BULLETIN No. 2

OF THE

Department of Mechanical Engineering,

UNIVERSITY OF CALIFORNIA.

HYDRAULIC STEP.

by
F. G. Hesse

BERKELEY:
1887.

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Hesse, Friedrich Godfrey

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gift

Analysis of a Hydraulic Step

The machinery which was used in connection with a certain pumping system for irrigation contained a vertical shaft which supported a heavy load and rotated in water at a high velocity. The whole plant was required to be of such construction as to dispense with the necessity of supervision.

Under these conditions the footstep of the shaft claimed careful consideration. Lubrication, except with water, was out of the question. The use of lignum vitæ was also of doubtful utility on account of the grit, of which the water carried considerable quantities. Girard's method of forcing water between the surfaces of contact by means of a force-pump* was too complicated and lacked permanency.

In this dilemma it occurred to me that a permanent water pressure might be established through the agency of a forced vortex, created by the rotation of the shaft itself, which, in acting against a disc attached to the shaft, could be made to balance a portion of or the whole load.

Fig. 1 is a sectional view of the footstep, through the axis of shaft, S. It represents a cylindrical vessel, composed of a number of identical compartments m, m, etc. (the drawing shows three). The concentric shaft S, passes freely through these and rests on an ordinary footstep, which is attached to the bottom of the lowest compartment. The shaft carries for each division a disc of a diameter just sufficient to clear the surrounding face of the cylinder, and midway between top and bottom. The upper part of each disc is provided with radial ribs or blades r, r, and the bottom of each vessel is provided with stationary ribs r, r.

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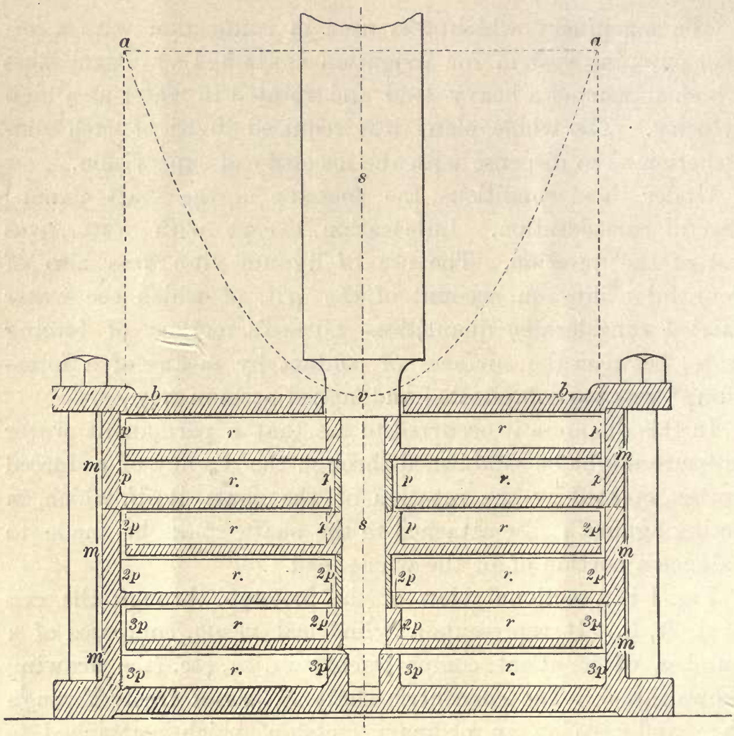
*See Armengaud, "Vignole des Mecaniciens," p. 13, and Note "Sure les Experiences de," etc., in *Comptes Rendues* de l'Academie des Sciences a Paris, T. 55.

For our purpose, and, as will be seen also in common practice, it is sufficient to consider only one compartment. If the shaft rotates and the vessel is filled with water, the wings, r, will cause the latter to rotate with the same angular velocity. A forced vortex is created, and the pressure of the water will increase from the center towards the circumference. The water in the space below the rotating disc will be under a

Fig. 1



uniform pressure, equal to the maximum pressure above; hence the disc, and with it the shaft, will be under the influence of an upward pressure, equal to the difference of the total pressures acting upon both sides of the disc.

Let ω denote the angular velocity of rotation;

r, the radius of the rotating disc in feet;

r_o, the radius of the hub in feet;

n, the number of revolutions per minute;

v, velocity of rotation at any given distance;

v_o, the velocity of rotation at r_o;
g, the acceleration of gravity;
γ, the density of the fluid;
m, the number of rotating discs;
P_1, the total pressure upon the upper face of the disc;
P_2, the total pressure upon the lower face of the disc;
$P = P_2 - P_1$, the resulting upward pressure.

Assuming r_0 to be zero, and neglecting fluid friction, we may obtain the pressure P as follows:

The hight to which the fluid would rise at a distance x, from the axis, measured from the plane bVb, is equal to $\frac{v^2}{2g} = \frac{\omega^2 x^2}{2g}$. The ordinates representing these heads terminate, therefore, in the surface of a paraboloid of revolution, with its vertex at V, and its volume aVa, is equal to one-half that of the cylinder $baab$, hence $P_1 = \gamma\ \pi\ r^4 \frac{\omega^2}{4g}$, and considering that the maximum specific pressure, $\gamma\, r^2 \frac{\omega^2}{2g}$ acts upon the entire lower face of the disc, we have $P_2 = \gamma\ \pi\ r^4 \frac{\omega^2}{2g}$, hence

$$P = \gamma\ \pi\ r^4 \frac{\omega^2}{2g}.$$

For any other value of r_0 we find

$$P_1 = \gamma\ \pi\ (r^2 - r_0^2)^2 \frac{\omega^2}{4g} \text{ (see note below)},$$

and since the maximum specific pressure acts upon the entire area of the disc, we have:

$$P_2 = \gamma\, \pi\ (r^2 - r_0^2)\ r^2 \frac{\omega^2}{2g}, \text{ and}$$

$$P = \gamma\, \pi\ (r^4 - r_0^4) \frac{\omega^2}{4g}.$$

If we express ω in terms of n and r, r_0, in terms of d, d_0 (diameters), we find:

$P = 0.00104\ (d^4 - d_0^4)\ n^2$, and for m rotating discs
$P = 0.00104\ (d^4 - d_0^4)\ n^2\ m.$

NOTE.—The specific pressure p, at distance x, is equal to $\gamma\ (x^2 - r_0^2)\ \frac{\omega^2}{2g}$.

hence $P_1 = 2\ \pi\ \gamma \frac{\omega^2}{2g} \int_0^r (x^2 - r_0^2)\ x\ dx = \pi\ \gamma\ (r^2 - r_0^2)^2\ \frac{\omega^2}{4g}$.

The work in the laboratory has to deal with two subjects of investigation, viz.: Pressure, and work consumed by frictional resistance of the disc rotating in water.

The value for pressure, found by the above formula, gives the same result as that found in practice, provided fluid friction does not exist; its presence will not only decrease the maximum pressure, $\gamma\, r^2 \frac{\omega^2}{2g}$, but also the total pressure under the disc, which is now no longer the resultant of a uniform maximum pressure under the disc. The cause of this is to be found in the clearance or space between the movable and stationary ribs and surfaces.

The rotating ribs do not impart the same motion to all the water above the disc. There remains a film of fluid next to the upper stationary surface, either wholly stationary, or if slippage takes place, partially so. Within the free space, or clearance, inner friction causes the water to assume gradually, according to some law, the full velocity of rotation. The pressures due to the normal forces being thus modified, radial currents towards the axis of rotation are induced, while a compensating flow from the axis, in conformity to the law of continuity, takes place between the rotating ribs. In other words, potential energy is converted into kinetic energy. The same action, but with currents reversed, takes place in the compartment below the disc.

The problem mapped out in the foregoing was given to the students as laboratory work. It calls for mechanical appliances, instruments of precision for measurement, etc., of varied character; it appeals to their knowledge acquired in the lecture-room to reach correct conclusions, and the results are not merely of temporary value but find important applications.

The students made the experiments and assisted in the designing and drawing of the apparatus, all of which was built, with the assistance of the students, by Jos. A. Sladky, Superintendent of the Machine Shops. His skill and ready comprehension of the cardinal points aimed at, have largely contributed to the successful results of the experiments.

Experiments for Pressure.

The footstep constructed to suit experimental convenience is shown in Fig. 2^b, which represents a section through the axis of the shaft. The inner diameter of the cylinder is $12\frac{1}{4}$ inches in order to admit a 12-inch disc; but discs of less diameter can be inserted together with corresponding rings V, and stationary ribs R', R'.

The shaft S', passes through another shaft, S, which has its bearings in the hub F, of the frame, and on which are mounted the driving pulleys P, P. Rotation is communicated to S' by means of a couple C. In this way the tension of the belt is entirely taken up by the shaft S, and its bearings F, and since the resisting forces upon the disc occur only in couples, the shaft S', is left free to slide in its bearings without friction.

The upper end of the shaft S', engages one end of the balance lever L, through the medium of the socket or step D, which is prevented from rotating, as seen in the figure, in order to protect the pointed pivot which acts upon L. The other end of lever L, is supported by the platform of a scale C, Fig. 2, which measures the pressure upon the disc.

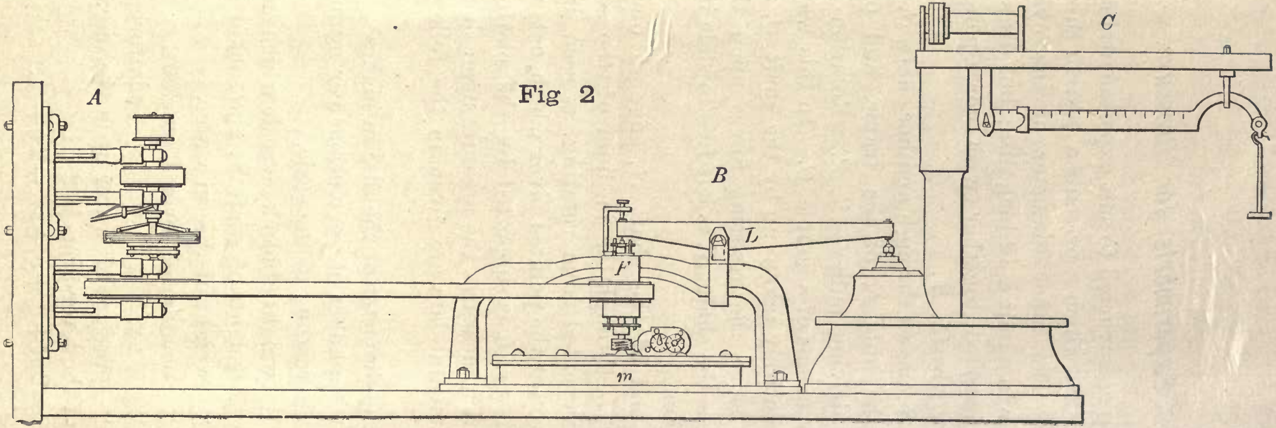
The experimental data are given in the following table, in which

d denotes the diameter of the disc in inches,
n " " number of revolutions per minute,
R " " reading of the scale,
g " " weight of block on scale which engages lever L.
g_1 " " weights of shaft S', and socket D,
g_2 " " weight of disc in water,
m " " reaction of lever L on shaft,
m_1 " " " " " " platform,
P " " pressure of water upon the disc.

