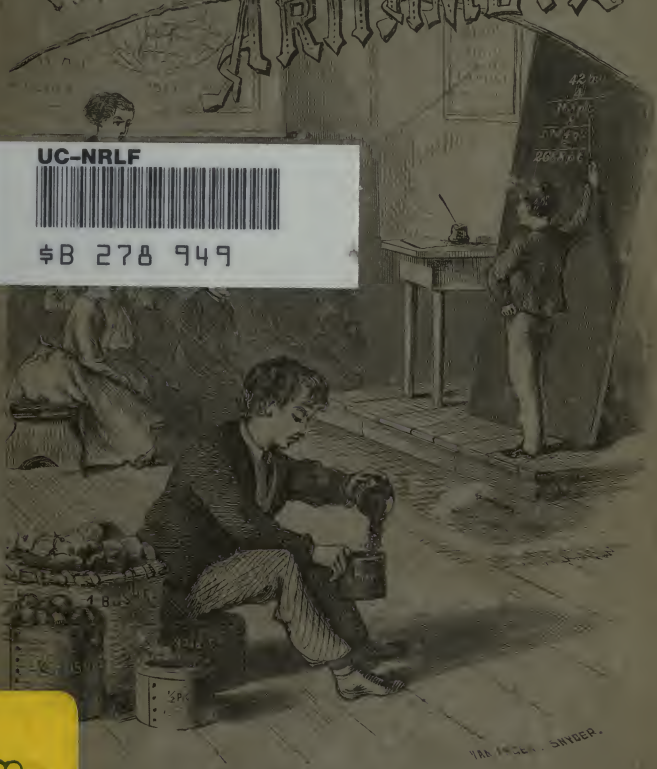
White's Graded School Series

Intermediate Arithmetic



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WILSON, HINKLE & CO.
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WHITE'S GRADED-SCHOOL SERIES.

AN

INTERMEDIATE

ARITHMETIC,

UNITING

MENTAL AND WRITTEN EXERCISES

IN A

NATURAL SYSTEM OF INSTRUCTION.

BY E. E. WHITE, M. A.

CINCINNATI:

WILSON, HINKLE & CO.

PHIL'A: CLAXTON, REMSEN & HAFFELFINGER.

NEW YORK: CLARK & MAYNARD.

PUBLISHERS' NOTICE.

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PREFACE.

It is claimed for this treatise that it possesses three very important characteristics, to wit:

1. *It is specially adapted to the grade of pupils for which it is designed.* It is not an abridgment of the Complete Arithmetic. It presents only those operations and principles which can be mastered by intermediate classes, and each subject is treated as thoroughly as the advancement of the pupils will permit. It is also believed that the subjects are introduced in the best possible order. There are reasons in favor of placing United States Money before Fractions, but stronger reasons favor the reverse order of arrangement in this work.

2. *It combines Mental and Written Arithmetic in a practical and philosophical manner.* This is done by making every mental exercise preparatory to a written one; and thus these two classes of exercises, which have been so unnaturally divorced, are united as the essential complements of each other. This union is natural and complete, and, as a consequence, the several subjects are treated in much less space than is possible when mental and written exercises are presented in separate books.

3. *It faithfully embodies the Inductive Method.* Instead of attempting to deduce a principle or rule from a single example, as is usually done, each process is developed inductively, and the successive steps are thoroughly mastered and clearly stated by the pupil before he is confronted with the author's generalization. See "Suggestions to Teachers." This method not only places "processes *before* rules," but

it teaches "rules *through* processes," thus observing two important inductive maxims. The work also observes the inductive method in not giving answers to questions and problems. For an illustration, see the method used in developing the idea of a Fraction (p. 94). Printed answers to the questions there asked, would clearly violate the wise maxim: "Never tell a pupil any thing which you can lead him to discover and express for himself."

Attention is also called to the use of *visible illustrations* (objects or pictures) *in developing new ideas and processes.* In the fundamental rules this illustrative or perceptive step is omitted, since it is fully presented in the Primary Arithmetic. The engraved cuts in Fractions, United States Money, and Denominate Numbers, are specially designed to be used as a means of developing and illustrating the subjects considered. It is hoped that they may be found as useful as they are beautiful.

Two other features, worthy of special notice, are *the great variety of exercises*, and *their preëminently progressive character.* Generally, each lesson contains both concrete and abstract examples, and every new process or combination is at once used in the solution of problems involving mental analysis. This arrangement avoids the mechanical monotony which characterizes long drills on a single class of exercises. The problems, all of which are original, are so graded that they present but one difficulty at a time, and all in their natural order. The pupil's progress is thus made easy and thorough.

It is hoped that these and other features may commend this work to all progressive teachers, and that it may prove as successful in the school-room as its plan is natural and simple.

Columbus, Ohio, May, 1870.

SUGGESTIONS TO TEACHERS.

In the preparation of this work two facts were kept in view, viz.: (1) that it is to be studied by pupils who must largely depend upon the living teacher for explanations, and (2) that those methods which are most natural and simple, are most successful in practice. Hence, its pages are not cumbered with long verbal explanations and peculiar methods, of little practical use to pupil or teacher. The author has left *something for the teacher to do*, and that this may be done wisely, he offers the following hints and suggestions:

1. *Mental Exercises.*—These exercises should be made a thorough intellectual drill. They should be recited mentally, that is, without writing the results, and, since the reasoning faculty is not trained by logical verbiage, the solutions should be concise and simple. See pages 25, 97, etc. They should also be made introductory to the Written Exercises, of which they are often a complete elucidation. The corresponding examples in the two classes of exercises should be recited together as well as separately.

2. *Written Exercises.*—The pupils should be required to solve every problem of the assigned lesson on the slate or paper, and the solutions should be brought to the recitation for the teacher's inspection and criticism. Since the answers are not given in the book, they should be obtained by a comparison of the pupils' results. From three to five minutes will suffice to ascertain the answers to twenty problems, and also to test the accuracy and neatness of each pupil's work. The time thus employed is more than made good by the increased interest, self-reliance, and study which the absence of answers secures. The mental problems may also be solved on the slate or paper in preparing the lesson, and then recited, not only mentally as above described, but also as a written exercise. This will increase the number of written problems, and, at the same time, it will secure a careful preparation of the entire lesson.

3. *Definitions and Principles.*—These should be deduced and stated by the pupils under the guidance of the teacher, and usually in connection with the solution of problems. Take for illustration the definition of multiplication. The pupils multiply 304 by 5. The teacher asks, What have you done? "I have multiplied 304 by 5." *T.* Do not use the word "multiplied." (If necessary the teacher shows what is meant by taking a thing one or more times.) "I have taken 304 five times." *T.* By what process have you taken 304 five

times? "By multiplying it." *T.* What then is multiplication? "Multiplication is the process of taking a number ——." *T.* How many times is the number taken in the above example? "It is taken five times, or as many times as there are units in the multiplier." *T.* Now complete your definition. "*Multiplication is the process of taking a number as many times as there are units in another number.*" These steps should be repeated with other examples until the definition is clearly reached and accurately stated. It should then be written and compared with the author's definition, which should be thoroughly memorized.

4. *Rules.*—These should also be deduced and stated by the pupils. The true order is this: 1. A mastery of the process without reference to the rule. 2. The recognition of the successive steps in order, and the statement of each. 3. The combination of these several statements into a general statement. 4. A comparison of this generalization with the author's rule. 5. The memorizing of the latter. Take for illustration the rule for adding fractions. *T.* What is the first step? "Write the fractions, separating them by the plus sign." (Pupils write an example.) *T.* What is the second step? "Reduce the fractions to a common denominator." *T.* What is the third step? "Add the numerators of the new fractions." *T.* The fourth step? "Under their sum write the common denominator." These questions should be repeated until the answers are promptly and accurately given, and then they should be united in a general statement. The first step may be omitted in the rule.

5. *Questions for Review.*—These are designed as a final test of the pupil's knowledge. Before they are reached, the definitions, principles, and rules should be thoroughly mastered, and the pupils should be able to make a topical analysis of them and recite each in order.

6. *Properties of Numbers.*—But one method of finding the greatest common divisor and least common multiple is given, namely, by *factoring*. The other methods are of no practical use to pupils of this grade, and this is introduced mainly to give practice in factoring numbers.

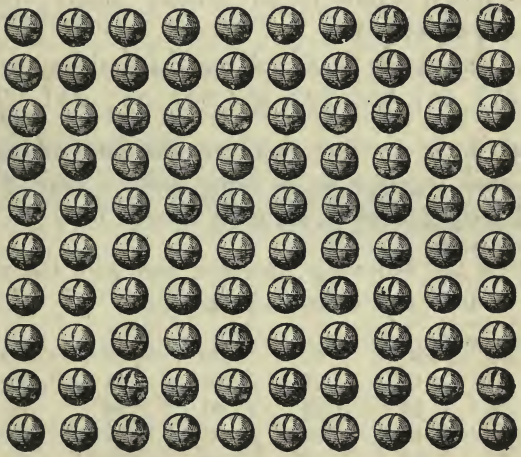
7. *Fractions.*—This section presents only the elements of Fractions, and these in the simplest manner. The subject will be more exhaustively treated in the Complete Arithmetic. The reduction of compound fractions is made introductory to the multiplication of fractions, as the two processes are best taught together.

CONTENTS.

INTERMEDIATE ARITHMETIC.

SECTION I.

NOTATION AND NUMERATION.



LESSON I.

ORAL EXERCISES.

Article 1.—1. Here are one hundred balls in ten rows. How many balls are there in one row? How many balls in two rows? In three rows? In five rows? In eight rows? In ten rows?

2. How many ones in ten? How many ones in two tens? In five tens? Eight tens? Ten tens?

3. How many tens in ten? How many tens in twenty? In thirty? Forty? Sixty? Seventy? Eighty? One hundred?

Art. 2. When a number is expressed by two figures, the first or right-hand figure denotes *Units*, and the second or left-hand figure denotes *Tens*.

4. Which figure in 25 denotes units? Which denotes tens?

5. How many tens and units are there in 37? In 57? 46? 33? 50? 45? 64? 88? 94? 99?

Art. 3. In reading numbers, the tens and units are read together as so many units. Thus, 45 is read forty-five units, or, more briefly, *forty-five*.

Read the following numbers, and give the number of tens and units in each:

(6)	(7)	(8)	(9)	(10)	(11)
1	11	21	20	14	67
3	13	23	40	34	83
5	15	25	60	55	75
9	19	29	80	95	72

Art. 4. When a number is expressed by three figures, the third or left-hand figure denotes *Hundreds*.

12. Which figure in 245 denotes hundreds? Which figure denotes tens? Which denotes units?

13. How many hundreds, tens, and units in 426? 708? 340? 235? 406? 560? 666?

Read the following numbers, and give the number of hundreds, tens, and units in each:

(14)	(15)	(16)	(17)	(18)
200	240	302	349	560
500	550	805	424	703
700	770	307	825	909
900	990	804	448	836

19. What is the greatest number that can be expressed by *one* figure? By *two* figures? By *three* figures?

20. When numbers are expressed by figures, in which place or order is the figure denoting units written? The figure denoting tens? The figure denoting hundreds?

Art. 5. The first three figures, viz.: units, tens, and hundreds, constitute the first or *Units' Period.*

WRITTEN EXERCISES.

1. Write in words, 4, 6, 8, 13, 14, 18, 20, 24, 30, 34.

2. Write in words, 40, 46, 60, 67, 70, 78, 80, 83, 87, 90, 95, 99.

Express in figures the following numbers:

(3)	(4)	(5)
Twelve;	Twenty-one;	Twenty-three;
Sixteen;	Thirty-two;	Twenty-four;
Eighteen;	Forty-two;	Forty-seven;
Twenty;	Sixty-five;	Sixty-five;
Sixty;	Eighty-five;	Seventy-nine;
Eighty.	Ninety-four.	Ninety-six.

6. Express in figures the numbers composed of three tens and four units; six tens and seven units; seven tens and six units; seven tens.

7. Express in figures the numbers composed of six tens and eight units; three tens and nine units; nine tens and no units; seven units.

8. Write in words, 100, 150, 200, 280, 300, 350, 390, 560, 607, 803, 340, and 908.

Express in figures the following numbers:

(9)	(10)
Two hundred;	Four hundred and five;
Five hundred;	Five hundred and six;
Seven hundred;	Six hundred and four;
Three hundred and forty;	Four hundred and forty-five;
Six hundred and seventy;	Eight hundred and thirty-seven;
Nine hundred and thirty.	Nine hundred and twenty-seven.

11. Express in figures the numbers composed of three hundreds, five tens, and four units; six hundreds, four tens, and three units; five hundreds, seven tens, and no units.

12. Express in figures the numbers composed of eight hundreds and six tens; five hundreds and four tens; seven hundreds and five units; two hundreds and six units; six tens.

13. What number is composed of 3 hundreds, 0 tens, and 6 units? 2 hundreds and 3 tens? 4 hundreds and 6 units? 5 hundreds and 8 tens?

14. What number is composed of 5 tens and 8 units? 6 hundreds and 5 units? 7 hundreds and 6 tens?

LESSON II.

ORAL EXERCISES.

Thousands' Period — Thousands, Ten-thousands, Hundred-thousands.

Art. 6. When a number is expressed by four figures, the fourth or left-hand figure denotes *Thousands.*

1. How many thousands in 4,635? 3,045? 6,309? 7,554? 5,384? 8,054? 5,006?

2. Read the units' period in 6,325; 5,080; 7,009; 3,406; 5,800; 6,370; 7,590; 8,008.

Read the following numbers:

(3)	(4)	(5)	(6)	(7)
1,000	2,200	1,020	2,007	3,432
3,000	4,400	3,040	4,001	4,568
5,000	6,600	5,060	5,003	5,608
7,000	8,800	7,090	6,005	7,893
9,000	9,900	9,070	8,009	9,890

Art. 7. When a number is expressed by five figures, the fifth or left-hand figure denotes tens of thousands, or *Ten-thousands.*

8. How many ten-thousands in 45,684? 50,480? 38,305? 15,056? 80,650?

9. How many ten-thousands and thousands in 36,308? 48,500? 60,070? 85,350? 90,308?

Art. 8. In reading a number expressed by five figures, the fifth and fourth figures are read together as so many thousands. Thus, 45,000 is read *forty-five thousand.*

Read the following numbers:

(10)	(11)	(12)	(13)
10,000	21,000	34,400	53,333
30,000	44,000	53,440	16,089
50,000	63,000	67,444	99,008
70,000	84,000	48,307	28,045
90,000	99,000	39,600	67,909

Art. 9. When a number is expressed by six figures, the sixth or left-hand figure denotes hundreds of thousands, or *Hundred-thousands.*

14. How many hundred-thousands in 534,000? 308,000? 650,430? 508,080?

15. How many hundred-thousands, ten-thousands, and thousands in 354,000? 607,800? 350,307? 193,240? 470,386?

Art. 10. In reading a number expressed by six figures, the sixth, fifth, and fourth figures are read together as thousands. Thus, 452,000 is read *four hundred and fifty-two thousand.*

Read the following numbers:

(16)	(17)	(18)	(19)
200,000	250,000	845,630	603,408
400,000	360,000	803,084	490,732
600,000	580,000	760,432	308,400
800,000	730,000	900,425	600,550
900,000	960,000	807,708	707,700

Art. 11. The fourth, fifth, and sixth figures of a number constitute the *Thousands' Period.*

20. Read the thousands' period in the 16th, 17th, 18th, and 19th examples.

21. How many orders in units' period? In thousands' period?

22. What are the names of the three orders in units' period? In thousands' period?

23. How may the two periods be separated? *Ans. By a comma.*

WRITTEN EXERCISES.

1. Write in words, 3000; 4060; 3580; 7086; 6606; and 8080.

2. Write in words, 4400; 5008; 6070; 8506; 5087; 7600; and 3003.

3. Express in figures, three thousand; seven thousand; nine thousand; four thousand five hundred; eight thousand nine hundred.

4. Express in figures, two thousand four hundred and forty; four thousand six hundred and sixty; five thousand eight hundred; six thousand five hundred and twenty-five.

5. Express in figures, seventy-five; two hundred and forty; three hundred and six; five hundred and forty-five; four thousand.

6. Express in figures four hundred and forty; five hundred and ninety; seven thousand eight hundred; eight thousand and fifty.

7. Write in words, 10000; 25000; 40500; 36000; 44000; 30400; 45080; 64008; 89800.

8. Express in figures, forty-five thousand five hundred and four; sixty thousand seven hundred and ninety; thirty-eight thousand and twenty; ninety-six thousand and eighty-four.

9. Express in figures, four hundred and twenty; seven hundred and eighty-nine; four thousand and fifty-seven; seventy-five thousand; sixteen thousand and ninety-eight.

10. Express in figures, as one number, 87 thousand 327 units; 60 thousand 405 units; 70 thousand 346 units; 4 thousand 40 units; 5 thousand 5 units; 95 thousand 406 units.

11. Express in figures, as one number, 88 thousand 88 units; 8 thousand 80 units; 65 thousand 60 units; 6 thousand 600 units; 60 thousand.

12. Write in words, 300000; 440000; 334000; 245500; 304800; 450340.

13. Express in figures, four hundred thousand; six hundred thousand; eight hundred and forty thousand; seven hundred and sixty thousand.

14. Express in figures, nine hundred and fifty thousand four hundred; four hundred and fifty-five thousand two hundred and eighty.

15. Separate the following numbers into periods: 3080; 44004; 400080; 20066; 109038; 160006; 809090; 706030; 40004; 30030.

LESSON III.

DEFINITIONS, PRINCIPLES, AND RULES.

Art. 12. ***Arithmetic*** is the science of numbers, and the art of numerical computation.

A ***Number*** is a unit or a collection of units.

A ***Unit*** is one thing of any kind.

An ***Integer*** is a whole number.

Art. 13. There are three methods of expressing numbers:

1. By *words;* as, five, fifty, etc.
2. By *letters*, called the *Roman* method. (Art. 23.)
3. By *figures*, called the *Arabic* method.

Art. 14. ***Notation*** is the art of expressing numbers by figures or letters.

Numeration is the art of reading numbers expressed by figures or letters.

The word Notation is commonly used to denote the *Arabic* method, which expresses numbers by *figures.*

Art. 15. In expressing numbers by figures, ten characters are used, viz.: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

The first of these characters, 0, is called *Naught,* or *Cipher.* It denotes nothing, or the absence of number.

The other nine characters are called *Significant Figures,* or *Numeral Figures.* They each express one or more units. They are also called *Digits.*

Art. 16. The successive figures which express a number, denote successive *Orders of Units.* These orders are numbered from the right; as, first, second, third, fourth, fifth, and so on.

A figure in units' place denotes *units of the first order;* in tens' place, *units of the second order;* in hundreds' place, *units of the third order*, and so on — the term *units* being used to express *ones* of any order.

Art. 17. Ten units make one ten, ten tens make one hundred, ten hundreds make one thousand; and, generally, *ten units of any order make one unit of the next higher order.*

NOTE.—The teacher can make this principle plain by means of the illustration given on page 9. It is easily shown that 10 ones or units equal 1 ten, and that 10 tens equal 1 hundred.

Art. 18. Figures have two values, called *Simple* and *Local.*

The *Simple Value* of a figure is its value when standing alone.

The *Local Value* of a figure is its value arising from the order in which it stands.

When 3, for example, stands alone, or in the first order, it denotes 3 *units;* when it stands in the second order, as in 34, it denotes 3 *tens;* when it stands in the third order, as in 354, it denotes 3 *hundreds.* Hence, *the local value of figures increases from right to left in a tenfold ratio.*

The local value of each of the successive figures which express a number, is called a *Term.* The terms of 325 are 3 *hundreds,* 2 *tens,* and 5 *units.*

Art. 19. The figures denoting the successive orders of units, are divided into groups of three figures each, called *Periods.* The first or right-hand period is called *Units;* the second, *Thousands;* the third, *Millions;* the fourth, *Billions;* the fifth, *Trillions;* the sixth, *Quadrillions;* the seventh, *Quintillions,* etc.

Art. 20. The three orders of any period, counting from the right, denote, respectively, *Units*, *Tens*, and *Hundreds*, as shown in the table:

Hundreds of Trillions	Tens of Trillions	Units of Trillions		Hundreds of Billions	Tens of Billions	Units of Billions		Hundreds of Millions	Tens of Millions	Units of Millions		Hundreds of Thousands	Tens of Thousands	Units of Thousands		Hundreds of Units	Tens of Units	Units of Units
5	5	5	,	4	4	4	,	3	3	3	,	2	2	2	,	1	1	1
5th Period, *Trillions.*				4th Period, *Billions.*				3d Period, *Millions.*				2d Period, *Thousands.*				1st Period, *Units.*		

The several orders may be named more briefly by calling the first order of each period *by the name of the period*, and omitting the word "of" after tens and hundreds, thus:

Hundred-trillions.	Ten-trillions.	Trillions.		Hundred-billions.	Ten-billions.	Billions.		Hundred-millions.	Ten-millions.	Millions.		Hundred-thousands.	Ten-thousands.	Thousands.		Hundreds.	Tens.	Units.
5	5	5	,	4	4	4	,	3	3	3	,	2	2	2	,	1	1	1
5th Period.				4th Period.				3d Period.				2d Period.				1st Period.		

Art. 21. RULE FOR NOTATION.—*Begin at the left, and write the figures of each period in their proper orders, filling all vacant orders and periods with ciphers.*

Art. 22. RULE FOR NUMERATION.—1. *Begin at the right, and separate the number into periods of three figures each.*

2. *Begin at the left, and read each period containing one or more significant figures as if it stood alone, adding its name.*

NOTE.—The name of the units' period is usually omitted.

WRITTEN EXERCISES.

1. Write in words, 20080406.

SUGGESTION.—Separate the number into periods, thus: 20,080,406. Then write each period, thus: *Twenty million eighty thousand four hundred and six.*

2. Write in words, 50038456.

3. Write in words, 300607008.

4. Write in words, 40000300400.

SUGGESTION.—Omit the third period, since it contains no significant figures, thus: *Forty billion three hundred thousand four hundred.*

5. Write in words, 3450000067.

6. Read 3000080040; 10080603400.

7. Read 15000407030; 5075803004.

8. Read 400440300500; 130030003003.

9. Express in figures, twelve billion forty-six million and nine.

PROCESS.—First, write 12, with a comma after it, to form the fourth or billions' period, thus: 12,; then write 46 in the next period, filling the vacant order with a cipher, thus: 12,046,; then, as there are no thousands, fill the next three orders with ciphers, thus: 12,046,000,; and, finally, write 9 in the units' period, filling the vacant orders with ciphers, thus: 12,046,000,009.

10. Express in figures, fifty million thirty-two thousand six hundred and forty.

11. Three hundred million nine thousand two hundred and six.

12. Forty-eight billion seventeen thousand and sixty-four.

13. Five million five thousand and five.

14. One million one hundred thousand and ten.

15. Three trillion three hundred million three hundred and three.

16. Sixty-two million three hundred thousand and forty-nine.

17. Five hundred million five thousand.

18. Four hundred and six thousand five hundred and seven.

19. Two million ten thousand and eighty.

20. Ninety million seven thousand four hundred and ninety.

21. Four hundred million forty thousand four hundred and four.

22. Thirty billion seventy-five thousand.

23. Nine billion nine thousand and nine.

24. Fifty-four million eighty-seven thousand and eighty-six.

25. Two hundred and two thousand five hundred and eighty.

26. Fifty billion fifty million five hundred thousand and seven.

27. Seventeen billion seven hundred thousand three hundred and six.

28. Ninety million ten thousand and fifty-five.

LESSON VII.

ROMAN NOTATION.

Art. 23. In the Roman Notation, numbers are expressed by means of seven *capital letters*, viz.: I, V, X, L, C, D, M.

I stands for one; V for five; X for ten; L for fifty; C for one hundred; D for five hundred; M for one thousand.

Art. 24. All other numbers are expressed by repeating or combining these letters.

1. When a letter is repeated, its value is repeated; thus: II represent 2; XX, 20; CCC, 300, etc.

2. When a letter is placed before one of greater value, the less value is taken from the greater; thus: IV stands for 4; IX for 9; XC for 90.

3. When a letter is placed after one of greater value, the less value is added to the greater, thus: VI stands for 6; XI for 11; CX for 110.

Art. 25. In the following table, numbers are expressed by letters and figures:

I, 1;	VIII, 8;	XV, 15;	XL, 40;
II, 2;	IX, 9;	XVI, 16;	L, 50;
III, 3;	X, 10;	XVII, 17;	LX, 60;
IV, 4:	XI, 11;	XVIII, 18;	LXX, 70;
V, 5;	XII, 12;	XIX, 19;	LXXX, 80;
VI, 6;	XIII, 13;	XX, 20;	XC, 90;
VII, 7;	XIV, 14;	XXX, 30;	C, 100.

WRITTEN EXERCISES.

Express the following numbers in figures:

(1)	(2)	(3)
XIV	CCL	MDCL
XXIV	DCXC	MDLX
XXXIX	CCXC	MDLIX
XCVI	DCCL	MDCCC
CXI	DCLIX	MDCCCLX
CIX	MCCL	MDCCCLXIX

Express the following numbers by letters:

(4)	(5)	(6)	(7)
45	156	210	1500
76	184	550	1650
90	345	700	1850
93	433	750	1868
99	555	880	1940

Express the following numbers by letters:

(8)	(9)	(10)	(11)
204	1200	1685	2000
409	1350	1944	2050
540	1408	1865	2550
675	1590	1909	3010

QUESTIONS FOR REVIEW.

What is arithmetic? What is a number? A unit? An integer?

In how many ways may numbers be expressed? How are numbers expressed in the Arabic method? In the Roman method? What is notation? What is numeration?

How many figures are used to express numbers? Which are called significant figures? Which has no numerical value?

What is meant by orders of units? How are the orders numbered? How many units of any order make one unit of the next higher order?

What is meant by the simple value of a figure? On what does the local value of a figure depend? What is the law of increase from right to left?

How many orders make a period? What are the names of these orders? Give the names of the first six periods. Give the rule for notation. Give the rule for numeration.

How are numbers expressed in the Roman notation? Name the letters used, and give the value of each. How are numbers expressed by these letters?

SECTION II.

ADDITION.

LESSON I.

Additive Numbers, 1, 2, and 3.

1. Four and 2 are how many? 8 and 2? 6 and 2? 7 and 2? 3 and 2? 9 and 2?

2. Two and 3 are how many? 5 and 3? 6 and 3? 7 and 3? 9 and 3? 11 and 3?

3. How many are 8 and 3? 18 and 3? 38 and 3? 47 and 3? 67 and 3? 87 and 3?

4. How many are 9 and 2? 39 and 2? 59 and 2? 48 and 3? 38 and 3? 88 and 3?

5. Frank has 5 marbles in one hand and 2 marbles in the other: how many marbles has he in both hands?

SOLUTION.—5 marbles and 2 marbles are 7 marbles: Frank has 7 marbles in both hands?

6. A drover bought 9 sheep of one farmer and 2 sheep of another: how many sheep did he buy?

7. Jane spelled 17 words correctly and misspelled 3: how many words did she try to spell?

8. A grocer sold 8 pounds of sugar to one customer, 3 pounds to another, and 2 pounds to another: how many pounds of sugar did he sell?

9. A man walked 4 miles the first hour, 3 miles the second, and 2 miles the third: how many miles did he walk in the three hours?

10. Begin with 1 and count to 45 by adding 2 successively, thus: 1, 3, 5, 7, 9, 11, 13, etc.

11. Begin with 2 and count to 50 by adding 3 successively.

NOTE.—This drill should be continued until the class can add by 2's or 3's with rapidity and accuracy.

WRITTEN EXERCISES.

Add the following numbers:

(1)	(2)	(3)	(4)	(5)
2	10	112	2112	12102
2	21	211	1201	21210
1	12	122	1122	10222
2	20	111	2021	11121
1	22	222	1212	21212
2	11	121	2221	12111

(6)

PROCESS.

121
233
123
332
231
323
123
232
333

2051, *Sum.*

Write the numbers so that the units shall form the first column; the tens, the second column; and the hundreds, the third column. Begin with the units' column, and add, naming results only, thus: 3, 5, 8, 11, 12, 14, 17, 20, 21—21 units equal 2 tens and 1 unit. Write the 1 unit under the units' column, and add the 2 tens with the tens' column, thus: 5, 8, 10, 12, 15, 18, 20, 23, 25—25 tens equal 2 hundreds and 5 tens. Write the 5 tens under the tens' column, and add the 2 hundreds with the hundreds' column, thus: 5, 7, 8, 11, 13, 16, 17, 19, 20—20 hundreds equal 2 thousands and 0 hundreds. Write the 0 hundreds under the hundreds' column, and write the 2 thousands in thousands' place. The sum is 2 thousands, 0 hundreds, 5 tens, and 1 unit, or 2051. To test the accuracy of the work, add the columns downward.

(7)	(8)	(9)	(10)
13	232	1323	3232
22	123	2112	2323
20	212	2131	23213
31	131	3213	13221
12	120	1301	32233
21	102	2222	232111
23	223	1111	323212
32	121	1323	232021

11. Add 213, 322, 203, 312, 222, 321, 231, 123, 303, 232, 311, 132.

12. What is the sum of 2132, 3113, 2323, 1313, 2132, and 3320?

13. $2021 + 12333 + 22031 + 332231 + 231323 =$ how many?

14. $3231 + 2302 + 2330 + 12332 =$ how many?

15. A grocer sold 12 pounds of sugar to one customer, 21 pounds to another, 32 pounds to another, and 30 pounds to another: how many pounds did he sell?

16. July has 31 days; August, 31; September, 30; October, 31; November, 30; and December, 31: how many days in the last six months of the year?

17. A farm contains 120 acres, another 212 acres, another 133 acres, and another 322 acres: how many acres do the four farms contain?

18. A man bought four loads of hay, the first weighing 2130 pounds, the second 2312 pounds, the third 2232 pounds, and the fourth 2322 pounds: how many pounds of hay in the four loads?

LESSON II.

MENTAL EXERCISES.

New Additive Numbers, 4 and 5.

1. Three and 4 are how many? 5 and 4? 6 and 4? 8 and 4? 7 and 4? 9 and 4?

2. Two and 5 are how many? 4 and 5? 6 and 5? 8 and 5? 7 and 5? 9 and 5?

3. How many are 18 and 4? 28 and 4? 48 and 4? 16 and 4? 36 and 4? 56 and 4?

4. How many are 17 and 5? 27 and 5? 47 and 5? 29 and 5? 49 and 5? 69 and 5?

5. There are 17 birds on one tree and 4 on another: how many birds on both trees?

6. A man gave 26 dollars for a coat and 5 dollars for a hat: how many dollars did he give for both?

7. A drover bought 19 cows of one man and 4 of another: how many cows did he buy?

8. James picked 27 peaches from one limb and 5 peaches from another: how many peaches did he pick from both limbs?

9. Mary has written 16 lines: if she write 5 lines more, how many lines will she then have written?

10. George gave 15 cents for a slate and 5 cents for a pencil: how many cents did he give for both?

11. Begin with 2 and count to 50 by adding 4 successively.

12. Begin with 3 and count to 48 by adding 5 successively.

WRITTEN EXERCISES.

