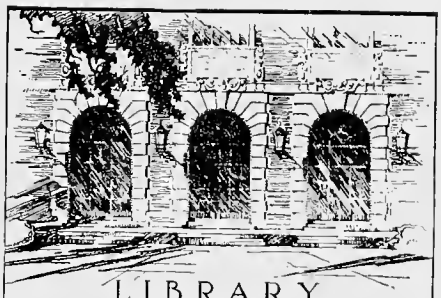
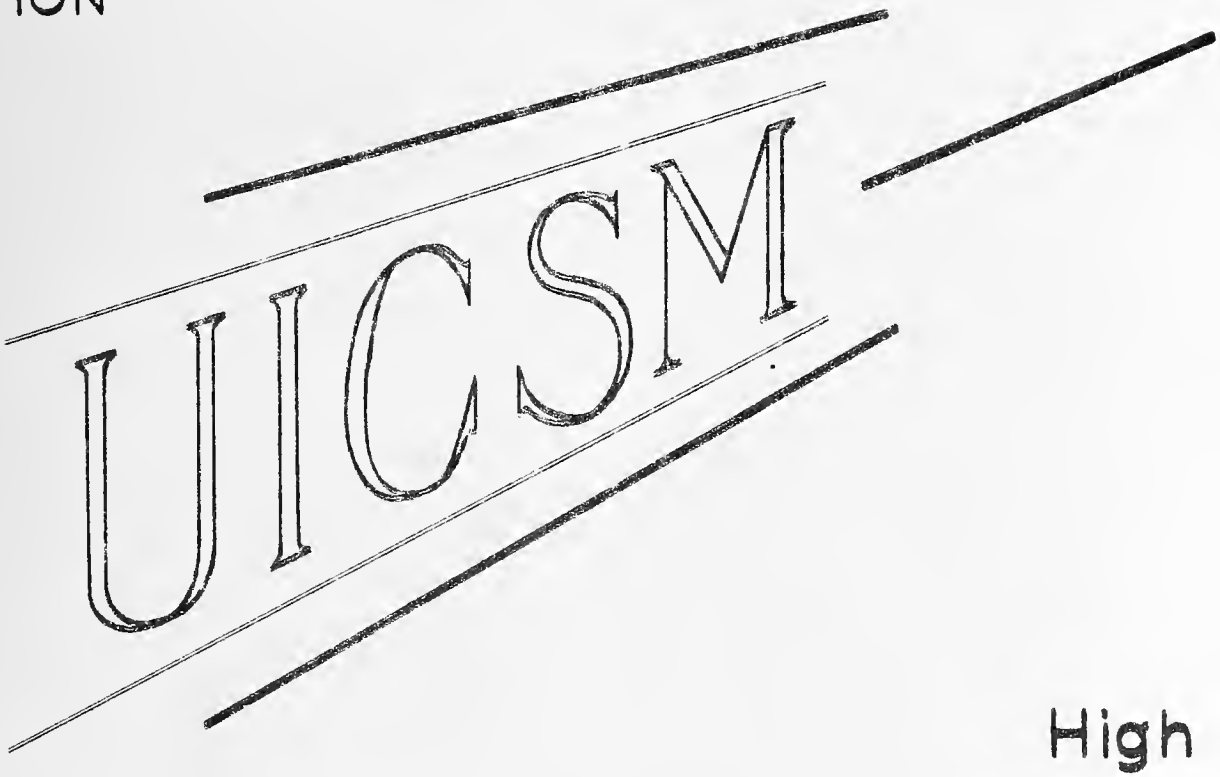


EXPERIMENTAL EDITION

PROGRAMED



High
School
Mathematics

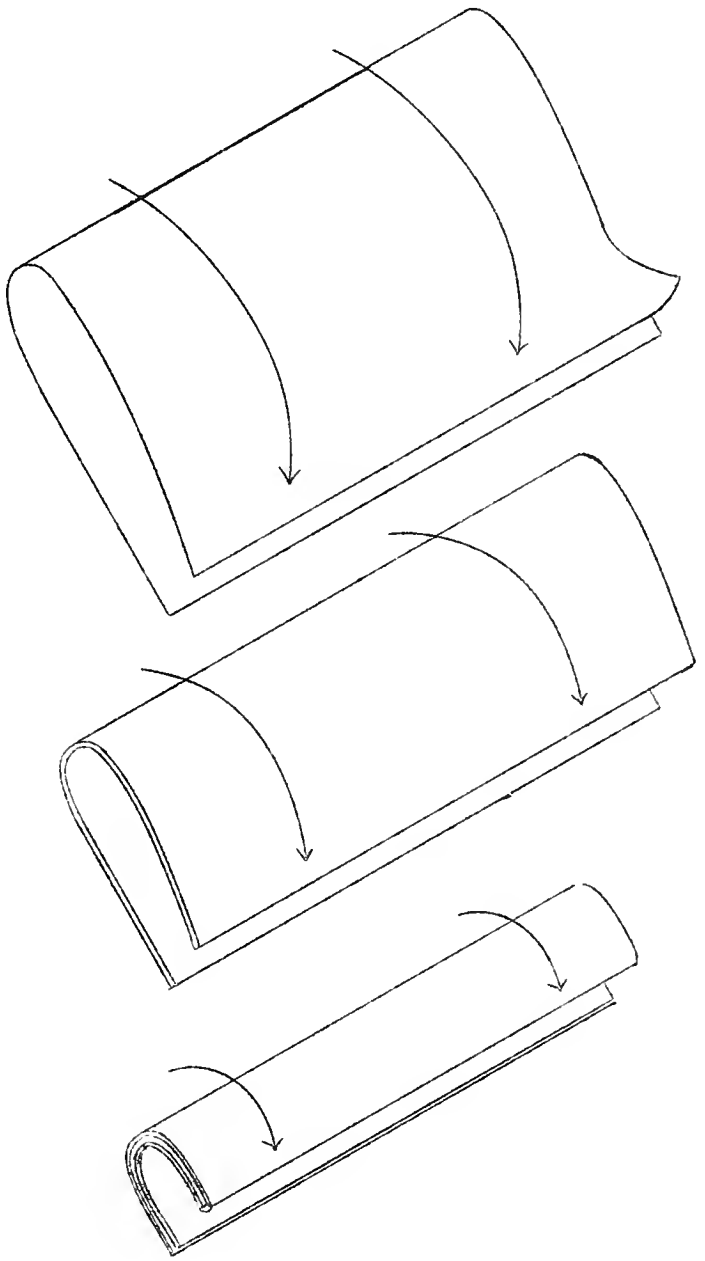
PART 43

Comparative Studies of Principles
for
Programing Mathematics
in
Automated Instruction

Supported by grants
from
U. S. Office of Education (Title VII)
National Science Foundation

Fill in the heading on your work sheet.

Some of the exercises in this book will ask you to draw a line segment between two dots. You will be able to do a neater job on these exercises if you have a ruler or some other kind of "straight-edge". If you don't have a wooden or plastic straight-edge with you, you can make a perfectly good straight-edge by folding a sheet of paper several times.



Turn to PAGE 2.

Imagine that you own a very smart grasshopper. This grasshopper is so smart that he has learned to play a game called a 'number plane lattice game'. In playing this game, your grasshopper needs this equipment:

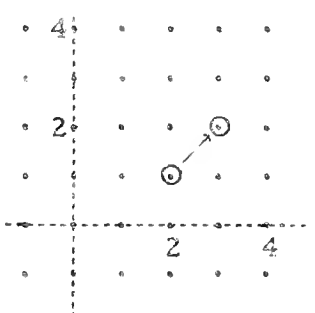
(a) a large picture of part of the number plane lattice,
(b) a pair of dice, one red and one green, and
(c) a pack of small cards, each card containing a rule and "jumping" instructions.

Here is how your grasshopper plays the game. First, he rolls the dice. [This is a big grasshopper, or he has very small dice!] Suppose that 2 comes up on the red die and 1 on the green die. This means that he is to start the game sitting on the dot corresponding to the point (2, 1) on the number plane lattice. Next, he turns a card face up and reads the rule and instructions. Suppose that the card says:

> Rule: A jump takes you from (x, y) to (x + 1, y + 1).
>
> Instructions: Make one jump.

The grasshopper will finish this game on the dot corresponding to (3, 2). He starts at (2, 1) and makes one jump according to the given rule. This takes him to (2 + 1, 1 + 1) or (3, 2). Since the instructions were to make just one jump, he finishes at (3, 2).

Here is a diagram showing his jump.



Turn to PAGE 3.

Here is how your grasshopper might play another game.

Start: (−4, −1)
Rule: A jump takes you from (x, y) to (x + 2, y + 1).
Instructions: Make 3 jumps.

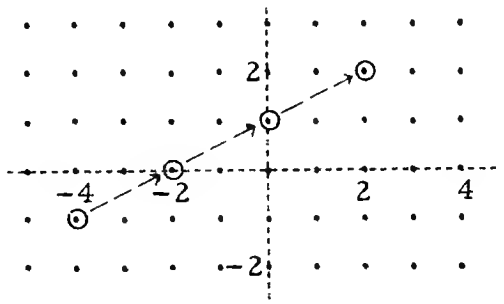
Where does he finish?

Solution.

First jump: From (−4, −1) to (−4 + 2, −1 + 1) or (−2, 0)
Second jump: From (−2, 0) to (−2 + 2, 0 + 1) or (0, 1)
Third jump: From (0, 1) to (2, 2)

So, after 3 jumps, he finishes on (2, 2).

Here is a diagram showing his jumps.



The exercises below are about a game your grasshopper played. Answer them on your work sheet.

Start: (4, 1)
Rule: A jump takes you from (x, y) to (x − 1, y + 1).
Instructions: Make 2 jumps.

(1) First jump: From (4, 1) to (3, 2)
Second jump: From (3, 2) to (? , ?)

(2) Where did he finish?

(3) Draw a diagram showing his jumps.

Turn to PAGE 4.

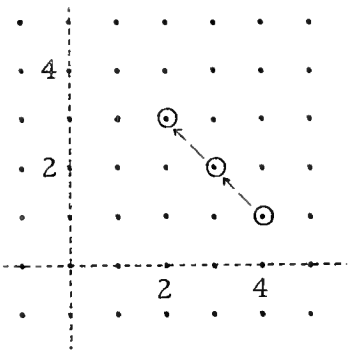
Check your answers.

Start: (4, 1)
Rule: A jump takes you from (x, y) to (x - 1, y + 1).
Instructions: Make 2 jumps.

(1) First jump: From (4, 1) to (3, 2)
Second jump: From (3, 2) to (2 , 3)

(2) Finish: (2,3)

(3) →



Record your results on your work sheet.

* * *

Do these exercises about another game the grasshopper played. Write your answers on your work sheet.

Start: (0, 1)
Rule: A jump takes you from (x, y) to (x + 3, y).
Instructions: Make 3 jumps.

(1) First jump: From (0, 1) to (3, 1)
Second jump: From (3, 1) to (? , ?)

(2) Third jump: From (? , ?) to (? , ?)

(3) Where did he finish?

(4) Draw a diagram showing his jumps.

Turn to PAGE 5.

Check your answers.

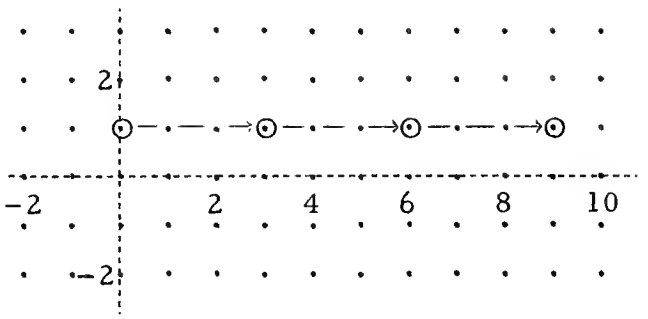
Start: (0, 1)
Rule: A jump takes you from (x, y) to (x + 3, y).
Instructions: Make 3 jumps.

(1) First jump: From (0, 1) to (3, 1)
Second jump: From (3, 1) to (6 , 1)

(2) Third jump: From (6 , 1) to (9 , 1)

(3) Finish: (9, 1)

(4)



Record your results on your work sheet.

Turn to PAGE 6.

Now, let's take over from the grasshopper and play some number plane lattice games.

We shall make "moves" instead of "jumps", and we shall use an abbreviated form for the rule. For example, the rule:

A move takes you from (x, y) to (x + 2, y - 3)

will be written:

$$(x, y) \rightarrow (x + 2, y - 3)$$

Do these exercises on your work sheet.

Rule: $(x, y) \rightarrow (x + 2, y - 3)$

Start at (3, 3) and make 3 moves.

(1) First move takes you to (3 + 2, 3 - 3) or (5, 0).
Second move takes you to (? , ?).

(2) Third move takes you to (? , ?).

(3) Where do you finish?

(4) Draw a diagram showing your moves.

Turn to PAGE 7.

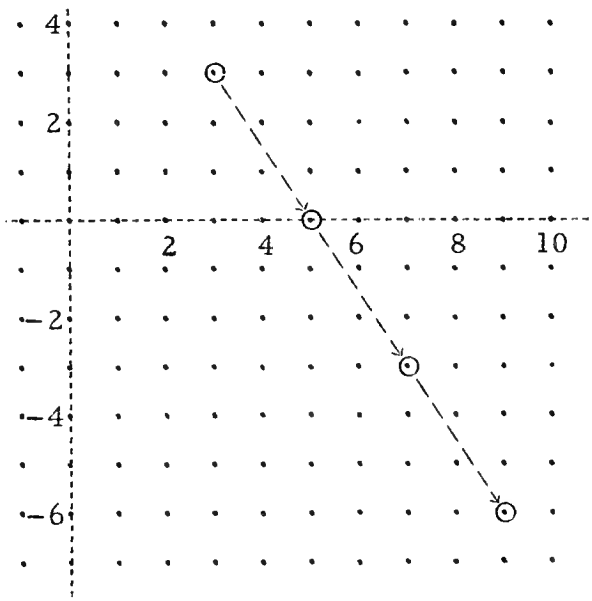
Check your answers.

Rule: $(x, y) \rightarrow (x + 2, y - 3)$
Start at (3, 3) and make 3 moves.

(1) First move takes you to (5, 0). Second move takes you to (7 , - 3).

(2) Third move takes you to (9 , -6). (3) Finish: (9, -6)

(4)



Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(x, y) \rightarrow (2x, 2y)$
Start at (1, 2) and make 2 moves.

(1) Where do you finish? [First move takes you to (2, 4).]

(2) Draw a diagram of your moves.

Turn to PAGE 8.

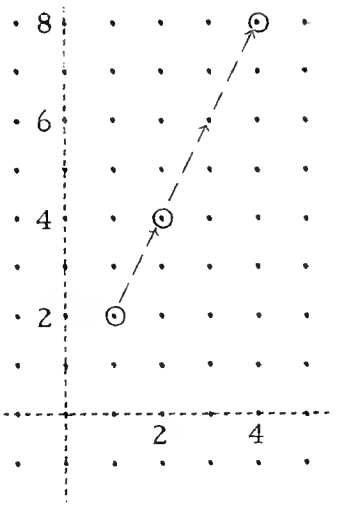
Check your answers.

Rule: $(x, y) \rightarrow (2x, 2y)$

Start at (1, 2) and make 2 moves.

(1) Finish: (4, 8)

(2)



Record your results on your work sheet.

* * *

Answer this question on your work sheet.

Rule: $(x, y) \rightarrow (3x, 2y)$

Start at (0, 0) and make 10 moves.

What is the final point?

Turn to PAGE 9.

Check your answer.

Rule: $(x, y) \rightarrow (3x, 2y)$
Start at (0, 0) and make 10 moves.

Finish: (0,0)

Record your results on your work sheet.

* * *

Here is a sample number plane lattice game where the rule is a bit more complicated.

Rule: $(x, y) \rightarrow (2x - 3, 3y + 1)$
Start at (2, 0) and make 2 moves. What is the final point?

Solution.

First move: From (2, 0) to $(2 \cdot 2 - 3, 3 \cdot 0 + 1)$, or (1, 1)
Second move: From (1, 1) to $(2 \cdot 1 - 3, 3 \cdot 1 + 1)$, or (−1, 4)

So, the final point is (−1, 4).

Do these exercises on your work sheet.

Rule: $(x, y) \rightarrow (3x - 5, y + 2)$
Start at (2, −2) and make 3 moves.

(1) First move takes you to (1, 0).
Second move takes you to (? , ?).

(2) Third move takes you to (? , ?).

(3) What is the final point?

(4) Make a diagram showing your moves.

Turn to PAGE 10.

Check your answers.

Rule: $(x, y) \rightarrow (3x - 5, y + 2)$

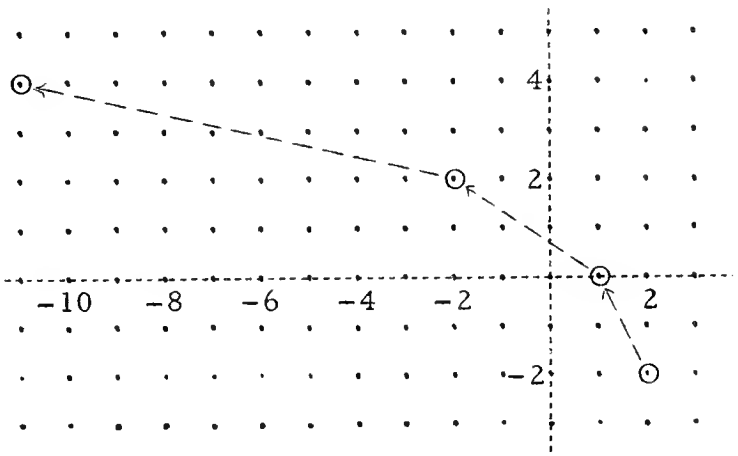
Start at $(2, -2)$ and make 3 moves.

(1) First move takes you to $(1, 0)$.
Second move takes you to $(-2, 2)$.

(2) Third move takes you to $(-11, 4)$.

(3) Final point: $(-11, 4)$

(4)



Record your results on your work sheet.

