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Improving Performance Through Cost Allocation

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Abstract

Considered is an intrafirm resource allocation model with a single principal and n agents. Each agent represents a distribution division and the principal represents an owner who is responsible for production of the "output" that is eventually "sold" by the agents. It is assumed that each agent (division manager) knows the local profit function for the division and has disutility for effort. The principal seeks to maximize firm-wide profits net of output costs and compensation to the agents. In this setting, which incorporates divergence of preferences and asymmetric information, it is shown that the principal and the n agents can strictly improve their welfare by moving from a set of compensation functions that do not include any allocation of costs to compensation functions that are based on cost allocation.

I. Introduction

In a review of the cost allocation literature, Biddle and Steinberg [1984] conclude that "... the process of explaining and ultimately improving cost allocations has just begun." (p. 35). Our goal in this paper is to extend this process by presenting an intrafirm resource allocation model in which the firm is better off allocating costs than not allocating costs.

Zimmerman [1979] addresses the positive question of why firms allocate costs for internal reporting. He presents two examples that suggest possible uses for cost allocation. The first looks at the use of fixed cost allocations to reduce the managers' consumption of perquisites. In this example, the preferences of the manager and the superior diverge with respect to expenditures valued by the manager as perquisites. Furthermore, there is asymmetry of information since the superior does not know the level of expenditures and/or the (marginal) value of these expenditures. Zimmerman claims that the cost allocation acts as a lump-sum tax, reducing the manager's wealth and hence the consumption of perquisites. Biddle and Steinberg [1984, p. 6], as well as Zimmerman [p. 509], note that using the cost allocation as a lump-sum tax may cause the manager to seek alternative employment, since the manager's utility has been reduced. This illustrates that a necessary condition for a cost allocation to be useful to the superior is that it does not decrease the welfare of the managers. In addition, Baiman [1981, p. 109] remarks that the lump-sum tax is not really a cost allocation in the true sense because it does not have any relationship to either the existence or size of a joint cost.

Zimmerman's second example looks at the use of cost allocations in a decentralized firm to motivate the subunit managers to efficiently use a centrally provided input. With this example, asymmetric information arises because a subunit's manager has more knowledge of its technology and market conditions. Zimmerman here ignores the problem of overconsumption of perquisites and does not model divergence of preferences in any other way. This second example suggests that cost allocations are useful because they approximate hard-to-observe costs of service degradation, delay and future expansion.

If cost allocations are to have a role in coordinating a decentralized firm, then clearly such allocations should dominate no cost allocation. Our paper demonstrates this dominance by presenting a model that formalizes and refines Zimmerman's examples. Our model incorporates both asymmetric information and divergence of preferences. We show conditions under which the use of a cost allocation mechanism will lead to a strict improvement in the welfare of both the principal (top management) and the agent(s) (managers). In our model, cost allocations are useful because they signal scarcity of the centrally provided input to the subunit managers and encourage managers to increase their effort levels, thereby reducing a moral hazard problem.

We prove two main results. First, we show that there exist full-cost allocations that will leave the firm better off than with no allocation of costs. This first result is merely an existence theorem, and does not provide a mechanism for the firm to reach this superior position. Second, we present a cost allocation mechanism under which the dominant strategy of each subunit manager results in higher net profits for the firm than under no-cost allocation. In other words, we show that the firm can use cost allocations to reach a position of superior profits. The mechanism that we provide is <u>not</u> based on a full or "tidy" allocation of costs; however, it may be thought of as approximate full-cost allocation mechanism.

It should be pointed out that full-cost allocations are not necessarily optimal. Baiman and Noel [1983] have found a setting where cost allocations are optimal. Their model is of a principal and a single agent and looks at the allocation of a fixed cost over time. In contrast, our model is of a principal and many agents and looks at the allocation of total costs in a single period. Our results are consistent with the results in Demski [1981], who showed that cost allocation can have a use as a means of coordinating agents.

In the next section we describe our model of the integrated firm. Section III contains the definition of "no-cost allocation" and the properties of the no cost allocation reward functions. In Section III we define a solution concept and state and prove our existence result. In Section IV we present and analyze our dominant strategy cost allocation mechanism. A brief summary is contained in Section V.

II. The Model

The focus on this paper is on presenting some (sufficient) conditions under which cost allocation dominates no cost allocation, not on presenting an optimal organizational structure. Consider an integrated firm that centrally produces a product that is transferred to and either sold by several distribution divisions, or transformed into a final product that is then sold. Assume that the center must meet the product demand of each division. The situation we are modelling is one in which the divisions make binding contracts with third parties; the contract must be fulfilled by the firm. Divisional profits gross of the costs of the product transferred from the center depend on the division manager's level of effort and the quantity of the product transferred. All the division managers and the center know the cost function faced by the center, but only division managers know their own profit and utility functions ex ante, and can observe their own levels of effort. The center observes only the realized profits of each division ex post. Finally, every division observes the amount of product transferred and the realized profits of every other division ex post; that is, each division has access to all of the information that the center has ex post. Thus, our model differs from the standard principal-agent models, where the principal (the center) is assumed to know the utility function(s) of the agent(s). In addition, we assume that the principal does not have a prior distribution over the profit and utility functions; that is, over the agents' types.

This last assumption is the one that differentiates our model dramatically from the standard principal-agent models in the economics and accounting literatures. However, this assumption is a common one in the public goods literature; for example, see Groves and Ledyard [1977a]. The firm we are modelling is one in which the division managers have private information about their divisions that the center cannot acquire. This information has so many dimensions that it is not possible for the center to form a prior distribution; that is, the division managers' "types" cannot be parametrized in a way that would allow the formation of a probability distribution over these types.

We are thus modelling decision making under uncertainty rather than decision making under risk. Given the paradigm that we have chosen, maximization of expected value is no longer an appropriate decision-making criterion.

We assume that each division manager seeks to maximize his or her own utility, which is a function of the reward (or incentive compensation) paid by the center and the manager's effort level. In general, the reward function may depend on whatever the principal can observe *ex post*: the jointly observable transferred product, divisional realized profits, and the firm's realized costs, but not on managerial effort. As in the standard principal-agent models, the center (as the residual claimant) seeks reward functions that result in maximum net profits, where net profits equal the sum of divisional gross profits less the center's cost of producing the transferred product and less the total compensation paid to all the division managers.

We capture the essential elements of the situation described above by looking at a model with a single principal and n agents. Although we model asymmetric information, our model may be viewed as deterministic, since the only "uncertainty" concerns the private information held by the divisions, and not a random state of the world that must be realized at some point in the process. That is, each division manager knows what his or her division's output would be for every combination of effort and resource.

Our interest lies in coordination in the integrated firm, in which the center is the residual claimant. Since the center must meet all of the demands of the divisions, coordination has a slightly different interpretation here. We assume that the divisions of the firm are such that their only connection is via the center. That is, the only externalities that exist are those created by the fact that the cost function faced by the center is not separable, so that the marginal cost generated by the ith division depends upon what the jth division demands. Note that the situation we are modelling is similar to the situation modelled by Groves and Loeb [1979]. Their model deals with a similar coordination issue, is also deterministic, and allows for asymmetric information. However, their model does not take into account managerial disutility of effort nor does it examine the maximization of profits net of incentive payments made to divisional managers. Therefore, the critical element of divergence of preferences is missing from their model.