(1)	(2)	(3)	(4)	(5)
15	251	15215	23512	52134
25	153	14343	30425	34445
35	354	45046	41341	53054
45	452	50350	23301	44052
55	355	33432	41545	25253
45	254	43543	43453	34545
35	555	23343	25445	41534
25	444	45452	41505	22335

6. What is the sum of four hundred and four; four thousand and forty; forty thousand four hundred; and four million four hundred thousand?

7. A grain dealer bought 2350 bushels of wheat on Monday, 4215 bushels on Tuesday, 3245 bushels on Wednesday, 1500 bushels on Thursday, 2424 bushels on Friday, and 1350 bushels on Saturday: how many bushels did he buy?

8. In a city containing five wards, there are 345 voters in the first ward, 443 in the second, 213 in the third, 523 in the fourth, and 425 in the fifth: how many voters in the city?

9. A father gave to his eldest son 225 acres of land, to the second 155 acres, to the third 145 acres, and to the youngest 124 acres: how many acres did he give to all?

10. The first three cars of a freight train contain 35240 pounds each; the next four cars, 25345 pounds each; the next two cars, 31540 pounds each; and the last car, 25432 pounds: how many pounds of freight in the ten cars?

LESSON III.

MENTAL EXERCISES.

New Additive Number, 6.

1. Two and 6 are how many? 4 and 6? 3 and 6? 5 and 6? 7 and 6? 9 and 6? 8 and 6?

2. How many are 17 and 6? 28 and 6? 48 and 6? 68 and 6? 58 and 6? 78 and 6?

3. How many are 19 and 6? 29 and 6? 59 and 6? 39 and 6? 69 and 6? 49 and 6?

4. Begin with 3 and count to 63 by adding 6 successively.

5. Mary's father gave her 5 peaches and her mother gave her 6: how many peaches did both give her?

6. John solved 18 problems before school and 6 problems in school: how many problems did he solve in all?

7. A farmer bought a cow for 27 dollars and a calf for 6 dollars: how many dollars did he pay for both?

8. The head of a fish is 5 inches long, its body 16 inches, and its tail 6 inches: how long is the fish?

9. In a certain orchard there are 29 apple trees, 5 pear trees, and 6 peach trees: how many trees in the orchard?

10. William gave a blind boy 19 cents, John gave him 15 cents, and Charles 6 cents: how many cents did they all give him?

WRITTEN EXERCISES.

(1)	(2)	(3)	(4)	(5)
3640	24137	43260	35260	305129
2566	16126	32345	16165	224603
1654	20050	16606	32542	350164
2366	16654	46060	36344	255234
3456	33456	50050	24030	145344
5634	44162	16566	33246	242456
4565	23206	24656	21438	145346
5656	36562	32562	44546	200500

6. Add thirty-six thousand three hundred and twenty-five; fourteen thousand and forty-six;

twenty-three thousand four hundred and five; fifteen thousand and sixteen; and three hundred and six thousand three hundred and four.

7. What is the sum of three million one thousand and fifty-six; six hundred thousand six hundred and twenty-five; four million forty-two thousand and four; forty-five million six hundred and fifty thousand?

LESSON IV.

MENTAL EXERCISES.

New Additive Number, 7.

1. Two and 7 are how many? 5 and 7? 3 and 7? 6 and 7? 8 and 7? 7 and 7? 9 and 7?

2. How many are 18 and 7? 48 and 7? 68 and 7? 88 and 7? 28 and 7?

3. Fifteen and 7 are how many? 35 and 7? 65 and 7? 45 and 7? 75 and 7?

4. Begin with 4 and count to 53 by adding 7 successively.

5. Charles had 6 marbles and his father gave him 7: how many marbles had he then?

6. A garden contains 19 pear trees and 7 peach trees: how many trees in the garden?

7. A man bought a set of harness for 37 dollars and a saddle for 7 dollars: how much did he pay for both?

8. Mr. Jones gave 8 plums to John, 6 to Henry, and 7 to George: how many plums did he give to the three boys?

9. Frank gave 10 cents for a lead-pencil, 5 cents for a piece of rubber, and 7 cents for paper: how much did the three articles cost?

10. A gentleman gave 36 dollars for a suit of clothes, 7 dollars for a pair of boots, and 5 dollars for a hat: how much did he pay for all?

WRITTEN EXERCISES.

(1)	(2)	(3)	(4)
10640	24045	32620	7121365
14075	14036	75437	2171634
26507	25507	50743	1237773
16021	46364	64017	7143656
34412	54563	32516	2674467
53452	16057	18416	6734765
26123	72027	13673	6574636
16021	47735	31654	7147347

5. What is the sum of sixteen million four thousand and sixty-five; three hundred thousand two hundred and fifty-six; seven thousand and forty; and five million five thousand and seven?

6. What is the sum of forty-five million seven thousand and seventy; six million sixty-five thousand two hundred and six; and seventy-five thousand and forty-four?

7. January has 31 days; February (except in leap year), 28; March, 31; April, 30; May, 31; and June 30: how many days in the first six months of the year?

8. A gentleman owns five farms, containing, respectively, 285 acres, 345 acres, 146 acres, 438

acres, and 248 acres: how many acres of land does he own?

9. A newsboy sold 327 papers in April, 465 in May, 318 in June, and 278 in July: how many papers did he sell in the four months?

10. The first ward of a city contains 1675 youth of school age; the second, 2357 youth; the third, 2347; the fourth, 3270; and the fifth, 2677: how many youth of school age in the city?

LESSON V.

MENTAL EXERCISES.

New Additive Number, 8.

1. Two and 8 are how many? 5 and 8? 3 and 8? 6 and 8? 4 and 8? 9 and 8?

2. How many is 16 plus 8? 36 plus 8? 56 plus 8? 25 plus 8? 45 plus 8? 65 plus 8?

3. How many is 13 + 8? 33 + 8? 53 + 8? 29 + 8? 49 + 8? 69 + 8?

4. Begin with 3 and count to 51 by adding 8 successively.

5. Jane solved 17 problems in the morning and 8 in the evening: how many problems did she solve?

6. A farmer raised 16 loads of wheat in one field and 8 loads in another: how much wheat did he raise?

7. Kate spelled 38 words correctly and misspelled 8: how many words did she try to spell?

8. Charles gave 25 cents for a speller and 8

cents for a pencil: how much did he give for both?

9. A lady paid 27 dollars for a shawl, 8 dollars for a bonnet, and 3 dollars for a pair of shoes: how much did she pay for all?

10. A merchant sold 18 yards of muslin to one customer, 7 yards to another, and 8 yards to another: how many yards did he sell?

WRITTEN EXERCISES.

(1)	(2)	(3)	(4)	(5)
308	2617	19864	42764	5868
280	4565	34687	38768	4384
667	6387	46768	34187	5065
444	7836	65837	63506	6008
555	5666	80040	24483	4873
371	4084	18608	43832	8345
736	8168	36084	41608	6654
644	7846	45687	37860	5636

6. Add thirty thousand six hundred and fifty; fifty thousand and eighty-five; four hundred thousand six hundred and seven; and three hundred and forty thousand and seventy.

7. Add eight million eight thousand and eight; eighteen million eighteen thousand and eighteen; and eight hundred million eight hundred thousand eight hundred.

8. The distance by railroad from Philadelphia to Harrisburg is 106 miles; from Harrisburg to Pittsburg, 249 miles; from Pittsburg to Crestline, 188 miles; from Crestline to Fort Wayne, 132

miles; from Fort Wayne to Chicago, 148 miles: how far is it from Philadelphia to Chicago?

9. One of the wards of a certain city contains 1384 houses; another, 2868 houses; another, 857 houses; and another, 1486 houses: how many houses in the city?

10. A steamship sailed 217 miles the first day; 265 miles the second; 227 miles the third; 187 miles the fourth; and 168 miles the fifth: how many miles did it sail in the five days?

LESSON VI.

MENTAL EXERCISES.

New Additive Number, 9.

1. Three and 9 are how many? 7 and 9? 9 and 7? 8 and 9? 9 and 8? 5 and 9?

2. How many is $14 + 9$? $24 + 9$? $44 + 9$? $16 + 9$? $36 + 9$? $56 + 9$?

3. How many is $17 + 9$? $37 + 9$? $57 + 9$? $23 + 9$? $43 + 9$? $63 + 9$?

4. Begin with 3 and count to 57 by adding 9 successively.

5. A farmer sold 6 hogs to his neighbor and 9 to a drover: how many hogs did he sell?

6. Andrew sold 8 bunches of grapes and had 9 bunches left: how many bunches had he at first?

7. There are 17 cows in one field and 9 cows in another: how many cows in both fields?

8. A pole is 7 feet in the water and 9 feet in the air: how long is the pole?

9. A man paid 23 dollars for a coat, 9 dollars for a pair of pants, and 8 dollars for a vest: how much did he pay for the suit?

10. A boy paid 45 cents for a ball, 8 cents for marbles, and 7 cents for an orange: how much did he pay for all?

WRITTEN EXERCISES.

(1)	(2)	(3)	(4)	(5)
57384	4369	45566	48	4868
5834	13846	806	76	3769
691	3482	9376	287	1804
2637	691	2038	80	786
13484	5873	4056	409	5863
596	578	8705	96	4836
43486	509	6508	378	3988

6. What is the sum of nine billion nine million and nine; nine hundred million nine hundred thousand nine hundred; and ninety million nine hundred thousand and ninety?

7. The State of Maine contains 31766 square miles; New Hampshire, 9280 square miles; Vermont, 10212; Massachusetts, 7800; Connecticut, 4674; and Rhode Island, 1306. How many square miles in all of the New England States?

8. The distance by railroad from Boston to Springfield is 98 miles; from Springfield to Albany, 103 miles; from Albany to Buffalo, 298 miles; from Buffalo to Cleveland, 183 miles; from Cleveland to Chicago, 355 miles. How far from Boston to Chicago?

LESSON VIII.

REVIEW.

1. An orchard contains 25 apple trees and 8 peach trees: how many trees in the orchard?

2. A gardener sold 17 quarts of strawberries in market and 9 quarts to a grocer; how many quarts did he sell?

3. A lady gave 15 cents for thread, 8 cents for needles, and 7 cents for pins: how many cents did she spend?

4. James gave 8 cherries to George, 7 to William, 6 to Thomas, 9 to Harry, and kept 5: how many cherries had he at first?

5. A gentleman gave 95 dollars for a horse, 15 dollars for a saddle, and 5 dollars for a bridle: how much did he pay for all?

6. Begin with 2 and add to 72 by 7's, thus: 9, 16, 23, 30, 37, etc.

7. Begin with 5 and add to 61 by 8's.

8. Begin with 3 and add to 69 by 6's.

9. Begin with 4 and add to 67 by 9's.

WRITTEN EXERCISES.

1. $32545 + 8607 + 11709 + 50063 =$ how many?

2. A man paid \$3575 for a lot, \$5450 for a house, \$875 for a stable, and \$675 for other improvements: what did the property cost him?

NOTE.—This character (\$) denotes *dollars*, and is called the *dollar sign:* \$35 is read 35 *dollars;* \$1 is read 1 *dollar.*

3. The first book of a series contains 328 pages; the second, 392 pages; the third, 400 pages; and the fourth, 432 pages: how many pages in the series?

4. Ohio contains 39964 square miles; Michigan, 56243 square miles; Indiana, 33809 square miles; and Illinois, 55409 square miles: what is the area of these four States?

5. The distance by railroad from Pittsburg to Columbus is 193 miles; from Columbus to Cincinnati, 120 miles; from Cincinnati to St. Louis, 340 miles: how far is it from Pittsburg to St. Louis?

6. A father divided his estate between two sons and three daughters, giving to each son $3250 and to each daughter $2750: what was the value of the estate?

7. A farmer raised in one year 380 bushels of wheat, 245 bushels of oats, 87 bushels of rye, and as many bushels of corn as of wheat, oats, and rye together: how many bushels of grain did he raise?

DEFINITIONS, PRINCIPLES, AND RULE.

Art. 26. ***Addition*** is the process of finding the sum of two or more numbers.

The number obtained by adding two or more numbers is called the *Sum* or *Amount*.

The ***Sum*** contains as many units as all the numbers added, taken together.

Numbers are either *Concrete* or *Abstract.*

A ***Concrete Number*** is applied to a particular thing or quantity; as, 4 pears, 7 hours, 30 steps.

An ***Abstract Number*** is not applied to any particular thing or quantity; as, 4, 7, 30.

Fourteen balls and 13 balls are numbers of the *same kind;* and 6 tens and 3 tens are numbers of the *same order.* Numbers of the same kind or order are called *Like Numbers.* Only like numbers can be added.

Art. 27. The *Sign of Addition* is $+$. It is called *plus* meaning *more.* When placed between two numbers, it shows that they are to be added. Thus, $8 + 5$ is read 8 *plus* 5, and it shows that 5 is to be added to 8.

The *Sign of Equality* is $=$. It is read *equals* or *is equal to.* Thus $7 + 8 = 15$ is read 7 *plus* 8 *equals* 15.

Art. 28. RULE FOR ADDITION.—1. *Write the numbers to be added so that figures denoting units of the same order shall be in the same column, and draw a line underneath.*

2. *Beginning with units, add each column, and write the sum, when less than ten, underneath.*

3. *When the sum of any column exceeds nine, write the right-hand figure under the column added, and add the number denoted by the left-hand figure or figures with the next column.*

4. *Write the entire sum of the left-hand column.*

PROOF.—*Add the columns downward.*

SECTION III.

SUBTRACTION.

LESSON I.

Subtrahend Figures, 1, 2, 3.

1. How many is 4 less 3? 6 less 3? 8 less 3? 10 less 3? 12 less 3? 11 less 3?

2. How many is 11 less 2? 21 less 2? 41 less 2? 19 less 2? 29 less 2? 49 less 2?

3. Three from 12 leaves how many? 3 from 22? 3 from 42? 3 from 52? 3 from 32? 3 from 20? 3 from 40? 3 from 50?

4. Begin with 50 and count back to 0 by subtracting 2 successively, thus: 50, 48, 46, 44, 42, etc.

5. Begin with 40 and count back to 1 by subtracting 3 successively.

6. Charles bought 12 sticks of candy and ate 3 of them: how many sticks were left?

7. The teacher pronounced 21 words to Henry, and he misspelled 2 of them: how many words did he spell correctly?

8. A lesson in arithmetic consists of 15 examples, and Charles has solved all but 3 of them: how many examples has he solved?

9. James is 11 years old and his brother Henry is 3 years younger: how old is Henry?

WRITTEN EXAMPLES.

1. From 345 take 123.

PROCESS.

Minuend,	345
Subtrahend,	123
Difference,	222

Write 123 under 345, placing units under units, tens under tens, and hundreds under hundreds. Subtract 3 units from 5 units, and write 2 units, the difference, below; subtract 2 tens from 4 tens, and write 2 tens, the difference, below; subtract 1 hundred from 3 hundreds, and write 2 hundreds, the difference below. The difference, or remainder, is 222.

(2)	(3)	(4)	(5)	(6)	(7)	(8)
57	46	88	75	685	409	967
43	24	65	53	343	307	645

(9)	(10)	(11)	(12)	(13)	(14)	(15)
246	487	507	718	563	485	560
132	231	302	312	330	212	320

16. From four thousand and sixty-five take two thousand and thirty-one.

17. A grocer bought 585 pounds of sugar and sold 231 pounds: how many pounds had he left?

18. In a graded school, there are 345 boys and 321 girls: how many more boys than girls in the school?

LESSON II.

MENTAL EXERCISES.

New Subtrahend Figures, 4 and 5.

1. How many is 7 less 4? 6 less 4? 9 less 4? 8 less 4? 10 less 4? 11 less 4?

2. How many is 7 less 5? 9 less 5? 8 less 5? 10 less 5? 12 less 5? 15 less 5?

3. How many is 13 less 4? 23 less 4? 43 less 4? 63 less 4? 83 less 4? 53 less 4? 93 less 4?

4. How many is 14 less 5? 44 less 5? 34 less 5? 54 less 5? 64 less 5? 74 less 5? 94 less 5?

5. Begin with 60 and count back to 0 by subtracting 4 successively.

6. Begin with 53 and count back to 1 by subtracting 4 successively.

7. A man gave $12 for a saddle and $4 for a bridle: how much did the saddle cost more than the bridle?

8. Charles earned 21 cents by selling papers and gave 4 cents for a comb: how many cents had he left?

9. Kate is 15 years old and her sister is 4 years younger: what is her sister's age?

10. There are 21 passengers in a car: if 5 of them leave at a station, how many will remain?

11. There are 13 men in one coach and 5 men in another: how many men in the first coach more than in the second?

WRITTEN EXERCISES.

(1)	(2)	(3)	(4)	(5)	(6)
335	2036	308	1565	3683	7863
214	1034	205	1433	2542	4552

7. From five thousand and seventy-six take three thousand and fifty.

8. A farm contains 358 acres of land: if 155 acres should be sold, how many would be left?

9. A man bought a house for $4320 and sold it for $6450: how much did he gain?

10. A man bought 3487 bushels of wheat and sold 1425 bushels: how many bushels had he left?

11. A ship-builder sold a vessel for $24350: if the vessel cost him $27585, how much did he lose?

12. The number of school-houses in Ohio, in 1867, was 11353; in Pennsylvania, 11453: how many more school-houses in Pennsylvania than in Ohio?

13. A wool dealer having bought 23437 fleeces of wool, shipped 12322 fleeces to Boston: how many fleeces had he left?

LESSON III.

MENTAL EXERCISES.

New Subtrahend Figures, 6 and 7.

1. How many is 8 less 6? 10 less 6? 12 less 6? 13 less 6? 15 less 6? 14 less 6?

2. How many is 12 less 7? 13 less 7? 15 less 7? 16 less 7? 18 less 7? 22 less 7?

3. How many is 14 less 6? 24 less 6? 44 less 6? 64 less 6? 34 less 6? 74 less 6?

4. How many is 14 less 7? 24 less 7? 44 less 7? 16 less 7? 36 less 7? 46 less 7?

5. Begin with 56 and count back to 0 by subtracting 7 successively.

6. Begin with 60 and count back to 0 by subtracting 6 successively.

7. Ella was absent from school 7 days in a term of 75 days: how many days was she present?

8. John earned 25 cents by selling oranges and gave 6 cents for a lead-pencil: how many cents had he left?

9. A boy was carrying home 21 eggs; he fell and broke 7 of them: how many were left?

10. A man having 23 dollars, gave 6 dollars for a hat: how many dollars had he left?

11. A teacher pronounced 25 words to an idle pupil, who misspelled 7 of them: how many words did he spell correctly?

WRITTEN EXERCISES.

1. From 5334 take 2726.

PROCESS.

Min., 5334
Sub., 2726
Dif., 2608

Since 6 units can not be taken from 4 units, add 10 units to the 4 units, making 14 units; then subtract 6 units from 14 units, and write 8 units, the difference, below. To balance the 10 units (equal 1 *ten*) added to the minuend, add 1 ten to the 2 tens of the subtrahend; then subtract 3 tens from 3 tens, and write 0 tens, the difference, below.

Add 10 hundreds to the 3 hundreds of the minuend, making 13 hundreds; subtract 7 hundreds from 13 hundreds, and write 6 hundreds, the difference, below. To balance the 10 hundreds (equal 1 *thousand*) added to the minuend, add 1 thousand to the 2 thousands of the sub-

trahend; subtract 3 thousands from 5 thousands, and write 2 thousands, the difference, below. The difference is 2608.

This process may be shortened, thus: 6 units from 4 units plus 10 units, or 14 units, leave 8 units; 2 tens and 1 ten are 3 tens, and 3 tens from 3 tens leave 0 ten; 7 hundreds from 3 hundreds plus 10 hundreds, or 13 hundreds, leave 6 hundreds; 1 thousand and 2 thousands are 3 thousands, and 3 thousands from 5 thousands leave 2 thousands. The difference is 2608.

NOTE.—The teacher should show that the adding of 10 to a term of the minuend and 1 to the next higher term of the subtrahend, increases both minuend and subtrahend equally, and does not affect the difference.

(2)	(3)	(4)	(5)	(6)
44	63	272	1385	5754
26	46	147	1276	3457

(7)	(8)	(9)	(10)	(11)
3416	3041	14406	20670	30401
2507	2637	7345	17356	20576

12. From fourteen thousand and forty-four take six thousand and sixteen.

13. A man whose income is $1850 expends annually $1365: how much does he lay up?

14. The number of youth of school age in a certain city is 1234, and only 756 pupils are enrolled in the schools: how many youth do not attend school?

15. The number of pupils enrolled in the public schools of Ohio, in 1867, was 704767; in Pennsylvania, 789389: how many more pupils were enrolled in Pennsylvania than in Ohio?

LESSON IV.

MENTAL EXERCISES.

New Subtrahend Figures, 8 and 9.

1. How many is 9 less 8? 11 less 8? 13 less 8? 10 less 8? 14 less 8? 12 less 8?

2. How many is 16 less 8? 26 less 8? 56 less 8? 17 less 8? 27 less 8? 67 less 8?

3. How many is 11 less 9? 13 less 9? 15 less 9? 12 less 9? 16 less 9? 17 less 9?

4. How many is 16 less 9? 26 less 9? 36 less 9? 15 less 9? 25 less 9? 45 less 9?

5. Begin with 50 and count back to 2 by subtracting 8 successively.

6. Begin with 57 and count back to 3 by subtracting 9 successively.

7. A school has enrolled 65 pupils, and 8 are absent: how many are present?

8. Mr. Smith is 44 years of age and his youngest son is 8 years of age: what is the difference in their ages?

9. A school contains 9 more girls than boys: if there are 56 girls, what is the number of boys?

WRITTEN EXERCISES.

1. From 800000 take 238.

2. From forty million take eighty thousand.

3. A nursery contains 705 peach trees and 428 plum trees: how many more peach trees than plum trees in it?

4. The Pilgrims landed at Plymouth in 1620, and our National Independence was declared in 1776: how many years between the two events?

5. The first steamboat was made in 1807, and the Atlantic Cable was laid in 1866: how many years between the two events?

6. Mont Blanc in Europe is 15668 feet high, and Mount Sorata in South America is 21286 feet high: what is the difference is the height of these two mountains?

7. A man who owned 3408 sheep, sold 1897 of them: how many sheep had he left?

8. Mt. Etna is 10874 feet high, and Mt. Vesuvius 3948 feet: how much higher is Etna than Vesuvius?

9. America was discovered in 1492, and the Pilgrims landed at Plymouth in 1620: how many years intervened?

10. The population of the State of New York in 1860 was 3881000, and that of Ohio, 2340000: how many more people in New York than in Ohio?

LESSON V.

REVIEW.

1. From a cask containing 45 gallons of molasses, 39 gallons were sold: how many gallons remained unsold?

2. An orchard contains 56 apple trees and 48 peach trees: how many more apple trees than peach trees in the orchard?

3. A grocer sold 57 pounds of butter from a firkin containing 65 pounds: how many pounds remained in the firkin?

4. In a school, 63 pupils are enrolled and 54 are present: how many pupils are absent?

5. If a man earns $45 a month, and spends $36: how much does he lay up?

6. A man gave $75 for a watch and $22 for a chain: how much did the watch cost more than the chain?

7. Charles has 17 marbles and John 8: how many more marbles has Charles than John?

8. A teacher asked his class 52 questions, and 8 were answered incorrectly: how many were answered correctly?

9. In a term of 64 days, Charles attended school 55 days: how many days was he absent?

10. Subtract by 4's from 62 back to 2, thus: 62, 58, 54, 50, 46, etc.

11. Subtract by 6's from 75 back to 3.

12. Subtract by 9's from 68 back to 5.

13. Subtract by 7's from 59 back to 3.

14. Subtract by 8's from 48 back to 0.

WRITTEN EXERCISES.

1. From 202380 take 165436.

2. 4308560 — 1674805 = how many?

3. Illinois contains 55409 square miles, and Missouri 67380 square miles: how much more area has Missouri than Illinois?

4. By the census of 1850 the entire population of the United States was 23191876, and by the census of 1860 it was 31224885: what was the increase in 10 years?

5. In 1862 there were 10869 miles of railroad in Great Britain, and 33222 miles in the United States: how many more miles in the United States than in Great Britain?

6. An army of 30340 men lost 7568 in battle: how many men did it then contain?

7. In 1862 Ohio produced 35442858 pounds of butter and 20637235 pounds of cheese: how much more butter than cheese was produced?

8. A merchant having $11315 in bank, drew out $978: how much remained in the bank?

DEFINITIONS, PRINCIPLES, AND RULE.

Art. 29. ***Subtraction*** is the process of finding the difference between two numbers.

The ***Difference*** or ***Remainder*** is the number found by subtracting one number from another.

The ***Minuend*** is the number diminished.

The ***Subtrahend*** is the number subtracted.

Art. 30. Only *Like Numbers* can be subtracted. We can not take 3 pencils from 7 books, nor 3 units from 7 tens.

Art. 31. The *Sign of Subtraction* is —. It is read *minus* or *less*. It shows that the number after it is to be subtracted from the number before it.

Art. 32. RULE FOR SUBTRACTION.—1. *Write the subtrahend under the minuend, placing units under units, tens under tens, hundreds under hundreds, etc.*

2. *Begin at the right, and subtract each term of the subtrahend from the term above it, and write the difference underneath.*

3. *When any term of the subtrahend is greater than the term above it, add* 10 *to the upper term, and then subtract, and write the difference as before.*

4. *When ten has been added to the upper term, add* 1 *to the next higher term of the subtrahend before subtracting.*

PROOF.—*Add the remainder and subtrahend. If their sum is equal to the minuend, the work is correct.*

NOTE.—Instead of adding 1 to the next term of the subtrahend, 1 may be subtracted from the next term of the minuend.

LESSON VI.

Problems combining Addition and Subtraction.

1. Robert picked 21 peaches, and gave 7 to his sister and 8 to his brother: how many peaches had he left?

2. A garden contains 17 pear trees, 8 plum trees, and a certain number of peach trees: if there are 33 trees in the garden, what is the number of peach trees?

3. A grocer bought 35 bushels of apples, and

sold 17 bushels to A, 9 bushels to B, and the rest to C: how many bushels did he sell to C?

4. Jane is 8 years old and Lucy 13, and the sum of Jane's and Lucy's ages, less 7 years, is the age of Mary: how old is Mary?

5. A man bought a firkin of butter for $17, a crock of lard for $8, and a barrel of flour for $9; but he had not money enough by $7 to pay for them: how much money had he?

6. A man earned $45, and paid $15 for house rent, $8 for flour, $7 for shoes, and $10 for groceries: how much had he left?

7. A man sees 15 pigeons on one branch of a tree, and 9 pigeons on another branch: if 7 should fly away, how many would be left on the tree?

8. A farmer had 23 chickens, but 7 of them were stolen and 5 were carried off by a hawk: how many chickens had he left?

9. A drover bought 17 sheep of one farmer, 9 sheep of another, and 8 of another, and then sold 7 of them to a butcher: how many sheep had he left?

10. A man gave a watch and $9 in money for a horse valued at $75: what did he get for his watch?

WRITTEN EXERCISES.

1. From a piece of carpeting containing 150 yards, a merchant sold 3 carpets, containing respectively 27, 39, and 42 yards: how many yards were left?

2. Rhode Island contains an area of 1306 square miles; Delaware, 2120; Connecticut, 4674; New Jersey, 8320; Maryland, 9356; and New York, 47000: how many more square miles has New York than the other five States named?

3. A regiment entered the service with 1088 men: 150 were killed in battle, 65 died from disease, 24 deserted, and 250 were discharged: how many remained?

4. A grain dealer bought 1250 bushels of wheat on Monday, 2145 bushels on Tuesday, and 3240 bushels on Wednesday, and on Thursday, fearing a decline in price, he sold 5450 bushels: how much wheat had he left?

5. A man deposited $175, $141, $75, $304, and $250 in a bank, and then drew out $480 and $225: how many dollars remained in the bank?

6. A railroad train left Cincinnati for St. Louis with 336 passengers, and during the trip 145 passengers came aboard, and 208 passengers left: how many were in the cars when the train reached St. Louis?

7. In 1860 the population of Maine was 628279; New Hampshire, 326073; Vermont, 315098; Massachusetts, 1231066; Connecticut, 460147; Rhode Island, 174620; and New York, 3880735. How many more inhabitants in New York than in the six New England States?

8. A man gave to his eldest son $2380; to the second, $245 less than to the eldest; and to

the youngest, $450 less than to the second: how much did he give to all?

9. From the sum of 2348 and 1864 subtract their difference.

10. From the sum of 506703 and 340067 take their difference.

SECTION IV.

MULTIPLICATION.

LESSON I.

Multiplicand Figures, 1, 2, and 3.

1. Twice 2 are how many? 4 times 2? 6 times 2? 5 times 2? 8 times 2? 9 times 2?

2. Twice 3 are how many? 3 times 3? 5 times 3? 4 times 3? 7 times 3? 9 times 3?

3. How many are 5 times 1? 5 times 3? 7 times 1? 7 times 2? 8 times 1? 8 times 3?

4. A boy has 2 hands: how many hands have 6 boys? 8 boys? 10 boys?

5. There are 3 feet in a yard: how many feet in 2 yards? 4 yards? 6 yards?

6. If 3 bushels of apples fill a barrel, how many bushels will fill 3 barrels? 5 barrels? 7 barrels?

7. If a man earn 3 dollars a day, how many dollars will he earn in 6 days?

8. If a boy walk 3 miles a day in attending school, how many miles will he walk in 10 days?

WRITTEN EXERCISES.

1. Multiply 232 by 3.

PROCESS.

Multiplicand, 232
Multiplier, 3
Product, 696

Write the multiplier 3 under the units' figure of the multiplicand, and multiply, thus: 3 times 2 units are 6 units; 3 times 3 tens are 9 tens; 3 times 2 hundreds are 6 hundreds. The product is 696.

(2)	(3)	(4)	(5)	(6)
3212	10202	23321	202122	303203
3	3	3	4	3

7. Multiply 230321 by 2. By 3.

8. Multiply 320201 by 3. By 4. By 2.

9. If a gold watch is worth $220, what is the worth of 4 gold watches?

10. A drover bought 3 horses at $133 apiece: what did they cost?

11. There are 320 rods in a mile: how many rods are there in 3 miles? In 4 miles?

LESSON II.

MENTAL EXERCISES.

New Multiplicand Figures, 4 and 5.

1. Twice 4 are how many? 3 times 4? 5 times 4? 4 times 4? 6 times 4? 8 times 4? 7 times 4? 9 times 4? 10 times 4?

2. Twice 5 are how many? 5 times 5? 6 times 5? 8 times 5? 7 times 5? 9 times 5?

3. How many are 7 times 4? 7 times 5? 9 times 4? 9 times 5? 10 times 5?

4. If a lemon cost 4 cents, what will 6 lemons cost? 8 lemons? 10 lemons?

5. How much will a man earn in 7 days, at $4 a day? In 8 days? 9 days?

6. There are 5 cents in a half-dime: how many cents in 3 half-dimes? 5 half-dimes?

7. If you write 5 lines a day, how many lines will you write in 4 days?

8. If 5 boys can sit on one bench, how many boys can sit on 8 benches?

9. What will 10 quarts of currants cost, at 5 cents a quart?

10. If there are 5 school-days each week, how many school-days are there in 6 weeks? In 8 weeks? 10 weeks?

WRITTEN EXERCISES.

1. Multiply 434 by 6.

PROCESS.

