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# Hydraulic Engineering 

A Practical Treatise on the PRINCIPLES OF WATER PRESSURE AND FLOW AND THEIR APPLICATION TO.<br>THE DEVELOPMENT OF WATER POWER, INCLUDING THE CALCULATION, DESIGN, AND CONSTRUCTION OF WATER WHEELS, TURBINES, AND OTHER DETAILS OF<br>HYDRAULIC POWER PLANTS

## PART I-HYDRAULICS

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## ILLUSTRATED

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## Foreword

ONE of the most striking features of modern engineering development is the extent to which rapids, waterfalls, and other natural resources of water power formerly allowed to run to waste are being harnessed and turned to economic use. Water power is fast supplanting steam for the generation of electric current, especially for long-distance transmission. The immense works that have been carried out and are still in process of development around Niagara Falls, on the slopes of the Western Cordillera, and elsewhere, are among the most remarkable engineering achievements that the world has ever seen. The importance of these essentially modern developments, commercially and industrially, as well as from the engineering point of view, is of itself sufficient to indicate the necessity for an authoritative treatise on the principles and mechanical details embodied in their construction and as developed under the most advanced modern conditions. It is the purpose of the present volume to fill this acknowledged need.
(1. After explaining the fundamental principles of water flow and pressure, this volume takes up their various applications to the development and utilization of water power as based on the latest experience in the calculation, design, construction, and installation of water wheels and turbines, flumes, penstocks, and other details of hydraulic and hydro-electric power plants.

1. Special stress is laid on the practical as distinguished from the merely theoretical or descriptive form of treatment, so that the work will be found especially adapted for purposes of selfinstruction and home study. It is designed not only to meet the requirements of a manual of practical instruction for the beginner, but also to serve as a reference work replete with information and suggestions of the utmost practical value to the most advanced and experienced engineer.
(1. The method adopted in the preparation of this volume is that which the American School of Correspondence has developed and employed so successfully for many years. It is not an experiment, but has stood the severest of all tests - that of practical use-which has demonstrated it to be the best method yet devised for the education of the busy workingman.
(1. For purposes of ready reference, and timely information when needed, it is believed that this volume will be found to meet every requirement.


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## HYDRAULICS.

1. Hydraulics is that branch of Mechanics which treats of the laws governing the pressure and motion of water. Hydrostatics is that particular branch of hydraulics which treats of water at rest, and hydrodynamics is that branch which treats of water in motion.
2. Units of Measure. The unit of length most frequently used in hydraulics is the foot. The unit of volume is the cubic foot or the United States gallon. The unit of time usually employed in hydraulic formulas is the second, but in many water-supply problems the minute, the hour, and the day are also often used. The unit of weight is the pound, and that of energy the foot-pound.

1 U.S. gallon $=231$ cubic inches $=0.1337$ cubic foot;
1 cubic foot $=7.481 \mathrm{U}$. S. gallons;
1.2 U. S. gallons $=1$ Imperial gallon.
3. Weight of Water. The weight of distilled water at different temperatures is given in Table No. 1.

The weight of ordinary water is greater than that of distilled water on account of the impurities contained. For ordinary purposes the weight of a cubic foot of fresh water may be taken equal to 62.5 pounds. Sea water will weigh about 64 pounds per cubic foot.

TABLE NO. 1.
Weight of Distilled Water.

| Temperature, <br> Fahrenheit. | Weight, Pounds <br> per Cubic Foot. | Temperature, <br> Fahrenheit. | Weight, Pounds <br> per Cublc Foot. |
| :---: | :---: | :---: | :---: |
|  | 62.42 | $140^{\circ}$ | 61.39 |
| 39.3 | 62.424 | 160 | 61.01 |
| 60 | 62.37 | 180 | 60.59 |
| 80 | 62.22 | 200 | 60.14 |
| 100 | 62.00 | 212 | 59.84 |
| 120 | 61.72 |  |  |

As will be seen from this table, water is heaviest at a temperature of about $39.3^{\circ} \mathrm{F}$., or as is commonly stated, about $40^{\circ} \mathrm{F}$.
4. Atmospheric Pressure. As has already been explained in the papers on Elementary Mechanics, the atmosphere everywhere exerts a pressure upon all objects uniform in every direction, and is itself compressed to the same degree. At sea level the average pressure of the atmosphere is sufficient to balance a column of mercury in a closed tube (a barometer) about 30 inches high, which is equivalent to a pressure of 14.7 pounds per square inch. A corresponding water barometer would be 34 feet high, the weight of water being much less than that of


Fig. 1: mercury. At points higher than sea level the air pressure is less, and hence the height to which a mercury or water barometer will be raised will be less. Since we depend upon air pressure to raise water into "suction" pipes it is important to know how much this pressure is when designing such pipes.

The following table gives, for dif. ferent elevations above sea level, the pressure of the atmosphere, expressed, first, in pounds per square inch, second, in the height of the mercury barometer and, third, in the height of the water barometer:

TABLE NO. 2.
Atmospheric Pressure at Different Elevations.

| Ele vation <br> above Sea Level. <br> Feet. | Pressure in <br> Pounds per <br> Square Inch. | Height of <br> Mercury <br> Barometer. <br> Inches. | Height of <br> Water <br> Barometer. <br> Feet. |
| :---: | :---: | :---: | :---: |
| 0 | 14.7 | 30.00 | 34.0 |
| 500 | 14.5 | 29.47 | 33.3 |
| 1,000 | 14.2 | 28.94 | 32.8 |
| 2,000 | 13.7 | 27.92 | 31.6 |
| 4,000 | 12.7 | 25.98 | 29.4 |
| 6,000 | 11.8 | 24.18 | 27.4 |
| 8,000 | 11.0 | 22.50 | 25.5 |
| 10,000 | 10.3 | 20.93 | 23.7 |

## PRESSURE OF WATER-AT REST.

5. Transmission of Pressure. If AB , Fig. 1, be a tight vessel containing water, and a close fitting piston C be heavily loaded with a weight $P$ the entire body of water will be subjected to a pressure corresponding to the weight $P$. The water will not be compressed into a smaller space as would a gas likeair, because water is almost incompressible, but whatever pressure is exerted by the weight P will be transmitted through the water equally in all directions so that the pressure of the water against the walls of the vessel will be the same per square inch as that of the weight $P$ upon the water (neglecting the small effect of the weight of the


Fig. 2.
water in the vessel). Thus if the area of the piston $=10$ square inches and the weight $\mathrm{P}=1,000$ pounds, the pressure per square inch will be 100 pounds, and this will be the pressure in every part of the liquid and upon the walls of the vessel, Furthermore, the pressure of the water upon the walls of the vessel is perpendicular to the surface at all points. The pressure at $a$ is upwards, on the bottom of the vessel it is downwards and on the sides it is horizontal.

As a further illustration of the foregoing principle, let B and C, Fig. 2, be two vessels connected by a pipe, and let P be a loaded piston exerting a heavy pressure in the small vessel $B$. In accordance with the principle above stated, this pressure will be transmitted equally to the larger vessel where the water will exert the same pressure per square inch upon the vessel and upon any piston $W$ which may be inserted in any opening of the vessel C. By making the area of P small and of W large a small load P will balance a large load W.

If $a$ is the area of the piston P and A that of the piston W , then the pressure per square inch produced by the weight $P$ will be $\frac{\mathrm{P}}{a}$. This will also be the pressure per square inch on W , and


Fig. 3.
hence the weight $W$ which will be sustained will be equal to the area A multiplied by the pressure per square inch, $\frac{\mathrm{P}}{a}$, or

$$
\begin{equation*}
\mathrm{W}=\mathrm{A} \frac{\mathrm{P}}{a} \tag{I}
\end{equation*}
$$

The principle above stated is utilized in the hydraulic press shown in Fig. 3. In this apparatus a pump on the right with
small plunger feeds a large plunger $p$ underneath the movable plate of the press on the left. The pump planger corresponds to the piston P in Fig. 2, and the press to the piston W. By making the pump very small and the plunger under the press very large, enormous pressures can be exerted even by means of a hand punp. The pressure produced is given by formula (1) above. It is to be noted that the pressure per square inch on the interior of the apparatus, the pump, piping and press, is the same at all points.

Examples. 1. If the area of the pump plunger be 2 sq . in. and that of the press 1 sq . ft ., what pressure will be exerted by the press when the load on the pump is 100 lb ?

Using equation 1 we have $a=2 \mathrm{sq}$. in., $\mathrm{A}=144 \mathrm{sq}$. in., and $P=100 \mathrm{lb}$., whence $\mathrm{W}=144 \times \frac{100}{2}=7,200 \mathrm{lb}$. Ans.
2. If a pressure of 10 tons be desired and the area of the press plunger be 200 sq . in., and the available pressure on the pump plunger be 150 lb ., what area must be given to the pump plunger ?

Here $W=10 \times 2,000=20,000 \mathrm{lb} ., \mathrm{A}=200$ and $\mathrm{P}=150$. Using equation 1 and letting $x=$ desired area, we have $20,000=$ $\frac{200 \times 150 .}{x}$ Solving for $x$ we have $x=\frac{200 \times 150}{20,000}$
$=1.5 \mathrm{sq} . \mathrm{in}$. Ans.
6. Pressure Due to the Weight of Water. Let Fig. 4 represent a vessel of water. Consider a vertical column of the water of height $h$ and a cross-section of one square foot. Its volume will be $h$ cubic feet and it will weigh $62.5 \times h$ pounds. As it is supported entirely by the water underneath, it therefore exerts a pressure upon that water of $62.5 \times h$ pounds. Likewise the pressure at any other point in the vessel at a distance $h$ below the surface is $62.5 \times h$ pounds per square


Fig. 4. foot. Furthermore, since the water exerts equal pressures in all directions it follows that the pressure against the sides of the vessel at this depth, or against any object immersed in the water, will also be $62.5 \times h$ pounds per square foot.

Since the weight of water is so nearly constant we may conveniently use the depth $h$ as a measure of the pressure. When so used it is called the pressure head or simply the "head " acting on the given surface. For each foot of head the pressure will be 62.5 pounds per square foot, but in expressing pressure in pounds it is customary to use the square inch. A pressure of 62.5 pounds per square foot being equal to $\frac{62.5}{144}$ or .434 pounds per square inch, it follows that one foot of head gives a pressure of . 434 pounds per square inch. Conversely, a pressure of one pound per square inch requires a head of $\frac{1}{.434}$ or 2.304 feet.

Rule. To convert feet of head to pounds pressure multiply by .434. To convert pounds pressure to feet of head multiply by 2.304.
Examples. 1. What will be the pressure per square inch in the vessel of Fig. 5 at a point $a 10$ feet below the water surface ? Assume the vessel to be round with a diameter of bottom $=6$ feet and of upper part $=2$ feet.

Here the head is 10 feet, and by the above rule the pressure per square inch $=10 \times .434=4.34$ pounds. It acts equally in all directions and is independent of the shape of the vessel.
2. What will be the total pressure on the bottom of the vessel ?

The area of the bottom in sq. ft. $=\frac{3.14 \times 6^{2}}{4}=28.26 \mathrm{sq} . \mathrm{ft}$. The head is 14 feet and hence the pressure per sq. $\mathrm{ft} .=14 \times 62.5$ $=875$ pounds. The total pressure on the bottom $=875 \times 28.26$ $=24,728$ pounds.
3. What will be the total upward pressure on the portion AB ?

The area of this portion is the difference between the two circles respectively 6 feet and 2 feet in diameter. This is equal to $\frac{\left(6^{2}-2^{2}\right) \times 3.14}{4}=25.12 \mathrm{sq} . \mathrm{ft}$. The head is 8 feet and the pressure, therefore, $8 \times 62.5=500$ pounds per sq. ft. Total upward pressure $=500 \times 25.12=12,560$ pounds.
4. What is the entire weight of water in the vessel ?


## NEW CROTON DAM UNDER CONSTRUCTION

Largest masonry dam in the world, part of the waterworks system of New York City. Took 14 years to build, requiring about $850,000 \mathrm{cu}$. yds. of masonry. Completed in 1906. Cost, $87,700,000$. Length, 2,400 feet; height, 301 feet; thickness, 216 feet at base, tapering to ten feet at top of spillway (at left) and 21 feet at top of main dam. Capacity, $30,000,000,000$ gals., and with auxiliary dams, $100,000,000,000$ gals. Water at dam, 160 feet deep, the impounded river forming a lake 20 miles long and 2 miles in extreme width, burying under 30 feet of water the old dam 3 miles upstream.

CURVE IN CHICAGO DRAINAGE CANAL NEAR ROMEO, ILLINOIS
The white streak along the bank is all stone excavated from the channel. The limestone taken out in the construction of this waterway, which
is practically a 25 -mile extension of the harbor of Chicago, has a market value, for concreting, paving, etc., of about $\$ 30,000,000$.

The volume of the lower part of the vessel $=\frac{6 \times 6^{2} \times 3.14}{4}$ $169.56 \mathrm{cu} . \mathrm{ft}$. , and of the upper part $=\frac{8 \times 2^{2} \times 3.14}{4}=25.12$ cu. feet. Total volume $=194.68 \mathrm{cu} . \mathrm{ft}$., and weight of water $=$ $194.68 \times 62.5=12,167$ pounds.

Note that the difference between the downward pressure on the bottom and the upward pressure on $\mathrm{AB}=12,168 \mathrm{lbs}$., which is equal to the total weight of the water, or the net pressure of the vessel upon-its support, if we neglect the weight of the vessel itself.


Fig. 5.


Fig. 6.
7. Pressure of Water upon Plane Areas in General. In the preceding articles it has been shown that the pressure per square inch upon any submerged body is equal to $.434 h$ where $h$ is the head in feet; furthermore, that this pressure is at right angles to the surface of the body. Let Fig. 6 represent a vessel of rectangular shape containing water of a depth $h_{1}$. The pressure on the bottom is then $h_{1} \times .434$ pounds per square inch. If the area of the bottom be $\mathrm{A}(=b d)$, then the total pressure on the bottom is $\Lambda \times h_{1} \times .434$ pounds. In this case the pressure is the same per square inch at all points of the surface considered.

Consider now the pressure on one of the sides, as BC. In this case the pressure per square inch is not uniform, varying from nothing at B to a maximum at C where it is equal to $h_{1} \times$ .434 pounds per square inch, the same as on the bottom. At any depth $h$ the pressure is $h \times .434$ pounds per square inch. This variation in pressure is represented in Fig. 6 by the variation in length of the arrows acting against BC. From an inspection of the figure it is evident that the average length of these arrows is equal to one-half the length of the one at the bottom, or in other words, the average pressure per square inch against $B C$ is equal to one-half the maximum, or $\frac{1}{2} \ell_{1} \times .434$, which is the same as the pressure at the center of BC. The total pressure on the entire surface is then equal to this average pressure multiplied by the total area, or equal to $\frac{1}{2} h_{1} \times .434 \times h_{1} d$.

If the area in question be a plate $\mathrm{B}^{\prime} \mathrm{C}^{\prime}$ immersed in the water to a depth $h_{2}$ the result is the same, except in this case there is an equal pressure on each side. As before, the pressure on either side of the plate is equal to $\frac{1}{2} h_{2} \times .434 \times$ (area of submerged portion of plate).

If the plate be wholly submerged, as BC, Fig. 7, the pressure per square inch at B will be $h_{1} \times .434$, and that at C will be $h_{2} \times .434$, and the variation in pressure will be represented by a trapezoid of arrows instead of a triangle. The average pressure will now be $\frac{h_{1}+h_{2}}{2}$ which is again the same as the pressure at the center of BC. The total pressure will be this average pressure multiplied by the area of the plate.

In all the above cases it will be seen that the average pressure found is the same as the pressure at the center of the plate. In a similar way it can be shown that for plates of any shape the average pressure is equal to the pressure at the center of gravity of the area, hence the following:

Rule. The total pressure on a submerged vertical plane surface is equal to the pressure per unit area at its center of gravity multiplied by its area.
Suppose now the plate BC, Fig. 8, be an inclined plate immersed in water. From the principles already explained the
pressure per square inch will be the same at any given depth as if the plate were vertical: Hence at $B$ the pressure is $h_{1} \times .434$ and that at C is $h_{2} \times .434$. The average pressure is again $\frac{h_{1}+h_{2}}{2}$ $\times .434$, or the pressure at its center, and the total is equal to this pressure multiplied by the area of the plate. Whence the more general rule,--

Rule. The total pressure on any submerged plane surface is equal to the pressure per unit area at its center of gravity multiplied
by its area. Such pressure always acts at right angles to the surface.
8. Pressure in a Given Direction. In the above discussion we have considered only the total pressure of the water, which always acts perpendicular to the surface of the body. In Fig. 9 let $P$ represent this total pressure on the surface BC, which has a

length $l$ and a width $d$ (its area equals $l d$ ). Suppose it is desired to find the horizontal and vertical components $\mathrm{P}_{\mathrm{h}}$ and $\mathrm{P}_{\mathrm{v}}$ of this pressure. Since $P$ is perpendicular to $B C$ the inclination of $P$ from the horizontal is the same as that of BC from the vertical. Call this angle $\theta$. From Mechanics we have at once, $\mathrm{P}_{\mathrm{h}}=\mathrm{P}$ cos $\theta$ and $\mathrm{P}_{\mathrm{v}}=\mathrm{P} \sin \theta \quad$ From the foregoing articles we also have $\mathrm{P}=h \times .434 \times l d$, in which $h$ is the depth of the center of gravity of BC. Hence we have $\mathrm{P}_{\mathrm{h}}=\mathrm{P} \cos \theta=.434 h \times \cos \theta$ $\times l d$, and $\mathrm{P}_{\mathrm{v}}=\mathrm{P} \sin \theta=.434 h \times \sin \theta \times l d$. From the figure we see that the area of the vertical projection of the plate $\mathrm{BC}=$ $m \times d=l \cos \theta \times d$, and the horizontal projection $=n \times d=l$ $\sin \theta \times d$. Whence we have $\mathrm{P}_{\mathrm{h}}=.434 \hbar \times m d$ and $\mathrm{P}_{\mathrm{v}}=.434$
$h \times n d$. That is, $\mathrm{P}_{\mathrm{h}}=.434 k \times$ (vertical projection of plate $)$, and $\mathrm{P}_{\mathrm{v}}=.434 h \dot{\times}$ (horizontal projection of plate). Whence the general

Rule. The horizontal component of the pressure on a plate is equal to the pressure per square inch at its center of gravity multiplied by the area of its vertical projection, and the vertical component of the pressure is equal to the pressure at its center of gravity multiplied bg its horizontal projection.
Examples. 1. What will be the horizontal and vertical components of the pressures on a plate, BC, as in Fig. 9, which is inclined at an angle of $10^{\circ}$ to the vertical, the length $l$ of the plate

being 30 in . and the width 10 in ., and the center being 2 feet below the water surface.

Here the pressure per sq. in. at the center is that due to a head of 2 ft ., or is equal to $2 \times .434=.868 \mathrm{lb}$. per sq. in. The vertical projection of the plate is equal to $30 \times \cos 10^{\circ}$ and its horizontal projection $=30 \times \sin 10^{\circ}$. Cos. $10^{\circ}=.985$ and $\sin 10^{\circ}$ $=.174$, hence by rule 5 the required horizontal component $=$ $.868 \times 30 \times .985 \times 10=256 \mathrm{lb}$., and the vertical component $=.868 \times 30 \times .174 \times 10=45 \mathrm{lb}$. Ans.
2. Required the horizontal and vertical components of the pressures on the three faces of the wedge shown in Fig. 10, the length of the wedge perpendicular to the paper being 12 in .

Face BC. The depth of the center of BC below the surface is evidently 16 in . The pressure per sq. in. at this depth $=\frac{16}{12}$ $\times .434=.579 \mathrm{lb}$. The vertical projection of $\mathrm{BC}=20 \times 12$
$=240 \mathrm{sq}$. in., and its horizontal projection $=4 \times 12=48 \mathrm{sq}$. in., whence the desired components are: Horizontal component $=240 \times .579=13.89 \mathrm{lb}$., vertical component $=48 \times .579$ $=27.8 \mathrm{lb}$.

Face BD. The pressures are the same as on BC, the horizontal component acting towards the left and the vertical component acting downwards. The total downward pressure $=2 \times 27.8$ $=55.6 \mathrm{lb}$.

Face CD. The pressure per sq. in. at this depth $=.434 \times \frac{26}{12}$ $=.940 \mathrm{lbs}$. Total upward pressure $=8 \times 12 \times .940=90.2 \mathrm{lb}$.
9. Pressure on Curved Surfaces. If we are dealing with a curved surface as BC, Fig. 11, the pressure is still at all points normal to the surface, but the varying direction of the pressures makes it difficult to determine readily the resultant pressure. The results of the preceding article will, however, enable us to solve the problem sufficiently accurate for all purposes. Suppose the pressure on each square inch be resolved into vertical and


Fig. 11. horizontal components. Each of these components will equal the normal pressure at the center of the square inch multiplied by the horizontal or vertical projection of the inch of area. Adding all together we find that the total horizontal pressure will equal a certain average horizontal pressure multiplied by the vertical projection of the entire area, and the vertical pressure will equal a certain average vertical pressure multiplied by the horizontal projection. It can be shown that the average value of the horizontal pressure is equal to the pressure at the center of gravity of the vertical projection, but the average value of the vertical pressure cannot be readily determined with accuracy. It may always be estimated by taking as near as may be a pressure corresponding to the average depth of the area below the water surface. Where the body is submerged a great distance, or is under a great pressure in a closed vessel, the error will be unimportant.
10. Bursting Pressure in Pipes and Cylinders. Let BECD, Fig. 12, be the cross-section of any pipe of diameter $d$ and length $l$ and containing water under a head $h$. The figure shows the pipe connected to an open vessel with water standing at a height $l l$ above the center. This free surface of water may represent a reservoir at a height $h$ above the pipe, or the pipe may be entirely closed and the pressure head $h$ exerted upon the water by means of a force pump or a pumping engine. The pressure per square inch at the center of the pipe will be $h \times .434$ pounds. The pressure against the pipe BEDC will be perpendicular to the surface at all points, and if the diameter is small compared to the height 7, this pressure will be practically the same at all points and equal to $h \times .434$ pounds per square inch.

Suppose we wish to find the total hori-


Fig. 12. zontal force acting against the half BDC. By the foregoing article we may consider the pressure on its vertical projection BC. The center of gravity of this vertical projection will be at the center of the pipe and the pressure per square inch at that point will be $h \times .434$. The area of the projection BC is equal to $d \times 7$. Hence the total horizontal pressure against BDC will equal $h \times .434 \times d l$. The pressure against the side BEC will be the same, but opposite in direction.

The action of the pressures on BDC and BEC tends to burst the pipe at points $B$ and C. This is resisted by the stress in the pipe, the amount of which at each of these points is one-half the total horizontal pressure on BDC or BEC, or equal to $\frac{1}{i} \pi \times .434 \times \mathrm{dl}$. If we consider a length of pipe of only one inch then $l=1$ and we have the important formula for the bursting stress in a pipe :

$$
\begin{equation*}
\mathrm{S}=\frac{1}{2} \pi \times .434 \times d \tag{6}
\end{equation*}
$$

in which $\quad S=$ stress per lineal inch of pipe

$$
h=\text { head of water in feet }
$$

and

$$
d=\text { diameter of pipe in inches. }
$$

By expressing the pressure-head in pounds per square inch instead of feet head we have

$$
\begin{equation*}
\mathrm{S}=\frac{p d}{2} \tag{7}
\end{equation*}
$$

in which $p=$ pressure per square inch at center of pipe.
If $t=$ thickness of pipe in inches and $s=$ stress on the metal per square inch then

$$
\begin{equation*}
s=\frac{p d}{2 t} \tag{8}
\end{equation*}
$$

For large pipes and low heads the stress at C will be a little larger than at $B$.
11. Longitudinal Stress in Closed Pipes and Cylinders. Let Fig. 13 represent a side view of a short pipe or cylinder, closéd at


Fig. 13.


Fig. 14.
the ends like a steam boiler and containing water under a pressure $p$ per square inch. Consider the portion to the left of a section BC. (See Fig. 14a.) The cross-section of the cylinder at BC will be a circle of diameter $d$ on which there will be a stress S due to the horizontal water pressure on the end of the cylinder at D. This total horizontal pressure may be found as in the preceding article. It is equal to the average pressure $p$ multiplied by the vertical projection of the area of the end. This projection, Fig. 14b, is equal to the area of the circle of diameter $d$, or to $\frac{1}{4} \pi l^{2}$. Hence the total horizontal force is $p \times \frac{1}{4} \pi d^{2}$, and hence

$$
\text { Total stress }=p \times \frac{1}{4} \pi d^{2}
$$

This stress is distributed entirely around the circumference of the cylinder, or over a distance equal to $\pi d$. The stress per inch of circumference is then equal to

$$
\begin{equation*}
\mathrm{S}^{\prime}=\frac{p \times \frac{1}{4} \pi d^{2}}{\pi d}=\frac{p d}{4} \tag{9}
\end{equation*}
$$

This is seen to be just one-half of the stress in a circumferential direction, as given by formula 7 .

If $t$ equal thickness of cylinder, then the horizontal stress per square inch of metal is

$$
\begin{equation*}
s^{\prime}=\frac{p u l}{4 t} \tag{10}
\end{equation*}
$$

Examples 1. What will be the stress per lineal inch in a pipe 30 in . in diameter under a water pressure of 40 feet?

The pressure per lineal inch is equal to, by equation $6, \frac{1}{2} \times 40$ $\times .434 \times 30=260.4 \mathrm{lb}$. Ans.
2. If the safe strength of the metal of a pipe in example 1 is $2,000 \mathrm{lb}$. per sq. in., what will be the necessary thickness of the pipe wall?

The stress per lineal inch is 260.4 lb ., and if the safe stress is $2,000 \mathrm{lb}$. per sq. in., the necessary thickness will be equal to $260.4 \div 2,000=.130$ inch. Ans.

## EXAMPLES FOR PRACTICE.

1. What is the bursting stress per square inch in a pipe $\frac{1}{2}$ inch thick and 4 feet in diameter under a pressure head of 400 feet? 8,330 lb. Ans.
2. What is the stress per square inch in a boiler plate 1 inch thick, the boiler being 6 feet in diameter working under a pressure of 150 lb . per sq. in. (Use equation 8.) 5,400 lb. Ans.
3. What is the horizontal stress per sq. in. in the boiler of example 2? 2,700 lb. Ans.
4. Center of Pressure on Rectangular Areas. In the preceding discussion of Arts. 7 and 8 the total pressure was the quantity determined. In the case of the plate reaching to the surface, Fig, 6, the variation in the pressure was represented by a triangle of forces, and where the plate was wholly submerged, Fig. 7, it was represented by a trapezoid. In either case the "center of pressure," or the point where the resultant of the pressure forces would be applied, will be opposite the center of gravity of the
pressure area. In the case of the triangle the center of gravity is two-thirds the distance from the apex to the base, hence,

> The center of pressure against a rectangular plate which reaches to the surface, or projects above it, is two-thirds the distance from the surface to the lower edye of the plate.

In the case of the wholly submerged plate, with trapezoidal pressure diagram BC, Fig. 15, the pressure head at B is equal to $h_{1}$ and that at $\mathrm{C} h_{2}$, which heads will be equal to the heights BD and CE of the trapezoid representing the pressures.

By the method explained in the paper on Strength of Materials, Art. 48, we find the center of gravity of the trapezoid of length $l$ to be at a distance from $B$ equal to

$$
\begin{equation*}
x=\frac{2}{3} l \frac{\frac{h_{1}}{2}+h_{2}}{h_{1}+h_{2}} \tag{12}
\end{equation*}
$$

Another form of expression can be obtained by noting that if the point $A$ be the intersection of the plane BC, produced, with the surface of the water, the line DE, produced, will also pass through $A$, since a plate $A C$ would have a triangle of pressures


Fig. 15.
represented by AEC. Then by proportion we would have $\frac{h_{1}}{h_{2}}=\frac{\mathrm{AB}}{\mathrm{AC}}$, or $h_{1}=h_{2} \frac{\mathrm{AB}}{\mathrm{AC}}$. Substituting this value for $h_{1}$ in equation 12 , and reducing, we have

$$
\begin{equation*}
x=\frac{2}{3} l \frac{\frac{\mathrm{AB}}{2}+\mathrm{AC}}{\frac{\mathrm{AB}+\mathrm{AC}}{}} \tag{13}
\end{equation*}
$$

13. Center of Pressure on Plane Areas of Any Form. The center of pressure of irregular plane areas can be found by the following rule, the demonstration of which is here omitted. Let BC, Fig. 15, represent a plane area of any form, then

> The distance $A F$ from the surface to the center of pressure is equal to the moment of inertia of the given area about an axis at $A$ divided by the product of the area times the distance from $A$ to its center of gravity.

Examples. 1. What force $S$ will be required to lift a sluice gate BC, Fig. 16, placed on the sloping face of a dam and hinged at B? The gate is 3 feet wide, 4 feet long from B to C , and has such a slope that the vertical projection $\mathrm{BD}=3.5 \mathrm{ft}$., and the


Fig. 16.
horizontal projection CD $=1.93 \mathrm{ft}$. The depth of $B$ below the surface is 10 ft .

We will first find the total pressure $P$ against the gate. By Art. 7 this will be the pressure per sq. in. at the center multiplied by the area. The depth of the center is $10+\frac{1}{2} \mathrm{BD}=10+\frac{3 . \overline{0}}{2}$ $=11.75$ feet. The pressure per sq. ft. $=11.75 \times 62.5=734$ lb. The total pressure $=734 \times 3 \times 4=8,808 \mathrm{lb}$.

The center of pressure will be found next. This is at a distance from B given by formula 12 , in which $h_{1}=10$ and $h_{2}$ 10
$=13.5$. We have then $x=\frac{2}{3} \times 4 \times \frac{\bar{z}+13.5}{10+13.5}=2.1$ feet.
Now taking moments about B we have $\mathrm{S} \times 1.93=\mathrm{P} \times 2.1$ or $S=8,808 \times \frac{2.1}{1.93}=9,590 \mathrm{lb}$. Ans.
2. Find the water pressure on a gate AB, Fig. 17, one foot long, when the heads on the two sides are different; also find the reactions $R_{1}$ and $R_{2}$ of the gate against sills at $A$ and $B$.


Fig. 17.
By Art. 7 the total pressure $P_{1}$ of the water on the left side of the gate is equal to the pressure at the half depth $\frac{h_{1}}{2}$ multiplied by its submerged area. Taking here the square foot as the unit and letting $w=$ weight of a cubic foot of water, the pressure at a depth $\frac{h_{1}}{2}$ is $\frac{l_{1}}{2} \times w$ pounds per square foot, and as the exposed area is $h_{1} \times 1$ the total pressure $\mathrm{P}_{1}=\frac{h_{1}}{2} \times w \times h_{1}=\frac{1}{2} h_{1}^{2} \times w$. The center of pressure, or point of application of $\mathrm{P}_{1}$, is $\frac{2}{3} /_{1}$ below the surface (Art. 12).

In like manner the pressure $P_{\dot{2}}^{\dot{2}}=\frac{1}{2} h_{2}^{2} w$, and its point of application is $\frac{2}{3} h_{2}$ below the water surface on that side.

The forces $P_{1}$ and $P_{2}$ being known, the reaction $R_{1}$ may be found by taking moments about $B$ as explained in Mechanics.

There results the equation

$$
\mathrm{R}_{1} \times d-\mathrm{P}_{1} \frac{h_{1}}{3}+\mathrm{P}_{2} \frac{h_{2}}{3}=0
$$

whence

$$
\mathrm{R}_{1}=\frac{1}{3}-\frac{\mathrm{P}_{1} h_{1}+\mathrm{P}_{2} h_{2}}{d}
$$

Substituting the values of $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ above given, we have

$$
\begin{align*}
\mathrm{R}_{1}= & \frac{1}{3} \times w \times \frac{1}{2} \frac{l_{1}^{3}+l_{2}^{3}}{d} \\
& =\frac{1}{6} w \frac{h_{1}^{3}+h_{2}^{3}}{d} \tag{15}
\end{align*}
$$



Fig. 18.
in which all dimensions are to be expressed in feet, and the result will be for a gate one foot long. For other lengths the value of $\mathrm{R}_{1}$ will be proportional to the length.
3. Find the pressure P on a dam AB , Fig. 18. Let $h=$ depth of water against the dam. Consider a length of dam of one foot. By Art. 7 the total pressure $P$ is equal to the pressure at the half depth multiplied by the area of AB or

$$
\mathrm{P}=\frac{h}{2} \times w \times(\text { length of } \mathrm{AB}) \times 1
$$

The center of pressure, by Art. 12 , is two-thirds of the distance from $A$ to $B$.

The horizontal component of the pressure is, by Art. 8, equal to the pressure per square foot at mid-depth multiplied by the vertical projection of the face AB , or

$$
\begin{equation*}
\mathrm{P}_{\mathrm{h}}=\frac{h}{2} \times w \times h \times 1=\frac{1}{2} w h^{2} \tag{16}
\end{equation*}
$$



Fig. 19.
The vertical component is likewise

$$
\mathrm{P}_{\mathrm{v}}=\frac{h}{2} \times w \times(\text { length } \mathrm{BD})
$$

but we can write $\mathrm{BD}=l_{l} \tan \theta$. Hence

$$
\begin{equation*}
P_{\mathrm{v}}=\frac{1}{2} w h^{2} \tan \theta \tag{17}
\end{equation*}
$$

If the dam is submerged, as shown in Fig. 19, then the method employed in example 1 of this Article must be used.

## EXAMPLES FOR PRACTICE.

1. What is the horizontal pressure on a dam one foot long on which the water has a depth of 80 feet ; and where is the center of pressure? $200,000 \mathrm{lb}$., and 26 ft .8 in . from the bottom. Ans.
2. What is the vertical component of the pressure in example 1 if the face of the dam slopes 1 inch horizontally to 1 foot vertically ? (The horizontal projection $=\frac{1}{12} \times 80=6 \frac{2}{3} \mathrm{ft}$.)
16,670 lb. Ans.
3. In Fig. 19 if $h_{1}=10 \mathrm{ft}$., $h_{2}=40 \mathrm{ft}$., $\mathrm{AD}=30 \mathrm{ft}$., and $\mathrm{BD}=10 \mathrm{ft}$., what will be the horizontal and vertical components
of the pressure P? The center of gravity of the area is 30 ft . deep. Use rule 5.

$$
\text { Hor. comp. }=46,875 \mathrm{lb} . ; \text { Vert. comp. }=15,62 \mathrm{~J} \mathrm{lb} .
$$

Ans.
4. How far from A is the center of pressure in example 3? The length of $\mathrm{AB}=31.62 \mathrm{ft}$. Use equation (12). 18.97 ft . Ans.
14. Buoyant Effect of Water on Submerged Bodies. If a body AB, Fig 20, be submerged, the water exerts an uplift upon it owing to the fact that the pressure upwards on the bottom of the body is greater than the pressure downwards on the top. The net upward force, or buoyant effect, is exactly equal to the weight


Fig. 20.
of a volume of water equal to that of the body $A B$. It is plain that this must be so, for if AB be replaced by water, the water would tend neither to rise nor fall, that is, it would be just suppoirted by the surrounding pressures. Hence the following well-known law:

The weight of a body in water is less than its weight in air by an amount equal to the weight of an equal volume of water.
15. The Specific Gravity of a substance is the ratio of its weight to that of an equal volume of water. The specific gravity " is found by weighing a body in air and then in water. The difference is the weight of an equal volume of water. Then if W equals weight in air; and $W^{\prime}$ equals weight in water, then $W$ $\mathrm{W}^{\prime}=$ weight of water displaced, and

$$
\begin{equation*}
\text { Specific gravity }=\frac{W}{W-W^{\prime}} \tag{18}
\end{equation*}
$$

as explained in Elementary Mechanics.

## EXAMPLES FOR PRACTICE.

1. If a body weighs 100 lb . in air and 40 lb . in water, what is its specific gravity? 1.67. Aus.
2. If a body of .6 cu . ft. in volume weighs 75 lb ., what is its specific gravity, the weight of water being 62.5 lb . per cu. ft.? 2.0. Ans.
3. If a body of $3 \mathrm{cu} . \mathrm{ft}$. in volume has a specific gravity of .75 , what force is necessary to submerge it? Here the buoyant effect is greater than the weight of the body. 46.9 lb . Ans.

## FLOW OF WATER THROUGH ORIFICES.

16. Velocity of Flow Through Orifices. If AB, Fig. 21, be a vessel containing water of depth $h$, and C and D are any open tubes connected therewith, the water will stand in these tubes at


Fig. 21.
the same height $l$ above the base level AB as in the large vessel, and the pressure in the tubes at any given depth is the same as in the large vessel at the same depth. If we now make an opening at E so that the water will issue in a vertical direction it has been experimentally demonstrated that the water will rise very nearly to the same level as it will in the tubes. The discrepancy is due to the air resistance and a slight friction at the opening. Neglecting this discrepancy the velocity of the water at E can be determined on the principle that it must be sufficient to canse the water to rise the distance $h_{1}$. In Mechanics it was shown that, neglecting air resistance, the velocity a body must have to cause it to rise against gravity a distance $h_{1}$ is the same as the velocity acquired by a body falling through the same distance. This velocity is given by formula

$$
v=\sqrt{2 g h}
$$

in which $v=$ velocity in feet per second

$$
\begin{aligned}
g & =\text { acceleration of gravity } \\
& =32.2 \text { feet per second }
\end{aligned}
$$

and $h=$ height of fall, or the height a body will rise when started with a velocity $v$.

Applying this to the jet issuing from E we find that the theoretical velocity of efflux is

$$
v=\sqrt{2 g l_{1}}
$$

If the orifice be in the side of the vessel, as at $F$, on the same level as E , it is plain that the water will issue with the same force as at E , since the pressure is the same. Hence in general:

The theoretical velocity of efflux from an orifice in any direction is

$$
\begin{equation*}
v=\sqrt{2 g h} \tag{19}
\end{equation*}
$$

where $h$ is the pressure head in feet at the orifice.
In practice the velocity is a little less than that given by the formula, the actual velocity being from $97 \%$ to $99 \%$ of the theoretical. This ratio of .97 to .99 is called the coefficient of velocity.

- 17. Use of Orifices for Measuring Water. In making use of an orifice for measuring water it is desirable, for the sake of accuracy, that the orifice be constructed in such a way that the water in passing out will touch the inner edge only. This may be done by making the orifice of a very thin plate, or cutting it on a bevel so that the water will not come in contact with the side, as shown in Fig. 22. To get accurate results an orifice should be made of metal, such as brass, and fastened to the inside of the tank as in Fig. 22, but in many cases sufficiently accurate results can be obtained by cutting a beveled hole in the side of a tank.

To give reliable results the orifice should be located a distance from the nearest side or the bottom of the tank not less than three times the width of the orifice. The tank or channel should also have a cross-section much larger than that of the orifice so

POWER HOUSE ON CHICAGO DRAINAGE CANAL AT LOCKPORT, ILLINOIS
; in front of it is the forebay, with screens protecting the conduits leading to the
Courtesy of R. Isham Randolph. Ites Power House in backgrou is the arched concrete

PART OF DAM CONSTRUCTION ON CHICAGO DRAINAGE CANAL AT LOCKPORT, ILL.
The temporary dam of stone, clay, and concrete here shown diverts water from the "Butterfly" dam and the new channel, to the "Bear Trap"
dam and the Desplaines River.
that the velocity of the water as it approaches the orifice will be sinall, otherwise the discharge will be affected by this "velocity of approach ". If the cross-section of the tank is as much as twenty times that of the orifice this effect is of no consequence.
18. Discharge Through Small Orifices. When water flows through an orifice, such as shown in Fig. 22, the direction of the flow at the edges is such as to canse the water vein to contract as it issues from the orifice. The area of the contracted vein at its smallest section is only 60 to 70 per cent of the full area of the orifice, the exact value depending upon the size of the orifice, and the pressure. Now the discharge through any orifice, pipe, or channel is equal to the area of the cross-section of the stream of water multiplied by its velocity at that point. If we measure the sross-section in square feet and the velocity in feet per second, then the discharge will be expressed in cubic feet per second. In the case of the orifice, then, to determine the discharge per second we would need to multiply the area of the cross-section of the vein of water by its velocity. In Art. 16 it was shown that the actual velocity of the jet was about 97 to 99 per cent of the theoretical velocity $v$, which refers to the velocity of the vein at the contracted section where the velocity is a maximum. The discharge will then be found by multiplying this actual velocity by the actual area at the point of contraction. Thus if we take the coefficient of velocity as .98 and the coefficient of contraction as .65 , the discharge would be

$$
\mathrm{Q}=.98 v \times .65 \mathrm{~A}
$$

where $\quad Q=$ discharge in cubic feet per second
$v=$ theoretical velocity in feet per second by equation
19, and $\Lambda=$ area of orifice in square feet.
If we substitute for $v$ its value $\sqrt{2 g / h}$ we have

$$
\begin{aligned}
\mathbb{Q} & =.98 \times .65 \mathrm{~A} \sqrt{2 g h} \\
& =.637 \mathrm{~A} \sqrt{2 g h},
\end{aligned}
$$

The coefficient . 637 in this case is called the coefficient of dis. charge, and as it varies with different conditions, it is desirable to use the more general formula

$$
\begin{equation*}
\mathrm{Q}=c \mathrm{~A} \sqrt{2 g h} . \tag{20}
\end{equation*}
$$

in which $c=$ coefficient of discharge, which varies in value from about .66 to .60 . This coefficient is the product of the "coefficient of velocity" and the "coefficient of contraction."

TABLE NO. 3.
Coefficients for Circular Vertical Orifices,

| Head, $h$, in Feet. | Diameter of Orifice in Feet. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.02 | 0.04 | 0.07 | 0.10 | 0.2 | 0.6 | 1.0 |
| 0.4 |  | 0.637 | 0.624 | 0.618 |  |  |  |
| 0.6 | 0.655 | . 630 | . 618 | . 613 | 0.601 | 0.593 |  |
| 0.8 | . 648 | . 626 | . 615 | . 610 | . 601 | . 594 | 0.590 |
| 1.0 | . 744 | . 623 | . 612 | . 608 | . 600 | . 595 | . 591 |
| 1.5 | . 637 | . 618 | . 608 | . 605 | . 600 | . 596 | . 593 |
| 2.0 | . 632 | . 614 | . 607 | . 604 | . 599 | . 597 | . 595 |
| 2.5 | . 629 | . 612 | . 605 | . 603 | . 599 | . 598 | -. 596 |
| 3 | . 627 | . 611 | . 604 | . 603 | . 599 | . 598 | . 597 |
| 4 | . 623 | . 609 | . 603 | . 602 | . 599 | . 597 | . 596 |
| 6 | . 618 | . 607 | . 602 | . 600 | . 598 | . 597 | . 596 |
| 8 | . 614 | . 605 | . 601 | . 600 | . 598 | . 596 | . 596 |
| 10 | . 611 | . 603 | . 599 | . 598 | . 597 | . 596 | . 595 |
| 20 | . 601 | . 599 | . 597 | . 596 | . 596 | . 596 | . 594 |
| 50 | . 596 | . 595 | . 594 | . 594 | . 594 | . 594 | . 593 |
| 100 | . 593 | . 592 | . 592 | . 592 | . 592 | . 592 | . 592 |

TABLE NO. 4.
Coefficients for Square Vertical Orifices.

| Head, $h$, in Feet. | Side of the Square in Feet. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.02 | 0.04 | 0.07 | 0.1 | 0.2 | 0.6 | 1.0 |
| 0.4 |  | 0.643 | 0.628 | 0.621 |  |  |  |
| 0.6 | 0.660 | . 636 | . 623 | . 617 | 0.605 | 0.598 |  |
| 0.8 | . 652 | . 631 | . 620 | . 615 | . 605 | . 600 | 0.597 |
| 1.0 | . 648 | . 628 | . 618 | . 613 | . 605 | . 601 | . 599 |
| 1.5 | . 641 | . 622 | . 614 | . 610 | . 605 | . 602 | . 601 |
| 2.0 | . 637 | . 619 | . 612 | . 608 | . 605 | . 604 | . 602 |
| 2.5 | . 634 | . 617 | . 610 | . 607 | . 605 | . 604 | . 602 |
| 3 | . 632 | 616 | . 609 | . 607 | . 605 | . 604 | . 603 |
| 4 | . 628 | . 614 | . 608 | . 606 | . 605 | . 603 | . 602 |
| 6 | . 623 | . 612 | . 607 | . 605 | . 604 | . 603 | . 602 |
| 8 | . 619 | . 610 | . 606 | . 605 | . 604 | . 603 | . 602 |
| 10 | . 616 | . 608 | . 605 | . 604 | . 603 | . 602 | . 601 |
| 20 | . 606 | . 604 | . 602 | . 602 | . 602 | . 601 | . 600 |
| 50 | . 602 | . 601 | . 601 | . 600 | . 600 | . 599 | . 599 |
| 100 | . 599 | . 598 | . 598 | . 598 | . 598 | . 598 | . 598 |

TABLE NO. 5.
Coefficients for Rectangular Orifices 1 Foot Wide.

| Head, $h$, <br> in Feet. | Depth of Orifice in Feet. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | .125 | .25 | .50 | .75 | 1.0 | 1.5 | 2.0 |
|  |  |  |  |  |  |  |  |
| .4 | .634 | .633 | .622 |  |  |  |  |
| .6 | .633 | .633 | .619 | .614 |  |  |  |
| .8 | .633 | .633 | .618 | .612 | .608 |  |  |
| 1. | .632 | .632 | .618 | .612 | .606 | .626 |  |
| 1.5 | .630 | .631 | .618 | .611 | .605 | .626 | .628 |
| 2. | .629 | .630 | .617 | .611 | .605 | .624 | .630 |
| 2.5 | .628 | .628 | .616 | .611 | .605 | .616 | .627 |
| 3. | .627 | .627 | .615 | .610 | .605 | .614 | .619 |
| 4. | .624 | .624 | .614 | .609 | .605 | .612 | .616 |
| 6. | .615 | .615 | .609 | .604 | .602 | .606 | .610 |
| 8. | .609 | .607 | .603 | .602 | .601 | .602 | .604 |
| 10. | .606 | .603 | .601 | .601 | .601 | .601 | .602 |
| 20. |  |  |  | .601 | .601 | .601 | .602 |
|  |  |  |  |  |  |  |  |

19. Experimental Coefficients of Discharge. Many experiments have been made on different kinds of orifices to determine the value of $c$, equation 20 , so that by means of this formula and a table of coefficients, orifices could readily be used for measuring water. The accompanying tables give these coefficients for circular, square, and rectangular orifices in vertical planes, the rectangular orifices all being one foot wide.

Example 1. What is the discharge from a circular orifice 3 in. in diameter under a pressure head of 10 feet?

By Table No. 3 the coefficient of discharge for an orifice of a diameter of .25 ft . and under a head of 10 feet is .597 . The area of the orifice $=\frac{1}{4} \times 3.14 \times .25^{2}=.049 \mathrm{sq} . \mathrm{ft}$. Then by equation 20 the discharge will be $.597 \times .049 \times \sqrt{2 \times 32.2 \times 10}=.743$ cu. ft. per sec. Ans.
2. What will be the velocity of flow in example 1 , the coefficient of velocity being taken equal to .97 ?

By equation 19 the velocity $=.97 \sqrt{2 \times 32.2 \times 10}=24.6$ ft. per sec. Ans.

## EXAMPLES FOR PRACTICE.

1. What will be the discharge from an orifice 4 in . square under a head of 16 feet? 2.14.cu. ft. per sec. Ans.
2. What must be the diameter of a circular orifice acting under a head of $2 \check{5}$ feet to discharge $1 \mathrm{cu} . \mathrm{ft}$. per sec.? (Assume $c=.6$ for a trial solution.) 2.76 in. Ans.
3. A pipe discharges 1.5 cm . ft. per sec. into a tank from - which the water escapes through an orifice 6 in . square. How


Fig. 23a.


Fig. $23 b$.
deep will the tank be filled above the orifice when the outflow is just equal to the inflow? 1.53 ft . Ans.

## FLOW OF WATER OVER WEIRS.

20. General Explanation. The term weir is usually given to a notch cut in the side of a tank or reservoir through which water may flow and be measured. The notch is usually rectangular and may have a width less


Fig. 24. than that of the tank, as shown in Fig. 23a, or equal to that of the tank, as in Fig. 233. Such weirs are often used for measuring the flow of a small stream by building a small dam and leading all the water through a notched plank or timber wall. For accurate work weirs should be sharp-crested (the "crest" is the lower edge over which the water passes). so that the water will touch the inner corner only as in the case of the standard orifice described in Art. 18. The back side of the weir should be smooth and vertical for a considerable distance downwards from the crest:

If the weir is made as in Fig. 23a the water in passing out will cause a contraction of the stream laterally, but if made as in Fig. $23 b$ the water will pass out parallel to the sides of the tank and there will be no lateral, or, as it is called, "end contraction". In either case reliable results may be obtained by the use of the proper coefficients, but if the form of Fig. $23 a$ be used, the distance of the notch from the side of the tank or channel should be at least three times the depth of the water on the weir in order that the contraction may be complete.

The measurement of water flowing over a weir is accomplished by merely measuring the depth of the water flowing over it. Then knowing this and the length of the weir the discharge can be calculated. In measuring this depth of water, or "height" of water on the weir, as it is commonly called, it is necessary to take the level of the water some distance back from the weir, as at A, Fig. 24, in order to avoid the effect of the curvature of the water surface. The difference between the level of the water and that of the weir is then the desired height $H$. The necessary distance back from the weir may be taken as 2 or 3 feet for small weirs to 8 or 10 feet for large ones.

A common and accurate way of determining the level of the water at A is by means of a submerged hook, shown at G, Fig. 24, called a hook gauge, arranged to be easily moved vertically along a scale. Fig. 25 shows such a gauge in detail. The gauge is set by moving it until the hook comes to the surface of the water. The scale is then read and the level of the water determined.
21. Formulas for Discharge. If the weir were a rectangular orifice at a considerable depth below the


Fig. 25. surface its discharge would be given by the formula

$$
\begin{equation*}
\mathrm{Q}=c \times b \times d \sqrt{2 g h} \tag{21}
\end{equation*}
$$

as in equation 20 of Art. 18. In this expression $b=$ breadth and $d=$ height of orifice, and $h=$ average depth of orifice below the
surface, or the average pressure head. In the case of the weir the depth $d$ is the height $H$, and the average pressure head, $h$, is something less than H, varying from nothing for the water at the surface to the full value H for the water at the crest. For a case like this where the square root of a quantity, $h$, is taken, that varies from zero to a given value $H$, the average value of this square root is two-thirds the square root of the maximum limit H. That is, for $h$ in equation 20 we may substitute $\frac{2}{3} \sqrt{\mathrm{H}}$, giving for the discharge

$$
\begin{align*}
& \mathrm{Q}=c b \times \frac{2}{3} \mathrm{H} \sqrt{2 g \mathrm{H}} \\
& =c \times \frac{2}{3} b \sqrt{2 g} \times \mathrm{H}^{\frac{3}{2}} \tag{22}
\end{align*}
$$

in which $c$ is the coefficient of discharge and equal to about .60 to . 65 as for orifices.

If the channel is small the "velocity of approach" will have an appreciable effect upon the discharge, increasing it somewhat above what it otherwise would be. This is taken account of by calculating approximately the velocity of the water in the channel of approach at the place where the level of the water is measured, and determining the head $h$ corresponding to this velocity by the formula

$$
h=\frac{v^{2}}{2 g} .
$$

Then the discharge will be, for weirs with end contractions,

$$
\begin{equation*}
\mathrm{Q}=c \times \frac{2}{3} b \sqrt{2 g}(\mathrm{H}+1.4 h)^{\frac{3}{2}} \tag{23}
\end{equation*}
$$

and for weirs without end contractions

$$
\begin{equation*}
\mathrm{Q}=c \times \frac{2}{3} \sqrt{2 g}\left(\mathrm{H}+1 \frac{1}{3} h\right)^{\frac{3}{2}} \tag{24}
\end{equation*}
$$

The coefficient $c$ should, in all cases, be selected according to the character of the weir.

In calculating " velocity of approach ", it is necessary first to get an approximate value for the discharge $Q$ by omitting the term $h$. The resulting discharge, divided by the cross-section of the
tank or channel will be, with sufficient accuracy, the desired velocity of approach.
22. Coefficients of Discharge. Tables Nos. 6 and 7 give val. ues of the coefficient $c$ for the above formulas for rectangular sharp-crested weirs.

TABLE NO. 6.
Coefficients for Contracted Weirs.

| Effective Head in Feet, $h$. | Length of Weir in Feet, $\boldsymbol{b}$. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.66 | 1 | 2 | 3 | 5 | 10 | 19 |
| 0.1 | 0.632 | 0.639 | 0.646 | 0.652 | 0.653 | 0.655 | 0.656 |
| 0.15 | . 619 | . 625 | . 634 | . 638 | . 640 | . 641 | . 642 |
| 0.2 | . 611 | . 618 | . 626 | . 630 | . 631 | . 633 | . 634 |
| 0.25 | . 605 | . 612 | . 621 | . 624 | . 626 | . 628 | . 629 |
| 0.3 | . 601 | . 608 | . 616 | . 619 | . 621 | . 624 | . 625 |
| 0.4 | . 595 | . 601 | . 609 | . 613 | . 615 | . 618 | . 620 |
| 0.5 | . 590 | . 596 | . 605 | . 608 | . 611 | . 615 | . 617 |
| 0.6 | . 587 | . 593 | . 601 | . 605 | . 608 | . 613 | . 615 |
| 0.7 |  | . 590 | . 598 | . 603 | . 606 | . 612 | . 614 |
| 0.8 |  |  | . 595 | . 600 | . 604 | . 611 | . 613 |
| 0.9 |  |  | . 592 | . 598 | . 603 | . 609 | . 612 |
| 1.0 |  |  | . 590 | . 595 | . 601 | . 608 | . 611 |
| 1.2 |  |  | . 585 | . 591 | . 597 | . 605 | . 610 |
| 1.4 |  |  | . 580 | . 587 | . 594 | . 602 | . 609 |
| 1.6 |  |  |  | . 582 | . 591 | . 600 | . 607 |

TABLE NO. 7.
Coefficients for Weirs without Contractions.

| Effective Head in Feet, $h$. | Length of Weir in Feet, b. . |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 19 | 10 | 7 | 5 | 4 | 3 | 2 |
| 0.1 | 0.657 | 0.658 | 0.658 | 0.659 |  |  |  |
| 0.15 | . 643 | . 644 | . 645 | . 645 | 0.647 | 0.649 | 0.652 |
| 0.2 | . 635 | . 637 | . 637 | . 638 | . 641 | . 642 | . 645 |
| 0.25 | . 630 | . 632 | . 633 | . 634 | . 636 | . 638 | . 641 |
| 0.3 | . 626 | . 628 | . 629 | . 631 | . 633 | 636 | . 639 |
| 0.4 | . 621 | . 623 | . 625 | . 628 | . 630 | . 633 | . 636 |
| 0.5 | . 619 | . 621 | . 624 | . 627 | . 630 | . 633 | . 637 |
| 0.6 | . 618 | . 620 | . 623 | . 627 | . 630 | . 634 | . 638 |
| 0.7 | . 618 | . 620 | . 624 | . 628 | . 631 | . 635 | . 640 |
| 0.8 | . 618 | . 621 | . 625 | . 629 | . 633 | . 637 | . 643 |
| 0.9 | . 619 | . 622 | . 627 | . 631 | . 635 | . 639 | . 645 |
| 1.0 | . 619 | . 624 | . 628 | . 633 | . 637 | . 641 | . 648 |
| 1.2 | . 620 | . 626 | . 632 | . 636 | . 641 | . 646 |  |
| 1.4 | . 622 | . 629 | . 634 | . 640 | . 644 |  |  |
| 1.6 | . 623 | . 631 | . 637 | . 642 | . 647 |  |  |

23. The Francis Formula. The most widely used weir formula for large weirs without end contractions is that derived by Mr. James B. Francis from an extensive series of experiments on weirs 10 feet long. His formula is

$$
\begin{equation*}
\mathrm{Q}=3.33 b \mathrm{H}^{\frac{3}{2}} \tag{25}
\end{equation*}
$$

in which the unit of length must be the foot. This is equivalent to the use of a constant value of the coefficient $c$ of equation 22 , equal to .623. It gives results sufficiently close for most purposes. With end contractions the length $b$ is to be reduced by .1 H for


Fig. 26.,
one end contracted and by . 2 H for both ends contracted. The formula is further modified to allow for velocity of approach, but where this element enters, use may be made of the other formula.
24. Submerged Weirs. Where the water on the downstream side of a weir is higher than the crest, as in Fig. 26, the discharge is closely given by the formula

$$
\begin{equation*}
\mathbf{Q}=3.33 \zeta(n \mathrm{H})^{\frac{3}{2}}, \tag{26}
\end{equation*}
$$

where H is the height of the water on the upper side and $n$ is a coefficient depending on the ratio of the head on the lower side, $\mathrm{H}^{1}$, to the head H . The values of $n$ are as follows:

TABLE NO. 8.
Values of $n$ for Submerged Weirs.

| $\frac{H^{\prime}}{H}$ | $n$ | $\frac{H^{\prime}}{H}$ | $n$ | $\frac{H^{\prime}}{H}$ | $n$ | $\frac{H^{\prime}}{H}$ | $n$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| .00 | 1.000 | .20 | 0.985 | .45 | 0.912 | .70 | 0.787 |
| .02 | 1.006 | .25 | 0.973 | .50 | 0.892 | .75 | 0.750 |
| .05 | 1.007 | .30 | 0.959 | .55 | 0.871 | .80 | 0.703 |
| .10 | 1.005 | .35 | 0.944 | .60 | 0.846 | .90 | 0.574 |
| .15 | 0.996 | .40 | 0.929 | .65 | 0.819 | 1.00 | 0.000 |

25. Weirs of Irregular Section. In many cases it is desirable to determine the flow of a stream by measurements taken of the height of water flowing over some dam or weir; and, on the other hand, in the design of waste-weirs some method of estimating their capacity is essential. The law of flow over such weirs

TABLE NO. 9.
Values of the Coefficient $C$ in the Formula

$$
\mathrm{Q}=\mathrm{C} b \mathrm{H}^{\frac{3}{2}} \text { for irregular weirs. }
$$


is very complicated, and the only accurate way of determining the constants for any particular case is by means of experiments on a section of the same form as the one in question. If this is impos.
sible, the best substitute for it is to use constants which have been determined for a weir agreeing as closely in form as may be to the one under consideration.

In Table No. 9 are given several sets of coefficients for five forms of dams, as determined by experiment. This coefficient is to be used in place of the value 3.33 in equation 25.

It will be noted by comparing Nos. 1 and 3 that the discharge falls off considerably by using a flat slope for the back of the dam.

Examples. 1. What will be the discharge of a sharp-crested weir 4 ft . long with $\mathrm{H}=6$ inches, there being contraction at both ends?

By Table No. 6 the coefficient may be taken at .610 . Then by equation $22, \mathrm{Q}=.61 \times \frac{2}{3} \times 4 \times \sqrt{64.4} \times\left(\frac{1}{2}\right)^{\frac{3}{2}}=4.6 \mathrm{cu}$. ft. per sec. Ans.
2. If the channel of approach in example 1 be 6 feet wide by $2 \frac{1}{2}$ feet deep, what will be the effect of the "velocity of approach"?

Assuming the same discharge as above, the velocity of flow in this channel will be $\frac{4.6}{6 \times 2 \frac{1}{2}}=.11 \mathrm{ft}$. per sec. The head $h$ corresponding to this velocity $=\frac{v^{2}}{2 g}=\frac{.11^{2}}{64.4}=.0002 \mathrm{ft}$. Introducing this value for $h$ in equation 23 , it is seen that the additional term $1.4 h$ is too small to be of any practical consequence.

## FLOW OF WATER THROUGH PIPES.

26. Discharge Through Pipes for Different Velocities. The rate of discharge through a pipe is equal to the average velocity of the flowing water multiplied by the cross-section of pipe. Velocities are usually expressed in feet per second and discharge in cubic feet per second or gallons per minute. The diameter of a pipe is always given in inches. These differences in units make it desirable to have a table at hand giving for a velocity of one foot per second the discharge of pipes of various diameters expressed both in cubic feet per second and in gallons per minute. Such a table is given below :



TABLE NO. 10.
Discharge of Pipes in Cubic Feet Per Second and in Gallons Per Minute for a Velocity of One Foot Per Second.
(For other velocities multiply the discharge here given by the velocity expressed in feet per second.)

| Diameter of Pipe in Inches. | Discharge. |  |
| :---: | :---: | :---: |
|  | Cubic Feet Per Second. | Gallons Per Minute. |
| 1 | . 0055 | 2.4 |
| 2 | . 0218 | 9.8 |
| 3. | . 0491 | 22.0 |
| 4 | . 0873 | 39.1 |
| 6 | . 1964 | 88.1 |
| 8 | . 3491 | 157 |
| 10 | . 5454 | 245 |
| 12 | . 7854 | 352 |
| 14 | 1.069 | 480 |
| 16 | 1.396 | 627 |
| 20 | 2.182 | 978 |
| 24 | 3.142 | 1410 |
| 30 | 4.909 | 2200 |
| 36 | 7.069 | 3155 |
| 42 | , 9.621 | 4317 |
| 48 | 12.568 | 5639 |

## EXAMPLES FOR PRACTICE.

1. What will be the discharge in gallons per minute of a 6 -inch pipe for a velocity of 4.5 feet per sec.? 396 gallons per min. Ans.
2. What velocity will be required to discharge $1,000,000$ gallons per day through an 8 -inch pipe? 4.4 ft . per sec. Ans.
3. What diameter of pipe will be required to discharge 1,000 gals. per min. at a velocity of 5 feet per sec.?

An 8 -inch pipe will discharge 785 gals. per min. and a 10 inch pipe will discharge 1,225 gals. per min. A 10 -inch pipe would therefore be necessary if no intermediate size is available. Ans,
27. General Principles Governing the Flow of Water Through Pipes. Let ABCD, Fig. 27, be any pipe leading from a reservoir and having a stop valve at D . Also suppose $\mathrm{B} b$ and $\mathrm{C} c$ are tubes connected with the pipe at $B$ and $C$ and open at the top.