The ith agent (division manager) knows the ith division's profit function $\Pi_i(x_i, e_i)$, where $x_i \geq 0$ is the quantity transferred to the ith division and $e_i \in [0, \bar{e}_i]$ is the effort level of division manager i. The profit function $\Pi_i(x_i, e_i)$ represents the division's profits gross of costs of x_i, and is hereafter referred to as the division's gross profit function. The center does not know the gross profit functions and can only observe realized gross profits ex post. It is assumed that a division cannot earn positive gross profits without selling any output, if x_i is a final good, or without any of the input when x_i is interpreted as an intermediate good. For positive levels of x_i we assume that gross profit is additively separable in transferred good and effort. We can rewrite the gross profit function as:

$$\Pi_i(x_i, e_i) = \begin{cases} 0 & \text{if } x_i = 0 \\ f_i(x_i) + h_i(e_i) & \text{if } x_i > 0 \text{ and } 0 \leq e_i \leq \bar{e}_i \end{cases} \quad (1)$$

The interpretation of these assumptions is straightforward. Effort is a shift parameter that changes the local profit function of the division. For example, we could view effort as the negative of perquisite consumption--as effort increases, perquisite consumption decreases. Therefore, local fixed costs (i.e., costs that are independent of the amount of product transferred) decrease as effort increases. This interpretation is consistent with the standard principal-agent models found in the literature. In those models, it is necessary to assume that the agents' utility functions are additively separable in output and effort (that is, the cross-partials are zero) to get the desired results. The assumption that local profit is additively separable in x_i and e_i helps to perform the same function in our model; that is, this assumption helps to alleviate the problems created by positive cross-partial derivatives. In addition, the assumption that x_i must be positive for positive local profit is equivalent to assuming that the division cannot generate profits just by the provision of effort (and this without the "input" x_i). The local profit in our model is analogous to the output produced by the agent in those models. In the usual model, the output produced by the agent is assumed to be a function of the action (effort) taken by the agent, where all other arguments of the output generating function (except for the random state of the world) are ignored. In our model the ith agent's output depends upon effort <u>and</u> the transferred product; thus we have posited a somewhat different situation than appears in the usual agency models.

We assume that f_i is continuously twice differentiable[2] for positive x_i, h_i is continuously twice differentiable, $h_i(0) = 0$, and h_i is increasing in e_i at a decreasing rate (i.e., $h_i'(e_i) > 0$ and $h_i''(e_i) < 0$). Additionally, we assume that for $x_i \geq 0$, profits are strictly concave in x_i and that for all e_i, $0 \leq e_i \leq \bar{e}_i$, there exists an $\tilde{x}_i$ such that $\Pi_i(\tilde{x}_i,e_i) > \Pi_i(x_i,e_i)$ for all $x_i > 0$. Hence, for all $x_i > 0$, $f_i''(x_i) < 0$, and at $\tilde{x}_i$, $f_i'(\tilde{x}_i) = 0$. Let P_i be the set of all possible divisional profit functions which meet the restrictions detailed above. We assume that the corporate center does not have any prior beliefs about the set P_i. For example, suppose that the divisions were salespeople in individual territories selling a product produced centrally. Then Π_i can be viewed as revenues minus the local costs associated with selling the product in the ith territory. In contrast to the standard principal-agent formulation, we assume that the principal does not have a prior distribution over P_i. However, we assume that the center knows that the demand and local curve costs faced by the divisions are well-behaved enough so that local profit is a jointly concave function of x_i and e_i, and increasing in e_i, and has an interior maximum in x_i for each e_i. For expository purposes only, we will refer to x_i as "output."

Each division manager is assumed to have a utility function separable in monetary reward, r_i, and own effort e_i:

$$U_i(r_i,e_i) = r_i - g_i(e_i) \tag{2}$$

where g_i is the disutility of effort. It is assumed that $g_i(e_i)$ is an increasing, strictly convex, continuously twice differentiable function of e_i and that $g_i(0) = 0$. Also, it is assumed that as effort approaches the maximum level, disutility of effort approaches infinity: i.e., $\lim g_i(e_i) \to \infty$, as $e_i \to \bar{e}_i$. We let G_i denote the set of all disutility of effort functions that meet the above conditions. In the standard principal-agent framework, the principal knows the functional form of the disutility of effort function g_i for each agent. In the world where the agent knows the profit function Π_i with certainty and where the Π_i are parametrized by a single variable, the principal can observe x_i, infer agent i's "type" (i.e., infer the function Π_i) and use a penalty contract to force the agent to provide the optimal amount of effort. We explicitly assume that the g_i functions are not known by the principal and that the principal has no distribution over the set G_i. Thus, there are many pairs of Π_i and g_i which would lead to the same x_i (given an R_i). The center, therefore, cannot infer division i's type and its unobservable effort.

The principal (corporate center) receives the gross profits from all the divisions and must pay for the production of the transferred product and the incentive compensation of the n division managers. The principal learns the realized gross profits of the ith division, $\Pi_i(x_i,e_i)$, ex post (a number not a function). The center must meet the demands of all the divisions, $x_1, x_2, \ldots, x_n$. The cost of production is assumed to depend only on the total amount produced, $x \equiv x_1 + x_2 + \ldots + x_n$, and is assumed to be known by the center and all the divisions. In addition, we shall use $x - x_i = \sum_{j \neq i} x_j$ to denote the total amount demanded by all the divisions except division i. At this stage of the analysis, we only restrict the cost function C(x) to be strictly increasing.[3] The most interesting cases occur when variable costs are not linear or when the cost function has discrete jumps. For these cases, the incremental production costs caused by the demand of one division depends upon the demands of all other divisions.[4] We do not exclude piecewise linear cost functions or cost functions that have discrete steps (although they must have a positive variable cost component). Also, note that a reason why the firm would want to centralize production in the first place is that there are economies of scale--for example, large fixed production costs.

Although the center does not know the profit functions of the divisions or the managers' disutility of effort functions, the center knows the sets of admissable profit and disutility of effort functions. We restrict attention to the case where real moral hazard problems arise. Consider the extreme case where the ith division manager is rewarded the full gross profits of division i. An interesting moral hazard problem would exist if, in this case, the manager would select an interior effort level; i.e., $0 < e_i < \bar{e}_i$. Restricting the admissable set of profit and disutility of effort functions for manager i to be in the subset, call it M_i, of $P_i \times G_i$ such that $h_i'(0) > g_i'(0)$, is a sufficient condition for the existence of an interior solution. We are assuming that the center knows that for sufficiently small levels of effort the marginal gross profitability of effort is greater than the marginal disutility of effort for the division manager.

In the next section, we define the no-cost allocation benchmark, state the center's problem, and give the properties of the no-cost allocation reward functions.

III. No-Cost Allocation Problem

In order to define the no-cost allocation reward functions, it is useful for us to give the exact chronology of the process we are modelling. The center decides on a contract (based on what it can observe) and offers this contract to the division manager. The division manager must then decide whether or not to accept the contract; that is, whether or not to accept employment. If the division manager decides to accept employment, he or she then chooses a level of transferred product, which is then provided by the center. Profit is then realized and transferred to the center. The center rewards the division manager and keeps the residual.

We can now define what we mean by the no-cost allocation benchmark. Clearly, no-cost allocation must mean that we are excluding $C(x)$ as an argument of the reward function. In addition, we exclude reward functions that are nonincreasing in the transferred product. Our rationale is that any reward function that is decreasing in the transferred product implicitly defines a cost allocation (but not necessarily a *full*-cost allocation). In contrast, with a full-cost allocation reward is decreasing in transferred product and the sum of the costs allocated to all of the divisions is exactly the firm's total cost. Let $R_i(\Pi_1,\ldots,\Pi_n,x_1,\ldots,x_n)$ be the no-cost allocation reward function for division i. Then, the division manager will choose (x_i,e_i) to solve the following:

$$\max_{x_i, e_i} R_i(\Pi_1,\ldots,\Pi_n,x_1,\ldots,x_n) - g_i(e_i)$$
$$\text{s.t. } R_i(\Pi_1,\ldots,\Pi_n,x_1,\ldots,x_n) - g_i(e_i) \geq W_i$$

Here W_i is the managèr's reservation wage. We assume that the division manager can only observe the realized profits of the other divisions ex post, and does not have a probability distribution over the possible realizations. Therefore, the manager would not be able to decide whether or not to accept the contract ex ante if the contract depended upon the realized profits of the other divisions. Given the informational assumptions that we have made, the center will restrict attention to reward functions that depend only on (Π_i,x_i) and are nondecreasing in x_i. Note that x_i is still an argument of the reward function. The previous discussion only eliminates reward functions that are nonincreasing in x_i; that is, reward functions that implicitly charge for x_i.