Multiplicand, 434
Multiplier, 6
Product, 2604

Multiply the number denoted by each figure of the multiplicand by 6. Thus: 6 times 4 units are 24 units, which equal 2 *tens* and 4 *units;* write the 4 units in units' place in the product, and reserve the 2 tens. Six times 3 tens are 18 tens, and 18 tens plus the 2 tens reserved are 20 tens, which equal 2 *hundreds* and 0 *tens;* write the 0 tens in tens' place in the product, and reserve

the 2 hundreds. Six times 4 hundreds are 24 hundreds, and 24 hundreds plus the 2 hundreds reserved are 26 hundreds, which equal 2 *thousands* and 6 *hundreds;* write the 6 hundreds in hundreds' place in the product, and the 2 thousands in thousands' place. The product is 2604.

(2)	(3)	(4)	(5)	(6)	(7)
453	2524	4545	3545	13545	25245
8	6	7	8	4	6

8. If there are 324 pins on a paper, how many pins are there on 3 papers? 5 papers?

9. If a train of cars run 425 miles a day, how far will it run in 8 days?

10. If 135 tons of iron rails will make one mile of railroad, how many tons will make 7 miles?

11. What will 6 horses cost, at $152 apiece?

12. A father divided his estate between four sons, giving to each $3545: what was the value of the estate?

13. There are 1440 minutes in a day: how many minutes in 7 days, or a week?

14. If it take 15520 shingles to cover a house, how many shingles will cover 8 houses?

LESSON III.

MENTAL EXERCISES.

New Multiplicand Figure, 6.

1. Twice 6 are how many? 4 times 6? 3 times 6? 5 times 6? 7 times 6? 6 times 6? 8 times 6? 9 times 6? 10 times 6?

2. There are 8 rows of trees in an orchard, and 6 trees in each row: how many trees in the orchard?

3. What will 7 lead-pencils cost, at 6 cents apiece?

4. What will 6 oranges cost, at 8 cents apiece?

5. There are 6 days for labor in each week: how many days for labor in 6 weeks? 9 weeks?

6. John caught 6 fishes and Harry 7 times as many as John: how many did Harry catch?

7. If a horse travel 6 miles an hour, how far will it travel in 5 hours? In 10 hours?

8. There are 6 feet in a fathom: how many feet in 7 fathoms? 9 fathoms?

WRITTEN EXERCISES.

1. Multiply 456 by 43.

PROCESS.

Multiplicand,	456
Multiplier,	43
Partial Products,	1368
	1824
Product,	19608

Write the multiplier under the multiplicand, placing units under units and tens under tens. First multiply by the 3 units, as in the preceding lesson, which gives 1368 for the *first* partial product. Next multiply by the 4 tens, observing that units multiplied by tens (or tens by units) produce *tens*, that tens by tens produce *hundreds*, and that hundreds by tens produce *thousands*, &c. This gives, for the *second* partial product, 4 *tens*, 2 *hundreds*, 8 *thousands*, and 1 *ten-thousand*, which are to be written in their proper orders, since unlike orders can not be added. Then add the two partial products, and their sum, which is 19608, is the product required.

NOTE.—The teacher should show that units multiplied by tens produce *tens;* tens by tens, *hundreds,* etc. This may be done, in the above example, by changing the 4 tens into 40 units. 40 times 6 units = 240 units, or 24 *tens;* and 40 times 5 tens = 200 tens, or 20 *hundreds,* etc. The first figure of each partial product is written under the multiplier which produces it.

(2)	(3)	(4)	(5)	(6)	(7)	(8)
606	562	653	1446	2306	4636	40563
54	67	86	234	726	67	143

9. If a ship sail 216 miles a day, how far will it sail in 38 days?

10. What will 27 carriages cost, at $165 apiece?

11. If a web of flannel contain 46 yards, how many yards in 397 webs?

LESSON IV.

MENTAL EXERCISES.

New Multiplicand Figure, 7.

1. Three times 7 are how many? 5 times 7? 7 times 7? 9 times 7? 8 times 7?

2. There are 7 days in a week: how many days in 2 weeks? 4 weeks?

3. How many hills of potatoes in 6 rows, if there are 7 hills in each row?

4. If Charles earn 7 dollars a week, how much will he earn in 5 weeks?

5. If a horse travel 7 miles in an hour, how far will he travel in 8 hours?

6. If 5 men can build a wall in 7 days, how long will it take one man to build it?

7. If a box of crackers will last 8 men 7 days, how long will it last one man?

8. An orchard contains 10 rows of trees, and there are 7 trees in each row: how many trees in the orchard?

WRITTEN EXERCISES.

1. Multiply 2745 by 306.

PROCESS.

	2745
	306
Partial Products,	16470
	8235
Product,	839970

Multiply successively by the first and third figures of the multiplier, observing that units multiplied by hundreds produce *hundreds,* and hence writing the first figure of the second partial product in hundreds' order. In 306 there are no tens to be used as a multiplier.

(2)	(3)	(4)	(5)	(6)	(7)
4086	32607	7908	8099	60772	86507
4008	4009	909	1088	1019	9003

8. Enos lived 905 years: how many days did he live, allowing 365 days to the year?

9. A planter raised 208 bales of cotton, each bale weighing 475 pounds: how many pounds of cotton did he raise?

10. If a garrison of soldiers consume 4865 pounds of bread a day, how many pounds will supply the garrison 408 days? 606 days?

11. What will 508 horses cost, at $125 apiece?

12. What will it cost to build 705 miles of railroad, at $7525 a mile?

LESSON V.

MENTAL EXERCISES.

New Multiplicand Figure, 8.

1. Three times 8 are how many? 5 times 8? 7 times 8? 9 times 8? 8 times 8?

2. How many are 5 times 8? 8 times 5? 6 times 8? 8 times 6? 7 times 8? 8 times 7?

3. There are 8 quarts in one peck: how many quarts in 3 pecks? 5 pecks? 7 pecks?

4. There are 8 pints in a gallon: how many pints in 4 gallons? 6 gallons? 8 gallons?

5. If 5 men can mow a field of grass in 8 days, how long would it take one man to do it?

6. If a quantity of provisions will last 7 men 8 days, how long will it last one man?

7. If 4 pipes will empty a cistern in 8 hours, how long will it take one pipe to empty it?

8. If a man earn 8 dollars a week, how much will he earn in 9 weeks? 11 weeks?

9. A railroad car has 8 wheels: how many wheels has a train of 7 cars? 9 cars?

10. If a horse eat 8 quarts of oats each day, how much will he eat in 6 days? 10 days?

11. If a pint of oil cost 8 cents, what will 8 pints cost?

12. James has 8 marbles, and John has 6 times as many: how many marbles has John?

13. What will 8 pounds of beef cost, at 10 cents a pound?

WRITTEN EXERCISES.

Multiplicand or Multiplier ending with Ciphers.

1. Multiply 148000 by 47.

PROCESS.

148000
47

Partial Products, 1036, 592

Product, 6956000

To shorten the process, write the multiplier under the significant figures of the multiplicand, and, omitting the ciphers in forming the partial products, annex them to the product obtained. The result will be the true product.

NOTE.—The teacher should show that the use of the ciphers in forming the partial products would produce the same result.

(2)	(3)	(4)	(5)
48000	308000	295	4306
36	405	43000	245000

6. There are 5280 feet in a mile: how many feet in 805 miles?

7. The earth moves in its orbit at an average rate of 68000 miles in an hour: how far does it move in 24 hours? In 48 hours?

8. If a carriage-wheel revolve 280 times in running a mile, how many times will it revolve in running 68 miles? 75 miles?

9. A canal-boat was loaded with 245 bales of hay, weighing 280 pounds each: what was the weight of the cargo?

10. There are 480 sheets of paper in a ream: how many sheets are there in 604 reams?

11. If an acre of land produce 380 pounds of cotton, how many pounds will a plantation of 248 acres produce?

12. A steamboat makes 145 trips in a season, and carries, on an average, 280 passengers each trip: how many passengers does she carry during the season?

LESSON VI.

MENTAL EXERCISES.

New Multiplicand Figure, 9.

1. Three times 9 are how many? 4 times 9? 6 times 9? 8 times 9? 7 times 9? 9 times 9?

2. How many are 5 times 9? 9 times 5? 7 times 9? 9 times 7? 10 times 9? 9 times 10?

3. How many are 5 times 10? 10 times 5? 7 times 10? 10 times 7? 9 times 10? 10 times 9?

4. A man gave 7 boys 9 raisins each: how many raisins did he give them all?

5. If a man earn 10 dollars a week, how much will he earn in 8 weeks?

6. Jane writes 9 lines each day at school: how many lines does she write in 8 days?

7. Charles receives 9 dollars a month as errand-boy: how much will he earn in 10 months?

8. If 7 men can do a piece of work in 9 days, how many men will it take to do the same work in one day?

9. If a quantity of provisions will supply 10 men 9 days, how long will it supply one man?

10. What will 6 barrels of flour cost, at $9 a barrel?

WRITTEN EXERCISES.

Both Multiplicand and Multiplier ending with Ciphers.

1. Multiply 198000 by 8900.

PROCESS.

198000
8900

Partial Products, 1782, 1584

Product, 1762200000

Write the significant figures of the multiplier under the significant figures of the multiplicand, and multiply, omitting the ciphers in forming the partial products, but annexing them to the product obtained for the true product.

(2)	(3)	(4)	(5)
94000	90800	470000	950000
1600	370000	1900	360000

6. There are 3600 seconds in one hour: how many seconds are there in 630 hours?

7. Light moves 192000 miles in a second: how far does it move in one hour?

8. A ship has provisions enough to allow the crew 130 pounds a day for 90 days: how many pounds of provisions are aboard?

9. What will 1700 tons of railroad iron cost, at $250 a ton?

10. An army is composed of 54 regiments,

containing, on an average, 670 men each: how many men in the army?

11. If a steamer can run 260 miles a day, how far can it run in 10 days? In 100 days?

12. In a field of corn there are 70 rows, and each row contains 280 hills, and each hill 3 stalks: how many stalks of corn in the field?

LESSON VII.

REVIEW.

1. What will 4 oranges cost, at 5 cents apiece?

2. What will 5 barrels of flour cost, at $9 a barrel?

3. If an orange is worth 5 apples, how many apples are 7 oranges worth?

4. If there are 8 pints in a gallon, how many pints are there in 6 gallons?

5. Two men start from the same place, and travel in opposite directions, one at the rate of 3 miles an hour and the other 4 miles an hour: how far will they be apart in 8 hours?

6. If an orange is worth 2 lemons and a lemon is worth 5 plums, how many plums are worth 6 oranges?

7. If 7 men can do a piece of work in 5 days, how long would it take one man to do it?

8. If 6 men can cut a field of grass in 8 days, how many men will it take to cut it in one day?

9. If 3 pipes fill a cistern in 10 hours, in how many hours will one pipe fill it?

WRITTEN EXERCISES.

1. What is the product of 4894×37?
2. What is the product of 5680×340?
3. $6084 \times 3008 =$ how many?
4. $704000 \times 4800 =$ how many?
5. Multiply forty-eight thousand by sixty-five thousand.
6. A freight train consists of 37 cars, and each car contains 9850 pounds of freight: how much freight in the entire train?
7. If 980 pounds of bread will supply the inmates of the State Prison one day, how many pounds will supply them one year, or 365 days?
8. If a sack of salt contain 168 pounds, what will be the weight of 1600 sacks?
9. A merchant bought 18 firkins of butter, each weighing 32 pounds, at 37 cents a pound: what did it cost?
10. A train of 27 cars is loaded with iron; each car contains 48 bars, and each bar weighs 365 pounds; what is the weight of the cargo?

DEFINITIONS, PRINCIPLES, AND RULE.

Art. 33. ***Multiplication*** is the process of taking one number as many times as there are units in another.

The ***Multiplicand*** is the number taken or multiplied.

The ***Multiplier*** is the number denoting how many times the multiplicand is taken.

The ***Product*** is the number obtained by multiplying.

The multiplicand and multiplier are called the *Factors* of the product.

Art. 34. The *Sign of Multiplication* is $\times$, and is read *multiplied by*. When placed between two numbers, it shows that the number before it is to be multiplied by the number after it. Thus: 6×3 is read 6 *multiplied by* 3.

NOTE.—Since a change in the order of the factors does not change the product, 6×3 may also be read 6 *times* 3.

Art. 35. Multiplication is a short method of addition. The sum of $5 + 5 + 5 + 5$ is the same as 4 times 5.

Art. 36. RULE FOR MULTIPLICATION.—1. *Write the multiplier under the multiplicand, placing units under units, tens under tens, etc.*

2. *When the multiplier consists of but one term, begin at the right and multiply successively each term of the multiplicand, writing the right-hand term of each result in the product and adding the left-hand term to the next result.*

3. *When the multiplier consists of more than one term, multiply the multiplicand successively by each significant term of the multiplier, writing the first term of each partial product under the term of the multiplier which produces it.*

4. *Add the partial products thus obtained, and the sum will be the true product.*

Art. 37. 1. When the multiplier or multiplicand or both end with one or more ciphers, omit the ciphers in the partial products and annex them to the product obtained.

2. Any number may be multiplied by 10, 100, 1000, etc, *by annexing to it as many ciphers as there are ciphers in the multiplier.*

LESSON VII.

MENTAL EXERCISES.

Problems combining Addition, Subtraction, and Multiplication.

1. $6 \times 7 + 4 + 5 + 8 + 7 - 6 =$ how many?
2. $8 \times 4 + 6 - 3 + 2 - 5 + 6 =$ how many?
3. A grocer bought 10 barrels of apples, at $4 a barrel, and sold them so as to gain $15: for how much did he sell them?
4. John has 6 marbles, and Willis has 4 times as many less 9, and Charles has as many as both John and Willis: how many marbles has Charles?
5. A lady teacher receives $9 a week, and spends $6 for board and washing: how much can she save in 8 weeks?
6. Two men start from the same place and travel in opposite directions, one at 7 miles an hour and the other at 5 miles an hour: how far will they be apart in 8 hours?

7. Two stages start from the same place and go in the same direction, one at 9 miles an hour and the other at 6 miles an hour: how far will they be apart in 5 hours?

8. When oranges are sold at 7 cents apiece and lemons at 5 cents apiece, how many cents will buy 6 oranges and 8 lemons?

9. If a man earn $8 a week and a boy $3, how much will they both earn in 7 weeks?

10. A pedestrian left a city and walked 9 hours at the rate of 4 miles an hour; he then returned at the rate of 3 miles an hour, but in 4 hours stopped to rest: how far was he from the city?

11. If a man earn $12 a week and spend $7, how much will he save in 9 weeks?

WRITTEN EXERCISES.

1. From 4080×26 take 2024×16.

2. A grocer bought 275 barrels of flour for $2475, and sold it at $12 a barrel: what did he gain?

3. A clerk receives $125 a month, and his expenses are $68 a month: how much does he lay up each year?

4. An agent sold 48 sets of outline maps, at $16 a set; the maps cost him $10 a set: how much did he make?

5. If a steamer carry, on an average, 75 passengers each trip, how many passengers will it carry in 12 weeks, making 3 trips a week?

6. A book contains 288 pages, each page contains 42 lines, and each line 13 words: how many words in the book?

7. A man bought a farm for $4780; he sold 80 acres at $33 an acre, and the remaining portion for $2560. How much did he make by the transaction?

8. A regiment contains 960 men, exclusive of the commissioned officers; the men receive $16 a month, and the aggregate salary of the officers is $2800 a month. What is the monthly pay of the regiment?

9. A drover bought 480 head of cattle in Ohio, at $45 a head, shipped them to New York, at an expense of $6 a head, and then sold them at $56 a head. How much did he make?

10. A miller manufactured 560 barrels of flour, and sold it at $9 a barrel; the wheat cost $2750, and the expense of running the mill was $960: how much did he make?

11. A man sold 5 horses at $87 apiece, and received $350 in cash and a note for the balance: what was the value of the note?

12. The President's salary is $25000 a year: if his expenses are $1500 a month, how much can he save during his term of 4 years?

13. If a quantity of provisions will supply 960 soldiers 27 days, how many soldiers will it supply one day?

SECTION V.

DIVISION.

LESSON I.

Divisor Figures, 1, 2, 3.

1. How many times is 2 contained in 6? 2 in 12? 2 in 16? 2 in 18? 2 in 20?

2. How many times is 3 contained in 9? 3 in 12? 3 in 15? 3 in 18? 3 in 21? 3 in 27?

3. How many times is 2 contained in 12? 3 in 12? 2 in 18? 3 in 18? 2 in 24? 3 in 24?

4. Two boys sit at one desk: how many desks will seat 8 boys? 16 boys?

5. If a man walk 3 miles an hour, how long will it take him to walk 15 miles? 21 miles?

6. William having 20 plums, divided them among his companions, giving 2 to each: how many companions had he?

WRITTEN EXERCISES.

1. Divide 848 by 2.

PROCESS.

Divisor, 2)848, *Dividend*,
424, *Quotient*.

Write the divisor at the left of the dividend, and draw a curved line between them, and a straight line under the dividend. Begin at the left, and divide successively each term of the dividend by the divisor. The quotient is 424.

(2)	(3)	(4)	(5)	(6)
2)482	2)8642	3)6936	3)9369	3)3696

7. Divide 3609 by 3. 8084 by 2.

8. Divide 4684 by 2. 6309 by 3.

9. At $3 a bushel, how many bushels of wheat can be bought for $963? For $639?

10. In how many hours can a man walk 396 miles, if he walk at the rate of 3 miles an hour?

11. If a man earn $2 a day, how long will it take him to earn $360?

LESSON II.

MENTAL EXERCISES.

New Divisor Figures, 4 and 5.

1. How many times is 4 contained in 8? 4 in 12? 4 in 20? 4 in 28? 4 in 36?

2. How many 5's in 15? 30? 40? 25? 50? 35? 45? 20?

3. In an orchard there are 16 trees, standing in rows of 4 trees each: how many rows of trees in the orchard?

4. How many ranks of 4 soldiers each will 24 soldiers make? 32 soldiers? 40 soldiers?

5. A man planted 30 peach trees in rows, setting 5 trees in each row: how many rows did they make?

6. A school-room contains 35 desks, arranged with 5 desks in each row: how many rows of desks in the room?

7. How many chairs, at $4 apiece, can be bought for $36?

8. How many pairs of boots, at $5 a pair, can be bought for $35?

9. Mary is reading 5 chapters a day: how long will it take her to read 45 chapters?

10. A boy had 50 peach-stones, which he planted in rows of 5 each: how many rows did he plant?

WRITTEN EXERCISES.

1. Divide 784 by 4.

PROCESS.

Divisor, 4)784, *Dividend*,
196, *Quotient*.

Write the divisor at the left of the dividend. Begin at the left-hand term of the dividend, and divide, thus: 4 is contained in 7 hundreds 1 *hundred* times, with 3 hundreds remaining. Write the 1 hundred in hundreds' place in the quotient, and reduce the 3 hundreds remaining to 30 tens, which, with the 8 tens added, make 38 tens. Four is contained in 38 tens 9 *ten* times, with 2 tens remaining. Write the 9 tens in tens' place in the quotient, and reduce the 2 tens remaining to 20 units, which, with the 4 units added, make 24 units. Four is contained in 24 units 6 times. Write the 6 units in units' place in the quotient. The quotient is 196.

(2)	(3)	(4)	(5)	(6)
4)764	4)936	5)640	5)870	5)765

7. Divide 1128 by 2; by 3; by 4.

8. Divide 8740 by 2; by 4; by 5.

9. Divide 18480 by 2; by 3; by 4; by 5.

10. A manufacturer packed 372 clocks in boxes, placing 4 clocks in each box: how many boxes were required?

11. If 4 bushels of wheat will make a barrel of flour, how many barrels will 972 bushels make?

12. If a man earn $4 a day, how many days will it take him to earn $1584?

LESSON III.

MENTAL EXERCISES.

New Divisor Figures, 6 and 7.

1. Six is contained in 12 how many times? 6 in 24? 6 in 36? 6 in 48? 6 in 54?

2. How many times 7 in 7? 7 in 21? 7 in 35? 7 in 49? 7 in 63? 7 in 42? 7 in 56?

3. How many 6's in 42? 7's in 42? 6's in 30? 5's in 30? 7's in 28? 4's in 28? 7's in 35? 5's in 35? 7's in 56?

4. If 6 chairs make a set, how many sets will 36 chairs make? 48 chairs? 60 chairs?

5. There are 7 days in a week: how many weeks in 49 days? In 56 days? 63 days?

6. There are 6 feet in a fathom: how many fathoms in 54 feet? In 60 feet?

7. An orchard contains 56 trees in rows of 7 trees each: how many rows of trees in the orchard?

8. How many plows, at $6 each, can be bought for $48? For $54?

9. If a horse travel 6 miles an hour, how long will it take him to travel 48 miles?

WRITTEN EXERCISES.

1. Divide 1608 by 67.

PROCESS.

67)1608(24, *Quotient.*
134
268
268

Draw a curved line at the right of the dividend, to separate it from the quotient. Since 67 is not contained in the number denoted by the first two left-hand figures of the dividend, take 160 *tens* for the *first partial dividend.* Divide 160 tens by 67, and write the result (2 tens) at the right, for the tens' figure of the quotient. Multiply 67 by this quotient figure, and subtract the product (134 tens) from 160 tens. The remainder is 26 tens, to which annex the 8 units for a *second partial dividend.* Divide 268 by 67, and write the result (4 units) for the units' figure of the quotient. Multiply 67 by this quotient figure, and subtract the product (268 units) from 268 units. The quotient is 24.

NOTE.—It is sometimes difficult to tell how many times the divisor is contained in a partial dividend. When this is the case, take the number denoted by the first left-hand figure, or first two left-hand figures, of the divisor for a *trial divisor*, and the number denoted by the proper number of left-hand figures in the partial dividend as a *trial dividend.* If the divisor, multiplied by the value of the quotient figure thus found, gives a product *greater* than the partial dividend, the quotient figure is *too large*, and should be reduced. The trial divisor in the above example is 6, the first trial dividend is 16, and the second 26. The teacher should make this process plain to the pupil.

2. Divide 312 by 24. 374 by 17.
3. Divide 792 by 36. 1625 by 65.

4. Divide 2520 by 36. 3024 by 63.

5. Divide 64347 by 267. 49179 by 507.

6. There are 36 inches in a yard: how many yards are there in 792 inches?

7. A bushel of corn weighs 56 pounds: how many bushels of corn in 24416 pounds?

8. A hogshead of molasses contains 63 gallons: how many hogsheads in 4788 gallons?

9. If 72 books can be packed in a box, how many boxes will it take to hold 17496 books?

10. How many farms of 156 acres each, can be sold from a tract of land containing 7332 acres?

11. If a vessel sail, on an average, 47 miles a day, how long will it take it to sail 2303 miles?

12. There are 365 days in a common year: how many years are there in 90155 days?

LESSON IV.

MENTAL EXERCISES.

New Divisor Figure, 8.

1. How many times is 8 contained in 8? 8 in 24? 8 in 40? 8 in 56? 8 in 72?

2. How many 8's in 56? 7's in 56? 8's in 48? 6's in 48? 8's in 72? 9's in 72?

3. $32 \div 8 =$ how many? $49 \div 7$? $54 \div 6$? $64 \div 8$? $56 \div 7$? $56 \div 8$?

4. There are 8 quarts in a peck: how many pecks in 72 quarts?

5. If a steamer run 8 miles an hour, in how many hours will it run 80 miles?

6. There are 8 furlongs in a mile: how many miles in 56 furlongs?

7. At $8 a barrel, how many barrels of flour can be bought for $64?

8. If a man work 8 hours a day, in how many days will he work 96 hours?

WRITTEN EXERCISES.

The Quotient containing One or more Ciphers.

1. Divide 34137 by 84.

PROCESS.

84) 34137 (406
336
537
504
33, *Remainder.*

Since the divisor is not contained in the second partial dividend (53 tens), write 0 in the tens' place in the quotient, and annex the 7 units for a *third* partial dividend. As there is no figure of the dividend left to annex to 33 to form a new partial dividend, 33 remains undivided, and is called the remainder.

2. Divide 24399 by 48. 467034 by 806.

3. Divide 2845007 by 5728. 215607 by 18036.

4. Divide 1423685 by 6785. 1604083 by 2088.

5. In one week there are 168 hours: how many weeks in 36248 hours?

6. A drover went West with $23450 to buy cattle: how many cattle could he buy, at $58 a head?

7. If a garrison consume 648 pounds of bread in a day, how long will 19608 pounds last it?

8. If the average daily receipts of a ferry-boat are $275, in how many days will the receipts amount to $10000?

9. How long will it take a pipe, discharging 158 gallons of water in an hour, to empty a cistern containing 7584 gallons?

10. A cord of wood contains 128 solid feet: how many cords in a pile containing 5280 solid feet?

LESSON V.

MENTAL EXERCISES.

New Divisor Figure, 9.

1. How many times 9 in 18? 9 in 27? 9 in 36? 9 in 54? 9 in 72? 9 in 90?

2. How many 9's in 45? 5's in 45? 9's in 63? 7's in 63? 9's in 72? 8's in 72?

3. How long will it take a steamer to make a trip of 81 miles, if it run 9 miles an hour?

4. If 9 words fill a line, how many lines will 72 words fill? 81 words?

5. If a man can do a piece of work in 90 days, how many men can do it in 9 days?

6. If a quantity of provisions will last 72 men one day, how long will it last 9 men?

7. How many sheep, at $9 a head, can be bought for $54? For $63?

8. A copy-book contains 100 lines with 10 lines on each page: how many pages in the book?

9. If a man earn $10 a week, how long will it take him to earn $100?

10. How many tons of hay, at $10 a ton, can be bought for $90.

WRITTEN EXERCISES.

The Divisor ending in One or more Ciphers.

1. Divide 350 by 10.

FIRST PROCESS.

10)350(35
30
50
50

SECOND PROCESS.

1|0)35|0
35, *Quotient.*

By comparing these two processes, it is seen that 350 is divided by 10, by cutting off the right-hand figure. The reason is obvious. The cutting off of the right-hand figure removes each of the other figures one place to the right, and thus decreases their value *ten-fold.* In like manner, it may be shown that cutting off the two right-hand figures divides a number by 100; cutting off three right-hand figures, by 1000, etc.

2. Divide 2865 by 100.

1|00)28|65
28, *Quotient.*
65, *Remainder.*

3. Divide 45600 by 10. By 100.
4. Divide 187000 by 1000. By 100.
5. Divide 384050 by 100. By 1000.
6. Divide 230045 by 1000. By 10000.
7. Divide 450860 by 10000. By 1000.

8. Divide 196800 by 4800.

PROCESS.

48|00)1968|00(41, *Quotient.*
192
48
48

First divide both divisor and dividend by 100, which is done by cutting off the two right-hand figures. Then divide 1968, the new dividend, by 48, the new divisor. The quotient is 41.

NOTE.—The teacher can easily show that *both divisor and dividend may be divided by* 100 (or any other number) *without affecting the value of the quotient.*

9. Divide 63200 by 7900.
10. Divide 116087000 by 2900.
11. Divide 70338000 by 75000.
12. Divide 58864 by 4500.

PROCESS.

45|00)588|64(13
45
138
135
364, *Remainder.*

First divide both divisor and dividend by 100, which, in the case of the dividend, leaves a remainder of 64. Next divide 548 by 45, leaving a remainder of 3 *hundreds*, to which annex the first remainder (64), obtaining 364 for the true remainder.

NOTE.—The true remainder is found by *annexing* the first remainder to the second. The reason for this can be easily given by the teacher.

13. Divide 466384 by 3900. 220345 by 940.
14. Divide 99990 by 5400. 172800 by 14400.
15. A barrel of beef contains 200 pounds: how many barrels will contain 12800 pounds?
16. There are 480 sheets of paper in a ream: how many reams will 129600 sheets make?

17. There are 3600 seconds in an hour: how many hours in 172800 seconds?

18. How many city lots, at $1600 each, can be bought for $25600?

19. How many cars, each carrying 1800 pounds, will transport 723690 pounds of hay?

20. How many barrels, each holding 196 pounds, will hold 8450 pounds of flour?

21. How many regiments, averaging 750 men each, will make an army of 35000 men?

22. A peach orchard contains 6758 trees, and there are, on an average, 62 trees on each acre: how many acres in the orchard?

23. A pipe discharges 94 gallons in an hour: in how many hours will it empty a cistern holding 3384 gallons of water?

24. What number multiplied by 98 will produce 15288?

25. The dividend is 5292 and the divisor is 63: what is the quotient?

26. The divisor is $1500 and the dividend $564000: what is the quotient?

DEFINITIONS, PRINCIPLES, AND RULES.

Art. 38. ***Division*** is the process of finding how many times one number is contained in another.

The ***Dividend*** is the number divided.

The ***Divisor*** is the number by which the dividend is divided.

The ***Quotient*** is the number of times the divisor is contained in the dividend.

The ***Remainder*** is the part of the dividend which is left undivided. When the dividend contains the divisor an exact number of times, there is no remainder.

Art. 39. The *Sign of Division* is $\div$, and is read *divided by*. When placed between two numbers, it shows that the number before it is to be divided by the number after it. Thus: $16 \div 4 = 4$ is read 16 *divided by* 4 *equals* 4.

Division is also expressed by writing the dividend above and the divisor below a short horizontal line. Thus: $\frac{18}{3}$ is read 18 *divided by* 3.

Art. 40. One number is contained in another number as many times as it can be taken from it. Hence division may be regarded as a short method of subtraction.

A number is contained in another as many times as it must be taken to produce it. Hence division may be regarded as the reverse of multiplication. The divisor and quotient are factors of the dividend.

Art. 41. There are two methods of division, called *Short Division* and *Long Division*.

In *Short Division,* the partial products and partial dividends are not written, but are formed mentally. This method is generally used when the divisor does not exceed 12.

In *Long Division,* the partial products and partial dividends are written.

Art. 42. RULE FOR SHORT DIVISION.—1. *Write the divisor at the left of the dividend, and draw a curved line between them and a straight line under the dividend.*

2. *Find how many times the divisor is contained in the left-hand term or terms of the dividend, taken as a partial dividend, and write the quotient under the last figure of the dividend used.*

3. *Multiply the divisor by the quotient term found, and subtract the product from the partial dividend used, performing each process mentally.*

4. *Prefix the remainder, if there be one, to the next term of the dividend for a second partial dividend, and divide, multiply, and subtract as before.*

5. *Proceed in this manner until all the terms of the dividend have been used.*

PROOF.—*Multiply the divisor by the quotient, to the product add the remainder, if there be any, and if the result equals the dividend, the work is correct.*

Art. 43. RULE FOR LONG DIVISION.—1. *Write the divisor at the left of the dividend, and draw a curved line between them, and also at the right of the dividend, to separate it from the quotient.*

2. *Take as many of the left-hand terms of the dividend as will contain the divisor, for a partial dividend; find how many times this will contain the divisor, and write the quotient at the*

right of the dividend for the first left-hand term of the quotient.

3. *Multiply the divisor by the quotient term found, write the product under the partial dividend used, and subtract.*

4. *To the remainder annex the next term of the dividend for a second partial dividend, and divide, multiply, and subtract as before.*

5. *Proceed in this manner until all the terms of the dividend have been used.*

NOTE. — When any partial dividend does not contain the divisor, write a cipher in the quotient, and annex another term of the dividend to form a new partial dividend.

