FACULTY WORKING PAPER 92-0178

An Integrated Approach to Product Design and Process Selection

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ABSTRACT

The conventional approach to new product design involves a sequential flow of decisions. At the first step, the marketing function determines the product profile by specifying the required product attributes based upon consumer research. Manufacturing is then expected to determine the appropriate processes capable of delivering these attributes efficiently. In practice, such a sequential approach leads to substantial delays in product introduction and frequent product redesigns long after it has been introduced. Increasing competitive pressure necessitates an alternative, integrated approach in which the product design and the process selection decisions are made simultaneously.

This study models such an integrated approach for the objective of maximizing the producer's incremental profit as an integer program. Process selection is merged with product design by explicitly considering the alternative processes available, and the associated fixed and variable costs, for providing an attribute at a given level. The overall decision involves simultaneously determining the levels at which each attribute will be present in the new product as well as the processes used for providing these attribute levels. In addition, we treat product price as an extrinsic decision variable. In view of the problem complexity, we propose a heuristic solution method that decomposes the overall problem into the product design and the process selection subproblems; the solution algorithm essentially iterates between these two subproblems.

Computational studies indicate that the suggested algorithm performs effectively with respect to the optimal solution. Furthermore, the relative effectiveness of the integrated approach is seen to depend strongly on process related problem parameters. In particular, the integrated approach is increasingly superior as the producer faces more intensive competition, greater product complexity, and high fixed and variable process costs.

1 INTRODUCTION

Manufacturing and service firms need to introduce new products frequently in order to remain competitive. Product design involves the consideration of the needs and preferences of individual market segments, products offered by the competition, as well as the company's own existing products in view of the possible cannibalization. The marginal profitability of the new product clearly depends upon the fixed and variable production costs in addition to the incremental net revenue generated. Thus, the process selection decision plays an important role in determining the overall success of the new product. The traditional approach to product design and process selection, also known as "over the wall approach" and " relay race", involves a sequential flow of decisions. Marketing function determines the "product profile" by specifying the required product attributes. Subsequently, the production function determines the processes capable of delivering these attributes. While such a delineation of decisions appears to be logical conceptually, in practice it leads to substantial delays in product introduction, bureaucracy, high cost, and frequent product redesigns long after it has been introduced.

Increasing competitive pressure has necessitated a significant reduction in the lead time required for introducing new products. This trend has generated increasing interest in an integrated approach in which both product and process are designed concurrently. Such an integrated approach has been operationalized in a variety of ways such as simultaneous engineering (Evans 1988), concurrent engineering (Brazier and Leonard 1990), quality function deployment, house of quality (Hauser and Clausing 1988). In particular, the house of quality provides a good framework for an objective analysis as well as a tool for integrating the product design process. However, decisions on many design alternatives are still made without considering the entire system. While there is substantive empirical evidence suggesting the potential of the integrated approach, the literature on modeling this problem is relatively sparse.

The bulk of the existing literature on product design focuses exclusively on consumer preferences. Two approaches to this problem have been proposed – Multidimensional Scaling (MDS) and Conjoint Analysis. In the MDS approach, each customer's ideal product is represented as a point in a continuous multiattribute space whose dimensions are differentially weighted across customers. The relative preference of any customer for a given product is inversely related to the "distance" of the product from the customer's ideal point in this space. Product selection is either deterministic, first-choice or probabilistic with the probability of a particular product being chosen is related to its distance from the customer's ideal point. Product design optimization based on the MDS approach was introduced by Shocker and Srinivasan (1974), has since been pursued by several other researchers (see, for example, Zufreyden 1979, Hauser and Simmie 1981, Gavish, Horsky and Srikanth 1983, Sudharshan, May and Shocker 1987, and Eliashberg and Manrai 1989). The objectives commonly considered include maximizing incremental profit and maximizing market share. Albers (1976) and Sudharshan, May and Gruca (1988) extend this line of research to a product line.

In the typical Conjoint Analysis approach, which is the methodology used in this study, each attribute is specified at one of several potential discrete levels. In most formulations, the overall utility of the product for a given customer is given by the sum of the idiosyncratic *part worth* of the level at which each attribute is active. The bulk of this research employs a deterministic first-choice selection criterion. A customer chooses the product with highest utility; alternatively, if price is considered as an extrinsic attribute, the product which provides the maximum surplus – utility net of price, is selected. Optimal product design based on conjoint analysis was first developed by Zufreyden (1977) for the objective of maximizing market share which has since been extended by Green, Carroll and Goldberg (1981). Other objectives that have been studied in the context of conjoint analysis include maximizing seller's return (Green and Kreiger 1985, Kohli and Krishnamurthi 1987, Dobson and Kalish 1988, McBride and Zufreyden 1988) and maximizing buyer's welfare (Green and Kreiger 1985). In their recent work, Kohli and Sukumar (1990) extend the approach proposed by Kohli and Krishnamurthi (1987) to include all three objectives for introducing a product line.

Much of the existing research on product design does not explicitly consider the cost of providing the product. As Johnson (1974) notes, these studies "..assume that these versions (of product) are all feasible from a manufacturing and pricing standpoint, that we could produce any one of them, and we wish to choose the "best" version." A notable exception is Dobson and Kalish (1988) who consider both fixed and variable manufacturing costs of each product, and also consider price to be an extrinsic variable. They construct a general model for selecting a line of products from a prespecified set of candidate products, and construct a two stage heuristic to obtain the approximate solution. However, Dobson and Kalish do not explicitly consider the process selection decision. In imputing the manufacturing costs to a product, they assume, in effect, that this decision is made independently and the optimal process is used for each product.

The model developed in this study addresses the introduction of a *single* new product for the objective of maximizing the producer's incremental profit. The significant point of departure of this work from previous research on product design lies in integrating the product design and process selection decisions. Unlike the Dobson-Kalish model, we consider each attribute level explicitly, and build the optimal (or near-optimal) product profile directly from these levels. Process selection is integrated into the product design problem by explicitly considering the alternative processes available, and the associated fixed and variable costs, for providing an attribute at a given level. The overall decision involves simultaneously determining the levels at which each attribute will be present in the new product(s) as well as the processes used for providing these attribute levels. In addition, akin to the Dobson-Kalish model, we treat product price as an extrinsic decision variable.

The incorporation of manufacturing costs into the model enables us to consider the tradeoff between using the processes currently available within the company and buying new equipment. Because the new product is likely to share a number of common features with existing products (Srinivasan and Shocker 1979), the current manufacturing facility may be capable of providing certain attribute levels with negligible increase in fixed costs, while on the other hand, new equipment has the potential of reducing variable costs. Furthermore, the model provides a basis for evaluating the economic worth of process flexibility. For example, consider the decision of selecting between a (relatively inflexible) process that can provide only a limited number of attributes, and an alternative process capable of providing a wider range of attributes, albeit less efficiently. The outcome of this decision clearly depends upon the mix of attributes under consideration, as well as the demand volume generated by the customers who switch to the new product. Similarly, whether or not an attribute will be present at a given level depends upon the associated manufacturing cost, in addition to the additional revenue that it will generate.

The paper is organized as follows. §2 discusses the major characteristics and assumptions of the model. We formulate the problem as a nonlinear mixed integer program, and show that it is NP-hard. In §3, we present an iterative heuristic solution procedure that decomposes the problem into product design and process selection subproblems. The performance of this heuristic is evaluated in §4. In addition, we demonstrate the merit of adopting the integrated approach over the sequential approach to product design and process selection, and relate it to various model parameters. §5 summarizes the main results of this study. Proofs of mathematical results stated in the paper can be found in Appendix 1.