In order to determine the properties of the no-cost allocation reward function (in addition to the aforementioned), we must state the center's problem. The center would like to find reward functions that solve:

$$\max_{\langle R_i \rangle_{i=1}^{n}} \sum_{i=1}^{n} [\Pi_i(x_i,e_i) - R_i(\Pi_i(x_i,e_i),x_i)] - C(\sum_{j=1}^{n} x_j)$$
$$\text{s.t. } (x_i,e_i) \in \operatorname{argmax} \{R_i(\Pi_i(x_i,e_i),x_i) - g_i(e_i)\}$$
$$\text{and } R_i(\Pi_i(x_i,e_i),x_i) - g_i(e_i) \geq W_i$$
$$\text{for all } i = 1,\ldots,n$$

However, the center does not know each division's (Π_i, g_i) pair, nor does it have a prior distribution over the set of possible profit and disutility of effort functions, M_i, for all divisions i. Thus, there may not exist optimal reward functions in the usual sense. In the standard principal-agent setting with unknown types, the principal has a distribution over the agents' possible types, and selects reward functions that maximize expected net profits, subjected to expected utility being greater than or equal to the managers' respective reservation wages (for the first best), and the maximization of expected utility by the managers (for second best), where expectations are taken with respect to the unknown types. The R_i selected in such a manner are ex ante optimal but not necessarily ex post optimal.

For our model we must abandon the usual notion of first or second best optimal reward functions, as well as expected utility maximization. Given the lack of information at the center when the reward functions must be designed, the center has a much more difficult problem than in the standard principal-agent models.

When expected utility maximization is no longer a feasible strategy, other criteria must be used. We assume that the center chooses reward functions that are ex post "rational." Specifically, the center wants it to be the case that whenever a division manager accepts the contract offered by the center: (1) ex post the firm would never be better off without that division in the firm, and (2) ex post the division is not operating on the downward sloping part of its local profit function.[5] Since x_i is costly to the center, the center would always be better off (ex post) with an x_i chosen so that its marginal contribution is at least zero. Clearly, some division managers may decide not to accept the contract with the above defined kind of reward function; and there may be situations in which the center would be better off with a different kind of contract that does not have that property given the actual types of managers. However, since the center does not know the manager's types, nor have a probability distribution over them, the center uses a "minimum regret" type strategy.

Formally, we define an admissible reward function R_i to be such that for all $(\Pi_i, g_i) \varepsilon M_i$, if (x_i^*, e_i^*) is chosen by division manager i (using the reward function R_i), then

$$\Pi_i(x_i^*, e_i^*) - R_i(\Pi_i(x_i^*, e_i^*), x_i^*) \geq 0; \tag{3}$$

that is, the net contribution of division i is nonnegative; and

$$\frac{\partial \Pi_i}{\partial x_i}(x_i^*, e_i^*) \geq 0. \tag{4}$$

The incentive compensation is therefore a real-valued function of Π_i and x_i. Given the above assumptions, we show in the Appendix that when there is no allocation of costs, the center will choose reward functions for the divisions that are concave, increasing functions of local profits alone, and such that $R_i(\Pi_i) < \Pi_i$ for all $\Pi_i > 0$. We denote the set of such real valued functions by R*.[6]

For the case where costs are not allocated, net profits are:

$$\sum_{i=1}^{n} \Pi_i(x_i^*, e_i^*) - C(x^*) - \sum_{i=1}^{n} R_i(\Pi_i(x_i^*, e_i^*)). \tag{5}$$

In the next section, we show that for all admissable reward functions that the center could select, the center and the division managers do strictly better by shifting to a full allocation of costs and adjusting the reward functions to guarantee that each division manager suffers no loss in utility.

III. Dominance of Full Cost Allocations

Suppose that the division managers that have decided to accept the no-cost allocation contracts have done so, and that the firm is now comprised of those managers only. The question then arises: are there other reward functions for the managers such that both the firm as the residual claimant and the managers can all be made better off?[7] That is, given that a manager has decided to accept the no-cost allocation contract and accept employment with the firm, can the firm move to a Pareto superior outcome via a revised set of contracts? In this section we show the existence of contracts that make both the firm and the managers better off. Clearly, there are some managers who may have decided not to accept the no-cost allocation contracts, but who would now accept employment with the revised contracts. Our purpose is to show that once the firm has been formed with the no-cost allocation contracts, a Pareto improvement can be made by using contracts that depend on a full allocation of costs. We do not show that these contracts are optimal; to do so we would have to consider those managers that did not accept the no-cost allocation contracts.

It should be noted that our results on the dominance of full-cost allocations hold for any reward functions that are (i) concave; (ii) increasing; (iii) functions of local profits only; and (iv) strictly less than their arguments. Thus any firm whose technology looks like the technology of the firm we have modelled and who is currently using a reward function of with properties (i)-(iv) can move to a Pareto superior outcome by fully allocating costs.

With the no-cost allocation contracts, it was not necessary to define an equilibrium concept, since the rewards of the individual division managers were independent of the actions of the other division managers. When we move to the full-cost allocation, a division manager's reward <u>will</u> depend on the transferred product demanded by the other division managers (but not on the unobservable effort of the other division managers); we therefore need to define the kind of equilibrium concept we are using and how we are using it. Since our model is deterministic we will use the Nash equilibrium as our solution concept.[8]

A Nash equilibrium is a best-replay equilibrium; that is, if you manage to reach it, given that the other players use their Nash strategies, your best response is to play the Nash strategy. In the tradition of much of the economics literature, we do not describe how the Nash strategy is reached.[9] We do show that if the firm gets to the Nash equilibrium (with the full-cost allocation contracts), then that solution strictly dominates the no-cost allocation solution. Given the notion of Nash as a best replay equilibrium, it is not inconsistent with our previous analysis to have the reward functions depend on the realized transferred products of all of the divisions. Since the divisions can observe every other division's realized profits and transferred product ex post, we have not violated the spirit of the Nash equilibrium concept.

To summarize, we show existence of reward functions such that at equilibrium the firm and the managers are strictly better off.

Let $R_i(y)$ be an arbitrary reward function in R*. We show that if y is gross divisional profits, the principal (center) and the n agents (divisions) can all do strictly better by adjusting y downward by a full allocation of costs, and adding a constant to the reward function. We first show that a division's profits net of the full-cost allocation are strictly greater when the argument of R_i includes the cost allocation than when the argument of R_i does not include the cost allocation.

