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U.S. Army
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INTERACTIONS OF THE BEACH - OCEAN - ATMOSPHERE SYSTEM AT VIRGINIA BEACH, VIRGINIA

TECHNICAL MEMORANDUM NO. 7

DEPARTMENT OF THE ARMY
CORPS OF ENGINEERS

INTERACTIONS OF THE BEACH-OCEAN-ATMOSPHERE SYSTEM AT VIRGINIA BEACH, VIRGINIA

by

W. Harrison
and
W. C. Krumbein

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An understanding of the changes taking place in a beach environment requires the sorting out and ranking in importance of the various processes acting upon each of the environmental factors being analyzed. Also required is a knowledge of the time lag between the inception of a group of "processes" and the moment of their maximum effect on the "response" being analyzed.

The approach to these two problems followed in this study involves the use of some 27 variables of the beach-ocean-atmosphere system at Virginia Beach, Virginia. The sorting out and ranking of the variables in a given analysis is approached through linear and quadratic multiregression analysis, as programmed for high-speed computers. One of the variables in the system is selected as the dependent variable and studied in relation to several controlling independent variables, by taking the latter one at a time, two at a time, and so on until all of the independent variables are included simultaneously. Thus the technique of sequential multiregression analysis is the tool used in the investigation. In addition, the dependent variable at time t_o is studied in its relation to the independent variables as measured at successive lag periods t_1, t_2, ... , backwards in time.

This report was prepared by Wyman Harrison, formerly Associate Professor of Marine Science, Virginia Institute of Marine Science, in pursuance of Contract DA-49-055-CIV-ENG-64-5 with the Coastal Engineering Research Center, in collaboration with W. C. Krumbein, Professor of Geology, Northwestern University, and consultant to the Coastal Engineering Research Center.

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INTERACTIONS OF THE BEACH-OCEAN-ATMOSPHERE SYSTEM AT VIRGINIA BEACH, VIRGINIA

by

W. Harrison
U. S. Coast and Geodetic Survey, Washington, D. C.[(1)] and
Virginia Institute of Marine Science, Gloucester Point, Virginia

and

W. C. Krumbein
Northwestern University, Evanston, Illinois

ABSTRACT

A number of interactions among beach variables are investigated by sequential linear multiregression analysis, as programmed for high-speed computers. The study includes the influence of beach geometry, wave characteristics, tidal effects, and local wind conditions on the velocity of longshore currents, deposition and erosion on the lower foreshore, and the response of grain size and beach slope to shore processes.

Results show that if about six variables are segregated out of any group of about a dozen, these six account for essentially all of the variability that is explained by all twelve. Thus, the regression method serves to condense relatively large data matrices to more compact form.

The most-influential combinations of variables arbitrarily designated as "process" variables are in general agreement with significant variables of wave-tank experimentation, and substantiate intuitive judgments regarding the relative importance of these variables on natural beaches. The results suggest that certain additional variables, seldom examined under controlled conditions, be studied in combination with variables normally examined in wave tanks.

The combination of six variables found to be most influential in the determination of longshore-current velocity in the study area is made up of wave period, wave height, lower-foreshore slope, wind velocity onshore, wind velocity offshore, and angle of wave approach, in order of decreasing importance. The significance of wind velocity on and offshore is believed to lie mainly in the ability of the wind to alter the form of incoming swells. A special regression analysis for quadratic effects reveals that water density is highly non-linear in its effect on longshore-current velocity.

Bottom slope in the shoaling-wave zone, some 250 feet seaward of the breakers, is found to be controlled primarily by average mean grain size of the bottom materials, wave period, wave length, wave steepness, water depth, and tidal-current velocity. This combination exerts its

(1) Present address. Study completed while at the Virginia Institute of Marine Science.

maximum influence on the slope through a lag in time of between 4 and 8 hours, and apparently to a lesser extent between 16 and 20 hours. Mean grain size of the beach slope in the shoaling-wave zone is found to depend upon the combination: mean bottom slope, wave period, wave steepness, wind velocity parallel to shore, angle of wave approach, and tidal-current velocity, and this combination is most influential after an 8-12 hour lag in time. When mean slope, the most-dominant independent variable, is removed from the analysis, water density and tidal-current velocity appear as the most influential variables on mean grain size. Wind velocity parallel to shore is believed important because it will influence the velocities of the tidal currents that flow parallel to the shore in the study area. Angle of wave-front approach may at times significantly augment or decrease tidal-current velocities near the bed and thereby the sizes of particles moved. Wind velocity onshore and offshore at times interlocks with water density, as density varies when stratified shelf waters undergo turnover. Fluid drag velocities vary as water density varies, and differing sizes of particles will be moved.

Net deposition on the lower foreshore during June and July is most influenced by slope of the foreshore. Wave period, wave height, wind velocity on-shore, angle of wave approach, and water density are variables that form the most-influential combination when in conjunction with lower foreshore slope. This combination expresses itself 8 to 12 and 20 to 24 hours prior to the low-tide time of measurement of net deposition; that is, during times of rising tide. The regression analysis suggests that net erosion on the foreshore is not nearly as much influenced by beach slope angle as is net deposition, but that lower-foreshore slope is still the most consistently important variable to net erosion through time. The combination of five variables suggested as most influential to net erosion during June and July includes lower-foreshore slope, wave period, wind velocity offshore, angle of wave approach, and depth to the water table at the top of the uprush. Of secondary importance, and manifesting at times of falling tide, is the combination made up of lower foreshore slope, wave period, wave height, and angle of wave approach.

Problem areas reviewed in the study are related to redundant and noisy data and to the linear model used. Descriptions of the regression techniques for the linear and higher-order models are also given.

INTRODUCTION

General Considerations

Study of the beach-ocean-atmosphere system under natural field conditions progresses from initial descriptive studies, consisting mainly of masses of seemingly unrelated observations, to the analysis of "cause-and-effect" (process-and-response) relationships between oceanic and atmospheric forces and their resulting products, the beaches. While in the simplest sense it is possible to choose some intuitively rational group of environmental processes that act as causal factors, and a like group of environmental responses that act as effects, this simple picture is complicated by numerous interrelationships among the various causal factors as well as among the response elements.

This study is mainly a "search procedure" for identifying interactions among subsets of the numerous variables that operate in the natural beach environment. A number of "causes" and "effects" common to the general environment are measured and evaluated, consistent with available time and resources. Least-squares techniques are employed in this search for relationships between the various individual "effects" and their attendant "causes."

When the beach-ocean-atmosphere complex is considered as a whole, it is evident that no single functional equation can at present fully express the complex interrelations that occur in nature. The tendency has been to "fragment" the system into portions, either in controlled wave-tank experiments or studies of a limited number of variables in the field, that may help explain some facets of the many phenomena composing the whole. It is one purpose of this paper to refrain from expressing the data to be presented, either within the framework of process-response models (Krumbein and Sloss, 1963, Chapter 7; Whitten, 1964), or as more formal deterministic models. Rather, the intent is to "sort out" sets of variables to see whether they may provide a basis on which more formal models can be erected.

The method used here is a form of sequential multiple linear regression to be described later. It is recognized that this is only one of several search procedures that may be used, and it is anticipated that opportunity may arise for extending the study in later publications by using alternative methods, such as factor analysis and discriminant functions, to see whether some optimum model for search can be identified.

Area of Investigation

This study was conducted at Virginia Beach, Virginia, an oceanic beach situated near the entrance to Chesapeake Bay, at the center of the mid-Atlantic Bight (fig. 1A). The study was concentrated along a strip of shoreline between 6 1/4 and 8 miles south of Cape Henry (fig. 1B). A relatively small, controlled inlet occurs near the center of the study strip. (The strip is bounded by the northern and southern transects of figure 1C). A study by Harrison, Krumbein, and Wilson (1964) indicates that the influence of the inlet on the adjacent beach is confined to a zone 500-900 feet to either side of its mouth and that it exerts no measureable influence at the northern or southern transects where all of the measurements for this study were made.

Surveys by the U. S. Army Corps of Engineers show that beach slopes in the foreshore area range from about 1:17 to 1:28, while beach slopes between roughly the 6 and 20-foot contours range between 1:50 and 1:60 (U. S. Congress, 1953, p. 13). Details of beach profile modifications in the study area are presented by Harrison and Wagner (1964), along with several maps of the nearshore bottom that show the presence of

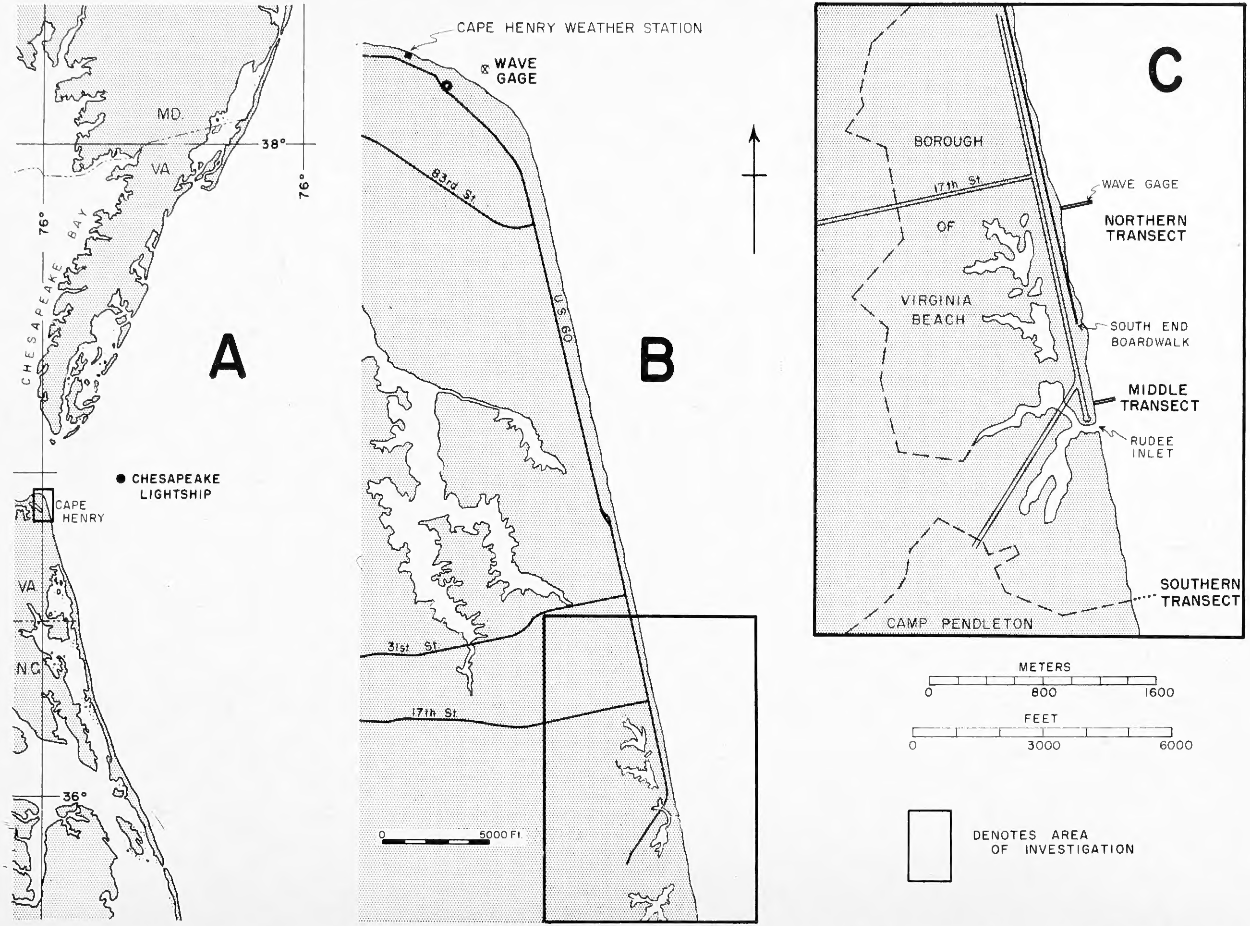

FIGURE 1. MAPS SHOWING AREA OF INVESTIGATION AND TRANSECTS AT WHICH MEASUREMENTS WERE TAKEN

bar-trough rhythms. Because an artificial beach-nourishment project has been in operation within the study area since 1956, the measurements for this study were taken during times of little or no pumping of sand.

Physical and dynamic properties of 219 sand samples from the area have been studied by Harrison and Morales-Alamo (1964), and the averages of several properties for various dynamic zones of the beach may be summarized as follows:

Zone	Mean Size in mm		Sorting Coefficient	Reynolds Number (Under Average Sea-Water Conditions)
	Nominal Dia.	Sieve Dia.*		
Shoaling-wave	0.25	0.22	0.70	4.8
Breaking-wave	0.30	0.27	0.70	8.1
Swash	0.28	0.25	0.55	6.2
Swash-berm	0.37	0.33	0.54	12.0

*Converted from nominal diameter (Int.-Agcy Comm. Wat. Res., 1957, fig. 5)

Sand samples with mean grain sizes approximating the above average values exhibit about 10% (by number) of heavy minerals and 2% or less of rock or shell fragments. Thus, the beach is composed largely of medium to fine quartz sand. Small samples of the average sand of the beach exhibit relatively low Reynolds numbers under average temperature and salinity conditions of the seawater.

Tides at Virginia Beach are of the semi-diunal (equal) type and have a mean range of 3.0 feet, as against a spring range of 3.6 feet. Tidal currents in the study area are related to the ebb and flood through the entrance to Chesapeake Bay. They are generally reversing in nature, and are usually parallel to the shore. Harrison, Brehmer, and Stone (1964, fig. 4) present evidence from which it may be calculated that peak tidal-current velocities some 3,000 feet from shore and a few feet above the bottom do not exceed roughly 0.5 knot (0.85 ft /sec) in the study area. Peak velocities measured one meter above the bottom at the end of the 15th Street fishing pier (fig. 1C, "northern transect") approximated 0.68 ft/sec. This pier marks the northern terminus of the study strip, and it is here that the tidal currents are at a maximum for the study area.

The wave climate for the study area may be estimated from an analysis by Harrison of four years of wave records from a stepped-resistance wave gage (fig. 1B) maintained by the U. S. Navy in 20 feet of water off Cape Henry and from five years of wave observations at Chesapeake Lightship (fig. 1A), in 60 feet of water. The brief summary that follows indicates that roughly two-thirds of the waves that could be expected to strike Virginia Beach come from only three directions and that mean heights and periods are relatively low.

Significant Wave Analysis				Visual Swell Observation	
Wave Period		Wave Height		Direction in Range 05° -164° (True N) From Which Swells Come	
Mean (sec)	1st Mode (sec)	Mean (ft.)	1st Mode (ft.)	Direction	Percent of Total Swells
5.3	4.0	1.7	2.0	45°-54° 85°-94° 135°-144°	30.0 20.0 18.0

A breakdown of the 15 most-frequent associations of wave period, height, and angle of approach at the Chesapeake Lightship is given in Harrison and Wilson (1965), together with details of the refraction of these wave fronts as they move into the study area.

Surf statistics at the Virginia Beach Life Boat Station, 1.6 miles north of Rudee Inlet (fig. 1C), have been complied by Helle (1958). Three years' records, for observations every four hours, reveal that surf is 4 feet or higher 10% of the year, 3 feet or higher 50% of the year, and 2 feet or higher 95% of the year. Surf tends to be highest in early fall when the angle of wave approach tends to be from the east (or ENE). Surf is also high in January, when it is largely out of the northeast. The average period of the surf tends to be greatest April through July (around 6.0 seconds).

Wind data from the U. S. Weather Bureau's Cape Henry station (fig. 1B) for a 16-year period is summarized in the Virginia Beach, Va., Erosion Control Study (U. S. Congress, 1953, plate 5). Analysis of these data shows that whereas prevailing winds are from the southern quadrants, velocities and total wind movements are greater from the northern quadrants. Northeast winds tend to be most common in September.

METHODS

Measured Variables

A list of the variables measured in the beach-ocean-atmosphere system at Virginia Beach is presented in table 1, and descriptions of the techniques used in their measurement are given in Appendix A. Of all of the measurements used in this study, most were taken in the months of February, June, and July. Some were taken in March and April. Thus, a winter and a summer set of data were available for combination and study. (Only summer data were used in the evaluation of the dependent variables J_f and K_f).

Table 1.--Variables Used in Analysis of Interactions of Beach-Ocean-Atmosphere System. (See Appendix A for Descriptions and Units Used.)

Symbol	Dimensions	Variable Description
B_d	L	Depth of water at wave breaking
C	LT^{-1}	Velocity of tidal current
C_o	LT^{-1}	Velocity of tidal current flowing opposite to longshore current
C_s	LT^{-1}	Velocity of tidal current flowing in same direction as longshore current
D	L	Depth of water table at top of uprush
h	L	Still-water depth
H_b	L	Height of breaking wave
H_o	L	Height of wave (expressed as deep-water height)
H_o/L_o	0	Wave steepness of wave (expressed as deep-water wave)
J_f	L	Net deposition at lower foreshore stations in 24.5-hour period
K_f	L	Net erosion at lower foreshore stations in 24.5-hour period
L_o	L	Wave length (expressed as length in deep water)
$(\overline{M_z})_s$	L	Average mean nominal grain diameter over bottom in shoaling-wave zone
R	L	Range of tide over one tidal cycle
$\overline{S}_f$	0	Mean slope of lower foreshore of beach
$\overline{S}_s$	0	Mean slope of beach over inner portion of shoaling-wave zone
T	T	Wave period
$\overline{U}_a$	LT^{-1}	Mean wind velocity directed against the longshore current
$\overline{U}_{of}$	LT^{-1}	Mean wind velocity in an offshore direction
$\overline{U}_{on}$	LT^{-1}	Mean wind velocity in an onshore direction
$\overline{U}_p$	LT^{-1}	Mean wind velocity parallel to shore
$\overline{U}_s$	LT^{-1}	Mean wind velocity in same direction as longshore current
$\overline{V}$	LT^{-1}	Mean velocity of longshore current
α	0	Angle of wave approach
η_r	LT^{-1}	Rate of rise of still-water level
η_f	LT^{-1}	Rate of fall of still-water level
ρ	ML^{-3}	Water density

Wind, tide, and most wave measurements were made every two hours, both day and night. Some of the other variables listed in table 1 (B_d, D, H_b, α, and ρ) were measured four times a day at 0700, 1100, 1500, and 1900 hours. (Values at 2300 and 0300 hours were arrived at by linear interpolation.) The variables J_f, K_f, $(\overline{M_z})_s$, $\overline{S}_f$, and $\overline{S}_s$ were usually measured once each day at low tide, while $(\overline{M_z})_s$ and $\overline{S}_s$ were measured every 4 hours during the February field period.

Approach

Designation of "cause" and "effect", or "stimulus" and "response", is rather arbitrary in a system of complexly interlocked variables (like those of table 1), all of which may vary simultaneously. In a general way it is possible to say that relationships between the variables in a specific beach environment are conditioned by the natural ranges in magnitude of the variables and by their natural frequencies of occurrence. Ranges in the magnitude of wave height, tidal-current velocity, wind velocity in various diections, and grain size distribution in the various littoral zones are all characteristic for a given beach. Also characteristic are the frequency of occurrence of waves of a given height, currents of a given velocity, and so on.

Ideally then, to understand the interaction of a variable singled out as an "effect" with a number of other variables designated as "causes", one must measure both "causes" and "effects" over the range of values that they assume in the study area. Understanding of erosion of the foreshore of a beach as it is "caused" by winds, and currents, for example, can come only when foreshore slope, wind velocity, wave height, longshore-current velocity, etc., are measured through their expectable range of values and over a long enough time period so that adequate representations of natural frequencies are obtained.

It is to be noted that consideration must be given to the fact that the moment of maximum interaction of an effect with its several causes may be influenced by a time delay. The well-documented delay in the response of laboratory beach slopes to unvarying wave trains in wave-tank experimentation is a good example of the delay factor in this cause-and-effect relationship.

It is also to be noted that a given variable designated as an "effect" in the beach-ocean-atmosphere system may in fact be little influenced by some of the variables that are considered as "causes". Wind velocity parallel to the shore taken as an effect, for example, is intuitively independent of water density, angle of wave approach, and mean grain size on the foreshore taken as causes. For this reason it becomes intuitively "unprofitable" to investigate wind velocity parallel to the shore as a function of such improbable causes. Where it appears, however, that one of the measured variables is significantly dependent upon most of the other measured variables, it becomes desirable to investigate that inferred dependency.

Method of Analysis Used in This Report

Among several methods available for analyzing observational data that involve interrelationships among independent variables, the one chosen here is that of sequential multiple linear regression. It involves measures of the relationships of a given dependent variable ("effect") in terms of several controlling environmental "causes" (independent variables), by taking the latter one at a time, two at a time, and so on, until all of the environmental processes are included simultaneously.

This sort of analysis is commonly called stepwise regression, and it may be conducted with regression techniques or multiple and partial correlation techniques. These methods also permit study of interrelations among the independent variables themselves, and they are useful for evaluation of data redundancy.

Redundant variables, that is, variables that in large part restate what some other variable has already measured, are common in early stages of quantification in the observational sciences, especially when physical models are not clearly discernible in the complex of observations. In these cases a method for "sorting out" a set of independent variables in terms of their importance or meaningfulness in controlling the response of some dependent variable, Y, helps to reduce the number of variables in the set to more manageable proportions.

We shall illustrate the method with a subset of data from a larger example to be treated more fully in a later section of this paper. The problem here is to examine the nearshore bottom slope just seaward of the zone of breaking waves, as it responds to several shore process elements.

The full example includes a dozen independent variables, designated as X1, X2, ..., X12. We select five of these, retaining the same number designations that they have in the larger example, for ease of later comparsion. As set up, our introductory example includes the following variables:

Bottom Slope In Shoaling-wave Zone	Y
Mean grain size	X1
Wave period	X2
Wave height	X4
Angle of wave approach	X9
Still-water depth at time of slope measurement	X10

In conventional step-wise regression the "strongest" single X-variable is first obtained, and this is then "held constant," statistically, to identify the second strongest variable. Multiple and partial correlation is commonly used, and some step-wise computer programs have built-in provisions that fix the relative importance of a given X for all subsequent stages of analysis. This sometimes leads to spurious results, in that a variable that may be weak in combinations of two or three Xs may rise in relative importance as combinations of four or five Xs are considered. Hence, our analysis is based on a sequential regression analysis of all possible combinations of Xs, so that every X has a chance to enter into every possible combination. The reader is referred to Kemeny, and others (1958, chap. 5), and to Rao (1952, chap. 1) for reviews of the matrix algebra that follows.

The procedure adopted uses the computational form for the Y and five Xs of our illustration, in terms of the general linear model:

$$Y_i = \beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_4 X4 + \beta_9 X9 + \beta_{10} X10 + e_i$$

which is expressed computationally in matrix form as:

$$\underline{S}\,\underline{\hat{\beta}} = \underline{g} \tag{1}$$

where $\underline{g}$ is a 6 x 1 vector-Y, $\underline{S}$ is a 6 x 6 matrix of squares and cross-products of the Xs, and $\underline{\hat{\beta}}$ is a 6 x 1 vector of the estimated βs. In detail, the matrix equation is:

$$\begin{bmatrix} N & \Sigma X1 & \Sigma X2 & \cdot\cdot & \Sigma X10 \\ \Sigma X1 & \Sigma X1^2 & \Sigma X1X2 & \cdot\cdot & \Sigma X1X10 \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\ \Sigma X5 & \Sigma X5X1 & \Sigma X5X2 & \cdot\cdot & \Sigma X10^2 \end{bmatrix} \bullet \begin{bmatrix} \hat{\beta}_0 \\ \beta_1 \\ \cdot\cdot \\ \hat{\beta}_{10} \end{bmatrix} = \begin{bmatrix} \Sigma Y \\ \Sigma X1Y \\ \cdot\cdot \\ \Sigma X10Y \end{bmatrix} \tag{2}$$

The computer first inverts the entire matrix and post-multiplies the inverse by $\underline{g}$, to obtain $\underline{\hat{\beta}}$. The proportion of the total sum of squares of **Y** attributable to all five Xs is then computed and expressed as a percentage. This was found to be 78.7% in our example, which suggests that the set of five Xs taken together yields a fairly satisfactory predictor equation for nearshore bottom slope.

In extracting all possible combinations of Xs from the matrix in equation (2), the computer program is so arranged that it always starts with N in the upper left corner of $\underline{S}$, and always has ΣY as the first element of $\underline{g}$ in equation (1). Thus, in the first computer loop the Xs are taken one at a time, yielding the following as the first two sub-matrices:

$$\begin{bmatrix} N & \Sigma X1 \\ \\ \Sigma X1 & \Sigma X1^2 \end{bmatrix} \bullet \begin{bmatrix} \hat{\beta}_0 \\ \\ \hat{\beta}_1 \end{bmatrix} = \begin{bmatrix} \Sigma Y \\ \\ \Sigma X1Y \end{bmatrix} \text{ and } \begin{bmatrix} N & \Sigma X2 \\ \\ \Sigma X2 & \Sigma X2^2 \end{bmatrix} \bullet \begin{bmatrix} \hat{\beta}_0 \\ \\ \hat{\beta}_2 \end{bmatrix} = \begin{bmatrix} \Sigma Y \\ \\ \Sigma X2Y \end{bmatrix}$$

When all such combinations are computed, the next computer loop takes the 3 x 3 matrices having pairs of Xs, again starting with N as the upper left element of S, and ΣY as the first element of g. The first part of this loop has the submatrix for X1 and X2, the second has X1 and X4, and so on for all combinations of our example. The first submatrix in this stage is:

$$\begin{bmatrix} N & \Sigma X1 & \Sigma X2 \\ \Sigma X1 & \Sigma X1^2 & \Sigma X1X2 \\ \Sigma X2 & \Sigma X2X1 & \Sigma X2^2 \end{bmatrix} \bullet \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} = \begin{bmatrix} \Sigma Y \\ \Sigma X1Y \\ \Sigma X2Y \end{bmatrix}$$

Of course, the individual $\hat{\beta}$s change in value with each stage, but by continuing the looping process until all five Xs are used, it is possible to scan the computer output to identify the strongest individual X, the strongest pair of Xs, and so on.

