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Hours and Employment

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Hours and Employment
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## Abstract

With real wage rate and hours given to them by collective agreement, firms are assumed to supply whatever output is most profitable to them. With a well-functioning capital market such supply will create its own demand. Using the empirically robust Cobb-Douglas production function the paper finds that a compensated shortening of hours of men and machines alike will have the same unfavorable effect upon employment as a direct increase of the hourly real wage rate. A compensated shortening of hours of men but not of machines will have a less unfavorable effect.

# HOURS AND EMPLOYMENT ARBEITSZEIT UND BESCHÄFTIGUNG 

By Hans Brems, Urbana-Champaign, USA

WENN'T Abend ward, Und still de Welt un still dat Hart; Wenn möd up't Knee di liggt de Hand, Un ut din Husklock an de Wand Du hörst den Parpendikelslag, De nich to Woort keem över Dag

Theodor Storm (1817-1888), "An Klaus Groth," Forster (1957: 363)

1. Hours

Hours of work per man per year display an unmistakable downward trend, very similar between the United States and Germany: According
to Waddison (1987: 686), over the period 1870-1984 the German hours of work per man per year declined from 2,941 to 1,676 , and the United States hours from 2,964 to 1,632.

The historical parallel between our countries, then, is close enough. But in the mid-eighties an American observer of the German scene noticed a lively public debate on an issue that in the United States has remained a nonissue, i.e., shorter hours. Shorter hours were seen as a cure for unemployment, thus reviving the durable fallacy that a given pile of work is waiting to be done, and the shorter the hours the more men it would take to do it: if seven men working for five days @ 8 hours can dig a ditch, how many men would it take to dig it in five days @ 7 hours? Back in junior high school that is how most of us were introduced to algebra.
2. A Simple :Oodel

Scholars know better and have already said so, not the least in the present journal. All my own modest contribution can hope to do is to reduce the issue to its essence and to express that essence in simple mathematics.

The first thing to do is to abandon the Keynesian notion that demand always creates its own supply. Instead, with real wage rate
and hours given to them by collective agreement, let firms respond by supplying whatever physical output is most profitable to the individual firm. With a well-functioning capital market such supply will always create its own demand: a flexible rate of interest will be the equilibrating variable between saving and investment. In that case the propensities to consume or save will be of no direct consequence Eor the aggregate demand for output. A higher propensity to save will simply reduce one use, i.e., consumption, of output but expand another, i.e., investment, by as much as Smith ${ }^{1}$ [1776 (1805: 78-79)] and Ricardo ${ }^{2}$ [1815-1823 (1951, IV: 179-180)] said it would. The New Classical economics is indeed classical!

The second thing to do is to use the empirically robust CobbDouglas production function having the constant elasticity of substitution between labor and capital equaling one. With great ease such a function will handle the practically important distinction between a shortening of hours of men alone and a shortening of hours of men and machines alike.

Let us use the following notation.

## 3．Variables

L $\equiv$ employment，number of men
P ミprice
w $\equiv$ money wage rate，dollars per man－hour X ミ physical output

4．Parameters
a $\equiv$ joint factor productivity
$\alpha \equiv$ elasticity of physical output with respect to labor
$B \equiv$ elasticity of physical output with respect to capital stock
$h$ 引hours of work per year of men and machines alike
$h_{L} \equiv$ hours of work per man per year
$h_{S} \equiv$ hours of operation per machine per year S 引 physical capital stock，number of machines

The subscript i refers to the ith firm．

## 5. Men and Machines Work Same Hours Per Year

Consider first the simple case that men and machines alike work $h$ hours per year.

We must begin at the firm level and write the production function of the ith firm common to all $n$ firms:

$$
\begin{equation*}
X_{i}=a\left(h L_{i}\right)^{\alpha}\left(h S_{i}\right)^{\beta}=a h L_{i} S_{i}^{\beta} \tag{1}
\end{equation*}
$$

where $0<\alpha<1,0<\beta<1, \alpha+\beta=1$, and a is what growth measurement [Maddison (1987: 658)] calls "joint factor productivity." In (1) the elasticity of physical output $X_{i}$ with respect to hours $h$ per man or machine per year is 1 . The elasticity with respect to number of men $L_{i}$ is $a$.