Suppose that costs are fully allocated so that the argument of the reward function is now realized net divisional profits; i.e., $y = \Pi_i(x_i, e_i) - C_i(x_i; x-x_i)$ where $C_i(x_i; x-x_i) \geq 0$ and $\sum_{i=1}^{n} C_i(x_i; x-x_i) = C(x)$. We require that for each i the C_i function be increasing, lower semi-continuous, and quasi-convex in x_i, and nondecreasing in $x-x_i$.[10] Note that if the cost function C is lower semi-continuous and quasi-convex, and $C_i = C/n$, then these criteria are met. Also, if there is a "dual" allocation $C_i = (1/n)FC + (x_i/x)VC$, where FC and VC are fixed and variable costs, respectively, and where VC is either linear or quadratic in x, then the criteria on the C_i are again met. Finally, if C is convex, $C_i = (x_i/x)C$, and the center knows that the firm will be operating in that portion of the cost function where marginal costs are greater than or equal to average cost, C_i will again meet the requirements.[11] Thus, for common (full) allocation schemes, and reasonable cost functions, the C_i functions will be increasing, lower semi-continuous, and quasi-convex in x_i, and nondecreasing in $x-x_i$. Moreover, we only seek to demonstrate the <u>existence</u> of conditions whereby the firm is strictly better off fully allocating costs.[12]

As in the previous section, we let (x_i^*, e_i^*) denote the output and effort levels selected by the ith division's manager when costs are not allocated. When costs are allocated, let the n-tuple $((\hat{x}_1, \hat{e}_1), \ldots, (\hat{x}_n, \hat{e}_n))$ of output and effort define a Nash equilibrium. Then $(\hat{x}_i, \hat{e}_i)$ maximizes:

$$R_i(\Pi_i(x_i, e_i) - C_i(x_i; x-x_i)) - g_i(e_i). \tag{6}$$

In addition, for an accepted contract, division managers must receive positive compensation, and thus $\hat{x}_i > 0$. Hence, $(\hat{x}_i, \hat{e}_i)$ maximizes

$$R_i(f_i(x_i) + h_i(e_i) - C_i(x_i; \hat{x}-\hat{x}_i)) - g_i(e_i). \tag{7}$$

For tractability, it is assumed that $(\hat{x}_i, \hat{e}_i)$ satisfies the first-order necessary conditions for an interior maximum.

We now show that for each division, net profits are higher with cost allocations than with no allocation. That is, we prove that:

$$f_i(\hat{x}_i) + h_i(\hat{e}_i) - C_i(\hat{x}_i; \hat{x}-\hat{x}_i) > f_i(x_i^*) + h_i(e_i^*) - C_i(x_i^*; x^*-x_i^*). \tag{8}$$

<u>PROPOSITION 1</u>: For every $R_i \in R^*$, for i = 1, 2, ..., n, the net profit (net of allocated costs) in equilibrium for division i is higher with an increasing, lower semi-continuous, quasi-convex full cost allocation than with no cost allocation.

<u>Proof</u>:

By definition, (x_i^*, e_i^*) satisfies the following first-order conditions:

$$R_i'(f_i(x_i^*)+h_i(e_i^*))f_i'(x_i^*) = 0 \tag{9}$$

and

$$R_i'(f_i(x_i^*)+h_i(e_i^*))h_i'(e_i^*) - g_i'(e_i^*) = 0. \tag{10}$$

Since $R_i' > 0$, (9) implies that $f_i'(x_i^*) = 0$. Note that the choice of x_i^* is independent of e_i. Also by definition, $(\hat{x}_i, \hat{e}_i)$ satisfies the following first order conditions:

$$R_i'(f_i(\hat{x}_i)+h_i(\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i))[f_i'(\hat{x}_i)-C_i'(\hat{x}_i;\hat{x}-\hat{x}_i)] = 0 \tag{11}$$

and

$$R_i'(f_i(\hat{x}_i)+h_i(\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i))h_i'(\hat{e}_i) - g_i'(\hat{e}_i) = 0, \tag{12}$$

where C_i' is defined as the first derivative of C_i from the right. (Since C_i is lower semi-continuous, this derivative exists). Since $R_i' > 0$, (11) yields

$$f_i'(\hat{x}_i) - C_i'(\hat{x}_i;\hat{x}-\hat{x}_i) = 0. \tag{13}$$

Thus, $\hat{x}_i$ is also selected independently from $\hat{e}_i$. Since $f_i'(x_i^*) = 0$, $C_i' > 0$ for all x_i, and f_i is strictly concave, (13) implies that $x_i^* > \hat{x}_i$. From concavity of R_i, we have R_i' nonincreasing in its argument. This with $x_i^* > \hat{x}_i$ gives us:

$$R_i'(f_i(\hat{x}_i)+h_i(e_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i)) \geq$$

$$R_i'(f_i(\hat{x}_i)+h_i(e_i)) \geq$$

$$R_i'(f_i(x_i^*)+h_i(e_i)) \qquad \text{for all } e_i > 0. \tag{14}$$

When $e_i = \hat{e}_i$, we have:

$$R_i'(f_i(\hat{x}_i)+h_i(\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i)) \geq R_i'(f_i(x_i^*)+h_i(\hat{e}_i)). \tag{15}$$

Since $h_i' > 0$, (15) implies that:

$$0 = R_i'(f_i(\hat{x}_i)+h_i(\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i))h_i'(\hat{e}_i) - g_i'(\hat{e}_i) >$$

$$R_i'(f_i(x_i^*)+h_i(\hat{e}_i))h_i'(\hat{e}_i) - g_i'(\hat{e}_i). \tag{16}$$

As $R_i'(f_i(x_i^*)+h_i(e_i))h_i'(e_i) - g_i'(e_i)$ is decreasing in e_i (from concavity), (10) and (16) together imply that $e_i^* < \hat{e}_i$.

By strict concavity (in x_i) of f_i, quasi-convexity of C_i and (13) we have $\hat{x}_i$ as a unique maximizer of $f_i - C_i$. Therefore,

$$f_i(\hat{x}_i) - C_i(\hat{x}_i;\hat{x}-\hat{x}_i) > f_i(x_i^*) - C_i(x_i^*;\hat{x}-\hat{x}_i). \tag{17}$$

As $x_j^* > \hat{x}_j$ for all j and C_i is nondecreasing in $x-x_i$, (17) then gives us:

$$f_i(\hat{x}_i) - C_i(\hat{x}_i;\hat{x}-\hat{x}_i) > f_i(x_i^*) - C_i(\overset{*}{x}_i;\overset{*}{x}-\overset{*}{x}_i). \tag{18}$$

Finally, using $\hat{e}_i > e_i^*$ and h_i increasing, we get:

$$f_i(\hat{x}_i) + h_i(\hat{e}_i) - C_i(\hat{x};\hat{x}-\hat{x}_i) > f_i(x_i^*) + h_i(e_i^*) - C_i(x_i^*;x^*-x_i^*). \tag{19}$$

We have therefore shown that net divisional profits are higher with an *ex ante* cost allocation.

$\square$

The next step in the demonstration that there exist reward functions based on full-cost allocations that dominate reward functions based on no allocation, is to show that divisional profits net of cost allocation *and* managerial disutility of effort are higher at $(\hat{x}_i, \hat{e}_i)$ than at (x_i^*, e_i^*), $i = 1, 2, \ldots, n$.

PROPOSITION 2: For each $i=1, 2, \ldots, n$,

$$f_i(\hat{x}_i) + h_i(\hat{e}_i) - C_i(\hat{x}_i; \hat{x}-\hat{x}_i) - g_i(\hat{e}_i) >$$

$$f_i(x_i^*) + h_i(e_i^*) - C_i(x_i^*; x^*-x_i^*) - g_i(e_i^*). \tag{20}$$

Proof:

Rewriting $f_i(x_i) + h_i(e_i)$ as $\Pi_i(x_i, e_i)$, using Proposition 1 and the fact that $y > R_i(y)$ for $y > 0$, we have:[15]

$$[\Pi_i(\hat{x}_i,\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i)] - [\Pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)] >$$

$$R_i([\Pi_i(\hat{x}_i,\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i)] - [\Pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)]). \tag{21}$$

From concavity of R_i for $y > 0$ (and $R_i' > 0$), we get:

$$R_i([\Pi_i(\hat{x}_i,\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i)] - [\Pi_i(x_i^*,e_i^*)-C_i(x_i^*;x-x_i^*)]) \geq$$

$$R_i(\Pi_i(\hat{x}_i,\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i)) - R_i(\Pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)). \tag{22}$$