The data for this example are given in table A and the complete output is given in table B. Before discussing the results, some remarks on the method of computation are appropriate. The total number combinations obtained by this procedure is 2^k-1, where k is the total number of Xs used with a given Y. In our example k = 5, and we obtain 31 elements of output. This rises rapidly as k increases: for 12 Xs the output has 4095 elements and for 13 Xs it is 8191. Thus there are practical limitations on the size of problem that can be economically handled, to say nothing of the sheer bulk of output for large sets of Xs. For many problems with k of the order of 10 or more, the sequence may be carried as far as say six Xs at a time, which commonly yields most of the information in the set of data. The computational procedure may be simplified by using deviation matrices, but the present procedure is given here, to agree with the computer program as described in Krumbein, Benson, and Hempkins (1964).

The Sum of Squares Criterion

Once the $\hat{\beta}$s are estimated for each combination of Xs, the raw sum of squares of the computed Y values is obtained by multiplying the transpose of the $\hat{\beta}$-vector by the vector g:

$$SSY_{raw} = \underline{\hat{\beta}}^T \underline{g}$$

The sum of squares of the observed values of Y is computed as:

$$SSY_{obs} = \Sigma\ (Y_i - \overline{Y})^2 = \Sigma Y_i^2 - \overline{Y}\Sigma Y_i$$

Table A.- Subset of Data From Virginia Beach Study

Sample Number	Beach Slope $(\overline{S}_s)$	Mean Grain Size $(\overline{M_z})_s$	Wave Period (T)	Wave Height (H_o)	Angle of Wave Approach (α)	Water Depth (h)
01	0.68	0.79	7.80	1.82	30.00	12.40
02	0.85	0.65	8.00	8.84	25.00	11.40
03	0.66	0.81	9.03	5.12	35.00	10.70
04	0.50	0.74	6.06	5.43	40.00	11.60
05	1.86	0.22	5.90	1.42	30.00	11.30
06	2.33	0.23	8.40	1.09	30.00	10.70
07	2.17	0.25	12.00	1.15	25.00	11.10
08	1.83	0.26	4.80	8.53	25.00	12.80
09	1.68	0.41	10.80	6.11	10.00	13.30
10	2.05	0.55	10.40	1.60	30.00	13.30
11	1.83	0.47	10.80	1.04	30.00	14.10
12	1.84	0.59	7.90	1.02	35.00	13.40
13	1.87	0.47	4.30	1.11	30.00	13.50
14	1.82	0.50	10.80	0.62	35.00	13.30
15	1.85	0.52	3.80	1.69	30.00	14.40
16	1.75	0.47	4.10	1.22	20.00	14.10
17	1.51	0.42	4.50	2.13	30.00	15.30
18	1.38	0.37	6.10	1.47	20.00	14.00

Table B.- All Combinations of Xs for Beach Data of Table A.

Independent Variable Combinations					Percentage of Sum of Squares of Y Accounted for:
X1 (Mean Grain Size)					63.1
X2 (Wave Period)					1.1
X4 (Wave Height)					23.7
X9 (Angle of Wave Approach)					5.6
X10 (Water Depth)					5.2
1	2				65.5
1		4			74.1
1			9		64.2
1				10	66.4
	2	4			24.4
	2		9		6.5
	2			10	9.1
		4	9		36.1
		4		10	24.4
			9	10	8.9
1	2	4			75.9
1	2		9		66.8
1	2			10	71.8
1		4	9		74.1
1		4		10	74.8
1			9	10	68.6
	2	4	9		36.5
	2	4		10	25.9
	2		9	10	12.0
		4	9	10	36.2
1	2	4	9		75.9
1	2	4		10	78.1
1	2		9	10	75.3
1		4	9	10	74.9
	2	4	9	10	36.5
1	2	4	9	10	78.7

where the last term on the right is the so-called "correction term", which is then used to obtain the sum of squares of the computed Ys:

$$SSY_{comp} = SSY_{raw} - \bar{Y}\Sigma Y_i$$

Thus, the proportion of the total sum of squares attributable to linear regression, expressed as a percentage, can now be computed directly as 100 SSY_{comp} / SSY_{obs}. For example, if SSY_{obs} = 250, and SSY_{comp} = 150, then the linear relation "accounted for" 60% of the total sum of squares of Y_{obs}.

The quantity 100 SSY_{comp} / SSY_{obs} is informally referred to as the "percent reduction in the sum of squares", inasmuch as in the above example, if the total sum of squares of Y is 250, the 150 units attributable to Y_{comp} leave, in effect, only 40% of the original variability "unaccounted" for. Hence, the reference to a percent reduction in SSY_{obs} means that the regression relation has in effect "reduced" the initial variability by 60%. The sum of squares criterion is generally most useful when the number of samples (or times of observation of process elements) is several times as great as the number of process elements measured. Thus, for six Xs the minimum number of times of observation should be 15 to 18. As the number of Xs measured approaches the number of observations, the sum of squares reduction tends to be forced closer to 100%, and may give an impression of explaining a greater part of the variability in Y than is correct. (The unusually high percent reductions in the lower part of table B49, described later, may be attributable to such effects).

The reduction of the sum of squares is a measure of the mathematical association between variables, and is not necessarily the measure of a physical relationship. Where the independent variable has physical meaningfulness in the problem, however, it is not extreme to infer that the strength of the mathematical relation is also a measure of the strength of the physical relation.

We return to the output for this example, as given in table B. It is interesting to note the wide range in the percentage of the sum of squares of Y accounted for by the various combinations of Xs. In pre-computer days, when only a few variables could be handled feasibly, it is not surprising that different researchers, using different sets of variables, might well have made different inferences, depending upon the particular two or three variables that might have been used,

The material in table B can be conveniently arranged as in table C, which lists only the three strongest combinations for each computer loop. There seems to be no doubt in the present subset of data that the effect of mean particle size is by far the strongest single variable in slope response, even though mean grain size is itself influenced by the process elements. For Xs taken two at a time, it is noteworthy that mean grain size (X1) is consistently an element of each pair; and that the combination of mean grain size and wave height (X4) is the strongest pair.

Table C. - The Three Strongest Combinations of Xs Taken 1, 2, 3, and 4 at a Time From Table A

X1	X2	X4	X9	X10	Percentage of Sum of Squares of Y accounted for:
1					63
		4			24 One at a time
			9		6
1		4			74
1				10	66 Two at a time
1	2				65
1	2	4			76
1		4		10	75 Three at a time
1		4	9		74
1	2	4		10	78
1	2	4	9		76 Four at a time
1	2		9	10	75
1	2	4	9	10	79 Five at a time

NET CONTRIBUTIONS OF RANKED VARIABLES

X1	63.1 %
X4	11.0 %
X2	1.8 %
X10	2.2 %
X9	0.6 %
	78.7 %

This is borne out by the next stage, in which variables X1 and X4 are consistently present, but now the added contributions by the other three variables does not cover a very wide range. Thus, the three strongest combinations differ by only about 1%. Similarly, combinations of four Xs at a time show a limited range of contributions, with perhaps a suggestion that variables X2 (wave period) and X10 (water depth) are slightly stronger than angle of wave approach, X9.

In considering the implications of the analysis, we may estimate the relative importance of the several variables in the following way: select X1 first, with 63.1% of the total sum of squares of Y attributed to it. Then, because the gap between the pair (X1, X4) and the next competitor (X1, X10) is about 10%, choose X4 as the second strongest variable. Its contribution, in the presence of X1, is (74.1 - 63.1) = 11.0%. From here on the choice is less clear, but if we tentatively accept wave period (X2) as the third strongest, we obtain a contribution of (75.9 - 74.1) = 1.8% from it. Similarly, if we tentatively accept water depth (X10) as the fourth strongest, we obtain (78.1 - 75.9) = 2.2% from it. Lastly, the contribution of angle of wave approach (X9) is found by the relation (78.7 - 78.1) = 0.6%. The relatively small contributions attributable to X4, X9, and X10 suggest that, except for the two strongest variables, there is little to choose from among the other three, which add on the average about 1.5% each, in contrast to 63% by mean grain size and 11% by wave height.

Examination of the weakest variable in the set is also illuminating. As table C shows, wave period "accounts for" only 1.1% of the sum of squares of Y. Yet on the strictly least-squares basis of choice used here, this becomes of about equal rank with X9 and X10, which individually contribute something more than 5% when taken alone.

The IBM 1620 and 709 programs used in this study compute the linear coefficients and the sums of squares reduction for all combinations of Xs, as stated, and the output lists the Xs involved and the corresponding percent SS reduction. If intermediate output (such as the coefficients) is desired, the program produces this by way of a control card. Details are given in Krumbein, Benson, and Hempkins (1964).

Implications of Linear Regression Analysis

An important aspect of the present method of analysis is that the general linear model, as used here, examines only the linear relations among the variables, although the model itself can be extended to some non-linear cases, providing that the βs always remain linear. It is sometimes instructive to examine the matrix of linear correlation coefficients along with the multiple regression output, to see whether additional light is shed on the regression by the linear relations (interlock) among the various Xs used in the study.

Table D is the upper right half of the Pearson product-moment-correlation-coefficient matrix for all pairs of variables in table A. Inasmuch as r_{ij} is the same as r_{ji} the matrix is symmetrical and only half of it need be examined. The diagonal in table D carries the value 1.0 for all entries, and merely means that any variable correlated with itself always has r = 1. The top row of the table has the correlation coefficients of Y with each X in turn, and the relation between the values in table B and the corresponding r's in table D is that the 63.1% listed opposite mean grain size in table B is the same as 100 $r_{YX1}{}^2$ = 100 $(-.795)^2$ = 63.2%, which agrees within rounding error.

Table D.-Correlation Matrix For All Pairs of Variables In Table A.

r_{ij}

Slope	Mean Grain Size	Wave Period	Wave Height	Angle of Wave Approach	Still-Water Depth	
1.00	-.795	.104	-.487	-.238	.228	Slope
	1.000	.062	.205	419	-.060	Mean Grain Size
		1.000	-.049	-.035	-.356	Wave Period
			1:000	-.217	-.305	Wave Height
				1.000	-.222	Angle Wave App.
					1.000	Still-Water Depth

Note: Attention is called to negative correlation between $(\overline{M}_z)_s$ and $\overline{S}_s$ which is explained on p.45

The most interesting parts of table D are the linear relations among the Xs in this subset of data. For example, r_{X1X9}, between grain size and angle of wave approach, is + 0.419, the largest r in any row other than the first. This correlation with at least one of the process elements is perhaps to be expected, considering that mean grain size is itself in general dependent in part on shore process elements. For a set of data having only 18 observations, any r less than about 0.40 is of doubtful significance.

The lack of strong correlations among the variables (except for mean grain size and slope) merely means that the linear relations among the variables are weak, and neither the correlation table nor the regression results gives any direct information on non-linear relations that may occur. The problem of non-linearity is always present in linear analysis, and it will be examined in the next section of this paper, where an extension of the linear model is described that permits examination of the second degree (quadratic) effects that may be present.

LONGSHORE-CURRENT VELOCITY

Longshore-current velocity typifies a variable measured in this study that depends rather obviously upon most of the other measured variables. We may expect from theoretical and laboratory studies (cf. Brebner and Kamphuis, 1963, for example) that the following variables will be significant ones in their influence on longshore-current velocity: beach slope, wave period, wave height, and angle of wave approach. Other measured variables that are not customarily considered in laboratory studies may be expected to exert lesser, but possibly significant, effects upon longshore-current velocity. Among these might be local wind velocities in onshore or offshore directions, which would affect the form of the incoming swells, and water density, which would affect fluid drag and the quantity of sediment transported. Tidal currents opposed to or in the same direction as the longshore currents might also be of significance.

Of the greatest importance in the evaluation of the influence of the several variables considered as causes, on the one considered an effect, is the assessment of the simultaneous influence of each of the various possible combinations of the causal variables. The method used in investigation of this simultaneous influence was described in the previous section. Here we take longshore-current velocity as a dependent variable.

Results of Analysis

The results of the first stage of the regression analysis are shown in table 2. This table shows the extent to which 13 environmental factors (independent variables, or Xs) reduce the sum of squares of longshore-current velocity when the factors are taken one at a time. The total % SS reduction by all 13 Xs taken simultaneously is 69.97. (The number of longshore-current measurements used was 53.) An initial inference is that some 70% of the interaction of longshore-current velocity with the environment is "explained" by these 13 measured environmental variables. While it is possible that certain important variables may have been omitted from the analysis, it is also likely that a relatively high "noise-level" may be present, due in part to local fluctuations in the phenomena studied, as well as to errors of measurement. Mean longshore-current velocity, $\overline{V}$, for example, shows a great variation about the true mean velocity because the point of actual observation relative to the upstream and downstream rip-current boundaries was never know. Thus the value used for $\overline{V}$ actually ranged from zero velocity, up through the true mean velocity, and beyond to the maximum expectable one.

Again, noise may have been introduced into the analysis owing to the way that the independent variables were measured through time, relative to the dependent variable. Wave height, for example, could have been measured at the instant that the longshore-current measurement was taken or up to 4 hours before its measurement. In view of these difficulties

Table 2.-Per Cent Reduction in Longshore Current Velocity Sum of Squares Attributable to Thirteen Independent Variables, Taken Individually

Variable	Symbol	Position	% Reduction in SS
Mean Lognshore Current Velocity	$\overline{V}$	Y	---
Mean Slope Angle, Lower Foreshore	$\overline{S}_f$	X1	5.11
Wave Period	T	X2	21.98
Wave Length (deep water value)	L_o	X3	26.46
Wave Height (deep water value)	H_o	X4	17.89
Wave Steepness (deep water value)	H_o/L_o	X5	47.42
Mean Wind Velocity Onshore	$\overline{U}_{on}$	X6	6.92
Mean Wind Velocity Offshore	$\overline{U}_{of}$	X7	5.63
Mean Wind Velocity Directed with Longshore Current	$\overline{U}_s$	X8	0.68
Mean Velocity Directed against Longshore Current	$\overline{U}_a$	X9	0.00
Angle of Wave Approach	α	X10	1.19
Water Density	ρ	X11	0.65
Tidal Current Velocity with Longshore Current	C_s	X12	1.25
Tidal Current Velocity against Current	C_o	X13	0.23

with the noise content of the data, it becomes more understandable that the total % SS reduction by all 13 variables simultaneously was only 69.97%. In actual fact it would be slightly lower than the 69.97 figure if variables X3 and X5, which exhibit considerable "data redundancy" with variables X2 and X4, had been left out of the analysis. Data redundancy is brought up again later.

Discussion

Table 2 shows that the variable X5, wave steepness, has the greatest influence upon longshore-current velocity, followed by the wave factors of L_o, T, and H_o. As noted earlier, the fundamental variables of T (X2) and H(X4) could be expected from theoretical (Putman, Munk, and Traylor, 1949) and laboratory (Brebner and Kamphuis, 1963) studies to be among the most-influential in determining longshore-current velocity. Also of expected importance would be beach slope and angle of wave approach, but both of these variables are outranked by wind velocity onshore and offshore when the variables are taken one at a time.

As stated, when the independent variables are taken one at a time, their relative rank cannot, in general, be determined directly from the degree to which they reduce the sum of squares of the dependent variable, Some of the variables may be redundant. That such redundancies are present in the longshore-current example can be seen by adding the individual reductions in the sum of squares of X_0 in table 2. This sum is 135.41%, indicating that some variables, when taken one at a time, show stronger relations than they show when taken in combination with other independent variables. Thus, one may say that there must be, as a minimum, at least 35% of data redundancy and (or) noise in these observations.

Before going further it is well to note that variables X3 and X5 (table 1) will exhibit a certain amount of artifically introduced data redundancy because they repeat information found in variables X2 and X4. Thus, wave length L, being directly proportional to the square of the wave period T, might be expected to exhibit a closely similar % SS reduction to T. Because X3 gives a 4.48% SS reduction (table 2) over that for X2, however, the relation between $\overline{V}$ and T may be non-linear. More will be said of non-linearity later. Variable X5, then, repeats information found in the "fundamental" variables X2 and X4. It is stronger than either, however, and by combining these two attributes wave steepness be comes a useful variable to measure in studies involving prediction of $\overline{V}$. Its non-dimensionality may also enter, inasmuch as H_o/L_o is free of any scale factor.

The next step is to examine the computer output for the strongest combinations of variables when they are taken in combinations of two, three, four, and so on. Table 3 summarizes certain of the strongest combinations for Xs two through nine at a time; it reveals that wave steepness retains its dominant role in all of the strongest combinations. The strongest doublet is not X5 and X3 (H_o/L_o and L_o), as might be ex-

Table 3.--The Strongest Per Cent Reductions in Longshore Current Velocity Sum of Squares Attributable to Each of Several Combinations of Thirteen Independent Variables. (Total Percent Reduction, All Xs = 69.97).

	Independent Variable Combinations													Percent Reduction in SS
Two Xs					5		7							54.21
at a time	1				5									53.91
					5					10				50.77
			3		5									50.09
					5	6								49.86
Three Xs	1				5		7							60.69
at a time	1		3		5									57.41
	1				5					10				57.03
					5		7				11			56.02
	1	2			5									56.02
Four Xs	1		3		5		7							62.23
at a time	1				5		7			10				61.96
	1				5		7				11			61.85
	1	2			5		7							61.50
	1				5		7					12		61.37
	1		3		5					10				61.19
	1			4	5		7							60.90
	1				5		7						13	60.72
Five Xs	1	2	3		5					10				64.53
at a time	1		3		5		7			10				64.07
	1		3		5		7				11			63.52
	1	2	3		5		7							63.35
	1				5		7			10	11			63.02
Six Xs	1	2	3		5		7			10				66.39
at a time	1		3		5		7			10	11			65.25
	1	2	3		5					10			13	65.02
	1	2	3		5			8		10				64.95
	1	2	3	4	5					10				64.92
Seven Xs	1	2	3		5	6	7			10				67.56
at a time	1	2	3		5		7			10		12		66.72
	1	2	3		5		7			10	11			66.67
	1	2	3	4	5		7			10				66.60
	1	2	3		5		7			10			13	66.53

pected from table 1, but rather is X5 and X7 (H_o/L_o and $\overline{U}_{of}$). The third strongest doublet is X5 and X10 (α). Although these differ by only a small percentage, they are being taken here as ranked by the least-squares procedure.

The practice of inferring the relative rank of combinations of variables from their rank when taken individually is generally not a sound procedure. Slope angle of the lower foreshore (X1), for example, does not appear (table 2) to contribute as much as fully six other variables (Xs 2-7). The method of sequential multiregression analysis shows, however, that $\overline{S}_f$ is present in the strongest triplet and remains prominent in several of the stronger triplets. From an initial SS reduction of 5.11%, when considered individually, it is seen that in the presence of variables X5 and X7 it contributes fully 6.48% (table 4), for an increase in its original value of only 1.37%. Thus, the strongest individual variable may be influenced by other variables. However, when the grounds for accepting the strongest variable on a physical basis are sound, it is conventionally taken at face value, and the effects of other independent variables grouped with it are expressed in terms of the added reduction contributed by the combined variable.

Continuing in table 3, we noted that the strongest triplet (X1, X5, X7) now includes beach slope (X1), another fundamental variable, which ranked fully seventh in order of importance when the variables were considered individually (table 2). Table 4 indicates the change in the original % SS reduction value of X1 when in the presence of X5 and X7. The value of X1 has increased some 1.37%.

The strongest combinations of independent variables taken four at a time is composed of X1, X3, X5, and X7. Interestingly, the original SS-reduction value of X3 (26.46%, table 2) has been reduced by 24.92% (table 4), in the presence of X1, X5, and X7. This is largely because it already occurs in variable X5.

At this point it is instructive to rewrite the empirical laboratory expressions found by Brebner and Kamphuis (1963, p. 22) to represent the mean longshore-current velocity as they computed it: a) using Airy wave theory, b) Snell's law for wave refraction (assuming a gently sloping plane beach), and c) by expressing the wave parameters in terms of deep-water values.

$$\overline{V} = 1.9\ (g\ \overline{S}_f)^{1/3}\ H_o^{\ 1/2}\ (H_o/L_o)^{1/6}\ (\sin 1.65\alpha_o + 0.1 \sin 3.30\alpha_o) \quad \text{Energy Approach} \quad (1)$$

$$\overline{V} = 4.0\ (g\ \overline{S}_f^{\ 2})^{1/4}\ H_o^{\ 1/2}\ (H_o/L_o)^{1/4} (\sin 1.65\alpha_o + 0.1 \sin 3.30\alpha_o) \quad \text{Momentum Approach} \quad (2)$$

Assuming for the moment that there is a reasonable relationship between the major factors that determine longshore-current velocity in the laboratory and those that influence it in nature, we may look for the position of the combination of X1, X3, X5, and X10 in the ranking of % SS

Table 4. - Contributions by Additional Variables Among the Seven Strongest Independent Variables Taken Jointly

Variable	% SS Accounted For Individually (See Table 2)	Xs In Combination	% SS Accounted For By Combination	Contribution of New X	Increase or Decrease Relative to Original Contribution
X5	47.42	X5	47.42	———	———
X7	5.63	X5,X7	54.21	6.79	Increase of 1.16%
X1	5.11	X5,X7,X1	60.69	6.48	Increase of 1.37%
X3	26.46	X5,X7,X1,X3	62.23	1.54	Decrease of 24.92%
**X10	1.19	X5,X7,X1,X3 X10	64.07	1.84	Increase of 0.65%
X2	21.98	X5,X7,X1,X3 X10,X2	66.39	2.32	Decrease of 19.66%
X6	6.92	X5,X7,X1,X3 X10,X2,X6	67.56	1.17	Decrease of 5.76%

** This combination is second strongest (table 3) in the combinations of 5 Xs at a time.

reductions for variables taken four at a time (table 3). This combination ranks sixth in importance, but is separated from the most-important combination by only 1.02%. The suggestion, then, is that the method of data analysis reasonably duplicates the expectable influence of combinations of important variables as they have been suggested by theoretical and laboratory approaches, when studied by a straight forward least-squares procedure. Then, an important corollary to this inference is that the data set itself is a reasonably faithful representation of the environmental interactions under study.

Exmination of the results (table 3) for five Xs at a time again shows that the variables found to be of significance in the laboratory exert the greatest % SS reduction on $\overline{V}$. It is at this point that the variable of angle of wave approach (X10) enters into the strongest combination. X10 exhibits an increase in its original SS-reduction value of 0.65%, when in the presence (table 4) of X1, X3, X5, and X7. What effect expressing α and α_o (using Snell's law and Airy wave theory) would have had in bringing the relative ranking of X1, X3, and X5 in table 2 more in line with the powers in expression (1) and (2) is unkown. It may be unimportant in this set of data, owing to complex wave-refraction patterns that may be present.

The highest-ranking combinations of six variables at a time include X7, wind velocity offshore, which probably has a significant effect upon wave steepness, X5. Thus the interlocking nature of the variables again enters the picture. On several occasions during the measurement periods wind shifts from onshore to offshore were noticed in connection with passage of cold fronts. Waves were observed being "knocked down," as spray blew from their crests in a seaward direction. Because of the apparent "corroboration" of the method of analysis for the fundamental variables mentioned above, it seems fair to attach significance to the additional atmospheric variables that turn out to be significant in the least-squares analysis when they combine with these fundamental variables. That is, the indicated importance of variables X7 and possibly X11 (table 3) to longshore-current velocity may be believed to have true physical significance, <u>when in combination</u> with the variables just mentioned. Once physical significance is attached to these variables it is realistic to enquire into the precise physical relationships involved. As mentioned, for example, strong offshore winds will often rather effectively alter the wave form, and water density enters into fluid drag in the longshore-current trough, the amount of sediment entrained, and thereby the velocity of the longshore current. Bruun (1963) and Inman and Bagnold (1963) give excellent reviews of the problems in understanding the generation of longshore currents. Any inferences advanced here are not capable of immediate verification based upon field studies but may serve as a basis for the design of field measurements.

Because of the strictly artificial introduction of data redundancy in variables X3 and X5 (table 2), a new summary (table 5) was prepared from the computer output. This new table lists the strongest combinations when variables X3 and X5 are removed from the analysis. As expected, variables X2 and X4 are dominant throughout all of the combinations, and variable X1 through all but one combination. Variable X6, wind velocity onshore, takes the place occupied by X10 in the five Xs at a time combination of table 3. Angle of wave approach, X10, then appears in the combination of six Xs at a time. Thus, wind velocity again appears important in influencing lonshore-current velocity.

Table 5.-The Strongest Percent Reductions in Longshore Current Velocity Sum of Squares Attributable to Combinations of Eleven Independent Variables (X3 and X5 Removed From The Original Thirteen Xs).

	Independent Variable Combinations							Percent Redcution in SS
Two Xs at a time		2	4					41.10
Three Xs at a time	1	2	4					47.23
Four Xs at a time	1	2	4		7			51.98
Five Xs at a time	1	2	4	6	7			57.10
Six Xs at a time	1	2	4	6	7		10	59.77
Seven Xs at a time	1	2	4	6	7	8	10	62.04

Table 6 summarizes the combinations of independent variables that least influence $\overline{V}$. The table reveals certain variables that contribute to both the weakest and to the strongest combinations. It is instructive to refer to the weakest (table 6) and the strongest (table 2) combinations of independent variables taken six at a time. Intuitively, we might expect that none of the six variables that appear in the strongest combination (Xs 1, 2, 3, 5, 7, 10) would appear in the weakest combination (Xs 8, 9, 10, 11, 12, 13). It is found, however, that X10 does appear in both combinations.

When the number of variables is fairly large, as in this study, it is not unusual to find that as the number of Xs per combination is increased, a variable whose net contribution is of the order of 1 or 2 %, may "rise to the top" when combined with a very strong combination of Xs, whereas when combined with very weak Xs, its own slight contribution is not sufficient to raise the total contribution by any significant amount.

Table 6.-The Five Weakest Per Cent Reductions in Longshore Current Velocity Sum of Squares Attributable to Each of Several Combinations of Thirteen Independent Variables.