From the laws of pressure explained in Art. 6 we know that if the valve D be closed so that there will be no motion of the water the water will rise in the tubes $\mathrm{B} b$ and $\mathrm{C} c$ to the same level as that in the reservoir. The pressure at A will be represented by the head $h_{1}$ and that at B and C by the heads $h_{2}$ and $h_{3}$ respectively, which heads may be greater or less than the head at A according as the pipe slopes downwards or upwards from A.

Now let the valve at D be opened partly so as to permit the water to escape slowly. It will be found that the pressures at B and C will immediately decrease and that the water in the tubes will fall to some lower levels $b^{\prime}$ and $c^{\prime}$. This decrease in pressure


Fig. 27.
is due to two causes. First, a part of the pressure head has been used to give the water some velocity in the pipe, and second, a part has been consumed in the friction of the water in passing from A to B and C. The portion used in giving the water its velocity is the same as the head required to produce a given velocity of efflux, $v$, from an orifice, and is found from the formula $v=\sqrt{2 g h}$. Solving for $h$ we have

$$
\begin{equation*}
h=\frac{v^{2}}{2 g} \tag{27}
\end{equation*}
$$

in which $v$ is the actual velocity of flow in the pipe.
The pressure head lost in friction is usually much greater than that used in velocity and is the most important as well as the most difficult part of the problem of determining the flow in pipes.

If H represents the total loss of pressure head between the reservoir and any point $B, h_{\mathrm{v}}$ the head necessary to produce the given velocity at B ("velocity head") and $h_{\mathrm{f}}$ the pressure lost by friction between A and B we then have in general

$$
\begin{equation*}
\mathrm{H}=h_{\mathrm{v}}+h_{\mathrm{f}} \tag{28}
\end{equation*}
$$

or from equation 27

$$
\begin{equation*}
\mathrm{H}=\frac{v^{2}}{2 g}+h_{\mathrm{f}} \tag{29}
\end{equation*}
$$

In the figure, $b b^{\prime}$ represents the head II for point B and $c c^{\prime}$ that for point C. Between B and C the loss in head is the difference between $b b^{\prime}$ and $c c^{\prime}$ and is all due to friction, since the velocity is the same at the two points, the pipe being of uniform size.

If now we open the valve $D$ farther so as to give the water a higher velocity the level of the water in the tubes $b \mathrm{~B}$ and $c \mathrm{C}$ will fall still more, that is, there will be a greater loss of pressure head, H , than before. This increase in loss of pressure is due mainly to the increased friction loss $h_{\mathrm{f}}$ cansed by the higher velocity, but to a small extent also to the increased energy transformed into velocity head.

In any case that part of the head HI needed to produce the velocity $v$, which is equal to $\frac{v^{2}}{2 y}$, can readily be calculated or can be obtained from the following table:

## TABLE NO. 11.

Velocity Heads

$$
h=\frac{v^{2}}{2 g}
$$

Corresponding to Various Values of $v$.

|  | $\begin{gathered} h \\ \mathrm{ft} . \end{gathered}$ | $\begin{gathered} v \\ \mathrm{ft} . \text { per sec. } \end{gathered}$ | $\begin{gathered} h \\ \mathrm{ft} . \end{gathered}$ | $v$ ft. persec. | $h$ ft. | $v$ ft. per sec. | $\begin{gathered} h \\ \mathrm{ft} . \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.0 | 0.06 | 4.0 | 0.25 | 6.0 | 0.56 | 8.0 | 0.99 |
| 2.2 | 0.08 | 4.2 | 0.28 | 6.2 | 0.60 | 8.2 | 1.04 |
| 2.4 | 0.09 | 4.4 | 0.30 | 6.4 | 0.64 | 8.4 | 1.10 |
| 2.6 | 0.10 | 4.6 | 0.33 | 6.6 | 0.68 | 8.6 | 1.15 |
| 2.8 | 0.12 | 4.8 | 0.36 | 6.8 | 0.72 | 8.8 | 120 |
| 3.0 | 0.14 | 5.0 | 0.39 | 7.0 | 0.76 | 9.0 | 1.26 |
| 3.2 | 0.16 | 5.2 | 0.42 | 7.2 | 0.80 | 9.2 | 1.31 |
| 3.4 | 0.18 | 5.4 | 0.45 | 7.4 | 0.85 | 9.4 | 1.37 |
| 3.6 | 0.20 | 5.6 | 0.49 | 7.6 | 0.90 | 9.6 | 1.43 |
| 3.8 | 0.22 | 5.8 | 0.52 | 7.8 | 0.94 | 9.8 | 1.49 |

The usual problem in practice consists in calculating the friction loss $h_{f}$ between any two given points in a pipe for a given velocity $v$; or, conversely, to determine the velocity which will occur with a given loss of head $h_{\mathrm{f}}$.
28. Formulas for Friction Loss in Pipes. A great number of experiments have been made to determine the friction loss in the flow of water through pipes. The results show great variations due to many causes, chief of which is the variation in the character of the pipe as to material, degree of roughness of the interior, diameter, etc. Consequently much less accuracy is possible in the estimation of the flow of water through pipes than through orifices or over weirs. Theory is of very little assistance here, and the only practicable method of calculation is to express by some formula the approximate law of variation in friction, and then use coefficients as determined from experiments.

Results of experiments show that the friction loss in a pipe is approximately proportional to the length of the pipe and to the square of the velocity of the water, and is inversely proportional to the cross-section of the pipe divided by its circumference. If we, let
$l_{\mathrm{f}}=$ loss by friction between any two points;
$l=$ length of pipe between same two points;
$v=$ velocity of water in pipe;
$r=$ ratio of cross-section to circumference
$\quad$ of pipe, called the "hydraulic mean
$\quad$ radius",
we then have, according to the above law,

$$
h_{\mathrm{f}}=\frac{v^{2} l}{r} \times k
$$

where $k$ is some coefficient.
It is usual to write this formula so as to express directly the value of $v$. By solving for $v$ we have

$$
v=\sqrt{r \frac{h}{l}} \times \sqrt{k}
$$

Putting C for $\sqrt{\bar{k}}$ we may write

$$
\begin{equation*}
v=\mathrm{C} \sqrt{r \frac{h}{l}} \tag{30}
\end{equation*}
$$


"BUTTERFLY" DAM, CHICAGO DRAINAGE CANAL, LOCKPORT, ILL.

versisham Randolph, Chief Engineer.


## - HOLw 'TTGMNIVTd LV

which is known as the Chezy formula. The values of $r, h$, and $l$ are to be expressed in feet, and the result will give $v$ in feet per second.

The above formula may be used for all kinds of pipe by using a suitable value of C as determined by experiments on similar pipe. For ordinary cast iron pipe the value of C varies from about 100 for pipes 1 or 2 inches in diameter to 140 or 150 for pipes 4 or 5 feet in diameter. Various diagrams and formulas for C have been devised for cast iron pipe, all of which are more or less unsatisfactory. Mr. Hamilton Smith has constructed a diagram which is probably as satisfactory as any now in use. This diagram is not entirely convenient in form, and instead of it we give below an extended table giving the actual velocities of flow $v$ for various diameters of pipe and various losses of head for a length of 100 feet for pipes from $\frac{3}{4} \mathrm{in}$. to 3 in . in diameter, and for a length of 1,000 feet for larger pipes. This table is very convenient to use in calculations, as the desired velocity or loss of head can be seen at a glance.

TABLE NO. 12.
Discharge, Friction Head, and Velocity of Flow Through Smooth Pipes such as Cast Iron.

|  | 3/4-inch Pipe. |  | 1-inch Pipe. |  | 11/2-inch Plpe. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Discharge, Gals. per Minute. | Loss of Head, Feet per 100 Feet. | Veloclty, Feet per Second. | Loss of Head, Feet per 100 Feet. | Velocity, Feet per Second. | Loss of Head, Feet per 100 Feet. | Velocity, Feet per Second. |
| 1 | 0.5 | 0.72 | 0.02 | 0.41 |  |  |
| 2 | 2.0 | 1.4 | 0.6 | 0.82 |  |  |
| 3 | 4.0 | 2.2 | 1.1 | 1.2 |  |  |
| 4 | 7.2 | 2.9 | 1.8 | 1.6 |  |  |
| 5 | 11.0 | 3.6 | 2.6 | 2.0 |  |  |
| 6 | 15.0 | 4.3 | 3.6 | 2.4 |  |  |
| 7 | 20.4 | 5.1 | 4.8 | 2.9 |  |  |
| 8 | 25.5 | 5.8 | 6.2 | 3.3 |  |  |
| 9 | 32.0 | 6.5 | 7.7 | 3.7 |  |  |
| 10 | 39.0 | 7.2 | 9.4 | 4.1 | 1.1 | 1.8 |
| 12 |  |  | 13.0 | 4.9 | 1.6 | 2.2 |
| 14 |  |  | 17.1 | 5.7 | 2.2 | 2.5 |
| 16 |  |  | 21.8 | 6.5 | 2.8 | 2.9 |
| 18 |  |  | 27.1 | 7.3 | 3.5 | 3.3 |
| 20 |  |  | 33.0 | 8.2 | 4.3 | 3.6 |
| 30 |  |  |  |  | 9.5 | 5.4 |
| 40 |  |  |  |  | 16.0 | 7.2 |
| 50 |  |  |  |  | 24.0 | 9.1 |
| 60 |  |  |  |  | 34.0 | 10.9 |
| 70 |  |  |  |  | 45.0 | 12.7 |

TABLE NO. 12.-Continued.

| Dicharge, Gals. per Minute. | 2-Inch Pipe. |  | 21/2-inch Pipe. |  | 3-inch Pipe. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Loss of Head, Feet per 100 Feet. | Velocity, Feet per Second. |  | Velocity, Feet per Second. | Loss of Head, Feet per 100 Feet. | Velocity, Feet per Second. |
| 10 | . 4 | 1.0 | 0.1 | 0.65 | 0.05 | 0.45 |
| 20 | 1.2 | 2.0 | 0.4 | 1.3 | 0.2 | 0.90 |
| 30 | 2.4 | 3.1 | 0.8 | 1.9 | 0.4 | 1.4 |
| 40 | 4.0 | 4.1 | 1.4 | 2.6 | 0.7 | 1.8 |
| 50 | 6.1 | 5.1 | 2.1 | 3.3 | 1.0 | 2.3 |
| 60 | 8.6 | 6.1 | 2.9 | 3.9 | 1.4 | 2.7 |
| 70 | 11.5 | 7.1 | 3.9 | 4.6 | 1.8 | 3.2 |
| 80 | 14.8 | 8.2 | 5.0 | 5.2 | 2.3 | 3.6 |
| 90 | 18.4 | 9.2 | 6.3 | 5.9 | 2.8 | 4.1 |
| 100 | 22.2 | 10.2 | 7.7 | 6.5 | 3.4 | 4.5 |
| 120 |  |  | 10.8 | 7.8 | 4.8 | 5.4 |
| 140 |  |  | 14.3 | 9.1 | 6.3 | 6.3 |
| 160 |  |  | 18.3 | 10.4 | 8.0 | 7.2 |
| 180 |  |  | 22.7 | 11.8 | 9.9 | 8.1 |
| 200 | . |  | 27.5 | 13.1 | 12.0 | 9.0 |
| 250 |  |  |  |  | 18.0 | 11.3 |
| 300 |  |  |  |  | 25.0 | 13.6 |

TABLE NO. 12.-Continued.

| Discharge,Gals. per Minute. | 4-inch Pipe. |  | 6-nnch Pipe. |  | 8-inch Pipe. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Loss of } \\ \text { Head, Feet } \\ \text { per 1,000 } \\ \text { Feet. } \end{gathered}$ | Velocity, Feet per Second. | Loss of Head, Feet per 1.000 Feet. | Velocity Feet per Second. | Loss of Head, Feet per 1,000 Feet. | Velocity, Feet per second. |
| 50 | 2.3 | 1.3 |  |  |  |  |
| 75 | 5.2 | 1.9 |  |  |  |  |
| 100 | 8.7 | 2.5 | 1.2 | 1.1 |  |  |
| 125 | 13.1 | 3.2 | 1.8 | 1.4 |  |  |
| 150 | 18.3 | 3.8 | 2.5 | 1.7 |  |  |
| 175 | 24.3 | 4.5 | 3.3 | 2.0 |  |  |
| 200 | 31.0 | 5.1 | 4.2 | 2.3 | 1.1 | 1.3 |
| 250 | 46.5 | 6.4 | 6.3 | 2.8 | 1.6 | 1.6 |
| 300 | 65.0 | 7.7 | 8.9 | 3.4 | 2.2 | 1.9 |
| 350 |  |  | 11.9 | 4.0 | 2.9 | 2.2 |
| 400 |  |  | 15.1 | 4.5 | 3.7 | 2.6 |
| 450 |  |  | 18.7 | 5.1 | 4.6 | 3.9 |
| 500 |  |  | 22.7 | 5.7 | 5.6 | 3.2 |
| 600 |  |  | 31.8 | 6.8 | 7.9 | 3.8 |
| 700 |  |  | 42.2 | 7.9 | 10.5 | 4.5 |
| 800 |  |  | 54.0 | 9.1 | 13.4 | 5.1 |
| 900 |  |  |  |  | 16.6 | 5.8 |
| 1000 |  |  |  |  | 20.2 | 6.4 |
| 1100 |  |  |  |  | 24.1 | 7.0 |

TABLE NO 12.-Continued.

| Discharge, Gals. per Minute. | 10-inch Pipe. |  | 12-inch Pipe. |  | 16-Inch Plpe. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Loss of Head, Feet per 1,000 Feet. | Velocity, Feet per Second. | $\left\lvert\, \begin{gathered} \text { Loss of } \\ \text { Head, Feet } \\ \text { per } 1,000 \\ \text { Feet. } \end{gathered}\right.$ | Velocity, Feet per Second. | Loss of Head, Feet per 1,000 Feet. | Velocity, Feet per Second. |
| 200 | . 35 | . 82 | . 14 | . 57 |  |  |
| 300 | . 73 | 1.2 | . 30 | - 85 |  |  |
| 400 | 1.24 | 1.6 | . 51 | 1.1 |  |  |
| 500 | 1.87 | 2.0 | . 78 | 1.4 | . 18 | . 80 |
| 600 | 2.6 | 2.4 | 1.10 | 1.7 | . 26 | . 96 |
| 700 | 3.5 | 2.9 | 1.45 | 2.0 | . 34 | 1.1 |
| 800 | 4.4 | 3.3 | 1.82 | 2.3 | . 43 | 1.3 |
| 900 | 5.5 | 3.7 | 2.3 | ${ }_{2} .6$ | . 54 | 1.4 |
| 1000 | 6.7 | 4.1 | 2.8 - | 2.8 | . 66 | 1.6 |
| 1100 | 8.0 | 4.5 | 3.3 | 3.1 | . 78 | 1.8 |
| 1200 | 9.4 | 4.9 | 3.9 | 3.4 | . 92 | 1.9 |
| 1300 |  |  | 4.5 | 3.7 | 1.06 | 2.1 |
| 1400 |  | 5.7 | 5.1 | 4.0 | 1.22 | 2.2 |
| 1500 |  |  | 5.8 | 4.2 | 1.38 | ${ }_{9} .4$ |
| 1600 1700 |  |  | 6.5 | 4.5 | 1.55 | 2.6 |
| 1800 1900 |  |  | 8.1 9.0 | 5.1 | ${ }_{2.1}^{1.93}$ | 2.9 3.0 |
| 2000 |  |  | 9.9 | 5.7 | 2.3 | 3.2 |
| 2200 |  |  | 11.7 | 6.2 | 2.8 | 3.5 |
| 2400 |  |  |  |  | 3.3 | -3.8 |
| 2600 |  |  |  |  | 3.8 | 4.2 |
| 2800 |  |  |  |  | 4.4 | 4.5 |
| 3000 |  |  |  |  | 5.0 | 4.8 |
| 3500 |  |  |  |  | 6.6 | 5.6 |

Examples. 1. What is the head lost in friction due to the flow of 800 gallons per minute in a 6 -inch pipe ?

From Table No. 12 we see that the friction head in a 6 -inch pipe for a flow of 800 gals. per min. is 54.0 ft . for each $1,000 \mathrm{ft}$. of pipe. Ans.
2. What size of pipe will be requiired to convey 700 gallons of water per minute a distance of 8,000 feet with a total loss of head of 40 feet?

The loss of head per $1,000 \mathrm{ft}$. is $40 \div 8=5 \mathrm{ft}$. From the table we find that for a discharge of 700 gallons per min. the loss of head in an $8 . \mathrm{in}$. pipe is 10.5 ft . per 1,000 , and in a $10-\mathrm{in}$. pipe it is 3.5 ft . A $10-\mathrm{in}$. pipe wonld then be required if the assumed loss is not to be exceeded. Ans.
3. If a town is supplied with water from an elevated reservoir through a pipe line 15,000 feet long, how high must the

TABLE NO. 12.-Continued.

| Discharge, Gals. per Minute. | 20-inch Pipe. |  | 24-inch Pipe. |  | 30-inch Pipe. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Loss of Head, Feet per 1,000 Feet. | Velocity, Feet per Second. | Loss of Head, Feet per 1,000 Feet. | Velocity, Feet per Second. | Loss of Head, Feet per 1,000 Feet. | Velocity, <br> Feet per Second. |
| 1000 | 23 | 1.0 | . 08 | . 71 |  |  |
| 1200 | . 32 | 1.2 | . 12 | . 85 |  |  |
| 1400 | . 42 | 1.4 | . 16 | . 99 |  |  |
| 1600 | . $52{ }^{\circ}$ | 1.6 | . 20 | 1.1 |  |  |
| 1800 | . 64 | 1.8 | . 25 | 1.3 |  |  |
| 2000 | . 77 | 2.0 | . 31 | 1.4 | . 10 | . 91 |
| 2200 | . 92 | 2.2 | . 37 | 1.6 | . 12 | 1.00 |
| 2400 | 1.08 | 2.5 | . 43 | 1.7 | . 14 | 1.09 |
| 2600 | 1.25 | 2.7 | . 50 | 1.8 | . 17 | 1.18 |
| 2800 | 1.43 | 2.9 | . 58 | 2.0 | . 19 | 1.27 |
| 3000 | 1.62 | 3.1 | . 66 | 2.1 | . 22 | 1.36 |
| 3200 | 1.82 | 3.3 | . 74 | 2.3 | . 24 | 1.45 |
| 3400 | 2.04 | 3.5 | . 83 | 2.4 | . 27 | 1.55 |
| 3600 | 2.27 | 3.7 | . 92 | 2.5 | . 30 | 1.64 |
| 3800 | 2.51 | 3.9 | 1.02 | 2.7 | . 33 | 1.73 |
| 4000 | 2.76 | 4.1 | 1.12 | 2.8 | . 36 | 1.82 |
| 4500 | 3.43 | 4.6 | 1.39 | 3.2 | . 46 | 2.05 |
| 5000 | 4.16 | 5.1 | 1.68 | 3.5 | . 56 | 2.27 |
| 5500 | 4.96 | 5.6 | 2.00 | 3.9 | . 67 | 2.50 |
| 6000 | 5.80 | 6.1 | 2.35 | 4.3 | . 78 | 2.73 |
| 6500 |  |  |  |  | . 90 | 2.96 |
| - 7000 |  |  |  |  | 1.03 | 3.18 |
| 7500 |  |  |  |  | 1.17 | 3.41 |
| 8000 |  |  |  |  | 1.32 | 3.64 |
| 9000 |  |  |  |  | 1.64 | $4.09$ |
| 10000 |  |  |  |  | 2.00 | 4.55 |

reservoir be above the town and what must be the size of the pipe line so that the pressure of water in the distributing pipes be not less than 60 pounds per sq. in., equivalent to $60 \times 2.3=138 \mathrm{ft}$. head. The amount of water required is 1,800 gals. per min.

This problem has several solutions since various sizes of pipe may be assumed and the reservoir placed at the elevation to furnish the necessary pressure. An examination of Table No. 12 shows that to deliver 1,800 gals. per min. a $12-\mathrm{in}$. pipe would consume in friction 8.1 ft . of head per $1,000 \mathrm{ft}$., a $16-\mathrm{in}$. pipe would consume only 1.93 ft . per $1,000 \mathrm{ft}$., and a $20-\mathrm{in}$. pipe only 64 ft . No value is given for a $10-\mathrm{in}$. pipe, but it would evidently be 20 feet or more per 1,000 , which would give a total loss for $15,000 \mathrm{ft}$. of 300 feet, a loss which would ordinarily be impracticable.

If we use a $12-\mathrm{in}$. pipe the total loss in friction will be $8.1 \times$ $15=121.5 \mathrm{ft}$. The velocity of flow will be 5.1 ft . per sec. and the necessary velocity head, $h_{\mathrm{v}}$, will be, by Table No. 11, . 4 ft . The total head $=121.5+.4=121.9 \mathrm{ft}$., and the necessary elevation of the reservoir $=138+121.9=259.9 \mathrm{ft}$. above the town.

If a 16 -inch pipe be assumed, the friction loss $=1.93 \times 15=$ 28.9 ft ., the velocity head $=.1 \mathrm{ft}$., and the total head $=29 \mathrm{ft}$. Elevation of reservoir $=29.0+138=167 \mathrm{ft}$.

If a 20 -inch pipe be used, the friction head $=.64 \times 15=$ 9.6 ft ., the velocity head is less than .1 ft . and may be neglected. The required height of reservoir $=147.6 \mathrm{ft}$.

Still larger sizes will give still lower elevations for the reservoir, but it is evident that the reservoir in any case must have an elevation somewhat greater than 138 ft .

From the above results we see that a $12-\mathrm{in}$. pipe requires the reservoir to be at an elevation of 259.9 ft ., a $16-\mathrm{in}$. pipe requires an elevation of 167 ft ., and a $20-\mathrm{in}$. pipe an elevation of 147.6 ft . The proper size to use would be that size which would give the cheaper construction for the pipe and reservoir combined.
29. The Hydraulic Grade Line. Referring again to Fig. 27 , it will be seen that the drop in pressure between B and C will be proportional to the length of the pipe from $B$ to $C$, and if we have a long pipe with several open tubes attached to it like $b \mathrm{~B}$ and $c \mathrm{C}$, the level of the water in them would be lower and lower as we proceed along the pipe, the drop being uniform so long as the pipe is of the same size and kind, the amount of the drop per 1,000 feet being given in Table No. 12. If now a line were drawn from E through the points $b, c$, etc., so that the height of this line above the pipe would represent the pressures in it, this line would be called the "hydraulic grade line" for the pipe under the given conditions. It is convenient in various problems to construct such a grade line. Its position will evidently vary with the velocity of the flow and will be a horizontal line when the water is still, and always a straight line for a pipe of uniform conditions.
30. Siphons. If in any case a pipe line rise above this hydraulic grade line, as shown in Fig. 28, the pressure in such portion of the pipe will be less than atmospheric, the pressure measured by the grade line as described above referring in all cases to the pres-
sure in excess of the usual atmospheric pressure. That portion of the pipe BC lying above the grade line is called a siphon. The greatest height above the grade line which it is practicable to operate a siphon is considerably less than the height of the water barometer given in Art. 4. Evidently since the velocity of flow, and hence the hydraulic grade line, can be varied by varying the opening at $D$, a pipe which may act as a siphon at one time may not so act at another. Thus in the figure, if valve D be nearly closed so that the flow is reduced and hence also the frictional loss,


Fig. 28.
the grade line will rise to some position such as AE and there will be pressure in excess of atmospheric at all points.
31. Flow Through Special Forms of Pipes. Riveted Pipe. The friction loss in riveted pipes depends úpon the thickness of the plates and the manner of making the joints. Experiments on this class of pipes are not sufficiently numerous to enable any general expression to be formulated, so that in the design of such pipes the selection of coefficients must be made by reference to the experimental data. In general it is found that the coefficient C , of equation 30, changes little with change in diameter or velocity, and in this respect exhibits considerable difference from its variation in cast-iron pipe. For ordinary velocities the value of C, for new pipe appears to range between 100 and 115. A value of 100 is as great as it is well to use.
32. Wood Stave Pipe. Few experiments have been made on this class of pipe although it has been used quite extensively in the West. The pipe is usually quite smooth and not subject to deterioration on the interior, so that its discharging capacity is high. For ordinary velocities the value of C , equation 30 , may be taken at 110.
33. ${ }^{-}$Fire Hose. In making provisions for fire protection it becomes necessary to estimate the effectiveness of a stream of water when led through a given length of hose for a given pressure at the hydrant, or to find what pressure is required to throw a stream a given height or a given distance. The usual size of fire hose is $2 \frac{1}{2}$ inches. At the end of the hose is attached a nozzle of a diameter usually of 1 in ., $1 \frac{1}{8} \mathrm{in}$., or $1 \frac{1}{4} \mathrm{in}$., which partly controls the amount and pressure of the water discharged. If there were no friction in the hose the water could be thrown nearly to a height corresponding to the pressure head at the hydrant, but the hose friction is very great, and two or three hundred feet of hose will cut down the effective pressure often more than one-half. Evidently the more rapid the flow through the hose the greater the friction loss, hence if the nozzle is small so that the discharge will be small, the effective pressure near the nozzle will be greater than with a large nozzle and large discharge. Hence a higher stream can be thrown through a small nozzle with a given hydrant pressure and length of hose than through a large nozzle, although the stream is not so effective in quenching a fire as the larger stream.

In Table No. 13 are given the necessary data for estimating the loss of head and effectiveness of fire streams for various pressures and for three sizes of nozzles

In the table, page 44, the pressure given is that at the nozzle instead of at the hydrant. To get the latter, it is necessary to add to the nozzle pressure the head lost in the hose. The result will be the hydrant pressure, providing nozzle and hydrant are at same level. If not, then a correction would need to be made for this difference in elevation. The vertical height and horizontal distances are to be measured from the nozzle. The heads are given in pounds per square inch, which is the customary unit in this class of work. To reduce to feet of head multiply pounds pressure by 2.3.

Examples. 1. What hydrant pressure will be required to throw a stream of water 75 feet vertically through a $1 \frac{1}{8}-\mathrm{in}$. nozzle and 300 feet of hose.

In the table for the $1 \frac{1}{8}$-in. nozzle we see that for a height of 75 feet the loss of head per 100 feet of hose is 20 pounds, and the pressure at the nozzle is (in first column of table) 50 pounds. The

TABLE NO. 13.
Hose and Fire=Stream Data.

|  | 1-inch Smooth Nozzle. |  |  |  |  | 11/8-inch Smooth Nozzle. |  |  |  |  | 14/4-inch Smooth Nozzle. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 b |  | 1 b . | ft . | ft. | t |  | b. | ft. | ft . |  |  | 1 b. | ft. | ft. | ft. |
|  | 132 | 5 | 35 | 37 | 77 | 168 | 8 | 36 | 38 | 80 | 209 | 12 | 37 | 40 | 83 |
|  | 161 | 7 | 51 | 47 | 109 | 206 | 12 | 52 | 50 | 115 | 256 | 19 | 53 | 54 | 119 |
|  | 186 | 10 | 64 | 55 | 133 | 238 | 16 | 65 | 59 | 142 | 296 | 25 | 67 | 63 | 148 |
|  | 208 | 12 | 73 | 61 | 152 | 266 | 20 | 75 | 66 | 162 | 331 | 31 | 77 | 70 | 169 |
|  | 228 | 15 | 79 | 67 | 167 | 291 | 24 | 83 | 72 | 178 | 363 | 37 | 85 | 76 | 186 |
|  | 246 | 17 | 85 | 72 | 179 | 314 | 28 | 88 | 77 | 191 | 392 | 43 | 91 | 81 | 200 |
|  | 263 | 20 | 89 | 76 | 189 | 336 | 32 | 92 | 81 | 203 | 419 | 49 | 95 | 85 | 213 |
| 90 | 279 | 22 | 92 | 80 | 197 | 356 | 36 | 96 | 85 | 214 | 444 | 55 | 99 | 90 | 225 |
| 100 | 295 | 25 | 96 | 83 | 205 | 376 | 40 | 99 | 89 | 224 | 468 | 62 | 101 | 93 | 236 |

hydrant pressure will then be $50+(20 \times 3)=110$ pounds per square inch. The discharge will be about 266 gallons per minute.
2. With a hydrant pressure of 100 pounds, what will be the discharge through 250 feet of hose with a $1 \frac{1}{4}-\mathrm{in}$. nozzle, and how high can such a stream be thrown with effectiveness?


Fig. 29.

This problem must be solved by trial. In the table for $1 \frac{1}{4}-\mathrm{in}$. nozzles, we see that for a discharge of 269 gallons the nozzle pressure is 40 pounds, and the loss of head per 100 feet of hose is 25 pounds; for a discharge of 331 gallons the nozzle pressure is 50 pounds, and the loss of head per 100 feet is 31 pounds, etc. We have given the head of 100 pounds, which must equal the sum of the nozzle pressure and the loss in the hose. If we try the first value for discharge, we have a nozzle pressure of 40 pounds and a total loss in the hose of $25 \times 2.5=62.5$ pounds, or a total of $40+62.5=102.5$ pounds. This being a little more than the total available head, it is evident that we have assumed too high a
discharge. The next lower value is 256 gallons, giving a nozzle pressure of 30 pounds and a total hose loss of $19 \times 2.5$ or 47.5 pounds, giving a total of $30+47.5=77.5$ pounds. Evidently the correct value is somewhere between 296 and 256 , and further that it is but very little below the former value. For a total change in discharge of 40 gallons we have a change in total head of 102.5 -77.5 or 25 pounds. Hence for a change of 2.5 pounds the discharge will vary about $\frac{1}{10}$ of 40 gallons, or 4 gallons. The discharge may then be taken as 292 gallons per minute. The effective height will be between 67 feet and 53 feet, but only a little less than the former value, say 65 feet. This is as close an estimate as the conditions of the problem will warrant, since the hose friction is a factor that varies greatly according to the character of the hose.
34. Minor Losses of Head in Pipes. In most of the following formulas the quantity $\frac{v^{2}}{2 y}$ occurs. For given values of $v$ this quantity can readily be taken from Table No. 11.

Loss of Head at Entrance. This is expressed by the formula

$$
\begin{equation*}
h=\left(\frac{1}{c^{2}}-1\right) \frac{v^{2}}{2 y}, \tag{3I}
\end{equation*}
$$

where $v=$ velocity in the pipe, and $c$ is the coefficient of discharge. For various forms at entrance, as shown in Fig. 29, we have the following values:

$$
\text { c. } \quad \frac{1}{c^{2}}-1 .
$$

Pipe projecting into reservoir, Fig. (a) .72 . 93
End of pipe flush with reservoir, Fig. (b) . 82 . 49
Conical or bell-shaped mouth, Fig. (c) or (d) . 93 to . 98.15 to .04
Loss of Head at Bends. For $90^{\circ}$ bends this is equal to

$$
\begin{equation*}
h=n \frac{v^{2}}{2 y} \tag{32}
\end{equation*}
$$

in which $n$ has the following values according to the ratio of the radius of the pipe $r$ to the radius of curvature $R$ :

| r | . 1 | . 2 | . 3 | . 4 | . 5 | . 6 | . 7 | . 8 |  | . 9 | 1.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $n$ | . 13 | . 14 | . 16 | . 21 | . 29 | . 44 | . 66 | . 98 |  | 1.41 |  |

Loss of Head in Valves. Weisbach's experiments on small gate-valves gave values for $n$ in the expression $h=n \frac{v^{2}}{2 y}$ as follows: Ratio of height of opening to diameter. $\begin{array}{llllllll}\frac{7}{8} & \frac{3}{4} & \frac{5}{8} & \frac{1}{2} & \frac{3}{8} & \frac{1}{4} & \frac{1}{8}\end{array}$ Values of $n$........................ . . 07 . 26 . 812.15 .51798

In applying the above formula $v$ is the velocity in the pipe.
For a throttle-valve placed at various angles $\theta$ with the axis of the pipe, Weisbach found the following values of $n$ :

| $\theta \ldots$ | $5^{\circ}$ | $10^{\circ}$ | $20^{\circ}$ | $30^{\circ}$ | $40^{\circ}$ | $50^{\circ}$ | $60^{\circ}$ | $65^{\circ}$ | $70^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $n$. | .24 | .52 | 1.5 | 3.9 | 11 | 33 | 118 | 256 | 750 |

Experiments on large gate-valves have been made by Kuichling and by J. W. Smith. The following table gives values of the coefficient $c$ in the expression $\mathrm{Q}=c \mathrm{~A} \sqrt{2 y h}$. In this expression A is the area of the opening, $h$ is the head lost in the valve, Q is the rate of discharge.

TABLE NO. 14.

## Coefficients for Large Gate=Valves.

| $\left.\begin{array}{l}\text { Ratio of height of opening } \\ \text { to diameter }\end{array}\right\}$ | .05 | .1 | .2 | .3 | .4 | .5 | .6 | .7 | .8 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left.\begin{array}{l}\text { Ratio of area of opening } \\ \text { to total area }\end{array}\right\}$ | .05 | .10 | .53 | .36 | .48 | .60 | .71 | .81 | .89 |  |
| Coefficient $c$ for 24-in. valve <br> Coefficient $c$ for 30-in valve | 1.7 | 1.2 | . .0 | .72 | .70 | .77 | .92 | 1.2 | 1.6 |  |

Example. If a pump draws water from a pipe projecting into a reservoir what will be the loss of head at entrance, the velocity of water in the pipe being 6 feet per second

Using equation 31 of Art. 34 the value of $\left(\frac{1}{c^{2}}-1\right)$ is, for this case, about. 93 . The loss of head is then $.93 \times \frac{v^{2}}{2 y}$ which by Table No. $11=.93 \times .56$ or .52 feet. Ans.

If the pipe is flush with the reservoir the loss of head will be only $.49 \times .56$, or .27 feet.

Finally, if the pipe is enlarged to a bell-month or conical form the loss of head will be very small, say $.10 \times .56$ or .056 feet.



## FLOW OF WATER IN OPEN CHANNELS.

35. General Formula. Where water flows in an open channel like a ditch, or a concrete, brick or tile sewer flowing less than full, the inclination of such channel is what furnishes the necessary fall or head to the water for overcoming friction. In this case there is no pressure at any point, and the loss of head from point to point will be the difference in level of the water surface between the given points. This difference in level, or head, after the flow has become steady is equal to the loss of head due to friction in the same distance.

The frictional loss in open channels is expressed by the same general formula as that used for pipes in Art. 28. It is

$$
\begin{equation*}
v=c \sqrt{r s} \tag{33}
\end{equation*}
$$

in which as before

$$
\begin{aligned}
& v=\text { velocity in feet per second, } \\
& c=\text { a coefficient, } \\
& r=\text { hydraulic mean radius = the cross-section of the actual } \\
& \quad \text { stream of water divided by that part of the perim- } \\
& \quad \text { eter that is under water ("wetted perimeter"). } \\
& s=\text { slope of channel, or ratio of fall to length }=\frac{h}{l}
\end{aligned}
$$

For open channels the value of $c$ varies much more than for pipes, as the nature of the channel varies more. Thus the channel may be a smooth tile sewer where $c$ may be 100 or more, which is about the same as for iron pipe; or the channel may be a rough natural water-course for which the value of $c$ will be only 30 or 40 . Estimates of flow in very rough channels are obviously subject to great uncertainties, but for sewers and open masonry drains or conduits, estimates may be quite closely made, as the values of $c$ have been quite well determined.

For convenience the value of $c$ has been expressed in a formula, called Kutter's formula, in which the condition of the channel is taken account of by a special coefficient $n$, called the coefficient of roughness. This formula for ordinary cases is

$$
\begin{equation*}
c=\frac{\frac{1.8}{n}+45}{1+\frac{45 n}{\sqrt{r}}} \tag{34}
\end{equation*}
$$

in which $r=$ hydraulic mean radius in feet, and $n=$ coefficient of ronghness, varying from a value of about . 009 for smooth plank to . 0:30 for natural channels full of stone, etc.

The following are the values of $n$ usually assumed for the various surfaces mentioned: $n$

Channels of well-planed timber . . . . . . . . . . . . . . . . . . . . . . . . 009
. " neat cement or of very smooth pipe . . . . . . . . . . . 010
.. .. unplaned timber or ordinary pipe. . . . . . . . . . . . . 012
.. $\quad$. smooth ashlar masonry or brickwork . . . . . . . . . 013
." "t ordinary brickwork . . . . . . . . . . . . . . . . . . . . . . . . 01 j
". " rubble masonry . . . . . . . . . . . . . . . . . . . . . . . . 017
" in earth free from obstructions. . . . . . . . . . 020 to .025
" with detritus or aquatic plants . . . . . . . . . . . . . . . . 030
After selecting the value of $n$, the value of $c$ can readily be obtained from Table No. 15.

TABLE NO. 15.
Values of $c$ in Kutter's Formula, for Various Values of $n$.

| $r$ in Feet. | Values of $n$. |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | . 009 | . 010 | . 011 | . 012 | . 013 | . 015 | . 017 | . 020 | .025 | . 030 |
| . 1 | 108 | 94 | 82 | 73 | 65 | 53 | 45 | 35 | 26 | 20 |
| . 2 | 129 | 113 | 100 | 89 | 80 | 66 | 56 | 45 | 34 | 26 |
| . 3 | 112 | 121 | 111 | 99 | 90 | 75 | 63 | 52 | 38 | 30 |
| . 4 | 1.0 | 132 | 118 | 106 | 96 | 80 | 69 | 56 | 42 | 34 |
| . 5 | 157 | 139 | 124 | 111 | 101 | 85 | 73 | 60 | 45 | 36 |
| . 6 | 169 | 143 | 128 | 116 | 105 | 89 | 76 | 63 | 48 | 38 |
| . 7 | 166 | 147 | 132 | 119 | 109 | 92 | 79 | 65 | 50 | 40 |
| . 8 | 170 | 151 | 135 | 122 | 112 | 95 | 82 | 68 | 52 | 42 |
| . 9 | 173 | 154 | 138 | 125 | 114 | 97 | 84 | 70 | 54 | 43 |
| 1.0 | 175 | 156 | 140 | 127 | 116 | 99 | 86 | 71 | 55 | 45 |
| 1.2 | 180 | 160 | 145 | 131 | 120 | 103 | 89 | 74 | 58 | 47 |
| 1.4 | 184 | 161 | 148 | 135 | 124 | 106 | 92 | 77 | 60 | 49 |
| 1.6 | 187 | 167 | 151 | 137 | 126 | 108 | 94 | 79 | 62 | 51 |
| 1.8 | 189 | 169 | 153 | 140 | 129 | 110 | 97 | 81 | 64 | 53 |
| 2.0 | 191 | 172 | 155 | 142 | 130 | 112 | 98 | 83 | 65 | 54 |
| 2.5 | 196 | 176 | 160 | 146 | 135 | 116 | 02 | 86 | 69 | 57 |
| 3.0 | 199 | 179 | 163 | 149 | 138 | 119 | 105 | 89 | 71 | 59 |
| 3.5 | 202 | 182 | 166 | 152 | 140 | 122 | 107 | 91 | 73 | 61 |
| 4.0 | 201 | 184 | 168 | 154 | 143 | 124 | 110 | 93 | 75 | 63 |
| 4.5 | 206 | 186 | 170 | 156 | 144 | 126 | 111 | 95 | 77 | 64 |
| 50 | 208 | 188 | 172 | 158 | 146 | 127 | 113 | 97 | 88 | 66 |

36. The Hydraulic Mean Radius $r$. As before explained, this is a name given to the quotient found by dividing the actual crosssection of a stream of water by the "wetted perimeter," or that part of the perimeter of the cross-section of the channel that is under water. In the case of a pipe flowing full, of diameter $d$, the cross-section is $\frac{1}{4} \pi d^{2}$ and the perimeter is $\pi d$, hence the value of $r$ is $\frac{1}{4} \pi d^{2} \div \pi d=\frac{1}{4} d$. For a pipe flowing half full it is, similarly, $\frac{1}{8} \pi d^{2} \div \frac{1}{2} \pi d$ or $\frac{1}{4} d$, the same as when flowing full. When less than half full the cross-section of the stream falls off more rapidly than the wetted perimeter, so that the valne of $r$ decreases. Hence we see from equation 33 that the velocity also falls off.

For any given form of channel filled to a given point the value of $r$ can readily be found by plotting the cross-section to a large scale and measuring the area and the wetted perimeter.

Example. What will be the velocity and discharge of water flowing in a concrete channel 4 ft . wide and 3 ft . deep and having a slope of 1 ft . per $1,000 \mathrm{ft}$.?

Equation 33 must be used. We will first get the values of $r$ and $s$. The value of $r$ is equal to the cross-section of the stream of water divided by the wetted perimeter $=\frac{3 \times 4}{4+3+3}=1.2 \mathrm{ft}$. The slope $s=\frac{1}{1,000}=.001$. The value of $c$ is to be obtained from Table No. 15, $n$ being taken at .013 , say, the same as for brickwork. For $n=.013$ and $r=1.2$ Table No. 15 gives $c=120$. Substituting then in equation 33 we have $v=120 \times \sqrt{1.2 \times .001}$ $=4.16 \mathrm{ft}$. per sec. The discharge will be $4.16 \times 4 \times 3=49.92$ cu. ft. per sec. Ans.
37. Flow Through Ordinary Sewers. Sewers are usually constructed of vitrified earthen pipe or of brick or concrete. For the former material the value of $n$ in equation 34 is usually taken at .013 , and for brick and concrete about .015 . If the concrete is smoothly finished $n$ may be taken at .013 .

The following Table No. 16 gives the velocities and discharges for circular sewers flowing full. For sewers flowing half full the velocity will be the same and the discharge one-half of the given values.

TABLE NO. 16.
Velocity and Discharge for Pipe Sewers ( $n=$.or3);

## Velocity in Feet per Second ( $V$ ); Discharge in Cubic Feet Per Second ( $Q$ ).

(For $n=.011$ add 20 per cent.)
(For $n=.015$ subtract 16 per cent.)

| Fall of | 4-inch. |  | 6 -inch. |  | 8-inch. |  | 10-inch. |  | 12-inch. |  | 15-inch. |  | 18-inch. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { per } \\ 100 \mathrm{ft} . \end{gathered}$ | V | Q | V | Q | V | Q | V | Q | V | Q | V | Q | V | Q |
| 10. | 5.75 | . 50 | 7.99 | 1.57 | 10.04 | 3.50 | 11.94 | 6.51 | 13.73 | 10.78 | 16.24 | 19.93 | 18.59 | 32.86 |
| 5. | 4.06 | . 35 | 5.64 | 1.11 | 7.09 | 2.48 | 8.43 | 4.60 | 9.70 | 7.62 | 11.48 | 14.08 | 13.13 | 23.23 |
| 4. | 3.63 | . 32 | 5.05 | . 99 | 6.34 | 2.21 | 7.54 | 4.11 | 8.65 | 6.80 | 10.26 | 12.59 | 11.74 | 20.73 |
| 3. | 3.15 | . 27 | 4.25 | . 83 | 5.49 | 1.92 | 6.53 | 3.56 | 7.51 | 5.90 | 8.89 | 10.91 | 10.17 | 17.97 |
| 2. | 2.57 | . 22 | 3.56 | . 70 | 4.48 | 1.56 | 5.33 | 2.91 | 6.13 | 4.82 | 7.25 | 8.90 | 8.30 | 14.67 |
| 1. | 1.82 | . 16 | $2.5 \%$ | . 49 | 3.17 | 1.11 | 3.77 | 2.06 | 4.33 | 3.40 | 5.13 | 6.30 | 5.87 | 10.38 |
| . 8 | 1.61 | . 14 | 2.25 | . 44 | 2.83 | . 99 | 3.37 | 1.84 | 3.87 | 3.04 | 4.59 | 5.63 | 5.25 | 9.28 |
| . 6 | 1.38 | . 12 | 1.95 | . 38 | 2.45 | . 86 | 2.92 | 1.59 | 3.35 | 2.64 | 3.97 | 4.89 | 4.55 | 8.04 |
| . 4 |  |  | 1.59 |  | 2.00 | . 69 | 2.38 | 1.30 | 2.74 | 2.15 | 3.24 | 3.97 | 3.70 | 6.55 |
| .2 |  |  |  |  | 1.40 |  | 1.67 | . 91 | 1.91 | 1.51 | 2.27 | 2.79 | 2.60 | 4.60 |
| . 1 |  |  |  |  |  |  | 1.17 |  | 1.35 | 1.06 | 1.60 | 1.96 | 1.83 | 3.24 |
| . 09 |  |  |  |  |  |  |  |  |  |  | 1.51 | 1.86 | 1.73 | 3.06 |
| . 08 |  |  |  |  |  |  |  |  |  |  |  |  | 1.63 | 2.88 |
| . 07 |  |  |  |  |  |  |  |  |  |  |  |  | 1.53 | 2.69 |

TABLE NO. 16.-Continued.

| Fall of Sewer, in Feet per 100 ft . | 20-inch. |  | 22-inch. |  | 24-inch. |  | 30-inch. |  | 33-inch. |  | 36-inch, |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | V | Q | V | Q | V | Q | V | O | V | Q | V | Q |
| 10. | 20.08 | 43.8 | 21.51 | 56.8 | 22.91 | 72.0 | 26.84 | 131.7 | 28.69 | 170.3 | 30.46 | 215.3 |
| 5. | 14.18 | 30.9 | 15.20 | 40.1 | 16.19 | 50.9 | 18.97 | 93.1 | 20.27 | 120.4 | 21.54 | 152.3 |
| 4. | 12.69 | 27.7 | 13.59 | 35.9 | 14.47 | 45.5 | 16.96 | 83.3 | 18.13 | 107.7 | 19.26 | 136.5 |
| 3. | 10.98 | 24.0 | 11.77 | 31.1 | 12.53 | 39.4 | 14.69 | 72.1 | 15.70 | 93.6 | 16.68 | 118.0 |
| 2. | 8.97 | 19.6 | 9.61 | 25.4 | 10.23 | 32.2 | 11.99 | 58.9 | 12.82 | 76.1 | 13.62 | 96.3 |
| 1. | 6.34 | 13.8 | 6.79 | 17.9 | 7.24 | 23.3 | 8.48 | 41.6 | 9.c6 | 53.8 | 9.63 | 68.1 |
| . 8 | 5.67 | 12.4 | 6.07 | 16.0 | 6.47 | 20.3 | 7.58 | 37.2 | 8.11 | 48.1 | 8.61 | 60.9 |
| . 6 | 4.91 | 10.7 | 5.26 | 13.9 | 5.60 | 19.6 | 6.57 | 32.2 | 7.62 | 41.7 | 7.46 | 52.7 |
| . 4 | 4.00 | 8.7 | 4.29 | 11.3 | 4.56 | 14.3 | 5.35 | 26.3 | 5.72 | 34.0 | 6.08 | 43.0 |
| . 2 | 2.81 | 6.1 | 3.01 | 7.9 | 3.21 | 10.1 | 3.76 | 18.5 | 4.62 | 23.9 | 4.28 | 30.2 |
| . 1 | 1.98 | 4.3 | 2.12 | 5.6 | 2.26 | 7.1 | 2.66 | 13.0 | 2.84 | 16.9 | 3.02 | 21.3 |
| . 09 | 1.87 | 4.1 | 2.01 | 5.3 | 2.14 | 6.7 | 2.51 | 12.3 | 2.69 | 16.0 | 2.86 | 20.2 |
| . 08 | 1.76 | 3.8 | 1.89 | 5.0 | 2.02 | 6.3 | 2.37 | 11.6 | 2.53 | 15.0 | 2.69 | 19.0 |
| . 07 | 1.64 | 3.6 | 1.76 | 4.6 | 1.88 | 5.9 | 2.20 | 10.8 | 2.36 | 14.0 | 2.51 | 17.7 |
| . 06 | 1.51 | 3.3 | 1.63 | 4.3 | 1.73 | 5.4 | 2.04 | 10.0 | 2.18 | 12.9 | 2.32 | 16.4 |
| . 05 |  |  | 1.48 | 3.9 | 1.58 | 5.0 | 1.86 | 9.1 | 1.99 | 11.8 | 2.11 | 14.9 |
| . 04 |  |  | 1.32 | 3.5 | 1.40 | 4.4 | 1.65 | 8.1 | 1.77 | 10.5 | 1.88 | 13.3 |
| . 03 |  |  |  |  | 1.20 | 3.8 | 1.40 | 6.9 | 1.52 | 9.0 | 1.62 | 11.4 |
| . 02 |  |  |  |  | 0.96 | 3.1 | 1.13 | 5.6 | 1.22 | 7.2 | 1.30 | 9.2 |

TABLE NO. 17.
Velocity and Discharge for Brick and Concrete Sewers ( $n=.015$ ); Velocity in Feet per Second $(V)$; Discharge in Cubic Feet

Per Second ( $Q$ ).
(For $n=.013$ add 19 per cent.)
(For $n=.017$ subtract 13 per cent.)

| Fall of Sewer in Feet per 100 ft . | 33-inch. |  | 36-inch. |  | 42-nch. |  | 4-foot. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | V | Q | V | Q | V | Q | V | Q |
| . 5 | 17.17 | 102.0 | 18.27 | 129.2 | 20.37 | 196.1 | 22.36 | 281.1 |
| . 4 | 15.36 | 91.2 | 16.34 | 115.5 | 18.21 | 175.3 | 20.00 | 251.3 |
| . 3 | 13.30 | 79.0 | 14.15 | 100.0 | 15.77 | 151.8 | 13.31 | 217.6 |
| . 2 | 10.85 | 64.5 | 11.55 | 81.7 | 12.88 | 123.9 | 14.13 | 177.6 |
| . 1 | 7.68 | 45.6 | 8.16 | 57.7 | 8.90 | 87.6 | 9.99 | 125.6 |
| . 8 | 6.86 | 40.7 | 7.30 | 51.6 | 8.14 | 78.3 | 8.93 | 112.3 |
| . 6 | 5.94 | 35.2 | 6.32 | 44.6 | 7.04 | 67.8 | 7.73 | 97.2 |
| . 4 | 4.84 | 28.8 | 5.15 | 36.4 | 5.75 | 54.0 | 6.31 | 79.3 |
| . 2 | 3.41 | 20.3 | 3.63 | 25.7 | 4.05 | 39.0 | 4.45 | 55.9 |
| . 1 | 2.40 | 14.3 | 2.52 . | 18.1 | 2.85 | 27.5 | 3.13 | 39.4 |
| . 09 | 2.27 | 13.5 | 2.42 | -17.1 | 2.70 | 26.0 | 2.97 | 37.3 |
| . 08 | 2.14 | 12.7 | 2.28 | 16.1 | 2.55 | 24.5 | 2.80 | 35.2 |
| . 07 | 2.00 | 11.9 | 2.13 | 15.0 | 2.38 | 2).9 | 2.61 | 32.9 |
| . 06 | 1.85 | 11.0 | 1.97 | 13.9 | 2.20 | 21.1 | 2.42 | 30.4 |
| . 05 | 1.68 | 10.0 | 1.79 | 12.6 | 1.95 | 18.8 | 2.20 | 27.6 |
| . 04 | 1.49 | 8.9 | 1.59 | 11.3 | 1.78 | 17.1 | 1.96 | 24.6 |
| . 03 | 1.28 | 7.6 | 1.37 | 9.7 | 1.53 | 11.7 | 1.68 | 21.2 |
| . 02 |  |  |  |  | 1.23 | 11.9 | 1.36 | 17.1 |
| .15 |  |  |  |  |  |  | 1.16 | 14.6 |

TABLE NO. 17.-Continued.

| Fall of Sewer, in Feet per 100 ft . | 5-foot. |  | 6-foot. |  | 8-foot. |  | 10-foot. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | V | - Q | V | $Q$ | V | Q | V | Q |
| 5. | 26.05 | 512 |  |  |  |  |  |  |
| 4. | 23.30 | 457 | 26.34 | 745 |  |  |  |  |
| 3. | 20.17 | 396 | 22.81 | 645 |  |  |  |  |
| 2. | 16.47 | 323 | 18.62 | 527 | 22.53 | 1133 | 26.03 | 2045 |
| 1. | 11.64 | $2: 8$ | 13.17 | 37: | 15.93 | 801 | 18.41 | 1446 |
| . 6 | 10.41 | 204 | 11.78 | 333 | 14.25 | 716 | 16.46 | 1293 |
| . 8 | 9.01 | 177 | 10.19 | 288 | 12.33 | 620 | 14.25 | 1119 |
| . 4 | 7.36 | 144 | 8.32 | 235 | 10.07 | 506 | 11.63 | 914 |
| . 2 | 5.19 | 102 | 5.87 | 166 | 7.10 | 357 | 8.21 | 645 |
| . 1 | 3.66 | 72 | 4.14 | 117 | 5.02 | $25 \%$ | 5.81 | 456 |
| . 09 | 3.47 | 68 | 3.92 | 111 | 4.76 | 239 | 5.51 | 433 |
| . 08 | 3.27 | 64 | 3.70 | 105 | 4.49 | 226 | 5.19 | 408 |
| . 07 | 3.05 | 60 | 3.46 | 98 | 4.20 | 211 | 4.86 | 38. |
| . 06 | 2.82 | 55 | 3.20 | 90 | 3.88 | 195 | 4.50 | 353 |
| . 05 | 2.57 | 50 | 2.92 | 82 | 3.54 | 178 | 4.10 | 322 |
| . 04 | 2.29 | 45 | 2.60 | 74 | 3.16 | 159 | 3.66 | 288 |
| . 03 | 1.97 | 39 | 2.24 | 63 | 2.73 | 137 | 3.17 | 249 |
| .02 | 1.60 | 31 | 1.82 | 51 | 2.23 | 112 | 2.58 | 203 |
| . 015 | 1.37 | 27 | 1.56 | 44 | 1.93 | 97 | 2.23 | 175 |
| . 012 |  |  | 1.39 | 39 | 1.70 | 86 | 1.99 | 156 |
| . 010 |  |  |  |  | 1.55 | 78 | 1.81 | 143 |
| $.0095$ |  |  |  |  | 1.25 | , 63 | 1.77 | 139 |
| . 0090 |  |  |  |  |  |  | 1.72 | 135 |

## MEASUREMENT OF THE FLOW OF STREAMS.

38. General Methods. For measuring the flow of a small stream the best method is by the use of a weir constructed of plank and built into a temporary dam of earth. Such weirs can readily be used for streams up to 3 or 4 feet in depth and 40 or 50 feet wide, although streams normally of such size would have flood flows many times greater and which could not be so measured. Where a dam already exists in a stream, observations of the flow over such a dam will give fairly good results when the coefficient of discharge is carefully selected as noted in Art. 25.

Where a weir cannot be used, then the flow must be measured by actually determining the mean velocity of the flow at a given


Fig. 30.
section and the area of such cross-section, then the discharge will be equal to the product of these quantities.
39. Variations in Velocity. Owing to the disturbing effect of the bottom and sides of a channel, the velocity of the water will not be the same at all points in a given cross-section. In general the velocity will be greater near the center of a stream than near the edges, and will be greater where the water is deep than where it is shallow. Thus if Fig. 30a represents the cross-section of a stream, the velocity of flow along the surface will vary in some such way as is represented in Fig. 7 , being greatest near the deep-
est parts and very small near the banks. Likewise if we consider the velocities along the vertical section 2 they will vary somewhat as shown in Fig. $c$, and at section 6 they will be as shown in Fig. $d$. In both Figs. $d$ and $c$ the maximum velocity is shown to be a little below the surface. This is usually the case, although it depends somewhat on the effect of the wind.

From these statements it will be seen that there are great variations in the velocity thronghout the cross-section, and therefore the determination of the average velocity is not readily accomplished.

Instead of trying to get the average velocity through the entire cross-section, it is usual to divide the section of the stream into several vertical strips as shown in Fig. $a$. Then get the average velocity and discharge of each strip separately. In doing this a place should be selected where the flow is as uniform and the channel as regular as possible. In case floats are used to get velocities, as described later, it is necessary to establish two sections 100 feet apart or more, between which points the velocities are measured. In either case careful soundings must be taken and an accurate plot made of the cross-section, and the area of each division $a_{1}, a_{2}$, etc., determined. The divisions of the section may be marked by knots or tags on a rope stretched across the channel. The sections having been divided off, it remains to determine the average velocity in each.
40. Use of the Current Meter. The most accurate method of finding the velocity is by means of the current meter, one form of which is illustrated in Fig. 31.

The essential part of the current meter consists in the series of cups mounted on a wheel with vertical axis shown at the left of the vertical rod. This wheel being submerged, is rotated by the current, and the number of revolutions is recorded by an electrical device which may be held in a boat or on shore. The long vane attached to the wheel is to keep the meter always parallel with the current. A heavy weight is attached to the bottom of the rod to keep the meter steady, the whole apparatus being suspended by means of a rope from a boat or bridge. The number of revolutions per minute of the wheel being known, the velocity of the water at the wheel is calculated by multiplying by a coefficient determined by previous experiments with the meter.

The average velocity for any given strip is determined either by getting the velocity along the center of the strip at several different depths and taking the average, or by moving the meter slowly from top to bottom and then back to the top and taking a single reading. Whichever way determined the resulting velocity multiplied by the area of the strip in question equals the discharge of that strip. Then the total discharge equals the sum of the discharges of all the strips.

The coefficient to use in calculating actual water velocities


Fig. 31.
from meter readings is determined by a "rating" of the meter. This rating is done by moving the meter at various known velocities through still water in a reservoir, pond, or canal. Then knowing the velocity of the meter through the water and its readings, a rating curve or table of coefficients can be worked out.

4i. Use of Floats. Very often a meter is not at hand, and a less accurate method must be employed. That most often used is by means of floats. These are of three kinds-surface floats,
subsurface floats, and rod floats. The best form is the rod float.
The rod float is a rod of wood, or a tube of tin, which is weighted at one end so that it will float in an upright position and as near to the bottom of the stream as practicable. The float is then placed in the stream at the desired point, and far enough up stream from the upper of two measured cross-sections so that it will acquire the same velocity as the water by the time it reaches such section. The time of its passage from the upper to the lower section is then observed and its velocity deduced therefrom. In this way observations are made for each of the vertical strips in which the stream section is divided. The average velocity of each strip is taken equal to that of the rod itself.

The surface float may be made of any convenient form which will be readily seen from the point of observation. Its use will give only the surface velocities of the several strips and not the desired average velocities. To get the average velocity, we may use the approximate formula,

$$
\text { Average velocity }=.9 \times \text { surface velocity }
$$

whence the discharge of the several strips can be calculated as before. This method is not so accurate as the use of rod floats and is not to be recommended except for very rough determinations. It is much influenced by the wind, and observations should, if possible, be made on still days.

Sometimes a very rough determination is desired from one or two measurements of velocity. If the surface velocity is measured at a point where it is a maximum (near the center of the stream), then the average velocity for the entire stream may be taken at about $\frac{8}{10}$ of the measured velocity, although the exact value of this coefficient will vary between quite wide limits. The discharge then equals the total cross-section multiplied by the average velocity.

The sub-surface float consists of a submerged body a little heavier than water that is attached by means of a fine cord to a surface float of much smaller size. The sub-surface float can be adjusted to float at any desired depth. By setting it at mean depth the observed velocity will be approximately the average velocity of the vertical strip. The use of such floats is not looked upon with much confidence. Rod floats are much better.

## WATER-POWER DEVELOPMENT

## PART I

1. Introduction. One of the fundamental teachings of science is that all energy in the solar system is derived from the sun. Through the agency of that luminary, water from the earth's oceans, seas, and lakes is transformed into vapor, and in this condition is diffused throughout the atmosphere, transported by the winds-themselves created by this same solar energy-over long distances and wide areas, and finally precipitated over land and water, hills and valleys, mainly in the form of rain and snow. Of the total precipitation on the continents, part-is evaporated from land and water surfaces, vegetation, etc.; part runs off more or less rapidly as surface flow into the nearby drainage channels, and thence, more or less directly, to the ocean; and part sinks into the ground. Of this last, a portion is retained by capillary attraction within reach of vegetation, to be taken up slowly by the rootlets and transpired through the leaves; the balance percolates downward until it reaches the surface of the underground water flow, which it joins in its relatively slow motion to some nearby stream, lake, or other drainage course, or directly to the ocean. It is then again evaporated into the atmosphere, with a continuous repetition of the cycle described above.

Thus every elevated body of water, every running stream, is a source of power whose energy has been derived or borrowed from the sun; and under proper conditions, a large proportion of this energy may be transformed into useful work.
2. Unit of Work. For industrial purposes, the unit of work most generally adopted is the foot-pound (ft.-lb.), which represents the quantity of work done in lifting a mass of one pound through a height of one foot against the opposing force of gravity-or in raising a weight of one pound through a height of one foot. Since the force of gravity, and therefore the weight of a given mass, is not constant for
all points on the surface of the earth, it follows that the foot-pound, or gravitation-measure of work, is not a constant unit. Its variation, however, is so small as to be negligible for ordinary purposes; and, being much simpler than the theoretically accurate units which must occasionally be employed in scientific investigation, it has remained in very general use. Thus the work done in raising 20 pounds of water through a height of 1 foot, or 1 pound of water through a height of 20 feet, or 5 pounds of water through a height of 4 feet, is said to be 20 foot-pounds.
3. Power. In the preceding definition, the element of time was not considered; thus, in the above example, 20 foot-pounds of work were done, whether the indicated operation took one minute to perform or extended over a period of one hour, or longer. The term power is defined as the amount of energy that can be exerted, or work done, in a given time.
4. Unit of Power. For industrial purposes, the unit most commonly employed is the horse-power (h.p.), which represents the capacity to perform 33,000 foot-pounds of work in one minute, or 550 foot-pounds of work in one second; it thus indicates the rate of work.

