HYDRAULIC TURBINES

HYDRAULIC TURBINES

WITH A CHAPTER ON

CENTRIFUGAL PUMPS

BY

R. L. DAUGHERTY
ASSISTANT PROFESSOR IN SIBLEY COLLEGE, CORNELL UNIVERSITY

McGRAW-HILL BOOK COMPANY, INC.
239 WEST 39TH STREET, NEW YORK
6 BOUVERIE STREET, LONDON, E. C.
1913

THE·MAPLE·PRESS·YORK·PA

PREFACE

The design of hydraulic turbines is a highly specialized industry, requiring considerable empirical knowledge which can be learned only through experience; but it is a subject in which comparatively few men are interested as a relatively small number are called upon to design turbines. But with the increasing use of water power many men will find it necessary to be familiar with the construction of turbines, understand their characteristics, and be able to make an intelligent selection of a type and size of turbine for any given set of conditions. To this latter class this book is largely directed. However, a clear understanding of the theory, as here presented, ought to be of interest to many designers, as it is desirable that American designs be based more upon a mathematical analysis, as in Europe, and less upon the old cut and try methods.

The broad problem of the development of water power is treated in a very general way so that the reader may understand the conditions that bear upon the choice of a turbine. Thus the very important items of stream gauging and rating, rainfall and runoff, storage, etc., are treated very briefly, the detailed study of these topics being left for other works.

The purpose of the text is to give the following: a general idea of water-power development and conditions affecting the turbine operation, a knowledge of the principal features of construction of modern turbines, an outline of the theory and the characteristics of the principal types, commercial constants, means of selection of type and size of turbine, cost of turbines and water power and comparison with cost of steam power. A chapter is also added on centrifugal pumps. It is hoped that the book may prove of value both to the student as a text and to the practicing engineer as a reference.

R. L. D.

ITHACA, N. Y.
August, 1913.

CONTENTS

HYDRAULIC TURBINES

CHAPTER I

INTRODUCTION

1. Historical.—Water power was utilized many centuries ago in China, Egypt, and Assyria. The earliest type of water wheel was a crude form of the current wheel, the vanes of which dipped down into the stream and were acted upon by the impact of the current (Fig. 1). A large wheel of this type was used to pump the water supply of London about 1581. Such a wheel could utilize but a small per cent. of the available energy of the stream. The current wheel, while very inefficient and limited in its scope, is well suited for certain purposes and is not yet obsolete. It is still in use in parts of the United States,

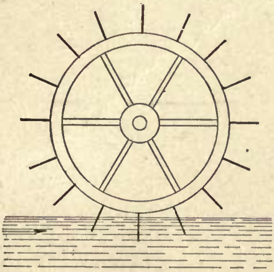


FIG. 1.—Current wheel.

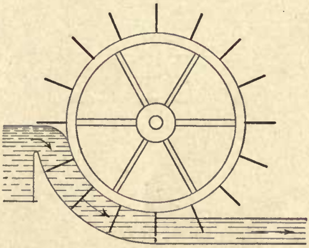


FIG. 2.—Breast wheel.

in China, and elsewhere for pumping small quantities of water for irrigation.

The undershot water wheel was produced by confining the channel so that the water could not escape under or around the ends of the vanes. This form of wheel was capable of an efficiency of 30 per cent. and was in wide use up to about 1800.

The breast wheel (Fig. 2) utilized the weight of the water rather than its velocity due to the same fall with an efficiency as high as 65 per cent. It was used up to about 1850.

The overshot water wheel (Fig. 3) also utilized the weight of the water. When properly constructed it is capable of an efficiency of between 70 and 90 per cent. which is as good as the modern turbine. The overshot water wheel was extensively used up to 1850 when it began to be replaced by the turbine, but it is still used as it is well fitted for some conditions.

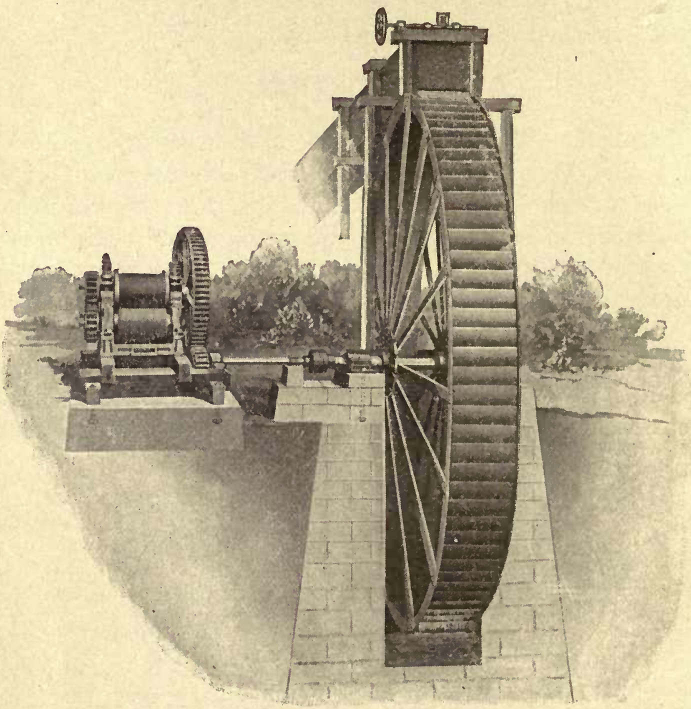


FIG. 3.—I. X. L. steel overshot water wheel. (*Made by Fitz Water Wheel Co.*)

2. The Turbine.—The turbine will be more completely described in a later chapter but in brief it operates as follows: A set of stationary guide vanes direct the water flowing into the rotating wheel and, as the water flows through the runner, its velocity is changed both in direction and in magnitude. Since a force must be exerted upon the water to change its velocity in any way, it follows that an equal and opposite force must be exerted by the water upon the vanes of the wheel. A turbine may be defined as a water wheel in which a motion of the water relative to its buckets is essential to its action.

The original inward flow turbine of J. B. Francis is shown in Fig. 4 and in Fig. 5.

3. Advantage of Turbine over Water Wheel.—The water wheel has been supplanted by the turbine because:

1. The latter occupies smaller space.
2. A higher speed may be obtained.
3. A wider choice of speed is possible.

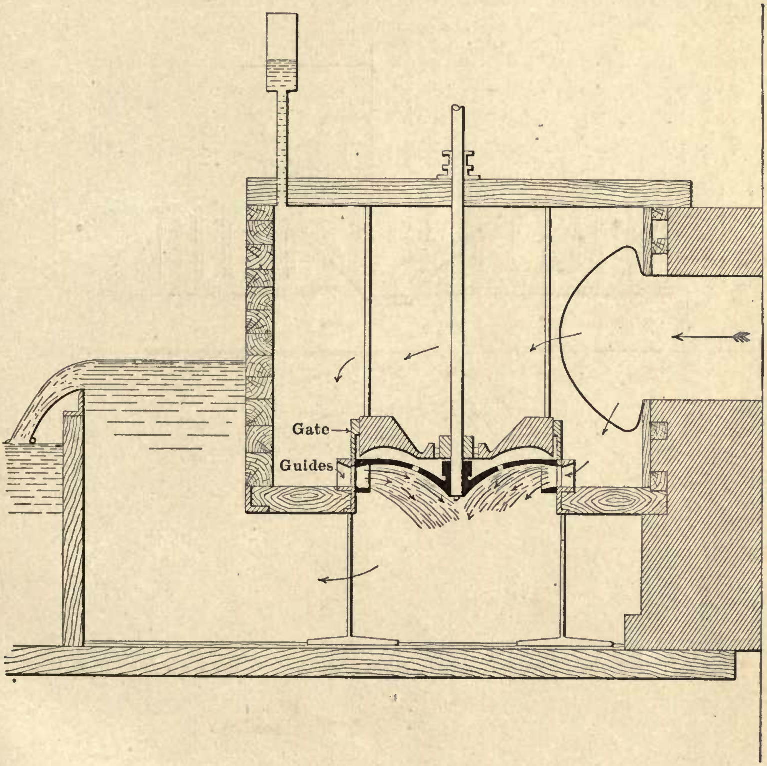


Fig. 4.—Francis turbine in flume.

4. It can be used under a wide range of head, whereas the head for an overshot wheel should be only a little more than the diameter of the wheel.
5. A greater capacity may be obtained without excessive size.
6. It can work submerged.
7. There is less trouble with ice.
8. It is usually cheaper.

4. Advantages of Water Wheel over Turbine.—For small plants the turbine is often poorly designed, cheaply made, unwisely selected, and improperly set. It may thus be very inefficient and unsatisfactory. In such cases the overshot water wheel may be better. The latter has a very high efficiency when the water supply is much less than its normal value. It is adapted for heads which range from 10 to 40 ft. and for quantities of water from 2 to 30 cu. ft. per second.[1]

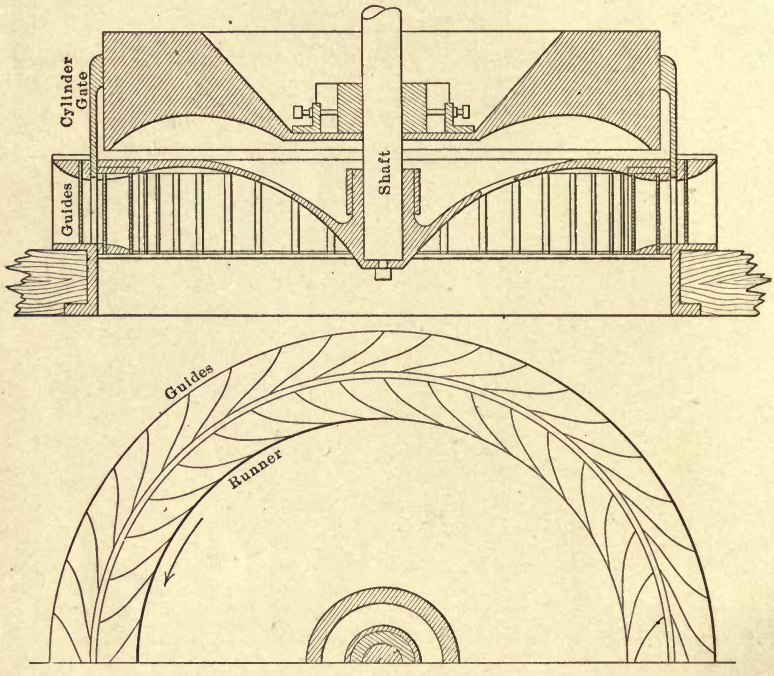


FIG. 5.—Francis turbine.

An overshot wheel on the Isle of Man is 72 ft. in diameter and develops 150 h.p. Another at Troy, N. Y., is 62 ft. in diameter, 22 ft. wide, weighs 230 tons, and develops 550 h.p.

5. Essentials of a Water-power Plant.—A water-power plant requires some or all of the following:

A dam which may create most of the head available for the plant or it may merely create a small portion of it and be erected

[1] See "Test of Steel Overshot Water Wheel," by C. R. Weidner, *Eng. News*, Jan. 2, 1913, Vol. LXIX, No. 1. A later test of this I.X.L. wheel after ball bearings were substituted gave an efficiency of 92 per cent.

primarily to provide a mill pond and to furnish a suitable intake for a pipe line. This body of water is sometimes called the fore-bay but is more usually called the head-water.

The water may be conducted from the head-water to the turbine by means of an open channel called a canal or flume, or by means of a closed pipe under pressure which is called a penstock.

Racks and screens are usually employed at the intake to the flume or penstock to keep trash from being carried down to the wheels. The water at this point may be shut off by means of head gates.

The turbine with its case or pit and draft tube, if any, comprise the setting.

The body of water that the turbine discharges into is called the tail water. The channel conducting the water away is the tail race.

CHAPTER II

TYPES OF TURBINES AND SETTINGS

6. Classification of Turbines.—Turbines are classified according to:

1. Action of Water
   (a) Impulse (or pressureless).
   (b) Reaction (or pressure).
2. Direction of Flow
   (a) Radial outward
   (b) Radial inward
   (c) Axial (or parallel)
   (*d*) Mixed (radial inward and axial).
3. Position of Shaft
   (*a*) Vertical.
   (*b*) Horizontal.

7. Action of Water.—In the impulse turbine the wheel passages are never completely filled with water. Throughout the flow the water is under atmospheric pressure. The energy of the water leaving the stationary guides and entering the runner is all in the form of kinetic energy. During flow through the wheel the absolute velocity of the water is reduced as the water gives up its kinetic energy to the wheel. In Europe a type of impulse turbine commonly used is called the Girard. In the United States practically the only impulse turbine is the tangential water wheel, more popularly known as the Pelton wheel. (See Fig. 6.)

In the reaction turbine the wheel passages are completely filled with water under a pressure which varies throughout the flow. The energy of the water leaving the stationary guide vanes and entering the runner is partly in the form of pressure energy and partly in the form of kinetic energy.[1] During flow through the wheel both the pressure and the absolute velocity of the water are reduced as the water gives up its energy to the wheel.

8. Direction of Flow.—Radial flow means that the path of a particle of water as it flows through the runner lies in a plane which

[1] Strictly speaking, the water possesses only kinetic energy but transmits pressure energy.

is perpendicular to the axis of rotation. If the water enters at the inner circumference of the runner and discharges at the outer circumference we have an outward flow type known as the Fourneyron turbine. (See Fig. 52.)

If the water enters at the outer circumference of the runner and discharges at the inner circumference we have an inward flow type which is known as a Francis turbine. Fig. 5 (page 4) shows this type.

If a particle of water remains at a constant distance from the axis of rotation as it flows through the runner we have what is known as axial or parallel flow. The type of turbine falling in

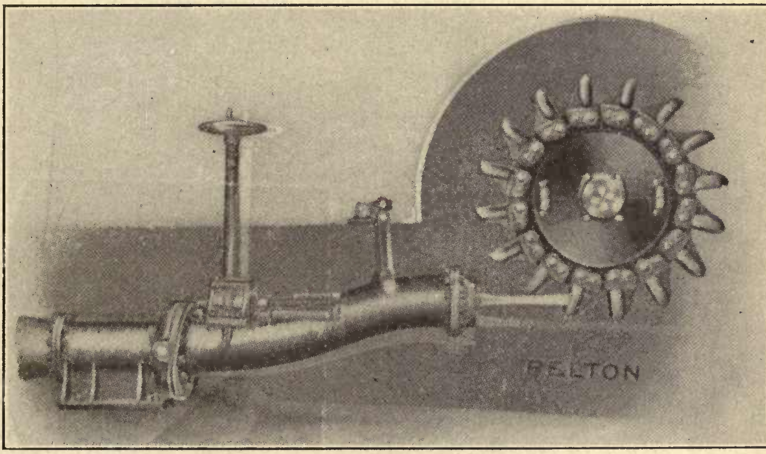


FIG. 6.—Tangential water wheel with deflecting nozzle.

this class is commonly called the Jonval and is used to some extent in Europe.

If the water enters a wheel radially and then during its flow through the runner turns and discharges axially we have a mixed flow turbine. This is known as the American type of turbine or is also called a Francis turbine.

The reaction turbines used in this country are nearly all Francis turbines of the radial inward flow or the mixed flow type and to them our discussion will be confined.

9. Position of Shaft.—The distinction as to position of shaft is obvious. The vertical shaft turbines are, however, further classified as right-hand or left-hand turbines according to the direction of rotation. If, in looking down upon the wheel from above, the rotation appears clockwise it is called a right-hand turbine. The reverse of this is a left-hand turbine.

So far as efficiency is concerned there is little difference between vertical or horizontal turbines. Other things being equal, the hydraulic losses should be identical in either case, but there might be some difference in the mechanical friction of the bearings. As

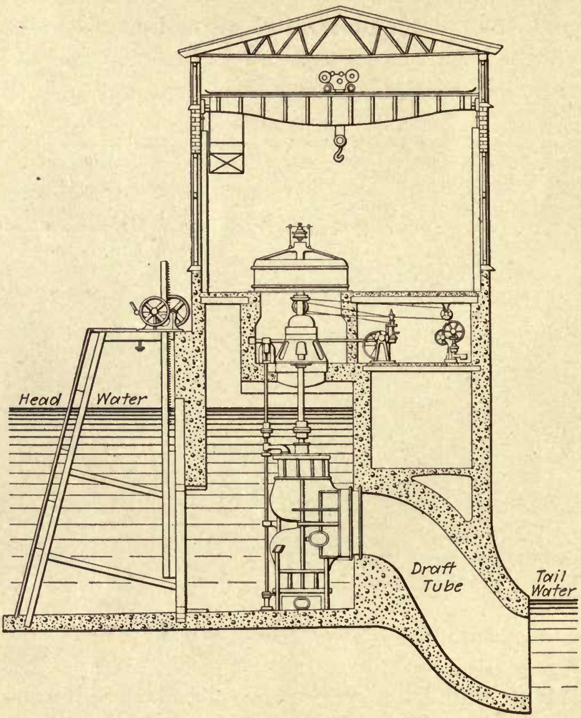


FIG. 7.—Pair of vertical shaft turbines in open flume.

the latter is a relatively small item anyway, a reasonable variation in its value would have but slight effect on the efficiency.

10. Choice of Shaft.—In general a horizontal shaft is more desirable from the standpoint of the station operator on account of greater accessibility and less bearing trouble. However a vertical shaft turbine occupies less floor space, but often requires more excavation and a deeper building.

The vertical shaft turbine is used where it is necessary to set the turbine down by the water while the generator or other machinery that it drives must be above. Since such conditions are usually met with in low-head plants, it will be found that ordinarily the vertical setting is used only for low heads. (See Fig. 7.)

The horizontal shaft turbine is used where the turbine can be set above the tail water level and if the generator or other machinery that it drives can be set at the same elevation. This is almost always the case with a high-head plant and quite frequently the case with a low-head plant also. (See Fig. 8.)

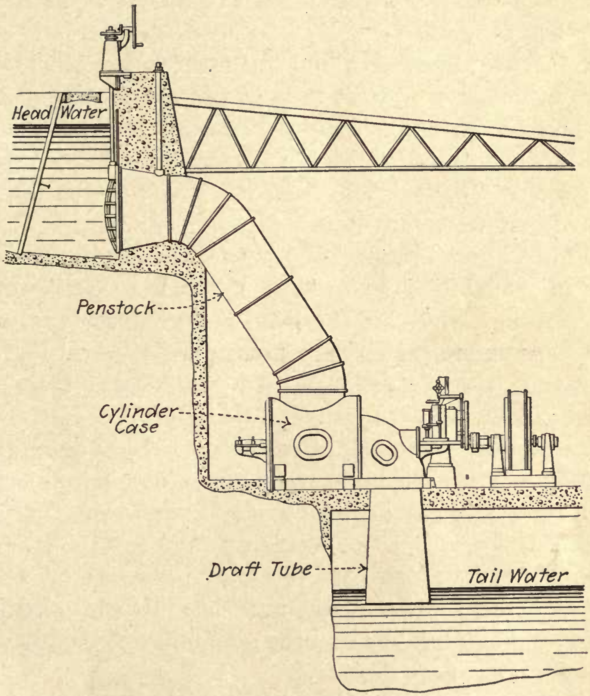


FIG. 8.—Horizontal shaft turbine in case.

These statements are purely general and there are many exceptions to this rule.

11. The Draft Tube.—Occasionally reaction turbines have been set so as to discharge above the tail water; in such cases the fall from the point of discharge to the water was lost. To avoid this loss turbines have been submerged below the tail water level as in Fig. 4, page 3. By the use of a draft tube (or suction tube), as in Fig. 7 and Fig. 8, it is possible to set the turbine above the tail water without suffering any loss of head. This is due to the fact that the pressure at the upper end of the draft tube is less than the atmospheric pressure. This suction compensates for the loss of pressure at the point of entrance to the turbine guides.

As will be shown later, when the theory is presented, the use of a draft tube that diverges or flares may result in a small increase in efficiency. The big advantage of the draft tube, however, is that it allows the turbine to be set above the tail water where it is more accessible and yet does not cause any sacrifice in head. It is this that permits a horizontal shaft turbine to be installed without any loss of head.

Since the wheel passages must be open to the air, it is readily seen that the use of a draft tube in the usual sense of the word is not possible with an impulse turbine. However, as will be seen later, the impulse turbine is better suited for comparatively high heads so that the loss from the wheel to the tail water is a relatively unimportant item.

12. Flumes and Penstocks.—If the turbine be used under a head of about 30 ft. or less a flume may conduct the water to an open pit as in Fig. 7. If the head is much greater than this it becomes uneconomical and a penstock is used as in Fig. 8. The turbine must then be enclosed in a water-tight case. Various forms of cases will be described in Chapter V.

For penstocks where the pressure head is less than about 230 ft. (100 lb. per square inch) wood-stave pipe is frequently used. It is cheaper than metal pipe for similar service.

Cast-iron pipe is used for heads up to about 400 ft. It is not good in large diameters nor for high pressures on account of porosity, defects in casting, and low tensile strength. Its advantages are durability and the possibility of readily obtaining odd shapes if such are desired.

For high heads, steel pipe, either riveted or welded, is used. It is cheaper than cast iron in large sizes but it corrodes more rapidly.

CHAPTER III

WATER POWER

13. Investigation.—Before a water-power plant is erected a careful study should be made of the stream to determine the horse-power that may be safely developed. It is important to know not only the average flow but also both extremes. The extreme low-water stage and its duration will determine the amount of storage or auxiliary power that may be necessary. The extreme high-water stage will fix the spillway capacities of dams, determine necessary elevations of machines, and other facts essential to the safety and continuous operation of the plant.

14. Rating Curve.—The first step in such an investigation is the establishment of a rating curve. (See Fig. 9.) To determine

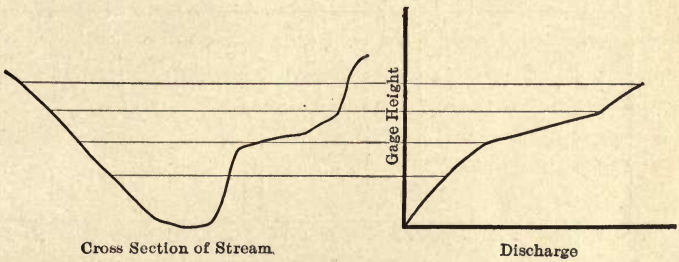


FIG. 9.—Rating curve.

the discharge of the stream a weir, current meter, floats, or other means may be employed according to circumstances.[1]

By measuring the flow of the stream for different stages a rating curve is readily drawn. This will not be a smooth curve if there are abrupt changes in the area of the section. A given gage height may really represent a range of flows depending upon whether the river is rising or falling, the flow being greater if the stream is rising and less if it is falling. This is because the hydraulic gradient is different in the two cases.[2] If possible,

[1] Hoyt and Grover, "River Discharge."
Water Supply Papers No. 94 and No. 95 of the U. S. G. S.

[2] Mead, "Water-power Engineering," p. 201.

the points for the rating curve should be taken when the river is neither rising nor falling. The discharges from the rating curve for gage readings taken under all conditions will be more or less in error, but in the end such errors will usually balance each other and be unimportant.

15. The Hydrograph.—When gage readings are taken regularly and frequently for any length of time and the corresponding discharges secured from the rating curve a history of the flow may be plotted as in Fig. 10. Such a curve is called a hydrograph. This curve is extremely useful in the study of a water-power proposition. To be satisfactory it should cover a period of several years since the flow will vary from year to year. Since

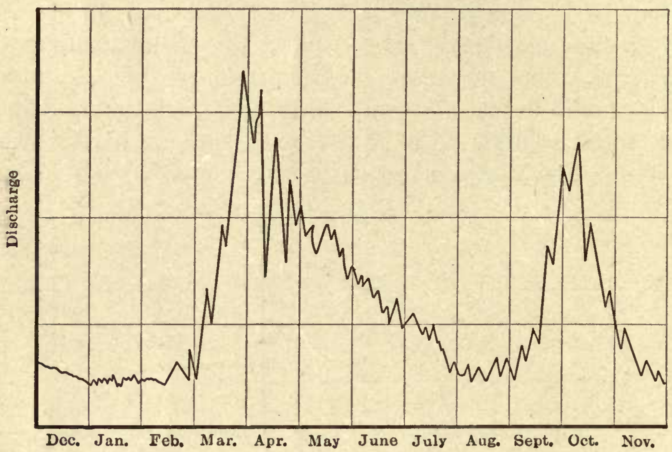


Fig. 10.—Hydrograph.

it is very important to know the extremes also, it should cover both a very dry year and a very wet one as well as the more normal periods.

16. Rainfall and Run-off.—Rainfall records are usually available for many years back and are a valuable aid in extending the scope of the hydrograph taken, provided a relation between rainfall and run-off can be estimated. If the ground be frozen, or the slopes steep and stony, or the ground saturated and the rain violent nearly all the water that falls upon the drainage basin may appear in the stream as run-off. On the other hand, if the soil be dry and the land such that opportunity is given it, all the rain may be absorbed and none of it appear in the stream.

Usually the conditions are such that the relation is between these two extremes. In a general way it may be said to lie between the two curves shown in Fig. 11.[1]

The relation between rainfall and run-off is very complicated and only partially understood at present. For more information consult Water Supply Papers of the U. S. G. S. and other sources.

17. Absence of Satisfactory Hydrograph.—If no hydrograph of the stream is available and there is no time to secure one, a study of the stream may be made by comparison with the hydrographs of adjacent streams. It is well, however, to take a hydrograph for a year, if possible, in order to be able to check the comparison.

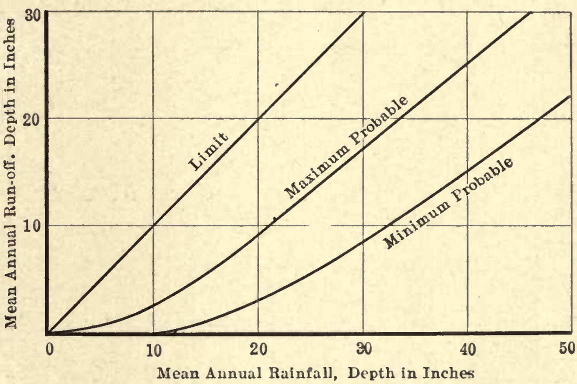


FIG. 11.—Relation of rainfall to run-off.

If no hydrographs of adjacent streams are available, it is necessary to use the rainfall records and make a thorough study of the physical conditions of the water shed. If the relation between rainfall and run-off can be estimated, then fairly satisfactory conclusions may be drawn, provided a hydrograph for one year can be used to work from. Where there is not time to take a year's record it is well to be very conservative and provide for future extension of power if it is later found to be warranted.

18. Variation of Head.—Since the discharge of any stream is usually a widely varying quantity, it follows that the water level at any point must vary. If the turbine be of the reaction type set in the usual way, the total head acting upon the wheel will be the fall from the surface of the head water to the surface

[1] F. H. Newell, Proc. Eng. Club of Phila., Vol. XII, 1895.

of the tail water with the pipe line loss deducted. If, in times of high water, the head water level rose the same amount as the tail water level the net head under which the turbine operated would remain constant. But, under the usual conditions, the tail water level rises more than the head water level and the net head under which the turbine operates becomes less. This is illustrated in Fig. 12 where three rates of flow are shown.

At high water the horse-power of the stream may be large even though the fall be reduced, owing to the increased quantity of water. But the horse-power of the turbine may be seriously diminished. A turbine is only a special form of orifice and therefore the discharge through it is proportional to the square root of the head. If then the discharge through it be reduced due to the

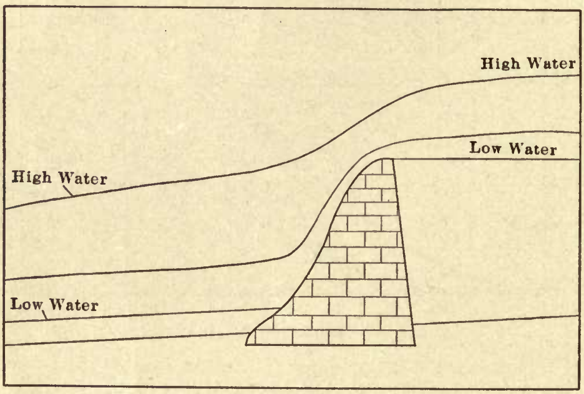


FIG. 12.—Decrease of available head at high water.

lower head, the horse-power input to the turbine is decreased. If the best efficiency is to be obtained, the speed also should vary as the square root of the head. But usually the turbine is compelled to run at constant speed and this causes a further reduction of the power of the turbine since the efficiency is lowered. (The speed should be the best for low water because economy of water is then important.) It is thus seen that the decrease of the head at high water causes a loss of power and a drop in efficiency. This change of head will be an insignificant item for a high-head plant but may be very serious for a low-head plant.

19. Power of Stream.—If the conditions are such that there is no appreciable change in head, the hydrograph with a suitable scale may represent the power of the stream also. But if the head varies to any extent with the flow then the power curve must be computed from the hydrograph by using the heads that would be obtained at various stages of flow. Or the hydrograph itself may still be used as a power curve if the power scale that is used is made to vary as the head varies instead of being uniform.

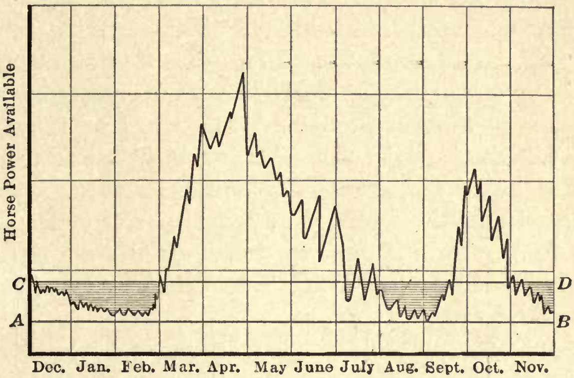


Fig. 13.—Power curve of a stream.

If Fig. 13 represents the power curve of a stream then *A–B* represents the greatest power that the stream can be counted upon to furnish at all times.

20. Pondage and Load Curve.—By pondage is meant the

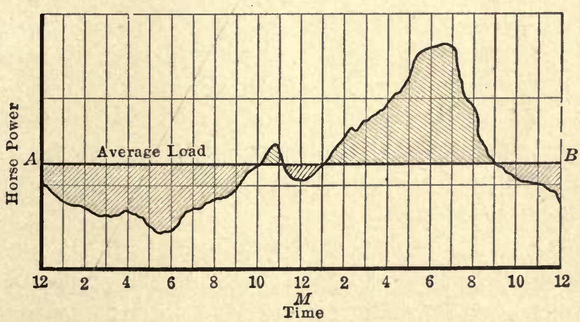


Fig. 14.

storing of a limited amount of water. If the plant be operated 24 hours on a steady load then pondage is of little value except for equalizing the flow of water when the stream is low. But if the plant be operated for only a portion of the 24 hours or if the load be variable as shown by the load curve in Fig. 14, then the water that is not used when the load is light may be stored and used when the load is heavy. If the pondage be ample, the average load carried by the plant may then be equal to *A–B* in Fig. 13, while the peak load may be much greater.

21. Storage.—By storage is meant the storing of a considerable quantity of water, so that it varies from pondage in degree only. Pondage merely indicates sufficient capacity to supply water for a few hours or perhaps a few days, but storage implies a capacity which can supply water needed during a dry spell of several weeks or months or more. The effect of storage is to enable the minimum power of the stream to be raised from *A–B* to *C–D* (Fig. 13). The greater the storage capacity the higher *C–D* is placed until it equals the average power of the stream. The water for the turbines may be drawn direct from the storage reservoir (in which case the head varies) or the reservoir may be used as a stream feeder only.

A plant operating under a low head requires a relatively large amount of water for a given amount of power. A storage basin for such a plant would require a very large capacity if it were to furnish power for any length of time. But a low head is usually found in a fairly flat country where it is possible to construct a storage reservoir of limited capacity only, and often none at all, on account of flooding the surrounding country. But for a high head the conditions are different as only a relatively small amount of water is required so that the capacity of the storage reservoir need not be excessive. The higher the head, the more valuable a cubic foot of water becomes. The topography of a country where a high head can be developed is usually such that storage reservoirs of large capacity can be constructed at reasonable cost. A low-head plant usually possesses pondage only—a high-head plant usually possesses storage.

22. Storage and Turbine Selection.—If a plant possesses neither storage nor pondage, or the stream flow may not be interrupted because of other water rights, the economy of water when the turbine is running under part load is of no importance. The efficiency at full load is all that is of interest. But if the plant does have pondage or storage in any degree the economy of water under all loads is of importance. The more extensive the pondage the more valuable a high efficiency on all loads becomes. Thus the question of storage has an important bearing in turbine selection.

23. Power Transmitted through Pipe Line.—Suppose that a nozzle, whose area can be varied, is placed at the end of a pipe line B–C (Fig. 15). With the nozzle closed we have a pressure head at C of CX which is equal to the static head. The hydraulic gradient is then a horizontal line. If the nozzle be partially opened, so that flow takes place, the losses in the pipe line as well

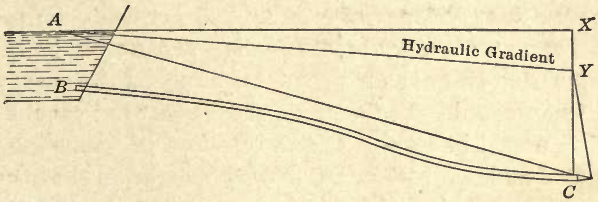


FIG. 15.—Varying rates of flow in pipe line.

as the velocity head in the pipe cause the pressure to drop to CY. A further opening of the nozzle would cause the pressure to drop to a lower value. If the nozzle were removed the pressure at C is then atmospheric only, which we ordinarily call zero pressure. The hydraulic gradient is then A–C.

Head is the amount of energy per pound of water. The head at

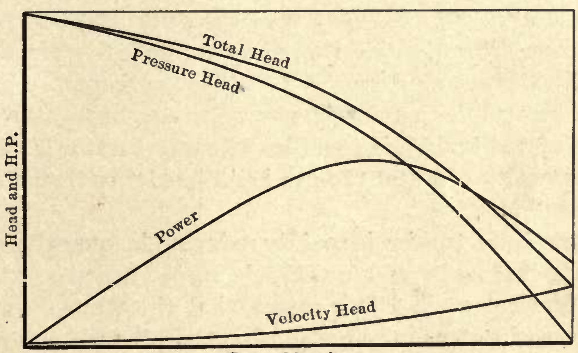


FIG. 16.—Head and power at end of pipe line.

C is the elevation head, taken as zero, plus the pressure head, plus the velocity head. When the discharge is zero the head is a maximum, being equal to CX. When the nozzle is removed the discharge is a maximum but the head at C is a minimum, being only the velocity head. For any intermediate value of discharge the head will be intermediate between these two extremes.

The power transmitted through the pipe line and delivered at C is a function of both the quantity of water and the head. It is zero when the discharge is zero and very small when the discharge is a maximum. The power becomes a maximum for a discharge between these two extremes (Fig. 16). It can be shown that the power is a maximum when the flow is such that approximately one-third the static head is used up in pipe friction. However this means a pipe line efficiency of only 66 2/3 per cent. If economy in the use of water is any object the discharge would be kept at a lower value than this so as to prevent so much of the energy of the water being wasted. For a given discharge that means that a larger pipe would be used. It is a question of balancing cost of pipe on one hand against the value of the water on the other.

The most economical size of pipe, financially, could be determined by the application of Kelvin's law. According to that, the annual interest on the cost of the pipe line should equal the annual value of the power wasted.

24. Pipe Line and Speed Regulation.—A fundamental proposition in mechanics is that

$$\text{input} = \text{output} + \text{losses} + \text{change in energy.}$$

If the speed of a turbine is to remain constant it follows that the input must always be equal to the power output plus the losses. As the power output varies, therefore, the quantity of water supplied to the turbine must vary. It is thus apparent that a turbine does not run under an absolutely constant head at all loads. By referring to Fig. 16 it is seen that when the turbine is using only a small quantity of water the head will be higher than when it is carrying full load.

If the load on a turbine is rapidly reduced the quantity of water supplied to it must be very quickly decreased in order to keep the speed variation small. This means that the momentum of the entire mass of water in the penstock and draft tube must be suddenly diminished. If the penstock be long a big rise in pressure may be produced so that momentarily the pressure may be greater than the static pressure. This increase in pressure may be sufficient to even cause an increase in the power input for a very brief interval of time. On the other hand, if the load on the turbine be suddenly increased, the water in the penstock and draft tube must be accelerated and this causes a temporary drop in pressure below the normal value, and for the time being the power input to the turbine may be diminished below its former valve. The longer the pipe line and the higher the maximum velocity of flow, the worse these effects become. It is thus seen that the speed regulation depends upon the penstock and draft tube as well as upon the governor and the turbine.[1]

If the velocity of the water is checked too suddenly a dangerous water hammer may be produced. In order to avoid an excessive rise in pressure, relief valves are often provided. Automatic relief valves are analagous to safety valves on boilers; they do not open until a certain pressure has been attained. Mechanically operated relief valves are opened by the governor at the same time the turbine gates are closed and afford the water a by-pass so that there is no sudden reduction of flow. To prevent waste of water these by-passes may be slowly closed by some auxilliary device. Another means of equalizing these pressure variations is to place near the turbine a stand pipe or a surge chamber, with compressed air in its upper portion. These have the advantage over the relief valves that they are not only able to prevent the pressure increase from being excessive but they are able to supply water in case of an increasing demand and thus prevent too big a pressure drop.[2]

[1] A case may be cited where the length of a conduit was 7.76 miles, the average cross-section 100 sq. ft., and the maximum velocity 10 ft. per second. The amount of water in the conduit was, therefore, 128,125 tons and with the velocity of 10 ft. per second there would be in round numbers 200,000 ft.-tons of kinetic energy.

