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A SERIES OF TEXTBOOKS FOR PERSONS ENGAGED IN THE ENGINEERING PROFESSIONS AND TRADES OR FOR THOSE WHO DESIRE INFORMATION CONCERNING THEM. FULLY ILLUSTRATED AND CONTAINING NUMEROUS PRACTICAL EXAMPLES AND THEIR SOLUTIONS

HYDRAULICS
PNEUMATICS
GRAPHICAL STATICS
HEAT
REFRIGERATION

SCRANTON:
INTERNATIONAL TEXTBOOK COMPANY

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1156
BURR PRINTING HOUSE,
FRANKFORT AND JACOB STREETS,
NEW YORK.

PREFACE

The International Library of Technology is the outgrowth of a large and increasing demand that has arisen for the Reference Libraries of the International Correspondence Schools on the part of those who are not students of the Schools. As the volumes composing this Library are all printed from the same plates used in printing the Reference Libraries above mentioned, a few words are necessary regarding the scope and purpose of the instruction imparted to the students of—and the class of students taught by—these Schools, in order to afford a clear understanding of their salient and unique features.

The only requirement for admission to any of the courses offered by the International Correspondence Schools, is that the applicant shall be able to read the English language and to write it sufficiently well to make his written answers to the questions asked him intelligible. Each course is complete in itself, and no textbooks are required other than those prepared by the Schools for the particular course selected. The students themselves are from every class, trade, and profession and from every country; they are, almost without exception, busily engaged in some vocation, and can spare but little time for study, and that usually outside of their regular working hours. The information desired is such as can be immediately applied in practice, so that the student may be enabled to exchange his present vocation for a more congenial one, or to rise to a higher level in the one he now pursues. Furthermore, he wishes to obtain a good working knowledge of the subjects treated in the shortest time and in the most direct manner possible.

In meeting these requirements, we have produced a set of books that in many respects, and particularly in the general plan followed, are absolutely unique. In the majority of subjects treated the knowledge of mathematics required is limited to the simplest principles of arithmetic and mensuration, and in no case is any greater knowledge of mathematics needed than the simplest elementary principles of algebra, geometry, and trigonometry, with a thorough, practical acquaintance with the use of the logarithmic table. To effect this result, derivations of rules and formulas are omitted, but thorough and complete instructions are given regarding how, when, and under what circumstances any particular rule, formula, or process should be applied; and whenever possible one or more examples, such as would be likely to arise in actual practice—together with their solutions—are given to illustrate and explain its application.

In preparing these textbooks, it has been our constant endeavor to view the matter from the student's standpoint, and to try and anticipate everything that would cause him trouble. The utmost pains have been taken to avoid and correct any and all ambiguous expressions—both those due to faulty rhetoric and those due to insufficiency of statement or explanation. As the best way to make a statement, explanation, or description clear is to give a picture or a diagram in connection with it, illustrations have been used almost without limit. The illustrations have in all cases been adapted to the requirements of the text, and projections and sections or outline, partially shaded, or full-shaded perspectives have been used, according to which will best produce the desired results. Half-tones have been used rather sparingly, except in those cases where the general effect is desired rather than the actual details.

It is obvious that books prepared along the lines mentioned must not only be clear and concise beyond anything heretofore attempted, but they must also possess unequaled value for reference purposes. They not only give the maximum of information in a minimum space, but this information is so ingeniously arranged and correlated, and the indexes are so full and complete, that it can at once be made available to the reader. The numerous examples and explanatory remarks, together with the absence of long demonstrations and abstruse mathematical calculations, are of great assistance in helping one to select the proper formula, method, or process and in teaching him how and when it should be used.

The two papers in this volume that deal with the subject of refrigerating and ice-making machinery give the practical engineer enough of the theory to enable him to handle all the problems that he is likely to meet. They also give him those practical pointers regarding the operating of a plant that will enable him to obtain economically the maximum refrigerating effect and show him how to tell from general observation the capacity and efficiency of his plant. Considerable information is also given regarding the erection, operation, and installation of plants, together with general rules for remodeling an old plant or designing a new one. Both the absorption and compression types of machines are treated. As a preliminary to the discussion of ice-making and refrigerating machinery, short papers on hydraulics, pneumatics, and heat are included in this volume.

As mentioned above, this volume is printed from the plates used in printing the Reference Libraries of the International Correspondence Schools. On account of the omission of certain papers, the material contained in which is given in better form elsewhere, there are several breaks in the continuity of the page numbers, formula numbers, article numbers, etc. This, however, does not impair the value of the volume, as the index has been reprinted and made to conform to the present arrangement.

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INDEX

HYDRAULICS.

HYDROSTATICS.

967. Hydrostatics treats of liquids at rest under the action of forces.

Liquids are very nearly *incompressible.* A pressure of 15 pounds per square inch compresses water less than $\frac{1}{20000}$ of its volume.

Fig. 157 represents two cylindrical vessels of exactly the same size. The vessel *a* is fitted with a wooden block of the same size as the cylinder, and can move in it; the vessel *b* is filled with water, whose depth is the same as the length of the wooden block in *a*. Both vessels are fitted with air-tight pistons *P* whose areas are each 10 square inches.

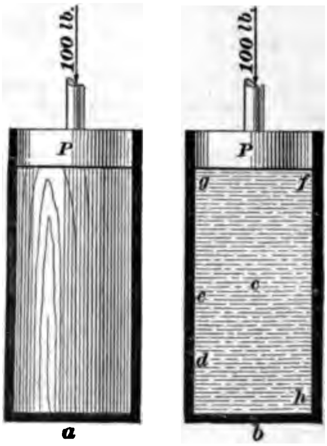


FIG. 157.

Suppose, for convenience, that the weights of the cylinders, pistons, block, and water be neglected, and that a force of 100 pounds be applied to both pistons. The pressure per square inch will be $\frac{100}{10} = 10$ pounds. In the vessel *a*, this pressure will be transmitted to the bottom of the vessel, and will be 10 pounds per square inch; it is easy to see that there will be no pressure on the sides. In the vessel *b*, an entirely different result is obtained. The pressure on the bottom will be the same as in the other case—that is, 10 pounds per square inch—but, owing to the fact that the molecules of the water are perfectly free to move, this pressure of 10 pounds per square inch is ***transmitted in every direction with the same intensity;*** that is to say, the pressure at any point, *c*, *d*, *e*, *f*, *g*, *h*, etc., due to the force of 100 pounds, is exactly the same, and equals 10 pounds per square inch.

This may be easily proven experimentally by means of an apparatus like that shown in Fig. 158. Let the area of the piston *a* be 20 square inches; of *b*, 7 square inches; of *c*, 1 square inch; of *d*, 6 square inches; of *e*, 8 square inches; and of *f*, 4 square inches.

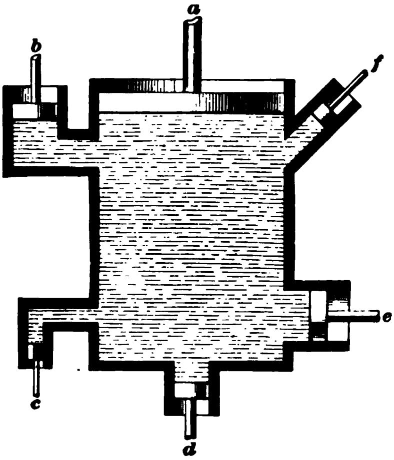


FIG. 158.

If the pressure due to the weight of the water be neglected, and a force of 5 pounds be applied at *c* (whose area is 1 square inch), a pressure of 5 pounds per square inch will be transmitted in all directions; and in order that there shall be no movement, a force of $6 \times 5 = 30$ pounds must be applied at *d*, 40 pounds at *e*, 20 pounds at *f*, 100 pounds at *a*, and 35 pounds at *b*.

If a force of 99 pounds were applied to *a*, instead of 100 pounds, the piston *a* would rise, and the other pistons *b*, *c*, *d*, *e*, and *f* would move inwards; but, if the force applied to *a* were 100 pounds, they would all be in equilibrium. Had 101 pounds been applied at *a*, the pressure per square inch would be $\frac{101}{20} = 5.05$ pounds, which would be transmitted in all directions; and, since the pressure due to *c* is only 5 pounds per square inch, it is now evident that the piston *a* will move downwards, and the pistons *b*, *c*, *d*, *e*, and *f* will be forced outwards.

The whole may be summed up as follows:

968. Rule.—*The pressure per unit of area exerted anywhere upon a mass of liquid is transmitted undiminished in* *all directions, and acts with the same force upon all surfaces in a direction at* ***right angles*** *to those surfaces.*

This law was first discovered by Pascal, and is the most important in Hydromechanics. Its meaning should be thoroughly understood.

EXAMPLE.—If the area of a piston *e* in Fig. 158 were 8.25 square inches, and a force of 150 pounds were applied to it, what forces would have to be applied to the other pistons to keep the water in equilibrium, assuming that their areas were the same as given before?

SOLUTION.—$\frac{150}{8.25} = 18.182$ lb. per sq. in., nearly.

$$\left.\begin{array}{l} 20 \times 18.182 = 363.64 \text{ lb.} = \text{force to balance } a. \\ 7 \times 18.182 = 127.274 \text{ lb.} = \text{force to balance } b. \\ 1 \times 18.182 = 18.182 \text{ lb.} = \text{force to balance } c. \\ 6 \times 18.182 = 109.092 \text{ lb.} = \text{force to balance } d. \\ 4 \times 18.182 = 72.728 \text{ lb.} = \text{force to balance } f. \end{array}\right\} \text{Ans.}$$

969. ***The pressure due to the weight of a liquid*** *may be downwards, upwards, or sideways.*

970. Downward Pressure.—In Fig. 159, the pressure on the bottom of the vessel *a* is, of course, equal to

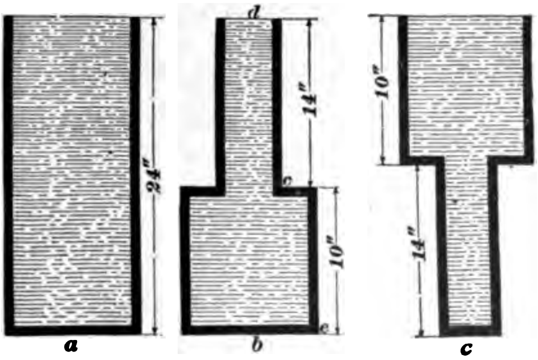


FIG. 159.

the weight of the water it contains. If the area of the bottom of the vessel *b*, and the depth of the liquid contained in it, are the same as in the vessel *a*, the pressure on the bottom of *b* will be the same as on the bottom of *a*. Suppose the bottoms of the vessels *a* and *b* are 6 inches square, and that the part *c d*, in the vessel *b*, is 2 inches square, and that both vessels are filled with water. Then, the weight of 1 cubic inch of water being $\frac{62.5}{1,728} = .03617$ pound, and the number of cubic inches in a, $6 \times 6 \times 24 = 864$ cubic inches, the weight of the water is $864 \times .03617 = 31.25$ pounds. Hence, the total pressure on the bottom of the vessel a is 31.25 pounds, or $\frac{31.25}{36} = .868$ pound per square inch.

The pressure in b, due to the weight contained in the part $c\,c$, is $6 \times 6 \times 10 \times .03617 = 13.02$ pounds.

The weight of the part contained in $c\,d$ is $2 \times 2 \times 14 \times .03617 = 2.0255$ pounds, and the weight per square inch of area in $c\,d$ is $\frac{2.0255}{4} = .5064$ pound.

According to Pascal's law, this weight (pressure) is transmitted equally in all directions; therefore, every square inch of the large part of the vessel b will be subjected to a pressure of .5064 pound. The area of the part $c\,c$ is $6 \times 6 = 36$ square inches, and the total pressure due to the weight of the water in the small part will be $.5064 \times 36 = 18.23$ pounds. Hence, the total pressure on the bottom of b will be $13.02 + 18.23 = 31.25$ pounds, the same result as in the case of the vessel a.

If an additional pressure of 10 pounds per square inch were applied to the upper surface of both vessels, the total pressure on their bottoms would be $31.25 + (6 \times 6 \times 10) = 31.25 + 360 = 391.25$ pounds.

In case this pressure were obtained by means of a weight placed on a piston, as shown in Figs. 157 and 158, the weight for the vessel a would be $6 \times 6 \times 10 = 360$ pounds, and for the vessel b, $2 \times 2 \times 10 = 40$ pounds.

971. The General Law for the Downward Pressure upon the Bottom of any Vessel:

Rule.—*The pressure upon the bottom of a vessel containing a fluid is independent of the shape of the vessel, and is equal to the weight of a prism of the fluid whose base has the same area as the bottom of the vessel, and whose altitude is the dis-*

tance between the bottom and the upper surface of the fluid, plus the pressure per unit of area upon the upper surface of the fluid multiplied by the area of the bottom of the vessel.

Suppose that the vessel *b*, in Fig. 159, is inverted, as shown at *c*; the pressure upon the bottom in this case will also be .868 pound per square inch, but it will require a weight of 3,490 pounds to be placed upon the piston at the upper surface to make the pressure on the bottom 391.25 pounds, instead of a weight of 40 pounds, as in the other case.

EXAMPLE.—A vessel filled with salt water, having a specific gravity of 1.03, has a circular bottom 13 inches in diameter. The top of the vessel is fitted with a piston 3 inches in diameter, on which is laid a weight of 75 pounds; what is the total pressure on the bottom, if the depth of the water is 18 inches?

SOLUTION.—The weight of 1 cubic inch of the water is $\frac{62.5 \times 1.03}{1,728} =$ .037254 lb.

$13 \times 13 \times .7854 \times 18 \times .037254 = 89.01$ pounds = the pressure due to the weight of the water.

$\frac{75}{3 \times 3 \times .7854} = 10.61$ lb. per sq. in. due to the weight on the piston.

$13 \times 13 \times .7854 \times 10.61 = 1,408.29$ lb. = pressure on the bottom due to the weight.

Total pressure $= 1,408.29 + 89.01 = 1,497.3$ lb. Ans.

972. Upward Pressure.—In Fig. 159 the vessel *b* is of exactly the same size as that shown at *c*. There is no upward pressure on the surface *c* due to the weight of the water in the large part *e c*, but there is an upward pressure on *c* due to the weight of the water in the small part *c d*. The pressure per square inch due to the weight of the water in *c d* was found to be .5064 pound (see Art. **970**); the area of the upper surface *c* of the large part *e c* is evidently $(6 \times 6) - (2 \times 2) = 36 - 4 = 32$ square inches, and the total upward pressure due to the weight of the water is $.5064 \times 32 = 16.2$ pounds.

If an additional pressure of 10 pounds per square inch were applied to a piston fitting the top of the vessel, the total upward pressure on the surface *c* would be $16.2 + (32 \times 10) = 336.2$ pounds.

973. General Law for Upward Pressure:

Rule.—*The upward pressure on any submerged horizontal surface equals the weight of a prism of the liquid whose base has an area equal to the area of the submerged surface, and whose altitude is the distance between the submerged surface and the upper surface of the liquid, plus the pressure per unit of area on the upper surface of the fluid multiplied by the area of the submerged surface.*

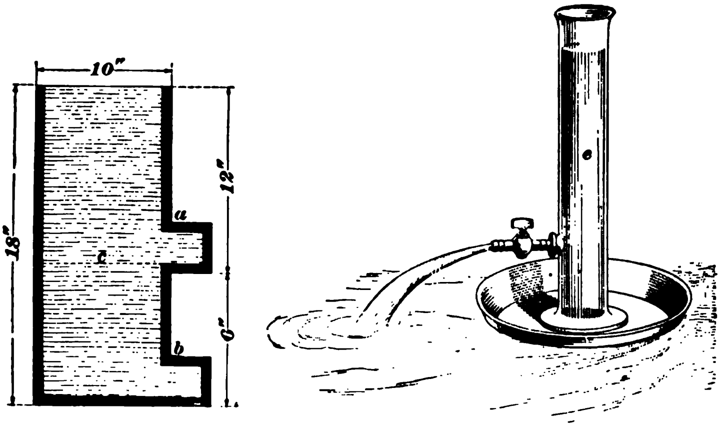
EXAMPLE.—A horizontal surface 6 inches by 4 inches is submerged in a vessel of water 26 inches below the upper surface. If the pressure on the water is 16 pounds per square inch, what is the total upward pressure on the horizontal surface?

SOLUTION.— $6 \times 4 \times 26 \times .03617 = 22.57$ lb., or the upward pressure due to the weight of the water.

$6 \times 4 \times 16 = 384$ lb., or the upward pressure due to the outside pressure of 16 lb. per sq. in.

The total upward pressure $= 384 + 22.57 = 406.57$ lb. Ans.

974. Lateral (Sideways) Pressure.—Suppose the top of the vessel shown in Fig. 160 (*a*) is 10 inches square,



(*a*) (*b*)

FIG. 160.

and that the projections at *a* and *b* are 1 inch × 1 inch, and 10 inches long.

The pressure per square inch on the bottom of the vessel, due to the weight of the liquid, is $1 \times 1 \times 18 \times$ the weight of a cubic inch of the liquid.

The pressure at a depth equal to the distance of the upper surface *b* below the top of the vessel is 1 × 1 × 17 × the weight of a cubic inch of the liquid.

Since both of these pressures are transmitted in every direction, they are also transmitted laterally (sideways), and the *pressure per unit of area on the projection b is a mean between the two*, and equals 1 × 1 × 17½ × the weight of a cubic inch of the liquid.

To find the lateral pressure on the projection *a*, imagine that the dotted line *c* is the bottom of the vessel; then the conditions will be the same as in the preceding case, except that the depth is not so great.

The lateral pressure on *a* is thus seen to be 1 × 1 × 11½ × the weight of a cubic inch of the liquid.

General Law for Lateral Pressure:

975. Rule.—*The pressure upon any vertical surface due to the weight of a liquid is equal to the weight of a prism of the liquid whose base has the same area as the vertical surface, and whose altitude is the depth of the center of gravity of the vertical surface below the level of the liquid.*

Any additional pressure is to be added as in the previous cases.

EXAMPLE.—A well 3 feet in diameter and 20 feet deep is filled with water; what is the pressure on a strip of the wall 1 inch wide, the center of which is 1 foot from the bottom? What is the pressure on the bottom? What is the upward pressure per square inch, 2 feet 6 inches from the bottom?

SOLUTION.— 1 × 36 × 3.1416 = 113.1 sq. in. = area of the strip. Depth of center of gravity = 20 − 1 = 19 ft.

113.1 × 19 × 12 × .03617 = 932.71 lb. = total pressure upon the strip. Ans.

The pressure per square inch is $\frac{932.71}{113.1}$ = 8.247 lb., nearly.

36 × 36 × .7854 × 20 × 12 × .03617 = 8,836 lb. = the pressure on the bottom. Ans.

20 − 2.5 = 17.5. 1 × 17.5 × 12 × .03617 = 7.596 lb. = the upward pressure per square inch, 2 ft. 6 in. from the bottom, since 2 ft. 6 in. = 2.5 ft. Ans.

The effects of the lateral pressure are illustrated in Fig.

160 (*b*). A tall vessel *e* has a stop-cock near its base, and is arranged to float upon the water, as shown. When this vessel is filled with water, the lateral pressures at any two points of the surface of the vessel, and opposite to each other, are equal. Being equal, and acting in opposite directions, they destroy each other (see Art. **881**), and no motion can result; but, if the stop-cock be opened, there will be no resistance to that pressure acting on the surface equal to the area of the opening, the pressure which formerly acted causing the water to flow out, while its equal and opposite pressure will cause the vessel to move backwards through the water in a direction opposite to that of the spouting water.

Since the pressure on the bottom of a vessel due to the weight of the liquid is dependent only upon the height of the liquid, and not upon the shape of the vessel, it follows that if a vessel has a number of radiating tubes, as Fig. 161, the water in each tube will be on the same level, no

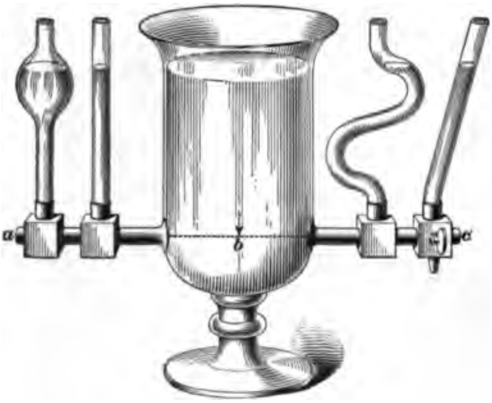


FIG. 161.

matter what may be the shape of the tubes. For, if the water were higher in one tube than in the others, the downward pressure on the bottom due to the height of the water in this tube would be greater than that due to the height of the water in the other tubes.

Consequently, the upward pressure would also be greater;

the equilibrium would be destroyed, and the water would flow from this tube into the vessel, and rise in the other tubes until it was at the same level in all, when it would be in equilibrium. This principle is expressed in the familiar saying, *Water seeks its level.*

This explains why city water reservoirs are located on high elevations, and why water on leaving the hose-nozzle spouts so high.

If there was no resistance by friction and air, the water would spout to a height equal to the level of the water in the reservoirs. If a long pipe, whose length was equal to the vertical distance between the nozzle and the level of the water in the reservoir, was attached to the nozzle, the water would just reach the end of the pipe. If the pipe was lowered slightly, the water would trickle out.

Fountains, canal locks, and artesian wells are examples of the application of this principle.

EXAMPLE.—The water level in a city reservoir is 150 feet from the level of the street; what is the pressure of the water per square inch on the hydrant?

SOLUTION.— $1 \times 150 \times 12 \times .03617 = 65.106$ lb. per sq. in. Ans.

976. The weight of a column of water 1 inch square and 1 foot high is $.03617 \times 12 = .434$ pound, nearly. Hence, if the depth (head) is given, the pressure per square inch may be found by multiplying the depth in feet by .434. The constant .434 is the one ordinarily employed in practical calculations.

977. In Fig. 162, let the area of the piston *a* be 1 square inch; of *b*, 40 square inches. According to Pascal's law, 1 pound placed upon *a* will balance 40 pounds placed upon *b*.

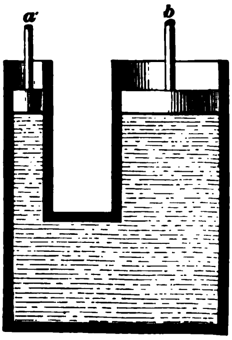


FIG. 162.

Suppose that *a* moves downwards 10 inches; then, 10 cubic inches of water will be forced into the tube *b*. This will be distributed over the entire area of the tube *b* in the form of a cylinder, whose cubical contents must be 10 cubic inches, whose base has an area of 40 square inches, and whose altitude must be $\frac{10}{40} = \frac{1}{4}$ of an inch; that is, a movement of 10 inches of the piston *a* will cause a movement of $\frac{1}{4}$ of an inch of the piston *b*.

Here is the old principle of machines: *The power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.*

Hence, if 1 pound on the piston *a* represents the power P, the equivalent weight W on *b* may be obtained from the equation $P \times 10 = W \times \frac{1}{4}$, or $10 = \frac{1}{4} W$, and $W = 40$.

Another familiar fact is also recognized, for the velocity ratio of P to W is $10 : \frac{1}{4}$, or 40. Since in any machine the weight equals the power multiplied by the velocity ratio, $W = P \times 40$, and when $P = 1$ pound, $W = 40$ pounds.

978. This principle is made use of in the hydraulic press

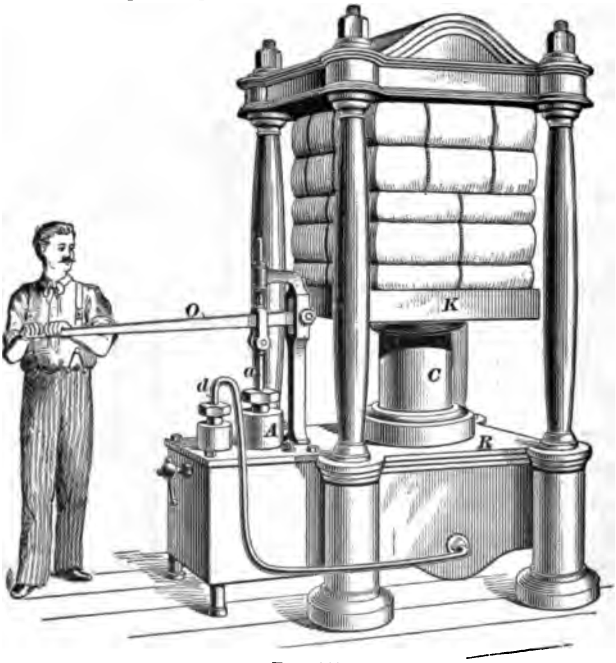


FIG. 163.

represented in Fig. 163. As the man depresses the lever O, he forces down the piston *a* upon the water in the cylinder *A*. The water is forced through the bent tube *d* into the cylinder in which the large piston *C* works, and causes it to rise, thus lifting the platform *K*, and compressing the bales. If the area of *a* be 1 square inch, and that of *C* be 100 square inches, the velocity ratio will be 100.

If the length of the lever between the hand and the fulcrum is 10 times the length between the fulcrum and the piston *a*, the velocity ratio of the lever will be 10, and the total velocity ratio of the hand to the piston *C* will be 1,000.

Hence, a force of 100 pounds applied by the hand will raise $100 \times 1,000 = 100,000$ pounds. But, if the average movement of the hand per stroke is 10 inches, it will require $\frac{1,000}{10} = 100$ strokes to raise the platform 1 inch, and it is again seen that what is gained in power is lost in speed.

Applications of this principle are seen in the hydraulic machines used for forcing locomotive drivers on their axles, etc., and for testing the strength of boiler shells.

EXAMPLE.—A suspended vertical cylinder is tested for the tightness of its heads by filling it with water. A pipe whose inside diameter is ¼ of an inch, and whose length is 20 feet, is screwed into a hole in the upper head, and then filled with water; what is the pressure per square inch on each head, if the cylinder is 40 inches in diameter and 60 inches long?

SOLUTION.—Area of heads $= 40^2 \times .7854 = 1,256.64$ sq. in.

Pressure per square inch on the bottom head, due to the weight of the water in the cylinder $= 1 \times 60 \times .03617 = 2.17$ lb. $(\frac{1}{4})^2 \times .7854 = .04909$ sq. in., the area of the pipe.

$.04909 \times 20 \times 12 \times .03617 = .426$ lb. = the weight of water in pipe = the pressure on a surface area of .04909 sq. in.

The pressure per square inch due to the water in the pipe is $\frac{1}{.04909} \times .426 = 8.68$ lb. per sq. in. upon the upper head. Ans.

The pressure per square inch on the lower head is $8.68 + 2.17 = 10.85$ lb. per sq. in. Ans.

EXAMPLE.—In the last example, if the pipe is fitted with a piston weighing ¼ of a pound, and a 5-pound weight is laid upon it, what is the pressure per square inch upon the upper head?

SOLUTION.—In addition to the pressure of .426 pound on the area of

.04909 sq. in., there is now an additional pressure upon this area of $5 + \frac{1}{4} = 5.25$ lb., and the total pressure upon this area is $.426 + 5.25 = 5.676$ lb.

The pressure per square inch is $\frac{1}{.04909} \times 5.676 = 115.6$ lb. Ans.

979. Pressure Upon Oblique Surfaces.—Heretofore, the pressure upon horizontal and vertical surfaces only has been considered. The pressure upon sides which are oblique to the bottom will now be considered.

According to the law of Pascal, the pressure which a fluid exerts upon a surface is at right angles to the surface; hence, in Fig. 164, the pressure acts in a direction indicated by the arrow-head on the line $C\ D$, and the lateral pressure on every square unit of area of the side $F\ H$ will be in the direction $C\ D$. According to the law for lateral pressure (Art. **975**), the total perpendicular pressure upon the side $F\ H$ will be: Area of side $F\ H \times \frac{1}{2}\ A\ H \times$ the weight of a cubic inch of the fluid.

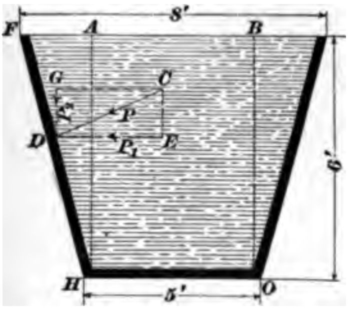


FIG. 164.

Let the line $C\ D$ represent this perpendicular pressure, and call it P. Now, resolve P into two components, one, P_1, acting horizontally, and the other, P_2, acting vertically. The angle $C\ D\ E = F\ H\ A$, for $C\ D$ is perpendicular to $F\ H$, and $E\ D$ is perpendicular to $A\ H$. (See Art. **692.**) Therefore, $P_1 = P \times \cos\ C\ D\ E = P \times \cos\ F\ H\ A$; but the cosine of $F\ H\ A$ is numerically equal to $A\ H$, which equals the projection of $F\ H$ on a line at right angles to $E\ D$.

The angle $G\ D\ C = E\ C\ D = A\ F\ H$, since $C\ D$ is perpendicular to $F\ H$, and $G\ D$ is perpendicular to $A\ F$; consequently, the vertical component $P_2 = P \cos\ G\ D\ C = P \cos\ A\ F\ H$. But the cosine $A\ F\ H$ is numerically equal to $A\ F$, which equals the projection of $F\ H$ upon a line at right angles to $G\ D$. Hence, the rule for finding the pressure exerted by a fluid in any direction upon a plane surface is:

Rule.—*The pressure exerted by a fluid in any direction upon* **any** *plane surface is equal to the weight of a prism of the fluid whose base is the projection of the surface at right angles to the direction considered, and whose height is the depth of the center of gravity of the surface below the level of the liquid.*

980. When the pressure per unit of area is so great that the pressure may be regarded as uniformly distributed over the area of the surface pressed against, the preceding rule holds good for any surface. Consequently, if a cylinder is filled with water, and a pressure is applied, the pressure upon any half section of the cylinder, as $A\,C\,B$ (Fig. 165), tending to separate one half from the other, is equal to the *projected area of the half cylinder (or the inside diameter times the length of the cylinder), multiplied by the depth of the center of gravity of the cylinder (or $\frac{1}{2}\,d$), multiplied by the weight of a cubic unit of water, plus the diameter of the shell multiplied by the pressure per square inch multiplied by the length of the cylinder.*

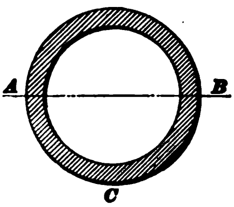


FIG. 165.

If d = the inside diameter in inches and l = the length of the cylinder in inches, the pressure due to the weight of the water (when the cylinder is horizontal and filled with water) upon the half cylinder $A\,C\,B = d \times l \times \frac{d}{2} \times$ the weight of a cubic inch of water $= l \times \frac{d^2}{2} \times$ the weight of a cubic inch of water.

The pressure due to an additional pressure P_1 pounds per square inch $= l \times d \times P_1$. Thus, if a cylinder 60 inches long and 40 inches in diameter, lying horizontally, is filled with water, the pressure on any half section, as $A\,C\,B$, due to the weight of the water, will be found as follows:

$$60 \times \frac{40^2}{2} \times .03617 = 1{,}736.16 \text{ pounds.}$$

If there were an additional pressure per square inch of 50 pounds, the total pressure would be $60 \times 40 \times 50 + 1,736.16 = 121,736.16$ pounds.

If the cylinder were vertical instead of horizontal, the depth of the center of gravity would evidently be $\frac{1}{2} l$ below the surface, and the pressure tending to separate one half from the other, due to the weight of the water, would be $d \times l \times \frac{l}{2} \times$ weight of a cubic inch of water $= d \times \frac{l^2}{2} \times$ weight of a cubic inch of water.

Any additional pressure should be calculated as in the previous case.

EXAMPLE.—What would be the pressure due to the weight of the water if the cylinder in the last example were vertical?

SOLUTION.— $40 \times \frac{60^2}{2} \times .03617 = 2,604.24$ lb. Ans.

In the case of a sphere, the projected area is evidently the area of a circle whose diameter is the same as the diameter of the sphere. Hence, the pressure upon a hemisphere due to the weight of the water will be $d^2 \times .7854 \times \frac{d}{2} \times$ the weight of a cubic inch of water $= \frac{d^3}{2} \times .7854 \times$ the weight of a cubic inch of water.

The pressure upon a hemisphere whose diameter is 20 inches, when filled with water, due to the weight of the water only, will be $\frac{20^3}{2} \times .7854 \times .03617 = 113.63$ pounds.

For an additional pressure P in pounds per square inch, the pressure on the hemisphere due to this will be $d^2 \times .7854 \times P$.

EXAMPLE.—If the ends of the vessel shown in Fig. 164 make right angles with the bottom, and the distance between them, or length of the vessel, is 12 feet, what are the horizontal, vertical, and perpendicular pressures against the sides, both making equal angles with the bottom?

SOLUTION.—Apply rule, Art. **979.** In finding the horizontal pressure, the *direction considered* is that of the line ED; that is, horizontal. The projected area of the surface whose edge is FH projected at right angles to ED is $AH \times$ length of vessel $= 6 \times 12 = 72$ sq. ft.

Depth of center of gravity $= 6 \div 2 = 3$ ft. Hence, horizontal pressure $= 6 \times 12 \times 3 \times 62.5 = 13{,}500$ lb. Ans.

NOTE.—We multiply by 62.5, because all dimensions are in feet.

In a similar manner, the vertical pressure is found. Thus, *direction considered* is that of the line GD. Projected area of surface FH, when projected at right angles to GD, $= FA \times$ length of vessel. $FA = \frac{8-5}{2} = 1\frac{1}{2}$ ft. Vertical pressure $= 1\frac{1}{2} \times 12 \times 3 \times 62.5 = 3{,}375$ lb. Ans.

To find the perpendicular pressure, first find the length of the side FH. $FH = \sqrt{6^2 + (1\frac{1}{2})^2} = 6.1847$ ft. Perpendicular pressure $= 6.1847 \times 12 \times 3 \times 62.5 = 13{,}916$ lb., nearly. Ans.

EXAMPLE.—A sphere having a diameter of 30 inches is filled with water and subjected to an additional pressure of 75 pounds per square inch; what is the total pressure tending to separate one vertical half section of the sphere from its opposite half?

SOLUTION.—The pressure due to the weight of the water is $\frac{30^3}{2} \times .7854 \times .03617 = 383.5$ lb.

$30^2 \times .7854 \times 75 = 53{,}014.5$ lb. = additional pressure.

Total pressure tending to separate any two halves $= 53{,}014.5 + 383.5 = 53{,}398$ lb. Ans.

BUOYANT EFFECTS OF WATER.

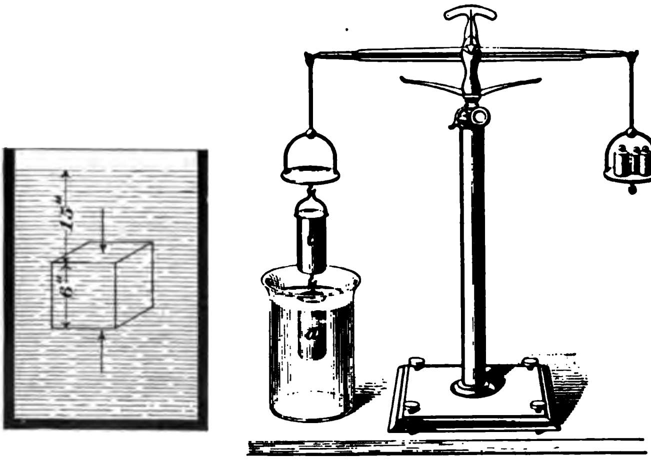
981. In Fig. 166 (*a*) is shown a 6-inch cube entirely submerged in water. The lateral pressures are equal, and act in opposite directions. The upward pressure $= 6 \times 6 \times 21 \times .03617$; the downward pressure $= 6 \times 6 \times 15 \times .03617$, and the difference $= 6 \times 6 \times 6 \times .03617 =$ the volume of the cube in cubic inches $\times$ the weight of 1 cubic inch of water. That is, the upward pressure exceeds the downward pressure by the weight of a volume of water equal to the volume of the body.

This excess of upward pressure over the downward pressure acts against gravity; consequently, *if a body be immersed in a fluid, it will lose in weight an amount equal to the weight of the fluid it displaces.* This is called the **principle of Archimedes,** because it was first stated by him.

Archimedes' principle may be experimentally demonstrated with the beam scales, as shown in Fig. 166 (*b*).

From one scale pan suspend a hollow cylinder of metal *t*, and below that a solid cylinder *a* of the same size as the hollow part of the upper cylinder. Put weights in the other scale pan until they exactly balance the two cylinders. If *a* be immersed in water, the scale pan containing the weights will descend, showing that *a* has lost some of its weight. Now, fill *t* with water, and the volume of water that can be poured into *t* will equal that displaced by *a*. The scale pan that contains the weights will gradually rise until *t* is filled, when the scales balance again.

If the immersed body is lighter than the liquid, the up-



(*a*) (*b*)

Fig. 166.

ward pressure will cause it to rise and extend partly out of the liquid, until the weight of the body and the weight of the liquid displaced are equal. If the immersed body is heavier than the liquid, the downward pressure plus the weight of the body will be greater than the upward pressure, and the body will fall downwards until it touches bottom or meets an obstruction. If the weights of equal volumes of the liquid and the body are equal, the body will remain stationary, and be in equilibrium in any position or at any depth beneath the surface of the liquid.

An interesting experiment in confirmation of the above facts may be performed as follows: Drop an egg into a glass jar filled with fresh water. The mean density of the egg being a little greater than that of water, the egg will fall to the bottom of the jar. Now, dissolve salt in the water, stirring it so as to mix the fresh and salt water. The salt water will presently become denser than the egg, and the egg will rise. Now, if fresh water be poured in until the egg and water have the same density, the egg will remain stationary in any position that it may be placed below the surface of the water.

EXAMPLES FOR PRACTICE.

1. The diameter of the plunger of a hydraulic press used in an engineering establishment is 12". Water is forced into the cylinder of the press by means of a small pump having a plunger whose diameter is ¾", and stroke is 4". What pressure is exerted by the large plunger, when the force acting on the small plunger is 125 pounds?

Ans. 32,000 lb.

2. If the small plunger, in the last example, makes 96 working strokes per minute, (*a*) how long will it take the large plunger to move 8"? (*b*) What is the velocity ratio?

Ans. { (*a*) 5⅓ min.
(*b*) 256 : 1.

3. A vertical pipe, 88 feet high, is filled with water; (*a*) what is the pressure on bottom? (*b*) If the diameter of the pipe is 8", what is the total pressure on a strip, 2½" high, whose center of gravity is 21 feet from the bottom?

Ans. { (*a*) 38.2 lb. per sq. in., nearly.
(*b*) 1,827.03 lb.

4. A sphere, 16" in diameter, is submerged in water with its center of gravity 43 ft. 8" below the surface. What is (*a*) the total lateral pressure? (*b*) the total pressure?

Ans. { (*a*) 7,620.8 lb.
(*b*) 15,241.6 lb.

SPECIFIC GRAVITY.

982. In Art. **963** it was stated that the specific gravity of a body was the ratio between the weight of the body and the weight of an equal volume of water, but no methods were given for finding this ratio. Some of these methods will now be explained.

Archimedes' principle affords an easy and accurate method of finding the specific gravity of solids not easily soluble in water. Weigh the body in air; then, weigh the body in water, suspending it by a string, and attaching the string to a scale pan in place of the two cylinders shown in

Fig. 166 (*b*). *The difference between the two weights will be the weight of an equal volume of water. The ratio of the weight in air to the difference thus found will be the specific gravity.* The abbreviation for specific gravity is Sp. Gr.

Let W be the weight of the solid in air and W' the weight in water; then $W - W'$ = the weight of a volume of water equal to the volume of the solid, and

$$\text{Sp. Gr.} = \frac{W}{W - W'}. \qquad (27.)$$

EXAMPLE.—A body in air weighs $36\frac{1}{4}$ ounces and in water 30 ounces; what is its specific gravity?

SOLUTION.—Substituting in formula **27**,

$$\text{Sp. Gr.} = \frac{W}{W - W'} = \frac{36\frac{1}{4}}{36\frac{1}{4} - 30} = \frac{36\frac{1}{4}}{6\frac{1}{4}} = 5.8. \quad \text{Ans.}$$

983. If the body is lighter than water, a piece of iron or other heavy substance must be attached to it, sufficiently heavy to sink both. *Then, weigh both bodies in air and both in water. Weigh both separately in air, and weigh the heavier body in water. Subtract the weights of the bodies in air and in water, and the result will be the weight of a volume of the water equal to the volume of the two bodies. Find the difference of the weights of the heavy body in air and in water, and the result will be the weight of a volume of water equal to the volume of the heavy body. Subtract this last result from the former, and the result will be the weight of a volume of water equal to the volume of the light body. The weight of the light body in air divided by the weight of its equal volume of water is the specific gravity of the light body.*

Let W = weight of both bodies in air;
W' = weight of both bodies in water;
w = weight of light body in air;
W_1 = weight of heavy body in air;
W_2 = weight of heavy body in water.

Then, the specific gravity of the light body is given by

$$\text{Sp. Gr.} = \frac{w}{(W - W') - (W_1 - W_2)}. \qquad (27a.)$$

EXAMPLE.—A piece of cork weighs 4.8 ounces in air. A piece of cast iron weighs 36 ounces in air and 31 ounces in water. The weight of the iron and cork together in water is 15.8 ounces; what is the specific gravity of the cork? of the cast iron?

SOLUTION.—Substituting in formula **27**a the values given,

$$\text{Sp. Gr.} = \frac{w}{(W - W') - (W_1 - W_2)} = \frac{4.8}{(40.8 - 15.8) - (36 - 31)} = \frac{4.8}{20} =$$
.24, the specific gravity of the cork. Ans.

By formula **27**, Sp. Gr. $= \frac{W}{W - W'} = \frac{36}{36 - 31} = 7.2$, the specific gravity of the iron. Ans.

984. To find the specific gravity of a liquid:

Weigh an empty flask; fill it with water, then weigh it and find the difference between the two results; this will equal the weight of the water. Then, weigh the flask filled with the liquid, and subtract the weight of the flask; the result is the weight of a volume of the liquid equal to the volume of the water. The weight of the liquid divided by the weight of the water is the specific gravity of the liquid.

Let W = the weight of the flask and liquid;
W' = the weight of the flask and water;
w = the weight of the flask.

Then, $$\text{Sp. Gr.} = \frac{W - w}{W' - w}. \qquad (\mathbf{27}b.)$$

EXAMPLE.—If the weight of the flask is 8 ounces, the weight when filled with water is 33 ounces, and when filled with alcohol 28 ounces, what is the specific gravity of the alcohol?

SOLUTION.—Substituting in formula **27**b,

$$\text{Sp. Gr.} = \frac{W - w}{W' - w} = \frac{28 - 8}{33 - 8} = .8. \quad \text{Ans.}$$

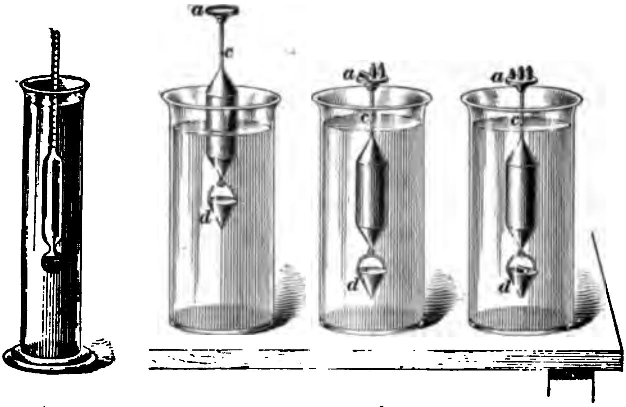
The method of finding the specific gravity of gases is about the same as that just given for liquids, the air being pumped out of the flask by an air pump. As there are great difficulties attending the operation, the method will not be described.

985. Instruments called **hydrometers** are in general use for determining quickly and accurately the specific gravities of liquids and some forms of solids. They are of two kinds, viz.:

1. *Hydrometers of constant weight*, *as* **Beaume's.**
2. *Hydrometers of constant volume*, *as* **Nicholson's.**

A hydrometer of constant weight is shown in Fig. 167 (*a*). It consists of a glass tube, near the bottom of which are two bulbs. The lower and smaller bulb is loaded with mercury or shot, so as to cause the instrument to remain in a vertical position when placed in the liquid. The upper bulb is filled with air, and its volume is such that the whole instrument is lighter than an equal volume of water.

The point to which the hydrometer sinks when placed in



(*a*) (*b*)
FIG. 167.

water is usually marked, the tube being graduated above and below in such a manner that the specific gravity of the liquid can be read directly. It is customary to have two instruments, one with the zero point near the top of the stem for use in liquids heavier than water, and the other with the zero point near the bulb for use in liquids lighter than water.

These instruments are more commonly used for determining the degree of concentration or dilution of certain liquids, as acids, alcohol, milk, solutions of sugar, etc., rather than their actual specific gravities. They are then known as *acidometers*, *alcoholometers*, *lactometers*, *saccharometers*, etc., according to the use to which they are put.

986. Nicholson's Hydrometer.—This instrument is shown in Fig. 167 (*b*). It consists of a hollow cylinder carrying at its lower end a basket *d*, heavy enough to keep the apparatus upright when placed in water. At the top of the cylinder is a vertical rod, to which is attached a shallow pan *a* for holding weights, etc. The cylinder must be of such size that the apparatus may be so much lighter than water that a certain weight W must be placed in the pan to sink it to a given point *c* on the rod. The body whose specific gravity it is desired to find must weigh less than W. It is placed in the pan *a*, and enough weight w is added to sink the point *c* to the water-level. It is evident that the weight of the given body is $W - w$. The body is now removed from the pan *a* and placed in the basket *d*, an additional weight being added to sink the point *c* to the water-level. Represent the weight now in the pan by W'. The difference $W' - w$ is the weight of a volume of water equal to the volume of the body. Hence,

$$\text{Sp. Gr.} = \frac{W - w}{W' - w}. \qquad (27c.)$$

EXAMPLE.—The weight necessary to sink the hydrometer to the point *c* is 16 ounces; the weight necessary when the body is in the pan *a* is 7.3 ounces, and when the body is in the basket *d*, 10 ounces; what is the specific gravity of the body?

SOLUTION.—$\text{Sp. Gr.} = \frac{W - w}{W' - w} = \frac{16 - 7.3}{10 - 7.3} = \frac{8.7}{2.7} = 3.222.$ Ans.

987. Archimedes' principle gives a very easy and accurate method of finding the volume of an irregularly shaped body. Thus, subtract its weight in water from its weight in air, and divide by .03617; the result will be in cubic inches, or divide by 62.5 and the result will be in cubic feet.

If the specific gravity of the body is known, its cubical contents can be found by dividing its weight by its specific gravity, and then dividing again by either .03617 or 62.5.

EXAMPLE.—A certain body has a specific gravity of 4.38 and weighs 76 pounds; how many cubic inches are there in the body?

SOLUTION.— $\frac{76}{4.38 \times .03617} = 479.72$ cu. in. Ans.

Since the weight of a cubic foot of water varies for different temperatures, and with the amount of impurities it contains, it is necessary to have some standard when getting the specific gravity. This standard is pure distilled water at its maximum density, which occurs at a temperature of 39.2° Fahrenheit. At this temperature water weighs 62.425 pounds per cubic foot; but for ordinary calculations it is customary to take it as weighing 1,000 ounces, or 62.5 pounds, per cubic foot.

CAPILLARY ATTRACTION.

988. If a clean glass rod be placed vertically in water, the water will be drawn up around the tube. (See *a*, Fig. 168.) If the same rod be placed in mercury, the liquid will be depressed instead of raised. On examination, it will be found that water *wets the glass, while mercury does not.* If the rod be greased and placed in water, the surface of the

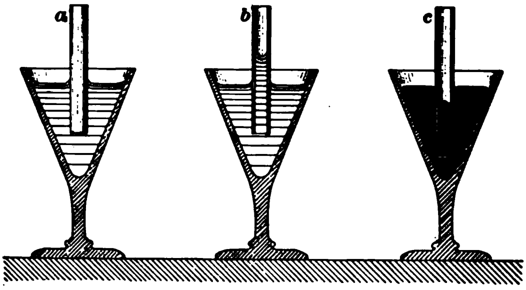


FIG. 168.

water will be depressed about the rod. If a clean lead or zinc strip be placed in mercury, the surface of the mercury will be *raised* about the strip.

In the last two cases, the greased rod came out *dry*, no water adhering to it, while the mercury did adhere to the lead or zinc strip, which came out *wet*.

In general, *all liquids that will wet the solids placed in* *them will be lifted, while those that do not will be pushed down.*

These phenomena are called **capillary attraction,** because they are best shown in very fine or hair-like tubes. In Fig. 168, *b* is a glass tube in water, and *c* is a glass tube in mercury. The surface of the water in the tube *b* is *concave*, while the surface of the mercury in the tube *c* is *convex*.

The amount which a liquid will ascend or be depressed varies inversely as the diameter of the tube. Thus, water will rise twice as far in a tube $\frac{1}{32}$ of an inch in diameter as in one $\frac{1}{16}$ of an inch in diameter.

There are many illustrations of capillary action. It is capillary attraction that aids the ascent of sap in the pores of plants. It lifts the oil between the fibers of a lamp wick to the place of combustion. It enables cloth and sponges to take up moisture. It causes blotting paper to absorb ink; but when the paper is *sized*, its pores are filled, and the ink dries by evaporation. It is capable of exerting great force, as is shown in the effects produced by the swelling of wood and other substances when kept wet. Dry wooden wedges driven into a groove cut around a cylinder of stone, and occasionally wet, will cause it to break asunder. As the pores between the fibers of a rope run around it in spiral lines, the swelling produced by wetting a tight rope will cause the fibers to shorten, and to contract the rope with immense force.

EXAMPLES FOR PRACTICE.

1. If a certain quantity of red lead weighs 5 pounds in air, and 4.441 pounds in water, what is its specific gravity? Ans. 8.94 +.

2. A piece of iron weighing 1 pound in air and .861 pound in water is attached to a piece of wood weighing 1 pound in air. When both bodies are placed in water they weigh .2 pound. What is (*a*) the specific gravity of the iron? (*b*) of the wood? Ans. { (*a*) 7.194. (*b*) .602.

3. An empty flask weighed 13 oz.; when filled with water, it weighed 22 oz., and when filled with sulphuric acid, 29.56 oz. What was the specific gravity of the acid? Ans. 1.84.

4. How many cubic feet of brick, having a specific gravity of 1.9, are required to weigh 260 pounds? Ans. 2.19 cu. ft., nearly.

HYDROKINETICS.

THE MEAN VELOCITY.

989. Hydrokinetics, also called **hydrodynamics** and **hydraulics,** treats of water in motion. The velocity is not the same at all points of the flow, unless all cross-sections of the pipe, or canal, are equal. That *velocity* which, being *multiplied by the area of the cross-section of the stream,* will equal the total quantity *discharged* is called the **mean velocity.**

Let Q = the quantity in cubic feet which passes any section in 1 second;
A = the area of the section in square feet;
v_m = the mean velocity in feet per second.

Then, $$Q = A\,v_m, \qquad (28a.)$$

and $$v_m = \frac{Q}{A}. \qquad (28b.)$$

EXAMPLE.—The area of a certain cross-section of a stream is 27.9 square inches; the mean velocity of the water through this section is 51 feet per second; what is the quantity discharged, in cubic feet?

SOLUTION.— $Q = A\,v_m = \frac{27.9}{144} \times 51 = 9.9$ cu. ft. per sec. Ans.

NOTE.— 1 square foot = 144 square inches.

EXAMPLE.—In the last example, what would the mean velocity have been had the area of the cross-section been 36 square inches, to discharge the same quantity?

SOLUTION.— $v_m = \frac{Q}{A} = \frac{9.9}{\frac{36}{144}} = \frac{9.9 \times 144}{36} = 39.6$ ft. per sec. Ans.

VELOCITY OF EFFLUX.

990. If a small aperture is made in a vessel containing water, the velocity with which the water issues from the vessel is the same as if it had fallen from the level of the surface to the level of the aperture, all resistances being neglected. This velocity is called the **velocity of efflux.**

The *vertical height* of the level surface of the water above the horizontal line through the center of the aperture is called the **head.** In Fig. 169, a is the head for the aperture A; b is the head for the aperture B, and c is the head for the aperture C.

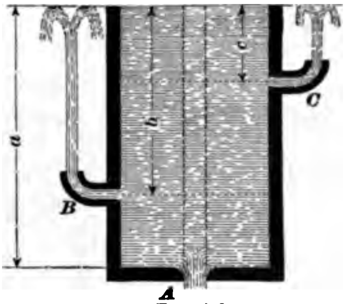


FIG. 169.

Let v = the velocity of efflux in feet per second;

h = the head in feet at the aperture considered;

W = the weight of the water in pounds flowing through this aperture per second.

Were it not for the resistance of the air, friction, and the effect of the falling particles, the issuing water would spout to the level of the water in the vessel; that is, to a height equal to its head. The kinetic energy of the issuing water will be expressed by $\frac{Wv^2}{2g}$. The work it can do will be Wh.

Since the kinetic energy equals the work, $\frac{Wv^2}{2g} = Wh$, or $v = \sqrt{2gh}$; that is, *the velocity of efflux is the same as if the same weight of water had fallen through a height equal to its head.*

EXAMPLE.—A small orifice is made in a pipe 50 feet below the water level; what is the velocity of the issuing water?

SOLUTION.— $v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 50} = 56.7$ ft. per sec. Ans.

From the above formula, as in the laws of falling bodies, $h = \frac{v^2}{2g}$. Here, h is called the *head due to the velocity* v. Consequently, if the velocity of efflux is known, the head can be found.

EXAMPLE.—An issuing jet of water has a velocity of 60 feet per second; what is the head that causes it to flow with this velocity?

SOLUTION.— $h = \frac{v^2}{2g} = \frac{60^2}{2 \times 32.16} = 55.97$ feet. Ans.

991. Suppose that a tall vessel be fitted with a piston, and that it has an orifice near the bottom fitted with a stop-cock. If an additional pressure be applied to the piston, it is evident that the velocity of efflux will be increased.

Let p be the pressure per unit of area at the level of the water, due to the additional pressure on the piston. If the unit of area is one square inch, the height of a column of water that will cause a pressure equal to p is $\frac{p}{.434}$ feet.

If the unit of area is 1 square foot, the height of a column of water is $\frac{p}{62.5}$ feet. Denote this height corresponding to the additional pressure by h_1. The original head of the water in the vessel is h; hence, $h_1 + h =$ the total head, and the velocity of efflux, when the cock is opened, will be

$$v = \sqrt{2g(h_1 + h)}. \qquad \textbf{(29.)}$$

The total head, $h_1 + h$, is called the **equivalent head**, and must, in all cases, be reduced to feet before substituting in the formula.

EXAMPLE.—The area of a piston fitting a vessel filled with water is 27.36 square inches. The total pressure on the piston is 80 pounds; the weight of the piston is 25 pounds, and the head of the water at the level of the orifice is 6 feet 10 inches; what is the velocity of the efflux, assuming that there are no resistances?

SOLUTION.— $80 + 25 = 105$ lb. = the total pressure on the upper surface of the liquid. $\frac{105}{27.36} = 3.8377$ lb. per sq. in.

$\frac{3.8377}{.434} = 8.8426$ feet = head in feet due to the pressure of 105 pounds = h_1. 6 ft. 10 in. = 6.8333 ft. = h.

$v = \sqrt{2g(h_1 + h)} = \sqrt{2g(8.8426 + 6.8333)} = \sqrt{2 \times 32.16 \times 15.6759} =$ 31.75 ft. per sec. Ans.

992. When water issues from the side of a vessel, it will be subjected to the same laws that govern projectiles. The range may be calculated in the same manner by taking the *velocity of efflux* as the *initial velocity* of the projectile.

The range may be calculated more conveniently by the following formula :

$$R = \sqrt{4hy}, \qquad \textbf{(30.)}$$

in which R is the range, h is the head, or equivalent head at the level of the orifice, and y is the vertical height of the orifice above the point where the water strikes, all dimensions being in feet. In Fig. 170, the upper surface of the

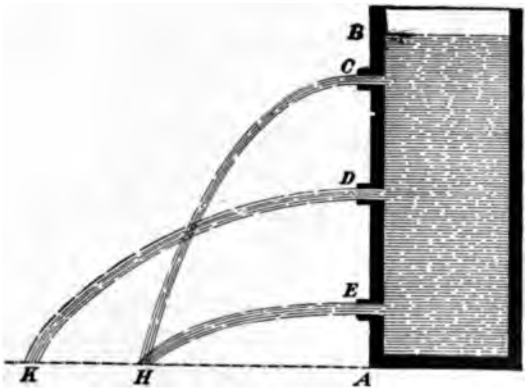


FIG. 170.

water is free. For the orifice E, $h = BE$ and $y = EA$; for the orifice C, $h = BC$ and $y = CA$.

The greatest range is obtained when $h = y$; that is, when the orifice is half way between the upper surface of the water and the level of the place where the stream strikes. If two orifices are situated equally distant from the middle orifice, giving the greatest range, as C and E in Fig. 170, the ranges of the water issuing from them will be equal.

EXAMPLE.—The vertical height above the ground of the surface of the water in a vessel is 12 feet. If an orifice is situated 4 feet from the upper surface, what is the range? What is the greatest range? Where is the other point of equal range?

SOLUTION.— $R = \sqrt{4hy} = \sqrt{4 \times 4 \times 8} = 11.31$ ft., nearly. Ans.

Greatest range $= \sqrt{4 \times 6 \times 6} = 12$ feet. Ans.

$6 - 4 = 2$; hence, the point of equal range is $6 + 2 = 8$ feet below the surface of the water.

PROOF.—Range $= \sqrt{4hy} = \sqrt{4 \times 8 \times 4} = 11.31$ ft., as before.

993. When the water flows through an orifice of large size in the bottom of the vessel, compared with the area of the base, a different rule must be used from that given above. In Fig. 171, suppose that the area of the orifice in the bottom of the vessel is a, and that the area of the bottom is A; then, the velocity v is expressed by the formula

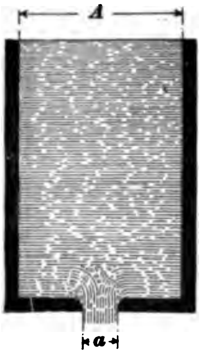


FIG. 171.

$$v = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}}. \qquad \textbf{(31.)}$$

That is, *the velocity of efflux from the bottom of a vessel, in feet per second, equals the square root of 2 g times the head, divided by 1 minus the ratio of the square of the area of the orifice to the square of the area of the bottom.*

The value of v given in the last formula is always greater than the theoretical velocity that would be produced by the head h, where h is the depth of the orifice below the surface of the water in the vessel. This increase is due to the fact that, in order to keep the vessel full, that is, in order to keep the head h constant, a quantity of water must flow in at the top equal to the quantity discharged at the orifice. The head equivalent to the velocity of this entering water must be added to the head h in order to obtain the actual head producing flow from the orifice, and the value of v given by the formula is the velocity produced by the sum of the two heads.

If the area of the cross-section of the base is more than 20 times the area of the orifice, use the formula $v = \sqrt{2gh}$. That is,

Rule.—*The velocity of efflux from a small orifice, when the cross-sectional area of the vessel is equal to, or more than, twenty times the area of the orifice, equals the square root of 2 g times the head.*

EXAMPLE.—A vessel has a rectangular cross-section, 11 × 14 inches, and the upper surface of the water is 14 feet above the bottom. If an orifice 4 inches square is made in the bottom of the vessel, what is the velocity of the efflux?

SOLUTION.—Area of the cross-section is $14 \times 11 = 154$ sq. in. Area of orifice is $4 \times 4 = 16$ sq. in. Since $154 \div 16 = 9\frac{5}{8}$, the area of the base is less than 20 times the area of the orifice; hence, using formula **31**,

$$v = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}} = \sqrt{\frac{2 \times 32.16 \times 14}{1 - \frac{16^2}{154^2}}} = 30.17 \text{ ft. per sec. Ans.}$$

EXAMPLE.—If the orifice had been 2 inches square in the above example, what would the velocity of efflux have been? Also, if it had been 8 inches square?

SOLUTION.— 2 inches $\times$ 2 inches = 4 sq. in., or the area of the orifice. Since $154 \div 4 = 38\frac{1}{2}$, the area of the base is greater than 20 times the area of the orifice; hence, using the last rule,

$$v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 14} = 30.008 \text{ feet per second. Ans.}$$

8 inches $\times$ 8 inches = 64 square inches, or the area of the orifice in the second case.

$$v = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}} = \sqrt{\frac{2 \times 32.16 \times 14}{1 - \frac{64^2}{154^2}}} = 32.99 \text{ feet per second; practi-}$$

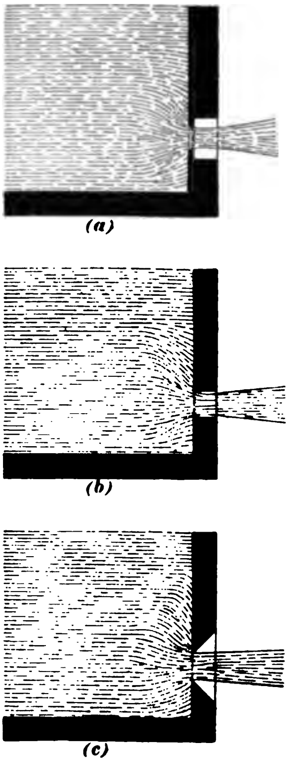
cally, 33 feet per second. Ans.

THE STANDARD ORIFICE.

994. An orifice in the side or bottom of a vessel or reservoir, and at a distance below the surface of the water, is called a **standard orifice** when the flow through it takes place in such a manner that the jet touches the opening on the inside edge only. A hole in a thin plate, as at (*a*), Fig. 172, is such an orifice, as is also a square-edged hole in the side of the vessel, as at (*b*), when the thickness of the side is not so great that the jet touches it beyond the inner edge. If the sides of the reservoir are very thick, a standard orifice can be made by beveling the outer edges, as shown at (*c*).

When a jet issues from such an orifice, it *contracts* so that the diameter is least at a distance from the edge equal to about one-half the diameter of the orifice. Beyond this point the jet gradually enlarges, and gradually becomes broken by the effect of the resistance of the air.

The **coefficient of contraction** is the number by which the area of the orifice is to be multiplied in order to obtain the least cross-section of the jet. Experiments on jets from standard orifices have given values for this coefficient varying from .57 to .71. The most probable mean value is about .62.


(a) (b) (c)
FIG. 172.

The **coefficient of velocity** is the number by which the theoretical velocity must be multiplied in order to obtain the actual maximum velocity, or velocity where the cross-section of the jet is least.

If v is the theoretical velocity, v' the actual velocity, and c' the coefficient of velocity, we have the formula

$$v' = c'v = c'\sqrt{2gh}. \qquad \textbf{(32.)}$$

It is found that c' is greater for high heads than for low, and values ranging from .975 to nearly 1 have been obtained by different experimenters. An average value usually taken is .98.

EXAMPLE.—What is the actual velocity of discharge from a small standard orifice in the side of a vessel, if the head is 20 feet?

SOLUTION.—

$$v' = c'\sqrt{2gh} = .98\sqrt{2 \times 32.16 \times 20} = 35.15 \text{ ft. per second. Ans.}$$

Use of Logarithms.—Most of the problems occurring in hydraulics involve the operations of multiplication, division, involution, and evolution in such a way that they are most readily solved by the use of logarithms. The student should, therefore, review the section on logarithms carefully and learn to apply them at once in the solution of the problems given in this section.

EXAMPLE.—Solve the last example by the use of logarithms.

SOLUTION.—First find the logarithm of the product of the numbers under the radical sign from the table of logarithms, as follows:

$$\begin{aligned} \log 2 &= .30103 \\ \log 32.16 &= 1.50732 \\ \log 20 &= 1.30103 \\ \hline & \quad 3.10938 \end{aligned}$$

The logarithm of the square root of the above product is found by dividing its logarithm by 2; thus,

$$\log \sqrt{2 \times 32.16 \times 20} = 3.10938 \div 2 = 1.55469.$$

Finally, the logarithm of the product of .98 multiplied by the quantity under the radical sign is the sum of the logarithm of .98 and 1.55469, or $\bar{1}.99123 + 1.55469 = 1.54592$. From the table of Logarithms, the number corresponding to this logarithm is found to be 35.15. Ans.

It was shown in Art. **989** that the theoretical discharge from an orifice whose area is a is $Q = a\,v_m$, where v_m is the theoretical mean velocity due to the head.

The actual discharge is, in the same way, $Q' = a'v'$, in which a' is the area and v' the mean velocity where the cross-section of the jet is least.

The **coefficient of discharge** is the number by which the theoretical discharge is to be multiplied to give the actual discharge. Let c'' be this factor; then $Q' = c''\,Q = a'v' = .62\,a \times .98\,v = .6076\,a\,v = .6076\,a\sqrt{2gh}$.

Since the coefficients of contraction and velocity vary, it is evident that the coefficient of discharge varies also. As an average value we may take $c'' = .61$, and we then have

$$Q' = c''\,Q = c''\,a\sqrt{2gh} = .61\,a\sqrt{2gh}. \qquad \textbf{(33.)}$$

EXAMPLE.—What will be the actual discharge from a circular standard orifice 3 inches in diameter under a head of 25 feet?

SOLUTION.—The area of a 3-inch circle $= .7854 \times 3^2 = 7.0686$ square inches $= \frac{7.0686}{144} = .049$ square foot.

$Q' = c''\,a\sqrt{2gh} = .61 \times .049\sqrt{2 \times 32.16 \times 25} = 1.1985$ cu. ft. per second. Ans.

995. Standard orifices are sometimes used to measure the quantity of water flowing in a stream or from a pipe or other channel. When used for this purpose they are either **circular, square,** or **rectangular.** The coefficient of discharge varies for each of these forms, and is also different for different heads and areas of orifice.

996. For a circular orifice the discharge in cubic feet per second may be obtained from the formula

$$Q = .7854\, d^2 c \sqrt{2gh} = 6.299\, d^2 c \sqrt{h}. \qquad (34a.)$$

Here d is the diameter of the orifice in feet, h the head on the center of the orifice in feet, and c a coefficient that depends on d and h and is to be taken from the table of Coefficients for Circular Vertical Orifices.

For values of d and h between those given in the table the value of c may be found by interpolation, but for most cases the value of c corresponding to the nearest values of d and h will be near enough for practical purposes.

EXAMPLE.—What is the discharge from a circular orifice $1\frac{1}{2}$ inches in diameter, if the head is 7 feet?

SOLUTION.—The diameter of the orifice is .125 foot; from the table of Coefficients for Circular Vertical Orifices, the coefficient is found to be .600 for an orifice .10 foot in diameter under a head of 6 feet, and the same for a head of 8 feet. In the same way, the coefficient for a diameter of .2 foot is .598 from 6 feet to 8 feet head. For an orifice .125 foot in diameter, the coefficient is $.600 - (.002 \times .25) = .5995$.

From the formula

$$Q = 6.299 \times .125^2 \times .5995 \sqrt{7} = .15611 \text{ cubic foot per second.}$$

If we use the nearest value of c given in the table, the result is

$$Q = 6.299 \times .125^2 \times .6 \sqrt{7} = .15623,$$

a result that differs from the first by about .08 of 1 per cent., which is much less than the probable errors in measuring h and d.

NOTE.—The values of the constants and coefficients used in the formulas for the flow of water are average values determined from experiment, and are, therefore, only approximately correct. In practice it is useless to use more than four decimal places, and in most cases three are enough.

997. The discharge for a square vertical orifice is given by the formula

$$Q = c\, d^2 \sqrt{2gh} = 8.02\, c\, d^2 \sqrt{h}. \qquad (34b.)$$

Here d is the length in feet of one side of the orifice, h is the head in feet on the center of the orifice, Q is the discharge in cubic feet per second, and c is a coefficient that depends on

h and d. The values of c for different values of h and d are to be taken from the table of Coefficients for Square Vertical Orifices.

998. For rectangular orifices the discharge in cubic feet per second is given by the formula

$$Q = c \times \tfrac{2}{3} b \sqrt{2g}\,(\sqrt{h_2^3} - \sqrt{h_1^3}), \qquad \textbf{(34}c.\textbf{)}$$

in which b is the breadth of the orifice, h_1 the head measured from the upper edge of the orifice, and h_2 the head measured from the lower edge, all in feet.

If d is the depth of the orifice in feet, and h the head in feet, on its center, the discharge, when h is greater than 4 d, may be taken from the formula

$$Q = c\,b\,d\sqrt{2gh} = 8.02\,c\,b\,d\sqrt{h}. \qquad \textbf{(34}d.\textbf{)}$$

For approximate computations the value of c that may be used in formulas **34**c and **34**d is $c = .615$. The table of Coefficients for Rectangular Vertical Orifices gives average values of c for orifices 1 foot wide with different values of d and h.

EXAMPLE.—A rectangular orifice has a depth $d = 2$ feet and a breadth $b = 3$ feet. The depth of the top edge below the surface of the water is 5 feet; what is the discharge in cubic feet per minute?

SOLUTION.—Use formula **34**c; $h_1 = 5$; $h_2 = 5 + 2 = 7$; $Q = .615 \times \tfrac{2}{3} b \sqrt{2g}(\sqrt{h_2^3} - \sqrt{h_1^3}) = .615 \times \tfrac{2}{3} \times 3 \times \sqrt{2 \times 32.16}(\sqrt{7^3} - \sqrt{5^3}) = 72.41$ feet per second $= 72.41 \times 60 = 4{,}344.6$ cu. ft. per minute. Ans.

EXAMPLE.—A dam across a stream has an opening closed by a sluice gate, see Fig. 173 (a), in such a way that the gate when opened forms a practical standard orifice of rectangular cross-section. The width of the opening is 1 foot, and it is found that the dam is kept just filled when the gate is opened 9 inches. What is the rate of flow of the stream, if the center of the opening is 6 feet below the surface of the water in the dam?

SOLUTION.—From the table of Coefficients for Standard Rectangular Vertical Orifices, we find that the value of c' for an orifice 1 foot wide and .75 foot in depth to be .604 when the head is 6 feet. Since the head is more than four times the depth, formula **34**d may be used, from which we have

$$Q = 8.02 \times .604 \times 1 \times .75\sqrt{6} = 8.9 \text{ cu. ft. per second. Ans.}$$

999. If the orifice is made at the side of the reservoir, as shown at *a*, Fig. 173 (*b*), or at the bottom, as shown at *b*, the contraction of the steam is reduced and the discharge increased. Experiments show an increase in the discharge

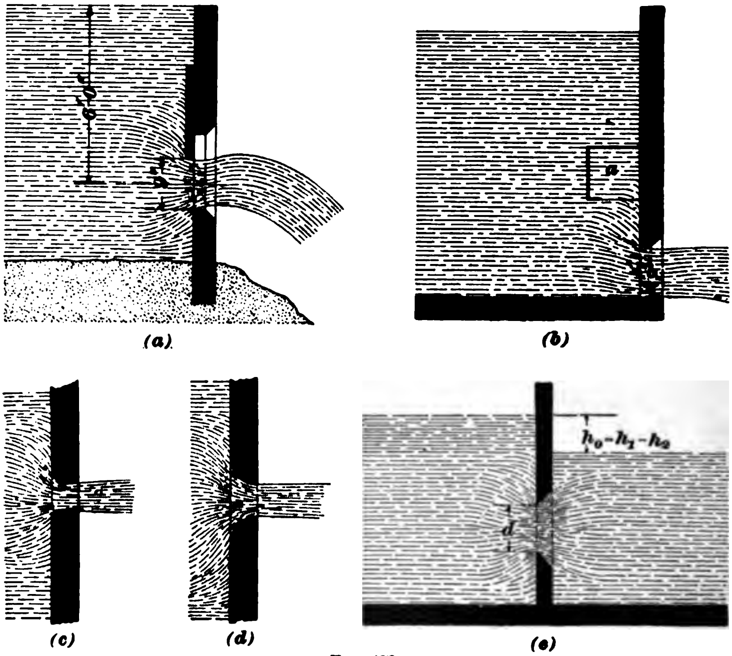


FIG. 173.

of about 3.5 per cent. for *a*, and from 6 to 12 per cent. for *b*. These values have not been accurately determined, and where accurate measurements are to be made the orifice should always be arranged as shown in Fig. 172.

If the inner edge of the orifice is rounded, as shown in Fig. 173 (*c*) and (*d*), the coefficient of discharge is increased, and may be made nearly 1, if the edge is rounded as shown at (*d*). Orifices with rounded edges should never be used for measuring water.

An example of a **submerged rectangular orifice** is shown in Fig. 173 (*e*). The discharge is given by the formula

$$Q = .615\, b\, d \sqrt{2 g h_0}, \qquad (34c.)$$

where h_0 is the difference in level of the water on the two sides of the orifice, b is the breadth of the orifice, and d its depth, all in feet.

EXAMPLES FOR PRACTICE.

1. What is the discharge in cubic feet per minute from a standard circular orifice whose diameter is 2¼ inches, if the head is 20 feet? Ans. 43.71 cu. ft. per min.

2. A square orifice in the side of a reservoir measures .2 foot on each side, and the head on the center is 22 feet; what is the discharge in cubic feet per second? Ans. .9058 cu. ft. per sec.

3. What is the discharge from a rectangular orifice 1 foot wide, if the head on the upper edge is 2¼ feet and the depth of the orifice 10½ inches? Ans. 7.302 cu. ft. per sec.

4. What is the approximate discharge from a rectangular gate in the side of a dam when the breadth is 15 inches, the depth 6 inches, and the head on the upper edge 4½ feet? Use the approximate coefficient of discharge, $c = .615$. Ans. 6.72 cu. ft. per sec.

5. What is the discharge from a submerged rectangular orifice 1½ feet wide and 1 foot deep, if the difference in the level of the water on the two sides of the orifice is 3¼ feet? Ans. 13.84 cu. ft. per sec.

WEIRS.

1000. A **weir** is an obstruction placed across a stream for the purpose of diverting the water, so as to make it flow through a desired channel. This channel may be a notch or opening in the obstruction itself, and it has been found that, when properly constructed and carefully managed, such a weir forms one of the most convenient and accurate devices for measuring the discharge of streams.

Many careful experiments have been made to determine the quantity of water that will flow over different forms of weirs under varying conditions. As the result of these experiments two forms have come to be generally used, and the amount of flow in any particular case is determined by simple formulas and tabulated coefficients that depend on observed conditions.

1001. A **weir with end contractions** is shown in Fig. 174 (*a*). The notch is narrower than the channel through which the water flows; this causes a contraction at the bottom and at the two ends of the issuing stream.

1002. A **weir without end contractions,** also called a **wier with end contractions suppressed,** is

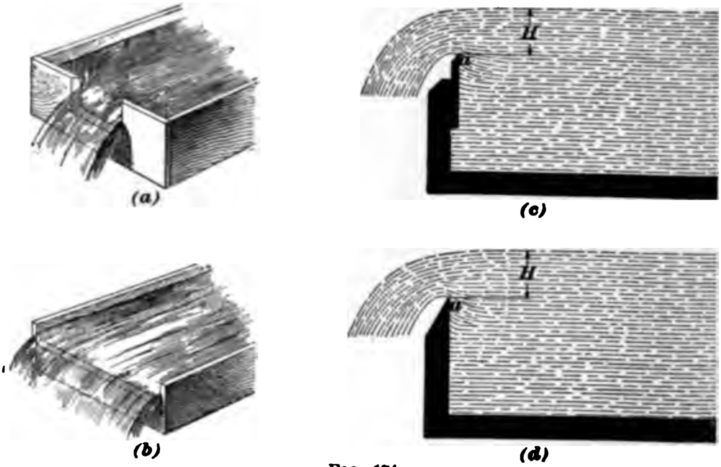


FIG. 174.

shown in Fig. 174 (*b*). In this case the notch is as wide as the channel leading to it; consequently, the issuing stream is contracted at the bottom only.

1003. The edge *a* of the notch, Fig. 174 (*c*) and (*d*). is called the **crest of the weir.** The inner edges of the notch are made sharp, so that the water in passing through it touches only along a line. For very accurate work the edges, both vertical and horizontal, should be made with a thin plate of metal having a sharp inner edge, as shown in Fig. 174 (*c*); but for ordinary work, the edges of the board in which the notch is cut may be chamfered off to an angle of about 30°, as shown at (*d*).

The bottom edge of the notch must be straight and set perfectly level, and the sides must be set carefully at right angles to the bottom.

The head *H* producing the flow, Fig. 174 (*c*) and (*d*), is the vertical distance from the **crest** to the surface of the water. It must be measured at a point so far from the crest that the curvature of the flowing water will not affect the measurement.

The distance from the crest of the weir to the bottom of the feeding canal or reservoir should be at least three times the head; and, with a weir having end contractions, the distance from the vertical edges to the sides of the canal should also be at least three times the head.

The water must approach the weir quietly, and with little velocity. It is sometimes necessary to provide means for reducing the velocity of the water as it approaches the weir.

1004. Fig. 175 shows a simple form of weir located in a small stream for the purpose of measuring its discharge.

A plank dam is put across the stream at a convenient point, care being taken to prevent any leakage under or

FIG. 175.

around the dam. The length of the notch is great enough to provide for the flow with a head of between .5 and 1.5 feet. A stake *E* is driven firmly into the ground at a point about 6 feet up the stream from the weir and near the bank, as shown. The stake is driven until its top is at exactly the same level as the crest *B*. The head is the vertical distance from the top of this stake to the surface of the water and it may be measured by a square or two-foot rule, as shown in the figure.

THE HOOK GAUGE.

1005. For accurate weir measurements, such as are made in testing the efficiency of water-wheels, the head on the crest is measured with an instrument called a **hook gauge**; see Fig. 176. A hook *a* is attached to the lower end of a sliding scale *b*. The scale is graduated to hundredths of a foot and is provided with a vernier by means of which it can be read to thousandths. The scale and hook can be raised or lowered by means of the screw *s*. The instrument is fastened securely to solid and substantial beams or masonry, at a point over the water a few feet up-stream from the weir, and where the surface of the water is quiet and protected from the influence of wind or eddies. The gauge is set so that the scale will read zero when the point of the hook is at the same level as the crest of the weir. When the point of the hook is raised to the surface of the water, it lifts the surface slightly before breaking through. To use

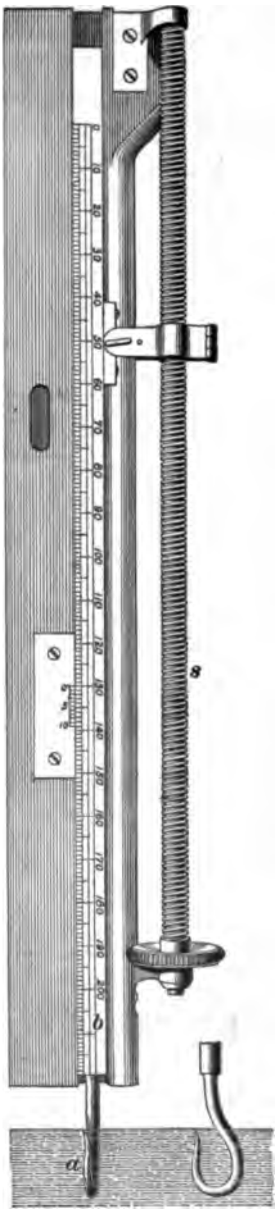


FIG. 176.

the gauge, start with the hook below the surface of the water and raise it slowly until a slight pimple, caused by the lifting of the surface, appears over the point; the reading of the scale for this position of the hook gives the head on the crest.

THE DISCHARGE OF WEIRS.

1006. When the dimensions of the notch and the head on the crest of a weir are known, the discharge can be computed by means of the following formulas and tables of coefficients:

Let l = length of the weir;
H = measured head;
v = velocity with which the water approaches the weir;
h = head equivalent to the velocity with which the water approaches the weir;
c = coefficient of discharge;
Q = theoretical discharge;
Q' = actual discharge.

The theoretical discharge per second is

$$Q = \tfrac{2}{3}\sqrt{2g}\, l\, (H + h)^{\frac{3}{2}}. \qquad \textbf{(35}a.\textbf{)}$$

If there is no velocity of approach, this becomes

$$Q = \tfrac{2}{3}\sqrt{2g}\, l\, H^{\frac{3}{2}}. \qquad \textbf{(35}b.\textbf{)}$$

The actual discharge for weirs without end contractions is given by the formulas

$$Q' = c\left(\tfrac{2}{3}\sqrt{2g}\right) l\, (H + \tfrac{4}{3}h)^{\frac{3}{2}} = 5.347\, c\, l\, (H + \tfrac{4}{3}h)^{\frac{3}{2}}, \qquad \textbf{(36}a.\textbf{)}$$

and $$Q' = c\left(\tfrac{2}{3}\sqrt{2g}\right) l\, H^{\frac{3}{2}} = 5.347\, c\, l\, H^{\frac{3}{2}}. \qquad \textbf{(36}b.\textbf{)}$$

For weirs with end contractions, the formulas are

$$Q' = c\,\tfrac{2}{3}\sqrt{2g}\, l\, (H + 1.4\, h)^{\frac{3}{2}} = 5.347\, c\, l\, (H + 1.4\, h)^{\frac{3}{2}}, \qquad \textbf{(37}a.\textbf{)}$$

and $$Q' = \tfrac{2}{3}\sqrt{2g}\, l\, H^{\frac{3}{2}} = 5.347\, c\, l\, H^{\frac{3}{2}}. \qquad \textbf{(37}b.\textbf{)}$$

EXAMPLE.—What is the discharge of the stream in Fig. 175, if the length of the weir is 5 feet, the head $10\frac{1}{2}$ inches, the coefficient of discharge .603, and the velocity of approach = 0?

SOLUTION.—Applying formula **36b**, we have

$Q' = .603 \times 5.347 \times 5 \times .875^{\frac{3}{2}} = 13.1934$ cu. ft. per second. Ans.

1007. The **velocity of approach** is the mean velocity with which the water flows through the canal leading to the weir. If A is the area of the cross-section of the water in this canal, we have $v = \frac{Q'}{A}$, from which we see that Q' must be determined approximately by assuming $v = 0$, and then use this value of Q' to find v. V may also be measured approximately by means of a float on the water in the canal or stream.

Having found v, we have the equivalent head, $h = \frac{v^2}{2g} = .01555\ v^2$. See Arts. **990** and **991**. Since v is small with a properly constructed weir, it is usually neglected, unless great accuracy is required.

The table of Coefficients for Weirs with End Contractions gives values of the coefficient of discharge c for weirs with end contractions and different values of H and l.

The table of Coefficients for Weirs Without End Contractions gives values for c for weirs without end contractions.

Weirs with end contractions are more often used and are to be recommended in most cases.

Values of c for values of H and l between those given in the tables can be found by interpolating, assuming that the variation is uniform between the values given.

EXAMPLE 1.—What is the discharge from a weir with end contractions under the following conditions: The length of the weir is 4 feet $1\frac{1}{2}$ inches, and the measured head $10\frac{1}{16}$ inches? Assume that there is no velocity of approach.

SOLUTION.—The length l of the weir = 4 feet $1\frac{1}{2}$ inches = 4.125 feet, and the head $H = 10\frac{1}{16}$ inches = .84 foot. From the table of Coefficients for Weirs with End Contractions, we find the coefficient $c = .600$ for a weir 3 feet long and a head of .8 foot and $c = .604$ for a weir 5 feet long with the same head. This is an increase in the coefficient of $(.604 - .600) \div 2 = .002$ for each increase of 1 foot in length. The coefficient for a weir 4.125 feet long is, therefore, $.600 + (1.125 \times .002) = .60225$. The rate of increase for a head of .9 foot is $(.603 - .598) \div 2 = .0025$, and the coefficient for a weir 4.125 feet long is $.598 + (1.125 \times .0025) = .60081$. The decrease in coefficient for an increase in head of .1 foot is $.60225 - .60081 = .00144$ and for an increase in head of .04 foot the decrease is $.00144 \times \frac{.04}{.1} = .000576$. This subtracted from the coefficient for .8 foot gives $.60225 - .000576 = .601674$ as the coefficient of discharge for a weir 4.125 feet long and a head of .84 foot. Using but four decimal places, the discharge is

$$Q' = 5.347 \times .6017 \times 4.125 \times .84^{\frac{3}{2}} = 10.22 \text{ cu. ft. per second. Ans.}$$

EXAMPLE 2.—If the canal leading to the weir of example 1 is 10 feet wide and 3 feet deep below the crest of the weir, what is the head equivalent to the velocity of approach ?

SOLUTION.—The depth of water in the canal is the depth below the crest plus the head = 3.84 feet. The area of the cross-section of the water in the canal is $A = 3.84 \times 10 = 38.4$ square feet, and the velocity is $v = \frac{10.22}{38.4} = .266$ foot per second. The head h equivalent to the velocity v is

$$h = \frac{v^2}{2g} = \frac{.266^2}{64.32} = .0011 \text{ foot. Ans.}$$

NOTE.—This value of h is so small that its influence on the discharge is much less than the probable errors in measuring the head H, and so need not be considered in finding the discharge.

FLOW THROUGH TUBES.

THE STANDARD TUBE.

1008. A **standard tube** or an **adjutage** is a tube whose length is equal to $2\frac{1}{2}$ or 3 times its diameter. When water flows from a reservoir through such a tube, as shown in Fig. 177 (h), the jet contracts when it first leaves the reservoir, then expands again until it fills the tube near its outer end, this contraction and expansion resembling that of the jet from a standard orifice.

Owing to the contraction, an annular space is left between the jet and the tube. When the expansion of the jet is sufficient to fill the outer end of the tube, as shown in the figure, the current of water carries some of the air from the annular space along with it, thus producing a partial vacuum.

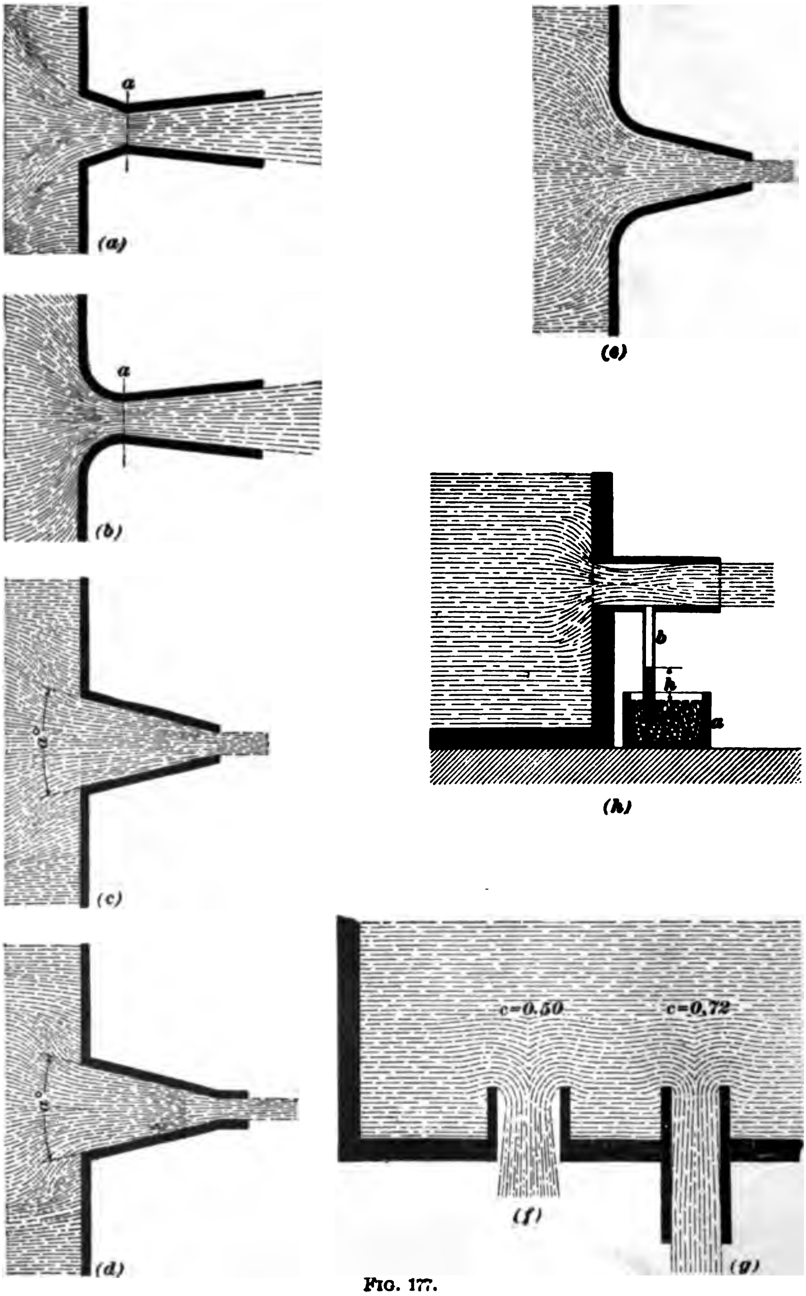


FIG. 177.

If a small branch *b* is carried down into a cup of mercury *a*, the pressure of the atmosphere will force mercury into *b* to a height *h* that depends on the vacuum in the annular space around the jet, and if a small hole be made in the tube it will be found that air is drawn in through the hole. On account of the difficulty in maintaining uniform conditions, which makes the value of the coefficient of discharge uncertain, tubes are seldom used for measuring the flow of water.

The *coefficient of discharge* for a standard tube is greater than for a standard orifice. An average value is

$$c = .82.$$

The jet as it issues from the tube has the same area as the tube; hence, the coefficient of velocity is the same as the coefficient of discharge.

CONICAL TUBES.

1009. For **conical tubes,** see Fig. 177 (*c*) and (*d*), the coefficient of discharge reaches a maximum value of .946 when the angle $a°$ of the cone is 13° 24′. The coefficient of velocity increases with the angle of the cone until it becomes about the same as the coefficient for the standard orifice. If the inner edge of the tube is well rounded, as at (*e*), Fig. 177, the coefficient of discharge is still further increased and may be made nearly 1.

COMPOUND TUBES.

1010. Examples of **compound tubes** are shown in Fig. 177 (*a*) and (*b*). Experiments have shown that the velocity through the minimum section *a* is greater than the theoretical velocity due to the head. The values of the coefficient of discharge for the section *a* vary greatly under different conditions of head and proportions of tubes. Under certain conditions values as high as 2.43 have been obtained.

1011. Inward Projecting Tubes.—When a tube projects into a reservoir, as shown in Fig. 177 (*f*) and (*g*), the contraction is increased and the discharge greatly reduced. The coefficient of discharge for the inwardly projecting orifice, Fig. 177 (f), is about 0.50, and for the tube, Fig. 177 (g), 0.72.

THE ENERGY OF A JET.

1012. The **energy in a jet of water** is a measure of the theoretical work which the jet can do.

If W is the weight of water per second passing a given cross-section of the jet, and v the velocity in feet per second at that cross-section, the energy K is expressed by the formula

$$K = W h = W \frac{v^2}{2g}. \qquad \textbf{(38}a.\textbf{)}$$

Here h is the head that would produce a velocity v if the water were to fall freely through a vacuum.

If a is the area of the jet and w the weight of a cubic unit of water, we have $W = w\,a\,v$, and the equation for K becomes

$$K = W \frac{v^2}{2g} = w\,a\,v \frac{v^2}{2g} = \frac{w\,a\,v^3}{2g}. \qquad \textbf{(38}b.\textbf{)}$$

These formulas show that with a given weight of water passing per second the energy of the jet is proportional to the square of the velocity, and with a given area of jet the energy is proportional to the cube of the velocity.

EXAMPLE 1.—What is the theoretical work that can be done by a jet through which $2\frac{1}{2}$ cubic feet of water passes with a velocity of 8 feet per second?

SOLUTION.—From formula **38**a, we have

$$K = 2.5 \times 62.5 \times \frac{8^2}{2 \times 32.16} = 155.47 \text{ ft.-lb. per second. Ans.}$$

EXAMPLE 2.—What is the theoretical work that can be done by a jet whose area is .5 square foot, if the velocity of flow is 12 feet per second?

SOLUTION.—

$$K = w\,a\,v \frac{v^2}{2g} = 62.5 \times .5 \times 12 \times \frac{12^2}{2 \times 32.16} = 839.55 \text{ ft.-lb. per second.}$$

Ans.

EXAMPLE 3.—What is the horsepower equivalent to the energy in the jets in examples 1 and 2?

SOLUTION.—One horsepower equals 33,000 foot-pounds of work done in one minute, or $\frac{33,000}{60} = 550$ foot-pounds in one second; hence, the theoretical horsepower in the jet in example 1 is $\frac{155.47}{550} = .282$; and in example 2, $\frac{839.55}{550} = 1.526$. Ans.

Since the energy in a given weight of water passing through a jet is proportional to the square of the velocity, it is evident that when the water is to be used for doing work by the action of a jet on the vanes of a water-wheel, the velocity should be as great as possible.

The theoretical velocity that can be obtained is $v = \sqrt{2 g h}$, where h is the head of the orifice from which the water flows. It was shown in Art. **994** that the actual velocity of a jet is always less than the theoretical velocity, and is found by multiplying the theoretical velocity by a coefficient that depends on the form of the orifice from which the water flows; this coefficient is always less than 1.

EXAMPLE 1.—What is the energy in the jet from a standard orifice from which 20 cubic feet of water flows per second, if the head on the orifice is 16 feet, and the coefficient of velocity .98 ?

SOLUTION.—The actual velocity is $v = .98 \sqrt{2 g h}$, and the energy

$$K = W\frac{v^2}{2 g} = W \times .98^2 h = 20 \times 62.5 \times .98^2 \times 16 =$$

19,208 ft.-lb. per second. Ans.

EXAMPLE 2.—Water, under a head of 8 feet, flows from a circular standard orifice whose diameter is 6 inches. What is the energy of the jet if the coefficient of velocity is .98 ?

SOLUTION.—The diameter of the orifice is .5 foot, and from the table of Coefficients for Standard Vertical Orifices, the coefficient of discharge for a head of 8 feet is found to be .598 for an orifice whose diameter is .2 foot, and .596 when the diameter is .6 foot. The coefficient, therefore, decreases at a rate equal to $\frac{.598 - .596}{4} = .0005$ for each increase of .1 foot in the diameter of the orifice, and the coefficient for an orifice whose diameter is .5 foot is $.598 - (.0005 \times 3) = .5965$. The discharge is $Q = 6.299\, d^2 c \sqrt{h} = 6.299 \times .5^2 \times .5965 \times \sqrt{8} = 2.657$ cubic feet per second. The energy is

$$K = W\frac{v^2}{2 g} = 2.657 \times 62.5 \times .98^2 \times 8 = 1,275.9 \text{ ft.-lb. per second. Ans.}$$

EXAMPLES FOR PRACTICE.

1. A weir with end contractions is 5 feet long, and the measured head is .55 foot; if the water approaches the weir with a velocity of $1\frac{1}{4}$ feet per second, what is the discharge? Ans. 7.497 cu. ft. per sec.

2. A weir without end contractions is 6 feet long, and the head is .25 foot; what is the discharge, neglecting the velocity of approach? Ans. 2.541 cu. ft. per sec.

3. A jet flows from a standard circular orifice $1\frac{1}{4}$ inches in diameter under a constant head of 75 feet; assuming the coefficient of velocity to be .98, what is the energy of the jet? Ans. 2,275.43 ft.-lb. per sec.

4. What is the theoretical horsepower in the jet in the last example? Ans. 4.137 H. P.

NOZZLES.

1013. Nozzles are used when it is desired to deliver water with a high velocity for any purpose. Their most common application is in connection with hose for fire purposes, etc. By means of nozzles a very high coefficient of velocity is obtained, and the energy of the jet is therefore great. The theoretical height to which a stream from a nozzle can be thrown is equal to the head that would produce the velocity with which the jet flows from the nozzle if the water were to fall freely through a vacuum. If v is this velocity, the theoretical height to which the stream will go is

$$h = \frac{v^2}{2g}.$$

The resistance of the air always reduces this height.

EXAMPLE.—A stream issues from the nozzle of a fire hose under a pressure of 60 pounds per square inch. If the coefficient of velocity of the nozzle is .99, what is the theoretical height to which the stream will rise?

SOLUTION.—A pressure of 60 pounds per square inch corresponds to a head of $\frac{60}{.434} = 138.25$ feet. The velocity due to this head is $v = .99\sqrt{2g \times 138.25}$, and the height corresponding to the velocity v is

$$h' = \frac{v^2}{2g} = \frac{.99^2 \times 2g \times 138.25}{2g} = .99^2 \times 138.25 = 135.5 \text{ feet. Ans.}$$

THE MINER'S INCH.

1014. The **miner's inch** is an arbitrary unit for measuring water by its flow through an orifice. The orifice and the head vary so much in different localities that no definite value can be given for the unit. Its chief use is for measuring the water sold for irrigating and mining purposes.

The following list illustrates the general meaning of the term, and also shows the great variation in its value as used in different localities:

At Smartsville, California, the miner's inch is the amount of water discharged per square inch of area by an orifice four inches deep, having its lower edge level with the bottom of the box or sluice and its top edge seven inches below the surface of the water. The length of this orifice is made to correspond with the number of miner's inches it is to discharge. Under these conditions the Smartsville inch corresponds to a discharge of about 1.76 cubic feet of water per minute, or 2,534 cubic feet per 24 hours.

The Park Canal Mining Company furnishes water under such conditions that a miner's inch corresponds to a discharge of about 1.39 cubic feet per minute. The South Yuba Canal Company computes a miner's inch from the discharge through a two-inch aperture over a one and one-half inch plank with a head on the center of the aperture of six inches.

FLOW THROUGH PIPES.

PRESSURE HEAD AND VELOCITY HEAD.

1015. In Fig. 178 is shown a short pipe with a varying cross-section leading from a reservoir to a small nozzle. If the pipe is closed at the nozzle, so that no water can flow through it, the water will rise in each of the tubes *a*, *b*, and *c*, until it reaches the same level as the surface of the water in the reservoir. When there is a flow through the pipe, the water falls in the tubes and stands lowest in the tubes connected where the cross-section of the pipe is least.

This result is due to the difference in the velocities of the water in the different sections of the pipe. The same quantity flows through each section in a given time and, since the mean velocity is equal to the quantity flowing divided by the area of the section, i. e., $v = \frac{Q}{a}$, the velocity must increase as the section is reduced. It was shown in Art. **990** that a given velocity of flow v is equivalent to the velocity that would be produced if the water fell in a vacuum a distance equal to the head h which produces the given flow.

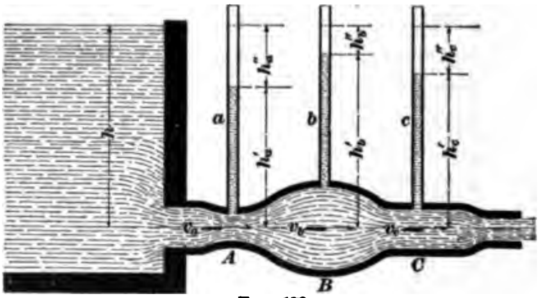


FIG. 178.

The water in flowing through the section A has a certain velocity v_a equivalent to the velocity that would be produced by a head we may call h_a'', so that $v_a^2 = 2gh_a''$. That is, part of the pressure due to the head h is exerted in producing the velocity whose equivalent head is $h_a'' = \frac{v_a^2}{2g}$, and the rest exists as pressure shown by the head h_a'. In the same way at the section B we have the **velocity head** $h_b'' = \frac{v_b^2}{2g}$ required to produce the velocity v_b and the **pressure head** h_b'; and at the section C the velocity head $h_c'' = \frac{v_c^2}{2g}$ and the pressure head h_c'.

1016. Neglecting the effects of friction, we have the following principle:

The sum of the velocity head plus the pressure head for any given section is equal to the total head h due to the water in the reservoir.

This principle can be proved as follows: The energy in the water as it leaves the reservoir is equal to Wh. Since there are to be no losses in energy from friction or otherwise, and the quantity of water passing is the same for each section, the energy must be the same at all sections. The total energy in the water at any section is the sum of the potential energy due to the pressure and the kinetic energy due to the velocity.

Hence,
$$Wh = W\left(h_a' + \frac{v_a^2}{2g}\right) = W(h_a' + h_a'') =$$
$$W\left(h_b' + \frac{v_b^2}{2g}\right) = W(h_b' + h_b''), \text{ etc.},$$

from which
$$h = h_a' + h_a'' = h_b' + h_b'' = h_c' + h_c''.$$

This principle is expressed by the theorem: *The pressure head plus the velocity head equals the hydrostatic head.*

EXAMPLE.—If, in Fig. 178, the hydrostatic head h is 20 feet and the pressure head at the section A is $h_a' = 12$ feet, what is the velocity v_a in the section A?

SOLUTION.—The velocity head $h_a'' = 20 - 12 = 8$ feet; hence, the velocity $v_a = \sqrt{2gh_a''} = \sqrt{2 \times 32.16 \times 8} = 22.684$ ft. per second. Ans.

LOSSES OF HEAD.

1017. In practice it is found that the pressure shown by a gauge on a pipe through which water flows and the

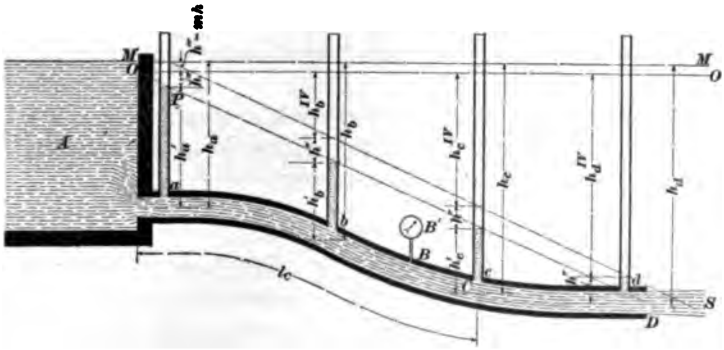


FIG. 179.

velocity of flow are always less than the theoretical pressure and velocity; i. e., the velocity head plus the pressure head is less than the hydrostatic head.

This difference is due to friction and other resistances that absorb part of the energy of the moving water. In Fig. 179, let *A* be a large reservoir from which a long pipe leads to some point below the surface of the water in *A*, and suppose that at various points in the pipe, as at *a*, *b*, *c*, and *d*, we insert tubes that extend above the water surface in *A*. Now, if the pipe is closed at *D*, the water will flow into it and rise in each of the branches until its level reaches the level of the water in the reservoir.

1018. The **hydrostatic head** on the center of any section of the pipe is the vertical distance in feet from the center of the pipe to the level of the water in the tube attached at that point, when there is no flow; or, in other words, the vertical distance from the center of the pipe to the level of the surface of the water in the reservoir. Since the pressure can be directly determined from the head, the level of the water in the tube is also a measure of the pressure of the water at the point in the pipe at which it is attached. Instead of a tube that rises to the height of the water in the reservoir, it is evident that we can use a pressure gauge, as at *B'*, which may be graduated to show either the pressure in pounds per square inch or the head in feet. If the gauge shows only the pressure in pounds per square inch, this pressure must always be reduced to its equivalent head in feet for use in problems and formulas pertaining to the flow of water.

The horizontal line *M M*, to which the water in the tubes *a*, *b*, *c*, *d* will rise when no water flows through the pipe, is called the **hydrostatic grade line.**

PIEZOMETERS.

1019. A gauge or tube inserted in a pipe to show the pressure of the water is called a **piezometer.** When a piezometer is to be placed on a pipe through which water is flowing, the tube should always be so inserted as to be at right angles to the current in the pipe, as shown at *a*,

Fig. 180. If the tube is so inclined that the current flows against the end as shown at *b*, the action of the current will force the water into the tube and cause it to rise higher than the head due to the pressure; and if inclined in the opposite direction, as at *c*, the action of the current will reduce the indicated pressure.

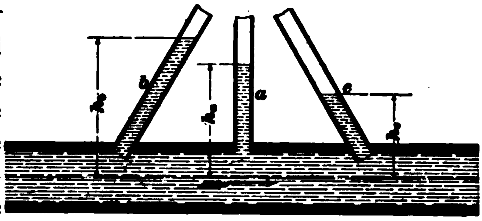


FIG. 180.

EXAMPLE.—A pressure gauge attached to a water main shows a pressure of 76 pounds per square inch. What is the equivalent head?

SOLUTION.—The head $h = \frac{76}{.434} = 175.1$ feet. Ans.

LOSS OF HEAD AT ENTRANCE.

1020. Where water flows from a reservoir into a pipe, a piezometer attached to the pipe near the reservoir shows a certain pressure whose head is h_a'. The velocity at which the water flows is equal to a head h'', and, if there were no losses, $h_a' + h''$ would equal the total hydrostatic head h_a. The water on entering the pipe meets with resistances, due to friction, contraction, etc., that absorb part of its energy, and this causes a loss of head similar to the loss when water flows through an orifice or a short tube.

This loss is proportional to the mean velocity with which the water flows through the pipe, and also depends on the form of the end of the pipe where it enters the reservoir; hence, the lost head can be expressed by the equation

$$h''' = m\,h'' = m\frac{v^2}{2g}, \qquad (39.)$$

where m is a factor that depends on the form of the end of the pipe.

Fig. 181 shows average values of the coefficient m as determined by experiment for different cases. In practice a water main generally projects into the reservoir and terminates in a bell-shaped end, as shown at (*d*), so that the coefficient m for long water mains may generally be taken as .5.

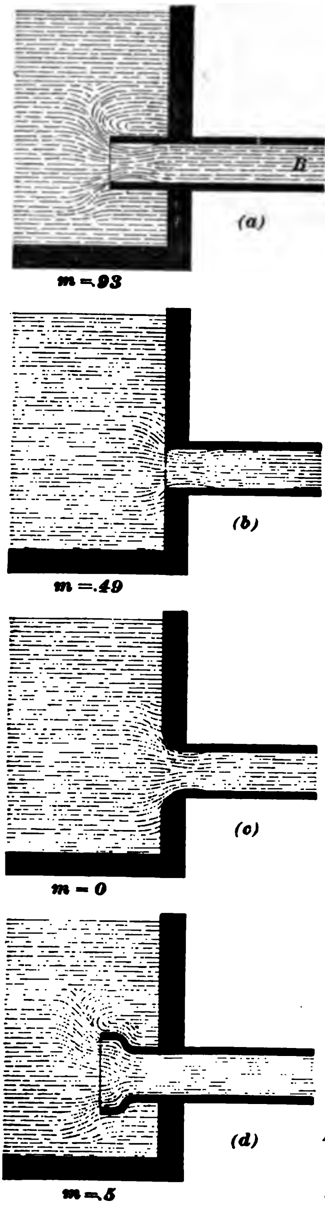


FIG. 181.

EXAMPLE.—A 12-inch pipe discharges 12 cubic feet of water per second. If the pipe enters the reservoir, as shown at (*d*), Fig. 181, what is the loss of head at entrance?

SOLUTION.—The mean velocity of flow is

$$v = \frac{12}{.7854} = 15.279 \text{ feet per second.}$$

The loss of head is

$$h''' = m\frac{v^2}{2g} = .5 \times \frac{15.279^2}{2 \times 32.16} =$$

1.815 feet. Ans.

LOSS OF HEAD FROM FRICTION IN THE PIPE.

1021. When water flows through a pipe, it meets with resistances, due to the friction of the particles on the sides of the pipe and on each other. These resistances absorb energy and cause a further loss in head.

Experiments have shown that the friction of water flowing through a pipe depends, approximately, on the following laws:

1. *The loss in friction is proportional to the length of the pipe.*

2. *It varies nearly as the square of the velocity.*

3. *It varies inversely as the diameter of the pipe.*
4. *It increases with the roughness of the pipe.*
5. *It is independent of the pressure in the pipe.*

In accordance with these laws, the friction head h^{IV} is expressed by the equation

$$h^{IV} = f \frac{l}{d} \frac{v^2}{2g}, \qquad (40a.)$$

in which l is the length of the pipe, d its diameter, and f a coefficient that depends on the roughness of the pipe.

It has been found that f varies with the diameter of the pipe and the velocity of flow. The table of Coefficients for Pipes gives values of f for clean cast-iron pipes well laid.

EXAMPLE.—What is the loss of head due to friction in a 10-inch pipe 1,000 feet long, if the mean velocity of flow is 8 feet per second?

SOLUTION.—From the table of Coefficients for Pipes, the coefficient f for a 10-inch pipe is found to be .0197, when the velocity of flow is 8 feet per second; therefore,

$$h^{IV} = f \frac{l}{d} \frac{v^2}{2g} = .0197 \times \frac{1,000}{.83\frac{1}{3}} \times \frac{8^2}{2 \times 32.16} = 23.522 \text{ feet. Ans.}$$

LOSSES OF HEAD DUE TO CHANGE OF SECTION AND BENDS.

1022. When water flows from a small section to a larger one, see Fig. 182 (*a*), energy is absorbed in producing eddies among the water particles just at the enlargement. The change from a large to a smaller section, as at (*b*), causes a contraction in the mouth of the smaller section. The result in both cases is a loss of head. If the change in section in the pipe is made gradually, as at Fig. 182 (*c*) and (*a*), the loss is made small and may be neglected when computing the flow. In practice a change in section is usually made by means of a **reducer,** see Fig. 182 (*c*).

1023. When there are sudden bends in the pipe there will be a loss, due partly to shock and eddies, and partly to the contraction in the flow, as shown in Fig. 182 (*f*) and (*g*).

Experiments made with bends like Fig. 182 (*f*) show that

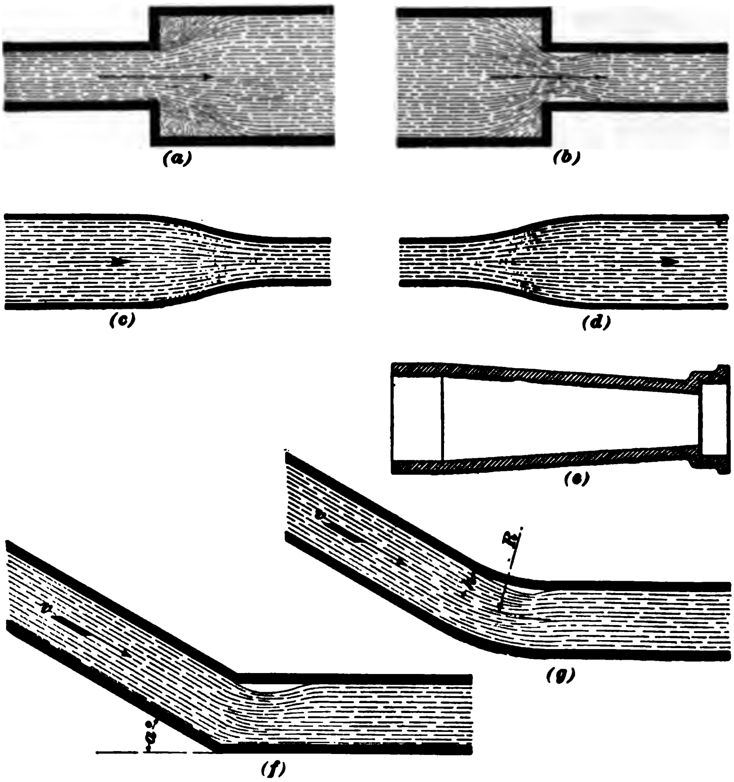


FIG. 182.

the loss of head may be expressed in terms of the mean velocity by the formula

$$h^v = c\frac{v^2}{2g}. \qquad (40b.)$$

Table 18*a* gives values of *c* for different values of the angle a°.

TABLE 18*a*.

$a^\circ =$	10°	20°	40°	60°	80°	90°	100°	110°	120°	130°	140°	150°
$c =$	.017	.046	.139	.364	.74	.984	1.26	1.56	1.86	2.16	2.43	2.81

For a bend like Fig. 182 (g), the loss of head is expressed by the formula

$$h^{VI} = c' \frac{v^2}{2g}, \qquad (40c.)$$

in which c' depends on the ratio between the radius r of the pipe and the radius R of the bend.

The following table gives values of c' corresponding to various values of the ratio $\frac{r}{R}$:

TABLE 18b.

$\frac{r}{R} =$	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
$c' =$	.131	.138	.158	.206	.294	.440	.661	.977	1.408	1.978

From the table it is seen that when R is made large in comparison to r, the value of c', and hence the loss in head, is small.

There may be other resistances, such as valves, which change the direction of flow of the water, or suddenly change the area through which the water flows. If the pipe is carefully designed and laid, however, these losses may be made so small in comparison with the other losses named above as to be neglected in the formulas for head and velocity.

TOTAL HEAD AND VELOCITY OF FLOW.

1024. The **total head** at any section of the pipe is equal to the sum of the pressure head, the velocity head, and the losses in head due to the energy absorbed in overcoming the resistances in the pipe between the reservoir and the given section.

Thus, at the point c in Fig. 179, the total head is $h_c = h_c' + h'' + h''' + h_c^{IV}$.

At the end of the pipe the pressure head becomes 0, and the total head h is given by the formula

$$h = h'' + h''' + h^{IV}. \qquad (41.)$$

If we suppose that there are n bends similar to Fig. 182 (g), there will be a further loss of head equal to $n\,h^{\text{vi}} = n\,c'h'' = n\,c'\frac{v^2}{2g}$, and we have the total head

$$h = h'' + h''' + h^{\text{iv}} + h^{\text{vi}} =$$

$$\frac{v^2}{2g} + f\frac{l}{d}\,\frac{v^2}{2g} + m\frac{v^2}{2g} + n\,c'\frac{v^2}{2g}. \qquad \textbf{(41}a.\textbf{)}$$

By solving formula **41**a for v, we get the following formula for the velocity of flow:

$$v = \sqrt{\frac{2gh}{1 + f\frac{l}{d} + m + n\,c'}} = 8.02\sqrt{\frac{h}{1 + f\frac{l}{d} + m + n\,c'}}. \qquad \textbf{(42.)}$$

Formula **42** will give the mean velocity of flow through a pipe in which the total head is h, when the length l, and the diameter d, and the values of the coefficients f, m, and c' are known.

The table of Coefficients for Pipes gives mean values of f that may be used for clean iron pipes, either smooth or coated with coal tar. Since f depends on v, which is unknown, it is first necessary to take from the table a mean value of f depending on the diameter of the pipe, and then solve for v. This gives an approximate value for v from which to find a new value of f and solve again for v. If the last value of f is nearly the same as would be given in the table for the value of v last found, the result is satisfactory. If not, the last value of v must be taken as an approximation from which a new value of f is to be found, and the process repeated.

If m has the value .5 given in Fig. 181 (d), Art. **1020**, for the common case of a pipe with a bell end, and there are no sharp bends or similar resistances, the formula for v becomes

$$v = \sqrt{\frac{2gh}{1.5 + f\frac{l}{d}}} = 8.02\sqrt{\frac{h}{1.5 + f\frac{l}{d}}}. \qquad \textbf{(43.)}$$

LONG PIPES.

1025. For pipes in which the length l is greater than 4,000 d, the velocity head and loss of head at entrance become so small in comparison to the loss due to friction that they may be neglected, and the formula for velocity becomes

$$v = \sqrt{\frac{2gdh}{fl}} = 8.02\sqrt{\frac{dh}{fl}}. \qquad \textbf{(44.)}$$

For d in inches formula **43** becomes

$$v = 2.315\sqrt{\frac{hd}{fl + .125\,d}}, \qquad \textbf{(43}a.\textbf{)}$$

and instead of formula **44** we have

$$v = 2.315\sqrt{\frac{hd}{fl}}. \qquad \textbf{(44}a.\textbf{)}$$

EXAMPLE 1.—A 12-inch pipe, 3,000 feet long, with a bell end, enters the reservoir in such a way that the coefficient m may be taken as .5. The pipe has two 45° bends, each with a radius of 2 feet. If the head on the discharge end of the pipe is 35 feet, what will be the velocity of flow?

SOLUTION.—The loss of head from the bends depends on the ratio between the radius of the pipe and the radius of the bend. This ratio is $\frac{6}{24} = .25$. From the table of Coefficients for Pipes, the coefficient c' for a radius of .2 is .138, and for the ratio .3 $c' = .158$; therefore, for .25 the coefficient is $.138 + (.158 - .138) \times .5 = .148$. Assuming .022 as an approximate value for f for use in this case, and substituting the values of the coefficients in formula **42**, there results

$$v = 8.02\sqrt{\frac{35}{1 + .022 \times \frac{3000}{1} + .5 + 2 \times .148}} = 5.762 \text{ feet per second.}$$

From the table the value of f for a velocity of 5.762 feet per second is

$$.0207 - (.0007 \times .762) = .0202.$$

Using this value of f, the velocity becomes

$$v = 8.02\sqrt{\frac{35}{1 + .0202 \times \frac{3000}{1} + .5 + 2 \times .148}} = 6.007 \text{ feet per second.}$$

Since the coefficient f for a velocity of 6 feet per second is .020, this value of v may be assumed as practically correct. Ans.