2 Problem Description

The model is based on the conjoint analysis approach to estimating consumer preferences. The product to be designed is comprised of K attributes. A given attribute $k \in \mathcal{K}$ is realized in the product at exactly one of J_k discrete levels. Some of these attributes can be options which occur at two levels – 'present' and 'absent'. The set of levels for attribute k is denoted by $\mathcal{J}_k$. A product profile is denoted by $\mathbf{X} = (X_1, \ldots, X_K)$, where $X_k = (x_{1k}, \ldots, x_{J_k k})^T$.

$\mathcal{I} = (1, 2, \ldots, I)$ denotes the set of customers. A given customer $i \in \mathcal{I}$ represents either an individual or a segment. The weight p_i is a measure of segment population and product purchase frequency for customer i. Let w_{ijk} denote the part worth of attribute k at level j for customer i. Consistent with the most frequently used formulations of conjoint analysis, we assume that the utility U_i of any product $\mathbf{X}$ for customer i, $i = 1, \ldots, I$ is the sum of the part worths of individual attributes; i. e., $U_i = \sigma_k \sigma_j w_{ijk} x_{jk}$. We also assume that the customer employs a deterministic first-choice selection criterion based on maximizing his/her surplus. Therefore, given the utility U_i, price π and surplus $u_i (= U_i - \pi)$ of the new product, customer i will switch if $u_i \geq u_i^0$ where u_i^0 refers to the surplus of the product currently purchased by customer i. [U_i^0 and π_i^0 are similarly defined.] l_i denotes the marginal contribution made by customer i to the firm; it is nonzero only if i is a customer of one of the existing products made by the firm.

The model developed in this study does not address the competitive reaction to product design decision made by the producer. Thus, the situations modeled here are that of no competition or passive competition in the sense of Dobson and Kalish (1988).

The set of candidate processes is denoted by $\mathcal{M}$, and $|\mathcal{M}| = M$. Each process $m \in \mathcal{M}$ has a fixed cost of f_m, and a variable cost of v_{jkm} for producing attribute k at level j. In this paper, a process denotes an entity that can deliver an attribute at a given level. Thus, a process can be a single workcenter, a manufacturing cell or even an entire plant; it could also denote a subcontractor. We assume that each process has unlimited capacity. While, as we discuss later in §3.2, it is possible to include these constraints in the overall solution approach, doing so introduces discontinuities that obscure the useful insights otherwise available from a basic model.

Aside from the assumptions made above, the model is quite general. No structure is imposed on the part worths which can be defined arbitrarily in the multiattribute space for the individual customers. The model allows for the cannibalization of existing customers by the new product. Similarly, in regard to the process, no restriction is placed on the values that the fixed and variable costs can take. The formulation also permits process flexibility, i. e., a given process can be used for producing more than one attribute.

The integrated product design and process selection problem (**PDPS**) is formulated as the following mathematical program:

PDPS

$$Z = \max \quad \sum_{i\in\mathcal{I}} (\pi p_i - l_i) s_i - \sum_{m\in\mathcal{M}} f_m q_m - \sum_{i\in\mathcal{I}} p_i s_i \left(\sum_{m\in\mathcal{M}} \sum_{k\in\mathcal{K}} \sum_{j\in\mathcal{J}_k} v_{jkm} z_{jkm} \right) \tag{1}$$

subject to

$$\sum_{k\in\mathcal{K}} \sum_{j\in\mathcal{J}_k} w_{ijk} x_{jk} - \pi - u_i^0 \le B s_i \quad \forall i \tag{2}$$

$$\pi - \sum_{k\in\mathcal{K}} \sum_{j\in\mathcal{J}_k} w_{ijk} x_{jk} + u_i^0 \le B(1 - s_i) \quad \forall i \tag{3}$$

$$\sum_{j\in\mathcal{J}_k} x_{jk} = 1 \quad \forall k \tag{4}$$

$$\sum_{m\in\mathcal{M}} z_{jkm} \ge x_{jk} \quad \forall j, k \tag{5}$$

$$z_{jkm} \le q_m \quad \forall j, k, m \tag{6}$$

$$\pi \ge 0; s_i, z_{jkm}, x_{jk}, q_m \in (0, 1) \quad \forall i, j, k, m \tag{7}$$

where $s_i = 1$ if customer i switches to the new product, 0 otherwise; $x_{jk} = 1$ if attribute k is provided at level j in the new product, 0 otherwise; $q_m = 1$ if process m is selected, 0 otherwise; and $z_{jkm} = 1$ if level j of attribute k is assigned to process m, 0 otherwise.

Expression (1) states the objective of maximizing the difference between net revenue – total revenue less loss due to cannibalization, and incremental fixed and variable costs. Disjunctive constraints (2) and (3) enforce the product selection criterion for each customer; the first term in (2) is the total utility U_i provided by the product to customer i. Constraint (4) requires each attribute to be specified at exactly one level. Constraints (5) and (6) guarantee that the selected level of each attribute is assigned to at least one process and the appropriate variable and fixed costs are taken into account. Finally, constraints (7) specify the nature of the variables.

This formulation can be used to answer questions such as:

- Should a new product be introduced at all?
- Is it better to buy a flexible machine that is capable of providing a number of attributes or should dedicated machines be purchased?
- Should a new option, favored by a large number of customers, be offered even if it requires significant fixed expenditure?
- If a company is thinking about expanding its current line of products, which of the attribute levels currently being offered should be duplicated in order to minimize incremental fixed costs?

The following result indicates that it is unlikely that an efficient procedure for solving this problem exactly can be constructed.

Remark 1. **PDPS** *is NP-hard in the strong sense.*

In view of the computational complexity of **PDPS**, we construct a heuristic solution approach for solving it.

3 Heuristic Solution Procedure

The formulation of **PDPS** given by (1)–(7) indicates that it comprises two subproblems: (1)–(5) and (7) define the problem of determining attribute levels, while (1), (5)–(7) specify the problem of determining the processes. (5) provides the coupling constraints between these two subproblems; also, variables s_i appear in both subproblems. The proposed heuristic solution procedure exploits this structure by decomposing **PDPS** into the subproblems which are solved iteratively.

3.1 The Basic Algorithm

A brief statement of the algorithm is given below.

Algorithm **BasicDesign**

Step 1: Initialization: Determine initial product profile $\mathbf{X}^1$ by selecting appropriate values of $x_{jk}, k = 1, \ldots, K,\ \ j = 1, \ldots, J_k$. Set $n = 1, Z^0 = 0$.

Step 2: Process Selection Problem: For the given product $\mathbf{X}^n$ at iteration n, determine the optimal set of processes, the optimal set of switching customers and the optimal price. Let the set of processes selected be $\mathcal{M}^n$. Determine the profit Z^n corresponding to $\mathbf{X}^n$ and $\mathcal{M}^n$. Stop if $(Z^n - Z^{n-1})/Z^n \leq \epsilon$. Else, set $n \leftarrow n+1$, and go to step 3.

Step 3: Attribute-Level Selection Problem: For the given set of processes $\mathcal{M}^{n-1}$, find the best product profile $\mathbf{X}^n$. Stop if $\mathbf{X}^n = \mathbf{X}^{n-1}$. Else, go to step 2.

