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FACULTY WORKING PAPER NO. 89-1558

Implicit Cost Allocation and Bidding for Contracts

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WORKING PAPER SERIES ON THE POLITICAL ECONOMY OF INSTITUTIONS
NO. 26

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Implicit Cost Alloction and Bidding for Contracts

Susan I. Cohen, Associate Professor
Department of Business Administration

Martin Loeb
University of Maryland

*Associate Professor, College of Commerce and Business Administration, University of Illinois, Champaign, Il 61820.

**Associate Professor, College of Business and Management, University of Maryland, College Park, MD 20742.

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ABSTRACT

The question of how, or even whether, indirect costs should be allocated for pricing decisions has been controversial and unresolved. This paper takes a step toward answering this question by examining the special case of a firm that must incur incremental fixed costs to complete any or all of the several projects for which it is submitting simultaneous bids. An independent private-values bidding model is employed to endogenously determine an optimal cost allocation; we term such a cost allocation "implicit." The optimal implicit fixed cost allocation is shown to fully allocate fixed costs **ex ante**, although the fixed costs may be under, over, or exactly allocated **ex post**.

I. Introduction

A flourishing line of recent research in management accounting has focused on explaining the use of cost allocations as part of a firm's internal control system. This literature began in earnest with Zimmerman [1979], and was a reaction to earlier studies that either advocated abandoning cost allocations or adopting complicated new cost allocation methods.[1]

The controversy surrounding the allocation of fixed costs for use in pricing goods or services in external markets has an even longer history, and has been associated with the full cost versus marginal cost or absorption costing versus variable cost debate.[2] Giving a motivation that was a precursor to one given by Zimmerman [1979] for allocating fixed costs for internal control, Devine [1950] suggests that fixed costs proxy for some unobservable opportunity costs that are not captured in variable costs alone.[3] Thus, the widespread use of full costs in pricing (Gordon et al., 1981; Govindarajan and Anthony, 1983) may be partially reconciled with economists prescriptions for pricing so as to equate marginal revenue with marginal cost. More recently, Dickhaut and Lere [1983] and Lere [1986] present more formal analyses to explain the use of allocated fixed costs for pricing. In Dickhaut and Lere [1983], the use of absorption costs for pricing results from systematic measurement error in the accounting system. In Lere [1986], bounded rationality and costly information provide the rationale for using allocated fixed costs in pricing.[4]

In this paper we use a bidding model to help explain the fact that allocations of fixed costs are used in pricing decisions. Considered is a firm preparing bids for a number of projects. We assume that if the firm wins one or more of these projects, it will have to incur F dollars of additional fixed costs for the purchase of specialized equipment or personnel training. We assume that this equipment or training will have no value to the firm other than for the projects for which the firm is now bidding. While there is general agreement that incremental fixed costs are relevant for pricing decisions (e.g., Horngren and Foster 1987, p. 304), the question of how such a joint incremental cost should be allocated for pricing has not been answered. Indeed, the prevailing view appears to be that there is no optimal method of allocating the fixed costs - the choice among methods is arbitrary.[5] To our knowledge, our model is the first in which a cost allocation arises endogenously in a firm's pricing (bidding) decision. We show that the firm's optimal implicit cost allocation is tidy in an **ex ante** sense, but not in an **ex post** sense.[6]

Allocation in our model is driven by the incremental nature of the fixed costs combined with the stochastic nature of demand. This is in contrast to the motivations for allocating fixed costs (measurement error and bounded rationality) given in Dickhaut and Lere [1983] and Lere [1986]. In our model, optimal bids include an allocation of fixed (capacity) costs, even though there is a likelihood that there will be excess capacity. Although new projects do not impose externalities on other new projects, the additional capacity cost, F, that the group of possible new projects taken together impose on existing projects may be viewed as a type of congestion cost. Thus, the motivation for allocating fixed costs in our model is similar to that provided by Banker et al.

[1988], Devine [1950], Miller and Buckman [1987] and Zimmerman [1979].