[2] See "Control of Surges in Water Conduits," by W. F. Durand, *Journal A. S. M. E.*, June, 1911.

CHAPTER IV

THE TANGENTIAL WATER WHEEL

25. Development.—The development of this wheel was begun in the early days in California but the present wheel is a product of the last 20 years. For the purpose of hydraulic mining in 1849 numerous water powers of fairly high head were used, some of the jets being as much as 2000 h.p. When the gold was exhausted many of these jets were then used for power purposes. The first wheels were very crude affairs, often of wood, with flat plates

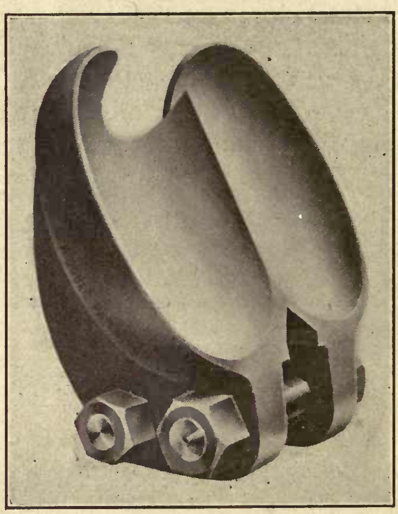


Fig. 17.—Pelton-Doble ellipsoidal bucket. (*Made by Pelton Water Wheel Co.*)

upon which the water impinged. The ideal maximum efficiency of a wheel with flat vanes is only 50 per cent. The next improvement was the use of hemispherical cups with the jet striking them right in the center. A man by the name of Pelton was running one of these wheels one day when it came loose on its shaft and slipped over so that the water struck it on one edge and was discharged from the other edge. The wheel was observed to pick up in power and speed and this led to the development of the split bucket.

26. Buckets.—An illustration of the best type of bucket is found in Fig. 17. The jet strikes the dividing ridge and is split into two halves. The better buckets are made of bronze or steel, the cheaper ones of cast iron. They are all polished inside and the "splitter" ground to a knife edge so as to reduce friction and eddy losses. The buckets may weigh as much as 430 lb. apiece and be

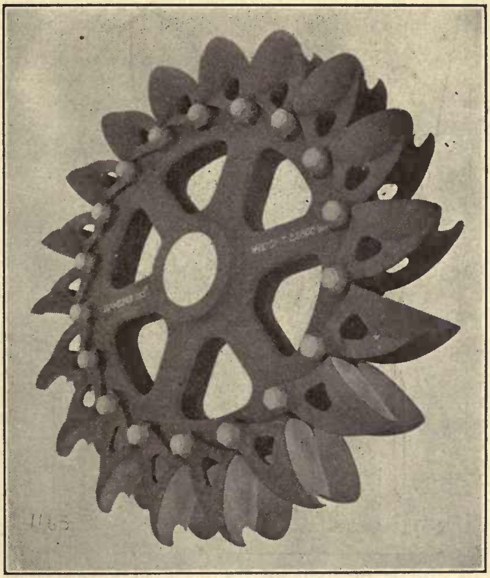


Fig. 18.—Pelton-Doble tangential water wheel runner showing interlocking chain-type construction. (*Made by Pelton Water Wheel Co.*)

from 24 to 30 in. in width. They are bolted onto the wheel. The latest "chain-type" construction is shown in Fig. 18.

27. General Proportions.—It has been found that for the best efficiency the area of the jet should not exceed 0.1 the projected area of the bucket, or the diameter of the jet should not exceed 0.3 the width of the bucket.[1] If this ratio is exceeded the buckets are crowded and the hydraulic friction loss becomes excessive. It is evident also that there must be some relation between size of jet and the size of the wheel. For a given size jet there is no upper limit as to size of wheel so far as the hydraulics is concerned.

[1] W. R. Eckart, Jr., Proc. of Inst. of Mech. Eng. (London), Jan. 7. 1910.

In special cases, where a low r.p.m. was desired, diameters as large as 35 ft. have been used when the diameter of the jet was only a few inches. But there is a lower limit for the ratio of wheel diameter to jet diameter. Obviously the wheel could not be as small as the jet for instance. The considerations which influence this matter will be further considered in Chapter VII, but for the present it will be sufficient to state that a ratio as

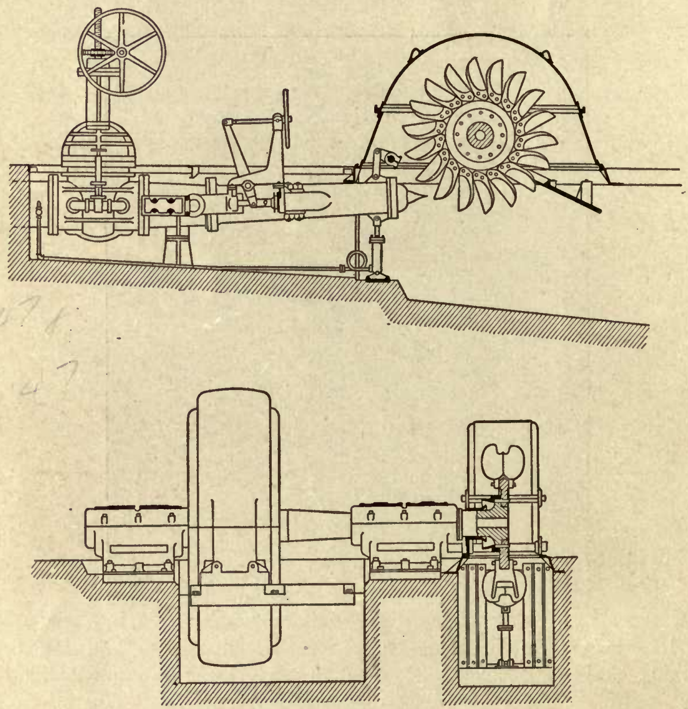


FIG. 19.—Tangential water wheel unit with deflecting nozzle.

low as 9 may be used without an excessive loss of efficiency.[1] (The nominal diameter is that of a circle tangent to the center line of the jet.) The more common value, and one which involves no sacrifice of efficiency, is 12. From that we get a very convenient rule that the diameter of the wheel in feet equals the diameter of the jet in inches. The size of jet necessary to develop a given amount of power under any head may be com-

[1] S. J. Zowski, "Water Turbines," published by Engineering Society, Univ. of Mich., 1910.

puted and then the diameter of wheel necessary is known at once.

The r.p.m. of the wheel can be computed by taking the peripheral speed as 0.47 of the jet velocity or 0.45 $\sqrt{2gh}$.

The use of one jet only upon a single wheel is to be preferred if it is possible. However, two jets are often used upon one wheel though at some sacrifice of efficiency. For a given size wheel the horse-power of one jet is limited by the maximum size of the jet that may be employed. If a greater horse-power is desired it is necessary to use two or more jets upon the one wheel or to use a larger wheel with a single jet. The larger wheel means a lower r.p.m. and a higher cost both of the wheel and the generator if a direct connected unit is used. In case this additional expense is not justified by the increased efficiency of the single jet wheel the duplex nozzle would be used.

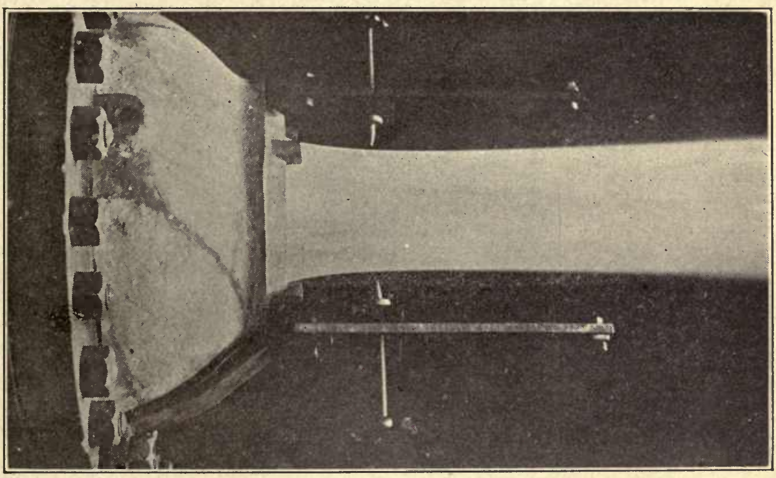


FIG. 20.—5286 h.p.Jet, from 7 1/2 in. needle nozzle. Head = 822 ft. Jet velocity = 227.4 ft. per second.

The tangential water wheel is almost always set with a horizontal shaft and, if direct connected to a generator, is overhung so that the unit has only two bearings (Fig. 19). It is quite common for two wheels to drive a single generator mounted between them in which case we have the double overhung type.

28. Speed Regulation.—Various means have been adopted to regulate the power input to the tangential water wheel but the following are the only ones that are of any importance. The use of any throttle valve in the pipe line is wasteful as it destroys a portion of the available head and thus requires more water to be used for a given amount of power than would otherwise be the case. The ideal mode of governing, so far as economy of water is concerned, would not affect the head but would merely vary the water used in direct proportion to the power demanded.

The needle nozzle (Fig. 21) accomplishes this result very nearly. As the needle is moved back and forth it varies the area of the opening and thus varies the amount of water discharged. The coefficient of velocity is a maximum when the nozzle is wide open but it does not decrease very seriously for the smaller nozzle openings. (See Fig. 62.) Thus the velocity of the jet is very nearly the same for all values of discharge. The efficiency of a well-constructed needle nozzle is very high, being from 95 to 98

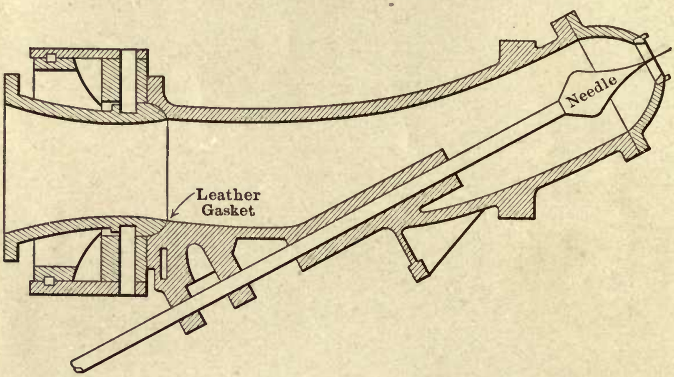


FIG. 21.—Deflecting needle nozzle. (*After drawing by Prof. W. R. Eckart, Jr.*)

per cent.[1] The needle nozzle is nearly ideal for economy of water but may not always permit close speed regulation. If the pipe line is not too long, the velocity of flow low, and the changes of load small and gradual, the needle nozzle may be very satisfactory. In case it is used the penstock is usually provided with a standpipe or a surge tank.

If the pipe line is long, the velocity of flow high, and the changes of load severe, dangerous water hammer might be set up if the discharge were changed too quickly. It might therefore be difficult to secure close speed regulation with the needle nozzle as the governors would have to act slowly. The deflecting nozzle, shown in Fig. 6, page 7, is much used for such cases. The

[1] W. R. Eckart, Jr., Inst. of Mech. Eng. (London), Jan. 7, 1910. *Bulletin* No. 6, Abner Doble Co.

nozzle is made with a ball-and-socket joint so that the entire jet can be deflected below the wheel if necessary. The governor sets the nozzle in such a position that just enough water strikes the buckets to supply the power demanded. The rest of the water passes below the buckets and is wasted. Since there is no change in the flow in the pipe line the governor may accomplish any degree of speed regulation desired as there is little limit to the rapidity with which the jet may be deflected. Such a nozzle is usually provided with a needle also which is regulated by hand. Fig. 21 is really of this type. The station attendant sets the needle from time to time according to the load that he expects to carry. However, the device is wasteful of water even with the best of attention. If other water rights prevent the flow of a stream from being interfered with it may be satisfactory.

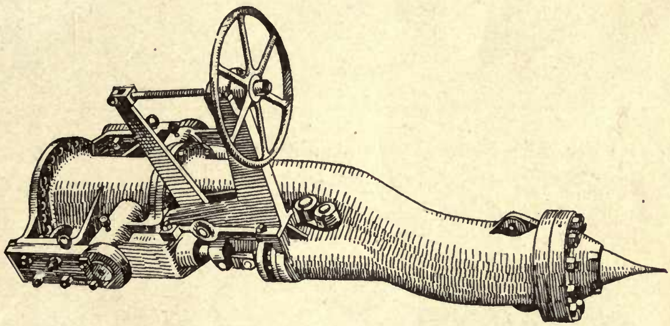


FIG. 22.—Deflecting needle nozzle for 8000 h.p. wheel.

The combined needle and deflecting nozzle may possess the advantages of both of the above types, by having the needle automatically operated. If the load on the wheel is reduced the governor at once deflects the jet thus preventing any increase of speed. Then a secondary relay device slowly closes the needle nozzle and, as it does so, the nozzle is gradually brought back to its original position where all the water is used upon the wheel. Thus close speed regulation is accomplished with very little waste of water.

The needle nozzle with auxiliary relief shown in Fig. 23 and Fig. 24. accomplishes the same results as the above. When the needle of the main nozzle is closed the auxiliary nozzle underneath it is opened at the same time. This discharges an equivalent amount of water which does not strike the wheel. This auxiliary nozzle

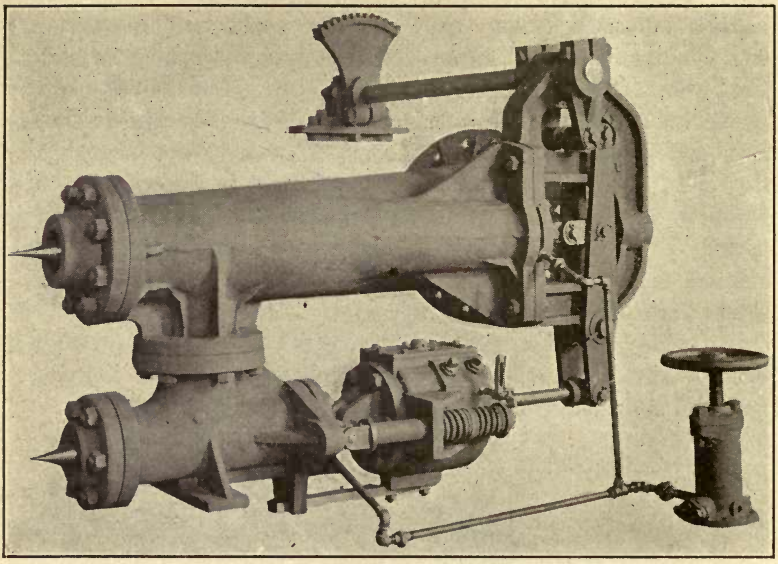


FIG. 23.—Pelton-Doble auxiliary relief needle nozzle.
(*Made by Pelton Water Wheel Co.*)

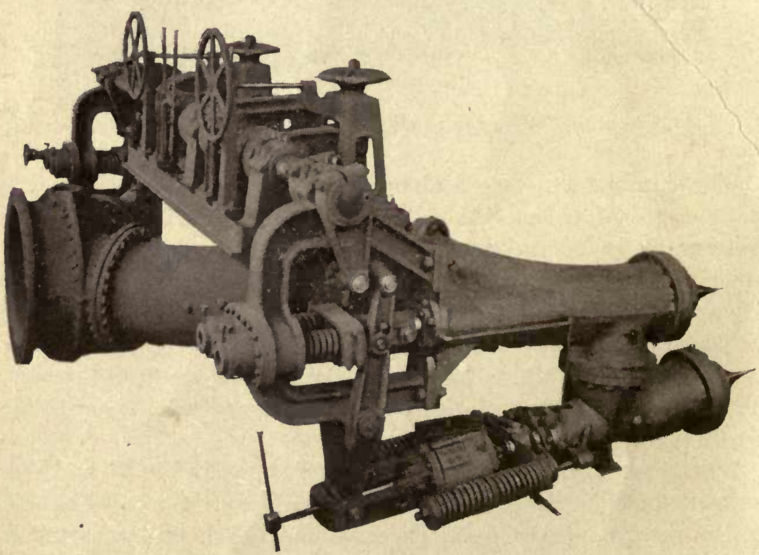


FIG. 24.—Pelton-Doble auxiliary relief needle nozzle for use with 10,000 kw. tangential water wheel. (*Made by Pelton Water Wheel Co.*)

is then slowly closed by means of a dash-pot mechanism. While both of these types relieve the pressure in case of a decreasing load they are unable to afford any assistance in the case of a rapid demand for water. The deflecting nozzle alone is the only type that is perfect there.

29. Conditions of Use.—The tangential water wheel is best adapted for high heads and relatively small quantities of water. By that is meant that the choice of the type of turbine is a function both of the capacity as well as the head. For a given head the larger the horse-power, the less reason there is for using this type of wheel.

In Switzerland a head as high as 3200 ft. has been used for a comparatively small horse-power.[1] In this country the highest head is 2100 ft. effective. This plant, which is in Colorado, has three 1800 h.p. Pelton-Doble tangential water wheels running at 400 r.p.m. In California are five Pelton-Doble wheels of 1800 h.p. under a head of 1960 ft. There are numerous cases of heads between 1000 and 2000 ft. but probably the majority of the installations are for heads of less than 1000 ft.

The largest power developed by a single jet upon a single wheel is 10,000 h.p. This is a Pelton-Doble wheel in California running at 400 r.p.m. under a head of 1528 ft. There are two such wheels in the plant.

The largest jet employed upon any Pelton-Doble wheel is 10 1/4 in. in diameter. The net head is 506 ft. in this case. There are a number of large jets of 9 in. or over used for heads from 900 to 1500 ft.

30. Efficiency.—The efficiency of the tangential water wheel is about the same as that of the average reaction turbine. From 75 to 85 per cent. may reasonably be expected though lower values are often obtained, due to poor design.

[1] A plant is now being built in Switzerland to utilize a head of 5412 ft. for 15,000 h.p. *Power*, May 27, 1913.

CHAPTER V

THE REACTION TURBINE

31. Development.—The primitive type of reaction turbine known as Barker's Mill is shown in Fig. 25. The reaction of the jets of water from the orifices causes the device to rotate. In order to improve the conditions of flow the arms were then curved and it became known in this form as the Scotch turbine. Then three or more arms were used in order to increase the power, and with still further demands for power more arms were added and the orifices made somewhat larger until the final result was a complete wheel. In 1826 a French engineer, Fourneyron, placed stationary guide vanes within the center to direct the water as it flowed into the wheel and we then had the outward flow turbine. In 1843 the first Fourneyron turbines were built in America.

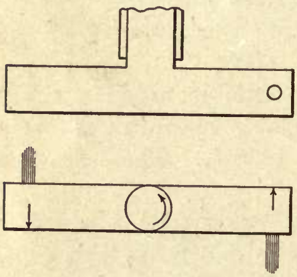


FIG. 25.—Barker's mill.

The axial flow turbine commonly called the Jonval was also an European design introduced into this country in 1850.

The first inward flow turbines were built in the United States in 1838 but they were very crude. The turbine designed by J. B. Francis (Fig. 5, page 4) in 1849 was such a vast improvement upon any others that he is rightly credited with being the originator of the modern inward flow turbine.

Modern turbines, however, differ somewhat from the original design of Francis. Since the water has to turn and flow axially after its discharge (Fig. 4, page 3), it was very natural to give the bucket the shape shown in Fig. 26, *I*. The flow is not purely radial but a trifle mixed.

The mixed flow turbine, often known as the American type, arose as the result of a demand for higher speed and power under the low falls first used in this country. Higher speeds could be obtained only by using runners of smaller diameter, but a smaller

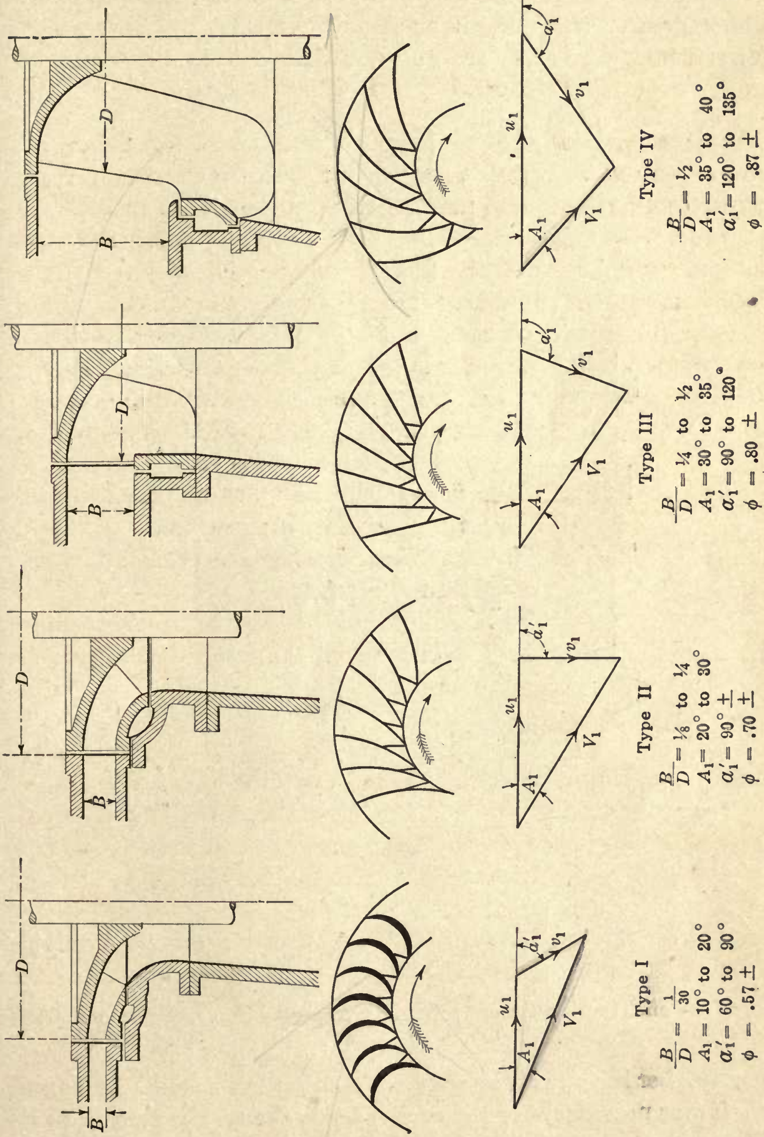


FIG. 26.—Types of runners.

diameter meant less power as long as the original designs were adhered to. So in order to increase the capacity of a runner of given diameter the design was altered by making the depth of the runner greater (*i.e.*, the dimension *B*, Fig. 26, was increased). Fewer vanes were also used and it was then desirable to extend

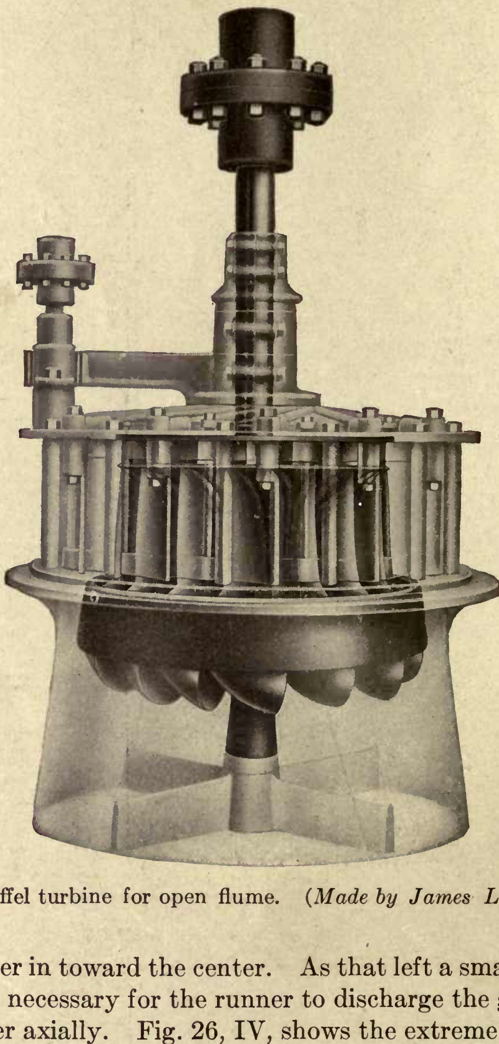


FIG. 27.—Leffel turbine for open flume. (*Made by James Leffel and Co.*)

them further in toward the center. As that left a small discharge area it was necessary for the runner to discharge the greater part of the water axially. Fig. 26, IV, shows the extreme high-speed, high capacity mixed-flow type of to-day.

As civilization moved from the valleys, where the low falls

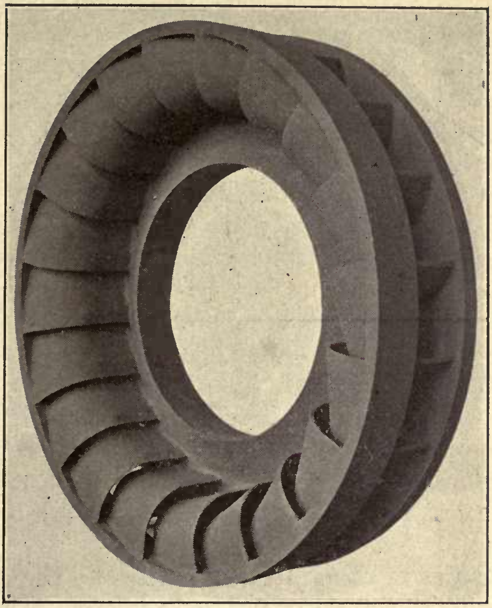


FIG. 28.—42″ Francis runner. 8000 h.p., 600 ft. head. (*Made by Platt Iron Works Co.*)

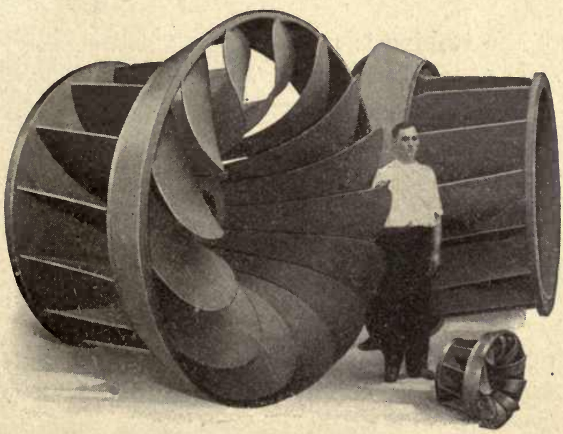


FIG. 29.—Turbine runners of the Allis-Chalmers Mfg. Co.

were found, up into the more mountainous regions, and as means of transmitting power were introduced, it became desirable to develop higher heads, and in 1890 a demand arose for high-head wheels which American builders were not able to supply. For a time European designs were used and then it was seen that the original Francis type was well suited for those conditions.

At present American practice is covered by the various types of runners shown in Fig. 26, all of which are known as Francis runners. American turbines have been developed by "cut and try" methods, European turbines largely by mathematical analysis. While good turbines have been produced by "cut and try" methods, yet they have often been sadly defective. One trouble has been that turbines have been used for certain speeds and powers when they were better suited for other speeds and powers. Europeans are now adopting our Pelton wheels and Francis turbines, while we are adopting their methods of applying mathematical analysis and designing the turbine to fit the plant.

32. Advantages of Inward Flow Turbine.—The Fourneyron turbine has a high efficiency on full load and is useful in some cases where a low speed is desired but it has been supplanted by the Francis turbine for the following reasons:

1. The inward flow turbine is much more compact, the runner can be cast in one piece, and the whole construction is better mechanically.

2. Since the turbine is more compact and smaller, the construction will be much cheaper. The smaller runner will permit of a higher r.p.m. and that means a cheaper generator can be used.

3. The gates for governing are more accessible and it is easier to construct them so as to minimize the losses. Thus the efficiency of the turbine on part load is better than is the case with the outward flow type.

4. It is easier to secure the converging passages that are necessary through the runner.

5. A draft tube can be more conveniently and effectively used.

33. Runners.—Runners may be cast solid or built up but the majority are cast solid as the construction is more substantial. Built up wheels have the vanes shaped from steel sheets and the crown, hubs, and rings are cast to them. The best runners are made of cast bronze. Cast steel is used for some high heads.

Cheaper runners are made of cast iron. Turbines are rated according to the diameter of the runner in inches. This diameter is easily fixed in many cases, but in the case of the type shown in Fig. 30 either one of four dimensions may be used. Different makers follow different practices in this regard.

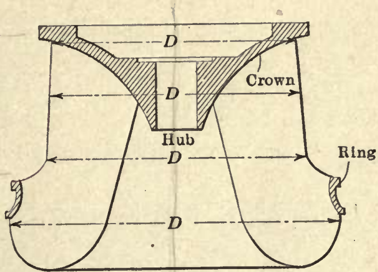


FIG. 30.—Methods of specifying runner diameter.

34. General Proportions.—For the notation used see Art. 42, page 49. The peripheral speed, u_1, may be expressed as $\phi\sqrt{2gh}$. For the reaction turbine the best wheel speed is given by values of ϕ ranging from 0.57 to 0.87[1] according to the design. If the value of ϕ is above 0.87 the speed is either higher than the best speed or the nominal diameter is higher than the real diameter.

The values of the angles used are indicated in Fig. 26. A further discussion of these points will be given in Chapter VIII. Other commercial constants will be given in Chapter X.

35. Comparison of Types of Runners.—As a means of illustrating the differences between the various types of runners the following tables are presented:

TABLE 1.—COMPARISON OF 12-IN. WHEELS UNDER 30-FT. HEAD

Type	Discharge, cu. ft. per minute	H.p.	R.p.m.
Tangential water wheel	7.9	0.37	380
Reaction turbines:			
Type I	99.0	4.3	460
Type II	329.0	14.9	554
Type III	741.0	33.4	600
Type IV	1209.0	55.5	730

[1] "Water Turbines," by S. J. Zowski is the source of much of Fig. 26.

TABLE 2.—COMPARISON OF WHEELS TO DEVELOP 15 H.P. UNDER 30-FT. HEAD

Type	Diameter	R.p.m.
Tangential water wheel	60 in.	55
Reaction turbines:		
Type I	21 in.	274
Type II	12 in.	554
Type III	8 in.	900
Type IV	6 in.	1460

It will be seen that the tangential water wheel is a low-speed, low-capacity type, while the reaction turbine of Type IV is a high-

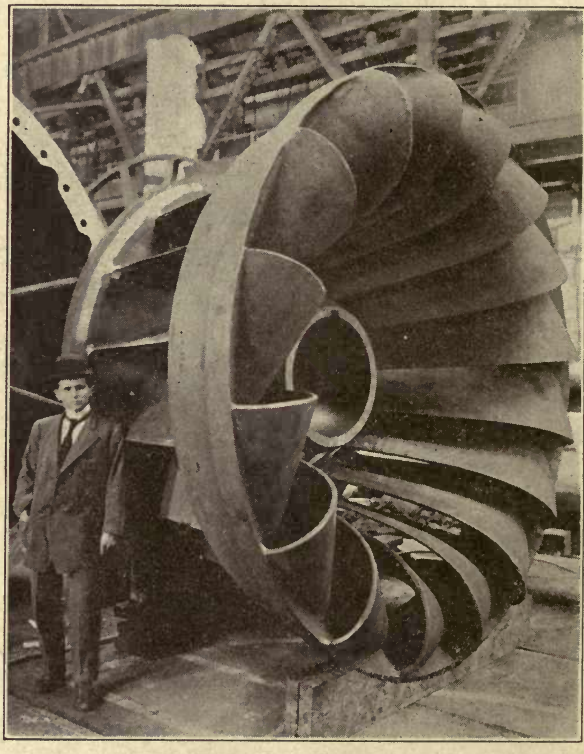


FIG. 31.—13,500 h.p. runner. Head = 53 ft., speed = 94 r.p.m. (*Made by I. P. Morris Co.*)

speed, high-capacity runner. This may be contrary to the popular impression but these terms as used here have only relative meanings. Under high heads where the r.p.m. would be naturally high the relatively lower speed of the tangential water wheel is of advantage, while under the low heads the relatively higher speed of the reaction turbine is of advantage. This difference of speed exists even when the runners are of the same diameter as

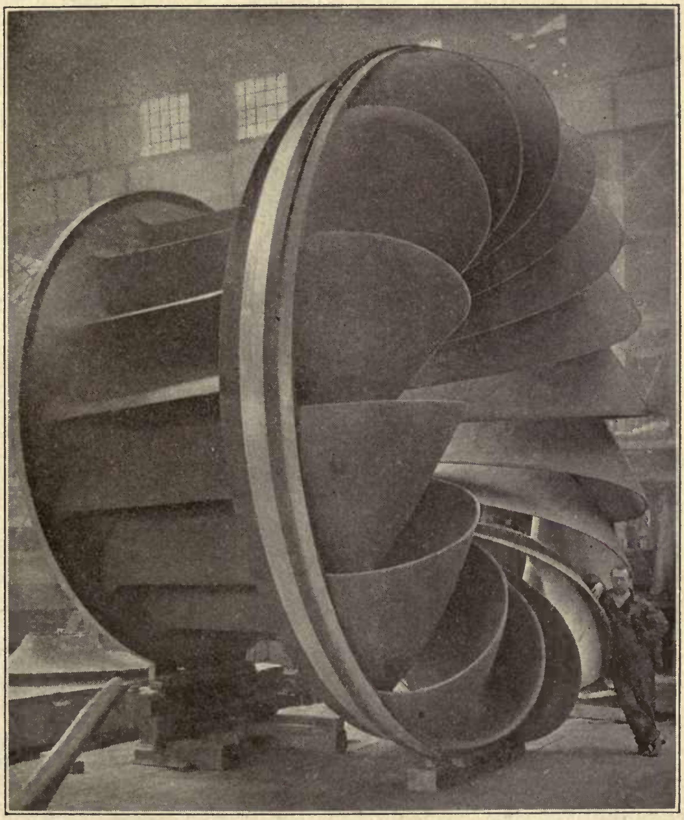


Fig. 32.—10,000 h.p. runner at Keokuk, Ia. Head = 32 ft., speed = 57.7 r.p.m. (*Made by Wellman-Seaver-Morgan Co.*)

seen by the first table. But when the diameters are made such as to give the same power as in the second table the difference becomes much greater. It must be understood that these tables do not prove one type of wheel to be any better than another but merely show what may be obtained. If the tangential water wheel or Type I of the reaction turbines appear in an unfavorable light it is only because the head and horse-power are not suitable for them.

36. Speed Regulation.—The amount of water supplied to the reaction turbine is regulated by means of gates of which there are three types.

The cylinder gate is shown in Fig. 5, page 4. It is the simplest and cheapest form of gate and also the poorest. When the gate is partially closed there is a big shock loss in the water entering the turbine runner due to the sudden contraction and the sudden expansion that must take place. With this type of gate the efficiency on part load is relatively low and the maximum efficiency is obtained when the gate is completely raised.

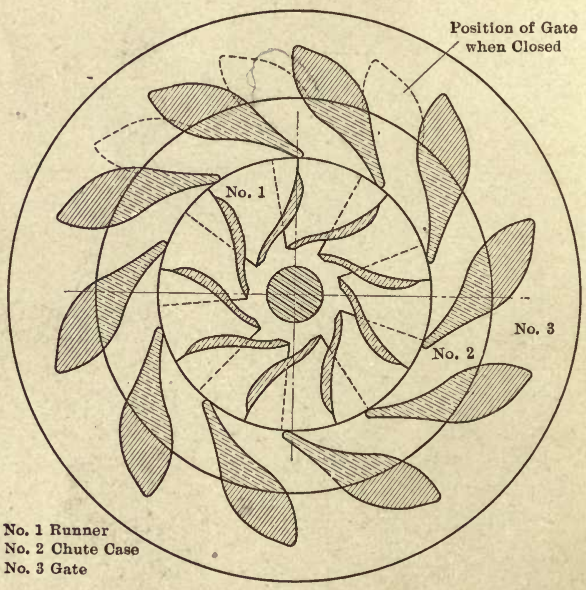


Fig. 33.—Register gate.

A better type of gate is the register gate shown in Fig. 33. With this type the guide vanes are made in two parts, the inner portion next to the runner is stationary, the outer portion is on a ring which may be rotated far enough to shut the water off entirely, if necessary, as shown by the dotted lines. While this is more efficient than the preceding type there is still a certain amount of eddy loss that cannot be avoided.

The wicket gate, also called the pivoted guide vane, is shown

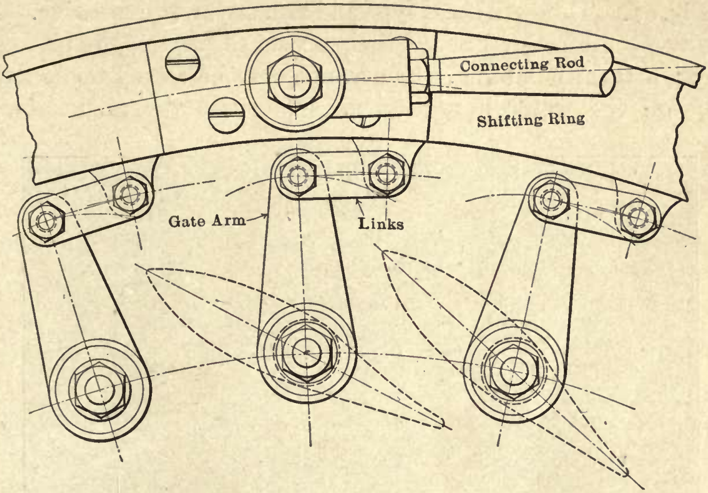


FIG. 34.—Wicket gate with all operating parts outside.