	Independent Variable Combinations										Percent Reduction in SS
Two Xs at a time						9				13	0.23
						9		11			0.65
					8	9					0.68
					8					13	0.77
								11		13	0.87
Three Xs at a time					8	9				13	0.77
						9		11		13	0.87
						9			12	13	1.28
					8	9		11			1.41
					8			11		13	1.50
Four Xs at a time					8	9		11		13	1.50
						9		11	12	13	1.68
					8	9	10			13	1.88
					8	9			12	13	2.00
						9	10	11		13	2.20
Five Xs at a time					8	9		11	12	13	2.45
					8	9	10	11		13	2.61
						9	10	11	12	13	3.13
					8	9	10		12	13	3.23
					8	9	10	11	12		3.61
Six Xs at a time					8	9	10	11	12	13	3.66
				7	8	9	10	11		13	6.02
				7	8	9		11	12	13	6.16
				7	8	9	10	11	12		6.44
				7	8	9	10		12	13	6.44
Seven Xs at a time				7	8	9	10	11	12	13	6.51
	1				8	9	10	11	12	13	9.23
	1			7	8	9	10	11	12		11.26
	1			7	8	9	10	11		13	11.88
	1			7	8	9	10		12	13	11.92
Eight Xs at a time	1			7	8	9	10	11	12	13	12.59
	1		6		8	9	10	11	12	13	19.33
		4	6		8	9	10	11	12	13	25.76
	1	4	6		8	9		11	12	13	26.95
	1	4	6		8	9	10		12	13	27.74

For example, the combination 1, 5, 7, 10 in table 3 accounts for 61.96 %, an increase of 1.27 % over combination 1, 5, 7, In table 6, the combination 8, 9, 13 accounts for 0.77 %, and the combination 8, 9, 10, 13 accounts for 1.88 %, an increase of 1.10 %. Thus the position of X10 depends very largely on the strength of the combination in which it occurs, and X10 seems to play a minor role in this set of data in that it first shows up as weak or strong in combinations of 4 Xs at a time. It will be recalled from table 2 that X10 accounts for 1.19 % of the SS of Y, a contribution that seems to remain much the same throughout the analysis.

Summary

Based on an analysis involving eleven independent variables, it is possible to say that the combination of six factors (table 5) that is most influential in the determination of longshore-current velocity in the study area at Virginia Beach is the combination made up of wave period, wave height, lower foreshore slope, wind velocity onshore, wind velocity offshore, and angle of wave approach, in that order. (When the derived variables of wave length and wave steepness are added to the analysis, the six most-important factors are $\overline{S}_f$, T, L_o, H_o/L_o, $\overline{U}_{on}$, and $\overline{U}_{of}$). The wave factors and the beach slope are variables agreed upon by workers in the field of longshore-current generation to be of fundamental importance. The significance of winds on and offshore is believed to lie in their ability to alter the wave form prior to breaking.

The least-squares relations developed by this analysis appear to adequately represent the combinations of variables of most significance in nature. It is to be recalled that the framework of analysis is linear, but it is not uncommon, when a large number of variables is involved, that a linear approximation yields reasonable results, even though some relations may be known to be non-linear. Moreover, non-dimensional ratios among some of the original variables, such as H_o/L_o in this example, may rise to greater relative importance than the original variables themselves, as shown in table 2. This suggests that further analysis by use of non-dimensional variables alone (perhaps as derived from use of Buckingham's Pi theorem) may be useful and informative. We hope to extend the analysis in this direction.

As a summary of the foregoing linear analysis, it seems fair to say that the least squares techniques used here have helped "sort out" the relative interplay of a group of variables as they affect longshore-current velocity on a particular beach during a given time-span. Statistically our model is "fixed", and extensions of our findings to generalizations about other beaches is valid mainly in that the underlying variables are perhaps the same, even though their relative rankings may vary from beach to beach, or on the same beach from time to time. That non-linearity is also a problem is discussed next.

THE PROBLEM OF NON-LINEARITY

The occurrence of non-linear relations among beach-ocean-atmosphere variables was touched upon above, and it is discussed here in connection with the study of longshore-current velocity. Non-linearity can be detected in several ways, the simplest perhaps being the examination of scatter diagrams of each pair of variables in the entire set. Another search method is to run the regression analysis first with the "raw" data and then with all or some Xs transformed to logarithms. In this example we shall use an extension of the linear model to seek for quadratic effects among the variables.

When the linear regression of Y on some single X is studied, the computational model reduces to the following form:

$$Y = \hat{\beta}_0 + \hat{\beta}_1 X$$

In terms of regression analysis, this involves the 2 x 2 matrix, vector-Y, and β-vector discussed in connection with equation (2), and yields the "sum of squares reductions" associated with one X at a time. However, this model can be extended as follows to include higher powers of X:

$$Y = \hat{\beta}_0 + \hat{\beta}_1 X + \hat{\beta}_2 X^2 \qquad (3)$$

where the coefficient $\hat{\beta}_2$ is now associated with the quadratic form of X. The coefficients are still linear, and hence the same general technique may be used, even, if desirable, to include such powers as X^3, X^4, etc.

The procedure used here is first to take each X by itself in terms of Y, to obtain the values in table 2. Then, for each X, its square is also included as in equation (3), to obtain a corresponding sum of squares of Y attributable to the combined linear and quadratic effects of the X. The difference between these two "sum of squares reductions" gives an estimate of the second degree non-linearity associated with each X. This was done with a computer program that computed the linear and quadratic relations between Y and each of the 13 Xs, as well as all interlocks between the Xs themselves. In this latter analysis the order of entry of the Xs is involved, in that the expression:

$$X1 = \hat{\beta}_0 + \hat{\beta}_1 X2 + \hat{\beta}_2 X2^2$$

is not the same as the expression:

$$X2 = \hat{\beta}_0 + \hat{\beta}_1 X1 + \hat{\beta}_2 X1^2$$

even though, as was mentioned earlier, r_{X1X2} is the same as r_{X2X1}. Thus, the complete quadratic output for a problem involving 12 Xs is voluminous, and we shall here emphasize mainly the analysis in equation (3) that contains Y directly.

Table 2A shows the linear and quadratic counterpart of table 2. It is apparent that in absolute terms, variables X5, X6, and X11, representing wave steepness, wind velocity onshore, and water density, have the largest quadratic components. In relative terms, the lower foreshore slope angle,

Table 2A.-Linear And Quadratic Relations Between Longshore Current Velocity And The Independent Variables In Table 2.

Variable	Positions	% SS Reduc. Attributable to Linear	% SS Reduc. Attributable to Linear and Quadratic	% Added by Quadratic
Mean Longshore Current Velocity	Y	--	--	--
Slope Angle Lower Foreshore	X1	5.11	10.13	5.02
Wave Period	X2	21.98	25.60	3.62
Wave Length (deep water)	X3	26.46	27.43	0.97
Wave Height (deep water)	X4	17.89	19.97	2.08
Wave Steepness (deep water)	X5	47.42	72.71	25.30
Wind Velocity Onshore	X6	6.92	25.01	18.09
Wind Velocity Offshore	X7	5.63	6.26	0.64
Wind Velocity Directed with Longshore Current	X8	0.68	1.13	0.46
Wind Velocity Directed against Longshore Current	X9	0.00	0.00	0.00
Angle of Wave Approach	X10	1.19	2.38	1.19
Water Density	X11	0.65	12.42	11.77
Tidal Current Velocity with Longshore Current	X12	1.25	1.31	0.06
Tidal Current Velocity against Longshore Current	X13	0.23	0.37	0.14

X1, has doubled, from 5.11% to 10.13%. Thus, these several variables may enter the least-squares relations more effectively as logarithms or as variables raised to some power. Of particluar interest is water density, which in the linear sense is quite negligible (only 0.65%), but in the quadratic has risen to nearly 12%. This is an illustration of one limitation of a strictly linear analysis: some variables that have virtually no linear effect may become quite strong in a model that explicity includes non-linear effects.

DEPOSITION AND EROSION ON THE LOWER FORESHORE

Time Lag In Peak Interaction

In a sequential multiregression analysis of the interaction of the Mission Beach, California, foreshore slope to the four independent variables of wave height, wave period, angle of wave approach, and longshore-current velocity, Krumbein (1961, p. 45) found that the maximum effect of these combined independent variables occurred (in a least-squares sense at least) sometime between 6 and 12 hours prior to the time of the measurement of foreshore slope. In regard to wave period, the analysis indicated that the greatest effect on foreshore slope was exerted some 30 hours prior to the time of measurement of beach slope. Because of the laboratory and field evidence for the delay in time in the peak interaction of certain dependent variables like beach slope and the independent variables that influence them significantly, the four interaction studies at Virginia Beach that follow were investigated over five or six "lag periods."

In the case of modification of the segment of the lower foreshore that is covered and uncovered by the tide, it is necessary to standardize the times of slope measurement so that the measurement times are representative of similar dynamic conditions on the beach. Low-tide time is a convenient reference time and was adopted here. Figure 2 shows the scheme adopted at Virginia Beach for making foreshore measurements in June and July of 1963. The ticks marks designated P_1, P_2, P_3,......... P_{149} represent the standard times during the 25-day period when the independent variables were measured or for which interpolated values were obtained. The tick marks designated R_1, R_2, R_3........R_{25} represent the times of low tide when measurements of the altitudes of the lower foreshore stations were made. This measurement time (R_n) is seen (fig. 2) to progress through the times of measurement of the other variables. Because each lag period is 4 hours in length, the measurements for all of the independent variables for lag period 1 will have been made 0 to 4 hours prior to the foreshore-altitude measurements, for lag period 2 they will have been made 4 to 8 hours prior, and so on. Because the precise time of peak interaction between a dependent variable (J_f, K_f) measured at low-tide time and one or more independent variables is unknown, it is probably just as well that bias has not been introduced by setting up lag times of fixed numbers of hours prior to measurement of the dependent variable.

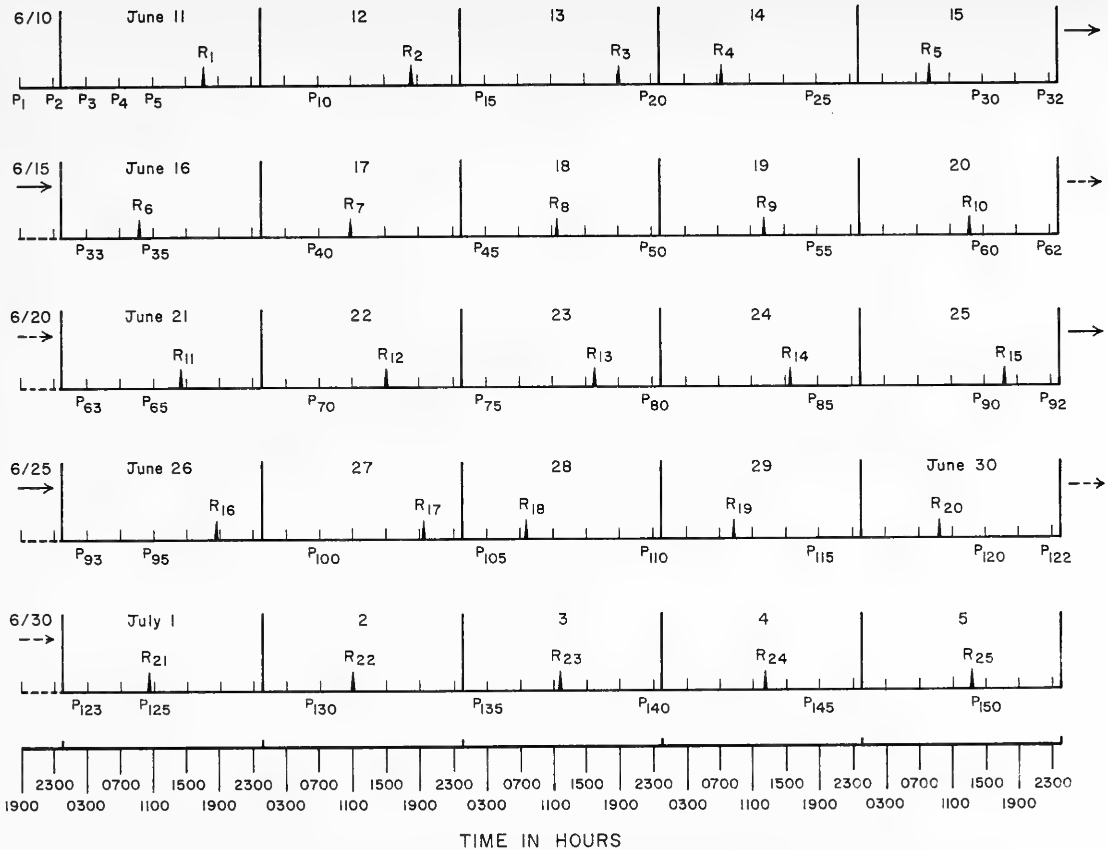

FIGURE 2. DIAGRAM SHOWING TIMES OF MEASUREMENT OF PROCESS ELEMENTS (P_1, P_2 .... P_{152}) AND RESPONSE ELEMENTS (R_1, R_2 .... R_{25})

Measurement of the Dependent Variable

An attempt is made to understand the changes in the overall altitude of the lower-foreshore surface along a 100-125 foot segment running perpendicular to the shoreline (fig. 3). The dependent variable has the dimensions of a length, the magnitude of which is believed sufficient to adequately reflect the environmental processes producing the foreshore changes. This length is the net thickness of material added to or subtracted from the foreshore surface at five or six stations in a 24.5-hour period; i.e., over two tidal cycles.

The section of the "lower foreshore" referred to here includes the zone of breaking waves somewhere in its lower half (fig. 4) at the time of measurement of the altitude of the beach surface (time of low tide). The remainder of the lower-foreshore surface studied here consists of the region covered by the breaker and swash-backwash zones of the previous two tidal cycles. Measurements of the altitude of the beach surface were made at six stations at 15th Street (northern transect, fig. 1) and five stations at the Camp Pendleton line (southern transect, fig. 1), at the low-tide times designated by the letter "R" on figure 2. Thus, the measurements cover 25 consecutive days in June and July.

Precision of the sounding-line or leveling-rod measurement is estimated at ± 0.10 foot. Altitudinal variations at the stations were summed over the five or six stations and the net erosion or deposition recorded. Unfortunately, the large sampling distance involved (25 feet) undoubtedly missed minor irregularities in the profiles that could have been significant. Figure 4 shows the range of changes in lower-foreshore altitudes at the two transects during the 25 consecutive days (fig. 2) of measurements. Clearly, the beaches were not anywhere near an equilibrium condition, such as might be expected (cf. Strahler, 1964) during the late summer. Daily values for the increment of deposition, J_f, averaged about 0.7 foot at 15th Street; for erosion, K_f, averaged about 0.5 foot. Respective average values at the Camp Pendleton line were 0.5 foot (K_f) and 0.4 foot (J_f). The values compare with daily net fluctuations of 0.2-0.3 foot on Cape Cod beaches (Zeigler and Tuttle, 1961).

For this part of the study, the variations in the causal independent variables were followed over six lag periods (two tidal cycles), as shown below:

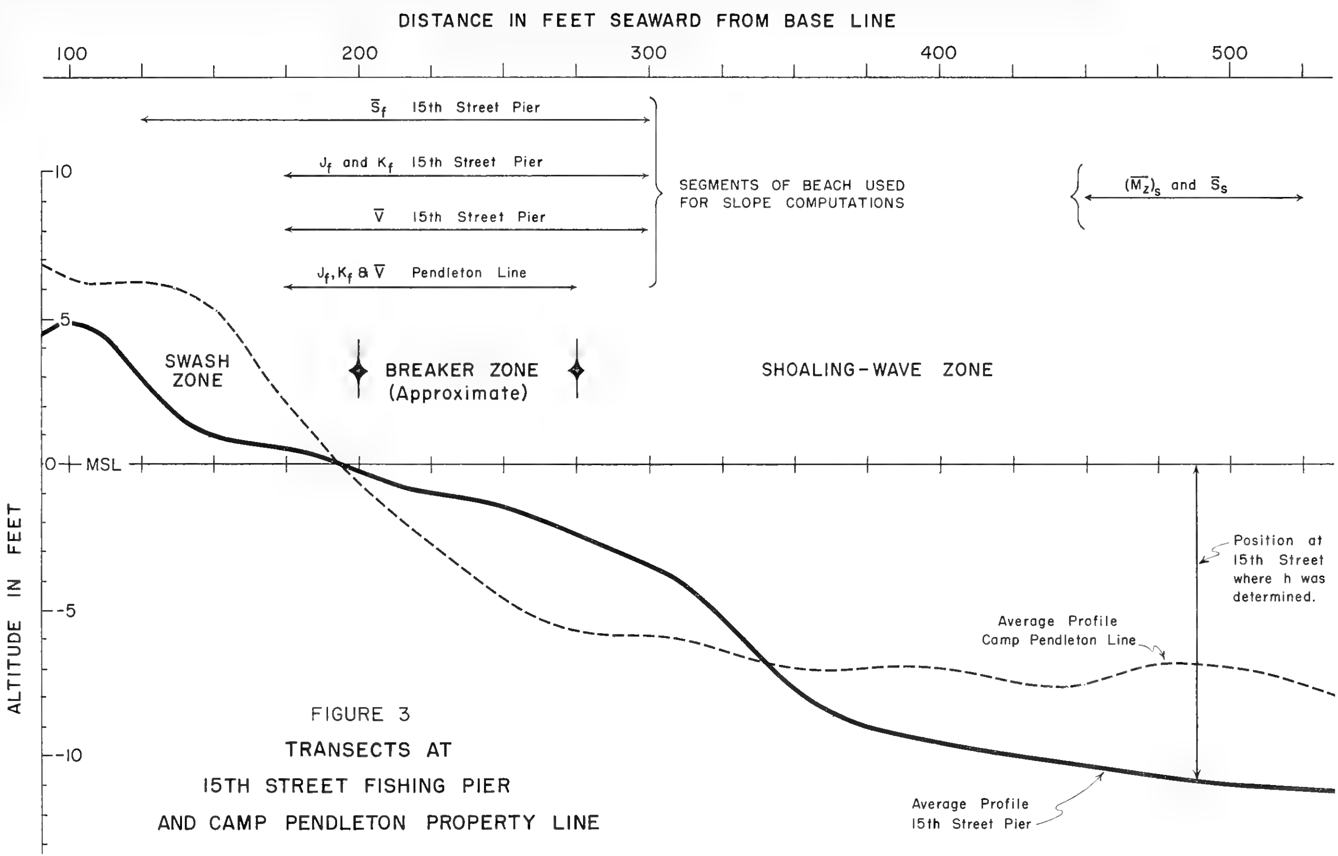

FIGURE 3
TRANSECTS AT
15TH STREET FISHING PIER
AND CAMP PENDLETON PROPERTY LINE

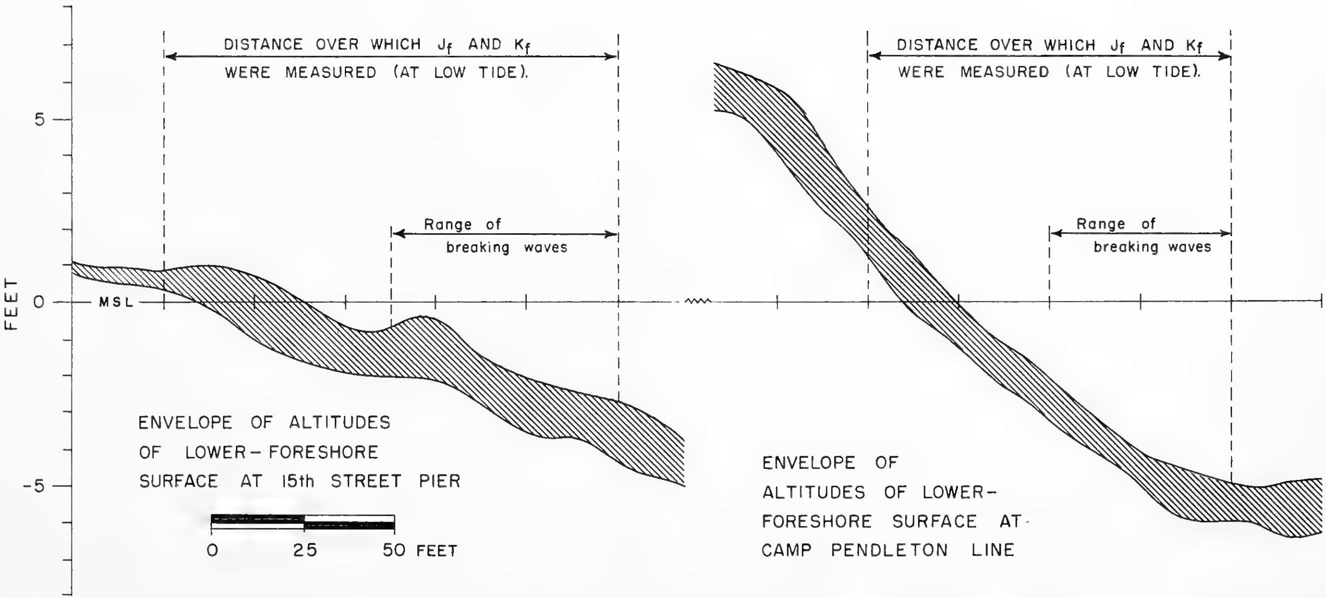

FIGURE 4. ENVELOPES OF ALTITUDINAL VALUES AT LOWER-FORESHORE STATIONS

Lag Period Number	Hours Prior To Measurement Of J_f or K_f	Tide Stage
1	0-4	Falling (LW at 0 hrs)
2	4-8	Rising and falling about HW (HW at 6.12 hrs)
3	8-12	Rising
----------	----------	Low (LW at 12.25 hrs)
4	12-16	Falling
5	16-20	Rising and falling about HW (HW at 18.37 hrs)
6	20-24	Rising
----------	----------	Low (LW at 24.5 hrs)

The interval of two tidal cycles duration was chosen for convenience only, as it is far easier to measure the dependent variable in daylight hours than at night. It is true, however, that the single tidal cycle is the more basic dynamic unit and is preferable to the two-cycle interval. Thus, a small amount of erosion of the foreshore might occur during the first tidal cycle under one set of independent variables only to be followed by a greater amount of deposition during the second tidal cycle under a different set of independent variables. The dependent variable, however, would be measured as an increment of deposition and it would be impossible to realistically interpret the correlation between the set of independent variables that resulted in erosion and the dependent variable that showed deposition. Alternatively, a large amount of deposition might occur during the first tidal cycle, and a lesser amount of erosion during the second cycle. Misinterpretations could again arise.

The difficulties just mentioned are believed to be rather minimal in the data set used for the regression analyses. The values for J_f and K_f usually occur in runs of two to seven successive days. The inference is that J_f and K_f values for intermediate low-tide times would have shown similar tendencies of net erosion and deposition to those at the actual measurement times. For a number of reasons, then, the data sets used in evaluating J_f and K_f are relatively noisy. This will generally be the case when part of the work is done in the breaker zone and times of measurement are not keyed to successive times of low tide.

Measurement of The Independent Variables

The variables now mentioned (and described in Appendix A) were measured for the analyses of both the increment of net erosion (J_f) and net deposition (K_f).

Slope of the lower foreshore was determined by first passing a smooth curve through the plotted altitudes of the sample stations at which J_f and K_f were determined (fig. 3). A line of mean slope was then passed through this curve by eye, and the line's angle with the horizontal was taken as the mean slope angle ($\overline{S}_f$) of the "lower foreshore." This angle was relatively easy to measure. The two transects (fig. 4) provided a good range of slope angles for the regression analysis, although the values were somewhat clustered. Wind, wave, water density, and tide data were assumed constant at both transects. In addition to $\overline{S}_f$, only the values for longshore-current velocity and depth to the water table at the top of the uprush varied between the two transects. Thus, the following relationship could be postulated:

$$J_f \text{ or } K_f = f\,(\overline{S}_f,\ T,\ L_o,\ H_o,\ H_o/L_o,\ \overline{U}_{on},\ \overline{U}_{of},\ \overline{U}_p,\ \alpha,\ \overline{V},\ \rho,\ \eta_r,\ \eta_f,\ D)_{t_{1-6}}$$

where t = a "lag period," 0-4 hours long, prior to measurement of the dependent variables or prior to another lag period of equal duration.

Average mean size on the lower-foreshore slope was not included in the analysis because of the considerable data redundancy that would have been introduced. Also, the noise content of such data would have been quite high, because grain-size variability is at a maximum when crossing the breaker zone.

Net Deposition (J_f)

Results.--Table 7 shows the results of the first stage of the analysis, in which the independent variables are studied one at a time. Total %-SS-reduction values are relatively high, considering the noise content of the variables. The least-squares analysis shows that lag periods 1, 4, and 5 have the most influence on net deposition on the lower foreshore. If the total %-SS-reductions for lag periods for corresponding tide stages are added together, lag periods 1 and 4, 2 and 5, and 3 and 6 have total %-SS-reduction values of 173.38, 169.85, and 163.27, respectively. Thus, the variables that act during falling tide levels appear to exert a somewhat greater influence on foreshore deposition than those acting during the time of rising tide.

Slope angle of the lower foreshore (X1) emerges as the most consistently important variable, reaching its maximum influence 0-4 hours before measurement of J_f. Other less-influential variables show relatively less consistency by lag periods, with the possible exception of wave height (X4), which increases in influence backward in time. The five strongest % SS reductions by independent variables taken in combinations of two to

Table 7.- Per Cent Reductions in Net Deposition at Lower Foreshore Stations Sum of Squares Attributable to Fourteen Independent Variables, Taken Individually, for Lag Periods 1-6.