EXAMPLE 2.—A 10-inch pipe, 8,000 feet long, is so laid that there is practically no loss of head from bends or valves. If the head is 150 feet, what is the mean velocity of flow?

SOLUTION.—Since the length is more than 4,000 times the diameter, formula **44** may be used. Taking .02 as a mean value of f, the approximate velocity of flow is

$$v = 8.02\sqrt{\frac{\frac{5}{6} \times 150}{.02 \times 8,000}} = 7.089 \text{ feet per second.}$$

By comparison with the table it is seen that the assumed value of f corresponds very nearly with the value for a velocity of 7 feet per second; therefore, the result found is nearly correct. Ans.

EXAMPLE 3.—What would have been the value of v in the above example if formula **43** had been used?

SOLUTION.—

$$v = 8.02\sqrt{\frac{150}{1.5 + .02 \times \frac{8,000}{.8\frac{1}{3}}}} = 7.061 \text{ ft. per second. Ans.}$$

From the last two examples it is seen that for long pipes the effect of resistances at entrance may be neglected without affecting the practical accuracy of the result.

HEAD REQUIRED TO PRODUCE A GIVEN VELOCITY.

1026. A formula for the head required to produce a given velocity of flow v can be found from the formulas given above by solving for h. Thus, from formula **42,** the value of the head is

$$h = \frac{v^2\left(1 + f\frac{l}{d} + m + n\,c'\right)}{64.32}. \qquad \textbf{(45.)}$$

For a straight cylindrical pipe in which the influence of bends disappears and m is taken as .5, the formula becomes

$$h = \frac{f\,l\,v^2}{64.32\,d} + .0233\,v^2. \qquad \textbf{(45}a\textbf{.)}$$

Formulas **45** and **45***a* apply when d is in feet; for d in inches, formula **45***a* becomes

$$h = \frac{f\,l\,v^2}{5.36\,d} + .0233\,v^2. \qquad \textbf{(45}b\textbf{.)}$$

EXAMPLE.—An 8-inch pipe, 2,500 feet long, has three bends whose radius equals the diameter of the pipe. If the coefficient for loss at entrance is $m = .5$ and $f = .02$, what must be the head to produce a velocity of flow of 7 feet per second?

SOLUTION.—The ratio between the radius of a bend and the radius of a pipe is $\frac{4}{8} = .5$; therefore, from the table of Coefficients for Pipes, the coefficient c' is .29.

Substituting in formula **45**, we have

$$h = \frac{7^2\left(1 + .02\frac{2{,}500}{\frac{8}{12}} + .5 + 3 \times .29\right)}{64.32} = 58.94 \text{ feet. Ans.}$$

THE QUANTITY DISCHARGED FROM PIPES.

1027. The formulas just given are made use of in ascertaining the quantity of water that will be discharged from a pipe in a given time, with a given head. This is readily found from the formula $Q = A\,v$, where A equals the area of the cross-section of the pipe and v the mean velocity as determined by the above formulas.

Where the diameter is given in feet, the discharge in cubic feet per second is

$$Q = .7854\,d^2\,v. \qquad \textbf{(46.)}$$

Since one cubic foot contains 7.48 gallons, when d is in feet the discharge in gallons per second is

$$Q = .7854\,d^2\,v \times 7.48. \qquad \textbf{(46}a.\textbf{)}$$

And when d is in inches,

$$Q = .0408\,d^2\,v. \qquad \textbf{(46}b.\textbf{)}$$

EXAMPLE.—What is the discharge in gallons per minute from a 6-inch pipe, if the mean velocity of efflux is 5.6 feet per second?

SOLUTION.— $Q = .0408\,d^2\,v = .0408 \times 36 \times 5.6 = 8.225$ gallons per second. $8.225 \times 60 = 493.5$ gallons per minute. Ans.

EXAMPLE.—The length of a pipe is 6,270 feet, its diameter is 8 inches, and the total head at the point of discharge is 215 feet. How many gallons are discharged per minute?

SOLUTION.—First find the approximate value of v from formula **44**a, taking the value of $f = .025$.

$$v = 2.315\sqrt{\frac{hd}{fl}} = 2.315\sqrt{\frac{215 \times 8}{.025 \times 6{,}270}} = 7.67 \text{ feet per second, nearly.}$$

From the table of Coefficients for Pipes, the value of f for an 8-inch pipe and a velocity of 7.67 feet per second is

$$.0213 - \frac{(.001 \times 1.67)}{2} = .0205.$$

With the value of f used in formula **43a**, the velocity is

$$v = 2.315\sqrt{\frac{215 \times 8}{.0205 \times 6,270}} = 8.47 \text{ feet per second.}$$

The value of f for $v = 8.47$ feet per second is $.0203 - \left(\frac{.001 \times .47}{2}\right) =$ .0201, and this value gives

$$v = 2.315\sqrt{\frac{215 \times 8}{.0201 \times 6,270}} = 8.552 \text{ feet per second.}$$

The discharge is

$$Q = .0408 \times 8^2 \times 8.552 = 22.33 \text{ gallons per second.}$$
$$22.33 \times 60 = 1,340 \text{ gallons per minute. Ans.}$$

TO COMPUTE THE DIAMETERS OF PIPES.

1028. The diameter of a pipe that will furnish a certain quantity of water with a given head and length can be found as follows:

With h, l, and d in feet and the quantity Q in cubic feet per second, the formula for the diameter of a pipe without sharp bends is

$$d = 0.479\left[(1.5\,d + f\,l)\frac{Q^2}{h}\right]^{\frac{1}{5}}. \qquad \textbf{(47.)}$$

In using this formula take the approximate value of f as .02, and compute an approximate value for d, neglecting the term 1.5 d in the second member of the formula. With this value of d find the value of v from the formula $v = \frac{Q}{.7854\,d^2}$, and find the corresponding value of f from the table of Coefficients for Pipes.

Repeat the computation for d by placing the approximate values of d and f just found in the second member of the formula. One or two repetitions of this process will give a near approximation of d from which to select the pipe from the standard market sizes.

For pipes whose length is more than 4,000 times their diameter the following formula may be used;

$$d = 0.479\left(\frac{f\,l\,Q^2}{h}\right)^{\frac{1}{5}}. \qquad (47a.)$$

EXAMPLE 1.—What must be the diameter of a pipe to discharge 1,000,000 gallons of water per 24 hours, if the length is 1,250 feet, and the head 75 feet?

SOLUTION.—The discharge in cubic feet per second is $\frac{1,000,000}{86,400 \times 7.48} =$ 1.5473. The approximate diameter of the pipe is

$$d = 0.479\left(\frac{.02 \times 1,250 \times 1.5473^2}{75}\right)^{\frac{1}{5}} = .4579 \text{ foot.}$$

The velocity corresponding to this value of d is

$$v = \frac{1.5473}{.7854 \times .4579^2} = 9.395 \text{ feet per second.}$$

From the table of Coefficients for Pipes the coefficient f for a pipe 6 inches (= .5 foot) in diameter, and a velocity of flow of 10 feet per second, is .02.

Since these values of d and v correspond closely with the approximate values just found, the value $f = .02$ may be used again. Using the approximate value of d in formula **47**, there results

$$d = 0.479\left[(1.5 \times .4579 + .02 \times 1,250)\frac{1.5473^2}{75}\right]^{\frac{1}{5}} = .4603 \text{ foot.}$$

The nearest market size to this diameter is a 6-inch pipe; hence, that may be taken. Ans.

EXAMPLE 2.—A water main 17,320 feet long must supply a city with 10,000,000 gallons of water per 24 hours under a steady flow. If the head is 120 feet, what must be the diameter of the pipe?

SOLUTION.—The flow in cubic feet per second is

$$Q = \frac{10,000,000}{86,400 \times 7.48} = 15.473$$

The approximate value of d is, therefore,

$$d = 0.479\left(\frac{.02 \times 17.320 \times 15.473^2}{120}\right)^{\frac{1}{5}} = 1.771 \text{ feet.}$$

The velocity of flow corresponding to this diameter is

$$v = \frac{15.473}{.7854 \times 1.771^2} = 6.28 \text{ feet per second.}$$

From the table of Coefficients for Pipes, the coefficient f for a pipe 20 inches in diameter, and a velocity of flow of 6 feet per second, is .0173.

Since the length of the pipe is more than 4,000 times the approximate diameter, formula **47*a*** may be used; hence,

$$d = 0.479\left(\frac{.0173 \times 14{,}320 \times 15.478^2}{120}\right)^{\frac{1}{5}} = 1.721 \text{ feet} = 21 \text{ inches, nearly.}$$

Therefore, the nearest available market size above this may be used. Ans.

THE EFFECTIVE HEAD.

1029. The head that produces the flow in a pipe is always the difference between the total head and the pressure head on the end where discharge takes place. If the pipe discharges directly into the atmosphere, as in Fig. 179, the pressure head becomes zero, and flow is produced by the total head h_a.

In most cases, however, the discharge is into a reservoir,

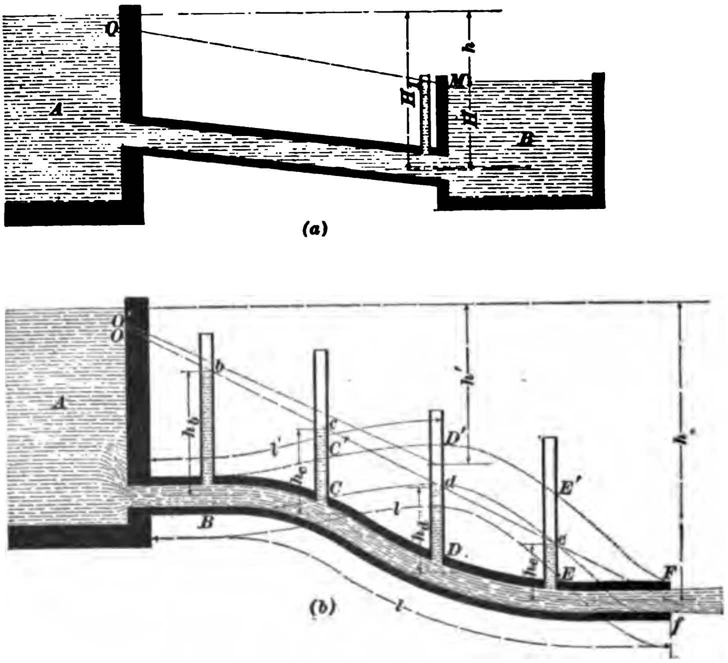


FIG. 183.

as in Fig. 183 (*a*), or the pipe is required to furnish water to a water-wheel or to a system of distributing pipes at a certain pressure per square inch.

In Fig. 183 (*a*) the effective head h is the difference between the total head H_1 and the head H of the water in the reservoir B.

EXAMPLE.—A pipe must supply water at a pressure of 25 pounds per square inch, under a total head of 75 feet. What is the effective head?

SOLUTION.—The head required to produce a pressure of 25 pounds per square inch is $25 \div .434 = 57.6$ feet; then, the effective head is

$$h = 75 - 57.6 = 17.4 \text{ feet. Ans.}$$

THE HYDRAULIC GRADIENT.

1030. If, in Fig. 183 (*b*), we draw a line through the points b, c, d, etc., at which the water would stand in piezometer tubes attached to a pipe through which water flows, this line is called the **hydraulic grade line,** also the **hydraulic gradient.** If the section of the pipe is uniform and without sharp bends or other obstructions, the line so drawn will be a straight line extending from the reservoir to the center of the end of the pipe where the water discharges into the atmosphere; and the above formulas for velocity, quantity of flow, etc., apply so long as no part of the pipe rises above this line. If, however, the pipe is so laid that it rises above the hydraulic gradient at any part, as shown by the dotted line, there is an important change in the result. If the pipe is flowing full, a piezometer attached at any point which rises above the hydraulic gradient will show a pressure less than that of the atmosphere. This causes air to collect at the point D' that rises highest above the hydraulic gradient, and the flow becomes broken, until finally the pipe will be filled only to D'. Flow now takes place in the same manner that it would if the pipe were opened to the air at D', and from that point to F the pipe is only partly filled and acts as a channel to carry the water discharged at D'.

This results in a change in the hydraulic gradient to $O' D'$, and the flow is to be computed for a pipe with the head h' and length l', instead of the head h and length l. If, however, means are provided for preventing the collection of air at D' and the pipe is kept filled from D' to F, the velocity and discharge will be the same as for a pipe all of which is laid below the hydraulic gradient.

When a pipe discharges under pressure, as shown in Fig. 183 (*a*), the hydraulic gradient is the line $O\,M$ from the reservoir A to the surface of the water in the reservoir B, or to a point that represents the head due to the pressure where discharge takes place.

1031. When part of a pipe rises above the hydraulic gradient it is called a **siphon.**

The principles on which the action of a siphon depends are explained in the section on Pneumatics; see Art. **1074.** If the siphon is kept filled, the flow through it will take place in accordance with the laws given in Arts. **1015** to **1029** for pipes laid below the hydraulic gradient, and the same formulas apply.

The total head producing the flow in a siphon is the vertical distance h from the discharge end F of the pipe to the level of the water in the reservoir, see Fig. 183 (*b*). If the siphon is of uniform section, without sharp bends or obstructions, the hydraulic gradient will be a straight line $O\,F$ from the reservoir to the discharge end F, and the pressure in all parts of the pipe that rise above the line will be less than the atmospheric pressure. Air always tends to collect in the highest point of a siphon, and means must be provided for its removal, in order to keep up the flow.

EXAMPLES FOR PRACTICE.

1. What will be the velocity of discharge from a 12-inch pipe, 800 feet long, under a head of 28 feet, if the pipe has a bend whose radius is 2 feet? Ans. 10.57 ft. per sec.

2. How much would the velocity of discharge be in example 1 if the pipe had no sharp bend? Ans. 10.69 ft. per sec.

3. What is the increase in the discharge, in gallons per minute, without the bend, in example 1? Ans. 42.29 gal.

NOTE.—From the preceding examples it is seen that the influence of easy bends on the discharge is slight, even in comparatively short pipes. For pipes of considerable length, the influence of a few such bends is hardly perceptible.

4. What will be the velocity of discharge from a 16-inch pipe, 10,640 feet long, with a head of 175 feet? Ans. 9.06 ft. per sec.

5. What will be the discharge in gallons per minute in example 4? Ans. 5,677.4 gal.

6. What head will be required to give a velocity of flow of 8 feet per second in an 8-inch pipe 1,350 feet long? Ans. 42.37 ft.

7. What must be the diameter of a pipe that must furnish a maximum discharge of 1,400 gallons per minute, if the length of the pipe is 6,240 feet and the head 125 feet?

Ans. { The diameter must be about .75 foot.
The standard market size next greater is 10 inches.

FLOW OF WATER IN CONDUITS AND CHANNELS.

1032. Water is often conveyed from the source of supply to the point where it is to be used in **conduits** formed of masonry or timber work, or in open channels, called **ditches** or **canals.**

The flow in these cases is caused entirely by the **slope,** or fall, of the conduit or channel, which must be so graded that the fall will be continuous and nearly uniform.

The **slope** is the ratio of the fall to the length in which it occurs. Thus if S is the slope, h the fall, and l the length in which the fall occurs, the slope is given by the formula

$$S = \frac{h}{l}. \qquad (48.)$$

EXAMPLE.—If a canal has a fall of $2\frac{1}{8}$ inches in 500 feet, what is the slope?

SOLUTION.—The head is $2\frac{1}{8}$ inches = .17708 foot; therefore, the slope is

$$S = \frac{.17708}{500} = .00035416.$$

The **wetted perimeter** of the cross-section of any channel or conduit is the part of its boundary in contact with the water. Thus, if a circular conduit whose diameter is 4 feet is half filled with water, its wetted perimeter is equal to one-half its circumference, or $\frac{1}{2} \times 3.1416 \times 4 = 6.2832$ feet.

The **hydraulic radius** is the ratio of the area of the cross-section of the water in a channel to the wetted perimeter. If the wetted perimeter is denoted by p, the area of the water cross-section by a, and the hydraulic radius by r, then

$$r = \frac{a}{p}. \qquad (49.)$$

r is sometimes called the **hydraulic mean depth.**

EXAMPLE.—What is the hydraulic radius of the circular conduit in the last example?

SOLUTION.—The area $= \frac{1}{2} \times .7854 \times 4^2 = 6.2832$, and the wetted perimeter is $p = \frac{1}{2} \times 3.1416 \times 4 = 6.2832$; therefore, $r = \frac{a}{p} = \frac{6.2832}{6.2832} = 1$.

The **mean velocity** is the average velocity of flow for the whole cross-section of the water in a channel. Owing to the friction along the sides, the water filaments nearest the walls of a channel move slowest; hence, the velocity is different in different parts of the cross-section.

The **discharge** is the product of the area of the water cross-section of any part of a channel multiplied by the mean velocity of the same cross-section. If Q is the discharge and v the mean velocity, we have, as in the case of orifices and pipes,

$$Q = a v.$$

FORMULA FOR MEAN VELOCITY.

1033. The mean velocity of flow in a conduit or channel of any kind is given by the formula

$$v = c\sqrt{r s}. \qquad (50.)$$

Here c is a coefficient that depends on the value of r and s and on the degree of roughness of the surface of the channel.

Kutter's formula is a formula that gives the value of c when the nature of the interior of the conduit is known. The most simple form of Kutter's formula is the following:

$$c = \frac{23 + \frac{1}{n} + \frac{.00155}{s}}{.5521 + \left(23 + \frac{.00155}{s}\right)\frac{n}{\sqrt{r}}}. \qquad (51.)$$

In the above formula n is a coefficient, called the **coeffi-**

cient of roughness. Its value depends on the condition of the interior surface of the channel, and great care and judgment are required in selecting the value to be used when applying the formula.

The following table gives the values that may be used under the conditions most often met with in practice :

VALUES OF THE COEFFICIENT OF ROUGHNESS.

Character of Channel.	Value of *n*.
For clean, well-planed timber	.009
For clean, smooth, glazed iron and stoneware pipes	.010
For masonry smoothly plastered with cement, and for very clean, smooth, cast-iron pipe	.011
For unplaned timber, ordinary cast-iron pipe, and selected pipe sewers, well laid and thoroughly flushed	.012
For rough iron pipes and ordinary sewer pipes laid under the usual conditions	.013
For dressed masonry and well-laid brickwork	.015
For good rubble masonry and ordinary rough or fouled brickwork	.017
For coarse rubble masonry and firm, compact gravel	.020
For well-made earth canals in good alinement	.0225
For rivers and canals in moderately good order and perfectly free from stones and weeds	.025
For rivers and canals in rather bad condition and somewhat obstructed by stones and weeds	.030
For rivers and canals in bad condition, overgrown with vegetation and strewn with stones and other detritus, according to condition	.035 to .050

EXAMPLE 1.—What is the value of c for a rough plank sluice 24 inches wide, when the depth of water in the sluice is 15 inches, and the fall 3 inches in 100 feet ?

SOLUTION.—The slope is $s = \frac{.25}{100} = .0025$; the wetted perimeter $p = 2 + (2 \times 1.25) = 4.5$ feet, and the area of the water cross-section $a = 2 \times 1.25 = 2.5$ square feet. The hydraulic radius is, therefore, $r = \frac{2.5}{4.5} = .5556$. From the table, the value of n for unplaned timber is found to be .012; therefore,

$$c = \frac{23 + \frac{1}{.012} + \frac{.00155}{.0025}}{.5521 + \left(23 + \frac{.00155}{.0025}\right) \times \frac{.012}{\sqrt{.5556}}} = 114.7. \quad \text{Ans.}$$

EXAMPLE 2.—Compute (*a*) the velocity of flow, and (*b*) the quantity of discharge for the sluice of example 1.

SOLUTION.—(*a*) The velocity is

$$v = c\sqrt{r s} = 114.7\sqrt{.5556 \times .0025} = 4.275 \text{ ft. per second.} \quad \text{Ans.}$$

(*b*) The discharge is

$$Q = a v = 2.5 \times 4.275 = 10.69 \text{ cu. ft. per second.} \quad \text{Ans.}$$

FLOW IN BROOKS AND RIVERS.

1034. Weirs and orifices furnish the best and most accurate means of measuring the discharge of pipes, conduits, brooks, rivers, or channels of any kind whose volume is too great to be measured in a tank. When these methods are too expensive or the discharge too great, as in the case of large rivers, Kutter's formula may be used as described in the last article. For brooks and rivers, however, the channel is seldom regular enough to make an accurate determination of the values of n, r, and s possible; hence, the results obtained by this method are very uncertain.

The best method of finding the discharge of streams where weirs can not be used is to make a careful measurement of the cross-section and then find the mean velocity at that cross-section by one of the methods described below.

Experiments have shown that if a cross-section of a stream, as shown at (*a*), Fig. 184, is taken, the maximum velocity is at some distance below the surface where the stream is deepest. The velocity of any film surrounding the point of maximum velocity is less the greater its distance from that point, and the least velocity is in the film bordering on the bed of the stream.

At (*b*) is shown a plan of the stream, with a curve $a\ b\ c$ showing the relative velocities at different points on the surface; and at (*c*) a longitudinal section in the deepest part, with a curve $d\ e\ f$ showing the relative velocities from the surface to the bottom.

In order to measure the average velocity, select a part of the stream where the flow is regular and not too rapid, and where the channel is as smooth and regular as possible, for [illegible] feet or more. Divide the cross-section into a convenient number of parts, as shown in Fig. 184 (*d*), and find the average depth of each of these parts by sounding.

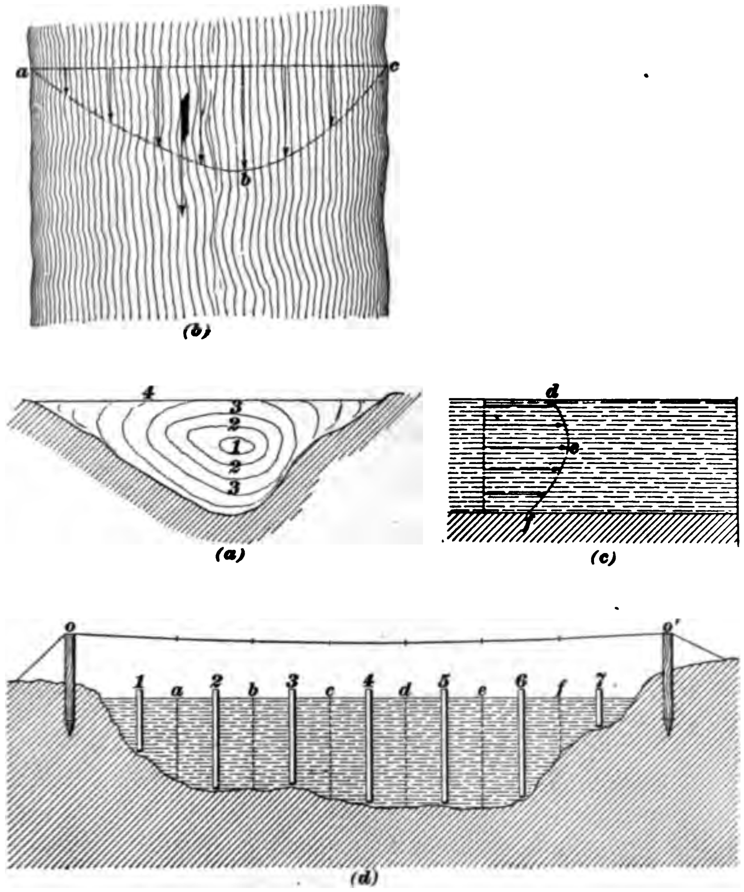


FIG. 184.

If possible, it is well to mark the cross-section by a wire *o o'* stretched across the stream, with bits of cloth or string tied to it to mark the points of division.

The area of each of the parts of the cross-section is its breadth multiplied by its average depth, and the quantity of water passing a division is the product of its area and its mean velocity of flow.

THE CURRENT-METER.

1035. The best method of measuring the velocity is by means of a **current-meter,** an instrument provided with vanes like a windmill that turn when the instrument is held in a current of water. Fig. 185 shows the original current-meter of this class, known as **Woltmann's tachometer.**

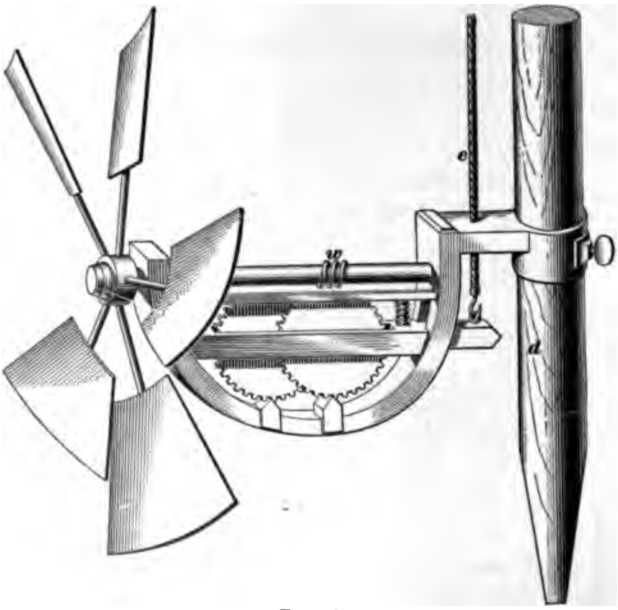


FIG. 185.

The instrument is fastened to a rod *d*, by which it can be held in the stream where the velocity of the current is to be measured. The vanes are attached to a spindle which is provided with a worm *w*. A frame carries a train of wheels that can be made to gear with the worm by pulling the cord *e*.

To use the instrument, it must be held steadily in the water with the wheel facing the current. When it is in the proper position, throw the recording device into gear by pulling the cord *e*, and hold it there for a certain time, then release it, taking great care to note the exact time during which the cord is held. The instrument can then be taken out of the water and a record made of the time and the number of revolutions shown by the recording device.

Fig. 186 shows one of the latest forms of current-meter, in which the number of revolutions is shown by an electric

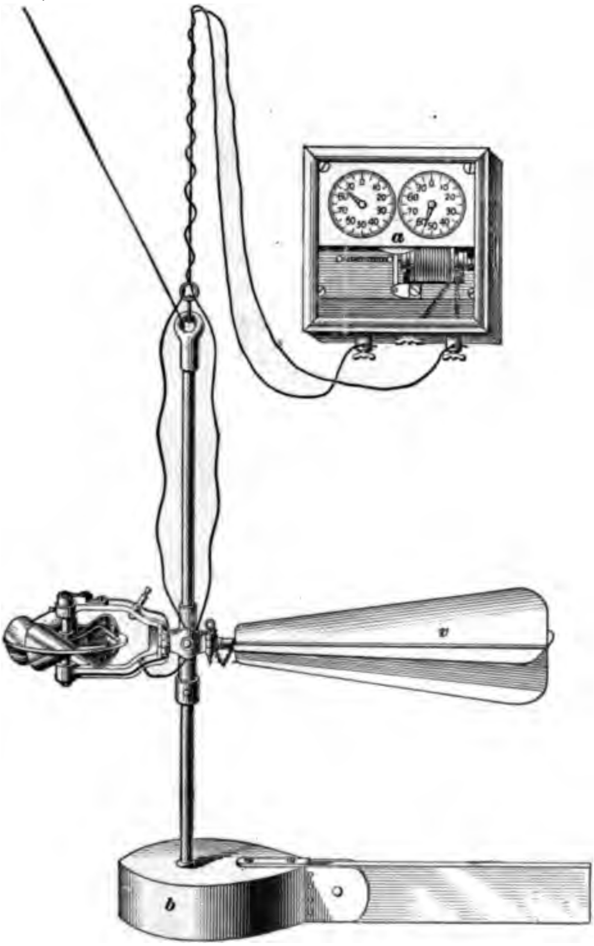


FIG. 186.

register *a*, which is connected to the instrument by an insulated wire. The register may be placed on the shore or in a boat. A vane *v* holds the wheel to the current, and for deep, swift currents, a heavy weight *b* is used to sink and hold the meter in place. For shallow streams the instrument is best used on a rod similar to that shown for Woltman's tachometer.

RATING THE CURRENT-METER.

1036. Before using a current-meter, it must be **rated** by passing it through still water at different known velocities and noting the number of revolutions for each velocity.

USE OF THE CURRENT-METER.

1037. To find the mean velocity for a given division of any cross-section of a stream (for example, the division marked 2 in (*d*), Fig. 184), hold the meter for a given length of time at different successive depths in the middle of the division and note the velocity for each position; then take the average of these velocities as the mean velocity of the given section. Thus, suppose the mean depth of division 2 in Fig. 184 (*d*) is 8 feet, and the meter shows a velocity of 1.8 feet per second at the bottom, 2.1 feet per second at 2 feet, 2.5 feet per second at 4 feet, and 2.3 feet per second at 6 feet from the bottom, and 2.2 feet per second just below the surface; then the average of the five readings is 2.18 feet per second, which may be taken as the mean velocity for this division. If this division is 6.25 feet wide, the quantity of water flowing through it is $8 \times 6.25 \times 2.18 =$ 109 cubic feet per second. The quantity flowing in each division can be found in the same way and the sum of the values found for all the divisions will be the total discharge of the stream.

A good method for finding the mean velocity for any division is to start with the meter at the bottom of the stream in the middle of the division and move it slowly and uniformly to the surface, noting the time and number of revolutions in the period during which the meter is being raised.

MEASURING THE VELOCITY BY MEANS OF FLOATS.

1038. If a current-meter can not be had, the velocity of a stream may be measured by means of **floats.** For this purpose the best is the **rod float**, which consists of a rod of wood or a closed tin tube, with one end so weighted that it will float in the water in a nearly vertical position, and with the lower end as near the bottom as possible, without touching at any point. See 1, 2, 3, etc., Fig. 184 (*d*). It is best to use a number of tin tubes about two inches in diameter and of such lengths that each will float in the division whose velocity is to be measured with only enough above the surface to be plainly seen. Fill the lower ends of the tubes with sand or shot until they float at the required depth. Mark two stations on the stream at least 100 feet apart in such a manner that the exact moment at which a tube passes each station can be noted. A good way is to mark the stations by a wire, as shown in Fig. 184 (*d*). Another and more accurate method is to have a transit at each station and note the time when each tube passes the cross-hair. Good results may be obtained by range stakes placed on the opposite banks.

Start the tubes far enough up the stream from the first station, so that they will have the velocity of the water when they pass that station, and carefully note the time it takes each tube to pass over the distance between the two stations. The distance between the stations in feet, divided by the time in seconds, gives the velocity in feet per second for the division in which the tube floated. From this velocity and the area of the division, the quantity of flow can be computed in the same manner as described for the current-meter.

Surface floats are sometimes used for obtaining the velocity of flow. Find the average velocity in feet per second with which the float passes between two stations, in the same way as has been described for rod floats. Then, if v' is the observed velocity for any division of the stream, the mean velocity of that division may be taken as

$$v = .9\,v'. \qquad \textbf{(52.)}$$

Example.—If two stations on a stream are 200 feet apart, and a surface float in a given section passes over the distance between the two stations in 7 minutes and 25 seconds, what is the mean velocity of flow in that section?

Solution.—The velocity of the float is

$$v' = \frac{200}{445} = .449 \text{ foot per second;}$$

therefore, according to the formula, the mean velocity is

$$v = .9\,v' = .9 \times .449 = .404 \text{ ft. per second.} \quad \text{Ans.}$$

For a rough approximation a single surface float passing along the axis of the stream may be used. For this case the mean velocity of the whole stream may be taken as .8 of the velocity of the float. Surface floats should be of such a form as to rise but little above the surface of the water, so as to be but little affected by currents of air, and they should be used only when there is a calm, since the wind has a great influence on the surface velocity of a stream.

Submerged Floats.—A float that gives better results than the surface float is shown in Fig. 187. A body heavy enough to sink, and at the same time present as large a surface to the water in proportion to its weight as possible, is attached to a float on the surface by a fine cord or wire, whose length is just sufficient to allow the submerged body to float half way between the surface and the bottom. The surface float *B* should be made to offer as little resistance as possible to its passage through the water, the object being to get the velocity at mid-depth, as nearly as possible. In the figure the submerged float *A* is made of two strips of tin or sheet metal fastened together in the form of a cross, as shown in plan at *C*. A small weight *D* is attached to the bottom, as shown, to assist in keeping the float in a vertical position.

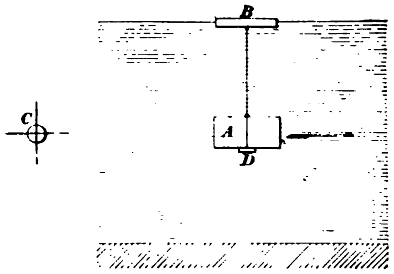


Fig. 187.

This float is used in the same manner as has been described for rod floats. The mean velocity for any division may be taken as .98 of the velocity of the float.

The principles involved in the above methods of measuring the velocities of streams are simple, but in order to get good results great care must be taken in making the various measurements and observations. In most cases it is best to repeat the observations several times and take the mean of the results obtained as the probable real value.

EXAMPLES FOR PRACTICE.

1. A circular brick sewer 3 feet in diameter falls 3.75 feet in a length of 2,500 feet. What is the slope? Ans. .0015.

2. The sewer in example 1 flows half full. (*a*) What is its wetted perimeter? (*b*) What is its hydraulic radius? (*c*) What is the value of *c* from Kutter's formula, using the value of *n* for fouled sewers? (*d*) What is the mean velocity of flow? (*e*) What is the discharge in cubic feet per second?

Ans. { (*a*) 4.7124 ft. (*b*) .75. (*c*) 80.9. (*d*) 2.71 ft. per sec. (*e*) 9.58 cu. ft. per sec.

3. The following table gives the results of a survey for the purpose of finding the flow of a river:

Divisions of the Cross-Section.	Width of Divisions.	Mean Depth of Divisions.	Mean Velocity in each Division as Determined by Current-Meter.
No. 1	6 feet	2.12 feet	0.315 feet per second
No. 2	10 feet	5.17 feet	1.227 feet per second
No. 3	10 feet	8.27 feet	2.080 feet per second
No. 4	10 feet	7.46 feet	2.049 feet per second
No. 5	10 feet	4.72 feet	1.156 feet per second
No. 6	5.25 feet	3.35 feet	0.720 feet per second

What is the discharge (*a*) in each division, and (*b*) in the whole stream?

Ans. {
(*a*) {
Division 1, 4.0068 cu. ft. per sec.
Division 2, 63.4359 cu. ft. per sec.
Division 3, 172.0160 cu. ft. per sec.
Division 4, 152.8554 cu. ft. per sec.
Division 5, 54.5632 cu. ft. per sec.
Division 6, 12.6630 cu. ft. per sec.
(*b*) 459.5403 cu. ft. per sec.

PNEUMATICS.

PROPERTIES OF AIR AND GASES.

1039. Pneumatics is that branch of Mechanics which treats of the properties of gases.

1040. The most striking feature concerning gases is that, *no matter how small the quantity may be, they will always fill the vessels which contain them.* If a bladder or football is partly filled with air and placed under a glass jar (called a **receiver**), from which the air has been exhausted, the bladder or football will immediately expand, as shown in Fig. 188. The force which a gas always exerts when confined in a limited space, is called **tension.** The word tension in this case means pressure, and is only used in this sense in reference to gases.

FIG. 188.

1041. As *water* is the most common type of fluids, so *air* is the most common type of gases. It was supposed by the ancients that air was imponderable, i. e., that it weighed nothing, and it was not until about the year 1650 that it was proven that air really had weight. A cubic inch of air, under ordinary conditions, weighs .31 grain, nearly. The ratio of the weight of air to water is about 1 : 774; that is, air is only $\frac{1}{774}$ as heavy as water. In Art. **989** it was shown that if a body was immersed in water, and weighed less than the volume of water displaced, the body would rise and extend partly out of the water. The same is true to a certain extent of air. If a vessel made of light material is filled with a gas lighter than air, so that the total weight of the vessel and gas is less than the weight of the volume of air which they displace, the vessel will rise. It is on this principle that balloons are made.

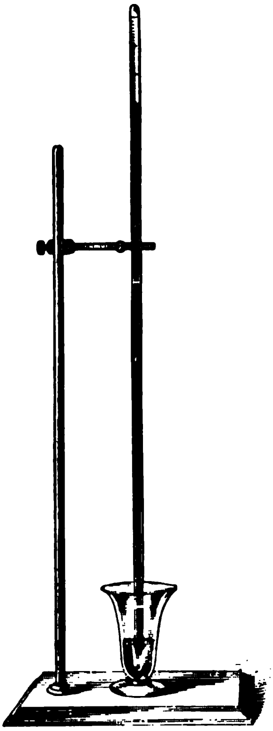


FIG. 189.

1042. Since air has weight, it is evident that the enormous quantity of air that constitutes the atmosphere must exert a considerable pressure upon the earth. This is easily proven by taking a long glass tube, closed at one end, and filling it with mercury. If the finger is placed over the open end, so as to keep the mercury from running out, and the tube is inverted and placed in a cup of mercury, as shown in Fig. 189, the mercury will fall, then rise, and after a few oscillations will come to rest at a height above the top of the mercury in the glass equal to about 30 inches. This height will always be the same under the same atmospheric conditions (allowance being made for the effects of capillary attraction). Now, if the atmosphere has weight, it must press upon the upper surface of the mercury in the glass with equal intensity upon every square unit, except upon that part of the surface occupied by the tube. According to Pascal's law (see Art. **970**), this pressure is transmitted in all directions. There being nothing in the tube, except the mercury, to counterbalance the upward pressure of the air, the mercury falls in the tube until it exerts a downward pressure on the upper surface of the mercury in the cup sufficiently great to counterbalance the upward pressure produced by the atmosphere. In order that there shall be equilibrium, the pressure of the air per unit of area on the upper surface of the mercury in the glass must equal the pressure (weight) exerted per unit of area by the mercury inside of the tube. Suppose that the area of the inside of the tube is one square inch ; then, since mercury is 13.6 times as heavy as water, the weight of the mercurial column is $.03617 \times 13.6 \times 30 = 14.7574$ pounds. The actual height of the mercury is a little less than 30 inches, and the actual weight of a cubic inch of distilled water is a little less than .03617 pound. When these considerations are taken into account, the average weight of the mercurial column at the level of the sea is 14.69 pounds, or, as it is usually expressed, 14.7 pounds. Since this weight, exerted upon 1 square inch of the liquid in the glass, just produced equilibrium, it is plain that the pressure of the outside air is 14.7 pounds upon every square inch of surface.

1043. Vacuum.—The space between the upper end of the tube and the upper surface of the mercury is called a **vacuum,** meaning that it is an entirely empty space, and does not contain any substance, solid, liquid, or gaseous. If there was a gas of some kind there, no matter how small the quantity might be, it would expand, filling the space, and its tension would cause the column of mercury to fall and become shorter, according to the amount of gas or air present. The space is then called a **partial vacuum.** If the mercury fell 1 inch, so that the column was only 29 inches high, we should say, in ordinary language, that there were *29 inches of vacuum.* If it fell 8 inches, we would say that there were 22 inches of vacuum ; if it fell 16 inches, we would say that there were 14 inches of vacuum, etc. Hence, when the vacuum gauge of a condensing engine shows 26 inches of vacuum, there is enough air in the condenser to produce a pressure of $\frac{30 - 26}{30} \times 14.7 = \frac{4}{30} \times 14.7 = 1.96$ pounds per square inch. In all cases where the mercurial column is used to measure a vacuum, the height of the column in inches gives the number of inches of vacuum. Thus, if the column were 5 inches high, or the vacuum gauge showed 5 inches, the vacuum would be 5 inches.

If the tube had been filled with water instead of mercury, the height of the column of water to balance the pressure of the atmosphere would have been $30 \times 13.6 = 408$ inches = 34 feet. This means that if a tube were filled with water, inverted and placed in a dish of water in a manner similar to the experiment made with the mercury, that the resulting height of the column of water would be 34 feet.


FIG. 190.

1044. The **barometer** is an instrument used for measuring the pressure of the atmosphere. There are two kinds in general use—the mercurial barometer and the aneroid barometer. The **mercurial barometer** is shown in Fig. 190. The principle is the same as in the case of the inverted tube shown in Fig. 189. The tube and cup at the bottom are protected by a brass or iron casing. At the top of the tube is a graduated scale which can be read to $\frac{1}{1000}$ of an inch, by means of a vernier. Attached to the casing is an accurate thermometer for determining the temperature of the outside air at the time the barometric observation is taken. This is necessary, since mercury expands when the temperature is increased, and contracts when the temperature falls; for this reason a standard temperature is assumed, and all barometer readings are reduced to this temperature. This standard temperature is usually taken at 32° F., at which temperature the height of the mercurial column is 30 inches. Another correction is made for the altitude of the place above sea level, and a third correction for the effects of capillary attraction.

1045. In Fig. 191 is shown a cut of an **aneroid barometer.** These instruments are made in various sizes, from the size of a large watch up to an 8 or 10 inch face. They consist of a cylindrical box of metal, with a top of thin, elastic, corrugated metal. The air is removed from the box. When the atmospheric pressure increases, the top is pressed inwards, and when it is diminished, the top is pressed outwards by its own elasticity, aided by a spring beneath. These movements of the cover are transmitted and multiplied by a combination of delicate levers which act upon an index hand and

FIG. 191.

cause it to move either to the right or left over a graduated scale. These barometers are self-correcting (compensated) for variations in temperature. They are very portable, occupying but a small space, and are so delicate that they are said to show a difference in the atmospheric pressure when transferred from the table to the floor. They must be handled with care, as they are easily injured. The mercurial barometer is the standard.

1046. With air, as with water, the lower we get, the greater the pressure, and the higher we get, the less the pressure. At the level of the sea, the height of the mercurial column is about 30 inches; at 5,000 feet above the sea, it is 24.7 inches; at 10,000 feet above the sea, it is 20.5 inches; at 15,000 feet above the sea, it is 16.9 inches; at 3 miles, it is 16.4 inches, and at 6 miles above the sea level, it is 8.9 inches.

The density also varies with the altitude; that is, a cubic foot of air at an elevation of 5,000 feet above the sea level will not weigh as much as a cubic foot at sea level. This is proved conclusively by the fact that at a height of $3\frac{1}{2}$ miles the mercurial column measures but 15 inches, indicating that half the weight of the entire atmosphere is below that. It is known that the height of the earth's atmosphere is at least 50 miles; hence, the air just before reaching the limit must be in an exceedingly rarefied state. It is by means of barometers that great heights are measured. The aneroid barometer has the heights marked on the dial, so that it can be read directly. With the mercurial barometer, the heights must be calculated from the reading.

1047. The atmospheric pressure is everywhere present, and presses all objects in all directions with equal force. If a book is laid upon the table, the air presses upon it in every direction with an equal average force of 14.7 pounds per square inch. It would seem as though it would take considerable force to raise a book from the table, since, if the size of the book were 8 inches by 5 inches, the pressure upon it is $8 \times 5 \times 14.7 = 588$ pounds; but there is an equal pressure beneath the book to counteract the pressure on the top. It would now seem as though it would require a great force to open the book, since there are two pressures of 588 pounds each, acting in opposite directions, and tending to crush the book; so it would but for the fact that there is a layer of air between each leaf acting upwards and downwards with a pressure of 14.7 pounds per square inch. If two metal plates be made as perfectly smooth and flat as it is possible to get them, and the edge of one be laid upon the edge of the other, so that one may be slid upon the other, and the air thus excluded, it will take an immense force, compared with the weight of the plates, to separate them. This is because the full pressure of 14.7 pounds per square inch is then exerted upon each plate with no counteracting equal pressure between them.

If a piece of flat glass be laid upon a flat surface that has been previously moistened with water, it will require considerable force to separate them; this is because the water helps to fill up the pores in the flat surface and glass, ana thus creates a partial vacuum between the glass and the surface, thereby reducing the counter pressure beneath the glass.

1048. Tension of Gases.—In Fig. 189 the space above the column of mercury was said to be a vacuum, and that if any gas or air was present, it would expand, its tension forcing the column of mercury downwards. If enough gas is admitted to cause the mercury to stand at 15 inches, the tension of the gas is evidently $\frac{14.7}{2} = 7.35$ pounds per square inch, since the pressure of the outside air of 14.7 pounds per square inch only balances 15 inches, instead of 30 inches, of mercury; that is, it balances only half as much as it would if there were no gas in the tube; therefore, the pressure (tension) of the gas in the tube is 7.35 pounds. If more gas is admitted until the top of the mercurial column is just level with the mercury in the cup, the gas in the tube has then a tension equal to the outside pressure of the atmosphere. Suppose that the bottom of the tube is fitted with a piston, and that the total length of the inside of the tube is 36 inches. If the piston be shoved upwards so that the space occupied by the gas is 18 inches long, instead of 36 inches, the temperature remaining the same as before, it will be found that the tension of the gas within the tube is 29.4 pounds per square inch. It will be noticed that the volume occupied by the gas is only half that in the tube before the piston was moved, while the pressure is twice as great, since $14.7 \times 2 = 29.4$ pounds. If the piston be shoved up, so that the space occupied by the gas is only 9 inches, instead of 18 inches, the temperature still remaining the same, the pressure will be found to be 58.8 pounds per square inch. The volume has again been reduced one-half, and the pressure increased 2 times, since $29.4 \times 2 = 58.8$ pounds. The space now occupied by the gas is 9 inches long, whereas, before the piston was moved it was 36 inches long ; as the tube was assumed to be of uniform diameter throughout its length, the volume is now $\frac{9}{36} = \frac{1}{4}$ of its original volume, and its pressure is $\frac{58.8}{14.7} = 4$ times its original pressure. Moreover, if the temperature of the confined gas remains the same, the pressure and volume will always vary in a similar way. The law which states these effects is called *Mariotte's Law*, and is as follows:

1049. Mariotte's Law.—*The temperature remaining the same, the volume of a given quantity of gas varies inversely as the pressure.*

The meaning of this is : If the volume of the gas is diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, etc., of its former volume, the tension will be increased 2, 3, 5, etc., times, or if the outside pressure be increased 2, 3, 5, etc., times, the volume of the gas will be diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, etc., of its original volume, the temperature remaining constant. It also means that if a gas is under a certain pressure, and the pressure is diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{10}$, etc., of its original pressure, that the volume of the confined gas will be increased 2, 3, 10, etc., times—its tension decreasing at the same rate.

Suppose 3 cubic feet of air to be under a pressure of 60 pounds per square inch in a cylinder fitted with a movable piston; then, the product of the volume and pressure is $3 \times 60 = 180$. Let the volume be increased to 6 cubic feet, then the pressure will be 30 pounds per square inch, and $30 \times 6 = 180$, as before. Let the volume be increased to 24 cubic feet, it is then $\frac{24}{3} = 8$ times its original volume, and the pressure is $\frac{1}{8}$ of its original pressure, or $60 \times \frac{1}{8} = 7\frac{1}{2}$ pounds, and $24 \times 7\frac{1}{2} = 180$, as in the two preceding cases. It will now be noticed that if a gas be enclosed within a confined space, and allowed to expand without losing any heat, *the product of the pressure, and the corresponding volume for one position of the piston, is the same as for any other position of the piston.* If the piston were to compress the air, the same result would be obtained.

Let p = pressure for one position of the piston;
p_1 = pressure for any other position of the piston;
v = volume corresponding to the pressure p;
v_1 = volume corresponding to the pressure p_1.

$$\text{Then, } p\,v = p_1\,v_1. \qquad \textbf{(53.)}$$

1050. Knowing the volume and the pressure for any position of the piston, and the volume for any other position, the pressure may be calculated, or, if the pressure is known for any other position, the volume may be calculated.

EXAMPLE.—If 1.875 cubic feet of air be under a pressure of 72 pounds per square inch (*a*) what will be the pressure when the volume is increased to 2 cubic feet? (*b*) to 3 cubic feet? (*c*) to 9 cubic feet?

SOLUTION.—Solving formula **53,** for p_1, the unknown pressure,

(*a*) $p_1 = \frac{p\,v}{v_1} = \frac{72 \times 1.875}{2} = 67\frac{1}{2}$ lb. per sq. in. Ans.

(*b*) $p_1 = \frac{72 \times 1.875}{3} = 45$ lb. per sq. in. Ans.

(*c*) $p_1 = \frac{72 \times 1.875}{9} = 15$ lb. per sq. in. Ans.

EXAMPLE.—Ten cubic feet of air have a tension of 5.6 pounds per square inch; (*a*) what is the volume when the tension is 4 pounds? (*b*) 8 pounds? (*c*) 25 pounds? (*d*) 100 pounds?

SOLUTION.—Solving formula **53,** for v_1,

(*a*) $v_1 = \frac{p\,v}{p_1} = \frac{5.6 \times 10}{4} = 14$ cu. ft. Ans.

(*b*) $v_1 = \frac{5.6 \times 10}{8} = 7$ cu. ft. Ans.

(*c*) $v_1 = \frac{5.6 \times 10}{25} = 2.24$ cu. ft. Ans.

(*d*) $v_1 = \frac{5.6 \times 10}{100} = .56$ cu. ft. Ans.

1051. NOTE.—There are two ways of measuring the pressure of a gas: by means of an instrument called a **manometer**, and by means of a **gauge.** The manometer generally used is practically the same as a mercurial barometer, except that the tube is much longer, so that pressures equal to several atmospheres may be measured, and is enlarged and bent into a U shape at the lower end; both lower and upper ends are open, the lower end being connected to the vessel containing the gas whose pressure it is desired to measure. The gauge is so common that no description of it will be given here. With both the manometer described above and the gauge, the pressures recorded are the amounts by which they exceed the atmospheric pressure, and are called the **gauge pressures.** To find the real pressure, called the **absolute pressure,** the atmospheric pressure must be added to the gauge pressure. In all formulas in which the pressure of a gas or steam is used, the absolute pressure must be used, unless the gauge pressure is distinctly specified as being the proper pressure to use. For convenience, all pressures given in Arts. **1039** to **1088,** inclusive, and in the questions referring to these articles, will be absolute pressures, and the word "absolute" will be omitted to avoid its constant repetition.

1052. As a necessary consequence of Mariotte's law, it may be stated that *the density of a gas varies directly as the pressure, and inversely as the volume; that is, the density increases as the pressure increases, and decreases as the volume increases.*

This is evident, since if a gas has a tension of 2 atmospheres, or $14.7 \times 2 = 29.4$ pounds per square inch, it will weigh twice as much as the same volume would if the tension was 1 atmosphere, or 14.7 pounds per square inch. For, let the volume be increased until it is twice as great as the original volume, the tension will then be 1 atmosphere. The total weight of the gas has not been changed, but there are now 2 cubic feet for every 1 cubic foot of the original volume, and the weight of 1 cubic foot now is only half as great as before. Thus, the density decreases as the volume increases, and as an increase of pressure causes a decrease of volume, the density increases as the pressure increases.

Let D be the density corresponding to the pressure p and volume v, and D_1 be the density corresponding to the pressure p_1 and volume v_1; then,

$$p : D = p_1 : D_1, \text{ or } p\,D_1 = p_1\,D, \qquad \textbf{(54.)}$$

and $$v : D_1 = v_1 : D, \text{ or } v\,D = v_1\,D_1. \qquad \textbf{(55.)}$$

Since the weight is proportional to the density, the weights may be used in place of the densities in formulas

54 and **55**. Thus, let W be the weight of a cubic foot of air or other gas, whose volume is v and pressure is p; let W_1 be the weight of a cubic foot when the volume is v_1 and pressure is p_1; then,

$$pW_1 = p_1 W. \qquad \textbf{(56.)}$$

$$vW = v_1 W_1. \qquad \textbf{(57.)}$$

EXAMPLE.—The weight of 1 cubic foot of air at a temperature of 60° F., and under a pressure of one atmosphere (14.7 pounds per square inch), is .0763 pound; what would be the weight per cubic foot if the volume were compressed until the tension was 5 atmospheres, the temperature still being 60° F.?

SOLUTION.—Applying formula **56**, $p W_1 = p_1 W$, or $1 \times W_1 = 5 \times .0763$. Hence, $W_1 = .3815$ lb. per cu. ft. Ans.

EXAMPLE.—If in the last example the air had expanded until the tension was 5 pounds per square inch, what would have been its weight per cubic foot?

SOLUTION.—Applying formula **56**, $p W_1 = p_1 W$. Here $p = 14.7$, $p_1 = 5$ and $W = .0763$. Hence, $14.7 \times W_1 = 5 \times .0763$, or $W_1 = \frac{.3815}{14.7} = .02595$ lb. per cu. ft. Ans.

EXAMPLE.—If 6.75 cubic feet of air, at a temperature of 60° F., and a pressure of one atmosphere, are compressed to 2.25 cubic feet (the temperature still remaining 60° F.), what is the weight of a cubic foot of the compressed air?

SOLUTION.—Applying formula **57**, $v W = v_1 W_1$, or $6.75 \times .0763 = 2.25 \times W_1$; hence, $W_1 = \frac{6.75 \times .0763}{2.25} = .2289$ lb. per cu. ft. Ans.

1053. In all that has been said before, it has been stated that the temperature was constant; the reason for this will now be explained. Suppose five cubic feet of air to be confined in a cylinder placed in a vacuum, so that there will be no pressure due to the atmosphere, and suppose the cylinder to be fitted with a piston weighing say 100 pounds, and having an area of 10 square inches. The tension of the gas will be $\frac{100}{10} = 10$ pounds per square inch. Suppose that the temperature of the air is 32° F., and that it is heated until the temperature is 33° F., i. e., the temperature is 1°, it will be found that the piston has risen a certain amount, and, consequently, the volume has increased, while the pressure is the same as before, or 10 pounds per square inch. If more heat is applied until the temperature of the gas is 34° F., it will be found that the piston has again risen, and the volume again increased, while the pressure still remains the same. It will be found that for every increase of temperature there will be a corresponding increase of volume. The law which expresses this change, is called *Gay-Lussac's Law*, and is as follows:

1054. Gay-Lussac's Law.—*If the pressure remains constant, every increase of temperature of 1° F. produces in a given quantity of gas an expansion of $\frac{1}{492}$ of its volume at 32° F.*

If the pressure remains constant, it will also be found that every decrease of temperature of 1° F., will cause a decrease of $\frac{1}{492}$ of the volume at 32° F.

Let v = original volume of gas;
v_1 = final volume of gas;
t = temperature corresponding to volume v;
t_1 = temperature corresponding to volume v_1.

Then, $$v_1 = v\left(\frac{460 + t_1}{460 + t}\right). \qquad (58.)$$

That is, *the volume of gas after heating (or cooling) equals the original volume, multiplied by 460, plus the final temperature, divided by 460, plus the original temperature.*

EXAMPLE.— 5 cubic feet of air at a temperature of 45°, are heated under constant pressure up to 177°; what is its volume?

SOLUTION.—Applying formula **58**,

$$v_1 = v\left(\frac{460 + t_1}{460 + t}\right) = 5\left(\frac{460 + 177}{460 + 45}\right) = 6.307 \text{ cu. ft.} \quad \text{Ans.}$$

1055. Suppose that a certain volume of gas is confined in a vessel so that it cannot expand; in other words, suppose that the piston of the cylinder before mentioned to be fastened so that it cannot move. Let a gauge be placed on the cylinder so that the tension of the confined gas can be registered. If the gas is heated, it will be found that for every increase of temperature of 1° F., there will be a corresponding increase of $\frac{1}{492}$ of the tension. That is, the volume remaining constant, the tension increases $\frac{1}{460}$ of the original tension for every degree rise of temperature.

Let p = the original tension;
t = the corresponding temperature;
p_1 = final tension;
t_1 = final temperature.

Then, $$p_1 = p\left(\frac{460 + t_1}{460 + t}\right). \qquad \textbf{(59.)}$$

That is, *if a certain quantity of gas be heated (or cooled) from $t°$ to $t_1°$, the volume remaining constant, the resulting tension p_1 will be equal to the original tension, multiplied by 460, plus the final temperature, divided by 460, plus the original temperature.*

EXAMPLE.—If a certain quantity of air is heated under constant volume from 45° to 177°, what is the resulting tension, the original tension being 14.7 pounds per square inch?

SOLUTION.—Applying formula **59**,

$$p_1 = p\left(\frac{460 + t_1}{460 + t}\right) = 14.7\left(\frac{460 + 177}{460 + 45}\right) = 18.542 \text{ lb. per sq. in. Ans.}$$

1056. According to the modern and now generally accepted theory of heat, the atoms and molecules of all bodies are in an incessant state of vibration. The vibratory movement in the liquids is faster than in the solids, and in the gases, faster than in either of the other two. Any increase of heat increases the vibrations, and a decrease of heat decreases them. From experiments and calculations based upon higher mathematics, it has been concluded that at 460° below zero, on the Fahrenheit scale, all these vibrations cease. This point is called the **absolute zero,** and all temperatures reckoned from this point are called the **absolute temperatures.** The point of absolute zero has never been reached, the lowest recorded temperature being about 393° F. below zero, but, nevertheless, it has a meaning, and is used in many formulas, being nearly always denoted by T. The ordinary temperatures are denoted by t. When the word temperature alone is used, the meaning is the same as ordinarily used, but when absolute temperature is specified, 460° F. must be added to the temperature.

The absolute temperature corresponding to 212° F. is $460 + 212 = 672°$ F. If the absolute temperature is given, the ordinary temperature may be found by subtracting 460 from the absolute temperature. Thus, the absolute temperature being 520° F., what is the temperature?

$$520° - 460° = 60°.$$

Let p = pressure in pounds per square inch;
V = volume of air in cubic feet;
T = absolute temperature;
W = weight in pounds.

Then, $$pV = .37052\ T. \qquad \textbf{(60.)}$$

That is, *the pressure in pounds per square inch, multiplied by the volume of the air in cubic feet, equals .37052 times the absolute temperature corresponding to the pressure p and volume V.*

In this formula, the weight of the air is 1 pound.

EXAMPLE.—The pressure upon 9 cubic feet of air weighing 1 pound is 20 pounds per square inch; what is the temperature?

SOLUTION.—Applying formula **60**, $pV = .37052\ T$, or $20 \times 9 = .37052\ T$; hence, $T = \frac{180}{.37052} = 485.8°$, nearly. $485.8° - 460 = 25.8°$, the temperature. Ans.

EXAMPLE.—What is the volume of 1 pound of air whose temperature is 60° F. under a pressure of one atmosphere?

SOLUTION.—Applying formula **60**, $pV = .37052\ T$. Substituting, $14.7 \times V = .37052 \times (460 + 60) = .37052 \times 520$, or $V = \frac{.37052 \times 520}{14.7} = 13.107$ cubic feet. Ans.

1057. If the weight of the air be greater or less than 1 pound, the following formula must be used:

$$pV = .37052\ W\ T. \qquad \textbf{(61.)}$$

That is, *the pressure in pounds per square inch, multiplied by the volume in cubic feet, equals .37052 times the weight in pounds multiplied by the absolute temperature.*

EXAMPLE.— 3 cubic feet of air weighing .35 pound, are under a pressure of 48 pounds per square inch; what is the temperature of the air?

SOLUTION.—Applying formula **61**, $pV = .37052\ WT$. Substituting, $48 \times 3 = .37052 \times .35 \times T$, or $T = \frac{48 \times 3}{.37052 \times .35} = 1{,}110.4°$. Then, $1{,}110.4° - 460° = 650.4°$. Ans.

EXAMPLE.—What is the weight of 1 cubic foot of air at a temperature of 32°, and under a pressure of one atmosphere?

SOLUTION.—Applying formula **61**, $pV = .37052\,WT$. Substituting, $14.7 \times 1 = .37052 \times (460 + 32) \times W$, or

$$W = \frac{14.7}{.37052 \times 492} = .0806382 \text{ lb. Ans.}$$

If the pressure be taken as 14.69856 pounds per square inch, and the absolute zero as 459.4°, instead of 460° below zero, and if .370514 be used, instead of .37052, more exact values, the weight of 1 cubic foot would be $\frac{14.69856}{.370514 \times 491.4} = .08073$ lb., nearly.

EXAMPLE.—What is the exact volume of 1 pound of air at a temperature of 32°, and at a pressure of one atmosphere? Take absolute zero at 459.4, and the pressure as 14.69856 pounds per square inch.

SOLUTION.— $pV = .370514\,WT$, or $14.69856 \times V = .370514 \times 1 \times (459.4 + 32)$. $V = \frac{.370514 \times 491.4}{14.69856} = 12.387$ cu. ft. Ans.

1058. If in the formula $pV = .37052\,WT$, both sides of the equation be divided by T (which, of course, does not alter the equality), there results the expression $\frac{pV}{T} = .37052\,W$. Let p_1, V_1 and T_1 represent the pressure, volume and temperature of the same weight of air in another state; then, $p_1V_1 = .37052\,WT_1$. Dividing both sides by T_1, $\frac{p_1V_1}{T_1} = .37052\,W$. Therefore, since $\frac{pV}{T}$ and $\frac{p_1V_1}{T_1}$ are equal to the same thing (i. e., $.37052\,W$), they are equal to each other, and

$$\frac{pV}{T} = \frac{p_1V_1}{T_1} \qquad \textbf{(62.)}$$

This very important formula is the complete expression of Gay-Lussac's law, and is true for any of the so-called permanent gases. It was from this formula that formulas **58** and **59** were derived. Thus, let the pressure be constant; then, $p = p_1$, and $\frac{pV}{T} = \frac{pV_1}{T_1}$, or $V_1 = \frac{VT_1}{T} = V\left(\frac{460 + t_1}{460 + t}\right)$. Similarly, letting the volume be constant, $V = V_1$, and $\frac{pV}{T}$

$= \frac{p_1 V}{T_1}$, or $p_1 = \frac{p T_1}{T} = p\left(\frac{460 + t_1}{460 + t}\right)$. So, also, by letting the temperature be constant, $T = T_1$ and $\frac{pV}{T} = \frac{p_1 V_1}{T}$, or $pV = p_1 V_1$, which is the same as formula **53.**

1059. In formulas **53, 62, 63,** and **64,** it matters not with what units the pressures and volumes are measured, except that they must be the same throughout the same example, and the pressures must always be ***absolute pressures.***

EXAMPLES FOR PRACTICE.

1. A vessel contains 25 cubic feet of gas at a pressure of 18 pounds per square inch; if 125 cubic feet of gas having the same pressure are forced into the vessel, what will be the resulting pressure?
Ans. 108 lb. per sq. in.

2. A pound of air has a temperature of 126°, and a pressure of 1 atmosphere; what volume does it occupy? Ans. 14.77 cu. ft.

3. The volume of steam in the cylinder of a steam engine at cut-off is 1.35 cubic foot, and the pressure is 85 pounds per square inch; if the pressure at the end of the stroke is 25 pounds per square inch, what is the new volume? Ans. 4.59 cu. ft.

4. A certain quantity of air has a volume of 26.7 cubic feet, a pressure of 19.3 pounds per square inch, and a temperature of 42°; what is the weight? Ans. 2.77 lb.

5. A receiver contains 180 cubic feet of gas at a pressure of 20 pounds per square inch; if a vessel holding 12 cubic feet, to be filled from the receiver until its pressure is 20 pounds per square inch, what will be the pressure in the receiver? Ans. $18\frac{2}{3}$ lb. per sq. in.

6. 10 cubic feet of air having a pressure of 22 pounds per square inch, and a temperature of 75°, are heated until the temperature is 300°; the volume remaining the same, what is the new pressure?
Ans. 31.25 lb. per sq. in.

7. If a spherical shell whose outside diameter is 18 inches, has a part of the air within it removed until the pressure is 5 pounds per square inch, what is the total pressure due to the atmosphere tending to crush the shell? Ans. 9,873.42 lb.

THE MIXING OF GASES.

1060. If two liquids which do not act chemically upon each other are mixed together and allowed to stand, it will be found that after a time the two liquids have separated, and that the heavier has fallen to the bottom. If two equal vessels, containing gases of different densities, be put in communication with each other, they will be found to have mixed in equal proportions after a short time. If one vessel be higher than the other, and the heavier gas be in the lower vessel, the same result will occur. The greater the difference of the densities of the two gases, the quicker they will mix. It is assumed that no chemical action takes place between the two gases. When the two gases have the same temperature and pressure, the pressure of the mixture will be the same; this is evident, since the total volume has not been changed, and unless the volume or temperature changes, the pressure cannot change. This property of the mixing of gases is a very valuable one, since, if they acted like liquids, carbonic acid gas (the result of combustion), which is $1\frac{1}{2}$ times as heavy as air, would remain next to the earth, instead of dispersing into the atmosphere, the result being that no animal life could exist.

1061. Mixtures of Equal Volumes of Gases Having Unequal Pressures.—*If two gases having equal volumes and temperatures, but different pressures, be mixed in a vessel whose volume equals one of the equal volumes of the gas, the pressure of the mixture will be equal to the sum of the two pressures, provided that the temperature remains the same as before.*

EXAMPLE.—Two vessels containing 3 cubic feet of gas, each at a temperature of 60°, and subjected to pressures of 40 pounds and 25 pounds per square inch, respectively, are placed in communication with each other, and all the gas is compressed into one vessel. If the temperature of the mixture is also 60°, what is the pressure?

SOLUTION.—According to the rule just given, the pressure will be $40 + 25 = 65$ pounds per square inch. This may be proven by applications of Mariotte's law; thus, compress the gas whose pressure is 25 pounds per square inch until its pressure is 40 pounds; its volume may be found thus: $p v = p_1 v_1$, or $25 \times 3 = 40 \times v$; whence, $v = 1.875$ cubic feet. Let communication be established between the two vessels, the pressure will evidently be 40 pounds and the total volume $3 + 1.875 = 4.875$ cubic feet. If this be compressed until the volume is 3 cubic feet, the temperature remaining at 60° throughout the whole operation, the final pressure may be found by formula **53**, $p v = p_1 v_1$.

Thus, $40 \times 4.875 = p_1 \times 3$, and $p_1 = \frac{40 \times 4.875}{3} = 65$ pounds per square inch, as before.

1062. Mixture of Two Gases Having Unequal Volumes and Pressures.

Let v and p be the volume and pressure, respectively, of one of the gases.

Let v_1 and p_1 be the volume and pressure, respectively, of the other gas.

Let V and P be the volume and pressure, respectively, of the mixture. Then, if the temperature remains the same,

$$V P = v p + v_1 p_1. \qquad \textbf{(63.)}$$

That is, *if the temperature is constant, the volume after mixture, multiplied by the resulting pressure, equals the volume of one gas before mixture multiplied by its pressure, plus the volume of the other gas multiplied by its pressure.*

EXAMPLE.—Two gases of the same temperature, having volumes of 7 cubic feet and 4½ cubic feet, and whose pressures are 27 pounds and 18 pounds per square inch, respectively, are mixed together in a vessel whose volume is 10 cubic feet. The temperature of the two gases and of the mixture being 60° F., what is the resulting pressure?

SOLUTION.—Applying formula **63**, $P V = p v + p_1 v_1$, or $P \times 10 = 27 \times 7 + 4\frac{1}{2} \times 18$. Hence, $P = \frac{189 + 81}{10} = 27$ lb. per sq. in. Ans.

1063. Mixture of Two Volumes of Air Having Unequal Pressures, Volumes, and Temperatures.

If a body of air having a temperature t_1, a pressure p_1, and a volume v_1 be mixed with another volume of air having a temperature t_2, a pressure p_2, and a volume v_2, to form a volume V having a pressure P and a temperature t, then, either the new temperature t, the new volume V, or the new pressure P may be found, if the other two quantities are known, by the following formula, in which T_1, T_2, and T are the absolute temperatures corresponding to t_1, t_2, and t:

$$P V = \left[\frac{p_1 v_1}{T_1} + \frac{p_2 v_2}{T_2}\right] T. \qquad \textbf{(64.)}$$

EXAMPLE.—Five cubic feet of air having a tension of 30 pounds per square inch, and a temperature of 80° F., are required to be compressed together with 11 cubic feet of air having a tension of 21 pounds per square inch, and a temperature of 45° F., in a vessel whose cubical contents are 8 cubic feet. The new pressure is required to be 45 pounds per square inch. What is the temperature of the mixture?

SOLUTION.—Substituting in formula **64**,

$45 \times 8 = \left[\frac{30 \times 5}{540} + \frac{21 \times 11}{505}\right] \times T$, or $360 = .7352\ T$. Hence, $T = \frac{360}{.7352} = 489.66°$, nearly, and $t = 29.66°$. Ans.

EXAMPLES FOR PRACTICE.

1. Two vessels contain air at pressures of 60 and 83 pounds per square inch. The volume of each vessel is 8.47 cubic feet. If all of the air in both vessels is removed to another vessel, and the new pressure is 100 pounds per square inch, what is the volume of the vessel, the temperature being the same throughout?

   Ans. 12.11 cu. ft.

2. A vessel contains 11.83 cubic feet of air at a pressure of 33.3 pounds per square inch. It is desired to increase the pressure to 40 pounds per square inch by supplying air from a second vessel which contains 19.6 cubic feet of air at a pressure of 60 pounds per square inch. What will be the pressure in the second vessel after the pressure in the first has been raised to 40 pounds per square inch?

   Ans. 55.96 lb. per sq. in.

3. If 4.8 cubic feet of air having a tension of 52 pounds per square inch and a temperature of 170° are mixed with 13 cubic feet having a tension of 78 pounds per square inch and a temperature of 265°, what must be the volume of the vessel containing the mixture in order that the tension of the mixture may be 30 pounds per square inch and the temperature 80°?

   Ans. 32.31 cu. ft.

PNEUMATIC MACHINES.

THE AIR PUMP.

1064. The **air pump** is an instrument for removing air from an enclosed space. A section of the principal parts is shown in Fig. 192, and the complete instrument in Fig. 193. The closed vessel R is called the **receiver**, and the space which it encloses is that from which it is desired to remove the air. The receiver is usually made of glass, and the edges are ground so as to be perfectly air-tight. When made in the form shown, it is called a **bell jar receiver.**

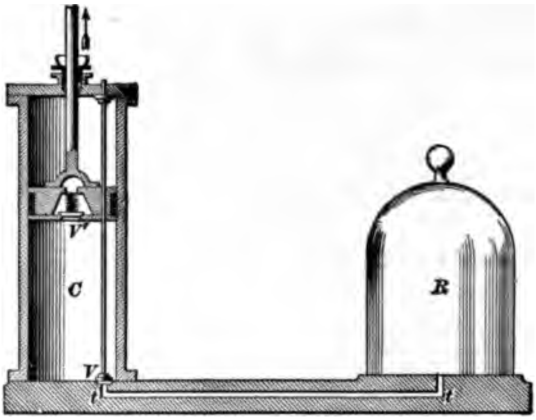


FIG. 192.

The receiver rests upon a horizontal plate in the center of which is an opening communicating with the pump cylinder *C* by means of a bent tube *t*. The pump piston fits the cylinder accurately, and has a valve *V'* opening upwards. At the junction of the tube with the cylinder is another valve *V* also opening upwards. When the piston is raised the valve *V'* closes, and, since no air can get into the cylinder from above, the piston leaves a vacuum behind it. The pressure on top of *V* being now removed, the tension of the air in the receiver *R* causes *V* to rise; the air in the receiver then expands and occupies the

FIG. 193.

space displaced by the piston, the space in the tube t and in the receiver R. The piston is now pushed down, the valve V closes, the valve V' opens, and the air in C escapes. The lower valve V is sometimes supported, as shown in Fig. 192, by a metal rod passing through the piston and fitting it somewhat tightly. When the piston is raised or lowered, this rod moves with it. A button near the upper end of the rod confines its motion to within very narrow limits, the piston sliding upon the rod during the greater part of the journey.

1065. Degrees and Limits of Exhaustion.—Suppose that the volume of R and t together is four times that of C, and that there are, say, 200 grains of air in R and t, and 50 grains in C, when the piston is at the top of the cylinder. At the end of the first stroke, when the piston is again at the top, 50 grains of air in the cylinder C will have been removed, and the 200 grains in R and t will occupy the spaces R, t, and C. The ratio between the sum of the spaces R and t and the total space $R+t+C$ is $\frac{4}{5}$; hence, $200 \times \frac{4}{5} = 160$ grains = the weight of air in R and t after the first stroke. After the second stroke, the weight of the air in R and t would be $(200 \times \frac{4}{5} \times \frac{4}{5} = 200 \times (\frac{4}{5})^2 = 200 \times \frac{16}{25} = 128$ grains. At the end of the third stroke, the weight would be $[200 \times (\frac{4}{5})^2] \times \frac{4}{5} = 200 \times (\frac{4}{5})^3 = 200 \times \frac{64}{125} = 102.4$ grains. At the end of n strokes, the weight would be $200 \times (\frac{4}{5})^n$. It is evident that *it is impossible to remove all of the air that is contained in R and t by this method.* It requires an exceedingly good air pump to reduce the tension of the air in R to $\frac{1}{50}$ of an inch of mercury. When the air has become so rarefied as this, the valve V' will not lift, and, consequently, no more air can be exhausted.

1066. Sprengel's Air Pump.—In Fig. 194, $c\,d$ is a glass tube longer than 30 inches, open at both ends, and connected by means of India rubber tubing with a funnel A filled with mercury and supported by a stand. Mercury is allowed to fall into this tube at a rate regulated by a clamp at c. The lower end of the tube $c\,d$ fits in the flask B, which has a spout at the side a little higher than the lower end of *c d*; the upper part has a branch at *x* to which a receiver *R* can be tightly fixed. When the clamp at *c* is opened the first portions of the mercury which run out close the tube and prevent air from entering from below. These drops of mercury act like little pistons, carrying the air in front of them and forcing it out through the bottom of the tube. The air in *R* expands to fill the tube every time that a drop of mercury falls, thus creating a partial vacuum in *R*, which becomes more nearly complete as the process goes on. The escaping mercury falls into the dish *H*, from which it can be poured back into the funnel from time to time. As the exhaustion from *R* goes on, the mercury rises in the tube *c d* until, when the exhaustion is complete, it forms a continuous column 30 inches high; in other words, it is a barometer, whose Torricellian vacuum is the receiver *R*.

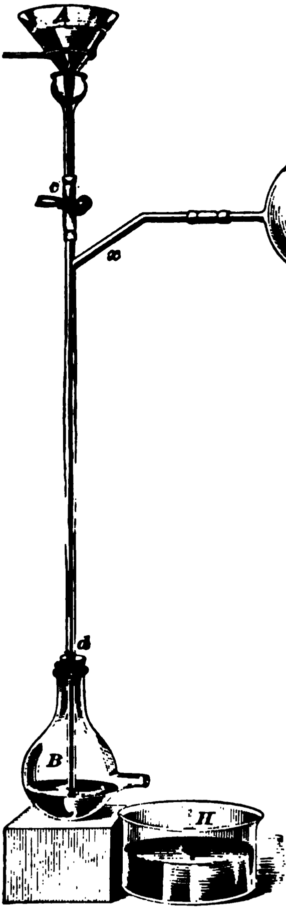


FIG. 194.

This instrument necessarily requires a great deal of time for its operation, but the results are very complete, a vacuum of $\frac{1}{43000}$ of an inch of mercury being sometimes obtained. By use of chemicals in addition to the above, a vacuum of $\frac{1}{650000}$ of an inch of mercury has been obtained.

1067. NOTE.—A theoretically perfect vacuum is sometimes called a **Torricellian vacuum.**

1068. Magdeburg Hemispheres.—By means of the two hemispheres shown in Fig. 195, it can be proven that the atmosphere presses upon a body equally in all directions. They were invented by Otto Von Guericke, of Magdeburg, and are called the **Magdeburg hemispheres.** One of the hemispheres is provided with a stop-cock, by which it can be screwed on to an air pump. The edges fit accurately and are well greased, so as to be air-tight. As long as the hemispheres contain air, they can be separated with ease; but when the air in the interior is pumped out by means of an air pump, they can be separated only with great difficulty. The force required to separate them will be equal to the area of the largest circle of the hemisphere (projected area) in square inches, multiplied by 14.7 pounds.

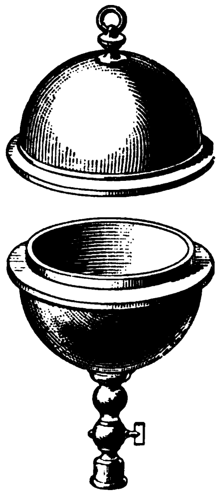


FIG. 195.

This force will be the same in whatever position the hemisphere may be held, thus proving that the pressure of air upon it is the same in all directions.

1069. The Weight Lifter.—The pressure of the atmosphere is very clearly shown by means of an apparatus like that illustrated in Fig. 196. Here, a cylinder fitted with a piston is held in suspension by a chain. At the top of the cylinder is a plug *A*, which can be taken out. This plug is removed, the piston pushed up (the force necessary being equal to the weight of the piston and rod *B*) until it touches the cylinder head. The plug is then screwed in, and the piston will remain at the top until a weight has been hung on the rod equal to the area of the piston, multiplied by 14.7 pounds, less the weight of the piston and rod. If a force was applied to the rod sufficiently great to force the piston downwards, it would raise any weight less than the above to the top of the cylinder. Suppose the weight to be removed, and the piston to be supported, say midway of the length of the cylinder. Let the plug be removed and air admitted above the piston, then screw the plug back into its place; if the piston be shoved upwards, the farther up it goes, the greater will be the force necessary to push it, on account of the compression of the air. If the piston is of large diameter, it will also require a great force to pull it out of the cylinder, as a little consideration will show. For example, let the diameter of the piston be 20 inches, the length of the cylinder 36 inches, plus the thickness of the piston, and the weight of the piston and rod 100 pounds. If the piston is in the middle of the cylinder, there will be 18 inches of space above it, and 18 inches of space below it. The area of the piston is $20^2 \times .7854 = 314.16$ square inches, and the atmospheric pressure upon it is $314.16 \times 14.7 = 4,618$ pounds, nearly. In order to shove the piston upwards 9 inches, the pressure upon it must be twice as great, or 9,236 pounds, and to this must be added the weight of the piston and rod, or $9,236 + 100 = 9,336$ pounds. The force necessary to cause the piston to move upwards 9 inches would then be $9,336 - 4,618 = 4,718$ pounds. Now, suppose the piston to be moved downwards until it is just on the point of being pulled out of the cylinder. The volume above it will then be twice as great as before, and the pressure one-half as great, or $4,618 \div 2 = 2,309$ pounds. The total upward pressure will be the pressure of the atmosphere less the weight of the piston and rod, or $4,618 - 100 = 4,518$ pounds, and the force necessary to pull it downwards to this point will be $4,518 - 2,309 = 2,209$ pounds.

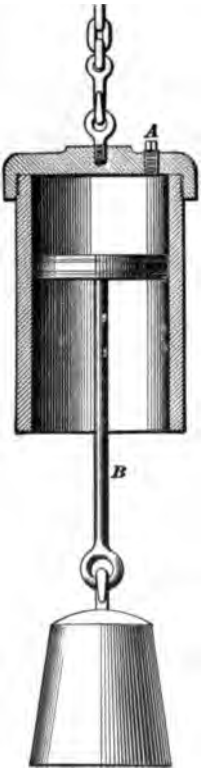


FIG. 196.

1070. The Baroscope.—The buoyant effect of air is very clearly shown by means of an instrument called the **baroscope,** shown in Fig. 197. It consists of a scale beam, from one extremity of which is suspended a small weight, and from the other a hollow copper sphere. In air they exactly balance each other; but when placed under the receiver of an air pump and the air exhausted, the sphere sinks, showing that it is really heavier than the small weight. Before the air is exhausted, each body is buoyed up by the weight of the air it displaces, and since the sphere displaces the most air, it loses more weight by reason of this displacement than the small weight. Suppose that the volume of the sphere exceeds that of the weight by 10 cubic inches; the weight of this volume of air is 3.1 grains. If this weight be added to the small weight, it will overbalance the sphere in air, but will exactly balance it in a vacuum.

FIG. 197.

AIR COMPRESSORS.

1071. For many purposes compressed air is preferable to steam or other gas for use as a motive power. In such cases **air compressors** are used to compress the air. These are made in many forms, but the most common one is to place a cylinder, called the *air cylinder*, in front of the cross-head of a steam engine, so that the piston of the air cylinder can be driven by attaching its piston rod to the cross-head, in a manner similar to a steam pump. A cross-section of the air cylinder of a compressor of this kind is shown in Fig. 198, in which *A* is the piston and *B* is the piston rod, driven by the cross-head of a steam engine not shown in the figure. Both ends of the lower half of the cylinder are fitted with inlet valves *D* and *D'*, which allow the air to enter the cylinder, and both ends of the upper half are fitted with discharge valves F and F', which allow the air to escape from the cylinder after it has been compressed to the required pressure.

Suppose the piston A to be moving in the direction of the arrow; then the inlet valves D in the left-hand end of the cylinder from which the piston is moving will be forced inwards by the pressure of the atmosphere, which over-

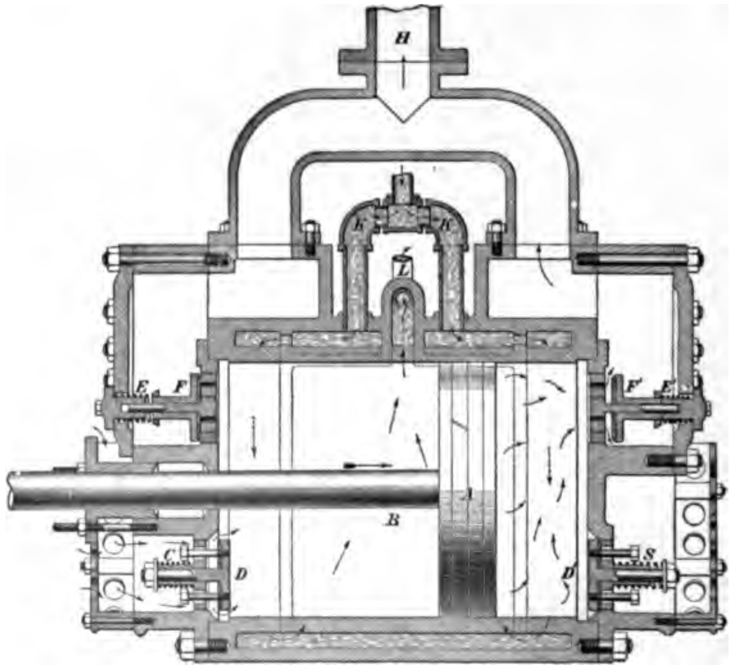


FIG. 198.

comes the resistance of the light spring C, thus allowing the air to flow in and fill the cylinder. On the other side of the piston, the air is being compressed, and, consequently, it acts with the springs S to force the inlet valves D' in the right-hand end of the cylinder to their seats. In the right-hand end of the cylinder, the discharge valves F' are opened when the pressure of the air in the cylinder is great enough to overcome the resistance of the light springs E'

and the tension of the air in the passages leading to the discharge pipe *H*, and the discharge valves *F* are pressed against their seats by the springs *E* and the tension of the air in the passages. Suppose it is desired to compress the air to 59 pounds per square inch, and we wish to find at what point of the stroke the discharge valves will open. Now, 59 pounds per square inch equals a pressure of 4 atmospheres, very nearly; hence, when the pressure in the cylinder becomes great enough to force air out through the discharge valves, the volume must be one-quarter of the volume at atmospheric pressure, or the valves will open when the piston has traveled three-quarters of its stroke, provided the air be compressed at constant temperature.

The air, after being discharged from the cylinder, passes out through the delivery pipe *H*, and from thence is conveyed to its destination. It was shown in the early part of this paper that when air or any other gas was compressed its temperature was increased. For high pressures this increase of temperature becomes a serious consideration, for two reasons: 1st. When the air is discharged at a high temperature, the pressure falls considerably when it has cooled down to its normal temperature, and this represents a serious loss in the economical working of the machine. 2d. The alternate heating and cooling of the compressor cylinder by the hot and cold air is very destructive to it, and increases the wear to a great extent. To prevent the air from heating, cooling devices are resorted to, the most common one being the so-called **water jacket.** This is effected in the following manner: The cylinder walls are hollow, as shown in the cut; the cold water enters this hollow space in the cylinder wall through the pipe *K' K'*, and flows around the cylinder, finally passing out through the discharge pipe *L*. The water tends to keep the cylinder walls cold, and these cool the air as it is compressed.

1072. The Cartesian Diver. — The instrument shown in Fig. 199, called the **cartesian diver,** illustrates the elasticity of air and the transference of pressure in all directions in water. It consists of a glass jar filled with water, having a rubber bulb at the top filled with air. The image in the jar is made of glass and is hollow, the weight being less than an equal volume of water, so that it will float at the top of the jar. The tail of the image has a hole in it, the water being prevented from getting inside of the image by the tension of the air within it. If the bulb be squeezed, the air in it will be forced out, creating a pressure upon the water which, being transferred in all directions, causes the water to flow into the tail of the image, compressing the air inside, and thus causing it to fall to the bottom of the jar. When the bulb is released, the air flows back into it; the pressure upon the water is removed, the air within the image expands; the image, again becoming lighter than water, rises to the top of the jar.

FIG. 199.

1073. Hero's Fountain.—**Hero's fountain** derives its name from its inventor, Hero, who lived at Alexandria 120 B. C. It is shown in Fig. 200. It depends for its operation upon the elastic properties of air. It consists of a brass dish *A*, and two glass globes *B* and *C*. The dish communicates with

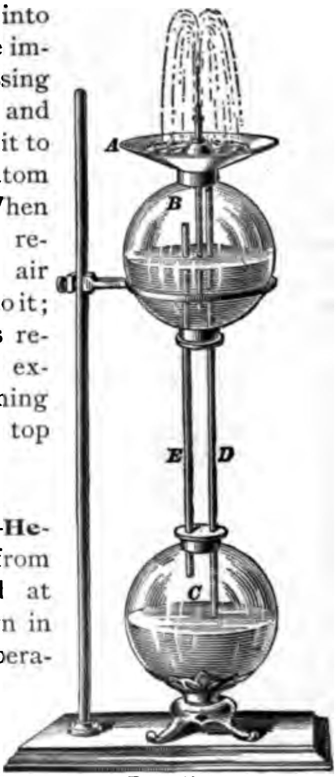


FIG. 200.

the lower part of the globe C by a long tube D, and another tube E connects the two globes. A third tube passes through the dish A to the lower part of the globe B. This last tube being taken out, the globe B is partially filled with water; the tube is then replaced and water is poured into the dish. The water flows through the tube D into the lower globe, and expels the air, which is forced into the upper globe. The air thus compressed acts upon the water and makes it jet out through the shortest tube, as represented in the figure. Were it not for the resistance of the atmosphere and friction, the water would rise to a height above the water in the dish equal to the difference of the level of the water in the two globes.

THE SIPHON.

1074. The action of the **siphon** illustrates the effect of atmospheric pressure. It is simply a bent tube of unequal branches, open at both ends, and is used to convey a liquid from a higher point to a lower, over an intermediate point higher than either. In Fig. 201, A and B are two vessels, B being lower than A, and $A\ C\ B$ is the bent tube or siphon. Suppose this tube to be filled with water and placed in the vessels, as shown, with the short branch $A\ C$ in the vessel A. The water will flow from the vessel A into B, so long as the level of the water in B is below the level of the water in A, and the level of the water in A is above the lower end of the tube $A\ C$. The atmospheric pressure upon the surfaces of A and B tends to force the water up the tubes $A\ C$ and $B\ C$. When the siphon is filled with water, each of these pressures is counteracted in part by

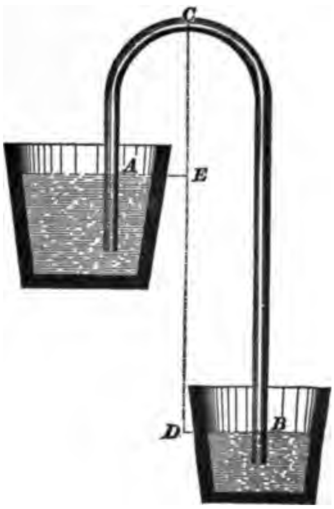


FIG. 201.

the pressure of the water in that branch of the siphon which is immersed in the water upon which the pressure is exerted. The atmospheric pressure opposed to the weight of the longer column of water will, therefore, be more resisted than that opposed to the weight of the shorter column; consequently, the pressure exerted upon the shorter column will be greater than that upon the longer column, and this excess pressure will produce motion.

Let A = the area of the tube in square inches.

$h = D\,C$ = the vertical distance in inches between the surface of the water in B and the highest point of the center line of the tube.

$h_1 = E\,C$ = the distance in inches between the surface of the water in A and the highest point of the center line of the tube.

The weight of the water in the short column is $.03617\,A\,h_1$, and the resultant atmospheric pressure, tending to force the water up the short column, is $14.7 \times A - .03617\,A\,h_1$. The weight of the water in the long column is $.03617\,A\,h$, and the resultant atmospheric pressure, tending to force the water up the long column, is $14.7\,A - .03617\,A\,h$. The difference between these two is $(14.7\,A - .03617\,A\,h_1) - (14.7\,A - .03617\,A\,h) = .03617\,A(h - h_1)$. But $h - h_1 = E\,D$ = the difference between the levels of the water in the two vessels. To find the discharge from a siphon, use the difference $h - h_1$, reduced to feet, as the head, and the total length of the siphon between the two water levels, as the length of the pipe; the discharge may then be calculated by formula **50**, Art. **1033**.

It will be noticed that the short column must not be higher than 34 feet for water, or the siphon will not work, since the pressure of the atmosphere will not support a column of water that is higher than 34 feet; 28 feet is considered to be the greatest height for which a siphon will work well.

1075. Intermittent Springs.—Sometimes a spring is observed to flow for a time and then cease; then, after an interval, to flow again for a time. The generally accepted explanation of this is that there is an underground reservoir fed with water through fissures in the earth, as shown in Fig. 202. The outlet for the water is shaped like a siphon, as shown. When the water in the reservoir reaches the same height as the highest point of outlet, it flows out until the

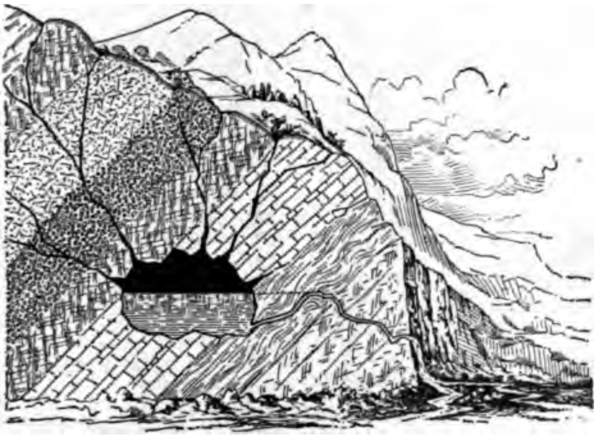


FIG. 202.

level of the water in the reservoir falls below the mouth of the siphon, the water flowing out of the reservoir faster than it is supplied to it. This flow then ceases until the water in the reservoir has again reached the level of the highest point of the siphon.

THE INJECTOR.

1076. A section of an injector is shown in Fig. 203. There are many different kinds of these instruments, but the principle is the same in all. When they are used for lifting water from a point below the discharge orifice and forcing it into the boiler of a steam engine or locomotive, they depend for their lifting action upon the creation of a partial vacuum by the action of steam. In the injector shown in Fig. 203, F is the connection for the steam pipe from the boiler, P is the connection for the pipe from the water supply, N is the connection to which the discharge pipe leading to the boiler is attached, and the waste water and steam are discharged through the **overflow** nozzle *O*.

The method of operation is as follows: The valve *B* is first opened by turning the wheel *W*; the **primer** valve *R* is then opened by the handle *J*, thus permitting steam to flow through the passage *E* and a connection, not shown in the figure, to the nozzle *u*. From *u* the jet of steam rushes out through *O*. A passage connects the chamber surrounding *u* with the space above the valve *L*. The jet of steam from *u* out through *O* carries with it the air in the chamber to

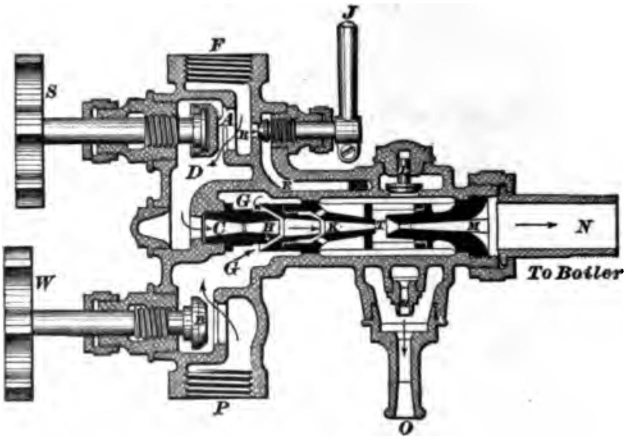


FIG. 203.

which *O* is connected, thus forming a partial vacuum in the space above *L*; the air in the passages *D*, *C*, *G*, *H*, *K*, *T*, and in the water pipe connected at *P* is thus drawn out through the valve *L*, and a partial vacuum is formed, which permits the pressure of the atmosphere to force water through *P* until it finally fills the passages and flows out through *L* and the overflow nozzle *O*. As soon as water appears at *O*, the valve *R* is closed and the main steam valve *A* is opened by the wheel *S*, thus admitting steam to the passages *C*, *H*, *K*. This steam draws water from *G* through the opening surrounding *H* and discharges it through *K* with such a high velocity that it rushes past the opening T into the nozzle M and thence into the boiler.

THE LOCOMOTIVE BLAST.

1077. Fig. 204 shows the front end of a locomotive. E is the exhaust pipe, the center of which is directly in line with the center of the smokestack S. T, T are the tubes through which the hot furnace gases are discharged. The

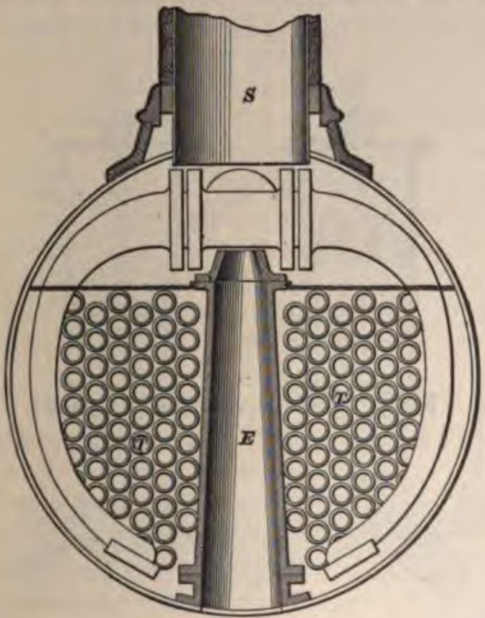


FIG. 204.

exhaust steam has a pressure of about two pounds above the atmosphere, and rushes through the exhaust pipe E and up the smokestack S with a very high velocity, taking the air out with it, and producing a partial vacuum in the space in front of the tubes. No air can get in this space except through the grates of the fire-box; consequently, this partial vacuum created in front of the tubes as described causes an influx of air through the grate, and produces the **forced draft, or blast.** The faster the engine runs, the greater the quantity of air drawn through the grate.

PUMPS.

1078. The Suction Pump.—A section of an ordinary suction pump is shown in Fig. 205. Suppose the piston to be at the bottom of the cylinder and to be just on the point of moving upwards in the direction of the arrow. As the piston rises it leaves a vacuum behind it, and the atmospheric pressure upon the surface of the water in the well causes it to rise in the pipe *P*, for the same reason that the mercury rises in the barometer tube. The water rushes up

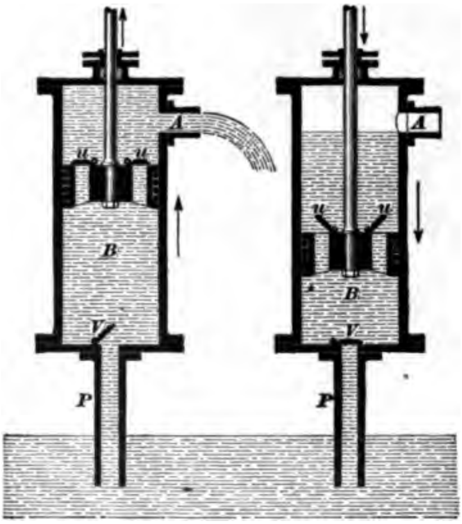


FIG. 205.

the pipe and lifts the valve *V*, filling the empty space in the cylinder *B* displaced by the piston. When the piston has reached the end of its stroke, the water entirely fills the space between the bottom of the piston and the bottom of the cylinder and also the pipe *P*. The instant that the piston begins its down stroke, the water in the chamber *B* tends to fall back into the well, and its weight forces the valve *V* to its seat, thus preventing any downward flow of the water. The piston now tends to compress the water in the chamber *B*, but this is prevented through the opening of the valves *u*, *u* in the piston. When the piston has reached the end of its downward stroke, the weight of the water above closes the valves *u*, *u*. All the water resting on the top of the piston is then lifted with the piston on its upward stroke, and discharged through the spout *A*, the valve *V* again opening, and the water filling the space below the piston as before.

It is evident that the distance between the valve *V* and the surface of the water in the well must not exceed 34 feet, the highest column of water which the pressure of the atmosphere will sustain, since otherwise the water in the pipe would not reach to the height of the valve *V*. In practice this distance should not exceed 28 feet. This is due to the fact that there is a little air left between the bottom of the piston and the bottom of the cylinder, a little air leaks through the valves which are not perfectly air-tight, and a pressure is needed to raise the valve against its weight, which, of course, acts downwards. There are many varieties of the suction pump, differing principally in the valves and piston, but the principle is the same in all.

1079. The Lifting Pump.—A section of a **lifting pump** is shown in Fig. 206. These pumps are used when water is to be raised to greater heights than can be done with the ordinary suction pump. As will be perceived, it is essentially the same as the pump previously described, except that the spout is fitted with a cock and has a pipe attached to it, leading to the point of discharge. If it is desired to discharge the water at the spout, the cock may be opened; otherwise, the cock is closed, and the water is lifted by the piston up through the pipe *P′* to the point of discharge, the valve *c* preventing it from falling back into the pump, and the valve *V*

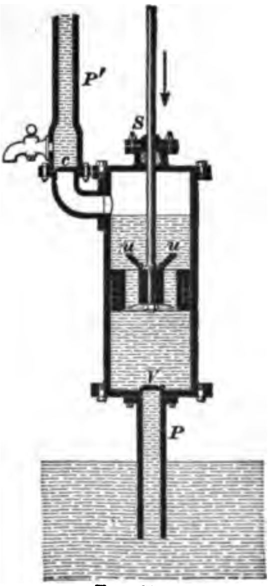


FIG. 206.

preventing the water in the pump from falling back into the well. It is not necessary that there should be a second pipe P', as shown in the figure, for the pipe P may be continued straight upwards, as shown in Fig. 207.

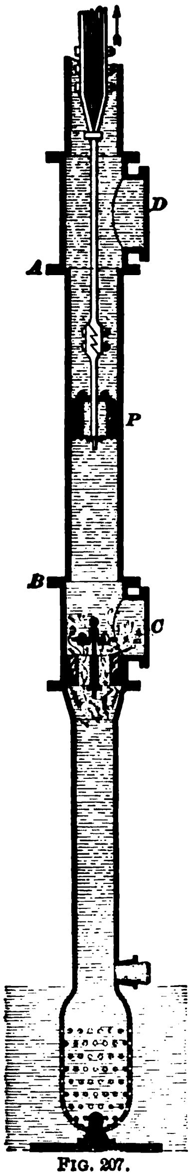


FIG. 207.

1080. In the figure is shown a section of a lifting pump for raising water from great depths, as from the bottom of mines to the surface. This pump consists of a series of pipes connected together, of which the lower end only is shown in the cut. That part of the pipe included between the letters A and B forms the pump cylinder in which the piston P works. That part of the pipe above the highest point of the piston travel, through which the water is discharged, is called the **delivery pipe,** and the part below the lowest point of the piston travel is called the **suction pipe.** The lower end of the suction pipe is expanded, and has a number of small holes in it, to keep out the solid matter. C is a plate covering an opening, and which may be removed to allow the suction valve to be repaired. D is a plate covering a similar opening through which the piston and piston valves may be repaired. The piston rod, or rather the piston stem, is made of wrought iron, inserted with wood, and connected with the piston. The only limit to the height to which a pump of this kind can raise water is the strength of the piston rod. Lifting pumps of this kind are used to raise water from great depths to the earth's surface; hence, a very long piston rod is necessary. In the lifting pump shown in Fig. 206 the water is raised from a point a few feet below the earth's surface to a point considerably higher. This requires the piston rod to move through a stuffing-box, as shown at *S*, and also necessitates the rod being round, in order that the water may not leak out.

1081. Force Pumps.—The **force pump** differs from the lifting pump in several important particulars, but chiefly in the fact that the piston is solid; that is, it has no valves. A section of a *suction and force pump* is shown in Fig. 208. The water is drawn up the suction pipe as before, when the piston rises; but when the piston reverses, the pressure on the water caused by the descent of the piston

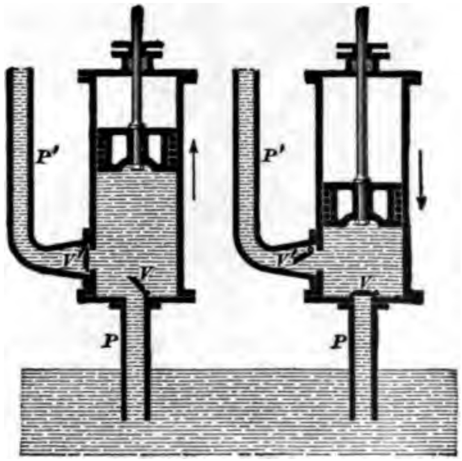


FIG. 208.

opens the valve *V'* and *forces* the water up the delivery pipe *P'*. When the piston again begins its upward movement, the valve *V'* is closed by the pressure of the water above it, and the valve *V* is opened by the pressure of the atmosphere on the water below it, as in the previous cases. For an arrangement of this kind, it is not necessary to have a stuffing-box. The water may be forced to almost any desired height. The force pump differs again from the lifting pump in respect to its piston rod, which should not be longer than is absolutely necessary in order to prevent it from *buckling*, while in the lifting pump the length of the piston rod is a matter of indifference.

1082. Plunger Pumps.—When force pumps are used to convey water to great heights, the pressure of the water in the cylinder becomes so great that it becomes extremely difficult to keep the water from leaking past the piston, and the constant repairing of the piston packing becomes a nuisance. To obviate this difficulty the piston is made very long, as shown in Fig. 209, and is then called a **plunger.** The suction valve in this case consists of two clack valves inclined to each other and resting upon a square pin *A*; they are prevented from flying back too far during the up stroke of the plunger by the two uprights *I*, *I*. During the down stroke of the plunger the valves at *A* are closed and the delivery valve at *B* is open. A little air is always carried into the cylinder of a pump with the entering of the water. In force pumps this fact becomes a serious consideration, since, after repeated strokes, the air accumulates, and during the down stroke of the plunger it is compressed. After a time it would become sufficiently compressed to entirely prevent the water from entering through the suction valve, the pressure on the top of the valve being greater than that of the atmosphere below. In the pump shown in the figure, the plunger is a trifle smaller than the cylinder, and the air collects around the plunger below the stuffing-box. To remove this air a narrow passage *C*, shown by the dotted lines, that can be closed at its upper end by the cock *D*, connects the interior of the pump with the atmosphere when the cock is open. It is evident that this

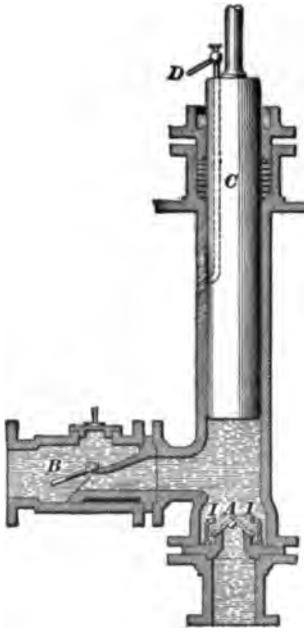


FIG. 209.

cock must not be opened, except during the down stroke of the plunger; for, if it were open during the up stroke, the pressure below the plunger being less than the pressure of the atmosphere above, the air would rush in instead of being expelled.

1083. Double-Acting Pumps.—In the pumps previously described, the discharge was intermittent; that is, the pump could only discharge when the piston was moving in one direction. In some cases it is necessary that there should be a continuous discharge; in all cases it takes more

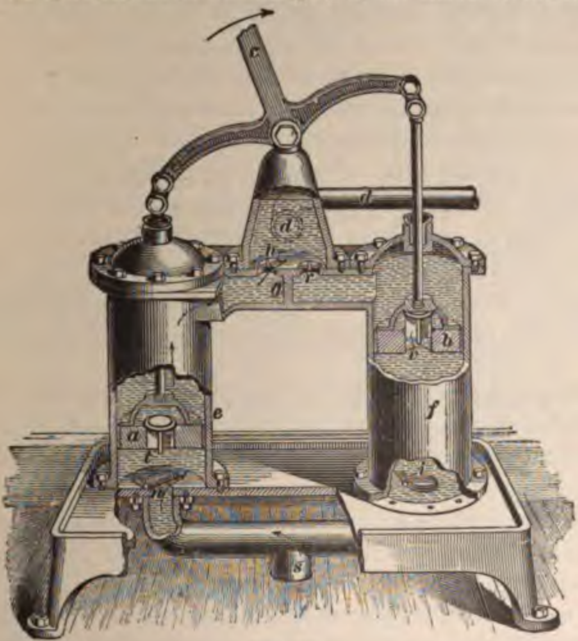


FIG. 210.

power to run the pump with an intermittent discharge, as a little consideration will show. If the height that the water is to be raised is considerable, its weight will be very great, and the entire mass must be put in motion during one stroke of the piston.

In order to obtain the advantage of a more continuous discharge, double-acting pumps are used. Fig. 210 shows a part sectional view of such a pump. Two pistons *a* and *b* are used, which are operated by one handle *c* in the manner shown. The pump has one suction pipe *s* and one discharge pipe *d*. The cylinders *e* and *f* are separated by a diaphragm *g*, so that they cannot communicate with each other above the pistons. In the figure, the handle *c* is moving to the right, the piston *a* upwards, and the piston *b* downwards. As the piston *a* moves upwards, it lifts the water above it and causes it to flow through the delivery valve *h* into the discharge pipe *d*. This upward movement of the piston creates a partial vacuum below it in the cylinder *e*, and causes the water to rush up the suction pipe *s* into the cylinder, as shown by the arrows. In the cylinder *f*, the downward movement of the piston *b* raises the piston valve *v*, and the weight of the water on the suction valve *i* keeps it closed. When the handle *c* has completed its movement to the right and begins its return, all of the valves on the right-hand side open except *v*, and those on the left-hand side close except *t*; water is then discharged into the delivery pipe by the cylinder *f*, and only at the instant of reversal is the flow into the delivery pipe *d* stopped.

1084. Air Chambers.—In order to obtain a continuous flow of water in the delivery pipe, with as nearly a uniform velocity as possible, an **air chamber** is usually placed on the delivery pipe of force pumps as near to the pump cylinder as the construction of the machine will allow. The air chambers are usually pear-shaped, with the small end connected to the pipe. They are filled with air which the water compresses during the discharge. During the suction, the air thus compressed expands and acts as an accelerating force upon the moving column of water, a force which diminishes with the expansion of the air, and helps to keep the velocity of the moving column more nearly uniform. An air chamber is sometimes placed upon the suction pipe. These air chambers not only tend to promote a uniform discharge, but they also equalize the stresses upon the pump, and prevent shocks due to the incompressibility of water.

They serve the same purpose in pumps that a fly-wheel does to the steam engine. Unless the pump moves very slowly, it is absolutely necessary to have an air chamber on the delivery pipe.

1085. Steam Pumps.—Steam pumps are force pumps operated by steam acting upon the piston of a steam engine, directly connected to the pump, and in many cases cast with the pump. A section of a double-acting steam pump showing the steam and water cylinders, with other details, is illustrated in Fig. 211. Here *G* is a steam piston,

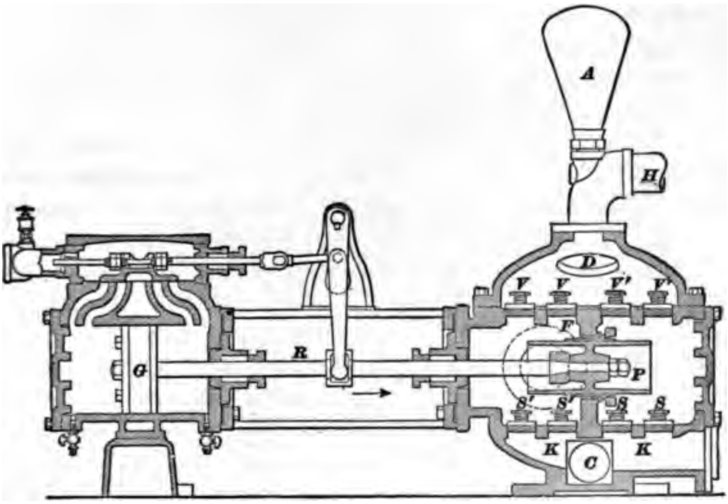


FIG. 211.

and *R* the piston rod, which is secured at its other end to the plunger *P*. *F* is a partition cast with the cylinder, which prevents the water in the left-hand half from communicating with that in the right-hand half of the cylinder. Suppose the piston to be moving in the direction of the arrow. The volume of the left-hand half of the pump cylinder will be increased by an amount equal to the area of the circumference of the plunger, multiplied by the length of the stroke, and the volume of the right-hand half of the cylinder will be diminished by a like amount. In consequence of this, a volume of water in the right-hand half of the cylinder equal to the volume displaced by the plunger in its forward movement will be forced through the valves V', V' into the air chamber A, through the orifice D, and then discharged through the delivery pipe H. By reason of the partial vacuum in the left-hand half of the pump cylinder, owing to this movement of the plunger, the water will be drawn from the reservoir through the suction pipe C into the chamber K, K, lifting the valves S', S', and filling the space displaced by the plunger. During the return stroke the water will be drawn through the valves S, S into the right-hand half of the pump cylinder, and discharged through the valves V, V in the left-hand half. Each one of the four suction and four discharge valves is kept to its seat, when not working, by light springs, as shown.

There are many varieties and makes of steam pumps, the majority of which are double-acting. In many cases two steam pumps are placed side by side, having a common delivery pipe. This arrangement is called a **duplex pump.** It is usual to so set the steam pistons of duplex pumps that when one is completing the stroke the other is in the middle of its stroke. A double-acting duplex pump made to run in this manner, and having an air chamber of sufficient size, will deliver water with nearly a uniform velocity.

In mine pumps for forcing water to great heights, the plungers are made solid, and in most cases extended through the pump cylinder. In many steam pumps pistons are used instead of plungers, but when very heavy duty is required plungers are preferred.

1086. Centrifugal Pumps.—Next to the direct-acting steam pump, the **centrifugal pump** is the most valuable instrument for raising water to great heights that has yet been described. As the name denotes, the effects produced by centrifugal force are made use of. Fig. 212 represents one with half of the casing removed. The hub S is hollow, and is connected directly to the suction pipe. The curved arms a, called **vanes** or **wings,** are revolved with a high velocity in the direction of the arrow, and the air enclosed between them is driven out through the discharge passage and delivery pipe *D D*. This creates a partial vacuum in the casing and suction pipe, and causes the water to flow in through *S*. This water is also made to revolve with the vanes, and, of course, with the same velocity. The centrifugal force of the revolving water causes it to fly outwards towards the end of the vanes, and becomes greater the farther away it gets from the center. This causes it to leave the vanes, and finally to leave the pump by means of the discharge passage and delivery pipe *D D*. The height to which the water can be forced depends upon the velocity of the revolving vanes. In the construction of the centrifugal pump, particular care is required in giving the correct form to the vanes; the efficiency of the machine depends greatly upon this point being attended to. What is required is to raise the water, and the energy used to drive the pump should be devoted as far as possible to this one purpose. The water when it is raised should be delivered with as little velocity as possible, for any velocity which the water then possesses, has been produced at the expense of the energy used to drive the pump. The form of the vanes is such that the water is delivered at the desired height with the least expenditure of energy.

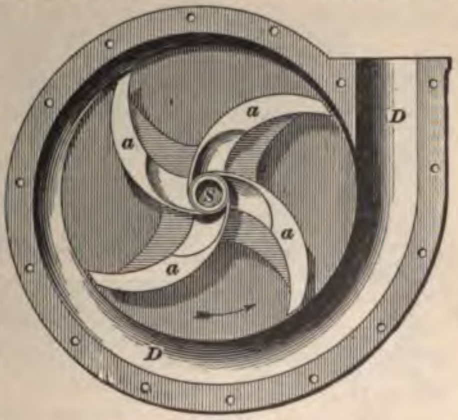


FIG. 212.

The number of vanes depends upon the size and capacity of the pump. It will be noticed that, in the pump shown in the figure, the vanes have sharp edges near the hub. The object of this is to provide for a free ingress of the water, and also to cut any foreign substance that may enter the pump and prevent it from working properly.

Almost any liquid can be raised with these pumps, but when used for pumping chemicals, the casing and vanes are made of materials that the chemicals will not affect.

1087. The Hydraulic Ram.—The construction of a hydraulic ram is shown in Fig. 213. This machine is used for raising water from a point below the level of the water in a spring or reservoir to a point considerably higher, with no power other than that afforded by the inertia of a moving column of water. In the figure, *a* is a pipe called the *drive pipe*, connecting the ram with the reservoir; the valve *b* slides freely in a guide, and is provided with lock-nuts to regulate the distance that the valve can fall below its seat. When

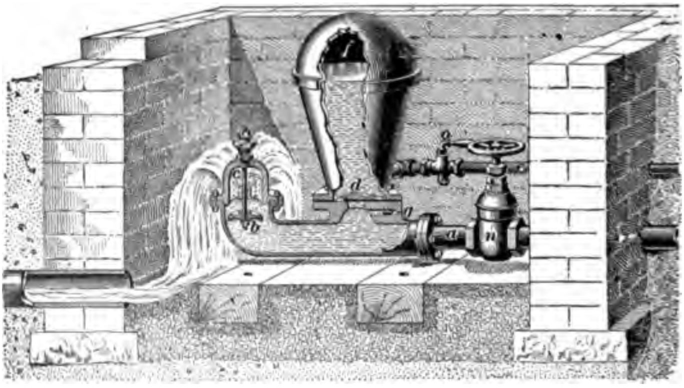


FIG. 213.

the water is first turned on by opening the valve *n*, the valve *b* is already opened, and the water flows out through *c*, as shown. As the discharge continues, the velocity of the water in the drive pipe will increase until the upward pressure against the valve *b* is sufficient to force the valve to its seat. The actual closing of the valve takes place very suddenly, and the momentum of the column of water, which was moving with an increasing velocity through the drive pipe *a*, will very rapidly force some water through the valve *d* into the air-chamber *f*. Immediately after this, a rebound takes place, and for a short interval of time the water flows back up the drive pipe *a* and tends to form a vacuum under the air chamber valve *d*; this opens the snifter valve *g* and admits a little air, which accumulates under the valve *d* and is forced into the air chamber with the next shock. This air keeps the air chamber constantly charged; otherwise, the water, being under a greater pressure in the air chamber than it is in the reservoir, would soon absorb the air in the chamber and the ram would cease to work until the chamber was recharged with air. The rebound also takes the pressure off the under side of valve *b* and causes it to drop, and the above-described operations are repeated. The delivery pipe is shown at *e*; a steady flow of water is maintained through it by the pressure of the air in the chamber *f*; this air also acts as a cushion when valve *b* suddenly closes, and prevents undue shock to the parts of the ram.

The height to which water can be raised by the hydraulic ram depends upon the weight of the valve *b* and the velocity of the water in *a*.

