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Acknowledgment:

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PAUL BECKERMAN UNIVERSITY OF ILLINOIS SEPTEMBER 1978

Abstract: This essay reviews James Tobin's argument that the introduction of index-linked government bonds would render monetary policy more efficient. A macro-economic model incorporating inflationary expectations and uncertainty is used to show that the effects of a given monetary policy would probably be no different in a regime with index-linked government bonds than they would be in a regime with nominal government bonds. Monetary policy will have different effects in the two regimes when the model is complicated to allow monetary policy to generate changes in the state of expectations and uncertainty; it is not possible, however, to predict in which regime monetary policy would be the more efficient.

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Introduction

In a well-known 1963 treatise, "An Essay on the Principles of Government Debt Management," (Tobin, 1963) James Tobin suggested that the monetary authority could increase the "efficiency" of its open-market operations if it dealt in index-linked government bonds. The introduction of index-linked long-term government debt, Tobin argued, would make open-market operations more efficient in the sense that a given change in the target variable—which Tobin took to be "the supply price of capital," i.e., the ratio of the financial-market valuation of equity to the replacement cost of the capital represented by the equity—could be achieved with a smaller change in the maturity composition of the outstanding government debt.

Let us first review Tobin's argument. In the analytical model of his treatise, if the state of financial anticipations and investor preferences is given, the relative supplies of the different kinds of government debt (money, short-term debt, and long-term debt) and private capital assets determine the valuation of the capital assets. All other things being equal, the higher is the proportion of money and government short-term debt held by the public, or the lower is the proportion of long-term debt held by the public, the higher will be the real valuation placed by the public on capital assets. Open-market operations work by manipulating the relative proportions of outstanding government debt to change the supply price of capital—the higher is the supply price of capital, the lower will be the private cost of capital, and hence the higher the level of investment should be. Now, all other things being equal, the effect of any shift in the relative supplies of government on the supply price of capital will be the stronger, the stronger is the substitutability of long-term government debt and capital assets, and the weaker is the substitutability of long-term government debt

and money. The essential assumption behind Tobin's claim for index-linked long-term government debt was that such debt would be regarded by financial markets as a stronger substitute for equities and a weaker substitute for money than nominal long-term government debt would be. Tobin put it this way:

> A substantial part of the independence of risk between current debt instruments and capital equity arises from their difference in status with respect to uncertainties of the future purchasing power of money. A purchasing power bond would share the role of capital equity as a hedge against changes in the price level. It would therefore be a much better substitute than existing debt instruments for ownership of capital. There would remain, of course, the additional risks of capital ownership, for which capital would command a premium over the rate of return on purchasing power bonds. This premium would vary with, among other things, the relative supplies of purchasing power bonds and real capital. For its part, the marketable purchasing power bond would involve risks of interest rate change in the same manner as conventional bonds. But there would be less reason to expect its interest rate to move together with other government debt interest rates. The purchasing power bond would be substantially independent of other debt instruments in risk. It would be a much poorer substitute for other government obligations than long-term bonds are at present for short-term obligations and cash. (Tobin, 1963, p. 204.)

Professor Tobin's claim for index-linked government bonds has been questioned on various grounds. A number of commentators have doubted that index-linked bonds really would be regarded by the financial markets as a closer substitute for equities than nominal bonds. For example, investors might in fact consider the *ex ante* (i.e., "before" market valuation) expected real rates of return on nominal bonds and capital to be subject to negatively correlated random variables: the view of the financial market might be that, whatever happens in the future, higher inflation is likely to be favorable to the real rate of return on capital assets, lower inflation less favorable. If this is really how financial-market participants feel, risk-averse financial investors might prefer to hold

hedged portfolios of nominal bonds and equities, and nominal bonds might in effect be better substitutes for capital assets than index-linked bonds. [See Fischer (1975), Siegel (1974).] Another point is that in certain contexts, e.g., in Brazil, as described in Beckerman ("Index-linked Financial Assets...," 1978), the rates for short-term finance may be a more important determinant of (or constraint on) the level of investment spending than the supply price of capital or the long-term bond rate. Indeed, in some places short-term instruments might even be a closer substitute for capital assets traded in the stock markets than long-term instruments. In such circumstances the principal target for monetary policy must then be the short-term rates, and it may be easier to affect these if the long-term bonds themselves are nominal.

In this essay, however, we question Tobin's analysis on more fundamental grounds. Even supposing that index-linked bonds are regarded by the financial markets as a closer substitute for equities than nominal bonds, and that the supply price of capital is the appropriate target variable for monetary policy, it is not clear that monetary policy will be more efficient if government bonds are index-linked. In the passage from Tobin's 1963 essay that we have cited above, Tobin really makes two assumptions: first, that index-linked bonds are closer substitutes for equities than nominal bonds; and second, that index-linked bonds are weaker substitutes for cash and near-money assets than nominal bonds. In other words, Tobin assumes that the introduction of index-linked bonds turns the "demand-for-equity" function more sensitive, and the demand-for-money function less sensitive, to changes in the interest rate. As we shall see below, both assumptions are necessary if we are to be certain that monetary

policy will be more efficient with index-linked bonds. If we are careful, however, to think about the demand-for-equity and the demand-for-money functions as well as the interest rate in <u>real</u> terms, it seems unlikely that the two assumptions would hold simultaneously. For if the introduction of index-linked bonds turns the demand-for-real-equity function more sensitive to changes in the real interest rate, it is likely also to turn the demand-for-real-balances function more sensitive to changes in the real interest rate. The effects of introducing index-linked bonds on the two functions would then be contrary. If a fall in the real interest rate has an enhanced expansionary effect on the demand for equity in a regime with index-linked bonds, it also has an enhanced contractionary effect on the demand for money. In our analysis in Section I, along with Appendices A and B, we shall show (contrary, perhaps, to intuition) that there is no reason to believe that the effect of introducing index-linked bonds on the demand-for-equity function would dominate the effect on the demand-for-money function.

In general, we shall argue that there is no reason to expect monetary policy to differ systematically in regimes with nominal and with index-linked bonds. In Section II, however, we discuss a possible exception to this generalization. It is possible that monetary policy, through changes in the price level and other system parameters of the model we develop, generates systematic changes in the state of expectations and uncertainty. Since, as we will show in Section I, the state of expectations and uncertainty may play a different structural role in macro-economic systems with nominal and with index-linked bonds, monetary policy might have different consequences through this channel in the two regimes. We will

argue, however, that even when this "expectations-and-uncertainty" channel is taken into account, there is no way to predict on theoretical grounds whether monetary policy would be more efficient with index-linked or nominal bonds.

