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Pension Policy and Capital Structure
Decisions: A Liability-based Approach**

*Michael J. Alderson
Cheng F. Lee*

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Capital Structure Decisions: A Liability-based Approach

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Abstract

Following Arnott and Gersowitz [1980] and DeAngelo and Masulis [1980], this paper deals with corporate pension funding decisions as part of the overall capital structure decision. A liability-based model is derived in accordance with the assumption that the tax system is in different ways biased in favor of both pension income and income from equity securities. The conditions for obtaining optimal pension policy for both whole economy and individual firms are derived. Four analytical propositions for normative pension management also are derived.

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1. Introduction

Extant theories of corporate pension funding policy offer two different sets of implications for the management of pension funds. On the one hand, firms should minimize their contributions to the fund and maximize the investment risk of the contributed assets in a world of incorrectly priced pension insurance premiums as shown by Sharpe [1976]. Under Miller's [1977] model of capital structure, however, the absence of marginal gains to corporate leverage results in an optimal pension policy which is characterized by full funding and 100 percent investment in bonds, as shown by Black [1980] and Tepper [1980]. Some application of these recommendations can be observed in the financial community. For example, the recent willingness of pension funds to provide debt financing for leveraged buyouts appeals to aspects of both approaches (see Forbes [1984]). A few corporations, most notably Chrysler, have even committed their pension investments fully to an "All Bonds" strategy. Nevertheless, there is for the most part a wide gap between theory and practice in the pension funding area. The implications of the literature call for corner solutions of some sort, while in practice firms subscribe to a continuum of funding strategies. By investigating the implications of the joint effects of insurance and taxes for optimal corporate strategy, Bicksler and Chen [1985] have shown that optimal corporate pension strategy in both asset-allocation and funding decisions can be a noncorner interior solution.

This paper provides an alternate approach to the pension funding problem which potentially avoids the extreme implications of most of

prior models. A model is presented which integrates corporate pension funding policy with the capital structure decision. By viewing the unfunded vested liability as debt substitute, it will be shown that in a world of differential taxation optimal corporate pension funding policies exist which are unique to firms with different levels of non-pension tax shields. The approach taken is thus a liability-based one; whereas prior models have focused in the properties of pension assets as part of the corporate entity, this analysis is concerned with the role of the net vested liability in the financial structure. In the second section, the assumptions and definitions of the model are explained. The characteristics of the demand for pensions which follow from these assumptions are discussed in third the section. The one period valuation model and the supply function which it implies are explained in the fourth section. Equilibrium with no tax shield risk is discussed in section five. An optimal corporate pension funding policy under tax shield risk is present in section six. The development of four analytical proportions which are implied by the model is explored in section seven. Finally, summary and conclusion remarks are indicated in section eight.

2. Assumptions and Definitions

This model incorporates two instances of differential taxation which contribute to the complexities of the U.S. tax system. The first and simplest of these concerns the nature of the Social Security tax. Under current law, a FICA (Federal Insurance Corporation of America) tax of 6.70% is levied against the first \$38,700 of income earned in a given

year. Employers are required to match the contribution of their labor pool on roughly a dollar-for-dollar basis. The statutes exclude pension income from the FICA tax base for both employers and employees. This creates a preference for pension income which will be driving force in the equilibrium process.

The second differential present in the model accounts for the heterogeneous nature of the federal personal income tax. Individual taxpayers are allowed to exclude 60% of their long-term capital gains from gross income. If $t(pd,i)$ and $t(pe,i)$ represent the constant marginal tax rates on debt and equity income for individual i , respectively, this exclusion causes $t(pd,i) > t(pe,i) \forall i$. The relationship between $t(pd,i)$ and $t(pe,i)$ creates a personal tax bias in favor of equity investment which will ultimately constitute a constraining influence in the model.

Some additional assumptions are made with regard to the tax system. First, wage income, whether deferred or not, is subject to a flat income tax of $f\%$. This simplification has the effect of streamlining the analysis, while preserving the components critical to a meaningful result.¹ The corporate tax rate is a constant $t(c)$. Marginal tax rates on investment income are assumed to be progressive in nature, with at least one investor in each of the following mutually exclusive and exhaustive tax brackets:²

- Bracket 1 $(1-f)(1-t(pd,i)) > (1-t(c))(1-t(pe,i))$,
- Bracket 2 $(1-f)(1-t(pd,i)) = (1-t(c))(1-t(pe,i))$,
- Bracket 3 $(1-f)(1-t(pd,i)) < (1-t(c))(1-t(pe,i))$.

At the end of the one period time frame, corporations are assumed to receive state dependent operating earnings of $X(s)$ in state s ; $X(s)$ is monotonically increasing in s with $0 < X(S1) < X(S5)$.³ The face value of debt (B), certain non-cash charges (D) such as depreciation and depletion, and pension contributions required (P) are excludable from taxable income. Firm specific tax credits of G dollars exist, but consistent with the tax code only $\theta\%$ of a firm's tax liability can be offset by these credits.

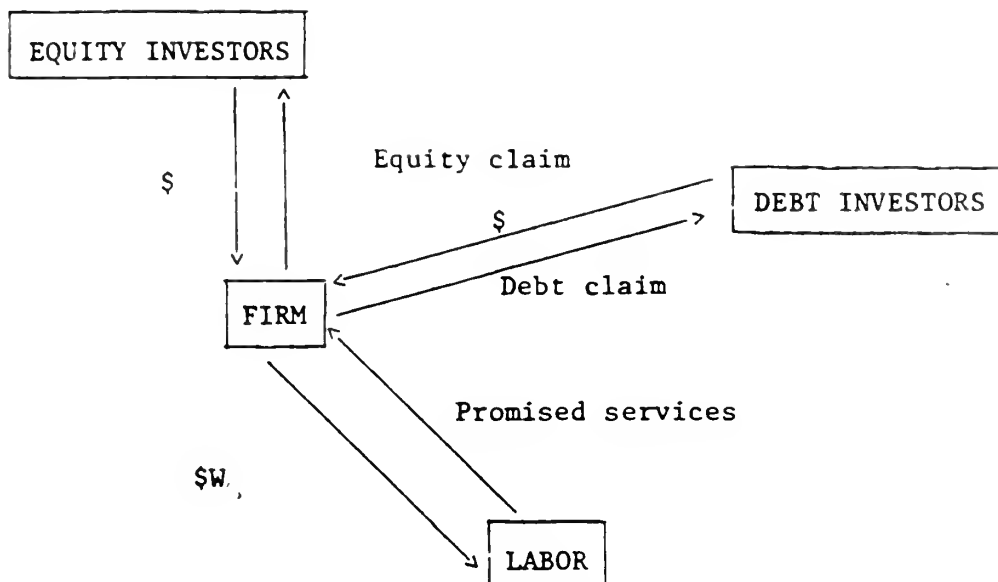
There are two mutually exclusive and exhaustive groups in the economy: capitalists and labor. Only capitalists may initiate firm formation and maintain non-neutral investment positions. Accordingly, labor is the only group which may sell its services. At the beginning of the period, firms negotiate an agreement with labor, calling for the payment of w dollars immediately in exchange for future services. Simultaneously, firms issue E dollars worth of equity and R dollars worth of pure discount bonds. The bonds issued are assumed to be riskless.