Variable	Symbol	Position	Percent Reduction in SS By Lag Periods					
			Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Total % SS Reduction			85.54	82.19	80.49	87.84	87.66	82.78
Net Deposition Lower Foreshore	J_f	Y						
Slope Angle Lower Foreshore	$\overline{S}_f$	X1	36.45	35.44	34.40	33.24	18.72	33.21
Wave Period	T	X2	18.00	0.25	16.83	18.86	0.00	0.77
Wave Length	L_o	X3	18.09	1.12	19.16	21.29	0.25	1.34
Wave Height	H_o	X4	7.95	10.07	13.21	16.56	18.88	23.43
Wave Steepness	H_o/L_o	X5	21.11	0.40	0.62	0.77	32.42	5.18
Wind Velocity Onshore	$\overline{U}_{on}$	X6	16.70	11.70	1.85	6.74	1.63	0.00
Wind Velocity Offshore	$\overline{U}_{of}$	X7	3.80	0.05	1.01	1.10	2.78	8.22
Wind Velocity Parallel to Shore	$\overline{U}_p$	X8	4.26	12.93	2.91	5.58	2.20	2.48
Angle of Wave Approach	α	X9	2.83	0.48	0.55	0.02	0.51	5.70
Longshore Current Velocity	$\overline{V}$	X10	9.78	0.01	6.40	15.84	4.49	0.65
Water Density	ρ	X11	10.33	10.33	16.28	0.00	5.39	0.01
Rate of Rise, Still-water Level	η_r	X12	0.00	9.88	0.08	0.00	6.72	2.96
Rate of Fall, Still-water Level	η_f	X13	3.46	5.62	0.00	1.81	6.71	0.00
Water Table Depth, Top of Uprush	D	X14	7.46	22.64	13.68	9.49	2.57	23.96

six at a time are shown in appendix B (tables B1-B6), and the five weakest, for lag periods 2, 3, 5, and 6, in tables B7, B8, B9 and B10. Frequency tables of the computer output for lag periods 2, 3, 5, and 6 are shown in appendix tables B11, B12, B13, and B14.

Discussion.--Considering the strongest combinations of Xs taken six at a time (tables B1-B6), it is noted that variable X1, lower foreshore slope, always contributes to these strongest combinations. In an analysis (not presented here) of the interactions of the lower foreshore slope with twelve variables of the environment, it was noted that the strongest combinations of five Xs at a time, in the most-influential lag period, consisted of H_o, α, $\overline{V}$, ρ, and R. It would seem reasonable to expect that some or all of these variables would also be of significance in this analysis for J_f. (The corresponding Xs in the analysis for J_f are X4, X9, X10, X11, and perhaps X12 and X13.) Tables B1-B6 show a tendency for most of these variables to enter into the strongest combinations, but most also enter into the weakest, and there is little consistency from one lag period to the next.

Assuming physical validity for an analysis such as this--for changes in a surface that crosses two differing dynamic zones and using a data set that is noisy and redundant--the following few points should be noted. Beach slope is a controlling factor in deposition on the lower foreshore, as net deposition is measured at low tide, after two tidal cycles. (It is understood that grain size of the slope material is also of great importance.) The rate of fall of the still-water level is of some significance during the two periods of falling tide(lag periods 1 and 4). The rate of rise of still-water, however, has no significance during the two periods of rising tide. Depth to the water table has the greatest effect on net deposition around the time of high tide or during falling tide. The effect of the longshore current enters into the strongest combinations of variables taken six at a time during only one lag period. This is not surprising, inasmuch as the longshore current serves largely to transport material parallel to the shoreline. Wind velocity appears to have significance to net deposition during the falling tide immediately prior to the time of low-tide measurements. Wind blowing offshore may set up a weak sea-surface current that will aid in the removal of fine sand suspended in the breaker zone but, at the same time, not cause an equal quantity of sand to be transported onshore in the corresponding return flow on the bottom. As a result of wind blowing onshore, large particles may be transported out of the breaker zone by the seaward return flow of water (cf. King, 1959, p. 207-213) on the bottom. Local winds in onshore and offshore directions will also influence the form of the incoming swells, as discussed in the section on the longshore currents.

Among the wave variables, H_o, and occasionally H_o/L_o, are the most important to net deposition. Statistically, water density plays a role (tables B2 and B3) during the first part of the tidal cycle immediately prior to measurement of J_f. Presumably, ρ affects the rates of particle movement by its effect on fluid drag velocities and turbulence at the bed.

Table 8 shows the results of the analysis with redundant variables X3 and X5 removed. This procedure results in the disclosure that lag periods 3 and 6, corresponding to periods of rising tide, are the most influential on net deposition, and that the variables $\overline{S}_f$, T, H_o, and α are the four most influential variables in combination.

Summary.--Based upon an analysis involving 12 independent variables, it is possible to say that the 6 most-influential variables in determining net deposition on the lower foreshore are $\overline{S}_f$, T, H_o, $\overline{U}_{on}$, α, and ρ, and that this combination of variables expresses itself 8 to 12 and 20 to 24 hours prior to the low-tide time of measurement of net deposition; i.e., this combination of variables expresses itself during times of rising tide.

Net Erosion (K_f)

Results.--Table 9 shows the results of the first stage of the analysis. Total %-SS-reductions for the six lag periods are considerably lower than those obtained for J_f (table 7). Taken individually, lag periods 1 and 2 exert the greatest influence on net erosion of the lower foreshore, as net erosion is measured at low tide after two tidal cycles. If the total %-SS-reduction values for lag periods for correlative tide stages (p.35) are added together, however, lag periods 1 and 4, 2 and 5, and 3 and 6 have total %-SS-reduction values of 113.41, 104.26, and 72.60, respectively. Thus, the variables acting during times of high tide or falling tides exert the greatest influence on foreshore erosion.

It is noted that variable X12, rate of rise of still-water level, shows slight percent reductions in SS during lag periods 1 and 4, times of falling tide level. This is due to the fact that the time of measurement of K_f was based upon the predicted time of low water. But because the actual time of low water, used in computing η_r, sometimes occurred slightly before predicted low water, K_f was sometimes measured just after the still-water level had begun to rise. The effect on the analysis in lag periods 1 and 4 is believed negligible.

The influence (table 9) of lower foreshore slope, X1, on net erosion tends to be relatively uniform over the six lag periods, and amounts to about one-third of its observed influence on net deposition (table 7). The only variables exceeeding slope angle in influence in a given lag period are η_r in lag period 3, and $\overline{U}_{of}$, α, and $\overline{V}$ in lag period 2.

The five strongest % SS reductions by combinations of variables two to six at a time are shown in tables B15-B20, and the five weakest in tables B21-B26. Frequency tables of the computer output are not presented for K_f.

Discussion.--The strongest combinations of independent variables taken six at a time, for the two most-influential lag periods (1 and 2), include $\overline{S}_f$, L_o, H_o/L_o, $\overline{U}_{of}$, α, ρ, and D (tables B15 and B16). Because variables X11 and X14 contribute also to the weakest combinations (tables

Table 8.-The Five Strongest Per Cent Reductions in Net Deposition at Lower Foreshore Stations Sum of Squares Attributable to Combinations of Six Independent Variables (X3 and X5 of The Original 14 Xs Not Used) for Each of Six Lag Periods.

	Independent Variable Combinations												Percent Reductions in SS
Lag 1	1	2		6			9	10			13		73.31
	1	2		6			9		11		13		72.85
	1	2		6			9				13	14	72.77
	1	2	4	6			9				13		72.61
	1	2	4	6			9	10					72.23
Lag 2	1			6				10	11	12		14	69.40
	1			6			9	10	11			14	68.04
	1			6					11	12	13	14	67.76
	1			6			9		11	12		14	67.62
	1		4	6					11	12		14	66.95
Lag 3	1	2	4				9		11	12			72.41
	1	2	4				9		11			14	70.96
	1	2	4			8	9		11				70.78
	1	2	4						11	12		14	70.56
	1	2	4	6					11	12			70.45
Lag 4	1	2		6					11		13	14	61.26
	1			6			9		11		13	14	61.02
	1	2	4						11		13	14	60.80
	1			6				10	11		13	14	60.39
	1	2	4			8			11			14	59.46
Lag 5	1	2	4	6			9	10					68.98
	1	2	4	6				10			13		68.80
	1	2	4	6				10		12			68.75
	1	2	4	6		8		10					68.56
	1	2	4			8			11		13		68.54
Lag 6	1	2	4		7	8	9						76.14
	1	2	4	6		8	9						75.64
	1		4		7	8	9		11				72.36
	1		4	6	7	8	9						72.34
	1		4		7	8	9	10					72.06

Table 9.-Per Cent Reductions in Net Erosion at Lower Foreshore Stations Sum of Squares Attributable to Fourteen Independent Variables, Taken Individually, for Lag Periods 1-6.

Variable	Symbol	Position	Percent Reduction in SS By Lag Periods					
			Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Total % SS Reduction			58.78	61.88	33.75	54.63	42.38	38.85
Net Erosion Lower Foreshore	K_f	Y						
Slope Angle Lower Foreshore	$\overline{S}_f$	X1	10.47	10.57	10.69	10.78	13.56	13.39
Wave Period	T	X2	0.63	4.16	0.00	0.22	0.43	3.00
Wave Length	L_o	X3	5.38	4.48	0.15	0.63	0.58	3.13
Wave Height	H_o	X4	1.65	3.65	0.36	0.29	1.05	0.57
Wave Steepness	H_o/L_o	X5	1.66	1.09	1.40	0.73	0.04	2.74
Wind Velocity Onshore	$\overline{U}_{on}$	X6	0.58	3.14	6.74	7.72	2.12	2.77
Wind Velocity Offshore	$\overline{U}_{of}$	X7	7.86	23.66	4.93	2.16	1.18	0.71
Wind Velocity Parallel to Shore	$\overline{U}_p$	X8	5.83	8.50	1.28	3.35	0.21	0.02
Angle of Wave Approach	α	X9	5.49	13.80	7.55	8.16	4.39	0.10
Longshore Current	$\overline{V}$	X10	0.25	21.06	3.11	4.40	0.00	1.07
Water Density	ρ	X11	0.05	0.05	0.05	6.43	4.39	3.20
Rate of Rise Still-water Level	η_r	X12	1.66	0.81	10.75	0.86	0.58	3.96
Rate of Fall Still-water Level	η_f	X13	1.91	1.55	0.00	2.38	0.65	0.00
Water Table Depth Top of Uprush	D	X14	0.59	0.06	1.38	0.03	2.64	10.14

B21 and B22), it is difficult to assess their true influence. It appears, however, that these variables will logically combine with wave steepness (X5, tables B15 and B16) to erode the lower foreshore during falling tide. Variable X11, water density, will affect the rate of sand-grain transport by fluid drag. The variable D, X14, will be of importance in determination of the magnitude of erosion or deposition during rising and falling tides as it affects the infiltration of swashes and the ability of the backwash to transport sand back toward the breaker zone; i.e., the ability to scour the lower foreshore. The significance to net erosion of variable X7, wind velocity offshore, is believed to lie in three areas: 1) in its interlock with α and $\overline{V}$ (table B9), noted earlier in the section on longshore-current velocity, 2) in the postulated mechanism whereby offshore wind of sufficient magnitude is capable of aiding in the offshore tranport of finer particles thrown into suspension in the breaker zone at the time (lag period 2) that high tide covers the lower foreshore, and 3) in its interlock with water density, wherein offshore winds cause denser water to move shoreward in the lower layers, as mentioned in the section treating $(\overline{M_z})_s$. Angle of wave approach, X9, although interlocked to a degree with $\overline{V}$, may influence circulation over the lower foreshore at high tide in such a way that rip currents are more effective in removing sand from the foreshore for certain values of α. The rate of rise of the still-water level (X12) is of importance to net foreshore erosion in lag periods 3 and 6, when the tide is rising. The rate of rise and fall (X13, table B15) of the still water level will determine the time over which any of the other independent variables will be able to act at a given point.

Table 10 presents the results of the regression analysis for the strongest variables taken six at a time after redundant variables X3 (L_o) and X5 (H_o/L_o) have been removed. If the maximum %-SS-reduction values for lag periods for correlative tide stages are added together, lag periods 1 and 4, 2 and 5, and 3 and 6 have values of 67.21, 84.82, and 58.09, respectively. Thus, the influence of variables acting about the time of high tide is highest on net erosion of the foreshore, as it is measured in this study. And the four variables most influential at high tide will be (table 10) $\overline{S}_f$, $\overline{U}_{of}$, α, and D.

Summary.--Based upon an analysis involving 12 independent variables, it is possible to say that the combination of 5 variables that shows the most influence on net erosion of the lower foreshore is composed of variables $\overline{S}_f$, T, $\overline{U}_{of}$, α, and D. This combination is most-influential about the times of high tide. During times of falling tide, variables $\overline{S}_f$, T, H_o and α compose the most-influential combination of variables. The rate of rise of the still-water level, depth to the water table, and beach slope are judged to be the most important variables during time of rising tide.

Table 10.-The Five Strongest Per Cent Reductions in Net Erosion at Lower Foreshore Stations Sum of Squares Attributable to Combinations of Six Independent Variables (X3 and X5 of The Original 14 Xs Not Used) for Each of Six Lag Periods.

	Independent Variable Combinations												Percent Reductions in SS
Lag 1	1	2	4		7		9		11				32.47
	1	2	4	6		8	9						32.42
	1		4	6		8	9	10					31.82
	1		4		7	8	9	10					31.27
	1		4	6		8	9			12			31.12
Lag 2	1	2			7		9		11			14	53.26
	1	2				8	9		11			14	51.41
	1					8	9		11	12		14	50.19
	1		4		7		9		11			14	48.59
	1					8	9		11		13	14	48.49
Lag 3	1		4				9		11	12		14	24.95
	1		4	6			9		11	12			24.87
	1	2	4				9		11	12			24.66
	1		4		7		9		11	12			24.66
	1		4			8	9		11	12			24.65
Lag 4	1	2	4	6			9			12			34.74
	1	2	4	6	7		9						34.71
	1	2	4	6			9		11				34.66
	1	2		6			9		11	12			34.43
	1	2	4	6		8	9						34.33
Lag 5	1		4		7		9	10				14	31.56
	1			6		8	9	10				14	31.36
	1				7	8	9	10				14	31.10
	1	2			7	8	9					14	30.57
	1	2	4		7		9					14	30.52
Lag 6	1			6	7			10		12		14	33.14
	1			6				10	11	12		14	32.90
	1			6		8		10		12		14	32.79
	1		4	6				10		12		14	32.75
	1			6				10		12	13	14	32.47

BOTTOM SLOPE AND PARTICLE SIZE IN THE SHOALING-WAVE ZONE

Mean Slope in the Shoaling-Wave Zone ($\overline{S}_s$)

Also studied was the interaction of 12 environmental variables with a portion of the beach slope roughly 250 feet beyond the breaker zone. In the case of measurement of the mean slope in the shoaling-wave zone ($\overline{S}_s$), or average mean grain size on the slope [$(\overline{M_z})_s$], the measurements of the independent variables were nearly always made exactly 4, 8, 12, 16, or 20 hours prior to measurement of the dependent variable. This exact lag-time convention was considered more realistic than the 4-hour-long one used for lower-foreshore measurements, because the measurement of the dependent variable was not tied to a dynamic condition, such as obtained at the time of low tide for lower-foreshore measurements.

Variables measured.--Recent laboratory work on the equilibrium characteristics of sand beaches beyond the breaker zone (cf. Eagleson, Glenn, and Dracup, 1963) commonly utilizes measurement or determination of the following variables, where one is concerned with the mechanics of slope alteration: bottom slope itself at the point in question, particle size on the slope, wave period, wave length, wave height, water depth, specific gravity of the sand particles and the fluid, and kinematic viscosity of the fluid. The variables described in the paragraphs below were measured in this study in an effort to approximate the variables found useful in laboratory studies. The subset of data used earlier in the description of the regression method was taken from the fourth lag period of this section of the study.

The segment of the shoaling-wave-zone slope selected for study here was one which was reasonably flat and yet which exhibited a relatively large range of slope values during the periods of observation. As seen in figure 3, the slope was determined from soundings at four stations at the 15th Street pier, each of which was 25 feet apart. The slope was determined by plotting the sounding values for these four, and for several surrounding stations, on cross-section paper and then drawing an estimated mean slope through the four stations by eye. Still-water depth (h) at the midpoint of this mean slope (fig. 3) was determined for each lag period from precision tide data or direct measurement by sounding line. Mean grain size, M_z, determined as explained in appendix A, was obtained for each of the four stations used in slope determination. The four values were averaged to give $(\overline{M_z})_s$ values for the regression analyses. Wave parameters were again estimated from wave-spectrum-analyzer records furnished by the Coastal Engineering Research Center. Specific gravity of the particles was assumed constant (2.65, or that of common quartz). Rather than determine kinematic viscosity, we instead determined water density, from precision temperature and salinity measurements (see appendix A for details).

In addition to the above variables that are similar or identical to those commonly measured in laboratory studies, we utilized measurements of wind velocity (onshore, offshore, and parallel to shore), angle of wave-front approach (which is not constantly parallel to the shoreline in nature, as it usually is in the laboratory), and tidal-current velocity. Finally, we inserted wave steepness as a variable in the least-squares analysis and ran five lag periods.

Thus:

$$\overline{S}_s = f\ [\ (\overline{M_z})_s,\ T,\ L_o,\ H_o,\ H_o/L_o,\ \overline{U}_{on},\ \overline{U}_{of},\ \overline{U}_p,\ \alpha,\ h,\ \rho,\ C\ ]_{t_{1-5}}$$

Results.--Table 11 presents the results of the first stage of regression analysis. The highest %-SS-reduction is found at lag period 5, perhaps because of the dominance of X4 (H_o) and X7 ($\overline{U}_{of}$). It is also noted in passing that lag periods 2 and 5, which show the highest %-SS-reductions are lag periods that coincide with times of high tide for 6 of the 18 slope observations used in the analysis.

Discussion.--It is seen in table 11 that when the variables are considered individually, mean grain size and water density (Xs1 and 11) generally have the greatest effect on shoaling-wave zone slope, out-ranking the wave parameters considerably. The significantly high effect of $(\overline{M_z})_s$ on $\overline{S}_s$ could easily have been predicted from considerations of the mechanics of slope formation and from the known relation between slope and particle size, usually found on foreshore slopes (cf. Bascom, 1951). A difference exists in the mechanics of alternation of the lower foreshore slope, however, where permeability of the sloping surface assumes considerable importance in the process of transportation and deposition. Fluid drag forces are of greater significance to grain movement on the shoaling-wave slope.

Water density, a factor of unexpected importance, undoubtedly influences the shoaling-zone slope through its effect on threshold drag velocities and turbulence at the bed. Wave tank experiments on beach slope modification at the Coastal Engineering Research Center, using warm and cold water, have revealed that under constant incoming wave energies the slope modification was more rapid under cold-water conditions. (The final slopes under both conditions were closely similar).

The strongest combinations of variables taken six at a time make an interesting study. Referring again to lag periods 2 and 5 (tables B28 and B31), which show the greatest percent reductions in total SS, it is seen that average grain size, wave period, wave steepness and tidal-current velocity, appear in the two combinations of six variables for these lag periods. Absent, but occurring in all of the other strongest combinations of six (tables B27, B29, and B30) is wave height (X4), while wind velocity offshore and parallel to shore appears in two of the other combinations. This may be evidence that the slope in this region is shaped by tidal currents in conjunction with wave characteristics at high tide (lag periods 2 and 5), but more by wave height and winds during half tides or low tide. More observations for lag periods keyed to times of low tide will be needed to settle this point.

Table 11.-Per Cent Reduction in Shoaling-wave Zone Slope Sum of Squares Attributable to Twelve Independent Variables, Taken Individually, for Lag Periods 1-5.

Variable	Symbol	Position	Percent Reduction in SS By Lag Periods				
			Period 1	Period 2	Period 3	Period 4	Period 5
Total % SS Reduction			93.15	95.92	92.82	92.06	97.13
Slope Angle Shoaling-wave Zone	$\overline{S}_s$	Y	-----	-----	-----	-----	-----
Mean Grain Size Shoaling-wave Zone	$(\overline{M_z})_s$	X1	63.20	65.61	65.52	63.15	66.92
Wave Period	T	X2	0.00	6.56	1.66	1.07	0.95
Wave Length	L_o	X3	0.84	11.63	3.77	3.15	3.07
Wave Height	H_o	X4	9.16	20.23	7.05	23.74	25.77
Wave Steepness	H_o/L_o	X5	1.05	30.53	1.66	2.96	23.43
Wind Velocity Onshore	$\overline{U}_{on}$	X6	1.91	0.58	8.84	2.11	0.28
Wind Velocity Offshore	$\overline{U}_{of}$	X7	4.10	0.07	8.99	4.68	10.65
Wind Velocity Parallel to Shore	$\overline{U}_p$	X8	2.60	7.56	7.95	2.93	0.01
Angle of Wave Approach	α	X9	10.26	42.52	21.71	5.64	0.52
Still-water Depth	h	X10	12.25	8.45	4.72	5.21	9.56
Water Density	ρ	X11	50.54	54.86	59.65	59.55	57.89
Tidal Current Velocity	C	X12	0.70	5.89	2.41	0.24	7.57

Removing the contribution of the dominant variable (X1) from the analysis (tables B32-B36) reveals the new dominance of another geometric variable, X10 (water depth), which may be considered as a mediator for the various process elements in the environment.

Thus, ignoring X10, it is seen that lag periods 2 and 5 are still the most-influential on shoaling-zone slope, when the variables are taken six at a time. Because variables X3 and X5 exhibit data redundancy with those of X2 and X4, as explained earlier, it is seen that wave period is the important variable for measurement times weighted with those of high-tide measurements (lag periods 2 and 5; tables B33 and B36), while wave height (X4) is most important in altering the slope for measurement times weighted with those of low-tide measurements (tables B32, B34, B35). This result of the least-squares analysis, if truly representative of high and low-tide conditions, would be in keeping with the knowledge that waves of a given period and height will exert a greater effect at a fixed point on the bottom when the tide is low than when it is high (cf. Inman and Nasu, 1956, p. 30). The wave-height effect might then be diminished at high tide to the extent that the only effect that is felt is the effect of wave length (wave period) as it influences drag over the bottom.

A study of the weakest combinations of the variables taken six at a time shows (tables B37-B41) that wave steepness and tidal-current velocity, which are both present in four of the strongest combinations for the five lag periods are also present in these weakest combinations. The only variables that are present in the strongest combinations of six at a time (tables B27-B31), and present in none of the weakest combinations for the corresponding lag periods, are mean size (X1), wave length (X3), wave height (X4), water depth (X10), and water density (X11).

Finally, reference to frequency tables B42-B46, for variables considered in combinations of six at a time, reveals that the distributions of numbers of combinations by %-SS-reduction classes are polymodal for lag periods 1, 3, 4, and 5, but essentially unimodal for lag period 2. Lag periods 3 and 4 exhibit four modes, while 1 and 5 exhibit three each. One inference is that sub-groups of variables that influence the slope to dissimilar degrees are somewhat segregated during lag periods 3 and 4. These sub-groups, however, if they are in fact semi-discrete in a physical sense, each seem to influence the shoaling-wave-zone slope to about the same degree during lag period 2. If true, it might be said that the combination X2, X3, X5, X9, X10, and X12 (table B33) is the most-significant combination of variables to influence the foreshore slope and that this influence is exerted between 4 and 8 hours prior to slope measurement. Additional work with such frequency tables is planned. They are presented in appendix B for the interested reader and for future reference.

Summary.--The bottom slope of the beach at 15th Street, some 250-feet seaward of the breaker zone, may be thought of as being controlled mostly by the following combination of six variables: average mean grain size of the bottom materials, wave period, wave length, wave steepness, water depth, and tidal-current velocity. This combination of variables exerts its maximum influence on the slope through a lag in time of between 4 and 8 hours, and apparently to a lesser extent between 16 and 20 hours. This delay may reflect the influence exerted on the slope during the time of the previous two high tides. Wave height, angle of wave approach, and water density become more influential through delays in time amounting to 0-4, 8-12, or 12-16 hours, and coinciding with or approaching times of low tide.

Average Mean Grain Size in the Shoaling-wave Zone $(\overline{M_z})_s$

Variables measured.--The measurements used in this analysis were the same ones as were used in the previous analysis for $\overline{S}_s$, average mean grain size being interchanged with $\overline{S}_s$ as the dependent variable. Thus:

$$(\overline{M}_z)_s = f\,(\overline{S}_s,\ T,\ L_o,\ H_o,\ H_o/L_o,\ \overline{U}_{on},\ \overline{U}_{of},\ \overline{U}_p,\ \alpha,\ h,\ \rho,\ C)_{t_{1-5}}$$

The analysis was run for five periods, as was done for $\overline{S}_s$.

Results.--Table 12 presents the results of the first stage of the analysis where it is seen that the most-influential lag period is number 3, which occurs 8-12 hours prior to measurement of the independent variable. Lag periods 2, 4, and 5 are of about equal influence, while lag period 1 is the least influential. Variable X1 is seen to be among the dominate variables when they are taken individually, and this is to be expected inasmuch as a high interdependence between $\overline{S}_s$ and $(\overline{M_z})_s$ was noted in the previous analysis. Because of the possible masking influence of this dimensionless variable, X1, two sets of tables have been prepared, both of which show the strongest combination of independent variables influencing $(\overline{M_z})_s$. One set (tables B47-B51) includes áll 12 Xs; the other set (tables B52-B56) does not include X1. Finally, a set of tables for the combination of independent variables showing the weakest influence (tables B57-B61), and a set of frequency tables (tables B62-B66) have been prepared.

Discussion.--Turning to table B49, for lag period 3, one sees that the most-influential combination of variables taken six at a time consists of $\overline{S}_s$, T, H_o/L_o, $\overline{U}_p$, α, and C. Variables H_o/L_o, $\overline{U}_p$, and C also enter into the weakest combination, when they combine with L_o, $\overline{U}_{of}$, and h (table B59). Recognizing the importance of slope in its influence on particle-sizes transported, we turn our attention to the other variables in this strongest combination of six variables. The reader is reminded that T is redundant with H_o/L_o, through the relationship $L_o = 5.12T^2$, which relationship was used in this study for obtaining L_o. At any rate, the fluid forces at the water-sediment interface that are induced by varying wave periods and wave steepness will tend to move grains of various sizes over the shoaling-zone slope. Superimposed upon this mass transport by the wave-drift current will be the transport induced by tidal and wind-driven currents. For the bottom slope in question (fig. 3), the major current will be the tidal

Table 12.-Per Cent Reduction in Mean Grain Size (Shoaling-wave Zone) Sum of Squares Attributable to Twelve Independent Variables, Taken Individually, for Lag Periods 1-5.