The initial product profile can be determined in one of several ways. For example, any one of the existing products can serve as the starting profile. An approach to obtaining the initial solution that we found to be particularly effective in our computational studies was to solve the product design problem given in step 3 with all processes being available, i. e., $\mathcal{M}^0 = \mathcal{M}$. However, the performance of the algorithm depends more critically on solving Steps 2 and 3 efficiently. The solution approaches to these two problems are now discussed.

3.2 The Process Selection Problem

Consider the problem of determining the optimal set of processes for a given product $\mathbf{X}$. Let j_k denote the level at which attribute $k, k = 1, 2, \ldots, K$ is active in this product. Its utility for customer i is $U_i = \sum_{k \in \mathcal{K}} w_{ij_k k}$. The resulting problem is

PSP

$$Z(\mathbf{X}) = \max \sum_{i \in \mathcal{I}} (\pi p_i - l_i) s_i - \sum_{m \in \mathcal{M}} f_m q_m - \sum_{i \in \mathcal{I}} p_i s_i \left(\sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}_k} v_{jkm} z_{jkm} \right)$$

subject to

$$U_i - \pi - u_i^0 \leq B s_i \quad \forall i \tag{8}$$

$$\pi - U_i + u_i^0 \leq B(1 - s_i) \quad \forall i \tag{9}$$

$$\sum_{m \in \mathcal{M}} z_{j_k k m} \geq 1 \quad \forall k \tag{10}$$

$$z_{j_k k m} \leq q_m \quad \forall k, m \tag{11}$$

$$\pi \geq 0; s_i, z_{j_k k m}, q_m \in (0, 1) \quad \forall i, k, m \tag{12}$$

Now consider a restricted version of (**PSP**) with $\pi = \pi'$. The set of customers $\mathcal{I}_{\pi'}$ who switch to the new product is

$$\mathcal{I}_{\pi'} = \{i | U_i - u_i \geq \pi'\}. \tag{13}$$

Let $P_{\pi'} = \sum_{i \in \mathcal{I}_{\pi'}} p_i$ represent the total population of the switching customers. The restricted problem can be written as

RPSP

$$Z(\mathbf{X}, \pi') = \max \quad \left[\pi' P_{\pi'} - \sum_{i \in \mathcal{I}_{\pi'}} l_i\right] - \sum_{m \in \mathcal{M}} f_m q_m - P_{\pi'} \left(\sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} v_{j_k k m} z_{j_k k m} \right)$$

subject to

$$(10),\ (11),\ \text{and}\ \ z_{j_k k m}, q_m \in \{0, 1\} \quad \forall k, m.$$

The first term in the objective is a constant with respect to the problem variables; it can, therefore, be ignored for the purpose of finding the optimal solution. The remaining problem has the structure of the uncapacitated facility location problem (UFL) if we treat attribute levels j_k as the unit demand points and the processes m as the candidate location sites. f_m is the fixed cost of opening a facility at location m, and $P_{\pi'} v_{j_k k m}$ is the cost of satisfying the demand at j_k from location m. Although UFL is NP-hard, efficient solution procedures are available for obtaining strong bounds and for solving reasonably large problems within acceptable computation time. In our computational study, we use the dual ascent procedure DUALOC developed by Erlenkotter (1978).

Let $Z'(\mathbf{X}, \pi')$ denote the optimal solution to UFL, then

$$Z(\mathbf{X}, \pi') = \pi' P_{\pi'} - \sum_{i \in \mathcal{I}_{\pi'}} l_i - Z'(\mathbf{X}, \pi').$$

The solution to **PSP** is given by

$$Z(\mathbf{X}) = max_{\pi} Z(\mathbf{X}, \pi) = Z(\mathbf{X}, \pi^*).$$

Let $\delta_i = U_i - u_i^0$. The following result indicates that the search for π^* involves considering at most I values of π.

Proposition 1. $\pi^* \in \{\delta_1, \delta_2, \ldots, \delta_I\}$.

Consequently, at each iteration, we need to solve at most I uncapacitated facility location problems in order to determine the optimal set of processes. As shown in the proof of Proposition 1, this step generates the best price and the set of switching customers as well.

If we impose capacity limitations on individual processes then (11) is replaced by constraints of the form

$$P_{\pi'} z_{j_k km} \leq CAP_m q_m \quad \forall k, m$$

where CAP_m is the capacity of process m. In this case, the solution approach to **PSP** essentially requires solving a number of capacitated facility location (CFL) problems, instead of the UFL problems considered here. While CFL is harder to solve optimally, several heuristic solution methods exist for it so that from an algorithmic standpoint, capacity constraints can be included within the solution approach.

3.3 The Attribute-Level Selection Problem

Now consider the problem for determining the optimal set of product attributes given a set of processes $\mathcal{M}^0 = \{m | q_m = 1\}$. Let

$$f(\mathcal{M}^0) = \sum_{m \in \mathcal{M}^0} f_m, \text{ and}$$

$$v'_{jk} = min_{m \in \mathcal{M}^0} \{v_{jkm}\} = v_{jkm'}.$$

Clearly, for a given (j, k), z_{jkm} equals 1 for $m = m'$, and equals 0 for all other m.

The attribute-level selection problem can now be stated as

ASP

$$Z(\mathcal{M}^0) = \max \sum_{i \in \mathcal{I}} \left[\left(\pi - \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}_k} v'_{jk} x_{jk} \right) p_i s_i - l_i s_i \right] - f(\mathcal{M}^0)$$

subject to

$$(2), (3)\ (4), \text{ and } s_i, x_{jk} \in \{0, 1\} \quad \forall i, j, k.$$

ASP generalizes the model proposed by Kohli and Sukumar (1990) for the seller's return problem to include product price as an extrinsic decision variable. We construct a heuristic solution method that extends the Kohli-Sukumar algorithm to incorporate this generalization. This procedure uses a dynamic programming like approach in which each attribute is treated as a *stage*, and individual attribute levels are the *states*. Starting with attribute 1, this method successively performs a local optimization with respect to adjacent attributes that is followed by attribute augmentation. Specifically, at step k of the procedure, only attributes k and $k + 1$ are considered. Let $J_t = |\mathcal{J}_t|, \ t = 1, \ldots, K$. For a level $j \in \mathcal{J}_{k+1}$, J_k partial product profiles are constructed by associating each $r \in \mathcal{J}_k$ with j. Of these J_k profiles, the profile (r_j^*, j) that maximizes profit is selected, and r_j^* is merged into j. The next iterative step involving attributes $k+1$ and $k+2$ is then solved with respect to these augmented attribute levels for attribute $k+1$. Thus the heuristic forms partial product profiles of increasing cardinality in attribute space, with the profile at the end of stage k consisting of the first $k+1$ attributes. A formal description of the procedure now follows.

Algorithm **BuildProduct**

Step 1: Initialization: For each customer i, determine the "partial price" of attribute k as

$$\theta_{ik} = \pi_i^0 U_{ik}^0 / U_i^0.$$

where U_{ik}^0 is the part worth of attribute k to customer i in the product that i buys currently. Set $k = 1$.