I. Aggregate demand and monetary policy with nominal and index-linked bonds.

Our purpose in this section is to develop a model of aggregate demand capable of representing government bonds as nominal or index-linked. We then use this model to determine under what conditions monetary policy would be more efficient in the model with index-linked bonds than in the model with nominal bonds.

Our model of aggregate demand comprises three equations: an "investment-equals-savings" relation, a "money-demand-equals-money-supply" relation, and a "capital-valuation" relation. All these relations are quite conventional in macro-economic theory; only here, in order to represent the availability or non-availability of index-linked government bonds, we incorporate as shift parameters the expectations of and the uncertainties attaching to the real rates of return on money, bonds, and equities.

Suppose first that government bonds are nominal. We begin by considering the demand for real money balances. Let M represent the money supply, p the price level, y the real rate of national income, and W the stock of real wealth. Let L^n represent the demand-for-money function (as a fraction of real wealth) with nominal bonds (represented by the superscript n). Also, let $-\bar{\pi}$ represent the expected real rate of return on money for "the coming period," |2|, i the nominal rate of interest, $\bar{r}$ the expected real rate of return on bonds |2|, and $\bar{\kappa}$ the expected real rate of return on equities. |3|

And finally, let $\hat{\pi}$, $\hat{r}$, and $\hat{\kappa}$ represent some measure of the uncertainties perceived by the financial markets to apply to $\bar{\pi}$, $\bar{r}$, and $\bar{\kappa}$. We take the demand for nominal money balances to be given by

$$pL^n(y, -\bar{\pi}, \hat{\pi}, \bar{r}, \hat{r}, \bar{\kappa}, \hat{\kappa}) = W,$$

$$L^n_y, L^n_{-\bar{\pi}} > 0, L^n_{\bar{r}}, L^n_{\bar{\kappa}} < 0, L^n_{\hat{\pi}}, L^n_{\hat{r}}, L^n_{\hat{\kappa}} \gtrless 0,$$

$$\bar{r} = i - \bar{\pi}, \hat{r} = \hat{\pi}.$$

That is, the demand for money is a positive function of the rate of national income and of the expected real rate of return on money, and a negative function of the expected real rates of return on competing assets. The expected real rate of return on bonds is given by $i - \bar{\pi}$ on the assumption that the bonds carry no coupon, being simply sold at a discount. Intuitively, one might suppose that the demand for money is a negative function of the uncertainty attaching to the real rate of return on money and a positive function of the rates of return on competing assets. This is not clear a priori, however, because the subjective probability distributions held in the financial markets of π and κ may be either negatively or positively covariant, so that $L^n_{\hat{\pi}}$, $L^n_{\hat{r}}$, and $L^n_{\hat{\kappa}}$ may reasonably take any sign.

Next we consider the "demand-for-equity" function. Let K represent the real par value of outstanding equity, which we assume to be equal to the real replacement value of existing physical capital. Let qK represent the financial markets' real valuation of this equity (or capital), and let C^n be the demand-for-equity function with nominal bonds. The valuation of equity is given then by

$$qK = C^n(-\bar{\pi}, \hat{\pi}, \bar{r}, \hat{r}, \bar{\kappa}, \hat{\kappa}) \cdot W,$$

$$C^n_{\bar{\pi}},\ C^n_{\bar{r}} < 0,\ C^n_{\bar{k}} > 0,\ C^n_{\tilde{\pi}},\ C^n_{\tilde{r}},\ C^n_{\tilde{k}} \gtrless 0,$$

$$\bar{r} = i - \bar{\pi},\ \tilde{r} = \tilde{\pi}.$$

Demand for equity is taken to be related positively to the expected real rate of return on equity and negatively to the expected rates of return on competing assets. The signs of $C^n_{\tilde{\pi}}$, $C^n_{\tilde{r}}$, and $C^n_{\tilde{k}}$ are taken to be unknown a priori.

Finally, for the "IS" equation, we will write

$$I^n(q, \bar{r}, \tilde{r}) = S^n(y, \bar{r}, \tilde{r}) + [T(y) - G],$$

$$I^n_q > 0,\ I^n_{\bar{r}},\ I^n_{\tilde{r}} < 0,\ S^n_y > 0,\ S^n_{\bar{r}},\ S^n_{\tilde{r}} \gtrless 0,\ T_y > 0,$$

$$\bar{r} = i - \bar{\pi},\ \tilde{r} = \tilde{\pi},$$

where I is the real-rate-of-investment function, S the real-rate-of-savings function, T the real-rate-of-tax-revenue function, and G the real rate of government expenditure. Investment is considered to be encouraged by a higher supply price of capital q, and to be discouraged, all other things being equal, by a higher expected real rate of interest or by a higher uncertainty attaching to that rate. Savings is considered to be encouraged by a higher rate of national income, but the effects of increases in the expected real rate of interest or the uncertainty attaching to this rate on the savings rate cannot be said a priori: we should think that an increase in $\bar{r}$ would encourage savings, and that an increase in $\tilde{r} = \tilde{\pi}$, i.e., in inflationary uncertainty, would discourage savings; nevertheless, the supply schedule of savings with respect to $\bar{r}$ may be backward-bending, and it is possible that in the short-run an increase

in inflationary uncertainty actually induces people to save to maintain their wealth as best they can. Finally, the tax function is taken here to depend only on real income. This is not true, of course, for inflation can affect real tax receipts, but since the tax function is not affected by whether bonds are nominal or index-linked, the tax function is beside the point for our present purposes.