Corporations are assumed to recognize the debt substitution attributes which unfunded liabilities possess. It follows that they limit the amount of pure discount bonds which are issued, in order to employ unfunded vested pension obligations in their capital structures. The precise factors influencing the amount of pure discount bonds are exogenous to the model.⁴

Figure 1 illustrates the macro-structure of the model. The non-uniform application of the FICA tax encourages labor to seek deferred compensation arrangements as a means of avoiding the Social Security tax. In this model, labor can defer compensation by loaning (in pre-personal tax

Figure 1

PHASE I



PHASE II

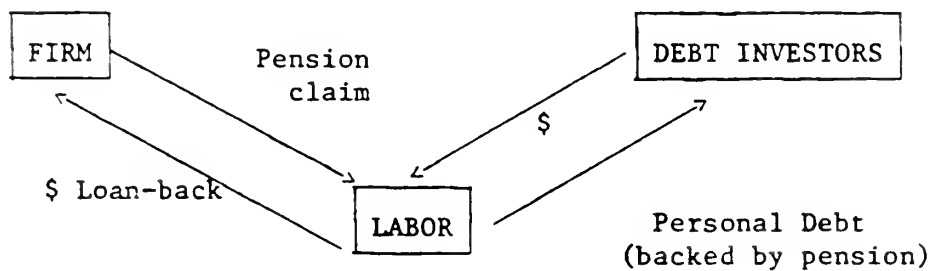


Fig. 1. In Phase I, the firm sells debt and equity claims on its after corporate tax cash flows. Phase II shows labor loaning back wages to the firm in exchange for an unfunded pension claim. This claim is then effectively sold in the capital markets.

dollars) all or part of its current time wage w back to the firm. A personal flat tax of $f\%$ and a FICA tax of $s\%$ are immediately paid on only the amount retained by the wage earner. When the pension is received at the end of the period, employees remit the personal tax of $f\%$ only.

Labor cannot by definition maintain a non-neutral investment position. To offset the effect which the pension obligations have on their portfolios, employees must sell personal debt. The amount of debt sold is equal to the present value of their unfunded pension claims. At the end of the period, when this personal debt matures, employees are assumed to service their obligations with the after personal tax (only) pension income which they receive.⁵

In summary, employees contract for pensions by loaning pre-tax wage income back to the firm in exchange for an unfunded obligation. Because they cannot hold debt, they in effect end up selling their vested pension benefits in the capital markets. The sale of these benefits allows for the determination of an optimal level of pensions in equilibrium. On the demand side, a constraint on the quantity of claims which investors desire will limit the amount of deferred claims which employees demand from the firm, given that they will request only what they can offset with personal debt. Limitations on the supply of pension income offered by each firm will then be shown to establish interior levels of unfunded pension liabilities which are unique to each firm when non-pension tax shields exist.

Following Alderson and Chen [1985], the relationship between pension assets and the assets of a firm might more suitably be explained by the separation hypothesis than the integration hypothesis. Hence,

the model is constructed without explicit consideration of pension assets. As such, the conflicting influences of Sharpe [1976] and Black [1980] are not accounted for. The fact that all firms are subject to these effects makes their exclusion unimportant, however, and allows the model to explore other factors which affect corporate pension funding policy

Notation

- $t(pd,i)$ = the marginal tax rate on debt for investor i ,
- $t(pe,i)$ = the marginal tax rate on equity for investor i ,
- $Pd(s)$ = the current market price per dollar of debt income to be delivered in the future state s ,
- $(1-t(pd,i))/Pd(s)$ = the after-tax yield on state s debt,
- $Pe(s)$ = the current market price per dollar of equity income to be delivered in the future state s ,
- $(1-t(pe,i))/Pe(s)$ = the after-tax yield on state s equity,
- $X(s)$ = earnings in state s , monotonically increasing in s ,
- B = the face value of debt (fully tax deductible),
- D = non-cash tax deductions,
- G = the dollar value of tax credits,
- $t(c)$ = the statutory marginal corporate tax rate,
- θ = the statutory maximum fraction of a firm's gross tax liability which can be offset by tax credits,
- P = the face value of unfunded pension benefits,
- $p(s)$ = the probability of a particular state s occurring,
- s = the FICA tax rate on earned income.

3. Aggregate Pension Demand and Differential Taxation

This section will deal with the mechanics of the demand for pension income in the model. Perhaps more appropriately stated, it will explain the demand for pension backed debt ("P-debt" by abbreviation), since the amount of P-debt demanded by investors will determine the quantity of pension income requested by employees. The market setting discussed will consider only P-debt and equity claims because conventional corporate debt, by assumption both riskless and insufficient to satisfy total demand, is irrelevant to this analysis in a manner not unlike that of municipal and Treasury issues. If bond is risky, then debtholder claims outrank those of unfunded pensions in a liquidation. This case will be considered in Section 6.

To begin, recall that $t(pd,i)$ and $t(pe,i)$ represent the constant marginal personal tax rates on debt and equity income, for the i -th individual. Since by assumption the personal tax code is biased so that $t(pd,i) > t(pe,i) > 0$ for all i , the state-contingent after personal tax cash flow per unit of state s equity income exceeds that per unit of state s debt income, so that $(1-t(pe,i)) > (1-t(pd,i))$ for all individuals.

Let $Pd(s)$ equal the current time ($t=0$) price in the market of one dollar of pre-tax state s income from an investment in P-debt. Likewise, $Pe(s)$ will represent the current time market price of one dollar of pre-tax state s income from an equity investment. Their reciprocals, $1/Pd(s)$ and $1/Pe(s)$, are pre-tax current yields; $(1-t(pd,i))/Pd(s)$ and $(1-t(pe,i))/Pe(s)$ are accordingly the after-tax yields to the i -th individual. Utility maximizing investors will adjust their portfolio

holdings of state s claims in order to maximize these after-tax yields. In making their portfolio selection, investors will prefer P-debt to equity if $(1-t(pd,i))/Pd(s) > (1-t(pe,i))/Pe(s)$; equity will be preferred to P-debt if the inequality is reversed; finally, the individual will be indifferent between the two when their after-tax yields are equal.

The cross sectionally constant corporate tax rate is represented by $t(c)$, and the cross sectionally constant tax on wage income is denoted by f . It is assumed that progressive personal tax rates are such that at least one investor is in each of the following mutually exclusive and exhaustive tax brackets:

- B.1 Debt Preferring $(1-f)(1-t(pd,i)) > (1-t(c))(1-t(pe,i))$,
- B.2 Debt Indifferent $(1-f)(1-t(pd,i)) = (1-t(c))(1-t(pe,i))$,
- B.3 Debt Adverse $(1-f)(1-t(pd,i)) < (1-t(c))(1-t(pe,i))$.

It is assumed that investors are risk neutral and homogeneous in their belief that each states will occur with probability $p(s)$. In the course of constructing a portfolio, every investor will select the security of their tax preference (debt or equity) with the highest expected yield. Given that all claims must be held in equilibrium, it follows that the markets must set prices for its state contingent claims so that pre-tax expected yields are equal. If PD and PE are the current market prices of the pre-tax cash flows from debt and equity respectively, this implies that $p(s)/Pd(s) = 1/PD$ and $p(s)/Pe(s) = 1/PE$ for all s . DeAngelo and Masulis [1980] have shown that under these circumstances it is possible to examine both (1) the P-debt and equity demand decisions of individuals, and (2) the pension and equity supply decisions of firms, by examining only two markets: the one for pre-tax cash flows to equity

(with current price PE) and the one for pre-tax cash flows to P-debt (with current price PD). Let the superscript u denote investors who are indifferent between investing in P-debt or equity. Exactly which investors are marginal in this sense is established by the market prices PD and PE, when they equate the pre-tax personal yields of P-debt and equity for certain groups of investors. This occurs when, for individual u

$$\begin{aligned} p(s)(1-t(pd,u))/Pd(s) &= (1-t(pd,u))/PD \\ &= (1-t(pe,u))/PE = p(s)(1-t(pe,u))/Pe(s) \end{aligned}$$

