

Turn to PAGE 34.



Now, let's see if you can apply the discovery mentioned on the previous page.

Do these exercises on your work sheet.

$$A = \{(x, y), x \text{ and } y \text{ integers: } y = 2 \text{ and } -3 < x < 3\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

Make one move from each point in set A. Call the new set 'X'.

(1) Give a brace description of set X. That is, complete this:

$$X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\quad ? \quad} \text{ and } \underline{\quad ? \quad}\}$$

(2) Plot the points in each set on the same diagram and indicate the moves.

Turn to PAGE 35.



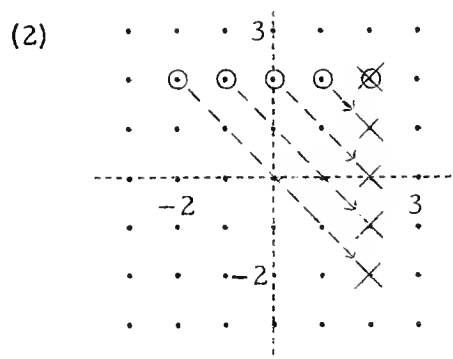
Check your answers.

$$A = \{(x, y), x \text{ and } y \text{ integers: } y = 2 \text{ and } -3 < x < 3\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

Make one move from each point in set A. Call the new set 'X'.

$$(1) X = \{(x, y), x \text{ and } y \text{ integers: } x = 2 \text{ and } -3 < y < 3\}$$



Remember to look for a "pattern" on the picture.

Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$A = \{(x, y), x \text{ and } y \text{ integers: } y = x - 3\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

Make one move from each point in set A. Call the new set 'X'.

(1) Give a brace description of set X.

(2) Plot the points in each set on the same diagram and indicate the moves.

Turn to PAGE 36.

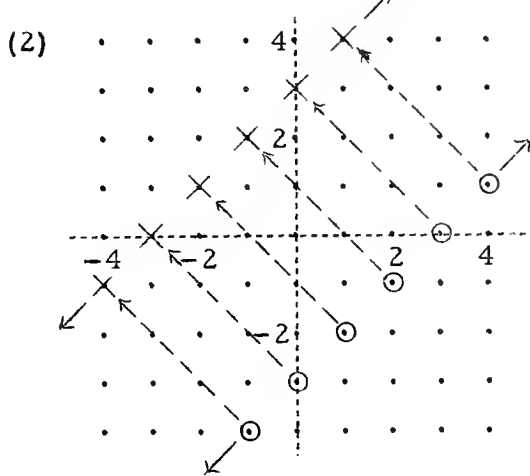


Check your answers.

$$A = \{(x, y), x \text{ and } y \text{ integers: } y = x - 3\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

(1)  $X = \{(x, y), x \text{ and } y \text{ integers: } x = y - 3\}$



[The "pattern" is particularly clear in this picture.]

Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

Set A: See the picture

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

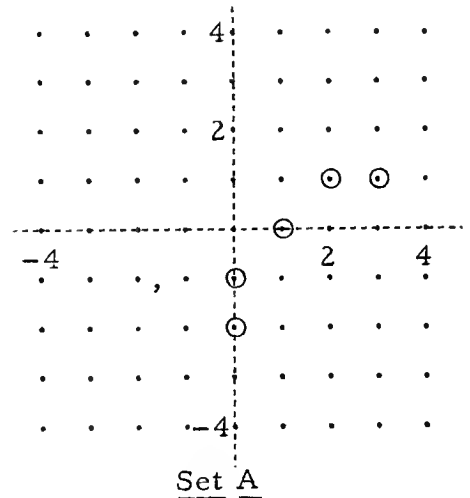
Make one move from each point in set A.

Call the new set 'X'.

- (1) Plot the points in set X on the same diagram with set A and indicate the moves. [Try to do this exercise first, but if you have trouble, do Exercises (2) and (3) first.]

(2)  $A = \{ \underline{\hspace{2cm}} ? \underline{\hspace{2cm}} \}$  [List the members of set A.]

(3)  $X = \{ \underline{\hspace{2cm}} ? \underline{\hspace{2cm}} \}$  [List the members of set X.]



Turn to PAGE 37.

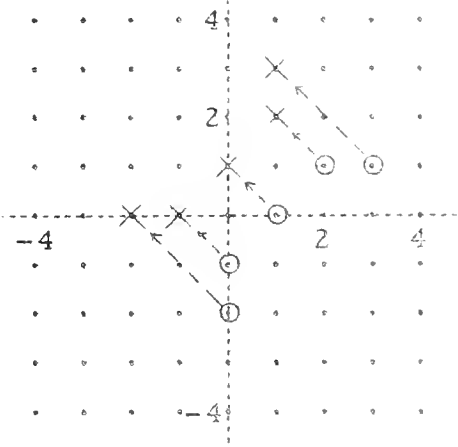




Check your answers.

Rule:  $(x, y) \rightarrow (y, x)$

Make one move from each point in Set A. Call the new set 'X'.

(1) 

(2)  $A = \{(0, -2), (0, -1), (1, 0), (2, 1), (3, 1)\}$

(3)  $X = \{(-2, 0), (-1, 0), (0, 1), (1, 2), (1, 3)\}$

Record your results on your work sheet.

\* \* \*

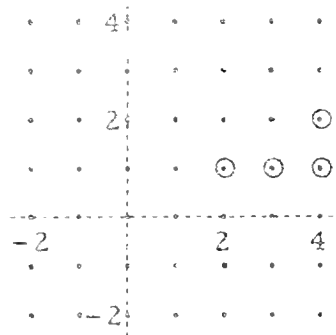
Do these exercises on your work sheet.

A: the set pictured at right

Rule:  $(x, y) \rightarrow (y, x)$

Make one move from each point in set A.

Call the new set 'X'.



Set A

(1) Plot the points in set X on the same diagram with set A and indicate the moves.

(2)  $A = \{ \underline{\quad ? \quad} \}$  [List set A.]

(3)  $X = \{ \underline{\quad ? \quad} \}$  [List set X.]

(4) Draw a dashed line through the dots corresponding to the points in set D, where

$$D = \{(x, y), x \text{ and } y \text{ integers: } y = x\}.$$

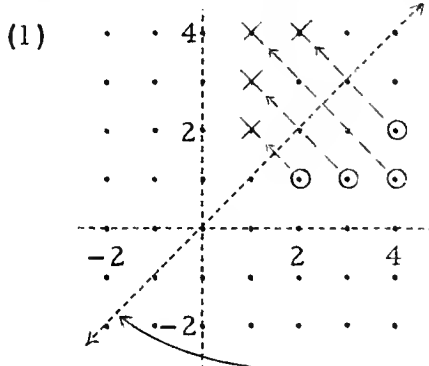
Turn to PAGE 38.



Check your answers.

Rule:  $(x, y) \rightarrow (y, x)$

Make one move from each point in set A. Call the new set 'X'.



(2)  $A = \{ (2,1), (3,1), (4,1), (4,2) \}$

(3)  $X = \{ (1,2), (1,3), (1,4), (2,4) \}$

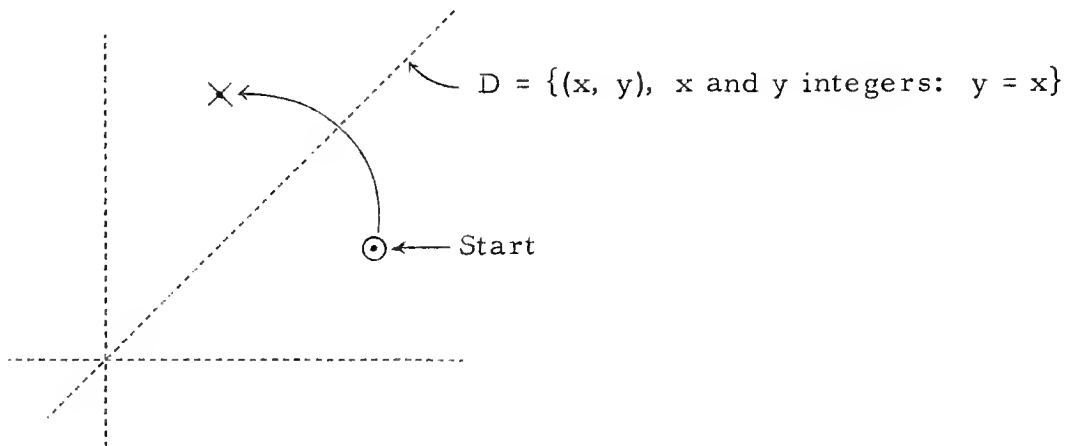
(4)  $[D = \{(x, y), x \text{ and } y \text{ integers: } y = x\}]$

Record your results on your work sheet.

\* \* \*

The exercises above should help you see the pattern involved in making a move according to the rule:

$(x, y) \rightarrow (y, x)$



Turn to PAGE 39.



Notice the dot with the loop around it in the picture below. If you make one move from this point according to the rule:

$$(x, y) \rightarrow (y, x)$$

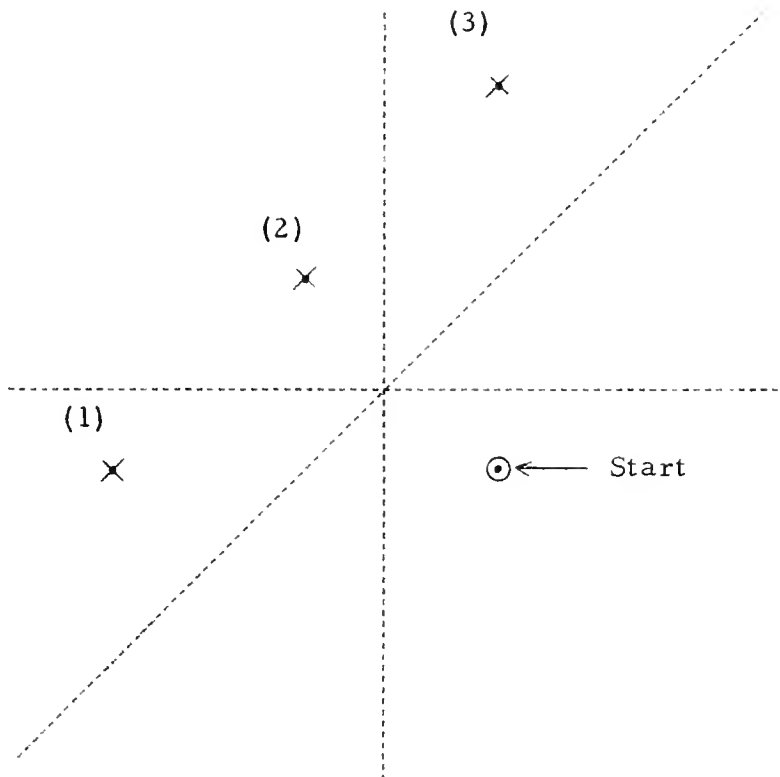
where do you think you would end up?

Point (1)?

Point (2)?

Point (3)?

Circle the answer on your work sheet.

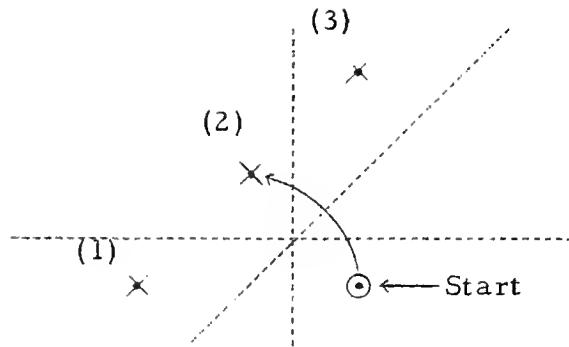


Turn to PAGE 40.



Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$



Record your result on your work sheet.

\* \* \*

Notice the dot with the loop around it in the picture below. If you make one move from this point according to the rule:

$$(x, y) \rightarrow (y, x)$$

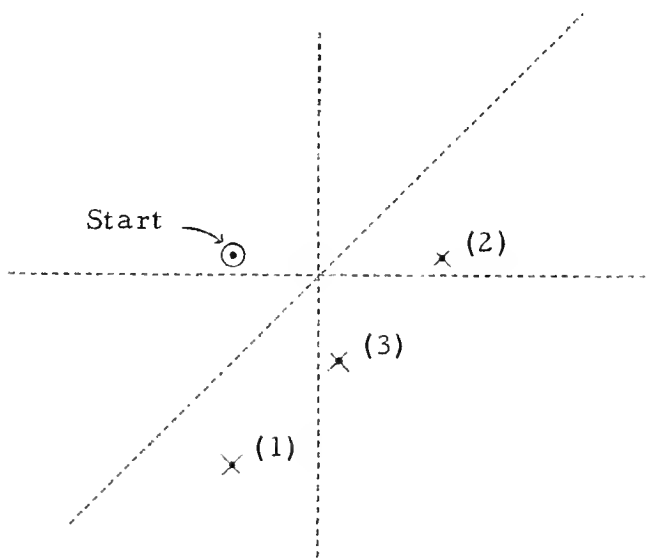
where do you think you would end up?

Point (1)?

Point (2)?

Point (3)?

Circle the answer on your work sheet.



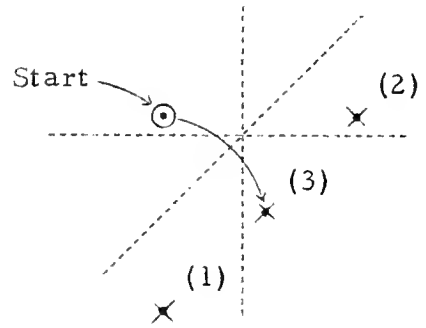
Turn to PAGE 41.





Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$



Record your result on your work sheet.

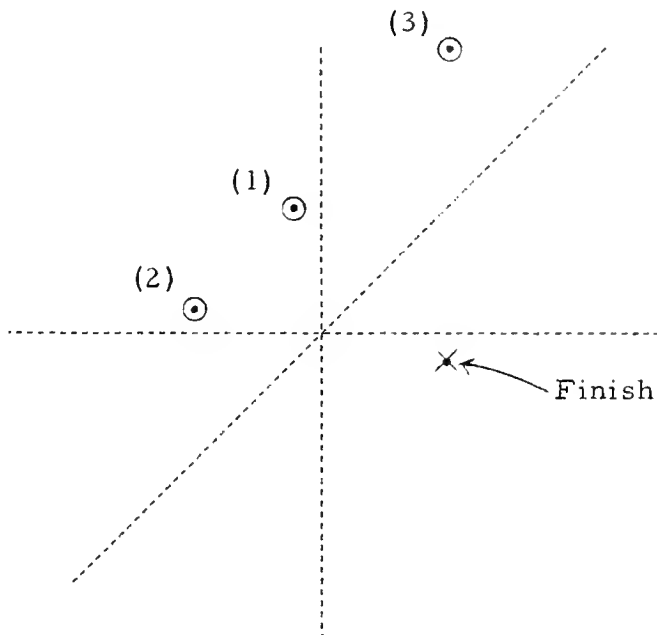
\* \* \*

Which of the marked points (1), (2), or (3) would be the starting point if you made one move according to the rule:

$$(x, y) \rightarrow (y, x)$$

and ended at the dot with the cross-mark through it?

Circle the answer on your work sheet.