The ratio of the lever-arms is 2 : 1.

We have $P = 2R + g_2 + g_1 + m_1 - 2g - m$.

Fig 2



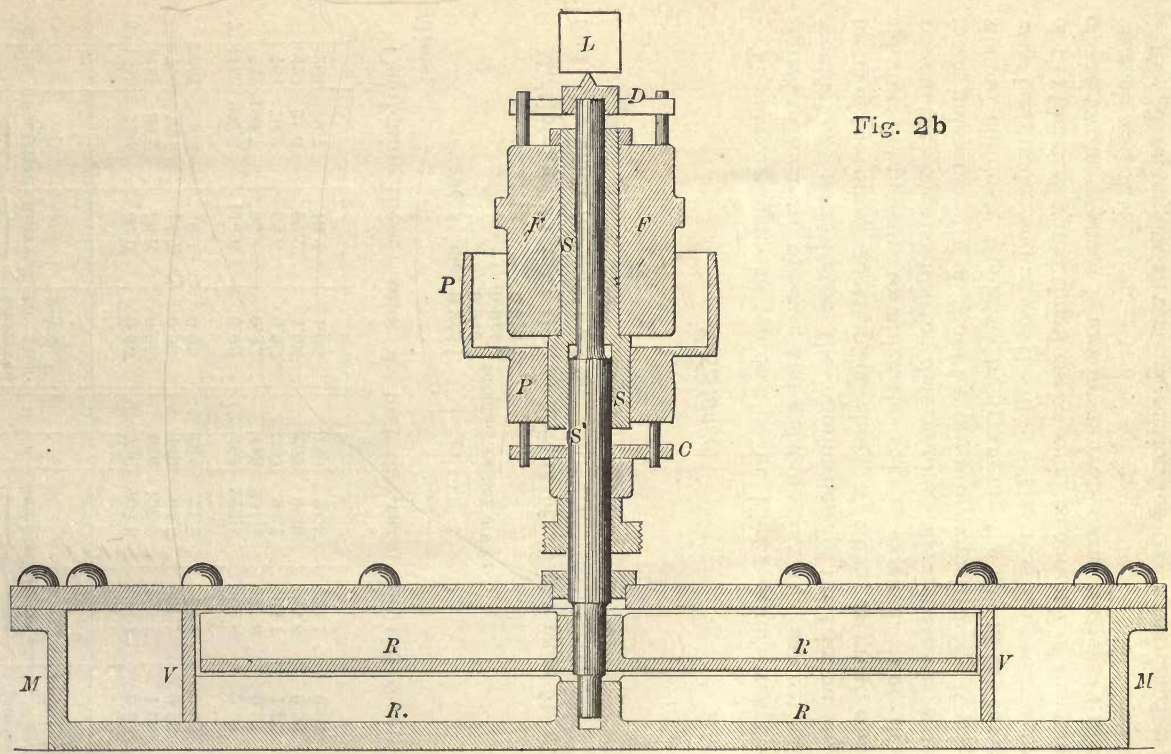


Fig. 2b

d	n	P Computed	P Observed.	Ratio of Observed to Computed P.	d	n	P Computed	P Observed.	Ratio of Observed to Computed P.
5″	1510	71.5	64.9	0 90	9″	786	205 6	186.1	0.91
	1128	38 2	36.5	0.96		626	130.2	130 1	1.00
	710	15.8	14.7	0.93		440	64.3	61.7	0 96
	557	9.8	9.1	0.93		254	21.2	20.1	0.95
7″	1155	160.5	148.6	0.93	12″	462	222.8	220.8	0.98
	932	105.7	96.1	0.92		443	204.1	197.8	0.97
	764	70.2	66 8	0.95		300	94.7	95.1	1.00
	424	21 6	22.2	1.03		252	68.0	65.6	0.96
	284	9.7	10.2	1.05		186	21.3	20.1	0.94

Combining these results, we find the ratio for the several discs:

$$\left.\begin{array}{lll} \text{For } 5'' & \text{disc} & 0.926 \\ 7 & \text{``} & 0.940 \\ 9 & \text{``} & 0.944 \\ 12 & \text{``} & 0\ 973 \end{array}\right\} \text{adopted value, 0.95.}$$

Multiplying .95 into the coefficient of P (see page 5), we find: $P=0.001\ (d^4-d_0^4)\ n^2\ m.$

Resistance.

The arrangement as represented in Fig. 2, which was designed to measure pressure and resistance at the same time, would have necessitated the elimination of the friction moment of the upper footstep of shaft S', which is great, considering the pressure to which the step is exposed. For this reason the experiments for determining the resistance were made independently of those for pressure, which made it possible to eliminate the latter by arranging the ribs in the same way both under and above the disc. The resulting resistance of the footstep would therefore be equal to one-half of the sum of the two resistances found for movable and stationary ribs on each side of the disc.

Fig. 3 represents a vertical section of the dynamometer, which was designed and built for these tests.

Fig. 4 is a front view and Fig. 4[a] a top view of the index plate.