* * *

Answer this question on your work sheet.

Rule: $(j, k) \rightarrow (2j - 5, 2k + 3)$

Start at $(5, -3)$ and make 7 moves.

What is the final point?

Turn to PAGE 11.

Check your answer.

Rule: $(j, k) \rightarrow (2j - 5, 2k + 3)$

Start at $(5, -3)$ and make 7 moves.

Final point: $(5, -3)$

[The final point would be $(5, -3)$ if you made 101 moves!]

Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(x, y) \rightarrow (x^2, 3y - 1)$

Start at $(2, 2)$ and make 3 moves.

(1) First move takes you to $(4, 5)$.
Second move takes you to $(16, \_?\_)$.

(2) Third move takes you to $(\_?\_, \_?\_)$.

(3) So, the final point is $(\_?\_, \_?\_)$.

Turn to PAGE 12.

Check your answers.

Rule: $(x, y) \rightarrow (x^2, 3y - 1)$

Start at (2, 2) and make 3 moves.

(1) First move takes you to (4, 5).
Second move takes you to (16, 14).

(2) Third move takes you to (256, 41).

(3) So, the final point is (256, 41).

Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(x, y) \rightarrow (x^2, 3 - y)$.

Start at (−1, −1) and make 3 moves.

(1) First move takes you to (1, 4). Do you agree? [Yes or No?]

(2) Second move takes you to ____?____.

(3) Third move takes you to ____?____.

(4) What would be the final point if you made 4 moves?

(5) What would be the final point if you made 20 moves?

(6) Draw a diagram showing your first 3 moves.

Turn to PAGE 13.

Check your answers.

Rule: $(x, y) \rightarrow (x^2, 3 - y)$

Start at $(-1, -1)$ and make 3 moves.

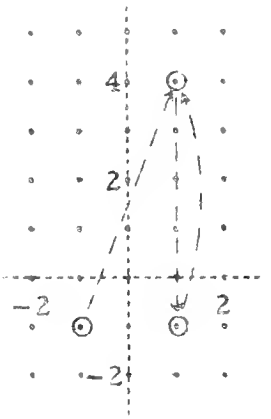
(1) First move takes you to (1, 4). Yes (6)

(2) Second move takes you to $(1, -1)$.

(3) Third move takes you to $(1, 4)$.

(4) The final point after 4 moves would be $(1, -1)$.

(5) The final point after 20 moves would be $(1, -1)$.



Record your results on your work sheet.

* * *

The exercise below introduces a type of number plane lattice game which might be a bit more challenging than the kind you have tried up to now. Be on the lookout for a system to use in solving this new type. Do this exercise on your work sheet.

Rule: $(x, y) \rightarrow (x + 1, y + 2)$

After making 1 move, the final point is (5, 8).

What was the starting point?

Turn to PAGE 14.

Check your answer.

Rule: $(x, y) \rightarrow (x + 1, y + 2)$

After making 1 move, the final point is (5, 8).

The starting point was $(4, 6)$.

If you had trouble with this exercise, here is a solution.

The first component of the final point is 5, and according to the rule, 1 was added to some number to obtain 5. That number must be 4 since $4 + 1 = 5$. So, the first component of the point immediately before the final point must be 4. Since only 1 move was made, the first component of the starting point must be 4.

The second component of the starting point must be 6, since according to the rule, 2 was added to some number to obtain 8, and $6 + 2 = 8$. So the starting point was (4, 6).

Record your result on your work sheet.

* * *

Do this exercise on your work sheet.

Rule: $(x, y) \rightarrow (x + 3, y - 2)$

After making 2 moves the final point is $(-1, 4)$.

Give the starting point.

Turn to PAGE 15.

Check your answer.

Rule: $(x, y) \rightarrow (x + 3, y - 2)$

After making 2 moves the final point is $(-1, 4)$.

Starting point: $(-7, 8)$ 

Solution.

After 2 moves, the first component of the final point is -1. The rule tells us that 3 was added to some number to obtain -1. That number, of course, was -4. So, the first component of the point reached after 1 move was -4. Again, the rule tells us that 3 was added to some number to obtain -4. In this case, the number was -7. So, the first component of the starting point was -7. By the same kind of reasoning, the second component of the starting point was 8.

Thus, the starting point was $(-7, 8)$.

In brief outline,

After 2 moves: $(-1, 4)$
After 1 move: $(-4, 6)$
Start: $(-7, 8)$

[We can check our solution by reading the brief outline from the bottom up to see if each move agrees with the rule.]

Record your result on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(x, y) \rightarrow (x - 2, y + 3)$

After 2 moves, the final point is $(1, 8)$.

(1) After 2 moves: $(1, 8)$
After 1 move: $(3, \_?\_)$

(2) Start: $(\_?\_, \_?\_)$

Turn to PAGE 16.

Check your answers.

Rule: $(x, y) \rightarrow (x - 2, y + 3)$

After 2 moves, the final point is (1, 8).

(1) After 2 moves: (1, 8)
After 1 move: (3, 5)

(2) Start: (5, 2)

Record your results on your work sheet.

* * *

Do these exercises on your work sheet. [Remember to be on the lookout for a systematic way to solve this kind of problem.]

Rule: $(x, y) \rightarrow (x - 3, y + 1)$

After 4 moves, the final point is (−6, 3).

(1) After 4 moves: (−6, 3)
After 3 moves: (−3, ___?___)

(2) After 2 moves: (___?___ , ___?___)

(3) After 1 move: (___?___ , ___?___)

(4) Start: (___?___ , ___?___)

(5) Make a diagram showing the moves which must have been made to reach the final point.

Turn to PAGE 17.

Check your answers.

Rule: $(x, y) \rightarrow (x - 3, y + 1)$

After 4 moves, the final point is $(-6, 3)$.

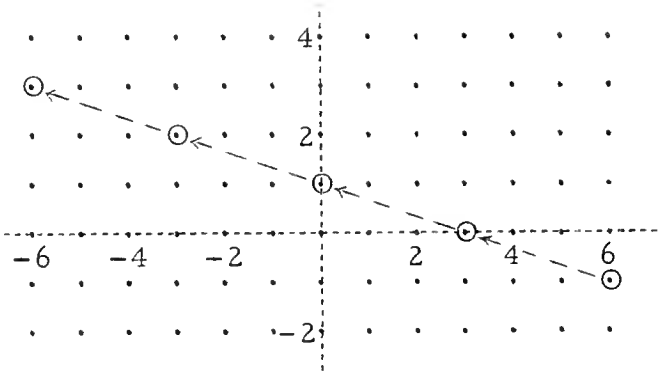
(1) After 4 moves: $(-6, 3)$
After 3 moves: $(-3, 2)$

(2) After 2 moves: $(0, 1)$

(3) After 1 move: $(3, 0)$

(4) Start: $(6, -1)$

(5)



Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(x, y) \rightarrow (2x + 1, y - 1)$

After 3 moves the final point is $(15, -3)$.

(1) After 3 moves: $(15, -3)$
After 2 moves: $(7, \underline{?})$

(2) After 1 move: $(\underline{?}, \underline{?})$

(3) Start: $(\underline{?}, \underline{?})$

Turn to PAGE 18.

Check your answers.

Rule: $(x, y) \rightarrow (2x + 1, y - 1)$

After 3 moves the final point is (15, −3).

(1) After 3 moves: (15, −3)
After 2 moves: (7, −2)

(2) After 1 move: (3, −1)

(3) Start: (1, 0)

Here is how part of the solution might be done.

The first component of the final point is 15. The rule tells us that some number was multiplied by 2 and then 1 was added to obtain 15. That number must have been 7 since $2 \cdot 7 + 1 = 15$. So, after 2 moves, the first component of the point reached was 7.

[You can check the completed solution by reading the answers from the starting point to the final point to see if each move followed the rule.]

Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(x, y) \rightarrow (2x + 1, y - 1)$

After 2 moves, the final point is (11, 3).

(1) After 2 moves: (11, 3)
After 1 move: (5, __?__)

(2) Start: (__?__, __?__)

Turn to PAGE 19.

Check your answers.

Rule: $(x, y) \rightarrow (2x + 1, y - 1)$

After 2 moves, the final point is (11, 3).

(1) After 2 moves: (11, 3)
After 1 move: (5, 4)

(2) Start: (2, 5) [2 · 2 + 1 = 5]

Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(x, y) \rightarrow (2x - 1, 2y + 1)$

After 3 moves, the final point is (9, 7).

(1) After 3 moves: (9, 7)
After 2 moves: (5, ___?___) [2 · ___?___ + 1 = 7]

(2) After 1 move: (___?___ , ___?___)

(3) Start: ___?___

Turn to PAGE 20.

Check your answers.

Rule: $(x, y) \rightarrow (2x - 1, 2y + 1)$

After 3 moves, the final point is (9, 7).

(1) After 3 moves: (9, 7)

After 2 moves: (5, 3) $[2 \cdot \underline{3} + 1 = 7]$

(2) After 1 move: (3, 1)

$[2 \cdot \underline{3} - 1 = 5, \quad 2 \cdot \underline{1} + 1 = 3]$

(3) Start : (2, 0)

$[2 \cdot \underline{2} - 1 = 3, \quad 2 \cdot \underline{0} + 1 = 1]$

Record your results on your work sheet.

* * *

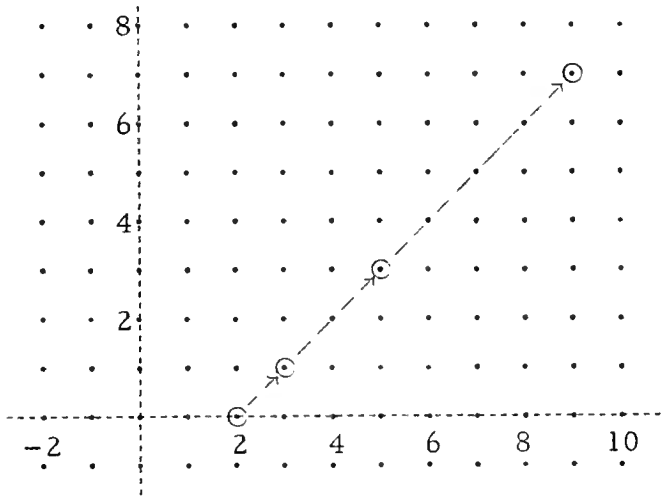
On your work sheet, draw a diagram showing the moves which must have been made to reach the final point in the game whose solution is given near the top of this page.

Turn to PAGE 21.

Check your answer.

Rule: $(x, y) \rightarrow (2x - 1, 2y + 1)$

After 3 moves, the final point is (9, 7).



Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(x, y) \rightarrow (3x + 2, 3 - 2y)$

After 3 moves, the final point is (53, −7).

(1) After 3 moves: (53, −7)

After 2 moves: (17, 5) [$3 \cdot \underline{17} + 2 = 53$. $3 - 2 \cdot \underline{5} = -7$.]

After 1 move : ___?___ [$3 \cdot \underline{?} + 2 = 17$. $3 - 2 \cdot \underline{?} = 5$.]

(2) Start : ___?___

Turn to PAGE 22.

Check your answers.

Rule: $(x, y) \rightarrow (3x + 2, 3 - 2y)$

After 3 moves, the final point is (53, −7).

(1) After 3 moves: (53, −7)

After 2 moves: (17, 5)

[$3 \cdot \underline{17} + 2 = 53$. $3 - 2 \cdot \underline{5} = -7$.]

After 1 move : (5, −1) ←

[$3 \cdot \underline{5} + 2 = 17$. $3 - 2 \cdot \underline{-1} = 5$.]

(2) Start: (1, 2) ←

[$3 \cdot \underline{1} + 2 = 5$. $3 - 2 \cdot \underline{2} = -1$.]

Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(x, y) \rightarrow (3x - 2, 4 - 2y)$

After 3 moves, the final point is (28, 12).

(1) After 3 moves: (28, 12)

After 2 moves: ___?___ [Remember, you are "backing up".]

(2) After 1 move : ___?___

(3) Start : ___?___

Turn to PAGE 23.

Check your answers.

Rule: $(x, y) \rightarrow (3x - 2, 4 - 2y)$

After 3 moves, the final point is (28, 12).

(1) After 3 moves: (28, 12)

After 2 moves: $(10, -4)$

(2) After 1 move : $(4, 4)$

(3) Start : $(2, 0)$

Record your results on your work sheet.

Turn to PAGE 24.

You have played several number plane lattice games where you were given the final point and asked to find the starting point. Perhaps you have discovered that you can use equations to help you "back up" in a lattice game.

Here is a sample showing how equations can help. [Perhaps you discovered a different method.]

Sample.

Rule: $(x, y) \rightarrow (3x + 4,\ 2 - 3y)$
After 2 moves, the final point is (79, 68).
Give the starting point.

Solution.

After 2 moves: (79, 68)

First Component	Second Component
$3x + 4 = 79$	$2 - 3y = 68$
$3x = 75$	$-3y = 66$
$x = 25$	$y = -22$

So, the point just before (79, 68) was (25, −22).

After 1 move: (25, −22)

$3x + 4 = 25$	$2 - 3y = -22$
$x = 7$	$y = 8$

So, the starting point was (7, 8).

Do this exercise on your work sheet.

Rule: $(x, y) \rightarrow (2x + 5,\ 3 - 2y)$

After 3 moves, the final point is (19, −15).

Give the starting point.

Turn to PAGE 25.

Check your answer.

Rule: $(x, y) \rightarrow (2x + 5, 3 - 2y)$

After 3 moves, the final point is $(19, -15)$.

First Component	Second Component
$2x + 5 = 19$	$3 - 2y = -15$
$x = 7$	$y = 9$
After 2 moves: $(7, 9)$	
$2x + 5 = 7$	$3 - 2y = 9$
$x = 1$	$y = -3$
After 1 move: $(1, -3)$	
$2x + 5 = 1$	$3 - 2y = -3$
$x = -2$	$y = 3$

Starting point: $(-2, 3)$

Record your results on your work sheet.

* * *

Do this exercise on your work sheet.

Rule: $(x, y) \rightarrow (2x - 5, 3 + 4y)$

After 4 moves, the final point is $(-27, -1)$.

Give the starting point.

Turn to PAGE 26.

Check your answer.

Rule: $(x, y) \rightarrow (2x - 5, 3 + 4y)$

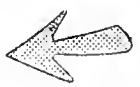
After 4 moves, the final point is $(-27, -1)$.

After 4 moves: $(-27, -1)$

After 3 moves: $(-11, -1)$

After 2 moves: $(-3, -1)$

After 1 move : $(1, -1)$

Start : $(3, -1)$ 

Record your result on your work sheet.

* * *

The exercises below introduce still another type of number plane lattice game. Do these exercises on your work sheet.

$A = \{(0, 0), (1, 1), (2, 2)\}$

Rule: $(x, y) \rightarrow (x + y, x - y)$

Make one move from each point in set A. Call the new set 'X'.

(1) From (0, 0), you move to $(0 + 0, 0 - 0)$, or $(0, 0)$.

From (1, 1), you move to $(1 + 1, 1 - 1)$, or $(2, 0)$.

From (2, 2), you move to (? + ? , ? - ?), or (? , ?).

(2) If the new set is called 'X' then X = { ? , ? , ? }.

Turn to PAGE 27.

Check your answers.

$A = \{(0, 0), (1, 1), (2, 2)\}$

Rule: $(x, y) \rightarrow (x + y, x - y)$

Make one move from each point in set A. Call the new set 'X'.

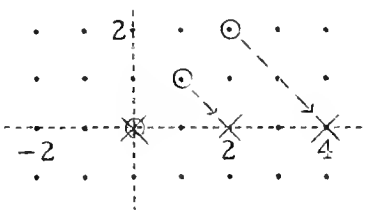
(1) From (0, 0), you move to (0, 0).
From (1, 1), you move to (0, 0).
From (2, 2), you move to (2 + 2 , 2 - 2), or (4 , 0).

(2) X = {(0,0),(2,0) ,(4,0) }

Record your results on your work sheet.

* * *

Now, let's plot the points in each of the sets A and X listed above, and show the moves from each point in A to the corresponding point in X.



Do these exercises on your work sheet.

$A = \{(0, 0), (1, -1), (2, -2), (3, -3)\}$

Rule: $(x, y) \rightarrow (x, |y|)$

Make one move from each point in set A. Call the new set 'X'.

(1) X = { ? , ? , ? , ? }

(2) Plot the points in each set on the same diagram and indicate each move by drawing a dashed line and an arrow. [See the diagram above.]

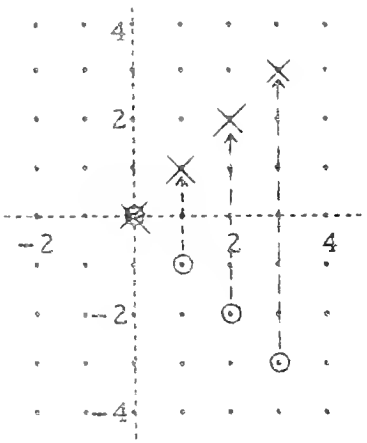
Turn to PAGE 28.

Check your answers.

$$A = \{(0, 0), (1, -1), (2, -2), (3, -3)\}$$

Rule: $(x, y) \rightarrow (x, |y|)$

(1) $X = \{(0,0), (1,1), (2,2), (3,3)\}$ (2)



Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

$$A = \{(2, 1), (3, 2), (4, 3), (5, 3)\}$$

Rule: $(x, y) \rightarrow (y, x)$

Make one move from each point in set A. Call the new set 'X'.

(1) $X = \{(1, 2), (2, \underline{\ ?\ }), \underline{\ ?\ }, \underline{\ ?\ }\}$.

(2) Plot the points in each set on the same diagram and indicate each move. [Remember, loops for A, cross-marks for X.]

Turn to PAGE 29.