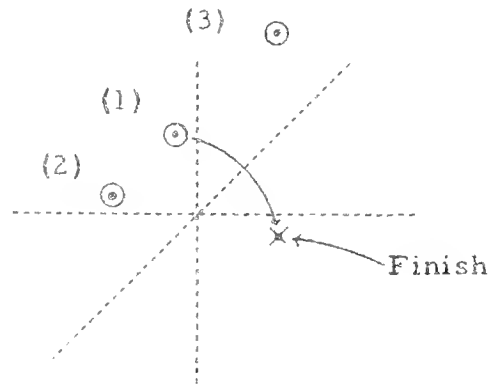


Turn to PAGE 42.



Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$



Record your result on your work sheet.

\* \* \*

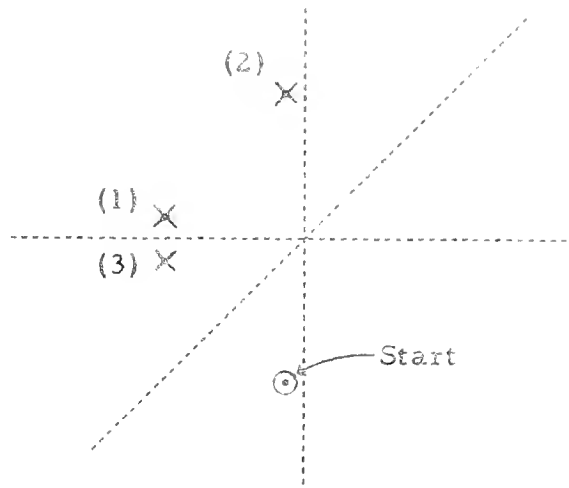
If you make one move from the point labeled as the start point according to the rule:

$$(x, y) \rightarrow (y, x)$$

where do you finish?

Point (1)?      Point (2)?      Point (3)?

Circle the answer on your work sheet.

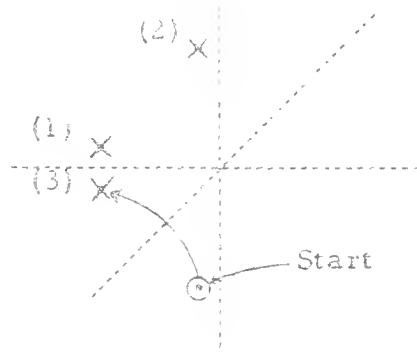


Turn to PAGE 43.



Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$

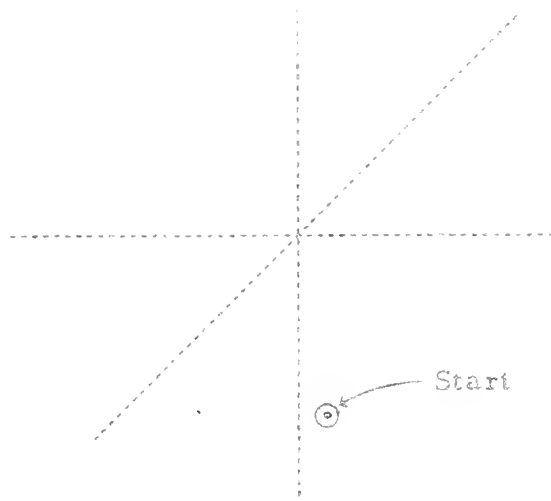


Record your result on your work sheet.

\* \* \*

Use your eye and mark a dot on the picture on your work sheet to show where you finish if you make one move from the point labeled Start according to the rule:

$(x, y) \rightarrow (y, x)$

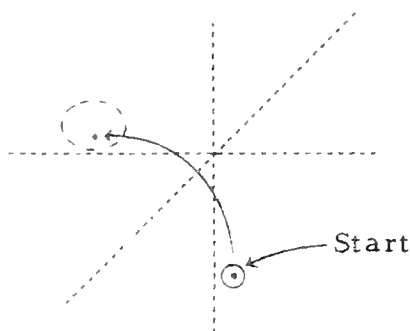


Turn to PAGE 44.



Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$



You are right if your mark would be within the boundary indicated.

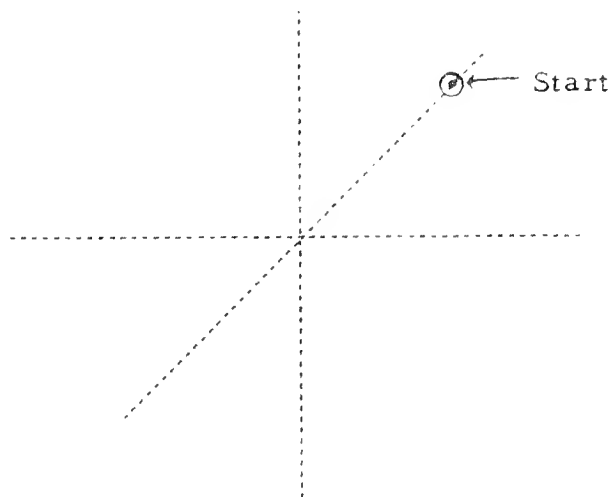
Record your result on your work sheet.

\* \* \*

Use your eye and mark a dot on the picture on your work sheet to show where you finish if you make one move from the start point according to the rule:

$(x, y) \rightarrow (y, x)$

[Hint. A "move" might not move you at all.]



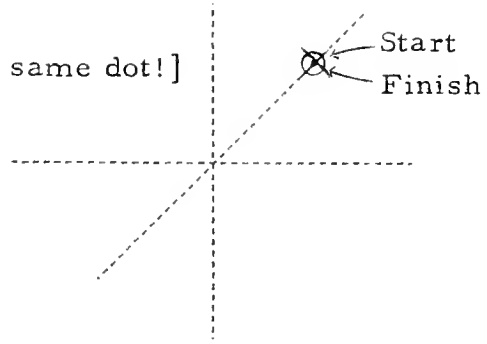
Turn to PAGE 45.





Check your answer.

[The same dot!]



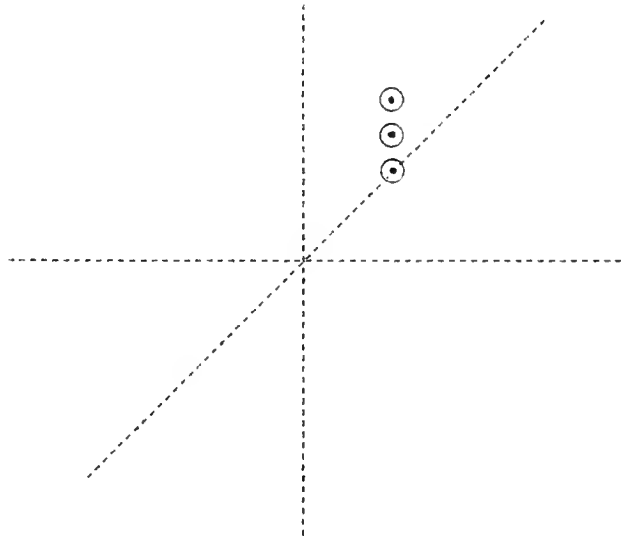
Rule:  $(x, y) \rightarrow (y, x)$

Record your result on your work sheet.

\* \* \*

Use your eye and mark dots on the picture on your work sheet to show where you finish if you make one move from each point indicated by a loop according to the rule:

$(x, y) \rightarrow (y, x)$

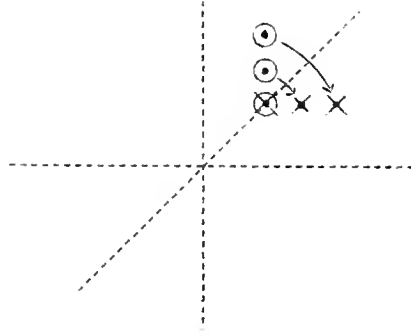


Turn to PAGE 46.



Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$

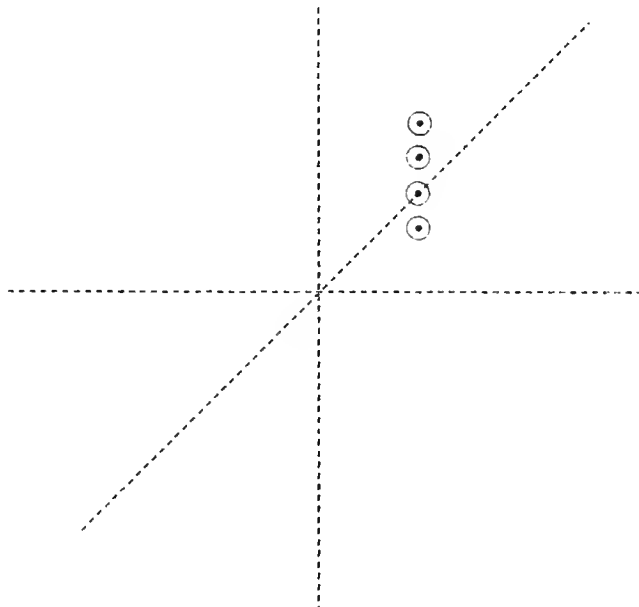


Record your result on your work sheet.

\* \* \*

Use your eye and mark dots on the picture on your work sheet to show where you finish if you make one move from each point indicated by a loop according to the rule:

$(x, y) \rightarrow (y, x)$

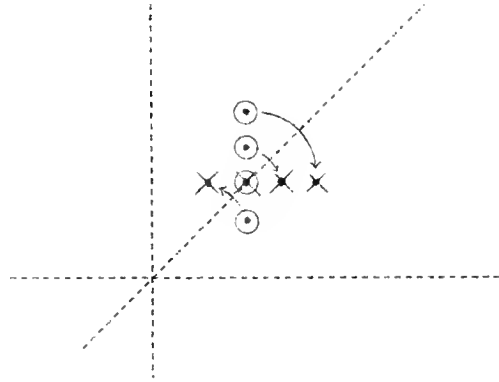


Turn to PAGE 47.



Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$

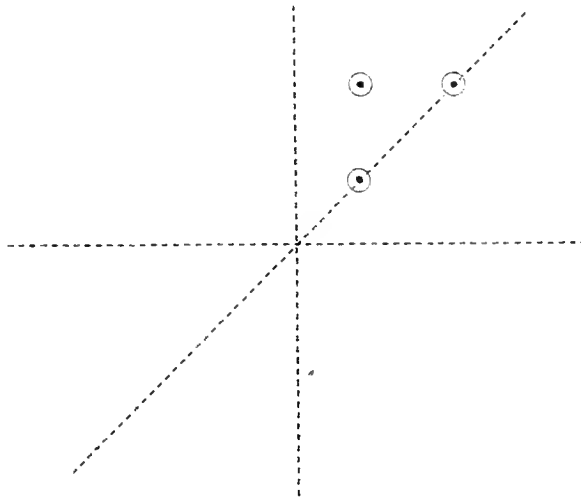


Record your result on your work sheet.

\* \* \*

Use your eye and mark dots on the picture on your work sheet to show where you finish if you make one move from each point indicated by a loop according to the rule:

$(x, y) \rightarrow (y, x)$

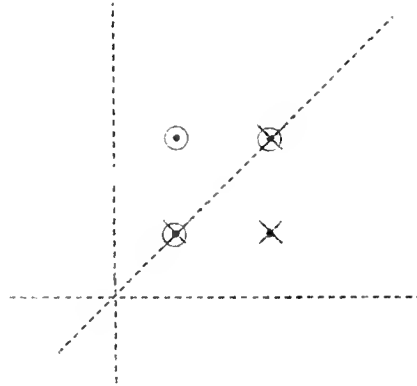


Turn to PAGE 48.



Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$

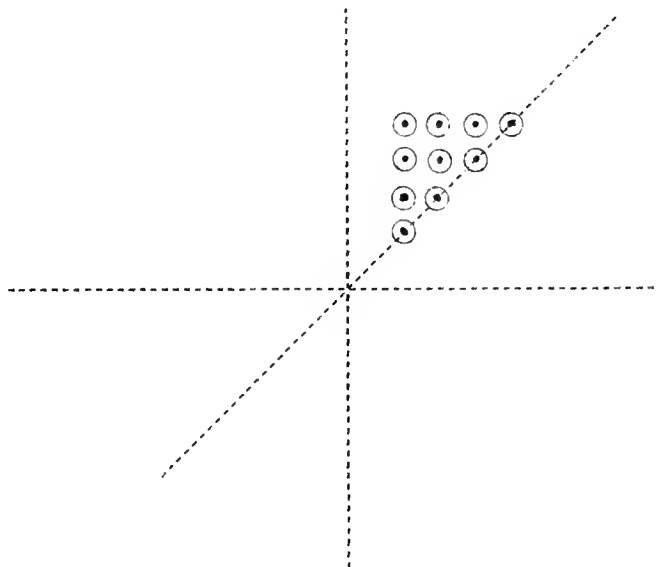


Record your result on your work sheet.

\* \* \*

Use your eye and mark dots on the picture on your work sheet to show where you finish if you make one move from each point indicated by a loop according to the rule:

$(x, y) \rightarrow (y, x)$



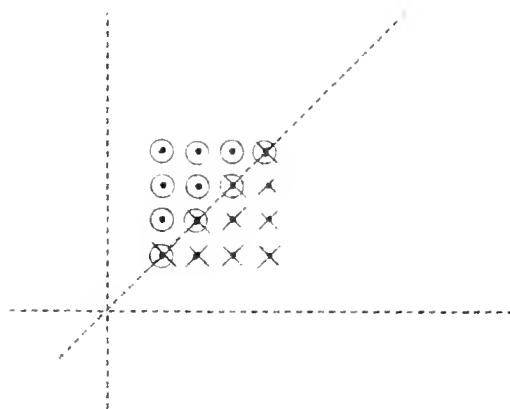
Turn to PAGE 49.





Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$

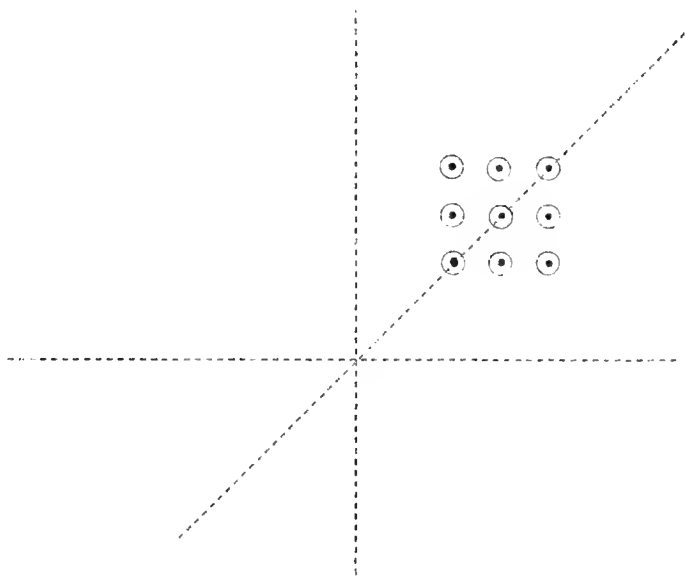


Record your result on your work sheet.

\* \* \*

Use your eye and mark dots on the picture on your work sheet to show where you finish if you make one move from each point indicated by a loop according to the rule:

$(x, y) \rightarrow (y, x)$

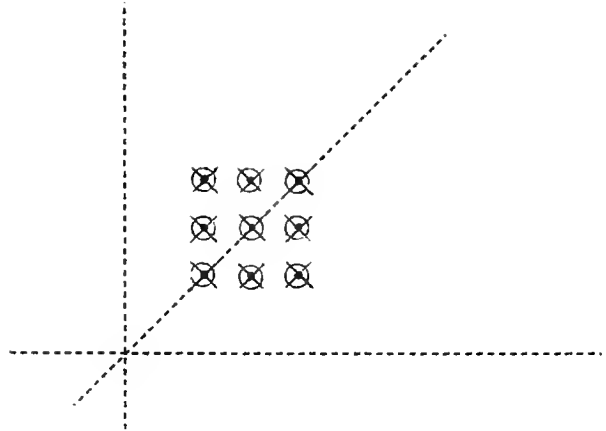


Turn to PAGE 50.



Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$



Record your result on your work sheet.

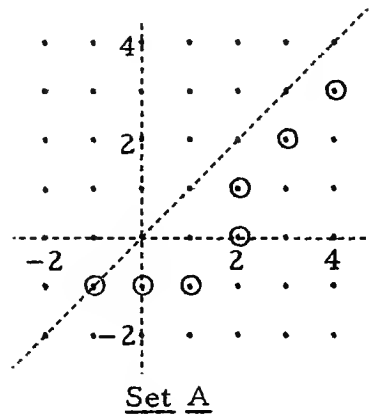
\* \* \*

Do this exercise on your work sheet.

Set A is the set pictured.

Rule:  $(x, y) \rightarrow (y, x)$

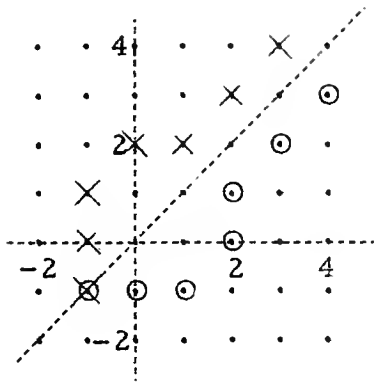
Make one move from each point in set A. Use cross-marks to indicate the new set.



Turn to PAGE 51.



Check your answer.



Rule:  $(x, y) \rightarrow (y, x)$

Record your result on your work sheet.

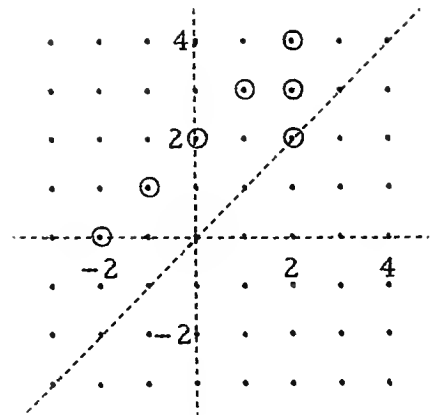
\* \* \*

Do this exercise on your work sheet.

Set A is the set pictured.

Rule:  $(x, y) \rightarrow (y, x)$

Make one move from each point in set A. Use cross-marks to indicate the new set.



Set A

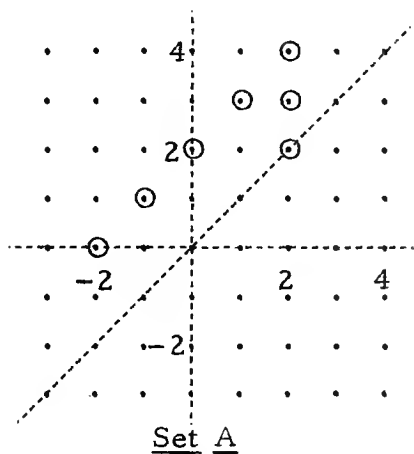
Turn to PAGE 52.



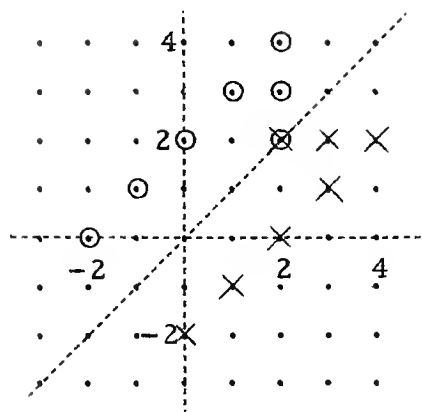
Check your answer.

Set A is the set pictured.

Rule:  $(x, y) \rightarrow (y, x)$



Solution.



Record your result on your work sheet.

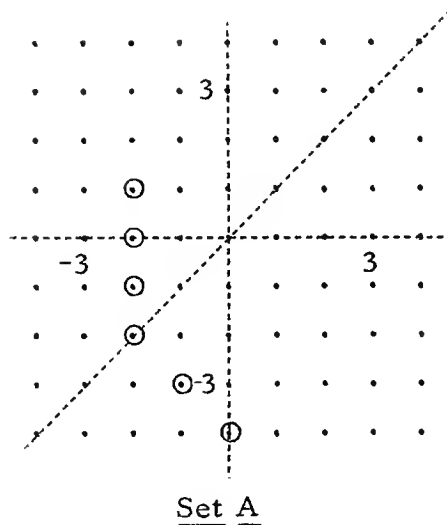
\* \* \*

Do this exercise on your work sheet.

Set A: see picture

Rule:  $(x, y) \rightarrow (y, x)$

Make one move from each point in set A. Use cross-marks to indicate the new set.



Turn to PAGE 53.

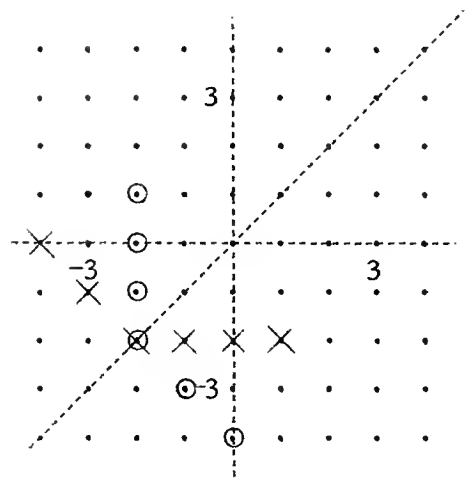
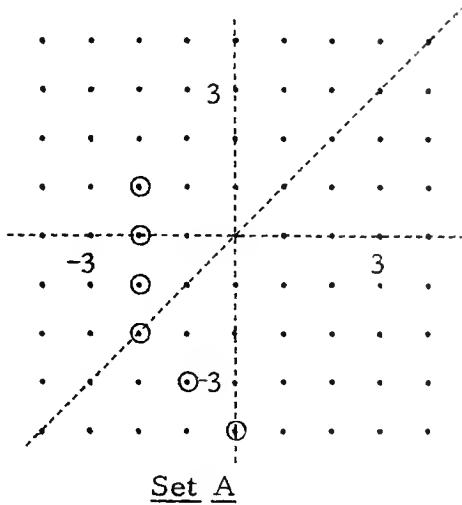




Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$

Solution



Record your result on your work sheet.

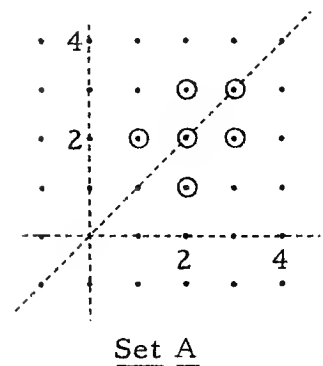
\* \* \*

Do this exercise on your work sheet.

Set A: see picture

Rule:  $(x, y) \rightarrow (y, x)$

Make one move from each point in set A. Use cross-marks to indicate the new set.

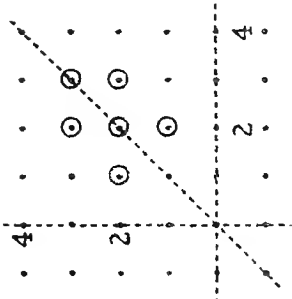


Turn to PAGE 54.

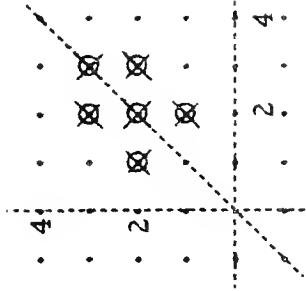


Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$



Set A



Solution

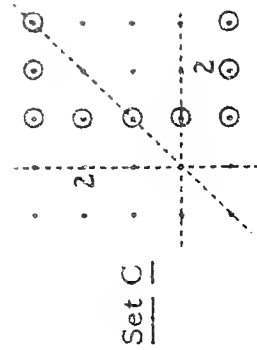
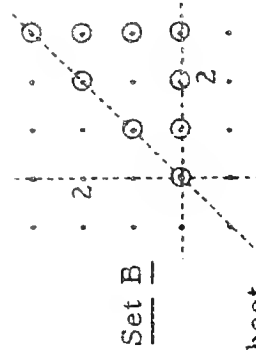
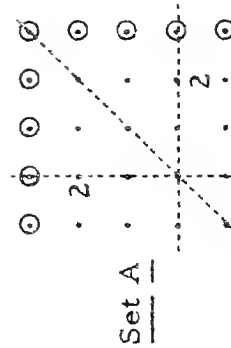
Record your results on your work sheet.

\* \* \*

Suppose that for each set pictured below, one move was made from each point in the set according to the rule:

$$(x, y) \rightarrow (y, x)$$

For which of these sets would a move from any point in the set take you to another point in the set?



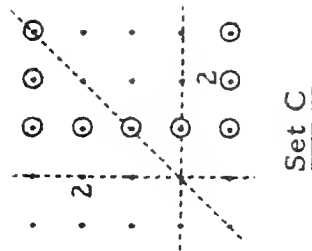
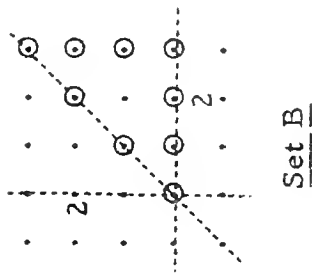
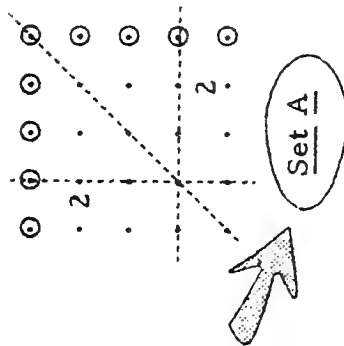
Circle the answer on your work sheet.

Turn to PAGE 55.



Check your answer.

Rule:  $(x, y) \rightarrow (y, x)$



Turn to PAGE 56.



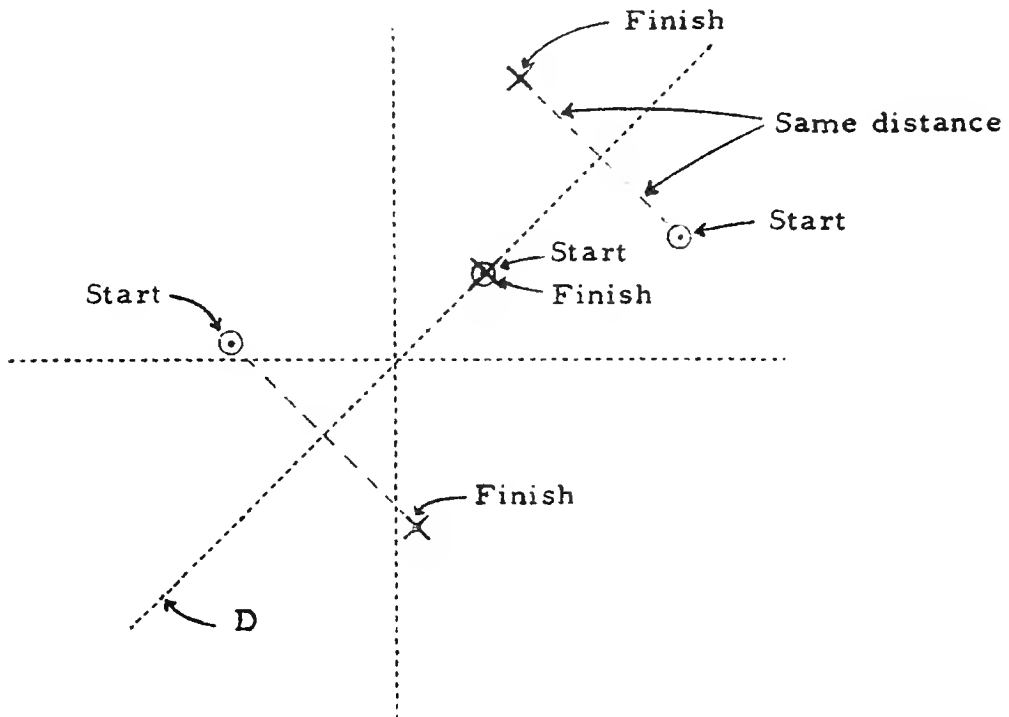
As you have probably seen from the preceding exercises, when you make one move from a point according to the rule:

$$(x, y) \rightarrow (y, x)$$

you go straight toward the graph of the line through set D, where

$$D = \{(x, y), x \text{ and } y \text{ integers: } x = y\}$$

and beyond it. The new point is the same distance from the line as the starting point is.



Turn to PAGE 57.





Do these exercises on your work sheet.

$$A = \{(2, 3), (3, 4)\}$$

$$B = \{(2, 4), (3, 4), (4, 4)\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

(1)  $A \cup B = \{ \underline{\quad ? \quad} \}$

(2) Make one move from each point in set A. Call the new set 'X'.  
Then  $X = \{ \underline{\quad ? \quad} \}$ .

(3) Make one move from each point in set B. Call the new set 'Y'.  
Then  $Y = \{ \underline{\quad ? \quad} \}$ .

(4) If one move is made from each point in set  $A \cup B$  then the new set is  $\{ \underline{\quad ? \quad} \}$ .

(5)  $X \cup Y = \{ \underline{\quad ? \quad} \}$

Turn to PAGE 58.



Check your answers.

$$A = \{(2, 3), (3, 4)\}$$

$$B = \{(2, 4), (3, 4), (4, 4)\}$$

$$\text{Rule: } (x, y) \rightarrow (y, x)$$

- (1)  $A \cup B = \{(2,3), (3,4), (2,4), (4,4)\}$ .
- (2) Make one move from each point in set A. Call the new set 'X'.  
Then  $X = \{(3,2), (4,3)\}$ .
- (3) Make one move from each point in set B. Call the new set 'Y'.  
Then  $Y = \{(4,2), (4,3), (4,4)\}$ .
- (4) If one move is made from each point in set  $A \cup B$  then the new set is  $\{(3,2), (4,3), (4,2), (4,4)\}$ .
- (5)  $X \cup Y = \{(3,2), (4,3), (4,2), (4,4)\}$   
[Compare the answers to Exercises (4) and (5).]

\* \* \*

Do these exercises on your work sheet.

$$A = \{(1, 1), (2, 2), (-3, -1)\}$$

$$B = \{(-1, -2), (2, 2), (-3, -1)\}$$

$$\text{Rule: } (x, y) \rightarrow (2x + 1, 3y)$$

- (1)  $A \cup B = \{ \underline{\hspace{2cm}} ? \underline{\hspace{2cm}} \}$
- (2) Make 2 moves from each point in set A. Call the new set 'X'.  
Then  $X = \{ \underline{\hspace{2cm}} ? \underline{\hspace{2cm}} \}$ .
- (3) Make 2 moves from each point in set B. Call the new set 'Y'.  
Then  $Y = \{ \underline{\hspace{2cm}} ? \underline{\hspace{2cm}} \}$ .
- (4) If 2 moves are made from each point in set  $A \cup B$  then the new set is  $\{ \underline{\hspace{2cm}} ? \underline{\hspace{2cm}} \}$ . [See Exercise (1) for  $A \cup B$ .]
- (5)  $X \cup Y = \{ \underline{\hspace{2cm}} ? \underline{\hspace{2cm}} \}$

Turn to PAGE 59.