The two journal bearings J J, support the shaft S', which carries the pulley P, the graduated disc D, and the cylindrical casing C. This shaft S', is bored to receive the shaft S, shown in dotted lines. Connection between the two is established through the medium of a spiral spring t, the ends of which are attached to the box C, and shaft S, respectively.

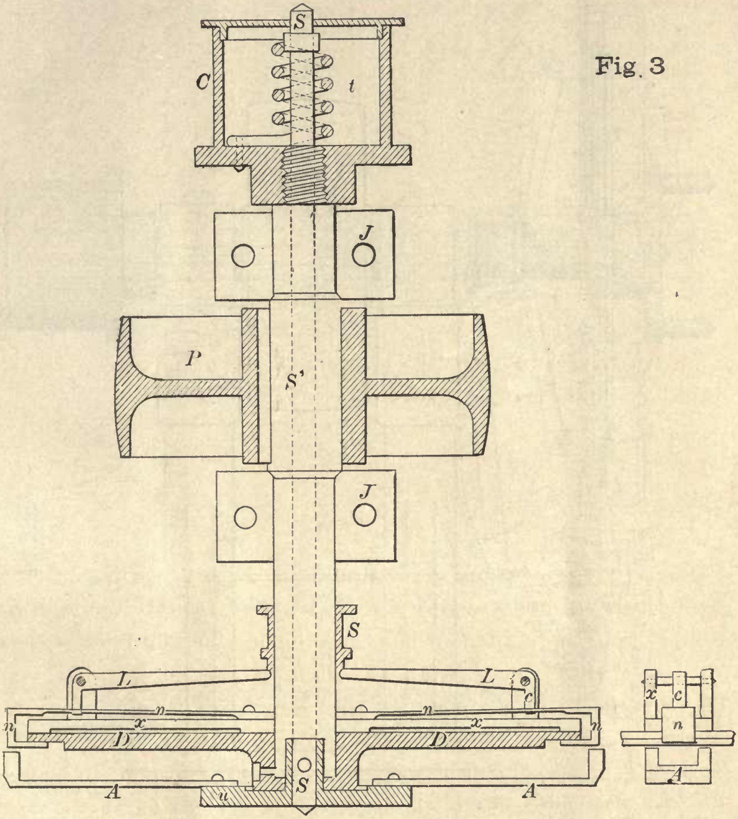


Fig. 3

The lower end of S is provided with a disc u, which carries two coupling pins for the purpose of conveying rotation to the driving pulley for the footstep A, Fig. 2. The relative position of the two shafts, that is, the tension of the spring t, measures the moment required to communicate motion to the footstep. Since precision of measurement was aimed at in the construction of the dynamometer, the recording apparatus was not designed for reading while in motion, because such an arrangement would have impaired its sensitiveness.

The bar x, which carries the index hand (see Fig. 4[a]), rests on the plate D, and is pivoted on the shaft S'. n, n, are two springs attached to x; their outer ends are bent over as seen in the drawings. If these springs are forced down so as to bear upon x, their outer ends enter the forked ends of the

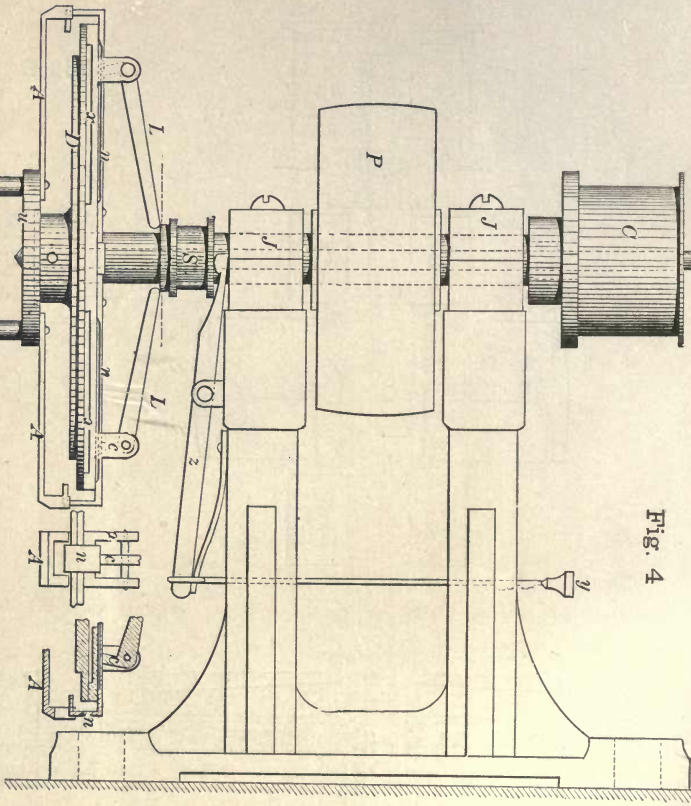


Fig. 4

arms A, A, which are connected with the disc u (see Fig. 4). If released, they become disengaged from A, A, but bind x to the disc D (see Fig. 3). L, L are two levers, pivoted to the bar x. In the position in Fig. 4, their short ends C, C, abut against the edge of a slot cut in the springs n, n, and keep them down. By a slight depression of L, L, however, the arms C, C, will release their hold, pass through the slots and the springs will fly up (see Fig. 3).