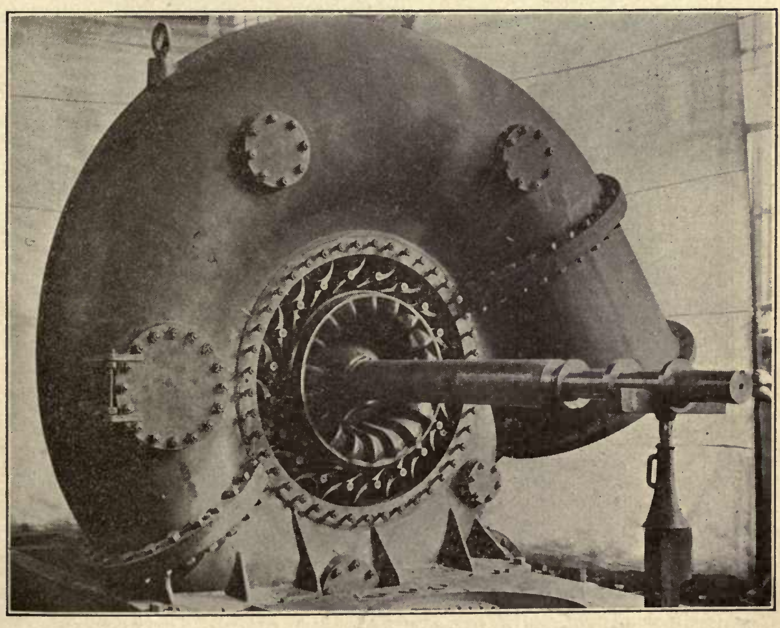


FIG. 35.—Wicket gates and runner in turbine made by Platt Iron Works Co.

in Fig. 34. This is the best type and also the most expensive. As the vanes are rotated about their pivots the area of the passages through them is altered. The vanes may be closed up so as to shut off the water if necessary. Of course the angle, A_1, is

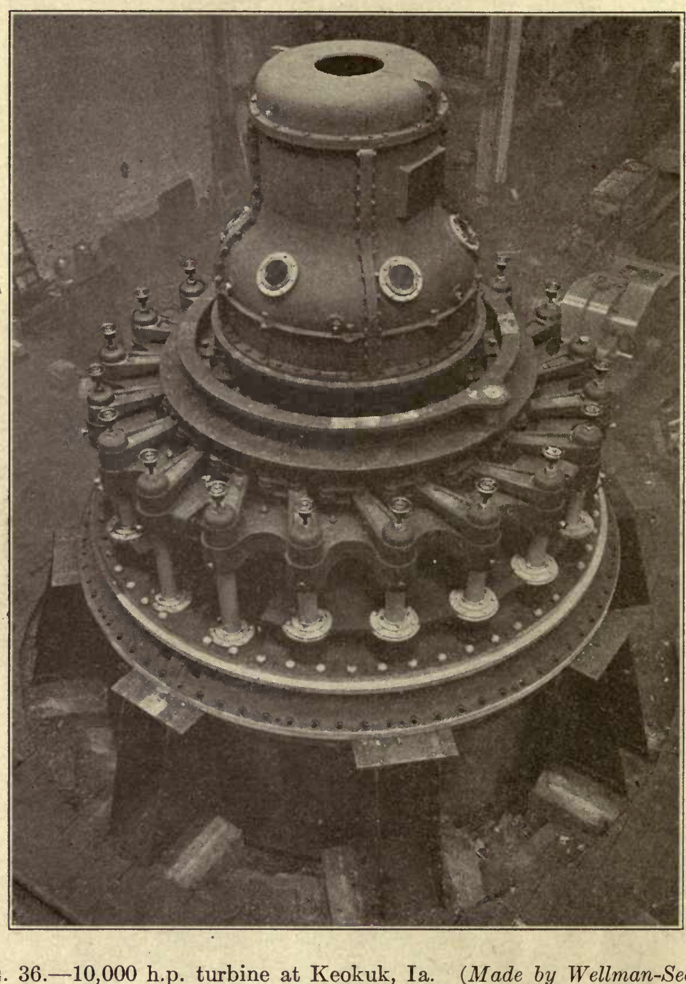


FIG. 36.—10,000 h.p. turbine at Keokuk, Ia. (*Made by Wellman-Seaver-Morgan Co.*)

altered and a certain amount of eddy loss may also result but it is less than occasioned by either of the other forms. The maximum efficiency is obtained before the gates are opened to the greatest extent.

The connecting rod from the relay governor operates a shifting ring. This in turn, by means of links, rotates the vanes. Often the shifting ring and links are inside the case. The better, though more expensive, type has the working parts outside the case. These links are shown in Fig. 36. and Fig. 37.

37. Bearings.—For small vertical shaft turbines a step bearing made of lignum vitæ is used under water. This wood gives good results for such service and wears reasonably well. It is also used for the outboard bearings of the propeller shafts of ships. For larger turbines a thrust bearing is usually provided to which oil is supplied under pressure. Such a bearing is often located at the

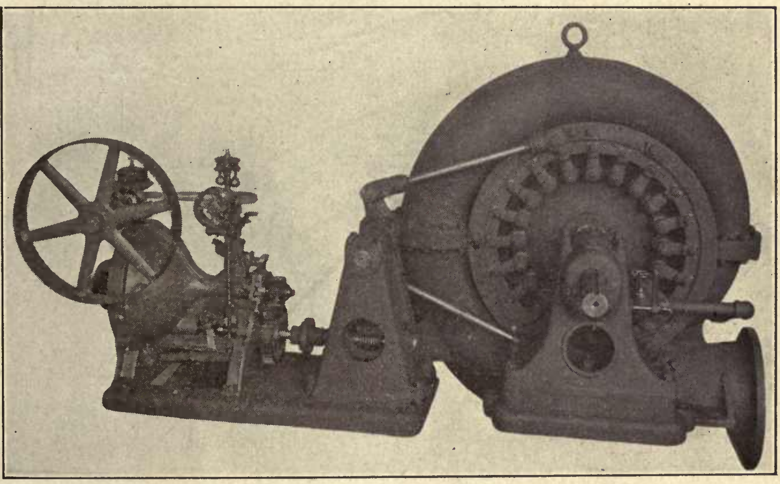


Fig. 37.—Shifting ring and links on a wicket gate spiral case turbine. (*Made by Platt Iron Works Co.*)

top of the shaft in which case it is called a suspension bearing. It is quite common also for a cylinder to be provided below the turbine, on the under side of which water is admitted under penstock pressure. The upward thrust on this cylinder is sufficient to take care of the greater portion of the weight of the revolving parts.

A horizontal turbine set in an open flume often has lignum vitæ bearings as the water is a sufficient lubricant. However the water must be clear; gritty water would destroy the bearings. If the turbine is in a case so that the bearings are accessible the usual types of bearings are used. It must not be forgotten that even though the shaft be horizontal a very considerable end thrust must be allowed for due to the reaction of the streams discharged from the runner. That is one reason for using runners in pairs. Also a single runner is often used which has a double discharge. (See Fig. 38.)

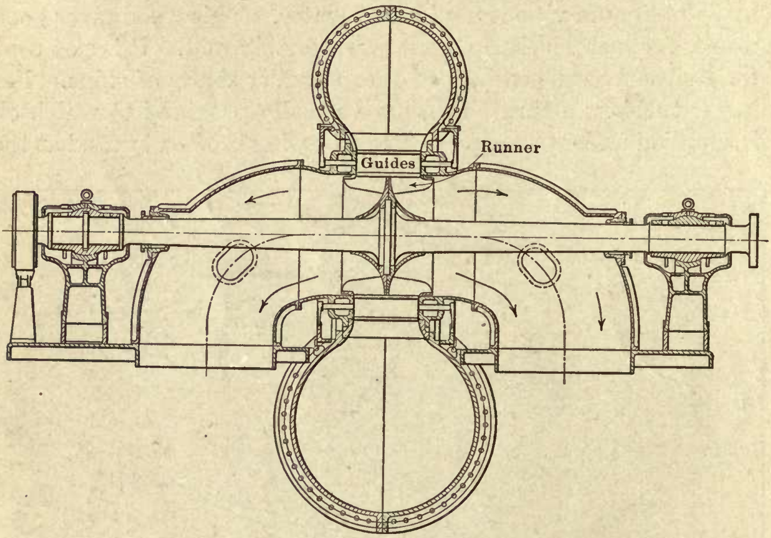


FIG. 38.—Double discharge runner in spiral case.

38. Cases.—For low heads turbines may be used in open flumes without cases. Fig. 39 shows such a type consisting of four wheels on a horizontal shaft.

Cases may also be used for very low heads and are always used for high heads. The cheapest cases are the cylinder cases (Fig. 8, page 9), and the globe cases (Fig. 40). These cases are undesirable because they permit of considerable eddy loss as the water flows into them and around in them to the guides. The cone case shown in Fig. 41 is a very desirable type. It can be seen that the water suffers no abrupt changes in velocity as it flows from the penstock to the guides, but instead is uniformly accelerated.

The spiral case, illustrated by Fig. 42, is considered the best type. The area of the waterway decreases as the case encircles the guides because only a limited portion of the water flows clear around to enter the further part of the circumference. Thus the

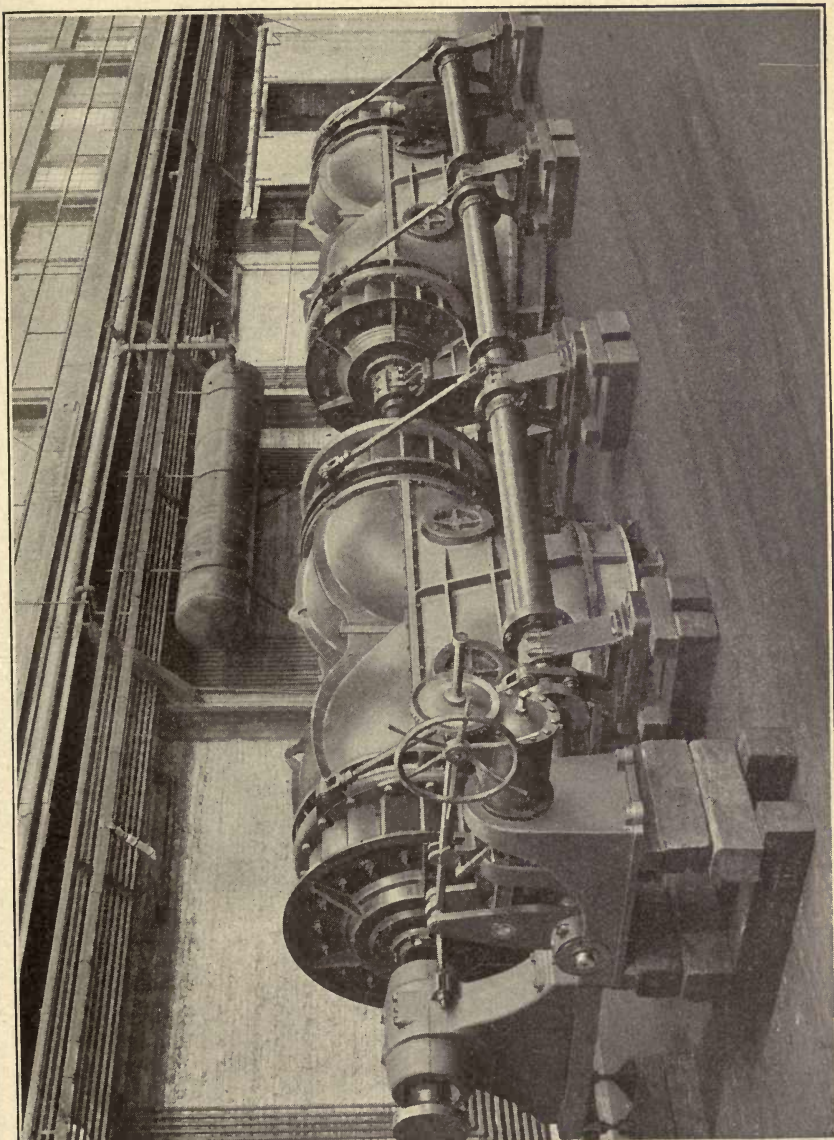


Fig. 39.—Allis-Chalmers turbine for open flume.

average velocity throughout the case is kept the same. The case is also designed to accelerate the water somewhat as it leaves the penstock and flows to the guides.

39. Velocities.—The velocities at different points are indicated by Fig. 44.[1] The velocity of flow in the penstock is determined by the consideration of the cost and other conditions in each

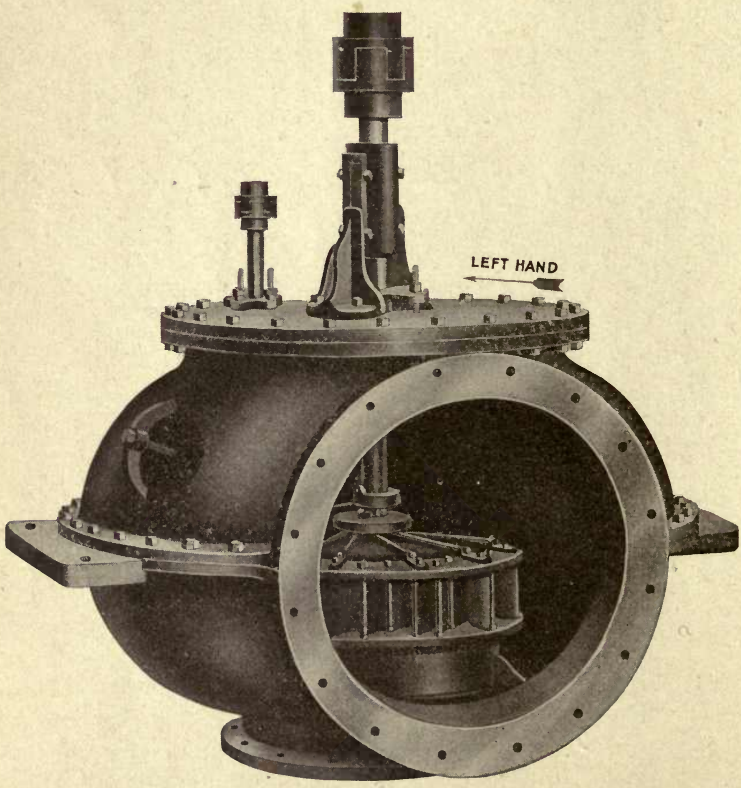


FIG. 40.—Turbine in globe case. (*Made by James Leffel and Co.*)

case. The mean velocity of flow allowable in the turbine case is as follows:

If the case is cylindrical the velocity should be as low as 0.08 to 0.12. $\sqrt{2gh}$ where h is the effective head. If a spiral case is used the velocity may be from 0.15 to $0.24\sqrt{2gh}$. For heads of several hundred feet the value of 0.15 is used to reduce wear on

[1] Mead's "Water Power Engineering."

the case, 0.20 is used for moderate heads, and 0.24 is used for low heads.

The velocity at entrance to the turbine runner, $V_1 = 0.6$ to

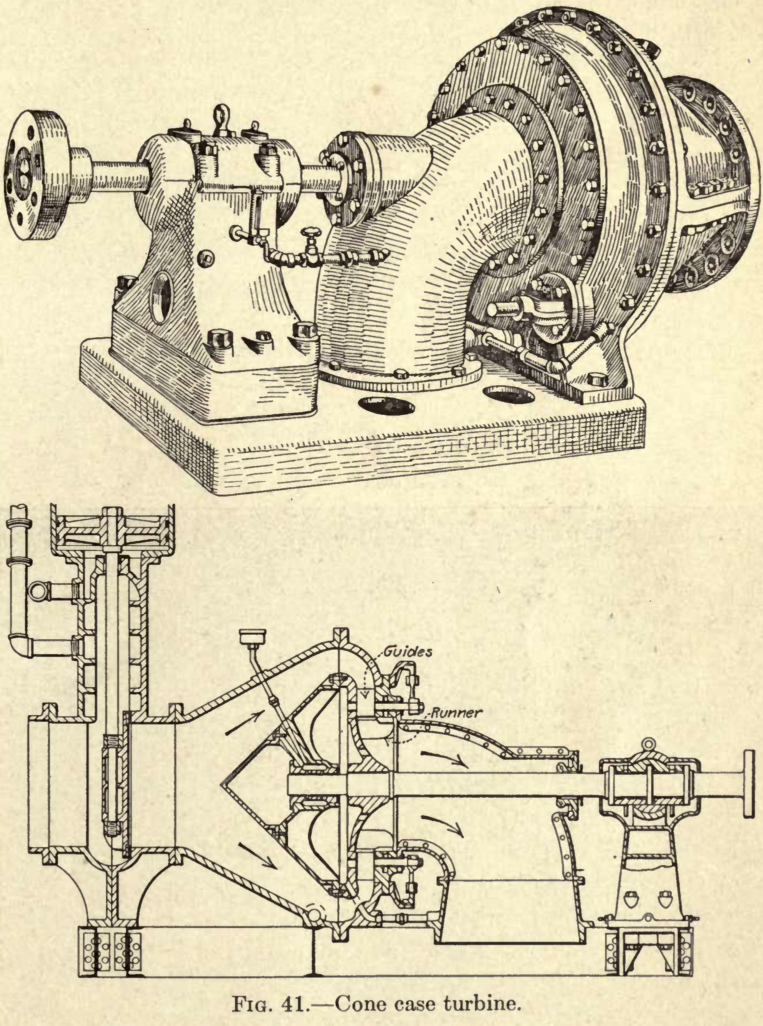


Fig. 41.—Cone case turbine.

$0.8\sqrt{2gh}$. The velocity at the point of discharge, $V_2 =$ from 0.22 to $0.30\sqrt{2gh}$. These values depend entirely upon the design of the turbine and are not arbitrarily assigned.

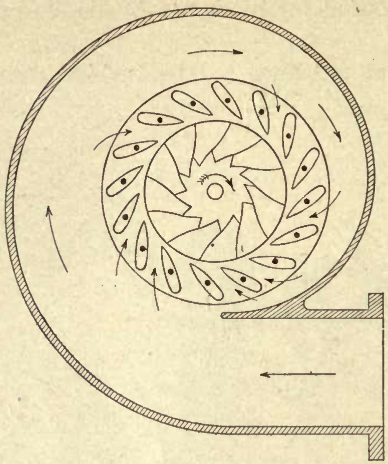


FIG. 42.—Spiral case turbine.

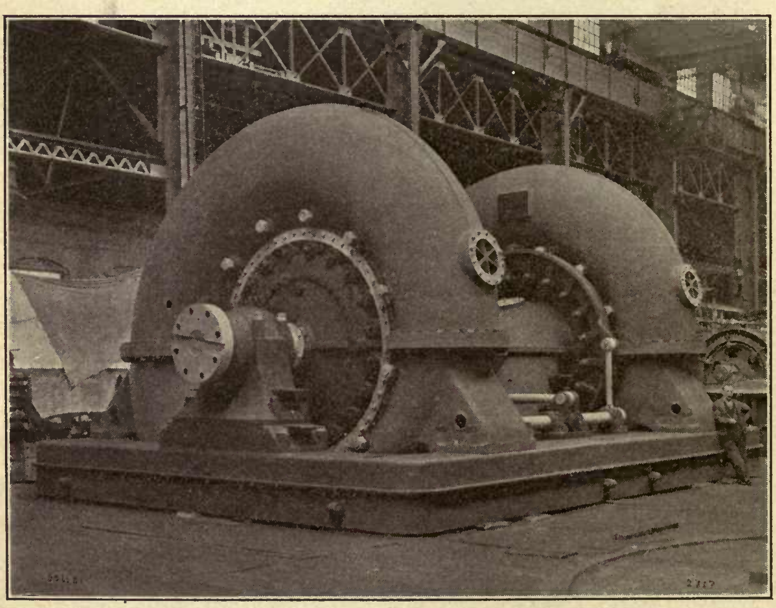


FIG. 43.—Cast steel spiral casings at Niagara Falls. 14,000 h.p. at 180 ft. head. (*Made by Wellman-Seaver-Morgan Co.*)

The velocity at entrance to the upper end of the draft tube should equal the velocity with which the water leaves the turbine, otherwise a sudden change in velocity will take place. Velocity of discharge from the lower end of the draft tube may be about 0.10 to $0.15\sqrt{2gh}$. The value of the latter is determined by the value of the velocity at the upper end and by the length and the amount of flare to be given the tube. The flare should not exceed 1 ft. in diameter for every 3 ft. in length.

40. Conditions of Use.—The reaction turbine is best adapted for a low head or a relatively large quantity of water. As was stated in Art. 29, the choice of a turbine is a function of capacity as well as head. For a given head the larger the horse-power the more reason there will be for using a reaction turbine.

The use of a reaction turbine under high heads is accompanied

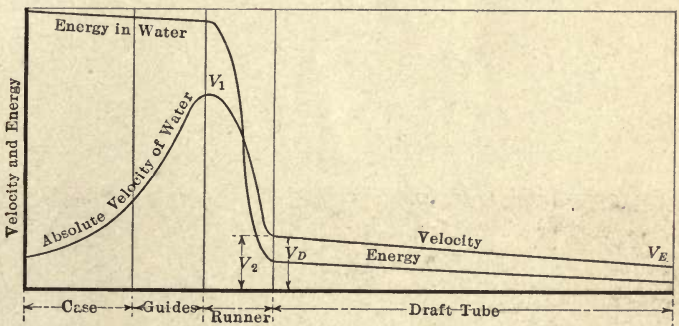


FIG. 44.—Velocity and energy transformations in turbine.

by certain difficulties. It is necessary to build a case which is strong enough to stand the pressure; also the case, guides, and runner may be worn out in a short time by the water moving at high velocities. This depends very much upon the quality of the water. Thus a case is on record where a wheel has been operating for six years under a head of 260 ft. with clear water and the turbine is still in excellent condition. Another turbine made by the same company and according to the same design was operated under a head of 160 ft. with dirty water. In four years it was completely worn out and was replaced with an impulse wheel. The tangential water wheel has the advantage that the relative velocity of flow over its buckets is less for the same head and thus the wear is less. Also repairs can be more readily made.

The conditions which cause the runners of reaction turbines

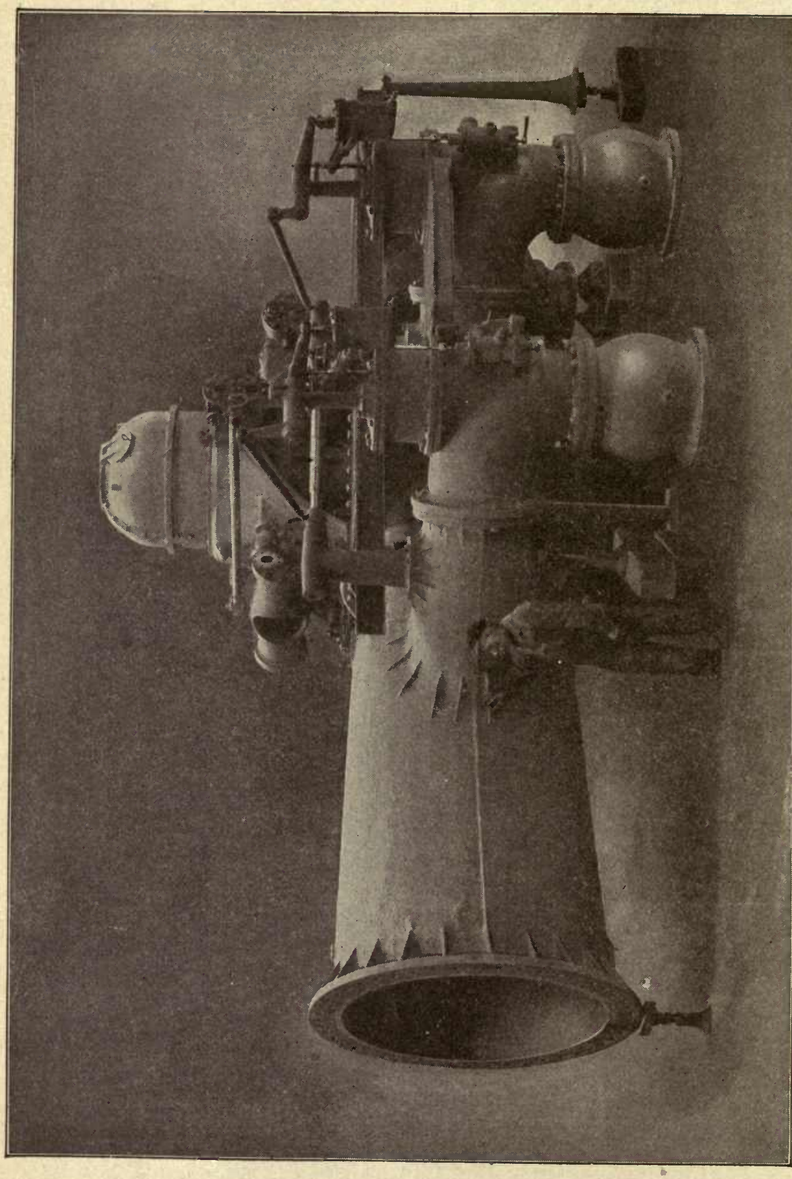


FIG. 45.—13,000 h.p. vertical shaft, spiral case turbine. (*Made by Allis Chalmers Mf. Co.*)

and the buckets of tangential water wheels to wear out are not entirely understood. In some cases it has been laid to electrolysis but careful investigations have eliminated that factor in other cases. In one instance where there was no electrolysis and the water was clear the runners lasted only a few months under a 600-ft. head. Both steel and bronze runners were tried. This pitting that takes place may be due to oxidation and will be favored if the design is not entirely correct. An improper design will cause a certain amount of eddying and this will tend to liberate the air in solution in the water. This erosion that takes place is chemical in its nature and is different from mechanical abrasion.

Reaction turbines are used under heads as low as 16 in. though several feet is the usual minimum. The highest head yet employed for a reaction turbine is 670 ft. The latter is used for two 6000 h.p. units installed by the I. P. Morris Co. in Mexico. The speed is 514 r.p.m.

The most powerful turbine unit so far built will develop 25,000 h.p. with the maximum gate opening. This unit, however, consists of two runners upon one shaft and discharging into a common draft tube. The head is 168 ft. and the speed 200 r.p.m. It was built by the I. P. Morris Co. for the Washington Water Power Co.

The greatest power developed by a single runner is 22,500 h.p. The head is 480 ft. and the speed 360 r.p.m. See Fig. 46. This wheel was built by the Allis-Chalmers Co. for the Pacific Coast Power Co. It is a double discharge turbine having two draft tubes.

The largest single discharge turbine is of 20,000 h.p. capacity. The I. P. Morris Co. are building several such units and the Pelton Water Wheel Co. are also building one of that power for operation at 514 r.p.m. under 487 ft. head. The unbalanced end thrust when a single draft tube is used is a big problem.

The power of a turbine depends not only upon its size but also upon the head under which it operates. The turbines listed above are the most powerful but they are not the largest in point of size. The largest turbines so far are the 10,000 h.p. turbines at Keokuk, Ia. The I. P. Morris Co. is building eight of these and the Wellman-Seaver-Morgan Co. seven. The diameter of the runners is about 16 ft., the height 12 ft., and the weight 140,000 lb. The total weight of the rotating parts, including the generator is 550,000 lb. The average head is 30 ft. and the speed 57.7 r.p.m. (Fig. 32 and Fig. 36).

The I. P. Morris Co. has recently received a contract for nine 10,800 h.p. turbines to run at 56 r.p.m. under 30 ft. head. These

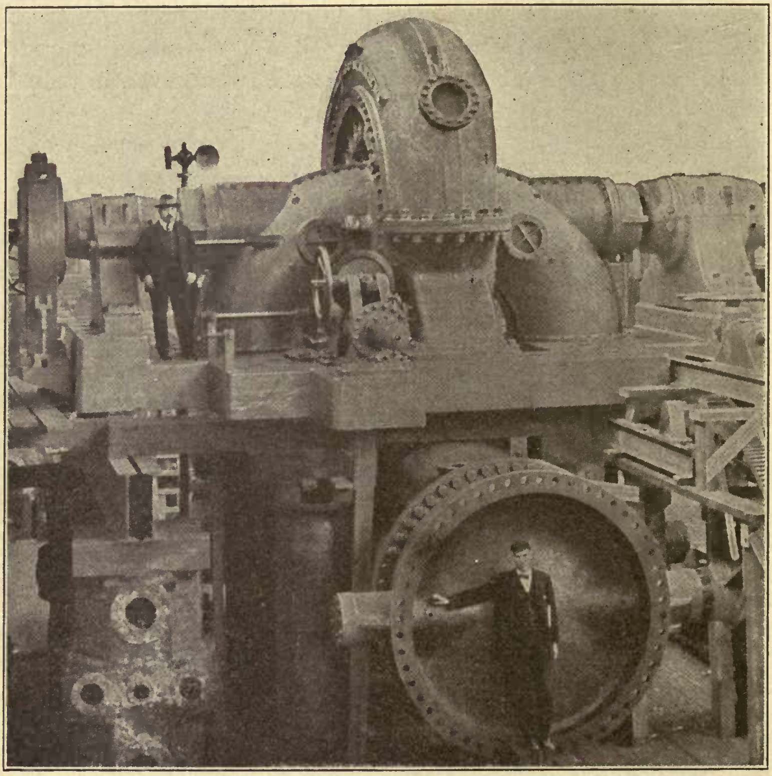


FIG. 46.—22,500 h.p. turbine for Pacific Coast Power Co. (*Made by Allis-Chalmers Mfg. Co.*)

will exceed in size those at Keokuk and will be the largest in the world. Both these cases are vertical shaft turbines.

41. Efficiency.—The efficiency obtained from the average reaction turbine may be from 80 to 85 per cent. Under favorable conditions with large capacities higher efficiencies up to about 90 per cent. may be realized. For small powers or unfavorable conditions 75 per cent. is all that should be expected.

CHAPTER VI

GENERAL THEORY

42. Notation.—The following notation will be employed:

V = absolute velocity of water (or relative to earth)
v = velocity of water relative to the wheel.
u = linear velocity of a point on the wheel.
r = radius to any point from axis of rotation (in feet).
z = elevation above any datum plane.
A = angle between V and u.
a = angle between v and u.
s = tangential component of $V = V \cos A$.
F = area of stream normal to the direction of flow in the stationary passages in square feet.
f = area of stream normal to the direction of flow in the rotating passages in square feet.
D = diameter of runner in inches.
q = cubic feet per second.
w = weight of a cubic foot of water (taken as 62.5 lb.).
W = pounds of water per second $= wq$.
p = pressure in pounds per square foot.
h = effective head on wheel.
h'' = head utilized by wheel.
h' = head lost in wheel.
H = total head $= z + \frac{V^2}{2g} + \frac{p}{w}$
x = ratio r_2/r_1
y = ratio F_1/f_2
ϕ = ratio of peripheral speed, u_1 to $\sqrt{2gh}$
N = revolutions per minute.
ω = angular velocity (radians per second.) $= 2\pi N/60 = \frac{u}{r}$

Suffix (1) refers to the stream leaving the guides and entering the runner, suffix (2) refers to the stream leaving the wheel.

By F or f will be meant the total area of all the parts into which the stream may be divided. Thus F_1 will equal the total area of all the streams in the guide passages normal to V_1, and f_2 will equal the total area of all the streams leaving the runner measured normal to v_2.

The equation of continuity holds throughout the stream so that $q = FV = fv$, and in particular

$$q = F_1V_1 = f_1v_1 = f_2v_2 \tag{1}$$

43. Relation between Absolute and Relative Velocities.—The absolute velocity of a body is its velocity relative to the earth. The relative veclocity of a body is its velocity relative to some other body which may itself be in motion relative to the earth. The absolute velocity of the first body is the vector sum of its velocity relative to the second body and the velocity of the second body. The relation between the three velocities u, v, V is shown

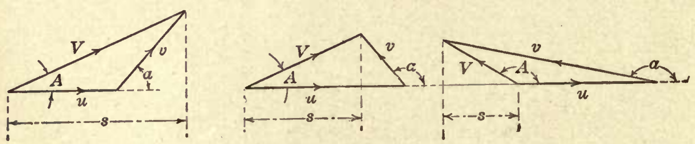


FIG. 47.—Relation between relative and absolute velocities.

by the vector triangles in Fig. 47. The tangential component of V is

$$s = V \cos A = u + v \cos a \tag{2}$$

44. The General Equation of Energy.—Energy may be transmitted across a section of a flowing stream in any or all of the three forms known as potential energy, kinetic enegy, or pressure energy.[1] Since head is the amount of energy per unit weight of water, the total head at any section

$$H = z + \frac{V^2}{2g} + \frac{p}{w} \tag{3}$$

There can be no flow without some loss of energy so that the total head must decrease in the direction of flow by the amount of head lost or

$$H_1 - H_2 = \text{Head lost} \tag{4}$$

Suffixes (1) and (2) may here denote any two points.

In flowing through the runner of a turbine the water gives up energy to the vanes in the form of mechanical work and a portion

[1] L. M. Hoskins, "Hydraulics." Chapter IV.

of the energy is lost in hydraulic friction and is dissipated in the form of heat. Thus the head lost by the water equals $h''+h'$. And if suffixes (1) and (2) are restricted to the meanings given them in Art. 42 equation (4) may be written

$$\left(z_1+\frac{V^2_1}{2g}+\frac{p_1}{w}\right)-\left(z_2+\frac{V^2_2}{2g}+\frac{p_2}{w}\right)=h''+h' \qquad (5)$$

45. Effective Head on Wheel.—Obviously the turbine should

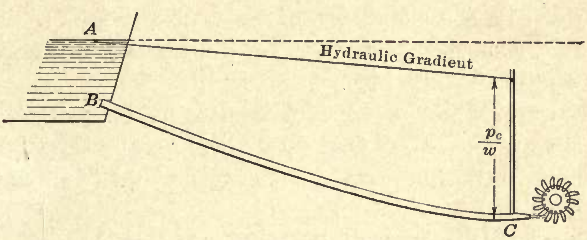


FIG. 48.—Effective head for tangential water wheel.

not be charged up with head which is lost in the pipe line, so the value of h should be the total fall available minus the penstock losses. In the case of the tangential water wheel (Fig. 48)

$$h=H_c=\frac{V^2_c}{2g}+\frac{p_c}{w} \qquad (6)$$

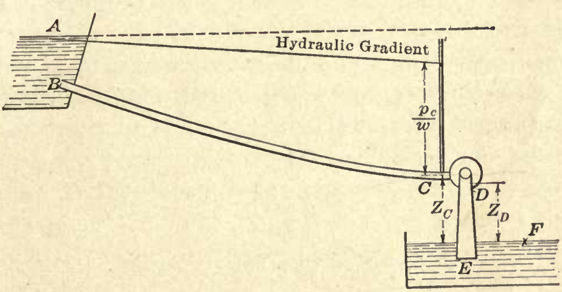


FIG. 49.—Effective head for reaction turbine.

In the case of the reaction turbine (Fig. 49) it may be taken as

$$h=H_c-H_D=\left(z_c+\frac{V^2_c}{2g}+\frac{p_c}{w}\right)-\left(z_D+\frac{V^2_D}{2g}+\frac{p_D}{w}\right) \qquad (7)$$

In general $\frac{p_D}{w}$ will be less than atmospheric pressure and will thus be negative if pressures are measured relative to the atmospheric pressure. This method gives the net head supplied to the runner but since the draft tube is usually regarded as much a part of the turbine as the case it is customary to charge the draft tube loss against the turbine. Thus

$$h = H_c - H_F = \left(z_c + \frac{V^2_c}{2g} + \frac{p_c}{w} \right) - 0 \qquad (8)$$

Since $H_D - H_F$ = Loss in draft tube + Discharge loss, it is evident that (8) gives a somewhat higher value of h then (7). However (8) is usually used in tests because the turbine maker generally designs the draft tube also.

46. Power.—Since head is the amount of energy per unit weight of water it follows that by multiplying by the total weight of water per unit time we have energy per unit time and this is power. Thus

$$\text{Power} = WH = \text{pounds per second} \times \text{feet} \qquad (9)$$

In this expression H may be interpreted as in (3) or it may be replaced by h'' or any other head according to what is wanted.

But also power equals force applied times the velocity of the point of application. Thus

$$\text{Power} = Pu = \text{pounds} \times \text{feet per second} \qquad (10)$$

where P represents the total force applied.

Torque, G, equals $P \times r$ and angular velocity $\omega = \frac{u}{r}$

Since then $Pu = G\omega$ it is evident that

$$\text{Power} = G\omega = \text{foot pounds per second} \qquad (11)$$

Any of these three expressions for power may be used according to circumstances. While (10) is the most obvious to many, it will be found that in hydraulics (9) is usually more convenient.

(The following simplifications for horse-power of a turbine are convenient. Using the h of Art. 45,

$$\text{h.p.} = 62.5\ qhe/550 = qhe/8.8.$$

For estimations, the value of the efficiency may be assumed as 0.80 in which case our expression becomes h.p. $= qh/11$.)