Example 1. A pump raising $7,500,000$ gallons of water* in 10 hours to an elevated tank 50 feet high, is performing:

$$
\begin{aligned}
\frac{7,500,000 \times 62.5 \times 50}{7.5} & =3,125,000,000 \mathrm{ft} .-\mathrm{lbs} . \text { of useful work; or, } \\
\frac{3,125,000,000}{10 \times 60} & =5,208,333 \mathrm{ft} .-\mathrm{lbs} . \text { per minute }
\end{aligned}
$$

which is equivalent to:

$$
\frac{5,208,333}{33,000}=157.8 \mathrm{~h} . \mathrm{p} .
$$

This amount of horse-power is the rate of work which, in the example above, must be continued for 10 hours in order to raise the total quantity of water. The entire problem may be conveniently performed in one operation, thus:

$$
\frac{7,500,000 \times 62.5 \times 50}{7.5 \times 10 \times 60 \times 33,000}=157.8 \mathrm{~h} . \mathrm{p} .
$$

5. Energy. The amount of energy existing in any agent is measured by the quantity of work it is able to do; energy and work

[^0]are therefore measured by the same unit. "When energy is exerted, work is done against resistance." As usually stated in Theoretical Mechanics, energy may exist as potential energy-energy of position; or kinetic energy - energy of motion; or partly in one form, and partly in the other. Thus (see Fig. 1) a cannon-ball weighing $W$ pounds, located in an elevated position $h$ feet above any plane of reference, possesses Wh foot-pounds of potential energy with respect to that plane, by virtue of its position. If it be allowed to fall to the plane, it will, at its lowest point, theoretically have acquired a velocity of $v(=2 g h)$ feet per second, and will therefore, at that level, possess kinetic energy to the amount of $W \frac{v^{2}}{2 g}(=W h)$ foot-pounds by reason of its motion. Further, if we analyze the conditions at some intermediate plane $h_{1}$ feet below its original position, and $h_{2}$ feet above the lower level, we shall find that the ball has acquired at this point a velocity of $v_{1}(=$ $\sqrt{2 g h_{1}}$ ) feet per second, and therefore possesses kinetic energy to the amount of $W \frac{v_{1}{ }^{2}}{2 g}$ ( $=W h_{1}$ ) foot-


Fig. 1. Illustrating Relation between Potential and Kinetic Energy. pounds due to its motion; but, by reason of its position $h_{2}$ feet above the lower plane, it still possesses $W h_{2}$ foot-pounds of potential energy; consequently, with respect to the lower plane, the ball possesses a total energy represented by $W\left(\frac{v_{1}{ }^{2}}{2 g}+h_{2}\right)=W\left(h_{1}+h_{2}\right)=W h=W \frac{v^{2}}{2 g}$ foot-pounds. Thus potential and kinetic energies are mutually convertible, theoretically without loss; practically, more or less energy will be transformed into heat during the conversion, and dissipated. But the great principle of the Conservation of Energy teaches that the total quantity of energy existing, or stored in the ball in any position, is theoretically a constant quantity.
6. Pressure $=$, Velocity $=$, and Gravity=Head. In hydraulic work, because of the nature of the medium dealt with-water being considered in this connection a perfect fluid, and incompressible - and because of the character of the problems presented, it is customary and convenient to consider the energy of water as capable of existing in three forms-Pressure, Velocity, and Gravity. Thus, in Fig. 2, with the conditions as represented (see also "Hydraulics," page 34), if the valve at $D$ be closed, the water will rise in tube $C C$ (called a piezometer tube) to the same level $E F$ as that existing in the reservoir, and the pressure in the pipe at $C$ will be represented by the head $h$


Fig. 2. Illustrating Relations of Pressure-, Velocity-, and Gravity-Head.
feet. Now, if the valve at $D$ be partially opened, so that there is some velocity of flow $v_{1}$, in the pipe at section $C$, the column of water in the tube $C C$ will sink to some lower level, as $C C^{1}$, and the pressure in the pipe at $C$ will be that due to the head $h_{1}$ feet. *Similarly, if the valve be now completely opened, so that the velocity of flow $v_{2}$, in the same section, becomes greater than $v_{1}$, the column of water in the tube will sink still lower, as $C C^{\prime \prime}$, indicating a pressure in the pipe at $C$ represented by the head $h_{2}$ feet. If the loss of head in friction, etc., in the two cases of flow indicated above be respectively represented by $h_{f}^{\prime}$ and $h_{f}^{\prime \prime}$, the important relations existing are clearly shown in this diagram. It is evident that at the end of the pipe, where the water discharges freely into the air, no pressurehead exists, all the energy possessed by the issuing water being kinetic.
7. Total Head. Now let $G G^{1^{1}}$ represent any horizontal plane of reference-for example, the level of the tail-race water in a hydraulic power plant. With reference to this plane, the total effective head existing in the pipe at the section $C$, is:
(a) For the case of no flow-
$z+h=H$ feet;
(b) For the case of partial flow-

$$
z+h_{1}+\frac{v_{1}^{2}}{2 g}=H-h^{\prime f} \text { feet }
$$

(c) For the case of full flow-

$$
z+h_{2}+\frac{v_{2}{ }^{2}}{2 g}=H-h_{f}^{\prime \prime} \text { feet. }
$$

The distance $z$ may be called the gravity-head (it corresponds to the head in potential energy already referred to); $\frac{v_{1}{ }^{2}}{2 g}$ and $\frac{v_{2}{ }^{2}}{2 g}$ are properly termed the velocity-heads (they correspond to the heads in kinetic energy already explained); $h_{1}$ and $h_{2}$ are known as the pressure-héads (see "Hydraulics," Article 6); $h_{f}^{\prime}$ and $h_{f}^{\prime \prime}$ represent the heads lost in overcoming the various resistances to flow, principally friction in the pipe for the usual cases; but in the general case they include losses of head due to entrance, valves, curves, etc. (see "Hydraulics," Articles 28 and 34).
8. Energy per Pound of Water. The quantities stated above as number of feet in $(a),(b)$, and (c) may be understood in another sense. Each may represent the total number of foot-pounds of energy existing in every pound of water in, or passing through, the pipe at section $C$; thus,
(a) $z+h=\quad H \quad$ foot-pounds per pound of water
(b) $z+h_{1}+\frac{v_{\mathrm{k}}^{2}}{2 g}=H-h_{f}^{\prime}$
(c) $z+h_{2}+\frac{v_{2}{ }^{2}}{2 g}=H-\mathrm{h}^{\prime \prime} f$
9. Total Energy. Now suppose $W_{1}$ and $W_{2}$ pounds of water per second respectively to pass the section $C$ in the two cases of flow considered; then, with respect to the plane $G G^{1}$, the total energy of the water as it passes this section is, for the one case:
(b) $W_{1}\left(z+h_{1}+\frac{v_{1}{ }^{2}}{2 g}\right)$ foot-pounds;
and for the other:
i(c) $W_{2}\left(z+h_{2}+\frac{v_{2}{ }^{2}}{2 g}\right)$ foot-pounds;
and these expressions represent, for the two cases considered, the total amount of energy possessed by the water, with respect to the plane $G G^{1}$, and theoretically capable of being delivered to a machine
or motor, by the descent of the water from the upper level EF to the lower level $G G^{1}$.

Where the water issues freely into the air from the extremity of the pipe, or through a nozzle at the end, no pressure exists; therefore, in the expression corresponding to (b) or (c), above, for such section, the term representing pressure-head disappears, leaving the two terms indicating gravity-head and velocity-head.

Further, if the plane of reference passes through the center of the end of the pipe or nozzle opening, the term representing gravityhead also disappears, leaving the velocity-head alone to indicate the energy of the stream at this point.

It is usually more convenient to express the sum of gravity-head and pressure-head in a single term: thus, $z+h_{1}=H_{1}$; and $z+h_{2}$ $=H_{2}$; here $H_{1}$ and $H_{2}$ may be called the piezometer heights.
10. Efficiency. The efficiency of any apparatus for utilizing the kinetic energy of moving water, or the potential energy of still water, is the ratio of the amount of work given out by the apparatus to the amount of work delivered to it; or, as it is sometimes stated, it is the ratio of the useful work to the theoretic energy. This topic will be treated more fully in a later article; for the present, if $e$ represent the efficiency of a motor, then,

$$
e=\frac{\text { Foot-pounds or horse-power given out by motor }}{\text { Foot-pounds or horse-power delivered to motor }}
$$

As will be seen later, the denominator does not represent the full theoretic energy of the waterfall, since more or less of this energy must be utilized in overcoming the resistances encountered in conducting the water to the motor.

Example 2. A motor is operated by a stream of water discharged through a 2 -foot pipe with a velocity of 10 feet per second. The motor gives out at its shaft 4.4 horse-power. What is the efficiency of the motor?

$$
\frac{3.1416 \times 10 \times 62.5 \times 100}{550 \times 64.4}=5.5 \text { horse-power delivered to motor }
$$

$e=\frac{4.4}{5.5}=80$ per cent efficiency.
Example 3. A small turbine wheel using 100 cubic feet of water per minute under a head of 45 feet, is found to give 6 horse-power. What is the efficiency of the wheel?

$$
\begin{aligned}
6 \text { Horse-Power } & =6 \times 33,000=198,000 \mathrm{ft} .-\mathrm{lbs} . \text { per min. } \\
e & \frac{198,000}{100 \times 62.5 \times 45}=70.4 \text { per cent efficiency. }
\end{aligned}
$$


INTERIOR OF GENERATING STATION OF THE ONTARIO POWER COMPANY, NIAGARA FALLS, ONT.

Theoretic Efficiency. If the efficiency of the motor actuated by the water were 100 per cent, it would give out at its shaft, as useful work, the same number of foot-pounds that were delivered to it. It is also interesting to note that if the efficiency of the hydraulic parts of the plant were 100 per cent-that is, if there were no hydraulic losses of head-the total energy of the water (see Fig. 2) represented by the total head $H$ feet, or $H$ foot-pounds per pound of water, would be available; and, if operating a motor of 100 per cent efficiency, the total energy of the water would be given out as useful work at the shaft of the motor. In practice these ideal conditions can never be fully realized, for there are certain hydraulic and mechanical losses of cnergy, which, while they may be reduced to the lowest limits by means of proper design, nevertheless, cannot be entirely eliminated.

Example 4. A pond containing $2,000,000$ cubic feet of water is at an average elevation of 50 feet above the lower level. How much potential energy does this theoretically represent at the lower level?

$$
2,000,000 \times 62.5 \times 50=6,250,000,000 \mathrm{ft} . \mathrm{llbs} .
$$

If this water is fed to a small motor at the rate of 100 cubic feet per minute, what horse-power does this represent, and how long may the motor be operated?

$$
\begin{aligned}
& \frac{100 \times 62.5 \times 50}{33,000}=9.5 \mathrm{~h} . \mathrm{p} \\
& \frac{2,000,000}{100 \times 60 \times 24}=13 \frac{7}{8} \text { days, or } 13 \text { days } 21 \text { hours. }
\end{aligned}
$$

Assuming that the motor has an efficiency of 75 per cent, how much power may be taken off at its shaft?

$$
9.5 \times .75=7.1 \mathrm{h.p}
$$

Example 5. The discharge of a stream is 1,000 cubic feet per second; its mean velocity is 3 feet per second. What horse-power does this represent?

$$
\frac{1,000 \times 62.5 \times(3)^{2}}{550 \times 64.4}=1,588.1 \mathrm{~h} . \mathrm{p}
$$

Example 6. Water issues from a nozzle at the rate of 50 feet per second; the area of the nozzle opening is 0.1 square foot How many foot-pounds of kinetic energy does this represent? How many horse-power? If this jet operates a motor of 80 per cent efficiency, what horse-power will the motor actually yield?

$$
\begin{aligned}
& 0.1 \times 50 \times 62.5 \times \frac{(50)^{2}}{64.4}=21,125 \mathrm{ft} . \text {-lbs. per second. } \\
& \frac{21,125}{550}=22 \mathrm{h.p.} \\
& 22 \times .80=17.6 \mathrm{~h} . \mathrm{p} .
\end{aligned}
$$

11. Pipe End with Nozzle. Pressure at Base of Nozzle. For many purposes - as in hydraulic mining, in the operation of certain types of water motor (described later), and at the extremity of fire-hose-water is delivered at considerable velocity through a nozzle attached to the end of a pipe. It is therefore desirable to develop a formula for velocity of flow, and quantity of discharge, for such' cases.

If the pressure-head $h_{1}$ (Fig. 3) at the entrance or base of a smooth nozzle be observed, either by a piezometer tube or by a pressure


Fig. 3. Pipe with Nozzle Attachment.
gauge, then, since the nozzle velocity $V$ is a consequence of the pressure-head $h_{1}$ and the velocity-head $\frac{v^{2}}{2 g}$ of the water in the pipe approaching the nozzle with a velocity of $v$ feet per second, the real or effective head on the nozzle is $h_{1}+\frac{v^{2}}{2 g}$; the theoretic velocity from the nozzle is:

$$
V=\sqrt{2 g\left(h_{1}+\frac{v^{2}}{2 g}\right)}
$$

and the actual velocity is:

$$
V=c_{\mathrm{i}} \sqrt{2 g\left(h_{1}+\frac{v^{2}}{2 g}\right)},
$$

in which $c_{1}$ denotes the coefficient of velocity, which, for smooth nozzles, is the same as the coefficient of discharge. In these equations, $h_{1}$ is expressed in feet; $V$ and $v$ in feet per second. Let $D$ and $d$ be the diameters, in feet, of the nozzle and pipe respectively.

Since the Discharge $q=$ Area $\times$ Velocity,

$$
q=\frac{\pi D^{2}}{4} V=\frac{\pi d^{2}}{4} v ;
$$

therefore,

$$
v=\left(\frac{D}{d}\right)^{2} V
$$

Substituting this value of $v$ in the equation above, and solving for $V$, there results:

$$
\begin{equation*}
V=\sqrt{\frac{2 g h_{1}}{\left(\frac{1}{c_{1}}\right)^{2}-\left(\frac{D}{d}\right)^{4}}} \tag{1}
\end{equation*}
$$

in feet per second; and the discharge (area $\times \mathrm{V}$ ) is: $\qquad$

$$
\begin{equation*}
q=0.7854 D^{2} \sqrt{\frac{2 g h_{1}}{\left(\frac{1}{c_{1}}\right)^{2}-\left(\frac{D}{d}\right)^{4}}} \tag{2}
\end{equation*}
$$

in cubic feet per second; and the velocity-head of the issuing jet is:

$$
\begin{equation*}
\frac{V^{2}}{2 g}=\frac{h_{1}{ }^{2}}{\left(\frac{1}{c_{1}}\right)^{2}-\left(\frac{D}{d}\right)^{4}} \tag{3}
\end{equation*}
$$

In many cases it is common to read the pressure at the base of the nozzle in pounds per square inch; then $h_{1}$ (in feet) equals $2.304 p_{1}$ (in pounds per square inch); and the discharge is frequently stated in gallons per minute; making these substitutions in Equation 2, above, we have:

$$
\begin{equation*}
q=29.83 D^{2} \sqrt{\frac{p_{1}}{\binom{1}{c_{1}}^{2}-\left(\frac{D}{d}\right)^{4}}} . \tag{4}
\end{equation*}
$$

in gallons per minute.
Example 7. The pressure-gauge at the base of a smooth $1 \frac{1}{4}$-inch nozzle reads 80 pounds per square inch; compute the velocity and discharge from the nozzle, the velocity-head of the issuing stream, and the mean velocity in the pipe, if the latter be $2 \frac{1}{2}$ inches in diameter. Assume 0.97 as the value of the coefficient.

Substituting the given numerical values in Equation 1, we have:

$$
\begin{aligned}
V & =\sqrt{\frac{64.4 \times(2.304 \times 80)}{\left(\frac{1}{.97}\right)^{2}-\left(\frac{1}{2}\right)^{4}}}=38.5 \text { foot per second. } \\
q & =\text { Area } \times V=\frac{0.7854 \times 1.25}{144} \times 38.5=0.33 \text { cubic foot per second. } \\
\frac{V^{2}}{2 g} & =\frac{(38.5)^{2}}{64.4}=22.7 \text { feet. } \\
v & =\left(\frac{D}{d}\right)^{2} V=\frac{1}{4} \times 38.5=9.6 \text { feet per second. }
\end{aligned}
$$

What horse-power does this represent?

$$
\frac{0.33 \times 62.5}{550} \times 22.7=0.85 \mathrm{~h} . \mathrm{p}
$$

With a motor of 80 per cent efficiency, how much useful work will be obtained?

$$
0.85 \times 0.80=0.68 \mathrm{~h} . \mathrm{p}
$$

12. Pipe Line with Nozzle. In Fig. 4 , let $h$ be the total head on the end of the nozzle, $D$ its smaller diameter in feet, and $V$ the velocity of the issuing stream in feet per second. Let $d$ and $v$ be the corre-


Fig. 4. Loss of Head in Pipe and Nozzle.
sponding quantities for the pipe or hose, and $l$ its length in feet. Of the total available head $h$ on the end of the nozzle, only $\frac{V^{2}}{2 g}$ remains; so that $h-\frac{V^{2}}{2 g}$ represents the head lost or dissipated in overcoming various resistances to flow, from the reservoir to the tip of the. nozzle. This lost head consists of several parts (see "Hydraulics," Article 34), we may therefore write:

$$
\begin{equation*}
h-\frac{V^{2}}{2 g}=\left\{\left(\frac{1}{c}\right)^{2}-1\right\} \frac{v^{2}}{2 g}+\frac{8 g l}{C^{2} d} \frac{v^{2}}{2 g}+m \frac{v^{2}}{2 g}+n \frac{v^{2}}{2 g}+m^{\prime} \frac{V^{2}}{2 g} \ldots \tag{7}
\end{equation*}
$$

in which,
$\left\{\left(\frac{1}{c}\right)^{2}-1\right\} \frac{v^{2}}{2 g}=$ Loss of head at entrance; $\frac{8 g l}{C^{2} d} \frac{v^{2}}{2 g}=$ Head lost in friction in the pipe (see "Hydraulics," Articles 28 and 36 ); $m \frac{v^{2}}{2 g}=$ Head lost in bends and curves; $n \frac{v^{2}}{2 g}=$ Head lost by the passage of the water through valves and gates; and, lastly, $m^{\prime} \frac{V^{2}}{2 g}=$ Head lostin passing through the nozzle.

The equation for the value of $m^{1}$ assumes a form similar to that for entrance loss into a pipe:

$$
m^{1}=\left\{\left(\frac{1}{c_{1}}\right)^{2}-1\right\}
$$

in which $c_{1}$ is the coefficient of velocity, which, for smooth nozzles, is the same as the coefficient of discharge; its value may be taken as 0.97 for such nozzles, with the small diameter between $\frac{3}{4}$ inch and $1 \frac{1}{2}$ inches, under ordinary range of pressures.

Since, in steady flow, the velocities $v$ and $V$ are inversely proportional to the areas of the corresponding cross-sections,

$$
V=v\left(\frac{d}{D}\right)^{2}
$$

Inserting this value of $V$ in Equation 5, and solving for $v$, there results:

$$
\begin{equation*}
v=\sqrt{\frac{2 g h}{\left\{\left(\frac{1}{c}\right)^{2}-1\right\}+\frac{8 g l}{C^{2} d}+m+n+\left(\frac{1}{c_{1}}\right)^{2}\left(\frac{d}{D}\right)^{4}}} \cdots \tag{6}
\end{equation*}
$$

for the velocity of flow in the pipe, in feet per second.
The velocity and discharge from the nozzle are then:

$$
\begin{equation*}
V=\left(\frac{d}{D}\right)^{2} v, \tag{7}
\end{equation*}
$$

and,

$$
\begin{equation*}
q=\frac{1}{4} \pi D^{2} V \tag{8}
\end{equation*}
$$

In many cases the sum of the losses at entrance, through valves and gates, and around bends and curves, is sufficiently small, in comparison with the loss in pipe friction, to be negligible; in such cases, Equation 6 reduces to

$$
\begin{equation*}
v=\sqrt{\frac{2 g h}{\frac{8 g l}{C^{2} d}+\left(\frac{1}{c_{1}}\right)^{2}\left(\frac{d}{D}\right)^{4}}} \cdots \cdots \cdots \cdot \tag{9}
\end{equation*}
$$

Example 8. A smooth nozzle with a small diameter of 1 inch is attached to a 3 -inch pipe 1,500 feet long; the tip of the nozzle is 64 feet below the surface of the water in an elevated reservoir. Assume $C=100$, and determine the velocity of flow in the pipe, and through the nozzle. Find also the discharge, and the efficiency of the pipe and nozzle.

Since in this case the entrance loss is relatively small, because the pipe is long in comparison with its diameter, and therefore pipe friction is relatively large, Equation 9 may be used:

$$
v=\sqrt{\frac{64.4 \times 64}{\frac{8 \times 32.2 \times 1,500}{(100)^{2} \times 0.25}+\left(\frac{1}{.97}\right)^{2}\left(\frac{3}{1}\right)^{4}}}=4.14 \text { feet per second, }
$$

for the velocity of flow in the pipe.

$$
V=v\left(\frac{d}{D}\right)^{2}=4.14 \times 9=37.26 \text { feet per second, }
$$

for the velocity of the jet issuing from the nozzle.

$$
q=\frac{\pi d^{2}}{4} v=\frac{3.1416 \times\left(\frac{1}{4}\right)^{2}}{4} \times 4.14=0.20 \mathrm{cu} . \mathrm{ft} . \text { per second. }
$$

The energy of the jet is:

$$
W \frac{V^{2}}{2 g}=\frac{.20 \times 62.5 \times(37.26)^{2}}{64.4}=269.5 \mathrm{ft.} . \mathrm{lbs} . \text { per second. }
$$

The theoretic energy is:

$$
W h=.20 \times 62.5 \times 64=800 \mathrm{ft} .-\mathrm{lbs} . \text { per second. }
$$

The efficiency of pipe and nozzle, therefore, is:

$$
\frac{269.5}{800}=33.7 \text { per cent }
$$

13. If, under the conditions just stated, we suppose the nozzle removed, the last term in the denominator of Equation 9 will disappear, and the equation will assume the form:

$$
\begin{equation*}
v=\frac{C}{2} \sqrt{\frac{h d}{l}}=C \sqrt{r \frac{n}{l}}=C \sqrt{r s} \tag{10}
\end{equation*}
$$

which is Equation 30 in "Hydraulics," for the case of a pipe of uniform diameter; or Equation 33, for flow in open channels.
14. Equation 7, taken in connection, with Equation 6 or its simpler form, Equation 9, shows that the smaller the nozzle diameter compared with that of the pipe, within ordinary practical limits, the greater will be the nozzle velocity; but the greatest discharge will occur (Equation 8) when the nozzle diameter is as large as possible; that is, when it is equal to the pipe diameter-in other words, when there is no nozzle attached.
15. Relation of Pipe and Nozzle Diameters. When the object of attaching a nozzle to a pipe is to utilize the velocity-head of the issuing jet $\left(=\frac{V^{2}}{2 g}\right)$ without regard to the quantity of water discharged, a large pipe and a relatively small nozzle should be employed. When the object is to obtain as large a discharge as possible, no nozzle should be used, and the pipe-should be as large as practical considerations will warrant. When the object is to utilize the energy of the jet in producing power by means of a water-motor, in which case both velocity-head and quantity of discharge are concerned, there is a definite relation existing between the diameters of nozzle and pipe that will render this a maximum.
16. Maximum Power Derivable from Nozzle Jet. From Equations 9 and 7, we derive:

$$
\begin{equation*}
V=\sqrt{\frac{2 g h}{\frac{8 g l}{C^{2} d}\left(C_{d}^{D}\right)^{4}+\left(\frac{1}{c_{1}}\right)^{2}}} \tag{11}
\end{equation*}
$$

Then, if $w$ be the weight in pounds of a cubic foot of water, we have, for the theoretical kinetic energy of the issuing jet in foot-pounds per second (weight of discharge in pounds per second $X$ velocity-head):

$$
\begin{equation*}
K=w \frac{1}{3} \pi D^{2} V \frac{V^{2}}{2 g}=\frac{w \pi D^{2} V^{3}}{8 g} . \tag{12}
\end{equation*}
$$

Substituting in this equation the value of $V$ above (Equation 11), and ascertaining, by the procedure usually adopted in such cases (differential calculus), the value of $D$ to render $K$ a maximum, we obtain:

$$
\begin{equation*}
D=\frac{1}{2}\left(\frac{C^{2} d^{5}}{g c_{1}{ }^{2} l}\right)^{\frac{3}{3}}, \tag{13}
\end{equation*}
$$

which is a formula for diameter of nozzle in terms of diameter and length of pipe (all in feet) to produce the maximum kinetic energy of the jet issuing from the nozzle.

With a nozzle of this diameter, the velocity of the issuing jet is obtained by placing the value of $D$ from Equation 13 in Equation 11, with the result:

$$
V=2 c_{1} \sqrt{\frac{g h}{3}}=c_{1} \sqrt{2 g\left(\frac{2}{3} h\right)}=0.816 c_{1} \sqrt{2 g h} \ldots \text { (14) }
$$

Since the value of $c_{1}$ for ordinary cases is about 0.97 , it may be said that the nozzle velocity necessary to produce the maximum power is about 80 per cent of the theoretic velocity due to the actual static head on the nozzle tip.
17. Relation between Total Head and Friction Head for Maxi= mum Power. The relation expressed by Equation 14 leads to some interesting conclusions. Since $V=.80 \sqrt{2 g h}$ for maximum power, $\frac{V^{2}}{2 g}=.64 h$; therefore, since the total head is $h, .36 h$ must be used in overcoming pipe and nozzle resistance, to give the most advantageous velocity for power purposes. Again, omitting nozzle resistance (as represented by $c_{1}$ ), $\frac{V^{2}}{2 g}=.667 h$; therefore $.333 h$ must be used in overcoming pipe friction alone. That is to say, with the conditions
arranged to furnish maximum power, $\frac{1}{3}$ of the total static head on the nozzle tip is being used to overcome pipe friction, and the remaining ${ }_{3}^{2} h$ is transformed into the velocity-head of the issuing stream after due deduction or allowance for nozzle resistance. The second value of $V$ (Equation 14) shows this directly. If no nozzle is attached, therefore, the conditions for maximum power obtain when $\frac{1}{3}$ the total static head is used in overcoming pipe friction, the remaining $\frac{2}{3}$ of the head being available as velocity-head, or as pressure-head, or partly in one form and partly in the other.
18. Usually the discharge in cubic feet per second $(q)$ is known; then, by simple substitution (Equations 8 and 14), the values for maximum work are:

$$
\begin{equation*}
D=\left(\frac{12 q^{2}}{\pi^{2} c_{1}^{2} g h}\right)^{1} . \tag{15}
\end{equation*}
$$

and, from Equations 13 and 15:

$$
\begin{equation*}
d=2\left(\frac{6 q^{2} l}{\pi^{2} C^{2} h}\right)^{\frac{2}{s}} \tag{16}
\end{equation*}
$$

in which $D$ and $d$ are the diameters in feet of nozzle tip and pipe to furnish maximum power. Being stated in terms of $q, l$, and $h$, these equations are occasionally the most convenient to use in solving problems.

Example 9. By damming a stream, an impounding reservoir was created, capable of supplying uniformly 5.92 cubic feet of water per second to a powerhouse below. The nozzle tip is to be 590 feet below the average water level in the reservoir; the length of pipe is 6,000 feet from reservoir to nozzle; the pipe being of riveted steel, and making due allowance for deterioration of surface with age, $C$ was assumed to have the low value 83 . What size pipe and nozzle should be used to give the maximum power? What will be the nozzle velocity? What horse-power will be developed at the nozzle? What efficiency does this represent for pipe and nozzle? What power may be derived from a wheel of 75 per cent efficiency, driven by the jet? What is the efficiency of the whole system?
From Equation 16:

$$
d=2\left\{\frac{6 \times(5.92)^{2} \times 6,000}{(3.1416)^{2} \times(83)^{2} \times 590}\right\}^{\frac{1}{b}}=1 \text { foot, pipe diameter. }
$$

From Equation 15:
$D=\left\{\frac{12 \times(5.92)^{2}}{(3.1416)^{2} \times(.97)^{2} \times 32.2 \times 590}\right\}^{\frac{1}{2}=2.67 \text { inches, nozzle diameter, }}$
or Equation 13 may be used to determine $D$.
From Equation 14:
$V=0.816 \times 0.97 \sqrt{64.4 \times 590}=152$ feet per second, nozzle velocity.

$$
\begin{aligned}
\text { Horsc-power }=\frac{W V^{2}}{2 g \times 550} & =\frac{5.92 \times 62.5 \times(152)^{2}}{64.4 \times 550}=241 \mathrm{~h} . \mathrm{p} . \\
\text { Theoretic horse-power } & =\frac{W h}{550}=\frac{5.92 \times 62.5 \times 590}{550}=397 \mathrm{~h} . \mathrm{p} . \\
\text { Efficiency } & =\frac{241}{397}=61 \text { per cent (nearly). } \\
\text { Useful work from wheel } & =241 \times .75=181 \mathrm{~h} . \mathrm{p} . \\
\text { Efficiency of whole system } & \left.=.75 \times .61=46 \text { per cent (or } \frac{18}{3} \frac{8}{9} \frac{1}{7}\right) .
\end{aligned}
$$

19. Multiple Nozzles. Sometimes an impulse wheel is driven by means of jets issuing from two or more nozzles of the same or of different diameters. Then, for maximum power, the sum of the areas of the several nozzles must equal the area corresponding to $D$, as computed for a single nozzle, on the assumption that the nozzle tips are at substantially the same level, and that the coefficient $c_{1}$ has the same value for each. Thus, if there be two nozzles with diameters $D_{1}$ and $D_{2}$,

$$
\begin{equation*}
D_{1}{ }^{2}+D_{2}{ }^{2}=\frac{1}{4}\left(\frac{C^{2} d^{5}}{g c_{1}{ }^{2} l}\right)^{\frac{1}{2}}=\frac{C d^{2}}{4 c_{1}} \sqrt{\frac{d}{g l}} . \tag{17}
\end{equation*}
$$

One diameter, as $D_{1}$, may be assumed, and the other computed from the above relation.

If the two nozzles are of equal diameter $D_{1}$,

$$
D_{1}{ }^{2}=\frac{1}{8}\left(\frac{C^{2} d^{5}}{g c_{1}{ }^{2} l}\right)^{\frac{1}{2}} ;
$$

therefore,

$$
\begin{equation*}
D_{1}=\frac{1}{2}\left(\frac{C^{2} d^{5}}{4 g c_{1}^{2} l}\right)^{\frac{1}{1}} . \tag{18}
\end{equation*}
$$

If the value of $D$ for one nozzle has already been determined, then, for two nozzles of equal diameter $D_{1}$, from the relation stated above,

$$
\frac{2 \pi D_{1}{ }^{2}}{4}=\frac{\pi D^{2}}{4}
$$

therefore,

$$
\begin{equation*}
D_{1}=\frac{D}{\sqrt{2}} \cdots \ldots \ldots . \tag{18a}
\end{equation*}
$$

With three or more nozzles, of the same or of different diameters, the relation of areas stated above will furnish a means of readily determining the diameters. Thus, for three nozzles of equal diameter $D_{1}$,

$$
\begin{equation*}
D_{1}=\frac{D}{\sqrt{3}} \tag{18b}
\end{equation*}
$$

If the discharge $q$ is known, an analysis similar in all respects to that above will give, in place of Equation 17:

$$
\begin{equation*}
D_{1}^{2}+D_{2}^{2}=\frac{2 q}{\pi c_{1}} \sqrt{\frac{3}{g h}} ; \tag{19}
\end{equation*}
$$

and, in place of Equation 18:

$$
\begin{equation*}
D_{1}=\left(\frac{3 q^{2}}{\pi^{2} c_{1}^{2} g h}\right)^{\frac{1}{4}} \tag{20}
\end{equation*}
$$

which will prove more convenient for use in some problems.
Example 10. If, in example 9, two nozzles of equal diameter were required, the diameter of each nozzle could be determined directly from Equation 18; or more simply, from Equation 18a, since the value of $D$ has already been found:

$$
D_{1}=\frac{D}{\sqrt{2}}=\frac{2.67}{1.41}=1.9 \text { inches for each nozzle. }
$$

If three equal nozzles were required, then, from Equation 18b:

$$
D_{1}=\frac{D}{\sqrt{3}}=\frac{2.67}{1.73}=1.5 \text { inches for each nozzle. }
$$

## I MPULSE, REACTION, AND DYNAMIC PRESSURE

20. Impulse and Reaction of Water in Motion. Let $W$ be the number of pounds of water discharged per second from an orifice, pipe, or nozzlé, or flowing in a stream, with a uniform velocity of $v$ feet per second; then,

$$
\begin{equation*}
F=W \frac{v}{g} \text { pounds } . \tag{21}
\end{equation*}
$$

is called the impulse of the moving water. It may be regarded as a continuous pressure in the direction of motion; and it will be exerted as such upon a surface placed in the path of the jet or stream, with an intensity varying with the conditions, and ranging to the maximum value $F$, above. The reaction, or back-pressure, is equal in value to the impulse, but opposite in direction. For example, if a vessel containing water be freely suspended at $A$ (Fig. 5), and water be allowed to flow out through an orifice at $B$, the pressure due to the head of water $h$ causes $W$ pounds of water per second to be discharged with the velocity $v(=$ theoretically $\sqrt{2 g h})$ feet per second. In the direction of the jet, the impulse produces motion; in the opposite direction, it produces an equal back-pressure (action and reaction being equal in amount and opposite in direction), causing the vessel to swing to the right. The first of these forces is the impulse, and the
second is the reaction of the jet; and if a force $R$ be applied as shown, of just sufficient intensity to prevent this motion of the vessel, its value is:

$$
\begin{equation*}
R=W \frac{v}{g}=F, . . \tag{22}
\end{equation*}
$$

which is the reaction of the jet.
21. The impulse or reaction of a jet issuing from an orifice is double the hydrostatic pressure on the area of the orifice. For, if $\boldsymbol{a}$ is the area of the orifice, and $w$ the weight of a cubic unit of water, the normal hydrostatic pressure on the area of the orifice when clostú (see "Hydraulics," Article 6) is:
Hydrostatic pressure
$=$ wah pounds.
When the orifice is


Fig. 5. Measuring the Reaction of a Jet by Weighing. opened, the weight oi: the discharge per second (see "Hydraulics," Article 18) is theoretically $W=$ wav; hence,

$$
\begin{equation*}
F=R=W \frac{v}{g}=\text { wav } \frac{v}{g}=\frac{2 w a v^{2}}{2 g}=2 \text { wah. } \tag{23}
\end{equation*}
$$

This conclusion has been verified ly many experiments (see Fig. 6).
Example 11. What must be the velocity of a jet of water 1 inch in diameter, issuing from a nozzle, in order that its impulse may be 100 pounds? What will be the discharge in cubic feet and in gallons per seeond?

$$
\begin{aligned}
F & =\frac{W v}{g}=\frac{w a v^{2}}{g}=100 \\
\therefore v & =\sqrt{\frac{100 \times 32.2}{62.5 \times .0054}}=97.7 \text { foot per second. } \\
q & =a v=.0054 \times 97.7=.53 \text { cubic foot per second. } \\
& .53 \times 7.5=4 \text { gallons per second. }
\end{aligned}
$$

22. Dynamic Pressure of Water in Motion. If a jet of water strike a stationary plane normally, it produces a dynamic pressure on that plane equal to the impulse of the jet; that is:

$$
P=F=W \frac{v}{g}
$$

If a jet moving with a velocity $v_{1}$ be retarded by a surface so that its velocity becomes $v_{2}$, without changing its direction, the impulse in the first case is:

$$
F_{1}=\frac{W v_{1}}{g} ;
$$

and in the second case:

$$
F_{2}=\frac{W v_{2}}{g} \text {; }
$$

and the difference,

$$
\begin{equation*}
P=F_{1}-F_{2}=W\left(\frac{v_{1}-v_{2}}{g}\right) \tag{24}
\end{equation*}
$$

is a measure of the dynamic pressure which has been developed in


Fig. 6. Illustrating Relation between Impulse and Hydrostatic Pressure.
the direction of motion by the retardation of the velocity. If a jet of water impinge upon a stationary surface which changes its direction of motion without changing its velocity, a dynamic pressure is developed, its amount depending upon the velocity and the change in direction. In all cases this pressure is exerted upon the surface causing the retardation of velocity or change in direction of flow.
23. Static and Dynamic Pressures. Dynamic pressure must be clearly distinguished from static pressure, the laws governing in the two cases being entirely different. A static pressure due to a given head will cause a jet of water to be discharged from an orifice with a velocity proportional to the head; if this jet impinge upon a surface, a dynamic pressure will be exerted upon it, which may be equal to, greater than, or less than the static pressure due to the heaa,


UNIT CONSISTING OF TWO PAIRS OF 34-INCH "IMPROVED NEW AMERICAN" TURBINES ON HORIZONTAL SHAFT, FOR USE IN
depending upon the circumstances. Again, at any point below the surface of water, static pressure is exerted with equal intensity in all directions; dynamic pressure is exerted with different intensities in different directions.
24. Definitions. From a comparison of Equations 21 and 24, we may now define the impulse of a jet or stream of water as the dynamic pressure which it is capable of producing in the direction of its motion when its velocity in that direction is entirely destroyed. This may be accomplished by carefully deflecting the jet 90 degrees to its original path by means of a smooth surface, so that, no energy being dissipated in overcoming frictional or other resistances, the velocity of the water is not changed, but its component in the original direction is zero; and the reaction of a jet or stream of water may be defined as the backward dynamic pressure, in the line of mo-

tion, which is exerted against a vessel out of which it issues, or against a surface away from which it moves.
25. Laboratory Experiments on Impulse, Reaction, and Dynamic Pressure. Fig. 5 shows how the reaction of a jet may be measured; the necessary weight in the scale pan to prevent motion of the vessel has been found to be very nearly:

$$
R=F=\frac{W v}{g}=2 w a \frac{v^{2}}{2 g} .
$$

Fig. 6 shows how the pressure due to the impulse of a jet may be made to balance the hydrostatic pressure due to twice the head causing the flow. $B$ is a loose plate with surface carefully finished to fit the mouthpiece so as to prevent leakage. Fig. 7 illustrates a simple device for measuring by weighing the dynamic pressure exerted upon a surface by the impulse of a jet impinging upon and gliding over it,
when its motion in the original direction has been entirely destroyed by being deflected 90 degrees. The result of the experiment is found to show very nearly that:

$$
P=W \frac{v}{g}=2 w a \frac{v^{2}}{2 g},
$$

as theory requires.
Fig. 8 illustrates a case of dynamic pressure exerted upon a curved surface, due to both impulse and reaction, the former being due


Fig. 8. Measuring Pressure from a Jet on a Curved Surface, by Weighing.


Fig. 9. Measuring Pressure from a Jet whose Direction is Completely Reversed. to the direct impact of the jet, the latter to the circumstance that the deflected stream leaves the surface in a direction which has a component of velocity parallel to the original path, but opposite in direction. Here experiment shows:
$P>W \frac{v}{g}<2 W \frac{v}{g}$, as theory requires.

Fig. 9 shows the case where the stream is deflected 180 degrees; that is, there is a complete reversal in the direction of motion; and we should expect the dynamic pressure exerted upon the surface to be equal to the sum of both impulse and reaction; namely,

$$
P=F+R=2 F^{\prime}=2 W \frac{v}{g}=4 \text { wa } \frac{v^{2}}{2 g},
$$

which agrees quite closely with the results of laboratory experiments.
Example 12. In Fig. 7 the diameter of the tube is 1 inch; there is no contraction of the jet; and the discharge is .5 cubic foot per second. What is the velocity, and the dynamic pressure against the plane? What would be the dynamic pressure in the case represented by Fig. 9?

$$
\begin{aligned}
v & =\frac{q}{a}=\frac{5}{.0054}=92.6 \text { feet per second } \\
P & =W \frac{v}{g}=\frac{(.5 \times 62.5 \times 92.6}{32.2}=90 \text { pounds } \\
P & =2 W \frac{v}{g}=2 \times 90=180 \text { pounds }
\end{aligned}
$$

## FIXED SURFACES

20. Dynamic Pressures on Fixed Surfaces. When a stream of water impinges with a uniform velocity $v$ on a smooth surface at rest, it glides over the surface and leaves it with the original velocity $v$, since there are supposed to be no frictional or other resistances, only its direction of motion being changed. The water, as it strikes the


Fig. 10. Illustrating Case of No Dynamic Pressure.
surface, exerts upon it an impulse $F$ in the direction of the path of entry; as it leaves the surface, it exerts on it an equal reaction $F$, in a direction opposite to its path of exit (see Figs. 11 to 14). The dynamic pressure thus developed depends upon velocity $v$, and change of direction of stream (angle $\theta$ ). The stream is assumed to be moving horizontally while in contact with the surface, so that its velocity is not affected by gravity.
27. Resultant Dynamic Pressure. From the principle of Composition of Forces (Mechanics), the resultant dynamic pressure upon a fixed surface struck by a jet may be readily found by constructing the parallelogram of the forces of impulse and reaction, as shown in Fig. 15, in which $a b=b c=F=R$; from which we deduce (Trigonometry) that the value of this resultant pressure is:

$$
\begin{equation*}
P_{\mathrm{R}}=F \sqrt{2(1-\cos \theta)}=2 \sin \frac{1}{2} \theta \cdot W^{v} . \tag{25}
\end{equation*}
$$

and that it makes an angle of $\left(90^{\circ}-\frac{1}{2} \theta\right)$ with the original direction of the jet. Its line of action passes through the intersection of $F$ and $R$, and it bisects the angle between them.
28. Dynamic Pressure Parallel to Initial Direction of Jet. This is simply the component of the Resultant Dynamic Pressure in the
desired direction. From Fig. 16, this is found to be (Resolution of Forces) $a b=b c \cos \left(90-\frac{1}{2} \theta\right)$; so that,

$$
\begin{equation*}
P_{\mathrm{J}}=P_{\mathrm{R}} \cos \left(90-\frac{1}{2} \theta\right)=(1-\cos \theta) W \frac{v}{g} \cdots \tag{26}
\end{equation*}
$$

If, in this equation, $\theta=0$, the stream glides over the surface without change of direction or retardation of velocity, and $P=0$;


Fig. 11.


Fig. 14.
Illustrating Dynamic Pressure of Jet on Various Fixed Surfaces.
that is, no dynamic pressure is exerted upon the surface (see Fig. 10).
If $\theta=90^{\circ}, \cos \theta=0$ (see Figs. 7 and 12); and therefore the dynamic pressure is:

$$
P=F=W \frac{v}{g}
$$

Here the escaping jet has no component of velocity normal to the surface; therefore the reaction has no influence on the pressure.

If $\theta=180^{\circ}$ (see Figs. 9 and 14), indicating a complete reversal


INTERIOR VIEW OF POWER PLANT OF THE DALTON POWER COMPANY, DALTON, MASSACHUSETTS

turbine installation for a paper mill


## Courtesy of S. Morgan Smith Co., York, Pa.

in the direction of the stream, $\cos \theta=-1$; hence the dynamic pressure is:

$$
P=2 F=2 W \frac{v}{g} .
$$



Fig. 15. Resultant Dynamic Pressure.
Here the pressure is a consequence of both impulse and reaction to their full amount.
29. Dynamic Pressure in Any Given Direction. It is frequently of importance to determine the dynamic pressure in a given direction exerted on a fixed surface by a stream of water. This may be ascertained by resolving the resultant dynamic pressure into its two components, parallel and at right angles to the required direction; the former represents the pressure in the required direction. Or the impulse and reaction may be separately resolved into their rectangular components, as above, and the algebraic sum taken of the two components parallel to the required direction. Thus, in Fig. 17, let it be required to find the dynamic pressure in a direction represented by the arrow $x$, which makes an angle $a$ with the


Fig. 16. Dynamic Pressure Parallel to Initial Direction of Jet. direction of the entering, and an angle $\theta$ with that of the departing stream. The components of the impulse and the reaction in the required direction, since $R=F$, are:

$$
P_{1}=F \cos \alpha ; \text { and } P_{2}=-F \cos \theta ;
$$

and therefore:

$$
P=P_{1}+P_{2}=F(\cos \alpha-\cos \theta)=(\cos \alpha-\cos \theta) W_{g}^{v}(\mathbf{2 7})
$$

If, in this general equation (27), $a=0^{\circ}$,

$$
P=(1-\cos \theta) W \frac{v}{g},
$$

as in Equation 26.
If $a=0^{\circ}$, and $\theta=90^{\circ}$,

$$
P=F \doteq W \frac{v}{g}, \text { as in Figs. } 7 \text { and } 12
$$

If $a=0^{\circ}$, and $\theta=180^{\circ}$,

$$
P=2 F=2 W \frac{v}{\mathrm{~g}}, \text { as in Figs. } 9 \text { and } 14 .
$$

If $a=0^{\circ}$, and $\theta=0^{\circ}, P=0$, as in Fig. 10 .
Example 13. Let the jet of Problem 7 impinge tangentially upon the fixed curved vane of Fig. 15, with $0=60^{\circ}$. What is the resultant dynamic pressure upon the vane, in intensity and direction? What is the dynamic pressure in a direction parallel to the jet? What is the dynamic pressure in a direction making an angle of 30 degrees with the direction of the jet?
From Equation 25 and Problem 7:

$$
\begin{aligned}
P_{\mathrm{R}} & =2 \sin \frac{1}{2} 0 W \frac{v}{g} \\
& =2 \times \frac{1}{2} \times \frac{.33 \times 62.5 \times 38.5}{32.2}=24.7 \text { pounds. }
\end{aligned}
$$

From Equation 26:

$$
\begin{aligned}
P_{\mathrm{J}} & =(1-\cos \theta) \mathrm{W} \frac{v}{g} \\
& =\left(1-\frac{1}{2}\right) \frac{.33 \times 62.5 \times 38.5}{32.2}=12.4 \text { pounds. }
\end{aligned}
$$

From Equation 27 (Fig. 17):

$$
\begin{aligned}
P & =(\cos \alpha-\cos \theta) W \frac{v}{g} \\
& =(.866-.500) \frac{.33 \times 62.5 \times 38.5}{32.2}=4.5 \text { pounds. }
\end{aligned}
$$

30. Weight of Water Impinging. In all the preceding equations, $W$ represents the weight of water in pounds per second impinging upon the surface; and, since the surface has in each case been assumed to be stationary, $W$ is also the weight of water in pounds per second issuing from the nozzle or orifice, or flowing in the stream. It is to be clearly kept in mind that this statement is not necessarily true if the surface is supposed to move; as, for example, in the case of a jet impinging upon the vanes or blades of a water wheel. Such cases will be considered later.
31. Force and Work. It must also be clearly realized that the dynamic pressures are forces; they are not expressed in terms of energy or work; just as a weight resting upon a table produces pressure thereon, but does not perform work. A force must be exerted against a resistance through a definite distance, in order that work may be done; the weight may be allowed to move, and thereby compress a spring, for example, thus doing work. Similarly, the above pressures must be exerted against resistances over some definite distances, in order that work may be done. In general, if $P$ is the


Fig. 17. Dynamic Pressure in Any Given Direction.
dynamic pressure on the surface in pounds, and if the surface is supposed to move a distance of $u$ feet per second while overcoming some resistance, then,

$$
\begin{equation*}
\text { Work }=P \times u \text { foot-pounds per second } \ldots \tag{28}
\end{equation*}
$$

It is by reason of the dynamic pressures defined and explained above, produced by a retardation in velocity, or a change in direction of flow, that turbine wheels and other water-motors are able to transform the kinetic energy of moving water into useful work-such pressures being exerted over definite distances against resistances.
32. Losses of Energy. In the above discussion, no frictional or other losses of energy were considered. It is clear that if the surfaces are rough, or if the jet impinges on the surface in such a way as to produce "shock" or "eddies" or "foam," some of the original energy of the jet will be dissipated as heat, and the resulting pressures will be correspondingly reduced below the values indicated by the fore-
going formulæ. These losses may be largely eliminated by having the surfaces smooth and properly curved, and by so directing the jet as to strike the surface tangentially.

## ABSOLUTE AND RELATIVE VELOCITIES

33. Definitions. While all velocities are in reality relative, it is convenient to define absolute velocity as the rate of speed of a moving object with respect to the surface of the earth; and relative velocity as the rate of speed of a moving object with respect to another moving body-or as the velocity the object would appear to have to a person standing upon, and viewing it from, the second moving body. In the one case, velocity is measured from, or referred to, the earth,


Fig. 18. Illustrating Absolute and Relative Velocities.
which is regarded as stationary; in the other case, the velocity is measured from, or referred to, the second moving body, regarded as stationary for this purpose. Thus, let Fig. 18 represent a tank so mounted that it may move horizontally to the right with a uniform absolute velocity of $u$ feet per second; and let water issue from the various openings as indicated. Theoretically, the following absolute and relative velocities will result:

| Orifice | Relative Velocity (to tank) | Absolute Velocity (referred to the earth) | 0 | Cos 0 |
| :---: | :---: | :---: | :---: | :---: |
| A | $V=\sqrt{2 g h}$ | $v=V+u$ | $0^{\circ}$ | 1 |
| $B$ | $V=$ " | $v=V-u$ | $180^{\circ}$ | -1 |
| C | $V=$ | $v=\sqrt{V^{2}+u^{2}}$ | $90^{\circ}$ | 0 |
| D | $V=$ " | $v=\sqrt{V^{2}+u^{2}+2 V u \cos 0}$. | 0 | $\begin{gathered} \cos 0 \\ \text { (positive) } \end{gathered}$ |
| E | $V=$ " | $v=\sqrt{V^{2}+u^{2}+2 V u \cos \theta}$ | 0 | (positive) $\cos 0$ (negative) |

The expression for absolute velocity from orifice $D$ or $E$ may be regarded as a general formula, and the formule for the other cases
may be simply derived from it by assigning the proper values to $\theta$. These considerations of absolute and relative velocities are of great importance in determining the dynamic pressures produced by a stream of water on the moving vanes or blades of water-motors. For example, consider Fig. 19, which represents a revolving wheel having an orifice from which water issues horizontally with the relative velocity $V$ (velocity relative to wheel), while the orifice itself is moving horizontally with an absolute velocity $u$ (velocity relative to the ground); then, from what has preseded.
$v=\sqrt{V^{2}+u^{2}+2 V u \cos \theta}$. (29)
is the absolute velocity of the


Fig. 19, Velocity of Stream Leaving or Striking Revolving Vane. water as it leaves the wheel (velocity with respect to the ground). In all cases, then, the absolute velocity of a stream of water striking or leaving a moving surface is represented in magnitude and direction by the diagonal of a parallelogram of which one side is the velocity of the stream relative to the moving surface, and the other side is the absolute velocity of that surface (with reference to the ground); i.e., it is the resultant of these two velocities.

If the directions of the component velocities lie in the same straight line, $\theta=0^{\circ}$ or $180^{\circ}$; and, applying Equation 29, we derive the special formulæ:

$$
\begin{equation*}
v=V+u ; \text { or, } v=V-u . \tag{29a}
\end{equation*}
$$

## SURFACES MOVING IN A STRAIGHT LINE

34. Dynamic Pressure on Moving Surfaces. When a stream of water impinges upon a moving surface, the conditions are essentially different from those just discussed for surfaces at rest. Because the surface is continually moving away from the stream, two important results follow-the stream does not strike the surface with its full or absolute velocity, and the quantity of water reaching the sur.. face per second is less than the stream discharge.
35. Case I. Jet Striking a Moving Flat Vane Normally. Let a jet (Fig. 20) whose absolute velocity is $v$, and cross-section $a$, impinge normally upon a smooth surface which is itself moving with a uniform absolute velocity $u$ in the same direction as the jet. The relative velocity of the jet, or the velocity with which it strikes the surface, is $v-u$; the weight of water leaving the orifice per second is


Fig. 20. Jet Striking a Moving Flat Vane Normally.


Fig. 21. Jet Striking Flat Radial Vanes of a Revolving Wheel.
$W=w a v$; the weight of water striking the surface per second is wa $(v-u)$, if $w$ represents the weight of a cubic unit of water; accordingly, the dynamic pressure exerted upon the surface, in the direction of motion, is:

$$
\begin{equation*}
P=w a(v-u) \frac{(v-u)}{g}=\frac{w a}{g}(v-u)^{2} \tag{30}
\end{equation*}
$$

which is equivalent to considering the surface stationary, and the stream moving with an absolute velocity of $(v-u)$ feet per second.
36. Work Done upon (or Given Up to) the Moving Body per Second. The work done in one second by the force $P$ (Force $\times$ Distance) is:

$$
\begin{equation*}
\text { Work }=P u=\frac{w a(v-u)^{2} u}{g} \tag{31}
\end{equation*}
$$

The work is zero if $u=v$; or $u=0$; and it is a maximum, and equal to:

$$
\begin{equation*}
\text { Work }(\text { Max. })=\frac{4}{27} \frac{w a v^{3}}{g}=\frac{8}{27} W \frac{v^{2}}{2 g} . \tag{32}
\end{equation*}
$$

when $u=\frac{1}{3} v$.
37. Efficiency. Since the theoretic energy of the impinging jet is $W \frac{v^{2}}{2 g}$, the efficiency in the case just considered is $\frac{8}{27}$, or about

30 per cent. It is evident, however, that no practical motor could be constructed on such a plan.

Case II. This represents a wheel (Fig. 21) provided with many flat radial vanes against which, in rapid succession, a jet of water impinges. The resultant action of the jet in this case is not precisely the same as in the preceding example; but if we assume that the jet impinges normally on the vanes, and that, as the vanes come in rapid succession under the influence of the jet, and several vanes are more or less under action at the same time, the quantity of water impinging is the same as the nozzle discharge ( $W=$ wav ); also, that the vanes move away from the jet in the direction of the latter while under impact, then we obtain for the approximate value of the dynamic pressure, if $u$ represents the linear absolute velocity of the vanes at the center of impact:

$$
\begin{equation*}
P=W \frac{v-u}{g}=\operatorname{wav} \frac{v-u}{g} \tag{33}
\end{equation*}
$$

38. Work Done upon (or Given Up to) the Wheel per Second.

$$
\begin{equation*}
\text { Work }=P u=\frac{w a v}{g}(v-u) u \text {. } \tag{34}
\end{equation*}
$$

The work is zero if $u=v$, or $u=0$; and it is a maximum and equal to:

$$
\text { Work }(\text { Max. })=\frac{1}{\frac{1}{2}} \frac{w a v^{3}}{g}=\frac{1}{2} W \frac{v^{2}}{2 g} \ldots(\mathbf{3 5})
$$

when $u=\frac{1}{2} v$.
39. Efficiency. Since the jet has a theoretic energy of $W \frac{v^{2}}{2 g}$ foot-pounds, it is seen that the highest efficiency that can theoretically be obtained by means of a jet impinging upon rotating flat vanes is 50 per cent.

The preceding analysis applies more directly to the case of a series of flat vanes moving in a straight line, as indicated in Fig. 20 , and coming in rapid succession under the influence of the jet. A motor constructed on this plan is, however, impracticable.
40. Case III. Jet Striking a Moving Curved Vane Tangentially. Fig. 22 represents a case in which the jet, with an absolute velocity $v$, impinges tangentially upon a vane which moves in the same direction with the uniform absolute velocity $u$. The velocity of the stream relative to the surface is $v-u$; and the dynamic pressure is the same as though the surface were at rest, and the stream

moving and impinging with the absolute velocity $v-u$. Hence, for the dynamic pressure in the direction of the jet, we may use Equation 26, substituting $v-u$ for $v$; so that,

$$
\begin{equation*}
P=(1-\cos \theta) W \frac{v-u}{g} \tag{36}
\end{equation*}
$$

While the dynamic pressure may be exerted with different intensities upon different parts of the vane, the total value, in the direction of motion, is that indicated by Equation 36.
41. Work Done. If $a$ is the area of the cross-section of the jet, the weight of water issuing from the nozzle per second is $W=$ wav; the weight striking the vane is wa $(v-u)$; and therefore the work is:

$$
\begin{equation*}
\text { Work }=P u=(1-\cos \theta) \frac{w a}{g}(v-u)^{2} u \tag{37}
\end{equation*}
$$

The work is zero when $v=u$, and when $u=0$; also when $\theta=0^{\circ}$; and it is a maximum, and equal to:

$$
\begin{gather*}
\text { Work }(\text { Max. })=\frac{4}{27}(1-\cos \theta) \text { wa } \frac{v^{3}}{g}=\frac{8}{27} \\
\quad(1-\cos \theta) W \frac{v^{2}}{2 g} \cdots \ldots \ldots(38) \tag{38}
\end{gather*}
$$

when $u=\frac{1}{3} v$.
42. Efficiency. Since the theoretic energy of the impinging jet is $W \frac{v^{2}}{2 g}$,
the efficiency is: the efficiency is:

$$
\begin{equation*}
e=\frac{8}{27}(1-\cos \theta) . \tag{39}
\end{equation*}
$$

If $0=0^{\circ}$, work $=0$, and $e=0$; in this case the vane is a flat surface whose plane is in the direction of the stream, which therefore glides over the surface without doing work.


hydro-electric power plant of the kalamazoo valley electric company at otsego, michigan
Consists of eight 56 -inch "Samson" turbines, all driving one powerful generator. $C$ Courteey of James Leffel \& Co., Sprinafield, ohio.

If $\theta=90^{\circ}$, the water leaves the vane at right angles to the direction of motion, and the maximum work, from Equation 38, is:

$$
\begin{equation*}
\text { Work }(\text { Max. })=\frac{8}{2 \tau} W_{\overline{v^{2}}}^{v^{2}} \tag{40}
\end{equation*}
$$

and the efficiency is $\frac{8}{27}$, or about 30 per cent. (Compare with Equation 32.)

If $\theta=180^{\circ}$, the stream is completely reversed. In this case, (since $\cos 180^{\circ}=-1$ ),

$$
\begin{equation*}
\text { Work (Max.) }=\frac{16}{8} W \frac{v^{2}}{2 g} \tag{41}
\end{equation*}
$$

and the efficiency is $\frac{1}{2} \frac{6}{7}$, or about 60 per cent.
43. Case IV. If, instead of a simple curved vane, as in the preceding case, we consider a wheel with a large number of such vanes, as in Fig. 23, and assume the jet to impinge tangentially, and the vanes to move in the direction of the jet while under its influence, and also the quantity of water impinging to be equal to the nozzle discharge, by an analysis similar to that which has preceded, we obtain:


Fig. ${ }^{\circ} 23$. Jet Striking Curved Vanes of a Revolving Wheel Tangentially.
44. The work is:

$$
\begin{equation*}
\text { Work }=(1-\cos \theta) W \frac{(v-u) u}{g} \tag{41a}
\end{equation*}
$$

This is zero when $u=0$, or when $u=v$; also when $\theta=0^{\circ}$; and it is a maximum, and equal to:

$$
\begin{equation*}
\text { Work }(\operatorname{Max} .)=\frac{1}{2}(1-\cos \theta) W \frac{v^{2}}{2 g} \tag{42}
\end{equation*}
$$

when $u=\frac{1}{2} v$.
45. Efficiency. The efficiency is:

$$
\begin{equation*}
e=\frac{1}{2}(1-\cos \theta) \tag{43}
\end{equation*}
$$

When $\theta=0^{\circ}$, the stream merely glides along the surface without doing work, and $e=0$.

When $\theta=90^{\circ}$, the jet is deflected normally to the direction of motion, and,

$$
\begin{equation*}
\text { Work }(\text { Max. })=\frac{1}{2} W \frac{v^{2}}{2 g} \tag{44}
\end{equation*}
$$

and efficiency is $e=\frac{1}{2}$, or 50 per cent, as for radial flat vanes.

When $\theta=180^{\circ}$, the stream is completely reversed, and

$$
\begin{equation*}
\text { Work }(\text { Max. })=W \frac{v^{2}}{2 g} \tag{45}
\end{equation*}
$$

in which case the efficiency is $e=1$, or 100 per cent. The preceding analysis applies more directly to the case of a series of curved vanes moving in a straight line parallel to the jet, and coming in rapid succession under its influence. Such a motor is evidently impracticable.
46. In applying these considerations to water wheels, we must bear in mind that losses due to impact and friction have not been considered. The conclusions are therefore, to that extent, theoretic; but they represent limiting values which may be approached more and more closely, as the frictional and other resistances are reduced by means of correct design and construction. In the case of the conditions represented by Equation 45, since the efficiency is theoretically 100 per cent, it is clear that all the energy of the jet has been given up to the wheel, which would indicate that the absolute velocity of the water leaving the vanes must be zero; for if the water thus leaving has any absolute velocity, it still possesses some energy after passing clear of the wheel, which represents a portion of the original energy of the jet which has not been imparted to the wheel; the efficiency then could not be 100 per cent. This conclusion may be readily reached from the preceding analysis; for, since the best absolute velocity of the vane is $\frac{1}{2} v$, the water upon its surface has the relative velocity $v-\frac{1}{2} v=\frac{1}{2} v$, which is the same as the velocity of the vane, but in the opposite direction; then, if $\theta=180^{\circ}$, as in the case under discussion, the absolute velocity of the water as it leaves the vane, is $\frac{1}{2} v-\frac{1}{2} v=0$.

While the above discussion shows that for maximum efficiency the velocity of the vanes should be one-half the velocity of the jet, the efficiency is not much lowered by slight variations of the vane velocity above or below the value indicated. It is also clear that to thus realize the full energy of the stream, we suppose the jet to both enter and leave the vanes in a direction tangential to the circumference, and a complete reversal is effected. It will be shown in a subsequent article that certain practical considerations render it impossible to fully realize these theoretic conditions.
47. If the vanes are plane radial surfaces, as in Fig. 21, the water passes from the wheel normally to the circumference, and
the highest obtainable efficiency is (theoretically) 50 per cent (Equation 35). In this case the water leaving the wheel still possesses absolute velocity to the extent of $\frac{v}{\sqrt{2}}$, the component of which, in the direction of motion of the vanes, is $\frac{1}{2} v$; this represents a dynamic pressure of $W \frac{\frac{1}{2} v}{g}$ pounds in that direction, or $W \frac{\frac{1}{2} v}{g} \times \frac{1}{2} v(=P \times u)$ $=\frac{1}{2} W \frac{v^{2}}{2 g}$ foot-pounds of work; that is, one-half of the original energy of the jet is carried away by the escaping water, and is thus lost to the wheel. Or, an absolute velocity of $\frac{v}{\sqrt{2}}$ represents kinetic energy to the amount of $\frac{W\left(\frac{v}{1 \overline{2}}\right)^{2}}{2 g}=\frac{1}{2} W \frac{v^{2}}{2 g}$. Equation 58 shows even more clearly that in order to realize the full theoretic energy of the stream, the absolute velocity of the departing water ( $v_{1}=\frac{v}{\sqrt{2}}$ for this case) must be zero.
48. Case V. General. In the usual case the direction of motion of the vane is not the same as that of the jet. In Fig. 24, let the arrow marked $v$ represent the direction of the jet as it impinges on the vane with an absolute velocity $v$; and let the arrow marked $u$ represent the direction of motion of the vane, as well as its absolute velocity. While this case can be analyzed and solved in a manner similar to that em-


Fig. 24. General Case of a Jet Impinging on a Moving Vane. ployed in the preceding cases, it will be well here to adopt another procedure illustrating an important and useful principle:

The difference between the components of the absolute impulses of the entering and departing streams, in the direction of motion, is the resultant dynamic pressure in that direction:
49. Dynamic Pressure in a Given Direction. The absolute velocity of entry $v$ being known, it remains to determine the absolute velocity of exit, $v_{1}$. By means of the principle enunciated in Article 33 , we first find the.relative velocity $V$ with which the jet strikes the surface at $A$, by drawing to scale the lines $v$ and $u$ (both known) and
completing the parallelogram. $V$ then represents, in intensity and direction, the relative velocity of the stream at $A$. The stream passes over the surface, and leaves it at $B$ with this same relative velocity, if not retarded by friction or shock. Now, by the principle just referred to and used for the point $A$, the absolute velocity of the stream as it leaves the vane at $B$ may be determined. Draw $u$ and $V$, and complete the parallelogram; $v_{1}$ then represents the absolute velocity of the escaping water at $B$.