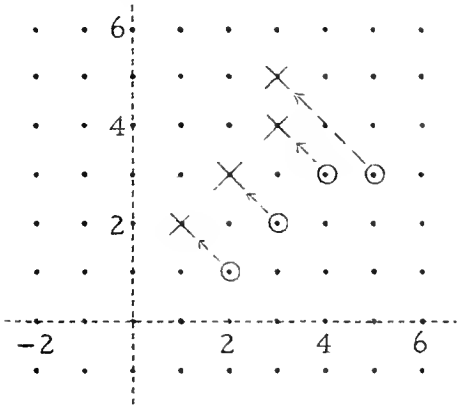
Check your answers.

A = {(2, 1), (3, 2), (4, 3), (5, 3)}

Rule: (x, y) → (y, x)

(1) X = {(1, 2), (2, 3), (3, 4), (3, 5)}

(2)



Record your results on your work sheet.

* * *

When <u>one</u> move is made from each point in a set according to the rule '(x, y) → (y, x)' and the points involved are plotted, we get an interesting picture. Study such pictures carefully and look for a "pattern".

Do these exercises on your work sheet.

A = {(x, y), x and y integers: 0 < x < 6 and y = 1}

Rule: (x, y) → (y, x)

Make one move from each point in set A. Call the new set 'X'.

(1) A = { ? } [<u>List</u> the members of set A.]

(2) X = { ? } [<u>List</u> the members of set X.]

(3) Plot the points in each set on the same diagram and indicate the moves.

(4) X = {(x, y), x and y integers: ?, and x = 1}

Turn to PAGE <u>30</u>.

Check your answers.

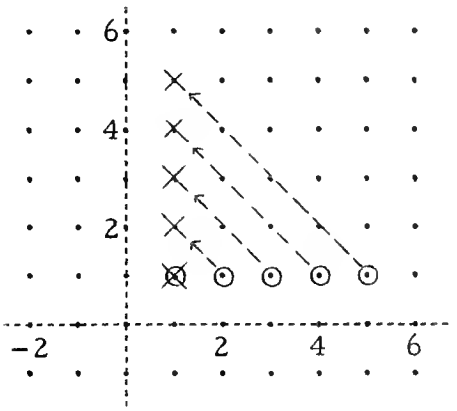
A = {(x, y), x and y integers: $0 < x < 6$ and $y = 1$}

Rule: $(x, y) \rightarrow (y, x)$

(1) A = { (1,1), (2,1), (3,1), (4,1), (5,1) }

(2) X = { (1,1), (1,2), (1,3), (1,4), (1,5) }

(3)



Remember, look for a "pattern".

(4) X = {(x, y), x and y integers: $0 < y < 6$ and $x = 1$}
[Compare this description of set X with the description of set A at the top of this page.]

Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

A = {(x, y), x and y integers: $-2 < x < 2$ and $-4 < y < -2$}

Rule: $(x, y) \rightarrow (y, x)$

Make one move from each point in set A. Call the new set 'X'.

(1) A = { _____?_____ } [List the members of set A.]

(2) X = { _____?_____ } [List the members of set X.]

(3) Plot the points in each set on the same diagram and indicate the moves. [Remember, loops for A, cross-marks for X.]

(4) X = {(x, y), x and y integers: $-2 < y < 2$ and _____?_____ }

Turn to PAGE 31 .

Check your answers.

A = {(x, y), x and y integers: $-2 < x < 2$ and $-4 < y < -2$}

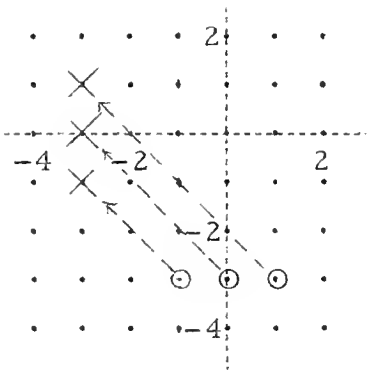
Rule: $(x, y) \rightarrow (y, x)$

Make one move from each point in set A. Call the new set 'X'.

(1) A = { $(-1,-3), (0,-3), (1,-3)$ }

(2) X = { $(-3,-1), (-3,0), (-3,1)$ }

(3)



(4) X = {(x, y), x and y integers: $-2 < y < 2$ and $-4 < x < -2$ }

[Compare this description of set X with the description of Set A at the top of this page.]

Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(x, y) \rightarrow (y, x)$

After making one move from each point in set A, the result in set X where

X = {(2, 7), (3, 7), (4, 7)}.

(1) A = {__?__, __?__, __?__}

(2) X = {(x, y), x and y integers: $1 < x < 5$ and ____?____}

(3) A = {(x, y), x and y integers: ____?____ and $x = 7$}

Turn to PAGE 32.

Check your answers.

Rule: $(x, y) \rightarrow (y, x)$

After making one move from each point in set A, the result is set X where

$$X = \{(2, 7), (3, 7), (4, 7)\}.$$

(1) A = { (7,2), (7,3), (7,4) }

(2) X = {(x, y), x and y integers: $1 < x < 5$ and $y = 7$ }

(3) A = {(x, y), x and y integers: $1 < y < 5$ and $x = 7$}

Record your results on your work sheet.

* * *

Do this exercise on your work sheet.

A = {(x, y), x and y integers: $2 < x < 5$ and $-3 < y < 0$}

Rule: $(x, y) \rightarrow (y, x)$

If you make one move from each point in set A, and call the new set 'X', then X = {(x, y), x and y integers: _____?_____ and _____?_____}.

[Try to complete the description of set X without listing the members of either set A or set X.]

Turn to PAGE 33.

Check your answers.

$A = \{(x, y),\ x \text{ and } y \text{ integers: } 2 < x < 5 \text{ and } -3 < y < 0\}$

Rule: $(x, y) \rightarrow (y, x)$

If you make one move from each point in set A, and call the new set 'X', then

$X = \{(x, y),\ x \text{ and } y \text{ integers: } 2 < y < 5 \text{ and } -3 < x < 0\}$.

[Of course, '$X = \{(x, y),\ x \text{ and } y \text{ integers: } -3 < x < 0 \text{ and } 2 < y < 5\}$' is also correct.]

Record your results on your work sheet.

* * *

If you were able to complete the description of set X above without listing the members of set A or set X, you have probably made an interesting discovery. We hope you have discovered that when you are given a brace-notation description of a set of points and you make one move from each point in the set according to the rule '$(x, y) \rightarrow (y, x)$', you can easily get a brace-notation description of the new set. You simply copy the description of the given set except that after the ':' you substitute the pronumeral which indicates second components for the pronumeral which indicates first components, and you substitute the first component pronumeral for the second component pronumeral. The exercise answered at the top of this page is a good example.

Turn to PAGE 34.

Now, let's see if you can apply the discovery mentioned on the previous page.

Do these exercises on your work sheet.

$A = \{(x, y),\ x \text{ and } y \text{ integers: } y = 2 \text{ and } -3 < x < 3\}$

Rule: $(x, y) \rightarrow (y, x)$

Make one move from each point in set A. Call the new set 'X'.

(1) Give a brace description of set X. That is, complete this:

$X = \{(x, y),\ x \text{ and } y \text{ integers: } \underline{\quad ? \quad} \text{ and } \underline{\quad ? \quad}\}$

(2) Plot the points in each set on the same diagram and indicate the moves.

Turn to PAGE 35.

Check your answers.

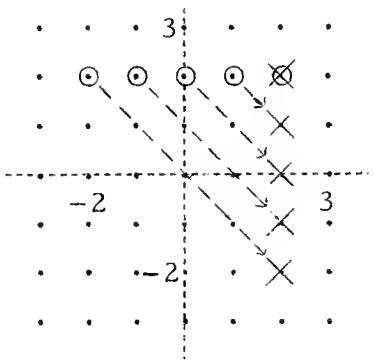
$A = \{(x, y),\ x \text{ and } y \text{ integers: } y = 2 \text{ and } -3 < x < 3\}$

Rule: $(x, y) \rightarrow (y, x)$

Make one move from each point in set A. Call the new set 'X'.

(1) $X = \{(x, y),\ x \text{ and } y \text{ integers: } x = 2 \text{ and } -3 < y < 3\}$

(2)



Remember to look for a "pattern" on the picture.

Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

$A = \{(x, y),\ x \text{ and } y \text{ integers: } y = x - 3\}$

Rule: $(x, y) \rightarrow (y, x)$

Make one move from each point in set A. Call the new set 'X'.

(1) Give a brace description of set X.

(2) Plot the points in each set on the same diagram and indicate the moves.

Turn to PAGE 36.

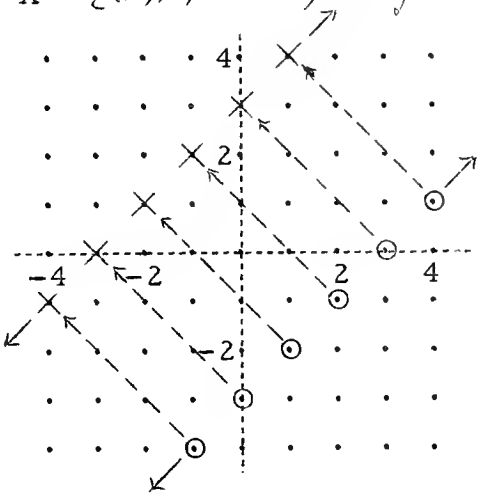
Check your answers.

A = {(x, y), x and y integers: y = x - 3}

Rule: (x, y) → (y, x)

(1) X = {(x, y), x and y integers: x = y - 3}

(2)



[The "pattern" is particularly clear in this picture.]

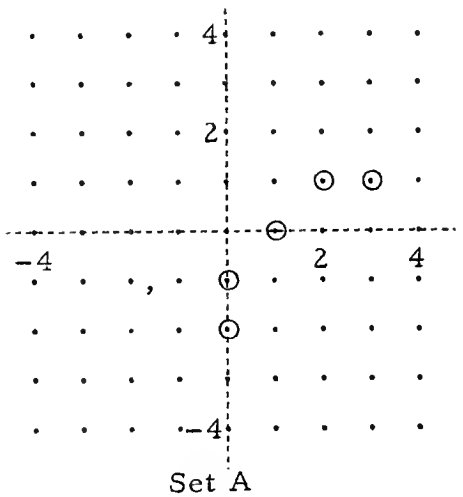
Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

Set A: See the picture

Rule: (x, y) → (y, x)



Make one move from each point in set A. Call the new set 'X'.

(1) Plot the points in set X on the same diagram with set A and indicate the moves. [Try to do this exercise first, but if you have trouble, do Exercises (2) and (3) first.]

(2) A = { ? } [List the members of set A.]

(3) X = { ? } [List the members of set X.]

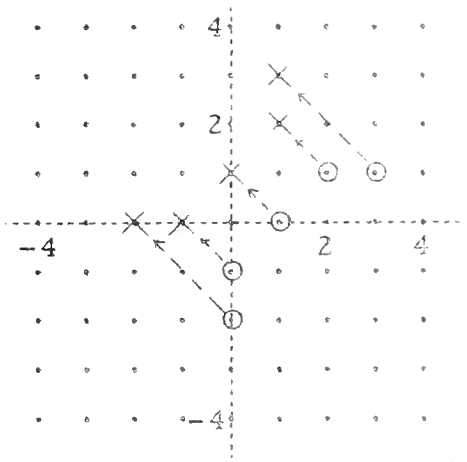
Turn to PAGE 37.

Check your answers.

Rule: $(x, y) \rightarrow (y, x)$

Make one move from each point in Set A. Call the new set 'X'.

(1)



(2) A = $\{(0,-2),(0,-1),(1,0),(2,1),(3,1)\}$

(3) X = $\{(-2,0),(-1,0),(0,1),(1,2),(1,3)\}$

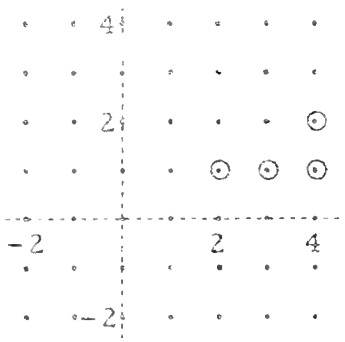
Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

A: the set pictured at right

Rule: $(x, y) \rightarrow (y, x)$



Set A

Make one move from each point in set A. Call the new set 'X'.

(1) Plot the points in set X on the same diagram with set A and indicate the moves.

(2) A = { ? } [List set A.]

(3) X = { ? } [List set X.]

(4) Draw a dashed line through the dots corresponding to the points in set **D**, where

$$\mathbf{D} = \{(x, y),\ x \text{ and } y \text{ integers: } y = x\}.$$

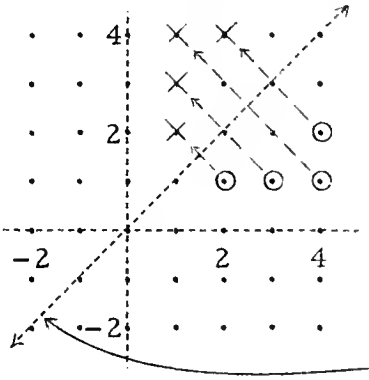
Turn to PAGE 38.

Check your answers.

Rule: $(x, y) \rightarrow (y, x)$

Make one move from each point in set A. Call the new set 'X'.

(1)



(2) A = { (2,1), (3,1), (4,1), (4,2) }

(3) X = { (1,2), (1,3), (1,4), (2,4) }

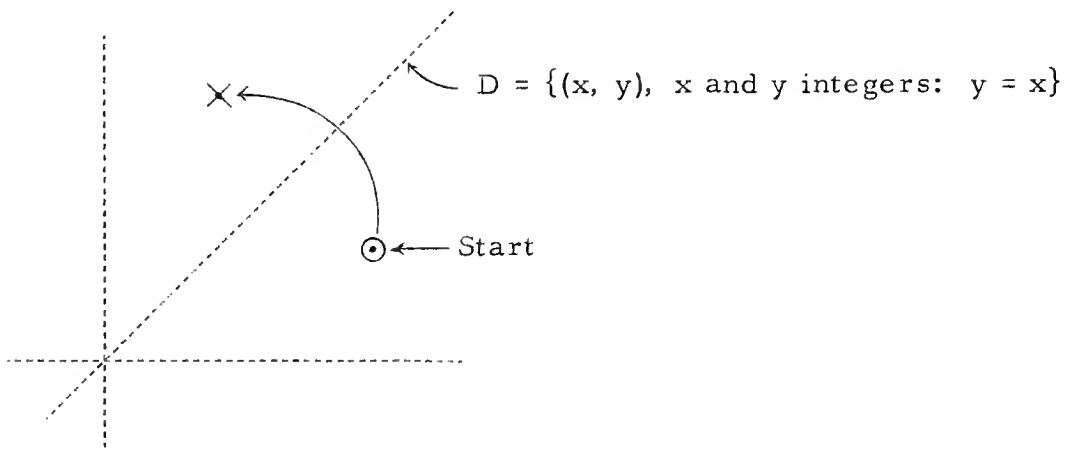
(4) [D = {(x, y), x and y integers: y = x}]

Record your results on your work sheet.

* * *

The exercises above should help you see the pattern involved in making a move according to the rule:

$(x, y) \rightarrow (y, x)$



Turn to PAGE 39.

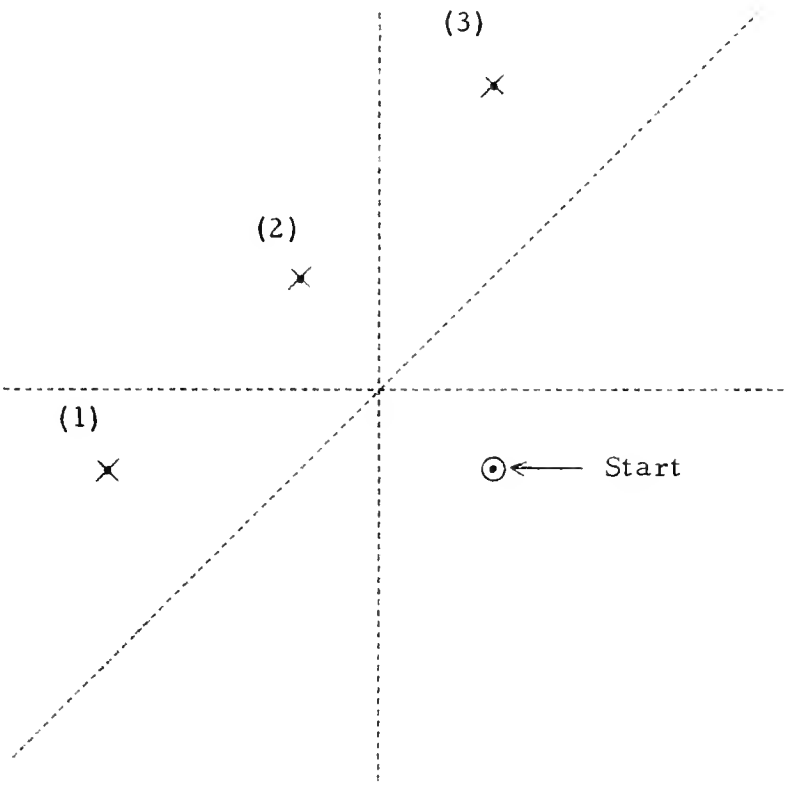
Notice the dot with the loop around it in the picture below. If you make one move from this point according to the rule:

$$(x, y) \rightarrow (y, x)$$

where do you think you would end up?

Point (1)? Point (2)? Point (3)?

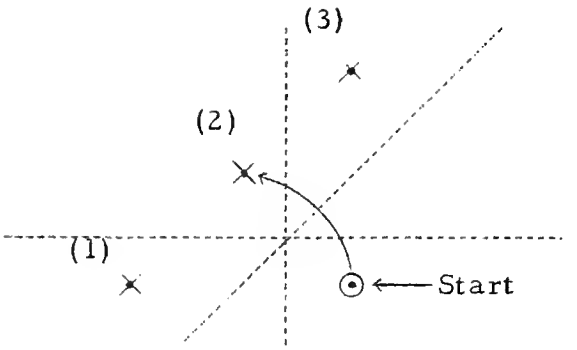
Circle the answer on your work sheet.



Turn to PAGE 40.

Check your answer.

Rule: $(x, y) \rightarrow (y, x)$



Record your result on your work sheet.