The complete model of aggregate demand with nominal bonds is given, then, by

$$M = pL^n(y, -\bar{\pi}, \tilde{\pi}, \bar{r}, \tilde{r}, \bar{\kappa}, \tilde{\kappa}) \cdot W,$$

$$L^n_y, L^n_{-\bar{\pi}} > 0, L^n_{\bar{r}}, L^n_{\bar{\kappa}} < 0, L^n_{\tilde{\pi}}, L^n_{\tilde{r}}, L^n_{\tilde{\kappa}} \gtrless 0; \tag{1n}$$

$$qk = C^n(-\bar{\pi}, \tilde{\pi}, \bar{r}, \tilde{r}, \bar{\kappa}, \tilde{\kappa}) \cdot W,$$

$$C^n_{-\bar{\pi}}, C^n_{\bar{r}} < 0, C^n_{\bar{\kappa}} > 0, C^n_{\tilde{\pi}}, C^n_{\tilde{r}}, C^n_{\tilde{\kappa}} \gtrless 0; \tag{2n}$$

$$I^n(q, \bar{r}, \tilde{r}) = S^n(y, \bar{r}, \tilde{r}) + [T(y) - G],$$

$$I^n_q > 0, I^n_{\bar{r}}, I^n_{\tilde{r}} < 0, S^n_y > 0, S^n_{\bar{r}}, S^n_{\tilde{r}} \gtrless 0, T_y > 0; \tag{3n}$$

$$\bar{r} = i - \bar{\pi}; \tag{4n}$$

and

$$\tilde{r} = \tilde{\pi}. \tag{5n}$$

This model can easily be converted to represent a macro-economic system incorporating index-linked bonds, as follows: (i) the functions L^n, C^n, I^n, and S^n are changed to L^i, C^i, I^i, and S^i, though their arguments remain unchanged; (ii) the real rate of interest is now certain, so

that $\bar{r} = r$ and $\hat{r} = 0$; and (iii) the signs of $L^i_{\hat{r}}$, $C^i_{\hat{r}}$, $I^i_{\hat{r}}$, and $S^i_{\hat{r}}$ are now irrelevant, since $\hat{r}$ is fixed at zero.

The model of aggregate demand with index-linked bonds is given, then, by

$$M = pL^i(y, -\bar{\pi}, \hat{\pi}, \bar{r}, \hat{r}, \bar{\kappa}, \hat{\kappa}) \cdot W,$$

$$L^i_y,\ L^i_{-\bar{\pi}} > 0,\ L^i_{\bar{r}},\ L^i_{\bar{\kappa}} < 0,\ L^i_{\hat{\pi}},\ L^i_{\hat{r}},\ L^i_{\hat{\kappa}} \gtrless 0; \tag{1i}$$

$$qK = C^i(-\bar{\pi}, \hat{\pi}, \bar{r}, \hat{r}, \bar{\kappa}, \hat{\kappa}) \cdot W$$

$$C^i_{-\bar{\pi}},\ C^i_{\bar{r}} < 0,\ C^i_{\bar{\kappa}} > 0,\ C^i_{\hat{\pi}},\ C^i_{\hat{r}},\ C^i_{\hat{\kappa}} \gtrless 0; \tag{2i}$$

$$I^i(q, \bar{r}, \hat{r}) = S^i(y, \bar{r}, \hat{r}) + [T(y) - G],$$

$$I^i_q > 0,\ I^i_{\bar{r}},\ I^i_{\hat{r}} < 0,\ S^i_y > 0,\ S^i_{\bar{r}},\ S^i_{\hat{r}} \gtrless 0,\ T_y > 0; \tag{3i}$$

$$\bar{r} = r; \tag{4i}$$

and

$$\hat{r} = 0. \tag{5i}$$

Each of the models (1n) - (5n) and (1i) - (5i) has four system parameters, p, y, q and $\bar{r}$, and eight shift parameters, G, M, K, W, $\bar{\pi}$, $\hat{\pi}$, $\bar{\kappa}$, and $\hat{\kappa}$. Each model may be solved for an aggregate-demand function,

$$y = D^n(p;\ G, M, K, W, \bar{\pi}, \hat{\pi}, \bar{\kappa}, \hat{\kappa}) \tag{6n}$$

and

$$y = D^i(p;\ G, M, K, W, \bar{\pi}, \hat{\pi}, \bar{\kappa}, \hat{\kappa}). \tag{6i}$$

To solve the system for p and y requires an aggregate-supply relation, such as

$$y = F(p;\ K,\ \bar{\pi},\ \hat{\pi},\ \bar{\kappa},\ \hat{\kappa}). \tag{7}$$

Here, however, we concentrate on aggregate demand alone. Furthermore, to simplify and focus our discussion of Tobin's argument, we consider the asset-demand relations of each model alone. We derive a pair of "LM" relations for the equations [(1n), (2n), (4n), (5n)] and [(1i), (2i), (4i), (5i)], that is, the two models without their respective "IS" equations. The "LM" relations are obtained by solving the equations for

$$q = Z^n(y,\ p;\ M,\ K,\ W,\ \bar{\pi},\ \hat{\pi},\ \bar{\kappa},\ \hat{\kappa}) \tag{8n}$$

and

$$q = Z^i(y,\ p;\ M,\ K,\ W,\ \bar{\pi},\ \hat{\pi},\ \bar{\kappa},\ \hat{\kappa}), \tag{8i}$$

eliminating $\bar{r}$ from the money-demand and the equity-demand equations.

In Appendix A we show that

$$Z^n_y = -WK^{-1}\ C^n_{\bar{r}}(L^n_{\bar{r}})^{-1}\ L^n_y < 0,$$

$$Z^n_p = -WK^{-1}\ C^n_{\bar{r}}(L^n_{\bar{r}})^{-1}\ p^{-1} < 0,$$

$$Z^n_{\bar{\pi}} = WK^{-1}\ [C^n_{\bar{\pi}} - C^n_{\bar{r}}(L^n_{\bar{r}})^{-1}\ L^n_{\bar{\pi}}] > 0,$$

$$Z^n_{\hat{\pi}} = WK^{-1}\ [(C^n_{\hat{\pi}} + C^n_{\hat{r}}) - C^n_{\bar{r}}(L^n_{\bar{r}})^{-1}\ (L^n_{\hat{\pi}} + L^n_{\hat{r}})] \gtrless 0,$$

$$Z^n_{\bar{\kappa}} = WK^{-1}\ [C^n_{\bar{\kappa}} - C^n_{\bar{r}}(L^n_{\bar{r}})^{-1}\ L^n_{\bar{\kappa}}] > 0,$$

$$Z^n_{\hat{\kappa}} = WK^{-1}\ [C^n_{\hat{\kappa}} - C^n_{\bar{r}}(L^n_{\bar{r}})^{-1}\ L^n_{\hat{\kappa}}] \gtrless 0,$$

$$Z^n_M = K^{-1}\ C^n_{\bar{r}}(L^n_{\bar{r}})^{-1}\ p^{-1} > 0,$$

$$Z^n_K = -qK^{-1} < 0,$$

$$\text{and} \quad Z^n_W = K^{-1}\,[C^n - C^n_{\underline{r}}(L^n_{\underline{r}})^{-1}\,L^n] \gtrless 0; \tag{8n'}$$