1088. Power Necessary to Work a Pump:

Rule I.—*In all pumps, whether lifting, force, steam, single- or double-acting, or centrifugal, the number of foot-pounds of power needed to work the pump is equal to the weight of the water in pounds, multiplied by the vertical distance in feet between the level of the water in the well, or source, and the point of discharge, plus the work necessary to overcome the friction and other resistances.*

Rule II.—*The work done in one stroke of a pump is equal to the weight of a volume of water equal to the volume displaced by the piston during the stroke, multiplied by the total vertical distance in feet through which the water is to be raised, plus the work necessary to overcome the resistances.*

A little consideration will make Rule II evident. Suppose that the height of the suction is 25 feet; that the vertical distance between the suction valve and the point of discharge is 100 feet; that the stroke of the piston is 15 inches, and that its diameter is 10 inches. Let the diameters of the suction pipe and delivery pipe be 4 inches each. The volume displaced by the pump piston or plunger in one stroke equals $\frac{10^2 \times .7854 \times 15}{1,728} = .68177$ cubic foot. The weight of an equal volume of water = .68177 × 62.5 = 42.61063 pounds. Now, in order to discharge this water, *all* of the water in the suction and delivery pipes had to be moved through a certain distance in feet equal to .68177 divided by the area of the pipes in square feet.

Four inches = $\frac{1}{3}$ of a foot. $(\frac{1}{3})^2 \times .7854 = \frac{.7854}{9} = .0872\frac{2}{3}$ square foot. $.68177 \div .0872\frac{2}{3} = 7.8125$ feet.

The weight of the water in the delivery pipe is $(\frac{1}{3})^2 \times .7854 \times 100 \times 62.5 = 545.42$ pounds.

The weight of the water in the suction pipe is $(\frac{1}{3})^2 \times .7854 \times 25 \times 62.5 = 136.35$ pounds.

545.42 + 136.35 = 681.77 pounds = the total weight of water moved in one stroke. The distance that it is moved in one stroke is 7.8125 feet. Hence, the number of foot-pounds necessary for one stroke is 681.77 × 7.8125 = 5,326.33 foot-pounds. Had this result been obtained by Rule II, the process would have been as follows: The weight of the water displaced by the piston in one stroke was found to be 42.61063 pounds. 42.61 × 125 = 5,326.33 pounds, which is exactly the same as the result obtained by the previous method, and is a great deal shorter.

EXAMPLE.—What must be the necessary horsepower of a double-acting steam pump if the vertical distance between the point of discharge and the point of suction is 96 feet? The diameter of the pump cylinder is 8 inches, the stroke is 10 inches, and the number of strokes per minute is 120. Allow 25% for friction, etc.

SOLUTION.—Since the pump is double-acting, it raises a quantity of water equal to the volume displaced by the plunger at every stroke. The weight of the volume of water displaced in one stroke $= (\frac{8}{12})^2 \times .7854 \times \frac{10}{12} \times 62.5 = 18.18$ pounds, nearly.

18.18 × 96 × 120 = 209,433.6 foot-pounds per minute.

Since 25% is to be allowed for friction, the actual number of foot-pounds per minute = 209,433.6 ÷ .75 = 279,244.8 foot-pounds per minute.

One horsepower = 33,000 foot-pounds per minute; hence, $\frac{279,244.8}{33,000} = 8.462$ H. P., nearly. Ans.

ELEMENTARY GRAPHICAL STATICS.

PROPERTIES OF FORCE.

1089. Before beginning the subject of Graphical Statics, it will be well to restate some of the fundamental principles of mechanics, and explain them somewhat more at length than has been done in former articles.

Force, so far as we know, always relates to bodies, and may be defined as the action of a body upon another, causing, or tending to cause, motion in the latter.

1090. If two forces act upon a body, tending to produce the same amount of motion, but in opposite directions, it is evident that there can be motion in neither direction. The forces are then said to be **balanced** or **in equilibrium.**

This condition is not limited to two forces; three or more forces may so act upon a body as to be balanced among themselves, as when a body hangs by two or more ropes; here the weight of the body, which is one of the forces, balances the pulls of the ropes, which are the other forces.

1091. When, on the contrary, a force acting upon a body is not resisted by an equal and opposite force, motion takes place, and the force is said to be **unbalanced.**

1092. The general laws of force and its effects are the subject of the science of **mechanics.**

Statics is that branch of mechanics which treats of balanced forces—that is, of the conditions of equilibrium.

1093. In order to determine the **effect** of a force, it is necessary to consider (1) its *point of application;* (2) its *direction*, and (3) its *magnitude.*

1094. The **point of application** of a force is the point at which the force is applied and upon which its effect is exerted. The point of application of a force is usually known.

1095. The **direction** of a force is the direction in which it tends to move its point of application; the line along which it tends to move the point of application is called the **line of action** of the force.

The motion produced by an unbalanced force will always be in a *straight line*, and the line of action of a force may always be represented by a straight line. For convenience, *any point in the line of action* of a force may be taken as its *point of application*.

If several forces have a common point of application, or their lines of action meet at a common point, they are called **concurring forces.** If their lines of action do not meet at a common point, they are called **non-concurring forces.**

It is evident that the *line of action* of a force does not fully define or fix its *direction*, for the force might act towards either end of the line; the direction in which a force acts along its line of action is called the **sense** of the force.

The direction of a *force* must not be confused with its *line of action*. The line of action *must pass through the point of application*; while the direction may be represented by *any line parallel to the line of action*.

The *direction* of a force includes its *line of action* and its *sense*.

The lines of action of all forces considered here will be understood to lie in the *same plane*.

1096. The **magnitude** of a force is measured by comparison with some *known force* or with some assumed *unit of force*. Unless otherwise specially stated, the unit of magnitude herein assumed for forces will always be *one pound*.

1097. Resultants and Components.—The **resultant** of any number of forces is that single force which will produce the same effect as that produced by the combined action of those forces, and, on the contrary, if several forces acting together can produce the same effect as that produced by a single force, they are called the **components** of that force.

The process (by whatever method) of finding the resultant of several forces is called **the composition of forces.**

The process of resolving a force into its components is called **the resolution of forces.**

In any investigation in statics, *the components may be replaced by the resultant*, or *the resultant by the components.*

1098. *Problems in statics are solved by assuming the condition of equilibrium.*

If several forces are in equilibrium and the magnitudes and directions of all but a certain number of them are known, the magnitudes and directions of the unknown forces can, in some cases, be ascertained by determining the relations between the known forces and those which are necessary to balance them.

The operation of finding the values of the unknown forces in a combination of forces which are in equilibrium is similar to the solution of an algebraic equation. In the latter case, the condition of equality is assumed between the members of the equation; in the former case, the condition of equilibrium is assumed between the forces; in either case, the values of the unknown quantities are ascertained by means of their relations to the known quantities.

THE GRAPHICAL REPRESENTATION OF FORCES.

1099. In the geometrical solution of mechanical problems every force is represented by a straight line that either coincides with, or is parallel to, the line of action of the force. The magnitude of the force is represented by the length of the line, and its sense by an arrow-head pointing in the direction in which the force tends to move its point of application.

In Fig. 214, the line *A B* represents a force of 300 pounds to a scale of 200 pounds to the inch; that is, every inch of length represents 200 pounds of force. The length of *A B* is, therefore, $\frac{300}{200} = 1\frac{1}{2}$ inches. The arrow-head indicates that the force acts from *A* towards *B*.

FIG. 214.

A decimally divided scale, i. e., a scale of 10, 20, 30, 40, or 50 divisions to the inch, is the most convenient to use in representing forces. As the larger the scale used, the more accurate are the results, it is always best to use as large a scale as may be expedient.

If the position of the line represents the line of action of the force, and the point of application is at *A*, then the force tends to *pull* the point of application *A* towards *B*; but, if *B* is the point of application, then the force tends to *push* it away from *A*. If the force be applied at any point along its line of action (between *A* and *B*, or in line beyond *A* or beyond *B*), its effect will be the same; namely, a tendency to move the point of application along the same straight line, *A B*, or *A B* produced, in the direction indicated by the arrow-head. (See Art. **1095.**) Usually, the point of application of a force is considered to be at one end of the line representing the force.

Hereafter, *the arrow-head, indicating the sense of a force, will be placed between that end which represents the point of application, and the center of the line.* This practice will here be conventionally followed; thus, in Fig. 214, the point of application of the force is understood to be at *A*.

NOTE.—When the direction of a force is given by its angle, it will be understood according to the note in Art. **879.**

1100. It is thus seen that forces may be correctly represented by lines; this is called the **graphical method** of representing forces. The method is made very valuable by the additional fact that *the relations necessary to the condition of equilibrium between the forces so represented, can be determined by applying the principles of geometry to the lines so used.*

A combination of forces may thus be analyzed; such geometrical or graphical analysis of balanced forces is called **graphical statics.**

Graphical Statics may, therefore, be defined as a method of solving statical problems by means of diagrams in which the lengths and directions of the lines represent, respectively, the magnitudes and directions of the forces treated.

CONCURRING FORCES.

1101. The Force Polygon.

Referring to Fig. 117, Art. **881**, it will be remembered that *g a* represents the resultant of *g f*, *f e*, *e d*, etc., which means that the forces *g f*, *f e*, etc., may be replaced by the single force *g a*. Now, if we apply a force equal to *g a*, but in opposite direction, the two will balance each other; or, what is the same thing, the new force will balance the combination of forces *g f*, *f e*, etc. This is indicated by simply changing the arrow-head on line *a g*. Hence the following principles:

(*a*) *If any number of forces are in equilibrium, lines representing the forces in magnitude and direction, drawn end to end consecutively, must form a closed polygon.*

Conversely: (*b*) *If any number of concurring forces can be represented by the sides of a closed polygon, in such a manner that the directions of the arrow-heads will follow each other around the polygon, the forces are in equilibrium.*

The special condition of equilibrium in drawing the force polygon is that it must *close*. The arrow-heads on the lines forming the sides of the polygon must be so placed that a pencil passed successively over each line in the direction indicated by the arrow-head will move continuously around the polygon to the place of beginning; *the arrow-heads on any two adjacent sides must not point towards the point of intersection of the two sides.*

If the lines of action of all the forces are parallel, the closed polygon will be a straight line; in this case, the pencil would travel from one end of the line to the opposite end and back again to the starting point.

1102. Lines representing forces in equilibrium, if drawn end to end, will always form a closed polygon, whether the forces do or do not meet at a common point; but forces which are represented by the sides of a closed polygon are not necessarily in equilibrium unless they are *concurring* forces. If the forces so represented are non-concurring forces, their lines of action may be so arranged that their resultants will form what is known as a *couple*.

ILLUSTRATION.—In Fig. 215, the lines $A\ O = 30$ lb., $O\ B = 80$ lb., $C\ O = 20$ lb., $D\ O = 60$ lb., $O\ E = 75$ lb., and

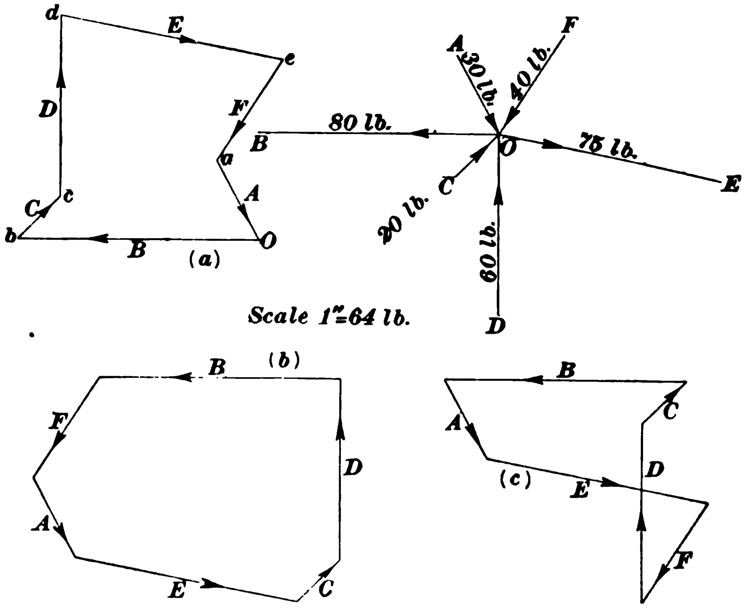


FIG. 215.

$F\ O = 40$ lb., represent forces which meet at the common point O. The lengths of the lines represent the magnitudes of the forces to a scale of 64 lb. = 1 in.; the directions and positions of the lines represent the lines of action of the forces; one end of each line is at the point of application of the force which it represents, while an arrow-head marked upon it indicates the sense of the force. All the forces are exerted in directions towards the common point of application O, except those represented by the lines $O\ B$ and $O\ E$, which act in directions away from O.

In Fig. 215 (a), the line $a\ O = 30$ lb. $= \frac{30}{64} = \frac{15}{32}$ in., is drawn equal and parallel to $A\ O$; as indicated by the arrowhead, it was drawn from a towards O, the movement being parallel to the *sense* of the force $A\ O$; in other words, the line $a\ O$ is drawn parallel, not simply to the line $A\ O$, but to the direction (i. e., line of action and sense) of the force represented by it; likewise, from O, the line $O\ b = 80$ lb. $= \frac{80}{64} = 1\frac{1}{4}$ in., is drawn equal and parallel to $O\ B$; from b the line $b\ c = 20$ lb. $= \frac{20}{64} = \frac{5}{16}$ in., is drawn equal and parallel to $C\ O$; from c the line $c\ d = 60$ lb. $= \frac{60}{64} = \frac{15}{16}$ in., is drawn equal and parallel to $D\ O$; from d the line $d\ e = 75$ lb. $= \frac{75}{64} = 1\frac{11}{64}$ in., is drawn equal and parallel to $O\ E$ and from e, the line $e\ a = 40$ lb. $= \frac{40}{64} = \frac{5}{8}$ in., is drawn equal and parallel to $F\ O$. The terminus of the last line falls just at the starting point a, forming a closed figure; therefore, these forces, which meet at a common point, must be in equilibrium. A pencil passed along each line successively in the direction indicated by the arrow-heads will return to the starting point.

In Fig. 215, (a) is a **force polygon** for the forces represented in Fig. 215. In drawing this force polygon, the forces were taken in order passing around the point O, in a direction opposite to the movement of the hands of a clock; practically the same results, i. e., the same relations between the lines, would have been obtained had the forces been taken in any order. In (b) and (c) the forces were not taken in regular order. *For the purposes of graphical statics, it makes no difference in what order the lines are drawn;* the relative lengths of the lines necessary to form a closed polygon are the same in any case. It is usually the most convenient, however, to take the forces in order around the point of application. It is not necessary to use the same scale to represent the forces in the force polygon, as (a) that was used to represent them in the original figure, as Fig. 215.

1103. In the force polygon, place the arrow-head in each case a little to one side of the center of the line and nearer to the end of the line corresponding to the point of application. (See Art. **1099.**) If the sense of the force is *towards* the point of application, the arrow-head should point towards the *shorter* portion of the line, but if the sense of the force is *away* from the point of application, the arrow-head should point towards the *longer* portion of the line. The advantage of this practice will be shown further along.

1104. *The point of application of the forces can not be represented in the force polygon by a single point.* In Fig. 215 each line not only represents the magnitude and direction of a force, but it also definitely locates its *line of action.* The point of application of each force must be some point in its line of action; in this case it is the common point O. But in the force polygon (a), (b), or (c), each line is simply drawn *parallel* to the direction of the corresponding force; it does not in any manner indicate either its line of action or its point of application; it represents the force in magnitude and direction only.

1105. The Triangle of Forces and the Equilibrant.

In Fig. 216, the lines $A\ O$ and $O\ B$ represent two forces whose lines of action meet at an angle of 30 degrees at the common point O. The force $A\ O =$ 35 pounds and $O\ B = 45$ pounds. The scale used is 32 pounds to the inch. As the lines $A\ O$ and $O\ B$ can not be so drawn as to form a closed figure, the forces can not be in equilibrium. (Arts. **1101** and **1102.**) But it is evident that a line drawn from B to A, as indicated by the dotted line $B\ A$, would fulfil this condition; for a pencil could then start at A, and, passing along each line in the

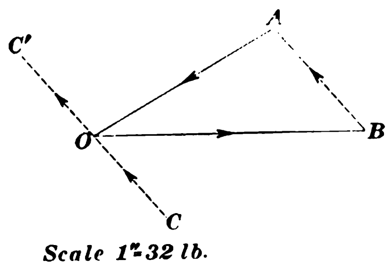


FIG. 216.

direction indicated by the arrow-heads, travel entirely around the figure and return to the starting point. The force $B A$ (meaning the force represented in magnitude and direction by the line $B A$) is, therefore, in equilibrium with the forces $A O$ and $O B$; measured to the scale used it equals 23 pounds. The position, with reference to the point of application, of the force represented by $B A$ may be represented by either $C O$ or $O C'$, each equal to $B A$.

As explained in Art. **1101**, a force equal to $B A$, but acting in the opposite direction (with the arrow-head reversed), would be the resultant of the two forces $A O$ and $O B$. In general, *if the direction of the arrow-head on any line of an equilibrium polygon be reversed, the force represented by that line will be the resultant of all the forces represented by the other lines of the polygon.*

1106. The complete triangle $A O B A$, Fig. 216, is called a **triangle of forces**; $B A$ is called the **closing line.**

A force which is in equilibrium with any combination of forces is called the **equilibrant** of the original forces; it must be equal in magnitude, but opposite in direction to their resultant, and is sometimes called the **anti-resultant.**

The equilibrant is the force which, when added to any combination of forces not in equilibrium, will produce equilibrium between all the forces.

In the usual language, a force or system of forces is said to be *equilibrated*, or *balanced*, by the equilibrant.

1107. From the foregoing may be deduced the following principles:

(**a**) *If two concurring forces are respectively represented in magnitude and direction by lines drawn end to end, a third line drawn from the unconnected end or terminus of the second to the beginning of the first line, completing the triangle, will represent in magnitude and direction the equilibrant of the original forces.*

(**b**) *If the third line is drawn in the opposite direction, from the beginning of the first line to the terminus of the second, it will represent the resultant of the two original forces.*

(**c**) *If three forces are in equilibrium, each force will be the equilibrant of the other two; each force will be equal in magnitude, but opposite in direction, to the resultant of the other two.*

(**d**) *The line representing the equilibrant of two forces may be changed to represent their resultant, or the line representing their resultant may be changed to represent their equilibrant by simply reversing the direction, of the arrowhead.*

EXAMPLE.—Two forces, $A\ O = 100$ pounds, and $O\ B = 122$ pounds (Fig. 216), act upon a body at the common point O, their lines of action meeting at an angle of 35 degrees; the sense of $A\ O$ is towards O, and the sense of $O\ B$ is away from O. What is the magnitude of their equilibrant?

SOLUTION.—Draw two lines of indefinite length making an angle of 35 degrees with each other. With any convenient scale, say 16 pounds to the inch, lay off $A\ O$ upon one line equal to 100 pounds $= \frac{100}{16} = 6\frac{1}{4}$ inches, and upon the other line lay off $O\ B$ equal to 122 pounds $= \frac{122}{16} = 7\frac{5}{8}$ inches. The arrows marked upon the lines, indicating the sense of each force with reference to the point O, must *follow* each other; one arrow must point towards, and one arrow must point away from, the intersection of the lines. The line connecting the extremities of the two lines, i. e., the closing line of the triangle, will to the same scale represent the magnitude of the equilibrant. The closing line $B\ A$ is found to equal $4\frac{3}{8}$ inches $= 4\frac{3}{8} \times 16 = 70$ pounds. The line of action of the equilibrant must be parallel to the closing line; an arrow marked upon the closing line following the direction of the other two arrows around the triangle will indicate the sense of the equilibrant.

1108. The Equilibrant and Resultant of any Number of Forces.

The statement that the line necessary to close the figure represents the equilibrant, while a line in the same position, but drawn in the opposite direction, represents the resultant of the forces represented by the original lines, is also true of lines representing any number of forces.

ILLUSTRATION.—In Fig. 217, let each side of the triangle *A O B* represent the same force as that represented by the corresponding lines in Fig. 216.

In Art. **1107**, these forces were shown to be in equilibrium. For instance, it was found that a line drawn from *B* to *A* represented a force in equilibrium with the forces *A O* and *O B*, and that a line in the same position, but drawn from *A* to *B*, represented the resultant of those forces. But the lines *A O* and *O B* may each represent the resultant of any number of forces. For the lines *A c* and *c O*, for instance, may represent two forces, of which the force *A O* is the resultant. But *A c* may represent the resultant of the forces *A b* and *b c*, while *c O* may represent the resultant of the forces *c d* and *d O*. Again, *A b*, *b c*, *c d*, and *d O* may each represent the resultant of other forces, each of which may be in turn the resultant of still other forces; and as this process may be extended indefinitely, it is plain that *A O* may represent the resultant of any number of forces. The same is true of *O B*; and, if instead of two original forces, as represented by the lines *A O* and *O B*, there were eight original forces, as represented by the lines *A b*, *b c*, *c d*, *d O*, *O e*, *e f*, *f g*, and *g B*, or any number of forces represented by lines starting at *A* and terminating at *B*, the closing line *B A* would still represent a force in equilibrium with, i. e., the equilibrant of, the entire system of original forces, and a line from *A* to *B* would represent their resultant.

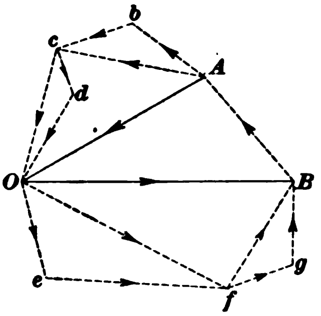


FIG. 217.

Hence the following principles:

(**a**) *If any number of concurring forces not in equilibrium are respectively represented in magnitude and direction by lines drawn successively end to end, the equilibrant of the original forces may be represented by a line drawn from the terminus to the starting point of the original lines, closing the polygon.*

(**b**) *A line drawn directly from the starting point to the terminus of the original lines will represent the resultant of the original forces.*

(**c**) *If any number of concurring forces are in equilibrium, each force will be the equilibrant of all the others; each force will be equal in magnitude, but opposite in direction, to the resultant of all the others.*

These principles could have been deduced directly from the principles stated in Art. **1101,** (*a*) and (*b*). For, if lines representing a combination of forces which are in equilibrium form a closed polygon, then, when the forces are not in equilibrium so that the lines representing them will not close, it is evident that the closing line, drawn from the terminus to the starting point of the original lines, will fulfil the conditions necessary for equilibrium and will represent a force in equilibrium with the original forces.

If the original forces are already in equilibrium, the force polygon will close, showing that they can have neither equilibrant nor resultant.

1109. Values of Forces Which May Be Determined by the Force Polygon.

The force polygon may be called a **geometrical equation,** for it expresses the relations between the values of the quantities (forces) involved that are necessary to a condition of equilibrium. If the equation contains an unknown quantity, its value may be found by solving the equation, i. e., by drawing the polygon.

It should be borne in mind that each line in the force polygon represents two distinct values—namely, the *magnitude* and the *line of action* of a force.

The values for each force necessary to satisfy the condition of equilibrium between the forces represented may be determined by drawing the force polygon:

(*a*) When both the magnitude and the line of action of one force are unknown.

(*b*) When the magnitude of one force and the line of action of another force are unknown, the line of action of the former and the magnitude of the latter being known.

(*c*) When the lines of action of two forces are unknown, their magnitudes being known.

(*d*) When the magnitudes of two forces are unknown, their lines of action being known.

NOTE.—The arrow-head marked upon each line to indicate the sense of the force along its line of action is to some extent independent of the above conditions; the sense of a force possessing *any* of the unknown conditions as above may always be determined in connection with the determination of the unknown condition.

1110. *Case* (*a*)—How to find the ***magnitude***, ***line of action***, and ***sense*** of the one unknown force necessary to fulfil the condition of equilibrium has been explained quite fully in Arts. **1105** to **1108**. The operation is substantially the same in all cases, and will require no further explanation.

1111. *Case* (*b*)—EXAMPLE.—In Fig. 218, the lines of action of the forces AO, BO, CO, and DO meet at the common point O; the magnitude and direction of each are known. $AO = 40$ pounds,

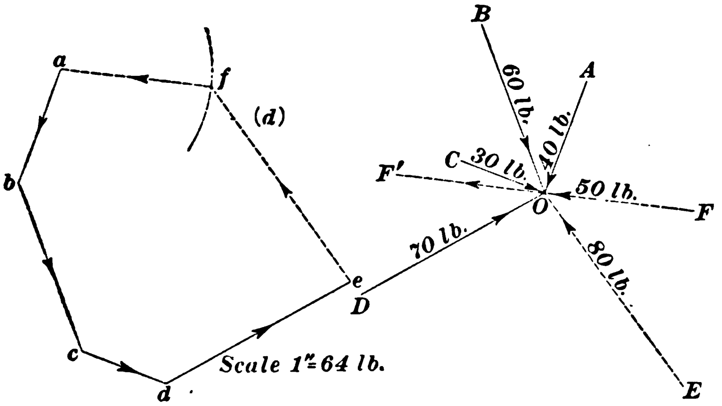


FIG. 218.

$BO = 60$ pounds, $CO = 30$ pounds, and $DO = 70$ pounds, and the direction of each force is indicated in the figure. A force EO whose ***magnitude*** is unknown acts along the line of action EO. A force of 50 pounds designated as FO also acts upon the point O, but its *line of* *action* is not known. The sense of neither force is known. It is desired to find the magnitude of the force $E O$ and the direction of the force $F O$.

SOLUTION.—Draw end to end consecutively, and to any scale, the lines representing the known forces $A O$, $B O$, $C O$, and $D O$, as $a b$, $b c$, $c d$, and $d e$ in (d), Fig. 218. Through e, draw a line of indefinite length, as $e f$, having the direction of the force $E O$; from the point a as a center, with a radius whose length represents 50 pounds (the magnitude of the force $F O$ whose direction is unknown), strike the arc of a circle intersecting the indefinite line $e f$ at f; draw the line $f a$, and mark upon it an arrow indicating a direction corresponding to the general directions of the other lines around the polygon. The direction of the force $F O$ is thus found to be the direction of the line $f a$; and the magnitude of the force $E O$ is represented by the length of the line between e and f; $E O = 80$ pounds. The same scale must be used for all the lines of the force polygon.

1112. *Case* (c)—EXAMPLE.—In Fig. 219, the forces $A O$, $B O$, $C O$, and $D O$, which meet at the common point O, have the same values as the corresponding forces represented in Fig. 218; namely,

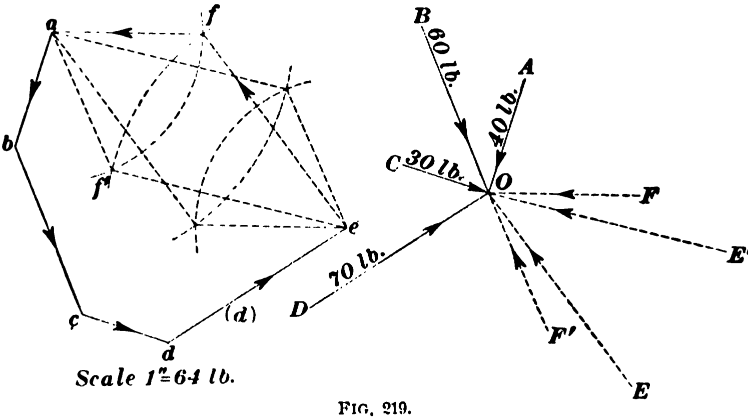


FIG. 219.

40, 60, 30, and 70 pounds, respectively. Also acting upon the same point are two forces, $E O = 80$ pounds and $F O = 50$ pounds, whose *directions*, i.e., *lines of action and sense*, are unknown. It is desired to find the directions of the two forces $E O$ and $F O$, in order that there may be equilibrium.

SOLUTION.—Draw end to end consecutively, and to any scale, the lines representing the known forces $A O$, $B O$, $C O$, and $D O$, as $a b$, $b c$, $c d$, and $d e$, in (d). From e as a center, strike the arc of a circle having a radius equal to $E O = 80$ pounds; and, from a as a center, strike the arc of a circle having a radius equal to $FO = 50$ pounds; or *vice versa*. If the forces are in equilibrium, the arcs will intersect, as at f and f'. Draw the lines ef and fa (or ef' and $f'a$); mark upon the lines arrows indicating directions corresponding to the general directions of the other lines around the polygon. Then, the directions of the lines ef and fa (or ef' and $f'a$) will be the directions of the forces EO and FO, respectively. If EO has the direction ef, then, FO must have the direction fa; but, if EO has the direction ef', as indicated by the line $E'O$, then, FO must have the direction $f'a$, as indicated by the line $F'O$. Or, if the center for the larger radius had been taken at a, and for the shorter radius at e, the figure enclosed by dotted lines would have been reversed, as indicated, but the results would have been the same.

1113. *Case* (*d*).—EXAMPLE.—In Fig. 220, the forces AO, BO, CO, and DO, which meet at the common point O, again have the values 40, 60, 30, and 70 pounds, respectively, as in Fig. 218. Two other forces, designated as EO and FO, act upon the point O. The *lines of*

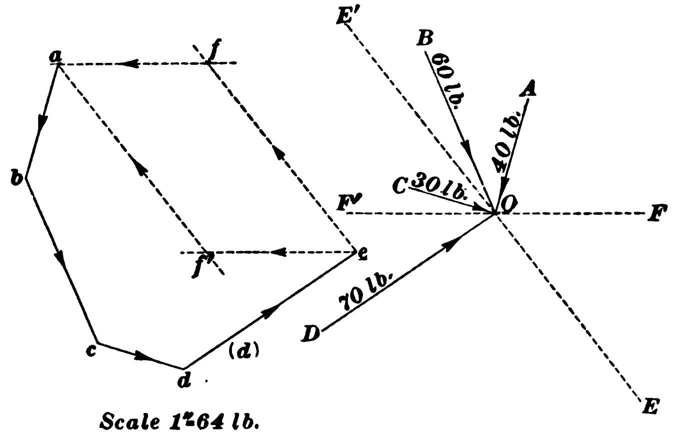


FIG. 220.

action of these two forces are the lines EE' and FF', but neither the *magnitude* nor the *sense* of either force is known. It is desired to find the magnitude and sense of the two forces EO and FO, in order that they may be in equilibrium.

SOLUTION.—Draw end to end consecutively, and to any scale, the lines representing the known forces AO, BO, CO, and DO, as ab, bc, cd, and de, in (*d*). Through e, draw a line of indefinite length parallel to EE', and through a, draw a line of indefinite length parallel to FF'; these lines will intersect at f. Mark arrow-heads upon them having the same general directions around the polygon as the lines representing the known forces. Then, the line $e f$ will represent the force whose line of action is $E E'$, and the line $f a$ will represent the force whose line of action is $F F'$. The same result will be obtained if the line parallel to $E E'$ is drawn through a, and the line representing $F F'$ is drawn through c. The force $E O$ may have the position $E O$ or $O E'$, and the force $F O$ may have the position $F O$ or $O F'$.

The geometrical equation or force polygon for the last condition (d) is very valuable; it is extensively used for determining the stresses in bridges, roofs, and other framed structures, when the lines of action of the two unknown forces are *not parallel.*

1114. Bow's Notation.

The following system of notation, devised by Mr. R. H. Bow, of Edinburgh, is of great assistance in the graphical solution of statical problems:

ILLUSTRATION.—In Fig. 221, let the lines represent a system of forces which meet at a common point. Instead of being designated by letters written at the ends of the lines,

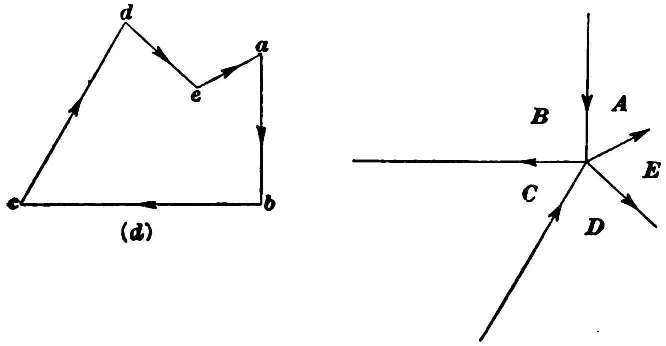


FIG. 221.

they are designated by letters placed in the spaces *between* the lines. Any line (or force) is designated by the two letters between which it is situated. For instance, the vertical line in the figure is designated as the line $A B$, the horizontal line as the line $B C$, etc. The force polygon (d), Fig. 221, is drawn in the same manner as has been explained; the lines are designated by letters placed at the angles or *ends* of the lines; and the line designated by any two letters represents the same force as that represented by the line that is designated by the corresponding letters in Fig. 221. Thus, in (*d*), Fig. 221, the line *a b* represents the same force that in Fig. 221 is represented by the line *A B*, the line *b c* represents the same force as the line *B C*, *c d* the same force as *C D*, etc.

This very convenient method of notation will be adopted; small letters placed at the angles of the force polygon will designate a force which in the original figure or frame is designated by the corresponding capital letters placed in the spaces and between which the line is situated. In the original figure, as Fig. 221, any *entire space* not crossed by a line representing a force will be designated by the *same letter*.

1115. Forces in a Frame—External and Internal Forces.

ILLUSTRATION.—Fig. 222 represents a triangular frame which is acted upon by three forces. The directions of the three forces are known, and if their lines of action were produced they would all meet at the common point *O*; therefore, they are concurring forces. (See Art. **1095.**) The magnitude of one force (*A B*) is known, and the condition of equilibrium between all the forces is assumed. The forces are designated by Bow's notation, each force being designated by the two letters between which the line is situated. The three forces which act upon the frame are *A B*, *B D*, and *D A*; as these forces are entirely external to the frame itself, they are called **external forces.** It is evident that they must be resisted and held in equilibrium by forces in the frame; for they are applied at different points, and equilibrium between them can only be produced by means of forces in the frame. These forces in the members of the frame are called **internal forces,** or **stresses.**

The lines of action of the internal forces must extend in the directions of the members *B C*, *A C*, and *C D*, of the frame. Each joint (*1*, *2*, and *3*) is the point of application of one external and two internal forces. The lines of action of all the forces are known, and, at the joint *2*, the magnitude and direction of the external force *A B* (= 1,000 pounds) are known. By beginning with this known force, the force polygon for the forces which meet at this joint may be drawn, and the values of the two internal forces which act upon this joint may be determined. As each internal force

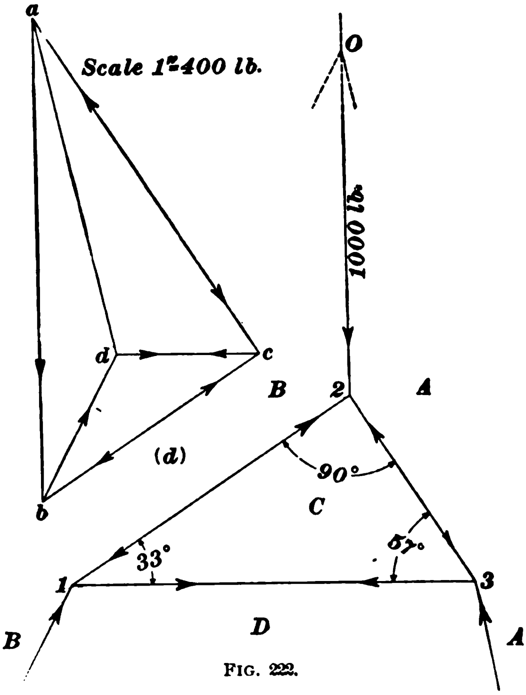


FIG. 222.

thus determined acts equally upon two joints, one force which acts upon each of the joints *1* and *2* becomes known; therefore, as but three forces act upon each joint, the force polygon may be drawn for each of those joints and the remaining forces determined. Thus it will be seen that by commencing at a joint at which not more than two forces which meet upon it are unknown, all the remaining forces may be determined by drawing the force polygons. The manner of drawing the polygons will now be explained.

1116. Consider first, the forces which act upon joint *2*; draw *a b*, Fig. 222, (*d*), parallel to the direction of the known force *A B*, making the length of the line equal to that force by any convenient scale. The scale used in the figure is 400 pounds to the inch and *a b* is made $\frac{1000}{400} = 2\frac{1}{2}$ inches long. Having thus drawn *a b*, draw *a c* and *b c* parallel, respectively, to *A C* and *B C*; *a c* and *b c* will then represent the stresses on *A C* and *B C*, respectively, and, with their arrow-heads properly placed (pointing from *b* towards *c*, and from *c* towards *a*), complete the polygon for joint *2*. The direction of the arrow-heads is of great importance, but very simply determined; for it will be remembered that the arrow-heads show the direction in which a pencil must travel around the polygon from *a*, through *b* and *c*, and back to *a*. On the line *a b*, the direction of the arrow-head is given by the known direction of *A B*, and is placed nearer *b*, the latter point corresponding to joint *2*, which is the point of application of the force *A B*. On the line *b c*, the arrow-head must point from *b* towards *c*, and be nearer to the end *c*; on the line *c a*, the arrow-head must point towards, and be nearer to, the end *a*. In this polygon, the arrow-head in each case points towards the *shorter* portion of the line, indicating that the sense of the force (which must always correspond to the direction in which the pencil travels) is in each case *towards* its point of application (joint *2*), or, in other words, indicating that each force *pushes* against the joint. Measuring with the same scale used for *a b*, the line *b c* = 544 pounds, and the line *c a* = 838 pounds.

It has been thus found that the force in *B C* pushes against joint *2* an amount equal to 544 pounds, represented by the line *b c*; but the member *B C* simply connects joint *2* with joint *1*; it is attached to joints *2* and *1* only, and the internal force in it is not affected by any other force except at those two joints; it simply transfers the push from one joint to the other. Therefore, the force in *B C* must push against joint *1* the *same amount* that it pushes against joint *2*, but in the *opposite direction*. Hence, the line *b c* (which represents the force in *B C*) must be common to two force polygons, but in the polygon for joint *1* it must be drawn in the direction opposite to that in which it was drawn for joint *2*. As shown in the figure, the second arrow-head marked upon it, which relates to its sense with reference to joint *1*, must point from *c* towards *b*, and be nearer the latter point, which in the new polygon corresponds to the point of application of the force.

For joint *1*, draw from *b* and *c*, the lines *b d* and *c d* parallel, respectively, to *B D* and *C D*; then will *b d* and *c d* represent the forces *B D* and *C D*, respectively. In going over the polygon to mark the arrow-heads, it must be remembered that the starting direction is from *c* towards *b*, and then through *d*, back to *c*. The direction in which the pencil must travel in returning to the starting point *c* indicates that the external force *B D* pushes against joint *1*, but that the internal force *D C* *pulls away* from the joint; in the latter case, the arrow-head points towards the *longer* portion of the line. Measuring with the scale used for *a b*, the line *b d* = 338 pounds, and the line *d c* = 296 pounds.

Now consider joint *3*. For the same reasons which were given in regard to the direction in which the internal force *B C* (= 544 pounds) acts upon joint *1*, the internal force *A C* (= 838 pounds) is known to act upon joint *3* in the opposite direction from that in which it acts upon joint *2*, and the internal force *C D* (= 296 pounds) to act upon joint *3* in the opposite direction to that in which it acts upon joint *1*. Therefore, in constructing the polygon for joint *3*, mark additional arrow-heads on *a c* and *c d*, near the opposite ends of the lines and pointing in the opposite directions from those that have been marked for joints *1* and *2*, and draw *d a*, marking its arrow-head to correspond with the new or reversed arrow-heads marked on *a c* and *c d*. The line *d a* represents the force *D A*, and its arrow-head indicates that *D A* *pushes* against joint *3*. By scaling *d a* it is found to equal 720 pounds. If the operations have been properly done, *d a* will be parallel to *D A*.

1117. It will be noticed that in (*d*), Fig. 222, each line which represents the internal force, or stress, in a member of the frame has two arrow-heads marked upon it. If these arrow-heads have been marked in accordance with the instructions previously given (see also Art. **1103**), they will in each case indicate whether the stress represented by the line upon which they are marked is compression (push) or tension (pull). If on any line of a force polygon, the two arrow-heads point *away* from each other, such relative directions indicate that the internal force or stress in the member is compression; but if the arrow-heads point *towards* each other, then *tension* is indicated. Thus, in Fig. 222 and in (*d*), Fig. 222, the stresses in the members *A C* and *B C*, which are represented by the lines *a c* and *b c*, are compression, for the arrow-heads on each of these lines point away from each other, while the arrow-heads upon the line *c d*, which represents the stress in the member *C D*, indicate that the stress in that member is tension, because they point towards each other.

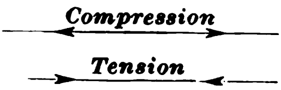


FIG. 223.

The relative positions of the arrow-heads to denote tension or compression are shown in Fig. 223.

1118. When the directions of the forces which act upon a body are towards each other, their combined action tends to shorten or compress the body; this character of stress is called **compressive stress,** or **compression.** It is evident that the forces of the body which resist compression must act outwards or away from each other, as indicated by the arrow-heads upon the line which denotes compression in Fig. 223.

When the forces which act upon a body are exerted in directions away from each other, tending to elongate or stretch the body, this character of the stress is called **tensile stress,** or **tension.** It is also evident that the forces in the body which resist tension must act inwards or towards each other, as indicated by the arrow-heads upon the line denoting tension in Fig. 223.

The subject of stresses will be more fully treated in connection with Strength of Materials.

1119. A system of force polygons, as (*d*), Fig. 222, for determining the forces in a frame, is called a **stress diagram**; it determines the relative values of the internal forces, or stresses, in the several members of the frame.

1120. A frame which is constructed for the purpose of resisting the action of force by transferring it from one position to another is called a **truss**.

ILLUSTRATION.—Fig. 224 represents a form of truss that is sometimes built; it supports at the center a load of 3,000 pounds, represented by the line *B F*. But the truss can

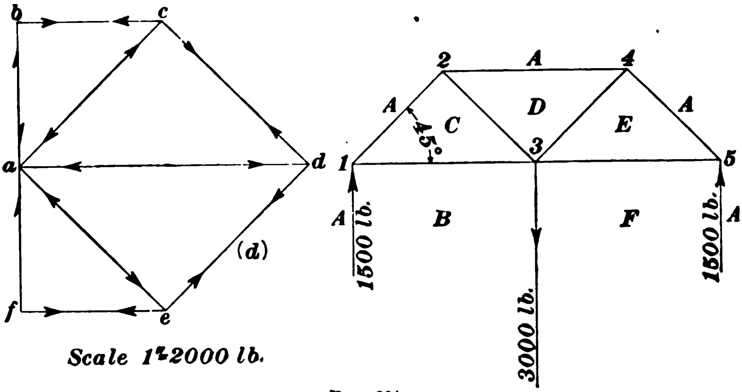


FIG. 224.

not support a load without being itself supported by other external forces balancing the load; for the office of the truss is simply to transfer the effect of the load from its position at the joint *3* to the supporting forces *A B* at joint *1*, and *F A* at joint *5*. As the load is midway between these supporting forces, it is transferred to, and supported by, the forces *A B* and *F A* in equal portions; that is, $A B = F A = \frac{1}{2} B F = 1,500$ pounds, and $A B + F A = 1,500 + 1,500 = 3,000$ pounds $= B F$. The subject of supporting forces will be noticed further along. The weight of the frame itself will be neglected in this case; the stress diagram will be drawn for the forces referring to the load only. For clearness, several letters *A* are written in Fig. 224; they all designate the same space. Since in the stress diagram each force polygon is drawn in succession for the forces which meet at a joint of the frame, the forces are in each case treated as concurring forces. The diagram (*d*), Fig. 224, is the stress diagram for the frame here considered. In drawing the stress diagram, it is necessary to begin at a joint at which not more than two quantities are unknown, with reference to the forces which meet at that joint (Art. **1109**). At joint *1*, three forces meet; namely, the supporting force *A B* and the internal forces in the two members of the frame which connect at this joint, *B C* and *C A*. The directions of all three forces are known, and the magnitude of one force (the supporting force *A B*) is known; therefore, at this joint but two quantities are unknown (the magnitudes of the internal forces in *B C* and *C A*), and the stress diagram may be begun by drawing the force polygon for the forces which meet at joint *1*.

Draw *a b*, Fig. 224, (*d*), parallel to the direction of the supporting force *A B*, and equal to that force, to any convenient scale. In (*d*), Fig. 224, a scale of 2,000 pounds to the inch is used; therefore, $a\,b = \frac{1500}{2000} = 0.75$ inch. In order that *a b* may have the same sense as the supporting force *A B*, it must be drawn upwards from *a* to *b*, parallel to *A B*. From *b* the pencil must be returned to the starting point *a*, by lines drawn parallel to the lines of action of the two remaining forces which act upon this joint, or parallel to *B C* and *C A*. The lines *b c* and *c a* are such lines; the pencil can travel from *b* to *c* and from *c* to the starting point *a*, thus passing completely around the force polygon *a b c a* without being raised from the paper. It is of course not expedient to do this in the actual operation of drawing the lines; the line *a b* is drawn equal and parallel to *A B*; then, through *b*, a line of indefinite length is drawn parallel to *B C*, and through *a*, a line of indefinite length is drawn parallel to *A C*; these two lines must be so drawn that they will intersect, and their intersection will be the point *c*. (Arts. **1113** and **1116.**) When the lines are thus drawn, and the arrow-head on *a b* marked, the arrow-heads on *b c* and *c a* are placed in the direction in which the pencil travels in moving around the polygon from *a* to *b*, then to *c*, and back to *a*. (See also Art. **1103.**) The direction in which the pencil travels along each line will represent the sense of each respective force *with reference to the joint for which the polygon is drawn*. Hereafter, half arrow-heads will usually be marked upon lines which represent external forces, as *A B*, *B F*, and *F A*, acting upon a truss or frame work, to distinguish them from the internal forces, or stresses, in the members which will be designated by full arrow-heads. The arrow-head on the line *a b* is marked nearer the upper end *b*, because the joint *1*, which is the point of application of the supporting force *A B*, is at the upper end of the line which represents that force; on the line *b c*, the arrow-head is marked nearer the left end *b*, because joint *1*, the point of application of the force, is at the left end of the line *B C*; on the line *c a*, the arrow-head is marked nearer the lower end *a*, because joint *1*, for which the polygon is drawn, is at the lower end of the line *C A*. It is found that the force in *B C* is exerted in a direction *away* from joint *1*, while the force in *C A* is exerted *towards* it. Measuring with the scale used for *a b*, the line *b c* is found to equal 1,500 pounds, and the line *c a* to equal 2,120 pounds. Had the scale been sufficiently large and the work been done with sufficient care, so that exceedingly accurate results had been obtained, it would have been found that the force in *C A*, or line *c a*, equals 2,121.3 pounds. Such degree of accuracy, however, is seldom necessary.

Next consider the forces which act upon joint *2*: Three forces meet at this point; the lines of action of all three forces are known, for they act in the three members of the truss which meet at this joint; the magnitude and sense of one force (the stress in *A C*, represented by the line *c a*) have been found. As the force in *A C* was found to push against joint *1*, it must also push against joint *2*, for it simply transfers the push from one joint to the other; it is evident that it must push against joint *2* in the opposite direction from which it pushes against joint *1*.

Therefore, mark an additional arrow-head on the opposite end of *a c*, pointing in the opposite direction; from *a* draw *a d* parallel to *A D*, and from *c* draw *c d* parallel to *C D*; mark the arrow-heads as usual, starting in the direction of the new arrow-head marked on *a c*, and this will complete the diagram for joint *2*. The line *c d* scales 2,120 pounds and *d a* 3,000 pounds.

For joint *4*, mark an arrow-head on *a d*, in a reversed position and direction, and draw *a e* and *d e* parallel to *A E* and *D E*, respectively. Mark the arrow-heads as in the preceding cases. The lines *a e* and *d e* are found to be each equal to 2,120 pounds.

For joint *5*, the force in *A E* has just been found; it equals $e\,a = 2{,}120$ pounds. Mark an arrow-head in a reversed position and direction on *a e*; draw *a f* and *e f* parallel to *A F* and *E F*, respectively, and mark arrow-heads as usual. The line $e\,f = 1{,}500$ pounds, and $f\,a = 1{,}500$ pounds $= \frac{1}{2}\,b\,f = a\,b$, as it should. It is seen that the value of *f a*, as found from the diagram, must equal the supporting force *F A*, already known. This is a good check upon the work.

The five forces in the members which meet at joint *3* have all been determined in the polygons for the joints *1*, *2*, *4*, and *5*, but in order that the stress diagram shall be in every way complete, it will be necessary to consider these forces with reference to the joint *3*. As each force is known, it will make no difference which is taken first. Start with any force, as the load *B F*; this force is equal in amount to the sum of the supporting forces *A B* and *F A*, but must have the opposite direction, i. e., it is exerted downwards, while the supporting forces are exerted upwards. Therefore, beginning at *b* in the stress diagram, pass downwards over the line *b f*, which is equal in amount, but opposite in sense, to the sum of the lines *f a* and *a b*, and mark arrow-heads in reversed positions and directions on

$f e$, $e d$, $d c$, and $c b$, returning to the starting point b, for the force in each of the members which meet at the joint *3* must act upon this joint in a direction opposite to that in which it is exerted upon the joint at the opposite end of the member. The reversed arrow-heads will indicate the sense of the forces with reference to joint *3*.

After the forces which act on joint *2* had been found, joint *3* might have been considered instead of joint *4*, since the directions of all of the five forces which act at *3* are known, and only the two forces $D E$ and $E F$ are unknown. (Art. **1113.**)

If the arrows have been properly marked upon the lines, they will, by comparison with Fig. 223, indicate the character of the stress in each member of the truss, that is, whether the stress is tension or compression. It is found that the members $A C$, $A D$, and $A E$ are in compression, while the members $B C$, $C D$, $D E$, and $E F$ are in tension. The stress $a c = c d = d e = e a = 2{,}120$ pounds; the stress $b c = f e = 1{,}500$ pounds, and the stress $a d = 3{,}000$ pounds $= b c + f e$.

1121. In (d), Fig. 224, the line $b f$, which represents the load, is called the **load line.** In drawing the stress diagram, the load line is usually drawn first and the loads laid off upon it.

NON-CONCURRING FORCES.

1122. Thus far force has been considered with reference to its direct action only, and, therefore, its treatment has been limited to concurring forces. Before treating of non-concurring forces the subject of **moments** must be explained.

A force can act in two ways: it can either produce a motion of translation, that is, cause all points of the body acted upon to move in parallel straight lines; or it can produce a motion of rotation, that is, turn the body. A **moment** measures the capacity of a force to produce rotation.

1123. The **moment of a force** *about a point is the product of the magnitude of the force by the perpendicular distance from the point to the line of action of the force.* (See Art. **906.**)

Thus, in Fig. 225, the moment of the force F about the point O equals $F\,l$.

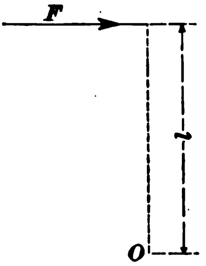


FIG. 225

1124. The following principles are of great importance, and should be thoroughly understood:

(*a*) *A moment always implies a tendency to* **rotation**; *if it is unresisted, rotation occurs.*

(*b*) *The point about which the moment is taken, i. e., the point about which rotation would occur under the action of the force, is variously known as the* **origin** *of* **moments,** *the* **center** *of* **moments,** *and the* **center of rotation.** The last name will here be used. In Fig. 225, the point O is the center of rotation of the moment $F\,l$.

(*c*) *The* **lever arm** *of a moment is the perpendicular distance from the center of rotation to the line of action of the force.* In Fig. 225, l is the lever arm of the moment $F\,l$.

(*d*) *The moment of a force about a point depends upon the lever arm and the magnitude of the force only; it is independent of the point of application of the force.*

ILLUSTRATION.—In Fig. 226, the force F might really act at P upon the material lever $P\,O$, which is inclined to the line of action $A\,B$ of the force. But, according to Art. **1095,** *any* point in the line of action of the force may be considered as its point of application; the point of application should always be considered to be such a point in the line of action, as P', that the lever arm $O\,P'$ will be perpendicular to the line of action. The inclined lever arm might have any inclination, and, therefore, any length between

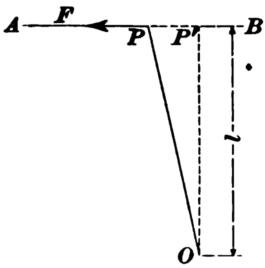


FIG. 226.

the same line of action and center of rotation, but the perpendicular lever arm can have but one length; the force, if applied to the perpendicular lever at P', will have the same rotative effect as if applied at P or upon any inclined lever.

1125. *The amount, magnitude, or numerical value of a moment is the measure of the rotative effect of the force at the center of rotation*, or, as said above (Art. **1122**), *of the capacity of the force to produce rotation about a given point.*

It is the product of the magnitude of the force (in pounds or tons) by the length of the perpendicular lever (in inches or feet), and is, therefore, usually expressed in foot-tons, foot-pounds, inch-tons, or inch-pounds.

In Fig. 226, if $F =$ the force and $l =$ the lever arm, the moment $= F\,l$; or, if $F = 10$ pounds and $l = 20$ inches, the moment $= 10 \times 20 = 200$ inch-pounds.

Either the inch-pound or the foot-pound will here be used as the unit of moments, according to the convenience of each special case. When the inch-pound is used, it represents the effect of a force of one pound acting upon a lever arm of one inch; and when the foot-pound is used as the unit of moments, it represents the effect of a force of one pound acting upon a lever arm of one foot. *To reduce inch-pounds to foot-pounds, divide by 12. To reduce foot-pounds to inch-pounds, multiply by 12.*

NOTE.—In Mechanics, the term "foot-pound" is used in a very different sense; namely, *the work performed in lifting* a weight of one pound through a *height* of one foot. The two meanings of the term should not be confused. In one case, it is the unit of work, and, in the other the unit of rotative effect.

1126. The **direction** of a moment is said to be to the *right* if its tendency is to cause rotation in a direction corresponding to the movement of the hands of a clock, and to the *left* if it tends to cause rotation in the opposite direction. Thus, in Fig. 225, the direction of the moment is to the right, while in Fig. 226, the direction of the moment is to the left.

1127. A moment will here be considered as **positive** when it tends to produce rotation to the *right*, and as **negative** when it tends to cause rotation to the *left*. Thus, the moment of the force in Fig. 225 is positive; that of the force in Fig. 226, negative.

In computations, *positive* moments are designated by the + and *negative* moments by the − sign. In Fig. 225, the moment is $+F\,l$, or simply $F\,l$ (+ sign being understood); but, in Fig. 226, the moment is $-F\,l$.

1128. *The moment of a force about any point in its line of action is zero*, for the lever arm of a moment is the perpendicular distance from the center of rotation to the line of action of the force; and, if this distance is zero, as in this case, then the moment of the force F will be $F \times 0 = 0$.

1129. *For the existence of an effective moment two forces are necessary: one, which we may call the applied force* (as F in Figs. 225 and 226), *tending to produce rotation; and another, which is of the nature of a resistance, acting at the center of rotation, equal in amount, but opposite in direction, to the applied force. The effect of the latter is to fix the center of rotation, which would otherwise be moved.*

ILLUSTRATIONS.—(*a*) Fig. 227 illustrates a very simple example of a moment; it represents a stick standing on end. If a steady horizontal force F, just sufficient to tip the stick, is applied to its upper end, the stick will rotate about O; that is, it will fall over, the top moving in the direction of the dotted line, while the bottom remains at O. When the force F is applied to the top of the stick, an exactly equal and opposite force F' is incited at the bottom; for, were it not for the force F', the stick would not rotate about O, but would simply slide along towards the

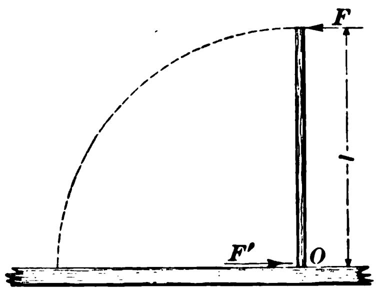


FIG. 227.

left, the point O (center of rotation) not being fixed, the motion would be one of translation, not of rotation. The force F', in this case, is the friction between the end of the stick and the body on which it rests.

(*b*) Fig. 228 represents a shorter stick or block which can not be so easily tipped. If it rests on smooth, slippery ice, and a steady horizontal force F is applied to the top, it will slide along on the ice and rotation will not occur. But, if it rests on some substance which will offer considerable frictional resistance, this resistance becomes an equal and opposite force, F', and the block will not slide, but tip over; or, in other words, a moment will be formed and rotation will take place.

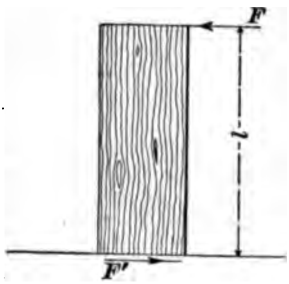


FIG. 228.

1130. *When two equal and opposite forces act at different points, thus tending to cause rotation, the combination is called* ***a statical couple.*** (See Arts. **907** to **909.**)

The **lever arm** of a couple is *the perpendicular distance between the forces.*

The **moment** of a couple is *the product of either of the equal forces by the lever arm.* The intersection of the line of action of either force with the common lever arm may be regarded as the center of rotation, and the moment of the couple as the moment of the other force about that center.

ILLUSTRATION.—Thus, in the couple represented in Fig. 229, the forces F and F' are equal, and act at the opposite ends of the same lever arm l. The moment of the couple is either $F\,l$ or $F'\,l$; for, as indicated in the figure, the force F tends to cause rotation about the point O, while F' also tends to cause rotation about O, and as O is a point midway between the two forces, the moment of $F = F \times \frac{1}{2}\,l$, and the moment of $F' = F' \times \frac{1}{2}\,l$. As the forces are equal and have equal lever arms, the combined moment of the two forces equals $\frac{1}{2} Fl + \frac{1}{2} F'l = Fl = F'l =$ the product of either force by the lever arm. If, then, either force of a couple be considered as tending to cause rotation about the point of application of the second force, and the effect of the second force is ignored, the result thus obtained will be the same as if the center of rotation were taken between the two forces, and the moments of the two forces taken about that center. In general, ***any point in the plane of a couple may be taken as the center of rotation.*** For, if through that point a perpendicular be drawn to the common direction of the forces, the algebraic sum of the moments of the forces about that point will always be equal to the constant product $F l$.

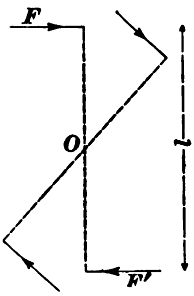


FIG. 229.

1131. *The effect of a statical couple, and the only effect, is tendency to cause rotation.*

ILLUSTRATION.— A good illustration of a statical couple is the pulley by which a line of shafting is driven by a belt, Fig. 230. By means of the belt a force F is continually applied to the rim of the pulley, causing it to rotate. Another force F' must be continually applied to the axis of the pulley by means of the journal, or the pulley would be drawn from its position by the belt. The moment at the axis or center of rotation, i. e., the measure of the effect of the applied force on the shaft, is equal to the force F (pull of the belt) multiplied by the radius of the pulley.

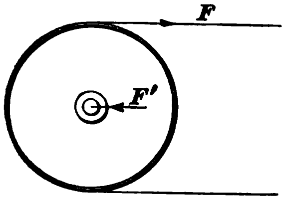


FIG. 230.

1132. *When rotation does not occur as the effect of a couple, then the moment of the couple is balanced by the opposite moment of a second couple, and the forces are thus held in equilibrium. A couple can not be balanced by a single force.*

ILLUSTRATION.—Suppose, in Fig. 231, that $A\ C$ is a lever 30 inches long, having a fulcrum at B, 10 inches from A. If a weight is suspended from C, it will cause the bar to rotate about B in the direction of the arrow. A weight suspended from A will cause it to revolve in the opposite direction, as indicated by the arrow. Suppose, for simplicity, that the bar itself weighs nothing. If equal weights of 12 pounds are hung at A and C, it is evident that the bar will revolve in the direction of the arrow at C, on account of the arm $B\ C$ being longer than the arm $A\ C$. Let the weight at A be increased until it equals 24 pounds; the bar will then balance exactly, and any additional weight at A will cause the bar to rotate in the opposite direction, as shown by the arrow at that point. When the lever is balanced, i. e., when the forces are in equilibrium, it will be found that $24 \times 10 = 12 \times 20$, or, considering B as the center of rotation, $24 \times$ perpendicular distance $A\ B = 12 \times$ perpendicular distance $B\ C$. In other words, the moment of W about B must equal the moment of P about B. Further, P tends to cause rotation in the direction in which the hands of a watch move, and is positive, or $+$; W tends to cause rotation in an opposite direction, and is negative, or $-$. Adding the two algebraically, $P \times B\ C + (-W \times A\ B) = P \times B\ C - W \times A\ C = 0$, since the two moments are equal.

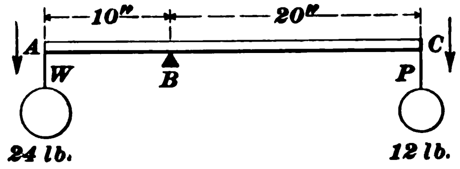


FIG. 231.

1133. When the moments of several forces are taken about the same point or center, the algebraic sum of all the moments is the **resultant moment** of the forces about that point.

In other words, the resultant (effective) moment at any point is the arithmetic difference between the sum of the (positive) moments which tend to cause rotation to the right, and the sum of the (negative) moments which tend to cause rotation to the left about the point.

In Fig. 231, the positive and negative moments were found to balance, leaving no resultant moment when the moments of *all* the forces were considered. In such cases the algebraic sum of all the moments about a point is zero.

ILLUSTRATION.—In Fig. 232 the moment of the force at a about the point $b = 400 \times 25 = 10{,}000$ foot-pounds to the right, or positive. The positive moment of the same force about the point $c = 400 \times 50 = 20{,}000$ foot-pounds, while the negative moment of the force at b about the same point $= 400 \times 25 = 10{,}000$ foot-pounds; therefore, the resultant moment at c from the forces at the left of that point $= 20{,}000 - 10{,}000 = 10{,}000$ foot-pounds to the right, or positive.

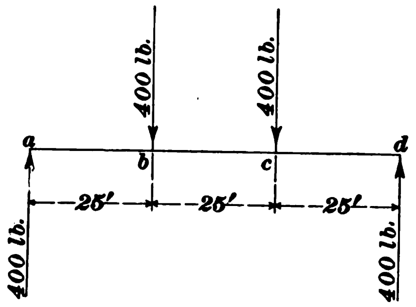


FIG. 232.

EXAMPLE.—What is the resultant moment at d, Fig. 232?

SOLUTION.—The positive moment of the supporting force at $a = 400 \times 75 = 30{,}000$ foot-pounds; the negative moment of the load at $b = 400 \times 50 = 20{,}000$ foot-pounds; and the negative moment of the load at $c = 400 \times 25 = 10{,}000$ foot-pounds. The resultant moment $= 30{,}000 - 20{,}000 - 10{,}000 = 0$.

1134. *In any structure, it is a necessary condition of equilibrium that the moments of* **all** *the forces about any point should balance; in other words, the algebraic sum of the moments of all the forces about any point must equal zero.*

If, at any point the moments did not balance, the structure, or some part of it, would rotate about that point, that is, it would be overturned, as when a dam is pushed down by the pressure of the water. But, as the structure, or any part of it, does not rotate, the moments of all the forces which act upon it must be balanced at every point.

1135. *In any structure, the algebraic sum of the moments of all the* **external forces** *about any point must equal zero.*

As the moments of the internal forces simply tend to keep the structure intact by resisting any tendency to rupture or distortion of the structure itself, and act entirely within the structure, they can not prevent the structure *as a whole* from rotating about any point. The entire structure would, therefore, rotate if the moments of the external forces, which always tend to cause rotation, did not balance about every point.

ILLUSTRATION.—It was found that the resultant moment of the external forces at the left of the point c, Fig. 232, about that point = 10,000 foot-pounds (positive). But the external force at d was not considered; the moment of this force about c is $-400 \times 25 = -10{,}000$ foot-pounds (negative). Therefore, the resultant moment of *all* the external forces about the point c is $10{,}000 - 10{,}000 = 0$.

1136. *In a structure, the resultant moment of the external forces on either side of any point is called the* ***bending moment*** *at that point.*

The bending moment is usually designated by the letter M. In computing the bending moment at any point, it is customary to consider the forces at the *left* of it; this custom will always be followed except when otherwise stated. Thus, at the point c in Fig. 232, $M = 400 \times 50 - 400 \times 25 = 20{,}000 - 10{,}000 = 10{,}000$ foot-pounds. The bending moment is the moment which causes bending stresses in a structure, and which at every point in the structure must be balanced or resisted by the moments of those stresses.

1137. *The moment of the internal forces or stresses*, which at any point in a structure *resists the bending moment*, is called the **resisting moment.**

NOTE.—The term **resisting moment,** as used here in a general sense, must not be confused with the term **moment of resistance** employed to designate the resisting moment of solid beams. The latter term relates to the Strength of Materials, and will be studied in connection with that subject.

1138. ***At every point in the structure, the resisting moment must equal the bending moment.***

When in a structure the moments of the external forces at one side of any point do not balance, but must be balanced by the moments of forces on the opposite side of the point, as in Fig. 231, it is evident that the opposing moments on the opposite sides of the point tend to bend the structure about the point. If the structure offered no resistance to this bending, it would close up like a jack-knife. The *resisting moment* of the internal forces at each point in the structure must be sufficient to resist the effect of the opposing moments, or the *bending moment* of the external forces on the opposite sides of the point.

ILLUSTRATION.—In Fig. 233, the forces F and G represent portions of a load situated at the center of a beam; the load is, for clearness, represented as though divided. A portion of the load (the force F) forms a couple with the supporting force F', while the other portion (the force G) forms a couple with the supporting force G'. The tendency of these two couples is to cause rotation, either about the point of application of the combined forces F and G, or about the points of application of the supporting forces F_1 and G_1. But the two halves of the beam do not rotate about the points of application of either the supporting forces or the load, because the moments of the couple formed by these (external) forces are resisted and equilibrated by the opposite moments of the couple formed by the internal forces of the beam c and t. These internal forces are compressive stress in the upper portion of the beam, and tensile stress in the lower portion, as indicated in the figure; their moments resist the moments of the external forces, and preserve the form of the beam.

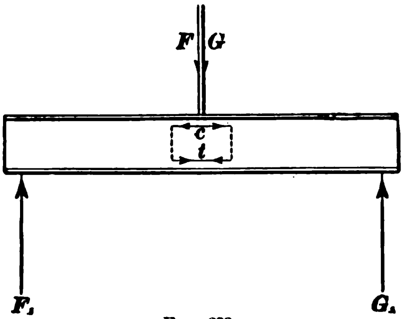


FIG. 233.

1139. The preceding principle forms the basis for the analysis of stresses by what is known as the Moment Method. The amount of the bending moment is found at each required point in the structure; and as at each point the resisting moment must equal the bending moment, if the bending moment at any point is divided by the lever arm of the resisting moment (the effective depth of the structure), the quotient will equal the resisting force or bending stress in the structure at that point. This principle will be used in the analysis of stresses.

EXAMPLES FOR PRACTICE.

NOTE.—The following examples refer to Fig. 232:

1. What is the moment at a point 10 feet to the right of *a*, caused by the supporting force at *a*? Ans. 4,000 foot-pounds.

2. What is the moment caused by the same force at a point 20 feet to the right of *a*? Ans. 8,000 foot-pounds.

3. What is the resultant moment at a point 10 feet to the right of *b*, caused by the forces at the left of this point?
Ans. 14,000 – 4,000 = 10,000 foot-pounds.

4. What is the resultant moment at a point midway between *b* and *c* caused by the forces at the left?
Ans. 15,000 – 5,000 = 10,000 foot-pounds.

5. What are the resultant moments at *b* and *c*, respectively? See Art. **1133,** Illustration.

NOTE.—It will be noticed that when the resultant moments are the same at any two points, and no forces are applied between the points, the resultant moment will be the same at all intermediate points.

6. What is the resultant moment at a point 5 feet to the right of *c* caused by the forces at the left?
Ans. 22,000 – 12,000 – 2,000 = 8,000 foot-pounds.

7. What is the resultant moment at a point 15 feet to the right of *c* caused by the forces at the left?
Ans. 26,000 – 16,000 – 6,000 = 4,000 foot-pounds.

8. In each of the preceding examples, compute the resultant moment for the forces at the *right* of the point, and compare both the amount and sign of the answers with the preceding answers.

REACTIONS OF SUPPORTS.

1140. The forces that support a structure, and, through the medium of the structure, resist the final effect of the loads upon it (including its own weight) are called **reactions.**

According to the third law of motion, every action has an equal and opposite reaction. (Art. **870.**) When a structure is acted upon by downward forces, the supports *react* upwards. The structure simply transfers the effect of the downward forces or loads to the points of support; but these downward forces would move the points of support downwards, were they not resisted and balanced by the equal effect of the supporting forces which *react* upwards, so that all the forces which act upon each point of support are in equilibrium. In order to ascertain the amounts of the internal forces in a structure necessary to resist the effect of the external forces, the value of each reaction must be found. If a beam which is merely supported at each end is loaded uniformly over its entire length, or is loaded in the middle only, it is evident that the reaction at each support is one-half the load plus one-half the weight of the beam. But when a load is placed in such a position upon a beam that the distances from the load to the two ends of the beam are unequal, the reactions of the two supports will be unequal. But, in all cases where the loads and reactions are vertical, *the sum of the reactions equals the sum of the loads* (*the weight of the structure included*). For simplicity, the weight of the structure itself will here be neglected. The upward reactions are considered *positive*, or +, and the downward forces *negative*, or −. The loads and reactions constitute the external forces acting upon the structure. (Art. **1115.**)

1141. In order that a structure may be in equilibrium, three conditions must be fulfilled by the forces acting upon it:

I. *The algebraic sum of all vertical forces = 0.*

II. *The algebraic sum of all horizontal forces = 0.*

III. *The algebraic sum of the moments of all forces about any point = 0.* See Art. **1134.**

1142. According to Art. **1136,** if a structure is supported at each end and does not extend beyond either support (which is the most common case), *there can be no bending moment at either reaction;* for, on one side of each reaction there is no force to cause moment, and the algebraic sum of the moments of the external forces on the other side of the reaction is zero, because all the external forces which cause moments about the point are considered. (Art. **1135.**)

If the center of moments is taken on the line of action of any force, the moment of that force with reference to this center of moments is zero, since the length of the lever arm is zero. (Art. **1128.**) Consequently, if there are but two unknown forces acting upon a structure (as, for example, the two reactions), one may be found by taking the center of moments on the line of action of the other, and adding, algebraically, the moments of all the other forces about this point. By thus taking the center of moments on the line of action of the other unknown force, the second unknown force may be found. This will be made clear in the two following articles.

1143. A Single Load in Any Position Upon the Span.

ILLUSTRATION.—In Fig. 234, the force W may represent *any* single load, in *any* position, upon *any* span l. To find the value of R_1, take moments about B, the point of application of R_2. The moment of the load W, tending to cause rotation to the left is $- Wb$, and the moment of reaction R_1 tending to cause rotation to the right is $R_1 l$.

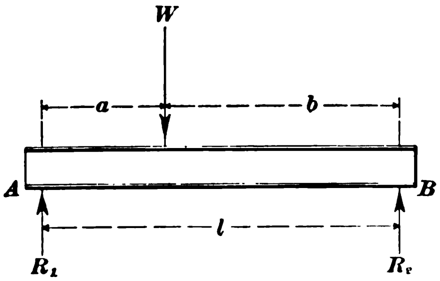


FIG. 234.

But the algebraic sum of these two moments must equal zero. Therefore, $R_1 l - Wb = 0$; transposing, $R_1 l = Wb$, and $R_1 = \frac{Wb}{l}$. In like manner, by taking moments about

A, $R_2 l - Wa = 0$, and $R_2 = \frac{Wa}{l}$. By Art. **1140,** the sum of the two reactions must equal the load; this is found to be the case, for $\frac{Wa}{l} + \frac{Wb}{l} = W\frac{a+b}{l} = W\frac{l}{l} = W.$

EXAMPLE.—A load of 24,000 pounds is supported upon a span $A\,B = 30$ feet, and is situated at a distance of 10 feet from A. What are the reactions at A and B, respectively?

SOLUTION.—If the load is at a distance of 10 feet from A, it is $30 - 10 = 20$ feet from B. The reaction at $A = 24{,}000 \times \frac{20}{30} = 16{,}000$ lb. Ans. The reaction at $B = 24{,}000 \times \frac{10}{30} = 8{,}000$ lb. Ans. The sum of the reactions $= 16{,}000 + 8{,}000 = 24{,}000 =$ the load. Art. **1140.**

EXAMPLES FOR PRACTICE.

1. A load of 16,000 pounds is supported upon a span $A\,B = 40$ feet, and is situated at a distance of 15 feet from A. What are the reactions at A and B, respectively? Ans. 10,000 lb. at A, and 6,000 lb. at B.

2. A weight of 18,000 pounds is situated at a distance of 12 feet from the end B of a span $A\,B = 90$ feet. What are the reactions at A and B, respectively? Ans. 2,400 lb. and 15,600 lb.

3. A load of 20,000 pounds is situated at a distance of 15 feet from the end B of a span $A\,B = 75$ feet. What are the reactions at A and B, respectively? Ans. 4,000 lb. and 16,000 lb.

1144. Any Number of Loads Upon the Span. The principle is applied in exactly the same manner when the span is loaded with any number of loads.

ILLUSTRATION.—In Fig. 235, the beam is shown loaded with four loads, W_1, W_2, W_3, and W_4. In this figure, the loads are represented as though they were equal, and the distances a, b, c, d, and e, are equal; but the principle is exactly the same whether the loads and distances are equal or unequal. If moments are taken about the point of application of R_2, the moment of R_1 will be positive and the moments of the loads will be negative. Then,

$$R_1 l - W_1(b+c+d+e) - W_2(c+d+e) - W_3(d+e) - W_4 e = 0;$$

transposing,

$$R_1 l = W_1(b+c+d+e) + W_2(c+d+e) + W_3(d+e) + W_4 e;$$

dividing by l,

$$R_1 = \frac{W_1(b+c+d+e) + W_2(c+d+e) + W_3(d+e) + W_4 e}{l}.$$

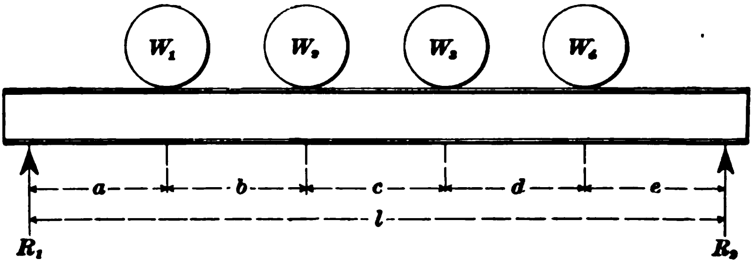


FIG. 235.

Taking moments about R_1,

$$-R_2 l + W_1 a + W_2(a+b) + W_3(a+b+c) + W_4(a+b+c+d) = 0,$$

and

$$R_2 = \frac{W_1 a + W_2(a+b) + W_3(a+b+c) + W_4(a+b+c+d)}{l}.$$

If W_1 is removed and the beam loaded with W_2, W_3, and W_4 only, the values of the reactions are:

$$R_1 = \frac{W_2(c+d+e) + W_3(d+e) + W_4 e}{l}, \text{ and}$$

$$R_2 = \frac{W_2(a+b) + W_3(a+b+c) + W_4(a+b+c+d)}{l}.$$

1145. Taking again the values of R_1 and R_2 found in Art. **1143,** we have

$$R_1 : R_2 :: \frac{Wb}{l} : \frac{Wa}{l}.$$

Or, $R_1 : R_2 :: b : a$.

Therefore,

Any load situated between its two supports is transferred to them in amounts inversely proportional to its distances from them.

Also, since $R_1 = \frac{W b}{l}$, and $R_2 = \frac{W a}{l}$,

Each reaction equals the load multiplied by the distance from the load to the opposite reaction and divided by the length of the span.

Finally, by observing the values of R_1 and R_2 in Art. **1144,**

The reaction at either end of the span from any number of loads equals the sum of the reactions at the same point from the several separate loads.

1146. Special Cases.—If, as indicated in Fig. 235, $a = b = c = d = e = p$, then, $a + b + c + d + e = 5\,p = l$, and $\frac{a}{l} = \frac{1\,p}{5\,p} = \frac{1}{5}$, $\frac{a+b}{l} = \frac{2\,p}{5\,p} = \frac{2}{5}$, etc. Therefore, with all loads upon the span,

$$R_1 = W_1 \frac{b+c+d+e}{l} + W_2 \frac{c+d+e}{l} + W_3 \frac{d+e}{l} + W_4 \frac{e}{l};$$

$$\text{or, } R_1 = \tfrac{4}{5} W_1 + \tfrac{3}{5} W_2 + \tfrac{2}{5} W_3 + \tfrac{1}{5} W_4.$$

$$R_2 = W_1 \frac{a}{l} + W_2 \frac{a+b}{l} + W_3 \frac{a+b+c}{l} + W_4 \frac{a+b+c+d}{l};$$

$$\text{or, } R_2 = \tfrac{1}{5} W_1 + \tfrac{2}{5} W_2 + \tfrac{3}{5} W_3 + \tfrac{4}{5} W_4.$$

Again, if $W_1 = W_2 = W_3 = W_4 = W$, then $R_1 = (\frac{4}{5} + \frac{3}{5} + \frac{2}{5} + \frac{1}{5})\,W = 2\,W$, and $R_2 = (\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5})\,W = 2\,W$. In this case, as in all cases, the sum of the reactions equals the sum of the loads: $2\,W + 2\,W = 4\,W$.

The conditions of equal loads and equal distances are very common in such structures as bridges and roof trusses. For any system of loads, the reactions are computed for those loads which are upon the structure in each condition of loading for which the reaction is required.

NOTE.—When computing moments or reactions, it is well, for the sake of clearness, to make a rough sketch and mark the loads and distances upon it.

EXAMPLE.—If, in Fig. 235, the load $W_1 = W_2 = W_3 = W_4 = 15{,}000$ pounds, and $a = b = c = d = e = 20$ feet $= \frac{1}{5}\,l$, what are the values of R_1 and R_2, respectively?

SOLUTION.—$R_1 = (\frac{4}{5} + \frac{3}{5} + \frac{2}{5} + \frac{1}{5})\,15{,}000 = \frac{10}{5} \times 15{,}000 = 30{,}000$ lb., and $R_2 = (\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5})\,15{,}000 = \frac{10}{5} \times 15{,}000 = 30{,}000$ lb.

EXAMPLES FOR PRACTICE.

1. Assuming the loads and distances to be the same as in the preceding example, but that the load W_1 is removed, and the span loaded only with W_2, W_3, and W_4, what are the values of R_1 and R_2, respectively? Ans. 18,000 lb. and 27,000 lb.

2. With loads and distances the same, but with W_2 also removed, and the span loaded with W_3 and W_4 only, what are the values of R_1 and R_2, respectively? Ans. 9,000 lb. and 21,000 lb.

3. With loads and distances the same, but with W_3 also removed, and the span loaded with W_4 only, what are the values of R_1 and R_2, respectively? Ans. 3,000 lb. and 12,000 lb.

4. With distances the same, but with the loads $W_1 = 10{,}000$ lb., $W_2 = 20{,}000$ lb., $W_3 = 30{,}000$ lb., and $W_4 = 40{,}000$ lb., what are the values of R_1 and R_2, respectively?

Ans. $\begin{cases} R_1 = 8{,}000 + 12{,}000 + 12{,}000 + 8{,}000 = 40{,}000 \text{ lb.} \\ R_2 = 2{,}000 + 8{,}000 + 18{,}000 + 32{,}000 = 60{,}000 \text{ lb.} \end{cases}$

5. With loads and distances as in the preceding example, what is the bending moment M at W_1? At W_2? At W_3? At W_4?

Ans. $\begin{cases} \text{At } W_1,\ M = 800{,}000 \text{ ft.-lb.} \\ \text{At } W_2,\ M = 1{,}400{,}000 \text{ ft.-lb.} \\ \text{At } W_3,\ M = 1{,}600{,}000 \text{ ft.-lb.} \\ \text{At } W_4,\ M = 1{,}200{,}000 \text{ ft.-lb.} \end{cases}$

6. With loads and distances the same as in the last two examples, but with W_1 removed, and the span loaded with W_2, W_3, and W_4 only, what are the values of R_1 and R_2, respectively?

Ans. $\begin{cases} R_1 = 32{,}000 \text{ lb.} \\ R_2 = 58{,}000 \text{ lb.} \end{cases}$

7. With all conditions the same as in the preceding example, what is the bending moment M at the position previously occupied by W_1? At W_2? At W_3? At W_4?

Ans. { At W_1, $M = 640,000$ ft.-lb.
At W_2, $M = 1,280,000$ ft.-lb.
At W_3, $M = 1,520,000$ ft.-lb.
At W_4, $M = 1,160,000$ ft.-lb.

1147. In the preceding articles it has been shown that the values of the reactions may be determined by very simple applications of the principle of moments; the use of these values, when obtained, will now be shown. For a structure, the magnitudes of the loads are either known or assumed, and the points of application of the loads are known; when the values of the reactions from the given loads are determined, all the *external* forces which act upon the structure are known, and the values of the *internal* forces or stresses can usually be determined by drawing the system of force polygons, i. e., the stress diagram, for the structure. All the internal forces which act at the point of application of each external force are unknown, and as the force polygon can only be drawn when not more than two quantities are unknown (Art. **1109**), it is necessary to begin the stress diagram by drawing the force polygon for a joint at which not more than two internal forces act; that is, at a joint at which not more than two members meet. The point of application of a reaction usually fulfils this requirement; it is customary to begin with the left reaction.

1148. Illustration.—Fig. 236 represents a frame or truss of the form known as a **Warren girder**; it is a form of truss often used for bridges. The distance between the reactions, or, in the usual language, the **length of span,** is assumed to be 60 feet, and is divided into four equal portions, or **panels,** B, E, H, and K, each 15 feet in length, as shown in the figure. The loads W_1, W_2, and W_3 are supported, respectively, at the joints *3*, *5*, and *7* (called **panel points**).

It is assumed that $W_1 = 4$ tons, $W_2 = 3$ tons, and $W_3 =$ 2 tons. Therefore (Art. **1144**), the left reaction

$$R_1 = \frac{4 \times 45 + 3 \times 30 + 2 \times 15}{60} = 5 \text{ tons},$$

$$\text{and } R_2 = \frac{4 \times 15 + 3 \times 30 + 2 \times 45}{60} = 4 \text{ tons}.$$

(*a*), Fig. 236, is the stress diagram, drawn to a scale of 4

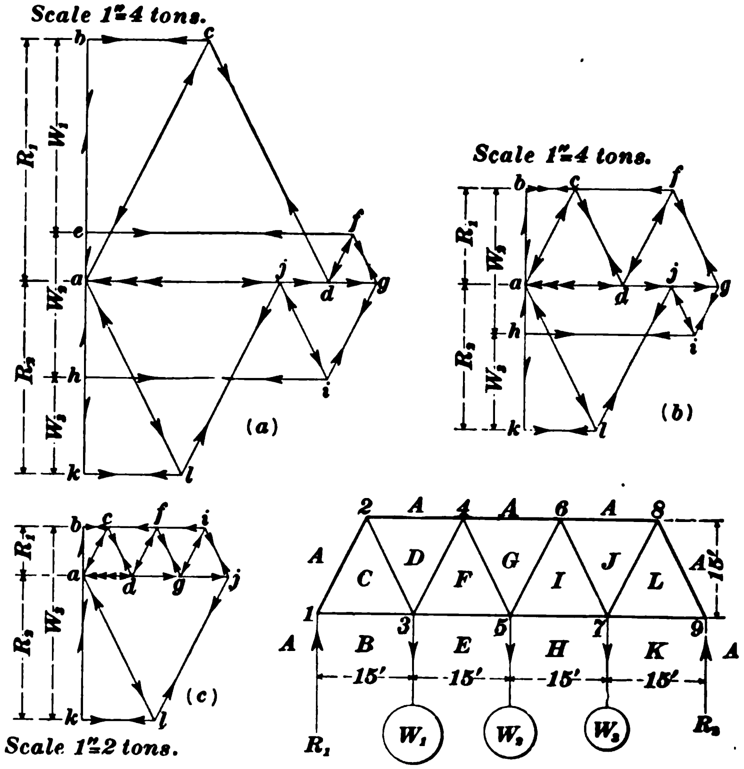


Fig. 236.

tons to the inch, by means of which are determined the internal forces or stresses in all the members of the truss for this system of loads. By the aid of the system of notation used, it will be readily understood and will require but little explanation. Each line upon which two full arrow-heads are marked represents the stress in that member of the truss which is situated between the two corresponding letters in the diagram of the truss. This stress diagram is constructed as follows:

Draw a vertical line *b k*; beginning at the *top*, lay off downwards, the loads $W_1 = B\ E = b\ e$, $W_2 = E\ H = e\ h$, and $W_3 = H\ K = h\ k$ to the scale used; the loads are laid off *downwards*, because they act downwards upon the truss, and they are taken *in order*, passing from left to right across the truss. Next, the reactions are laid off upon the same line, but the order of the operation is *reversed;* beginning at the *bottom* of the line, lay off the reactions *upwards* in the *reverse order*, to the same scale, or passing across the truss from right to left; that is, beginning at *k*, lay off upwards $R_2 = K\ A = k\ a$, and $R_1 = A\ B = a\ b$. It is evident that *b e h k a b* is the *force polygon* for the external forces which act upon the truss; since, through the medium of the truss, these forces are in equilibrium, they form a closed polygon, and, as the forces are all vertical, the polygon is a straight line. Half arrow-heads are marked upon the lines to indicate the directions of the forces. The line *k b*, which represents the sum of the reactions, equals the line *b k*, which represents the sum of the loads (Art. **1140**), or the algebraic sum of these vertical forces is zero. (Art. **1141.**) The line *b k* is called the **load line.** (Art. **1121.**)

Of the forces which act upon joint *1*, all conditions are known except two, the stresses in *B C* and *C A*, the lines of action of the stresses being known; therefore, the stress diagram may be begun by drawing the force polygon for the forces which act upon this joint. R_1 is represented by the line *a b*; therefore, from *b* and *a* draw *b c* and *a c* parallel, respectively, to *B C* and *A C*, and intersecting at *c*. Since the arrow-head on *a b* points from *a* towards *b*, pass the pencil around the polygon from *a* to *b*, then to *c*, and back to *a*, and mark the arrow-heads in the direction in which the pencil moves. These arrow-heads indicate the sense of each stress with reference *to the joint considered*, that is, joint *1*. On *b c* the arrow-head is nearer *b*, and on *c a* it is nearer *a*, because these ends of the lines correspond to the point of application of the respective forces, or, in other words, to the ends of the members *B C* and *C A*, which connect at this joint.

Of the forces which act upon joint *2*, the magnitude of one force, the stress in *A C*, has been determined. As explained in Art. **1120,** it is evident that the stress in this member acts upon joint *2* in a direction opposite to that in which it acts upon joint *1*. Therefore, an additional arrow-head is marked on *a c* in a reversed position and direction, so that it will point from *a* towards *c*; *a d* and *c d* are drawn parallel to *A D* and *C D*, respectively, and the arrow-heads marked on them, starting in the direction of the *reversed* arrow-head on *a c*.

Of the forces which act upon joint *3*, the magnitudes of two forces, the stresses in *B C* and *C D*, have been determined, and the magnitude of the external force W_1, or *B E*, is known; as the lines of action of all the forces are known, but two conditions remain unknown, namely: the magnitudes of the stresses in *E F* and *F D*. To determine these, mark additional arrow-heads in reversed positions and directions on *d c* and *c b*; trace *b e* equal to *W*, and from *e* and *d* draw *e f* and *d f* parallel, respectively, to the members *E F* and *D F*; they intersect at *f*. (See Art. **1113.**) Mark the arrow-heads as usual. For joint *4*, additional arrow-heads are marked in reversed positions and directions on *a d* and *d f*, and *f g* and *a g*, drawn parallel to *F G* and *A G*, respectively.

It is to be noticed that *g a*, not *g d*, represents the stress in *G A*. As has been repeatedly explained, the stress in any member is given by that line of the stress diagram situated between the same two letters between which the member is situated in the diagram of the truss.

For joint *5*, the polygon *g f e h i g* is drawn; for joint *6*, the polygon *a g i j a* is drawn; for joint 7, the polygon *j i h k l j* is drawn; for joint *8*, the triangle *a j l a* is drawn, and for the last joint *9*, the triangle *a l k a* is drawn, completing the diagram. If the work has been done correctly, the line *k a* will equal the reaction R_2 $(= K A) = 4$ tons $= 1$ inch, by the same scale (4 tons to the inch) by which the line *a b* was made equal to R_1.

Each line in the stress diagram which represents stress in a member of the truss forms a side of *two* polygons, because each member is connected at *two* joints.

1149. The correspondence of the large letters on the truss with the small ones in the force polygons is of great assistance; by noticing it carefully, the student will have no difficulty in tracing and checking the stress diagram. Consider, for instance, joint *6*, the polygon for which is *a g i j a*. Notice that the corresponding letters *A G I J A*, on the truss, follow one another around the joint in such a manner that in passing from one to another we move in a direction opposite to that in which the hands of a clock move, and by naming them all we go around an entire circle in the same direction, passing from *A* to *G*, from *G* to *I*, from *I* to *J*, etc. *This gives the directions of the corresponding lines in the diagram for the given joint.* Thus, in joint *6*, the lines go from *a* to *g*, from *g* to *i*, from *i* to *j*, from *j* to *a*, following the order *A G I J A*. This applies to any joint. The lines, of course, are not actually *drawn* in the order given; *j a* is not drawn from *j* to *a*, because the point *j* is not known; to locate it, a line is drawn from *i* parallel to *I J*, and another from *a* parallel to *A J*; their intersection gives the point *j*; but the direction of the stress is from *j* to *a*, according to the rule given above.

If from any cause it should be necessary to draw the first force polygon so that the direction of the letters is from right to left around the joint, all of the others must also be drawn in this order.

1150. If the load W_1 is removed from joint *3*, and the loads W_2 and W_3 are allowed to remain at joints *5* and *7*, then R_1 will equal $\frac{3 \times 30 + 2 \times 15}{60} = 2$ tons, and R_2 will equal $\frac{3 \times 30 + 2 \times 45}{60} = 3$ tons. The diagram (*b*), Fig. 236, is the stress diagram for this condition of loading. For joint *1*, the line *a b* is drawn equal and parallel to R_1 (= *A B*), and *b c* and *a c* drawn parallel to *B C* and *A C*, respectively; the directions of the arrow-heads on *b c* and *a c* are marked to correspond with that on *a b*, whose direction is known. For joint *2*, an additional arrow-head is marked in the opposite position and direction on line *a c*, and the triangle *a c d a* is completed as usual. For joint *3*, when starting at *d*, the reversed arrow-heads have been marked on lines *d c* and *c b*; the next line to be drawn would usually be a line *b e* to represent the load W_1; but as W_1 has been removed, there is no force *B E* acting at this joint, and, consequently, there is no line *b e* to be drawn. The space formerly denoted by *E* is now denoted by *B*, and, instead of the member *E F*, we have the member *B F*. From *b* draw *b f* parallel to *B F*, and from *d* draw *d f* parallel to *D F*; these lines give the stresses in *B F* (or *E F*) and *D F*. The student will follow through in detail the process of drawing the force polygons for the remaining joints; it is substantially the same as in (*a*), Fig. 236, and by the aid of the notation used will be readily understood.

If (*b*), Fig. 236, be compared with (*a*), Fig. 236, one fact is very noticeable: after removing the load W_1, which is the heaviest of the three loads, it is found that the stresses in the members *D F* and *F G* [(*b*), Fig. 236] are *greater* than when W_1 was upon the truss. This would be generally true of the stresses in these members with reference to the load W_1; it is characteristic regarding the positions of loads for maximum stress, and will be noticed again in the analysis of stresses.

1151. At (*c*), Fig. 236, is shown a stress diagram for the truss, loaded with the load W_3 only, the loads W_1 and W_2 both being removed. The entire space between R_1 and W_3 is now denoted by *B*, and, instead of the members *B C*, *E F*, and *H I*, we have *B C*, *B F*, and *B I*. For clearness this diagram is drawn to a larger scale, one of 2 tons to the inch. It will need no special explanation. The feature to be specially noticed is that by removing the load W, the *character* of the stresses in the members $G\,I$ and $I\,J$ is *reversed.* In (*a*) and (*b*), Fig. 236, the character of the stress in the member $G\,I$, as indicated by the arrows marked upon the line $g\,i$, is *tension*, while in (*c*), Fig. 236, the arrows marked upon the line $g\,i$ indicate that the stress is *compression.* Also, in (*a*) and (*b*), Fig. 236, the arrows upon the line $i\,j$ indicate that the stress in the member $I\,J$ is *compression*, while in (*c*) the arrows upon the line $i\,j$ indicate that the stress is *tension.*

1152. The following is a tabulation of the stresses in the members of the truss, as scaled from the stress diagrams (*a*), (*b*), and (*c*), Fig. 236, for the three conditions of loading explained above. They are expressed in tons:

Member.	(*a*)	(*b*)	(*c*)
A C	+ 5.59	+ 2.24	+ 0.56
C D	− 5.59	− 2.24	− 0.56
D F	+ 1.12	+ 2.24	+ 0.56
F G	− 1.12	− 2.24	− 0.56
G I	− 2.24	− 1.12	+ 0.56
I J	+ 2.24	+ 1.12	− 0.56
J L	− 4.47	− 3.35	− 1.68
L A	+ 4.47	+ 3.35	+ 1.68
B C	− 2.50	− 1.00	− 0.25
E F	− 5.50	− 3.00	− 0.75
H I	− 5.00	− 3.50	− 1.25
K L	− 2.00	− 1.50	− 0.75
A D	+ 5.00	+ 2.00	+ 0.50
A G	+ 6.00	+ 4.00	+ 1.00
A J	+ 4.00	+ 3.00	+ 1.50

When a stress is preceded by a + sign, the sign indicates that the stress is *compression*, and when preceded by a − sign, the sign indicates that the stress is *tension*. The signs are sometimes written after the stresses; when used with stresses they have no other meaning.

THE EQUILIBRIUM POLYGON.

1153. In Arts. **1107** and **1108**, it was shown that, by means of the force polygon, either the equilibrant or the resultant of any number of concurring forces can be found.

A method of finding the magnitude, direction, and line of action of either the equilibrant or the resultant of any number of forces, whose lines of action do not meet at a common point, will now be given. The forces are considered as acting in the same plane.

ILLUSTRATION.—In Fig. 237 three forces of 100, 40, and 60 pounds, respectively, are represented by the lines F_1, F_2,

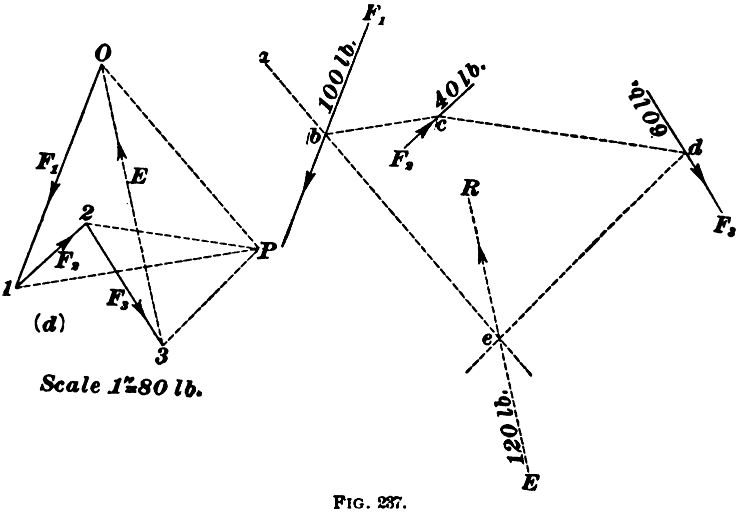


FIG. 237.

and F_3; their magnitudes are represented by the lengths, and their lines of action located, by the positions of the respective lines, while the sense of each force is indicated by an arrow-head marked upon the corresponding line. For these forces, construct the force polygon *0-1-2-3-0*, by drawing end to end the lines *0-1*, *1-2*, and *2-3*, respectively, parallel and equal to F_1, F_2, F_3, and complete the polygon by drawing the closing line *3-0*. The polygon is drawn in the figure to a scale of 80 pounds to the inch. The closing line = 120 pounds, if drawn in the direction *3-0*, represents in magnitude and direction the equilibrant of the forces F_1, F_2, F_3, and, if drawn in the opposite direction *0-3*, it represents the resultant of those forces [(*d*), Art. **1107**; also, (*a*) and (*b*), Art. **1108**]; but the *position* of the line along which the equilibrant or resultant must act is still unknown. To find the line of action of the equilibrant and resultant, proceed as follows:

Choose *any* point, as *P*, and draw the radial lines *P 0*, *P 1*, *P 2*, and *P 3*. Through any point, as *b*, on the line of action of the force F_1 (represented also by *0-1*) draw a line *a b e* of indefinite length and parallel to *P 0*; also, through the same point, draw a line *b c* parallel to *P 1* and intersecting the line of action of the force F_2 at *c*.

The line *b c* is drawn between the lines of action of F_1 and F_2, because the line *P 1*, to which *b c* is drawn parallel, is drawn from *P* to the point of intersection *1* of the line *0-1*, which represents F_1, and the line *1-2*, which represents F_2. Likewise, through *c*, draw a line *c d* parallel to *P 2*, intersecting the line of action of the force F_3 at *d*. Finally, through *d*, draw a line *d e* parallel to *P 3*, intersecting at *e* the line *b e* which was drawn parallel to *P 0*. The line of action of the equilibrant, or of the resultant, of the forces F_1, F_2, and F_3, must pass through the point *e*; this point is the intersection of the lines *b e* and *d e*, which are drawn parallel to the lines *P 0* and *P 3*. These latter lines are drawn from *P* to the extremities of the line *0-3*, which represents the equilibrant, or the resultant in the force polygon *0-1-2-3-0*.

If, through the point *e*, the line *E R* is drawn equal and parallel to *3-0*, it will represent the equilibrant, or, if the line *R E* is drawn in the opposite direction from *R* towards *E*, it will represent the resultant of the forces F_1, F_2, and F_3; the equilibrant and resultant are exactly equal and opposite.

This method of finding the magnitude, direction, and position of the equilibrant and resultant is applicable to any number of forces acting in the same plane. It may also be extended so as to apply to forces acting in different planes, but such application of the method will not here be considered.

1154. In (*d*), Fig. 237, the point *P* is called the **pole;** the lines *P 0*, *P 1*, *P 2*, and *P 3*, which join the pole with the vertexes of the force polygon, are called the **rays,** or **strings.** The complete figure, composed of the force polygon, the pole, and the rays, is called the **force diagram.** The polygon *b c d e b*, Fig. 237, which represents a system of intervening or connecting forces through the medium of which the system of original forces and their equilibrant may be held in equilibrium, is called the **equilibrium polygon.** A line of the equilibrium polygon included between the lines of action of two forces, as the line *b c* or *c d*, etc., is sometimes called a **line of resistance.**

As the pole *P* may be taken anywhere, any number of force diagrams, and, consequently, any number of equilibrium polygons, may be drawn, each of which will give the same value and locate the same line of action for the equilibrant (or for the resultant). To test the accuracy of the work, take a new position for the pole and proceed as before. If the work has been correctly done, the first and last lines of the equilibrium polygon, *b e* and *d e*, which are drawn respectively parallel to *P 0* and *P 3*, the first and last rays of the force diagram, will intersect somewhere on the line of action of the equilibrant and resultant, or *E R*, produced if necessary.

1155. The equilibrium polygon gives an easy method of resolving a force into two components.

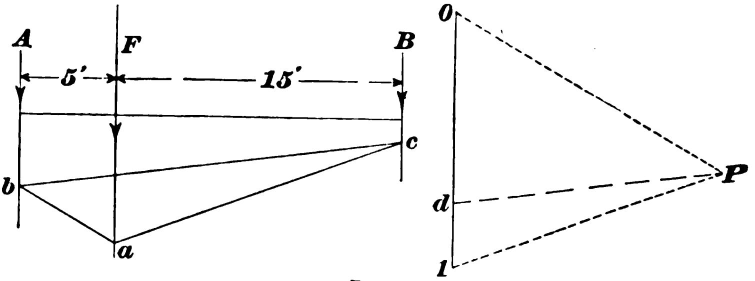


Fig. 238.

In Fig. 238, let $F = 16$ pounds be the force, and let it be required to resolve it into two *parallel* components *A* and

B, at distances, respectively, of 5 feet and 15 feet from F. The magnitudes of A and B are required. Draw *0-1* to represent $F = 16$ pounds; choose any convenient pole P and draw the rays $P0$ and $P1$. Take any point, a on F, and draw ab parallel to $P0$, intersecting A at b, and ac parallel to $P1$, intersecting B at c. Join b and c by the line bc. Through the pole P, draw Pd parallel to bc, intersecting *0-1* at d. Then, $0d$ is the magnitude of A measured with the scale to which *0-1* was drawn, and $d1$ is the magnitude of B measured with the same scale.

The operations may be checked by the method of moments. The process is similar to that for finding the reactions at A and B due to a load F in a corresponding position on the space AB; the components A and B will be equal in magnitude, though opposite in direction, to the respective reactions. Thus, taking moments about A, we have, $B \times 20 = F \times 5$, or $B = \frac{16 \times 5}{20} = 4$, and $A = F - B = 12$, since $A + B = F$; or, taking moments about B, $A \times 20 = F \times 15$; or, $A = \frac{16 \times 15}{20} = 12$, as before. The student should familiarize himself with all these methods and the principles involved.

1156. If the components are not parallel to the given force, they must intersect its line of action in a common point.

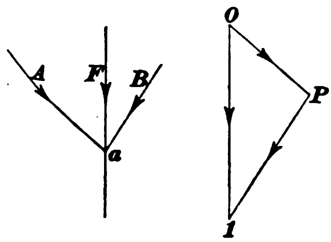


FIG. 239.

In Fig. 239, let $F = 16$ pounds be the force. It is required to resolve it into two components, A and B, intersecting at a, as shown. Draw *0-1* to some convenient scale to represent 16 pounds; then draw $1P$ and $0P$ parallel to A and B; $0P$ and $P1$ are the values of the components both in magnitude and direction.

EXAMPLE.—Let F_1, F_2, F_3, F_4, and F_5 (Fig. 240) be five forces whose magnitudes are 7, 10, 5, 12, and 15 pounds, respectively. It is required to find their resultant and to resolve this resultant into two components parallel to it and passing through the points *a* and *b*.

SOLUTION.—Choose any point, *0*, and draw *0-1* parallel and equal to F_1, *1-2* parallel and equal to F_2, etc.; *0-5* will be the value of the resultant, and its direction will be from *0* to *5*, opposed to the other forces acting around the polygon. Choose a pole *P*, and complete the force diagram. Choose a point *c* on F_1, and draw the equilibrium polygon *c d e f g h c*; the intersection of *c h*, parallel to *P 0*, and *g h*, parallel to *P 5*, gives a point *h* on the resultant *R*. Through *h*, draw *R* parallel

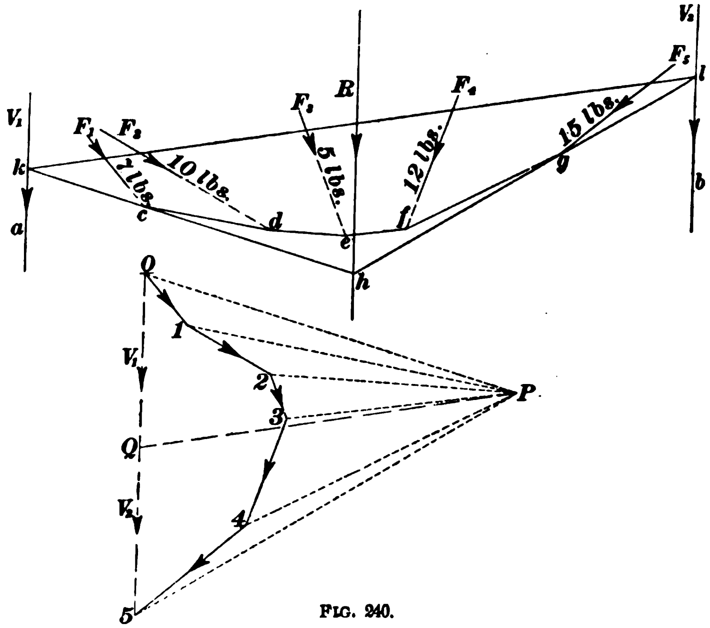


FIG. 240.

to *0-5*, and it will be the position of the line of action of the resultant of the five forces. The components must pass through the points *a* and *b*, according to the conditions; hence, draw V_1 and V_2 parallel to *R* through *a* and *b*. Since *0-5* represents the magnitude of *R*, draw *h k* and *h l* parallel to *P 0* and *P 5* (they, of course, coincide with *c h* and *g h*, since the same pole *P* is used), intersecting V_1 and V_2 in *k* and *l*. Join *k* and *l*, and draw *P Q* parallel to *k l*. Then $0\,Q = V_1$ and $Q\,5 = V_2$.

1157. By the principle explained in the preceding article, the reactions from any system of loads may be found. This will be shown by an example.

EXAMPLE.—Let R_1 be the reaction of the left support, and R_2 the reaction of the right support; let the distance between the two supports be 14 feet. Suppose that loads of 50, 80, 100, 70, and 30 pounds are supported at distances from the left support equal to 2, 5, 8, 10, and 12½ feet, respectively. The reactions of the supports are required, neglecting the weight of the beam. See Fig. 241.

SOLUTION.—The reactions may be found graphically by resolving the resultant of the loads, which in this case acts vertically downwards, into two parallel components passing through the points of support.

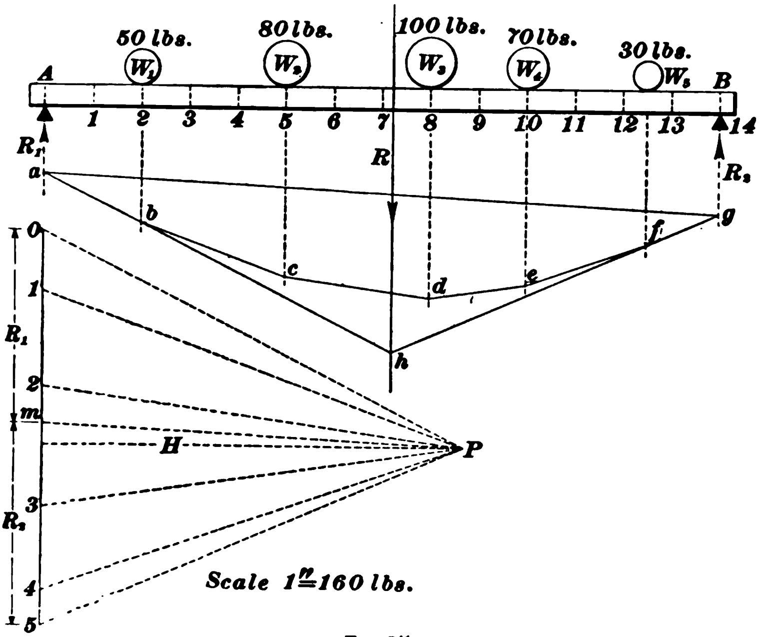


FIG. 241.

The reactions are equal and opposite to the components. Draw the beam to some convenient scale and locate the loads, as shown in the figure. Construct the force diagram, making *0-1* = 50 pounds, *1-2* = 80 pounds, etc., the line *0-5* representing the force polygon. Choose a point *b* on the line of action of the force W_1, and draw the equilibrium polygon *a b c d e f g a*; *a b* and *f g* intersect in *h*, the point through which the resultant *R* must pass. By drawing *P m* parallel to the closing line *a g* of the equilibrium polygon, the resultant *0-5* is resolved into the components *0 m* and *m 5*, which are equal and opposite to the reactions R_1 and R_2, respectively.

Measuring *0 m* and *m 5*, with the same scale that was used to lay off *0-5*, it is found that $R_1 = 160$ pounds, and $R_2 = 170$ pounds. By calculation, $R_1 = 160.4$ pounds, and $R_2 = 169.6$ pounds, which shows that the graphical method is sufficiently accurate for all practical purposes. The larger the scale used, the more accurate will be the results.

1158. In case it had not been desired for any purpose to ascertain the line of action of the resultant (or equilibrant), but simply to determine the values of the reactions

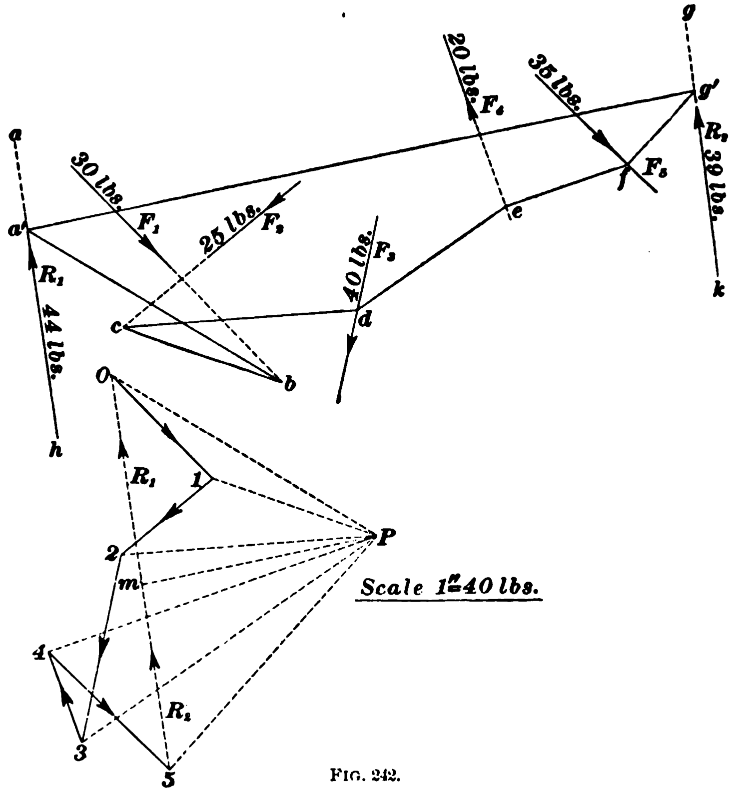


FIG. 242.

R_1 and R_2, which, when applied at the points *A* and *B*, would support or equilibrate the loads, the portions *b h* and *f h* of the lines *a h* and *g h* could have been omitted. The figure *a b c d e f g a* would then be the equilibrium polygon; the force diagram would have been the same.

The method of ascertaining the reactions without locating the line of action of the equilibrant and resultant will be illustrated by a very general

EXAMPLE.—The system of forces $F_1 = 30$ lb., $F_2 = 25$ lb., $F_3 = 40$ lb., $F_4 = 20$ lb., and $F_5 = 35$ lb. have the directions and positions shown in Fig. 242. It is desired to determine the magnitudes and directions of the reactions R_1 and R_2, which, if applied at *a* and *g*, respectively, will equilibrate the system of original forces.

SOLUTION.—Construct the force polygon *0-1-2-3-4-5-0*; the closing line *5-0* represents the equilibrant of the original forces, but its position need not be found. Choose any position for the pole *P* and draw the rays *P 0*, *P 1*, *P 2*, *P 3*, *P 4*, and *P 5*. Through *a* and *g*, which are to be the points of application of the reactions, draw the lines *a h* and *g k* of indefinite length and parallel to *5-0*. From *a*, or from some point on *a h*, as *a'*, draw *a' b* parallel to *P 0*, intersecting the line of action of the force F_1 (produced as far as necessary) at *b*; from *b*, draw *b c* parallel to *P 1*, intersecting the line of action of F_2 at *c*; from *c*, draw *c d* parallel to *P 2*, intersecting the line of action at F_3 at *d*; from *d*, draw *d e* parallel to *P 3*; from *e*, draw *e f* parallel to *P 4*, and from *f* draw *f g'* parallel to *P 5*, intersecting the line *g k* at *g'*. Draw *g' a'*, and through *P* draw *P m* parallel to it, intersecting *0-5* at *m*; $m\,0 = R_1 = 44$ pounds, and $5\,m = R_2 = 39$ pounds. The line *g' a'* is called the **closing line** of the equilibrium polygon.

EXAMPLES.—Solve by the graphical method the examples given in Art. 1143.

GRAPHICAL EXPRESSION FOR MOMENTS.

1159. Culmann's Principle.—The moment of a single force about any point may be expressed and determined graphically in the following manner:

EXAMPLE.—See Fig. 243. Let $F = 10$ pounds represent a force which tends to cause rotation about the point *c* as a center. The lever arm of the moment is $f\,c = 7\frac{1}{2}$ feet.

It is desired to ascertain the moment of *F* about *c*.

SOLUTION.—Draw *0-1* parallel to *F* and equal to 10 pounds to any convenient scale. Choose any point *P* as the pole, and draw the rays *P 0* and *P 1*; also, draw *P 2* perpendicular to *0-1*. Through any point, as *b*, on *F*, draw the lines *b a* and *b g* parallel, respectively, to *P 0* and *P 1*; the lines *b a* and *b g* correspond to the lines *c b* and *c d* of the equilibrium polygon, Fig. 237, through the intersection of which the resultant must pass, the force *F*, in the present case, corresponding to the resultant in Fig. 237. Prolong *a b*, and through *c* draw *c e* parallel to F, intersecting $b\,g$ and $a\,b$ in d and e. By geometry it can be shown that $0\text{-}1 \times f\,c = d\,e \times P\,2$, or, as $0\text{-}1 = F$ and $F \times f\,c$ is the

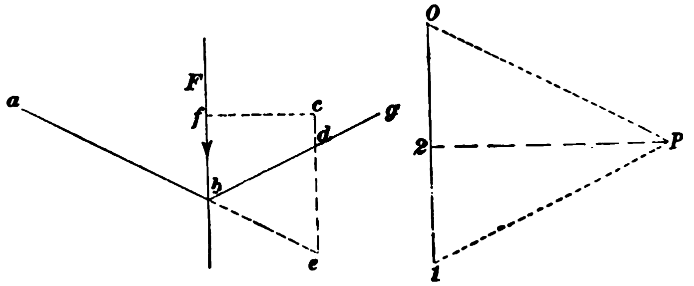


FIG. 243.

moment of F about c, the moment of F about $c = e\,d \times P\,2$, if $e\,d$ is measured to the same scale used for $f\,c$, and $P\,2$ is measured to the same scale used for $0\text{-}1$.

1160. The line $P\ 2$ is called the **pole distance;** it is always drawn perpendicular to the line which represents the force; it is usually designated by the letter H, and will so be designated hereafter. The line $d\ c$, which is intercepted between the two lines which meet upon the line of action of the force (one line, as $a\ b$, being prolonged if necessary), is called the **intercept.**

In the triangle $P\,0\text{-}1$, the lines $P\,0$ and $P\,1$ represent the components of the force F in the directions $a\ b$ and $g\ b$, respectively; while the lines of action of those components are $a\ b$ and $g\ b$, meeting at b. Since $d\ e$ is limited by $g\ b$ and $a\ b$ (the latter produced), we may give the following general definition: The **intercept** of a force whose moment about a point is to be found is the segment (or portion) which the two components (produced, if necessary) cut off from a line drawn through the center of moments parallel to the direction of the force. The principle explained above may be stated as follows:

The moment of a force about any point equals the intercept with respect to that point multiplied by the pole distance.

This is known as *Culmann's Principle;* it is one of the most important facts in Graphical Statics, and should be thoroughly understood.

Since, in Fig. 243, but one force F is represented, it is evident that, in this figure, the equilibrium polygon is not complete; for the action of a single force can not produce a condition of equilibrium. The lines *a b* and *g b*, however, may be any two lines of the equilibrium polygon which intersect upon the line of action of the force.

1161. If the principle given in the preceding article be applied to the resultant of any number of forces, it will give the resultant moment of all the forces about any given point; for the moment of the resultant of the given forces about any point will equal the resultant moment of the same forces about the same point. Culmann's principle thus becomes perfectly general, and is applicable to any number of forces.

Example.—Let $F_1 = 20$ pounds, $F_2 = 25$ pounds, and $F_3 = 18$ pounds be three forces acting as shown in Fig. 244; it is desired to find their resultant moment about the point C.