* * *

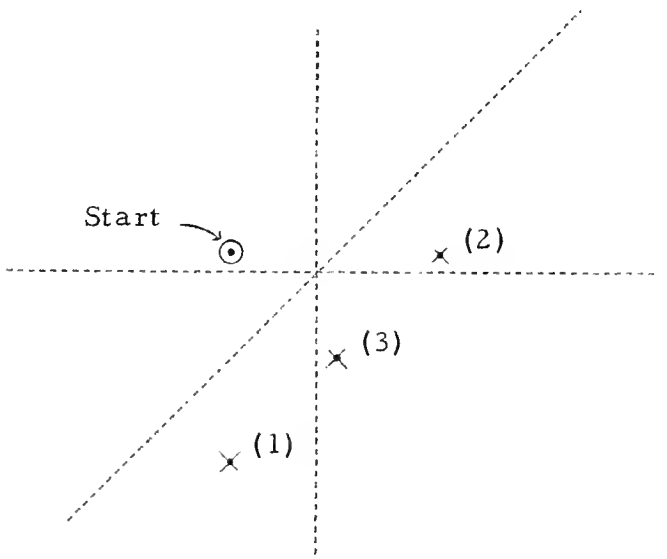
Notice the dot with the loop around it in the picture below. If you make one move from this point according to the rule:

$$(x, y) \rightarrow (y, x)$$

where do you think you would end up?

Point (1)? Point (2)? Point (3)?

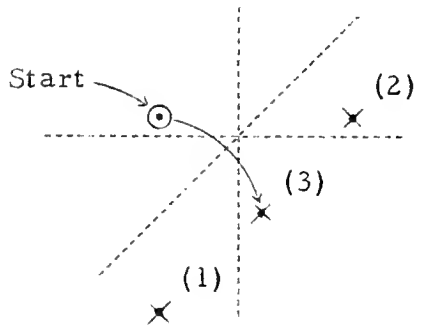
Circle the answer on your work sheet.



Turn to PAGE 41.

Check your answer.

Rule: $(x, y) \rightarrow (y, x)$



Record your result on your work sheet.

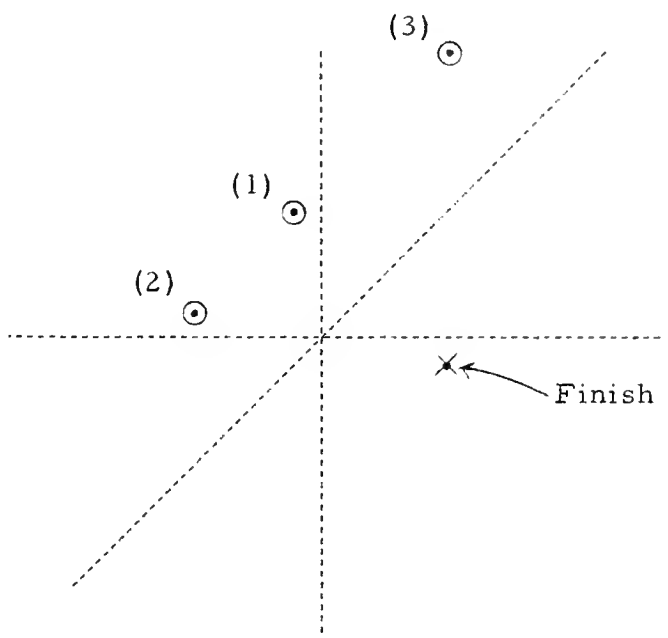
* * *

Which of the marked points (1), (2), or (3) would be the starting point if you made one move according to the rule:

$$(x, y) \rightarrow (y, x)$$

and ended at the dot with the cross-mark through it?

Circle the answer on your work sheet.

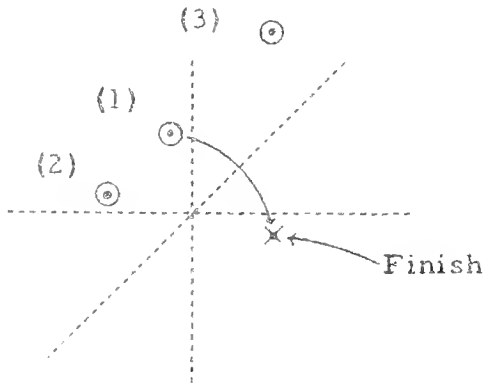


Turn to PAGE 42.

[from page 40]

Check your answer.

Rule: $(x, y) \rightarrow (y, x)$



Record your result on your work sheet.

* * *

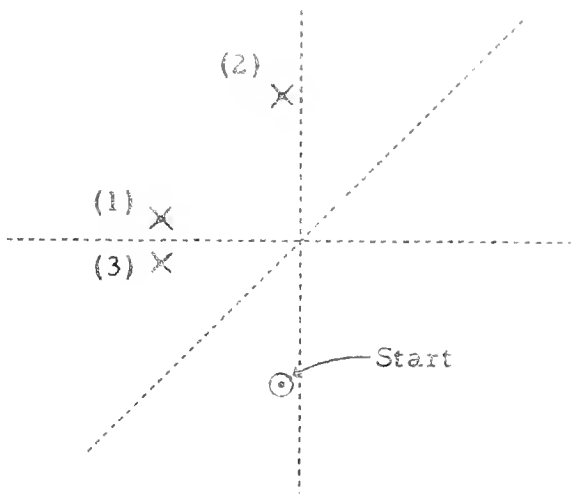
If you make one move from the point labled as the start point according to the rule:

$$(x, y) \rightarrow (y, x)$$

where do you finish?

Point (1)? Point (2)? Point (3)?

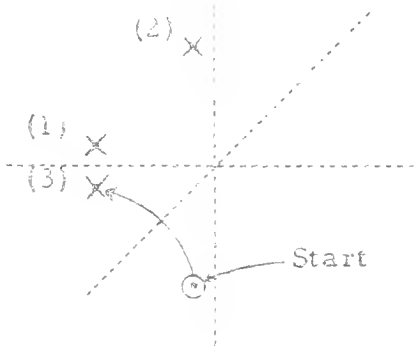
Circle the answer on your work sheet.



Turn to PAGE 43.

Check your answer.

Rule: $(x, y) \rightarrow (y, x)$

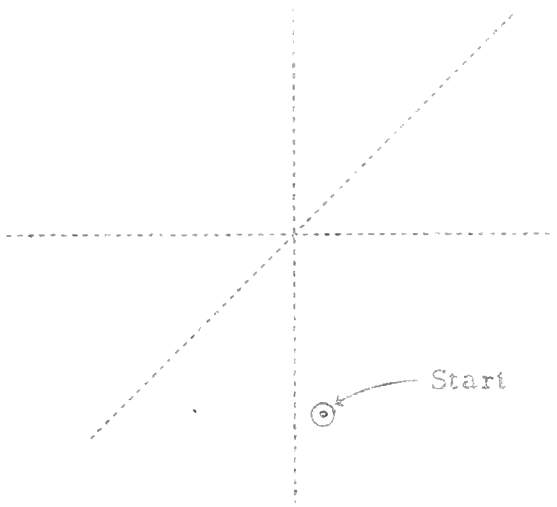


Record your result on your work sheet.

* * *

Use your eye and mark a dot on the picture <u>on your work sheet</u> to show where you finish if you make one move from the point labeled <u>Start</u> according to the rule:

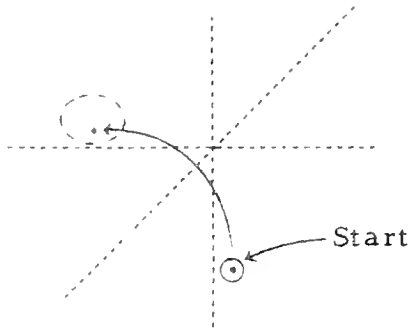
$(x, y) \rightarrow (y, x)$



Turn to PAGE <u>44</u>.

Check your answer.

Rule: $(x, y) \rightarrow (y, x)$



You are right if your mark would be within the boundry indicated.

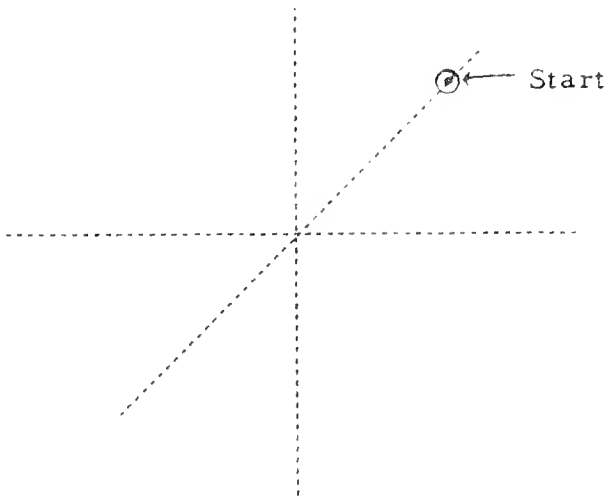
Record your result on your work sheet.

* * *

Use your eye and mark a dot on the picture on your work sheet to show where you finish if you make one move from the start point according to the rule:

$$(x, y) \rightarrow (y, x)$$

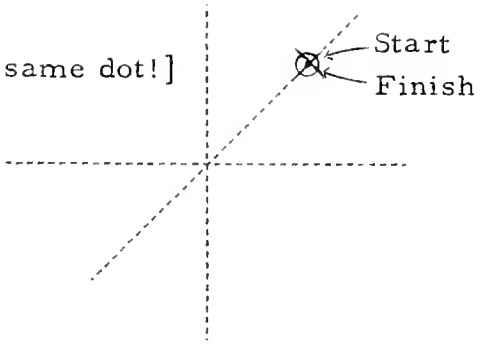
[Hint. A "move" might not move you at all.]



Turn to PAGE 45.

Check your answer.

[The same dot!]



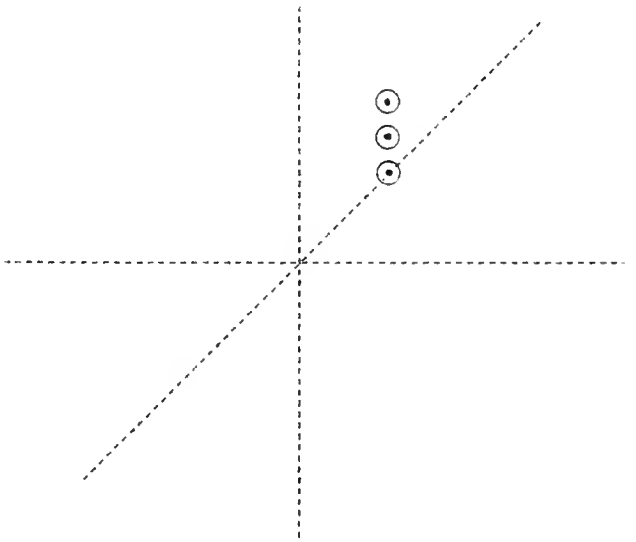
Rule: $(x, y) \rightarrow (y, x)$

Record your result on your work sheet.

* * *

Use your eye and mark dots on the picture on your work sheet to show where you finish if you make one move from each point indicated by a loop according to the rule:

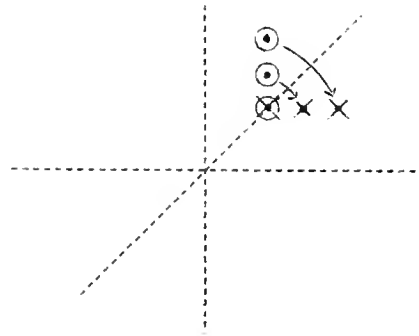
$$(x, y) \rightarrow (y, x)$$



Turn to PAGE 46.

Check your answer.

Rule: $(x, y) \rightarrow (y, x)$

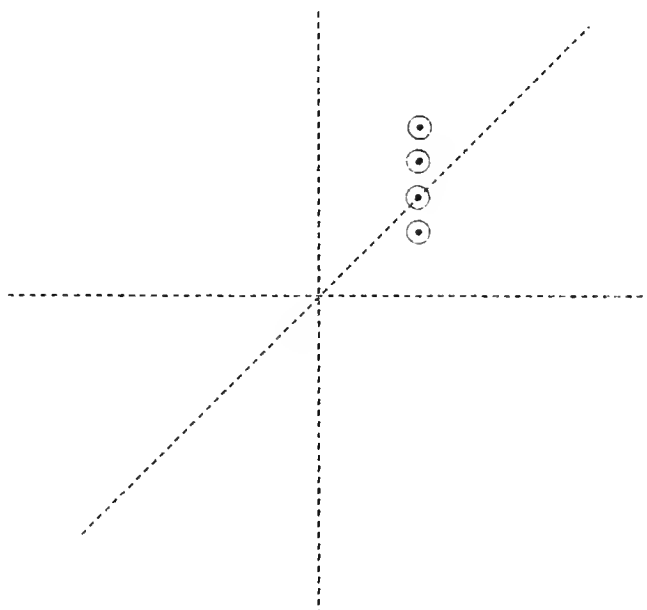


Record your result on your work sheet.

* * *

Use your eye and mark dots on the picture on your work sheet to show where you finish if you make one move from each point indicated by a loop according to the rule:

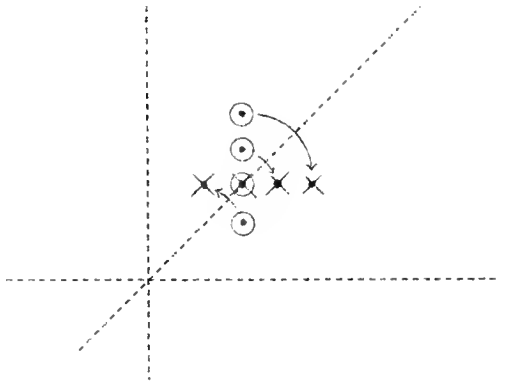
$$(x, y) \rightarrow (y, x)$$



Turn to PAGE 47.

Check your answer.

Rule: $(x, y) \rightarrow (y, x)$

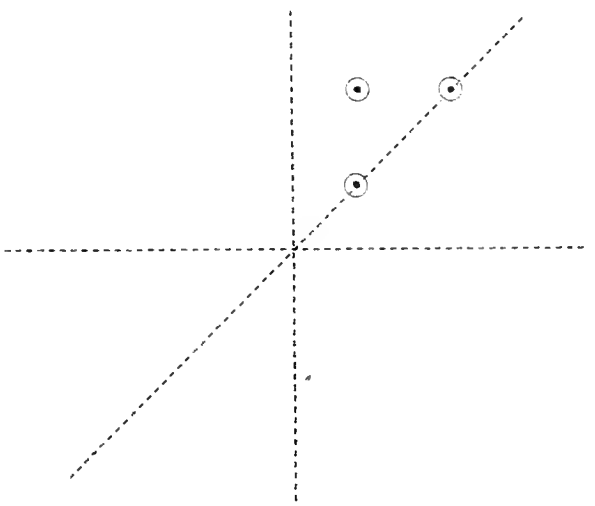


Record your result on your work sheet.

* * *

Use your eye and mark dots on the picture on your work sheet to show where you finish if you make one move from each point indicated by a loop according to the rule:

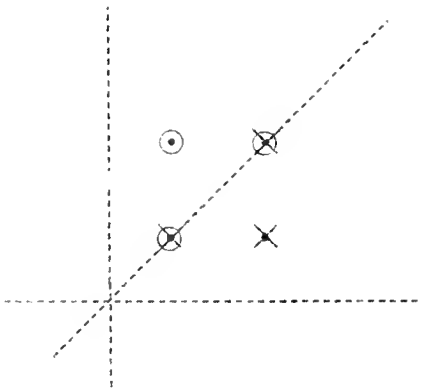
$$(x, y) \rightarrow (y, x)$$



Turn to PAGE 48 .

Check your answer.

Rule: $(x, y) \rightarrow (y, x)$

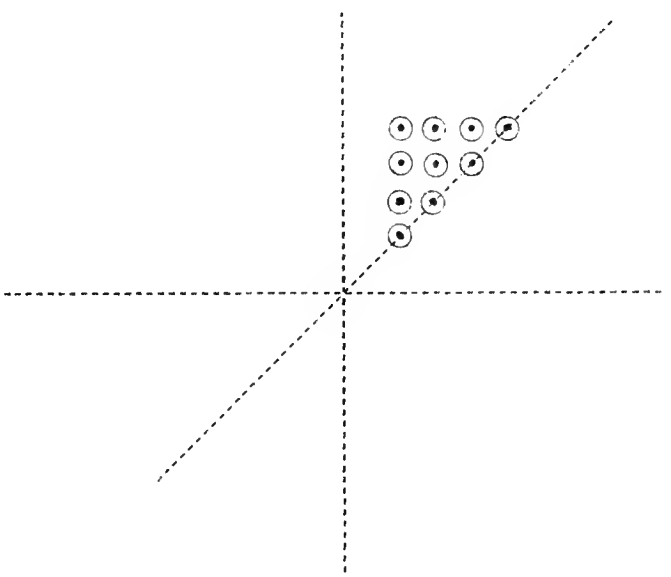


Record your result on your work sheet.

* * *

Use your eye and mark dots on the picture on your work sheet to show where you finish if you make one move from each point indicated by a loop according to the rule:

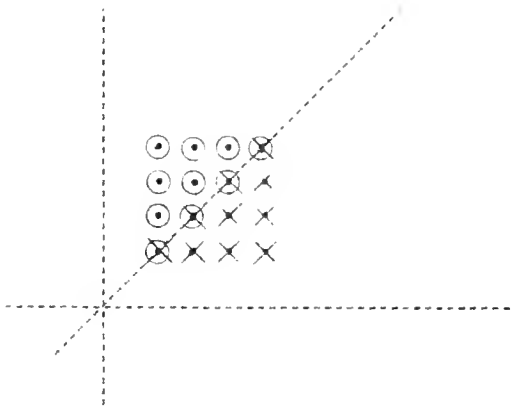
$$(x, y) \rightarrow (y, x)$$



Turn to PAGE 49.

Check your answer.