and also that

$$Z^i_y = -WK^{-1}\,C^i_r(L^i_r)^{-1}\,L^i_y < 0,$$

$$Z^i_p = -WK\,C^i_r(L^i_r)^{-1}\,p^{-1} < 0,$$

$$Z^i_{\underline{\pi}} = WK^{-1}\,[C^i_{\underline{\pi}} - C^i_r(L^i_r)^{-1}\,L^i_{\underline{\pi}}] > 0$$

$$Z^i_{\pi} = WK^{-1}\,[C^i_{\pi} - C^i_r(L^i_r)^{-1}\,L^i_{\pi}] \gtrless 0,$$

$$Z^i_{\underline{K}} = WK^{-1}\,[C^i_{\underline{K}} - C^i_r(L^i_r)^{-1}\,L^i_{\underline{K}}] > 0,$$

$$Z^i_R = WK^{-1}\,[C^i_R - C^i_r(L^i_r)^{-1}\,L^i_R] \gtrless 0,$$

$$Z^i_M = K^{-1}\,C^i_r(L^i_r)^{-1}\,p^{-1} > 0,$$

$$Z^i_K = -qK^{-1} < 0,$$

$$\text{and} \quad Z^i_W = K^{-1}\,[C^i - C^i_r(L^i_r)^{-1}\,L^i] \gtrless 0. \tag{8i'}$$

The conditions under which monetary policy will be more efficient with index-linked bonds can be determined readily from (8n') and (8i'). Since

$$Z^n_M = K^{-1}\,C^n_{\underline{r}}(L^n_{\underline{r}})^{-1}\,p^{-1}$$

$$\text{and} \quad Z^i_M = K^{-1}\,C^i_r(L^i_r)^{-1}\,p^{-1},$$

a change in the money supply will affect the supply price of capital by

more in a regime with index-linked bonds if (and only if)

$$C^i_r(L^i_r)^{-1} > C^n_{\underline{r}}(L^n_{\underline{r}})^{-1} . \qquad (9)$$

Since

$$Z^n_W = K^{-1} [C^n - C^n_{\underline{r}}(L^n_{\underline{r}})^{-1} L^n]$$

and $$Z^i_W = K^{-1} [C^i - C^i_r(L^i_r)^{-1} L^i] ,$$

a change in the quantity of bonds outstanding (i.e., an increase in W such that K and M remain unchanged) will affect the supply price of capital by more (in absolute terms) in a regime wth index-linked bonds if (and only if), again, condition (9) holds. (Since we are presumably making the comparative-statics change in W from the same initial position, $C^i = C^n$ and $L^i = L^n$.) Now C^i_r, L^i_r, $C^n_{\underline{r}}$, and $C^i_{\underline{r}}$ are all assumed to be less than zero, so we may speak in terms of their absolute value. If $|C^i_r|$ is greater than $|C^n_{\underline{r}}|$, <u>and</u> $|L^i_r|$ is less than $|L^n_{\underline{r}}|$, as Tobin assumes, then condition (9) clearly holds, and monetary policy is more efficient with index-linked bonds. It is not likely, however, that the introduction of index-linked bonds would increase the real-interest-rate sensitivity of the demand for equities without at the same time increasing the real-interest-rate sensitivity of the demand for real money balances. With the introduction of index-linked bonds, a certain real interest rate replaces an uncertain real interest rate in both asset-demand functions. A change in a certain real interest rate should have a stronger effect than a change in an uncertain expected real interest rate on <u>both</u> functions, not on the demand-for-equity function alone. If $|C^i_r|$ is greater than $|C^n_{\underline{r}}|$, then we should expect $|L^i_r|$ to be greater than $|L^n_{\underline{r}}|$. If this is the case, we cannot say for sure

whether condition (9) would hold, and therefore we cannot say for sure whether the introduction of index-linked bonds would increase the effectiveness of monetary policy.

In fact, we can support our claim that condition is not likely to hold with some "theoretical evidence." In Appendix B we derive demand-for-money and demand-for-equity functions incorporating the expectations and uncertainty parameters we have described by using the Mossin capital-asset pricing model. We make no claims for the empirical validity of the functions we derive, and indeed they are open to certain theoretical objections; nevertheless, they are plausible functions. Now if the demand-for-money and demand-for-equity functions take the form derived in Appendix B, then (i) $|L^i_r| \gtrless |L^n_{\underline{r}}|$ as $|C^i_r| \gtrless |C^n_{\underline{r}}|$, i.e., Tobin's two assumptions cannot both hold simultaneously, and (ii) condition (9) becomes an equality, i.e.,

$$C^i_r(L^i_r)^{-1} = C^n_{\underline{r}}(L^n_{\underline{r}})^{-1} \quad . \tag{10}$$

In this case we have a strong result, that monetary policy would be neither more nor less effective with index-linked bonds than with nominal bonds. This strong result depends on the particular forms of the asset-demand functions derived in Appendix B; hence we will not claim that it is any more than a "plausible possibility." At all events, it supports the essential point of this Section: that there is no reason to believe that condition (9) is certain to hold.

II. Monetary policy with nominal and index-linked bonds, taking the effects of monetary policy on the state of expectations into account.

Let us assume in this Section that condition (10) holds in fact as an equality, as suggested by the analysis in Appendix B. Inspection of

the expressions (8n') and (8i') shows that under this assumption Z_x^i is equal to Z_x^n for x = M, W, K, and p, but Z_x^i does not (necessarily) equal Z_x^n for x = y, $\bar{\pi}$, $\tilde{\pi}$, $\bar{\kappa}$, or $\tilde{\kappa}$. Since monetary policy can affect y through mechanisms represented by our model, and can probably affect $\bar{\pi}$, $\tilde{\pi}$, $\bar{\kappa}$, and $\tilde{\kappa}$ through mechanisms outside our model, monetary policy can therefore have different degress of efficiency in regimes with nominal bonds and with index-linked bonds. In this section, however, we will argue that it is still not possible to predict in which regime monetary policy would be the more effective.

Consider the consequences of an expansionary monetary policy on y, $\bar{\pi}$, $\tilde{\pi}$, $\bar{\kappa}$, and $\tilde{\kappa}$. First, in general, expansionary monetary policy leads to an increase in y. To the extent that this generates an increase in the demand for money, the expansionary effect of the monetary policy is reduced; through this channel, therefore, monetary policy will be more expansionary in a regime with index-linked bonds if and only if L_y^i is less than L_y^n. There is no particular reason, however, to believe a priori that L_y^i would be either less or greater than L_y^n.