Therefore,

$$[\Pi_i(\hat{x}_i,\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i)] - [\Pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)] >$$

$$R_i(\Pi_i(\hat{x}_i,\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i)) - R_i(\Pi_i(x_i^*;x^*-x_i^*)-C_i(x_i^*;x^*-x_i^*)). \quad (23)$$

By definition of $(\hat{x}_i,\hat{e}_i)$,

$$R_i(\Pi_i(\hat{x}_i,\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i)) - g_i(\hat{e}_i) \geq R_i(\Pi_i(x_i^*,e_i^*)-C_i(x_i^*;\hat{x}-\hat{x}_i)) - g_i(e_i^*). \quad (24)$$

Since $x_j^* > \hat{x}_j$ for all j, C_i nondecreasing in $x-x_i$, and $R_i' > 0$, we have:

$$R_i(\Pi_i(\hat{x}_i,\hat{e}_i)-C_i(\hat{x}_i,\hat{x}-\hat{x}_i)) - g_i(\hat{e}_i) \geq$$

$$R_i(\Pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)) - g_i(e_i^*). \quad (25)$$

Rewriting (25) and using (23) gives us:

$$[\Pi_i(\hat{x}_i,\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i)] - [\Pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)] >$$

$$g_i(\hat{e}_i) - g_i(e_i^*) \geq 0. \quad (26)$$

Finally, rearranging terms in (26), and noting that x_i^* and $\hat{x}_i$ are both positive so that we can substitute $f_i(x_i) + h_i(e_i)$ for $\Pi_i(x_i,e_i)$, we have our desired result that:

$$f_i(\hat{x}_i) + h_i(\hat{e}_i) - C_i(\hat{x}_i;\hat{x}-\hat{x}_i) - g_i(\hat{e}_i) >$$

$$f_i(x_i^*) + h_i(e_i^*) - C_i(x_i^*;x^*-x_i^*) - g_i(e_i^*). \quad (27)$$

$\square$

We can now use Propositions 1 and 2 to demonstrate our main result. That is, if the center currently uses reward functions R_1, R_2, ..., R_n based only on gross divisional profits, there exist reward functions

$\overline{R}_1, \overline{R}_2, \ldots, \overline{R}_n$ based on divisional profits net of a full allocation of costs such that the center is strictly better off <u>and</u> each division manager is strictly better off. It will therefore be in the best interests of the center and division managers to renegotiate the contracts to use a compensation scheme based on a full-cost allocation.

<u>THEOREM</u>: There exists $\overline{R}_1, \overline{R}_2, \ldots, \overline{R}_n$, which depend on divisional profits minus a full-cost allocation, that lead to an outcome that Pareto dominates the outcome realized with $R_1, R_2, \ldots, R_n$, when the R_i are concave, increasing functions that are less than their arguments.

<u>Proof</u>:

Let ε_i be the difference between gross profits net of allocated costs and disutility of effort at $(\hat{x}_i, \hat{e}_i)$ and (x_i^*, e_i^*). That is

$$\varepsilon_i \equiv [\Pi_i(\hat{x}_i, \hat{e}_i) - C_i(\hat{x}_i; \hat{x} - \hat{x}_i) - g_i(\hat{e}_i)]$$
$$- [\Pi_i(x_i^*, e_i^*) - C_i(x_i^*; x^* - x_i^*) - g_i(e_i^*)]. \tag{28}$$

By Proposition 2, $\varepsilon_i > 0$.

Since $\hat{e}_i \geq e_i^*$ and g_i is increasing, we have that:

$$[\Pi_i(\hat{x}_i, \hat{e}_i) - C_i(\hat{x}_i; \hat{x} - \hat{x}_i)] - [\Pi_i(x_i^*, e_i^*) - C_i(x_i^*; x^* - x_i^*)] > \varepsilon_i. \tag{29}$$

We can now define $\overline{R}_i$ as:

$$\overline{R}_i(\Pi_i(x_i, e_i) - C_i(x_i; x - x_i)) \equiv K_i + R_i(\Pi_i(x_i, e_i) - C_i(x_i; x - x_i)) \tag{30}$$

where

$$K_i \equiv R_i(\Pi_i(x_i^*,e_i^*)) - g_i(e_i^*) - R_i(\Pi_i(\hat{x}_i,\hat{e}_i) - C_i(\hat{x}_i;\hat{x}-\hat{x}_i) + g_i(\hat{e}_i) + \frac{\varepsilon_i}{2}. \tag{31}$$

Clearly, $(\hat{x}_i,\hat{e}_i)$, i=1, 2, ..., n form a Nash equilibrium when division managers are rewarded according to (30). At equilibrium, the division manager's utility is:

$$R_i(\Pi_i(x_i^*,e_i^*)) - g_i(e_i^*) + \frac{\varepsilon_i}{2}, \tag{32}$$

so that the division managers' utilities have all increased by a strictly positive amount. At the $(\hat{x}_i,\hat{e}_i)$ equilibrium, the center's profits are:

$$\sum_{i=1}^{n} [\Pi_i(\hat{x}_i,\hat{e}_i)-\overline{R}_i(\Pi_i(\hat{x}_i,\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i))] - C(\hat{x}) =$$

$$\sum_{i=1}^{n} [\Pi_i(\hat{x}_i,\hat{e}_i)-R_i(\Pi_i(x_i^*,e_i^*))-g_i(\hat{e}_i)+g_i(e_i^*) - \frac{\varepsilon_i}{2} -C_i(\hat{x}_i;\hat{x}-\hat{x}_i)]. \tag{33}$$

Without the cost allocation, the center's profits are:

$$\sum_{i=1}^{n} [\Pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)-R_i(\Pi_i(x_i^*,e_i^*))] \tag{34}$$

Therefore, the benefit of the cost allocation can be calculated as the difference between the r.h.s. of (33), and (34):

$$\sum_{i=1}^{n} \{[\Pi_i(\hat{x}_i,\hat{e}_i)-C_i(\hat{x}_i;\hat{x}-\hat{x}_i)-g_i(\hat{e}_i)]$$

$$- [\Pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)-g_i(e_i^*)]\} - \sum_{i=1}^{n} \frac{\varepsilon_i}{2} = \sum_{i=1}^{n} \frac{\varepsilon_i}{2} > 0. \quad (35)$$

$\square$

Thus, the center and all the divisions are strictly better off with a full allocation of costs and the reward functions $\bar{R}_i$, i=1, 2, ..., n. Although the center does not know the values of K_i a priori, the proof given above demonstrates that there exist reward functions that are Pareto superior to reward functions based on no allocation of costs. We have not shown that the $\bar{R}_i$ are optimal reward functions, only that they dominate reward functions that are optimal in the class R*.

IV. A DOMINANT STRATEGY COST ALLOCATION MECHANISM

In this section, we give an explicit formulation of compensation functions $\tilde{R}_1,\tilde{R}_2,\ldots,\tilde{R}_n$ that are based upon a cost allocation (although not on a full cost allocation) and that result in an outcome that Pareto dominates the outcome achieved with the $\bar{R}_i$'s. Furthermore, with the cost allocation that we present, each division's manager can explicitly calculate his or her best input and effort levels without knowledge of the current behavior of the other divisions' managers.