Rule: $(x, y) \rightarrow (y, x)$

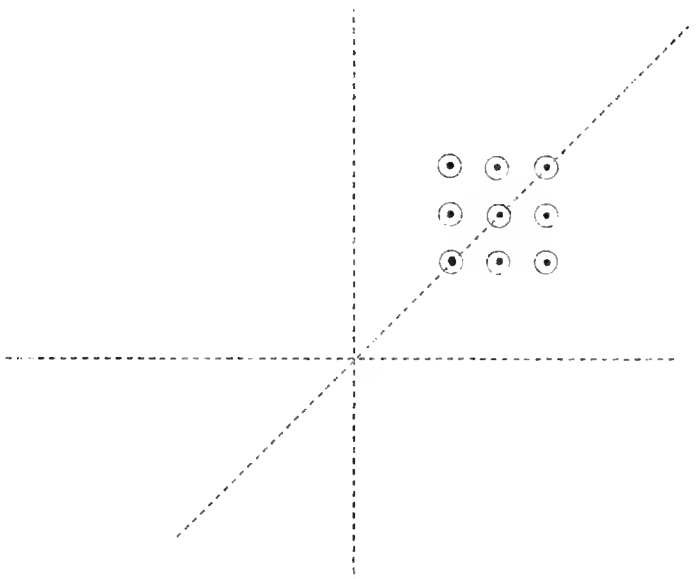


Record your result on your work sheet.

* * *

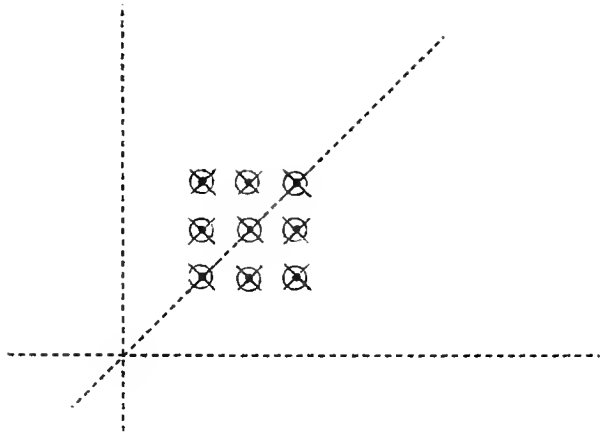
Use your eye and mark dots on the picture on your work sheet to show where you finish if you make one move from each point indicated by a loop according to the rule:

$$(x, y) \rightarrow (y, x)$$



Turn to PAGE 50.

Check your answer.



Rule: (x, y) → (y, x)

Record your result on your work sheet.

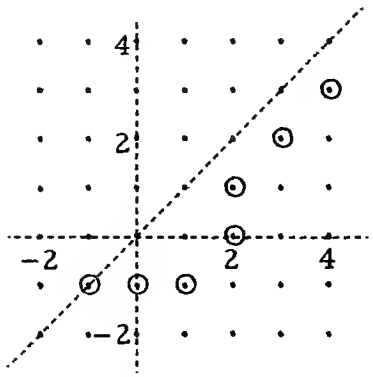
* * *

Do this exercise on your work sheet.

Set A is the set pictured.

Rule: (x, y) → (y, x)

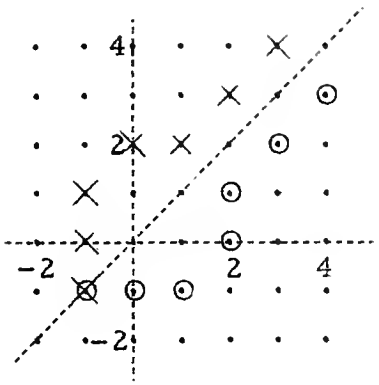
Make one move from each point in set A. Use cross-marks to indicate the new set.



Set A

Turn to PAGE 51.

Check your answer.



Rule: $(x, y) \rightarrow (y, x)$

Record your result on your work sheet.

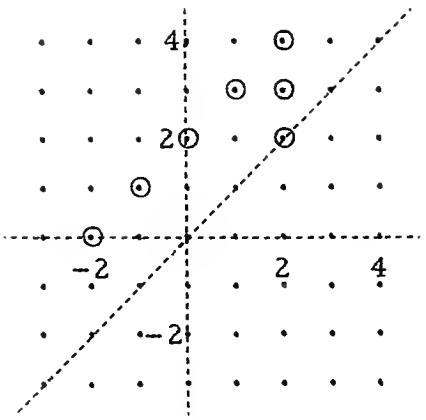
* * *

Do this exercise on your work sheet.

Set A is the set pictured.

Rule: $(x, y) \rightarrow (y, x)$

Make one move from each point in set A. Use cross-marks to indicate the new set.



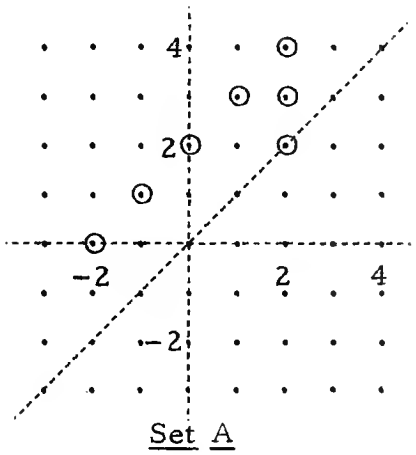
Set A

Turn to PAGE 52.

Check your answer.

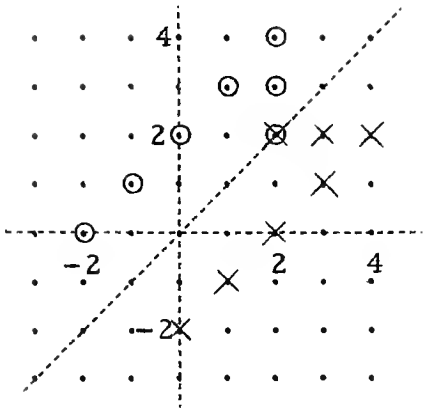
Set A is the set pictured.

Rule: (x, y) → (y, x)



Set A

Solution.



Record your result on your work sheet.

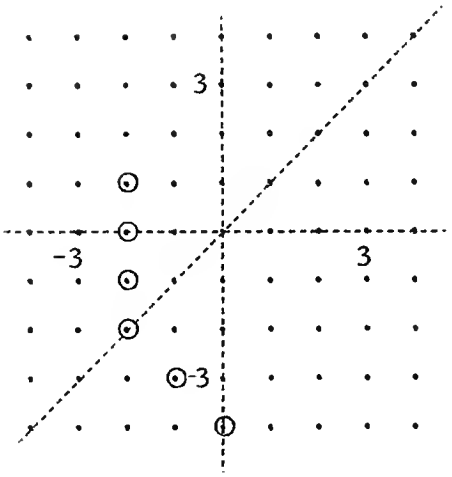
* * *

Do this exercise on your work sheet.

Set A: see picture

Rule: (x, y) → (y, x)

Make one move from each point in set A. Use cross-marks to indicate the new set.

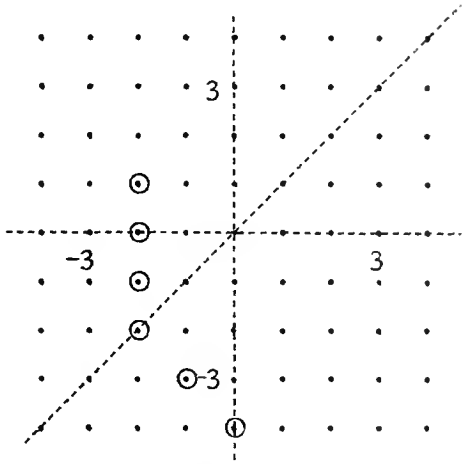


Set A

Turn to PAGE 53.

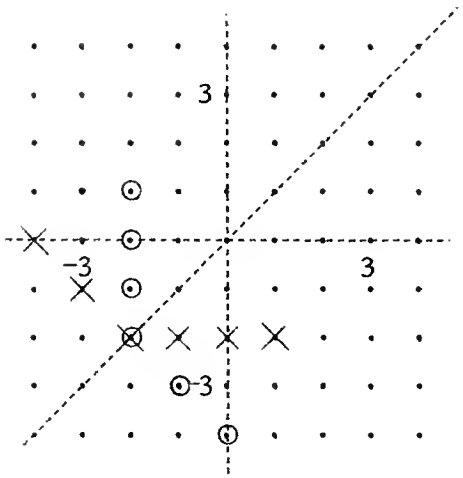
Check your answer.

Rule: (x, y) → (y, x)



Set A

Solution



Record your result on your work sheet.

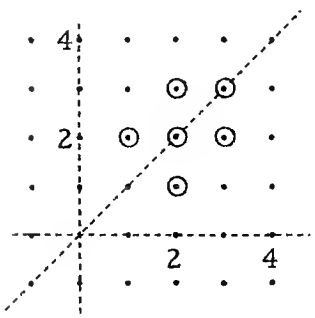
* * *

Do this exercise on your work sheet.

Set A: see picture

Rule: (x, y) → (y, x)

Make one move from each point in set A. Use cross-marks to indicate the new set.



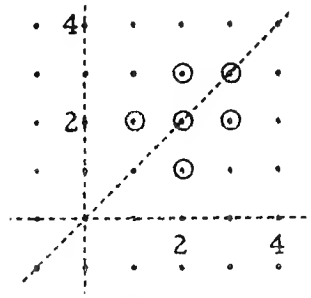
Set A

Turn to PAGE 54.

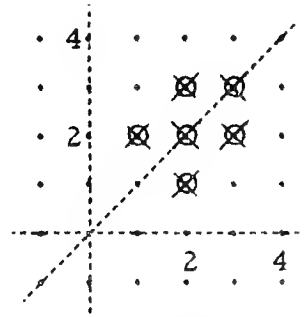
[from page 53]

Check your answer.

Rule: $(x, y) \rightarrow (y, x)$



Set A



Solution

Record your results on your work sheet.

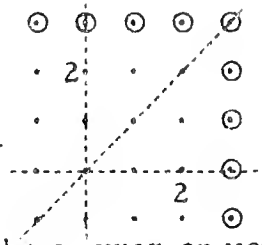
* * *

Suppose that for each set pictured below, one move was made from each point in the set according to the rule:

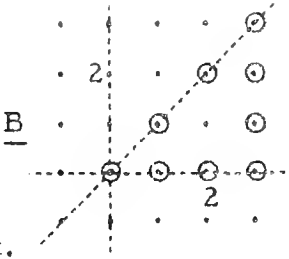
$$(x, y) \rightarrow (y, x)$$

For which of these sets would a move from any point in the set take you to another point in the set?

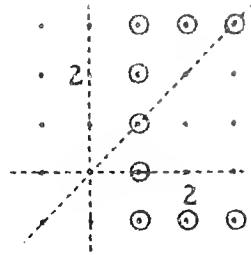
Set A



Set B



Set C



Circle the answer on your work sheet.

Turn to PAGE 55.

[Page 54]

[Part 43]

[Page 54]

Check your answer.

Rule: (x, y) → (y, x)

Turn to PAGE 56.

[from page 54]

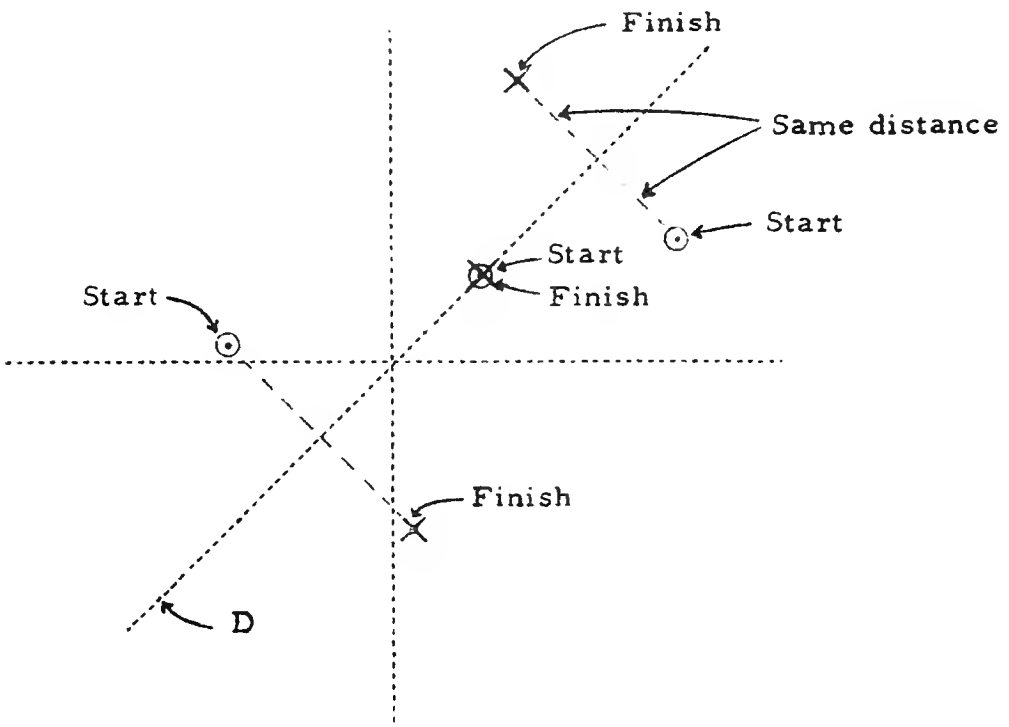
As you have probably seen from the preceding exercises, when you make one move from a point according to the rule:

$$(x, y) \rightarrow (y, x)$$

you go straight toward the graph of the line through set D, where

$$D = \{(x, y), \text{ x and y integers: } x = y\}$$

and beyond it. The new point is the same distance from the line as the starting point is.



Turn to PAGE 57.

Do these exercises on your work sheet.

$A = \{(2, 3), (3, 4)\}$

$B = \{(2, 4), (3, 4), (4, 4)\}$

Rule: $(x, y) \rightarrow (y, x)$

(1) $A \cup B = \{$? $\}$

(2) Make one move from each point in set A. Call the new set 'X'. Then $X = \{$? $\}$.

(3) Make one move from each point in set B. Call the new set 'Y'. Then $Y = \{$? $\}$.

(4) If one move is made from each point in set $A \cup B$ then the new set is $\{$? $\}$.

(5) $X \cup Y = \{$? $\}$

Turn to PAGE 58.

Check your answers.

$A = \{(2, 3), (3, 4)\}$

$B = \{(2, 4), (3, 4), (4, 4)\}$

Rule: $(x, y) \rightarrow (y, x)$

(1) $A \cup B = \{(2,3), (3,4), (2,4), (4,4)\}$.

(2) Make one move from each point in set A. Call the new set 'X'. Then $X = \{(3,2), (4,3)\}$.

(3) Make one move from each point in set B. Call the new set 'Y'. Then $Y = \{(4,2), (4,3), (4,4)\}$.

(4) If one move is made from each point in set $A \cup B$ then the new set is $\{(3,2), (4,3), (4,2), (4,4)\}$.

(5) $X \cup Y = \{(3,2), (4,3), (4,2), (4,4)\}$

[Compare the answers to Exercises (4) and (5).]

* * *

Do these exercises on your work sheet.

$A = \{(1, 1), (2, 2), (-3, -1)\}$

$B = \{(-1, -2), (2, 2), (-3, -1)\}$

Rule: $(x, y) \rightarrow (2x + 1, 3y)$

(1) $A \cup B = \{$ ____?____ $\}$

(2) Make 2 moves from each point in set A. Call the new set 'X'. Then $X = \{$ ____?____ $\}$.

(3) Make 2 moves from each point in set B. Call the new set 'Y'. Then $Y = \{$ ____?____ $\}$.

(4) If 2 moves are made from each point in set $A \cup B$ then the new set is $\{$ ____?____ $\}$. [See Exercise (1) for $A \cup B$.]

(5) $X \cup Y = \{$ ____?____ $\}$

Turn to PAGE 59.

Check your answers.

$$A = \{(1, 1), (2, 2), (-3, -1)\}$$
$$B = \{(-1, -2), (2, 2), (-3, -1)\}$$

Rule: $(x, y) \rightarrow (2x + 1, 3y)$

(1) $A \cup B = \{(1,1), (2,2), (-3,-1), (-1,-2)\}$

(2) Make 2 moves from each point in set A. Call the new set 'X'.
Then $X = \{(7,9), (11,18), (-9,-9)\}$.

(3) Make 2 moves from each point in set B. Call the new set 'Y'.
Then $Y = \{(-1,-18), (11,18), (-9,-9)\}$.

(4) If 2 moves are made from each point in set $A \cup B$ then the new set is $\{(7,9), (11,18), (-9,-9), (-1,-18)\}$.

(5) $X \cup Y = \{(7,9), (11,18), (-9,-9), (-1,-18)\}$.

Record your results on your work sheet.

* * *

Do this exercise on your work sheet.

Suppose that A and B are sets of points in the number plane lattice such that

$$A \cup B = \{(1, 3), (2, 4), (-7, -3), (-5, -4)\}.$$

Rule: $(x, y) \rightarrow (x + 3, 2y - 1)$

Make 3 moves from each point in set A. Call the new set 'X'.
Make 3 moves from each point in set B. Call the new set 'Y'.

Then, $X \cup Y =$ ____?____.

Turn to PAGE 60.

Check your answer.

Suppose that A and B are sets of points in the number plane lattice such that

$$A \cup B = \{(1, 3), (2, 4), (-7, -3), (-5, -4)\}.$$

Rule: $(x, y) \rightarrow (x + 3, 2y - 1)$

Make 3 moves from each point in set A. Call the new set 'X'.
Make 3 moves from each point in set B. Call the new set 'Y'.

Then, $X \cup Y = \{(10,17), (11,25), (2,-31), (4,-39)\}$.

Record your result on your work sheet.

* * *

Suppose that A and B are sets of points in the number plane lattice and that one of the points in $A \cup B$ is (5, 11).

Rule: $(x, y) \rightarrow (2x + 1, y - 3)$

Make 2 moves from each point in set A. Call the new set 'X'.
Make 2 moves from each point in set B. Call the new set 'Y'.

Answer this question your work sheet.