Art. 44. When one or more of the right-hand figures of the divisor are ciphers —

1. *Cut off the ciphers from the right of the divisor, and an equal number of figures from the right of the dividend.*

2. *Divide the new dividend thus formed by the new divisor, and the result will be the quotient.*

3. *Prefix the remainder, if there be one, to the figures cut off from the dividend, and the result will be the true remainder.*

Art. 45. To divide any number by 10, 100, 1000, etc., —

Cut off as many figures from the right as there are ciphers in the divisor. The figures cut off are the true remainder.

LESSON VI.

Miscellaneous Review Problems.

1. The sum of two numbers is 15 and one of the numbers is 6: what is the other?

2. The difference between two numbers is 8 and the smaller number is 9: what is the larger?

3. The product of two numbers is 56 and one of the numbers is 7: what is the other?

4. The quotient of two numbers is 6 and the divisor is 8: what is the dividend?

5. How many barrels of flour, at $8 a barrel, will pay for 24 yards of carpeting, at $2 a yard?

6. How many tons of coal, at $9 a ton, will pay for 15 cords of wood, at $6 a cord?

7. A grocer bought 7 barrels of flour, at $6 a barrel: for how much a barrel must he sell it to gain $14 on the lot?

8. If one man can build a wall in 36 days, how many men can build it in 4 days?

9. If 6 men can do a piece of work in 8 days, how many men can do it in 12 days?

10. Two steamers start from the same port and sail in the same direction, one sailing 12 miles an hour and the other 9 miles an hour: how far apart will they be in 10 hours?

WRITTEN EXERCISES.

1. The greater of two numbers is 4056 and their difference is 3650: what is the less number?

2. The subtrahend is 34203 and the remainder is 8706: what is the minuend?

3. The divisor is 534 and the quotient 43: what is the dividend?

4. The product of two numbers is 5328 and one of the numbers is 148: what is the other?

5. Multiply the sum of 486 and 392 by their difference.

6. Divide the product of 48 and 24 by their difference.

7. A merchant bought 35 yards of cloth at $56, and sold it at $2 a yard: how much did he gain?

8. A drover bought 240 sheep at $8 a head, and then sold 90 of them at $12 a head, 75 at $9 a head, and the rest at $6 a head. How much did he gain?

9. A farmer exchanges 65 bushels of wheat at $2 a bushel and 35 sheep at $6 a head for cows at $34 a head. How many cows did he receive?

10. A man's income is $3500 a year: he pays $450 a year for house-rent, $150 for taxes, $350 for hired help, and $45 a month for other expenses. How much has he left at the close of the year?

11. A man bought 80 acres of land at $35 an acre, paid $325 for improvements, and then sold it for $3750. How much did he gain?

12. A grain merchant having 3500 bushels of wheat, sold 1650 bushels, and then bought twice

as much as he had left. How many bushels did he buy?

13. A man left an estate to his wife and three children. The wife received $4500; the youngest child, $1500; the second child, $1850; and the eldest child as much as both of the others less $1350. What was the value of the estate?

14. A and B start together on a journey, A traveling 28 miles a day and B 33 miles: how far apart will they be in 12 days?

15. A and B start together and travel in opposite directions, A going 28 miles a day and B 33 miles: how far apart will they be in 12 days?

Questions for Review.

What is addition? What is meant by sum or amount? What does it contain? What is meant by like numbers? What numbers can be added? What is the sign of addition? What does it show? Give the rule for addition. What is the method of proof?

What is subtraction? The difference or remainder? The minuend? The subtrahend? What numbers can be subtracted? What does the sum of the remainder and subtrahend equal? What is the sign of subtraction? What does it show? Give the rule for subtraction. What is a method of proof?

What is multiplication? The multiplicand? The multiplier? The product? Of what are the multiplicand and multiplier factors? What is the sign of multiplication? What does it show? How may the product be obtained by addition?

Give the rule for multiplication. How may you mul-

tiply when either the multiplicand or multiplier, or both, end in ciphers? How may any number be multiplied by 10, 100, 1000, etc.?

What is division? The dividend? The divisor? The quotient? The remainder? What is the sign of division? What does it show? In what other way may division be expressed? How many times may the divisor be subtracted from the dividend? Of what is division the reverse?

What is short division? When is it used? Give the rule. What is long division? Give the rule. How do you proceed when a partial dividend will not contain the divisor? How may you divide when the divisor ends in ciphers? How may any number be divided by 10, 100, 1000, etc.?

SECTION VI.

PROPERTIES OF NUMBERS.

LESSON I.

Divisor, Greatest Common Divisor, and Factor.

NOTE.—The term number, used in this section, denotes an *integer*.

1. What numbers besides itself and 1 will exactly divide 15? 21? 25? 30? 56? 63?

2. What numbers besides itself and 1 will exactly divide 7? 11? 13? 17? 23? 37? 41?

3. What numbers will exactly divide 4? 5? 16? 19? 24? 29? 33? 31? 42?

4. What are the divisors of 10? 28? 31? 33? 43? 49? 53? 55? 70? 90? 99?

NOTE.—Since every number is exactly divisible by itself and 1, these divisors need not be given.

5. What number is a divisor of both 9 and 12? 15 and 20? 24 and 27? 42 and 56?

6. What divisor is common to 28 and 35? 27 and 36? 42 and 54? 63 and 81?

7. What is a common divisor of 15 and 30? 45 and 60? 50 and 75? 60 and 84?

8. What is the greatest divisor common to 48 and 72? 27 and 54? 50 and 75?

9. What is the greatest common divisor of 24 and 36? 32 and 48? 56 and 84?

10. What is a common divisor of 16, 32, and 48? 15, 30, and 45? 36, 54, and 72?

11. What is the greatest common divisor of 32 and 48? 15, 30, and 45? 36, 54, and 72? 18, 45, and 81?

Art. 46. A number that has no divisor except itself and 1, is called a *Prime Number.* A number that has other divisors besides itself and 1, is called a *Composite Number.*

12. Name all the prime numbers between 0 and 20. Between 20 and 30. 40 and 50.

13. Name all the composite numbers between 20 and 30. 40 and 50. 60 and 70.

14. What are the prime divisors of 15? 18? 22? 28? 33? 36? 37? 40? 43?

15. What are the prime divisors of 16? 20? 25? 27? 35? 44? 55? 60?

Art. 47. The divisors of a number are called its *Factors;* and prime divisors are called *Prime Factors.*

16. What are the prime factors of 21? 24? 35? 39? 42? 49? 54? 56? 63? 66? 72?

17. Of what number are 2, 3, and 5 prime factors? 2, 5, and 7? 2, 2, 3, and 5?

NOTE.—The product of the prime factors of a number equals the number.

18. Of what number are 2, 2, 3, and 3 prime factors? 2, 3, 5, and 7? 3, 5, 2, and 7?

WRITTEN EXERCISES.

1. What are the prime factors of 126?

PROCESS.

$$\begin{array}{r} 2)\,126 \\ 3)\,63 \\ 3)\,21 \\ \hline 7 \end{array}$$

$126 = 2 \times 3 \times 3 \times 7.$

Divide 126 by 2, a prime divisor; next divide the quotient 63 by 3, a prime divisor; and then divide the quotient 21 by 3, a prime divisor. The prime factors are 2, 3, 3, and 7.

What are the prime factors of

2. 63?	6. 175?	10. 264?	14. 440?
3. 72?	7. 147?	11. 200?	15. 500?
4. 84?	8. 275?	12. 256?	16. 648?
5. 96?	9. 325?	13. 250?	17. 900?

18. What is the greatest common divisor of 126 and 210?

PROCESS.

$126 = 2 \times 3 \times 3 \times 7$
$210 = 2 \times 3 \times 5 \times 7$

$2 \times 3 \times 7 = 42$, *Ans.*

Resolve 126 and 210 into their prime factors. The product of the factors common to both will be the greatest common divisor required.

NOTE.—This process and that for finding the least common multiple (Art. 54) may be easily explained by means of objects.

What is the greatest common divisor of

19. 54 and 90?
20. 72 and 96?
21. 75 and 90?
22. 84 and 108?
23. 81 and 135?
24. 63, 84, and 126?
25. 96, 144, and 192?
26. 128, 224, and 320?

Art. 48. A ***Divisor*** of a number is a number that will exactly divide it.

A ***Common Divisor*** of two or more numbers is a number that will exactly divide each of them.

The ***Greatest Common Divisor*** of two or more numbers is the greatest number that will exactly divide each of them.

Art. 49. A ***Prime Number*** is one that has no divisor except itself and one.

A ***Composite Number*** is one that has other divisors besides itself and one.

Art. 50. An ***Even Number*** is exactly divisible by 2; as, 2, 4, 6, 8, 10, 12, etc.

An ***Odd Number*** is not exactly divisible by 2; as, 1, 3, 5, 7, 9, 11, 13, etc.

All even numbers except 2 are composite.

Art. 51. RULES.—1. To resolve a composite number into its prime factors, *Divide it by any prime divisor, and the quotient by any prime divisor, and so continue until a quotient is obtained which is a prime number. The several divisors and the last quotient are the prime factors.*

2. To find the greatest common divisor of two or more numbers, *Resolve the given numbers into their prime factors, and select the factors which are common. The product of the common factors will be the greatest common divisor.*

LESSON II.

Multiple and Least Common Multiple.

Art. 52. When a number is multiplied by an integer, the product is called a *Multiple.* Thus, 36, or 12×3, is a multiple of 12.

1. What number is a multiple of 3? 4? 5? 7? 8? 10? 15? 20? 25? 30?

2. What number is a multiple of 18? 24? 35? 45? 44? 60? 100? 250?

NOTE.—The teacher should show that a number has any number of multiples.

3. What number is a common multiple of 3 and 4? 4 and 5? 6 and 8? 5 and 6?

4. What number is a common multiple of 7 and 5? 6 and 9? 3, 4, and 6? 4, 8, and 12?

NOTE.—The teacher should show that two or more numbers have any number of common multiples.

5. What is the least common multiple of 3 and 4? 5 and 6? 3, 6, and 12? 2, 4, and 8?

6. What is the least common multiple of 3, 5, and 10? 2, 5, and 10? 2, 3, 5, and 10?

WRITTEN EXERCISES.

1. What is the least common multiple of 12, 18, and 30?

PROCESS.

$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$
$30 = 2 \times 3 \times 5$

$2 \times 2 \times 3 \times 3 \times 5 = 180$, *L. C. M.*

Resolve the numbers into their prime factors, and select all the different factors, repeating each as many times as it is found in any number. The factor 2 is found twice in 12; the factor 3, twice in 18; and the factor 5, once in 30. The product of $2 \times 2 \times 3 \times 3 \times 5$ is the least common multiple required.

What is the least common multiple of

2. 12, 15, and 20?
3. 21, 24, and 42?
4. 32, 48, and 80?
5. 27, 54, and 108?
6. 24, 80, and 120?
7. 24, 72, 18, 48?
8. 15, 24, 18, 32?
9. 75, 150, 300?
10. 125, 250, 500?
11. \$48, \$72, \$144?

Art. 53. A ***Multiple*** of a number is any number which it will exactly divide.

NOTE.—Every number is an exact divisor of its product by an integer.

A ***Common Multiple*** of two or more numbers is any number which each of them will exactly divide.

The ***Least Common Multiple*** of two or more numbers is the least number which each of them will exactly divide.

Art. 54. RULE.—To find the least common multiple of two or more numbers, *Resolve the numbers into their prime factors, and then select all the different factors, taking each the highest number of times it is found in any number. Multiply the factors thus selected; their product will be the least common multiple.*

QUESTIONS FOR REVIEW.

What is meant by the divisor of a number? When is a divisor a *common* divisor? Define a common divisor. What is the greatest common divisor of two or more numbers? How is it found?

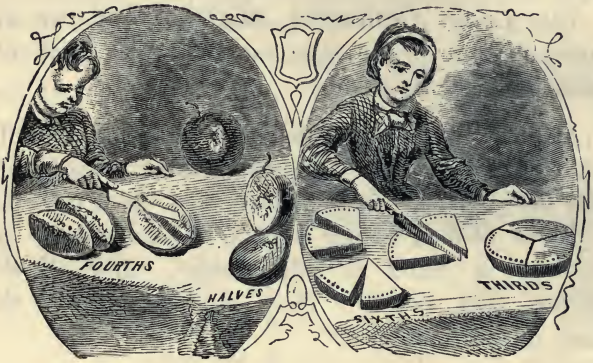
By what may every number be divided? What is a prime number? A composite number? What is an even number? An odd number?

What is meant by the factor of a number? A prime factor? A composite factor? How may a composite number be resolved into prime factors?

What is a multiple of a number? When is a multiple a *common* multiple? Define a common multiple. What is the least common multiple of two or more numbers? Give the rule for finding the least common multiple. What is the difference between a *divisor* and a *multiple?*

SECTION VII.

FRACTIONS.



LESSON I.

The Idea of a Fraction developed.

1. If a melon be cut into two *equal* pieces, what part of the melon will one piece be?

2. How many halves in a melon? How many halves in any thing?

3. If a melon be cut into four equal pieces, what part of the melon will one piece be? Two pieces? Three pieces?

4. How many fourths in an apple? How many fourths in any thing?

5. Which is the greater, one half or one fourth of an apple? How many fourths equal one half?

6. If a cake be cut into three equal pieces, what part of the cake will one piece be?

7. How many thirds in a cake? How many thirds in any thing?

8. If a cake be cut into six equal pieces, what part of the cake will one piece be? Two pieces? Three pieces? Four pieces? Five pieces?

9. How many sixths in any thing?

10. Which is the greater, one third or one sixth of a cake? How many sixths equal one third?

11. A single thing is a *unit.* How many halves in a unit? How many thirds? How many fourths? How many sixths?

12. What is meant by one third?

Ans. One third is one of the three equal parts of a unit.

13. What is meant by two thirds? One fourth? Three fourths? One sixth? Three sixths?

14. Which is the greater, two thirds or a unit? Five thirds or a unit? Three fourths or a unit? Four fourths or a unit?

Art. 55. Such parts of a unit as two thirds, three fourths, five sixths, etc., are called *Fractions.*

A fraction may be expressed by two numbers, one written under the other, with a horizontal line between them; as, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

The number below the line denotes the number of equal parts into which the unit is divided. It is called the *Denominator.*

The number above the line denotes the number of equal parts taken. It is called the *Numerator*.

Read the following fractions, and tell, in each case, into how many equal parts the unit is divided, and how many parts are taken:

(15)	(16)	(17)	(18)	(19)	(20)
$\frac{3}{4}$	$\frac{6}{7}$	$\frac{5}{12}$	$\frac{5}{17}$	$\frac{6}{25}$	$\frac{20}{33}$
$\frac{5}{6}$	$\frac{9}{13}$	$\frac{8}{15}$	$\frac{13}{18}$	$\frac{13}{30}$	$\frac{25}{40}$
$\frac{7}{8}$	$\frac{5}{9}$	$\frac{5}{11}$	$\frac{10}{19}$	$\frac{15}{20}$	$\frac{30}{50}$

Write the following fractions in figures:

(21)	(22)	(23)
Two fifths;	Seven twelfths;	Twenty-four fortieths;
Seven ninths;	Ten thirteenths;	Thirty-five fiftieths;
Nine tenths;	Forty fiftieths;	Twenty-two twelfths;
Ten ninths.	Twenty seventeenths.	Forty fifty-fifths.

DEFINITIONS.

Art. 56. A ***Fraction*** is one or more of the equal parts of a unit.

Art. 57. A fraction is expressed by two numbers, called the *Numerator* and the *Denominator*.

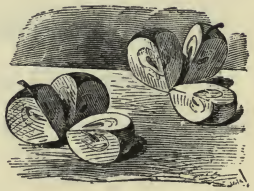
The ***Denominator*** of a fraction denotes the number of equal parts into which the unit is divided.

The ***Numerator*** of a fraction denotes the number of equal parts taken.

The numerator and denominator are called the *Terms* of a fraction.

LESSON II.

Integers and Mixed Numbers reduced to Fractions.



1. How many thirds in an apple? How many thirds in 2 apples?

2. How many fourths in 3 pears?

SOLUTION.—In 1 pear there are 4 fourths, and in 2 pears there are twice 4 fourths, which are 8 fourths. There are 8 fourths in 2 pears?

3. How many sixths in 3 oranges? In five oranges? 6 oranges? 8 oranges?

4. How many fifths in 3? 5? 8? 10?

5. How many eighths in 4? 6? 8? 10?

6. How many halves in 2 and 1 half oranges?

SOLUTION.—In 2 oranges there are twice 2 halves, which are 4 halves, and 4 halves and 1 half are 5 halves. There are 5 halves in 2 and 1 half oranges.

7. How many fourths in 2 and 3 fourths?

8. How many thirds in 5 and 2 thirds? 8 and 1 third? 7 and 2 thirds?

9. How many sixths in 5 and 2 sixths? 8 and 3 sixths? 12 and 5 sixths?

10. How many tenths in 6 and 3 tenths? 7 and 5 tenths? 8 and 7 tenths?

11. Read $6\frac{2}{3}$; $33\frac{1}{3}$; $45\frac{7}{8}$; $25\frac{1}{12}$; $50\frac{7}{15}$; $66\frac{5}{20}$.

12. How many fifths in $6\frac{2}{5}$? $8\frac{3}{5}$? $12\frac{4}{5}$?

WRITTEN EXERCISES.

13. Reduce 157 to ninths. $157\frac{7}{9}$ to ninths.

PROCESS.

157
9
1413
$\frac{1413}{9}$, *Ans.*

PROCESS.

$157\frac{7}{9}$
9
1413
7
1420
$\frac{1420}{9}$, *Ans.*

14. Reduce 96 to eighths. $96\frac{5}{8}$ to eighths.

15. Reduce $35\frac{1}{12}$ to twelfths. $46\frac{8}{9}$ to ninths.

16. Reduce $73\frac{6}{11}$ to elevenths. $63\frac{6}{7}$ to sevenths.

17. Reduce $53\frac{17}{20}$ to a fraction.

SUGGESTION. — Reduce the mixed number to twentieths.

18. Reduce $33\frac{4}{15}$ to a fraction.

Reduce to a fraction:

19. $85\frac{9}{14}$	22. $236\frac{4}{7}$	25. $48\frac{17}{30}$
20. $36\frac{11}{15}$	23. $49\frac{7}{16}$	26. $69\frac{27}{100}$
21. $48\frac{5}{13}$	24. $75\frac{9}{20}$	27. $93\frac{9}{200}$

Art. 58. A ***Mixed Number*** is an integer and a fraction united; as, $5\frac{1}{2}$, $16\frac{2}{3}$, $33\frac{4}{7}$.

Art. 59. RULES.—1. To reduce an integer to a fraction, *Multiply the integer by the given denominator, and write the denominator under the product.*

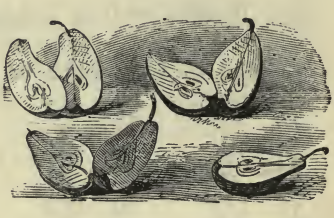
2. To reduce a mixed number to a fraction, *Multiply the integer by the denominator of the fraction, to the product add the numerator, and write the denominator under the result.*

LESSON III.

Fractions reduced to Integers or Mixed Numbers.

1. How many pears in six half-pears? In 7 half-pears?

2. How many days in 10 half-days? In 11 half-days?



SOLUTION.—In 11 half-days there are as many days as two half-days are contained times in 11 half-days, which is $5\frac{1}{2}$ times. There are $5\frac{1}{2}$ days in 11 half-days.

3. How many pints in 14 half-pints? In 17 half-pints? In 21 half-pints?

4. How many yards in 18 thirds of a yard? In 19 thirds of a yard? In 22 thirds of a yard?

5. How many weeks in 28 sevenths of a week? 30 sevenths of a week?

6. A mason was 17 half-days in building a wall: how many days did he work?

7. A boy earned 25 fourths of a dollar by selling papers: how many dollars did he earn?

8. A man walked 25 eighths of a mile in an hour: how many miles did he walk?

9. How many ones in $\frac{45}{9}$? $\frac{56}{9}$? $\frac{75}{9}$? $\frac{84}{9}$?

10. How many ones in $\frac{60}{10}$? $\frac{65}{10}$? $\frac{77}{10}$? $\frac{93}{10}$?

WRITTEN EXERCISES.

11. Reduce $\frac{177}{15}$ to a mixed number.

PROCESS: $\frac{177}{15} = 177 \div 15 = 11\frac{2}{15}$, *Ans.*

12. Reduce $\frac{207}{12}$ to a mixed number.

Reduce to an integer or mixed number:

13. $\frac{94}{15}$
14. $\frac{105}{12}$
15. $\frac{307}{20}$
16. $\frac{360}{60}$
17. $\frac{75}{24}$
18. $\frac{312}{30}$
19. $\frac{160}{80}$
20. $\frac{220}{20}$
21. $\frac{322}{16}$
22. $\frac{421}{21}$
23. $\frac{350}{35}$
24. $\frac{504}{41}$
25. $\frac{744}{35}$
26. $\frac{315}{35}$
27. $\frac{630}{22}$
28. $\frac{796}{45}$

Art. 60. A ***Proper Fraction*** is one whose numerator is less than its denominator; as, $\frac{3}{4}$, $\frac{5}{9}$, $\frac{1}{8}$.

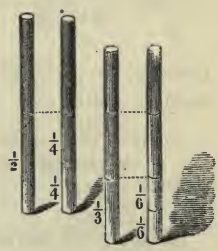
An ***Improper Fraction*** is one whose numerator is equal to or greater than the denominator; as, $\frac{5}{5}$, $\frac{6}{5}$, $\frac{9}{7}$.

The value of a proper fraction is less than one; and the value of an improper fraction is equal to or greater than one

Art. 61. RULE.—To reduce an improper fraction to an integer or mixed number, *Divide the numerator of the fraction by the denominator.*

LESSON IV.

Fractions reduced to Lower Terms.



1. How many half-inches in 2 fourths of an inch? In 4 fourths of an inch?

2. How many thirds of an inch in 2 sixths? In 4 sixths? In 6 sixths?

3. How many fourths in 6 eighths?

SOLUTION.—In 2 eighths there is 1 fourth, and in 6 eighths there are as many fourths as 2 eighths are contained times in 6 eighths, which is 3. There are 3 fourths in 6 eighths.

4. How many fifths in 2 tenths? 4 tenths? 6 tenths? 8 tenths? 12 tenths?

5. How many thirds in 6 fifteenths? 9 fifteenths? 12 fifteenths? 18 fifteenths?

6. How many sevenths in $\frac{6}{21}$? $\frac{12}{21}$? $\frac{15}{21}$?

7. How many eighths in $\frac{8}{32}$? $\frac{16}{32}$? $\frac{24}{32}$?

8. Reduce $\frac{9}{12}$, $\frac{8}{16}$, and $\frac{10}{20}$ each to fourths.

9. Reduce $\frac{14}{21}$, $\frac{25}{35}$, and $\frac{16}{28}$ each to sevenths.

10. Reduce $\frac{12}{24}$, $\frac{20}{32}$, and $\frac{18}{48}$ each to eighths.

11. Reduce $\frac{15}{30}$, $\frac{30}{36}$, and $\frac{24}{48}$ each to sixths.

12. Reduce $\frac{24}{40}$, $\frac{15}{50}$, and $\frac{36}{60}$ each to tenths.

WRITTEN EXERCISES.

13. Reduce $\frac{63}{84}$ to its lowest terms.

PROCESS.

$$\frac{63 \div 3}{84 \div 3} = \frac{21 \div 7}{28 \div 7} = \frac{3}{4}, \text{ Ans.}$$

Or: $$\frac{63 \div 21}{84 \div 21} = \frac{3}{4}.$$

Reduce $\frac{63}{84}$ to $\frac{21}{28}$ by dividing both terms by 3; next reduce $\frac{21}{28}$ to $\frac{3}{4}$ by dividing both terms by 7; $\frac{3}{4}$ can not be reduced to lower terms, and, hence, is in its *lowest terms*. Or, reduce $\frac{63}{84}$ to $\frac{3}{4}$ by dividing both terms by 21, the *greatest* number which will exactly divide each term.

NOTE.—The teacher should show that the value of a fraction is not changed by dividing both of its terms by the same number.

Reduce to lowest terms—

14. $\frac{64}{72}$	18. $\frac{72}{108}$	22. $\frac{105}{175}$	26. $\frac{2331}{3633}$
15. $\frac{72}{144}$	19. $\frac{32}{256}$	23. $\frac{13}{143}$	27. $\frac{144}{288}$
16. $\frac{96}{144}$	20. $\frac{96}{100}$	24. $\frac{225}{315}$	28. $\frac{1321}{2412}$
17. $\frac{56}{160}$	21. $\frac{84}{196}$	25. $\frac{182}{180}$	29. $\frac{3600}{4800}$

Art. 62. When a fraction is reduced to an equivalent fraction with smaller terms, it is reduced to *lower terms*.

A fraction is in its *Lowest Terms* when no integer except 1 will exactly divide both numerator and denominator.

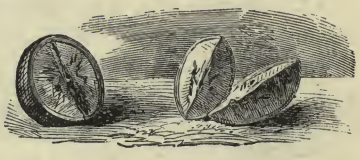
Art. 63. PRINCIPLE.—*The division of both terms of a fraction by the same number does not change its value.*

Art. 64. RULE.—To reduce a fraction to its lowest terms, *Divide both terms of the fraction by any common divisor; then divide both terms of the resulting fraction by any common divisor; and so on, until the terms of the resulting fraction have no common divisor except* 1. Or:

Divide both terms of the fraction by their greatest common divisor.

LESSON V.

Fractions reduced to Higher Terms.



1. How many fourths of an orange in 1 half? In 2 halves?

2. How many eighths of an orange in 1 fourth? In 3 fourths?

SOLUTION.—In one fourth there are 2 eighths, and in 3 fourths there are 3 times 2 eighths, which are 6 eighths. There are 6 eighths in 3 fourths.

3. How many ninths in 1 third? In 2 thirds? 3 thirds? 4 thirds?

4. How many tenths in $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$?

5. How many twelfths in $\frac{2}{6}$? $\frac{4}{6}$? $\frac{5}{6}$?

6. Change $\frac{3}{4}$ and $\frac{5}{6}$ each to twelfths.

7. Change $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{5}{9}$ each to eighteenths.

8. Change $\frac{5}{6}$, $\frac{7}{8}$, and $1\frac{1}{2}$ each to twenty-fourths.

9. Change $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{7}{15}$ each to thirtieths.

10. Change $\frac{3}{7}$, $1\frac{1}{14}$, and $\frac{1}{2}$ each to twenty-eighths.

WRITTEN EXERCISES.

11. Change $\frac{17}{35}$ to seventieths.

PROCESS.

$70 \div 35 = 2.$

$\frac{17 \times 2}{35 \times 2} = \frac{34}{70}$, *Ans.*

One thirty-fifth is as many seventieths as 35 is contained times in 70, which is 2 times, and 17 thirty-fifths are 17 times 2 seventieths, which are 34 seventieths. This is the same as multiplying both terms by the quotient of 70 divided by 35.

12. Change $\frac{19}{24}$ to ninety-sixths.

13. Change $\frac{11}{21}$ and $\frac{23}{42}$ each to eighty-fourths.

14. Change $\frac{7}{12}$, $\frac{13}{24}$, and $\frac{15}{36}$ each to seventy-seconds.

15. Reduce $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ to equivalent fractions having a common denominator.

PROCESS.

$\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$.

$\frac{20}{24}$, $\frac{21}{24}$, $\frac{22}{24}$.

Change the fraction to *twenty-fourths*, thus: $\frac{5}{6} = \frac{20}{24}$; $\frac{7}{8} = \frac{21}{24}$; $\frac{11}{12} = \frac{22}{24}$.

Reduce to a common denominator:

16. $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{12}$

17. $\frac{2}{3}$ $\frac{5}{9}$ $\frac{5}{6}$

18. $\frac{3}{5}$ $\frac{7}{10}$ $\frac{8}{15}$

19. $\frac{3}{4}$ $\frac{7}{8}$ $\frac{9}{16}$

20. $\frac{5}{2}$ $\frac{3}{7}$ $\frac{7}{14}$

21. $\frac{1}{6}$ $\frac{5}{12}$ $\frac{7}{24}$

22. $\frac{2}{5}$ $\frac{3}{10}$ $\frac{4}{15}$ $\frac{11}{30}$

23. $\frac{4}{9}$ $\frac{11}{12}$ $\frac{17}{18}$ $\frac{23}{36}$

24. $\frac{9}{10}$ $\frac{13}{20}$ $\frac{14}{25}$ $\frac{33}{50}$

Art. 65. When a fraction is changed to an equivalent fraction with greater terms, it is reduced to *Higher Terms.*

Several fractions having the same denominator, are said to have a *Common Denominator.*

Art. 66. PRINCIPLE.—*The multiplication of both terms of a fraction by the same number does not change its value.*

Art. 67. RULES.—1. To reduce a fraction to higher terms, *Divide the given denominator by the denominator of the fraction, and multiply both terms by the quotient.*

2. To reduce fractions to a common denominator, *Divide the least common multiple of the denominators by the denominator of each fraction, and multiply both terms by the quotient.*

LESSON VI.

Addition of Fractions.

1. A boy gave 1 fourth of a pine-apple to his brother, 1 fourth to his sister, and 1 fourth to a playmate: what part of it did he give away?

How many fourths are $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$?

2. A grocer sold 1 eighth of a cheese to one customer, 2 eighths to another, and 3 eighths to another: what part of it did he sell?

How much is $\frac{1}{8} + \frac{2}{8} + \frac{3}{8}$?

3. How many sixths in $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{5}{6}$? $\frac{3}{6}$, $\frac{4}{6}$, and $\frac{1}{6}$? $\frac{2}{6}$, $\frac{3}{6}$, and $\frac{5}{6}$?

4. A boy gave 1 half of his money for a knife, and 1 third of it for a ball: what part of his money did he spend?

SUGGESTION.—Change $\frac{1}{2}$ and $\frac{1}{3}$ to sixths.

5. How many tenths in $\frac{1}{2}$ and $\frac{3}{5}$? $\frac{4}{5}$ and $\frac{3}{10}$?

6. How many twelfths in $\frac{1}{3}$ and $\frac{1}{4}$? $\frac{3}{4}$ and $\frac{5}{6}$?

7. How many eighths in $\frac{1}{4}$ and $\frac{1}{8}$? $\frac{3}{4}$ and $\frac{3}{8}$?

8. How many fifteenths in $\frac{1}{3}$ and $\frac{1}{5}$? $\frac{2}{3}$ and $\frac{2}{5}$? $\frac{1}{3}$ and $\frac{3}{5}$? $\frac{2}{3}$ and $\frac{3}{5}$?

9. How many twentieths in $\frac{1}{4}$ and $\frac{3}{5}$? $\frac{2}{5}$ and $\frac{3}{4}$? $\frac{3}{5}$ and $\frac{3}{4}$? $\frac{4}{5}$ and $\frac{3}{4}$?

10. How many twenty-fourths in $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{8}$? In $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{8}$?

WRITTEN EXERCISES.