Step 2: Recursion:

a. Local Optimization: At step k, for each level of attribute $k+1$, determine the level r_j^* of attribute k that maximizes the overall contribution.

b. Attribute Augmentation: Set

$$\begin{aligned} w_{ij,k+1} &= w_{ij,k+1} + w_{ir_j^*k} \\ v'_{j,k+1} &= v'_{j,k+1} + v_{r_j^*k} \\ \theta_{i,k+1} &= \theta_{i,k+1} + \theta_{ik}. \end{aligned}$$

Step 3: Termination: If $k \leq K-1$, set $k = k+1$, and go to step 2. Else, determine the optimal level j_K^* for attribute K. Set $x_{jk} = 1$ if $j = j_K^*$; $x_{jk} = 0$, otherwise. Do a backward pass; for $k = K-1, \ldots, 1$, set $x_{rk} = 1$ if $r = r_j^*$ and $x_{j,k+1} = 1$; $x_{rk} = 0$, otherwise.

The major step in this algorithm involves local optimization at each stage. For a given level j of attribute $k+1$, this problem can be formulated as:

P1$(j, k+1)$

$$r_j^* = arg\ max_{r \in \mathcal{J}_k} \sum_{i \in \mathcal{I}} \left(p_i(\pi - v'_{j,k+1} - v'_{rk}) - l_i \right) s_i \tag{14}$$

subject to

$$w_{ij,k+1} + w_{irk} - \pi - \left(U_{i,k+1}^0 + U_{ik}^0 - (\theta_{ik} - \theta_{i,k+1} \right) \leq B s_i \tag{15}$$

$$\pi - (w_{ij,k+1} + w_{irk}) + \left(U^0_{i,k+1} + U^0_{ik} - (\theta_{ik} - \theta_{i,k+1}\right) \le B(1 - s_i) \tag{16}$$

$$s_i \in \{0, 1\} \ \forall i; \ \ \pi \ge 0.$$

Equation (14) enforces local optimization with respect to attributes k and $k+1$, while equations (15) and (16) insure that customer i will switch to the partial product if and only if it provides a higher surplus than the surplus offered by the current partial product.

To solve $\mathbf{P1}(j, k+1)$, we first find the optimal price π^*_{rj} corresponding to each $r \in \mathcal{J}_k$. The maximum price that customer i is willing to pay to switch to the partial product defined by levels r and j for attributes k and $k+1$, respectively, is given by

$$e^i_{rj} = w_{ij,k+1} + w_{irk} - [U^0_{i,k+1} + U^0_{ik} - \theta_{ik} - \theta_{i,k+1}].$$

The profit corresponding to a price of e^i_{rj} is given by

$$E_{ir} = \sum_{g \in \mathcal{G}} [p_g(e^i_{rj} - v'_{j,k+1} - v'_{rj}) - l_g]$$

where $\mathcal{G} = \{g | g \in \mathcal{I}, e^g_{rj} > e^i_{rj}\}$. Arguments similar to those used in the proof of Proposition 1 yield

$$\pi^*_{rj} \in \{e^1_{rj}, \cdots, e^I_{rj}\}.$$

Consequently, the maximal profit corresponding to level r of attribute k is given by

$$E^*_{rj} = max_{i \in \mathcal{I}}\{E_{ir}\}.$$

$\mathbf{P1}(j, k+1)$ is then solved by

$$r^*_j = arg \ \ max_{r \in \mathcal{J}_k}\{E^*_{rj}\}.$$

At step 3 of **BuildProduct**, the optimal attribute level is given by

$$j^*_K = arg \ \ max_{j \in \mathcal{J}_K}\{\overline{E}_j\}, \ \ \text{where} \ \ \overline{E}_j = E^*_{r^*_j j}.$$

Thus algorithm **BasicDesign** essentially iterates between the product and the process spaces, thereby dealing separately with problems of reduced dimensionality. While it is difficult to analyze its convergence properties theoretically, in all our computational experiments, it was found to converge quite rapidly. However, there are instances in which it may converge to a solution that is only locally optimal. In order to overcome this drawback, we imbed this algorithm within a simulated annealing based approach. This enhancement is now described.

3.4 Algorithm Refinement Based on Simulated Annealing

Simulated annealing has been successfully employed for solving some difficult problems such as graph partitioning (Johnson et al. 1989), pattern recognition (Geman and Geman 1984), VLSI design (Kirkpatrick, Gelatt and Vecchi 1983), single machine minimum tardiness schedule (Matsuo, Suh and Sullivan 1987a), job shop minimum makespan (Matsuo, Suh and Sullivan 1987b) and operation partitioning (Ahmadi and Tang 1991). We give below a brief description of this approach. For details, the interested reader is referred to Kirkpatrick et al. (1983), and Johnson et al. (1989).

Consider the following problem:

$$\max\ h(y), \quad \text{subject to } y \in \mathcal{Y}.$$

where $h(y)$ is a real-valued function on domain $\mathcal{Y}$. Each element $y \in \mathcal{Y}$ has an associated set of *neighbors* $\mathcal{N}(y)$ such that any element $y' \in \mathcal{N}(y)$ can be reached from y by a one-step perturbation. A typical ascent algorithm based on neighborhood search moves from a given y to its neighbor y' if $h(y') > h(y)$. If there is no such neighbor, then the algorithm terminates having returned y as the best solution. If $h(y)$ is not concave, such an algorithm will frequently yield solutions that are only locally optimal.

In contrast, simulated annealing permits occasional *downhill* moves, and thereby, provides a mechanism to escape from a local optima. Under simulated annealing, the probability that a move will be made from y to y' is given by

$$\alpha = exp[-\{h(y) - h(y')\}^+/\mathrm{TEMP}(g)]$$

where $[t]^+$ denotes $max(0, t)$, and $\mathrm{TEMP}(g)$ is a positive number. Thus, a profitable move will always be accepted; however, there is a nonzero probability of accepting an unprofitable move as well. α is a function of the difference in the objective function values at y and y', as well as the *temperature* $\mathrm{TEMP}(g)$ at a given *state* g. A simulated annealing algorithm goes through several states of gradually decreasing temperatures. A commonly-used sequence of temperatures follows a geometric series given by $\mathrm{TEMP}(g) = r * \mathrm{TEMP}(g-1)$ where r is the *cooling rate* such that $0 < r < 1$. Clearly, the probability of accepting a downhill move decreases with a reduction in temperature. The system is deemed to be *frozen* if there is virtually no change in the solution value at successive temperatures. Under certain assumptions, it is possible to show (Anily and Federgruen 1985; Geman and Geman 1984) the existence of certain values of r that guarantee that simulated annealing will obtain the optimal solution in the limiting case. Although it is conjectured that the rate of convergence may be slow, this approach has yielded good solutions for many practical problems within reasonable computation time.