Equivalence of after personal tax yields causes individuals to be indifferent between investing their next dollar of capital in P-debt or equity. Figure 2 illustrates the demand curve for P-debt.⁷ If it is the case that $PD > PE$, then $1/PD < 1/PE$, causing $(1-t(pd,i))/PD < (1-t(pe,i))/PE$ for all investors. Clearly, under these conditions the demand for P-debt is zero, since it is an inferior investment for everyone. If relative prices are such that $PE > PD(1-f) > PE(1-t(c))$, investors in bracket 1 will demand debt cash flows, and marginal rates will be such that $(1-t(pd,i))(1-f) > (1-t(pe,i))(1-t(c))$. Should PD fall relative to PE, larger amounts of pre-tax debt income will be demanded in the aggregate, causing the implied marginal personal tax rates to change correspondingly. This is because as relative debt yields rise, the personal tax indifference to debt is overcome for the most recent marginal investor, making that individual debt preferring, and causing the investor with the least debt aversion to become indifferent. When $PD(1-f) = PE(1-t(c))$, investors in bracket 1 demand debt only and those in bracket 2, with

Figure 2

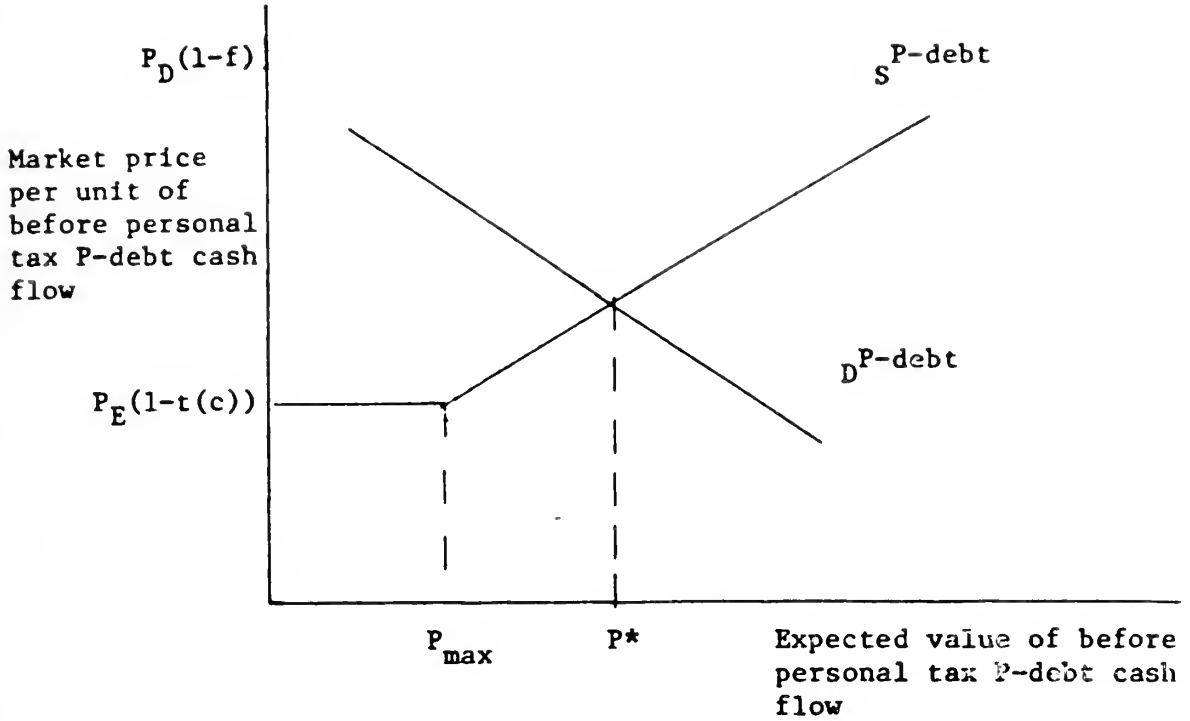


Fig. 2. Market equilibrium in a 'pension funding, taxation, and capital structure' world; $D^{P\text{-debt}} \equiv$ aggregate demand for P-debt; $S^{P\text{-debt}} \equiv$ aggregate supply for P-debt; $P^* \equiv$ equilibrium aggregate quantity of P-debt; $P_{\max} \equiv$ aggregate quantity of P-debt supplied when all firms are at the maximum P-debt level allowing full utilization of all corporate tax shields. Equilibrium occurs at $P_D(1-f) > P_E(1-t(c))$.

$(1-f)(1-t(pd,i)) = (1-t(pe,i))(1-t(c))$, are marginal. Finally, for prices $PD(1-f) < PE(1-t(c))$, marginal investors are those in bracket 3 with $(1-f)(1-t(pd,i)) < (1-t(pe,i))(1-t(c))$; all other individuals demand debt only.

4. Firm Valuation

This section describes the one period state preference valuation model which will be used in the subsequent discussion of optimal corporate pension policy. From this valuation model the supply function for pension income emerges.

Let $X(s)$ represent cash operating earnings before pension expenditures. This variable is monotonically increasing in s , with $X(S1) < X(s) < X(S5) < \infty$. From $X(s)$ certain non-cash charges (D), the interest expense on bonds (B), and pension contributions (P) are deducted to arrive at corporate taxable income. Firms pay a tax of $t(c)\%$ on taxable earnings. They can offset their liability with any tax credits (G) which they possess, subject to the limitation that the offset not to exceed $\theta\%$ of the total liability.⁸ The firm specific variables B , P , D , and G define the sub-intervals over the $[S1, S5]$ range which describe the potential pre-tax dollar returns which P-debtholders and shareholders face. Table 1 describes these returns, which will be referred to as $D(s)$ and $E(S)$, respectively.⁹ In the first interval, $[S1, S2]$, the operating stream is sufficient to compensate regular bondholders but insufficient or just sufficient to pay the required amount to P-debtholders, who hold a claim on the after-personal-tax pension income of labor. P-debtholders will receive from zero to P dollars. The firm will pay no taxes, thereby wasting all deductions and credits. Technically, $s \in [S1, S2]$. When

$X(s) - B < P$, or when $X(s) < (P+B)$. If $(B+D) < X(s) < (B+P+D)$ then $s \in [S2, S3]$. In this interval both regular and P-debtholders are paid in full. No corporate taxes are paid, however, as earnings fail to exceed total tax deductions. Excess deductions and all tax credits are consequently unutilized. When $(P+B+D) < X(s) < (P+B+D + G/\theta t(c))$, $s \in [S3, S4]$.

Table 1

State	D(s)	E(s)
S1 - S2	$[X(s)-B](1-f)$	0
S2 - S3	$P(1-f)$	$X(s)-B-P$
S3 - S4	$P(1-f)$	$X(s)-B-P - (1-\theta)t(c)[X(s)-B-P-D]$
S4 - S5	$P(1-f)$	$X(s)-B-P - t(c)[X(s)-B-P-D] + G$

This interval is similar to the prior one except that the firm now has earnings which exceed all available deductions, resulting in a tax liability which can be partially offset by available tax credits. The limitation on the offset exists because the total credits available exceed the maximum percentage of the gross liability allowed by law. Thus, all deductions and a fraction of available credits are utilized. In the final interval, when $s \in [S4, S5]$, the returns to both parties reflect the full utilization of all tax deductions and tax credits by the firm.

The optimal deferred wage occurs when the current time firm value is maximized; that is, when the market value (V) of the claims of labor (D) and equity (E) are greatest in total.¹⁰ D and E are valued by discounting their cash flows in every state nature by the appropriate state contingent unit price, $P_d(s)$ and $P_e(s)$, respectively. Thus,

$$(1) V = D + E$$

$$\begin{aligned}
 V &= \int_{S1}^{S5} D(s)Pd(s)ds + \int_{S1}^{S5} E(s)Pe(s)ds \\
 V &= [\int_{S1}^{S2} (X(s) - B)(1-f)Pd(s)ds + \int_{S2}^{S5} P(1-f)Pd(s)ds] \\
 &+ [\int_{S2}^{S3} (X(s) - B - P)Pe(s)ds \\
 &+ \int_{S3}^{S4} (X(s) - B - P) - (1-\theta)t(c) (X(s) - P - B - D) Pe(s)ds \\
 &+ \int_{S4}^{S5} (X(s) - B - P - t(c)(X(s) - B - P - D) + G)Pe(s)ds].
 \end{aligned}$$

The objective of the firm is to select the P* value which maximizes V. The appropriate first order condition as derived in Appendix A is:

$$\begin{aligned}
 (2) \partial V/\partial P &= \int_{S2}^{S3} (Pd(s)(1-f) - Pe(s))ds \\
 &+ \int_{S3}^{S4} (Pd(s)(1-f) - Pe(s)(1-(1-\theta)t(c)))ds \\
 &+ \int_{S4}^{S5} (Pd(s)(1-f) - Pe(s)(1-t(c)))ds.
 \end{aligned}$$

Equation (2) shows that the level of unfunded pension liabilities which a firm possesses is relevant to valuation, because $\partial V/\partial P$ is not equal to zero for all pension policies if D, B, and/or G are positive. This of course implies that some values of P are strictly preferred to others.