Check your answers.

$$A = \{(1, 1), (2, 2), (-3, -1)\}$$

$$B = \{(-1, -2), (2, 2), (-3, -1)\}$$

$$\text{Rule: } (x, y) \rightarrow (2x + 1, 3y)$$

- (1)  $A \cup B = \{(1, 1), (2, 2), (-3, -1), (-1, -2)\}$
- (2) Make 2 moves from each point in set A. Call the new set 'X'.  
Then  $X = \{(7, 9), (11, 18), (-9, -9)\}$ .
- (3) Make 2 moves from each point in set B. Call the new set 'Y'.  
Then  $Y = \{(-1, -18), (11, 18), (-9, -9)\}$ .
- (4) If 2 moves are made from each point in set  $A \cup B$  then the new set is  $\{(7, 9), (11, 18), (-9, -9), (-1, -18)\}$ .
- (5)  $X \cup Y = \{(7, 9), (11, 18), (-9, -9), (-1, -18)\}$ .

Record your results on your work sheet.

\* \* \*

Do this exercise on your work sheet.

Suppose that A and B are sets of points in the number plane lattice such that

$$A \cup B = \{(1, 3), (2, 4), (-7, -3), (-5, -4)\}.$$

$$\text{Rule: } (x, y) \rightarrow (x + 3, 2y - 1)$$

Make 3 moves from each point in set A. Call the new set 'X'.

Make 3 moves from each point in set B. Call the new set 'Y'.

Then,  $X \cup Y = \underline{\hspace{2cm} ? \hspace{2cm}}$ .

Turn to PAGE 60.



Check your answer.

Suppose that A and B are sets of points in the number plane lattice such that

$$A \cup B = \{(1, 3), (2, 4), (-7, -3), (-5, -4)\}.$$

$$\text{Rule: } (x, y) \rightarrow (x + 3, 2y - 1)$$

Make 3 moves from each point in set A. Call the new set 'X'.

Make 3 moves from each point in set B. Call the new set 'Y'.

$$\text{Then, } X \cup Y = \{(10, 17), (11, 25), (2, -31), (4, -39)\}.$$

Record your result on your work sheet.

\* \* \*

Suppose that A and B are sets of points in the number plane lattice and that one of the points in  $A \cup B$  is (5, 11).

$$\text{Rule: } (x, y) \rightarrow (2x + 1, y - 3)$$

Make 2 moves from each point in set A. Call the new set 'X'.

Make 2 moves from each point in set B. Call the new set 'Y'.

Answer this question your work sheet.

What is one of the points in  $X \cup Y$ ?

Turn to PAGE 61.






Check your answer.

One of the points in  $A \cup B$  is  $(5, 11)$ .

$$\text{Rule: } (x, y) \rightarrow (2x + 1, y - 3)$$

Make 2 moves from each point in set A. Call the new set 'X'.

Make 2 moves from each point in set B. Call the new set 'Y'.

Then one of the points in  $X \cup Y$  is  $(23, 5)$ . 

Record your result on your work sheet.

\* \* \*

Suppose that A and B are sets of points in the number plane lattice and R is a moving rule.

From each point in set A, make n moves according to rule R. Let X be the new set.

From each point in set B, make n moves according to rule R. Let Y be the new set.

It follows that n moves according to rule R from any point in  $A \cup B$  takes you to a point in \_\_\_\_\_?

Complete this last sentence on your work sheet.

Turn to PAGE 62.



Check your answer.

Suppose that  $A$  and  $B$  are sets of points in the number plane lattice and  $R$  is a moving rule.

From each point in set  $A$ , make  $n$  moves according to rule  $R$ . Let  $X$  be the new set.

From each point in set  $B$ , make  $n$  moves according to rule  $R$ . Let  $Y$  be the new set.

It follows that  $n$  moves according to rule  $R$  from any point in  $A \cup B$  takes you to a point in  $X \cup Y$ .



Record your result on your work sheet.

\* \* \*

Suppose that  $A$  and  $B$  are sets of points in the number plane lattice and that  $(-1, 2)$  and  $(3, -2)$  belong to  $A \cup B$ .

$$\text{Rule: } (x, y) \rightarrow (x + 3, 2y - 1)$$

Make 2 moves from each point in set  $A$ . Call the new set ' $X$ '.

Make 2 moves from each point in set  $B$ . Call the new set ' $Y$ '.

It follows that two points in  $X \cup Y$  are  $\frac{?}{(1)}$  and  $\frac{?}{(2)}$ .

Complete this last sentence on your work sheet.

Turn to PAGE 63.



Check your answers.

$(-1, 2)$  and  $(3, -2)$  belong to  $A \cup B$ .

$$\text{Rule: } (x, y) \rightarrow (x + 3, 2y - 1)$$

Make 2 moves from each point in set A. Call the new set 'X'.

Make 2 moves from each point in set B. Call the new set 'Y'.

It follows that two points in  $X \cup Y$  are  $\frac{(5, 5)}{(1)}$  and  $\frac{(9, -11)}{(2)}$ .

Record your results on your work sheet.

\* \* \*

Do this exercise on your work sheet.

Suppose that A and B are sets of points in the number plane lattice such that

$$A \cup B = \{(4, 3), (4, 4)\}.$$

$$\text{Rule: } (x, y) \rightarrow (x + 5, 2y - 1)$$

Make 2 moves from each point in set A. Call the new set 'X'.

Make 2 moves from each point in set B. Call the new set 'Y'.

It follows that  $X \cup Y = \underline{\quad? \quad}$ .

Turn to PAGE 64.



Check your answer.

$$A \cup B = \{(4, 3), (4, 4)\}$$

$$\text{Rule: } (x, y) \rightarrow (x + 5, 2y - 1)$$

Make 2 moves from each point in set A. Call the new set 'X'.

Make 2 moves from each point in set B. Call the new set 'Y'.

It follows that  $X \cup Y = \underline{\{(14, 9), (14, 13)\}}$ .

Record your result on your work sheet.

\* \* \*

Do this exercise on your work sheet.

Suppose that A and B are sets of points in the number plane lattice such that

$$A \cup B = \{(a, b), (e, f)\},$$

[Of course, a, b, e, and f are integers.]

$$\text{Rule: } (x, y) \rightarrow (x + 3, 2y + 1)$$

Make 2 moves from each point in set A. Call the new set 'X'.

Make 2 moves from each point in set B. Call the new set 'Y'.

Then,  $X \cup Y = \underline{\quad ? \quad}$ .

Turn to PAGE 65.





Check your answer.

Suppose that A and B are sets of points in the number plane lattice such that

$$A \cup B = \{(a, b), (e, f)\},$$

where a, b, e, and f are integers.

$$\text{Rule: } (x, y) \rightarrow (x + 3, 2y + 1)$$

Make 2 moves from each point in set A. Call the new set 'X'.

Make 2 moves from each point in set B. Call the new set 'Y'.

$$\text{Then, } X \cup Y = \{(a+6, 4b+3), (e+6, 4f+3)\}.$$

Record your result on your work sheet.

Turn to PAGE 66.



Do these exercises on your work sheet.

Rule:  $(m, n) \rightarrow (m, n + 1)$

$A = \{(x, y), x \text{ and } y \text{ integers: } x = 3\}$

Make one move from each point in set A. Call the new set 'X'.

- (1) Is set A finite?                      Yes or no?
- (2) Is set X finite?                      Yes or no?
- (3) Is (3, 7) in set A?                      Yes or no?
- (4) Is (3, 8) in set X?                      Yes or no?
- (5) Is (3, 92) in set X?                      Yes or no?
- (6) Is (3, -4) in set X?                      Yes or no?
- (7) Find a point in set A which is not in set X.
- (8) Find a point in set X which is not in set A.
- (9) Give a brace-notation description of set X.

Turn to PAGE 67.



Check your answers.

$$\text{Rule: } (m, n) \rightarrow (m, n + 1)$$

$$A = \{(x, y), x \text{ and } y \text{ integers: } x = 3\}$$

Make one move from each point in set A. Call the new set 'X'.

- (1) Is set A finite? no                      (2) Is set X finite? no  
 (3) Is (3, 7) in set A? yes                      (4) Is (3, 8) in set X? yes  
 (5) Is (3, 92) in set X? yes                      (6) Is (3, -4) in set X? yes  
 (7) [There is no point in set A which is not in set X.]  
 (8) [There is no point in set X which is not in set A.]  
 (9)  $X = \{(x, y), x \text{ and } y \text{ integers: } x = 3\}$



Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$\text{Rule: } (m, n) \rightarrow (m + 3, n)$$

$$A = \{(x, y), x \text{ and } y \text{ integers: } y = 5\}$$

Make one move from each point in set A. Call the new set 'X'.

- (1) Is set A finite?                      Yes or no?  
 (2) Is set X finite?                      Yes or no?  
 (3) Give a brace-notation description of set X.

Turn to PAGE 68.



Check your answers.

$$\text{Rule: } (m, n) \mapsto (m + 3, n)$$

$$A = \{(x, y), x \text{ and } y \text{ integers: } y = 5\}$$

Make one move from each point in set A. Call the new set 'X'.

(1) Is set A finite? no

(2) Is set X finite? no

(3)  $X = \{(x, y), x \text{ and } y \text{ integers: } y = 5\}$

Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$\text{Rule: } (m, n) \mapsto (m + 2, n + 1)$$

$$A = \{(x, y), x \text{ and } y \text{ integers: } x = 3\}$$

Make one move from each point in set A. Call the new set 'X'.

(1) Is set A finite?

(2) Is set X finite?

(3) Plot the points in each set on the same diagram and indicate the moves. [Remember, loops for A, cross-marks for X, small arrows.]

(4) Give a brace-notation description of set X.

Turn to PAGE 69.





Check your answers.

Rule:  $(m, n) \rightarrow (m + 2, n + 1)$

$A = \{(x, y), x \text{ and } y \text{ integers: } x = 3\}$

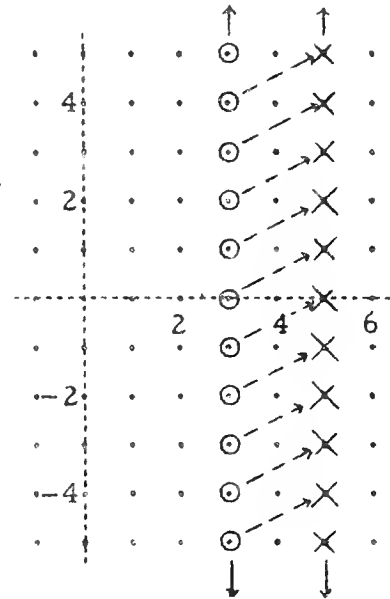
Make one move from each point in set A. Call the new set 'X'.

(1) Is set A finite? no

(2) Is set X finite? no

(3) .....  $\rightarrow$

(4)  $X = \{(x, y), x \text{ and } y \text{ integers: } x = 5\}$



Record your results on your work sheet.

\* \* \*

Do this exercise on your work sheet.

Rule:  $(m, n) \rightarrow (m + 2, n + 1)$

$A = \{(x, y), x \text{ and } y \text{ integers: } x = 6\}$

Make three moves from each point in set A. Call the new set 'X'.

Describe set X, using brace-notation.

Turn to PAGE 70.



Check your answer.

$$\text{Rule: } (m, n) \rightarrow (m + 2, n + 1)$$

$$A = \{(x, y), x \text{ and } y \text{ integers: } x = 6\}$$

Make 3 moves from each point in set A. Call the new set 'X'.

$$X = \{(x, y), x \text{ and } y \text{ integers: } x = 12\}$$



[After one move from each point in set A, the set selector of the description of the new set would be 'x = 8'. After 2 moves it would be 'x = 10'. So, after 3 moves it would be 'x = 12'.]

Record your result on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$\text{Rule: } (m, n) \rightarrow (m + 1, n + 1)$$

$$A = \{(x, y), x \text{ and } y \text{ integers: } x + y = y + x\}$$

Make one move from each point in set A. Call the new set 'X'.

Write 'true' in the blank if the statement is true. Write 'false' in the blank if the statement is false.

- (1) Set A is the number plane lattice itself.
- (2) Set A is infinite.
- (3) Set X is the number plane lattice itself.
- (4) Set X is infinite.
- (5) Give a brace-notation description of set X.

Turn to PAGE 71.



Check your answers.

$$\text{Rule: } (m, n) \rightarrow (m + 1, n + 1)$$

$$A = \{(x, y), x \text{ and } y \text{ integers: } x + y = y + x\}$$

Make one move from each point in set A. Call the new set 'X'.

- (1) Set A is the number plane lattice itself. true
- (2) Set A is infinite. true
- (3) Set X is the number plane lattice itself. true
- (4) Set X is infinite. true
- (5) [Any description which names the set of all ordered pairs of integers is correct.]

Record your results on your work sheet.

\* \* \*

For each point listed below in Exercises (1) - (6), write 'yes' if the point belongs to set A, and write 'no' if the point does not belong to set A, where

$$A = \{(x, y), x \text{ and } y \text{ integers: } x^2 + y^2 = 9\}.$$

Do these exercises on your work sheet.

- (1) (3, 0)                      (2) (0, -3)                      (3) (2, 7)
- (4) (-3, 0)                      (5) (5, 4)                      (6) (0, 3)
- (7)  $n(A) = \underline{\quad ? \quad}$  [Remember, 'n(A)' means the number of elements in A.]

Turn to PAGE 72.



Check your answers.

$$A = \{(x, y), x \text{ and } y \text{ integers: } x^2 + y^2 = 9\}$$

- (1) (3, 0) yes                      (2) (0, -3) yes                      (3) (2, 7) no  
 (4) (-3, 0) yes                      (5) (5, 4) no                      (6) (0, 3) yes  
 (7)  $n(A) = 4$

Record your results on your work sheet.

\* \* \*

For each point listed below in Exercises (1) - (6), write 'yes' if the point belongs to set A, and write 'no' if the point does not belong to set A, where

$$A = \{(x, y), x \text{ and } y \text{ integers: } x^2 + y^2 = 25\}.$$

Do these exercises on your work sheet.

- (1) (-5, 0)                      (2) (3, -4)                      (3) (25, 0)  
 (4) (-4, -3)                      (5) (0, 5)                      (6) (16, 9)  
 (7) Plot all of the points in set A. There are a total of 12 points in set A.

Turn to PAGE 73.