47. Force Exerted.—Whenever the velocity of a stream of water is changed either in direction or in magnitude a force is required. Since force equals mass times acceleration we may write

$$P = m \frac{\Delta V}{\Delta t}$$

where m denotes the mass and $\triangle V$ the average acceleration. If the acceleration is a variable quantity the above expression will merely give the average value of the force during the time $\triangle t$. If the acceleration is constant the average value will be its true value at any instant of time. In our work here we shall commonly deal only with cases of constant accelerations and conse-

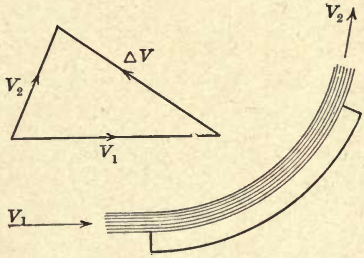


FIG. 50.

quently constant values of P. If the time $\triangle t$ is taken as one second, and if the mass, W/g, is taken as the mass of water per second we may write[1]

$$P = \frac{W}{g} \triangle V \tag{12}$$

48. Torque Exerted.—On account of difficulty in determining the point of application of the force in (12), the equation giving the value of the torque will be found more useful. Momentum mV is a vector quantity and by multiplying it by a distance its moment may be obtained. The moment of momentum is sometimes called angular momentum. By taking moments in (12) we have the moment of the force equal to the increment of angular momentum, or

$$Pr = \frac{W}{g} \triangle (Vr).$$

Let MN (Fig. 51) represent a curved vane which rotates about an axis O perpendicular to the plane of the figure. Water strikes vane at M with an absolute velocity V_1 at angle A_1, and leaves at N with an absolute velocity V_2 at angle A_2. Distances OM and ON will be denoted by r_1 and r_2, respectively, and the linear velocities of M and N by u_1 and u_2 respectively. The momentum of a

[1] L. M. Hoskins, "Hydraulics,." Chapter XIV.

particle of water of mass m striking the vane at M is mV_1. This may be resolved into two components, one radial and one tangential. The momentum about O of the radial component is zero, the moment of the tangential component is $mV_1 \cos A_1 r_1$. In like manner the angular momentum of the water leaving the vane at N is $mV_2 \cos A_2 r_2$. If the total mass of the water per

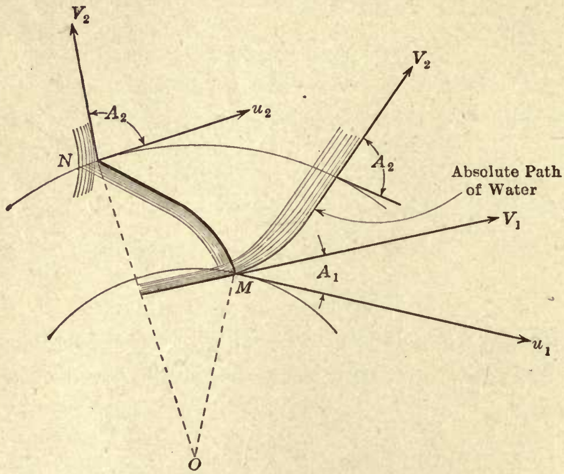


FIG. 51.

second equals W/g then the total moment of the forces exerted upon the water by all the vanes equals

$$\frac{W}{g}(V_2 \cos A_2 r_2 - V_1 \cos A_1 r_1).$$

The moment of the forces exerted by the water upon the vanes is opposite in sign to this, or

$$G = \frac{W}{g}(V_1 \cos A_1 r_1 - V_2 \cos A_2 r_2).$$

Since $s = V \cos A$ this may be expressed as

$$G = \frac{W}{g}(r_1 s_1 - r_2 s_2) \tag{13}$$

A little thought will show that s_2 is greatly affected by the speed of the wheel. It will then be seen that the torque has its greatest value when the wheel is at rest and as the speed increases the value of G continually decreases.

It is readily seen from (13) that s_1 will be the greatest when $A_1 = O^0$. However the area F_1 is proportional to the sine of A_1 so that if the angle were made zero the area would be zero. Therefore the angle is made as small as possible and still give the required area.

Since $s_2 = u_2 + v_2 \cos a_2$ it will be seen that s_2 becomes smaller (algebraically) as a_2 approaches 180°. However the angle cannot become 180° on account of the fact that a finite area f_2 must be provided.

Note that the water always flows from point (1) to point (2). The inner radius is r_1 only for the outward flow turbine; for the inward flow turbine r_1 would be greater than r_2.

49. Power Delivered to Runner.—If the flow is steady and the speed of the wheel uniform an expression for the power developed by the water may readily be obtained.

From (9) and (11)

$$\text{Power} = Wh'' = G\omega$$

Using the value of G given by (13)

$$Wh'' = \frac{W}{g}(r_1 \quad s_1 - r_2 \quad s_2)\omega$$

Eliminating the W and noting that $u = r\omega$ we have the head utilized by the wheel

$$h'' = \frac{1}{g}(u_1 s_1 - u_2 s_2) \tag{14}$$

The power delivered to the runner as mechanical work is

$$Wh'' = \frac{W}{g}(u_1 s_1 - u_2 s_2) \tag{15}$$

The power output of the turbine is less than this by an amount equal to the friction of the bearings and other mechanical losses. The power given by (15) corresponds to the indicated power of a steam engine.

50. Equation of Energy for Relative Motion.—Using the value of h'' given by (14) in (5) we have

$$\left(z_1 + \frac{V^2{}_1}{2g} + \frac{p_1}{w}\right) - \left(z_2 + \frac{V^2{}_2}{2g} + \frac{p_2}{w}\right) = \frac{1}{g}(u_1 s_1 - u_2 s_2) + h'$$

All of the absolute velocities will be replaced in terms of relative velocities as follows:

$$V^2_1=v^2_1+u^2_1+2u_1v_1 \cos a_1$$
$$V^2_2=v^2_2+u^2_2+2u_2v_2 \cos a_2$$
$$s_1=u_1+v_1 \cos a_1$$
$$s_2=u_2+v_2 \cos a_2$$

The substitution of these values gives us

$$\left(z_1+\frac{v^2_1-u^2_1}{2g}+\frac{p_1}{w}\right)-\left(z_2+\frac{v^2_2-u^2_2}{2g}+\frac{p_2}{w}\right)=h' \qquad (16)$$

This equation serves to establish a relation between points (1) and (2). If the wheel is at rest u_1 and u_2 become zero, v_1 and v_2 become absolute velocities and equation (16) becomes the equation of energy in its usual form as in (4).

51. Impulse Turbine.—The following numerical solution is given to illustrate the application of the foregoing principles. This impulse turbine is of the outward flow type known as the

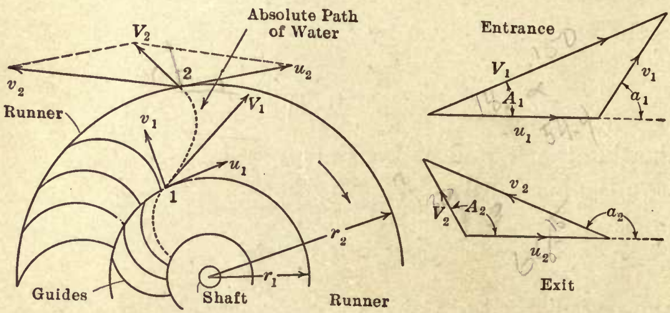


FIG. 52.—Outward flow turbine.

Girard. Obviously the direction of flow makes no difference in the theory.

By construction, $A_1=18°$, $a_2=165°$, $r_1=2.0$ ft., $r_2=2.5$ ft. The hydraulic friction loss in flow through the runner will be taken as proportional to the square of the relative velocity so that $h'=k\frac{v_2^2}{2g}$, where k is an empirical constant. Assume $k=0.4$. Suppose $h=350$ ft., $N=260$ r.p.m., $q=100$ cu. ft. per second. Find relative velocity at entrance to runner, relative velocity and magnitude and direction of absolute velocity at exit from runner, head utilized by wheel, hydraulic efficiency, losses, and the horse-power. (See Fig. 52.)

$V_1=\sqrt{2gh}=8.025\sqrt{350}=150$ ft. per second.
$u_1=2\pi r_1 N/60=54.4$ ft. per second.
$u_2=(r_2/r_1)u_1=68.0$ ft. per second.
By trigonometry $v_1=99.55$ ft. per second.

Suppose the flow is in a horizontal plane so that $z_1=z_2$. Since it is an impulse turbine the pressure throughout the runner will be atmospheric. Thus $p_1=p_2$.
Equation (16) then becomes

$$(1+k)v_2^2=v_1^2+u_2^2-u_1^2$$
$$1.4\ v_2^2=9910+4624-2960=11,574$$
$$v_2=90.9 \text{ ft. per second.}$$

By trigonometry $V_2=30.6$ ft. per second, $A_2=130°$
$s_1=V_1\cos A_1=150\times0.951=143$
$s_2=u_2+v_2\cos a_2=68.0-90.9\times0.966=-19.7$
(Also $s_2=V_2\cos A_2=30.6\times(-0.639)=-19.7$)

$$h''=\frac{1}{g}(u_1s_1-u_2s_2)$$
$$=\frac{1}{32.2}(54.4\times143+68\times19.7)=283 \text{ ft.}$$

Hydraulic efficiency $=h''/h=283/350=0.81$
Hydraulic friction loss $=k\frac{v_2^2}{2g}=0.4\frac{8270}{64.4}=51.3$ ft.
Discharge loss $=\frac{V_2^2}{2g}=\frac{940}{64.4}=14.6$ ft.
Power $=\frac{Wh''}{550}=\frac{100\times62.5\times283}{550}=3220$ h.p.

52. Reaction Turbine.[1]—Another numerical case will be given to illustrate the application of the foregoing principles to the reaction turbine. The turbine used here is the Fourneyron or outward flow type, though the theory applies to any type.

By construction, $A_1=18°$, $a_2=165°$, $r_1=2.0$ ft., $r_2=2.5$ ft., $F_1=1.36$ sq. ft., $f_2=1.425$ sq. ft. Assume $k=0.2\left(h'=k\frac{v_2^2}{2g}\cdot\right)$ Suppose $h=350$ ft., $N=525$ r.p.m., and $q=164.5$ cu. ft. per second. Find head utilized by turbine, hydraulic efficiency,

[1] See Art. 7. If the area f_2 is made small enough the wheel passages will be completely filled with water under pressure. We then have a reaction turbine. Note that

$$H_1=\frac{V_1^2}{2g}+\frac{p_1}{w}, \text{ so that } V_1 \text{ is not equal to } \sqrt{2gH_1}.$$

losses, pressure at guide outlets (entrance to turbine runner), and the horse-power.

Since the wheel passages are completely filled the areas of the streams, F_1 and f_2 are known, thus

$V_1 = q/F_1 = 164.5/1.36 = 121$ ft. per second.

$v_2 = (F_1/f_2)V_1 = 115.5$ ft. per second.

For the above r.p.m. $u_1 = 110$ ft. per second, $u_2 = 137.5$ ft. per second.

$s_1 = V_1 \cos A_1 = 115.$

$s_2 = u_2 + v_2 \cos a_2 = 137.5 - 115.5 \times 0.966 = 26.0.$

$h'' = \frac{1}{g}(u_1 s_1 - u_2 s_2) = \frac{1}{32.2}(110 \times 115 - 137.5 \times 26) = 282$ ft.

Hydraulic efficiency $= 282/350 = 0.805.$

Hydraulic friction loss $= k\frac{v_2^2}{2g} = 0.2\frac{13350}{64.4} = 41.5$ ft.

By trigonometry $V_2 = 41$ ft. per second.

Discharge loss $= \frac{V_2^2}{2g} = \frac{1680}{64.4} = 26$ ft.

Since v_2 is determined by the area f_2 we do not have the use for equation (16) that we did in the case of the impulse turbine. By it, however, we can compute the difference in pressure between entrance to and discharge from the runner. Thus from (16), taking $z_1 = z_2$,

$$\frac{p_1}{w} - \frac{p_2}{w} = \frac{(1+k)v_2^2 - u_2^2 - v_1^2 + u_1^2}{2g} = 122 \text{ ft.}$$

(If the turbine discharges into the air then $\frac{p_2}{w} = O$ and $\frac{p_1}{w} = 122$ ft.) This pressure difference may also be computed from equation (5).

$$\text{Power} = \frac{Wh''}{550} = \frac{62.5 \times 164.5 \times 282}{550} = 5270 \text{ h.p.}$$

53. Effect of Speed.—As the speed of the wheel, u_1, varies, v_1, v_2, V_2, h'', efficiency, and horse-power vary. Fig. 53 shows the velocity diagrams at entrance and discharge for three different speeds. It is usually assumed that the best speed (giving the highest efficiency) is such that the discharge loss is the least. It can be seen that V_2 is very small either when $v_2 = u_2$ or $A_2 = 90°$. A means of determining the speed necessary to accomplish this will be given later.

54. PROBLEMS

1. For the impulse turbine in Art. 51 it will be found that $v_2 = u_2$ when $u_1 = 68.4$ ft. per second. Find the r.p.m., efficiency, losses, and horse-power. Compare with values given in Art. 51.

Ans. 326 r.p.m., $e = 0.845$, 3365 h.p.

2. For the reaction turbine in Art. 52 it will be found that $A_2 = 90°$ if $u_1 = 86.3$ ft. per second. At that speed the rate of discharge will be found (by method given later) to be 159 cu. ft. per second. Find the r.p.m., efficiency, losses, and horse-power. Compare with values given in Art. 52.

Ans. 412 r.p.m., $e = 0.852$, 5380 h.p.

3. Compare the best r.p.m. of the impulse turbine with the best r.p.m. of the reaction turbine in Problems (1) and (2). Compare the values of v_2 in Problems (1) and (2).

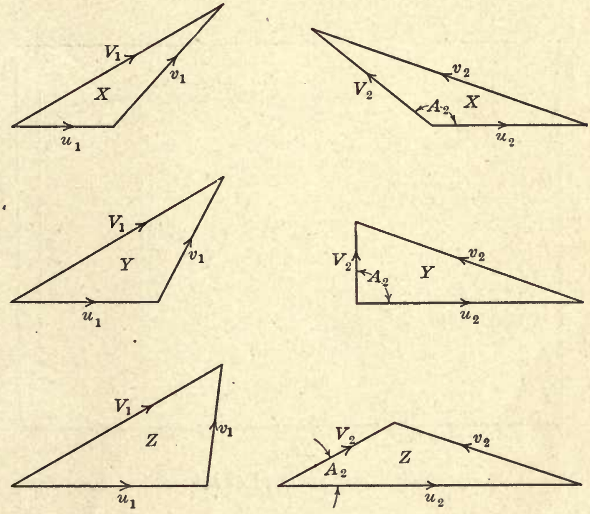


FIG. 53.—Velocity diagrams for three different speeds.

55. Relative Efficiencies.—For the best efficiency to be obtained the water must enter the runner without shock and leave with as little velocity as possible.

(a) In order to enter without shock the vane angle must agree with the angle a_1 and the quantity of water should be sufficient to fill the space in the runner if it is a reaction turbine.

(b) In order to leave with as little velocity as possible v_2 and u_2 should be approximately equal and nearly opposite in direction.

(c) With the reaction turbine V_2 can be reduced to some extent by a diverging draft tube. All hydraulic friction and eddy losses should be minimized by smooth vanes.

The first requirement as to vane angle does not apply in the case of the tangential water wheel as a consideration of the shape of the bucket will show. Neither does the other point of the quantity of water have any application in an impulse turbine since the passages are never completely filled. But with the reaction turbine an improper speed will cause shock loss due to an abrupt change in the direction of the water entering the runner, for the angle a_1 will vary with the speed but the vane angle, a'_1, is fixed. If the turbine runs at such a speed that there is no shock loss at full gate a partial closure of the gates will produce a loss. In all reaction turbines V_1 is increased by partially closing the gates and if u_1 is constant a_1 must change. With the wicket

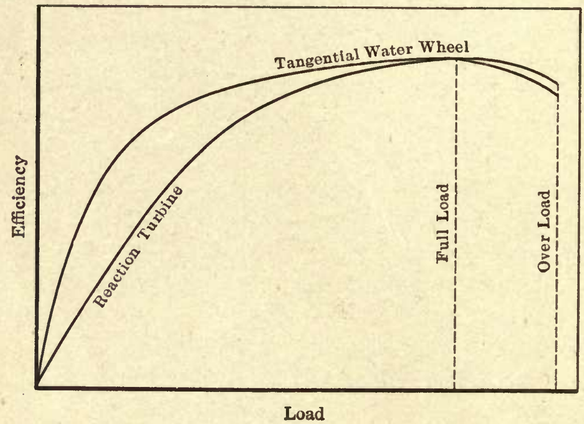


FIG. 54.—Relative efficiencies on part load of impulse and reaction turbines.

gates A_1 may change also. In addition to this a smaller quantity of water than normal must fill the usual bucket areas so that not only is a_1 suddenly changed to a'_1 but also v_1 must be suddenly changed to v'_1. It is thus apparent that the reaction turbine suffers shock losses when operated on part gate that cannot occur with the tangential water wheel.

With the tangential water wheel operated at constant speed the value of v_2 is approximately the same for all amounts of water supplied. But with the reaction turbine, since $q=f_2v_2$ and f_2 is fixed, it follows that on part load, when q is decreased, v_2 must be decreased also. If u_2 has its normal value it is evident that V_2 must be increased.

It is thus apparent that if a tangential water wheel and a reaction turbine have the same efficiency on full-load that the tangential water wheel will have the higher efficiency on part-load. (See Fig. 54.)

CHAPTER VII

THEORY OF THE TANGENTIAL WATER WHEEL

56. Introductory.[1]—The tangential water wheel has been classed as an impulse turbine with approximately axial flow. The term tangential is applied because the center line of the jet is tangent to the path of the center of the buckets. In this article the assumption will therefore be made that $A_1 = O^0$ and that $r_1 = r_2$. It will be shown later that these assumptions are not entirely correct.

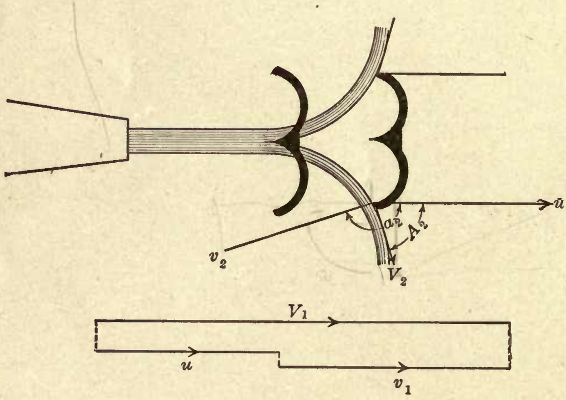


FIG. 55.

If the angle A_1 be assumed equal to zero then u_1 and V_1 are in the same straight line and $v_1 = V_1 - u_1$. The conditions at exit from the buckets are shown in Fig. 55. In applying equation (12) we desire to find only the component of the force tangential to the wheel since that is all that is effective in producing rotation. Therefore we shall find only the component of $\triangle V$ along the direction of u_1. Thus

$$P = \frac{W}{g}(V_1 - V_2 \cos A_2)$$

$$= \frac{W}{g}(V_1 - u_2 - v_2 \cos a_2)$$

[1] For notation see Art. 42.

By equation (16) since $z_1 = z_2$, $p_1 = p_2$, $u_1 = u_2$, $(1+k)v_2^2 = v_1^2$,

$$v_2 = \frac{v_1}{\sqrt{1+k}} = \frac{V_1 - u_1}{\sqrt{1+k}}$$

Substituting this value of v_2 we obtain

$$P = \frac{W}{g}\left(1 - \frac{\cos a_2}{\sqrt{1+k}}\right)(V_1 - u_1) \qquad (17)$$

A more exact value for the force exerted may be found in Art. 59. The above is only an approximation.

57. The Angle A_1.—The angle A_1 is usually not zero as can be seen from Fig. 56. One bucket will be denoted by B and the

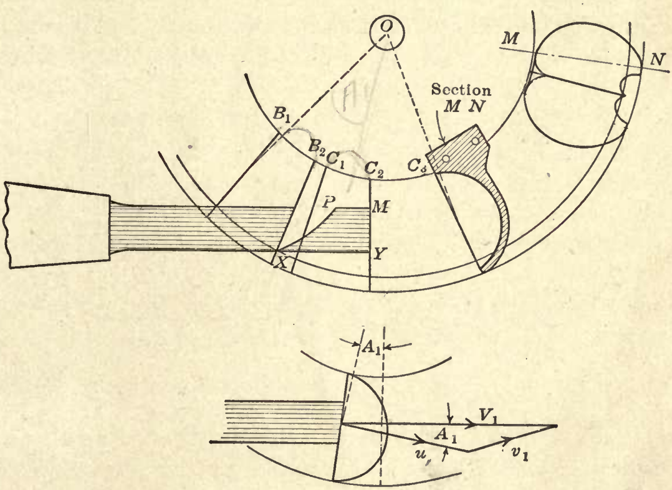
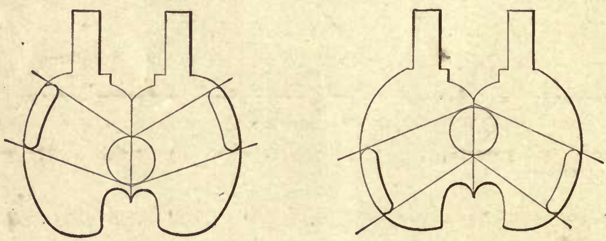


FIG. 56.

bucket just ahead of it by C. Different positions of these buckets will be denoted by suffixes. The bucket enters the jet when it is at B_1 and begins to cut off the water from the preceding bucket C_1. When the bucket reaches the position B_2 the last drop of water will have been cut off from C_2, but there will be left a portion of the jet, $MPXY$, still acting upon it. The last drop of water X will have caught up with this bucket when it reaches position C_3. Thus while the jet has been striking it the bucket has turned through the angle B_1OC_3. The average value of A_1 will be taken as the angle obtained when the bucket occupies the mean between these two extreme positions. It is evident that position C_3 will depend upon the speed of the wheel, and that the faster the wheel goes the farther over will C_3 be. Thus the angle A_1 decreases as the speed of the wheel increases. The variation in the value of A_1 as worked out for one particular case is shown in Fig. 58.

58. The Ratio of the Radii.—It is usually assumed that $r_1 = r_2$. However inspection of Fig. 57 (*a*) will show that when the bucket first enters the jet r_2 may be less than r_1. When the bucket has gotten further along r_2 may be greater than r_1. The value of $x(=r_2/r_1)$ depends upon the design of the buckets, and its determination is a drafting-board problem which is not within the scope of this book. It is evident that a value of x must be a mean in the same way that a value of A_1 is a mean. And just as A_1 varies with the speed, so also does x vary with the speed. A little thought will show that when the wheel is running slowly compared with the jet velocity that the value of x will be less than when the wheel is running at a high speed. This may be verified by actual observation. When the wheel is running at its proper speed it is probably true that x is very nearly equal to unity. In any case the variation of the value of x from unity cannot be very great. Nevertheless if it is desired to connect the theory up closely with the facts it is necessary to consider it.



(*a*) (*b*)

FIG. 57.—Radii for different bucket positions.

59. Force Exerted.—If the radii are not equal it will not be convenient to use equation (12) on account of the difficulty of locating the line of action of the force. However we can use equation (13) and by it determine a force at the radius r_1 which shall be the equivalent of the real resultant force. Dividing (13) by r_1 we obtain

$$P = \frac{W}{g}(s_1 - x s_2)$$

$$s_1 = V_1 \cos A_1$$

$$s_2 = u_2 + v_2 \cos a_2.$$

By equation (16)

$$(1+k)v_2^2 = v_1^2 + u_2^2 - u_1^2.$$

By trigonometry

$$v_1^2 = V_1^2 - 2V_1u_1 \cos A_1 + u_1^2.$$

Substituting this value of v_1, and with $u_2 = xu_1$,

$$(1+k)v_2^2 = V_1^2 - 2V_1u_1 \cos A_1 + x^2u_1^2.$$

Substituting this in the expression for s_2 we obtain

$$P = \frac{W}{g}\left[V_1 \cos A_1 - x^2u_1 - \frac{x \cos a_2}{\sqrt{1+k}}\sqrt{V_1^2 - 2V_1u_1 \cos A_1 + x^2u_1^2}\right] \quad (18)$$

Equation (18) is a true expression for the force exerted. No great error is involved, however, by taking $x = 1.0$. If that is done the expression under the radical becomes the value of v_1 and may be found graphically. For the sake of simplicity and ease in computation A_1 may be taken equal to zero and the equation then reduces to (17), but an exact value of P will not be obtained. There is little excuse for taking $k = 0$, as most writers do, for equation (17) is not simplified to any extent and the results are entirely incorrect.

60. Power.—With P as obtained from (18) the power is given by Pu_1. We may also compute h'' and obtain the power by multiplying by W.

Since $h'' = \frac{1}{g} u_1 (s_1 - xs_2)$ it is evident that the expression for h'' is the same as (18) if u_1 be substituted for W. Thus the expression for power has the same value no matter from which basis it is derived.

61. The Value of W.—W is the total weight of water striking the wheel per second. It is obvious that the weight of water discharged from the nozzle is

$$W = w F V_1.$$

Under normal circumstances all of this water acts upon the wheel. However for high values of the ratio u_1/V_1 a certain portion of the water may go clear through without having had time to catch up with the bucket before the latter leaves the field of action. It is apparent, for instance, that if the buckets move as fast as the jet none of the water will strike them at all. For all speeds less than that extreme case a portion of the water only may fail to act.

The third equation from the bottom of page 64 should read:

$$P = \frac{W}{g}(s_1 - xs_2)$$

Thus referring to Fig. 56, it can be seen that if the wheel speed is high enough compared to the jet velocity the water at X may not have time to catch up with bucket C. The variation of W with speed is shown in a particular case by Fig. 58.

It may also be seen that the larger the jet compared to the diameter of the wheel the sooner will this loss begin to occur and it is not desired to have it occur until the normal wheel speed is exceeded. Thus there is a limit to the size of jet that may be used as stated in Art. 27.

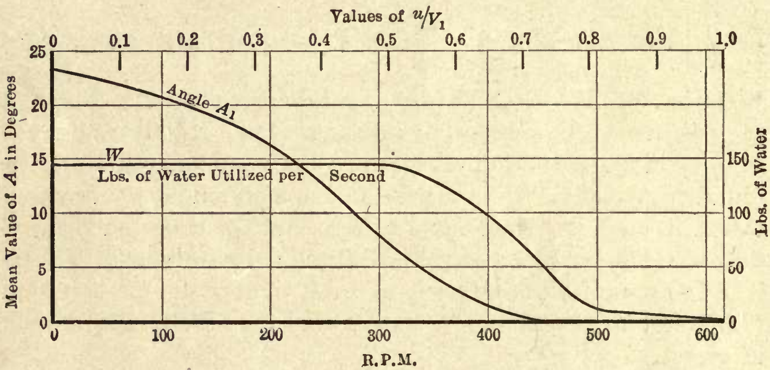


FIG. 58.—Values of A_1 and W for a certain wheel.

62. The Value of k.—The value of k is purely empirical and must be determined by experiment. If the dimensions of the wheel are known and the mechanical friction and windage losses are determined or estimated, then from the test of the wheel the horse-power developed by the water may be obtained. The value of k is then the only unknown quantity and may be solved for. The value of k is probably not constant for all values of u_1/V_1. Some theoretical considerations, which need not be given here, have indicated that it could scarcely be constant and an experimental investigation has shown the author that k decreased as u_1/V_1 increased. For a given wheel speed however it is nearly constant for various needle settings unless the jet exceeds the limit set in Art. 27. The crowding of the bucket then increases the eddy losses and would require a higher value of k.

The value of k may be as high as 2.0 but the usual range of values is from 0.5 to 1.5.

63. Constant Input—Variable Speed.—The way the torque and power vary with the speed for different needle settings is shown by Fig. 59 and Fig. 60. With the wheel at rest the torque may vary within certain limits as is shown by the curve for full nozzle opening. This is due to differences in A_1, and x for various positions of the buckets. With a given nozzle opening the horse-power input is fixed and constant. The horse-power output varies with the speed. It will be noticed that the maximum efficiency is attained at slightly higher speeds for the larger nozzle openings than for the smaller.

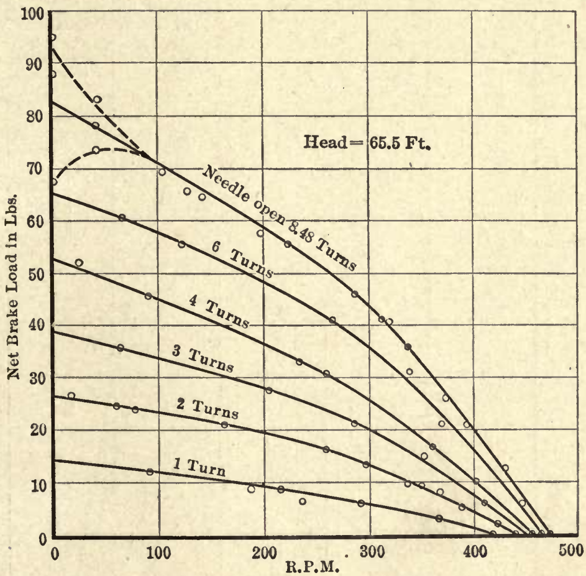


Fig. 59.—Relation between torque and speed.

Fig. 61 shows the variation of the various losses for a constant power input but a variable speed.[1]

64. Best Speed.—It is usually assumed that the best speed is the one for which the discharge loss is the least. As shown in Art. 53, that condition will be attained approximately either when $u_2 = v_2$ or when $A_2 = 90°$. In the case of the impulse turbine the former is the easier assumption. It will be found that $u_2 = v_2$ if u_1 is found from

$$k\,x^2 u_1^2 + 2V_1 u_1 \cos A_1 - V_1^2 = 0^2.$$

[1] The curves shown in this chapter are from the test of a 24-in. tangential water wheel by F. G. Switzer and the author.

[2] L. M. Hoskins, "Hydraulics," Art. 198, Art. 208.

An inspection of the curves in Fig. 61 will show that the highest efficiency is not obtained when the discharge loss is the least. So that, although the difference is not great, the above equation does not give the best speed. The hydraulic friction

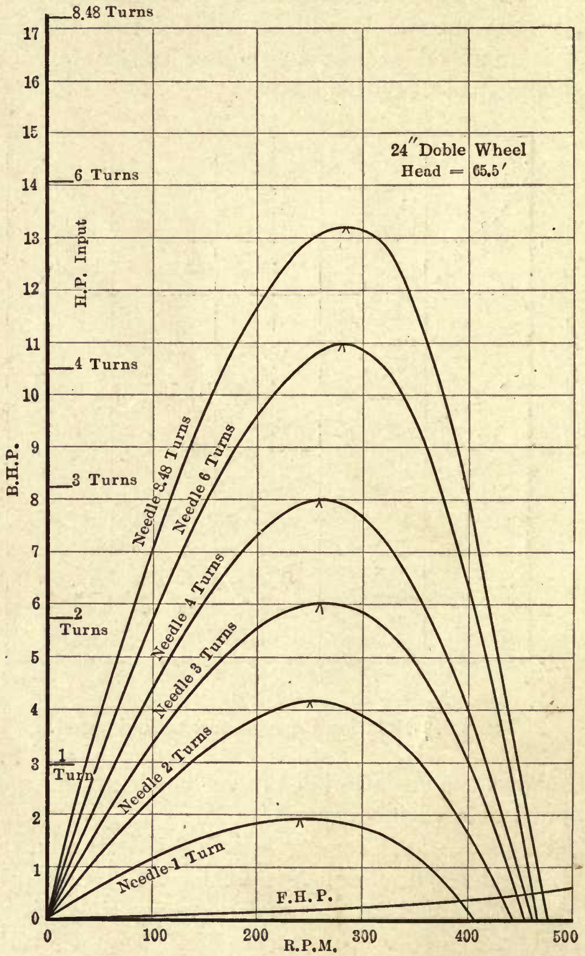


Fig. 60.—Relation between power and speed for different needle settings.

losses and the bearing friction and windage cause the total losses to become a minimum at a slightly higher speed. It does not seem possible to compute this in any simple way but it will be found that the best speed is usually such that $u_1/V_1 = 0.45$ to 0.49.

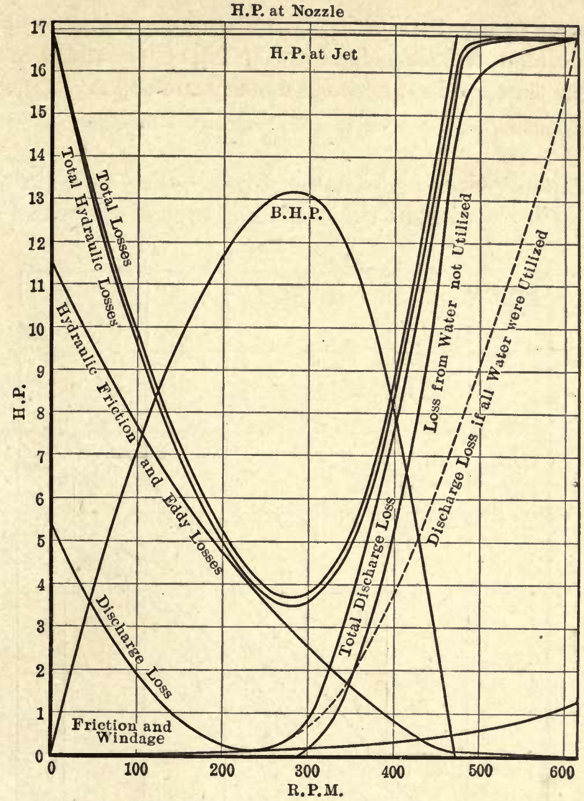


FIG. 61.—Segregation of losses for constant input and variable speed.

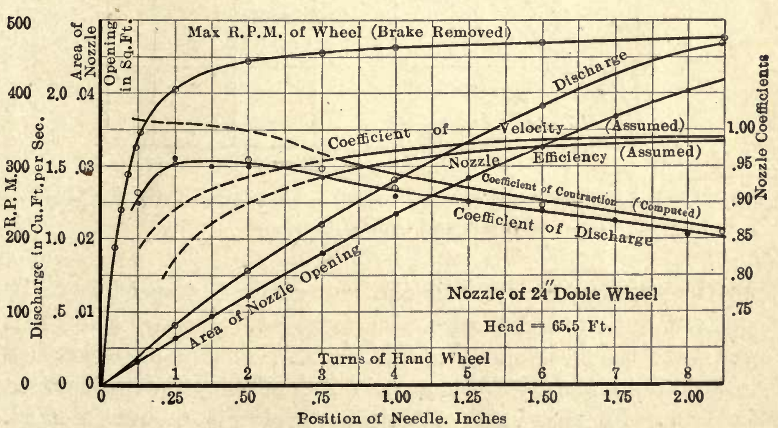


FIG. 62.—Nozzle coefficients and other data.

The speed of any turbine is generally expressed as $u_1 = \phi\sqrt{2gh}$. The coefficient of velocity of the nozzle will reduce the above values slightly, so that the best speed is usually such that

$$\phi = 0.43 \text{ to } 0.47$$

65. Constant Speed—Variable Input.—The case in Art. 63 is valuable in showing us the characteristics of the wheel but the

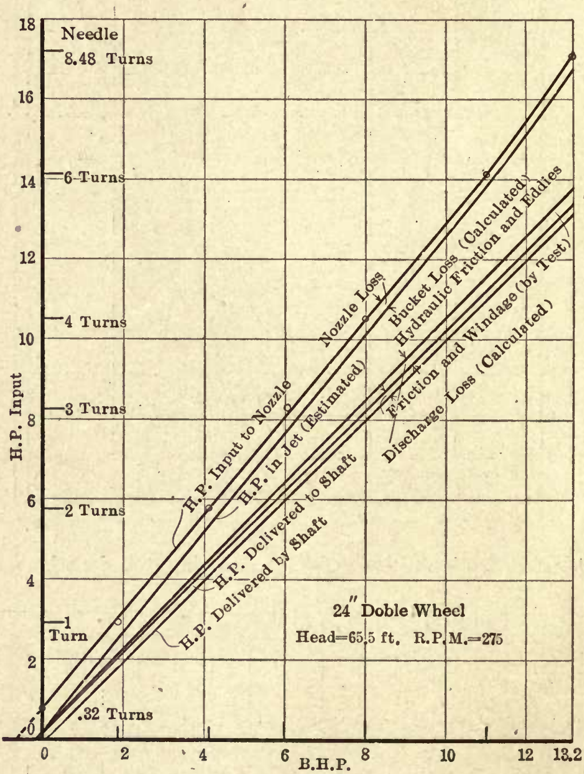


FIG. 63.—Relation of input to output and segregation of losses for variable input and constant speed.

practical commercial case is the one where the speed is to be constant but the input vary with the load. From Fig. 60 it is seen that the best speed is 275 r.p.m. That value was taken because the highest efficiency was obtained with the nozzle open six turns. For that value of N the curves in Fig. 63 were plotted. It will be noted that the relation between input and output is very nearly a straight line. Above six turns it bends up slightly because the wheel is then slightly overloaded.

The friction and windage was determined by a retardation run[1] and was assumed to be constant at all loads. The hydraulic

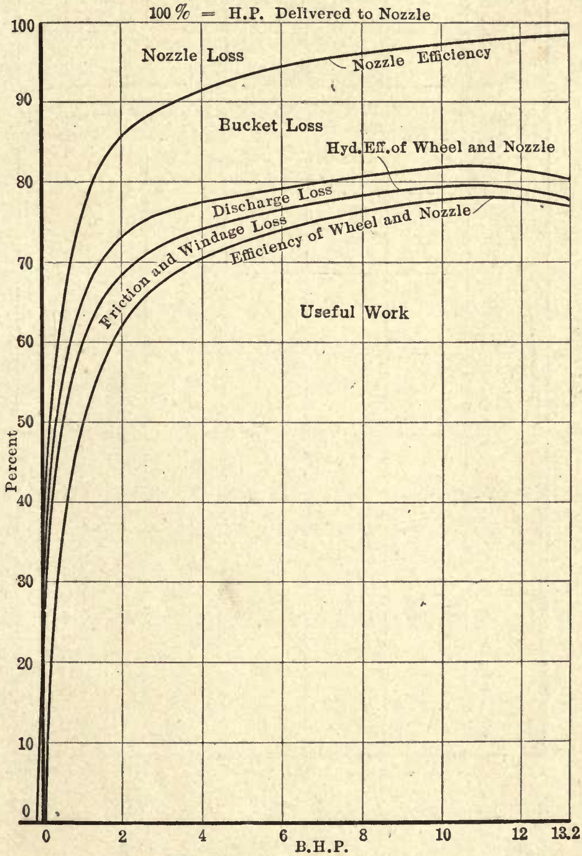


FIG. 64.—Efficiencies and percent losses at constant speed.

losses were segregated by the theory already given (Art. 51). These results plotted in per cent. are shown in Fig. 64 and Fig. 65.