The absolute impulse of the stream. before striking the vane at $A$ is $W \frac{v}{g}$; its component in the direction of motion is $W \frac{v}{g}$ $\cos a$. The absolute impulse of the stream as it leaves the vane at $B$ is $W \frac{v_{1}}{g}$; its component in the direction of motion is $W_{\frac{v_{1}}{g}}^{\cos } 0$. Hence the dynamic pressure in the direction of motion is:

$$
\begin{equation*}
P=W \frac{v \cos \alpha-v_{1} \cos \theta}{g} \tag{46}
\end{equation*}
$$

This is a general formula for the dynamic pressure in any given direction exerted by a jet of water upon a vane moving in a direction parallel to a straight line, if $a$ and $\theta$ be the angles between that direction and the directions of $v$ and $v_{1}$.

If the surface is at rest, $v=v_{1}$, and Equation 46 becomes $P=$ $(\cos a-\cos \theta) W \frac{v}{g}$, which is Equation 27.
50. Usually, in the case represented by Fig. 24, the angles $a$ and $\beta$ are known, or assumed, and $\theta$ is unknown; it therefore becomes desirable to express the angle $\theta$ in other and known terms. By taking the components of the velocities at $B$ in the direction of motion, it is evident that $v_{1} \cos \theta=u-V \cos \beta$; if this value be substituted in Equation 46, there will result:

$$
\begin{equation*}
P=W \frac{v \cos \alpha-u+V \cos \beta}{g}, \tag{47}
\end{equation*}
$$

in which,

$$
\begin{equation*}
\left.V^{2}=u^{2}+v^{2}-2 u v \cos \alpha \text { (Trigonometry, from the triangle } A u v\right) \ldots \tag{47a}
\end{equation*}
$$

51. Curvature of Vane at Entrance. In order that the stream may strike the vane without shock, the curve of the vane at $A$ should be tangent to the direction of $V$. It therefore becomes important to express the angle $\phi$ in known terms. From either triangle at $A$,
making use of the trigonometric principle that the sides of any plane triangle are proportional to the sines of their opposite angles, we obtain:

$$
\begin{equation*}
\frac{\operatorname{Sin}(\phi-) \alpha}{\operatorname{Sin} \phi}=\frac{u}{v} \tag{48}
\end{equation*}
$$

which may be reduced, by known trigonometric relations, to:

$$
\begin{equation*}
\cot \phi=\cot \alpha-\frac{u}{v \sin \alpha} \tag{48a}
\end{equation*}
$$

Equation $48 a$ determines the angle $\phi$, when $u$, $v$, and the angle $a$ are known; and this fixes the proper curvature of the vane at the point $A$.

Example 14. In Fig. 24, let $u=70.71, v=100, \alpha=45^{\circ}$, and $\beta=30^{\circ}$. What is the dynamic pressure on the vane in the direction of motion, when 1 cubic foot of water strikes the vane per second? What should be the value of the angle $\phi$ in order that no loss by impact may occur?

From Equation $47 a$ :

$$
V=\sqrt{70.71^{2}+100^{2}-2 \times 70.71 \times 100 \times .707}=70.71 \text { feet per second. }
$$

From Equation 47:

$$
P=62.5 \frac{100 \times .707-70.71+70.71 \times .866}{32.2}=1,356 \text { pounds. }
$$

From Equation 48a:

$$
\operatorname{Cot} \phi=1-\frac{70.71}{100 \times .707}=0 ; \quad \therefore \phi=90^{\circ}
$$

## REVOLVING SURFACES

52. Case VI. In the case of water motors, the vanes upon which the jet impinges revolve about an axis. The motion of every point on the vane is therefore circular; hence, at any instant, the direction of motion of any point is tangent to the circumference drawn through, or it is normal to the radius drawn to, that point. At any point, therefore, that portion of the dynamic pressure which is effective in producing motion is its component in the direction of motion of that point. Fig. 25 illustrates two cases of wheels with vertical axes, the vanes revolving in horizontal planes. In the one case $(B)$, the water, after impinging, passes outward, or away from the axis; in the other $(a)$, the stream passes inward, or toward the axis. The following analysis, however, is general, and therefore applies to both types. As heretofore, $v$ and $v$ represent the absolute,
and $V$ and $V_{1}$ the relative velocities of the entering and departing streams; $u$ and $u_{1}$ (drawn normal to the radii $r$ and $r_{1}$ ) represent the absolute velocities and directions of motion of the points $A$ and $B$ on the vane; the angles to be used in the analysis are sufficiently clear from the diagram, in view of what has preceded: Constructing the two parallelograms in the usual manner, there is obtained, at


Fig. 25. Wheels with Vertical Axes, the Vanes Revolving in Horizontal Planes.
the point $A, V$ as the relative velocity of the entering stream; and at the point $B, v_{1}$ as the absolute velocity of the departing stream. For the parallelogram at $B$, however, the value of $V_{i}$ must first be computed by means of Equation 54.
53. Components of Pressures in Direction of Motion. The total dynamic pressure exerted in the direction of motion will depend upon the impulses of the entering and the departing streams. The absolute impulse of the water on entering is $W \frac{v}{g}$; and that of the water on leaving is $W \frac{v_{1}}{g}$. The components of these in the directions of the motion of the vane at entrance and departure, are respectively:

$$
\begin{equation*}
P=W \frac{v \cos \alpha}{g} ; \text { and } P_{1}=W \frac{v_{1} \cos \theta}{g} \cdots \cdots \tag{49}
\end{equation*}
$$

Since their directions are not parallel, and the velocities of the points $A$ and $B$ are not equal, their difference cannot be taken to give the resultant dynamic pressure, as was done in Case V, which represented motion in a straight line; but this resultant pressure is not important. The two expressions in Equation 49, however, are useful in an analysis of the work that can be delivered by the vane.
54. Useful Formulæ. Since in any rotating body the linear velocities of points are directly proportional to their distances from the axis of rotation,

$$
\begin{equation*}
\frac{r}{r_{1}}=\frac{u}{u_{1}} \tag{50}
\end{equation*}
$$

The relative velocities $V$ and $V_{1}$ are connected with the velocities of rotation by the following simple relation:

$$
\begin{equation*}
V_{1}^{2}-V^{2}=u_{1}^{2}-u^{2} \tag{51}
\end{equation*}
$$

Ordinarily, for a revolving vane, the data given or assumed will be the angles $a$, $\phi$, and $\beta$; the radii $r$ and $r_{1}$; the absolute velocity of the jet, $v$; the number of revolutions per second, $n$; and the weight of water delivered to the vane per second, $W$. Then,

$$
\begin{equation*}
u=2 \pi r n ; \quad \text { and } u_{1}=2 \pi r_{1} n, \tag{52}
\end{equation*}
$$

from which $u$ and $u_{1}$ may be determined.
In the triangle $A u v$ (sides are proportional to sines of opposite angles),

$$
\begin{equation*}
V=\frac{v \sin \alpha}{\sin \phi} \tag{53}
\end{equation*}
$$

which determines the relative velocity of entrance, $V$.
From Equation 51:

$$
\begin{equation*}
V_{1}=\sqrt{u_{1}^{2}-u^{2}+V^{2}}, \tag{54}
\end{equation*}
$$

which gives the value of the relative velocity of exit, $V_{1}$. Finally, taking the components of the velocities at $B$ in the direction of motion of that point, there results:

$$
\begin{equation*}
v_{1} \cos \theta=u_{1}-V_{1} \cos \beta \tag{55}
\end{equation*}
$$

From the above equations, the numerical values of $P$ and $P_{1}$ of Equation 49 can be fully determined.

Exariple 15. In Fig. 25; suppose $r=2 \mathrm{ft}$.; $r_{1}=3 \mathrm{ft}$.; $\alpha=45^{\circ} ; \phi=90^{\circ}$; $v=100 \mathrm{ft}$. per second; $n=6$ revolutions per second. Compute the velocities $u, u_{1}, V$, and $V_{1}$.

From Equation 52:

$$
\begin{aligned}
u & =2 \times 3.1416 \times 2 \times 6
\end{aligned}=75.4 \text { feet per second. }
$$

From Equation 53:

$$
V=\frac{100 \times .707}{1}=70.71 \text { feet per second. }
$$

From Equation 54:

$$
V_{1}=\sqrt{\overline{113.1^{2}-75.4^{2}+7 \overline{70.71}^{2}}}=110 \text { feet per second. }
$$

55. Work Derived from Revolving Vanes. In the discussion of "Work" and "Efficiency" under Cases IV and V, it was assumed that all points of the vane move with the same velocity; and in Case IV, that the stream enters upon it in the same direction as that of motion, or that $a=0$. Considering the general case just discussed, it may be said that the work of a series of vanes arranged around a wheel may be regarded as that due to the absolute impulse of the entering stream in the direction of motion of the point of entrance, minus that due to the absolute impulse of the departing stream in the direction of motion of the point of exit; or,

$$
\begin{equation*}
\text { Work }=P u-P_{1} u_{1}, \tag{56}
\end{equation*}
$$

in which $P$ and $P_{1}$ are the components of the dynamic pressures due to the absolute impulses at $A$ and $B$, in the directions of motion of the points $A$ and $B$, respectively, as shown in Fig. 25 and Equation 49. Using the values of Equation 49, in Equation 56, there results:

$$
\begin{equation*}
\text { Work }=W \frac{u v \cos \alpha-u_{1} v_{1} \cos \theta}{g} \tag{57}
\end{equation*}
$$

This is a perfectly general formula, applicable to the work of all wheels with outward or inward flow. It shows that the useful work consists of two parts-one due to the entering, and the other to the departing stream.

Another very simple general expression for the work of a series of revolving vanes may be deduced as follows: The total absolute energy of the entering stream is $W \frac{v^{2}}{2 g}$; the total absolute energy of the departing stream is $W \frac{v_{1}{ }^{2}}{2 g}$; hence, neglecting friction and other resistances, the difference represents the energy imparted to, or taken up by, the wheel from the stream; that is:

$$
\begin{equation*}
\text { Work }=W \frac{v^{2}-v_{1}^{2}}{2 g} . \tag{58}
\end{equation*}
$$

which is a useful formula of wide applicability. From Equation 58, the efficiency is:

$$
\begin{equation*}
e=\frac{v^{2}-v_{1}^{2}}{v^{2}}=1-\left(\frac{v_{1}}{v}\right)^{2} \tag{59}
\end{equation*}
$$

Example 16. As a numerical example, consider the case of the outwardflow horizontal wheel driven by a jet from a fixed nozzle, shown in Fig. 26.

```
Let \(r=2\) feet;
    \(r_{1}=3\) feet;
    \(\alpha=45^{\circ}\) (approach angle);
    \(\phi=90^{\circ}\) (entrance angle);
    \(\beta=15^{\circ}\) (exit angle);
    \(v=100\) feet per second;
    \(q=2.2\) cubic feet per second;
    \(n=337.5\) revolutions per minute.
```

It is required to find the useful work of the wheel, and its efficiency.


Fig. 26. Horizontal Wheels Driven by Jets from Fixed Nozzles.

From Equation 52:

$$
u=2 \pi r n=2 \times 3.1416 \times 2 \times \frac{337.5}{60}=70.71 \text { feet per second; }
$$

and, from Equation 50:

$$
u_{1}=\frac{r_{1}}{r} u=\frac{3}{2} \times 70.71=106.06 \text { feet per second. }
$$

From Equation 53:

$$
V=\frac{v \sin \alpha}{\sin \phi}=\frac{100 \times \sin 45^{\circ}}{\sin 90^{\circ}}=100 \times 0.7071=70.71 \text { feet per second. }
$$

From Equation 54:
$V_{1}=\sqrt{u_{1}^{2}-u^{2}+V^{2}}=\sqrt{(106.06)^{2}-(70.71)^{2}+(70.71)^{2}}=106.06$ feet per second.
From Equation 55:

$$
v_{1} \cos \theta=u_{1}-V_{1} \cos \beta=106.06-106.06 \times \cos 15^{\circ}=3.61
$$

Then, from Equation 57:
Work $=2.2 . \times 62.5 \frac{70.71 \times 100 \times 0.707-106.06 \times 3.61}{32.2}=\begin{gathered}19,712 \mathrm{ft} .-\mathrm{lbs} . \\ \text { per second. }\end{gathered}$

$$
\frac{19,712}{550}=35.8 \text { horse-power. }
$$

The theoretic energy of the jet is:

$$
\begin{aligned}
W \frac{v^{2}}{2 g}=2.2 \times 62.5 \frac{(100)^{2}}{64.4} & =21,380 \mathrm{ft} . \text { libs. per second. } \\
\frac{21,380}{550} & =38.9 \text { horse-power. }
\end{aligned}
$$

Therefore the efficiency of the wheel is:

$$
e=\frac{19,712}{21,380} \text {, or } \frac{35.8}{38.9}=92.2 \text { per cent. }
$$

This would seem to indicate a very high efficiency; but it must be borne in mind that losses in friction, shock, etc., have not been considered in the preceding analyses. The effect of such resistances will be to reduce the computed efficiency.

Example 17. In the above example, assume the same data, except that $\beta=30^{\circ}$.

The values of $u, u_{1}, V$, and $V_{1}$ are not altered.

$$
v_{1} \cos \theta=106.06-91.85=14.21
$$

and,

$$
\begin{aligned}
\text { Work } & =14,910 \mathrm{ft} \text {.-lbs. per second, } \\
& =27.2 \text { horse-power. } \\
\text { Efficiency } & =70 \text { per cent. }
\end{aligned}
$$

In both of the above examples the work and efficiency may be simply computed from Equations 58 and 59, after the value of $v_{1}$ has been determined. From Fig. 24, parallelogram at $B$, since $u_{1}$ and $V_{1}$ are equal in the above examples, it follows that $\theta=\frac{1}{2}(180-\beta)$; therefore, from Equation 55:

$$
\begin{aligned}
v_{1} & =\frac{u_{1}-\mathrm{V}_{1} \cos \beta}{\cos \theta}=\frac{u_{1}-V_{1} \cos \beta}{\sin \frac{1}{2} \beta} \\
& =\frac{106.06(1-.966)}{.131}=27.52(\text { for example } 16) ;
\end{aligned}
$$

and,

$$
v_{1}=106.06 \frac{(1-.866)}{.259}=54.87 \text { (for example 17). }
$$

Substituting numerical values in Equations 58 and 59, the same results for the work and efficiency will be found as computed before.

## HYDRAULIC MOTORS

56. Definition. A hydraulic motor may be defined as a machine in which theenergy stored in water is utilized to produce motion and thus perform work. The energy of water, as was explained in Article 6,
may exist in the form of gravity, of pressure, or of velocity; of these, gravity and pressure are not essentially or fundamentally separate and distinct phenomena, but rather the result of considering the weight of the water from different points of view. In general, then, it may be said that a hydraulic motor is an apparatus (usually a wheel) which is caused to move (usually rotate) by reason of a weight of water falling from a higher to a lower level, or because of the dynamic pressure induced by a change of direction, or of velocity, or both, in a moving stream. The dynamic pressure may be due to impulse, or reaction, or both. Many wheels are actuated by a combination in varying proportion of the above agencies, which are but manifestations of the energy existing in the water.
57. General Requirements for High Efficiency. The efficiency of any motor should, if possible, be independent of the quantity of water supplied to it; or, if the efficiency does vary with the supply, it should, when possible, be greatest in time of low water. It has already been shown that when $W$ pounds of water fall through a height of $h$ feet, or are delivered with a velocity of $v$ feet per second, the theoretic energy in foot-pounds per second is:

$$
K=W h ; \quad \text { or } K=W \frac{v^{2}}{2 g} .
$$

58. The actual work performed, or that may be performed, per second is equal to the theoretic energy, minus all the losses of energy. It is convenient to subdivide these losses into four general classes:
(a) Lesses incidental to the conduction of the water from the supply to the motor, occasioned by friction and the various other resistances usually encountered, such as bends, changes of section, passages through orifices or other controlling devices which are not essentially parts of the apparatus itself, etc.;
(b) Losses in passage through the motor, which include friction, losses in eddies resulting from abrupt change in cross-section and improper entrance angle, and losses in passage through controlling devices which form part of the apparatus, etc.;
(c) The residual energy still possessed by the departing water flowing away with an absolute velocity $v_{1}$;
(d) Shaft and journal friction.

Sometimes the friction of the moving parts in the air or water is included, but will not here be considered.
59. Efficiency. Let $W h^{\prime}$ represent the energy lost in conduction; $W h^{\prime \prime}$, that lost in passage through the wheel; $W \frac{v_{1}{ }^{2}}{2 g}$, the energy
still remaining in the departing water; and $W h^{\prime \prime \prime}$, the energy lost in shaft and journal friction; then,

$$
k=W\left(h-h^{\prime}-h^{\prime \prime}-\frac{v_{1}^{2}}{2 g}-h^{\prime \prime \prime}\right) .
$$

represents the actual useful work per second that the wheel is capable of performing. Accordingly, if $v$ is the velocity due to the head $h$, the efficiency is:

$$
e=\frac{k}{K}=1-\frac{h^{\prime}}{h}-\frac{h^{\prime \prime}}{h}-\left(\frac{v_{1}}{v}\right)^{2}-\frac{h^{\prime \prime \prime}}{h} .
$$

This formula, being very general, leads to the four following broad statements of the conditions requisite for high efficiency:
(1) The water must be conducted to the motor, and
(2) The water must pass through the motor, with the minimum loss of energy.
(3) The water must reach the tail-race level with the minimum absolute velocity consistent with practical considerations, such as the necessity for quick and proper clearance of water from the buckets, etc.
(4) The friction and other mechanical resistances of the moving parts must be reduced to a minimum.
60. This analysis, with the corresponding formulæ, compares the energy of the entire waterfall with the ultimate output of the machine. In estimating the power and efficiency yielded by the motor itself, regarded as a user of water delivered to it with a definite amount of energy, certain of the above losses should be omitted. Thus, losses in the conduction of the water to the motor cannot properly be charged against the motor; nor should losses in journal and shaft friction, which are outside and independent of the wheel regarded as a water user; in fact, the overcoming of journal and shaft friction is part of the work performed by the wheel, though it is not useful work. The energy in the departing water is properly chargeable to the wheel, since it is directly dependent upon the design or construction of the wheel. Therefore the hydraulic efficiency of the wheel may be stated thus:

$$
\begin{equation*}
e=1 \doteq \frac{h^{\prime \prime}}{h}-\left(\frac{v_{1}}{v}\right)^{2} ; \tag{60}
\end{equation*}
$$

or, as popularly stated, for high efficiency "the water should enter the wheel without shock, and leave without velocity." When the actual power and efficiency of a water motor are practically measured as described in Articles 115 et seq., the shaft and journal friction
and air or water resistance are automatically included in the result. This explains why the results of actual tests of power and efficiency are always lower than the corresponding values computed from formulæ derived without consideration of such losses. It is therefore well to employ two terms, hydraulic efficiency and actual efficiency, in order to distinguish clearly between the two sets of conditions involved.
61. Classification. In the absence of a uniform or generally accepted classification, hydraulic motors may be divided into two general classes:
(a) Water-wheels, in which the water does not enter and actuate the wheel around the entire circumference.
(b) Turbines, in which the water enters and actuates the wheel around the entire circumference.

Each of these main divisions has several subdivisions.

## WATER=WHEELS

62. Overshot Wheel. In this form of wheel, the water enters at the top and acts mainly by its weight; nevertheless, in most forms, an appreciable amount of kinetic energy is likewise imparted to the wheel. Fig. 27 shows a vertical section of such a wheel. The buckets are formed by vanes or partitions made in two partsone part $a$ in line with the radius of the wheel, the other part $b$ inclined in a direction definitely determined by the design. The bottom of the bucket is formed by the rim or sole-plate $F$; the side pieces are made by two cheeks or shrouds $E$. The whole is bolted to arms assembled on the hub, and supported by the axle.

Let $h$ be the total fall from


Fig. 27. Vertical Section of Overshot the surface of the water in the head-race or flume to the surface of the water in the tail-race; and let $W$ be the weight of water delivered to the wheel per second. The theoretic energy of the waterfall per second is $W h$ foot-pounds. The total fall $h$ may be conveniently divided into three parts-namely,
$h_{0}$, the average head in filling the buckets; $h-h_{0}-h_{1}$, the average head of descent of the filled buckets; and $h_{1}$, that part of the head which remains between the empty buckets and the tail-race. The water strikes the buckets with a velocity $v_{0}$, approximately equal to $\sqrt{2 g h_{0}}$; the buckets themselves are moving with a tangential velocity $u$ approximately in the same direction as $v_{0}$; this occasions a loss of head in impact, $h^{\prime \prime}$ (Mechanics):

$$
h^{\prime \prime}=\frac{\left(v_{0}-u\right)^{2}}{2 g} .
$$

The water then descends through the average distance $h-h_{0}-h_{1}$, acting by its weight alone; finally it drops out of the kuckets, and reaches the level of the tail-race with the absolute velority $v_{1}$, which represents part of the original energy wasted. Accordiuyly, the efficiency of the wheel is:

$$
e=1-\frac{h^{\prime \prime}}{h}-\frac{v_{1}^{2}}{2 g h} .
$$

Since the water leaving the buckets has a velocity $u$ when commencing the descent through height $h_{1}$, its velocity at the level of the tail-race is:

$$
v_{1}=\sqrt{\overline{u^{2}}+2 g h_{1}} .
$$

Substituting the values $h^{\prime \prime}$ and $v_{1}$ in the equation of efficiency above,

$$
e=1-\frac{v_{0}{ }^{2}-2 v_{0} u+2 u^{2}+2 g h_{1}}{2 g h},
$$

and ascertaining by the usual procedure in such cases what value of $u$ will render the efficiency e a maximum, it is readily found that:

$$
\begin{equation*}
u=\frac{1}{2} v_{0} ; \tag{61}
\end{equation*}
$$

that is, theoretically, the velocity of the wheel should be one-half that of the entering water for maximum efficiency. With this value of $u$, the hydraulic efficiency is:

$$
\begin{equation*}
e(\text { Max. })=1-\frac{1}{2} \frac{h_{0}}{h}-\frac{h_{1}}{h} . \tag{62}
\end{equation*}
$$

and

$$
\begin{equation*}
\text { Work }(\text { Max. })=W h \times e=W\left(h-\frac{h_{0}}{2}-h_{1}\right) \ldots( \tag{63}
\end{equation*}
$$

for the maximum efficiency and work of the overshot wheel. This equation teaches that one-half of the entrance drop $h_{0}$, and the whole of the exit drop $h_{1}$, are lost. Therefore, in order that the efficiency should be as high as possible, both $h_{0}$ and $h_{1}$ should be as
small as practicable. The former requirement may be met by making the wheel of large diameter; but $h_{0}$ can never be zero, for in that case no water would enter the wheel; practically the size of wheel is usually such that $\theta$ equals 10 to 15 degrees. The fall $h_{1}$ is made small by giving to the buckets such a form that the water will be retained as long as possible, and by having as little clearance as practically advisable between the lowest point of the wheel and the tailrace level. In the design illustrated in Fig. 28, the buckets are deep in order to hold the water as long as possible; and moreover, they are shaped to conform to the direction of the entering water, thereby avoiding shock. Wheels of this description have been constructed 50 feet in diameter. In this case the power is taken from the axle of the small pinion, which is driven by a toothed ring attached to the circumference of the wheel. In other cases the power may be taken directly from the shaft of the water-wheel, through intermediate gearing, or by a crank-shaft. The method of regulating the supply of water to the wheel is also shown in the diagram. The theoretic advantageous velocity of the overshot wheel was shown to be $u=\frac{1}{2} v_{0}$; practically, this advantageous velocity is found to be about $u=0.4 v_{0}$; and the efficiency of the wheel is high, ranging from 70 to 85 per cent, or over. One great advantage of the overshot wheel is that its efficiency is highest in times of drought, when the supply is low, for then the buckets are but partly filled, they do not begin to empty at as high a point above tail-water as when they are full; hence $h_{1}$ becomes small, with corresponding increase in efficiency. The main disadvantage of the overshot wheel lies in its size and its cost of construction. Moreover, its speed being slow (commonly from 3 to 6 feet peripheral velocity), it often requires the installation of somewhat complicated and expensive transmission gearing in order to drive machinery at a suitable speed; it is therefore best
adapted to drive slow-moving machinery, usually with heads from 10 to 40 feet (though much larger heads have been used), and with a supply of from 100 to 350 gallons per second. A peripheral speed much greater than that commonly employed would result in a waste of water from the buckets due to centrifugal force.

The number of buckets and their depth are sometimes determined by formulæ, but they are largely matters of experience. If $r$ is the radius of the wheel in feet, the number of buckets is usually $5 r$ or $6 r$, and their radial depth 10 to 15 inches. The width of the wheel parallel to the shaft is governed by the quantity of water actuating the wheel; it should preferably be so great that the buckets will not be quite full, thus reducing the fall $h_{1}$. If the tail-water level is constant, the lowest part of the wheel should be set just clear of that level; if it is variable, just sufficient clearance should be allowed to prevent interference and resistance in times of high water.

These precautions are necessary, for it is clear that the direction of motion of the buckets in the lowest portion of the wheel is opposite to the stream flow in the tail-race; and even slight submergence, therefore, will offer great additional resistance to its motion. This difficulty is sometimes obviated, when for any reason the wheel is to be submerged 4 or 5 inches (as by reason of variable tail-race level), by adopting a reverse-feed arrangement at the end of the supply channel, by which means the water is introduced on the back instead of on the front of the wheel, causing it to revolve in the opposite direction, so that the lower buckets move in the same direction as the tail-water. Such a wheel is often called a back-pitch or back-shot wheel.

For shallow streams of water with fairly constant depth, the supply channel is usually open-ended, as in Fig. 27; for deeper streams, or greater falls, the supply channel is provided with a sluice-gate or other regulating device, as in Fig. 28. Such a supply-regulating device is especially necessary in case of variable stream-flow.

Perhaps the largest overshot wheel in existence is that at Laxey, Isle of Man (Fig. 29), off the west coast of England. It is 72 feet 6 inches in diameter, and is said to yield 150 to 200 horse-power useful work, which consists in draining a mine 1,200 to 1,380 feet
deep. The water for operating is conveyed to the wheel in an underground conduit, and is carried up the masonry tower by pressure, flowing over the top into the buckets of the wheel. Probably the


Fig. 29. Overshot Wheel at Laxey, Isle of Man.
Diameter of wheel, 72 ft .6 in . Water carried up masonry tower by pressure, then flowing into buckets of the wheel.
largest wheel of this type in the United States was erected at Troy, N. Y., with a diameter of 62 feet and a width of 22 feet, developing 550 horse-power.
63. Breast Wheel. This type of wheel is designed to receive the water on one side, about or a little above the level of the hori-
zontal diameter; its lower portion, therefore, moves in the direction of the tail-water stream; for this reason the wheel may be drowned, or submerged, to a depth of 4 to 6 inches, which makes it suitable for use when both head-race and tail-race levels and supply are subject to variation. It is also evident from the manner of arranging the supply water, that this type is applicable only to small falls, from about 8 to 15 feet; for larger falls, the size of wheel would become impracticable. It-is clear that the water acts both by impulse and by weight; therefore, to prevent the escape of the water before the buckets reach their lowest position, the lower quarter of the wheel is encased in a circular breast which encloses the buckets, thius prac-


Fig. 30. Breast-Wheel with Supply Controlled through Size of Orifice. tically compelling the water, or most of it, to remain therein until the lowest point is reached. In Fig. 30, water is conducted from the source in a channel or trough to and through an orifice $A$, which controls the supply to the wheel through regulation of the size of the orifice. In Fig. 31 the control of the supply is accomplished by means of a shuttle-gate arrangement which consists of a number of openings $J J$ in the inclined end of the trough, one or more of which may be closed by shifting the sliding gate $B$. The guide-pieces are for the purpose of causing the water to enter the buckets in a direction most favorable for good efficiency. With the arrangement indicated in Fig. 31, considering the way in which the water enters the buckets, and observing that the mouths of the buckets are practically covered by the extension of the guide-pieces, it is evident that vents or airholes $F F$ in the sole-plate are necessary. Or the sole-plate may be dispensed with entirely, and the buckets formed of polygonal pockets, as $b a c$, in which the vents are naturally formed by the spaces left between the inner sides of consecutive buckets; these
being at the top, the buckets may be completely filled with water.
Work and Efficiency. In Fig. 30, the water is admitted through the orifice $A$, under a head $h_{0}$; it therefore strikes the wheel with a velocity $v_{0}$, which is approximately equal to $\sqrt{2 g h_{0}}$, and actually equal to $c_{1} \sqrt{2 g h_{0}}$, where $c_{1}$ is the coefficient of velocity for the orifice at $A$. The water, being then confined between the vanes and the curved breast, acts by its weight alone through the distance $h_{2}$, which is approximately equal to $h-h_{0}$; finally it escapes at the level of the tail-race with the velocity $u$, or the velocity of the circumference of the wheel. The reasoning in the article on overshot wheels may be applied to this case, by making the fall $h_{1}$ equal to zero, and the resulting conclusions may be considered to apply approximately to the case of breast wheels. Accordingly, the following relations are approximately true:

The most advantageous theoretic velocity is

$$
u=\frac{1}{2} v_{0}=\frac{1}{2} \sqrt{2 g h_{0}} \ldots . . .
$$

The maximum efficiency is theoretically:

$$
\begin{equation*}
e(\text { Max. })=1-\frac{1}{2} \frac{h_{0}}{h} \tag{65}
\end{equation*}
$$

The maximum work is theoretically:

$$
\begin{equation*}
\text { Work }(\text { Max. })=W\left(h-\frac{1}{2} h_{0}\right) . \tag{66}
\end{equation*}
$$

Practically, the coefficient of velocity of the entrance orifice should be considered, as well as loss due to the clearance between wheel and breast, which will always exist; for any attempt to prevent this entirely by making the clearance less than about $\frac{3}{18}$ inch would
result in a considerable increase in circumferential friction, and also, if the wheel is slightly off center, in repeated shocks. For these reasons the efficiency of the breast wheel is materially less than that of the overshot wheel, the usual values ranging from about 50 per cent for small wheels to about 75 per cent for large, well-designed wheels.

When the fall is not great, the wheel is sometimes designed to receive the supply water at a point appreciably below the horizontal diameter; in this case it is frequently termed a side wheel. Its efficiency is lower than that of the regular breast wheel. The best wheels of this type have been constructed with diameters ranging between 12 and 24 feet, running with circumferential velocities between 6 and 10 feet per second. They may be regarded as a type intermediate between the regular breast wheel and the undershot wheel. Breast wheels are sometimes provided with some simple automatic governing device controlled by the speed of the wheel, whereby the feed-water orifice is partially throttled when the speed of rotation exceeds a definite predetermined amount.
64. Undershot Wheel. The common undershot wheel is provided with plane radial vanes, and the wheel is so set that the water impinges on the lower vanes only, in an almost horizontal direction. In one sense, then, the undershot wheel may be regarded as a special kind of breast wheel, which is operated entirely by the impulse of the moving water. The formulæ developed for the case of breast wheels may therefore be applied approximately to the case of undershot wheels by changing $h_{0}$ to $h$, and $v_{0}$ to $v$; thus, for the most advantageous velocity of the wheel:

$$
\begin{equation*}
u=\frac{1}{2} v=\frac{1}{2} \sqrt{2 g_{i} ;} ; \tag{67}
\end{equation*}
$$

the maximum efficiency is:

$$
\begin{equation*}
e=(\text { Max. })=\frac{1}{2} \text {, or } 50 \text { per cent; } \tag{68}
\end{equation*}
$$

and the maximum work of the wheel is:

$$
\begin{equation*}
\text { Work }(\text { Max. })=\frac{1}{2} W h \tag{69}
\end{equation*}
$$

Here, also, the coefficient of velocity of the water in passing through the orifice should properly be considered. In this type, as well as in the last, for reasons set forth in a preceding article, the maximum efficiency and maximum work are practically less than indicated in the foregoing formulæ; also, the most advantageous speed of the
wheel is more nearly $u=.40 \sqrt{2 g h}$ than $.50 \sqrt{2 g h}$. In practice the efficiencies of such wheels are found to lie between 20 and 40 per cent. The lowest efficiencies are obtained from wheels placed in an unconfined current of water, such as a wheel attached to a barge anchored in a stream; and the higher efficiencies may be expected from well-constructed wheels, in which the actuating stream of water is properly confined, so that it cannot spread laterally.

Fig. 32 shows a simple type of radial-vane undershot wheel operating under a head of water. Here it is seen that the wheel is set in a circular channel constructed with a radius a trifle larger than that of the periphery of the wheel. The sliding gate for regulating the supply from the penstock is arranged at an angle of about $45^{\circ}$,


Fig. 32. Simple Type of Radial-Vane Undershot Wheel. which enables its lower edge to be set close to the wheel rim. By this means the vanes are kept from contact with the moving water until they are almost vertical. The slight drop in the channel below the wheel compensates to some extent for the friction loss in passing the orifice of entry. The circular channel is succeeded by a gently inclined bed, so that the water maintains its uniform velocity after leaving the wheel, until, at a point well away from the wheel, the channel bed is given a sudden, steep inclination.

The depth of opening at the orifice usually varies from about 8 inches as a minimum, to about 20 inches in flood. The number of blades, $N$, is sometimes calculated from the empirical formula:

$$
N=4 R,
$$

in which $R$ is the wheel radius. Then $N$ and $R$ will determine the spacing between the blades. In practice, this spacing may vary between 18 and 24 inches.

The undershot wheel is a relatively high-speed wheel; hence it
may be made more compact than the types described before; its construction and installation are extremely simple, and, from these points of view, it is economical. But its efficiency is lower than that of the other types; it is suitable only for very simple installations, to drive machinery at relatively high speed, where an ample supply of water is available, under a low head.
65. Poncelet Wheel. In this wheel (Fig. 33), the vanes are curved in such a way that the water enters through the regulating orifice or opening without shock. Let $v$ be the absolute velocity


Fig. 33. Poncelet Wheel, Water Entering without Shock. of the entering stream, and $u$ the peripheral speed of the wheel. The stream, entering with the absolute velocity $v$, impinges tangentially on the smooth vanes, which are themselves moving in the same general direction with an absolute velocity $u$. The relative velocity of the water is therefore $v-u$; and it glides smoothly up the curved vane in the general direction of motion of the stream to a height corresponding to this velocity; when at its uppermost point, it is at rest relatively to the vane; it then falls, exerting pressure as it falls, gliding along the vane in the general direction opposite to the motion of the stream, attaining the velocity $v-u$ at the lowest point or extremity of the vane, and passing from the vane tangentially. Its dynamic pressure is therefore due to both impulse and reaction:

$$
P=F+R=2 F=2 W \frac{v-u}{g} ;
$$

the work of the wheel is $(P \times u)$;

$$
\text { Work }=2 W\left(\frac{v-u}{g}\right) u \text {; }
$$

and this is a maximum, and equal to:

$$
\begin{equation*}
\text { Work (Max.) }=W \frac{v^{2}}{2 g}=W h . \tag{70}
\end{equation*}
$$

when $u=\frac{1}{2} v$.
Since the theoretic energy of the stream is $W \frac{v^{2}}{2 g}=W h$,

$$
\begin{equation*}
e(\text { Max. })=1, \text { or } 100 \text { per cent } . \tag{71}
\end{equation*}
$$

This follows from the fact that with the advantageous velocity $u=\frac{1}{2} v$, the absolute velocity of exit is zero; hence the stream "enters. without impact, and departs without velocity."

The preceding analysis and the conclusions are theoretic, since they do not consider the various losses of head or energy which must take place. Practically, the efficiency lies between 65 and 75 per cent.

The curved form is given to the bed of the channel of approach, in order to direct the entering stream of water so as to avoid shock. The depth of the vane should be such that the entering water may run up its length (due to its relative velocity) without interference. The spacing of the blades usually ranges between 10 and 18 inches.

The Poncelet wheel, like other undershot wheels, has a relatively high speed; its efficiency is almost independent of the flow, and also of the speed, when a curved channel of approach is used. Moreover, this speed does not vary much, in spite of considerable variations of head. This form of wheel may be used to advantage with a head not exceeding about 6 feet, when the application of power does not call for a high velocity, as for pumping, grinding, etc.
66. In the foregoing cases, the analytical relations have been deduced largely by comparison and analogy, resulting in conclusions more or less approximately true. In each case, however, these relations may be developed quite independently, giving theoretically accurate results. For example, take the case of the breast wheel represented in Fig. 30. In the figure, let $A v_{0}$ and $A u$ represent in intensity and direction the velocities, respectively, of the entering water and of the vanes, inclined to each other at an angle $a$. The dynamic pressure exerted by the water on the vanes, in the dircction of motion, is:

$$
P=W \frac{v_{0} \cos \alpha-u}{g} ;
$$

and the work per second is:

$$
\begin{equation*}
K=W \frac{\left(v_{0} \cos \alpha-u\right)}{g} u \tag{72}
\end{equation*}
$$

The work $K$, of the dynamic pressure alone, is a maximum, and equal to:

$$
\begin{equation*}
\text { Work (Max.) }=W \frac{v_{0}{ }^{2} \cos ^{2} \alpha}{4 g} \tag{73}
\end{equation*}
$$

when $u=\frac{1}{2} v_{0} \cos a$.
To this value of $K$ must be added the term $W h_{2}$, representing the work done by the weight of water in the buckets falling the distance $h_{2}$; this term is theoretically independent of the speed; accordingly,

$$
\begin{equation*}
\text { Total work (Max.) }=\mathrm{W}\left(\frac{v_{0}{ }^{2} \cos ^{2} \alpha}{4 g}+h_{2}\right) \tag{74}
\end{equation*}
$$

but $v_{0}=c_{1} \sqrt{ } 2 g h_{0}$, where $c_{1}$ is the coefficient of velocity for the orifice at $A$. 'Therefore,

$$
\begin{equation*}
\text { Total work }(\text { Max. })=W\left(\frac{1}{2} c_{1}^{2} \cos ^{2} \alpha . h_{0}+h_{2}\right) ; \tag{75}
\end{equation*}
$$

and the maximum hydraulic efficiency is:

$$
\begin{equation*}
e(\text { Max. })=\frac{1}{2} c_{1}^{2} \cos ^{2} \alpha \frac{h_{0}}{h}+\frac{h_{2}}{h} . \tag{76}
\end{equation*}
$$

If, in these equations (73, 75, and 76), $h_{2}$ be replaced by its equal $h-h_{0}$, and if $c_{1}$ equals unity, and the angle $a$ equals zero, there will result the approximate equations 64, 65, and 66, deduced in Article 63.

The angle $a$, however, cannot be zero; in fact it cannot practically be made less than about 10 degrees, for then little or no water would enter the wheel; it should, nevertheless, be as small as practicable, and is usually found between 10 and 25 degrees. The value of the coefficient $c_{1}$ is rendered large by well rounding the edges of the orifice; in this way $c_{1}$ may be made equal to .95 or even .98. In a manner similar to the above, formulæ for the other cases discussed may also be developed, with a greater degree of accuracy, theoretically considered. It is evident, however, that the approximate formulæ are sufficiently exact for most purposes, since the losses due to improper entry, foam, and leakage, cannot be algebraically expressed.

## SPECIAL FORMS OF WHEELS

Water wheels in great variety have been in use from very early times, some of them operating with a fair degree of efficiency. A few of these forms will be very briefly described.
67. Sagebien Side Wheel. The buckets of this wheel (Fig. 34) are formed by flat vanes which are tangent to the horizontal cylinder $O$, whose axis is concentric with the shaft of the wheel. The depth of the bucket-ring is relatively large, and there is no sole-plate, each bucket forming a sort of vessel open on top and bottom. The wheel turns in a circular channel, prolonged upstream by a suitable iron casing, sometimes called a swan's neck. The side cheeks of the


Fig. 34. Sagebien Side Wheel.
channel are continued downstream to the point where the wheel leaves the tail-race. There is very little work done by the water on the vanes beyond a point vertically below the center of the wheel. The inclination of the blades is not favorable for their easy emergence on the downstream side; but, as the speed of the wheel is rarely so great as 3 feet per second, being more usually between 1 and 2 feet, this resistance is small. The efficiency of this wheel, on account of its low speed (since resistances increase more or less rapidly with the speed), is very high, ranging from 80 to 90 per cent according to the height of the fall and the diameter, which varies between 20 and 40 feet, depending upon the fall available, the variability of the supply, and the fluctuation in the tail-race level. The number of revolutions per minute is often less than 1 , and rarely exceeds $2 \frac{1}{2}$. The penstock speed is usually 1 to 2 feet per second, and this is about the velocity with which the water enters the wheel. The spacing of the blades, measured on the outside of the wheel is about 15 inches. This type of wheel is used for small falls, from 2 to 9 feet, and is suitable for large
flows. On account of its slow speed, it is adaptable only for installations where the machinery runs slowly and opposes uniform resistance to driving.
68. Millot Wheel. This is a form of breast wheel (Fig. 35) in which the breast is not needed. The supply channel divides into two branches, which pass around to the inner side of the wheel, so that the water enters at the inner cireumference. This wheel is difficult to construct, and can be used only for small powers, since, by reason of the feed-water arrange-


Fig. 35. Millot Wheel. ment, the arms must be placed in the middle section of the wheel, instead of being fixed to the flanges; for thisreason the breadth is limited to about 5 feet.
69. Floating Wheel, or Cur= rent Wheel. This type is simply an undershot wheel with flat, radial vanes, erected on a scow or barge intended to be anchored in a stream, or mounted on some suitable framework built up from the stream bed. The flat blades are attached to an inner circle, but are not enclosed in shrouds, so that the water has very free entry. As the barge rises and falls with the changes of the stream level, the depth of blade immersion is constant. The efficiency is theoretically a maximùm, and equal to 50 per cent, when the peripheral speed of the wheel is one-half the velocity of the current; actually, it rarely reaches 40 per cent. When such a wheel is required to drive stationary machinery-that is, machinery so mounted that it does not follow the fluctuations in the surface level-some special device must be employed to insure the required condition of constant depth of paddle immersion. These wheels are extremely simple, but require to be of large size in order to develop even a moderate amount of power.

Wheels of this type have been used for operating dredges on the river Rhine, Germany; they have also been used to a limited extent, principally for irrigation purposes, in the western part of the

United States. One at Fayette Valley, Idaho, was said to be 28 feet in diameter, with 28 paddles, each 16 feet long and $2 \frac{1}{2}$ feet wide. 70. Tympanium. This is an ancient form of circular openframe wheel (Fig. 36), fitted with radial partitions so directed as to point upward on the rising side of the wheel, and downward on the descending side. The wheel is mounted in such a way that its lowest parts are submerged to a convenient depth, and it may be turned by the impulse of the current impinging on radial vanes arranged around its circumference. The partitions scoop up a quantity of water, which, as the wheel revolves, runs back toward the axis, where it is discharged into a trough that conveys it away. A very evident disadvantage of this form of wheel is the fact that the water has to be


Fig. 36. Tympanium.


Fig. 37. Scoop Wheel.
raised at the extremity of each radius, so that its lever arm, and therefore its resistance, increases as it is raised to a horizontal plane. This defect does not exist in the next type.
71. Scoop Wheel. As this wheel (Fig. 37) revolves, the partitions dip into, and scoop up the water; and as they ascend, the water is discharged into a trough placed under one end of the shaft, which is arranged in as many compartments as there are partitions or scoops.

An improved form of scoop wheel is shown in Fig. 38, which consists of four curved scrolls or channels suitably mounted on the wheel body. The water is conveyed to the central chambers by the scrolls, and it then flows away in a channel or trough.

Many other forms of water motor might be shown, most of them ancient and obsolete, which were mainly used for the purpose of raising water; but the above examples serve to indicate some of the principal devices employed for the purpose.
72. Ocean Waves. Many attempts have been made to develop useful power from the almost ceaseless motion of the ocean waves. The essential mechanism usually consists of some form of float which is constrained by a fixed shaft, or a series of such shafts, fastencd to a suitable foundation, to move in a vertical direction under the influence of the motion of the waves. The float, by its motion, operates a system


Fig. 38. Scoop Wheel, Improved Type. of levers and wheels, or ropes and pulleys, which may be made by suitable connections to compress. air, or to raise water from a lower to a higher level. In some such way, the irregular or intermittent character of the wave motion may be made to store up power, which, in turn may be released uniformly. Fig. 39 is a diagrammatic representation of such a device.
73. Tides. The ocean tide furnishes a more reliable means of developing power under suitable conditions. Particularly in the vicinity of tidal rivers, and more rarely along shore, the physical configuration of the land may afford opportunity for impounding large volumes of water during the rising of the tide, which may be made to develop power at ebb by flowing out through a suitable channel and operating one or more wheels. Since the wheels must necessarily remain idle during the rising of the tide, some suitable means must be provided for storing power, so that the machinery dependent upon this power may be in continuous operation, or may operate at any time, irrespective of the tidal conditions. Where power is used intermittently-as in some pumping plants which operate only a certain number of hours each day of 24 hours- a system of power storage, while convenient and advisable to provide against the contingency of a breakdown or other mishap, is not so necessary.
74. Water=Pressure Engine. This is a hydraulic motor which
performs work by reason of the static pressure of water acting upon a piston or a revolving disc. The cylinder and piston type of motor has a reciprocating motion identical with that of the steam engine; and the operation is very similar, the water entering and leaving through ports which are opened and closed by valves properly connected with the piston-rod. The useful work is due to the difference in the pressure of admission and discharge. As in the case of the steam engine, the reciprocating motion is generally changed by suitable mechanism into rotary motion before being applied to drive machinery. In the other type, the rotary motion is obtained directly from the shaft of the rotating discs or vanes. This latter type has not been widely used, as in practice there are many inherent difficulties in this mode of transmitting high power.


Fig. 39. Device for Utilizing Power of Wave Motion.

## I MPULSE WHEELS

75. The term impulse wheel is sometimes used to include only those special forms of hydraulic motor which are driven by a jet of water issuing from a nozzle and impinging upon vanes or buckets of special shape attached to the circumference of the wheel. This definition would improperly exclude such motors as the undershot wheel, which is nevertheless a true impulse wheel actuated by a broad stream of water; and also several other types of true impulse wheels.
76. Horizontal Impulse Wheels. When a wheel operated by a stream of water issuing from a nozzle and impinging on its vanes is so placed that its plane of rotation is horizontal (the axis being vertical), it is called a horizontal impulse wheel.

There are two general classes of such wheels, the outward-flow, and the inward-flow, as described in Article 52 and illustrated dia-
grammatically in Figs. 25 and 26. In order to deduce the conditions or relations for maximum efficiency, consider Fig. 26, in which both types are represented, so that the following analysis and the resulting conclusions will be generally applicable to such wheels. The construction of the parallelograms, and the notation, being the same as heretofore, further explanation will be unnecessary.

In order that the water may enter the wheel without shock or foam, the relative velocity $V$ should be tangent to the vane at $A$ as explained before. This condition of tangency will obtain when $u$ and $v$ are proportional to the sines of their opposite angles, in the triangle Auv (as in Article 51, Equations 48 and 48 $\alpha$ ); that is:

$$
\frac{u}{v}=\frac{\sin (\phi-\alpha)}{\sin \phi} ; \text { or, } \cot \phi=\cot \alpha-\frac{u}{v \sin \alpha}
$$

The absolute velocity of exit $v_{1}$ should be very small (Equations 58 and 59), for the energy represented by this velocity is not given to the wheel, but wasted. 'Theoretically it should be zero for maximum efficiency, as has already been shown; but practically, if this were the case, the vanes would be unable to clear themselves of the contained water. This absolute velocity $v_{1}$ will be small when

$$
\begin{equation*}
u_{1}=V_{1} \tag{77}
\end{equation*}
$$

These two equations are usually given as representing the conditions of maximum hydraulic efficiency. Equation 77, however, is only approximately true, the real minimum value of $v_{1}$ is found when $V_{1}=u_{1} \cos \beta$, in which case $v_{1}=u_{1} \sin \beta$; but this equation leads to very complex formulæ. Hence the simpler relation of Equation 77 , which is sufficiently accurate, will be used.

Referring to Equation 51, it is clear that if $u_{1}$ equals $V_{1}, u$ must equal $V$. Then, from the parallelogram at $A$, Fig. 26, it is seen that when $u=V$, the diagonal bisects the angle $\dot{\phi}$; or,

$$
\begin{equation*}
\phi=2 a \tag{78}
\end{equation*}
$$

Using this value of $\phi$ in Equation 48, there results:

$$
\begin{equation*}
u=\frac{v}{2 \cos \alpha} \tag{79}
\end{equation*}
$$

Equations 78 and 79 state the conditions involved in Equations 48 and 77 , for maximum hydraulic efficiency, in terms sometimes more convenient for use. When a wheel constructed according to this
condition (Equation 78) is running with the advantageous velocity $u$ of Equation 79, the absolute velocity of exit is:

$$
\begin{equation*}
v_{1}=v \frac{r_{1}}{r} \frac{\sin \frac{1}{2} \beta}{\cos \alpha} \tag{80}
\end{equation*}
$$

and the corresponding hydraulic efficiency (Equation 59) is:

$$
\begin{equation*}
e=1-\left(\frac{r_{1}^{\cdot}}{r} \frac{\sin \frac{1}{2} \beta}{\cos \alpha}\right)^{2} \tag{81}
\end{equation*}
$$

77. An analysis of this formula teaches that, for high efficiency, both the approach angle $\alpha$ and the exit angle $\beta$ should be small; but


Fig. 40. Faulty Design of Vane. they cannot be zero, otherwise water would not pass into and out of the wheel. Values of 15 to 30 degrees are common. Since, for small angles, the sine varies much more rapidly than the cosine, the equation of efficiency also shows that $\beta$ is more important than $a$; so that if $\beta$ be very small, $a$ may be as large as 40 or 45 degrees, with high efficiency. The equation further shows that for given values of $a$ and $\beta$, the inward-flow wheel, in which $r_{1}$ is less than $r$, has a higher efficiency than the outward-flow wheel.

The actual curve between the entrance and exit points of a vane is not of importance, provided it be smooth and gradual, as abrupt changes of direction lead to shock and to consequent loss of energy
78. Vertical Impulse Wheels.


Fig. 41. Good Type of Vane, with Double Cups and Dividing Central Rib.

Of this type of wheel (frequently called a hurdy-gurdy when the vanes are flat planes, and sometimes a tangent or tangential impulse wheel), there are several forms in the
market, differing merely in details, and known by various trade names, such as Pelton, Doble, Cascade, etc. Essentially this type consists of a wheel mounted on a horizontal shaft, which transmits the power received from a jet or several jets of water acting upon

This wheel, 9 ft .10 in . in diameter, is capable of developing $5,000 \mathrm{~h} . \mathrm{p}$. at $325 \mathrm{r} . \mathrm{p} . \mathrm{m}$. under 865 feet of effective head.

Fig. 42. A 5,000-H.P. Pelton Water-Wheel Runner.
 a series of cupshaped vanes attached to its periphery. The simplest type would be a wheel with flat radial vanes, as in Fig. 21; but, as has already been shown, the efficiency in such a case would be low, so that in practice curved vanes are invariably used.

In Fig. 40 is shown a faulty design of vane, for the water, after striking the outer lip, is abruptly changed in direction at the corners $a$ and $b$, with consequent shock and loss of energy; also, after leaving a vane, the stream strikes the back of the one adjoining, thus producing back-pressure, with further loss of energy. For these reasons the cups or vanes must be very carefully designed.

In the best forms, the vanes are double cups or buckets with a central rib designed to divide and turn the stream sidewise, while at


Fig. 43. Runner of 8,000-H.P. Doble Water-Wheel in DeSabla Power Plant. Velocity of jet, $20,000 \mathrm{ft}$. per minute.


Fig. 44. The "Cascade" (Leffel) Impulse Wheel.
Three-nozzle system.
the same time deflecting it backwards, opposite the direction of motion, as in Fig. 41.

Figs. 42 and 43 show the usual method of attaching the buckets to the wheel; it is clear that in these designs one or more buckets may be very easily and quickly removed and replaced when this is


Fig. 45. A 2,000-Horse-Power Double Unit, 500 Feet Head, for Direct Connection to Generator.
rendered necessary by reason of wear or breakage. The buckets in Fig. 42 are of the Pelton type.

In the Doble vane, Fig. 43, the outer portion of the lip is dispensed with for the purpose of preventing interference between the - jet and the approaching vane, though the central rib is retained for parting the stream sidewise.

In the Cascade (Leffel) wheel (Fig. 44), the lobes or half-buckets are set staggering, or breaking joint, on opposite sides of a thin circular disc, the sharp edge of which serves the same purpose as the central rib of the other forms in dividing the stream.
79. The analysis and conclusions of Articles 43 and 44, Fig. 23, apply in the case of these wheels; namely, the most advantageous velocity, theoretically, is:

$$
u=\frac{1}{2} v ;
$$

and at this velocity, the efficiency is a maximum, and equal to;

$$
e(\text { Max. })=1 \text {, or } 100 \text { per cen } i \text {, }
$$

when $\theta=180^{\circ}$-that is, when the stream is completely reversed. However, $\theta$ cannot be made equal to $180^{\circ}$, so as to completely revers $=$ the direction of the stream, without interference between the de-


Fig. 46. Interior of Power House of Puget Sound Power Company, Electron, Wash. Four wheel units aggregating $30,000 \mathrm{~h} . \mathrm{p}$. in this station; of the "double-overhung" type, coupled to 3,500 k.w. $225 \mathrm{r} . \mathrm{p} . \mathrm{m}$. generators. Each unit has an overload capacity of $7,000 \mathrm{~h}$.p.
parting water and the adjoining vane, as shown in Fig. 40, where the water is deflected vertically; and this is equally true when the stream is deflected sideways. The vane is therefore so shaped as to throw the divided stream just clear of the next vane, which condition makes it necessary that $\theta$ shall be less than 180 degrees, and consequently the efficiency will be less than 100 per cent, even theoretically. Nevertheless this form of wheel probably comes as near as any to realizing the theoretic condition for maximum efficiency.

As in all the other cases discussed, the theoretic conclusions derived from analyses are not quite true practically. Thus the most advantageous velocity of the wheel is somewhat less than . 5 of the jet velocity (though it is probably always considerably greater than .4 that velocity), while the maximum efficiency may be 90 per cent or somewhat higher.

80. The simplicity, cheapness, and high efficiency of this type of water motor commend it for use when the head of water is not less than about 50 feet-though many are in operation with heads of about 25 feet-especially when the supply of water is not abundant. It has the further advantage, due to its simplicity and cheapness, of allowing of almost indefinite extension of the existing installation, and of division of the power into groups or units, by placing a number of wheels on the same shaft, as in Figs. 45 and 46, or providing a wheel for each machine or group of machines.

Further, several wheels mounted on the same shaft may be operated by jets of water issuing from nozzles under different heads, by properly proportioning the diameters of the wheels and nozzles, as shown in Fig. 47. Here the center wheel is 33 feet in diameter, which is unusually large for this type of motor, and therefore special care was necessary in the design. The two side wheels are each 12 feet in diameter. The variation in heads in this case is about 10 to 1 .

For heads much lower than 50 feet, while this type of motor will, with proper regulation, still give a high efficiency, the construction is such that it cannot utilize a large quantity of water, and therefore the power output will not be great. This disadvantage may be obviated to some extent by mounting several wheels on the same shaft; but in the case of low heads, some form of turbine motor is to be preferred.

In setting up, this wheel must of necessity be placed above the tail-race level, and so high above it that there shall be no danger of interference from back-water. This means that a certain proportion of the total available head must be sacrificed to this condition; and unless the total head is sufficiently great to make the loss thus incurred relatively insignificant, this will not be the best type of motor for obtaining the greatest efficiency from the waterfall (see, however, article on "Draft-Tube"). These wheels are well adapted for running high-speed machinery, such as electric generators, air-compressors, etc., by direct connection, thus doing away with much belting or gearing with the attendant loss of power and expense of maintenance. These wheels have been used successfully with heads greater than 2,000 feet. They are manufactured in sizes from 6 inches in diameter to more than 30 feet for special cases, and two or more sizes of nozzle tips are usually provided for adjustment or regulation.
81. Regulation. In connection with the practical working of a water-wheel, an important matter is the quick and efficient control of the discharge from the nozzle in order to vary the power output of the wheel as the load varies, or to conform to fluctuations in the supply of water, so as to maintain a constant speed. Interchangeable nozzles of varying sizes have already been referred to; but this method requires hand manipulation, takes time, and requires attention. When the supply of water is adequate, and the power required sufficiently large, or the load variable, from two to five nozzles may


Fig. 48. Wheel Operated by Several Nozzles. be arranged to play simultaneously around the periphery of the wheel, as shown in Fig. 48. By this means, not only may much greater power be derived from one wheel; but, by shutting off one or more jets, the supply and power may be regulated to correspond to the load fluctuations with very little speed variation. Several wheels may be mounted upon the same shaft, each operated by its own jet or jets; and the regulation or control may be effected by shutting off the supply of one or more wheels, which would then run dead. In cases where the supply of water is abundant, so that waste is immaterial, good results can be obtained, especially with the smaller wheels, by mounting the two halves of the vanes on separate wheels (practically dividing the ordinary wheel, with its vanes, into two equal portions by a vertical plane at right angles to the axis). When the wheel is working at full power, the two halves are kept together, and thus form an ordinary wheel of this type; when, however, the speed increases, a governor
causes the two wheels to separate more or less, and thus some of the water is allowed to escape between. Several other ingenious devices have been developed for the purpose of accomplishing the same end; a description of some of them, taken mainly from manufacturers' catalogues, follows:
82. Under average conditions of operation, a governor is not necessary, as, with a constant load, the speed of the wheel is absolutely uniform. When slight and infrequent changes occur-such as are caused by hanging up stamps of a battery, for example-the wheel can be regulated by hand, by means of the main stop-gate, as shown in Fig. 45; but this would occasion considerable loss of energy, on account of the sudden change of section of the stream. It some-


Fig. 49. Section of a Needle Nozzle.
times happens, however, especially when operating electric plants, that the fluctuations in speed are sudden and severe; and in these cases an automatic regulator is essential. In such cases the speed of the wheel may be controlled by means of various devices, among which may be described the following:

The deflecting nozzle is a cast-iron nozzle provided with a ball and socket joint, which permits it to be raised or lowered, thus throwing the stream on or off the buckets; the power of the wheel is consequently increased or diminished to correspond to the change of load, and a constant speed is maintained. A steel deflecting plate, which deflects the stream itself-the nozzle remaining stationary-is sometimes used to accomplish the same results when the design will not admit of a deflecting nozzle. Both these devices are wasteful of water; but they effectually prevent water-hammer, which would result from a sudden decrease of velocity in the pipe.

The stream cut-off is a spherical plate fitting tightly over the end
of the nozzle tip, which, by varying its position, changes the discharge area of the nozzle, and thus influences the power of the wheel.

The needle nozzle (Figs. 49 and 50) consists of a nozzle body in which is inserted a concentric tapered needle. A change of position of this needle


Fig. 50. Stream of Water from Pelton Needle Nozzle Operating under 390 -Foot Head and Developing 1,500 H.P.
Note the shadow of needle showing through stream, and the perfect form of jet. produces a corresponding change of discharge area of the nozzle; the amount of water used is thus varied, and the power of the wheel influenced proportionally.
Theneedleregulating and deflecting nozzle (Figs. 51 and 52 ) is a most valuable combination, consisting of a deflecting nozzle swinging on a pair of trunnions, with which is incorporated a needle nozzle, with means for operating either the needle or deflecting nozzle simul-


Fig. 51. Doble Needle Regulating and Deflecting Nozzle for 8,000-H.P. Wheel.
taneously or separately. This accomplishes a twofold object-accurate regulation, and water economy without water ram. The deflecting nozzle is a most sensitive means of regulation when actuated by an automatic governor, but does not save water. On the other hand,
the needle nozzle, while it is extremely economical in the use of water, is difficult to control quickly by means of the governor. The operation of the combination is as follows:

Assuming the full load to be on the water-wheel, and the nozzle in position of greatest efficiency, a decrease in load, tending to cause increase in speed, will cause the nozzle to be suddenly deflected


Fig. 52. Pelton Needle Regulating and Deflecting Nozzle in Operation. by the automatic governor. Simultaneously, the needle portion of the nozzle will be actuated by hand, or by another automatic device, tending to close the needle gradually


Fig. 53. Various Sized Jets from Small Doble Needle Regulating Nozzle.
and decrease the flow. The governor then raises the nozzle to accommodate the decreased flow of water (and consequent decrease of power), and the nozzle is then brought back to the position of greatest efficiency, having, at the same time, controlled the speed within the required limits.


Fig. 54. Hydro-Electric Unit with Needle Nozzle Operated by "Woodward" Governor.
Such a device is essential where water is valuable, and where economy is necessary to carry over the peak load. The needle portion need not necessarily be operated by an automatic device, but may be controlled by hand, and the same results obtained, although necessarily in a longer period of time. In Fig. 52, the upper and lower lines indicate the limits of deflection. Fig. 53 shows how the size of the jet may be varied by means of the needle nozzle.
83. The conditions as to head, power, and character of load determine which device or combination is best suited to any individual case. These various mechanisms are actuated, through a proper system of rock-shafts and levers, by an automatic governor (Figs. 48 and 54), which, for ordinary machinery, may be a mechanical governor of the plain, centrifugal-ball type, the power to move the regulating device being furnished directly by the wheel itself; but where close regulation is required, as in driving electrical machinery, a
more sensitive device is necessary. Fig. 55 represents a Lombard automatic governor of the hydraulic type, using direct water-pressure to actuate the pistons, which are controlled by balanced valves. Fig. 54 represents a hy-dro-electric unit in which the needle nozzle, instead of being arranged for hand control, is directly operated by a Woodward compensating governor - mounted upon the nozzle body and geared to the needle shaft, which is threaded, and moves in a nut which forms part of the nozzle body, so that the action of the governor regulates directly the position of the needle. It is readily seen that the ball governor is the ultimate device, which actuates or sets in motion the controlling and regulating apparatus. This topic will be further considered under "Turbines" (see Articles 179 et seq.).