For **PDPS**, we define the neighborhood of a given solution individually for the process and the product spaces. In the process space, the neighborhood $\mathcal{N}_1(\mathbf{Q}^r)$ of a given point $\mathbf{Q}^r = (q_1^r, \ldots, q_M^r)$ consists of all points $\mathbf{Q}^s = (q_1^s, \ldots, q_M^s)$ such that

$$\begin{aligned} q_m^s &= q_m^r \quad \text{for } m = 1, \ldots, M; \;\; m \neq n, \;\; and, \\ q_n^s &= 1 - q_n^r. \end{aligned}$$

It can be seen that the neighborhood of $\mathbf{Q}^r$ consists of M points. Similarly, the neighborhood $\mathcal{N}_2(\mathbf{X}^r)$ of a point $\mathbf{X}^r = (X_1^r, \ldots, X_K^r)$ in the product space comprises all points $\mathbf{X}^s$ such that

$$\begin{aligned} X_k^s &= X_k^r \quad \text{for } k = 1, \ldots, K \;\; k \neq n, \\ x_{jn}^s &= x_{jn}^r \quad \text{for } j = 1, \ldots, J_k \;\; j \neq g, \;\; and, \\ x_{gn}^s &= 1 - x_{gn}^r. \end{aligned}$$

It can be seen that the neighborhood of $\mathbf{X}^r$ consists of $\sum_{k=1}^{K}(J_k - 1)$ points. For a given pair $(\mathbf{X}^r, \mathbf{Q}^s)$ of points from the product and the process spaces, the optimum objective function value $Z(\mathbf{Y}^{rs})$ can be obtained as follows. For each (j, k) such that $x_{jk}^r = 1$, set $z_{jkm'} = 1$ where $m' = arg\ min_m\{v_{jkm} | q_m^s = 1\}$ is the machine with the minimum variable cost for producing attribute k at level j among all the open machines, and set all other $z_{jkm} = 0$. Next use the approach given in §3.2 to determine the optimal values of s_i and π. It follows, therefore, that a solution $\mathbf{Y}$ to **PDPS** is completely determined by the selection of the product profile and the set of open machines.

The parameters required for implementing this algorithm are the starting temperature INITTEMP; the cooling rate r; the termination threshold ϵ; and the maximum values N_1 and N_2 that the counters which control the number of neighbors scanned can take. A formal statement of the simulated annealing algorithm is given below. The various steps are described in detail subsequently. In the following, $Z(\cdot)$ represents the optimal objective function corresponding to solution $(\cdot)$, $\mathbf{Y}^{inc}$ denotes the incumbent solution, and Z_g is the best solution value obtained until temperature g.

Algorithm **IntegratedDesign**

Step 1: Initialization

Set the initial solution $\mathbf{Y}^0 = (\mathbf{X}^0, \mathbf{Q}^0)$ to the solution obtained from algorithm **BasicDesign**. Set the incumbent solution $\mathbf{Y}^{inc} = \mathbf{Y}^0$. Also set TEMP(0)=INITTEMP; $n_1 = 0$; $g = 0$; and $Z_0 = Z(\mathbf{Y}^0)$. Go to Step 2.

Step 2: Annealing

If $n_1 \geq N_1$, go to Step 5. Else, set $g = g - 1$; $\text{TEMP}(g) = r{*}\text{TEMP}(g+1)$; and $n_2 = 0$. Go to Step 3.

Step 3: Neighborhood Scan

a) Set $n_2 \leftarrow n_2 + 1$. If $n_2 < N_2$, fix $m = 1$, and go to Step 3b). Otherwise, set $Z_g = Z(\mathbf{Y}^{inc})$. If $Z_g \leq (1 + \epsilon)Z_{g-1}$, then set $n_1 \leftarrow n_1 + 1$. Else, set $n_1 = 0$. Go to Step 2.

b) If $m > M$, go to Step 3a). Else, determine the neighbor $\mathbf{Q}^1$ of $\mathbf{Q}^0$ by setting $q_m^1 = 1 - q_m^0$. Determine $\mathbf{X}^1$ and $\mathbf{Y}^1$. Go to Step 4.

Step 4: Neighbor Selection

If $Z(\mathbf{Y}^1) > Z(\mathbf{Y}^0)$, set $\mathbf{Y}^0 = \mathbf{Y}^{inc} = \mathbf{Y}^1$, set $m \leftarrow m + 1$, and go to step 3b). Otherwise, compute

$$\alpha = exp[-\{Z(\mathbf{Y}^1) - Z(\mathbf{Y}^0)\}/\text{TEMP}(g)].$$

Generate a random number a from the uniform distribution [0,1]. Set $\mathbf{Y}^0 = \mathbf{Y}^1$ if $\alpha \geq a$. Set $m \leftarrow m + 1$. Go to step 3b).

Step 5: Reoptimization

Determine the optimal solution $\mathbf{Y}^*$ corresponding to the incumbent product profile $\mathbf{X}^{inc}$.

The algorithm is initialized with the solution obtained from **BasicDesign** in Step 1. Computational experience (see, for example, Johnson et al. 1989, and Ahmadi and Tang 1981) indicates that good starting solutions usually enhance the performance of simulated annealing. Step 2 implements the specified cooling schedule; it also determines whether the frozen state is reached. This algorithm maintains a counter n_1 that is incremented by one each time the incumbent solution value obtained at the end of the neighborhood search at any temperature does not exceed the incumbent solution value at the previous temperature by the threshold ϵ. The system is deemed to be frozen if the counter value equals N_1.

At each temperature, Step 3 controls the generation of neighbors. This is done by considering the neighborhood of the partial solution $\mathbf{Q}^0$ through N_2 cycles. In any cycle, the algorithm generates each of the M neighbors of $\mathbf{Q}^0$ in turn. For any neighbor $\mathbf{Q}^1$, the approach given in §3.3 is used to determine the corresponding product profile $\mathbf{X}^1$. Subsequently, $\mathbf{Y}^1$ is determined from $\mathbf{Q}^1$ and $\mathbf{X}^1$. Step 4 in the algorithm executes the logic pertaining to the acceptance of a neighboring solution, and if necessary, updates the current and the incumbent solutions. The solution obtained at the end of simulated annealing is reoptimized at Step 6. This is done by retaining the incumbent product profile $\mathbf{X}^{inc}$, and solving the process selection problem following the procedure given in §3.2 to determine the optimal processes, the optimal price and the optimal set of switching customers.

Several parameter values were tested for implementing the simulated annealing phase of algorithm **IntegratedDesign**. The values actually used in our computational study were $r = 0.90$; $N_1 = 5$; $N_2 = 10$; and INITTEMP$= 0.01 * Z_0$ where Z_0 is the solution value obtained from **BasicDesign**.

4 Computational Experience

We conduct two sets of computational experiments. The first set addresses the performance of algorithm **IntegratedDesign** with respect to the optimal solution, while the second set evaluates the relative merit of adopting the integrated approach over the sequential approach as a function of various problem parameters.

4.1 Experimental Details

In order to better focus on the impact of *process* characteristics, we keep certain customer-specific parameters fixed across all experiments. For example, the part worths w_{ijk} are generated initially through random sampling from the uniform distribution [60, 340]; the realized values are retained in all experiments. Similarly, in the second set of experiments, the total number of customers I is fixed at 20 (I is varied in the first set merely to generate a larger number of problem scenarios). Customer populations are sampled from the uniform distribution [200, 600] and the resulting values remain fixed subsequently. In all cases, the number of levels is the same for all attributes, i.e., $|\mathcal{J}_k| = \eta, \ \forall k$. Also, the marginal contributions $l_i = 0, \ \forall i$.

In each problem instance, we generate $I/5$ competing products; the active attribute levels for these products are assigned randomly. These products are assigned prices sampled from the uniform distribution $[aU_{ave}, bU_{ave}]$, where U_{ave} is a measure of the average utility across all customers and across all products currently available. $U_{ave} = Kw_{ave}$, where w_{ave} is the mean realized part worth given by

$$w_{ave} = \frac{\sum_i \sum_k \sum_j w_{ijk}}{IK\eta}$$

and K and η are, respectively, the number of attributes and the number of attribute levels considered in the problem instance. The best product and the resulting surplus u_i^0 for any customer i are determined from the profiles of the competing products and their prices, as well as the part worths w_{ijk}. We vary a and b in our experiments to yield various values of the mean price of competing products $\pi_{ave} = U_{ave}(a + b)/2$.