What is one of the points in $X \cup Y$?

Turn to PAGE 61.

Check your answer.

One of the points in $A \cup B$ is (5, 11).

Rule: $(x, y) \rightarrow (2x + 1, y - 3)$

Make 2 moves from each point in set A. Call the new set 'X'.
Make 2 moves from each point in set B. Call the new set 'Y'.
Then one of the points in $X \cup Y$ is (23, 5).

Record your result on your work sheet.

* * *

Suppose that A and B are sets of points in the number plane lattice and R is a moving rule.

From each point in set A, make n moves according to rule R. Let X be the new set.

From each point in set B, make n moves according to rule R. Let Y be the new set.

It follows that n moves according to rule R from any point in $A \cup B$ takes you to a point in ____?____.

Complete this last sentence on your work sheet.

Turn to PAGE 62.

Check your answer.

Suppose that A and B are sets of points in the number plane lattice and R is a moving rule.

From each point in set A, make n moves according to rule R. Let X be the new set.

From each point in set B, make n moves according to rule R. Let Y be the new set.

It follows that n moves according to rule R from any point in $A \cup B$ takes you to a point in $X \cup Y$.

Record your result on your work sheet.

* * *

Suppose that A and B are sets of points in the number plane lattice and that $(-1, 2)$ and $(3, -2)$ belong to $A \cup B$.

Rule: $(x, y) \rightarrow (x + 3, 2y - 1)$

Make 2 moves from each point in set A. Call the new set 'X'.

Make 2 moves from each point in set B. Call the new set 'Y'.

It follows that two points in $X \cup Y$ are $\frac{?}{(1)}$ and $\frac{?}{(2)}$.

Complete this last sentence on your work sheet.

Turn to PAGE 63.

Check your answers.

$(-1, 2)$ and $(3, -2)$ belong to $A \cup B$.

Rule: $(x, y) \rightarrow (x + 3, 2y - 1)$

Make 2 moves from each point in set A. Call the new set 'X'.
Make 2 moves from each point in set B. Call the new set 'Y'.

It follows that two points in $X \cup Y$ are $\underset{(1)}{(5, 5)}$ and $\underset{(2)}{(9, -11)}$.

Record your results on your work sheet.

* * *

Do this exercise on your work sheet.

Suppose that A and B are sets of points in the number plane lattice such that

$$A \cup B = \{(4, 3), (4, 4)\}.$$

Rule: $(x, y) \rightarrow (x + 5, 2y - 1)$

Make 2 moves from each point in set A. Call the new set 'X'.
Make 2 moves from each point in set B. Call the new set 'Y'.

It follows that $X \cup Y =$ ___?___.

Turn to PAGE 64.

Check your answer.

$$A \cup B = \{(4, 3), (4, 4)\}$$

Rule: $(x, y) \rightarrow (x + 5, 2y - 1)$

Make 2 moves from each point in set A. Call the new set 'X'.
Make 2 moves from each point in set B. Call the new set 'Y'.
It follows that $X \cup Y =$ $\{(14,9), (14,13)\}$.

Record your result on your work sheet.

* * *

Do this exercise on your work sheet.

Suppose that A and B are sets of points in the number plane lattice such that

$$A \cup B = \{(a, b), (e, f)\},$$

[Of course, a, b, e, and f are integers.]

Rule: $(x, y) \rightarrow (x + 3, 2y + 1)$

Make 2 moves from each point in set A. Call the new set 'X'.
Make 2 moves from each point in set B. Call the new set 'Y'.

Then, $X \cup Y =$ ____?____.

Turn to PAGE 65.

Check your answer.

Suppose that A and B are sets of points in the number plane lattice such that

$$A \cup B = \{(a, b), (e, f)\},$$

where a, b, e, and f are integers.

Rule: $(x, y) \rightarrow (x + 3, 2y + 1)$

Make 2 moves from each point in set A. Call the new set 'X'.
Make 2 moves from each point in set B. Call the new set 'Y'.
Then, $X \cup Y = \{(a+6, 4b+3), (e+6, 4f+3)\}$.

Record your result on your work sheet.

Turn to PAGE 66.

Do these exercises on your work sheet.

Rule: $(m, n) \rightarrow (m, n + 1)$

$A = \{(x, y),\ x \text{ and } y \text{ integers: } x = 3\}$

Make one move from each point in set A. Call the new set 'X'.

(1) Is set A finite? Yes or no?

(2) Is set X finite? Yes or no?

(3) Is (3, 7) in set A? Yes or no?

(4) Is (3, 8) in set X? Yes or no?

(5) Is (3, 92) in set X? Yes or no?

(6) Is (3, −4) in set X? Yes or no?

(7) Find a point in set A which is not in set X.

(8) Find a point in set X which is not in set A.

(9) Give a brace-notation description of set X.

Turn to PAGE 67.

Check your answers.

Rule: $(m, n) \rightarrow (m, n + 1)$

$A = \{(x, y),\ x \text{ and } y \text{ integers: } x = 3\}$

Make one move from each point in set A. Call the new set 'X'.

(1) Is set A finite? no (2) Is set X finite? no

(3) Is (3, 7) in set A? yes (4) Is (3, 8) in set X? yes

(5) Is (3, 92) in set X? yes (6) Is (3, −4) in set X? yes

(7) [There is no point in set A which is not in set X.]

(8) [There is no point in set X which is not in set A.]

(9) $X = \{(x, y),\ x \text{ and } y \text{ integers: } x = 3\}$

Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(m, n) \rightarrow (m + 3, n)$

$A = \{(x, y),\ x \text{ and } y \text{ integers: } y = 5\}$

Make one move from each point in set A. Call the new set 'X'.

(1) Is set A finite? Yes or no?

(2) Is set X finite? Yes or no?

(3) Give a brace-notation description of set X.

Turn to PAGE 68.

Check your answers.

Rule: $(m, n) \rightarrow (m + 3, n)$

$A = \{(x, y),\ x \text{ and } y \text{ integers: } y = 5\}$

Make one move from each point in set A. Call the new set 'X'.

(1) Is set A finite? no

(2) Is set X finite? no

(3) X = {(x, y), x and y integers: $y = 5$ }

Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(m, n) \rightarrow (m + 2, n + 1)$

$A = \{(x, y),\ x \text{ and } y \text{ integers: } x = 3\}$

Make one move from each point in set A. Call the new set 'X'.

(1) Is set A finite?

(2) Is set X finite?

(3) Plot the points in each set on the same diagram and indicate the moves. [Remember, loops for A, cross-marks for X, small arrows.]

(4) Give a brace-notation description of set X.

Turn to PAGE 69.

Check your answers.

Rule: $(m, n) \rightarrow (m + 2, n + 1)$

$A = \{(x, y),\ x \text{ and } y \text{ integers: } x = 3\}$

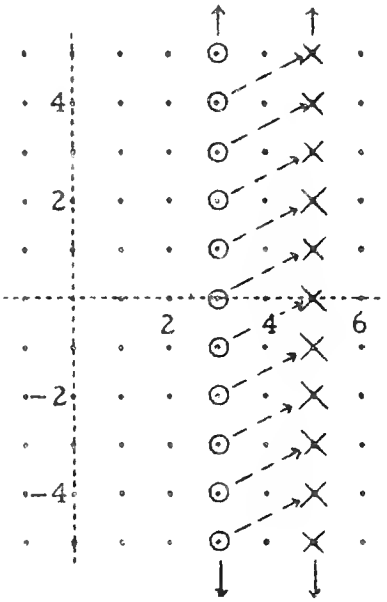
Make one move from each point in set A. Call the new set 'X'.

(1) Is set A finite? no

(2) Is set X finite? no

(3) .....>

(4) $X = \{(x, y),\ x \text{ and } y \text{ integers: } x = 5\}$



Record your results on your work sheet.

* * *

Do this exercise on your work sheet.

Rule: $(m, n) \rightarrow (m + 2, n + 1)$

$A = \{(x, y),\ x \text{ and } y \text{ integers: } x = 6\}$

Make three moves from each point in set A. Call the new set 'X'.

Describe set X, using brace-notation.

Turn to PAGE 70.

Check your answer.

Rule: $(m, n) \rightarrow (m + 2, n + 1)$

$A = \{(x, y), x \text{ and } y \text{ integers: } x = 6\}$

Make 3 moves from each point in set A. Call the new set 'X'.

$X = \{(x, y), x \text{ and } y \text{ integers: } x = 12\}$ 

[After one move from each point in set A, the set selector of the description of the new set would be 'x = 8'. After 2 moves it would be 'x = 10'. So, after 3 moves it would be 'x = 12'.]

Record your result on your work sheet.

* * *

Do these exercises on your work sheet.

Rule: $(m, n) \rightarrow (m + 1, n + 1)$

$A = \{(x, y), x \text{ and } y \text{ integers: } x + y = y + x\}$

Make one move from each point in set A. Call the new set 'X'.
Write 'true' in the blank if the statement is true. Write 'false' in the blank if the statement is false.

(1) Set A is the number plane lattice itself.

(2) Set A is infinite.

(3) Set X is the number plane lattice itself.

(4) Set X is infinite.

(5) Give a brace-notation description of set X.

Turn to PAGE 71.

Check your answers.

Rule: $(m, n) \rightarrow (m + 1, n + 1)$

$A = \{(x, y), x \text{ and } y \text{ integers: } x + y = y + x\}$

Make one move from each point in set A. Call the new set 'X'.

(1) Set A is the number plane lattice itself. true

(2) Set A is infinite. true

(3) Set X is the number plane lattice itself. true

(4) Set X is infinite. true

(5) [Any description which names the set of all ordered pairs of integers is correct.]

Record your results on your work sheet.

* * *

For each point listed below in Exercises (1) - (6), write 'yes' if the point belongs to set A, and write 'no' if the point does not belong to set A, where

$A = \{(x, y), x \text{ and } y \text{ integers: } x^2 + y^2 = 9\}$.

Do these exercises on your work sheet.

(1) (3, 0) (2) (0, −3) (3) (2, 7)

(4) (−3, 0) (5) (5, 4) (6) (0, 3)

(7) n(A) = ? [Remember, 'n(A)' means the number of elements in A.]

Turn to PAGE 72.

Check your answers.

$$A = \{(x, y),\ x \text{ and } y \text{ integers: } x^2 + y^2 = 9\}$$

(1) (3, 0) yes (2) (0, −3) yes (3) (2, 7) no

(4) (−3, 0) yes (5) (5, 4) no (6) (0, 3) yes

(7) n(A) = 4

Record your results on your work sheet.

* * *

For each point listed below in Exercises (1) - (6), write 'yes' if the point belongs to set A, and write 'no' if the point does not belong to set A, where

$$A = \{(x, y),\ x \text{ and } y \text{ integers: } x^2 + y^2 = 25\}.$$

Do these exercises on your work sheet.

(1) (−5, 0) (2) (3, −4) (3) (25, 0)

(4) (−4, −3) (5) (0, 5) (6) (16, 9)

(7) Plot <u>all</u> of the points in set A. There are a total of 12 points in set A.

Turn to PAGE 73.

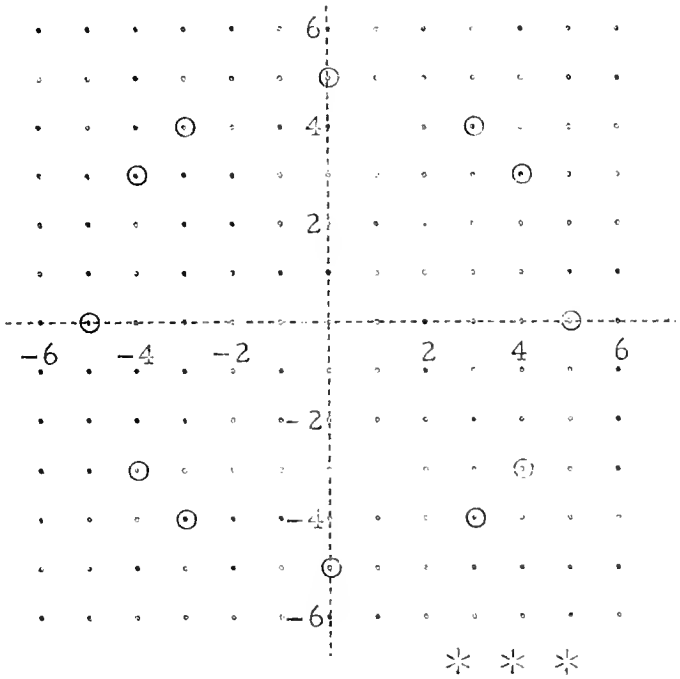
Check your answers.

$$A = \{(x, y),\ x \text{ and } y \text{ integers: } x^2 + y^2 = 25\}$$

(1) (−5, 0) yes (2) (3, −4) yes (3) (25, 0) no

(4) (−4, −3) yes (5) (0, 5) yes (6) (16, 9) no

(7)



Record your results on your work sheet.

Look at Answer (7) above. Notice that the points in set A are arranged on the circle with center at (0, 0) and radius 5.

Do these exercises on your work sheet.

$$A = \{(x, y),\ x \text{ and } y \text{ integers: } x^2 + y^2 = 25\}$$

Rule: $(m, n) \rightarrow (m + 2, n)$

Make one move from each point in set A. Call the new set 'X'.

(1) Plot the points in each set on the same diagram and indicate the moves. [Loops for A, cross-marks for X.]

(2) Which of the sets described below is set X?

$\{(x, y),\ x \text{ and } y \text{ integers: } (x-2)^2 + y^2 = 25\}$

$\{(x, y),\ x \text{ and } y \text{ integers: } (x+2)^2 + y^2 = 25\}$

(3) The points in set X are arranged on the circle with center at ___?___ and radius ___?___.

Turn to PAGE 74.

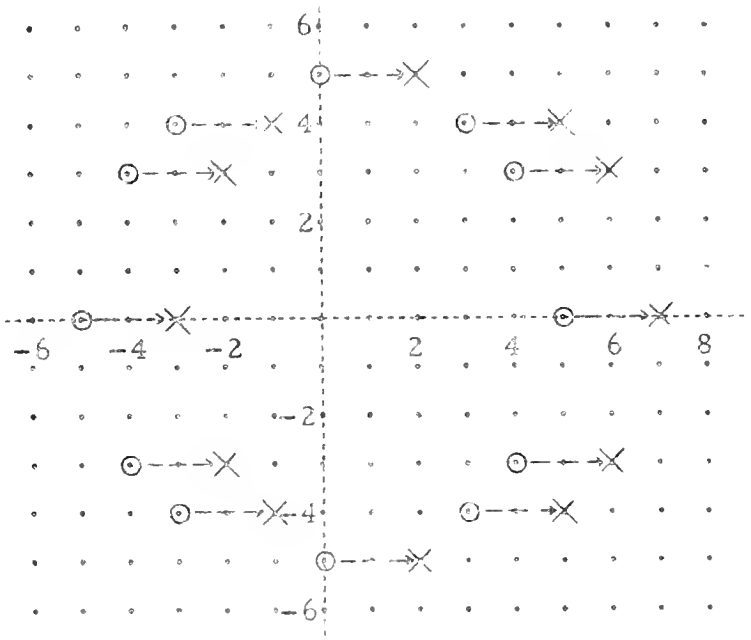
Check your answers.

$A = \{(x, y),\ x \text{ and } y \text{ integers: } x^2 + y^2 = 25\}$

Rule: $(m, n) \rightarrow (m + 2,\ n)$

Make one move from each point in set A. Call the new set 'X'.

(1)



(2) $X = \{(x, y),\ x \text{ and } y \text{ integers: } (x-2)^2 + y^2 = 25\}$

(3) The points in set X are arranged on the circle with center at $\underset{\text{(a)}}{(2,0)}$ and radius $\underset{\text{(b)}}{5}$.

Record your results on your work sheet.

* * *

Do these exercises on your work sheet.

$A = \{(x, y),\ x \text{ and } y \text{ integers: } x^2 + y^2 = 25\}$

Rule: $(m, n) \rightarrow (m + 2,\ n - 3)$

Make <u>two</u> moves from each point in set A. Call the new set 'X'.

(1) Plot the points in each set on the same diagram.

(2) The points in set X are arranged on the circle with center at ___?___ and radius ___?___.

☆(3) X = {(x, y), x and y integers: ___?___}

Turn to PAGE 75.

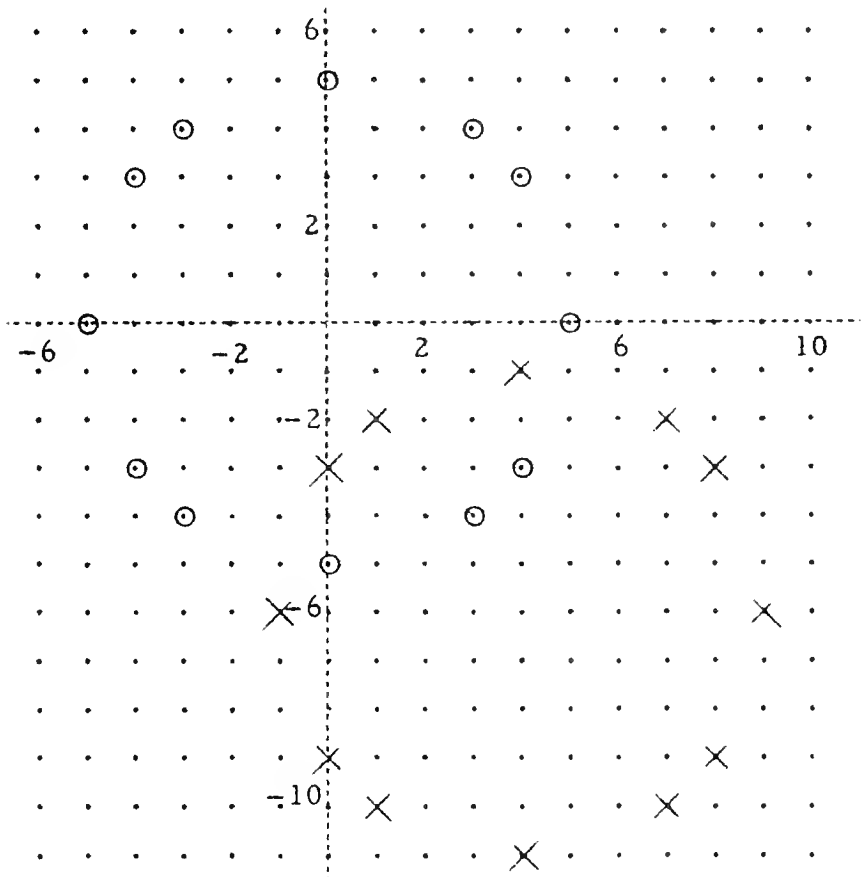
Check your answers.