Variable	Symbol	Position	Percent Reduction in SS By Lag Periods				
			Period 1	Period 2	Period 3	Period 4	Period 5
Total % SS Reduction			86.15	93.10	98.75	94.68	93.89
Mean Grain Size Shoaling-wave Zone	$(\overline{M_Z})_s$	Y	-----	-----	------	-----	-----
Slope Angle Shoaling-wave Zone	$\overline{S}_s$	X1	63.20	60.33	48.74	48.69	42.09
Wave Period	T	X2	1.59	2.49	1.29	2.08	1.91
Wave Length	L_o	X3	0.32	4.81	0.05	0.85	0.37
Wave Height	H_o	X4	1.27	9.06	0.72	11.58	16.82
Wave Steepness	H_o/L_o	X5	5.56	19.86	3.24	0.07	2.74
Wind Velocity Onshore	$\overline{U}_{on}$	X6	8.94	4.93	15.60	1.06	0.03
Wind Velocity Offshore	$\overline{U}_{of}$	X7	1.19	12.51	0.02	0.07	1.41
Wind Velocity Parallel to Shore	$\overline{U}_p$	X8	5.09	24.24	0.81	8.27	4.00
Angle of Wave Approach	α	X9	16.77	33.23	15.78	6.34	2.94
Still-water Depth	h	X10	1.88	0.10	14.86	0.33	1.66
Water Density	ρ	X11	31.60	26.07	22.48	18.53	14.38
Tidal-Current Velocity	C	X12	7.71	24.34	5.23	28.41	40.58

current and it will react a complex way with wind-generated bottom currents and wave-drift currents. Thus, it is possible to conceive of a situation in which the angle of wave approach, the direction and velocity of the wind, and the direction and velocity of the tidal current at a given time will interact to reinforce or impede one another. A wind moving opposite in direction to a tidal current, for example, will produce an increase in the tidal-current velocity near the bottom (Reid, 1957) and, should the wave fronts be traveling in the direction of flow of the tidal current, the net current velocity could have a velocity up to about 16 percent (cf. Collins, 1964) greater than the simple algebraic summation of the wave-induced current and the reinforcing tidal current. As noted by Collins (1964, p. 1051), "the effects of even very small currents on the mass transport [of fine material] by waves could be very large." D. R. Tuck (oral communication) has studied the interrelationships of wind velocity and direction, angle of wave approach, and tidal-current velocity on the average mean-grain-size values for the beach slope in question. In instances where the wave-drift current was reinforced by the tidal current, water velocities of 21-29 cm/sec were obtained near the bed. Mean particle sizes actually observed at the bed under these circumstances, and for the slopes in effect at the time, were within the range of particle sizes predicted by theory as being moved by the net current.

Thus, there is every reason to believe that a combination such as $\bar{S}_s$, T, H_o/L_o, $\bar{U}_p$, α, and C, mentioned at the outset as being the most-influential combination of six variables in the most-influential lag period, is in fact a valid combination for this area of the beach at Virginia Beach.

Table B54 shows that the most-influential combination (X1 ignored for 6 Xs at a time) includes: T, L_o, $\bar{U}_p$, α, h, and ρ. Variables X10 and X11 take the place of variables X5 and X12 of the analysis in which X1 was included. Water depth merely acts to mediate the various process elements, while water density will affect fluid drag at the bed. As seen in all of tables B52-B56, water density, X11, is a dominant variable in the strongest combinations.

Of further and more general significance is the rather persistent contribution of variables X6, X7, X8, and X11 and X12 in the strongest combinations of Xs taken six at a time (tables B52-B56). Wind velocities onshore (X6) and offshore (X7) have also been seen to be of importance in net erosion on the lower foreshore. In that section of the study it was postulated that when the offshore wind reaches a certain velocity it is important in producing a weak surface current beyond the breaker zone that is capable of transporting fine particles put into suspension in the breakers out into the shoaling-wave zone. The onshore wind, however, may produce a seaward return flow of water on the bottom that will transport somewhat coarser particles out of the breaker zone and onto the slope in the shoaling-wave zone. Winds parallel to shore will probably interact significantly with tidal currents, which are generally parallel to the shoreline in the area of investigation, and augment or decrease the current velocities in the lower layers.

Also of considerable significance in consideration of the role of onshore and offshore winds is their interlock with water density, X11. If the wind blows offshore long enough and strong enough, the wind stress at the water surface produces a seaward movement of the upper layers of the ocean and a shoreward return flow in the lower layers. (The procedure is more or less reversed with an onshore wind.) This sort of "pseudo-upwelling" results in the shoreward movement of colder, more-saline water in the summer months and relatively warmer, more-saline water in the winter months. This type of turnover of the continental-shelf waters has been undergoing documentation by the Virginia Institute of Marine Science for over two years, using sea-bed drifters and sea-surface drift bottles. In addition, the temperature and salinity measurements made at the 15th Street pier correlate well with observed offshore and onshore wind movements. Thus, there is a clear interlock between $\overline{U}_{of}$, $\overline{U}_{on}$, and ρ. And water density will clearly influence fluid drag velocities and turbulence at the bed and, thereby, the particle-size distribution in the shoaling-wave zone.

Summary.--Based upon an analysis involving 12 independent variables, average mean size on the shoaling-zone slope at 15th Street, 250 feet beyond the breakers, is most-influenced by the combination of variables (taken arbitrarily in a combination of six) composed of mean slope, wave period, wave steepness, wind velocity parallel to shore, angle of wave approach, and tidal-current velocity. This combination of variables is most influential on $(\overline{M_z})_s$ after a lag in time of 8-12 hours.

Mean slope is the most-dominant independent variable at all times. When removed from the analysis, it is in the main replaced by water density, and this variable is found to be of considerable importance in all of the strong combinations of variables.

The importance of wind velocity onshore and offshore in the analyses is found in their interlock with water density as it will change when shelf water in the lower layers is moved to the shore during offshore winds and out to sea under onshore winds. The shelf water in the upper and lower layers is stratified as to density, especially during the summer months, and the density affects the sizes of particles moved.

Wind velocity parallel to shore is significant in its ability to reinforce or decrease tidal-current velocities near the bottom, because tidal currents in the study area flow generally parallel to shore. The angle of wave approach is probably of significance as it interlocks with tidal-current velocity. Wave-drift and tidal currents interlock in their effect on particle movement at the bed.

FUTURE STUDY

A major aim of any search for significant interactions in a natural system is to provide a basis upon which more formal models may be erected. The authors are continuing their efforts in this direction, using the present data set. Additional techniques (for example, factor analysis and discriminant functions) will be examined. The data set will also be augmented with additional field measurements in the study area. Development of relatively informal "process-response" models is underway, while construction of stochastic process (simulation) models will be attemped in the near future. Expression and analysis of the present variables in non-dimensional form will also, it is hoped, lead to better integration of field and wave-tank studies of the beach-ocean-atmoshpere system.

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LIST OF FIGURES

APPENDIX A

Descriptions of the variables used in this study and explanations of how they were measured are given in this appendix, together with the ranges in the values that were employed in the regression analyses. Additional details of the measurement techniques may be found in two other reports in this series on Virginia Beach (Harrison and Wagner, 1964; Harrison and Morales-Alamo, 1964).

B_d

Depth of Water at Wave Breaking

The average vertical distance, in feet and hundredths, from the water surface immediately in front of a breaking wave to the bottom. This is an average of ten measurements, recorded four times each day. Values used ranged from 0.35 to 3.80 feet.

C, C_o, C_s

Tidal-current Velocity, C; Opposed to Longshore Current, C_o; And in Same Direction as Longshore Current, C_s.

The mean tidal-current velocity was measured one meter above the bottom at a point 850 feet from shore. A Price current meter was used, and measured or interpolated values were expressed to hundredths of feet per second. The respective ranges of values used were: C=0.00 to 0.68 ft/sec, C_o=0.00 to 0.66 ft/sec, and C_s=0.00 to 0.36 ft/sec. Some tidal-current-velocity values were estimated from a relationship established between the tidal range and the tidal-current velocity at the point.

D

Depth of Water Table at Top of Uprush

The depth to the water table at the top of the swash line was measured to feet and tenths four times daily. Values used ranged from 0.01 to 1.83 feet.

h

Still-water Depth

The still-water depth was determined from tide-gage data. The values used are for the depth from the still-water surface to the mid-point of the slope (fig. 3) in the shoaling-wave zone that was investigated for changes in inclination and sediment size. Values used in the analyses ranged from 10.6 to 16.1 feet.

H_b

Height of Breaking Wave

The breaker height was measured directly with a graduated rod and was taken as the average trough-to-crest distance of ten successive breaking waves. Breaker heights were measured four times daily. Values used in the analyses ranged from 0.69 to 4.50 feet.

H_o

Deep-water Wave Height (Significant)

Wave-height values were determined from wave-spectrum-analyzer records furnished by the Coastal Engineering Research Center, as they were obtained from the relay-type wave gage in 20 feet of water at the end of the 15th Street pier. The peak value on the linear-average curve for wave heights of the dominant wave train was multiplied by 2.22. This was done because J. M. Caldwell had found (written communication, 1963) the following relationship in a study of 92 simultaneous wave recordings made by both magnetic-tape and paper-tape methods:

$$\frac{\text{Average height on analyzer record}}{\text{Significant height on chart record}} = 0.45$$

Height values were converted to deep-water ones by entering tables. Thus conversion to deep-water wave height values did not involve consideration of wave refraction or special shoaling effects. Values for deep-water significant wave heights ranged from 0.56 to 12.97 feet. The absolute validity of the wave-height values so determined is not of importance, as all that was needed was a consistently objectively-determined measure of this variable.

H_o/L_o

Deep-water Wave Steepness

Values of wave steepness, expressed in terms of the ratio for deep-water waves H_o/L_o, ranged from 0.00100 to 0.07226.

J_f

Net Deposition at Lower Foreshore Stations in Previous 24.5-hour Period

Net deposition was determined from measured changes in altitude of stations occupied on successive days at low tide. Net deposition was measured over a distance of 125 feet, or at six stations (fig. 3, 4) at 15th Street and at five stations (Fig. 4) at the Camp Pendleton property line. Values used ranged from 0.00 to 1.50 feet.

K_f

Net Erosion at Lower Foreshore Stations in Previous 24.5-hour Period

The net erosion was determined in a like manner to that of the increment of deposition, J_f. Values used ranged from 0.00 to 1.30 feet.

L_o

Wave Length in Deep Water

Values for wave length (expressed as a deep-water value) ranged from 49.20 to 989.24 feet.

$(\overline{M_z})_s$

Average Mean Nominal Grain Diameter

Over Bottom in Shoaling-wave Zone

Mean-size values were averaged for four samples taken 25 feet apart in the shoaling-wave zone (fig. 3) where MLW depths ranged between 10.1 and 14.1 feet. The sampling device, a pipe dredge, permitted taking an integrated sample over a 15-foot distance parallel to shore, and so the average value for the four samples is presumed to represent the average mean size of a 15 x 100 foot area of the bottom. Individual $(M_z)_s$ values were determined using a Woods Hole Rapid Sand Analyzer (Zeigler, and others, 1960; Zeigler and Gill, 1959), and the procedure outlined in Harrison and Morales-Alamo (1964). The statistics used to estimate the mean nominal diameters were

$$M_z = \frac{P_{20} + P_{50} + P_{80}}{3} \quad \text{(for summer data)}$$

$$M_z = \frac{P_{10} + P_{30} + P_{50} + P_{70} + P_{90}}{5} \quad \text{(for winter data)}$$

Average $(M_z)_s$ values used in the analyses ranged form 0.234 to 0.843 mm.

R

Range of Tide Over One Tidal Cycle

The measured tidal range at 15th Street as taken from records furnished by the U. S. Coast and Geodetic Survey. Values used ranged between 2.4 and 4.7 feet. This was the only independent variable whose values were held constant over several lag periods, a practice which is to be discouraged.

$\overline{S}_f$

Mean Slope Over Lower Foreshore of Beach

The mean slope was determined for a 200-foot distance of the lower foreshore (fig. 3) from measurements at nine stations that were made daily at low tide. This measurement was for $\overline{S}_f$ taken as a dependent variable. Values ranged from 1.40 to 4.55 degrees. When $\overline{S}_f$ was taken as an independent variable, the slope was measured over the distances shown on figure 4.

$\overline{S}_s$

Mean Slope of Beach Over Inner

Portion of Shoaling-wave Zone

The mean slope was determined for a 100-foot length of beach in the shoaling-wave zone (fig. 3) where MLW depths ranged between 10.1 and 14.1 feet. Measurements of the altitude of five stations were made daily at low tide in the summer, and every four hours in the winter. Mean slope values used in the analyses ranged between 0.50 and 2.22 degrees.

T

Wave Period

Wave period was determined from wave-spectrum-analyzer records furnished by the Coastal Engineering Research Center. The period used was the one corresponding to the peak value on the linear-average curve of wave heights for the dominant wave train. (Period values thus obtained were nearly always larger than those obtained in a "significant-wave" chart analysis.) Values used ranged from 3.10 to 13.90 seconds.

$\overline{U}_a$, $\overline{U}_{of}$, $\overline{U}_{on}$, $\overline{U}_p$, $\overline{U}_s$

Mean Wind Velocity Against the Direction of Longshore Current, $\overline{U}_a$;

In An Offshore Direction, $\overline{U}_{of}$; In An Onshore Direction, $\overline{U}_{on}$;

Parallel to Shore $\overline{U}_p$; And in the Same Direction as

the Longshore Current, $\overline{U}_s$.

Mean wind velocities and directions were recorded at 2-hour intervals at the Cape Henry weather station (fig. 1B), some 7.5 miles north of the study area. The deviations in wind directions permitted when describing the direction as onshore, offshore, and so on, are given below:

$\overline{U}_a$	($\pm$ 10°	of wind vector directed opposite to longshore-current flow)
$\overline{U}_{of}$	($\pm$ 80°	of wind vector perpendicular to shore and in an offshore direction)
$\overline{U}_{on}$	($\pm$ 80°	of wind vector perpendicular to shore and in an onshore direction)

$\overline{U}_p$ (± 10° of wind vector directed in either direction along trend of shoreline)

$\overline{U}_s$ (± 10° of wind vector directed parallel to longshore-current flow, and in the same direction)

$\overline{U}_a$ values ranged from 0.00 to 16.00 M.P.H.

$\overline{U}_{of}$ values ranged from 0.00 to 34.00 M.P.H.

$\overline{U}_{on}$ values ranged from 0.00 to 26.75 M.P.H.

$\overline{U}_p$ values ranged from 0.00 to 17.50 M.P.H.

$\overline{U}_s$ values ranged from 0.00 to 18.00 M.P.H.

$\overline{V}$

Mean Velocity of Longshore Current

The current velocity and direction was measured four times daily by timing the movement of 2 or 3 fluorescene dye patches moving over (usually) a 100-foot distance. Values used in the analyses are absolute values, a zero-velocity value representing either a no-current condition or a rip-current condition. Values used ranged from 0.00 to 3.20 feet per second.

α

Angle of Wave Approach

The angle of wave-front approach of the dominant wave train as measured with a pelorus in a zone 1000 to 1300 feet from shore in water depths of 20-26 feet. Measurements were made 4 times daily. Values used in this study were absolute values only and ranged between 2 and 75 degrees.

η_r

Rate of Rise of Still-water Level

The instantaneous rate of rise of the tide, in hundredths of feet per hour, as determined from observed times and magnitudes of high and low water. Values used ranged from 0.00 to 0.67 feet per hour, and were computed for the mid-points of the 6 lag periods. Because times of low tide in nature did not always coincide with anticipated times ("R," fig. 2), the η_r and η_f computations sometimes resulted in values for rising tide during a lag period when it should only have been falling, or vice versa.

η_f

Rate of Fall of Still-water Level

The instantaneous rate of fall of the tide, in hundredths of feet per hour, as determined from observed times and magnitudes of high and low water. Values used ranged from 0.00 to 0.60 feet per hour, and were computed for the mid-points of the 6 lag periods.

ρ

Water Density

The "sigmatee" values for the sea water as determined (U. S. Navy Hydro. Office, 1962) from temperature ($\pm$ 0.05°C) and salinity ($\pm$ 0.02 ‰) data once daily at noon time in the surf zone, for the summer measurements, and three times daily in the shoaling-wave zone, for the winter measurements. Values used ranged from 1.0136 to 1.02500 gm/cm^3.

NOTE: Summary Tables of Appendix B are started on opposite page.

APPENDIX B

Summary tables of the computer output

are given in this appendix.

Table B1.-The Five Strongest Per Cent Reductions in Net Deposition at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 1.

	Independent Variable Combinations													Percent Reduction in SS
Two Xs	1				5									53.15
at a time	1									10				51.96
	1	2												49.18
	1										11			48.87
	1		3											47.95
Three Xs	1					6				10				61.30
at a time	1				5	6								61.23
	1				5							13		60.08
	1					6			9					59.33
	1									10		13		59.26
Four Xs	1					6			9	10				70.62
at a time	1				5	6			9					70.54
	1				5			8				13		67.55
	1	2				6			9					66.97
	1		3			6			9					66.85
Five Xs	1			4	5	6			9					72.89
at a time	1		3			6			9			13		72.53
	1	2				6			9			13		72.47
	1				5	6			9	10				72.33
	1				5	6		8	9					72.27
Six Xs	1	2		4	5		7	8						76.72
at a time	1		3	4	5		7	8						75.85
	1	2		4	5		7						14	74.81
	1			4	5	6			9			13		74.72
	1	2		4	5	6							14	74.69

Table B2.-The Five Strongest Per Cent Reductions in Net Deposition at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 2.

	Independent Variable Combinations										Per Cent Reduction in SS
Two Xs	1						11				47.83
at a time	1							12			42.37
	1		6								42.22
	1	4									39.78
	1					10					39.66
Three Xs	1			8			11				56.53
at a time	1						11	12			56.48
	1		6				11				55.20
	1	4					11				52.55
	1				9		11				51.71
Four Xs	1		6				11	12			60.75
at a time	1		6		9		11				60.24
	1						11	12		14	59.68
	1		6	8			11				59.54
	1	4					11	12			59.48
Five Xs	1		6				11	12		14	65.08
at a time	1		6		9		11	12			63.71
	1		6		9		11			14	63.71
	1		6				11	12	13		63.09
	1					10	11	12		14	62.93
Six Xs	1		6			10	11	12		14	69.40
at a time	1		6		9	10	11			14	68.04
	1		6				11	12	13	14	67.76
	1		6		9		11	12		14	67.62
	1	4	6				11	12		14	66.95

Table B3.-The Five Strongest Per Cent Reductions in Net Deposition at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 3.

	Independent Variable Combinations										Percent Reduction in SS
Two Xs	1							11			47.52
at a time	1		3								41.38
	1			4							40.65
	1					8					40.31
	1	2									39.54
Three Xs			3	4				11			60.66
at a time	1					8		11			56.53
	1			4				11			56.17
	1		3					11			55.32
	1	2						11			54.68
Four Xs	1		3		4			11			70.17
at a time	1	2		4				11			67.42
			3	4				11		14	66.42
			3	4				11	12		64.17
	1	2			5			11			63.57
Five Xs	1		3	4				11	12		73.14
at a time	1	2	3	4				11			72.06
	1		3	4			9	11			71.64
	1		3	4				11		14	71.37
	1		3	4		8		11			71.21
Six Xs	1	2	3	4				11	12		74.62
at a time	1		3	4	5			11	12		74.60
	1	2		4	5			11	12		74.50
	1		3	4			9	11	12		74.31
	1		3	4				11	12	14	73.84

Table B4.-The Five Strongest Per Cent Reductions in Net Deposition at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 4.

	Independent Variable Combinations										Percent Reduction in SS
Two Xs			3	4							48.21
at a time	1			4							42.60
		2		4							42.38
	1		3								41.67
	1	2									39.61
Three Xs	1		3	4							58.51
at a time	1	2		4							54.02
		2	3	4							51.69
	1	2			5						51.48
	1		3		5						49.25
Four Xs	1	2	3	4							62.91
at a time	1		3	4	5						60.66
	1		3	4		6					58.96
	1		3	4			7				58.91
	1		3	4				8			58.91
Five Xs	1	2	3	4	5						66.36
at a time	1	2	3	4						10	65.41
	1	2	3	4				8			64.45
	1	2	3	4			7				63.92
	1	2	3	4					9		63.92
Six Xs	1	2	3	4	5			8			71.66
at a time	1	2	3	4	5				9		68.11
	1	2	3	4	5		7				67.76
	1	2	3	4	5	6					67.20
	1	2	3	4	5					10	67.17

Table B5.-The Five Strongest Per Cent Reductions in Net Deposition at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 5.

	Independent Variable Combinations														Percent Reduction in SS
Two Xs					5					10					47.56
at a time				4	5										43.18
			3		5										39.89
	1				5										39.00
					5			8							38.53
Three Xs				4	5			8							58.54
at a time				4	5					10					57.08
			3		5					10					56.34
		2			5					10					54.95
	1				5					10					54.33
Four Xs				4	5			8		10					68.40
at a time	1	2			5								13		68.37
	1	2			5							12			68.31
	1	2	3	4											66.15
				4	5	6				10					65.00
Five Xs	1	2	3		5								13		74.23
at a time	1	2	3		5							12			74.15
	1			4	5			8					13		72.80
	1			4	5			8				12			72.74
	1	2		4	5								13		71.50
Six Xs	1	2	3		5				9				13		78.83
at a time	1	2	3		5				9			12			78.67
	1	2	3		5			8					13		78.30
	1	2	3		5			8				12			78.20
	1	2	3		5		7						13		76.56

Table B6.-The Five Strongest Per Cent Reductions in Net Deposition at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 6.

	Independent Variable Combinations														Percent Reduction in SS
Two Xs	1			4											53.23
at a time	1				5										40.10
				4										14	39.21
		2			5										37.82
	1													14	37.30
Three Xs	1	2			5										67.69
at a time	1		3		5										66.68
	1		3	4											57.41
	1			4					9						57.12
			3		5									14	56.66
Four Xs	1	2			5		7								71.00
at a time	1		3		5		7								70.59
	1	2			5				9						70.25
	1	2			5			8							70.18
	1	2			5									14	69.45
Five Xs	1	2			5			8	9						79.39
at a time	1		3		5			8	9						77.91
	1	2			5	6		8							74.92
	1	2			5				9					14	73.12
	1		3		5				9					14	72.97
Six Xs	1	2			5			8	9					14	79.87
at a time	1	2			5			8	9		11				79.75
	1	2			5		7	8	9						79.57
	1	2			5			8	9			12			79.54
	1	2		4	5			8	9						79.48

Table B7.-The Five Weakest Per Cent Reductions in Net Deposition at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 2.

	Independent Variable Combinations									Percent Reduction in SS
Two Xs at a time				7		10				0.07
	2			7						0.26
	2					10				0.33
	2		5							0.41
			5	7						0.43
Three Xs at a time	2			7			11			0.34
	2		5	7						0.46
	2		5			10				0.48
			5	7		10				0.52
	2				9	10				0.94
Four Xs at a time	2		5	7		10				0.54
	2			7	9	10				1.32
	2		5		9	10				1.37
		3	5	7		10				1.55
		3	5	7	9					1.75
Five Xs at a time		3	5	7	9	10				2.27
	2		5	7	9	10				3.15
	2		5	7		10			13	6.69
	2			7	9	10			13	7.98
			5	7	9	10			13	8.54
Six Xs at a time	2		5	7	9	10			13	8.94
	2		5	7		10		12	13	11.13
	2		5	7	9	10		12		12.06
	2		5		9	10		12	13	12.22
	2			7	9	10		12	13	12.31

Table B8.-The Five Weakest Per Cent Reductions in Net Deposition at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 3.

	Independent Variable Combinations									Percent Reduction in SS
Two Xs at a time							12	13		0.08
					9			13		0.55
	5							13		0.62
					9		12			0.63
	5						12			0.65
Three Xs at a time					9		12	13		0.63
	5						12	13		0.65
			7				12	13		1.09
	5				9				14	1.19
	5				9			13		1.21
Four Xs at a time	5				9		12	13		1.21
	5		7				12	13		1.62
			7		9		12	13		1.65
	5		7		9			13		2.17
	5		7		9		12			2.20
Five Xs at a time	5		7		9		12	13		2.20
	5	6	7				12	13		2.76
	5	6		8			12	13		4.88
	5	6	7		9			13		4.99
		6	7		9		12	13		6.79
Six Xs at a time	5	6	7		9		12	13		7.03
	5		7		9	10	12	13		8.40
	5	6	7	8			12	13		9.92
	5	6	7			10	12	13		9.98
	5	6	7	8	9			13		10.11

Table B9.-The Five Weakest Per Cent Reductions in Net Deposition at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 5.

	Independent Variable Combinations								Percent Reduction in SS
Two Xs at a time	2					9			0.53
		3				9			0.66
	2		6						1.65
			6			9			1.83
		3	6						2.04
Three Xs at a time	2		6			9			1.83
		3	6			9			2.15
	2				8	9			2.23
			6		8	9			2.40
	2		6		8				2.41
Four Xs at a time	2		6		8	9			2.42
		3	6		8	9			2.97
	2			7	8	9			3.93
		3		7	8	9			4.18
			6		8	9		14	4.70
Five Xs at a time	2		6	7	8	9			6.25
		3	6	7	8	9			6.31
	2			7	8	9	10		6.78
			6	7	8	9		14	7.23
		3		7	8	9	10		7.42
Six Xs at a time	2		6	7	8	9		14	8.92
	2	3	6	7	8	9			9.15
	2	3	6	7		9		14	9.42
		3	6	7	8	9		14	9.80
	2	3		7	8	9		14	10.48

Table B10.-The Five Weakest Per Cent Reductions in Net Deposition at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 6.