The problem parameters of interest are i) the number of attributes K, ii) the number of levels in each attribute η, iii) the number of processes M, iv) the mean process fixed cost μ_f, v) the coefficient of variation of process fixed cost ζ_f, vi) the coefficient of variation of variable cost ζ_v, vii) the ratio $\rho = \mu_v/w_{ave}$ of the mean variable cost to the mean part worth, and viii) the ratio π_{ave}/U_{ave} which is a measure of the surplus enjoyed by the customers currently. The base values of these parameters are given in Appendix 2.

The actual fixed costs f_m used for the various processes are obtained by sampling from a uniform distribution which has mean μ_f, and for which the upper and lower limits are derived from μ_f and ζ_f. Similarly, the actual variable costs v_{jkm} are obtained from a uniform distribution determined by μ_v and ζ_v.

The first experiment considers 27 different scenarios comprising 3 values of the number of segments, and 9 combinations of the number of attributes and the number of attribute levels as shown in Table 1. The other problem parameters are retained at their base values indicated in Appendix 2. 10 instances of each scenario are generated. For a given instance, the optimal solution value Z_{opt} is obtained by first explicitly enumerating all η^K profiles. For each of these profiles, the process selection problem is solved to determine the optimal set of processes and the optimal price corresponding to that profile, and consequently the profit value as well. The optimal solution for the overall problem is the best profit value across all these product profiles. For each problem instance, we compute the performance ratio $PR_{int} = (Z_{opt} - Z_{int})/(Z_{opt})$, where Z_{int} is the solution value obtained from **IntegratedDesign**. Table 1 reports the average value of PR_{int} across all 10 instances; also reported in parentheses is the number of times the optimal solution was obtained in these 10 instances.

For comparison purposes, we also report the corresponding performance ratio PR_{seq} for the sequential approach, where $PR_{seq} = (Z_{opt} - Z_{seq})/(Z_{opt})$ and Z_{seq} is the sequential solution value. In any problem instance, the sequential solution is obtained by first determining the product profile using algorithm **BuildProduct** ignoring the attribute level variable costs v_{jkm}. These costs are taken into account at the subsequent stage when we solve the process selection problem to yield the optimal set of processes as well as the optimal price. It can be seen that, while the sequential approach performs limited optimization with respect to the processes and the product price for the selected product profile, it lacks the integrated approach's ability to revise this profile subsequently.

The exponential growth in the number of product profiles to be generated explicitly limits the size of problems that can be considered in the first set of experiments. An alternative approach of evaluating the performance of **IntegratedDesign** through the use of upper bounds, based on Lagrangean relaxation, did not yield satisfactory results. We found that these bounds were quite weak even for small problems. The major reason for the weakness of these bounds appears to be the fact that the attribute-level selection problem has very little structure; it is essentially a general integer program.

The second set of experiments examines the impact of eight parameters, which are varied one at a time while the others are retained at their base values, on the relative performance of the integrated approach vis-a-vis the sequential approach. Algorithm **IntegratedDesign** is used for generating the integrated solution value while the sequential solution value is determined in the manner described above. Each of M, K and η are considered at the seven levels of 3, 5, 7, 9, 11, 13, 15. Similarly, ζ_f and ζ_v are tested at levels of 0.08, 0.16, 0.24, 0.32, 0.40, 0.48 and 0.56. [Note that the largest value that the coefficient of variation of a uniformly distributed nonnegative random variable can take is 0.58.] μ_f is considered at levels \$60000, \$120000, \$180000, \$240000, \$300000, \$360000, and \$420000. Mean fixed cost values higher than \$420000 result in many solutions with zero objective function values; consequently, they are not considered. We test ρ at the seven levels of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7. Finally, we generate problems for seven values of the ratio $\pi_{ave}/(U_{ave})$ – 0.60, 0.65, 0.70, 0.75, 0.80, 0.85 and 0.90.

The results of the second set of experiments are shown in Tables 2 through 9. As in the first set, 10 problem instances are generated randomly for each scenario. For reporting purposes, we take the average of the solution values Z_{seq} and Z_{int} across these instances under the sequential and integrated approaches, respectively. We use the measures $PR = Z_{int}/Z_{seq}$ and $\Delta = Z_{int} - Z_{seq}$ to evaluate the relative performance of the integrated approach. In addition, we also record the average optimum product prices π_{int} and π_{seq} obtained under the two approaches in each problem scenario.

4.2 Experimental Results

Table 1 indicates that algorithm **IntegratedDesign** frequently finds the optimal solution. Across the 27 scenarios and the 270 problem instances, it is 1% away from the optimal solution on average. More importantly, its performance does not appear to be data-dependent; the worst performance ratio for the integrated solution is 6.5% (averaged over 10 problems). It is also seen that, even for these small problems, the integrated approach yields substantially superior results to the sequential approach which is on average 16% away from the optimal solution.

INSERT TABLE 1 HERE

Tables 2 and 3 deal, respectively, with the impact of the mean variable cost and the mean fixed cost on solution values. As seen in Table 2, while both Z_{int} and Z_{seq} decrease with an increase in ρ $(= \mu_v/w_{ave})$, Z_{int} decreases more slowly. Consequently, except at very high values of ρ, ΔZ increases leading to an exponential increase in PR. Furthermore, π_{int} is more sensitive to variations in ρ. A similar result is obtained for μ_f as indicated in Table 3 although in this case, the solution values as well as PR are less sensitive to changes in μ_f.

INSERT TABLES 2 AND 3 HERE

Tables 4 and 5 deal with the impact of the coefficient of variation in the fixed and the variable costs ζ_f and ζ_v, respectively. In each case, both Z_{int} and Z_{seq} increase with an increase in the parameter value. Recall that both approaches find the optimal set of processes and the optimal price, albeit for (possibly) different product profiles. As ζ_v (ζ_f) increases, the probability of finding processes with lower variable (fixed) costs to manufacture the profile selected by the sequential approach increases; this, in turn, results in higher profit values. The integrated approach is similarly benefitted, so that in Table 4, ΔZ does not change appreciably with ζ_v. However, as Tables 5 shows, ΔZ tends to decrease with an increase in ζ_f. Also, any change in the value of ζ_v has a greater impact on the Z values relative to a similar change in ζ_f.

INSERT TABLES 4 AND 5 HERE

Table 6 considers the number of candidate processes M. Both Z_{seq} and Z_{int} increase with an increase in M which is attributable to the fact that as M increases, there is a increasing likelihood of finding processes with lower cost structures. Note that, while both Z_{seq} and Z_{int} attenuate with M, the incremental benefits of increasing the number of candidate processes are greater for the sequential approach which results in a decrease in both ΔZ and PR as M increases. Nonetheless, the initial difference in the profits under these two approaches is large enough in that the sequential approach requires much larger values of M to attain comparable profit levels.

INSERT TABLE 6 HERE

As seen in Table 7, the integrated approach is increasingly superior with an increase in the number of attributes, and the incremental benefit generally appears to be additive. Clearly, this approach is able to make a better use of the cost advantages resulting from higher process flexibility, i.e., the ability of any process to deliver an increasing number of attributes. Similarly, Table 8 indicates that the relative performance of the integrated approach, as measured by ΔZ, increases with an increase in the number of attribute levels η as well.