Solution.—Draw the force diagram and equilibrium polygon, and determine the position of the resultant R in the manner previously

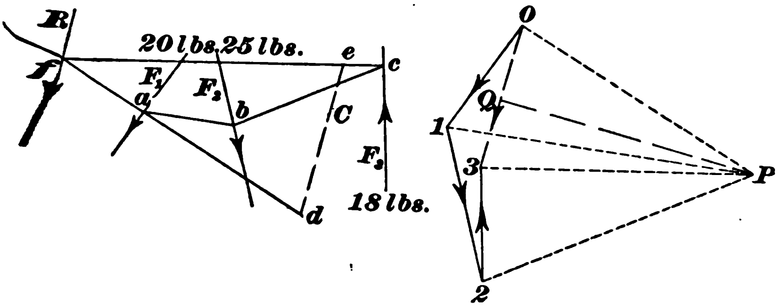


Fig. 244.

explained. Through C draw $C e$ parallel to R. The intercept $d e$, multiplied by the pole distance $P Q$, or H, equals the resultant moment of the given forces about C.

1162. That the moment of the resultant of a system of forces about any point equals the resultant moment of the same forces about the same point will be shown by an example in which the given forces are vertical. This renders the force diagram and equilibrium polygon simpler figures, and the representation of the moments of the forces will be readily understood.

EXAMPLE.—In Fig. 245, the forces $F_1 = 30$, $F_2 = 20$, and $F_3 = 20$ pounds are vertical. F_2 is situated between F_1 and F_3, at a distance of 50 feet from F_1, and of 30 feet from F_3, as shown in the figure. It is desired to ascertain (*a*) the moment of the resultant *R* about the point *C*, which is distant 24 feet horizontally from F_1, and (*b*) the resultant moment of the given forces about the same point.

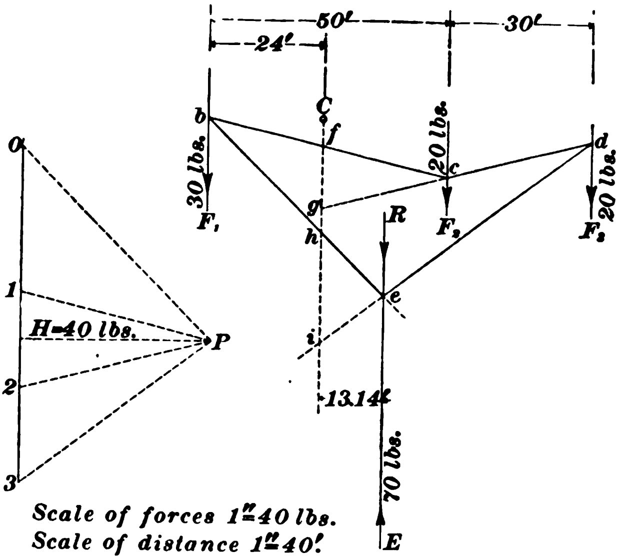


FIG. 245.

SOLUTION.—The force diagram which determines the magnitude and the equilibrium polygon which locates the position of the resultant *R* are constructed in the usual manner. *R* is found to equal 70 pounds, and its position to be at a distance of 13.14 feet horizontally to the right of *C*. By computation the following results are obtained:

Since the forces are parallel and have the same sense, the resultant $= 30 + 20 + 20 = 70$ pounds. Distance of F_2 from $C = 50 - 24 = 26$ ft., and of F_3, $50 + 30 - 24 = 56$ ft. If the sense of *R* be reversed so that it acts upwards, the four forces F_1, F_2, F_3, and *R* will be in equilibrium, and the sum of their moments about $C = 0$. Denoting the distance of *R* from *C* by x, $-30 \times 24 - 70x + 20 \times 26 + 20 \times 56 = 0$, or $x = 13\frac{1}{7}$ ft. Hence, the moment of *R* about $C = 13\frac{1}{7} \times 70 = 920$ foot-pounds.

(*a*) By Culmann's principle (Art. **1160**), the moment of *R* about $C = hi \times H = 23 \times 40 = 920$ foot-pounds to the right, or positive.

(*b*) By the same principle, the moment of F_2 about $C = fg \times H = 13 \times 40 = 520$ foot-pounds, and the moment of F_3 about $C = gi \times H = 28 \times 40 = 1{,}120$ foot-pounds, both positive; the moment of F_1 about $C = -fh \times H = -18 \times 40 = -720$ foot-pounds, to the left, or negative. Therefore, the resultant moment of F_1, F_2, and F_3 about $C = fg \times H + gi \times H - fh \times H = (fg + gi - fh) H = (fi - fh) H = hi \times H$, or $23 \times 40 = 920$ foot-pounds, the same as in (*a*); that is, the resultant moment of the given forces equals the moment of their resultant.

1163. If the resultant R (Fig. 245) is replaced by the equilibrant E, the moment of the latter will be equal in amount, but opposite in character, to the moment of R; that is, the moment of the equilibrant will be -920 foot-pounds. This added algebraically to the resultant moment of the given forces gives $920 - 920 = 0$, as it should be. (See Art. **1134.**)

1164. In Fig. 245, the forces F_1, F_2, F_3, and E do not meet at any common point, and, therefore, can only be in equilibrium through the medium of some structure, which is interposed between their lines of action in such a manner as to transfer the effect of each force in proper proportions to the points of application of the other forces.

If the forces F_1, F_2, and F_3 are equilibrated by the force E, then the effect of E must in some way be transferred to the points of application of F_1, F_2, and F_3, the portion of E transferred to each point of application being equal to the force acting upon that point. It is plain that this can only be accomplished through the medium of some structure in which sufficient resisting (internal) force or *strength* can develop to distribute the effect of the external forces. It is for the purpose of giving sufficient strength to the structure that the effect of the original system of forces on each of its parts is ascertained in making the design. The structure may be a bridge truss, roof truss, stand pipe, beam, column, piece of shafting, part of a steam engine, or any piece of material intended to resist force.

1165. It will be noticed that equilibrium exists between the forces represented in Fig. 245, only when the action of *all* the forces are considered. But, at any point as C, between F_1 and F_2, the forces on one side of the point, if considered separately (as F_1 on the left), have a resultant moment which tends to cause rotation in one direction; while the forces on the opposite side of the point (as F_2, F_3, and E on the right) have a resultant moment which tends to cause rotation in the *opposite* direction. Thus the moment of F_1 (the only force on the left) about $C = -fh \times H$, while the resultant moment of F_2, F_3, and E (on the right) about $C = fg \times H + gi \times H - hi \times H = fh \times H$. The former moment is negative and the latter moment is positive, and, as they are thus equal and opposite, they balance each other. But, as the forces which produce these opposite moments are situated upon opposite sides of the point C, they can only balance or resist each other through the rigidity of the structure. The moment of F_1 would cause that portion of the structure at the left of C to rotate downwards or to the left, while the resultant moment of F_2, F_3, and E would cause that portion of the structure to the right of C to rotate downwards or to the right. The effect of these opposite resultant moments would be to *bend* the structure about the point C (or some point in the line Ci) in much the same manner that you would bend a stick across your knee, if this tendency to bending were not resisted by the opposite moments developed by the internal forces (strength) of the structure. (See Art. **1138.**)

1166. The resultant moment, at any point in a structure, of all the forces upon either side of the point, tending to bend the structure about the point is the *bending moment* in the structure at that point. (See Art. **1136.**)

This bending moment is equal to the intercept between the lines of the equilibrium polygon (drawn through the point parallel to the forces), as fh, multiplied by the pole distance.

1167. In the case of a simple structure carrying vertical loads equilibrated by vertical reactions, the lines of action of all the external forces are vertical, and the intercepts in the equilibrium polygon, which are proportional to the bending moments, are vertical. This is the case of an ordinary bridge, girder, beam, joist, or other structure which carries vertical loads and has vertical supports; it is the most common, as well as the most simple case. The bending moment at any point in such a structure can be found by the following

Rule.—*The bending moment at any point equals the vertical intercept in the equilibrium polygon multiplied by the pole distance.*

1168. The equilibrium polygon drawn to determine the bending moments caused by vertical forces acting upon a structure is called a **moment diagram.**

EXAMPLE.—Fig. 246 represents four equal loads, $F_1 = F_2 = F_3 = F_4 = 200$ pounds, which are supported upon the horizontal beam, and are in turn supported, together with the beam, by the vertical reactions R_1 and R_2. The weight of the beam itself is not considered. It is desired to ascertain the resultant or bending moment at any point, as C, a point at the center of the span. The length of the span and all distances are shown in the figure.

SOLUTION.—The figure *0-4 P 0* is the force diagram in which the force polygon is the straight line *0-1-2-3-4-0*; the line *0-4*, which represents the loads, is the load line (Art. **1121**), while the line in the opposite direction *4-0*, which coincides with the load line and represents the equilibrant or the sum of the reactions, is the closing line of the force polygon. In the present case, the line *P m*, which divides the load line into the reactions *m 0* and *4 m*, coincides with the line *P 2*; the force diagram might have been so drawn that the line H would also have coincided with *P m* and *P 2*. The pole distance $H = 500$ pounds. The polygon *a b c d e f a* is the equilibrium polygon; *f a* is the closing line. It will be noticed that the closing line of the equilibrium polygon is simply a line drawn to the starting point, to *close the polygon;* its position is usually between the lines of action of the reactions. The vertical intercept *g h* between the lines of the equilibrium polygon, when multiplied by the pole distance $H_1 = 24 \times 500 =$ 12,000 foot-pounds, will equal the bending moment upon the beam from all the forces at the left of the point C. For it has been shown (Art. **1160**) that the bending moment of a force about any point is the product of the pole distance by the parallel intercept at that point between those two lines of the equilibrium polygon which meet upon the line of action of the force. Remembering this simple principle, it is plain that the positive moment of R_1 about $C = g\,k \times H = 40 \times 500 = 20,000$

foot-pounds, while the negative moment of F_1 about $C = -ik \times H = -12 \times 500 = -6,000$ foot-pounds, and the negative moment of F_2 about $C = -hi \times H = -4 \times 500 = -2,000$ foot-pounds. But $gk - ik - hi = gh$, and the algebraic sum of $gk \times H$, $-ik \times H$ and $-hi \times H = gh \times H$; the resultant moment of R_1, F_1, and F_2 about C is, therefore, $gh \times H = 24 \times 500 = 12,000$ foot-pounds, which is the same

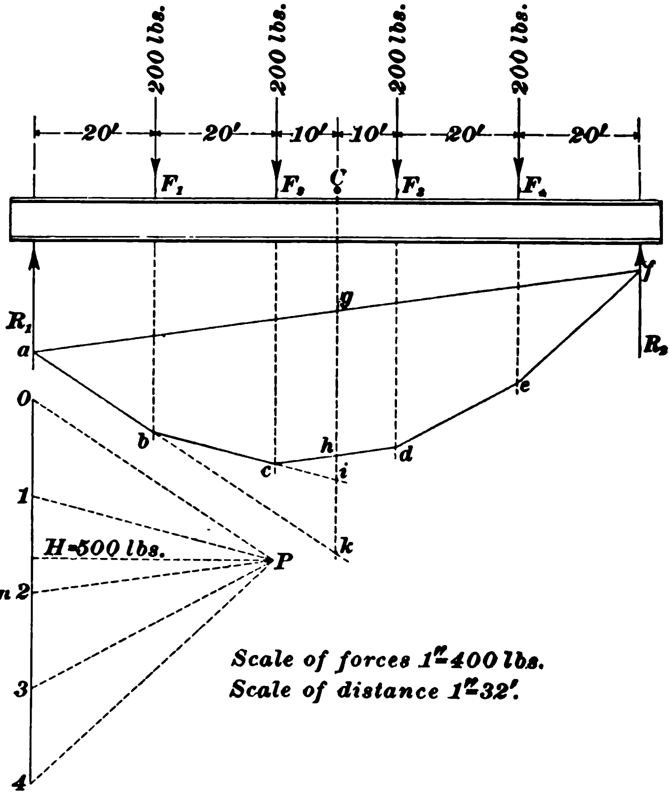


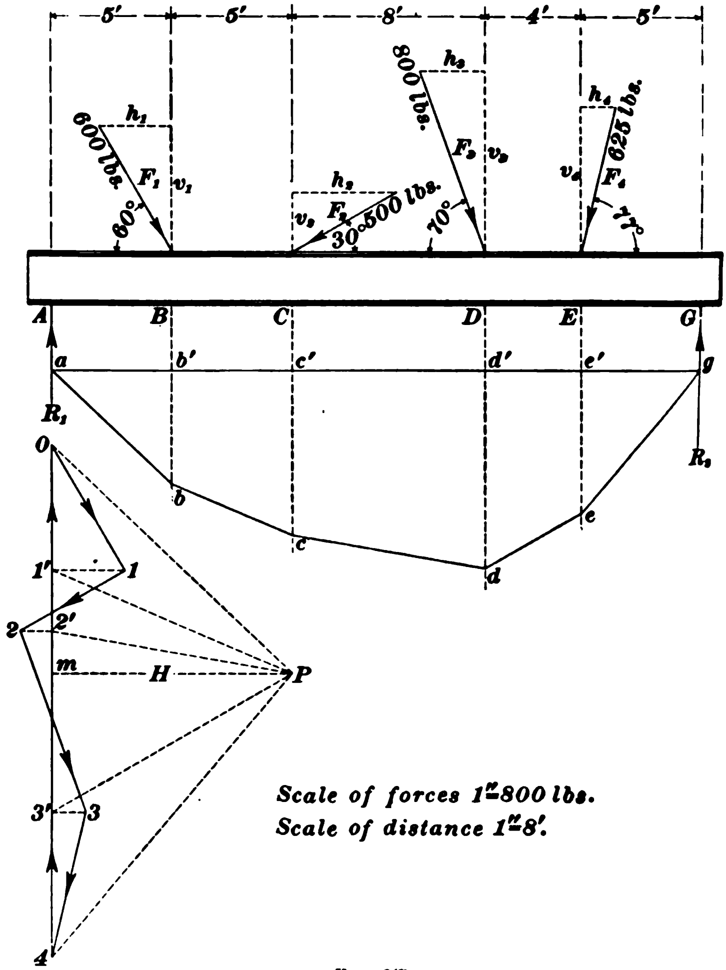
FIG. 246.

as 20,000 − 6,000 − 2,000 = 12,000 foot-pounds. It is thus seen that the resultant moment of the forces at the left of C about that point equals the vertical intercept in the equilibrium polygon at that point multiplied by the pole distance. The same can be shown of the moments of the forces at the right of C, except that the resultant moment would be negative instead of positive. As C may be any point, the principle is perfectly general.

The student may solve by the graphical method the xamples given in Art. **1146.**

1169. If a beam is acted upon by inclined forces, the ending moments resisted by the beam are caused by those omponents of the applied forces which are parallel to the eactions; the two reactions being parallel to each other, and he perpendicular distance between them being considered as the span of the beam. If the reactions are vertical the bending moments are caused by the vertical components of the pplied forces. In Fig. 247, the inclined forces $F_1 = 600$, $F_2 = 500$, $F_3 = 800$, and $F_4 = 625$, all in pounds, act upon the eam $A\ G$; as the reactions are vertical, the bending moments are caused by the vertical components v_1, v_2, v_3, and v_4. The horizontal components, h_1, h_2, h_3, h_4, of the forces do ot produce bending moment, but simply tend to move or lide the beam to the right or left, according to the inclination of each force; this tendency produces direct tension and compression along different portions of the beam. In the present case, the reactions are vertical, and in order that the beams shall not slide to the right or left, the horizontal components of the given forces must balance among themselves. In the figure, it is noticed that the sum of the horizontal components h_1 and h_3 ($= 300 + 273.6 = 573.6$ lb.) which tends to move the beam to the right is just balanced by the sum of h_2 and h_4 ($= 433 + 140.6 = 573.6$ lb.) which tends to move it to the left; consequently, the tendency to cause motion in one direction is just balanced by the opposite tendency, and the beam does not move. This condition of equilibrium between the components of the given forces perpendicular to the reactions will always be fulfilled, if the directions of the reactions are parallel to the equilibrant, or the closing line in the force polygon. In the force polygon *0-1-2-3-4-0*, the equilibrant *4-0* is vertical, indicating that, in this case, the reactions, or at least the resultant of the reactions, must be vertical. To draw the equilibrium polygon that will determine the bending moments upon the beam, proceed as follows:

Draw the force polygon 0-1-2-3-4-0, in the usual manner; the closing line 4-0 = 2,130.8 pounds is the equilibrant. Project each side 0-1, 1-2, 2-3, and 3-4 of the force polygon



Scale of forces 1"=800 lbs.
Scale of distance 1"=8'.

FIG. 247.

upon the equilibrant by lines drawn through the vertexes (1, 2, and 3) perpendicular to the equilibrant, as the lines 1-1', 2-2', and 3-3'. The lines projected upon the equilibrant,

i. e., the portions of the equilibrant cut off by these perpendicular lines, as *0-1'*, *1'-2'*, *2'-3'*, and *3'-4*, represent the components, parallel to the equilibrant, of the forces F_1, F_2, F_3, and F_4; these components are the forces which cause bending moment upon the beam, and *0-1'-2'-3'-4* is, therefore, the *load line* for the beam. These components are, in this case, vertical. Choose any pole *P*, and draw the rays *P 0*, *P 1'*, *P 2'*, *P 3'*, and *P 4* to the points thus determined upon the load line. Through the points of application of the forces and reactions R_1, F_1, F_2, F_3, F_4, and R_2, which act upon the beam, as through *A*, *B*, *C*, *D*, *E*, and *G* (written below the beam for convenience), draw lines parallel to the equilibrant and of indefinite length. These lines, *A a*, *B b*, *C c*, *D d*, *E e*, and *G g*, must be drawn *through the points of application of the forces and parallel to the equilibrant.* Construct the equilibrium polygon by drawing between these parallel lines the lines *a b*, *b c*, *c d*, *d e*, and *e g* parallel to the rays, and, finally, draw the closing line *g a* to the starting point. In the force diagram, the line *P m* which divides the equilibrant *4-0* into the reactions *4 m* and *m 0* is drawn from the pole to the equilibrant parallel to the closing line *g a* of the equilibrium polygon; by the scale of forces R_1, or *m 0* = 944.5 pounds, and R_2, or *4 m* = 1,186.3 pounds. *H* is perpendicular to the equilibrant; in this case, it happens to coincide with *P m*. The intercept between the lines of the equilibrium polygon, from which the bending moment at any point is obtained, must be drawn through the point and *parallel to the equilibrant;* the intercept thus drawn for any point, when multiplied by *H*, equals the bending moment at the point. Thus, the maximum intercept is *d d'*, and $d\,d' \times H = 8.24 \times 1{,}000 =$ 8,240 foot-pounds. The direct compression in the beam between F_1 and F_2 is represented by *1-1'*; *2-2'* represents the tension in the beam between F_2 and F_3, and *3-3'* represents the compression between F_3 and F_4.

1170. In Fig. 246, the closing line of the equilibrium polygon *f a* is not horizontal. The point *f* is simply a point in the line of action of the supporting force R_2; it has no definite position with reference to the starting point *a*, but may be any distance above or below a point horizontally opposite to *a*, and the closing line *f a* may have any inclination, according to the position of the pole in the force diagram. The same is true of the point *g* and the line *g a* in Fig. 247; it is true of the closing line in any equilibrium polygon. It is sometimes desired to draw the equilibrium polygon so that the closing line shall connect two given points in the lines of action of the supporting forces, or so that the closing line shall be horizontal or have some desired inclination. This may always be done by proceeding as in the following

EXAMPLE.—It is desired to construct the equilibrium polygon for the forces F_1, F_2, F_3, and their reactions, between the points *a* and *e*,

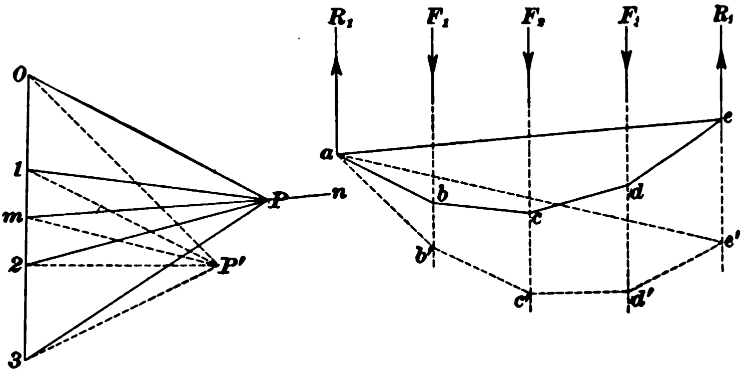


FIG. 248.

Fig. 248, which are points on the lines of action of the supporting forces R_1 and R_2; or, in other words, so that the closing line of the equilibrium polygon will coincide with *e a*.

SOLUTION.—Lay off the load line *0-1-2-3*; choose any pole, as *P'*, and draw the rays *P' 0*, *P' 1*, *P' 2*, and *P' 3*, thus forming the force diagram *0-1-2-3 P' 0*. From this construct the equilibrium polygon *a b' c' d' e' a* in the usual manner, and, in the force diagram, draw the line *P' m* parallel to the closing line *e' a*. The point *m* will divide the load line *0-3* into the reactions *m 0* and *3 m*, without regard to the *direction* of the line *P' m*, which must differ for different positions of the pole. Through the point *m*, draw a line of any length, as *m n*, parallel to the desired closing line *e a* of the equilibrium polygon. For a new pole, choose any point, as *P*, upon the line *m n*; draw the new rays *P 0*, *P 1*, *P 2*, and *P 3*, and construct the new equilibrium polygon in the usual manner by lines drawn parallel to the new rays. The line *e a* will be the closing line of the new equilibrium polygon *a b c d e a*.

EXAMPLES FOR PRACTICE.

In the examples of Art. **1168**, construct the equilibrium polygons so that the closing lines will be horizontal.

1171. When a load is distributed uniformly over the length of a structure, it is divided into sections of any convenient length, and the weight of each section is considered to be concentrated at the center of gravity of the section. The smaller the sections, i. e., the greater the number of sections into which the load is divided, the more accurate will be the results, but for all practical purposes it is not necessary to divide the load into an inconveniently large number of sections. The sections may be of equal or of unequal length, but it is better to make them of equal length when practicable.

EXAMPLE.—Fig. 249 represents a beam, 16 feet long between supports, carrying a uniform load of 960 pounds per foot. The weight of the beam itself is 40 pounds per foot. It is required to construct the equilibrium polygon determining the values of the reactions R_1 and R_2 and the maximum bending moment for the entire load.

SOLUTION.—The total load per foot supported by the beam is $960 + 40 = 1{,}000$ pounds. In order to construct the equilibrium polygon, this uniform load is considered to be divided into sections, each 2 feet long; the center of gravity of each section is at the center of the section, and the weight of each section is $(960 + 40) \times 2 = 2{,}000$ pounds. There are eight sections. The weight of each section is laid off to any convenient scale, upon the load line at *0-1*, *1-2*, ... *7-8*, the pole *P* is chosen and the rays *P 0*, *P 1*, etc., are drawn in the usual manner. The pole distance H is equal to 16,000 pounds. Beginning at any point on the vertical through R_1, as *a*, the line *a b* is drawn parallel to *P 0* to intersect the vertical through the center of gravity of the first section of the load; from *b*, the line *b c* is drawn parallel to *P 1* to intersect the vertical through the center of gravity of the next section of the load; from *c* the line *c d* is drawn parallel to *P 2* to intersect the vertical through the center of gravity of the third section of the load, etc. From *i* on the vertical through the last section of load, the line

ik is drawn parallel to the last ray $P8$, to intersect the vertical through R_2, and finally the closing line ka is drawn joining the verticals through the reactions. In the force diagram, Pm is drawn parallel to ka, dividing the load line into the reactions $8m = R_2$ and $m0 = R_1$. Measuring to the scale of forces, each reaction is found to equal 8,000 pounds. The maximum intercept is found to be at ee', ff' or any point between. This is an error, for, in the case of a really uniform load, the maximum intercept occurs only at the center. This

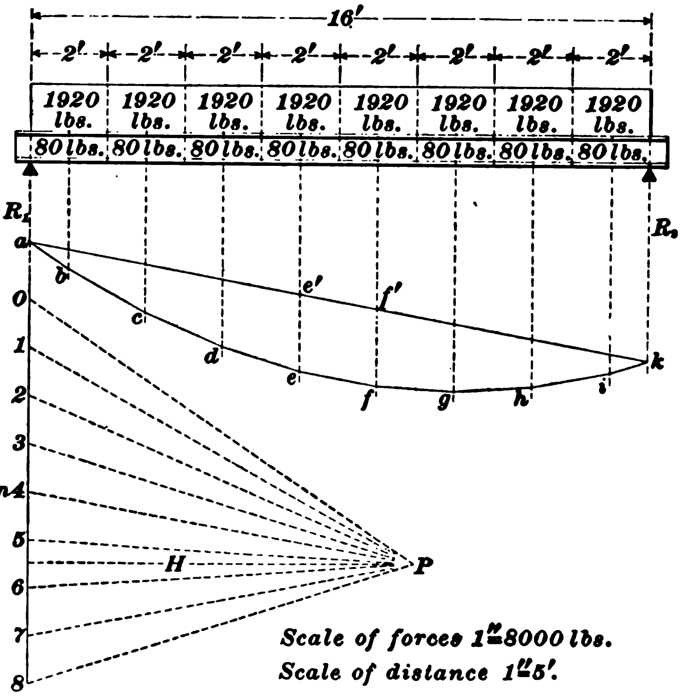


FIG. 249.

slight error is due to the load upon each section having been considered as concentrated at its center of gravity; if the number of sections were greater, the error would be less. Either ee' or ff' equals the maximum intercept. Measuring with the scale of distances $ee' = ff'$ is found to equal 2 feet; therefore, the maximum bending moment equals 2 × 16,000 = 32,000 foot-pounds.

From the preceding we derive the following general

Rule.—*When a beam (or any similar structure) carries a uniform load, divide the beam into any even number of equal* *parts (the more the better); divide the total load into half as many parts; consider the equal parts of the load to be applied alternately at the various points of division of the beam, and treat them as ordinary concentrated loads.*

In the example given, the beam is divided into 16 parts, the load into 8. The first point of division (adjacent to the support) carries a weight of 1,000 pounds; the next one carries no weight; the next one does, etc.

1172. The preceding results may be readily computed by the principles of moments. It is evident that each reaction equals one-half the total load. If the total load be considered concentrated at its center of gravity, either as a whole or as divided into sections, and the reactions computed by the principles of Art. **1144,** each reaction will be found to equal one-half the load; one-half the total load is $\frac{(960 + 40) 16}{2} = 8,000$ lb. For the bending moment at any point, take moments of the forces at the left about that point. At the center, the positive moment of the reaction is $8,000 \times 8 = 64,000$ foot-pounds. The negative moment of that portion of the load at the left of the center (considering it to be concentrated at its center of gravity) is $-8,000 \times 4 = -32,000$ foot-pounds. The algebraic sum of these moments is $64,000 - 32,000 = 32,000$ foot-pounds, which is the same as the result found graphically.

1173. A structure which supports its entire load or system of loads between two reactions, as in the preceding example, is usually known as a **simple structure.** It is evident that, in such a structure, the two reactions are at its respective ends. By far the greatest number of structures are of this class.

But it is sometimes necessary for a structure to extend beyond, and support loads beyond one or both reactions. The projecting or overhanging end of such a structure, that is, the portion of the structure which extends beyond a support, is known as a **cantilever.** Exactly the same principles apply in finding the bending moments in structures having cantilever ends, as in simple structures. The equilibrium polygon for a structure with cantilevers, although somewhat different in form from the equilibrium polygon drawn for a simple structure, is constructed according to the same principles and in practically the same manner.

ILLUSTRATION.—In Fig. 250 is represented a beam upon which are supported three loads, $W_1 = 1{,}000$ pounds, $W_2 = 800$ pounds, and $W_3 = 600$ pounds; the beam supports one load, W_3, beyond its right support. It is desired to ascertain the values of the reactions and bending moments for this condition of loading, neglecting the weight of the beam

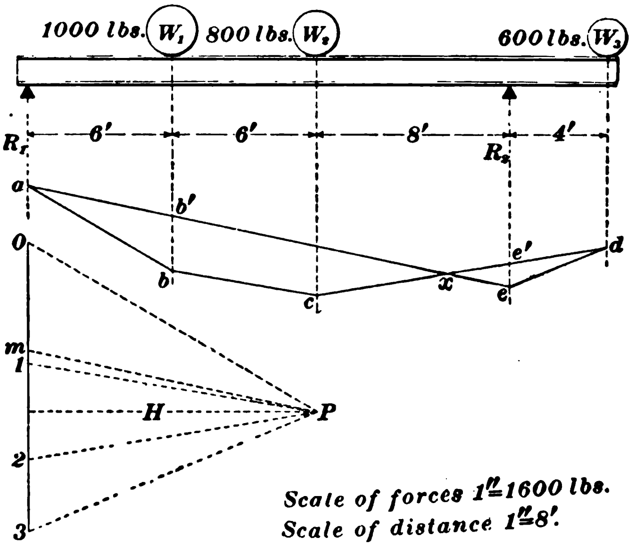


FIG. 250.

itself. Upon the load line *0-1*, *1-2*, and *2-3* are laid off equal to W_1, W_2, and W_3, respectively, to any convenient scale; *0-1-2-3-0* is the polygon of the external forces (loads and reactions) acting upon the beam. The pole P is chosen and the rays of the force diagram $P\,0$, $P\,1$, $P\,2$, and $P\,3$ are drawn in the usual manner. In the figure, the pole distance H is made equal to 2,400 pounds.

Beginning at any point on a vertical line drawn through the left reaction R_1, as the point *a*, draw *a b* parallel to *P 0*, intersecting at *b* the vertical through the first load W_1; from *b* draw *b c* parallel to *P 1*, intersecting at *c* the vertical through W_2; from *c* draw *c d* parallel to *P 2*, *intersecting the vertical through* W_3; from *d* draw *d e* parallel to *P 3*, intersecting the vertical through R_2; and finally draw the closing line *e a*, completing the equilibrium polygon. In the force diagram, the line *P m* is drawn parallel to *e a*, dividing the load line into the reactions $3\ m = R_2$ and $m\ 0 = R_1$. By the scale used, $R_2 = 1{,}500$ pounds, and $R_1 = 900$ pounds. $R_1 + R_2 = 2{,}400 = W_1 + W_2 + W_3$, as it should be.

1174. It will be well to notice quite particularly the construction of the equilibrium and force polygons for the preceding condition of loading. Upon the load line the loads are laid off downwards, beginning with the load furthest at the left and taking all the loads in order, passing across the structure to the right. Thus *0-1* is made equal to W_1, *1-2* is made equal to W_2, and *2-3* is made equal to W_3. It would have been as well to have begun with the load at the right, proceeding in order towards the left; and then to have begun the equilibrium polygon at some point on the vertical through the right reaction, drawing the first line to the vertical through the load furthest to the right. But it is customary to begin with the forces at the left, and, by following this custom and always proceeding in the same order, there will be less liability to confusion and error.

In constructing the equilibrium polygon, therefore, begin at the vertical through the left reaction R_1, and draw the first line *a b* parallel to the first ray *P 0*, intersecting the vertical through the first load W_1; *whether this load be situated at the right or at the left of the reaction.* The next line *b c* is drawn to the vertical through the load next at the right of W_1, or W_2, etc.; the line parallel to the last ray being drawn between the vertical through the load furthest at the right and the vertical through the right reaction. *The closing line of the equilibrium polygon is drawn between the verticals through the reactions.*

1175. In order that the equilibrium polygon may be properly drawn, the point on the load line to which a ray is drawn must be common to (lie between) the same two forces between the lines of action of which is drawn the line of the equilibrium polygon that is parallel to that ray. Thus, in Fig. 250, the point *0* on the load line is common to *m 0*, which represents R_1, and *0-1*, which represents W_1; and in the equilibrium polygon the line *a b*, which is drawn parallel to *P 0*, is drawn between the lines of action of (i. e., the verticals through) R_1 and W_1. Again, the point *m* is common to the two reactions, *3 m*, which represents R_2, and *m 0*, which represents R_1; the line *c a*, which is parallel to *P m*, is drawn between the lines of action of R_1 and R_2. The same is true of any line of this or any properly constructed equilibrium polygon; it connects the lines of action of the same two forces between which the point lies, to which the corresponding ray of the force diagram is drawn. By observing this principle, the equilibrium polygon, of whatever form, may be drawn correctly.

1176. The bending moment at any point of a cantilever or other structure is found by multiplying the intercept in the equilibrium polygon by the pole distance *H*. The position of the maximum intercept can usually be determined by inspection, aided, if necessary, by a few measurements. In a simple structure it is always under a load, but it may be under some portion of a uniformly distributed load. In a cantilever it is always at the support. In a structure having a cantilever end, as that shown at Fig. 250, measurement will determine whether the intercept is maximum at a reaction or under one of the loads. In the present case, it is found that $b\ b' = 2.25$ feet is the maximum intercept; therefore, the maximum bending moment is $2.25 \times 2{,}400 =$ 5,400 foot-pounds. The intercept $c\ c'$ at R_2 equals 1 foot, and the bending moment at that point is $1 \times 2{,}400 =$ 2,400 foot-pounds.

1177. It will be noticed that at *x*, Fig. 250, the lines of the equilibrium polygon cross; at this point there can be no intercept; therefore, the bending moment at this point is zero. At all points at the left of x the bending moment is positive; while at all points at the right of x the bending moment is negative. (Art. **1127.**) Imagine a straight line to be stretched along the top of an actual beam loaded as in the figure, between R_1 and x; the beam would be found to be bent or deflected in such a manner that all points on it between R_1 and x would be slightly *below* the straight line. The bending moment which tends to bend a beam in this manner is considered *positive*. But imagine the straight line to be stretched along the top of the beam, between x and W_2; the beam would be found to be slightly bent in the opposite direction, i. e., so that all points along the top of the beam between x and W_2 would be slightly *above* the straight line. The bending moment which tends to bend a beam in this manner is considered *negative*.

The sign of the bending moment at any point in a structure is the same as that of the resultant moment of the forces at the left of the point. Thus, when computed from the forces at the *left*, the bending moment at W_1 or W_2, Fig. 250, is found to be positive, while at R_2 it is found to be negative, as stated above.

The point x is sometimes called a point of no bending or point of zero moment, but it is more commonly known as a **point of contraflexure** (i. e., contrary flexure).

1178. When the equilibrium polygon is constructed according to the methods just given, it will be found that the tendency of the bending moments is to bend the structure in the direction in which it would be necessary to bend the closing line in order to make it coincide with the other sides of the polygon. Thus, in Fig. 250, the bending moment between R_1 and the point of contraflexure x tends to bend the beam downwards, which is the direction in which $a\,x$ must be bent in order to make it coincide with a, b, c, x; the bending moment between the point of contraflexure and W_2 tends to bend the beam upwards, which is the direction in which $x\,e$ must be bent to make it coincide with $x\,d\,e$.

The bending moment is *positive* where the closing line, or line drawn towards the left, is the *upper* line of the polygon, and *negative* where it is the *lower* line of the polygon. It is thus known that the bending moment, as found at W_1, is positive, or + 5,400 foot-pounds, while the bending moment, as found at R_2, is negative, or — 2,400 foot-pounds.

1179. The values of the reactions and of the bending moments for the above mode of loading can also be readily ascertained by applying the principles given in Arts. **1136** and **1142.** According to Art. **1142,** the algebraic sum of the moments about either reaction is zero. Therefore, if moments are taken about R_2, giving to each moment its proper sign (Art. **1127**), then,

$$R_1 \times 20 - 1{,}000 \times 14 - 800 \times 8 + 600 \times 4 = 0;$$

by performing each multiplication indicated and transposing,

$$20\ R_1 = 14{,}000 + 6{,}400 - 2{,}400, \text{ and } R_1 = \frac{18{,}000}{20} = 900 \text{ lb.}$$

Again, by taking moments about R_1,

$$-R_2 \times 20 + 1{,}000 \times 6 + 800 \times 12 + 600 \times 24 = 0;$$

by multiplying, transposing, and changing signs,

$$20\ R_2 = 6{,}000 + 9{,}600 + 14{,}400, \text{ or } R_2 = \frac{30{,}000}{20} = 1{,}500 \text{ lb.}$$

All the forces acting upon the structure are now known, and the bending moment at any point is simply the resultant moment of all the forces upon either side of that point. (Art. **1136.**) The bending moment at W_1, taking the moment of the force at the left, is $900 \times 6 = 5{,}400$ foot-pounds (positive). If, about the same point, the moments of the forces at the right of the point be taken, there will be $800 \times 6 + 600 \times 18 - 1{,}500 \times 14 = 4{,}800 + 10{,}800 - 21{,}000 = -5{,}400$ foot-pounds (negative). By taking moments about R_2, the resultant moment of the forces at the left is $900 \times 20 - 1{,}000 \times 14 - 800 \times 8 = 18{,}000 - 14{,}000 - 6{,}400 = -2{,}400$ foot-pounds (negative), while the moment of the force at the right about the same point is $600 \times 4 = 2{,}400$ foot-pounds (positive).

PRACTICAL EXAMPLES.

1180. EXAMPLE.—Fig. 251 represents a very common form of crane or derrick. The members are designated by Bow's system of notation. Assuming the dimensions to be as shown and the load W to be 8,000 pounds; construct the stress diagram, determining the stress in each member, the pull upon the guy rope $B\ D$, and the direction and amount of the inclined reaction $A\ D$.

SOLUTION.—Of the three forces which act upon joint *1*, one force, $A\ B$ or W, is known; the lines of action of the other two forces are

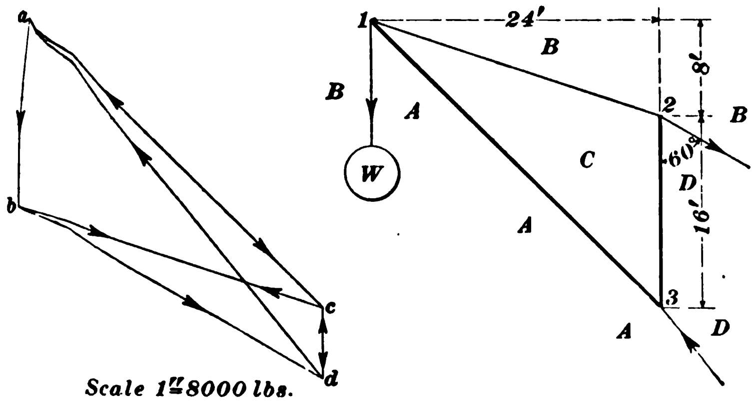


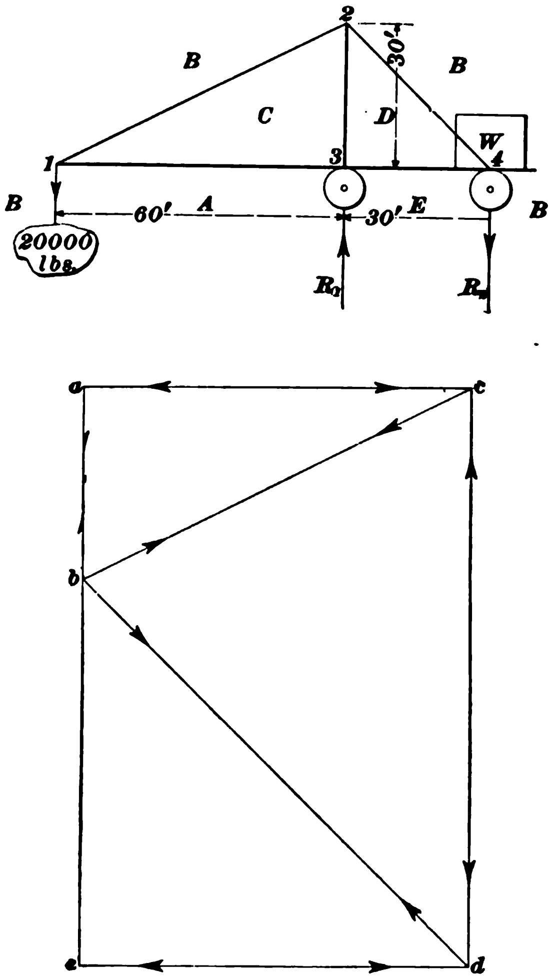
FIG. 251.

also known, for they are the internal forces in the members of the frame $B\ C$ and $C\ A$.

The line $a\ b$ is, therefore, drawn parallel to the direction in which the load $A\ B$ acts upon this joint, or downwards, and to any convenient scale its length is made equal to the magnitude of that force; from b and a the lines $b\ c$ and $a\ c$ are drawn parallel to $B\ C$ and $A\ C$, respectively, and the arrow-heads marked as usual from the known direction of the arrow-head on $a\ b$. For joint *2*, an additional arrow-head is marked in a reversed position and direction on $c\ b$; the lines $c\ d$ and $b\ d$ are drawn parallel, respectively, to $C\ D$ and $B\ D$, and the arrow-heads marked as usual. For joint *3*, additional arrow-heads are marked in reversed positions and directions on $a\ c$ and $c\ d$, and $d\ a$ is drawn. If everything has been done properly, $a\ d$ will be parallel to $A\ D$.

The lines $a\ b$, $b\ d$, and $d\ a$ form the polygon of the external forces; as each external force acts upon but one joint, therefore, but one arrow-head has been marked upon each of these lines. Measuring the lines of the stress diagram to the same scale to which $a\ b$ was made equal to W, and noticing the relative directions of the arrow-heads

upon each line which represents an internal force, the following resu are obtained, in which + indicates compression, and — tension:



Scale 1" = 20000 lbs.

FIG. 252.

Stress in $A\ C = a\ c = +$ 16,970 lb.
Stress in $B\ C = b\ c = -$ 12,650 lb.
Stress in $C\ D = c\ d = +$ 2,930 lb.

Pull upon $B\ D = b\ d =$ 13,860 lb.
Reaction $A\ D = a\ d =$ 19,150 lb.

EXAMPLE.—In Fig. 252 is represented a very simple form or traveling crane, which is sometimes used to hoist large position in erecting structures. It is so constructed that it d along upon a track, and the long arm projected beyond 'e.

; of a system of ropes and pulleys operated by an engine at eights are raised at the outer end as shown. The engine unterweight, and more counterweight is added if neces- ming the dimensions and load to be as shown in the figure, ·ess diagram, determining the stress in each member of the ,d the amount of counterweight W necessary to prevent · from overturning under the action of the load. The ıe traveler itself will be neglected.

ɪ.—In the figure, $a\ e\ d\ c$ is the stress diagram for the forces nd upon the frame, drawn to a scale of 1 inch = 20,000 t joint *1*, three forces act; the lines of action of all three nown, and the magnitude and sense of one force, the load own. Therefore, $a\ b$ is laid off vertically downwards, and , made equal to the load $A\ B$ = 20,000 pounds. The lines ,re drawn parallel to $A\ C$ and $B\ C$, respectively, and the ; marked, starting from a in the direction $a\ b$, and going triangle back to a.

. *2*, mark an additional arrow-head on $b\ c$ in a reversed ,d position, and draw $c\ d$ and $b\ d$, parallel to $C\ D$ and $B\ D$, '; starting from c, mark arrow-heads on the lines in the iven by the reversed arrow-head on $c\ b$.

4, mark a reversed arrow-head on $b\ d$, and draw $b\ e$ and $d\ e$ pectively, to $B\ E$ and $D\ E$, marking the arrow-heads as

nitudes of all the internal forces and of the counterweight w been found; but, in order that the diagram be in every te and the character of each stress determined by the rela- ›ns of the arrow-heads marked upon the lines, it will be › consider the forces which act upon joint *3* with reference t. The magnitudes of all the internal forces which act upon ıve been found in drawing the polygons for joints *1*, *2*, and orce in each member must act upon joint *3* in a direction that in which it acts upon the joint at the opposite end of '; therefore, additional arrow-heads must be marked in the ections and positions on the lines representing the forces ɔers at this meeting joint. The line $e\ a$, pointing upwards, :he reaction $A\ E$ or R_1, and the polygon $a\ c\ d\ e\ a$ is the ɔn for joint *3*.

The straight line $a\ b\ e\ a$ is the polygon of the external forces $A\ B$, $B\ E$, and $E\ A$, or the load line. The algebraic sum of the reactions, or $e\ a + (-b\ e)$, equals the load $a\ b$.

It is to be noticed that the force in $A\ C\ (= a\ c)$ and the force in $E\ D$ $(= e\ d)$ are equal. The other forces which act upon joint *3* are perpendicular to these forces and do not affect them; therefore, the forces in $A\ C$ and $E\ D$ must just balance each other.

By measuring the lines to the nearest hundred pounds with the scale used for $a\ b$, and noticing the relative directions of the two arrow-heads which have been marked upon each line representing an internal force, it is found that:

Stress in $A\ C = a\ c = +40{,}000$ lb.
Stress in $B\ C = b\ c = -44{,}700$ lb.
Stress in $C\ D = c\ d = +60{,}000$ lb.
Stress in $B\ D = b\ d = -56{,}600$ lb.
Stress in $D\ E = d\ e = +40{,}000$ lb.
Reaction $E\ A = e\ a = 60{,}000$ lb.
Counterweight $B\ E = b\ e = 40{,}000$ lb.

1182. EXAMPLE.—In Fig. 253 is represented a roof truss acted upon by the five forces $F_1 = F_2 = F_3 = F_4 = F_5 = 900$ pounds, and the

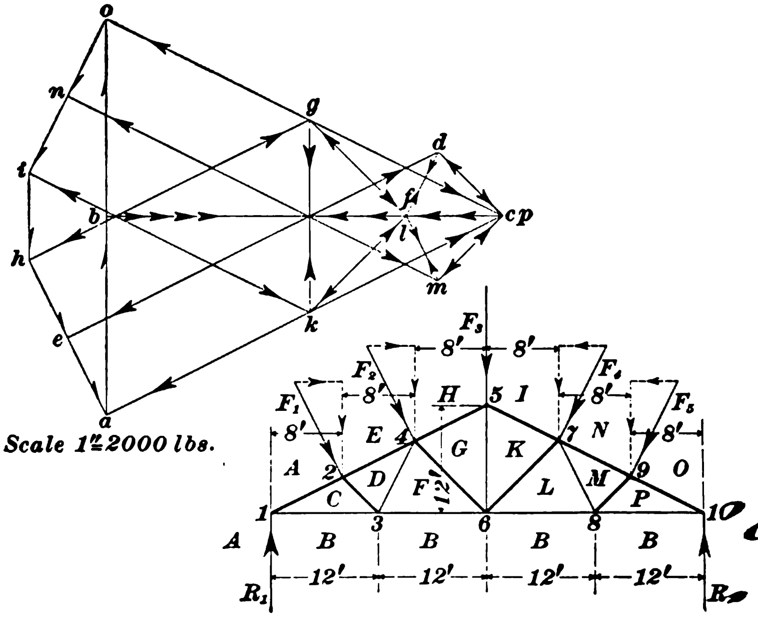


FIG. 253.

two vertical reactions R_1 and R_2. The horizontal distance between any two adjacent forces, as between R_1 and F_1, or between F_1 and F_2, etc., is 8 feet, and the center height of the truss is *5-6* = 12 feet. The truss is of the form known as the **triangular roof truss.** It is desired to find the values of the reactions and to determine the stress in each member of the truss.

SOLUTION.—A diagram of the truss and the directions and positions of the forces are shown in Fig. 253. In the diagram of the truss, the members are designated by Bow's system of notation; the stress diagram is shown at the left, the lines being designated by letters corresponding to the letters which in the diagram of the truss designate the respective members in which the forces act. The polygon of the external forces is first drawn, beginning, for convenience, with F_5, and laying off each force in order, *o n* equal and parallel to *O N*; *n i* equal and parallel to *N I*, etc. The closing line *a o* = 4,120 pounds represents the equilibrant, or the sum of the reactions. That component of each inclined force which affects the reactions (in this case vertical) may be obtained by projecting each inclined force upon the equilibrant by lines perpendicular to the latter, as explained in Art. **1169.** In order to avoid confusion in the stress diagram, this operation is not shown, but in the diagram of the truss each inclined force is represented as resolved into its horizontal and vertical components. (Art. **1156.**) As the horizontal components balance (that is, those whose directions are to the right balance those whose directions are to the left), it is evident that the values of the reactions resist the vertical components only. Hence, only the vertical components of the inclined forces and the vertical force F_3 are used in computing the reaction. The vertical component of each inclined force is found to be 805 pounds.

Therefore, $R_1 = 805\left(\frac{40}{48}+\frac{32}{48}\right) + 900 \times \frac{24}{48} + 805\left(\frac{16}{48}+\frac{8}{48}\right) = 2{,}060$ lb., and $R_2 = 805\left(\frac{8}{48}+\frac{16}{48}\right) + 900 \times \frac{24}{48} + 805\left(\frac{32}{48}+\frac{40}{48}\right) = 2{,}060$ lb. $R_1 + R_2 = 4{,}120$ lb. = the equilibrant *a o*. Therefore, upon the equilibrant, *a b* is laid off upwards equal to R_1 or *A B*, and *b o* equal to R_2, or *B O*; *o n i h e a b o* is the complete polygon of the external forces acting upon the truss. Although the horizontal components of the inclined forces do not affect the values of the reactions, they do affect the magnitudes of the stresses in the members of the truss. In drawing the force polygon for each joint acted upon by an external force, the original force must be used. (If both the horizontal and vertical components are used the effect will be the same.)

Of the three forces acting on joint *1*, the magnitude and direction of one force (the reaction) are known, and the lines of action of the other two forces are known, as they are the internal forces acting in the two members of the truss that connect at this joint; the reaction AB is represented by the line ab. Therefore, draw ac and bc parallel to AC and BC, respectively, and mark arrow-heads as usual.

For joint 2, the forces CA and $AE(=ca$ and $ae)$ are already known. Mark in the opposite direction and position an additional arrow-head on ac, and from c and e draw cd and ed parallel, respectively, to CD and ED (Art. **1113**), and mark the arrow-heads by moving in the directions ea (as shown by arrow-head already marked), ac (as shown by reversed arrow-head), cd, de.

Of the forces which act upon joint 3, the forces in BC and CD are now known; they are represented by the lines bc and cd. Therefore, additional arrow-heads are marked in reversed directions and positions on the lines dc and cb, and from b and d the lines bf and df are drawn parallel to BF and DF, respectively, and the arrow-heads marked by going over the polygon in the direction of the reversed arrow-heads on dc and cb.

Of the forces which act upon joint 4, the external force HE is known, and the forces in ED and DF have been found; these forces are represented by the lines he, ed, and df, respectively. Additional arrow-heads are marked in opposite positions and directions on ed and df; from h and f the lines hg and fg are drawn parallel, respectively, to HG and FG, and arrow-heads marked as usual.

For joint 5, the forces in $IH(=ih)$ and $HG(=hg)$ are known. An additional arrow-head is marked on hg with position and direction reversed; gk and ik are drawn parallel, respectively, to GK and IK, and the arrow-heads marked as usual.

Of the forces which act upon joint 6, only the forces in BL and LK remain unknown. Additional arrow-heads are marked in reversed positions and directions on kg, gf, and fb, and lines bl and kl are drawn parallel, respectively, to BL and KL, and arrow-heads marked as usual.

The operation of drawing the force polygon is substantially the same for each joint, and for each remaining joint of the truss the operation will be readily followed without being explained in detail.

For joint 7, $niklmn$ is the force polygon; for joint 8, $mlbpm$ is the polygon; for joint 9, $onmpo$ is the polygon, and $opbo$ is the polygon for joint 10. This last polygon completes the stress diagram; the last line in this polygon is the line bo that was laid off equal to the reaction R_2.

The arrow-head on each line is always marked nearer the end of the line which corresponds to the point of application of the force, according to Art. **1099**.

From the stress diagram, the character and magnitude of the stress in each member of the truss are found to be as follows:

Stress in $A\ C = a\ c = +\ 4{,}610$ lb.
Stress in $B\ C = b\ c = -\ 4{,}120$ lb.
Stress in $C\ D = c\ d = +\ 950$ lb.
Stress in $D\ E = d\ e = +\ 4{,}290$ lb.
Stress in $D\ F = d\ f = -\ 750$ lb.
Stress in $B\ F = b\ f = -\ 3{,}110$ lb.
Stress in $F\ G = f\ g = +\ 1{,}430$ lb.
Stress in $G\ H = g\ h = +\ 3{,}260$ lb.
Stress in $G\ K = g\ k = -\ 2{,}010$ lb.
Stress in $I\ K = i\ k = +\ 3{,}260$ lb.
Stress in $K\ L = k\ l = +\ 1{,}430$ lb.
Stress in $B\ L = b\ l = -\ 3{,}110$ lb.
Stress in $L\ M = l\ m = -\ 750$ lb.
Stress in $M\ N = m\ n = +\ 4{,}290$ lb.
Stress in $M\ P = m\ p = +\ 950$ lb.
Stress in $B\ P = b\ p = -\ 4{,}120$ lb.
Stress in $P\ O = p\ o = +\ 4{,}610$ lb.

It will be noticed that the stresses in all members to the right of $G\ K$ are the same as those in the corresponding members to the left. This might have been anticipated, since the loads are systematically distributed with respect to $G\ K$. In all cases of this kind it is sufficient to determine the stresses in one-half of the truss (the center member included).

1183. For the following examples, and for those given in the questions, results which do not vary more than one per cent. from the answers given will be considered sufficiently accurate. Usually, results between 10 and 100 pounds will be given to the nearest tenth of a pound; between 100 and 1,000 pounds, to the nearest pound; between 1,000 and 10,000 pounds, to the nearest ten pounds, and results above 10,000 pounds will be given to the nearest hundred pounds. For graphical work it is always best to use as large a scale as may be convenient, but in all practical work the results will be sufficiently accurate without using an inconveniently large scale, if the lines are carefully drawn. In order to obtain the required degree of accuracy in graphical constructions, the lines should be drawn with a hard and well sharpened pencil. All lines must be distinct and must give results within the above limits, for accuracy, but need not be drawn in ink. Hard pencil lines are preferable to ink lines for extended or complicated graphical work.

EXAMPLES FOR PRACTICE.

1. A beam rests upon two supports which are 12 feet apart. It supports a load W_1 4 feet to the left of the left support, a load W_2 at the center, or midway between the supports, and a load W_3 4 feet to the right of the right support; each load is equal to 600 pounds. Compute (*a*) the value of the left reaction R_1, (*b*) the value of the right reaction R_2, (*c*) the value (magnitude and sign) of the bending moment at R_1, (*d*) at the center, and (*e*) at R_2. The weight of the beam will be neglected in this and the following examples.

Ans. { (*a*) + 900 lb.
(*b*) + 900 lb.
(*c*) − 2,400 ft.-lb.
(*d*) − 600 ft.-lb.
(*e*) − 2,400 ft.-lb.

2. Solve the preceding example by constructing the equilibrium polygon.

3. In the above example, considering the load W_2 to be removed from the center of the span, and the loads W_1 and W_3 to remain upon the beam, compute the values (*a*), (*b*), (*c*), (*d*), and (*e*).

Ans. { (*a*) + 600 lb.
(*b*) + 600 lb.
(*c*) − 2,400 ft.-lb.
(*d*) − 2,400 ft.-lb.
(*e*) − 2,400 ft.-lb.

4. Construct the equilibrium polygon for the conditions given in the preceding example, obtaining the results graphically.

5. Considering the load W_3 to be removed, but the loads W_1 and W_2 to be upon the beam in their respective positions, compute the values (*a*), (*b*), (*c*), (*d*), and (*e*).

Ans. { (*a*) + 1,100 lb.
(*b*) + 100 lb.
(*c*) − 2,400 ft.-lb.
(*d*) + 600 ft.-lb.
(*e*) 0.

6. Construct the equilibrium polygon for the preceding example, obtaining the results graphically.

7. Considering the loads W_1 and W_3 removed, but the load W_2 to be upon the center of the beam; compute the values (*a*), (*b*), (*c*), (*d*), and (*e*).

Ans. { (*a*) + 300 lb.
(*b*) + 300 lb.
(*c*) 0.
(*d*) + 1,800 ft.-lb.
(*e*) 0.

8. Construct the equilibrium polygon for the preceding example, obtaining the results graphically.

9. Considering the loads W_2 and W_3 removed, but the load W_1 to be in its position upon the left end of the beam, compute the values (*a*), (*b*), (*c*), (*d*), and (*e*).

Ans. (*a*) + 800 lb.
(*b*) − 200 lb.
(*c*) − 2,400 ft.-lb.
(*d*) − 1,200 ft.-lb.
(*e*) 0.

10. Construct the equilibrium polygon for the preceding example, obtaining the results graphically.

11. A beam 15 feet long between supports weighs 40 pounds per foot. It carries, besides its own weight, a uniformly distributed load of 360 pounds per foot and a concentrated load $W = 3,600$ pounds, at a distance of 5 feet from the left reaction R_1. Construct the equilibrium polygon and determine the value of (*a*) the left reaction, (*b*) the right reaction, and (*c*) the bending moment under the load.

Ans. (*a*) 5,400 lb.
(*b*) 4,200 lb.
(*c*) 22,000 ft.-lb.

12. With the same beam and uniform load, but with the load W at a distance of 6 feet from the left reaction, find the values (*a*), (*b*), and (*c*) as before.

Ans. (*a*) 5,160 lb.
(*b*) 4,440 lb.
(*c*) 23,760 ft.-lb.

13. (*a*) What position of the load W upon the beam will give the greatest bending moment? (*b*) What is the value of this bending moment? (*c*) What is its sign? Ans. (*b*) 24,750 ft.-lb.

14. See Fig. 251. Assume the mast CD to be 24 feet high; the member BC to be 24 feet long and horizontal (having the position of the dotted line marked *2-4'*); the guy rope BD making an angle of 60 degrees with the vertical mast, as in the figure, and the load W remaining the same (8,000 pounds). Draw the stress diagram for the derrick under the above assumptions, determining the stress in each member, the pull in the guy rope BD, and the amount of the reaction AD.

Ans. Stress in $AC = +$ 11,310 lb.
Stress in $BC = -$ 8,000 lb.
Stress in $CD = +$ 4,620 lb.
Pull in $BD =$ 9,240 lb.
Reaction $AD =$ 14,940 lb.

HEAT.

THE PROPERTIES, SOURCES, AND MEASUREMENT OF HEAT.

089. The Nature of Heat.—As to the exact nature heat, scientists differ, but all modern thinkers and invesators agree that *heat is a form of energy*, and that it *is a d of motion.* It is not purposed here to enter into the erent theories regarding heat, but as much of the genlly accepted theory will be given as will be necessary to ke clear the principles which are to follow.

n Art. **831** it was stated that bodies were composed of lecules. Notwithstanding the extreme minuteness of the lecules, they play a very important part in the modern ory of heat. Each molecule attracts the molecules surnding it in a manner similar to the attraction between earth and bodies near its surface, only with an immensely ater force in proportion to their sizes. Without going o any theory regarding the precise nature of heat, it will taken for granted that each and every molecule has a id vibratory motion to and fro, and that the molecules kept from getting beyond a certain distance from one other by the attractive force between them. This atctive force is called **cohesion**; without it, everything roughout the universe would crumble instantly into the est dust.

In Art. **846** it was stated that the molecules were supsed to be round; it is likewise supposed that they are at a nsiderable distance apart, compared with the diameter of e molecule. When heat is applied to a body the number these vibrations is greatly increased, proportional to the nount of heat supplied. In consequence of this increase, e distance through which a molecule moves is increased, and the force of cohesion which binds them together is lessened. If enough heat is added to a solid, the force of cohesion is so far overcome that the body melts. If more heat is supplied in sufficient quantity, the melted body becomes a vapor, and so long as it is kept at this temperature the force of cohesion has no effect, in consequence of the number of vibrations having been so far increased and the distance between any two molecules having become too great for the force of cohesion to act. If the vapor be cooled, the number of vibrations will decrease, and also the distance between any two molecules; the force of cohesion begins to act, and the body becomes a liquid. If cooled further, and a sufficient quantity of heat is removed—in other words, if the number of vibrations is so far decreased that the molecules are comparatively near together—the body becomes a solid and remains so until the temperature is again increased to the melting point.

1090. If a body is heated and brought near the hand, the sensation of warmth is felt; if heat be removed from this same body, and it is again brought near the hand, the sensation of cold is felt. The heat which thus manifests itself is called **sensible heat,** because any change from any state to a hotter or colder state is indicated at once by the sense of feeling, or by the aid of instruments called **thermometers.** The more sensible heat a body possesses, the hotter it is; the more sensible heat that is taken away from it, the colder it is.

THERMOMETERS.

1091. The different states that a body is in according to the amount of sensible heat it possesses are indicated by the word **temperature,** and by comparison with some other body having the same amount of sensible heat. Thus, a piece of iron having exactly the same amount of sensible heat as a piece of melting ice, is said to have *the temperature of melting ice.* If a piece of lead has the same amount of sensible heat as a kettle of boiling water, it is said to have *the temperature of boiling water*, etc.

1092. Owing to the imperfection of our senses, it is impossible to determine by their aid, with any degree of accuracy, the temperature of different bodies; hence, for this purpose, thermometers are used. In these instruments the effects of heat upon bodies are made use of in obtaining the temperature, the most common method being to utilize the expansive effect of heat upon liquids. Liquids are used for ordinary purposes in place of solids or gases, because in the first the expansion is too small, and in the second too great. Mercury and alcohol are the only liquids used—the former because it boils only at a very high temperature, and the latter because it does not solidify at the greatest known cold produced by ordinary means.

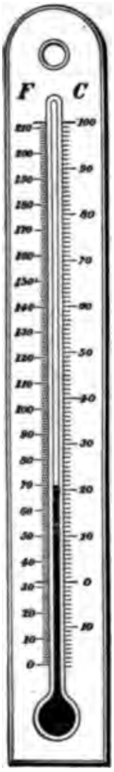


FIG. 214.

1093. In Fig. 214 is shown a mercurial thermometer with two sets of graduations on it. The one on the left, marked *F*, is the **Fahrenheit thermometer,** so named after its inventor, and is the one commonly used in this country and in England; the one on the right, marked *C*, is the **Centigrade thermometer,** and is used by scientists throughout the world, on account of the graduations being better adapted for calculations. As will be seen, the instrument consists of a glass tube having a bulb at one end and closed at the other, so as to keep the air out. Before closing the upper end, the tube is partially filled with mercury, and the air above it is driven out by heating the mercury to near its boiling point, when the tube above the mercury will be filled with mercurial vapor. It is now sealed, and, on cooling, the vapor condenses and a vacuum results. The expansion or contraction of the mercury, by applying or withdrawing heat from the body with which the bulb is in contact, causes the highest point of the mercury column to rise or fall, and, since for equal changes of temperature the mercury rises or falls equal distances, this instrument, when properly made and graduated, indicates any change in temperature with great accuracy.

1094. For a good thermometer, the inside diameter should be the same throughout its length. In order to graduate the thermometer, it is placed in melting ice, and the point to which the mercurial column falls is marked **freezing.** It is then placed in the steam rising from water boiling in an open vessel, and the point to which the mercurial column rises is marked **boiling.**

1095. There are now two fixed points, the freezing point and the boiling point. If it is desired to make a Fahrenheit thermometer, the distance between these two fixed points is divided into 180 parts, called degrees. The freezing point is marked 32°, and the boiling point 212°. Thirty-two parts are marked off from the freezing point downwards, and the last one is marked 0°, or zero. The graduations are carried above the boiling point and below the zero point as far as desired. This thermometer was invented in 1714, and was the first to come into general use.

1096. In graduating a Centigrade thermometer, the freezing point is marked 0°, or zero, and the boiling point 100°; the distance between the freezing and boiling points is divided into 100 equal parts; these equal divisions are carried as far below the freezing point and above the boiling point as desired. The reason that Fahrenheit placed the zero point on his thermometer 32° below freezing was because that was the lowest temperature he could obtain, and he supposed that it was impossible to obtain a lower one. Where there is any doubt as to the thermometer used, the first letter of the name is placed after the degree of temperature. For example, 183° F. means 183° above zero on the Fahrenheit instrument; 183° C. would mean 183° above zero on the Centigrade instrument.

1097. In Russia and a few other countries another instrument is used, called the **Reaumur**; the freezing point is marked 0°, or zero, and the boiling point 80°, the space between these two points being divided into 80 equal parts; 183° R. would mean 183° on the Reaumur thermometer.

1098. Of these three thermometers, the Centigrade is used the most; but, since the Fahrenheit instrument is the one in general use in this country, all temperatures given here will be understood to be in Fahrenheit degrees, unless otherwise stated. In order to distinguish the temperatures below the zero point from those above, the sign of subtraction is placed before the figures, indicating the number of degrees below zero. Thus, $-18°$ C. would mean that the temperature was 18° below the zero point on the Centigrade thermometer; $-25.4°$ F. would mean 25.4° below zero on the Fahrenheit thermometer. As was stated in Art. **1056,** the point of absolute zero, or $-460°$ F., is the point at which all vibratory motion of the molecules ceases. It is supposed that, at this temperature, and under a heavy pressure so as to bring the molecules close enough together, even hydrogen would be solidified. The absolute zero on the Centigrade scale is $-273\frac{1}{3}°$ C.

1099. The **absolute temperature** is the temperature measured above the point of absolute zero. Hence, on the Fahrenheit scale, the absolute temperature T is $460° + t°$ when $t =$ the ordinary temperature, and is above zero. If $t°$ is below zero, its value is negative, and the absolute temperature T is $460° + (-t°) = 460° - t°$.

Throughout the remainder of this volume, where temperatures are mentioned, t will denote the ordinary temperature indicated by the thermometer, and T the absolute temperature.

EXAMPLE.—What are the absolute temperatures of 212°, 32°, and $-39.2°$?

SOLUTION.—Since no scale is specified, the Fahrenheit is the one intended to be used.

$460° + 212° = T = 672°$. Ans.
$460° + 32° = T = 492°$. Ans.
$460° - 39.2° = T = 420.8°$. Ans.

1100. The absolute temperature on the Centigrade scale is $T = 273\frac{1}{3}° + t°$ when $t°$ is above zero, or $T = 273\frac{1}{3}° - t°$ when $t°$ is below zero.

EXAMPLE.—What are the absolute temperatures corresponding to 100°, 4°, and − 40° C.?

SOLUTION.—$273\frac{1}{3}° + 100° = T = 373\frac{1}{3}°$ C.
$273\frac{1}{3}° + 4° = T = 277\frac{1}{3}°$ C.
$273\frac{1}{3}° - 40° = T = 233\frac{1}{3}°$ C.

1101. It is frequently necessary to change from one scale to the other. For example, what would 80° C. be on the Fahrenheit scale?

Since the number of degrees between the freezing point and boiling point on the Centigrade scale is 100, and on the Fahrenheit 180, it is evident that if F = the number of degrees Fahrenheit, and C = the number of degrees Centigrade, that

$$F : C :: 180 : 100, \text{ or } F = \frac{180}{100} C = \frac{9}{5} C.$$

Also, $C = \frac{100}{180} F = \frac{5}{9} F$. Therefore,

1102. To change Centigrade temperatures into their corresponding Fahrenheit values:

Rule.—*Multiply the temperature Centigrade by $\frac{9}{5}$, and add 32°; the result will be the temperature Fahrenheit.*

1103. To change Fahrenheit temperatures into their corresponding Centigrade values:

Rule.—*Subtract 32° from the temperature Fahrenheit, and multiply by $\frac{5}{9}$, and the result will be the temperature Centigrade.*

1104. Expressing these two rules by means of formulas, let t_c = temperature Centigrade, and t_f = temperature Fahrenheit. Then,

$$t_f = \frac{9}{5} t_c + 32°, \qquad \textbf{(65.)}$$

$$\text{and } t_c = (t_f - 32°)\frac{5}{9}. \qquad \textbf{(66.)}$$

EXAMPLE.—Change (*a*) 100° *C.*, (*b*) 4° *C.*, and (*c*) —40° *C.* into Fahrenheit temperatures.

SOLUTION.—(*a*) $t_f = \frac{9}{5} t_c + 32 = \frac{9}{5} \times 100 + 32 = 212°$ *F.* Ans.
(*b*) $t_f = \frac{9}{5} \times 4 + 32 = 39.2°$ *F.* Ans.
(*c*) $t_f = \frac{9}{5} \times -40 + 32 = -40°$ *F.* Ans.

EXAMPLE.—Change (*a*) 60° *F.*, (*b*) 32° *F.*, and (*c*) −20° *F.* into their corresponding Centigrade temperatures.

SOLUTION.—(*a*) $t_c = (t_f - 32)\frac{5}{9} = (60 - 32)\frac{5}{9} = 15\frac{5}{9}°$ *C.* Ans.
(*b*) $t_e = (32 - 32)\frac{5}{9} = 0°$ *C.* Ans.
(*c*) $t_c = (-20 - 32)\frac{5}{9} = -28\frac{8}{9}°$ *C.* Ans.

1105. Since mercury freezes at −37.84° F. (this corresponds to −38.8° C.), some other means must be had to obtain temperatures below this point. For this purpose alcohol is used in place of mercury. This liquid has never been frozen until very recently, and then only at an extremely low temperature. Since alcohol vaporizes at 173° F., the boiling point of water cannot be marked on the alcohol thermometer by heating it to that point. The freezing point is determined as for mercury. An alcohol and a mercurial thermometer are placed in a vessel containing hot water or other liquid, and the point to which the alcohol column rises is marked. Suppose that the point to which the mercury column rises is marked 132°, then the distance between the point marked and the freezing point would be divided into 132 − 32 = 100 equal parts, and each one of these parts would correspond to one degree on the mercurial thermometer. These equal divisions are then carried below the zero point as far as it is desired.

1106. There are many other kinds of thermometers, some of which depend upon the expansion and contraction of different metals and gases when heated and cooled. For temperatures above 662° F., the point at which mercury vaporizes, other means are employed to obtain the temperatures.

EXPANSION OF BODIES.

1107. The volume of any body, solid, liquid, or gaseous, is always changed if the temperature is changed; nearly all of them expand when heated, and contract when cooled. In solids, which have definite figures, the expansion may be considered in three ways, according to the conditions: 1st.—The expansion in one direction, as the elongation of an iron bar; this is called **linear expansion.** 2d.—**Surface expansion,** where the area is increased. 3d.—**Cubical expansion,** where the increase in the whole volume is considered.

1108. In Fig. 215 is shown an apparatus for exhibiting the linear expansion of a solid body. A metal rod *A* is fixed at one end by a screw *B*, the other end passing freely

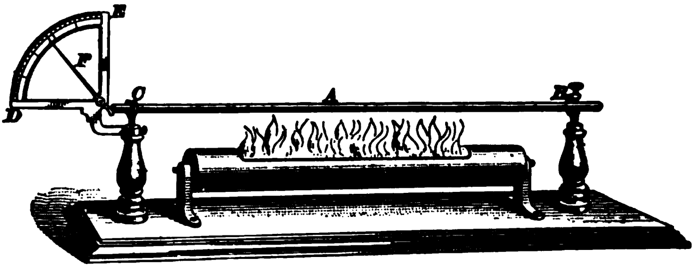


FIG. 215.

through the eye *C*, held in the post, and pressing against the short arm of the indicator *F*. The rod is heated as shown, and its elongation causes the indicator to move along the arc *D E*.

1109. An illustration of surface expansion is afforded nearly every day in machine shops, particularly in locomotive shops, where piston rods, crank-pins, etc., are "shrunk in" and tires shrunk on their centers. In shrinking on a tire, it is bored a little smaller than the wheel center. The tire is then heated until the area of its circumference is expanded enough to allow it to slide over the wheel center. It is then cooled with cold water, when it contracts, tending to regain its original area, but is prevented by reason of the wheel center being a trifle larger. This causes the tire to hug the center with immense force and prevents it from coming off.

1110. Cubic expansion may be illustrated by means of a *Gravesandes' ring*. This consists of a brass ball *a*, Fig.

216, which at ordinary temperatures passes freely through the ring *m*, of very nearly the same diameter. When the

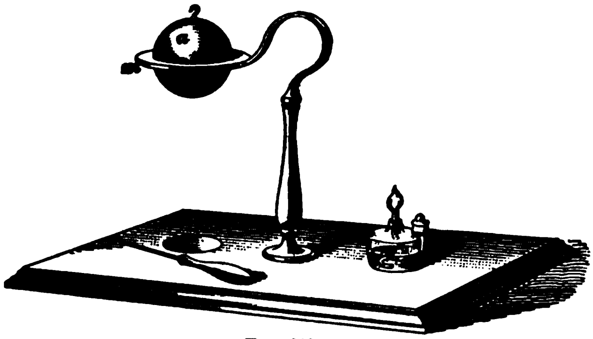


FIG. 216.

ball is heated, it expands so much that it will no longer pass through the ring.

1111. The expansion of liquids is clearly shown in the mercurial and alcohol thermometers. The expansion of gases was treated on to some extent in pneumatics.

COEFFICIENT OF EXPANSION.

1112. Suppose that the temperature of the metal rod, shown in Fig. 215, was 32° F. before heating, and exactly 10 feet long; that after the temperature had been raised 1°, or to 33°, the bar was 10 ft. $+ \frac{1}{1200}$ in. long. The linear expansion would then be (10 ft. $+ \frac{1}{1200}$ in.) $-$ 10 ft. $= \frac{1}{1200}$ in., and the ratio between this expansion and the original length of the bar would be

$$\frac{1}{1200} : 10 \times 12, \text{ or } \frac{1}{1200 \times 120} : 1, \text{ or } .000006944 : 1.$$

For every increase of temperature of 1° this rod would elongate .000006944 of its length. This number .000006944, which equals the expansion of the rod for one degree rise of temperature divided by the original length, is called the **coefficient of linear expansion.** Had the temperature of the rod been increased 100° instead of 1°, the amount of elongation would have been $.000006944 \times 100 = .0006944$, of its length, or $.0006944 \times 120 = .083328''$, or $\frac{1}{12}''$. Table 19 contains the coefficients of expansion for a number of solids, mercury, and alcohol, and the average cubical expansion of gases. No liquids are given except mercury and alcohol, for the reason that the coefficient of expansion for liquids is different for different temperatures.

TABLE 19.

Name of Substance.	Linear Expansion.	Surface Expansion.	Cubic Expansion.
Cast Iron	.00000617	.00001234	.00001850
Copper	.00000955	.00001910	.00002864
Brass	.00001037	.00002074	.00003112
Silver	.00000690	.00001390	.00002070
Bar Iron	.00000686	.00001372	.00002058
Steel (untempered)	.00000599	.00001198	.00001798
Steel (tempered)	.00000702	.00001404	.00002106
Zinc	.00001634	.00003268	.00004903
Tin	.00001410	.00002820	.00004229
Mercury	.00003334	.00006668	.00010010
Alcohol	.00019259	.00038518	.00057778
Gases	....	....	.00203252

1113. Let L = length of any body;
l = amount of expansion or contraction due to heating or cooling the body;
A = area of any section of the body;
a = increase or decrease of area of the same section after heating or cooling the body;
V = volume of the body;
v = increase or decrease in volume due to heating or cooling the body;
C_1 = coefficient of expansion taken from column 1, Table 19;
C_2 = coefficient taken from column 2, Table 19;
C_3 = coefficient taken from column 3, Table 19;
t = difference in degrees of temperature

between the original temperature and the temperature of the body after it has been heated or cooled.

Then, $l = L\,C_1\,t.$ **(67.)**

$a = A\,C_2\,t.$ **(68.)**

$v = V\,C_3\,t.$ **(69.)**

EXAMPLE.—How much will a bar of untempered steel, 14 ft. long, expand if its temperature is raised 80° ?

SOLUTION.—Since only one dimension is given, that of length, linear expansion only can be considered. From Table 19 the coefficient of linear expansion per unit of length for a rise in temperature of 1° is found to be .00000599 for untempered steel. Hence, using formula **67**, $l = L\,C_1\,t$, and substituting $14 \times .00000599 \times 80 = .0067088$ ft., or $.0067088 \times 12 = .0805056$ in.

This seems to be a very small amount, but in engineering constructions, where long pieces are rigidly connected, it must be taken into account. If the cross-section of the above bar were 2 in. square, and the bar was fitted tightly between two supports, an expansion of the above amount would exert a pressure against the supports of about 58,000 pounds.

Suppose that an iron rod 1½ inches in diameter and 100 feet long was used as a tie-rod in constructing a bridge; that it was put in place and securely fastened to two rigid supports during a warm day in summer when the temperature in the sunlight was, say, 110°. On a cold day in winter, when the thermometer registered zero, the amount that the bar would tend to shorten, owing to this change in temperature, would be, substituting these values in formula **67**,

$$.00000686 \times 100 \times 110 = .07546 \text{ ft.} = .90552 \text{ in.}$$

If this rod were rigidly secured, so that it could neither stretch nor shorten, it would then exert a pull on the supports of about 33,400 pounds.

EXAMPLE.—The wheel center of a locomotive driver is turned to exactly 50″ in diameter. If the steel tire be bored 49.94″ in diameter, to what temperature must the tire be raised in order that it may be easily shoved over the center ? Assume that the diameter of the tire is expanded to $\frac{1}{1000}$ of an inch larger than the center, and that the original temperature is 60°.

SOLUTION.—For this case formula **68** may be used. The original diameter of the tire is 49.94 in., and it is to be increased to 50.001". The area of a circle 49.94" in diameter is 1,958.79 sq. in.; area of a circle 50.001" in diameter is 1,963.58 sq. in. The difference between them is 1,963.58 − 1,958.79 = 4.79 sq. in. = a in formula **68.** Hence, since $C_2 = .00001198$, and $A = 1{,}958.79$, substitute these values in $a = A\,C_2\,t$, and $4.79 = 1{,}958.79 \times .00001198 \times t = .023466\,t$. Therefore, $t = \frac{4.79}{.023466} = 204.125°$, and $204.125° + 60° = 264.125°$. Ans.

NOTE.—Owing to the form of the equation here denoted by formula **68,** and to the manner in which the coefficients C_2 were determined, this example may be more easily solved by means of formula **67.** Thus, regard the diameter as a linear dimension and apply formula **67.** Increase in diameter = 1 = 50.001 − 49.94 = .061". $L = 49.94$ and $C_1 = .00000599$. Substituting $.061 = 49.94 \times .00000599 \times t$, or $t = \frac{.061}{49.94 \times .00000599} = 203.92°$, and $203.92° + 60° = 263.92°$. Ans. The slight difference in the two results is immaterial, and was to have been expected.

EXAMPLE.—What is the decrease in volume of a copper cylinder 30" long and 22" in diameter if cooled from 212° to 0°, the measurement being taken at a temperature of 70°?

SOLUTION.—212° − 70° = 142° = the increase in temperature above 70°. Use formula **69,** $v = V\,C_3\,t$.

$$V = 22^2 \times .7854 \times 30 = 11{,}404 \text{ cu. in.}$$
$$v = 11{,}404 \times .00002864 \times 142 = 46.38 \text{ cu. in.}$$
$$11{,}404 + 46.38 = 11{,}450.38 \text{ cu. in.} = \text{the volume at } 212°.$$
$$70° - 0° = 70° = \text{the difference in temperature.}$$
$$v = 11{,}404 \times .00002864 \times 70 = 22.86 \text{ cu. in., nearly.}$$
$$46.38 + 22.86 = 69.24 \text{ cu. in. Ans.}$$

The bars of a furnace must not be fitted tightly at their extremities, but must be free at one end; otherwise, in expanding they would split the masonry.

In laying the rails on railways, a small space is left between the successive rails; for, if they touched, the force of expansion would cause them to curve or to break the chairs. Water-pipes are fitted to one another by means of telescope joints, which allow room for expansion; so, also, are steam pipes, by means of the so-called expansion joints. If a glass vessel is heated or cooled too rapidly, it cracks, especially if it is thick; the reason for this is that, since glass is a poor conductor of heat, the sides become unequally heated, and, consequently, unequally expanded, which causes a fracture.

1114. It will be found, upon trial, that the three preceding formulas will not work back; i. e., if the length of a bar, after it has been heated, be found by formula **67**, and an attempt be made to reduce the bar to its original length by again applying formula **67**, and substituting for t the same value as in the first case, the value obtained for l will be slightly different in the two cases. The difference, however, is so slight that it is neglected in practice. If, however, the student desires to obtain exactly the same result in both cases, he must use the following more cumbersome formula, in which t_1, t_2, l_1, l_2, are, respectively, the original and final temperatures, the original and final lengths, and C_1 has the same value as in formula **67**:

$$l_2 = \left[\frac{1 + C_1\ (t_2 - 32)}{1 + C_1\ (t_1 - 32)}\right] l_1. \qquad \textbf{(70.)}$$

This formula is always used when calculating the expansion of gases by substituting V_1, V_2, and $\frac{1}{492}$ for l_1, l_2, and C_1, respectively. The results obtained will be exactly the same as those obtained by formula **58**, Art. **1054.** For, substituting the values as directed, the formula becomes

$$V_2 = \left[\frac{1 + \frac{1}{492}\ (t_2 - 32)}{1 + \frac{1}{492}\ (t_1 - 32)}\right] V_1 = \frac{\frac{492 + t_2 - 32}{492}}{\frac{492 + t_1 - 32}{492}} \times V_1 = \left(\frac{460 + t_2}{460 + t_1}\right) V_1.$$

1115. Although, as stated before, solids and liquids expand very nearly uniformly throughout all ranges of temperature, water is a marked exception to the general rule. If water is cooled down from its boiling point, it continually contracts until it reaches 39.2° F., when it begins to expand, until it freezes at 32° F. On the other hand, if water at 32° F. is heated, it contracts until it reaches 39.2° F., when it commences to expand. Therefore, the density of water is greatest where this change occurs. The importance of this exception is seen in the fact that ice forms on the *surface* of water, since it is lighter than the warmer body of water lying at varying depths below it. Were it not for this fact, all the large bodies of water would freeze solid, and would so affect the climate of the earth that it would be uninhabitable. The coefficient of expansion of water is such a very changeable quantity (varying with the temperature) that a special table is necessary.

1116. The effect of heat upon the expansion of gases was treated of in Arts. **1056,** etc., and will not be repeated here. It should be stated, however, that the constant .37052, used in formulas **60** and **61,** Arts. **1056** and **1057,** has that value only for air. For other gases it varies. If the value of this constant for any gas be represented by R, formula **61,** Art. **1057,** becomes

$$p V = R W T. \qquad (71.)$$

The value of R for several gases is given in Table 20.

TABLE 20.

Gas.	Volume of One Pound at 32° F. and a Tension of 1 Atmosphere (14.7 lb. per sq. in.).	Weight of One Cu. Ft. at 32° F. and a Tension of 1 Atmosphere (14.7 lb. per sq. in.).	R.
Air	12.388	.08073 lb.	.37052
Oxygen	11.2056	.08925 lb.	.33552
Nitrogen	12.7226	.0786 lb.	.38143
Hydrogen	178.891	.00559 lb.	5.34946

EXAMPLE.—What is the volume of 3 ounces of hydrogen gas having a tension of 20 pounds per square inch and a temperature of 80°?

SOLUTION.—3 ounces = $\frac{3}{16}$ of a pound. Since $t = 80°$, $T = 460 + 80 = 540°$. $R = 5.34946$ from Table 20. Hence, by formula **71,** $p V = R W T$, or $20 V = 5.34946 \times \frac{3}{16} \times 540 = 541.6328$, and $V = \frac{541.6328}{20} = 27.08164$ cu. ft.; say, 27.082 cu. ft. Ans.

EXAMPLE.—What is the weight of 10 cu. ft. of oxygen having a tension of one atmosphere and a temperature of 60° ?

SOLUTION.—By formula **71**, $p\ V = R\ W\ T$, or $10 \times 14.7 = .33552 \times W \times 520$. Hence, $147 = 174.4704\ W$,

$$\text{and } W = \frac{147}{174.4704} = .84255 \text{ lb. Ans.}$$

In Table 19 the coefficient of expansion for gases was given as .00203252; this is the fraction $\frac{1}{492}$ reduced to a decimal. This value of the coefficient of expansion is very nearly the same for all gases, particularly so for those which are very difficult to liquefy.

EXAMPLES FOR PRACTICE.

1. What are the absolute temperatures corresponding to (*a*) 120° R., (*b*) 120° C., and (*c*) 120° F. ?
Ans. (*a*) 338¼° R. (*b*) 393¼° C. (*c*) 580° F.

2. Change – 10° R. to the corresponding Fahrenheit and Centigrade readings. Ans. 9½° F.; – 12½° C.

3. (*a*) How much will an iron tie-rod 60 ft. long expand when the temperature is raised from 40° to 110° ? (*b*) Calculate, also, by formula **70.** (*c*) What is the difference of the two results ?
Ans. (*a*) .345744'. (*b*) .345725'. (*c*) .000019'.

4. To what temperature must a steel tire of 59.93" internal diameter be raised in order that its diameter may be 60.0015" ? Original temperature = 71°. Ans. 270°.

5. What is the volume of .68 lb. of nitrogen gas having a tension of 20 lb. per sq. in. and a temperature of 345° ? Ans. 10.44 cu. ft.

HEAT PROPAGATION.

1117. Heat is propagated through matter and space in three different ways—by *conduction*, by *convection*, and by *radiation*.

1118. Conduction is the slow progress of the vibratory motion from places of higher to places of lower temperature in the *same* body. The rate at which heat is conducted varies greatly with different substances, the *good conductors* being those in which conduction is most rapid, and the *bad* *conductors* being those in which it is very slow. The metals furnish the best conductors, and of these, silver stands first, and copper second. The fluids, both liquid and gaseous, are very poor conductors of heat. Water, for example, can be made to boil at the top of a vessel while a cake of ice is fastened within a few inches of the surface. If thermometers are placed at different depths, while *water boils at the top*, it is found that the conduction of heat downwards is very slight.

1119. Representing the conductivity of silver by 100, the following table shows the conducting power of a number of the metals:

Silver	100.0	Iron	11.9
Copper	73.6	Steel	11.6
Gold	53.2	Lead	8.5
Brass	23.1	Platinum	8.4
Zinc	19.0	Rose's Alloy	2.8
Tin	14.5	Bismuth	1.8

Organic substances conduct heat poorly. This enables trees to withstand great and sudden changes in the atmosphere without injury. The bark is a poorer conductor than the wood beneath it. Cotton, wool, straw, bran, etc., are all poor conductors. Rocks and earth are poorer conductors as the less dense and homogeneous is the mass; hence, the length of time required for the sun's rays to penetrate the earth. The mean highest temperature of the air near the ground in Central Europe is in the month of July, but at a depth of from 25 to 28 feet in the earth it is in the month of December.

1120. Convection is the transfer of heat by the motion of the heated matter itself. It can, therefore, take place only in fluids and gases. For example, as heat is applied to the *bottom* and *sides* of a vessel of water, the densities of the heated particles decrease, and they are crowded up by the heavier ones which take their places. There is thus a constant circulation going on, which tends to equalize the temperature of the whole by bringing the hot portions in contact with the colder, and also greatly facilitates the *conduction* of heat among the *molecules*.

1121. Radiation of heat is the communication of heat from a hot body to a colder one, across an intervening space between them. The best example of radiant heat is that received from the sun, the intervening space in this case being 93,000,000 miles. A person standing in front of a fire, but at some distance from it, feels a sensation of warmth which is not due to the temperature of the air, for, if a screen be interposed between him and the fire, the sensation immediately ceases, which would not be the case if the surrounding air had a high temperature. Hence, bodies can send out rays which excite heat and which penetrate the air without heating it. This, of course, is **radiant heat,** and *takes place in all directions around the body.*

1122. The intensity of heat radiation from a given source

1.—Varies as the temperature of the source.

2.—Varies inversely as the square of the distance from the source; and,

3.—Grows less as the inclination of the rays to the given radiant surface grows less.

The truth of all these laws has been established by careful experiment, and the second is still further verified by mathematical calculations.

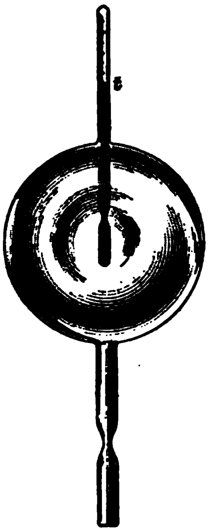


FIG. 217.

1123. *Radiant heat is transmitted in a vacuum as well as in air.* This is demonstrated by the following experiment:

In the top of a glass flask a thermometer is fixed in such a manner that its bulb occupies the center of the flask. See Fig. 217. The neck of the flask is carefully narrowed by means of the blowpipe; the flask is then attached to an air pump, and a vacuum produced in the interior. This being accomplished, the tube is sealed at the narrow part. On immersing in hot water, or on bringing the flask near some hot charcoal, the mercury is seen to rise at once. It can rise only by reason of the radiation through the vacuum in the interior, for glass is such a poor conductor that the heat could not travel with sufficient rapidity through the sides of the flask and the stem of the thermometer to cause this almost instantaneous rise.

1124. The **radiating power of heated surfaces** also depends very greatly upon the form, shape, and the material of which they are composed. If a cubical vessel, filled with hot water, has one of its vertical sides coated with polished silver, another with tarnished lead, a third with mica, and the fourth with lampblack, experiment has shown that the *radiating power* will be respectively about in the ratio of 2.5:45:80:100, or that bright surfaces radiate less heat than dark ones having the same temperature.

In the same way it is found that the heat-absorbing power of bodies varies in a similar manner. Lampblack *reflects* few of the heat rays which impinge upon it; nearly all are absorbed, while, on the other hand, polished silver reflects the greater part of the radiations, and absorbs only about 2½ per cent.

Some substances neither absorb nor reflect the heat rays to any extent, but transmit nearly all of them just as glass transmits light. For example, rock salt reflects less than 8 per cent. of the radiation it receives, absorbs almost none, and transmits 92 per cent.

1125. It will now be seen that there is a system of exchange going on between heated bodies at all times, which tends to an equalization of temperature. The hot bodies are always cooling, and the cold bodies are always tending towards a rise in temperature, so that heat is created only to be diffused and apparently lost. That it is *not* lost, however, will be shown in the subsequent pages.

1126. Dynamical Theory of Heat.—Before going any farther, it will be convenient to explain here the view now generally taken as to the mode in which heat is propagated.

On this subject, it is stated in Ganot's Physics: "A hot body is one whose molecules are in a state of vibration. The higher the temperature of a body, the more rapid are these vibrations, and a diminution in temperature is but a diminished rapidity of the vibrations of the molecules. The propagation of heat through a bar is due to a gradual communication of this vibratory motion from the heated part to the rest of the bar. A good conductor is one which readily takes up and transmits the vibratory motion from molecule to molecule, while a bad conductor is one which takes up and transmits the motion with difficulty. But, even through the best of the conductors, the propagation of this motion is comparatively slow. How, then, can be explained the instantaneous perception of heat when a screen is removed from a fire, or when a cloud drifts from the face of the sun? In this case, the heat passes from one body to another without affecting the temperature of the medium which transmits it. In order to explain these phenomena, it is imagined that all space, the space between the planets and the stars, as well as the interstices in the hardest crystal and the heaviest metal—in short, matter of any kind—is permeated by a medium having the properties of matter of infinite tenuity, called **ether.** The molecules of a heated body, being in a state of intensely rapid vibration, communicate their motion to the ether around them, throwing it into a system of waves which travel through space and pass from one body to another with the velocity of light. When the undulations of the ether reach a given body, the motion is given up to the molecules of that body, which, in their turn, begin to vibrate; that is, the body becomes heated. This process of this motion through the ether is termed radiation, and what is called a ray of heat is merely one series of waves moving in a given direction."

HEAT MEASUREMENT.

1127. The Unit of Heat.—There are three units in use for measuring the *quantity of heat* given up or absorbed by a body when heated or cooled.

1128. The British Thermal Unit.—*The amount of heat necessary to raise one pound of water one degree Fahrenheit is called a* ***British thermal unit.*** Instead of writing out the words British thermal unit in full, it is customary to abbreviate them to B. T. U. Thus, 7 pounds of water raised 15° F. would equal $7 \times 15 = 105$ B. T. U. The unit of heat used here will be the B. T. U.

1129. The Thermal Unit.—*The amount of heat necessary to raise one pound of water 1° C. is called a* ***thermal unit.*** Since 1° C. $= \frac{9}{5} \times 1°$ F., it follows that the thermal unit is $\frac{9}{5}$ times as large as a B. T. U. Hence, to change B. T. U. into thermal units, multiply the number of B. T. U. by $\frac{5}{9}$. To change thermal units into B. T. U., multiply the number of thermal units by $\frac{9}{5}$.

The thermal unit is used by American and British scientific writers, as being better adapted to their calculations.

1130. The Calorie.—*The amount of heat necessary to raise one kilogram of water 1° C. is called a* ***calorie.*** One kilogram = 2.2 pounds and 1° C. $= \frac{9}{5} \times 1°$ F.; hence, a calorie $= 2.2 \times \frac{9}{5} = 3.96$ B. T. U. The calorie is used in France and in other countries using the metric system of weights and measures.

SPECIFIC HEAT.

1131. When equal weights of two different substances, having the same temperature, are placed in similar vessels and subjected for the same length of time to the heat of the same lamp, or are placed at the same distance in front of the same fire, it is found that their final temperature will differ considerably; thus, mercury will be much hotter than water But as, from the conditions of the experiment, they have each been receiving the same amount of heat, it is clear that the quantity of heat which is sufficient to raise the temperature of mercury through a certain number of degrees will raise the same weight of water through a less number of degrees; in other words, it requires more heat to raise a certain weight of water one degree than it does to raise the same weight of mercury one degree. Conversely, if the same quantities of water and of mercury at 200° be allowed to cool down to the temperature of the room, the water will require a much longer time for the purpose than the mercury; hence, in cooling through the same number of degrees, water gives up more heat than does mercury.

1132. *The number of B. T. U., or parts of a B. T. U., required to raise the temperature of one pound of any substance 1° F. is called the* **specific heat** *of that substance.* It will be seen from the above definition that *the specific heat of a substance is the ratio between the amount of heat required to raise the temperature of the substance 1°, and the amount of heat required to raise the temperature of the same weight of water 1°.*

If the specific heat of lead were given as .0314, it would mean that the amount of heat required to raise a certain weight of lead 1° would raise the same weight of water only .0314 of 1°, or it would mean that .0314 B. T. U. would raise the temperature of one pound of lead 1° F.

EXAMPLE.—The specific heat of copper is .0951; how many B. T. U. will it take to raise the temperature of 75 pounds 180°?

SOLUTION.—Since it takes .0951 B. T. U. to raise 1 lb. of copper 1°, it will take $.0951 \times 75 \times 180$ to raise 75 lb. 180°. Hence, $.0951 \times 75 \times 180 = 1,283.85$ B. T. U. Ans.

1133. In the example just given, if it had been required to raise 75 lb. of water 180°—that is, from the freezing point to the boiling point—it would have taken $75 \times 180 = 13,500$ B. T. U., and $\frac{1,283.85}{13,500} = .0951 =$ the specific heat of copper. The following is the formula for finding the number of B. T. U. required to raise the temperature of a substance a given number of degrees, or for finding the number of B. T. U. given up by a body in cooling given number of degrees:

Let W = weight of body in pounds;
s = specific heat of substance composing the body;
t = original temperature of body;
t_1 = final temperature of body;
n = number of B. T. U. required, or given up, in changing the temperature of the body from $t°$ to $t_1°$.

Then,

$$n = W(t_1 - t)s. \qquad (72.)$$

EXAMPLE.—A piece of wrought iron weighing 31.8 lb., and having a temperature of 900°, is cooled to a temperature of 60°; how many units of heat did it give up? The specific heat of wrought iron is .1138.

SOLUTION.—Apply formula **72**, $n = W(t_1 - t)s$. Substituting n = 31.8 (900 − 60) .1138 = 2,992.03 B. T. U. Ans.

If a body be cooled from a temperature t down to a temperature t_1, the value of n will be negative, the minus sign indicating that the body was cooled.

1134. In the following table are given the specific heats of a number of substances under constant pressure:

TABLE 21.

SOLIDS.

Copper	0.0951	Cast Iron	0.1298
Gold	0.0324	Lead	0.0314
Wrought Iron	0.1138	Platinum	0.0324
Steel (soft)	0.1165	Silver	0.0570
Steel (hard)	0.1175	Tin	0.0562
Zinc	0.0956	Ice	0.5040
Brass	0.0939	Sulphur	0.2026
Glass	0.1937	Charcoal	0.2410

LIQUIDS.

Water	1.0000	Lead (melted)	0.0402
Alcohol	0.7000	Sulphur "	0.2340
Mercury	0.0333	Tin "	0.0637
Benzine	0.4500	Sulphuric Acid	0.3350
Glycerine	0.5550	Oil of Turpentine	0.4260

GASES.

	Constant Pressure.	Constant Volume.
Air	0.23751	0.16847
Oxygen	0.21751	0.15507
Nitrogen	0.24380	0.17273
Hydrogen	3.40900	2.41226
Superheated Steam...	0.48050	0.34600
Carbonic Oxide......	0.24790	0.17580
Carbonic Acid......	0.40400	0.15350

1135. The reason that there are two values for the specific heat of gases is that it takes less heat to raise the temperature of a gas when the volume is constant than when the pressure is constant but the volume varies. Thus, consider a closed cylinder filled with gas. If heat be applied, the pressure and temperature will both increase. Denoting the specific heat for constant pressure by s_p, and for constant volume by s_v, the number of heat units required to heat the gas from $t°$ to $t_1°$ will be $s_v W(T_1 - T)$. If, however, the cylinder be imagined to be fitted with a frictionless piston, free to move up or down, and heat be applied, the gas will expand, overcoming a resistance equal to the weight of the piston, plus the pressure of the atmosphere. Hence, in addition to the heat required to increase the vibratory movement of the molecules, heat is also required to overcome the outer pressure which remains constant in this case. The number of heat units necessary will then be $s_p W(T_1 - T)$. This subject will be more fully discussed later.

1136. Mixing Two Bodies of Unequal Temperatures.—If a certain quantity of water having a temperature of 40° be mixed with a like quantity having a temperature of 100°, it is evident that the temperature after mixing will be $\frac{40 + 100}{2} = 70°$. But, if 5 lb. of water having a temperature of 40° be mixed with 5 lb. of copper having a

EXAMPLES FOR PRACTICE.

1. How many units of heat are required to raise the temperature of 10 oz. of platinum from 80° to 2,000°? Ans. 38.88 B. T. U.

2. In order to determine the specific heat of a certain alloy, a piece weighing 12¼ oz. was heated to a temperature of 320°, and was then immersed in 2 lb. 6 oz. of water contained in a lead vessel weighing 4 lb. 7 oz. The temperature of the water and of the vessel being 70°, what was the specific heat of the alloy if the temperature of the mixture was 79°? Ans. .1202.

3. In order to determine the temperature of a chimney, a silver bar weighing 20 oz. is placed in it until it has attained the same temperature. It is then immersed in 1 lb. of water contained in a brass vessel weighing 10 oz. The temperature of the vessel and water being 65°, and of the mixture 98¼°, what was the temperature of the chimney? Ans. 596°.

4. An iron casting weighing 3 tons is cooled from 2,100° to 100°; (*a*) how many units of heat does it give up? (*b*) If all this heat could be utilized, how many pounds of coal would it be equivalent to, assuming that 1 lb. of coal gives out 14,500 B. T. U. during its combustion?

Ans. { (*a*) 1,557,600 B. T. U.
{ (*b*) 107.42 lb.

LATENT HEAT.

1138. In all that has been said in the preceding pages, only the phenomena relating to sensible heat have been considered. If a quantity of pounded ice at a temperature of 32° be put in a vessel and held over the flame of a spirit lamp, heat passes rapidly into the ice and melts it; but a thermometer resting in this mixture of ice and water shows no tendency to rise; it will remain at 32° until all of the ice has been melted. Where has the heat gone that was supplied to the ice? This question was first investigated by Dr. Black, of Edinburgh, in 1760, and is easily explained by the modern dynamical theory of heat.

Dr. Black took a pound of water and a pound of ice, both having a temperature of 32°, and placed them in two vessels suspended in a chamber which was kept at as nearly a uniform temperature as possible. At the end of half an hour the temperature of the water was 39.2°, but the ice did not reach that temperature until 10½ hours had passed, being melted, of course, in the meantime. Dr. Black reasonably

Solution.—Substituting the values given in formula **73**,

$$156 = \frac{1 \times .1138 \times t_1 + 2 \times 75 + .5 \times .0951 \times 75}{1 \times .1138 + 2 + .5 \times .0951}, \text{ or}$$

$$156 = \frac{.1138\, t_1 + 153.56625}{2.16135}, \text{ or}$$

$$156 \times 2.16135 = .1138\, t_1 + 153.56625.$$

Hence,

$$.1138\, t_1 = 183.60435, \text{ or}$$

$$t_1 = \frac{183.60435}{.1138} = 1{,}613.4°. \text{ Ans.}$$

1137. By means of formula **73**, the specific heat of a substance may be obtained.

$$\text{Thus, in } t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3 + etc.}{W_1 s_1 + W_2 s_2 + W_3 s_3 + etc.},$$

suppose that the specific heat s_3 be required and all of the other quantities, including t, are known.

Then, solving the above equation for s_3,

$t\,(W_1 s_1 + W_2 s_2 + etc.) + t\, W_3 s_3 = W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3 + etc.$, or $t\, W_3 s_3 - t_3\, W_3 s_3 = W_1 s_1 t_1 - W_1 s_1 t + W_2 s_2 t_2 - W_2 s_2 t + etc.$,

$$\text{or} \quad s_3 = \frac{W_1 s_1 (t_1 - t) + W_2 s_2 (t_2 - t) + etc.}{W_3 (t - t_3)}. \qquad (74.)$$

Example.—A silver vessel weighing 13 oz. is suspended by a string; 1 lb. 4 oz. of water having a temperature of 120° is poured into it, and in this is placed a piece of metal weighing 14 oz. and having a temperature of 100°. If the temperature of the vessel was 72°, and the temperature of the mixture is 117°, what is the specific heat of the piece of metal?

Solution.—Using formula **74**, and letting W_1, s_1, and t_1 represent, respectively, the weight, specific heat, and temperature of the silver vessel, W_2, s_2, and t_2 ditto for the water, and W_3, s_3, and t_3 the same for the piece of metal,

$$s_3 = \frac{W_1 s_1 (t_1 - t) + W_2 s_2 (t_2 - t)}{W_3 (t - t_3)} =$$

$$\frac{13 \times .057 (72 - 117) + 20 \times 1 (120 - 117)}{14 (117 - 100)} =$$

$$\frac{-33.345 + 60}{238} = .112. \text{ Ans.}$$

All weights must be reduced to either pounds or ounces before substituting.

value is only 5.09; that is, to change one pound of frozen mercury at its temperature of fusion (— 37.8°F.) into liquid mercury of the same temperature requires only 5.09 units of heat. Now, it is reasonable to suppose that if it requires 142.65 units of heat to convert a pound of ice at 32° into water at 32°, then the same number of heat units would be given up when water at 32° is changed into ice at 32°; experiment has verified this.

1139. If water be heated to its boiling point of 212° under a constant pressure of 14.69 lb. per sq. in., it has been found by experiment that it will require about 966 units of heat per pound of water to change it into steam at 212°. This extra number of units of heat necessary to convert a liquid into a gas, or, rather, vapor, of the same temperature and pressure is called the **latent heat of vaporization**, and the temperature at which this change of state takes place is called the **temperature of vaporization.**

1140. According to the modern theory of heat, the extra quantity of heat necessary for a change of state of a body is used in forcing the molecules of a body farther apart, and in overcoming the force of cohesion. This latent heat is not lost, but performs work in giving additional energy to the molecules of a body, and it always reappears when the body resumes its former state. Thus, for instance, a pound of steam under a pressure of one atmosphere contains $966 + 180 = 1,146$ units of heat more than does a pound of water at 32°. Hence, if 1 lb. of steam at 212° be mixed with $\frac{966}{180} = 5.37$ lb. of water at 32°, the temperature of the mixture will be exactly 212°, or the boiling point of water; in other words, the steam raised 5.37 lb. of water from the freezing point to the boiling point without lowering its own temperature, by merely changing from steam into water. If a pound of water at a temperature of 32° be changed into ice of the same temperature, 142.65 units of heat will be given up during this change of state.

1141. In the following table are given the temperatures of fusion and of vaporization, and the latent heats of fusion and vaporization, whenever they have been determined with sufficient accuracy:

TABLE 22.

Substance.	Temperature of Fusion.	Temperature of Vaporization.	Latent Heat of Fusion.	Latent Heat of Vaporization.
Water	32°	212°	142.65	966.6
Mercury	—37.8°	662°	5.09	157
Sulphur	228.3°	824°	13.26	
Tin	446°	....	25.65	
Lead	626°	....	9.67	
Zinc	680°	1,900°	50.63	493
Alcohol	Unknown	173°	....	372
Oil of Turpentine	14°	313°	....	124
Linseed Oil	....	600°		
Aluminum	1,400°			
Copper	2,100°			
Cast Iron	2,192°	3,300°		
Wrought Iron	2,912°	5,000°		
Steel	2,520°			
Platinum	3,632°			
Iridium	4,892°			

The following example will show the purpose of Tables 21 and 22; their use will be further illustrated in a later section.

EXAMPLE.—How many units of heat will it be necessary to use in changing 12 lb. of ice at a temperature of $-20°$ C. into steam of 212°?

SOLUTION.—By formula **65**, $t_f = \frac{9}{5} \times -20 + 32 = -4°$ F. This is equivalent to $32° + 4° = 36°$ F. below the freezing point. In Table 21, specific heat of ice was given as .504; hence, it will take $12 \times 36 \times .504 = 217.728$ B. T. U. to raise the temperature of 12 lb. of ice from $-4°$ to 32°. To convert this ice into water of 32° will require $142.65 \times 12 = 1{,}711.8$ B. T. U. To raise this water from 32° to a temperature of 212° will require $12 \times 180 = 2{,}160$ B. T. U. To convert it into steam of 212° will require $966.6 \times 12 = 11{,}599.2$ B. T. U. The total number of units of heat required will be $217.728 + 1{,}711.8 + 2{,}160 + 11{,}599.2 = 15{,}688.728$ B. T. U. Ans.

1142. A solid may be changed into a liquid, not only by melting it, but also by dissolving it, as salt or sugar is dissolved in water. Since the particles of the solid body must be torn asunder, in opposition to the forces which hold them together, it is reasonable to suppose that a certain amount of heat will be required to do this. That such is a fact may be easily proven by any one having a thermometer. Put a thermometer in a vessel of water, and leave it there until it indicates the temperature of the water, then put in some salt or sugar, and stir so as to make it dissolve more quickly, and it will be found that the mercury has fallen several degrees. In fact, if any solid be dissolved in a liquid that does not act chemically upon it, the temperature of the mixture will be lower than if the solid did not dissolve. It is this principle that is taken advantage of in the so-called freezing mixtures. A mixture of one part of nitrate of ammonia and one part of water will reduce the temperature from 50° to 4°, a fall of 46°. The effects are still more striking when both bodies are solids, one of which is already at the freezing point. Thus, a mixture of two parts of snow, or finely pounded ice, and one part of common salt, will reduce the temperature from 50° to 0°, a range of 50°, while a mixture of four parts of potash and three parts of snow, or pounded ice, will lower the temperature from 32° to — 51°, a range of 83°.

1143. Latent heat plays an important part in everyday life. It takes a long time and severe cold to freeze the water of a river to any depth, even though the thermometer goes far below the freezing point. This is because 142.65 units of heat must be given up by every pound of water, after being brought to the freezing point, before the ice can form. If it were not for this, the rivers, lakes, and other bodies of water would be frozen solid as soon as the water reached the freezing point, and would be melted as soon as the temperature went above that point. In the spring all of the snow on the hills would be melted during a warm day, and great floods would be the consequence. As it is, 142.65

units of heat must be supplied to every pound of snow at 32° to convert it into water at 32°.

EXAMPLE.—How many units of heat will it take to evaporate 25 lb. of mercury from a temperature of 70° ?

SOLUTION.—The temperature of vaporization of mercury is 662°, and the specific heat is .0333; the increase in temperature from 70° will be $662° - 70° = 592°$. The number of units of heat required will be $25 \times 592 \times .0333 = 492.84$ heat units. The latent heat of vaporization is 157; hence, $492.84 + 25 \times 157 = 4,417.84$ B. T. U. will be required. Ans.

EXAMPLES FOR PRACTICE.

1. If a pound of steam at 212° and 7 pounds of ice at 32° are mixed what will be the resulting temperature? Ans. 50.5°.
2. How many units of heat are required to vaporize 10 lb. of mercury from a temperature of 100 ? Ans. 1,757.146 B. T. U.
3. How many pounds of oil of turpentine at 60° can be vaporized by 1 lb. of coal, if the coal gives out 14,500 B. T. U. during combustion ? Ans. 62.56 lb.
4. How many pounds of water at 32° can be vaporized by 1 pound of coal ? Ans. 12.646 lb.
5. How many pounds of coal are required to raise 100 lb. of wrought iron from 85° to its melting point ? Ans. 2.219 lb.

SOURCES OF HEAT AND COLD.

1144. Different Sources of Heat.—Heat is derived from the following sources: 1.—**Physical sources**—that is, the radiation of heat from the sun, terrestrial heat, change of state in bodies and electricity. 2.—**Chemical sources,** or molecular combinations, more especially combustion. 3.—**Mechanical sources,** comprising friction, percussion, and pressure.

1145. Physical Sources.—(1) The most intense of all of the sources of heat is the sun. The majority of scientists are of the opinion that all of the heat received or given up by the earth has, or has had, its source in the sun. It would be out of place here to elucidate this theory fully, and the subject will be explained as subdivided above. It is the amount of heat radiated from the sun, and received by the earth, that causes the change of seasons; that causes the water in the rivers, lakes, and seas to evaporate and form the clouds, to be again precipitated as rain or snow. Without it, no living thing, animal or vegetable, could exist.

(2) The earth possesses a heat peculiar to itself, called **terrestrial heat.** When a descent is made below the surface, the temperature is found to gradually increase. This is not caused by the heat radiated from the sun, for the material comprising the earth is such a poor conductor that the heat of the sun's rays penetrates only a very short distance below the surface. The explanation usually given for this phenomenon is that the interior of the earth is in a molten condition. The terrestrial heat exerts but a slight effect, not raising the temperature of the surface more than $\frac{1}{20}$ of a degree.

(3) If a liquid be poured upon a finely divided solid, as a sponge, flour, starch, roots, etc., the temperature will be increased from 1° to 10°, according to conditions. This phenomenon might be called *heat produced by capillarity.*

(4) The heat produced by a change of state has already been described; it is the heat given off when a body is converted from a gas or liquid to a liquid or solid.

(5) Extremely high temperatures may be produced by the electric current. By means of it, quick-lime, firebrick, osmium, porcelain, and several other substances, which, until very recently, have resisted every attempt to melt them, may be made to run like water.

1146. Chemical Sources.—Whenever two or more substances which act chemically upon one another are brought together and allowed to combine, heat is evolved. When this phenomenon is produced by oxygen uniting with carbon, or other substance, and is accompanied by light, it is called **combustion.** This subject will not be treated of here, but will be considered by itself in connection with the subject of steam boilers.

1147. Mechanical Sources.—(1) The friction between any two bodies rubbed together produces heat. Rubbing one hand briskly against the other will soon make the hands too warm for comfort. The friction between a journal and its bearing causes heat; the heat causes the journal and bearing to expand, the journal expanding more rapidly on account of being smaller and being heated more quickly; the expansion causes a greater pressure on the bearing, producing more friction and heat. If the bearing is not properly oiled, the heat will become so intense in a short time that the soft metal in the bearings will melt. When shooting stars strike the earth's atmosphere their velocity is so great (sometimes as high as 150 miles a second) that the friction of the atmosphere causes them to take fire almost instantly. Wherever there is friction, there is heat.

(2) Heat is also generated by percussion. The repeated blows of a hammer upon a piece of iron, lead, or other metal, will soon make it quite hot.


FIG. 218.

(3) The generation of heat by pressure was spoken of in connection with gases—that is, the temperature rises when a gas is compressed. This is also true of solids and liquids, but the results are not so marked in their cases. The production of heat by the compression of gases is easily shown by means of the *pneumatic syringe* shown in Fig. 218. This consists of a glass tube with thick sides, hermetically closed with a leather piston. At the bottom is a small cavity in which a piece of cotton, moistened with ether or carbon disulphide, is placed. The tube being filled with air, the piston is suddenly plunged downwards. Thus compressed, the air generates so much heat that the cotton is ignited, which can be seen to burn when the piston is suddenly withdrawn. The

ignition of the cotton in this experiment indicates a temperature of at least 570°, since it will not ignite at a lower temperature.

THE PRODUCTION OF MECHANICAL WORK BY HEAT.

THE MECHANICAL EQUIVALENT OF HEAT.

1148. From what has been previously stated, it should now be evident that heat is a kind of energy, since, when a body is heated, the heat imparted to it manifests itself in giving the molecules a greater velocity, and in forcing them farther apart, in opposition to the force of cohesion, which tends to draw them together and reduce their velocity; but this requires energy, and heat is the form of energy used. Again, when a body is cooled, heat is given up; in other words, the energy of the molecules is lessened. The heat thus given up can be used to heat or impart energy to other bodies. Since heat is a kind of energy, it is reasonable to suppose that there is some relation between energy (or work) and heat. From careful experiment it has been found that one unit of heat (1 B. T. U.) is equivalent to 778 foot-pounds of work—that is, 778 foot-pounds of work would be required, to be expended by friction or otherwise, to raise a pound of water 1° in temperature under a pressure of one atmosphere, and that the heat given up by 1 pound of water in cooling 1°, if used as energy, could raise 1 pound to a height of 778 feet, or 778 pounds 1 foot. This number, 778, has been obtained in many ways, and is called **the mechanical equivalent of heat.** It is denoted in all formulas into which it enters, in books treating of heat, by the letter J, the initial letter in the name of Dr. Joule who first determined its value with any degree of accuracy.

1149. The First Law of Thermodynamics.—*Heat is energy, and has capacity for doing work; the number of units of work which can thus be performed by a given* *quantity of heat is proportional to the number of units of heat in that quantity.* This law is more concisely stated as follows: *Heat and mechanical energy are mutually convertible.*

For example, if a weight of 778 pounds be dragged 20 feet along a horizontal surface, and the coefficient of friction between the weight and the surface is .25, the work alone will be $778 \times 20 \times .25 = 3,890$ foot-pounds. If this had been done in such a manner that the entire movement could have taken place in water, say an upright shaft turning in a pivot bearing, the friction thus produced could raise the temperature 1° of $\frac{3,890}{778} = 5$ lb. of water; or 1 lb., 5°; or 10 lb., $\frac{1}{2}$°, etc. Here, the mechanical energy necessary to overcome the friction was converted into heat. The amount of heat obtainable from a given amount of mechanical energy is always the same, and is in the proportion of one British thermal unit to every 778 foot-pounds of work.

1150. A fine illustration of the conversion of mechanical energy into heat is given by the experiment shown in Fig. 219. A brass tube, about 7 in. in length and $\frac{3}{4}$ in. in diameter, is attached to a small wheel, by means of a cord

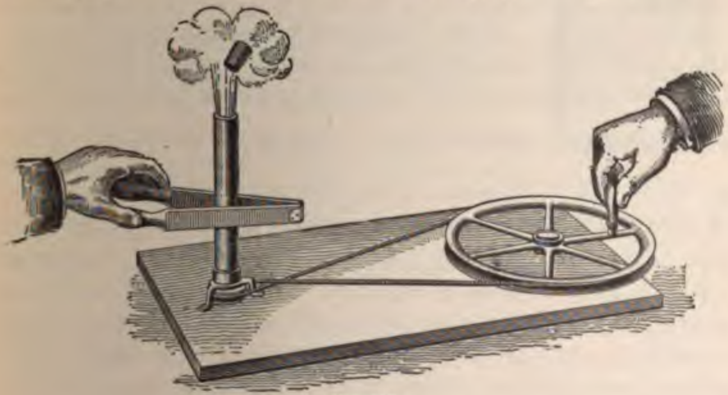


FIG. 219.

passing around this wheel and a larger one turned by a handle, as shown; the tube is three-fourths full of water, and is closed with a cork. The tube being held by the clamp and made to rotate rapidly by means of the larger wheel, considerable friction is generated, causing the water within the tube to be heated; the temperature rapidly increases, and part of the water is converted into steam, whose pressure becomes so great as to force out the cork. Suppose that the weight of the water is $1\frac{1}{2}$ oz., that its original temperature was 60°, and that a pressure of 10 lb. per sq. in. was necessary to force out the cork. From the steam and water tables, to be given in connection with the subject of Steam and Steam Engines, the number of heat units in a pound of water above a temperature of 32° from which steam of 10 lb. pressure is being given off is 209.39, and the number of heat units in a pound of steam of this pressure above 32° is 1,155.1. To create this pressure, it can be shown that the weight of the steam in the tube above the water will be about .0005 oz.; hence, the weight of the water will be $1.5 - .0005 = 1.4995$ oz. Since 1 oz. $= \frac{1}{16}$ of a pound, the number of heat units in the water will be $\frac{1.4995}{16} \times 209.39 = 19.6238$. The number of heat units in the steam will be $\frac{.0005}{16} \times 1{,}155.1 = .0361$, nearly. The sum is $19.6238 + .0361 = 19.6599$. From the steam tables above referred to, the number of heat units in a pound of water (above 32°) having a temperature of 60°, and under a pressure of one atmosphere, is 28.00626; in $1\frac{1}{2}$ oz. there would be $\frac{1.5}{16} \times 28.00626 = 2.6256$ heat units. Consequently, the number of heat units necessary to be supplied in order to blow out the cork is $19.6599 - 2.6256 = 17.0343$. The theoretical number of foot-pounds of work which would have to be exerted in turning the large wheel to accomplish this result would be $17.0343 \times 778 = 13{,}252.69$ foot-pounds. The actual amount of heat used would be greater than that just calculated, for the reason that some is lost by radiation and conduction, and in overcoming the friction of the bearings.