Under the assumption that investors are risk neutral and have homogeneous beliefs, equation (2) reduces to:¹¹

$$\begin{aligned}
 (3) \quad \partial V / \partial P &= (PD(1-f) - PE) \int_{S2}^{S3} p(s) ds \\
 &+ (PD(1-f) - (1-(1-\theta)t(c))PE) \int_{S3}^{S4} p(s) ds \\
 &+ (PD(1-f) - (1-t(c))PE) \int_{S4}^{S5} p(s) ds.
 \end{aligned}$$

In the aggregate, equation (3) represents the present value of the expected corporate after-tax cash flow which results at the margin when unfunded pension liabilities are substituted for equity in the firm's financial structure. Further, this marginal present value contains three distinct components whose magnitudes depend on the degree to which the marginal pension deduction is utilized. The first term, $PD(1-f) - PE(1-t(c))$, is the present value of the unfunded pension for equity substitution when the marginal corporate tax deduction resulting from the increased pension liability is fully utilized. The corresponding integral represents the probability of full utilization of the marginal pension deduction. The second term, $PD(1-f) - PE(1-(1-\theta)t(c))$, is a lower present value due to the partial loss of the corporate tax shield caused by the statutory θ ceiling on usable tax credits. The corresponding integral is the probability of the partial loss of the marginal pension deduction due to the θ ceiling. Finally, the third term, $PD(1-f) - PE$, represents the present value of the unfunded liability substitution when available deductions already exceed total earnings. The third integral is the probability of the total loss of the deduction for the marginal pension.

5. Equilibrium With No Tax Shield Risk

In order to show the contribution of this model to the body of pension theory, this section will characterize market equilibrium in a way which allows for the derivation of analytic results similar to those obtained by Sharpe in his no insurance world. When there are no tax shields other than the pension contribution, the partial or total loss of the marginal tax benefits of sponsoring a plan is impossible. If $D = B = G = 0$, then $S_2 = S_3 = S_4$, and equation (3) reduces to

$$(4) \quad \partial V / \partial P = (PD(1-f) - (1-t(c))PE) \int_{S_2}^{S_5} p(s) ds$$

Equation (4) allows for the derivation of the aggregate supply curve for P-debt. Figure 3 illustrates this curve. If relative prices are such that $PD(1-f) < PE(1-t(c))$, the $\partial V / \partial P < 0$ for all values of P, and the firm will supply no pension income to labor. If $PD(1-f) > PE(1-t(c))$, then $\partial V / \partial P > 0$ for all feasible pension decisions, and labor will completely take the place of shareholders in the capital structure. If $PD(1-f) = PE(1-t(c))$, $\partial V / \partial P = 0$ for all values of P, implying that the firm is indifferent among all feasible P-debt-equity alternatives. The supply curve is therefore perfectly elastic over the feasible set. Summing across all firms, it follows that the aggregate supply curve is also perfectly elastic at relative prices $PD(1-f) = PE(1-t(c))$. This result is the same as that obtained by Sharpe in a no insurance world; pension policy is irrelevant to firm value.

Turning briefly to the demand side, marginal investors must be in bracket 2 at market equilibrium. As pointed out in section 3, marginal

Figure 3

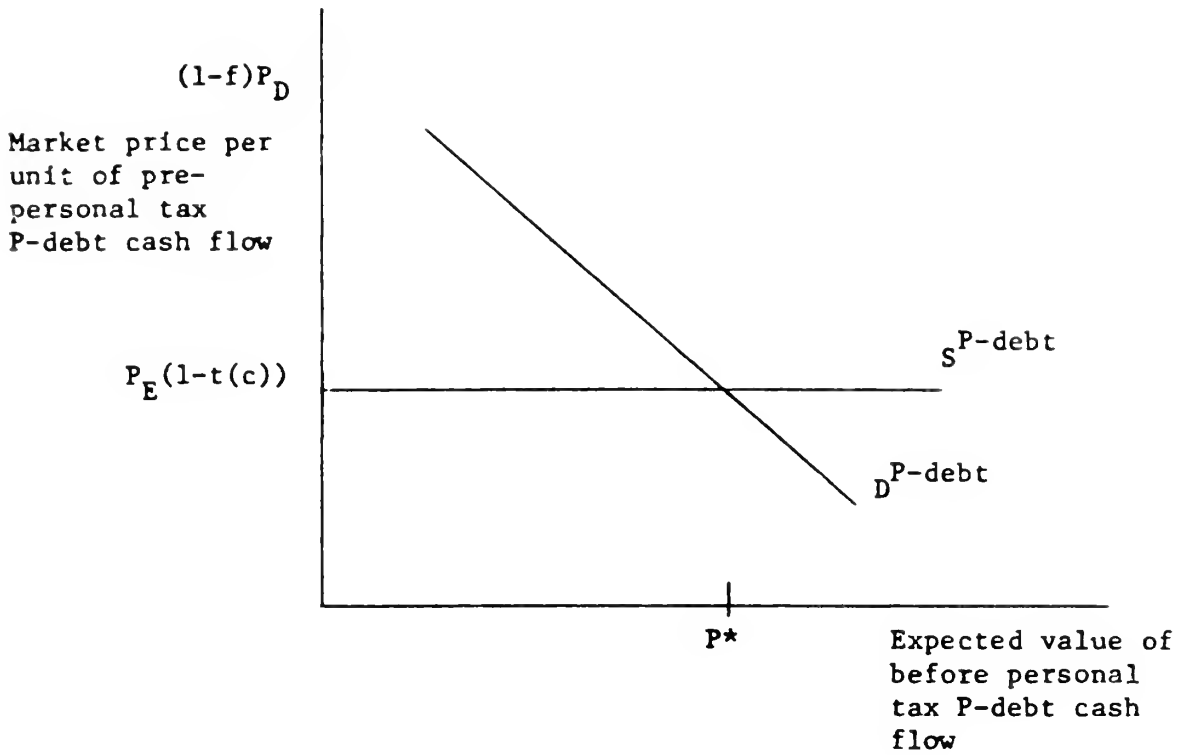


Fig. 3 , Market equilibrium in a world with no corporate tax shield substitutes. $D^{P-debt} \equiv$ aggregate demand for P-debt; $S^{P-debt} \equiv$ aggregate supply of P-debt; $P^* \equiv$ equilibrium aggregate quantity of P-debt. Equilibrium occurs at $P_D(1-f) = P_E(1-t(c))$.

investors are those for whom after personal tax expected yields on P-debt and equity are equal: $(1-t(pd,u))/PD = (1-t(pe,u))/PE$. When $PD(1-f) = PE(1-t(c))$, marginal investors are in bracket 2 with tax rates satisfying $(1-t(pd,i))(1-f) = (1-t(pd,i))(1-t(c,i))$.