Fig. 55. "Lombard" Automatic Hydraulic Goverrnor.
84. In the case of long pressure-pipes, especially when under high pressure, it is difficult and dangerous suddenly to vary the quantity of water delivered by the nozzle in such a manner as is necessary to regulate the speed of a hydro-electric generating unit subject to
sudden violent variations of load. Consequently it has become customary to regulate the speed of such units by deflecting the jet of water, so that all, or part of it, misses the water-wheel buckets, and is for the moment necessarily wasted. The water which is thus prevented from giving its energy to the water-wheel, is projected through the tail-race at a very high vclocity-in some cases exceeding 300 feet per second (18,000 feet per minute)-and becomes destruc-


Fig. 56. "Ensign" Vortex Baffle-Plate as Installed in a Tail-Race.
tive, particularly when the water unavoidably carries infinitesimal particles of sand. No masonry can long withstand the action of such a jet, and even iron and steel are rapidly worn away, as if by a terrific sand-blast.

The Ensign Vortex Bafle-Plate (patented), illustrated in Fig.56, is designed to divide such a jet in halves, and deflect the halves until they impinge upon each other, and harmlessly spend their force. The device is a trough-like structure with a sharp central vertical dividing wedge, made to be replaceable in case of wear. The device splits the impinging jet, and guides each half around the curved sur-
faces, spreading it out into two thin sheets which meet and harmlessly spend their force against each other. The water then falls by gravity into the tail-race with very little disturbance.
85. Downward=Flow Impulse Wheels. In this type of motor, the horizontal impulse wheel is driven by the jet from a nozzle inclined downward at a convenient angle, as in Fig. 57, which represents in outline the plan and the development of part of a cylindrical section of such a wheel. The water, in passing through the wheel, neither approaches nor recedes from the axis of rotation; it is therefore sometimes called a parallel-flow or axial-flow wheel.

The stream enters at $A$, as shown, with the relative velocity $V$; passes downward over the vane, always maintaining the same distance from the axis; and, neglecting the effect of friction and gravity, issues from the vane at $B$, with the same relative velocity $V$.

As before, to prevent impact losses at $A$, the direction of the relative velocity $V$ must be tangent to the vane at that point; and in order that the efficiency should be high, the absolute velocity of departure $v_{1}$ must be small, which later condition will be fulfilled if $u=V$ at $B$. Therefore, as explained in the preceding analyses, $\phi$ should be made equal to $2 a$, and the best speed of the wheel is $u=$ $\frac{v}{2 \cos a}$. The efficiency under these conditions is:

$$
\dot{e}=1-\left(\frac{\sin ^{\frac{1}{2}} \beta}{\cos \alpha}\right)^{2},
$$

which again shows that both $a$ and $\beta$, particularly the latter, should be small for high efficiency.

In the above analysis, no account was taken of the force of gravity acting as the water descends through the vertical distance between $A$ and $B$; this would increase the efficiency and the advantageous velocity above the values as found from the equations above.

It is evident that several nozzles might be employed also with this type of wheel, instead of one, where the supply of water is adequate.
(Articles 11 and 12 develop the hydraulic formulæ to be used in problems of nozzle discharge. Article 16 shows the proper relation between the diameters of nozzle and pipe to furnish maximum power; and Article 19 considers the case of multiple nozzles to fulfil the same condition.)
86. Girard Impulse Wheels. This type of wheel (Fig. 58) consists essentially of two flat, parallel, and concentric rings or crowns, between which are inserted the curved vanes or blades, the whole attached rigidly to the axle and forming the wheel proper, or runner. The feed or operating water issues from a nozzle placed inside the wheel as shown, in which case it is an outward-flow impulse wheel; or the nozzle may be placed outside, making it an inward-flow wheel; or several nozzles or groups of nozzles may be employed, located symmetrically around the circumference. The analyses and conclusions contained in the preceding articles apply to these cases.

Axial or parallel flow may be applied to this type of wheel, as explained in Article 85, under the heading, "Downward-Flow Impulse Wheels."

Occasionally, with the out-ward-flow type, the two crowns are made to diverge (i.e., their distance apart becomes greater) toward the outer circumference, constituting the so-called bellmouthed profile. In this way, choking of the passageways, due to excessive narrowness, is avoided: Openings in the crowns to facilitate the escape of air are frequently made with the same object in view. This type of wheel is widely used in Europe, a number of single motors of this kind developing 1,000 horse-power. Among the several wheels installed at the TTerni Steel Works, Italy, ranging from 50 to 1,000 horse-power each, under a head of about 600 feet, is a large 800 -horse-power wheel which drives the rolling-mill machinery; its outer diameter is 9 feet 5 inches; its inner diameter, 8 feet 2.4 inches; distance between crowns at entrance, 4.91 inches; at exit, 16.14 inches. The quantity of water used is 16 cubic feet per second; and the normal speed, 200 revolutions per minute.


CANAL AND DAM AT SPIER FALLS, NEW YORK

HEAD-GATE HOISTS OF THE HUDSON RIVER WATER POWER COMPANY AT SPIERS FALLS, NEW YORK

In the electric power station at Vernayaz, Switzerland, are six 1,000 -horse-power Girard wheels, working under a head of 1,640 feet; the outer diameter of each wheel is about 6.5 feet; and the normal speed, about 540 revolutions per minute. These wheels work with but one guide (the nozzle tube) each.

A turbine built for the Ouiatchouan Pulp Company (Quebec) has two sets of such guides spaced 180 degrees apart. This wheel develops 1,000 horse-power under a head of 240 feet, running at 225 revolutions per minute; it is enclosed in a cast-iron case and provided with a draft-tube and air-admission valve, both of which contrivances will be described in a later article.

Example 18. Let us assume a Girard outward-flow impulse wheel, with $a=25$ degrees; $\beta=20$ degrees; ratio $\frac{r_{1}}{r}=\frac{4}{3}$; supplied with 2 cubic feet per second-through 12 -inch pipe 2,000 feet long, with nozzle attached, having a coefficient of velocity of 0.95 . Total head over nozzle tip, 152.00 feet, of which 8.3 feet are consumed in pipe friction and entrance losses. Wheel to make 240 r.p.m.

Velocity in the pipe is:

$$
v_{\mathrm{p}}=\frac{q}{a}=\frac{2}{\pi^{\frac{1}{4}}}=2.55 \text { feet per second. }
$$

Velocity of jet is:

$$
v=0.95 \sqrt{2 g(152.00-8.3)+\overline{2.55^{2}}}=91.4 \text { feet per second. }
$$

The best speed for the inner rim is

$$
u=\frac{v}{2 \cos \alpha}=\frac{91.4}{.2 \times 0.906}=50.5 \text { feet per second }
$$

Since $2 \pi r n=50.5$.

$$
\begin{aligned}
& r=\frac{50.5}{2 \pi^{240} 60}=2.01 \text { feet } \\
& r_{1}=\frac{4}{3} \times 2.01=2.68 \text { feet. }
\end{aligned}
$$

The theoretic efficiency is:

$$
e=1-\left(\frac{r_{1}}{r} \frac{\sin \frac{1}{2} \beta}{\cos \alpha}\right)^{2}=1-\left(\frac{4}{3} \frac{0.174}{0.906}\right)^{2}=0.93 .
$$

The actual efficiency would probably have a value bẹtween 75 and 80 per cent.

The work imparted to the wheel is, theoretically:
Work $=W \frac{v^{2}}{2 q}=2 \times 62.5 \times \overline{\frac{91.4}{6}}^{24.4}=16,213 \mathrm{ft}$.-lbs. per second, if the nozzle be not considered a part of the motor, and if losses in the wheel itself be disregarded.
If the nozzle be considered part of the motor, the work imparted to it, disregarding wheel losses, is:
Work $=w^{q} q\left(152.00-8.3+\frac{\overline{2.55}_{2 g}^{2}}{}\right)$
$=62.5 \times 2\left(143.7+\frac{6.5}{64.4}\right)=17,975 \mathrm{ft}$. llbs. per second.
If the wheel, under the second assumption, have an efficiency of 75 per cent, the useful work of the wheel is:
Useful work $=17,975 \times 0.75=13,475 \mathrm{ft}$.-lbs. per second;
$\frac{13,475}{550}=24.5$ horse-power .
87. Rotating Vessels. As a preliminary to the study of the theory and operation of turbines, it will be necessary to consider very briefly some ofs the essential fea-


Fig. 59. Revolving Open Vessel. tures of the action of rotating fluids.

Flow from a Revolving Vessel with Free Surface. Let $A C$ (Fig. 59) be any open vessel, revolving about a vertical axis $X X$. It is shown in treatises on Hydraulics, that the water surface $E K$, which is horizontal before rotation, becomes, under the action of centrifugal force and gravity, a curved surface $A O B$, due to the rotation; that this surface is a paraboloid of revolution, so that any vertical section through the axis is a parabola with the vertex at $O$, and axis vertical and coincident with the axis of rotation; that the head of water over an orifice $F$ in the base or side, at any distance $r$ from the axis of rotation, is $h+\frac{u^{2}}{2 g}$, if $u$ be linear velocity of the center of the orifice, and $h$ its distance below
the lowest point $O$; and, therefore, that the relative velocity of efflux from $F$ is:

$$
\begin{equation*}
V=\sqrt{2 g\left(h+\frac{u^{2}}{2 g}\right)}=\sqrt{2 g h+u^{2}} \tag{82}
\end{equation*}
$$

Let $n$ be the number of revolutions per second; then $u=2 \pi r n$; and

$$
\begin{equation*}
V=\sqrt{2 g h+4 \pi^{2} r^{2} n^{2}} . \tag{83}
\end{equation*}
$$

This result is independent of the shape of the containing vessel; and the axis of rotation may lie within or without it, the axis of the paraboloid in any case coinciding with the axis of rotation.
88. Closed Vessel. The above formulæ apply equally well to the case of a closed rotating vessel in which the curved surface is wholly or partially prevented from forming, as in Fig. 60. Here also $h$ is the depth $M O$ in the axis of rotation; and the parabola $A O B$ represents the vertical section of the paraboloid of pressures. In both cases, then,

$$
\frac{u^{2}}{2 g}=\frac{2 \pi^{2} r^{2} n^{2}}{g}
$$

is the head $G H$, to be added to


Fig. 60. Revolving Closed Vessel. the minimum static head $M O$ at the axis, to obtain the total pressure head over the orifice. If the orifice is in the vertical bounding wall of the vessel, as at $F^{1}$, the pressure head is $M^{1} O^{1}+O^{1} B=H$.
89. Revolving Tubes. Fig. 61 represents the simple case of one or more hollow arms attached to a vessel, and rotating with it about a vertical axis. From what has preceded, it is clear that the static pressures at the points $A$ and $B$ in the tube, when rotation has been established, but when no flow occurs, are, respectively:

$$
\begin{aligned}
& O M+G H=h+\frac{u^{2}}{2 g}, \text { for the point } A ; \text { and, } \\
& O M+G_{1} H_{1}=h+\frac{u_{1}^{2}}{2 g} \text { for the point } B
\end{aligned}
$$

if $u$ and $u_{1}$ be the linear velocities of the points $A$ and $B$ respectively.
When the orifices are opened and flow talies place, the pressurehead in each case falls by an amount equal to the velocity head plus
the head lost in frictional resistances, as explained in Articles 6 and 7; and the line of pressure now assumes some other form, such as $L B$. Neglecting for the present the frictional losses, it is evident that the following relations must obtain, by reason of the principle of the conservation of energy:

$$
h+\frac{u^{2}}{2 g}=h^{\prime}+\frac{V^{2}}{2 g},
$$

which becomes, for the point $B$, since no pressure head exists at the end of the tube when it discharges freely into the air:

$$
\begin{equation*}
h+\frac{u_{1}{ }^{2}}{2 g}=0+\frac{V_{1}^{2}}{2 g}, \tag{83a}
\end{equation*}
$$

so that

$$
\begin{equation*}
h=h^{\prime}+\frac{V^{2}}{2 g}-\frac{u^{2}}{2 g}=\frac{V_{1}^{2}}{2 g}-\frac{u_{1}^{2}}{2 g} \ldots . \tag{83b}
\end{equation*}
$$

if $V$ and $V_{1}$ represent the relative velocities in the tube at the points $A$ and $B$ respertively. If the tube is submerged as in Fig. 76, there is static pressure at the end; so that, if $h^{\prime \prime}$ is the static pressure on the end (the depth of submergence), then,

$$
\begin{equation*}
h+\frac{u_{1}^{2}}{2 g}=h^{\prime \prime}+\frac{V_{1}^{2}}{2 g}, \tag{84}
\end{equation*}
$$

and therefore,

$$
h=h^{\prime}+\frac{V^{2}}{2 g}-\frac{u^{2}}{2 g}=h^{\prime \prime}+\frac{V_{1}{ }^{2}}{2 g}-\frac{u_{1}{ }^{2}}{2 g} .(\mathbf{8 4 a})
$$

The above equation ( $84 a$ ) expresses the relation between the pressure-head, velocity-head, and rotation-head at any point of a revolving tube. In case the tube is only partly full, as when a stream impinges and glides along a vane (or one side of a tube or bucket of a water-motor), there can be no static pressure, and the above becomes:

$$
\begin{equation*}
\text { - } \quad V_{1}{ }^{2}-V^{2}=u_{1}{ }^{2}-u^{2} . \tag{84b}
\end{equation*}
$$

which is Equation 51, for the case of a jet impinging on a vane.
Fig. 61 represents essentially a reaction wheel, since the dynamic pressure causing rotation is caused entirely by the reaction of the issuing jets.
90. In order to discuss the work and energy of such an apparatus, we may use Equation 57, which expresses the work of the impulse of the entering stream and the reaction of the departing stream, by simply omitting the term representing the former Accordingly, for the work of a reaction wheel:

$$
\begin{align*}
\text { Work } & =W \frac{-u_{1} V_{1} \cos \theta}{g} \cdots \cdots \ldots \ldots  \tag{85}\\
& =W \frac{u_{1} V_{1} \cos \beta-u_{1}{ }^{2}}{g h}(\text { from Equation } 55) \cdots \ldots  \tag{86}\\
& \left.=W \frac{u_{1} \cos \beta \sqrt{2 g h+u_{1}{ }^{2}}-u_{1}{ }^{2}}{g} \text { (from Equation } 83 a\right) \cdot \tag{87}
\end{align*}
$$

Dividing the expression for Work by the theoretic energy $W h$, we have:

$$
\begin{equation*}
\text { Efficiency }=\frac{u_{1} \cos \beta \sqrt{2 g h}+u_{1}^{2}-u_{1}{ }^{2}}{g h} . \tag{88}
\end{equation*}
$$

The work is zero when $u_{1}=0$-that is, when there is no rotation; also when $u_{1}^{2}=2 g h \cot ^{2} \beta$; and it is a maximum, and equal to Work $($ Mäx. $)=W h(1-\sin \boldsymbol{\beta})$
when,

$$
\begin{equation*}
u_{1}{ }^{2}=\frac{g h}{\sin \beta}-g h, \tag{90}
\end{equation*}
$$

the efficiency, in this case, being: $e($ Max. $)=1-\sin \boldsymbol{\beta}$

The work and efficiency, therefore, increase as the angle $\beta$ decreases. When $\beta=90$ degrees, the work and the efficiency both become zero, for the jet in such case issues radially; when $\beta=0$ degrees, the work is $W h$, and the efficiency is unity, or 100 per cent; but the velocity $u_{1}$ (and therefore also $V_{1}$ ) becomes infinitely great. It must be remembered that frictional and air resistances have not been considered in the above anal-


Fig. 61. Revolving Vessel with Hollow Arms Attached. ysis; both increase rapidly with increased speed of rotation. In general, however, it may be stated that within certain limits the efficiency of a reaction wheel increases with the speed and with the smallness of the angle $\beta$; and it is greatest in any given case, when the angle $\beta$ is zero-that is, when the water issues in a direction exactly opposite to that of rotation.
91. Reaction Wheel. Fig. 62 represents an apparatus com-
monly known as Barker's Mill. It is the reaction wheel described in the preceding article, with the direction of the issuing streams of water directly opposite to that of revolution, or $\beta=0$. Making $\beta=$ 0 in the preceding equations, we have:

$$
\begin{array}{r}
\text { Work }=W \frac{u_{1} \sqrt{2 g h+u_{1}{ }^{2}}-u_{1}{ }^{2}}{g} ; \\
\text { Efficiency }=\frac{u_{1} \sqrt{2 g h+u_{1}{ }^{2}}-u_{1}{ }^{2}}{g h} ; \ldots \\
\text { Work (Max.) }=W h ; \ldots \ldots \ldots \ldots \ldots \ldots \\
\text { Efficiency (Max.) }=\text { unity, or } 100 \text { per cent, } \tag{95}
\end{array}
$$

when $u_{1}$ =infinity; in which case also $v_{1}=$ infinity.
If $a_{1}$ be the area of the exit orifices, and $w$ the weight of a cubic unit of water, the weight of water discharged in one second is $w a v$,
 which becomes infinite when $u_{1}$ $=V_{1}=$ infinity. As stated before, frictional and air resistances increase rapidly with the speed, so that the above relations, in deriving which these resistances have not been considered, are theoretic. It is evident, however, that the efficiency of a reaction wheel of this type increases with the speed within certain limits; and that the discharge varies with the speed.
92. Effect of Friction. If $c_{v}$ be the coefficient of velocity representing the effect of friction in the arms and orifice, then,

$$
\begin{equation*}
V_{1}=c_{v} \sqrt{2 g h+u_{1}^{2}} \tag{96}
\end{equation*}
$$

instead of the theoretical expression,

$$
V_{1}=\sqrt{2 g h+u_{1}{ }^{2}}
$$

The expressions for the effective work of the wheel and the efficiency then become:

$$
\begin{align*}
\text { Work } & =W \frac{c_{r} u_{1} \sqrt{2 g h+u_{1}^{2}}-u_{1}^{2}}{g} \cdots  \tag{97}\\
\text { Efficiency } & =\frac{c_{v} u_{1} \sqrt{2 g h+u_{1}^{2}}-u_{1}^{2}}{g h}, \ldots  \tag{98}\\
\text { Efficiency (Max.) } & =1-\sqrt{1-c_{v}{ }^{2}} \cdots \cdots ? \tag{99}
\end{align*}
$$

when,

$$
\begin{equation*}
u_{1}^{2}=\frac{g h}{\sqrt{1-c_{v}{ }^{2}}}-g h . \tag{100}
\end{equation*}
$$

If $c_{\mathrm{v}}=1$-that is, when frictional loss is not considered- $e=1$; and $u_{1}=V_{1}=$ infinity, as before. When $c_{v}=.94$, the advantageous velocity $u_{1}=\sqrt{2 g h}$, and the efficiency is 65 per cent. Thus the effect of friction is greatly to decrease the theoretic efficiency. To render. $c_{\mathrm{v}}$ large, the tubes should be smooth and well rounded by means of easy curves. In addition to the above considerations, the air resistance, which has not been included in the above analysis, increases very rapidly with the speed of rotation,


Fig. 63. Scotch Wheel. and its effect is to reduce still further the computed efficiency. Because of the low actual efficiency resulting from the above factors, the reaction wheel is not used as a practical hydraulic motor.
93. The Scotch wheel (Fig. 63) is an improvement on the Barker's Mill; the three ori-


Fig. 64. Combe's Turbinc. fices are made adjustable in size by means of movable flaps, for the purpose of regulating the quantity of water and the power.
94. The next advance in hydraulic motor design consisted in the employment of a large number of issuing streams, as in the Combe's turbine (Fig. 64). Here the supply pipe furnishes water directly to the buckets, without the directing intermediary of guides; the water completely fills the passageways, and discharges into the atmosphere with a somewhat low absolute velocity. A modification of this type, with supply pipe above, was called the Cadiat turbine.
95. "In 1826 the French engineer Fourneyron improved the Cadiat turbine by placing fixed guide-blades just inside the wheelring, around the entire circumference, by means of which the water received a forward direction of motion before entering the channels of the moving turbine. This rendered attainable a very low value of the absolute velocity of the water at exit from the outer rim of the wheel-ring. Also, the wheel being operated under water, the complete filling of the wheel-channels was assured when properly designed. This was the first modern turbine - a motor which, as varied and improved by Fontaine, Henschel, Jonval, and others in Europe, and by Boyden and Francis, and their successors in America, has grown in popular favor, and, together with the impulse wheels already described, has almost entirely supplanted the old forms of vertical waterwheels so long considered as giving the highest efficiency."*

[^1]

## LARGEST TURBINE IN THE WORLD

Installed at the Shawinigan Falls Power Station on the St. Maurice river, about 84 miles northeast of Montreal, Que. Weight, ${ }^{3} 364,000$ pounds; height, 30 feet; weight of wheel, 5 tons; weight of 23 -in. shaft, 10 tons; intake, 10 feet in diarneter; amount of water passing through ver minute, 395,000 gallons.

UNIT CONSISTING OF TWO PAIRS OF 51-INCH "CYLINDER GATE NEW AMERICAN" TURBINES ON HORIZONTAL SHAFT Designed for direct connection to $1,500-\mathrm{K}$. W. generator, operating under a variable head of 25 to 35 feet.

## WATER-POWER DEVELOPMENT

## PART II

## TURBINES

96. A turbine consists essentially of a series of short, curved passageways or buckets divided from one another by vanes or blades, the whole forming a single rigid body attached to the axle, and called the runner. The water for operating the runner passes into the passageways through a set of fixed or stationary channels called guides, the feed-water being admitted around the entire circumference. For convenience and efficiency of operation, the turbine is provided with various controlling, regulating, and governing devices (Fig. 65).
97. Classification. Turbines are classified in several different ways, depending on the criterion used as the basis. Thus, with respect to the direction of flow of the water through the wheel, there are three classes-Radial-Flow, Axial-Flow, and Mixed-Flow turbines.

A radial-flow turbine is one in which the path of a particle of water within the wheel lies in a plane perpendicular to the axis of rotation. The direction of flow may be either outward or inward; that is, the turbine may have internal or external feed (Figs. 66


Fis. 65. Diagram of Typical Turbine. and 67). An axial- or parallel-flow turbinie is one in which the distance of a particle from the axis of rotation remains constant during its passage through the wheel (Fig. 68). Mixed flow is a combination of radial and axial flow; it is usually inward and axial.
98. Another classification divides these motors into impulse
and reaction turbines. If the wheel-passages are not completely filled with water, and if air enters freely so that the entire stream within each wheel-passage is under atmospheric pressure, the


Fig. 66. Section of Guides and Buckets, Fourneyron Turbine, Niagara Falls.


Fig. 67. Section of Runner of Francis Center-Vent Turbine.


Fig. 68. Axial-Flow or Parallel-Flow Turbine.


Fig- 69. Fourneyron Turbine. Radial outward flow.
motor is called an impulse turbine.
If the wheel-passages are completely filled by the water flowing through them under pressure, the motor is called a reaction turbine.

The limit turbine is a type intermediate between the impulse and the reaction turbine; but, though it combines many of the advantages of both types, it has not as yet received much attention. 99. The three typical classes of turbines above described are often called by the names of the eminent hydraulicians who invented or perfected them; thus the reaction turbine with radial outward flow
is frequently called the Fourneyron turbine (Fig. 69). The guides $G$ are rigidly attached to the fixed plate $P$, which is connected with the hollow pipe enclosing the shaft. In such a wheel the discharge may be either into the air or into a body of water; a suction or draft tube cannot very conveniently be used with this type of motor.

A reaction wheel similar to the above, but - with radial inward flow,


Fig. 70. Francis Turbine. Radial inward flow. is often called a Francis turbine (Fig. 70).

A reaction turbine with axial flow is generally termed a Jonval turbine (Fig. 71). The discharge from the two latter types of motor may take place into the air, directly into the tail-water, or into a suction (draft) tube. Limit turbines are sometimes called Haenel turbines; they may be considered impulse turbines without free deviation.
100. The most common forms of reaction turbines used in America, particularly for the smaller sizes, are of the mixedflow type, having radial inward


Fig. 71. Jonval Turbine. Axial flow. admission and axial downward discharge, as the Swain turbine; or of the plain-inward-flow type.

Turbines of the American pattern (inward, downward, and out-
ward), in which the water passes radially inward, then axially, and finally leaves the buckets at a slant between the axial and the radial outflow direction, are called American vortex turbines, types of which will appear in subsequent articles.
101. Any turbine may be made to act as either an impulse or a reaction turbine. If the conditions be such that the water, in passing through the vanes, fails to fill them completely, it is an impulse turbine; if the wheel be placed under water (or drowned), or if by any other means the water is compelled to completely fill all the passages under pressure, it acts as a reaction turbine.
102. In an impalse turbine, the energy of the water is wholly converted into kinetic energy at the inlet surface. Thus the water enters the wheel with a velocity due to the available head, and therefore without pressure; is received upon the curved vanes; and imparts to the wheel the whole of its energy through the agency of the dynamic pressure due to its impulse. Since special care must be exercised to insure that the water will be freely deviated on the curved vanes, such motors are sometimes called turbines with free deviation. For the above stated reasons, the water passages in such wheels should never be completely filled; and in order to insure an unbroken flow through the wheel-passages, and to prevent the formation of eddies at the backs of the vanes, ventilating holes are arranged in the wheel sides.

In a reaction turbine, only a portion of the available energy is converted into kinetic energy at the inlet surface of the wheel. Thus, such a wheel is driven by the dynamic pressure of flowing water which is at the same time under a certain degree of static pressure.
103. Turbine Development in America. "In 1834, M. Fourneyron, a French engineer, had brought out the radial outward-flow turbine known under his name; and in $1840, \mathrm{Mr}$. U. A. Boyden, of Massachusetts, commenced to study and to improve upon this type. M. Fourneyron's diffuser (described later) was also introduced in America by Mr. Boyden, and is therefore usually known as the Boyden diffuser. It should here be stated that the now obsolete diffuser was the forerunner of the conical draft-tube (described later) of the present day; and the same principles underlie the action of both.
"Mr. Boyden was soon followed in this work by Mr. James B. Francis. In 1849, however, Mr. Francis built a radial inward-flow
or vortex turbine for the Booth Cotton Mills, Lowell, Mass. This turbine, which worked under a head of 19 feet, and when tested showed an efficiency of 79.7 per cent, or practically 80 per cent, may be regarded as the prototype of all American turbines.
"The number of revolutions varies as the square root of the head employed; and for the same head, the number of revolutions of different turbines is inversely proportional to their diameters. As all the carly turbines were used with low heads (about 20 feet or less), and as even then (as now) the tendency was to increase the speed of shafting, machinery builders naturally reduced the turbine diameter.
"Mr. Francis's turbine was of the plain inward-flow type, with sufficient room in its interior for the water to turn and escape axially. With the continued reduction of the turbine diameter, this interior space became more and more reduced, so that it soon became necessary to turn the water in an axial direction while still in the runner-bucket-or, in other words, to curve the bucket from a radial to a more or less axial direction. This has been going on gradually, as can be seen, by comparing the early forms of the Humphrey and the Swain turbines with the present form of the Hercules, New American, Leffel, and other turbines, which have scarcely more interior space than is required to pass the shaft through, and have a much greater part of the runner-buckets in the axial- or parallel-flow direction than in the radial-flow direction; while the runner-buckets have assumed such an intricate shape that it is very difficult to analyze the action of the water while flowing through these buckets, or to predetermine mathematically their shape for given conditions."

Clemens Herschel writes:

[^2]outer circumference becomes so large- 6 inches and even 12 inches being not uncommon-that the buckets are unable properly to guide the water as the best efficiency would demand. The area through which the water enters the runner-being the outer circumference of the runner, multiplied by the axial dimension of the bucket en-trance-decreases, of course, with the diameter of the runner; and with it, and in the same ratio, decreases the quantity of water passed through, and the power developed by the turbine.
"To prevent this decrease in entrance area, and in power, builders have gradually increased the axial dimension of the bucket entrance. Thus the efforts made towards greater speed and power have transformed the plain inward-flow turbine-runner of fifty years ago into the shape now generally employed.'*
104. "The American type of turbine is thus distinguished by the great depth of its buckets, its great capacity in proportion to its diameter, and its high speed. It is also distinguished by the form of its buckets, which consist of a ring of curved vanes arranged parallel to the axis and inclosed within the guide-ring; while below the guidering, the buckets expand downward and outward, forming large cup-shaped outlets. The shape of the guide-buckets has not changed to the same extent.
105. "The type to be employed in each individual case should be in accordance with the height of the head to be utilized, as follows:

1. Low heads, say up to 40 feet: American type of turbine (i. e., of the "inward and downward" variety), with horizontal or vertical shaft in open flume or case, nearly always with draft-tube. For heads up to about this limit, the American type of turbine has the great advantage over all other turbine types in common use, that it gives the greatest number of revolutions for a given head and power developed, or the greatest power for a given head and diameter of runner; while the American system of manufacturing only one line of turbines from stock patterns has the great advantage of enabling the builders to fill orders cheaply and quickly.
2. Medium heads, say from 40 to 300 or 400 feet: Radial inward-flow reaction or Francis turbine, with horizontal shaft and concentric or spiral cast-iron case with draft-tube.
3. High heads, say above 300 or 400 feet: Impulse wheel of the usual type (Pelton); or radial outward-flow segmental-feed, free deviation (a Girard impulse wheel); or a combination of both, with horizontal shaft and cast-or wrought-iron case, often with draft-tube.

[^3]"Extremes in speed or power or both will, of course, often demand the use of a turbine type for a head outside of the range for which the type is here proposed.
"Turbines with horizontal shafts should be employed ${ }^{\text {in }}$ all cases except where the use of turbines with vertical shafts is either imperative or gives a decided advantage over turbines on horizontal shafts. This advantage may often be gained by using dynamos with vertical shafts, direct-connected to the turbines, which gives an excellent, compact, and neat arrangement, as in the plant of the Niagara Falls Power Company and many others. Horizontal turbines are not only more convenient in attendance and easier of access for adjustment or repairs; but most of the transmission of power is done by horizontal shafts, and nearly all standard patterns of direct-driven dynamos or other machinery are arranged to connect to a horizontal driving-shaft. With a very low total head or pressure-head above the turbine, it will often be necessary to use vertical turbines to be able to utilize such a head at all. In many locations, horizontal turbines, on account of the great rise of the tail-water during times of flood, would have to be set at so great a height above low tail-water that the head below the turbine would be utterly beyond the practical working limit of draft-tube during the low-water season, and part of the head would thus have to be sacrificed just at the time of least water. In such a case, vertical turbines are of great advantage, as they may be set at any elevation, because their being submerged during times of flood does not interfere with their operation."*

## ESTIMATES FOR WATER POWER $\dagger$

106. The methods of estimating the water power that can be derived by damming a stream, are similar to those for water supply. In the absence of gaugings, the records of rainfall and evaporation are to be collected and discussed; but a few gaugings will give much more definite information, if records of water stages during several years can be had. Here, also, the minimum flow of the stream must receive careful attention, particularly when the plant is to generate electric power for trolley and light service, for the interruption of such serv-

[^4]ice is a serious public inconvenience. It has frequently happened, indeed, that a water-power plant built without sufficient investigation has proved unable to furnish sufficient power during dry seasons, and it has been necessary to install an auxiliary steam plant to make good the deficiency.

Let $W$ be the weight of water delivered per second to a hydraulic motor, and $h$ be its effective head as it enters the motor, $h$ being due either to pressure or to velocity, or to pressure and velocity combined. The theoretic energy per second of this water is:

$$
K=W h ;
$$

and if $W$ be in pounds, and $h$ in feet, the theoretic horse-ppwer of the water as it enters the motor is:

$$
H P=\frac{W h}{550} ;
$$

and this is the power that can be developed by a motor of efficiency unity. The work $k$ delivered by the motor is, however, always less than $K$, owing to losses in impact and friction, and the horse-power $h p$ of the motor is less than $H P$. The efficiency of the motor is:

$$
e=\frac{k}{K}=\frac{\dot{k}}{\overline{W h}} ; \text { or, } e=\frac{h p}{H P} ;
$$

and the vålue of this for turbine wheels is usually about 0.75 ; that is, the wheel transforms into useful work about 75 per cent of the energy of the water that enters it.
107. In designing a water-power plant, it should be the aim so to arrange the forebays and penstocks which lead the water to the wheel that the losses in these approaches may be as small as possible. The entrance from the head-race into the forebay, from the forebay into the penstock, and from the penstock to the motor, should be smooth and well-rounded; sudden changes in cross-section should be avoided; and all velocities should be low, except that at the motor. If these precautions be carefully observed, the loss of head outside the motor can be made very small.

Let $H$ be the total head from the water level in the head-race to that in the tail-race below the motor. The total available energy per second is $W H$; and it should be the aim of the designer to render the losses of head in the approaches as small as possible, so that the effective head $h$ may be as nearly equal to $H$ as possible. Neglect
of these precautions may render the effective power less than that estimated.
108. The efficiency $e_{1}$ of the approaches is the ratio of the energy $K$ of the water as it enters the wheel, to the maximum available energy $W H$; or $e_{\mathrm{t}}=\frac{K}{W I I .}$ The efficiency $E$ of the entire plant, consisting of both approaches and wheel, is the ratio of the work $k$ delivered by the wheel to the energy $W H$; or,

$$
E=\frac{k}{W I I}=\frac{e K}{W K I}=e e_{1} ;
$$

or, the final efficiency is the product of the separate efficiencies. If the efficiency of the wheel be 0.75 , and that of the approaches 0.96 , the efficiency of the plant as a whole is 0.72 ; or only 72 per cent of the theoretic energy is utilized. Usually the efficiency of the approaches can be made higher than 96 per cent.

In making estimates for a proposed plant, the efficiency of turbine wheels may be taken at 75 per cent; the effective work is then 0.75 Wh , where $h$ is the actual effective head on the motor; and accordingly, if the wheels are required to deliver the work $k$ per second, the approaches are to be arranged so that $W h$ shall not be less than 1.33 k . Especially when the water supply is limited is it important to make all efficiencies as high as possible.
109. Water Delivered to a Motor. To determine the efficiency of a hydraulic motor by formula, $k$ is to be measured by the methods of Article 118, and $h$ found by Articles 112 to 114. In order to find the weight $W$ that passes through the wheel in one second, there must be known the discharge per second $q$, and the weight $w$ of a cubic unit of water; then,

$$
W=w q
$$

Here $w$ may be found by weighing one cubic foot of the water; or, in approximate computations, $w$ may be taken at 62.5 pounds per cubic foot: In precise tests of motors, however, its actual value should be ascertained as closely as possible.
110. The measurement of the flow of water through orifices, weirs, tubes, pipes, and channels is so fully discussed under "Hydraulics," that it only remains here to mention one or two simple methods applicable to small quantities, and to make a few remarks regarding the subject of leakage. In any particular case, that method of deter-
mining $q$ is to be selected which will furnish the required degree of precision with the least expense.

For a small discharge, the water may be allowed to fall into a tank of known capacity. The tank should be of uniform horizontal cross-section, whose area can be accurately determined; and then the heights alone need be observed in order to find the volume. These, in precise work, will be read by hook gauges; and in cases of less accuracy, by measurements with a graduated rod. At the beginning of the experiment, a sufficient quantity of water must be in the tank so that a reading of the gauge can be taken; the water is then allowed to flow in, the time between the beginning and end of the experiment being determined by a stop-watch, duly tested and rated. This time must not be short, in order that the slight errors in reading the watch may not affect the result. The gauge is read at the close of the test after the surface of the water becomes quiet; and the difference of the gaugereadings gives the depth which has flowed in during the observed time. The depth, multiplied by the area of the cross-section, gives the volume; and this, divided by the number of seconds during which the flow occurred, furnishes the discharge per second $q$.

If the discharge be very small, it may be advisable to weigh the water rather than to measure the depths and cross-sections. The total weight divided by the time of flow then gives directly the weight $W$. This has the advantage of requiring no temperature observation, and is probably the most accurate of all methods; but unfortunately it is not possible to weigh a considerable volume of water, except at great expense.

When water is furnished to a motor through a small pipe, a common water meter may often be advantageously used to determine the discharge. No water meter, however, can be regarded as accurate until it has been tested by comparing the discharge as recorded by it with the actual discharge as determined by measurement or weighing in a tank. Such a test furnishes the constants for correcting the results found by its readings, which otherwise are liable to be 5 or 10 per cent in error.
111. The leakage which occurs in the flume or penstock before the water reaches the wheel, should not be included in the value of $W$, which is used in computing its efficiency, although it is needed in order to ascertain the efficiency of the entire plant. The manner of deter-
mining the amount of leakage will vary with the particular circumstances of the case in hand. If it be very small, it may be caught in pails and directly weighed. If large in quantity, the gates which admit water to the wheel may be closed, and the leakage being then led into the tail-race, it may be there measured by a weir, or by allowing it to collect in a tank. The leakage from a vertical penstcek whose cross-section is known, may be ascertained by filing it with


Fig. 72. Weir for Modern Power Plant.
water, the wheel being still, and then observing the fall of the water level at regular intervals of time. In designing constructions to bring water to a motor, it is best, of course, so to arrange them that all leakage will be avoided; but this cannot often be fully attained, except at great expense.

The most common method of measuring $q$ is by means of a weir placed in the tail-race below the wheel, as in Fig. 72. This has the disadvantage that it sometimes lessens the fall which would be otherwise available, and that often the velocity of approach is high. It has,
however, the advantage of cheapness in construction and operation, and for any considerable discharge appears to be almost the only method which is both economical and precise. If the weir is placed above the wheel, the leakage of the penstock must be carefully ascertained.
112. Effective Head on a Motor. The total available head $I I$ between the surface of the water in the reservoir or head-race and that in the lower pool or tail-race, is determined by running a line of levels from one to the other. Permanent bench-marks being established, gauges can then be set in the head-race and tail-race, and graduated so that their zero points will be at some datum below the tailrace level. During the test of a wheel, each gauge is read by an observer at stated intervals; and the difference of the readings gives the head $H$. In some cases it is possible to have a floating gauge on the lower level, the graduated rod of which is placed alongside a glass tube that communicates with the upper level; the head $H$ is then directly read by noting the point of the graduation which coincides with the water surface in the tube. This device requires but one observer, while the former requires two; but it is usually not the cheapest arrangement, unless a large number of obseryations are to be taken.

From this total head $H$, are to be subtracted the losses of head in entering the forebay and penstock, and the loss of head in friction in the penstock itself, and these losses may be ascertained by the methods discussed in "Hydraulics." Then,

$$
h=H-h^{\prime}-h^{\prime \prime}
$$

is the effective head acting upon and chargeable to the wheel. In properly designed approaches, the lost heads $h^{\prime}$ and $h^{\prime \prime}$ are very small.
113. When water enters upon a wheel through an orifice which is controlled by a gate, losses of head will result, which can be estimated by the appropriate hydraulic formulx. If this orifice is in the head-race, the loss of head should be subtracted, together with the other losses, from the total head $H$. But if the regulating gates are a part of the wheel itself, as is the case in a turbine, the loss of head should not be subtracted, because it is properly chargeable to the construction of the wheel, and not to the arrangements which furnish the supply of water. In any event that head should be determined which is to be used in the subsequent discussions: if the efficiency of the fall
is desired, the total available head is required; if the efficiency of the motor, that effective head is to be found which acts directly upon it.
114. When water is delivered through a nozzle or pipe to an impulse wheel, the head $h$ is not the total fall, since a large part of this may be lost in friction in the pipe, but is merely the velocity-head $\frac{v^{2}}{2 g}$ of the issuing jet. The value of $v$ is known when the discharge $q$ and the area of the cross-section $a$ of the stream have been determined; and,

$$
h=\frac{v^{2}}{2 g}=\frac{q^{2}}{2 g a^{2}} .
$$

It is here assumed that the center of the nozzle is substantially at tailwater level. In the same manner, when a stream flows in a channel against-the vanes of an undershot wheel, the effective head is the veloc-ity-head; and the theoretic energy is, in either case:

$$
K=W h=W \frac{v^{2}}{2 g}=\frac{w q^{3}}{2 g a^{2}} .
$$

If, however, the nozzle be above the elevation of tail-water, and the water, in passing through the wheel, falls a distance $h_{0}{ }^{\prime}$ below the mouth of the nozzle, then the head which actually acts upon the wheel considered as a water motor merely, is given by:

$$
h_{1}=\frac{v^{2}}{2 g}+h_{0}^{\prime} ;
$$

but the effective head chargeable to the wheel as part of the installation is:

$$
h=\frac{v^{2}}{2 g}+h_{0},
$$

in which $h_{0}$ is the distance of the nozzle center above the tail-water level. In order to utilize the fall $h_{0}$ efficiently, it is plain that the wheel should be placed as near the level of the tail-race as possible.

Lastly, when water enters a turbine wheel through a pipe, a piezometer or a pressure-gauge may be placed near the wheel entrance, to register the pressure-head during the flow; if this pressure-head, measured from the water level in the tail-race, be called $h_{0}{ }^{\prime \prime}$, and if the velocity in the pipe be $v$, then,

$$
h=\frac{v^{2}}{2 g}+h_{0}^{\prime \prime}
$$

is the effective head chargeable to the wheel as part of the installation.

The head chargeable to the wheel itself, regarded merely as a watermotor, without reference to its installation, is:

$$
h_{1}=\frac{v^{2}}{2 g}+h_{0}{ }^{\prime},
$$

in which $h_{0}^{\prime}$ is the pressure-head measured upward from the lowest part of the exit orifices.

From the above discussion, it will be seen that a distinction is sometimes made between the efficiency of the motor itself, and the efficiency of the motor installed as a hydraulic machine. In the former case, only that part of the available head which is actually utilized by the motor should be used in the calculations; in the latter case, the entire available head. The case of discharge into a draft-tube is considered in a subsequent article.
115. Measurement of Effective Power. The effective work and horse-power delivered by a water-wheel or hydraulic motor are often required to be measured. Water-power may be sold by means of the weight $W$, or quantity $q$, furnished under a certain head, leaving the consumer to provide his own motor; or it may be sold directly by the number of horse-power. In either case, tests must be made from time to time, in order to insure that the quantity contracted for is actually delivered, and is not exceeded. It is also frequently required to measure the effective work, in order to ascertain the power and efficiency of the motor, either because the party who buys it has contracted for a certain power and efficiency, or because it is desirable to know exactly what the motor is doing, in order to improve if possible its performance.
116. The test of a hydraulic motor has for its object: First, the determination of the effective energy and power; second, the determination of its efficiency; and third, the determination of that speed which gives the greatest power and efficiency. If the wheel be still, there is no power; if it be revolving very fast, the water is flowing through it so as to change but little of its energy into work; and in all cases there is found a certain speed which gives the maximum power and efficiency. To execute these tests, it is not at all necessary to know how the motor is constructed, or the principle of its action, although such knowledge is very valuable, and is in fact indispensable, in order to enable the engineer to suggest methods by which its operation may be improved.
117. A method in which the effective work of a small motor may be measured, is to compel it to exert all its power in lifting a weight. For this purpose, the weight may be attached to a cord which is fastened to the horizontal axis of the motor, around which it winds as the shaft revolves. The wheel then expends all its power in lifting this weight $W_{1}$ through the height $h_{1}$ in $t_{1}$ seconds; and the work performed per second, then, is:

$$
k=\frac{W_{1} h_{1}}{t_{1}}
$$

This method is rarely used in practice, on account of the difficulty of measuring $t_{1}$ with precision.
118. The usual method of measuring the effective work of a hydraulic motor is by means of the friction brake or power dynamometer invented by Prony about 1780. In Fig. 73 is illustrated a simple method of applying the apparatus to a vertical shaft, the upper diagram being a plan, and the lower an elevation. Upon the vertical shaft is a fixed pulley; and placed against this, are seen two rectangular pieces of wood hollowed so as to fit it, and connected by two bolts. By turning the nuts on these bolts while


Fig. 73. Measuring Work of Motor by Prony Brake. the pulley is revolving, the friction can be increased at pleasure, even to the extent of stopping the motion; around these bolts, between the blocks, are two spiral springs (not shown in the diagram) which press the blocks outward when the nuts are loosened. To one of these blocks is attached a cord, which runs horizontally to a small movable pulley over which it passes, and supports a scale pan in which weights are placed. This cord runs in a direction opposite to the motion of the shaft, so that when the brake is tightened it is prevented from revolving by the tension caused by the weights. The direction of the cord in the horizontal plane must be such that the perpendicular let fall upon it from the center of the
shaft, or its lever-arm, is constant; this can be effected by keeping the small pointer on the brake at a fixed mark established for that purpose.
119. To measure the work done by the wheel, the shaft is disconnected from the machinery which it usually runs, and is allowed to revolve, transforming all its work into heat by the friction between the revolving pulley and the brake, which is kept stationary by tightening the nuts and at the same time placing sufficient weight in the scale-pan to hold the pointer at the fixed mark. Let $n$ be the number of revolutions per second, as determined by a counter attached to the shaft; $P$, the tension in the cord, which is equal to the weight of the scale-pan and its loads; $l$, the lever-arm of this tension with respect to the center of the shaft; $r$, the radius of the pulley; and $F$, the total force of friction between the pulley and the brake. Now, in one revolution, the force $F$ is overcome through the distance $2 \pi r$; and in $n$ revolutions through the distance $2 \pi r n$. Hence the effctive work done by the wheel in one second is:

$$
K=F .2 \pi r n=2 \pi n F r .
$$

The force $F$, acting with the lever-arm $r$, is exactly balanced by the force $P$ acting with the lever-arm $l$; accordingly the moments $F r$ and $P l$ are equal; and hence the work done by the wheel in one second is:

$$
\begin{equation*}
K=2 \pi n P l . \tag{101}
\end{equation*}
$$

If $I$ be in pounds, and $l$ in feet, the effective horse-power of the wheel is given by:

$$
\begin{equation*}
h p .=\frac{2 \pi n P l}{550} \text {. } \tag{101a}
\end{equation*}
$$

As the number of revolutions in one second cannot be accurately read, it is usual to record the counter readings every minute or half-minute. If $N$ be the number of revolutions per minute,

$$
\begin{equation*}
h p=\frac{2 \pi N P l}{33,000} . \tag{101b}
\end{equation*}
$$

It is seen that this method is independent of the radius of the pulley, which may be of any convenient size. For a small motor, the brake may be clamped directly upon the shaft; but for a large one a pulley of considerable size is needed, and a special arrangement of levers is used, instead of a cord.
120. The efficiency of the motor is now found by dividing the effective work per second by the theoretic work per second. Let $K$ be this theoretic work, which is expressed by $W h$; then,

$$
\begin{equation*}
e=\frac{k}{K} ; \text { or, } e=\frac{h p}{H P} . \tag{101c}
\end{equation*}
$$

The work measured by the friction brake is that delivered at the circumference of the pulley, and does not include that power which is required to overcome the friction of the shaft upon its bearings. The shaft or axis of every water-wheel must have at least two bearings, the friction of which consumes probably about 2 or 3 per cent of the power. The hydraulic efficiency of the wheel, regarded as a user of water, is hence 2 or 3 per cent greater than the computed value of $e$.
121. There are in use various forms and varieties of the friction brake; but they all act upon the principle and in the manner above described. For large wheels, they are made of iron, and almost completely encircle the pulley; while a special arrangement of levers is used to lift the large weight $P$. If the work transformed into friction be large, both the brake and the pulley may become hot, to prevent which a stream of cool water is allowed to flow upon them. To insure steadiness of motion, it is well that the surface of the pulley should be lubricated, which, for a wooden brake, is well done by the use of soap. It is important that the connection of the cord to the brake should be so made that the lever-arm $l$ increases when the brake moves slightly with the wheel; if this is not done, the wheel will be apt to cause the brake to revolve with it.

## TURBINE TESTING*

122. Holyoke Testing-Flume. At Holyoke, Mass., where the Connecticut River furnishes a large water power, falling some 60 feet, the Holyoke Water-Power Company controls the water rights, and leases power to the many mill operators of that city. The millowners pay a certain price per annum per mill-power, which, in that locality, is the right to use 38 cubic feet of water per second under a head of 20 feet, either for continuous use (a 24 -hour day) or for a definite fraction of each day.
[^5]In order that the rate at which any mill turbine uses water at any stage or position of its gate or regulating apparatus may become known by simply observing the position of the gate, each turbine, before being installed in the mill where it is to work, is tested at the testing-flume of the company, and thus becomes a water-meter, whose indications, when the motor is in final place, are noted from day to day by an inspector to the company. In the same test, its power, best speed, and efficiency are also determined.

The testing-flume occupies the lower part of a substantial building, and its main features are shown in vertical section in Fig. 74. The


Fig. 74. Vertical Section of Holyoke Testing-Flume.
walls of the wheel-pit $D D$, which is 20 feet square, are built of stone masonry, and lined with brick laid in cement. The water is admitted to it from the head canal through a trunk or penstock, and vestibule, which are not shown in the figure. Over an opening in the floor of the wheel-pit, the wheel $W$ to be tested is set in place, the water discharged from it finding its way through a large opening into the tail-race $C, 35$ feet long and 20 feet wide; and finally over a sharp-crested weir at $A$, into the lower canal. The whole head $h$ available for testing may be from 4 to 18 feet for the smaller wheels, and from 11 to 14 feet for large wheels, up to 300 horse-power. The measuring capacity of the weir, which may be used to its full length, 20 feet (and then would have no end-contractions), is about 230 cubic feet per second. The head $h$ becomes known in any test by observations of the water level in two glass tubes communicating with the respective bodies of water $W$ and $C$. The water in channel $C$, which is a channel of approach
for the weir $A$, communicates (at a point some distance back of the weir) by a lateral pipe with the interior of a vessel open to the air, in a side chamber. Water rises in this vessel, and finally remains stationary at the same level as that of the surface in the channel of approach. A hook gauge being used in connection with this vessel, observations and readings are taken, from which the value of $I$, or head on the weir, may be computed, for use in the proper weir formula for the discharge $q$.

Fig. 74 shows a turbine in position for testing, with a vertical shaft-the ordinary case. Upon the upper end of the shaft is secured a cast-iron pulley $P$, to the rim of which the Prony brake is fitted for purposes of test.
123. The procedure of testing was about as follows: The brake being carefully balanced and adjusted beforehand, a light weight was placed on the scale-pan, and the wheel started at full gate; sufficient friction was then produced to balance the weight, and the speed of wheel noted. The load was then increased at intervals of two or three minutes, by 25 lbs. at a time, until the speed of the wheel had fallen below that of maximum efficiency for the head; the weights were then reduced again, and the velocity of the wheel allowed to increase until the maximum was again passed. The same process was then repeated within a smaller range of speed and with smaller variations of load, until the speed of best work had been more exactly ascertained, and the performance of the turbine at maximum efficiency, under full head and at full gate, had been very precisely determined. This was repeated at each of the part gates, usually down to one-half maximum discharge.
124. Test of the Tremont Turbine. The test of the Tremont Turbine, a 160 -horse-power turbine of the radial outward-flow type (Fourneyron), made at Lowell, Mass., in 1855 by Mr. J.B.Francis, was an event of special interest in the history of hydraulic science, and has become classic. Though the test is by no means recent, it was carried out so thoroughly as to make its details highly instructive to the student of hydraulics. The main features of this test will now be presented and commented on.

The inner and outer radii of the turbine were 3.37 and 4.14 feet respectively; height between crowns, 0.937 foot at entrance, and 0.931 foot at exit. There were 33 guide-blades and 44 turbine-vanes.

As to angles, $a=28^{\circ}, \phi=90^{\circ}$, and $\beta=22^{\circ}$; and the head $h$ on the wheel varied from about 12.5 to about 13.5 feet. The gate was a thin cylinder, movable vertically between the guides and the wheel. There were no horizontal partitions dividing up the wheel-channelsin fact, no special device for preventing the loss of head usually arising at part gate with this kind of regulating apparatus.
125. The annexed table (page 105) gives the principal data and results of Mr. Francis's test of the Tremont turbine, arranged in the order of the speed of wheel. In Experiments Nos. 1 to 15 (see column 1), the cylindrical gate was fully open ("full gate"); while in Experiments 16 to 20, it was in a single fixed position, leaving open, at the wheel-entrance, about one-quarter of the vertical height between crowns; in other words, the gate was drawn up about one-quarter of its full range of height. In this special "part-gate" position, however, the quantity of water passing per second was much greater than one-quarter of that passing at "full gate," as is seen from the values of $q$ in column 4. For example, in Experiment 18, in which (for this position of the gate) the efficiency was a maximum, the value of $q$ is about one-half of the $q$ used in Experiment 6, which gives the maximum efficiency at full gate. It would be said, therefore, that in Experiment 18 the wheel was working at about "half gate." The heading of each column of the table shows clearly the nature of the quantity given in that column, and the units of measurement involved in its numerical value.
126. The rate of flow, or discharge in cubic feet per second, was measured by two weirs at the end of the tail-race, using the Francis weir formula; and the useful power was measured by means of the Prony brake, which in this case consisted of a large and strong friction brake with ares of wood rubbing on the cast-iron pulley which was keyed to the turbine shaft, and arranged with a bell-crank lever and "dashpot" to "cushion" the motion of the lever. In this brake, $r=2.75$ feet, and $l=10.83$ feet.
127. To explain the computations connected with these investigations, Experiment No. 6 will be selected. In this test, 1,524 lbs. was placed in the scale pan, and the nuts tightened up, until the wheel raised this weight and held it just balanced. When the speed of the wheel had adjusted itself to the load, the speed counter indicated $n=0.851$ revolution per second. Hence,

## TEST OF THE TREMONT TURBINE

Selected Experiments

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No OF Exper. | $\begin{gathered} h \\ \text { (FEET.) } \end{gathered}$ | (Revs PER SEC.) | $\begin{aligned} & q \\ & \text { (CUB, FT. } \\ & \text { PER SEC. } \end{aligned}$ | $\begin{aligned} & 2 \pi n P l \\ & (\text { FT, -LBS. } \\ & \text { PER SEC. } \end{aligned}$ | $\underset{\text { (EFFIC.) }}{e}$ | h.p. |

FULL GATE

| 1 | 12.80 | 0.00 | 135.6 | 0 | 0.00 |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 12.95 | 0.45 | 133.4 | 73,160 | .68 |  |
| 3 | 12.97 | 0.53 | 133.7 | 78,490 | .72 |  |
| 4 | 12.97 | 0.60 | 134.8 | 82,110 | .75 |  |
| 5 | 12.94 | 0.64 | 135.1 | 83,960 | .77 |  |
| 6 | 12.90 | 0.85 | 138.2 | 88,210 | .794 | 160.3 |
| 7 | 12.90 | 0.88 | 139.0 | 88,190 | .788 |  |
| 8 | 12.90 | 0.90 | 139.6 | 88,076 | .784 |  |
| 9 | 12.85 | 1.00 | 141.9 | 86,310 | .75 |  |
| 10 | 12.85 | 1.06 | 142.5 | 83,970 | .73 |  |
| 11 | 12.80 | 1.18 | 144.8 | 77,150 | .67 |  |
| 12 | 12.70 | 1.31 | 147.3 | 66,840 | .57 |  |
| 13 | 12.65 | 1.46 | 152.3 | 51,680 | .43 |  |
| 14 | 12.55 | 1.60 | 156.6 | 33,350 | .27 |  |
| 15 | 12.54 | 1.79 | 162.3 | 0 | 0.00 |  |
|  |  |  |  |  |  |  |
|  |  |  |  | PART GATE |  |  |
| 16 | 13.51 | 0.00 | 60.3 | 0 |  |  |
| 17 | 13.55 | 0.46 | 67.8 | 24,460 | 0.00 | .43 |
| 18 | 13.48 | 0.67 | 71.8 | 27,980 | .46 | 50.9 |
| 19 | 13.39 | 0.96 | 76.6 | 21,250 | .33 |  |
| 20 | 13.34 | 1.25 | 80.4 | 0 | .00 |  |

$K=2 \pi n P l$
$=2 \times 3.1416 \times 0.851 \times 1,524 \times 10.83$
$=88,214$ foot-pounds per second $=160.3$ horse-power.
For computing the value of the discharge $q$, it is to be observed that the water passed over two contracted weirs with a combined length $l=16.98$ feet; the number of end contractions was therefore $n=4$; the head over the weir crest was $I=1.87$ feet; velocity of approach was not considered. Therefore,

$$
\begin{aligned}
q & =3.33(b-0.1 n H) H^{\frac{3}{2}} \\
& =3.33(16.98-.75)(1.87)^{\frac{3}{2}} \\
& =138.2 \text { cubic feet per second. }
\end{aligned}
$$

The difference in elevation between head and tail-water levels in this experiment was 12.90 feet; consequently the total available energy was:

$$
K=138.2 \times 62.5 \times 12.90=111,400 \mathrm{ft} .-\mathrm{lbs} . \text { per second. }
$$

Therefore the efficiency was:

$$
e=\frac{88,214}{111,400}=79.4 \text { per cent. }
$$

128. Discussion of Test of Tremont Turbine. In the experiments with full gate, Nos. 1 to 14 inclusive (see table, page 105), on account of the progressive lessening of the weight $P$ in the scale pan (the brake friction being regulated each time to correspond), the uniform speed to which the wheel adjusts itself in successive experiments increases progressively from the zero value, or state of rest, of Experiment 1 (when the friction was so great as to prevent any motion), up to a maximum rate of 1.79 revolutions per second, attained when no brake friction whatever (no load) was present. In this last experiment, there being no useful work done, all the energy of the mill-site is wasted, partly in axle friction, but chiefly in fluid friction (eddying of the water, and finally, heat), both in the wheel-passages and also in the tail-race, where the water which has left the wheel with high velocity soon has its velocity extinguished. The same statement is true also for Experiment No. 1, except that axle friction is wanting. In both experiments the efficiency is, of course, zero.

The quantity of water discharged per second, $q$, is seen to increase slowly (after Experiment 2) from 133.4 to 162.3 cubic feet per second, though not differing from the average by more than ten per cent. This may be accounted for, in a rude way, as an effect of centrifugal action (as in a centrifugal pump), since the Tremont turbine is an outward-flow wheel. The reverse is found to be true for inward-flow turbines, notably the Thompson vortex wheel, which is therefore to some extent self-regulating in the matter of speed, since a less discharge at a speed higher than the normal diminishes the power, and hence the tendency to further increase of speed.

In the succession of experiments Nos. 1 to 15 (all at full gate and under practically the same head $h$ ), the efficiency is seen to have a zero value both at beginning and end of this series, and to reach its maximum at about the sixth experiment, in which the speed is noted as being about one-half that at which the turbine runs when entirely "unloaded" (Experiment 15). This is roughly true in nearly all turbine tests; but a notable feature of considerable practical advantage is that a fairly wide deviation from the best speed affects the efficiency but slightly. For instance, a variation of speed by 25 per
cent either way from the best value (of 0.85 revolution per second) causes a diminution in the efficiency of only about four per cent.

It should be remembered, also, in this connection, that since the water used per second $(q)$ is somewhat different at different speeds (at full gate), the speed of maximum power differs slightly from that of maximum efficiency.
129. In the five "part-gate" experiments, Nos. 16 to 20, the gate remains fixed in a definite position (about one-quarter raised, although the discharge is about one-half that of full gate) through all these five runs. The head is practically constant. At first the wheel is prevented from turning. The power and efficiency are then, of course,


Fig. 75. Curves Showing Results of Test of Tremont Turbine.
zero; but $q=60.3$ cubic feet per second. As the turbine is permitted to revolve under progressively diminishing friction, the speed of steady motion becomes greater, reaching its maximum ( 1.25 revolutions per second) when the wheel runs "unloaded," in Experiment 20; but the power reaches a maximum and then diminishes. The same is true of the efficiency, whose maximum (in Experiment 18) is seen to be about 46 per cent only. This forms a striking instance of the disadvantage and wastefulness of a cylindrical gate unaccompanied by other mitigating features, when in use at part gate. This defect, however, may be largely remedied by the use of horizontal partitions in the wheel-channels, as in Fig. 81, or by employing curved upper crowns, as in the American "inward and downward" turbines.
130. Fig. 75 is a graphic representation of the results of the test as set forth in the table (page 105).

## REACTION TURBINES

131. Formula for Inward or Outward Flow. Discharge. The following analysis applies equally well with either direction of flow. The discharge from a reaction turbine, unlike that from an impulse turbine, depends on the speed of revolution, as well as on the orifice areas, as in the case of the reaction wheel already discussed. Let Fig. 76 represent diagrammatically an outward-flow turbine with the customary notation as shown in Fig. 77 and explained in preceding analyses. In addition, let $a_{1}, a$, and $a_{0}$ be the areas of the respective orifices or water-passages, measured normal to the directions of $V_{1}, V$, and $v_{0}$; and let $H$ represent the pressure head on the guide-orifices at the gate openings, as would be indicated by piezometer tubes or pressure gauges if they were inserted at such points.

From Article 6, neglecting frictional losses,

$$
H+\frac{v_{0}{ }^{2}}{2 g}=h+H_{1}
$$

also, from Article 89, neglecting frictional losses,

$$
H_{1}+\frac{V_{1}^{2}}{2 g}-\frac{u_{1}^{2}}{2 g}=H+\frac{V^{2}}{2 g}-\frac{u^{2}}{2 g} .
$$

The addition of these two equations results in the following formula:

$$
V_{1}^{2}-V^{2}+v_{0}^{2}=2 g h+u_{1}^{2}-u^{2} \ldots \ldots . .(102)
$$

Being a reaction turbine, the buckets are completely filled; therefore the same quantity of water must pass per second through each of the areas $a_{1}, a$, and $a_{0}$; from which condition the following relations are obtained :

$$
V_{1}=\frac{q}{a_{1}} ; \quad V=\frac{q}{a} ; \quad \mathrm{v}_{0}=\frac{q}{a_{0}} .
$$

Substituting these values in the last formula above, and solving for $q$, there results:

$$
q=c_{d} \frac{\sqrt{\frac{2 g h+u_{1}{ }^{2}-u^{2}}{1}} \frac{1}{a_{1}{ }^{2}}-\frac{1}{a^{2}}+\frac{1}{a_{0}{ }^{2}}}{}=m \sqrt{2 g h+u_{1}{ }^{2}-u^{2}} \ldots(\mathbf{1 0 2 a})
$$

which is a formula for discharge through a reaction turbine, for either inward or outward flow. The coefficient $c_{d}$ is introduced to take account of losses through leakage and friction, which factors were not
considered in the above analysis. For an outward-flow turbine, $u_{1}$ is greater than $u$, consequently the discharge increases with the speed; for an inward-flow turbine $u_{1}$ is less than $u$, and therefore the discharge varies inversely with the speed.