We define new compensation functions based on an allocation of costs by:

$$\tilde{R}_i[\pi_i, x_i] \equiv R_i[\pi_i(x_i,e_i)-C_i(x_i;x^*-x_i^*)] + R_i[\pi_i(x_i^*,e_i^*)]$$

$$- R_i[\pi_i(x_i^*,e_i^*) - C_i(x_i^*;x^*-x_i^*)]. \tag{35}$$

Let $\tilde{K}_i$ and $(\tilde{x}_i,\tilde{e}_i)$ be defined as follows:

$$\tilde{K}_i \equiv R_i[\pi_i(x_i^*,e_i^*)] - R_i[\pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)] \tag{36}$$

and

$$(\tilde{x}_i,\tilde{e}_i) \in \text{argmax}\{R_i[\pi_i(x_i,e_i)-C_i(x_i;x^*-x_i^*)] - g_i(e_i) + \tilde{K}_i\}. \tag{37}$$

The following theorem shows that for R_i's in R^*, the associated $\tilde{R}_i$'s will result in higher total profits for the firm, while causing no loss in utility for the divisions' managers. Note that with the $\tilde{R}_i$ compensation functions, the manager of division i is "charged" a portion of total input costs that <u>would have been</u> incurred if the other divisions continued to demand their no cost allocation level of inputs. The $\tilde{K}_i$ term, which can be explicitly computed by the Center, is included to ensure that the manager of division i suffers no loss in utility in the switch from no-cost allocation to cost allocation.

<u>THEOREM</u>: The compensation functions $\tilde{R}_1,\tilde{R}_2,\ldots,\tilde{R}_n$ lead to an outcome that Pareto dominates the outcome achieved with the compensation functions $R_1,R_2,\ldots,R_n$, when the R_i's are concave, increasing functions of divisional profits alone and are less than their respective arguments.

<u>Proof</u>: For the outcome achieved with the $\tilde{R}_i$'s to be Pareto superior the outcome achieved with the R_i's it is sufficient that:

$$\sum_{i=1}^{n} \{\pi_i(\tilde{x}_i,\tilde{e}_i)-\tilde{R}_i[\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)]-\tilde{K}_i\} - C(\tilde{x}) >$$

$$\sum_{i=1}^{n} \{\pi_i(x_i^*,e_i^*)-R_i[\pi_i(x_i^*,e_i^*)]\} - C(x^*), \tag{38}$$

and

$$\tilde{R}_i[\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)] + \tilde{K}_i - g_i(\tilde{e}_i) \geq$$

$$R_i[\pi_i(x_i^*,e_i^*)] - g_i(e_i^*) \qquad \text{for all } i=1,2,\ldots,n. \tag{39}$$

From the definition of $\tilde{K}_i$, the left-hand-side of (39) equals the right-hand-side of (39) when $(x_i,e_i) = (x_i^*,e_i^*)$. Since $(\tilde{x}_i,\tilde{e}_i)$ is chosen to solve:

$$\underset{(x_i,e_i)}{\text{MAX}} \quad \tilde{R}_i[\pi_i(x_i,e_i)-C_i(x_i;x^*-x_i^*)] + \tilde{K}_i - g_i(e_i),$$

the inequality in (39) is immediate.

Recall that $\pi_i(x_i,e_i) \equiv f_i(x_i) + h_i(e_i)$. The first-order conditions for the above problem are:

$$\tilde{R}_i'[f_i(\tilde{x}_i)+h_i(\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)][f_i'(\tilde{x}_i)-C_i'(\tilde{x}_i;x^*-x_i^*)] = 0 \tag{40}$$

and

$$\tilde{R}_i'[f_i(\tilde{x}_i)+h_i(\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)]h_i'(\tilde{e}_i) - g_i'(\tilde{e}_i) = 0. \tag{41}$$

Since $\tilde{R}' > 0$, (40) yields:

$$f_i'(\tilde{x}_i) - C_i'(\tilde{x}_i;x^*-x_i^*) = 0. \tag{42}$$

Thus $\tilde{x}_i$ is selected independently from $\tilde{e}_i$. Since $f_i'(x_i^*) = 0$ and $f_i(\cdot) - C_i(\cdot)$ is quasi-concave, (42) implies that $x_i^* > \tilde{x}_i$. From concavity of $R_i(\cdot)$, we have R_i' non-increasing in its argument; this along with $x_i^* > \tilde{x}_i$ gives us:

$$R_i'[f_i(\tilde{x}_i)+h_i(e_i)-C_i(\tilde{x}_i;x^*-x_i^*)] \geq R_i'[f_i(\tilde{x}_i)+h_i(e_i)] \geq$$

$$R_i'[f_i(x_i^*)+h_i(e_i)] \qquad \text{for all } e_i > 0. \tag{43}$$

When $e_i = \tilde{e}_i$, we have:

$$R_i'[f_i(\tilde{x}_i)+h_i(\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)] \geq R_i'[f_i(x_i^*)+h_i(\tilde{e}_i)]. \tag{44}$$

Since $h' > 0$, (44) and (41) imply that

$$0 = R_i'[f_i(\tilde{x}_i)+h_i(\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)]h_i'(\tilde{e}_i) - g_i'(\tilde{e}_i) \geq$$

$$R_i'[f_i(x_i^*)+h_i(\tilde{e}_i)]h_i'(\tilde{e}_i) - g_i'(\tilde{e}_i). \tag{45}$$

From the definition of (x_i^*,e_i^*) we know that:

$$R_i'[f_i(x_i^*)+h_i(e_i^*)]h_i'(e_i^*) - g_i'(e_i^*) = 0. \tag{46}$$

From R_i concave, h_i concave and g_i convex, the right-hand-side of (46) is decreasing in e_i. Thus, (45) and (46) together imply that $e_i^* \leq \tilde{e}_i$.

By quasi-concavity in x_i of $f_i(\cdot) - C_i(\cdot)$, and (38) we have x_i as the unique maximizer of $f_i(\cdot) - C_i(\cdot)$. Therefore,

$$f_i(\tilde{x}_i) - C_i(\tilde{x}_i;x^*-x_i^*) > f_i(x_i^*) - C_i(x_i^*;x^*-x_i^*). \tag{47}$$

Since $e_i^* \leq \tilde{e}_i$, and $h_i(\cdot)$ is increasing, we have

$$f_i(\tilde{x}_i) + h_i(\tilde{e}_i) - C_i(\tilde{x}_i;x^*-x_i^*)$$

$$> f_i(x_i^*) + h_i(e_i^*) - C_i(x_i^*;x^*-x_i^*). \tag{48}$$

Rewriting (48) results in:

$$\pi_i(\tilde{x}_i,\tilde{e}_i) - C_i(\tilde{x}_i;x^*-x_i^*) > \pi_i(x_i^*,e_i^*) - C_i(x_i^*;x^*-x_i^*) \tag{49}$$

Using $R_i(y) < y$ for all $y > 0$ and (49), we get:

$$R_i[\{\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)\}-\{\pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)\}]$$

$$< \{\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)\}-\{\pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)\}] \tag{50}$$

Concavity of $R_i(\cdot)$ and $R_i' > 0$ together imply that:

$$R_i[\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)]-R_i[\pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)]$$

$$\leq R_i[\{\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)\}-\{\pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)\}]. \tag{51}$$

Combining (50) and (51) gives:

$$R_i[\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)]-R_i[\pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)]$$

$$\leq \{\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)\}-\{\pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)\}. \tag{52}$$

Rearranging terms in (52) yields:

$$\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)-R_i[\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)]$$

$$+\ R_i[\pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)] > \pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*). \tag{53}$$

Subtracting $R_i[\pi_i(x_i^*,e_i^*)]$ from both sides of (53) and using the definition of $\tilde{K}_i$ gives us:

$$\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)-R_i[\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)] - \tilde{K}_i$$

$$> \pi_i(x_i^*,e_i^*)-C_i(x_i^*;x^*-x_i^*)-R_i[\pi_i(x_i^*,e_i^*)]. \tag{54}$$

Since (54) holds for all $i = 1,2,\ldots,n$ and since $C(\cdot) \equiv \sum_{i=1}^{n} C_i(\cdot)$, we get:

$$\sum_{i=1}^{n} \{\pi_i(\tilde{x}_i,\tilde{e}_i)-R_i[\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)] - \tilde{K}_i\}$$

$$- \sum_{i=1}^{n} C_i(\tilde{x}_i;x^*-x_i^*) >$$

$$\sum_{i=1}^{n} \{\pi_i(x_i^*,e_i^*)-R_i[\pi_i(x_i^*,e_i^*)]\} - C(x^*). \tag{55}$$