With a real wage rate $w / ?$ and hours $h$ given to it by collective agreement, the ith firm will maximize its profits by hiring man-hours until in real terms the last man-hour costs as much as it contributes. Under pure competition, then, the real wage rate equals the physical marginal productivity of labor:

$$
\begin{equation*}
\frac{w}{P}=\frac{\partial X_{i}}{\partial\left(h L_{i}\right)}=\frac{1}{h} \frac{\partial X_{i}}{\partial L_{i}}=a \alpha L_{i}^{\alpha-1} S_{i}^{\beta} \tag{2}
\end{equation*}
$$

from the right-hand side of which hours h have disappeared. Number of men $L_{i}$ is raised to the power $\alpha-1=-\beta$. Raise both sides of (2) to the power - $1 / B$, rearrange, and write number of men demanded as a function of the hourly real wage rate $w / P$ :

$$
L_{i}=(a \alpha)^{1 / B}{\underset{P}{-1 / B}}_{-1 /} S_{i}
$$

easily aggregated into

$$
\begin{equation*}
L=(a \alpha)^{1 / \beta}{\underset{\mathrm{P}}{ }}_{\frac{\mathrm{w}^{-1 / \beta}}{} \mathrm{S}} \tag{3}
\end{equation*}
$$

where

$$
\begin{aligned}
L & \equiv \sum_{i=1}^{n} L_{i} \\
S & \equiv \sum_{i=1}^{n} S_{i}
\end{aligned}
$$

## 6. Shorter Hours at Same Hourly Real Wage Rate

Because hours h appear nowhere in (3), its elasticity with respect to them is

$$
\begin{equation*}
\frac{\partial \log _{e} L}{\partial \log _{e} h}=0 \tag{4}
\end{equation*}
$$

In other words, the number of men demanded $L$ remains the same regardless of the number of hours h. Puzzling? Not at all: in a linearly homogeneous production function like our (1) physical marginal productivities of factors are known to depend solely upon the factor proportion. Shortening the number of hours $h$ for men and machines alike will affect neither the factor proportion (hS $\left.i_{i}\right) /\left(h L_{i}\right)$ nor the physical marginal productivity of labor $\partial X_{i} / \partial\left(h L_{i}\right)$. Since the hourly real wage rate is not affected either, the two can remain equal at the same number of men $L$ as before the shortening.

While employment $L$ remains unaffected, physical output is seriously affected by the shorter hours: the elasticity of physical output of the firm (1) with respect to hours was seen to be 1 . A percent
shortening of hours, in other words, will reduce firm output by 1 percent. There is indeed no given pile of work waiting to be done!
7. A Higher Hourly Real Wage Rate at Same Hours

Because the hourly real wage rate $w / P$ does appear in (3), the real-wage-rate elasticity of employment is

$$
\begin{equation*}
\frac{\partial \log _{e} L}{\partial \log _{e}(w / P)}=-\frac{1}{B} \tag{5}
\end{equation*}
$$

In other words, if $\beta=1 / 4$ and the hourly real wage rate $w / p$ is up by 1 percent the number of men demanded $L$ is down by 4 percent.

## 8. A Compensated Shortening of Hours of Men and Machines Alike

Now consider an increase in the hourly real wage rate in disguise: shorten the hours $h$ for men and machines alike but offer labor a compensation for lost annual real income by letting the hourly real wage rate vary in inverse proportion to hours $h$ :

$$
\frac{w}{P}=\frac{K}{h}
$$

where K is a constant. The production function is still (1). With a real wage rate $w / P$ and hours $h$ given to it by collective agreement, the ith firm will again be hiring man-hours until in real terms the last man-hour costs as much as it contributes. As before, then:

$$
\begin{equation*}
\frac{w}{P}=\frac{K}{h}=\frac{\partial X_{i}}{\partial\left(h L_{i}\right)}=\frac{1}{h} \frac{\partial X_{i}}{\partial L_{i}}=a \alpha L_{i} \alpha-I_{S}{ }_{i}^{\beta} \tag{7}
\end{equation*}
$$

from the right-hand side, but not from the left-hand side, of which hours $h$ have disappeared. Number of men $L_{i}$ is raised to the power $\alpha-1=-3$. Raise both sides of (7) to the power $-1 / B$, rearrange, and write number of men demanded as a function of hours of work per man per year h