	Independent Variable Combinations								Percent Reduction in SS
Two Xs at a time			6					13	0.00
			6			11			0.01
						11		13	0.01
					10			13	0.65
					10	11			0.65
Three Xs at a time			6			11		13	0.01
					10	11		13	0.65
	2					11		13	0.77
	2		6			11			0.85
	2		6					13	0.85
Four Xs at a time	2		6			11		13	0.85
	2				10	11		13	1.19
		3	6			11		13	1.44
	2		6		10			13	1.63
	2		6		10	11			1.63
Five Xs at a time	2		6		10	11		13	1.63
		3	6		10	11		13	2.13
	2		6	8		11		13	3.85
		3	6	8		11		13	4.44
			6	8	10	11		13	4.58
Six Xs at a time	2		6	8	10	11		13	5.06
		3	6	8	10	11		13	5.54
	2		6		10	11	12	13	7.13
		3	6		10	11	12	13	7.90
	2		6	8	10		12	13	7.95

Table B11.-Net Deposition on Lower Foreshore vs 14 Xs (Lag Period 2)

SS Reduction	Number of Independent Variables					
Class %	1	2	3	4	5	6
0.0-4.9	6	14	18	9	2	
5.0-9.9	2	10	17	19	14	2
10.0-14.9	4	29	78	97	67	19
15.0-19.9		9	61	143	139	64
20.0-24.9	1	7	41	127	212	175
25.0-29.9		5	25	81	172	218
30.0-34.9		3	27	109	198	250
35.0-39.9	1	11	43	112	236	355
40.0-44.9		2	31	141	336	459
45.0-49.9		1	16	97	311	592
50.0-54.9			4	32	134	324
55.0-59.9			3	32	145	332
60.0-64.9				2	35	199
65.0-69.9					1	14
Σ	14	91	364	1001	2002	3003

Total SS Reduction = 82.19%

Table B12.-Net Deposition on Lower Foreshore vs 14 Xs (Lag Period 3)

SS Reduction	Number of Independent Variables					
Class %	1	2	3	4	5	6
0.0-4.9	7	19	25	17	4	
5.0-9.9	1	8	25	36	22	4
10.0-14.9	2	13	32	41	31	12
15.0-19.9	3	20	55	92	92	52
20.0-24.9		7	51	133	172	121
25.0-29.9		4	19	67	110	136
30.0-34.9	1	10	42	96	117	106
35.0-39.9		6	46	158	232	208
40.0-44.9		3	49	166	441	538
45.0-49.9		1	11	85	262	434
50.0-54.9			4	48	202	455
55.0-59.9			4	40	168	398
60.0-64.9			1	19	109	313
65.0-69.9				2	32	149
70.0-74.9				1	12	77
Σ	14	91	364	1001	2002	3003

Total SS Reduction = 80.49%

Table B13.-Net Deposition on Lower Foreshore vs 14 Xs (Lag Period 5)

SS Reduction	Number of Independent Variables					
Class %	1	2	3	4	5	6
0.0-4.9	8	21	19	5		
5.0-9.9	3	24	47	45	21	4
10.0-14.9		8	59	92	48	8
15.0-19.9	2	8	34	98	121	56
20.0-24.9		7	25	69	112	100
25.0-29.9		6	36	64	99	98
30.0-34.9	1	11	55	128	155	124
35.0-39.9		4	29	149	279	250
40.0-44.9		1	31	84	211	314
45.0-49.9		1	15	106	228	303
50.0-54.9			11	92	267	360
55.0-59.9			3	51	256	496
60.0-64.9				13	128	434
65.0-69.9				5	64	327
70.0-74.9					13	119
75.0-79.9						10
Σ	14	91	364	1001	2002	3003

Total SS Reduction = 87.66%

Table B14.-Net Deposition on Lower Foreshore vs 14 Xs (lag Period 6)

SS Reduction	Number of Independent Variables					
Class %	1	2	3	4	5	6
0.0-4.9	8	26	38	25	6	
5.0-9.9	3	20	55	73	48	14
10.0-14.9		4	29	75	91	54
15.0-19.9		3	21	46	54	42
20.0-24.9	2	7	13	28	46	34
25.0-29.9		12	55	112	129	91
30.0-34.9	1	8	44	123	185	158
35.0-39.9		9	58	157	227	198
40.0-44.9		1	27	160	382	417
45.0-49.9			7	65	234	438
50.0-54.9		1	9	42	138	299
55.0-59.9			6	64	247	487
60.0-64.9				9	86	308
65.0-69.9			2	18	80	222
70.0-74.9				4	47	210
75.0-79.9					2	31
Σ	14	91	364	1001	2002	3003

Total SS Reduction = 82.78%

Table B15.-The Five Strongest Per Cent Reductions in Net Erosion at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 1.

	Independent Variable Combinations	Percent Reduction in SS
Two Xs at a time	3 7	18.47
	1 7	15.61
	1 8	15.51
	1 9	15.19
	1 3	15.06
Three Xs at a time	3 7 11	29.16
	3 5 7	29.12
	3 4 5	26.59
	3 5 9	25.05
	1 3 5	24.71
Four Xs at a time	3 5 7 11	37.33
	1 3 4 5	36.50
	3 5 7 9	36.21
	1 3 5 7	35.59
	3 7 9 11	35.02
Five Xs at a time	3 5 7 9 11	47.42
	1 3 4 5 9	43.07
	1 3 5 7 9	42.55
	1 3 5 7 11	42.13
	3 5 6 8 9	41.05
Six Xs at a time	1 3 5 7 9 11	51.79
	3 5 6 7 9 11	48.95
	3 4 5 7 9 11	48.85
	2 3 5 7 9 11	48.31
	3 5 7 9 11 13	48.05

Table B16.-The Five Strongest Per Cent Reductions in Net Erosion at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 2.

	Independent Variable Combinations	Percent Reduction in SS
Two Xs at a time	1 7	33.22
	3 7	31.80
	9 10	30.40
	2 7	29.36
	7 10	29.29
Three Xs at a time	1 3 7	39.79
	3 5 7	39.65
	1 2 7	37.98
	1 7 14	37.48
	1 4 7	37.11
Four Xs at a time	1 3 5 7	47.81
	1 2 5 7	44.62
	1 3 7 14	44.09
	3 5 7 10	43.06
	1 3 4 7	42.99
Five Xs at a time	1 3 5 7 14	50.49
	1 3 5 7 11	50.49
	1 3 5 7 10	49.04
	1 3 5 7 9	48.66
	1 3 5 7 13	48.64
Six Xs at a time	1 3 7 9 11 14	55.70
	1 3 5 7 11 14	54.11
	1 3 5 7 9 11	53.39
	1 2 7 9 11 14	53.26
	1 3 8 9 11 14	52.58

Table B17.-The Five Strongest Per Cent Reductions in Net Erosion at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 3.

	Independent Variable Combinations											Percent Reduction in SS
Two Xs	1										12	20.81
at a time	1					6						20.40
	1								9			18.00
	1						7					15.85
									9		12	13.01
Three Xs	1								9		12	23.07
at a time	1						7				12	22.81
	1					6					12	22.35
	1					6			9			21.84
	1				5						12	21.58
Four Xs	1								9	11	12	23.99
at a time	1						7		9		12	23.96
	1			4					9		12	23.92
	1					6			9		12	23.59
	1							8	9		12	23.57
Five Xs	1			4	5				9		12	24.78
at a time	1			4					9	11	12	24.63
	1				5				9	11	12	24.59
	1	2		4	5				9			24.51
	1			4		6			9		12	24.40
Six Xs	1	2	3	4	5				9			31.64
at a time	1	2	3	4	5	6						27.50
	1	2	3	4	5						12	27.17
	1	2		4	5	6			9			25.77
	1		3	4	5				9		12	25.75

Table B18.-The Five Strongest Per Cent Reductions in Net Erosion at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 4.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs	1					6							22.03
at a time	1								9				17.86
									9	10			16.29
						6					11		15.57
	1							8					14.98
Three Xs	1					6			9				27.75
at a time	1					6					11		25.57
	1	2				6							25.54
	1		3			6							25.11
	1				5	6							23.42
Four Xs	1	2				6			9				32.04
at a time	1		3			6			9				32.03
	1			4		6			9				29.73
	1	2				6					11		29.11
	1				5	6			9				29.10
Five Xs	1	2		4		6			9				33.87
at a time	1		3	4		6			9				33.82
	1	2				6			9			12	33.15
	1		3			6			9			12	33.01
	1	2				6			9		11		32.86
Six Xs	1	2	3	4	5	6							40.83
at a time	1	2	3	4	5		7						38.28
		2	3	4	5		7		9				38.06
	1	2	3	4	5				9				38.02
	1	2		4	5	6			9				36.19

Table B19.-The Five Strongest Per Cent Reductions in Net Erosion at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 5.

	Independent Variable Combinations											Percent Reduction in SS
Two Xs at a time	1										14	19.24
	1								9			17.83
	1					6						17.04
	1			4								15.59
	1						7					15.22
Three Xs at a time	1								9		14	26.44
	1				5						14	21.25
	1			4							14	20.82
	1									10	14	20.50
	1						7				14	20.43
Four Xs at a time	1				5				9		14	28.99
	1						7		9		14	28.23
	1								9	10	14	27.07
	1		3						9		14	27.06
	1			4					9		14	27.02
Five Xs at a time	1				5		7		9		14	31.71
	1				5			8	9		14	31.70
	1						7		9	10	14	30.15
	1	2						8	9		14	29.84
	1			4			7		9		14	29.53
Six Xs at a time	1				5	6		8	9		14	34.73
	1				5		7	8	9		14	33.05
	1		3		5		7		9		14	32.79
	1	2			5		7		9		14	32.78
	1				5		7		9	10	14	32.65

Table B20.-The Five Strongest Per Cent Reductions in Net, Erosion at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 6.

	Independent Variable Combinations														Percent Reduction in SS
Two Xs at a time	1														19.98
	1													14	17.54
	1					6									17.14
	1				5										17.14
	1	2													16.70
Three Xs at a time	1											12		14	25.34
	1	2										12			22.35
	1					6						12			22.34
	1				5							12			22.29
										10		12		14	22.17
Four Xs at a time	1									10		12		14	29.89
	1					6						12		14	26.54
	1	2										12		14	26.45
	1		3									12		14	26.18
	1					6				10				14	25.87
Five Xs at a time	1					6				10		12		14	32.47
	1								9	10		12		14	30.63
	1									10	11	12		14	30.60
	1			4						10		12		14	29.99
	1						7			10		12		14	29.94
Six Xs at a time	1					6	7			10		12		14	33.14
	1					6				10	11	12		14	32.90
	1					6		8		10		12		14	32.79
	1			4		6				10		12		14	32.75
	1					6				10		12	13	14	32.47

Table B21.-The Five Weakest Per Cent Reductions in Net Erosion at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 1.

	Independent Variable Combinations								Percent Reduction in SS
Two Xs at a time				10	11				0.28
	2			10					0.63
					11			14	0.66
			6		11				0.71
				10				14	0.80
Three Xs at a time	2			10	11				0.81
				10	11			14	0.88
			6	10	11				1.02
	2			10				14	1.17
	2				11			14	1.17
Four Xs at a time	2			10	11			14	1.17
			6	10	11			14	1.82
			6	10	11	12			1.93
	2		6	10		12			2.24
	2			10	11	12			2.25
Five Xs at a time	2	4		10	11			14	2.51
			6	10	11	12		14	2.55
	2			10	11	12		14	2.57
	2		6	10	11	12			2.89
	2		6	10	11			14	2.90
Six Xs at a time	2		6	10	11	12		14	3.23
			6	10	11	12	13	14	3.28
	2			10	11	12	13	14	3.38
	2		6	10		12	13	14	3.78
	2		6	10	11	12	13		3.83

Table B22.-The Five Weakest Per Cent Reductions in Net Erosion at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 2.

	Independent Variable Combinations									Percent Reduction in SS
Two Xs at a time						11			14	0.10
							12		14	0.82
						11	12			0.86
				5					14	1.11
				5		11				1.32
Three Xs at a time						11	12		14	0.86
				5		11			14	1.33
				5			12		14	1.54
							12	13	14	1.57
						11		13	14	1.58
Four Xs at a time						11	12	13	14	1.59
				5		11	12		14	1.74
				5			12	13	14	3.24
					6	11		13	14	3.25
				5		11		13	14	3.27
Five Xs at a time					6	11	12	13	14	3.27
				5		11	12	13	14	3.47
				5	6	11	12		14	4.57
				5	6		12	13	14	4.63
				5	6	11		13	14	4.77
Six Xs at a time				5	6	11	12	13	14	4.81
			4		6	11	12	13	14	7.09
	2	3		5		11	12		14	7.12
	2		4	5		11	12		14	7.56
		3	4	5		11	12		14	8.51

Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 3.

	Independent Variable Combinations									Percent Reduction in SS
Two Xs at a time	2							13		0.00
	2						11			0.05
							11	13		0.05
		3						13		0.15
		3					11			0.23
Three Xs at a time	2						11	13		0.05
		3					11	13		0.23
	2		4					13		0.36
			4				11	13		0.47
	2		4				11			0.48
Four Xs at a time	2		4				11	13		0.48
		3	4				11	13		0.66
	2				8		11	13		1.34
	2						11	13	14	1.43
		3			8		11	13		1.71
Five Xs at a time	2				8		11	13	14	1.97
	2		4		8		11	13		2.19
	2		4				11	13	14	2.25
		3			8		11	13	14	2.47
		3	4		8		11	13		2.58
Six Xs at a time	2		4		8		11	13	14	3.03
		3	4		8		11	13	14	3.57
	2	3		5		10	11	13		4.73
	2	3	4			10	11	13		4.74
	2			5	8		11	13	14	5.35

Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 4.

	Independent Variable Combinations									Percent Reduction in SS
Two Xs at a time	2							13		0.32
			4						14	0.37
	2		4							0.49
				5					14	0.73
		3							14	0.88
Three Xs at a time	2		4						14	0.67
		3	4						14	1.24
			4				12		14	1.25
	2						12		14	1.41
				5			12		14	1.59
Four Xs at a time	2		4				12		14	1.68
		3	4				12		14	2.31
			4	5			12		14	2.44
				5	7		12		14	2.47
	2				7		12		14	2.73
Five Xs at a time	2	3	4				12		14	4.30
	2	3		5	7		12			4.72
	2	3			7		12		14	4.76
	2		4		7		12		14	4.81
	2	3		5	7				14	5.06
Six Xs at a time	2	3		5	7		12		14	5.33
	2	3	4		7		12		14	6.65
			4	5	7	10	12		14	6.94
	2	3			7	10	12		14	7.14
	2	3		5	7	10			14	7.15

Table B25.-The Five Weakest Per Cent Reductions in Net Erosion at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 5.

	Independent Variable Combinations								Percent Reduction in SS
Two Xs				5		10			0.04
at a time					8	10			0.21
				5	8				0.23
	2					10			0.52
	2				8				0.66
Three Xs				5	8	10			0.25
at a time	2				8	10			0.70
				5	8		12		0.77
				5		10		13	0.82
					8	10	12		0.82
Four Xs				5	8	10	12		0.98
at a time				5	8		12	13	1.02
	2				8	10	12		1.05
				5	8	10		13	1.12
				5		10	12	13	1.17
Five Xs				5	8	10	12	13	1.39
at a time	2	3			8	10	12		1.64
	2				8	10	12	13	1.75
			4	5	8	10	12		1.85
		3			8	10	12	13	1.95
Six Xs			4	5	8	10	12	13	2.38
at a time	2	3			8	10	12	13	2.47
	2	3	4		8	10	12		2.47
		3	4	5	8	10	12		2.73
	2		4		8	10	12	13	2.73

Table B26.-The Five Weakest Per Cent Reductions in Net Erosion at Lower Foreshore Stations Sum of Squares Attributable to Each of Several Combinations of Fourteen Independent Variables, for Lag Period 6.

	Independent Variable Combinations								Percent Reduction in SS
Two Xs					8			13	0.02
at a time						9		13	0.10
					8	9			0.11
		4						13	0.57
		4			8				0.61
Three Xs					8	9		13	0.11
at a time		4			8			13	0.61
				7	8			13	0.74
		4				9		13	0.78
		4			8	9			0.80
Four Xs		4			8	9		13	0.80
at a time		4		7	8			13	1.00
				7	8	9		13	1.26
		4		7		9		13	1.46
		4		7	8	9			1.54
Five Xs		4		7	8	9		13	1.54
at a time		4		7	8		10	13	1.90
		4			8	9	10	13	2.08
		4		7		9	10	13	2.62
		4		7	8	9	10		2.63
Six Xs		4		7	8	9	10	13	2.63
at a time	3	4	5	7	8			13	3.84
		4	5	7	8		10	13	3.89
	3	4		7	8		10	13	3.92
	3	4	5		8		10	13	3.93

Table B27.-The Five Strongest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 1.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs at a time	1										11		73.39
	1											12	73.27
	1						7						71.67
	1									10			69.12
	1			4									67.79
Three Xs at a time	1			4							11		76.38
	1			4								12	76.22
	1		3							10			76.07
	1	2								10			75.13
	1						7			10			75.11
Four Xs at a time	1			4	5		7						81.06
	1			4		6	7						79.97
	1			4		6					11		79.79
	1		3	4	5								78.71
	1			4	5						11		78.55
Five Xs at a time	1			4		6	7	8					87.80
	1		3	4	5		7						85.32
	1	2		4	5		7						84.50
	1		3	4	5						11		84.02
	1		3	4	5							12	83.40
Six Xs at a time	1			4		6	7	8		10			88.99
	1			4	5	6	7	8					88.94
	1			4		6	7	8	9				87.85
	1			4		6	7	8			11		87.81
	1			4		6	7	8				12	87.81

Table B28.-The Five Strongest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 2.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs	1										11		77.50
at a time											11	12	75.08
	1						7						73.18
	1									10			73.10
	1			4									71.82
Three Xs	1			4						10			84.89
at a time									9		11	12	83.85
	1			4							11		82.36
	1			4			7						80.41
	1								9		11		80.08
Four Xs				4					9		11	12	87.30
at a time	1			4					9	10			86.49
	1			4	5					10			86.47
	1	2		4						10			86.14
	1			4						10		12	86.01
Five Xs				4					9	10	11	12	91.01
at a time	1	2	3		5					10			90.94
	1	2	3	4						10			90.68
			3	4					9		11	12	89.48
		2		4					9		11	12	89.05
Six Xs	1	2	3		5					10		12	93.44
at a time	1			4					9	10	11	12	91.93
	1	2	3	4						10		12	91.91
	1	2	3	4			7			10			91.87
		2	3		5				9	10		12	91.87
Seven Xs	1	2	3	4			7			10		12	94.33
at a time	1	2	3		5		7			10		12	94.05
	1	2	3		5				9	10		12	94.02
		2	3		5				9	10	11	12	93.74
		2	3	4					9	10	11	12	93.65
Eight Xs	1	2	3	4			7		9	10		12	95.78
at a time	1	2	3		5		7		9	10		12	95.04
	1	2	3	4	5		7			10		12	94.54
	1	2	3	4					9	10	11	12	94.47
	1	2	3		5				9	10	11	12	94.39

Table B28.-The Five Strongest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 2.

	Independent Variable Combinations												Percent Reduction in SS
Nine Xs	1	2	3	4			7		9	10	11	12	95.85
at a time	1	2	3	4	5		7		9	10	11		95.83
	1	2	3	4		6	7		9	10		12	95.78
	1	2	3	4			7	8	9	10		12	95.78
	1	2	3		5		7	8	9	10		12	95.06
Ten Xs	1	2	3	4			7	8	9	10	11	12	95.89
at a time	1	2	3	4	5		7		9	10	11	12	95.87
	1	2	3	4		6	7		9	10	11	12	95.85
	1	2	3	4	5		7	8	9	10		12	95.83
	1	2	3	4	5	6	7		9	10		12	95.83

Table B29.-The Five Strongest Per Cent Reductions In Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 3.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs	1										11		81.13
at a time	1											12	78.77
	1				5								76.44
	1			4									73.76
	1						7						70.41
Three Xs	1			4							11		86.92
at a time	1										11	12	86.25
	1				5						11		85.36
	1		3									12	84.97
	1	2										12	84.91
Four Xs	1				5						11	12	89.11
at a time	1			4							11	12	88.93
	1		3								11	12	87.71
	1	2		4							11		86.97
	1			4						10	11		86.95
Five Xs	1	2		4							11	12	89.83
at a time	1		3	4							11	12	89.69
	1				5				9		11	12	89.39
	1			4						10	11	12	89.27
	1				5					10	11	12	89.26
Six Xs	1	2		4			7				11	12	90.13
at a time	1	2		4					9		11	12	90.08
	1	2		4						10	11	12	90.03
	1	2		4		6					11	12	90.00
	1		3	4						10	11	12	89.95

Table B30.-The Five Strongest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 4.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs	1										11		81.42
at a time	1			4									74.11
											11	12	73.64
	1											12	70.20
				4							11		69.86
Three Xs	1			4						10			88.83
at a time	1								9		11		84.86
	1				5						11		84.50
				4							11	12	83.98
	1							8			11		83.10
Four 'Xs	1			4							11	12	89.65
at a time	1			4					9		11		89.63
	1			4			7				11		89.45
	1			4				8			11		89.29
	1			4						10	11		89.27
Five Xs	1			4				8		10	11		91.14
at a time	1			4			7		9		11		90.56
	1			4					9		11	12	90.41
	1			4	5					10	11		90.28
	1			4				8	9		11		90.26
Six Xs	1			4	5			8		10	11		91.85
at a time	1			4				8	9	10	11		91.54
	1			4		6		8		10	11		91.38
:	1			4			7	8		10	11		91.35
	1		3	4				8		10	11		91.32

Table B31.-The Five Strongest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 5.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs at a time	1										11		83.61
											11	12	81.95
	1								9				77.72
	1			4									74.13
	1				5								73.44
Three Xs at a time	1								9		11		91.50
				4							11	12	90.20
	1			4							11		88.57
					5						11	12	87.16
	1										11	12	87.08
Four Xs at a time	1								9		11	12	94.05
	1								9	10	11		93.91
	1			4							11	12	92.96
	1	2							9		11		92.31
	1		3						9		11		92.29
Five Xs at a time	1			4					9		11	12	94.95
	1								9	10	11	12	94.76
	1					6			9	10	11		94.54
	1				5				9		11	12	94.49
	1						7		9		11	12	94.33
Six Xs at a time	1	2			5				9		11	12	95.39
	1		3		5				9		11	12	95.34
	1					6			9	10	11	12	95.26
	1				5				9	10	11	12	95.07
	1			4	5				9		11	12	95.07
Seven Xs at a time	1	2			5	6			9		11	12	95.71
	1		3		5	6			9		11	12	95.69
	1		3		5		7		9		11	12	95.59
	1	2			5		7		9		11	12	95.58
	1	2	3			6			9	10	11		95.56
Eight Xs at a time	1		3		5		7	8	9		11	12	96.33
	1	2			5		7	8	9		11	12	96.28
	1		3	4	5	6			9	10	11		96.12
	1	2	3			6			9	10	11	12	96.06
	1		3		5	6			9	10	11	12	95.99

Table B31.-The Five Strongest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 5.

	Independent Variable Combinations												Percent Reduction in SS
Nine Xs at a time	1		3	4	5	6	7		9	10	11		96.86
	1	2		4	5	6	7		9	10	11		96.64
	1	2	3	4		6	7		9	10	11		96.50
	1		3		5		7	8	9	10	11	12	96.48
	1	2		4	5		7	8	9		11	12	96.42
Ten Xs at a time	1		3	4	5	6	7	8	9	10	11		97.02
	1	2	3	4	5	6	7		9	10	11		96.92
	1		3	4	5	6	7		9	10	11	12	96.90
	1	2		4	5	6	7	8	9	10	11		96.80
	1	2		4	5	6	7		9	10	11	12	96.75

Table B32.-The Five Strongest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Eleven Independent Variables (Variable X1 Not Used) for Lag Period 1.

	Independent Variable Combinations											Percent Reduction in SS
Two Xs at a time						7				11		58.49
										11	12	56.81
					6					11		54.90
							8			11		53.56
			4							11		53.51
Three Xs at a time			4	5						11		65.54
	2	3								11		61.83
			4			7				11		61.33
					6		8			11		61.08
			4							11	12	59.85
Four Xs at a time			4		6	7	8					77.12
			4		6		8			11		75.91
			4		6			9		11		74.78
		3	4	5						11		74.40
			4	5	6					11		73.39
Five Xs at a time			4		6	7	8		10			82.99
			4		6	7	8	9				80.75
			4		6	7	8			11		80.52
		3	4		6	7	8					79.73
	2		4		6	7	8					79.47
Six Xs at a time			4		6	7	8	9	10			86.66
			4	5	6	7	8		10			85.78
			4		6	7	8		10	11		84.79
			4	5	6	7	8			11		84.23
			4	5	6	7	8	9				84.03

Table B33.-The Five Strongest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Eleven Independent Variables (Variables X1 Not Used) for Lag Period 2.

	Independent Variable Combinations											Percent Reduction in SS
Two Xs at a time										11	12	75.08
						7				11		68.93
								9		11		68.62
							8			11		65.56
			4							11		63.77
Three Xs at a time								9		11	12	83.85
			4							11	12	79.48
						7				11	12	76.63
					6					11	12	76.22
				5						11	12	76.15
Four Xs at a time			4					9		11	12	87.30
				5				9		11	12	85.51
		3						9		11	12	84.86
								9	10	11	12	84.64
	2							9		11	12	84.45
Five Xs at a time			4					9	10	11	12	91.01
		3	4					9		11	12	89.48
	2		4					9		11	12	89.05
			4				8	9		11	12	89.01
	2	3		5					10		12	88.29
Six Xs at a time	2	3		5				9	10		12	91.87
			4				8	9	10	11	12	91.24
	2		4					9	10	11	12	91.21
			4		6			9	10	11	12	91.07
			4			7		9	10	11	12	91.02

Table B34.-The Five Strongest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Eleven Independent Variables (Variable X1 Not Used) for Lag Period 3.

	Independent Variable Combinations											Percent Reduction in SS
Two Xs at a time			4							11		63.57
								9		11		62.62
	2									11		61.21
		3								11		60.37
					6					11		60.24
Three Xs at a time			4		6					11		70.19
			4	5						11		69.85
			4						10	11		66.25
			4					9		11		66.23
	2	3								11		65.73
Four Xs at a time			4	5	6					11		75.51
			4	5					10	11		72.31
			4		6				10	11		72.06
	2		4		6					11		71.81
	2		4	5						11		71.74
Five Xs at a time			4	5	6				10	11		77.27
	2		4	5	6					11		76.55
		3	4	5	6					11		76.32
			4	5	6		8			11		75.96
			4	5	6	7				11		75.89
Six Xs at a time			4	5	6				10	11	12	78.48
	2		4	5	6				10	11		77.66
		3	4	5	6				10	11		77.57
			4	5	6	7			10	11		77.38
			4	5	6		8		10	11		77.38

Table B35.-The Five Strongest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Eleven Independent Variables (Variable X1 Not Used) for Lag Period 4.