The value of the coefficient $c_{d}$ varies with the head of water and with the details of the wheel design. In one case of an outward-flow turbine, in which $r=2.67 \mathrm{ft}$.; $r_{1}=3.32 \mathrm{ft}$.; with total head of water varying between 17.16 and 17.34 ft ; number of revolutions per minute, between 63.5 and $100 ;$ and the discharge with full gate, from 117 to 127.7 cubic feet per second, the value of $c_{d}$ was found to range between .941 and .950 .

A formula for discharge from a turbine operating at part gate is difficult to formulate theoretically, because of the losses of head resulting from the partial closure, analytical expressions for which are not definitely known for turbines. The values of $q$ for any turbine operating at part or full gate may be obtained by measuring the quantity of water actually discharged, by any of the various methods described in works on hydraulies, and substituting theresulting values of $q$ inEquation 102a.


Fig. 76. Diagrammatic Representation of Outward-Flow Turbine.
132. Work and Efficiency. The following analysis is also valid for both directions of flow. In addition to the notation adopted and employed in the preceding article, and indicated on the corresponding diagrams, let $d_{1}, d$, and $d_{0}$ be the respective depths of the exit, the entrance, and the guide-orifices or water-passages. With gates fully open (in which case $d_{0}$ becomes equal to $d$ ), and neglecting the thickness of the vanes or passage walls,

$$
a_{0}=2 \pi r d \sin \alpha ; \quad a=2 \pi r d \sin \phi ; \quad a_{1}=2 \pi r_{1} d_{1} \sin \beta .
$$

Since, in a reaction turbine, the passageways are always completely full of water,

$$
q=v_{0} .2 \pi r d \sin \alpha=V .2 \pi r d \sin \phi=V_{1} 2 \pi r_{\mathbf{r}} d_{1} \sin \beta .(\mathbf{1 0 2 b})
$$

The general conditions previously established and discussed-that for
maximum efficiency the water must enter tangentially to the vanes, and the absolute velocity of the water at discharge must be as low as pos-sible-will be fulfilled for the first case, when $u$ and $v_{0}$ are proportional to the sines of their opposite angles-that is, when

$$
\begin{equation*}
\frac{u}{v_{0}}=\frac{\sin (\phi-\alpha)}{\sin \phi} \tag{103}
\end{equation*}
$$

and for the second case, approximately and simply, when

$$
u_{1}=V_{1} .
$$

(The theoretic condition is $V_{1}=u_{1} \cos \beta$, as explained in a preceding article.)


Fig. 77. Turbine Notation.
Substituting $u_{1}$ for $V_{1}$ in the third member of Equation 102b, and equating it to the first, there results:

$$
\frac{u_{1}}{v_{0}}=\frac{r d \sin \alpha}{r_{1} d_{1} \sin \beta} .
$$

Then, since $\frac{u}{u_{1}}=\frac{r}{r_{1}}$, the above equation may be reduced to:

$$
\begin{equation*}
\frac{u}{v_{0}}=\frac{r^{2} d \sin \alpha}{r_{1}^{2} d_{1}^{2} \sin \beta} . \tag{103a}
\end{equation*}
$$

From the trigonometric relations at the point $\Lambda$, in Fig. 77,

$$
V^{2}=u^{2}+v_{0}^{2}-2 u v_{0} \cos \alpha .
$$

Substituting this value of $V^{2}$ in Equation 102, and making $V_{1}=u_{1}$, as established above for one of the conditions of maximum efficiency, the following additional important relation immediately results:

$$
\begin{equation*}
u v_{0}=\frac{g h}{\cos \alpha} . \tag{104}
\end{equation*}
$$

From the above necessary relations, the following practical formule may be developed by combining Equations 103 and 104:

$$
\begin{equation*}
u=\sqrt{\frac{g h \sin (\phi-\alpha)}{\cos \alpha \sin \phi}} . \tag{105}
\end{equation*}
$$

and,

$$
v_{0}=\sqrt{\frac{g h \sin \phi}{\cos \alpha \sin (\phi-\alpha)}} \cdots \cdots \cdots(\mathbf{1 0 6})
$$

Equation 105 gives the advantageous velocity of the circumference at the point of wheel entrance, from which the advantageous velocity of the circumference at exit may be obtained from the relation $\frac{u}{u_{1}}=\frac{r}{r_{1}}$; and Equation 106 gives the value of the absolute velocity of entrance of the water into the wheel.

By combining Equations 103 and 103a, there is obtained:

$$
\begin{equation*}
\frac{\sin (\phi-\alpha)}{\sin \phi}=\frac{r^{2} d \sin \alpha}{r_{1}^{2} d_{1} \sin \beta^{\prime}} \tag{107}
\end{equation*}
$$

which establishes the necessary relations between the dimensions and angles of the wheel which must obtain in order that the above conclusions may be valid.

The work imparted to the wheel is, theoretically:

$$
\text { Work }=W \frac{h-v_{1}^{2}}{2 g} ; \cdots \ldots \ldots . . .(108)
$$

and the efficiency is theoretically:

$$
\begin{equation*}
e=1-\frac{v_{1}^{2}}{2 g h} . \tag{108a}
\end{equation*}
$$

in which $h$ indicates the available head properly chargeable to the machine as installed.

By means of Equations 105 and 107, and the relations at the point $B$, the above value of the efficiency may be reduced to:

$$
\begin{equation*}
e=1-\frac{d}{d_{1}} \tan \alpha \tan \frac{1}{2} \beta \tag{109}
\end{equation*}
$$

in which $d$ and $d_{1}$ are the entrance and exit depths, respectively.
The discharge is:

$$
q=a_{0} v_{0} ;
$$

and the useful work of the wheel is:

$$
\text { Work }=e \times w q h \text {. }
$$

It is to be observed that the foregoing formulae for work and efficiency do not take into account losses of energy incurred during the passage of the water to and through the guide- and wheel-buckets and those due to clearance.

If losses due to impact and friction in the runner buckets be neglected, then the work imparted to the wheel is:

$$
\text { Work }=W \frac{h^{1}-v_{1}^{2}}{2 g} ; \cdots \ldots . . . .(109 \mathbf{a})
$$

and the efficiency is:

$$
\begin{equation*}
e=\frac{h^{1}-v_{1}{ }^{2}}{2 g h} \tag{110}
\end{equation*}
$$

in which $h^{1}$ is the head representing the total energy of the water as it enters the runner buckets.
133. Downward=Flow or Parallel=Flow Reaction Turbine. Let $r$ be the mean radius, and $u$ the mean velocity, of the entrance and exit orifices of the wheel, and let $d$ and $d_{1}$ be the widths of the entrance and exit orifices respectively. The formule developed in the preceding articles for inward- and outward-flow reaction turbines may be adapted to this case by making $u_{1}=u$, and $r_{1}=r$.

Thus the advantageous velocity of entrance is:

$$
\begin{equation*}
v_{0}=\sqrt{\frac{g h \sin \phi}{\cos \alpha \sin (\phi-\alpha)}} . \tag{111}
\end{equation*}
$$

The advantageous speed is:

$$
\begin{equation*}
u=\sqrt{\frac{g h \sin (\phi-\alpha)}{\cos \alpha \sin \phi}} . \tag{112}
\end{equation*}
$$

The necessary relation between the vane angles and the wheel dimensions is:

$$
\begin{equation*}
\frac{\sin (\phi-\alpha)}{\sin \phi}=\frac{d \sin \alpha}{d_{1} \sin \beta} \tag{113}
\end{equation*}
$$

and the hydraulic efficiency is:

$$
\begin{equation*}
e=1-\frac{d}{d_{1}} \tan \alpha \tan \frac{1}{2} \beta \tag{114}
\end{equation*}
$$

## IMPULSE TURBINES

134. Formulæ. The velocity $v_{0}$ with which the water leaves the guide-orifices and enters the runner buckets, is, in the case of impulse turbines, theoretically equal to $V \underline{2 g h_{0}}$, where $h_{0}$ stands for the effective head on such orifices. The analysis and conclusions of Articles 76 and 77 apply directly to such motors. Accordingly, for a properly designed impulse turbine, the entrance angle should be double the approach angle; that is:

$$
\phi=2 \alpha .
$$

The advantageous speed is:

$$
u=\sqrt{\frac{g h_{0}}{\frac{\cos ^{2} \alpha}{}}} .
$$

When the motor is running at this best speed, the absolute velocity of exit is:

$$
v_{1}=v_{0} \frac{r_{1} \sin \frac{1}{2} \boldsymbol{\beta}}{r \cos \alpha} .
$$

The work imparted to the wheel is:

$$
\text { Work (Max.) }=W \frac{v_{0}{ }^{2}-v_{1}^{2}}{2 g}=W h_{0}\left\{1-\left(\frac{r_{1} \sin \frac{1}{2} \beta}{r \cos \alpha}\right)^{2}\right\} ;
$$

and the efficiency is:

$$
e(\text { Max. })=\frac{h_{n}}{h}\left\{1-\left(\frac{r_{1} \sin \frac{1}{2} \beta}{r \cos \alpha}\right)^{2}\right\},
$$

in which $h$ is the available head properly chargeable to the motor
It is clearly to be seen that both the approach angle $a$ and the exit angle $\beta$ should be small for high efficiency, and that the angle $\beta$ exercises a greater influence on the efficiency than the angle $a$, both of which conclusions have already been discussed in the articles referred to above.

The discharge is:

$$
q=a_{0} v_{0}=a_{0} \sqrt{2 g h_{0}} ;
$$

and the work of the turbine per second is:

$$
\text { Work }=e \times w_{q} h .
$$

With both reaction and impulse turbines, when the guide-buckets are considered part of the turbine, $h$ is the head representing the total available energy existing in the water as it enters the guides. When the turbine runner alone is considered to constitute the motor proper, and the guides part of the approach, $h$ is the head representing the total available energy existing in the water as it emerges from the guide-buckets. In the latter case, in the above analysis, $h$ may be put equal to $h_{0}$. In both cases, however, there is a certain amount of loss in the clearance.
135. General Definitions. Complete and Partial Admission. These terms signify that water may be admitted to all the wheel passages at once, or to a limited number of them.

Gate-Opening; Part Gate. The area left open for the clear passage of water by the regulating gate or gates, is called the gate-opening. It should not be confounded with the amount of water flowing through the gate-opening at any particular position of the gate. The term five-eighths gate opening, etc., is often employed to mean that position of the regulating apparatus which allows five-eighths, etc., of the
full discharge (i.e., discharge at full gate) to take place. Five-eighths discharge would better express the meaning in such a case.

Clearance. The clear space between the guide-ring and the runner is called the clearance.
136. Impulse and Reaction Turbines. Comparisons. Reaction wheels necessarily have complete admission; and partial closure of the gates (to all the wheel passages simultaneously) results in material loss of energy, due to expansion of section after the contraction; while, with impulse wheels, the admission may be complete, or the supply may be decreased by partial admission, with little, if any sacrifice of efficiency, Moreover, the regulating gates for impulse turbines are of much simpler construction than those for reaction turbines.

The speed of an impulse turbine for a given head is less than that of a reaction turbine; but the relative velocity of entrance is greater; hence there is greater liability to shock and eddies.

The dimensions of an impulse wheel may vary between wide limits, so that for high falls with a small supply of water, a comparatively large wheel may be employed, with a low speed. The speed of a reaction turbine under such conditions would be inconveniently great; and any considerable increase in diameter to reduce the speed would increase the fluid friction, and render more troublesome the proper proportioning of the vanes. When there is an ample supply of water, and the fall is not too great, the reaction turbine is usually to be preferred; but on high falls, on account of the resulting speed becoming inconveniently great, a turbine of the impulse type should be adopted, which permits without disadvantage an increase in diameter with corresponding decrease in speed. On very high falls, an impulse wheel would be preferable.

The advantageous speed of an impulse turbine remains the same for all positions of the gate; but with reaction turbines it is less at part gate than at full gate; and it has already been stated that the partial closing of the gates results in a material loss of energy in the case of reaction wheels; and since, for many industrial purposes, it is essential to maintain a constant speed in spite of variations in power or useful work, it follows that to maintain this constant speed with a reaction turbine involves considerable loss of efficiency. It is also evident that a turbine of the impulse type has a marked advantage in point of efficiency when the supply is low.
137. To partially prevent the loss of head, with consequent loss of power, incidental to the operation of a reaction turbine at "part gate," the turbine runner may be divided into several parts or stories by means of horizontal partitions, as described later (see Figs. 81, 82). During partial operation, one or more of these divisions would be entirely closed off by the gate, the remaining divisions only being in action; the expansion of the stream due to partial throttling being thus avoided, the efficiency at "part gate", when using less than the usual quantity of water, would not be materially altered.
138. 'The Duplex motor, a double tirbine of the parallel-flow type, consists of a pair of concentric runners made in one piece, supplied with water by a similar pair of concentric annular supply guidepassages. The supply for either division may be cut off independently, leaving the other runner in action without sacrifice of efficiency. Such motors must be distinguished on the one hand from double motors of the two-story type, as just described, and on the other hand from double turbines, in which two essentially different wheels are combined and mounted on the same shaft for the purpose of increasing the capacity of the turbine without increasing its diameter, as in the Leffel and other types.
139. The principal disadvantages of the impulse type are that the turbine must always work in air, for, under water, the runner buckets cannot be ventilated; or, if a draft-tube is used, an air-admission valve must be provided, both to supply air for ventilating the buckets, and to keep the surface of the water in the draft-tube below the exit orifices and moving parts of the motor, as explained in a subsequent article.
140. In impulse turbines, the entrance angle $\phi$ should be double the approach angle $a$; but in reaction turbines it is often greater than $3 a$, and its value depends on the exit angle $\beta$; hence the vanes in impulse turbines are of sharper curvature for the same values of $a$ and $\beta$. $\beta$ is usually greater for inward-flow than for outward-flow reaction wheels, in order that the exit orifices may be sufficiently large. If the entrance angle $\phi$ is $90^{\circ}$ (a good value), Equation 105 shows that the velocity $u$ is that due to one-half the head. Equation 109 shows that the efficiency is increased by making the exit depth $d_{1}$ greater than the entrance depth $d$ (Bell-mouthed profiles; Diffuser); but usually they do notdiffer very much, and frequently they are made equal.
141. Discharge. Impulse turbines always discharge into the air, at some distance above tail-water; consequently that part of the available head between the center of discharge and the tail-water level is lost, unless the motor is set to operate in the space above the "hanging column" of a draft-tube, in which case only part of the head is lost, as described later. A reaction turbine may discharge into the free air, in which case the same loss occurs; or it may be "drowned" -that is, set below the water level in the tail-race; or it may discharge into a suction or draft-tube; in these two latter cases the abovementioned loss will not take place.

## TURBINE ACCESSORIES

142. Diffuser. An apparatus for the purpose of providing a gradual enlargement of section for the passage of the discharge water


Fig. 78. Illustrating Operation of Diffuser.
as it leaves the runner-buckets of a radial outward-flow turbine, is called a diffuser. It usually consists of two fixed conical zones flaring out opposite the water edges of the turbine crowns, giving a bellmouthed or divergent profile to the walls of the passageway at that point of the flow. The object of the diffuser is to prevent part of the loss of energy incurred in the general case, due to the absolute velocity of the escaping water. In this case the absolute velocity of discharge at the extremities of the runner buckets is slowly decreased as the cross-section of the passageway through the diffuser increases; and the discharge water finally passes out at a much lower absolute velocity than that at the runner, with a consequent gain in efficiency (see Fig.78,
and $n K n$ of Fig. 71). The efficiency of a reaction turbine is increased by making the exit depth $d_{1}$ greater than the entrance depth $d$, as shown by Equation 109; the stationary diffuser produces the same result.
143. Draft=Tube. A wheel set above the level of the tail-race and discharging at an elevation $h$ feet above that level, loses $h$ feet of available head. If the discharge takes place through a (substantially) vertical pipe which is always full of water, and the lower extremity of which is below the level of the tail-water, most of this head becomes effective, since the draft-tube virtually adds this additional head to the static head. In order that the tube may always remain full of water, the internal fluid pressure must be greater than zero; and therefore the draft-tube must not be placed more than about 25 feet (the practical suction limit) above the tail-water level. In practice, these tubes are rarely made longer than about 18 feet, their principal use being to render the turbine easily accessible for examination and repairs, without the necessity of draining the wheel-pit, and without loss of head in setting the turbine above tail-water.
144. The draft-tube is a suction-tube or water-barometer; therefore, if $v$ be the velocity of the water in the tube, the balancing height of water in feet cannot be greater than $34-\frac{v^{2}}{2 g}$; and if the vertical length is greater than this, that portion of the tube above this height will contain a vacuum, which involves a loss of head equal to the length of this empty space. The above considerations do not take into account several additional losses, such as that due to entrance, friction, etc., as well as that carried away by the discharge water, the effect of which is to reduce the effective head; it is evident, therefore, that the total draft-head cannot be made available in the turbine. The water discharged at the lower end of the turbine should have a velocity of not less than 2 or 3 feet per second, in order that air bubbles may not rise in the tube and displace the water; and also to carry off any air that may be in the tube on starting the turbine.
145. In order to seal the draft-tube against an inrush of air, it should dip below the level of the tail-water from 6 to 12 inches for short and small draft-tubes, increasing to 20 to 24 inches for long and large tubes. The dip should be sufficient to insure an unbroken seal at all stages of tail-water level. To facilitate the escape of the water from a vertical draft-tube, the lower end should be flared outward,
trumpet-shape. These tubes are usually built-up of lap-riveted steel plates, of circular cross-section, as this form offers great resistance to collapse. They must be thoroughly air-tight, as any leakage of air will destroy the vacuum, with consequent loss of draft-head. When the power-house is built with a concrete substructure, it will often be found advisable to mould the draft-tube directly in the concrete, and so dispense with a metal-tube.

Example 19. A turbine (Fig. 79) receives a uniform supply of 20 cubic feet of water per second from a steel penstock 2 feet in diameter and 2,000


Fig. 79. Diagrammatic Representation of Turbine Installed, Showing Penstock, Head-Race, and Tail-Race.
feet long. The total drop from head-race to tail-race is 80 feet. The turbine is installed 10 feet above tail-water. Discuss its operation.

The mean velocity of flow in the pipe is:

$$
v=\frac{q}{\pi r^{2}}=\frac{20}{3.1416 \times(1)^{2}}=6.4 \text { feet per second. }
$$

The head lost in friction (taking the value of $c$ as 100 , from $v=$ c. $\sqrt{r s}=\mathrm{c} \sqrt{\left.\frac{d}{4} \frac{h_{f}}{l}\right) \text { is: }}$

$$
\left.h_{f}=15.8 \text { feet ( } F D \text { in the figure }\right) .
$$

The head lost at entrance (taking $m=0.5$, from $h_{e}=m \frac{v^{2}}{2 g}$ ), is:

$$
h_{e}=0.34 \text { foot. }
$$

The velocity-head of the moving water $\left(\frac{v^{2}}{2 g}\right)$, is:

$$
\frac{v^{2}}{2 g}=0.68 \text { foot }
$$

In the figure, $C F=0.34 \times 0.68=1.02$ feet; and $K D=70-(15.8$ $+1.02)=53.18$ feet. Consequently the total effective energy delivered to the turbine is represented by 53.18 feet of pressure and 0.68 foot of kinetic energy $=53.86$ feet; but it has an additional gravityhead of 10 feet chargeable to it, which is wasted if the motor discharges into the atmosphere, and practically all utilized if it discharges into a draft-tube. In either case the head $h$ chargeable to the installed motor, and to be used in computations of efficiency, is $53.86+10=$ 63.86 feet. At first sight it might appear that the head chargeable to the motor should be 80 feet, the total fall; but from what has preceded, it is evident that $15.8+0.34=16.14$ feet head is utilized in overcoming penstock resistances, and is not properly chargeable against the motor, either as a separate machine, or as installed. In the former case, the head $h$ should be 53.86 feet; in the latter, 63.86 feet.

Thus, with a diameter of 2 feet, the loss of head in the penstock is $15.8+0.34=16.14$ feet; and the power lost is $16.14 \times 20 \times 62.5$ $=20,175$ foot-pounds per second $=36.7$ horse-power.

If the turbine discharge into the atmosphere, the head of 10 feet is also lost; this would occasion a further loss of $10 \times 20 \times 62.5=$ 12,500 foot-pounds per second, or 22.7 horse-power. With a drafttube, this last loss would be avoided.

Suppose a 3 -foot penstock to be substituted for the 2 -foot, the discharge remaining as before.

The mean velocity in this case is 2.84 feet per second.
The head lost in friction (supposing $c$ to be 110 in this case) is:

$$
h_{f}=1.8 \text { feet. }
$$

The head lost at entrance is:

$$
h_{e}=0.06 \text { foot }
$$

The velocity-head is:

$$
\frac{v^{2}}{2 g}=0.12 \text { foot. }
$$

Thus the loss of head in the penstock is $1.8+0.06=1.86$ feet, which represents a loss of $1.86 \times 20 \times 62.5=2,325$ foot-pounds per second, or 4.2 horse-power.
146. Flaring or Conical Draft-Tubes. When draft-tubes are used, they should be of the flaring type, so as to change the speed of the water gradually; for when the tube is of the same cross-sectional area
for its entire length, much of the energy which should be made available owing to the lower velocity of discharge, is lost in shock when the water issuing from the runner at a relatively high speed strikes the water in the draft-tube moving with a lower speed. It has already been shown that the head corresponding to the absolute velocity with which the water leaves a wheel is in the general case entirely lost; but when the discharge takes place into a draft-tube of conical or flaring shape (the cross-sectional area increasing gradually from the turbine to the tail-water), the velocity of the water is gradually reduced below that of discharge at the extremities of the runner-buckets, so that the amount lost to the turbine is less, as in the case of the diffuser. Since the reduction of velocity should be gradual, the change in section must be gradual ; and it is therefore sometimes advantageous to increase the total length of the draft-tube without increasing the drafthead, by curving or inclining the tube; this procedure may sometimes be adopted to save tail-race excavation.
147. Pulsation or oscillation of the water is frequently noticed, especially in connection with high draft-head, particularly if the turbine is subject to sudden changes of load, and is controlled by a quickacting governor. This is extremely detrimental to good speed regulation; and in extreme cases, may even seriously injure the motor. Conical draft-tubes are not subject to these pulsations to the same extent as cylindrical tubes; they are also more efficient in expelling air when the turbines are started, and are better able to retain the draft-head when the motors are running with light loads. A draft-tube with diffuser is shown in Fig. 71.
148. The use of draft-tubes in recent years has marked a notable advance in turbine practice; it has made practicable the employment of turbines on horizontal shafts, the turbine being connected to the tube by means of a draft-tee or elbow.

Draft-tubes may be employed with greater or less convenience in connection with any class of turbine, though the Jonval and Francis types, with their modifications, are best adapted to their use. Even impulse wheels of the Pelton and other types have been fitted with draft-tubes; but in such cases the upper surface of the water in the tube must be maintained automatically at an elevation just below the lowest point reached by the revolving buckets, which thus move in rarified air within a strong casing forming the top of the draft-tube.

This gives the advantage, not only of added draft-head, as just described, but also of decreased air friction for the running parts. In such cases, that portion of the draft-head is necessarily lost which is represented by the distance between the water surface in the drafttube and the center of the nozzle discharge opening; or, in case two or more nozzles are used, it is the mean vertical distance. In the case of an impulse turbine fitted with a draft-tube in the manner described above, the head lost is the vertical distance, or the mean vertical distance, from the water level in the draft-tube to the center of the guide-bucket discharge openings.
149. The principle of the air-admission valve for automatically maintaining a constant height of "hanging column" in the draft-tube, is very simple. A vertical pipe is placed at the side of the turbine, its upper end connected to the turbine case, and its lower end to the draft-tube, just as a water-gauge is connected to a boiler. In this pipe floats a copper ball connected with an air-admission valve in the turbine case; when the water level rises, the float rises with it and opens the valve, admitting air, which causes the water level and the copper ball to fall, thus closing the valve.
150. In European practice, gates are occasionally employed to close the lower end of draft-tubes for the purpose of filling them with water before starting the turbines (as in priming pumps), or for decreasing the feed when running under light load, thus reducing the tendency, under the above conditions, of the water to crowd to one side, or to drop through the draft-tube without expelling the air and producing suction, with consequent loss of draft-head. Curved or inclined draft-tubes are more likely to suffer in this way than vertical tubes; and large and long tubes more than small and short. But such gates are costly and cumbersome for large turbines, and their employment in American practice to any great extent is improbable.
151. Fall=Increaser. Under the name of Fall-Increaser, Mr. Clemens Herschel has patented an apparatus designed to increase the fall acting on hydraulic turbines by the use of freshet water otherwise going to waste. In this way the normal output of power of a hydraulic power-plant may be maintained at a constant normal quantity in spite of back-water, and for all those days in the year when there is water enough flowing in the river to produce so much as the normal output. In some cases of low fall, when there is an abundance of
water to be had, also for certain cases of tide-mills, the fall acting on the turbines may be increased throughout the year, over and above the natural fall, so as to produce a greater speed, and thus render the location more fit for generating electricity; while at the same time it will diminish the cost of the plant-generators, turbines, and building -per horse-power produced.

The fall-increaser, shown underneath the turbines, and operating at a time when the direct discharge of the turbines has been shut off,


Fig. 80. Application of a Fall-Increaser to a Power Plant.
exhausts the turbine discharge from the vacuum-box, and also produces a partial vacuum in this vacuum box, thus increasing the fall that would otherwise act upon the turbines. Fig. 80 shows diagrammatically the application of a fall-increaser to a power plant.
152. Regulating Gates. For many industrial purposes the power required of a turbine is variable, as when the number of machines operated in a factory is not constant, or when running dynamos to supply electric current to meet the fluctuating demands of lighting or of transportation service; and since the speed of the tur-
bine should be fairly constant, the variation in power must be provided by varying the quantity of water supplied to the motor. Under these conditions, it is evident that the average position of the turbine gate is not that of full gate, and the problem of design to secure high efficiency at part gate, and also at full gate, is not a simple one. To regulate the quantity of water supplied to the motor without serious loss of efficiency, is more difficult in the case of reaction turbines than of impulse turbines, for, with the latter type, it is only necessary to vary the cross-section of the guide-passages at the place of discharge, which can be done with relatively little loss of energy. With a reaction turbine, however, since the wheel passages are always filled, a throttling of the stream at any point causes not only a contraction at that point, but a subsequent expansion, with a resulting loss of energy. Perhaps the most wasteful methods for regulating the discharge are by throttling the flow by means of the gate in the penstock or supplypipe, or in the draft-tube; or by the use of a cylindrical gate encircling the lower end of the draft-tube; in such cases, losses invariably result from subsequent enlargement of section, or from impact. The plain cylindrical gate moving axially is open to the same objection, unless the turbine channels are provided with partitions, as already described, or have an upper crown which curves downward.
153. Theoretically the most perfect regulator for a radial-flow turbine is the device due to Nagel and Kaemp, in which the roofs of the guide-passages and the crown of the turbine are together movable, so that, in consequence of the crown and roof being always even and opposite, sudden enlargement at entrance is avoided for all positions of regulation. The design is, however, expensive, and involves difficulties of a practical nature.
154. The regulation of the Jonval or parallel-flow (axial-flow) turbine is usually accomplished by sliding (register) plates or swinging flaps for closing the guide-passages. It is found that the entire closure of a number of the guide-passages, instead of the partial closure of all, is conducive to higher efficiency, since in the former case the absolute velocity of entrance has the same value as when all guide-passages are open.
155. Of the great variety of gate arrangements that have been tried, only the following three hạve come into general use:

1. Cylinder Gate. The cylinder gate, moving in an axial direc-
tion, as used by Mr: Francis with his early turbines, is now by far the most extensively employed gate. It may be placed on the inlet or on the outlet circumference of the runner, and it regulates by cutting off the supply of water from the upper sections of the bucket orifices. To partially prevent the loss of energy incidental to the use of this regulator when operated at "part-gate," the width of the guide and runner-buckets (the distance between the crowns) may be divided into two or more spaces by additional crowns or partitions, thus virtually forming two or more turbines which are regulated by one common cylinder gate. Thus, for example, a triple or three-story turbine, when working at one-third or two-thirds gate-opening, has two or one turbine working at full gate with full-gate efficiency; while the

one turbine or two turbines remaining are entirely shut off. The cylinder gate in nearly all cases works between the guides and runner. Thus Fourneyron divided his turbine (Fig. 81) into three stories by means of horizontal partitions; and the guide-passages may also be divided to correspond, as in Fig. 82.

As examples of recent design illustrating this principle, may be mentioned a 700 -horse-power horizontal-inflow turbine of Swiss design, with a width of bucket of about 30 inches, divided into five stories, all regulated by a single gate. The 5,000-horse-power double (or twin) turbines of the Niagara Falls Power Company in PowerHouse No. 1, are examples of three-story outflow turbines, with cylinder gates on the discharge (outer) side of the runners. The guidepassages are also divided into three stories.
2. Register Gate. The register gate (so named from its sim-
ilarity to a common hot-air register) may be of the plate or the cylindrical type, according to the kind of turbine to which it is applied. The latter type consists of a rotating cylinder having slots which correspond with the outlet openings of the guide-buckets. The axis of the cylinder is coincident with the axis of the turbine shaft; its motion is circumferential, so that it cuts off the supply from the sides of the guide-passages. When applied to parallel-flow turbines, the plate type of register gate must be used. Register gates are sometimes placed outside, and sometimes inside the guide-ring. This form of gate is no longer widely used. Many ingenious forms of this type of gate have been designed; in one form, a part of each of the vanes which form the guide-buckets is separate from the rest of the vane, being attached at each crown to a movable ring, so that by rotating these rings by a suitable device, the size of each of the clear openings between the vanes can be altered simultaneously in accordance with fluctuations in the load on the turbine. The movable part of the vanes may be either at the entrance or at the discharge side of the guidebuckets, though the former plan is now rarely used, as the shape of the bucket is thereby too much distorted when the gate is partly closed ; whereas, with the latter arrangement, the shape of the bucket is much better maintained.
3. Wicket- or Pivot-Gate. In this type of gate, the whole guidevane swings on pivots so located as to balance the vane as nearly as possible in every position. Here also the vanes move, and thus alter the size of the discharge openings of the buckets, simultaneously. Such gates maintain the correct shape of the guide-buckets at part gate, better than any other gate.
156. The Case. This term is usually applied to the fixed parts sustaining the guides and gate or gates, which the maker furnishes with the wheel, being regarded as part of the same. It includes the plate or dise which supports the guides, and a plate which relieves the wheel of the pressure of the water; in some types these two functions are performed by the same plate. The case usually carries the mechanism for operating the gate, and often the step on which the shaft runs. The term is sometimes used to signify the iron casing or vessel inclosing the entire mechanism, into which the water passes by connection with the penstock. Closed turbine-chambers would perhaps better express the meaning in the latter case.
157. Turbine=Chamber, or Flume. In order that the water may act upon a motor, the latter is placed in a chamber communicating with the upper level by a penstock or head-race, and with the lower level by a tail-race (with or without the intervention of a draft-tube), into which it discharges after passing through the motor. This chamber is frequently called a flume; but it is preferable to call such a contrivance an open or closed turbine-chamber (or the latter, a turbine casing), as the case may be, and to restrict the use of the term flume to mean a water conductor carrying water not under pressure. The turbine-chamber may be built


Fig. 83. Vertical Section of Lower Part of a Risdon Wheel-Shaft and Wooden Step-Bearing. of wood, iron, stone, or concrete, or a combination of these, the choice depending principally upon the head.

The best and simplest arrangement for a single or multiple horizontal turbine with draft-tube or tubes, is an open turbine-chamber, built of wood (for temporary purposes), masonry, concrete, or concrete and steel, forming a direct continuation or branch of the head-race or forebay; and this plan has been adopted in recent years for many important powerplants.
Open turbine-chambers have three advantages over closed turbine casings or chambers-namely, the friction of the water flowing to the motor is reduced to a minimum ; the mechanism is readily accessible; and the arrangement is very convenient for speed regulation. The minimum depth of water above the highest point of the entrance rim of the guide-bucket for turbines operating with low draft-head should be 3.5 to 4.5 feet for open chambers, to avoid the formation of funnels, and the suction of air into the turbine.
158. Step, Suspension, and Thrust Bearings. Most vertical shafts of turbines run on wooden ${ }^{\text {r step-bearings, the block forming }}$ this bearing being sometimes arranged so as to be free to revolve also in its socket, so that if the upper surface becomes heated to such an
extent as to greatly increase its frictional resistance, the block will turn on its lower surface until the upper surface has cooled. Lignum vitx, maple, and oak are the woods commonly employed for stepbearings; they are thoroughly dried, and then boiled to impregnation in linseed oil; they become, therefore, in a measure, self-lubricating. Fig 83 shows a vertical section of the lower part of a Risdon wheelshaft $(S)$ and wooden step-bearing $(U)$. In some cases the shaft and attachments are suspended from a collar-bearing above, as in Fig. 87.
159. Thrust or Balancing Piston. This arrangement is principally used for horizontal inward- and outward-flow turbines, and takes the thrust both of the action of the water and the weight of the rotating parts. Fig. 84 (Niagara Falls Power Company, Power-House No. 1) represents an example of twinflow wheels on the same shaft. The weight of the dynamo, shaft, and turbine (about 70 tons) is balanced, when the wheels are in motion, by the upward pressure of the water in the wheel-case on a balancing piston


Fig. 84. Thrust Piston with Twin-Flow Wheels Mounted on SameShaft. Niagara Falls Power Co., Power-House No. 1. or disc $B$, placed above the upper wheel and rigidly attached to the shaft. The upper dise containing the guides is perforated so that the water pressure existing in the penstock can be transmitted directly through to the lower side of the balancing piston, while the upper side is open to atmospheric pressure. The lower disc is not perforated; and the weight of the water upon it is carried by inclined rods upward
to the wheel-case, which, together with the penstock, is supported upon several girders. At the upper end of the shaft is a special thrust-bearing designed to receive the excess of vertical pressure, which may act either upward or downward, under different conditions of power and speed.

Fig. 85 shows diagrammatically the device applied to their singleinflow turbines by the Niagara Falls Power Company, in Power-House No. 2. The water from the penstock fills the annular chamber $A$ under nearly hydrostatic pressure; passes through the guide-passages at $G$; and enters the wheel channels at $C$ under reduced pressure and at high velocity. The revolving turbines, shaft, and attachments are shown in black. The water, leaving the turbine-channels $W$, enters the space $D$ with low absolute velocity and low pressure. At the lower end of the shaft, while lateral support is provided by the stepbearing, a great lifting force is furnished by the admission of water directly from the head-race by an independent pipe under the full head-race pressure to the space $U U$ on the under side of the conical shell or balancing disc $V V$, which is keyed upon the shaft and revolves with it. The pressure on the upper surface of the piston is small, being that of the water in the upper end of the draft-tube. In this way the larger part of the weight of the wheel, shaft, and armature of the electric generator is supported by fluid friction. The diameter of the balancing dise is 4.9 feet; the weight of the revolving mass is 71 tons, of which 66 tons are supported by the upward pressure of the disc, leaving five tons to be sustained by a suspension or collar bearing at the upper end of the shaft.
160. Thrust-Chamber. 'This arrangement is principally used to take the end thrust of vertical inflow turbines, and consists of an annular chamber, formed by the cast-iron turbine case, and open towards the runner, which revolves in front of it. The water pressure in the chamber is supplied by a pipe connected with the penstock, and provided with a valve for regulating the pressure. By means of this valve, the end thrust can be so regulated that the shaft will press against the step-bearing with just enough force to prevent any end motion.

According to Thurso, the use of wood with water lubrication for bearings and steps. located under water, has been practically abandoned by European manufacturers; and metal bearings and
steps, with forced oil lubrication, are employed instead, using a pressure and return pipe for circulating the oil. Frizell shows a type of such oil step-bearing (Fig. 86). The shaft passes through a stuffingbox, and rests on the revolving plate $a$; the surfaces of contact between $a$ and $b$ are dressed to an exact fit, and wear keeps them in that condition. Oil for lubrication passes through the pipe $e$, being forced in under pressure by a pump; it fills the space $f$, and exerts a lifting


Fig. 85. Thrust Piston Device for SiagleInflow Turbine.


Fig. 86. Oil Step-Bearing. Niagara Falls Power Co., Power-House No. 2.
pressure on the plate $a$, nearly equal to the weight of the shaft and its attachments, so that the contact surfaces sustain very moderate friction. The oil passes between these surfaces into the space $c$, and therefore the upward pressure on plate $a$ would be neutralized, were it not that another pipe $d$, communicating with this space, conveys the oil back to the tank from which the pump draws its supply.
161. A metal collar thrust-bearing is shown in Fig. 87. Such metal bearings should never be located in the water, but are usually placed on the end of the shaft opposite to that from which the power
of the turbine is taken off. Both the straight and the collar bearings of the main turbine shaft should be adjustable, and should be lined with bronze as a base, and the bronze in turn lined with an antifriction metal, or babbitt, well hammered and bored.
162. In double turbines working on the same shaft, the end thrusts balance and neutralize each other, provided both work at


Fig. 87. Metal Collar Thrust-Bearing. the same gate opening. Thurso recommends the employment of thrust-chambers for single turbines working under a high head, say several hundred feet; and in case the runners are of such size or shape as to preclude the use of such chambers, the thrust-piston should be used instead, placing the runner at one end of the case, and the piston at the other.
Turbines having thrust-chambers or pistons, or double turbines so mounted that the two end thrusts balance, should nevertheless be provided with a small collar-bearing, to take care of unavoidable variations in the end thrusts.
163. Stop=Valves. It is a matter of practical convenience to have a separate stop-valve in the supply-pipe near the turbine, in order that the water may be quickly shut off and the turbine stopped. They should also be placed in convenient locations, dividing the pipeline into sections, so that single sections may be shut off for inspection, repairs, renewals, etc.

The turbine may be shut down by closing the regulating gates; but this method will not permit the turbine to be taken apart or repaired; and under high head the regulating gates are seldom so tight that their closure will bring the wheel to a standstill. Where several units are supplied by the same penstock, stop-valves are a necessity; for, otherwise, the stoppage of one unit for repairs by means of closing down the head-gate would involve the stoppage of all the turbines connected with that penstock. In American practice, gatevalves are most commonly used for this purpose. The small sizes are fitted with screw-spindles to be operated by hand; but for the larger sizes some kind of motor must be provided, to furnish sufficient power
for moving the heavy gates. Electric, hydraulic, and pneumatic power have all been used. In the case of hydraulic power, where the head employed is high enough, the pressure water may be taken directly from the penstock; but for lower heads a pressure-pump and weighted accumulator are required. Sometimes a by-pass is employed in connection with a gate-valve for the purpose of partially equalizing or neutralizing the pressure.
164. Air=Valves. At summits of a pipe-line and near stopvalves, air-valves should be placed for the purpose of permitting the escape of air in filling, the entrance of air on emptying, and occasion-


Fig. 88.


Fig. 89.
"Victor" High-Pressure Runners.
Fig. 88-For Heads of 100 to 2,000 feet; Fig. 89 -Runner with Scoop-Shaped Vanes.
ally the escape of air which may gradually accumulate at summits. They are usually designed to operate automatically.
165. Blow=Off Valves. These should be placed at all depressions of a pipe-line, for the purpose of cleaning out or of emptying sections of the line.
166. Automatic Stop=Valves. 'These should be placed at critical points of a line, so that, in case of accident to the pipe, the valves will gradually close, and thus prevent the loss of water and possible damage to property.
167. Illustrations. Some illustrations of turbine parts, accessories, and details will now be presented, with very brief explanation when necessary.

Fig. 88 shows a Victor high-pressure runner, intended for heads of from 100 to 2,000 feet. Fig. 89 shows another type of Victor run-


Fig. 90. Upright-Shaft "Samson"' Runner.


Fig. 91. Upright "Samson" Turbine Complete.
Courtesy of James Leffel \& Co., Springfield, Ohio.
ner, with characteristic scoop-shaped vanes. These wheels are cast in one piece, of cast iron or bronze. Fig. 90, the Samson runner, made by James Leffel \& Company, Springfield, Ohio, is a double wheel, the upper portion of the wheel-passages being partitioned off by a diaphragm. Each wheel or set of buckets receives its separate quantity of water from one and the same set of guides, each portion of the water, however, acting on its own buckets. Fig. 91 shows the turbine complete, in its case. The small shaft to the left is for the purpose of operating the balanced wicket-gates. The horizontal projecting rim or flange serves the purpose of supporting the mechanism upon the floor of the turbine-chamber; the conical shell below the rim is a short, flaring draft-tube. Fig. 92 shows another type of Leffel turbine without and within a globe casing. The shaft and gearing for operating the regulating gates are clearly indicated. The penstock is supposed to be bolted to the flange of the casing. Fig. 93 shows a simple type of wheel with a quarter-turn draft-tube through which the water discharges, first horizontally, and then vertically downward. The shaft and gate-rods extend through stuffing-boxes in one side of the turbine-chamber (not shown); and the horizontal iron base-plate is frequently set directly upon its floor, usually with a draft-tube extending below tail-water. Fig. 94 represents two complete wheels on a horizontal shaft, one at each end of the cylindrical steel-plate case Both wheels discharge horizontally toward each other, the water then passing downward through the central discharge pipe into a draft-tube. The large iron base-plate is set upon the floor of the turbine chamber, and the shaft may be extended in one or both directions, passing through stuffingboxes in the sides of the turbine-chamber. The thin horizontal shaft on top of the cylinder has rigidly fastened to it two pinions, so that, by the turning of this shaft, the regulation of both gates is effected simultaneously through the action of the rods.

The types represented in Figs. 93 and 94 are supposed to be placed on the floor of a simple open or decked turbine-chamber, of wood or other simple construction. For high heads, particularly when economy in space is necessary, iron casings are employed. Fig. 95 shows the simplest style of horizontal-shaft wheel in an iron casing. The pulley shown is intended for rope transmission; but a flat pulley for belting may be substituted. The governing and controlling de-

Fig. 93. Single-Discharge, Horizontal-Shaft "Samson" Turbine for Open Penstock, with Quarter-Turn Draft-Tube. Courtesy of Tames Leffel \& Co., Springfield, ohia


Fig. 93. Single-Discharge, Horizontal-Shaft "Samson" Turbine for Open Penstock, with Quarter-Turn Draft-Tube.



Fig. 96. Double-Discharge, Horizontal-Shaft "Samson".Turbine with Casing. Courtesy of James Leffel \& Co., Springfteld, Ohio.


Fig. 97. Double-Discharge, Horizontal-Shaft Turbine with Globe Casing. $\quad$ Courtesy of James Letfel \& Co., Springfield, Ohio.
vices are seen on the right. Fig. 96 shows a casing containing two wheels on the same horizontal shaft, discharging in opposite directions through the curved elbows into the wheel-pit or tail-race.* The Samson turbines are of the horizontal-shaft type, usually having one runner built with two similar sets of buckets, taking the water equally divided from one set of guides, and discharging in opposite directions. Fig. 97 represents a "Niagara" type of turbine, the illustration showing it to be strongly built and very compact.

The Risdon-Alcott Turbine Company, of Mount Holly, New Jersey, manufactures a line of turbines of distinctive character. In Figs. 98, 99, and 100, are seen three types of these runners, showing very decidedly the peculiar spoon- or scoop-shaped vanes; they are cast in one piece, of cast iron or bronze. The top of the wheel is formed by a bell-shaped crown, which extends to the inner bottom of the buckets and forms their inner boundary. A band cast around the lower, outer portion of the buckets forms their outer boundary; the warped surfaces forming the vanes connect bell and band. In Fig. 100, the band isrepresented as transparent for the sake of clearness; and in Fig. 98, part of the band is supposed to have been removed for the same purpose. Three styles of gate are used with these wheels -the cylinder, the register, and the hinged. In Fig. 101 is seen an outside view of the wheel-case, etc.; $B-B$ are the guide-vanes rigidly attached to the plate $R ; C$ is the vertical cylinder gate, to which are rigidly attached the horizontal projections $D-D$, fitting between the guide-vanes. These projections necessarily move with the cylinder in its vertical travel, and thus form in effect movable roofs for the guidebuckets. $S$ is a short discharge tube, which may dip a few inches below tail-water, or to which may be attached a draft-tube; $V$ is the turbine-shaft; and $W$ is the shaft which operates the gate $C$.through the intervention of the gearing, as shown; the supporting rim $R$ rests on the floor of the turbine-chamber.

Figs. 102 and 103 show two turbines fitted with outside and inside register gates. The register gate consists essentially of a cylinder containing slots arranged parallel to the shaft; in one position of its circumferential motion, the passage of water is entirely cut off by reason of the slots coming opposite the guide-vanes, which are of consid-

[^6]

Fig. 98. Revolving Part of Risdon-Alcott Standard Turbine.
erable thickness; while a comparatively small movement from this position will leave the guide-passages fully open.

Fig. 104 shows two Risdon-Alcott turbines mounted on the same horizontal shaft, fitted with balanced hinged gates, discharging centrally into a common draft-tube. The variouscasings,etc., and different methods of mounting described in connection with the Leffel wheels, may also be found in the catalogues of the


Fig. 99. Risdon-Alcott Double-Capacity Wheel.

Risdon-Alcott Turbine Company, and therefore need not be further illustrated.

Fig. 105 represents a vertical-shaft turbine with wicket-gate, built by the S. Morgan Smith Co., of York, Pa.; it is shown set in a


Fig. 100. Alcott Special Runner.
Risdon-Alcott Turbine Co.
steel turbine-chamber. Fig. 106 is an outside view of a turbine case, etc., from the same manufacturers, showing the cylinder regulating gate fully raised; the mechanism for operating the gate is clearly seen.

Figs. 107 to 110 show some of the usual methods of installing turbines.

Besides those already mentioned, there are several well-known American manufacturers of turbines and accessories-such as the Dayton Globe Iron Works Company, Dayton, Ohio; the Holyoke

Machine Company, and others. Figs. 111, 112, and 113 show some patterns of gate-valves; Fig. 114, a gate-valve with by-pass; and Fig. 115, a wicket-gate. Several other forms are in the market.


Fig. 101. Risdon Cylinder-Gate Turbine.
Risdon-Alcott Turbine Co.
Fig. 116 represents one form of safety relief-valve designed to operate at a pressure slightly greater than the normal; in the event of the water-flow being suddenly checked by the closing of the gate or by the operation of the governor, the excess pressure is relieved by the


Fig. 102. Risdon Register-Gate Turbine.
Outside Register Pattern.
Risdon-Alcott Turbine Co.
momentary opening of the valves. They may be placed singly or in a battery.

Fig. 117 shows two types of safety air-valves designed to open


Fig. 103. Alcott High-Duty Turbine.
Inside Register Pattern.
Risdon-Alcott Turbine Co.
automatically in case the pipe-line should be emptied suddenly, thus permitting the air to rush in, and preventing possible collapse of the pipe due to the formation of a vacuum.

Fig. 118 illustrates a compensator consisting of a hydraulic plun-

ger connected to the pipe-line, and balanced by suitable steel springs. The compensator is designed to take care of shocks in the pipe-line


F'ig. 105. Vertical-Shaft Turbine in a Steel Flume.
resulting from checking the flow of water through sudden action of the governor or sudden closing of a gate-valve.
168. Gauges. In order to be able to obtain, without inconvenience, certain information relative to the operation of a water-power installation, gauges should be set up at various points of the works-
for example, a pressure-gauge, to indicate the pressure of the water near the entrance of the guide-buckets; a vacuum-gauge, to show the amount of draft or suction near the discharge openings of the runnerbuckets; a gauge to show the pressure between the runner-dise and


Fig. 106. Smith Turbine with Cylinder Regulating Gate Fully Raised.
Courtesy of S. Morgan Smith Co., York, Pa.
the head of the case or the dome; and, where a thrust-chamber or thrust-piston is employed, a gauge showing the pressure in the chamber, or behind the piston. A gauge should also be placed at the lower end of a long penstock, to indicate water-hammer, and the pressure fluctuations due to speed regulation of the turbine. The speed of
the turbine at any moment should be indicated by a tachometer or gauge; and the gate-opening, or position of the regulating gates, should be shown by means of a dial and pointer.


Fig. 107. Pair of Register-Gate Wheels in Wooden Penstock, Each Discharging through an Independent Elbow into its Own Draft-Tube.


Fig. 108. Pair of Turbines in Wooden Penstock, Discharging through Single Draft-Tube.
169. Transmission of Power from Turbine Shaft. The simplest case is presented when the electric generators can be placed directly upon and revolve with the turbine shafts, as at the Niagara

Power Company's installations; in such cases the revolving armatures may be made heavy enough to act as fly-wheels. In other cases the power may be transmitted to other shafts by means of spur or bevel gearing, or by belt or rope transmission-with considerable loss of power. See Fig. 119.
170. Connecting and Disconnecting Mechanism. Mechanical devices for the purpose of throwing a wheel or shaft into or out of connection with the general system, involve problems which differ in


Fig. 109. A 2,000-Horse-Power Installation.
Comprises four 43 -inch cylinder-gate wheels under 48 feet head, giving 1,600 horse-power; and two 36 -inch cylinder-gate wheels under 27 feet head, giving 400 horse-power.
no essential particular from those met with in general mechanical engineering practice, and will therefore not be considered here.

## GOVERNORS AND SPEED REGULATION

171. "Industrial operations require a uniform speed of shafting, although the quantity of work or the number of machines in operation may vary greatly from hour to hour, or from minute to minute. This condition necessitates an automatic device for controlling the admission of water to the wheels, diminishing the quantity when the velocity exceeds the normal, and vice vers $\hat{a}$. The essential part of such a controlling and regulating device consists of an organ which moves in one direction and sets in motion the mechanism for partially closing the gate when the velocity exceeds the normal limit, and which moves in the opposite direction and sets in motion the mechanism for further opening the gate when the velocity falls below that limit.'**

[^7]"The greatest difficulty encountered by the hydraulic-power engineer is the speed regulation of turbines under variable loads; and it has only been during the last few years that engineers have been


Fig. 110. An Installation of Vertical Turbines in Iron Cases with DraftTubes. Combined Power, 635 H. P.

Courtesy of Risdon-Alcott Turbine Co.
able to regulate the speed of turbines supplied by long penstocks as closely as is common in steam-engineering practice. The reason for this is that the weight of the moving column of steam will rarely exceed 100 lbs ., while the velocity may be about 100 feet per second; on
the other hand, the weight of the moving column of water may be millions of pounds, while the velocity is rarely over 10 feet per second. Thus the energy represented by the moving water-column may be hundreds or even thousands of times the energy represented by the moving steam-column.
"Every change of load or power developed requires a change in the engine cut-off or in the gate-opening of the turbine; and this, in turn, requires a change in the velocity of flow in the supply pipe or penstock, which means a change in the amount of energy represented by the moving column Moreover, steam is compressible and elastic; water is incompressible and inelastic; and if the load of the turbine, and thus the velocity required in the penstock, is suddenly decreased, the excess energy in the moving column must find some outlet, otherwise either the penstock or the turbine will be wrecked; while, if the load and the required velocity in the penstock are suddenly increased, the lack of energy must be supplied from some outside source.
"Any decrease in the gate-opening, and consequent decrease in the velocity of the water in the penstock, will thus produce a tem-• porary increase in the penstock pressure; and with the gates closing quickly, this increase in pressure may rise to the force and suddenness of a blow, usually called water-hammer. On the other hand, any increase in the gate-


Fig. 111. Straightway Valve of Single-Disc Type, with Rising Spindle,Intended for Pressure on One Side Only. Disc rises clear of opening, giving a perfectly Imo way Seats are of bronze, to be replaced when worn. opening and in the velocity of the water in the penstock, will produce a temporary decrease in the penstock pressure. Such changes in gate-opening will frequently cause longdrawn pulsations in the penstock pressure, or surging of the water; and this action is often favored by badly arranged penstocks, relief-valves, standpipes, air-chambers, and connections, and aided by wave motion and eddies in the head-race near the penstock entrance.
" To obtain a good speed regulation, it is often necessary, espe-


Fig. 112. A 24-Inch Gate-Valve of Single-Disc Type, with Outside Screw and Yoke and Rising Spindle. Arranged for operating by electric motor, and provided with roller bearings to take thrust from stem.

Pelton Water Wheel Co., San Francisco, Cal.


Fig. 113. Straightway Gate-Valve of Large Size, of Screw-Lift Pattern. Smaller sizes have a single instead of a double screw.

Dayton (rlobe Iron Works Co., Dayton, Ohio.


Fig. 114. Special High-Pressure Gate-Valve with By-Pass, and Roller Bearings on Stem. Pelton Water Wheel Co., San Francisco, Cal.


Fig. 115. Wicket-Gate.


Fig. 116. Battery of Safety Relief-Valves. Pelton Water Wheel Co.


Fig. 117. Safety Air-Valve for Pipe-Lines.
cially in connection with turbines supplied by long penstocks, to use some auxiliary device or devices for the escape and supply of energy, or for the escape at least, which may be briefly considered as follows:
172. "The pressure-relief valve serves for the escape of energy


Fig. 119. Diagram Showing Various Methods Used for Transmitting Power.
when the pressure rises beyond a certain limit, in exactly the same manner as the safety-valve of a boiler. Relief-valves are well known; but nearly all of them are of poor design, being held closed by a single, short, helical spring of a few turns, and therefore can open only to a very small extent. A good spring relief-valve should have 3 to 6 helical springs, according to the size of the valve; and these springs should be long and have many turns, so as to permit the valve to open suffi-
ciently, without requiring too great a rise in pressure in the penstock. Provision should be made for ascertaining at any time whether the valve is in working condition and set for the proper pressure.
173. "The Lombard pressure-relief valve, shown diagrammatically in Fig. 120, is a great improvement over the ordinary relief-valve. Its action is as follows: $A$ is the end of a penstock, or a nozzle of a penstock, in which the pressure is to be relieved when a certain limit has been reached. The dise of the relief-valve $c$ is held to its seat against the waterpressure in the penstock by the waterpressure behind the piston $e$, the pres-sure-water behind the piston being supplied from the penstock through pipe $f$. As the pressures per unit-area against the valvedisc and behind the piston are equal, the


Fig. 120. Diagrammatic Representation of "Lombard" Pressure-Relief Valve. piston is made larger in diameter than the dise, so that the total pressure behind the piston will not only overcome the total pressure against the valve-dise, but also hold the latter firmly to its seat. The space behind the piston $e$ is also connected through pipe $i$ to the waste-valve $D$. This is a balanced valve, held closed by means of the spring $P$; while the water-pressure in the penstock, communicated through pipe $r$, and acting behind the piston $n$, tends to open the waste-valve. The force of the spring $P$ can be regulated so that the water-pressure will overcome the force of the spring and open the valve at any desired pressure in the penstock.
"With the relief-valve closed and the water-pressure in the pen-
stock rising above the normal to the pressure for which the spring $\dot{P}$ is set, the piston $n$ will open the waste-valve, which will relieve the pressure behind the relief-valve piston $e$, and allow the pressure-water to escape. While water will begin to flow through pipe $f$ as soon as the waste-valve is opened, yet the area of pipe $f$ is so much smaller than the area of pipe $i$ and the waste-valve opening, that the pressure behind the piston $e$ will at once fall below the pressure which exists in the penstock; and therefore the pressure in the penstock forces the relief-valve disc $c$ and piston $e$ back, or, in other words, opens the relief-valve.
"The greater the rise in pressure is in the penstock, the greater will be the extent to which the waste-valve opens, and consequently the greater will be the reduction in pressure behind the piston $e$, and therefore the greater the extent to which the relief-valve opens.
"As soon as the pressure in the penstock has fallen to the pressure for which the spring $p$ is set, the latter closes the waste-valve, the pipe $f$ restores the full penstock pressure behind the piston $e$, and the latter closes the relief-valve. To prevent any surging in the penstock due to the closing of the relief-valve and the consequent retardation of the water, the relief-valve is made to close slowly, the rate of closing being adjustable by means of the valve $h$.
"Relief-valves must be prevented from freezing or from becoming incrusted with ice, as otherwise they may be rendered entirely useless.
174. "The by-pass, which may be employed where economy in water consumption is not demanded, consists of a valve or gate of sufficient area to pass the entire volume of water required by the turbine at full-gate opening, and moved in conjunction with the speedregulating gates. With the turbine-gate fully open, the by-pass is closed; but when the regulating gates commence to close, the by-pass opens, and its passage area is increased in the same proportion as the gate-opening decreases, so that the combined area of the gate-opening and the by-pass is always sufficient to pass the entire volume of water required by the turbine at full-gate opening; thus the velocity of the water in the penstock and the amount of water discharged remain always the same, the discharge of the by-pass being run to waste. This arrangement not only permits the closest speed regulation with violently fluctuating loads, but also relieves the penstock from shock or water-hammer, and is therefore often used in connection
with impulse turbines working under high heads and supplied by very long penstocks.
"European engineers have abandoned the ordinary by-pass, on account of the great waste of water which its use implies, but frequently use the temporary by-pass, which is essentially the same device, except that the speed-regulating gates and the by-pass are connected in such a way that the temporary by-pass will open while the speed-regulating gate closes; but as soon as the closing movement of the regulating gate ceases, the by-pass at once starts automatically to close again slowly, being actuated by a spring, counterweight, or hydraulic pressure. The temporary by-pass does not open at all when the regulating gate closes very slowly. It will be seen that the temporary by-pass is similar in its effect to the relief-valve, except that the by-pass opens before a rise in the penstock pressure, due to the closing of the regulating-gates, takes place.
175. "The standpipe is frequently employed to aid the governor, and thus to improve the speed regulation of turbines; and is simply an open reservoir which, to a limited extent, will absorb or store energy, when the gate-opening is decreased in consequence of a reduction in the load of the turbine, and will supply energy when the gate-opening is increased in consequence of an increase in the load of the turbine. The standpipe is the best possible relief-valve, and should have its top edge a few feet above the high-water level in the head-race, and its diameter or capacity should be in accordance with the volume of water discharged by the turbine at full-gate opening, and with the length of the penstock.
"When the gate-opening of the turbine is suddenly reduced, the excess of water flows into the standpipe, causing the water therein to rise, and perhaps to escape over the top edge of it, until the watercolumn in the penstock has slowed down; while, when the gate-opening is suddenly enlarged, the additional water required is supplied from the standpipe, causing the water therein to fall until the speed of the water-column has increased to meet the demand. In connection with high heads, standpipes are rarely used, as they are not only very expensive but also less effective on account of the inertia of the watercolumn in the standpipe.
"Standpipes must be carefully protected from freezing, as otherwise they may be rendered entirely useless. A waste-pipe should be
provided to carry off the water escaping over the top edge of the standpipe.
176. "Air-chambers are often used on penstocks; but while they may be useful to protect the penstock against the effects of water-hammer, they are of little or no value as an aid to the regulation of the turbines. To cushion the shocks in a penstock, an air-chamber should be of ample capacity; and as air is readily absorbed by the water, an air-pump should be provided to replace the air thus carried off. Gauge-glasses and try-cocks should be placed on each air-chamber, so that it may at once be seen whether it is effective.
177. "The blows struck by water in a penstock (or water-hammer), are first and most violently felt at the lower end of the penstock, and in the direction in which the water-column moves; and from there back up, so to say, with diminishing strength, toward the upper end of the penstock. Therefore all such safety devices as the relief-valve, by-pass, standpipe, and air-chamber should be at the extreme lower end of the penstock, and their discharge or connection should be in the direction in which the water-column moves. A standpipe should be connected with the penstock by a short, straight pipe of large diameter; and an air-chamber, by a short neck, also of large diameter.
178. "The fly-wheel is frequently employed in Europe, and to some extent in America, to aid the governor and thus to improve the speed regulation, especially in connection with turbines working under high heads; but a fly-wheel cannot, of course, protect a penstock against water-hammer. A turbine-runner has very little fly-wheel capacity; and the use of a fly-wheel will therefore eliminate the small variations in speed due to slight but sudden fluctuations in load, to water-hammer, to the surging of the water in the penstock and drafttube, and to other causes, which momentary variations even the best governor cannot prevent. The amount of energy which a fly-wheel can absorb or give out is small, but it will at least retard the changes in speed of the turbine with changes of load. Where the turbines are used to drive dynamos, sufficient fly-wheel capacity may be given to the armature or revolving field to make a separate fly-wheel unnecessary. This plan was adopted by the Niagara Falls Power Company for its 5,000 -horse-power alternating machines."*
179. Mechanical and Hydraulic Governors. The power to

[^8]operate the turbine gates is sometimes furnished by the turbine itself; but on account of the size and weight of these gates in a large installation, considerable force must be exerted to operate them; and for this reason, they are not directly actuated by the sensitive centrifugal governor. An auxiliary relay motor, using either mechanical or - hydraulic power, which is itself controlled by the centrifugal governor. is therefore interposed for the purpose of actuating the gates.

The hydraulic relay consists essentially of a piston and cylinder operated by pres-sure-water from the penstock, oroil from a pressure tank. The governor proper consists of the usual type of revolving centrifugal balls in gear with the turbine shaft, and so connected with the valves or devices of the motor or other mechanism which actuates the gates, that any change in the relative position of the balls brought about by a variation


Fig. 121. Diagrammatic Representation of "Replogle" Centrifugal Governor and Return.
in the turbine speed, will bring into play the mechanism which operates the gates. (See also Article 83.)
180. Over=Governing or Racing. When a change of speed due to a change in load takes place, the governor will set the regulating gate in motion; but, owing to the inertia of the water, a certain amount of time is required for the turbine to return to the proper speed; so that the gate would have traveled in the interval beyond the required position, and in a short time, under the influence of the governor, it would start moving in the opposite direction. To obviate this difficulty, the motion of the relay, and with it the movement of the regulating gate,
must be arrested before the latter has traveled too far, by a return device.
181. Time of Closure. Mechanical governors effect an entire

closure of gate usually in 15 to 25 seconds, although some types have been constructed that require only about 3 seconds.