$$
L_{i}=(a \alpha)^{1 / \beta_{h}} 1 / \beta_{K}-1 / \beta_{S}
$$

easily aggregated into

$$
\begin{equation*}
\mathrm{L}=(\mathrm{a} \alpha)^{1 / \beta_{h}} \mathrm{~h}^{1 / \beta_{K}-1 / \beta_{S}} \tag{8}
\end{equation*}
$$

whose elasticity with respect to hours of work per man per year is

$$
\begin{equation*}
\frac{\partial \log _{e} L}{\partial \log _{e} h}=\frac{1}{\beta} \tag{9}
\end{equation*}
$$

In other words, if $\beta=1 / 4$ and hours per man year $h$ are shortened by 1 percent the number of men demanded $L$ is down by 4 percent. The elasticities (5) and (9) are equal with opposite signs: the 1 percent compensated shortening of hours has exactly the same effect upon employment as a 1 percent direct increase of the hourly real wage rate. The reason is easy to see: in a linearly homogeneous production function like our (1) physical marginal productivities of factors are known to depend solely upon the factor proportion. Shortening the number of hours $h$ for men and machines alike will affect neither the factor proportion (hS $\left.{ }_{i}\right) /\left(h L_{i}\right)$ nor the physical marginal productivity of labor $\partial X_{i} / \partial\left(h L_{i}\right)$. The only way to raise the latter to equality with the new higher hourly real wage rate is to reduce employment $L_{i}$.
9. A Compensated Shortening of Hours of Men But Not of Machines

Finally consider the possibility that men and machines may work different hours per year, i.e., $h_{L}$ and $h_{S}$, respectively. The production function of the ith firm will then be

$$
\begin{equation*}
x_{i}=a\left(h_{L} L_{i}\right)^{\alpha}\left(h_{S} S_{i}\right)^{\beta} \tag{10}
\end{equation*}
$$

Here the elasticity of physical output $X_{i}$ with respect to hours $h_{L}$ per man per year is $\alpha$. The elasticity with respect to number of men $\mathrm{L}_{\mathrm{i}}$ is also $\alpha$.

With this possibility we may consider an increase in the hourly real wage rate in a different disguise: shorten the hours $h_{L}$ for men but not the hours $h_{S}$ for machines and, as before, offer labor a compensation for lost annual real income by letting the hourly real wage rate vary in inverse proportion to hours $h_{L}$ :

$$
\begin{equation*}
\frac{w}{p}=\frac{K}{h_{L}} \tag{11}
\end{equation*}
$$

where $K$ is a constant. The new production function is (10).

With a real wage rate $\mathrm{w} / \mathrm{P}$ and hours $h_{\mathrm{L}}$ given to it by collective agreement, the ith firm will still be hiring man-hours until in real terms the last man-hour costs as much as it contributes. Consequently:

$$
\begin{equation*}
\frac{w}{p}=\frac{K}{h_{L}}=\frac{\partial X_{i}}{\partial\left(h_{L} L_{i}\right)}=\frac{1}{h_{L}} \frac{\partial X_{i}}{\partial L_{i}}=a \alpha h_{L}^{\alpha-1} L_{i}^{\alpha-1}\left(h_{S} S_{i}\right)^{\beta} \tag{12}
\end{equation*}
$$

Here number of men $L_{i}$ is raised to the power $\alpha-1=-\beta$. Raise both sides of (12) to the power - $1 / \beta$, rearrange, and write number of men demanded as a function of hours of work per man per year $h_{L}$ :

$$
L_{i}=(a \alpha)^{1 / \beta_{h_{L}}} 1 / \beta-I_{K}-1 / \beta_{h_{S}} S_{i}
$$

easily aggregated into

$$
\begin{equation*}
L=(a \alpha)^{1 / \beta_{h_{L}}} 1 / \beta-1_{K}-1 / \beta_{h_{S}} S \tag{13}
\end{equation*}
$$

whose elasticity with respect to hours of work per man per year is

$$
\begin{equation*}
\frac{\partial \log _{e} L}{\log _{e} h_{L}}=\frac{1}{3}-1 \tag{14}
\end{equation*}
$$