	Independendnet Variable Combinations											Percent Reduction in SS
Two Xs at a time										11	12	73.64
			4							11		69.86
							8			11		64.43
	2									11		61.47
		3								11		61.06
Three Xs at a time			4							'11	12	83.98
				5						11	12	75.59
								9		11	12	75.26
							8			11	12	74.56
			4	5						11		74.41
Four Xs at a time			4	5						11	12	86.04
			4		6					11	12	84.65
	2		4							11	12	84.44
			4						10	11	12	84.42
		3	4							11	12	84.36
Five Xs at a time			4	5					10	11	12	88.27
	2		4	5						11	12	86.42
		3	4	5						11	12	86.42
			4	5			8			11	12	86.30
	2		4						10	11	12	86.30
Six Xs at a time			4	5				9	10	11	12	88.90
			4	5			8		10	11	12	88.60
			4	5	6				10	11	12	88.36
			4	5		7			10	11	12	88.28
	2		4	5					10	11	12	88.28

Table B36.--The Five Strongest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Eleven Independent Variables (Variable X1 Not Used) for Lag Period 5.

	Independent Variable Combinations										Percent Reduction in SS
Two Xs at a time									11	12	81.95
			4						11		68.69
								10	11		65.58
	2								11		62.99
						8			11		62.08
Three Xs at a time			4						11	12	90.20
				5					11	12	87.16
							9		11	12	85.45
	2								11	12	82.32
						8			11	12	82.24
Four Xs at a time			4					10	11	12	91.18
			4	5					11	12	90.96
			4			8			11	12	90.59
			4		7				11	12	90.50
	2		4						11	12	90.41
Five Xs at a time			4	5				10	11	12	91.61
			4			8		10	11	12	91.32
			4		7			10	11	12	91.28
			4	5	7				11	12	91.27
		3	4					10	11	12	91.21
Six Xs at a time	2	3		5				10	11	12	92.08
	2		4	5				10	11	12	91.91
		3	4	5				10	11	12	91.81
			4	5	7			10	11	12	91.74
			4	5		8		10	11	12	91.69

Table B37.-Weakest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 1.

	Independent Variable Combinations										Percent Reduction in SS
Two Xs at a time	2									12	0.75
		3								12	1.21
	2			5							1.46
				5	6						1.92
	2				6						1.93
Three Xs at a time	2			5	6						1.94
	2			5						12	2.41
				5	6					12	3.67
	2				6					12	3.68
		3		5	6						3.98
Four Xs at a time	2			5	6					12	3.95
		3		5	6					12	4.56
		3		5	6	7					10.13
		3		5		7				12	10.42
	2			5		7				12	10.78
Five Xs at a time		3		5	6	7				12	12.44
		3		5	6			9		12	12.56
	2			5	6			9		12	13.30
		3		5		7		9		12	14.75
	2			5		7		9		12	15.04
Six Xs at a time		3		5	6	7		9		12	16.48
	2			5	6	7		9		12	21.90
	2		4			7	8		10	12	22.55
	2			5		7	8	9		12	23.34
				5		7	8	9	10	12	24.93

Table B38.-The Five Weakest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 2.

	Independent Variable Combinations											Percent Reduction in SS
Two Xs at a time					6	7						0.86
					6						12	6.14
	2				6							6.57
	2					7						7.10
							8				12	8.27
Three Xs at a time	2				6	7						7.29
	2				6				10			9.15
					6		8				12	9.97
		3			6				10			12.49
					6	7			10			12.56
Four Xs at a time	2				6	7			10			13.39
		3			6	7			10			16.73
	2				6	7					12	20.93
			4		6		8		10			25.03
		3			6	7					12	26.99
Five Xs at a time	2				6	7			10		12	31.12
		3			6	7			10		12	33.91
	2		4	5	6	7						35.46
		3	4	5	6	7						37.61
		3	4	5	6				10			42.05
Six Xs at a time	2		4	5	6	7					12	45.27
		3	4	5	6	7					12	48.88
	2		4	5	6		8				12	49.11
	2		4	5	6	7			10			50.34
		3	4	5	6		8				12	51.72
Seven Xs at a time	2		4	5	6	7	8				12	61.78
		3	4	5	6	7			10		12	63.10
	2		4	5	6	7			10		12	64.00
		3	4	5	6	7	8				12	64.65
	2		4	5	6	7		9	10			66.40
Eight Xs at a time	2		4	5	6	7	8		10		12	73.71
		3	4	5	6	7	8		10		12	73.84
	2		4	5	6	7	8	9	10			75.63
		3	4	5	6	7	8	9	10			76.42
	2		4	5	6	7	8		10	11		77.49

Table B38.-The Five Weakest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 2.

	Independent Variable Combinations												Percent Reduction in SS
Nine Xs at a time		2		4	5	6	7	8	9	10	11		80.84
			3	4	5	6	7	8	9	10	11		81.30
	1		3		5	6	7	8	9	10		12	82.38
	1		3		5	6	7	8		10	11	12	82.51
		2	3	4	5	6	7	8	9		11		83.02
Ten Xs at a time	1		3		5	6	7	8	9	10	11	12	86.62
		2	3	4	5	6	7	8	9	10	11		86.92
	1	2			5	6	7	8	9	10	11	12	87.22
	1	2	3	4	5	6	7	8			11	12	89.13
	1	2	3	4	5	6	7	8	9		11		89.14

Table B39.-The Five Weakest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Independent Variables, for Lag Period 3.

	Independent Variable Combinations								Percent Reduction in SS
Two Xs at a time	2			5					2.25
				5				12	3.10
		3		5					3.93
	2							12	4.90
	2						10		5.39
Three Xs at a time	2			5				12	4.92
		3		5				12	8.08
		3		5		8			8.37
	2			5		8			8.73
	2		4					12	9.28
Four Xs at a time	2			5	7			12	9.91
	2			5		8		12	11.11
	2			5	7	8			11.69
		3		5	7	8			11.90
		3		5	7			12	12.13
Five Xs at a time	2			5	7	8		12	12.46
		3		5	7	8		12	13.63
		3		5	7	8	10		15.26
	2			5	7	8	10		16.80
	2		4		7	8		12	17.25
Six Xs at a time		3		5	7	8	10	12	21.28
	2			5	7	8	10	12	21.74
		3	4		7	8	10	12	33.34
	2		4		7	8	10	12	33.40
	2	3	4	5	7			12	33.64

Table B40.-Weakest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 4.

	Independent Variable Combinations									Percent Reduction in SS
Two Xs at a time	2								12	1.71
				6					12	2.58
	2			6						2.93
						8			12	2.94
	2		5							2.99
Three Xs at a time	2		5	6						3.35
	2		5						12	3.71
	2			6					12	3.86
			5	6					12	3.86
				6		8			12	4.06
Four Xs at a time	2		5	6					12	4.21
	2		5	6	7					5.78
		3	5	6					12	6.24
		3	5	6	7					6.48
			5	6		8			12	6.75
Five Xs at a time	2		5	6	7				12	8.46
	2		5	6		8			12	9.17
		3	5	6	7				12	9.73
	2		5	6			9		12	11.05
			5	6		8		10	12	11.52
Six Xs at a time	2		5	6		8	9		12	14.36
	2		5	6	7		9		12	14.87
		3	5	6	7		9		12	15.63
	2		5	6	7		9	10		15.86
	2		5	6		8	9	10		15.97

Table B41.-The Five Weakest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 5.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs at a time						6		8					0.37
						6			9				0.55
								8	9				0.55
		2						8					1.06
		2							9				1.63
Three Xs at a time						6		8	9				0.61
		2						8	9				1.75
		2				6		8					1.85
		2				6			9				1.96
			3					8	9				4.34
Four Xs at a time		2				6		8	9				2.02
			3			6		8	9				4.92
			3			6			9	10			10.15
		2				6	7		9				11.02
						6		8	9			12	11.14
Five Xs at a time			3			6		8	9	10			12.62
		2				6		8	9	10			13.06
			3			6	7		9	10			13.61
		2				6	7		9	10			13.70
		2				6		8	9			12	17.77
Six Xs at a time			3		5	6		8	9	10			30.92
			3		5	6	7		9	10			31.76
			3			6	7	8	9	10			34.39
		2		4	5	6		8		10			38.49
			3	4	5	6		8		10			38.88
Seven Xs at a time			3		5	6	7	8	9	10			40.86
		2		4	5	6	7	8		10			45.23
		2		4	5	6		8	9	10			46.51
			3	4	5	6	7	8		10			47.09
			3	4	5	6		8	9	10			47.96
Eight Xs at a time		2		4	5	6	7	8	9	10			51.12
			3	4	5	6	7	8	9	10			53.38
		2	3	4		6	7	8	9	10			61.87
		2	3	4	5	6	7	8	9				62.68
		2	3	4	5	6		8	9	10			62.98

Table B41.-The Five Weakest Per Cent Reductions in Shoaling-wave Zone Slope Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 5.

	Independent Variable Combinations												Percent Reduction in SS
Nine Xs at a time		2	3	4	5	6	7	8	9	10			65.00
		2	3	4	5		7	8	9	10	11		74.23
		2	3	4	5	6	7	8	9			12	74.71
		2	3	4	5	6	7	8		10	11		75.58
			3	4	5	6	7	8	9	10	11		75.79
Ten Xs at a time		2	3	4	5	6	7	8	9	10	11		77.47
	1	2	3	4	5	6	7	8	9			12	83.94
	1	2	3	4	5	6	7	8	9	10			84.94
	1	2		4	5	6	7	8	9	10		12	85.09
	1		3	4	5	6	7	8	9	10		12	85.39

Table B42.-Slope Angle Shoaling-wave Zone vs 12 Xs (Lag Period 1)

SS Reduction Class %	Number of Independent Variables 1	2	3	4	5	6
0.0-4.9	7	12	7	1		
5.0-9.9	1	11	12	1		
10.0-14.9	2	13	36	23	4	
15.0-19.9		5	21	32	13	1
20.0-24.9		1	16	34	24	6
25.0-29.9		2	5	27	30	12
30.0-34.9			7	14	22	11
35.0-39.9			3	9	14	11
40.0-44.9		1	6	9	10	9
45.0-49.9			6	24	34	14
50.0-54.9	1	8	20	38	38	27
55.0-59.9		2	21	56	84	47
60.0-64.9	1	5	12	39	93	109
65.0-69.9		3	14	33	47	63
70.0-74.9		3	29	83	118	79
75.0-79.9			5	69	228	378
80.0-84.9				1	31	137
85.0-89.9					2	20
Σ	12	66	220	495	792	924

Total SS Reduction = 93.15%

Table B43.-Slope Angle Shoaling-wave Zone vs 12 Xs (Lag Period 2)

SS Reduction Class %	Number of Independent Variables 1	2	3	4	5	6	7	8	9	10
0.0-4.9	2	1								
5.0-9.9	4	7	3							
10.0-14.9	1	5	5	1						
15.0-19.9		2	2	1						
20.0-24.9	1	4	6	1						
25.0-29.9		2	4	2						
30.0-34.9	1	8	17	13	2					
35.0-39.9		4	13	9	2					
40.0-44.9	1	5	17	23	9					
45.0-49.9	1	3	19	32	19	4				
50.0-54.9		4	11	24	19	3				
55.0-59.9		4	13	19	14	5				
60.0-64.9	1	2	22	61	63	4				
65.0-69.9		9	26	52	78	44	4			
70.0-74.9		4	35	88	123	97	31	2		
75.0-79.9		2	22	118	250	268	108	10		
80.0-84.9			5	40	150	274	255	99	12	
85.0-89.9				11	60	198	294	211	62	6
90.0-94.9					3	27	100	171	138	49
95.9-99.9								2	8	11
Σ	12	66	220	495	792	924	792	495	220	66

Total SS Reduction = 95.92%

Table B44.-Slope Angle Shoaling-wave Zone vs 12 Xs (Lag Period 3)

SS Reduction	Number Independent Variables					
Class %	1	2	3	4	5	6
0.0-4.9	5	4	1			
5.0-9.9	4	15	8	1		
10.0-14.9		9	25	15	2	
15.0-19.9		5	17	15	5	
20.0-24.9	1	6	13	12	7	2
25.0-29.9		4	25	35	10	
30.0-34.9		1	16	50	37	4
35.0-39.9		1	10	36	45	15
40.0-44.9			3	27	57	35
45.0-49.9			1	11	47	50
50.0-54.9					13	41
55.0-59.9	1	4	7	9	12	16
60.0-64.9		6	33	81	119	113
65.0-69.9	1	6	14	29	59	80
70.0-74.9		2	16	46	71	91
75.0-79.9		2	15	51	104	127
80.0-84.9		1	13	44	65	53
85.0-89.9			3	33	139	283
90.0-94.9						4
Σ	12	66	220	495	792	914

Total SS Reduction = 92.82%

Table B45.- Slope Angle Shoaling-wave Zone vs 12 Xs (Lag Period 4)

SS Reduction	Number of Independent Variables					
Class %	1	2	3	4	5	6
0.0-4.9	7	13	6	2		
5.0-9.9	2	18	31	17	3	
10.0-14.9		4	32	44	20	2
15.0-19.9			7	31	34	12
20.0-24.9	1	5	8	11	27	19
25.0-29.9		3	13	23	11	16
30.0-34.9		1	11	29	35	8
35.0-39.9		1	8	27	46	42
40.0-44.9			2	6	19	24
45.0-49.9			2	12	18	18
50.0-54.9				6	18	17
55.0-59.9	1	2	1		9	22
60.0-64.9	1	8	26	37	25	16
65.0-69.9		7	18	25	39	29
70.0-74.9		3	39	103	117	74
75.0-79.9			5	68	202	263
80.0-84.9		1	10	34	73	131
85.0-89.9			1	20	89	176
90.0-94.9					7	45
Σ	12	66	220	495	792	914

Total SS Reduction = 95.92%

Table B46.-Slope Angle Shoaling-wave Zone vs 12 Xs (Lag Period 5)

SS Reduction	Number Independent Variables									
Class %	1	2	3	4	5	6	7	8	9	10
0.0-4.9	5	9	12	2						
5.0-9.9	2	5	9							
10.0-14.9	1	9	26	15						
15.0-19.9		2	6	4	1					
20.0-24.9	1	4	10	3	1					
25.0-29.9	1	7	25	22	6					
30.0-34.9		6	24	26	17					
35.0-39.9		2	17	38	26	5				
40.0-44.9			11	33	45	22	2			
45.0-49.9		1	10	26	32	22	6			
50.0-54.9			1	14	29	20	9	2		
55.0-59.9	1	4	11	11	28	29	8			
60.0-64.9		3	15	23	29	35	22	7		
65.0-69.9	1	6	19	55	75	38	16	3	1	
70.0-74.9		5	7	62	119	150	107	33	2	
75.0-79.9		1	4	66	95	72	31	18	8	1
80.0-84.9		2	7	32	106	213	211	122	32	2
85.0-89.9			4	44	102	122	99	59	31	9
90.0-94.9			2	19	81	188	230	165	71	19
95.0-99.9						8	51	86	75	35
Σ	12	66	220	495	792	924	792	495	220	66

Total SS Reduction = 97.13%

Table B47.-The Five Strongest Per Cent Reduction in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 1.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs	1											12	75.16
at a time	1						7						70.81
	1					6							66.84
	1								9				65.87
	1				5								65.60
Three Xs	1										11	12	79.76
at a time	1								9			12	79.15
	1						7				11		76.56
	1			4								12	76.41
	1						7		9				76.28
Four Xs	1								9		11	12	82.28
at a time	1					6					11	12	81.17
	1						7				11	12	81.02
	1						7		9		11		80.98
	1			4							11	12	80.63
Five Xs	1						7		9		11	12	84.31
at a time	1						7		9	10	11		83.60
	1					6			9		11	12	83.33
	1				5				9		11	12	82.82
	1								9	10	11	12	82.68
Six Xs	1						7		9	10	11	12	85.43
at a time	1						7	8	9		11	12	85.29
	1					6	7		9		11	12	84.66
	1	2					7		9		11	12	84.41
	1			4			7		9		11	12	84.39

Table B48.-The Five Strongest Per Cent Reduction in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 2.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs	1						7						77.10
at a time							7				11		72.90
	1											12	72.66
	1							8					69.29
											11	12	67.98
Three Xs							7				11	12	81.14
at a time	1				5		7						80.69
	1						7				11		80.23
	1	2					7						78.80
	1			4			7						78.28
Four Xs	1			4	5		7						85.70
at a time	1	2	3				7						84.76
					5		7				11	12	84.04
							7		9		11	12	82.93
	1				5		7				11		82.48
Five Xs	1		3	4	5		7						90.07
at a time	1	2		4	5		7						89.76
	1			4	5		7			10			88.18
	1			4	5		7	8					87.98
	1	2	3				7			10			87.48
Six Xs	1		3	4	5		7		9				91.34
at a time	1	2		4	5		7		9				90.99
	1		3	4	5		7	8					90.68
	1		3	4	5		7			10			90.50
	1		3	4	5		7				11		90.48

Table B49.-The Three Strongest Per Cent Reductions in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 3.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs	1				5								72.66
at a time	1											12	69.19
	1									10			64.19
Three Xs	1				5							12	84.45
at a time	1		3									12	80.29
	1	2										12	79.80
Four Xs	1				5				9			12	90.79
at a time	1		3		5							12	86.51
	1	2			5							12	86.19
Five Xs	1	2			5				9			12	93.56
at a time	1		3		5				9			12	93.49
	1	2		4					9			12	92.52
Six Xs	1	2			5			8	9			12	94.89
at a time	1		3		5			8	9			12	94.34
	1	2			5	6			9			12	94.29
Seven Xs	1	2			5		7	8	9			12	96.91
at a time	1		3		5		7	8	9			12	96.46
	1			4	5		7	8	9			12	96.06
Eight Xs	1	2			5	6	7	8	9			12	98.12
at a time	1		3		5	6	7	8	9			12	97.91
	1	2	3		5		7	8	9			12	97.39
Nine Xs	1	2			5	6	7	8	9	10		12	98.32
at a time	1	2	3		5	6	7	8	9			12	98.28
	1	2		4	5	6	7	8	9			12	98.15
Ten Xs	1	2	3		5	6	7	8	9	10		12	98.74
at a time	1	2	3		5	6	7	8	9		11	12	98.28
	1	2	3	4	5	6	7	8	9			12	98.28
Eleven Xs	1	2	3		5	6	7	8	9	10	11	12	98.75
at a time	1	2	3	4	5	6	7	8	9	10		12	98.74
	1	2		4	5	6	7	8	9	10	11	12	98.41

Table B50.-The Five Strongest Per Cent Reduction in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 4.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs	1											12	83.38
at a time											11	12	75.89
	1									10			55.84
	1										11		54.02
	1		3										53.15
Three Xs	1						7					12	89.36
at a time	1					6						12	86.94
	1			4								12	84.87
	1				5							12	84.84
	1										11	12	84.66
Four Xs	1		3				7					12	90.38
at a time	1	2					7					12	90.26
	1					6	7					12	90.20
	1			4			7					12	90.19
	1					6		8				12	89.55
Five Xs	1		3			6		8				12	92.85
at a time	1	2				6		8				12	92.77
	1		3		5		7					12	91.48
	1	2			5		7					12	91.30
	1		3			6	7					12	91.17
Six Xs	1		3			6	7	8				12	93.32
at a time	1	2				6	7	8				12	93.24
	1	2		4		6		8				12	93.04
	1		3			6		8		10		12	93.01
	1		3		5	6		8				12	93.00

Table B51.–The Five Strongest Per Cent Reductions in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 5.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs	1											12	75.60
at a time											11	12	72.49
					5							12	58.00
	1									10			57.18
				4								12	56.57
Three Xs				4							11	12	79.29
at a time	1						7					12	78.77
	1			4								12	77.91
	1				5							12	77.24
	1					6						12	76.41
Four Xs				4					9		11	12	83.53
at a time	1			4			7					12	81.31
	1						7	8				12	80.20
	1				5		7					12	80.08
	1			4							11	12	79.91
Five Xs				4			7		9		11	12	86.31
at a time	1			4			7		9			12	85.76
				4		6			9		11	12	84.46
			3	4			7		9			12	84.32
				4				8	9		11	12	83.81
Six Xs			3	4		6	7		9			12	88.99
at a time		2		4		6	7		9			12	88.18
			3	4			7	8	9			12	87.62
			3	4			7		9		11	12	87.23
		2		4			7		9		11	12	87.11

Table B52.–The Five Strongest Per Cent Reductions in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Eleven Independent Variables, (Variable X1 Not Used) For Lag Period 1.

	Independent Variable Combinations											Percent Reduction in SS
Two Xs										11	12	65.30
at a time						7				11		62.98
					6					11		44.24
				5						11		37.77
							8			11		37.15
Three Xs						7		9		11		69.28
at a time					6					11	12	69.23
						7				11	12	68.98
								9		11	12	68.62
		3								11	12	65.50
Four Xs						7		9		11	12	73.64
at a time			4		6					11	12	73.41
					6			9		11	12	71.81
						7		9	10	11		71.13
					6	7				11	12	71.01
Five Xs			4		6			9		11	12	79.14
at a time			4	5				9		11	12	74.88
					6	7		9		11	12	74.75
			4		6	7		9		11		74.65
			4		6		8			11	12	74.42
Six Xs			4		6		8	9	10	11		80.89
at a time			4		6		8	9		11	12	80.35
			4		6	7		9		11	12	79.87
			4	5	6			9		11	12	79.82
	2		4		6			9		11	12	79.57

Table B53.--The Five Strongest Per Cent Reductions in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Eleven Independent Variables (Variable X1 Not Used) For Lag Period 2.

	Independent Variable Combinations										Percent Reduction in SS
Two Xs at a time						7			11		72.90
									11	12	67.98
								9		12	55.20
				5			8				55.08
							8		11		54.06
Three Xs at a time						7			11	12	81.14
				5		7			11		76.26
								9	11	12	76.18
					6	7			11		73.47
	2					7			11		73.36
Four Xs at a time				5		7			11	12	84.04
						7		9	11	12	82.93
					6	7			11	12	82.35
	2					7			11	12	82.31
						7	8		11	12	81.79
Five Xs at a time				5		7		9	11	12	86.14
			4	5		7			11	12	85.37
				5	6	7			11	12	84.68
	2					7		9	11	12	84.59
		3				7		9	11	12	84.23
Six Xs at a time			4	5		7		9	11	12	87.58
				5	6	7		9	11	12	86.61
				5		7	8	9	11	12	86.37
	2		4	5		7			11	12	86.36
	2			5		7		9	11	12	86.32

Table B54.--The Five Strongest Per Cent Reductions in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Eleven Independent Variables (Variable X1 Not Used) For Lag Period 3,

	Independent Variable Combinations											Percent Reduction in SS
Two Xs at a time										11	12	33.00
	2									11		30.36
									10	11		30.13
			4		6							29.79
					6					11		29.73
Three Xs at a time	2	3							10			50.38
	2	3					8					47.09
	2									11	12	46.89
	2	3								11		46.64
			4	5	6							46.60
Four Xs at a time			4	5	6				10			62.30
	2	3					8		10			62.14
	2	3							10	11		61.49
			4	5						11	12	61.18
			4	5					10	11		58.62
Five Xs at a time			4	5	6				10	11		68.03
	2	3					8		10	11		66.81
			4	5		7			10	11		65.40
			4	5	6					11	12	65.35
	2	3	4		6				10			65.25
Six Xs at a time	2	3					8	9	10	11		70.91
	2	3	4		6				10	11		69.77
			4	5		7	8		10	11		69.44
			4	5	6				10	11	12	69.25
			4	5	6	7			10	11		68.82

Table B55.-The Five Strongest Per Cent Reductions in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Eleven Independent Variables (Variable X1 Not Used) For Lag Period 4.

	Independent Variable Combinations											Percent Reduction in SS
Two Xs at a time			4								12	43.53
						7					12	36.98
									10			34.80
								9			12	33.28
					6						12	31.51
Three Xs at a time			4							11	12	82.16
	2									11	12	78.48
		3								11	12	78.21
					6					11	12	76.84
						7				11	12	76.70
Four Xs at a time	2		4							11	12	84.58
		3	4							11	12	84.48
			4	5						11	12	84.48
			4			7				11	12	82.62
			4					9		11	12	82.19
Five Xs at a time			4	5		7				11	12	86.22
	2		4						10	11	12	86.17
	2		4				8			11	12	86.01
		3	4						10	11	12	85.82
		3	4				8			11	12	85.57
Six Xs at a time			4	5	6		8			11	12	87.45
			4	5		7		9		11	12	86.96
			4	5		7			10	11	12	86.60
	2		4			7			10	11	12	86.59
	2	3	4				8			11	12	86.58

Table B56.-The Five Strongest Per Cent Reductions in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Eleven Independent Variables (Variable X1 Not Used) For Lag Period 5.

	Independent Variable Combinations											Percent Reduction in SS
Two Xs at a time										11	12	72.49
				5							12	58.00
			4								12	56.57
						7					12	50.01
									10		12	48.80
Three Xs at a time			4							11	12	79.29
				5						11	12	75.09
									10	11	12	73.89
							8			11	12	73.15
						7				11	12	72.86
Four Xs at a time			4					9		11	12	83.53
			4			7				11	12	79.88
			4			7		9			12	79.64
		3	4							11	12	79.48
	2		4							11	12	79.44
Five Xs at a time			4			7		9		11	12	86.31
			4		6			9		11	12	84.46
		3	4			7		9			12	84.32
			4				8	9		11	12	83.81
	2		4					9		11	12	83.77
Six Xs at a time		3	4		6	7		9			12	88.99
	2		4		6	7		9			12	88.18
		3	4			7	8	9			12	87.62
		3	4			7		9		11	12	87.23
	2		4			7		9		11	12	87.11

Table B57.-The Five Weakest Per Cent Reductions in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 1.