Substituting the marginal investor's tax rate condition $PD = PE(1-t(pe,u))/(1-t(pd,u))$ into the marginal value of P-debt expression (4) yields:

$$(5) \partial V / \partial P[P^*] = (PD(1-f)/(1-t(pd,u))) \times [((1-t(pd,u)) - (1-t(pe,u))(1-t(c))) \int_{S2}^{S5} p(s)ds]$$

In equilibrium, the expression in brackets is zero, because marginal investors are in bracket 2. The market in an intuitive sense endogenously determines the relative marginal personal tax rates on P-debt and equity so that the corporate pension advantage exactly offsets the P-debt disadvantage to investors, at any P-debt level.

The liability-based model therefore shows that pension funding policy will be irrelevant to firm value when pension contributions are always deductible. This is because at the margin, the constant corporate tax advantage of providing more pension income is offset by the personal tax disadvantage of doing so. While an optimal level of unfunded pensions will exist in the economy, there are no individual optimums which obtain that are unique to each firm. In this instance, the equilibrium characteristics are the same as those of the asset-based models of Sharpe [1976] and Tepper [1981]. The next section will explain how the existence of non-pension tax shields affects the analysis.

6. Tax Shield Risk and Optimal Corporate Pension Policy

When corporate tax shield substitutes exist ($D, B, G > 0$), corporate pension policy is no longer irrelevant. Instead, relative market prices will adjust until each firm has a unique interior optimum pension level, in equilibrium. Similar to the previous section, there is a constant expected marginal personal tax disadvantage to P-debt. The existence of tax shield substitutes, however, causes the expected marginal corporate tax benefit to decline as more pension income is contracted. At P^* , the expected marginal corporate tax benefit just equals the expected marginal corporate tax benefit just equals the expected marginal personal tax disadvantage of debt. Unique interior optimums exist when firms have different amounts of debt substitutes. (See Appendix B for mathematical proof.)

To begin, the aggregate supply curve for pension income will be derived, under the assumption that the pension income stream is riskless.¹² When pension contributions are not the firm's only tax shield, both individual and aggregate supply curves will consist of a perfectly elastic section, after which they will smoothly slope upward (Figure 2). The curve is perfectly elastic at relative prices $PD(1-f) = PE(1-t(c))$ over the levels of pension income which allow for the full utilization of all corporate tax shields (D, B, G , and P). The supply curve is upward sloping beyond the full utilization level because firms are willing to supply more pension income to labor only if they are compensated (for the greater likelihood of a partial or total loss of the additional tax shield) with higher unit P-debt prices $PD(1-f) > PE(1-t(c))$. The supply curve represents the schedule of $(PD(1-f), P)$ combinations at which the firm has unique interior optimum pension levels. In market

equilibrium, the point at which the aggregate demand curve intersects the aggregate supply curve (in its upward sloping section, under weak assumptions on the personal tax code) sets relative prices $PD(1-f) > PE(1-t(c))$. These prices in turn determine the unique interior optimum pension policy for each firm.

A formal derivation of the supply curve proceeds as follows. When $D, B,$ and $G > 0, S_4 > S_3 > S_2 > 0,$ and the first order condition is once again represented by equation (3). If relative prices are such that $PD(1-f) < PE(1-t(c)), \partial V/\partial P < 0$ for all values of $P,$ and the firm will supply no pension income. When $PD(1-f) = PE(1-t(c)),$ the supply curve is perfectly elastic to the point where the selected pension contribution just allows for the full utilization of all tax shields in every state of nature. Referring to this amount of $P_{MAX},$ and letting $X(S_1)$ represent the lowest possible earnings, P_{MAX} occurs when θ times the gross tax liability is greater than or equal to $G.$ This is written in equation form as:

$$\theta t(c)[X(S_1) - D - B P_{MAX}] > G,$$

which, when solving for P_{MAX} implies

$$P_{MAX} = X(S_1) - D - B - G/\theta t(c) < X(S_1).$$

When P is such that $0 < P < P_{MAX}, S_2 = S_3 = S_4,$ because all corporate tax shields are fully utilized in every state of nature. The first order condition reduces to:

$$\partial V/\partial P = PD(1-f) - PE(1-t(c)) \text{ for } 0 < P < P_{MAX}.$$

At relative prices $PD(1-f) = PE(1-t(c))$, the firm is indifferent to all pension levels which allow for the full utilization of all corporate tax shields. Consequently, the supply curve is perfectly elastic at relative prices $PD(1-f) = PE(1-t(c))$ over the interval $0 < P < P_{MAX}$.

No pension income will be provided beyond P_{MAX} at relative prices $PD(1-f) = PE(1-t(c))$. This is because when $P_{MAX} < P < X(S_1)$, $S_4 > S_3 > S_2$. If $PD(1-f) = PE(1-t(c))$. This is because when $P_{MAX} < P < X(S_1)$, $S_4 > S_3 > S_2$. If $PD(1-f) = PE(1-t(c))$, $\partial V/\partial P < 0$, because the second and third terms are negative, while the first is zero. Firms will only provide pension income beyond P_{MAX} if $\partial V/\partial P > 0$. In the context of equation (3), this is possible only if $PD(1-f) > PE(1-t(c))$. From an economic perspective, the higher pension price is necessary in order to compensate shareholders for the loss of the corporate tax shield which will occur on marginal units of pension contribution when $s \in [S_3, S_4]$. For all prices $PD(1-f) > PE(1-t(c))$, there exists a P^* which sets $\partial V/\partial P = 0$. As PD rises relative to PE , equation (3) will become positive, and the firm will find it advantageous to supply more pension income. The supply curve is thus smoothly upward sloping beyond P_{MAX} .

Market equilibrium will occur on the upward sloping portion of the pension supply curve, under a set of weak assumptions. First, the perfectly elastic section of the supply curve must be short. This is reasonable because there are few pension levels which are both riskless to the employee and which have a zero probability of tax shield loss, as the range $0 < P < P_{MAX}$ does. Second, investors in bracket 1 must be sufficient in number to demand larger quantities of P -debt than P_{MAX} in the aggregate, at relative prices $PD(1-f) = PE(1-t(c))$.¹⁴ If both

of these conditions hold, then the aggregate demand curve will intersect the aggregate supply curve at relative prices $PD(1-f) > PE(1-t(c))$.¹⁵ These prices will dictate a unique interior optimum pension policy for each firm, and imply that investors are in bracket 1 with tax rates satisfying $(1-t(pd,i)) (1-f) > (1-t(pd,i)) (1-t(c))$.

There is a duality relationship which exists between relative market prices and relative personal tax rates. This relationship can be used to intuitively show the trade-off which determines the optimal pension policy. For the sake of simplifying the analytics, it is assumed that $t(pe,i) = t(pe,u) = 0$,¹⁶ and that P^* occurs over the range $P_{MAX} < P^* < X(S1)$. Evaluating the first order condition at P^* and substituting the simplified marginal personal tax rate condition $PD = PE/(1-t(pe,u))$ yields:¹⁷

$$(6) \partial V / \partial P [P^*] = (PD(1-f) / (1-t(pd,u))) \times \\ [t(c) \left(\int_{S4}^{S5} p(s) ds + (1-\theta) \int_{S3}^{S4} p(s) ds \right) - t(pd,u)]$$

The first term in square brackets represents the expected corporate tax advantage of an additional dollar of pension income substituted for equity. Notice that this term is higher when the probability of losing any or all of the marginal tax shield is lower. The second term in square brackets, $t(pd,u)$, quantifies the expected marginal personal tax disadvantage of riskless P-debt. This is because under the simplifying assumptions made, an increase in P of one unit raises the personal tax liability by $t(pd,u)$ in all states of nature. At low levels of pension income, the first term exceeds the second term because there is a higher

probability that the additional pension contribution can be fully utilized to reduce the firm's tax liability, and investors purchasing the corresponding P-debt are in the lower tax brackets. As the quantity of pension income provided rises, so does the likelihood that any or all of the marginal tax benefit will be lost.¹⁸ At equilibrium, the declining marginal corporate advantage of pension contributions just equals the marginal personal disadvantage of P debt. At P*, it is not advantageous for firms to supply more pension income or for labor to demand more.