66. Illustrative Problem.—Referring to Fig. 66 let the total fall to the mouth of the nozzle be 1000 ft. Suppose $B-C=$ 5000 ft. of 30-in. riveted steel pipe and at C a nozzle be placed whose coefficient of velocity$=0.97$. Suppose the diameter of

[1] See Art. 86.

the jet from the nozzle = 6 in. This jet acts upon a tangential water wheel of the following dimensions: Diameter = 6 ft.,

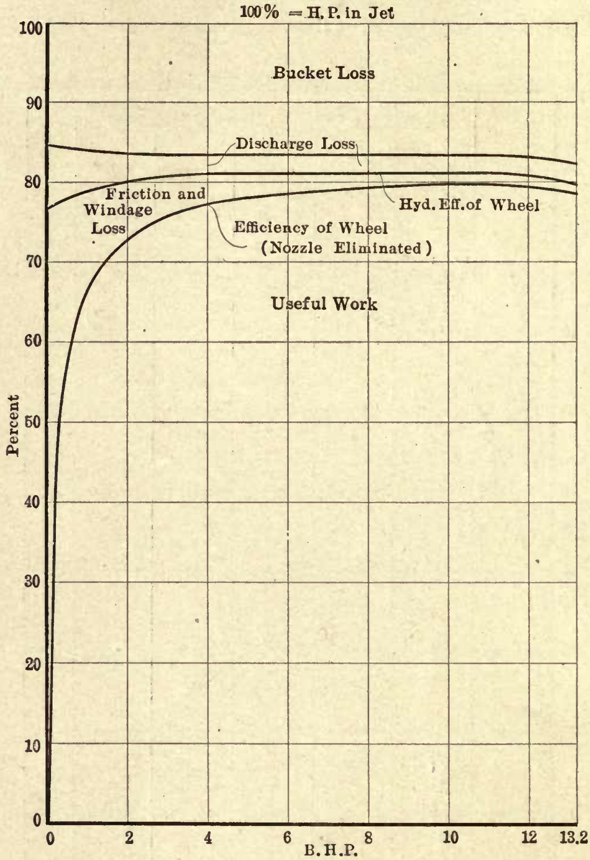


FIG. 65.—Efficiencies and percent losses at constant speed based upon power in jet.

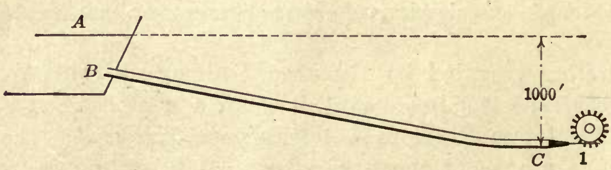


FIG. 66.

$A_1 = 12°$, $a_2 = 170°$. Assume $k = 0.6$, $\phi = 0.465$, and assume bearing friction and windage = 3 per cent of power input to shaft.

The problem of the pipe line is a matter of elementary hydraulics and a detailed explanation will not be given of the steps here employed. The coefficient of loss at B will be taken as 1.0, the coefficient of loss in the pipe will be assumed 0.03. The loss in the nozzle will be given by $\left(\frac{1}{c'^2}-1\right)\frac{V_1^2}{2g}$, where c' = the coefficient of velocity and V_1 the velocity of the jet. If V_c = the velocity in the pipe then the losses will be

$$\left(1+0.03\frac{5000}{2.5}\right)\frac{V_c^2}{2g}+0.063\ \frac{V_1^2}{2g}$$

Taking $H_A = 1{,}000$ ft. and $H_1=\frac{V_1^2}{2g}$ then by equation (4) we may solve for $\frac{V_c^2}{2g}=1.38$ ft. or $\frac{V_1^2}{2g}=861$ ft.

Thus $V_c = 9.42$ ft. per second and $V_1 = 235.5$ ft. per second.

Rate of discharge, $q = 4.62$ cu. ft. per second.

The pressure head at nozzle, $\frac{p_c}{w}=914.5$ ft.

The wheel speed $u_1 = 0.465 \times 8.025\sqrt{915.88} = 113$ ft. per second.

Therefore $N = 360$ r.p.m.

By methods illustrated in Art. 51, $v_1 = 126.7$ ft. per second, $v_2 = 100$ ft. per second, and $s_2 = 14.5$, assuming $x = 1.0$. Thus, $h''=\frac{u_1}{g}(s_1-s_2)=757$ ft.

The means of obtaining the following answers will doubtless be obvious.

Total head available,	$H_A = 1000$ ft.
Head at nozzle,	$H_c = 915.88$ ft.
Head in jet,	$H_1 = 861$ ft.
Head utilized by wheel,	$h'' = 757$ ft.

Total power available at A	= 5250 h.p.
Power at nozzle (C)	= 4800 h.p.
Power in jet	= 4520 h.p.
Power input to shaft	= 3970 h.p.
Power output of wheel	= 3851 h.p.

Hydraulic efficiency of wheel	= 0.878
Mechanical efficiency of wheel	= 0.970
Gross efficiency of wheel	= 0.852
Efficiency of nozzle	= 0.941

Gross efficiency of wheel and nozzle	=0.801
Efficiency of pipe line *B–C*	=0.915
Overall efficiency of plant	=0.733

67. PROBLEMS

1. Suppose the dimensions of a tangential water wheel are: $A_1=20°$, $a_2=165°$, $\phi=0.45$, $k=0.5$, and coefficient of velocity of nozzle $=0.98$. If the diameter of the jet $=8$ in. and the head on the nozzle 900 ft., compute the value of the force exerted upon the wheel, assuming $x=1.00$.

2. Compute the force on the wheel in problem (1), assuming $A_1=0$.

3. Compute the hydraulic efficiency of the wheel in (1).

4. Suppose it is desired to develop 2000 h.p. at a head of 600 ft. Assuming an efficiency of 80 per cent., what will be the size of jet required, and what will be the approximate diameter and r.p.m. of the wheel?

CHAPTER VIII

THEORY OF THE REACTION TURBINE

68. Introductory.[1]—A difficult problem in connection with the theory of the reaction turbine has been the determination of an expression for the amount of water discharged. Many writers on hydraulics do not attempt it at all beyond stating, of course, that it is proportional to the square root of the head. Others propose expressions which are very incomplete and are true for the best speed only. The equation which the author presents will be found to be more general in its application.

As an introductory illustration let us consider a pipe with a nozzle as shown in Fig. 67. The velocity of the jet, $V_1 = c'\sqrt{2gh}$. where c' is a coefficient of velocity. If the areas at (1) and (2) are denoted by F_1 and F_2 respectively, then since $q = F_1V_1 = F_2V_2$,

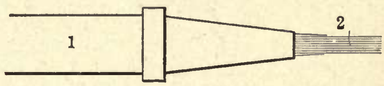


FIG. 67.

$$V_1 = (F_2/F_1)V_2 = c\sqrt{2gh}$$

It is evident that c is less than c' and therefore less than unity. The value of c depends upon the friction losses, since they decrease the flow, and also upon the ratios of the areas F_1 and F_2.

If, in the turbine, V_1 = the velocity of the water leaving the guide vanes and entering the runner, we may also write

$$V_1 = c\sqrt{2gh}$$

In that case also the value of c depends upon the losses and upon the dimensions of the turbine. Since the losses vary with the speed, evidently c will vary with the speed. Its usual value is from 0.6 to 0.8.

69. Losses.—The net head supplied the turbine is used up in two ways; in hydraulic losses and in mechanical work delivered to the runner. The head utilized in mechanical work is $h'' = \frac{1}{g}(u_1 s_1 - u_2 s_2)$. In accordance with the usual method in hy-

[1] For notation, see Art. 42.

draulics we may represent hydraulic friction loss in the runner by $k\,v_2^2/2g$, k being an experimental constant. If the turbine discharges into the air or directly into the tail race the discharge loss is $V_2^2/2g$.[1] In addition there may be a shock loss at entrance to the runner. The term *shock* is commonly applied here but the phenomena are rather those of violent turbulence. This turbulent vortex motion causes a large internal friction or eddy loss.

Referring to Fig. 68, the value of v_1 and its direction are determined by the vectors u_1, and V_1. Since the wheel passages are filled in the reaction turbine, the relative velocity just after the water enters the runner is determined by the area f_1 and its direction by the angle of the wheel vanes at that point. If all loss is to be avoided, these values should agree with those determined by the vector diagram; but that is possible for only one value of u_1 for a given head. For any other condition the velocity

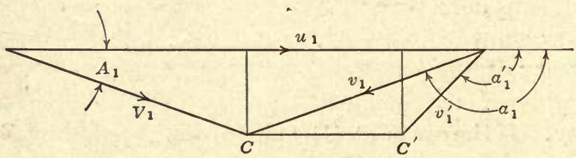


FIG. 68.

v_1 at angle a_1 is forced to become v'_1 at angle a_1'. This causes a loss of head which will be assumed to be equal to $(CC')^2/2g$. Since the area of the stationary guide outlets normal to the radius should equal the area of the wheel passages at entrance normal to the radius, the normal component (*i.e.*, perpendicular to u_1) of v_1, should equal that of V_1. Therefore CC' is parallel to u_1 and its value is easily found to be

$$CC' = u_1 - \frac{\sin\,(a'_1 - A_1)}{\sin\,a'_1} V_1$$

If $k' = \sin\,(a'_1 - A_1)\,/\sin\,a'_1$, then

$$\text{shock loss} = \frac{(u_1 - k'V_1)^2}{2g}.$$

70. Relation between Speed and Discharge.—Equating the net head to the sum of all these items we have

$$h = k\frac{v_2^2}{2g} + \frac{V_2^2}{2g} + \frac{(u_1 - k'V_1)^2}{2g} + \frac{2(u_1 s_1 - u_2 s_2)}{2g}.$$

[1] See Art. 78.

All velocities can be expressed in terms of u_1 and V_1 as follows:

$$u_2 = xu_1,\ v_2 = yV_1,\ s_1 = V_1 \cos A_1,$$
$$s_2 = u_2 + v_2 \cos a_2 = xu_1 + yV_1 \cos a_2,$$
$$V_2^2 = u_2^2 + v_2^2 + 2u_2v_2 \cos a_2 = x^2u_1^2 + y^2V_1^2 + 2xyu_1V_1 \cos a_2.$$

Making these substitutions and reducing we obtain,

$$[(1+k)y^2 + k'^2]V_1^2 + 2(\cos A_1 - k')V_1u_1 + (1-x^2)u_1^2 = 2gh.$$

From this equation V_1 may be computed for any value of wheel speed, u_1. It is customary to express the wheel speed as $u_1 = \phi\sqrt{2gh}$, and we may also say $V_1 = c\sqrt{2gh}$. The use of these

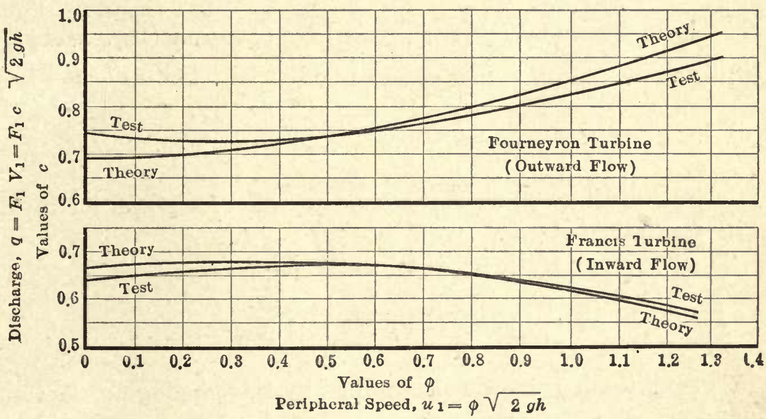


FIG. 69.—Comparison of the relation between c and ϕ as determined by theory and by test.

factors is more convenient in general. Introducing them our equation becomes

$$[1+k)y^2 + k'^2]c^2 + 2(\cos A_1 - k')c\phi + (1-x^2)\phi^2 = 1. \qquad (19)$$

From this equation c may be computed for any value of ϕ. For the outward flow turbine (19) becomes a hyperbola concave upward, for the inward flow turbine it becomes an ellipse concave downward.

A comparison between the values of c as determined by this equation and as determined by experiment is shown in Fig. 69. One turbine was an outward flow turbine and the other was a radial inward flow turbine. Considering the imperfections and limitations of the theory, the agreement is remarkably close.

71. Torque Exerted.—By equation (13) the torque exerted is

$$G=\frac{W}{g}(r_1s_1-r_2s_2).$$

Substituting the values of s_1 and s_2 as in Art. 70 we obtain,

$$G=\frac{W}{g}[(\cos A_1-xy\cos a_2)V_1-x^2u_1]r_1.$$

It is apparent that the torque decreases as the speed increases.

72. Power and Efficiency.—The power is $G\omega$, and since $r\omega=u$,

$$\text{Power}=\frac{W}{g}[(\cos A_1-xy\cos a_2)V_1u_1-x^2u_1^2].$$

The head utilized $h''=\frac{1}{g}(u_1s_1-u_2s_2)$ by equation (14). Substituting the values of s_1 and s_2 we obtain

$$h''=\frac{1}{g}[(\cos A_1-xy\cos a_2)V_1u_1-x^2u_1^2].$$

Multiplying h'' by W we obtain an expression for power identical with that given above.

The hydraulic efficiency is h''/h, therefore

$$e=\frac{1}{gh}[(\cos A_1-xy\cos a_2)V_1u_1-x^2u_1^2].$$

This equation is somewhat simplified by introducing our factors c and ϕ. We then have

$$e=2(\cos A_1-xy\cos a_2)c\phi-2x^2\phi^2 \qquad (20)$$

Assumptions regarding losses are necessary in order to compute the discharge as can be seen by equation (19), but when the discharge is determined the actual hydraulic efficiency can be computed without dependence upon any arbitrary coefficients of loss. Thus equation (20) involves no factors except the turbine dimensions. If the relation between speed and discharge is known, as by experiment, the actual hydraulic efficiency may be computed, provided the proper wheel dimensions are known.

72. Variable Speed—Constant Gate Opening.—Since c varies with the speed the input for a fixed gate opening will not be constant for all speeds as it is in the case of the impulse turbine. The variation of the losses at full gate with the speed ranging from zero up to its maximum value is shown by Fig. 70. The horse-power in each case is obtained by multiplying $wq/550$ by the head lost as given by Art. 69

The curves for the impulse turbine in Fig. 61 may also represent percentages by the use of a proper scale since the input is constant. But for the reaction turbine the percentage curves will be slightly different from those in Fig. 70. It will thus be true that the speed at which the efficiency is a maximum will be slightly different from the speed for which the power is the greatest.

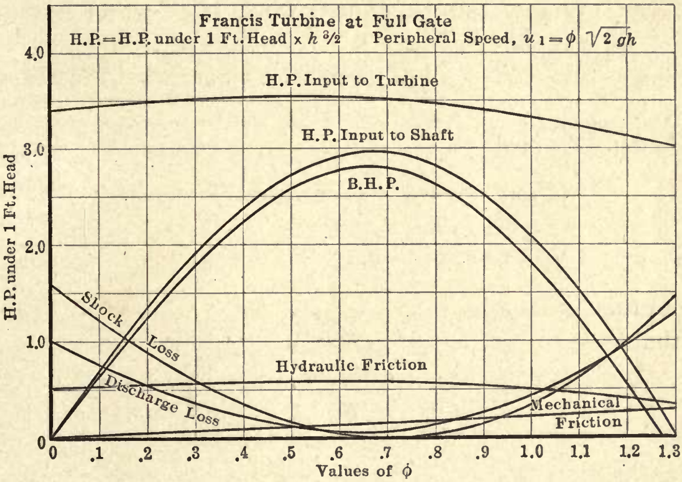


FIG. 70.—Losses at full gate and variable speed.

The Francis turbine for which the curves in Fig. 69 and Fig. 70 were constructed had the following dimensions:

$$A_1 = 13°,\ a'_1 = 115°,\ a_2 = 165°,\ F_1 = 5.87 \text{ sq. ft.},\ f_2 = 6.83 \text{ sq. ft.},\ r_1 = 4.67 \text{ ft.},\ r_2 = 3.99 \text{ ft.}$$

From this data $x = 0.855$, $y = 0.860$, $k' = 1.08$, and $k = 0.5$ (assumed). Attention is called to the fact that the horse-power output was determined by an actual brake test while the horse-power input to shaft was computed from the theory given in the preceding article. The two differ by the amount of power consumed in bearing friction and other mechanical losses.

73. Best Speed.—By means of calculus the value of ϕ giving the maximum value of e may be determined. However the resulting equation is somewhat lengthy to use and will not be given here. In view of the imperfections of the theory a simpler approximate method will be employed, by assuming that the best speed is the one for which the discharge loss is the least. If the turbine is properly designed the vane angle at entrance will have such a value that the shock loss becomes zero at this same speed. If the shock loss is not very small when the discharge loss is the least then the approximate method cannot be used and resort must be had to the longer calculus method, but that would not arise with any rational design.

The discharge loss will be approximately the least when $A_2=90°$. If a draft tube is used there is a further reason for assuming $A_2=90°$ and that is that otherwise the water would enter the tube with a whirling motion which would increase the losses[1].

If $A_2=90°$

$$s_2=V_2 \cos A_2=u_2+v_2 \cos a_2=0$$
$$=xu_1+yV_1 \cos a_2=0$$

or
$$x\phi+yc \cos a_2=0.$$

From this

$$c=-\left(\frac{x}{y \cos a_2}\right)\phi=\alpha\phi. \qquad (21)$$

where
$$\alpha=-\left(\frac{x}{y \cos a_2}\right).$$

Substituting this value of c in equation (19) the value of ϕ may be obtained. The resulting equation is

$$[(1+k)y^2+k'^2)\alpha^2+2(\cos A_1-k')\alpha+(1-x^2))]\phi^2=1 \qquad (22)$$

For an existing turbine for which a'_1 is known so that k' can be determined this equation can be used to find the value of ϕ necessary to make $A_2=90°$, if α is determined by equation (21).

In designing, however, the value of a'_1 would not be known until after the other factors had been determined. In solving for the best speed, then, it would be allowable to consider the shock loss zero. If $u_1-k'V_1=0$, $k'=\frac{u_1}{V_1}=\frac{\phi}{c}=\frac{1}{\alpha}$.

Introducing this value of k' in (22) we obtain

$$[(1+k)y^2\alpha^2+2 \cos A_1\alpha-x^2]\phi^2=1 \qquad (23)$$

[1] For contradiction of this see Trans. A. S. C. E., Vol. LXVI, p. 378 (1910).

This equation will determine the best speed providing the vane angle is to be obtained afterward.

The methods given enable us to determine the value of ϕ giving the maximum hydraulic efficiency. However an inspection of the curves in Fig. 70 shows that the bearing friction and other mechanical losses cause the gross efficiency to be attained at a slightly lower speed than that giving the maximum hydraulic efficiency. It is really the gross efficiency that we are interested in, and that is what is meant when the word "efficiency" without qualification is used.

With reaction turbines values of ϕ for best efficiency range from 0.57 to 0.87 according to the design.

74. Vane Angle.—Since the vane angle a'_1 should agree with the angle a_1 determined by the vector diagram, it is readily seen from trigonometry that if there is to be no shock loss

$$\tan a'_1 = \frac{c \sin A_1}{c \cos A_1 - \phi} \qquad (24)$$

The value of ϕ would be determined by equation (23) and then by (21) $c = \alpha\phi$. It should be noted that the relation expressed by (21) is true for this one value of ϕ only. For any other value of ϕ it is necessary to determine c by (19).

75. Constant Speed—Variable Input.—The relation between input and output and the segregation of losses for a cylinder gate turbine at constant speed is shown in Fig. 71. In the four tests the gate was raised 1/4, 1/2, 3/4, and 4/4 of its opening. With the turbine running on full gate but at an incorrect speed there is a shock loss at entrance as shown in Art. 69. This loss is due to an abrupt change in the direction of the relative velocity of the water. When the turbine is running at the normal speed but with the gate partially closed there is a shock loss of a slightly different nature. A partial closure of the gates increases the value of V_1 and the angle A_1 may be affected somewhat. However q will be reduced while f_1 remains the same and thus the velocity v_1 must be suddenly reduced to v'_1. The loss of head due to this may be roughly represented by $(v_1 - v'_1)^2/2g$. While this expression may not give the exact value of the loss, yet it must be true that it will be of the nature shown by the curves.

There will aways be a slight leakage through the clearance spaces and such a loss is indicated in Fig. 71 though it was not possible to compute it with any exactness. It was merely added to show that it exists but it was not accounted for in Fig. 70. These results on a percentage basis are shown in Fig. 72.

The cylinder gate turbine is rather inefficient on light loads due to the big shock loss. The wicket gates do not occasion such large shock losses and hence reduce the input curve to a line more nearly parallel to the output line in Fig. 71, and thus improve the part load efficiency of the turbine. Also the cylinder gate turbine gives its best efficiency when the gate is completely raised and the

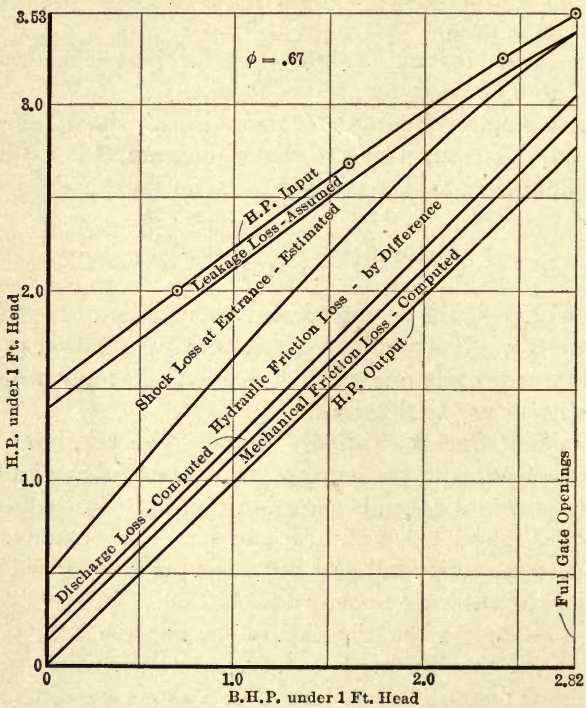


FIG. 71.—Losses for cylinder gate Francis turbine at constant speed.

power output has its greatest value. But the wicket gate turbine usually develops its best efficiency before the gates are fully open. There is thus left some overload capacity.

76. Limitations of Theory.—The defects of this theory or any theory are as follows: In order to apply mathematics in any simple way it is necessary to idealize the conditions of flow by assuming that all the particles of water at any section move in the same direction and with the same velocity. Such is very far from being the case so what we use in our equations is the average direction and the average velocity of all the particles of water. That in itself could easily cause a discrepancy between our theory and the fact, because the theory is incomplete.

But even to determine accurately these average dimensions that are used in the equations is a matter involving some difficulty. Thus, though the direction of the streams leaving the runner is influenced by the vane angle at that point, it cannot be said that the angle a_2 is exactly equal to the vane angle at exit. In fact the author has roughly proved by study of a test where some

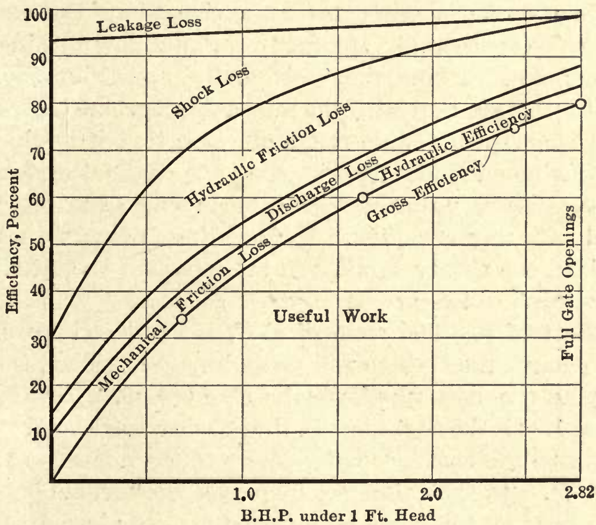


FIG. 72.—Cylinder gate turbine at constant speed.

special readings were observed that the two may differ by from 5 to 10 degrees, and that a_2 varied regularly for different values of ϕ. The same thing may be said about the area f_2. Some recent experiments in Germany[1] have shown that there may be a certain amount of contraction of the streams and that this contraction varies for different speeds. Thus the true value of f_2 may be slightly less than the area of the wheel passages. These observations concerning a_2 and f_2 apply equally well to other angles and areas.

In computing the results plotted in Fig. 69 the coefficients of c

[1] *Zeitschrift des Vereins deutscher Ingenieure,* May 13, 1911.

and ϕ in equation (19) were treated as constant. It has just been shown that the real values of the angles and areas may vary slightly with the speed. Also it is stated in Art. 62 that the value of k is not constant at all speeds for the impulse turbine. While the conditions with the reaction turbine are very different, yet it is doubtless true that k is not strictly constant here. If it were known just how k and the dimensions used varied with the speed, the theoretical curve could be made to more nearly coincide with the actual curve. In addition the expression for shock loss is only an approximation. But even as it is the discrepancy is not serious.

By the use of the proper average dimensions the equations given may be successfully applied to a radial flow turbine. For the mixed flow turbine they will apply approximately. The reasons for this are that with the mixed flow turbine the radius r_2 varies through such a wide range of value that it is difficult to fix a proper mean value; likewise the vane angle at exit and also the area varies so radically that a mean value can scarcely be obtained with any accuracy. Even if these mean values could be obtained the theory would still be imperfect as stated in the first paragraph of this article.

77. Effect of y.—The ratio of F_1/f_2 is expressed by y. If y is small enough the turbine will be an impulse turbine, the value of ϕ giving the best speed will be about 0.45, $p_1/w=0$, and c will equal 1.00 if the slight loss in the nozzle is neglected. (Actually c will be the coefficient of velocity of the nozzle and will be about 0.97). As the value of y increases, the turbine becomes a reaction turbine, the value of ϕ increases, p_1/w increases, and c decreases. The general tendency of these factors is shown in Fig. 73.

It is thus seen that the design of a reaction turbine can be varied so as to secure quite a range of results.

78. Effect of Draft Tube.—If the turbine discharges into the air or directly into the tail water then the discharge loss will be $\frac{V_2^2}{2g}$ as stated in Art. 69. If, however, the turbine discharges into a diverging draft tube the theory will have to be modified slightly since the kinetic energy of the water leaving the runner may not be wholly lost. If the turbine is running at its proper speed and the draft tube is correctly designed there will be a gradual reduction of the velocity with which the water leaves the runner.

Referring to Fig. 49 it will be seen that $V_2 = V_D$ and if the tube diverges the water will finally be discharged with a velocity V_E which will be less than V_D. Thus the discharge loss may be represented by $m\frac{V_2^2}{2g}$, where m is less than unity. If loss in the tube be neglected the value of m will depend only upon the ratio of the areas F_D and F_E and will equal $(F_D/F_E)^2$.

If the turbine is not running at its proper speed the value of V_2 will be higher than the normal value and, since A_2 will no longer be 90°, the water will enter the draft tube with a whirling

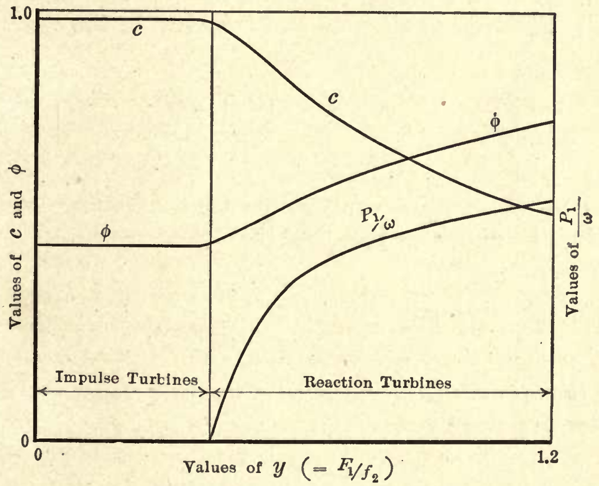


FIG. 73.

motion, which will tend to increase the draft tube losses. However, since the quantity of water will not be radically different, the value of V_E will be approximately the same as before. If the discharge loss is about the same the value of m would have to be less since V_2 is greater. Actually the whirling motion in the draft tube together with the fact that a high value of V_2 will be rather abruptly reduced to V_D will increase the draft-tube losses to such an extent that m may be considered as approximately constant in our theory. If anything it is probable that m increases and approaches 1.00 as the speed departs widely from the normal value.

Introducing then the discharge loss as $m\frac{V_2^2}{2g}$ in the equation of Art. 70 we get the following as a substitute for equation (19).

$$[(m+k)y^2+k'^2]c^2+2\,[\cos\ A_1-k'-(1-m)xy\ \ \cos\ a_2]c\phi \\ -[(2-m)x^2-1]\phi^2=1 \quad (25)$$

It will be seen that equation (25) will give a slightly higher value of c than (19), and this higher value of c will give a slightly higher efficiency when substituted in (20). A higher value of c also means a greater quantity of water discharged, so that the diverging draft tube produces a gain in power due both to the greater quantity of water discharged and to the increased efficiency.

Applying the general equation of energy to the draft tube in Fig. 49,

$$H_D = z_D+\frac{V_D^2}{2g}+\frac{p_D}{w}$$
$$H_F = O.$$

The losses between these two points are the friction losses in the tube plus the discharge loss, $V_E^2/2g$.

The pressure at D is

$$\frac{p_D}{w}=-z-\frac{V_D^2}{2g}+\text{Losses.} \quad (26)$$

If the pressure at D be assigned a limiting value the allowable elevation is given by

$$z=-\frac{p_D}{w}-\frac{V_D^2}{2g}+\text{Losses.} \quad (27)$$

The minimum value of the pressure can never be below absolute zero. Actually on account of air in suspension in the water, leakage of air into the draft tube, and other factors it cannot reach that value. It should not be made less than -25 ft. and for large draft tubes it is usually not allowed to be as low as that.

79. ILLUSTRATIVE PROBLEMS

1. The Francis turbine for which the curves shown in this chapter were constructed had the following dimensions: $A_1=13°$, $a'_1=115°$, $a_2=165°$, $F_1=5.87$ sq. ft., $f_2=6.83$ sq. ft., $r_1=4.67$ ft., $r_2=3.99$ ft. From this data $x=0.855$, $y=0.860$, $k'=1.08$, and $k=0.5$ (assumed). Equation (19) then becomes

$$2.275c^2-0.210c\phi+0.27\phi^2=1$$

By equation (21) $c = 1.028\,\phi$. Substituting $1.028\,\phi$ for c in (19) we get $\phi = 0.637$. This value of ϕ will give approximately the maximum hydraulic efficiency. The exact calculus method gives $\alpha = 0.95$ and this in (19) gives $\phi = 0.688$. The mechanical losses will cause the speed for the maximum gross efficiency to be somewhat lower than that for the maximum hydraulic efficiency. Thus the test of this turbine gave the maximum gross efficiency when $\phi = 0.67$. It is thus seen that the approximate method is really about as good as the longer exact method.

The value of c is $1.028 \times 0.637 = 0.655$. With the values of ϕ and c the speed and discharge under any head can be computed and the rest of the problem is identical in method with the illustration in Art. 52. The equations given in Art. 71 are merely algebraic simplifications of the general equations given in Chapter VI.

If the angle a'_1 were not known, equation (23) would have to be used. It would give

$$(1.173 + 2.005 - 0.730)\phi^2 = 1.$$

From this $\phi = 0.637$ as before. The value of a'_1 would then be determined by (24). In this particular case the value of a'_1 as determined for $\phi = 0.637$ and $c = 0.655$ will not quite agree with the value of a'_1 given. The curves in Fig. 63 will show that the shock loss is not quite zero at this speed.

2. If the turbine in problem (1) is used under a head of 30 ft. find the r.p.m. for $A_2 = 90°$, the quantity of water discharged, the hydraulic efficiency, and the horse-power.

$u_1 = 0.637 \times 8.025\sqrt{30} = 28$ ft. per second.

From this $N = 57.3$ r.p.m.

$V_1 = 0.655 \times 8.025\sqrt{30} = 28.8$ ft. per second.

$q = 5.87 \times 28.8 = 169$ cu. ft. per second.

By equation (20)

$$e = 2(0.975 + 0.855 \times 0.860 \times 0.966)c\phi - 2 \times 0.73\phi^2$$
$$= 3.37 \times 0.655 \times 0.637 - 1.46 \times 0.405 = 0.813$$

$$\frac{6.25 \times 169 \times 30 \times 0.813}{550} = 468 \text{ h.p.}$$

3. If the turbine in the preceding problem were to be run at the same speed of 57.3 r.p.m. while the head decreased to 18 ft., find the discharge, hydraulic efficiency, and horse-power.

Ans. $\phi = 0.823$, by (19) $c = 0.638$, $q = 127.5$ cu. ft. per second, $e = 0.782$, 204 h.p.

4. Suppose the turbine specified in problem (1) were to discharge into a draft tube, the area of which would be 12.5 sq. ft. at the upper end and 25.0 sq. ft. at the mouth. If the turbine is used under a head of 30 ft., as in problem (2), find the discharge, hydraulic efficiency, and horse-power and compare with answer to (2).

The velocity heads would be inversely proportional to the squares of the areas, or $V_E^2 = 0.25 V_D^2$. To allow for draft-tube losses, assume $m = 0.3$. Equation (25) then becomes

$$1.759c^2 + 0.784c\phi - 0.245\phi^2 = 1.$$

Since A_2 is to equal 90°, we use $\alpha = 1.028$ as before. Thus gives us $\phi = 0.642$ and $c = 0.660$. $q = 170.5$ cu. ft. per second. By (20), $e = 0.827$, the power $= 481$ h.p.

5. A low speed, low capacity turbine has the following dimensions: $A_1 = 20°$, $a_2 = 162°$, $F_1 = 1.074$ sq. ft., $f_2 = 1.552$ sq. ft., $r_1 = 2.0$ ft., $r_2 = 1.6$ ft. Assume $k = 0.3$. Find best ϕ, c, vane angle, hydraulic efficiency.

Ans. $\alpha = 1.215$, $\phi = 0.625$, $c = 0.759$, $a'_1 = 71° 17'$, $e = 0.894$.

6. A high speed, high capacity turbine has the following dimensions: $A_1 = 35°$, $a_2 = 155°$, $F_1 = 3.61$ sq. ft., $f_2 = 3.83$ sq. ft., $r_1 = 1.0$ ft., $r_2 = 0.6$ ft., assume $k = 0.3$. Find best ϕ, c. vane angle, hydraulic efficiency. Compare with (5).

Ans. $\alpha = 0.703$, $\phi = 0.856$, $c = 0.602$, $a'_1 = 136° 20'$, $e = 0.845$.

7. Suppose the turbines in problems (5) and (6) were used under a head of 25 ft., find the r.p.m., discharge, and horse-power.

Ans. 119.6 r.p.m., 32.7 cu.ft. per second, 82.7 h.p., 328 r.p.m., 87.0 cu. ft. per second, 209 h.p.

CHAPTER IX

TURBINE TESTING

80. Importance.—Testing is necessary to accompany theory in order that the latter may be perfected until it becomes reliable enough to be useful. Unless the theory agrees with the facts it is not true theory but only an incorrect hypothesis. Only by means of theory and testing working hand in hand can improvements in design be readily brought about. Thus the ease of testing is a measure of the rate of development of any machine.

Again, if we are to thoroughly understand turbines, it will be necessary to make a thorough study of test data in order to appreciate the differences between different types. Unfortunately there is a scarcity of good and thorough test results.

The only public testing flume in the United States is the one at Holyoke, Mass. Nearly 2000 runners have been tested there and it has been an important factor in the development of modern turbines. The maximum head obtainable there is about 17 ft., also it is scarcely possible to test runners above 42 in. in diameter because of the limitations imposed by the depth of the flume.

An acceptance test should always be made when a turbine is purchased if it is possible to do so. Otherwise the purchaser will have no assurance that the specifications have been fulfilled. Thus a case may be cited where the power and efficiency of a tangential water wheel were both below that guaranteed as can be seen by the following:

	Efficiency	Normal h.p.	Maximum h.p.
Guarantee........	0.800	3500	5225
Test.............	0.720	2300	3500

In this table the normal horse-power means the power at which the maximum efficiency is obtained, any excess power over that being regarded as an overload. The actual efficiency is 8 per cent less than that guaranteed and the wheel is really a 2300-h.p. wheel instead of a 3500-h p. wheel. It is true that the wheel could deliver 3500 h.p. but at an efficiency of only 67 per cent. Since that is the maximum power the 5225-h.p. overload could not be attained.

Another case may also be given where the facts are of a different nature.[1] A comparison of the guaranteed and test results for a reaction turbine is shown in Fig. 74. The efficiency secured was higher than that guaranteed, but it was also attained at a much higher horse-power. If the turbine were then run on the load specified it would be operating on part gate all the time and at a correspondingly low efficiency. This is a common failing in "cut and try" practice. A turbine of excess capacity is provided; it never lays down under any load put upon it and the owner is satisfied. Quite frequently also a turbine which must run at a certain speed is really adapted for a far different speed.