Suppose that enough heat were lost to bring the total number of foot-pounds of work up to 15,000, and that it took 10 minutes to cause the cork to be blown out; the number of foot-pounds per minute would be $\frac{15{,}000}{10} = 1{,}500$ and the horsepower exerted would be $\frac{1{,}500}{33{,}000} = \frac{1}{22}$ H. P.

1151. Having shown that mechanical work can be changed into heat, it will now be demonstrated that heat can be changed into mechanical work. Fig. 220 represents a cylinder *A B* partly filled with gas or air confined within the cylinder by means of the piston *P*. The gas is then under a pressure of the atmosphere, and has also an additional pressure due to the weight of the piston. If heat be applied to the bottom of the cylinder, the piston will gradually rise in proportion to the amount of heat supplied. In expanding, it will have to do work in order to raise the piston. Suppose a rope, fastened to the piston and passed over a pulley, to have a weight on the other end a trifle less than the total pressure of the atmosphere plus the weight of the piston. Now, if the gas within the cylinder be cooled, the piston will fall, owing to the combined weight of the piston and the pressure of the atmosphere, and raise the weight, thus performing work. In the first case, a certain amount of heat was supplied to the gas to do work; in the second case, heat was *taken away* from the gas (cooled) in order that work might be done. In both cases the amount of work done was proportional to the amount of heat supplied or taken away, and, had the work done been the same, the amount of heat supplied or taken away would also have been the same.

FIG. 220.

1152. When a body free to expand is heated, two operations are performed: 1. The temperature is raised and its volume is increased. 2. The body, in expanding, overcomes the outer pressure, and thus does work. The coefficient of cubic expansion of mercury is .0001001, or say .0001. Suppose that 1 cubic foot of mercury be confined in a non-expanding vessel, having a diameter corresponding to a circle whose area is 1 sq. ft. The height of the mercurial column will then be 1 ft. Let the mercury be heated until its temperature is 100° higher than before; the volume will be increased .0001 × 100 = .01 cu. ft. Since the area could become no larger (being confined in a non-expanding vessel), the column of mercury must be .01 ft. longer than it was before being heated. In expanding, the pressure of the atmosphere (equaling a weight of 144 × 14.7 = 2,116.8 lb.) was overcome through this distance, and work was done equivalent to 2,116.8 × .01 = 21.168 foot-pounds. The greater part of the heat went to increase the temperature, and to push the molecules farther apart against the force of cohesion tending to pull them together. This is called the **inner work.** The work of overcoming the outside pressure through a certain distance, by expanding, is called the **outer work.** The outer work for the above case was found to be 21.168 foot-pounds; the inner work may be found as follows: The specific heat of mercury under constant pressure, taken from Table 21, is .0333; hence, to raise the temperature of 1 cu. ft. (= 850 lb.) 100° will require 850 × 100 × .0333 = 2,830.5 heat units, equivalent to a total work of 2,830.5 × 778 = 2,202,129 foot-pounds. Subtracting the outer work, the inner work equals 2,202,129 − 21.168 = 2,202,107.832 foot-pounds. In the case of a solid body, this difference would be still more marked. In fact, the outer work is so slight, compared with the inner work in solid and liquid bodies, that it is usually neglected, except in the case of water.

1153. In the case of gases, however, the outer work plays a very important part, as a little consideration will show. Thus, suppose that air was substituted for the mercury in the previous case, and was prevented from escaping from the vessel by a piston without weight, as shown in Fig. 221. Let the original temperature of the air be 70°, and let ted until the temperature is 100° higher, or 170°. volume is determined by formula **58,** Art. **1054,**

$$= \frac{460 + 170}{460 + 70} = 1.19 \text{ cu. ft., nearly.}$$ Hence, the in- ..19 — 1 = .19 cu. ft., and the piston is raised .19 outer work will be 2,116.8 × .19 = 402.192 foot- The weight of a cu. ft. of air having a temper- '0° is found by formula **71,** Art. **1116,** $pV =$: 14.7 × 1 = W × .37052 × 530, or

$$W = \frac{14.7}{.37052 \times 530} = .07486 \text{ lb., nearly.}$$

ific heat of air for constant pressure is .23751; ; total number of heat units required is .07486 × 751 = 1.778 heat units. 1.778 × 33.284 foot-pounds. Since the outer .ired 402.192 foot-pounds, the inner l require 1,383.284 — 402.192 = ›ot-pounds. This shows that, in the · and gases, the outer work is a little half the inner work. Since the ɔhesion has no perceptible effect in ·f gases, the inner work tends only he temperature, or, in other words, ;e the vibratory movement of the . Consequently, if the piston in were fastened down, so that the : the gas would remain the same, ıld be no outer work, and the total ıired to raise the temperature 100° 981.092 foot-pounds, or to raise the

FIG. 221.

ıre 1°, 9.81092 foot-pounds. The inner work may lculated by using the specific heat for constant vol- directed in Art. **1135.** Thus, inner work = - T) × 778 = .16847 × .07486 × 100 × 778 = 981.18 .ds. The slight difference in results is due to

. Some bodies do not always expand when heat is ıs, for example, water. When *water* of 32° is heated to 33°, its volume decreases, and this continues until a temperature of 39.2° has been reached; beyond this point, the volume increases with the temperature, and, consequently, the density decreases, since when the weight remains the same, and the volume grows larger, the density decreases. The reason of this apparent contradiction to the heat theory is that the increase in density is caused by the pressure of the atmosphere; the heat added to the water until the temperature of 39.2° is reached gives a freer movement to the molecules, and enables the atmospheric pressure to force them nearer together. If it were not for this pressure, it would take more heat to raise the temperature of a pound of water 1° than now; this is evident from analogy to the cylinder and piston in Fig. 221; when the piston was fastened down, it took far less heat to increase the temperature of the air than when it was free to move. In the case of water, the specific heat varies somewhat with the pressure, while in the case of gases it is practically constant. In consequence of this variation, the latent heat of ice is diminished by heavy pressures—that is, its melting point is lowered. Under a pressure of 13,000 atmospheres (191,100 lb. per sq. in.), ice will melt at 0° instead of at 32°.

WORK DONE BY EXPANSION OF AIR AND GASES.

1155. Isothermal Expansion. — When a gas expands, it does work; when it is compressed, work is required to be done upon the gas to compress it. Suppose that a certain quantity of air is confined in a vessel having an area of 1 sq. ft., and whose length is 5 ft., plus the thickness of the piston, so that the piston can move 5 ft. Suppose the piston to be in the position shown in Fig. 222, and that the absolute pressure of the volume of air enclosed in the cylinder is 100 lb. per sq. in. on the piston, and that the temperature is 150°. Since the area of the piston is 1 sq. ft., the volume of the enclosed air is 1 cu. ft. Now, let this air expand, and keep the temperature constant by adding heat to it. The piston will move ahead; the atmospheric pressure upon it will be overcome through the distance it moves; the volume of the air will increase and the pressure decrease, according to Mariotte's law. When the piston has moved 1 ft., the volume will be 2 cu. ft., and the pressure is found by the formula $p_1 v_1 = p_2 v_2$, to be $100 \times 1 = p_2 \times 2$, or $p_2 = 50$ lb. per sq. in. When the piston has moved 2 ft., the pressure is $\frac{100}{3} = 33\frac{1}{3}$ lb. per sq. in., etc.

FIG. 222.

1156. To show the effects of this expansion upon the pressure and volume graphically, two indefinite straight lines are drawn at right angles to each other, as $O\,Y$ and $O\,X$, in Fig. 223. Any line drawn from $O\,X$ parallel to $O\,Y$ is called an **ordinate.** Choose a convenient scale, say 1 in. = 1 cu. ft., and lay off $O\,L$ = 1 in. = 1 ft. of cylinder length = 1 cu. ft. of cylinder volume = the volume of air admitted at full pressure before expanding. Make $O\,F$ = 5 in. = the total travel of the piston = the total volume after the piston has reached the end of the cylinder. Now, choose another scale to represent the pressures, say 1 in. = 20 lb. The length of a line representing 100 lb. would be $\frac{100}{20} = 5$ in. Lay off this distance on $O\,Y$, thus locating the point H. The pressure is 100 lb. per sq. in. throughout the distance $O\,L$; hence, drawing $H\,M$ parallel to $O\,X$, it is evident that any ordinate measured from $O\,X$ to this line, with a scale of 1″ = 20 lb., will equal 100 lb. pressure per sq. in. When the piston reaches the point L, no more air is admitted, and as it begins to move away from the position $A\,L$, the pressure begins to fall, the volume increasing in proportion to the distance of the piston from $O\,Y$. The pressures corresponding to a number of different positions of the piston, calculated by the formula $p\,v = p_1\,v_1 = p_2\,v_2$ etc., are as follows:

When piston has moved	$\frac{1}{2}$	ft., or to	*d*,	pressure	$= 66\frac{2}{3}$	lb.
" "	"	1	"	*K*,	"	$= 50$	"
" "	"	$1\frac{1}{2}$	"	*f*,	"	$= 40$	"
" "	"	2	"	*I*,	"	$= 33\frac{1}{3}$	"
" "	"	$2\frac{1}{2}$	"	*h*,	"	$= 28\frac{4}{7}$	"
" "	"	3	"	*G*,	"	$= 25$	"
" "	"	$3\frac{1}{2}$	"	*k*,	"	$= 22\frac{2}{9}$	"
" "	"	4	"	*F*,	"	$= 20$	"

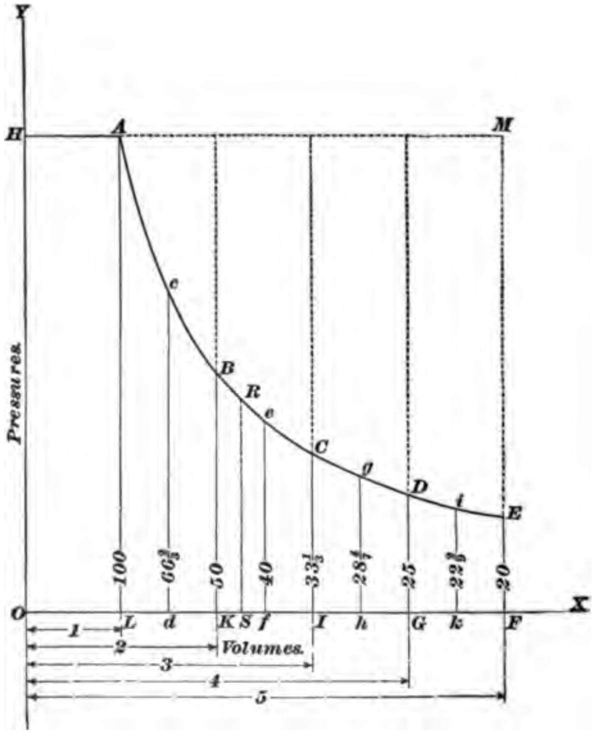


FIG. 223.

At the points *d*, *K*, *f*, *I*, *h*, *G*, *k*, *F*, erect ordinates, and make them equal in length to the pressure at that point, to the scale of 1 in. = 20 lb.—that is, make $c\ d = 66\frac{2}{3}$ lb., $B\ K =$ 50 lb., etc., and through the points A, c, B, e, C, g, D, i, E, draw the curve shown in this figure. If care has been taken in drawing this figure, any ordinate drawn from a point on the line $O\ X$, and limited by the curve, will indicate the pressure of the air in the cylinder when the piston is at that point. Thus, suppose it is desired to know the pressure when the piston is at the point S. Erect the ordinate $S\ R$, and measure it with the same scale that was used to draw the curve; the reading on the scale will be the pressure at that point.

1157. In order to find the work done by the air while the piston was traveling from L to F, and during which time the pressure fell from $A\ L$, or 100 lb. per sq. in., to $E\ F$, or 20 lb. per sq. in., the average pressure, or **mean ordinate,** must be known. This can be found by dividing the area of $A\ E\ F\ L$ by its length $L\ F$. That this statement may be clearly understood, suppose a semicircle to be drawn as shown in Fig. 224, having a diameter of 6 in. Its area will be $6^2 \times .7854 \div 2 =$ 14.1372 sq. in. Divide this area by the length, 6 in., thus $14.1372 \div 6 = 2.3562$ in. On the diameter as a side, and with 2.3562 in. for another side, construct the rectangle $A\ B\ C\ D$; the area of this rectangle will evidently be the same as the area of the semicircle.

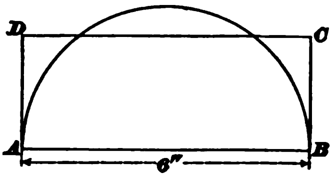


FIG. 224.

1158. **Rule.**—*No matter what the shape of an area may be, if any line be drawn through it and limited by lines perpendicular to it, and tangent to the bounding line of the area, the product of the length of this line, and the mean ordinate drawn from this line to the bounding line, will be equal to the area of that part included by the line, the tangents, and the bounding line included between the points of tangency.*

Thus, in Fig. 225, if the length of $A B$ is known, and $B D$, perpendicular to it, is tangent to the bounding line $A E D I$, and $E F$ is the mean perpendicular (mean ordinate) from $A B$ to the bounding line $A E D B$, $A B \times E F =$ the area of $A E D B A$; if $G H$ is the mean ordinate from $A C$ to $A H C$ of the area $A H J C$, $A C \times G H =$ area of $A H J C$, and if $L K$ is the mean ordinate from $I B$ to $I D B$, $I B \times L K =$ the area $I L D B$. Conversely, if an area is given, and the length of a line in it, so located that perpendiculars to the extremities of the line are tangent to the bounding line of the area, the mean ordinate to this line equals the area divided by the length of the line. Thus, the mean ordinate of $A E D B = \frac{\text{area of } A E D B}{A B} = E F$, etc. Returning now to Fig. 223, if the area $A E F L$ is known, the mean ordinate (mean pressure) can be found by dividing this area by the length $L F$.

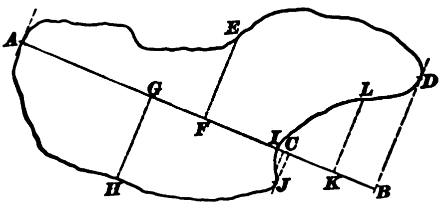


FIG. 225.

1159. The area may be found in two ways: 1. Approximately, by dividing the figure into a number of small areas, adding the ordinates at the center of each of these small areas, and dividing the sum by the number of areas; this result, multiplied by the length $L F$, is the area $A E F L$. 2. Exactly, by using the planimeter, an instrument for measuring plane areas.

The first method, as applied to Fig. 223, is shown in Fig. 226. $L F$ is divided into 8 equal parts, and ordinates are erected at the points of division, thus dividing the area $A E F L$ into 8 small areas. At the *middle points* of these areas, the ordinates *1-1*, *2-2*, *3-3*, etc., are drawn and measured, the lengths (measured to the same scale used to lay off $A L$) being marked on the drawing. The sum of these ordinates is $80 + 57.1 + 44.4 + 36.4 + 30.8 + 26.7 + 23.5 + 21 = 319.9$ lb. Hence, the mean pressure $= 319.9 \div 8 =$

39.99 lb. per sq. in. The second method, by using the planimeter, will not be described, since instructions always go with the instrument. Calling the mean pressure 40 lb. per sq. in., the work which the air could do in expanding from L to F at a constant temperature would be equal to the area

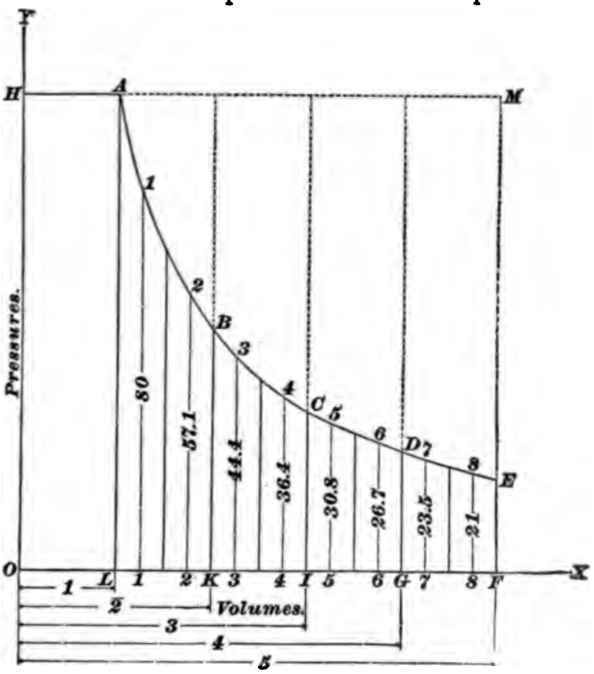


FIG. 226.

of the piston in square inches, multiplied by the mean pressure per sq. in., multiplied by the distance through which it moves or works = 144 × 40 × 4 = 23,040 foot-pounds.

1160. The work can also be calculated directly, without constructing the diagram, by means of the following formula, in which

L = the work in foot-pounds;
P = the total initial pressure in pounds per square foot;
P_1 = the total final pressure in pounds per square foot;
V = the initial volume in cubic feet;
V_1 = the final volume in cubic feet.

$$L = 2.3026\, P V \log \frac{V_1}{V}. \qquad \textbf{(75.)}$$

Since $P V = P_1 V_1$, $\frac{P}{P_1} = \frac{V_1}{V}$ and formula **75** might be written

$$L = 2.3026\, P V \log \frac{P}{P_1}. \qquad \textbf{(76.)}$$

Whichever formula is used, it must be kept in mind that the fraction $\frac{V_1}{V}$ or $\frac{P}{P_1}$ must *always be greater than 1—that is, the numerator must always be greater than the denominator.*

Substituting the values used in Fig. 223, formula **75** or **76** gives

$$L = 2.3026 \times (144 \times 100) \times 1 \times \log \frac{5}{1} =$$

$2.3026 \times 144 \times 100 \times .69897 = 23{,}176$ foot-pounds, nearly.

This is the actual value, and shows that the approximate method used in the previous work was very close.

Suppose that the number of parts had been doubled—that is, that the line $L F$ had been divided into 16 equal parts, instead of 8—the sum of the ordinates drawn at the middle of these parts would then have been

$88.9 + 72.7 + 61.5 + 53.3 + 47.1 + 42.1 + 38.1 + 34.8 + 32 + 29.6 + 27.6 + 25.8 + 24.2 + 22.9 + 21.6 + 20.5 = 642.7$. $642.7 \div 16 = 40.17$ lb. per sq. in. $144 \times 40.17 \times 4 = 23{,}138$ foot-pounds, nearly.

Where a table of logarithms is not at hand, a sufficiently close result for all practical purposes can be obtained by dividing $A\ E\ F\ L$ into 10 parts.

1161. The curve shown in Fig. 223 is called the **isothermal expansion curve,** or the **expansion curve of constant temperature.** It is known in mathematics as the **equilateral hyperbola,** and hence, when used on indicator diagrams, is sometimes called the **hyperbolic expansion curve.** If the initial volume, pressure, and final volume are known, the curve may be constructed graphically without calculating the different points, as was done in Fig. 223. Thus, in Fig. 227, let $O\ Y$ and $O\ X$ be two lines at right angles to each other. These lines are known in mathematics as the **coordinate axes,** the line $O\ Y$ being called the **axis of ordinates,** or **axis of** Y, and the line $O\ X$, the **axis of abscissas,** or **axis of** X. Let $O\ A$ represent the absolute initial pressure, and $O\ B$ the initial volume. Through A draw the indefinite straight line $A\ M$ parallel to the axis of X, and through B draw the

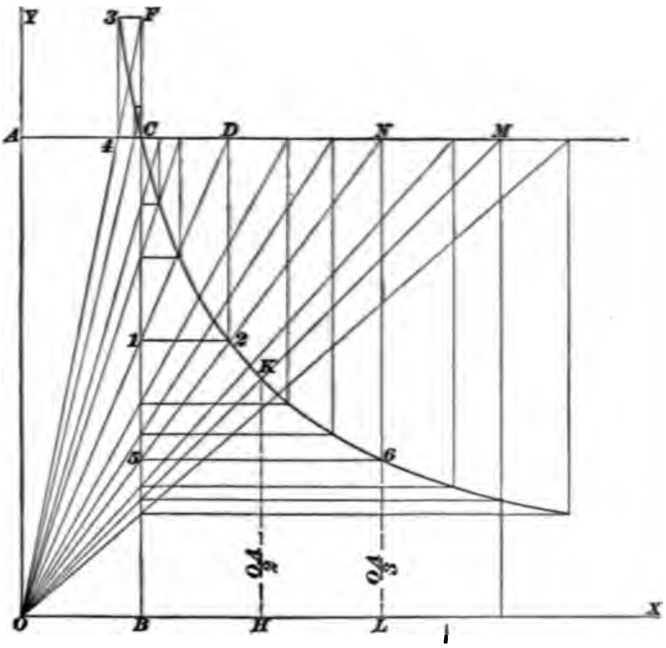


FIG. 227.

indefinite straight line $B\ F$ parallel to the axis of Y. The point C, where these two lines meet, is the point where the expansion is to begin; consequently, it is one point on the curve. Through the point O, called the **origin,** and which is point of no volume and no pressure, draw a number of lines, $O\ F$, $O\ D$, $O\ N$, $O\ M$, etc., cutting $B\ F$ at F, *1*, *5*, etc., and $A\ M$ at *4*, D, N, etc. Through the points F, *1*, *5*, etc., draw lines parallel to the axis of X, and through *4*, D,

N, etc., draw lines parallel to the axis of Y. These lines intersect in the points *3*, *2*, *6*, etc., which are points on the required isothermal expansion line. To prove this, lay off $B\,H$ equal to $O\,B$, and draw $H\,K$ parallel to the axis of Y, intersecting the curve in K. Now, if K is a point on the isothermal expansion line, $H\,K$ must be equal in length to one-half of $O\,A$, since, when the volume is twice as great, the pressure is only half as great. Similarly, if $H\,L = B\,H = O\,B$, $L\,6$ must be one-third as long as $O\,A$. By measurement this will be found to be the case. This curve and method of constructing it is much used in "working up" indicator diagrams, and will be further explained in connection with the subject of Steam and Steam Engines.

1162. If the air or gas be compressed, the action will be exactly the reverse of the expansion. Heat would have to be abstracted instead of added; the pressure would increase instead of decreasing, and the volume decrease instead of increasing.

In Fig. 228, which is Fig. 223 repeated, let $E\,F$ represent the initial pressure = 20 lb. per sq. in., $O\,F$ the initial volume = 5 cu. ft. As the volume decreases, the pressure will increase, as indicated by the isothermal curve $E\,D\,C\,B\,A$, when the temperature is kept constant. The curve may be constructed as shown in Fig. 227, by taking O as the point from which to draw the lines $O\,D$, $O\,N$, etc., in Fig. 227. The point F could not be taken from which to draw these lines, for they must always be drawn from the point of no pressure and no volume. F is a point of no pressure, but it indicates a volume of 5 cu. ft. The work required to compress the air under these conditions may be calculated by formula **75** or **76**, remembering that the larger volume or pressure must be in the numerator of the fraction.

Formulas **75** and **76** then become

$$L = 2.3026\,P\,V \log \frac{V}{V_1} \qquad \textbf{(77.)}$$

and

$$L = 2.3026\,P\,V \log \frac{P_1}{P}, \qquad \textbf{(78.)}$$

in which the letters have the same meaning as before.

1163. Formulas **75, 76, 77,** and **78** will be easier to use if the pressure be taken in pounds per square inch, and $144 \times 2.3026 = 331.5744$ be substituted for 2.3026. As before, the volume must always be taken in cubic feet. Formulas **75** and **76** then become

$$L = 331.5744\, p\, V \log \frac{V_1}{V} \qquad \textbf{(79.)}$$

$$\text{and } L = 331.5744\, p\, V \log \frac{p}{p_1}. \qquad \textbf{(80.)}$$

in which p = pressure in pounds per square inch.

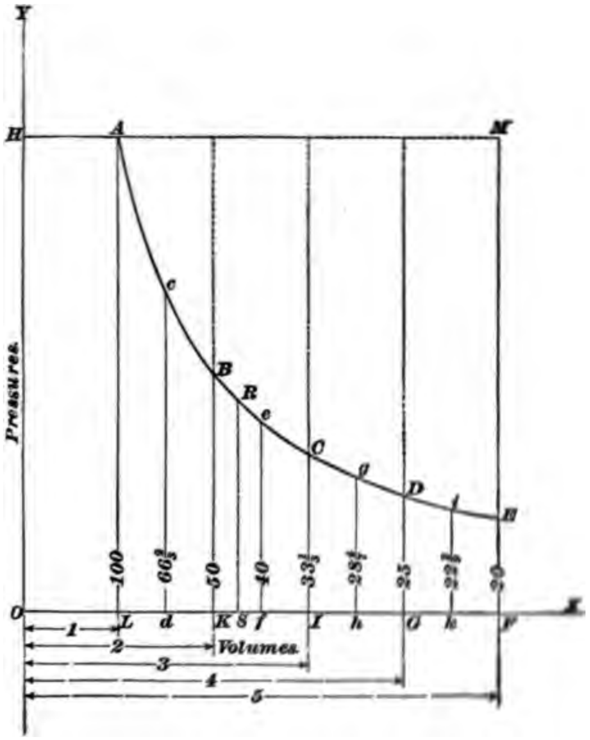


FIG. [illegible].

EXAMPLE.—The initial volume of a body of gas which is to be compressed is 6.78 cu. ft. The initial pressure is 16 lb. per sq. in. If this gas be compressed until its tension is 144 lb. per sq. in., what work will have to be expended, the temperature being kept constant?

SOLUTION.—Using formula **80**, and remembering that the greater pressure must be in the numerator,

$$L = 331.5744\,p\,V \log \frac{p_1}{p} = 331.5744 \times 16 \times 6.78 \times \log \frac{144}{16} = 331.5744$$
$$\times 16 \times 6.78 \times .95424 = 34{,}323.24 \text{ foot-pounds. Ans.}$$

1164. Adiabatic Expansion.—Suppose that a volume of air expands from the same initial volume and pressure as in the case of Fig. 223, but that no heat is added or taken away. The temperature will fall during expansion, and rise during compression. The pressure will fall much faster than in the case of isothermal expansion, and increase much faster than in isothermal compression for the same increase or decrease in volume. The air expands no longer, according to the law $p\,v = p_1\,v_1 = p_2\,v_2$, etc., but according to another law which can only be proven by the use of higher mathematics; this law is for air:

$$p\,v^{1.41} = p_1\,v_1^{1.41} = p_2\,v_2^{1.41}, \text{ etc.} \qquad \textbf{(81.)}$$

In other words, the pressure multiplied by the 1.41 power of the corresponding volume is a constant, and is equal to the product of the pressure and 1.41 power of the volume at any other part of the stroke. The initial volume is 1 cu. ft., and the initial pressure is 100 lb. per sq. in. in Fig. 223. Using these values in the present case, $p\,v^{1.41} = 100 \times 1^{1.41} = 100$; hence, $p_1\,v_1^{1.41} = 100$, $p_2\,v_2^{1.41} = 100$, etc.

Assuming the different volumes, the pressures may be calculated as follows:

Let $v = 2\frac{1}{2}$ cu. ft.; then, $p \times 2.5^{1.41} = 100$, or $p = \frac{100}{2.5^{1.41}}$;

$$\log p = \log 100 - 1.41 \log 2.5 =$$
$$2 - 1.41 \times .39794 = 2 - .56110 = 1.43890,$$
$$\text{or } p = 27.47 \text{ lb., nearly}$$

Calculating in this manner the pressures corresponding to the different values of the volumes for points correspond-g to the volume points in Fig. 223, the following results e obtained:

Pressure corresponding to volume		cd =	56.5 lb.
"	"	" BK =	37.63 "
"	"	" ef =	27.47 "
"	"	" CI =	21.25 "
"	"	" gh =	17.1 "
"	"	" DG =	14.16 "
"	"	" ik =	12.— "
"	"	" EF =	10.34 "

Making the different ordinates equal in length to these 'essures, and using the same scale as before—1 in. = 20 lb.

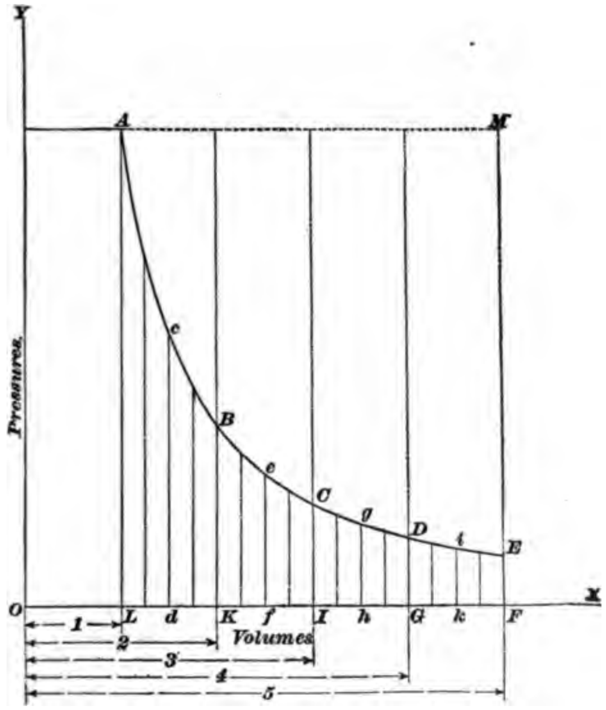


FIG. 229.

-the curve shown in Fig. 229 is produced by tracing the ne $A\ B\ C\ D\ E$ through these points. It will be noticed that the area of $A\ E\ F\ L$ in Fig. 229 is considerably smaller than in Fig. 223; consequently, the mean pressure is less, and the work done in expanding is less. This was to be expected, since, no heat being added, the temperature must fall and with it the pressure also. Erecting ordinates at the middle points of these divisions, and measuring them in a manner similar to the approximate method of finding the mean pressure followed in Fig. 226, the mean pressure is found to be

$$\frac{73 + 45.5 + 31.9 + 24 + 19 + 15.5 + 13 + 11.1}{8} = 29\tfrac{1}{8} \text{ lb. per}$$

sq. in.

The work done is evidently $144 \times 29\frac{1}{8} \times 4 = 16{,}776$ foot-pounds.

1165. The mathematical formula which gives the work directly when the initial and final volumes and the initial pressure are known is

$$L = 2.44\,PV\left[1 - \left(\frac{V}{V_1}\right)^{.41}\right]. \qquad \textbf{(82.)}$$

By means of formula **82**, just given, the work is found to be 16,974 foot-pounds. Thus, substituting the values given and remembering that P = pressure in pounds per square foot,

$$L = 2.44 \times (100 \times 144) \times 1\left[1 - \left(\frac{1}{5}\right)^{.41}\right] = 2.44 \times 14{,}400$$

$(1 - .51691) = 16{,}974$ foot-pounds.

1166. If the initial and final pressures and the initial volume are given, to find the work a formula may be derived as follows:

From formula **81**, $PV^{1.41} = P_1 V_1^{1.41}$; hence, $\frac{V^{1.41}}{V_1^{1.41}} = \frac{P_1}{P}$, or $\frac{V}{V_1} = \left(\frac{P_1}{P}\right)^{\frac{1}{1.41}}$. Affecting both sides of this last equation with an exponent of .41, there results $\left(\frac{V}{V_1}\right)^{.41} = \left(\frac{P_1}{P}\right)^{\frac{.41}{1.41}}$, or $\left(\frac{V}{V_1}\right)^{.41} = \left(\frac{P_1}{P}\right)^{.29078}$, since $.41 \div 1.41 = .29078$.

Substituting the right-hand member of the last equation in formula **82**,

$$L = 2.44\,PV\left[1 - \left(\frac{P_1}{P}\right)^{.29078}\right]. \qquad \textbf{(83.)}$$

1167. If the pressure be taken in pounds per square inch, **82** and **83** become

$$L = 351.36 p\,V\left[1 - \left(\frac{V}{V_1}\right)^{.41}\right], \qquad \textbf{(84.)}$$

$$\text{and } L = 351.36 p\,V\left[1 - \left(\frac{p_1}{p}\right)^{.29078}\right]. \qquad \textbf{(85.)}$$

In both formulas, p and V are the initial pressure and volume, respectively. When a gas expands without receiving or losing any heat, the pressure falls, as shown by Fig. 229, and it is said to expand **adiabatically.** The curved line $A\ B\ C\ D\ E$ is called the **adiabatic curve.**

1168. Formulas **82** and **83** (and, of course, **84** and **85**) may be used for compression as well as for expansion, the letters P and V representing the initial pressure and volume, and P_1 and V_1 the final pressure and volume, in both cases. To show that such is the case, proceed as follows:

Dividing both sides of formula **82** by 2.44,

$$\frac{L}{2.44} = PV\left[1 - \left(\frac{V}{V_1}\right)^{.41}\right].$$

Now, if the formula is true for both cases, the work done during adiabatic expansion must equal the work required to immediately compress the air back to its pressure before expansion. But, if the final pressure and volume after expansion be represented by P_1 and V_1, these letters will represent the initial pressure and volume during compression. Consequently, in order to prove that the formula holds good for both cases, it must be proved that

$PV\left[1 - \left(\frac{V}{V_1}\right)^{.41}\right] = P_1 V_1\left[1 - \left(\frac{V_1}{V}\right)^{.41}\right]$. By formula **81**, $PV^{1.41} = P_1 V_1^{1.41}$, or $P_1 = \frac{PV^{1.41}}{V_1^{1.41}}$. Representing the exponent 1.41 by m, for convenience, $P_1 = \frac{PV^m}{V_1^m}$. Substituting this value of P_1 in $P_1 V_1 \left[1 - \left(\frac{V_1}{V}\right)^{.41}\right]$, it becomes, since $.41 = m - 1$,

$$\frac{P V^m}{V_1^m} \times V_1 \left[1 - \left(\frac{V_1}{V}\right)^{m-1}\right] = P V \times \frac{V^{m-1}}{V_1^{m-1}} \left[1 - \frac{V_1^{m-1}}{V^{m-1}}\right] =$$

$$P V \left[\frac{V^{m-1}}{V_1^{m-1}} - 1\right] = -P V \left[1 - \frac{V^{m-1}}{V_1^{m-1}}\right] = -P V \left[1 - \left(\frac{V}{V_1}\right)^{.41}\right],$$

which was to be proved.

When applying the formula for cases of compression, it will be found that the result is negative. The minus sign merely indicates compression, the numerical value being the same as in the case of expansion.

1169. From formula **81** two other formulas may be derived, which are of great importance in investigations pertaining to the theory of heat. They are derived as follows:

Since $P V^{1.41} = P_1 V_1^{1.41}$, $\left(\frac{V}{V_1}\right)^{1.41} = \frac{P_1}{P}$.

Representing 1.41 by m, as before,

$$\left(\frac{V}{V_1}\right)^m = \frac{P_1}{P}, \text{ or } \frac{V}{V_1} = \left(\frac{P_1}{P}\right)^{\frac{1}{m}}. \quad (a)$$

Multiplying both sides of equation (a) by $\frac{P}{P_1}$,

$$\frac{P V}{P_1 V_1} = \frac{P}{P_1}\left(\frac{P_1}{P}\right)^{\frac{1}{m}} = \frac{P}{P_1} \times \frac{P_1^{\frac{1}{m}}}{P^{\frac{1}{m}}} = \frac{P_1^{\frac{1}{m}-1}}{P^{\frac{1}{m}-1}} = \left(\frac{P_1}{P}\right)^{\frac{1-m}{m}}.$$

Substituting for m its value, $1 - 1.41 = -.41$, and

$$\frac{P V}{P_1 V_1} = \left(\frac{P_1}{P}\right)^{-\frac{.41}{1.41}} = \left(\frac{P_1}{P}\right)^{-.29078}.$$ According to the theory of exponents, see Arts. **529** and **530**, $\left(\frac{P_1}{P}\right)^{-.29078} = \left(\frac{P}{P_1}\right)^{.29078}$.

Hence, $\frac{P V}{P_1 V_1} = \left(\frac{P}{P_1}\right)^{.29078} \quad (b)$

1170. According to formula **62**, Art. **1058**,

$$\frac{P V}{T} = \frac{P_1 V_1}{T_1}, \text{ or } \frac{P V}{P_1 V_1} = \frac{T}{T_1}.$$

Substituting this value of $\frac{PV}{P_1 V_1}$ in equation (b)

$$\left(\frac{P}{P_1}\right)^{.29078} = \frac{T}{T_1}. \qquad (86.)$$

Likewise, since $.29078 = \frac{1.41 - 1}{1.41} = \frac{m - 1}{m}$, formula **86** may be written $\left(\frac{P}{P_1}\right)^{\frac{m-1}{m}} = \frac{T}{T_1}$. ($c$)

But, since $\frac{P}{P_1} = \left(\frac{V_1}{V}\right)^m$, from formula **81**, equation (c) may be written $\left[\left(\frac{V_1}{V}\right)^m\right]^{\frac{m-1}{m}} = \frac{T}{T_1}$, or $\left(\frac{V_1}{V}\right)^{m-1} = \frac{T}{T_1}$.

Substituting for m its value,

$$\left(\frac{V_1}{V}\right)^{.41} = \frac{T}{T_1}. \qquad (87.)$$

1171. Formulas **86** and **87** may be used to compute the temperature of the air after adiabatic expansion or compression when the initial and final pressure or the initial and final volume are known and the initial temperature has been noted.

In formulas **86** and **87**, the pressures, volumes, and temperatures may be expressed in any units desired, remembering that the pressures and temperatures are *absolute.* In other words, the pressures may be in pounds per square inch, pounds per square foot, inches of mercury, etc.; the volumes may be in cubic feet, cubic inches, cubic meters, etc., and the temperatures may be in Fahrenheit, Centigrade, or Reaumur degrees.

EXAMPLE.—The temperature of the air as it enters the cylinder of an air compressor is 60°; what is its final temperature after being compressed to 100 pounds per square inch, absolute, the compression being adiabatic?

SOLUTION.—The initial pressure is, of course, 14.7 lb. per sq. in.; hence, substituting in formula **86** the values of P, P_1, and T, $T_1 = T\left(\frac{P_1}{P}\right)^{.29078} = 520\left(\frac{100}{14.7}\right)^{.29078} = 908.1°$. Therefore, final temperature $= 908.1 - 460 = 448.1°$. Ans.

1172. If the volume of air were 5 cu. ft. and the pressure were 10.34 lb. per sq. in.—that is, if the piston were at *E F*, Fig. 229, and the air were compressed to 1 cu. ft., no heat being lost—the final pressure would be 100 lb., as before; the curve of pressures would be the adiabatic curve *E D C B A*, as in the case of expansion. The work which the air would do when it expanded isothermally, or at constant temperature, was found to be 23,176 foot-pounds, and when it expanded adiabatically, 16,974 foot-pounds, a

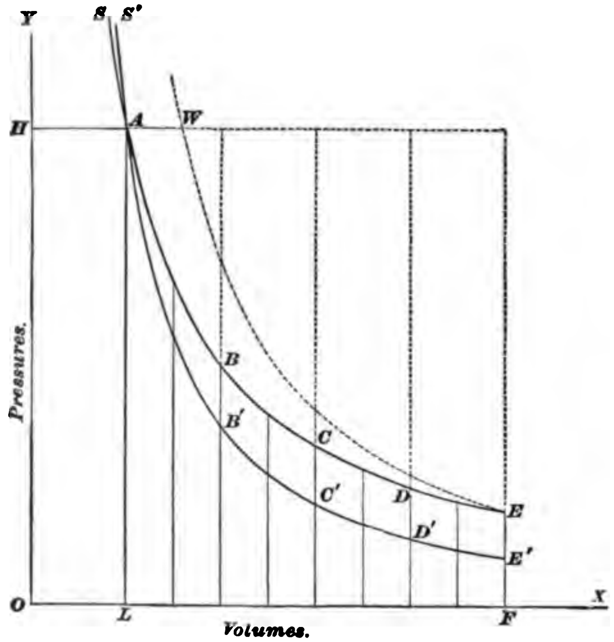


FIG. 230.

result considerably less. This was to be expected, since, as no heat was added, the heat required to do the work of expansion had to be taken from the gas, thus reducing its energy and the amount of work that it could do. To better show the effects of isothermal and adiabatic expansion, the two curves shown in Figs. 223 and 229 are drawn together in Fig. 230. Here *S A B C D E* is the isothermal curve of expansion or compression, and $S' A B' C' D' E'$ is the corresponding adiabatic curve. If 5 cu. ft. of air having a tension of 20 lb. per sq. in. be compressed isothermally, the curve of compression would follow the line $E D C B A S$, while, if compressed adiabatically, the initial tension and volume being the same, it would follow the dotted line $E S'$. Hence, if the air were thus compressed to 1 cu. ft., it is easy to see that the work required would be far more for adiabatic compression than for isothermal compression. To obtain the area $A C' E' F L$, the following formula may be used, which gives it directly for air when p and p_1 are the greater and lesser pressures, respectively, and V and V_1 their corresponding volumes:

$$\frac{p V - p_1 V_1}{.41} = \text{area.} \qquad (88.)$$

1173. By means of this formula, the mean ordinate may be calculated directly, without drawing the curve and measuring the mean ordinates of the equal parts. The pressure corresponding to a volume of 5 cu. ft., and denoted by the ordinate $E' F$, was found to be 10.34 lb. per sq. in. The greater pressure was 100 lb. per sq. in., and the corresponding volume 1 cu. ft.; hence, the area $A B' C' D' E' F L A$ is

$$\frac{p V - p_1 V_1}{.41} = \frac{100 \times 1 - 10.34 \times 5}{.41} = 117.805 \text{ sq. in. This,}$$

divided by the length $L F = 4$, gives $\frac{117.805}{4} = 29.45125$ lb. per sq. in. = mean ordinate. Since the area of the piston is 144 sq. in., and the piston moved 4 ft., the work it would do is $29.45125 \times 144 \times 4 = 16,964$ foot-pounds. The previous calculation gave 16,974 foot-pounds, a difference of 10 foot-pounds. Both methods would have given the same result had the calculation for the final pressure, 10.34 lb. per sq. in., been carried out to a sufficient number of decimal places, and 7-figure logarithms used instead of those of 5 figures. The difference is so slight that the results are practically the same.

1174. A little thought will show that the work done is directly proportional to the areas, and that the areas themselves may be considered as representing the work done on the piston during one stroke. For the mean pressure was just now found to be 29.45 lb. per sq. in. Since every inch of length on any ordinate in Fig. 230 represents a pressure of 20 lb. per sq. in., the actual length in inches of the mean ordinate is $29.45 \div 20 = 1.4725$ in. The length of the area is 4 in., and the actual area is $1.4725 \times 4 = 5.89$ sq. in. Now, since the ordinates are so drawn that 1 in. = 20 lb. pressure per sq. in., and the area of one sq. ft. is 144 sq. in., $5.89 \times 20 \times 144 =$ work $= 16,963$ foot-pounds, the same result as before. Therefore, if in any diagram of this kind the actual area be multiplied by the vertical scale of pressures in pounds per square inch (in this case, 1 in. = 20 lb. per sq. in.) and by the horizontal scale of volumes in cu. ft. (in this case, 1 in. = 1 cu. ft.), and then multiplied by 144, the result is the work. The work is represented by the area, and the ratio of any two areas is the same as the ratio of the works.

1175. A study of the curves $EDCBA$ and EW, in Fig. 230, will show why the walls of air compressors are cooled. Suppose that EF represents a pressure of 14.7 lb. per sq. in., instead of 20 lb., as formerly. This is the pressure of the atmosphere, and, consequently, the initial pressure in the air compressor cylinder. If the air were not cooled while being compressed, the pressures corresponding to the various volumes would be given by the dotted adiabatic curve EW.

If the air thus compressed could be used at once, there would be no loss, since the heat imparted to it would be utilized in doing work, and it would make no difference whether the compression was adiabatic or isothermal. Such is not the case, however. The air, after leaving the compressor, is stored in a large reservoir called a **receiver,** from which it is conveyed in pipes to the engines, pumps, or other machines which it operates. These are situated sometimes 5 miles or more from the compressor, and when the air reaches them, its temperature has been reduced to that of the atmosphere. As a consequence of this reduction in temperature, the pressure falls to a point determined by the intersection of an ordinate drawn through the point *W* and the isothermal curve *E B A*. Fig. 231 shows the curves when applied to an air compressor. *O L* represents the initial volume, say 5 cu. ft.; *L B* the atmospheric pressure 14.7 pounds per square inch, and *O H* the final pressure, say 100 pounds per square inch. *B C* represents the adiabatic, and *B A* the isothermal, compression curve, respectively.

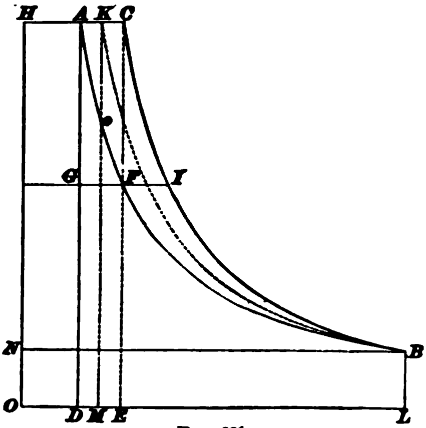


FIG. 231.

The point *F*, where the ordinate through *C* intersects the isothermal *B A*, indicates the pressure of air, when compressed adiabatically after it has cooled to the temperature of the outside air. Measuring the ordinate *E F*, the pressure at the point is found to be 57.26 pounds per square inch.

The work done in compressing the air adiabatically, and in forcing it out of the cylinder is proportional to the area *B L O H C*, while, for isothermal compression, the work is proportional to the area *B L O H A*. The work lost through adiabatic compression is the difference of these two areas, or the area *A B C*. By the use of some cooling device, such as the water jacket described in Art. **1071**, the compression curve will lie between the curves *B C* and *B A*, and the subsequent fall of pressure due to the cooling will be greatly reduced. In the best types of modern air compressors, this curve will lie about half way between *B C* and *B A*, as shown by the dotted curve *B K*, and the fall of pressure will then be Ko, instead of CF, as in the former case where no cooling took place.

1176. The *efficiency* of the cooling device is determined as follows: Suppose that $B\ K\ H\ N$ represents an actual indicator diagram taken from the air cylinder of an air compressor. Lay off $N\ O$ equal to the pressure of the atmosphere, as determined by a barometer, or equal to 14.7 pounds per square inch, if no barometer reading has been taken, and draw the isothermal and adiabatic curves $B\ A$ and $B\ C$ in the usual manner. Then, the efficiency of the cooling device $= \frac{\text{area of } CBK}{\text{area of } CBA}$.

The student who approaches the subject of cooling devices for air compressors for the first time is apt to reason fallaciously in the following manner: He argues that, although the air is cooled, the work done on the air is the same in either case, the work not shown by the card being turned into heat and carried away by the cooling water. The fallacy of this reasoning may be proved by taking an indicator diagram from the steam cylinder. It will always be found that the work shown by the steam diagram will always equal that shown by the air diagram, plus the work needed to overcome the friction of the moving parts, no more and no less. The student may reason himself out of the fallacy, thus: During the compression of a given weight of air, there are four quantities which are liable to vary: the pressure P, the volume V, the temperature T, and the total quantity of heat Q which the air possesses. During adiabatic compression, the total quantity of heat in the air remains the same; i. e., Q is constant while P, V, and T vary. During isothermal compression, on the contrary, P, V, and Q vary, T remaining constant. If Q represents the total amount of heat in the gas before compression, and Q_1 the total amount of heat after compression, $Q - Q_1 = 0$, in the case of adiabatic compression, while, in the case of isothermal compression, $778\ (Q - Q_1)$ is exactly equal to the work represented by the area $A\ B\ C$ in Fig. 231.

EXAMPLES FOR PRACTICE.

1. If 5.68 cu. ft. of air having a temperature of 50° is compressed adiabatically to a volume of 1.3 cu. ft., what is the final temperature?
Ans. 473.56°.

2. In the above example, if the initial tension is 14.7 lb. per sq. in., what is the final tension? Ans. 117.57 lb. per sq. in.

3. With the same data as above, calculate the work required to compress the air when the compression is adiabatic? Ans. 24,365 ft.-lb.

4. With the conditions the same as in the preceding example, calculate the work required when the compression is isothermal?
Ans. 17,729 ft.-lb.

5. Eight-tenths cu. ft. of air, at a temperature of 120° and a pressure of 45 lb. per sq. in., expands adiabatically to the pressure of the atmosphere. (*a*) What is the final volume? (*b*) The final temperature? (*c*) The work done during expansion?
Ans. { (*a*) 1.769 cu. ft. (*b*) −41.07°. (*c*) 3,513 ft.-lb.

THE IDEAL HEAT ENGINE.

1177. Second Law of Thermodynamics.—*Heat cannot pass from a cold to a hot body by a self-acting process unaided by external agency.*

1178. The Reversible Cycle Process.—In Fig. 232, suppose *S S* to be the cylinder of a single-acting engine; i. e., one which does work only when the piston is moving in one direction, and, for simplicity, assume that the engine is a hot-air engine. Call the fire which heats the air, or *source of heat*, the **hot body**; the atmosphere into which the hot air exhausts, and which absorbs the heat, the *refrigerator*, or **cold body**, and the air in the cylinder which does the work, owing to the expansion, the **intermediate body**. Suppose, further, that the cylinder is made of a perfect non-conducting heat material and that the head (which call *a*) can be removed and replaced, whenever it is desired, by one that is a perfect conductor of heat. Call this head *b*. All of the above conditions regarding the construction of the cylinder are, of course, impossible; the only reason for making these assumptions is that the action of the intermediate body may be considered under perfect conditions.

Let $O\,Y$ and $O\,X$, Fig. 232, be the coordinate axes, and let $O\,V_1$ represent the volume $S\,1$ of the air in the cylinder, whose absolute temperature is T_1; pressure, P_1, and volume, V_1. When the piston is at *1*, the line $V_1\,A$ represents the pressure P in pounds per square foot.

1. Suppose the head *b* to be in place, and to be in contact with the hot body, which is always kept at a uniform temperature T_1, any heat abstracted being immediately supplied by the fire. Then, so long as the head *b* is in contact with the hot body, the temperature of the air in the cylinder will remain constant. Suppose the air to expand until the piston has reached another position, as *2*, overcoming a resistance at every point just equal to the tension of the expanding air. Heat is supplied by the hot body and the temperature remains constant; in other words, the expansion is isothermal. The work done will be represented by the area $A\,B\,V_2\,V_1$.

2. Replace head *b* with head *a*, and let the air expand further until the piston has reached the extreme position *3*. No heat can now enter or leave the cylinder, and this expansion will be adiabatic. The position *2* should be so chosen that, at the end of the adiabatic expansion, the temperature T_2, corresponding to the pressure $C\,V_3$ and volume $O\,V_3$, which denote by P_2 and V_2, respectively, will be the same as that of the cold body. The work done during this period is represented by the area $B\,C\,V_3\,V_2$, and the total work done during expansion from *1* to *3* by $A\,B\,V_2\,V_1 + B\,C\,V_3\,V_2 = A\,B\,C\,V_3\,V_1$

3. Replace the head *a* with head *b*, and, supposing head *b* to be in contact with the cold body, move the piston to position *4*. The air will then be compressed, and, since the temperature of the cold body is assumed to remain at T_2, the compression is isothermal. Consequently, a certain quantity of heat must be abstracted from the air and rejected to the cold body. The work done *upon* the air will be represented by the area $C\,V_3\,V_4\,D$. The position *4* should be so chosen that if the air be compressed adiabatically from *4* to *1*, the volume, pressure, and temperature of the air, when

the piston reaches position *1*, will be the same as at the beginning.

4. Replace the head *b* with head *a*. No heat can then enter or leave the cylinder, and the air will be compressed adiabatically to the original volume, pressure and temperature, provided the position *4* has been rightly chosen. The work done *upon* the air is represented by the area $D V_4 V_1 A$, and the total work done *upon* the air is $C V_3 V_4 D + D V_4 V_1 A = C V_3 V_1 A D$.

The excess of work done by the air over that done upon it; i. e., the excess of heat in foot-pounds, taken from the hot body over that rejected to the cold body, is determined by difference of the areas $A B C V_3 V_1$ and $C V_3 V_1 A D$, or $A B C D$. It should be noted that the only means by which the piston could be returned from *3* to *1* was through the application of an *external force*. It will also be noticed that the condition of the intermediate body is now exactly the same as regards pressure, volume, and temperature as in the beginning.

1179. A series of operations similar to that described above is called a **cycle process**, and when the last operation leaves the intermediate body in the same state as in the beginning, the process is called a **closed cycle**; otherwise, it is an *open* cycle. Thus, the process represented by the lines $A B$, $B C$, $C D$, and $D A$ is a *closed cycle*, while that represented by the lines $A B$, $B C$, $C F$, and $F E$ is an *open cycle*, and heat must be added to the intermediate body to bring it into the same conditions that governed it in the beginning.

1180. Every closed cycle process is **reversible**; that is, the operations described in connection with it may be reversed. Thus, let the air expand adiabatically from *1* to *4*, the pressures being represented by the curve $A D$; then, isothermally from *4* to *3*, the pressures following the curve $D C$; then, compress it adiabatically from *3* to *2*, the pressures following the curve $C B$, and, lastly, compress it isothermally from *2* to *1*, the pressures following the curve $B A$.

The work done by the air in this case is ***negative;*** that is, the work done by the air in expanding is less than that performed upon it during compression, and the amount of this negative work is the area $ABCD$. The whole process is the exact reverse of the preceding one. In other words, work is done upon the intermediate body instead of by it, as, for example, in an air compressor.

It will be noticed that, in this reverse process, heat is taken from the cold body and rejected into the hot body, through the aid of an external force; while, in the direct process, heat was taken from the hot body and rejected into the cold body.

For reasons which will be shown later, any engine which operates through a reversible cycle, like that just described, is a *perfect engine* of its kind.

1181. Calculation of the Efficiency of a Perfect Heat Engine.—It is first necessary to show how the points B and D, Fig. 232, are determined. The absolute temperatures of the air (intermediate body) at the points A and B are the same; i. e., T_1, since AB is an isothermal. During the subsequent adiabatic expansion from B to C, the temperature of the intermediate body falls to T_2, the temperature of the cold body, and remains at that temperature during the following isothermal compression until the point D is reached, which must be so chosen that the adiabatic compression from D to A will just raise the temperature to T_1 again. From formula **87**, $\left(\frac{V_2}{V_1}\right)^{.41} = \frac{T_1}{T_2}$ for cases of adiabatic expansion or compression. Extracting the .41 root of both sides of this equation, $\frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{.41}}$. Letting $OV_3 = V_2$ and $OV_2 = V_1$, $\frac{V_2}{V_1} = \frac{OV_3}{OV_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{.41}}$ for adiabatic expansion. Considering the air to expand from A to D instead of compressing from D to A, $OV_4 = V_2$ and $OV_1 = V_1$, or $\frac{V_2}{V_1} = \frac{OV_4}{OV_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{.41}}$. Therefore, $\frac{OV_3}{OV_2} = \frac{OV_4}{OV_1}$, since both are equal to $\left(\frac{T_1}{T_2}\right)^{\frac{1}{.41}}$. In other words, *the ratio of adiabatic expansion equals the ratio of adiabatic compression.* For example, in Fig. 232, $O V_3$ represents 6 cubic feet; $O V_2$, 3 cubic feet, and $O V_1$, 1 cubic foot: then, to find $O V_4$,

$$\frac{6}{3} = \frac{O V_4}{1}, \text{ or } O V_4 = 2 \text{ cubic feet.}$$

Since $\frac{O V_3}{O V_2} = \frac{O V_4}{O V_1}$, it follows that $\frac{O V_3}{O V_1} = \frac{O V_2}{O V_4}$ by a simple transposition of the terms $O V_1$ and $O V_2$ from one side of the equation to the other side; i. e., *the ratio of isothermal expansion equals the ratio of isothermal compression.*

1182. The efficiency of any machine may be defined as the ratio of the work done to the work expended. During the first operation of the reversible cycle of Fig. 232, all of the heat taken from the hot body is expended in doing external work, since, as the temperature of the air (intermediate body) remains constant, the vibratory movement of the molecules remains constant also, and no inner work is done. The heat supplied in foot-pounds of work is, by formula **75**, $L = 2.3026 \, P_1 V_1 \log \frac{V_2}{V_1}$. For convenience, substitute for $P_1 V_1$, $c \, T_1$ and for $\frac{V_2}{V_1}$, r_1; then, $2.3026 \, c \, T_1 \log r_1$ = work represented by the area $A B V_2 V_1$.

NOTE.—That this substitution may be made is easily shown by means of formula **62**, Art. **1058**. Thus, $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$. Represent the actual value of $\frac{P_1 V_1}{T_1}$ by c; then, $\frac{P_1 V_1}{T_1} = c$, or $P_1 V_1 = c \, T_1$ and $P_2 V_2 = c \, T_2$.

During the second operation, no heat is supplied to the intermediate body, but part of its heat is converted into work in order to overcome the external resistances. The amount of heat in foot-pounds which is thus converted is, by formula **82**, $2.44 \, P_2 V_2 \left[1 - \left(\frac{V_2}{V_3}\right)^{.41}\right] = 2.44 \, c \, T_1 \, (1 - r_2^{.41})$ = area $B C V_3 V_2$, since $P_2 V_2 = P_1 V_1$ in this case.

During the third operation, work is done upon the air (intermediate body), and heat is abstracted by the cold body equal to this work in foot-pounds. The amount of this work is $2.3026\ c\,T_2 \log \frac{V_3}{V_4} = 2.3026\ c\,T_2 \log r_2 =$ area $C V_3 V_4 D$. During the fourth operation, the temperature is raised, owing to the conversion of work upon the air into heat, and the amount of this work is (see last equation, Art. **1168**) $2.44\,c\,T_1\left[1-\left(\frac{V_1}{V_4}\right)^{.41}\right] = 2.44\,c\,T_1\left(1-r_4^{\,.41}\right) =$ area $D V_4 V_1 A$. It was shown during the demonstration of the determination of the points B and D that $\frac{V_2}{V_3} = \frac{V_1}{V_4}$. Hence, $r_3 = r_4$, and the work done during adiabatic expansion, or the area $B C V_3 V_2$, equals the work done during adiabatic compression, or the area $D V_4 V_1 A$. Since, in the first case, heat in the intermediate body is converted into work, and, in the second case, work from some external source is converted into heat, the two works, being equal, neutralize each other, and the total work done by the machine and represented by the area $A B C D A$ equals the difference of the work done by the intermediate body during *isothermal expansion* over that done during *isothermal compression;* i. e., work done $= 2.3026\,c\,T_1 \log r_1 - 2.3026\,c\,T_2 \log r_2$.

Consequently, $r_1 = r_2$, and the work accomplished during the cycle $= 2.3026\,c\,T_1 \log r_1 - 2.3026\,c\,T_2 \log r_2 = 2.3026\,c \log r_1 (T_1 - T_2)$. Hence, the efficiency of a perfect heat engine $= \frac{2.3026\,c \log r_1 (T_1 - T_2)}{2.3026\,c\,T_1 \log r_1} = E = \frac{T_1 - T_2}{T_1}$. **(89.)**

That is, *for a perfect heat engine operating through a reversible cycle process, the efficiency of the machine is the ratio of the difference of the absolute temperatures of the sources of heat and of cold to the absolute temperature of the source of heat.*

Since, according to the first law, heat and mechanical energy are mutually convertible, it follows that the fraction $\frac{T_1 - T_2}{T_1}$ represents the percentage of the heat taken from the hot body, which was utilized in doing work.

1183. The efficiency of the engine can become equal to unity, or 100%—i. e., the engine can turn the whole of the heat supplied to it into work—only when $T_2 = 0$, and this can only occur when the cold body has the absolute zero of temperature. The absolute temperature T_2 can not be made 0, nor even approached; in fact, it is impracticable to reduce the temperature below that of the surrounding air; hence, in order to obtain a comparatively high efficiency, the initial temperature must be very high. Suppose that the temperature of the air at the beginning of expansion was 540°, and at the beginning of adiabatic compression was 32°; the absolute temperatures would be $540 + 460 = 1000°$, and 492°, respectively. The efficiency would be $\frac{T_1 - T_2}{T_1} = \frac{1000 - 492}{1000} = 50.8\%$. Such a high temperature could not be used in actual practice. In a practical working engine, the efficiency would be even less than that indicated by the fraction $\frac{T_1 - T_2}{T_1}$, since work is required to overcome the loss due to friction, a part of the heat supplied is radiated, etc. The terms heat and work are here considered to be synonymous.

1184. It is easy to see that a closed cycle is more efficient than an open cycle. For, referring to Fig. 232, let *A B C F E A* represent an open cycle. Then, the work done by the air is the area $A\ B\ C\ V_2\ V_1$ as before, while the work done upon the air when the point *E* has been reached is $C\ V_2\ V_1\ E\ F$. The gain in area over that obtained in the closed cycle is the area *A D F E*. But in order that the temperature of the intermediate body may be the same as that of the hot body, an amount of heat must be imparted equal to the work represented by the area *E G Y A*, and, since this area is evidently greater than the area *A D F E*, it follows that there is a loss over that of the previous cycle.

1185. It is now easy to prove that an engine operating through a cycle between the temperatures T_1 and T_2 can not have a greater efficiency than $\frac{T_1 - T_2}{T_1}$. For, suppose that an engine could be devised having a greater efficiency than the one operating, as indicated by Fig. 232 (which call No. 1); call this engine No. 2, and let it drive No. 1 through a reverse cycle. (For example, suppose engine No. 2 to be a

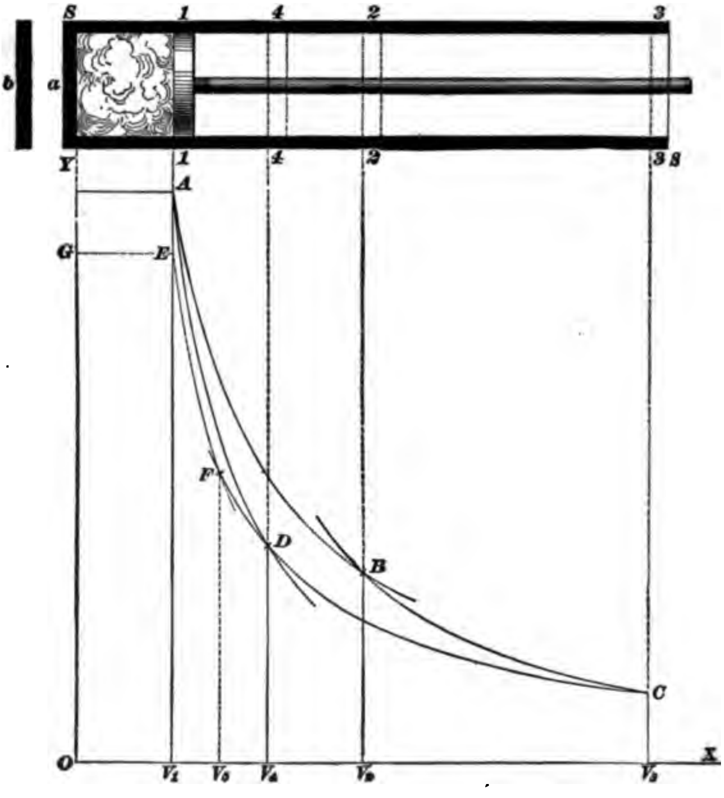


FIG. 232.

hot-air engine, and No. 1 an air compressor.) Then, engine No. 2 takes heat from the hot body and rejects it into the cold body, while engine No. 1, operating in a reverse cycle, takes heat from the cold body and rejects it into the hot body. Suppose the horsepower of both engines to be the same. Then, in engine No. 2, work is done *by* the intermediate body by aid of the heat received from the hot body; and, in Engine No. 1, work is done *upon* the intermediate body by aid of the heat taken from the cold body through the agency of engine No. 2. If friction be neglected and both engines are perfect engines, it is evident that this combination could go on running forever.

Since the power of both engines is the same, and engine No. 2 was assumed to be more efficient than engine No. 1, it is evident that engine No. 2 will reject less heat into the cold body than engine No. 1 takes from it. From this, it follows that if the engines be kept to work long enough, the whole of the heat in the cold body could be taken out of it and transferred to the hot body—that is to say, heat could be transferred from a cold body to a hot body by means of a self-acting contrivance—a result contrary to all experience, and contradicting the second law of thermodynamics. It is easy to see that the result would be a perpetual motion machine—an impossibility.

1186. The conclusion is thus evident: *No heat engine operating between the temperatures T_1 and T_2 can have a greater efficiency than the reversible cycle engine.* Likewise, *the ideal thermal efficiency of any heat engine may be determined by the fraction $\frac{T_1 - T_2}{T_1}$, where T_1 is the highest and T_2 the lowest absolute temperatures of the intermediate body.*

If the student is not satisfied by the above reasoning that no engine can have a greater efficiency than $\frac{T_1 - T_2}{T_1}$, he may assume the intermediate body to be subjected to any process whatever; then, calculate the work done by it and the work done upon it. If between the same limits of temperature he can obtain a greater amount of work for the same quantity of heat taken from the hot body, then the above reasoning is not true.

1187. It was previously shown that a closed cycle had a greater efficiency than an open one. Now, take a cycle process like that illustrated in Fig. 233. Here the air has a pressure $A\ V_1 = 100$ pounds per square inch, a temperature T_1 of say 425°, and a volume of say 1 cubic foot. It expands isothermally to a volume of 4 cubic feet, doing work represented by the area $A\ B\ V_2\ V_1$ equivalent to $331.5744 \times 100 \times 1 \log \frac{4}{1} = 19{,}963$ foot-pounds. To restore the air to its original volume, pressure and temperature, it might now be compressed isothermally, in which case the work done upon the air would be the same as that done by it; i. e., the work obtained from the machine would be zero. Hence, in order to obtain useful work from the machine, it is necessary to lower the pressure, and, in so doing, the temperature as well. The pressure $B\ V_2$ is evidently $\frac{100 \times 1}{4} = 25$ pounds per square inch. Suppose it be lowered to the pressure of the atmosphere, 14.7 pounds per square inch. This may be done in two ways: either by removing a portion of the air from the cylinder (reducing its weight) or by cooling it (removing some of its heat). Suppose, for convenience, that the latter method is employed. Then, the resulting temperature will be, using formula **59**, Art. **1055**, $T_2 = \frac{14.7 \times 885}{25} = 520°$, corresponding to a thermometer temperature of 60°. Now, compressing it isothermally, it will follow the curve $C\,D$, and the pressure corresponding to a volume of 1 cubic foot will be $\frac{14.7 \times 4}{1} = 58.8$ pounds per square inch. The work done upon the air is 331.5744×14.7

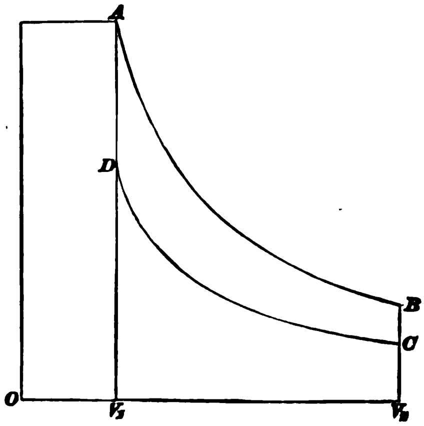


FIG. 233.

$\times 4 \times \log \frac{4}{1} = 11{,}738$ foot-pounds. The heat energy required to raise the temperature and pressure to the original temperature and pressure is $778 s_v W (T_1 - T_2)$. The weight of 1 cubic foot of air at the temperature T_2 of 520° and a pressure of 58.8 pounds per square inch is, by formula **61**, Art. **1057**, $W = \frac{58.8}{.37052 \times 520} = .3052$ pound, nearly. Hence, the heat energy required $= 778 \times .16847 \times .3052 (885 - 520) = 14{,}601$ foot-pounds. Hence, the work accomplished during the cycle $A B C D A = 19{,}963 - 11{,}738 = 8{,}225$ foot-pounds, while the heat energy expended was $19{,}963 + 14{,}601 = 34{,}564$ foot-pounds. Consequently, the efficiency $= \frac{8{,}225}{34{,}564} = 23.79\%$. Had the engine operated through a reversible cycle, the efficiency would have been $\frac{885 - 520}{885} = 41.24\%$.

Since a similar result may be obtained for any process which the student may apply the reasoning to, it follows that, under the theoretical conditions governing the reversible cycle process, the reversible cycle is the most efficient.

The foregoing description of the ideal heat engine, and conclusions derived from the consideration of it, comprise the most important laws and generalizations to be found in the science of thermodynamics. The student should study it with extreme care, and review it after finishing the subject of Steam and Steam Engines.

1188. NOTE.—The following application of the foregoing principles to the indicator diagram of a steam engine should not be read until the subject of Steam and Steam Engines has been studied.

In Fig. 234, *B C D E F G* represents a diagram taken from a perfect steam engine; i. e., an engine which admits steam at full boiler pressure, cuts off instantly, exhausts at the end of the stroke, the pressure falling instantly to that of the atmosphere, has no back pressure, exhaust closes instantly at the proper point, and which neither radiates nor absorbs heat from the cylinder walls. Since the isothermal of saturated steam is a straight line parallel to the atmospheric line $I\ E$, the clearance, or initial, volume $O\ V_1$ may be regarded as if filled with a mixture of steam and water having the absolute pressure $O\ A$, say 100 pounds per square inch, and the temperature 327.625° corresponding to that pressure. Now, let the piston move to the position $C\ V_2$, the volume increasing to $O\ V_2$, and all the water turning into steam. By addition of heat, the temperature (consequently, the pressure) may be kept constant, and $B\ C$ is the isothermal curve. At C, the supply of heat is stopped and the adiabatic expansion begins, continuing to the end of the stroke, or until the point D is reached.

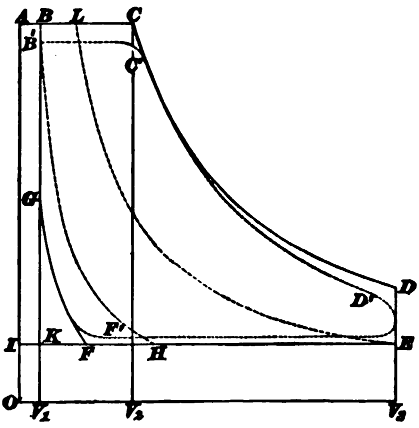


FIG. 234.

Here the exhaust-valve opens, and the greater part of the steam is allowed to escape into the atmosphere or into the condenser; suppose, for convenience, that it escapes into the atmosphere. The pressure immediately falls to $V_3\ E$. The engine now reverses its stroke and pushes the steam out of the cylinder at the constant pressure $V_3\ E$ until the point F is reached. Since the pressure is constant, the temperature is constant, and the line $E\ F$ corresponds to the isothermal compression line of Fig. 232. At F, the exhaust port closes, and the steam is compressed adiabatically during the remainder of the stroke $F\ K$, following the curve $F\ G$. Now, by adding heat, the pressure is raised to $V_1\ B$, and the above operations may be repeated.

It is evident that the cycle just described is not reversible, being open at both ends; but, nevertheless, it has a greater efficiency than could be obtained from an actual engine. The thermal efficiency of the process just described is easily found. The temperature corresponding to a pressure of 100 pounds per square inch is, from the steam table, 327.625°, and to 14.7 pounds per square inch, 212°; whence $T_1 = 787.625°$ and $T_2 = 672°$. Therefore, the efficiency $= \frac{787.625 - 672}{787.625} = 14.68\%$. Since this 14.68% represents the efficiency when the steam operates through a reversible cycle, it is evident that no non-condensing steam engine operating with a boiler pressure of 100 pounds, absolute, can attain an efficiency as high as 14.68%, for perfect conditions can never be obtained, there being no substance which is a perfect non-conductor of heat. The dotted outline $B' C' D' F' G$ shows a very good diagram supposed to be taken from an actual engine. Here the initial pressure is $V_1 B'$, 5 pounds less than the boiler pressure. The back pressure is a little over 2 pounds, say enough to make it 17 pounds, absolute. It will be noticed that all of the corners, except B', are rounded, and that the expansion line $C' D'$ falls below the theoretical expansion line $C D$. In consequence of this, the engine operates as though the boiler pressure were 95 pounds (corresponding to a temperature of 323.936°) and the back pressure 17 pounds (corresponding to a temperature of 219.452°). Hence, the theoretical thermal efficiency is $\frac{783.936 - 679.452}{783.936} = 13.33\%$.

To show what the conditions must be in order that the steam engine may operate through a reversible cycle, consider Fig. 234 again. It is absolutely necessary that the cycle be closed; hence, the steam must be cut off at some point L so chosen that, during the succeeding adiabatic expansion, the pressure will fall to $V_2 E$ at the end of the stroke; a point H must be chosen for the point of exhaust closure such that, at the end of the subsequent adiabatic compression, the pressure will be $V_1 B$. In other words, the diagram must be $B\ L\ E\ H\ B$. With these conditions fulfilled, and with a cylinder which is a perfect non-conductor of heat, the cycle would be reversible, provided there were no rounded corners.

PRINCIPLES
OF
REFRIGERATION.

FUNDAMENTAL PRINCIPLES OF REFRIGERATION.

THE MEANS OF PRODUCING REFRIGERATION.

1331. Refrigeration may be defined as the process of lowering the temperature of a body or of keeping the temperature below that of the atmosphere.

1332. Production of Cold.—Cold may be produced by one of the following processes:

1. A transfer of heat from a warmer to a colder body.

2. A chemical action, as exemplified by the so-called freezing mixtures. (See Instruction Paper, *Heat*, Art. **1142.**)

3. The adiabatic expansion of a gas. As explained in *Heat*, Art. **1164,** no heat is added to or abstracted from a gas during adiabatic expansion. If, therefore, the gas does work in pushing a piston, this work must be performed at the expense of the energy contained in the gas; the temperature of the gas will therefore fall, or, in other words, the gas will be cooled. It is to be carefully noted that if a gas does no work during adiabatic expansion, its temperature does *not* fall.

4. Evaporation of liquids having low boiling points. It was shown in *Heat*, Art. **1139,** that when a liquid is changed to a vapor, a certain quantity of heat, called the *latent heat of*

vaporization, must be added to the liquid to effect the change. When this process of vaporization or evaporation takes place in the presence of other bodies, the heat required for it is drawn from these bodies, and they are thereby cooled.

1333. The third method of producing cold mentioned in the preceding paragraph suggests a mechanical process of refrigeration. Referring to *Heat*, Art. **1178** and Fig. 232, let the volume of the gas in the cylinder be represented by the abscissa $O V_3$ and the pressure by the ordinate $V_3 C$. Let the gas be compressed adiabatically until it has the volume represented by $O V_2$ and the pressure represented by $V_2 B$. During this adiabatic compression no heat is added to or abstracted from the gas. A certain amount of work—represented by the area $B C V_3 V_2$—is done on the gas by the piston and is stored up in the gas, thus causing a rise of temperature. When the gas is in the state represented by the point B, conceive the head a to be replaced by the conducting head b, and let the cylinder be placed in contact with a body which we will call the *hot body*. As the gas is compressed, heat passes freely from it to the hot body, the gas will remain at constant temperature, and the compression will be isothermal. Suppose the state of the gas at the end of compression to be represented by the point A; then the work done by the piston is represented by the area $B A V_1 V_2$. Since the temperature of the gas does not change during this isothermal process, the gas neither receives nor loses energy, and the heat given up to the hot body must be the exact equivalent of the work done on the gas and represented by the area $B A V_1 V_2$. Let the cylinder be removed from the hot body, and let the gas expand adiabatically from the state A to the state represented by D. Since the gas receives no heat during this expansion, the work done by it upon the piston—represented by the area $A D V_4 V_1$—must be supplied by the gas itself; that is, the gas parts with enough of its energy to do this work, and as a result its temperature falls. We will assume that the expansion proceeds until the temperature of the gas is the same as at the initial state, represented by the point C. Suppose, now, that the cylinder is placed in contact with a second body, which we will designate the *cold body*, and that the head of the cylinder is again a perfect conductor. Let the gas now expand isothermally from the state D to the initial state C. During this expansion, the gas does the work represented by the area $D C V_3 V_4$ upon the piston; but since the temperature of the gas remains constant, the energy stored up in it remains constant also; therefore the body in contact with the cylinder must give up to the gas the heat equivalent of the work done on the piston.

1334. It is obvious that an engine operating through the cycle just described is a refrigerating-machine. It takes a certain quantity of heat Q_2 from the cold body and adds a certain greater quantity of heat Q_1 to the hot body. The difference $Q_1 - Q_2$ represents the heat equivalent of the net work done by the piston upon the gas, and is represented by the area $A B C D$.

The refrigerating-machine, it will be observed, is a heat engine reversed. The latter goes through the Carnot cycle (see Fig. 232) in the order $A B C D$; the former in the order $A D C B$. The heat engine takes a quantity of heat Q_1 from the hot body, gives up a smaller quantity Q_2 to the cold body, and changes the difference $Q_1 - Q_2$ into mechanical work; the refrigerating-machine, on the contrary, takes the heat Q_2 from the cold body, changes the mechanical work into the heat $Q_1 - Q_2$, and delivers the heat Q_1 to the hot body.

1335. The Carnot cycle of a refrigerating-machine using a saturated vapor as a working fluid is shown in Fig. 317. Suppose the vapor to be in the state represented by the point A; that is, the volume is represented by $O V_1$ and the pressure by $V_1 A$. The vapor is compressed adiabatically from the state A to the state B, and then isothermally from B to C. The pressure of any saturated vapor depends only upon the temperature; therefore, if the temperature remains constant, as in isothermal expansion, the pressure also remains constant, and the isothermal line BC is a straight line parallel to the axis OX. (See *Steam and Steam Engines*, Art. **1210.**) During the compression from B to C, some or all of the vapor is condensed. The

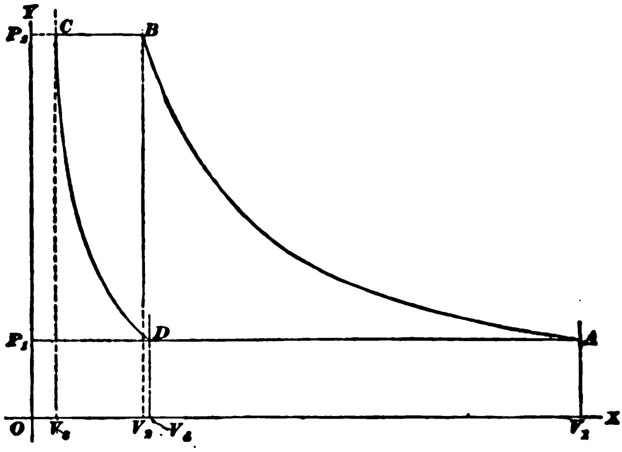


FIG. 317.

latent heat liberated by this change of state is absorbed by the hot body. The next stage of the process is adiabatic expansion from the state C to the state D, and this is followed by isothermal expansion from the state D to the original state A. During this isothermal expansion, the liquid changes back to a vapor, and the heat required to effect this vaporization is taken from the cold body with which the cylinder is in contact.

As in the previous case, a quantity of heat Q_1 is given up to the hot body, a smaller quantity Q_2 is abstracted from the cold body, and mechanical work equivalent to the difference $Q_1 - Q_2$ is done by the machine upon the working fluid. It is to be noted that in the present instance the refrigeration is produced by the vaporization of a volatile liquid.

[illegible] In actual operation, the cycle of the refrigerating-machine differs materially from that of the ideal Carnot cycle. In the ideal cases just described, the working fluid is assumed to remain in the cylinder. In practice this is not the case. The steam engine receives steam from the boiler, the source of heat, and rejects the exhaust steam into the condenser after extracting a certain portion of it in the form of work. Likewise the refrigerating-machine takes a supply of the working fluid from the cold body, does a certain amount of work upon it, rejects it at a higher temperature to the condenser, which we have heretofore called the hot body, and takes a new supply from the cold body.

In practice the cold body is either a room with non-conducting walls, in which are placed the articles to be chilled, or a non-freezing liquid called a **brine,** which after being cooled may be pumped through coils of pipes in the rooms to be cooled. The hot body is a condenser, through the coils of which the working fluid is forced by the compressor piston. Water is kept continually flowing over the coils of the condenser and carries away the heat generated by the isothermal compression BA, Fig. 232, or BC, Fig. 317.

1337. It is instructive to note the relation of the refrigeration process to the second law of thermodynamics. (See Art. **1177,** *Heat.*) This law asserts that *heat can not pass from a cold to a hot body by a self-acting process unaided by external agency.* In other words, whenever heat is transferred from a body to a warmer body, there must be some work expended to effect the transfer. In the process described above, the refrigerating-machine is the agent used in transferring the heat from the refrigerating room to the condensing water, and the work expended by the machine on the fluid is that required in the transfer. The transfer of heat may be expressed algebraically as follows:

Let Q_1 = heat delivered to condenser in B. T. U.;
Q_2 = heat taken from body cooled in B. T. U.;
W = work in foot-pounds done by engine on working fluid;
J = 778 = mechanical equivalent of heat.

Then, $$Q_1 = Q_2 + \frac{W}{J}. \qquad \textbf{(108.)}$$

CAPACITY AND EFFICIENCY OF REFRIGERATING-MACHINES.

1338. Refrigerating Capacity.—The **capacity** of a refrigerating-machine is a measure of its ability to abstract heat.

The unit of *refrigerating*, or *ice-melting*, *capacity* is the quantity of heat required to melt one ton (2,000 pounds) of ice at 32° F. to water at 32° F. Since 142.65 B. T. U. are required to melt one pound of ice at 32° F. (see Table 22), the unit of refrigerating capacity is equivalent to 142.65 B. T. U. × 2,000 = 285,300 B. T. U. The refrigerating capacity of a machine expressed in tons is given by the following formula:

Let H = B. T. U. abstracted from cold body in 24 hours;
h = B. T. U. abstracted from cold body in 1 hour;
F = refrigerating capacity expressed in tons.

Then,
$$F = \frac{H}{285,300}, \qquad \textbf{(109.)}$$
or
$$F = \frac{h}{11,887.5}. \qquad \textbf{(110.)}$$

ILLUSTRATION.—A refrigerating-machine abstracts 2,375,241 B.T.U. in 24 hours. The ice-melting capacity is

$$\frac{2,375,241}{285,300} = 8.33 \text{ tons.}$$

1339. Ice-Making Capacity.—The ice-making capacity is the number of tons of ice that a machine is capable of freezing in twenty-four hours. As the temperature of the water from which the machine freezes the ice varies from 50° to 95°, and as it is necessary to cool this water to 32° before any ice can be made, it will be seen that the ice-making capacity is variable and is largely affected by the conditions under which the machine operates. Owing to the necessity of cooling the water from which the ice is made from its initial temperature to a temperature below the freezing point, and owing to other losses, such as radiation, etc., the *ice-making* is only about 50 or 60 per cent. of the *ice-melting* capacity.

1340. Efficiency. — The theoretical maximum efficiency of a heat engine is given by the expression $\frac{T_1 - T_2}{T_1}$. (See *Heat*, Art. **1182.**) If we denote by Q_1 and Q_2, respectively, the heat given up by the hot body and the heat rejected to the cold body, it can readily be shown that this efficiency is also given by the ratio $\frac{Q_1 - Q_2}{Q_1}$. Referring to *Heat*, Art. **1182,** the heat equivalent of the work performed during isothermal expansion is $\left(\frac{2.3026\, c \log r_1}{778}\right) T_1$, and the heat equivalent of the work of isothermal compression is $\left(\frac{2.3026\, c \log r_2}{778}\right) T_2$. It is shown in Art. **1181** that $\frac{O\,V_2}{O\,V_1} = \frac{O\,V_3}{O\,V_4}$, or $r_1 = r_2$; hence, denoting the constant $\frac{2.3026\, c \log r_1}{778}$ by k, we have $Q_1 = k\,T_1$, $Q_2 = k\,T_2$, and $\frac{T_1 - T_2}{T_1} = \frac{k\,Q_1 - k\,Q_2}{k\,Q_1} = \frac{Q_1 - Q_2}{Q_1}$. This expression might have been written at once, for the efficiency of a heat engine is clearly the ratio of the heat transformed into mechanical work to the whole quantity of heat supplied.

In the case of the refrigerating-machine, the useful work is measured by the quantity of heat Q_2 removed from the cold body, and the work expended, that is, the work done by the machine upon the working fluid, expressed in heat-units, is $Q_1 - Q_2$ B. T. U. The ratio $\frac{Q_2}{Q_1 - Q_2}$ of the work obtained to the work expended is called the *efficiency* of the refrigerating-machine. If T_2 and T_1, respectively, denote the absolute temperatures at which the heat is abstracted from the cold body and delivered to the condenser, then, as in the case of the heat engine, $Q_2 = k\,T_2$ and $Q_1 = k\,T_1$; hence $\frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$. Denoting the theoretical maximum efficiency by E, we have the following formulas:

$$E = \frac{Q_2}{Q_1 - Q_2}. \qquad \textbf{(111.)}$$

$$E = \frac{T_2}{T_1 - T_2}. \qquad \textbf{(112.)}$$

$$E = \frac{J Q_2}{W}. \qquad \textbf{(113.)}$$

In the last formula, J and W have the same significance as in Art. **1337.**

In the case of the steam engine, the efficiency increases with the range of temperature, and the effort of the engineer is to make the difference $T_1 - T_2$ as large as possible by increasing boiler pressure and lowering the temperature of the exhaust steam. With the refrigerating-machine, on the other hand, the expression $E = \frac{T_2}{T_1 - T_2}$ shows that the efficiency is increased by making the difference $T_1 - T_2$ smaller; that is, *the efficiency increases as the temperature of the cold body increases and the temperature of the condenser (hot body) decreases.* It is of course impossible to attain the maximum theoretical efficiency in practice on account of the losses of various kinds in the process, such as the friction of the machine, heat losses by radiation, etc. The work actually delivered to the compressor is always greater than the mechanical equivalent of the heat $Q_1 - Q_2$.

EXAMPLE.—If, per cubic foot of ammonia, 64.5 B. T. U. are carried from the cold room and 73.6 B. T. U. are delivered to the condenser, what is the theoretical maximum efficiency?

SOLUTION.— $E = \frac{Q_2}{Q_1 - Q_2} = \frac{64.5}{73.6 - 64.5} = \frac{64.5}{9.1} = 7.09.$ Ans.

1341. In general it is found that the efficiency as defined above is greater than 1, which is apparently a contradiction of the fundamental law that the efficiency of a machine is always less than 1. (See *Elementary Mechanics*, Arts. **949** and **950.**) This is due to the fact that in ordinary machines a certain amount of energy is delivered to the machine and a certain percentage of that energy appears as useful work, the remainder being expended in overcoming losses due to friction; thus the energy obtained is a part of that expended. In the refrigerating-machine, however, the energy obtained, that is, the heat abstracted from the cold body, is not a part of the energy supplied to the machine, and, in fact, has no direct connection with it. The work to be done by the compressor is *not* the mechanical equivalent of the heat abstracted, but represents only the difference between that heat and the heat delivered to the condenser. If, under exceptional circumstances, the cooling water used with the condenser should have a temperature lower than the temperature of the cold body, heat would of itself flow from the cold body to the condenser, and no compressor or working fluid would be required. This consideration shows that there is no necessary connection between the work of the compressor and the heat abstracted from the cold body, and as a consequence the ratio of one to the other may be either greater or less than 1.

1342. A refrigerating-machine is generally driven by a steam engine; therefore the energy delivered to the machine is contained primarily in the fuel fed to the furnace, usually coal. For this reason, it is customary in commercial work to measure the commercial efficiency or the *economy* of a refrigerating-machine by the pounds of ice-melting effect per pound of coal used. For every pound of coal consumed in the boiler to produce steam to operate the refrigerating-machine, a quantity of heat is abstracted from the cold body sufficient to melt a definite number of pounds of ice at 32° F. into water at 32° F. (See Art. **1338.**) This quantity of ice is a measure of the commercial efficiency of the machine.

EXAMPLE.—A refrigerating-machine having an actual capacity of 23.5 tons requires 4,350 pounds of coal per 24 hours to operate it. Required, the efficiency expressed in ice per pound of coal.

SOLUTION.— 23.5 tons = 47,000 lb.; 47,000 ÷ 4,350 = 10.8; hence, 10.8 pounds of ice are melted per pound of coal burned. Ans.

THE AIR REFRIGERATING-MACHINE.

1343. The air machine utilizes the fall of temperature that occurs when compressed air expands adiabatically and performs work. The principles underlying the operation of this machine are stated in Art. **1333.**

The general arrangement of an air refrigerating-machine is shown in Fig. 318. The machine consists essentially of a

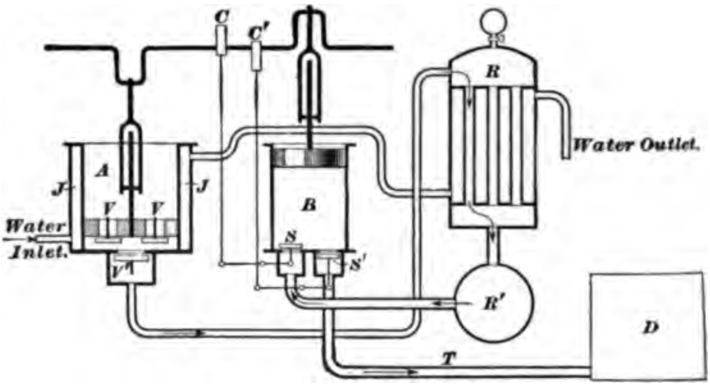


FIG. 318.

single-acting compression cylinder *A*, expansion cylinder *B*, also single-acting, a condenser *R*, and a cooler or refrigerator box *D*. The piston of the cylinder *A* is provided with suction valves *V*, *V* opening inwards, a discharge-valve *V'*, and also with a water-jacket *J*. The diameter of the cylinder *B* is slightly less than that of *A*. The piston is solid, but the cylinder-head is provided with two valves, an inlet valve *S* and an outlet valve *S'*, which are operated by the eccentrics *C* and *C'*. The pistons are driven by cranks set at 180°. The condenser *R* is a surface condenser and receives a current of cold water from the water-jacket *J* of the compression cylinder *A*. A receiver *R'* is connected with the condenser and also communicates with the inlet valve *S* of the expansion cylinder *B*.

The air at ordinary pressure is taken into the cylinder *A* through the valves *V*, *V*, and is compressed adiabatically until the pressure becomes sufficient to open the valve *V'*. The air then passes into the condenser *R*, where it comes in contact with the cold surfaces of that vessel. The adiabatic compression has raised the temperature of the air, but in passing through the condenser, sòme of the heat contained in the air is given up to the cold water circulating through the condenser, and the temperature is lowered nearly to that of the surrounding air. During this time the valve *S* of the expansion cylinder *B* opens and permits an amount of air equal in weight to that expelled from *A* to pass from the receiver *R'* into the cylinder. The valve *S* closes and the air in the cylinder *B* expands, forcing the piston forwards and doing a certain amount of work, which may be deducted from the work of the compression. This expansion of the air in the cylinder *B* and the performance of work in forcing the piston forwards is at the expense of the energy stored in the air. The air therefore gives up sufficient heat to do the mechanical work, and as a result its temperature falls. As the air on entering *B* was at a normal temperature, the expansion brings the temperature below that of the surrounding objects. In other words, the air is cooled.

When the piston in *B* reaches the upper limit of its stroke, the valve *S'* opens, and as the piston descends, the cooled air escapes by means of the pipe *T* into the refrigerator box *D*.

The difference between the work done on the air in the compression cylinder and that done by the air in the expansion cylinder, and, in addition, the work required to overcome the friction of the entire machine, must be supplied by a steam engine or other motor.

1344. The operation of the actual air machine as just outlined differs in some degree from the action of the ideal machine as described in Art. **1333**, and the cycle of the actual machine differs from the Carnot cycle. The cycle has, as usual, four operations; they are: compression, cooling, expansion, and refrigeration. Let $O\ V_3$, Fig. 319, represent the volume of air taken into the compression cylinder per stroke. During compression the volume decreases and the pressure rises, as indicated by the curve *A B*. The water circulating through the jacket carries away some of the heat developed, so that the compression is not strictly adiabatic, and the temperature at the end of compression is less than it would be if the curve *A B* were an adiabatic.

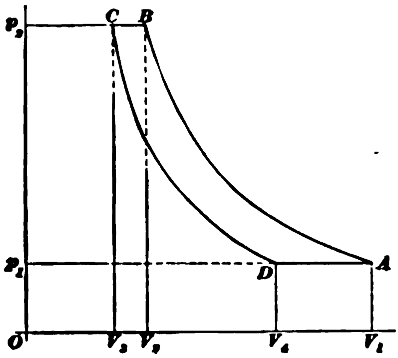


Fig. 319.

As the air from *A*, Fig. 318, is pushed into the condenser *R*, an equal weight is delivered to the cylinder *B*; consequently the weight of air in *R*, *R'* and the conducting pipes is practically constant, and the pressure remains practically constant also. During the second operation, therefore, the air is cooled in the condenser at constant pressure. According to Gay-Lussac's law (*Pneumatics*, Art. **1054**), the cooling is accompanied by a decrease in volume. If $O V_2$, Fig. 319, represents the volume before cooling, $O V_3$ will represent the volume after cooling; and the operation will be represented by the constant-pressure line *B C*. The temperature of the air in the state represented by *C* is not much above that of the surrounding atmosphere.

The air now enters the cylinder *B* and expands adiabatically; the temperature, which was normal at the beginning of the expansion, must fall much below the temperature of surrounding objects; the volume increases from $O V_3$ to $O V_4$. The cylinder *B* is made of such diameter that when the piston reaches the end of its stroke, the pressure of the air will be just that of the atmosphere. This third operation is represented by the curve *C D*.

The air is now pushed into the cooler *D* by the return stroke of the piston. Being colder than the surrounding objects, it absorbs heat from them and expands at the constant pressure $V_4 D$. If the supply to the compressor had been of the same temperature as the cooler; for example, if the compressor had drawn its supply from the cooler, the volume in the cooler would increase from $O V_4$ to the original volume $O V_1$, as indicated by the line $D A$, and the cycle would be closed. Usually, however, it is more convenient to reject the air into the cooler D and draw the fresh supply from the atmosphere, which has a much higher temperature. In this case, it is evident that the air does not return to its original state in the cooler, and the cycle is not closed.

1345. General Theory.—In the following discussion, it will be assumed, for the sake of simplicity, that compression and expansion are adiabatic and that the air is drawn into the compressor from the cooling chamber, so that the machine works through a closed cycle. The clearance of the two cylinders will be neglected. Referring to the diagram, Fig. 319, let V_a, V_b, V_c, V_d denote the volumes, T_a, T_b, T_c, T_d the absolute temperatures, and t_a, t_b, t_c, t_d the ordinary temperatures of the air when in the states represented by the points A, B, C, and D, respectively; and let p_1 and p_2 denote the pressures of the air during the operations $D A$ and $B C$, respectively; also let s_p and s_v denote, respectively, the specific heat of air at constant pressure and constant volume, and let M denote the weight of air used per stroke of the compressor.

The temperatures t_a of the cooling chamber and t_c of the air as it leaves the condenser are known or assumed, and the temperatures t_b and t_d can be obtained by formula **86**, *Heat*. Thus,

$$\frac{T_b}{T_a} = \left(\frac{p_2}{p_1}\right)^{.29078} \text{ and } \frac{T_d}{T_c} = \left(\frac{p_1}{p_2}\right)^{.29078} = \frac{T_a}{T_b}. \quad (a)$$

To find the volume of the air at the end of compression, we have, from formula **81**,

$$p_1 V_a^{1.41} = p_2 V_b^{1.41},$$

or

$$V_b = V_a \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.41}} \quad (b)$$

The volume of the air during the cooling in the condenser decreases from V_b to V_c at constant pressure. According to formula **71**,

$$p_2 V_b = RMT_b,$$

and $$p_2 V_c = RMT_c.$$

Dividing, $$\frac{V_b}{V_c} = \frac{T_b}{T_c},$$

or $$V_c = V_b \frac{T_c}{T_b}. \quad (c)$$

Finally, $$p_2 V_c^{1.41} = p_1 V_d^{1.41}.$$

or $$V_d = V_c \left(\frac{p_2}{p_1}\right)^{\frac{1}{1.41}}.$$

Since from (b) $$\left(\frac{p_1}{p_2}\right)^{\frac{1}{1.41}} = \frac{V_b}{V_a},$$

we have $$V_d = V_c \frac{V_a}{V_b}, \quad \text{or} \quad \frac{V_d}{V_c} = \frac{V_a}{V_b}. \quad (d)$$

The preceding formulas enable us from the assumed data to calculate the volume and temperature of the gas at each of the four points A, B, C, and D.

The heat given up by the air to the condenser is

$$Q_1 = s_p M (t_b - t_c). \quad (e) \quad \text{(See Art. } \mathbf{1135}, \textit{ Heat.})$$

The heat absorbed by the air from the cooler as it expands from D to A is

$$Q_2 = s_p M (t_a - t_d). \quad (f)$$

The specific heat s_p is used for the reason that the air passes through the condenser and through the cooler at constant pressure, as shown by the lines BC and DA.

The heat equivalent of the work done on the air—represented by the area $ABCD$—must be precisely the difference between the heat delivered to the condenser and that abstracted from the cooler. Hence, denoting the net work by W, we have

$$\frac{W}{J} = Q_1 - Q_2 = s_p M[(t_b - t_c) - (t_a - t_d)],$$

or

$$W = J s_p M[(t_b - t_c) - (t_a - t_d)]$$
$$= J s_p M[(T_b - T_c) - (T_a - T_d)]. \quad (g)$$

The theoretical efficiency is therefore

$$E = \frac{J Q_2}{W} = \frac{J s_p M(T_a - T_d)}{J s_p M[(T_b - T_c) - (T_a - T_d)]}$$
$$= \frac{T_a - T_d}{(T_b - T_c) - (T_a - T_d)}. \quad (h)$$

Since $\frac{T_d}{T_c} = \frac{T_a}{T_b}$, it can readily be shown that the expression reduces to

$$E = \frac{T_a}{T_b - T_a} = \frac{T_d}{T_c - T_d}. \quad (i)$$

Since the net work per stroke is W, if we denote the number of strokes per minute by n, the horsepower required to drive the machine is

$$H = \frac{n W}{33{,}000}. \quad (j)$$

The gross horsepower required will of course be much greater, on account of the friction of the two pistons and of the other parts of the machine.

If desired, the work W may be expressed in terms of the pressures and volumes instead of temperatures. Thus, using formula **88**, *Heat*,

$$W_{ab} \text{ (area } A\,B\,V_b\,V_a) = \frac{144\,(p_2 V_b - p_1 V_a)}{.41}.$$

$$W_{bc} \text{ (area } B\,C\,V_c\,V_b) = 144\,p_2\,(V_b - V_c).$$

$$W_{cd} \text{ (area } C\,D\,V_d\,V_c) = \frac{144\,(p_2 V_c - p_1 V_d)}{.41}.$$

$$W_{da} \text{ (area } D\,A\,V_a\,V_d) = 144\,p_1\,(V_a - V_d).$$

$$W = W_{ab} + W_{bc} - W_{cd} - W_{da} =$$
$$144 \times \frac{1.41}{.41}\,[p_2(V_b - V_c) - p_1(V_a - V_d)]. \quad (k)$$

The factor 144 is used to reduce the pressures from pounds per square inch to pounds per square foot.

The heat Q_1 is given up by the air in the condenser. If G denote the weight of cooling water used per stroke, and t_e and t_f the temperature of the water on entering and leaving the condenser, the heat absorbed by the water per stroke is $G\ (t_f - t_e)$ B. T. U.

Hence, $$G\,(t_f - t_e) = Q_1 = s_p M\,(t_b - t_c),$$

or $$G = \frac{s_p M\,(t_b - t_c)}{t_f - t_e}. \quad (l)$$

The capacity of the compression cylinder (neglecting clearance) is V_a and that of the expansion cylinder is V_d. To find the ratio of these volumes, we have

$$p_1 V_d = R M T_d,$$

and $$p_1 V_a = R M T_a;$$

whence, dividing one equation by the other,

$$\frac{V_d}{V_a} = \frac{T_d}{T_a}. \quad (m)$$

1346. To illustrate the application of the equations developed in the preceding article, the horsepower and approximate cylinder dimensions for an air refrigerating-machine will be calculated from the following data: The machine is required to have an ice-melting capacity of 300 pounds of ice per hour. The pressure (absolute) in the cold chamber is 14.7 pounds per square inch, and the temperature is 40° F. The pressure in the expansion cylinder at cut-off is 50 pounds gauge, or 64.7 pounds absolute. The initial and final temperatures of the cooling water are 65° and 85°, and the temperature of the air as it leaves the cooler is 95° F. The machine is single-acting and makes 75 strokes per minute.

The heat Q_2 abstracted per hour is

$$142.65 \text{ B. T. U.} \times 300 = 42,795 \text{ B. T. U.}$$

The heat abstracted per stroke is $\dfrac{42,795}{75 \times 60} = 9.51$ B. T. U.

The absolute temperature at the end of compression is

$$T_b = T_a\left(\frac{p_2}{p_1}\right)^{.29078} = (40° + 460°) \times \left(\frac{64.7}{14.7}\right)^{.29078} = 769.33°;$$

hence, $t_b = 769.33° - 460° = 309.33°$ F.

The temperature $T_c = 95° + 466° = 555°$; according to formula (a),

$$T_d = T_c \frac{T_a}{T_b} = 555 \times \frac{500}{769.33} = 360.703°;$$

therefore, $t_d = 360.703° - 460° = -99.297°$.

Substituting known values in formula (f),

$$Q_2 = s_p M (t_a - t_d),$$

$$9.51 = .23751 \times M \times [40 - (-99.297)],$$

or $$M = \frac{9.51}{.23751 \times (40 + 99.297)} = .2874 \text{ pound.}$$

The volume V_a, which is the capacity of the compressor cylinder, can now be found from formula **71**, *Heat*. The weight of air per stroke is .2874 pound, and the absolute temperature T_a at which the air is admitted to the compressor is $40° + 460° = 500°$; hence the volume is

$$V_a = \frac{R M T_a}{p_1} = \frac{.37052 \times .2874 \times 500}{14.7} = 3.622 \text{ cu. ft.}$$

For the volume V_d of the expansion cylinder, we have, from formula (m),

$$V_d = V_a \frac{T_d}{T_a} = 3.622 \times \frac{360.703}{500} = 2.613 \text{ cu. ft.}$$

These volumes should be increased about 20 per cent. to allow for the loss due to clearance and to fall of pressure caused by the resistance of valves and passages, and for various imperfections in the operation of the machine. Making this allowance, the cylinders may have the following dimensions:

Diameter of compression cylinder, 22 inches ;
Diameter of expansion cylinder, 18½ inches ;
Length of stroke of both cylinders, 20 inches.

The work per stroke is

$$W = J\, s_p\, M\,[(T_b - T_c) - (T_a - T_d)] =$$

$$778 \times .23751 \times .2874 \times [(769.33 - 555) - (500 - 360.703)] =$$

$$3{,}984.7 \text{ foot-pounds.}$$

The net horsepower required is

$$H = \frac{n\,W}{33{,}000} = \frac{75 \times 3{,}984.7}{33{,}000} = 9.06 \text{ H. P.}$$

The gross horsepower will be about $\frac{4}{3}$ of the net horsepower, or 12.08 H. P.

The cooling water required per minute is

$$75\,G = 75\,\frac{s_p\,M\,(t_b - t_c)}{t_f - t_e} =$$

$$\frac{75 \times .23751 \times .2874 \times (309.33 - 95)}{85 - 65} = 54.86 \text{ pounds.}$$

The theoretical efficiency of the machine is

$$E = \frac{T_a}{T_b - T_a} = \frac{500}{769.33 - 500} = 1.856.$$

Since the machine transfers heat from the cold room, where the temperature is 40°, or 500° absolute, to the condensing water, the final temperature of which is $T_f = 85° + 460° = 545°$, the maximum possible efficiency of a perfect engine working between these temperatures through a Carnot cycle is

$$E_m = \frac{T_a}{T_f - T_a} = \frac{500}{545 - 500} = 11.11. \quad \text{(See Art. } \mathbf{1340.}\text{)}$$

The theoretical efficiency is therefore only about $\frac{1}{6}$ of the maximum theoretical efficiency.

1347. Capacity of Air Machine.—A formula for the theoretical ice-melting capacity of an air refrigerating-machine may be derived as follows: Let T_1 denote the temperature of the air entering the compressor; if the cycle is closed, T_1 is the same as T_a, Art. **1345**, but in general this is not the case. As in the previous discussion, V_a and V_d are the volumes of the compression and expansion cylinders,

T_d the temperature of the air at the end of expansion, and T_a the temperature of the cooling chamber. Let p_1 denote the pressure at the end of expansion; this pressure should be that of the atmosphere, 14.7 pounds, if the expansion cylinder is properly proportioned and the valves are set correctly.

The weight of air M entering the compressor per stroke is given by the formula

$$p_1 V_a = RMT_1,$$

whence $$M = \frac{p_1 V_a}{RT_1}.$$

Provided there is no loss of air in passing through the machine, the same weight of air must be delivered by the expansion cylinder; that is,

$$M = \frac{p_1 V_d}{RT_d}.$$

The heat abstracted per stroke is

$$Q_1 = s_p M (T_a - T_d) = \frac{s_p p_1 V_a}{RT_1}(T_a - T_d).$$

Using the second value of M,

$$Q_1 = \frac{s_p p_1 V_d}{RT_d}(T_a - T_d) = \frac{s_p p_1 V_d}{R}\left(\frac{T_a}{T_d} - 1\right).$$

If n denotes the number of strokes per minute, the heat H extracted in 24 hours is given by the expression

$$H = 24 \times 60 \times \frac{n s_p p_1 V_a}{RT_1}(T_a - T_d) =$$
$$24 \times 60 \times \frac{n s_p p_1 V_d}{R}\left(\frac{T_a}{T_d} - 1\right).$$

Now, making use of formula **109**, the ice-melting capacity is

$$F = \frac{H}{285,300} = \frac{24 \times 60}{285,300} \times \frac{n s_p p_1 V_a}{RT_1}(T_a - T_d) =$$
$$\frac{24 \times 60}{285,300} \times \frac{n s_p p_1 V_d}{R}\left(\frac{T_a}{T_d} - 1\right).$$

Substituting the numerical values .23751 and .37052 for s_p and R, we obtain finally

$$\left.\begin{aligned} F &= .003235\, n p_1 \frac{V_a}{T_1}(T_a - T_d), \\ \text{or} \quad F &= .003235\, n p_1 V_d \left(\frac{T_a}{T_d} - 1\right). \end{aligned}\right\} \qquad \textbf{(114.)}$$

If the weight of air is found from the delivery of the expansion cylinder, the pressure p_1 may be found by means of an indicator. If an indicator is not available, the pressure may be taken as 14.7 lb. per square inch. The temperature T_d and T_a must also be determined.

EXAMPLE.—The diameter of the expansion cylinder is 21 inches and the stroke is 24 inches. The temperature of the air after expansion is − 52°, the temperature of the refrigerating chamber is 33°, and the pressure in the chamber is 14.7 lb. per sq. in. The machine makes 70 strokes per minute. Required, the theoretical ice-melting capacity.

SOLUTION.—The volume V_d of the cylinder is $\frac{.7854 \times 21^2 \times 24}{1{,}728} =$ 4.81 cu. ft.

$$T_a = 460° + 33° = 493°, \text{ and } T_d = 460° - 52° = 408°.$$

Substituting these values in formula **114,**

$$F = .003235 \times 70 \times 14.7 \times 4.81 \times \left(\frac{493}{408} - 1\right) = 3.335,$$

the ice-melting effect in tons per 24 hours. Ans.

1348. Economy of the Air Machine.—The ideal air refrigerating-machine, like the ideal hot-air engine, has theoretically a high efficiency. In practice, however, the air machine has proved to be a wasteful and uneconomical machine, and it is at present rarely used, save under exceptional circumstances. The principal reasons for the inefficiency of the actual air machine are as follows:

1. *Friction.*—As the cooling effect is accomplished by means of the increase of the sensible heat in the air and not by the latent heat of evaporation, as in the case of the ammonia machine, it follows that the compression cylinder

must be large. In practice, the cylinder of the air machine has about twenty times the capacity of that of an ammonia machine of the same tonnage; the usual quantity of air delivered per minute is about *one hundred cubic feet per ton of ice-melting capacity.* This increase in the size of the cylinders as compared with those of other classes of machines means a much greater expenditure of work in friction. In actual practice, this friction loss amounts to about 25 per cent. of the total work.

2. *Clearance.*—In the case of the air machine there are two cylinders, a compression and an expansion cylinder, instead of the single compression cylinder of the ammonia machine. There is, therefore, at least double the clearance space. The air machines are usually built with a short stroke, which also increases the clearance per cubic foot of air pumped.

Clearance spaces greatly decrease the efficiency of any machine that has to compress a gas, for if the piston does not come up close to the head at the end of its stroke, it can not expel all the gas or air, and the portion that is left expands again in the cylinder and virtually cuts down the capacity of the machine by that amount. It has been, therefore, the study of all builders of machines for compressing gases to do away with clearance spaces as much as possible.

3. *Cylinder Superheating.*—When air is compressed, it becomes heated and naturally heats the walls of the cylinder. The cylinder being hot, the air drawn in during the next stroke also becomes heated. This heating reduces the density of the air, which affects both capacity and economy. In order to overcome this difficulty, the cylinder was jacketed, but this only partly remedied the defect, owing to the slow transmission of heat through the walls of the cylinder. The Bell-Coleman Co. then added to their machine water injection during compression. The injection did away largely with the cylinder superheating, but augmented another defect, namely:

4. *Moisture.*—Air at any ordinary temperature can hold a certain amount of water vapor in suspension. The limit, or point of saturation, that is, the point at which the air can hold no more water vapor, is called the **dew point.** When this point is reached, the excess of moisture above that which the air is able to hold is precipitated in the form of dew. The weight of moisture contained in a given volume of air at the dew point is not the same for all temperatures; in fact, air will hold in suspension four times the weight of moisture at 72° that it will at 32°. Assume that the air on entering the expansion cylinder is at a temperature of 72° and is saturated with moisture. As the temperature falls during expansion, the water is gradually precipitated out and condenses on the walls of the cylinder. The water cools as the expansion goes on until it reaches 32°, and then it freezes. The condensation, cooling, and freezing of the water take a great deal away from the useful effect of the machine. Besides, the snow, which is the result of freezing the moisture, often gives trouble by clogging the valves.

The Haslam Foundry and Engineering Co., Derby, England, make what is known as a "dry-air" system. They

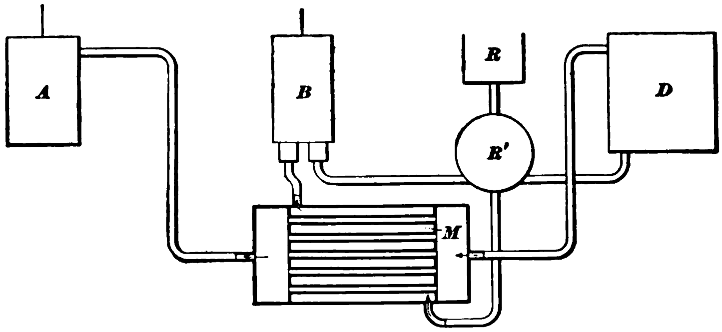


FIG. 320.

place a *drier* in the suction-pipe from the condenser to the expansion cylinder. The compression cylinder *A*, Fig. 320, takes the cold air from the refrigerator box *D*. On its way to *A*, this cold air passes through the pipes of the drier *M*. The cold strikes through these pipes and cools the air surrounding the pipes on its way from the receiver *R'* to the expansion cylinder *B*. The air gives up a large percentage of its moisture in the drier, and the frosting in the cylinder *B* is much diminished.

LATENT-HEAT REFRIGERATING-MACHINES.

FLUIDS USED AS REFRIGERATING AGENTS.

1349. The air refrigerating-machine produces its refrigerating effect by means of the fall of temperature incident to adiabatic expansion. In all other refrigerating-machines, the abstraction of heat is brought about by the vaporization of some liquid having a low boiling point. Such machines may be classed as **latent-heat** refrigerating-machines.

1350. Theoretically, any volatile liquid may be used as a working fluid in a latent-heat machine; there are, however, various considerations of a practical nature that govern the choice of the liquid. The chief requisites of the fluid used are: (1) It should vaporize at a low temperature when at ordinary atmospheric pressure. (2) It should have a high latent heat.

The fluids that have been used in compression machines are ether, sulphur dioxide, anhydrous ammonia, and Pictet's fluid.

1351. The following table gives the boiling points and latent heats of various substances at atmospheric pressure, 14.7 pounds per square inch.

TABLE 23.

Substance.	Temperature of Boiling Point.	Latent Heat, B. T. U.	Specific Heat of Liquid.
Nitric Acid	248° F.	....	....
Saturated Brine	226° F.	....	....
Water	212° F.	966	1.0000
Alcohol	173° F.	....	....
Chloroform	140° F.	....	....
Ether, Sulphurous	95° F.	170	.5299
Ether, Methyl	– 10° F.	....	....
Sulphur Dioxide	14° F.	168.7	.4100
Anhydrous Ammonia	– 28.5° F.	573	1.0058
Carbon Dioxide	– 140° F.	141	.9550

Under a pressure of 342 pounds per square inch, carbon dioxide boils at a temperature of 5°. Its latent heat under the same conditions is 121.5.

ETHER.

1352. Early in the history of ice-making and refrigerating machines, ether was almost universally used as the working fluid. This was due to its high condensing temperature and consequent low condensing pressure. This low condensing pressure made it possible to use compression pumps of ordinary construction, very much after the style and pattern of air-pumps. However, the disadvantages in the use of ether were found to be very great; the first cost of ether is considerable; but the greatest objection to it is its inflammability and its liability to explode when mixed with air.

SULPHUR DIOXIDE.

1353. The objections to ether led to further investigation. Sulphur dioxide was found to be more efficient than ether, for, though it required a higher condensing pressure, it did not require to be evaporated under a vacuum. Consequently the compression pumps were made somewhat smaller for a given capacity, but were built stronger and more attention was given to the elimination of clearance spaces. The temperatures produced with sulphur dioxide, though lower than that obtained with ether, were not sufficiently low.

Table 24 gives the leading properties of sulphur dioxide.

TABLE 24.

PROPERTIES OF SATURATED SULPHUR DIOXIDE.