INSERT TABLES 7 AND 8 HERE

In Table 9 we use the ratio of the average price π_{ave} to the average utility $U_{ave} = (Kw_{ave})$ of the products currently available as a surrogate to measure the intensity of the prevailing market competition. Lower this ratio, higher is the surplus enjoyed currently by the average customer, and therefore, less likely (s)he is to switch to the new product. As seen from the table, while increased competition clearly erodes the profits obtained under both approaches, the integrated approach is more robust especially at low values of π_{ave}/U_{ave} signifying extensive competition.

INSERT TABLE 9 HERE

As seen from Tables 2 through 9, a common outcome across all the different scenarios tested in the second set is that the optimum product price π_{int} under the integrated approach is consistently higher than the price π_{seq} under the sequential approach. This implies that the utility of the new product for the average switching customer is higher under the integrated approach.

In summary, these experiments reveal that the integrated approach is increasingly preferable as the problem instance becomes more *tightly constrained.* Such a tightness could be brought about by the intensity of competition in the market – measured by π_{ave}/U_{ave}, that results in large consumer surpluses. It also results from higher product complexity, expressed in terms of the number of attributes and the number of attribute levels. In regard to the process, this tightness is caused by high variable and fixed process costs, and from having a limited number of alternative candidate processes.

5 Discussion and Summary

This paper models an integrated framework in which the decision on new product design is made jointly with the selection of optimal processes required to manufacture the product. This problem is formulated as nonlinear integer program. We construct a solution approach that has the conceptually appealing property of decomposing the problem into the product design and process selection subproblems that represent the marketing and the manufacturing aspects of this decision, while providing a mechanism to link these two subproblems together. Our emphasis in this paper is on constructing a basic model for the integrated approach, and using this model to gain useful insights into product-process interactions. To this extent, we makes certain simplifying assumptions. As discussed elsewhere in this paper, algorithmic extensions within such an approach are certainly possible; it is also possible, at a cost, to relax some of its assumptions. We have chosen not to do so in order to avoid introducing discontinuities that detract from the insights that we obtain from our computational experience.

The significance of product-process interactions at the system design level has been emphasized in the literature on concurrent engineering, simultaneous engineering, and quality functional deployment. We believe that this study is one of the earliest attempts at modeling these interactions and proposing an integrated solution approach. Furthermore, our computational experience also identifies conditions that heighten these interactions, and consequently, result in greater benefits from adopting the integrated approach. These conditions relate to *more intensive market competition, greater product complexity, low contribution margins*, and *high fixed process costs.*

With an increase in market competition, product-process design integration becomes increasingly important in order to introduce products that sell well and that are profitable. In a niche market where competition is non-existent, the need for integrated design may not be paramount. On the other hand, intensive competition is forcing the major automobile manufacturers to integrate their design and manufacturing activities. Chrysler, for example, has built a billion dollar center in order to locate experts from different functional areas in the same facility (Woodruff and Lasly 1992). Similarly, Nissan claims that simultaneous engineering and close interaction between the design and manufacturing personnel played critically important roles in introducing Quest in the crowded minivan market (Narita 1991).

In regard to product complexity, the computational study suggests that the integrated approach is more desirable for products with more attributes and/or more attribute levels. As these two parameter values increase, product and process decisions become more interrelated, and techniques that consider the overall decision in totality become more useful. The results of this study also show that the integrated approach is able to deliver a product with higher utility and is consequently able to charge a higher price as well.

The results also suggest that when variable costs are high, there is a greater need to carefully select the attribute levels. A number of new products with novel, well-conceived attributes have failed during the growth stage of their life cycles primarily because the firms have underestimated the costs associated with providing these attributes. A case in point is that of Ultrasofts Diapers (Swasy 1990). While the firm touted its design features such as the superabsorbent pulp material used in these diapers to keep babies dry, they underestimated the associated processing costs and the manufacturing problems that these features might entail. Consequently, in spite of strong customer acceptance, this product flopped. The difference in the values of π_{int} and π_{seq} indicates that, by explicitly accounting for the manufacturing costs, the integrated approach provides a more realistic assessment of the optimal price.

Similarly, adopting the integrated approach becomes more important for larger investments resulting in high fixed costs. Again, there is empirical evidence to suggest that firms that have carefully evaluated both product design and process selection alternatives before making major investments have been successful. For example, before introducing its Sensor line of razors and blades, Gillette spent considerable time and effort in insuring that the state-of-the-art technology in laser welding could indeed meet the required output rates without adversely affecting blade hardness and sharpness (Ingrassia 1992). The success of Sensor (43% of the nondisposable razor market in less than 3 years) and the higher price that it commands in the market testify to the wisdom of having considered the product design and the manufacturing decisions simultaneously.

In highlighting the above conditions, we do not mean to suggest that the benefits of adopting the integrated approach is limited to these conditions only. Rather, under these conditions, the integrated approach is so significantly superior that its adoption is critical for achieving any measure of competitiveness.

Acknowledgements

The authors thank Don Erlenkotter for providing the DUALOC code, and Marilyn Lucas for the extensive computational support. This research was partly supported by a grant from the Research Board at the University of Illinois.

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Appendix 1

Proofs

Remark 1. **PDPS** *is NP-hard in the strong sense.*

PROOF: The result is proved by restriction: we show that a special case of **PDPS** is NP-hard in the strong sense. Consider the case in which $I = 1,\ l_1 = 0,\ p_i = 1,\ u_1^0 = 0,\ J_k = 1,$ and $w_{11k} = 1,\ \forall k$. Clearly, in an optimal solution, $\pi = K$, and $s_1 = 1$. **PDPS** then reduces to

$$Z = \max \quad K - \sum_{m \in \mathcal{M}} f_m q_m - \sum_{m \in \mathcal{M}} v_{1km} z_{1km}$$

subject to

$$\sum_{m \in \mathcal{M}} z_{1km} \geq x_{1k} \quad \forall k$$

$$z_{1km} \leq q_m \quad \forall k, m$$

$$z_{1km}, q_m \in (0, 1) \quad \forall k, m$$

This problem has the structure of the uncapacitated facility location (UFL) problem [see, for example, Erlenkotter 1974]. The result follows from the observation that the vertex cover problem that is known to be NP-hard in the strong sense (see, for example, Garey and Johnson 1979) can be transformed to UFL (Cornuejols, Nemhauser and Wolsey 1990). □.

Proposition 1. $\pi^* \in \{\delta_1, \delta_2, \ldots, \delta_I\}$.