11. What is the sum of $\frac{5}{13}$, $\frac{6}{13}$, $\frac{7}{13}$, $\frac{9}{13}$?

PROCESS: $\frac{5}{13} + \frac{6}{13} + \frac{7}{13} + \frac{9}{13} = \frac{27}{13} = 2\frac{1}{13}$, *Ans.*

12. What is the sum of $\frac{13}{24}$, $\frac{23}{24}$, $\frac{5}{24}$, and $\frac{19}{24}$?

13. What is the sum of $\frac{17}{30}$, $\frac{13}{30}$, $\frac{23}{30}$, and $\frac{29}{30}$?

14. What is the sum of $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{5}{12}$?

PROCESS.

$\frac{5}{6} + \frac{7}{8} + \frac{5}{12} =$

$\frac{20}{24} + \frac{21}{24} + \frac{10}{24} = \frac{51}{24}$.

$\frac{51}{24} = 2\frac{3}{24} = 2\frac{1}{8}$, *Ans.*

Change the fractions to twenty-fourths; add the numerators of the new fractions; and reduce the resulting improper fraction to a mixed number.

15. What is the sum of $\frac{3}{5}$, $\frac{5}{6}$, and $\frac{7}{15}$?

16. Add $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$.

17. Add $\frac{1}{6}$, $\frac{7}{12}$, and $11\frac{1}{24}$.

18. Add $\frac{2}{5}$, $\frac{3}{10}$, and $13\frac{3}{15}$.

19. Add $\frac{1}{4}$, $\frac{3}{5}$, and $12\frac{7}{10}$.

20. Add $\frac{2}{5}$, $\frac{3}{10}$, $\frac{3}{4}$, and $12\frac{3}{10}$.

21. Add $\frac{5}{6}$, $\frac{5}{8}$, $\frac{5}{12}$, and $12\frac{7}{24}$.

22. Add $\frac{3}{7}$, $\frac{9}{14}$, $\frac{20}{21}$, and $34\frac{1}{2}$.

23. Add $\frac{9}{10}$, $\frac{11}{15}$, $\frac{3}{20}$, and $23\frac{3}{30}$.

24. What is the sum of $16\frac{2}{3}$, $18\frac{3}{4}$, and $37\frac{1}{2}$?

PROCESS.

$16\frac{2}{3}$ $\frac{8}{12}$
$18\frac{3}{4}$ $\frac{9}{12}$
$37\frac{1}{2}$ $\frac{6}{12}$
$72\frac{11}{12}$, *Ans.*

First add the fractions and then the integers. $\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$, $\frac{1}{2} = \frac{6}{12}$. $\frac{8}{12} + \frac{9}{12} + \frac{6}{12} = \frac{23}{12} = 1\frac{11}{12}$. Write the $\frac{11}{12}$ under the fractions and add the 1 with the integers.

25. Add $45\frac{1}{2}$, $67\frac{3}{4}$, and $62\frac{5}{8}$.
26. Add $37\frac{2}{3}$, $18\frac{3}{4}$, $33\frac{1}{3}$, and $25\frac{7}{12}$.
27. Add $30\frac{1}{4}$, $66\frac{2}{3}$, $84\frac{3}{4}$, and $133\frac{1}{3}$.
28. Add $75\frac{1}{2}$, 108, $160\frac{3}{5}$, and 207.

Art. 68. RULES.—To add fractions, *Reduce the fractions to a common denominator, add the numerators of the new fractions, and under the sum write the common denominator.*

2. To add mixed numbers, *Add the fractions and the integers separately, and combine the results.*

LESSON VII.

Subtraction of Fractions.

1. Albert had 2 thirds of an orange and he gave 1 third to his sister: how many thirds had he left?

How much is $\frac{2}{3}$ less $\frac{1}{3}$? $\frac{3}{3}$ less $\frac{2}{3}$?

2. Charles bought 3 fourths of a pound of raisins, and then gave 1 fourth of a pound to his playmate: what part of a pound had he left?

How much is $\frac{3}{4}$ less $\frac{1}{4}$? $\frac{3}{4}$ less $\frac{2}{4}$?

3. A farmer bought $\frac{2}{3}$ of a bushel of flax-seed,

and sold $\frac{1}{2}$ of a bushel to a neighbor: what part of a bushel had he left?

SUGGESTION. — Change $\frac{2}{3}$ and $\frac{1}{2}$ to sixths.

4. How much is $\frac{3}{4}$ less $\frac{1}{2}$? $\frac{3}{4}$ less $\frac{1}{3}$?
5. How much is $\frac{7}{8}$ less $\frac{3}{4}$? $\frac{7}{8}$ less $\frac{2}{3}$?
6. How much is $\frac{7}{12}$ less $\frac{1}{3}$? $\frac{7}{12}$ less $\frac{1}{2}$?
7. How much is $\frac{9}{10}$ less $\frac{3}{4}$? $\frac{9}{10}$ less $\frac{4}{5}$?
8. How much is $\frac{15}{16}$ less $\frac{5}{8}$? $\frac{15}{16}$ less $\frac{3}{4}$?
9. How much is $\frac{7}{9}$ less $\frac{1}{6}$? $\frac{8}{15}$ less $\frac{3}{10}$?

WRITTEN EXERCISES.

10. From $\frac{11}{24}$ take $\frac{7}{24}$.

PROCESS: $\frac{11}{24} - \frac{7}{24} = \frac{4}{24} = \frac{1}{6}$, *Ans.*

11. From $\frac{29}{54}$ take $\frac{11}{54}$.
12. From $\frac{67}{120}$ take $\frac{43}{120}$.
13. From $\frac{11}{12}$ take $\frac{5}{8}$.

PROCESS: $\frac{11}{12} - \frac{5}{8} = \frac{22}{24} - \frac{15}{24} = \frac{7}{24}$, *Ans.*

14. Take $\frac{3}{4}$ from $\frac{7}{12}$; $\frac{3}{5}$ from $\frac{5}{9}$.
15. Take $\frac{7}{15}$ from $\frac{9}{10}$; $\frac{7}{10}$ from $\frac{13}{15}$.
16. Subtract $\frac{12}{25}$ from $\frac{33}{50}$; $\frac{13}{50}$ from $\frac{23}{25}$.
17. Subtract $\frac{5}{18}$ from $\frac{7}{12}$; $\frac{7}{12}$ from $\frac{7}{8}$.
18. Subtract $\frac{5}{14}$ from $\frac{8}{21}$; $\frac{11}{21}$ from $\frac{13}{14}$.
19. Subtract $\frac{5}{12}$ from $\frac{7}{15}$; $\frac{11}{15}$ from $\frac{11}{12}$.
20. From $33\frac{1}{3}$ take $18\frac{3}{4}$.

PROCESS.

$33\frac{1}{3}$ $\quad \frac{4}{12}$

$18\frac{3}{4}$ $\quad \frac{9}{12}$

$14\frac{7}{12}$, *Ans.*

First subtract the fractions and then the integers. Since $\frac{9}{12}$ is greater than $\frac{4}{12}$, add $\frac{12}{12}$ to $\frac{4}{12}$, making $\frac{16}{12}$, and then take the $\frac{9}{12}$ from $\frac{16}{12}$, writing $\frac{7}{12}$ under the fractions, and adding 1 to the 8 units before subtracting the integers.

21. Take $30\frac{1}{4}$ from $66\frac{2}{3}$; $45\frac{3}{5}$ from $66\frac{2}{3}$.
22. Take $112\frac{3}{4}$ from $145\frac{1}{2}$; $90\frac{1}{3}$ from $108\frac{1}{2}$.
23. Subtract $250\frac{3}{5}$ from 300; $105\frac{3}{4}$ from $261\frac{1}{3}$.
24. Subtract $130\frac{5}{8}$ from $241\frac{2}{3}$; $166\frac{2}{3}$ from $233\frac{1}{3}$.

Art. 69. RULES.—1. To subtract fractions, *Reduce the fractions to a common denominator, subtract the numerator of the subtrahend from the numerator of the minuend, and under the difference write the common denominator.*

2. To subtract mixed numbers, *First subtract the fractions and then the integers.*

LESSON VIII.

Problems involving the Addition and Subtraction of Fractions.

1. A boy spent $\frac{1}{3}$ of his money for a sled, and $\frac{2}{5}$ of it for a pair of skates: what part of his money had he left?

2. John bought a knife for $\frac{3}{8}$ of a dollar and a ball for $\frac{1}{4}$ of a dollar, and then sold both of them for $\frac{7}{8}$ of a dollar. What part of a dollar did he gain?

3. Jane having $\frac{5}{6}$ of a quart of plums, gave $\frac{1}{3}$ of a quart to her brother and $\frac{1}{4}$ of a quart to her sister: how much had she left?

4. A farmer bought $1\frac{1}{2}$ bushels of clover-seed, and then sold $\frac{2}{5}$ of a bushel to one neighbor and $\frac{3}{4}$ of a bushel to another: what part of a bushel had he left?

5. A student spends $\frac{2}{5}$ of his time in study, $\frac{1}{10}$ of it in labor, and $\frac{1}{3}$ of it in sleep: what part has he left?

6. One sixth of a pole is in the ground, $\frac{3}{5}$ of it in water, and the rest in the air: what part is in the air?

7. A man bequeathed $\frac{1}{3}$ of his estate to his wife, $\frac{1}{4}$ of it to each of his two sons, and the rest to his daughter: what part did the daughter receive?

8. A man did $\frac{1}{4}$ of a piece of work the first day, $\frac{1}{3}$ of it the second day, and then completed it the third: what part of the work did he do the third day?

WRITTEN EXERCISES.

9. From the sum of $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{2}{3}$ take $1\frac{1}{2}$.

10. From the sum of $\frac{4}{5}$ and $\frac{3}{7}$ take their difference.

11. A man owning $\frac{1}{3}\frac{5}{2}$ of a vessel, sold $\frac{1}{4}$ and $\frac{1}{8}$ of the vessel: what part had he left?

12. A farmer bought $240\frac{5}{8}$ acres of land, and sold $90\frac{2}{3}$ acres and $75\frac{1}{4}$ acres: how many acres had he left?

13. From a piece of broadcloth containing $20\frac{3}{4}$ yards, a merchant sold $5\frac{1}{2}$ yards, $4\frac{1}{3}$ yards, and $8\frac{1}{4}$ yards: how many yards were left?

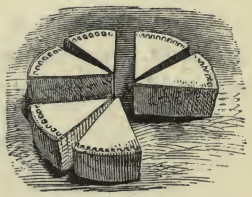
14. A man earned $\$56\frac{3}{8}$ one month and $\$70\frac{5}{6}$ the next, and then gave $\$85\frac{1}{3}$ for a horse: how much money had he left?

15. From the sum of $47\frac{3}{5}$ amd $33\frac{1}{3}$ take their difference.

16. A pedestrian walked $\frac{4}{15}$ of his journey the first day, $\frac{5}{12}$ of it the next day, and completed it the third day: what part of the journey did he travel the third day?

LESSON IX.

Fractions multiplied by Integers.



1. What part of a cake is twice 2 eighths of it? 3 times 2 eighths of it?

2. A father gave 3 fourths of an orange to each of 4 children: how many fourths did they all receive?

3. How much is 4 times $\frac{3}{4}$? 6 times $\frac{3}{4}$?

4. If a boy earn 2 thirds of a dollar in a day, how much will he earn in 3 days?

5. How much is 3 times $\frac{2}{3}$? 5 times $\frac{2}{3}$?

6. How much is 6 times $\frac{3}{5}$? 9 times $\frac{3}{5}$?

7. How much is 7 times $\frac{5}{8}$? 8 times $\frac{7}{9}$?

8. How much is 5 times $6\frac{2}{3}$? 7 times $8\frac{1}{2}$?

SUGGESTION.— Multiply the integer and the fraction separately, and add the products.

9. How much is 3 times $6\frac{2}{5}$? 8 times $7\frac{3}{4}$?

10. How much is 6 times $4\frac{5}{6}$? 9 times $8\frac{2}{7}$?

11. How much is 8 times $6\frac{2}{3}$? 8 times $7\frac{3}{5}$?

WRITTEN EXERCISES.

Multiply:

12. $\frac{5}{12}$ by 8. 15. $\frac{21}{25}$ by 25. 18. $\frac{15}{32}$ by 16.
13. $\frac{7}{10}$ by 12. 16. $\frac{11}{45}$ by 16. 19. $16\frac{2}{3}$ by 4.
14. $\frac{13}{20}$ by 24. 17. $\frac{13}{22}$ by 33. 20. $18\frac{3}{4}$ by 12.

Art. 70. PRINCIPLE.—*A fraction is multiplied by multiplying its numerator or dividing its denominator.*

Art. 71. RULES.—1. To multiply a fraction by an integer, *Multiply the numerator or divide the denominator.*

2. To multiply a mixed number by an integer, *Multiply the fraction and the integer separately, and add the products.*

LESSON X.

Fractional Parts of Integers.

1. If 6 pears be divided equally between 2 boys, what part of the whole will each receive?

What is $\frac{1}{2}$ of 6 pears? $\frac{1}{2}$ of 10 pears?

2. A father divided 5 melons equally between two children: what part of the whole did each receive?

What is $\frac{1}{2}$ of 5 melons?

SUGGESTION.—First take 1 half of 4 melons and then 1 half of 1 melon.

3. Charles divided 12 plums equally between 3 boys: what part of the whole did each receive?

What is $\frac{1}{3}$ of 12 plums? $\frac{1}{3}$ of 13 plums?

4. What is $\frac{1}{3}$ of 9? $\frac{1}{3}$ of 12? $\frac{1}{3}$ of 16?
5. What is $\frac{1}{4}$ of 20? $\frac{1}{4}$ of 28? $\frac{1}{4}$ of 30?
6. What is $\frac{1}{5}$ of 25? $\frac{1}{5}$ of 26? $\frac{1}{5}$ of 37?
7. What is $\frac{1}{6}$ of 24? $\frac{5}{6}$ of 24?

SOLUTION.—$\frac{1}{6}$ of 24 is 4, and $\frac{5}{6}$ of 24 is 5 times 4, which is 20. $\frac{5}{6}$ of 24 is 20.

8. What is $\frac{1}{10}$ of 40? $\frac{5}{10}$ of 40? $\frac{7}{10}$ of 40?
9. What is $\frac{3}{7}$ of 63? $\frac{3}{7}$ of 64? $\frac{6}{7}$ of 65?
10. What is $\frac{4}{9}$ of 45? $\frac{5}{6}$ of 37? $\frac{7}{8}$ of 58?
11. What is $\frac{5}{8}$ of 33? $\frac{7}{8}$ of 58? $\frac{5}{12}$ of 50?

WRITTEN EXERCISES.

12. What is $\frac{3}{8}$ of 659?

PROCESS.

$$\begin{array}{r} 8)659 \\ \hline 82\frac{3}{8} \\ 3 \\ \hline 247\frac{1}{8}, \textit{ Ans.} \end{array} \qquad \textit{Or:} \qquad \begin{array}{r} 659 \\ 3 \\ \hline 8)1977 \\ \hline 247\frac{1}{8} \end{array}$$

13. What is $\frac{2}{9}$ of 191? $\frac{5}{6}$ of 367?
14. What is $\frac{7}{8}$ of 508? $\frac{5}{12}$ of 243?
15. What is $\frac{6}{7}$ of 466? $\frac{5}{11}$ of 4648?
16. What is $\frac{9}{22}$ of 906? $\frac{13}{40}$ of 6070?

Integers multiplied by Fractions.

17. Multiply 256 by $\frac{3}{4}$.

PROCESS.

$\frac{1}{4}$) 256
64
3
192, *Ans.*

Or:

256
3
4) 768
192

$\frac{3}{4}$ is 3 times $\frac{1}{4}$, and hence $\frac{3}{4}$ times 256 is 3 times $\frac{1}{4}$ of 256. Or, $\frac{3}{4}$ is $\frac{1}{4}$ of 3, and hence $\frac{3}{4}$ times 256 is $\frac{1}{4}$ of 3 times 256.

18. 48 by $\frac{5}{12}$.
19. 65 by $\frac{7}{8}$.
20. 59 by $\frac{6}{7}$.
21. 87 by $\frac{4}{5}$.
22. 163 by $\frac{8}{11}$.
23. 300 by $\frac{13}{15}$.
24. 257 by $\frac{11}{16}$.
25. 305 by $\frac{21}{25}$.
26. 248 by $\frac{12}{25}$.
27. 406 by $\frac{5}{21}$.
28. 856 by $\frac{13}{45}$.
29. 794 by $\frac{23}{33}$.

30. Multiply 324 by $16\frac{2}{3}$.

PROCESS.

324
$16\frac{2}{3}$
1944
324
216
5400, *Ans.*

First multiply by the integer and then by the fraction, and add the products.

31. 48 by $16\frac{1}{2}$.
32. 72 by $18\frac{3}{4}$.
33. 96 by $23\frac{7}{8}$.
34. 246 by $12\frac{3}{4}$.
35. 324 by $17\frac{1}{2}$.
36. 406 by $33\frac{1}{3}$.
37. 108 by $56\frac{3}{7}$.
38. 524 by $72\frac{3}{5}$.
39. 684 by $66\frac{2}{3}$.

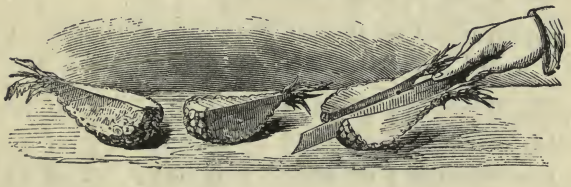
Art. 72. RULES.—1. To find the fractional part of an integer, or to multiply an integer by a fraction, *Divide the integer by the denominator and multiply the quotient by the numerator.* Or:

Multiply the integer by the numerator and divide the product by the denominator.

2. To multiply an integer by a mixed number, *Multiply by the integer and the fraction separately, and add the products.*

LESSON XI.

Compound Fractions reduced to Simple Fractions.



1. If each third of a pine-apple be cut into 2 equal pieces, what part of the pine-apple will one piece be?

What is $\frac{1}{2}$ of $\frac{1}{3}$?

2. A boy having $\frac{1}{4}$ of a melon, gave $\frac{1}{2}$ of it to a playmate: what part of the melon did the playmate receive?

What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{5}$?

3. What is $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{3}$?

4. What is $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{2}$ of $\frac{1}{6}$? $\frac{1}{2}$ of $\frac{1}{8}$?

5. What is $\frac{1}{5}$ of $\frac{1}{4}$? $\frac{1}{5}$ of $\frac{1}{7}$? $\frac{1}{5}$ of $\frac{1}{9}$?

6. A girl having $\frac{3}{4}$ of an orange, divided it equally between her 2 brothers: what part of the orange did each receive?

SUGGESTION.— Divide each fourth into 2 equal pieces, and then give 3 pieces to each.

7. What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{3}{4}$?

8. What is $\frac{1}{3}$ of $\frac{1}{8}$? $\frac{1}{3}$ of $\frac{3}{8}$? $\frac{1}{3}$ of $\frac{5}{8}$?
9. What is $\frac{1}{5}$ of $\frac{5}{6}$? $\frac{1}{4}$ of $\frac{3}{7}$? $\frac{3}{4}$ of $\frac{3}{5}$?

SOLUTION.—$\frac{1}{4}$ of $\frac{3}{5}$ is $\frac{3}{20}$, and $\frac{3}{4}$ of $\frac{3}{5}$ is 3 times $\frac{3}{20}$, which is $\frac{9}{20}$. $\frac{3}{4}$ of $\frac{3}{5} = \frac{9}{20}$.

10. What is $\frac{3}{4}$ of $\frac{5}{6}$? $\frac{2}{3}$ of $\frac{5}{8}$? $\frac{3}{8}$ of $\frac{2}{7}$?
11. What is $\frac{3}{7}$ of $\frac{7}{9}$? $\frac{3}{4}$ of $\frac{6}{7}$? $\frac{5}{6}$ of $\frac{7}{12}$?
12. What is $\frac{1}{3}$ of $12\frac{1}{2}$? $\frac{1}{3}$ of $13\frac{1}{2}$?

SOLUTION.—$\frac{1}{3}$ of $13\frac{1}{2} = \frac{1}{3}$ of $12 + \frac{1}{3}$ of $1\frac{1}{2}$ or $\frac{3}{2}$. $\frac{1}{3}$ of 12 is 4, and $\frac{1}{3}$ of $\frac{3}{2}$ is $\frac{3}{6}$ or $\frac{1}{2}$. Hence, $\frac{1}{3}$ of $13\frac{1}{2}$ is $4\frac{1}{2}$.

13. What is $\frac{1}{5}$ of $18\frac{1}{3}$? $\frac{1}{5}$ of $21\frac{2}{3}$? $\frac{1}{5}$ of $31\frac{3}{4}$?
14. What is $\frac{1}{7}$ of $22\frac{1}{5}$? $\frac{1}{8}$ of $42\frac{1}{2}$? $\frac{1}{9}$ of $46\frac{3}{5}$?
15. What is $\frac{1}{8}$ of $33\frac{1}{3}$? $\frac{1}{9}$ of $64\frac{1}{5}$? $\frac{1}{10}$ of $62\frac{1}{3}$?

WRITTEN EXERCISES.

16. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ to a simple fraction.

PROCESS: $\frac{2}{3}$ of $\frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \frac{6}{15} = \frac{2}{5}$, *Ans.*

Or: $\frac{2}{3}$ of $\frac{3}{5} = \frac{2 \times \cancel{3}}{\cancel{3} \times 5} = \frac{2}{5}$.

NOTE.—The examples should be solved by both methods. The teacher should explain the process of cancellation.

Reduce to a simple fraction:

17. $\frac{3}{5}$ of $\frac{5}{7}$.
18. $\frac{5}{6}$ of $\frac{3}{5}$.
19. $\frac{6}{7}$ of $\frac{7}{12}$.
20. $\frac{3}{8}$ of $\frac{7}{12}$.
21. $\frac{5}{9}$ of $\frac{9}{11}$.
22. $\frac{6}{7}$ of $\frac{14}{15}$.
23. $\frac{4}{11}$ of $\frac{5}{12}$.
24. $\frac{7}{12}$ of $\frac{12}{21}$.
25. $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{2}{3}$.
26. $\frac{2}{5}$ of $\frac{3}{4}$ of $2\frac{1}{2}$.
27. $\frac{3}{7}$ of $2\frac{1}{3}$ of $\frac{3}{5}$.
28. $\frac{4}{5}$ of $\frac{6}{7}$ of $1\frac{2}{3}$.

Fractions multiplied by Fractions.

29. Multiply $\frac{4}{5}$ by $\frac{3}{4}$.

PROCESS.

$$\frac{4}{5}\times\frac{3}{4}=\frac{4\times3}{5\times4}=\frac{12}{20}=\frac{3}{5}.$$

Since $\frac{3}{4}$ is $\frac{3}{4}$ of one, $\frac{3}{4}$ times $\frac{4}{5}=\frac{3}{4}$ of once $\frac{4}{5}$, or $\frac{3}{4}$ of $\frac{4}{5}$, which equals $\frac{3\times4}{4\times5}$. Hence,

$$\frac{4}{5}\times\frac{3}{4}=\frac{4\times3}{5\times4}=\frac{3}{5}.$$

Or: $\frac{4}{5}\times\frac{3}{4}=\frac{\cancel{4}\times3}{5\times\cancel{4}}=\frac{3}{5}$, *Ans.*

29. $\frac{5}{8}$ by $\frac{2}{5}$.	33. $\frac{13}{16}$ by $\frac{8}{13}$.	37. $2\frac{1}{2}$ by $2\frac{1}{2}$.
30. $\frac{4}{7}$ by $\frac{7}{6}$.	34. $\frac{10}{17}$ by $\frac{34}{30}$.	38. $3\frac{1}{3}$ by $3\frac{1}{3}$.
31. $\frac{3}{15}$ by $\frac{5}{6}$.	35. $\frac{5}{9}$ by $\frac{9}{20}$.	39. $6\frac{1}{2}$ by $2\frac{1}{3}$.
32. $\frac{5}{6}$ by $\frac{6}{13}$.	36. $\frac{9}{25}$ by $\frac{15}{3}$.	40. $6\frac{1}{4}$ by $12\frac{1}{2}$.

Art. 73. A ***Simple Fraction*** is a fraction not united with an integer or another fraction; as, $\frac{3}{4}$.

A ***Compound Fraction*** is a fraction of a fraction; as, $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{2}{5}$ of $3\frac{1}{3}$.

Art. 74. RULE.—To reduce a compound fraction to a simple fraction, or to multiply one fraction by another, *Multiply the numerators together, and also the denominators.*

NOTE.—The process may often be shortened by canceling common factors before multiplying.

LESSON XII.

Fractions divided by Integers.

1. A father divided $\frac{3}{4}$ of a melon equally between 3 boys: what part of the melon did each receive?

SOLUTION.—If 3 boys receive $\frac{3}{4}$ of a melon, each will receive $\frac{1}{3}$ of $\frac{3}{4}$, which is $\frac{1}{4}$.

2. A woman divided $\frac{4}{5}$ of a pound of crackers equally between 3 poor children: what part of a pound did each receive?

How much is $\frac{4}{5}$ divided by 3?

3. If 5 pounds of cheese cost $\frac{3}{4}$ of a dollar, what will 1 pound cost?

4. How much is $\frac{3}{4} \div 5$? $\frac{3}{7} \div 5$? $\frac{3}{8} \div 5$?

5. If 6 men can build $\frac{7}{8}$ of a wall in a day, what part of the wall can one man build?

6. How much is $\frac{7}{8} \div 6$? $\frac{7}{8} \div 5$? $\frac{7}{8} \div 9$?

7. A grocer put $16\frac{1}{2}$ pounds of sugar into 5 equal parcels: how much sugar was put into each parcel?

8. Divide $16\frac{1}{2}$ by 5. $12\frac{1}{2}$ by 5. $18\frac{1}{2}$ by 5.

9. Divide $20\frac{1}{4}$ by 3. $30\frac{2}{3}$ by 4. $31\frac{3}{4}$ by 6.

WRITTEN EXERCISES.

10. Divide $\frac{7}{12}$ by 3.

PROCESS: $\frac{7}{12} \div 3 = \frac{1}{3}$ of $\frac{7}{12} = \frac{7}{36}$.

Or: $\frac{7}{12} \div 3 = \frac{7}{12 \times 3} = \frac{7}{36}$.

11. $\frac{7}{11}$ by 7.
12. $\frac{6}{7}$ by 12.
13. $\frac{7}{12}$ by 10.
14. $\frac{8}{13}$ by 6.
15. $\frac{14}{15}$ by 7.
16. $\frac{16}{17}$ by 8.
17. $\frac{20}{21}$ by 5.
18. $\frac{21}{25}$ by 3.
19. $24\frac{1}{3}$ by 6.
20. $29\frac{3}{5}$ by 7.
21. $46\frac{3}{4}$ by 5.
22. $66\frac{2}{3}$ by 8.

Art. 75. PRINCIPLE.—*A fraction is divided by*

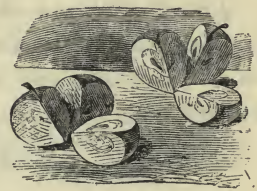
dividing the numerator or multiplying the denominator.

Art. 76. RULES.—1. To divide a fraction by an integer, *Divide the numerator or multiply the denominator.*

2. To divide a mixed number by an integer, *Divide the integral part and then the fraction.*

LESSON XIII.

Integers divided by Fractions.



1. How many times is $\frac{1}{3}$ of an apple contained in 2 apples? $\frac{2}{3}$ of an apple in 2 apples?

2. How many times are $\frac{3}{4}$ of a yard contained in 4 yards?

SOLUTION.—4 yards = 12 fourths, and 3 fourths are contained in 12 fourths 4 times.

3. If a basket holds $\frac{2}{5}$ of a bushel, how many baskets will hold 4 bushels?

4. How many times is $\frac{2}{5}$ contained in 4? $\frac{2}{3}$ in 4? $\frac{2}{7}$ in 4?

5. If $\frac{5}{8}$ of a yard of silk will trim a hat, how many hats will 6 yards trim?

6. How many times is $\frac{3}{8}$ contained in 3? $\frac{3}{8}$ in 6? $\frac{3}{4}$ in 6? $\frac{3}{4}$ in 3?

7. Divide 12 by $\frac{3}{5}$. 15 by $\frac{3}{4}$. 20 by $\frac{2}{3}$.

8. Divide 8 by $\frac{4}{7}$. 10 by $\frac{5}{9}$. 12 by $\frac{6}{11}$.

WRITTEN EXERCISES.

9. Divide 14 by $\frac{3}{5}$.

PROCESS.

$14 = \frac{70}{5}$. $\frac{70}{5} \div \frac{3}{5} = 70 \div 3 = 23\frac{1}{3}$, *Ans.*

NOTE.—Since 14 is reduced to fifths by multiplying it by 5, the process may be shortened by omitting the denominators, thus: $13 \div \frac{3}{5} = 14 \times 5 \div 3 = 23\frac{1}{3}$.

10. 16 by $\frac{2}{3}$.
11. 20 by $\frac{4}{4}$.
12. 45 by $\frac{3}{5}$.
13. 60 by $\frac{7}{8}$.
14. 21 by $\frac{3}{7}$.
15. 42 by $\frac{3}{5}$.
16. 30 by $2\frac{1}{2}$.
17. 40 by $3\frac{1}{3}$.
18. 16 by $5\frac{1}{3}$.

Art. 77. RULE.—To divide an integer by a fraction, *Multiply the integer by the denominator of the fraction, and divide the product by the numerator.*

LESSON XIV.

Fractions divided by Fractions.

1. If $\frac{1}{3}$ of a barrel of flour will supply a family one month, how many months will $\frac{2}{3}$ of a barrel last?

2. How many times is $\frac{1}{3}$ contained in $\frac{2}{3}$? $\frac{1}{3}$ in $\frac{3}{3}$?

3. If $\frac{2}{5}$ of a yard of cloth will make a vest, how many vests will $\frac{4}{5}$ of a yard make?

4. How many times $\frac{2}{5}$ in $\frac{4}{5}$? $\frac{2}{5}$ in $\frac{6}{5}$?

5. If a pound of butter cost $\frac{1}{4}$ of a dollar, how many pounds can be bought for $\frac{6}{8}$ of a dollar?

SUGGESTION.—Change $\frac{1}{4}$ to eighths.

6. How many times $\frac{1}{4}$ in $\frac{4}{8}$? $\frac{1}{3}$ in $\frac{4}{6}$?

7. How many times $\frac{1}{4}$ in $\frac{1}{2}$? $\frac{1}{3}$ in $\frac{1}{2}$? $\frac{1}{5}$ in $\frac{1}{2}$?
8. How many times $\frac{1}{3}$ in $\frac{2}{5}$? $\frac{1}{3}$ in $\frac{3}{4}$? $\frac{3}{5}$ in $\frac{2}{3}$?

WRITTEN EXERCISES.

9. Divide $\frac{3}{4}$ by $\frac{2}{3}$.

PROCESS: $\frac{3}{4} = \frac{9}{12}$. $\frac{2}{3} = \frac{8}{12}$. $\frac{9}{12} \div \frac{8}{12} = \frac{9}{8} = 1\frac{1}{8}$.