Temperature of Ebullition in Deg. F.	Absolute Pressure in Lb. per Sq. In.	Total Heat Reckoned from 32° Fahr.	Heat of Liquid Reckoned from 32° Fahr.	Latent Heat of Vaporization.	Density of Vapor or Weight of 1 Cubic Ft.
Deg. F.	Lb.	B. T. U.	B. T. U.	B. T. U.	Lb.
−40	3.16	155.22	−17.76	172.98	.048
−31	4.23	156.39	−16.55	172.94	.062
−22	5.56	157.55	−15.05	172.60	.079
−13	7.23	158.69	−13.26	171.95	.099
− 4	9.27	159.82	−11.18	171.00	.124
5	11.76	160.93	− 8.82	169.75	.154
14	14.75	162.02	− 6.17	168.19	.190
23	18.31	163.10	− 3.23	166.33	.232
32	22.53	164.16	0.00	164.16	.282
41	27.48	165.21	3.52	161.69	.341
50	33.26	166.24	7.32	158.92	.410
59	39.93	167.25	11.41	155.84	.491
68	47.62	168.25	15.79	152.46	.584
77	56.39	169.23	20.45	148.78	.692
86	66 37	170.20	25.41	144.79	.819
95	77.64	171.15	30.65	140.50	.965
104	90.32	172.08	36.18	135.90	1.131

PICTET FLUID.

1354. It was found by Prof. Pictet, a Swiss physicist, that a mixture of 97% of sulphur dioxide and 3% carbon dioxide, commonly known as carbonic acid gas, gives a boiling point 14° F. lower than pure sulphur dioxide. This liquid, or rather mixture, has been since known as **Pictet Fluid.** Its latent heat has never been closely determined, but is very nearly the same as that of pure sulphur dioxide.

CARBON DIOXIDE.

1355. This liquid has the lowest boiling point of any of the fluids employed in refrigeration. Under a gauge pressure of 200 pounds per square inch, it will have a

TABLE 25.

PROPERTIES OF SATURATED CARBON DIOXIDE.

Temperature of Ebullition in Deg. F.	Absolute Pressure in Lb. Per Sq. In.	Total Heat from 32° F.	Heat of Liquid from 32° F.	Latent Heat of Vaporization.	Density of Vapor, or Weight of 1 Cu. Ft.
− 22	210	98.35	− 37.80	136.15	2.321
− 13	249	99.14	− 32.51	131.65	2.759
− 4	292	99.88	− 26.91	126.79	3.265
5	342	100.58	− 20.92	121.50	3.853
14	396	101.21	− 14.49	115.70	4.535
23	457	101.81	− 7.56	109.37	5.331
32	525	102.35	0.60	102.35	6.265
41	599	102.84	8.32	94.52	7.374
50	680	103.24	17.60	85.64	8.708
59	768	103.59	28.22	75.37	10.356
68	864	103.84	40.86	62.98	12.480
77	968	103.95	57.06	46.89	15.475
86	1,080	103.72	84.44	19.28	21.519

temperature of about — 22° F. Its condensing pressure is correspondingly high, being about 900 pounds per square inch for a water temperature of 70° F.

Table 25 gives the properties of carbon dioxide at different temperatures.

AMMONIA.

1356. Chemical Composition.—One atom of nitrogen combines with three atoms of hydrogen to form one molecule of **ammonia**; this is the only combination of these two elements. The ordinary ammonia of commerce is a **solution** of ammonia gas in water, and is properly known as **aqua ammonia.** The gas which passes off from the aqua ammonia is the ammonia formed by the combination of nitrogen and hydrogen. When this gas is **entirely free** from vapor of water, it is called **anhydrous** ammonia gas.

1357. Physical Properties.—Ammonia gas, when liquefied under a high pressure and allowed to evaporate under atmospheric pressure, gives a temperature of 28.5° F. below zero. Liquid anhydrous ammonia when subjected to a temperature of — 115° F. freezes and forms a solid. In this state it is almost odorless and is heavier than the liquid.

Ammonia has no effect on either iron or steel, but rapidly corrodes copper and brass. It is therefore necessary to make the parts of ammonia machines out of the former metals.

At a temperature of 900° F., the gas is resolved into its constituent elements. But it is probable that this dissociation occurs, to a limited degree, at much lower temperatures.

Ammonia is not inflammable at ordinary temperatures, but if mixed with oxygen will burn with a pale-yellow flame. The liquid will not explode, but when run into drums or flasks, room should be left for expansion. Like almost all liquids, ammonia expands when heated, and if sufficient space is not left for expansion, the flask is likely to burst if exposed to a high temperature.

TABLE 26.

PROPERTIES OF SATURATED AMMONIA.

Temperature. Degrees F.	Pressure, Absolute. Lb. per Sq. In.	Heat of Vaporization, Thermal Units.	Volume of Vapor per Lb., Cu. Ft.	Volume of Liquid per Lb., Cu. Ft.	Weight of a Cu. Ft. of Vapor, Pounds.
t	p	r	v	v_1	w
− 40	10.69	579.67	24.3700	.0234	.0410
− 35	12.31	576.69	21.2900	.0236	.0467
− 30	14.13	573.69	18.6600	.0237	.0535
− 25	16.17	570.68	16.4100	.0238	.0609
− 20	18.45	567.67	14.4800	.0240	.0690
− 15	20.99	564.64	12.8100	.0242	.0779
− 10	23.77	561.61	11.3600	.0243	.0878
− 5	26.93	558.56	10.1200	.0244	.0988
0	30.37	555.50	9.0400	.0246	.1109
+ 5	34.17	552.43	8.0600	.0247	.1241
+ 10	38.55	549.35	7.2300	.0249	.1384
+ 15	42.93	546.26	6.4900	.0250	.1540
+ 20	47.95	543.15	5.8400	.0252	.1712
+ 25	53.43	540.03	5.2600	.0253	.1901
+ 30	59.41	536.92	4.7500	.0254	.2105
+ 35	65.93	533.78	4.3100	.0256	.2320
+ 40	73.00	530.63	3.9100	.0257	.2588
+ 45	80.66	527.47	3.5600	.0260	.2809
+ 50	88.96	524.30	3.2500	.0260	.3076
+ 55	97.93	521.12	2.9600	.0260	.3378
+ 60	107.60	517.93	2.7000	.0265	.3704
+ 65	118.03	514.73	2.4800	.0266	.4034
+ 70	129.21	511.52	2.2700	.0268	.4405
+ 75	141.25	508.29	2.0800	.0270	.4808
+ 80	154.11	504.66	1.9100	.0272	.5236
+ 85	167.86	501.81	1.7700	.0273	.5649
+ 90	182.80	498.11	1.6400	.0274	.6098
+ 95	198.37	495.29	1.5100	.0277	.6623
+ 100	215.14	491.50	1.3900	.0279	.7194
+ 105	232.98	488.72	1.2890	.0281	.7757
+ 110	251.97	485.42	1.2030	.0283	.8312
+ 115	272.14	482.41	1.1210	.0285	.8912
+ 120	293.49	478.79	1.0410	.0287	.9608
+ 125	316.16	475.45	.9699	.0289	1.0310
+ 130	340.42	472.11	.9051	.0291	1.1048
+ 135	365.16	468.75	.8457	.0293	1.1824
+ 140	392.22	465.39	.7910	.0295	1.2642
+ 145	420.49	462.01	.7408	.0297	1.3497
+ 150	450.20	458.62	.6946	.0299	1.4396
+ 155	481.54	455.22	.6511	.0302	1.5358
+ 160	514.40	451.81	.6128	.0304	1.6318
+ 165	549.04	448.39	.5765	.0306	1.7344

1358. The leading properties of ammonia that are dependent upon the pressure or temperature are given in Table 26, which is taken from Wood's "Thermodynamics." The table is calculated from the following formulas, which are based partly on experimental data and partly on thermodynamic principles:

Let p = absolute pressure of gas or vapor in pounds per square inch;
t = temperature of vapor, Fahrenheit;
v = volume of one pound of vapor;
v_1 = volume of one pound of ammonia liquid;
w = weight of one cubic foot of vapor;
r = latent heat of vaporization in B. T. U.

$$\text{Then, } \log p = 6.2495 - \frac{2{,}196}{460.66 + t}. \qquad \textbf{(115.)}$$

$$v_1 = \frac{.016}{.6502 - .000778\,t}. \qquad \textbf{(116.)}$$

$$v = v_1 + .00107\frac{r}{p}t + .4923\frac{r}{p}. \qquad \textbf{(117.)}$$

$$w = \frac{1}{v}.$$

$$r = 555.5 - .613\,t - .000219\,t^2. \qquad \textbf{(118.)}$$

It will be noted that with ammonia vapor, as with all other saturated vapors, the temperature depends only upon the pressure, and *vice versa;* the relation between the temperature and pressure is given by formula **115.**

A comparison of the properties of ammonia with those of other refrigerating fluids shows that the latent heat of vaporization of ammonia is much greater than that of other fluids. This property makes ammonia especially desirable as a refrigerating agent, because, on account of the high latent heat, a greater refrigerating effect per pound of fluid circulated can be obtained with ammonia than with the other agents.

1359. Specific Heat.—The specific heat of liquid anhydrous ammonia has been variously computed to be from 1 to 1.2 that of water. The latest determinations show it to be 1.0058, virtually the same as water. The specific heat of ammonia gas is .508.

1360. Saturated and Superheated Gas.—When the temperature of the ammonia gas is the same as that of the boiling point of liquid anhydrous ammonia due to the pressure of the gas, the gas, or rather vapor, is at its greatest density and is said to be saturated. If heat is applied to this saturated gas so that its temperature is increased while the pressure remains constant, the gas becomes superheated and approaches very nearly a perfect gas in its properties. (See *Steam and Steam Engines*, Arts. **1191** and **1192**.)

The relation between the pressure volume and temperature of superheated ammonia gas has not yet been accurately determined. For ordinary purposes, however, the following equation is sufficiently exact:

$$p\,v = .62\,M\,T, \qquad \textbf{(119.)}$$

where M is the weight of gas and T is the absolute temperature.

Corresponding to formulas **81, 86,** and **87,** *Heat*, we have the following approximate formulas for the *adiabatic* expansion of superheated ammonia:

$$p_1\,V_1^{1.3} = p_2\,V_2^{1.3}. \qquad \textbf{(120.)}$$

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{.3}{1.3}} = \left(\frac{p_1}{p_2}\right)^{.2308}. \qquad \textbf{(121.)}$$

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{.3}. \qquad \textbf{(122.)}$$

1361. Aqua Ammonia.—As already stated (Art. **1356**), aqua ammonia, known also as **ammonia liquor,** is a solution of ammonia gas in water. At 32° F. and under atmospheric pressure, water will absorb 1,140 times its volume of ammonia gas. The amount of gas held in solution affects the specific gravity of the solution; the more gas absorbed the less the density. The amount of ammonia that can be absorbed by water is governed by the temperature of the water and the pressure of the gas. The colder the water and greater the pressure, the greater the quantity of ammonia taken up.

1362. The strength of a solution of anhydrous ammonia in water is determined by an instrument called a **hydrometer**—Beaumé's hydrometer—which is used for determining the densities of various liquids, as shown in Fig. 321. It consists of a glass tube with a bulb near the middle and a second bulb near the end, partly filled with mercury. A graduated scale is marked on the stem.. When this instrument is placed in a liquid, it is evident that it will sink deeper the less the density of the liquid; hence the density will be indicated by the mark on the scale at the level of the liquid. For liquids lighter than water, the point to which the instrument sinks when placed in a solution of 10 parts of salt to 90 of water is marked 0°, and the point to which it sinks in distilled water is marked 10°. The space between the two marks is divided into 10 parts, and the division is continued to the top of the stem. The hydrometer thus graduated is generally used for ammonia solutions, though there is another graduation in which the reading for pure water is 0° instead of 10°.

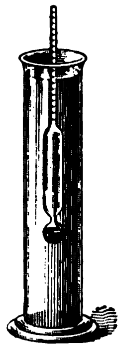


FIG. 321.

In Table 27, the first column gives the number of parts of ammonia gas in 100 parts of the solution, the second column gives the specific gravity of the solution, and the third column gives the corresponding reading on the Beaumé hydrometer. For example, if the hydrometer reading is 16°, the solution consists of 10 parts, by weight, of ammonia to 90 parts of water, and the specific gravity of the solution is .960.

1363. Heat of Absorption.—As stated in *Heat*, all chemical actions are accompanied by an increase or decrease

TABLE 27.

STRENGTH OF AMMONIA LIQUOR.

Percentage of Ammonia by Weight.	Specific Gravity.	Degrees Beaumé.	
		Water 10°.	Water 0°.
0	1.000	10.0	0
1	.993	11.0	1.0
2	.986	12.0	2.0
4	.979	13.0	3.0
6	.972	14.0	4.0
8	.966	15.0	5.0
10	.960	16.0	6.0
12	.953	17.1	7.0
14	.945	18.3	8.2
16	.938	19.5	9.2
18	.931	20.7	10.3
20	.925	21.7	11.2
22	.919	22.8	12.3
24	.913	23.9	13.2
26	.907	24.8	14.3
28	.902	25.7	15.2
30	.897	26.6	16.2
32	.892	27.5	17.3
34	.888	28.4	18.2
36	.884	29.3	19.1
38	.880	30.2	20.0

in the temperature of the mixture. This is especially true of solutions. In the case of ammonia absorbed in water, 925.7 B. T. U. is given up for each pound of ammonia gas absorbed under atmospheric pressure. Though no very exhaustive experiments have been made on this subject, results deduced from the practical running of refrigerating-machines show that this figure is practically constant.

Since heat is given up when ammonia gas is absorbed, heat will be absorbed when the gas is again liberated from the water. The quantity of heat necessary to liberate one pound of anhydrous gas is 925.7 B. T. U., the same amount of heat that is given out by the solution when the gas is being absorbed.

1364. Tests for Ammonia.—If it is desired to test the purity of liquid anhydrous ammonia, draw some out into a flask having a cork with a bent tube inserted in it. Wrap the flask up in dry waste or cloth before drawing off the ammonia, or the fingers are liable to be frozen fast to the flask. The liquid ammonia evaporates slowly, the gas passing out of the bent tube. If an accurate low-temperature thermometer is obtainable and is immersed in the boiling liquid, it should indicate a temperature of $-28.5°$ F., with normal barometric pressure. If the liquid is pure anhydrous ammonia, there should be no residue left in the flask. A deposit of oil or water indicates impure ammonia.

To detect a leak in piping in case the odor does not betray it, hold a glass rod moistened with muriatic acid near the supposed leak. A white fume rising from the rod indicates an escape of ammonia.

To detect ammonia leaks in piping under water or brine, add to a sample of the suspected liquid a few drops of *Nessler's Reagent;* a yellow coloring indicates traces of ammonia, but if the quantity of ammonia is large, the color changes to a dark brown.

1365. To Prepare "Nessler's Reagent."—Dissolve 17 grams (0.6 oz.) of mercuric chloride in about 300 cubic centimeters (10½ fluid oz.) of distilled water; dissolve 35 grams (1¼ oz.) of potassium iodide in 100 cubic centimeters (3½ oz.) of water; add the former solution to the latter, with constant stirring, until a slight permanent red precipitate is produced. Next dissolve 120 grams (4¼ oz.) of potassium hydrate in about 200 cubic centimeters (7 oz.) of water; allow the solution to cool; add it to the above solution, and make up with water to one liter (33¾ oz.), then add mercuric chloride solution until a permanent precipitate again forms; allow to stand till settled, and decant off the clear solution for use; keep it in glass-stoppered blue bottles, and set away in a dark place to keep it from decomposing.

RELATIVE EFFECT OF REFRIGERATING FLUIDS.

1366. The density of the gas at the evaporating temperature and the latent heat of the liquid determine the size of the compression cylinder necessary for any required capacity. The same machine working between 5° and 64.4° will give the following cooling effects per *cubic foot* of compressor piston displacement under theoretically perfect conditions:

Carbon Dioxide	248.18 B. T. U.
Ammonia	62.75 B. T. U.
Sulphur Dioxide	22.88 B. T. U.
Sulphuric Ether	3.68 B. T. U.

THE AMMONIA COMPRESSION SYSTEM.

GENERAL DESCRIPTION.

1367. Suppose a flask or ordinary bottle *B*, Fig. 322, supplied with a cork having a bent tube *G* inserted, be partially filled with anhydrous ammonia. This can be done easily, as the evaporation of the ammonia is comparatively slow, owing to its high latent heat. As the ammonia enters the flask, frost will begin to gather on the outside. If, now, we place this flask into a pail *A* partially filled with water *C*, in a short time ice *D* will begin to gather on the outside of the flask. This is the simplest form of ice-machine, but in this form the liquid ammonia, when it evaporates, passes out of the flask and is lost.

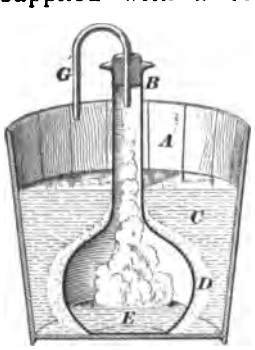


FIG. 322.

As with all volatile vapors, the temperature at which vaporization (or condensation) occurs rises as the pressure of the vapor increases. To prove this, insert a thermometer into the flask so that the bulb is immersed in the boiling ammonia. The temperature will fall rapidly, and if the thermometer is correct, it should register 28.5° below zero. Take a piece of pipe; weld or plug one end and fit the other with a cap *B*, Fig. 323. Arrange a stuffing-box *C* about the thermometer *T* in the cap; also provide an opening connecting with the pressure gauge *P*. Unscrew the cap and pour the contents of the flask into the pipe *A* and screw on the cap *B*. If the gauge points to zero, the thermometer should still read —28.5° F. Watch the gauge and thermometer carefully. The ammonia evaporating in the pipe liberates gas. As this gas can not escape, it creates a pressure in the pipe, which will be shown on the gauge, and a corresponding increase in the temperature of the boiling ammonia will become apparent. This will continue until the temperature of the liquid will be identical with the surrounding objects. Assume this temperature to be about 70° F.; the gauge should then show a pressure of 130 pounds per square inch. If, therefore, we keep the temperature of the pipe at 70° by immersing it in water at that temperature, and arrange to keep a pressure slightly in excess of 130 pounds per square inch in the pipe, no further evaporation will take place, and the remaining liquid ammonia will lie quietly in the pipe.

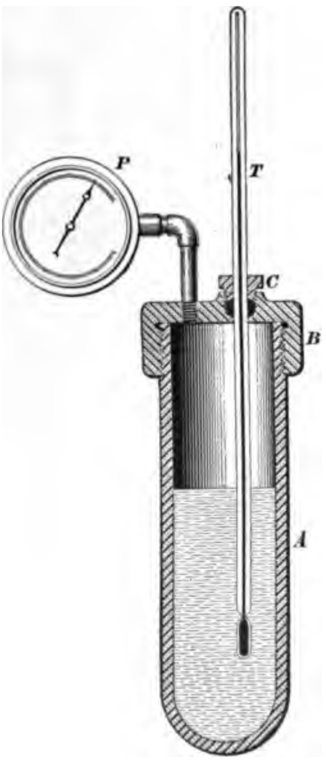


FIG. 323.

1368. From the foregoing, it is apparent that if some means be devised of taking the evaporating gas as it leaves the flask in Fig. 322 and transfer this gas into the pipe of Fig. 323, it would be possible to save the gas. In place of a short piece of pipe *A*, Fig. 323, take a large coil of pipe *A*, Fig. 324, submerged in a tank of water *C*. The water enters by means of the pipe *F*, and the overflow passes out of the pipe *F'*; the continuous flow tends to keep the temperature of the coil *A* constant. Replace the flask *B* in Fig. 322 with a coil of pipe *B*, Fig. 324, immersed in a

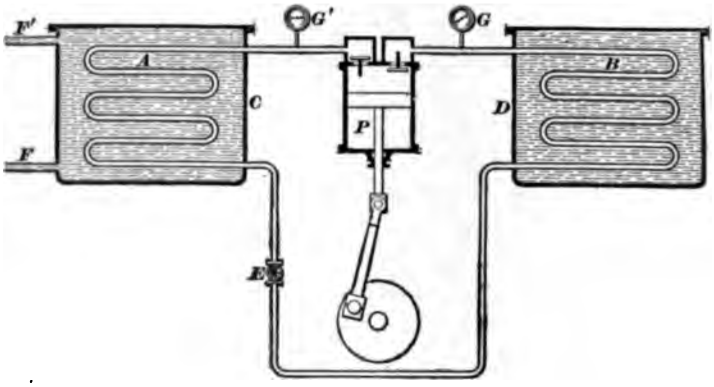


FIG. 324.

water-tank *D*. Provide a pump capable of working against a high pressure, and connect the suction of the pump with the coil *B* and the discharge with the coil *A*; also provide pressure gauges *G'* and *G* on each of these lines. Connect the bottom of the two coils together, and place a valve *E* in the line. Partially fill the coil *A* with anhydrous ammonia. If the temperature of the water in *A* is about 70°, the gauge *G'* will show a pressure of 130 pounds. Open the valve *E* slightly and leave it open. The pressure denoted by the gauge *G* will gradually rise, and ice will begin to form on the lower pipes of *B*. When the pressure shown by *G* has reached 15 pounds, start the pump *P*. This pump will draw the gas out of the coil *B*, compress it, and deliver it to the coil *A*. The gas entering *A*, which has been heated by the compression, comes in contact with the cold pipe surface, and is first cooled until its temperature is but little above that of the condensing water flowing out through *F'*. The gas then condenses and falls to the bottom of the coil in the form of liquid anhydrous ammonia. As the valve *E* is open, the coil *A* is prevented from filling up. The withdrawal of a quantity of the gas in the coil *B* tends to decrease the pressure in that coil; however, a quantity of the liquid passes from *A* to *B* through the expansion-valve *E*, vaporizes, and supplies an amount of gas equal to that withdrawn by the pump.

THE CYCLE OF THE AMMONIA COMPRESSION MACHINE.

1369. In Fig. 325 let the length $O V_1$ represent to scale the volume of the compressor cylinder *P*, Fig. 324,

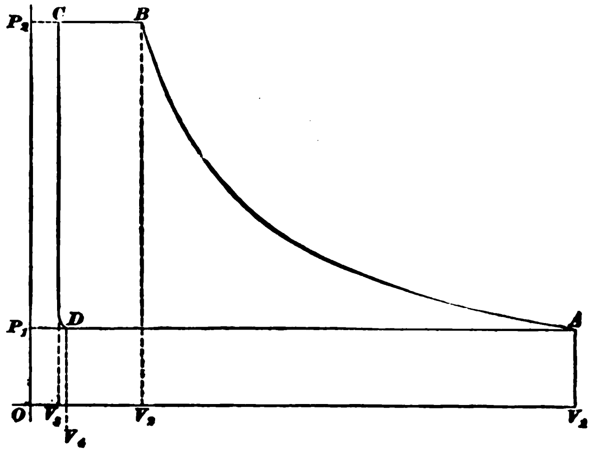


FIG. 325.

and let the ordinate $O P_1$ ($= V_1 A$) represent the pressure of the ammonia gas in the coil *B*. Then we may say that the point *A* represents the state of the gas in the cylinder when the piston is at the lower end of the cylinder. As the piston rises, the gas is compressed until it reaches the state represented by the point *B*; that is, the pressure is represented by $O P_2$ and the volume by $O V_2$. The character of the compression depends upon the method of cooling the compression cylinder, to be explained later. We will assume for the present that the compression is adiabatic. If the vapor is dry and saturated at the state A, it will be superheated at the state B, after the adiabatic compression. When the pressure in the cylinder reaches P_2, the valve opens and the compressed vapor is delivered to the coil A at a practically constant pressure. During this operation the temperature of the superheated vapor is reduced, and then the vapor is wholly or partially condensed to a liquid. The volume is reduced from $O\,V_2$ to $O\,V_3$. The ammonia liquid now runs slowly through the valve E into the coil B. Since the pressure in the coil B is much lower than that in A, there will be a drop in pressure in passing the valve, and this will be accompanied by a drop in temperature. The slight increase in volume from V_3 to V_4 is due to the fact that a small quantity of the liquid is vaporized before the pressure has dropped to P_1. The liquid ammonia in the state represented by the point D now vaporizes, and the volume increases from $O\,V_4$ to $O\,V_1$. The gas returns to its original state represented by A, and the cycle is closed.

The cycle just described differs from the ideal cycle described in Art. **1335** in having the line CD, representing the third operation, nearly straight instead of curved, as in Fig. 317. To attain the ideal conditions shown in Fig. 317, it would be necessary to allow the liquid from the condenser A to expand and do work in an expansion cylinder instead of passing through the regulating valve E. For constructive reasons, the valve is preferred, as the cylinder would be small, and the work recovered by the expansion would in any case be insignificant.

GENERAL THEORY.

1370. Heat Transfers.—Let T_a denote the absolute temperature of the gas in the state A, Fig. 325, which is practically the temperature of the coil B, and let T_b

denote the temperature of the gas at the end of compression. Since the gas is superheated during the compression, the final temperature T_b is given by formula **121:**

$$\frac{T_b}{T_a} = \left(\frac{p_2}{p_1}\right)^{.2308},$$

or

$$T_b = T_a\left(\frac{p_2}{p_1}\right)^{.2308}.$$

Let T_c denote the final temperature that the gas or vapor attains in the condenser; that is, T_c is the temperature of condensation. Let r_2 denote the latent heat of vaporization at the temperature T_c. The specific heat of ammonia gas being .508, the heat given up by a pound of the gas in cooling from T_b to T_c is .508 $(T_b - T_c)$ B. T. U. In condensing, a pound of the gas gives up further r_2 B. T. U. If M denote the weight in pounds of gas used per stroke, then the heat given up to the condenser per stroke is evidently

$$\left.\begin{aligned} Q_1 &= M[.508\,(T_b - T_c) + r_2]\text{ B. T. U.} \\ &= M[.508\,(t_b - t_c) + r_2]\text{ B. T. U.} \end{aligned}\right\} \quad (a)$$

If t_e and t_f denote, respectively, the temperature of the cooling water as it enters and as it leaves the condenser, then the weight of cooling water required per stroke is

$$G = \frac{M[.508\,(t_b - t_c) + r_2]}{t_f - t_e}. \quad (b)$$

For the sake of simplicity, we will assume that the pressure of the liquid ammonia drops from P_2 to P_1 before vaporization begins; while this is not precisely the case, the difference in the result is not appreciable. During the fall of temperature from T_c to T_a, the original temperature of the gas in the coil B, a pound of the liquid gives up $s\,(T_c - T_a)$ heat-units, where s denotes the specific heat of liquid ammonia. During the vaporization in the coil B, each pound absorbs r_1 B. T. U., where r_1 denotes the latent heat of vaporization corresponding to the pressure P_1 and temperature T_a. The net heat abstracted from the refrigerator per stroke is therefore

$$\left.\begin{aligned} Q_2 &= M[r_1 - s(T_c - T_a)] \text{ B. T. U.} \\ &= M[r_1 - s(t_c - t_a)] \text{ B. T. U.} \end{aligned}\right\} \quad (c)$$

Since the specific heat of liquid ammonia is practically 1, we may write the equation:

$$\left.\begin{aligned} Q_2 &= M[r_1 - (T_c - T_a)] \text{ B. T. U.} \\ &= M[r_1 - (t_c - t_a)] \text{ B. T. U.} \end{aligned}\right\} \quad (d)$$

The work of the compressor per stroke in foot-pounds is

$$W = J(Q_1 - Q_2) = J\,M\,[r_2 - r_1 + .508\,(t_b - t_c) + t_c - t_a].$$

If n denote the number of strokes per minute, the theoretical horsepower of the compressor is

$$H = \frac{nJM[r_2 - r_1 + .508\,(t_b - t_c) + t_c - t_a]}{33{,}000}. \quad (e)$$

1371. Efficiency.—The theoretical efficiency of the refrigerating-machine is

$$E = \frac{JQ_2}{W} = \frac{r_1 - (T_c - T_a)}{r_2 - r_1 + .508\,(T_b - T_c) + T_c - T_a}. \quad (f)$$

If we denote by T_t the temperature of the cold room or of the brine, if the latter be used, then the effective range of temperature is $T_f - T_t$, where T_f, as before, denotes the absolute temperature of the cooling water as it leaves the condenser. In practice T_t is always a little higher (5° to 10°) than the temperature T_a of the ammonia in the refrigerating coils, and the temperature T_c of the ammonia in the condenser coils is always higher than T_f. The theoretical maximum efficiency for this temperature range is

$$E_m = \frac{T_t}{T_f - T_t}. \quad (g) \qquad \text{(See Art. 1346.)}$$

The economy of a machine may be judged by comparing the actual efficiency with this ideal efficiency E_m.

1372. Volume of Compressor Cylinder.—Let v denote the volume of a pound of ammonia vapor at the pressure p_1 in the coil B; then, since M pounds of vapor are used per stroke, the cubic capacity of the cylinder must be

$$C = Mv \text{ cubic feet.} \quad (h)$$

To allow for clearance and imperfections in the operation, the volume as thus calculated should be increased 10 per cent. or more.

1373. Capacity.—In 24 hours the heat abstracted from the cold body is

$$(24 \times 60 \times n \times Q_2) \text{ B. T. U.} =$$
$$24 \times 60 \times n M [r_1 - (t_c - t_a)] \text{ B. T. U.}$$

Hence, the ice-melting capacity in tons per 24 hours is

$$F = \frac{24 \times 60 \times n M [r_1 - (t_c - t_a)]}{285,300} =$$
$$.00505 n M [r_1 - (t_c - t_a)]. \quad \textbf{(123.)}$$

1374. Problem.—As an application of the theory developed in the preceding paragraphs, the dimensions of an ammonia refrigerating-machine will be worked out from the following data:

Ice-melting capacity.............. 20 tons in 24 hours;
Temperature in condenser......... 90° F.;
Temperature in refrigerating coil.. 15° F.;
Initial and final temperatures of condensing water, 60° and 85° F.

The compressor is double-acting and makes 56 revolutions per minute. Required, also, the horsepower of the compressor, the theoretical efficiency, and the quantity of cooling water used per minute.

Referring to the table of the properties of ammonia, the pressures corresponding to the temperatures 15° and 90° F. are, respectively, $p_1 = 42.93$ pounds and $p_2 = 182.8$ pounds, absolute, per square inch. The latent heats are, respectively, $r_1 = 546.26$ B. T. U. and $r_2 = 498.11$ B. T. U. The absolute temperature T_a is $460° + 15° = 475°$; the absolute temperature T_b of the superheated gas at the end of compression is

$$T_b = T_a\left(\frac{p_2}{p_1}\right)^{.2308} = 475° \times \left(\frac{182.8}{42.93}\right)^{.2308} = 663.62°;$$

hence, $t_b = 663.62° - 460° = 203.62°$.

The capacity being 20 tons, we have

$$20 = .00505\, n M [r_1 - (t_c - t_a)],$$

or

$$M = \frac{20}{.00505\, n [r_1 - (t_c - t_a)]} =$$

$$\frac{20}{.00505 \times 112 \times [546.26 - (90 - 15)]} = .075.$$

The quantity of ammonia used per stroke is .075 pound, and the quantity circulated per minute is .075 lb. $\times$ 112 = 8.4 pounds.

The volume of 1 pound of ammonia vapor at a temperature of 15° is, according to the table, 6.49 cubic feet; hence, the theoretical capacity of the cylinder is

$$C = Mv = .075 \times 6.49 \text{ cu. ft.} = .48675 \text{ cu. ft.} = 841.2 \text{ cu. in.}$$

Adding $\frac{1}{8}$ to allow for imperfections in the operation, the actual volume is 841.2 cu. in. $\times 1\frac{1}{8} = 946.5$ cu. in. If the stroke is made double the piston diameter, this volume will require a diameter of $8\frac{1}{2}$ inches and a stroke of 17 inches.

The horsepower required to drive the compressor is, approximately,

$$H = \frac{nJM[r_2 - r_1 + .508(t_b - t_c) + t_c - t_a]}{33,000} =$$

$$\frac{112 \times 778 \times .075 \times [498.11 - 546.26 + .508(203.62 - 90) + 90 - 15]}{33,000} =$$

16.7, nearly.

The horsepower of the steam cylinder should be at least $\frac{1}{3}$ greater than that of the compressor; in the present case it will be about 16.7 H. P. $\times \frac{4}{3} = 22.93$, say 23 H. P.

The heat given up by the ammonia to the cooling water per stroke is

$Q_1 = M[.508(t_b - t_c) + r_2]$ B. T. U. =

.075 [.508 (203.62 − 90) + 498.11] B. T. U. = 41.687 B. T. U.

The cooling water required per minute is therefore

$$G = \frac{n\,Q_1}{t_f - t_e} = \frac{112 \times 41.687}{85 - 60} = 187 \text{ lb., nearly.}$$

187 pounds is equivalent to 3 cubic feet, or about 22½ gallons.

The theoretical efficiency is

$$E = \frac{r_1 - (t_c - t_a)}{r_2 - r_1 + .508\,(t_b - t_c) + t_c - t_a} = \frac{471.26}{84.569} = 5.57.$$

If we assume the temperature of the brine or of the cold room to be 5° higher than that of the ammonia in the condenser, $T_i = T_a + 5° = 460° + 15° + 5° = 480°$. The effective temperature range is therefore $T_f - T_i = 85° + 460° - 480° = 65°$, and the maximum efficiency is

$$E_m = \frac{T_i}{T_f - T_i} = \frac{480}{65} = 7.38.$$

1375. Influence of Suction Pressure.—The *suction* pressure, that is, the pressure of the ammonia vapor as it passes from the refrigerating coil to the compressor cylinder, exerts a marked influence upon the capacity and also upon the economy of the refrigerating-machine. Consider the formula for capacity, $F = .00505\,n\,M\,[r_1 - (t_c - t_a)]$. Since the n remains constant, it is evident that the capacity depends upon the factors M and $[r_1 - (t_c - t_a)]$. Referring to the table of properties of ammonia, it is seen that as the suction pressure p_1 is increased, the corresponding latent heat r_1 becomes less and the corresponding temperature t_a rises; consequently the factor $[r_1 - (t_c - t_a)]$ changes but little with different suction pressures, and *the capacity is practically proportional to the weight of ammonia circulated*, that is, to M. Now, since the same *volume* of ammonia vapor passes through the compressor in a given time, whatever the pressure may be, it is evident that the *weight* of ammonia circulated is exactly proportional to the weight of the vapor per cubic foot. Thus, referring to the table, a cubic foot of vapor at a pressure of 20.99 pounds per square inch absolute weighs .0779 pound, while at a pressure of 42.93 pounds absolute a cubic foot weighs .154 pound, or practically twice as much. With the greater suction pressure, therefore, the machine will circulate double the ammonia that it will with the lower suction pressure, and consequently the capacity at the higher pressure will be nearly double that at the lower pressure.

The power required to operate the compressor is of course greater with the higher suction pressure and greater capacity, but the increase in power is not proportional to the increase in capacity. It can readily be shown that the ratio of the power consumed to the capacity decreases as the suction pressure is raised. The power is proportional to $Q_1 - Q_2$ (see Art. **1370**), and the capacity is proportional to Q_2 (see Art. **1373**); hence the ratio $\frac{H}{F}$ is proportional to $\frac{Q_1 - Q_2}{Q_2}$. Now, taking the expressions for Q_1 and Q_2, $Q_1 = M[.508(t_b - t_c) + r_2]$ and $Q_2 = M[r_1 - (t_c - t_a)]$, it is seen that the only effect on Q_1 of raising the suction pressure is to lower the temperature t_b at the end of compression. The effect on Q_2 is to decrease r_1 and increase t_a; but since t_a increases faster than r_1 decreases, the net effect of raising the suction pressure is to increase Q_2. Since Q_1 is decreased and Q_2 is increased, the fraction $\frac{Q_1 - Q_2}{Q_2}$, and consequently the ratio $\frac{H}{F}$, is decreased by raising the suction pressure. We have, therefore, the important fact that *increasing the suction pressure increases both the capacity and economy of the machine.*

Fig. 326 is the indicator-diagram taken from an ammonia cylinder of a compressor running under 151 pounds head pressure and 28 pounds suction pressure. The tonnage in this case is 74.8, while the ammonia cylinder horsepower is but 65.7 or .88 of a horsepower per ton of work done.

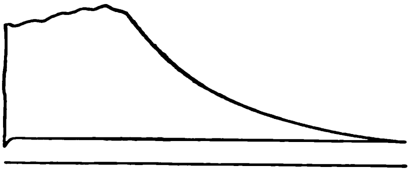


FIG. 326.

Fig. 327 shows a diagram taken from the same compressor under the following conditions: Head pressure 135 pounds, back pressure 2 pounds, ammonia cylinder horsepower 46.02; tonnage 25.9 or 1.8 horsepower per ton of refrigerating effect. From this it will be seen that while the capacity is about a third, the expenditure of power per ton of refrigerating effect is approximately double. These diagrams and the data connected with them are taken from Prof. J. E. Denton's report of a test of a refrigerating plant. (Transactions A. S. M. E., Vol. XII.)

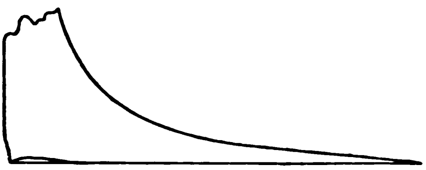


FIG. 327.

The upper limit of suction pressure is fixed by the temperature at which it is required to keep the cold room or the circulating brine. Suppose, for example, that the average temperature of the brine is 33° F.; the temperature in the refrigerating coils must be, say, 10° lower, or 23° F., and the pressure corresponding is 51.25 pounds per square inch absolute, or 36.55 pounds gauge.

1376. Influence of Condensing Pressure.—The pressure in the condenser does not affect the weight of ammonia circulated, and has therefore no influence on the capacity of the refrigerating-machine. The lower the temperature of the ammonia in the condenser can be kept, the lower will be the pressure, and consequently the less will be the work of compression. It is advantageous, therefore, to have the initial temperature of the condensing water as low as possible and to use as much water as is consistent with economy. Prof. Denton, in the report previously mentioned, states that "maximum economy with a given type of engine, where water must be bought at average city prices, is obtained at 28 pounds suction pressure and 150 pounds condensing pressure." This condensing pressure requires about a gallon of water per minute per ton of ice-melting capacity, taking the initial temperature of the water at 56° F. If the supply of water is 25 per cent. less, the condensing pressure rises to 190 pounds and the work of compression is increased about 20 per cent. On the other hand, if the water-supply is increased to three gallons per minute per ton capacity, the condensing pressure will fall to about 105 pounds. The work of compression is reduced about 25 per cent., but the saving in power thus effected is more than counterbalanced by the cost of the extra water used to lower the pressure.

THE COMPRESSOR OR PUMP.

1377. The vital part of a compression refrigerating-machine is the compressor or pump; upon the design and construction of this part chiefly rests the commercial advantage or disadvantage of any particular make of machine.

1378. Pump Cylinder.—Compressor cylinders are usually made of a fine, close-grained charcoal iron or semi-steel. It is necessary to have a cylinder heavy enough to withstand a test pressure of at least 350 pounds per square inch. Heads and bolts should be made in proportion. The joint between the cylinder and head is usually of a tongue and groove, male and female type, lead being used as a packing material in the groove. Other makers use a smooth face on the flanges of the head and cylinder and a very thin lead or other metallic packing. In this case, the flange should be scored with three or four concentric grooves between the bore of cylinder and bolt-holes, to hold the gasket in place and prevent it from blowing out. The bore or inside walls of the cylinder should be smooth, and sufficient oil must be used in running the machine to prevent groaning and consequent wear.

Some makers line the cylinder with a wrought-iron or steel bushing, but the majority use cast iron. There is more friction with the former metals, while with care and a proper mixture, a cast-iron cylinder that will hold ammonia can be made.

The stroke should in all cases be as long as possible, so as to get a comparative fast piston speed with the least number of revolutions. Another reason for making the stroke long is that the clearance spaces are smaller with a long stroke and small bore than a shorter stroke and longer bore. A well-proportioned compressor should have a stroke of at least double the diameter of the bore, and two and one-half times is better practice. The following cylinder capacities will be found convenient for ready reference: With a suction or back pressure of 25 pounds, a single-acting compressor should displace $3\frac{1}{2}$ cubic feet per ton and a double-acting compressor 4 cubic feet per ton. With 15 pounds back pressure, these figures should be 5 cubic feet for the former and $5\frac{1}{2}$ cubic feet for the latter type of machine. These figures are for well-built compressors having comparatively small clearances, and are based upon actual tests of refrigerating-machines.

1379. Compressor Piston.—The pistons of all double-acting compressors are necessarily solid. Those of the single-acting type are often made solid, while others are fitted with suction-valves. Cast-iron spring rings are used for packing-rings to prevent leakage; at least 4 or 5 rings should be employed. Sectional rings and others similar to those used in high-class engines and air-compressors are found in ammonia pumps.

1380. Heat of Compression.—It was stated in Art. **1369** that if ammonia vapor be compressed adiabatically, it will be superheated, and the work done on the vapor by the piston will be stored up in the vapor in the form of heat. This heat must be gotten rid of during the period of compression, otherwise it must be absorbed by the condensing water before the vapor condenses. It is quite evident that it will be most economical to remove the heat, as far as possible, as fast as it is generated, and keep the temperature of the cylinder comparatively low throughout the compression. In fact, it is absolutely necessary to employ some method of keeping the cylinder cool, otherwise the excessive heat developed in compression would soon become so great that the gas would enter the cylinder in a greatly superheated state, which would lessen its density. This decrease in density would naturally cause a corresponding decrease in the weight of gas pumped in a given time, thus affecting both capacity and economy.

A number of expedients have been tried with this object in view. The simplest of these is jacketing the cylinder with water. This method of cooling the compression cylinder is known as the **dry-compression system.** The gas enters the cylinder in perhaps a saturated state, though usually somewhat superheated; the instant compression begins, however, the vapor is immediately superheated.

In the second method, **wet compression,** the cylinder is not jacketed, but a certain amount of liquid anhydrous ammonia is allowed to enter the cylinder with each stroke of the compressor; the mixture of vapor and liquid remains saturated while it is compressed, the heat equivalent of the work of compression is taken up by the vaporization of a part of the liquid, and the vapor remains at the temperature due to the pressure.

The third method employed is a modification of wet compression. Instead of permitting anhydrous ammonia to enter the cylinder, a certain quantity of oil is injected during the stroke; the purpose of the oil is to cool the gas during compression and seal the valves so as to cut down the clearance space.

1381. Dry Compression.—The majority of compressors built in the United States are of the water-jacketed, dry-compression type. In the case of vertical compressors, the water-jackets are merely small tanks enclosing the walls of the cylinder, and are sufficiently high so that the top head of the cylinder is also immersed. They are open at the top and the water passes off by gravity. Horizontal compressors are usually water-jacketed on the cylinder walls only, the heads being unjacketed. With the inlet water at 60° F., the quantity of water required for a water-jacket is about $\frac{1}{8}$ of a gallon per minute per ton of refrigerating effect.

1382. Wet Compression.—The injection of a small quantity of anhydrous ammonia to cool by its evaporation the walls of the cylinder was the invention of Prof. C. P. G. Linde, of Munich, Germany. The machines built under this system bear his name and are of the horizontal, double-acting type. The temperature of the gas leaving the compressor in case of the Linde machine is much lower than that in the dry-compression system, and the theoretical economy is somewhat higher than that of the latter system. An objection sometimes urged against wet compression is the necessity of introducing a small quantity of liquid ammonia, which if increased in any degree by carelessness is liable to act somewhat like water carried over in the steam of a steam engine, and may blow out the cylinder-head. The chances of such an occurrence are so small that the objection does not appear to be a serious one.

1383. Oil Injection.—The third method of cooling the cylinder during compression is that of the De La Vergne Company. In the earlier machines a small quantity of oil was admitted into the compression cylinder during suction and was expelled at compression. The mixture of oil and gas at a somewhat high temperature passed to the oil separator, where the oil was separated from the ammonia gas. The gas passed on to the condenser in its regular cycle; the oil was taken from the separator, passed through a cooling coil immersed in running water, and was then allowed to run into a receiver, from which it again passed into the suction-pipe.

It will be seen from this description that as a certain quantity of oil was allowed to enter with the gas, the volume of the gas entering the cylinder at each stroke was decreased in proportion to the amount of oil injected; in case a considerable quantity of oil was fed in, this would cut down the capacity appreciably. In order to obviate this difficulty, the De La Vergne Company in their new compressors inject oil by means of a small pump after the work of compression has set in, and not during suction as formerly. This also permits the oil to be kept fully charged with ammonia.

1384. Single and Double Acting Compressors.— The single-acting ammonia compressor is usually of the vertical type and compresses gas on one side of the piston only, viz., the upper side, or, in other words, the side farthest away from the stuffing-box. The gas has at all times access to the space in the cylinder below the piston; here only the suction pressure acts constantly on the under side of the piston. In this form of compressor, the pressure on the stuffing-box side is always low, and it is therefore easy to keep the piston rod packed. As the work of compression is done on one side of the piston only, the water-jacket has a greater opportunity of reducing the temperature of the cylinder walls. The colder the compressor cylinder is kept, the greater will be the weight of gas pumped per revolution, as explained in previous paragraphs. It will therefore be readily seen that the loss due to superheating is less in this form of compressor than in the double-acting type. Experiment and test have proven this to be the case, the loss due to superheating in single-acting compressors being from 15 to 22 per cent.

The double-acting compressor is usually of the horizontal type and has suction and delivery valves on both sides of the piston. Such a compressor is cheaper to build than the single-acting one, and with the same cylinder capacity will deliver double the quantity of ammonia. The loss due to cylinder superheating, however, is greater than in the single-acting type, this loss amounting to 25 or 30 per cent.

The clearance spaces in a single-acting are usually much smaller than those of the double-acting compressor. In the former type it is merely necessary to adjust the piston so that it just clears the upper head of the compressor; this can be done, no matter what the stroke of the pump may be. In the case of the double-acting machine, however, the adjustment is a much more difficult task, for if the clearance is cut down on one side of the piston, it will increase by a like amount on the other side. It is therefore necessary to be very careful, in the selection of gaskets, etc., to get them of the proper thickness so that the clearance spaces will be reduced to the minimum on both sides of the piston.

1385. Vertical and Horizontal Compressors.— In all types of compressors except those using oil injection, as little oil is fed into the compression cylinder as possible. This small quantity of oil, which is much less than that used in the case of steam engines, makes it necessary to select a compressor in which the wear on the moving parts is equal all around. In case of a vertical compressor, the weight of the piston does not come on the walls of the cylinder, as is the case with the horizontal machine. The valves are also usually arranged so that they work up and down, and will therefore not wear their stems in such a way as to get them out of line with their seats. The vertical machine, however, is more expensive to build than the ordinary horizontal compressor, the bed-plate of which has the form of that of an ordinary horizontal engine. A still cheaper construction is that of the direct-acting horizontal machine, in which the compressor is attached tandem with the engine cylinder on an extension of the steam-engine bed-plate. The advantages of horizontal compressors are that they can be placed in low headroom and that all parts are accessible from the floor. A *direct-acting* machine is one in which the compressor and steam cylinders are in line, and the compressor piston and steam piston are attached to the same piston rod. In the *indirect* machine, the steam and compressor cylinders have separate piston rods and connecting-rods; the compressor is driven from a main shaft, which is in turn driven by the steam engine in the ordinary manner.

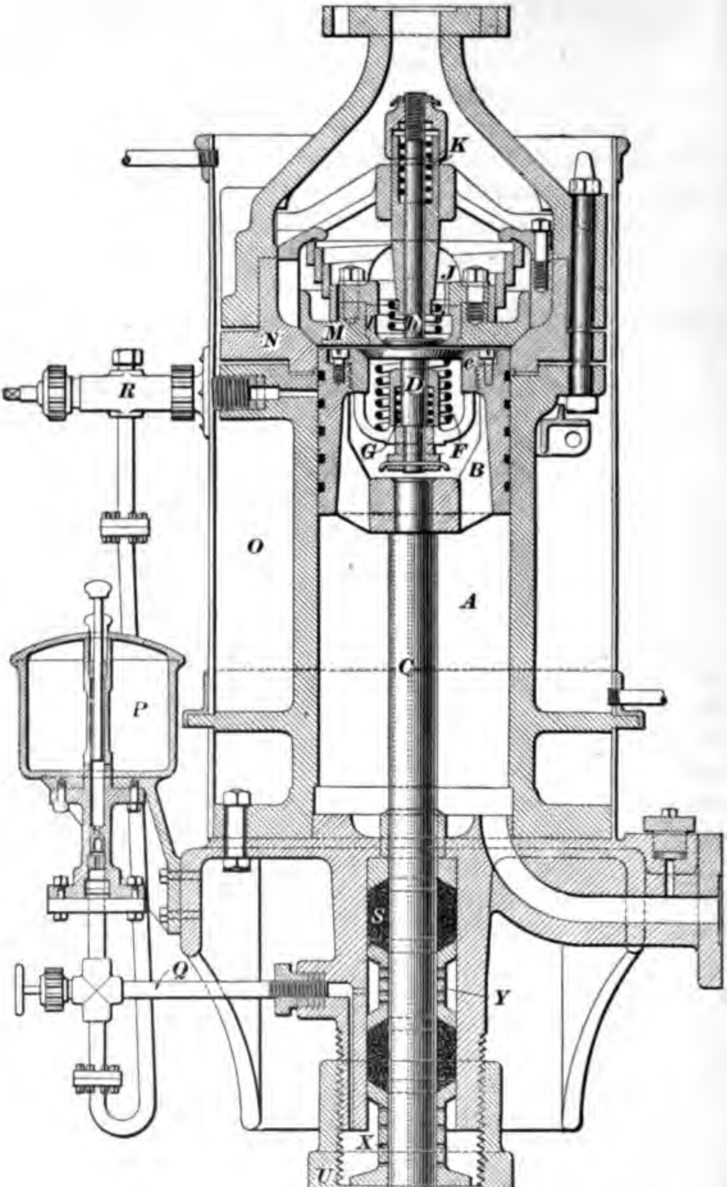


FIG. 328.

1386. Clearance Spaces.—When a pump piston is at the end of its stroke, the space that is left between the piston, cylinder-head, and valves is known as the clearance space. If this space is reduced to nothing, that is, if the valves are so ground that they come flush with the head, and the piston is allowed to come up until it just touches the head, all gas between the piston and head will be expelled through the delivery valve. Such a construction is hardly practical. It will be seen, however, that it is necessary to make this space as small as possible, since the gas that is left in the cylinder will expand as the piston travels back and will keep a corresponding quantity of gas from entering the cylinder.

Well-made compressors are run with a clearance of $\frac{1}{32}$ inch between the piston and cylinder-head. In order to determine this clearance, one of the valves should be taken out and a piece of fuse wire or lead laid on top of the piston. The machine is then turned by hand until the piston has completed its stroke. The fuse wire or lead will have been flattened, and its thickness will be the exact distance between the cylinder-head and piston, and will thus indicate the amount of clearance. The piston rod can be adjusted if the first test shows an unsatisfactory clearance, and the experiment may be repeated until the clearance space is cut down to a minimum.

1387. Suction and Discharge Valves.—A section through a compressor cylinder as constructed by the Frick Company is shown in Fig. 328. The suction-valve is shown at *D*. As the piston moves downwards, the valve rises from its seat *e*, and the gas from the refrigerating coil, which has at all times free access to the lower end of the cylinder *A*, passes through the open valve to the upper end of the cylinder. As soon as the piston again starts upwards, the valve *D* closes and compression begins. The valve is fitted with balanced springs *F* and *G*, which may be adjusted so as to prevent the valve from rattling.

Ice-machine builders have for a long time been striving to design a suction-valve so arranged that in case either the valve-stem or valve breaks, the pieces will not fall into the cylinder of the compressor. Compressors built in former years were greatly troubled with imperfect workmanship on their suction-valves. Machines were occasionally wrecked because of a broken valve. The fragments dropped into the cylinder, and the piston on the next stroke drove them against the head, thus causing serious damage.

The discharge-valve is an ordinary valve with a spindle *k* and with balanced springs *J* and *K*. In the construction shown in Fig. 328, the whole head *M* is a delivery-valve. Under ordinary circumstances, the gas passes through the small discharge-valve, but in case the suction-valve is displaced by any accident, the head *M* will rise from its seat. This action prevents damage to the head.

The suction and delivery valves and also the valve-seats are made of steel. The valves should be well ground to their seats, and they should be inspected occasionally to ascertain if either valve or seat is cut or defective in any way. A spare set of valves should always be carried in stock to replace any that may be found defective.

The area of the suction-valve should be at least equal to the area of the suction-pipe. The lift of the valve should be at least $\frac{1}{4}$ of the diameter, in order that the area of the annular opening may be equal to the area of the valve.

1388. The Stuffing-Box.—As shown in Fig. 328, the stuffing-box *S* is made very long in order that a large amount of packing may be used. The stuffing-box is usually provided with a thimble or oil-gland *Y* placed about midway between the two ends. A certain amount of packing is first inserted in the stuffing-box, and then the oil-gland is set in place. A second packing is then inserted, and this is followed by a second oil-gland *X*. The gland *U* is then screwed on. The oil-gland is filled by means of a connecting line from the oil separator or a hand oil-pump *P*. There is usually a by-pass connection *R* from the hand-pump to the compressor cylinder, so that oil may be fed directly into the cylinder in case the piston packing-rings start to groaning. Garlock and Crandal packings are the favorite soft packings used on ammonia compressors; metallic packings with soft backing are also used with considerable success.

The piston-rod packing of a double-acting compressor requires greater attention than that of a single-acting machine. The wet system is particularly hard on packing, as there are few soft packings that will stand the freezing action of the liquid anhydrous ammonia without becoming hard and causing leaky stuffing-boxes. All piston rods should be packed carefully, but should not be packed at all if they are scored or in any way imperfect, as any imperfection on the rod will soon cut the packing. When possible, the rod should be ground perfectly circular, but if this is not possible, a very fine finishing cut should be taken, and the rod should then be thoroughly smoothed off with a fine grade of emery-cloth. Too much care can not be taken in packing the piston rods with metallic packing. No old rods should be thus packed without being thoroughly trued beforehand. Rods should be calipered at the end of the season, and any repairs on them should be made at that time. Care in this respect will soon pay for itself in the ammonia bill.

THE MOTOR.

1389. Speed of Compressor.—The maximum speed at which a compressor should run, even in the case of very small machines, is 100 revolutions per minute. If the compressor is speeded up above this point, the valves will not act quickly enough, and while the capacity is but slightly increased, the power required to drive the machine is directly in proportion to the number of revolutions. Compressors of capacities ranging from 10 to 25 tons should not exceed 70 revolutions per minute, while those of larger sizes and greater piston speed make from 50 to 60 revolutions per minute.

1390. Steam Engine.—Compressors operated by a steam engine as the motive power are usually direct connected with engines about as follows: Machines of 10 tons capacity or under, plain slide-valve engine; 10 to 25 tons inclusive, adjustable cut-off engine of the Meyer type; 30 tons and over, Corliss engine. A number of the very large machines with capacities ranging from 200 to 500 tons have in late years been provided with compound condensing engines. From this it will be seen that the relative efficiency of a small compression machine is much lower than that of the larger sizes. Owing to the low piston speed of the engine and consequent great amount of cylinder condensation, the steam consumption of the various classes is about as follows: Slide-valve engines, 50 pounds of steam per horsepower, 90 pounds of steam per hour per ton of refrigerating effect; Meyer cut-off, 40 pounds of steam, 65 pounds of steam per hour per ton of refrigerating effect; Corliss engine, 30 pounds of steam per horsepower, 50 pounds of steam per hour per ton of refrigerating effect.

The above are based upon a condensing pressure, or head pressure, of 150 pounds per square inch and a suction or back pressure of 15 pounds per square inch, both gauge pressures.

1391. Gas Engine. — Small compressors are often driven by gas or gasoline engines. Owing to the relatively high speed of the gas engine, it is necessary to belt such engines to the compressors. In case of gasoline engines, the economy is much better than that of a slide-valve engine, the cost of operating amounting to about 1 cent per horsepower per hour, or 2 cents per hour per ton of refrigerating effect. This is equivalent to coal at $2.50 per ton, not including attendance. For each ton of refrigerating effect, 2 to 2½ gas-engine horsepower should be allowed.

1392. Electric Motor. — Compressors driven by means of an electric motor are usually geared, rawhide pinions being used to decrease the noise. The motor should be of ample capacity to drive the compressor, so as to prevent the armature from burning out. 1½ to 2 kilowatts per ton of refrigerating effect is the usual practice.

1393. Water-Power.—Under high heads, the Pelton water-wheel is found to be the most efficient and best adapted water-wheel for driving ammonia compression machines. It is usually direct connected to the shaft, and is made with a large diameter so as to get a high circumferential velocity with 60 or 70 revolutions per minute. Under heads of less than 50 feet, a turbine is generally used. In most cases, it must be connected to the compressor by a belt or gearing which will reduce the speed to the proper number of revolutions.

1394. Position of Cranks.—The cranks of the compressor and direct-connected steam engine are set at 90° to each other. With this arrangement, the steam engine encounters its greatest resistance when the piston is at the middle of its stroke. If the cranks connected at 180°, or directly in line, the engine would encounter the greatest resistance when the piston is at the end of its stroke, and consequently has a low steam pressure to urge it forwards. Such a construction, though much cheaper, brings a much greater strain upon the cranks, shaft, and fly-wheels, and makes these latter parts do the work of compression with the energy stored in them. The fly-wheels should be well balanced and heavy. They should be carried on a shaft of ample size, and if possible out-board bearings should be provided to prevent deflection.

THE AMMONIA CONDENSER.

1395. Atmospheric Condensers.—The atmospheric condenser consists of a coil made up of straight pipes and return bends. The hot gas enters the coil at the top. Water is permitted to trickle over the coil in a thin film and condenses the gas within it, which after liquefying runs to the bottom of the coil and enters a receiver placed at that point. It will be seen that the hottest gas and the coldest water come in contact with the same part of the coil. This has a tendency to increase the head or condensing pressure, but it is necessary to let the gas in at the top in order that the liquefied gas will run to the bottom of the coil by gravity.

The De La Vergne condenser shown in Fig. 329 is constructed with a series of return bends *B* having pockets *P*.

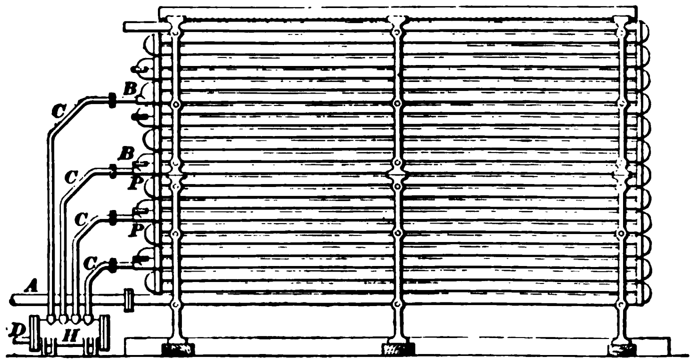


FIG. 329.

These pockets are connected to a header *H* at the bottom of the condenser by means of pipes *C*, *C*. The header in turn is connected by a pipe *D* with the receiver. By this arrangement, the hot gas is enabled to enter the bottom of the coil at *A* and gradually work up towards the top. The liquid anhydrous ammonia is trapped out by the pockets and does not run back to mix with the incoming hot gas. Atmospheric condensers are usually made of 1¼, 1½, or 2 inch pipe, the latter size being used in the De La Vergne condenser. The coils are about 20 feet long and 18 to 24 pipes high; 22 to 30 square feet of surface per ton is the usual allowance, depending upon the temperature of the condensing water, but 40 square feet will give more satisfactory results. The principal objection to atmospheric condensers is that they are least efficient when most needed. In hot weather, when the humidity is great, their action is very sluggish and the efficiency of the machine is diminished to a considerable degree. They are, however, low in first cost, easy to keep clean, and easily repaired.

Atmospheric condensers require from 3 to 4 gallons of water per minute per ton of refrigerating effect in hot summer weather, with water at 75° to 80°.

1396. Submerged Condensers. — The submerged condenser consists of a coil or coils of pipe submerged in a tank of water. The incoming gas enters at the top of the coil, and the outlet for the condensed liquor is at the bottom of the coil, the number of coils in the tank depending upon the size of pipe used and the capacity of machine. 1, 1¼, and 1½ inch pipe is used in these condensers, and 30 square feet of surface per ton is allowed in usual practice. The tanks should be deep rather than wide, so that the circulating water, which enters at the bottom of the tank and overflows at the top, will have an opportunity to reach a high temperature before leaving the tank. Condenser coils of 1¼-inch pipe should not exceed 350 feet in length. The inlets and outlets to all coils should be provided with valves or cocks, so that any coil may be turned off in case of leakage, without necessitating the shutting down of the plant. The outlets of the various coils are connected to a receiver, usually made of wrought iron with heads welded in. This receiver should have a capacity of about one-half gallon per ton of refrigerating effect, and should be provided with a pair of gauge-cocks and an air-valve for drawing off air or other impure gases. The water economy of a submerged condenser varies directly with the temperature of the water. With condensing water at 60°, 1½ gallons per ton is good practice, but 2 gallons should be used when the temperature is as high as 75°.

1397. The Surface Condenser. — The Hendrick surface condenser consists of a heavy cast-iron shell provided with water coils of extra heavy pipe. The action of this condenser is exactly the reverse of that of the submerged condenser. The water circulates through the coils; the gas enters the shell at the top, condenses, and falls to the bottom. The lower portion of the condenser forms the receiver of the anhydrous liquid and is provided with gauge-cocks and necessary fittings. On account of the large gas space and rapid water circulation, this condenser is very efficient; it gives a very low condensing pressure for any given water temperature and is also economical in the use of water.

GAUGES, VALVES, AND OTHER DETAILS.

1398. Pressure Gauges.—The pressure gauges used for ammonia are in construction similar to steam-pressure gauges except that the tube is made of steel for the reason that brass will not withstand the action of ammonia. The gauges register pressure only, or pressure and vacuum both, as may be desired. Cheap diaphragm gauges are made, but they are not accurate, and are seldom used by reputable builders.

1399. Liquid Level Gauges.—The anhydrous receiver and the oil trap should each be provided with a pair of liquid level gauges. These are of heavy cast iron or steel, and should be so arranged that they can be quickly closed in case of the glass breaking. Fig. 330 shows a section of the Hiller automatic safety gauge-cock, which is designed to shut off automatically in case of the glass breaking. It is also arranged so that the valve can be ground without taking the pressure off the vessel. The valve *A* is opened by a fork *D* attached to the valve-stem *B*. The stem *C* is used for grinding the valve.

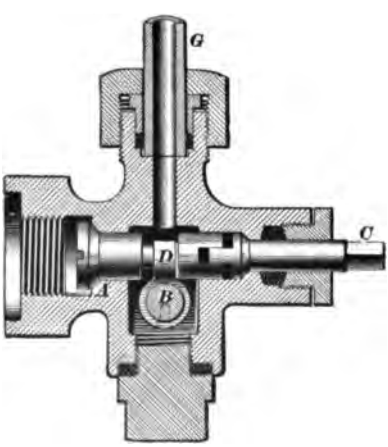


FIG. 330.

1400. Pipe.—All pipe used in ammonia work should be extra heavy, even for the low-pressure lines. Such pipe should be strictly of wrought iron, steel pipe being unsuitable for this purpose. In case any bent-pipe coils are used, they should be made of lap-welded, redrawn, or tuyère pipe.

1401. Fittings and Valves.—Fittings used on ammonia work are made of cast iron, malleable iron, or steel. They are protected as follows: (1) By means of a jam or lock-nut bearing against a rubber gasket which sets in a recess turned in the fitting; in case of a leak, a slight turn of the jam-nut forces the gasket well into the counterbore and thread and stops the leak. (2) By means of a gland fitting similar to the lock-nut, save that a gland is held against the gasket by means of bolts, which are fast to the flange of the fitting. (3) By means of either malleable iron or semisteel fittings, in which there is a slight recess or counterbore. After the pipe is made up, solder is run into the counterbore with a blowpipe, and a perfectly tight joint is made. Such joints are very satisfactory, except in lines that are subjected to both excessive heat and cold, in which case the solder is liable to crack.

Unions used in this work are flange unions with male and female joints, with lead packing in the groove.

Valves are of extra heavy pattern, all iron, and of either gate, globe, or angle type. The seats of such valves are usually soft metal, which can readily be repacked. Hard-metal valves with steel seats are not effective, as the liquid anhydrous ammonia appears to cut the steel.

Cocks used in this work are of the Fairbanks asbestos-packed pattern, the plug being packed with asbestos packing under a heavy hydraulic pressure. Cocks should be returned to the factory for repairs.

1402. Oil Trap and Strainer.—Between the compressor and condenser an oil trap is placed for the purpose of eliminating any oil that might pass over from the compressor cylinder with the ammonia gas. As shown in Fig. 331, it consists of a cylinder having an inlet *D* for the gas on the side, about half way up from the bottom, and an outlet *B* at the top. The lower portion of the cylinder acts as an oil receiver, and is provided with a pair of gauge-cocks and a glass *C* for determining the quantity of oil in the receiver. Some builders place the inlet opening at the top, with a pipe on the inside, leading the gas well down into the receiver; but such an arrangement is not as effective for eliminating the oil as the one above described, where the incoming gas impinges on a flat surface. The strainer consists of a large pair of flanges, with a piece of very fine perforated iron between them. The aggregate area of the perforations is greater than that of the pipe. The strainer is placed in the suction line and is intended to prevent any scale, dirt, or grit from entering the compressor cylinder. It should be cleaned at least once a season.

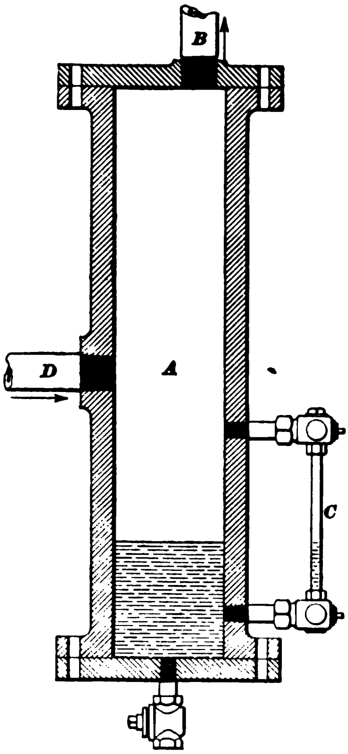


FIG. 331.

1403. Drier.—The best equipped compression machines are provided with **driers.** A drier consists of a vertical pipe 10 or 12 inches in diameter and 4 to 6 feet long, provided with removable heads. The shell is partly filled with quicklime or caustic soda and placed in the suction line of the compressor. The ammonia gas passing to the compressor must traverse the drier and there give up any moisture that it may contain.

1404. By-Pass. — It is often necessary, in making repairs, to empty the condenser into the expansion coil. For this purpose, the compressor is provided with a by-pass, which permits the ammonia to be taken from the condenser and delivered to the expansion coils, where it is stored until after the repairs are completed. A by-pass for a single-acting machine is shown in Fig. 332. When the condenser is to be pumped out, valves *A* and *B* are closed and valves *X* and *Y* are opened. To pump out the pipe *A C*, valves *A*, *X*, and *Y* are closed and valve *B* is opened.

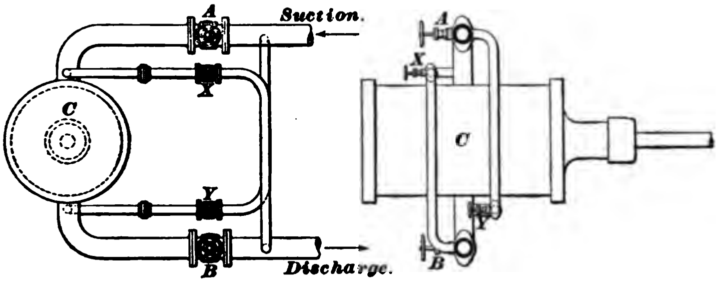


FIG. 332.

The by-pass on the Frick duplex compressor is shown in Fig. 333. The angle-valves *D* and *D'* are so arranged that when one of them, say *D'*, is open and the valve *C'* is shut, compressed gas will pass from the cylinder *B* through the discharge-pipe; when, however, *D'* is closed and *C'* is open, the gas will flow from *B* through the pipe *M*. The valves *S* and *S'* act in a similar manner. The valves *E* and *E'* are so arranged that when *E* is closed, the gas will flow through the pipe *M* from *C'* to the valve *G*, but when *E* is open, the gas will flow through the pipe *K* to the suction-pipe without passing to the valve *G*.

To pump out the compressor *B*, the valves *C'*, *G*, and *D* are opened and the valves *S*, *S'*, *D'*, *C*, *E*, *E'*, and *G'* are closed. The closing of the valve *S* shuts off the suction-pipe from the cylinder *A*, and the gas is pumped from cylinder *B* instead. It passes through the open valve *C'*, the pipe *M*, and the open valve *G*, and is expelled after compression into the discharge through the open valve *D*.

To pump out the condenser, the valves *D*, *C*, *G'*, *C'*, and *E* are opened and all others are closed. The valve *S'* being closed, the gas can not be drawn from the suction-pipe. As the piston in *B* makes its compression stroke, the gas will flow from the condenser through the discharge, the open valve *C*, the pipe *N*, the open valve *G'*, and thence into the cylinder *B*. The valve *D'* being closed, the compressed gas will flow through *M*, through the open valve *E*, and through

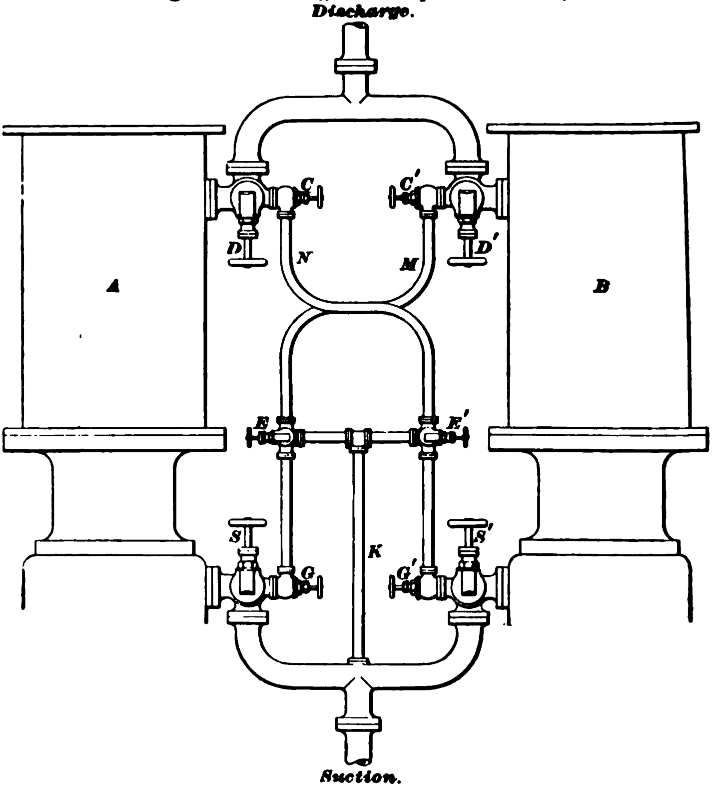


FIG. 333.

the pipe *K* into the suction line. With the valves thus arranged, pump *B* takes gas from the condenser and discharges it into the suction line. The pump *A* does no work whatever except to churn the gas back and forth.

CONSIDERATIONS AFFECTING THE ECONOMY OF THE COMPRESSION SYSTEM.

1405. Friction.—The total work delivered to the refrigerating-machine, including the steam engine or other motor that drives it, is the work done by the steam in the engine cylinder. The net or useful work is that done on the gas in the compressor cylinder. The difference between the two is the work done against the frictional resistances of the engine and compressor. To determine the friction, therefore, it is only necessary to determine by means of an indicator the indicated horsepower of the steam cylinder and that of the compressor cylinder; the difference will be the power absorbed in overcoming friction.

The ratio of the friction horsepower to the horsepower of the steam cylinder depends in some degree upon the type of machine. In this respect, the direct-acting has an advantage over the indirect machine; based on the horsepower of the steam cylinder, the friction of the former type varies from 12 to 20 per cent., while that of the latter type is from 18 to 30 per cent., the increase being due to the increased number of bearings and wearing surfaces. In general, the friction of a vertical machine is less than that of a horizontal machine.

1406. Clearance.—This very important factor in the economy of the compression system often cuts down the capacity and economy of the pump. In a first-class compressor, the clearance has been known to be as low as $\frac{3}{10}$ of 1 per cent. In actual practice, however, it probably amounts to about 1 per cent. The method of ascertaining the clearance space between the cylinder-head and piston has already been explained in Art. **1386.** That in the recess of the suction and discharge valves may be calculated as follows:

Rule.—*Multiply the area of the valve in inches by the distance between the face of the valve and the head when the valve is seated. Divide this product by the volume of the cylinder, and the result is the per cent. of clearance for the valve.* Applying this rule to each of the valves, and adding the results to the clearance space between the head and piston, the sum will be the total clearance for the compressor.

1407. Cylinder Superheating.—As stated in Art. **1380,** the greater the difference of temperature between the inlet and outlet gas, the greater the amount of cylinder superheating; therefore the warmer the condensing water and colder the brine, the more the vapor is superheated. For single-acting machines the loss due to cylinder superheating amounts to about 22 per cent. and for double-acting machines about 30 per cent. when the machine is running with a head pressure of 150 pounds and a back pressure of 15 pounds per square inch. If the head pressure is increased or the back pressure decreased, the loss will be greater.

1408. Compound Compression.—In order to overcome the loss due to cylinder superheating, compound ammonia compressors have been built. The gas is first compressed in a large cylinder and is then discharged into a smaller one, where it is compressed to the final pressure. There are, however, no authentic tests on such compressors, and it is an open question whether the increased friction does not more than make up for the gain due to the lessened superheating.

1409. Effect of Pressures.—Neglecting the loss due to superheating, it was shown in Arts. **1375** and **1376** that for maximum theoretical economy and capacity, the suction pressure should be as high as possible and the condensing pressure as low as possible.

It is evident also that these are the conditions that reduce the loss due to superheating. It follows, therefore, that the back pressure should be high, not only in order to increase the capacity, but also to reduce the superheating.

THE AMMONIA ABSORPTION SYSTEM.

GENERAL DESCRIPTION.

1410. The action of a refrigerating system of the absorption type is based upon the affinity of a vapor, usually ammonia vapor, for water. As stated in Arts. **1361** and **1363,** water will absorb 1,140 times its volume of ammonia gas, and to liberate one pound of ammonia gas thus absorbed requires the expenditure of 925.7 B.T.U. Suppose we have a strong solution of aqua ammonia; if heat be applied to the solution, the ammonia gas or vapor will be driven off at relatively high pressure, and when passed through a condensing coil, will condense to the liquid state. The liquid can now, just as in the compression system, be admitted to a refrigerating coil through an expansion-valve. In this coil it will vaporize and withdraw heat from the surrounding objects. The vapor may now be again absorbed by water, thus regaining its original state and closing the cycle of operations.

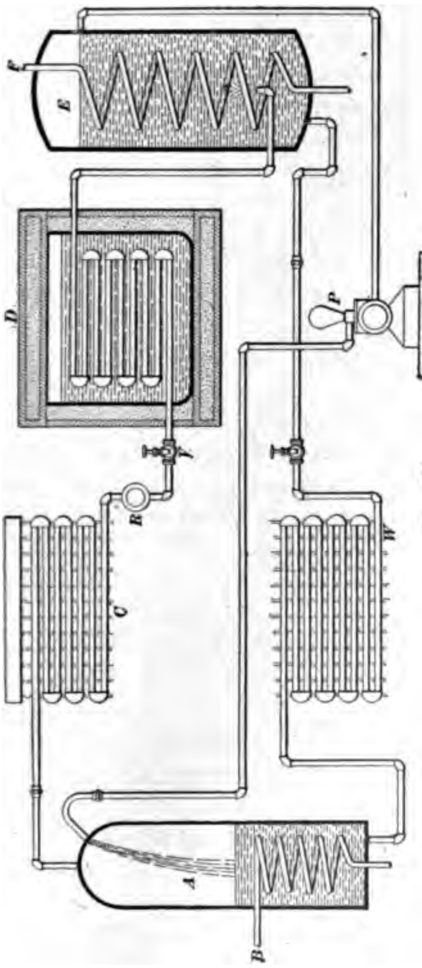


FIG. 334.

1411. The essential features of the absorption system are shown in Fig. 334. Steam is admitted at a pressure of about 40 pounds per square inch, gauge, to a coil *B*, submerged in a strong solution of aqua ammonia contained in the vessel *A*. The temperature of the solution will be raised nearly to that of the incoming steam, say to about 270° F., and the heat absorbed will cause the ammonia gas to be driven off

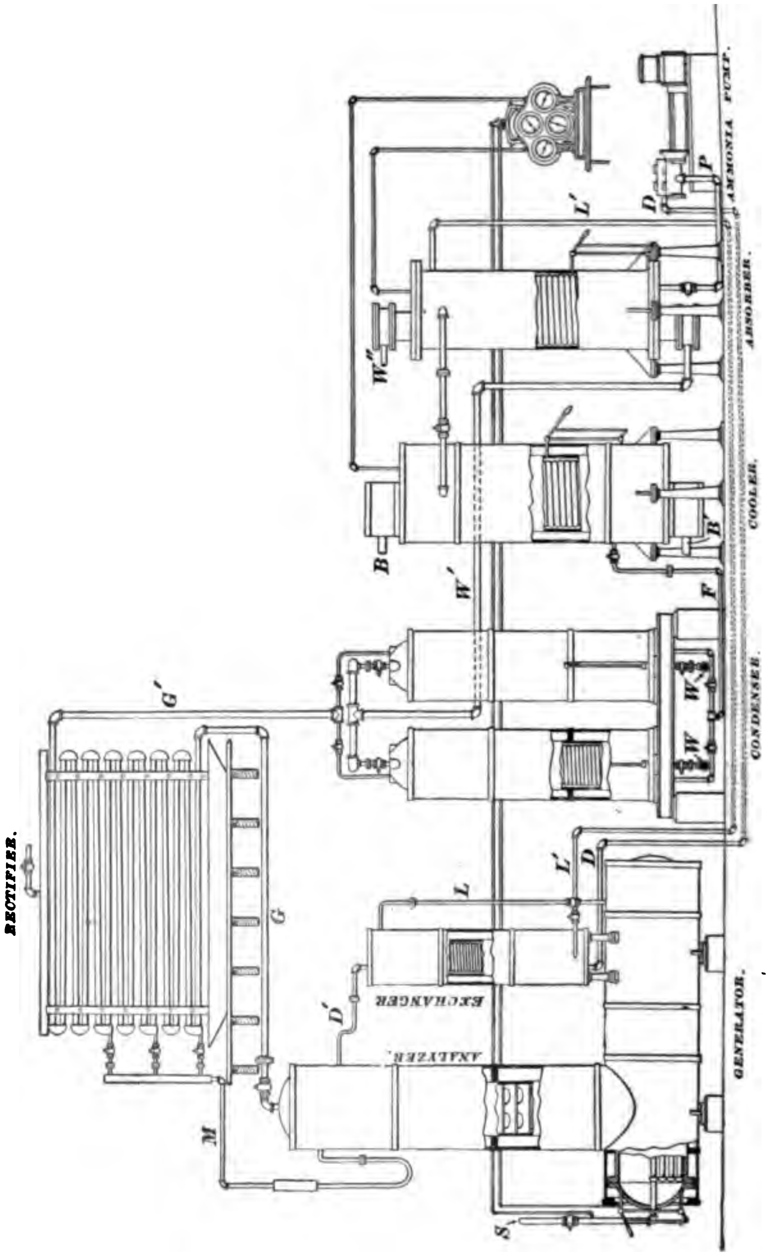

RECTIFIER.
ANALYZER.
EXCHANGER
GENERATOR.
CONDENSER.
COOLER.
ABSORBER.
AMMONIA PUMP.
G'
G
M
D'
L
L'
D
S
W'
B
W''
F
B'
W
W
L'
D
P

Fig. 335.

at a pressure of say 160 pounds per square inch. The pressure corresponding to any given temperature depends upon the strength of the solution; the stronger the solution the higher the temperature, and *vice versa*. As the temperature of the solution is below the boiling point of water for this pressure, no water will evaporate, and only ammonia gas will pass over into the condenser *C*. The construction of the condenser is similar to that of the compression system. The cold water flowing over the condensing coils absorbs heat from the gas, and the combined effect of the high pressure and the cooling action of the water is to liquefy the gas. It is to be noted that the pressure is not produced mechanically, as in the compression machine, but by chemical action.

As the ammonia liquid passes through the expansion-valve to the expansion coils, the pressure is reduced, reevaporation begins in the expansion coils, and heat is absorbed from the brine or other substance in the chamber *D*.

In the compression system, the ammonia pump draws the gas from the expansion coils, but in the absorption system, the removal of the gas is effected by allowing the gas from the expansion coils to mingle with the weak solution of ammonia from which the gas was expelled in the **still** or **generator** *A*.

During the process of generating the ammonia gas in the still *A*, the strong solution rises to the top on account of its smaller specific gravity, and the weaker solution settles to the bottom and flows through a pipe to the vessel *E* called the **absorber.** Here it meets the gas as it comes from the expansion coil and absorbs it. Since a low temperature is required for efficient absorption, the weak liquor on its way to the absorber passes through a coil *W*, which is cooled by running water. The absorber *E* is also provided with a water coil *F*. A small pump *P* takes the strong liquid from near the top of the absorber and forces it back into the generator. This completes the cycle of operations.

1412. Fig. 335 shows a sectional elevation of a Pontifex-Hendrick absorption machine. The various parts and their functions are as follows: The **generator, still,** or **retort,** is the cylinder containing the charge of aqua ammonia; it is supplied with a steam coil to drive the ammonia gas from the solution. The steam inlet is shown at *S*. The ascending gas passes through the trays of the **analyzer,** which act as baffle-plates, and cool the gas by means of the incoming strong liquor which they contain. From the analyzer the gas passes through the pipe *G* to the **rectifier;** this is a coil of the atmospheric type, supplied with pockets on some of the return bends to catch any moisture that may pass from the generator and be mixed with the ammonia gas. The gas on leaving the rectifier is strictly anhydrous, all water vapor having been removed. The water returns to the analyzer through the drip *M*. The gas then enters the **condenser,** where it is liquefied, then passes to the **cooler,** where, after abstracting the heat from the brine, it is revaporized and passes off to the **absorber.** The weak liquor in the generator is forced by the high pressure in the generator into the shell of the **exchanger,** whence it passes to the absorber. After absorbing the gas and being cooled by the water coils in the absorber, the liquor is forced by means of the ammonia pump through the coils of the exchanger and into the analyzer. The hot weak liquor entering the shell and the cool strong liquor entering the coil of the **exchanger,** transfer or exchange their heat, so that the weak liquor becomes cooled and the strong liquor becomes heated. The use of the exchanger effects a considerable saving of both water and coal. The strong liquor entering the analyzer gradually works down from tray to tray, and in its descent is heated by the ascending gas.

The various parts indicated by the reference letters are as follows:

B, brine inlet to cooler.
B', brine outlet from cooler.
S, steam inlet.
W, *W*, water inlet to condenser.
W', pipe conveying water from condenser to absorber.

W'', waste-water outlet from absorber.

L, pipe conveying weak liquor from generator to exchanger.

L', pipe conveying weak liquor from exchanger to absorber.

F, pipe conveying gas from condenser to cooler.

G, pipe conveying gas from analyzer to rectifier.

G', pipe conveying gas from rectifier to condenser.

D, pipe conveying strong liquor from pump to exchanger.

D', pipe conveying strong liquor from exchanger to analyzer.

P, pipe conveying strong liquor from absorber to pump.

DETAILS OF THE ABSORPTION SYSTEM.

1413. The **generator,** sometimes called the still or retort, is the vessel in which the aqua ammonia is placed and from which the ammonia gas is evaporated. There are two classes of generators, viz., direct fired and those indirectly heated by means of a steam coil. The former class is now seldom seen. Generators provided with steam coils are subdivided into two classes, viz., vertical and horizontal. Vertical stills, used largely on absorption machines in the South, are usually made of boiler plate with riveted seams. The internal coils are of the helical type, nested one inside of the other. The steam inlets pass through the side of the shell, and the outlet for the condensed steam through the bottom head. These tails are protected by either lock-nuts or glands. The same objection applies to the vertical still as to the vertical boiler; it does not make dry gas. The evaporating surface being comparatively small, depending upon the diameter of the shell, the boiling is very rapid, and it is difficult to keep the steam coils always submerged. If these coils are exposed to the action of the gas and liquor when uncovered, they will become pitted and will have to be renewed in the course of two or three years. Vertical stills are also liable to boil over when forced, the ebullition being so strong at such a time as to cause the complete charge to pass over into the condenser. Another objection is the excessive height necessary to accommodate stills of this class.

With a view to the elimination of these difficulties, the horizontal still was constructed. The ebullition in this case is very quiet, no action being noticeable in a gauge-glass. The liberating surface of the liquid is large, and there is no difficulty in keeping the coils covered with liquor. Coils in use in this type of machine have lasted for fourteen years without any appreciable pitting. The shells of horizontal generators are usually made of cast iron. The first cost of a horizontal generator is greater than that of the vertical still.

1414. Analyzer.—In the case of the vertical still, the upper portion of the shell is provided with a series of wrought-iron plates so arranged that the returning strong liquor will pass over these plates and come in contact with the ascending gas. In the case of the horizontal still, the analyzer is attached as shown in Fig. 335. It is provided with cast-iron pans, one set above the other in such a manner that though the descending strong liquor comes in contact with the gas, the latter does not pass through the liquor as in the former case; the gas is thereby left much drier. Cast-iron plates are also more durable than the wrought-iron ones.

1415. Rectifier.—The rectifier is a coil or series of coils arranged like those in the condenser. Its office is to precipitate on the sides of the pipes any moisture that may have passed over with the gas. This moisture is then collected in a trap, like water in an ordinary steam separator, and the liquid is then allowed to pass back into the analyzer by gravity or enter the receiver of the absorber. The rectifier is a very necessary part of the absorption machine, as upon it the efficiency of this type of machine largely depends.

1416. Exchanger and Weak-Liquor Cooler.—The exchanger is a cast or wrought iron shell provided with coils. The inlet for the hot, weak liquor from the still enters the shell near the top, and the outlet is near the bottom. The ammonia pump delivers the strong liquor into the bottom of the coils. After being heated, the strong liquor passes out of the top of the coils into the analyzer. The inlets and outlets of these coils are connected by means of a header. Some makers reverse this action and allow the weak liquor to pass through the coils and the strong liquor through the shell. One method has no particular advantage over the other. The tails of the coils are provided with either lock-nuts or glands, to prevent ammonia leakage. Well-proportioned machines should have at least 6 square feet of exchanging surface per ton of refrigerating effect. Machines employing vertical stills seldom have over 2 square feet per ton, as better exchanging increases the liability of boiling over. In such machines, it is necessary to provide other means for cooling the weak liquor before it enters the absorber, as the surface is not sufficiently large in the absorber to cool the weak liquor below 130° or 140° F. For this purpose, a *weak-liquor cooler* is provided. It consists of either an atmospheric or submerged coil of pipe through which the weak ammonia liquor passes on its way from the exchanger to the absorber. The surface depends largely upon how much it is desired to cool the liquor, and may be from 3 to 10 square feet per ton.

1417. Condenser.—Condensers used for this system are similar to those already described in case of the compression machine in Arts. **1395** to **1397**.

1418. Absorber.—There are four classes of absorbers, viz., the *submerged* or *tank* absorber, the *full* absorber, *empty* absorber, and *atmospheric* absorber. The first consists of a series of coils arranged in a water-tank. The gas enters the top of the coils, and the weak liquor sprayed in at the top gradually works down through the coil, absorbs the gas in its descent, and enters the receiver at the bottom, into which all the coils are connected. The water in the tank enters at the bottom, passes off at the top, and carries away the heat of absorption. Such absorbers should contain from 30 to 40 square feet of surface per ton of refrigerating effect. The *full* absorber consists of a cylindrical shell provided with water coils nested one within the other, or else with straight tubes similar to the tubes of a boiler. The latter is preferable on account of ease of cleaning, but the liability of leakage is greater, owing to the number of joints. The gas and weak liquor enter the shell near the bottom, the gas passing up through the liquor contained in the shell and being absorbed by it. The liquor overflows near the top of the shell and passes to the pump suction, the water entering the top of the coils or tubes and out at the bottom. The objection to this type is that it is necessary for the gas entering the absorber to have sufficient pressure to pass through the liquor. This takes away from the absorber the pressure equal to the head due to the height of liquor above the inlet gas. For this reason, the *empty* absorber is much more efficient, particularly in the case of low temperatures. The construction of this absorber is similar to that of the full absorber, except that the weak liquor is sprayed in over the coils at the top of the shell and the gas enters below this spray. The bottom of the absorber takes the place of the receiver. The *atmospheric* absorber is not a commercial success, and is only provided by one or two builders.

1419. Ammonia Pump.—The aqua-ammonia pump consists of a small well-proportioned steam or power pump, amply strong to run against a pressure at 350 to 400 pounds per square inch. This is not the regular service of the pump, but such pressure is used for the purpose of testing the various parts of an absorption machine and the piping for the purpose of detecting leaks before the machine is started. Direct-acting steam-pumps are built either with or without fly-wheels. Duplex ammonia pumps are not satisfactory. Ammonia pumps should have a capacity of 150 cubic inches per minute per ton of refrigerating effect.

1420. Castings and Coils.—All castings used for ammonia work should be made in either loam or dry sand and should be tested to a hydraulic pressure of 500 pounds per square inch. The iron used should be of fine grain and free from imperfections, sand, or blow-holes. All coils used for this work should be made of extra heavy lap-welded redrawn tuyère pipe. They should be continuously welded and tested to a hydraulic pressure of 800 pounds per square inch, or an air-pressure of 300 pounds per square inch under water. The coils should be well hammered while being tested, so as to dislodge all scale, dirt, etc.

CONSIDERATIONS AFFECTING THE ECONOMY OF THE ABSORPTION SYSTEM.

1421. Generation of Anhydrous Ammonia.—Few of the absorption machines now built are able to generate pure anhydrous ammonia gas without some trace of moisture. In order to get economical results, the absorption machine must make anhydrous ammonia. Any adulteration, even if as low as 5 per cent., decreases the economy of the machine considerably. Machines having vertical stills are more subject to this trouble than those with horizontal stills. The analyzer of a horizontal still condenses out some of the water on the under side of the analyzer trays, the gas leaving with probably not over 5 per cent. entrained moisture. To extract this remaining 5 per cent., the rectifier is used. The most efficient form of rectifier is that containing a number of drip pockets for the purpose of trapping out the moisture by a differential process; the gas leaves the last pocket in a practically dry, anhydrous state.

The method of testing samples of anhydrous ammonia for impurities has already been explained in Art. **1364.** If it is not convenient to draw out a sample of anhydrous ammonia, its condition may be approximately determined by noting the temperature of the brine, or, better yet, that of the ammonia in the expansion coils, if this is possible, and the back pressure. After determining these two readings, turn to Table 26, giving the properties of saturated ammonia gas. The temperature of the ammonia due to the back pressure as given by the table should be nearly identical with the temperature in the expansion coils. If the latter temperature is any higher than that given in the table for the corresponding pressure, one may be assured that the variation is due to water in the anhydrous ammonia. The ice-machines of the South are much troubled in this particular, as few of them carry a back pressure above that of the atmosphere, even with high brine temperatures in the freezing tank. This is due to the fact that the machines do not make anhydrous ammonia. The makers and operators of such machines think that it is necessary to carry a low back pressure; this is a mistake, and the trouble should be looked for in the still, analyzer, or rectifier of the machine; when these are constructed correctly, the back pressure will rise of its own accord.

1422. Efficiency of Absorber. — Naturally the higher the back pressure the stronger will be the strong liquor leaving the absorber; and the stronger the liquor the less quantity will have to be pumped per ton of work done. From this it will be seen that a high back pressure is conducive to economy, in that it tends to secure a strong solution in the absorber and thus diminishes the quantity of liquor to be pumped. The absorber that will give the strongest solution for any given back pressure and the coolest pump delivery for the same amount of water is the most efficient. Absorbers run on the empty principle are more efficient in this respect than either the submerged or full absorber. An absorber should have sufficient surface and so distributed that it will be economical in the quantity of cooling water used. It should be designed with a view to using the water after it has passed through the condenser and has done its work in condensing the ammonia gas. Water should never be run through the absorber first and then through the condenser, as this invariably increases the head pressure.

1423. Efficiency of Exchanger.—The exchanger should be of ample size and have large surfaces, so that it is unnecessary to add on a weak-liquor cooler. A large exchanger naturally gives cool weak liquor to the absorber and warm strong liquor to the analyzer. This economizes both the steam and water consumption of the machine.

1424. Comparison of Absorption and Compression Systems.—Recent tests made by Professor J. E. Denton on a 50-ton Pontifex-Hendrick machine have demonstrated the following facts: That with excessively low brine temperatures, such as 20° below zero F., the capacity of the machine was only diminished by about 10 per cent. The economy of the machine at these low temperatures was better than that of a compression machine driven by a compound Corliss engine that would not use over 15 pounds of steam per horsepower per hour. Its economy with high brine temperatures was better than that of a compression machine driven by an ordinary Corliss engine. The water consumption of this machine with low brine temperatures was from two to three gallons per ton per minute, but the temperature of the water in this case was 70° F. This is probably considerably more than would be used by a compression machine.

The relative economy of the two systems is still an open question. Under certain conditions, it is likely that the absorption machines give better results than compression machines; for example, a small absorption machine will probably be more economical than a compression machine driven by an ordinary slide-valve engine. On the other hand, tests of large compression machines under favorable conditions show an economy higher than that of the ordinary absorption system of the same capacity.

CARBON-DIOXIDE REFRIGERATING-MACHINES.

1425. Compression machines using carbonic acid as a refrigerating fluid are coming into quite extensive use, especially in Germany, where liquid carbonic acid is manufactured very cheaply as a by-product of the brewing industry.

The characteristic feature of the carbon-dioxide system is the high pressure required for the condensation of the gas. Referring to Table 25, Art. **1355,** it is seen that at a temperature of 68° F., the pressure is 864 pounds per square inch, absolute. The back pressure is correspondingly high; for a temperature of 5° F. in the expansion coil, the pressure is 342 pounds, absolute.

1426. Reference to Table 25 shows also that at ordinary refrigerating temperatures, the density of carbon dioxide is great compared with ammonia at the same temperature. Thus, at 77° F., a cubic foot of carbon dioxide (vapor) weighs 15.475 pounds, while a cubic foot of ammonia at that pressure weighs only .5 pound. Though the latent heat of carbon dioxide—and in consequence the refrigerating effect per pound—is less than that of ammonia, the greater density permits the use of a compression cylinder of much smaller dimensions than the cylinder of the ammonia compressor. This may be illustrated by a numerical example. In the problem, Art. **1374,** the theoretical volume of the compressor cylinder for a capacity of 20 tons was found to be a little less than ½ cubic foot. Suppose we have a compressor using carbon dioxide, the condenser temperature being 68° and the temperature in the refrigerating coil 14° F. The heat abstracted *per pound* of vapor is given by equation (*c*), Art. **1370:**

$$Q_2 = r - s\,(t_c - t_a).$$

The specific heat of liquid carbon dioxide being 1, we have

$$Q_2 = r - (t_c - t_a).$$

At 14° F., the latent heat is 115.7 B. T. U. (see Table 25); hence

$$Q_2 = 115.7 - (68 - 14) = 61.7 \text{ B. T. U.}$$

For a capacity of 20 tons, the heat abstracted per minute must be

$$\frac{20 \times 285{,}300}{24 \times 60} = 3{,}962.5 \text{ B. T. U.}$$

At 112 strokes per minute, the weight of carbonic acid circulated per stroke must be

$$\frac{3,962.5}{61.7 \times 112} = .57 \text{ lb.}$$

At a temperature of 14° F., a cubic foot of carbon dioxide weighs 4.535 lb.; hence the cylinder volume required is $.57 \div 4.535 = \frac{1}{8}$ cubic foot, about $\frac{1}{4}$ of the volume required for ammonia.

1427. The small volume of the compressor is an undoubted advantage of the carbon-dioxide machine. Other advantages claimed are the following:

Carbon dioxide, or carbonic acid, is cheaper than ammonia, is non-corrosive, non-explosive, and is not dangerous to life when diluted with air. Another advantage of carbon dioxide is that it has no harmful effect on beer, meat, water, or any articles in cold storage.

The objections to the carbon-dioxide machine are as follows: The high pressures employed render the construction and operation of the machine difficult. It is also difficult to construct the piping and coils so that the joints will withstand the pressure. Gauge-glasses will not stand the pressures. Owing to the small size of the compressor, the prejudicial effect of the clearance is more marked in the carbon-dioxide machine. Carbon dioxide being colorless and odorless, it is difficult to detect leaks. The usual test for leaks is lime-water, which becomes milky in the presence of carbon dioxide.

1428. In the expression for the heat abstracted,

$$Q_2 = r - s\,(t_c - t_a),$$

the subtractive part, $s\,(t_c - t_a)$, is due to the incompleteness of the cycle (see Art. **1369**). The latent heat of ammonia being relatively great, 500 B. T. U. and more per pound at ordinary temperatures, the loss $s\,(t_c - t_a)$ due to fall of the temperature of the liquid is small in comparison. In the case of carbon dioxide, however, the latent heat is much less, and this loss is relatively much greater; thus, in the example, Art. **1426,** the loss, 68 — 14 = 54, is nearly one-half of the latent heat, 115.7.

To decrease the loss of efficiency due to the incompleteness of the cycle, it has been proposed to introduce a motor between the condenser and refrigerator, upon which the expanding liquid will do work, thus rendering the cycle complete.

With an expansion-valve, the efficiency of a carbon-dioxide machine is about two-thirds that of an ammonia compression machine.

SULPHUR-DIOXIDE REFRIGERATING-MACHINES.

1429. Refrigerating-machines using sulphur dioxide as a refrigerating fluid are in use to some extent. However, a reference to Table 24, giving the properties of saturated sulphur dioxide, shows that this gas is not especially adapted for a refrigerating fluid. Its latent heat per pound is less than $\frac{1}{3}$ of the latent heat of ammonia, while its density is but little greater than that of ammonia. Hence, for the same ice-making capacity, the volume of the compressor cylinder must be about three times the volume of an ammonia compression cylinder. This naturally increases the first cost of the plant, and even allowing that such a machine is as economical as an ammonia machine, its first cost is sufficient to debar its sale. Another serious objection to the sulphur-dioxide machine is the fact that the utmost care must be taken that no gas shall escape from the condenser and other coils, as this gas combining with water forms sulphurous acid, which rapidly attacks iron. The machines first built had copper condensing coils, but the first cost of these was so great that they were soon abandoned. The utmost caution must be exercised in keeping the piston rod perfectly dry, to prevent corrosion. Sulphur dioxide is poisonous when inhaled.

For low refrigerating temperatures, the pressure in the refrigerating coils is less than atmospheric pressure (see

Table 24). As a result, air is likely to leak into the coils, and this in combination with sulphur dioxide will form sulphur trioxide, which in turn will in contact with water form sulphuric acid.

At very low refrigerating temperatures, the efficiency of the sulphur-dioxide machine is low; at higher temperatures, the efficiency compares fairly well with that of the ammonia compression machine.

VACUUM REFRIGERATING-MACHINES.

1430. The vacuum machine employs water vapor as the refrigerating fluid. From the fact that water vapor in order to have a low temperature must have a very low tension arises the name "vacuum" machine.

The operation of the vacuum machine is precisely similar to that of an ammonia compression machine. The vacuum is formed by a pump, which withdraws the vapor from the refrigerator, where the pressure is about .1 pound per square inch or less, and compresses it into a condenser at a pressure of about 1.5 pounds. The evaporation of a part of the water in the refrigerator withdraws enough heat from the remainder to turn it to ice; or, if the refrigerator contains brine, the heat absorbed by evaporation lowers the temperature of the brine.

In a vacuum machine of this type, the vapor cylinder must have a capacity of about 150 times that of an ammonia compression machine for the same tonnage. The number of gallons of condensing water per ton of ice-melting capacity, assuming a range of 30° F. in the condensing water, is 340. The ice-melting capacity per pound of coal, assuming three pounds of coal per hour per horsepower, is about 25 pounds.

It is evident that the enormous size of a vacuum machine of this type puts it out of competition with other refrigerating-machines.

1431. In another form of vacuum machine, the use of the large compressor is avoided by the use of sulphuric acid as an absorbent. Fig. 336 is a diagram showing the principle of operation of this machine. By means of the air-pump *P*, a nearly perfect vacuum is produced in the chamber *A*, and the water in the bottom of this chamber begins to vaporize. The vessel *B* contains sulphuric acid, which is delivered to the vessel *C* in the form of a spray. The acid having a great affinity for water absorbs the vapor in the chamber, and the dilute acid flows out into the vessel *D*. Fresh water enters the chamber through the water injector *F*, and carries with it some of the salt solution that has been

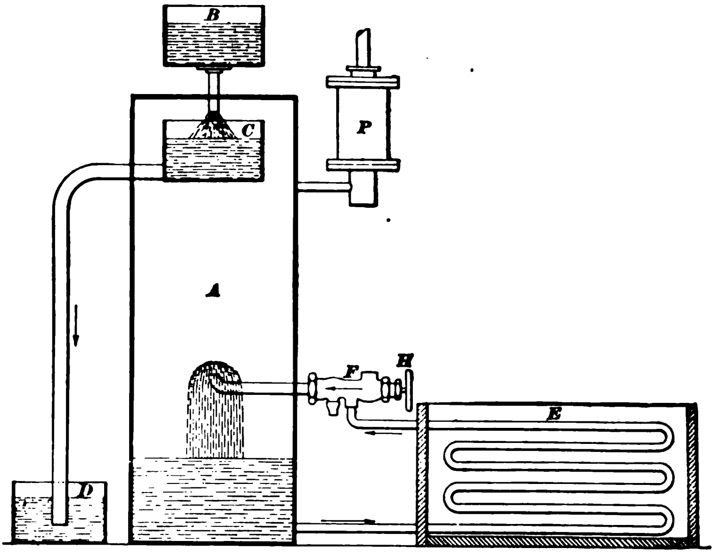


FIG. 336.

passing through the refrigerating coil *E*. The vaporization of some of the water chills the remainder, which falls to the bottom of the chamber, and passing to the coil *E*, absorbs a fresh supply of heat. A pump, not shown in the figure, is required to pump the acid after it is reconcentrated into the vessel *B*.

The principal objection to the machine just described is that the sulphuric acid is an inconvenient liquid to handle. The vessels and pipes containing it must be of lead or be lead-lined.

APPLICATION OF REFRIGERATION.

REFRIGERATING SYSTEMS.

1432. There are two principal systems of refrigeration, viz., the **brine** system and the **direct-expansion** system. In the former system, the expansion coils (*B*, Fig. 324) are immersed in a tank of brine; this brine, which is a non-freezing solution of salt, gives up its heat to the ammonia evaporating in the coils, and is then pumped through coils of pipe placed on the sides or ceiling of the room to be cooled. The circulating brine thus continually absorbs heat from the cold room and gives it to the ammonia, and the latter carries it to the condenser.

1433. In the direct-expansion system, the ammonia is admitted through the expansion-valve directly into the coils in the rooms to be refrigerated. The heat of the cold room is taken up by the ammonia directly, and the intermediate agent, brine, is not employed. The difference between the two systems may be explained as follows: In Fig. 324, suppose the expansion coil *B* to be a comparatively short or compact coil, and let the vessel *D* be a tank containing brine; with this arrangement, we have the brine system. On the other hand, suppose the vessel *D* to represent the room or rooms to be cooled, and suppose the expansion coil *B* to be a long coil, divided into many branches, and located on the ceilings or sides of the rooms; this arrangement constitutes the direct-expansion system.

1434. Advantages of the Systems.—Each of the systems just described has certain advantageous features, which are, however, dependent to some extent on the conditions under which the system is operated. Advocates of the brine system urge the following points in favor of this system and against the direct-expansion system:

1. The total weight of ammonia required by the brine system is less than that required by the other system.

2. The whole of the ammonia part of the system is

located in one room, under the direct care of the attendant; with the direct-expansion system, on the other hand, the close attention of the engineer is required in every room.

3. On account of the large quantity of circulating brine, the temperatures are more easily regulated with the brine system than with the other; also, the brine will circulate several hours after the machine has stopped and still keep the rooms cold. In the direct-expansion system, this reservoir of cold—as the brine may be called—is lacking, and as soon as the machine stops, the refrigeration also stops, and the temperature of the rooms begins to rise. With direct expansion, therefore, it is necessary to keep the machine running continually 24 hours per day,.while with the brine system the machine can, if desired, be shut down at night and started the next morning. This is a decided advantage for the brine system in the case of small plants, where the work required is not sufficient to justify the employment of a night attendant. In large systems, also, where the machines run continuously, it is of some advantage to be able to stop the machine for some hours to make needed repairs.

4. In cold-storage refrigeration, the danger of damaging goods by a leaky ammonia coil is a serious objection to the direct-expansion system.

The points advanced by those who favor the direct-expansion system are as follows:

1. The system is simpler and cheaper to install; there being no intermediate agent to circulate, the brine tank, expansion coils in the tank, and brine-pump are dispensed with.

2. The economy of the direct-expansion system is superior to that of the brine system. In the first place, the power required to drive the brine-pump is saved. Then the temperature of the ammonia in the expansion coil must be lower in the brine system than in the direct-expansion system. Suppose, for example, that the cold room is to be kept at a temperature of 32° F. By using a sufficient amount of piping, the temperature of the ammonia in the expansion coils may be say 22°, the drop of 10° being necessary

to effect the transfer of heat. The back pressure corresponding to 22° is about 35 pounds gauge. If brine is used at a temperature of 22°, it is evident that the temperature of the ammonia in the expansion coils must be lower in order that the brine may give up its heat to the ammonia. Allowing a drop of 10°, the temperature in the expansion coils is 12° and the corresponding suction pressure is about 25 pounds per square inch, gauge. It was shown in Art. **1375** that the capacity and economy of a machine depend very largely on the suction pressure; hence, since with the brine system the suction pressure is necessarily lower than with the direct-expansion system, it follows that the latter system is the more economical, other things being equal.

The brine system is generally preferred for small installations and the refrigeration of cold-storage boxes in markets, hotels, etc., where the large number of boxes would require too close attention with direct expansion. For large installations, the superior economy of the direct-expansion system is a strong point in its favor.

THE DIRECT-EXPANSION SYSTEM.

1435. The pipe used in direct-expansion coils varies from 1 inch to 2 inches in diameter. The coils are preferably made of continuously welded, extra heavy pipe, but this is not possible in the case of 2-inch pipe coils when made up with steel return bends and soldered joints. The length of coils used depends entirely upon the builder, as there are no regular rules or formulas for this work. About 400 feet for 1-inch and 1¼-inch pipe and 600 feet for 1½-inch and 2-inch pipe is a maximum. Rules for piping for direct expansion are entirely empirical. The following is the average practice:

High-temperature work, such as brewery refrigeration, packing-house work, and all work above 32° F., 1 lineal foot of 1¼-inch pipe to every 16 cubic feet of space. For freezing rooms to be held at a temperature of 15° F. or under, 1 foot of 1¼-inch direct-expansion pipe for 6 cubic feet of space.

Pipes of larger diameter will carry more space in proportion to their diameter. The quantity of piping thus specified is based upon a back pressure of 15 pounds per square inch, gauge, which is equivalent to a temperature of 0° F. If the back pressure is 30 pounds, double the quantity of pipe should be used, and if as low as atmospheric pressure, one-half the quantity is all that is necessary. Flat metal disks are at times attached to direct-expansion coils for the purpose of increasing the radiating surface. Opinions differ as to the efficiency of these disks. Details of the piping of cold-storage warehouses, breweries, etc., will be given in a subsequent section.

All coils should be provided at the inlet with expansion-valves, that is, valves for regulating the amount of anhydrous ammonia entering the coil, sometimes known as *feed* valves. The outlet valves are the full size of the coil and act as stop-valves for shutting it off.

The feed and return lines should have the following sizes:

Capacity.	Main Feed Line.	Main Return Line.
5 tons.	½ inch.	1¼ inches.
10 "	½ "	1½ "
25 "	½ "	2 "
40 "	¾ "	2½ "
75 "	1 "	3 "
120 "	1¼ "	4 "

The above table is for the maximum tonnage that can be secured with any given size of pipe, the velocity of the gas being 80 feet per second. In case of return lines of any great length, say over 100 feet, a pipe one size larger should be used. All direct-expansion coils should be provided with drip pans for catching the melted frost.

THE BRINE SYSTEM.

VARIETIES AND PROPERTIES OF BRINE.

1436. Salt Brine.—There are two salts commonly used for making the brine used in brine circulation. The first is Liverpool salt (chloride of sodium), which forms the ordinary brine capable of withstanding a temperature of about 0° F. This salt is cheap in first cost, but has a corrosive action on iron.

The following table gives the percentage by weight of the salt in a brine solution of a given hydrometer reading, and the freezing point of that solution:

TABLE 28.

PROPERTIES OF SALT BRINE.

Degrees Beaumé, 60° Fah.	Degrees on Salometer 60° Fah.	Specific Gravity, 60° Fah.	Per Cent. of Salt by Weight.	Weight of One Gallon.	Weight of One Cubic Foot.	Freezing Point, Degrees Fah.	Specific Heat.
0	0	1.000	0	8.35	62.40	32.00	1.000
1	4	1.007	1	8.40	62.80	31.80	0.992
5	20	1.037	5	8.65	64.70	25.40	0.960
10	40	1.073	10	8.95	66.95	18.60	0.892
15	60	1.115	15	9.30	69.57	12.20	0.855
19	80	1.150	20	9.60	71.76	6.86	0.829
23	100	1.191	25	9.94	74.26	1.00	0.783

Care should always be exercised in selecting a good salt for making brine. Ordinary desert salt will not answer for this purpose, as it will not give a strong enough solution on account of being impure. Such salt always gives trouble by having ice form on the expansion coils, and in case of ice-making, by freezing the cans fast to the coils.

1437. Chloride of Calcium Brine.—The other salt used for making brine is the chloride of calcium. It has all the most essential properties which ordinary salt lacks. It has no corrosive action on iron, which makes it unnecessary to have the brine-pump lined with brass. It has, in fact, an oily nature, and for that reason has a strong tendency to leak if the piping is in any way imperfect; care should therefore be exercised in the pipework of a chloride of calcium brine circulation. It is possible to obtain much lower temperatures by the use of chloride of calcium than common salt brine, −50° F. being the limit.

The following table gives the properties of chloride of calcium brine:

TABLE 29.

PROPERTIES OF CALCIUM BRINE.

Degrees Beaumé, 60° Fah.	Specific Gravity, 60° Fah.	Per Cent. of Calcium.	Freezing Point, Degrees Fah.	Degrees Beaumé, 60° Fah.	Specific Gravity, 60° Fah.	Per Cent. of Calcium.	Freezing Point, Degrees Fah.	Specific Heat.
1	1.007	1	+31.10	21	1.169	19	+ 1.76	
2	1.015	2	30.38	22	1.179	20	− 1.48	
3	1.024	3	29.48	23	1.189	21	− 4.90	.76
4	1.032	4	28.58	24	1.199	22	− 8.68	
5.5	1.041	5	27.68	25	1.209	23	−11.64	.75
6.5	1.049	6	26.60	26	1.219	24	−17.14	
8	1.058	7	25.52	27	1.229	25	−21.82	
9	1.067	8	24.26	28	1.240	26	−27.04	
10	1.076	9	22.82	29	1.250	27	−32.62	
11	1.085	10	21.38	30	1.261	28	−39.28	
12	1.094	11	19.76	31	1.272	29	−46.30	.693
13	1.103	12	18.14	32	1.283	30	−54.40	
14.5	1.112	13	16.34	33	1.294	31	−52.42	
15.5	1.121	14	14.36	34	1.305	32	−39.28	
17	1.131	15	12.20	35	1.316	33	−25.24	
18	1.140	16	10.04	35.5	1.327	34	− 9.76	
19	1.150	17	7.52	36.5	1.338	35	+ 2.84	
20	1.159	18	4.64	37.5	1.349	36	+14.36	

The cost of chloride of calcium is about double that of salt. The quality is extremely variable; insist upon having *fused* chloride of calcium. The salt is excessively deliquescent, that is, it is capable of absorbing a large quantity of water; for this reason, it is often used as a drier. This great avidity of calcium chloride for water renders adulteration by the absorption of water very easy. Even the fused salt contains as much as 20 per cent. of water, whereas the unfused salt, though still in solid form, contains upwards of 50 per cent. of water. Care should therefore be used in selecting the salt.

When it is desired to purchase chloride of calcium, request samples. Dissolve a certain weight of each sample of the salt in the same quantity of water; take a hydrometer reading of each one of the samples after the salt is thoroughly dissolved; the one giving the highest reading is the best sample.

MAKING BRINE.

1438. When a plant is first charged with brine, the brine-pump delivery should be brought over the top of the brine tank, even though a temporary connection has to be made, and allowed to enter a crib. This crib should be about 2 ft. by 4 ft., and be hung from two stringers laid across the top of the tank. The bottom of the crib should be formed of slats and a piece of burlaps or bagging laid over them. The brine-pump is then slowly started, delivering a stream of brine into the crib. Into this crib, bags of salt are then dumped. In case chloride of calcium is used, the iron drums in which it comes should be cracked open by means of a sledge-hammer. It is best not to remove the iron until after the calcium is broken, the iron casing preventing pieces of the calcium from flying about. By keeping the pump running slowly, the salt in the crib will gradually dissolve. Samples of the brine should be drawn out occasionally and tested with a hydrometer until the required gravity is reached, when the addition of salt or chloride of calcium should be discontinued. The crib can then be removed and the temporary pump delivery disconnected. In case of chloride of calcium brine, all that is necessary to strengthen the brine is to throw some of the broken calcium into the brine tank, near the pump suction. The affinity of this salt for water is so great that it will readily dissolve, even when the brine is at a comparatively low temperature. Do not attempt to strengthen salt brine with chloride of calcium, expecting to gradually work a calcium brine into the system. The calcium will readily dissolve, but as it has a greater affinity for water than salt, it will precipitate the salt out of the brine, thus clogging the pipes in the circulation. If it is desired to make the change, pump out all the salt brine and mix up a fresh batch of calcium brine.

THE BRINE TANK.

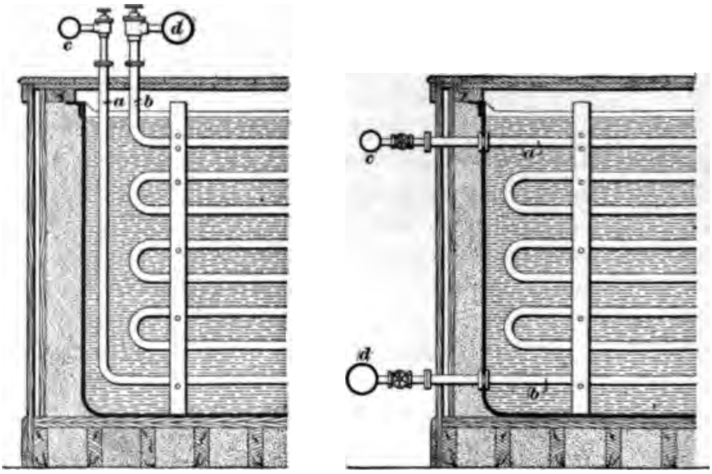
1439. The shape of the brine tank is preferably rectangular, and deep rather than wide. Sufficient headroom should be left above the tank for replacing the expansion coils. Depth is given to the tank so that the brine will have an opportunity of falling as it becomes colder, on account of its greater density. This arrangement permits the warm brine from the circulation to enter the tank near the top and the pump suction to take it out near the bottom, where the brine is the coldest. The tank should also be provided with a drain on the bottom for emptying it. The pump suction should be several inches above the bottom, so that any dirt, sediment, etc., may be able to settle on the bottom of the tank without entering the suction-pipe. It is also well to provide the suction with a strainer having a removable screen that can be drawn up through the top of the tank, cleaned, and replaced. Tanks 6 feet deep should be built of $\frac{3}{16}$-inch iron, riveted with $\frac{3}{8}$-inch rivets, $1\frac{1}{4}$ inches center to center. There should be a 3″ × 3″ angle-iron rim around the top, and one girth brace consisting of two 3″ × 3″ angle-irons with a $\frac{1}{2}$″ × 6″ plate riveted in between. This brace should be placed 30 inches from the bottom. The tank should be braced across the top every 6 feet. In case the tank is 8 feet deep, it should be constructed of ¼-inch iron, and it should be provided with two girth braces of 3″ × 3″ angle-iron with ½″ × 6″ plate. Brine tanks of the depths above specified should be provided with good foundations, as their weight when full exceeds the safe limit of the average warehouse floor. Preferably they should stand on brick or stone piers, carried up from hard-pan and covered with 2-inch planks. On these planks lay 3″ × 10″ joists, 12 inches center to center, well bridged, and fill the space between the joists with ground cork, or if that is too expensive, planing-mill shavings packed in tightly. Lay a ¾-inch pine floor over the cork, then two layers of good water-proof paper, over which lay a 1¼-inch floor. The tank is then set on this floor and hot pitch is run in under it. The description of side-wall insulation is given in a subsequent section.