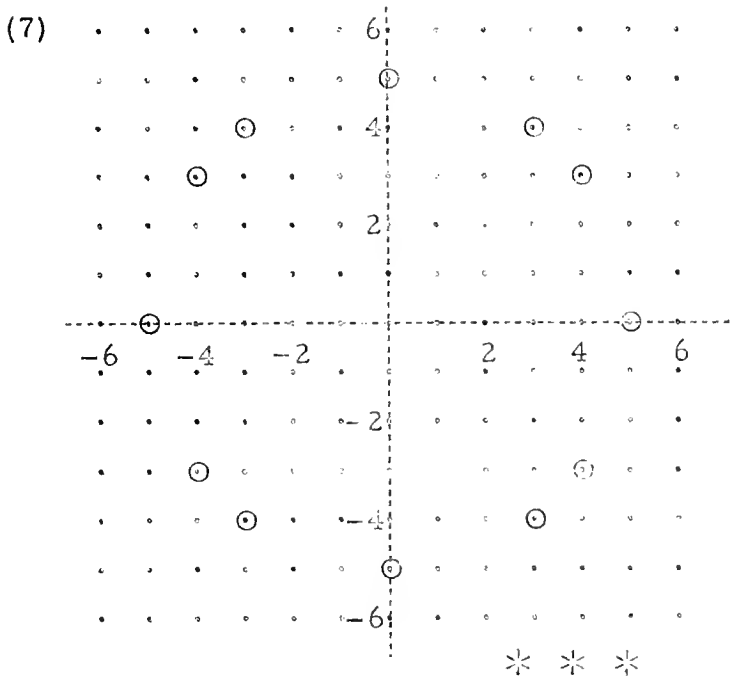




Check your answers.

$$A = \{(x, y), x \text{ and } y \text{ integers: } x^2 + y^2 = 25\}$$

- (1) (-5, 0) yes                      (2) (3, -4) yes                      (3) (25, 0) no  
 (4) (-4, -3) yes                      (5) (0, 5) yes                      (6) (16, 9) no



Record your results on your work sheet.

Look at Answer (7) above. Notice that the points in set A are arranged on the circle with center at (0, 0) and radius 5.

Do these exercises on your work sheet.

$$A = \{(x, y), x \text{ and } y \text{ integers: } x^2 + y^2 = 25\}$$

$$\text{Rule: } (m, n) \rightarrow (m + 2, n)$$

Make one move from each point in set A. Call the new set 'X'.

- (1) Plot the points in each set on the same diagram and indicate the moves. [Loops for A, cross-marks for X.]  
 (2) Which of the sets described below is set X?

$$\{(x, y), x \text{ and } y \text{ integers: } (x - 2)^2 + y^2 = 25\}$$

$$\{(x, y), x \text{ and } y \text{ integers: } (x + 2)^2 + y^2 = 25\}$$

- (3) The points in set X are arranged on the circle with center at    ?    and radius    ?   .

Turn to PAGE 74.

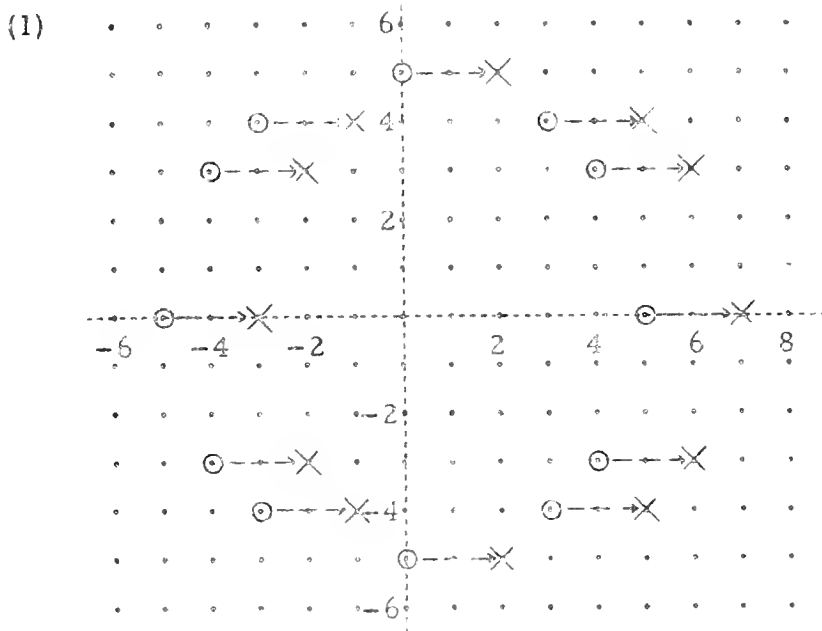


Check your answers.

$$A = \{(x, y), x \text{ and } y \text{ integers: } x^2 + y^2 = 25\}$$

$$\text{Rule: } (m, n) \rightarrow (m + 2, n)$$

Make one move from each point in set A. Call the new set 'X'.



(2)  $X = \{(x, y), x \text{ and } y \text{ integers: } (x-2)^2 + y^2 = 25\}$

(3) The points in set X are arranged on the circle with center at  $\frac{(2, 0)}{(a)}$  and radius  $\frac{5}{(b)}$ .

Record your results on your work sheet.

\* \* \*

Do these exercises on your work sheet.

$$A = \{(x, y), x \text{ and } y \text{ integers: } x^2 + y^2 = 25\}$$

$$\text{Rule: } (m, n) \rightarrow (m + 2, n - 3)$$

Make two moves from each point in set A. Call the new set 'X'.

(1) Plot the points in each set on the same diagram.

(2) The points in set X are arranged on the circle with center at    ? and radius    ?.

☆(3)  $X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2cm}} \text{ ? } \underline{\hspace{2cm}}\}$

Turn to PAGE 75.

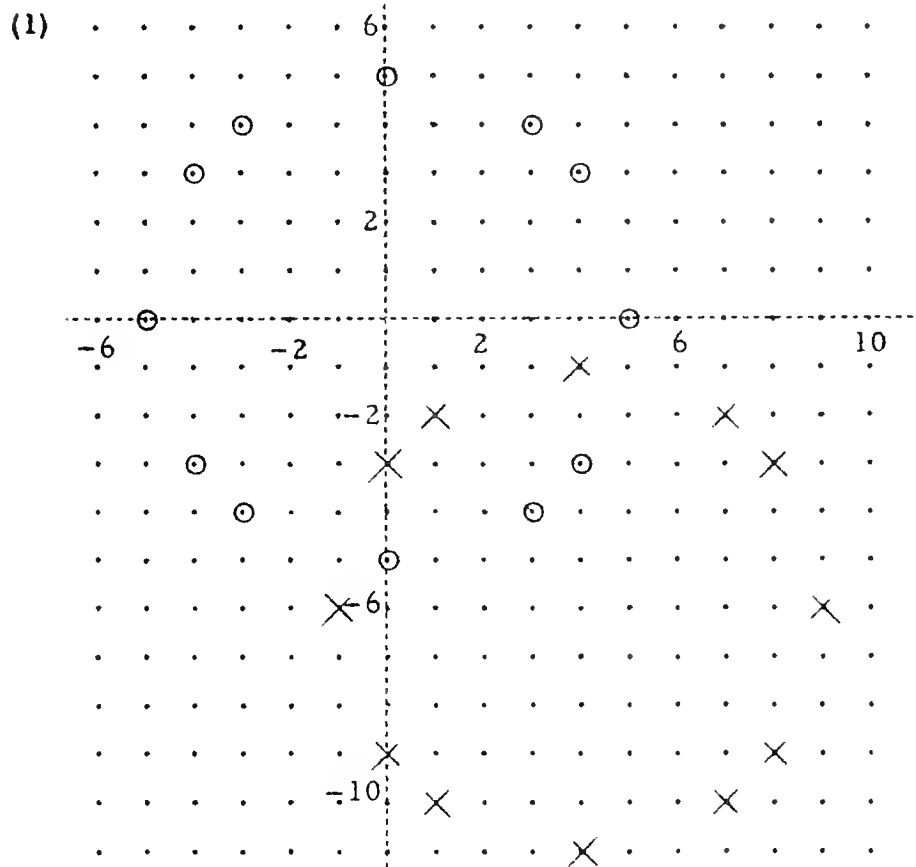


Check your answers.

$$A = \{(x, y), x \text{ and } y \text{ integers: } x^2 + y^2 = 25\}$$

$$\text{Rule: } (m, n) \rightarrow (m + 2, n - 3)$$

Make two moves from each point in set A. Call the new set 'X'.



(2) The points in set X are arranged on the circle with center at  $\frac{(4, -6)}{(a)}$  and radius  $\frac{5}{(b)}$ .

★(3)  $X = \{(x, y), x \text{ and } y \text{ integers: } (x-4)^2 + (y+6)^2 = 25\}$

If you tried Exercise (3) and got it right, turn to PAGE 76.

Otherwise, this is the end of Part 43. Put your work sheet under the front cover of this booklet, and return it to your teacher.



Do this exercise on your work sheet.

$$A = \{(x, y), x \text{ and } y \text{ integers: } 2x^2 + 7xy + 6y^2 = 0\}$$

Rule:  $(x, y) \mapsto (x + a, y - b)$ ,  $a$  and  $b$  are integers

Make  $n$  moves from each point in set  $A$ . Call the new set 'X'.

Write a brace-notation description of set  $X$ .

Turn to PAGE 77.





Check your answer.

$$A = \{(x, y), x \text{ and } y \text{ integers: } 2x^2 + 7xy + 6y^2 = 0\}$$

$$\text{Rule: } (x, y) \rightarrow (x + a, y - b), a \text{ and } b \text{ integers}$$

Make  $n$  moves from each point in set  $A$ . Call the new set 'X'.

$$X = \{(x, y), x \text{ and } y \text{ integers: } 2(x-na)^2 + 7(x-na)(y+nb) + 6(y+nb)^2 = 0\}$$

Put your work sheet under the front cover of this booklet, and return it to your teacher.

















# PART 43

WORK SHEET

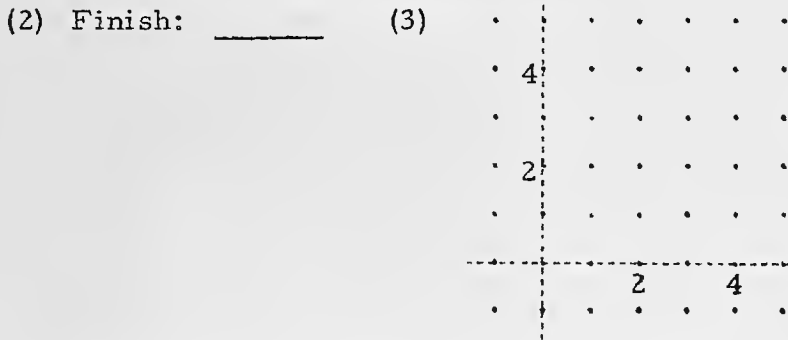
Name \_\_\_\_\_  
 School \_\_\_\_\_  
 Date \_\_\_\_\_

## Answers

## Result Check

### Page 3

- (1) First jump: From (4, 1) to (3, 2)  
 Second jump: From (3, 2) to (\_\_\_\_, \_\_\_\_)

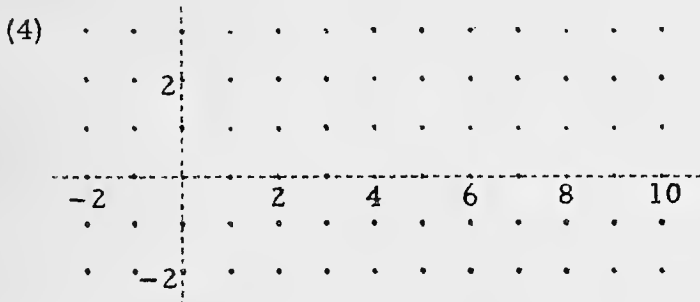


### Page 3

1  
 2 3  
 OK

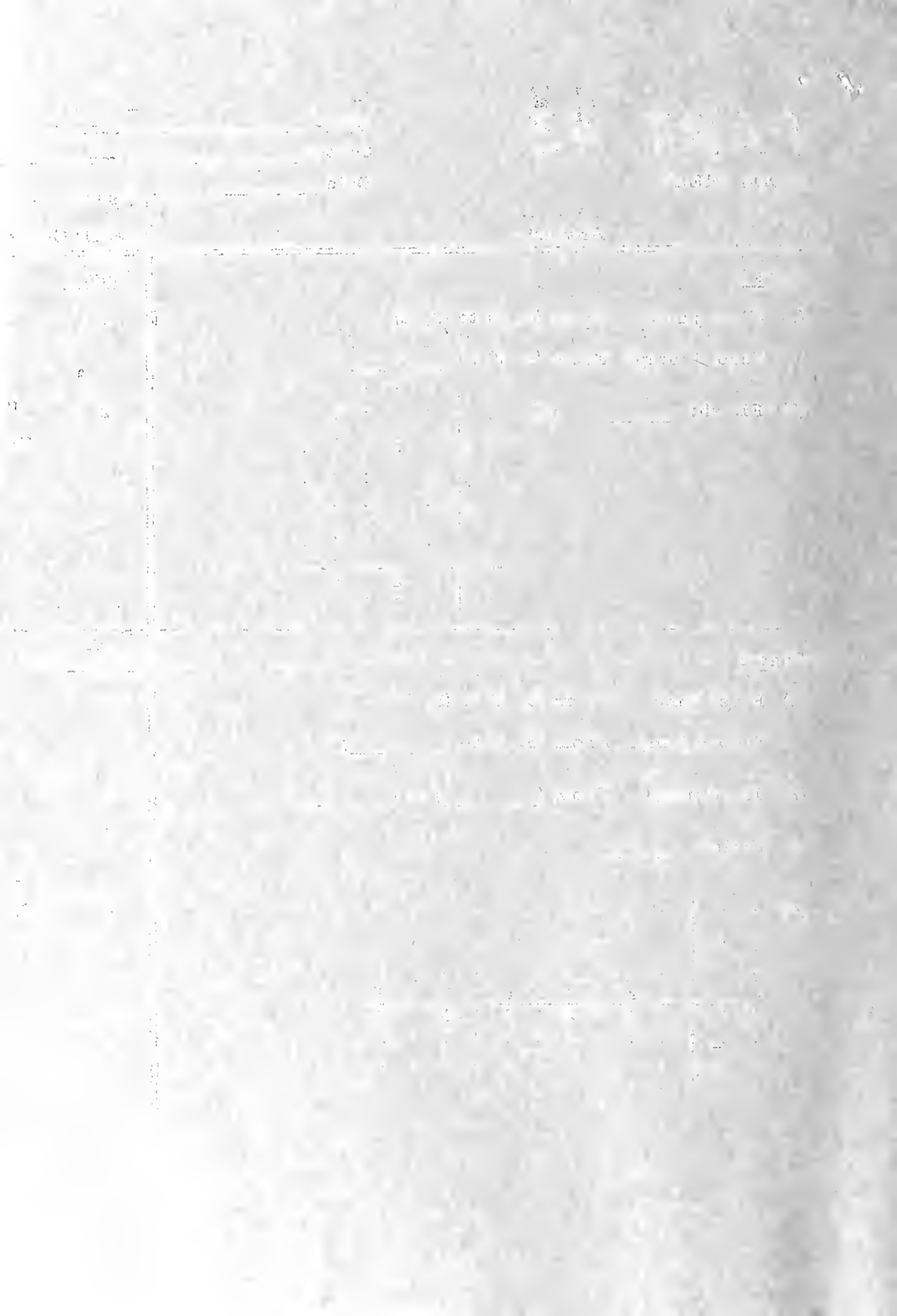
### Page 4

- (1) First jump: From (0, 1) to (3, 1)  
 Second jump: From (3, 1) to (\_\_\_\_, \_\_\_\_)
- (2) Third jump: From (\_\_\_\_, \_\_\_\_) to (\_\_\_\_, \_\_\_\_)
- (3) Finish: \_\_\_\_\_