Second, an expansionary monetary policy could generate increases in $\bar{\pi}$ and $\bar{\kappa}$, if the public revised its expectations upward after an increase in p and/or y. At least through the LM sector of the model, increases in $\bar{\pi}$ and $\bar{\kappa}$ would cause added expansionary effects, since $Z_{\bar{\pi}}$ and $Z_{\bar{\kappa}}$ are both greater than zero. Once again, however, it is simply not possible to say, a priori, whether $Z_{\bar{\pi}}^i$ would be greater than $Z_{\bar{\pi}}^n$, or whether $Z_{\bar{\kappa}}^i$ would be greater than $Z_{\bar{\kappa}}^n$.

Finally, the consequences of an expansionary monetary policy on $\tilde{\pi}$ and $\tilde{\kappa}$ might be either to increase them or decrease them; it is simply not

possible to say what a given monetary policy would be likely to do to $\tilde{\pi}$ and $\tilde{R}$. In any event, we cannot even predict whether increases in $\tilde{\pi}$ or $\tilde{R}$ would be expansionary or contractionary (i.e., we cannot say what the signs of $Z_{\tilde{\pi}}$ or $Z_{\tilde{R}}$ would be); and there is certainly no way to determine, <u>a priori</u>, whether $Z^{i}_{\tilde{\pi}}$ would be greater than $Z^{n}_{\tilde{\pi}}$ or whether $Z^{i}_{\tilde{R}}$ would be greater than $Z^{n}_{\tilde{R}}$.

Thus, even if we must admit that the quantitative effects of monetary policy, through the channels of changes in the rate of national income and in the state of expectations and uncertainty, might be <u>different</u> in regimes with index-linked and with nominal bonds, we still cannot predict with any confidence in which regime monetary policy would be <u>more efficient</u>.

At this point it may help the reader if we provide an illustrative example of the kind of point that we are making in this section. Assume that in a given economy a short-term increase in the money supply always leads rapidly to an increase in the uncertainty attaching to the real rates of return on money and nominal bonds. Now consider the consequences of an open-market purchase of bonds with money by the monetary authority, first in a system with nominal bonds and then in a system with index-linked bonds. According to our analysis in the previous section, the impact effect of the purchase should be (roughly) the same in the two regimes. The value of outstanding bonds will be increased and their real rate of return reduced, and in both cases investors will prefer to move out of bonds and into equity, thus bidding up the supply price of capital. In view of our assumption, however, this expansionary monetary policy will also be accompanied by an increase in inflationary uncertainty, i.e., an increase in the uncertainty attaching to real rate of return on money. This increase in the uncertainty

attaching to the real rate of return on money will, in itself, cause shifts in the asset-demand functions in both systems. But where bonds are nominal, the uncertainty attaching to the real rate of return on bonds will be increased; in contrast, where bonds are index-linked, there will be no effect on the uncertainty attaching to the real rate of return on bonds. Consequently, there should be further shifts in asset demands out of nominal bonds, whereas there should not be any such shifts out of index-linked bonds. We cannot say, however, whether the shifts out of nominal bonds would diminish or augment the impact effect of the monetary policy. If the demand for money is increased, the impact effect of the monetary policy would be diminished; if the demand for equity is increased, the impact effect of the monetary policy would be augmented. In the former case, monetary policy would be more efficient with index-linked bonds; in the latter case, monetary policy would be more efficient with nominal bonds. We cannot say *a priori*.

Conclusion.

Our conclusion may be stated simply: there is no *a priori* reason to believe that monetary policy would be more efficient with index-linked bonds than it would be with nominal bonds. This rather stands to reason: an index-linked financial asset is simply a financial asset denominated in a constant-purchasing-power unit of account; thus, to introduce an index-linked financial asset in an economy is essentially like introducing a second unit of account into the economy along with the monetary unit.[4] As long as the rate of exchange of the two units of account is fixed and expected to remain unchanged, the use of a second unit makes no difference; consequently, our analysis finds no difference in the efficiency of

monetary policy between regimes with index-linked and nominal bonds. There will be differences between the two regimes if the monetary policy generates changes in the state of expectations and uncertainty--that is, if the monetary policy alters the expectation of and the uncertainty attaching to the exchange rates between the two units of account. But in this case we cannot predict in which regime monetary policy would be the stronger.

Footnotes.

1. The real rate of return on money is equal to the negative of the rate of inflation only if these two rates are understood to be the instantaneous rates. If the unit of time of these rates is longer than infinitesimal, they are not equal, and it is inappropriate to use the rate of inflation instead of the real rate of return on money in estimating the demand-for-money function (unless the rate of inflation is very low, in which case they are approximately equal). This is explained as follows. Let the unit of time run from t=0 to t=1, and let p_o and p_1 give the price level at these two instants. Let

$$\dot{p} = \frac{p_1 - p_o}{p_o}$$

give the percentage increase in the price level over the period, i.e., the rate of inflation over the period. For argument's sake, suppose that the price of apples is perfectly correlated with (i.e., faithfully accompanies) the price level through the period. If one dollar purchases an apple at time t=0, then $(1+\dot{p})$ dollars purchase one apple at time t=1; therefore, at time t=1, one apple purchases $1/(1+\dot{p})$ dollars. This means that, measured by its power to purchase apples, the real rate of return on one dollar held from t=0 to t=1 is given by

$$-\pi = \frac{\frac{1}{1+\dot{p}} - 1}{1} = -\frac{\dot{p}}{1+\dot{p}} = \frac{p_1 - p_o}{p_1}.$$

From the formula it is clear that π does not equal $\dot{p}$, though when $\dot{p}$ is very small they are approximately equal. More intuitively, the point here is that if there is, say, a 100% percent increase in the price level

over the period, a dollar held over the period loses only 50 percent, not 100 percent, of its value. Where an economic theory treats money as an asset whose real rate of return is compared by economic actors with the real rates of return offered by other assets, it is appropriate to use $-\overline{\pi}$, not $\dot{p}$, as the real rate of return on money. The real rate of return on money exactly equals minus the rate of inflation if these rates are understood to be the instantaneous rates: in this case, the price level at t=1 is given by

$$p_1 = p_o \lim_{n\to\infty} (1 + \frac{\dot{p}}{n})^{n(t_1 - t_o)} = p_o \, e^{\dot{p}(t_1 - t_o)} \quad ,$$

and the real value of one dollar at the end of the period is given by

$$1 \cdot e^{-\pi(t_1 - t_o)} = \frac{1}{1 + \frac{\Delta p}{p}}$$

$$= \frac{1}{1 + \frac{p_1 - p_o}{p_o}} = \frac{p_o}{p_1} = e^{-\dot{p}(t_1 - t_o)} \quad ,$$

hence $\pi = \dot{p}$.