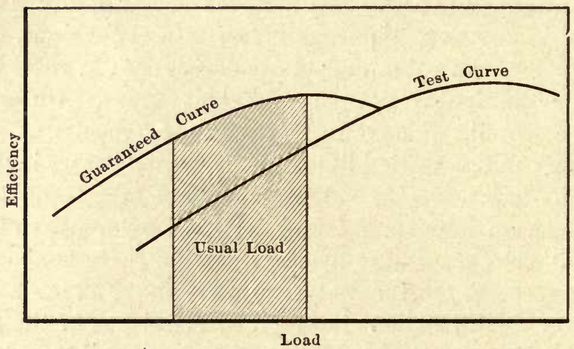


FIG. 74.

Thus under the given conditions its efficiency may be very low, when the runner might really be excellent if operated at its proper speed. A test would show up these defects, otherwise they may remain unknown.

Another reason for making tests would be to determine the condition of the turbine after length of service. The effect of seven years' continuous operation upon a certain tangential water wheel is seen in Fig. 75. This drop in efficiency is due to roughening of the buckets, to wear of the nozzle, and to the fact that end play of the shaft together with the worn nozzle caused the jet to strike upon one side of the buckets rather than fairly in the center. It might be noted however that a 7-ft. wheel of the same make in the same plant showed no change in efficiency after

[1]Trans A. S. C. E., Vol. LXVI, p. 357.

the same length of service. With reaction turbines the guides and vanes become worn and the clearance spaces also wear so as to permit the leakage loss to increase.

As to whether efficiency is important or not depends upon circumstances. If there is an abundance of water in excess of the demand the only requirement is that the turbine deliver the power demanded. But where the water must be purchased, as it is in some cases, or where vast storage reservoirs are constructed at considerable expense it is desirable that water be used with the utmost economy.

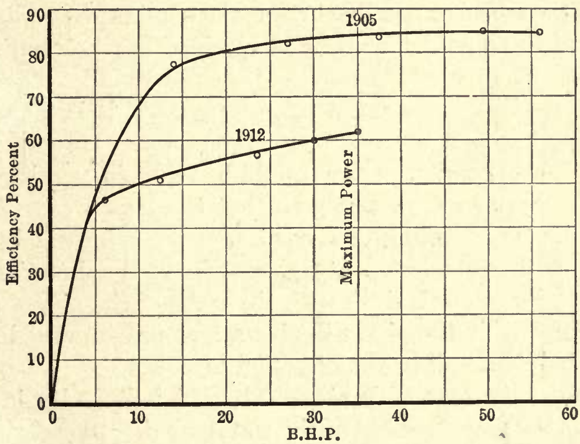


FIG. 75.—Effects of service upon a 42″ tangential water wheel.

81. Purpose of Test.—The nature of the test will depend upon the purpose for which it is made. In a general way there are four purposes as follows:

1. *To find results for particular specified conditions.* This will usually be an acceptance test to see if certain guarantees have been fulfilled. The guarantee will usually specify certain values of efficiency obtained at certain loads at a fixed speed under a given head. Occasionally several values of the head will be specified.

2. *To find best conditions of operation.* Such a test will cover a limited range of speed, load, and head; all of them, however, being in the neighborhood of the maximum efficiency point. A test of this nature will show what a given turbine is best fitted for.

3. *To determine general principles of operation.* This test is similar to the above except that it is more thorough. It should cover all speeds from zero to the maximum possible under no load. Various gate openings may be used and the head may also be varied. Such a test will enable one to understand the turbine better and could also be used to verify the theory.

4. *To investigate losses.* This test would be similar to the preceding except that a number of secondary readings of velocities, pressures, etc., at various points might be taken. Such a test will be of interest chiefly to the designer.

82. Measurement of Head.—The head should be measured as close to the wheel as possible in order to eliminate pipe-line losses. The head to be used should be as specified in either equation (6) or (8) of Art. 45, according to the type of turbine. The pressure may be read by means of a pressure gage if it is high enough. For lower heads, a mercury column or a water column will give more accurate results. Care should be taken in making connections for the pressure reading so that the true pressure may be obtained. The reading of any piezometer tube will be correct only when the tube leaves at right angles to the direction of flow and when its orifice is flush with the walls of the pipe. No tube projecting within the pipe will give a true pressure reading, even though it be normal to the direction of flow.[1]

83. Measurement of Water.—The chief difficulty in turbine testing is the measurement of the water used. In some commercial plants the circumstances are such that it is scarcely possible to measure the water at all and in others the expense is prohibitive. The necessity for cheap and accurate means of determining the amount of water discharged is imperative.

The standard method of measurement is by means of a weir. For large discharges, however, the expense of constructing a suitable weir channel may be excessive, and, in case the turbine discharges directly into a river, it may be almost impossible to construct it. In the case of a turbine operating under a low head the increase in the tail water level caused by the weir may cause a serious decrease in head below that normally obtained. This would make the test of little value. However, where it is feasible, the use of a weir is a very satisfactory method and should be provided for when the plant is constructed. It should be remembered, though, that all weir formulas and coefficients are purely

[1] Hughes and Safford, "Hydraulics," p. 104.

empirical in their nature and that the discharge as determined by them may be as much as 5 per cent. in error, unless standard proportions are carefully adhered to.[1]

In order to avoid the increase in the tail water level the use of submerged orifices may be desirable in low-head plants. A submerged orifice will produce a certain elevation of the tail water level, but it will not be as great as the weir. At present enough experimental data has not been gathered to make this method applicable in general, but perhaps in the future it may be used with fair success.

Either in the tail race or in the head race a Pitot tube, current meter, or floats may be used. These methods involve no disturbance of the head under which the turbine ordinarily operates, but they do require a suitable channel in which the observations can be taken. These instruments should be in the hands of a skilled observer who understands the sources of error attendant upon their use.[2]

The Pitot tube consists of a tube with an orifice facing the current. The impact of the stream against this orifice produces a certain pressure which is proportional to the square of the velocity. If h is this reading in feet of water and K an experimental constant, then

$$V=K\sqrt{2gh}.$$

The value of K may be 1.00, it may be greater than unity or less than unity according to the design of the tube and the manner of determining h. Since it would be very difficult to determine accurately the height of the column of water in a tube above the level of the stream it is customary to use two tubes and read the difference between the two. For convenience in reading, the instrument is made so that valves may be closed and the device lifted out of the water without changing the levels of the columns, or sometimes both columns may be drawn up to a convenient place. The orifice for this second tube is usually in a plane parallel to the direction of flow and will thus give a lower reading than the other. It does not, however, give the value of the pressure at that point, as stated in Art. 82. For low velocities it is desirable to magnify this difference in the two readings and for that purpose the orifice of the second tube may be directed down

[1] See "Weir Experiments, Coefficients, and Formulas," by R. E. Horton, U. S. G. S. Water Supply and Irrigation Paper No. 150, Revised, No. 200.

[2] See Hoyt and Grover, "River Discharge."

stream. Its reading will then be less than for the one parallel to the direction of flow. Such an instrument is called a pitometer, and the value of K for it is always less than 1.0.

The current meter is an instrument having a little wheel which is rotated by the action of the current, the speed of rotation being proportional to the velocity of flow.

The Pitot tube may also be used in a pressure pipe. Since the reading of the impact tube alone will be the sum of the pressure head plus the velocity head it will be necessary to use two tubes in the same manner as in the case of the open channel. The value of h will be the difference between these two readings, and the value of K must be determined experimentally. If, however, only one orifice is used and the pressure is determined by a piezometer tube with its orifice lying flush with the walls of the pipe the difference between these two readings may be considered equal to the velocity head, that means the value of $K = 1.0$.

For the tangential water wheel the Pitot tube may also be used to determine the jet velocity. In such a case only the impact tube is required. While it is well to determine the value of K experimentally, yet if the tube is properly constructed it may be taken as 1.0. A check on this may be obtained as follows: It is probably true that the maximum velocity obtained at any point in the jet is the ideal velocity. The latter can be computed from the head back of the nozzle and the value of K should be such as to make the two agree. Either in the pipe or the jet it is desirable to take a velocity traverse across each of two diameters at right angles to each other. In computing the average velocity it is necessary to weight each of these readings in proportion to the area affected by them.[1]

84. Measurement of Output.—The determination of the power output of a turbine is also a matter of some difficulty. Perhaps the most satisfactory method is to use some form of a Prony brake or absorption dynamometer.[2]

[1] See "Application of Pitot Tube to Testing of Impulse Water Wheels," by Prof. W. R. Eckart, Jr., *Institution of Mechanical Engineers*, Jan. 7, 1910. Also printed in *Engineering* (London), Jan. 14, 21, 1910.

Engineering News, Vol. LIV, Dec. 21, 1905, p. 660. See also *Zeitschrift des ver. deut. Ing.*, Mar. 22, 29, and Apr. 5, 1913. For useful information regarding all the methods of measurement given here see Hughes and Safford, Hydraulics.

[2] C. M. Allen, "Testing of Water Wheels after Installation," *Journal A. S. M. E.*, April, 1910.

The use of a simple brake is restricted to comparatively small powers. For large powers it becomes rather expensive and difficult. A good absorption dynamometer may be used satisfactorily for fairly large powers but the drawback is one of initial expense. In many cases also where turbines are direct connected to electric generators it may be impossible to attach a brake of any kind.

In such cases it is necessary to supply an electrical load for the generator and determine the generator efficiency. However, this method of testing involves a number of instrument readings which may be more or less in error and a rather complicated process of computation. Nevertheless it can be done with very satisfactory results. One drawback about it is that the speed cannot be varied through the same range of values as in the brake test. The output of the generator may be absorbed by a water rheostat which will furnish an absolutely constant load. If it is a direct-current machine this rheostat may simply consist of a number of feet of iron wire wound on a frame and immersed in water to keep it cool. This water should be running water or a large pond so that its temperature may not change. The current is shorted through this coil; the load is varied by changing the length of wire in use. For a three-phase alternator the rheostat may consist of three iron pipes at the vertices of an equilateral triangle with a terminal connected to each. The load is varied by changing the depth of immersion of the pipes in water.[1]

85. Working Up Results.—In figuring up the results of test data it should be borne in mind that any single reading may be in error but that all of them should follow a definite law. Thus a smooth curve should be drawn in all cases. Also if any readings should follow a law which is any approach to a straight line it is better to work from values given by that line rather than from the experimental values themselves. Thus if a turbine be tested at constant gate opening and at all speeds, the curve showing the relation between speed and efficiency may be drawn at once from the experimental data. However, a more accurate curve can be constructed by noting that the relation between speed and brake reading is a fairly straight line. See Fig. 59. This is not a straight line but the curvature is not very marked so that it may be drawn readily and accurately. Values given by this curve may then be used for constructing the efficiency curve. Again, when a turbine is tested at constant speed, it should be noted that the re-

[1] *Power*, Vol. XXXVII, June 17, 1913, p. 857.

lation between input and output is not a straight line absolutely, but it is approximately so. If any point falls decidedly off from a straight line it is probably in error. From the line giving the relation between input and output the efficiency curve may be constructed.

In computing the true power in a jet it might be also noted that it is not that given by using the square of the average velocity but something 1 or 2 per cent. higher than that. The reason is that the velocity throughout the jet varies and the summation of the kinetic energy of all the particles is not that obtained by using the average velocity.[1]

86. Determination of Mechanical Losses.—With the tangential water wheel the mechanical losses will consist of bearing friction

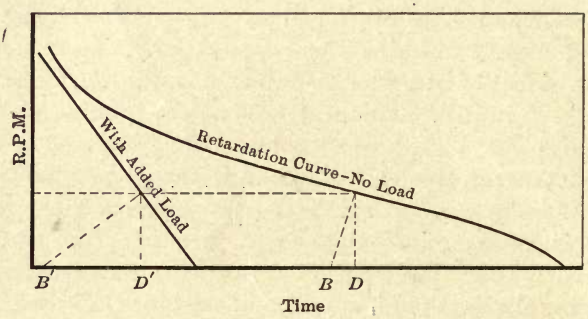


FIG. 76.—Retardation curves.

and windage. With the reaction turbine they will consist of the bearing friction and the disk friction due to the drag of the runner through the water in the clearance spaces. There are several ways of determining this but the retardation method is probably as satisfactory as any. The turbine is brought up to as high a speed as is possible or desirable and the power shut off. As the machine slows down readings of instantaneous speed are taken every few seconds and a curve plotted between instantaneous speed and time as shown in Fig. 76. Instantaneous speed may be determined by a tachometer, by a voltmeter, or by an ordinary continuous revolution counter. With the latter the total revolutions are read every few seconds without stopping it; the difference

[1] L. M. Hoskins, "Hydraulics," p. 119.
L. F. Harza, *Engineering News*, Vol. LVII, Mar. 7, 1907, p. 272.
See also Prof. Eckart's paper previously mentioned.

between two consecutive readings will then enable us to find the average speed corresponding to the middle of this time interval.

The power lost at any speed is equal to a constant times the subnormal to the curve at that speed. If L equals the power lost then

$$L = K \times BD.$$

(The proof of this may be readily worked out or can be found in "Experimental Electrical Engineering" by V. Karapetoff, Vol. I, page 315.)

To determine the value of the constant K a second run is necessary with a definite added load. This load, which may be small, may be obtained by closing the armature circuit on a resistance if a generator is used in the test or by applying a known torque if a Prony brake is used. With the first method a watt-meter should be used and the load kept constant for a limited range of speed, with the second method the torque should be kept constant for a limited range of speed. If this known added load be denoted by M we then have

$$L + M = K \times B'D'.$$

Since L is the only unknown quantity except K it may be eliminated from these two equations and we have

$$K = \frac{M}{B'D' - B\,D}$$

In some work that the author has done this method has proven to be very reliable and has checked with values of friction and windage losses as determined by other methods.

CHAPTER X

GENERAL LAWS AND CONSTANTS

87. Head.—The theory that had been presented has made it clear that the speed and power of any turbine depends upon the head under which it is operated. The peripheral speed of any runner may be expressed as $u_1 = \phi\sqrt{2gh}$. It has also been shown that for the best efficiency ϕ must have a certain value depending upon the design of the turbine. It is thus apparent that the best speed of a given turbine varies as the square root of the head.

The discharge through any orifice varies as the square root of the head, and a turbine is only a special form of discharge orifice. Since $V_1 = c\sqrt{2gh}$, and since a definite value of c goes with the best value of ϕ as given above, it follows that the discharge of a given turbine varies as the square root of the head.

Since the power of each unit volume of water varies as the head, and since the amount of water discharged varies as the square root of the head it must then be true that the power input varies as the three halves power of the head.

In reality the discharge through any orifice is not strictly proportional to the square root of the head, that is, the coefficient of discharge is not strictly a constant but varies slightly with the head. However, the variation in the coefficient is small and inappreciable except for very large differences in the head. Therefore the above statements are accurate enough for most practical purposes.

The theory has also shown that the losses of head in any turbine vary as the squares of the various velocities concerned. This rests upon the assumption that the coefficient of loss k is constant for all values of h as long as ϕ remains constant. That is probably not true, but may be assumed as true for all practical purposes. Since these velocities vary as the square root of the head their squares will vary as the first power of the head. The amount of water varies as the square root of the head and, since the power lost is the product of these two items, it follows that the various hydraulic losses vary as the three-halves power of the head. As the hydraulic losses vary in just the same proportion as the power input, the hydraulic efficiency will be independent of the value of the head. If the mechanical losses followed the same law then the gross efficiency would also remain unchanged. The mechanical losses really follow a law which changes at different speeds, as can be seen in Fig. 77. The factors which influence this are rather complicated and it does not seem possible to lay down any rule to express mechanical losses as a function of the speed. It is probably true, however, that these losses increase faster than the first power of the speed but not much faster than the square of the speed. Since the speed varies as the square root of the head it is seen that if the friction losses vary at the same rate as the hydraulic losses they must increase

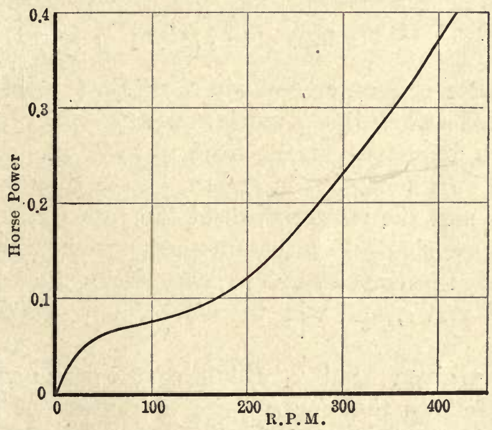


FIG. 77.—Friction and windage of a 24″ tangential water wheel.

as the cube of the speed. As they do not do so, it is apparent that the gross efficiency will be higher the higher the head under which the turbine operates.

The mechanical losses are really comparatively small being only from 2 to 5 per cent. usually and thus the change in gross efficiency cannot be very great. For moderate changes in head we may then state that the efficiency of a turbine remains constant as long as the speed varies so as to keep ϕ constant. Therefore the power output of a given turbine varies as the three-halves power of the head.

This proposition is rather important because it is often possible to test a turbine under a certain head which is different from the head under which it is to be run. The question may then arise as to how far the test results can be applied to the new head. As long as the two heads are not radically different we may state that they will apply directly. If there is a big difference in head we may expect that the efficiency under the higher head may be one or two or more per cent. higher. This is borne out by some tests made by F. G. Switzer and the author where the head was varied from 30 ft. to 175 ft.

It is customary to state the performance of a turbine under one foot head. Then by means of the above relations we may easily tell what it will do under any head. If the suffix (1) denotes a value for one foot head we may then write,

$$N = N_1\sqrt{h} \qquad (28)$$

$$q = q_1\sqrt{h} \qquad (29)$$

$$\text{h.p.} = \text{h.p.}_1\, h^{3/2}* \qquad (30)$$

88. Diameter of Runner.—When a certain type of runner has been perfected a whole line of stock runners of that type may then be built with diameters ranging from 10 to 70 in. All of these runners will be homologous in design, that is they will have the same angles and the same values of the ratios x, y, and B/D. Each runner will simply be an enlargement or reduction of another. They will then have the same characteristics, that is the same values of ϕ and c, and will therefore follow certain laws of proportion.

Since for a given head u_1 will have the same value for all of them, it follows that for a series of runners of homologous design the best r.p.m. will be inversely proportional to the diameter.

Since the discharge through any runner is equal to $F_1c\sqrt{2gh}$, and since c will have essentially the same value for all the runners of such a series, the discharge will be proportional to the area F_1. But if the runners are strictly homologous the area F_1 will be proportional to the square of the diameter. It will therefore be true that the discharge of any turbine of the series will be proportional to the square of the diameter.

Since the power is directly related to the discharge it also follows that the power of the turbine is proportional to the square of the diameter.

These relations are of practical value because, if the speed,

*The easiest way to find $h^{3/2}$ is to note that $h^{3/2} = h.\sqrt{h}$. This may be found in one setting of the slide rule.

discharge, and power of any runner is known by accurate test, predictions may then be made regarding the performance of any other runner of the series. These laws may not hold absolutely in all cases because the series may not be strictly homologous, that is the larger runners may differ slightly from the smaller ones. Also it will no doubt be true that the efficiency of the larger runners will be somewhat higher than that of the smaller ones. It may also be found that careful tests of two runners made from the same patterns will not give exactly the same results due to difference in finish or other imperceptible matters. Despite these factors, however, the relations stated are true enough to be used for most purposes.

89. Commercial Constants.—For a given turbine the maximum efficiency will be obtained only for a certain value of ϕ. All tables in catalogs of manufacturers as well as all values given in this chapter are based upon the assumption that the speed will be such as to secure this value of ϕ. Substituting values of N and D for u_1 in the expression $u_1 = \phi\sqrt{2gh}$, we obtain

$$N = \frac{1840\phi\sqrt{h}}{D} \tag{31}$$

where D is the diameter of the runner in inches. From this may also be written

$$\phi = 0.000543\frac{DN}{\sqrt{h}} \tag{32}$$

Since ϕ is constant for any series of runners of homologous design, it follows from (31) that the expression $\frac{D\,N}{\sqrt{h}}$ must remain a constant. If, then, the best r.p.m. of any diameter of runner under any head is determined, the proper r.p.m. of any other runner of the series under any head may be readily computed.

For the tangential water wheel:

$$\phi = 0.43 \text{ to } 0.47$$

$$\frac{D\,N}{\sqrt{h}} = 790 \text{ to } 870.$$

For the reaction turbine:

$$\phi = 0.57 \text{ to } 0.87$$

$$\frac{D\,N}{\sqrt{h}} = 1050 \text{ to } 1600.$$

If values outside these limits are met with it is because the speed is not the best or because the nominal value of D is not the true value.

90. Diameter and Discharge.—Since the discharge of any runner is proportional to the square of its diameter and to the square root of the head, we may write

$$q = K_1 D^2 \sqrt{h} \tag{33}$$

The value of K_1 depends upon the velocity V_1 and the area F_1. The former depends upon the value of c (Art. 68), and the latter depends upon the diameter D, the height of the runner B (Fig. 26), the value of the angle A_1, and also the number of buckets and guides.

Since there are so many factors involved, it will be seen that a given value of K_1 can be obtained in several ways. For some purposes it might be convenient to express these items by separate constants but for the present purpose it will be sufficient to cover all of them by the one constant.

The lowest value of K_1 will be obtained for the tangential water wheel with a single jet. For this type of wheel there is evidently no minimum value of K_1 below which we could not go. The maximum value of K_1 is, however, fixed by the maximum size of jet that may be used. (See Art. 27 and Art. 61.) Using this maximum size of jet we obtain a value of $K_1 = 0.0005$. However the more usual value is about $K_1 = 0.0003$. There is seldom any reason for using a large diameter of wheel with a small jet and so much lower values are rare.

With the reaction turbine the lowest values of K_1 would be obtained with type *I* in Fig. 26 and the highest with type *IV*. The value of the area F_1 is proportional to the sine of A_1 and normally small values of A_1 go with small values of the ratio B/D. Taking the usual values that go with either extreme we get a minimum value of $K_1 = 0.0014$ and a maximum value $K_1 = 0.0360$. These are not absolute limits but they cannot be exceeded very much and to do so at all would mean to extend our proportions of design beyond present practice. For the usual run of stock turbines values of K_1 vary from 0.005 to 0.025. To summarize:

For the tangential water wheel $K_1 = 0.0002$ to 0.0005
For the reaction turbine $K_1 = 0.0014$ to 0.0360.

91. Diameter and Power.—Since the power of any runner is proportional to the square of the diameter and to the three-halves power of the head, we may write

$$h.p. = K_2 D^2 h^{\frac{3}{2}} \tag{34}$$

As the power is directly dependent upon the discharge it is evident that the discussion in the preceding article will apply equally well here. K_2 may be computed directly from K_1 if the efficiency is known, or it may be determined independently by test.

For the tangential water wheel $K_2 = 0.000018$ to 0.000045
For the reaction turbine $K_2 = 0.00012$ to 0.0033

92. Specific Speed.—In Art. 89 we have the relation between diameter and r.p.m.; in Art. 91 we have the relation between diameter and power. It is now desirable to establish the relation between r.p.m. and power as follows:
From (31)

$$D = \frac{1840\,\phi\sqrt{h}}{N}$$

From (34)

$$\sqrt{K_2}\,D = \frac{\sqrt{h.p.}}{h^{\frac{3}{4}}}$$

Substituting the above value of D in the second expression we have

$$\sqrt{K_2}\,\frac{1840\,\phi\sqrt{h}}{N} = \frac{\sqrt{h.p.}}{h^{\frac{3}{4}}}$$

Letting N_s stand for the constant factors and rearranging we have

should read

$$N_s = \frac{N\sqrt{h.p.}}{h^{\frac{5}{4}}}{}^* = \frac{N\sqrt{h.p.}}{h^{5/4}}{}^* \tag{35}$$

This expression is a very useful factor and is called the specific speed. It is also called unit speed or type characteristic by various writers. Its physical meaning can be seen as follows: If the head be reduced to 1 ft. then $N_s = N\sqrt{h.p.}$ By then varying the diameter of the runner the value of N will change in an inverse ratio, but the square root of the horse-power varies directly as D. Thus the product of the two or N_s remains con-

* $h^{\frac{5}{4}} = h \times h^{\frac{1}{4}} = h\sqrt{\sqrt{h}}$.

stant for all values of D as long as the series is homologous. If a value of D be chosen which will make the h.p. = 1.0 when $h=1$ ft., we then have $N_s = N$.

That is, the specific speed is the speed at which a turbine would run under one foot head if its diameter were such that it would develop 1 h.p. under that head. The specific speed is also an excellent index of the class to which a turbine belongs and hence the term type characteristic is very appropriate. There is no standard symbol used by all to denote this constant though N_s is quite common. Other notations are N_u, K_T, and numerous others. In Europe the specific speed will be expressed in metric units; to convert from one to the other multiply N_s in English units by 4.45.

It will be seen that N_s involves both ϕ and K_2; thus it will have its limits for the same reasons that these other factors have. Since the power is the h. p. output, the efficiency is also included, though it does not appear directly.

The usual value of N_s for the tangential water wheel is from 3.5 to 4.5. This may run up as high as 5 or 6, though the latter involves some sacrifice of efficiency because the diameter of the jet will then be too large for the size of the wheel. With the reaction turbine the minimum value of N_s that has been attained so far is about 10, while the maximum value is 100. With the usual line of stock turbines N_s varies from 20 or 30 up to about 85. It will be seen that there is a gap between the tangential water wheel and the reaction turbine, as values of N_s between 6 and 10 do not exist with either type. Similar gaps are also seen with values of ϕ, K_1, and K_2. By the use of two or more nozzles on one tangential water wheel this value of N_s can be increased. In Europe a few two-stage radial inward-flow reaction turbines have been built and these could have lower values of N_s than 10. Thus the entire field can be covered.

To recapitulate:

For the tangential water wheel $N_s = 3.5$ to 4.5 (6 max.)
For the reaction turbine $N_s = 10$ to 100.[1]

93. Determination of Constants.—The constants given in this chapter may be computed from theory, but for practical

[1] Tables of values of ϕ, K_1, K_2, and K_5 computed from catalogs for the different stock turbines made by various firms in this country will be found in Mead's "Water Power Engineering." p. 326–350. The $K_5 = (N_s)^2$.

use should be secured from test data. The catalogs of turbine manufacturers usually contain tables giving the discharge, power, and speed of different diameters of runners under various heads. As these tables are supposed to be based upon tests they may be used for the determination of these factors. If all the runners of the series were strictly homologous it would be necessary to compute these constants for one case only. Actually variations will exist with different diameters of runners and thus there will be some variation in the values secured. Since each manufacturer usually makes several lines of runners so as to cover the field to better advantage, there will be as many distinct values of these constants as he makes types of runners. If the catalog tables are purely fictitious then the computations based upon them will not be very reliable.

94. Illustrative Case.—In order to illustrate the preceding article the following tables are given. For the sake of comparison only two firms out of many are chosen for this case. The values given are based upon catalog tables. Since K_2 depends upon K_1 it has been omitted to save space.

TABLE 3.—JAMES LEFFEL AND CO.

Type	ϕ	K_1	N_s
Standard	0.722–0.727	0.0061–0.0064	30.8–32.6
Special	0.750–0.779	0.0094–0.0097	41.6–43.2
Samson	0.838–0.844	0.0170–0.0171	61.5–61.9
Improved Samson	0.856–0.886	0.0220–0.0220	71.0–73.5

TABLE 4.—DAYTON GLOBE IRON WORKS CO.

Type	ϕ	K_1	N_s
High head type	0.578–0.585	0.0051–0.0064	22.8–26.0
American	0.662–0.704	0.0054–0.0080	25.0–32.3
Special New American	0.697–0.727	0.0175–0.0205	50.0–57.4
Improved New American	0.886–0.944	0.0233–0.0263	78.2–80.5

This table shows the variation in constants that might be expected, and shows also how each firm attempts to cover the ground. It will be noticed, however, that the two do not agree in all respects. Thus suppose a turbine was desired whose specific speed was 42. The "Special" turbine of the Leffel Co. would fulfill the conditions, but the Dayton Globe Iron Works Co. have no line of turbines that would exactly answer the requirement. The latter firm might furnish a turbine that would have the required specific speed but it would have to be a special design—it would not be a stock turbine, and would therefore be more expensive.

95. Uses of Constants.—After these factors are determined it will then be easy to find what results may be secured for any size turbine of the same design under any head. Another use for them is that when the limits are fixed they will enable one to tell what is possible and what is not. In the next chapter it will be shown how they are of direct use also in the selection of a turbine.

96. NUMERICAL ILLUSTRATIONS

1. The test of a 16-in. runner under a 25-ft. head gave the following as the best results: $N=400$, $q=17.5$ cu. ft. per second, h.p. $=39.8$. Find the constants.

$$\text{From (32) } \phi = 0.000543\frac{16\times400}{5} = 0.696$$

$$\text{From (33) } K_1 = \frac{17.5}{16^2\times5} = 0.01368$$

$$\text{From (34) } K_2 = \frac{39.8}{16^2\times125} = 0.00124$$

$$\text{From (35) } N_s = \frac{400\times6.32}{55.8} = 45.2$$

2. Suppose that a 40-in. runner of the same design as in problem (1) is used under a 150-ft. head. Compute the speed, discharge, and horse-power.

$$\text{From (31) } N = \frac{1840\times0.696\times12.25}{40} = 392 \text{ r.p.m.}$$

$$\text{From (33) } q = 0.01368\times1600\times12.25 = 268 \text{ cu. ft. per second}$$

$$\text{From (34) } 0.00124\times1600\times1838 = 3650 \text{ h.p.}$$

3. Suppose that turbines of the type in problem (1) were satisfactory for a certain plant but that the number of the units (and consequently the power of each) and the speed has not been decided upon. If the head is 150 ft., then by (35)

$$N\times\sqrt{h.p.} = 45.2\times525 = 23{,}730.$$

By the use of different diameters of runners of this one type the following results can be secured:

14,100 h.p. at 200 r.p.m.
6,250 h.p. at 300 r.p.m.
3,520 h.p. at 400 r.p.m.
2,250 h.p. at 500 r.p.m.
1,560 h.p. at 600 r.p.m.
1,150 h.p. at 700 r.p.m.
878 h.p. at 800 r.p.m.
695 h.p. at 900 r.p.m.

If the capacity of the plant were 25,000 h.p. it might then have 4 units at 300 r.p.m., 16 units at 600 r.p.m., or 36 units at 900 r.p.m. If none of the possible combinations were suitable it would be necessary to use another type of turbine—that is one with a different value of N_s.

By equation (34) the diameters are found to be 52.3 in., 26.2 in., and 17.5 in. for 300, 600, and 900 r.p.m. respectively.

CHAPTER XI

SELECTION OF A TURBINE

97. Speed.—For a given diameter of runner the possibilities as to speed regardless of the power can be determined by the values of the constants given in Art. 89. Or if a definite speed be selected the diameter of the runner can be found. Thus if the head be 60 ft. and the speed 300 r.p.m. the diameter of a tangential water wheel would be about 21 in. and the diameter of a reaction turbine might be from 27 in. to 42 in.

98. Capacity.—For a given diameter of runner the limits of capacity regardless of the speed can be determined by the methods given in Art. 90. Or if a definite capacity be selected the diameter of the runner may be found. Thus suppose that the water available is 30 cu. ft. per second under a 60-ft. head, the diameter of a tangential water wheel would be 88 in. or more while the diameter of a reaction turbine might range from 53 in. to 10 in.

The power instead of the discharge may be used as given in Art. 91.

99. Specific Speed an Index of Type.—Both the elements of speed and capacity are involved in the specific speed. It was stated in Art. 35 that both speed and capacity were merely relative terms; that is, a high-speed turbine is not necessarily one which runs at a high r.p.m., but one whose speed is high compared with other turbines of the same power under the same head. In like manner a high-capacity turbine is not necessarily one of great power but merely one whose power is high compared with others at the same speed under the same head. Since $N_s = \frac{N\sqrt{h.p.}}{h^{\frac{5}{4}}}$ it is evident that a low-speed low-capacity turbine will be indicated by a low value of N_s and a high-speed high-capacity turbine by a high value of N_s. As stated in Art. 92, values of N_s for the tangential water wheel may run up as high as 5 or 6, for the reaction turbine they range from 10 to 100. Values in the neighborhood of 20 indicate a runner such as Type I in

Fig. 26, while values in the neighborhood of 80 indicate Type IV. Thus when the speed and horse-power of any turbine under a given head are specified the type of turbine necessary is fixed.

Other things being equal, it is seen that a high head means a comparatively low value of N_s while a low head means a high value. Aside from any structural features it is apparent that a high head calls for a tangential water wheel or a low-speed reaction turbine, while a low head demands a high-speed reaction turbine. However, the head alone does not determine the value of N_s. So far as the r.p.m. is concerned there may be considerable variation, yet neither a very low nor a very high r.p.m. is desirable and for the present purpose we may suppose that it is restricted within narrow limits. The value of N_s will thus be affected by the power of the turbine as well as the head. If the head is high the value of N_s may still be high enough to require a reaction turbine. Or if the head is very low and the power is likewise low a high value of specific speed may result. It is thus clear that the choice of the type of turbine is a function of the power and speed as well as the head.

100. Illustrations of Specific Speed.—For a turbine of 2000 h.p. at 1000 r.p.m. under 1600 ft. head the value of N_s is 4.42. Thus a very low-speed turbine, the tangential water wheel, is required. The actual r.p.m., however, is high.

For a 5000 h.p. turbine at 100 r.p.m. under 36 ft. head N_s equals 80.3 Thus a high-speed reaction turbine is indicated, though the actual r.p.m. may be relatively low.

Suppose that a 12-h.p. turbine is to be run at 100 r.p.m. under a 36-ft. head, the value of the specific speed is 3.95, which means a tangential water wheel. For the larger power under the same conditions in the preceding example a reaction turbine was required. If the speed were 600 r.p.m., however, a low-speed reaction turbine would be necessary for N_s would equal 23.6.

Suppose that a 20-h.p. turbine is to run at 300 r.p.m. under a 60-ft. head. The value of N_s is 8.04 and that would require a tangential water wheel with two nozzles.

If 10,000 h.p. is required at 300 r.p.m. under a 60-ft. head, the value of N_s would be 179.5. As this is an impossible value it would be necessary to reduce the speed or to divide the power up among at least 4 units of 2500 h.p. each.

101. Selection of a Stock Turbine.—The choice of the type of turbine will be taken up in the next chapter. For the present suppose that required values of speed and power under the given head are determined. The value of the specific speed can then be computed and will indicate the type necessary. If the turbine is to be built as a special turbine nothing more is to be done except to turn the specifications over to the builders.

If, however, the turbine is to be selected from the stock runners listed in the catalogs of the various makers, it will be necessary to find out what firms are prepared to furnish that particular type of runner. It would be a tedious matter to search through a number of tables in numerous catalogs to find the particular combination desired, but the labor is avoided by the use of the constants given in the preceding chapter. It will be necessary merely to compute values of specific speeds of turbines made by different manufacturers. This can be quite readily done and such a table will always be available for future use.

A make of turbine should then be selected having a value of N_s very near to the value desired. The value of N_s ought to be as large as that required, otherwise the turbine may prove deficient on power, and for the best efficiency under the usual loads it should not greatly exceed the desired value. Having selected several suitable runners in this way, bids may be called for. These bids should be accompanied by official signed reports of Holyoke tests of this size of wheel or the nearest sizes above and below, if none of that particular size are available. This is to enable us to check up the constants obtained from catalog data and to verify the efficiencies clamed. Holyoke test data is very essential if the conditions of the installation are such that an accurate test is not feasible. In making a final choice other factors would be considered such as efficiency on part load, and efficiency and power under varying head.

102. Illustrative Case.—Suppose a turbine is required to develop 480 h.p. at 120 r.p.m. under 20-ft. head. The value of N_s is then 62.2. There are four makes of turbines which approach this as follows:

TABLE 5.

Maker	Type	N_s	K_2
Camden Water Wheel Wks.	United States Turbine....	64.7	0.00190
James Leffel and Co......	Samson..................	61.8	0.00158
Platt Iron Works.........	Victor Standard..........	63.0	0.00205
Trump Mfg. Co..........	Standard Trump.........	61.5	0.00210

It is thus apparent that any one of these manufacturers could supply a turbine from their present designs which would nearly fill the requirements. A number of other firms in this country could not fit the case except with a special design or a modification of an existing design. Thus inspection of the table for the Dayton Globe Iron Works Co. in Art. 94 will show that the nearest approach they have to it is their Special New American with an average value of N_s of 53.7. They could supply a turbine to run at 120 r.p.m. under the head specified, but it would develop only 358 h.p. Or if they supplied a turbine capable of delivering 480 h.p. it would run at 103.5 r.p.m.

Turning to the four cases presented in the table, it is apparent that the Camden wheel is a little over the required capacity, but it may not be enough to be objectionable. The Platt Iron Works wheel is very little over the required capacity and the Leffel and Trump are a trifle under it. If there is a little margin allowable in the power, any of these might be used. The value of N_s according to which the wheel is rated should be the value for the speed and power at which it develops its best efficiency. In any plant the variation in the head produces a deviation from the best value of ϕ, if the wheel be run at constant speed, and thus causes a drop in efficiency. The power of the wheel may increase or decrease according to the way the head changes. Thus in actual operation the conditions depart so much from those specified in the determination of N_s that small discrepancies in its value such as exist in the table are of little importance.

If desired, the diameters of the runners may be determined by means of K_2. For the four cases in the order given they will be 53.2 in., 58.2 in., 51.2 in., 50.3 in. Actually standard sizes will not agree precisely with these figures and thus a further modification may be brought about. However, mathematical exactness must not be expected in work of this nature. What we attempt to do is merely to select a turbine the peak of whose efficiency falls as near as possible to the conditions of head, speed, and power chosen. Although our conditions may be such that we may rarely realize the very highest efficiency of which the turbine is capable, yet we should be very close to it.