### Page 4

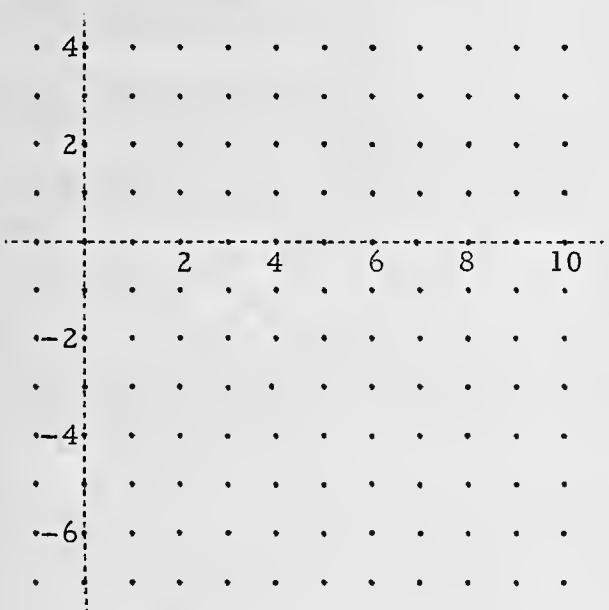
1  
 2  
 3  
 4  
 OK

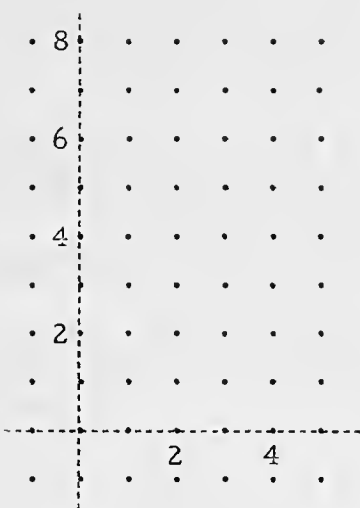


WORK SHEET

Name \_\_\_\_\_

Part 43

Answers	Result Check
<u>Page 6</u>	<u>Page 6</u>
(1) First move takes you to (5, 0). Second move takes you to (____, ____).	1
(2) Third move takes you to (____, ____).	2
(3) Finish: _____	3
(4) 	4      OK

<u>Page 7</u>	<u>Page 7</u>
(1) Finish: _____	1 2
(2) 	OK

QUESTION 1

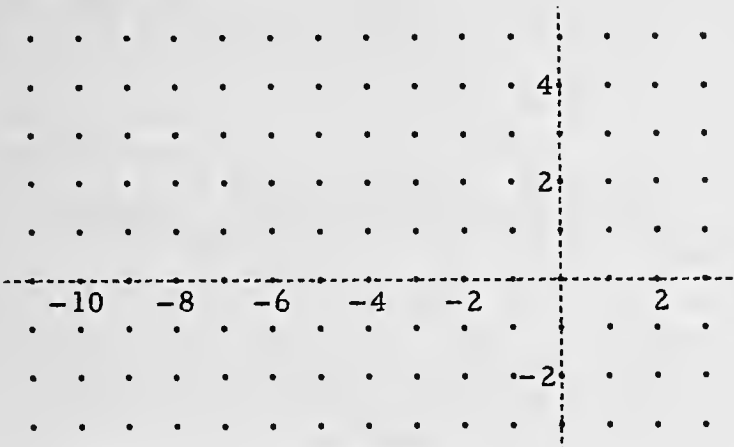
1.1. The following table shows the number of students who took part in a school sports day. (1)

Age Group	Number of Students
10-11	15
12-13	20
14-15	25
16-17	30
18-19	35
20-21	40
22-23	45
24-25	50
26-27	55
28-29	60
30-31	65
32-33	70
34-35	75
36-37	80
38-39	85
40-41	90
42-43	95
44-45	100
46-47	105
48-49	110
50-51	115
52-53	120
54-55	125
56-57	130
58-59	135
60-61	140
62-63	145
64-65	150
66-67	155
68-69	160
70-71	165
72-73	170
74-75	175
76-77	180
78-79	185
80-81	190
82-83	195
84-85	200
86-87	205
88-89	210
90-91	215
92-93	220
94-95	225
96-97	230
98-99	235
100-101	240
102-103	245
104-105	250
106-107	255
108-109	260
110-111	265
112-113	270
114-115	275
116-117	280
118-119	285
120-121	290
122-123	295
124-125	300
126-127	305
128-129	310
130-131	315
132-133	320
134-135	325
136-137	330
138-139	335
140-141	340
142-143	345
144-145	350
146-147	355
148-149	360
150-151	365
152-153	370
154-155	375
156-157	380
158-159	385
160-161	390
162-163	395
164-165	400
166-167	405
168-169	410
170-171	415
172-173	420
174-175	425
176-177	430
178-179	435
180-181	440
182-183	445
184-185	450
186-187	455
188-189	460
190-191	465
192-193	470
194-195	475
196-197	480
198-199	485
200-201	490
202-203	495
204-205	500
206-207	505
208-209	510
210-211	515
212-213	520
214-215	525
216-217	530
218-219	535
220-221	540
222-223	545
224-225	550
226-227	555
228-229	560
230-231	565
232-233	570
234-235	575
236-237	580
238-239	585
240-241	590
242-243	595
244-245	600
246-247	605
248-249	610
250-251	615
252-253	620
254-255	625
256-257	630
258-259	635
260-261	640
262-263	645
264-265	650
266-267	655
268-269	660
270-271	665
272-273	670
274-275	675
276-277	680
278-279	685
280-281	690
282-283	695
284-285	700
286-287	705
288-289	710
290-291	715
292-293	720
294-295	725
296-297	730
298-299	735
300-301	740
302-303	745
304-305	750
306-307	755
308-309	760
310-311	765
312-313	770
314-315	775
316-317	780
318-319	785
320-321	790
322-323	795
324-325	800
326-327	805
328-329	810
330-331	815
332-333	820
334-335	825
336-337	830
338-339	835
340-341	840
342-343	845
344-345	850
346-347	855
348-349	860
350-351	865
352-353	870
354-355	875
356-357	880
358-359	885
360-361	890
362-363	895
364-365	900
366-367	905
368-369	910
370-371	915
372-373	920
374-375	925
376-377	930
378-379	935
380-381	940
382-383	945
384-385	950
386-387	955
388-389	960
390-391	965
392-393	970
394-395	975
396-397	980
398-399	985
400-401	990
402-403	995
404-405	1000
406-407	1005
408-409	1010
410-411	1015
412-413	1020
414-415	1025
416-417	1030
418-419	1035
420-421	1040
422-423	1045
424-425	1050
426-427	1055
428-429	1060
430-431	1065
432-433	1070
434-435	1075
436-437	1080
438-439	1085
440-441	1090
442-443	1095
444-445	1100
446-447	1105
448-449	1110
450-451	1115
452-453	1120
454-455	1125
456-457	1130
458-459	1135
460-461	1140
462-463	1145
464-465	1150
466-467	1155
468-469	1160
470-471	1165
472-473	1170
474-475	1175
476-477	1180
478-479	1185
480-481	1190
482-483	1195
484-485	1200
486-487	1205
488-489	1210
490-491	1215
492-493	1220
494-495	1225
496-497	1230
498-499	1235
500-501	1240
502-503	1245
504-505	1250
506-507	1255
508-509	1260
510-511	1265
512-513	1270
514-515	1275
516-517	1280
518-519	1285
520-521	1290
522-523	1295
524-525	1300
526-527	1305
528-529	1310
530-531	1315
532-533	1320
534-535	1325
536-537	1330
538-539	1335
540-541	1340
542-543	1345
544-545	1350
546-547	1355
548-549	1360
550-551	1365
552-553	1370
554-555	1375
556-557	1380
558-559	1385
560-561	1390
562-563	1395
564-565	1400
566-567	1405
568-569	1410
570-571	1415
572-573	1420
574-575	1425
576-577	1430
578-579	1435
580-581	1440
582-583	1445
584-585	1450
586-587	1455
588-589	1460
590-591	1465
592-593	1470
594-595	1475
596-597	1480
598-599	1485
600-601	1490
602-603	1495
604-605	1500
606-607	1505
608-609	1510
610-611	1515
612-613	1520
614-615	1525
616-617	1530
618-619	1535
620-621	1540
622-623	1545
624-625	1550
626-627	1555
628-629	1560
630-631	1565
632-633	1570
634-635	1575
636-637	1580
638-639	1585
640-641	1590
642-643	1595
644-645	1600
646-647	1605
648-649	1610
650-651	1615
652-653	1620
654-655	1625
656-657	1630
658-659	1635
660-661	1640
662-663	1645
664-665	1650
666-667	1655
668-669	1660
670-671	1665
672-673	1670
674-675	1675
676-677	1680
678-679	1685
680-681	1690
682-683	1695
684-685	1700
686-687	1705
688-689	1710
690-691	1715
692-693	1720
694-695	1725
696-697	1730
698-699	1735
700-701	1740
702-703	1745
704-705	1750
706-707	1755
708-709	1760
710-711	1765
712-713	1770
714-715	1775
716-717	1780
718-719	1785
720-721	1790
722-723	1795
724-725	1800
726-727	1805
728-729	1810
730-731	1815
732-733	1820
734-735	1825
736-737	1830
738-739	1835
740-741	1840
742-743	1845
744-745	1850
746-747	1855
748-749	1860
750-751	1865
752-753	1870
754-755	1875
756-757	1880
758-759	1885
760-761	1890
762-763	1895
764-765	1900
766-767	1905
768-769	1910
770-771	1915
772-773	1920
774-775	1925
776-777	1930
778-779	1935
780-781	1940
782-783	1945
784-785	1950
786-787	1955
788-789	1960
790-791	1965
792-793	1970
794-795	1975
796-797	1980
798-799	1985
800-801	1990
802-803	1995
804-805	2000
806-807	2005
808-809	2010
810-811	2015
812-813	2020
814-815	2025
816-817	2030
818-819	2035
820-821	2040
822-823	2045
824-825	2050
826-827	2055
828-829	2060
830-831	2065
832-833	2070
834-835	2075
836-837	2080
838-839	2085
840-841	2090
842-843	2095
844-845	2100
846-847	2105
848-849	2110
850-851	2115
852-853	2120
854-855	2125
856-857	2130
858-859	2135
860-861	2140
862-863	2145
864-865	2150
866-867	2155
868-869	2160
870-871	2165
872-873	2170
874-875	2175
876-877	2180
878-879	2185
880-881	2190
882-883	2195
884-885	2200
886-887	2205
888-889	2210
890-891	2215
892-893	2220
894-895	2225
896-897	2230
898-899	2235
900-901	2240
902-903	2245
904-905	2250
906-907	2255
908-909	2260
910-911	2265
912-913	2270
914-915	2275
916-917	2280
918-919	2285
920-921	2290
922-923	2295
924-925	2300
926-927	2305
928-929	2310
930-931	2315
932-933	2320
934-935	2325
936-937	2330
938-939	2335
940-941	2340
942-943	2345
944-945	2350
946-947	2355
948-949	2360
950-951	2365
952-953	2370
954-955	2375
956-957	2380
958-959	2385
960-961	2390
962-963	2395
964-965	2400
966-967	2405
968-969	2410
970-971	2415
972-973	2420
974-975	2425
976-977	2430
978-979	2435
980-981	2440
982-983	2445
984-985	2450
986-987	2455
988-989	2460
990-991	2465
992-993	2470
994-995	2475
996-997	2480
998-999	2485
1000-1001	2490
1002-1003	2495
1004-1005	2500
1006-1007	2505
1008-1009	2510
1010-1011	2515

WORK SHEET

Name \_\_\_\_\_

Part 43

Answers	Result Check
<p><u>Page 8</u></p> <p>The final point, or the finish is _____.</p>	<p><u>Page 8</u></p> <p>✓ OK</p>
<p><u>Page 9</u></p> <p>(1) First move takes you to (1, 0)  Second move takes you to (____, ____).</p> <p>(2) Third move takes you to (____, ____).</p> <p>(3) Final point: _____</p> <p>(4) </p>	<p><u>Page 9</u></p> <p>1</p> <p>2</p> <p>3</p> <p>4</p> <p>OK</p>
<p><u>Page 10</u></p> <p>The final point is _____.</p>	<p><u>Page 10</u></p> <p>✓ OK</p>
<p><u>Page 11</u></p> <p>(1) First move takes you to (4, 5).  Second move takes you to (16, ____).</p> <p>(2) Third move takes you to (____, ____).</p> <p>(3) So, the final point is (____, ____).</p>	<p><u>Page 11</u></p> <p>1</p> <p>2</p> <p>3 OK</p>

1950-1951

1950-1951

1950-1951

1950-1951

1950-1951

1950-1951

1950-1951

1950-1951

1950-1951

1950-1951

1950-1951

1950-1951

1950-1951

1950-1951

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1950-1951

1950-1951

WORK SHEET

Name \_\_\_\_\_

Part 43

Answers	Result Check
<u>Page 12</u>	<u>Page 12</u>
(1) First move takes you to (1, 4). _____	1
(2) Second move takes you to _____.	2
(3) Third move takes you to _____.	3
(4) The final point after 4 moves would be _____.	4
(5) The final point after 20 moves would be _____.	5
(6) $\begin{array}{ccc ccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 4 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \hline -2 & & & & & 2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -2 & \cdot & \cdot & \cdot \end{array}$ [Show <u>first 3</u> moves only.]	6    OK
<u>Page 13</u>	<u>Page 13</u>
The <u>starting</u> point was _____.	✓    OK
<u>Page 14</u>	<u>Page 14</u>
The <u>starting</u> point was _____.	✓    OK
<u>Page 15</u>	<u>Page 15</u>
(1) After 2 moves: (1, 8) After 1 move: (3, ___)	1
(2) Start: (___, ___)	2    OK

Section 1: Introduction

The purpose of this document is to provide a comprehensive overview of the project's objectives and scope.

The project aims to develop a robust system that can handle complex data processing tasks efficiently.

The system will be designed to be scalable and maintainable, ensuring long-term usability.

The following sections will detail the system architecture, implementation details, and testing procedures.

Section 2: System Architecture

The system architecture is based on a modular design, allowing for easy integration and updates.

The architecture consists of several key components, including the user interface, data storage, and processing engine.

Each component is designed to work seamlessly together to achieve the project's goals.