EXPANSION COILS.

1440. The shape of the brine tank governs the style of expansion coils to be used. In case of a round brine tank, it is necessary to use helical coils, one nested within the other. Such coils are unsatisfactory, as the inside coil is much shorter than the outside one, which makes it necessary to regulate the feed of each coil by itself, so as to properly distribute the ammonia between them. It is also difficult to clean these coils and repair them in case of leaks.

The simplest and most effective expansion coil is the flat return-bend coil. It is inexpensive to build, easily handled and shipped, and should be made of continuously welded pipe, with no joints in the tank. The tails of these coils should project through the side or top of the brine tank and should be fitted with expansion and stop valves and headers, as shown in Fig. 337.

The size of the main feed line supplying the flow header and that of the return or suction line from the return header should be the same as given in Art. **1435** for direct-expansion feeds and returns. The individual feed for each coil should be ½ inch. This is a convenient size, as it does not readily clog and is sufficiently small for fine adjust-



Bottom Feed. (a) *Top Feed.* (b)

FIG. 837.

ment; 1-inch pipe is a very satisfactory size to use for expansion coils, though larger sizes are often used.

To get economical results, coils of this size pipe should not exceed 150 feet in length. Fifty square feet per ton of refrigerating capacity and 90 square feet per ton of ice-making capacity should be allowed. In other words, one coil of 1-inch pipe 150 feet long will give 1 ton of refrigerating effect.

1441. There are three methods of supplying the expansion coils with anhydrous ammonia. They are:

1. The *bottom feed*, in which the liquid ammonia is admitted at the bottom of the coil and the suction is taken from the top. By this method, it is necessary to very nearly fill the coil with anhydrous ammonia, in order to get the benefit of all the pipe. Furthermore, the liberating area is equal to only the cross-sectional area of the pipe, so that the ammonia can not evaporate very rapidly.

2. The *top feed*, in which the liquid ammonia enters the top of the coil and the gas is taken from the bottom. This method does away with two of the important defects in the former method. A slight feed at the top of the coil allows a stream of liquid ammonia to gradually work down through the coil and evaporate along the full length of the coil. This gives a long evaporating area, and the benefit of the full length of the coil is obtained; further, the coldest ammonia is delivered at the top of the tank where it is most needed. The principal objection to this method is that there is a liability of overfeeding, which permits some of the liquid anhydrous ammonia to pass down into the return, and so to the compressor or absorber, without having evaporated.

The arrangement for the bottom-feed system is shown in Fig. 337 (*a*) and that for the top-feed system in Fig. 337 (*b*). In each figure *a* represents the feed-pipe, *b* the return pipe, *c* the flow header, and *d* the return header.

3. *Top feed* and *bottom expansion*, which is a combination of the two preceding methods, retains the good points of both. In this method, one coil is provided with a feed valve at the top, and the anhydrous ammonia is allowed to work through, as in case of the top-feed system; but instead of having the bottom tail of this coil connected with the return, this tail is connected with the bottom tail of the adjoining coil. Any liquid that is overfed in the first coil passes into the second, and there it is completely evaporated and passes out at the top into the return suction line. This last method, though more expensive than either of the others, gives very satisfactory results.

THE BRINE-PUMP.

1442. If chloride of sodium (common salt) brine is used, the brine-pump should be bronze-lined throughout; that is, it should have a brass cylinder, rods and glands on stuffing-boxes. This bronze lining is not necessary with chloride of calcium brine, iron working parts being perfectly admissible. Tupper's square flax packing is a good packing to use with chloride of calcium brine. Any ordinary soft packing will answer for salt brine.

The size of the brine-pump depends upon the quantity of brine circulated per minute and upon the range of temperature between the warm brine entering the expansion tank and the cold brine leaving it. A very good rule of thumb for capacity, in case of brine circulation, is as follows: *Twenty-five gallons of brine cooled one degree in one minute is equivalent to one ton of refrigerating effect per twenty-four hours;* hence 5 gallons of brine cooled five degrees or six gallons cooled four and one-sixth degrees are approximately equal to that tonnage.

If, therefore, it is desired to select a brine-pump for a 10-ton plant, and the range of temperature between the brine at the inlet and that at the outlet is 5°, a brine-pump of 50 gallons capacity should be selected. The maximum piston speed at which brine-pumps should run is 60 feet per minute; 40 feet per minute is better, and is a fair average. The pressure against which a brine-pump usually has to work is comparatively low, and the ordinary low service or tank pump is the type usually used. Both single and duplex pumps are used, the latter class predominating.

BRINE MAINS.

1443. In running brine mains, it is important to have the lines as straight as possible and to avoid risers and downtakes. Where these are unavoidable and form a loop, it is best to provide the highest point of the riser with an air-cock for bleeding it. This is particularly important in case of the secondary mains, where an air trap will cause the brine to flow through the other secondary mains at the expense of the one having the air trap. This gives rise to the supposition that the coils or main are blocked and that the circulation is prevented on that account. Water fittings of long radius or sweep are preferable for use in extended brine circulations; such fittings are considerably more expensive than the ordinary steam fittings, but are much better on account of the decreased friction. Unions used on brine work should in all cases be flanged unions or those having ground joints. A $\frac{1}{16}$-inch 2-ply rubber gasket is all that is necessary for the flange unions.

1444. In laying out a brine circulation, it is very important to distribute the flow of brine in such a way that the quantity sent to each portion of the house should return from all those parts at nearly the same return temperature. It is first necessary to estimate the tonnage required to cool each section; from this the quantity of brine is estimated. Knowing the quantity of brine, the next step is to ascertain the sizes of the various mains needed to conduct this quantity of brine with a friction head* of say 10 pounds per square inch. The method of calculating the tonnage will be given in a subsequent section under the different heads of cold-storage refrigeration, breweries, packing houses, etc The second step, namely, that of estimating the quantity of brine required, is based upon the rule of thumb given in Art. **1442.** The initial and terminal temperatures having been decided on, 25 divided by their difference gives the quantity of brine required per ton of refrigerating effect. This multiplied by the tonnage of any given section to be cooled gives the quantity of brine required to refrigerate that space.

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* When a liquid flows through a horizontal pipe with a uniform velocity, there is a loss of pressure due to the friction between the liquid and the pipe; thus, if the pressure of the liquid as it enters the pipe is 40 pounds per sq. in., it may be only 30 pounds on leaving the pipe, the loss being 10 pounds per sq. in. A pressure in pounds per square inch may be changed to a head of water as follows: A cubic foot of water weighs 62.42 pounds; therefore, a column of water one foot high with one square inch cross-section weighs $62.42 \div 144 = .43 +$ pound. It follows that a column of water produces at its base a pressure of .43 pound per sq. in. for each foot of height. The head of water corresponding to the loss of pressure when water or other liquid flows through a pipe is called the **friction head.** A friction head of say 20 feet corresponds to a loss of pressure of $20 \times .43 = 8.6$ pounds per sq. in.

This process may be expressed by the following formula:

$$G = \frac{25\,T}{t_2 - t_1}, \qquad (124.)$$

in which

G = gallons of brine required per minute;
T = tonnage of section to be cooled;
t_2 = temperature of brine inlet;
t_1 = temperature of brine outlet.

1445. The determination of the size of the pipe necessary to carry a given quantity of brine is based on D'Arcy's formula for the flow of water through clean cast-iron pipes. This formula is practically correct for wrought-iron pipes. In case there are very many elbows in the work, it is best to deduct 10% from the results given by the formula.

Let Q = gallons of brine delivered per minute;
h = head in feet required to overcome friction;
l = length of pipe in feet.

Then, for 1-inch pipe, $Q = 28.5\sqrt{\frac{h}{l}}$. **(125.)**

The formula as given applies only to 1-inch pipe. If it is desired to ascertain the quantity of brine that will flow through pipes of other diameters, it is necessary to multiply the amount that a 1-inch pipe will deliver by the factor in the following conversion table opposite the given diameter:

Diameter.	Factor.
1 inch	1.00
1¼ inches	1.84
1½ "	3.02
2 "	6.53
2½ "	10.23
3 "	19.10
4 "	40.50
5 "	72.00
6 "	115.00

If we denote by G the actual quantity of brine used per minute and by Q the quantity delivered per minute by a

1-inch pipe, then the factor required will be given by the quotient $\frac{G}{Q}$. The size of pipe required can then be determined from the value found for the factor.

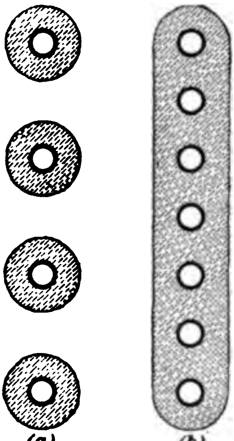
EXAMPLE.—Two houses are supplied with refrigeration from a central station. The quantity supplied to house A is equivalent to the melting of 12 tons of ice, or, in ordinary language, A requires 12 tons of refrigeration; house B requires 14 tons. The range of temperature is 5°, the friction head from the station to A must not exceed 15 feet, and that from A to B must not exceed 12 feet. The distance of house A from the station is 150 feet and B is 200 feet beyond A. What size of pipe is required from the central station to A, and what size from A to B?

SOLUTION.—According to formula **124**, the quantity of brine that must pass per minute to house A is $G = \frac{25 \times 12}{5} = 60$ gallons, and for house B, $G = \frac{25 \times 14}{5} = 70$ gallons. Since the pipe to A must carry the brine for both A and B, it must carry 130 gallons per minute. To find the quantities delivered per minute by a 1-inch pipe under the given conditions, we use formula **125**. For house A, $Q = 28.5\sqrt{\frac{h}{l}} = 28.5\sqrt{\frac{15}{150}} = 9$ gallons, nearly. For B, $Q = 28.5\sqrt{\frac{12}{200}} = 7$ gallons, nearly. For house A the factor is $\frac{G}{Q} = \frac{130}{9} = 14.44$. The next higher factor given in the table is 19.1; hence a 3-inch pipe will be used. For B, $\frac{G}{Q} = \frac{70}{7} = 10$, which calls for a $2\frac{1}{2}$-inch pipe between A and B.

BRINE COILS.

1446. The majority of brine coils are constructed of $1\frac{1}{4}$-inch pipe. The pipes are run in return-bend flat coils and hung on either walls or ceiling. The coils are built of common black steam or water pipe, and are provided with either cast or wrought iron return bends at the ends of the runs. Wrought-iron return bends are preferable, as there is less danger of leaks, due to sand holes, etc. Cast-iron couplings are used on well-built brine circulations, as pipe can be made up much tighter into these couplings without stretching them than in case of the wrought-iron couplings.

The return bends for 1¼-inch pipe should be at least 6 inches center to center, so as to give a good circulation of air between the pipes and prevent their freezing over into one solid mass. It will be seen that style (*a*), Fig. 338, gives a considerable more surface per lineal foot of pipe than style (*b*), where the runs are frozen together.



(*a*) FIG. 338. (*b*)

Coils should be provided at inlet and outlet with heavy steam-cocks. Gate-valves are not advisable, as the flow can be better regulated by a cock, and one can tell at a glance exactly how much the cock is open. Flange unions should be provided at both ends of the coil, just inside of the cocks, so that the coil can be readily disconnected in case of necessity. The inlet is usually connected at the bottom of the coil and the outlet at the top. A pet or air cock should be placed in the coil near the outlet cock for drawing off any air that may accumulate. In case side-wall coils are used, they should be hung on furring strips at least 6 inches deep, so as to keep the coil that distance from the wall. This will insure a good circulation of air and prevent the frost from reaching the wall. These furring strips should be 3″ × 6″ and be placed about 8 feet center to center. A drip pan or trough should be hung under the coil to catch the drip coming from the melted snow when the coil is turned off. It is best to have tubs at the ends of these troughs, as drain-pipes are liable to freeze up and cause trouble. Coils are usually run in lengths of about 300 feet of 1¼-inch pipe. Such a coil will absorb in twenty-four hours a quantity of heat equivalent to the melting of about ½ ton of ice. With a range of 4° between the inlet and outlet of the coil, about 3 gallons of brine a minute will be required to give this capacity. To secure this flow, the friction head need not exceed 2 feet.

1447. Testing.—When the brine circulation is completed, it should be subjected to a thorough test with hydraulic pressure equal to about double the running pressure of the brine. Leaks should be carefully repaired. When chloride of calcium brine is used, it is especially important to have a tight job of pipe fitting, as calcium brine never rusts up a leak. After the work has been completed, but before it has been accepted, the circulation should be subjected to a "frost test," as it is called. The brine is refrigerated as in regular service, and the brine pipes in the circulation including the mains are allowed to frost over. If there are any leaks, they will become apparent, as the leaky points will not frost and can be readily detected. It is best to leave off the insulation of the mains until a frost test has been applied, so that there is no chance for leakage and the consequent ruin of insulation.

COOLING THE BRINE.

1448. Ordinarily the brine is cooled in a brine tank containing expansion coils. This consists essentially of either a round or rectangular tank in which is placed one or more coils of pipe. The tank contains the brine solution, and the pipe coils the liquefied anhydrous ammonia, which expands in these coils and abstracts the heat from the brine. Two improvements on this system are known as the *Hendrick brine-cooler system* and the *British Linde circulating system.*

1449. Hendrick Brine Cooler.—The purpose of the brine cooler is to give a more efficient means of cooling brine than that of the ordinary brine tank. It consists of a cylindrical shell set on end and supplied with helical pipe coils continuously welded throughout their length. The tails of the coils pass through the top and bottom heads of the shell, and the ends of these tails are connected into a manifold or header. The shell is sufficiently strong to withstand a working pressure due to the back pressure of the machine. In effect the brine cooler is the reverse of the brine tank, the ammonia being on the outside and the brine inside of the coils. In this arrangement, the brine is pumped continuously through the coils, thus creating a very rapid circulation in them and giving a much higher efficiency to the surface than that in the brine tank, where the coils are submerged in a sluggish liquid, and the circulation depends upon the difference in the density of the brine as it becomes cold. Furthermore, the expansion of the ammonia in the shell insures the highest back pressure possible for any given temperature, and there is no chance of any throttling action in the coils, as in case of the expansion tank. This is a very important factor, especially in the case of compression machines, where the higher the back pressure the greater the efficiency of the machine.

Care is taken in designing coils for brine coolers of this type, so that the pressure required to overcome friction does not exceed ten pounds per square inch. Owing to the rapid circulation of both the ammonia, which boils around the coils, and the brine inside, it is possible to get an outlet brine temperature within 3° of the ammonia temperature. This insures a brine temperature nearly as low as the temperature of the ammonia itself, and brings up the efficiency of the brine system practically to that of the direct-expansion system.

The principal objection to the cooler is the friction in the coils, but this can be reduced to a minimum by proper design.

The circulation is similar to that of an ordinary brine circulation, but the connections are as follows: A brine-storage tank of any required capacity is used ; the brine-pump takes the brine from this tank, and delivers it to the brine cooler. The cold brine leaves the bottom of the brine cooler and passes to the circulation coils, and the warm brine returns from the coils to the brine-storage tank.

1450. The British Linde System. — The *British Linde air-circulating system* consists of a series of flat return-bend direct-expansion coils. Brine is taken from a catch pan under these coils and is pumped over them. The air to be cooled is forced by means of a blower through these coils and leaves any moisture, together with any impurities, that it may contain in the brine. The arrangement is, in fact, the reverse of the atmospheric condenser, the air being cooled instead of heated, and brine taking the place of water. It is claimed for this system that it not only gives very dry air, but also that the brine absorbs any odors that air may be infected with. The air leaves the coils entirely free from any impurities—in fact so much so that eggs and lemons can be stored on the same system of air circulation. The principal objections to this system are the rapid corrosion of the pipes and the necessity of continually strengthening the brine charge. This is necessary, since the moisture that is abstracted from the atmosphere weakens the brine, and an addition of salt is necessary to keep it from freezing on the coils. The operation of this system is about as expensive as that of the ordinary brine-tank circulation.

TESTS OF REFRIGERATING-MACHINES.

GENERAL REMARKS.

1451. The primary object of a test of a refrigerating-machine is the determination of its efficiency—that is, the ratio of the refrigerating or ice-melting capacity to the fuel consumed. The test involves, therefore, the accurate determination of three principal data: (1) The capacity of the machine. (2) The heat given up to the condensing water. (3) The power consumed by the machine.

1452. An exact determination of the ice-melting capacity can not be made unless the machine is made to cool a liquid during the test. This restriction presents no difficulty in the case of machines working with the brine system, since the cooling of the brine is actually the work which the machine does. In direct-expansion machines, the capacity must be calculated from the weight and temperature of the ammonia circulated, though such a determination can not be considered accurate.

The heat given up to the condenser can readily be determined by weighing the condensing water and observing the temperatures at the inlet and outlet. For testing purposes, the condenser should be of the submerged type.

The work of compression may be determined by taking indicator-diagrams from the ammonia cylinder. Since, however, the work thus obtained would not include the work spent in overcoming the friction of the machine, it is not customary to base the consumption on the work of compression. In most compression machines, the engine and compressor are coupled together. The work shown by indicator-diagrams taken from the steam cylinder will include the work of compression and the friction both of compressor and engine. To determine the actual consumption of work by the refrigerating-machine, the work of the engine running empty may be determined by means of the indicator, and this subtracted from the total work of the steam cylinders will give the net work delivered to the machine.

In the case of absorption machines, the steam fed to the generator is measured, and furnishes a basis for comparison.

1453. Heat Balance.—The accuracy of the results of any test of a refrigerating-machine may be checked by forming a balance between the heat received and given up by the machine. It is clear that, on the whole, the heat received, including the heat equivalent of the work consumed, must be exactly equal to the heat rejected. In the case of the compression machine, the heat received from the brine or cold room is Q_2, the heat equivalent of the work of the compressor is $\frac{W}{J}\left(=\frac{W}{778}\right)$, and the heat given to the condenser is Q_1. These three quantities are measured directly, the work W in 24 hours being obtained from the indicator-diagrams taken from the compressor. Were it not for losses due to radiation, convection, etc., the observed quantities should satisfy the relation

$$Q_2 + \frac{W}{J} = Q_1.$$

If, however, we denote by Q_3 the total loss due to all causes, the heat balance is

$$Q_2 + \frac{W}{J} = Q_1 \pm Q_3.$$

In a carefully conducted test, the difference between the two members of this equation should be small. A large discrepancy indicates either some error in observation or gross inaccuracies in the apparatus or in the method of conducting the test.

TEST OF COMPRESSION MACHINE.

CAPACITY OF DIRECT-EXPANSION SYSTEM.

1454. As stated in Art. **1452,** an accurate determination of the quantity of refrigeration produced by a direct-expansion system is not possible, and it is necessary to resort to calculation to obtain an approximate result. From formula **123,** the refrigeration in 24 hours expressed in tons of ice is $F = .00505\, n\, M\, [r_1 - (t_c - t_a)]$, and the weight M of ammonia circulated per stroke is $M = \frac{C}{v} = C\, w$, where C denotes the volume of the compressor cylinder in cubic feet, v the volume of a pound of ammonia vapor at the pressure in expansion coil, and w the weight of a cubic foot of the vapor at the same pressure. Substituting, we have

$$F = .00505\, n\, C\, w\, [r_1 - (t_c - t_a)]. \qquad \textbf{(126.)}$$

The factor n is the number of compression strokes per minute; in single-acting compressors n will be equal to the number of revolutions per minute, and in double-acting machines to double that number. This formula gives the theöretical tonnage of the machine. To allow for such losses as clearance and cylinder superheating, deduct 25 per cent. from the result in case of a single-acting machine and 30 per cent. for a double-acting machine.

To obtain a rough approximation to the capacity, the following rule of thumb is sometimes used: With a suction pressure of 15 pounds, gauge, and a head pressure of 150 pounds, a well-made single-acting compressor with small clearance spaces gives an ice-melting effect of one ton for 4½ cubic feet of piston displacement per minute. With double-acting compressors, allow 5 to 6 cubic feet per minute for each ton.

EXAMPLE.—A single-acting compressor having two cylinders with 12-inch bore and 30-inch stroke is running at 60 revolutions per minute. The head pressure is 150 pounds and the back pressure 15 pounds. What capacity is the compressor developing? What would be the capacity of a single-cylinder double-acting compressor under the same conditions?

SOLUTION.—Referring to the table of the properties of ammonia, it is seen that t_a, the temperature in the expansion coil, is slightly below 0° F., and the temperature t_c, corresponding to the head pressure, is slightly below 85° F. It will be sufficiently exact to take these temperatures. The latent heat r, corresponding to 0°, is 555.5 B. T. U., and the weight of a cubic foot of ammonia vapor at 0° F. is .1109 pound. The volume of the compressor cylinder in cubic feet is $.7854 \times 1^2 \times 2\frac{1}{2} = 1.9635$. Substituting in formula **126**, the capacity for one cylinder is

$$F = .00505 \times 60 \times 1.9635 \times .1109 \times [555.5 - (85 - 0)] = 31.04.$$

For both cylinders the theoretical capacity is 62.08 tons, and deducting 25 per cent., the approximate actual capacity is 62.08 tons $\times$ .75 = 46.56 tons. Ans.

Evidently the theoretical capacity of the double-acting single-cylinder compressor would also be 62.08 tons; the probable actual capacity, deducting 30 per cent., is 62.08 tons $\times$ .70 = 43.46 tons. Ans.

1455. Direct Measurement of Ammonia.—The formula for capacity applies to compressors working under the most favorable conditions, with small clearance losses and no leak about valves or pistons. The cylinder superheating is based upon a suction pressure of 15 pounds and a back pressure of 150 pounds. This loss is found in case of wet or dry compression or oil injection, the exact percentage differing with make of machine. To make even an approximately exact test upon a compression machine running with a direct-expansion system, it is necessary to measure directly the quantity of ammonia circulated by means of an ammonia meter in the feed line, between the receiver of the condenser and the expansion-valve.

The meter should be calibrated in cubic feet by means of water before being placed in the feed line, care being taken to thoroughly eliminate any remaining water before the meter is connected up. The weight in pounds per cubic foot of the liquid anhydrous ammonia for any required head pressure is given in the table of Properties of Saturated Ammonia.

Let P denote the weight in pounds of anhydrous ammonia passed through the meter in one hour. Then the heat absorbed by the ammonia per hour is $P\,[r - (t_c - t_a)]$ B. T. U. and the capacity in tons per 24 hours is

$$F = \frac{24}{285,300} P[r - (t_c - t_a)] = .000084\,P[r - (t_c - t_a)]. \qquad \textbf{(127.)}$$

EXAMPLE.—If an ammonia meter shows a delivery of 25 cubic feet of liquid anhydrous ammonia per hour, the machine running with a back pressure of 15 pounds and a head pressure of 150 pounds, what is the actual capacity of the machine?

SOLUTION.—Taking the temperatures t_a and t_c at 0° and 85°, as in the solution of the example of Art. **1454**, the volume of a pound of liquid at the given head pressure is .0273 cubic foot; therefore, 25 cubic feet will weigh $\frac{25}{.0273}$ lb. = 916 lb. The latent heat r is 555.5 B. T. U.; hence, substituting in formula **127**,

$$F = .000084 \times 916 \times [555.5 - (85 - 0)] = 36.2 \text{ tons.}$$

CAPACITY OF BRINE SYSTEM.

1456. When the refrigerating-machine is used to cool a liquid—as, for example, brine—the quantity of refrigeration produced can be determined by direct observation. If we denote by G the weight of brine circulated in a given time, by s the specific heat of the brine, and by t_1 and t_2, respectively, the temperatures of the brine at the outlet and inlet, then, according to formula **72,** *Heat*, the heat given up by the brine in the given time is

$$G s (t_2 - t_1) \text{ B. T. U.}$$

The heat given up in 24 hours is readily calculated, and from this the capacity in tons follows at once.

ILLUSTRATION.—In one of Prof. Denton's tests previously mentioned, the average temperature at the brine inlet was 36.76° F., that at the outlet was 28.86° F., the specific heat of the brine was .82, and the weight of brine circulated per minute was 2,281 pounds. The heat given by the brine to the ammonia per minute was, therefore,

$$2{,}281 \times .82 \times (36.76 - 28.86) = 14{,}776 \text{ B. T. U.}$$

1457. Arrangement for Test.—Fig. 339 shows the general arrangement for making a capacity test on machines of any type that use brine as a carrier of cold.

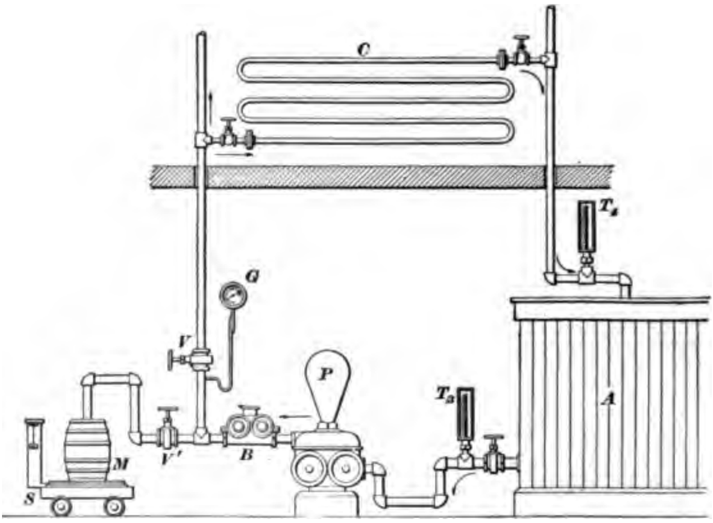


FIG. 339.

The brine tank is shown at *A*, the brine-pump at *P*, the brine meter at *B*, and at *C* the coils in the cooling rooms. A tee should be placed in the delivery line between the brine meter and coils *C*, and cocks or valves should be placed on each side of this tee. These cocks are denoted by V and V'. A pressure gauge G indicates the pressure of the brine. A tank or large barrel M is placed on the scale S, and is so arranged that the pipe from V' leads to it. A thermometer T_4 is placed in the brine line returning to the tank, and another T_3 is placed in the pump-suction line leaving the tank.

1458. Thermometers and Mercury Wells.—Thermometers used in making tests on refrigerating work should be very accurate; they should be graduated to tenths of a degree, so that they can be read to hundredths of a degree. Iron sockets, as shown in Fig. 340, having a ¾-inch pipe thread near the top, should be placed in the pipe wherever temperatures are required. The socket is filled with mercury and the bulb of the thermometer is placed in it. The iron around the socket should be thin, so that the heat may be readily transmitted. The socket should be at least 3 inches long, and if the pipe is not of sufficient diameter, larger size tees with outlets bushed should be provided to receive them. Where it is desirable to get an approximate temperature, portable sockets or wells made of wood

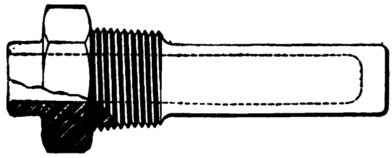


FIG. 340.

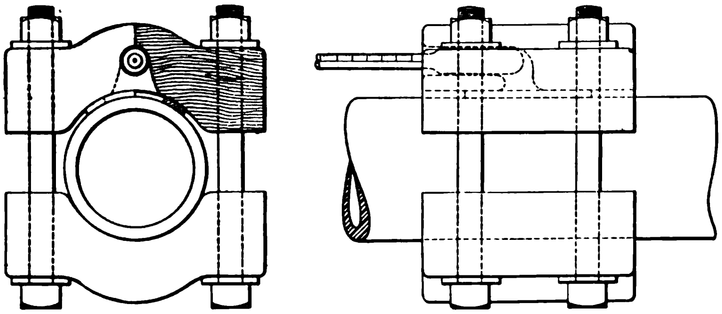


FIG. 341.

can be used with very good results. Fig. 341 shows a sketch of such a socket for vertical pipes.

1459. Calibrating the Brine Meter.—The specific gravity of the brine should be determined by a specific-gravity hydrometer at 62° Fahrenheit. The corresponding specific heat for brine at this gravity can be found by referring to Table 28 or 29, according to the kind of brine used. There now remains to be determined the number of pounds of brine that the meter discharges per cubic foot. To do this, the brine is circulated through and the reading of the pressure gauge *G*, Fig. 339, is noted. The line from the valve *V'* to the barrel should be arranged with a swinging joint, so that it can be swung over the barrel or beyond it at will. Stop the brine-pump, close the valve *V*, and swing the pipe beyond the barrel. Start the brine-pump and open the valve *V'* until the gauge *G* shows the same pressure as in regular running. Throw the pipe into the barrel, and at the same instant take the reading of the meter *B*. When the barrel is nearly full and the reading of the meter has come to an even number of cubic feet, throw the pipe clear of the barrel and weigh the amount of brine held by the barrel. The barrel should be weighed empty before the test is started. The difference between these two weighings will give the weight of the number of cubic feet of brine shown by the meter reading. For example, if the net weight of the brine is 736 pounds and the meter reading is 10 cubic feet, 73.6 pounds is the weight of 1 cubic foot according to the meter. Such a determination does away with the necessity of multiplying the meter reading by the specific gravity of the brine, and also detects any error that may exist in the meter.

DETERMINATION OF HEAT REJECTED.

1460. The procedure adopted in measuring the heat rejected by the ammonia to the condenser is exactly similar to that employed in measuring the heat absorbed from the brine. The quantity of water passing through the condenser is indicated by a water-meter, previously calibrated. The temperatures of the water at the entrance and at exit are measured by accurate thermometers. Evidently the product of the weight of water passing through the meter in a given time and the range of temperature give the heat absorbed in that time in B. T. U. If the compressor has a water-jacket, the heat delivered to the water of the jacket is determined in a similar manner.

ILLUSTRATION.—In Denton's test, quoted in a preceding paragraph, the average initial temperature of the condensing water was 44.65° F., the average final temperature was 83.66° F., and the quantity circulating through the condenser per minute was 2,281 pounds. The average temperature of the water entering the jackets was 44.65°, that of the water leaving the jackets was 69°, and the weight passing through the jackets per minute was 25 pounds. The heat rejected to the condenser per minute was

$$2,281 \times (83.66 - 44.65) = 88,981.8 \text{ B. T. U.}$$

The heat rejected to the water in the jackets per minute was

$$25 \times (69 - 44.65) = 609 \text{ B. T. U.}$$

MEASUREMENT OF THE WORK OF COMPRESSION.

1461. The work of compression and the power consumed by the compressor are determined from indicator-diagrams taken from the compressor cylinder. All the directions for taking diagrams given in *Steam and Steam Engines*, Arts. **1259-1261**, apply equally well to refrigerating-machines.

For ammonia work, it is preferable to use a special indicator, the working parts of which are made of steel instead of brass. Ammonia has no effect on steel, but rapidly corrodes brass. In case it is not possible to procure an ammonia indicator, an ordinary steam indicator will answer the purpose, provided the piston is removed after every series of cards is taken and both cylinder and piston are wiped dry and well covered with oil. This will prevent the ammonia gas from attacking the portions of the indicator made of brass.

In the case of a vertical, single-acting compressor, the indicator pipe is screwed into the upper head of the cylinder.

If the compressor is double-acting, it is of course preferable to use two indicators, one at each end of the cylinder; but if only one indicator is at hand, the pipes from the two ends may be joined by a three-way cock, as shown in Art. **1259**, *Steam and Steam Engines*.

If the condensing pressure is below 150 pounds, a 100-pound spring will do the work; if above that pressure, a 125-pound spring should be used.

The diagrams may be worked up as explained in *Steam and Steam Engines*, Arts. **1265** to **1268**. Having obtained the mean effective pressure P, the indicated horse-power is given by the formula

$$\text{I. H. P.} = \frac{PLAN}{33,000};$$

the work per minute in foot-pounds is expressed by the product $PLAN$; the work in 24 hours in foot-pounds is

$$W = 24 \times 60 \times PLAN = 1,440\, PLAN,$$

and the heat equivalent of this work is

$$\frac{W}{J} = \frac{W}{778} = \frac{1,440\, PLAN}{778} \text{ B. T. U.}$$

The factor N in the preceding formulas denotes the number of *compression* strokes per minute; in the case of single-acting compressors, N is equal to the number of revolutions per minute, and in the case of the double-acting compressors, it is double that number.

1462. Examples of Indicator-Diagrams. — The diagrams shown in Figs. 326 and 327 are excellent examples of indicator-diagrams for a compressor having very small clearance spaces. It will be noticed that, though there is some clearance space, the heel of the diagram is very sharp, there being no curve similar to the compression curve on a steam-engine diagram. This is due to the fact that the ammonia gas in the clearance space is cooled sufficiently by the water-jacket to drop to the suction pressure by its own contraction, due to the loss in temperature, before the piston has receded any appreciable amount. The pressure at the end of compression shows that the compressor is obliged to compress the gas a little above the condenser pressure before the valve opens; the same is true in the case of the suction-valve, as the heel of the card falls below the suction pressure for a short distance until the suction-valve opens.

SCHEDULE OF TEST.

1463. In making a complete efficiency test of a compression refrigerating-machine, the data and results contained in the items of the following schedule should be obtained. In making a capacity test only, the items marked * need not be observed.

General Data.

1. Date.
2. Duration of test.
3. Name of machine.
4. Class of machine.
5. Nominal capacity of machine.
6. Diameter of steam cylinder.
7. Stroke of steam cylinder.
8. Diameter of ammonia cylinder.
9. Stroke of ammonia cylinder.
10. Diameter of brine-pump, steam end.
11. Diameter of brine-pump, brine end.
12. Stroke of brine-pump.

Observations.

13. Average high ammonia pressure, gauge.
14. Average back ammonia pressure, gauge.
15. Average temperature of the brine inlet.
16. Average temperature of the brine outlet.
17. Average range of temperature of brine.
18. Weight of brine circulated per minute.

*19. Average temperature of condensing water at inlet.

*20. Average temperature of condensing water at outlet.

*21. Average range of temperature of condensing water.

*22. Weight of water circulated per minute through condenser.

*23. Weight of water circulated per minute through jackets.

*24. Average temperature of water entering jackets.

*25. Average temperature of water leaving jackets.

*26. Average range of temperature in jackets.

27. Average temperature in engine room.

28. Specific gravity of brine.

29. Specific heat of brine.

30. Revolutions per minute.

*31. Mean effective pressure, steam cylinder.

*32. Mean effective pressure, ammonia cylinder.

RESULTS.

*33. Average horsepower of steam cylinder.

*34. Average horsepower of ammonia cylinder.

*35. Average friction horsepower.

*36. Friction horsepower in per cent. of steam horsepower.

*37. Condensing water, gallons per minute per ton.

38. Ice-melting capacity, tons per twenty-four hours.

*39. Refrigerating effect, pounds of ice per pound of coal.

HEAT BALANCE.

40. Heat given to ammonia by brine per minute, B. T. U.

41. Heat given to ammonia by compressor per minute, B. T. U.

42. Total heat received by ammonia per minute, B.T.U.

43. Heat delivered to condenser by ammonia per minute, B. T. U.

44. Heat delivered to jackets by ammonia per minute, B. T. U.

45. Total heat rejected by ammonia per minute, B.T.U.

46. Difference between heat received and rejected, B. T. U.

CONDUCTING AND WORKING UP THE TEST.

1464. Before a test is started, the plant must be properly arranged. Meters must be standardized, thermometer sockets must be placed, and accurate low-temperature thermometers must be obtained and compared with a standard. For accurate determination of the capacity, the test should not be shorter than ten hours, while official tests are seldom shorter than twenty-four hours, and are often continued for several days.

The frequency of the observations depends somewhat upon the length of the test and upon the uniformity of the conditions. Brine temperatures, items 15 and 16, should be read every 15 minutes; all temperatures should be read to tenths of a degree, and closer if possible. The necessity of this will be easily seen when one considers that the usual range of temperature is four or five degrees Fahrenheit, and the discrepancy of a tenth of a degree would mean from two to three per cent. variation in the results.

Readings of pressure gauges and meters may be made every half hour. The inital and final readings of the meters should be made with particular care. It is best to have two persons make these readings where possible. Indicator-diagrams should be taken every half hour or hour.

The specific gravity of the brine, item 28, is determined by means of an accurate hydrometer; the specific heat, item 29, may be found from Table 28 or Table 29, according as the salt or calcium chloride brine is used.

After the test is completed and the observations are recorded, the *average* of all the observations on each item should be found, and the items included under "Results" should be calculated. Items 33 and 34 are obtained by measuring the indicator-diagrams; item 35 is the difference between items 33 and 34, and item 36 is obtained by dividing item 35 by item 33. To find item 38, we have the equation

$$\text{Item } 38 = \frac{\text{item } 18 \times \text{item } 17 \times \text{item } 29 \times 60 \times 24}{285{,}300}.$$

(See Art. **1456.**)

Item 37 is readily obtained from items 22 and 38.

Items 40 to 46 can readily be calculated from the principles already explained. Should there be a considerable discrepancy between items 42 and 45, that is, should item 46 be relatively large, this fact would indicate either inaccurate apparatus or errors in the observations.

CALCULATION OF EFFICIENCY.

1465. In order to determine the commercial efficiency of a refrigerating plant, it is necessary to make a capacity determination, and at the same time take into account the amount of fuel consumed to produce the work. The ratio between these quantities will give the efficiency of the *plant*, as explained in Art. **1342**; but it will not determine the efficiency of the *machine*, as it takes into account all the steam consumed by the auxiliaries, such as the water-pump, brine-pump, boiler-feed pump, etc. To determine the efficiency of the machine, including the steam engine, only one of the following plans may be adopted:

1466. Approximate Commercial Efficiency.—The average horsepower of the steam cylinder is determined as indicated in item 33, and the fuel consumption per horsepower is judged from the type of the engine and the conditions under which it is working. The consumption of engines ordinarily used to drive refrigerating-machines varies from $2\frac{1}{2}$ to 4 pounds of coal per horsepower per hour, the lower figure applying to condensing engines. It is quite customary to assume a consumption of 3 pounds per horsepower. Using this figure, we have the following formula:

$$\text{Efficiency (item 39)} = \frac{\text{item 18} \times \text{item 17} \times \text{item 29} \times 60}{\text{item 33} \times 3 \times 142.65}.$$

EXAMPLE.—In an actual test, the following data were recorded:

Brine circulated per minute	1,030 pounds.
Average range of brine temperature	8.62°.
Specific heat of brine	.78.
Average horsepower of steam cylinder	61.79.

Assuming a fuel consumption of 3 pounds of coal per horsepower per hour, find the efficiency of the combined refrigerating-machine and steam engine, expressed in pounds of ice melted per pound of coal.

SOLUTION.—

$$\text{Heat abstracted per hour} = 1{,}030 \times 8.62 \times .78 \times 60 \text{ B. T. U.}$$

$$\text{Ice melted per hour} = \frac{1{,}030 \times 8.62 \times .78 \times 60}{142.65} \text{ pounds.}$$

$$\text{Coal used per hour} = 61.79 \times 3 \text{ pounds.}$$

$$\text{Ice melted per pound of coal} = \frac{1{,}030 \times 8.62 \times .78 \times 60}{61.79 \times 3 \times 142.65} = 15.71 \text{ pounds.}$$

Ans.

1467. Direct Measurement of Steam and Fuel. —The result obtained by assuming the coal consumption for the engine can only be approximate, and a comparison between two machines based upon such an assumption might easily be unfair to one or the other of them.

In order to make an accurate efficiency test on any particular plant, the exhaust of the compressor engine should

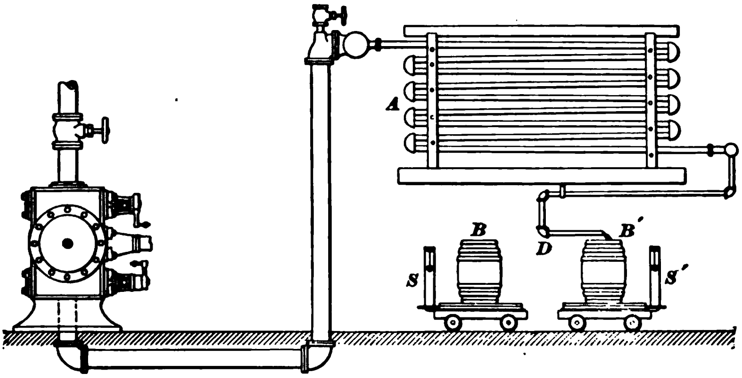


FIG. 342.

be condensed and weighed. To do this, an arrangement similar to the one shown in Fig. 342 will be found convenient. *A* is a steam condenser of any convenient form, and *B* and *B'* are barrels placed upon the scales *S* and *S'*. The outlet from the condenser *A* is provided with a pipe having a swinging joint *D*, so that it can be readily shifted from one barrel to the other. Before the test, the barrels should be filled with warm water, so that they will absorb all moisture possible, and then should be weighed empty. When the test is started, the water is allowed to enter the barrel *B* until that barrel is full. The pipe *D* is then quickly swung to barrel *B'*, and while that is filling, barrel *B* is weighed. The barrels are thus alternately filled, weighed, and emptied, until the end of the test. The total amount of water condensed and weighed in the barrels is calculated, and if the test is for a shorter time than twenty-four hours, the proportionate amount for the full twenty-four hours is estimated. The weight of water used in twenty-four hours divided by the capacity in tons gives the weight of steam consumed per ton of refrigerating effect per twenty-four hours. If a boiler test is carried on at the same time that the refrigerating-machine is being tested, the evaporation of the boilers can be determined. The total amount of steam used in twenty-four hours divided by the weight of coal required to evaporate a pound of water will give the number of pounds of coal required to operate the refrigerating-machine for that space of time. The efficiency in pounds of ice per pound of coal can then readily be calculated.

1468. Power Plants.—Plants run by belt or water-power can best be tested by indicating the ammonia cylinder and estimating from the resulting cards the horsepower required to operate the plant. From this, the capacity per horsepower expended can be easily estimated. An allowance of at least 10 per cent. may be made for the friction of the machine.

TEST OF ABSORPTION MACHINE.

1469. As there are very few absorption machines on the market that make perfectly anhydrous ammonia, the only satisfactory way to make a test on machines of this class is by means of brine circulation. The arrangement of the connections will be the same as for the compression machine, and the data required and the method of calculation are the same for both systems. In case an absorption machine is used in connection with direct expansion, and it is not possible to make a thermal-heat test, one of the following methods may be used for the purpose of establishing approximate results.

1470. Measuring the Anhydrous Ammonia.—The anhydrous ammonia may be measured by a meter in a manner similar to that described in Art. **1455**, but samples of this ammonia should be frequently drawn off and tested by allowing the sample to evaporate in a flask, as described in Art. **1364**. If any moisture is found, the test will be of little value, as such a small quantity as 5 per cent. of entrained moisture will cut down the efficiency of the machine 20 per cent. and the capacity 10 per cent.

1471. Measuring the Pump Delivery.—The other method consists in measuring the pump delivery. If, however, it is not convenient to provide an ammonia meter, the ammonia-pump strokes can be counted, and the amount of ammonia actually discharged can thus be estimated.

Before starting this test, see that the packing-rings in the ammonia end of the pump are tight, so that there is no leakage from one side of the pump to the other; also that all the valves are seated and tight on their seats. The exact stroke of the pump should then be taken every few minutes, while the pump is in operation, so as to see if there is any variation, which is often the case with steam-pumps. After the exact length of stroke has been determined, the cubical contents of the pump can readily be computed. A revolution counter should then be attached to the piston rod so as to record the number of displacements made by the pump during the test.

1472. A general arrangement of the plant is shown in Fig. 343, in which *A* is the absorber of the machine to be tested and *B* is the strong-liquor receiver connected with the ammonia pump *P* by the pipe *S*. A tee is provided in this pipe, the opening of which is controlled by the valve *C*. The outlet of this valve is connected with a small coil D immersed in a pail of ice-water E. If the pump suction S is very warm, a sample drawn out of E will be cooled in passing through the coil D. It is necessary to cool this sample so that the strong liquor in the absorber under several pounds pressure will not lose any of the gas when reduced to atmospheric pressure. If the liquor is chilled some 15° or 20°, it will retain this gas, whereas if the sample be drawn out at the temperature

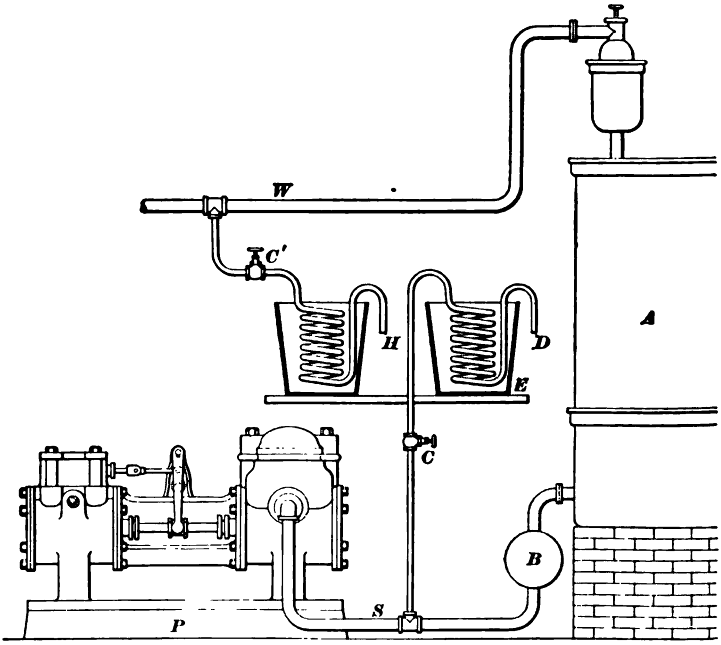


FIG. 343.

of the absorber, the gas will leave the liquid as it is being drawn out, and the sample thus obtained will be much weaker than the liquor in the absorber. The same is true with the weak-liquor line W, which is also arranged with a pipe coil having an outlet H. In drawing samples from either of these two sources, care should be taken that all the liquor left in the coil should first be run out before a new sample is taken.

1473. In making hydrometer readings, the following points should be observed: In filling the hydrometer jar, let the inlet pipe *D* or *H* enter the jar so that it nearly touches the bottom. This will prevent any foaming or escape of gas from that cause. When the jar is full, let a little of the liquid run over the edge for the purpose of removing any foam, air-bubbles, etc., from its surface. The hydrometer is then carefully lowered and a reading is taken. Depress the hydrometer half an inch or an inch after making the first reading, and then allow it to rise. Then make another reading, and the mean of these two readings should be the correct reading of the liquid. The object of the double reading is to eliminate the effect of capillary attraction.

The temperature of the liquid should be taken at the time of the hydrometer reading, and a correction should be made as follows: If the temperature of the liquid is above 55° F., 1 degree Beaumé should be deducted for every 17° above, and if the liquid is colder than 55°, 1 degree Beaumé should be added for every 17° below 55°.

If the machine is running with a comparatively steady load, samples of liquid should be taken from the strong and weak liquor lines every 15 minutes; their specific gravities should be determined, and at the end of the test the average gravity of each of the two liquors should be computed. Table 27 should then be consulted to determine the per cent. of ammonia in both the strong and the weak liquor. The difference between these readings will give the per cent. by weight of ammonia in each pound of strong liquor pumped. This per cent. of ammonia multiplied by the total weight of strong liquor pumped in an hour will give the weight of anhydrous ammonia circulated through the system in that time. Formula **127** can then be used for determining the capacity. In making the test, it is assumed that the machine is making anhydrous ammonia and that there is no entrained moisture.

1474. Economy of the Absorption Machine.—Having measured the capacity of the absorption machine, preferably by brine circulation, the efficiency or economy of the system remains to be determined. We have in this case two sources of steam consumption, namely, the generator and the ammonia-circulating pump. The consumption of the latter, in case it is a steam-pump, can be determined by an arrangement similar to that used in connection with the steam engine of the compression machine, and shown in Fig. 342. If, however, the pump is driven by an engine that also does other work, the best way to determine its steam consumption is as follows: Indicate the engine with the pump running when it is doing normal work, and indicate it again under the same conditions, but with the ammonia pump disconnected. Compute the horsepower from each set of diagrams; the difference between the two will give the power necessary to drive the ammonia pump. From this the steam consumption can be estimated more or less closely from the class of engine which drives the pump.

To ascertain the amount of steam taken by the generator, the arrangement shown in Fig. 344 may be adopted. The

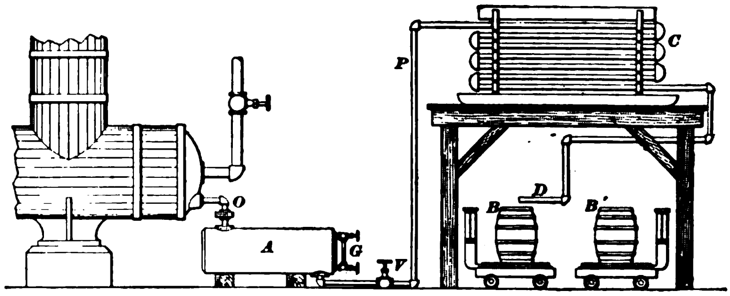


FIG. 344.

generator coils are connected by a pipe O to the receiver A, which is supplied with the gauge-glass G. The bottom of A is connected with a cooling coil C by a pipe P, in which is placed a valve V. The outlet D of this coil is arranged to swing over the barrels B and B', which are placed on scales. The condensed steam entering A can be seen in the glass G. This is kept at a constant level by means of the valve V, so that only the amount of steam used in the generator will pass into the coil C, and from there into the barrels B and B'. The barrels are then manipulated as explained in Art. **1459** in connection with the compression machine. The weight of steam consumed being known, the calculation for efficiency proceeds as explained in Art. **1467.**

1475. Heat Balance. — The, various quantities of heat with which we are concerned in the test of an absorption machine are the following: (1) The heat given up to the condensing water by the condenser, absorber, rectifier, and weak-liquor cooler; let this heat be denoted by Q_c. (2) The heat absorbed in the generator, Q_g. (3) The heat abstracted from the brine, Q_b. (4) The heat equivalent of the work of the pump, which may be denoted by Q_p.

To determine the quantity Q_c it is necessary to measure the weight of condensing water flowing per minute or per hour and the temperature range. (Items 17 and 18, Schedule of Test.) The heat Q_g may be determined as follows:

Let S = weight of steam passing through the condenser per minute;
L = latent heat of steam at given pressure;
t_1 = temperature of steam at given pressure;
t_2 = temperature of water leaving generator.

In changing to water, each pound of steam gives up L B.T.U. and the water gives up in addition $t_1 - t_2$ B.T.U. in passing from the temperature t_1 to t_2 (assuming, as is customary, that the specific heat of water is 1).

Therefore,

$$Q_g = S(L + t_1 - t_2). \qquad \textbf{(128.)}$$

The heat Q_b is determined as in the test of the compression machine, and the heat Q_p is found from the pump indicator-diagrams.

The total heat given to the machine is evidently

$$Q_g + Q_b + Q_p,$$

and Q_c is the heat delivered by the machine to the condensing water; hence we have

$$Q_c = Q_g + Q_b + Q_p.$$

In practice, Q_p is so small that it may be neglected in forming the heat balance, and the equation becomes

$$Q_c = Q_g + Q_b.$$

REFRIGERATING
AND
ICE-MAKING MACHINERY.

INSTALLATION AND MANAGEMENT OF REFRIGERATING-MACHINES.

COMPRESSION MACHINE.

ERECTION.

1476. Foundations.—The foundation for the compression machine consists of a regular engine foundation for the compressor, engine, and pump, and the necessary floor for the condenser, on which is placed the catch pan, if the condenser is of the atmospheric type. In the case of a submerged condenser, a stone foundation or brick piers should be provided. For brine circulation, the brine tank should also be set upon a strong brick or stone foundation. The footing course of the engine or compressor foundation should be of large stones or of a grout foundation well tamped. The upper courses of the foundation may be of either concrete or stone, the latter being preferable. In setting the foundation bolts, it is well to set them in 1½ or 2 inch wrought-iron pipe, which gives a limited play to the foundation bolt and allows it to enter the base of the engine or compressor more readily, particularly if the template is not true. Care should be taken to wedge the head of the bolt that remains in the foundation, to prevent its turning.

After the masonwork is carried to the proper height, a 1½ or 2 inch hardwood plank should be placed between the stone foundation and the bed-plate of the machine. This plank acts as a cushion and greatly diminishes the injurious effects of shocks or pounding. This end would not be attained if the machine were set directly on the stone foundation, owing to the lack of elasticity ot the latter.

1477. Lining Up and Cleaning.—In case of a self-contained machine, where the engine and compressor are tandem and on the same bed-plate, there is no need of lining up, as these parts are usually shipped from the shop in perfect alinement. But in case of the cross machine, with the engine parallel with the compressor, great care should be taken in lining up both the compressor and the engine with the main shaft and in making the foundation of equal strength throughout, so as to prevent any possibility of either part settling more than the other. A slight settling of the engine or compressor will throw it completely out of line with the other parts, and this causes a great deal of trouble and annoyance, to say nothing of excessive oil bills.

After the engine and compressor are practically in line, all bearings should be thoroughly cleaned with kerosene to remove all grit and then keyed up to a loose fit. It is better to have the compressor knock a little at the start rather than run any risk of burning out the brasses or melting the babbit. The valve bonnets and valves should be removed and the cylinder thoroughly inspected. The castings in the suction and discharge ports should be kept perfectly clean, as the smallest particle of any substance lodging under the valves naturally causes a leak and is likely to cut either the valve, the cylinder, or both.

1478. Placing the Condenser.—After the engine and compressor are in place and bolted down and all bearings protected, so as to prevent dirt entering them, the next thing is to put the condenser in place. Whether the latter is of the submerged or of the atmospheric type, it is essential that the foundation or floor on which it is to stand should be practically level. Any inequality in the floor causes a like unevenness in the pipes or coils of the condenser. This naturally makes a trap in which the anhydrous liquid can collect, thereby interrupting the work of the condenser. The floors of an atmospheric condenser are often made to pitch in a certain direction, at right angles to the run of the coils. In such cases, the coils are practically level, but each coil is propped up a certain amount so as to allow for the pitch in the floor. With submerged condensers, it is best to have very deep tanks, as the deeper the tank the greater is the range in the temperature of the water. Such condensers always have the water-supply near the bottom, the overflow or waste being at the top. In this case, it is necessary, as already stated, to have very strong foundations for the condenser tank. These should be brick piers, placed on good stone or concrete bases, and sufficiently close on centers to prevent any sagging in the floor beams that support the condenser tank. The coils in the tank should also be carefully leveled, so as to prevent any traps or pockets in the pipes, and the best practice recommends valves or cocks at the ends of each coil, outside of the tank, as in this way any leaking coil may be shut off independently of the others.

1479. Setting the Brine Tank.—As the brine tank is usually made deep and has considerable insulation, it is advisable to set it on a brick or stone foundation, similar to the one described in the preceding article. On top of this foundation, the insulation for the bottom of the brine tank is built in the form of a floor. The best thing to use for the filling between the joists is granulated cork; but this is very expensive, and planing-mill shavings are often used instead. Care should be taken not to use sawdust, owing to the possibility of spontaneous combustion in case it becomes dampened, and also because of its great avidity for moisture.

A well-insulated brine tank is shown in Fig. 345. It consists of the main joists resting on the foundations, two air spaces, and one-half to 1 inch of pitch, in which the brine tank is bedded. In case it is not possible to procure pitch, the floor on which the brine tank is to rest may be sprinkled with either ground cork or planing-mill shavings about 1 inch thick, and the brine-tank bed set in them. When the brine tank is rectangular and is to be placed in a corner, the walls round it should be insulated before putting the tank in place.

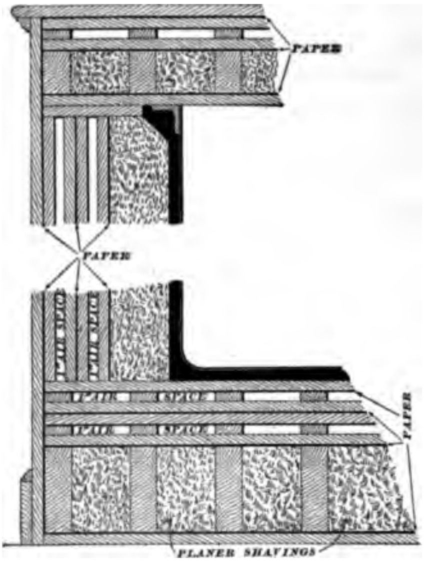


FIG. 345.

The tank should be thoroughly tested for leaks before it is lowered on the foundation, and if any leaks are discovered, they should be stopped.

After the tank is lowered, the expansion coils are placed in it, the same care being taken to keep them level as in the case of the condenser coils. Sometimes cheap plants are built without cocks or valves in the expansion coils; but the better class of machine builders put expansion and shut-off valves in each separate coil. This facilitates regulation and saves time and money in case of accident, as each coil can be shut off independently.

The insulated cover of the brine tank need not be as heavily insulated as the bottom and sides. It should be provided with a manhole or trap-door to give access to the inside of the tank. It is preferable to make the cover in sections if possible, and secure it in such a manner as will allow it to be easily removed, so that the coils can be taken out and replaced if necessary. Such work, of course, makes it necessary to have sufficient headroom over the brine tank. The same thing applies to the submerged condenser.

1480. Pipe Connections.—After all the principal parts of the plant are in place, the anhydrous-ammonia receiver can be set up. This receiver should always be placed below the level of the condenser, so that the liquid from the latter vessel can drain into it. The oil separator between the compressor and condenser should also be located, and the gauge frame with the pressure gauges should be secured in position. The brine and water circulating pumps, boiler-feed pump, and the rest of the steam-boiler plant should be located and placed according to good engineering practice.

The steam main, from which the pipe to the engine of the compressor branches out, should be provided with a stop-valve to shut off the steam in case of necessity; this in addition to the throttle-valve of the engine. The steam-pipe to the engine should be taken from the top of the steam main, so as to get perfectly dry steam. Bleeders should be provided along the bottom of the steam main to drain any water of condensation that may collect in it, the pipe being pitched in the direction of the bleeder. In large plants, it is advisable to connect with these bleeders a trap which will work automatically and remove all water of condensation from the mains. In small plants, the steam for the pumps is often taken from the under side of the main; this brings all the water of condensation to the pumps, but as these are slow-moving parts with plenty of clearance, there is not the same danger from water as in Corliss engines.

The exhaust steam from the compressor engine can be led into an exhaust main and used for heating, making distilled water, or some other purpose. A valve should be placed in the main exhaust from the compressor engine and one in the exhaust of each pump, to prevent steam backing up into the cylinders when they are not in use or are being packed.

The water should be taken from the water-pump to the ammonia condenser, and there should be a branch pipe to the water-jacket of the compressor. In case it is desired to run the engine as a condensing engine, the overflow water from the ammonia condenser can be utilized as circulating water for the steam condenser. This is quite often done at a very slight additional first cost and an increased economy of from 15 to 20%.

The ammonia connections consist of a discharge-pipe from the compressor to the oil trap, and one from the oil

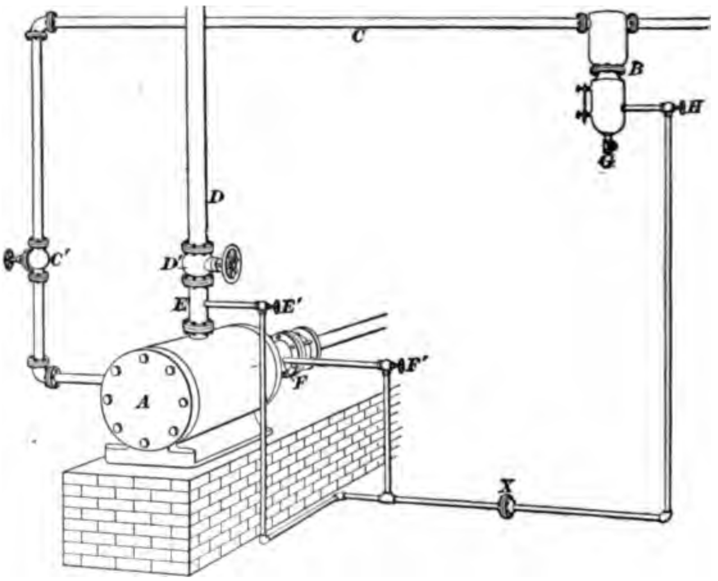


FIG. 346.

trap to the ammonia condenser; a drain-pipe from the ammonia condenser to the ammonia receiver; the expansion-pipe from the receiver to the cooler or expansion tank; a gas suction-pipe from the expansion coils to the compressor; and the gauge pipes commonly known as the "head" pressure and the "back" pressure, these names being given, respectively, to the condensing and evaporating pressure of the ammonia. The head-pressure pipe is usually taken from the discharge-pipe, and the back-pressure from the suction-pipe to the compressor.

Ordinarily the oil from the oil separator is drawn out by means of a cock *G* (Fig. 346) at the bottom of the vessel; though in some systems a connection is made between the oil separator *B* and the gas suction-pipe *D*, and piston-rod stuffing-box *F* of the compressor. By this means a small quantity of oil can be fed into the chamber of the stuffing-box, and if the piston rings begin to groan, some oil can be allowed to enter the cylinder through the suction-pipe. If this arrangement is used, care should be taken to fit the pipe running from the oil separator *B* to the suction-pipe *D* and stuffing-box *F* with a strainer *X* for straining out any impurities, such as scale, that may lodge in the oil separator. The discharge-pipe from the compressor to the condenser should be fitted with an air-valve, or cock, at its highest point, for the purpose of removing air from the condenser. A drain valve should be fitted to the under side of the bottom header of the expansion coils for removing oil from them, and a charging inlet to the suction side of the compressor for taking in anhydrous ammonia for charging the machine. It is also customary to provide an air-valve on the top of the cylinders in single-acting machines and on the discharge-pipe in double-acting machines for the removal of air when the system is being pumped out before charging. In making the suction and discharge connections to and from the compressor, the by-pass connections described in Art. **1404** should not be neglected, as these appliances are indispensable for a successful operation of the plant. Besides the stop-valves on the compressor, a valve should be placed in the discharge main near the condenser, one in the suction-pipe near the expansion coils, and one in the gauge pipes. All pipes and fittings, before being placed in position and after being cut to length, should be well hammered and thoroughly blown out by means of a steam hose for the removal of dirt, scale, etc.

TESTING.

1481. Kinds of Tests.—Refrigerating-machines of the compression type are usually subjected to two classes of tests, viz., the pressure test and the vacuum test. The object of the former is to ascertain the strength of the various parts of the refrigerating-machine and connections, and also the tightness of all joints, etc., and that of the latter is to determine if such joints, together with the packing of the compressor rod, work tight in a vacuum. It often occurs in practice that a piece of mechanism that is perfectly tight under severe pressures leaks in a vacuum.

1482. Testing Water and Steam Pipes.—Before subjecting the machine to the air-pressure test, all the steam, water, waste, and exhaust connections should be tested by turning the live steam in the steam-pipes and subjecting the exhaust-pipes to a slight back pressure, by either partially closing some of the exhaust-valves or by setting the back-pressure valve, if there is one. The water-pipes are usually tested by throttling the inlet water-valves to the condenser and water-jackets of the compressor, whereby a pressure exceeding the ordinary working pressure by about 30% is obtained.

1483. Packing and Inspecting the Compressor.—Before the compressor is started, its cylinder should have a careful inspection, and all grummets which were put on when the plant was being erected should be removed from the various bearings. As a rule, many of the compressors are assembled and run in the shops of the builder. There they are usually subjected to an air-pressure test and all clearances and valves are adjusted; but transportation and subsequent erection often cause derangements, and so it is necessary to examine the different parts of the compressor before the latter is even turned over by hand. The valve covers are first removed from both suction and discharge valves and the cages and valves are taken out. The valve-seats can then be easily seen and examined, and if found perfectly true and clean, are left in place. If, however, they are cut in any way, or if the valves do not lie true in them, they should be removed and ground to a fit, or new seats should be put in. Through the openings of the suction and discharge valves, the inside of the cylinder of the compressor can be seen. It should be carefully examined and cleaned, the cylinder-head being removed if necessary for this purpose. If this is neglected, there is danger of the inner walls of the cylinder being cut before the parts wear down to a bearing. If, however, the cylinder is found to be in comparatively good condition, the compressor should be pinched around by hand until the piston is on the dead-center nearest the removed valve covers. Adjustment for clearance can then be made as described in Art. **1386**, after which the valves and cages can be replaced and the covers can be bolted into place. In case the compressor is double-acting, the clearance at both ends of the cylinder should be examined and the clearance space divided up between them. When the crank is on one of the dead-centers, a mark should be made on the cross-head and also on the guide, for

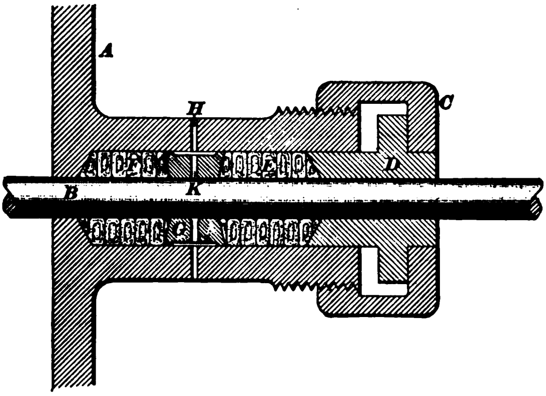


FIG. 347.

the purpose of identifying the location of the piston in the cylinder in case it is necessary to take up the main crank or wrist-pin bearings.

The next thing is to pack the piston rod of the engine and compressor. The engine piston rod may be packed

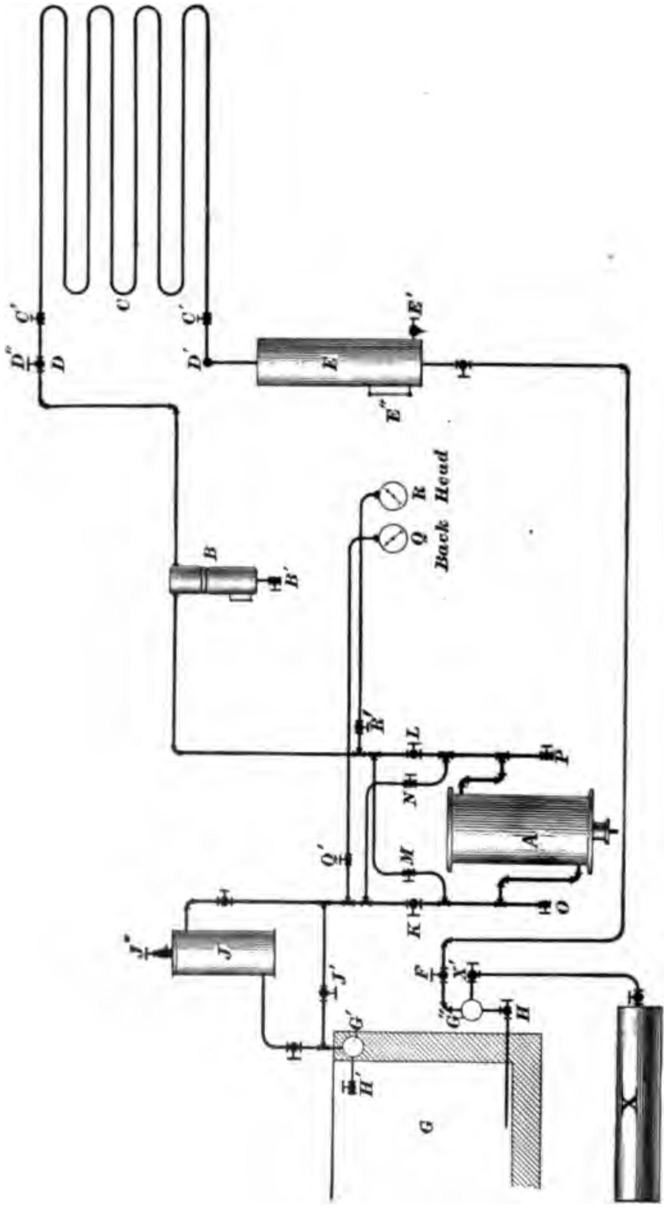


Fig. 348.

with any ordinary good packing. The compressor piston rod, however, should be packed with special ammonia packing. If metallic packing is used, that known as "Common Sense" will be found to give very good results. For a soft packing, Garlock's, Knowlton's, or Crandall's ammonia packing may be used. In packing these compressors, an oil thimble or spool *G*, Fig. 347, is usually provided, which divides the stuffing-box into two parts. This appliance is used more particularly with double-acting compressors. The oil spool *G* is connected with the oil supply of the compressor by the hole *H*, and a certain amount of oil is allowed to enter at this point for the purpose of lubricating the rod. Some of this oil finds its way naturally into the cylinder and serves to lubricate the piston of the compressor.

Seldon's packing has been found very satisfactory on compressors of the wet-compression type. As freezing does not harden or disintegrate it to the same extent that it does packings of a different make, this packing is less likely when frozen to cut the rod.

1484. The Compressor Pressure Test.—After the compressor cylinder has been thoroughly inspected and the piston rods of both engine and compressor packed, the machine is ready for the pressure test of compressed air. This test is made by simply turning the ammonia pump or compressor into an air-compressor; in other words, making the pump suck in air from the engine room and discharge it into the condenser of the ice-machine. Before steam is turned on the engine, however, it is advisable to turn the compressor two or three times by hand, for the purpose of seeing that all parts are in order and do not interfere in any way with one another. The valves of the steam engine should be set according to good engineering practice.

Fig. 348 shows the general arrangement of a compression machine for brine circulation. *A* denotes the compressor, *G* the brine tank, and *C* the condensing coils. The various stop-valves on the pipe, such as *L*, *K*, *C'*, should be inspected and all of them opened, with the exception of the air-valves J', X, E', D' on the various parts of the apparatus. The main suction-valve K at the compressor should then be closed and the compressor started. It will have no source from which to get air, and after a few revolutions will exhaust the air from the cylinders. The air-cock O on the main suction-pipe is now slightly opened and a little air is admitted to the suction side of the compressor. This will tend to slacken up the speed of the compressor, which is now getting air and doing work in compressing it. After the compressor has been running a few minutes, the head-pressure gauge will begin to indicate an increase of pressure. When the pressure has risen to 75 or 100 pounds, the drain valve E' on the receiver of the condenser should be opened quickly. This will allow the compressed air that has accumulated in the condenser to expand and blow out any dirt, scale, chips, etc., that may have collected in the coils of the condenser. This operation should be repeated several times, and the oil-separator valve B' should be used alternately with E'. When the compressor is first started, the oil feed to the suction and that to the packing should be opened. The water to the jacket should be turned on. It is essential to run the compressor very slowly at the start, and care should be taken to run sufficient water through the jacket to keep the delivery pipe comparatively cool. The object of this is to prevent the oil that enters the cylinder of the compressor from vaporizing. If this vaporization takes place when the machine is worked with ammonia, the oil will condense again in the coils of the condenser, or it may be taken out by means of the oil separator; but when the machine is run as an air-compressor, there is danger of the vapor from the oil combining with the oxygen of the air and forming a very explosive mixture. If, however, the delivery pipe of the compressor is kept cool by running sufficient water through the water-jacket and keeping the speed of the compressor as low as possible, there is little danger of any chemical combination taking place. The West Virginia black oil recommended for lubricating compressor cylinders has a very high fire test, as well as extremely low chill test, and consequently it is not very liable to vaporize in the compressor, even when the latter is used as an air-compressor.

After the coils of the condenser are thoroughly blown out, the expansion-valve *F* is opened wide and the pressure is allowed to rise in the expansion and condenser coils. The back-pressure gauge *Q* should now be shut off, the connection between it and the stop-valve broken, and the stop-valve *Q'* opened quickly several times, so as to blow out the pipe. The same should be done with the high-pressure gauge *R* while the low-pressure gauge is in operation.

After the various parts of the machine, including the condenser coils, expansion coils, oil trap, etc., have been thoroughly blown out, the pressure is gradually increased on the various parts up to 300 pounds, which is the ordinary air-test pressure. When the pressure has reached this limit, one of the assistants should be sent around to inspect all joints, etc.

The air-inlet valve *O* of the compressor is now closed and the machine is shut down; the main suction-valve *K* is opened, thus allowing the full 300-pound pressure to be exerted on all parts of the machine. A careful inspection should be made of the jacket water to see if there are any air-bubbles due to leaks in the flanges of the compressor heads, valve covers, etc. The various joints may now be inspected by painting them with soap-suds. The slightest leak can then be readily seen by means of soap-bubbles. The machine is now allowed to stand for several hours with the test pressure on, the pressure gauge being carefully watched to see if there is any drop. If all the joints are found in good order and the apparatus holds the pressure for a considerable time, the vacuum test may be made next.

1485. Vacuum Test.—The main delivery valve *L*, Fig. 348, is first closed and the air-cock *P* is opened. The various air-cocks in the other parts of the apparatus are opened to relieve the pressure. After the pressure has dropped to that of the atmosphere, all the air-cocks are closed except *P*. The compressor is then started slowly. It will naturally draw the air through the main suction-valve, and as the main delivery valve is closed, will discharge it through the air-cock *P*. Both the back-pressure and high-pressure gauge pipes are now opened, and in a short time the gauges will register a drop of pressure of several inches. The machine is now allowed to run slowly until a vacuum of 30 inches is indicated on both gauges. With a well-constructed machine, it is not difficult to obtain such a vacuum; but with cheap machines having large clearances, the vacuum obtained is seldom more than 26 or 28 inches. As the gauge pressure becomes lower, which indicates a more perfect vacuum, the amount of air discharged from the air-cock on the delivery valve will be less and less; finally, it will reduce to but a slight breadth, and when this point is reached, it will be seen that the compressor ceases to lower the vacuum. The air-cock *P* on the delivery line is now closed, the delivery valve *L* is opened, and the compressor is allowed to stand. If the machine is shut down over night, the gauges should still indicate 30 inches of vacuum on the following morning.

The gauge pipes are then shut off at *Q'* and *R'*, the connections near the gauges are broken, and the pipes to the gauges are filled with oil. The oil protects the gauges and renders their readings more accurate.

CHARGING AND OPERATING.

1486. Charging the Connections.—After the machine has stood the vacuum test for several hours, and all joints, packings, etc., have been found to be perfectly tight, it is ready for charging with anhydrous ammonia and brine. An ammonia drum which has either a $\frac{3}{8}$ or a $\frac{1}{2}$ inch opening is connected with the system at any convenient point of the expansion-pipe or the expansion-coil header *G'*, Fig. 348, by means of the air-valve *X'*. After this connection is made, the compressor is started with the main valves *K* and *L* wide open and the by-pass valves *M* and *N* closed; all other valves are left open except the drier by-pass valve *J'* and the various air-valves, such as *O*, *P*, *D'*, *E'*, etc. The valve on the ammonia drum *X* is then slightly opened, and the pressure in the pipe connecting *X* with the valve *X'* is allowed to rise. If this pipe is found to be tight, the valve *X'* is opened slightly. The ammonia then enters the expansion header *G'*, passes through the expansion coils in the expansion tank *G*, and travels by the various pipes to the compressor *A*. From there it is discharged through the oil separator *B* and stored in the condenser *C*. The gas can go no farther, as the expansion-valve *F* is closed. The feed on the valve *X'* can be so regulated as to keep a pressure of from 10 to 15 pounds on the back-pressure gauge *Q*.

Before the compressor is started, the water should be turned on the condenser *C* and through the jacket of the compressor. Care should be taken that this is always done before starting the machine. With each stroke of the compressor, a certain quantity of ammonia gas is taken from the expansion coil and delivered to the condenser. The pressure in the condenser coils, indicated by the head-pressure gauge *R*, gradually rises until the pressure reached is sufficiently high to condense the ammonia in the coils of the condenser. This pressure depends, of course, on the temperature and quantity of the condensing water. After this pressure is reached, liquid anhydrous ammonia will be seen to accumulate in the gauge-glass *E'* of the receiver *E*. When the ammonia drum, or flask, becomes empty, the pressure on the back-pressure gauge will gradually fall, and the drum will begin to frost on the under side. The valve on the drum is then closed, and after a minute the valve *X'* is also closed; the connection between the drum and *X'* can then be broken, with the loss of but little ammonia. A second drum can then be connected; the valve on it is opened slightly for testing the connecting pipe, then the valve *X'* is opened, and the operations just described are repeated.

1487. Handling Ammonia Drums.—Flasks or drums for anhydrous ammonia are made of steel with heads welded in. They are tested to a heavy pressure, and usually hold one hundred pounds of the liquid. They are made sufficiently large to hold this quantity of ammonia and still leave some room for the expansion of the liquid when subjected to heat. Care should be taken, however, not to place them where they will be exposed to excessive heat, as in the boiler room, or where they can receive the direct rays of the sun. There are several different makes of these drums, each having a different style of connection. The rules for connecting the drums are given by the manufacturer and should be carefully followed by the user.

1488. If at any time it is necessary to remove the ammonia charge from a compression machine, this may be done as follows: An empty ammonia drum is connected with the liquid-anhydrous receiver on the condenser, and the ammonia drum is then packed in ice or snow and set on scales. The cock on the condenser and the one on the ammonia drum are then opened, when the pressure in the condenser will force the liquid into the drum. The latter should be allowed to fill until the scales indicate that about 95 pounds of ammonia have entered the flask. It should then be shut off and another drum connected in its place. As all ammonia drums in common use hold 100 pounds of the anhydrous liquid, there will be room enough left in the drum for the ammonia to expand.

1489. Ammonia Test.—While the pressure is being raised in the condenser, a careful man should be sent to inspect all the connections and note any leaks that the ammonia pressure may show. The water-supply over the condenser should then be diminished so as to raise the head pressure to at least 200 pounds, and all joints should be again carefully examined. If any leaks of a serious nature are discovered, the charging of the machine should be stopped at once by closing the valve X', Fig. 348; the expansion-valve F should then be opened so that the gas may escape into the expansion coil *G*, thus lowering the pressure in the condenser. If the pressure is then sufficiently low to take up the leak, the process of charging can be resumed after the necessary repair has been made. If, however, the leak is so serious as to necessitate the taking out of a fitting or a length of pipe, it is necessary to pump all gas out of the condenser. This is done by closing the two main valves *K* and *L* in the compressor and opening the by-pass valves *M* and *N*. This makes the condenser the suction side of the system and the expansion coils the high-pressure side. The compressor is then kept running until the head-pressure gauge *R* indicates a slight vacuum. The necessary repair can then be made and the expansion-valve *F* opened, by which the pressures in *C* and *G* are again equalized. This pressure will be sufficient in all probability to test the repair.

If, however, no serious leaks have been discovered, the process of charging can be continued until the receiver *E* is about two-thirds full of liquid anhydrous ammonia. The expansion-valve *F* is then gradually opened after the valve *X'* has been closed, and the pressure in the expansion coils is allowed to increase. An experienced man is then sent to examine all the coils and connections on the suction side of the compressor, and a careful test is given these connections, similar to that made for the condenser. A pressure of 150 pounds is usually considered sufficient for the ammonia test on the suction side. After all leaks have been stopped, the machine is ready for service. If the system is of the direct-expansion type, the machine can be started at once on its regular work; but if it is of the brine-circulating type, it is necessary first to charge it with chloride of calcium or with salt.

1490. Brine Charge.—The ammonia test having been successful and the expansion coils having been found tight, the tank is filled about two-thirds full of water and a crib is placed under the return pipe. Into this crib the broken chloride of calcium or the salt is dumped in small quantities. The brine-pump is then started and a stream of water turned on the chloride in the crib. This stream will at the start dissolve the chloride very rapidly until the water in the tank has become well saturated. If the expansion coils remain uncovered after the brine has

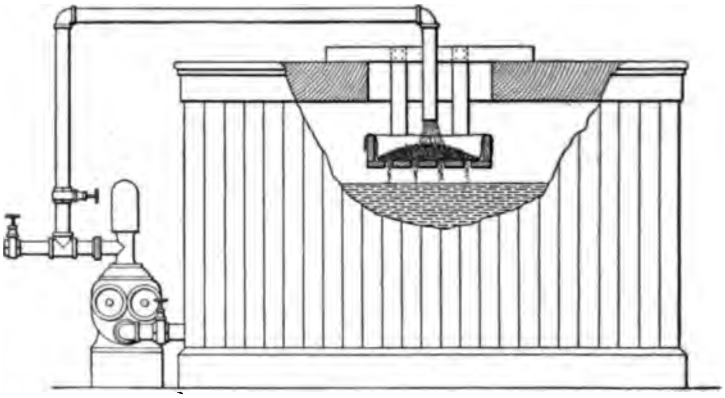


FIG. 349.

reached the desired density, sufficient water should be run into the tank to cover them, and enough chloride added to bring the brine to the required degree of saturation. The method of connecting the brine-pump, crib, etc., is shown in Fig. 349.

1491. Starting the Machine.—Now that the brine charge has been made, the machine charged with ammonia, and all connections tested, the apparatus is ready for service. Before any cooling is possible in the room, it is necessary to cool the brine charge below the freezing point. The brine-pump is stopped, the water-pump is started, and water is run over the condensers and through the water-jacket of the compressor. As the temperature of the brine is probably in the neighborhood of 55°, a comparatively high back pressure can be kept in the expansion coils. As the compressor is started when the pressure in the expansion coils is comparatively high—100 to 150 pounds—it is necessary to reduce this pressure to 30 or 40 pounds before the brine shows any perceptible cooling. The main suction and discharge valves *K* and *L* (Fig. 348) are opened, the by-pass valves *M* and *N* are closed, *F* and *J'* are also closed, and the compressor is started. The back-pressure gauge will begin to indicate less and less pressure. When a pressure of 30 pounds is reached, the expansion-valve *F* is slightly opened, and care is taken to regulate it so as to keep the back pressure between 30 and 35 pounds. After some time the brine temperature will have fallen to about 25°.

The brine-pump can now be started. This will start the circulation in the room and also that in the brine tank, which will help to cool the brine more rapidly. The expansion-valve *F* can be closed a little, so that the back pressure will drop gradually as the temperature of the brine falls. The machine is now in full operation.

If at any time the compressor cylinder begins to groan, some oil should be pumped into the suction-pipe. Care should also be taken that the piston-rod packing is well lubricated; if it begins to heat, the gland on the stuffing-box should be slackened and some oil pumped in. This will cool the rod. It is better to have the piston rod leak a little the first day or two than to have the packing so tight as to cut the rod.

1492. Expansion.—When the brine has cooled to a temperature of 15°, an inspection of the charge should indicate at least 6 inches of liquid anhydrous ammonia in the receiver of the condenser and a back pressure of 20 to 25 pounds. If, however, the back pressure is lower and there is a small quantity of anhydrous ammonia in the receiver, another drum should be connected up and its contents pumped into the machine. As shown in Fig. 348, there is one main expansion-valve *F* and a number of feed-valves *H*, one on each coil, for the purpose of regulating the amount of anhydrous ammonia being fed to the separate coils. These valves *H* should be adjusted once for all, so as to proportion the amount of anhydrous ammonia supplied to each coil; then they should be left alone. The total amount of anhydrous ammonia expanded should be regulated by the main expansion-valve *F*. If the compressor is working properly, the whole action of the machine hinges on the proper manipulation of the expansion-valve. As shown in Art. **1375**, a low back pressure is detrimental to the economical working of a compression machine. Therefore, care should be taken to carry as high a back pressure as possible, and at the same time avoid overfeeding. The best indication of overfeeding is a heavy frost on the suction-pipe of the compressor. Not all of the liquid anhydrous ammonia is evaporated in the expansion coils, and consequently some of it passes over to the compressor. The evaporation in the suction end of the cylinder causes the packing of the piston rod to freeze, and it is liable to leak as soon as it thaws out again. The greatest danger from overfeeding, however, arises from liquid ammonia or oil entering the cylinder of the compressor. The liquid being incompressible, its presence may result in the breakage of a cylinder-head or of a shaft, or the derangement of some part of the compressor. The action in this case is the same as that of a steam engine taking water in with the steam.

1493. Shutting Down the Machine.—To shut down a compression machine, the main throttle-valve of the steam engine is closed, care being taken that the compressor does not stop on a dead-center. The valve on the oil feed is then closed, together with the main suction and delivery valves *K* and *L* and the expansion-valve *F*. The other valves may be left open. The water-pump is then stopped and all the drips are opened; the same is done with the brine-pump. The refrigerating plant is now shut down, and the steam-boiler plant may be shut down as in ordinary practice.

1494. Practical Points Relating to the Starting of the Machine.—After steam is up in the boiler, the first thing to do is to start the water-pump and see that there is a plentiful supply of water going over the condensers. The brine-pump is next started and all its drips are closed. The starting of the brine and water pumps relieves the main steam-pipe of any water that may be in it and prevents the boiler from priming when the compressor is started. The throttle-valve is opened slowly to allow the engine cylinder to heat up, and then the various valves, except the expansion-valve, are opened. After making sure that the water is running over the condenser and through the water-jackets, the compressor is slowly started by opening its throttle. If, however, the engineer is a little awkward in starting the machine and gets it on a dead-center, it is necessary to pry it off center and give it a little lead before the compressor can start off. In case of a big machine, it will be found difficult for one man to pry it over center. The best thing to do is to equalize the pressure on both sides of the compressor piston by closing the main valve *K*, Fig. 348, and opening the by-pass valve *M*. There are notches cast in the rim of the fly-wheel at regular intervals for the purpose of inserting the pinch-bar; they provide an easy means of prying the machine around.

1495. Supplies.—Crude West Virginia black oil, known as Mount Farm oil, is a good lubricant for compressor cylinders, as it has a low cold test, a high fire test, and is inexpensive. Albany grease may be used on the main-shaft bearings and on the other parts of the machine, but it is particularly suited for the low speed and large bearing surfaces of the main shaft. If a composition bearing gives any trouble, the bronze may be cut out and replaced by hard babbit of a good quality. The anhydrous ammonia used should be practically free from foreign substances, among them oil and water. In testing the liquid for impurities, it is preferable to take a sample from a drum that has been connected with the machine and has run nearly empty, as all the impurities, being generally heavier than the liquid ammonia, will then be found in a comparatively small quantity of the liquid.

1496. Operating Details.—A head pressure that is too high is usually due to one of four causes, viz., (1) an insufficient water-supply; (2) too small a condenser; (3) dirty condenser coils; (4) air in the machine. The last-mentioned cause is the most common source of trouble. To remove the air from the condenser, it is first necessary to shut down the compressor. After a few minutes, the air will have risen to the highest point in the system and can be readily drawn off through the valve *D'* (Fig. 348). To do this, run a small piece of pipe from the valve *D'* into a pail of water. As long as large bubbles come to the surface, the gas being drawn out is either air or some impure hydrocarbon, both of which are detrimental to the successful operation of the machine and should be removed. When, however, a sharp, rattling noise is heard, with only a few small bubbles rising to the surface, the valve should be closed, as those are the indications of gaseous ammonia. If the machine is then started, the condensing pressure will be found to have dropped considerably.

The condenser coils should be kept perfectly clean by sweeping them off with a broom. In case it is necessary to use muddy water coming from a stream or brook, these coils should be cleaned at least once every twenty-four hours. Keep the brine charge strong enough to prevent the formation of ice on the coils, since this greatly affects the efficiency of the machine.

ABSORPTION MACHINE.

ERECTION.

1497. Generator.—The generator or retort should be set on a good foundation that will not settle under the weight it has to support. If horizontal, it should be perfectly level. The analyzer should be carefully plumbed after it is put in place. The analyzer trays or plates should then be placed in the analyzer and each tray leveled, so as to insure a good, even surface for the liquor to travel over. The generator should be provided with a pair of gauge-cocks for indicating the quantity of aqua ammonia in it, and all openings, such as gas and liquor connections, should be provided with valves or cocks for closing them. Upon completion, the generator is usually covered with some non-conducting material to prevent radiation.

1498. Condenser and Expansion Tank. — The general rule for the erection of the condenser and expansion tank of the compression machine applies also to those of the absorption type. The only difference consists in the method of connecting the expansion coils, which, as a rule, feed in at the top, the suction being taken from the bottom header. It is the usual practice to put in a gravity connection from the bottom header of the expansion coils to the receiver of the absorber, for the purpose of draining them of any water that may have been entrained with the anhydrous gas and accumulated there.

1499. Exchanger.—The exchanger is usually set on top of the generator, in case this is horizontal, or alongside of it if it is a vertical still. This should be covered, as well as the generator, with some non-conducting material, such as magnesia blocks, asbestos, or hair felt.

1500. Absorber and Aqua-Ammonia Pump.— The absorber should be set as near the expansion tank as practicable, so as to decrease the length of the suction-pipe. It should rest on a good foundation. If the absorber is of the tank type, the foundation should be made similar to that of a submerged condenser; but if it is of the cylindrical type, with either through tubes or helical coils, the foundation should be a good bed of concrete well tamped, so as to prevent the absorber from settling and tipping. It is quite important that the ammonia pump should be placed so that the receiver of the absorber is higher than the suction of the pump; this is to permit the liquid or strong aqua ammonia to run to the pump by gravity instead of being lifted by it. The liquor is charged with ammonia gas, and consequently any lifting action on the part of the pump would create a slight vacuum and liberate the gas; the pump would thus become gas-bound and fail to fill with liquor. The aqua-ammonia pump does not require any elaborate foundation, but merely such a one as an ordinary duplex or single-acting steam-pump needs. A cast-iron bed-plate or pan is a useful addition, as it catches any ammonia or steam that may leak from the pump. Relief-valves should be provided on the caps of the suction and discharge valves, to drain the cylinder.

1501. Rectifier.—The absorption machine of the present day is usually provided with a rectifier, for the purpose of removing any entrained moisture that may pass off from the analyzer on its way to the condenser. The rectifier is usually placed above the analyzer with a gravity drain to the analyzer. Any moisture entrained thus collects in the rectifier and runs through the gravity drain back to the analyzer. To prevent any gas from passing up through the drain-pipe, this pipe is usually provided with a trap or seal. Care should be taken that there is but one trap in this pipe and that the rest of the drip-pipe drains back to the analyzer. The two connecting pipes of the rectifier, viz., the gas line from the analyzer to the rectifier and that from the rectifier to the condenser, should drain, respectively, towards the analyzer and the condenser.

1502. Gauges and Connections.—Pressure gauges should be attached to the generator (or still), to the absorber, and to the expansion coils of the machine. These gauges are steel-spring Bourdon gauges, similar to those used in the compression machine. There should be a valve on the connection between the gauge and the vessel, so that the former may be shut off in case it is necessary to repair it. The generator, condenser, and absorber should be supplied with liquid level gauges, the one on the absorber being usually run open. Air-valves for drawing off air or foreign gases should be located on top of the condenser and absorber and on the absorber receiver. All pipes between the various vessels should have valves, so that any one can be disconnected from the others.

The class of fittings commonly used on the absorption machine are either gland or lock-nut fittings. Soldered-joint fittings are not satisfactory, as the wide ranges of temperature cause the soldered joints to crack. A connection is always run from the bottom of the expansion coils to the receiver of the absorber, so that it is possible to drain the expansion coils into the absorber in case any entrained moisture passes into the condenser and from there to the coils. This connection is usually known as a *blow-out* connection or *purge*.

The generator is connected with the live-steam pipe from the boiler. If there are more coils than one in the generator, they are connected by means of a header, a separate valve being placed on each coil for the purpose of shutting it off separately in case it leaks. The outlets of these coils are also connected to a header with separate valves. This outlet header is usually attached to an ordinary steam-trap, so that the condensed steam passing from the still is discharged by means of the trap. The steam-pipes to the ammonia pump, brine-pump, and water-pump are run as in ordinary practice, and the exhaust is taken to a feed-water heater. Where an absorption machine is used for refrigerating purposes, the steam-trap may be dispensed with, the condensed steam being taken to a boiler-return trap of the Bundy, Pratt & Cady, or Albany type. These traps, being automatic, return the condensed steam to the boiler without requiring any attention on the part of the engineer.

The water connections taken from the water-pump are usually made as follows: In case of excessively warm weather, as in the southern States, a separate supply of water is taken to the condenser, absorber, and rectifier of the machine. If, however, the water is comparatively cool, the condenser and absorber are connected *in series;* that is, the condensing water taken from the condenser is passed through the absorber. This makes a much more economical arrangement of the water consumption of the plant, which in this case is not greater than that of a good compression machine, the amount of water run from the rectifier being about equal to that used over the water-jacket of the compression machine. The waste water from the absorber, condenser, and rectifier is run to the sewer, or, in case of the closed system, it can be discharged into a tank for future distribution.

1503. Coils.—The various coils in the absorber, if they are water coils, should be arranged so that they can be easily blown out with steam or compressed air, to cleanse them of any mud incrustation or scale that may form in them. With comparatively good water, such cleansing is not necessary oftener than once a month, but where river or brook water is used, a frequent blowing out is necessary, especially in rainy weather.

TESTING AND CHARGING.

1504. Cold-Water or Hydraulic-Pressure Test.—As it is impossible to get air-pressure on an absorption machine without the aid of an air-compressor, and as the absorption machine is so constructed and connected that the various parts will drain naturally into the absorber, hydraulic pressure is used instead of air-pressure for testing a machine of this kind. The machine is filled with water by means of the aqua-ammonia pump; then the pump is allowed to run slowly until a pressure of at least 300 pounds is shown by the pressure gauge of the machine. Any leaks are then made tight. The various parts of the machine, however, are usually tested at the shop of the builders to a hydraulic pressure of 500 pounds, the test on the ground being merely for the purpose of determining the tightness of the various connections rather than the actual strength of the machine. Besides, by filling the machine with water, its various parts are thoroughly cleaned.

In order to fill an absorption machine with water, a connection is made such as the one shown in Fig. 350. The main suction-pipe *C* is provided with a tee *G*, from which a temporary suction-pipe *B* is run nearly to the bottom of a barrel *F*. A hose or direct hydrant connection, fitted with a cock *E*, serves to fill the barrel. The valve *C'* is first closed and the valves *A'* and *B'* are opened; then the pump is slowly started and the water is pumped into the machine. The cock *E* is so regulated as to keep the water in the barrel at a constant level. The pump delivering the water into the machine will first fill the coil of the exchanger *G* (Fig. 351), from which the water will flow through the pipe *R* into the analyzer and from there into the generator. Having filled the generator, the water fills the analyzer and rises through the pipe *S* and the drip-pipe *M* into the rectifier *B*. The air-cock *B'* on the rectifier is closed when water is found to run out of it. After filling the rectifier, the water will run to the condenser *C*, from there to the expansion tank *D*, and then to the absorber *E*. When water begins to run out of the upper air purge *E'* on the absorber, the system is completely filled, the exchanger having been filled by opening the valves *Q'* and *G'*. All the air-valves are now closed, and the absorber pressure gauge *U* and the cooler pressure gauge *Y* are shut off by the cocks *U'* and *Y'*. The aqua-ammonia pump is now run very slowly, and with every stroke the pressure will be seen to rise, as shown by the gauge *Z*. When this pressure has reached 150 pounds, a man is sent around to examine all joints, etc., for leaks. If any are found, the pump is shut down, the pressure is relieved by opening one of the air-cocks, and the joint is tightened. After it has been ascertained that all the joints are tight at a pressure of 150 pounds, the pump is again started, and the pressure is gradually raised until the gauge *Z* indicates 300 pounds. As most ammonia gauges are graduated up to only that pressure, it is not advisable to

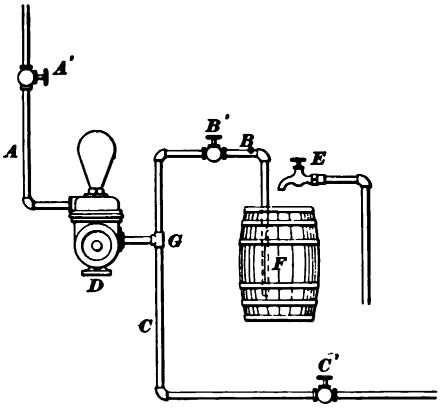


FIG. 350.

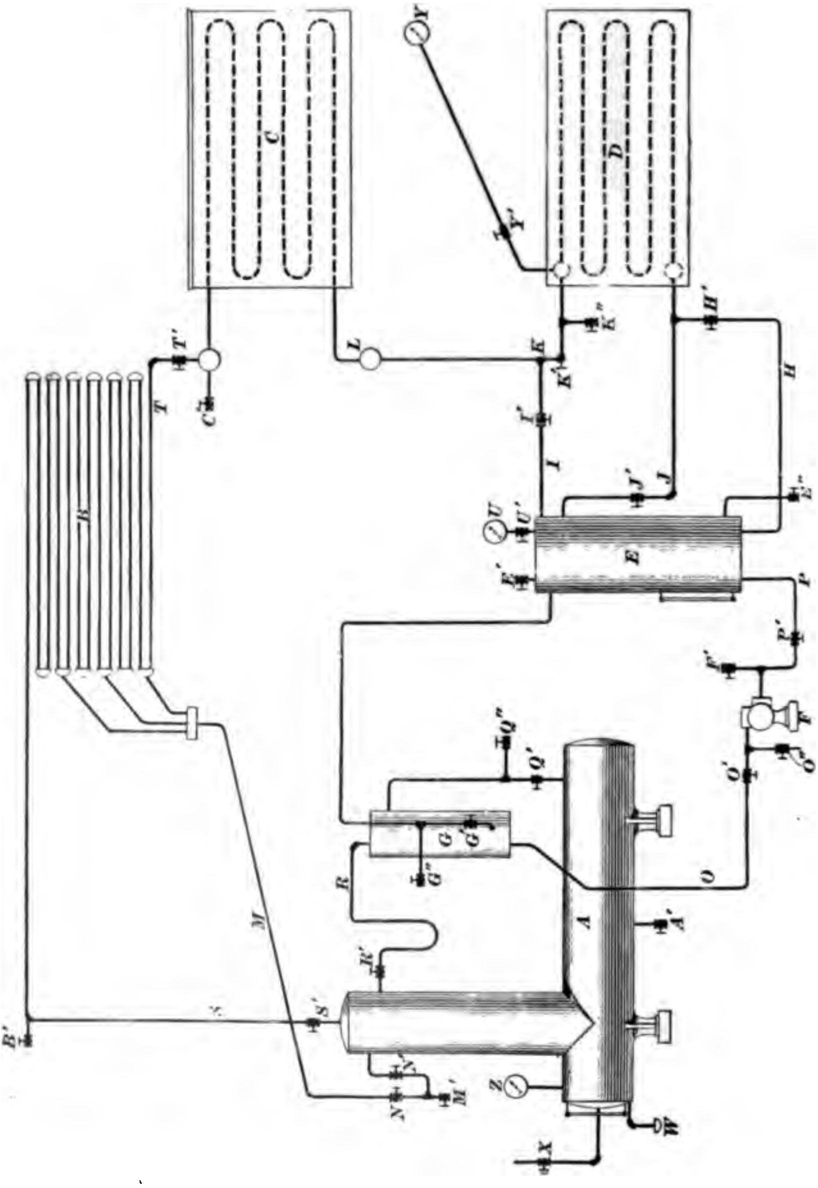


FIG. 351.

attempt to raise the pressure any higher. When the pressure has reached this point, the main delivery valve *O'* is closed, and the pressure is allowed to remain on the machine for at least half an hour. If no leaks are discovered, the machine is ready for the next operation.

1505. Steaming Out.—While the pressure is still on the machine, the gauge connections are broken and the gauges are blown out to clean them. The purpose of steaming the machine is to clean it thoroughly, to heat the joints, so that they can be easily taken up, and to drive out the air. In order to effect this, the following method is usually adopted: The various air-cocks *B'*, *C'*, *E'*, *F'* (Fig. 351) and the drain-cock *A'* are opened, and the water is allowed to run out of the machine until the various vessels and the coils in *C*, *D*, *E*, *G*, and *B* are empty and the generator *A* is about half full. The air-cocks are then closed and steam is gradually admitted to the generator coil by means of the valve *X*; this heats the water in *A* until it gives off steam. The valve *X* is gradually opened. The steam generated in *A* passes up through the analyzer and into the rectifier *B*, and from there it passes to the condenser *C*, driving out through the various air-cocks any air there may be in these vessels. The expansion-pipe *K*, which was broken to drain the condenser of any water that it might contain, is again connected, and the steam is allowed to enter the expansion coils *D*; from there the steam travels through the gas-pipe *J* into the absorber *E*. It will be necessary to carry a steam pressure of about 50 pounds in the generator coil, which will give about 30 pounds pressure in the generator, as indicated by the gauge *Z*; but by the time the steam gets to the absorber *E*, there will be but little pressure, owing to condensation and wire drawing. If the valves *Q'* and *G'* are opened, the boiling water in *A* will find its way through the exchanger *G* into the absorber *E*, as the difference in pressure between the generator and the absorber will be sufficient to force the water into the latter. The ammonia pump is now started with the main suction-valve *P'* opened; it will take the condensed steam and hot water from the bottom of the absorber E and discharge it into the exchanger G, and thence into the analyzer. The pump should be kept running slowly and the water circulating for an hour or more, so as to cleanse the absorber, exchanger, generator, and analyzer thoroughly. The air-valves can be left open on most of the other vessels, and the connection on the expansion-pipe K can be partially broken, so that any condensed steam will run out from these various drains. This will insure the expulsion of all air and the washing out of dirt, scale, and other foreign substances.

After the machine has been under steam for several hours, and when it does not show any more dirt in the steam, the various air-valves are left open, the absorber is pumped empty, and the water remaining in the generator is blown out from the drain valve A'. The flange connection on the line G' also is broken, so as to drain the exchanger of any water that it may contain. After all the water is out of the system, steam is still left in the generator coil, which superheats the steam in the generator and prevents any air from entering. When no more steam escapes from the various air-valves, which indicates that the pressure in the machine has dropped to that of the atmosphere, those valves should be closed. The various stop and expansion valves, such as N', S', B', T', K', H', Q', O', and G' should be closed, so that after the steam is condensed and a vacuum formed, the air that may leak into any leaky part may not fill the other parts. When the vessels cool down, the various rubber joints should be drawn up, as the heat has softened the rubber and makes it possible to draw such joints up easily. The machine is now ready for charging.

1506. Charging the System.—In order to charge the machine with aqua ammonia, the same arrangement of piping is used as that shown in Fig. 350, but the barrel F is replaced by an aqua-ammonia drum. Aqua ammonia comes in wrought-iron glycerine drums holding 750 pounds of ammonia. Each drum has a 2½-inch bung screwed in.

Having removed the bung of one of these drums, a suction-pipe *B* is passed through the bung-hole and is then attached to the elbow of the auxiliary suction-pipe. The pipe *B* reaches to within an inch of the bottom of the drum. As there is a vacuum in the machine, when the valve *B'* in the auxiliary suction-pipe, the delivery valve *A'*, and the valve *C'* in the main suction-pipe are all opened, aqua ammonia will be drawn into the absorber and exchanger of the machine. Care should be taken to sound the drum for the purpose of determining the level of the liquor in it. This may be done by means of a small stick passed down through the bung-hole. By placing one's ear on the suction-pipe near the drum, one can easily tell when the drum is empty by the gurgling sound due to incoming air.

The auxiliary suction-valve *C'*, Fig. 350, is now closed, the pipe entering the drum is detached from the elbow, and another drum is put in place of the first. Then the various stop-valves, such as *S'*, *T'*, etc. (Fig. 351), are opened and as much ammonia is let into the absorber and generator as the vacuum will permit. The main suction-valve to the absorber is then closed and the ammonia pump is started. When the gauge-glass on the generator shows that the latter is full of ammonia liquor, the pump is stopped, the auxiliary suction-valve is closed, and the main suction-valve is opened. The machine is then ready for generating ammonia gas.

1507. The liquid ammonia in the generator is gradually heated by turning steam on through the valve *X*. When the generator is heated to the boiling point of the liquid, the pressure gauge *Z* will indicate from 30 to 60 pounds. As all the main stop-valves are open, this will be the pressure in all the vessels of the machine. Before proceeding with the test, a person is sent around to inspect each joint carefully. If all joints are tight, the steam pressure in the generator coil is allowed to rise, thereby increasing the pressure of the ammonia in the generator. If the temperature of the condensing water, which is now turned on to the rectifier, condenser, and absorber, does not exceed 60°, liquid anhydrous ammonia will be seen to accumulate in the receiver *L* of the condenser when the pressure reaches 120 pounds. After the pressure has reached 60 pounds, as indicated by *Z*, the cocks *U'* and *Y'* are opened and the expansion-valve *K'* is closed. The closing of this valve prevents any of the liquid ammonia from passing into the expansion coils. The valve *Q'* is opened full, and the valve *G'* is opened one or two turns, so as to allow a small quantity of the aqua ammonia, from which the gas has partially evaporated, to pass over into the absorber. When enough liquor has entered the absorber so that it can be seen in the absorber gauge-glass, the ammonia pump is started and kept running at such speed as to keep the level of the liquor in the absorber constant. The valves *J'* and *H'* are now closed. The pressure denoted by *U* will gradually drop to that of the atmosphere, but is not likely to fall below atmospheric pressure, as there will be sufficient air left in *E* to prevent the pressure from dropping below that point. Samples for testing the specific gravity of the ammonia can now be taken at *F'*.

The steam pressure in the generator coil is next gradually increased until it reaches the full boiler pressure. When it has reached that point, the ammonia pump is stopped and the valve *G'* is closed. The machine is now allowed to generate gas from the liquor that is left in the generator. The nearer the steam pressure approaches the condensing pressure, the more complete will be the distillation—that is, the less ammonia gas will be left in the aqua ammonia in the generator. After this process of distilling has gone on for an hour, the specific gravity of the liquor is tested by means of the cooling apparatus shown in Fig. 352, which is attached to the valve *A'*. It consists of a coil *C* of small pipe placed in a bucket *B* of ice-water, and a hydrometer jar *D*. The latter is slowly filled, care being taken that the temperature of the liquor does not exceed 60°. This can be done by running the liquor very slowly out of *A'*. When several samples are taken, the coil *C* should be emptied before taking each sample, so as to obtain the liquor for each test more nearly in the same state as that of the liquor in the generator.

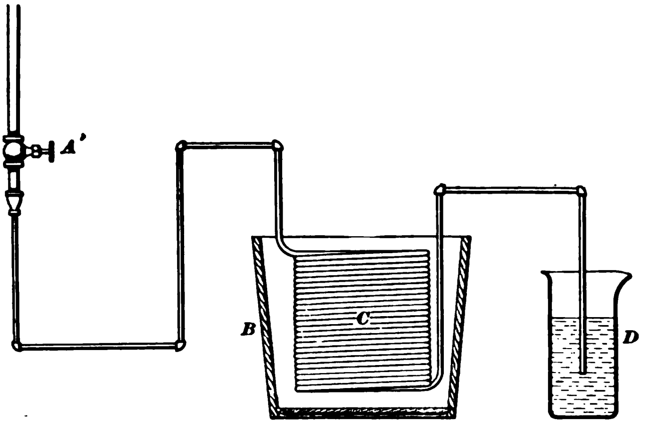


FIG. 352.

Table 30 gives the lowest possible specific gravity to which a solution of ammonia can be reduced under various steam and condensing pressures. Thus, for a maximum steam pressure of 90 pounds and a condensing pressure of 150 pounds, 13½° Beaumé is the lowest possible specific gravity that can be obtained.

TABLE 30.

LOWEST SPECIFIC GRAVITY OF AQUA AMMONIA, IN DEGREES BEAUMÉ.

Steam Pressure in Generator, Pounds.	Ammonia Condensing Pressure, Pounds.					
	100	120	135	150	165	180
60	13.5	15.0	16.0	16.5	17.0	18.0
70	12.5	14.0	15.0	16.0	16.5	17.0
80	12.0	13.0	14.5	15.0	15.5	16.0
90	11.0	12.0	13.0	13.5	14.5	15.0
100		11.5	12.5	13.5	14.0	14.5
120			11.5	12.0	13.0	13.5

1508. After the minimum specific gravity for the given steam pressure has been attained, the weak liquor remaining in the generator is allowed to pass into the absorber. If there is any air remaining in the absorber, its pressure will be shown by the gauge *U*, which will rise as the weak liquor fills the absorber, thereby compressing the air. When the generator is entirely or nearly empty, as shown by the gauge-glass, the valve *G'* is closed. The liquor that has entered the absorber has now been cooled down so that it can readily be drawn out of the valve *F'* and allowed to run to waste, or collected in drums, if it can be sold or used for any purpose. In case, however, the air has been all expelled from the absorber *E* (Fig. 351), it will be impossible to run the liquor out of the valve *F'*, as there is a vacuum in the absorber *E*. It is then necessary to start the ammonia pump, closing the valves *F'* and *O'* and opening *P'* and *O"*. When the pump is started, the liquor will be drawn from the absorber and delivered through the valve *O"*, which can be connected with a sewer or with an ammonia drum.

At this stage, an inspection of the different parts of the machine will show the conditions to be as follows: absorber empty, generator with 1 or 2 inches in the glass, condenser receiver full of anhydrous ammonia. A fresh drum of strong aqua ammonia is now connected with *F'* and pumped into the generator. This process is continued, drum after drum being pumped in, until the generator coil is covered with 4 or 6 inches of strong aqua ammonia. The steam pressure is turned on to the coil, when ammonia gas will be liberated from the strong solution in the generator. When the level has dropped 2 or 3 inches, some weak liquor is allowed to enter the absorber until its level is shown by the gauge-glass of the absorber. It will then be found that there is still sufficient aqua ammonia in the generator to cover the generator coil by 1 or 2 inches. It is necessary to keep the generator coil covered to prevent pitting, for the same reason that the tubes of a boiler are kept covered with water.

1509. Charging with Anhydrous Ammonia.—At any time during the summer season, when the machine is found to need more ammonia and can not be shut down long enough to distill off anhydrous from aqua ammonia as described in the previous article, the charge may be strengthened by using anhydrous ammonia. This is done while the machine is in regular operation. The anhydrous drum is connected with the valve K'' (Fig. 351) and the expansion-valve K' is closed. The cock on the drum is then opened wide and K'' is used as a temporary expansion-valve. This allows the ammonia in the condenser to accumulate in the receiver L. After the first drum is empty, a second one is attached, and so on, until the gauge-glass on the receiver L shows that there is a good supply of anhydrous ammonia in the machine. The valve K'' is then closed, the expansion-valve K' opened, and the machine is allowed to feed as before.

1510. Expelling the Air.—Before the machine is ready for work, it is necessary to expel all air and other extraneous gases, whose presence is ascertained by filling the absorber with liquid ammonia, as explained in the preceding article. If, when this is done and the liquor is allowed to cool, the gauge U still indicates a pressure, the air (or other gases) should be allowed to escape through the valve E'. To know when all the air and extraneous gases have been expelled, a small pipe is connected with the purge valve E'', the other end being immersed in a bucket of water. A sharp rattling sound and an absence of bubbles indicate the pressure of ammonia and show that the air and other gases are all out; so long as the latter remain, the water is very much agitated and the bubbles are abundant.

1511. Starting the Machine.—After the air and other gases have been expelled from the absorber, the ammonia pump is started and the steam is gradually turned on to the generator. When a pressure of 120 or 130 pounds has been reached, the valves S' and N' are opened and the water is turned on to the rectifier, condenser, and absorber.

These valves should be opened slightly so as not to create too strong a current between the rectifier and the generator, as the difference between the pressures in these two vessels may be considerable. Next, the valve *T'* is opened slightly. The generator pressure now extends to all the high-pressure parts of the machine. The valves *Q'* and *J'* are next opened; the expansion-valve *K'* is opened a very little until the gauge *Y* indicates 15 to 20 pounds; then *G'* is slightly opened, allowing some weak liquor to enter the absorber. As soon as the liquor is seen in the gauge-glass of the absorber, the suction-valve *P'* and the delivery valve *O'* are opened and the ammonia pump *F* is started. The pump is run at such a rate as to keep the liquor in the gauge-glass on the absorber at a constant level. The machine is now doing regular work, cooling the brine in the brine tank *D*.

1512. Limitations of Capacity.—With an absorption machine well charged, in good condition, and making anhydrous ammonia, there will be found two causes limiting the capacity of the machine: viz., (1) the steam pressure in the generator, and (2) the quantity of liquor circulated between the generator and the absorber. For horizontal generators, the pressure of steam carried on the generator is usually about $\frac{1}{3}$ of the ammonia condensing pressure. For instance, if the condensing pressure is 150 pounds, the steam pressure in the generator will be about 50 pounds. There is no fixed law or rule for machines having vertical stills, and great care must be exercised in not having too strong a charge or too high a steam pressure, or there is danger of "boiling over"—that is, the generation of the gas being very rapid in the still and the liberating area being comparatively small, the whole charge in the still may be blown into the condenser. The quantity of aqua ammonia circulated should not exceed one gallon a minute per ton of ice made per 24 hours, or half a gallon per ton of refrigerating effect.

Having started the ammonia pump at the proper speed, the weak-liquor valve G' is so regulated that the quantity of ammonia, as shown by the absorber gauge-glass, will remain constant. The amount of feed depends largely upon the frost on the suction-pipe J. When the brine is cooled down to 15° or 20°, this frost should come close to the absorber, without, however, entering the absorber itself.

Example.—An absorption machine has a double-acting ammonia pump, the cylinder of which is $2\frac{1}{2}$ inches in diameter and the stroke is 7 inches. The capacity of the machine is 10 tons. How fast should the pump run?

Solution.—The number of gallons of ammonia circulated per minute is $\frac{1}{2} \times 10 = 5$, which is equal to $5 \times 231 = 1{,}155$ cubic inches. The volume of the cylinder $= \frac{\pi}{4} \times (2\frac{1}{2})^2 \times 7 = 34.36$ cu. in. As this is the quantity of ammonia pumped per stroke, the number of strokes necessary to pump 1,155 cubic inches, that is, the number of strokes per minute, is $\frac{1{,}155}{34.36} = 33.6$. The number of revolutions per minute is half of this, or 16.8. Ans.

1513. Defective Working.—If, after the machine has been running some time, the pressure on the gauge Y (Fig. 351) gradually falls, while the temperature of the brine remains constant, this is an indication that the machine is not making anhydrous ammonia. The usual remedy for this is to run more water over the rectifier coil R, so that more ammonia will condense in the coil and return to the analyzer through the line M.

Another indication of poor working is the fall of the level of the liquid in the generator while remaining constant in the absorber. This means that some of the aqua ammonia in the generator is working over into the condenser, and from there into the cooler expansion coils, where it remains. In order to drain out these coils, the valve H' is opened slightly until the pipe H is frosted all over. This valve should not be opened too much, or the ammonia pump F will begin "kicking" on account of the liquor getting too strong in the absorber; but if opened slightly, the liquor will gradually drain out of the expansion coils and be carried over into the generator by means of the aqua-ammonia pump.

QUANTITY OF WATER USED.

1514. *Rectifier.*—The amount of water delivered to the rectifier should be governed by the temperature of the pipe *T*, which is kept from 15° to 20° higher than that of the condensing water.

Condenser and Absorber.—Sufficient water should be run over the condenser to keep the head pressure as low as possible, and enough over the absorber to keep the outlet water below 110°. If, however, it is not possible to get enough water to produce a sufficiently low temperature, a water-cooling tower or gradier may be installed with economy, as the fuel consumption will increase as the quantity of water is decreased. In a well-designed absorption machine, two gallons of water at 60° per ton of refrigerating effect, or four gallons per ton of ice made, is usually sufficient. This quantity must be increased as the temperature of the water increases.

Weak-Liquor Cooler.—If either the absorber or the exchanger of a machine is too small, and on this account the weak liquor entering through the exchanger *G* is very hot, a cooling coil may be inserted between the exchanger and the absorber. The weak liquor from the exchanger enters the bottom of this coil, the outlet of which at the top is connected to the absorber. Water is allowed to trickle over this coil, the arrangement being similar to that of an atmospheric condenser. Such a coil will increase the capacity of the machine from 10 to 15%.

POINTS ON OPERATING.