As $x_i^* > \tilde{x}_i$ for all $i = 1,2,\ldots,n$ and C_i is non-decreasing in $x-x_i$, we have

$$C(\tilde{x}) = \sum_{i=1}^{n} C_i(\tilde{x}_i;\tilde{x}-\tilde{x}_i) \leq \sum_{i=1}^{n} C_i(\tilde{x}_i;x^*-x_i^*). \tag{56}$$

Using (55) and (56) together we get:

$$\sum_{i=1}^{n} \{\pi_i(\tilde{x}_i,\tilde{e}_i)-R_i[\pi_i(\tilde{x}_i,\tilde{e}_i)-C_i(\tilde{x}_i;x^*-x_i^*)] - \tilde{K}_i\} - C(\tilde{x})$$
$$> \sum_{i=1}^{n} \{\pi_i(x_i^*,e_i^*)-R_i[\pi_i(x_i^*,e_i^*)]\} - C(x^*), \tag{57}$$

which is just (9), what we set out to prove.

□

Note that the cost allocated to division i is based on the amount of input that division i actually demands and the total amount of input that the other divisions would have demanded without allocated costs. This is necessary in order for the division managers to be able to explicitly compute their input demands and effort levels. If we view the input demands of the divisions when costs are not allocated as "historical" in some sense, then division i is being allocated costs based on its current input demand and the historical demand of the other divisions.

If we compute the total of all the allocated costs, and compare that to actual total cost, we see that with this mechanism more than total costs are allocated to the divisions. Specifically, since $x_i^* > \tilde{x}_i$ and $C_i(\cdot)$ is nondecreasing in $x-x_i$, it is straightforward to demonstrate that

$$C(\tilde{x}) \equiv \sum_{i=1}^{n} C_i(\tilde{x}_i;\tilde{x}-\tilde{x}_i) \leq \sum_{i=1}^{n} C_i(\tilde{x}_i;x^*-x_i^*)$$
$$< \sum_{i=1}^{n} C_i(x_i^*;x^*-x_i^*) \equiv C(x^*). \tag{58}$$

V. Summary

Suppose a principal (the center) compensates agents (division managers) according to reward functions that are based on divisional profits gross of any allocation of costs; and suppose that these reward functions are concave, increasing, and less than their respective arguments. We have demonstrated that for all such reward functions that the center could select, it is always in the interests of the center and the divisions to renegotiate a new set of compensation functions based on a full allocation of costs. Thus, we demonstrated the existence of conditions under which full cost allocations dominate no allocations.

Our analysis may be viewed as a formal demonstration of Zimmerman's [1979] assertion that cost allocations can dominate no cost allocation. Our formalization explicitly considered the welfare of division managers as well as the welfare of the principal, and explicitly modelled both divergence of preferences and asymmetric information. We allowed the cost function to have several jumps so that an increase in output may impose additional (capacity) "fixed" costs as well as variable costs. Hence, the implications of allocating total costs differ from those of allocating only variable costs. Our results are robust in the sense that most common full cost allocation methods can lead to Pareto improvements in welfare, in a wide range of environments.

In proving the existence of full cost allocations that dominate no cost allocation, we relied on the concept of a Nash equilibrium. However, the issue of the actual computation of the Nash equilibrium input and effort levels by the division managers was not resolved.

Cost allocation mechanisms that resolve this computation problem were given. With these mechanisms, the ith division manager's compensation is a function of the ith division's current input demand and the historical demand of the other n-1 divisions. These mechanisms result in a dominant strategy equilibrium, thereby solving the computation problems, but the allocation of costs is only approximate. This suggests that in order to resolve the computational problems associated with the full cost allocation model, one must be willing to sacrifice the notion of a tidy allocation of costs in the current period based only on the actions (demands) in the current period.

The intuition underlying our results is as follows: at the no-cost allocation equilibrium the manager could trade a small amount of additional effort for a small amount of transferred product and leave local profits unchanged, costs lower, and firm profits gross of reward greater. However, since effort is costly to the manager and the transferred product is not, the manager has no incentive to do so. From the firm's point of view, both the transferred product and effort are costly (the latter because the firm must pay the manager for effort in order to meet the manager's reservation wage). When the division is charged for the transferred product, the manager's relative prices for effort and transferred product change, and the manager increases effort and reduces consumption of the transferred product, giving the firm larger profits gross of rewards but net of costs. For this cost allocation scheme to Pareto dominate the no-cost allocation scheme, the decrease in costs must be greater than the increased reward paid to the manager to compensate for the increase in effort. Since R_i is concave, the firm saves more in costs of producing the transferred product than the manager incurs as a result of increased effort. Thus, a transfer from the center to the manager exists that makes both parties better off.

Finally, we again point out that we have not shown that cost allocations are an optimal method of coordinating decentralized units of an organization. Certainly for cost allocations to be optimal it is necessary that they dominate no-cost allocation. We have seen that this necessary condition is met.

Appendix

We prove that in the no cost allocation case, the center chooses reward functions that are concave, increasing functions of local profits alone, and such that $R_i(\Pi_i) < \Pi_i$ for all $\Pi_i > 0$. We demonstrate this by proving four separate assertions.

ASSERTION 1: Let $R_i(\Pi_i, x_i)$ be the reward function for division i. Without loss of generality, we can restrict R_i's domain to R_+^2 such that R_i is increasing in its first argument.

Proof:

By contradiction. Assume there exists $(a,b) \varepsilon R_+^2$ such that R_i is non-increasing in its first argument at (a,b).

CASE 1

There exists no $(\overline{\Pi}_i, \overline{g}_i) \varepsilon M_i$ such that $R_i(a,b) = R_i(\Pi_i(\overline{x}_i, \overline{e}_i), \overline{x}_i)$ for some $(\overline{x}_i, \overline{e}_i)$. Then (a,b) is not part of the relevant domain of R_i and may be excluded without loss of generality.

CASE 2

There exists $(\overline{\Pi}_i, \overline{g}_i) \varepsilon M_i$ such that for some $(\overline{x}_i, \overline{e}_i)$, $R_i(a,b) = R_i(\overline{\Pi}_i(\overline{x}_i, \overline{e}_i), \overline{x}_i)$. We claim that (x_i, e_i) will never be chosen by division manager i. Using the fact that $\overline{g}_i$ is strictly increasing in e_i, $\overline{\Pi}_i$ is strictly increasing in e_i, and $R_i(a,b)$ is non-increasing in its first argument at (a,b), we have that there exists some $\tilde{e}_i < \overline{e}_i$ such that:

$$R_i(a,b) - \bar{g}_i(\bar{e}_i) = R_i(\bar{\Pi}_i(\bar{x}_i,\bar{e}_i),\bar{x}_i) - \bar{g}_i(\bar{e}_i)$$

$$< R_i(\bar{\Pi}_i(\bar{x}_i,\tilde{e}_i),\bar{x}_i) - \bar{g}_i(\tilde{e}_i).$$

Therefore, since there exists $(\bar{x}_i,\tilde{e}_i)$ that makes division manager i strictly better off than at $(\bar{x}_i,\bar{e}_i)$, the latter will never be chosen by the division manager. We may therefore, without loss of generality, exclude (a,b) from the relevant domain of R_i.

$\square$

For an accepted contract, the following self-selection constraint must be met:

$$(x_i^*,e_i^*) \in \underset{(x_i,e_i) \in T_i}{\text{Argmax}}\ R_i(\Pi_i(x_i,e_i),x_i) - g_i(e_i) \tag{A1}$$

where $T_i = \{(x_i,e_i) \mid 0 \leq x_i \text{ and } 0 \leq e_i \leq \bar{e}_i\}$.