At any time, during the action of the dynamometer, the position of the index hand may be secured by pressing upon a knob y of a rod, which, by means of lever z, sleeve S, and levers L, disengages C, C. During the use of the dynamometer the arms A, of shaft S, are connected with the spring,

Fig. 4[a]

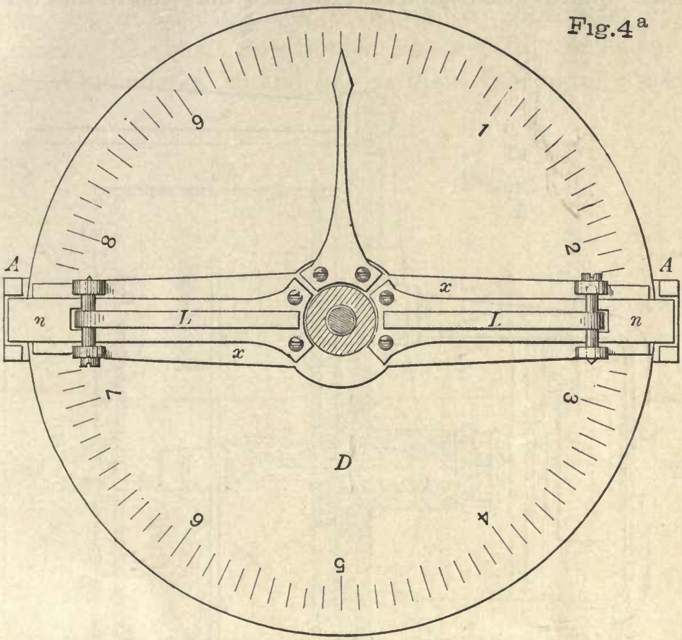


plate x, and index hand, Fig. 4, which relative position is secured as explained.

Resistance Determined by Dynamometer.

Let α=index reading of dynamometer.

α_1=index reading of dynamometer without water in the footstep, to eliminate frictional resistance and the correction for zero point of the graduated disc.

M=corrected moment of resistance of footstep.

ε=0.02298 value of one division of index plate.

r=radius of pulley on footstep shaft.

r_1=radius of pulley on dynamometer.

$\frac{r}{r_1}=0.494$

Then we have $M=\varepsilon\frac{r}{r_1}(\alpha-\alpha_1)=0.01135(\alpha-\alpha_1)$.

One hundred and thirty-four tests for moment of resistance of movable and stationary wings, were made.

The motive power was derived from a small Pelton wheel, but the water supply proved insufficient to obtain the desired velocities. The very laborious computations by the method of least squares also established the fact that the results could

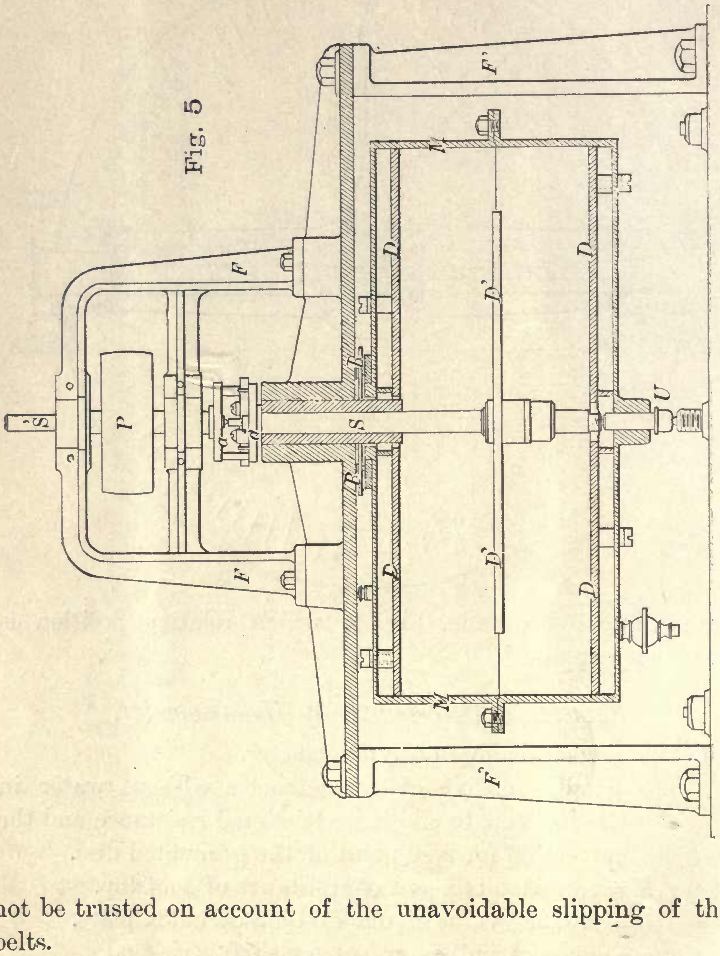


Fig. 5

not be trusted on account of the unavoidable slipping of the belts.

For these reasons I concluded to apply a different method, applicable not only for the step, which was the original object, but in general for the determination of fluid resistance to rotating discs, cylinders, etc.

Resistance Measured by Reaction.

The apparatus built for this purpose is shown in Fig. 5 in vertical section.