WORK SHEET

Name \_\_\_\_\_

Part 43

Answers	Result Check
<p><u>Page 16</u></p> <p>(1) After 4 moves: <math>(-6, 3)</math>                      After 3 moves: <math>(-3, \underline{\quad})</math></p> <p>(2) After 2 moves: <math>(\underline{\quad}, \underline{\quad})</math></p> <p>(3) After 1 move: <math>(\underline{\quad}, \underline{\quad})</math></p> <p>(4) Start: <math>(\underline{\quad}, \underline{\quad})</math></p> <p>(5) <math>\begin{array}{cccccccc} \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; 4 &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot \\ \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot \\ \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; 2 &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot \\ \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot \\ \hline -6 &amp; -4 &amp; -2 &amp; &amp; &amp; &amp; 2 &amp; 4 &amp; 6 &amp; &amp; &amp; &amp; \\ \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot \\ \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; -2 &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot &amp; \cdot \end{array}</math></p>	<p><u>Page 16</u></p> <p>1</p> <p>2</p> <p>3</p> <p>4</p> <p>5 OK</p>
<p><u>Page 17</u></p> <p>(1) After 3 moves: <math>(15, -3)</math>                      After 2 moves: <math>(7, \underline{\quad})</math></p> <p>(2) After 1 move: <math>(\underline{\quad}, \underline{\quad})</math></p> <p>(3) Start: <math>(\underline{\quad}, \underline{\quad})</math></p>	<p><u>Page 17</u></p> <p>1</p> <p>2</p> <p>3 OK</p>
<p><u>Page 18</u></p> <p>(1) After 2 moves: <math>(11, 3)</math>                      After 1 move: <math>(5, \underline{\quad})</math></p> <p>(2) Start: <math>(\underline{\quad}, \underline{\quad})</math></p>	<p><u>Page 18</u></p> <p>1</p> <p>2 OK</p>
<p><u>Page 19</u></p> <p>(1) After 3 moves: <math>(9, 7)</math>                      After 2 moves: <math>(5, \underline{\quad})</math> <math>[2 \cdot \underline{\quad} + 1 = 7]</math></p> <p>(2) After 1 move: <math>(\underline{\quad}, \underline{\quad})</math></p> <p>(3) Start: _____</p>	<p><u>Page 19</u></p> <p>1</p> <p>2</p> <p>3 OK</p>

4.  $\vec{a} = (1, 2, 3)$

$\vec{b} = (2, 1, 1)$

$\vec{c} = (3, 1, 1)$

$\vec{d} = (1, 1, 1)$

$\vec{e} = (1, 1, 1)$

$\vec{f} = (1, 1, 1)$

$\vec{g} = (1, 1, 1)$

$\vec{h} = (1, 1, 1)$

$\vec{i} = (1, 1, 1)$

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WORK SHEET

Name \_\_\_\_\_

Part 43

Answers		Result Check																				
<u>Page 20</u>		<u>Page 20</u>																				
<table border="1"> <tr><td>• • 8</td><td>• • • • • • • • • •</td></tr> <tr><td>• •</td><td>• • • • • • • • • •</td></tr> <tr><td>• • 6</td><td>• • • • • • • • • •</td></tr> <tr><td>• •</td><td>• • • • • • • • • •</td></tr> <tr><td>• • 4</td><td>• • • • • • • • • •</td></tr> <tr><td>• •</td><td>• • • • • • • • • •</td></tr> <tr><td>• • 2</td><td>• • • • • • • • • •</td></tr> <tr><td>• •</td><td>• • • • • • • • • •</td></tr> <tr><td>-2</td><td>2 4 6 8 10</td></tr> <tr><td>• •</td><td>• • • • • • • • • •</td></tr> </table>	• • 8	• • • • • • • • • •	• •	• • • • • • • • • •	• • 6	• • • • • • • • • •	• •	• • • • • • • • • •	• • 4	• • • • • • • • • •	• •	• • • • • • • • • •	• • 2	• • • • • • • • • •	• •	• • • • • • • • • •	-2	2 4 6 8 10	• •	• • • • • • • • • •		✓ OK
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• •	• • • • • • • • • •																					
<u>Page 21</u>		<u>Page 21</u>																				
<p>(1) After 3 moves: (53, -7)</p> <p>After 2 moves: (17, 5)</p> <p>After 1 move: _____</p> <p>[3 • ____ + 2 = 17. 3 - 2 • ____ = 5.]</p>		1																				
<p>(2) Start: _____</p>		2 OK																				
<u>Page 22</u>		<u>Page 22</u>																				
<p>(1) After 3 moves: (28, 12)</p> <p>After 2 moves: _____</p>		1																				
<p>(2) After 1 move: _____</p>		2																				
<p>(3) Start: _____</p>		3 OK																				



Part 43

Answers

Result Check

Page 24

Page 24

Rule  $(x, y) \rightarrow (2x + 5, 3 - 2y)$

After 3 moves, the final point is  $(19, -15)$ .

First Component

Second Component

$$2x + 5 = 19$$

$$3 - 2y = -15$$

$$x = \underline{\quad}$$

$$y = \underline{\quad}$$

After 2 moves:  $(\underline{\quad}, \underline{\quad})$

$$2x + 5 = \underline{\quad}$$

$$3 - 2y = \underline{\quad}$$

$$x = \underline{\quad}$$

$$y = \underline{\quad}$$

After 1 move:  $(\underline{\quad}, \underline{\quad})$

$$2x + 5 = \underline{\quad}$$

$$3 - 2y = \underline{\quad}$$

$$x = \underline{\quad}$$

$$y = \underline{\quad}$$

Starting point: \_\_\_\_\_



✓ OK



WORK SHEET

Name \_\_\_\_\_

Part 43

Answers	Result Check																		
<p><u>Page 25</u></p> <p>After 4 moves:</p> <p>After 3 moves:</p> <p>After 2 moves:</p> <p>After 1 move:</p> <p>Start: _____</p>	<p><u>Page 25</u></p> <p>✓ OK</p>																		
<p><u>Page 26</u></p> <p>(1) From (2, 2), you move to (___ + ___, ___ - ___), or (___, ___).</p> <p>(2) X = { _____, _____, _____ }</p>	<p><u>Page 26</u></p> <p>1</p> <p>2 OK</p>																		
<p><u>Page 27</u></p> <p>(1) X = { _____, _____, _____, _____ }</p> <p>(2)</p> <table style="border-collapse: collapse; margin-left: 20px;"> <tr> <td style="border-right: 1px dashed black; padding: 2px 10px;">. . 4</td> <td style="padding: 2px 10px;">. . . .</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 10px;">. .</td> <td style="padding: 2px 10px;">. . . .</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 10px;">. . 2</td> <td style="padding: 2px 10px;">. . . .</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 10px;">. .</td> <td style="padding: 2px 10px;">. . . .</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 10px;">-2</td> <td style="padding: 2px 10px;">. . . .</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 10px;">. .</td> <td style="padding: 2px 10px;">. . . .</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 10px;">. . -2</td> <td style="padding: 2px 10px;">. . . .</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 10px;">. .</td> <td style="padding: 2px 10px;">. . . .</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 10px;">. . -4</td> <td style="padding: 2px 10px;">. . . .</td> </tr> </table>	. . 4	. . . .	. .	. . . .	. . 2	. . . .	. .	. . . .	-2	. . . .	. .	. . . .	. . -2	. . . .	. .	. . . .	. . -4	. . . .	<p><u>Page 27</u></p> <p>1</p> <p>2 OK</p>
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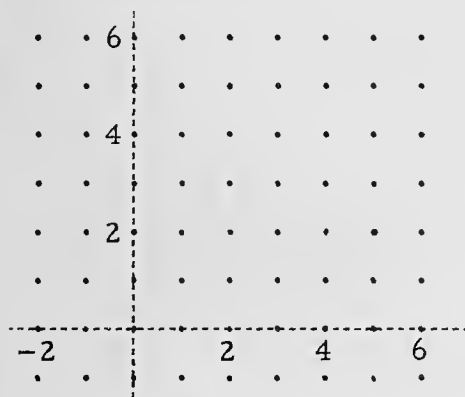
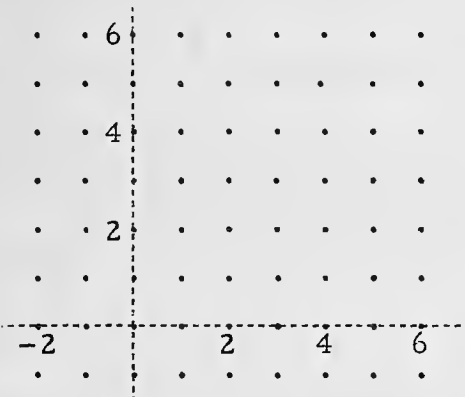




WORK SHEET

Name \_\_\_\_\_

Part 43

Answers	Result Check
<p><u>Page 28</u></p> <p>(1) <math>X = \{(1, 2), (2, \underline{\quad}), \underline{\quad}, \underline{\quad}\}</math></p> <p>(2) </p>	<p><u>Page 28</u></p> <p>1</p> <p>2 OK</p>
<p><u>Page 29</u></p> <p>(1) <math>A = \{ \underline{\hspace{10em}} \}</math></p> <p>(2) <math>X = \{ \underline{\hspace{10em}} \}</math></p> <p>(3) </p> <p>(4) <math>X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2em}} \text{ and } x = 1\}</math></p>	<p><u>Page 29</u></p> <p>1</p> <p>2</p> <p>3</p> <p>4 OK</p>

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2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.

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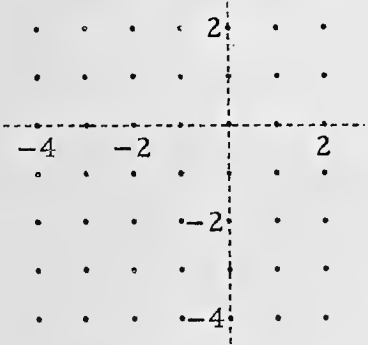
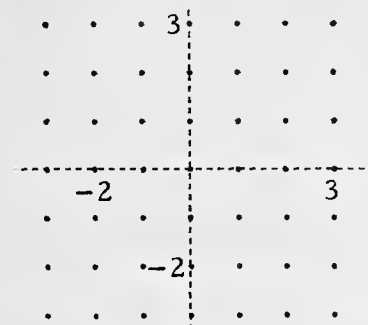
602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700.

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Part 43

| Answers  | Result Check  |
|--|---|
| <p><u>Page 30</u></p> <p>(1) <math>A = \{ \underline{\hspace{10em}} \}</math></p> <p>(2) <math>X = \{ \underline{\hspace{10em}} \}</math></p> <p>(3) </p> <p>(4) <math>X = \{(x, y), x \text{ and } y \text{ integers: } -2 &lt; y &lt; 2 \text{ and } \underline{\hspace{10em}} \}</math></p>                  | <p><u>Page 30</u></p> <p>1</p> <p>2</p> <p>3</p> <p>4      OK</p> |
| <p><u>Page 31</u></p> <p>(1) <math>A = \{ \underline{\hspace{2em}}, \underline{\hspace{2em}}, \underline{\hspace{2em}} \}</math></p> <p>(2) <math>X = \{(x, y), x \text{ and } y \text{ integers: } 1 &lt; x &lt; 5 \text{ and } \underline{\hspace{2em}} \}</math></p> <p>(3) <math>A = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2em}} \text{ and } x = 7 \}</math></p> | <p><u>Page 31</u></p> <p>1</p> <p>2</p> <p>3      OK</p>          |
| <p><u>Page 32</u></p> <p><math>X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2em}} \text{ and } \underline{\hspace{2em}} \}</math></p>  | <p><u>Page 32</u></p> <p>✓      OK</p>                            |
| <p><u>Page 34</u></p> <p>(1) <math>X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2em}} \text{ and } \underline{\hspace{2em}} \}</math></p> <p>(2) </p>   | <p><u>Page 34</u></p> <p>1</p> <p>2      OK</p>                   |



Part 43

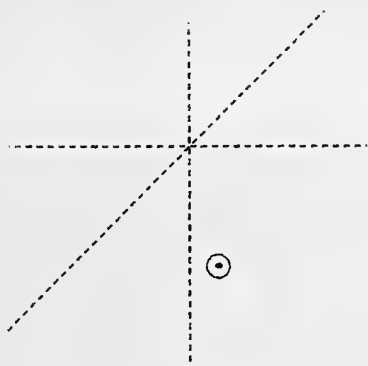
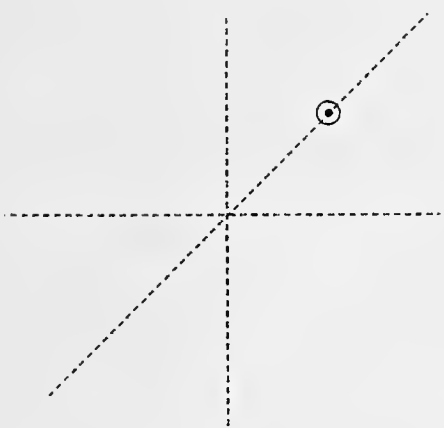
| Answers   | Result Check  |
|---|---|
| <p><u>Page 35</u></p> <p>(1) <math>X =</math> _____</p> <p>(2) </p>   | <p><u>Page 35</u></p> <p>1</p> <p>2 OK</p>          |
| <p><u>Page 36</u></p> <p>(1) </p> <p>(2) <math>A = \{ \text{_____} \}</math></p> <p>(3) <math>X = \{ \text{_____} \}</math></p>   | <p><u>Page 36</u></p> <p>1</p> <p>2</p> <p>3 OK</p> |
| <p><u>Page 37</u></p> <p>(1) </p> <p>(4) [Use diagram in Exercise (1).]</p> <p>(2) <math>A = \{ \text{_____} \}</math></p> <p>(3) <math>X = \{ \text{_____} \}</math></p> | <p><u>Page 37</u></p> <p>1</p> <p>2</p> <p>3 OK</p> |



WORK SHEET

Name \_\_\_\_\_

Part 43

| Answers        |           |   | Result Check   |    |
|----------------|-----------|---|----------------|----|
| <u>Page 39</u> |           |   | <u>Page 39</u> |    |
| Point (1)      | Point (2) | Point (3)   | ✓              | OK |
| <u>Page 40</u> |           |   | <u>Page 40</u> |    |
| Point (1)      | Point (2) | Point (3)   | ✓              | OK |
| <u>Page 41</u> |           |   | <u>Page 41</u> |    |
| Point (1)      | Point (2) | Point (3)   | ✓              | OK |
| <u>Page 42</u> |           |   | <u>Page 42</u> |    |
| Point (1)      | Point (2) | Point (3)   | ✓              | OK |
| <u>Page 43</u> |           |   | <u>Page 43</u> |    |
|                |           |   | ✓              | OK |
| <u>Page 44</u> |           |   | <u>Page 44</u> |    |
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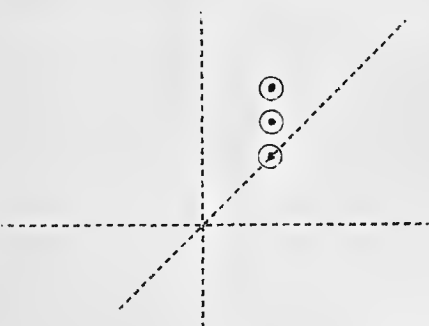
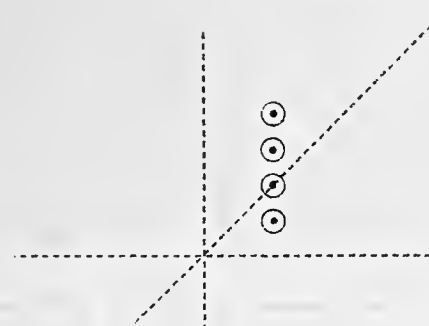
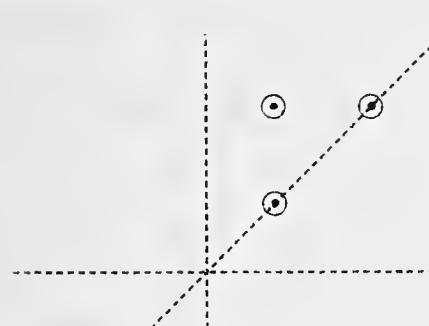
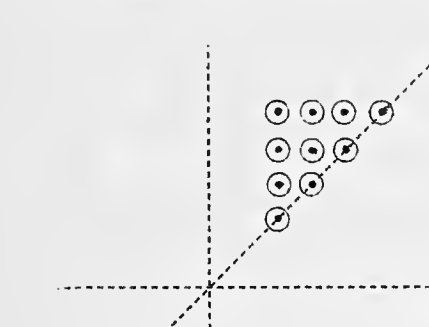
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| Answers  | Result Check                      |
|--|-----------------------------------|
| <p><u>Page 45</u></p>  <p>A scatter plot on a coordinate system with dashed axes. A dashed line with a positive slope passes through the origin. Three data points, represented by small circles with a dot in the center, are plotted along this line in the first quadrant.</p>   | <p><u>Page 45</u></p> <p>✓ OK</p> |
| <p><u>Page 46</u></p>  <p>A scatter plot on a coordinate system with dashed axes. A dashed line with a positive slope passes through the origin. Five data points, represented by small circles with a dot in the center, are plotted along this line in the first quadrant.</p>   | <p><u>Page 46</u></p> <p>✓ OK</p> |
| <p><u>Page 47</u></p>  <p>A scatter plot on a coordinate system with dashed axes. A dashed line with a positive slope passes through the origin. Three data points, represented by small circles with a dot in the center, are plotted along this line in the first quadrant.</p> | <p><u>Page 47</u></p> <p>✓ OK</p> |
| <p><u>Page 48</u></p>  <p>A scatter plot on a coordinate system with dashed axes. A dashed line with a positive slope passes through the origin. Ten data points, represented by small circles with a dot in the center, are plotted along this line in the first quadrant.</p>   | <p><u>Page 48</u></p> <p>✓ OK</p> |