The use of a one period model necessarily precludes the consideration of the carryback and carryforward (CB-CF) provisions in the tax code. The effect of the inclusion in a multi-period model is not difficult to predict. Both reduce the probability that a corporate tax shield will be lost. CB provisions allow any unused shields to be applied to reduce the tax liability of up to three prior years. CF provisions allow the firm to apply any shields which cannot be carried back to up to 15 years in the future. Because both involve a time-value loss, however, they reduce, but do not eliminate the expected value of the corporate tax shield loss on the marginal unit of pension. Firms would therefore require less price compensation for increasing the supply of pension income. As a result, the supply curve in a multi-period model would be more elastic in its upward sloping portion than the one presented here. Finally, it should be noted that our analysis of pension policy can be generalized to the case when corporate conventional debt is risky. When corporate conventional debt is risky, the returns to the

contributors of capital are described in Table 2, given that by law debtholder claims outrank those of unfunded pensions in a liquidation.

Table 2

State Interval	B(s)	D(s)	E(s)
S1 - S2	X(s)	0	0
S2 - S3	B	X(s)-B	0
S3 - S4	B	P(1-f)	X(s)-B-P
S4 - S5	B	P(1-f)	X(s)-B-P-(1-θ)t(c)(X(s)-B-P-D)
S5 - S6	B	P(1-f)	X(s)-B-P-t(c)(X(s)-B-P-D + G

Following Table 2, the valuation expression can be written as

$$\begin{aligned}
 V = & \int_{S1}^{S2} [X(s)Pb(s)]ds + \int_{S2}^{S3} [X(s)-B]Pd(s)ds + \int_{S5}^{S6} BPd(s)ds \\
 & + \int_{S3}^{S6} P(1-f)Pd(s)ds + \int_{S3}^{S4} [X(s)-B-P]Pe(s)ds \\
 & + \int_{S4}^{S5} [X(s)-B-P-(1-\theta)t(c)(X(s)-B-P-D)]Pe(s)ds \\
 & + \int_{S5}^{S6} [X(s)-B-P-t(c)(X(s)-B-P-D)+G]Pe(s)ds
 \end{aligned}$$

and the appropriate first order condition is

$$\begin{aligned} \partial V / \partial P &= \int_{S5}^{S6} [Pd(s)(1-f) - Pe(s)(1-t(c))] ds \\ &+ \int_{S4}^{S5} [Pd(s)(1-f) - Pe(s)(1-(1-\theta)t(c))Pe(s)] ds \\ &+ \int_{S3}^{S4} [Pd(s) - Pe(s)] ds \end{aligned}$$

Under the special conditions of risk neutrality and homogeneous beliefs,

$$\begin{aligned} \partial V / \partial P &= [PD(1-f) - PE] \int_{S3}^{S4} p(s) ds + [PD(1-f) - PE(1-(1-\theta)t(c))] \int_{S4}^{S5} (p(s)) ds \\ &+ [PD(1-f) - PE(1-t(c))] \int_{S5}^{S6} (p(s)) ds \end{aligned}$$

The first order condition is of the same form as in the case of riskless debt, but differs with regard to the probabilities of the respective state intervals.

This is because $\int_{S1}^{S2} p(s) ds > 0$ in the case of risky conventional debt,

implying that the supply curve has a greater slope than when conventional debt is riskless.

7. Development of Analytical Propositions

The integrated model of corporate pension policy and capital structure presented in the previous sections demonstrates that in a equity biased

world, the existence of corporate tax shield substitutes for pension contributions implies an interior optimum pension policy which is unique to each firm. The purpose of this section is to discuss four analytical propositions which originate from the liability-based model derived in this paper.

Let "a" denote a dummy parameter which represents G, D, t(c), or B, and define T_d to be the total differential of an expression. Totally differentiating the first order condition (equation 3) evaluated at P^* yields:

$$\begin{aligned} T_d &= \frac{\partial(\partial V/\partial P)}{\partial P^*} \partial P^* + \frac{\partial(\partial V/\partial P)}{\partial a} \partial a = 0 \\ &= (\partial^2 V/\partial P^2) \partial^2 V/\partial P \partial a \partial a = 0 \end{aligned}$$

Solving for $\partial P/\partial a$,

$$(\partial P^*/\partial a) = -(\partial^2 V/\partial P \partial a) / \partial^2 V/\partial P^2.$$

If the second order condition is satisfied ($\partial^2 V/\partial P^2 < 0$), then the sign of $\partial P^*/\partial a$ is determined by the sign of the cross partial,

$\partial^2 V/\partial P \partial a$. From (2) it follows that:

$$\partial P^*/\partial D = \partial^2 V/\partial P \partial D = -\theta t(c) - Pe(S4)(\partial S4/\partial D) - t(c)(1-\theta)Pe(S3)(\partial S3/\partial D) < 0$$

$$\partial P^*/\partial B = \partial^2 V/\partial P \partial B = \partial^2 V/\partial P \partial D - [(1-f) Pd(S2) - Pe(S2)] \partial S2/\partial B$$

$$\partial P^*/\partial G = \partial^2 V/\partial P \partial G = -\theta t(c) - Pe(S4)(\partial S4/\partial G) < 0$$

$$\partial P^*/\partial X(s) = \partial^2 V/\partial P \partial X(s) = [(1-f)Pd(S5) - (1-t(c))Pe(S5)] \partial S5/\partial X(s) > 0.$$

The sign of the first derivative which is presented, $\partial P^*/\partial D$, is unambiguously negative. This is because θ , $t(c)$, $Pe(S4)$, $\partial S4/\partial D$, and $\partial S3/\partial D$ are all positive. The sign of the $\partial P^*/\partial B$ term is uncertain,

because under the equilibrium condition $PE > PD(1-f) > PE(1-t(c))$, the term in brackets is negative. The sign of the expression will therefore depend on the relationship between $\partial^2 V / \partial P \partial D$ and $[(1-f)Pd(S2) - Pe(S2)]$. The third derivative term, $\partial P^* / \partial G$, is always negative. Finally, because $\partial S5 / \partial X(s) > 0$ and $(1-f)Pd(S5) > (Pe(S5)(1-t(c)))$, the $\partial P^* / \partial X(s)$ term is strictly positive.

These cross partial derivatives can be translated into three analytical proportions:

P.1 Ceteris paribus, the level of unfunded pension liabilities which a firm possesses will be inversely related to its quantity of investment related tax shields, i.e., depreciation, depletion, and investment tax credits. Within the framework of the model, firms will offer less uncertain deferred compensation the greater the amount of competing investment related tax shields which they are entitled to. This is because the non-pension tax shields reduce the amount of operating income which can be sheltered by the pension contribution. More competing tax shields imply a greater risk that the marginal pension contribution will not yield its full potential tax benefit, reducing the incentive to offer pensions. It is expected that the capital-labor ratio of the firm would have a direct bearing on this relationship, through its impact on wage-depreciation substitution. This proposition can be combined with Arnott and Gersowitz's [1980] findings to investigate how investment and production policies can affect the employment contracts.

P.2.1 Ceteris paribus, in a cross sectional analysis of firms, the level of unfunded pension liabilities will vary inversely with the quantity of

interest on conventional debt outstanding if $\partial^2/\partial P\partial D < [(1-f)Pd(S2) - Pe(S2)]$. This condition from equation (2) will exist when the tax advantage of the marginal pension contribution outweighs the cost advantage of additional unfunded pension liabilities in the capital structure.

P.2.2 Ceteris paribus, in a cross sectional analysis of firms, the level of unfunded pension liabilities will vary directly with the quantity of interest on conventional debt outstanding if $\partial^2/\partial P\partial D > [(1-f)Pd(S2) - Pe(S2)]$. This condition from equation (2) will exist when the tax advantage of the marginal pension contribution is less than the cost disadvantage of additional unfunded pension liabilities in the capital structure. Propositions 2.1 and 2.2 might be used to modify the empirical model used by Feldstein and Seligmen [1981], Lee, Wei and Alderson [1983] and others.