M, *M*, represents a closed cylinder, 15″ by 7″, in two sections bolted together, accurately turned and balanced and resting on a conical pivot *U* which is adjustable by a set screw. The shaft *S* rests upon an inverted conical pivot attached to cylinder *M*, and has its upper bearing in a sleeve,

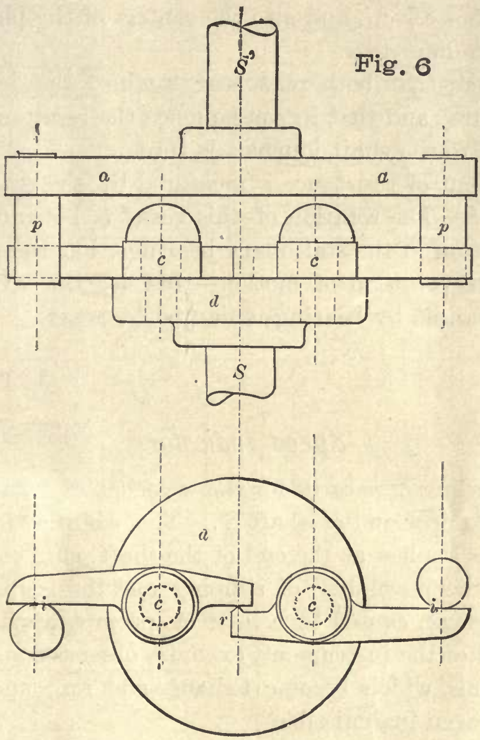


Fig. 6

which is inserted in the hub of the frame *M*. This shaft is provided with a shoulder and set screws for attaching objects to be experimented with. *D*, *D*, represent plates or discs capable of being adjusted to any position required for the test. The upper journal bearing of the shaft passes through a central opening in the cylinder, just large enough not to touch. The position of the cylinder is maintained by four friction rollers *R*, *R*.

The frame F is provided with bearings for a shaft S', which carries the driving pulley P. Motion is transmitted to shaft S by means of a coupling which is represented in Fig. 6 on a larger scale.

The disc or crosspiece a of the driving shaft S' carries two pins p, p. The disc d of the driven shaft S has pivoted to it at c, c, two levers, which engage each other at the center of the shaft r, while the outer ends receive the thrust of the pins p, p, at i, i.

The points $i\ c\ r\ c\ i$, and also the centers of the pins p, p, are contained in diameters.

It is evident that both pins bear against the levers with equal pressure* and that in consequence the resultant pressure transmitted to the shaft journals is zero.

The moment of resistance is measured by the reaction upon the vessel M. The rotation of this vessel is not influenced by journal friction in the stationary bearings, but the point to be aimed at was precision of motion—that is, the avoidance of vibrations caused by bearings enlarged by wear.

Speed Indicator.

The recording apparatus for the velocity of rotation is applied at the upper end of shaft S'. The counter in common use which is applied at the end of the shaft, etc., could not be trusted, as the possibility of slipping and the source of error in time-interval, caused by a lack of prompt application and withdrawal of the instrument, excludes observations for small time-intervals, which become tedious, and are, under certain conditions, even inadmissible.

The following is a description of the recording apparatus designed and built to obviate these defects:

N, N, Figs. 8 and 9, represent two identical speed indicators, which are pivoted at O, O. In the position shown they

*In an ordinary pin coupling skill may produce practically an equal contact, but an almost imperceptible change of the centers of the shafts will produce an unequal distribution of the pressures. In practice these centers cannot remain in line, hence the above result. In the coupling described the equality of pressure is not interfered with by such derangement.

are in gear with a pinion *P*, Fig. 8, which is attached to the end of shaft *S′*. These positions are maintained by springs *S*, *S*, and checks. The position which corresponds to contacts of the armatures *A* with their electromagnets *M*, *M*, leaves the spur wheels and pinion *P* out of gear. The contact is maintained by a projection on each indicator, which

Fig. 7

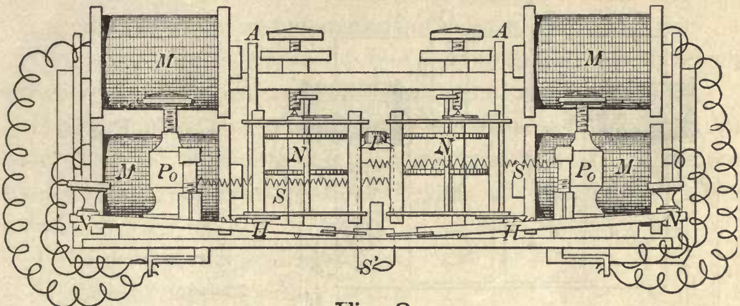
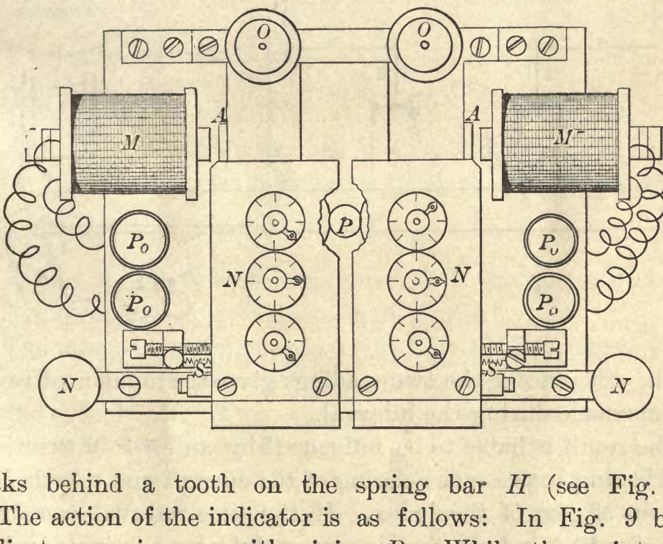


Fig. 8



locks behind a tooth on the spring bar *H* (see Fig. 7).

The action of the indicator is as follows: In Fig. 9 both indicators are in gear with pinion *P*. While the pointer of the pendulum is between the two contact springs, and just preceding that reading of the dial-plate of the clock, from which it is intended to count, the push button *R*1 must be depressed.