103. Variable Load and Head.—In any plant the load is usually not constant but varies over a considerable range. In comparing turbines for certain situations the average operating efficiency may be more important than the efficiency on full load only. If the turbine is to run on full load most of the time or if the installation is such that the pondage is limited or lacking altogether, then the efficiency on part load is of little importance. But if the load is variable and if water can be stored up during the time the wheel may be running under a light load then the efficiency at all times becomes of interest.

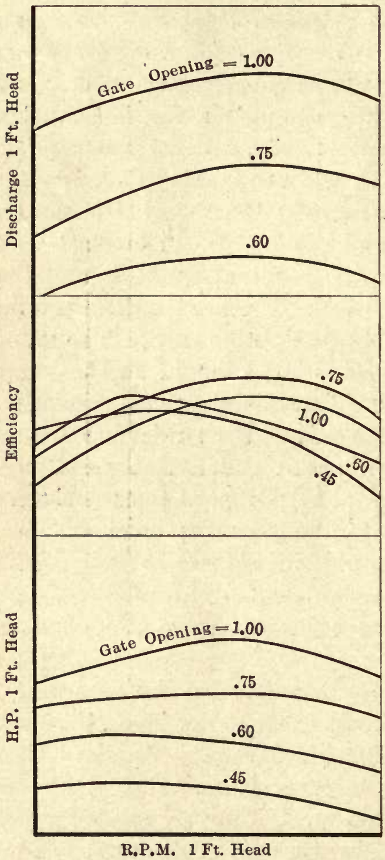


Fig. 78.

In most low head plants the variation in head is a serious item also and the turbines submitted should be compared as to their efficiency and power under the range of head anticipated.

All turbines having the same specific speed are not necessarily equally well suited for the same conditions. A detailed study of the characteristics of each one is essential before the final choice can be made. In many cases the best efficiency will be the deciding factor. In others the average operating efficiency will be more important, and sometimes capacity under varying head will be the chief item.

These factors can be studied by means of curves such as are shown in Fig. 78. Efficiency, discharge, and power for various gate openings reduced to 1-ft. head are plotted against ϕ or the r.p.m. under 1-ft. head. The normal speed and power should be that corresponding to the maximum efficiency. If the wheel is run at constant speed a variation in head causes a change in ϕ.

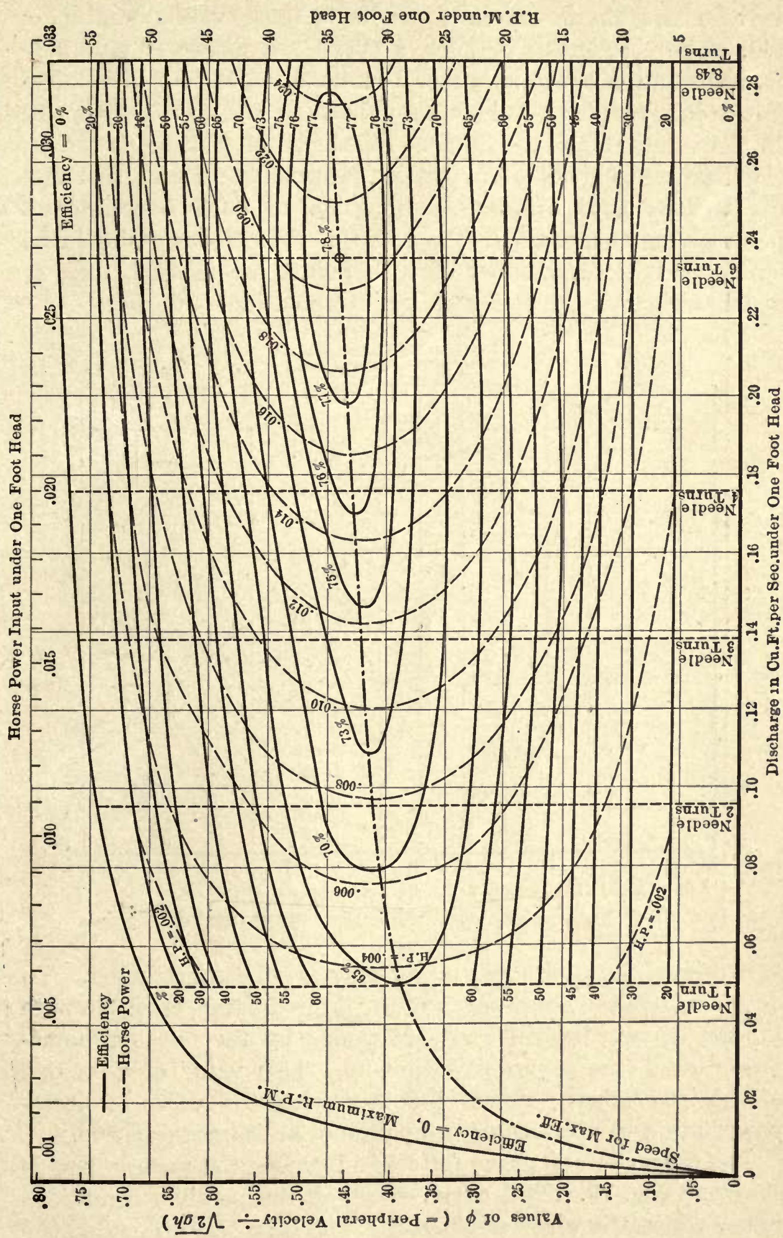


FIG. 79.—Characteristic curve for a 24″ tangential water wheel.

104. Characteristic Curve.—For a thorough study of a turbine the characteristic curve is a most valuable graphic aid. The coordinates of such a curve are discharge under 1-ft. head and ϕ or r.p.m. under 1-ft. head. Values of the horse-power input under 1-ft. head should also be laid off to correspond to the values of the discharge. Lines should then be drawn on the diagram to indicate the relation between speed and discharge for various gate openings. Alongside of each experimental point giving this relation, the value of the efficiency should be written. When a number of such points are located, lines of equal efficiency may be drawn by interpolation.

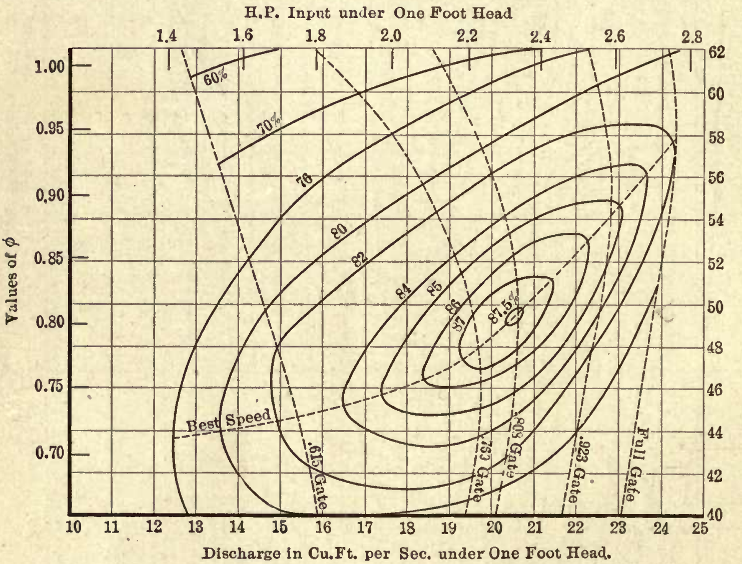


FIG. 80.—Characteristic curve for a high speed reaction turbine.

If desired, lines of equal power may also be constructed. To do so, assume the horse-power of the desired curve, then compute the horse-power input for any efficiency by the relation, horse-power input = horse-power output $\div e$. This value of e on one of the iso-efficiency curves together with the value of horse-power input locates one or two points of an iso-power curve.

The characteristic curve for a 24-in. tangential water wheel is shown in Fig. 79. This curve covers all the possible conditions under which the wheel might run. The only way to extend the field would be to put on a larger nozzle. Since the discharge of a tangential water wheel is independent of the speed the lines for the various gate openings will be straight. For the reaction turbine they will be curved as seen in Fig. 80. The latter is a portion of a characteristic curve for a high-speed turbine.

Any marked irregularities in the characteristic curve are indications of errors in the test. It is possible for there to be only one peak in the efficiency curves and an indication of two, as sometimes occurs, is due to incorrect data.

105. Use of Characteristic Curve.—From the characteristic curve it is apparent, at a glance, at what speed the turbine should run for the best efficiency at any gate opening. The best efficiency in Fig. 79 is obtained when $\phi = 0.457$ or $N_1 = 34$, and with the needle open 6 turns. With full nozzle opening the best value of N_1 is 35, with the needle open 3 turns the best speed is such that $N_1 = 32$. (With the reaction turbine these differences would be greater.)

From the characteristic curves any other curves may be constructed. For constant speed follow along a horizontal line, for a fixed gate opening follow along the curve for that relation.

If it is desired to investigate the effect of change of head when the speed is kept constant, compute the new values of ϕ or N_1. Thus the curve in Fig. 79 was determined by a test under a head of 65.5 ft. and the best speed was 275 r.p.m. That corresponded to $\phi = 0.457$ or $N_1 = 34$. (The value of D used for computing ϕ was slightly different from the nominal diameter.) If the speed is maintained at 275 r.p.m. when the head is 74 ft., then $\phi = 0.429$ and $N_1 = 32$. If $h = 55$ ft., $\phi = 0.497$ and $N_1 = 37$. In the last case the best efficiency would be 77 per cent., a drop of 1 per cent.

CHAPTER XII

SELECTION OF TYPE OF TURBINE

106. Possible Choice.—It has been shown that, if the speed and power under a given head are fixed, the type of turbine necessary is determined. If there is some leeway in these matters it may be possible to vary the specific speed through a considerable range of values. Suppose turbines of a given power may be run at 120 r.p.m., at 600 r.p.m., or at 900 r.p.m. Each one of these would given us a different specific speed and thus a different type of runner. Or, if the speed be fixed, the power, such as 20,000 h.p. may be developed in a single unit, in two units of 10,000 h.p. each, or in eight units of 2,500 h.p. each. Again we have different types of runners demanded. Both the speed and power may be varied in some cases and the choice is wider still. As an example, it may be required to develop 500 h.p. under 140-ft. head. Suppose this power is to be divided up between two runners and the speed to be 120 r.p.m. The value of N_s is then 4.12, showing that a double overhung tangential water wheel is required. Or if the power be developed in a single runner at 600 r.p.m., the value of N_s would be 29.2, which would call for a reaction turbine.

It is customary to choose a speed between certain limits, as neither a very low nor a very high r.p.m. is desirable. Also the number of units into which a given power is divided is limited. Nevertheless considerable latitude is left. It remains to be seen what considerations would lead us to choose such values of speed and power as would permit the use of a certain type of runner.

107. Maximum Efficiency.—The best efficiency developed by a turbine will depend, to some extent, upon the class to which it belongs. The impulse and reaction turbines are so different in their construction and operation that the difference in efficiency between them can be determined solely by experiment. However, abstract reasoning alone will lead to certain conclusions as to the relative merits of different types within each of these two main divisions.

For the tangential water wheel it has been shown that, if the highest efficiency is to be obtained, certain proportions must not be exceeded. If we desire a specific speed higher than 4, it is necessary to pass beyond these limits and thus a wheel whose specific speed is as high as 5 or 6 will not have as high an efficiency as the normal type.

A low-speed reaction turbine such as type *I* in Fig. 26 will have small values of the angle A_1. A consideration of the theory shows that this conduces to high efficiency. However, since the bucket passages are narrow, one would expect a large amount of hydraulic friction loss. Type *II* will have slightly larger values of the angle A_1, but the larger areas of the passages will probably more than offset this so that type *II* might be expected to be more efficient than type *I*. When we turn to type *IV* we find that the angle A_1 is given large values in order to gain area of

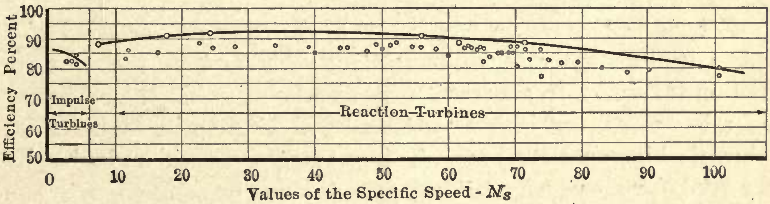


FIG. 81.

guide passages. Since the radius of the discharge varies through such a wide range with a corresponding range of peripheral speed, it does not seem possible to make the discharge loss as small as it is with the lower speed types. These two items will probably more than offset the low amount of friction in flow through the runner due to the large passages, so that we might expect the efficiency of the high-speed type to be lower than that of the medium-speed type. In other words we have sacrificed efficiency to capacity just as we have done in the case of the high-speed impulse turbine.

This reasoning is borne out by the facts, as can be seen by Fig. 81, where efficiency is plotted as a function of specific speed.[1] A number of test points were located and the curve shown was drawn through the highest on the sheet. It shows what has actually been accomplished and it also shows how the maximum efficiency varies with the type of turbine. It is apparent that if

[1] L. F. Moody, Trans. A. S. C. E., Vol. LXVI, p. 347.

one desires the best efficiency possible a specific speed should be chosen between 25 and 50.

It must not be thought that this curve represents the results that one should expect in every case. It merely shows the relative merits of the different types. The actual efficiency obtained depends not only upon the specific speed but also upon the capacity of the turbine and the head and other factors. The larger the capacity of a turbine the higher the efficiency will be. In a given case the efficiency obtained for a specific speed of 30, say, might be only 83 instead of the 93 shown by the curve. But if the specific speed had been 95 instead of 30 the efficiency realized might have been only 73.

108. Efficiency on Part-load.—Full-load will be defined as the load under which a turbine develops its maximum efficiency. Anything above that will be called an overload and anything less than that will be known as part-load.

For reasons given in Art. 55, the efficiency on part-load of a tangential water wheel will be relatively higher than that of a reaction turbine. In general it may also be said that the efficiency of a low-speed turbine on part-load will tend to be higher than that of a high-speed turbine. This last statement will undoubtedly be true if, as is usually the case, the turbine be compelled to run at constant speed.

For the tangential water wheel in Fig. 79 it can be seen that the best speed is slightly different for different gate openings and that it increases as the latter increases. This is also true with the reaction turbine, but in a more marked degree as can be seen in Fig. 80. If the speed is selected so as to give the best efficiency at a certain gate opening it will not be correct for any other gate opening and thus efficiency will be sacrificed at all gates except one.

This variation of the best speed with different gate openings is found in all turbines, but not in the same degree. With the low-speed reaction turbine it is small, approaching the tangential water wheel in that regard. With the high-speed reaction turbine it is very marked. There seems to be little difference beween turbines, in this regard, for specific speeds less than 60; but for specific speeds above that, it increases rapidly.[1]

If, then, a constant speed be selected which is the best for full-load, there will be a sacrifice of efficiency on part-load, and this

[1] C. W. Larner, Trans. A. S. C. E., Vol. LXVI, p. 341 (1910).

sacrifice will be greater the higher the specific speed of the turbine. These considerations, together with the facts given in the preceding article, imply efficiency curves for the various types such as are shown in Fig. 82. To prevent confusion the efficiency curve for a low-speed turbine is not shown, but its efficiency on full-load would be about the same as that for the tangential water wheel and a little less on part-load.

It may also be noticed that there is less overload capacity with the high-speed turbine than with the other types. This is because the point of maximum efficiency is nearer full gate than with the other types. If the customary 25 per cent. overload

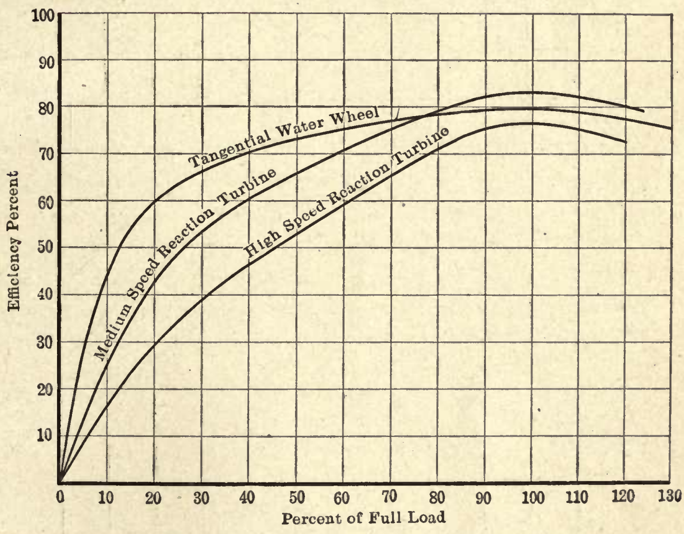


FIG. 82.—Typical efficiency curves.

must be allowed, then the normal load must be less than the rated power with a further decrease in operating efficiency.

109. Overgate with High-speed Turbine.—Full gate will be defined as the point where the turbine develops about its maximum power. If the gate be opened any wider than that there will be no further increase in power and, due to decrease in effiency, there may actually be a drop in power.

The discussion presented here will be based upon the valuable paper of Mr. Larner's on Characteristics of Modern Hydraulic Turbines previously referred to.

If the normal speed be taken as the speed at which the wheel develops its maximum efficiency it may be seen in Fig. 80 that the power at high gates increases as the speed increases above normal. This is a peculiarity of the high-speed turbine. In Fig. 83, full gate is denoted by 1.00 and a wider opening or overgate by 1.077. The normal speed is 52 r.p.m. At normal speed opening up the gate wider than full gate would be wasteful because the increase in power is very small and is accompanied by a big drop in efficiency. In some cases the power might even

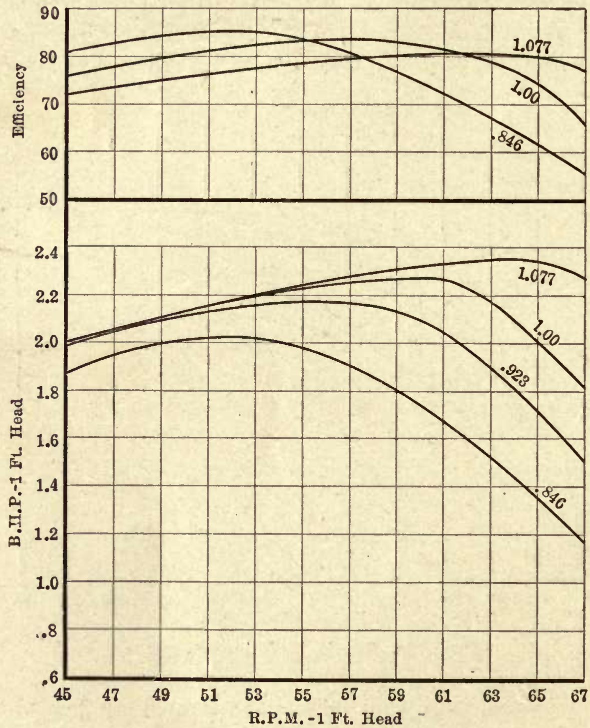


FIG. 83.—High speed turbine.

be less. But at higher speeds or higher values of ϕ, overgating gives higher values of the power. This is a feature that is not possessed by lower speed runners.

If the turbine is run at constant speed and the head falls below its normal value, ϕ will increase. Due to the decrease in head the power of the turbine will decrease. Under such circumstances the gate can then be opened up wider than its normal position with a resultant increase in power. The efficiency may also be higher than for the normal gate opening for the high value of ϕ. This feature fits the high-speed turbine for low head service where relatively large reductions of head may be expected.

It must not be thought that under the lower head overgating increases the actual power of the turbine; it merely increases the power under 1-ft. head for the high values of ϕ. Since the head is less the actual power will be less than normal, but it will be greater than it would be if it were not for this overgate feature.

The differences between the low- and high-speed runners are brought out in the following table. The normal head is 15 ft. and the wheels develop 100 h.p. In time of high water the head will decrease to 10 ft., while the wheels are kept at their normal speed.

TABLE 6.—HEAD = 15 FT.

Type	N_s	R.p.m.	ϕ	Gate	H.p.	Efficiency
Low speed.............	35.2	104	0.70	1.00	100	85
High speed............	77.7	232	0.81	1.00	100	82

TABLE 7.—HEAD = 10 FT.

Type	N_s[1]	R.p.m.	ϕ	Gate	H.p.	Efficiency
Low speed.............	35.2	104	0.86	1.00	47.3	78
High speed............	77.7	232	0.99	1.00	49.3	76
High speed............	......	232	0.99	1.077	58.0	80

Under the lower head the high-speed turbine is seen to be capable of developing 22 1/2 per cent. more power than the low-speed turbine by means of the overgate feature. The efficiency is also seen to be greater.

110. Choice of Type for Low Head.—No definite rules can be laid down for universal use because each case is a separate problem. Neither is it possible to draw any line between a high and a low head. All that can be done is to assume cases that are typical and establish broad general conclusions. In any particular case the engineer can then decide what considerations have weight and what have not.

The average low-head plant has very little, if any, storage capacity. In times of light load the water not used is generally being discharged over the spillway of the dam. Economy of

[1] These values of specific speed apply only when the turbine is developing its best efficiency. Under the reduced head with an incorrect value of ϕ, the real specific speed is different. But as an index of the type the values given are always appropriate.

water on part-load is thus of very little importance. The efficiency on full load is of value as it determines the amount of power that may be developed from the flow available.

Under a low head the r.p.m. is normally low and it is desirable to have a runner with a small diameter and a high value of ϕ, in order to secure a reasonable speed. A high speed means a cheaper generator and, to some extent, a cheaper turbine. These were the factors that brought about the development of the high-speed turbine.

A low-head plant is also usually subjected to a relatively large variation in the head under which it operates. When the head falls below its normal value the overgate feature of the high-speed turbine, enabling it to hold up the power, to some extent, at a good efficiency, is a very valuable characteristic.

The only disadvantage of the high-speed turbine for the typical low-head plant is that its maximum effiency under normal head is not as good as that of the lower speed turbines. However, the other advantages outweigh this so that it is undoubtedly the best for the purpose.

111. Choice of Type for Medium Head.—With a somewhat higher head a limited amount of storage capacity usually becomes available and thus the efficiency on part-load becomes of interest as well as the efficiency on full-load. The r.p.m. also approaches a more desirable value so that the necessity for a high-speed runner disappears. The variation in head will generally be less serious also, so that the overgate feature of the high-speed turbine becomes of less value. The high efficiency of the medium-speed turbine fits it for the case. The high-speed turbine should not be used unless the interest on the money saved is more than the value of the power lost through the lower efficiency.

112. Choice of Type for High Head.—For high heads the possibility of extensive storage increases and the average operating efficiency then becomes of more interest than the maximum efficiency, especially if the turbine is to run under a variable load. Since the normal speed under such a head is high, a runner with a large diameter and low value of ϕ may be desirable, as it keeps the r.p.m. down to a reasonable limit. The choice lies between a medium-speed turbine, a low-speed turbine, or the tangential water wheel.

If the wheel is to run on full-load most of the time, the high full-load efficiency of the medium-speed turbine fits it for the place. If the load is apt to vary over a wide range and be very light a considerable portion of the time, the comparatively flat efficiency curve of the tangential water wheel renders it suitable. There is little difference between the characteristics of the low- and medium-speed wheels. The choice between them is largely a matter of the r.p.m. desired, although there is some slight difference in efficiency.

113. Choice of Type for Very High Head.—Within certain limits there is a choice between the low-speed reaction turbine and the tangential water wheel. The former might be chosen in some cases because of its higher speed with a consequently cheaper generator and the smaller floor space occupied by the unit. The latter has the advantage of greater freedom from breakdowns and the greater ease with which repairs may be made. This consideration is of more value with the average high-head plant than with the average low-head plant, since the former is usually found in a mountainous region where it is comparatively inaccessible, and is away from shops where machine work can be readily done.

For extremely high heads there is no choice. The structural features necessary are such that the tangential water wheel is the only type possible. Also the relatively low speed of the tangential water wheel is of advantage where the speed is inherently high.

CHAPTER XIII

COST OF TURBINES AND WATER POWER

114. General Considerations.—Since there are so many factors involved, it is rather difficult to establish definite laws by which the cost of a turbine may be accurately predicted. No attempt to do so will be made here, but a discussion of the factors involved and their effects will be given and the general range of prices stated. A few actual cases are cited as illustrations.

A stock turbine will cost much less than one that is built to order to fulfill certain specifications. This fact is illustrated by the comparison of two wheels of about the same size and speed. The specifications of the stock turbine were as follows: 550 h.p. at 600 r.p.m. under a head of 134 ft., 26-in. double discharge bronze runner, cast steel wicket gates, cast-iron split globe casing 5 ft. in diameter, and riveted steel draft tube. Weight about 11,500 lb. Price $1750. The special turbine was as follows: 500 h.p. at 514 r.p.m. under a head of 138 ft., bronze runner, spiral case, riveted steel draft tube, connections to header, relief valve, and vertical type 5000 ft.-lb. Lombard governor. Price $4000. The latter includes a governor, relief valve, and some connections which the former did not, but the difference in cost is more than the price of these.

The cost of the turbine is also affected by the quality and quantity of material entering into it, the grade of workmanship, and the general excellence of the design. With the $4000 turbine cited in the preceding paragraph another may be compared which is of superior design. The specifications for the latter were as follows: 550 h.p. at 600 r.p.m. under 142-ft. head, single discharge bronze runner, spiral case with 30-in. intake, cast steel wicket gates, bronze bushed guide vane bearings, riveted steel draft tube, lignum vitæ thrust bearing, oil pressure governor sensitive to 0.5 per cent. The guaranteed efficiencies were

83 per cent. at 410 h.p.
84 per cent. at 500 h.p.
83 per cent. at 550 h.p.

(Nothing was said about efficiency in the preceding case.) Weight of turbine 30000 lb., of governor 3000 lb. Price $6000.

The turbine just quoted was similar to one previously built and the patterns required only slight modification. Where an entirely new design is called for the cost will be greater still, as is evidenced by the bid of another firm, as follows: 550 h.p. at 600 r.p.m. under 142-ft. head, single discharge cast iron runner, spiral case, cast steel guide vanes, cast steel flywheel, oil pressure governor, connections to header, 30-in. hand-operated gate valve, riveted steel draft tube, and relief valve. The guaranteed efficiencies were

	Per cent. of max. h.p.
81.5 per cent. at	100
84.5 per cent. at	90
84.5 per cent. at	85
82.5 per cent. at	75
79.5 per cent. at	60

Weight of turbine complete 38,000 lb. Price $8740. This last turbine includes a few items that the former does not, but the difference in cost cannot be accounted for by them. It will be noted that a flywheel was deemed necessary here, while it was not used on any of the others. Compare the weights and costs of these last two turbines with the weight and cost of the stock turbine first mentioned.

115. Cost of Turbines.—The cost of a turbine depends upon its size and not upon its power. Since the power varies with the head, it is apparent that the cost per h.p. is less as the head increases. Thus a certain 16-in. turbine (weight = 7000 lb.) without governor or any connections may be had for $1000. Under various heads the cost per horse-power would be as follows:

Head	H.p.	Cost per h.p.
30 ft	52	$19.20
60 ft	148	6.75
100 ft	318	3.14

One would not be warranted in saying, however, that under 10-ft. head a turbine would cost $100 per horse-power because the above would develop only 10 h.p. under that head. Neither would one be justified in saying that, since this turbine would develop 1650 h.p. under 300-ft. head, that the cost per horse-power might be only $0.605. Under a 10-ft. head a much lighter and cheaper construction would be entirely reasonable, while under a 300-ft. head the turbine would have to be built stronger and better than this one was.

For a given head, the greater the power of the turbine the less the cost per horse-power will be. Also for a given head and power, the higher the speed, the smaller the wheel and consequently the less the cost. Compare the 600-r.p.m. reaction turbines in Art. 114 with the following, which is a double over-hung tangential water wheel at 120 r.p.m. The horse-power is 500 under 134-ft. head. Oil pressure governor is included, but no connections to penstock are furnished. Weight 80,000 lb. Price $8900.

These last differences are very much magnified if we combine the cost of the generator with that of the turbine. The following are some generator quotations. The first is that of a generator at a special speed. The second is that of a generator of somewhat better construction than the first but of a standard speed. The others are all standard speeds.

150 kv.-a., 2400 volts, 3-phase, 60-cycle, 124 r.p.m.	$4850.
150 kv.-a., 2400 volts, 3-phase, 60-cycle, 120 r.p.m. (Weight 17,210 lb.)	$3300.
300 kv.-a., 2400 volts, 3-phase, 60-cycle, 120 r.p.m. (Weight 25,520 lb.)	$4700.
350 kv.-a., 2400 volts, 3-phase, 60-cycle, 514 r.p.m.	$2330.
350 kv.-a., 2400 volts, 3-phase, 60-cycle, 600 r.p.m.	$2100.

Taking the highest priced 600-r.p.m. turbine and combining it with the 350-kv.-a. generator we get a total of $10,850. Adding the cost of the 120-r.p.m. turbine to that of the 300-kv.-a. generator we get a total of $13,600 for a smaller amount of power.

Prof. F. J. Seery has derived the following empirical formula based upon the list prices of 35 wheels made by 20 manufacturers.

Log $X = A + D/B$, in which X is the cost in dollars for a single stock runner with gates and crown plates suitable for setting in a flume. The value of A ranges from 1.09 to 2.17, but the usual value is about 1.9. The value of B varies from 40 to 83 with a usual value of about 50. These prices are subject to discounts also. The cost of a draft chest for a twin runner will be given by

$X = 0.045\ D^{2.25}$, in which X is in dollars and D is the diameter of the runners in inches.

The cost of the casing increases these values very greatly, as some spiral cases may cost much more than the runner. A single case may be cited of a pair of 20-in. stock runners in a cylinder case with about 30 ft. of 5 ft. steel penstock. Each runner discharges into a separate draft tube about 3 ft. long. The power is 150 h.p. under 30-ft. head. The cost was $2000.

A few quotations are here given. A reaction turbine to develop 4000 h.p. at 600 r.p.m. under 375-ft. head and weighing 90,000 lb. would cost $14,000. Another reaction turbine of 10,000 h.p. under 565-ft. head cost $37,000. In the latter case the governor, pressure regulator, and the generator were included. The building, crane, transformer room, etc., cost $20,000 for this installation. A tangential water wheel of 2500 h.p. under 1200-ft. head cost $12,000, while another of 4500 h.p. under 1700-ft. head cost $8,000.

As has been stated, the cost of a turbine varies between fairly wide limits due to difference in design, workmanship, and commercial conditions. The cost is also less the higher the head or the greater the power. In a general sense it can be said to vary between $2 and $30 per horse-power and according to the following table:

Head	Cost per h.p.	Cost of building per h.p.
— -60 ft.	$30–$7	$30–$4
100–600 ft.	$12–$2	$ 7–$2
500–2000 ft.	$ 8–$2	$ 7–$2

The cost of the turbine is usually only about 6 per cent. of the total cost of the power plant. It, therefore, scarcely pays to buy a cheap turbine when the money saved is such a small portion of the entire investment.

116. Capital cost of Water Power.—The capital cost of water power includes the investment in land, water rights, storage reservoirs, dams, head races or canals, pipe lines, tail race, power house, equipment, transmission lines, interest on money tied up before plant can be put into operation, and often the cost of an auxiliary power or heating plant.

The capital cost per horse-power is less as the capacity of the plant is greater. This is shown by the following table from the report of the Hydro-Electric Power Commission of the

Province of Ontario. The proposed plant was to be located at Niagara Falls.

TABLE 8.

Items	50,000 h.p.	100,000 h.p.
Tunnel tail race	$1,250,000	$1,250,000.
Headworks and canal	450,000	450,000
Wheel pit	500,000	700,000
Power house	300,000	600,000
Hydraulic equipment	1,080,000	1,980,000
Electric equipment	760,000	1,400,000
Transformer station and equipment	350,000	700,000
Office building and machine shop	100,000	100,000
Miscellaneous	75,000	75,000
	$4,865,000	$7,255,000
Engineering, etc., 10 per cent	485,000	725,000
	$5,350,000	$7,980,000
Interest, 2 years at 4 per cent	436,560	651,168
Total capital cost	$5,786,560	$8,631,168
Capital cost per horse-power	$114	$86

The cost per unit capacity is usually less as the head increases. This is illustrated by the following table taken from Mead's "Water Power Engineering."

Capacity horse-power	Head	Capital cost per h.p.			
		Without dam	With dam	With dam and electrical equipment	With dam, electric equipment, and transmission line
8000	18	$63.50	86	115	150
8000	80	21.00	39	60	90

The capital cost may range from $40 to $200 per horse-power, but the average value is about $100.[1]

117. Annual Cost of Water Power.—The annual cost of water power will be the sum of the fixed charges and the operating expenses. The former will cover interest on the capital cost, taxes, insurance, depreciation, and any other items that are con-

[1] For specific cases see Mead's "Water Power Engineering," p. 650.

stant. The latter includes repairs, supplies, labor, and any other items that vary according to the load the plant carries. The annual cost per horse-power is the total annual cost divided by the horse-power capacity of the plant.

The total annual cost will vary with the number of hours the plant is in service and also with the load carried. The cost will be a maximum when the plant carries full load 24 hours per day and 365 days per year. It will be a minimum when the plant is shut down the entire year, being then only the fixed charges. (See Fig. 84.) It is evident that the annual cost per horse-power depends upon the conditions of operation.

However, under the usual conditions of operation, the annual cost may be said to vary from $10 to $30 per horse-power.

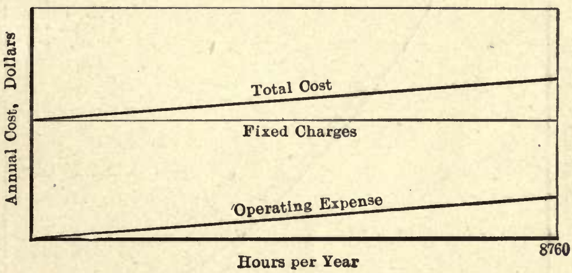


FIG. 84.

118. Cost of Power per Horse-power-hour.—In order to have a true value of the cost of power it is necessary to consider both the load carried and the duration of the load. While the annual cost per horse-power will be a maximum when the plant carries full load continuously throughout the year, the cost per horse-power-hour will be a minimum. Thus suppose the annual cost per horse-power of a plant in continuous operation on full-load is $20. The cost per horse-power-hour is then 0.228 cents. Suppose that the plant is operated only 12 hours per day and that the annual cost per horse-power then becomes $17, the cost of power will be 0.388 cents per horse-power-hour. So far the load has been treated as constant; we shall next assume that it varies continuously and that it has a load factor of 25 per cent. By that is meant that the average load is 25 per cent. of the maximum. If the plant be operated 12 hours per day as before, the annual cost per maximum horse-power may still be $17, but the annual cost per average horse-power will be $68. This latter divided by

4380 hours gives 1.55 cents per horse-power-hour. It is clear, then, that the cost of power per horse-power-hour depends very greatly upon the load curve. It may range anywhere from 0.40 cents to 1.3 cents per horse-power-hour and more if the load factor is low. (See Fig. 85.)

119. Sale of Power.—If power is to be sold, one of the first requirements generally is that the output of the plant should be continuous and uninterrupted. Such a plant should possess at least one reserve unit so that at any time a turbine can be shut down for examination or repair. This adds somewhat to the cost of the plant. The larger the units the more the added cost of this extra unit will be. On the other hand small units are

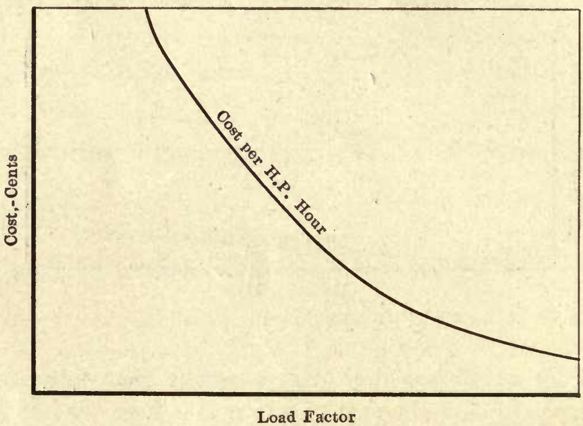


FIG. 85.

undesirable since a large number of them make the plant too complicated. Also the efficiency of the smaller wheels will be less than that of the larger sizes. Unless the water supply is fairly regular, storage reservoirs will be necessary and often auxiliary steam plants are essential in order that the service may not be suspended either in time of high or low water.

A market for the power created is essential. If the demand for the power does not exist at the time the plant is projected, there should be very definite assurance that the future growth of industry will be sufficient to absorb the output of the plant.

If the plant is to be a financial success, the price at which power is sold should exceed the cost of generation by a reasonable margin of profit. The price for which the power may be sold is usually fixed by the cost of its production in other ways. This point should be carefully investigated and, if the cost from other sources is less than the cost of the water power plus the profit, the proposition should be abandoned.

120. Comparison with Steam Power.—It is necessary to be able to estimate the cost of other sources of power in order to tell whether a water-power plant will pay or not. Also it is often essential to figure on the cost of auxiliary power. As steam is the most common source of power and is typical of all others, our discussion will be confined to it.