2. If the nominal interest rate is i, the real rate of interest is given by $i(1-\pi) - \pi$, not by $i-\dot{p}$. The explanation for this is as follows. Consider a nominal bond whose principal is one dollar and which pays interest at a rate i (in nominal terms) for the period t=0 to t=1; the bond is assumed to be sold for one dollar at t=0 and redeemed at t=1 for (1+i) dollars. In terms of "t=0 dollars," the deflated value of the redemption payment is given by $(1+i)/(1+\dot{p})$. The real rate of interest, measured using t=0 dollars (the correct unit of measurement because presumably economic decisions

will be taken at time $t=0$ on the basis of the real rate of interest), is thus given by

$$(1+i)/(1+\mathring{p}) - 1 = (i-\mathring{p})/(1+\mathring{p}) = i(1-\pi) - \pi \ ;$$

in this formula, $i(1-\pi)$ represents the deflated value of the coupon of the bond, and $-\pi$ represents the loss of purchasing power suffered by the one-dollar principal of the bond. If there is no coupon, i.e., if the bond is simply discounted, the real rate of return on the bond is $i-\pi$. Any good or service whose unit of measure is the monetary unit suffers a loss in purchasing power of π times its initial value over the period $t=0$ to $t=1$.

3. There are certain conceptual problems concerning $\bar{\kappa}$. If we consider the asset-demand functions to have a one-period time-horizon, then $\bar{\kappa}$ represents the expected real rate of return on the real current replacement value of the capital stock, K, or

$$\kappa = \frac{y - wN}{K} ,$$

where w represents the <u>real</u> wage and N represents the quantity of labor employed. As is customary, we fudge the capital-measurement problem. A reader who prefers not to think of K as the replacement value of the capital stock should think of K in units of installed electrical capacity.

4. I am grateful to Professor Adroaldo Moura da Silva of the Universidade de São Paulo for háving convinced me of this point.

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Appendix A. Comparative-statics analysis of the models [(1n), (2n), (4n), (5n)] and [(1i), (2i), (4i), (5i)].

We take advantage of the fact that equations (1n) and (1i) and equations (2n) and (2i), respectively, are the same except for the changes from L^n to L^i and from C^n and C^i.

$$M = pL(y, \bar{\pi}, \tilde{\pi}, \bar{r}, \tilde{r}, \bar{\kappa}, \tilde{\kappa}) \cdot W; \tag{1}$$

$$qK = C(\bar{\pi}, \tilde{\pi}, \bar{r}, \tilde{r}, \bar{\kappa}, \tilde{\kappa}) \cdot W. \tag{2}$$

Fully differentiating (1) and (2):

$$dM = W\{Ldp+p[L_y dy+L_{\bar{\pi}} d\bar{\pi}+L_{\tilde{\pi}} d\tilde{\pi}+L_{\bar{r}} d\bar{r}+L_{\tilde{r}} d\tilde{r}+L_{\bar{\kappa}} d\bar{\kappa}+L_{\tilde{\kappa}} d\tilde{\kappa}]\}+pLdW; \tag{1'}$$

$$qdK+Kdq = W\{C_{\bar{\pi}} d\bar{\pi}+C_{\tilde{\pi}} d\tilde{\pi}+C_{\bar{r}} d\bar{r}+C_{\tilde{r}} d\tilde{r}+C_{\bar{\kappa}} d\bar{\kappa}+C_{\tilde{\kappa}} d\tilde{\kappa}\}+CdW. \tag{2'}$$

From (1'),

$$\begin{aligned} d\bar{r} = & -L_{\bar{r}}^{-1} L_y dy - L_{\bar{r}}^{-1} L p^{-1} dp \\ & -L_{\bar{r}}^{-1} L_{\bar{\pi}} d\bar{\pi} - L_{\bar{r}}^{-1} L_{\tilde{\pi}} d\tilde{\pi} - L_{\bar{r}}^{-1} L_{\tilde{r}} d\tilde{r} - L_{\bar{r}}^{-1} L_{\bar{\kappa}} d\bar{\kappa} - L_{\bar{r}}^{-1} L_{\tilde{\kappa}} d\tilde{\kappa} \\ & +W^{-1} L_{\bar{r}}^{-1} p^{-1} dM - L_{\bar{r}}^{-1} L W^{-1} dW. \end{aligned}$$

Substituting this expression for $d\bar{r}$ into (2'),

$$\begin{aligned} dq = & -WK^{-1} \, C_{\bar{r}} L_{\bar{r}}^{-1} L_y dy - WK^{-1} \, C_{\bar{r}} L_{\bar{r}}^{-1} p^{-1} dp \\ & +WK^{-1} \, [C_{\bar{\pi}} - C_{\bar{r}} L_{\bar{r}}^{-1} L_{\bar{\pi}}] \, d\bar{\pi} + WK^{-1} \, [C_{\tilde{\pi}} - C_{\bar{r}} L_{\bar{r}}^{-1} L_{\tilde{\pi}}] d\tilde{\pi} \\ & +WK^{-1} \, [C_{\tilde{r}} - C_{\bar{r}} L_{\bar{r}}^{-1} L_{\tilde{r}}] d\tilde{r} \end{aligned}$$

$$+WK^{-1}[C_{\underline{\bar{\kappa}}} - C_{\underline{r}}L_{\underline{r}}^{-1}L_{\underline{\bar{\kappa}}}]\,d\bar{\kappa} + WK^{-1}[C_{\tilde{\kappa}} - C_{\underline{r}}L_{\underline{r}}^{-1}L_{\tilde{\kappa}}]\,d\tilde{\kappa}$$

$$+K^{-1}C_{\underline{r}}L_{\underline{r}}^{-1}p^{-1}dM - qK^{-1}dK + K^{-1}[C - C_{\underline{r}}L_{\underline{r}}^{-1}L]\,dW.$$