As soon as the pendulum has completed its stroke and produced contact, a current from battery B passes in the direction of the arrows, the electromagnet attracts the armature, and the indicator N 1 is thrown out of gear against the spring and retained as described. Just preceding the specified interval of time as before, the push button R2 must be depressed, which causes the indicator N 2 to be thrown out of gear and held in that position as explained.

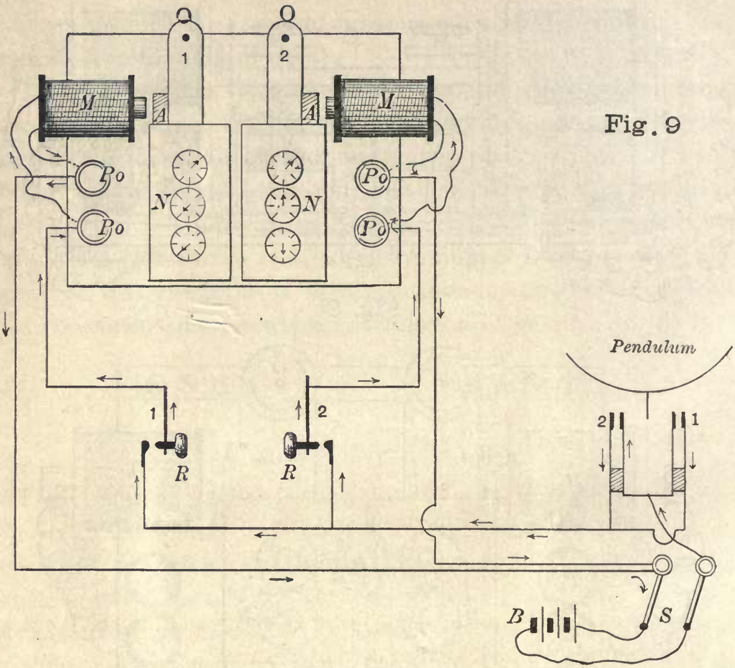


Fig. 9

The difference of the two readings gives the number of revolutions made during the interval.

The result is liable to be influenced by an error in the time intervening between the closing of the current and the throwing out of gear of the index. If the time-interval were the same for both indicators, this error would be eliminated; but such can hardly be expected on account of difference in friction, distribution of masses, intensity of electromagnets, etc.

This error will influence the result the more, the smaller the clock interval is. We can, however, from observed data, compute a correction which will make it possible to obtain excellent results for very small intervals.

Let the time intervening between the closing of the currents and the release of the spur wheel from the pinion be represented respectively by t_1 and t_2, for counters 1 and 2.

Let r_1 denote the first reading of clock;

r_2 denote the second reading of clock;

T denote the difference of these readings;

then r_1+t_1=time at instant of release of counter 1;

and r_2+t_2=time at instant of release of counter 2.

Hence, $r_2+t_2-(r_1+t_1)=T+t_2-t_1$=true time interval, which corresponds to the difference of readings of the dial.

If we repeat the observations for the same T, but in order reversed regarding the push buttons, and if we denote by d and d_1, respectively, the difference of dial readings, we have:

$T+(t_2-t_1)$ corresponding to d;

$T-(t_2-t_1)$ corresponding to d_1;

then the number of revolutions n per second equals

$$\frac{d}{T+(t_2-t_1)}=\frac{d_1}{T-(t_2-t_1)}$$

Hence $t_2-t_1=T\dfrac{d-d_1}{d+d_1}=\varepsilon\, T$

Now, if we always make one observation for speed in the order 1—2 of the push buttons, we find the true number of revolutions per minute $n=60\dfrac{d}{T\,(1+\varepsilon)}$

In our experiments the time-interval T was generally equal to five seconds and yielded excellent results.

The moments were measured (see Fig. 10) by weights V, V, attached to cords which fitted a groove turned on the flange of the cylinder M. Two weights were used to counteract the pressure on the journal U.

In order to relieve the friction upon the step pivot U, the cylinder, etc., were balanced by means of a weight W and lever. The distance between the points of suspension and the apparatus was over 14 feet, so that practically torsion could not be felt. The upper face of the cylinder was adjusted horizontally to relieve the friction rollers R, R, from lateral pressure.

For each of the 5, 7, 9 and 12 inch discs, two sets of experiments were made to ascertain the moment of resistance for stationary and movable wings. The resistance offered to a solid moving in a fluid is practically proportional to the square of the velocity. I found this to be the case for each individual set of tests, hence we have for the same disc $M = \beta n^2$, M representing moment of resistance and β a con-