2. $\frac{7}{8}$ by $\frac{3}{4}$.
3. $\frac{3}{5}$ by $\frac{1}{2}$.
4. $\frac{3}{12}$ by $\frac{5}{8}$.
5. $\frac{6}{7}$ by $\frac{2}{3}$.
6. $\frac{5}{9}$ by $\frac{2}{3}$.
7. $\frac{5}{6}$ by $\frac{3}{8}$.
8. $2\frac{1}{2}$ by $\frac{2}{3}$.
9. $3\frac{1}{3}$ by $\frac{3}{5}$.
10. $\frac{3}{5}$ by $2\frac{1}{2}$.

Art. 78. RULE. — To divide a fraction by a fraction, *Reduce the fractions to a common denominator, and divide the numerator of the dividend by the numerator of the divisor.*

NOTE. — When pupils are familiar with this method, they may be taught to divide by inverting the terms of the divisor and multiplying. This method is fully explained in the author's *Complete Arithmetic.*

LESSON XV.

Numbers Fractional Parts of Other Numbers.

1. 5 is $\frac{1}{3}$ of what number?

SOLUTION. — 5 is $\frac{1}{3}$ of 3 times 5, or 15.

2. 7 is $\frac{1}{6}$ of what number?
3. 12 is $\frac{1}{10}$ of what number?
4. $12\frac{1}{2}$ is $\frac{1}{8}$ of what number?
5. $16\frac{2}{3}$ is $\frac{1}{6}$ of what number?
6. 10 is $\frac{2}{3}$ of what number?

SOLUTION. — If 10 is $\frac{2}{3}$ of a number, $\frac{1}{3}$ of the number is $\frac{1}{2}$ of 10, which is 5; if 5 is $\frac{1}{3}$ of a number, $\frac{3}{3}$ of it will be 3 times 5, which is 15.

7. 12 is $\frac{3}{4}$ of what number?
8. 25 is $\frac{5}{8}$ of what number?
9. 30 is $\frac{6}{7}$ of what number?
10. 33 is $\frac{3}{5}$ of what number?
11. 44 is $\frac{2}{7}$ of what number?
12. A man spent $\frac{2}{5}$ of his money and had $21 left: how much money had he at first?
13. A boy gave 24 cents for a slate, which was $\frac{4}{7}$ of his money: how much money did he have?
14. A man pays $25 a month for house-rent, which is $\frac{5}{12}$ of his monthly salary: what is his salary?
15. A farmer sold a cow for $45, which was $\frac{1}{4}$ more than he paid for her: what was the cost of the cow?
16. A man sold $\frac{3}{7}$ of a farm for $1500: at this rate, what was the value of the farm?

LESSON XVI.

Miscellaneous Problems.

1. Reduce $18\frac{3}{4}$ to an improper fraction.
2. Reduce $\frac{113}{20}$ to a mixed number.
3. Reduce $\frac{3}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$ to a common denominator.
4. Add $\frac{1}{3}$, $\frac{1}{6}$, $\frac{3}{4}$, and $\frac{5}{12}$.
5. Add $28\frac{3}{5}$, $40\frac{1}{2}$, $63\frac{2}{3}$, and $19\frac{3}{10}$.
6. From $\frac{5}{7}$ take $\frac{2}{5}$. From $28\frac{2}{5}$ take $16\frac{2}{3}$.
7. Multiply $\frac{5}{8}$ by 7. 13 by $\frac{3}{5}$. $\frac{3}{4}$ by $\frac{6}{7}$.
8. Multiply $137\frac{2}{3}$ by 15. 256 by $21\frac{5}{8}$.
9. Divide 12 by $\frac{2}{3}$. $\frac{2}{3}$ by 12. $\frac{3}{5}$ by $\frac{6}{7}$.

10. Divide $243\frac{3}{3}$ by 11. 256 by $5\frac{1}{2}$.

11. $\frac{7}{8} + \frac{2}{3} =$ what? $\frac{7}{8} - \frac{2}{3}$? $\frac{7}{8} \times \frac{2}{3}$? $\frac{7}{8} \div \frac{2}{3}$?

12. There are 5280 feet in a mile: how many feet in $\frac{9}{40}$ of a mile?

13. A vessel is worth \$6000, and the cargo is worth $\frac{5}{8}$ as much as the vessel: what is the value of the cargo?

14. A man sold $\frac{2}{5}$ of his farm to one neighbor and $\frac{3}{8}$ of it to another: what part of the farm has he left?

15. A man owning $\frac{5}{8}$ of a factory, sold $\frac{2}{5}$ of his share: what part of the factory did he sell? What part does he still own?

16. There are $16\frac{1}{2}$ feet in a rod: how many feet in 66 rods?

17. There are $5\frac{1}{2}$ yards in a rod: how many rods in 66 yards?

18. At $\$6\frac{1}{4}$ a barrel, how many barrels of flour can be bought for \$150?

19. If $\frac{4}{7}$ of a ship is worth \$12000, what is the whole ship worth?

20. A man owning $\frac{5}{7}$ of an estate, sells $\frac{2}{5}$ of his share for \$2400: at this rate, what would be the value of the estate?

21. A farmer had two fields of wheat. The first yielded 840 bushels, which was $\frac{5}{12}$ of the amount yielded by the second field. How many bushels did the second field yield?

22. A man owning $\frac{5}{8}$ of a ship, sells $\frac{2}{3}$ of his share for \$4400: at this rate, what is the value of the ship?

23. The value of a certain ship is $9760, and $\frac{3}{5}$ of the value of the ship is $\frac{6}{7}$ of the value of the cargo: what is the value of the cargo?

Questions for Review.

What is a fraction? What does the denominator denote? The numerator? What are the terms of a fraction?

What is a mixed number? What is meant by $18\frac{3}{4}$? *Ans.* $18 + \frac{3}{4}$. When is a fraction called proper? When improper? When is the value of an improper fraction equal to 1?

How is an integer reduced to a fraction? How is a mixed number reduced to a fraction? What kind of a fraction is the result? Give examples. How is an improper fraction reduced to a whole or mixed number? Give examples.

How is a fraction reduced to lower terms? On what principle does the process depend? How may a fraction be reduced to its *lowest* terms by one division? How is a fraction reduced to higher terms? State the principle.

How are fractions having a common denominator added or subtracted? When fractions have different denominators, how are they added or subtracted? How may mixed numbers be added? How may they be subtracted?

In what two ways may a fraction be multiplied by an integer? How may an integer be multiplied by a fraction? Give the rule for multiplying a fraction by a fraction. What is a simple fraction? What is a compound fraction? How is a compound fraction reduced to a simple fraction?

In what two ways may a fraction be divided by an integer? How may an integer be divided by a fraction? How may a fraction be divided by a fraction?

SECTION VIII.

UNITED STATES MONEY.

LESSON I.

Art. 79. ***United States Money*** is the legal currency of the United States. It is also called *Federal Money.*

The denominations are *mills, cents, dimes,* and *dollars.*

Table.

10 mills (*m.*) . . are 1 cent, . . *c.* or *ct.*
10 cents . . . . are 1 dime, . . *d.*
10 dimes . . . . are 1 dollar, . $.

$\$1 = 10d. = 100c. = 1000m.$

NOTES.—1. United States money consists of *Coin* and *Paper Money.* Coin is called *Specie Currency,* or *Specie,* and paper money is called *Paper Currency.*

2. The principal gold coins are the eagle ($10), half-eagle ($5), quarter-eagle ($2½), three-dollar piece, and dollar.

The silver coins are the dollar, half-dollar, quarter-dollar, dime, half-dime, and three-cent piece.

The nickel coins are the five-cent piece, three-cent piece, and cent.

The copper coins (old) are the two-cent piece and the cent.

3. Paper money consists of notes issued by the United States, called treasury notes, and bank notes issued by banks.

4. Treasury notes of a value less than $1, as fifty cents, twenty-five cents, fifteen cents, ten cents, five cents, and three cents, are called *Fractional Currency.*

1. How many mills in 1 cent? In five cents? 4 cents? 7 cents? 9 cents?

2. How many cents in 1 dime? In 4 dimes? 5 dimes? 8 dimes? 10 dimes?

3. How many dimes in 1 dollar? In 3 dollars? 6 dollars? 4 dollars? 8 dollars?

4. How many cents in 10 mills? In 50 mills? 40 mills? 60 mills? 80 mills?

5. How many dimes in 40 cents? In 90 cents? 60 cents? 70 cents? 100 cents?

6. How many dollars in 50 dimes? In 70 dimes? 60 dimes? 80 dimes? 100 dimes?

7. How many dimes in 25 cents? In 75 cents? 15 cents? 35 cents? 95 cents?

8. How many cents in 35 mills? In 65 mills? 25 mills? 75 mills? 95 mills?

9. How many cents in 5 dimes? In 15 dimes? 45 dimes? 30 dimes?

10. How many dimes in 15 cents? In 95 cents? 85 cents? 65 cents?

11. How many cents in 1 dollar? In 5 dollars? 7 dollars? 9 dollars? 8 dollars?

12. How many dollars in 200 cents? In 500 cents? 400 cents? 900 cents?

WRITTEN EXERCISES.

Art. 80. Accounts are kept in dollars, cents, and mills. The figures denoting dollars are preceded by the sign, $. The first two figures at the right of dollars denote cents, and the third figure denotes mills. The figures denoting dollars are separated from those denoting cents by a period (.), called a *Separatrix*. Thus, $45.307 is read 45 *dollars*, 30 *cents*, 7 *mills*.

NOTE.—The pupil should here be taught that the first figure at the right of the separatrix denotes *tenths* of a dollar; the second, *hundredths;* the third, *thousandths*. He should also be taught to read the following numbers decimally, and to write similar numbers, when dictated, decimally. The first decimal period may thus be mastered.

Copy and read the following:

(13)	(14)	(15)	(16)
$3.45	$0.075	$40.045	$100.
$3.506	$0.005	$15.15	$405.
$1.055	$3.08	$10.015	$704.50
$0.75	$9.009	$60.60	$800.08

17. Write in figures 4 dollars 40 cents.
18. Write 12 dollars 33 cents 5 mills.
19. Write 60 dollars 6 cents 4 mills.
20. Write 75 cents 5 mills.
21. Write 30 cents; 40 cents 7 mills.
22. Write 300 dollars 3 cents 7 mills.
23. Write 500 dollars 5 mills.
24. Write 25 cents 7 mills; 6 cents 1 mill.
25. Write 10 dollars 3 cents 8 mills.
26. Write 1000 dollars; 50 dollars 5 cents.
27. Write 25 dollars 1 cent 5 mills.
28. Write 500 dollars 3 mills.
29. Write 5 mills; 5 cents 5 mills.
30. Write 60 dollars 60 cents 6 mills.

LESSON II.

Reduction of United States Money.

MENTAL AND WRITTEN EXERCISES.

1. How many cents in $15? *Ans.* 1500*c.*
2. How many mills in $15? *Ans.* 15000*m.*
3. How many cents in $5.25? *Ans.* 525*c.*
4. How many mills in $1.375? *Ans.* 1375*m.*
5. How many mills in $.62½? *Ans.* 625*m.*
6. Reduce $75 to cents. $108 to cents.
7. Reduce $125 to cents. $230 to cents.
8. Reduce $12.65 to cents. $5.60 to cents.
9. Reduce $1.08 to cents. $8.01 to cents.
10. Reduce $25 to mills. $40 to mills.
11. Reduce $.62½ to mills. $3.12½ to mills.

12. Reduce $.375 to mills. $.105 to mills.
13. Reduce $4.50 to mills. $3.03 to mills.
14. Reduce $.45 to mills. $.05 to mills.
15. Reduce $102 to cents. $10 to cents.
16. Reduce $120 to mills. $45 to mills.
17. Reduce $.25 to cents. $.01 to cents.
18. How many dollars in 7500 cents?

Ans. $75.

19. How many dollars in 7550 cents?

Ans. $75.50.

20. How many dollars in 3125 mills?

Ans. $3.125.

21. How many dollars in 4000 mills?
22. Reduce 1507 cents to dollars.
23. Reduce 1001 cents to dollars.
24. Reduce 1500 mills to dollars.
25. Reduce 10250 mills to dollars.
26. Reduce 5000 cents to dollars.
27. Reduce 5000 mills to dollars.
28. Reduce 375 cents to dollars.
29. Reduce 375 mills to dollars.
30. Reduce $4.50 to mills.
31. Reduce 4500 mills to dollars.
32. Reduce $10.10 to cents.
33. Reduce 1010 mills to dollars.

Art. 81. RULES. — 1. To reduce dollars to cents, *Annex two ciphers.*

2. To reduce dollars to mills, *Annex three ciphers.*

3. To reduce cents to mills, *Annex one cipher.*

4. To reduce dollars and cents to cents, or dollars, cents, and mills to mills, *Remove the separatrix and the dollar sign.*

5. To reduce cents to dollars, *Place the separatrix before the second right-hand figure.*

6. To reduce mills to dollars, *Place the separatrix before the third right-hand figure.*

NOTE.—Annexing two ciphers is multiplying by 100, and pointing off two figures from the right is dividing by 100. (See Arts. 37, 45.)

LESSON III.

Addition and Subtraction.

WRITTEN EXERCISES.

1. What is the sum of \$50, \$16.50, \$3.333, and \$.87½?

PROCESS.

\$50.
16.50
3.333
.875
\$70.708, *Ans.*

Write the several numbers to be added so that units of the same denomination may stand in the same column, and then add as in simple numbers. The dollar sign need not be written but once.

2. What is the sum of \$1.20, \$5, \$10.15, \$.85, and \$.62½?

3. What is the sum of \$9, \$12.50, \$4.37½, \$40.08, \$6.33, and \$25.?

4. Add 45.37½, \$100.50, \$16.12½, \$37, \$9.05, \$.87½, \$4.44, and \$95.

5. From \$37.50 take \$5.62½.

PROCESS.

\$37.500
5.625
\$31.875, *Ans.*

6. From \$6.37½ take \$5.87¾.

7. From \$100. take \$1.256.

8. From \$10. take \$.10.

9. A man sold a carriage for \$160.75, a horse for \$125, a set of harness for \$26.37½, and a saddle for \$15.62½: what was the amount received?

10. A grocer buys flour at \$8.62½ a barrel, and sells it at \$10. a barrel: what is his gain?

11. A merchant paid \$32.50 for a barrel of sugar, and sold it for \$35.: how much did he gain?

12. A laborer earns \$17.50 a week, and his expenses are \$12.62½ a week: how much can he save each week?

13. A man bought a house and lot for \$3506.75, and sold it for \$4000: what was his gain?

14. A man bought a carriage for \$160, paid \$22.75 for repairing it, and then sold it for \$180: how much did he lose?

15. Mr. Smith bought a house and lot for \$4500, and paid \$40.50 for a front fence, \$105.65 for painting the house, \$47.12 for papering several rooms, and \$25 for other improvements: what will he make if he sell the property for \$5000?

Art. 82. RULE.—To add or subtract sums of money, *Write units of the same denomination in the same column, add or subtract as in simple numbers, and separate dollars and cents by a period and prefix the dollar sign.*

LESSON IV.

Multiplication and Division.

WRITTEN EXERCISES.

1. What will 9 cords of wood cost, at $\$3.62\frac{1}{2}$ a cord?

PROCESS.

$\$3.625$
9
$\$32.625$, *Ans.*

If 1 cord of wood cost $3.625, nine cords will cost 9 times $3.625, which is $32.625.

2. What will 16 barrels of flour cost, at $7.50 a barrel?

3. What will 40 yards of cloth cost, at $\$1.12\frac{1}{2}$ a yard?

4. What will $12\frac{3}{5}$ tons of hay cost, at $11.50 a ton?

5. At $18\frac{3}{4}$ cents a dozen, what will 12 dozen of eggs cost?

6. If a boy earn $\$4.37\frac{1}{2}$ a week, how much will he earn in 20 weeks?

7. A drover sold 36 cows at $\$33.33\frac{1}{3}$ a head: how much did he receive for them?

8. What will 90 bushels of wheat cost at $\$1.62\frac{1}{2}$ a bushel?

9. If 9 cords of wood cost $\$32.62\frac{1}{2}$, what will 1 cord cost?

PROCESS.

$9)\$32.625$
$\$3.625$, *Ans.*

If 9 cords cost $32.625, one cord will cost $\frac{1}{9}$ of $32.625, which is $3.625. or $\$3.62\frac{1}{2}$.

10. If 12 pounds of sugar cost $2.16, what will 1 pound cost?

11. A man paid $1687.50 for 45 acres of land: what was the price an acre?

12. A grocer paid $135 for 18 barrels of flour: what was the cost a barrel?

13. A man earned $91. in 8 weeks: how much did he earn a week?

14. At $.12½ a dozen, how many dozens of eggs can be bought for $5?

PROCESS.

125 *m.*) 5000 *m.* (40, *Ans.*
500
0

$5 = 5000 mills, and $.12½ = 125 mills. Hence, $5. ÷ $.12½ = 5000 mills ÷ 125 mills, which is 40.

NOTE. — When both divisor and dividend are denominate numbers, they must be reduced to the same denomination before dividing.

15. At $1.25 a bushel, how many bushels of corn can be bought for $75.

16. At 3⅓ cents a piece, how many lemons can be bought for $7.

17. If a boy earn 75 cents a day, in how many days will he earn $24?

18. At 37½ cents a bushel, how many bushels of oats can be bought for $57.75.

19. A farmer sold 35 pounds of butter at 20 cents a pound, and received in payment muslin at 12½ cents a yard: how many yards of muslin did he receive?

20. A farmer exchanged 16 cows, at $27.50 a

head, for sheep at $5.50 a head: how many sheep did he receive?

21. How many lemons, at $2\frac{1}{2}$ cents each, can be bought for 20 oranges, at 5 cents each?

22. Multiply $\$12.62\frac{1}{2}$ by 15, and divide the product by $2.525.

23. Multiply $1.25 by 18, and divide the product by $\$.62\frac{1}{2}$.

24. Multiply $5.75 by 25, and divide the product by $\$.57\frac{1}{2}$.

Art. 82. RULES.—1. To multiply or divide sums of money by an abstract number, *Multiply or divide as in simple numbers, separate dollars and cents in the result by a period, and prefix the dollar sign.*

2. To divide one sum of money by another, *Reduce both numbers to the same denomination, and divide as in simple numbers.*

LESSON V.

Miscellaneous Written Problems.

1. What is the sum of $13.45, $9.87, $100., $.87, $1.40, and $14.?

2. From $10 take 5 mills.

3. From $500 take 500 cents.

4. Multiply $\$15.33\frac{1}{3}$ by 33.

5. Divide $50 by 50 cents.

6. A man's income tax in 1868 was $55.75, his State and city tax $68.35, and his other taxes $7.50: what was the amount of his taxes?

7. A man bought a house and lot for $5400, and, after expending $1500 for improvements, sold the property for $7500: how much did he gain?

8. What will 60 pounds of butter cost, at $33\frac{1}{3}$ cents a pound?

9. What is the cost of 35 reams of paper, weighing 44 pounds each, at 18 cents a pound?

10. How many yards of carpeting, at $1.75 a yard, can be bought for $350?

11. A fruit dealer makes a net profit of 20 cents on each bushel of apples he sells: how many bushels must he sell to make $80?

12. A widow is to receive one third of an estate of $12000, and the remainder is to be divided equally between 5 children: what is the share of each child?

13. A fruit dealer sold 144 baskets of peaches for $252: what was the price per basket?

14. If 40 acres of land cost $1400, how many acres can be bought for $1750?

15. A man sold 15 cords of wood, at $4.50 a cord, and received in payment 10 barrels of flour: what did the flour cost him a barrel?

16. If 8 barrels of salt cost $36, what will 13 barrels cost?

17. A grain dealer bought 15000 bushels of wheat at $1.35 a bushel, and sold it the next week for $1.48 a bushel: what was his gain?

18. A workman receives $1.50 a day, and his living costs him $.75 a day: how much can he lay up in a year, if he work 310 days?

19. A drover bought 240 sheep at $4.50 a head, drove them to market at an expense of $75, and then sold them at $6.50 a head: how much did he make?

20. A farmer exchanged 40 pounds of butter, at 22 cents a pound, and 8 dozen of eggs, at $12\frac{1}{2}$ cents a dozen, for cotton cloth at 10 cents a yard: how many yards of cloth did he receive?

21. A farmer exchanged 8 cows, valued at $37.50 a head, for sheep valued at $7.50 a head: how many sheep did he receive?

22. If a boy pays $2.50 a hundred for papers, and sells them at 5 cents a piece, how much does he make on 100 papers?

23. A farmer sold, one year, 200 bushels of wheat, at $1.80 a bushel; 500 bushels of corn, at $1.15 a bushel; 65 bushels of potatoes, at 80 cents a bushel; 12 tons of hay, at $16.50 a ton; and 225 pounds of butter, at 20 cents a pound. What was the amount of his annual product?

24. A man bought 250 bushels of coal, at 15 cents a bushel; 7 cords of wood, at $5.50 a cord; 18 bushels of potatoes, at $.90 a bushel; and 9 barrels of apples, at $2.75 a barrel. How much did he pay for all?

25. A bookseller sold 12 geographies, at $1.75; 20 readers, at $.85; 30 arithmetics, at $.65; and 45 spellers, at $.30. What was the amount of the bill?

26. The annual expenses of a man's family are as follows: provisions, $350; clothing, $400; fuel,

$95; books and periodicals, $50; house-rent, $240; and all other expenses, $150. If he receive an annual salary of $1500, how much can he lay up each year?

LESSON VI.

BILLS.

1. COLUMBUS, O., JUNE 10, 1869.

MR. CHARLES WILSON

Bought of JAMES COOPER & CO.:

13 lbs.	Coffee,	@ 30*c.*, ...	$3.90
4 lbs.	Butter,	@ 35*c.*, ...	1.40
10 lbs.	Bk't Flour,	@ 6*c.*, ...	.60
12 lbs.	Dr'd Beef,	@ 24*c.*, ...	2.88
25 lbs.	Sugar,	@ 18*c.*, ...	4.50
3 lbs.	Starch,	@ 20*c.*, ...	.60
			$13.88

Received payment,

JAMES COOPER & CO.

---

2. CHICAGO, JAN. 3, 1869.

JOSEPH MASON

Bought of PETER & BROTHERS:

27 yds.	Brussels carpeting,	@ $2.60, ...	
23 yds.	Ingrain "	@ 1.75, ...	
8¾ yds.	Oil cloth,	@ 1.20, ...	
32 yds.	Curtains,	@ .60, ...	
			$

Received payment,

PETER & BROTHERS,

Per SMITH.

What is the amount of the above bill?

3. NASHVILLE, TENN., OCT. 8, 1868.

SAMUEL MILLS

To JONES, SMITH & CO., *Dr.*

To	7	yds. Broadcloth,	@	$6.50,	. . . . .
"	3½	yds. Doeskin,	@	2.75,	. . . . .
"	7¾	yds. Linen,	@	.90,	. . . . .
"	2½	doz. Handkerchiefs,	@	1.50,	. . . . .
"	12½	yds. Muslin,	@	.18,	. . . . .
"	9	yds. do. bleached,	@	.33,	. . . . .
"	12	yds. Silk,	@	1.60,	. . . . .
"	19	yds. Binding,	@	.08,	. . . . .

Received payment,

JONES, SMITH & CO.

What is the amount of the above bill?

4. ST. LOUIS, MAY 23, 1869.

HENRY WILLIAMS

1869. *Bought of* ISAAC CLARKE:

Mch.	10,	5	Pair	Calf Boots,	@	$5.75, . .
"	"	8	"	Laidies' Gaiters,	@	3.10, . .
"	"	7	"	Children's Shoes,	@	1.75, . .
Apr.	4,	8	"	Coarse Boots,	@	2.75, . .
"	"	6	"	Calf Shoes,	@	3.25, . .
"	"	7	"	Ladies' Slippers,	@	1.20, . .
May	23,	3	"	Calf Boots,	@	5.75, . .

$

Received payment,

What is the amount of the above bill?

5. PITTSBURG, PA., DEC. 15, 1868.

ANDREW WILSON

Bought of SMITH & WARING:

1868.					
July	5,	7	gross	Shirt Buttons,	@ $4.50,
"	"	10	doz.	Linen Napkins,	@ 2.75,
Aug.	12,	8	"	Pair Kid Gloves,	@ 12.50,
"	"	$3\frac{1}{2}$	"	Linen Handk'fs,	@ 6.75,
"	"	$4\frac{2}{3}$	"	Shirt Bosoms,	@ 6.00,
Dec.	15,	$3\frac{3}{4}$	"	Silk Gloves,	@ 9.00,
"	"	8	"	Pair Socks,	@ 5.50,

$

What is the amount of the above bill?

---

6. INDIANAPOLIS, IND., AUG. 18, 1869.

THOS. M. COCHRANE

Bought of JONES, DUNLAP & CO.:

12	doz. Scythes,	@ $15,	. . . . .
$12\frac{1}{2}$	doz. Scy. Snaths,	@ 16.50,	. . . . .
6	doz. Rakes,	@ 2.25,	. . . . .
5	doz. Hoes,	@ 5.75,	. . . . .
$8\frac{2}{3}$	doz. Whetstones,	@ 1.50,	. . . . .

$

Cr.

June 20,	By Cash, ...	$75.00
Aug. 1,	By $2\frac{1}{2}$ doz. Scythes returned, .	37.50
		$112.50

$

Received payment,

JONES, DUNLAP & CO.

What is the amount due?

7. COLUMBUS, O., JULY 1, 1869.

SMITH & BELL

1868. *In account with* GEORGE STATIONER.

Feb.	1,	To	2½	M. Envelopes,	@	$5.75,
"	"	"	1½	reams Cap Paper,	@	8.00,
"	"	"	3	Blank Books,	@	1.25,
Mch.	9,	"	5	doz. Pencils,	@	1.25,
"	"	"	50	lbs. Wrapping Paper,	@	.10,
"	"	"	6	vols. Dickens,	@	1.75,
						$

Cr.

June	20,	By printing 1500 Circulars, .	$5.50
"	"	By printing Letter Heads, .	3.75
"	25,	By 33 tokens Press-work, . .	.50
			$
			$

What is the amount due?

DEFINITIONS.

Art. 84. A ***Bill of Goods*** is a written statement of goods sold, with the price of each article and the entire cost. It also gives the date and place of the sale, and the names of the buyer and seller.

A bill is drawn against the buyer, or *Debtor,* and in favor of the seller, or *Creditor.*

A bill is receipted by writing the words "*Received payment*" at the bottom, and affixing the

seller's name. A bill may be receipted by a clerk, agent, or any other authorized person, as in bill 2.

Art. 85. When sales are made at different times, the dates may be written at the left, as in bills 4, 5, and 7.

A bill presenting a debit and credit account between the parties, may be written and receipted as in bill 6.

Questions for Review.

What is United States money? What is it also called? Of what does United States money consist? What is each kind of money called?

What are the principal gold coins? Silver coins? Nickel coins? Copper coins? Name the two kinds of paper money. What is meant by "fractional currency"?

What are the principal denominations of United States money? Repeat the table. In what denominations are accounts kept? What use is made of the dollar sign? How are dollars and cents separated? Is the separatrix a period or a comma? Where is the figure denoting mills written?

How are dollars reduced to cents? Dollars to mills? Cents to mills? How are dollars and cents reduced to cents? Dollars, cents, and mills to mills? How are cents reduced to dollars? Mills to dollars?

Give the rule for adding or subtracting sums of money. Give the rule for multiplying or dividing a sum of money by an abstract number? By another sum of money?

What is a bill of goods? What does it contain? Against whom is it drawn? How is a bill receipted? By whom? Where are the dates of sales written? Where are items of credit written?

SECTION IX.

DENOMINATE NUMBERS.

LESSON I.

DRY MEASURE.

Art. 86. ***Dry Measure*** is used in measuring grain, fruit, most vegetables, coal, and many other dry articles.

The denominations are *pints, quarts, pecks,* and *bushels.*

Table.

2 pints (*pt.*) . .	are 1 quart, ...	*qt.*
8 quarts	are 1 peck, ...	*pk.*
4 pecks	are 1 bushel, . . .	*bu.*

1 bu. = 4 pk. = 32 qt. = 64 pt.

NOTES.—1. The standard bushel is $18\frac{1}{2}$ inches in diameter and 8 inches deep. It contains $2150\frac{2}{3}$ cubic inches.

2. In measuring grain, seeds, and small fruits, the measure must be *even* full; but in measuring corn in the ear, potatoes, apples, and other large articles, the measure must be *heaping* full.

1. How many pints in 3 quarts?

SOLUTION.—In 3 quarts there are 3 times 2 pints, which are 6 pints.

2. How many pints in 5 quarts? In 8 quarts? In 10 quarts?

3. How many quarts in 10 pints?

SOLUTION.—In 10 pints there are as many quarts as 2 pints are contained times in 10 pints, which is 5 times.

4. How many quarts in 8 pints? In 14 pints? In 16 pints? In 20 pints?

5. How many quarts in 3 pecks? In $5\frac{1}{2}$ pecks? In $7\frac{1}{4}$ pecks? In $10\frac{3}{4}$ pecks?

6. How many pecks in 16 quarts? In 20 quarts? In 32 quarts? In 56 quarts?

7. How many pecks in 5 bushels? In $7\frac{1}{2}$ bushels? In $9\frac{3}{4}$ bushels? In 11 bushels?

8. How many bushels in 12 pecks? In 20 pecks? In 32 pecks? In 40 pecks?

9. How many quarts in 8 pecks? In 12 pecks?

10. How many pints in 8 quarts? In 12 quarts?

11. What part of a quart is 1 pint? 2 pints?

12. What part of a peck is 1 quart? 3 quarts?

13. What part of a bushel is 1 peck? 2 pecks? 3 pecks? 4 pecks? 5 pecks?

14. How many pecks in 17 quarts? In 27 quarts? In 33 quarts?

15. How many bushels in 13 pecks? In 23 pecks? In 33 pecks?

16. What will $5\frac{1}{2}$ quarts of plums cost, at 4 cents a pint?

17. A man carried $3\frac{3}{4}$ pecks of cherries to market, and sold them at 10 cents a quart: how much did he receive?

18. If beans are worth $1.60 a bushel, how much are they worth a quart?

19. When apples sell at 20 cents a peck, what are they worth a bushel?

20. A boy bought half a bushel of chestnuts for $1.00, and sold them at 8 cents a quart: how much did he make?

WRITTEN EXERCISES.

21. How many pecks in 12 bushels? How many quarts? How many pints?

22. Reduce 12 bu. 3 pk. 1 pt. to pints.

1st PROCESS.

bu. pk. qt. pt.
12 + 3 + 0 + 1.
4
48 pk.
3
51 pk.
8
408 qt.
2
816 pt.
1
817 pt., *Ans.*

2d PROCESS.

bu. pk. qt. pt.
12 + 3 + 0 + 1.
4
51 pk.
8
408 qt.
2
817 pt., *Ans.*

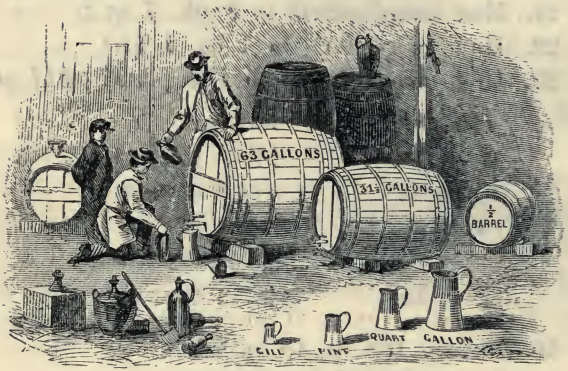
23. Reduce 5 bu. 2 pk. 7 qt. to pints.
24. Reduce 15 bu. 5 qt. 1 pt. to pints.
25. Reduce 8 bu. 3 pk. to quarts.
26. How many pints in 3 pk. 5 qt. 1 pt.?
27. How many quarts in 3 pk. 7 qt.?
28. How many pints in 1 bu. 1 qt.?
29. How many bushels in 768 pints? 817 pt.?