1515. Specific Gravity of Liquor.—In the majority of absorption machines, the weak liquor leaves the generator at from 18 to 20 degrees Beaumé, and the strong liquor leaves the absorber at from 25 to 30 degrees Beaumé. The specific gravity of the liquor in the generator depends upon the steam pressure, the condensing pressure of the ammonia, and the amount of steam-coil surface; that of the liquid in the absorber upon the temperature and quantity of the condensing water, the surface in the absorber, and the back pressure. Oftentimes the specific gravity of the strong liquor is taken in order to ascertain whether the machine is sufficiently charged. But such specific gravity does not indicate the strength of the charge. For example, if with a heavy charge in the machine and a light feeding at the expansion-valve, the back pressure is made to run low, the specific gravity of the liquor in the absorber becomes correspondingly low. If there is plenty of water passing through the absorber, a good charge of aqua ammonia in the machine, and the strong liquor is not so strong as it should be, the cause is probably either air or foul gas in the machine or dirt in the coils of the absorber. These coils should be kept clean by blowing them out occasionally with steam or compressed air, the latter being preferable if available. If very hard water is used, such coils should be blown out at least once a week. After giving the absorber a thorough cleaning and purging it free of air, the specific gravity of the strong liquor will be found to have increased by several degrees.

1516. Leaky Coils.—A slight leak in the generator, condenser, absorber, or exchanger coils, although it may not be discovered on inspection, causes a falling off in the capacity of the machine and a loss of ammonia. A leak in the coils of the generator is easily detected by opening the vent valve on the distilled water leaving the generator coil, when the odor of gaseous ammonia will be noticed, as all the ammonia will vaporize in coming in contact with the air, owing to the high temperature of the latter. Another way is to cool some of the condensed steam from the generator coils and apply the Nessler reagent test.

To determine a leak in the condenser or absorber coils, draw some of the overflow water from either one of these vessels and then apply the Nessler test. In case the absorber has water coils, a sample of water should be taken from each coil independently and tested. Then the leaky coil should be capped or plugged and the machine allowed to run with the other coils until the water cools off sufficiently to permit the machine being shut down for replacing the defective coil.

A leak in the exchanger coil is not so easily detected, particularly as there is no loss of ammonia noticed in the machine, the only loss being in the capacity. To test this coil, a tee with a valve Q' (Fig. 351) is placed in the weak-liquor pipe between the generator and exchanger, and another one at G' in the weak-liquor pipe between the exchanger and absorber. A cooling coil (see Fig. 352) is then connected with both of these valves. The specific gravity of samples of ammonia taken from one of the valves should be the same as that of samples taken from the other. If the sample from G' shows a higher specific gravity than that from Q', there must be a leak in one of the exchanger coils.

HOW TO REDUCE THE PRESSURE IN ANY VESSEL.

1517. Absorber.—The absorber is to the absorption machine what the suction of the compressor is to the compression machine. It is that part of the absorption machine by which a pressure may be reduced almost instantaneously, if the machine is in good running order and the absorber free from air or foul gas. By shutting the valve J' (Fig. 351), the absorber pressure should drop to 10 or 15 inches of vacuum inside of 5 minutes. It will therefore be seen that if it is desired to reduce the pressure in the absorber, all that is necessary is to close the valve J' and pump out the absorber the same as in ordinary running.

1518. Expansion Coils or Cooler.—If it is desired to reduce the pressure in the expansion coils or cooler, the expansion-valve K' is closed, and after the machine has been running for half an hour the drain-pipe valve H' is opened so as to permit any liquor that is left in the expansion coils to pass into the absorber, the valve J' being closed. While this has been going on, the machine will have been generating anhydrous ammonia, which is stored in the condenser.

1519. Condenser.—It is very convenient to make a by-pass connection *I* between the absorber and the expansion-pipe *K*. If it is desired to make a vacuum in the condenser coils, all the anhydrous ammonia is fed into the expansion coils by opening *K'* wide and closing *T'* until the pressure in the condenser coils has been reduced to that of the expansion coils, viz., 15 to 20 pounds. The expansion-valve *K'* is then closed, the by-pass valve *I'* opened, and *J'* and *H'* are closed. The gas that remains in the condenser will then be drawn into the absorber, and the pressure in those parts will be reduced.

If it is desired to reduce the pressure in the rectifier, the same thing is done, except that the valve *T'* is left open and the valves *S'* and *N'* are closed.

In case there is no by-pass connection on the machine, the steam is shut off the generator, and when the condensing pressure drops slightly, the valves *N'* and *S'* are closed, the other valves being left open as in regular running. The pressure in the rectifier, condenser, cooler, and absorber is then gradually worked down, the anhydrous ammonia being gradually stored in the generator. This, however, is a very tedious operation, it being much quicker to run all the anhydrous ammonia from the condenser into the cooler, and then, when the condenser is under cooler pressure, open up the cocks *C'* and purge the condenser of the remaining pressure. It will be found, as a rule, that only about one-half of the pressure left in the condenser is due to ammonia, the other half being due to air. Consequently, the loss of ammonia is not very great.

1520. Generator.—To reduce the pressure in the generator, turn on a high steam pressure so as to distill off as much ammonia gas as possible, then close the valve *T'* and shut down the machine, leaving the valves *S'* and *N'* open and water running over the rectifier coil; break the steam connection to the generator coil and run a water hose inside of this coil, turning on a very small stream of water. Care should be taken not to chill the steam coil too rapidly, as it is then liable to crack or be strained. The pressure in the generator will be found to drop very rapidly, and in about an hour the generator will be accessible for repairs.

1521. Ammonia Pump.—The suction and delivery valve chambers of the ammonia pump should be fitted with vent-cocks or valves, so that they may be emptied whenever it is desirable to examine the condition of the pump-valves. When this is being done, the main suction and delivery valves *P′* and *O′* should be closed, and the liquor from the suction and discharge chambers of the pump drawn out through the vent-cocks into a pail of water. The caps covering the valves can then be taken off and the valves examined. If trouble is found in keeping the piston rod of the ammonia pump packed tight, a stuffing-box arranged like that shown in Fig. 347 can be put on the pump. The spool should have a bore equal to the diameter of the rod, or a trifle larger, and the outside diameter of the spool should be $\frac{1}{64}$ inch less than the inside diameter of the stuffing-box. The length of this spool should be from 1 to $2\frac{1}{2}$ inches, according to the diameter of the rod. Care should be taken that the spool is drilled with several holes (marked *K*), so that any leak along the piston rod will easily pass up through these holes to the outer circumference of the spool. The stuffing-box is drilled and tapped at about one-half its length for $\frac{1}{4}$ or $\frac{1}{2}$ inch pipe at *H*. This pipe connects with the suction side of the pump. All that is necessary in this case is to pack the portion *E* between the spool and the gland, so that it will hold against the suction pressure. The portion *F* that has to hold against the discharge pressure is better packed with a packing having very little friction. Though this packing may not be perfectly tight, any leakage will pass along the rod and through the holes *K* to the suction side of the pump, through the pipe connection at *H*.

APPLICATIONS OF REFRIGERATION.

1522. The principles and methods of refrigeration explained in this Course find their most important application in the manufacture of beer, the preservation of perishable articles (especially victuals) in cold-storage rooms or boxes, and the production of artificial ice.

BREWERY REFRIGERATION.

GENERAL DESCRIPTION OF THE PROCESS.—PROPERTIES OF WORT.

1523. How Beer Is Made.—The making of beer consists of three different operations, viz.: (*a*) the preparation of malt from barley; (*b*) the preparation of wort from malt; (*c*) the fermentation of wort to convert it into beer.

By washing specially prepared malt with hot water, a dilution is obtained known as *clear wort.* This is boiled with hops in a copper vessel, and the resulting product, which is the *beer wort*, or simply *wort*, is first cooled and then converted into beer by adding yeast, which causes the chemical decomposition of the wort. This decomposition is called *fermentation*, and is effected in large vessels placed in rooms cooled by refrigerating machinery.

1524. Properties of Wort.—Wort consists mainly of saccharine and dextrine dissolved in water. The **strength** of the solution is measured by the amount of solid matter (that is, dextrine and saccharine) it contains. It is determined by a kind of hydrometer known as a **Balling saccharometer,** so graduated that, when immersed in the liquid, it registers the percentage of solid matter the liquid contains.

The following table gives the specific gravity and the specific heat of wort of different strengths:

TABLE 31.

SPECIFIC GRAVITY AND SPECIFIC HEAT OF WORT AT 60° F.

Strength, by Balling Saccharometer.	Specific Gravity.	Specific Heat.
8	1.0320	.944
9	1.0363	.937
10	1.0404	.930
11	1.0446	.923
12	1.0488	.916
13	1.0530	.909
14	1.0572	.902
15	1.0614	.895
16	1.0657	.888
17	1.0700	.881
18	1.0744	.874
19	1.0788	.867
20	1.0832	.861

For any other temperature t, the specific gravity s is given by the formula

$$s = s_{60} + .00015\,(60 - t), \qquad (129.)$$

in which s_{60} is the specific gravity at 60°, as given in the table. If t is greater than 60, the factor $60 - t$ is negative, which means that $t - 60$ should be multiplied by .00015, and the result subtracted from s_{60}.

EXAMPLE.—What is the specific gravity of a wort of 18% strength, (*a*) at a temperature of 45°? (*b*) at a temperature of 70°?

SOLUTION.—(*a*) From the second column in the table we find $s_{60} =$ 1.0744. Here $t = 45$, and formula **129** gives

$s = 1.0744 + .00015\,(60 - 45) = 1.0744 + .00015 \times 15 = 1.0767$. Ans.

(*b*) Here $t = 70°$, and $60 - t = -10$. The same formula gives $s =$ $1.0744 - .00015 \times 10 = 1.0729$. Ans.

THE COOLING OPERATIONS.

1525. Ends to be Attained.—In the refrigeration of breweries, there are three things to be accomplished, viz.: (1) cooling the wort, (2) removing the heat of fermentation, and (3) cooling off the cellars, fermenting rooms, etc., to a temperature of between 34° and 38° F.

1526. Cooling the Wort.—The wort is taken from the brew kettle at a temperature of about 200° and drawn into large shallow iron tanks *C* (Fig. 353) open to the air,

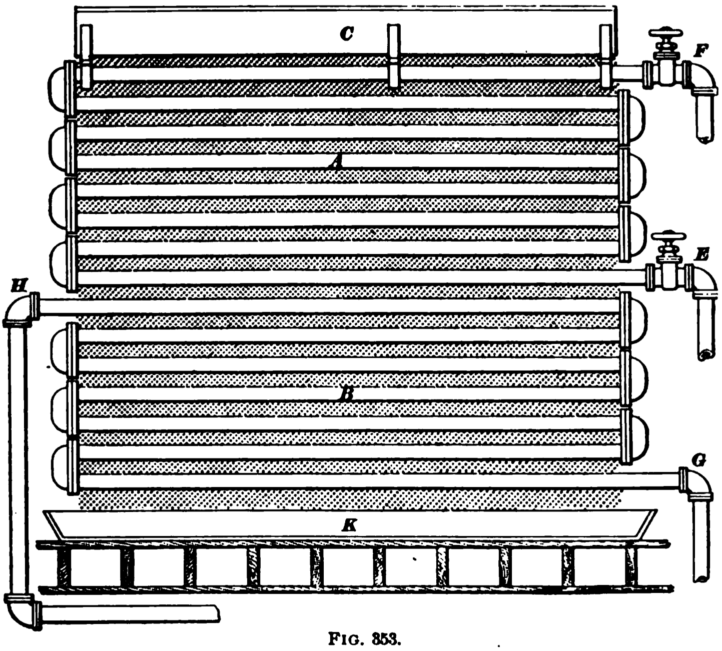


FIG. 353.

where it cools to a temperature of about 110° in from 2 to 3 hours. From there it is allowed to trickle over a **Baudelot cooler,** which consists of two coils *A* and *B*, one under the other, kept at a low temperature in order to cool the wort. Water at a temperature of about 60° is kept circulating in the upper coil *A*. The lower coil is kept at a still lower temperature by the circulation of either ice-water or brine, taken from the brine tank through the pipe G and returned to the same tank through a pipe H. Another way of cooling the coil B is to connect it with the refrigerating-machine through the pipe G and use it as an expansion coil. When, after running over the two coils of the cooler, the wort reaches the pan K, where it is collected, its temperature is reduced to about 40°.

When brine is used in the Baudelot cooler, the coils are usually made of copper tinned over. If the lower coil is used as an expansion coil, it is made of polished steel pipe.

1527. Refrigeration Required for Cooling Wort.—The capacity of a brewery is usually expressed by the number of 31.5-gallon barrels brewed per day of 24 hours. Let this number be denoted by b. Also, let g be the specific gravity of the wort, s its specific heat, T its temperature after leaving the upper coil of the cooler, t the temperature to which it must be reduced in trickling over the lower coil, and H the number of B. T. U. required to effect this reduction of temperature. The value of T may be taken equal to the temperature of the water in the upper coil. The weight of 31.5 gallons of water being 262.4 pounds, that of one barrel of wort is $262.4\,g$. Therefore,

$$H = 262.4\ g\ b\ s\ (T - t). \qquad \textbf{(130.)}$$

If F is the number of tons of refrigeration corresponding to this heat, we have (formula **109**)

$$F = \frac{H}{285,300} = \frac{262.4\,g\,b\,s\,(T-t)}{285,300} = .0009197\,g\,b\,s\,(T-t). \qquad \textbf{(131.)}$$

The values of g and s are taken from tables, assuming a mean temperature $= \frac{1}{2}\ (T + t)$. Taking $T = 70°$, $t = 40°$, $g = 1.05$, and $s = .91$, the preceding formula becomes

$$F = .0009197 \times 1.05 \times .91 \times 30\ b = .02636\ b = \tfrac{1}{38}\ b, \text{ nearly}. \qquad \textbf{(132.)}$$

If in this formula we make $F = 1$, we get $b = 38$; that is, 1 ton of refrigerating effect will cool 38 barrels of wort from 70° to 40°.

EXAMPLE.—(*a*) How many barrels of wort can be cooled in 24 hours from 70° to 40°, the refrigerating effect being 6.5 tons? (*b*) What must be the refrigerating effect of a cooler that it may cool 275 barrels of wort a day from 75° to 40°?

SOLUTION.—(*a*) From formula **132** we get

$$b = 38\,F = 38 \times 6.5 = 247 \text{ barrels. Ans.}$$

(*b*) Taking $g = 1.05$ and $s = .9$, formula **131** gives

$$F = .0009197 \times 1.05 \times 275 \times .9\,(75 - 40) = 8.4 \text{ tons. Ans.}$$

1528. Heat of Fermentation.—After leaving the Baudelot cooler, the wort is pumped into large vats or tanks placed in the fermenting room, where yeast is added to it in order to start and keep the process of fermentation. This room is kept at a temperature of about 40°. As the wort in the tubs ferments, it gives off a certain amount of heat. This heat is removed by means of **attemperators,** which are coils of iron pipe suspended in the fermenting tubs and kept at a low temperature by cooled water or brine made to circulate in them. The diameter of each coil is about two-thirds the diameter of the fermenting tub. The coils should have enough turns to give about 12 square feet of cooling surface for every 100 barrels of wort.

1529. Refrigeration Required to Remove the Heat of Fermentation.—The number H of B. T. U. necessary to dispose of the heat of fermentation of n barrels of wort is determined by the following formula:

$$H = 3\,n\,(s - s')\,(259 + s). \qquad \textbf{(133.)}$$

In this formula, s is the strength of the unfermented wort and s' the strength of the fermented wort (the beer), both as given by a Balling saccharometer. The corresponding number F of tons of refrigeration is

$$F = \frac{3\,n\,(s - s')\,(259 + s)}{285{,}300} = .00001052\,n\,(s - s')\,(259 + s). \qquad \textbf{(134.)}$$

A rough approximation, which is sufficiently close for many purposes, as for general estimates, is obtained by taking $s = 14$ and $s' = 4$, in which case the value of F becomes

$$F = .028\,n = \frac{n}{36}, \text{ nearly.} \qquad \textbf{(135.)}$$

If in this formula we make $F = 1$, we get $n = 36$, which shows that to remove the heat of fermentation 1 ton of refrigerating capacity is required for every 36 barrels of beer. This applies to strong beers. For weak beers, 1 ton of refrigerating capacity may suffice for 50 or 60 barrels. For rough and preliminary estimates, 25 barrels are usually allowed per ton of refrigerating capacity.

1530. Cooling the Cellars.—After the process of fermentation is completed, the beer is drawn off from the various fermenting tubs into the storage cellars, where it is ready to settle and age. These cellars are cooled by means of coils of pipe placed on either the ceiling or the side walls, and whose temperature is kept down either by using them as expansion coils or by allowing a current of brine to circulate through them. The amount of refrigeration required for this purpose depends greatly on the manner in which the rooms are insulated, as will be explained presently. With the ordinary insulation generally used, 1 ton of refrigeration may be allowed for every 10,000 to 14,000 cubic feet of storage room, where the temperature is kept at between 32° and 38°. This applies to large breweries. In breweries of 50,000 cubic feet or under, about 5,000 to 7,000 cubic feet per ton of refrigeration should be allowed.

INSULATION OF ROOMS.

1531. Object of Insulation.—The object of insulating a cold-storage or freezing room is to prevent it from taking in heat, either by radiation, by conduction, or by convection. The less the heat is that enters a room from the outside, the less will be the work required to cool it.

Consequently, if a room is well insulated, a smaller refrigerating capacity is required to keep it at the necessary temperature than when the room is poorly insulated. This means that less fuel will have to be expended to cool the space. The economic importance of insulation is therefore manifest.

1532. Insulators Used.—For the purpose of insulation, rooms are divided into damp rooms and dry rooms. Brewery refrigerating rooms and ice-storage houses are examples of the former class of rooms; while cold-storage and freezing warehouses would come under the latter class.

The ordinary insulating materials are wood and paper. The latter, however, is not well adapted to the insulation of damp rooms, where it is not durable. In late years this class of rooms has been insulated largely by means of hollow brick walls containing from one to three air-spaces. To make the air-spaces more perfect, the walls are covered with a coat of either pitch or paraffin wax. This method of insulating is found to be very efficient for comparatively high temperatures, such as are required in breweries and packing houses, and is not nearly so liable to contract mold or mildew as wood is.

Dry-storage rooms have always been insulated with wood and paper, with air-spaces, or air-spaces filled with some filling material, such as sawdust, planing-mill shavings, mineral wool, cork, wood-ashes, cinders, etc.

1533. Air-Spaces.—The best and cheapest non-conductor is air, but in order to make it efficient, it is necessary to make a "dead air-space," that is, so to inclose the air on all sides as to prevent its motion; otherwise there would be air-currents, by which a great deal of heat would be conveyed from the outside to the rooms. For this reason, brick walls are pitched or covered with paraffin, and when wood is used instead of brick, paper is laid between boards so as to prevent the escape of the air confined in the air-spaces.

In making an air-space with boards, it is always best to use double boards with paper in between, the double boarding with paper making it almost impossible for the air to pass through. Such construction, however, is comparatively expensive, and a single thickness of board with the air-space filled with some good non-conducting material answers the purpose fairly well. In choosing a filling material, the points to be considered are the following: (1) That it should be a good non-conductor; (2) that it should not be too heavy, as then it is liable to settle and leave a portion of the top insulation unprotected; (3) that it should not be affected by dampness to any appreciable extent; and (4) that it should not be too expensive.

1534. Filling Material.—The best non-conductor and insulator for filling an air-space is granulated cork, but this, being very expensive, is seldom used; it is light, is not affected by dampness, and consequently does not mold or rot. Several years ago, mineral wool, sometimes called slag wool, was used extensively for filling air-spaces. This is a mineral fiber and is made by blowing steam through iron slag when it is in a molten state. The product is a fluffy, woolly substance, quite similar in appearance to very coarse cotton. It forms an excellent non-conductor, and if the best quality is purchased is quite light, weighing about 7 pounds per cubic foot, and does not readily settle; but it is easily affected by moisture, and then becomes quite soggy and settles rapidly. Where high headroom is available, it is always best to put horizontal strips half way up the filling-in spaces, so as to divide the filling into two sections. This will take half of the weight off the filling and prevent its settling. For practical purposes, planing-mill shavings form the best insulating filling. These are pressed in bales and can be bought at very reasonable rates. They are thoroughly dry and are little affected by dampness. Planing-mill shavings are much better than sawdust, being a poorer conductor, less easily affected by moisture, and less liable to settle.

1535. Table 32 gives a list of the best non-conductors and their relative values as insulators.

TABLE 32.

HEAT TRANSMITTED THROUGH MATERIALS OF THE SAME AREA AND THICKNESS.

Pine	100
Mineral Wool	80
Granulated Cork	65
Wood-Ashes	50
Sawdust	55
Charcoal (powdered)	65
Cotton	35
Paper	25

1536. Insulating Paper.—There are two first-class insulating papers now made in this country, one known as the P. and B. paper, made by the Standard Paint Company, and the other known as the Neponset paper. These papers are water-proof and entirely odorless. This latter point is quite essential, particularly in cold-storage warehouses, where the slightest odor is liable to affect the goods in storage. The P. and B. paper is made in three qualities, known as the "Universal P. and B.," the "Giant," and the "Two-ply P. and B." The latter paper is found to be quite satisfactory for insulating purposes, and is not so expensive as the Giant.

1537. Details of Insulation.—Fig. 354 shows details of a system of insulation which has been found quite satisfactory for cold-storage work. The various parts and materials are properly marked, and the figure does not require any explanation.

Spruce is usually found to be the most satisfactory wood for insulating cold-storage rooms, as it does not possess the odor of white or yellow pine.

Upon completion, insulated walls, particularly in beef rooms, are often given a coat of varnish, but in cold-storage rooms whitewash is used, as a second coat of whitewash can always be easily applied, thereby thoroughly cleansing the walls and sweetening the atmosphere of the room.

In buildings where iron construction is used, the wrought-iron or hollow cast-iron columns which pass through the

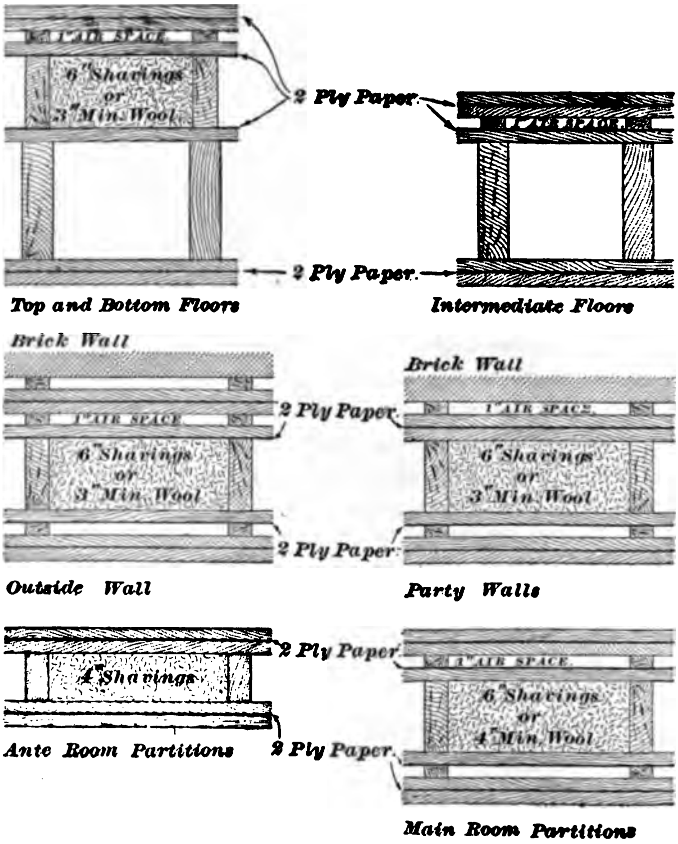


FIG. 854.

rooms should be insulated very carefully to prevent any conduction.

In all the foregoing illustrations it will be noticed that in case of brick walls an air-space is generally made by tacking strips to the wall. The object is to allow any moisture that may accumulate on the wall to run down the face of the wall all the way through to the cellar, without injuring the insulation of the building. It is advisable to have a sill or plate at the foot of the insulator, around the walls of the room, to keep the insulation from getting wet when the floor is washed.

1538. Insulation of Exposed Surfaces.—Exposed pipes or other surfaces through which a refrigerating medium is passing will soon be covered with frost. This is due to the moisture in the atmosphere being deposited on the pipe and then frozen. To thoroughly insulate such surfaces, it is not only necessary to keep the heat from passing through them, but to keep the air out; in other words, to make the insulation perfectly air-tight. It is also necessary to have the insulation consist of such substance as will not readily be injured by moisture. Hair felt was largely used in former years; this was wrapped around the pipe and then covered with paper; then more felt and more paper, and so on for several layers; then the whole was sewed in a canvas envelope and covered with several coats of good water-proof paint. Such insulation is found to be comparatively efficient and will last several years, but eventually the air will work through the hair felt, and then the pipes will begin to sweat, and if left long enough, freeze.

The "Voorhees" covering is a much better insulation. It is made by soaking the hair felt in boiling resin and is applied to the pipe while the resin is still hot enough, so that the sheets of felt can be easily molded to the shape of the pipe. These sheets are tied fast with heavy twine; two layers of the covering are applied, and after the resin has cooled and set the string is cut off. This insulation is probably the best on the market, but is very expensive, except in large quantities, as it is necessary to prepare it on the spot.

The best sectional insulation is the Nonpareil Cork covering, which consists of granulated cork soaked in a composition and molded up in the form of sectional insulation similar to magnesia sectional steam-pipe covering. Two layers of this are applied so as to break joints, and the whole is given a coat of rubber cement, which makes it practically air-tight.

In case of large brine mains that are boxed or laid under ground, a mixture of ground cork and pitch is found to give very satisfactory results. The pitch is first melted, then the cork is mixed with it, and the mixture is poured over the pipes and well tamped around them.

COLD STORAGE.

COLD-STORAGE WAREHOUSES.

1539. Object of Cold-Storage Warehouses.—Cold-storage warehouses are used for the preservation of perishable goods by means of a low temperature, which prevents decay. The goods are placed in rooms, and there kept sometimes for months, separate rooms being usually necessary for different kinds of goods, not only because different substances often act on and affect one another, but also because, as experience shows, they require different temperatures for their preservation.

1540. Conditions to be Maintained. — For the proper preservation of goods, it is necessary (1) that the air should be often renewed; (2) that the air should have the proper amount of moisture; (3) that the temperature should remain within certain limits.

The first requisite is obtained by a proper system of ventilation, to be described later; the second by a careful use of the hygrometer and psychrometer for ascertaining the relative humidity of the air, care being taken not to have the air too dry, as this may result in the evaporation of the goods, nor too damp, as this will cause mold or must. The amount of moisture that air can contain increases with the temperature. If when the air enters the room it is very damp, some of the moisture will be precipitated as the temperature falls, and adhere to the pipes, where it will freeze. Care should be taken to keep the pipes as free from this frost as possible.

The temperature is controlled by the refrigerating-machine. For regular public cold storage, the brine-circulation system has in general been found somewhat superior to the direct-expansion system. The latter has often been tried, but has been abandoned in some cases, owing to the risk of the ammonia leaking and to the difficulty in maintaining a steady temperature.

1541. General Design.—The majority of public cold-storage warehouses are built of brick or stone and preferably of the slow-burning mill construction, the best design being that of the perfect cube, as this gives the least exposed wall and roof area for the greatest cubical contents. In order to avoid the heat from the boiler and engine, they are placed in a separate building adjoining the main storage building. As a rule, the freezing rooms are placed together on the lower floors of the building, the cellar being often used for keeping frozen fish, etc. Such rooms have concrete floors, and the ground under these floors is frozen through for several feet, owing to the constant low temperature maintained in the rooms.

It is bad practice to place the freezing rooms above cold-storage rooms, as, in spite of the best insulation, the ceiling of the cold-storage rooms will sweat, owing to the cold striking through the floor of the freezing room and the moisture in the air of the cold-storage room naturally condensing on these cold surfaces.

REFRIGERATION REQUIRED.

1542. The refrigeration required for cold-storage rooms may be divided into two parts, viz.: (1) that required to keep the room at the required temperature, by preventing radiation through the walls; (2) that required to cool the articles brought in the room from their temperature to the temperature of the room.

1543. Refrigeration Required to Keep the Temperature of the Room.—The refrigeration required to keep the room at a constant temperature is computed by means of the following general formula:

$$H = c\,A\,(t - t_1), \qquad \textbf{(136.)}$$

in which H = B. T. U. of refrigeration required to maintain a given space at a certain temperature t_1, when this space is separated from another, in which the temperature is t, by a surface whose area in square feet is A; and c = constant depending upon the material and thickness of the substance separating the two spaces. The value of the constant generally varies between 2 and 5. For rough estimates, it may be taken as equal to 3.

The preceding refrigeration may be reduced to tons by dividing by 285,300; that is, the amount F of refrigeration, expressed in tons, is

$$F = \frac{c\,A\,(t - t_1)}{285{,}300} = .000003505\,c\,A\,(t - t_1). \qquad \textbf{(137.)}$$

1544. Table 33 contains values of c as given in Siebel's *Mechanical Refrigeration.*

For double floors and ceilings, air-tight and well filled, so as to prevent the ingress of air, c may be taken as 2. When a room is separated from the outside by a hermetically closed air-space between two walls, the value of c for the outside wall may be used in the formula, but instead of the temperature t_1, a mean should be taken between t_1 and t, which is equivalent to using $\frac{1}{2}(t - t_1)$ instead of $t - t_1$.

For a wall consisting of several materials, the coefficient c may be found from the formula

$$c = \frac{1}{\frac{b_1}{c_1} + \frac{b_2}{c_2} + \frac{b_3}{c_3} + \ldots}, \qquad \textbf{(138.)}$$

in which b_1, b_2, b_3, etc., are the thicknesses, and c_1, c_2, c_3, etc., the corresponding values of c for the several materials composing the wall.

In large cold-storage warehouses of 250,000 cubic feet or over, 1 ton of refrigeration will maintain 10,000 cubic feet of well-insulated space at a temperature of 30°, and 5,000 cubic

TABLE 33.

VALUES OF THE COEFFICIENT *c* IN FORMULA 137.

Partition.	Thickness, Inches.	*c.*
Single Windows		12.0
Double Windows		7.0
Pine Wood	12	2.0
Mineral Wool	12	1.6
Granulated Cork	12	1.3
Wood-Ashes	12	1.0
Sawdust	12	1.1
Charcoal (powdered)	12	1.3
Cotton	12	0.7
Soft Paper Felt	12	0.5
Brick	4½	5.5
"	9	4.5
"	14	3.6
"	18	3.0
"	27	2.6
"	36	2.2
Stone (masonry)	6	6.2
"	12	5.5
"	18	5.0
"	24	4.5
"	30	4.3
"	36	4.1

feet at 15°. In small warehouses of 50,000 to 100,000 cubic feet capacity, 1 ton of refrigerating effect will maintain 6,000 cubic feet at cold-storage temperatures and 3,000 at freezing temperatures. These figures do not include the refrigeration required to cool the goods.

1545. Refrigeration Required to Cool the Articles.—The amount of refrigeration required to reduce the goods from their temperature t to the temperature t_1 of the room is given by the formulas

$$H = (w_1 s_1 + w_2 s_2 + w_3 s_3 + \ldots) (t - t_1), \qquad (139.)$$

$$F = \frac{H}{385,300} = .000003505 (w_1 s_1 + w_2 s_2 + w_3 s_3 + \ldots)(t - t_1), \qquad (140.)$$

in which w_1, w_2, etc., are the weights in pounds of the different kinds of produce to be cooled, s_1, s_2, etc., their corresponding specific heats, and H and F are refrigeration units in B. T. U. and tons of refrigeration, respectively.

1546. The following table contains the specific heats of several articles, together with their latent heats of freezing:

TABLE 34.

SPECIFIC HEAT AND LATENT HEAT OF FREEZING OF VICTUALS.

Substance.	Specific Heat.	Latent Heat of Freezing, B. T. U.
Beef	.80	110
Veal	.70	90
Mutton	.80	110
Pork	.60	72
Eggs	.75	100
Vegetables	.90	125
Cream	.70	90
Milk	.90	125
Fish	.85	115
Lobster	.80	100
Oysters	.85	115
Chicken	.80	110

EXAMPLE.—Find the refrigeration required for a room 35′ × 50′ × 12′ in which are placed 25,000 pounds of beef daily, the temperature of the meat being 95°, that of the atmosphere 70°, and that of the rooms 35°.

SOLUTION.—(1) The refrigeration required to keep the room at 35° is given by formula **137.** Here we have

$$\text{Area of walls} = 2 \times 35 \times 12 + 2 \times 50 \times 12 = 2{,}040 \text{ sq. ft.}$$
$$\text{Area of floor and ceiling} = 2 \times 35 \times 50 = 3{,}500 \text{ sq. ft.}$$
$$A = 5{,}540 \text{ sq. ft.}$$

The values of t and t_1 are 70° and 35°, respectively, and therefore $t - t_1 = 35$. Taking the value of $c = 3$, the formula gives

$$F = .000003505 \times 3 \times 5{,}540 \times 35 = 2.04 \text{ tons.}$$

(2) The amount of refrigeration required to cool the beef to the temperature of the room is given by formula **140.** Here $t = 95°$, $t_1 = 35°$, $t - t_1 = 60$, $w_1 = 25{,}000$, $s_1 = .8$ (see Table 34), and since there is no other article, the terms $w_2 s_2$, $w_3 s_3$, etc., are left out; hence,

$$F = .000003505 \times 25{,}000 \times .8 \times 60 = 4.21 \text{ tons.}$$

The total amount of refrigeration required is, therefore,

$$2.04 + 4.21 = 6.25 \text{ tons. Ans.}$$

METHODS OF COOLING STORAGE ROOMS.

1547. General Considerations — The Three Methods of Cooling Generally Used. — The first requirement of a cold-storage room is good insulation. The best methods and materials for insulation have already been described. All windows and other openings by which light is admitted to the room should be closed with light-tight shutters, so as to prevent any daylight from entering. If this is not done, radiant heat will enter the room. Such windows can be closed with large shutters on the inside, similar to ice-house doors, which can be opened for ventilation when desired. A cold-storage room should be provided with some means of ventilation, so that the rooms can be thoroughly sweetened and the air renovated between seasons, when the rooms are not in use. Modern houses are equipped with a ventilating device, to be presently described, which permits of the ventilation of a cold-storage room while in use.

There are three methods employed at present for maintaining the temperatures of cold-storage and freezing rooms. The first is by means of direct radiation, where the brine pipes are run in the room and the brine is allowed to circulate through them. This was the original system and is the simplest and cheapest one to install. The second is by indirect radiation, where the pipe coils are place in a coil bunker on the upper floor and the air is allowed to fall to the floors below by gravity, and on being warmed is returned to the coil bunker by means of flues or ducts arranged for that purpose. The third and most approved method of cooling is by means of a fan or blower, which sets the air in circulation. This system is quite similar to the blower system of heating and ventilating. It consists of a fan or blower, which draws the air from the cold-storage rooms, blows-it through a system of pipe coils, and then delivers the cold air back to the cold-storage room by means of ducts arranged specially for the purpose.

1548. Direct Radiation.—In this method of cooling,

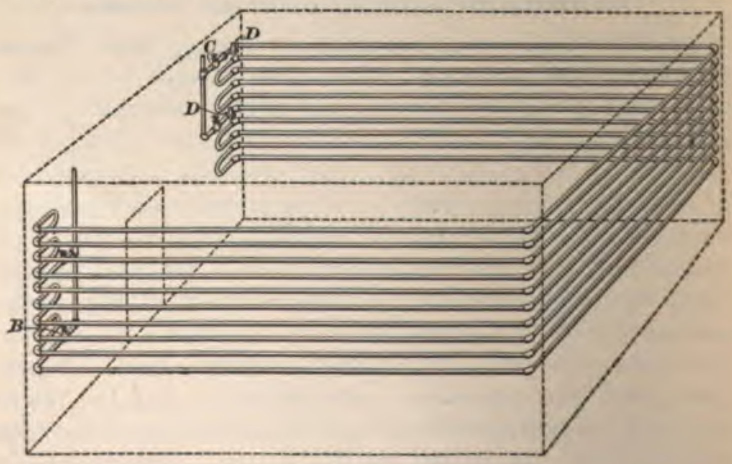


FIG. 355.

the most important feature is the arrangement of the pipe coils. These coils should be kept away from the wall at least 6 inches by means of furring studs nailed fast, on which the pipe hangers carrying the coils are fastened. This allows a circulation of air between the coils and the wall and prevents the frost from reaching the insulation in case of a heavy coating. There should be at least two coils to a room, and if possible more, so that the temperature can readily be regulated by means of cocks or valves provided on the inlets and outlets of these coils. The usual method is to allow the brine to enter the bottom of the coil and come out of the top, the outlet near the top being provided with an air-cock for the removal of air.

Fig. 355 shows a cold-storage room with the door at one end and provided with two coils of the return-bend pattern. The inlet to the coils is shown at *B*. The outlet is

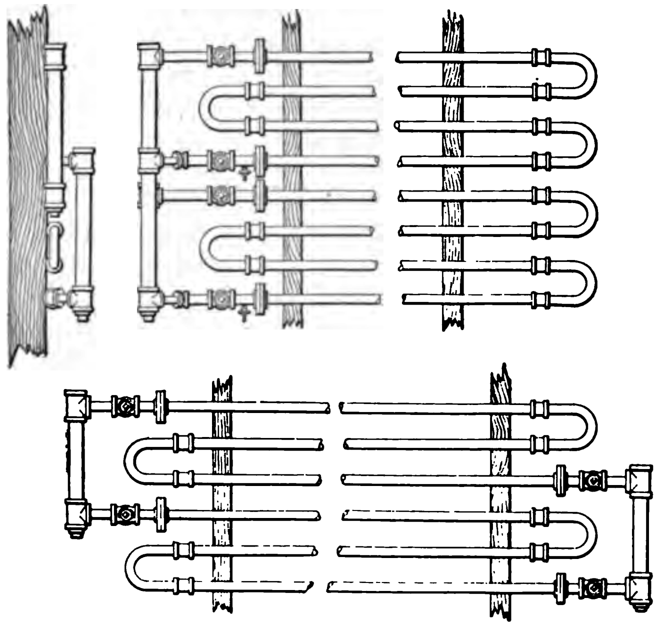


FIG. 356.

provided with a cock *C* and an air-valve *D*. These are both quite near the door, so that if the room is well filled with goods, the floors can be readily cut out for the purpose of regulating the temperatures and also closing off the coils in case of a leak. Fig. 356 shows several methods of coil connections in detail. The upper coil should be as close to the ceiling as possible, so as to prevent any chances of a warm-air pocket lying next to the ceiling. In case a girder passes through the room, it is best to elbow the first and second pipes under it, so as to thoroughly cool the upper portion of the room. The arrangement of the coils should be such that all portions of the room will be equally cooled by each coil. In very large rooms this is impossible, but ordinarily the coils run around all the walls of the room so as to permit this. Drip-pans should be provided under the coils for catching the melted frost when the coils are shut off. The latter should drain into a tub or barrel at one end. It is best not to connect these drip-pans into a series of drain-pipes, as these connections are liable to freeze up in being carried through freezing rooms; and even if kept clear, they let in from outside a certain amount of warm air laden with moisture. The temperature in the room is regulated by either closing off or partially throttling the outlets to the various coils. In throttling the outlet, the coil is kept full of brine, and there is no opportunity for air to accumulate, as there is in case of the inlet being throttled. 1¼-inch full-weight black iron pipe is commonly used for brine circulations. If the brine temperature is 0°, 1 lineal foot of pipe per 15 cubic feet will give a temperature of 30°; 1 lineal foot of pipe per 6 cubic feet will give a temperature of 10°, and 1 lineal foot of pipe per 3 cubic feet will give a temperature of 7°. With 15° brine, 1 lineal foot of pipe per 10 cubic feet will give 30°, and 1 lineal foot per 4 cubic feet will give 20°.

As a rule, all freezing rooms are equipped with direct piping, as there is no objection to the goods being near the pipes, and an exact temperature is not necessary, two or three degrees variation having no effect on the quality of the goods. For ordinary purposes, such as the keeping of tierced meats, butter, etc., direct-piped rooms may be used; but for good results in keeping fruit, cheese, eggs, etc., the indirect method or forced-air circulation is recommended. In case a house is already piped with direct radiation, this can be modified so that it will give the same results as indirect radiation, by putting aprons over the coils, as shown in Fig. 357, which represents a section of the room, on the walls of which are placed 2″ × 6″ planks *A* carrying the coils of the pipe *B*. These are provided with a drip-pan *C*. In front of these coils, a number of 2″ × 4″ studs *E* are fastened, and to these are nailed the boards *D*; the studs are toenailed to the ceiling. A false ceiling *F*, slanted slightly towards the coils, is built a few inches below the ceiling of the room. This arrangement creates a natural circulation or draft, as shown by the arrows. The warm air passing to the top of the room is cooled by means of the coils, falls back of the aprons *D*, and goes out of the opening at the bottom. With this arrangement and proper temperatures, eggs may be carried with results almost as good as those of a forced-circulation house, and better than those of an indirect-radiation house, the only disadvantage being that the coils are in the same room; but if these are carefully attended to and the drip-pans are kept clean and empty, the results will be as satisfactory as those obtained by the indirect method.

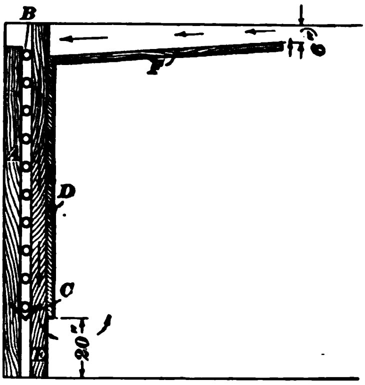


FIG. 357.

1549. Indirect Radiation.—Fig. 358 shows the section and Fig. 359 a plan of a cold-storage warehouse arranged for indirect radiation. It consists of the coil-bunker room *A*, in which are placed the coils *B*, on posts. These are provided with a drip-pan *C*. The cold-air duct *D* communicates with the room *E*. Alongside of *D* is another cold-air duct *D′* communicating with the room *G*, and a third *D″*

communicating with the room *F*. (See Fig. 359.) These ducts are provided with dampers *H*, *H*. The warm air from the room *E* passes up the duct *K*, and so to the top of the

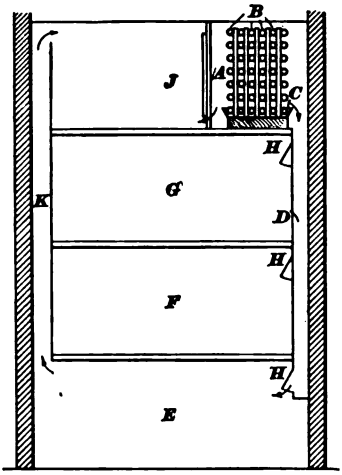


FIG. 358.

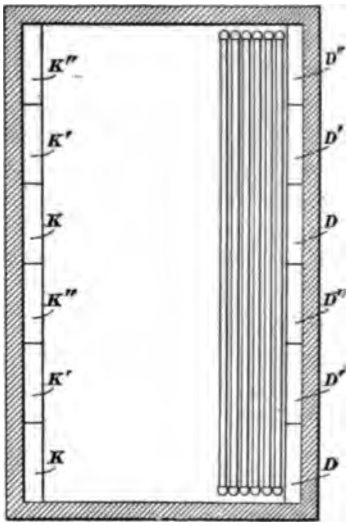


FIG. 359.

room *J* into the coil bunker *A*, where it is cooled and again circulates through the various rooms. The path of the air is shown by arrows.

By this arrangement, any of the rooms may be closed off by shutting the dampers *H*. The duct *K* should be 25 per cent. larger than the duct *D*; and the duct *D* should have an area of 3 square inches per square foot of floor area of the room. This applies to cold-storage rooms having a clear ceiling height of about 10 feet. This proportion between the area of the duct and that of the room will give a good circulation of air for a difference of temperature of 2° between the floor of the room *E* and the ceiling of the room *J*.

Fig. 359 gives a plan of the coil bunker and the room *J*, showing the subdivision of the ducts for the various lower rooms, as *D*, *D'*, *D''*, and *K*, *K'*, *K''*. By having all the coils in one coil bunker, as shown here, the advantage is gained that the temperature may be ascertained without going in more than one room, and therefore without letting heat into the other rooms by opening them. In this way it is comparatively easy to keep a constant temperature. Besides, the complete turning off of one coil will only have one-quarter of the effect on the temperature that it would have if each room had its independent series of coils, for if each room had two coils, the shutting off of one would cut down the radiating surface in the room 50 per cent. If, on the other hand, the coil bunker is supplied with 8 coils, the shutting off of any one of them cuts down the radiating surface but 12½ per cent. Furthermore, all the drip from the coils is concentrated in the coil bunker, and when the frost is scraped off the pipes, it falls into the drip-pan *C* and can easily be removed.

The disadvantage of this system is that all the rooms should be devoted to the carrying of one product; consequently, if there are not enough goods to fill all the rooms, some space is lost, and no other goods can be placed in the vacant rooms. This is specially true in the case of eggs, for which this system is particularly adapted. The quantity of pipe required to maintain a certain temperature is essentially the same as that required with direct radiation.

1550. Forced-Air Circulation.—The general arrangement of an air-circulating system is shown in Fig. 360. The circulation is accomplished by means of a fan or blower *B*, which is placed in the cooling room *A*. The air is drawn from the various cold-storage rooms through the duct *E* into the central opening in the fan case, and is driven by the

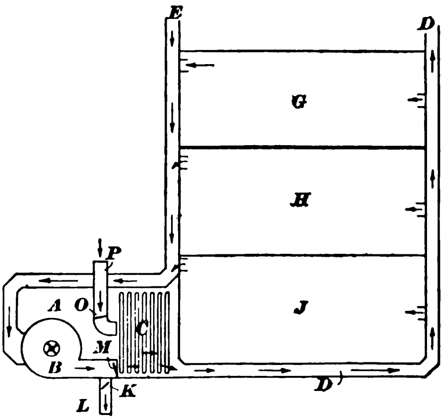


FIG. 360.

action of the fan through the brine coils *C* and out through the delivery duct *D* into the rooms *G*, *H*, and *J*.

An advantage of this system is that the bunker room *A* can be placed on the first floor or in the basement of the building, as it is not necessary to have this room either directly above or below the various cold-storage rooms. The drip from the coils of the bunker will in no way injure the goods stored in the rooms, as in the case of the indirect-radiating system, when the catch pan leaks.

Another advantage of this system is that the rooms may be easily ventilated. This is done in the following manner: The fan *B* is provided with an extra outlet pipe *L*, in which is a damper *K*. When the fan is running, the air is drawn from the various rooms, and by opening the damper *K* and closing the damper *M*, the air, instead of passing through the coils *C*, is blown outdoors through the pipe *L*. The damper *O* in the pipe *P* is then slightly opened; this permits fresh air to enter the coil bunker *A*. The air passes through the coils *C* and is cooled by means of them to the required temperature after it enters the rooms *G*, *H*, and *J*. This arrangement allows all the dead air to be drawn from these various rooms and fresh air to be admitted. In some houses, a separate series of coils and ducts, entirely independent of the main circulating system, are arranged for the purpose of ventilating the various rooms.

This indirect forced-draft system is especially good for storing eggs; it creates a circulation in the rooms, and consequently keeps the air thoroughly fresh, besides keeping the temperature very nearly uniform throughout the room. Care should be taken, however, not to run the fan or blower too fast, as too much air circulation tends to evaporate the eggs. The best way to regulate the amount of air is to run the fan at such a speed as to keep a difference of about 2° between the temperature of the inlet air to the room and that of the air returning to the bunker. This difference should not be less than 1°, or a good deal of evaporation will be noticed in the eggs.

The quantity of pipe required with a forced draft is about the same as for direct radiation, or, approximately, 1 foot of pipe for every 30 cubic feet of space. The coils of pipe should be so arranged in the bunker as to break the currents of air and yet be far enough apart so that there is no danger of a solid wall of snow forming between the coils.

1551. Sharp Freezing.—For the purpose of freezing fish rapidly, a **Sharp Freezer,** such as is shown in Fig. 361, is employed. This consists of a series of return-

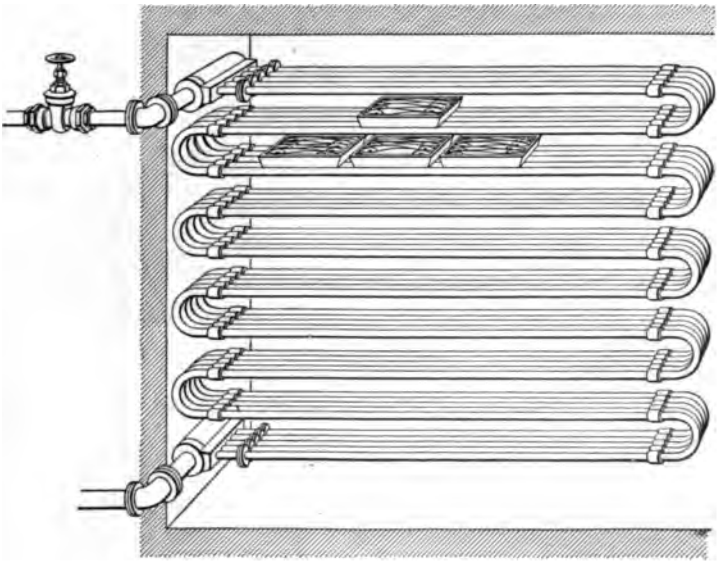


FIG. 361.

bend coils, one back of the other, the pipes of which form a series of shelves, one under the other. The brine or direct-expansion ammonia is allowed to enter the top header connecting these various coils, and the return is taken from the bottom header. On the shelves formed by the pipes are placed galvanized-iron pans, 20″ × 36″ at the bottom and about 4 inches deep, filled with fish. The doors covering these coils are then closed and the fish are allowed to remain in the freezer for 15 to 20 hours. They are then removed from the pans and packed in boxes about the size of the pan. Before the fish are placed in the pans, they are usually dipped in cold water, so as to remove any salt water or fish oil that may adhere to them; this also forms a thin coat of ice, which helps to preserve the fish. If brine circulation is used, a temperature of 10° to 15° below zero is required to obtain a temperature of 0° in the freezer.

If a plant is insulated for the sole purpose of freezing fish or meat, an allowance should be made at the rate of one ton of refrigerating effect per ton of meat or fish frozen in every twenty-four hours, in addition to the space cooled.

STORING VICTUALS.

1552. Storage of Eggs.—For the successful cold storage of eggs, a room should be equipped with either the indirect-radiating or the forced-air-circulating system. If this is not possible and direct radiation is already installed, the coils should be provided with aprons, as explained in Art. **1548.** A temperature of 30° to 31° should be maintained in the rooms, and the relative humidity of the air should be kept at between 65 and 70 per cent. of saturation. The eggs should be placed in new cases which have been thoroughly dried, together with their fillers, by placing them in a dry house or over the boilers. The eggs should also be thoroughly candled before being put in the house, and only those that are perfectly sound should be stored. When the eggs are taken out, they should be placed in a comparatively cold room, so that they will get warm gradually, which will prevent them from sweating to any great extent.

1553. Storage of Dairy Products.—Milk and cream can be kept for a considerable length of time if properly cooled before storing. This cooling is effected by means of a cooler built on the same general plan as the Baudelot cooler used in breweries, but having tinned-copper pipes instead of iron pipes for the circulation of brine.

When the milk is allowed to run over these pipes, all the animal odor is eliminated, and the milk preserves its sweetness much longer than when placed in cans while warm and then put in a cold-storage room.

Cheese is usually kept at a temperature of 34° to 36°. The room should be kept comparatively dry, with a relative humidity not exceeding 70 per cent., so as to avoid molding. A temperature lower than 34° is likely to make the cheese granular, and does not help to preserve the quality any better than the higher temperature.

Butter is now frozen and kept at a temperature of 10°. This temperature has been found to preserve the flavor much better than the higher temperatures used in former days. Butter is usually stored in 60-pound firkins. Small packages are not carried successfully, as it is very difficult to prevent the outer crust of the butter from losing its flavor and becoming somewhat stale. It is therefore better to use large firkins and make prints after the butter is taken from the storage rooms than to carry the prints themselves in storage.

1554. Storage of Fruit.—Winter apples are carried at temperatures as low as 28°, which is just above their freezing point. Care should be taken that the fruit is of a good quality and perfectly sound, in order to prevent decay, which causes fermentation, with the production of carbonic-acid gas and a disengagement of heat. This heat will raise the temperature of the room, thereby aiding in the decomposition of other apples in the immediate neighborhood. The carbonic-acid gas resulting from fermentation is also rather detrimental to the proper keeping of fruit.

Bartlett pears should be picked green and stored in boxes or small barrels. They can be carried successfully from 60 to 90 days, if care is taken to keep the room dark and at a temperature of 32° to 34°.

All citrus-fruits, such as lemons and oranges, should be carried at a temperature of 33° or 34°, and should be entirely isolated from any dairy products and eggs. If possible, rooms should be arranged that have separate entrances from those going into the egg and butter rooms. The odor from lemons and oranges is very penetrating and gives a very disagreeable flavor to eggs, making them almost unsalable.

1555. Temperature Required by Different Articles.—Table 35 gives the temperatures at which several goods are usually carried. The usual rule is to carry an article just above its freezing point. There are a few exceptions to this rule, such as cheese; but if care is taken to carry the room just above the freezing point and keep the air sweet and dry, good results are almost always attained.

TABLE 35.

STORAGE TEMPERATURES OF VARIOUS PRODUCTS.

Article.	Storage Temperature.
Apples	30°
Berries	34°
Butter	10°
Beer	36°
Cheese	34°
Dried Fruit	36°
Eggs	30°
Fresh Meat	33°
Frozen Meat	20°
Fish	15°
Furs	5°
Grapes	33°
Lemons	33°
Oranges	33°
Peaches	34°
Pears	33°

REFRIGERATION FOR PACKING HOUSES.

1556. Chilling, Storage, and Freezing Rooms.— In packing houses there are two kinds of rooms to be cooled, viz., **chilling rooms** and **storage rooms.**

Chilling rooms are those in which the carcasses are placed immediately after the animals have been slaughtered, in order to cool them from 95° (blood heat) to the storage temperature, which is about 35°. The temperature of chilling rooms is kept at about 28°.

After the meat has cooled to about 35° or 32°, it is taken to the storage rooms, where it is kept. The temperature of these rooms is about 35°.

It is often desirable to freeze the meat. This is done in special rooms called **freezing rooms,** which are kept at a temperature of 10° or under.

1557. Refrigeration Required.—To determine the exact amount of refrigeration required, the chill rooms, cellars, storage rooms, etc., should be carefully measured; the amount of refrigeration is then found by the rules given under cold-storage refrigeration. To this should be added the refrigerating effect required to cool the meat, as given by formula **140.** Beef, after being slaughtered, is usually hung in a well-ventilated room for several hours. The temperature of the meat, however, is reduced but little, and so it is customary to figure on the supposition that the beef enters the chill room at a temperature of about 95° (this is the value of t in formula **140**). The beef must then be cooled to at least 35° (this is the value of t_1 in the same formula). The same applies to other animals.

If the weights of carcasses of different kinds are represented by w_1, w_2, w_3, etc., and the number of carcasses of each class by n_1, n_2, n_3, etc., the weights to be cooled will be $n_1 w_1$, $n_2 w_2$, $n_3 w_3$, etc. Substituting these values for w_1, w_2, etc., in formula **140,** and putting $t - t_1 = 95° - 35° = 60°$, we get, for the tons of refrigeration required,

$$F = .000003505\,(n_1 w_1 s_1 + n_2 w_2 s_2 + \ldots .)\times 60$$
$$= .0002103\,(n_1 w_1 s_1 + n_2 w_2 s_2 + \ldots .). \qquad \textbf{(141.)}$$

EXAMPLE.—In a certain abbatoir there are 5,000 cubic feet of chill-room space, 10,000 of pickling space, and 20,000 of storage. 100 hogs and 30 beeves are killed daily, the average weight of the hogs being 240 pounds and that of the beeves 750 pounds. How much refrigeration is required?

SOLUTION.—Total space = 5,000 + 10,000 + 20,000 = 35,000 cu. ft. According to Art. **1544,** 1 ton of refrigeration will be required for 6,000 cu. ft. of space. Therefore, the refrigeration required to cool 35,000 cubic feet is

$$\frac{35,000}{6,000} = 5.83 \text{ tons.}$$

To apply formula **141,** we have $n_1 = 100$, $n_2 = 30$, $w_1 = 240$, $w_2 = 750$, $s_1 = .6$, $s_2 = .8$ (see Table 34). Substituting these values in the formula, we get

$$F = .0002103\,(100 \times 240 \times .6 + 30 \times 750 \times .8) = 6.81 \text{ tons.}$$

The total amount of refrigeration required is, then,

$$5.83 + 6.81 = 12.6 \text{ tons, nearly.} \quad \text{Ans.}$$

1558. For rough calculations, 1 ton of refrigeration may be taken to cool about 4,000 cubic feet of chill-room space and 8,000 to 10,000 cubic feet of storage or pickling room space.

In chilling, 1 ton of refrigeration will suffice for

20 hogs (average weight 250 pounds),
or 7 beeves (average weight 700 pounds),
or 50 calves (average weight 90 pounds),
or 70 sheep (average weight 75 pounds).

For freezing rooms, 1 ton of refrigeration per ton of meat frozen, including the space in which the meat is carried, is a good allowance.

1559. Chill Room.—The indirect system should be used in piping chill rooms for beeves, hogs, sheep, etc. If there is sufficient headroom, the pipes can be placed overhead in a bunker formed by means of drip-pans, as shown in Fig. 362, the beams for carrying the meat track forming the supports for the pans. If, however, the chill rooms were originally on the ice-bunker system, having the ice chamber overhead, the latter can be piped, and the drip from the coils will then be caught by the ice pans. By this arrangement, the steam which passes off from the meat is condensed in the ice chamber or coil bunker overhead, and to a large extent takes away the animal odor. Chill rooms

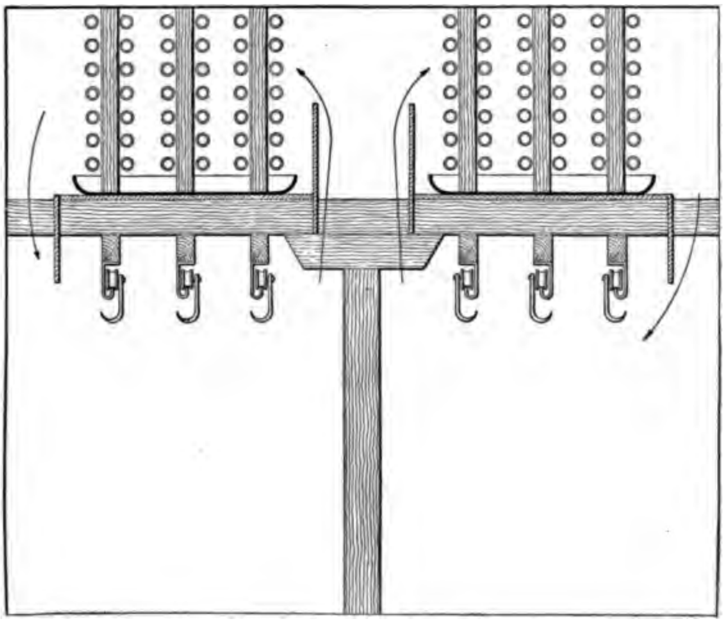


FIG. 362.

piped on the sides do not accomplish this, and often the meat coming from them is sour.

The quantity of pipe required to cool chill rooms is about as follows: For direct expansion, 1 foot of 2-inch pipe will suffice for 13 cubic feet of chill-room space, and 1 foot of 1¼-inch pipe will suffice for 9 cubic feet. In case of brine circulation with 15-degree brine going to the chill room, 1 foot of 1¼-inch pipe will suffice for 5 cubic feet of space. If the brine is at 0° or colder, the amount of pipe required will be the same as that given for direct expansion.

1560. Storage Rooms.—Storage or pickling rooms are piped overhead, if there is sufficient headroom; otherwise, on the side walls or along the posts. Their temperature is seldom below 5°, as pickling is prevented if the temperature goes below that point. These rooms are often kept at temperatures as high as 42° and 45°.

The quantity of piping required in case of direct expansion is as follows: 1 foot of 2-inch pipe will suffice for about 40 cubic feet of well-insulated space; 1 foot of 1¼-inch pipe will suffice for 30 cubic feet of space. For brine circulation with 15-degree brine, 1 foot of 2-inch pipe will cool 15 cubic feet of space, and 1 foot of 1¼-inch pipe will cool 10 cubic feet of space.

ICE MAKING.

SYSTEMS USED.

1561. There are now two systems in use for making ice, viz., the **can system** and the **plate system.** The can system is the more common of the two, being cheaper in first cost and requiring less attention in manipulation. The plate system, however, has the advantage of being more economical in the end and of giving a clearer ice.

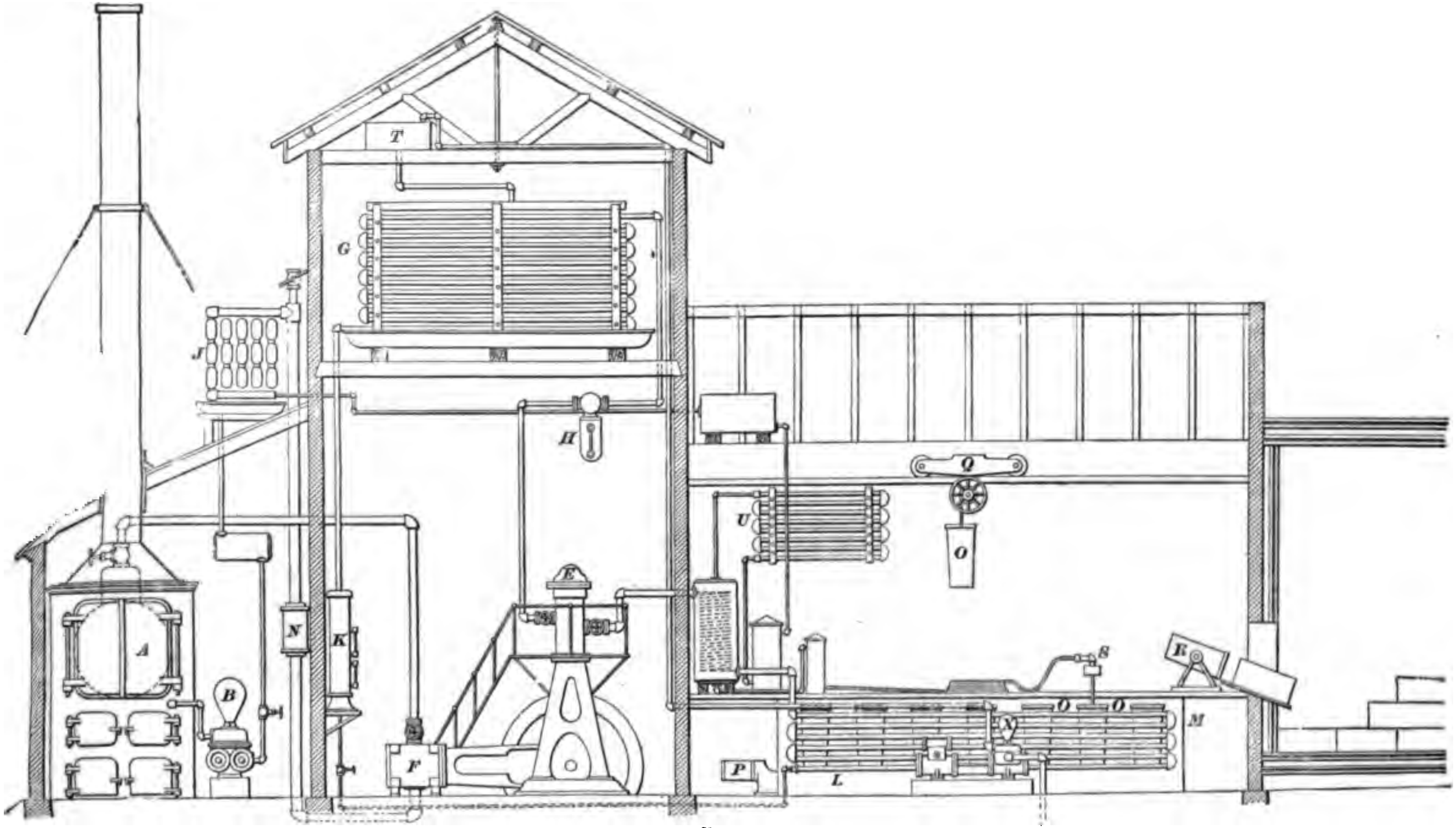
1562. The Can System.—The apparatus used in the can system consists of a large rectangular wood or iron tank containing the expansion coils or pipes. Galvanized-iron cans are placed between the rows of expansion coils. These cans are filled with distilled water, and when the brine is chilled below the freezing point, the water in the cans freezes. If the temperature of the brine is not allowed to fall below 25° and ordinary well-water is used in the cans, the ice produced will be comparatively clear on the outside and rather snowy in the center. If, however, the brine temperature is allowed to fall to about 15°, the ice will be entirely opaque.

To get good, clear ice, distilled water is used. This makes a bright, clear ice, with the exception of a small core or feather in the center. It is necessary, however, to have a good distilling apparatus. The white appearance in ice made from non-distilled water is due to minute air-bubbles which are held in suspension in the water and frozen in the ice, forming a sort of snow. In case of distilled water, this air is eliminated by boiling and subsequent evaporation. From this it will be seen that it is necessary to make enough distilled water in every twenty-four hours for the nominal ice-making capacity of the plant.

In the ordinary ice factory, the distilled water is usually made by condensing the exhaust steam of the engine operating the compressor in case of a compression plant, or by cooling the condensed steam that leaves the generator, still, or retort of the absorption plant. Ordinarily the steam that is required by a first-class machine of either kind is considerably less than the amount of ice that the machine can make in a given time. It is therefore necessary to draw live steam from the boiler and condense it, in order to make up the deficiency. It follows from this that the economy of the plant depends entirely upon the economy of the boiler.

The ordinary yield of a can plant is about 6 pounds of ice per pound of coal. There is a certain loss due to exhaust thrown away from auxiliaries, steam leaks, etc., that brings the boiler evaporation down to this figure, though there are some tests on record where a plant has done as well as 8 pounds of ice per pound of coal. The ordinary water consumption for condensing purposes, boiler, etc., is 4 gallons per minute per ton of ice made in 24 hours, the water being well-water at 60° or colder. With warmer water, a larger quantity has to be used.

1563. Plate System.—In the plate system, the refrigerating fluid is circulated through vertical cast or wrought iron hollow plates set on edge in a tank of water. These plates are usually about 16 feet long and 8 feet high. Ice begins to form on the plate and gradually extends out into the tank. If the temperature of the plate is kept comparatively high at the start, until 2 or 3 inches of ice is formed, and then gradually reduced as the ice formation increases, a clear cake or plate of ice will be formed on each


T
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side of the plate. The refrigerating fluid is then drained from the plate, and warm water is introduced, which thaws the ice adhering to the sides. As soon as the ice is detached, it floats to the surface of the water in the tank, and is then drawn up by means of a large traveling crane and cut up on a special table fitted for the purpose into cakes of the proper size.

From the above description, it will be seen that ordinary filtered water can be used in the cells of the plate system, and the expense of distilled water is avoided. Great care, however, must be exercised to prevent the plate from freezing too rapidly at the start, in order to avoid the formation of white ice.

Plate ice, in the average ice-making plant, requires from 8 to 12 days to freeze, according to the thickness of the cake and the quality of the watér from which the ice is made. In the can system, 24 to 60 hours is all the time required to freeze a cake of ice, according to the size of the cake. The difference in time in the two systems is due to the fact that in the plate system, freezing proceeds from one side only, whereas in the can system it proceeds from the two sides and the end.

The first cost of a plate plant is greatly in excess of that of a can plant. The floor area is also greater for the plate plant, but a good plant of this kind, properly put up and operated, is, once installed, by far more economical than a can plant, and gives much better ice.

COMPRESSION ICE-MAKING PLANT WITH CAN SYSTEM.

1564. Division of the Plant.—A complete can plant consists of the following parts:

(*a*) Steam-boiler plant.
(*b*) Refrigerating or ice-making machine.
(*c*) Freezing tank with accessories.
(*d*) Distilled-water system.

A complete can-system compression plant is shown in Fig. 363.

1565. Steam Plant.—For the purpose of getting good, clear can ice, it is necessary to have a steam-boiler of ample capacity, so that there will be no danger of the boiler foaming. A boiler that does not give good, dry steam invariably causes trouble in an ice plant, the ice becoming discolored and cloudy on account of the priming. Five horsepower per ton of ice to be made is a good allowance for the steam-boiler plant.

A well-equipped ice-factory boiler plant consists of the following parts (see Fig. 363):

(*a*) A good boiler with fixtures and stack.

(*b*) A boiler-feed pump.

(*c*) An injector, to be used in case of accident to the feed-pump.

(*d*) A feed-water heater.

In countries where the water is bad and contains alkalies, the steam-boiler plant should be in duplicate, and in case the water contains sulphates, a live-steam purifier can be used with advantage. Under these conditions, it is best to change boilers once every week or two, giving the spare boiler a thorough cleaning, removing all scale, etc.

1566. Refrigerating-Machine (see Fig. 363).—This consists of the ammonia compressor *E* with engine *F*, ammonia condenser *G*, oil trap *H*, receiver *K*, and expansion coils *L* in the ice tank *M*. Usually the ice-machine is placed in a room by itself, to prevent the dust from the boiler affecting the refrigerating machinery. The compressor is placed upon a solid stone foundation, and the condenser, usually of the atmospheric type, is placed above the compressor in a latticework tower, sufficient height being given to the condenser so that the water passing from it will flow to the exhaust-steam condenser *J*, and from there to the boiler-feed pump *B*. This allows the water to get quite warm before it is sent through the feed-water heater to the boiler.

To get economical results with a compression machine, the engine should preferably be equipped with some form of releasing valve-gear, as the Corliss. A plain slide-valve engine will use more steam to operate the compressor than the compressor would be able to freeze into ice. The common practice is to allow 2 tons of refrigerating effect per ton of ice to be made. The exhaust from the engine *F* is piped into the exhaust-steam condenser *J*, as shown in the figure. It is from this exhaust steam that the ice is made after the grease has been extracted by means of a separator *N*. If the engine requires more steam to operate it than it can freeze into ice, the machine is not economical, as a certain amount of exhaust steam has to be allowed to go to waste. With the usual allowance of 2 tons of refrigerating effect per ton of ice, and assuming 1½ horsepower in the steam engine to be required per ton of refrigeration, the power required will be 3 horsepower per ton of ice. This gives one-third of a ton, or about 667 pounds of steam per horsepower in 24 hours, or 28 pounds of steam per horsepower per hour. As an ordinary slide-valve engine uses from 40 to 50 pounds of steam per horsepower per hour, it appears that an engine with a releasing gear or some form of expansion-valve must be used, if the best effect is desired.

1567. Ice Plant.—The ice-making plant comprises the following parts (Fig. 363):

(*a*) Ice-making tank *M*.
(*b*) Ice cans or molds *O*, *O*.
(*c*) Grating for holding cans in position.
(*d*) Ice-can covers.
(*e*) Insulation of bottom and sides of tank.
(*f*) Brine agitator *P*.
(*g*) Crane *Q* with geared hoist and can lift.
(*h*) Ice-can dump *R*.
(*i*) Can filler *S* with hose.
(*j*) Expansion coils *L* with headers and valves.
(*k*) Brine hydrometer and thermometer.

1568. Ice tanks are usually made of steel, though in some plants wood is used. Steel tanks should be made of

$\frac{1}{4}$-inch steel for tanks 3 feet or more in depth, $\frac{3}{16}$-inch steel being heavy enough for tanks 30 inches deep. The tank should be properly braced and reinforced and have an angle-iron rim punched for bolt-holes around the top, so that the grating can be securely attached thereto.

1569. Expansion coils should be of extra heavy pipe and should run the full length of the tank, one coil between each row of cans. The coils should be continuously welded throughout their lengths, with tails carried through the end of the tank, stuffing-boxes being provided on the tank to prevent brine leakage. The coils are usually strapped in such a manner as to permit the grating to be supported by the straps, as shown in Fig. 364. There should be 100 square

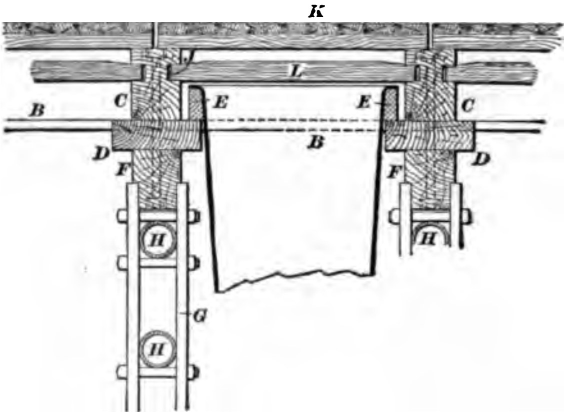

FIG. 364.

feet of coil surface for each ton of ice made. Each coil should be provided on its inlet with an expansion-valve and the outlet should be provided with a stop-valve.

1570. Ice Cans.—The ice cans or molds in which the ice is frozen are usually made of various sizes, and the cakes of ice formed in them usually weigh between 50 and 300 pounds.

To find the net weight in pounds of the cake formed in a can, find the volume of the can in cubic feet; multiply the result by 57.2 (weight of a cubic foot of ice) and take 5 per cent. off the product (this to allow for thawing). The result will be the required net weight.

The can should generally be made of 16-pound galvanized iron, although for small ones 18-pound galvanized iron may be used. The best cans are made of what is known as "Best Bloom stretched and patent leveled galvanized iron." Such iron is perfectly flat, and has no inequalities in it. Ice cans are usually made with a slight taper from the bottom upwards, which permits the ice to slide out easily after it has been thawed free of the can. The smaller sizes of cans have one joint along the sides, near one of the corners, and the larger sizes one or two joints. These joints should be riveted on 1-inch centers, with all rivet heads driven close. The seams should then be soaked with solder and floated flush. The bottoms are usually flanged and inverted into the body, the joint being made by riveting and soldering in place. The top of the can should be reinforced with a band of flat iron, and the edge of the can should be wired over. Two holes are punched in the long side of the can, under the band, for lifting.

1571. Tank Surface.—With brine at 15° in the ice tank, and a good circulation, an 8-inch ice can will freeze solid in about 24 hours, an 11-inch can in 30 to 36 hours, and a 14-inch can in 40 to 48 hours. In very warm weather, the brine sometimes rises to a temperature higher than 15°. Ice plants are therefore usually constructed with a real capacity greater than their nominal capacity; the ratio of the former capacity to the latter is called the **surface** of the tank. For example, a 10-ton plant having $2\frac{1}{2}$ surfaces would indicate that the total capacity of the ice cans was $2\frac{1}{2} \times 10 = 25$ tons. Plants having 8-inch cans are constructed with surfaces of $1\frac{1}{2}$ to 2; those having 11-inch cans with surfaces of 2 to $2\frac{1}{2}$, and those having 14-inch cans with surfaces of $2\frac{1}{2}$ to 3. These proportions permit of a slight rise in the brine temperature and also allow a certain quantity of ice to be carried frozen in the cans, which can be pulled in case of great demand.

1572. Grating and Covers.—The ice-can grating is usually made of hard wood, such as oak or ash, firmly bolted together with galvanized-iron cross-straps. Such a grating is shown in Fig. 364. It will be seen that the rim *E* of the ice cans rests on the galvanized-iron cross-strap *B*, which is mortised into the oak strip *D*. The upper strip *C* on which the can covers *K* rest is mortised out with a groove *J*. In this is placed a stick or button *L*, which firmly buttons the can down, so that it can not float up. The oak strip *F* is supported by means of the expansion-coil straps *G*, which are mortised into it, and which in turn support the expansion coil *H*. Through bolts firmly bolt *C*, *D*, *F*, and *B* together, making a very rigid framework. The can covers are usually made of two pieces of oak, well nailed together, with two layers of good insulating paper in between. The ends of these covers are sometimes hollowed out with a recess, which permits the cover to be raised easily, while other covers are provided with regular plates and handles for lifting.

1573. Insulation.—The ice tank should be insulated in the manner described in Art. **1479**, the grating being allowed to project over the edge of the tank and the insulation being nailed fast thereto.

1574. Agitator.—In order to get a high efficiency from the expansion coils in the tank and freeze the ice rapidly, agitators are used. They are of three classes, viz., centrifugal pumps, displacement pumps discharging into distributing pipes, and propellers.

The object of the centrifugal pump is to circulate a large quantity of brine, and thereby keep a comparatively even temperature. The usual method is to take the brine from one corner of the tank and discharge it into a header along the opposite side. This insures a comparatively rapid circulation and causes the ice to form much faster in the ice cans.

When, however, it is desired to have some cold storage in connection with the ice plant, and it is necessary to use a displacement pump, such as an ordinary steam-pump, to circulate the brine throughout the cold-storage rooms, the centrifugal pump can be done away with by properly arranging distributing pipes in the brine tank. Such arrangement is shown in Fig. 365. The brine-pump *B*,

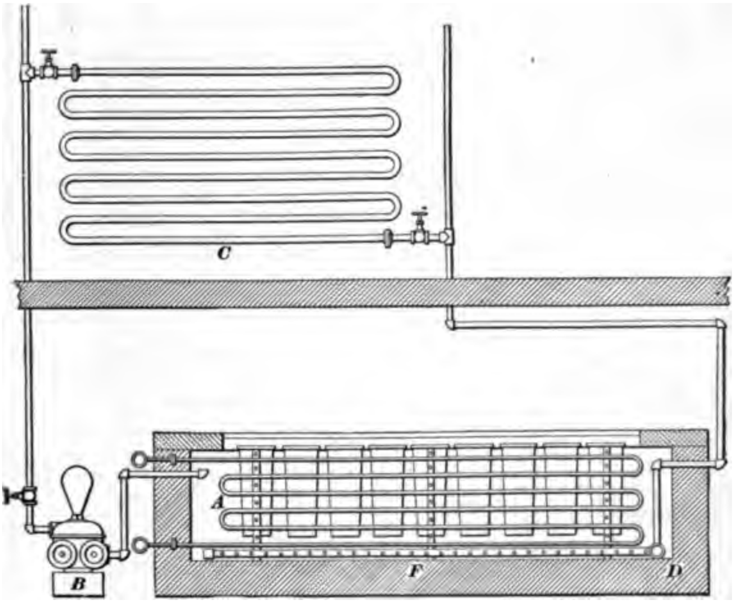

FIG. 365.

drawing the brine from near the top of the ice tank *A*, discharges it through the coils in the cold-storage room *C*. After passing through these coils, it returns to the brine tank and enters the header *D*; from there it is distributed through the distributing pipes *F*, which are perforated with small holes. It is advisable to place one of these distributing pipes under each line of expansion coils, as this permits the brine to impinge on the coils and increases their efficiency.

A propeller arrangement is shown in Fig. 366. The propeller *B* is driven by a belt or direct-connected engine at *A* and runs in a casing *E*. Wood partitions are built in the tank between the rows of cans and along the expansion coils *C*, *C*, *C*, as shown at *F*, *F*, *F*, the casing being made fast in the end of the partition. When the propellers are in operation, the brine is circulated as shown by the arrows. A 12-inch

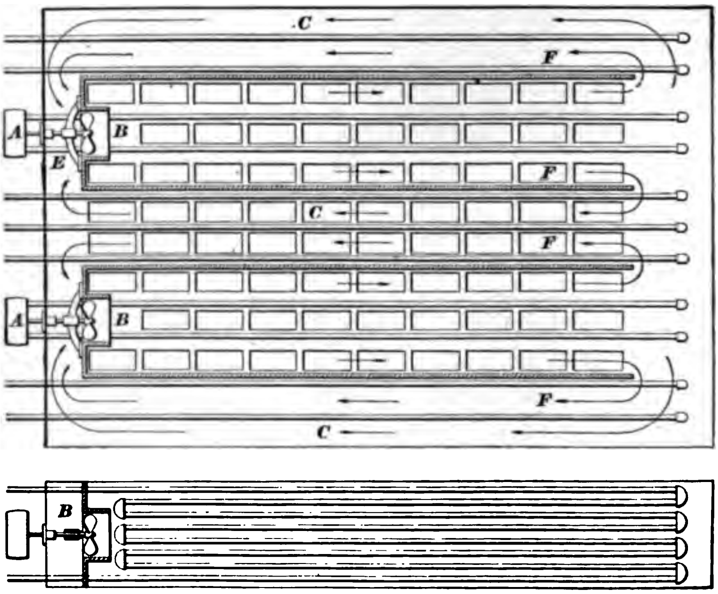

FIG. 366.

propeller will suffice for a 10-ton ice tank and an 18-inch propeller for a 15-ton tank. This arrangement makes a very efficient method of circulating the brine, though it requires more power to operate it than either of the two systems described above.

1575. Crane and Hoist.—Ice plants are usually provided with hand cranes of the traveling type. A crane of this kind consists of a light channel-iron frame on which moves a 4-wheel trolley provided with a geared hoist. On the drum of the hoist, a rope or chain is run, and to this is fastened a can latch, a common form of which is shown in Fig. 367. The latches *A*, *A* are provided with hooks on the under side; these hooks engage into the holes in the sides of the can *B*. When the ends of the latches are moved in the direction of the arrows, the hooks disengage and the can is released. With a traveling crane and a geared hoist, one man can handle from 15 to 20 tons of ice in 12 hours. If, however, the plant is larger in capacity and it

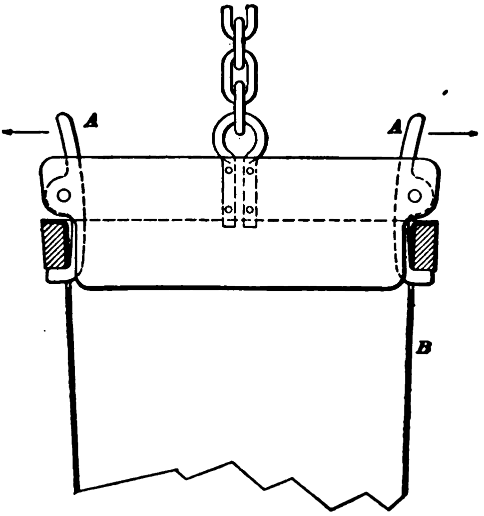

FIG. 367.

is desired to economize labor, a pneumatic hoist is recommended in place of the geared hoist. Pneumatic hoists are now made with a latch attached to the hook of the hoist that raises two cans from the tank at once. With such a device, one man can pull from 40 to 50 tons a day, or more than double the quantity he can pull with a geared hoist.

1576. Can Dump and Filler.—The can dump, or tilting frame, is shown in Fig. 368. It consists of a drip pan or tank *A*, to which are riveted fast a pair of supporting brackets *B*. These supporting brackets are provided with a hollow trunnion *C*, to which the water connection *D* is made. The box *E* is hung on the trunnions, and in it is placed the ice can *F*. The spray pipes *G* are also connected with the trunnion *C*. The arrangement of the trunnion is such that when the tilting box E is in the position shown by the full lines, no water enters the perforated pipes G; but when it is rocked on the trunnions and takes the position E', the water is automatically admitted into the pipes G, from which it flows over the ice can and thaws the ice out of it. The box E is so weighted that when the ice leaves the can, the box returns by itself to the first position and shuts the water off automatically.

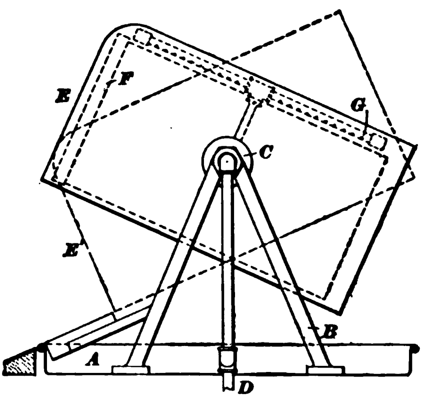

FIG. 368.

In some of the smaller plants, a dipping tank is used, which is kept partially filled with warm water. The can is immersed in this tank and then drawn out again, the ice being freed by this means.

1577. Filling the Cans.—After the ice is dumped out of the can, the latter is brought back to its original place in the tank. It is then necessary to refill it with water for freezing. This is accomplished by means of an automatic can filler. A hose is connected with the filters of the distilled-water system, and to this hose the can filler is attached. After the can is placed in its position in the ice tank, the can filler is inserted and a trigger with which it is provided is pulled. This starts the water running into the can. When the water has reached the desired level in the can, a ball float strikes the trigger and shuts off the water-supply. In this manner, any number of ice cans may be filled, the operation requiring little attention. All the cans are filled to exactly the same level.

1578. Ice-Storage Piping.—In an ice plant, it is advisable to have an ice-storage room that will hold a two or four week run, as this will permit the plant to be started several weeks in advance of the ice season, so that there may be a good supply on hand when the season opens. Then it will be possible to shut down the machine in case it is necessary to make slight repairs, and the increased demand during very hot weather may be accommodated. Care should be taken to properly insulate the room, to prevent radiation as much as possible. It should be sufficiently high to store two tiers of ice on end, and the piping that keeps the room cool should be arranged overhead, with a space of at least 2 feet between the top of the upper tier and the brine pipes. The walls should be furred out to a distance of 6 or 8 inches, and slats should be nailed on the furring pieces to prevent the ice from coming in contact with the warm side walls. In case 1¼-inch pipe is used for the piping, 1 lineal foot of pipe will suffice for about 10 cubic feet of space. These rooms should be kept at a temperature of about 28°, no lower temperature being advisable, as the ice is liable to check or honeycomb. The ice should be placed on end and a space of about ½ inch left between the cakes, and slate or boards should be placed over the first layer before the second is put in. The temperature of the ice-storage room should not be allowed to go above 30°, as the ice is liable to melt, and if this happens, the cakes will freeze together when the room cools down again and it will be necessary to quarry out the ice.

1579. Water-Supply.—An ice plant should be provided with an independent water-supply, such as a well, river, or lake, and have its own water-pump *X* (Fig. 363) for the purpose of circulating the condensing water. The water is usually piped first to a receiving tank *T*, from there to the ammonia condenser *G*, then to the steam condenser *J*, to the boiler-feed pump *B*, from which it is passed through the feed-water heater to the boiler *A*. A separate supply of water direct from the pump is usually taken to the distilled-water cooler *U*. With the water at about 60°, 4 or 5 gallons per minute are usually allowed for each ton

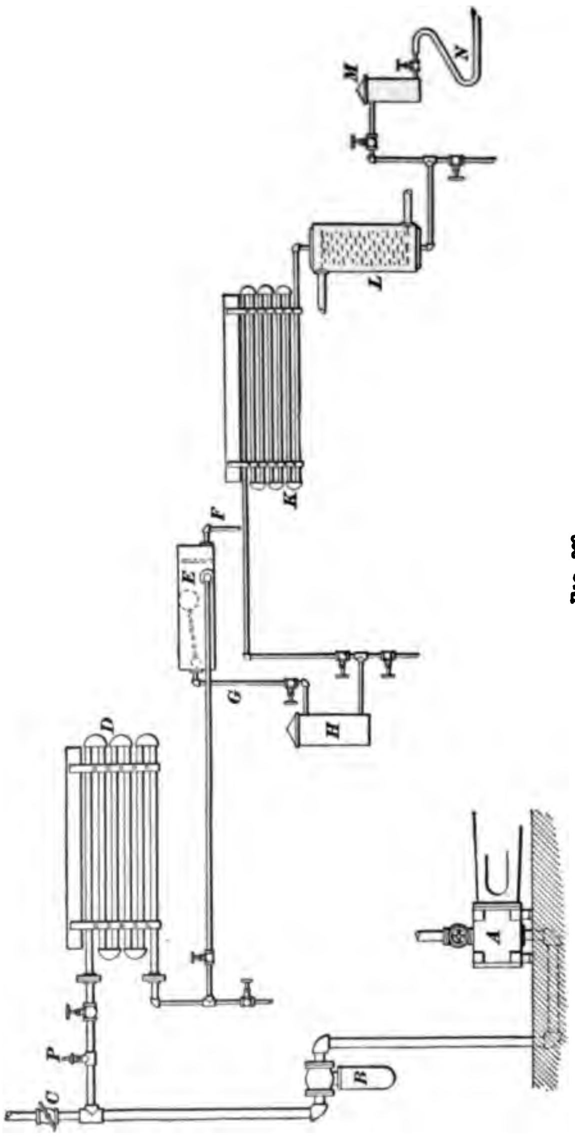

FIG. 309.

of ice made in 24 hours. In case the water is warm, say 80° or over, 6 to 8 gallons per minute should be allowed.

1580. Distilled-Water Apparatus.—The distilled-water system consists of the following parts:

(*a*) Oil separator.
(*b*) Back-pressure, or relief, valve.
(*c*) Exhaust-steam condenser.
(*d*) Reboiler and skimmer.
(*e*) Hot filter.
(*f*) Cooling coil.
(*g*) Gas for cooler.
(*h*) Cold filter.

The usual process of distillation is as follows (see Fig. 369): The exhaust steam on leaving the engine passes to the oil separator *B*. Here all the oil and any priming water that the steam may contain are separated out. The steam then passes to the exhaust-steam condenser *D*. Any surplus steam that is not immediately condensed in *D* is taken care of by the back-pressure valve *C*, which is usually set at about 2 or 3 pounds above the atmosphere. A small vent-cock is also provided in the pipe at *P* for the purpose of relieving the steam-pipe of any air or other extraneous gases. The condensed steam then passes to the reboiler *E*, where it is reboiled by means of live steam. The reboiler is arranged with a skimming diaphragm, so that any light impurities may float to the surface and be skimmed off and go to waste through the pipe *F*. The reboiled condensed steam now passes through the pipe *G* to the hot filter *H*, whence it goes to the distilled-water cooler *K*. Here it is cooled down to the temperature of the condensing water and then enters the top of the forecooler *L*. In this vessel it is cooled down to a temperature of 45° or 50°; then it passes through the cold filter *M*, and finally to the cans through the hose *N*. The object of distillation is, in the first place, to eliminate all oil and other foreign matter, and then to expel any air or gases that may be contained in the water when it enters the boiler. Most of these gases are blown out of the cock *P*, those remaining being driven off in the reboiler *E*.

1581. Oil Separator.—It is very important to get a first-class oil separator on the exhaust-pipe. The separator should be made the full size of the exhaust-pipe, as a smaller separator is insufficient to separate out all the oil carried by the steam.

1582. Exhaust-Steam Condenser.—The exhaust-steam condenser is usually of the atmospheric type; it is made of 2-inch pipe, enough coils being used to give an area of $1\frac{1}{2}$ to 2 times the area of the main exhaust-pipe. Enough water is run over these coils to condense the steam, but not sufficient to cool the condensed steam to any extent. About 8 square feet of surface per ton of ice made per 24 hours is a good proportion.

1583. Reboiler and Skimmer.—The reboiler and skimmer is shown in detail in Fig. 370. This form is com-

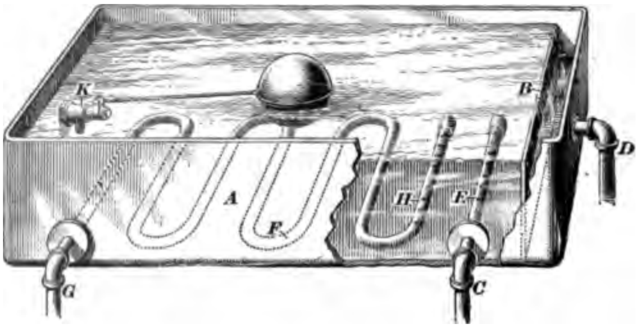

FIG. 370.

monly used and is quite popular, as it works well and is cheap. It consists of a rectangular tank *A*, which is provided with a heavy galvanized-iron diaphragm *B*, reaching to within 3 or 4 inches of the top of the tank. The end of the tank nearest the diaphragm is provided with a pipe flange *D* for taking off the water that is skimmed over the diaphragm. The water inlet at *C* is connected with the exhaust-steam condenser and is provided with a perforated pipe *E* inside the tank. The water coming from the condenser enters through this pipe and is distributed across the whole width of the tank. Any trace of oil that may have escaped the separator will float to the top and be skimmed off by means of the diaphragm *B*.

The reboiling is accomplished by means of the galvanized coil *F*, which is connected with the live-steam pipe at *G* and is provided with perforated openings at *H*. The live steam entering *G* heats the water near that end of the tank to the boiling point and keeps it boiling throughout the whole length of the coil; the condensed live steam passes out through the perforations in *H*. This allows the whole body of distilled water that is contained in the tank *A* to be thoroughly freed from air, etc., while it is passing from the inlet *C* to the outlet *K*. The outlet *K*, which connects with a hot filter, is provided with a float-valve, which is so arranged that when the tank is full the valve is wide open; it shuts gradually as the water level drops in the tank *A*, and entirely closes when the level is some 2 or 3 inches above the valve. This arrangement prevents the distilled water from being charged with air, as the water level can not drop down to that of the outlet, and permits the skimmer to work during the time that the can filler is not in use and when the tank has reached its level and is overflowing. All parts of the distilled-water apparatus, including coils, etc., should be heavily galvanized or else made of copper or brass.

1584. Filters.—The filters are usually heavy sheet-iron or cast-iron galvanized cylinders provided with a perforated false bottom. On this perforated bottom the filtering material is placed. In ice making, filters usually filter from top down, though some engineers prefer to filter from bottom up. All connections should be made in the side of the cylinder, so that the covers can be readily removed for cleaning or recharging. Stop-cocks or valves should also be provided on the distilled-water connections from and to the filters, and by-pass connections should be made so that the filter can be shut off, the by-pass opened, and the running of the plant not interfered with when the filter is being recharged or cleaned. As the distilled water that is used in an ice plant is comparatively free from impurities, the filters do not have much work to do, and it is usually sufficient to charge a filter once a season. For this reason, wash-out filters of different patent makes are seldom used in distilled-water plants.

When the filter is recharged, it should be thoroughly rinsed with distilled water so as to drive out all air and wash the filtering material.

It is advisable to let the distilled water run to waste for some time before filling any cans, so as to be sure that all air and other gases are eliminated.

For hot filters, crushed quartz is found very efficient. For cold filters, quartz, sand, or maple charcoal may be used. Filters between the forecooler and the ice tank are found very effective for eliminating certain red cores in ice, as this red does not precipitate except at a temperature very near the freezing point.

For either sand or quartz filters, 1 square foot of filtering surface is sufficient for a 15-ton ice plant. In filters that filter from top down, if the cans are found to fill slowly, the cover of the filter can be removed and the top of the filtering material skimmed off with a shovel, which will remove with it most of the foreign matter without the necessity of recharging the filter.

1585. Water-Cooling Coils.—The cooling coils are built like the steam condenser, but usually of pipes of a smaller size. The inlet is at the top of the coil and the outlet at the bottom. Four square feet of pipe surface should be allowed for each ton of ice.

1586. Gas Forecooler.—The gas forecooler consists of a cylindrical or rectangular tank, in which are placed galvanized-iron pipe coils. These coils are connected at the top and bottom with the compressor suction-pipe and the expansion coils from the tank, respectively. The cold gas leaving the expansion coils in the ice tank passes through these coils and cools the water contained in the body of the forecooling tank. The area of these coils should in no case be less than that of the main suction-pipe of the machine. The best practice makes them from 1½ to 2 times the area of that pipe. The coils should be kept well away from the body of the tank, so that ice may form on them without freezing against the walls of the tank and bursting it. The tank should be provided with a cover and should be air-tight, as it is subjected to the pressure of the water from the reboiler.

The usual allowance for forecooling coils is 4 to 5 square feet per ton of ice.

A forecooler added to the plant allows the water to come to the ice cans at a temperature of 40° to 50° and increases the capacity of the plant considerably.

1587. Distilled-Water Connections.—All connections and shells of all vessels should be either of block tin or galvanized iron. All valves should be of composition, no iron-body valves being used, after the exhaust steam once enters the steam condenser. The various parts of the distilled-water system should be connected with live-steam pipes, so as to permit of their being blown out and thoroughly cleansed. Blow-off cocks should be provided at short intervals on the various pipes, and all filters and other closed vessels should be provided with them. If the distilled water begins to run slowly into the cans, this indicates that the filters need cleaning or recharging.

ABSORPTION ICE-MAKING PLANT WITH CAN SYSTEM.

1588. General Description.—The absorption machine is well adapted for making ice, as it does not produce oil in the exhaust steam as does the compression machine. The usual arrangement is to take the condensed steam that leaves the generator to the reboiler, then pass it through the water-cooler to the filters, and finally to the ice cans. This arrangement makes it unnecessary to use any oil traps, skimmers, etc.

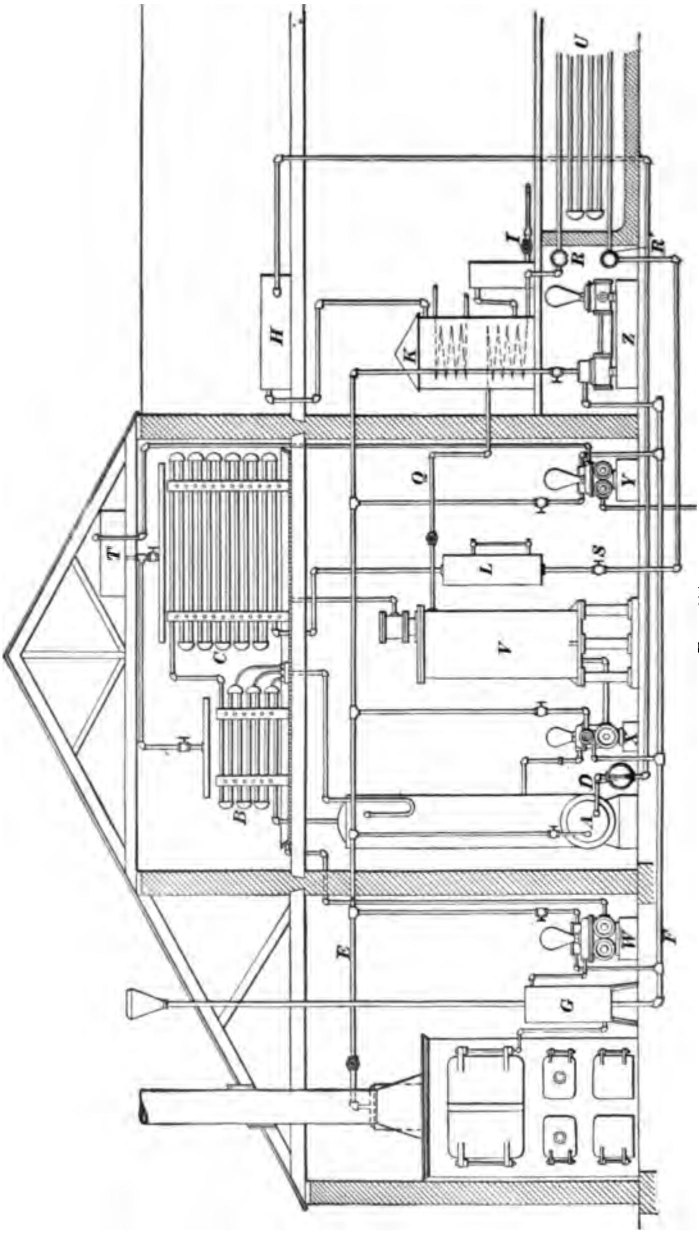

Fig. 371.

The general arrangement of an absorption ice-making plant is shown in Fig. 371. The steam main *E* leading from the boiler furnishes steam to the generator *A* and pumps *W*, *X*, *Y*, and *Z*. The exhaust from the four pumps is led to the main *F*, and thence passes through the feed-water heater *G*. The steam from the generator passes to the small receiver *D* and from there to the reboiler *H*. The course of the ammonia may readily be followed; the gas from the generator passes to the rectifier *B*, thence to the condenser *C*. The liquid gathers in the receiver *L* and is admitted to the brine coil *U* through the expansion-valve *S*. The gas from the coil *U* is carried by the pipe *Q*, which passes through the cooler *K*, to the absorber *V*. The pump *X* takes the liquor from the absorber and delivers it to the exchanger, which in Fig. 371 is hidden behind the analyzer. The student will find it advantageous, in tracing the course of the gas, to compare Fig. 371 with Fig. 351.

The pump *Y* takes water from some source, as a well or city water-supply, and delivers it to the tank *T* in the top of the building. From this tank it flows over the rectifier and condenser into a pan below them. From this pan is drawn the water for the boiler-feed pump *W* and for the absorber *V*. The pump *Z* is used to circulate the brine.

1589. The Distilled-Water System.—The course of the distilled water is more clearly shown in Fig. 372. The steam enters the generator *A* through the pipe *B*, and after condensing, passes out at *C* to a small receiver *D* having a gauge-glass *E* at one end. The outlet of this receiver is at the bottom; from it a pipe *F*, which can be throttled at *G*, leads to the reboiler *H*. Here the distilled water is thoroughly reboiled, then it passes successively to the cooler *K*, the filter *I*, and finally to the ice cans.

The upper portion of the distilled-water cooler *K* is fitted with a helical coil *N*, whose tails *L* and *M* project through the side of the tank. The gas-forecooling coil *O* is in the lower portion of the tank, with its tails *P* and *Q* passing through the shell, as shown. The water used for cooling the distilled water enters the bottom of the coil *N* through the tail *M* and passes out at *L*. The absorber suction-pipe *Q*

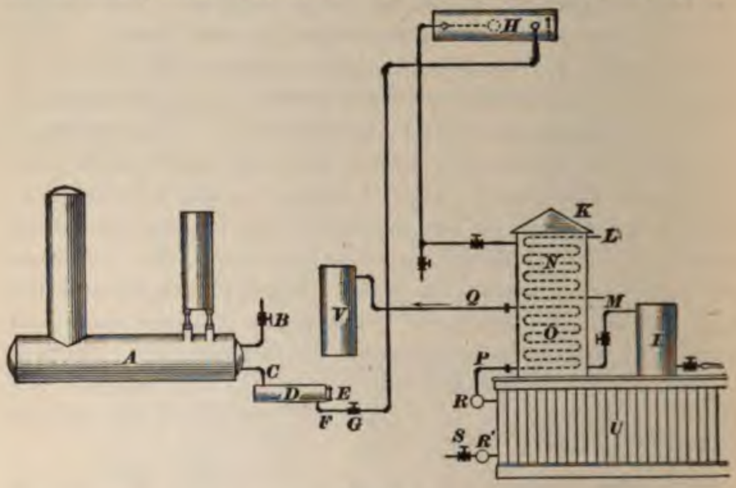

FIG. 372.

is connected with the upper tail, and the return-gas header *R* of the expansion coil is connected with the lower tail of the forecooling coil *O*.

The method of operation is as follows: After the machine has been started and distilled water begins to accumulate in the receiver *D*, as shown by the glass *E*, the valve *G* is opened slightly so as to keep the level of the liquid in *E* constant. This permits the distilled water to enter *H* and pass to waste over the skimming diaphragm. After the mass of water in *H* has become thoroughly heated up and reboiled, it is allowed to follow the course above described to the ice cans.

1590. Operating the Plant.—When a can plant is first started, live steam should be allowed to blow for several hours through the various parts of the distilled-water system. The distilled water is then turned on and allowed to run to waste for several days before any cans are filled. As the plant is designed to keep the brine at a certain temperature while freezing the water in the cans, and as it is the heat given off by the water in freezing that prevents the brine from falling below that temperature, it is advisable not to run the plant with the cans empty, as otherwise the temperature may drop so much as to freeze the brine. In order to have the cans full when the plant is first started, ordinary city or well water may be used to fill them with, as it is of course impossible to get distilled water before the machinery has been running for some time. The ice from this water is very poor and need not be used except in case of necessity. It takes from 4 to 5 days to freeze the batch of ice from this water, but the pulling of the ice may be begun at the end of the third day, even though the cakes are not thoroughly frozen. Then the cans are filled with distilled water.

1591. The capacity of an ice plant is governed more particularly by the number of cans filled than by those pulled in a given time. Some unscrupulous builders of ice machinery, taking advantage of this fact, freeze up the initial, or first, batch of ice, which gives them as a start 2½ times the nominal capacity of the plant. The plant is then operated for a week and is able to turn out from 5 to 10 per cent. more ice than its nominal capacity. This is due to the fact that they start with a tank full of ice and end up with one nearly empty. A 30-day run is the minimum time on which the capacity of an ice plant should be based.

1592. If it is found necessary to shut a can plant down for any length of time, care should be taken to see that all cans in which the ice has freed itself from the side of the can and floated up are emptied and refilled with fresh water. If this is not done, the cans will be bulged and the bottoms may burst when the plant is started again. In the case of an absorption plant, the generator coil should be thoroughly blown out every 24 or 36 hours so as to eliminate all rust, etc.

PLATE ICE PLANTS.

1593. The Two Systems Used.—There are two systems of plate ice plants, viz.: the **dry-plate** and the

wet-plate system. In the dry-plate system, the ammonia gas is expanded into a return-bend coil, such as is shown in Fig. 373, the space between the runs being filled in with wood or iron, so as to separate the two cakes forming on the sides of the coil. In the wet-plate system, the freezing is accomplished by means of a tank or cell

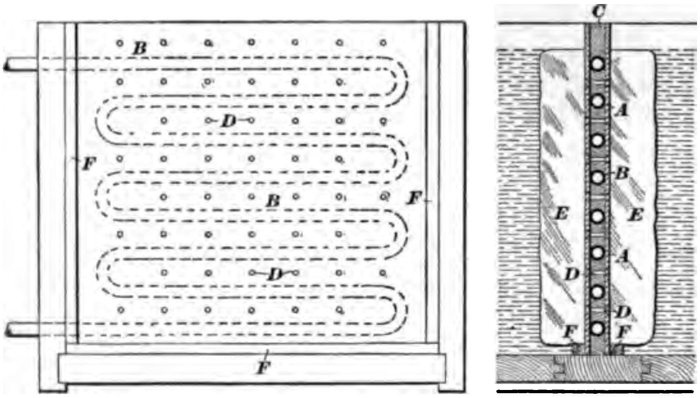

FIG. 373.

several inches thick, in which brine circulates. This tank either contains the direct-expansion coil or else the brine is piped from a separate cooling tank or brine cooler to the cells. In either case the ice adheres to the side of the plate, and when the right thickness is reached, it is thawed off by running either warm brine or water into the cells, or, in the dry system, by turning the hot ammonia gas direct from the condenser into the coils.

1594. Dry-Plate System.—The dry-plate system most commonly used is that shown in Fig. 373. It consists of a continuously welded return-bend or zigzag coil *B*, between the runs of which are placed blocks of wood *C*. On each side of this coil are laid plates of sheet steel *A*, *A*, and these are bolted or riveted through by means of the bolts or rivets *D*. This makes a comparatively inexpensive arrangement, and a plant constructed with such plates is not much more expensive than a can plant. These plates, which are about 30 inches on centers, are placed in a long wooden tank. Water is run in between the plates to within about 9 inches from the top, and the expansion ammonia is then turned on. Great care must be taken, in feeding this kind of plate, that the freezing does not take place too rapidly, or the air-bubbles will not have time to disengage, and white ice will be the result. After a certain amount of ice has accumulated on the coils, say about 4 inches, care must be taken not to feed too rapidly, or there is a liability of the coil frosting through and liquid ammonia getting over to the compressor.

The great objection to a dry plate is that if the expansion is insufficient, the ice will not form evenly over the whole plate, but will be thicker where the ammonia is fed in, gradually tapering down to a thin cake at the outlet. Plate plants of this type usually have forecooling coils of large surface, so as to take up any excessive expansion.

On the whole, a dry-plate plant is very difficult to operate with any degree of success.

1595. Wet-Plate System.—A wet-plate system very commonly used is shown in section in Fig. 374. It consists of wrought-iron cells filled with brine, in which are placed the direct-expansion coils. The wrought-iron cell *B B* contains the ammonia expansion coil *A* and is filled with brine. The ice *E E* forms on the sides of the cells. With this arrangement, it is possible to get a very clear ice out of ordinary well, river, or city water without the necessity of distillation. The ice forms gradually on the sides of the plate, and the temperature of the brine can always be kept at any desired point by the proper

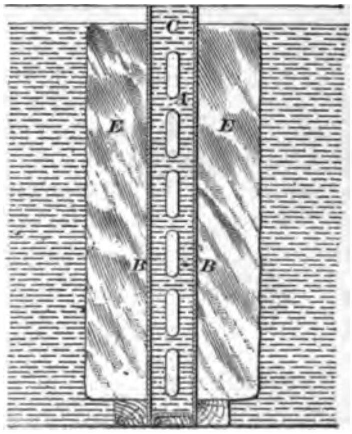

FIG. 374.

manipulation of the expansion-valve. After a little practice, it is easy to ascertain just the temperature of brine that is necessary for ice of different thicknesses. This has to be determined by experiment, as no two samples of water contain the same ingredients.

The principal objection to the wet-plate system is its large first cost and the liability of the cells leaking. If the cells are not properly made in the first place, the expense of maintaining them is considerable, the smallest leak of brine injuring the quality and appearance of the ice.

1596. Plate Plant Apparatus.—A plate ice plant usually consists of the following parts:

(*a*) Ice tank with plates.
(*b*) Forecooling tank with coils.
(*c*) Crane with hoists.
(*d*) Tilting table.
(*e*) Cutting-up saws.
(*f*) Water filters.
(*g*) Water connections.

In case of a wet-plate system, there are also

(*h*) Brine-storage tank.
(*i*) Brine-transfer pump.
(*j*) Brine connecting mains.

1597. Lifting and Cutting the Plates.—In order to lift the plates from the tank, two tension bolts, which have eyes at the top to which the crane is fastened, may be frozen in the ice. In other plants, a chain is slipped under the cake. This is accomplished by first passing a copper wire under the cake, after the latter, having been detached from the plate, floats to the surface; then the chain is made fast to one end and then drawn up around the cake by means of the wire. After the cake is drawn from the cell, it is necessary to lay it down. This is done by means of the tilting table *A*, Fig. 375, which is tilted by means of a worm-gear. The table is placed in a position such as shown by the full lines. The crane *D* is then run over and the cake C is allowed to lie on this table. The chains are taken off, and the table is then lowered into the position shown by the dotted lines.

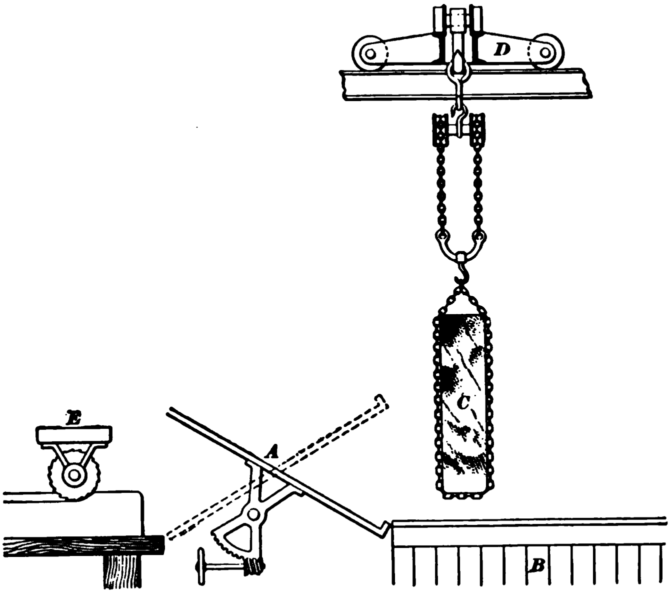

FIG. 375.

It is now necessary to cut the cake into merchantable-sized ice, which is usually accomplished in large plants by a gang of steam saws E. These are driven by power and the cake is fed through under them. The saws cut a groove about 3 inches deep in the cake, which is then split by hand with chisel bars. In small ice plants, what is known as a steam ice-cutter is used. This consists of a blade of steel with a $\frac{1}{8}$-inch copper pipe brazed fast to one end. This is placed on edge on top of the cake of ice F to be cut, as shown in Fig. 376, the steel blade A being held vertical and steam turned on into the copper pipe B. This rapidly thaws the ice and cuts down through. In other plants, an ordinary hand-saw is used to score the ice and chisel bars to separate the cakes.

The method of thawing the ice in a wet-plate plant is as follows: When the ice in a cell has reached very nearly the required thickness, the feed valve to that cell is shut off, but the suction-valve is left open. This draws all the ammonia gas from the coil. The thickness of the ice is also increased thereby ½ to ¾ inch. The ice is now ready for thawing. The brine contained in the cell is drawn out by means of the brine-pump and the cell is filled with cold water. Water warmer than 60° should not be used, as the ice is liable to shiver. If, however, the thawing water gets as low as 36°, it is best to run it off into the draining tank

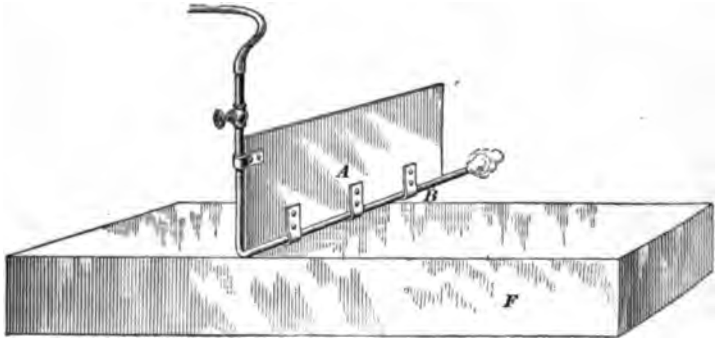

FIG. 376.

and refill the cell. After the water has stood in the cell for 4 or 5 hours, a chisel bar can be inserted along the top between the cake and the cell. If it is thawed free of the cell, the cake when gently pried will float up in the water. The crane is then made fast and the cake is drawn out. The thawing water may now be run out of the cell, and the ice space may be thoroughly cleansed of any dirt that may have been precipitated during the process of freezing by opening the waste-cock and washing the cell clean with a hose. The cell is then filled with brine and the tank with water to within 9 inches of the top; the expansion is turned into the coil, and the process of freezing is started again.

MISCELLANEOUS USES OF REFRIGERATION.

1598. Hotel Refrigeration.—The work required in hotels is the refrigeration of a large number of small boxes used for various purposes about the establishment, such as wine and beer boxes, general storage, meat, cook stores, oysters, fish, ice cream, etc.; also the freezing of caraffes and the making of small quantities of ice. Such work is out of all proportion to the space cooled, owing to the fact that the boxes are small and usually scattered throughout the building. For small refrigerating boxes for these purposes, namely, those of 100 cubic feet or less, about 500 cubic feet should be allowed for each ton of refrigerating effect. For boxes ranging between 100 and 1,000 cubic feet, 1 ton will suffice for about 1,200 cubic feet of space. In addition to this, the amount of ice that is to be made should be carefully considered, 2 tons of refrigerating effect being allowed for every ton of ice made.

Direct expansion is out of the question for hotel refrigeration, on account of the large number of boxes or rooms that would demand attention.

In calculating the capacity required for hotel refrigeration, it is always better to err on the safe side and get a machine rather too large than too small.

1599. Chocolate-Factory Refrigeration. — The setting of chocolate requires a cool, dry atmosphere in the cooling rooms. These rooms are usually kept at a temperature of about 68°. The indirect system of cooling and ventilating should be used, so as to prevent the possibility of any moisture from drip coils, etc. After allowing for the amount of refrigeration required to cool the given space to 68°, the number of persons in the coating room and the amount of chocolate to be cooled daily should be considered. One ton of refrigeration will be required to absorb the heat given off by 7 persons. It takes about 1,200 B. T. U. per pound to condense and freeze the moisture in the atmosphere, of which a great deal is exhaled by the persons in the room. The conveying of the cold air from the bunker and the taking away of the vitiated air from the rooms should be effected by means of a series of air-ducts or flues, the exact size and location of which should be determined by some expert ventilating engineer. This is quite important, in order to prevent drafts and the consequent colds to the persons in the coating room.

1600. Air Refrigeration.—Air is often cooled for various purposes, such as drying photographic plates, ets. When the object of the cooling is to dry the air, the amount of refrigeration required depends largely upon the amount of moisture that has to be eliminated by condensing and freezing it, the cooling of the air itself being of secondary importance. As stated in the preceding paragraph, 1,200 B. T. U. may be allowed for every pound of moisture taken from the air, in addition to the refrigeration required to cool the air to the temperature of condensation of the moisture it contains.