$A = \{(x, y),\ x \text{ and } y \text{ integers: } x^2 + y^2 = 25\}$

Rule: $(m, n) \rightarrow (m + 2,\ n - 3)$

Make two moves from each point in set A. Call the new set 'X'.

(1)



(2) The points in set X are arranged on the circle with center at $\frac{(4,-6)}{(a)}$ and radius $\frac{5}{(b)}$.

☆(3) $X = \{(x, y),\ x \text{ and } y \text{ integers: } (x-4)^2+(y+6)^2 = 25\}$

If you tried Exercise (3) and got it right, turn to PAGE 76.

Otherwise, this is the end of Part 43. Put your work sheet under the front cover of this booklet, and return it to your teacher.

Do this exercise on your work sheet.

$A = \{(x, y),\ x \text{ and } y \text{ integers: } 2x^2 + 7xy + 6y^2 = 0\}$

Rule: $(x, y) \rightarrow (x + a,\ y - b)$, a and b are integers

Make n moves from each point in set A. Call the new set 'X'.

Write a brace-notation description of set X.

Turn to PAGE 77.

Check your answer.

$A = \{(x, y),\ x \text{ and } y \text{ integers: } 2x^2 + 7xy + 6y^2 = 0\}$

Rule: $(x, y) \rightarrow (x + a,\ y - b),\ a \text{ and } b \text{ integers}$

Make n moves from each point in set A. Call the new set 'X'.

$X = \{(x, y),\ x \text{ and } y \text{ integers: } 2(x-na)^2 + 7(x-na)(y+nb) + 6(y+nb)^2 = 0\}$

Put your work sheet under the front cover of this booklet, and return it to your teacher.

PART 43

WORK SHEET

Name _____

School _____

Date _____

Answers

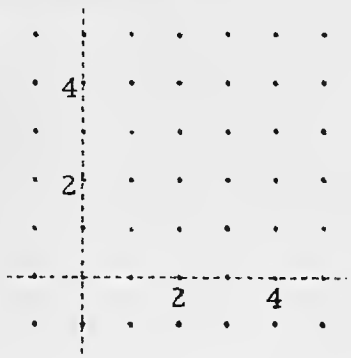
Page 3

(1) First jump: From (4, 1) to (3, 2)

Second jump: From (3, 2) to (___ , ___)

(2) Finish: _____

(3)



Page 4

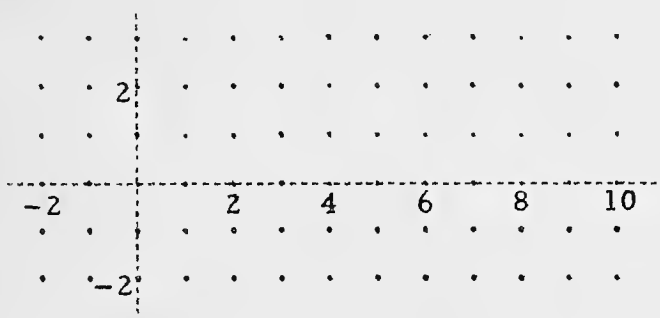
(1) First jump: From (0, 1) to (3, 1)

Second jump: From (3, 1) to (___, ___)

(2) Third jump: From (___, ___) to (___, ___)

(3) Finish: _____

(4)



Result Check

Page 3

1

2 3

OK

Page 4

1

2

3

4 OK

WORK SHEET Name _____

Part 43

Answers

Page 6

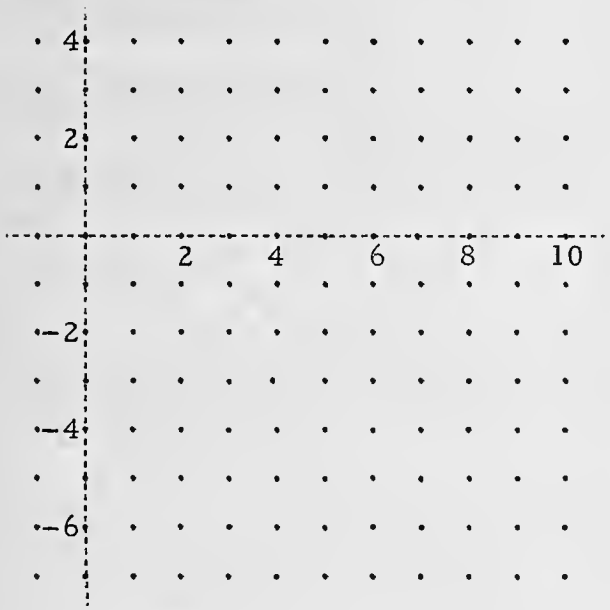
(1) First move takes you to (5, 0).

Second move takes you to (___, ___).

(2) Third move takes you to (___, ___).

(3) Finish: _____

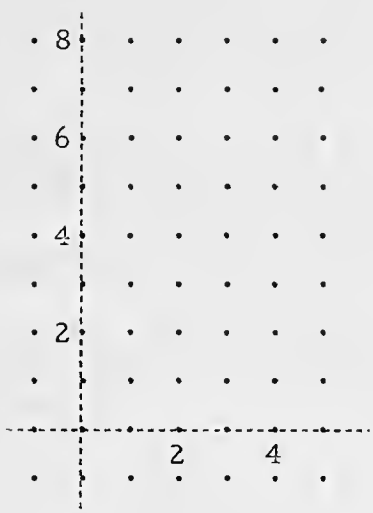
(4)



Page 7

(1) Finish: _____

(2)



Result Check

Page 6

1

2

3

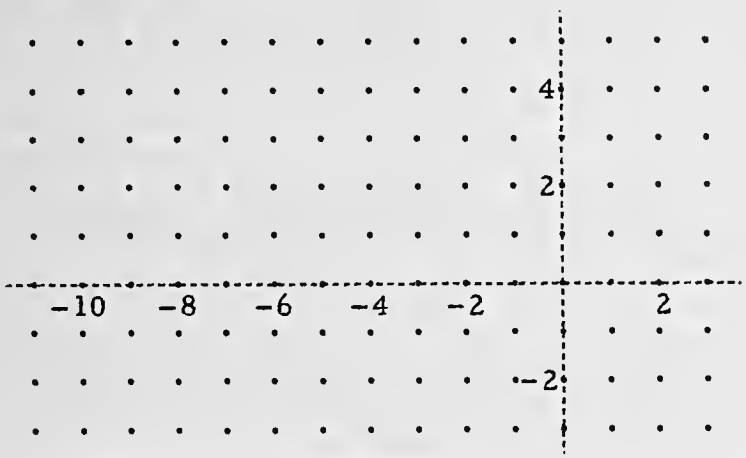
4 OK

Page 7

1 2 OK

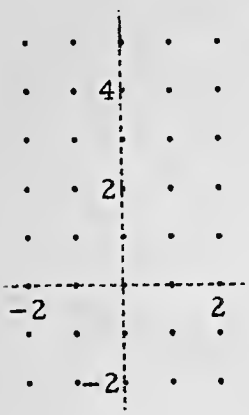
WORK SHEET Name _____

Part 43

Answers	Result Check
Page 8	**Page 8**
The final point, or the finish is _____.	✓ OK
Page 9	**Page 9**
(1) First move takes you to (1, 0)	1
Second move takes you to (___, ___).	
(2) Third move takes you to (___, ___).	2
(3) Final point: _____	3
(4) 	4 OK
Page 10	**Page 10**
The final point is _____.	✓ OK
Page 11	**Page 11**
(1) First move takes you to (4, 5).	1
Second move takes you to (16, ___).	
(2) Third move takes you to (____, ____).	2
(3) So, the final point is (____, ____).	3 OK

WORK SHEET Name _____

Part 43

Answers	Result Check
Page 12	Page 12
(1) First move takes you to (1, 4). _____	1
(2) Second move takes you to _____.	2
(3) Third move takes you to _____.	3
(4) The final point after 4 moves would be _____.	4
(5) The final point after 20 moves would be _____.	5
(6)  [Show first 3 moves only.]	6 OK
Page 13	Page 13
The starting point was _____.	✓ OK
Page 14	Page 14
The starting point was _____.	✓ OK
Page 15	Page 15
(1) After 2 moves: (1, 8)	1
After 1 move: (3, ___)	
(2) Start: (___, ___)	2 OK

WORK SHEET Name _____

Part 43

Answers

Page 16

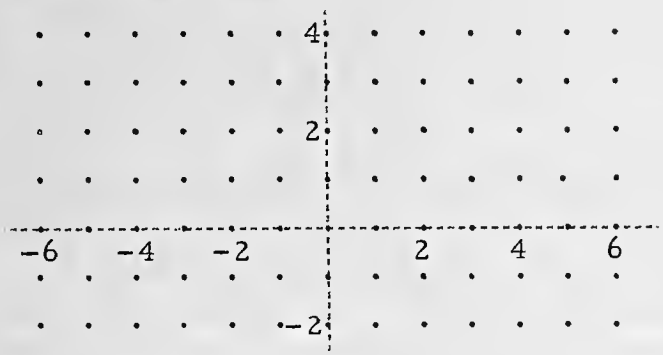
(1) After 4 moves: (-6, 3)
After 3 moves: (-3, ___)

(2) After 2 moves: (___, ___)

(3) After 1 move: (___, ___)

(4) Start: (___, ___)

(5)



Page 17

(1) After 3 moves: (15, -3)
After 2 moves: (7, ___)

(2) After 1 move: (___, ___)

(3) Start: (___, ___)

Page 18

(1) After 2 moves: (11, 3)
After 1 move: (5, ___)

(2) Start: (___, ___)

Page 19

(1) After 3 moves: (9, 7)
After 2 moves: (5, ___) [2 · ___ + 1 = 7]

(2) After 1 move: (___, ___)

(3) Start: _____

Result Check

Page 16

1

2

3

4

5 OK

Page 17

1

2

3 OK

Page 18

1

2 OK

Page 19

1

2

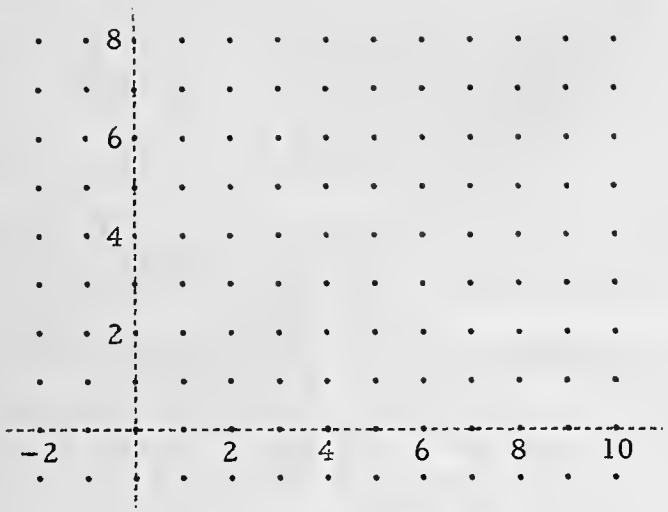
3 OK

WORK SHEET Name _____

Part 43

Answers

Page 20



Page 21

(1) After 3 moves: (53, −7)

After 2 moves: (17, 5)

After 1 move: _____

[3 · ___ + 2 = 17. 3 − 2 · ___ = 5.]

(2) Start: _____

Page 22

(1) After 3 moves: (28, 12)

After 2 moves: _____

(2) After 1 move: _____

(3) Start: _____

Result Check

Page 20

✓ OK

Page 21

1

2 OK

Page 22

1

2

3 OK

WORK SHEET Name _____

Part 43

Answers | Result Check

Page 24 | Page 24

Rule $(x, y) \rightarrow (2x + 5, 3 - 2y)$

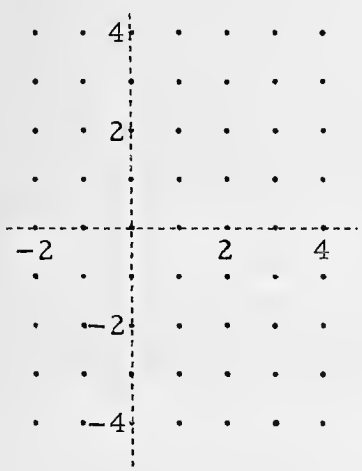
After 3 moves, the final point is $(19, -15)$.

First Component	Second Component
$2x + 5 = 19$	$3 - 2y = -15$
$x =$ ___	$y =$ ___
After 2 moves:	(___ , ___)
$2x + 5 =$ ___	$3 - 2y =$ ___
$x =$ ___	$y =$ ___
After 1 move:	(___ , ___)
$2x + 5 =$ ___	$3 - 2y =$ ___
$x =$ ___	$y =$ ___
Starting point:	_____

✓ OK

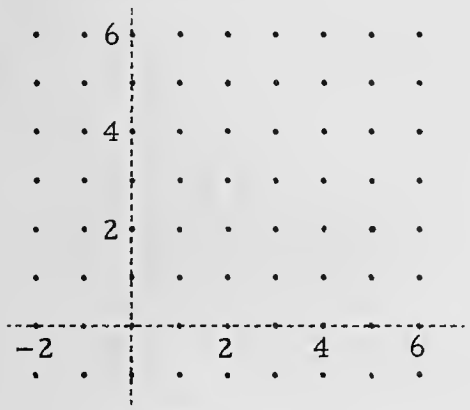
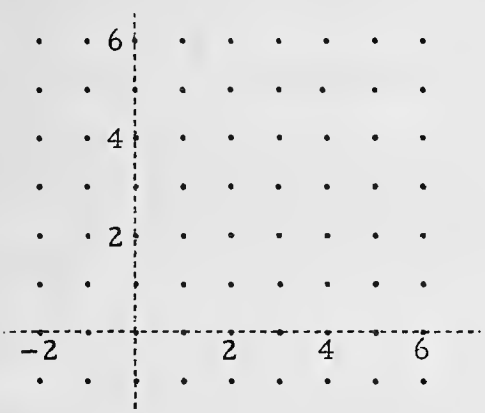
WORK SHEET Name _____

Part 43

Answers	Result Check
Page 25	Page 25
After 4 moves:	✓ OK
After 3 moves:	
After 2 moves:	
After 1 move:	
Start: _____	
Page 26	Page 26
(1) From (2, 2), you move to (___ + ___, ___ - ___), or (___, ___).	1
(2) X = {_____, _____, _____}	2 OK
Page 27	Page 27
(1) X = {_____, _____, _____, _____}	1
(2) 	2 OK

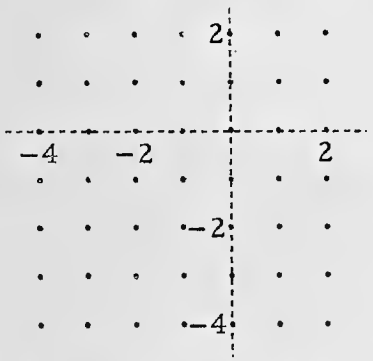
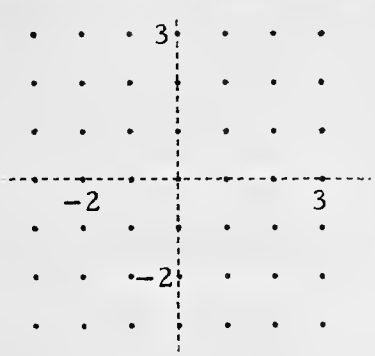
WORK SHEET Name _____

Part 43

Answers	Result Check
Page 28	Page 28
(1) X = {(1, 2), (2, ___), _____, _____}	1
(2) 	2 OK
Page 29	Page 29
(1) A = {_____}	1
(2) X = {_____}	2
(3) 	3
(4) X = {(x, y), x and y integers: _____ and x = 1}	4 OK

WORK SHEET Name _____

Part 43

Answers	Result Check	
Page 30	Page 30	
(1) A = {_____}	1	
(2) X = {_____}	2	
(3) 	3	
(4) X = {(x, y), x and y integers: $-2 < y < 2$ and _____}	4	OK
Page 31	Page 31	
(1) A = {_____, _____, _____}	1	
(2) X = {(x, y), x and y integers: $1 < x < 5$ and _____}	2	
(3) A = {(x, y), x and y integers: _____ and $x = 7$}	3	OK
Page 32	Page 32	
X = {(x, y), x and y integers: _____ and _____}	✓	OK
Page 34	Page 34	
(1) X = {(x, y), x and y integers: _____ and _____}	1	
(2) 	2	OK

WORK SHEET Name _____

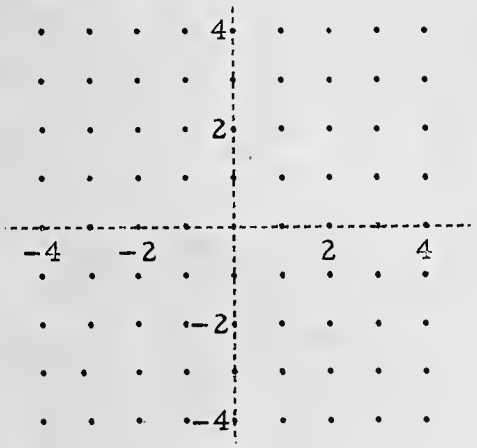
Part 43

Answers

Page 35

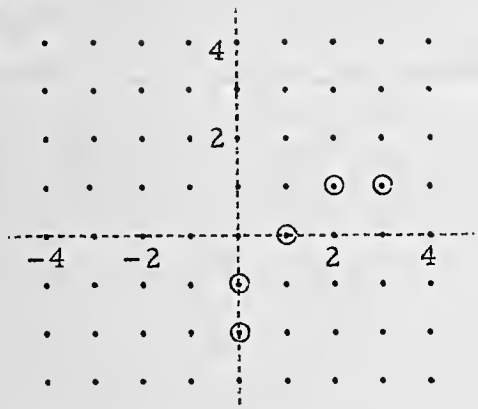
(1) X = _____

(2)



Page 36

(1)

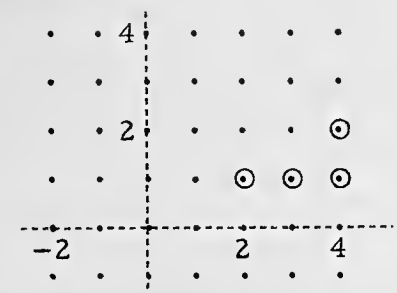


(2) A = {_____}

(3) X = {_____}

Page 37

(1)



(4) [Use diagram in Exercise (1).]