We examine the optimal pricing behavior of a risk-neutral firm participating in a number of simultaneous auctions in which competitors submit sealed bids, and in which each project is awarded to the firm submitting the lowest bid in that auction. In the terminology of the auction literature (Englebrecht-Wiggins, 1980; McAfee and McMillan, 1987) the firm participates in a number of sealed-bid first-price auctions. In our model a firm does not face the same bidders in each of the auctions in which it participates; some of the bidders may be the same across auctions, but not all. A firm, therefore may win any number (not just none or all) of the projects for which it submits bids.

In the next section, we derive the optimal bidding policy of firms. The firm's optimal bid for a project is shown to be composed of the firm's direct costs of the project plus an implicit allocation of fixed costs plus a bidding competition term; this latter term represents the rents earned by the firm as a result of its private information about costs. We also show that the implicit cost allocation is exact in an ex **ante** sense, and we examine conditions under which **ex post** allocated costs are less than, equal to, or greater than total fixed costs. A brief summary appears in section three.

II. The Firm's Bidding Problem

Consider the case of a risk-neutral firm preparing to submit simultaneous sealed bids for a number of projects. Each project is to be awarded to the lowest bidder. For each project that the firm wins, the firm will incur some direct costs associated with the specific project. In addition to the direct costs of the projects won, the firm will incur incremental fixed costs of F, if it wins any of the projects. The incremental fixed costs may arise from the purchase of specialized equipment or specialized training of personnel that only is useful for the projects for which the firm is now bidding. How should such incremental fixed costs be incorporated into the firm's bids? In order to address this question, we use an independent-private-values model (Milgrom and Weber [1982]) in which bidders differ only with respect to a single parameter representing an independent draw from a known probability distribution. This parameter will reflect the production efficiency of the firms.

In the standard independent-private-values models, each firm participates in only one auction, and the optimal bid is studied by examining the symmetric equilibrium. In our model, firms participate in a number of auctions, thus it is necessary to make additional assumptions in order to study optimal bids using the symmetric equilibrium. Suppose there are N potential risk-neutral bidders and J_0 projects to be awarded. We assume that there are $n < N$ bidders in each auction, that each bidder participates in $J < J_0$ auctions, and the set of J auctions in which the firm participates is exogenously determined and known to the firm.[7] Furthermore, we suppose that each of the N firms knows the parameters N,

n, J_0, and J, and that each firm believes that the n-1 other firms competing with the firm in any one auction are randomly chosen from the remaining N-1 firms.

In addition to the incremental fixed cost F incurred if a bidder wins at least one project, bidder i will incur direct costs of $c_j + v_i$, if the firm wins project j, where c_j is an idiosyncratic cost associated the project known by all the firms, and v_i is an idiosyncratic cost parameter known only by firm i. All other firms $k \neq i$ know that v_i represents an independent draw from the distribution $G(v)$, where $v \in [v^-, v^+]$ Thus, v_i is the single efficiency or cost parameter differentiating firms.[8]

All the firms face the identical problem of selecting J bids given a value of their cost parameter, V. We therefore may focus on a generic bidding firm. We suppose without loss of generality, that the generic firm submits bids for project 1,2,...,J. Let $b_{ji} = B_j(V_i)$ be firm i's symmetric Nash equilibrium bidding strategy. Thus firm i's optimal bid is $b_{ji} = B_j(V_i)$ for project j, given that all other firms $k \neq i$ bid $b_{jk} = B_j(V_k)$ if they participate in the j th auction, for $j = 1,2,...,J$.