In the case of hydraulic governors, this time may be reduced to one second; such governors therefore afford very close speed regulation.
182. Fig. 121 represents diagrammatically the Replogle Centrifugal Governor and Return, a device well known in American practice. $S$ is a toothed segment operated by the turbine-gate shaft $G$, which is also toothed for the purpose. $L$ is a lever, tilted up or down by the governor-ballswhen a variation of speed occurs; this lever, in its motion,


Fig. 123. Compensating Type "Woodward" Governor, Suitable for Electric and Heavy Mill Work. Risdon-Alcott Turbine Co.
completes an electrical contact by touching points $C$ or $O$. Such contact energizes one of two electromagnets in the circuit, by which means an auxiliary machine, or relay (not shown), is tripped, and begins to open or close the turbine-gate by turning $G$ in the proper direction. The turning of $G$ carries the rack $S$ with it, which, by a suitable cam arrangement, breaks the contact between the lever and the point $C$
or $O$. This interruption of the electrical circuit cuts the auxiliary power out of action, thus stopping the motion of the gate before the increase or decrease of power due to the gate motion has given velocity to the turbine above or below the normal.

Later types of this governor have been still further refined.
Figs. 122 and 123 illustrate two types of Woodward Governor.


A SPLENDID EXAMPLE OF A SIDE-HILL FLUME


PENSTOCKS LEADING WATER DOWN MOUNTAIN SIDE TO TURBINES Plant of the Puget Sound Power Company, Electron, Washington.

## WATER-POWER DEVELOPMENT

## PART III

## CONDUCTION OF WATER

183. Head=Race and Tail=Race. In general, comparatively narrow and deep races are preferable to wide and shallow ones, because of the smaller loss of head in the former case, particularly in localities where ice is likely to form in the winter months, which not only reduces the area of waterway, but offers considerable additional frictional resistance. In such localities it is advisable to protect that part of the tail-race which is under the power-house, together with the tail-race opening itself, against freezing and consequent accumulation of ice, by boarding up the upper part of the opening to within 1 or 2 feet of normal tail-race level, and attaching a floating strip of canvas or tarpaulin, or a hinged board, to the lower edge of this partition, the bottom of the board reaching to low tail-water level.

The velocity of the water in the races is usually between 2 and 3 feet per second, which is sufficiently low to allow the water to freeze over the surface, and thus prevent the formation and serious accumulation of anchor-ice and frazil.

The location and direction of the entrance to a head-race should be such as to prevent the carriage of sand in suspension into the headrace; and in order to keep out logs, ice, and floating debris, a heavy boom, or a crib with openings for the passage of water below the surface, should be placed across the entrance in such a direction that floating matter will have no tendency to lodge and accumulate against it, but will be deflected and carried downstream. Where possible, bends and curves should be avoided, because of the loss of head they occasion; when necessary, they should have long radii.

Where much sand is carried in suspension, the water may be allowed to flow through a sand-settler, which is sometimes merely a basin formed by an enlargement of the head-race, the increase in cross-:
section inducing a corresponding decrease in velocity, resulting in a deposition of the sand. Frequently grooves, or narrow upright boards, are placed in the bottom at right angles to the flow, to arrest sand rolling along the bottom. With open timber flumes, these sand-


Fig. 124. Head-Gate Hoist.
Operation of opening and lclosing is performed by means of a lever inserted in a winch-wheel on the end of the pinion-shaft.

Dayton Globe Iron Works Co., Dayton, O.
settlers may be merely large, shallow wooden boxes. When large volumes of water are handled, a ditch is often placed near the water-racks to catch the sand rolling along the bottom; and by means of a pipe, screened to keep out coarse material, and provided with a gate, the accu-mulated sand may from time to time be flushed out. With very large quantities of water carrying much sand, itwill frequently be found more eco- nomical to allow the sand to pass through the turbines, and renew the worn guides and runners when necessary, than to attempt to free the water of its burden of sand.

Near the power-house end of the head-race, should be located a wasteway with sluice-gate, to discharge ice and other matter, and a boom to guide to this gate such floating matter as may pass down the head-race.

The tail-race should be designed to afford easy discharge of the water, so as to prevent the possibility of backing up around the turbine or draft-tube. For this reason the walls of the tail-race should be so shaped as to deflect the water with the greatest directness in the proper direction. When draft-tubes are employed, they should be carefully curved or inclined in the direction of flow; when several single


Fig. 125. Cumpound-Geared Head-Gate Hoist for Double-Stem Gate. Shafts carrying spur gears and pinions are substantially mounted on cast-iron stands; wrought iron cranks are used for operating the hoist. Holst of similar pattern may be used for a single-stem gate.

Dayton Globe Iron Works Co., Dayton, 0.
draft-tubes discharge into the same tail-race, they should be located on one side. Double draft-tubes may be placed, one on each side; when so arranged, the obstruction to the flow of water will be much less than if they were placed in the center of the tail-race.
184. Water=Racks. These are screens through which the water is compelled to pass on its way to the turbines, so that all floating or suspended material larger than the clear opening between bars will be caught, and thus choking or damage to the wheels will be prevented.


Fig. 126. Single-stem Head-Gate Horst.
With pilot wheel and worm-wheel operation.
Dayton Globe Iron Works C'o., Dayton, $Q$.


Fig. 127. Double-Stem Worm-Wheel Hoist.

To insure this result, it is necessary that the clear opening between bars be somewhat less than the smallest dimension of the water passages in the guide- or runner-buckets; but this may not prevent temporary choking when the regulating gates are partially closed, a condition, however, quickly remedied by simply opening the gates. Double racks are sometimes employed, consisting of a coarse rack placed in front of the fine rack; this procedure is particularly advisable in the absence


Upper end of gate stem is threaded to match a bronze nut attached to bevel gear shown in cut. Ball bearings are used to insure easy operation. Dayton Globe Iron Works Co., Dayton, 0.
of a protecting boom or crib. For the fine racks, the clear space is usually $\frac{3}{4}$ to $1 \frac{1}{2}$ inches, the bars being of wrought iron or steel, $\frac{1}{4}$ to $\frac{3}{8}$ inch thick, by 3 to 4 inches wide. For coarse racks, the clear space is about 3 inches, the bars being of the same material, $\frac{1}{2}$ to 1 inch thick, by 4 to 5 inches wide.

In order that the water may find free passage, even though the rack be partially clogged, as well as because of the frictional resistance, the total clear area of rack should be considerably in excess of the
total area of the penstock inlets; for this reason, and also for greater convenience in cleaning, the racks are usually placed in an inclined position. For greater ease in handling, repairing, etc., a rack built


Fig. 129. Filler Gate.
With frame for bolting to a timber head-gate. Allows passage of sufficient water to equalize pressure, so that headgate need not be lifted under total pressure. Dayton Globe Iron Works Co., Dayton, 0.


Fig. 130. Head-Gate and Hoist.
Square, cast-iron gate. Dayton Globe Iron Works Co., Dayton, 0.
of removable sections 3 or 4 feet wide is frequently used, instead of a continuous rack.
185. Head=Gates. The purpose of the head-gate (see Figs. 124 to 136) is to control or shut off the water from a penstock, flume, open turbine-chamber, or forebay. It is usually a vertical gate sliding in guides, and, until quite recently, was constructed of wood
held together and braced by iron bolts and straps. Head-gates are now frequently built up of steel plate and structural steel-more particularly those of large size. Such gates are actuated by hand, by means of rack-and-pinion or screw-spindles, or by special devices operating under mechanical, electric, or hydraulic power. They are frequently counterweighted; and for large gates a by-pass or balanceport is usually employed, by means of which the waterpressure on the two faces may be balanced before moving the gate. In some cases, friction roller-bearings are employed to reduce the friction; and in others, by an ingenious contrivance, the gate is lifted from its seat in the preliminary action of opening. In order to reduce the wear, the gate is sometimes designed to slide on special guide-bearings, instead of on its seat, which it does not touch until reaching its position of complete closure.

A cylindrical gate, built up of plate and structural steel, and so pivoted that it


Fig. 131. Head-Gate and Hoist.
Circular, cast-iron gate. Dayton Globe Iron Works Co., Dayton, 0. is practically balanced, is shown in Fig. 133; it is also shown in position in the power house, Fig. 80.

Another form of head-gate which has not come into general use, though possessing many positive advantages, consists of a cast-iron cylinder, double-seated, the lower seat being formed by a ring attached to the floor of the head-race, and the upper seat by the edge or rim of a dome (forming the head), which is fastened by steel rods


Fig. 132. Worm-Geared Swing Gate.
to the lower ring, as in Fig. 134. The cylinder itself is raised or lowered by means of a chain. In the center of the dome is a small filling-gate operated by a separate chain. It is thus practically balanced, and hence requires but little power in operation; moreover, the lift required to give a clear waterway equal in area to that of the circular opening which it controls, is but one-quarter of the diameter of such opening.

Where the necessary headroom is not available, wicket-gates turning on a vertical spindle may be used, as in Fig. 135. Such a gate requires less power to operate than a sliding gate, but is generally liable to greater leakage. When open, it presents its edge to the current, and so offers some, though no great obstruction to the current. It is advisable to design gates intended to be used in cold climates, so that they may be entirely below the surface ice when closed, and entirely clear of it when open.
186. Gate= Houses. In many important waterpower projects, the flow into the canal


Fig. 133. Cylindrical Balanced Gate Built Up of Plate and Structural Steel. is controlled by a series of gates, with their hoisting gear and appurtenances, all enclosed in a covered building.
187. The Penstock. This term is applied to the pipe which
brings the water from the canal or other source of supply, to the tur-bine-chamber. When the source of supply is near the motor, it is a relatively unimportant detail of the system. On the other hand, it sometimes happens that this pipe is several miles long; in which case it assumes a position of primary importance; in fact it may become one of the controlling features or elements of the design.

Penswocks, or feeder-pipes, may be made of riveted wrought-iron or steel, of wooden staves, or of concrete-steel. They should always


Fig. 134. Cast-Iron Cylindrical Double- Seated Head-Gate.


Fig. 135. Wicket-Gate Turning on Vertical Spindle.
be as short as possible, even when a shorter penstock involves a greater expenditure for excavation, etc. The shorter the penstock, the better it is for the speed regulation of the turbines, and the less steel-plate work has to be kept painted and repaired.

The following rules should be observed when determining the cross-sectional area of the conductors which convey the water to and from the turbines:
"The speed of the water should be gradually increased from the speed in the head-race, usually 2 or 3 feet per second, to the penstock speed, by means of a cone or taper piece, as in Fig. 136. Near the lower end of the penstock, the speed should again be gradually increased, so that the water will arrive at the guide-buckets with a speed equal to that with which it has to enter these guide-buckets. At the
entrance of the draft-tube, or draft-tube elbow or tee, the water should have a speed equal to the absolute velocity with which it leaves the runner-buckets, and should then gradually decrease to a speed of about 2 or 3 feet at the lower end of the draft-tube. A speed of 2 or 3 feet is also usually chosen for the tail-race.
"In general it should be stated: Avoid changes of speed of the


Fig. 136. Entrance Taper and Head-Gate in Flume. Pelton Water Wheel Company, San F'rancisco, Cal. water where possible; but where súch changes are necessary, make them gradually; also, avoid changes of direction of water; but where such changes are necessary, use curves of long radii.
"The arrangement often employed, of having at the lower end of the penstock, and at right angles, to the same, a drum or receiver of much larger diameter than the penstock itself, from which drum a number of turbines are supplied by branches set at right angles to the drum, must be condemned on account of the resulting abrupt changes in speed and direction of the water.
"All nozzles or branches of penstocks should be at an angle of not over 45 degrees to the penstock; or, in other words, the directions of flow of the water in the penstock and in the nozzle or branch should form an angle of not over 45 degrees with each other. Directly beyond each nozzle or branch, the diameter of the penstock should be reduced, to keep the speed of the water uniform.
"When determining the speed for the water in the penstock, all conditions should be carefully considered, and it should also be borne
in mind that the friction loss in a penstock varies with the square of the speed.
"Conditions making a low speed advisable are: Low head, large diameter of penstock, great length of penstock, many bends in penstock, variable loads on the turbines, regulation of speed of turbines by changing the amount of water used.
"Conditions making a high speed permissible are: High heads, small diameter of penstock, short penstock, few or no bends in penstock, steady loads on the turbines, regulation of speed of turbines by by-pass.
"Many hydraulic engineers employ in all cases a penstock speed of 3 feet per second; but it is often of advantage greatly to exceed this velocity. From a great number of well-designed water-power plants constructed in America and Europe during recent years, the writer has deduced the following table of highest permissible speeds of water in penstocks of a length of 1,000 feet or less, with easy bends, and provided with proper arrangements for the protection of the penstocks against water-hammer:

## PERMISSIBLE SPEEDS OF WATER IN PENSTOCKS

Diameter of Penstock

| (in feet) | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| SPEED OF WATER <br> (in feet per second) | 12 | 11.5 | 11 | 10.5 | 10 | 9.5 | 9 | 8.5 | 8 |

"In penstocks of 1 or 2 feet diameter, speeds as high as 20 to 30 feet have been used. With very low heads, the penstock speed is often limited by the amount of head that it is permissible to lose in the penstock.
"The principal losses in the head of the water while entering the penstock and flowing through the penstock and draft-tube, are due to the following causes:
"(1) Entrance Loss. This loss may be kept low by having a large entrance connected to the penstock by an easy cone or taper piece. With the usual head-gate arrangement, such large entrance openings require very heavy and cumbersome gates for penstocks of large diameter; but there is no reason why this taper piece could not be partly or wholly in front of the gate and inside the head-race or forebay. The penstock entrance should always be as much below the surface of the water as circumstances will permit.
"(2) Friction Loss. This loss may be kept down by a low speed of water, and by smooth interior of the penstock and draft-tube.
"(3) Loss Due to Changes in Direction of Flow. This loss may be kept down by using as few and as easy bends as possible.
"(4) Loss Caused by Changes in Speed of the Water. This loss is due to the conversion of part of the energy in the water into another form, and may be kept low by having as few and as gradual changes as possible.
"(5) Loss Due to the Speed of the Water While Leaving the Lower End of the Draft-Tube. This loss is equal to the velocity-head, corresponding to the speed with which the water leaves the draft-tube, and may be kept down by making this speed low.
"Long penstocks, carrying water at high speed, should be provided with a safety-head, besides the usual devices for the protection of the penstock against water-hammer. For this purpose, a cast-iron or angle-bar flange is riveted to the lower end of the penstock, to which flange the head closing the lower end is bolted. The flange-bolts should have a factor of safety of not more than about half the factor employed for the rest of the penstock. Between flange and head a packing of dry white pine should be used, which, when water is admitted to the penstock, swells and makes a tight joint. Where the end cannot be used for this purpose, large nozzles may be riveted to the penstock, located as nearly as possible in the line of the waterhammer, and closed by heads secured as already described. The end of the penstock, or the nozzles, should be so situated that, should the heads blow out, no damage will be done by the jet of water issuing from the opening. This arrangement will not only save the penstock and turbines from being wrecked in case of severe waterhammer, but also the power-house from being demolished by the water set free.
"Ample air-inlets should be provided at the upper end of the penstock, as otherwise-should the safety-head by some chance give way, or the turbine-gates or turbine stop-valves, if such are employed, be opened while the head-gate is closed, but the penstock full of water-the penstock might collapse by the vacuum created in its interior. Care must be taken to prevent the water in the vents or air-inlets from freezing, as this would render them useless.
"A penstock which is carried for a considerable distance at about
the same elevation as that of its inlet, and with so little slope as to be nearly horizontal, and then descends to the power-house on a steep grade, is liable to collapse when the turbine-gates are opened quickly, as the water in the inclined part has the tendency to increase its speed more quickly than the water in the horizontal part, and may thus break away from the latter and cause a vacuum in the penstock. An air-inlet valve will prevent this, but it is better to have a small compensating or equalizing reservoir (or stand-pipe) at the junction of the horizontal and inclined part of the penstock. Such a reservoir may be built of steel plate, concrete, or masonry, and will not only prevent the collapse of the penstock from the cause above named, but will also greatly improve the regulation of the turbines; and decrease the water-hammer in the penstock, acting, in fact, in the same manner as a standpipe.
"Expansion-joints in steel penstocks are not as important as is often asserted, since most penstocks contain bends which permit of a limited movement large enough to compensate for expansion and contraction; but in a straight steel penstock rigidly held at each end, the strains due to changes in temperature are very heavy, and in such case expansion-joints must be provided.
"The lower end of a penstock should be held very securely in all cases, to prevent forces due to temperature changes and other causes from throwing the turbines out of alignment, cracking the powerhouse walls, etc.
"Steel-plate penstocks are usually made in small and large courses, and lap-riveted. Butt-strap joints, with a single butt-strap on the outside, offer less frictional resistance to the flow of the water, but are more expensive. A manhole should be provided at the upper end of a penstock, and at the lower end also, if required. When repainting the inside of a penstock, or.repairing the same, the water leaking through the head-gate should be prevented from running down the penstock; and for this purpose a small outlet-nozzle, about 6 inches in diameter, and closed by a blank flange, is provided at the lower side of the upper end of the penstock; and by building a small dam of clay in the penstock, just beyond this nozzle, the leakage is prevented from flowing down the penstock. All openings in penstocks for large nozzles, branches, manholes, etc., should be reinforced by steel-plate
rings riveted around the openings, to make up for the material cut away by the opening.
"Penstocks should be calked both inside and outside, and the plates thoroughly cleaned by scrapers and wire brushes before painting.
'"Masonry piers are often damaged by the expansion and contraction of the penstock they support, and the paint is rubbed off the penstock where it rests on the piers. Such unprotected places are hidden from view by the masonry, and are apt to corrode very quickly, as water is always retained between the surfaces of contact of the masonry and the penstock. It is therefore preferable to use steel piers on concrete or masonry bases. Such steel piers are cheaper than concrete or masonry piers; they leave every part of the penstock accesssible for painting and repairs, and are free to swing on their bases, like inverted pendulums, to accommodate themselves to any movements of the penstock caused by changes in temperature. The uprights or posts of these piers are provided with bolt-holes, to fasten to them the studs for a housing over and around the penstock when desired.
"Except where the distance between the penstock and the ground varies considerably, the steel piers are all made the same; and the variations in the height of the penstock above the rock or solid ground are made up in the height of the concrete or masonry bases. The uprights of the steel piers are anchored to the bases, or, if the latter are of small height, through the bases to the rock below.
"A penstock running down a steep mountain-side must be prevented from sliding down the slope. Where concrete or masonry piers are employed, it is often sufficient to rivet short pieces of heavy angle-bars to the penstock, and to have these bars bear against the up-hill side of the piers; but with steel piers the penstock must be anchored to the rock or to special anchor-piers. It is well to have, in any case, a specially heavy concrete or masonry pier at the lower end of the penstock, to prevent the latter from throwing the turbines out of alignment.
"In a climate like that of the northern part of the United States and of Canada, penstocks must be covered or boxed in, to protect them from the extreme cold; otherwise ice will form on their inner surfaces.
"During midsummer, the heat of the sun's rays acting on an
empty penstock will often injure the paint, cause it to blister off, and perhaps overstrain the penstock itself; and a covering will therefore prove an advantage both in cold and in hot weather. Even in a wellprotected penstock, ice will be formed in severe weather, when the water in it is allowed to remain stationary for more than a few hours at a time.
"Where the ground under a penstock consists of earth, it is pref-


Fig. 13\%. Redwood Stavepipe under 160 Feet Head, Santa Ana Canal, California.
erable to bury the penstock below the frost-line, like the water mains in a city street.
"A buried penstock is free from the bending strains occasioned in a penstock carried on piers by reason of the length between the piers being unsupported; but a buried penstock of large diameter will require stiffening angles to be riveted to the upper half of its circumference, to prevent it from collapsing by the weight of the earth above it.
"The cost of burying a penstock will be about the same as when masonry piers are used.
"Under the penstock, in the center of the ditch, should be a drain-age-ditch about one foot square in cross-section, and filled with pebbles or broken stone, as used for concrete-making. The penstock should rest on short wooden blocks; and the main ditch should be left open during the first year, or for one winter season at least; after which the
penstock is carefully inspected, recalked where leaky, and repainted inside and outside; and then the earth is packed under and around the penstock, and the ditch filled in, the wooden blocks being removed as the work proceeds.
"Wooden Penstocks. Wooden penstocks or stavepipes deserve a wider application than they have so far found in the Eastern States. Wooden penstocks are cheaper and will last longer than steel penstocks, need less protection against extremes in temperature, and require no painting. Their interior surfaces are smoother than those of steel penstocks, and therefore offer less resistance to the flow of the water. Such pipes have been built up to 9 feet diameter.
"Wooden penstocks are made of staves from 2 to 4 inches thick and from 6 to 8 inches wide, planed to the proper shape, and held together by round or oval iron or steel rods, connected by hooplocks of various designs. The staves must be thick enough, or the hoops spaced closely enough, to prevent the staves from bulging out between the hoops. Thick staves are usually provided on one edge with a bead of from $\frac{1}{16}$ to $\frac{1}{8}$ inch in height, by $\frac{1}{2}$ to $\frac{3}{4}$ inch in width, located next to the inner side of the stave, as with such a bead it will require less strain in the hoops to make the penstock water-tight.
"The joints at the ends of the staves are usually made by steel tongues driven into kerfs. These joints must be well broken.
"Curves in wooden penstocks require a long radius, and therefore their horizontal and vertical alignment must be located on the ground, like a railroad line. The minimum radius, in feet, that can be used in a wooden penstock is about $R=12.5 \times D_{p} \times t_{s}$, in which $D_{p}$ is the inside diameter of the penstock in feet, and $t_{s}$ the thickness of the staves in inches. Where a smaller radius is required, a section of steel penstock has to be inserted in the wooden one for the purpose.
"The wood employed should be clear and sound, and free from pitch, so that the staves will become saturated by the water. The wood used for such stavepipes is, in the order of its value for the purpose: California redwood, Douglas spruce (also called Douglas fir), spruce, white pine, southern pine, and cypress.
"The staves of a wooden penstock that is not left empty long enough to allow the wood to dry, will last much longer than the hoops; and the hoops may be renewed, when destroyed or weakened by rust, by placing new ones between the old hoops, if the soundness of the



Fig. 139. Side View of Sterling Flume in Provo Canyon, Utah. Courtesy of U. S. Geological Survey.


Fig. 140. Reinforced-Concrete Pipes as Made and Tested by U. S. Geological Survey.
staves will warrant it. A stop-valve should be used at the lower end of a wooden penstock, with no head-gate at the upper end, to insure the penstock being always full of water, which may be shut out by the use of stop-logs in case of necessity. A wooden penstock buried in the ground may be left empty for some time, without danger of the staves drying out.
"In heads of 200 feet and more in height, wooden penstocks are not economical, as the hoops require as much metal as the plates for a steel penstock.
"Penstocks constructed of concrete and steel also deserve a wide application, and should outlast both the steel and wooden penstocks, as the steel rods are protected by the concrete.
"Instead of welding together the ends of the embedded hoops, these ends may be run past each other for a distance of from 30 to 40 times the diameter of the hoop-rod; or an inch or so of each end of the hoop-rod may be bent back flat on itself, and the ends run past each other for a distance of from 20 to 30 times the diameter of the hoop-rod. For small concrete penstocks, steel wire wound spirally can be used to form the hoops.
"For heads of 200 feet and more in height, penstocks built of concrete and steel are not economical, as the hoops require as much metal as the plates for a steel penstock.
"Standpipes may be built either of steel plate or of concrete and steel. An excellent arrangement is to have a concrete base straddling the penstock, and the stand-pipe placed on top of this base, like a steel chimney or stack."*

Figs. 137, 138, and 139 are illustrations of wood stavepipes.
Figs. 140, 141, and 142 illustrate steel concrete pipes as made and tested by the United States Geological Survey.

Riveted-steel pipes are shown in several of the illustrations of power plants appearing in subsequent articles.
188. Canals; Flumes. The older method of applying the power of water to industrial purposes consisted in conducting the water by means of canals or flumes to the various establishments in which the power was to be utilized. This method required special physical conditions of a very favorable nature, in order that great expense in construction might be avoided. The later method is to convert the

[^9]
A. Pipe under Test.

B. Method of Raising End of Pipe.

Fig. 141. Reinforced-Concrete Pipes as Made and Tested by U. S. Geological Survey.
energy of the water into mechanical power, and the latter into a form suitable for transmission, in a single power-house, from which central station the power may be conveyed long distances, and applied to various machines located to the best advantage without reference to the waterfall itself.

Shafting, wire rope, compressed air, and water under pressure, have all been more or less utilized for the transmission of power;


Fig. 142. Construction of Pipe by Movable Form. Courtesy of U. S. Geological Survey.
but, since the great development of electricity in comparatively recent times, this agent is most generally employed for that purpose.

It does not usually happen that a fall available for power. pro-duction (either in natural fall, or one created by damming a stream) occurs in a single vertical plunge; usually there are several rapids of greater or less extent between the dam or actual waterfall and the proposed site of the power-house, necessitating the construction of a canal or flume of greater or less extent, if the entire fall is to be utilized. Even with a waterfall in which the total available head occurs in a single drop, a canal or flume is generally employed to convey the water to the several wheels or groups of wheels.

In a canal system, whether to be used for power purposes,


Fig. 143. Concrete-Lined Section of Truckee Canal, Nevada.


Fig. 144. Concrete-Tined Section of Truckee Canal, Nevada.
Ninth section, rounding Wadsworth Point at mouth of canyon.


Fig. 147. Santa Ana Canal, California. Gravel concrete lining.


Fig. 148. Riverside Canal, Callfornia.
Sand deposit left on bottom after a year's service. Courtesy of "Engineering News."
water supply, or irrigation, there are many important features requiring very special attention, some of which have already been referred to-such as the headworks, with the corresponding regulator-or headgates for controlling the supply of water into the canal head; the diversion dam or weir, of greatly varied construction, to raise the level of the water adjacent to the headworks and thus induce a proper flow into the canal; escape-heads (wasteways) and their gates, to empty


Fig. 149. Bear Valley Canal, Redlands, California. Boulder-lined and plastered.

Courtesy of "Engineering News."
the canal quickly in case of accident or danger, or to dispose of surplus water in times of flood or excessive rains; sand-gates, for scouring out deposits of sand or silt; vertical falls and inclined chutes, to compensate for excessive grade; and other features controlled by local conditions. Such features are treated in detail in connection with the subject of Irrigation.

The phenomena of erosion of bed and banks, of sedimentation of suspended matter, of capacity and velocity of flow with various cross-sections, and influence of kind of lining, together with their inter-


Fowler Switch Canal, California, Showing Effect of High Velocities.


Bear River Canal, Utah, Looking North.
Fig. 150. Views of Western Irrigation and Power Canals.
Courtesy of U. S. Geological Survey.



Fig. 152. Santa Ana Canal (Cement-Lined), California; Capacity, 240 Second-Feet. $\begin{gathered}\text { Courtesy of U. S. Geological survey. }\end{gathered}$


Fig. 153. End View of Sterling Flume in Provo Canyon, Utah.


Fig. 154. Flume of Kern Valley Power Development Works, California.
Courtesy of U. S. Geological Survey.





Fig. 160. Wasteway No. 1, Truckee Canal, Nevada. Showing mechanism for operating Taintor gates.


Fig. 161. Diversion Dam and Gates, Heading of Main Truckee Canal, Truckee-Carson Project, Nevada. View looking south along the dam, showing front face of dam; also the gate-operating mechanism.


Fig. 162. Map of Niagara Falls and Vicinity, Showing Location of Power Development Enterprises.
relations and mutual dependence, are discussed in connection with Hydraulics and Water Supply, and will not here be considered further than to present some typical illustrations (see Figs. 133 to 147).

## HYDRAULIC POWER INSTALLATIONS

189. Niagara Falls.* Along the boundary between Canada and the United States, there exists a chain of great lakes having a surface area of some 90,000 square miles, which receive the drainage from a catchment area of about 240,000 square miles. Between Lakes Superior and Huron there is a drop in elevation of the water surface of about 18 feet at the Sault Ste. Marie; between Lakes Erie and Ontario, which are connected by the Niagara River, there is a total drop in elevation of 326 feet. The distance between these two lakes is about 30 miles, but almost the entire fall occurs in the last 15 miles. There is a fall of about 56 feet in the rapids above the Niagara Falls; about 160 feet at the Falls; and about 110 feet below the Falls. The entire drainage of the upper lakes flows from Lake Erie through the Niagara River into Lake Ontario, and thence, by the St. Lawrence River, into the Atlantic Ocean.

These lakes form great natural storage reservoirs, so that the volume of flow and the levels in the Niagara River are remarkably uniform. In extreme cases the river level above the Falls varies $3{ }_{2}^{1}$ feet, the variation being chiefly due to wind holding back the outflow from the lakes. Below the Falls, the river level varies at most 15 feet, due chiefly to ice-blocks formed in the lower river.

The minimum flow of the Niagara River, as given by the government engineers, is 178,000 cubic feet per second; the mean flow is 250,000 cubic feet per second. The minimum flow, with the total fall of 326 feet, represents about $6,594,000$ gross horse-power; with a fall of 216 feet (i.e., from the upper river to the foot of the Falls); this would be about $4,369,000$ gross horse-power. With the mean flow of 250,000 cubic feet per second, the corresponding figures would be about $9,261,000$ and $6,136,000$.

The Falls comprise the Horseshoe Fall, about one-third of a mile wide, on the Canadian side; and the American Fall, about 600 feet wide, on the opposite side, the two being separated by Goat Island.

[^10]Below the Falls, the river flows through a gorge or ravine 600 to 1,200 feet wide, and 200 to 300 feet deep, eroded by the action of the river itself. Fig. 162 represents the conditions and locations of the various installations.

The importance of the Falls as a source of energy was recognized from an early period. The first important effort to obtain power was made in 1853, when construction on the so-called "Hydraulic Canal," 36 feet in width, 8 feet in depth, and 4,400 feet in length, was begun from a point above the upper cataracts to a basin at the top of the bluff, lceated on the side below the Falls. This canal was completed in 1861. On the bluff were constricted mills, having turbines supplied with water from the basin and discharging it through short tunnels on the face of the bluff. In these cases, only part of the available fall was utilized, water being plentiful and the cost of excivating pits for the turbines consicierable. In 1885, about 10,000 horse-power was utilized in this way, or the whole available supply of the hydraulic canal as then construeted.
190. Niagara Falls Power \& Manufacturing Company. In 1877 the Hydraulic Canal and all its appurtenances were purchased by the present owners. In 1892 the company commenced an enlargement of its canal, and it has made notable improvements from time to time. The plan adopted at that time was to widen the original channel to 70 feet, and to make the new part 14 feet deep, thus providing an additional capacity of about 3,000 cubic feet of water per second, giving a surplus power, after supplying the old leases, of about 40,000 horse-power. In 1895-96, a second power-house was erected for the purpose of supplying power to customers. For this new plant a branch canal was taken to a forebay, 30 feet wide and 22 feet deep, near the edge of the bank. From this forebay, penstock pipes of flange steel, 8 feet in diameter, conduct the water down over the high bank a vertical distance of 210 feet, to the site of the powerhouse on the sloping bank at the edge of the water in the lower river.

The first portion of the power-house, 60 by 100 feet, was completed in 1896. Because of the fluctuations of the water in the lower river, it was necessary to place the floor of the station on which the generators stand, about 20 feet above the ordinary water level. As it was desired to couple the generators directly to the ends of the water-wheel shafts, it was necessary to place the water-wheels also at
this elevation, and to employ draft-tubes, in order to obtain the full head available. It was also required that the wheels should run at a given speed suited to the speed desired for the generators. To fulfil all these conditions, turbine wheels mounted on horizontal axes were adopted. 'The specifications for these wheels required that each should furnish 1,900 horse-power, measured on the shaft of the wheel, when run at a speed of 300 revolutions per minute. The head under which the wheels work is generally 210 feet; but they were required to have sufficient capacity to deliver 1,000 effective horsepower under a head of 205 feet; and all parts were to have sufficient strength to withstand the pressure due to a head of 220 feet without undue strain. They were required to show a percentage of useful effect of not less than 78 per cent, at any point between full and threequarters discharge, under any head from 205 to 225 feet and running at a constant speed of 300 revolutions per minute; and not less than 60 per cent under the same conditions, from three-quarters to onehalf discharge.

The apparatus for regulating the speed of the wheels consists of a hydraulic piston which applies its force in either direction to a rack connected with a pinion in the gate-rigging of the turbine. The force which operates the hydraulic piston is air, compressed inder about fifteen atmospheres. This compressed air is contained in a cylinder, and the pressure is maintained by a pump which constitutes a part of the machine. The machine is provided with a high-speed ball governor actuating a balanced piston-valve. The governor has an "anti-racing" appliance by which the governing machine is checked before it has carried the gate too far in either direction.

The second section of this power station was completed in 1900, making the present size 120 by 100 feet. This portion of the station contains five turbines, each having a capacity of 2,500 horse-power (Fig. 163). They are fed by a new 11 -foot penstock consisting of a vertical portion about 200 feet high, with an arm on each end, that at the top having a length of about 68 feet, and that at the bottom about 115 feet. The penstock is built up in sections of 5 feet, the sections lapping inside and outside altcrnately. The thickness of the plates varies from $5-16$ inch at the top, to $1 \frac{1}{8}$ inches at the bottom. Most of the sections are made up of the two plates. The thinner plates have lap joints with two rows of rivets; while the thicker plates have
butt joints and double cover splice-plates with three rows of rivets on each side.

A vertical recess about 15 feet square and 50 feet high was cut out of the solid rock at the base of the cliff, and in it was set the bottom


Fig. 163. General Dimensions of Penstock of the Niagara Falls Hydraulic Power \& Manufacturing Company.
of the vertical portion of the penstock. At the lower elbow, the penstock increases to 13 feet in diameter, and then decreases, as it passes the turbines, to a diameter of 7 feet at the outer end. The upper end has a bell-shaped inlet, 22 feet wide, built into the masonry of the forebay at an oblique angle. Heavy cast-iron brackets are riveted to the top of the vertical portion of the penstock on each side, and
support one end of a pair of plate-girders 30 inches deep and 49 feet long, which have 8-inch transverse I-beams across their top flanges to support the horizontal portion. About 6 feet below the bottom of the plate-girders, the penstock is encircled by a pair of bent 10 -inch


Fig. 165. Iron Work in Tail-Race, Plant of Niagara Falls Hydraulic Power \& Manufacturing Company.

I-beams with horizontal webs, between which plates are riveted to afford a pin connection for two sets of eye-bar anchors which guy the penstock horizontally to 2 -inch eye-bolts drilled and cemented into the side of the cliff. At the lower elbow, the horizontal end of the pipe is seated on a bed of cut-stone masonry, and is bedded in cement mortar. The convex side rests on flanged cast-iron angle-blocks
riveted to the pipe and seated on stone piers. Large right-angled brackets of cast iron, reinforced by heavy angles, are riveted around the pipe just above the elbow; and the one on the convex side rests on solid masonry, while the one on the concave side is supported by two inclined braces whose lower ends rest on castings let into the


Fig. 166. Anchor at Top of Penstock, Plant of Niagara Falls Hydraulic Power \& Manufacturing Company. vertical rock wall.

The tail-race is a rock cut 17 to 21 feet wide and 19 feet deep below the center line of the lower arm of the penstock, which is supported on steel columns. The low er horizontal part of the penstock is made in sections which have alternate parallel and tapered sides. The former are uniformly about 10 feet long, and are fitted on top with vertical branches 5 feet in diameter. These branchesareflanged to the inside of the penstock and riveted to it and to outside reinforcement collars, which are 8 inches wide and $2 \frac{1}{8}$ to 35 inches thick. A cross-section near the end of the penstock shows the knee-brace supports of the beam platform, the suspension rods by which the penstock is tied to the beams each side of the vertical branch, the anchorages of the double draft-tubes to the masonry, and the transverse and longitudinal adjustable rods with which they are tied together just above high-water level. See Figs. 164, 165, 166, and 167.

The valve-gates are set horizontally, with their axes inclined 45 degrees to that of the penstock, and are operated by pneumatic


Fig. 167. Anchor Connections, End of Penstock, Plant of Niagara Falls Hydraulic Power \& Manufacturing Company. pressure. The interposition of the tapered sections between the turbine connections reduces the diametcr of the penstock in proportion to the diminished flow of water required as the successive turbines are passed, and brings it down to 7 feet at the last turbine, which is supplied through two side tubes. At this point the penstock is guyed laterally by four 5 by 1-inch horizontal eyebars on each side, which are anchored to steel bars drilled and cemented into the solid rock. The penstock terminates here with a conical section 7 feet long, which tapers to 18 inches, and conncets by a cast-iron elbow with a vertical air chamber about 11 feet in extreme length, and 4 feet in internal diámeter. It has an air-valve . on top, and gaugeglasses on the side. The tapering portion is fitted with thirty spring reliefvalves, set in three groups, having 6inch openings and set to open at a pressure of 100 pounds persquare inch (Fig. 168).


Fig. 168. Relief-Valves on End of Penstock.

The turbine
wheels are made of bronze, and are located in the draft-tube casing, one on each side of the casing proper; each pair weighs about 5,000 pounds. From the sides of the turbines, the discharge pipes

project laterally and then downward to connect with the drafttubes, which are 22 feet 8 inches long. A third power-house capable of developing 100,000 horse-power, also situated at the foot of the bluff, completes the installation of the Niagara Falls Power \& Manufacturing Company.
191. Niagara Falls Power Company. This company began work in 1890, and first delivered power in 1895. The installation comprises two powerhouses. PowerHouse No. 1 has ten 5,000 horsepower units, each unit consisting of twin outwardflow reaction turbines with vertical shafts, on the
upper ends of which are placed theelectric generators. PowerHouse No. 2 is in its main features similar to Power-House No. 1, but it contains eleven 5,500 horsepower simpleinwardflow turbines with draft-tubes.

The engineering history of the power company began in 1889, when the Cataract Construction Company proposed to divert water from the upper river into an open canal at a point somewhat over a mile above the Falls, to deliver the water to wheels in a pit at the side of the canal, and to conduct the water from the wheels to the river below the Falls through a tunnel 7,000 feet long, driven through the rock at a distance of nearly 200 feet beneath the city of Niagara Falls.


Fig. 170. Transverse Section of Wheel-Pit, Power-House No. 1, Niagara Falls Power Company.
192. General Considerations. The canal is located on the New York shore about $1 \frac{1}{4}$ miles above the crest of the American Fall, at a point where the mean water elevation is 561.2 feet. The discharge
portal of the tunnel is about 1,100 feet below the American Fall, at the mean elevation of 343.4 feet. This makes a gross head of 217.8 feet. The canal, which was excavated to give an average depth of 12 feet of water, and projected 600 feet beyond the original shore line into the river with embankments formed from the excavated material, has


Fig. 171. Longitudinal Section through Portion of WheelPit of Power-House No. 1, Niagara Falls Power Company. now a length of about 1,500 feet, and a width of 180 feet at the mouth, and 100 feet at the inner end. The wheel-pits-in which are located the penstocks conducting the water to the turbines, and the shafting connecting generators with turbines, and over which are built the power-housesare located on opposite sides of the canal, as shown in the accompanying plan(Fig. 169). The pit forPower-House No. 1 is 178 feet deep, 18 feet wid.e and 425 feet iong, and houses 10 units of 5,000 horsepower (Figs. 170 and 171); the pit for Power-House No. 2 (Fig. 172) is 178.5 feet deep, 20 feet wide, and 468 feet long, and accommodatcs 11 units of 5,500 horse-power. In both cases, about 27.2 cubic fcet of wheel-pit excavation were made per horse-power developed.

Each of the ten turbines of the first plant passes an average amount of 430 cuhic feet of water per second; and each of the second


Fig. 173. Cross-Section of Power-House No. 2 and Wheel-Pit Niagara Falls Power Company.

plant; 445 cubic feet. Each unit being supplied from a $7 \frac{1}{2}$-foot penstock, the velocity in the penstock is 9.75 feet in the first case, and 10 feet in the second. With 10 wheels in operation in each plant at one and the same time, the total quantity of water to be taken through them is over 10,000 cubic feet per second, equivalent to a velocity in the canal at its entrance of 4 feet per second. The tail-race tunnel, through which all the water must pass, has a maximum height of 21 feet, and a width of 18 feet 10 inches, and a net cross-sectional arca of 335 square feet. The mean velocity through it with the given quantity of water is, therefore, about 29 feet per second. The tunnel being about 7,000 feet long, a considerable portion of the gross head must necessarily be employed in effecting the removal of the water at the required rate; it was accordingly given a slope which averages 6 feet in 1,000 , placing the floor of the tunnel at the wheel-pit at the elevation of 387 feet, or 43.6 feet higher than the river level at the portal. A further reduction due to the location of the turbines in the first plant about 37 feet above the wheel-pit bottom (no drafttubes seeming applicable at the time), together with a minor loss in the flow through the canal and intake racks, reduced the gross head to 136 feet. In the second plant, the effective head based on the use of draft-tubes was taken at 145 feet.
193. The Canal and Forebays. The canal was excavated to allow a depth of 12 feet at low water, as already stated. The walls of the canal are of solid masonry 17 feet high, 8 feet thick at the base and 3 feet at the top, laid in an ordinary Portland cement mortar composed of 1 part cement and 2 parts sand. Besides the two powerhouses which have to be supplied with water, the canal is also tapped for a supply to a separate wheel-pit owned by the International Paper Company, whose property adjoins that of the Niagara Falls Power Company. Hydraulic power is sold in this case, as, at the inception of the work, electric power could not be furnished in time; and the power company provides for the disposal of the water through a 7 -foot tunnel running from the wheel-pit of the International Paper Company to the main tail-race tunnel.

The points of intake to the power-houses, or outflow from the canal, are distributed in two groups to avoid local high currents as much as possible, the two power-houses being, partly for this reason, located diagonally across the canal. There is a separate opening
for each penstock in each of these power-houses, but the intakes are materially different. Racks of the usual flat-iron bar construction guard the entrances to the penstocks; but in the later installation they are enclosed by a portion of the power-house, in a covered forebay (Fig. 173). While the extra cost of a covered forebay is considerable, its provision was deemed advisable as a means of fighting ice. Under ordinary circumstances, there is a floating timber boom extending


Fig. 174. Arched Entrances to Inner Forebay of Power-House No. 2, Niagara Falls Power Company. Entrances in lower wall, submerged when in operation. across the mouth of the canal; but cakes of ice, especially with a large flow of water, find their way under the boom and into the canal. During the present year the boom has been located a considerable distance in front of the canal, to divert ice more readily toward the Falls.

Much trouble used to result from allowing floating ice to enter the canal, and a large corps of men was constantly required in cold weather, several shifts per day, cleaning the racks at Power-House No. 1. These racks consisted of three parts-a bottom section; a top section, dropping from the top of the canal wall to a level about one foot below water; and a middle, removable section entirely submerged, which could be temporarily hauled out of position for purposes of cleaning, etc.

- Though a secondary boom was provided, extending into the canal a few feet in front of the rack, and across the entire series of openings, cakes of ice would succeed in getting under this boom, and would frequently also be drawn under the top section of the racks, cwing to the stuction of the inflowing water. As the wheels of the first power-house were not designed to allow for an accumulation of
ice, as will be explained, it can readily be seen that the passage of much ice into any penstock was likely to be a serious affair.

It was these general considerations that affected the design of the wheels of the second power-house, and the arrangement of the intake at that building. Here the secondary boom is replaced by the outside wall of the covered forebay, the water being admitted into the forebay through arched openings (Fig. 174), the crowns of the arches being about 4 feet below normal water level, this distance being assumed great enough to prevent blocks of ice diving through the arches.

In the design of the new power-house, moreover, an ice-run has been provided; and blocks of ice collecting in the canal are directed toward the run, and drawn by the current of water in the ice-run cown a shaft and into the tail-race tunncl. In a similar way, there is now an ice-run for Power-House No. 1, secured by utilizing a shaft that had been sunk in connection with the driving of the extension of the tunnel to the second power-house. It will thus be seen that much -has been done to overcome the ice troubles, and that in this connection the tail-race tunnel has to take care of the additional amount of water, if it is necessary to prevent any great lodgment of ice in the intake canal.
194. General Comparison of the Tuo Plants. A comparison of the design of the two stations can be made from the accompanying cross-section sketches. The main difference is the covered forebay in the case of the new plant, as already stated. At the normal level of the headwater, the width of the power-house is the same in each case, about 110 feet; but in the older plant, the outlying position of the racks makes the total width at the foot of the racks somewhat greater. Otherwise the general features of the two plants are the same; the width of the two buildings is in each case about 70 feet, their height to the ridge of the roof, $€ 0$ feet; the distance from floor to under side of the roof trusses, 40 feet; the width of the finished wheelpit, 17 feet 6 inches; and the general arrangement of the main machinery the same. Each turbine in both plants is supplied from a 90 -inch penstock, and drives a long vertical shaft, which is a tubular affair direct-connected to the elcetric generator shaft in the powerhouse.

The notable difference in the hydraulic machinery exists in the
turbines, which are of the inward-flow type in the new plant, and of the outward-flow type in the old; in the thrust bearings, which are of a dise type in the new, as against the usual collar type in the old; and in the exciter equipment, which comprises a group of dynamos direct-connected to turbines in an underground chamber in the new plant, as compared with direct-connected units having the exciters in the power-house on the top of long shafts, and the exciter turbines in the bottom of the wheel-pit, in the old plant. While the governors are different in the two plants, the scheme of levers, rods, and counterweights of the governing system is essentially the same.
195. Intake of Power-House No. 2. The arches through which the water enters the forebay of the new plant from the canal are sprung from 5 by 6 -foot piers, spaced 20 feet apart, and leaving openings 14 feet wide and 10 feet to the crown of the arch. This gives an area of flow of about 125 square feet; and as there are two arched openings per unit, the inlet area is 5.7 times that of the penstock, not allowing for the water taken from the forebay for the exciter turbines and other purposes. Immediately inside the arches are grooves on each side for stop-logs for shutting out water from the forebay at any time. A short distance beyond these, the water passes through the racks, which are of the usual construction, inclined 30 degrees with the vertical. The rack bars are of mild steel, 3 by $\frac{3}{8}-$ inch in size, separated by pieces of $\frac{3}{4}$-inch gaspipe into spaces $113-16$ inches wide. They are built in three sections as regards height--the upper, with rounded top, being about 8 feet long and extending 3 feet below normal water level; the middle, 10 feet long; and the bottom 2 feet 10 inches long. The center sections are removable, sliding between I-beams and channels; and for handling them, there is a 5 -ton Niles electric crane traveling the entire length of the forebay. The rack structure is supported on 15 -inch I-beams on 8 -foot centers, and the bars are divided into two groups between them. The total wetted area of openings between the rack bars is substantially equal to that of the aggregate area of the arched openings. Between the racks and the head-gates, which are in the power-house proper, are two sets of grooves in the buttresses against which the racks are supported. These allow for closing the inlet to the corresponding penstock with stop planks, in the event of a need for repairs to the gate.
196. Head-Gates. The gates in the new power-house, like those in Power-House No. 1, may be operated both by hand and by electric motor. They are lifted by screws; but the new gates are provided with wicket-gates for passing the water into the penstock and relieving pressure before opening, and are not provided with the roller bearings used instead in the old plant.

Ice-Runs. The ice-run from the canal at Power-House No. 2 is also provided with gates, so that no water may be wasted in this way during summer, as it is undesirable to have more water than necessary discharged into the tail-race tunnel. The ice-run has direct connection with the canal, and also with the covered forebay, the latter inlet to the ice-run being provided to take care of any ice which may succeed in entering the forebay. These gates are also designed to be lifted either by motor or by hand. They are of the lifting-screw type, and the motor drives a horizontal shaft which carries at each end a pair of bevel gears. The horizontal gear in each case revolves on ball bearings, and the screw is lifted and lowered through it.
197. Penstocks and Wheel-Pits. The intake proper to each penstock, that guarded by the head-gate, is 14 feet wide, and under normal conditions carries 14 feet of water. The mouthpiece to the penstock starts a few feet back of the gate, with a flaring elliptical entrance, and, pitching 2 inches in a foot, joins the 30 -inch circular penstock 14 feet beyond. The elliptical entrance is $12 \frac{1}{2}$ feet wide and $8 \frac{1}{2}$ feet high. The inclined portion of the penstock, and the mouthpiece, are bedded in concrete, and are arched over by brickwork which supports the substructure of the power-house proper. The junction between the walls of the chamber behind the gate and the penstock mouth is made with cement mortar, and the joint between the inside masonry wall of the chamber and the brick wall of the cable conduit behind is waterproofed with asphalt.

From the mouthpiece to the turbine, the penstock comprises a short length of straight riveted-steel pipe; a riveted-steel elbow at the top of the wheel-pit; six vertical sections of straight pipe, each 15 feet $5 \frac{1}{2}$ inches long and 90 inches in inside diameter; and a cast-iron elbow at the bottom connecting into the turbine. The upper elbow is provided with cast-iron brackets, and the portion of the penstock above it is supported from it by the brackets, which bear on steel
beams bridging the wheel-pit. The rest of the penstock is carried by the cast-iron elbow at the bottom. There is a stuffing-box of cast iron in the pipe immediately below the upper elbow, to allow for expansion; it consists of two rings of cast iron bolted to the abutting ends of the penstock, one sliding on the other, as shown in an accompanying detail (Fig. 175). The stuffing-box was required to stand


Fig. 175. Penstock Expansion Joint, Niagara Falls Power Company. a hydraulic test of 25 pounds. An idea of the size of the bottom supporting elbow of the penstock can be obtained from the fact that the metal is $2 \frac{1}{4}$ inches thick, and it is reinforced by circumferential ribs 4 by 4 inches in size.

The steel of the penstock is $\frac{3}{8}$ inch thick in the upper part, and $\frac{1}{2}$ inch in the lower; and each section has two 18 -inch manholes. There is a series of floors or decks and intermediate platforms in the wheel-pit, as indicated in the drawings, and the penstocks and shafts are thus accessible throughout their height. The pit is lined with brick throughout, and cast-iron brackets are laid in the walls to support the various beams. Drainage pipes emptying into the tail-race were provided behind the wall, laid in broken stone and wherever ground-water was likely to collect.
198. The Tail-Race Tunnel. The tail-race tunnel, from a point near the wheel-pit of Power-House No. 1 to the portal at the lower river, was built in a straight line for construction and hydraulic reasons. Surveys for this purpose, which were started in the latter part of March, 1890, marked the beginning of actual work on the Niagara Falls Power Company's plant. The work was executed from the discharge portal, and from two shafts which were sunk at points 2,600 and 5,200 feet from the portal.

The tunnel is lined throughout with brickwork, partly on account of the poor character of the rock, and also because of the decreased friction to the flow of water thereby secured. The invert was laid last, and has a face of vitrified paving brick. The sides and arched top are usually four rings of brick 16 inches in thickness, but are sometimes six and eight rings thick; and the space behind was filled with rubble masonry. The bricks were laid in a mortar of 1 part Portland cement to 3 of sand, except where the flow of water is very great, in which case the proportions are $1: 2$, and in some cases 1:1.

One of the most interesting points in connection with the tunnel is the provision of an ogee discharge. It provides for lowering the grade of the invert about 11 feet below the average low water of the river, so as to allow. aboutone-half of the flow from the tunnel to discharge kelow the surface. The


Fig. 176. Lower Part of 5,000 -Horse-Power Turbine, PowerHouse No. 1, Niagara Falls Power Company. ogee surface starts at a point 95 feet from the portal, and the $10 \frac{1}{2}$-foot drop takes place in this distance. This portion of the tunnel, to the elevation of the spring line, is lined with steel boiler-plate riveted to steel ribs 3 to 4 feet in depth, which are bedded in Portland cement concrete. For the last 25 feet of the tunnel, granite masonry is used instead of the brickwork, and the arch and the face of the portal are also of granite masonry, carried to 38 feet below the water surface to a ledge of white sandstone.
199. The Turbines. The distinct difference in the turbines of the two plants is that in the new plant the turbines are of the single inward-flow or Francis type, while those of the old plant are of the twin outward-flow or Fourneyron type, each wheel divided into three parts by horizontal partitions. In the former, as already stated, the design has been made to utilize draft-tubes, while in the older machines the water was discharged freely into the air directly above the
tail-race. The new wheels were designed by Messrs. Escher, Wyss \& Company, to give 5,500 horse-power each, under a head of 145 feet at a speed of 250 revolutions, this capacity of the turbine to provide for overload in a $5,000-$ horse-power generator. The wheels of the


Fig. 177. Detail of 5,500-H.P. Turbine in Power-House No. 2, Niagara Falls Power Company.
first power-house were designed by Messrs. Faesch \& Piccard (now Piccard \& Pictet), of Geneva, Switzerland; and, under a head of 136 feet (the distance from the surface of the headwater to the center between the upper and lower wheels of the turbine), give 5,000 horsepower, corresponding to a discharge of 430 cubic feet per second at an efficiency of 75.5 per cent.

In both cases there is a balancing disc or piston which may take up practically the entire weight of the revolving parts; and there is a thrust bearing near the top by which the revolving.parts are hung, this bearing taking the difference between the total weight of the rotating parts and the thrust on the balancing disc. In the first plant the balancing piston is above the upper of the twin turbines, and is in communication with the turbine chamber; but in the second plant a special piston below each of the single turbines is provided, and there -is a separate supply of pressure water for this purpose. In the old plant a type of collar thrust bearing is used, but this has required careful watching whenever the upward water-pressure thrust is reduced. The bearings for the new plant are of the disc type. The chief source of trouble with the bearings in the old plant arose from a diminution of the upward thrust on the shaft, due to a throttling of the supply of water in the penstock by the collection of ice in it. It was this source of trouble that led


Fig. 178. Arrangement of 'Turbines and Draft-Tubes, Power-House No. 2, Niagara Falls Power Company. to a design of balancing in the new plant from a separate, positive supply of water. In the new wheels there are 25 blades in the fixed or guide wheel, and 21 blades in the running wheel, which is of manganese bronze cast in a single piece and weighing nearly 4,000 pounds. It has a diameter of $5_{\frac{1}{1}}^{1}$ feet; but the total diameter of the turbine, which should include the casing around the guide-wheel, is 12 feet. The guide-wheel of the Faesch \& Piccard turbine has 36 buckets, and the turbine wheel 32 , of bronze cast solid with the rim. The outside diameter of the wheel of this turbine is about 7 feet.
200. The accompanying drawings (Figs. 176 to 182) will serve to show some of the details of the design of the wheels in both powerhouses. In the first three units installed in Power-House No. 1,
the cast-iron elbow at the foot of the penstock was carried on builtup girders, these in turn resting on brickwork walls 2 to $2 \frac{1}{2}$ feet thick; but the vibration to which they were subject was found so great that the succeeding units in Power-House No. 1, and those in Power-


Fig. 179. One of the 5,500 -Horse-Power Turbines, with Balancing Piston Showing Double Draft-Tubes, Power-House No. 2, Niagara Falls Power Company. House No. 2, all have the cast-iron penstock elbows provided with feet cast solid with the elbows, and seated on brackets embedded in the walls of the wheel-pit. This plan of bridging the tail-race is necessary in order that the bottom of the wheel-pit shall be as little obstructed as possible; and on this account, also, the draft-tube from each turbine in Power-House No. 2 bifurcates, the two branches passing down in recesses in the walls, discharging into the tail-race at theoppositesides. The pair of drafttubes for each turbine starts from a Y-piece; and they are of heavy ribbed cast-iron anchored to the rock, and serve as supports for the turbine casing and such an amount of the dead weight of the water as is not carried by the elbow at the bottom of the penstock.

Another feature of the new turbines is the enlarged casing around
the guide-blades, which allows for collecting stones that may be carried down the penstock, and from which they may be discharged into the tail-race as desired, through a pipe dropping from the bottom. In Power-House No. 1, it has sometimes been necessary to shut down the wheel to remove ice collecting in the turbine and in the lower part of the penstock.

The manipulation of turbine gates is similar in the two plants, in that the governors are connected to the gates by a series of levers and suspender rods extending from the governors in the power-house to the gates at the turbines.

The turbine shaft in the new plant, like that in the old, is built up of sections of pipe 3 feet 2 inches


Fig. 180. 'Thrust or Hanging Bearing. in diameter, with walls $\frac{3}{8}$ inch thick, connected by solid pieces of shaft 11 inches in diameter at the points where bearings are employed. Besides the two bearings at the turbine, and those at the generator at the top of the shaft, there are three guide-bearings, and the thrust or hanging bearing (Fig. 180). The sections of tubular shaft are fixed to the solid pieces by hollow steel cones bolted around the 11 -inch shaft, and to a ring of angle iron riveted to the tubular shaft. The guide-bearings are located roughly 40 feet apart, and are 20 inches long. They are lubricated with oil under slight pressure; and. a feature of their design is the use of oil shedders and oil catchers, to prevent oil from dripping downward along the shaft.
201. Thrust Bearings. The choice of a thrust bearing has been the subject of a considerable amount of experimental work largely carried on at the power-house. The plans of Messrs. Escher, Wyss \& Company called for a dise type of bearing, in which the shaft was hung by a revolving disc carried on a stationary disc. Three different types were constructed and placed on the shafts of the first three units of Power-House No. 2. The discs were all made of closegrained charcoal iron, of 25,000 pounds tensile strength, and with the mating faces scraped to a bearing. Their differences arise from the method of lubrication. For wheel No. 11, the first in the new plant, forced lubrication was adopted; for No. 12, a combination of forced and self-lubrication; and for No. 13, self-lubrication alone.

In the combination bearing, arrangements are made to utilize two different pressures, the low-pressure oil to come from a general lubricating system, and the high-pressure from an individual pump on the thrust deck, of 25 gallons capacity per minute at 400 pounds pressure. The oil is introduced into the stationary dise at two diametrically opposite points, and the oil forced between the surfaces. Each disc has two circumferential grooves, one near the outer, and the other near the inner edge. Connecting these in the stationary dise, are grooves $\frac{3}{8}$ inch deep, and branching grooves $\frac{5}{16}$ inch deep. The connecting grooves of the rotating dise are bent backward as regards the direction of rotation, and are $\frac{1}{2}$ inch deep near the inner ring (Fig. 181).

The thrust dises of the bearings in all cases are enclosed in a casing which is provided with two sight-holes in diametrically opposite positions, the sight-holes being fitted with $\frac{1}{4}$-inch plate glass to allow for observing both the condition of the bearing and the temperature of the oil by means of a thermometer hanging in the oil, illumination being secured by suspending an incandescent lamp within the casing. The stationary dise is fastened by steel dowel-pins to a third dise which has a spherical seat scraped to fit a support which in turn is bolted to the thrust girder. This spherical disc makes it possible to take up slight deviations from the vertical, where a more rigid construction with the thrust dise bearing might cause trouble. The radius of the spherical dise and ball seat is 3 feet 4 inches. The bottom of this is grooved at six points to allow the oil to pass from the outer or discharge chamber to the space where it can reach the bear-


Fig. 182. Brake on Main Shaft under Thrust-Bearing Girders.
ing surfaces at the inner ring. As a result of the trials, all units were equipped with the combination bearing.
202. Brake. The brake on the shafts of the new units is different from that in use in Power-House No. 1. It is hung from the thrustbearing girder, and clasps a flanged collar, or brake wheel, bolted to the end of the adjacent section of the tubular shaft. It is operated either by compressed air at 100 pounds pressure, or by hand. Its office, in addition to providing a means of stopping the machines quickly in case of accident, is to bring the rotating members to a stop on $8 . \mathrm{n}$ ordinary shut-down, in a reasonable length of time. For example, without applying the brake, the machines would tend to rotate for 30 to 45 minutes after the closing of the gates to the turbine; while, with the brake, full stop is obtained in less than a minute.

The brake consists of two levers on opposite sides of the brake-wheel, and a mechanism for drawing the levers toward each other to bring two oppositely located brake-shoes to a bearing on the brake wheel. Each lever has the fulcrum at one end, the point of power application at the other, and the shoe at the mid-point. The shoes are lined with maple, and together have a bearing surface of 180 degrees of the circumference of the wheel, or 7.2 square feet. The air-piston has a diameter of 13 inches and a total stroke of 6 inches, and the force exerted to draw the levers together is multiplied by two at the shoes. The brake-wheel is 5 feet in diameter, and 13 inches wide. The compressed air is controlled from the generator floor, as is also the hand operation of the brake. For the latter a sprocket-and-chain transmission turns a screw-shaft to bring the levers together. Coil springs on the shaft keep the brake-shoes normally free from the brake-wheel. See Fig. 182.
203. Balancing Pistons. As already explained, the weight of each turbine wheel and shaft is about 35 tons, and that of the revolving-field ring of the dynamo about 35 tons-altogether about 70 tons, which could not be carried on any pivot or collar-bearing at the speed of the turbines. In Power-House No. 1, water-pressure acts on the balancing piston, or cover of the upper turbine, and is relieved from acting on the lower turbine, giving an upward force of 65 to 70 tons to balance the weight wholly or partially. The excess pressure is taken by a collar-bearing. In Power-House No. 2, another arrangement was necessary. A special piston is here provided,

4 feet 6 inches in diameter, the water-pressure on which, due to the head, partially balances the weight. The excess or unbalanced load in this case is taken by the oil bearing shown, oil being forced between the bearing surfaces.

Speed Regulation. In Power-House No. 1, this is accomplished by means of a sensitive governor acting on a ratchet wheel connected


Fig. 183. Detail of $10,000-\mathrm{H} .-\mathrm{P}$. Turbine, Canadian Niagara Power Company.
with the sluices. In Power-House No. 2, the relay is of the hydraulic type. In this case the sensitive governor in action opens a valve, thus actuating a ram driven by oil from an oil reservoir, at a pressure of $1,200 \mathrm{lbs}$. per square inch.
204. Canadian Power Company. The installation of this company will be worked in combination with that of the Niagara Falls Power Company, so that, when necessary, it can assist the works on the other side of the river. The works were not started on the Canadian side until it was thought that all the problems involved had
been satisfactorily solved bygexperience on the American side. When complete, the power-house will have eleven units of 10,250 horsepower each (Fig. 183), on 133 feet effective fall. The adoption of these large units has resulted in a material reduction in the size of the wheel-pit, canal, and power-house, for a given power development, as compared with the American plant.