To derive expressions (9n') and (9i') in the text, we now need only write in the appropriate superscripts and note that where bonds are nominal

$$d\tilde{r} = d\tilde{\pi}$$

while where bonds are index-linked

$$\tilde{r} = 0, \text{ hence } d\tilde{r} = 0.$$

Thus:
$$dq = WK^{-1}\{-C^n_{\underline{r}}(L^n_{\underline{r}})^{-1}L^n_y dy - C^n_{\underline{r}}(L^n_{\underline{r}})^{-1}p^{-1}dp$$

$$+ [C^n_{\underline{\bar{\pi}}} - C^n_{\underline{r}}(L^n_{\underline{r}})^{-1}L_{\underline{\bar{\pi}}}]\,d\bar{\pi}$$

$$+[(C^n_{\tilde{\pi}}+C^n_{\tilde{r}}) - C^n_{\underline{r}}(L^n_{\underline{r}})^{-1}(L^n_{\tilde{\pi}}+L^n_{\tilde{r}})]\,d\tilde{\pi}$$

$$+ [C^n_{\underline{\bar{\kappa}}} - C^n_{\underline{r}}(L^n_{\underline{r}})^{-1}L^n_{\underline{\bar{\kappa}}}]\,d\bar{\kappa}$$

$$+ [C^n_{\tilde{\kappa}} - C^n_{\underline{r}}(L^n_{\underline{r}})^{-1}L^n_{\tilde{\kappa}}]\,d\tilde{\kappa}\}$$

$$+ K^{-1}C^n_{\underline{r}}(L^n_{\underline{r}})^{-1}p^{-1}dM - qK^{-1}dK + K^{-1}[C^n - C^n_{\underline{r}}(L^n_{\underline{r}})^{-1}L^n]dW; \qquad (9n')$$

and
$$dq = WK^{-1}\{-C^i_{\underline{r}}(L^i_{\underline{r}})^{-1}L^i_y dy - C^i_{\underline{r}}(L^i_{\underline{r}})^{-1}p^{-1}\}dp$$

$$+ [C^i_{\underline{\bar{\pi}}} - C^i_{\underline{r}}(L^i_{\underline{r}})^{-1}L^i_{\underline{\bar{\pi}}}]\,d\bar{\pi}$$

$$+ [C^i_\pi - C^i_r (L^i_r)^{-1} L^i_\pi] \, d\pi$$

$$+ [C^i_{\bar{\kappa}} - C^i_r (L^i_r)^{-1} L^i_{\bar{\kappa}}] \, d\bar{\kappa}$$

$$+ [C^i_\kappa - C^i_r (L^i_r)^{-1} L^i_\kappa] \, d\kappa \}$$

$$+ K^{-1} C^i_r (L^i_r)^{-1} p^{-1} dM - qK^{-1} dK + K^{-1} [C^i - C^i_r (L^i_r)^{-1} L^i] \, dW. \qquad (91')$$

Appendix B. Demand-for-money and demand-for-capital functions based on the Mossin version of the capital-asset pricing model.

In his book Theory of Financial Markets Jan Mossin presented a well-known version of the "capital-asset pricing model." In this appendix, we describe how Mossin's results might be applied, using some special assumptions, to derive macro-economic demand-for-equity and demand-for-money functions incorporating expected real rates of return and the uncertainties attaching to these rates. We then indicate some properties of these functions that are relevant to the argument of the main part of this essay.

The basic Mossin capital-asset pricing model assumes a financial market with various investors and securities. It is a "portfolio" model in the sense that the investors are assumed to determine their security portfolios on the basis of a one-period time horizon. The model shows how the values of the securities are interdependently determined in a competitive securities market, and it allows uncertainty to be taken into account in a rudimentary way. The principal assumptions of Mossin's basic model are that (1) each investor i has the same expected-utility function, given by

$$U_i = f(\varepsilon_i, \sigma_i^2), \tag{B-1}$$

where ε_i is the expected final value of investor i's portfolio and σ_i^2 is the variance of the subjective probability distribution held by the investor regarding that final value; (2) all investors have the same perception of the expected return and uncertainty characterizing each security available; (i.e., all investors have the same subjective probability

distribution of the final value of each asset); (3) there are no taxes, and no impediments to free trade in securities; and (4) there exists a riskless asset, with investors and the firms issuing the securities able to borrow at the riskless rate of return.

Let v_j represent the market value of security j, r the riskless lending rate, μ_j the investors' expectation of the final value of security j, and σ_{jk}^2 the investors' perception of the covariance of the final value of security j with that of security k (if j = k, the investors' perception of the variance of security j). Mossin shows (pp. 64-76) that

$$v_j = \frac{1}{1 + r} \quad \{\mu_j - R\sum_k \sigma_{jk}^2\} \quad , \tag{B-2}$$

where R is a constant, is the first-order condition for the investors' utility maximization. That is, each security is valued at the discounted value of its expected final value, less the discounted value of the risk term $R\sum_k \sigma_{jk}^2$. (R may be interpreted as the market's implicit valuation of risk, while $\sum_k \sigma_{jk}^2$ gives the total covariance of security j with all the available securities.)

In one extension of the model, Mossin drops the assumption that there exists a riskless asset and lending rate. If the lending rate is now uncertain, with expected value $\bar{r}$, and the final value of the lending asset has covariance σ_{rk}^2 with each security j, Mossin shows (pp. 94-96) that

$$v_j = \frac{\mu_j - R\sum_k \sigma_{jk}^2}{(1+\bar{r}) - R\sum_k \sigma_{jk}^2} \tag{B-3}$$

is the first-order condition for investors' utility maximization.

Note that $\sigma_{jk}^{\ 2} = \rho_{jk}\sigma_j\sigma_k$, where ρ_{jk} is the correlation coefficient of the final values of securities j and k, and $\sigma_j^{\ 2}$ and $\sigma_k^{\ 2}$ are the respective variances of these final values.

To apply the capital-asset pricing model to the demand-for-money and demand-for-equity functions, we assume that money and equity are regarded by the public as risky, return-bearing securities. The public values these assets, M and K, by adjusting the price level, p, and the supply price of capital, q, i.e., by adjusting M/p and qK.