	Independent Variable Combinations									Percent Reduction in SS
Two Xs		3			7					1.19
at a time		3	4							1.67
	2				7					1.88
		3						10		1.88
	2							10		2.36
Three Xs		3	4					10		2.39
at a time		3	4		7					2.74
	2		4					10		3.11
	2		4		7					3.35
	2				7			10		3.93
Four Xs	2		4		7			10		4.39
at a time	2		4			8		10		5.38
	2				7	8		10		5.47
			4		7	8		10		5.47
		3	4		7			10		5.51
Five Xs	2		4		7	8		10		5.61
at a time		3	4		7	8		10		7.00
	2			5	7	8		10		13.04
		3	4		7	8			12	14.84
	2		4			8		10	12	15.14
Six Xs	2			5	7	8		10	12	24.84
at a time	2	3	4		7	8		10		27.80
	2			5	7	8	9	10		28.16
	2		4		7	8	9	10		28.84
		3	4		7	8	9	10		29.26

Table B58.-The Five Weakest Per Cent Reduction in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 2.

	Independent Variable Combinations										Percent Reduction in SS
Two Xs					6			10			5.27
at a time	2				6						5.81
	2							10			5.97
		3			6						7.40
			4					10			9.34
Three Xs	2				6			10			8.51
at a time		3			6			10			11.91
	2		4					10			15.17
			4		6			10			15.84
	2		4		6						17.89
Four Xs	2		4		6			10			18.71
at a time		3	4		6			10			20.93
	2				6	7		10			22.91
		3	4	5				10			23.68
	2		4	5				10			24.27
Five Xs		3	4	5	6			10			28.39
at a time	2	3	4		6			10			29.34
	2		4	5	6			10			31.22
	2		4		6	7		10			33.06
	2	3			6	7		10			33.55
Six Xs	2	3	4		6	7		10			38.01
at a time	2	3	4		6		9	10			47.67
	2	3			6	7		10		12	48.32
	2	3			6	7	9	10			49.70
	2	3	4		6		9		11		49.74

Table B59.-The Five Weakest Per Cent Reduction in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 3.

	Independent Variable Combinations												Percent Reduction in SS
Two Xs at a time			3				7						0.09
				4			7						0.72
			3	4									0.79
							7	8					0.92
		2					7						1.46
Three Xs at a time			3	4			7						0.80
			3				7	8					1.64
				4			7	8					1.92
		2		4			7						2.09
			3	4				8					2.41
Four Xs at a time			3	4			7	8					2.88
			3		5		7	8					5.41
		2		4			7	8					6.56
		2			5		7	8					7.15
			3				7	8				12	7.88
Five Xs at a time			3		5		7	8				12	8.61
		2			5		7	8				12	12.21
			3	4			7	8				12	12.64
					5		7	8		10		12	15.01
			3				7	8		10		12	16.13
Six Xs at a time			3		5		7	8		10		12	16.41
			3		5		7	8	9			12	21.20
		2			5		7	8		10		12	22.11
			3	4			7	8	9			12	22.89
		2			5		7	8	9			12	23.43
Seven Xs at a time			3		5		7	8	9	10		12	27.81
			3			6	7	8	9	10		12	31.33
		2			5		7	8	9	10		12	31.48
			3		5	6	7	8	9			12	31.85
					5	6	7	8	9	10		12	32.25

Table B59.-The Five Weakest Per Cent Reduction in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 3.

	Independent Variable Combinations												Percent Reduction in SS
Eight Xs at a time			3		5	6	7	8	9	10		12	36.57
					5	6	7	8	9	10	11	12	39.65
		2			5	6	7	8	9	10		12	41.81
			3		5	6	7	8	9	10	11		43.39
			3		5	6	7	8	9		11	12	44.21
Nine Xs at a time			3		5	6	7	8	9	10	11	12	45.96
		2			5	6	7	8	9	10	11	12	51.16
			3	4		6	7	8	9	10	11	12	60.94
		2	3	4	5	6	7	8	9			12	62.91
		2		4		6	7	8	9	10	11	12	63.45
Ten Xs at a time		2	3	4	5	6	7	8	9		11	12	71.11
		2		4	5	6	7	8	9	10	11	12	71.85
			3	4	5	6	7	8	9	10	11	12	72.17
		2	3	4	5	6	7	8		10	11	12	72.91
		2	3	4	5	6	7	8	9	10		12	73.63
Eleven Xs at a time		2	3	4	5	6	7	8	9	10	11	12	77.23
	1	2	3	4	5	6	7	8		10	11	12	92.93
	1	2		4	5	6	7	8	9	10	11	12	93.87
	1	2	3	4	5	6	7	8	9		11	12	95.23
	1		3	4	5	6	7	8	9	10	11	12	97.00

Table B60.--The Five Weakest Per Cent Reduction in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variable Combinations, for Lag Period 4.

	Independent Variable Combinations								Percent Reduction in SS
Two Xs at a time			5		7				0.10
					7			10	0.40
			5					10	0.41
		3						10	0.96
				6	7				1.10
Three Xs at a time			5		7			10	0.44
		3			7			10	1.26
				6	7			10	1.28
			5	6	7				1.30
			5	6				10	1.40
Four Xs at a time			5	6	7			10	1.42
		3	5		7			10	1.77
		3		6	7			10	2.15
		3	5	6				10	2.16
		3	5	6	7				2.16
Five Xs at a time		3	5	6	7			10	2.17
	2		5	6	7			10	3.89
			5	6	7		9	10	9.27
		3	5	6		8		10	9.78
	2		5	6		8		10	9.87
Six Xs at a time		3	5	6	7		9	10	11.19
	2		5	6	7		9	10	13.25
		3	5	6	7	8		10	16.39
		3	5	6	7	8	9		17.52
		3	5	6		8	9	10	17.67

Table B61.--The Five Weakest Per Cent Reduction in Mean Grain Size Sum of Squares Attributable to Each of Several Combinations of Twelve Independent Variables, for Lag Period 5.

	Independent Variable Combinations									Percent Reduction in SS
Two Xs at a time		3			6					0.55
					6	7				1.69
		3							10	1.69
					6				10	1.72
		3				7				1.98
Three Xs at a time		3			6				10	1.73
	2				6				10	2.60
		3			6	7				2.79
				5	6	7				3.46
		3			6			9		3.98
Four Xs at a time		3			6		8		10	4.69
		3			6			9	10	4.79
	2				6		8		10	5.25
	2				6			9	10	5.58
		3			6	7		9		6.86
Five Xs at a time		3			6		8	9	10	7.37
	2				6		8	9	10	7.89
	2				6	7		9	10	11.39
		3			6	7		9	10	11.40
		3		5	6		8		10	11.43
Six Xs at a time		3		5	6		8	9	10	17.85
	2		4	5	6		8		10	19.12
		3	4	5	6		8		10	20.15
		3		5	6	7		9	10	21.18
	2			5	6		8	9	10	22.13

Table B62.- Mean Size vs 12 Xs (Lag Period 1)

SS Reduction	Number of Independent Variables					
Class %	1	2	3	4	5	6
0.0-4.9	5	9	6	1		
5.0-9.9	4	18	23	10	2	
10.0-14.9		5	20	15	1	
15.0-19.9	1	4	11	18	13	2
20.0-24.9		6	23	25	6	1
25.0-29.9		3	26	51	34	5
30.0-34.9	1	3	8	37	57	22
35.0-39.9		4	14	27	39	24
40.0-44.9		1	10	26	36	35
45.0-49.9			3	18	31	31
50.0-54.9			2	16	40	23
55.0-59.9			1	11	36	50
60.0-64.9	1	5	12	23	26	47
65.0-69.9		6	39	99	122	83
70.0-74.9		1	10	57	169	218
75.0-79.9		1	12	52	135	243
80.0-84.9				9	45	128
85.0-89.9						2
Σ	12	66	220	495	792	914

Total SS Reduction = 86.15%

Table B 63.- Mean Size vs 12 Xs (Lag Period 2)

SS Reduction	Number of Independent Variables					
Class %	1	2	3	4	5	6
0.0-4.9	4					
5.0-9.9	1	5	1			
10.0-14.9	1	4	1			
15.0-19.9	1	4	6	1		
20.0-24.9	2	8	10	4		
25.0-29.9	1	9	14	8	2	
30.0-34.9		4	12	12	4	
35.0-39.9	1	7	20	10	3	1
40.0-44.9		5	22	34	10	
45.0-49.9		3	19	40	36	5
50.0-54.9		2	15	44	44	22
55.0-59.9		2	16	35	47	21
60.0-64.9	1	7	29	74	94	52
65.0-69.9		3	12	36	69	69
70.0-74.9		2	29	101	136	99
75.0-79.9		1	11	68	217	290
80.0-84.9			3	37	110	273
85.0-89.9				1	19	69
90.0-94.9					1	13
Σ	12	66	220	495	792	914

Total SS Reduction = 93.10%

Table B64.- Mean Size vs 12 Xs (Lag Period 3)

SS Reduction	Number of Independent Variables											
Class %	1	2	3	4	5	6	7	8	9	10	11	12
0.0-4.7	6	11	7	1								
5.0-9.9	1	8	14	8	1							
10.0-14.9	1	4	10	9	2							
15.0-19.9	2	16	35	29	9	1						
20.0-24.9	1	4	17	37	26	4						
25.0-29.9		9	26	41	44	19	1					
30.0-34.9		3	37	58	43	26	8					
35.0-39.9			8	59	87	53	14	2				
40.0-44.9			5	25	56	44	19	3				
45.0-49.9	1	3	8	37	62	63	29	8	1			
50.0-54.9		3	8	25	63	69	33	5	1			
55.0-59.9			5	14	40	71	50	15				
60.0-64.9		3	7	10	34	76	77	31	5			
65.0-69.9		1	13	29	23	42	81	56	11			
70.0-74.9		1	15	55	79	45	23	43	27	5		
75.0-79.9			3	32	101	111	54	11	8	6	1	
80.0-84.9			2	16	63	126	122	44	5			
85.0-89.9				9	48	115	167	128	48	5		
90.0-94.9				1	11	47	106	121	66	18	2	
95.0-99.9							8	28	48	32	9	1
Σ	12	66	220	495	792	914	792	495	220	66	12	1

Total SS Reduction = 98.75%

Table B65.-Mean Size vs 12 Xs (Lag Period 4)

SS Reduction	Number of Independent Variables					
Class %	1	2	3	4	5	6
0.0-4.9	6	14	16	9	2	
5.0-9.9	2	11	20	14	4	
10.0-14.9	1	7	20	27	14	2
15.0-19.9	1	6	17	29	23	7
20.0-24.9		4	15	25	27	13
25.0-29.9	1	7	23	45	40	23
30.0-34.9		3	18	40	52	33
35.0-39.9		1	14	41	52	42
40.0-44.9		1	9	21	45	38
45.0-49.9	1	5	13	31	42	39
50.0-54.9		4	22	55	77	61
55.0-59.9		1	10	46	108	126
60.0-64.9			2	11	30	55
65.0-69.9			2	13	32	41
70.0-74.9				8	40	67
75.0-79.9		1	8	26	42	59
80.0-84.9		1	9	19	36	58
85.0-89.9			2	31	99	148
90.0-94.9				4	27	102
Σ	12	66	220	495	792	914

Total SS Reduction = 94.68%

Table B66.- Mean Size vs 12 Xs (Lag Period 5)

SS Reduction	Number of Independent Variables					
Class %	1	2	3	4	5	6
0.0-4.9	8	17	12	2		
5.0-9.9		8	23	14	2	
10.0-14.9	1	4	10	14	5	
15.0-19.9	1	9	27	34	17	2
20.0-24.9		3	8	18	15	5
25.0-29.9		2	17	24	12	4
30.0-34.9		1	9	40	39	9
35.0-39.9			6	27	53	30
40.0-44.9	2	11	24	44	65	48
45.0-49.9		4	26	54	81	103
50.0-54.9		2	12	42	78	84
55.0-59.9		3	15	44	87	119
60.0-64.9			9	35	55	53
65.0-69.9		1	2	15	42	47
70.0-74.9			7	25	49	57
75.0-79.9		1	13	59	137	183
80.0-84.9				4	53	139
85.0-89.9					2	31
Σ	12	66	220	495	792	914

Total SS Reduction = 93.89%

ADDENDUM

ALTERNATIVE MULTIREGRESSION TECHNIQUE FOR OBTAINING PREDICTOR EQUATIONS

by

W. Harrison

U.S. Coast and Geodetic Survey, Washington, D.C.

and

N. A. Pore

U.S. Weather Bureau, Washington, D.C.

ABSTRACT

The data used in the study of interactions in the beach-ocean-atmosphere system are subjected to a multiple-regression screening procedure which is programmed so that all Xs, from all time lags are subjected to a single screening. Examples of predictor equations suggested by the analysis are given for the following predictands: mean longshore current velocity, mean bottom slope and mean grain size in the shoaling-wave zone, and net deposition and net erosion on the lower foreshore during June and July. The screening technique yields comparable or somewhat stronger predictor equations than are obtained in the sequential multiregression analysis, where combinations of Xs are restricted to specific lag periods.

INTRODUCTION

The computer program for the least-squares search procedure used in the first part of this report permitted identification of the strongest combinations of 10-14 Xs (predictors) for each Y (predictand), when the combinations of Xs were restricted to specific lag periods. This scheme was adopted because of a program restriction that limited to 25 the number of Xs that could be studied at any one time, and because of the great numbers of combinations that would be generated if more than a few tens of Xs were considered.

It is desirable, nevertheless, to have available an extension of the method for searching large numbers of predictors so that each of many Xs may be considered, regardless of its position in time. One such search procedure - called here the screening procedure - identifies "optimum" sets of predictors for given predictands. The data to which the screening procedure will be applied will be identical in every way with those of the first part of this report, to which the Krumbein, Benson, and Hempkins (1964) computer program was applied.

The computer program for the screening procedure is similar to that presented by R. G. Miller (1958). Analysis of the data will proceed without the inclusion of variables X3 and X5 (L_o and H_o/L_o), in part because of their redundancy with each other and with T. (The reader is referred to the list of symbols in Table 1 for definitions of the various notations used for the variables).

METHOD OF ANALYSIS

As in the main part of this report, the procedure adopted for selecting predictors involves expression of a predictand Y as a linear function of a number of predictors X_n (n=1,...,N).

Thus:

$$Y = A_o + A_1X_1 + A_2X_2 + \ldots A_nX_n + \ldots A_NX_N$$

where the coefficients A_n(n=o, ...,N) are determined using the method of least squares.

Because of the large numbers of predictors involved, the screening procedure required the use of a high-speed, large-memory computer. The IBM 7030 was used. Basically, the manner in which the predictors were screened is shown below:

1) $Y = A_1 + B_1X_1$

2) $Y = A_2 + B_2X_1 + C_1X_2$

.
.
.

3) $Y = A_n + B_nX_1 + C_{n-1}X_2 \ldots . NX_n$

where Y is a predictand, the As are constants, the Xs are predictors, and B_1, B_2, C_1, C_2, etc. are regression coefficients.

The procedure is to first select the best single predictor (X_1) for regression equation 1. The second regression equation contains the first predictor (X1) and the predictor (X2) that contributes most to reducing the residual after the first predictor is considered, regardless of its lag position. This means that the second regression equation contains the best set of two predictors that includes the X selected at the first screening step. The procedure will not necessarily select the best set of r predictors out of the original set containing P predictors. In the closely analogous field of meteorology, however, it has been shown that the screening procedure can select a highly reliable set of predictors when applied to problems that involve redundant, interrelated variables.

Generally, the significance of the improvement attained at each step of the screening is tested and the screening discontinued when the amount of improvement is found not to be significant. Near that point the addition of many more predictors usually lowers the predictive ability of the system on independent data. As pointed out by Panofsky and Brier (1958), however, objective standard significance tests may be misleading on data such as those of this study, because the underlying assumptions may be violated. The predictors used here are certainly interdependent in time and space. Often the most practical and convincing test of significance can be an application of the result to an independent set of data. This will be the method used by the present investigators for "significance" tests in future work with predictor equations for data of this sort.

RESULTS

Predictors Selected

As mentioned earlier, variables X3 and X5 (L_o and H_o/L_o) were not used when screening for predictor equations, because of their redundance with each other and with T (wave period). The results of the screening procedure for 1 through 4 predictors are given in table A1, where the order of the first four selected predictors is shown along with the lag and correlation coefficients. For example, in run 1 the first predictor selected by screening was (T) with a lag of 0-4 hours and a correlation of 0.47. The second predictor selected (H_o, with lag of 0-4 hours) increases the correlation to 0.64. Four predictors bring the correlation to 0.72.

Predictor equations, each containing the four predictors of table A1, are presented below for the five screening runs.

(1) $\bar{V} = 1.73 - 0.26(T)_{-1} + 0.18(H_o)_{-1} + 0.14\ (\bar{S}_f)_{-1} + 0.02\ (\bar{U}_{of})_{-1}$

(2) $\bar{S}_s = -0.63 - 1.58 \left[(\bar{M}_z)_s\right]_{-5} + 0.10\ (\rho)_{-5} + 0.03\ (\alpha)_{-5} + 0.64\ (C)_{-2}$

(3) $(\bar{M}_z)_s = 0.64 - 0.17\ (\bar{S}_s)_{-1} + 0.40\ (C)_{-4} - 0.02\ (\rho)_{-1} + 0.03\ (h)_{-2}$

(4) $J_f = -0.03 - 0.18\ (\bar{S}_f)_{-1} + 4.70\ (\eta_r)_{-1} + 0.31\ (\bar{V})_{-1} + 0.09\ (T)_{-6}$

(5) $K_f = 0.12 + 0.02\ (\bar{U}_{of})_{-2} - 0.10\ (\bar{S}_f)_{-4} + 0.01\ (\alpha)_{-1} + 0.20\ (\bar{V})_{-6}$

where $\dot{\rho} = (\rho-1) \times 10^3$ and the other variables and their units are as defined and explained in Appendix A. The numbers subscripting the variable symbols designate lag periods, each of which is of four hours duration.

Time Lag in Predictor-Predictand Relationships

As is amply demonstrated in the main part of this report, there is a time lag in the peak interaction of the most-important combination of independent variables, as they influence the dependent variables. The program used for the screening procedure is written to print out all of the simple correlation coefficients between predictors and predictands. This permits an informative analysis of the time variance of specific predictors selected by the screening procedure. Figure A1 is a graphical presentation of one such analysis, in which the simple correlation coefficient, r, for the first four predictors selected in run 5 (table A1) is plotted versus time.

DISCUSSION

It is apparent that a great amount of useful information may be derived from diagrams such as that of figure A1. The high positive correlation of wind velocity offshore during the second lag period (4-8 hours prior to measurement of the predictand) indicates the lag in this factor's influence on beach erosion. The alternating negative and positive correlations of mean longshore-current velocity with beach erosion are to a certain degree a mirror image of the curve for $\overline{U}_{of}$, which would be expected if one considers that onshore winds create waves that produce strong longshore currents. The relatively constant value of the negative correlation between slope of the lower foreshore and beach erosion suggests that the slope itself, although having a real influence on beach erosion (table A1, run 5), does not need to vary significantly for erosion to take place. The correlation between the angle of wave approach is seen to be highest in the immediate past and it does not show the clear lag tendency that is shown by $\overline{U}_{of}$. As the data matrix from Virginia Beach is enhanced, a repeat of the present screening procedure will permit the plotting of more meaningful diagrams such as that of figure A1.

Turning to the predictor equations once again, it would be well to remind the reader that these are best tested by their application to a set of independent data. This will be done in the near future when additional data become available. It is also noted that the number of cases available for the screening procedure (table A1, "N=..") needs considerable augmentation.

A comparison between the best four-variable equations selected by the two multiregression computer programs is presented in table A2. In the case given for $\overline{V}$ as the predictand, both programs happened to select the same combination of Xs and, because Xs from just one lag period were involved, the "%-SS-reduction" values were also identical. The results for $\overline{S}_s$ are not quite the same. Although both computer programs selected the same kinds of Xs for the best 4-variable predictor equations, the total variance reduction ("%-SS-reduction") is slightly greater for the four predictors selected by the screening procedure. This is due to the fact that the screening procedure is able to select variable X12 from lag period 2 where it is stronger than it is in lag period 5, the lag period shown by the Krumbein, Benson and Hempkins (1964) program to be the strongest combination for the given lag period.

More pronounced differences exist in the kinds of variables chosen by the two programs in the remaining runs, and the variance reduction is perhaps significantly higher for the runs involving $(\bar{M}_z)_s$ and J_f, where the screening procedure is used.

SUMMARY REMARKS

Although the alternative search procedure described in this addendum will not necessarily select the best set of r predictors out of an original set containing P predictors, the screening procedure does allow rapid consideration of many more predictors than the Krumbein, Benson and Hempkins (1964) procedure and, in terms of variance reductions, appears to yield comparable predictor equations. Thus, the screening procedure appears to be worthy of application in studies such as this in which large numbers of interrelated predictors are to be considered, and in which the data extend through time.

ACKNOWLEDGMENT

We wish to thank W. C. Krumbein for review of this addendum.

TABLE A1

SELECTION OF PREDICTORS BY SCREENING PROCESS
(Lag interval is expressed in hours)

Predictand:	$\overline{V}$ RUN 1 (N=53)			$\overline{S}_s$ RUN 2 (N=18)			$(\overline{M}_z)_s$ RUN 3 (N=18)			J_f RUN 4 (N=20)			K_f RUN 5 (N=34)		
	Var.	Lag.	r	Var.	Lag.	r	Var.	Lag.	r	Var.	Lag.	r	Var.	Lag.	r
Eq. 1	T	0-4	0.47	$(\overline{M}_z)_s$	16-20	0.82	$\overline{S}_s$	0-4	0.79	$\overline{S}_f$	0-4	0.60	$\overline{U}_{of}$	4-8	0.44
Eq. 2	H_o	0-4	0.64	ρ	16-20	0.91	C	12-16	0.88	η_r	0-4	0.73	$\overline{S}_f$	12-16	0.56
Eq. 3	$\overline{S}_f$	0-4	0.69	α	16-20	0.96	ρ	0-4	0.92	$\overline{V}$	0-4	0.82	α	0-4	0.61
Eq. 4	$\overline{U}_{of}$	0-4	0.72	C	4-8	0.98	h	4-8	0.97	T	20-24	0.87	$\overline{V}$	20-24	0.65

TABLE A2

BEST FOUR-VARIABLE PREDICTOR EQUATIONS
(Variables X3 and X5 not used)

Predictand (Y)	Sequential Multiregression Method		Screening Procedure	
	Predictors (Xs)	%-SS Reduction	Predictors (Xs)	%-SS Reduction
$\overline{V}$	1,2,4,7 (Lag 1)	52.0	1,2,4,7 (Lag 1)	52.0
$\overline{S}_s$	1,9,11,12 (Lag 5)	94.1	1,9,11,12 (Lags 5,5,5,2)	95.2
$(\overline{M}_z)_s$	1,2,7,12 (Lag 4)	90.3	1,10,11,12 (Lags 1,2,1,4)	95.0
J_f	1,6,9,10 (Lag 1)	70.62	1,2,10,12 (Lags 1,6,1,1)	75.1
K_f	1,2,7,14 (Lag 2)	42.5	1,7,9,10 (lags 4,2,1,6)	43.0

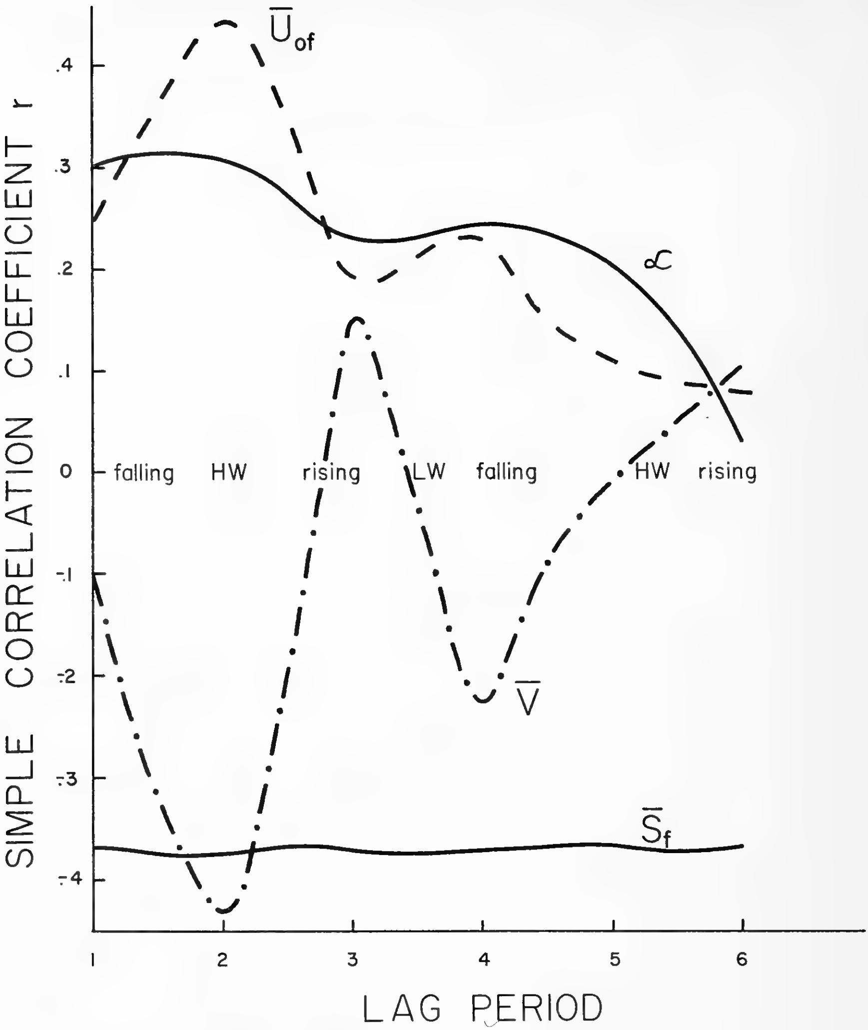

FIGURE A1. TIME VARIATION OF r FOR THE PREDICTAND K_f (BEACH EROSION) AND THE FOUR PREDICTORS $\overline{U}_{of}$ (WIND VELOCITY OFFSHORE), α (ANGLE OF WAVE APPROACH), $\overline{V}$ (MEAN LONGSHORE - CURRENT VELOCITY), AND $\overline{S}_f$ (MEAN SLOPE OF THE LOWER FORESHORE). DESIGNATIONS ALONG THE 0-INTERCEPT REFER TO TIDE CONDITIONS.

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