In 1906, five turbines of the double inward-flow type with drafttubes, had been installed. The tail-race tunnel is 2,200 feet long, 21 feet high by 19 feet wide, and the water in it will flow with a velocity of 27 feet per second. The wheel-pit is 570 feet long, 165 feet deep, and 18 feet wide. The weight of turbine-wheel, shaft, and field-ring is 120 tons, and this weight is carried by a balancing piston, on which water acts at the pressure due to the fall.
205. Electrical Development Company of Ontario. The success of the Niagara Falls Power Company has stimulated other enterprises of similar magnitude. The Electrical Development Company of Ontario obtained rights, and is erecting an installation of 125,000 horse-power on the Canadian side (see general plan, Fig. 162). In order to construct a masonry intake dam in the river, a cribwork cofferdam 600 feet long was built in some of the worst of the upper rapids, laying bare eleven acres of the river-bed. This temporary dam was, at the worst part, in water 24 feet deep, and flowing at probably 30 miles an hour. Its construction was an engineering feat of the greatest boldness. Within it, is now (1908) being erected a concrete gathering dam with granite coping, to direct water into the intake, while floating ice will pass over the dam and back into the river. The water, before entering the power-house, must pass through submerged arches and screens. The tail-race tunnel, 26 feet high, $23 \frac{1}{2}$ feet wide, and 1,900 feet long, passes right under the upper rapids and discharges underneath the Horseshoe Fall. A drift-way to the mouth of the tunnel was first driven, and then the tunnel excavated back from the mouth. The excavated material was thrown down from the mouth of the tunnel into the lower river, where it has disappeared. The wheel-pit is 416 feet long, 27 feet wide, and 150 feet deep. It is to receive eleven double turbines of 12,500 horse-power each. A feature of this installation is the two-branch tail-race tunnel, one branch on each side of the wheel-pit, five turbines discharging into the one, and six into the other.
206. Ontario Power Company. The plans of this company differ essentially from the others, and are no doubt partly conditioned by the fact that the ground nearer the Falls is already occupied. The intention is to develop 200,000 horse-power. The in-take-at the top of the rapids on the Canadian side, near the Dufferin Islands-is specially designed with reference to ice difficulties. The openings in the intake dam have a curtain dipping 9 feet into the water, below which the flow to the turbines takes place, the floating ice being carried past. A second curtain on the same principle is constructed between the forebay and inner basin, and the ice in the outer basin is carried forward over the lower part of the outer dam. The ice in winter is a serious difficulty at Niagara. Cake ice floats down from the upper lakes, and "mush" ice is formed in the turbulent rapids, primarily by the freezing of spray and foam. For ice in this latter form, there are screen frames.

From the intake, three great steel conduits, 18 feet and 20 feet in diameter, convey the water round the other power-houses to the top of the bluff below the Falls. These conduits, of which one is already (1908) constructed, are of $\frac{1}{2}$-inch steel plates, stiffened with bulb irons, and encased in concrete. The velocity in the conduits will be 15 feet per second. There is a spillway at che end, formed by a weir, to prevent water-hammer in the pipes. The flow over the weir passes down through a helical culvert or spillway in the rock to the lower river. From the conduits the water will be taken down to the turbines through twenty-two steel pipes, 9 feet in diameter, passing down the face of the bluff.

The power-house is on a platform at the foot of the bluff, and just above the level of the lower river. The turbines are to be in-ward-flow, twin turbines, each of 12,000 horse-power, under 175 feet head. The axis of the turbines is horizontal, and the shaft is 24 inches in diameter. The turbine wheels are 78 inches in diameter, and have movable guide-blades, which are more efficient than the cylindrical sluices used in the other power-houses, though of course economy of water is not of great importance at Niagara.
207. Snoqualmie Falls Power Company.* Within the past few years a number of plants have been established on the Pacific slope, to utilize natural water powers for generating electricity to be trans-

[^11]mitted to distant points, and there used for lighting and power purposes. Among the most interesting and important of these plants is that at the Snoqualmie Falls, in Washington. For this plant, no long flume or pipe-line is required to develop the necessary head of water, as the Snoqualmie River has at the Falls a vertical drop of 270 feet, giving an available energy of 30,000 to 100,000 horse-power. In this respect the plant resembles those at Niagara Falls. In the placing of the electric machinery, however, there is an essential difference; for, while the Niagara Falls plants have this placed in a building above ground, the Snoqualmie Falls plant has the water-wheels and electrical machinery all installed together in a large underground chamber whose floor is directly above the tail-race tunnel, which extends to the river below the Falls. The force of the water is used to drive impulse wheels on horizontal shafts, instead of turbines on vertical shafts, as at Niagara Falls. Another notable feature of the plant is the use of aluminum wire for the long-distance transmission lines. The entire plant represents an investment of about $\$ 1,000,000$.
208. The great fall of the Snoqualmie River is about 34.5 miles northeast from Tacoma (in a straight line), the same distance southeast from Everett, and 25 miles west from Seattle, being situated in the foothills of the Cascade Range. The river proper commences about three miles above the fall, at the junction of three forks which flow westward down the slopes of the range. Below the fall the river runs almost due north, and finally flows into Puget Sound near the city of Everett. The flow of the river is about 1,000 cubic feet per second at its lowest stage, increasing to over 10,000 cubic feet per second at its flood periods. The river does not freeze during the winter, and there is neither floating ice nor anchor ice to be dealt with.

An investigation showed that by the construction of dams or dikes, some of the large lakes on the watershed could be utilized as impounding reservoirs, so as to ensure a uniform flow sufficient to develop nearly 100,000 horse-power throughout the year, should a demand for so much power eventually be found. It was also determined that by the erection of a 50 -foot dam above the headworks, a reservoir could be formed, having an area of 15 square miles and an average depth of 25 feet. This would almost double the power, should the growth of the industries served make this desirable in the future.


The rock at the Falls is basaltic, with no regular cleavage. It is hard and non-absorbent, and is apparently divided by seams into great ledges. These conditions led to the adoption of the plan of placing the machinery in an underground chamber, as already noted. It was at one time proposed to build a power-house near the base of the fall; but this would have been at a disadvantage, on account of the clouds of spray, which keep everything damp, and which coat all of the surroundings with ice in cold weather.
209. A plan of the headworks above the Falls is shown in Fig. 184, which also shows the position of the underground power chamber and the tail-race tunnel. The intake bay is a rectangular chamber, about 60 feet long (parallel with the river), and 20 feet wide. It has walls, and a center pier of concrete masonry 6 feet thick and 25 feet high, built upon solid rock formation, its floor being on a submerged reef about 5 feet above the river-bed. This bay is protected from the river by a timber grating across the opening, supported by a steel girder construction bearing against the walls and pier. The timbers are 12 by 12 -inch, laid horizontally with 12 -inch spaces between them, through which the water flows into the intake. This grating protects the works from floating trees and logs; while just inside the intake are inclined steel screens made of flat bars on edge, which serve to exclude the smaller debris. The intake thus has two head-bays, separated by the pier.
210. A rudder boom 300 feet long is moored above the intake, and extends beyond it. By turning the capstan at the head of the boom, the rudders are thrown out, and cause the boom to swing out into mid-stream, so that it serves as a fender to deflect floating logs, etc., from the intake. The river is 150 feet wide from the head-bay to the opposite shore, and about 15 feet deep at ordinary stages. The face of the intake was continued 400 feet upstream and 200 feet downstream, in the shape of heavy retaining walls built of sawed cedar timber, tarred. The space behind them is filled with excavated rock, and has a top dressing of soil for a lawn and shrubbery.

At the end of the lower bulkhead, a submerged concrete dam is built across the river, resting on the rock bottom, and this raises the low-water elevation of the river 6 feet at the intake. This dam, whose location is shown in the plan, Fig. 184, is of the form shown
by the elevation and section, Fig. 185. It was first framed of heavy timbers sheeted over with 6 -inch planking, and then filled in solid with concrete. It was built about the time of lowwater flow, portions of the river-bed being laid bare by cofferdams. Preparatory to the construction of the dam, the river-bed was thoroughly cleaned of loose rock, and was roughened by occasional blasts so as to afford a gocd footing. In addition to this, pieces of steel rail were driven in holes drilled 2 feet deep, the rails extending up into the concrete body of the dam. Old railway cables were also embedded in the concrete to perfect the bond. The dam has a batter of 2 on 1 upstream, and $\frac{1}{2}$ on 1 downstream, with a level crest 8 feet wide. At each end of the dam is an abutment pier of concrete 8 feet square, these being 210 feet apart. The d:m was built on natural rock ledge, about 3 feet above the river bottom. It is always submerged from 2 to 10 feet, according to the stage of the river, and varies in height from 3 to 10 feet, and in width on bed rock from 16 to 35 feet, according to the conformation of the river bottom. . The lower bulkhead is only 5 feet higher than the dam, so that flood waters have a very considerable increased sectional area of discharge, as shown by the plan. The capacity of this spillway is such as to insure the complete discharge of an extreme flood without the river backing up to an unusual elevation. The top of the upper builkhead is above flood level.
211. The general arrangement of the plant, with its underground power chamber, is shown in Fig. 186. About 300 feet above the Falls, a shaft 10 by 27 feet was sunk in the bed of the river on the south side descend-


ing 270 feet to the level of the river below the Falls. While this shaft was being excavated, a tunnel 12 feet wide and 24 feet high, with a fall of 2 feet in its entire length, was drifted in from the face of the ledge below the Falls, to an intersection with the bottom of the shaft, a distance of 650 feet. Beginning at the foot of the shaft, and extending over and along the tunnel, a chamber 200 feet in length, 40 feet wide, and 30 feet high, with the floor at the elevation of high water below the Falls, was excavated out of the solid rock (Fig. 187).


Fig. 187. Interior of Power Chamber at Snoqualmie Falls.
This chamber forms the power-house, or machinery room, in which the water-wheels and electric generators have been installed. At average stages of the river, the water is about 12 feet deep in the tunnel; while during flood seasons the tunnel is nearly filled. The tunnel extends under the floor of the chamber, forming a tail-race with concrete roof 5 feet thick, The walls of the chamber have been left rough and whitewashed, while the floor is covered with concrete.

The chamber is ventilated by natural draft through the tail-race and up the shaft, the draft being so strong that it was necessary to curb it. The chamber is said to be cool and perfectly dry, the temperature remaining the same (about $55^{\circ} \mathrm{F}$.) throughout the year.

This low and uniform temperature contributes to the high efficiency of the generators.

Each intake bay contains a massive head-gate, moving vertically, which controls the flow of water through an opening 8 by 12 feet through the shore wall into the penstock. The gate is raised and lowered by mechanism connected with the piston-rod of a hydraulic cylinder. The shaft is 10 by 27 feet, and at the top has three compartments; the two end compartments are for the penstocks; while the center one, enclosed at the top by a steel bulkhead, forms a shaft 8 by 10 feet for the hydraulic elevator and the main cables forming the outgoing conductors, and also for raising and lowering machinery, etc. The steel bulkhead which encloses this center shaft extends from the bottom of the intake bay to the surface of the ground. It is built up of stcel plates, and is stiffened by horizontal frames of I-beams on the outside, riveted to the plates and to each other at the ends. Below it, the elevator shaft is timbered and sheathed with plank. At the surface, this shaft is surmounted by a small building. The penstock already built is steel pipe $7 \frac{1}{2}$ feet in diameter, passing through a concrete roof which keeps the shaft water-tight. The plates are in 8foot courses, and are 1 inch thick for the lower half of the pipes; in the upper half, the thickness decreases from $\frac{7}{8}$ to $\frac{1}{2}$ inch at the top. The joints are heavily riveted, and calked water-tight. At a depth of 250 feet, the penstock reaches the chamber, and connects with a horizontal cylindrical receiver which rests on a rock bench in the north side of the chamber, 12 feet above the floor. This receiver extends almost the full length of the chamber. Its diameter is 10 feet for half its length; and then reduces to 8 feet. It is built up of 1 -inch plates 8 feet wide. The penstock and receiver weigh 225 tons, and the weight of the water column in the penstock is 340 tons. A small independent penstock supplies water to the elevator machinery, as shown in Fig. 186.

At four points in the length of the receiver are 4 -foot branches extending from the side, each branch being fitted with a gate-valve: These valves weigh $23,000 \mathrm{lbs}$. each, and are said to be the largest valves in the world operated under such high pressure.: Each branch has a cast-iron elbow turning downward and opening into the hori-, zontal cylindrical receiver of a water motor. These elbows have an inside diameter of 4 feet, and the metal is 2 inches thick, each casting



[^12]

Side Elevation

weighing $8,000 \mathrm{lbs}$. Owing to the size and form, special care had to be taken to avoid internal strains in the castings which might lead to rupture, especially in view of the high pressure which they have to withstand.
212. The main generating plant consists of four electric generators, each driven by a 45 -inch Doble water-motor of 2,500 horscpower, coupled directly to it. Each motor consists of a shaft carrying six tangential-jet wheels, with two nozzles to each wheel. Each of the four elbows above referred to is bolted to a flanged ring on a horizontal cylindrical receiver, 48 inches in diameter and 20 feet 8 inches long. This is made of two $\frac{1}{2}$-inch steel plates 10 feet wide, and of sufficient length to make the shell with only one longitudinal seam, which is double-riveted. The heads are of dished steel plates. The receiver is supported by six pipes, each of which carries two nozzles delivering jets at right angles to each other, the nozzles entering the side and bottom of the wheel-casing. The use of the receiver effects an even distribution of the flow from the elbow to the several nozzlepipes, and also a steady and uniform rate of flow. From the buckets of the motors, the water falls through draft openings in the floor directly into the tail-race channel.

To handle the volume of water necessary to develop the power in each unit, requires 12 jets 37 . inches in diameter, discharging against six wheels. For convenience of bearing and shaft design, these wheels are divided into two groups of three wheels each, each group being in a separate housing, with a bearing between. This arrangement makes two groups of three vertical nozzle-pieces each.
213. One of the special features of this plant is the needleregulating tips used on the nozzles. They not only throw a perfect and unbroken stream, but give absolute control over the quantity of water applied to the wheels, and therefore over the power output of the unit. As these tips are controlled by the governor, the arrangement gives an excellent degree of speed regulation with variable load, at high efficiency. The full size of the jet is 3 inches; it is a solid, smooth stream, delivered with a head of 253 feet, entircly free from swirling or other disturbance. This form of nozzle maintains the same condition from full-jet size to $1-10$ of the jet area. The regulating nozzles are operated from two long rocker-shafts, one controlling the upper, and the ot'arr controlling the lower nozzle-tips. Both
rocker-shafts are operated by a Lombard governor, which is connected to the rocker-shaft by cranks and connections so arranged with clutches that either or both rocker-shafts can be disconnected from the governor and operated or regulated by the hand-wheel on the pedestal stand. By this governor arrangement, with the regulating nozzle-tips, the wheels use water in proportion to the power developed, so that they operate with high efficiency at part as well as at full load.

The wheels are encased in sheet steel housings with cast-iron fronts, three wheels in each, so that there are two housings to each unit. The housings are made with the upper half removable, to provide access to the wheels when desired. The cast-iron f:ont of each housing is made of such form as to provide a deflector guard, which takes care of the water thrown from the wheels by the centrifugal action, and directs this water into the tail-race, and thus prevents its being driven around the housing by the air currents created by the rapidly revolving wheels. In the top housing is a guarded opening to permit the indraft of air to replace that driven out of the housing and down the tail-race by the rush of the water and the action of the wheels as centrifugal blowers. To prevent water from splashing out where the shaft passes through the side of the housing, the opening is protected with patented centrifugal dises and guardframes. Although this arrangement prevenis the outflow of water, it permits a large and free indraft of air at this point also to replace that driven out by the action of the wheels and the water.

Each wheel unit weighs about $100,000 \mathrm{lbs}$., in addition to the weight of water in the distributing receiver and the nozzles; and in view of the high speed of the parts, and the power developed, careful design and construction were required for the foundations, which are of concrete, built solidly into the floor and one side wall of the chamber; and the lower part of the steel wheel-housing is firmly built into the concrete walls. The waste water drops from the wheels directly into the tail-race.

A tunnel is provided under the governor platform for the lower rocker-shaft and connections that operate the adjustable tips of the lower nozzles, and thus makes this operating gear accessible. The foundation for each unit is divided into two compartments corresponding to the two wheel housings, and a 2 by 3 -feet doorway is
formed in the front wall of each. Four steel rails are built into the concrete across the opening into the tail-race, to support a temporary floor, when it is desired to enter the foundation for the purpose of inspecting the wheels or rozzle-tips without removing the top wheelhousing.
214. Hydraulic Plant at Vauvry, Switzerland.* This installation is situated on the left bank of the river Rhone, a short distance above the point where that stream empties into Lake Leman. The water is taken from Lake Tanay, at an elevation of $4,644.5$ feet, and is delivered to the wheels in the power-plant at an elevation of $1,528.8$ feet, which represents a gross head of $3,115.7$ feet-probably greater than that of any other hydraulic plant in the wor'd. The installation is intended to supply electricity for lighting and power purposes to a large number of Swiss towns and villages in the valley of the Rhone.

Lake Tanay, the source of the water supply, has an area of about 112 acres, and receives the drainage of an area of about 1,875 acres, which, it was estimated, would yield a steady stream of about 12.2 cubic feet per second throughout the year. The main structural features of this plant comprise the supply pipe-line, and the powerhouse, with its machinery. The head of the pipe-line is at an elevation of 4,559 feet, which is 65.6 feet below the normal level of the lake, and 84.2 feet below its maximum level; and it terminates in a vertical shaft about 41.7 feet deep. From the shaft a short gallery or tunnel 984 feet long, with a sectional area of 10.75 square feet, is built on an almost level grade. This tunnel is provided with bulkheads, pipes, valves, and other apparatus for regulating and controlling the supply of water taken from the lake; and at its end the pressure pipe-line begins. For 328 feet, this line is a steel pipe 2.62 feet in diameter; then, for 984 feet, it consists of a masonry tunnel; and finally, for 3,936 feet, it is again a steel pipe, 2.62 feet in diameter. At the end of the last section, the pipe branches into three pipes, each of 1.64 feet diameter. One of these pipes extends to the powerplant; but the others are plugged, and will be built to the powerhouse only when the demand for power necessitates their construction. At the point of junction of the single and triple pipes, the

[^13]head of water is only 689 feet; but from this point on, the descent is very abrupt. At this point, also, there is a retaining valve, and a vertical standpipe (regulator) 1.31 feet in diameter, and 82 feet high, which relieves the water-hammer in the pipe-line above.
215. The steep-grade pipe-line, from the junction point just mentioned to the power-house, is 6,363 feet long, and has a fall of 2,952 feet. As stated, the pipe is 1.64 feet in diameter at the junction point, and it continues with this diameter for $2,682.8$ feet, varying in thickness of shell from about .275 to .45 inch, finally terminating in a Y, each branch of which is provided with a valve. From the branch of the Y, two 1.12 -foot


Fig. 188. Detail of Joint Connections for Pipe-Line of Water-Power Plant at Vauvry, Switzerland. pipes extend for a distance of 4,264 feet, with shells varying in thickness from about .3 to .7 inch: The transverse joints are made as shown at $a$ in Fig. 188; and in order that the joints may be tight, a suitable gasket is inserted before tightening up the bolts. As the pipe-line lies in a trench following the surface grades, it has many bends; and to provide for these, the wedgeshaped pieces shown at $b$ in Fig. 188 are inserted at the joints. The sections of pipe used varied in length from 16.4 feet to 32.8 feet, and weighed from about 1,760 pounds to about 2,500 pounds. Each branch of the pipe-line has at its lower end a slide-valve provided with a by-pass, to permit it to be operated by hand.
216. The power-house is a steel building 45.9 by 216.5 feet in plan. The two pipe-lines described terminate underneath its main floor; and each supplies water to two wheels, all of which are of the impulse type. Two makes of wheels were installed, as shown in Figs. 189 and 190. Each wheel is supplied with water through two nozzles, one above the other, the upper one of which is provided with a device actuated by a governor, for controlling the supply of water. A vertical diaphragm.divides the end of each nozzle into two openings, as shown. The pipe from which the nozzles are supplied rises vertically from the main, and is opened and closed by a hand-valve, above which it divides into two branches, one leading to each nozzle,
and each opened and closed by a hydraulic valve controlled from the operator's platform on the floor above.

Each wheel is mounted on the shaft of an alternator, and operates at 1,000 revolutions per minute. The dynamos are each of 500 horse-power.
217. The Mill Creek No. 3 Power-Plant of the Edison Electric Company, Los Angeles, Cal., which went into service in March, 1903,


Fig. 189. Detail of Duvillard Wheel for Vauvry Water-Power Plant, Switzerland.
is remarkable for the high head uscd. All the water usually flowing in Mill Creek at Akers Narrows, is diverted by a masonry dam, and conducted through 5 miles of pipe to a reservoir 1,960 feet above the powcr-house in Mill Creek Canyon. The conducting pipe slopes 0.2 foot per 100 feet, and is designed to carry 20 cubic feet of water per second. It contains 5 inverted siphons of steel pipe, aggregating 3,585 feet in length, and 25,190 feet of concrete pipe 3 inches thick and 32 inches in inside diameter, and passes through 10 tunnels having an aggregate length of 7,500 feet.

From the reservoir the water descends through a steel pressurcpipe, varying in diameter from 26 to 24 inches, and in thickness from No. 14 B. W. G. to $\frac{7}{8}$ inch. The pipe is protected from rust by a heavy coat of asphaltum, applied by dipping. At the lower end it branches, leading the water through 18 -inch and 14 -inch lap-welded
pipe to the four generating units, which are housed in a concrete building with steel roof-trusses and galvanized-iron roof. Of these generating units, three were made by the Abner Doble Company.
218. Each Doble unit consists of a 1,300 -horse-power Doble


Fig. 190. Detail of Wheels Supplied by the Société de c'onstructions Mécaniques de Vevey, for Vauvry Water-Power Plant, Switzerland. tangential water-wheel and a 750-kilowatt generator, mounted on a single shaft. This shaft has a speed of 430 revolutions per minute, and is mounted in three bearings which rest on a single cast-iron base-frame set in concrete. Each wheel is provided with a Doble needle-regulating and deflectingnozzle, with hand-operated balanced needle. With this apparatus the station attendant can set the needle by hand every half-hour at the most economical point in order to carry the load which from experience he is led to expect during the next halfhour. The governor takes care of all sudden fluctuations of load by deflecting the nozzle momentarily, so that all or part of the water issuing passes under the water-wheel, and wastes its energy against the "Vortex" baffle-plate installed in the tail-race.
219. The static pressure due to the head of 1,960 feet is over 850 pounds per square inch, and the spouting velocity of the jet is about 4 miles per minute. The generating units deliver current at 750 volts to the switchboard, whence it passes through transformers, and out over the 33,000 -volt 86 -mile transmission line to Los Angeles.


BULKHEAD AND INLET PIPES FOR TURBINE PLANT
Conveying water to four pairs of 30 -inch turbines at the power station of the Sacramento Electric Gas \& Railway Company, Folsom, California. The turbines operate under a head of 55 feet, and are direct-connected to generators.

Courtesy of S. Morgan Smith Co., York, Pa.


Fig. 190A. Exterior of Power-House, Showing Penstocks Delivering Water to Wheels under an Effective Head of 1,528 Feet.


Fig. 190B. Interior of Power-Honse.
Views of the De Sabla Power Plant of the Valley Counties Power Company, Butte County, California. This plant holds (1908) the record for long-distance transmission, 378 miles. Among the units installed is a Doble tangential water wheel delivering $8,000 \mathrm{~h}$. p. from a single jet of water, at $400 \mathrm{r} . \mathrm{p}$. m., and direct-connected to a $5,000-\mathrm{kw}$. alternator.
220. The De Sabla Power Plant, in Butte County, California, was erected by the Valley Counties Power Company in 1903, and is now an important source of supply for the California Gas \& Electric Corporation's extensive transmission system.

Water is taken from Butte Creek, through a 12 -mile ditch, and also from a branch of the Feather River, both conduits discharging into a regulating reservoir at the head of the pressure line. Besides taking up variations in load, this reservoir, containing over a day's supply, will permit repairs to be made on the ditch without shutting down the wheels. From this reservoir, two 30 -inch steel pressurepipes over 6,000 feet in length conduct the water down to the powerhouse, the total effective head being 1,528 feet. One pressure line supplies two 2,000 -kilowatt hydro-electric units, and the second line supplies a 5,000 -kilowatt unit. Hydraulically operated piston gate-valves of a special design are installed in the branch pipes leading to the units.
221. Each of the 2,000-kilowatt units consists of an alternator directly driven by a 3,700 -horse-power Doble tangential water-wheel, the speed being 240 revolutions per minute. The 5,000 -kilowatt alternator is direct-connected to an 8,000 -horse-power Doble tangential water-wheel, the speed being 400 revolutions per minute. Regulation of this plant is secured by hydraulic governors, which deflect or raise the nozzles as the load varies.

All three units are of the two-bearing type, the water-wheel being mounted on the extended end of the generator shaft, and overhanging one bearing. Each water-wheel is provided with a Doble needleregulating and deflecting nozzle.
222. The large wheel was the most powerful single water-wheel constructed at the time it was placed in operation, in September, 1904. It delivers 8,000 horse-power from a single jet of water, the jet having a spouting velocity of approximately 20,000 feet per minute. The general design of this unit is shown in Fig. 191. The shaft is 20 inches in diameter in the middle portion and 16 inches in the bearings, the latter being 60 inches long and of ring-oiling and watercooled construction.
223. The transmission voltage is 55,000 volts; and current has been delivered from this plant, over the lines of the California Gas \& Electric Corporation, a distance of 378 miles from the power-house-the present record (1908) for long-distance transmission.

224. Considerable interest was aroused in a new hydroelectric unit to be installed in the De Sabla Plant, the hydraulic end of which was to consist of a 9,000 -horse-power Doble tangential water-wheel driven by a single jet of water, the whole to embody the same general features of design as the 8,000 -horse-power De Sabla wheel.
225. Centerville Power Plant.* The California Gas \& Electric Corporation has recently placed in service in the hydro-electric generating station at Centerville, Cal., a 9,700-horse-power hydraulic reaction turbine, designed to operate under an effective head. of 550 feet (Fig. 192). The station in which this unit is installed is a part of the extensive generating system of that corporation, from which pewer is distributed to practically all of the cities within a radius of upwards of 100 miles from San Francisco. The development is about 200 miles northeast of that city, on Butte Creek. The 14,000 -kilowatt De Sabla hydro-electric station of the system, of which the one at Centerville is also a part, is approximately 8 miles

[^14]upstream from the latter, and has been in service for several years.
226. The flow of Butte Creek at the De Sabla station, although comparatively large, has been increased considerably by the construction, on the watershed above that point, of several reservoirs for impounding a part of the flood waters. The water diseharged from this station is diverted into an open canal about 8 miles long, having


Fig. 192. Section through Centerville Station of the California Gas \& Electric Corporation.
a capacity of 175 cubic feet per second, which leads to the Centerville station. It is so located that a difference in elevation of 591 feet is obtained between the end of the canal and the tail-race at the Centerville station.
227. An overflow concrete dam was erected across the creek at the upper end of the canal, to divert water into an approach to the latter. The flow into this approach is controlled by two large sluicegates; it is 150 feet long, and is built with converging sides, its width being somewhat greater than that of the canal at the upper end. The
bottom of the approach is also built on a sufficient grade to bring it to a level 4 feet below the bottom of the canal at the end of the approach, thus forming a settling basin in which the coarser materials carried by the stream during certain flood conditions are deposited. These materials are readily sluiced out of this pocket, through a gate in one side of the lower end of the approach. In order to facilitate this sluicing, the opposite side of the approach is built on a curve which throws the force of the stream through the gate when the latter is open.

Some of the clay and fine sand are not intercepted in this basin, and a certain amount of sand may be blown, or will fall, into the canal. Provision is made to remove most of these materials from the water by means of sand-boxes built in the bottoms of the flumes which carry the canal over the water-courses that are crossed. These boxes are merely depressions in the bottom of the flume, and are placed at intervals of approximately three-quarters of a mile. Each box is arranged with a sluice-gate in order that the sand and debris that collect in it may be removed easily.

A settling basin, 18 by 50 feet in plan, and 9 feet deep, is placed about 300 feet upstream from the lower end of the canal, to remove finally all the grit that is carried in the water, which enters this basin through a screen of iron bars placed across the canal at an angle of 30 degrees. These bars are spaced 2 inches apart, so that they intercept practically all floating matter. The lower end of the bars is 1 foot above the bottom of the canal, thus permitting sand and gravel to flow along into the basin, which is built by offsetting one side of the canal, and is lined with rubble masonry laid in cement mortar. The bottom drops away from the canal bottom gradually to a depth of 9 feet, the downstream end of the basin being vertical up to the level of the canal. The materials deposited in the basin are flushed out through a sluice-gate placed in the downstream end of its outer side; and the opposite side of the basin is curved toward the gate to induce a current toward the latter, which delivers into a natural water-course.
228. Three steel-pipe pressure lines lead from the end of the canal, down to the power station. The upper ends of these lines open in a reinforced-concrete forebay, 18 by 25 feet in plan; and the connections are made trumpet-shaped, to accelerate the water gradually in order to reduce the entrance losses.

The Centerville station was originally installed to supply power to mines in the vicinity, and at first contained a 400-kilowatt generator, direct-connected to a tangential water-wheel. A 900 -kilowatt generator, direct-connected to a 1,500-horse-power Doble tangential water-wheel was afterward added to the station. The 400 -kilowatt unit was removed at the time the large turbine was recently installed. As a result of the increase in capacity at different times, three pressure lines have been laid to convey the water from the end of the canal to the generating station, all three of which lines were built separately to meet the increases in the capacity of the station. These pipes, one of which is 42 and 36 inches, and the other two each 24 inches in diameter, are 2,565 feet long, and are laid down the side of the mountain, which rises quite abruptly above the station building. They are all of riveted steel, the thickness of the plates varying with the difference in head. The upper end of each of the three pipes is controlled separately by means of a gate-valve at the forebay. The lower ends of the pipes are joined at the rear of the station building by a three-way connection. A gate-valve is placed in each pipe just back of this junction, so that any pipe may be cut out of service without interfering with the other two.
229. Turbine. The turbine is of the radial inward-flow, single axial-discharge, Francis type, with a horizontal shaft direct-connected to a 5,500-kilowatt alternating-current generator revolving at a speed of 400 revolutions per minute. It is joined by means of a 45 -inch gate and taper-piece to the $Y$-casting which is the confluence of the three pipe-lines previously mentioned. The rated capacity is 9,700 horse-power; but the wheel has never exceeded 8,200 horse-power, owing to the limited capacity of the generator. It has a cast-steel spiral casing, made in two parts, and provided with a quarter-turn discharge to the draft-tube, and with a pressure-regulator. The runner is of cast steel, with 20 vanes; 24 pivoted guide-vanes are provided, and are connected to a shifting ring, which is in turn connected to the governor rocker-shaft by lever arms. A thrust bearing takes up the end thrust which arises with sudden changes of load, and is supplied with oil under pressure. A ring-oiling bearing of the ordinary pedestal type on the discharge side of the wheel, supports the other end of the shaft. The pressure-regulator or relief-valve is governor-operated, and is designed to relieve the pipe-line and wheel-casing of excessive
pressure and water-hammer when the vanes are closed. If the guide-vanes are suddenly closed, the pressure-regulator discharge is opened, but has the general tendency to close, and doés so gradually through the agency of a relay valve and dashpot arrangement. If the vanes are closed slowly, the regulator does not operate. The dashpot can be cut out, and the pressure-regulator will then act as a by-pass, being closed when the vanes are open, and vice versầ. Tests were made of the relief-valve at the time the turbine was first put in operation. The machine was running at full load, discharging about 155 second-feet; the vanes were suddenly closed, the reliefvalve opened at the same time, and closed after a period of 30 seconds. The total rise in pressure was 15 pounds above the static, or 28 pounds above the working pressure. With conditions at 1,000 -kilowatt load, the time of closing was 5 seconds, and the rise in pressure above static was 41 pounds, or 42 pounds above the working pressure. The guide-vanes and relief-valve are operated by a "Type N" Lombard governor, which is connected to the rocker-shaft by suitable pinions and segment, the relief-valve being lever-connected to the bell-crank of the shifting-ring lever.
230. From the hydraulic standpoint the striking feature of the Centerville plant is the use of a regular reaction type of turbine for the 5,500 -kilowatt unit at an effective head as great as 550 feet. The ordinary practice under such conditions has been to use impulse wheels of one form or another, on account partly of the speed conditions, and partly because of the difficulty of getting a turbine to stand up under the high velocity of the water, often carrying some sand in suspension. For the development of moderate power at any convenient speed, the impulse wheel is extremely well suited; but as the output rises, there comes a necessity for more and bigger nozzles, met commonly by the use of double or triple nozzles, and often by the addition of another wheel on the same shaft. By the time this is done, it is fairly evident that further increase could well be made by making the entire periphery of the wheel active, and this leads naturally to regular turbine construction, as it has in this case. Given the requirement of large output, the turbine meets the needs of the case admirably. The next question is the endurance of the turbine at such heads. The older turbines of cast iron proved generally inadequate. The Centerville turbine, in all essential
parts, is of steel, the runner being a solid steel casting on a forged steel shaft. Governing is accomplished by shifting the guide-blades, which are forged solid with their pivots and held between steel rings. Especial care has been taken in the hydraulic works, to free the water from grit; and there seems to be no reason why the wheel should not stand up in service quite as well as an ordinary impulse wheel.
231. Plant at Electron. The Puget Sound Power Company's hydro-electric plant (Fig. 193) is located on the Puyallup River, 32 miles from Tacoma and 48 miles from Seattle, Washington. This river has its origin in the glaciers and snow peaks of Mt. Rainier, the highest mountain in the United States; consequently an unfailing source of water is assured from the melting snow and ice.

The water-power scheme consists of the diversion of the Puyallup River, and the conduction of its flow by means of a flume 10 miles long to a reservoir located on a high plateau, and thence by steel pipes to the Pelton wheels, which operate under a head of 865 feet. The flume and reservoir are constructed with a view to the ultimate development of 60,000 horse-power; and the present equipment (1908) consists of four direct-connected Pelton wheels, each driving a 3,500-kilowatt General Electric generator at 225 revolutions per minute; and two Pelton wheels, each direct-connected to a 150 -kilowatt exciter. Each wheel unit has an overload capacity of 7,500 horse-power, making the present output of the station 30,000 horsepower. This power is transmitted to Tacoma and to Seattle, being used for the various industrial enterprises in that section, and particularly for operating the extensive system of suburban electric roads in the vicinity of Seattle.
232. The ultimate installation is to consist of eight units, and the entire equipment is so arranged as to provide for complete pilot control of both water-wheel and electrical apparatus, which is accomplished from the switchboard at one end of the building. The complete equipment of eight units will require a building considerably over 200 feet in length. It was found necessary to reduce the length of each unit to the minimum, and for this and other reasons the Pelton "double-overhung" construction was adopted. This construction consists of one water-wheel overhanging each end of the shaft beyond the bearings, of which there are two for each unit,


placed one on each side of the engine-type generator, which is in the center of the bed-plate; it enables bearings to be more nearly equally loaded than if entire output were obtained from one wheel. Each of the two wheels on each unit is required to develop 3,750 horsepower capacity when operating under an effective head of 865 feet, at 225 revolutions per minute. Each wheel is provided with a 24 -inch single-dise, bronze-mounted gate-valve, with 5 -inch by-pass arranged for operating normally by electric motors from the switchboard, and also provided with gear and worm-wheel for quick and slow motion by hand. .There is one combination needle and deflecting nozzle, with ball-and-socket joint, for each wheel, the weight of the swinging portion being suitably counterbalanced, and the position of the needle controlled by a hand-wheel. The power developed on each wheel is controlled in two different ways-first, by the deflecting portion of the nozzle, which is actuated by an automatic governor, thus limiting the quantity of water impinging on the wheel; and second, by varying the flow of water through the nozzle by means of the needle device above mentioned. Sudden changes of load are taken care of by means of the governor and deflecting nozzle; and consequently there is no variation in velocity of water in the main pipes; hence, no danger of water-ram. The adjustment of the needle to vary the flow through the nozzle is a comparatively slow operation and therefore cannot.injure the pipe-line.
233. The Pike's Peak Hydro-Electric Company, of Colorado Springs, Colorado, has the distinction of operating a water-wheel plant under the highest head available in the United States. In fact, there is but one installation in the world utilizing a higher head, and that for only a small amount of power-namely, the Vaurry plant in Switzerland, already described.

The plant in question is located on the outskirts of the town cf Manitou, Colorado, and consists of three Pelton units, each dircctconnected to a 750 -kilowatt electric generator running at 450 revclutions per minute. The net head on the Pelton wheels is 2,150 feet, equivalent to the enormous pressure of 935 pounds per square inch.
234. The wheels are mounted in the pulley compartment of the generator, and are provided with combination needle and deflecting nozzles operated by hydraulic governors. The gates, nozzles,
and other pressure parts are of cast steel, designed with a large safety factor, and were subjected to a cold-water test of 2,000 pounds per square inch before installing. The wheels proper consist of caststeel dises with gun-metal buckets, fine-ground and machined inside. Each wheel has an overload capacity of 1,500 horse-power.

Current is transmitted to Colorado Springs for power and lighting purposes, and is also largely consumed by the many mines and mills in that vicinity.
235. The Rio das Lazes* hydro-electric station of the Rio de Janeiro Tramway Light \& Power Company, is to have an initial installation of 54,000 horse-power, consisting of six 9,000 horse-power units.

A concrete dam, 115 feet high and 92 feet thick at the base, was built at a series of falls on the Rio das Lazes, 56 miles from Rio de Janeiro, Brazil, to develop a storage reservoir for the station. Two 8 -foot riveted-steel supply lines about 6,000 feet long, lead from this dam to a cross-receiver near the power-house; one 12 -inch and six 36 -inch pipes extend from this receiver to the station, at which the total head varies from 950 to 1,000 feet. These pipes are all of welded steel plates; the thickness of the plates for the 36 -inch pipes varies from 0.4 to 0.7 inch, and that of the 12 -inch pipe, from 0.25 to 0.3 inch, depending on the head. The receiver is connected with a small service reservoir at the same elevation as the main storage reservoir, this small reservoir acting as an equalizer to maintain and regulate the flow in the supply lines.

At the power-house, each 36 -inch pipe-line connects with the penstock of a 9,000 -horse-power vertical-shaft impulse wheel; the 12 -inch pipe connects with the penstocks of two 500 -horse-power horizontal shaft impulse wheels. The main wheels are each direct-connected to a 6,000-horse-power Westinghouse generator, the units being designed to operate at 300 revolutions per minute. The 500 -horse-power wheels are each direct-connected to direct-current generators, which supply excitation current to the main units, and operate at 500 revolutions per minute.
236. Each main wheel has four variable-orifice needle nozzles, through which water is supplied to the runner of the wheel. The operation of these nozzles is controlled by a special type of oil-pres-

[^15]sure governor of the fly-ball type. This governor is geared to the shaft of the unit, and actuates a valve in a pipe supplying oil to a cylinder that directly controls the position of the needle-nozzles. Each wheel is also provided with a relief-valve, which is connected to the governor in such manner that it is opened only when all or a relatively large percentage of the load on the generator suddenly drops off. Dependence for speed and pressure regulation is placed entirely on the governor and the relief-valve, respectively, as the nozzles are not deflected. The whole arrangement is therefore watersaving, since the relief-valve is wasting water for a few seconds only when the load is decreasing rapidly.
237. The wheels of the main units have a guaranteed efficiency of 82 per cent at full load. The governor is guaranteed to prevent a speed variation of more than 10 per cent when the full load is suddenly thrown off, the difference in speed between full load and no load being only 2 per cent. The relief-valves will prevent an increase of more than 3 per cent in the water-pressure in the penstocks.

The runners of the impulse wheels of the exciter units are mounted on an overhang of the shaft of the direct-current generators. Each of these wheels has a single needle-nozzle, controlled by an oilpressure governor similar to those for the main units.

## COST OF WATER=POWER*

238. Water-power is generally, though not necessarily, the cheapest form of power development. The water itself costs nothing, but it cannot be used as nature provides it. The cost of preparing it for proper use in wheels, and of providing for its control, will involve considerable expense, as a rule; and the interest on this expense will correspondingly be the largest charge against power cost, and will, so far as its importance is concerned, be commensurate with the fuel charges in gas, steam, or oil engines. The cost of developing water-power depends almost entirely on the local situation, and cannot be reduced to any formula or rule. In the early days of water-power development-say over 1,000 horse-power-only low heads were utilized, and the wheels were located at the side of the dam, making the total development involve little more than a timber

[^16]and stone dam, with a house at one end. As the demand grew for more power from the same stream, it was necessary to construct a canal for the purpose of bringing the water around the original power-house to some other house. Thus the expense began to increase materially, from the difficulty of bringing the greatest quantity of water to the wheels under the largest heads by long pipe-lines, or combinations of pipe-lines, canals, flumes, tunnels, and vertical shafts. .The simpler development complete cost about $\$ 40.00$ per horse-power, without electrical equipment. This has now increased so that a minimum of $\$ 75.00$ per kilowatt is considered a very good proposition; while a maximum of $\$ 200.00$ per kilowatt is not by any means prohibitive, including electrical equipment. The mean is somewhere near $\$ 100$ for very large installations favorably placed, like those at Niagara Falls. Taking these two limits, and a gross interest charge of 5 per cent, with an average depreciation of 4 per cent, and with insurance and taxes at 1 per cent, there is a total charge of 10 per cent on the first-cost, or $\$ 7.50$ per kilowatt-year, as a low limit, and $\$ 20$ per kilowatt-year as a high limit. The operating expenses, labor, oil, waste, repairs, etc., may be expected to cost from $\$ 1$ to $\$ 5$ per kilowatt-year, which places the cost of electric power at the bus-bars between the rare low limit of $\$ 8.50$ per kilo-watt-year, and the high limit of $\$ 25$ per kilowatt-year for full load, 24 -hour power, for these rates of charging expenses.

Besides the increase in development cost, there is another item that sometimes enters; and that is an increase in land expense or l.nd damage, which may be large in settled communities. The increase in cost is not all due to increase in cost of machinery; this has probably decreased, not only from better methods of manufacture and more competition, but also from the use of higher heads and better wheels and electric generators.
-239. Just how much can be paid profitably for the development of water-power, either now or in the future, will be measured solely by the power cost of that one of the competing systems-steam, gas or oil-most available in the same locality, or, if not in the same locality, at some point within the limits of electrical transmission. To the cost of generating water-power, must be added transmission cost, involving fixed charges on lines, transformers, switchboard, and other equipment, together with their maintenance and operating
charges. All of these together may add to first cost $\$ 30$ per kilowatt, and increase the power cost for a 150 -mile transmission $\$ 5$ per kilo-watt-year.

After analyzing the details of power cost for oil, gas, and steam development, Professor Lucke continues:
240. These costs may now be summarized for comparison, as follows:

## COMPARISON OF POWER COSTS

Conditions Assumed: Stations consisting of six units, two in reserve, and four working on 24 hours rated load, with the exception of the water-power. First cost and fixed charges are based on the capacity of 150 per cent of the output.

|  | Water Power | - Oil Enaines | Gas Engine and Producer | Steam Plant |
| :---: | :---: | :---: | :---: | :---: |
| First cost per k.w.-rating... | \$75.00-\$200.00 | $\begin{gathered} \text { 160-k.w. units } \\ \$ 217.00 \end{gathered}$ | $\begin{gathered} \text { 600-k.w. units } \\ \$ 270.00 \end{gathered}$ | 5,000-k.w. units \$110.00-\$150.00 |
| Fixed charges, rate per cent . | 10 per cent | 10 per cent | 10 per cent | 10 per cent |
| Fixed charges, per k.w.-year . . | \$7.50-\$20.00 | \$21.70 | \$27.00 | \$16.50-\$22.50 |
| Operating and mfg. costs per k.w.-year ........ | \$1.00-\$5.00 | \$56.94 | \$38.54 | \$52.50 |
| Total power costs per k.w.year | \$8.50-\$25.00 | \$78.64 | \$65.54 | \$69.00-\$75.00 |

From these figures it appears that we have not yet reached the limit of cost of development of water-powers which may be advisable. It apparently would pay to spend even more money than $\$ 200$, the present maximum per kilowatt for water-power development, if there were no other considerations entering. Among the chief considerations of this kind, may be set down that of transportation of products from the works, and of raw material to the works; but this must be considered against the question of transmission of current from the waterfall to a convenient point of transportation.
241. Load Factor; Peak Load. The discussion and the figures presented above refer to a 24 -hour continuous full load, a condition which frequently is practically fulfilled in the case of electro-chemical works, but rarely in other branches of industry. The term load factor may be defined as the ratio of the actual power output of an installation during a 24 -hour period of operation to its 24-hour
capacity at full load; or, in the case of the consumer, it may be defined as the ratio of average to maximum load. The term, however, is occasionally employed with a somewhat different significance.

The peak load represents the maximum output, corresponding to the maximum demand for power from all the industries served at any one time during a 24 -hour period of operation; or, in the case of the consumer, it represents the maximum use of power during that period. So short a time as a one- or two-minute interval may be used in estimating peak loads.

The value of the load factor varies between very wide limits in the different industries, depending on their nature and peculiarities. Thus, from the Census report of the year 1902, the average load factor of all United States lighting systems was akout 23 per cent, while the street-railways had a load factor of about 30 per cent, and an average of all electric light and railway systems amounted to about 26 per cent. From these low values the load factor varies, for the different industries or combinations of industries, up to nearly 100 per cent, as stated above.

It is evident that in the case of an installation furnishing power to an industry using that power fairly uniformly, the load factor will be higher than under the reverse conditions. . It is also clear that if the power is furnished to a combination of different industries so selected as to power demand that the times of high requirement in some occur simultancously with the times of low requirement in others, the result of this correspondence will be a more or less uniform demand for power from the power company, and the load factor will be high.

The general effect of the load factor on the cost of power production is evident, for the plant must be designed and installed to meet peak load conditions, even though these conditions obtain for short periods only during each 24 hours. Accordingly, during the balance of the time, part of the plant must lie idle or work at less than full load, whereas certain fixed charges on the total investment, such as interest, taxes, depreciation, and insurance, continue uniformly. Unless the entire plant representing the investment is put to its normal maximum use every hour in the day and every day in the year, it is not working at its theoretic maximum
efficiency, and the cost of power production will be correspondingly increased.

To carry over the peak load, the motor units may be designed with sufficient overload capacity; storage batteries of sufficient capacity may be installed, to take the excess over the average load; auxiliary steam or gas units may be employed, or one or more main units may be held in reserve, for the same purpose.
242. Cost to Consumers.* "Since the day when the first commercial electric light entered the field of artificial illuminants, there have been endeavors to find an equitable way of charging for energy supplied in the form of electricity. At first, in the absence of any measuring instruments, the flat rate was the only method. This was soon found to be impracticable for most cases, and the ampèrehour meter, followed by various types of integrating and recording wattmeters, soon brought into use the idea of paying for the exact amount of energy used, at a given price per ampère-hour or per kilowatt-hour. This method is still in very general use in its simplest form, but there has been dissatisfaction with it from the time it started. The fact of the matter is that ne ther the straight flat rate nor the straight kilowatt-hour rate is equitable, except when applied in connection with a definite load factor; and even then it may not be entirely so, due to uncertainty as to the number of hours per day that full-load conditions prevail, with corresponding high efficiency, and to the hours during which operation continues at light loads, with resultant low efficiency.
"It is fully recognized now, however, that the load factor is the root of the trouble; and unless a system of charging gives due consideration to it, there will always be inequality of rates and dissatisfaction on the part of the power company or of its customers, or of both. This has been shown in all classes of service -incandescent and arc lighting, heating and power purposes, including railway lines-and in power companies and consumers of all sizes."

The ratio of the use actually obtained to the theoretical or possible maximum use, is the load factor of the manufacturing establishment and of the railway line, just as it is of the powerhouse or of the transmission system.

[^17]243. "The effect of load factor on cost of power is thoroughly understood where steam plants are concerned; but it might be supposed, in the case of hydraulic power, where no furnaces have to be banked, and inefficiency at light loads becomes unimportant, that the conditions would be different. Hydraulic turbines of modern design, however, usually have such characteristics that their overload capacity is very slight; and it therefore becomes necessary, if peak loads are to be handled, to provide extra machinery to take care of these. With no provision for peaks, it is still necessary to hold at least one generating unit in reserve, and a margin of capacity must be left unused in the operating turbines for gate travel in regulation, and to allow for partial clogging of distributors by refuse which accidentally enters the penstocks. As the water is available and costs no more if used to the full capacity of the plant, it is plain that the power-selling company will strive vigorously for a uniform load as high as is practicable for the installed machinery to carry. This results in making peaks a prohibitive element to power deals where the hydraulic plant has been some time in the field and has been able to discriminate in the choice of its customers. The plants now operating at Niagara Falls have been particularly fortunate in this respect, one of the oldest having a 24 -hour loadline of about 26,000 horse-power, and fluctuations not exceeding 5 per cent of the average load. Needless to say, the portion of this power supplied for railway and lighting purposes is very small. The Niagara conditions are unique on account of the electro-chemical plants, which provide an ideal load and consume the greatest part of the power now developed.
"The typical street-railway load necessarily has prominent peaks; and if these cannot be smoothed down by adjustments of service, it is still possible, where a fair price is asked for the water power, to carry the heaviest part of the all-day load by means of this, and the remainder by steam engines, gas engines, or storage batteries, or combinations of engines and batteries.
244. "The point is frequently raised that power companies undertaking to supply customers of any sort should be equipped to take care of all requirements of these customers, including peak loads. This is done in some cases, the power companies going so far as to provide steam plants for reserves and peak purposes. The
character of local demands for power will usually determine this matter; and if the power companies eventually install auxiliary steam plants, it will be only because they are forced to it by periodic shortages of water, or inability to obtain customers whose aggregate use of power results in a high yearly load factor. The power company wants to sell all of its power all of the time; and in a thriving, progressive community it is probable that it finally will come very near doing this. The load factor will improve as customers increase in number; and as the load approaches the full capacity of the plant, the power company will become more discriminating about closing new contracts, or renewing old ones, that involve conditions tending toward poor load factor.
"If it is possible to make contracts for full power requirements, it is usual for power companies to place some penalty rate on the peak power, or to arrange the terms of charge so that there are distinct advantages to the purchaser in keeping the load line as nearly straight as possible. The most common method is to sell a solid block of 'firm' power which can be used at a load factor of 70 per cent to 80 per cent, or better, charging the minimum flat rate for this, and providing power above the firm amount on a kilowatt-hour basis, at rates gradually increasing with the height of the peaks. Sometimes provision is made for charging extreme rates for possible peaks of such height that the railway company has no expectation of ever reaching them. These clauses should be avoided, if possible, as the unexpected is constantly happening in the operation and growth of a railroad. Where measurement of peaks is dealt with at all, it should be specified that they are not to be counted unless they continue for two minutes or longer. (In some cases one minute is specified.) Uncontrollable occurrences, such as the partial grounding of a feeder, or the performance of a defective car, may produce peaks of short duration which are of small consequence to the power company, but might be very costly to the railway company under an unreasonable power agreement.
"The purchaser should be allowed, without charge, swings of about 10 per cent (or 5 per cent) above the firm line of purchased power, provided the kilowatt-hours used above the line do not exceed those unused below it; since it is impossible to always carry the
load directly on the limiting line, even with the aid of batteries and the most approved regulating devices."
245. A fair method of charging for power is on a sliding rate depending on the monthly load factor. The maximum two-minute (or one-minute) peaks are recorded in kilowatts each day, and averaged for the month, giving the average maximum demand for the month. The total number of kilowatt-hours used during the month, divided by the number of hours in a month, gives the average hourly rate of consumption for the month. Then the monthly load factor is obtained by dividing this average by the maximum demand; and this factor is used as follows in determining the charge for the month:
"Assuming that a manufacturer has made a contract to buy 400 horse-power for the operation of his factory, and that the rate per horse-power-year varies between the limits of $\$ 16$ and $\$ 43$, depending on the load factor, the determination of his rate per horse-power per year for any given month would be as follows:
"If the kilowatt-hours consumed during a thirty-day month are 43,200 , then the average demand for power is 43,200 divided by 720 (the number of hours in the month), equal to 60 kilowatts or 80 horse-power. Assuming further that his maximum demand each day was just 400 horse-power, then, of course, his average maximum demand for the month will be the same amount, and the load factor is 80 divided by $400=.2$, or, as commonly expressed, 20 per cent. If the rate per horse-power-year varies between $\$ 16$ and $\$ 43$, it will be evident that the variable quantity is their difference, or $\$ 27$. The rate is therefore equal to the minimum rate $(\$ 16)$, plus the load factor (.2) multiplied by the variable ( $\$ 27$ ). In the present case, this will amount to $16+.2 \times 27=\$ 21.40$. The total charge for the month would therefore be $\$ 21.40 \times 400 \div 12=$ $\$ 713.33$, which is equivalent to 1.65 cents per kilowatt-hour, or $\$ 107.00$ per horse-power-year,* for power actually used.
"If the monthly load factor had been 30 per cent (.3) instead of 20 per cent (.2), the rate per horse-power per year would have increased to $\$ 24.10$; but the equivalent cost per kilowatt-hour would have decreased to 1.24 cents, a reduction of almost 25 per cent in cost per kilowatt-hour due to increasing the load factor to 30 per cent.

[^18]"This may readily be put in the form of an equation which, if the desired rate per horse-power per year is $R$, the minimum rate limit is $A$, the maximum rate limit is $B$, and the load factor is $L$, is expressed by:
$$
R=A+L(B-A)
$$
"This method is much more equitable than that sometimes used, of selling all the power on a kilowatt-hour basis with a guarantee from the consumer of a specified load factor." With this system of


Fig. 194. Curve Sheet Showing Load Curves of One of the Niagara Companies.
charging, the method of establishing equitably the limiting values for horse-power per year. (corresponding to $\$ 16$ and $\$ 43$, in the example above) will require very careful consideration.
246. "If power companies cannot entertain peak propositions at all, or if they place prohibitive rates thereon, the purchaser must then provide the steam plant, or storage battery, or both, to care for a part of the load. Railway systems supplied with purchased hydro-electric power afford ideal opportunities for the application of storage batteries. The batteries can be charged at night with power that otherwise could not be used; and the discharge of the load peak provides power at an extremely low load factor which costs only the fixed charges, operation, and maintenance of the battery.
"Very careful consideration must be given to proportioning the
division of load between water power and steam power. The cost of hydro-electric power at 100 per cent load factor should be somewhere in the neighborhood of one-third the cost of steam-generated


Fig. 195. Curve Sheet Showing Railway Load Line Represented as $C \cdot C$ on Curve Sheet, Fig. 194.
power at 100 per cent load factor, assuming reasonable first cost of plant and moderate distance of transmission in the first case, and average cost of coal and labor in the second. Obviously the bulk of the load should be carried by the purchased power; but the higher the limiting firm line of this power is raised, the lower will
the load factors of both steam power and purchased power become, and the cost per kilowatt-hour of each will increase. In each case, however, there is a certain critical point to which the firm purchased power line may be raised before the total cost (which is of prime importance) of combined purchased power and steam power will commence to increase. In raising the firm line of purchased power to this point, the total cost will be decreasing."
247. Curve sheet, Fig. 194, shows at $A A$ the remarkably straight local load line of one of the Niagara power companies.


Fig. 196. Method of Plotting Costs per Horse-Power per Year in Terms of Load Factor, and Rate per Horse-Power per Year.

At $B \quad B$ is shown the total load line, including the long-distance load, of the same company. $C C$ shows a railway load, the shaded portion of which is carried by the railway company's steam engines and storage batteries. The unevenness of the power company's total load is not contributed to by the railway company, except to the extent of a dip during the early morning hours. The peaks of the railway load would, if included in the power company total load, distort it considerably, in an undesirable way. The curves are all plotted from the same base line, and represent the same day."

Curve sheet, Fig. 195, shows on a more open vertical scale the same railway load that is represented at $C C$ in Fig. 194. The firm line of purchased power is here located lower, with reference to the
total load, than has been described as the economical point. This is partly for the reason that the chart represents a winter day (the heavy load season of the year). The total load drops below the firm purchased power line during the middle of the day at some seasons, and, as the firm line cannot be shifted back and forth, there are necessarily times


Fig. 19\%. Rates of Fig. 196 Reduced to Cost per Horse-Power per Year, of Power Used. when the proportions of purchased power and steam are not the most economical, as in the instance of this particular day."

Fig. 196 gives the rates per horse-power in terms of load factor. "The curve marked 'Hydro-electric' is intended to represent rate of power cost after transmission for some distance, while that marked 'Steam Plant' represents rate at the powerhouse switchboard. These curves are about the best obtainable from any powerhouses of 5,000 to 10,000 horse-power capacity, with coal from $\$ 2.50$ to $\$ 2.75$ per ton."

Fig. 197 shows the same rates as represented in Fig. 196, reduced to cost per horse-power per year, of power actually used; while Fig. 198 gives these same costs in terms of kilowatt-hours, of power actually used.

Since there are approximately 8,640 hours in a year, and since one horse-power is approximately equal to $\frac{3}{4}$ of one kilowatt, the values in Figs. 197 and 198 are mutually convertible by means of the conversion factor $\frac{3}{4}$ of $8,640=6,480$. Or Fig. 198 may be constructed directly from the values taken from Fig. 196, by means of the conversion diagram, Fig. 199. Fig. 200 is interesting in that it shows in a general way how the fixed charges and operating charges, and therefore the


Fig. 198. Rates of Fig. 196 Reduced to Cost per Kilowatt-Hour, of Power Used.


Fig. 199. Method of Changing Rate per Horse-Power per Year at Various Load Factors, to Equivalent Cost per Kilowatt-Hour, and Vice-Versa.
total cost, vary with the load factor. While the per cent of the total cost chargeable to operation increases with the load factor, the actual cost of operation per kilowatt-year decreases.
248. General.* "Whenever the development of a water-power for the purpose of selling water or mechanical or electrical energy is under consideration, the most important question to be decided is: What is the limit of cost per horse-power that may be expended for a developmentandstill leave the plant a financial success; or what is a reasonable price to be charged per horsepower per year?
"Agreatamount of data has been published in regard to the cost of hydraulic power and power-plants; but, as waterpowers present an infinite variety of conditions, such prices of other plants should be used only with the greatest precaution. A few general figures, intended to apply to conditions at present prevailing in the northern part of the United States and in Canada, may be given here.
249. "A water-power electric plant, including transmission line and substation, where such are required, but without the local dist:ibution, should not cost more than $\$ 100$ per electrical horse-

[^19]power, if situated in a remote location or in a farming district; but $\$ 150$ to $\$ 200$ may be expended per electrical horse-power for power-plant, transmission, and substation, if the power can be sold in a large city or industrial district.
"The price charged for power-water per gross horse-power per year, delivered at or near the customer's turbines, may be taken at from $\$ 5$ to $\$ 15$, the lower figure being for remote locations, low heads, and large powers, and the higher figure for the reverse conditions. The price of $\$ 15$ to $\$ 25$ per mechanical horse-power per year at the power-house, or of $\$ 25$ to $\$ 50$ per electrical horse-power per year delivered to the customer, may be taken as the limits paid at present. Here, again, the lower rate is for remote locations and large powers, and the higher for the reverse conditions.
"It is also safe to state that in a climate such as that of the northern part of the United States, and in Canada, with the long and severe winters, it does not pay to develop a water-power if the power produced will cost more than 75 per cent of the amount for which steam power could be produced in the same locality.
"In Canada, with the great number of water-powers yet undeveloped or only partly utilized, it must be regarded as poor policy to install a larger plant than can be run at all stages of the water, or to have a great proportion of power dependent upon storage lakes during the months of low water.
"A water-power requiring an auxiliary steam plant during the low-water season, can only pay if either the cost of the development. is exceptionally low, or the locality very favorable for the sale of power, or both."

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[^0]:    * One cubic foot of water weighs 62.5 lbs . and contains 7.5 gallons (approximately).

[^1]:    *Church, "Hydraulic Motors."

[^2]:    "American turbines are mostly of a complex nature, as regards the action of the water on the buckets of the wheels, and have been perfected in efficiency by test, or, as it is irreverently called, by the 'cut and try' method of procedure. A wheel would be built on the inspiration of the inventor, then tested in a testing flume, changed in a certain part, and retested, until no further change in that particular could effect an improvement. Another part would then undergo the same process of reaching perfection; and thus, in course of time, the whole wheel would be brought up to the desired high standard of efficiency."
    "Another consequence of the reduction of the diameter is that the inner ends of the buckets, which closely approach the center of the turbine, are located on a very small circle, which limits their number and gives them a very close spacing, while the spacing on the

[^3]:    *Thurso, "Modern Turbine Practice."

[^4]:    *Thurso, "Modern Turbine Practice."
    $\dagger$ Articles 106 to 121 inclusive have been taken, with slight changes, from Prcf. Mansfield Merriman's "Treatise on Hydraulics."

[^5]:    *Articles 122 to 130 inclusive have been taken, with slight changes, from Professor Church's "Hydraulic Motors."

[^6]:    * The types illustrated in Figs. 90-96 are manufactured by the James Leffel \& Company, Springfield, O., who also manufacture the Niagara type of Samson turbines.

[^7]:    *Frìzell," Water-Power."

[^8]:    * Thurso, "Modern Turbine Practice."

[^9]:    *Articles 183 to 187 adapted from Thurso's "Modern Turbine Practice."

[^10]:    * Proceedings of the Institute of Mechanical Engineers, Feb., 1906. Also Engineering Record, Jan., 1900; Nov., 1901; Nov. and Dec., 1903; Feb. and Oct., 1904; Apr., 1905.

[^11]:    * Engineering News, December, 1900.

[^12]:    Plan

[^13]:    * Engineerin J New $\%$, November, 1902.

[^14]:    * Engineering Record, March and April, 1908; Engineering News, March, 1908.

[^15]:    *Engineering Record, October, 1907.

[^16]:    * Articles 238 to 240 are abstracted from a paper by Prof. Charles E. Lucke, of Columbia University, New York, on "Power Costs."

[^17]:    * Street Railway Journal, June 30 and July 7, 1906.

[^18]:    * Cost per kilowatt-hour $=$ Cost per horse-power-year $\div 6,480$.

[^19]:    *Thurso, "Modern Turbine Practice:"