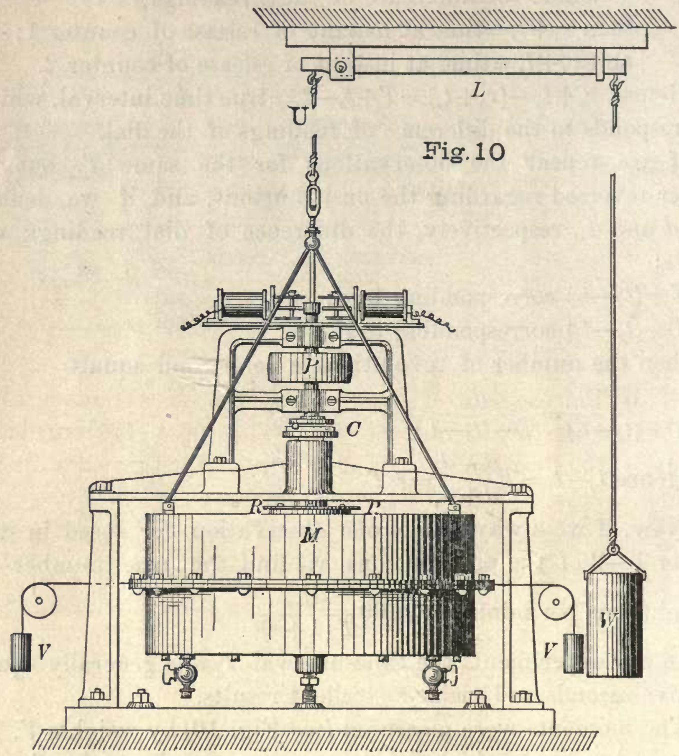


Fig. 10

stant. These values for β were computed by the method of least squares, and are given in the following table:

d Inches.	Number of Observ.	β For Stationary Ribs.	Number of Observ.	β For Movable Ribs.
5	19	.000000044	23	.000000122
7	27	.000000208	26	.000000569
9	27	.000000692	50	.000001900
12	24	.000002801	16	.000011830

The computed results agree very well with the observed values, except for small velocities, which, however, do not occur in the application for the footstep. The value of β is a function of d. To establish an empirical relation, I endeavored to obtain it in the form $\beta=\lambda\ \psi\ (d)$, in which $\psi\ (d)$ is a simple known function of d, and λ a variable coefficient which changes very little with d, so that its proper numerical value may be obtained with sufficient accuracy, from a small table.

β was found to be well represented by $\lambda\ d^5$; hence

$$M=\lambda d^5 n^2.$$

d Inches.	Values of λ for Stationary Ribs.	Values of λ for Movable Ribs.
5	.000003503	.000009713
7	3080	8415
9	2915	8007
12	2801	7690

One-half of the sum of these values of λ for stationary and movable wings represents the true value of λ for our step (see page 10, Resistance). The following table furnishes interpolated values of λ for d expressed in feet :

d Feet.	λ	d Feet.	λ
.4	.00000680	.75	.00000546
.45	.00000635	.8	.00000541
.5	.00000605	.85	.00000535
.55	.00000585	.9	.00000531
.6	.00000571	.95	.00000528
.65	.00000561	1	.00000525
.7	.00000553		

Recapitulation of the Results for Footstep.

$P=.001\ d^4 n^2 m$ neglecting the term d_0

$$d=\sqrt[4]{\frac{1000\ P}{n^2\ m}}$$

$L=.1047\ n^3 \lambda d^5 m$=work lost by friction in foot-pounds per second.

The following tables offer a comparative estimate between the hydraulic step and those in common use. The latter have been computed with a view to durability, from the data given by Reuleaux in the fourth edition of the "Konstrukteur:"

HYDRAULIC STEP.

P	n	d	m	L
2000	2500	0.75	1	2140
2000	2500	0.53	4	1530
2000	400	1.88	1	780
2000	400	1.33	4	553
8000	2500	0 75	4	8600
8000	1400	1.00	4	6000

FLAT PIVOT. STEEL ON BRONZE.

P	n	Coefficient of Friction.	d Outer.	d Inner.	L	Lubricant.
2000	2500	0.1	1	0.33	17000	Oil.
8000	2500	0.1	2	0.66	136000	

COLLAR BEARINGS; 10 COLLARS. STEEL ON BRONZE.

2000	2500	.1	0.25	0.15	5100	Oil.
8000	2500	.1	0.50	0.30	40800	

PIVOT OF LIGNUM VITÆ ON BRONZE.

2000	2500	0.24	0.13	0	5400	Water
2000	400	0.24	0 13	0	830	
8000	2500	0.24	0.26	0	43300	
8000	1400	0.24	0.26	0	24200	

The hydraulic step is perfect regarding wear, since the load is completely balanced by the water pressure upon the disc. This superiority is also maintained with reference to to the loss of work by friction. (See tables.)

Fig. 11

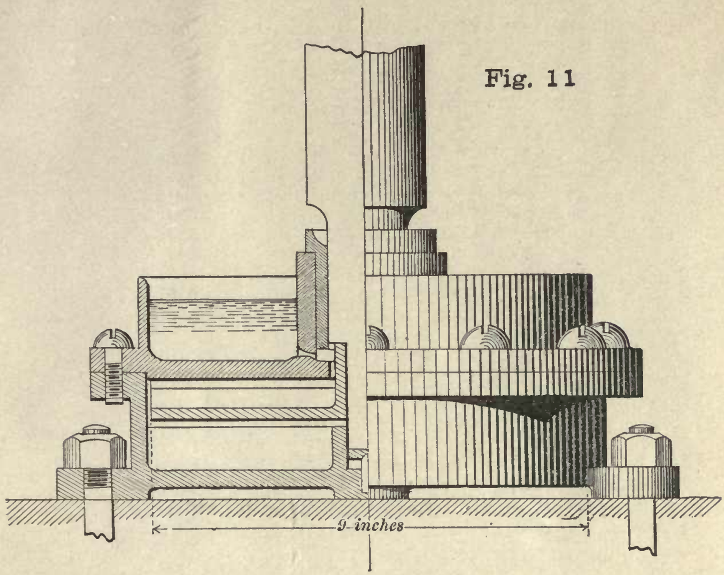


Fig. 11 represents a step for $P=2000$ and $n=2500$, and $d=9$ inches.

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Berkeley, July 15, 1887.