P.3 Ceteris paribus, the level of unfunded liabilities will be directly related to the expected operating earnings ($X(s)$) of the firm before depreciation and pension expense. The reason for this is obvious. Greater levels of operating earnings provide a greater assurance that the full tax benefits of the marginal pension will be recognized.

Equations (1) through (3) show that a key determinant of corporate pension policy is the probability of full utilization of the marginal pension contribution. Hypotheses P.1, P.2, and P.3 result from the fact that the state intervals in equation (1) are defined by the parameters $B, D, G, \theta, t(c),$ and $P,$ as they relate to the state contingent variable $X(s)$. It is implicit in the first order condition (3) that an increase in the right skewness of $X(s)$ will also increase the probability of full

utilization of the marginal pension contribution. A fourth proposition might therefore be articulated as follows:

P.4 Ceteris paribus, firms with relatively greater right skewness in the distributions of their state contingent operating earnings $X(s)$ will possess a greater level of unfunded pension liabilities.¹⁹

The possible influences suggested by the fourth proposition should affect the implications of P.3, since skewness and expected values are positively correlated.

8. Summary and Conclusions

The bulk of research performed on corporate pension policy has approached the issue from an asset-based perspective, concentrating on the merits of contributing assets to the pension fund. The implications of these models therefore tended to be of an extreme nature. Following the precedent of Arnott and Gersowitz [1980], this paper has dealt with the corporate pension funding decision as a part of the overall capital structure decision. The liability-based model assumes that the tax system is in different ways biased in favor of both pension income and income from equity securities. The assumed environment leads employees to seek deferred compensation arrangements with their employers. If employees are assumed to sell their deferred compensation benefits in the capital markets, the model shows that an optimal level of unfunded pensions will exist in the economy. When firms have different amounts of non-pension tax shields, an optimal pension policy will obtain for each firm, in equilibrium.

The model offers an important contribution to the pension funding literature because it justifies possible interior optimum funding levels

in a value maximization framework. This result differs from prior research, which advocated either full or restricted funding policies. A pension policy model in terms of Senbet and Taggart [1984] and Brock and Turnovsky (1981) models will be investigated in the future research.

Footnotes

1. If progressive rates were to apply to wage income, individuals in higher tax brackets would have less pension income, and would consequently sell less P-debt. This would not reduce the total amount of P-debt supplied, however, as long as the flat tax rate is set at a level sufficient to collect the same dollar revenue as a progressive tax system would.
2. The after-tax return from one dollar of pre-tax corporate income is $(1-t(c))(1-t(pe,i))$ to an investor who buys equity securities. The comparable return to P-debt is $(1-f)(1-t(pd,i))$. The latter return accounts for the fact that labor pays a flat tax before the loan of $f\%$ on wage income which it receives at time 0.

The brackets are defined on the basis of the relationship between the respective after-tax returns on P-debt and equity. The first bracket consists of individuals whose personal tax rates, $t(pd,i)$, and $t(pe,i)$, are such that the return from one dollar of pre-tax corporate income is greatest when received through P-debt rather than equity ownership. Bracket 2 individuals are indifferent to either form of investment, because both P-debt and equity vehicles deliver the same net dollar amount. In a similar vein, bracket 3 investors prefer equity investment to P-debt because they pay less taxes on pre-tax corporate income when it is received through equity.

3. In this model, operating earnings are defined as earnings before interest, taxes, and pension expense.
4. There is a wealth of literature which rationalizes the conventional debt decision in terms of prospective agency costs, management risk preferences, and/or other non-tax motivations. In order to make the point of this research more clearly, limitations on the amount of bonds are attributed to these factors.
5. The assumption that labor offsets unfunded liabilities with personal borrowing can be justified with a "Debt and Taxes" (Miller, 1977) argument. If unfunded liabilities truly are borrowing, then they should take up some of the limited capacity for debt holdings in the economy. Assuming that employees sell personal debt in an amount equal to the magnitude of unfunded liabilities is merely a means of operationalizing the debt capacity absorption. One must also consider the fact that labor negotiations are typically concerned with both current and deferred compensation issues. From the standpoint of the sponsoring firm, pension costs originate from two sources: normal cost, that portion attributed to current services, and prior service cost, the portion of benefits granted (1) for service prior to the adoption of the pension plan (past service cost) and (2) as a result of amendments to an existing plan. While annual contributions are required to cover normal costs, prior service costs can be amortized over a period of 30 to 40 years, depending on their origin. When labor accepts an agreement granting pension benefits for past services

(and/or increases thereof) which are currently unfunded, they are presumably doing so in lieu of cash wages, and as such are extending credit to the firm. This credit has a cost, because underfunding at the current time requires the firm to contribute a greater amount in the future, to make up for lost interest equivalents. These tax deductible interest equivalents are not unlike the interest expense on conventional debt, justifying their treatment as such in the model.

6. Because financial liabilities are financial assets simultaneously, it follows that unfunded obligations should be considered to absorb some of the economy's limited capacity for debt.
7. In a world without differential taxation, the single price law of markets requires that $P_d(s) = P_e(s)$ in equilibrium. For otherwise to be the case would introduce arbitrage profit opportunities in a world with unlimited short selling. When differential taxes are introduced, however, debt and equity securities are no longer perfect substitutes. In the absence of mechanisms to remove the effect of differential taxes, $P_d(s) = P_e(s)$ is perfectly consistent with equilibrium.
8. The Internal Revenue Code of 1954 as amended currently limits firm's use of tax credits to \$25,000 plus 90% of their pre-credit liability in excess of \$25,000.
9. The cash flows to P-debtholders are of course net of the flat personal income tax paid by the beneficiaries. The total cash flow to labor at time 0 consists of two parts. The first part consists of their gross wage less the present value of the loan and applicable taxes (a). The second part comes from the proceeds of the personal debt which they sell. That amount is equal to the present value of the expected pension benefit after taxes (b). Defining the total of the two as L,

(a)

$$L = [[w - [(X(s) - B) \int_{S1}^{S2} p(s)ds + P \int_{S2}^{S5} p(s)ds] (1-f)PD] (1-f-s)]$$

(b)

$$+ PD(1-f) [(X(s) - B) \int_{S1}^{S2} p(s)ds + P \int_{S2}^{S5} p(s)ds]]$$

If the labor markets are efficient, one might expect them to adjust w for a given P so that a desired expected value of L was attained. Differentiating L with respect to P and re-arranging,

$$\begin{aligned} \partial L / \partial P &= (1-f)(f+s)PD \int_{S2}^{S5} p(s)ds > 0 && S5 > S2 \\ &= 0 && S5 = S2 \end{aligned}$$

This shows that labor will never be worse off by loaning back funds to the firm, because the process allows the firm and labor to effect a transfer of wealth from the Social Security system to themselves.

10. It might at first seem odd that firms would want to maximize the market value of P -debt. Further thought shows that this is quite rational, since higher P -debt market values increase the potential flow of funds from labor to the firm.

Note that in this model labor is assumed to pay the employer portion of the FICA tax. This assumption is justified by the fact that employers can easily pass this tax on to employees in the form of reduced wages.

11. Chapter 3 of Alderson [1] deals with the characteristics of the model without the assumptions of risk neutrality and homogeneous beliefs.
12. As DeAngelo and Masulis point out, the personal tax disadvantage of P -debt is constant only for riskless debt. When P -debt is risky, equilibrium will still obtain in a unique interior sense, but the explanation is that the corporate advantage of pension income declines at the margin faster than the personal P -debt disadvantage. Further the assumption that pension benefits are riskless is consistent with the terms of ERISA (Employee Retirement Income Security Act), which established that corporations are liable for their unfunded liabilities up to 30% of net worth.
13. Proof of supply curve elasticity can be found in Appendix C.
14. It is not unreasonable to expect these conditions to hold. First, firms do default on their pension obligations and/or lose tax shields. Also, bracket 1 investors should be relatively important in the U.S. markets. This is because in the U.S. the corporate tax rate is 46%, which implies in Miller's special case that all individuals with a personal tax rate on debt of less than 46% would always prefer debt when $PD(1-f) > PE(1-t(c))$.