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Page 13

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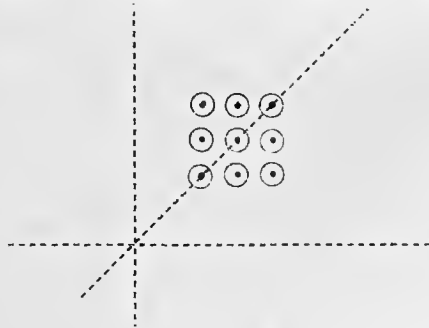
Page 25



Answers

Result Check

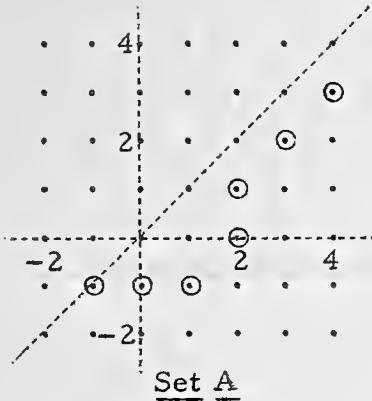
Page 49



Page 49

✓ OK

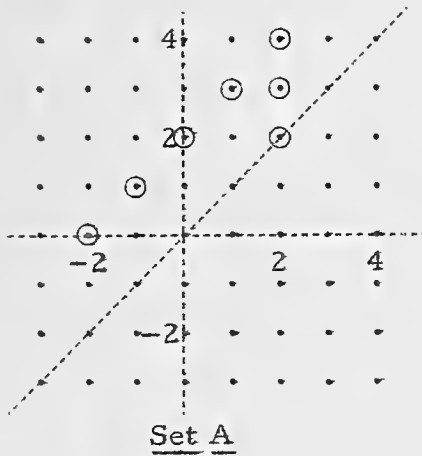
Page 50



Page 50

✓ OK

Page 51



Page 51

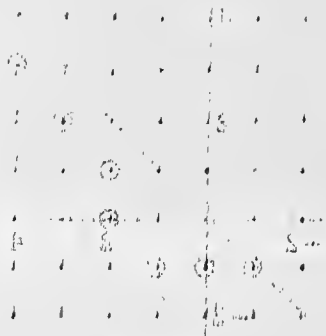
✓ OK

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A 10E

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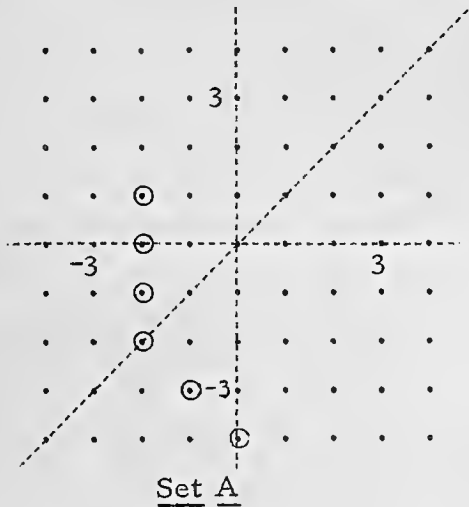
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Part 43

Answers

Result Check

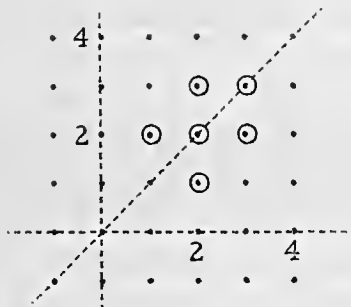
Page 52



Page 52

✓ OK

Page 53



Page 53

✓ OK

Page 54

Set A

Set B

Set C

Page 54

✓ OK

Page 57

- (1)  $A \cup B = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$
- (2)  $X = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$
- (3)  $Y = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$
- (4) If one move is made from each point in set  $A \cup B$  then the new set is  $\{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$
- (5)  $X \cup Y = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$

Page 57

1  
2  
3  
4  
5 OK



Part 43

| Answers  | Result Check  |
|--|---|
| <p><u>Page 58</u></p> <p>(1) <math>A \cup B = \{ \underline{\hspace{10em}} \}</math></p> <p>(2) <math>X = \{ \underline{\hspace{10em}} \}</math></p> <p>(3) <math>Y = \{ \underline{\hspace{10em}} \}</math></p> <p>(4) If 2 moves are made from each point in set <math>A \cup B</math> then the new set is <math>\{ \underline{\hspace{10em}} \}</math></p> <p>(5) <math>X \cup Y = \{ \underline{\hspace{10em}} \}</math></p> | <p><u>Page 58</u></p> <p>1</p> <p>2</p> <p>3</p> <p>4</p> <p>5 OK</p> |
| <p><u>Page 59</u></p> <p><math>X \cup Y = \underline{\hspace{10em}}</math></p>   | <p><u>Page 59</u></p> <p>✓ OK</p>                                     |
| <p><u>Page 60</u></p> <p>One of the points in <math>X \cup Y</math> is <u>          </u>.</p>  | <p><u>Page 60</u></p> <p>✓ OK</p>                                     |
| <p><u>Page 61</u></p> <p>It follows that n moves according to rule R from any point in <math>A \cup B</math> take you to a point in <u>          </u>.</p>   | <p><u>Page 61</u></p> <p>✓ OK</p>                                     |
| <p><u>Page 62</u></p> <p>It follows that two points in <math>X \cup Y</math> are <math>\frac{\hspace{1em}}{(1)}</math> and <math>\frac{\hspace{1em}}{(2)}</math>.</p>  | <p><u>Page 62</u></p> <p>1 2 OK</p>                                   |
| <p><u>Page 63</u></p> <p>It follows that <math>X \cup Y = \underline{\hspace{10em}}</math>.</p>  | <p><u>Page 63</u></p> <p>✓ OK</p>                                     |
| <p><u>Page 64</u></p> <p><math>X \cup Y = \underline{\hspace{10em}}</math>.</p>  | <p><u>Page 64</u></p> <p>✓ OK</p>                                     |





Part 43

| Answers   | Result Check   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |       |  |  |  |  |  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|-------|--|--|--|--|--|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|----|---|---|---|---|---|---|---|---|---|---|---|---|---|----|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <p><u>Page 66</u></p> <p>(1) _____ (2) _____</p> <p>(3) _____ (4) _____</p> <p>(5) _____ (6) _____</p> <p>(7) _____ is a point in set A which is not in set X; [or]<br/>There is <u>no</u> point in set A which is not in set X.</p> <p>(8) _____ is a point in set X which is not in set A; [or]<br/>There is <u>no</u> point in set X which is not in set A.</p> <p>(9) <math>X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2cm}}\}</math></p>   | <p><u>Page 66</u></p> <p>1 2</p> <p>3 4</p> <p>5 6</p> <p>7</p> <p>8</p> <p>9 OK</p> |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |       |  |  |  |  |  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |
| <p><u>Page 67</u></p> <p>(1) _____ (2) _____</p> <p>(3) <math>X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2cm}}\}</math></p>   | <p><u>Page 67</u></p> <p>1 2</p> <p>3 OK</p>   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |       |  |  |  |  |  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |
| <p><u>Page 68</u></p> <p>(1) _____ (2) _____</p> <p>(3)</p> <table border="0" style="margin-left: 20px;"> <tr><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>4</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>2</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td colspan="7" style="text-align: center;">-----</td></tr> <tr><td>.</td><td>.</td><td>2</td><td>.</td><td>4</td><td>.</td><td>6</td></tr> <tr><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>-2</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>-4</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> <tr><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr> </table> <p>(4) <math>X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2cm}}\}</math></p> | .  | . | . | . | . | . | . | . | 4 | . | . | . | . | . | . | . | . | . | . | . | . | . | 2 | . | . | . | . | . | . | . | . | . | . | . | . | ----- |  |  |  |  |  |  | . | . | 2 | . | 4 | . | 6 | . | . | . | . | . | . | . | . | -2 | . | . | . | . | . | . | . | . | . | . | . | . | . | -4 | . | . | . | . | . | . | . | . | . | . | . | . | <p><u>Page 68</u></p> <p>1 2</p> <p>3</p> <p>4 OK</p> |
| .   | .  | . | . | . | . | . |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |       |  |  |  |  |  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |
| .   | 4  | . | . | . | . | . |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |       |  |  |  |  |  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |
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| .   | 2  | . | . | . | . | . |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |       |  |  |  |  |  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |
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| -----   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |       |  |  |  |  |  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |
| .   | .  | 2 | . | 4 | . | 6 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |       |  |  |  |  |  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |
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| .   | -2   | . | . | . | . | . |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |       |  |  |  |  |  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |
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| .   | -4   | . | . | . | . | . |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |       |  |  |  |  |  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |
| .   | .  | . | . | . | . | . |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |       |  |  |  |  |  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |

max

1000 - 200x

0 = 1000 - 200x

1000 - 200x = 0

1000 = 200x

$1000 = 200x$

$x = \frac{1000}{200}$

$x = 5$  (1)

$x = 5$  (2)

$x = 5$  (3)

1000 - 200(5) = 1000 - 1000 = 0

$x = 5$  (4)

1000 - 200(5) = 1000 - 1000 = 0

$x = 5$  (5)

$x = 5$  (6)

$x = 5$  (7)

$x = 5$  (8)

$x = 5$  (9)

$x = 5$  (10)

$x = 5$  (11)

$x = 5$  (12)

$x = 5$  (13)

$x = 5$  (14)

$x = 5$  (15)

$x = 5$  (16)

$x = 5$  (17)

$x = 5$  (18)

$x = 5$  (19)

$x = 5$  (20)

$x = 5$  (21)

$x = 5$  (22)

WORK SHEET

Name \_\_\_\_\_

Part 43

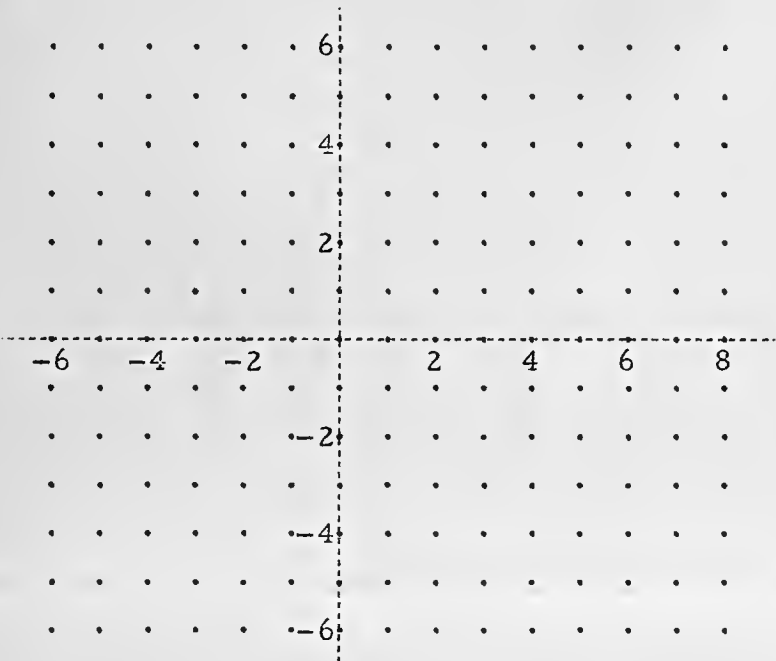
| Answers  | Result Check   |
|--|--|
| <p><u>Page 69</u></p> <p><math>X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2cm}}\}</math></p> <p>[After one move, the set selector of the description of the new set would be 'x = 8'.]</p>   | <p><u>Page 69</u></p> <p style="text-align: center;">✓      OK</p>                     |
| <p><u>Page 70</u></p> <p>(1) _____ (2) _____</p> <p>(3) _____ (4) _____</p> <p>(5) <math>X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2cm}}\}</math></p>   | <p><u>Page 70</u></p> <p style="text-align: center;">1 2<br/>3 4      OK<br/>5</p>     |
| <p><u>Page 71</u></p> <p>(1) (3, 0) _____ (2) (0, -3) _____ (3) (2, 7) _____</p> <p>(4) (-3, 0) _____ (5) (5, 4) _____ (6) (0, 3) _____</p> <p>(7) <math>n(A) = \underline{\hspace{1cm}}</math></p>  | <p><u>Page 71</u></p> <p style="text-align: center;">1 2 3<br/>4 5 6<br/>7      OK</p> |
| <p><u>Page 72</u></p> <p>(1) (-5, 0) _____ (2) (3, -4) _____ (3) (25, 0) _____</p> <p>(4) (-4, -3) _____ (5) (0, 5) _____ (6) (16, 9) _____</p> <p>(7)</p> <div style="display: flex; align-items: center;"> <div style="border-right: 1px dashed black; padding-right: 10px;"> <p style="margin: 0;">. . . . . 6</p> <p style="margin: 0;">. . . . . 4</p> <p style="margin: 0;">. . . . . 2</p> <p style="margin: 0;">-6   -4   -2</p> <p style="margin: 0;">. . . . . -2</p> <p style="margin: 0;">. . . . . -4</p> <p style="margin: 0;">. . . . . -6</p> </div> <div style="padding-left: 10px;"> <p style="margin: 0;">. . . . .</p> <p style="margin: 0;">. . . . .</p> <p style="margin: 0;">. . . . .</p> <p style="margin: 0;">. . . . .</p> <p style="margin: 0;">. . . . .</p> <p style="margin: 0;">. . . . .</p> <p style="margin: 0;">. . . . .</p> </div> </div> | <p><u>Page 72</u></p> <p style="text-align: center;">1 2 3<br/>4 5 6<br/>7      OK</p> |



WORK SHEET

Name \_\_\_\_\_

Part 43

| Answers  | Result Check                   |
|--|--------------------------------|
| <p><u>Page 73</u></p> <p>(1) </p>  | <p><u>Page 73</u></p> <p>1</p> |
| <p>(2) <math>X = \{(x, y), x \text{ and } y \text{ integers: } \underline{\hspace{2cm}}\}</math></p>   | <p>2</p>                       |
| <p>(3) The points in set X are arranged on the circle with center at <math>\frac{\hspace{1cm}}{(a)}</math> and radius <math>\frac{\hspace{1cm}}{(b)}</math>.</p> | <p>3a 3b<br/>OK</p>            |









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High school mathematics /



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