The probability that firm i wins auction j is just the probability that $b_{ji} \leq b_{jk}$ for all $k \neq i$. Since we are talking about symmetric Nash equilibrium strategies, we may now drop the subscript i that refers to the firm. Formally the probability that a firm wins auction j can be written as:

$$[1 - G(B_j^{-1}(b_j))]^{n-1}. \tag{1}$$

For ease of exposition, we define $H(v;n) \equiv [1 - G(v)]^{n-1}$. If the firm wins the auction (project) its profits gross of the fixed costs will be equal to $b_j - c_j - v$. Since the firm incurs the fixed cost F **only** if it wins at least one of the projects, we can write the firm's **ex ante** expected profits as:

$$P(v,b_1,\ldots,b_J) = \sum_{j=1}^{J} H(B_j^{-1}(b_j);n)[b_j - c_j - v]$$

$$- F\{1 - \prod_{j=1}^{J} (1 - H(B_j^{-1}(b_j)))\}. \quad (2)$$

where the expression in curly brackets multiplying F is the firm's probability of winning at least one project.

From the symmetry of the J auctions in which each bidder participates, we know that in equilibrium $B_j(v) - c_j$ will be independent of j. This results from the fact that there are n bidders in each of the auctions and c_j is the only parameter idiosyncratic to auction j. Define $B(v) \equiv B_j(v) - c_j$. In equilibrium we can write expected profits as $P^*(v)$, where

$$P^*(v) = \sum_{j=1}^{J} H(v;n)[B_j(v) - c_j - v] - F\{1 - (1 - H(v;n))^J\}. \quad (3)$$

$$= JH(v;n)[B(v) - v] - F\{1 - (1 - H(v;n))^J\}$$

From the envelope theorem, we know that in equilibrium the derivative of $P^*(v)$ with respect to v is just the partial derivative of $P(v;b_1,\ldots,b_j)$ with respect to v evaluated at $b_j = B_j(v)$ for all j. Thus, we have:

$$P^{*\prime}(v) = -JH(v;n). \quad (4)$$

Since $P^*(v^+) = 0$ in equilibrium, expected profits are then:

$$P^*(v) = \int_v^{v^+} -P^{*\prime}(s)ds$$
$$= \int_v^{v^+} JH(s;n)ds \tag{5}$$

The symmetric Nash equilibrium bidding strategies must therefore satisfy, using (3), (5) and the definition of B(v):

$$B_j(v) - c_j = v + \frac{P^*(v)}{JH(v;n)} + F\frac{\{1 - (1 - H(v;n))^J\}}{JH(v;n)}$$
$$= v + \frac{1}{H(v;n)}\int_v^{v^+} H(s;n)ds + F\frac{\{1 - (1 - H(v;n))^J\}}{JH(v;n)} . \tag{6}$$

Hence, the firm bids its direct costs of the project, $c_j + v$, plus a positive term that represents a payment to induce bidding competition, plus an allocated portion of fixed costs (as the coefficient of F is less than one) The bidding competition term is generally interpreted as the rents that the bidders earn as a result of their private information about v.

It is straightforward that $B_j(v)$ is increasing in v for all j, since costs are increasing in v and the auction form assumes that lower bids have a higher probability of winning. Thus, each auction is won by the most efficient (least cost) bidder participating in that auction.

In equilibrium, expected profits are decreasing in v (from equation (4)): as costs increase, the bidder expects to earn lower profits. We also see from (5) that expected **ex ante** profits do not depend on fixed costs F; profits are the total informational rents that can be captured by the bidder. Thus, we can conclude that the bidder expects to recoup all of fixed costs via the bids **ex ante**.

The term multiplying F in equation (6) represents the implicit cost allocation share, i.e. the fraction of fixed costs that are allocated to a particular project. This fraction is strictly decreasing in H, the probability of winning a particular project. Since H is decreasing in both n (the number of bidders in an auction) and v, we have that the fraction is strictly increasing in v and n. When the probability of winning a particular project decreases, the expected number of projects won also decreases.[9] The bidder recoups a larger fraction of fixed costs from each project, since the bidder expects to have a smaller number of projects from which to recoup fixed costs. In particular, we have:

$$F \frac{\{1 - (1 - H(v^{+};n))^{J}\}}{JH(v^{+};n)} = 1$$

and (7)

$$F \frac{\{1 - (1 - H(v^{-};n))^{J}\}}{JH(v^{-};n)} = \frac{1}{J}.$$

The bidder with the highest possible costs (v^{+}) allocates all of fixed costs to each of the projects since the bidder expects to win no projects. The bidder with the lowest possible costs (v^{-}) allocates 1/J to each project since the bidder expects to win all of the projects, and in this way will exactly recoup fixed costs.