In other words, if $\beta=1 / 4$ and hours per man per year $h_{L}$ are shortened by 1 percent the number of men demanded L is down by only 3 percent. The elasticities (5) and (14) are not equal with opposite signs: the 1 percent compensated shortening of hours no longer has the same effect upon employment as a 1 percent direct increase of the hourly real wage rate. The reason is easy to see: in a linearly homogeneous production function like our (10) physical marginal productivities of factors are known to depend solely upon the factor proportion. Shortening the number of hours $h_{L}$ of men but not the number of hours $h_{S}$ of machines will affect the factor proportion $\left(h_{S} S_{i}\right) /\left(h_{L} L_{i}\right)$ and raise the physical marginal productivity of labor $\partial X_{i} / \partial\left(h_{L} L_{i}\right)$ some of the way towards equality with the new higher hourly real wage rate. Only the rest of the way will require a curtailment of employment $L_{i}$.

Such alleviation of the contractive effect upon employment will require flexible work patterns such as staggering, overtime, Saturday work or the like. The extra costs of such patterns are ignored by the present paper but not by Kraft (1988). But even in our Eavorable case
of ignoring them, 3 percent less men will be employed than before hours were shortened.
10. Conclusion

British and German unions are currently pressing for a fully compensated ("mit vollem Lohnausgleich") shortening of the working week. We have found negative effects of such shortening upon employment and must ask, as Booth-Schiantarelli (1987) do, are unions acting irrationally?

Perhaps they are, or perhaps the explanation is the distinction between insiders and outsiders. Lindbeck and Snower (1986) and Blanchard and Summers (1988) distinguish between "insiders," who are employed hence decision-making, and "outsiders," who are unemployed hence disenfranchised. Facing demand-for-labor functions like our (3), (8), or (13) the decision-making insiders can have a higher real wage rate for themselves by accepting less employment of the disenfranchised outsiders. The resulting unemployment is involuntary to the outsiders but voluntary to the insiders, who accept it with the better conscience the more generous the unemployment insurance benefits [Casson (1984)]--and the more unemployment is perceived to be Keynesian!
$1_{\text {"Whatever }}$ a person saves from his revenue he adds to his capital, and either employs it himself in maintaining an additional number of productive hands, or enables some other person to do so, by lending it to him for an interest...
"What is annually saved is as regularly consumed as what is annually spent, and nearly in the same time too; but it is consumed by a different set of people."
"There is ... no danger that ... accumulated capital ... would not find employment. ... There are always to be found in a great country, a sufficient number of responsible persons, with the requisite skill, ready to employ the accumulated capital of others, and to pay them a share of the profits, and which, in all countries, is known by the name of interest for borrowed money."

## REFERENCES

Blanchard, Oliver J., and Sumers, Lawrence H., "Hysteresis and the European Unemployment Problem," in Cross, Rod (ed.) Unemployment, Hysteresis and the Natural Rate Hypothesis, Oxford: Blackwell, 1988.

Booth, Alison, and Schiantarelli, Fabio, "The Employment Effects of a Shorter Working Week," Economica, May 1987, 54, 237-248.

Casson, Mark, Economics of Unemployment, Cambridge, Mass.: MIT Press, 1984.

Forster, Leonard, German Verse, Baltimore: Penguin Books, 1957.

Kraft, Kornelius, "Die Effekte einer Arbeitszeitverkürzung bei Berücksichtigung von Fixkosten und Überstunden," Jahrbücher für Nationalökonomie und Statistik, Apr. 1988, 204, 316-325.

Lindbeck, Assar, and Snower, Dennis J., 'Wage Setting, Unemployment, and Insider-Outsider Relations," Amer. Econ. Rev., May 1986, 76, 235-239.

Maddison, Angus, "Growth and Slowdown in Advanced Capitalist Economies: Techniques of Ruantitative Assessinent," J. Econ. Lit., June 1987, 25, 649-698.

Ricardo, David, Pamphlets and Papers, $1815-1823$ reprinted in The Works and Correspondence of David Richardo, P. Sraffa and M. H. Dobb (eds.), vol. IV, New York: Cambridge U. Press, 1951.

Smith, Adam, An Inquiry into the Nature and Causes of the Wealth of Nations, London: W. Strahan, 1776, "new" edition, Glasgow: R. Chapman, 1805 .

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