(2) A = {_____}

(3) X = {_____}

Result Check

Page 35

1

2 OK

Page 36

1

2

3 OK

Page 37

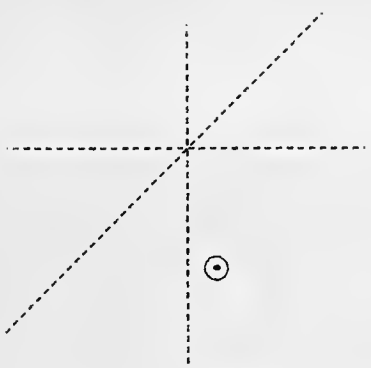
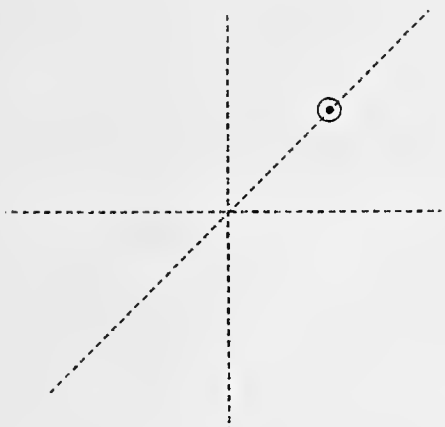
1

2

3 OK

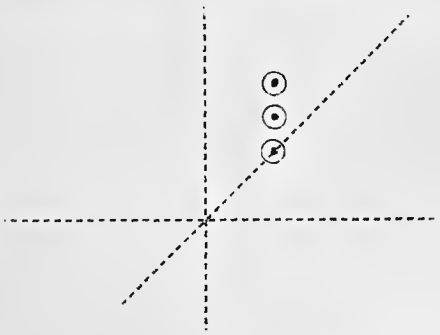
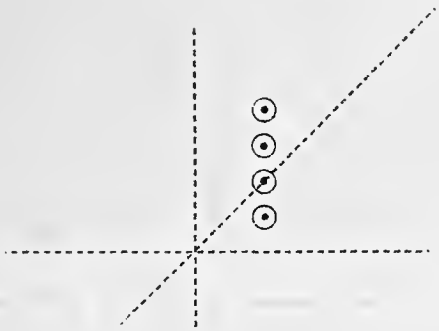
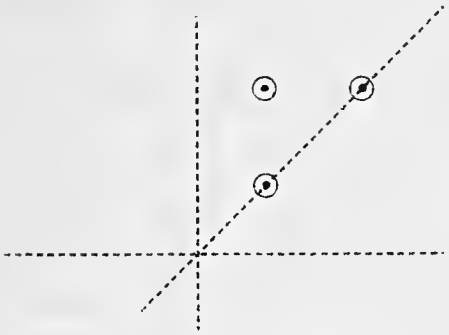
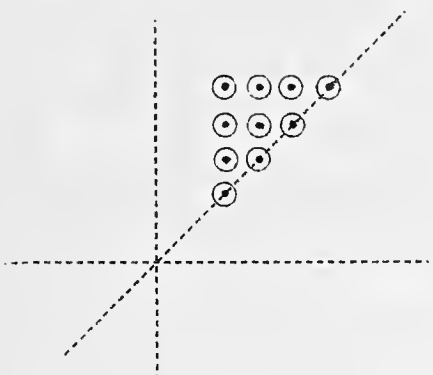
WORK SHEET Name _____

Part 43

Answers	Result Check
Page 39	Page 39
Point (1) Point (2) Point (3)	✓ OK
Page 40	Page 40
Point (1) Point (2) Point (3)	✓ OK
Page 41	Page 41
Point (1) Point (2) Point (3)	✓ OK
Page 42	Page 42
Point (1) Point (2) Point (3)	✓ OK
Page 43 	Page 43 ✓ OK
Page 44 	Page 44 ✓ OK

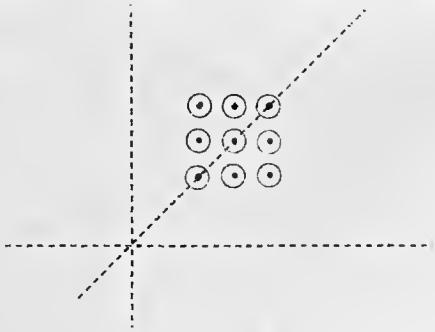
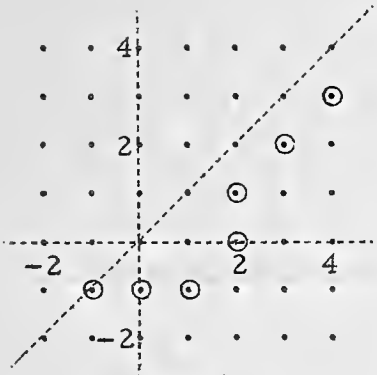
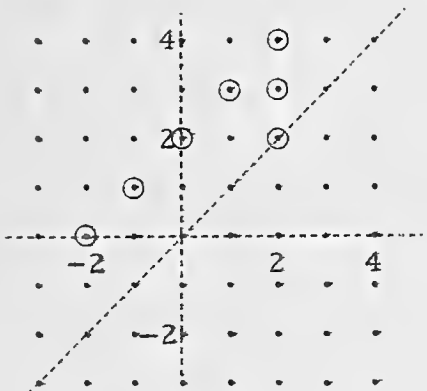
WORK SHEET Name _____

Part 43

Answers	Result Check
Page 45 	Page 45 ✓ OK
Page 46 	Page 46 ✓ OK
Page 47 	Page 47 ✓ OK
Page 48 	Page 48 ✓ OK

WORK SHEET Name _____

Part 43

Answers	Result Check
Page 49 	Page 49 ✓ OK
Page 50  Set A	Page 50 ✓ OK
Page 51  Set A	Page 51 ✓ OK

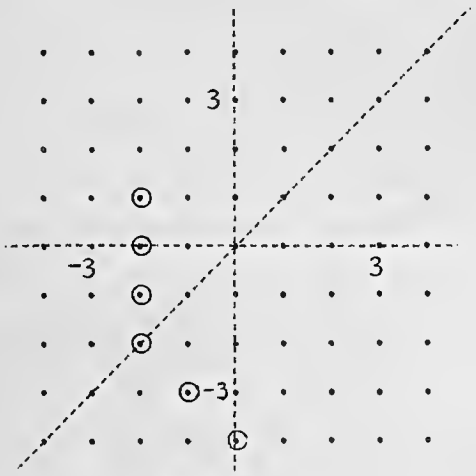
WORK SHEET Name _____

Part 43

Answers

Result Check

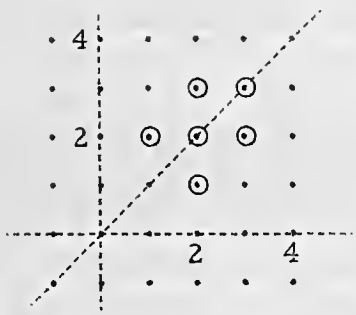
Page 52



Set A

Page 52 ✓ OK

Page 53



Page 53 ✓ OK

Page 54

Set A Set B Set C

Page 54 ✓ OK

Page 57

(1) $A \cup B = \{\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_\}$ 1

(2) $X = \{\_\_\_\_\_, \_\_\_\_\_\}$ 2

(3) $Y = \{\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_\}$ 3

(4) If one move is made from each point in set $A \cup B$ then the new set is $\{\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_\}$ 4

(5) $X \cup Y = \{\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_\}$ 5 OK

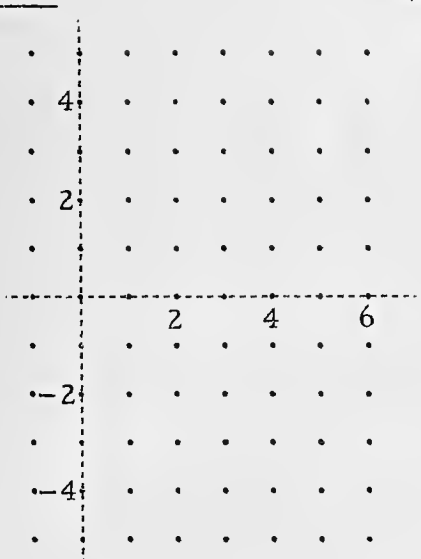
WORK SHEET Name _____

Part 43

Answers	Result Check
Page 58	Page 58
(1) $A \cup B = \{$ _____ $\}$	1
(2) $X = \{$ _____ $\}$	2
(3) $Y = \{$ _____ $\}$	3
(4) If 2 moves are made from each point in set $A \cup B$ then the new set is $\{$ _____ $\}$	4
(5) $X \cup Y = \{$ _____ $\}$	5 OK
Page 59	Page 59
$X \cup Y =$ _____	✓ OK
Page 60	Page 60
One of the points in $X \cup Y$ is _____.	✓ OK
Page 61	Page 61
It follows that n moves according to rule R from any point in $A \cup B$ take you to a point in _____.	✓ OK
Page 62	Page 62
It follows that two points in $X \cup Y$ are _____ (1) and _____ (2).	1 2 OK
Page 63	Page 63
It follows that $X \cup Y =$ _____.	✓ OK
Page 64	Page 64
$X \cup Y =$ _____.	✓ OK

WORK SHEET Name _____

Part 43

Answers	Result Check
Page 66	Page 66
(1) _____ (2) _____	1 2
(3) _____ (4) _____	3 4
(5) _____ (6) _____	5 6
(7) _____ is a point in set A which is not in set X; [or] There is no point in set A which is not in set X.	7
(8) _____ is a point in set X which is not in set A; [or] There is no point in set X which is not in set A.	8
(9) X = {(x, y), x and y integers: _____}	9 OK
Page 67	Page 67
(1) _____ (2) _____	1 2
(3) X = {(x, y), x and y integers: _____}	3 OK
Page 68	Page 68
(1) _____ (2) _____	1 2
(3) 	3
(4) X = {(x, y), x and y integers: _____}	4 OK

WORK SHEET Name _____

Part 43

Answers

Page 69

X = {(x, y), x and y integers: _____}

[After one move, the set selector of the description of the new set would be 'x = 8'.]

Page 70

(1) _____ (2) _____

(3) _____ (4) _____

(5) X = {(x, y), x and y integers: _____}

Page 71

(1) (3, 0) _____ (2) (0, −3) _____ (3) (2, 7) _____

(4) (−3, 0) _____ (5) (5, 4) _____ (6) (0, 3) _____

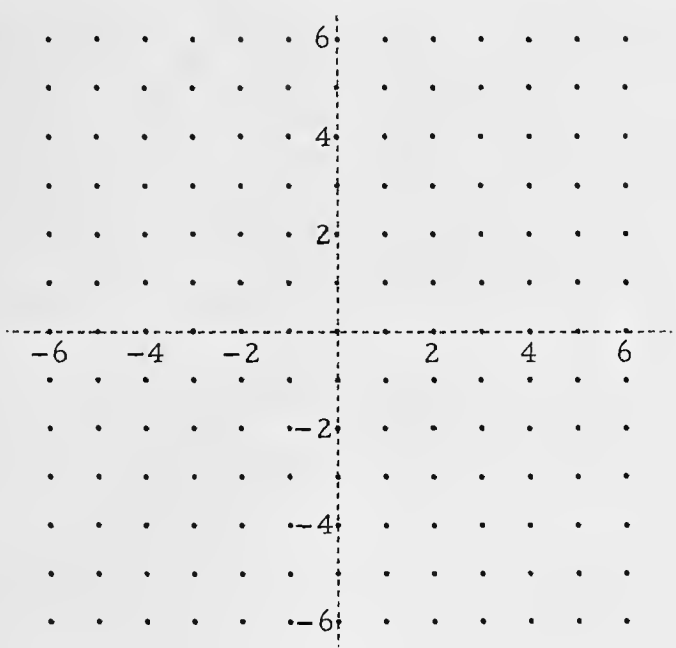
(7) n(A) = _____

Page 72

(1) (−5, 0) _____ (2) (3, −4) _____ (3) (25, 0) _____

(4) (−4, −3) _____ (5) (0, 5) _____ (6) (16, 9) _____

(7)



Result Check

Page 69

✓ OK

Page 70

1 2

3 4 OK

5

Page 71

1 2 3

4 5 6

7 OK

Page 72

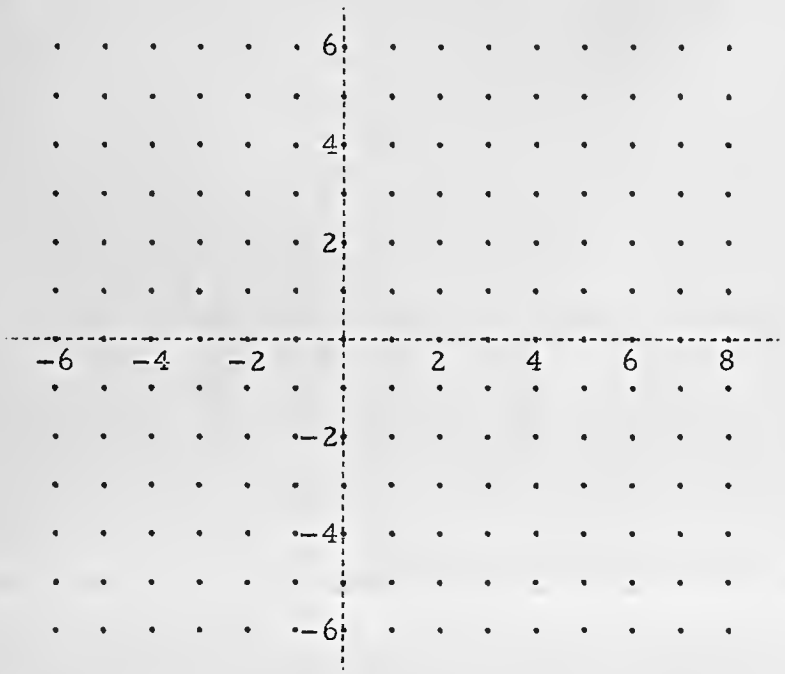
1 2 3

4 5 6

7 OK

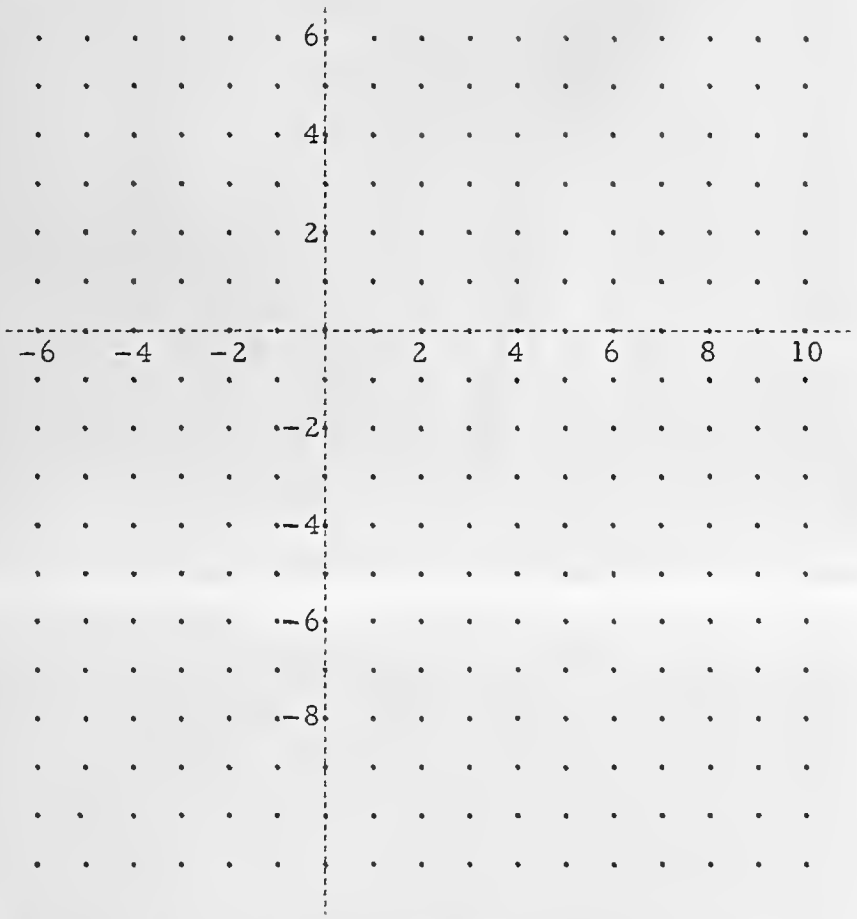
WORK SHEET Name _____

Part 43

Answers	Result Check
Page 73	Page 73
(1) 	1
(2) X = {(x, y), x and y integers: _____}	2
(3) The points in set X are arranged on the circle with center at _____ (a) and radius _____ (b).	3a 3b OK

WORK SHEET Name _____

Part 43

Answers	Result Check
Page 74	Page 74
(1) 	1
(2) The points in set X are arranged on the circle with center at _____ (a) and radius _____ (b)	2a 2b
☆(3) X = {(x, y), x and y integers: _____}	3 OK
☆Page 76	Page 76
X = {(x, y), x and y integers: _____ _____}	✓ OK