PROCESS.	PROCESS.
2)768 pt.	2)817 pt.
8)384 qt.	8)408 qt. + 1 pt.
4)48 pk.	4)51 pk.
12 bu.	12 bu. + 3 pk.
Ans. 12 bu.	*Ans.* 12 bu. 3 pk. 1 pt.

30. Reduce 168 qt. to bushels.
31. Reduce 342 pt. to bushels.
32. Reduce 51 pt. to pecks.
33. How many pecks in 37 pt.?
34. How many bushels in 151 qt.?
35. What will 3 pk. 5 qt. of cherries cost, at 5 cents a pint?
36. A man sold 1 bu. 3 pk. 5 qt. of clover-seed at 8 cents a quart: what did he receive?
37. A fruit dealer paid $7 for 3 bu. 3 pk. of peaches, and sold them at 75 cents a peck: what was his gain?
38. How many bushels of chestnuts can be bought for $15.50, at 5 cents a quart?
39. A fruit dealer put 3 bu. 2 pk. of strawberries into quart baskets: how many baskets were filled?

LESSON II.

LIQUID MEASURE.



Art. 87. ***Liquid Measure*** is used in measuring liquids; as, oil, milk, alcohol, etc.

The denominations are *gills, pints, quarts,* and *gallons.*

Table.

4 gills (*gi.*) . .	are 1 pint, ...	*pt.*
2 pints	are 1 quart, . . .	*qt.*
4 quarts	are 1 gallon, . . .	*gal.*

1 gal. = 4 qt. = 8 pt. = 32 gi.

NOTES.—1. The standard liquid gallon contains 231 cubic inches.

2. Beer, ale, and milk were formerly sold by Beer Measure, the gallon of which contains 282 cubic inches. Beer is still sometimes sold by this measure.

3. The size of casks for liquids is variable. Barrels generally contain 31½ gallons, and hogsheads 63 gallons.

1. How many gills in 3 pints? In 10 pints? In 20 pints? In 32 pints?

2. How many pints in 5 quarts? In $8\frac{1}{2}$ quarts? In 12 quarts? In $10\frac{1}{2}$ quarts?

3. How many quarts in 5 gallons? In $7\frac{3}{4}$ gallons? In 11 gallons?

4. How many pints in 16 gills? In 24 gills? In 32 gills? In 36 gills? In 40 gills?

5. How many quarts in 12 pints? In 16 pints? In 22 pints?

6. How many gallons in 20 quarts? 32 quarts? 28 quarts? 36 quarts? 40 quarts?

7. How many quarts in 8 pints? 15 pints? 19 pints? 13 pints? 21 pints?

8. How many pints in 8 quarts? 11 quarts? 16 quarts? $15\frac{1}{2}$ quarts? 20 quarts?

9. How many gallons in 8 quarts? 13 quarts? 21 quarts? 24 quarts? 29 quarts?

10. How many quarts in 6 gallons? $9\frac{3}{4}$ gallons? 11 gallons?

11. What part of a gallon is 1 quart? 2 quarts? 3 quarts? 4 quarts?

12. How many quarts in $\frac{3}{4}$ of a gallon?

13. What will 10 quarts of milk cost, at $5\frac{1}{2}$ cents a pint?

14. If a gallon of wine cost \$6, what will 1 pint cost?

15. If maple syrup cost \$1.60 a gallon, what will 1 quart cost?

16. At 4 cents a pint, what will 5 gallons of milk cost?

WRITTEN EXERCISES.

17. How many pints in 21 gallons?

18. How many gills in 7 gal. 3 qt. 1 gi.?

19. How many pints in 34 gal. 1 pt.?

20. Reduce 9 gal. 2 qt. 1 pt. to pints.

21. Reduce 38 pints to gallons.

22. Reduce 245 gills to gallons.

23. Reduce 130 gills to quarts.

24. Reduce 547 gills to gallons.

25. Reduce $45\frac{1}{2}$ gallons to gills.

26. Reduce 56 gal. 1 pt. to pints.

27. Reduce 305 pints to gallons.

28. What will 256 pints of maple syrup cost, at $1.30 a gallon?

29. How many vials, holding 2 gills each, can be filled from a gallon of alcohol?

30. How many jugs, each containing 1 gal. 2 qt., can be filled from a barrel of vinegar containing $31\frac{1}{2}$ gallons?

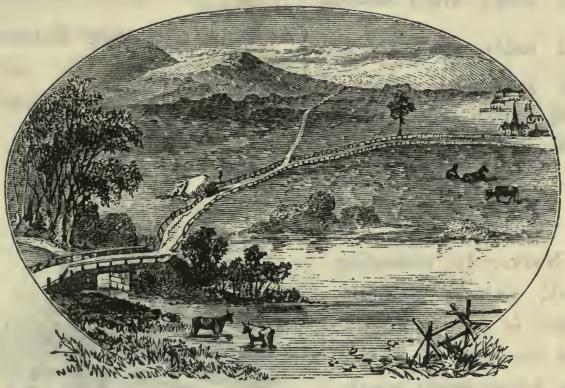
31. A grocer bought 25 gallons of maple syrup at $1.20 a gallon, and sold it at 40 cents a quart: how much did he gain?

32. A grocer bought 6 barrels of vinegar, containing $31\frac{1}{2}$ gallons each, at $6.50 a barrel, and sold it at 10 cents a quart. How much did he make?

33. A merchant bought a hogshead of molasses, containing 63 gallons, and sold $\frac{2}{3}$ of it at 75 cents a gallon, and the rest at 20 cents a quart: what did he receive for it?

LESSON III.

LONG MEASURE.



Art. 88. ***Long Measure*** is used in measuring lines or distances. It is also called *Linear Measure.*

The denominations are *inches, feet, yards, rods, furlongs,* and *miles.*

Table.

12	inches (*in.*)	are 1	foot,	. . . .	*ft.*
3	feet	are 1	yard,	. . . .	*yd.*
5½	yards . . .	are 1	rod,	. . . .	*rd.*
40	rods	are 1	furlong,	. . . .	*fur.*
8	furlongs . .	are 1	mile,	. . . .	*mi.*

1 mi. = 8 fur. = 320 rd. = 1760 yd. = 5280 ft. = 63360 in.

The following denominations are also used:

4 inches are 1 hand, { used in measuring the height of horses.

3 feet are 1 pace.

6 feet are 1 fathom, { used in measuring the depth of water.

3 miles are 1 league, { used in measuring distances at sea.

60 geographic miles, or, $69\frac{1}{2}$ statute miles (nearly), } are 1 degree at the equator.

360 degrees (°) make the circumference of the earth.

NOTE. — In measuring cloth, ribbons, etc., the width is not considered, and the yard is divided into *halves*, *fourths*, *eighths*, etc. The old table of Cloth Measure is seldom used.

1. How many inches in 1 foot? In 5 feet?
2. How many feet in 36 inches? In 72 inches?
3. How many feet in 4 yards? In 9 yards?
4. How many yards in 15 feet? In 21 feet?
5. How many yards in 2 rods? In 6 rods?
6. How many yards in 5 rods? In 9 rods?
7. How many rods in 2 furlongs? In 5 furlongs? In 8 furlongs?
8. How many furlongs in 80 rods? In 120 rods?

9. How many furlongs in 6 miles? In 9 miles?

10. How many miles in 32 furlongs? In 56 furlongs? In 72 furlongs?

11. How many rods in 66 paces?

12. A ditch is 28 furlongs long: how many miles long is it?

13. A vessel sunk in water 9 fathoms deep: what was the depth of the water in feet?

14. A steamer sails 3 leagues an hour: how many hours will it take it to sail 90 miles?

15. A horse is 15 hands high: what is its height in feet?

16. How many feet in a rod?

17. How many rods in a mile?

18. What part of a foot is 9 inches?

19. What part of a yard is 2 feet?

20. What part of a mile is 5 furlongs?

WRITTEN EXERCISES.

21. How many feet in 16 yards? How many inches?

22. How many inches in 3 fur. 20 rd. 3 yd.?

23. How many feet in 2 fur. 30 rd. 4 ft.?

24. Reduce 3 mi. 5 fur. 20 rd. to yards.

25. Reduce 1650 rods to miles.

26. Reduce 32274 inches to higher denominations.

27. Reduce 4 mi. 27 rd. 2 ft. 10 in. to inches.

28. How many steps of 2 ft. 6 in. each will a man take in walking 2 miles?

29. How many times will a wheel 6 feet in circumference turn round in going $2\frac{1}{2}$ miles?

30. Sound travels 1090 feet a second: how many miles will it travel in 60 seconds?

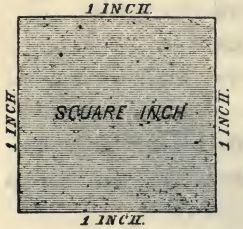
31. How many rods of fence will inclose a farm which is $\frac{1}{4}$ of a mile long and $\frac{1}{8}$ of a mile wide?

LESSON IV.

LAND OR SQUARE MEASURE.

Art. 89. ***Land*** or ***Square Measure*** is used in measuring surfaces. It is also called *Superficial Measure.*

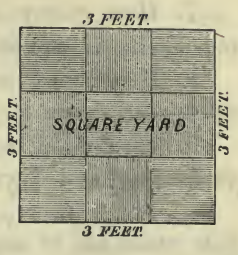
The denominations are *square inches, square feet, square yards, square rods* or *perches, roods, acres,* and *square miles.*



Art. 90. A ***Square Inch*** is a square, each side of which is an inch in length.

The figure at the left represents a square inch of real size.

A ***Square Yard*** is a square, each side of which is a yard, or three feet, in length. It contains 9 square feet.



NOTE.—The teacher should explain and define a right angle, a square, a rectangle, etc.

Table.

144	square inches (*sq. in.*)	are 1	square foot, . .	*sq. ft.*
9	square feet . . .	are 1	square yard, . .	*sq. yd.*
30¼	square yards . . .	are 1	square rod or perch,	*P.*
40	perches	are 1	rood, ...	*R.*
4	roods	are 1	acre, ...	*A.*
640	acres	are 1	square mile, . .	*sq. mi.*

NOTES.—1. Land Surveyors use Gunter's Chain, which is 4 rods or 66 feet long, and consists of 100 links, each link being $7\frac{92}{100}$ inches long. A square chain is 16 square rods, and 10 square chains are 1 acre.

2. Glazing and stone-cutting are estimated by the square foot; painting, plastering, paper-hanging, ceiling, and paving, by the square yard; and flooring, roofing, tiling, and bricklaying, by the square of 100 feet. Brick-laying is also estimated by the square yard, and by the 1000 bricks.

1. How many square feet in 5 square yards? In 7 square yards?

2. How many square yards in 36 square feet? In 72 square feet? In 90 square feet?

3. How many perches in 2 roods? In 5 roods?

4. How many roods in 80 perches? In 120 perches?

5. How many roods in 8 acres? In 12 acres?

6. How many acres in 16 roods? In 40 roods?

7. How many square chains in 32 square rods? In 64 square rods? In 80 square rods?

8. How many acres in 20 square chains? In 40 square chains? In 80 square chains?

9. How many square yards in a pavement 10 yards long and 4 yards wide?

SOLUTION.—In a pavement 10 yards long and 1 yard wide, there are 10 square yards, and in a pavement 10 yards long and 4 yards wide, there are 4 times 10 square yards, which are 40 square yards. There are 40 square yards in the pavement.

10. How many square yards in a ceiling 8 yards long and 6 yards wide?

11. How many square feet in a board 16 feet long and $1\frac{1}{2}$ feet wide?

12. How many perches in a field 30 rods long and 10 rods wide? How many roods?

13. How many square inches in a piece of tin 15 inches long and 4 inches wide?

14. How many square yards in a floor 15 feet long and 12 feet wide?

WRITTEN EXERCISES.

15. How many square yards in 16 perches? How many square inches?

16. How many perches in 5 A. 2 R.?

17. Reduce 1 A. 2 rd. 20 P. 10 sq. yd. 7 sq. ft. to square feet.

18. Reduce 70882 sq. ft. to higher denominations.

19. Reduce 5280 perches to higher denominations.

20. Reduce 5184 square inches to square yards.

21. How many acres in a field 56 rods long and 40 rods wide?

22. How many acres in a street $2\frac{1}{2}$ miles long and 4 rods wide?

23. How many square yards in a ceiling 72 feet long and $40\frac{1}{2}$ feet wide?

24. What will it cost to pave a walk 60 feet long and 15 feet wide, at $1.25 a square yard?

25. How many peach trees can be planted in an orchard containing 3 acres, if a tree be planted on each square rod?

26. If 1000 shingles will cover 100 square feet, how many shingles will cover a roof 40 feet long and 25 feet wide?

27. How many acres of land in a township 5 miles square?

28. How many acres in a township 7 miles long and 6 miles wide?

29. How many yards of carpeting, a yard wide, will carpet a room $20\frac{2}{3}$ feet long and 18 feet wide?

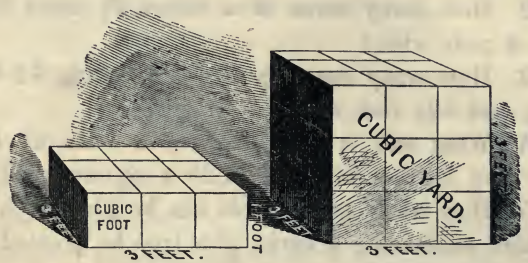
30. How many bricks, 8 in. long and 4 in. wide, will pave a walk 60 feet long and $12\frac{3}{4}$ feet wide?

31. What will it cost to plaster the walls and ceiling of a room 15 feet long, 12 feet wide, and 9 feet high, at 50 cents a square yard?

Art. 91. RULE.—To find the area of a rectangle, *Multiply the length by the width.*

LESSON V.

CUBIC MEASURE.



Art. 92. ***Cubic Measure*** is used in measuring solids. It is also called *Solid Measure.*

The denominations are *cubic inches, cubic feet,* and *cubic yards.*

A cubic inch is a cube whose edges are each one inch long. A cubic yard is a cube whose edges are each one yard long.

NOTE.—The teacher should explain and define a cube; also its faces and edges.

Table.

1728 cubic inches (*cu. in.*) are 1 cubic foot, *cu. ft.*
27 cubic feet . . . . are 1 cubic yard, *cu. yd.*

1 cu. yd. = 27 cu. ft. = 46656 cu. in.

NOTE.—A cubic yard of earth is called a load, and 24¾ cubic feet of stone or of masonry make a perch.

WRITTEN EXERCISES.

1. How many cubic inches in 5 cubic feet? In 12 cubic feet? 32 cubic feet?

2. How many cubic feet in 15552 cubic inches?

3. How many cubic feet in 120 cubic yards?

4. How many cubic yards in 405 cubic feet?

5. Reduce 15 cu. yd. 16 cu. ft. and 1305 cu. in. to inches.

6. Reduce 1473462 cubic inches to higher denominations.

7. How many cubic feet in a block of marble 15 feet long, 12 feet wide, and 5 feet thick?

PROCESS.

15 ft.
12
180 sq. ft.
5
900 cu. ft.

A block 15 feet long and 12 feet wide has 180 sq. ft. in its lower face, and since each foot of thickness gives 180 cubic feet, five feet of thickness will give 5 times 180 cu. ft., which are 900 cu. ft.

8. How many cubic feet in a rock 18 feet long, 13 feet wide, and 8 feet high?

9. How many cubic feet in a pile of wood 24 feet long, 3 feet wide, and 8 feet high?

10. How many cubic yards in a bin $9\frac{1}{2}$ feet long, 6 feet wide, and $4\frac{3}{4}$ feet deep?

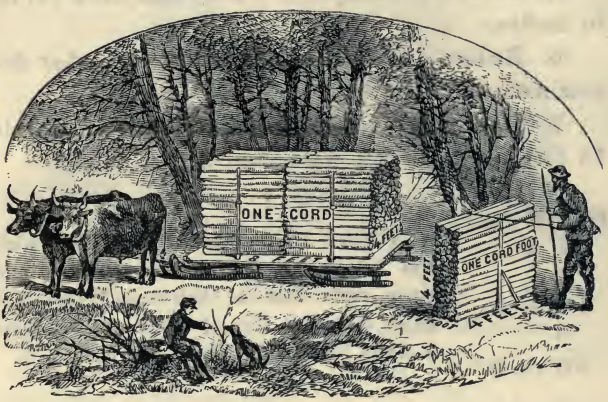
11. How many cubic feet of earth must be removed to make a cellar 44 feet long, 27 feet wide, and 5 feet deep? How many cubic yards?

12. How many cubic yards of earth must be removed to make a reservoir 120 feet long, 54 feet wide, and 9 feet deep below the surface?

13. What will it cost to dig a cellar 36 feet long, $18\frac{3}{4}$ feet wide, and $6\frac{1}{2}$ feet deep, at \$2.50 a cubic yard?

LESSON VI.

WOOD MEASURE.



Art. 93. ***Wood Measure*** is used in measuring wood and rough stone.

The denominations are *cubic feet, cord feet,* and *cords.*

Table.

16 cubic feet, . .	are 1 cord foot, .	*cd. ft.*
8 cord feet, or, 128 cubic feet,	are 1 cord, . . .	*cd.*

1 cd. = 8 cd. ft. = 128 cu. ft.

NOTES.—1. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains 1 cord; and 1 foot in length of such a pile contains one cord foot. (See cut above.)

2. Formerly 40 feet of round timber, or 50 feet of hewn timber, were called a ton, but this distinction is now seldom observed.

WRITTEN EXERCISES.

1. How many cord feet in a pile of wood 4 feet long, 4 feet wide, and 5 feet high?

2. How many cubic feet in 6 cord feet?

3. How many cord feet in $5\frac{1}{2}$ cords of wood?

4. How many cords of wood in 128 cord feet?

5. How many cords of wood in a pile containing 1536 cubic feet?

6. How many cords of wood in a pile 20 feet long, 4 feet wide, and 6 feet high?

7. How many cords of wood in a pile 48 feet long, $2\frac{1}{2}$ feet wide, and $5\frac{1}{2}$ feet high?

8. A man bought a pile of wood 36 feet long, 4 feet wide, and 8 feet high, and paid \$5.50 a cord. What did the wood cost him?

9. How many cords of stone in a wall 40 rods long, 2 feet thick, and 4 feet high?

10. At \$4.50 a cord, what is the value of a pile of wood, 40 feet long, $3\frac{1}{2}$ feet wide, and $6\frac{1}{2}$ feet high?

LESSON VII.

CIRCULAR MEASURE.

Art. 94. ***Circular Measure*** is used in measuring arcs of circles, and angles, and in estimating latitude and longitude. It is also called *Angular Measure.*

The denominations are *seconds, minutes, degrees, signs,* and *circumferences.*

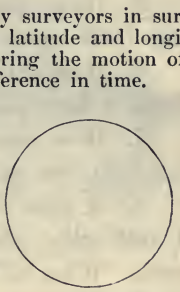
Table.

60 seconds (″)	are 1 minute,	. .	′
60 minutes . .	are 1 degree,	. . .	°
30 degrees . .	are 1 sign,	. . .	*S.*
12 signs, or 360 degrees,	are 1 circumference,		*C.* or *cir.*

1 cir. = 12 S. = 360° = 21600′ = 1296000″.

NOTES.—1. Circular Measure is used by surveyors in surveying land; by navigators in determining latitude and longitude at sea; and by astronomers in measuring the motion of the heavenly bodies, and in computing difference in time.

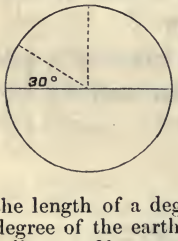
2. The portion of surface represented by the annexed figure is a *circle.*



The curved line which bounds the circle is its *circumference.*

Any portion of a circumference is an *arc.*

3. One-half of a circumference is called a *semi-circumference.*



One-fourth of a circumference is called a *quadrant.*

One-third of a quadrant is called a *sign.*

A semi-circumference contains 180°; a quadrant, 90°; and a sign, 30°.

4. Every circumference is divided into 360 equal parts, called degrees, and, hence, the length of a degree depends upon the size of the circle. A degree of the earth's surface at the equator contains 69½ statute miles, or 60 geographical miles—a minute of space being a geographical or nautical mile.

1. How many minutes in 5 degrees?
2. How many signs in 3 quadrants?
3. How many degrees in $\frac{1}{4}$ of a quadrant?
4. How many degrees in $3\frac{1}{3}$ signs?
5. How many signs in $\frac{1}{6}$ of a circumference?

WRITTEN EXERCISES.

6. How many seconds in 15° 30′?
7. Reduce 15° 33′ to minutes.
8. Reduce $5\frac{1}{2}$ signs to minutes.
9. Reduce 10800″ to signs.
10. The sun appears to revolve around the earth once a day: how many degrees does it appear to pass over in an hour? In 6 hours?

LESSON VII.

TIME MEASURE.

Art. 95. ***Time Measure*** is used in measuring time or duration.

The denominations are *seconds, minutes, hours, days, years* and *centuries.*

Table.

60	seconds (*sec.*)	are 1	minute, . . .	*min.*
60	minutes . .	are 1	hour, . . .	*h.*
24	hours . . .	are 1	day, ...	*d.*
365	days . . .	are 1	common year, .	*c. yr.*
366	days . . .	are 1	leap year, . .	*l. yr.*
100	years (365¼ d.)	are 1	century, . . .	*C.*

1 d. = 24 h. = 1440 min. = 86400 sec.

The following denominations are also used:

7 days	are 1 week, . . .	*w.*
4 weeks	are 1 lunar month,	*lr. m.*
13 lr. m. 1 d. 6 h., or 365¼ days,	are 1 Julian year,	*J. yr.*
12 calendar months	are 1 civil year, .	*c. yr.*

NOTES.—1. The exact length of a solar year is 365 d. 5 h. 48 min. 48 sec., which is nearly 6 hours, or ¼ of a day, longer than the common year. Since the common year lacks ¼ of a day of the true time, an additional day is added to every fourth year, making *leap year*. This additional day is given to February, and hence this month in leap year contains 29 days. The leap years are exactly divisible by 4; as, 1860, 1864, 1868, 1872, etc.

2. The names and order of the calendar months and the number of days in each are as follows:

January,	1st	month,	31 days.	July,	7th	month,	31 days.
February,	2d	"	28 or 29.	August,	8th	"	31 "
March,	3d	"	31 days.	September,	9th	"	30 "
April,	4th	"	30 "	October,	10th	"	31 "
May,	5th	"	31 "	November,	11th	"	30 "
June,	6th	"	30 "	December,	12th	"	31 "

3. The following couplet will assist in remembering the months which have 30 days each:

Thirty days hath September,
April, June, and November.

4. In most business transactions, 30 days are considered a month, and 360 days a year.

5. The year is divided into four seasons of three months each, as follows:

SPRING,	March, April, May.	AUTUMN or FALL,	September, October, November.
SUMMER,	June, July, August.	WINTER,	December, January, February.

1. How many seconds in 5 minutes? In 10 minutes? In 20 minutes?

2. How many minutes in 4 hours? In 8 hours?

3. How many hours in 120 minutes? In 240 minutes? In 300 minutes?

4. How many hours in 3 days? In 5 days?

5. How many days in 48 hours? In 72 hours? In 240 hours? In 480 hours?

6. How many days in 6 weeks? In 8 weeks? In 10 weeks? In 15 weeks?

7. How many weeks in 35 days? In 49 days?

8. How many weeks in 5 lunar months? In 12 lunar months?

9. How many lunar months in 16 weeks? In 32 weeks? 44 weeks? 60 weeks?

10. How many calendar months in 5 years? In 7 years? 10 years? 12 years?

WRITTEN EXERCISES.

11. How many seconds in 15 hours?

12. How many hours in 28800 seconds?

13. Reduce 5 d. 13 h. 40 min. to seconds.

14. Reduce 31 d. 30 min. 45 sec. to seconds.

15. Reduce 30600 minutes to higher denominations.

16. Reduce 52560 hours to common years.

17. How many minutes in a leap year?

18. How many seconds in the solar year, which contains 365 d. 5 h. 48. min. 48 sec.?

19. How many seconds in a common year?

20. The age of a certain man is 64 yr. 45 d. 12 h.: how many hours has he lived, allowing $365\frac{1}{4}$ days to the year?

21. How many hours in the three Spring months? In the three Summer months?

22. How many minutes will there be in the month of February, 1876? In February, 1878?

23. If your pulse beat 75 times a minute, how many times will it beat in 5 weeks?

24. How many days will it take a steamship to sail 3744 miles, if it sail at the rate of 12 miles an hour?

LESSON VIII.

AVOIRDUPOIS WEIGHT.

Art. 96. ***Avoirdupois Weight*** is used in weighing all articles except gold, silver, and the precious stones.

The denominations are *drams, ounces, pounds, hundred-weights,* and *tons.*

Table.

16 drams (*dr.*) . are 1 ounce, ... *oz.*
16 ounces . . . are 1 pound, . . . *lb.*
100 pounds . . . are 1 hundred-weight, *cwt.*
20 hundred-weights are 1 ton, ... *T.*

1 T. = 20 cwt. = 2000 lb. = 32000 oz. = 512000 dr.

---

196 pounds of flour, } . . . . are 1 barrel.
200 lb. pork or beef, }
100 lb. of fish . . . . . are 1 quintal.
14 lb. lead or iron . . . . . are 1 stone.
56 lb. of corn, rye, or flax-seed, }
60 lb. of wheat or clover-seed, } are 1 bushel.
32 lb. of oats, }

NOTES.—1. In wholesaling and freighting coal and in invoicing English goods at the United States custom-houses, the hundred-weight is divided into 4 quarters, of 28 pounds each, and the ton contains 2240 pounds. This is called the *long* or *gross* ton, while the ton of 2000 pounds is called the *short* or *net* ton.

2. The dram is seldom used in business transactions, and the quarter of 25 pounds is never used.

1. How many drams in 2 ounces? In $5\frac{1}{2}$ ounces? 10 ounces? 15 ounces?

2. How many ounces in 48 drams? In 64 drams? 96 drams? 160 drams?

3. How many ounces in 4 pounds? In $6\frac{3}{4}$ pounds? $10\frac{5}{8}$ pounds? $12\frac{1}{2}$ pounds?

4. How many pounds in 80 ounces?

5. How many pounds in 5 hundred-weight? In 8 cwt.? $12\frac{3}{5}$ cwt.? 25 cwt.?

6. How many hundred-weight in 4 tons? In 6 tons? $8\frac{2}{5}$ tons? $12\frac{1}{2}$ tons?

7. What will $\frac{3}{4}$ of a pound of candy cost, at 2 cents an ounce?

8. What will $\frac{2}{5}$ of a hundred-weight of flour cost, at 5 cents a pound?

WRITTEN EXERCISES.

9. Reduce 5 tons to ounces.

10. Reduce 3 T. 14 cwt. 56 lb. to pounds.

11. Reduce 5 cwt. 77 lb. 13 oz. to ounces.

12. Reduce 34920 pounds to tons.

13. Reduce 4560 ounces to higher denominations.

14. Reduce 11 T. 38 lb. 15 oz. to drams.

15. What will a barrel of flour cost, at 6 cents a pound?

16. What will 3 barrels of pork cost, at 15 cents a pound?

17. How many barrels will 3920 pounds of flour make?

18. A farmer sold 3600 pounds of wheat at $1.75 a bushel: how much did he receive?

19. A hay-stack contains 9000 pounds of hay: what is it worth at $12 a ton?

20. What will it cost to transport 50 T. 15 cwt. 75 lb. of freight, at $\frac{1}{2}$ cent a pound?

21. A farmer exchanged $45\frac{3}{4}$ pounds of butter, at 20 cents a pound, for sugar, at 15 cents a pound: how much sugar did he receive?

LESSON IX.

TROY WEIGHT.

Art. 97. ***Troy Weight*** is used in weighing gold, silver, and precious stones, and also in philosophical experiments.

The denominations are *grains, pennyweights, ounces,* and *pounds.*

Table.

24 grains (*gr.*) .	are 1 pennyweight,	*pwt.*
20 pennyweights .	are 1 ounce, . . .	*oz.*
12 ounces . . .	are 1 pound, . .	*lb.*

1 lb. = 12 oz. = 240 pwt. = 5760 gr.

NOTES.—1. Diamonds are weighed by carats and fractions of carats. A carat is 4 Troy grains.

2. The purity of gold is also expressed in carats, a carat meaning $\frac{1}{24}$ part. Gold that is 22 carats fine contain 22 parts of pure gold and 2 parts of alloy.

1. How many grains in 5 pennyweights? In 3 pwt.? 8 pwt.? 10½ pwt.?

2. How many pennyweights in 3 ounces? In 6 ounces? 9 ounces? 10 ounces?

3. How many ounces in 40 pennyweights? In 80 pwt.? 100 pwt.? 120 pwt.?

4. How many ounces in 4 pounds? In $7\frac{1}{2}$ pounds? $12\frac{3}{4}$ pounds? 20 pounds?

5. How many pounds in 36 ounces? In 60 ounces? 84 ounces? 96 ounces?

6. What part of a pound is 1 ounce? 6 ounces? 8 ounces? 9 ounces?

WRITTEN EXERCISES.

7. Reduce 44 lb. 3 oz. 13 pwt. to pennyweights.
8. Reduce 7 oz. 15 pwt. to grains.
9. Reduce 56 lb. 13 pwt. to grains.
10. Reduce 13486 pwt. to higher denominations.
11. Reduce 40408 grains to higher denominations.
12. Reduce 5680 ounces to pounds.
13. Reduce 5280 grains to ounces.
14. A lady bought a pearl necklace, weighing 8 oz. 15 pwt., at 75 cents a grain: what did it cost?

LESSON X.

APOTHECARIES WEIGHT.

Art. 98. ***Apothecaries Weight*** is used by physicians in prescribing and by apothecaries in mixing medicines.

The denominations are *grains, scruples, drams, ounces,* and *pounds.*

Table.

24 grains (*gr.*)	. are 1 scruple,	. . .	℈.
3 scruples	. . are 1 dram,	. . . .	ʒ.
8 drams	. . are 1 ounce,	. . . .	℥.
12 ounces	. . are 1 pound,	. . . .	lb.

NOTE.—Medicines are bought and sold in quantities by avoirdupois weight.

1. How many grains in 2 scruples?

2. How many scruples in 5 drams? In 7 drams? 9 drams? 12 drams? 20 drams?

3. How many drams in 21 scruples? In 27 scruples? 33 scruples? 40 scruples?

4. How many drams in 5 ounces? In 8 ounces? 10 ounces? 12 ounces?

5. How many pounds in 36 ounces? In 72 ounces? 96 ounces? 120 ounces?

6. How many ounces in 5 pounds? In 8 pounds? $10\frac{1}{3}$ pounds? 12 pounds?

WRITTEN EXERCISES.