In general the capital cost of a steam plant is less than that of a water-power plant. It varies from $40 to $100 per horse-power,

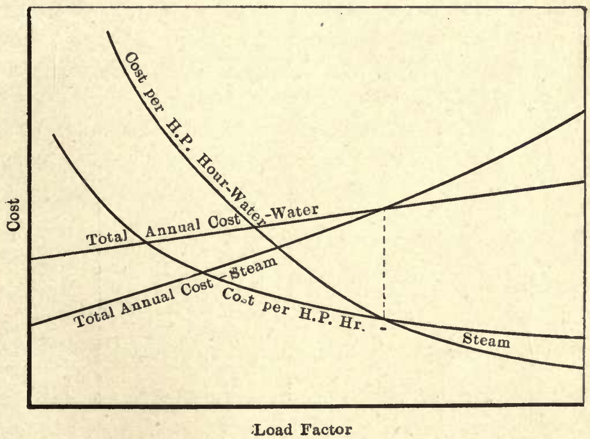


FIG. 86.—Comparison of costs of steam and water power.

with an average value of about $60 per horse-power. Depreciation, repairs, and insurance are at a somewhat higher rate but, nevertheless, the fixed charges are less than for water power.

The amount of labor necessary is greater and this, together with the cost of fuel amd supplies, causes the operating expenses to be higher than with the water power. The total cost of power for the two cases is compared in Fig. 86. As to whether the cost of steam power in a given case is greater or less than that of water power at 100 per cent load factor it is impossible to state without a careful investigation. But it is clear that, as a rule, the cost of steam power is less when the plant is operated but a portion of the year or when the load factor is low. Thus a water-power plant is of the most value when operated at high load factor throughout the year.

The annual cost of steam power per horse-power is very high for small plants but for capacities above 500 h.p. it does not vary so widely. Its value depends upon the capacity of the plant, the load factor, and the length of time the plant is operated. It may be anywhere from $20 to $70, though these are by no means absolute limits.

Since the operating expenses are of secondary importance in a water-power plant, the annual cost per horse-power will not be radically different for different conditions of operation. But with a steam plant the annual cost per horse-power varies widely for different conditions of operation on account of the greater effect of the variable expenses. It is much better to reduce all costs to cents per horse-power hour. The accompanying table gives the usual values of the separate items that make up the cost of steam power.

Items	Min.	Max.
Fuel	0.20	0.75
Supplies	0.03	0.06
Labor	0.07	0.14
Administration	0.02	0.15
Repairs	0.05	0.10
Fixed charges	0.30	0.45
Total cost per horse-power hour	0.67 cents	1.65 cents

The following comparison is made by C. T. Main in Trans. A. S. M. E., Vol. XIII, p. 140. The location was at Lawrence, Mass. Fixed charges were estimated on the following basis:

	Steam	Water
Interest	5	5
Depreciation	3.5	2
Repairs	2	1
Insurance	2	1
Total	12.5 per cent.	9 per cent.

For a steam plant at that location the capital cost was taken as $65 per horse-power. The annual cost of power was as follows:

Fixed charges 12.5 per cent	$8.13
Fuel	8.71
Labor	4.16
Supplies	0.80
Total annual cost per horse-power	$ 21.80

For a water plant the cost of the power house and equipment was taken as $65 while the cost of dams and canals at that place averaged $65 also, making a total capital cost of $130 per horse-power. The annual cost of power was as follows:

Fixed charges 9 per cent	$11.70
Labor and supplies	2.00
Total annual cost per horse-power	$ 13.70

However, for the case in question, a steam-heating plant was necessary and its cost was divided by the horse-power of the plant giving the capital cost of the auxiliary steam plant as $7.50 per horse-power of the power plant. The cost of its operation based upon the power plant would be,

Fixed charges at 12.5 per cent	$0.94
Coal	3.26
Labor	1.23
Total cost of heating	$5.43

Adding this to the cost of the power we obtain the total cost of the water power to be $19.13 per horse-power per year. Evidently the difference in favor of the water power will be $2.67 per horse-power.

The cost of any kind of power will evidently vary in different portions of the country and it is impossible to lay down absolute facts of universal application. In places near the coal fields the cost of steam power will be a minimum and it may be impossible for water power to compete with it. However where the cost of fuel is high water power may be a paying proposition even though its cost may be relatively high.

121. Value of Water Power.—The value of a water power is somewhat difficult to establish as it depends upon the point of view. However, the following statements seem reasonable:

An undeveloped water power is worth nothing if the power, when developed, is not more economical than steam or other power. If the power, when developed, can be produced cheaper than other power, then the value of the water rights would be a sum the interest on which would equal the total annual saving due to the use of the latter. Thus, referring to the case of Mr. Main cited in the preceding article, suppose the water supply is capable of developing 10,000 h.p. The annual saving then due to its use would be $26,700 as compared with steam. Its value is then evidently a sum the interest on which would be $26,700 per year.

A power that is already developed must be considered on a different basis. If the power cannot be produced cheaper than that from any other available source, the value of the plant is merely its first cost less depreciation. If the water power can be produced cheaper than any other, the value of the plant will be its first cost less depreciation added to the value of the water right as given in the preceding paragraph.

CHAPTER XIV

CENTRIFUGAL PUMPS

122. Definition.—Centrifugal pumps are so called because of the fact that centrifugal force or the variation of pressure due to rotation is an important factor in their operation. However, as will be shown later, there are other items which enter in.

The centrifugal pump is closely allied to the reaction turbine and may be said to be a reversed turbine in many respects. Therefore it will be found that most of the general principles given in Chapter VI will apply here also with suitable modifications. Energy is now given up by the vanes of the impeller

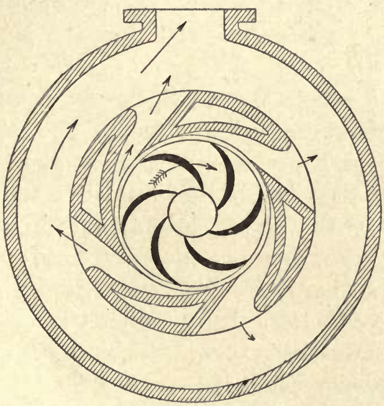


FIG. 87.—Turbine pump.

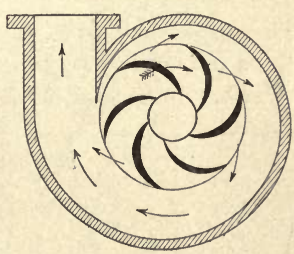


FIG. 88.—Volute pump.

to the water and we have to deal with a *lift* instead of a *fall*. The direction of flow through the impeller is radially outward. During this flow both the pressure and the velocity of the water are increased and when the water leaves the impeller it possesses a large proportion of kinetic energy. It is necessary in any efficient pump to conserve this kinetic energy and transform it into pressure.

123. Classification.—Centrifugal pumps are broadly divided into two classes:

1. Turbine Pumps.
2. Volute Pumps.

While there are other types besides these, the two given are the most important and will be all that are considered in this chapter.

The turbine pump is one in which the impeller is surrounded by a diffusion ring containing diffusion vanes. These provide gradually enlarging passages whose function is to reduce the velocity of the water leaving the impeller and efficiently transform velocity head into pressure head. The casing surrounding the diffuser may be either circular as shown in Fig. 87 or it may be of a spiral form. This latter arrangement would be similar to that of the spiral case turbine shown in Fig. 42.

The volute pump is one which has no diffusion vanes, but, instead, the casing is of a spiral type so made as to gradually reduce the velocity of the water as it flows from the impeller to the discharge pipe. (See Fig. 88.) Thus the energy transformation is accomplished in a different way. The spiral curve for such a case is usually called the volute and thus the pump receives its name.

The discussion given of the volute pump will apply equally well to all other types without diffusion vanes. The only difference will be that these other types are less efficient and also it will probably be impossible to express the shock loss at exit in any satisfactory way. Some of these other types have circular cases with the impeller placed either concentric or eccentric within them. Their only merit is cheapness.

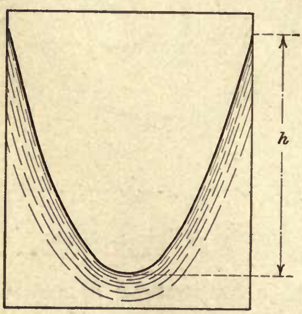


Fig. 89.

124. Centrifugal Force.—If a vessel containing water or any liquid is rotated at a uniform rate about its axis, the water will tend to rotate at the same speed and the surface will assume a curve as shown in Fig. 89. This curve can be shown to be a parabola such that $h = u_2^2/2g$, where u_2 = linear velocity of vessel at radius r_2. If the water be confined so that its surface cannot change, then the pressure will follow the same law.

If, as in Fig. 90, we have the water in a closed chamber set in motion by a paddle wheel, the pressure in an outer chamber communicating with it will be greater than that in the center by the amount $u_2^2/2g$. If a piezometer tube be inserted in this chamber, water will rise to a height h in it such that $h = u_2^2/2g$. This then is the height of a column of water sustained by centrifugal force. If the height of the tube be somewhat less than this, water will flow out, and we would have a crude centrifugal pump.

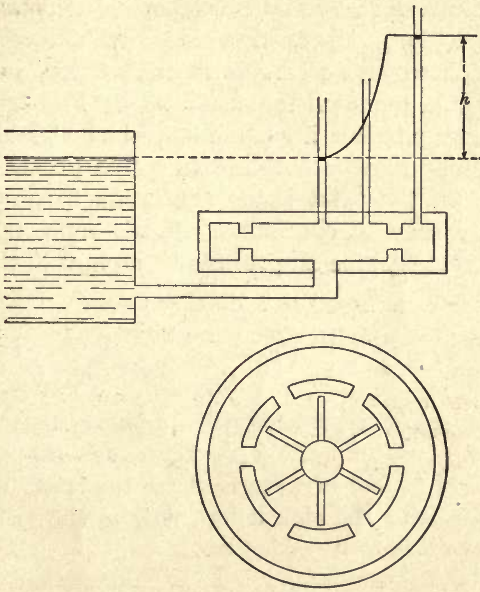


Fig. 90.

125. Notation.—The notation used will be essentially the same as that given in Art. 42. To that will be added A'_2 as the angle the diffusion vanes make with u_2. Also suffix (3) will denote a point in the casing. The actual lift of the pump will be denoted by h, while the head that is imparted to the water by the impeller will be denoted by h''. If h' represents all the hydraulic losses within the pump, then $h = h'' - h'$. Since we are dealing with head imparted to the water instead of given up by the water, we shall reverse signs in equation (14) and have

$$h'' = \frac{1}{g}(u_2 s_2 - u_1 s_1) \qquad (A)$$

It will also be found to be more convenient to express all velocities in terms of u_2 and v_2.

Whereas turbines are rated according to the diameter of the runner, centrifugal pumps are rated according to the diameter of the discharge pipe in inches. The usual velocity of flow at the discharge is 10 ft. per second. From this the size of pump necessary for a given capacity may be approximately estimated. In some cases, however, the velocity may be twice this value.

126. Definition of Head and Efficiency.—In all cases the head h under which the pump operates is the actual vertical height the water is lifted plus all losses in the suction and discharge pipes. It should be noted that the velocity head at the mouth of the discharge pipe is a discharge loss which should be added.

The head may also be obtained in a test by taking the difference between the total heads (Equation 3) on the suction and discharge sides of the pump. If the suffix (s) signifies a point in the suction pipe and suffix (D) a point in the discharge pipe we have

$$h=\frac{p_D}{w}+\frac{p_S}{w}+z_D-z_S+\frac{V_D^2}{2g}-\frac{V_S^2}{2g} \qquad (B)$$

In this case p_D/w represents the pressure gage reading reduced to feet of water while p_S/w represents the vacuum gage reading reduced to feet of water. Since the latter is essentially a negative quantity the sign before it is made positive in this equation, since it is to be added in.

The word efficiency without any qualification will always denote gross efficiency, that is the ratio of the power delivered in the water to the power necessary to run the pump. The hydraulic efficiency is the ratio of the power delivered in the water to the power necessary to run the pump after bearing friction, disk friction, and other mechanical losses are deducted. The hydraulic efficiency is therefore equal to Wh/Wh'' or h/h''. This latter expression is termed manometric efficiency by some and is treated as something essentially different from hydraulic efficiency. If the true value of h'' could be computed, the value of the hydraulic efficiency so obtained would be the same as that obtained experimentally by deducting mechanical losses from the power necessary to drive the pump. Actually the ratio of h/h'' will usually be less than this value but that is due to the fact that our theory is imperfect. (Art. 132.)

127. Head Imparted to Water.—We have seen in Art. 125 that the expression for the head imparted to the water by the vanes of the impeller is

$$h'' = \frac{1}{g}(u_2 s_2 - u_1 s_1).^1$$

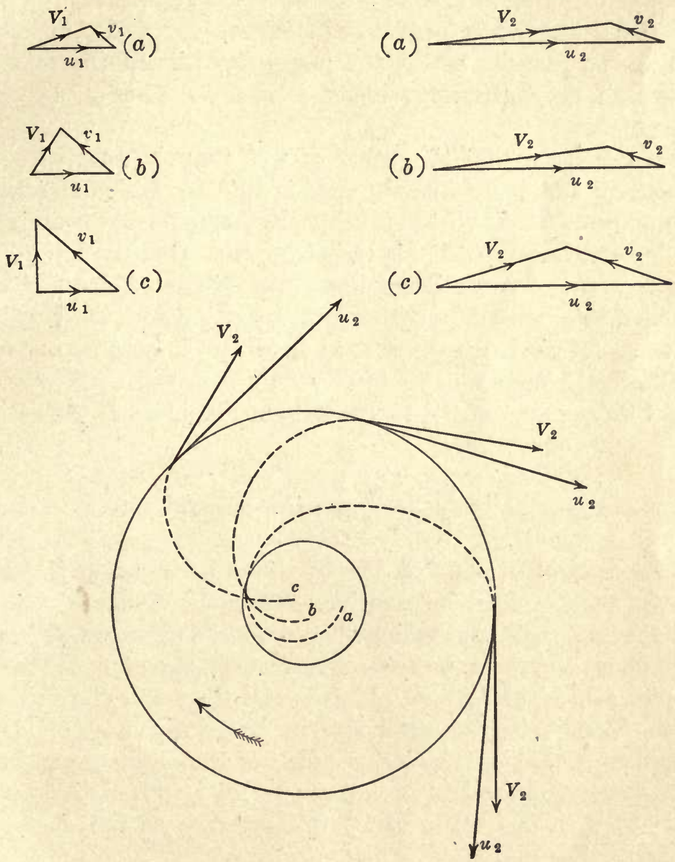


FIG. 91.—Velocity diagram for three rates of discharge.

[1] By another method of reasoning an equivalent equation may be obtained as follows:

$$h'' = \left(\frac{u_2^2 - u_1^2}{2g}\right) + \left(\frac{v_1^2 - v_2^2}{2g}\right) + \left(\frac{V_2^2 - V_1^2}{2g}\right)$$

The one given above will be found more convenient to use. See Investigation of Centrifugal Pumps by C. B. Stewart, *Bulletin of Univ. of Wis*, Eng. Series, Vol. III, No. 6, Vol. V, No. 3.

In Fig. 91 we have the paths of the water for three rates of flow shown, (*a*) representing a small discharge, (*b*) a larger discharge, and (*c*) the discharge for which the pump was designed. When there is no discharge the water in the eye of the wheel will be set in rotation the same as if the vanes extended into the center. The real value of r_1 and thus of u_1 will consequently be zero for such a case. As the discharge increases this rotation decreases until at the flow for which the pump is designed the rotation ceases and the entrance velocity is radial. Thus s_1 becomes zero.

It is thus seen that the value of u_1s_1 may be zero when no flow occurs and also when the flow is that for which the pump was designed. It is customary for the pump to be so designed that the entrance velocity is radial for the rated discharge. If the pump is not so designed there will still be some discharge for which the flow will be radial as the water enters the impeller. For values of discharge other than these two it may be assumed that the real effective value of either u_1 or s_1 will be very small so that the product of the two may be negligible. If the second term is omitted we have

$$h'' = \frac{u_2}{g} s_2 = \frac{u_2}{g} (u_2 + v_2 \cos a_2) \qquad (C)$$

Inspection of this equation shows that if the pump is to do positive work s_2 must be positive. Thus the absolute velocity of the water leaving the impeller must be directed forward. If the pump speed, u_2, be assumed constant, the above equation will plot as a straight line. If a_2 is less than 90°, the value of h'' will increase as the discharge increases above zero. If a_2 is equal to 90°, h'' will be independent of the discharge and will plot as a horizontal line for all values of v_2. If a_2 is greater than 90°, then the value of h'' will decrease as the discharge increases. See Fig. 93.

Since it is difficult to transform velocity head into pressure head without considerable loss, it is desirable to keep the absolute velocity of the water leaving the impeller as small as possible. For that reason the best pumps have vane angles as near 180° as possible in order that the relative velocity may be nearly opposite to the peripheral velocity of the impeller.

128. Losses.—In accordance with the usual methods in hydraulics, the friction loss in flow through the impeller may be represented by $k\ v_2^2/2g$, where k is an experimental constant. A study of Fig. 91 would indicate that there is no abrupt change of velocity at entrance to the impeller under any rate of flow; there is then no marked shock loss at entrance that would require the use of a separate expression as it may be covered by the value of k. However, where the water leaves the impeller, there is an important shock loss that follows a different law from the friction loss.

For the turbine pump this shock loss is similar to that in the case of the reaction turbine in Art. 69. Referring to Fig. 92, it may be seen that the velocity V_2 and the angle A_2 will be determined by the vectors u_2 and v_2. Since the vane angle A'_2 is fixed there can be only one value of the discharge that does not

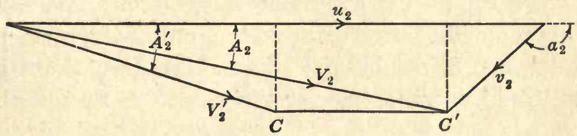


FIG. 92.

involve a shock loss. For any other value of the discharge the velocity V_2 will be forced to become V'_2 with a resultant loss which may be represented by $(CC')^2/2g$. Since the area of the diffusion ring normal to the radius should equal the area of the impeller normal to the radius, the normal component (*i.e.*, perpendicular to u_2) of V_2 should equal that of V'_2. Therefore CC' is parallel to u_2 and its value may be found to be

$$CC' = u_2 - \frac{\sin (a_2 - A'_2)}{\sin A'_2} v_2.$$

If $k' = \frac{\sin (a_2 - A'_2)}{\sin A'_2}$, then for the turbine pump the shock loss is approximately equal to

$$\frac{(u_2 - k'v_2)^2}{2g} *$$

For the turbine pump the total hydraulic loss may be represented by

$$h' = k\frac{v_2^2}{2g} + \frac{(u_2 - k'v_2)^2}{2g} \qquad (D)$$

* L. M. Hoskins, "Hydraulics," p. 237.

Since the volute pump has no diffusion vanes, there will be no abrupt change in the direction of the water at exit from the impeller, but there may be an abrupt change in the magnitude of the velocity. The water leaves the impeller with a velocity V_2 and enters the body of water in the case which is moving with a velocity V_3. In accordance with the usual law in hydraulics this shock loss may be represented by

$$\frac{(V_2-V_3)^2}{2g}$$

For the usual type of pump V_2 will decrease as the discharge increases, and in any case V_3 must increase as the quantity of water becomes greater. If the discharge becomes such that the two are equal then there will be no shock loss. The value of V_2 may be expressed in terms of u_2 and v_2, and if the ratio of (f_2/F_3) be denoted by n, we have $V_3=nv_2$. Making these substitutions the total hydraulic loss for the volute pump may be represented by

$$h'=k\frac{v_2^2}{2g}+\frac{(\sqrt{u_2^2+2u_2v_2\cos a_2+v_2^2}-nv_2)^2}{2g} \qquad (E)$$

Though the values of k may be different for the two types and though the expressions for shock loss are unlike in appearance, yet it can be seen that the losses in each case follow the same general kind of a law. In the turbine pump we have a gradual reduction of velocity but, except for one value of discharge, a sudden change in direction as the water leaves the impeller. With the volute pump we have no abrupt change of direction but a sudden change of velocity. The transformation of kinetic energy into pressure energy is incomplete in either case, but it is generally believed that the loss is somewhat greater in the volute pump that in the turbine pump. In order that the efficiency may be different in the two cases we have merely to assume a different value of k.

For an infinitesimal discharge the value of the velocity in the case, V_3, would be practically zero. Therefore a particle of water leaving the impeller with a velocity V_2 and entering a body of water at rest would lose all its kinetic energy. For such a case, however, the value of v_2 would be also practically zero so that V_2 would equal u_2. Therefore for a very slight discharge the shock loss would be $h' = u_2^2/2g$. Such a value of h' may be obtained from either (D) or (E) by putting $v_2 = 0$.

129. Head of Impending Delivery.—The head developed by the pump when no flow occurs is called the shut-off head or the head of impending delivery. We are then concerned only with the centrifugal head or the height of a column of water sustained by centrifugal force. In Art. 124 this was shown to be equal to $u_2^2/2g$. The same result may be obtained from the principles of Art. 127 and Art. 128. If v_2 becomes zero, then by equation (C), $h'' = u_2^2/g = 2\ u_2^2/2g$. But, as was shown in Art. 128, the loss of head, $h' = u_2^2/2g$ Therefore $h = h'' - h' = u_2^2/2g$.

Although ideally the head of impending delivery equals $u_2^2/2g$, we find that various pumps give values either above or below that. This may be accounted for in a number of ways. In any pump we never have a real case of zero discharge; for a small amount of water, about 5 per cent. of the total rated capacity perhaps, will be short circuited through the clearance spaces. A pump is said to have a rising characteristic if, when run at constant speed, the head increases as the discharge increases above zero until a certain value is reached when it begins to decrease. If the head continually decreases as the discharge increases above zero, the pump is said to have a falling characteristic. Thus the leakage through the clearance spaces will tend to make the shut-off head greater or less than $u_2^2/2g$ according to whether the pump has a rising or a falling characteristic. The more the vanes are directed backward, the more tendency there is for internal eddies to be set up and these tend to decrease the head. Also if the water in the eye of the impeller is not set in rotation at the same speed as the impeller the head may be further reduced. There is also a tendency for the water surrounding the impeller to be set in rotation and this, on the other hand, helps to increase the head since the real value of r_2 is greater than the nominal value.

It will usually be found that actually the head of impending delivery may be from 0.9 to $1.1 \frac{u_2^2}{2g}$.

130. Relation between Head, Speed and Discharge.—When flow occurs the above relation no longer holds, for other factors besides centrifugal force enter in. Due to conversion of velocity head into pressure head when water flows, a lift may be obtained which is greater than $u_2^2/2g$. (See Fig. 93.)

The actual lift of the pump h may be obtained by subtracting the losses h' from the head imparted by the impeller h''. The value of h'' will be taken as $\frac{u_2 (u_2+v_2 \cos a_2)}{g}$ and the values of h' are given in equations (D) and (E).

Making these substitutions for the turbine pump we obtain after reduction

$$u_2^2+2 (k'+\cos a_2) u_2 v_2-(k+k'^2) v_2^2=2gh. \qquad (F)$$

For the volute pump we obtain after rearranging

$$u_2^2+2nv_2\sqrt{u_2^2+2 u_2v_2 \cos a_2+v_2^2}-(1+k+n^2) v_2^2=2gh \qquad (G)$$

These equations involve the relation between the three variables u_2, v_2, and h. Any one of these may be taken as constant and the curve for the other two plotted. If the pump is to run at various speeds under a constant head, the latter will then be fixed and we may determine the relation between speed and discharge. The more common case is for the pump to run at a constant speed. For that case values of h may be computed for different values of v_2. The curves for a turbine pump run at constant speed are shown in Fig. 93.

Although it will not be done here, it will be found convenient to introduce ratios or factors as was done in the case of the turbine. We may write $u_2=\phi\sqrt{2gh}$ and $v_2=c\sqrt{2gh}$ and using these in equations (F) and (G) we obtain relations between c and ϕ similar to equation (19). As in the case of the turbine it will be found that the best efficiency will be obtained for a certain value of ϕ and c. It will thus be clear that the speed of the pump should vary as the square root of the lift and the best value of the discharge will also be proportional to the square root of the lift. Since $h=\frac{1}{\phi^2}\left(\frac{u_2^2}{2g}\right)$, it is apparent that the lift varies as the square of the speed. If this value of h be substituted in $v_2=c\sqrt{2gh}$ we obtain $v_2 = \left(\frac{c}{\phi}\right)u_2$, and this shows that the best value of the discharge varies directly as the speed.

Curves between c and ϕ will be of the same appearance as those drawn for a constant value of h. To construct curves of the same shape as those drawn for a constant speed it will be necessary to plot values of $\left(\frac{1}{\phi^2}\right)$ and of $\left(\frac{c}{\phi}\right)$.

The best value of ϕ depends upon the design of the pump. By choosing different values of a_2 and either A'_2 or n the pump may be given flat or steep characteristics. A flat characteristic is one where the head changes but little as the discharge increases at a constant speed. With a steep characteristic the head falls

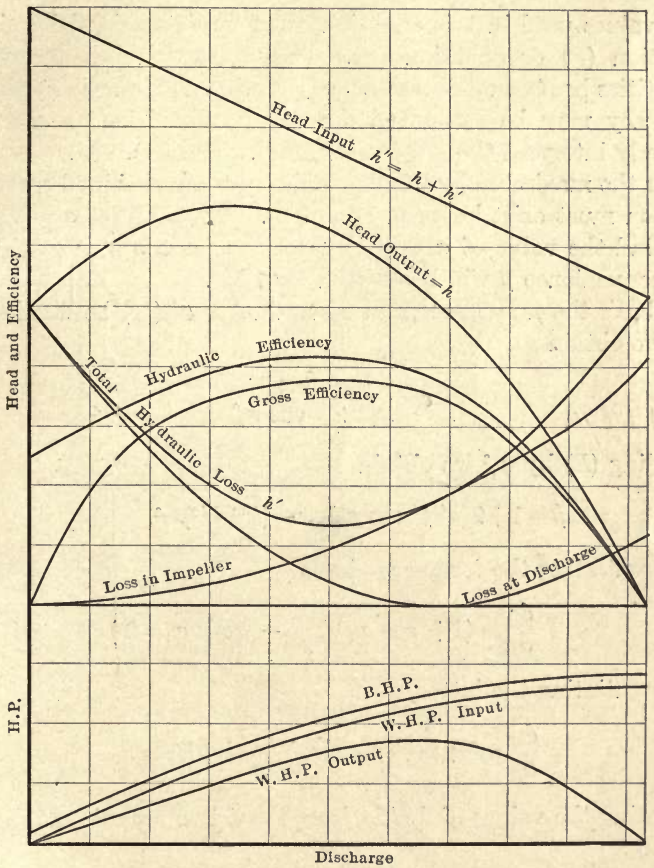


FIG. 93.—Ideal curves for a turbine pump.

off rapidly as the discharge increases. The values of ϕ for the best efficiency range from about 1.30 down to about 1.00. This means that the head for the best efficiency is usually

$$h = 0.6 \text{ to } 1.0 \frac{u_2^2}{2g}.$$

131. Efficiency.—The hydraulic efficiency has been defined in Art. 126. If our assumption regarding the value of h'' were correct, we would have

$$e=\frac{h}{h''}=\frac{gh}{u_2(u_2+v_2 \cos a_2)} \tag{H}$$

The values used in this equation must be determined by either equation (F) or equation (G). If the relation between these quantities has been determined by experiment then the hydraulic efficiency may be computed directly provided we are able to properly interpret the dimensions used. See Art. 132.

For the maximum hydraulic efficiency to be obtained a certain relation must exist between v_2 and u_2. We shall let $\alpha=(v_2/u_2)$, and find the value of α that is necessary for the best efficiency. For convenience β will be written for $1/\phi^2$.

If both the numerator and denominator of (H) be divided by u_2^2, we obtain

$$e=\frac{\beta}{2(1+\alpha \cos a_2)} \tag{I}$$

Dividing (F) by u_2^2 we obtain

$$\beta=1+2(k'+\cos a_2)\,\alpha-(k+k'^2)\alpha^2 \tag{J}$$

Differentiating the former we have

$$\frac{de}{d\alpha}=(1+\cos a_2\,\alpha)\frac{d\beta}{d\alpha}-\beta \cos a_2=0$$

Differentiating the latter,

$$\frac{d\beta}{d\alpha}=0+2(k'+\cos a_2)-2(k+k'^2)\alpha.$$

Equating the values of $d\beta/d\alpha$ from these two equations and reducing,

$$\alpha_2+2 \sec a_2\alpha-\frac{1+2k' \sec a_2}{k+k'^2}=0 \tag{K}$$

This equation applies only to the turbine pump. If α is determined by it then the best discharge for a given speed is known at once, since $v_2=\alpha u_2$. The lift corresponding to this discharge may then be determined by (F). If the lift is given and the speed is to be determined then this expression may be substituted for v_2 in (F) and u_2 solved for. Only one value of α, the smaller, will give positive values of h.

A similar process could be gone through with for the volute pump but the resulting equation would be very lengthy and its solution difficult. In view of the imperfection of the theory it hardly seems worth while. However the point of maximum hydraulic efficiency as given by the equations may be determined by trial if desired.

132. Defects of Theory.—The discussion of the defects of the theory of the reaction turbine in Art. 76. applies equally

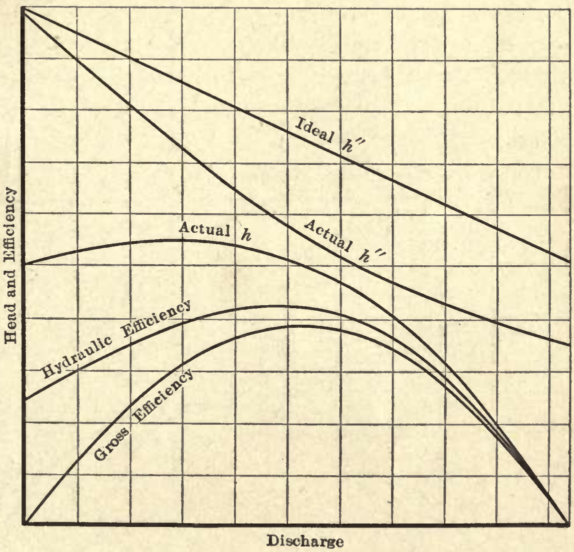


FIG. 94.—Actual curves for turbine pump.

well to the centrifugal pump. Probably one of the greatest sources of error lies in the assumptions made regarding losses. In particular the expressions for shock loss for either the turbine or the volute pump must be regarded as only rough approximations.

While actual tests have shown curves similar to the ideal curves given in Fig. 93, it is more common to find the relation between head and discharge to be like that in Fig. 94. In many cases also the pump has a falling characteristic so that the head for any delivery is less than the shut-off head. But at the same time the gross efficiency will be high and the hydraulic efficiency must be higher still. Since $h'' = h/e$ it will be seen that the actual h'' must be of the form shown in Fig. 94. This accounts for the discrepancy between the so-called manometric efficiency and the true hydraulic efficiency.

The reasons for this are the same as those given for the reaction turbine. In addition there is strong reason for believing that there is a dead water space on the rear of each vane, thus the

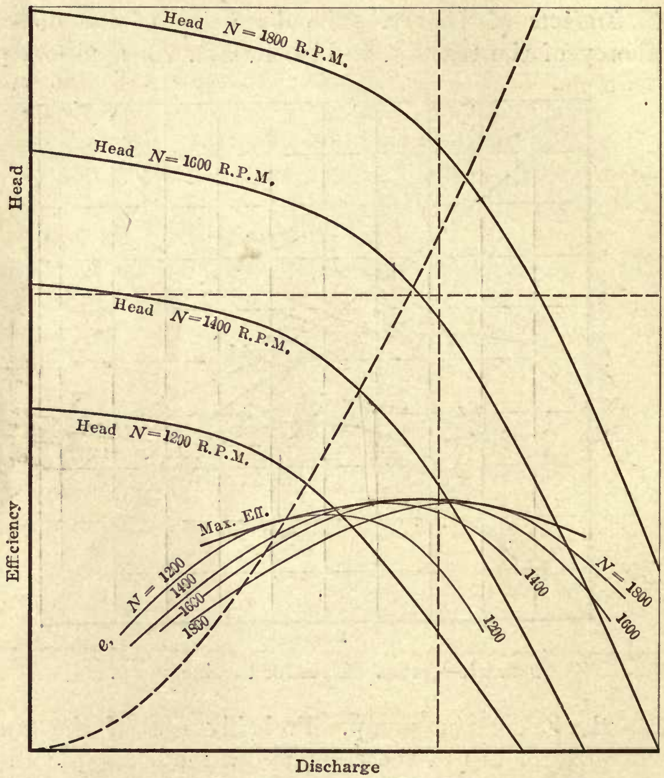


FIG. 95.—Curves for pump run at different speeds.

actual area f_2 will be less than the nominal area used in the computations. This is probably a bigger item that the contraction of the streams mentioned in connection with the turbine. The ordinary pump has no guide vanes at entrace to the impeller and the conditions of flow at that point are uncertain. It may be that it is not allowable to assume u_1 s_1 to be neglible.

The more vanes the impeller has the more perfectly the water is guided and the more nearly the actual curves approach the ideal. It is necessary to have enough vanes to guide the water fairly well but too many of them cause an excessive amount of hydraulic friction. Within reasonable limits—say 6 to 24—the efficiency is but little affected. If the use of few vanes lowers the value of h, the value of h'' is lowered at about the same rate so that the ratio of the two is but little altered.

133. Efficiency of a Given Pump.—If a given pump is run at different speeds the lift should vary as the square of the speed, the discharge as the speed, and the water h.p. as the cube of

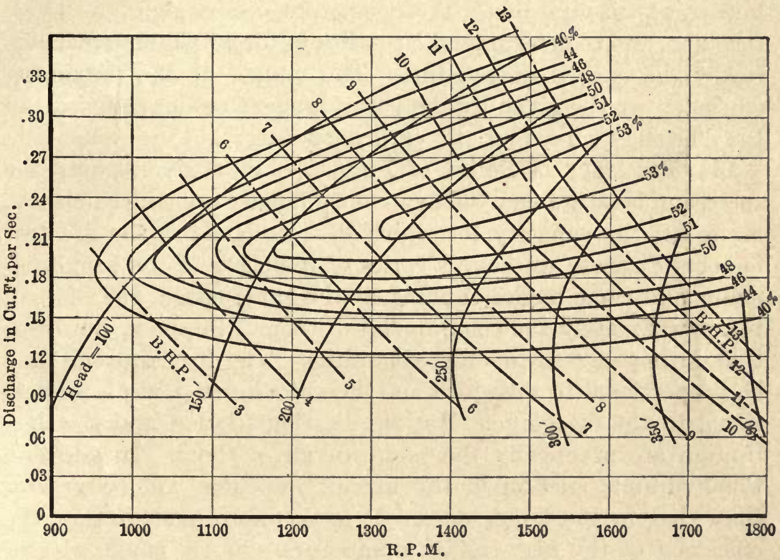


FIG. 96.—Characteristic curve of a 4-stage turbine pump.

the speed. If the efficiency of the pump remained constant the horse-power necessary to run the pump would also vary as the cube of the speed. It is probable that the hydraulic efficiency is reasonably independent of the speed. The mechanical losses, however, do not vary as the cube of the speed. For low speeds the mechanical losses do not increase so fast and thus the gross efficiency of the pump will improve as it is used under higher heads at higher speeds. After a certain limit is reached, however, the mechanical losses follow another law and for very high speeds they will increase faster than the hydraulic losses and the efficiency will begin to decline. Thus for a given pump run at increasing speeds the maximum efficiency will increase and then decrease again. It is thus clear that the head which may be efficiently developed with a single stage is limited. For higher heads it is necessary to resort to multistages.

These conclusions regarding efficiency are borne out by the curves shown in Fig. 95 and Fig. 96. In the latter the highest speed attained was not sufficient for the efficiency to begin to decrease again, though it had evidently reached its limit.

In a set of curves such as are given in Fig. 95, the operation of the pump under a constant head can be determined by following a horizontal line. For a constant discharge follow a vertical line, and to determine the conditions for maximum efficiency follow the curved dotted line. The values of the maximum efficiency will be given by the curve tangent to the peaks of all the efficiency curves for the various speeds.

134. Efficiency of Series of Pumps.—For a given pump the speed and head are seen to have some influence upon its efficiency. However, the capacity for which it is designed is the greatest factor. Suppose we have a series of impellers of the same diameter and same angles running at the same speed, the lift will be approximately the same for all of them. Suppose, however, that the impellers are of different widths. The discharge will then be proportional to the width and the water horse-power is proportional to the discharge. But the bearing friction and the disk friction are practically the same for all of them. In addition the hydraulic friction in the narrow impellers will be greater than that in the larger ones. It is therefore evident that the efficiency of the high-capacity impellers will be much greater than that of the low-capacity impellers. This is true to such an extent that the efficiency of a centrifugal pump may be said to be a function of the capacity. The following table may serve as a rough guide to the efficiency that should be expected.[1]

Capacity g.p.m.	Efficiency per cent.
100	45
300	55
900	70
3,000	75
10,000	80

[1] C. G. De Laval, Centrifugal Pumping Machinery.

Very large pumps have given efficiencies around 90 per cent. Single suction pumps have slightly lower efficiencies than double suction pumps.

135. Comparison of Turbine and Volute Pumps.—The comparative merits of these two types of pumps is an unsettled matter and sufficient test data has not been made public to enable a conclusive verdict to be reached. One thing is certain and that is that the turbine pump is more expensive. The relative sizes of the two types both having the same size of impeller can be seen by comparing Figs. 87 and 88. It is probable that for high heads the turbine pump gives somewhat better efficiencies, but this advantage is not so apparent with low heads. However this point of superiority is confined to a rather narrow range of operating conditions. It appears that the efficiency curve of a volute pump is apt to be somewhat flatter, thus though the maximum efficiency may not be so high, the average operating efficiency may be better.

INDEX