15. DeAngelo and Masulis point out that it is unnecessary to assume an equity biased tax code, as long as the subset of investors in bracket 1 not having equity biased personal rates is small so that those investors will never be marginal in their debt-equity decision. $P_d(s)(1-f) < P_e(s)$ will therefore imply disequilibrium.
16. This is Miller's special case in which the potential to defer capital gains perpetually results in tax free equity returns.
17. When $t(pe,i) > 0$ and $S_2 \neq 0$, the first order condition evaluated at P^* becomes:

$$\partial V / \partial P [P^*] =$$

$$\begin{aligned} & (PD(1-f)/1-t(pd,u)) [t(pe,u) \int_{S_2}^{S_5} p(s)ds \\ & + (1-t(pe,u)t(c)) (1-\theta) \int_{S_3}^{S_4} p(s)ds \\ & + (1-t(pe,u)t(c)) \int_{S_4}^{S_5} p(s)ds - t(pd,u) \int_{S_2}^{S_5} p(s)ds \end{aligned}$$

When $t(pe,i) = 0$ and $S_2 = 0$, $p(s)ds = 1$, and the expression reduces to equation (6).

18. This can be shown by differentiating the term in brackets with respect to P ,

$$- \theta (d(S_4)/\partial P)p(S_4) - (1-\theta)(d(S_3)/\partial P)p(S_3) < 0$$

19. The fourth hypothesis deals with skewness because there is an obvious link between the magnitude of the third moment and the probability of the highest state interval. No such linkage can be established with regard to the second moment without strong assumptions on the form of the distribution of operating earnings.

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Appendixes

These appendixes provides detail on the mathematical techniques used in the liability-based model of corporate pension policy.

A. First Order Conditions

$$\text{If } G = \int_{f(a,1)}^{f(a,2)} g(a,x)dx, \text{ then}$$

$$\partial G/\partial a = (\partial f(a,2)/\partial a) \times g(a, f(a,2)) - (\partial f(a,1)/\partial a) \times g(a, f(a,1))$$

$$\int_{f(a,1)}^{f(a,2)} \partial(g(a,x)/\partial a)dx$$

Accordingly, from equation (1)

$$\partial V/\partial P = (\partial(S2)/\partial P[(X(S2)-B)(1-f)Pd(S2)] - (\partial(S1)/\partial P)[(X(S1)-B)(1-f)Pd(s)]$$

$$+ \int_{S1}^{S2} Ods + (\partial S3/\partial P)[P(1-f)Pd(S3) + (X(S3)-B-P)Pe(S3)]$$

$$- (\partial S2/\partial P)[P(1-f)Pd(S2)+X(S2)-B-P)Pe(S2)] + \int_{S2}^{S3} [Pd(s)(1-f)-Pe(s)]ds$$

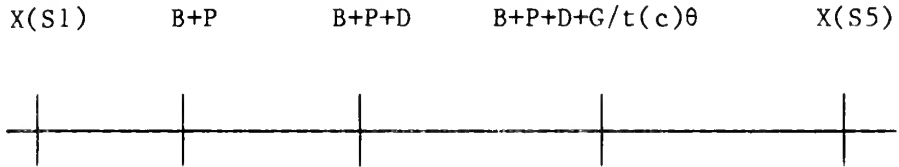
$$+ (\partial S4/\partial P)[P(1-f)Pd(S4)+X(S4)-B-P-(1-\theta)t(c)(X(S4)-B-P-D))Pe(S4)]$$

$$- (\partial S3/\partial P)[P(1-f)Pd(S3)+X(S3)-B-P-(1-\theta)t(c)(X(S3)-B-P-D))Pe(S3)]$$

$$+ \int_{S3}^{S4} [Pd(s)(1-f) - (1-(1-\theta)t(c))Pe(s)]ds]$$

$$\begin{aligned}
 &+ (\partial S5/\partial P)[P(1-f)Pd(S5)+X(S5)-B-P-t(c)(X(S5)-B-P-D)+G)Pe(S5)] \\
 &- (\partial S4/\partial P)[P(1-f)Pd(S4)+X(S4)-B-P-t(c)(X(S4)-B-P-D)+G)Pe(S4)] \\
 &+ \int_{S4}^{S5} [Pd(s)(1-f) - (1-t(c))Pe(s)]ds
 \end{aligned}$$

The state intervals are:



Accordingly, $(\partial S2/\partial P) = (\partial S5/\partial P) = 0$, and all terms involving the limits of integration cancel out. Therefore, the expression reduces to:

$$\begin{aligned}
 V/P = & \int_{S2}^{S3} [Pd(s)(1-f)-Pe(s)]ds + \int_{S3}^{S4} [Pd(s)(1-f)-(1-(1-\theta)t(c))Pe(s)]ds \\
 & + \int_{S4}^{S5} [Pd(s)(1-f)-(1-t(c))Pe(s)]ds
 \end{aligned}$$

B. Proof of Interior Solution

In order to show that an interior solution exists, the left and right hand derivatives must first be examined. Evaluating $\partial V/\partial P$ at $P = X(S5)$,

$$\partial V/\partial P[P=X(S5)] = [PD(1-f)-PE] \int_{S2}^{S5} p(s)ds < 0$$

when $PE > PD(1-f) > PE(1-t(c))$. Repeating the procedure at $P=0$,

$$\partial V / \partial P [P=0] = [PD(1-f) - PE(1-t(c))] \int_{S1}^{S5} p(s) ds > 0$$

This shows that interior decisions strictly dominate corner solutions.

A unique interior optimum obtains if V is convex in P . Differentiating (2) yields:

$$\begin{aligned} \partial^2 V / \partial P^2 = & \theta t(c) PE (\partial S4 / \partial P) p(S4) - (1-\theta) t(c) PE (\partial S3 / \partial P) p(S3) \\ & + (PD(1-f) - PE) (\partial S2 / \partial P) p(S2) \end{aligned}$$

When pension benefits are riskless, then $\partial S2 / \partial P = 0$, causing the third term to vanish and leaving $\partial^2 V / \partial P^2$ unquestionably negative, since $\partial S4 / \partial P$ and $\partial S3 / \partial P > 0$. If pension benefits are risky, the $\partial^2 V / \partial P^2 < 0$ as long as the first two terms dominate the third.

C. Proof of Supply Curve Elasticity

If SP is defined as the expected personal before tax cash flow supplied as pension benefits, then the supply curve is upward sloping if:

$$\partial SP / \partial PD = (\partial SP / \partial P^*) (\partial P^* / \partial PD) > 0$$

That this is true can be seen by realizing that

$$\partial SP / \partial P = \int_{S1}^{S5} p(s) ds > 0.$$

Further, totally differentiating the first order condition when set equal to zero and allows P^* and PD to vary,

$$Td(\partial V/\partial P) = \frac{\partial(\partial V/\partial P)}{\partial P} \partial P^* + \frac{\partial(\partial V/\partial P)}{\partial PD} \partial D = 0$$

Solving for $\partial P^*/\partial PD$,

$$\begin{aligned} \partial P^*/\partial PD &= (\partial^2 V/\partial PD \partial P) / -(\partial^2 V/\partial P^2) \\ &= \int_{S1}^{S5} p(s) ds / -(\partial^2 V/\partial P^2) > 0 \end{aligned}$$

Therefore, $\partial SP/\partial PD > 0$, and the supply of P -debt is smoothly upward sloping in $PD(1-f)$.

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