7. Reduce 16 lb. 11 ℥ 5 ʒ 2 ℈ 10 gr. to grains.

8. Reduce 10 ℥ 3 ʒ to grains.

9. Reduce 356 ʒ to pounds.

10. Reduce 26484 gr. to higher denominations.

11. How many pounds in 5760 ℈?

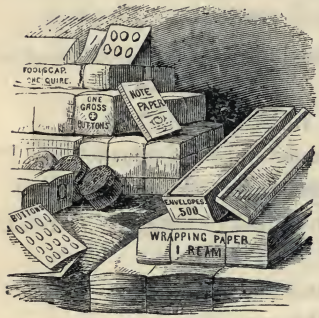
12. How many doses, of 18 gr. each, in 5 ʒ 2 ℈ of tartar emetic?

13. How many pills, of 5 gr. each, can be made from 1 ℥ 2 ʒ 2 ℈ of calomel?

14. How many ounces of calomel will make 480 pills, each weighing 6 grains?

LESSON XI.

MISCELLANEOUS TABLE.



PAPER.

24 sheets	are 1	quire.
20 quires	are 1	ream.
2 reams	are 1	bundle.
5 bundles	are 1	bale.

12 things	are 1	dozen.
12 dozen	are 1	gross.
12 gross	are 1	great gross.
20 things	are 1	score.

NOTE.—A sheet of paper folded in 2 leaves is called a *folio;* in 4 leaves, a *quarto,* or 4*to;* in 8 leaves, an *octavo,* or 8*vo;* in 12 leaves, a *duodecimo,* or 12*mo;* in 18 leaves, an 18*mo.*

1. How many sheets of paper in $5\frac{1}{2}$ quires?

2. How many quires of paper in 4 reams? In 8 reams? $12\frac{3}{5}$ reams? 15 reams?

3. How many bundles of paper in 6 reams? In 12 reams? 18 reams? 32 reams?

4. How many eggs in 5 dozen? In $7\frac{3}{4}$ dozen? $8\frac{1}{3}$ dozen? 12 dozen? 20 dozen?

5. How many years are 4 score years? 3 score years and 10?

WRITTEN EXERCISES.

6. How many sheets of paper in $12\frac{1}{2}$ reams?

7. Reduce 6 rm. 15 qu. 12 sheets to sheets.

8. What will 7200 sheets of paper cost, at $8.50 a ream?

9. How many crayons are there in 36 boxes, if each box contains one gross?

10. If a shirt require 6 buttons, how many shirts will 12 gross of buttons trim?

11. What will 44 gross of lead-pencils cost, at 75 cents a dozen?

12. A stationer bought 15 reams of letter-paper at $3.50 a ream, and sold it at 25 cents a quire. How much did he gain?

LESSON XII.

DEFINITIONS, PRINCIPLES, AND RULES.

Art. 99. A ***Denominate Number*** is a number composed of concrete units of one or several denominations.

Art. 100. Denominate Numbers are either *Simple* or *Compound.*

A ***Simple Denominate Number*** is composed of units of the same denomination; as, 7 quarts.

A ***Compound Denominate Number*** is composed of units of several denominations; as, 5 bu. 3 pk. 7 qt.

NOTE. — Compound Denominate Numbers are properly called *Compound Numbers*, since every compound number is necessarily denominate.

Art. 101. Denominate numbers express *Currency, Measure,* and *Weight.*

Currency is the circulating medium used in trade and commerce as a representative of value.

Measure is the representation of extent, capacity, or amount.

Weight is a measure of the force called gravity, by which bodies are drawn toward the earth.

Art. 102. The following diagram represents the three general classes of denominate numbers, their subdivisions, and the tables included under each:

- 1. Currency,
  - 1. Coin,
  - 2. Paper Money,

  } United States Money.
- 2. Measure,
  - 1. Of extension,
    - 1. Lines, or arcs,
      - 1. Long Measure,
      - 2. Circular Measure.
    - 2. Surfaces: Square Measure.
    - 3. Capacity,
      - 1. Cubic Measure,
      - 2. Wood Measure,
      - 3. Dry Measure,
      - 4. Liquid Measure.
  - 2. Of duration: Time Measure.
- 3. Weight,
  - 1. Avoirdupois Weight.
  - 2. Troy Weight.
  - 3. Apothecaries Weight.

Art. 103. The ***Reduction*** of a denominate number is the process of changing it from one denomination to another without altering its value.

Art. 104. Reduction is of two kinds, *Reduction Descending* and *Reduction Ascending.*

Reduction Descending is the process of changing a denominate number from a higher to a lower denomination. It is performed by multiplication.

Reduction Ascending is the process of changing a denominate number from a lower to a higher denomination. It is performed by division.

Art. 105. RULE FOR REDUCTION DESCENDING.—

1. *Multiply the number of the highest denomination by the number of units of the next lower which equals a unit of the higher, and to the product add the number of the lower denomination, if any.*

2. *Proceed in like manner with this and each successive result thus obtained, until the number is reduced to the required denomination.*

NOTE.—The successive denominations of the compound number should be written in their proper order, and the vacant denominations, if any, filled with ciphers.

Art. 106. RULE FOR REDUCTION ASCENDING.—

1. *Divide the given denominate number by the number of units of its own denomination, which equals one unit of the next higher, and place the remainder, if any, at the right.*

2. *Proceed in like manner with this and each successive quotient thus obtained, until the number is reduced to the required denomination.*

3. *The last quotient, with the several remainders annexed in proper order, will be the answer required.*

Questions for Review.

What is a number? What is an abstract number? What is a denominate number? Into what two classes are denominate numbers divided? Define each.

By what other name are compound denominate numbers usually called? Why may the word "denominate" be omitted?

What is currency? Of how many kinds of money is United States currency composed?

What is meant by measure? Name the two kinds of measures. How are the measures of extension divided?

What tables are used in measuring lines? In measuring surfaces? In measuring contents? In measuring capacity? What table is used in measuring duration? What tables are used in measuring the weight of bodies?

What is Reduction? Name the two kinds of reduction. Define Reduction Descending. Repeat the rule. Define Reduction Ascending. Repeat the rule.

For what is Dry Measure used? Name the denominations. Repeat the table. For what is Liquid Measure used? Name the denominations. Repeat the table. For what is Long Measure used? Name the denominations. Repeat the table.

For what is Square Measure used? Name the denominations. Repeat the table. What is a square inch? A square yard? How is the area of a rectangle found?

For what is Cubic Measure used? Name the denominations. Repeat the table. What is a cubic inch? A cubic yard? For what is Wood Measure used? Name the denominations. Repeat the table.

For what is Circular Measure used? Name the denominations. Repeat the table. For what is Time Measure used? Name the denominations. Repeat the table.

Name the calendar months in their order, and give the number of days in each. How many days has February in

leap years? Name the four seasons of the year and the months of each.

For what are the three weights respectively used? Give the denominations and repeat the table of each. Repeat the miscellaneous table.

LESSON XIII.

Miscellaneous Review Problems.

1. How many quarts in $\frac{5}{8}$ of a bushel?
2. How many pints in $3\frac{3}{4}$ gallons?
3. How many hours in $\frac{3}{7}$ of a week?
4. How many ounces in $2\frac{1}{2}$ pounds of sugar? In $2\frac{1}{2}$ pounds of silver?
5. What will $\frac{2}{5}$ of a cwt. of sugar cost, at 15 cents a pound?
6. What will $\frac{3}{8}$ of a gallon of oil cost, at 25 cents a pint?
7. What will $\frac{3}{5}$ of a ream of paper cost, at 20 cents a quire?
8. What will $\frac{2}{5}$ of a ton of hay cost, at 75 cents a cwt.?
9. A boy picked 3 pecks of cherries, and sold them at 10 cents a pint: how much did he receive?
10. If a ship sail 3 leagues an hour, in how many hours will it sail 63 miles?
11. How many half-pint bottles will a gallon of sweet oil fill?
12. How many quart baskets will 3 pk. 5 qt. of strawberries fill?
13. How many leap years in every century?
14. How many calendar months in 20 years?

WRITTEN EXERCISES.

15. A fruit dealer bought 24 barrels of apples, containing $2\frac{3}{4}$ bushels each, at \$2.50 a barrel, and sold them at \$1.25 a bushel: what was his gain?

16. What will 20 yd. 2 ft. of iron railing cost, at \$1.25 a foot?

17. What will 40 miles of telegraph wire cost, at 25 cents a yard?

18. How many times will a carriage-wheel 11 feet in circumference turn round in running 2 miles?

19. How many times will a car-wheel 5 feet in circumference turn round in running from Columbus to Cincinnati, the distance being 120 miles?

20. How many acres in a township 6 miles square?

21. What will a piece of land 40 rods long and 32 rods wide cost, at \$75 an acre?

22. How many hills of corn can be planted on 5 acres, allowing 1 hill to every square yard?

23. How many people can stand on a terrace 250 feet long and 120 feet wide, allowing 4 persons to each square yard?

24. What will it cost to gravel a street 129 rods long and 60 feet wide, at 75 cents a square yard?

25. How many square yards of plastering in the walls and ceiling of a room 21 feet long, 18 feet wide, and 9 feet high?

26. How many yards of carpeting, a yard wide, will carpet a room $18\frac{1}{3}$ feet long and 15 feet wide?

27. If 1000 shingles will cover 100 square feet, how many shingles will cover a roof each side of which is 48 feet long and 15 feet wide?

28. A park containing 40 acres is 50 rods wide: how long is it?

29. How many cubic feet in a bin 12 feet long, 8 feet wide, and $3\frac{1}{2}$ feet deep?

30. How many perches of stone in a wall 99 feet long, 8 feet high, and $1\frac{1}{2}$ feet thick?

31. What will it cost to dig a ditch 80 rods long, $4\frac{1}{2}$ feet wide, and 2 feet deep, at 15 cents a cubic yard?

32. At $4.50 a cord, what will be the cost of a pile of wood 48 feet long, 6 feet high, and 4 feet wide?

33. How many times will a clock that ticks seconds, tick in the month of June?

34. If a person read a half hour each day, how many hours will he read in 40 years, of $365\frac{1}{4}$ days each?

35. How many gold rings, each weighing 4 pwt., can be made from a bar of gold weighing 1 lb. 4 oz.?

36. A car contains 80 barrels of pork, and another 80 barrels of flour: what is the difference in the freight of the two cars?

37. How many gross of pens will supply 4320 pupils one year, if each pupil require 4 pens?

38. If 10 sheets of paper will make a 16mo book of 320 pages, how many reams will it take to publish an edition of 2000 copies?

SECTION X.

COMPOUND NUMBERS.

LESSON I.

Addition of Compound Numbers.

1. What is the sum of 5 bu. 3 pk. 6 qt. 1 pt.; 8 bu. 2 pk. 1 pt.; 10 bu. 1 pk. 3 qt.; and 3 pk. 5 qt. 1 pt.?

PROCESS.

bu.	pk.	qt.	pt.
5	3	6	1
8	2	0	1
10	1	3	0
	3	5	1
25 bu.	2 pk.	7 qt.	1 pt.

Write the compound numbers so that terms of the same denomination shall stand in the same column. Add first the column of pints. The sum is 3 pints, which equal 1 qt. 1 pt. Write the 1 pt. under the pints, and add the 1 qt. with the column of quarts. The sum of the quarts is 15 quarts, which equal 1 pk. 7 qt. Write the 7 qt. under the quarts, and add the 1 pk. with the column of pecks. The sum of the pecks is 10 pecks, which equal 2 bu. 2 pk. Write the 2 pk. under the pecks, and add the 2 bu. with the column of bushels. The sum of the bushels is 25 bushels. The sum of the four compound numbers added is 25 bu. 2 pk. 7 qt. 1 pt.

(2)

bu.	pk.	qt.	pt.
16	2	6	1
23	1	4	0
40	3	0	1
9	0	2	0

(3)

gal.	qt.	pt.	gi.
21	3	1	3
16	0	1	2
48	2	0	0
35	0	1	3

(4)

mi.	fur.	rd.	yd.	ft.	in.
19	7	39	5	2	10
27	3	24	3	1	6
45	4	33	0	0	7
6	0	17	2	1	0

(5)			
cwt.	lb.	oz.	dr.
15	63	11	13
18	85	0	10
6	15	15	0
0	75	8	7
19	36	14	15

(6)			
lb.	oz.	pwt.	gr.
9	11	19	23
13	6	13	20
7	10	8	11
	9	15	16
23	0	10	9

(7)				
lb.	℥.	ʒ.	℈.	gr.
44	11	7	2	19
23	9	6	0	8
	10	5	2	16
27	7	6	1	14
16	3	0	0	18

8. What is the sum of 15 w. 5 d. 22 h. 45 min. 34 sec.; 8 w. 6 d. 13 h.; 3 w. 20 h. 52 min.; 4 d. 22 h. 33 min. 55 sec.; 1 w. 2 d. 3 h. 30 min.?

9. Add 14° 30′ 46″; 53° 16′ 49″; 26° 34′ 15″; 18° 44′ 33″; 62° 36′; and 43° 45″.

10. Add 5 sq. mi. 625 A. 3 R. 35 P.; 14 sq. mi. 546 A. 2 R. 28 P.; 486 A. 1 R. 27 P.; 94 A. 24 P.; and 14 sq. mi. 300 A. 3 R. 36 P.

11. A wood dealer bought 5 piles of wood, the first containing 21 cd. 5 cd. ft. 15 cu. ft.; the second, 45 cd. 12 cu. ft.; the third, 18 cd. 7 cd. ft.; the fourth, 50 cd. 6 cd. ft. 14 cu. ft.; and the fifth, 16 cd. 5 cd. ft. How much wood did he purchase?

12. A printer used 3 bundles 1 ream 16 quires of paper on Monday; 2 bundles 1 ream on Tuesday; 4 bundles 16 quires on Wednesday; 3 bundles 1 ream 18 quires on Thursday; 5 bundles on Friday; and 3 bundles 1 ream on Saturday. How much paper did he use during the week?

13. The four quarters of an ox weighed respectively 2 cwt. 84 lb. 10 oz.; 3 cwt. 1 lb. 14 oz.; 2 cwt. 76 lb. 4 oz.; and 2 cwt. 98 lb. 14 oz. What was the weight of the four quarters?

14. A garden has four unequal sides. The first is 4 rd. 3 yd. 2 ft. 8 in.; the second, 5 rd. 1 ft. 10 in.; the third, 4 rd. 5 yd. 4 in.; and the fourth, 3 rd. 4 yd. 2 ft. 9 in. What is the distance round the garden?

15. A cistern full of water was emptied by 3 pipes. The first discharged 45 gal. 3 qt.; the second, 54 gal. 1 pt.; and the third, 61 gal. 2 qt. 1 pt. How much water did the cistern contain?

DEFINITIONS, PRINCIPLE, AND RULE.

Art. 107. A ***Compound Number*** is a number composed of units of several denominations.

Art. 108. The numbers expressing the successive denominations of a compound number, are called its *Terms*.

Compound numbers are of the *same kind* when their corresponding terms express units of the same denomination; as, 3 bu. 2 pk., and 6 bu. 3 pk. 5 qt.

Art. 109. ***Compound Addition*** is the process of finding the sum of two or more compound numbers of the same kind.

Art. 110. PRINCIPLE.—In both simple and compound addition, *the sum of each column is divided by the number of units of that denomination, which equals one of the next higher denomination.* In simple addition this divisor is 10; in compound addition it is a varying number, since the several denominations are expressed on a *varying scale.*

Art. 111. RULE.—1. *Write the compound numbers to be added so that units of the same denomination shall stand in the same column.*

2. *Add first the column of the lowest denomination, and divide the sum by the number of units of that denomination, which equals a unit of the next higher denomination; write the remainder under the column added, and add the quotient with the next column.*

3. *In like manner add the remaining columns, writing the sum of the highest column under it.*

LESSON II.

Subtraction of Compound Numbers.

1. From 13 lb. 5 oz. 16 pwt. 21 gr. take 9 lb. 4 oz. 18 pwt. 15 gr.

PROCESS.

lb.	oz.	pwt.	gr.
13	5	16	21
9	4	18	15
4 lb.	0 oz.	18 pwt.	6 gr.

Write the subtrahend under the minuend, placing terms of the same denomination in the same column. Subtract 15 gr. from 21 gr., and write 6 gr., the difference, under the grains. Since 18 pwt. are greater than 16 pwt., add 20 pwt. to 16 pwt., making 36 pwt. Subtract 18 pwt. from 36 pwt., and write 18 pwt., the difference, under the pennyweights. Since 20 pwt. were added to the minuend, add 1 oz. (which equals 20 pwt.) to the 4 oz. of the subtrahend, making 5 oz. Subtract 5 oz. from 5 oz., and write 0 oz., the difference, under the ounces. Subtract 9 lb. from 13 lb., and write 4 lb., the difference, under pounds. The difference is 4 lb. 18 pwt. 6 gr.

	(2)				(3)					(4)			
	cwt.	lb.	oz.	dr.	lb.	℥.	ʒ.	℈.	gr.	w.	d.	h.	min.
From	48	73	10	15	7	10	7	1	14	13	1	13	45
Take	29	47	14	9	3	11	5	2	16	8	6	17	33

	(5)				(6)				(7)			
	mi.	fur.	rd.	yd.	rd.	yd.	ft.	in.	gal.	qt.	pt.	gi.
From	405	5	25	4	35	5	2	10	44	3	1	2
Take	384	6	37	5	27	4	1	11	26	3	1	3

8. A farmer raised 7 bu. 1 pk. 4 qt. of clover-seed, and sold 5 bu. 6 qt. 1 pt. How much had he left?

9. A man bought a farm containing 356 A. 2 R. 25 P., and sold 148 A. 3 R. 36 P. How much land had he left?

10. Washington is 77° 2′ 48″ W. longitude, and San Francisco 122° 26′ 15″ W. longitude. How much farther west is San Francisco than Washington?

11. From a stack of hay, containing $5\frac{1}{4}$ tons, a farmer sold 3 T. 12 cwt. 65 lb. How much hay remained unsold?

12. From a hogshead of molasses, containing 63 gallons, a grocer sold 38 gal. 3 qt. 1 pt. How much molasses remained in the hogshead?

13. A silversmith bought a bar of gold weighing 1 lb. 5 oz. 12 pwt., and a bar of silver weighing 3 lb. 8 oz. 16 pwt. 10 gr. How much more silver than gold did he buy?

14. A company contracted to build 65 mi. 4 fur. of railroad, and completed the first year

27 mi. 7 fur. 20 rd. How much remained to be built?

15. A note was given July 23, 1863, and paid Sept. 15, 1869. How long did it run?

PROCESS.

	mo.	d.
1869,	9	15
1863,	7	23
6 yr.	1 mo.	22 d.

Write the earlier date under the later, writing the number of the year, month, and day in proper order, and subtract, allowing 30 days to a month and 12 months to a year.

16. What is the difference of time between Oct. 23, 1856, and June 15, 1866?

17. How long from April 12, 1861, to May 22, 1865.

18. Abraham Lincoln was born Feb. 12, 1809, and died April 15, 1865. What was his age?

19. The American Revolution began April 19, 1775, and ended Jan. 20, 1783. How long did it continue?

20. America was discovered Oct. 14, 1492, and the Declaration of Independence was signed July 4, 1776. How much time elapsed between these two events?

21. The laying of the Atlantic Cable was consummated July 28, 1866, and the Pacific Railroad was completed May 10, 1869. How much earlier was the first event than the second?

22. Andrew Jackson died at Nashville, Tenn., June 8, 1845, aged 78 yr. 2 mo. 23 days. What was the date of his birth?

DEFINITION AND RULE.

Art. 112. ***Compound Subtraction*** is the process of finding the difference between two compound numbers of the same kind.

Art. 113. RULE.—1. *Write the subtrahend under the minuend, placing terms of the same denomination in the same column.*

2. *Beginning at the right, subtract each successive term of the subtrahend from the corresponding term of the minuend, and write the difference beneath.*

3. *If any term of the subtrahend be greater than the corresponding term of the minuend, add to the term of the minuend as many units of that denomination as equal one of the next higher, and from the sum subtract the term of the subtrahend, writing the difference beneath.*

4. *Add one to the next term of the subtrahend, and proceed as before.*

NOTE.—Instead of adding one to the next term of the subtrahend, one may be subtracted from the next term of the minuend.

LESSON III.

Multiplication of Compound Numbers.

1. Multiply 34 gal. 3 qt. 1 pt. by 9.

PROCESS.

34 gal. 3 qt. 1 pt.
9

313 gal. 3 qt. 1 pt.

Write the multiplier under the lowest denomination of the multiplicand. 9 times 1 pt. are 9 pt., equal to 4 qt. 1 pt. Write the 1 pt. under pints, and reserve the 4 qt.

to add to the product of quarts. 9 times 3 qt. are 27 qt., and 4 qt. added are 31 qt., equal to 7 gal. 3 qt. Write the 3 qt. under quarts, and reserve the 7 gal. to add to the product of gallons. 9 times 34 gal. are 306 gal., and 7 gal. added are 313 gal. Hence, 9 times 34 gal. 3 qt. 1 pt. = 313 gal. 3 qt. 1 pt.

(2)	(3)	(4)
15 lb. 6 oz. 13 pwt.	7 yd. 2 ft. 11 in.	15 mi. 3 fur. 22 rd.
8	12	6

(5)	(6)	(7)
8 ℔. 10 ℥ 4 ʒ 2 ℈.	27 bu. 3 pk. 5 qt.	4 w. 6 d. 13 h.
4	9	7

8. If a barrel of sugar weighs 2 cwt. 45 lb. 8 oz., how much will 6 barrels weigh?

9. How much gold will make a dozen rings, each weighing 7 pwt. 15 gr.?

10. If a pupil studies 4 h. 30 min. each day, how many hours will he study in 12 school weeks, of 5 days each?

11. If a ship sail 3° 25′ 33″ in one day, how far will it sail in 15 days?

12. If one man can build 5 rd. 4 yd. 2 ft. of fence in a day, how many yards can 8 men build?

13. How much wheat in 12 bins, if each bin contains 50 bu. 2 pk. 5 qt.?

14. John's age is 7 yr. 9 mo. 16 d., which is one-fifth of the age of his father: how old is his father?

15. If a load of wood contains 6 cd. ft. 12 cu. ft., how much wood will 15 loads make?

16. If a family use 2 gal. 3 qt. 1 pt. of milk a week, how much will it use in a year?

17. How much hay is there in 6 stacks, each containing 4 T. 16 cwt. 70 lb.?

18. What is the distance round a square field, each side of which is 24 rd. 3 yd. 2 ft.?

19. If a printer uses 3 reams 15 quires 12 sheets of paper each day, how much paper will he use in 4 weeks, of 6 days each?

20. If a man walk 2 mi. 7 fur. 32 rd. an hour, how far will he walk in 12 hours?

21. A field contains 25 rows of corn. If each row yield 5 bu. 3 pk., how much corn will the field yield?

DEFINITION AND RULE.

Art. 114. ***Compound Multiplication*** is the process of taking a compound number a given number of times.

Art. 115. RULE.—1. *Write the multiplier under the lowest denomination of the multiplicand.*

2. *Beginning at the right, multiply each term of the multiplicand in order, and reduce each product to the next higher denomination, writing the remainder under the term multiplied, and adding the quotient to the next product.*

NOTE.—In both simple and compound multiplication *the successive products are each divided by the number of units of their denomination, which equals one of the next higher denomination.*

LESSON IV.

Division of Compound Numbers.

1. Divide 15 w. 6 d. 13 h. 12 min. by 12.

PROCESS.

12) 15 w. 6 d. 13 h. 12 min.
1 w. 2 d. 7 h. 6 min.

Write the divisor at the left of the dividend, as in simple division. $\frac{1}{12}$ of 15 w. = 1 w. with 3 w. remaining. Write the 1 w. under weeks. The 3 w. remaining equal 21 d., and 21 d. and 6 d. equal 27 d. $\frac{1}{12}$ of 27 d. = 2 d., with 3 d. remaining. Write the 2 d. under days. The 3 d. remaining equal 72 h., and 72 h. and 13 h. equal 85 h. $\frac{1}{12}$ of 85 h. = 7 h., with 1 h. remaining. Write the 7 h. under hours. The 1 h. remaining equals 60 min., and 60 min. and 12 min. equal 72 min. $\frac{1}{12}$ of 72 min. = 6 min. Write the 6 min. under minutes. The quotient is 1 w. 2 d. 7 h. 6 min.

(2) 8) 14 lb. 12 oz. 15 dr.

(3) 10) 53 yd. 2 ft. 8 in.

(4) 5) 52 A. 3 R. 30 P.

(5) 6) 9 cwt. 73 lb. 12 oz.

(6) 11) 65 w. 1 d. 1 h. 58 min.

(7) 7) 14 lb. $5\bar{3}$ $6\bar{3}$.

8. If a man sleep 52 h. 30 min. in a week, how long does he sleep, on an average, each day?

9. A man bought a stack of hay, containing 6 T. 19 cwt. 86 lb., and drew it home in 7 equal loads. How much hay did he draw at each load?

10. If a dozen silver spoons weigh 8 oz. 15 pwt., what is the weight of each spoon?

11. A farm of 345 A. 3 R. 24 P. was divided equally between 6 heirs: how much did each receive?

12. Five equal casks of vinegar contain 218 gal. 2 qt.: how much vinegar in each cask?

13. If a man can dig a ditch 36 rd. 4 yd. 2 ft. long in 8 days, how much can he dig in 1 day?

14. If 9 men can pave 22 sq. rd. 25 sq. yd. in a day, how much can 1 man pave in a day?

15. A ship sailed 48° 24′ 45″ in 15 days: how far did it sail each day?

16. How many goblets can be made of 5 lb. 6 oz. 12 pwt. of silver, if each goblet weighs 7 oz. 8 pwt.?

PROCESS.

5 lb. 6 oz. 12 pwt. = 1332 pwt.
7 oz. 8 pwt. = 148 pwt.
1332 pwt. ÷ 148 pwt. = 9, *Ans.*

Reduce both dividend and divisor to pennyweights, and divide as in simple division.

17. How many bottles, holding 3 qt. 1 pt. each, can be filled from a cask containing 45½ gallons?

18. How many baskets of peaches, containing 3 pk. 4 qt. each, will make 3½ bushels?

19. How many lengths of fence, each 10 ft. 4 in., will make 28 rd. 3 ft. of fence?

20. How many castings, weighing 12 lb. 8 oz. each, can be made from 5 cwt. 50 lb. of iron?

21. How many times will a wagon wheel, 11 ft. 8 in. in circumference, turn round in going 2 mi. 4 fur.?

22. How many rings, weighing 5 pwt. 16 gr. each, can be made from a bar of gold weighing 1 lb. 8 oz.?

23. How many steps, of 2 ft. 4 in. each, will a man take in walking $\frac{3}{4}$ of a mile?

24. If a man can walk 2 m. 6 fur. in an hour, how long will it take him to walk 22 miles?

DEFINITION AND RULES.

Art. 116. ***Compound Division*** is the process of dividing a compound number into equal parts.

Art. 117. RULE I.—1. *Write the divisor at the left of the dividend, as in simple division.*

2. *Beginning at the left, divide each term of the dividend in order, and write the quotient under the term divided.*

3. *If the division of any term give a remainder, reduce it to the next lower denomination, to the result add the number of that denomination in the dividend, and then divide as above.*

NOTE.—When the divisor is a large number, the successive terms of the quotient may be written at the right of the dividend, as in long division.

Art. 118. RULE II.—To divide a compound number by another of the same kind, *Reduce both compound numbers to the same denomination, and then divide as in simple division.*

NOTE.—This is not properly compound division, since the compound numbers are reduced to *simple* numbers before dividing.

LESSON V.

Miscellaneous Problems.

1. If 5 sheets of copper contain 28 lb. 10 oz. 8 dr., how much copper is there in each sheet?

2. How much silver will it take to make 4 dozen spoons, each spoon weighing 15 pwt. 12 gr.?

3. If a milk dealer sells daily 7 cans of milk, each holding 12 gal. 2 qt., how much milk does he sell in 4 weeks?

4. From the sum of 15 lb. 8 oz. 15 pwt. and 9 lb. 10 oz. 18 pwt. take their difference.

5. John Jones was born Aug. 8, 1856, and on Jan. 1, 1862, his age was just $\frac{1}{6}$ of the age of his father. How old was his father?

6. Two small casks, each holding 21 gal. 3 qt., were filled from a cask of cider containing 56 gal. 2 qt. How much cider remained in the large cask?

7. A father owning a farm of 256 A. 3 R. 24 P., gave 100 A. to his son and then divided what remained equally between his two daughters. What was each daughter's share?

8. A farmer having cut 12 T. 15 cwt. of hay from a meadow, sold 6 loads of 1 T. 3 cwt. 75 lb. each, and then put the rest in a stack. How much hay was in the stack?

9. A merchant bought 3 chests of tea, each weighing 2 cwt. 45 lb., and in one month sold 4 cwt. 80 lb. 12 oz. How much tea had he left?

10. A publisher bought 20 bundles of paper,

and used daily 3 reams 15 quires 12 sheets: how much paper had he left at the close of 12 days?

11. A railroad company bought 145 cords of wood, piled in three ranks; the first rank contained 36 cd. 5 cd. ft.; and the second, 64 cd. 6 cd. ft. 12 cu. ft. How much wood was in the third rank?

12. A man bought 3 loads of hay, which, with the wagon, weighed respectively 1 T. 8 cwt. 40 lb.; 1 T. 11 cwt. 80 lb.; and 1 T. 9 cwt. 60 lb.; and the wagon alone weighed 10 cwt. 90 lb. How much hay did he buy?

13. If 4 horses eat 15 bu. 3 pk. 4 qt. of oats in 12 days, how much will they eat in one day?

14. If 5 horses eat 21 bu. 1 pk. 6 qt. of oats in 4 weeks, how much will 3 horses eat in the same time?

15. How many steps, of 2 ft. 6 in. each, will a man take in walking 4 fur. 20 rd.?

16. How many times will a carriage wheel, 11 ft. 4 in. in circumference, turn round in running 10 miles?

17. How many yards of carpeting, a yard wide, will carpet a room 18 ft. by 21 ft.?

18. How many yards of Brussels carpeting, $\frac{3}{4}$ of a yard wide, will carpet a room 20 ft. by 28 ft.?

19. Three men, A, B, and C, bought a hogshead of sugar, weighing 13 cwt. 60 lb.; A received $\frac{1}{2}$ of it, B $\frac{2}{3}$ of the remainder, and C what was left. How much sugar did each receive?

20. A company graded 25 mi. 5 fur. 36 rd. of road; $\frac{1}{3}$ of the job was completed the first month, $\frac{1}{4}$ of it the second month, $\frac{3}{8}$ of it the third month, and the rest the fourth month. How many miles of road were graded each month?

Questions for Review.

What is a simple number? What is a compound number? When is a simple number denominate? When concrete? When abstract?

What is meant by the terms of a compound number? When are two or more compound numbers of the same kind? Give examples.

What is compound addition? In what respect does it differ from simple addition? Give the rule.

What is compound subtraction? Repeat the rule.

How do you find the difference of time between two dates? How many days to the month are allowed?

What is compound multiplication? How does it differ from simple multiplication? Repeat the rule.

What is compound division? How does it differ from simple division? Repeat the rule?

How is one compound number divided by another? Why must the two compound numbers be of the same kind? Is the process simple or compound division?

[30m-6,'11]

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