Even though **ex ante** fixed costs are fully allocated, **ex post** fixed costs may be under, exactly or over allocated, depending on how many auctions are actually won. If the bidder wins an average number of projects, which is just JH(v;n), the **ex post** costs recouped from the bids are just:

$$F\{1 - (1 - H(v;n))^J\}. \tag{8}$$

The term in curly brackets in (8) represents the **ex ante** probability that the firm will win at least one project. Hence, expression (8) also represents the firm's expected expenditures for fixed costs. Thus, a bidder winning an average number of projects given v (JH(v;n)) will indeed exactly allocate fixed costs **ex ante**. However, if the bidder wins an average number of projects, the firm's **ex post** incremental fixed costs will be F which is strictly larger than the expression in (8). Therefore when the bidder wins an average number of projects given v, the bidder will under allocate fixed costs **ex post**.

In general, the total amount of fixed costs recouped by the bidder, where X is the number of projects won, equals:

$$FX\frac{\{1 - (1 - H(v;n))^J\}}{JH(v;n)}. \tag{9}$$

Since costs are not fully recouped when the number of projects actually won equals E[X], we know that there exists an X(v;n) > E[X] such that for all x greater than (less than) (equal to) X(v;n) the bidder recoups more than (less than) (exactly) fixed costs **ex post**.

Even though the bidder expects to fully recoup all fixed costs **ex**

ante, the bidder must win a larger than "average" number of projects to fully recoup fixed costs **ex post**.

III. Summary

We have examined the situation in which a firm participates in a number of simultaneous auctions for projects and faces indirect costs that are incremental and fixed. In our model, the symmetric Nash equilibrium bid consists of direct costs plus a bidding competition term plus a term representing an implicit allocation of the incremental fixed costs. The cost allocation that is implicit in the firm's bids was shown to fully allocate the incremental fixed costs in an **ex ante** sense. Although the **ex ante** allocation results in an exact allocation of fixed costs, the incremental fixed costs may be under allocated, exactly allocated, or over allocated **ex post**. Although the value of allocating indirect costs for pricing has been debated for some time, we know of no other model in which an allocation of indirect costs arises endogenously in the determination of a firm's optimal prices.

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FOOTNOTES

1. See Biddle and Steinberg [1984] for a review of this literature. Some of the more recent papers that seek to provide a rationale for cost allocations include Baiman and Noel [1985], Balachandran et. al. [1987], Cohen and Loeb [1988], Magee [1988], Miller and Buckman [1987], and Suh [1987,1988].

2. This debate has carried over to normative prescriptions in popular accounting texts. For example, Anthony and Reece [1979, p. 547] state that "...each product should bear a **fair share** of the total cost of the business," while Horngren and Foster [1987, p. 306] have a section in their pricing chapter entitle "Superiority of the contribution approach."

3. Recent papers by Banker et. al. [1988] and Miller and Buckman [1987], in which there is a stochastic demand for manufacturing facilities provide additional support for allocating fixed costs as a means of dealing with difficult to observe congestion costs.

4. Hilton et. al. [1988], using a laboratory experiment, find partial support for the theory proposed by Lere [1986].

5. This view of cost allocations as arbitrary is largely due to the influential monographs of Thomas [1969,1974].

6. The term "tidy", as used by Demski [1981], indicates that the sum of allocated costs exactly equals the costs to be allocated.

7. Although not all combinations of the parameters N, J_0, n and J are feasible, it is easy to find combinations that are feasible. Such a determination is outside of the scope of the present paper.

8. Note that v^- and v^+ may be minus and plus infinity, respectively.

9. The distribution of X, the number of projects won, is binomial with parameters J and H(v;n). Since E[X] = JH(v;n), this result is immediate.