The existence of a solution to the problem in (A1) leads to our second assertion.

<u>ASSERTION 2</u>: Let $R_i(\Pi_i,x_i)$ be the reward function for division manager i. Then R_i is concave in its argument.

<u>Proof</u>:

A necessary condition for a solution to (A1) to exist for a particular $(\Pi_i,g_i) \in M_i$ is quasi-concavity of the maximand in (x_i,e_i). A necessary condition for quasi-concavity of the maximand in (A1) for all $(\Pi_i,g_i) \in M_i$ is concavity of R_i. Therefore, a necessary condition for a solution to (A1) to exist for all $(\Pi_i,g_i) \in M_i$ is concavity of R_i.[14]

$\square$

The third assertion depends directly on our definitions of no-cost allocation, and "reasonable" reward functions.

ASSERTION 3: Let $R_i(\Pi_i, x_i)$ be the reward function for division i. Then for the no cost allocation benchmark, R_i depends only on Π_i.

Proof:

The self-selection constraint can be rewritten, at (x_i^*, e_i^*), as:

$$\frac{\partial R_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial x_i} + \frac{\partial R_i}{\partial x_i} = 0.$$

and

$$\frac{\partial R_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial e_i} - g_i' = 0.$$

By definition of no-cost allocation, we know that $\frac{\partial R_i}{\partial x_i}$ is non-negative. Assume that $\frac{\partial R_i}{\partial x_i}$ is strictly positive. Then (x_i^*, e_i^*) will be chosen such that

$$\frac{\partial \Pi_i}{\partial x_i} = - \frac{\partial R_i}{\partial x_i} / \frac{\partial R_i}{\partial \Pi_i} .$$

From assertion 1, $\frac{\partial R_i}{\partial \Pi_i} > 0$; from assumption, $\frac{\partial R_i}{\partial x_i} > 0$. Thus, $\frac{\partial \Pi_i}{\partial x_i} < 0$ at (x_i^*, e_i^*), for all Π_i. From our definition of admissable reward functions, we excluded all R_i where $\frac{\partial \Pi_i}{\partial x_i} < 0$ at (x_i^*, e_i^*), for all Π_i. This gives us $\frac{\partial R_i}{\partial x_i} = 0$, or R_i depending only on Π_i.

$\square$

The center will choose reward functions R_i that depend only on Π_i and that are strictly increasing in Π_i. Our fourth assertion is that the division's reward will be positive only if it produces positive profits, and that the division's reward will always be less than its profits.

<u>ASSERTION 4</u>: Let $R_i(\Pi_i)$ be the reward function for division i. Then, for all $(\Pi_i, g_i) \varepsilon M_i$,

(i) $R_i(a) > 0$ only if $a > 0$; and

(ii) $R_i(a) < a$ for all $a > 0$.

<u>Proof</u>:

(i) Let (x_i^*, e_i^*) denote the output level and effort level chosen by division manager i, if the contract is accepted. A necessary condition for the contract (employment under the compensation plan) to be accepted is $R_i(\Pi_i(x_i^*, e_i^*)) \geq w_i$, where $w_i > 0$ is the division manager's reservation utility level. Since a division manager <u>knows</u> $\Pi_i(x_i, e_i)$ prior to accepting the contract, and reward functions are chosen for all $(\Pi_i, g_i) \varepsilon M_i$, the contract must have the property that $R_i(\Pi_i(x_i^*, e_i^*)) > 0$. If it were the case that $R_i(\Pi_i(x_i^*, e_i^*)) > 0$ and $\Pi_i(x_i^*, e_i^*) < 0$, then the center would not want the division manager to accept the contract. Thus, the center will restrict its attention to reward functions such that $R_i(\Pi_i(x_i^*, e_i^*)) > 0$ if $\Pi_i(x_i^*, e_i^*) > 0$, and $R_i(\Pi_i(x_i^*, e_i^*)) = 0$ if $\Pi_i(x_i^*, e_i^*) \leq 0$. Again, as reward functions are chosen for all admissible profit and disutility of effort functions, it follows that the center can restrict attention to reward functions such that $R_i(y) > 0$ only if $y > 0$, and $R_i(y) = 0$ if $y \leq 0$. Since

$\Pi_i(x_i,e_i) > 0$ only if $x_i > 0$, it must be the case that for an accepted contract, $x_i^* > 0$.[15] For an accepted contract, the self-selection constraint (A1) can be written as

$$(x_i^*,e_i^*) \in \underset{(x_i,e_i) \in T_i}{\text{Argmax}}\ R_i(f_i(x_i)+h_i(e_i)) - g_i(e_i)$$

(ii) Suppose that for some $a > 0$, $R_i(a) \geq a$, and that there exists $(\Pi_i,g_i) \in M_i$ such that $\Pi_i(x_i^*,e_i^*) = a$. From the definition of admissable reward functions, such an R_i is not admissable.

$\square$

Footnotes

[1] See Harris, Kriebel and Raviv [1982] for more on this point.

[2] We denote the first and second derivatives by single and double primes, respectively.

[3] Further restrictions are placed on the cost functions in Section III.

[4] See Zimmerman [1979, p. 515] for more on this point.

[5] This is similar in spirit, but not the same as the minimum regret strategy defined in Luce and Raiffa [1957].

[6] Note also that the properties of the reward functions do not exclude a simple sharing of a division's gross profits.

[7] Clearly, the game we model is one in which the managers accept the contract myopically; that is, they (the managers) assume that the contract is not going to be changed in the future. Therefore, those potential managers who have not joined the firm have no interest in any change in the reward scheme.

[8] It should be noted that the Nash equilibrium is ex post rational, in the same sense as the center's strategy in choosing the no-cost allocation reward functions.

[9] Groves and Ledyard [1985] provide a cogent defense of the use of Nash equilibria as the solution concept in the type of situation we have modelled:

> ...we turn to the Nash equilibrium of the complete information game. We do not suggest that each agent knows all of [the environment] when they compute [their strategies], just as in real markets no auctioneer knows the excess demand function when equilibrium prices are calculated. We do suggest, however, that the Complete Information Nash game-theoretic equilibrium [strategies] may be the possible "equilibrium" of [some] iterative process, i.e., the stationary messages, just as the demand-equal-supply price is thought of as the "equilibrium" of some unspecified market dynamic process. [p. 30]

[10] For our proof to go through, we do not need C_i to be quasi-convex in x_i, but the somewhat weaker condition that $f_i - C_i$ be strictly quasi-concave in x_i.

[11] It should be noted that this condition is analagous to Zimmerman's conditions on page 518 for his Case 1. He points out that if marginal cost is greater than average cost, allocating overhead dominates no overhead allocation.

[12] When $C_i = (x_i/x)C$, we are using a transfer pricing scheme where the price is average cost. We will thus demonstrate that there exist conditions under which average cost transfer pricing dominates no cost allocation.

[13] All of our results on dominance in full cost allocations can be obtained for compensation functions in the larger class of increasing, quasi-concave functions whose first derivatives are strictly less than one. However, optimal reward functions for the no allocation problem must meet the additional restriction that they are less than their arguments.

[14] Since R_i is concave and defined over all of its domain, it is necessarily continuous. In addition, there exists only a finite number of points where R_i is not differentiable [see Rockafellar, p. 83 and p. 246]. Therefore, without loss of generality we can let the partial derivatives be from either the right or left, as long as we are consistent. In that case, the first and second order necessary conditions have their usual interpretation.

[15] We can further restrict the set of reward functions to be ones that would be accepted for some $w_i > 0$. Thus, we need only concern ourselves with the properties of a reward function $R_i(y)$ over positive values; i.e., where $y > 0$.

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