PROOF: Order δ_i such that $\delta_{[1]} \leq \delta_{[2]} \leq \cdots \leq \delta[I]$, and let $\delta_{[0]} = 0$. When $\pi = \delta_{[i]}$, it follows from (13) that $s_j = 1$ for $j \in \{[i], [i+1], \ldots, [I]\}$, and $s_j = 0$ for all other j. s_j remains unchanged for any value of π in the interval $(\delta_{[i-1]}, \delta_{[i]}]$. Consequently,

$$z(\mathbf{X}, \pi) \leq z(\mathbf{X}, \delta_i) \quad \text{for} \quad \pi \in (\delta_{[i-1]}, \delta_{[i]}].$$

Repeating this argument for each of I such intervals yields the desired result. □

Appendix 2

Base Parameter Values

Parameter	*Base Value*
ρ	0.4
μ_f (\$)	60K
ζ_v	0.4
ζ_f	0.4
K	5
η	5
M	5
π_{ave}/U_{ave}	0.8

TABLE 1 – Performance of Algorithm **BuildProduct**

Number of Customers	*Number of Attributes*	*Number of Levels*	$(Z_{opt} - Z_{seq})/Z_{opt}$	$(Z_{opt} - Z_{int})/Z_{opt}$
10	2	2	0.098 (9)	0.000 (10)
		3	0.042 (4)	0.000 (10)
		4	0.096 (3)	0.007 (4)
	3	2	0.027 (3)	0.000 (10)
		3	0.107 (2)	0.000 (10)
		4	0.073 (2)	0.005 (8)
	4	2	0.044 (3)	0.010 (9)
		3	0.114 (3)	0.001 (8)
	5	2	0.053 (2)	0.005 (8)
20	2	2	0.025 (5)	0.000 (10)
		3	0.214 (6)	0.000 (10)
		4	0.174 (5)	0.065 (9)
	3	2	0.311 (8)	0.002 (9)
		3	0.313 (3)	0.006 (9)
		4	0.215 (5)	0.029 (9)
	4	2	0.302 (4)	0.022 (8)
		3	0.392 (4)	0.024 (9)
	5	2	0.364 (3)	0.048 (9)
30	2	2	0.037 (8)	0.000 (10)
		3	0.107 (4)	0.000 (10)
		4	0.172 (2)	0.000 (10)
	3	2	0.221 (2)	0.000 (10)
		3	0.209 (2)	0.001 (9)
		4	0.234 (3)	0.003 (9)
	4	2	0.153 (3)	0.018 (6)
		3	0.149 (3)	0.002 (9)
	5	2	0.188 (1)	0.017 (6)

TABLE 2 – Impact of ρ on the Relative Performance of the Integrated Approach

ρ	*Seq. App.*		*Int. App.*		*Rel. Perf.*	
	Z_{seq} (million \$)	π_{seq} (\$)	Z_{int} (million \$)	π_{int} (\$)	ΔZ (million \$)	PR
0.1	2.275	403	2.351	427	0.076	1.033
0.2	1.749	403	1.867	450	0.118	1.067
0.3	1.267	403	1.565	487	0.298	1.236
0.4	0.832	438	1.226	492	0.394	1.475
0.5	0.470	478	0.941	538	0.471	2.001
0.6	0.182	544	0.726	601	0.544	3.998
0.7	0.043	613	0.545	652	0.502	12.724

TABLE 3 – Impact of μ_f on the Relative Performance of the Integrated Approach

μ_f (\$)	*Seq. App.*		*Int. App.*		*Rel. Perf.*	
	Z_{seq} (million \$)	π_{seq} (\$)	Z_{int} (million \$)	π_{int} (\$)	ΔZ (million \$)	PR
60K	1.189	425	1.590	492	0.401	1.338
120K	1.040	425	1.439	492	0.399	1.384
180K	0.926	429	1.321	492	0.395	1.427
240K	0.832	438	1.226	492	0.394	1.475
300K	0.744	438	1.123	499	0.379	1.510
360K	0.662	441	1.026	544	0.364	1.549
420K	0.586	441	0.959	554	0.373	1.638

TABLE 4 – Impact of ζ_v on the Relative Performance of the Integrated Approach

ζ_v	*Seq. App.*		*Int. App.*		*Rel. Perf.*	
	Z_{seq} (million \$)	π_{seq} (\$)	Z_{int} (million \$)	π_{int} (\$)	ΔZ (million \$)	PR
0.08	0.336	522	0.729	675	0.393	2.167
0.16	0.384	509	0.816	633	0.432	2.125
0.24	0.501	472	0.896	620	0.395	1.790
0.32	0.653	464	1.036	530	0.383	1.586
0.40	0.832	438	1.226	491	0.394	1.475
0.48	1.040	422	1.437	456	0.397	1.382
0.56	1.270	404	1.690	434	0.420	1.333

TABLE 5 – Impact of ζ_f on the Relative Performance of the Integrated Approach

ζ_f	*Seq. App.*		*Int. App.*		*Rel. Perf.*	
	Z_{seq} (million \$)	π_{seq} (\$)	Z_{int} (million \$)	π_{int} (\$)	ΔZ (million \$)	PR
0.08	0.770	438	1.230	487	0.460	1.596
0.16	0.782	438	1.228	487	0.446	1.571
0.24	0.798	438	1.206	492	0.408	1.510
0.32	0.815	438	1.216	492	0.401	1.492
0.40	0.832	438	1.226	492	0.394	1.475
0.48	0.848	438	1.237	492	0.389	1.458
0.56	0.865	438	1.252	513	0.387	1.447

TABLE 6 – Impact of M on the Relative Performance of the Integrated Approach

M	*Seq. App.*		*Int. App.*		*Rel. Perf.*	
	Z_{seq} (million \$)	π_{seq} (\$)	Z_{int} (million \$)	π_{int} (\$)	ΔZ (million \$)	PR
3	0.649	447	1.196	491	0.547	1.843
5	0.832	438	1.226	491	0.394	1.475
7	1.053	408	1.409	489	0.356	1.339
9	1.040	414	1.402	518	0.362	1.349
11	1.100	408	1.450	479	0.350	1.318
13	1.238	408	1.558	480	0.320	1.258
15	1.368	417	1.527	468	0.159	1.116

TABLE 7 – Impact of K on the Relative Performance of the Integrated Approach

K	*Seq. App.*		*Int. App.*		*Rel. Perf.*	
	Z_{seq} (million \$)	π_{seq} (\$)	Z_{int} (million \$)	π_{int} (\$)	ΔZ (million \$)	PR
3	0.505	324	0.648	326	0.143	1.282
5	0.832	438	1.226	491	0.394	1.475
7	0.948	568	1.500	644	0.552	1.580
9	1.299	750	2.217	791	0.918	1.707
11	1.934	826	2.817	921	0.883	1.457
13	2.099	944	3.468	1112	1.369	1.654
15	2.968	1102	4.323	1250	1.355	1.457

TABLE 8 – Impact of η on the Relative Performance of the Integrated Approach

η	*Seq. App.*		*Int. App.*		*Rel. Perf.*	
	Z_{seq} (million \$)	π_{seq} (\$)	Z_{int} (million \$)	π_{int} (\$)	ΔZ (million \$)	PR
3	0.454	448	0.699	544	0.245	1.538
5	0.573	474	1.020	524	0.447	1.781
7	0.832	438	1.226	491	0.394	1.475
9	1.047	446	1.500	509	0.453	1.433
11	0.990	461	1.508	519	0.518	1.524
13	1.173	475	1.722	524	0.549	1.468
15	1.270	490	1.923	547	0.653	1.514

TABLE 9 – Impact of π_{ave}/U_{ave} on the Relative Performance of the Integrated Approach

π_{ave}/U_{ave}	*Seq. App.*		*Int. App.*		*Rel. Perf.*	
	Z_{seq} (million \$)	π_{seq} (\$)	Z_{int} (million \$)	π_{int} (\$)	ΔZ (million \$)	PR
0.60	0.541	451	0.924	482	0.383	1.704
0.65	0.625	452	1.062	487	0.437	1.701
0.70	0.832	438	1.226	491	0.394	1.475
0.75	1.110	462	1.378	511	0.268	1.241
0.80	1.325	471	1.571	525	0.246	1.186
0.85	1.470	490	1.784	536	0.314	1.213
0.90	1.640	515	2.010	544	0.370	1.225