The expected value of money at the end of the period is taken to consist of two components: a non-stochastic component depending on y, the non-pecuniary return to money holdings consequent on the usefulness of money as a medium for transactions; and a stochastic component depending on the real rate of return on money, $-\pi$. The expected final value of money will be taken to be given by

$$[\theta(y) + (1 - \bar{\pi})]M, \quad \theta_y \geq 0 . \tag{B-4}$$

Let $\sigma_\pi^{\ 2}$ represent the variance of $1 - \pi$; let $\sigma_\kappa^{\ 2}$ represent the variance of $(1 + \kappa)$; and let $\sigma_{\pi\kappa}^{\ 2}$, $\sigma_{r\pi}^{\ 2}$, and $\sigma_{r\kappa}^{\ 2}$ represent the covariances of $(1 - \pi)$ and $(1 + \kappa)$; and $(1 + r)$ and $(1 - \pi)$; and $(1 + r)$ and $(1 + \kappa)$, respectively. We will take $\sigma_\pi^{\ 2}$ and $\sigma_\kappa^{\ 2}$ to be the equivalents of $\hat{\pi}$ and $\hat{\kappa}$ in the main part of the text. Note that

$$\sigma_{\pi\kappa}^{\ 2} = \rho_{\pi\kappa}\sigma_\pi\sigma_\kappa .$$

We will take the variance of the final value of the money stock to be $M^2\sigma_\pi^{\ 2}$, the variance of the final value of equity to be $K^2\sigma_\kappa^{\ 2}$, the covariance of the final value of money and equity to be $MK\rho_{\pi\kappa}\sigma_\pi\sigma_\kappa$. (If α and

β are constants and x and y are stochastic variables, $var(\alpha x) = \alpha^2 var\ x$ and $cov(\alpha x, \beta y) = \alpha\beta cov(x,y)$.)

Now if the bonds are nominal, the real lending rate is subject to uncertainty, and the asset-demand relations are derived using the extension of Mossin's model. Let $\bar{r}$ represent the expectation rate. We take $\sigma_r = \sigma_\pi$, so that

$$\sigma_{r\pi}^2 = \sigma_\pi^2$$

$$\text{and} \quad \sigma_{r\kappa} = \rho_{\pi\kappa}\sigma_\pi\sigma_\kappa.$$

The covariance of the final values of the bonds and money stocks is then given by σ_π^2, and the covariance of the final values of the bonds and equities stocks is $\rho_{\pi\kappa}\sigma_\pi\sigma_\kappa$. Let

$$Q = \bar{r} - R(\sigma_\pi^2 + \rho_{\pi\kappa}\sigma_\pi\sigma_\kappa). \tag{B-5}$$

Then, applying formula (B-3),

$$(M/p)^n = L^n(y, \bar{\pi}, \sigma_\pi, \bar{r}, \sigma_r, \bar{\kappa}, \sigma_\kappa)$$

$$= \frac{1}{1+Q}\{[\theta(y) + (1-\bar{\pi})]M - R[M^2\sigma_\pi^2 + MK\rho_{\pi\kappa}\sigma_\pi\sigma_\kappa]\}; \tag{B-6}$$

and

$$(qK)^n = C^n(\bar{\pi}, \sigma_\pi, \bar{r}, \sigma_r, \bar{\kappa}, \sigma_\kappa)$$

$$= \frac{1}{1+Q}\{(1+\bar{\kappa})K - R[K^2\sigma_\kappa^2 + MK\rho_{\pi\kappa}\sigma_\pi\sigma_\kappa]\}. \tag{B-7}$$

If the bonds are index-linked, on the other hand, the real lending rate r is certain, and the asset-demand relations can be derived from Mossin's basic model:

$$(M/p)^i = L^i(y, \bar{\pi}, \sigma_\pi, r, 0, \bar{\kappa}, \sigma_\kappa)$$

$$= \frac{1}{1+r}\{[\theta(y) + (1-\bar{\pi})]M - R[M^2\sigma_\pi^{\ 2} + MK\rho_{\pi\kappa}\sigma_\pi\sigma_\kappa]\}; \qquad \text{(B-8)}$$

and

$$(qK)^i = C^i(\bar{\pi}, \sigma_\pi, r, 0, \bar{\kappa}, \sigma_\kappa)$$

$$= \frac{1}{1+r}\{(1+\bar{\kappa})K - R[K^2\sigma_\kappa^{\ 2} + MK\rho_{\pi\kappa}\sigma_\pi\sigma_\kappa]\}. \qquad \text{(B-9)}$$

We now determine the values of $L^n_{\underline{r}}$, $C^n_{\underline{r}}$, L^i_r and C^i_r. Let

$$N^M = [\theta(y) + (1-\bar{\pi})]M - R[M^2\sigma_\pi^{\ 2} + MK\rho_{\pi\kappa}\sigma_\pi\sigma_\kappa]$$

and

$$N^K = (1+\bar{\kappa})K - R[K^2\sigma_\kappa^{\ 2} + MK\rho_{\pi\kappa}\sigma_\pi\sigma_\kappa].$$

Then

$$L^n_{\underline{r}} = -\frac{1}{(1+Q)^2}N^M, \qquad \text{(B-10)}$$

$$C^n_{\underline{r}} = -\frac{1}{(1+Q)^2}N^K, \qquad \text{(B-11)}$$

$$L^i_r = -\frac{1}{(1+r)^2}N^M, \qquad \text{(B-12)}$$

and $$C^i_r = -\frac{1}{(1+r)^2}N^K. \qquad \text{(B-13)}$$

Several conclusions of interest for the main part of the essay follow from expressions (B-10) - (B-13). First, if $|C^i_r|$ is greater than $|C^n_{\underline{r}}|$, $(1+r)^{-2}$ must be greater than $(1+Q)^{-2}$; but if $(1+r)^{-2}$ is greater than $(1+Q)^{-2}$, $|L^i_r|$ must be greater than $|L^n_{\underline{r}}|$. Hence, in this model,

$|L^i_r| \gtrless |L^n_{\bar{r}}|$ as $|C^i_r| \gtrless |C^n_{\bar{r}}|$. Note, by the way, that expression (B-5) implies that if $(1 + r)^{-2}$ is greater than $(1 + Q)^{-2}$, and if we consider beginning from equilibrium positions with $r = \bar{r}$, then Q must be greater than r, which requires as a necessary (but not even sufficient) condition that $\rho_{\pi K}$ be less than zero. That is, in order for $|C^i_r|$ to be greater than $|C^n_{\bar{r}}|$, it is necessary (but not sufficient) that the expected final values of equities and nominal bonds be negatively correlated.

Finally, it is clear from inspection of expressions (B-10) - (B - 13) that

$$\frac{C^i_r}{L^i_r} = \frac{C^n_{\bar{r}}}{L^n_{\bar{r}}}$$